2016 Semester 1 Final Exam

Question 1

a)

i) $V = V_1 - V_2$

KCL at node 1:

$$1 = \frac{V_1}{2} + \frac{V_1 - V_2}{4}$$

$$4 = 2V_1 + V_1 - V_2$$

$$3V_1 - V_2 = 4 \quad (1)$$

KCL at node 2:

$$\frac{V_2 - V_1}{4} + \frac{V_2}{1} + \frac{V_2 - V_3}{3} = 0$$

$$3V_2 - 3V_1 + 12V_2 + 4V_2 - 4V_3 = 0$$

$$-3V_1 + 19V_2 - 4V_3 = 0 \quad (2)$$

Supernode:

$$V_3 - V_4 = 1$$

But $V_4 = 2(V_1 - V_2)$

$$-2V_1 + 2V_2 + V_3 = 1 \quad (3)$$

Voltage source:

$$V_4 = 2(V_1 - V_2)$$

$$= 2V_1 - 2V_2$$

$$2V_1 - 2V_2 - V_4 = 0 \quad (4)$$

ii) Solve for V_1, V_2, V_3 first then substitute for V_4 to make it easier

$$\begin{array}{c} \begin{array}{ccc} V_1 & V_2 & V_3 \end{array} \\ \left(\begin{array}{ccc|c} 3 & -1 & 0 & 4 \\ -3 & 19 & -4 & 0 \\ -2 & 2 & 1 & 1 \end{array}\right) \end{array}$$

$R_2 = R_2 + R_1$

$R_3 = 3R_3 + 2R_1$

$$\left(\begin{array}{ccc|c} 3 & -1 & 0 & 4 \\ 0 & 18 & -4 & 4 \\ 0 & 4 & 3 & 11 \end{array}\right)$$

$R_3 = 9R_3 - 2R_2$

$$\left(\begin{array}{ccc|c} 3 & -1 & 0 & 4 \\ 0 & 18 & -4 & 4 \\ 0 & 0 & 35 & 91 \end{array}\right)$$

$\therefore V_3 = \frac{91}{35} = 2.6V$

$V_2 = \frac{4 + 4(2.6)}{18} = 0.8V$ $\therefore V_4 = 2(1.6 - 0.8)$

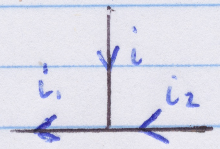
$= 1.6V$

$V_1 = \frac{4 + 0.8}{3} = 1.6V$

$$\boxed{\begin{array}{l} \text{Final answers:} \\ V_1 = 1.6V \\ V_2 = 0.8V \\ V_3 = 2.6V \\ V_4 = 1.6V \end{array}}$$

b)

i) $i = (i_1 - i_2)$



KVL at mesh 1:

$$-4 + 5(i_1 - i_3) + 6(i_1 - i_2) = 0$$

$$11i_1 - 6i_2 - 5i_3 = 4 \quad (1)$$

KVL at mesh 2:

$$6(i_2 - i_1) + 2(i_2 - i_3) + 12(i_1 - i_2) = 0$$

$$6i_1 - 4i_2 - 2i_3 = 0$$

$$3i_1 - 2i_2 - i_3 = 0 \quad (2)$$

Final answers:

$i_1 = 6A$

$i_2 = 7A$

$i_3 = 4A$

KVL at mesh 3:

$$4i_3 + 2(i_3 - i_2) + 5(i_3 - i_1) = 0$$

$$-5i_1 - 2i_2 + 11i_3 = 0 \quad (3)$$

$$\begin{array}{ccc} i_1 & i_2 & i_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 11 & -6 & -5 & 4 \\ 3 & -2 & -1 & 0 \\ -5 & -2 & 11 & 0 \end{array}\right)$$

$R_2 = 11R_2 - 3R_1$

$R_3 = 11R_3 + 5R_1$

$$\left(\begin{array}{ccc|c} 11 & -6 & -5 & 4 \\ 0 & -4 & 4 & -12 \\ 0 & -52 & 96 & 20 \end{array}\right)$$

