Copyrighted Materials
Copyright © 2011 Elsevier Retrieved from www.knovel.com

CHAPTER 6

Case Studies: Materials Selection

Image of rower on the Cam in Cambridge, UK, courtesy of Andrew Dunn.

Materials Selection in Mechanical Design. DOI: 10.1016/B978-1-85617-663-7.00006-0
© 2011 Michael F. Ashby. Published by Elsevier Ltd. All rights reserved.

CONTENTS

6.1 INTRODUCTION AND SYNOPSIS

Here we have a collection of case studies illustrating the selection methods of Chapter 5. They are deliberately simplified to avoid obscuring the method under layers of detail. In most cases little is lost by this: The best choice of material for the simple example is the same as that for the more complex, for the reasons given in Section 5.3.

Each case study is laid out in the following way:

- *The problem statement*, setting the scene
- *The translation*, identifying function, constraints, objectives, and free variables, from which emerge the attribute limits and material indices

- *The selection*, in which the full menu of materials is reduced by screening and ranking to a shortlist of viable candidates
- *The postscript*, allowing a commentary on results and philosophy

The first few examples are straightforward, chosen to illustrate the method. Later examples are less obvious and require clear thinking to identify and distinguish objectives and constraints. Confusion here can lead to bizarre and misleading conclusions. Always apply common sense: Does the selection include the traditional materials used for that application? Are some members of the subset obviously unsuitable? If they are, it is usually because a constraint has been overlooked or an objective misapplied. The answer is to rethink them.

Most of the case studies use the hard-copy charts of Chapter 4; those at the end illustrate computer-based methods.

6.2 MATERIALS FOR OARS

Credit for inventing the rowed boat seems to belong to the Egyptians. Boats with oars appear in carved relief on monuments built in Egypt between 3300 and 3000 B.C. Boats, before steam power, could be propelled by poling, by sail, or by oar. Oars gave more control than the other two, the military potential of which was well understood by the Romans, the Vikings, and the Venetians.

Records of rowing races on the Thames in London extend back to 1716. Originally the competitors were watermen, rowing the ferries used to carry people and goods across the river. Gradually gentlemen became involved (notably the young gentlemen of Oxford and Cambridge), sophisticating both the rules and the equipment. The real stimulus for development of boats and oars came in 1900 with the establishment of rowing as an Olympic sport. Since then both have drawn to the fullest on the craftsmanship and materials of their day. Consider, as an example, the oar.

The translation Mechanically speaking, an oar is a beam, loaded in bending. It must be strong enough to carry, without breaking, the bending moment exerted by the oarsman; it must have a stiffness to match the rower's own characteristics; and it must give the right "feel." Meeting the strength constraint is easy. Oars are designed on *stiffness*, that is, to give a specified elastic deflection under a given load.

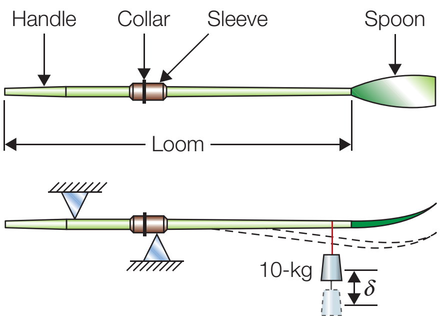

FIGURE 6.1
An oar. Oars are designed on stiffness, measured in the way shown in the *lower* figure, and they must be light.

Figure 6.1 (*top*) shows an oar: A blade or "spoon" is bonded to a shaft or "loom" that carries a sleeve and collar to give positive location in the rowlock. The lower part of the figure shows how the oar stiffness is measured: A 10-kg weight is hung on the oar 2.05 m from the collar and the deflection δ at this point is measured. A soft oar will deflect nearly 50 mm; a hard one only 30. A rower, ordering an oar, will specify how hard it should be.

In addition, the oar must be light; extra weight increases the wetted area of the hull and the drag that goes with it. So there we have it: an oar is a beam of specified stiffness and minimum weight. The material index we want was derived in Chapter 5 as Equation (5.15). It is that for a light, stiff beam:

$$M = \frac{E^{1/2}}{\rho} \tag{6.1}$$

where E is Young's modulus and ρ is the density. There are other obvious constraints. Oars are dropped, and blades sometimes clash. The material must be tough enough to survive this, so brittle materials (those with a toughness G_1c less than 1 kJ/m^2) are unacceptable. Given these requirements, summarized in Table 6.1, what materials would you choose to make oars?

The selection Figure 6.2 shows the appropriate chart: that in which Young's modulus is plotted against density ρ. The selection line for the index M has a slope of 2, as explained in Section 5.4; it is positioned so that a small group of materials is left above it. They are the materials with the largest values of M and represent the best choice, provided they satisfy the other constraint (a simple attribute limit on toughness). They contain three classes of material: woods, carbon-reinforced polymers, and certain ceramics (Table 6.2). Ceramics are brittle; the toughness-modulus chart in Figure 4.7 shows that all fail to meet the requirements of the design. The recommendation is clear. Make your oars out of wood or—better—out of CFRP.

Postscript Now we know what oars should be made of. What, in reality, is used? Racing oars and sculls are made either of wood or of a high performance composite: carbon-fiber–reinforced epoxy.

Wooden oars are made today, as they were 100 years ago, by craftsmen working mainly by hand. The shaft and blade are of Sitka spruce from the

Table 6.1 Design Requirements for the Oar

Function	Oar—meaning light, stiff beam
Constraints	Length L specified Bending stiffness S^* specified Toughness $G_{1c} > 1$ kJ/m^2
Objective	Minimize the mass m
Free variables	Shaft diameter Choice of material

