Continuous Random Variables

Continues random variable

- A random variable is called **continuous** if its possible values contain a whole interval of numbers. These typically arise from a measurement. New possible examples:

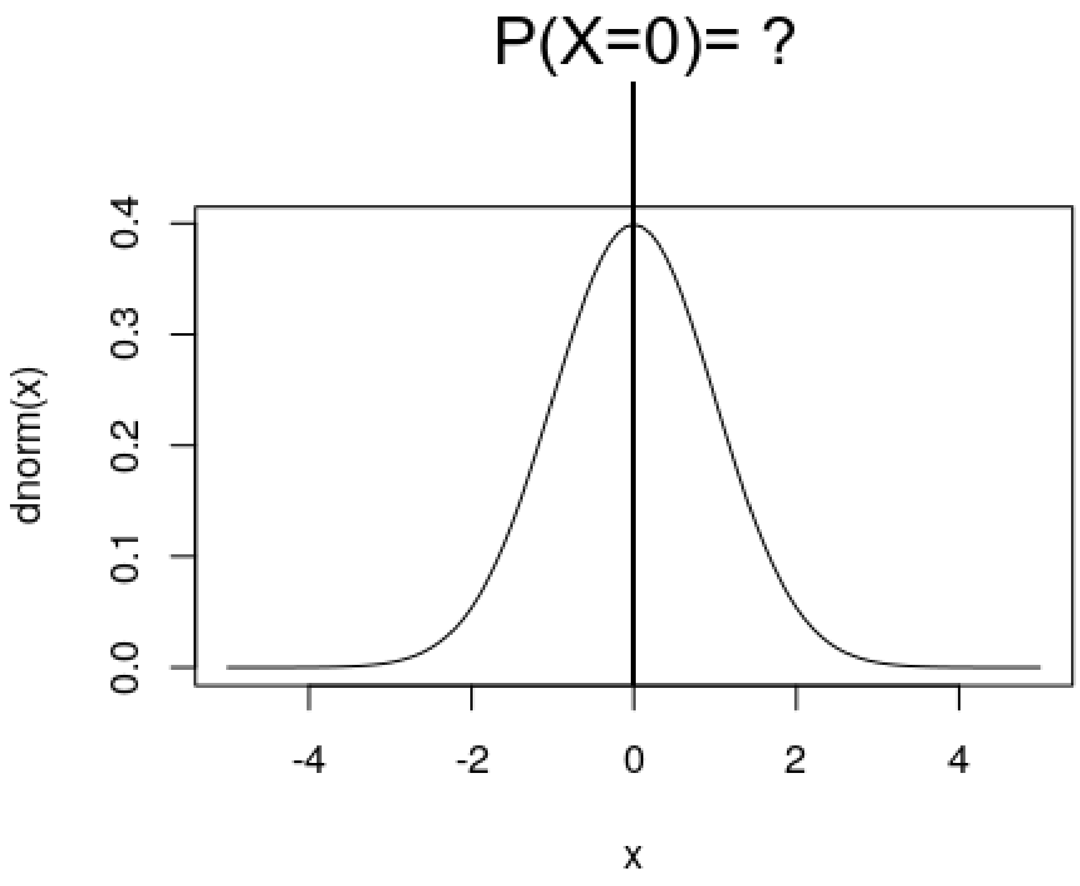
Experiment	Number *X*	Possible Values of *X*
Body height	Measurement in cm	$55 \leq x < 310$
A person's annual income	Measurement in money	$0 \leq x < \infty$

The probability distribution of a continuous random variable

- A discrete random variable X has a probability for each outcome of the experiment, continues random variables does not.
- A person waits for a bus that leaves every 30 minutes. How long does he have to wait.
- The probability of exactly 7.211916 minutes is pretty slim.
- Intervals are more meaning full. For example, the waiting time is less than 10 minutes, or is between 5 and 10 minutes

