Family Name ....

Given Name ....

Student No. ....

Signature ....

THE UNIVERSITY OF NEW SOUTH WALES

School of Electrical Engineering & Telecommunications

MID-SEMESTER EXAMINATION

Semester 2, 2018

ELEC1111
Electrical and Telecommunications Engineering

TIME ALLOWED:	**75 min**
TOTAL MARKS:	**100**
TOTAL NUMBER OF QUESTIONS:	**5**

THIS EXAM CONTRIBUTES 25% TO THE TOTAL COURSE ASSESSMENT

Reading Time: 5 minutes.

This paper contains 4 pages.

Candidates must **ATTEMPT ALL** questions.

Answer each question in a **separate answer booklet**.

Marks for each question are indicated beside the question.

This paper **MAY NOT** be retained by the candidate.

Print your name, student ID and question number on the front page of each answer book.

Authorised examination materials:

Candidates should use their own UNSW-approved electronic calculators.

This is a closed book examination.

Assumptions made in answering the questions should be stated explicitly.

All answers must be written in ink. Except where they are expressly required, pencils **may only be used** for drawing, sketching or graphical work.

QUESTION 1 [15 marks]

For the circuit shown in Figure 1,

a. **(6 marks)** Calculate the equivalent resistance R_{eq} as seen from terminals *a*-*b*.

b. **(3 marks)** Find voltage v using the result of part (a).

c. **(6 marks)** Use voltage division to find voltage v_1 from voltage v.

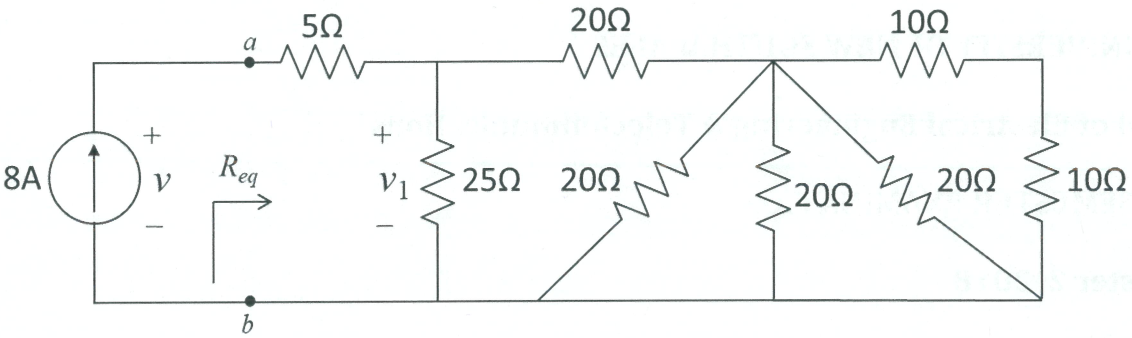
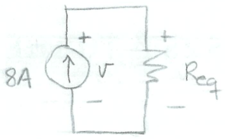


Figure 1

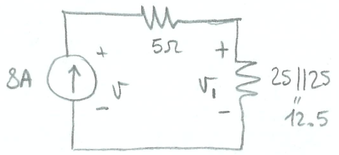
a) $R_{eq} = \left[\left[\left[\underbrace{(10+10)\|20\|20\|20}_{5}\right]+20\right]\|25\right]+5$

$\boxed{R_{eq} = 25\|25+5 = 12.5+5 = 17.5\Omega}$

b) $\boxed{V = R_{eq} \times i = 17.5\Omega \times 8A = 140\,V}$



c) We partially unsimplify the circuit and use voltage division to calculate V_1:



$\boxed{V_1 = V_x \frac{12.5}{17.5} = 100\,V}$

QUESTION 2 [20 marks]

Use nodal analysis to calculate the power supplied/absorbed by the dependent voltage source in the circuit shown in Figure 2.

