# Excel Functions for the Standard Normal and T Distributions

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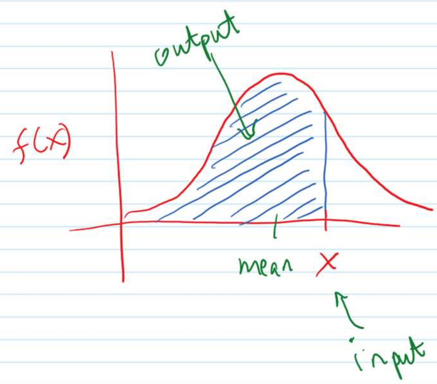
NOTE: The **.DIST** versions of each of the functions below will output probability as a function of percentage points (the "x-value", in other words) of the distribution. The **.INV** versions will output the x-value corresponding to probability. Most are left-tailed formulas, some are right-tailed formulas (usually indicated with a **.RT** at the end), and some are two-tailed (indicated with **.2T** at the end).

## *NORMAL DISTRIBUTION*

**NORM.DIST(x,mean,standard_dev,cumulative)** – Provides the cumulative frequency (left-tailed) (if **cumulative** is **TRUE**) corresponding to the normal distribution of mean **mean** and standard deviation **standard_dev** to the left of **x**. For this class, it is rare to use **FALSE** as the final argument unless you want to generate a plot of f(x) in Excel.



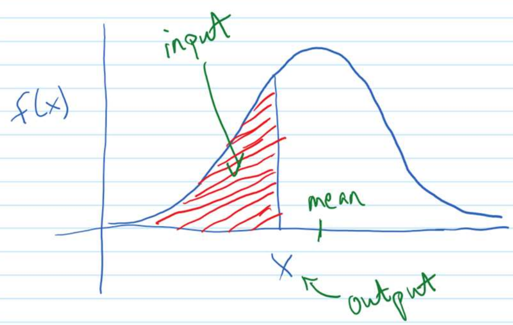

NORM.DIST(x, mean, standard_dev, cumulative)

If cumulative = FALSE
→ outputs f(x) (useful for plots)

If cumulative = TRUE
→ outputs probability/proportion to the left of x underneath f(x)

Example: The area of to the left of 5 beneath the normal distribution with mean 4 and standard deviation 3 is: **NORM.DIST(5,4,3,TRUE)** = 0.632

**NORM.INV(probability,mean,standard_dev)** – Outputs the x-value (percentage point) that has probability proportion to the left of it based on the normal distribution with mean **mean** and standard deviation **standard_dev**.



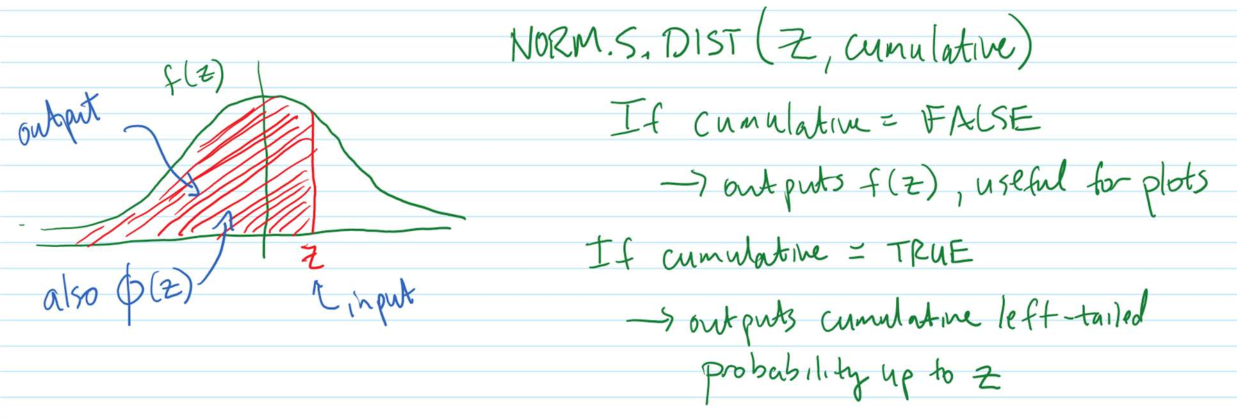

NORM.INV(probability, mean, standard_dev)