Trick question

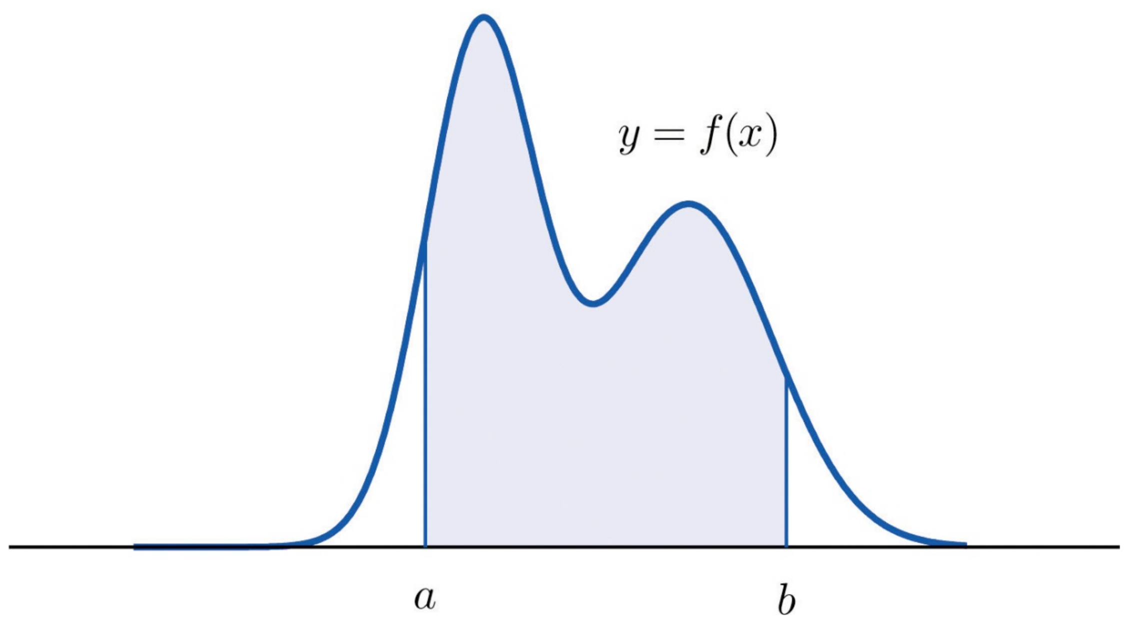
- Let X be a continues random variable, what is P(X=0) ?



Probability density function

- The probability distribution of a continuous random variable X is an assignment of probabilities to intervals of decimal numbers. The probability that X assumes a value in the interval [a,b] is the area under the curve in that interval.

$$P(a < X < b) = \text{area of shaded region}$$



1. For all numbers x, $f(x) \geq 0$
2. The area of the region under the graph of $y=f(x)$ and above the x-axis is 1.

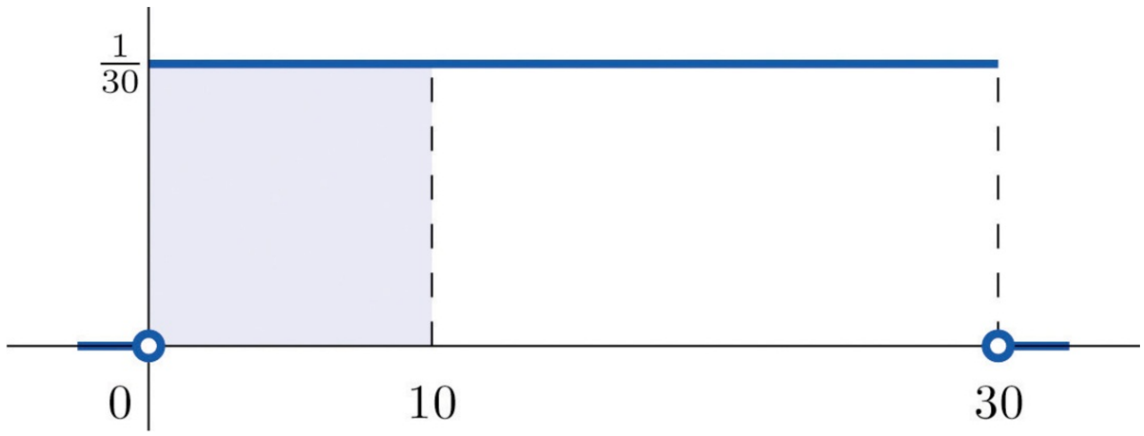
For any continuous random variable X: $P(a \leq X \leq b) = P(a < X \leq b) = P(a \leq X < b) = P(a < X < b)$

Bus stop example

- A person just arrived at the bus stop, for how long does the person has to wait until the next bus leaves. The bus leaves uniformly distributed every 30 minutes.
- Find the probability that a bus will come within the next 10 minutes.