$R_3 = R_3 - 13R_2$

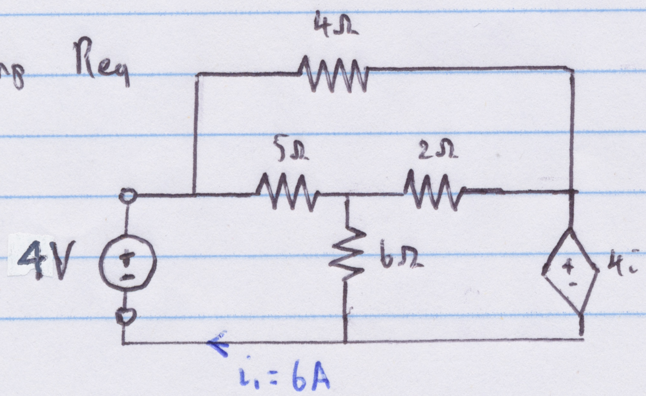
$$\left(\begin{array}{ccc|c} 11 & -6 & -5 & 4 \\ 0 & -4 & 4 & -12 \\ 0 & 0 & 44 & 176 \end{array}\right)$$

$\therefore\ i_3 = 4A$

$$i_2 = \frac{-12-16}{-4} = 7A$$

$$i_1 = \frac{4+6(7)+5(4)}{11} = 6A$$

ii) Finding Req

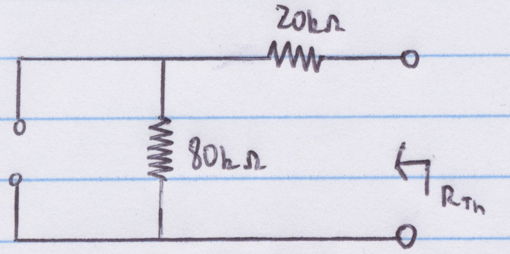


Attaching a 4V source at the terminals, we can use thevenin's theorem to calculate Req:

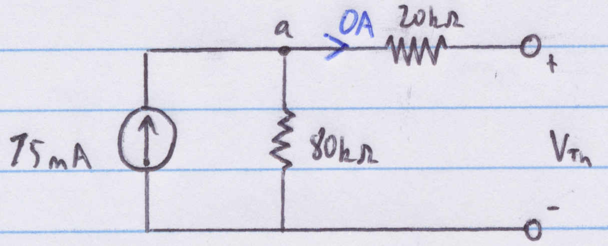
$$R_{eq} = \frac{4}{6} = \frac{2}{3}\,\Omega$$

Question 2

a)



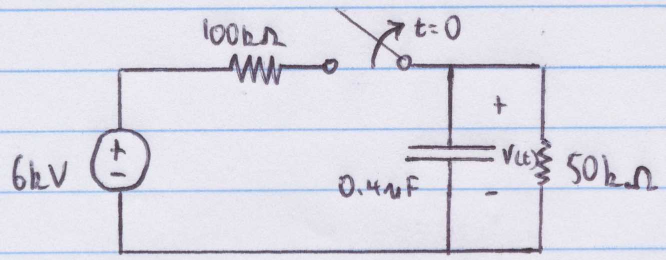
$R_{Th} = 100k\Omega$



KCL at node a:

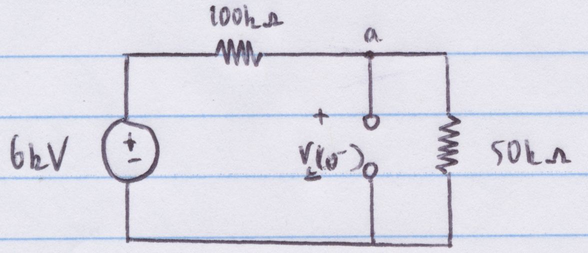
$$\frac{V_a}{80\times10^3} = 75\times10^{-3}$$

$$V_a = 6kV = V_{Th} \quad \text{since} \quad \frac{V_a - V_{Th}}{20k\Omega} = 0$$