Given left-tailed probability, will output the corresponding x-value based on normal distribution with specified mean and standard_dev

Example: The x-value corresponding to an area (probability) of 0.3 to the left of it based on a normal distribution with mean 4 and standard deviation 3 is**: =NORM.INV(0.3,4,3)** = 2.43

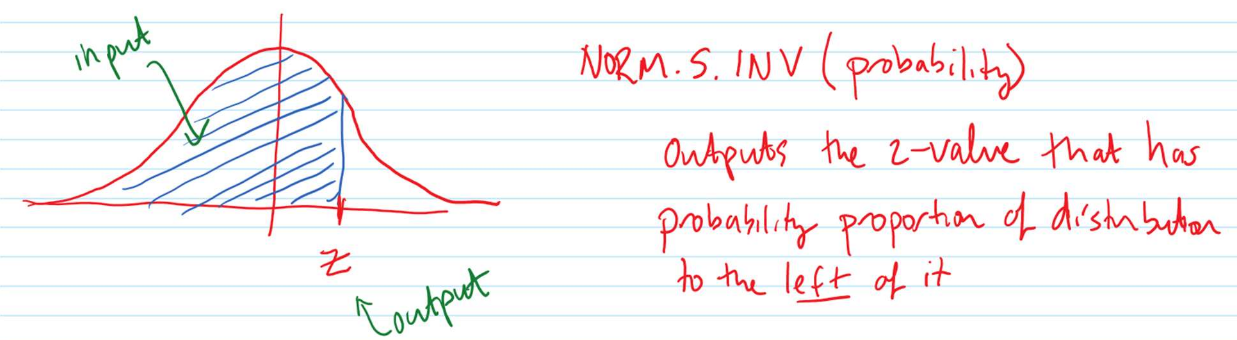
### ***STANDARD NORMAL DISTRIBUTION***

**NORM.S.DIST(z,cumulative)** – Outputs the cumulative frequency (left-tailed) (if **cumulative** = **TRUE**) corresponding to a z-value of **z** based on the standard normal distribution. If **cumulative** = **FALSE**, it outputs f(z), the probability density function. Note that this is the same as $\Phi(z)$ in the "Percentage Points of the Standard Normal Distribution" table.




Example: The area to the left of z = 0.7 of the standard normal distribution is given by:
**=NORM.S.DIST(0.7,TRUE)** = 0.758

**NORM.S.INV(probability)** – Outputs the z-value (percentage point) with **probability** proportion to the left of it (left-tailed) based on the standard normal distribution. Note that the output of this function is the same as $\Phi^{-1}(P)$, where $P$ is probability. In other words, the z-value corresponding to $P$ proportion of the distribution to the left of it.




Example: The z-value that has 80% of the distribution to the left of it is: **=NORM.S.INV(0.8)** = 0.842