Example continued

- X is uniformly distributed f(x)=1/30

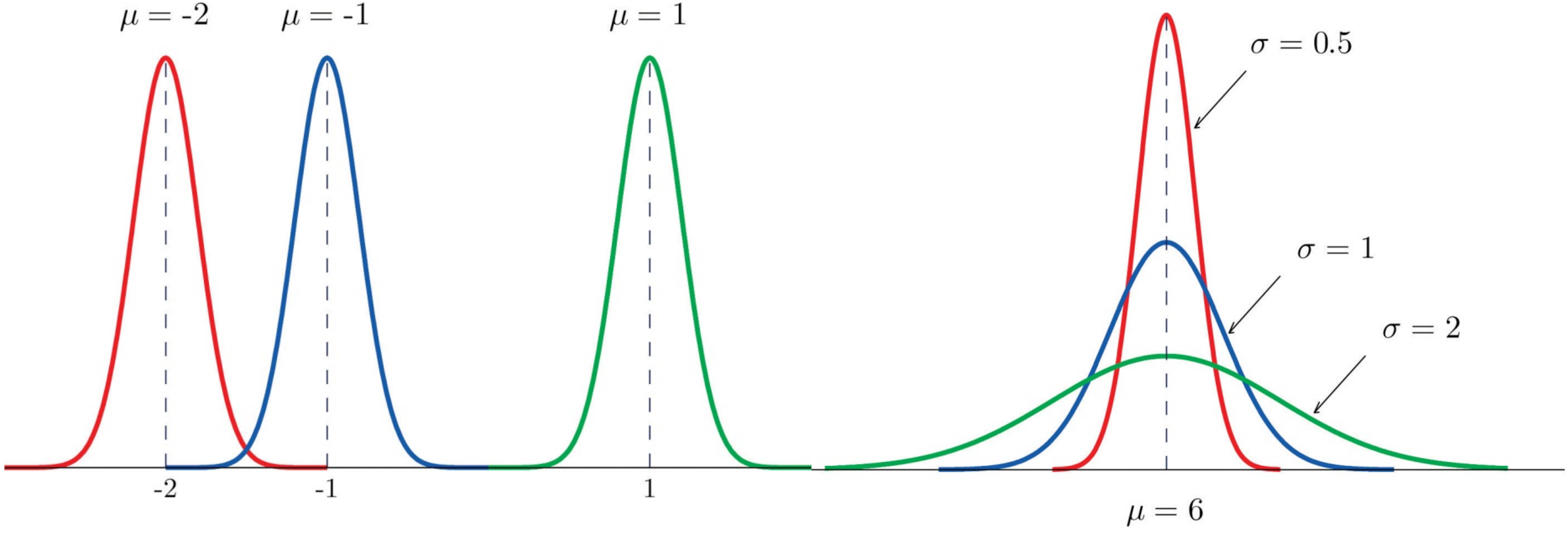


- We want to estimate P(0≤X≤10)

- We integrate over the probability density function in the limits 0 and 10. P(0≤X≤10) = 1/3

