Faculty of Engineering

School of Electrical Engineering and Telecommunications

ELEC 1111 – Topic 8

AC Analysis II

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Topic 8 Content

This lecture covers:

- Steady-state analysis of AC circuits in frequency domain.
  - Nodal analysis.
  - Mesh analysis.
- Application of superposition principle in frequency domain.
  - Circuits operating at a **single** frequency.
  - Circuits operating at **different** frequencies.
- Application of source transformation in frequency domain.
- Application of Thevenin's and Norton's theorems in AC circuits.
- AC Op Amp circuits.

Corresponds to Chapter 10 of your textbook

Analysing AC circuits

- When a circuit is operated by a **sinusoidal source**, its **steady-state response** can be obtained by using **phasors**.
- Transforming the circuit to phasor/frequency domain makes the analysis much simpler as we would no longer require to solve differential equations.
- Analyzing AC circuits usually require three steps:

1. **Transform** the circuit to **phasor/frequency domain.**
2. **Solve** the problem using **circuit analysis techniques** (nodal or mesh analysis, superposition, source transformation, etc.).
3. **Transform** the resulting phasors **back** to **time domain** (if required).

> **AC circuit analysis** is performed in the **same manner** as
> **DC circuit analysis** except that **complex numbers** are involved.

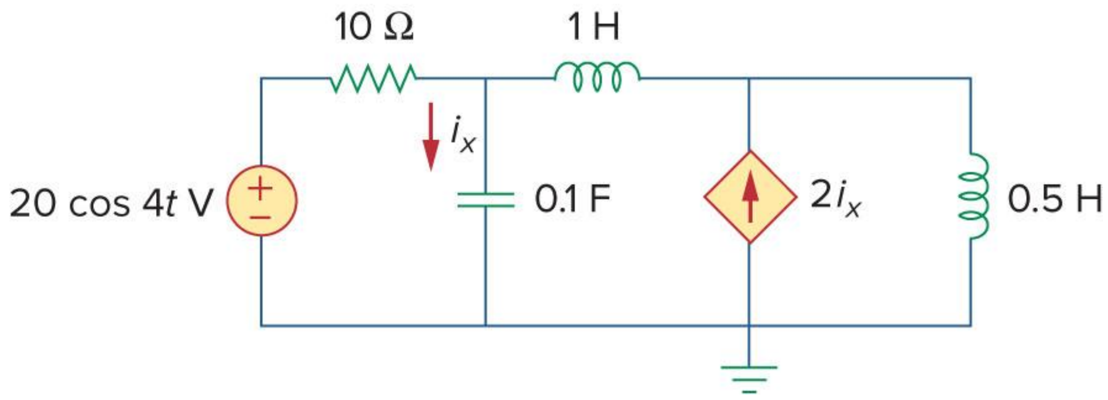
Nodal analysis in AC circuits

- We use **KCL** to write **nodal equations** in an AC circuits as we did in DC circuits.
  – The only difference is that **voltage phasors** at **each node** should be used.
- Always **convert a time-domain circuit** to **phasor/frequency domain** (if the circuit is not given in phasor domain) by calculating **all the impedances** of the circuit elements at the operating frequency and replacing the **sinusoids** (in <u>cosine</u> form) with their phasors.
- We deal with **voltage sources** in nodal analysis the same way as in DC:

1. Voltage source between non-reference node and reference node (i.e. ground):
   – Assign the **node voltage** to the **source voltage phasor.**
2. Voltage source between two non-reference nodes:
   – Form a **supernode.**
   – Apply KCL at the supernode using the already assigned phasor voltages at nodes inside supernode.
   – Write the extra equation relating node voltages inside supernode and the source voltage phasor.

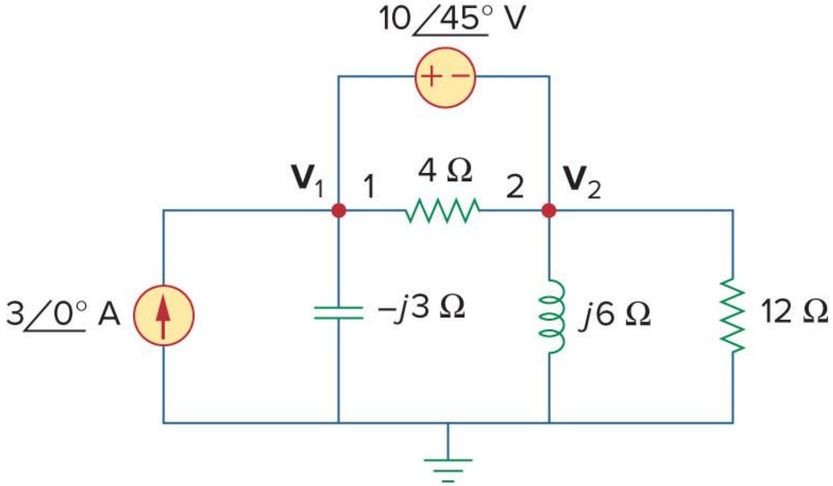
Exercise

Find i_x in the circuit given below using nodal analysis.



