DESN2000 MECH Workshop

Week 5 – Material Selection

Class overview

- **How this fits into your assignments** (5 min)
- **The process explained** (5 min)
- **Example 1: Minimalist table** (25 min)
- **Example 2: Satellite telescope mirror** (25 min)
- **Project time!** (55 min)
- **Summary** (5 min)

How this fits into your assignments

How this fits into your assignments	The process explained	Example 1: Minimalist table	Example 2: Satellite telescope mirror	Project time!	Summary
5 min	5 min	25 min	25 min	55 min	5 min
All	All	All	All	All	All

How this fits into your assignments

Relevant for both teams that want to write a technical analysis and those who want to prototype!

- ❑ Similar to fasteners, if your team wants to write a technical analysis:
  - This is one of 4 topics from which you must choose 2 to write about.
  - The tools covered in this workshop will enable you to explain in your report, **why** you decided on the materials you want your concept to be made from. Effective communication is an important industry skill!

- ❑ If your team wants to prototype instead:
  - This may help you choose what material you might want to make your prototype from, to maximise the chance of it working.

The process explained

How this fits into your assignments	The process explained	Example 1: Minimalist table	Example 2: Satellite telescope mirror	Project time!	Summary
5 min	5 min	25 min	25 min	55 min	5 min
All	All	All	All	All	All

The process explained

1. Determine what design variables you want to optimise.
   - Will likely come from CNs and/or FRs
2. Determine what design variables are constrained.
   - Determine from design constraints, FRs
3. Determine what free variables remain
   - Could be a length, radius, etc.
4. Create Material Index/s
   - Create using equations describing the physical (e.g., mass) and mechanical (beam deflection) properties of interest.
5. Use an Ashby chart to select materials
   - Book: Materials Selection In Mechanical Design, Michael F. Ashby

Example 1: Minimalist table

How this fits into your assignments	The process explained	Example 1: Minimalist table	Example 2: Satellite telescope mirror	Project time!	Summary
5 min	5 min	25 min	25 min	55 min	5 min
All	All	All	All	All	All

Example 1: Minimalist Table

✓ **Step 1: Decide what you want to optimise**

- ❑ We want to optimise the legs of the table.
- ❑ Specifically, we want to: *Minimise* mass $(\boldsymbol{m})$, *maximise* slenderness $(\boldsymbol{r})$.

✓ **Step 2: Determine constraints**

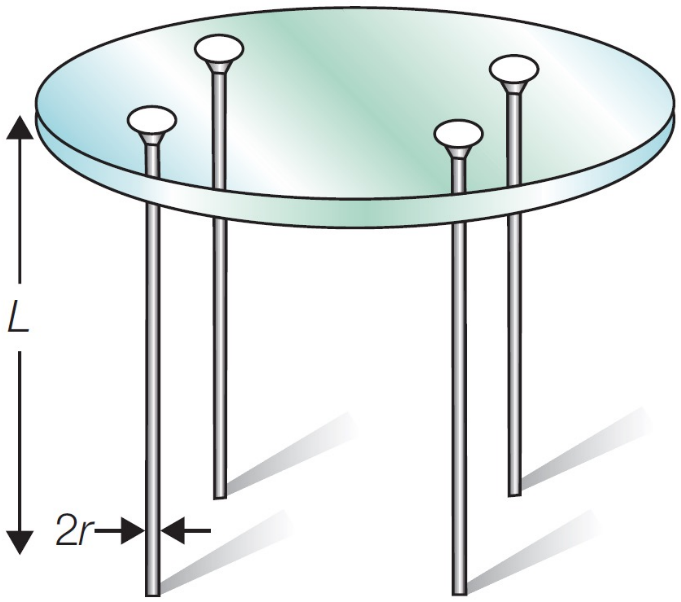
- ❑ Fixed leg length $(\boldsymbol{L})$, withstand buckling under design load $(\boldsymbol{F})$, resistant to fracture when struck.

✓ **Step 3: Identify free variables that remain**

- ❑ Diameter of table legs $(\boldsymbol{2r})$.