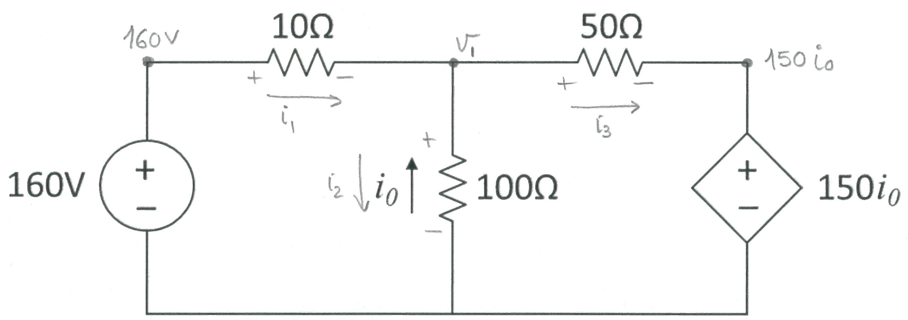


Figure 2

KCL @ node 1:

$$i_1 - i_2 - i_3 = 0$$

$$\frac{160 - V_1}{10} - \frac{V_1}{100} - \frac{V_1 - 150\, i_0}{50} = 0, \quad \text{where } i_0 = -i_2 = -\frac{V_1}{100}$$

Simplify equation

$$16 - \frac{V_1}{10} - \frac{V_1}{100} - \frac{V_1}{50} + \frac{15\cancel{0}}{5\cancel{0}}\left(\frac{-V_1}{100}\right) = 0; \quad 16 - V_1\left(\frac{1}{10} + \frac{1}{100} + \frac{1}{50} + \frac{3}{100}\right) = 0; \quad 16 - \frac{16}{100} V_1 = 0$$

$$\Downarrow$$

$$V_1 = 100\text{ V}$$

We calulate the power in the dependent source:

$$\boxed{P_{150 i_0}} = V \times i_3 = 150\, i_0 \times i_3 = 150\left(\frac{-V_1}{100}\right) \times \left(\frac{V_1 - 150\left(\frac{-V_1}{100}\right)}{50}\right) =$$

$$= 150 \times (-1) \times \left(\frac{100 - 150(-1)}{50}\right) = -150 \times \frac{25\cancel{0}}{5\cancel{0}} = \boxed{-750\text{ W}}$$

QUESTION 3 [15 marks]

Use a series of source transformations to find i_o in the circuit shown in Figure 3.

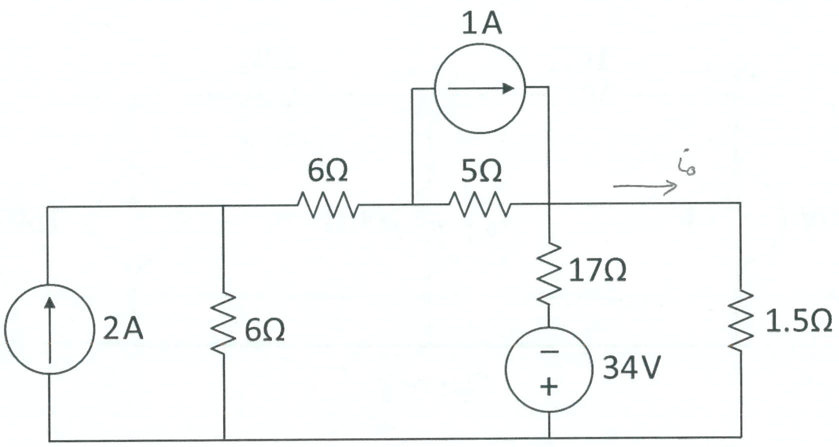
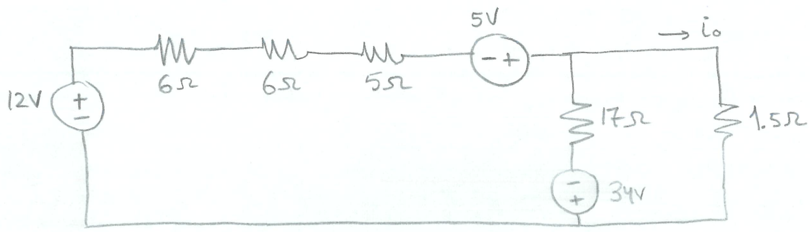
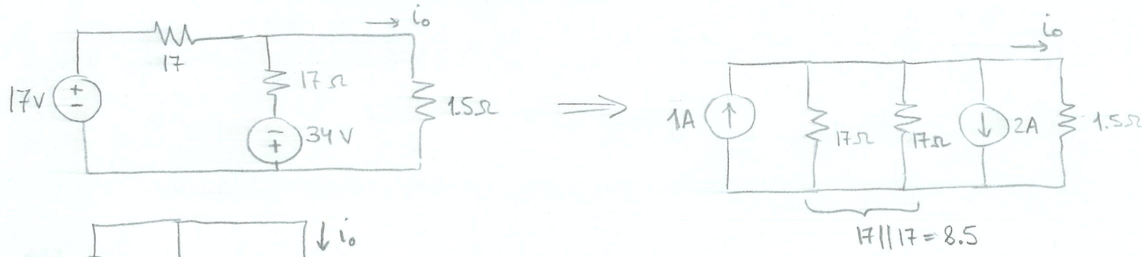
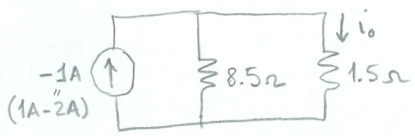