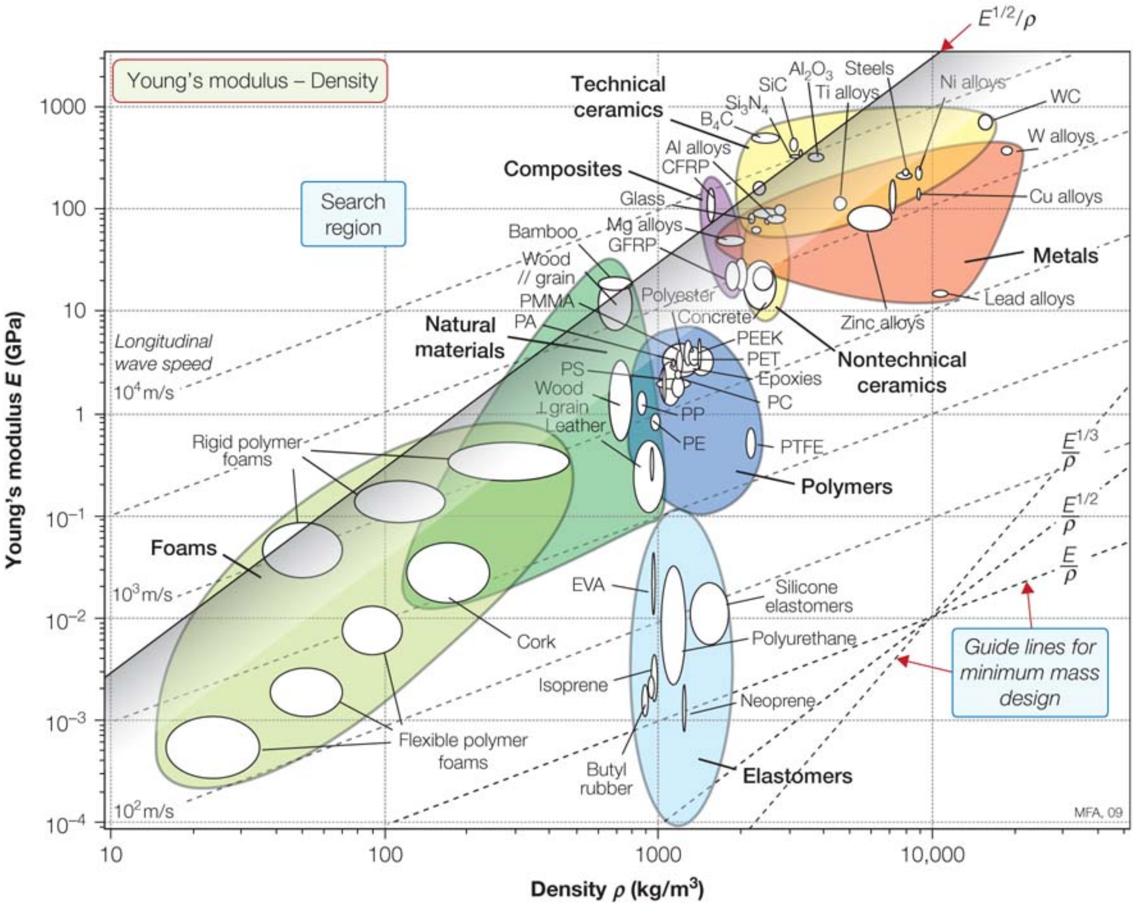

FIGURE 6.2

Materials for oars. CFRP is better than wood because the structure can be controlled.

Table 6.2 Materials for Oars

Material	Index M (GPa)$^{1/2}$/(Mg/m^3)	Comment
Bamboo	4.0–4.5	The traditional material for oars for canoes
Woods	3.4–6.3	Inexpensive, traditional, but with natural variability
CFRP	5.3–7.9	As good as wood, more control of properties
Ceramics	4–8.9	Good M but toughness low and cost high

northern United States or Canada, the further north the better because the short growing season gives a finer grain. The wood is cut into strips, four of which are laminated together to average the stiffness, and the blade is glued to the shaft. The rough oar is then shelved for some weeks to settle down, and finished by hand-cutting and polishing. When finished, a spruce oar weighs between 4 and 4.3 kg.

Composite blades are a little lighter than wood for the same stiffness. The component parts are fabricated from a mixture of carbon and glass fibers in an epoxy matrix, assembled and glued. The advantage of composites lies partly in the saving of weight (typical weight: 3.9 kg) and partly in the greater control of performance: The shaft is molded to give the stiffness specified by the purchaser. Until recently a CFRP oar cost more than a wooden one, but the price of carbon fibers has fallen sufficiently that the two cost about the same.

Could we do better? The chart shows that wood and CFRP offer the lightest oars, at least when normal construction methods are used. Novel composites, not shown on the chart, might permit further weight saving; and functional-grading (a thin, very stiff outer shell with a low-density core) might do it. But both appear, at present, unlikely.

Related reading

Redgrave, S. (1992). *Complete book of rowing*. Partridge Press.

Related case studies

6.3 “Mirrors for large telescopes”
6.4 “Materials for table legs”
10.2 “Spars for human-powered planes”
10.3 “Forks for a racing bicycle”

6.3 MIRRORS FOR LARGE TELESCOPES

There are some very large optical telescopes in the world. The newer ones employ complex and cunning tricks to maintain their precision as they track across the sky—more on that in the Postscript. But if you want a simple telescope, you make the reflector as a single rigid mirror. The largest such telescope is sited on Mount Semivodrike, near Zelenchukskaya in the Caucasus Mountains of Russia. The mirror is 6 m (236 inches) in diameter. To be sufficiently rigid, the mirror is made of glass about 1 m thick and weighs 70 tonnes.

The total cost of a large (236-inch) telescope is, like the telescope itself, astronomical—about US$300 m. The mirror itself accounts for only about 5% of this cost; the rest of the cost is the mechanism that holds, positions, and moves it as it tracks across the sky. This mechanism must be stiff enough to position the mirror relative to the collecting system with a precision about equal to that of the wavelength of light. It might seem, at first sight, that doubling the mass m of the mirror would require that the sections of the support structure be doubled too, so as to keep the stresses (and hence the strains and displacements) the same; but the heavier structure then deflects under its own weight. In practice, the sections have to increase as m^2, and so does the cost.

A century ago, mirrors were made of speculum metal (density: about 8000 kg/m^3). Since then, they have been made of glass (density: 2,300 kg/m^3), silvered on the front surface, so none of the optical properties of the glass are used. Glass is chosen for its mechanical properties only; the 70 tonnes of glass are just a very elaborate support for 100 nm (about 30 grams) of silver. Could one, by taking a radically new look at materials for mirrors, suggest possible routes to the construction of lighter, cheaper telescopes?

The translation At its simplest, the mirror is a circular disk with diameter $2R$ and mean thickness t, simply supported at its periphery (Figure 6.3). When horizontal, it will deflect under its own weight m; when vertical it will not deflect significantly. This distortion (which changes the focal length and introduces aberrations) must be small enough that it does not interfere with performance; in practice, this means that the deflection δ of the midpoint of the mirror must be less than the wavelength of light. Additional requirements are high dimensional stability (no creep) and low thermal expansion (Table 6.3).

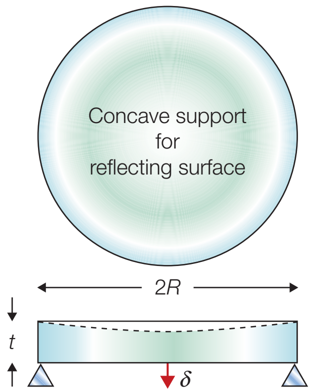

FIGURE 6.3
The mirror of a large optical telescope is modeled as a disk, simply supported at its periphery. It must not sag by more than a wavelength of light at its center.