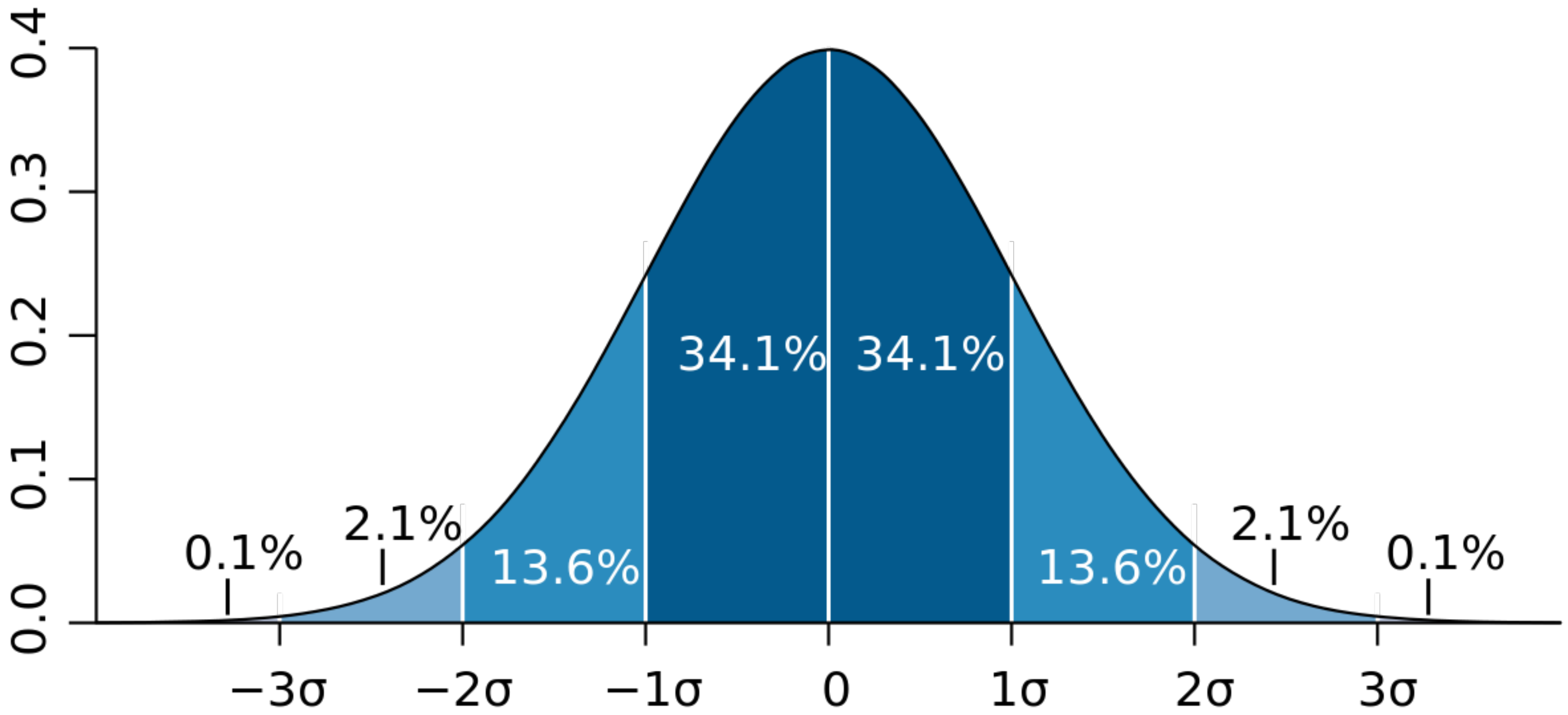
Normal Distribution

- Bell shaped curve with the probability density function= $\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
- Notated as: $\mathcal{N}(\mu, \sigma^2)$
- With the parameters μ for the mean and σ² for the variance



