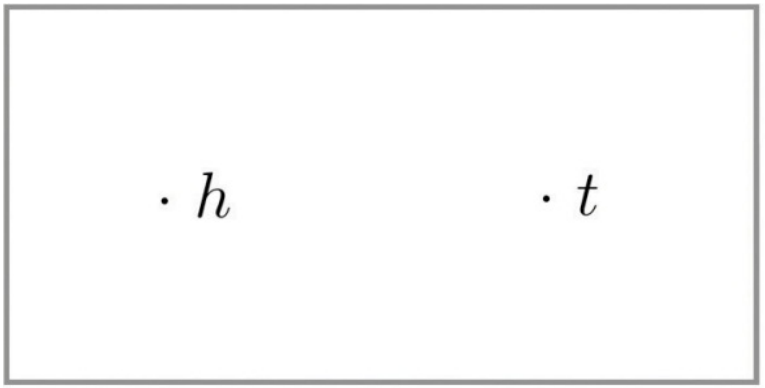
Basic Concepts of Probability

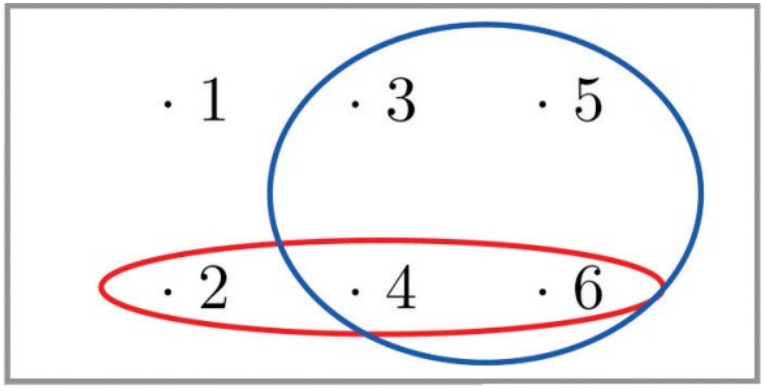
Rolling an ordinary six-sided die is a random experiment.
All possible outcomes can be listed, but for which the actual outcome on any given trial of the experiment
cannot be predicted with certainty.

Sample spaces and events

- Sample space of the die: S={⚀,⚁,⚂,⚃,⚄,⚅ }
- Rolling a particular a number is called the probability of the outcome.
- Event, or collection of outcomes, such as rolling an even number E={⚁,⚃,⚅ }



(a): One Coin Toss



(b): One Die Roll

Probability

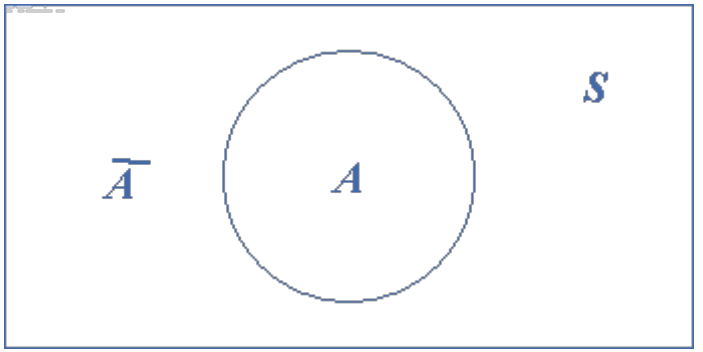
- The probability of an outcome e in a sample space S is a number p between 0 and 1.
- p = 0 corresponds to the outcome e being impossible.
- p = 1 corresponds to the outcome e being certain.
- The probability of an event A is the sum of the probabilities of the individual outcomes of which it is composed. It is denoted P(A).
- The sum over all probabilities has to be 1.

Question

- A box contains 10 white and 10 black marbles. Randomly, drawing out, with replacement, two marbles in succession and noting the color each time. (To draw “with replacement” means that the first marble is put back before the second marble is drawn.)
- What is the probability of drawing two marbles with the same color?

Complements, intersections, and unions

- The complement A^c or $\overline{A}$ are all others $P(A^c)=1-P(A)$



- A fair coin is tossed 5 times. What it the probability of observing at least one head?
- P(no heads) = 0.5^5
- P(at least one head) = 1 - P(no heads)

Intersection of events

- The intersection of events A and B, denoted A ∩ B, is the collection of all outcomes that are elements of both of the sets A and B. It corresponds to combining descriptions of the two events using the word “and”.
- A fair die is rolled, what is the

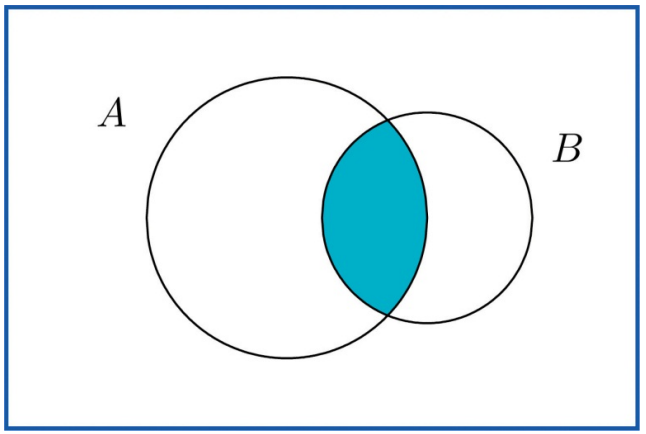
  probability the result is both even

  and greater than two ?

