Question 4 by Dan Nguyen (z5206032)

Let there be a flow network represented as a directed graph with the following properties:

- the flow is a pair of drink and dessert served to a customer;
- nodes are either a customer-drink pair, customer-dessert pair, type of drink, or type of dessert where there are two k customers, n types of drinks, and m types of desserts;
- the super-source is an arbitrary node with an edge to all types of drink nodes, where the capacity of the edges are $a_i \mid 1 \leq i \leq n$;
- the super-sink is an arbitrary node with an edge to all types of dessert nodes, where the capacity of the edges are $b_j \mid 1 \leq j \leq m$;
- there are k customer-drink nodes, where the r^{th} customer has p_r favourite drinks with unit-capacity edges to their favourite types of drinks: $(c_{r,1}, c_{r,2}, ..., c_{r,p_r})$;
- there are k customer-dessert nodes, where the r^{th} customer has q_r favourite drinks with unit-capacity edges to their favourite types of desserts: $(d_{r,1}, d_{r,2}, ..., d_{r,q_r})$; and
- a unit-capacity edge exists between the r^{th} customer-drink and r^{th} customer-dessert node for all $1 \leq r \leq k$;

Let the max flow, f, be the max number of pairs of drink and dessert served to as many customers as possible i.e. f is at most k.

There are at most $E = n + \sum_r p_r + k + \sum_r q_r + m$ edges.

There are at most $V = n + k + k + m$ nodes.

To find f, apply the Edmonds-Karp algorithm to the flow network which will have a time complexity $O(\min(|V||E|^2,\ |E|f)) = O(\min(V \cdot E^2,\ E \cdot k))$.

The minimum number of poor ratings can therefore be determined by $k - f$.