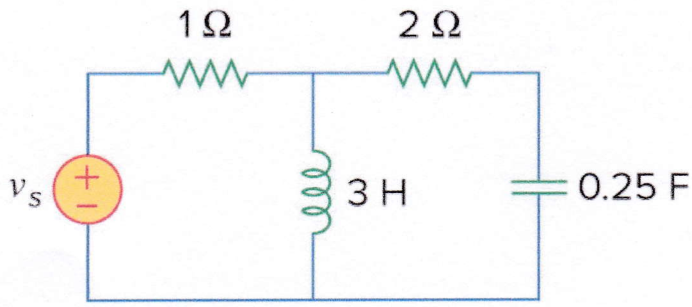
1. Assuming that $v_s = 8\cos(2t - 40°)$ V in the circuit below, find the average power delivered to each of the passive elements and supplied by the voltage source. Then, verify the conservation of energy principle for average power.



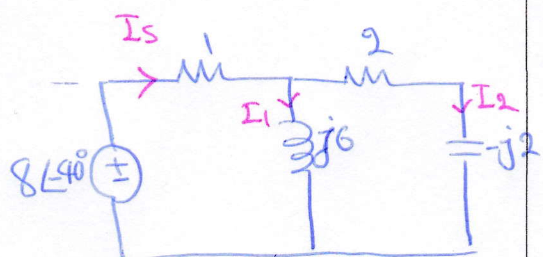
Answer: $P_{1\Omega} = 1.416$ W, $P_{2\Omega} = 5.097$ W, $P_{3H} = P_{0.25F} = 0$ W,
$P_{v_s} = 6.513$ W

Solution: Draw the circuit in phasor domain

$V_s = 8\cos(2t - 40°)^V \longrightarrow \boxed{8\angle -40 V}$

$\omega = 2 \frac{rad}{s} \longrightarrow 3H \longrightarrow j\omega L = \boxed{j6\,\Omega}$

$0.25F \longrightarrow -\frac{j}{\omega C} = \boxed{-j2\,\Omega}$

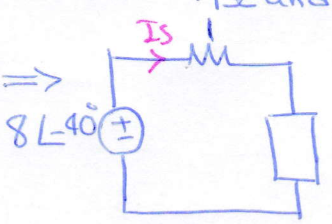


First of all, recall that inductor and capacitor do NOT absorb average power

$P_{3H} = P_{0.25F} = 0\,W$

To find $P_{1\Omega}$ and $P_{2\Omega}$, we need the current through them.

$\Rightarrow$



$j6 \| (2-j2) = \frac{j6 \times (2-j2)}{j6+2-j2} = 3.6 - j1.2\,\Omega$

$\Rightarrow I_S = \frac{8\angle -40°}{1 + 3.6 - j1.2} = \boxed{1.682\angle -25.38° A} \Rightarrow P_{1\Omega} = \frac{1}{2} R |I|^2 = \frac{1}{2} \times 1 \times (1.68)^2$

$\Rightarrow P_{1\Omega} = 1.416\,W$ absorbed

absorbed

To find the current through 2-Ω resistor, use current division:

$I_2 = \frac{j6}{j6+2-j2} \times I_S = \boxed{2.258\angle 0° A} \Rightarrow P_{2\Omega} = \frac{1}{2} \times 2 \times (2.258)^2 = 5.097\,W$

