different planning constraints, further formulations and solution techniques, applications to prediction, prescription and evaluation, and basic studies in participation and planning, including the effects of constraining freedom of choice.

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Notes* III

OPTIMAL STRATEGIES ON FOURTH DOWN†

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An attempted field goal on fourth down, with short yardage required for first down, is a considerably poorer strategy than is generally thought.
(RECREATION/SPORTS; PROBABILITY—APPLICATION)

In [1] and [2] we showed that, in American football, the expected value of having the ball with first down and ten yards to go varies from about -1.64 points at one's own goal line linearly (at the rate of about $1/14$ of a point per yard) up to about 10 yards from the opponent's goal line, with a somewhat larger increase per yard for the

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last 10 yards. We suggested that these numbers could be used for evaluating alternative strategies, and in particular that field goal attempts were less attractive than their usage indicated. This paper supports that suggestion quantitatively.

We consider only situations in which the team on offense has passed midfield and has fourth down with short yardage required. We then compute at several field positions the expected value of each of three decisions; namely, to go for the first down (we use the word "run" throughout this paper, although, of course, a pass might be attempted); to attempt a field goal; and a punt, which attempts to give the opponent possession very close to his own goal line (either the "coffin-corner" attempt to put the ball out of bounds near the goal line, or the "hanging" punt with offensive linemen attempting to down the ball near the goal line). To perform such computations we need the expected value of field position as determined in [2]. We also need the probabilities of various outcomes of an attempt to run on fourth down, a field goal attempt, and a punt.

In the 189 NFL games in 1971 there were only 92 attempts to go for first down with fourth down and one yard to go. 68 of these, or 74%, were successful, this probability of 0.74 having a standard error of 0.045. There were 451 attempts to make first down with third and one, and 320 of these or 71% were successful. These numbers are sufficiently close to confirm the intuitive belief that the probability of success in making first down with one yard to go is identical on third and fourth downs, and therefore these numbers are lumped to give $P = 0.715$ with a probable error of 0.019. There were virtually no attempts to go for first down with fourth down and more than one yard, except in extreme situations which probably do not yield good statistics. However, we again assume that third down is an adequate surrogate for fourth down, yielding the results shown in Table I.

To convert the probabilities of Table I into expected values we need to make some assumptions, and we make these conservatively; that is, in such a way as to favor the field goal over the run. Specifically we assume that the run gains exactly the required yards (no more) with the probability indicated in Table I, and with the complementary probability that the gain is 0 yards. The expected value is then the product of the probability of success times the expected value of having first and ten (or first and goal) at that point, minus the product of the probability of failure times the expected value of the opponent's field position at the original line of scrimmage.

For the expected point value of the first-and-ten we take from [2] the value $0.0726X - 1.64$, where $0 < X \leqslant 90$ is the distance from one's own goal line. Within ten yards of the opponent's goal line the data are less decisive, and we arbitrarily add 0.2 points per yard to the above; that is, $0.2X - 13.106$ for $90 \leqslant X < 100$ (first-and-goal).

To determine the probability of success in a field-goal attempt, we looked at all the attempts made in the NFL in the five years 1972–1976. The data for these five years may be lumped even though the goal posts were moved in 1973, because the distance

TABLE I

Probability of Achieving 1st Down with n Yards to Go
(1971 NFL season—189 games)

n	Trials	Successes	P	σ
1	543	388	0.715	0.019
2	327	186	0.569	0.027
3	356	146	0.410	0.026
4	302	97	0.322	0.027
5	336	91	0.271	0.024

is measured from the point where the ball was placed down to the crossbar. In this case, because of inadequacy of the sample size, all data in a given ten-yard strip are lumped; kicks from less than ten yards away are no longer possible under the new rules, and there were very few of them even before the rules changed, so these are lumped with kicks from distances from ten to twenty yards. The results are shown in Table II. These empirical data are not incompatible with data obtained from a model developed by Irving and Smith [3]. Their data used a collegiate who was probably not so good as the average professional kicker; on the other hand, he kicked in practice—that is, without a rush—which is notoriously easier. We interpolate in these lumped data of Table II to get the specific field positions of interest, with the results shown in Table III. Once again, to convert these probabilities to expected values, we compute the probability of success times three points (which is conservative because the actual kickoff from the 35-yard line has negative expected value) minus the probability of failure times the opponent's expected value given his position at the appropriate point. If the kick is attempted from less than 37 yards (i.e., assuming the line of scrimmage were less than 20 yards from the goal), the result of failure is assumed to be a touchback which, according to our formula, is worth -0.19 points to the opposition, or $+0.19$ points to the kicking team. If the kick fails farther out, the ball returns to the line of scrimmage, with correspondingly lower value to the kicking team.