✓ **Step 4: Create material index/s**

- ❑ We will start with minimising the mass…



Example 1: Minimalist Table

✓ **Step 4a: Create material index for minimising mass**

Good idea to begin with an equation which defines the variable that you are analysing. The table leg is a solid cylinder, whose mass can be defined as:

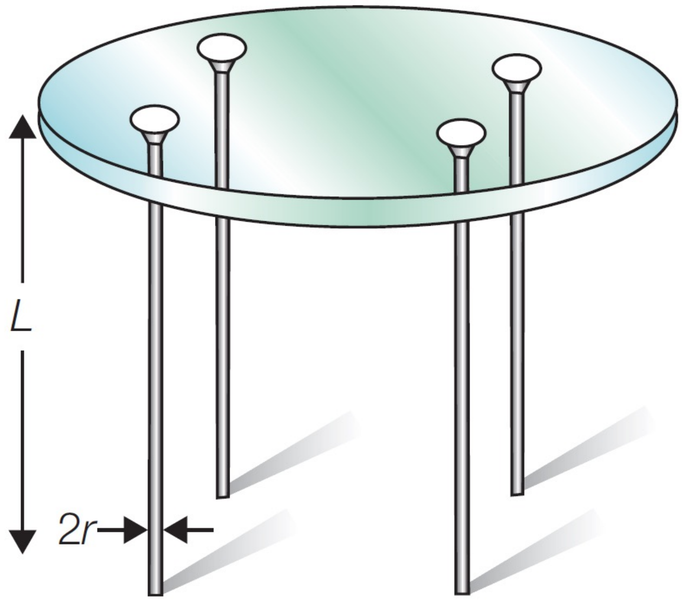
$$m = \pi r^2 L\rho \ (1)$$

With leg radius r, leg length L and density ρ.

The key constraint that affects mass is the maximum load F the leg can bear without buckling. The equation for elastic buckling of a column is given by

$$F_{crit} \geq \frac{\pi^2 EI}{L^2} = \frac{\pi^3 Er^4}{4L^2} \ (2)$$

With Young's modulus E, second moment of area for column $I = \frac{\pi r^4}{4}$.



We now want to try to combine F_{crit} and m into a new equation that can describe their relationship

Example 1: Minimalist Table

✓ **Step 4a continued…**

We can rearrange (2) to make $\boldsymbol{r}$ the subject in $\boldsymbol{F_{crit}} \geq \frac{\pi^3 E r^4}{4L^2}$.

$$r^4 \geq F_{crit}\frac{4L^2}{\pi^3 E}$$

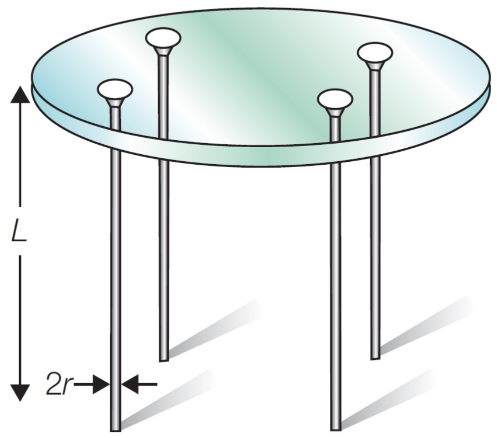
$$r^2 \geq \left(F_{crit}\frac{4L^2}{\pi^3 E}\right)^{\frac{1}{2}} \quad (3)$$

Substituting (3) into (1) $(\boldsymbol{m} = \boldsymbol{\pi r^2 L \rho})$,

$$m \geq \pi \times \left(F_{crit}\frac{4L^2}{\pi^3 E}\right)^{\frac{1}{2}} \times L\rho$$

Note that $\boldsymbol{F_{crit}}, \boldsymbol{L}$ and $\boldsymbol{\pi}$ are all constants. Only $\boldsymbol{E}$ and $\boldsymbol{\rho}$ are material properties that we can choose. Let's isolate them, to create an expression that relates mass to these properties.

$$m \geq \left(\frac{4Fcrit}{\pi}\right)^{\frac{1}{2}} L^2 \boxed{\left(\frac{\rho}{E^{\frac{1}{2}}}\right)} \quad (4)$$



$$\frac{\rho}{E^{\frac{1}{2}}}$$

This is what we need to create our material index.

Example 1: Minimalist Table

✓ **Step 4a continued…**

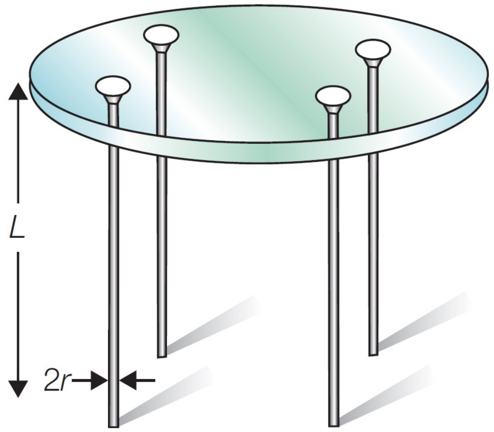
A common Ashby chart compares $\boldsymbol{E}$ and $\boldsymbol{\rho}$ in the form of $\frac{E^{\frac{1}{2}}}{\rho}$. We have the inverse, $\frac{\rho}{E^{\frac{1}{2}}}$.