Exercise

Compute $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit below using nodal analysis.

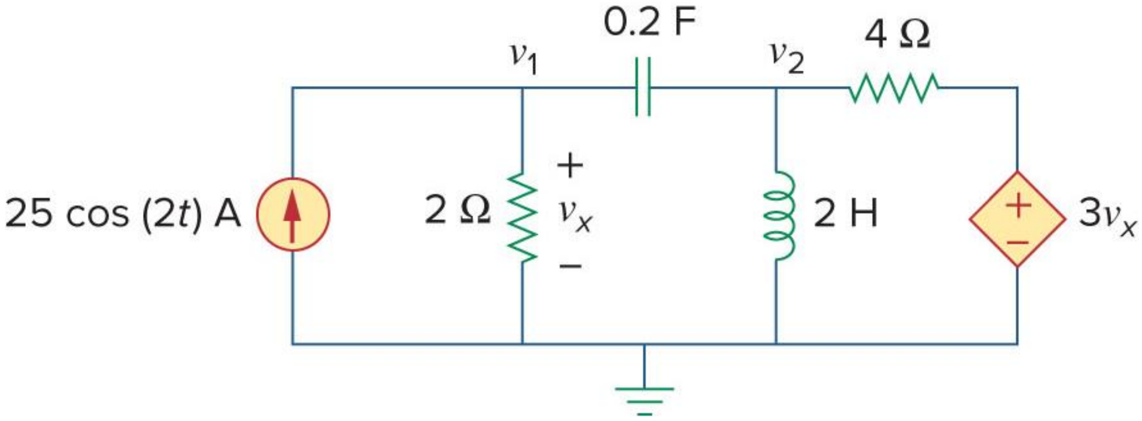


Exercise

Compute v_1 and v_2 in the circuit below using nodal analysis

- For practice!
- Answer: $v_1(t) = 28.31\cos(2t + 60.01°)\ \mathrm{V}$

  $v_2(t) = 82.56\cos(2t + 57.12°)\ \mathrm{V}$



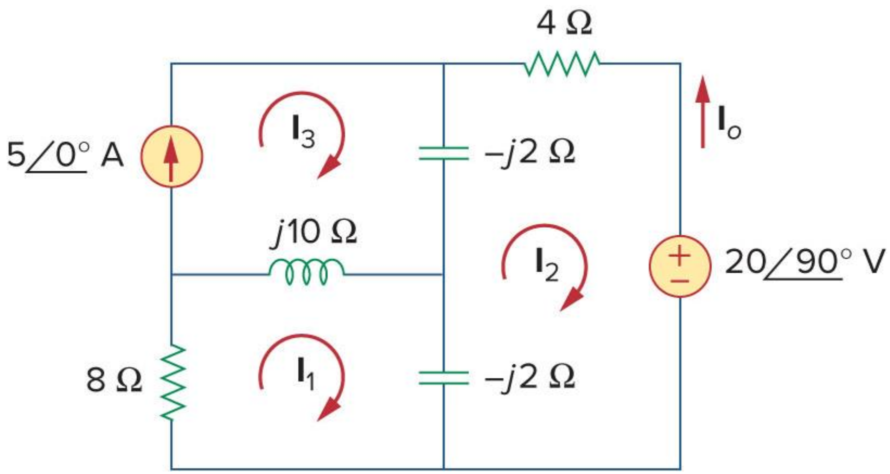
Mesh analysis in AC circuits

- We use **KVL** to write **mesh equations** in an AC circuits as we did in DC circuits.
  - The only difference is that **current phasors** in **each mesh** should be used.
- Always **convert a time-domain circuit** to **phasor/frequency domain** (if the circuit is not given in phasor domain) by calculating **all the impedances** of the circuit elements at the operating frequency and replacing the **sinusoids** (in <u>cosine</u> form) with their phasors.
- We deal with **current sources** in mesh analysis the same way as in DC:

1. Current source belonging to ONLY one mesh/loop:
   - The **mesh current** in that mesh/loop is **equal** to the **source current phasor.**
2. Current source shared between two meshes:
   - Form a **supermesh.**
   - Apply KVL in the supermesh using the already assigned mesh current phasors in the supermesh.
   - Write the extra equation relating mesh currents in the supermesh and the source current phasor.