## *Percentage Points of the Standard Normal Distribution*

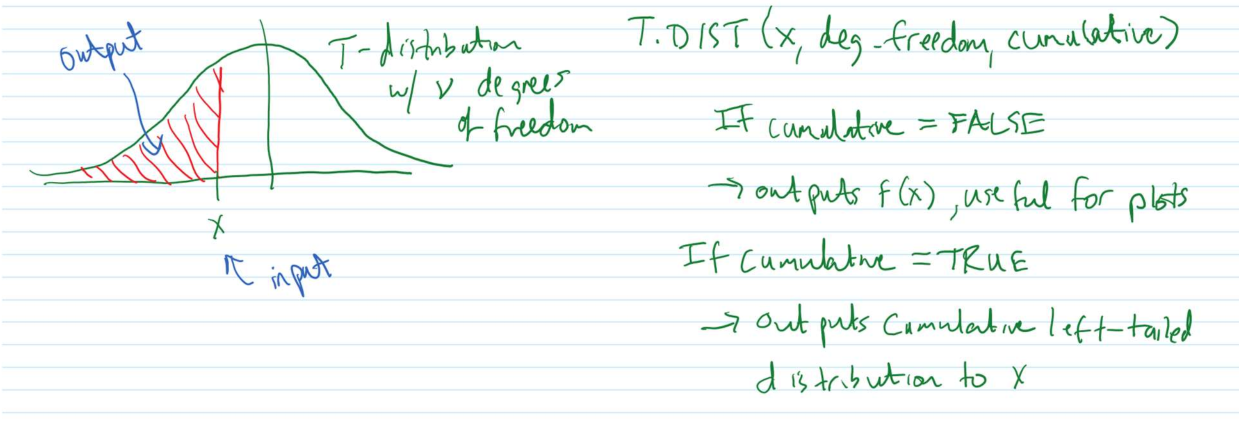
In setting up confidence intervals on the mean and performing hypothesis tests on the mean (variance known), we need to determine the parameter $z_\alpha$ or $z_{\alpha/2}$. These are the z-values that have $\alpha$ or $\alpha/2$ proportion of the distribution to the right of them, respectively. In order to calculate these "percentage points" of the standard normal distribution, we can use the NORM.S.INV function in Excel:

$z_\alpha$ **=NORM.S.INV(1-alpha)**

$z_{\alpha/2}$ **=NORM.S.INV(1-alpha/2)**

## *T DISTRIBUTION*

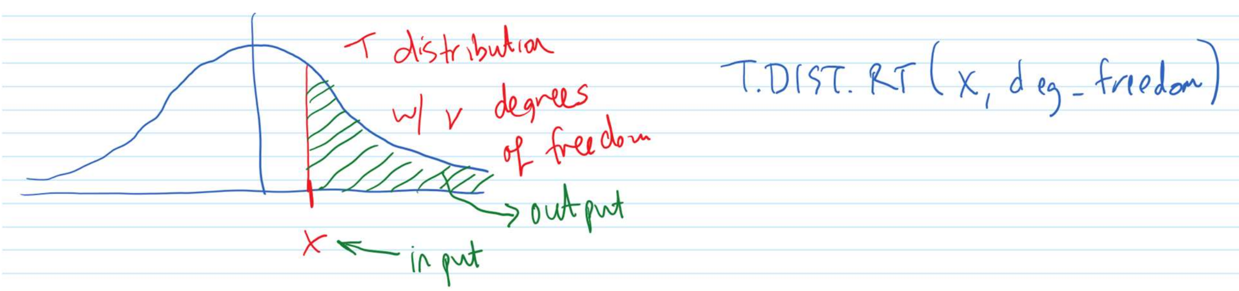
**T.DIST(x,deg_freedom,cumulative)** – Outputs the left-tailed, cumulative probability [F(x)] of the T distribution up to **x** (if **cumulative** = **TRUE**) based on **deg_freedom** degrees of freedom. If **cumulative** = **FALSE**, it outputs f(x), the probability density function.




Example: The area to the left of x = -0.5 of the T distribution with 9 degrees of freedom is:

**=T.DIST(-0.5,9,TRUE)** = 0.315

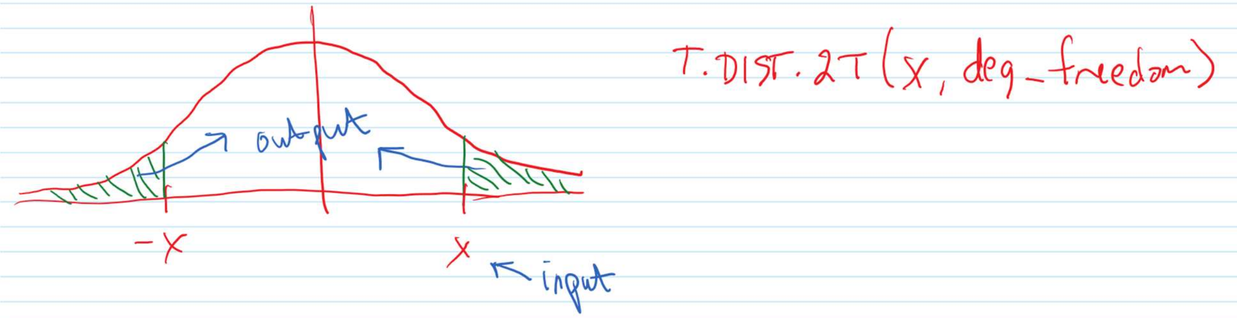
**T.DIST.RT(x,deg_freedom)** – Outputs the right-tailed cumulative probability of the T distribution with **deg_freedom** degrees of freedom.



