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Abstract. In this paper, the analysis of American football strategies is by applying Nash equilibrium. Up to the offensive or defensive team-role, each player usually practices the relevant plays for his role; each play is qualified regarding the benefit that could add to the team success. The team's strategies, that join the individual's plays, are identified by means of the strategy profiles of a normal game formal setting of American football, and valued by the each player's payoff function. Hence, the Nash equilibrium strategy profiles can be identified and used for the actions decision making in a match gaming.

Keywords: American football, Nash equilibrium, team's strategies.

1 Introduction

Recently, the formal modeling and strategic analysis for support the matches gaming of multi-player sports like American football (AF) or baseball [1-3], have led investigations in areas of sport science [4-6], computer science, game theory [7], operation research [1, 3], and simulation models [2, 8], among others. In American football gaming, the team members are encouraged to do the best individual actions, but they must cooperate for the best team's benefit. The strategies are indicated by the team manager regarding on each player's profile as well as the specific match circumstances to obtain the most benefit [9]. A planned-strategy should include both the individual and the team motivation. The selection of strategies is an essential aspect to be considered for a whole AF automation. In [10] a formal model for automated simulation of AF gaming, using a context-free (grammar) language and finite state machine, allows for precise simulation runs on this ever strategic multi-player game.

1.1 American football description

American football (AF) is one of the top strategic games, played by two teams on a rectangular shaped field, 120 yards long by 53.3 yards wide, with goalposts in the end of the field. Each team has 11 players and a match lasts 1 hour divided in four quarters. The offensive team goal is advance an oval ball, by running or passing toward the adversary's end field [11-13]. The ways to obtain points are by advancing the ball, ten yards at least, until reach to the end zone for touchdown scoring, or kicking the ball such that it passes in the middle of the adversary's goalposts for a field goal, or by the defensive tackling the ball carrier in the offensive end zone for a safety. The offensive team should advance the ball at least ten yards in at most four downs (opportunities) to get four additional downs; otherwise the defensive team that is avoiding the ten yards advance, changes to the offensive role. The current offensive team's advance starts from the last ball stop position. If the defensive catches the ball before a down is completed, it starts the offensive role at this position.

1.2 Selection of strategies

The Nash equilibrium (NE) [14] is a widely used mathematical concept in game theory, especially in non-cooperative games. The Nash equilibrium formal account for multi-player games follows. Let $P = \{1, \ldots, n\}$ be the set of players, $i \in P,\ a_x{}^i \in \Sigma^i$ be an element of the set of simple plays, and $s_x{}^i = a_1{}^i \ldots a_n{}^i$ be a strategy of player i is a sequence of actions , $s_x{}^i \in \mathrm{S}_i,\ \mathrm{S}_i$ the set of strategies for the i_{th} player. Let $(s_1, \ldots, s_n) \in S_1 \times \ldots \times S_n$ a strategy profile, one strategy per player, and let $\{u_1, \ldots, u_n\}$ be the set of every player payoff functions, such that $u_i\ (s_1, \ldots, s_n) = r \in \mathbb{R}$. Let $G = (S_1, \ldots, S_n; u_1, \ldots, u_n\ )$ be the game in normal form [14].

To identify the strategy profiles that satisfy the condition of Nash equilibrium, every strategy profile is evaluated with the payoff functions of the players, and the chosen profiles are those which, for every player, is the options that produces less loss for him regarding the other players' strategies; so, is the best option, for each player, but individually, in non-cooperative way. A NE strategy profile $(\mathrm{s_1}^*, \ldots, \mathrm{s_i}^*, \ldots, \mathrm{s_n}^*)$ maximizes the payoff function in equation (1):

$$\mathrm{u_i}(\,\mathrm{s_1}^*, \ldots, \mathrm{s_i}^*\,, \ldots, \mathrm{s_n}^*) \geq\ \mathrm{u_i}(\,\mathrm{s_1}^*, \ldots, \mathrm{s_i}\,, \ldots, \mathrm{s_n}^*)\ \ \forall \mathrm{i} \in \mathrm{P}, \mathrm{s_i}\ \in \mathrm{S_i} \tag{1}$$

We use the normal game formal account of American football in order to analyze the strategies of a team in both, offensive and defensive roles. Each strategy profile involves all the players' actions in certain moment of a match, and NE provides a formal way to measure and identify the strategy profile that satisfy the expectative of the players, both, given certain action and by regarding the other players' actions.