  Even={2,4,6} High={3,4,5,6}

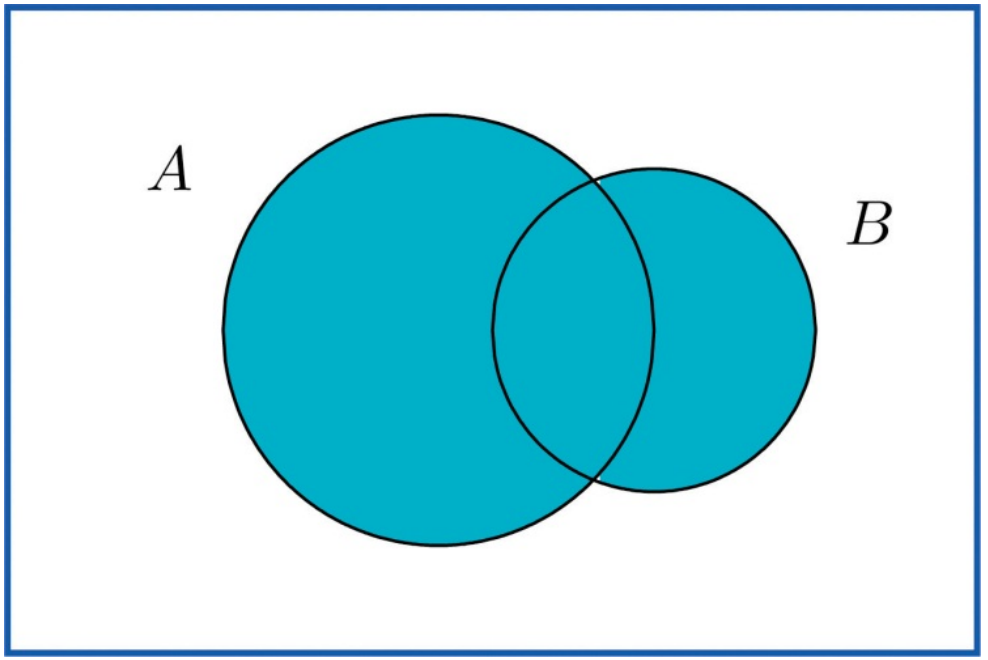
  Even∩High = {4,6}

  P(Even∩High) = 2/6



Union of events

- The union of events A and B, denoted A ∪ B, is the collection of all outcomes that are elements in the sets A and B. It corresponds to combining descriptions of the two events using the word "or".



Additive rule of probability

- Additive Rule of Probability:

  $P(A \cup B)=P(A)+P(B)-P(A \cap B)$

- Two fair dices are thrown. Find the probability that at least one die shows a four.

  $P(D1 \cup D2) = 6/36 + 6/36 – 1/36 = 11/36$

Multiplication rule of probability

- For dependent events: $P(A \cap B)=P(A)\cdot P(B \mid A)$

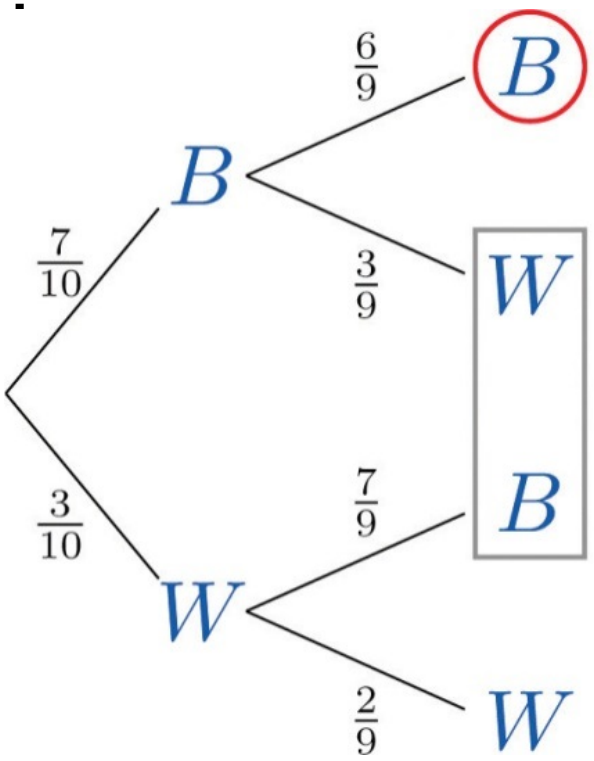
  For example, drawing without replacement.