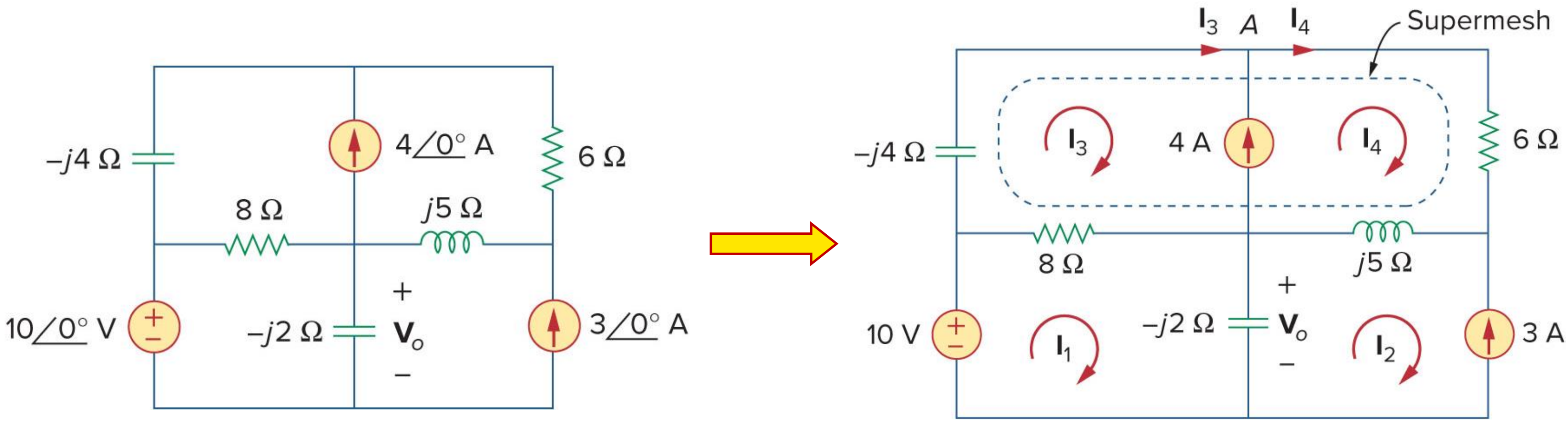
Exercise

Determine current $\mathbf{I}_o$ in the circuit below using mesh analysis.



Exercise

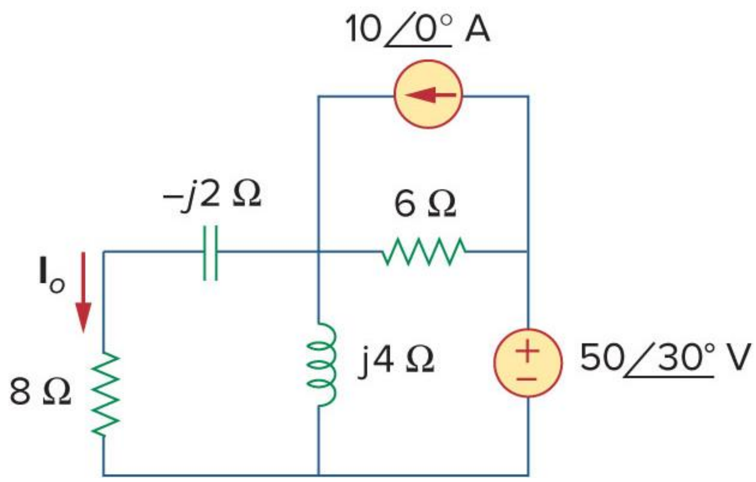
Find the voltage $\mathbf{V}_o$ in the circuit below using mesh analysis.



Exercise

Calculate $\mathbf{I}_o$ in the circuit below using mesh analysis.

- For practice!
- Answer: $\mathbf{I}_o = 5.97\angle 65.45° \text{ A}$



Superposition theorem in AC circuits

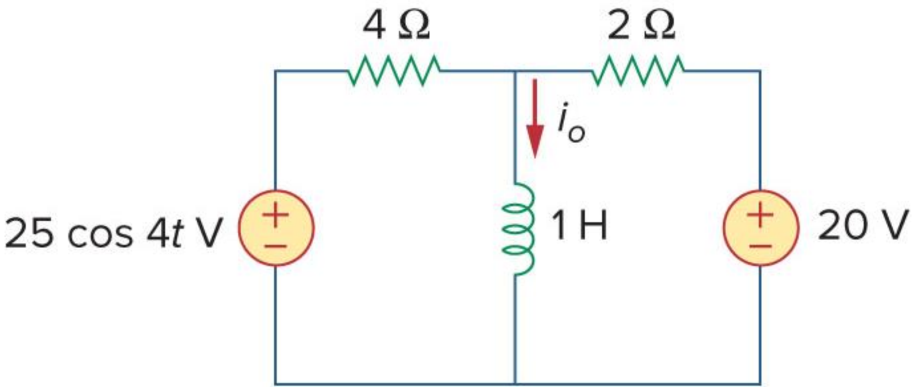
- Since AC circuits are **linear**, it is also possible to apply the principle of **superposition** in AC circuits with **multiple sources.**
- This becomes **particularly important** if the circuit has **sources** operating at **different frequencies.**
- The complication is that **each source** must have its **own frequency-domain** equivalent circuit because:
  - Impedances depend on frequency which means each element has a different impedance at different frequency ($\mathrm{Z} = R(\omega) + jX(\omega)\ \Omega$).
  - **Phasor voltages and phasor currents** resulting from each **different-frequency source CANNOT be added** to each other **in frequency domain**, instead they all must be converted back to time domain before being added.