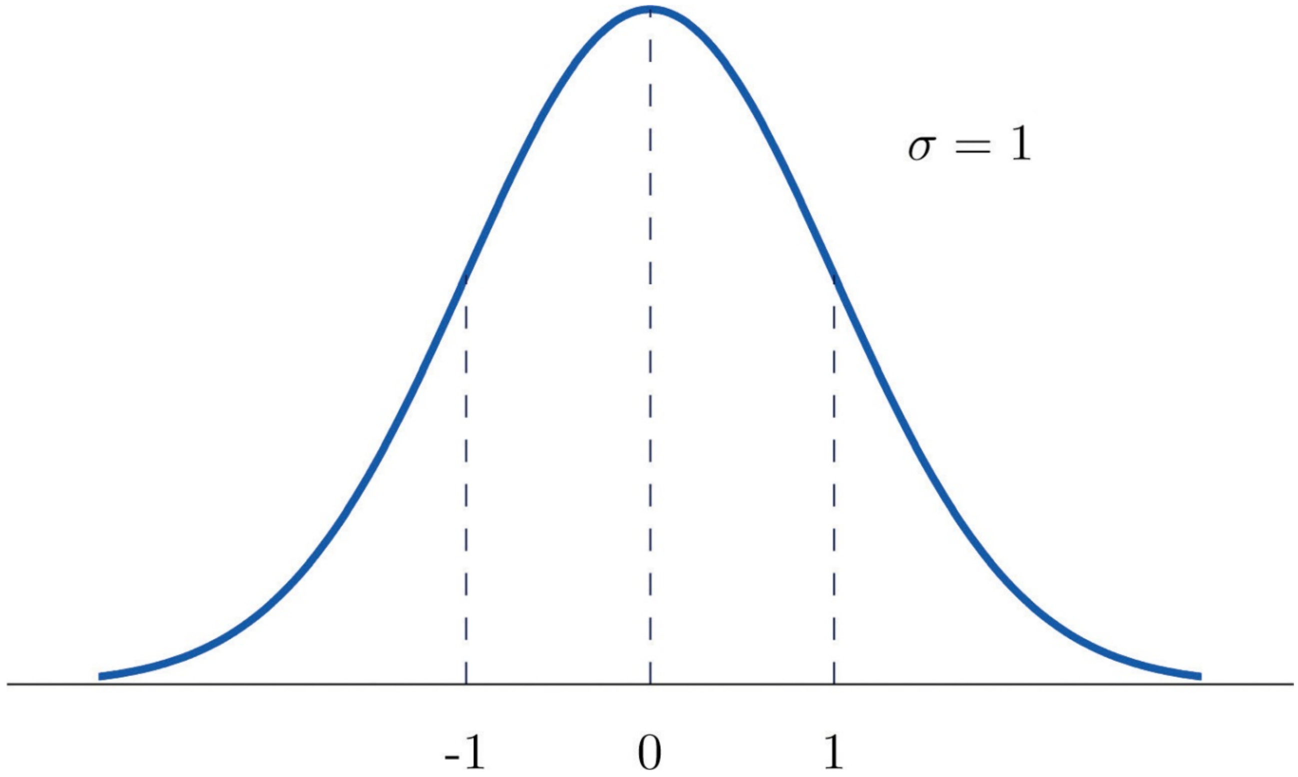
Normal Distribution

- The density curve for the normal distribution is symmetric about the mean.



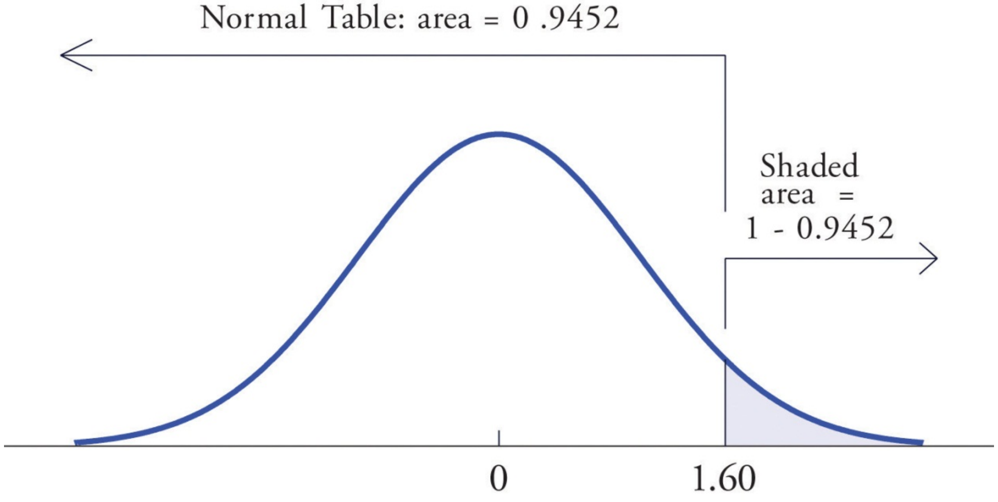
Standard normal random variable

- Standard normal random variable is a normally distributed random variable with mean μ = 0 and standard deviation σ = 1. Denoted as Z in the following.