Thus, we can rewrite equation $(\mathbf{4})$ $\boldsymbol{m} \geq \left(\frac{4Fcrit}{\pi}\right)^{\frac{1}{2}} \boldsymbol{L}^2 \left(\frac{\rho}{E^{\frac{1}{2}}}\right)$ as,

$$\boldsymbol{m} \geq \left(\frac{4Fcrit}{\pi}\right)^{\frac{1}{2}} \boldsymbol{L}^2 \frac{1}{\frac{E^{\frac{1}{2}}}{\rho}}$$

This is our material index $\boldsymbol{M1} = \frac{E^{\frac{1}{2}}}{\rho}$. By maximising it we will find materials that can minimise the weight.

Ok, now let's optimise for slenderness



Example 1: Minimalist Table

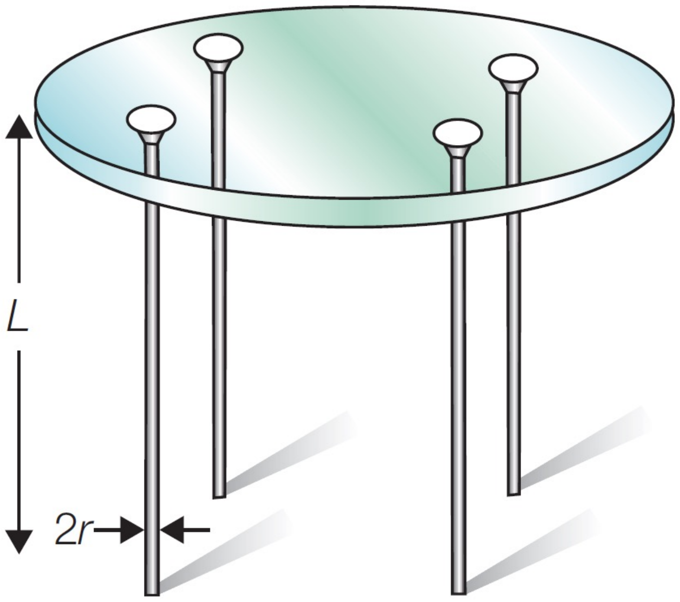
✓ **Step 5b: Create material index for maximising slenderness**

Let's go back to equation 2,

$$F_{crit} \geq \frac{\pi^3 E r^4}{4L^2} \quad (2)$$

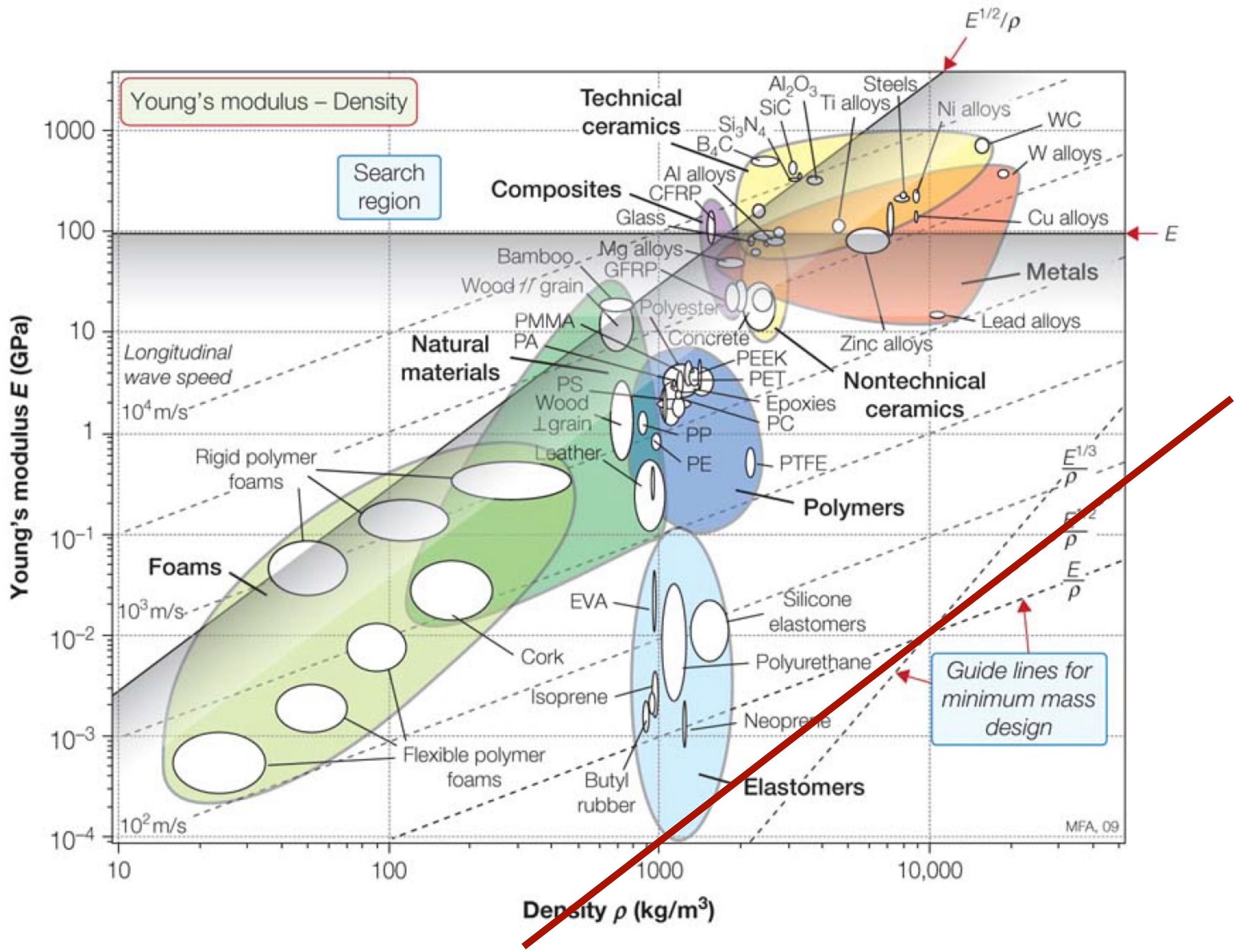
If we make $F_{crit} = F$ (the applied load) we can reverse the inequality (the applied load must always be less than or equal to the critical load). Then, we can rearrange to make r the subject. Thus,