Check the **frequency** of all **sources** in the circuit before applying **superposition** to make sure whether you need to **recalculate** the **impedances** for each frequency-domain equivalent circuit.

Exercise

Find i_o in the circuit shown below using superposition.

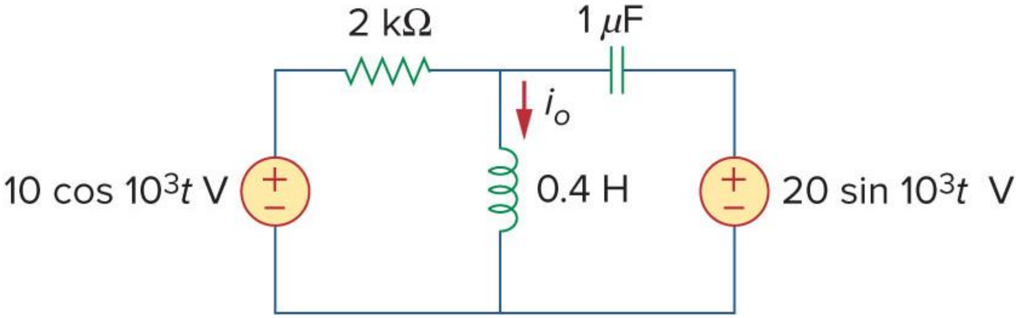
Note: This is an example of the use of superposition with sources operating at different frequencies (AC source at 4 rad/s and DC source at 0 rad/s).



Exercise

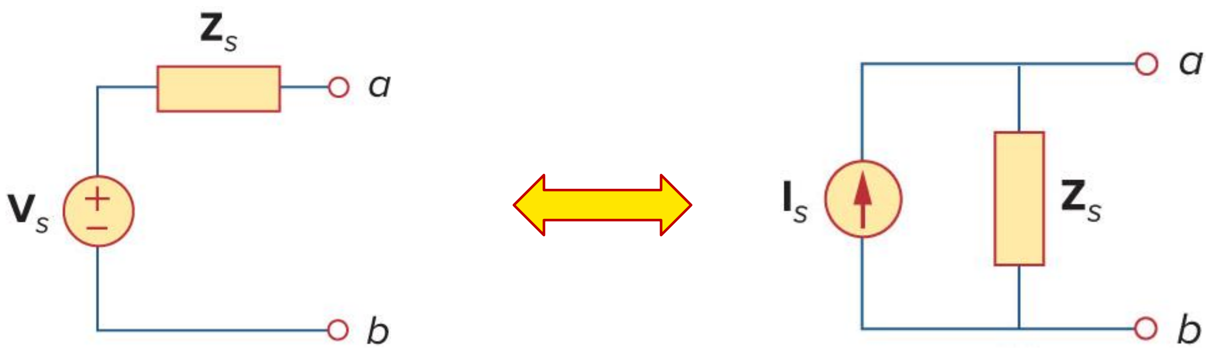
Use superposition to calculate i_o in the following circuit.

- For practice!
- Answer: $i_o = 39.5\cos(10^3 t - 18.43°)$ mA



Source transformation

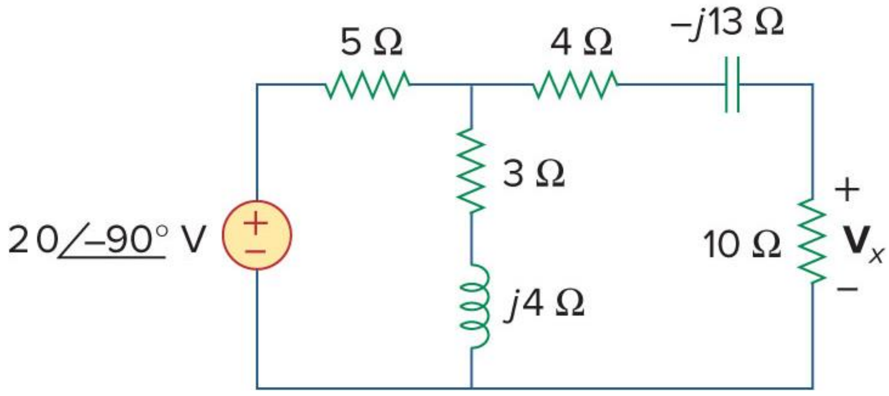
Source transformation in the frequency domain involves transforming a voltage source in series with an impedance to a current source in parallel with an impedance or vice versa.



$$\mathbf{V}_s = \mathbf{Z}_s \mathbf{I}_s \iff \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s}$$