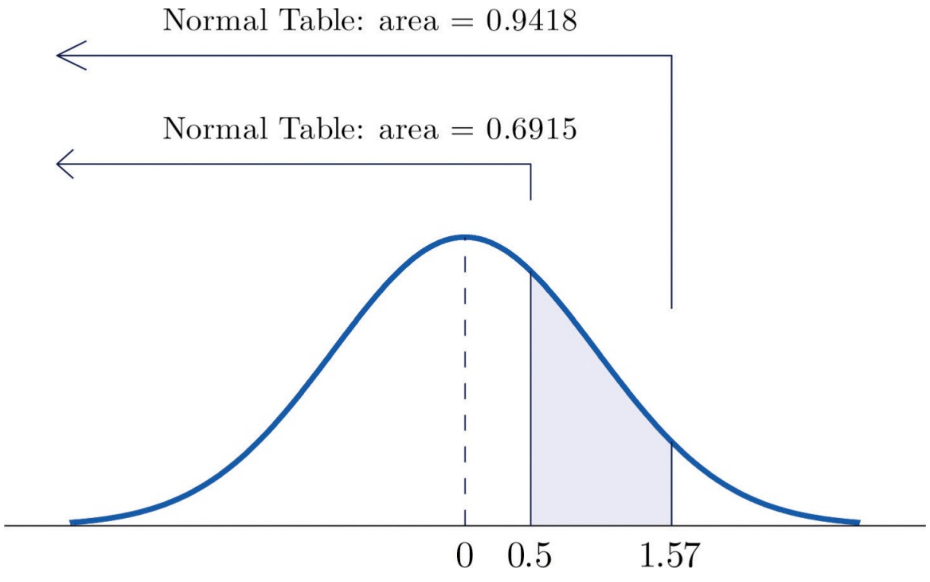
Standard normal random variable

- $P(Z<1.6)=0.9452$
- $P(Z>1.6)= 1-P(Z<1.6)$



- $P(0.5<Z<1.57)=$

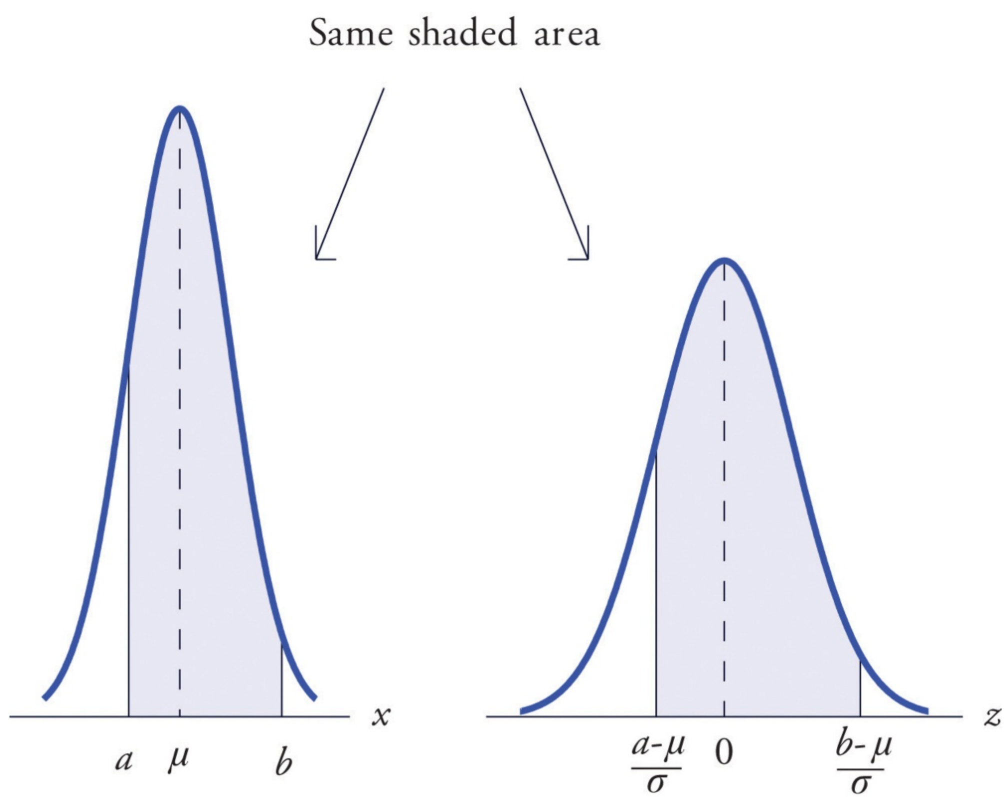
  $P(Z<1.57) -P(Z<0.5)$



Probability computations for general normal random variables

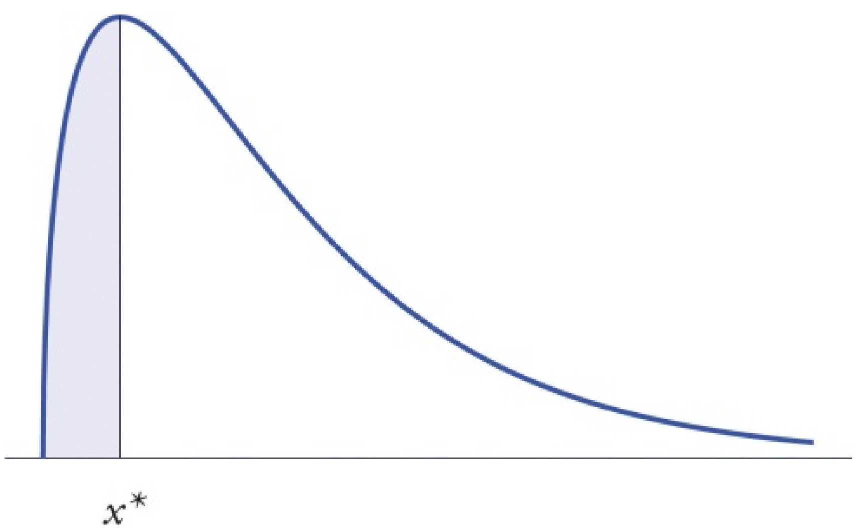