$$\boxed{\left[\frac{1}{E}\right]^{\frac{1}{4}}} \left(\frac{4F}{\pi^3}\right)^{\frac{1}{4}} L^2 \leq r$$



This is our second Material Index, $M2 = E$. We see that if E increases, r will decrease. Let's put this and $M1$ to use on our Ashby chart.

Example 1: Minimalist Table



✓ **Step 5: Use Ashby chart to select materials**

Our material index comprises of $\boldsymbol{E}$ and $\boldsymbol{\rho}$. Thus, we need an Ashby chart which compares these values.

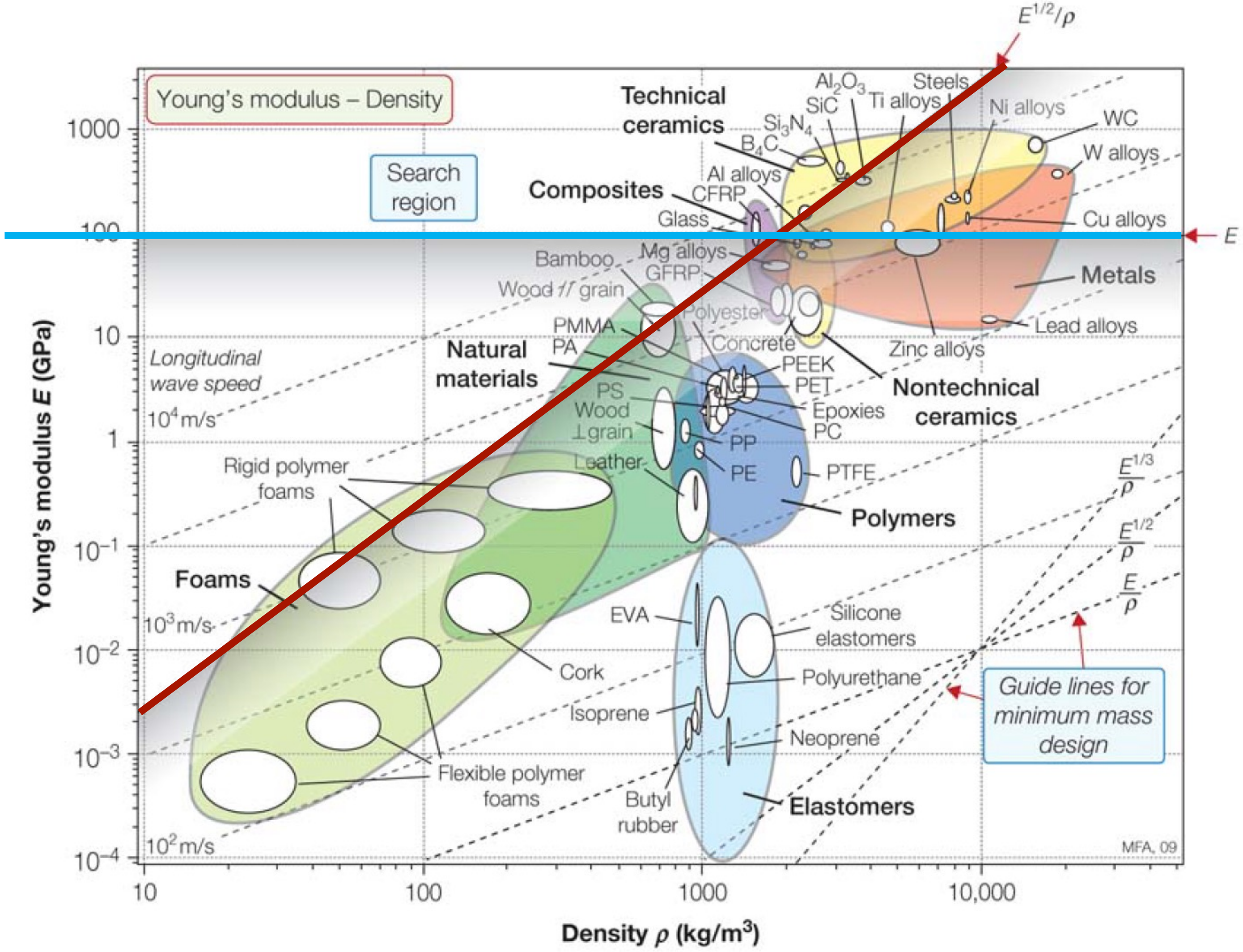
Let's start with $\boldsymbol{M1} = \frac{\boldsymbol{E}^{\frac{1}{2}}}{\boldsymbol{\rho}}$. Find the tie-line (dotted line) which matches our material index.