(Thevenin equilavent circuit)

b) At $t=0^-$



KCL at node a:

$$\frac{V_c(0^-) - 6\times10^3}{100\times10^3} + \frac{V_c(0^-)}{50\times10^3} = 0$$

$$3V_c(0^-) = 6\times10^3$$

$$V_c(0^-) = 2kV$$

c) $R_{Th} = 50\times10^3\,\Omega$ after switch has opened

$$\tau = R_{Th}C = 50\times10^3 \times 0.4\times10^{-6}$$

$$= 20\times10^{-3}\,s$$

d) $V_\infty = 0V$ (No source to supply voltage so capacitor discharges)

$$V_c(t) = 0 + (2\times10^3 - 0)e^{-\frac{t}{0.02}}\ V \quad \text{for} \quad t>1$$

$$= (2\times10^3)e^{-50t}\ V$$

e) $i(t) = C\frac{dv}{dt}$

$= (0.4\times10^{-6})\times(-50)\times(2\times10^{3})e^{-50t}$ A

$= -0.04e^{-50t}$ A

f) $W = \frac{1}{2}Cv_c(0^-)^2$

$= \frac{1}{2}\times 0.4\times10^{-6}\times(2\times10^{3})^2$

$= 0.8$ J

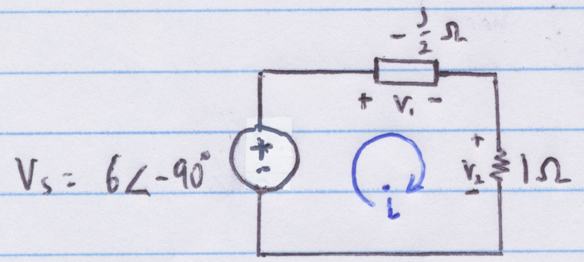
Question 3

a)

i) $6\sin 2t \Rightarrow 6\cos(2t-90^\circ)$

$\omega = 2$ rad/s

$Z_{eq}(1F) = -\frac{j}{\omega L} = \frac{-j}{2}\,\Omega$

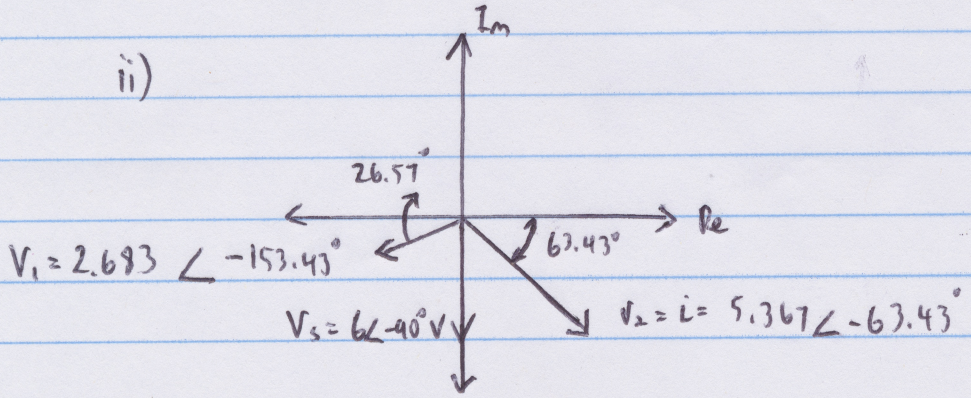


KVL: $i = \frac{6\angle-90^\circ}{1-\frac{j}{2}} = 5.367\angle-63.43^\circ$ A

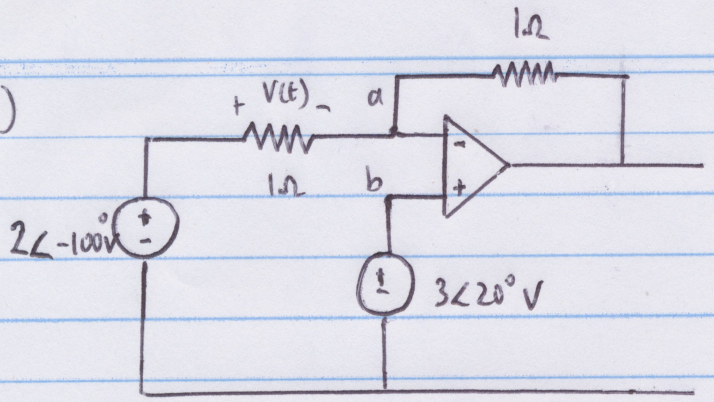
$\therefore V_1 = \left(-\frac{j}{2}\right)\times i = 2.683\angle-153.43^\circ$ V