Figure 3







$$i_o = -1 \times \frac{8.5 \| 1.5}{1.5} = -\frac{1.275}{1.5} = -0.85\ A$$

current divider

QUESTION 4 [20 marks]

The system shown in Figure 4 is being used to power a load.

a. **(15 marks)** Find the Thevenin equivalent of the system.
b. **(5 marks)** Find the power in the load using your Thevenin equivalent model.

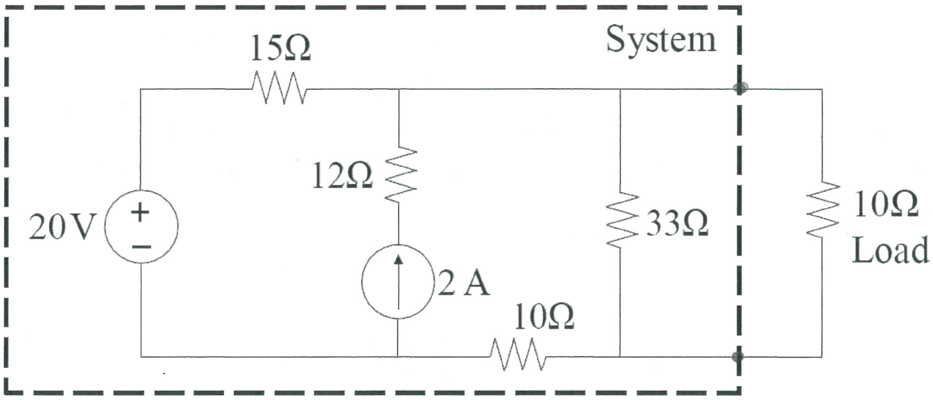
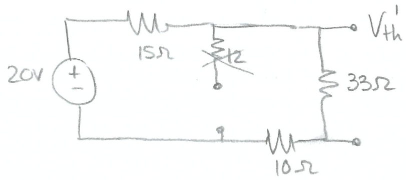


Figure 4

a)

→ V_{th} using superposition: (solutions for mesh and nodal analysis provided in next page).

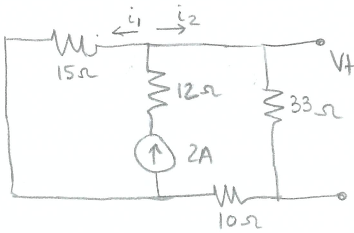
• V_{th}' due to 20V source:



$V_{th}' = 20 \frac{33}{33+15+10} = 11.38\text{ V}$

↑ voltage divider

• V_{th}'' due to 2A source:



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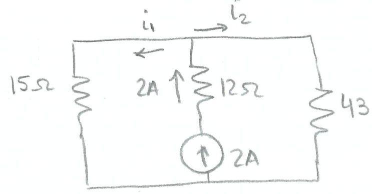


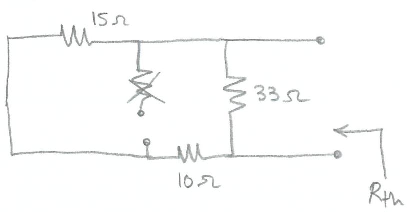
$i_2 = 2 \times \frac{43 \| 15}{43} = 0.517\text{ A}$

↑ current divider

$V_{th}'' = 33 i_2 = 33 \times 0.517 = 17.07\text{ V}$

$\boxed{V_{th} = V_{th}' + V_{th}'' = 28.45\text{ V}}$

→ R_{th}:

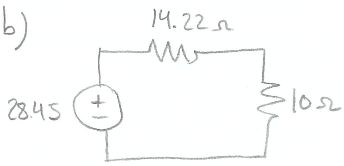


$\boxed{R_{th} = 33 \| 25 = 14.22\,\Omega}$

→ Note. Other options to calculate R_{th} are:

* $R_{th} = \frac{V_{th}}{i_{sc}}$
* $R_{th} = \frac{V_o}{i_o}$, where $V_o = 1V$ (or $i_o = 1A$) is a source connected to terminals a-b after turning off all the independent sources. The value is normally 1V/1A for simplicity.

b)



$\boxed{P_{10\Omega} = \frac{V_{10\Omega}^2}{10} = \frac{11.74^2}{10} = 13.8\text{ W}}$

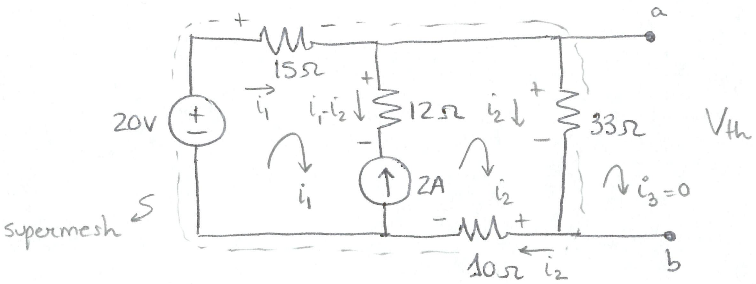
$V_{10\Omega} = 28.45 \frac{10}{24.22} = 11.74\text{ V}$

↑ voltage divider

OR $i = \frac{28.45}{24.22} \Rightarrow V_{10\Omega} = 10 i = 11.74\text{ V}$

QUESTION 4

→ Solution using mesh analysis



$$i_1 - i_2 = -2$$
$$-20 + 15 i_1 + 33 i_2 + 10 i_2 = 0$$

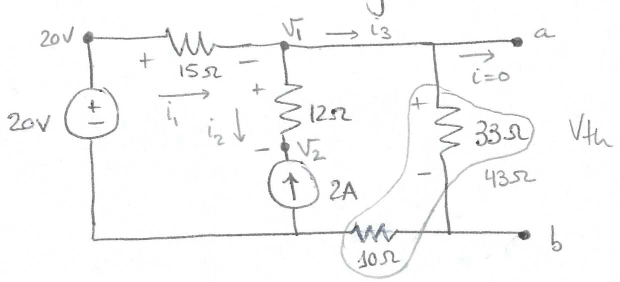
$$\left.\begin{aligned} i_1 - i_2 &= -2 \quad (\times -15) \\ 15 i_1 + 43 i_2 &= 20 \end{aligned}\right\}$$

$$\begin{aligned} -\cancel{15 i_1} + 15 i_2 &= 30 \\ \cancel{15 i_1} + 43 i_2 &= 20 \\ \hline 58 i_2 &= 50 \end{aligned}$$

$$\hookrightarrow i_2 = \frac{50}{58} = 0.8621 \text{ A}$$

$$\boxed{V_{th} = 33 i_2 = 33 \times 0.8621 = 28.44 \text{ V}}$$

→ Solution using nodal analysis



KCL@ node 1 : $i_1 - i_2 - i_3 = 0$

$$\frac{20 - V_1}{15} - (-2) - \frac{V_1}{43} = 0 \Rightarrow \frac{20}{15} - \frac{V_1}{15} + 2 - \frac{V_1}{43} = 0 \Rightarrow -\left(\frac{1}{15} + \frac{1}{43}\right) V_1 = -2 - \frac{20}{15}$$

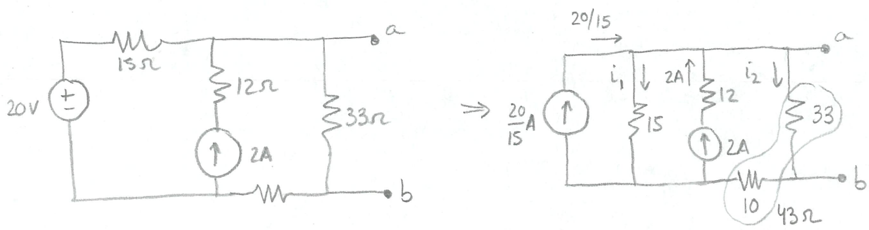
$$+0.08992 V_1 = +3.333$$

$$V_1 = 37.07 \text{ V}$$

$$\boxed{V_{th} = V_{33\Omega} = V_1 \frac{33}{43} = 37.07 \times \frac{33}{43} = 28.44 \text{ V}}$$