The mass of the mirror (the property we wish to minimize) is

$$m = \pi R^2 t \rho \tag{6.2}$$

where ρ is the density of the material of the disk. The elastic deflection, δ, of the center of a

Table 6.3 Design Requirements for the Telescope Mirror

Function	Precision mirror
Constraints	Radius R specified Must not distort more than δ under self-weight High dimensional stability: no creep, low thermal expansion
Objective	Minimize the mass, m
Free variables	Thickness of mirror, t Choice of material

horizontal disk due to its own weight is given, for a material with Poisson's ratio of 0.3 (Appendix B), by

$$\delta = \frac{3}{4\pi}\,\frac{m\,g\,R^2}{E\,t^3} \tag{6.3}$$

The quantity g in this equation is the acceleration due to gravity: 9.81 m/s^2; E, as before, is Young's modulus. We require that this deflection be less than (say) 10 μm. The diameter $2R$ of the disk is specified by the telescope design, but the thickness t is a free variable. Solving for t and substituting this into the first equation gives

$$m = \left(\frac{3\,g}{4\,\delta}\right)^{1/2} \pi\,R^4 \left[\frac{\rho}{E^{1/3}}\right]^{3/2} \tag{6.4}$$

The lightest mirror is the one with the greatest value of the material index

$$M = \frac{E^{1/3}}{\rho} \tag{6.5}$$

We treat the remaining constraints as attribute limits, requiring a melting point greater than 500°C to avoid creep, zero moisture take up, and a low thermal expansion coefficient ($\alpha < 20 \times 10^{-6}$/K).

The selection Here we have another example of elastic design for minimum weight. The appropriate chart is again that relating Young's modulus E and density ρ—but the line we now construct on it has a slope of 3, corresponding to the condition $M = E^{1/3}/\rho$ = constant (Figure 6.4). Glass lies at the value M = 1.7 (GPa)$^{1/3}$.m^3/Mg. Materials that have larger values of M are better; those with lower, worse. Glass is much better than steel or speculum metal (that is why most mirrors are made of glass), but it is less good than magnesium, several ceramics, carbon-fiber– and glass-fiber–reinforced polymers, or—an unexpected finding—stiff foamed polymers. The shortlist before applying the attribute limits is given in Table 6.4.

One must, of course, examine other aspects of this choice. The mass of the mirror, calculated from Equation (6.4), is listed in the table. The CFRP mirror is less than half the weight of the glass one, and its support structure could thus be as much as four times less expensive. The possible saving by using foam is even greater. But could they be made?

Some of the choices—polystyrene foam or CFRP—may at first seem impractical. But the potential cost saving (the factor of 16) is so vast that they are worth examining. There are ways of casting a thin film of silicone rubber or of epoxy onto the surface of the mirror backing (the polystyrene or the CFRP) to give an optically smooth surface that could be silvered. The most obvious obstacle is the lack of stability of polymers—they change dimensions with age, humidity, temperature and so on. But glass itself can be reinforced with carbon

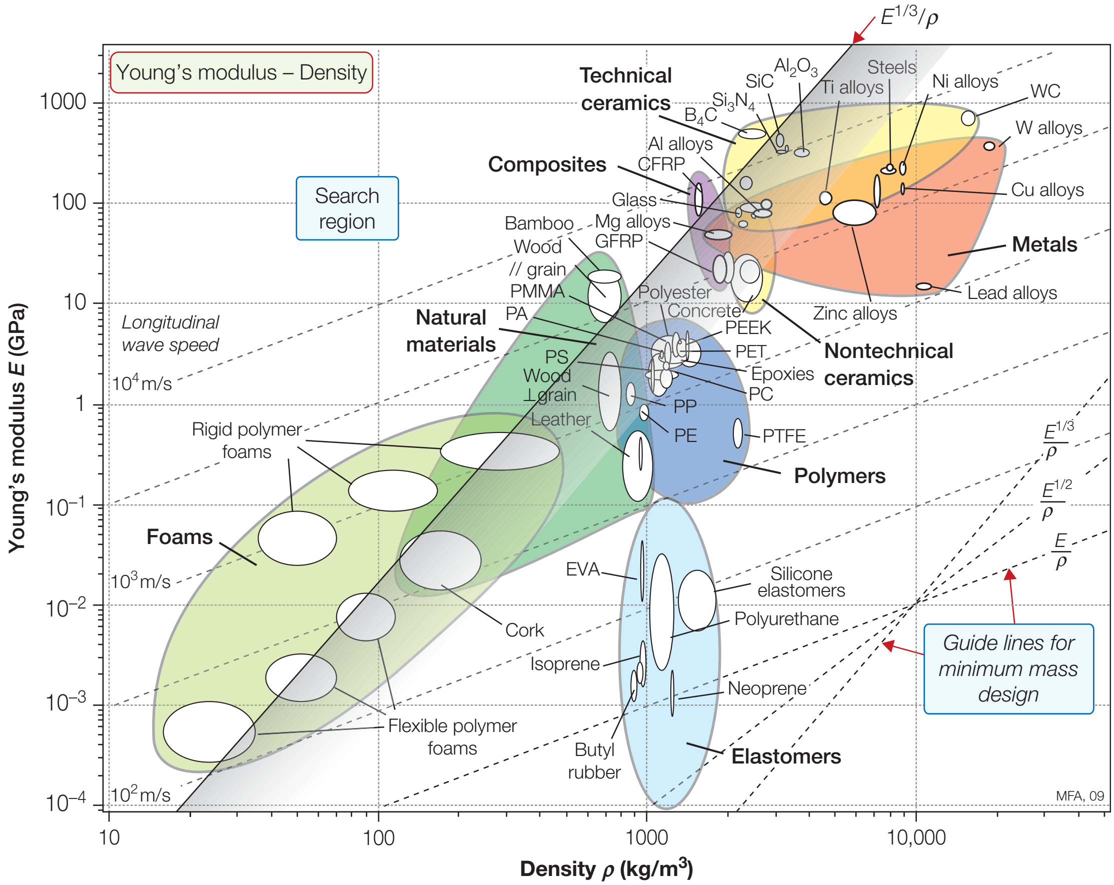

FIGURE 6.4

Materials for telescope mirrors. Glass is better than most metals, among which magnesium is a good choice. Carbon-fiber–reinforced polymers give, potentially, the lowest weight of all, but may lack adequate dimensional stability. Foamed glass is a possible candidate.

fibers; and it can also be foamed to give a material that is denser than polystyrene foam but much lighter than solid glass. Both foamed and carbon-reinforced glass have the same chemical and environmental stability as solid glass. They could provide a route to large cheap mirrors.