$V_2 = 1\times i = 5.367\angle-63.43^\circ$ V

ii)



b)



$2\sin(2t-10^\circ) \Rightarrow 2\cos(2t-100^\circ) = 2\angle -100^\circ$

$V_b = V_a = 3\angle 20^\circ$ (Principle of Op amps)

$$v(t) = 2\angle -100^\circ - 3\angle 20^\circ$$
$$= 4.359\angle -136.59^\circ$$
$$= 4.359\cos(2t - 136.59^\circ)\ V$$

$\Leftarrow$ Must be this form when asked for $v(t)$ as they are asking for voltage as a function of time.

c) $Z_{eq} = (R + jwL) \,//\, \left(-\frac{j}{wc}\right)$

$$= \frac{(R + jwL)\left(-\frac{j}{wc}\right)}{R + j\left(wL - \frac{1}{wc}\right)} = \frac{\frac{L}{c} - j\left(\frac{R}{wc}\right)}{R + j\left(wL - \frac{1}{wc}\right)} = \frac{r_1\angle\theta_1}{r_2\angle\theta_2} = \frac{r_1}{r_2}\angle\theta_1 - \theta_2$$

If $\theta_1 - \theta_2 = 0$, then Z_{eq} will be purely resistive $\Rightarrow \theta_1 = \theta_2$

$$\theta_1 = \tan^{-1}\left(-\frac{R}{wc} \div \frac{L}{c}\right) = \tan^{-1}\left(-\frac{R}{wL}\right)$$

$$\theta_2 = \tan^{-1}\left(\frac{wL - \frac{1}{wc}}{R}\right) = \tan^{-1}\left(\frac{w^2cL - 1}{wcR}\right)$$

$$\therefore -\frac{R}{wL} = \frac{w^2cL - 1}{wcR}$$

$$-wcR^2 = w^3cL^2 - wL$$

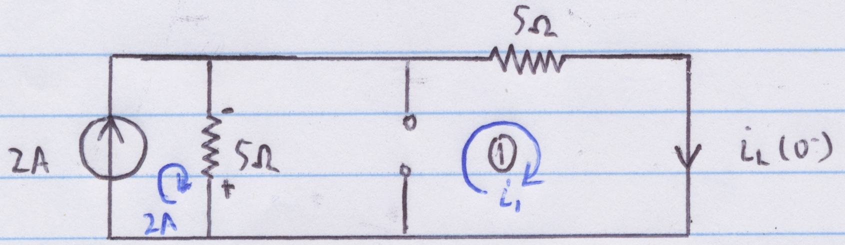
$$w(w^2cL^2 - L + R^2c) = 0 \Rightarrow w \neq 0$$

$$w^2cL^2 - L + R^2c = 0$$

$$w = \sqrt{\frac{L - R^2c}{L^2c}} = 1.278\times 10^3 \text{ rad/s}$$

Question 4

a) At $t = 0^-$

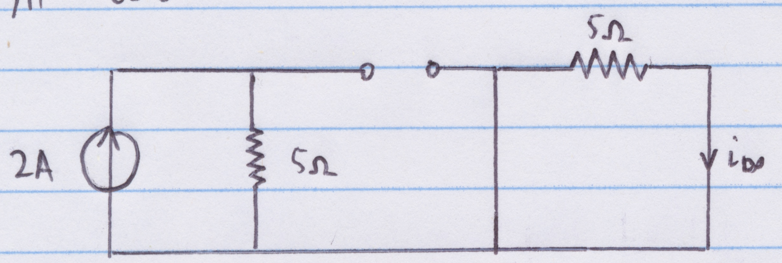


Note: $i_1 = i_L(0^-)$

KVL in mesh ①:

$$5(i_1 - 2) + 5i_1 = 0$$
$$10i_1 = 10$$
$$i_1 = 1A = i_L(0^-)$$

At $t = 0^+$



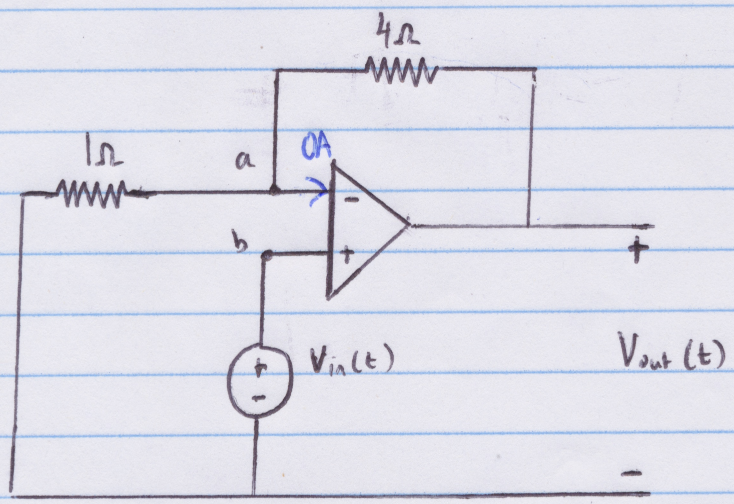
$i_\infty = 0A$

$R_{Th} = 5\Omega$

$\tau = \frac{L}{R} = \frac{100\times10^{-3}}{5} = 20\times10^{-3}$

$$i_L(t) = i_\infty + (i_L(0^-) - i_\infty)e^{-\frac{t}{\tau}}$$
$$= 1e^{-50t}\ A \quad \text{for } t > 0$$

b)



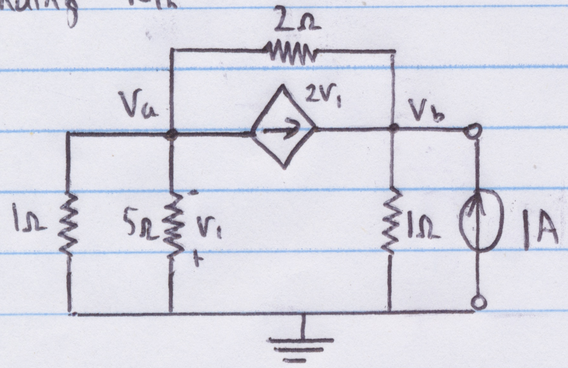
$V_a = V_b = V_{in}(t)$

KCL at node a:

$$\frac{V_a}{1} + \frac{V_a - V_{out}}{4} = 0$$
$$4V_a + V_a = V_{out}$$
$$5V_a = V_{out}(t)$$
$$\therefore 5V_{in}(t) = V_{out}(t)$$

Negative feedback is desirable so that ideal op amp principles can be used ($V_b = V_a$)

c) Finding R_{Th}



$V_1 = 0 - V_a = -V_a$

$R_{Th} = \frac{V_b}{1}$

KCL at node a:

$$\frac{V_a}{1} + \frac{V_a}{5} + \frac{V_a - V_b}{2} - 2V_a = 0 \Rightarrow 10V_a + 2V_a + 5V_a - 5V_b - 20V_a = 0$$

$$-3V_a - 5V_b = 0 \quad (1)$$

KCL at node b:

$$\frac{V_b - V_a}{2} + \frac{V_b}{1} = -2V_a + 1 \Rightarrow V_b - V_a + 2V_b = -4V_a + 2$$

$$3V_a + 3V_b = 2 \quad (2)$$

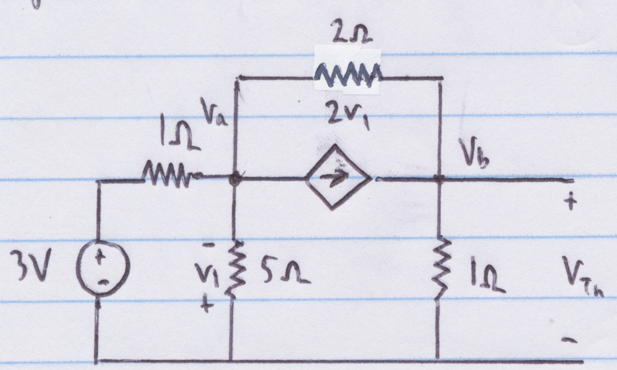
(1) + (2)

$$-2V_b = 2$$

$$\therefore V_b = -1$$

$$\therefore R_{Th} = -1\Omega$$

Finding V_{Th} (There are less nodes than there is meshes)