voltage divider

→ Solution using source transformation



$$i_2 = \left(\frac{20}{15} + 2\right) \frac{15 \| 43}{43} = 3.333 \; \frac{11.12}{43} =$$

current divider

$$= 0.862 \text{ A}$$

$$\boxed{V_{th} = 33 i_2 = 28.44 \text{ V}}$$

QUESTION 5 [30 marks]

a. **(15 marks)** The circuit shown in Figure 5 is used to test a "touch-switch" (Sw B), which detects the touch of a finger by the capacitance of the human body. The body can be modelled as a 100 pF capacitor relative to ground, and the resistance of the arm can be modelled as a 1.5 kΩ resistor.

   i. **(5 marks)** A person is charged to 2000 V as it walks across a carpet towards the touch-switch (Sw B). It stops in front of the touch-switch at time t = 0 s (Sw A opens). What energy is stored in the body?

   ii. **(10 marks)** At time t = 0 s, the person touches the touch-switch (Sw B closes). Calculate and plot the current through the 100 kΩ resistor for the first 50 microseconds. Ensure your plot is to scale and has at least three labelled values.

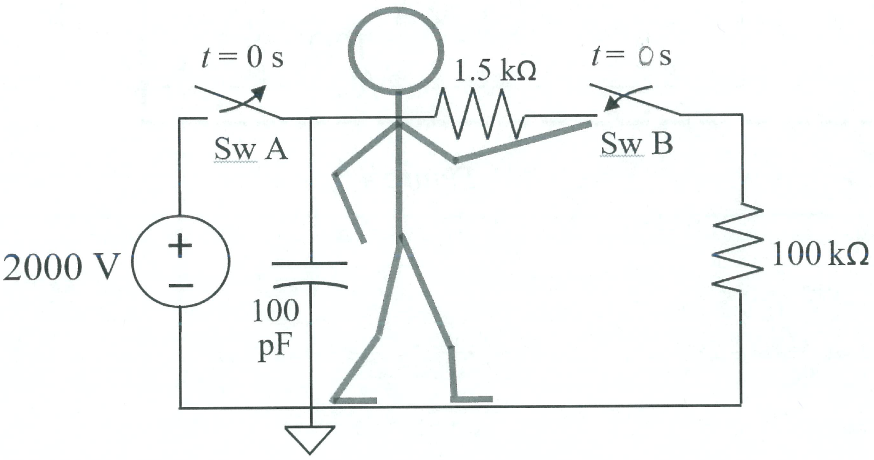
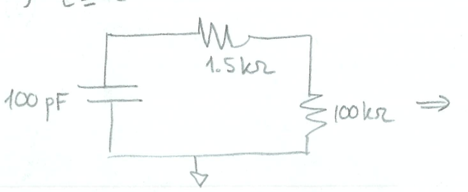


Figure 5

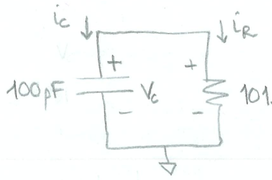
a)

i) $W_c = \frac{1}{2}CV^2 = \frac{1}{2} \times 100 \times 10^{-12} \times 2000^2 = 0.2\, mJ = 200\, \mu J$

ii) $t \geq 0s$

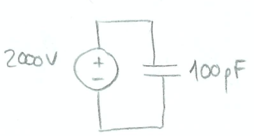


⇒

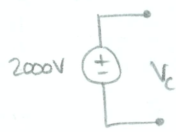


→ Natural response: $V_c(t) = V_0 e^{-t/\tau}$

- $t < 0$ ⇒ we calculate initial conditions $V(1) = V_0$



$\xrightarrow[\text{state}]{\text{steady}}$



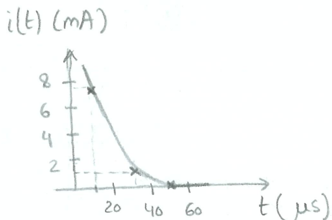
$V_c(0) = 2000V$

- $t \geq 0s$ ⇒ Natural response

$\tau = R \times C = 101.5 k\Omega \times 100\, pF = 1.015 \times 10^{-5} s = 10.15\, \mu s$