Exercise

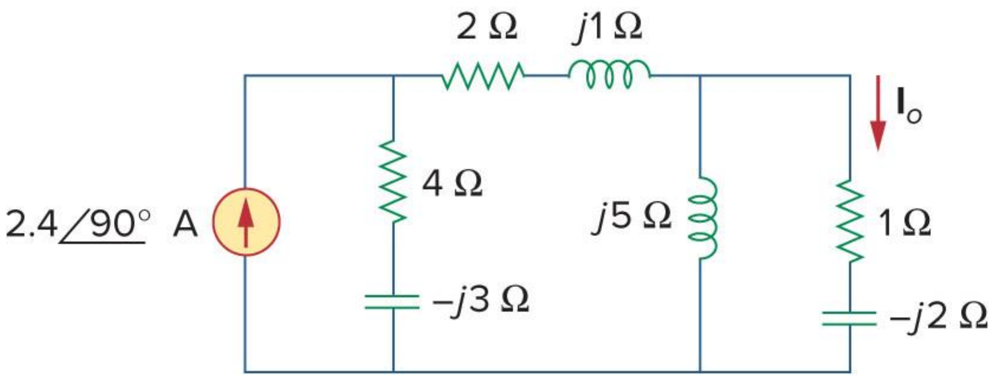
Calculate $\boldsymbol{V_x}$ in the circuit shown below using source transformation.



Exercise

Use source transformation to find $\mathbf{I}_o$ in the following circuit.

- For practice!
- Answer: $\mathbf{I}_o = 1.97\angle 99.46° \text{ A}$



Thevenin’s and Norton’s Theorems

- Both Thevenin and Norton’s theorems are applied to linear AC circuits the **same way** as in DC linear circuits.
  - The only **difference** is the fact that the calculated values will be **complex.**
- The two equivalent circuits are **related** through **source transformation.**

$$\mathbf{V}_{\mathrm{Th}} = \mathbf{Z}_N \mathbf{I}_N$$

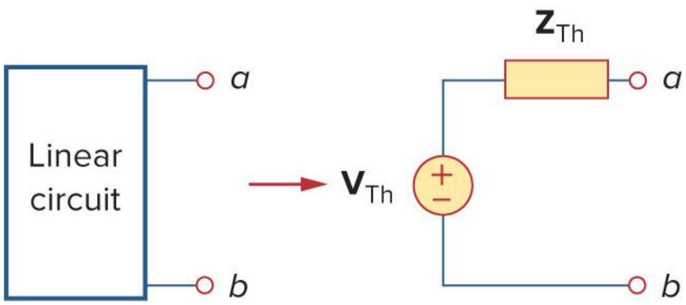
$$\mathbf{I}_N = \frac{\mathbf{V}_{\mathrm{Th}}}{\mathbf{Z}_{\mathrm{Th}}}$$

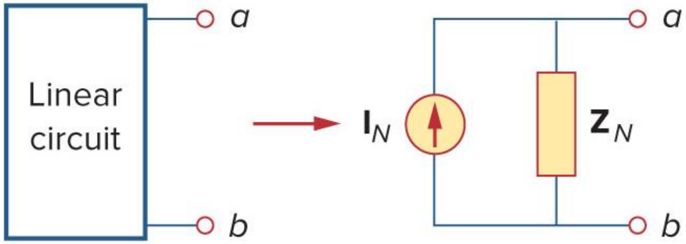
$$\mathbf{Z}_{\mathrm{Th}} = \mathbf{Z}_N$$

$\mathbf{V}_{\mathrm{Th}}$: Open-circuit voltage across terminals *a-b.*

$\mathbf{I}_N$: Short-circuit current through terminals *a-b.*

$\mathbf{Z}_{\mathrm{Th}} = \mathbf{Z}_N$: Equivalent or input impedance seen from terminals *a-b.*





Thevenin's and Norton's Theorems

- Finding the $\mathbf{Z}_{\mathrm{Th}} = \mathbf{Z}_N = \mathbf{Z}_{\mathrm{eq}} = \mathbf{Z}_{\mathrm{in}}$ is **the same** as in DC circuits:

1. Turn off all independent sources and calculate equivalent impedance from terminals (not possible if there is any dependent source).
2. Use definition of Thevenin resistance:

$$\boxed{\mathbf{Z}_{\mathrm{Th}} = \frac{\mathbf{V}_{\mathrm{Th}}}{\mathbf{I}_N}}$$