Move the line up until a reasonably small subset of materials is above the line (we want to maximise $\boldsymbol{M1}$ remember).

Any material along the line will at a minimum fulfil the criteria of withstanding $\boldsymbol{F_{crit}}$. Materials **above the line will perform better** than minimum, materials **below the line will fail to meet the requirements.**

From this, we see that woods, CFRP and ceramics are suitable. Metals and polymers are close, but they will be too large or heavy. Now let's consider $\boldsymbol{M2}$.

Example 1: Minimalist Table



✓ **Step 5: Use Ashby chart to select materials**

Plotting $\boldsymbol{E}$ on the Ashby chart is in this case, just a horizontal line since the $y - axis$ is Young's modulus.

As with minimising mass, we move the line up until a reasonably small subset of materials is above the line.

We see that applying $\boldsymbol{M2}$ eliminates wood (not strong enough if made very thin), leaving only **CFRP and ceramics**. Considering the constraint that the legs must not crack when struck, ceramics are eliminated, because they are too brittle.

Thus, based on our analysis, **CFRP is the best material** (though expensive!!!)

Example 2: Satellite telescope mirror

How this fits into your assignments	The process explained	Example 1: Minimalist table	Example 2: Satellite telescope mirror	Project time!	Summary
5 min	5 min	25 min	25 min	55 min	5 min
All	All	All	All	All	All

Example 2: Satellite Telescope Mirror

✓ **Step 1: Decide what you want to optimise**

- ❑ We want to optimise the mirror itself.
- ❑ Specifically, we want to *minimise* mass (m).

