MATH 23: 5% Assessment #2

Directions: Below are two problems that require a bit more computation and/or graphing than the usual homework problem. As such, you are encouraged to use whatever technology you can to do so. You are to turn in two *neat and organized* solutions to these problems, where it is evident both your methods, and your conclusions. This does not require a lot of text, but it does require some thought about organizing your results, and how best to present them.

This is due by the end of next week, Friday, June 5. You will receive other homework during this time. The best way is to get a little of this done each day. I will not be able to accommodate a lot of questions on this at the last moment, so you need to start soon.

1. We will study using Euler's method to study the nonlinear initial value problem:

$$y' = 5 - 3\sqrt{y}, \quad y(0) = 2.$$

Our goal is to compute approximate values to the equation at the points $t = 0.5, 1, 1.5, 2.0, 2.5$ and 3. Plot solutions using:

a) $h = 0.1$
b) $h = 0.05$
c) $h = 0.025$
d) $h = 0.01$

It will be necessary for you to utilize some sort of technological help to compute these. You may use whatever you like (python code, excel, matlab, Wolfram alpha), but cite what you use.

When done with your plots, comment on their behavior. Does it seem like, with smaller values of h, that you are getting closer to a single solution?

2. Consider the initial value problem:

$$y' = -ty + 0.1y^3, \quad y(0) = \alpha.$$

a) Draw direction fields for this equation with several different choices of α, for $1 \leq \alpha \leq 4$.

b) Observe that there seems to be a value α_0, where $2 \leq \alpha_0 \leq 3$, such that for $alpha \leq \alpha_0$, solutions seem to converge (when computing longer and longer approximations, the approximations get closer together), and for $\alpha_0 \leq \alpha$, the solutions diverge (the longer one computes them, the farther apart they get). Describe how this is made apparent from your vector field.

c) Use Euler's method with $h = 0.01$ to estimate α_0.