Assignment: 6.60, 6.61, 6.63, 6.64

40/40

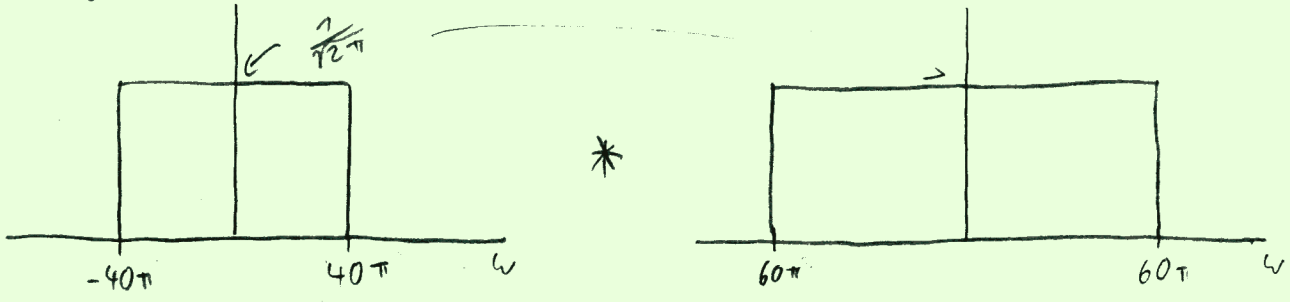
6.61 $x(t) = \dfrac{\sin(40\pi t)\sin(60\pi t)}{\pi^2 t^2}$ Find Bandwidth

$$= \frac{\sin(40\pi t)}{\pi t} \cdot \frac{\sin(60\pi t)}{\pi t} = 40\,\mathrm{sinc}(40\pi t)\cdot 60\,\mathrm{sinc}(60\pi t)$$

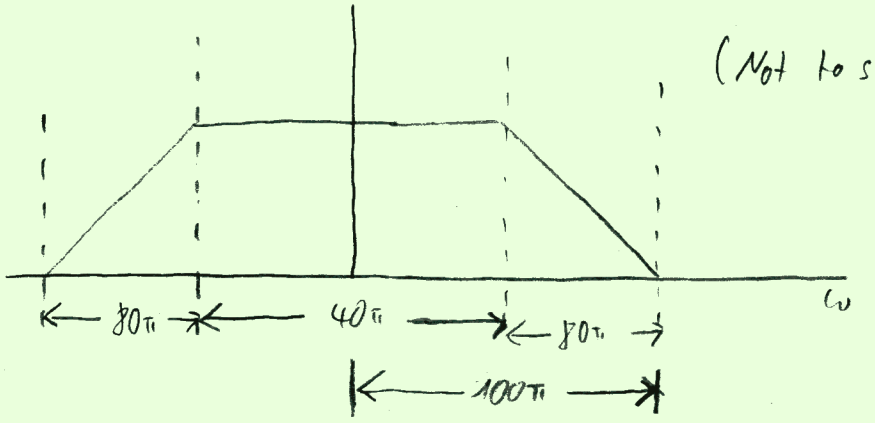
$$\mathrm{sinc}(at) \longleftrightarrow \frac{1}{\sqrt{2\pi a^2}}\,\mathrm{rect}\left(\frac{\omega}{2\pi a}\right)$$

$$\Rightarrow X(\omega) = 40\,\frac{1}{\sqrt{2\pi(40)^2}}\,\mathrm{rect}\left(\frac{\omega}{80\pi}\right) * 60\,\frac{1}{\sqrt{2\pi(60)^2}}\,\mathrm{rect}\left(\frac{\omega}{120\pi}\right)$$

$$= \frac{1}{\sqrt{2\pi}}\,\mathrm{rect}\left(\frac{\omega}{80\pi}\right) * \frac{1}{\sqrt{2\pi}}\,\mathrm{rect}\left(\frac{\omega}{120\pi}\right)$$



This will lead to something like this: no!