supplied

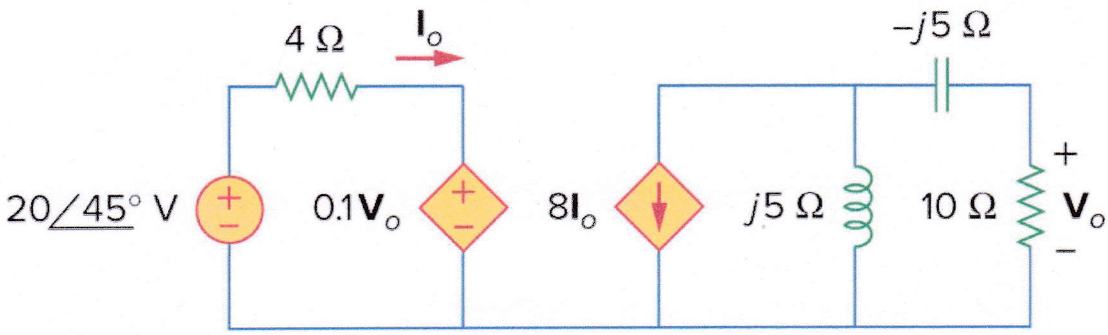
for Voltage Source $\rightarrow P_{V_S} = \frac{1}{2} V_{m_S} I_{m_S} \cos(\theta_V - \theta_i) = 6.513\,W \Rightarrow \boxed{P_{1\Omega} + P_{2\Omega} + P_{3H} + P_{0.25F} = P_{V_S} \rightarrow \sum P = 0}$

$= 0.5 \times 8 \times 1.682 \cos(-40 + 25.38)$

$\Rightarrow 1.416 + 5.097 = 6.513 = 6.513$

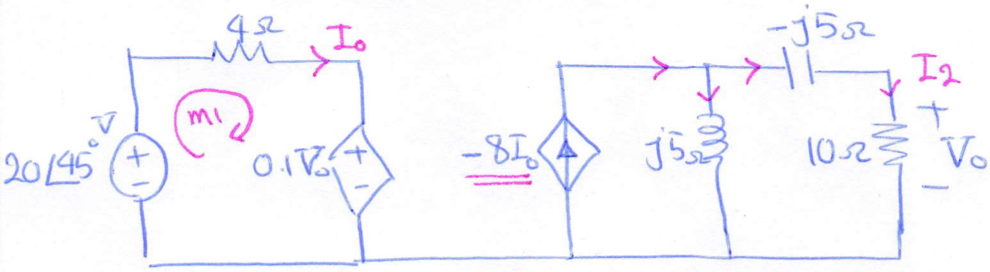
passive sign convention for active elements.

2. In the circuit given below, determine the average power absorbed by the $10\text{-}\Omega$ resistor.



Answer: $P_{10\Omega} = 1\ \mathrm{kW}$

Solution: We can redraw the circuit as follows



We can change the direction of the dependent current source and compensate that change by changing the sign of its value.

$\ast$ KVL in m1: $-20\angle 45^\circ + 4I_o + 0.1V_o = 0 \Longrightarrow \boxed{V_o = 200\angle 45^\circ - 40I_o}$ (I)

$\ast$ Current division in right hand side $\longrightarrow I_2 = \dfrac{j5}{10 - j5 + j5} \times (-8I_o) = \boxed{-j4I_o}$

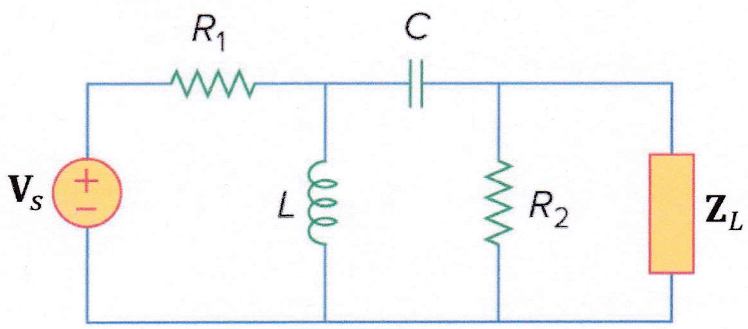
Now $V_o = 10I_2 = -j40I_o \longrightarrow \boxed{I_o = \dfrac{V_o}{-j40}}$ (II)

Substitute (II) into (I) $\longrightarrow V_o = 200\angle 45^\circ + \dfrac{jV_o}{j} \Longrightarrow (1+j)V_o = 200\angle 45^\circ$