Next, in Section 2 concerns an overview of American football, the team-roles, as well as the each role's most relevant plays. Section 3 describes the each role's utility function for worth the strategy profiles. Section 4 presents the strategy profiles that satisfy the Nash equilibrium condition; they are selected and used during actions decision making in a match. Some remarks feed the Discussion and Conclusion.

2 Strategies by team-role and the average occurrence of plays

In this section, we present some AF plays being divided according to the team-role, offensive or defensive (**Table 1**).

Table 1. Offensive and defensive plays

Off. plays	Description	Def. plays	Description
kb	Kick the ball	tl	Tackling
cb	Catch the ball by product of a pass	sf	Safety
rb	Run with the ball	sb	Stop the ball
pb	Pass the ball	in	Interception
fd	Scoring yards	qs	Tackling the quarterback
td	Touchdown	yb	Roll back the contraries
p	Extra point (1 point by product of a kick)	fb	Fumble the ball
re	Conversion (2 points)	fr	Turnover the ball
fg	Field goal	tb	Touchback

Using real statistical from NFL (National Football League) see http://gametheory.cs.cinvestav.mx/NFL_statistics.pdf, the probability of occurrence of each play above is calculated and listed in descending order in **Table 2**. These values come from performing statistical averages of values in tables showing data by player-role and not by specific player, but for a specific player his individual statistics can be used.

Table 2. Probability of ocurrence of AF plays

Play	Average	$p(play)$	Play	Average	$p(play)$
sb	1050.5	0.232884067	p	39.4375	0.008742851
yb	845	0.187327022	fg	26.96875	0.005978669
tl	775.6875	0.171961218	td	25.125	0.005569931
pb	566.75	0.125642118	in	15.6875	0.003477743
rb	348.484375	0.077255077	fb	15.09375	0.003346115
cb	346.9375	0.076912152	fr	9.625	0.002133755
fd	319.125	0.070746433	tb	5.875	0.001302422
kb	78.40625	0.017381786	re	1.03125	0.000228617
qs	40.46875	0.008971468	sf	0.625	0.000138555

2.1 Offensive team plays

- Offensive linemen players OL have two major tasks: 1) block the defensive team members which try to tackle to the quarterback (QB), and 2) open ways in order to runners can pass. The OL players are, the center, left guard, right guard, left tackle and right tackle. We defined these players as OL and the plays to consider are $OL_{\text{plays}} = \{tl, yb\}$.
- The quarterback (QB) is the offensive leader, whose plays follows, $QB_{\text{plays}} = \{rb, pb, fd, td, re, tb\}$.
- The backfield players BF are: the halfback, tailback the fullback. The BF plays follow, $BF_{\text{plays}} = \{rb, fd, td, re, tb, tl\}$.

- Receiver's role RC is to catch the ball passed by the QB; RC players are the tight end and wide. The RC plays follow, $RC_{\text{plays}} = \{cb, rb, fd, td, re\}$.

2.2 Defensive team plays

- The defensive linemen players DL are: the defensive end, defensive tackle and nose tackle, their main task is to stop running plays on the inside and outside, respectively, to pressure the QB on passing plays. The DL plays follow, $DL_{\text{plays}} = \{tl, sf, sb, qs, yb, fb, fr\}$.
- The linebacker players LB have several tasks: defend passes in shortest paths, stop races that have passed the defensive line or on the same line and attack the QB plays penetration; they can be three or four. The LB plays follow, $LB_{\text{plays}} = \{tl, sf, sb, qs, fb, fr\}$.
- The defensive backfield players DS are: the cornerbacks and safeties, which major task is to cover the receivers. The DS plays follow, $DS_{\text{plays}} = \{tl, in, fb, fr\}$.

