Testing Hypotheses

- A manufacturer of emergency equipment ensures that their oxygen tanks delivers pure air for 75 minutes on average.
- A government agency is more interested in testing this claim than on estimation of the population mean.

Null and alternative hypothesis

- A hypothesis about the value of a population parameter is an assertion about its value.
- The **null hypothesis**, denoted $\mathbf{H_0}$, is the statement about the population parameter that is assumed to be true.
- The **alternative hypothesis**, denoted $\mathbf{H_a}$, is a statement about the population parameter that is contradictory to the null hypothesis.
- **Hypothesis testing** is a statistical procedure in which a choice is made between the null hypothesis and alternative hypothesis based on information in a sample.

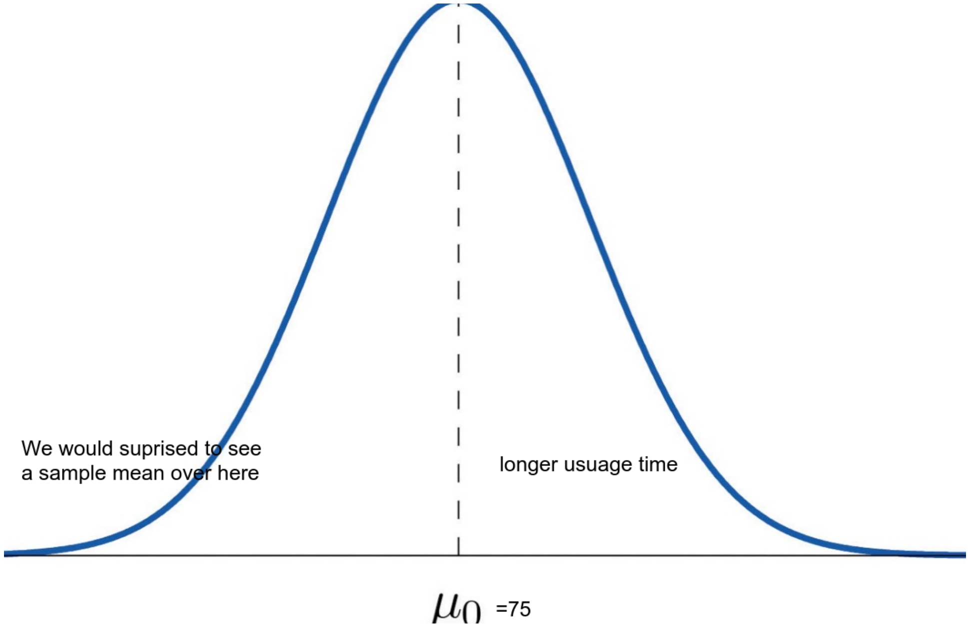
Result of a hypotheses test

- Te result of a hypotheses testing is:
  - 1. Reject H_0 (and therefore accept H_a), or
  - 2. Fail to reject H_0 (and therefore fail to accept H_a).

- The null hypothesis represents the status quo, or what has historically been true. In the previous example, null hypotheses is H_0:μ=75.
- The alternative hypothesis in the example is the contradictory statement H_a :μ<75.

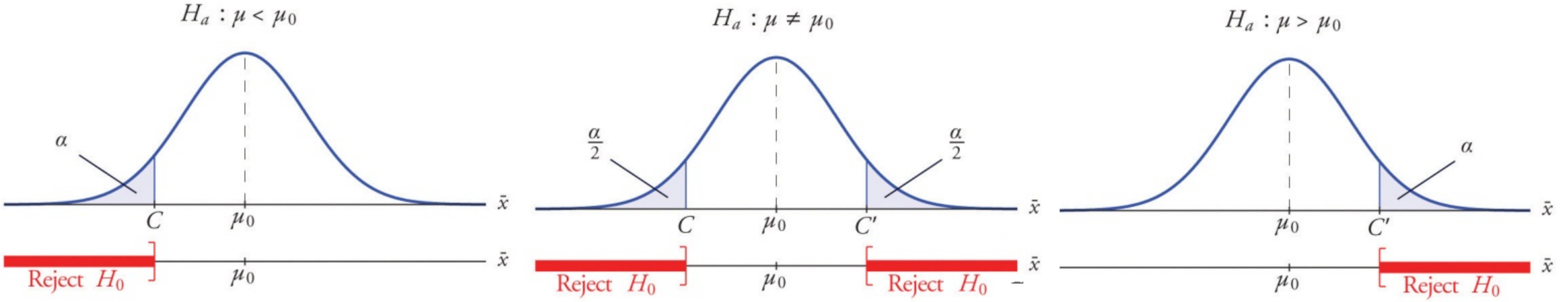
Hypotheses test

- For the example, we would analyses if the sample mean is close to the claimed mean $\mu_0 = 75$.
- If the sample mean is 75 or bigger, we would have no doubt in believing H_0.
- If the sample mean is much lower than 75, we had reason to believe that H_a is true.