$\Longrightarrow V_o = \dfrac{200\angle 45^\circ}{1+j} = \boxed{\dfrac{200}{\sqrt{2}}\angle 0^\circ\,\mathrm{V}}$

$\Longrightarrow P_{10\Omega} = \frac{1}{2}\dfrac{|V|^2}{R} = \frac{1}{2} \times \dfrac{4\times 10^4}{2\times 10} = 10^3\,\mathrm{W} = \boxed{1\,\mathrm{kW}}$

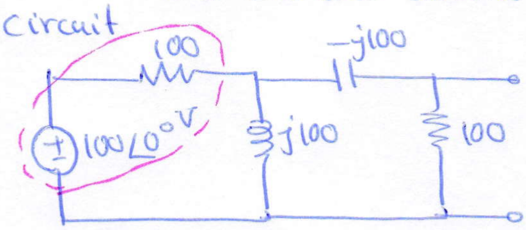
3. Determine the value of the load impedance $\mathbf{Z}_L$ such that the circuit can deliver maximum average power transfer if $R_1 = R_2 = 100\ \Omega$, $\mathbf{Z}_C = -j100\ \Omega$, $\mathbf{Z}_L = j100\ \Omega$, and $\mathbf{V}_s = 100\angle 0°\ \text{V}$. Then find the maximum average power absorbed by $\mathbf{Z}_L$



Answer: $\mathbf{Z}_L = \mathbf{Z}_{\text{Th}}^* = 40 + j20\ \Omega$, $P_{\max} = \dfrac{|\mathbf{V}_{\text{Th}}|^2}{8R_{\text{Th}}} = 6.25\ \text{W}$

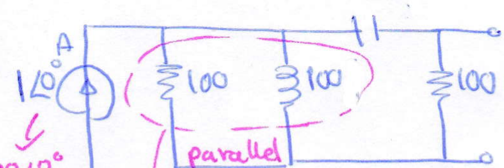
Hint: You need to find Thevenin equivalent circuit at the load terminals and use the maximum average power transfer formula given in the Answer

Solution: We can use source transformation to find the Thevenin equivalent circuit

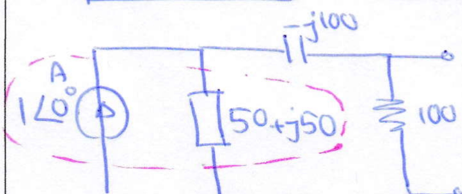


S.T →

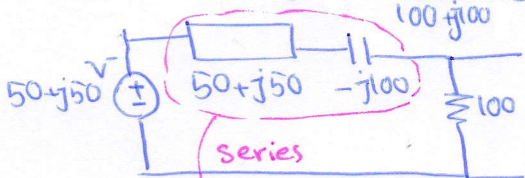
$\dfrac{100\angle 0°}{100}$



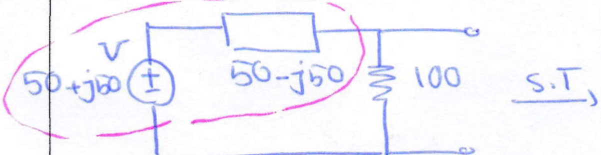
$100 \| j100 = \dfrac{100 \times j100}{100 + j100} = 50 + j50\ \Omega$



S.T →

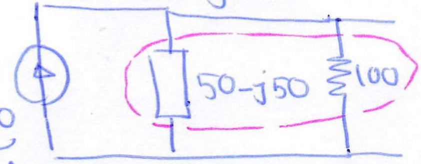


$50 - j50\ \Omega$

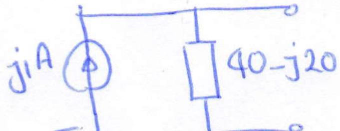


S.T →

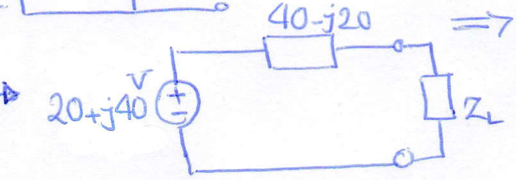
$\dfrac{50+j50}{50-j50} = j1$ A



$100 \| (50 - j50) = \dfrac{100 \times (50 - j50)}{150 - j50} = 40 - j20\ \Omega$