$V_c = 2000\, e^{\frac{-t}{1.015 \times 10^{-5}}} = 2000\, e^{-98522t}$

⇒ $i_{100k} = \frac{V_c}{R_{eq}} = \frac{V_c}{101.5 k\Omega} = 19.7\, e^{-98522t}$ mA



$V(\tau) = V(10.15 \mu s) = 2000 e^{-1} = 735.76\, V$ ⇒ $i(\tau) = 7.24\, mA$

$V(3\tau) = V(30.45 \mu s) = 2000\, e^{-3} = 99.59\, V$ ⇒ $i(3\tau) = 0.981\, mA$

$V(5\tau) = V(50.75 \mu s) = 2000\, e^{-5} = 13.47\, V$ ⇒ $i(5\tau) = 0.133\, mA$

↳ Also: $i_{100k} = -i_c = -C \frac{dv}{dt}$

$i_{100k} = -100 \times 10^{-12} \times 2000 \times (-98522)\, e^{-98522t} =$

$= 19.7\, e^{-98522t}$ mA

b. (15 marks) In the circuit shown in Figure 6, the switch has been closed for a long time before it is opened at $t = 0$.

- **i. (4 marks)** Find the initial voltage $v_c(0^-)$ across the capacitor.
- **ii. (4 marks)** Find the final voltage $v_c(\infty)$ across the capacitor.
- **iii. (4 marks)** Derive an expression for the voltage of the capacitor $v_c(t)$ for all time (i.e., for both $t < 0$ and $t \geq 0$).
- **iv. (3 marks)** Sketch the voltage $v_c(t)$ obtained in part (iii) as a function of time.

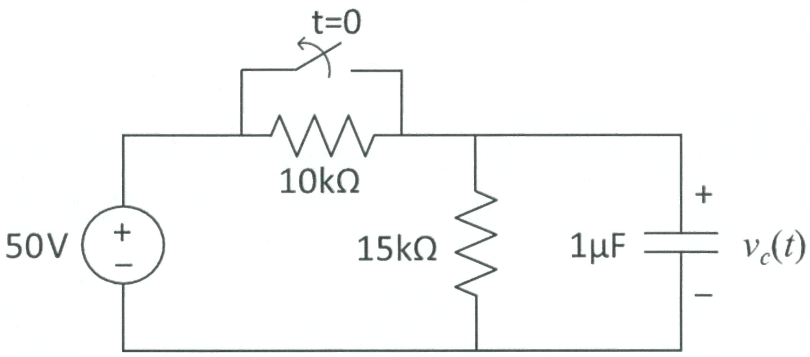


Figure 6

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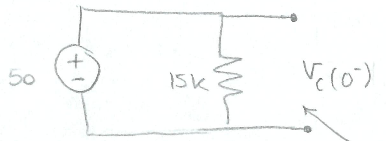
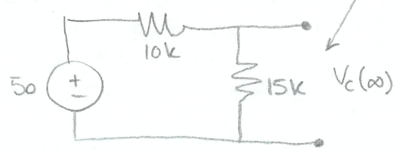
i) $t < 0$



$\boxed{V_c(0^-) = 50\text{ V}}$

Capacitor behaves as open circuit in DC steady state

ii) $t \geq 0$



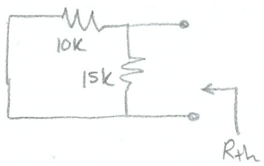
$\boxed{V_c(\infty) = 50 \dfrac{15k}{10k + 15k} = \dfrac{50 \times 15}{25} = 30\text{ V}}$

voltage divider

iii)

$$V_c(t) = V(\infty) + (V(0^-) - V(\infty))e^{-t/\tau}$$

where $\tau = R_{th}C$ in the circuit in part ii)



$R_{th} = 10k \| 15k = 6k$

$\tau = 6k \times 1\mu F = 6\text{ ms}$

$$V_c(t) = 30 + (50-30)e^{-t/6ms} = 30 + 20e^{-t/6ms}$$

$$\boxed{V_c(t) = \begin{cases} 50\text{ V}, & t < 0 \\ 30 + 20e^{-t/6ms}\text{ V}, & t \geq 0 \end{cases}}$$

iv)