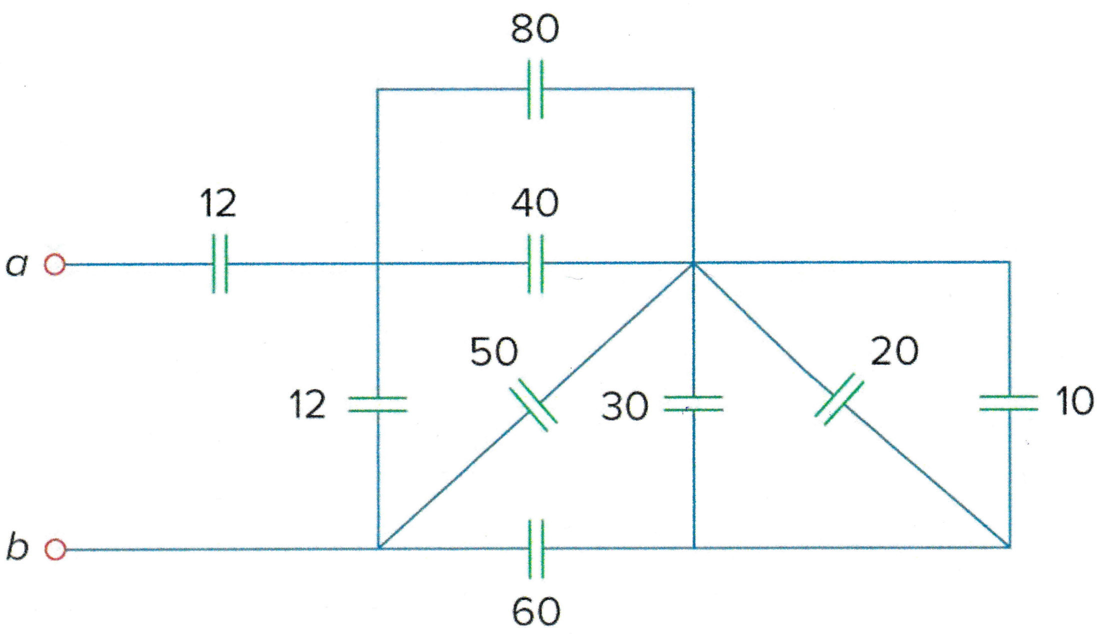
1. (**Final Exam – S2, 2015**) The circuit below shows a network of supercapacitors in a hybrid energy storage system where all capacitances are in μF

a) Find the equivalent capacitance at terminals a-b.

b) What is the energy stored if the voltage of terminals a-b is equal to 100 V.



Answer:

a) $C_{\text{eq}} = 10\ \mu\text{F}.$

b) $w = 50\ \text{mJ}$

Hint: since nothing is mentioned about initial conditions, they are assumed to be zero.

Solution: From right-hand-side $10 \parallel 20 \parallel 30 \rightarrow 10+20+30=60\ \mu F$

also from top $80 \parallel 40 \rightarrow 80+40=120\ \mu F$

a) Series: $\frac{60 \times 60}{60+60} = 30\ \mu F$

Series, parallel: $\frac{120 \times 80}{200} = 48\ \mu F$

Series, parallel:

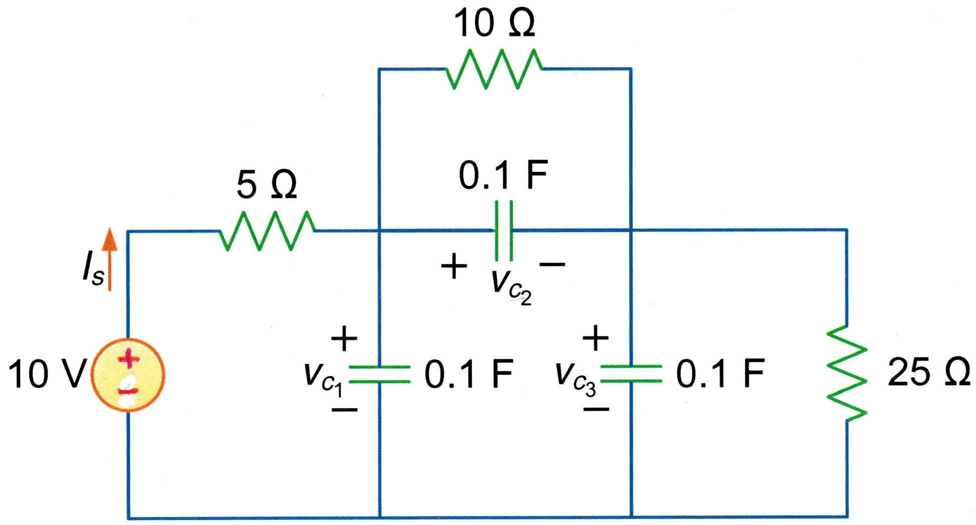
$C_{eq} = \frac{12 \times 60}{72} = \boxed{10\ \mu F}$