S.T →



$\Rightarrow Z_{Th} = 40 - j20\ \Omega$

$V_{Th} = 20 + j40\ V = 44.72\angle 63.43 V$ $\Rightarrow Z_{Th} = R_{Th} + jX_{Th}$

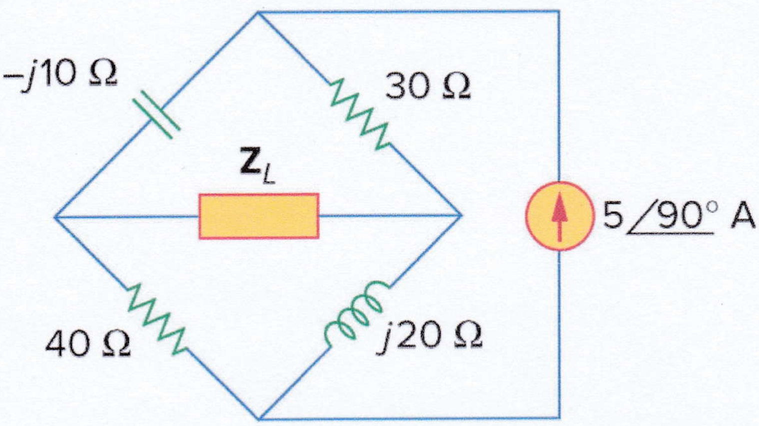
$R_{Th} = 40\ \Omega$

$X_{Th} = -20$

For maximum average power transfer $Z_L = Z_{Th}^*$

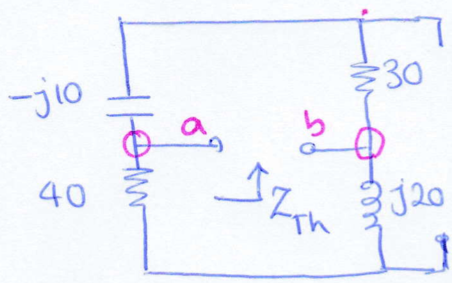
$Z_L = 40 + j20\ \Omega$ and $P_{max} = \dfrac{|V_{Th}|^2}{8 R_{Th}} = \dfrac{(44.72)^2}{8 \times 40} = 6.25 W$

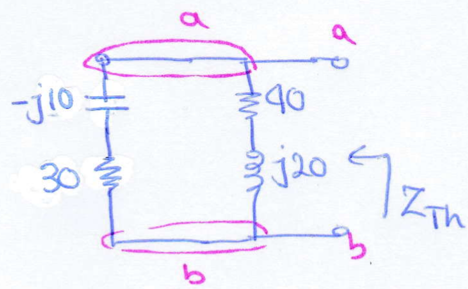
4. For the circuits shown below, find the value of load impedance $\mathbf{Z}_L$ for maximum average power transfer and then calculate the maximum average power absorbed by it.



Answer: $\mathbf{Z}_L = \mathbf{Z}_{Th}^* = 20\ \Omega$, $P_{max} = \dfrac{|\mathbf{V}_{Th}|^2}{8R_{Th}} = 31.25\ \text{W}$

Solution: Find Z_{Th} by setting the current source to zero

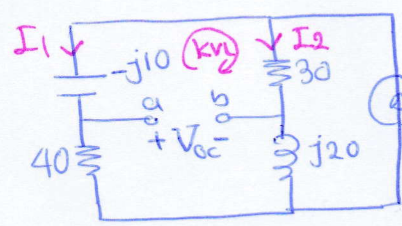




$$Z_{Th} = (30 - j10) \parallel (40 + j20) = \frac{(30-j10)(40+j20)}{30-j10+40+j20} = 20\ \Omega \Rightarrow R_{Th} = 20\Omega,\ X_{Th} = 0\ \Omega$$

