QUESTION 1 [25 marks]

(i) [12 marks] For the circuit shown in Figure 1,

a. **(10 marks)** Calculate the equivalent resistance R_{eq} as seen from terminals *a*-*b*.
b. **(2 marks)** Find the current i through the network using the result of part (a).

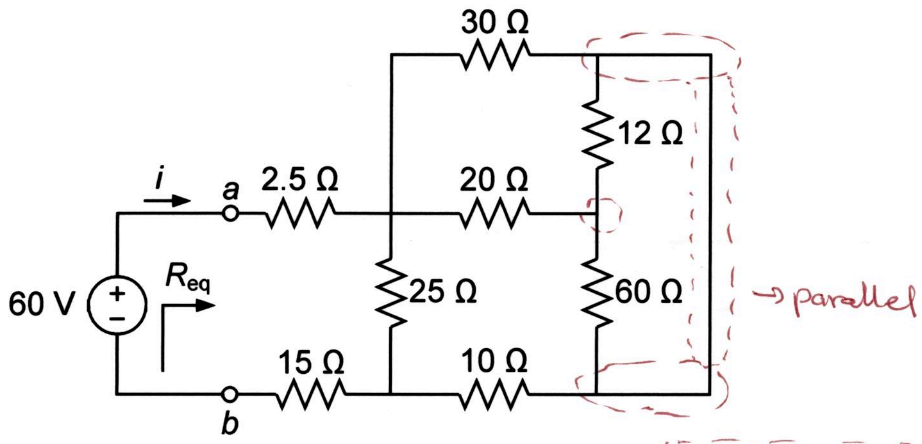
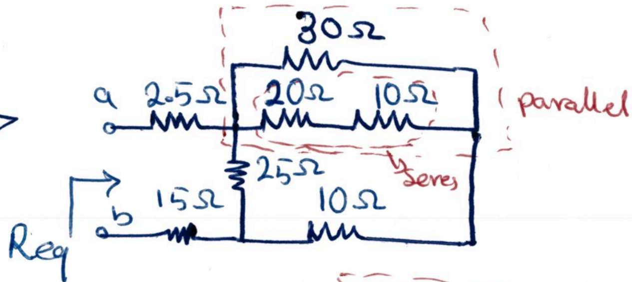
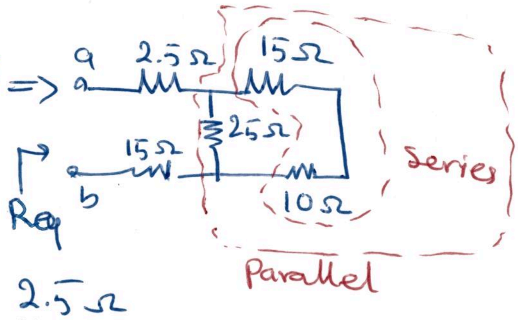


Figure 1

a) $12^{\Omega} \parallel 60^{\Omega} = \dfrac{12 \times 60}{12+60} = 10\Omega \Rightarrow$

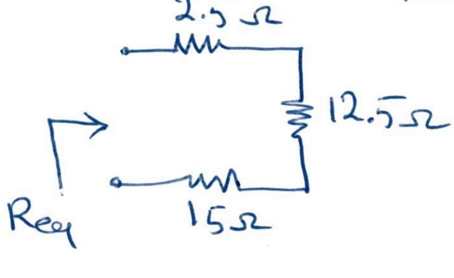


$20+10=30 \rightarrow 30 \parallel 30 = \dfrac{30\times 30}{30+30} = 15\Omega \Rightarrow$

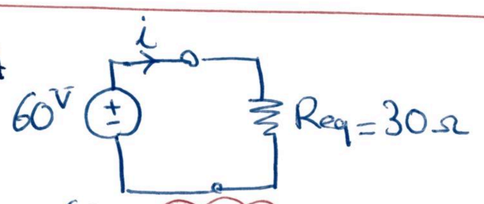


$\Rightarrow \quad 15+10=25 \rightarrow 25 \parallel 25 = \dfrac{25\times 25}{25+25} = 12.5$

$\Rightarrow R_{eq} = 2.5 + 12.5 + 15 = 30\Omega$



b) Equivalent circuit



$i = \dfrac{V}{R} = \dfrac{60}{R_{eq}} = \dfrac{60}{30} = 2A$

QUESTION 1 [25 marks] *Continued*

(ii) [13 marks] For the circuit shown in Figure 2,

a. **(10 marks)** Calculate all the powers absorbed and/or supplied by the elements.
b. **(3 marks)**, Verify the law of conservation of energy.