Postscript There are, of course, other things you can do. The stringent design criterion ($\delta < 10\,\mu$m) can be partially overcome by engineering design without reference to the material choice. The 8.2-m Japanese telescope on Mauna Kea, Hawaii, and the Very Large Telescope (VLT) at Cerro Paranal Silla in Chile each have a thin glass reflector supported by an array of hydraulic or piezo-electric jacks that exert distributed forces over the back surface,

Table 6.4 Mirror Backing for 200-inch (5.1-m) Telescope

Material	$M = E^{1/3}/\rho$ $(\text{GPa})^{1/3}.\text{m}^3/\text{Mg}$	m (tonne) $2R = 5.1$ m (from Eq. 6.4)	Comment
Steel (or speculum)	0.74	73.6	Very heavy—the original choice
GFRP	1.5	25.5	Not dimensionally stable enough—use for radio telescope
Al-Alloys	1.6	23.1	Heavier than glass, and with high thermal expansion
Glass	1.7	21.6	The present choice
Mg-Alloys	1.9	17.9	Lighter than glass but high thermal expansion
CFRP	3.0	9	Very light, but not dimensionally stable; use for radio telescopes
Foamed polystyrene	4.5	5	Very light, but dimensionally unstable. Foamed glass?

controlled to vary with the attitude of the mirror. The Keck telescope, also on Mauna Kea, is segmented, with each segment independently positioned to give optical focus. But the limitations of this sort of mechanical system still require that the mirror meet a stiffness target. While stiffness at minimum weight is the design requirement, the material-selection criteria remain unchanged.

Radio telescopes do not have to be quite as precisely dimensioned as optical ones because they detect radiation with a longer wavelength, about 0.25 mm rather than the 0.02 mm of light waves. But they are much bigger (60 m rather than 6) and they suffer from similar distortional problems. A recent 45 m radio telescope built for the University of Tokyo has a parabolic reflector made up of 6000 CFRP panels, each servo-controlled to compensate for macrodistortion. Radio telescopes are now routinely made from CFRP, for exactly the reasons we deduced.

Related case study

6.16 "Minimizing thermal distortion in precision devices"

6.4 MATERIALS FOR TABLE LEGS

Luigi Tavolino, furniture designer, conceives of a lightweight table of daring simplicity: a flat sheet of toughened glass supported on slender, unbraced cylindrical legs (Figure 6.5). The legs must be solid (to make them thin) and as light as possible (to make the table easier to move). They must support the table top and whatever is placed upon it without buckling (Table 6.5). What materials could one recommend?

The translation This is a problem with two objectives[1]: Weight is to be minimized and slenderness maximized. There is one constraint: resistance to buckling. Consider minimizing weight first.

The leg is a slender column of material of density ρ and modulus E. Its length, L, and the maximum load, F, it must carry are determined by the design: They are fixed. The radius r of a leg is a free variable. We wish to minimize the mass m of the leg, given by the objective function

$$m = \pi r^2 L \rho \tag{6.6}$$

subject to the constraint that it supports a load P without buckling. The elastic buckling load F_{crit} of a column of length L and radius r (see Appendix B) is

$$F_{crit} = \frac{\pi^2 E I}{L^2} = \frac{\pi^3 E r^4}{4 L^2} \tag{6.7}$$

using $I = \pi r^4/4$ where I is the second moment of the area of the column. The load F must not exceed F_{crit}. Solving for the free variable, r, and substituting it into the equation for m gives

$$m \geq \left(\frac{4F}{\pi}\right)^{1/2} (L)^2 \left[\frac{\rho}{E^{1/2}}\right] \tag{6.8}$$

The material properties are grouped together in the last pair of brackets. The weight is minimized by selecting the subset of materials with the greatest value of the material index

$$M_1 = \frac{E^{1/2}}{\rho}$$

(a result we could have taken directly from Appendix C).

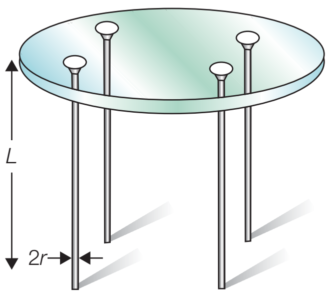

FIGURE 6.5
A lightweight table with slender cylindrical legs. Lightness and slenderness are independent design goals, both constrained by the requirement that the legs must not buckle when the table is loaded. The best choice is a material with high values of both $E^{1/2}/\rho$ and E.

Table 6.5 Design Requirements for Table Legs

Function	Column (supporting compressive loads)
Constraints	Length L specified Must not buckle under design loads Must not fracture if accidentally struck
Objectives	Minimize mass, m Maximize slenderness
Free variables	Diameter of legs, $2r$ Choice of material

[1] Formal methods for dealing with multiple objectives are developed in Chapter 7.

Now slenderness. Inverting Equation (6.7) with F_{crit} set equal to F gives an equation for the thinnest leg that will not buckle:

$$r \geq \left(\frac{4F}{\pi^3}\right)^{1/4} (L)^{1/2} \left[\frac{1}{E}\right]^{1/4} \tag{6.9}$$

The thinnest leg is that made of the material with the largest value of the material index

$$M_2 = E$$

The selection We seek the subset of materials that have high values of $E^{1/2}/\rho$ and E. We need the $E - \rho$ chart again (Figure 6.6). A guideline of slope 2 is drawn on the diagram; it defines the slope of the grid of lines for values of $E^{1/2}/\rho$. The guideline is displaced upward (retaining the slope) until a reasonably small subset of materials is isolated above it; it is shown at the position M_1 = 5 GPa$^{1/2}$/(Mg/m^3). Materials above this line have higher values of M_1. They are identified in the figure as *woods* (the traditional material for table legs), *composites* (particularly CFRP), and certain *engineering ceramics*. Polymers are out: They are not stiff enough; metals too: They are too heavy (even magnesium alloys, which are the lightest).

The choice is further narrowed by the requirement that, for slenderness, E must be large. A horizontal line on the diagram links materials with equal values of E; those above are stiffer. Figure 6.6 shows that placing this line at M_1 = 100 GPa eliminates woods and GFRP. If the legs must be really thin, then the shortlist is reduced to CFRP and ceramics: They give legs that weigh the same as the wooden ones but are barely half as thick. Ceramics, we know, are brittle: They have low values of fracture toughness. Table legs are exposed to abuse—they get knocked and kicked; common sense suggests that an additional constraint is needed, that of adequate toughness. This can be done using Figure 4.7; it eliminates ceramics, leaving CFRP. The cost of CFRP may cause Snr. Tavolino to reconsider his design, but that is another matter: He did not mention cost in his original specification.

It is a good idea to lay out the results as a table, showing not only the materials that are best but those that are second-best—they may, when other considerations are involved, become the best choice. Table 6.6 shows the way to do it.