$\Rightarrow Z_L = Z_{Th}^* = 20\Omega$ Pure resistive equivalent impedance

For V_{Th}



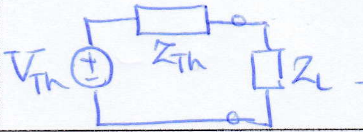
$5\angle 90^\circ A = j5\ A$

$\xrightarrow[\text{division}]{\text{Current}}$ $I_1 = \dfrac{30+j20}{70+j10} \times j5 = -1.1 + j2.3\ A$

$I_2 = \dfrac{40-j10}{70+j10} \times j5 = 1.1 + j2.7\ A$

*KVL

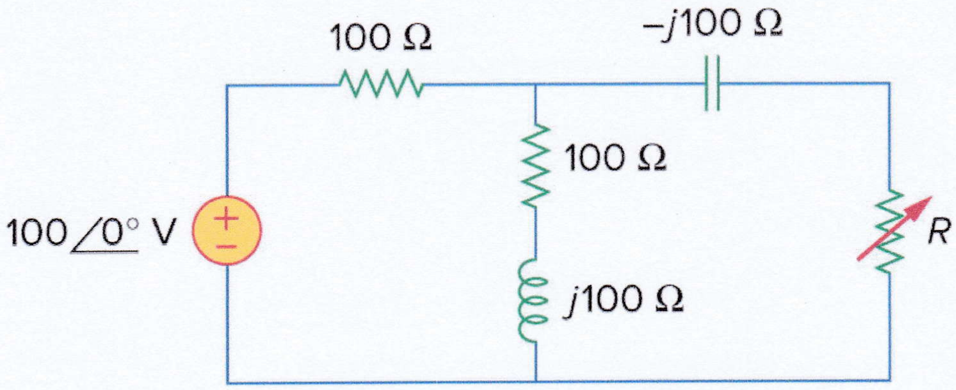
$\Rightarrow V_{oc} = V_{Th} = 30 I_2 + j10 I_1 = \boxed{10 + j70\ V = 70.71\angle 81.87^\circ V}$



$\rightarrow P_{max} = \dfrac{|V_{Th}|^2}{8R_{Th}} = \dfrac{(70.71)^2}{8 \times 20} = 31.25\ \text{W}$

$Z_{Th} = 20\Omega = R_{Th}$
$V_{Th} = 70.71\angle 81.87^\circ V$

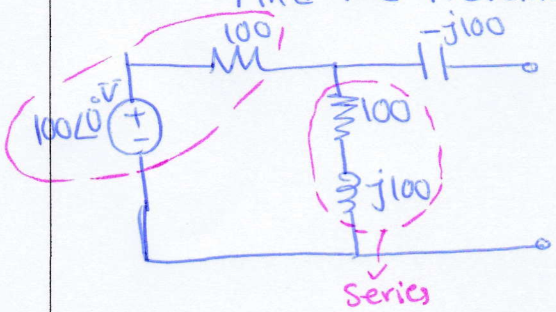
5. If the load is pure resistive R_L, what should be the value of R_L that it will receive the maximum average power from the circuit. Then calculate the power absorbed by R_L.



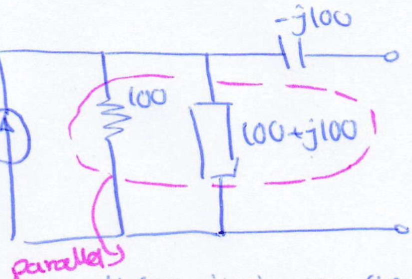
Answer: $R_L = |\mathbf{Z}_{\text{Th}}| = 100\ \Omega$, $P_{\text{max}} = \frac{1}{2} R_L |\mathbf{I}_L|^2 = 6.25\ \text{W}$

Hint: For a pure resistive load, the condition for maximum average power transfer is $R_L = |\mathbf{Z}_{\text{Th}}|$, Then, simply find the average power absorbed by R_L as the maximum average power.