b) $W(t) = \frac{1}{2} C V_c^2(t) \xrightarrow[\text{Constant}]{V=100} W = \frac{1}{2} \times 10 \times 10^{-6} \times 100^2 = 0.05\ J$ or $\boxed{50\ mJ}$

2. (**Final Exam – S2, 2015**) For the circuit below,

a) Determine the current I_s after the circuit has reached steady state.

b) Determine the capacitor voltages V_{C_1}, V_{C_2} and V_{C_3} after the circuit has reached steady state.



Answer:

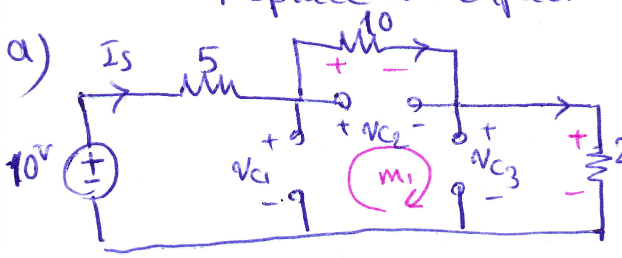
a) $I_s = 0.25\ \text{A}$

b) $V_{C_1} = 8.75\ \text{V}$, $V_{C_2} = 2.5\ \text{V}$, $V_{C_3} = 6.25\ \text{V}$

Hint: The steady state means that the capacitors are fully charged and under DC conditions.

Solution: Replace all capacitors with open circuit.

a)



KVL:

$\hookrightarrow I_s = \dfrac{10}{5+10+25} = \dfrac{10}{40} = \boxed{0.25A}$

b) V_{C_3}: It is parallel with 25 Ω $\longrightarrow V_{C_3} = 25 \times I_s = \boxed{6.25V}$ (I)

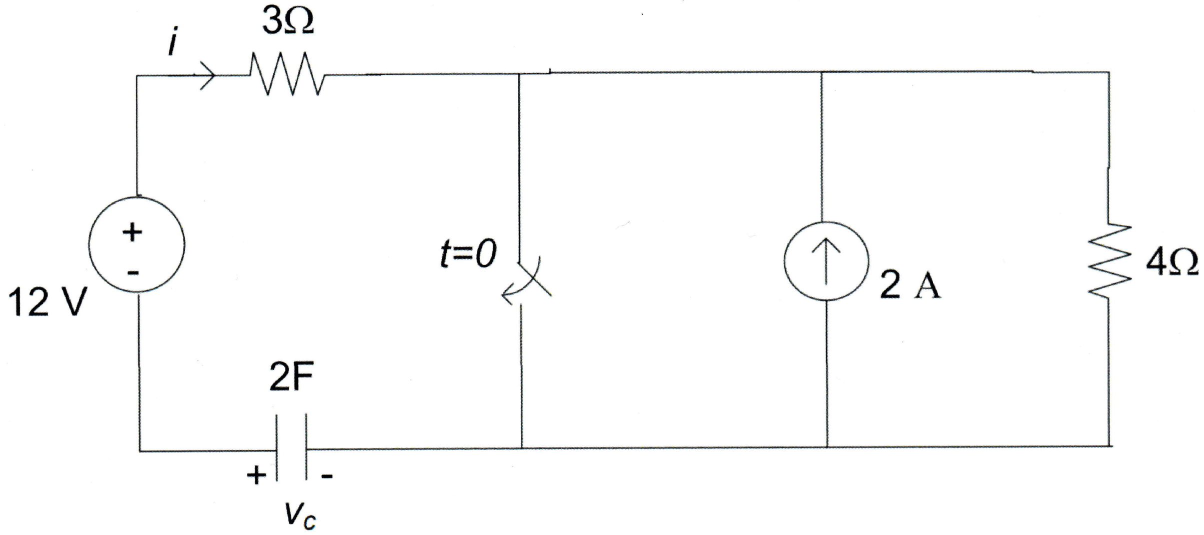
V_{C_2}: It is parallel with 10 Ω $\longrightarrow V_{C_2} = 10 \times I_s = \boxed{2.5V}$ (II)