3. Connect an external source (only possibility if there are ONLY dependent sources).

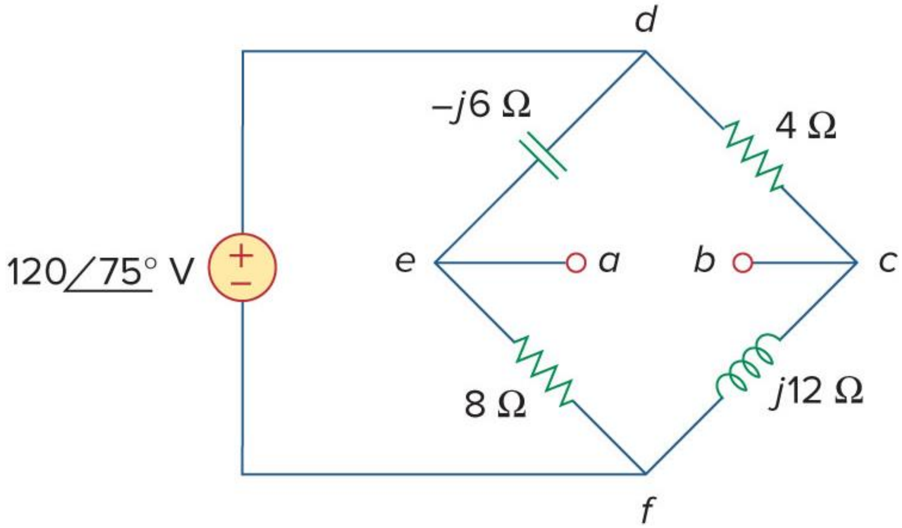
- Connect a **constant voltage source** in phasor form $\mathbf{V}_o = 1\angle 0°\ \mathrm{V}$ across the terminals (turn off independent sources if any).
- Find the phasor current $\mathbf{I}_o$ through the external voltage source
- $\mathbf{Z}_{\mathrm{eq}} = \frac{\mathbf{V}_o}{\mathbf{I}_o} = \frac{1\angle 0°}{\mathbf{I}_o}$ (Passive sign convention)

- Connect a constant current source in phasor form $\mathbf{I}_o = 1\angle 0°\ \mathrm{A}$ across the terminals (turn off independent sources if any).
- Find the phasor voltage $\mathbf{V}_o$ across the external current source
- $\mathbf{Z}_{\mathrm{eq}} = \frac{\mathbf{V}_o}{\mathbf{I}_o} = \frac{\mathbf{V}_o}{1\angle 0°}$ (Passive sign convention)

The **value** of the constant external source **does not** have to be **one.** It is just for simplicity.

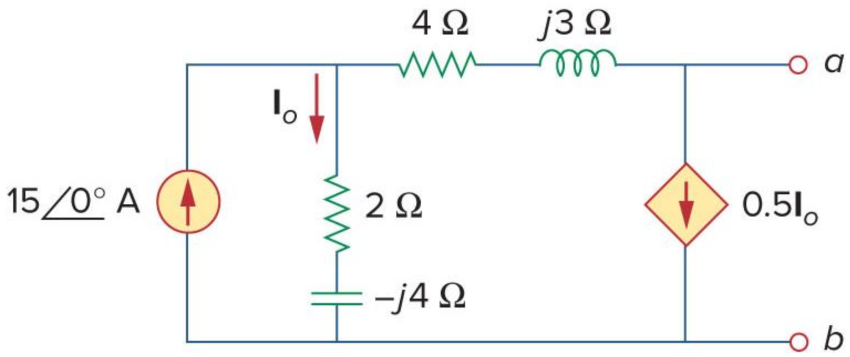
Exercise

Obtain the Thevenin equivalent circuit at the terminals *a*-*b* in the circuit below.



Exercise

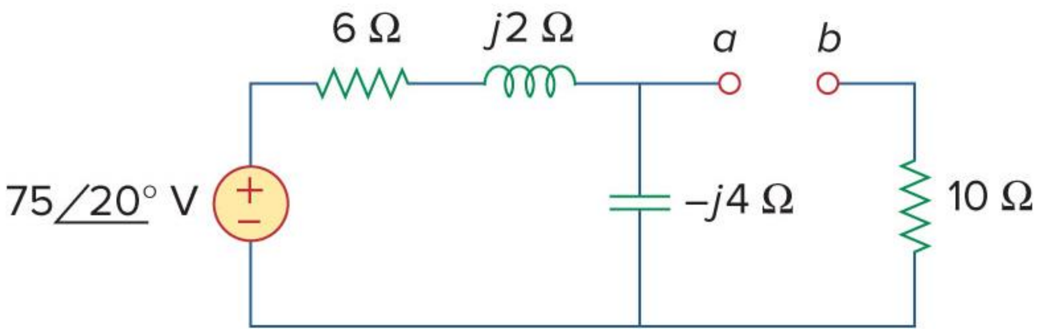
Find the Thevenin equivalent circuit as seem from the terminals *a*-*b* in the circuit below.



Exercise

Obtain the Norton equivalent circuit at the terminals *a*-*b* in the circuit below.