$V_b = V_{Th}$

KCL at node a:

$$\frac{V_a - 3}{1} + \frac{V_a - V_b}{2} - 2V_a + \frac{V_a}{5} = 0$$

$$-3V_a - 5V_b = 30 \quad (1)$$

KCL at node b:

$$\frac{V_b - V_a}{2} + \frac{V_b}{1} = -2V_a$$

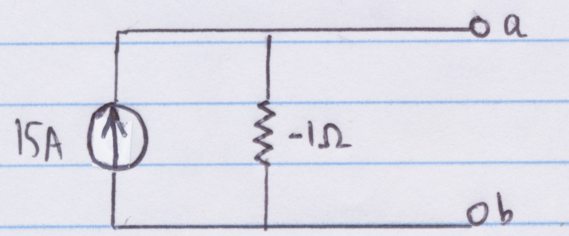
$$3V_a + 3V_b = 0 \quad (2)$$

$$(1) + (2)$$

$$-2V_b = 30$$

$$V_b = -15V = V_{Th}$$

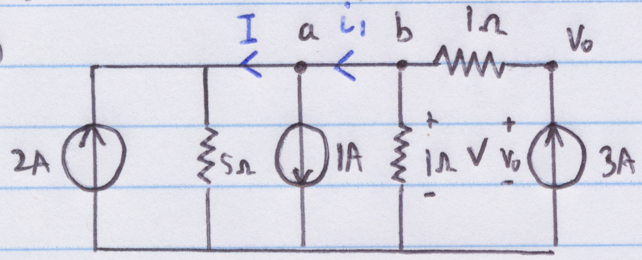
$$\therefore \quad I_N = \frac{V_{Th}}{R_{Th}} = 15A$$



Norton equilvaent circuit

Question 5

a)



KCL at node a:

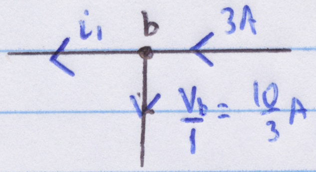
$$\frac{V}{1} + \frac{V}{5} + 1 = 2 + 3$$

$$5V + V + 5 = 25$$

$$6V = 20$$

$$\therefore V = \frac{10}{3} V$$

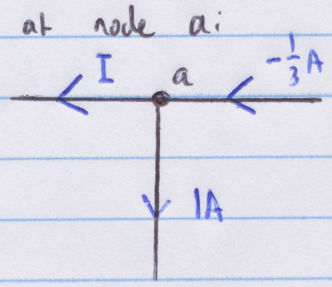
KCL at node b:



$$i_1 + \frac{10}{3} = 3$$

$$i_1 = -\frac{1}{3}$$

KCL at node a:



$$I + 1 = -\frac{1}{3}$$

$$\therefore I = -\frac{4}{3} A$$

b) Voltage across 1Ω resistor

$$\frac{V_0 - V_b}{1} = 3$$

$$V_0 = 3 + \frac{10}{3} = \frac{19}{3} V$$

$$P_{2A} = -\left(\frac{10}{3} \times 2\right) = -\frac{20}{3} W = \frac{20}{3} W \text{ supplied}$$

$$P_{1A} = \frac{10}{3} \times 1 = \frac{10}{3} W \text{ absorbed}$$

$$P_{3A} = -\left(\frac{19}{3} \times 3\right) = -19W = 19W \text{ supplied}$$

c)

$$f(A,B,C) = \overline{\overline{\overline{\overline{A.B} \cdot \overline{B.C}}} \cdot \overline{A.C}} \Rightarrow \text{Remove two lines}$$

$$= \overline{\overline{A.B} \cdot \overline{B.C} \cdot \overline{A.C}} \Rightarrow \text{Demorgan's theorem}$$

$$= \overline{\overline{A.B}} + \overline{\overline{B.C}} + \overline{\overline{A.C}} \Rightarrow \text{Remove two lines}$$

$$= A.B + B.C + A.C$$

d)

A	B	C	A.B	B.C	A.C	f(A,B,C)
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	1	0	1
1	0	0	0	0	0	0
1	0	1	0	0	1	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1

e)