V_{C_1}: write KVL in m1 $\longrightarrow -V_{C_1} + V_{C_2} + V_{C_3} = 0 \xrightarrow{\text{(I)}}_{\text{(II)}} V_{C_1} = \boxed{8.75V}$

3. (**Final Exam – S2, 2015**) For the circuit below, the switch has been in open position for a long time before changing position as shown at time $t = 0$.

a) Find the voltage v_c and the current i when $t < 0$, i.e., $v_c(0^-)$ and $i(0^-)$.

b) Find v_c and i when $t > 0$ and in steady state condition, i.e., $v_c(+\infty)$ and $i(+\infty)$.



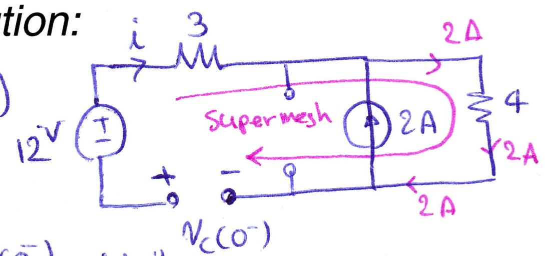
Answer:

a) $v_c(0^-) = -4\ \mathrm{V}$ and $i(0^-) = 0\ \mathrm{A}$

b) $v_c(+\infty) = -12\ \mathrm{V}$ and $i(+\infty) = 0\ \mathrm{A}$

Solution:

a)



$t = 0^-$ means just before zero when the switch is still open

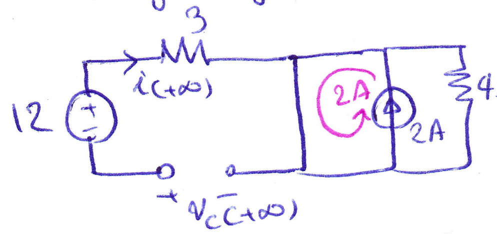
$\boxed{i(0^-) = 0}$ open circuit

$V_c(0^-)$: Write KVL in supermesh

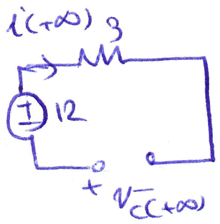
$-12 + 3\times0 + 4\times2 - V_c(0^-) = 0 \longrightarrow \boxed{V_c(0^-) = -4V}$

Note that 2A current source sents its current only through 4Ω resistor

b) $t > 0$ Switch is closed for a long time $t \to \infty$, thus capacitor is again fully charged with new voltage



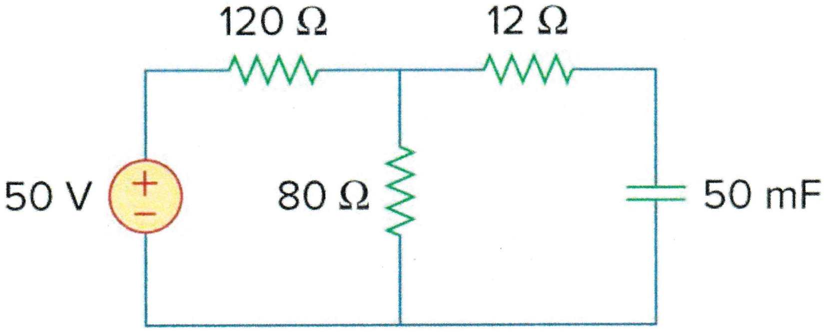
short circuit bypasses the right side



$\boxed{i(+\infty) = 0}$ again open circuit

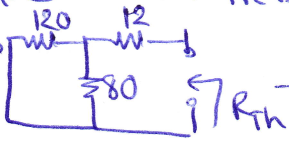
$\boxed{V_c(+\infty) = -12V}$

4. Find the time constant for the RC circuit given below.



Answer: $\tau = 3\ \mathrm{s}$

Solution: Time Constant $\tau = R_{Th} \cdot C$ where R_{Th} is the equivalent / thevenin resistance seen from Capacitor $\rightarrow$