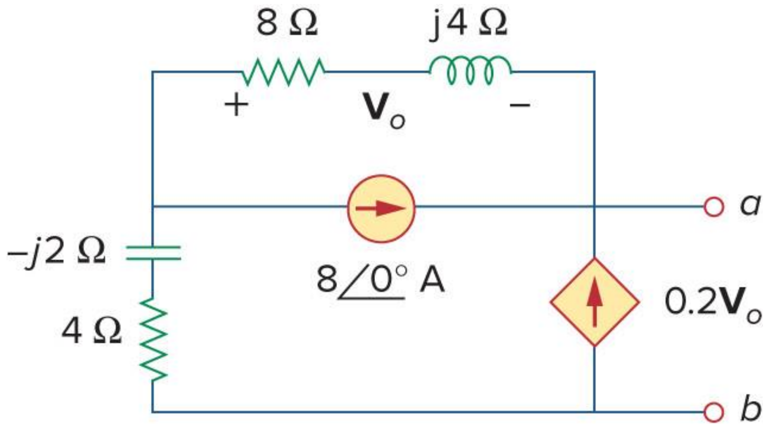
- For practice!
- Answer: $\mathbf{Z}_N = 12.4 - j3.2 = 12.8\angle - 14.47°\ \Omega$, $\mathbf{I}_N = 3.703\angle - 37.1°\ \text{A}$



Exercise

Determine the Thevenin equivalent circuit at the terminals *a*-*b* in the circuit below.

- For practice!
- Answer: $\mathbf{Z}_{\mathrm{Th}} = 4.47\angle -7.64° = 4.43 - j0.594\ \Omega,\ \ \mathbf{V}_{\mathrm{Th}} = 5.06\angle 145.31°\ \mathrm{V}$



Analysing AC Op Amp circuits

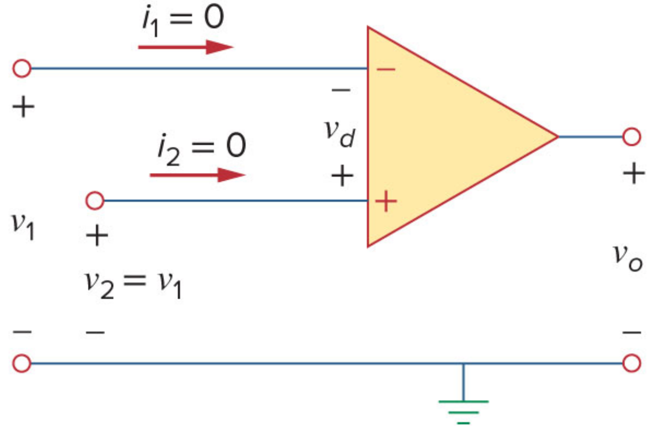
- If an Op Amp is connected to an AC source, we can use analysis in the phasor domain to find the steady state voltages and currents in the circuit.
- We use **nodal analysis** to solve AC Op Amp circuits in phasor domain.
- We consider **ideal Op Amp** and the following rules:

1. The **input phasor currents** to Op Amps terminal are **zero.**

$$\boxed{\mathbf{I}_1 = \mathbf{I}_2 = 0}$$

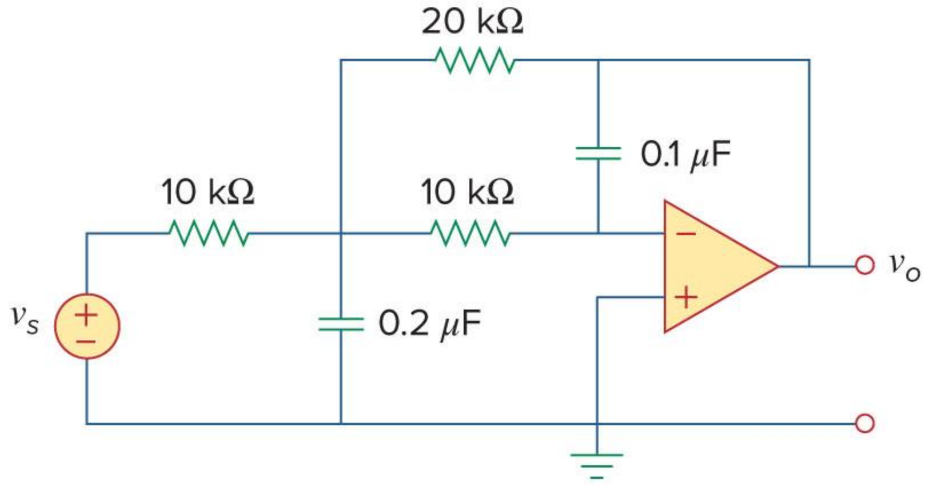
2. The input **phasor voltages** at **inverting** and **non-inverting** terminals are **equal** (only if there is a **negative feedback** connection from Op Amp's output to its inverting input).

$$\boxed{\mathbf{V}_1 = \mathbf{V}_2}$$



Exercise

Determine $v_o(t)$ for the Op Amp circuit shown below if $v_s(t) = 3\cos(1000t)$ V.