Example: The area to the right of x = 0.4 of the T distribution with 12 degrees of freedom is:

**=T.DIST.RT(0.4,12)** = 0.348

**T.DIST.2T(x,deg_freedom)** – Outputs the area underneath the T distribution with **deg_freedom** degrees of freedom that is to the left of **-x** and to the right of **x.** This is useful when calculating P-values for two-tailed hypothesis tests when variance is unknown.




Example: The area the right of x = 0.5 and to the left of -0.5 under the T distribution with 12 degrees of freedom is: **=T.DIST.2T(0.5,12)** = 0.626. Note that this is twice the area calculated using the **T.DIST** function or the **T.DIST.RT** function: **=T.DIST(-0.5,12)** = **T.DIST.RT(0.5,12)** = 0.313

Example: P-value of a two-tailed hypothesis test ($s = 1.5, \bar{x} = 11.8, n = 7$):

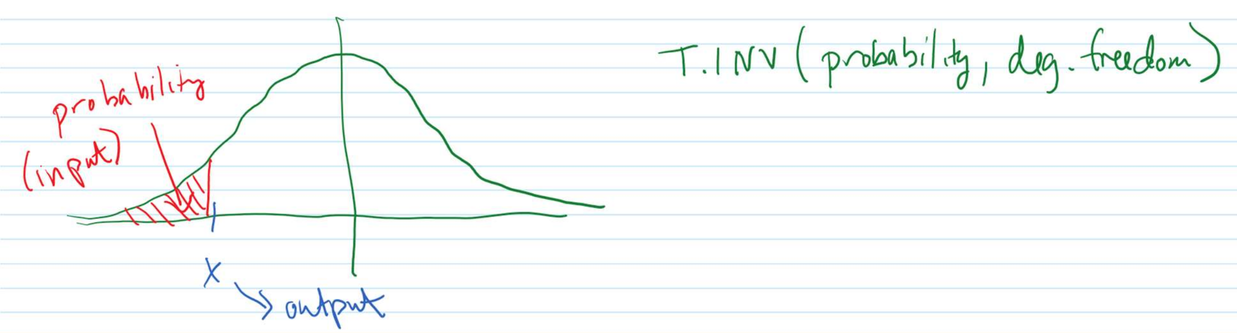
$H_0: \mu = 10$

$H_1: \mu \neq 10$

Convert $\bar{x}$ to test statistic: $t_0 = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{11.8-10}{1.5/\sqrt{7}} = 3.175$

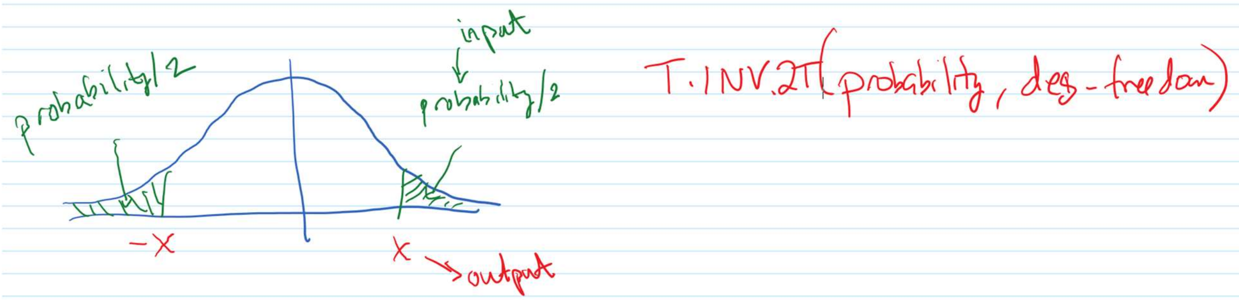
Then, use the **T.DIST.2T** function to determine the P-value: **=T.DIST.2T(3.175,6)** = 0.0192

**T.INV(probability,deg_freedom)** – Given the cumulative left-tailed **probability** (area), outputs the corresponding x-value (percentage point) of the T distribution with **deg_freedom** degrees of freedom.