Postscript Tubular legs, the reader will say, must be lighter than solid ones. True; but they will also be fatter. So it depends on the relative importance Snr. Tavolino attaches to his two objectives—lightness and slenderness—and only he can decide that. If he can be persuaded to live with fat legs, tubing

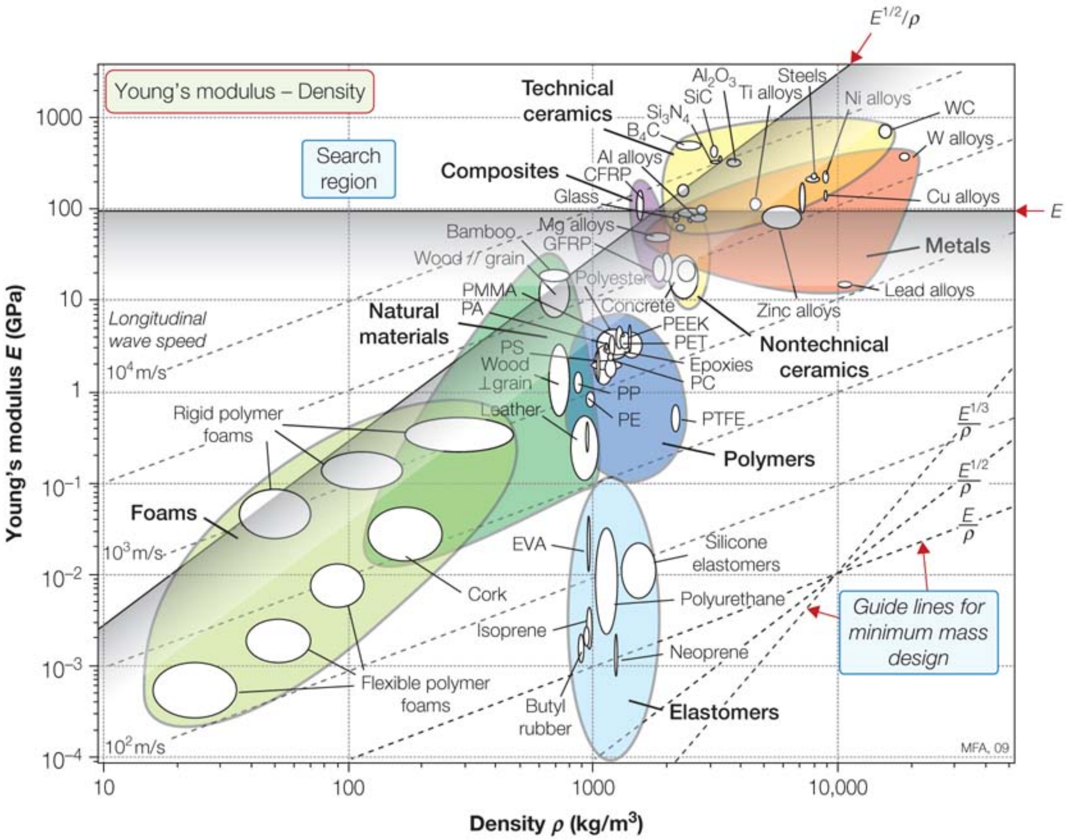

FIGURE 6.6

Materials for light, slender legs. Wood is a good choice; so is a composite such as CFRP, which, having a higher modulus than wood, gives a column that is both light and slender. Ceramics meet the stated design goals, but are brittle.

Table 6.6 Materials for Table Legs

Material	Typical M_1 ($GPa^{1/2}.m^3/Mg$)	Typical M_2 (GPa)	Comment
GFRP	2.5	20	Less expensive than CFRP, but lower M_1 and M_2
Woods	4.5	10	Outstanding M_1; poor M_2 Inexpensive, traditional, reliable
Ceramics	6.3	300	Outstanding M_1 and M_2 Eliminated by brittleness
CFRP	6.6	100	Outstanding M_1 and M_2, but expensive

can be considered—and the material choice may be different. Materials selection when section shape is a variable comes in Chapter 9.

Ceramic legs were eliminated because of low toughness. If (improbably) the goal is to design a light, slender-legged table for use at high temperatures, ceramics should be reconsidered. The brittleness problem can be bypassed by protecting the legs from abuse or by prestressing them in compression.

Related case studies

6.2 "Materials for oars"
6.3 "Mirrors for large telescopes"
8.5 "Conflicting objectives: Table legs again"
10.2 "Spars for human-powered planes"
10.3 "Forks for a racing bicycle"
10.5 "Table legs yet again: Thin or light?"

6.5 COST: STRUCTURAL MATERIALS FOR BUILDINGS

The most expensive thing that most people buy is the house they live in. Roughly half the expense of building a house is the cost of the materials of which it is made, and they are used in large quantities (family house: around 200 tonnes; large apartment block: around 20,000 tonnes). The materials are used in three ways: structurally to hold the building up; as cladding to keep the weather out; and as "internals" to insulate against heat and sound, and to decorate.

Consider the selection of materials for the structure (Figure 6.7). They must be stiff, strong, and cheap. Stiff, so that the building does not flex too much under wind and internal loads; strong, so that there is no risk of it collapsing. And cheap, because such a lot of material is used. The structural frame of a building is rarely exposed to the environment, and is not, in general, visible, so criteria of corrosion resistance or appearance are not important here.

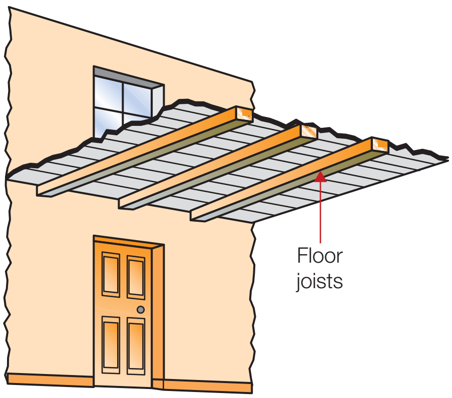

FIGURE 6.7
The materials of a building perform three broad roles. The frame gives mechanical support; the cladding excludes the environment; and the internal surfacing controls heat, light, and sound. The selection criteria depend on the function.

Table 6.7 Design Requirements for Floor Beams

Function	Floor beam
Constraints	Length L specified Stiffness: must not deflect too much under design loads Strength: must not fail under design loads
Objective	Minimize cost, C
Free variables	Cross-section area of beam, A Choice of material

The design goal is simple: strength and stiffness at minimum cost. To be more specific: Consider the selection of material for floor joists. Table 6.7 summarizes the requirements.

The translation Floor joists are beams; they are loaded in bending. The material index for a stiff beam of minimum mass, m, was developed in Chapter 5 (Equations (5.11) through (5.15)). The cost C of the beam is just its mass, m, times the cost per kg, C_m, of the material of which it is made:

$$C = m\,C_m = A\,L\rho\,C_m \tag{6.10}$$

which becomes the objective function of the problem. Proceeding as in Chapter 5, we find the index for a stiff beam of minimum cost to be

$$M_1 = \frac{E^{1/2}}{\rho\,C_m}$$

The index when strength rather than stiffness is the constraint was not derived earlier. Here it is. The objective function is still Equation (6.10), but the constraint is now that of strength: The beam must support F without failing. The failure load of a beam (Appendix B, Section B.4) is

$$F_f = C_2\frac{I\,\sigma_f}{y_m\,L} \tag{6.11}$$

where C_2 is a constant, σ_f is the failure strength of the material of the beam, and y_m is the distance between the neutral axis of the beam and its outer filament. We consider a rectangular beam of depth d and width b. We assume the proportions of the beam are fixed so that $d = \alpha b$ where α is the aspect ratio, typically 2 for wood beams. Using this and $I = bd^3/12$ to eliminate A in Equation (6.10) gives the cost of the beam that will just support the load F_f:

$$C = \left(\frac{6\sqrt{\alpha}}{C_2}\frac{F_f}{L^2}\right)^{2/3}(L^3)\left[\frac{\rho\,C_m}{\sigma_f^{2/3}}\right] \tag{6.12}$$

