Comparison of Means Scheme

Sample statistics from two data sets

$\bar{X}_1, n_1, S_1^2$ and $\bar{X}_2, n_2, S_2^2$

variance known?

σ_1^2, σ_2^2

yes

variance equal?

$\sigma_1^2 = \sigma_2^2$?

yes

sample size equal?

$n_1 = n_2$?

yes

$$Z_0 = \frac{\bar{X}_1 - \bar{X}_2}{\sigma\sqrt{\frac{2}{n}}}$$

no

$$Z_0 = \frac{\bar{X}_1 - \bar{X}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

no

sample size equal?

$n_1 = n_2$?

yes

$$Z_0 = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}}}$$

no

$$Z_0 = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

no

variance equal?

$\sigma_1^2 = \sigma_2^2$?

yes

sample size equal?

$n_1 = n_2$?

yes

$$T_0 = \frac{\bar{X}_1 - \bar{X}_2}{S_p\sqrt{\frac{2}{n}}}$$

no

$$T_0 = \frac{\bar{X}_1 - \bar{X}_2}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

no

sample size equal?

$n_1 = n_2$?

yes

$$T_0^* = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2 + S_2^2}{n}}}$$

no

$$T_0^* = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

For the two variance unknown but equal cases, the pooled sample variance (S_p^2) is given by:

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

and we compare the test statistic (T_0) to the t distribution with $n_1 + n_2 - 2$ degrees of freedom.

For the two T_0^* cases (variance unknown and unequal), the degrees of freedom (ν) are given by:

$$\nu = INT\left[\frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(s_1^2/n_1\right)^2}{n_1 - 1} + \frac{\left(s_2^2/n_2\right)^2}{n_2 - 1}}\right]$$

and we compare the test statistic (T_0^*) to the t distribution with ν degrees of freedom.