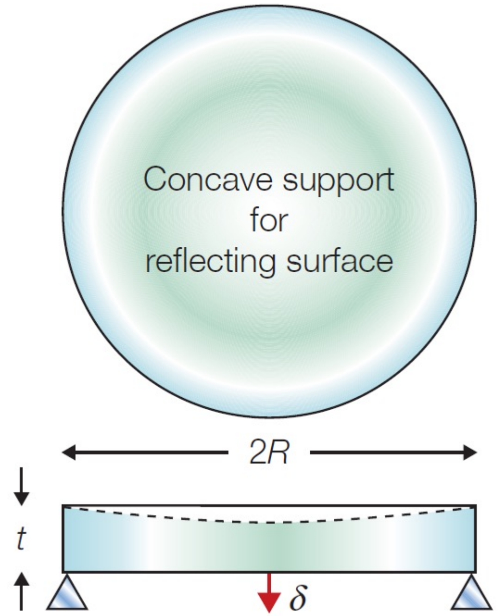
✓ **Step 2: Determine constraints**

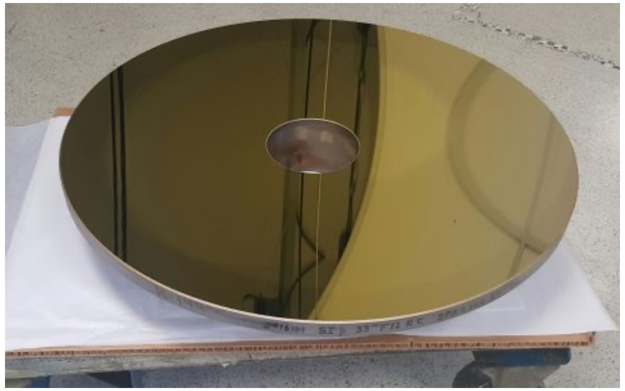
- ❑ Fixed mirror radius (R), mustn't distort more than δ under self-weight. High dimensional stability (retains shape well), low thermal expansion.

✓ **Step 3: Identify free variables that remain**

- ❑ Thickness (t).

✓ **Step 4: Create material index/s**





Example 2: Satellite Telescope Mirror

✓ **Step 4: Create material index/s**

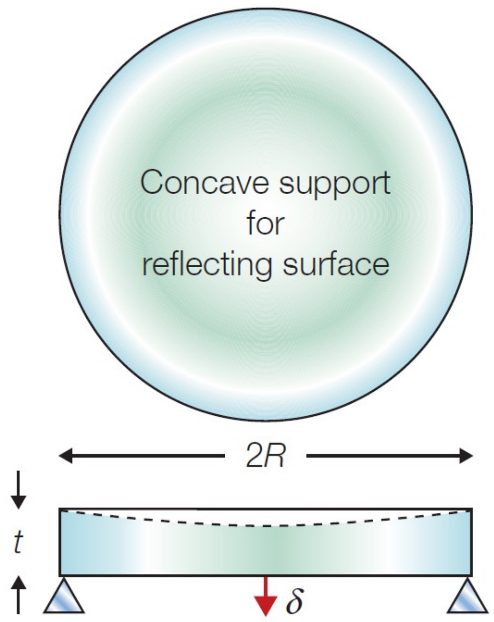
Let's begin with defining an equation for the mirror's mass

$$\boldsymbol{m = \pi R^2 t\rho}\ (\mathbf{1})$$

Where $\boldsymbol{R}$ = radius, $\boldsymbol{t}$ = thickness, $\boldsymbol{\rho}$ = density. (the concavity of the mirror is ignored for simplicity).

The mirror will be supported horizontally around its circumference, which means it will bend at the centre under its own weight. For a material with Poisson's ratio of 0.3, this mode of deflection – a horizontal disk at the centre, is given by,

$$\boldsymbol{\delta = \frac{3}{4\pi}\frac{mgR^2}{Et^3}}\ (\mathbf{2})$$



We can substitute $(\mathbf{2})$ into $(\mathbf{1})$ to ensure that our mass equation accounts for this constraint, $\boldsymbol{\delta}$.

Example 2: Satellite Telescope Mirror

✓ **Step 4: Create material index/s**

Before we substitute, we'll rearrange (2) making $\boldsymbol{t}$ the subject,

$$\boldsymbol{t} = \left(\frac{3}{4\pi}\right)^{\frac{1}{3}}\left(\frac{mgR^2}{E\delta}\right)^{\frac{1}{3}} \quad (3)$$

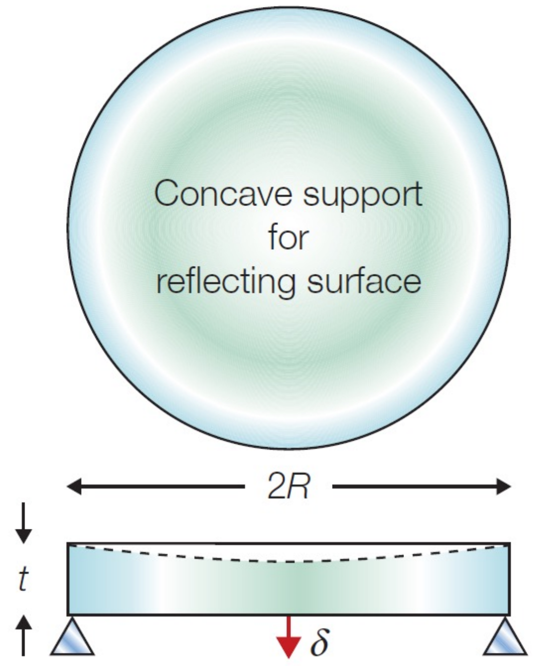
Substituting $(\mathbf{3})$ into $(\mathbf{1})$.

$$\boldsymbol{m} = \pi R^2 t\left(\frac{3}{4\pi}\right)^{\frac{1}{3}}\left(\frac{mgR^2}{E\delta}\right)^{\frac{1}{3}}\rho$$

If we redistribute the variables, we will find that,

$$\boldsymbol{m} = \left(\frac{3g}{4\delta}\right)^{\frac{1}{2}}\pi R^4\left[\frac{\rho}{E^{\frac{1}{3}}}\right]^{\frac{3}{2}}$$

Note the second bracket $\frac{\rho}{E^{\frac{1}{3}}}$ is close in format to what we used in the previous example. We just need to invert it.