The rejection region

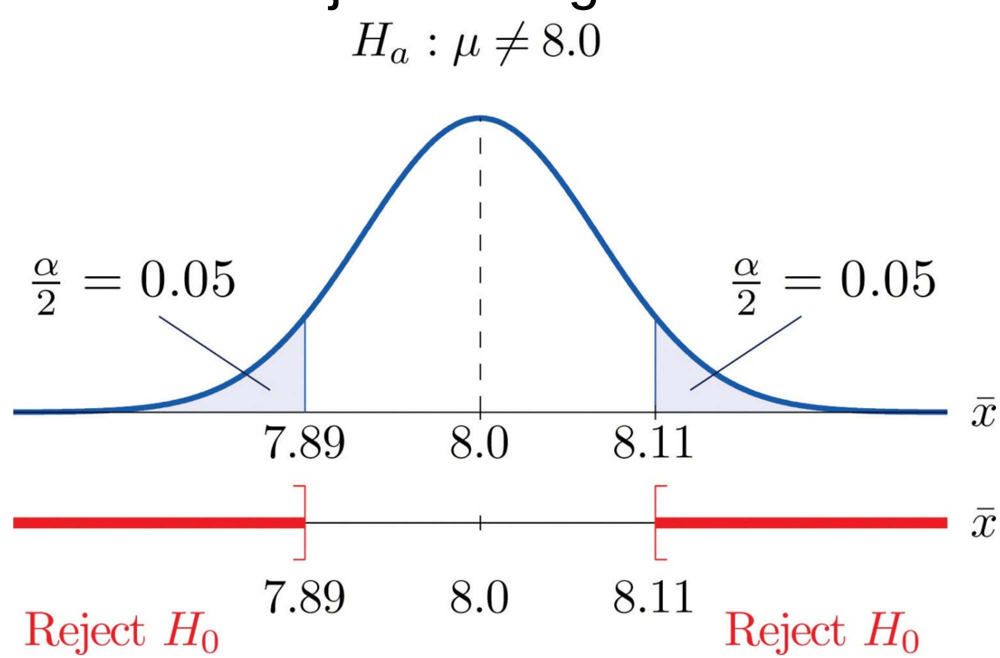
- Based on the assumption of the alternative hypothesis, we have different rejection regions.
- Rejection region is determined by the critical values C and C’.



Example

- The recipe for a bakery item is designed to result in a product that contains 8 grams of fat per serving. The quality control department samples the product periodically to insure that the production process is working as designed.
- Thus the null hypothesis is $H_0 : \mu = 8.0$.

  The alternative hypothesis is $H_a : \mu \neq 8.0$.
- Suppose it is known that the population is normal distributed with a standard derivation σ= 0.15 gram. Construct the rejection region for α=0.1 and a sample size of 5.
- $\mu_{\bar{x}} = \mu = 8.0 \quad \sigma_{\bar{x}} = \frac{0.15}{\sqrt{(5)}} = 0.067$



Hypothesis test

- Hypothesis tests are classified according to the form of the alternative hypothesis in the following way.

- If H_a has the form $\mu \neq \mu_0$ the test is called a **two-tailed test**.
- If H_a has the form $\mu < \mu_0$ the test is called a **left-tailed test**.
- If H_a has the form $\mu > \mu_0$ the test is called a **right-tailed test**.

Two types of errors

- The testing procedure will always result into either
- 1. reject the null hypothesis H_0 in favor of the alternative H_a presented, or
- 2. do not reject the null hypothesis H_0 in favor of the alternative H_a presented.
- There are four possible outcomes of hypothesis testing procedure:

		True State of Nature	
		H_0 is true	H_0 is false
Our Decision	Do not reject H_0	Correct decision	Type II error
	Reject H_0	Type I error	Correct decision

Type I and Type II error

- In a test of hypotheses, a Type I error is the decision to reject H_0 when it is in fact true. A Type II error is the decision not to reject H_0 when it is in fact not true
- We make a Type I error with probability α.
- α that is used to determine the rejection region and is called the level of significance of the test.
- To reduce both Type I error and Type II error probability, increase the sample size.

Standardizing the test statistic

- A **standardized test statistic** for a hypothesis test is the statistic that is formed by subtracting from the statistic of interest its mean and dividing by its standard deviation.

  Z or T = $\dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

- This will result into standard normal or Student's t-distribution for the estimation of the critical region.

Large sample tests for a population mean

- It is hoped that a newly developed pain reliever will more quickly produce perceptible reduction in pain to patients after minor surgeries than a standard pain reliever. The standard pain reliever is known to bring relief in an average of 3.5 minutes with standard deviation 2.1 minutes. To test whether the new pain reliever works more quickly than the standard one, 50 patients with minor surgeries were given the new pain reliever and their times to relief were recorded. The experiment yielded sample mean $\bar{x}=3.1$ minutes and sample standard deviation $s=1.5$ minutes. Is there sufficient evidence in the sample to indicate, at the 5% level of significance, that the newly developed pain reliever does deliver perceptible relief more quickly?