2.3 Special team plays

- Kicker player K kicks off the ball and do field goals and extra points. The kicker's plays follow, $K_{\text{plays}} = \{kb, p, fg\}$.
- The kickoff returner R is the player on the receiving team who catches the ball. The plays are $R_{\text{plays}} = \{rb, td, tb\}$.

3 Setting-up of payoff functions

The each role's payoff function to value the strategy profiles, selects the own convenience value by regarding:

- For QB is important to make a pass, his characteristic move, even with a touchdown scoring can generate a greater personal gain.
- The basic action of RC is to increase the score, but to make it happens he must catch the ball and run to the touchdown line.
- The OL main function is tackling the adversary to allow QB send pass; as well, open space for RC ball runs, or, in some cases, push back the opposing team.
- The BF preferred score is touchdown or conversion, and should run to get there. Other option is to get a first down, or tackling a player of the opposing team.
- The DL should be tackling the opposing QB, roll back yards to the opposing team or get a safety; in descent order of importance the following is to stop the ball, tackling and cause fumbles and try to recover it by the opponent.
- The main function of LB is to recover a lost ball and then could be to generate a safety.
- For DS intercepting a pass would be best, but it is also important to get the other team loses control of the ball.

- For K, the most important is to make a field goal, followed by an extra point and typically perform the corresponding kicks.
- For R, the best choice is to score a touchdown with the return of the kick, but usually just run until stopped, or perform touchback for time.

We propose that the player-roles' skills are qualified on the base of the player-roles' performance on certain plays, and the statistics resumes these qualifications. Let $u_i(x_1, \dots, x_i, \dots, x_n) = V_1(x_1) * p(x_1) + \cdots + V_i(x_i) * p(x_i) + \cdots + V_n(x_n) * p(x_n)$ be the payoff function of the player-role i, $(x_1, \dots, x_i, \dots, x_n)$ is a strategy profile such that x_i is one play of player-role i, The factors in the payoff function are: $V_i(x_i)$ represents the player-role i's preference on the play x_i, and $p(x_i)$ is the average statistics of the player-role on play x_i, by regarding the NFL statistics [15]; as well, the other elements in the formula are the contributions of the other player-roles whom directly share the play.

3.1 Offensive team

Let define the strategy profile for offensive team as (w, x, y, z), with $w \in QB_{\text{plays}}, x \in RC_{\text{plays}}, y \in OL_{\text{plays}}, z \in BF_{\text{plays}}$.

- For QB, we should consider the QB plays as well as the OL plays, the payoff function (2) follows.
$$u_{QB}(w, x, y, z) = V_{QB}(w) * p(w) + V_{OL}(y) * p(y) \quad (2)$$
- For RC, we should consider the RC plays, the QB as well as the OL plays, the payoff function (3) follows.
$$u_{RC}(w, x, y, z) = V_{RC}(x) * p(x) + V_{QB}(w) * p(w) + V_{OL}(y) * p(y) \quad (3)$$
- For BF, we should consider the BF plays, the QB plays as well as the OL plays, the payoff function (4) follows.
$$u_{BF}(w, x, y, z) = V_{BF}(z) * p(z) + V_{QB}(w) * p(w) + V_{OL}(y) * p(y) \quad (4)$$
- For OL, we should only consider the OL plays, the payoff function (5) follows.
$$u_{OL}(w, x, y, z) = V_{OL}(y) * p(y) \quad (5)$$

3.2 Defensive team

Let define the strategy profile for defensive team as (x, y, z) where $x \in DL_{\text{plays}}, y \in LB_{\text{plays}}, z \in DS_{\text{plays}}$.