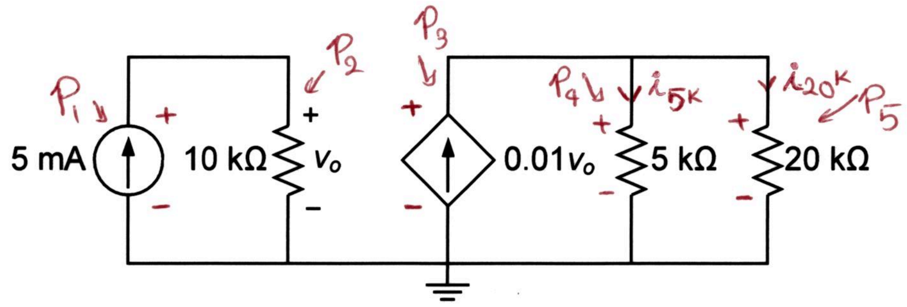


Figure 2

a) $P = v.i = \frac{v^2}{R} = Ri^2$ (passive sign Convention)

$V_o = 10^K \times 5^m = \boxed{50V}$ ← Ohm's Law

$i_{5K} = \frac{20^K}{5^K + 20^K} \times 0.01V_o = \frac{20 \times 0.5}{25} = \boxed{0.4\,A}$ ← Current division

$i_{20^k} = 0.01V_o - i_5 = 0.5 - 0.4 = \boxed{0.1\,A}$ ← KCL

$V_{5K} = V_{20K} = V_{\substack{dependent \\ source}} = 5^K \times 0.4\,A = 2^{KV} = \boxed{2000\,V}$

Now find powers of each element

* 5mA source: $P_1 = V_o 5^{mA} = 50^V \times 5^{mA} = \boxed{\begin{matrix} 250\,mW \\ = 0.25\,W \end{matrix}}$ Supplied (passive sign Convention for active elements)

* 10-kΩ resistor: $P_2 = \frac{V_o^2}{10k} = 10^k \times (5^m)^2 = \boxed{\begin{matrix} 250\,mW \\ = 0.25\,W \end{matrix}}$ absorbed

* $0.01V_o$ Source: $P_3 = V_{5K} \times 0.01V_o = 2000 \times 0.01 \times 50 = \boxed{\begin{matrix} 1000\,W \\ = 1\,KW \end{matrix}}$ Supplied

* 5-KΩ resistor: $P_4 = 5^K \times (i_{5K})^2 = 5^K \times (0.4)^2 = \boxed{800\,W}$ absorbed

* 20-KΩ resistor: $P_5 = 20^k \times (i_{20K})^2 = 20^k \times (0.1)^2 = \boxed{200\,W}$ absorbed

b) Conservation of energy → $\sum P_{supplied} = \sum P_{absorbed}$

$\Rightarrow 0.25^W + 1000W = 0.25^W + 800^W + 200W \Rightarrow 1000.25^W = 1000.25^W$

Confirmed

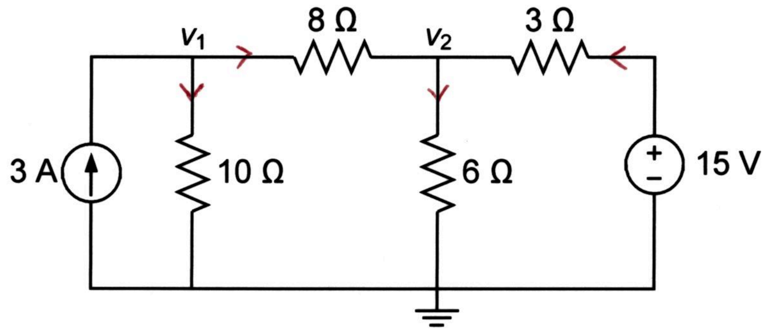
QUESTION 2 [30 marks]