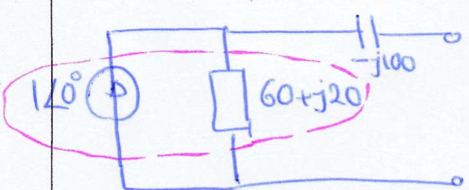
Solution: Find the Thevenin equivalent circuit using source transformation



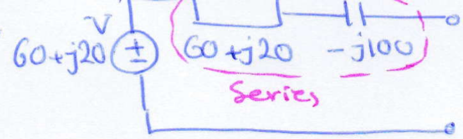
S.T. → $1\angle 0°\ A = \frac{100\angle 0°}{100}$

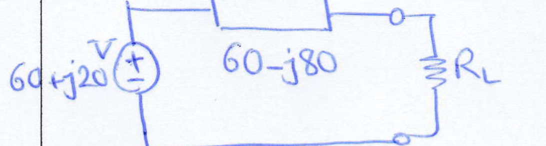


$100 \,\|\, (100+j100) = \frac{100\times(100+j100)}{200+j100} = 60+j20\ \Omega$



S.T. →





⇒ $Z_{Th} = 60 - j80\ \Omega \rightarrow |Z_{Th}| = 100$

$V_{Th} = 60 + j20\ V = 63.24\angle 18.43°\ V$

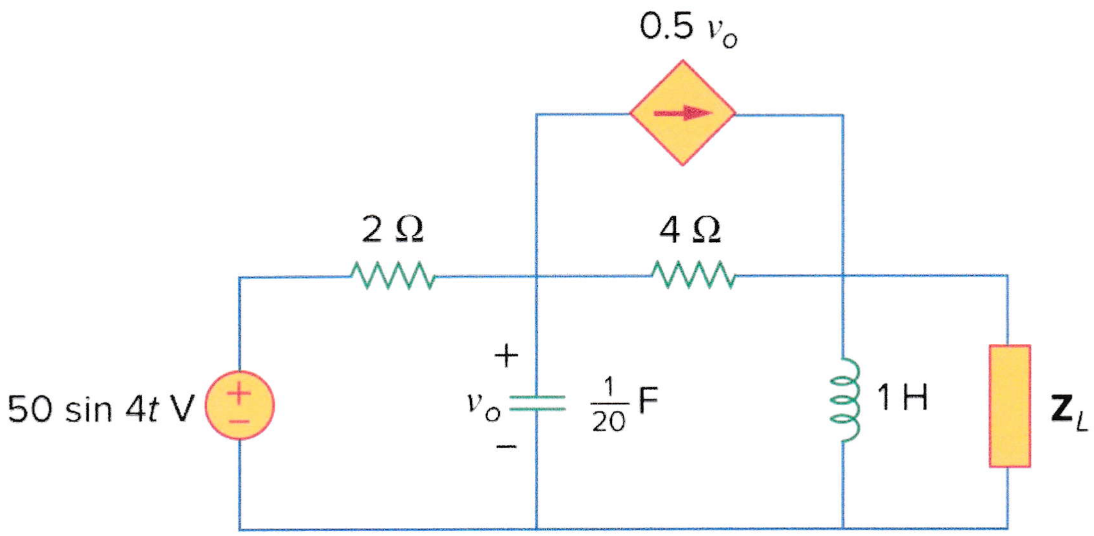
For maximum average power transfer when load is pure resistive

$R_L = |Z_{Th}| = 100\ \Omega$ → $P_{max} = \frac{1}{2} R_L |I_L|^2$ → $I_L = \frac{V_{Th}}{Z_{Th}+R_L} = \frac{60+j20}{60-j80+100}$

$I_L = 0.353\angle 45°\ A$ ⇒ $P_{max} = \frac{1}{2} \times 100 \times (0.353)^2$

$= 6.25\ W$

6. For the circuit shown below,
   a) Find the Thevenin equivalent circuit as seen from the terminals of the load impedance $\mathbf{Z}_L$, and draw the equivalent circuit.
   b) Determine the value of the load $\mathbf{Z}_L$ for maximum average power transfer.
   c) Calculate the maximum average power that can be delivered to the load $\mathbf{Z}_L$ from this circuit.