TABLE II

Probability of Success in Field Goal Attempt
m = distance from point of kick to goal post
(1972–6 NFL seasons)

m	Attempts	Successes	P	σ
50–60	230	46	0.200	0.026
40–50	1058	450	0.425	0.015
30–40	1015	665	0.655	0.015
20–30	821	650	0.792	0.014
0–20	354	335	0.946	0.012

TABLE III

Probability of Success in Field Goal Attempt
(From Interpolation in Table II)

Line of Scrimmage	Kick Distance	P
45	62	0
35	52	0.27
25	42	0.46
15	32	0.70
5	22	0.84

TABLE IV

Assumed Outcomes of "Coffin-Corner" Punt Attempt, Line of Scrimmage 35 Yards From Opponent's Goal

Result:	Touchback	0–10	10–20	> 20	Σ
Value (to kicker)	0.19	1.28	0.55	− 0.20	
Prob (assumed)	0.4	0.25	0.25	0.1	1.00
Prob x Value	0.08	0.32	0.14	− 0.02	0.52

For the punt, we have no data on the probabilities, and so we assume the values shown in Table IV. The probabilities are conservative (a good kicker should do better) and in any case, the results are not highly sensitive to them. The values are taken from the above formula, and the expected value as before is the summation of products of the probabilities of the opponent taking the ball at various points on the field, and the value to him of having the ball, first-and-ten, at that point on the field. Table IV shows that the expected value of attempting a coffin-corner punt (line of scrimmage the 35-yard line) is approximately half a point.

A typical calculation may now be shown. Given the ball on the opponent's 35-yard line with fourth down and one yard to go, the probability of making first down with the run is 0.715 and the resulting value of having first-and-ten with 34 yards to go is 3.152 points. The probability of turning the ball over to the opposition on the 35-yard line, where he has an expected value of 0.901 points, is 0.285. The expected value of this strategy is then 2.00 points. For the attempted field goal from 52 yards, the probability of success is 0.267 (see Table III). With probability 0.733 the opponent will gain the ball on the 35-yard line where it is worth to him 0.901 points. Hence the expected value of the attempted field goal is 0.141. In summary, the run is worth 2.00 points, the field goal attempt is worth 0.14 points, and the punt is worth 0.52 points. The run is clearly preferred, and the field-goal attempt is a poor third choice.

The results are summarized in Table V, which shows the expected values for each of the three strategies with the ball on the 35, 25, 15, and 5 yard lines. The run is the preferred decision in all cases for fourth and one or fourth and two, and on the five yard line a run is preferred even on fourth and four. In longer-yardage situations a kick is preferred, but on the 35-yard line a punt is preferable to a field-goal attempt.

TABLE V

Expected Point Value of Various Decisions
(R = run, F = field goal attempt, P = punt) on Fourth Down

Line of Scrimmage:		35	25	15	5
4th and 1	*R*	+ 1.99	+ 2.72	+ 3.45	+ 4.72
	F	+ 0.14	+ 1.21	+ 2.29	+ 2.71
	P	+ 0.52	—	—	—
4th and 2	*R*	+ 1.44	+ 2.17	+ 2.89	+ 4.13
	F	+ 0.14	+ 1.21	+ 2.29	+ 2.71
	P	+ 0.52	—	—	—
4th and 3	*R*	+ 0.82	+ 1.54	+ 2.27	+ 3.42
	F	+ 0.14	+ 1.21	+ 2.29	+ 2.71
	P	+ 0.52	—	—	—
4th and 4	*R*	+ 0.47	+ 1.20	+ 1.93	+ 3.02
	F	+ 0.14	+ 1.21	+ 2.29	+ 2.71
	P	+ 0.52	—	—	—
4th and 5	*R*	+ 0.28	+ 1.00	+ 1.73	+ 2.83
	F	+ 0.14	+ 1.21	+ 2.29	+ 2.71
	P	+ 0.52	—	—	—

The conclusions from Table V are remarkably different from current practice. They show, for example, that if one must kick because it is fourth-and-long, the punt is preferred to the field-goal attempt unless the latter has a success probability of about 0.5 or more. Inside the 30-yard line the run is preferred to the field goal attempt if there are one or two yards to go, and possibly with three. And inside the 10-yard line,

the run is preferred to the field goal attempt with four yards to go and perhaps even with five.

Several caveats are necessary in connection with the above recommendations. In the first place it assumes average kickers, runners, and the like. If one has an extraordinarily good field goal kicker and an extraordinarily poor fullback, the field-goal attempt would be more preferred than the above analysis would indicate. Conversely, with an offensive line capable of opening holes, a run could be even more preferable. In the second place, all of the above assumes that expected value is the appropriate measure of utility. This is doubtless valid in the first quarter with the score tied, but becomes questionable as the game proceeds. For example, if there were only two seconds to go in the game, then whether to run or kick would be determined by the score (e.g. the run being preferred if we were five points behind, and the kick if we were two points behind) and none of the above considerations would be relevant. Finally, as in all other management science studies, this is only a model. The individual manager must assess all the non-quantitative factors of personalities and the like, which we cannot take into consideration.

Having stated these caveats, it is still curious that the conclusions of this paper have not been implemented to date in NFL practice; that is, that NFL coaches continue to employ the field goal far more often than this analysis indicates is desirable. As indicated, all of the above analysis is conservative, so that the true relative value of the field goal is probably even less than shown above. We believe the reason for this paradox is that coaches do not have sufficient intuitive feel for the negative value imposed on the opposition team when they are given possession of the ball in the shadow of their own goal post. That is, the "failure" to obtain points by punting into the coffin-corner or by giving up the ball on an unsuccessful fourth-down run attempt actually has significant positive value because the chances are that the opposing team will have to kick without advancing the ball very far, and our team will again receive the ball in excellent field position with an opportunity to score.

In summary, we recommend that field goals not be attempted with average kickers in the normal course of a football game unless the line of scrimmage is quite close to the opponent's goal line and the number of yards required for first-down (or goal) is large.[1]

[1] This paper was presented at the ORSA/TIMS meeting, Atlanta, Georgia, November 8, 1977.

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