- For DL and LB, we should consider DL plays as well as LBplays, the payoff function (6) follows.
$$u_{DL|LB}(x, y, z) = V_{DL}(x) * p(x) + V_{LB}(y) * p(y) \quad (6)$$
- For DS, we should only consider the DS plays, the payoff function (7) follows.
$$u_{DS}(x, y, z) = V_{DS}(z) * p(z) \quad (7)$$

3.3 Special team

- For K, the payoff function (8) follows.

$$u_K(x) = V_K(x) * p(x) \text{ where } x \in K_{\text{plays}} \tag{8}$$

- For R, the payoff function (9) follows.

$$u_R(x) = V_R(x) * p(x) \text{ where } x \in R_{\text{plays}} \tag{9}$$

4 Experiments

We use the set of values in **Tables 3 – 4** that are assigned according to each player's preference values on each of the own plays, and are used to calculate the payoff functions, that in turns are used to find out the strategy profiles that fit the Nash equilibrium condition.

Table 3. Values of offensive plays by player

QB	RC	OL	BF
$V_{QB}(rb) = 0.5$	$V_{RC}(cb) = 0.7$	$V_{OL}(tl) = 0.5$	$V_{BF}(rb) = 0.7$
$V_{QB}(pb) = 0.7$	$V_{RC}(rb) = 0.7$	$V_{OL}(yb) = 0.6$	$V_{BF}(fd) = 0.4$
$V_{QB}(fd) = 0.6$	$V_{RC}(fd) = 0.6$		$V_{BF}(td) = 0.9$
$V_{QB}(td) = 0.8$	$V_{RC}(td) = 0.9$		$V_{BF}(re) = 0.8$
$V_{QB}(re) = 0.9$	$V_{RC}(re) = 0.8$		$V_{BF}(tl) = 0.5$
$V_{QB}(tb) = 0.5$			

Table 4. Values of defensive plays by player

DL	LB	DS
$V_{DL}(tl) = 0.5$	$V_{LB}(tl) = 0.5$	$V_{DS}(tl) = 0.5$
$V_{DL}(sf) = 0.8$	$V_{LB}(sf) = 0.8$	$V_{DS}(in) = 0.9$
$V_{DL}(sb) = 0.6$	$V_{LB}(sb) = 0.6$	$V_{DS}(fb) = 0.7$
$V_{DL}(qs) = 0.9$	$V_{LB}(qs) = 0.9$	$V_{DS}(fr) = 0.8$
$V_{DL}(yb) = 0.8$	$V_{LB}(fb) = 0.7$	
$V_{DL}(fb) = 0.5$	$V_{LB}(fr) = 0.8$	
$V_{DL}(fr) = 0.4$		

Now, we define the set of strategy profiles. The set of strategy profiles for offensive team, (w, x, y, z) where $w \in QB_{\text{plays}}$, $x \in RC_{\text{plays},}$ $y \in OL_{\text{plays}}, z \in BF_{\text{plays}}$ is $OffensiveT_{\text{profiles}} = \{(rb, rb, tl, rb), (rb, rb, tl, fd), \dots, (tb, re, yb, tl)\}$. The set of strategy profiles for defensive team, (x, y, z) where $x \in DL_{\text{plays},}$ $y \in LB_{\text{plays}}, z \in DS_{\text{plays}}$ is $DefensiveT_{\text{profiles}} = \{(tl, tl, tl), (tl, tl, in), \dots, (fr, fr, fr)\}$.

Using the payoff functions defined in Section 3, each strategy profile is valued by respective player's payoff function. Some illustrative examples follow.