The mass is minimized by selecting materials with the largest values of the index

$$M_2 = \frac{\sigma_f^{2/3}}{\rho C_m}$$

The selection Stiffness first. Figure 6.8(a) shows the relevant chart: modulus E against relative cost per unit volume, $C_m\,\rho$ (the chart uses a relative cost C_R, defined in Chapter 4, in place of C_m but this makes no difference to the selection). The shaded band has the appropriate slope for M_1; it isolates concrete, stone, brick, woods, cast irons, and carbon steels. Figure 6.8(b) shows strength against relative cost. The shaded band—M_2 this time—gives almost the same selection. They are listed, with values, in

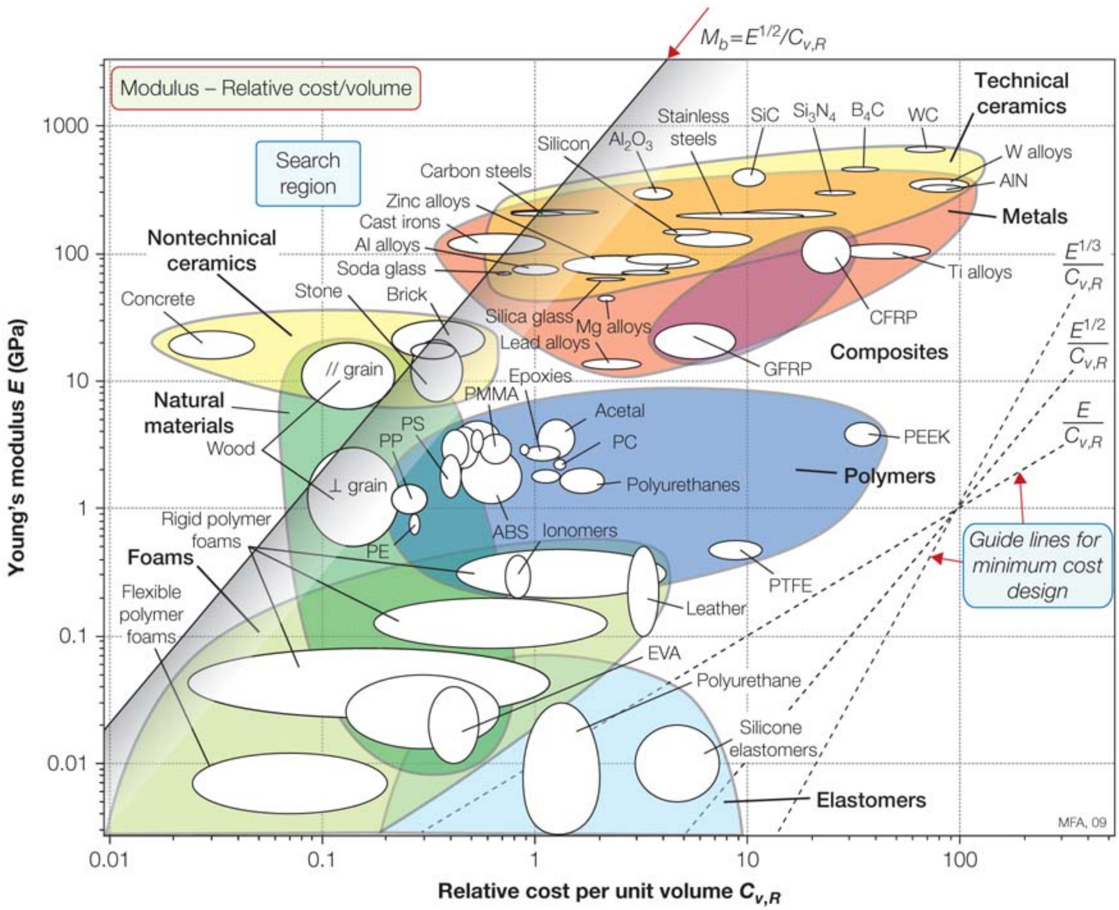

FIGURE 6.8(a)
The selection of cheap, stiff materials for the structural frames of buildings.

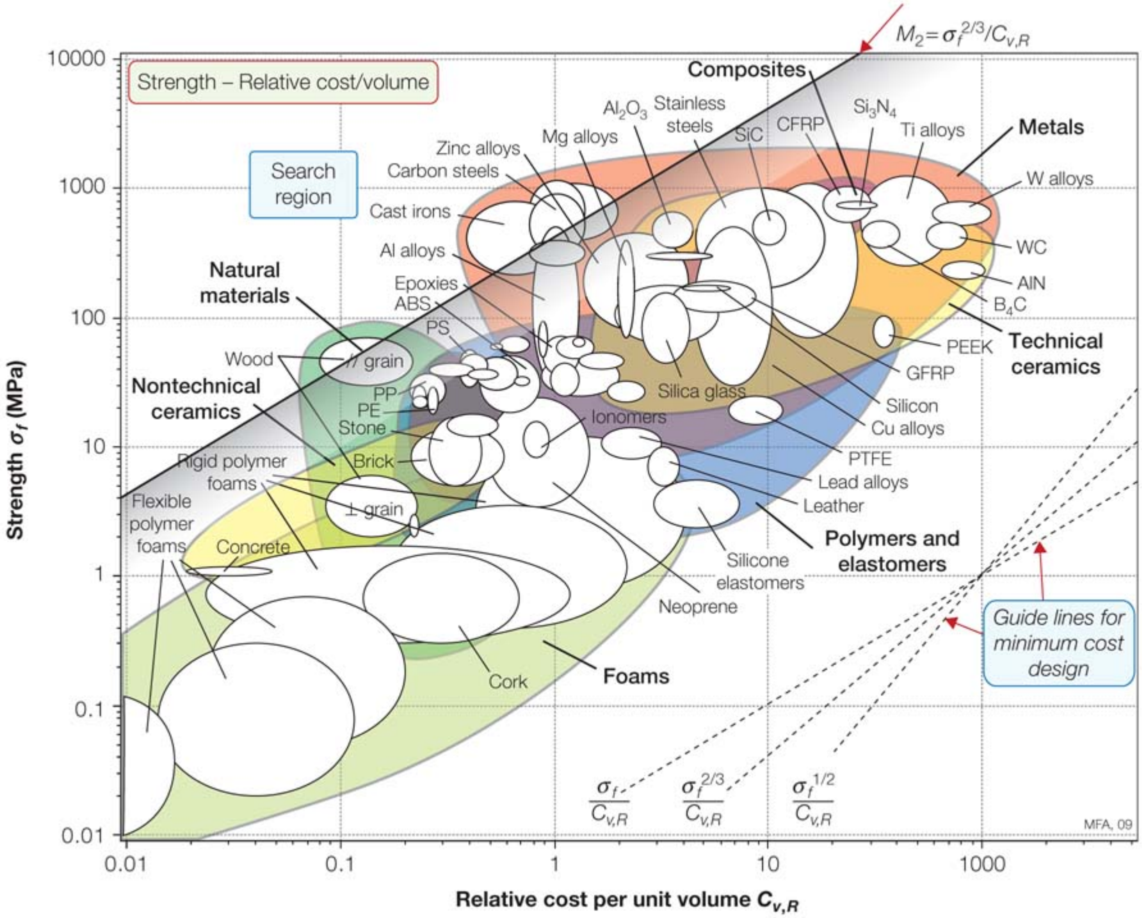

FIGURE 6.8(b)

The selection of cheap, strong materials for the structural frames of buildings.