Answer:

a) $\mathbf{V}_{\text{Th}} = 59.43\angle(-33.69°)\text{ V} = 49.45 - j32.96\text{ V}$,

$\mathbf{Z}_{\text{Th}} = 1.67 + j3.648\ \Omega = 4.0125\angle 65.4°\ \Omega$

b) $\mathbf{Z}_L = \mathbf{Z}^*_{\text{Th}} = 1.67 - j3.648\ \Omega = 4.0125\angle(-65.4°)\ \Omega$

c) $P_{\max} = \frac{|\mathbf{V}_{\text{Th}}|^2}{8R_{\text{Th}}} = 264.34\text{ W}$

Solution: Draw the circuit in phasor domain

$v_s = 50 \sin 4t \Rightarrow 50 \cos(4t - 90°)$

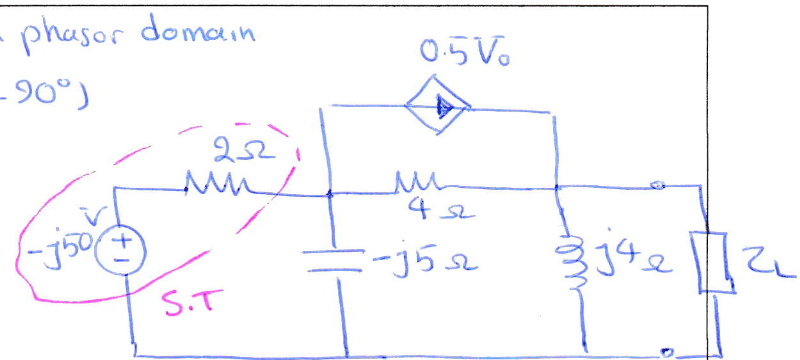
$\rightarrow \boxed{V_s = 50\angle{-90°}\,V = -j50\,V}$

$\omega = 4\ \frac{rad}{s}$

$\frac{1}{20}F \rightarrow \frac{-j}{\omega C} = -j5\,\Omega$

$1H \rightarrow j\omega L = j4\,\Omega$

$\longrightarrow$

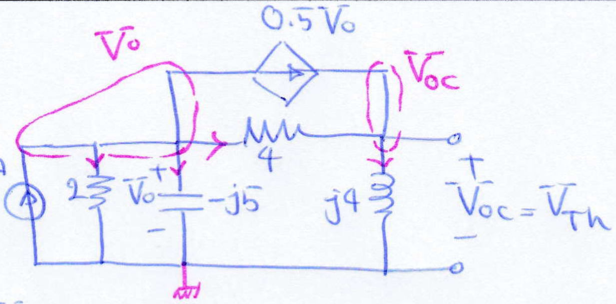


We can use source transformation and Nodal analysis to find V_{Th} first. Then we connect an external source to find Z_{Th}.

⟶ next page ⟶

*KCL at V_o:

$$-j25 = \frac{V_o}{2} + \frac{V_o}{-j5} + \frac{V_o - V_{oc}}{4} + 0.5V_o$$



$$\xrightarrow{\times 20} \quad -j500 = 10V_o + j4V_o + 5V_o - 5V_{oc} + 10V_o$$

$$\Rightarrow \boxed{(25+j4)V_o - 5V_{oc} = -j500} \quad \text{(I)}$$

* KCL @ V_{oc}:

$$0.5V_o + \frac{V_o - V_{oc}}{4} = \frac{V_{oc}}{j4} \xrightarrow{\times 4} 2V_o + V_o - V_{oc} = -jV_{oc}$$

$$\Rightarrow \boxed{3V_o = (1-j)V_{oc}} \longrightarrow \boxed{V_o = \frac{(1-j)V_{oc}}{3}} \quad \text{(II)}$$

$$\xrightarrow[\text{(II) into (I)}]{\text{Substitute}} \quad \frac{(25+j4)(1-j)}{3}V_{oc} - 5V_{oc} = -j500 \xrightarrow{\times 3}$$