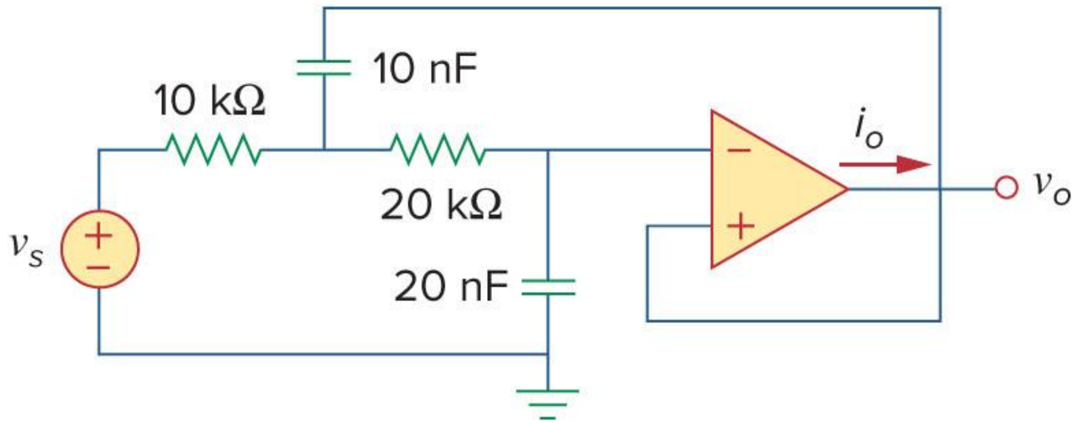


Exercise

Find v_o and i_o in the Op Amp circuit shown below if $v_s(t) = 12\cos(5000t)$ V

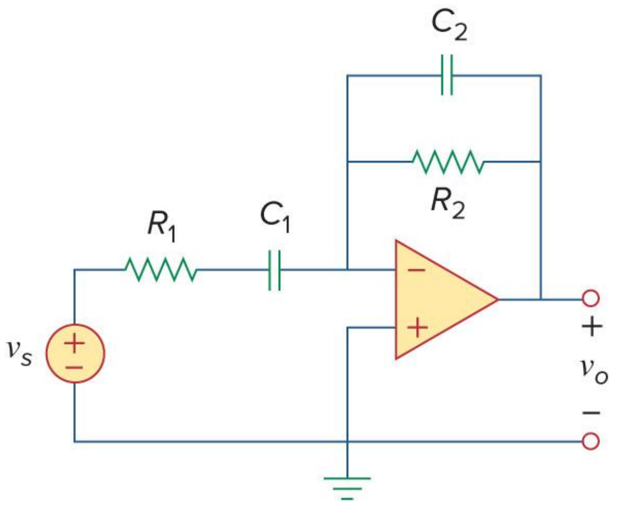
- For practice!
- Answer: $v_o(t) = 4\cos(5000t - 90°)$ V

  $i_o(t) = 0.4\cos(5000t - 90°)$ mA



Exercise

Compute the closed loop gain and the phase shift for the circuit below if $R_1 = R_2 = 10\ \text{K}\Omega$, $C_1 = 2\ \mu\text{F}$, $C_2 = 1\ \mu\text{F}$ and $\omega = 200\ \text{rad}/s$.

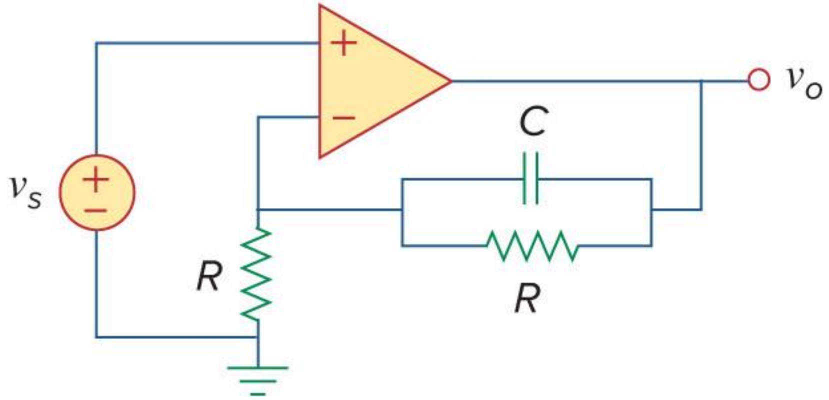


Exercise

Obtain the closed loop gain and the phase shift for the circuit below if $R = 10\ \text{K}\Omega$, $C = 1\ \mu\text{F}$, and $\omega = 1000\ \text{rad}/s$ (non-inverting amplifier).

- For practice!
- Answer: $\mathbf{G} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = 1.0147\angle(-5.6°)$

The gain is $|\mathbf{G}| = 1.0147$ and the phase shift is $\angle\mathbf{G} = -5.6°$.



Questions?