Table 6.8. They are exactly the materials with which buildings have been, and are, made.

Postscript Concrete, stone, and brick have strength only in compression; the form of the building must use them in this way (columns, arches). Wood, steel, and reinforced concrete have strength in bending and tension as well as compression; steel, additionally, can be given efficient shapes (I-sections, box sections, tubes, discussed in Chapter 9). The form of the building made from these has much greater freedom.

It is sometimes suggested that architects live in the past; that in the twenty-first century they should be building with carbon (CFRP) and fiberglass (GFRP) composites, aluminum and titanium alloys, and stainless steel. Some do, but

Table 6.8 Structural Materials for Buildings

Material	M_1 (GPa$^{1/2}$)/(kg/m^3)	M_2 (MPa$^{2/3}$)/(kg/m^3)	Comment
Concrete	160	14	Use in compression only
Brick	12	12	
Stone	9.3	12	
Woods	21	90	Can support bending and tension as well as compression, allowing greater freedom of shape
Cast iron	17	90	
Steel	14	45	

the last two figures give an idea of the penalty involved: The cost of achieving the same stiffness and strength is between 5 and 50 times greater. Civil construction (buildings, bridges, roads, and the like) is materials-intensive: The cost of the material dominates the product cost, and the quantity used is enormous. Then only the cheapest of materials qualify, and the design must be adapted to use them.

Related reading

Cowan, H.J. & Smith, P.R. (1988). *The science and technology of building materials*. Van Nostrand-Reinhold.

Doran, D.K. (1992). The construction reference book. Butterworth-Heinemann.

Related case studies

6.2 “Materials for oars”
6.4 “Materials for table legs”
8.2 “Floor joists again”
10.4 “Floor joists: wood, bamboo or steel?”

6.6 MATERIALS FOR FLYWHEELS

Flywheels store energy. Small ones—the sort found in children’s toys—are made of lead. Old steam engines and modern automobiles have flywheels too; they are made of cast iron. Flywheels have been proposed for power storage and regenerative braking systems for vehicles; a few have been built, some of high-strength steel, some of composites. Lead, cast iron, steel, composites—there is a strange diversity here. What *is* the best choice of material for a flywheel?

An efficient flywheel stores as much *energy per unit weight* as possible. As the flywheel is spun up, increasing its angular velocity ω, it stores more energy. But if the centrifugal stress exceeds the tensile strength of the flywheel, it flies apart. So strength sets an upper limit on the energy that can be stored.

Table 6.9(a) Design Requirements for a Maximum-energy Flywheel

Function	Flywheel for energy storage
Constraints	Outer radius, *R*, fixed Must not burst Adequate toughness to give crack tolerance
Objective	Maximize kinetic energy per unit mass
Free variable	Choice of material

Table 6.9(b) Design Requirements for Fixed Velocity

Function	Flywheel for child's toy
Constraint	Outer radius, *R*, fixed
Objective	Maximize kinetic energy per unit volume at fixed angular velocity
Free variable	Choice of material

The flywheel of a child's toy is not efficient in this sense. Its angular velocity is limited by the pulling power of the child and never remotely approaches the burst velocity. In this case, and for the flywheel of an automobile engine, we wish to maximize the *energy stored* at a given *angular velocity* in a flywheel with an outer radius, R, that is constrained by the size of the cavity in which it must sit.

The objective and constraints in flywheel design thus depend on its purpose. The two alternative sets of design requirements are listed in Tables 6.9(a) and 6.9(b).

The translation An efficient flywheel of the first type stores as much energy per unit weight as possible, without failing. Think of it as a solid disk of radius R and thickness t, rotating with angular velocity ω (Figure 6.9). The energy U stored in the flywheel (Appendix B) is

$$U = \frac{1}{2} J\omega^2 \qquad (6.13)$$

Here $J = \frac{\pi}{2}\rho R^4 t$ is the polar moment of inertia of the disk and ρ the density of the material of which it is made, giving

$$U = \frac{\pi}{4}\rho R^4 t\omega^2 \qquad (6.14)$$

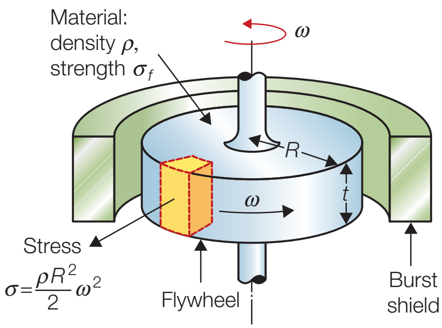

FIGURE 6.9
A flywheel. The maximum kinetic energy it can store is limited by its strength.

The mass of the disk is

$$m = \pi R^2 t \rho \tag{6.15}$$

The quantity to be maximized is the kinetic energy per unit mass, which is the ratio of the last two equations:

$$\frac{U}{m} = \frac{1}{4} R^2 \omega^2 \tag{6.16}$$

As the flywheel is spun up, the energy stored in it increases, but so does the centrifugal stress. The maximum principal stress in a spinning disk of uniform thickness (Appendix B again) is

$$\sigma_{max} = \left(\frac{3+\upsilon}{8}\right) \rho R^2 \omega^2 \approx \frac{1}{2} \rho R^2 \omega^2 \tag{6.17}$$

where υ is Poisson's ratio ($\upsilon \approx 1/3$). This stress must not exceed the failure stress σ_f (with an appropriate factor of safety, here omitted). This sets an upper limit to the product of the angular velocity, ω, and the disk radius, R (the free variables). Eliminating R_ω between the last two equations gives

$$\frac{U}{m} = \frac{1}{2} \left(\frac{\sigma_f}{\rho}\right) \tag{6.18}$$

The best materials for high-performance flywheels are those with high values of the material index

$$M = \frac{\sigma_f}{\rho} \tag{6.19}$$

It has units of kJ/kg.

