Question 5 by Dan Nguyen (z5206032)

Let there be a flow network represented as a directed graph with the following properties:

- the flow is the number of items in a warehouse or delivery;
- nodes are warehouses on a particular day i.e. the network can be represented by a d by n matrix with coordinates: $\{(D, w_i) \mid 1 \leq D < d \bigcap 1 \leq i \leq n\}$;
- warehouses have infinite item capacity;
- warehouses have self-delivery of infinite capacity on consecutive days (overnight-storage) i.e. an edge $\exists$ for $\{(Dw_i, (D+1)w_i) \mid 1 \leq D < d \bigcap 1 \leq i \leq n\}$.
- edges exists for each delivery k which departs node (t_k, w_k) and arrives at node (t_k^I, w_k^I), and has an item capacity, c_k;
- the super-source is an arbitrary node with edges to all warehouses on the first day where each edge has a capacity, $\{A_i \mid 1 \leq i \leq n\}$; and
- the super-sink is an arbitrary node with edges to all warehouses on the last day where each edge has a capacity, $\{B_i \mid 1 \leq i \leq n\}$.

Let the max flow, f, be the max number of items in a warehouse or delivery i.e. f is at most $\sum_i A_i$.

There are at most $E = k + (d-1)n$ edges.

There are at most $V = dn$ nodes.

Apply the Edmonds-Karp algorithm to the flow network which will have a time complexity of $O(\min(|V||E|^2,\ |E|f)) = O(\min(dn \cdot (k + (d-1)n)^2,\ (k + (d-1)n)\sum_i A_i))$.

If all edges connected to the super-sink is at max capacity, then it can be determined that it is possible to have at least B_i items present at each warehouse i at the end of day d.