(i) **[14 marks]** For the circuit shown in Figure 3,

a. **(10 marks)** Apply nodal analysis and show that the nodal equations are given as below,

$$\begin{cases} 9v_1 - 5v_2 = 120 \\ v_1 - 5v_2 = -40 \end{cases}$$

b. **(4 marks)** Given the values of node voltages as $v_1 = 20$ V and $v_2 = 12$ V, calculate the total power supplied by the sources.



a)

Figure 3

KCL@V_1: $3 = \frac{V_1}{10} + \frac{V_1 - V_2}{8} \xrightarrow{\times 40} 120 = 4V_1 + 5V_1 - 5V_2$

$\Rightarrow \boxed{9V_1 - 5V_2 = 120}$

KCL@V_2: $\frac{V_1 - V_2}{8} + \frac{15 - V_2}{3} = \frac{V_2}{6} \xrightarrow{\times 24} 3V_1 - 3V_2 + 120 - 8V_2 = 4V_2$

$\Rightarrow 3V_1 - 15V_2 = -120 \xrightarrow{\times 1/3} \boxed{V_1 - 5V_2 = -40}$

b) $P_{3A} = V_1 \times 3 = 20 \times 3 = 60^{W}$ supplied

$P_{15V} = 15 \times (\underbrace{\frac{15 - V_2}{3}}_{\text{current}}) = 5 \times (15 - 12) = 15$ W supplied

$\Rightarrow P_{Total} = 60 + 15 = \boxed{75 W}$ supplied

QUESTION 2 [30 marks] *Continued*

(ii) [16 marks] For the circuit shown in Figure 4,

a. **(12 marks)** Apply mesh analysis and show that the mesh equation are given as below,

$$\begin{cases} i_1 + 3i_2 - 2i_3 = 8 \\ i_1 + i_2 - 4i_3 = -1 \\ i_1 - i_2 = -4 \end{cases}$$

b. **(4 marks)** Given the values of mesh currents as $i_1 = -0.5$ A, $i_2 = 3.5$ A, and $i_3 = 1$ A, find the voltage v across 4-A current source.

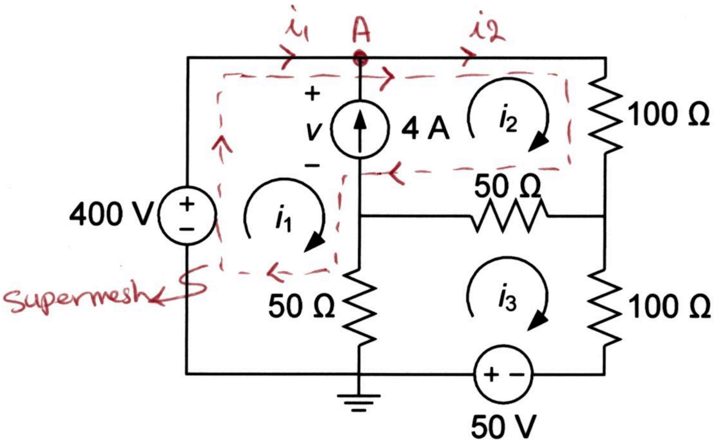


Figure 4

a) * KVL in Supermesh:

(Polarities are chosen based on passive sign convention for each KVL)