(Not to scale)

$\Rightarrow B = 100\pi \frac{\text{rad}}{\text{s}}$

$= 50$ Hz

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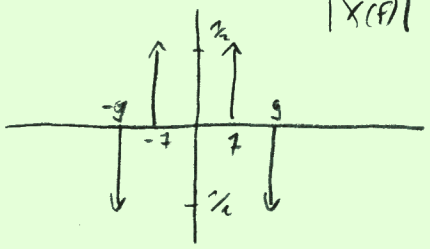
a Nyquist rate must be $\boxed{F_{min} = 100 \text{ Hz}}$ ✓

6.63 $x(t) = \cos(14\pi t) - \cos(18\pi t)$ $\quad f_s = 16\,\text{Hz}$

$$|H_I(f)| = \begin{cases} 1 & |f| \le 8 \\ 0 & |f| > 8 \end{cases} \qquad X_s(f) = f_s \sum_{k=-\infty}^{\infty} X(f - k f_s)$$

$$X(f) = \frac{1}{2}\left[\delta(f-7) + \delta(f+7) - \delta(f-9) - \delta(f+9)\right]$$



$$\Rightarrow X_s(f) = \frac{16}{2} \sum_{k=-\infty}^{\infty} \left[\delta(f - 16k - 7) + \delta(f - 16k + 7) - \delta(f - 16k - 9) - \delta(f - 16k + 9)\right]$$

(f_1, f_2, f_3, f_4)

k	f_1	f_2	f_3	f_4
0	7	-7	9	-9
1	23	9	25	7
-1	-9	-23	-7	-25
2	39	25	41	23
-2	-25	-39	-23	-41

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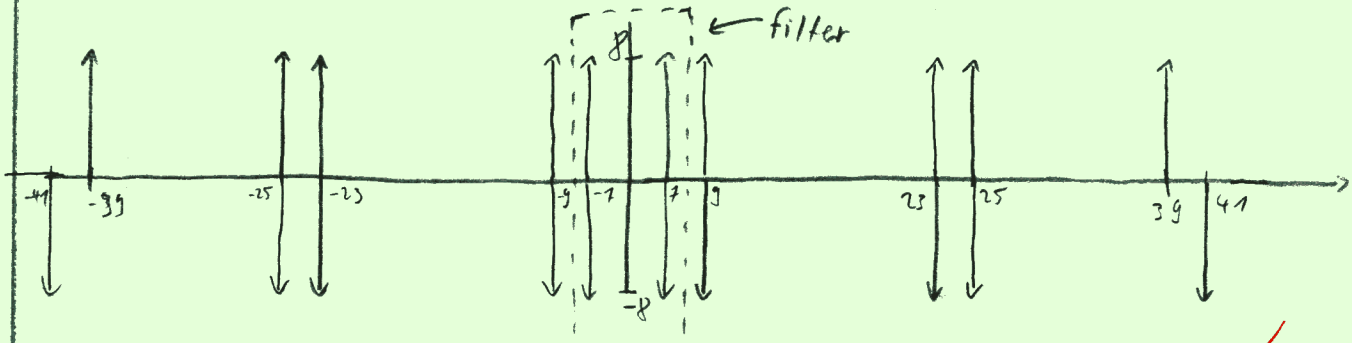
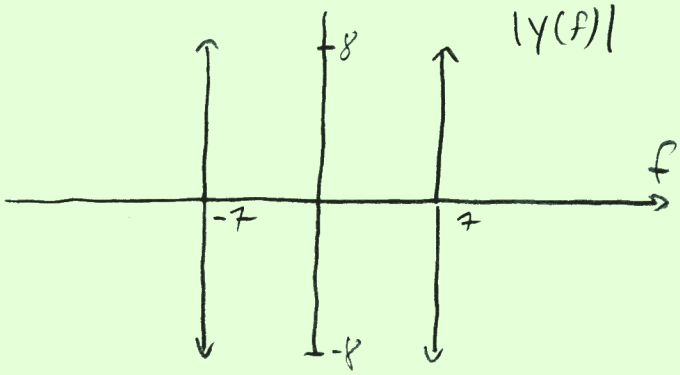


$$\Rightarrow Y(f) = 8\left[\delta(f-7) + \delta(f+7) - \delta(f-7) - \delta(f+7)\right] = 0$$ ✓