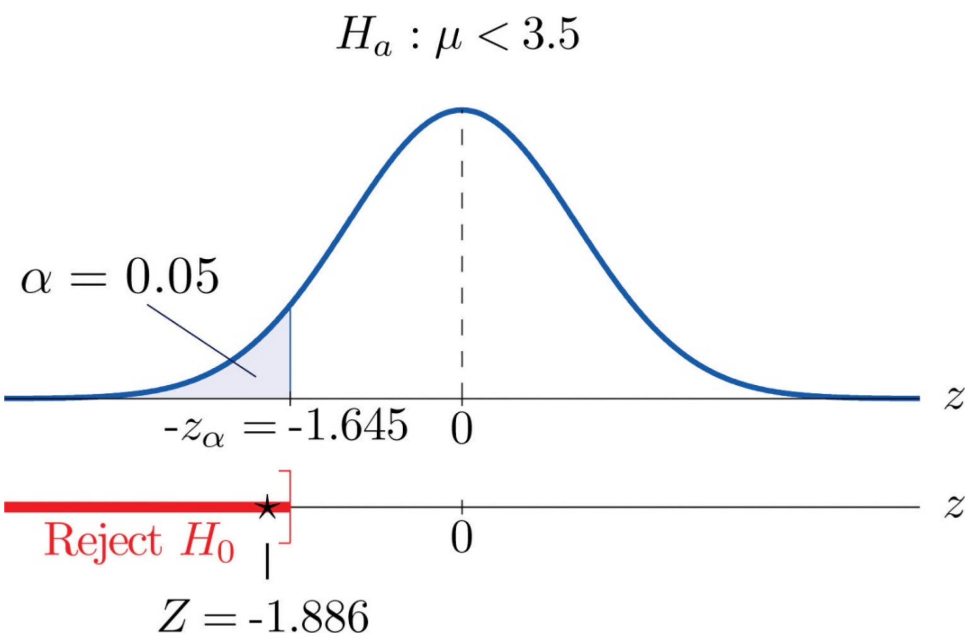
Example continued

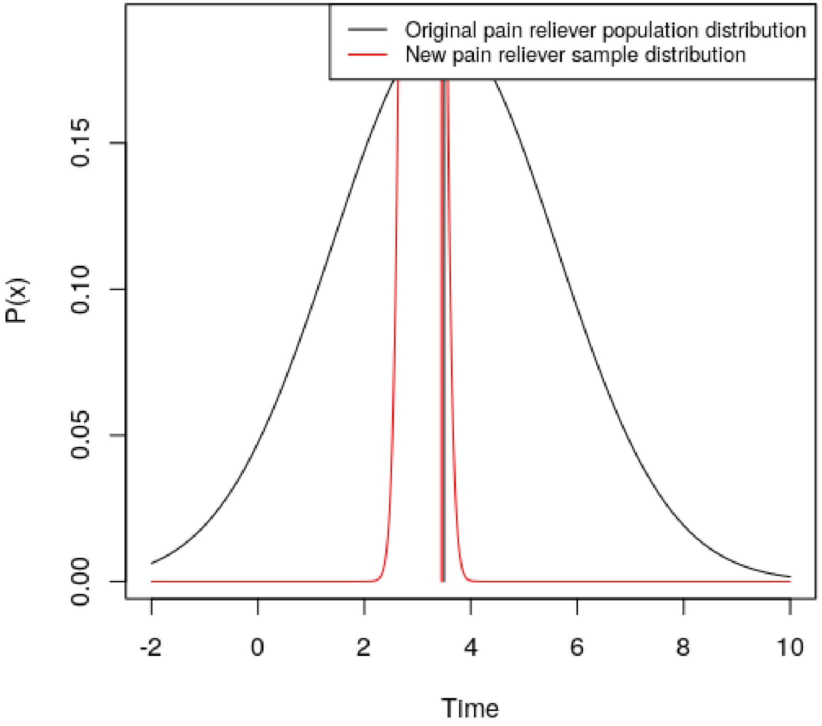
$H_0 = \mu = 3.5$

$H_a = \mu < 3.5$

- We have large sample, so we can use a normal test statistic.

$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{3.1 - 3.5}{1.5/\sqrt{(50)}} = -1.886$





p-value

- The area under the tail using the test statistic to cut of the tail is called p-value. For the previous example the p-value was 0.0294 or about 3%. Under repeated sampling from this population, if H_0 were true then only about 3% of all samples of size 50 would give a result as contrary to H_0 and in favor of H_a.
- The p-value can equally be used to decide weather H_0 is to be rejected or not.

Example

- The price of a popular tennis racket at a national chain store is \$179. Portia bought five of the same racket at an online auction site for the following prices:

  {155, 179, 175, 175, 161}.

  Assuming that the auction prices of rackets are normally distributed, determine whether there is sufficient evidence in the sample, at the 5% level of significance, to conclude that the average price of the racket is less than \$179 if purchased at an online auction.

Example

$H_0 = \mu = 179$

$H_a = \mu < 179$

- We have a small sample, and assume the population is normal distributed with its standard deviation unknown Therefore, we use the t distribution as test statistic with n-1 degrees of freedom.
- T= $\dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$ = $\dfrac{169-179}{10.29/\sqrt{(5)}} = -2.152$