(direction of currents in shared brancles are chosen based on the mesh for which KVL is written)

$$-400 + 100 i_2 + 50(i_2 - i_3) + 50(i_1 - i_3) = 0$$

$$\xrightarrow{\times 1/50} -8 + 2i_2 + i_2 - i_3 + i_1 - i_3 = 0$$

$$\Rightarrow \boxed{i_1 + 3i_2 - 2i_3 = 8}$$

* KVL in mesh 3: $-50 + 50(i_3 - i_1) + 50(i_3 - i_2) + 100 i_3 = 0$

$$\xrightarrow{\times 1/50} -1 - i_1 + i_3 - i_2 + i_3 + 2i_3 = 0 \Rightarrow \boxed{i_1 + i_2 - 4i_3 = -1}$$

* KCL @ node A: $i_1 + 4 = i_2 \Rightarrow \boxed{i_1 - i_2 = -4}$

b) * KVL in mesh 1: $v_1 + 50(i_1 - i_3) - 400 = 0$

$$\xrightarrow[i_3 = 1A]{i_1 = -0.5A} v = 50(1 + 0.5) + 400 = 475\ V$$

QUESTION 3 [25 marks]

(i) **[15 marks]** In the circuit of Figure 5,

a. **(12 marks)** Use only **source transformation** to reduce the circuit into a single resistor in series with a single voltage source as seen from terminals *a-b*, and then determine Thevenin voltage V_{Th} and Thevenin resistance R_{Th} at the terminals *a-b*.

b. **(3 marks)** Find the value of load resistance R_L for maximum power transfer, and then calculate the maximum power that can be delivered to R_L.

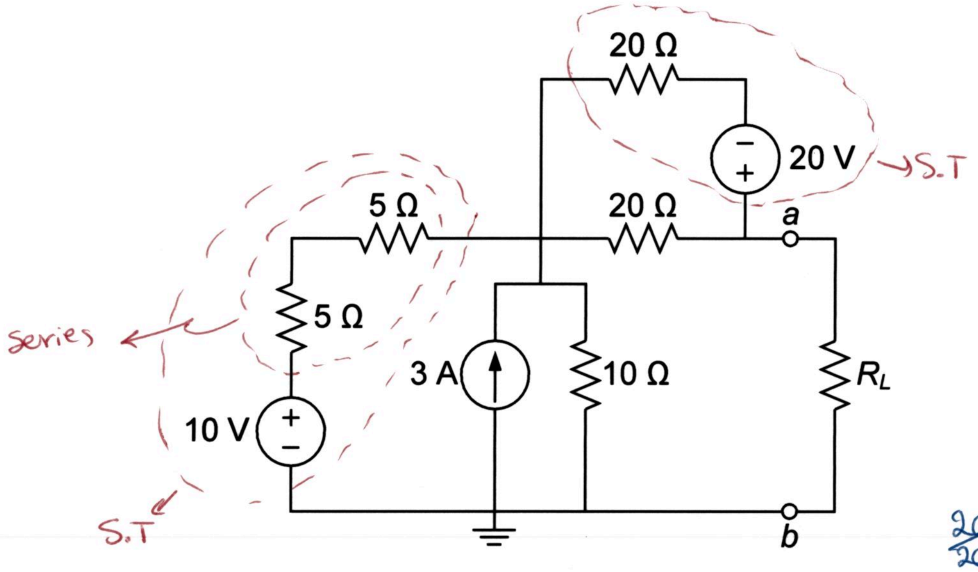


Figure 5

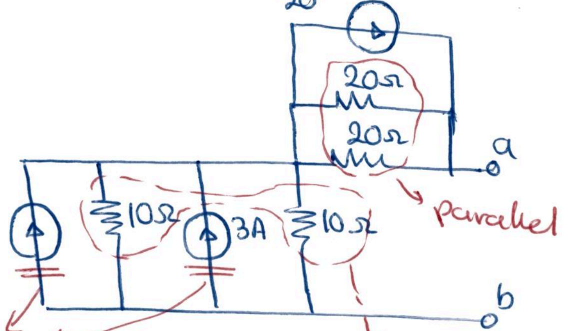
a) 2 source transformations (S.T.s)

