Question 4 by Dan Nguyen (z5206032)

Let a series of ants on a line have positions $x_{-n}, x_{-n+1}, ..., x_k, ..., x_{n-1}, x_n$, where $x_{-n} < x_{-n+1} < ... < x_k < ... < x_{n-1} < x_n$, and $x_k \in \mathbb{R}[-n, n]$. Let the series be X where its size is $2n + 1$.

Food is placed on the same line at a position x (where $x \in \mathbb{R}$) which minimises the total distance, D, that all ants along the line must travel to reach the food where the total distance is:

$$D(x) = \sum_{k=-n}^{n} |x - x_k| \tag{1}$$

Let $x_i \leq x \leq x_{i+1}$. Thus Equation 1 can be represented as:

$$D(x) = \sum_{k=-n}^{i} (x - x_k) + \sum_{k=i+1}^{n} (x_k - x) \tag{2}$$

Let $x = x + c$ into Equation 2:

$$\begin{aligned} D(x+c) &= \sum_{k=-n}^{i} ((x+c) - x_k) + \sum_{k=i+1}^{n} (x_k - (x+c)) \\ &= \sum_{k=-n}^{i} (x - x_k) + \sum_{k=-n}^{i} c + \sum_{k=i+1}^{n} (x_k - x) - \sum_{k=i+1}^{n} c \\ &= \sum_{k=-n}^{i} (x - x_k) + (n+i)c + \sum_{k=i+1}^{n} (x_k - x) - (n-i)c \\ D(x+c) &= 2cn + D(x) \end{aligned}$$

$$\therefore D(x+c) - D(x) = ci \tag{3}$$

It can be seen from Equation 3 that:

$$\begin{aligned} D(x+c) - D(x) < 0 \text{ for } i < 0 \\ D(x+c) - D(x) = 0 \text{ for } i = 0 \\ D(x+c) - D(x) > 0 \text{ for } i > 0 \end{aligned}$$

Therefore the total distance, D, is minimised when $i = 0$ i.e. when i is indexed at the median of X or x_0.

x is therefore the median, x_0, of all ants along the line.