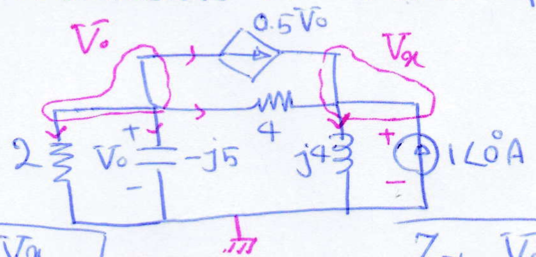
$$V_{oc}\left[(25+j4)(1-j) - 15\right] = -j1500 \Rightarrow V_{oc} = V_{Th} = \frac{-j1500}{(25+j4)(1-j)-15}$$

$$\boxed{V_{Th} = 59.43\angle -33.69^\circ\ V = 49.45 - j32.96\ V}$$

For Z_{Th}, Connect an external current source and turn off the independent source.

KCL@V_o:

$$\frac{V_o}{2} + \frac{V_o}{-j5} + \frac{V_o - V_x}{4} + 0.5V_o = 0$$



$$\boxed{(25+j4)V_o - 5V_x = 0} \longrightarrow \boxed{V_o = \frac{5V_x}{25+j4}} \quad \text{(I)}$$

$$\boxed{Z_{Th} = \frac{V_x}{1\angle 0^\circ}}$$

* KCL@V_x:

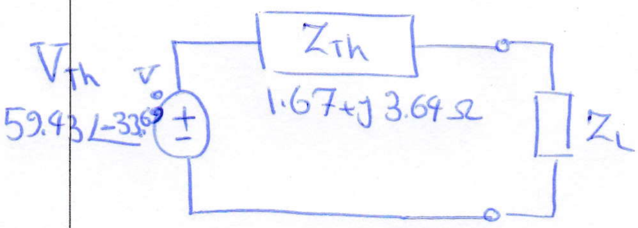
$$1\angle 0^\circ + 0.5V_o + \frac{V_o - V_x}{4} = \frac{V_x}{j4} \xrightarrow{\times 4} 4 + 2V_o + V_o - V_x = -jV_x$$

$$\boxed{3V_o + 4 = (1-j)V_x} \quad \text{(II)} \quad \xrightarrow[\text{(I) to (II)}]{\text{substitute}} \quad \frac{15V_x}{25+j4} + 4 = (1-j)V_x$$

$$\Rightarrow \Big[\underbrace{\frac{15}{25+j4} - (1-j)}_{-0.415+j0.906}\Big]V_x = -4 \Rightarrow V_x = \frac{-4}{-0.415+j0.906} = \boxed{\begin{aligned}&1.67 + j3.648\ V \\ &= 4.0125\angle 65.4^\circ\end{aligned}}$$

⟶ nextpage ⟶

Therefore $Z_{Th} = \dfrac{V_x}{1\angle 0^\circ} = 1.67 + j3.648\ \Omega = 4.0125\angle 65.4^\circ\ \Omega$



b) For maximum average power transfer

$Z_L = Z_{Th}^{*} = 1.67 - j3.648\ \Omega = 4.0125\angle -65.4^\circ\ \Omega$

$P_{max} = \dfrac{|V_{Th}|^2}{8R_{Th}}$ $\qquad Z_{Th} = 1.67 + j3.648\ \Omega \rightarrow \boxed{R_{Th} = 1.67\ \Omega}$

$X_{Th} = 3.64$

$\Rightarrow P_{max} = \dfrac{(59.43)^2}{8 \times 1.67} = 264.34\ W$