6.64 $x(t) = \sin(30\pi t) + \sin(70\pi t)$ $\quad f_s = 50$ Hz

$$|H_I(j2\pi f)| = \begin{cases} 1 & |f| \leq 25 \text{ Hz} \\ 0 & |f| > 25 \text{ Hz} \end{cases}$$

$$X_s = f_s \sum_{k=-\infty}^{\infty} X(j2\pi(f - k f_s))$$

$$X(f) = \frac{j}{2}\left[\delta(f+15) - \delta(f-15) + \delta(f+35) - \delta(f-35)\right]$$

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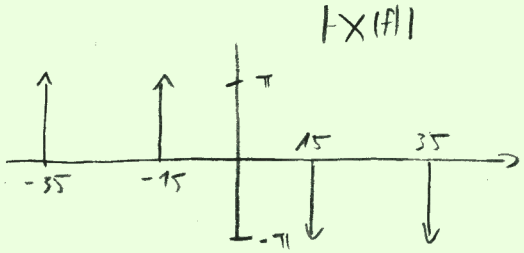
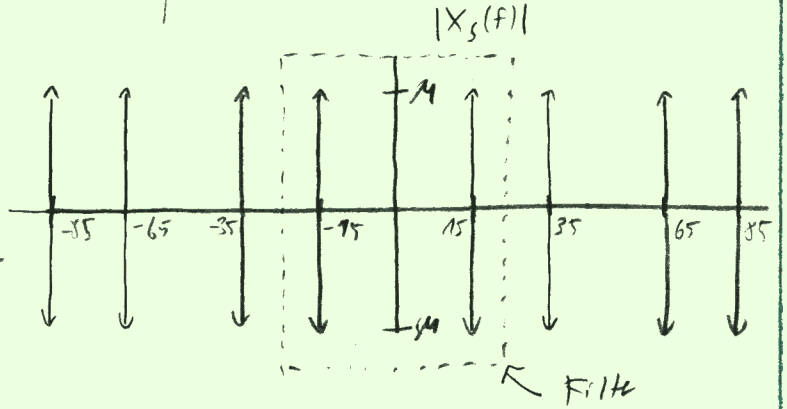
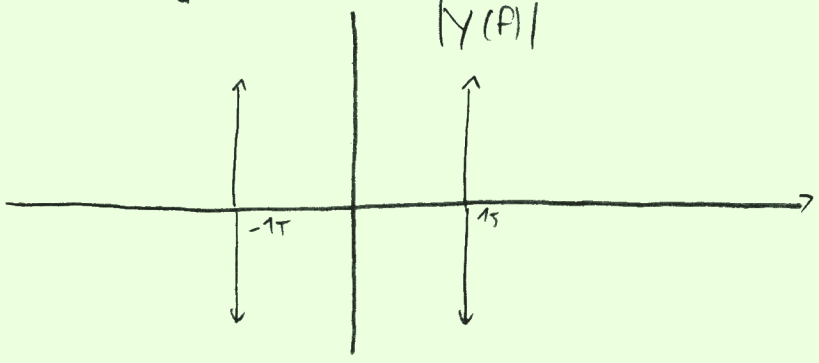


$$X_s(f) = \frac{50\pi}{2} \sum_{k=-\infty}^{\infty} \delta(f - 50k + 15) - \delta(f - 50k - 15) + \delta(f - 50k + 35) - \delta(f - 50k - 35)$$

$k=0$	$\pi, -35$	$15, 35$
$k=1$	$35, 15$	$65, 85$
$k=-1$	$-65, -85$	$-35, -15$
$k=2$	$85, 65$	$115, 135$
$k=-2$		



$$\Rightarrow Y(f) = M\left[\delta(f-15) + \delta(f+15) - \delta(f-15) - \delta(f+15)\right]$$



$= 0$ ✓