- If X is any normally distributed normal random variable then we can compute a probability of the form

$$P(a < X < b) = P\left(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right)$$

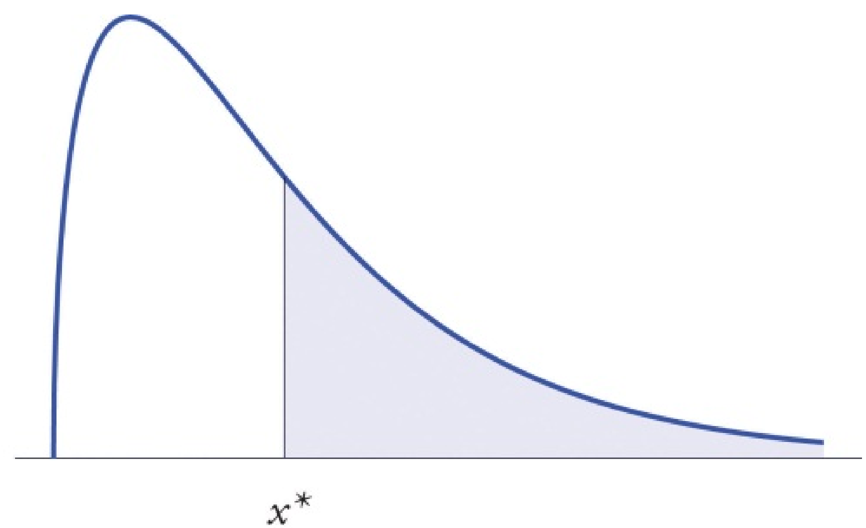


Areas of tails of distributions

- The **left tail** is the area cut of by x* from the left Figure a)
- The **right tail** cut off by x* from the right Figure b)

