Question 3 by Dan Nguyen (z5206032)

Let there be a flow network represented as a directed graph with the following properties:

- the flow is the number of spies moving to a stronghold;
- nodes are either a spy or stronghold coordinate i.e. $\{(x_i, y_j) \mid 1 \leq x, y \leq n\}$ and $\{(a_i, b_j) \mid 1 \leq i, j \leq m\}$ respectively;
- edges connect spies to strongholds i.e. an edge $\exists$ if $\sqrt{(x_i - a_j)^2 + (y_i - b_j)^2} \leq v_i \cdot T$; and
- strongholds have a capacity of 1.

At this point of construction, it is recognised that the flow network is a *max bipartite matching* problem since spies and strongholds can be divided into two disjoint sets. Therefore, to convert the problem into a *max flow* problem, consider the additional properties for the flow network:

- the super-source is an arbitrary coordinate with an edge to all spies;
- the super-sink is an arbitrary coordinate with an edge to all strongholds; and
- assign all edges in the flow network, a capacity of 1.

Let the max flow, f, be the max number of spies that can move from their original coordinate to a stronghold i.e. f is at most n.

To find f, apply the Edmonds-Karp algorithm to the flow network which will have a time complexity (for a *max bipartite matching* problem) of $O(|E||V|) = O(mn^2)$.