# An Investigation of the Simplified Navier Stokes-Based Equations with a consideration of various simplification strategies and numerical solution methodologies

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The flow of fluids in our world is governed by the Navier-Stokes equation, which describes, at its simplest, the conservation of mass and momentum within a flowing fluid [1]. Despite our knowledge of this equation and its importance, we remain surprisingly unable to obtain analytical solutions to it, due to (among other factors) the closed form nature of the solution, its non-linearity and our poor understanding of turbulence in general [1]. As such, numerical methods remain the only way to obtain solutions to the equation in all but the most simple flow cases [1].

To overcome these problems, various simplifications to the full Navier-Stokes equation have been proposed. These methods can vary considerably both in terms of the simplifications made, and the level of complexity reduction achieved but the general premise of all methods are similar [1]. They replace complicated terms in the Navier-Stokes equation with simplified models of those terms (including the removal of complete terms sometimes). By removing complicated terms of the equation which aren't needed to accurately model the flow, the complexity of the equation can be substantially reduced.

In this project, we will investigate a series of simplifications of the full Navier-Stokes equation, beginning with Euler flow [2]. We will investigate at least the effect of compressibility on the complexity of solving the equations. We will apply the various solution methodologies investigated in the homework (Euler, RK methods etc.) to the Euler equations, as well as others that were not studied within this class. We will then evaluate the performance of each of these methods by quantifying their accuracy, stability and convergence. This will allow us to gain an understanding of the effect of different solution methodologies on the solving of these important equations. If time allows, we will also repeat this analysis on other less simplified versions of Navier-Stokes based equations, allowing us to study how complicating the equation affects the process of solving them. In all cases, we will create animated solutions of the solved flow structure in order to better visualize the underlying flow behaviors and how they impact our ability to solve the equations.

[1] Temam, Roger. Navier-Stokes equations: theory and numerical analysis. Vol. 343. American Mathematical Soc., 2001.

[2] Constantin, Peter. "On the Euler equations of incompressible fluids." Bulletin of the American Mathematical Society 44.4 (2007): 603-621.