$5+5=10\Omega$

$i_S = \frac{V_S}{R}$ ⟹

$\frac{10}{10} = 1A$

$\frac{20}{20} = 1A$

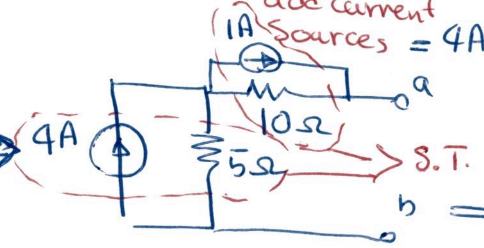


$10 \| 10 = \frac{10\times 10}{10+10} = 5\Omega$

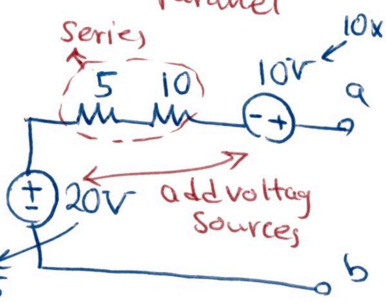
$20 \| 20 = \frac{20\times 20}{20+20} = 10\Omega$

$1A + 3A = 4A$

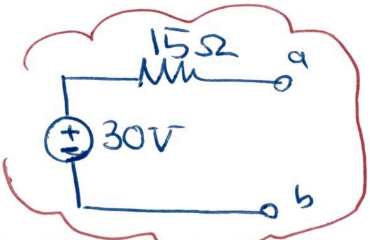
⟹



⟹



⟹



⟹ $V_{Th} = 30V$

$R_{Th} = 15\Omega$

b) $R_L = R_{Th} = 15\Omega$ for max power transfer

$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{30^2}{4\times 15} = 15W$

QUESTION 3 [25 marks] *Continued*

(ii) [10 marks] For the circuit shown in Figure 6, obtain the Norton equivalent circuit as seen from terminal *a-b* and draw the Norton equivalent circuit.

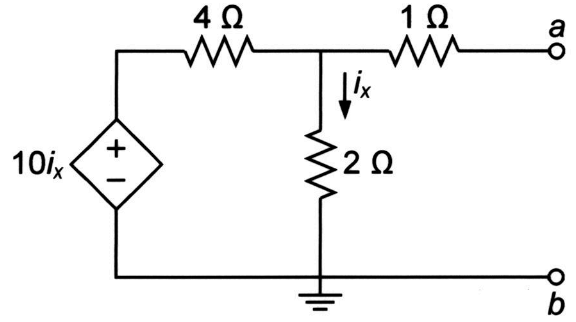
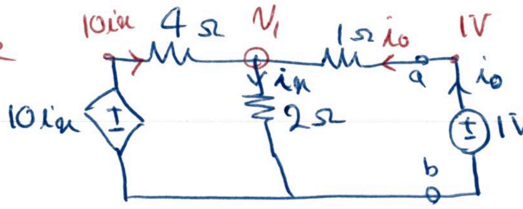


Figure 6

$\boxed{I_N = 0\,A}$ → No independent source

for $R_N = R_{eq}$, attach an external source

Method 1: Voltage Source



$\boxed{R_N = \frac{1}{i_o}\,\Omega}$

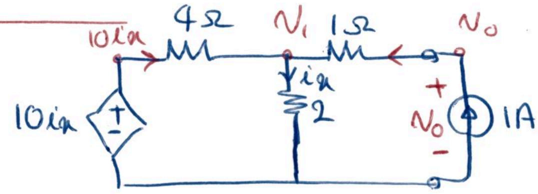
Nodal analysis:

find V_1 then find i_o to find R_N
$$\begin{cases} \dfrac{10i_x - V_1}{4} + i_o = \dfrac{V_1}{2} \\ \underbrace{i_o = \dfrac{1 - V_1}{1}}_{\mathrm{I}}, \quad i_x = \dfrac{V_1}{2} \end{cases} \Longrightarrow \frac{5V_1 - V_1}{4} + 1 - V_1 = \frac{V_1}{2}$$

$$\xrightarrow{\times 1/4} \cancel{4V_1} + 4 - \cancel{4V_1} = 2V_1 \Longrightarrow \boxed{V_1 = 2\,V} \xrightarrow{\mathrm{I}} \boxed{i_o = \frac{1-2}{1} = -1\,A}$$