Example: The x-value (percentage points) of the T distribution with 15 degrees of freedom corresponding to a left-tailed probability of 0.7 is: **=T.INV(0.7,15)** = 0.536

**T.INV.2T(probability,deg_freedom)** – Splits **probability** into two equal tails of the T distribution with **deg_freedom** degrees of freedom and outputs the corresponding positive x-value (percentage point).




Example: The x-value (percentage point) when we split probability = 0.6 equally into two tails of the T distribution with 6 degrees of freedom is: **=T.INV.2T(0.6,6)** = 0.553. The **T.INV.2T** function can be useful when calculating confidence intervals on the mean (variance unknown, see below).

**T.INV.RT** – You would expect Excel to have a **T.INV.RT** function, but this function does not exist!

## *Percentage Points of the T Distribution*

In setting up confidence intervals and performing hypothesis tests on the mean (variance unknown), we need to determine the parameter $t_\alpha$ or $t_{\alpha/2}$. These are the t-values that have $\alpha$ or $\alpha/2$ proportion of the distribution to the right of them, respectively. In order to calculate these "percentage points" of the T distribution, we can use the T.INV or T.INV.2T functions in Excel, shown here for the T distribution with 11 degrees of freedom:

$t_\alpha$ **=T.INV(1-alpha,11)**

$t_{\alpha/2}$ **=T.INV.2T(alpha,11)** [note that we could have also used **T.INV(1-alpha/2,11)**]

The above calculations can be used to calculate the data presented in the "Percentage Points of the T Distribution" table on the course website.

## *CONFIDENCE INTERVALS*

**CONFIDENCE.NORM(alpha,standard_dev,size)** – Outputs the half-interval for a two-sided confidence interval on the mean (variance known). In other words, for the (1-$\alpha$)-% confidence interval defined by the following, this function outputs the amount $z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$.

$$P\left[\bar{x} - z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right] = 1 - \alpha$$

Example: A sample of n = 7 items yields a sample average of 3.8. It is known that the population standard deviation is 1.6. What is a 95% confidence interval ($\alpha = 0.05$) on the mean of the population?

Lower 95% CI limit: **=3.8-CONFIDENCE.NORM(0.05,1.6,7)** = 2.61

Upper 95% CI limit: **=3.8-CONFIDENCE.NORM(0.05,1.6,7)** = 4.99

We are 95% sure that the population mean ($\mu$) lies between 2.61 and 4.99.

**CONFIDENCE.T(alpha,standard_dev,size)** – Outputs the half-interval for a two-sided confidence interval on the mean (variance unknown). In other words, for the (1-$\alpha$)-% confidence interval defined by the following, this function outputs the amount $t_{\frac{\alpha}{2},n-1} \cdot \frac{s}{\sqrt{n}}$.

$$P\left[\bar{x} - t_{\frac{\alpha}{2},n-1} \cdot \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\frac{\alpha}{2},n-1} \cdot \frac{s}{\sqrt{n}}\right] = 1 - \alpha$$

Example: A sample of n = 10 items yields a sample average of 12.9 and sample standard deviation of 2.4. Population standard deviation is unknown. What is a 90% confidence interval ($\alpha = 0.10$) on the mean of the population?

Lower 90% CI limit: **=12.9-CONFIDENCE.T(0.1,2.4,10)** = 11.51

Upper 90% CI limit: **=12.9-CONFIDENCE.T(0.1,2.4,10)** = 14.29

We are 90% sure that the population mean ($\mu$) lies between 11.51 and 14.29.