$\rightarrow R_{Th} = (120 \parallel 80) + 12 = 48 + 12 = 60\,\Omega$

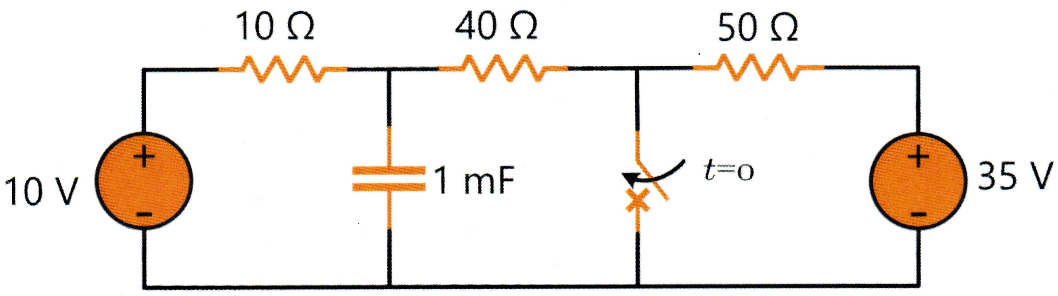
$\Rightarrow \tau = 60 \times 50 \times 10^{-3} = 3\ \mathrm{s}$

5. (**Mid-semester Exam - Summer 2017**) In the circuit below, the switch has been in the open position for a long time before closing at time $t = 0$.

a) Give an expression for the capacitor voltage $v_c(t)$ (i.e., as a function of time) for $t > 0$.

b) Give an expression for the capacitor current $i_c(t)$ for $t > 0$

c) Give an expression for the current of the 40-Ω resistor $i_{R40}(t)$ for $t > 0$.



Answer:

a) $v_c(t) = 8 + 4.5e^{\frac{-t}{8\times10^{-3}}}$ V

b) $i_{c(t)} = -\frac{4.5}{8} e^{\frac{-t}{8\times10^{-3}}} = -0.5625e^{\frac{-t}{8\times10^{-3}}}$ A

c) $i_{R40}(t) = \frac{8}{40} + \frac{4.5}{40} e^{\frac{-t}{8\times10^{-3}}} = 0.2 + 0.1125e^{\frac{-t}{8\times10^{-3}}}$ A

Solution:

$V_C(t) = V_C(\infty) + (V(0) - V(\infty))\, e^{-t/\tau}$

$V(0)$: initial voltage $t<0$

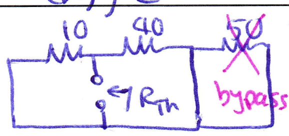
$V(\infty)$: final voltage at $t \rightarrow \infty$

τ: time constant

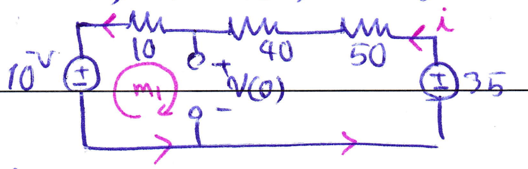
First find τ:

a) $\tau = R_{Th} \cdot C \rightarrow R_{Th}$ after the switch is closed

$\rightarrow \tau = 8 \times 1\times10^{-3}\ \mathrm{s}$



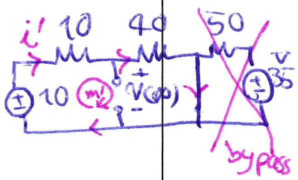
Then, find $V(0)$ when switch is open and capacitor is fully charged



Find i via KVL in loop : $i = \dfrac{35-10}{10+40+50} = \boxed{0.25A}$ (I)

*Now find $V(0)$ via KVL in m_1: $+V(0) - 10i - 10 = 0 \xrightarrow{\text{I}} \boxed{V(0) = 12.5V}$

*Close the switch and find $V(\infty)$ as capacitor is now fully charged with new voltage

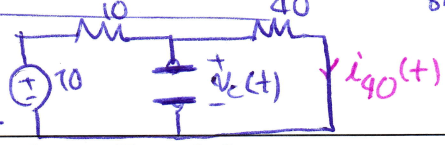


$i' = \dfrac{10}{10+40} = \boxed{0.2A}$ (II) $\rightarrow$ KVL in m_1' $-10 + 10i' + V(\infty) = 0 \xrightarrow{\text{II}} \boxed{V(\infty) = 8V}$