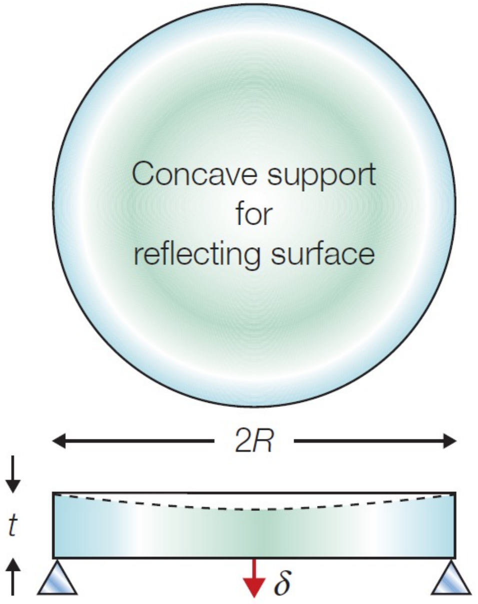
Example 2: Satellite Telescope Mirror

✓ **Step 4: Create material index/s**

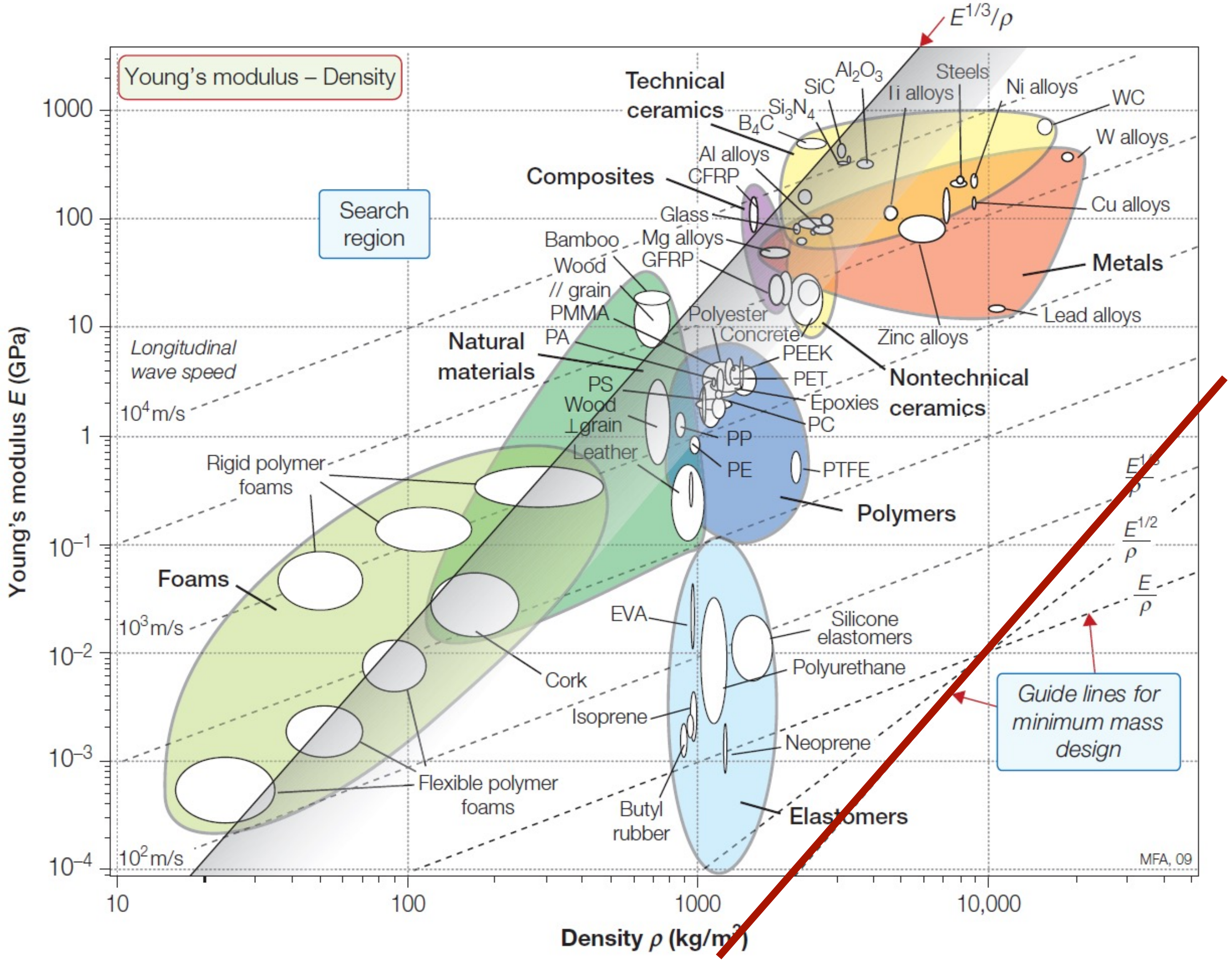
Therefore,

$$\boldsymbol{m = \left(\frac{3g}{4\delta}\right)^{\frac{1}{2}} \pi R^4 \boxed{\left[\frac{1}{\frac{E^{\frac{1}{3}}}{\rho}}\right]^{\frac{3}{2}}}}$$

This is our material index $\boldsymbol{M1 = \frac{E^{\frac{1}{3}}}{\rho}}$. By maximising it we will find materials that can minimise the weight. To the Ashby charts again!



Example 2: Satellite Telescope Mirror



Step 5: Use Ashby chart to select materials

We use the tie-line for $\frac{E^{\frac{1}{3}}}{\rho}$ this time.

Moving the line up until a reasonably small subset of materials is above the line, we see that **rigid foam polymers, glass, CFRP and a few ceramics** lie on or slightly above the line. Magnesium and aluminium alloys are also close contenders.

To narrow down our choice, we should consider some of the constraints.

Example 2: Satellite Telescope Mirror

Table 6.4 Mirror Backing for 200-inch (5.1-m) Telescope

Material	$M = E^{1/3}/\rho$ $(GPa)^{1/3}.m^3/Mg$	m (tonne) $2R$ = 5.1 m (from Eq. 6.4)	Comment
Steel (or speculum)	0.74	73.6	Very heavy—the original choice
GFRP	1.5	25.5	Not dimensionally stable enough—use for radio telescope
Al-Alloys	1.6	23.1	Heavier than glass, and with high thermal expansion
Glass	1.7	21.6	The present choice
Mg-Alloys	1.9	17.9	Lighter than glass but high thermal expansion
CFRP	3.0	9	Very light, but not dimensionally stable; use for radio telescopes
Foamed polystyrene	4.5	5	Very light, but dimensionally unstable. Foamed glass?

- Foam technically could be used and would be very light, but unlikely to retain shape well.
- Composites are also close in weight but are still impacted by dimensional instability.
- Metals are heavy and also vulnerable to heat expansion.