Offensive team

- For QB, $u_{QB}(rb, rb, tl, rb) = V_{QB}(rb) * p(rb) + V_{OL}(tl) * p(tl)$ is the payoff function that only embrace QB and OL plays. Reason is that RC and BF plays do not relevant impact the QB plays, so not the QB payoff function valuations. $u_{QB}(rb, rb, tl, rb) = 0.5 * 0.077255077 + 0.5 * 0.171961218$. $u_{QB}(rb, rb, tl, rb) = 0.124608$.
- For RC, $u_{RC}(rb, rb, tl, rb) = V_{RC}(rb) * p(rb) + V_{QB}(rb) * p(rb) + V_{OL}(tl) * p(tl)$ is the payoff function, that only embrace RC, QB and OL plays. Reason is that the BF plays do not relevant impact the RC plays, so not the RC payoff function valuations. $u_{RC}(rb, rb, tl, rb) = 0.7 * 0.077255077 + 0.5 * 0.077255077 + 0.5 * 0.171961218$. $u_{RC}(rb, rb, tl, rb) = 0.1786867014$.
- For BF, $u_{BF}(rb, rb, tl, rb) = V_{BF}(rb) * p(rb) + V_{QB}(rb) * p(rb) + V_{OL}(tl) * p(tl)$ is the payoff function, that only embrace BF, QB and OL plays. Reason is that the RC plays do not relevant impact the BF plays, so not the BF payoff function valuations. $u_{BF}(rb, rb, tl, rb) = 0.7 * 0.077255077 + 0.5 * 0.077255077 + 0.5 * 0.171961218$. $u_{RC}(rb, rb, tl, rb) = 0.1786867014$.
- For OL, $u_{OL}(rb, rb, tl, rb) = V_{OL}(rb) * p(rb)$ is the payoff function, that only embrace the OL plays. $u_{OL}(rb, rb, tl, rb) = 0.5 * 0.171961218$. $u_{OL}(rb, rb, tl, rb) = 0.085980609$.

Defensive team

- For $DL$and LB, $u_{DL|LB}(tl, tl, tl) = V_{DL}(tl) * p(tl) + V_{LB}(tl) * p(tl)$ is the payoff function, that only embrace the DL and LB plays. Reason is that the DS plays do not relevant impact the DL and LB plays, so not the DL and LB payoff function valuations. $u_{DL|LB}(tl, tl, tl) = 0.5 * 0.171961218 + 0.5 * 0.171961218$. $u_{DL|LB}(tl, tl, tl) = 0.171961218$.
- For DS, $u_{DS}(tl, tl, tl) = V_{DS}(tl) * p(tl)$ is the payoff function, that only embrace the DS plays. $u_{DS}(tl, tl, tl) = 0.5 * 0.171961218$. $u_{DS}(tl, tl, tl) = 0.085980609$.

We should calculate all payoff values on strategy profiles using the players' payoff functions. For the offensive team the strategy profile (pb, rb, yb, tl) satisfies the Nash equilibrium condition, and for the defensive team, the NE profile is (yb, sb, tl). So, the best combination of offensive plays is by combining a pass from QB, the RC running with the ball and the OL and BF opening the way and stopping their opponents. Notice that the Nash equilibrium strategy profile depends on the player's preference value as well as on the player's payoff function. For more examples with different preference values and payoff functions, please visit http://gametheory.cs.cinvestav.mx/Examples_AF_Analysis_of_Strategies.pdf.

5 Discussion

The analysis of strategies in American football gaming can be flexible done using both, the players' preference values on the plays and depending on specific circumstances on the match. Once the players' preference values in **Table 3 – 4** are modified, we can obtain different payoff values, therefore other strategy profiles that satisfy NE conditions for the different valuations. This is a real behavior approach: the individual player's preferences and assessments regulate the actions decision making during a match, at least partially. The strategy profiles can be approached by computer simulations of American football gaming [10] and measure how good are these strategy profiles to improve the team performance.

Conclusion

In simulations of American football gaming, the analysis of strategies can use the normal form game setting, so the strategy profiles are valued using the every player's payoff functions to find out those that fit the Nash equilibrium conditions for the whole team. These Nash equilibrium strategy profiles can be used for actions decision making on the basis of a full strategic reasoning, essential for a team's success.

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