

### Question 3 by Dan Nguyen (z5206032)

A string,  $S$ , of size  $n$  is constructed from an alphabet,  $A$ , of size  $k$ .  $S$  has all  $k$  characters of the alphabet appear at least once.

Let  $s$  be the *substring* of  $S$  which is a contiguous sequence of characters within  $S$ .

Use a modified sliding window algorithm which has an expected time-complexity  $O(n)$ .

Let there be a variable,  $U$ , to keep count of unique characters, and is initially zero.

Let the sliding window,  $W$ , have a capacity,  $m$ , with an initial size,  $k$ . For each character,  $c$ , in  $W$ , if  $c$  has not been encountered before in  $W$ , increment  $U$ . This will have an expected time-complexity of  $O(m) \leq O(n)$ .

When  $U$  is equal to  $k$ , then  $W$  will contain all  $k$  unique characters and the length of the shortest *substring* is  $m$ .

If  $S$  has been completely screened, and  $U$  is not equal to  $k$ , then increment the capacity of  $W$ , and re-screen  $S$  until  $m > n$ . This will have an expected time-complexity of  $O(n - m) \leq O(n)$ . Therefore, giving a final expected time-complexity of  $O(n)$ .