And now the other sort of flywheel—that of the child's toy. Here we seek the material that stores the most energy per unit volume V at constant velocity, ω. The energy per unit volume at a given ω is (from Equation (6.2)):

$$\frac{U}{V} = \frac{1}{4} \rho R^2 \omega^2.$$

Both R and ω are fixed by the design, so the best material is now that with the greatest value of

$$M_2 = \rho \tag{6.20}$$

The selection Figure 6.10 shows the strength–density chart. Values of M_1 correspond to a grid of lines of slope 1. One such is plotted as a diagonal line at the value M_1 = 200 kJ/kg. Candidate materials with high values of M_1 lie in search region 1, which is toward the top left. The best choices are

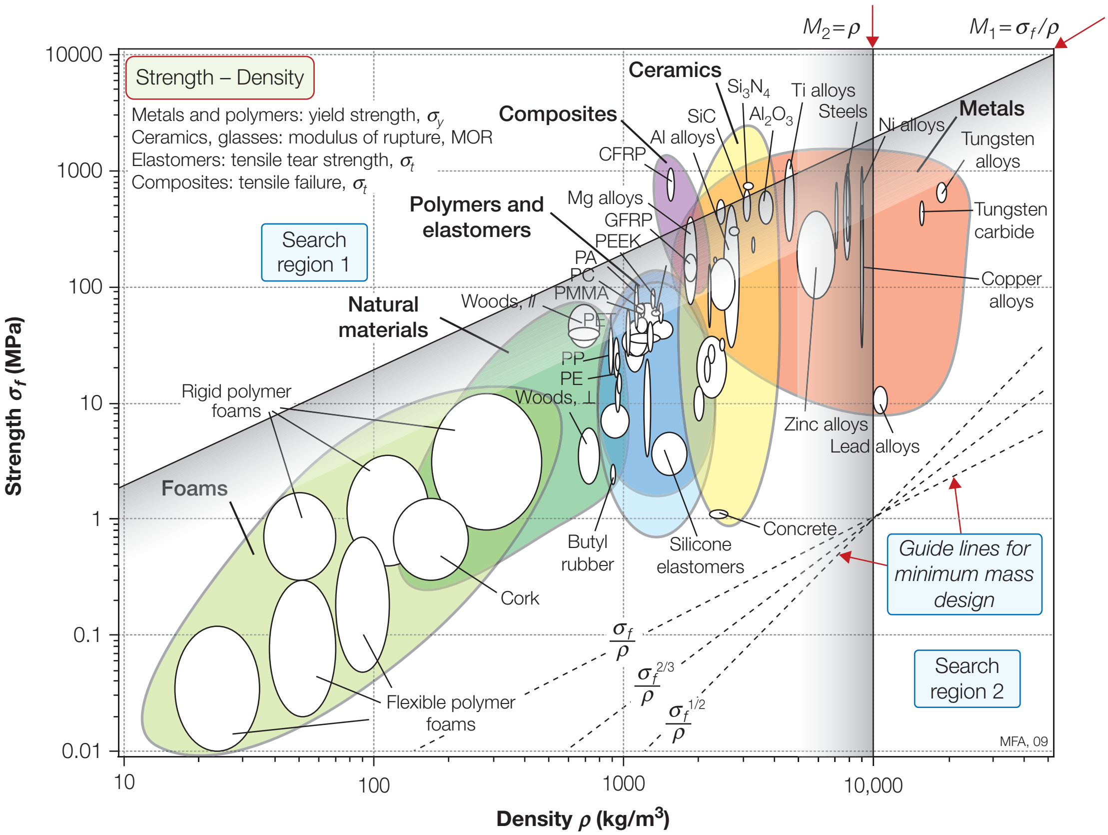

FIGURE 6.10

Materials for flywheels. Composites are the best choices. Lead and cast iron, traditional for flywheels, are good when performance is limited by rotational velocity, not strength.

unexpected ones: composites, particularly CFRP, high-strength titanium alloys, and some ceramics, but these are ruled out by their low toughness.

But what of the lead flywheels of children's toys? There could hardly be two more different materials than CFRP and lead: the one, strong and light; the other, soft and heavy. Why lead? It is because, in the child's toy, the constraint is different. Even a super-child cannot spin the flywheel of his or her toy up to its burst velocity. The angular velocity ω is limited instead by the drive mechanism (pull string, friction drive). Then, as we have seen, the best material is that with the largest density. The second selection line in Figure 6.10 shows the index M_2 at the value 10,000 kg/m^3. We seek materials in search region 2 to the right of this line. Lead is good. Tungsten is better, but more expensive. Cast iron is less good, but cheaper. Gold, platinum, and uranium (not shown on the chart) are the best of all, but may be thought unsuitable for other reasons.

Next Page

Table 6.10 Energy Density of Power Sources

Source	Energy Density kJ/kg	Comment
Gasoline	20,000	Oxidation of hydrocarbon—mass of oxygen not included
Rocket fuel	5000	Less than hydrocarbons because oxidizing agent forms part of fuel
Flywheels	Up to 400	Attractive, but not yet proved
Lithium-ion battery	Up to 350	Expensive, limited life
Nickel-cadmium battery	170–200	Less expensive than lithium-ion
Lead-acid battery	50–80	Large weight for acceptable range
Springs, rubber bands	Up to 5	Much less efficient method of energy storage than flywheel

Postscript A CFRP rotor is able to store around 400 kJ/kg. A lead flywheel, by contrast, can store only 1 kJ/kg before disintegration; a cast-iron flywheel, about 30. All of these are small compared with the energy density in gasoline: roughly 20,000 kJ/kg. Even so, the energy density in the flywheel is considerable; its sudden release in a failure could be catastrophic. The disk must be surrounded by a burst shield and precise quality control in manufacture is essential to avoid out-of-balance forces. This has been achieved in a number of composite energy-storage flywheels intended for use in trucks and buses, and as an energy reservoir for smoothing wind-power generation.

And now a digression: the electric car. Hybrid gas-electric cars are already on the roads, using advanced battery technology to store energy. But batteries have their problems: The energy density they can contain is low (see Table 6.10); their weight limits both the range and the performance of the car. It is practical to build flywheels with an energy density about equal to that of the best batteries.

Consideration is now being given to a flywheel for electric cars. A pair of counter-rotating CFRP disks are housed in a steel burst shield. Magnets embedded in the disks pass near coils in the housing, inducing a current and allowing power to be drawn to the electric motor that drives the wheels. Such a flywheel could, it is estimated, give an electric car an adequate range at a cost that is competitive with the gasoline engine and with none of the local pollution.

Related reading

Christensen, R.M. (1979). *Mechanics of composite materials* (p. 213 et seq.). Wiley Interscience.

Lewis, G. (1990). *Selection of engineering materials* (Part 1, p. 1). Prentice-Hall.

Medlicott, P.A.C., & Potter, K.D. (1986). The development of a composite flywheel for vehicle applications. In K. Brunsch, H-D. Golden, & C-M. Horkert (Eds.), *High Tech—the way into the nineties* (p. 29). Elsevier.