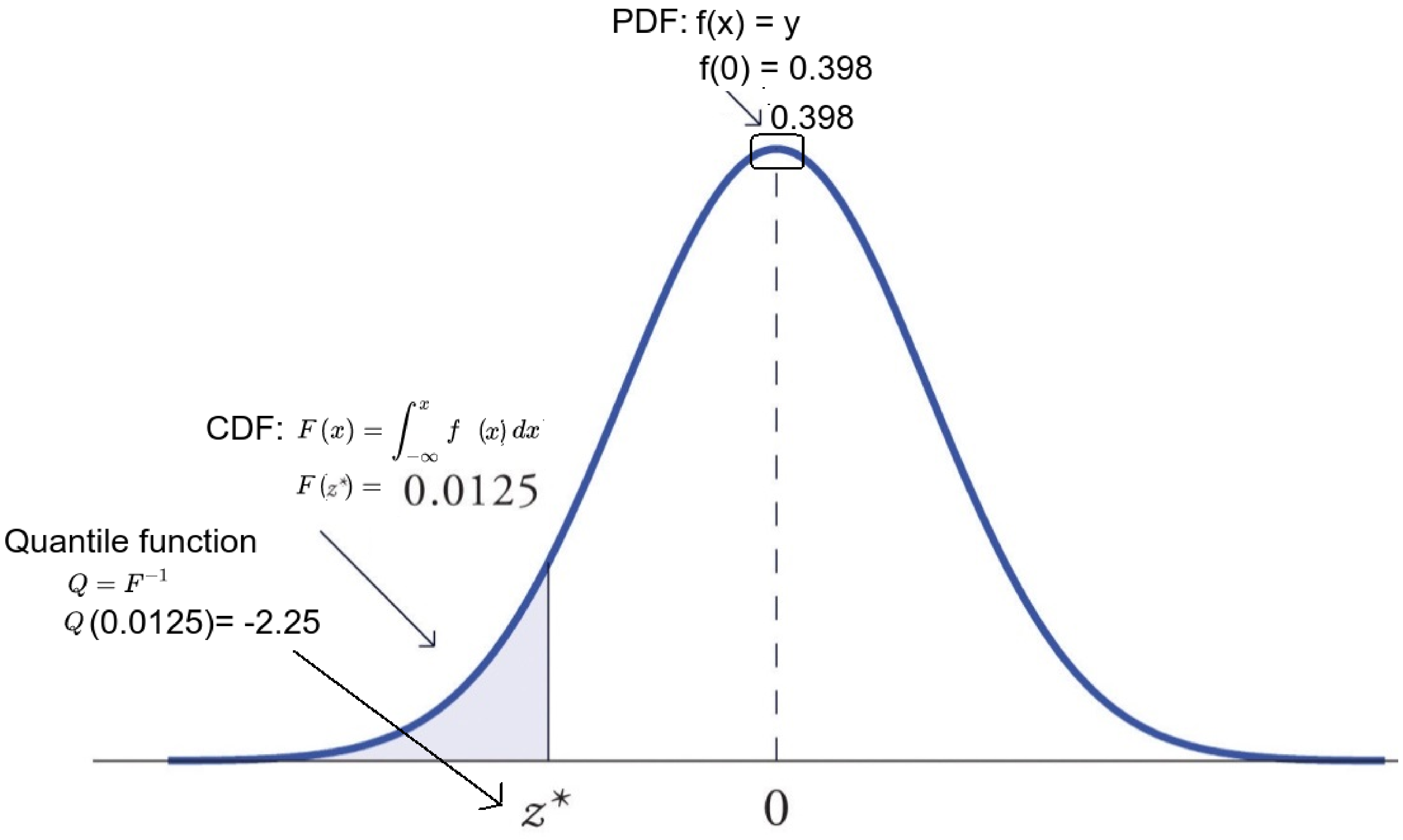


(a)



(b)

PDF, CDF, Quantile function



R Exercise

- How to create random samples from any distribution using random samples from the uniform distribution and the targets quantile function ?