Considering the choices, likely CFRP will be the best choice for a single material mirror. However, here is potential space for innovation.

Perhaps additional structural reinforcement in the mounting hardware could be used to account for the instability of CFRP.

The key message is, there is no perfect material. You will always have to account for some compromise.

Material Selection Process: Key Takeaways

- ❑ This is not an exact process. Physical testing would still be needed in the detail design phase to verify your choices (out of course scope).

- ❑ Formulating the material index is tricky. If unsure, go back and consider what is your key constraint. Is it that the part must support a certain load? Resist bending in a particular manner?

  - Often these are connected to equations involving material properties, which you can substitute into your equation which describes the variable you are trying to optimise for. This then creates a new expression which relates material properties to the variable you are designing for.

- ❖ If you'd like to see more case studies, Chapter 6 of the textbook (Materials Selection In Mechanical Design) contains the examples shown today and many more. You may be able to adapt some of them to your analysis (Don't forget to reference if you do use it ☺)

Project time !

How this fits into your assignments	The process explained	Example 1: Minimalist table	Example 2: Satellite telescope mirror	Project time!	Summary
5 min	5 min	25 min	25 min	55 min	5 min
All	All	All	All	All	All

Summary

How this fits into your assignments	The process explained	Example 1: Minimalist table	Example 2: Satellite telescope mirror	Project time!	Summary
5 min	5 min	25 min	25 min	55 min	5 min
All	All	All	All	All	All

Summary

Today we covered:

- ❑ Material selection

Reminder:

- ❑ **Design journal due Monday Week 7 at 0900.**

Next week:

- ❑ Sensor selection