Assignment 1

COMP3121/9101 21T3

Released September 15, due September 29

In this assignment we review some basic algorithms and data structures. There are *five problems*, for a total of 100 marks.

Your solutions must be typed, machine readable PDF files. *All submissions will be checked for plagiarism!*

For each question requiring you to design an algorithm, you *must* justify the correctness of your algorithm. If a time bound is specified in the question, you also *must* argue that your algorithm meets this time bound.

Partial credit will be awarded for progress towards a solution.

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1. You are given an array A of n positive integers. A pair of indices (i, j) where $i < j$ is said to be *consistent* if $A[j] - A[i] = j - i$.
   (a) (10 points) Design an algorithm which runs in *expected* $O(n)$ time and counts the number of pairs of consistent indices.
   (b) (10 points) Design an algorithm which runs in *worst case* $O(n \log n)$ time and counts the number of pairs of consistent indices.

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2. (20 points) You are given an array A of n positive integers, each no larger than m. You may assume that $m \leq n$. The *beauty* of an array is the fewest number of times that any integer from 1 to m inclusive appears in the array. For example, if $m = 3$ then the beauty of $\langle 1, 3, 1, 2, 3, 3, 2 \rangle$ is 2. However, if $m = 4$ the same array would have beauty 0.

   An index i is said to be *fulfilling* if $A[1..i]$ has strictly greater beauty than $A[1..i-1]$. Design an algorithm which runs in $O(n)$ and finds all fulfilling indices.

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3. (20 points) You are given a string s of length n, constructed from an alphabet with k characters. You may assume that all k different characters appear at least once in s.

   A *substring* of s is a contiguous sequence of characters within s.

   Design an algorithm which runs in $O(n)$ time and finds the length of the shortest substring of s which contains all k different characters.

   A solution for the case $k = 3$ will earn up to 10 points.

4. (20 points) You have $2n+1$ ants along a line. Their positions are described by the real numbers $x_{-n} < x_{-n+1} < \ldots < x_{n-1} < x_n$. You will place food at a point x on the line, and all ants will walk along the line to reach the food. Find the value of x which minimises the total distance walked by all ants.

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5. Determine whether:

   (I) $f(n) = O(g(n))$,

   (II) $f(n) = \Omega(g(n))$, i.e. $g(n) = O(f(n))$,

   (III) both (I) and (II), i.e. $f(n) = \Theta(g(n))$, or

   (IV) neither (I) nor (II)

   for the following pairs of functions, and justify your answer. Note that log denotes the natural logarithm, with base e.

   (a) (6 points) $f(n) = n^{1+\log n}$; $\quad g(n) = n \log n$;

   (b) (8 points) $f(n) = n^{1+\frac{1}{2}\cos(\pi n)}$; $\quad g(n) = n$;

   (c) (6 points) $f(n) = \log_2 n^{\log(n \log n)}$; $\quad g(n) = (\log n)^2$.