- For independent events: $P(A \cap B)=P(A)\cdot P(B)$

  For example, independent coin tosses or drawing with replacement.

Probabilities and tree diagrams

- A jar contains 10 marbles, 7 black and 3 white. Two marbles are drawn without replacement.
- What is the probability that both marbles are black?



$$P(B_1 \cap B_2) = \frac{7}{10} \cdot \frac{6}{9} = 0.47$$

$$P(B_1 \cap W_2) = \frac{7}{10} \cdot \frac{3}{9} = 0.23$$

$$P(W_1 \cap B_2) = \frac{3}{10} \cdot \frac{7}{9} = 0.23$$

$$P(W_1 \cap W_2) = \frac{3}{10} \cdot \frac{2}{9} = 0.07$$

Conditional probability and independent events

- What is the probability of rolling a 5.
- How would your estimation change if you know the result of the throw was odd?
- The conditional probability of A given B denoted: P(A|B), is the probability that event A has occurred for which it is known that event B has definitely occurred.
- Rule for Conditional Probability:

  $P(A|B)=P(A \cap B)/P(B)$

What is the probability?

- Probability of rolling a 5, given the result was odd

  Five={⚅} Odd={⚀,⚂,⚄ }

  P(Five| Odd)=P(Five ∩ odd) / P(odd)

  Five∩Odd ={⚅}∩{⚀,⚂,⚄}={⚅}, P(Five ∩ odd) = 1/6

  P(Odd)=3/6

  P(Five| Odd) = (1/6) / (3/6) = 1/3

R exercise

- Suppose you take out two cards from a standard pack of cards one after another, without replacing the first card. What is probability that the first card is the ace of spades, and the second card is a heart?

  The two events are dependent events because the first card is not replaced.

  There is only one ace of spades in a deck of 52 cards. So:

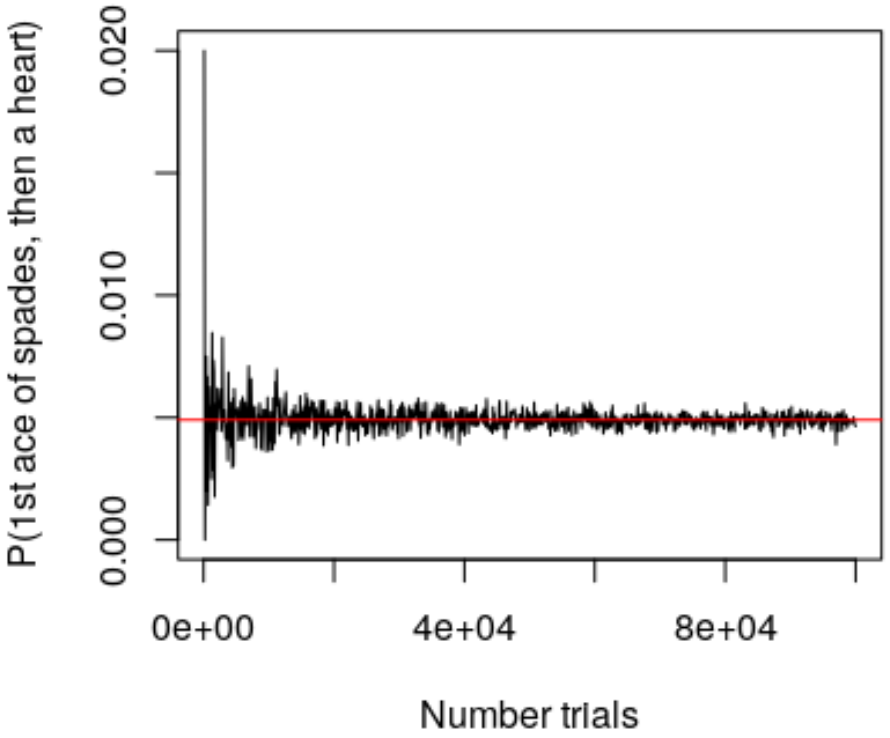
  P(1st card is the ace of spades)=1/52

  If the ace of spaces is drawn first, then there are 51 cards left in the deck, of which 13 are hearts:

  P(2nd card is a heart | 1st card is the ace of spades)=13/51

  So, by the multiplication rule of probability, we have:

  P(ace of spades, then a heart)=1/52 * 13/51= 1/204



- Simulate, and what is the influence of the number of trials