$\Longrightarrow V_c(t) = 8 + (12.5-8)e^{-t/\tau} = 8 + 4.5e^{-t/8\times10^{-3}} V \quad t>0$

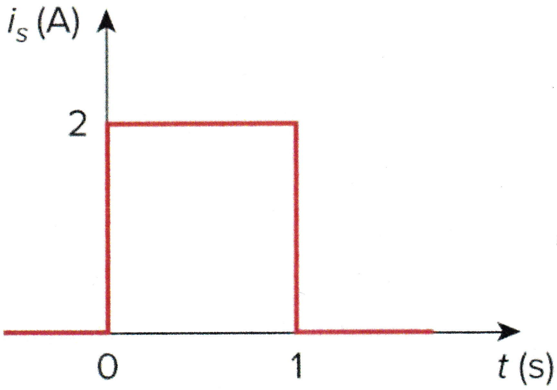
b) $i_c = C\dfrac{dV_c}{dt} \rightarrow i_c = 10^{-3} \times \dfrac{-1}{8\times10^{-3}} \times 4.5e^{-t/8\times10^{-3}} = -\dfrac{4.5}{8}e^{-t/8\times10^{-3}} A \quad t>0$

c) After switch is closed $\rightarrow$

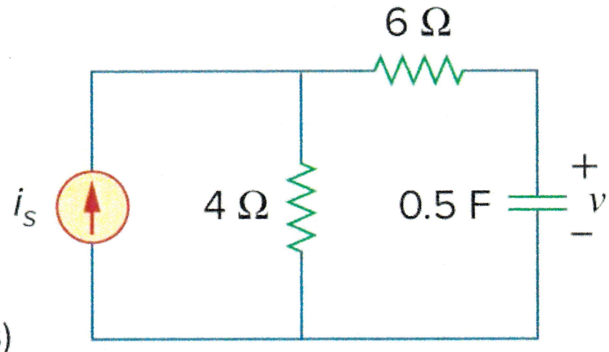


40Ω is parallel with capacitor $\rightarrow i_{40} = \dfrac{V_c(t)}{40} = \dfrac{8}{40} + \dfrac{4.5}{40}e^{-t/8\times10^{-3}} V \quad t>0$

6. If the waveform given in Fig. (a) is applied to the circuit of Fig. (b), find the expression for $v(t)$ assuming $v(0) = 0$.



(a)

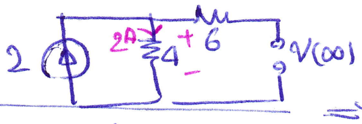


(b)

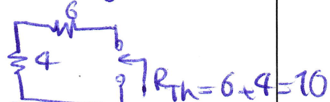
Answer: $v(t) = \begin{cases} 8\left(1 - e^{\frac{-t}{5}}\right) \text{V}, & 0 < t < 1 \\ 1.4502e^{\frac{-(t-1)}{5}} \text{V}, & t \geq 1 \end{cases}$

Solution: $V(t) = V(\infty) + (V(0) - V(\infty))e^{-t/\tau}$ Note that here we have a pulse as input, which means that for $0<t<1$ $V(t)$ will charge up to a voltage according to its time constant, initial voltage, and a final voltage values (if $i_s = 2$ was permanent). Then, it starts discharging its energy through resistors starting from initial voltage at $\boxed{t=1s}$

for $0<t<1$: $V(0) = 0 \longrightarrow V(\infty)$ assuming it would fully charge under $i_s = 2A$

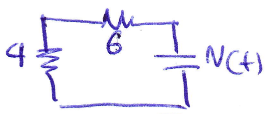


$V(\infty) = V_4 = 2\times4 = 8V$. Also



$\Longrightarrow V(t) = 8 + (0-8)e^{-t/\tau} = 8(1-e^{-t/5}) \quad 0<t<1$ $\quad \tau = 10\times0.5 = 5s$

for $t>1$:



$\boxed{\tau = 5s}$ the same

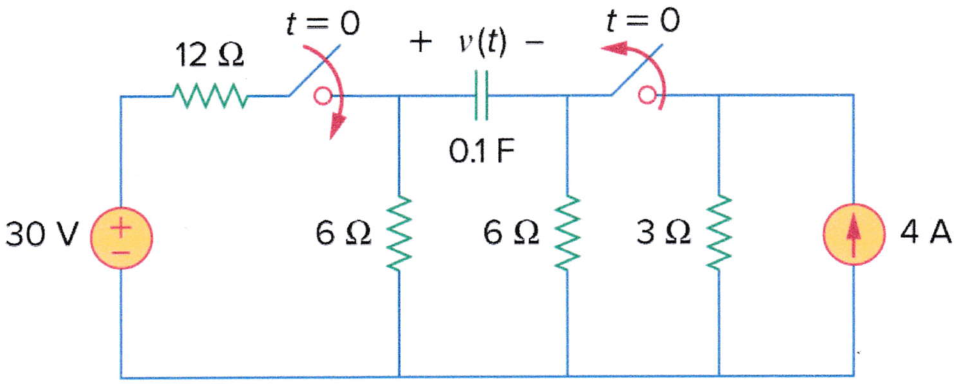
$V(0) \triangleq V(1) = 8(1-e^{-1/5}) = 1.4502$ intial voltage after current switch to zero

$V(\infty) = 0$ ~~fully discharged~~

$\Rightarrow V(t) = 0 + (1.45 - 0)e^{-t/5}$ But This is only valid for $t>1$

Thus: $V(t) = \begin{cases} 8(1-e^{-t/5}) V & 0<t<1 \\ 1.45e^{\frac{-(t-1)}{5}} V & t>1 \end{cases}$ Apply time shift because $V(1) = 1.45$ from second function.

7. In the circuit below, determine the response $v(t)$ for all time (i.e., for both $t < 0$ and $t > 0$), and sketch $v(t)$ waveform showing all critical points in the sketch.



(a)

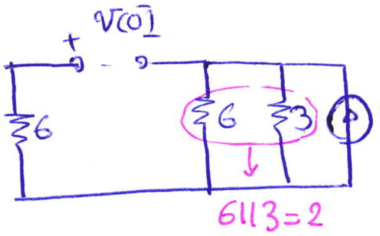
Answer: $v(t) = \begin{cases} -8 & t \leq 0 \\ 10 - 18e^{-t} & t \geq 0 \end{cases}$

Solution:

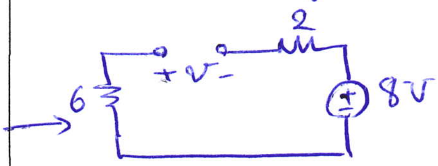
Before changes $t<0$

Switch on the left: opened

Switch on the right: closed



Source transformation



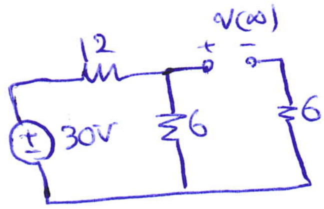
$V(0^-) + 8 = 0 \rightarrow \boxed{V(0^-) = -8\,V} = V(0^+) = V(0)$

KVL ↺

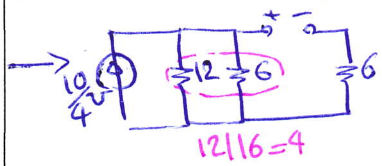
After the changes

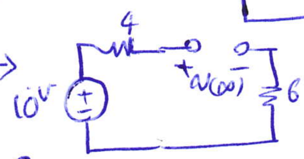
$t>0$ Switch on the left: closed

switch on the right: opened



Source transformation



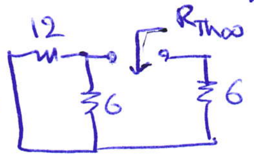


$\Rightarrow \boxed{V(\infty) = +10\,V}$

KVL ↺

Time Constant

$\tau = R_{Th_\infty} C \longrightarrow$



$\longrightarrow \boxed{R_{Th_\infty} = 6 + (6 \| 12) = 6 + 4 = 10\,\Omega}$

$\Rightarrow \boxed{\tau = 10 \times 0.1 = 1\,s}$

$\Rightarrow V(t) = V(\infty) + (V(0) - V(\infty))e^{-t/\tau} \quad t>0 \longrightarrow V(t) = 10 + (-8-10)e^{-t} = 10 - 18e^{-t} \quad t>0$

for all time $V(t) = \begin{cases} -8\,V & t<0 \\ 10 - 18e^{-t} & t \geq 0 \end{cases}$ or $v(t) = -8u(-t) + [10 - 18e^{-t}]u(t)$