$$\Rightarrow R_N = -1\,\Omega = \frac{1}{-1}$$

Method 2: Current Source



$\boxed{R_N = \frac{V_o}{1}\,\Omega}$

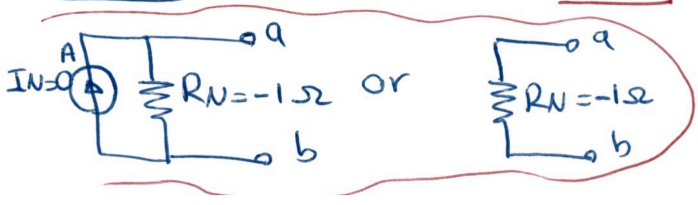
Nodal analysis:

find V_1 then find V_o to find R_N
$$\begin{cases} \dfrac{10i_x - V_1}{4} + 1 = \dfrac{V_1}{2} \\ i_x = \dfrac{V_1}{2}, \quad \underbrace{\dfrac{V_o - V_1}{1} = 1}_{\mathrm{I}} \end{cases} \Longrightarrow \frac{5V_1 - V_1}{4} + 1 = \frac{V_1}{2} \xrightarrow{\times 4} 4V_1 + 4 = 2V_1$$

$$= \boxed{V_1 = -2\,V} \xrightarrow{\mathrm{I}} V_o = 1 + V_1 = \boxed{-1\,V}$$

$$\Rightarrow R_N = -1\,\Omega = \frac{-1}{1}$$

* Norton equivalent circuit



QUESTION 4 [20 marks]

(i) [8 marks] In circuit shown in Figure 7, the switch has been in position *A* for a long time. At $t = 0$, the switch moves to position *B*.

a. **(4 marks)** Find the voltage $v_C(t)$ across the capacitor immediately after the switch changes to position *B*, $v_C(0^+)$, and its final voltage when $t \to \infty$, $v_C(\infty)$.
b. **(3 marks)** Derive an expression for the capacitor voltage $v_C(t)$ for $t > 0$.
c. **(1 marks)** Find the current $i(t)$ through 3-kΩ resistor for $t > 0$.

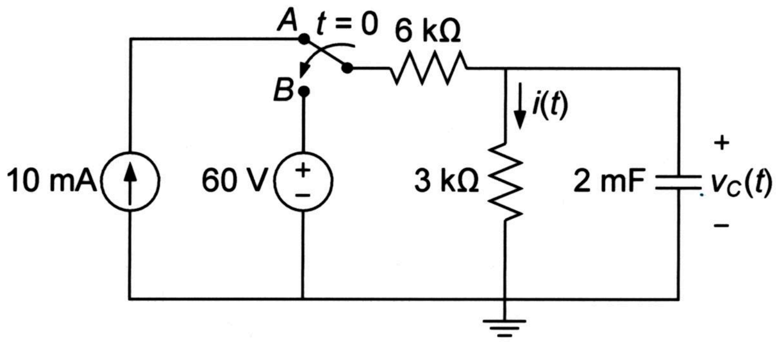
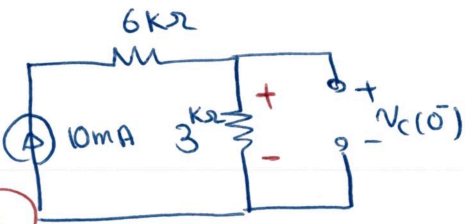


Figure 7

a) * for $t<0$

Switch is at A



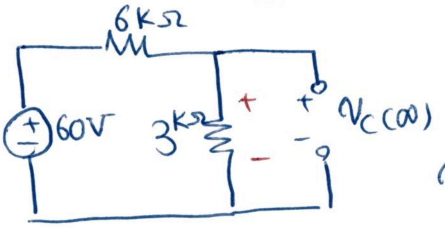
and $V_C(0^+) = V_C(0^-)$ No sudden change in $V_C(t)$

$$\Longrightarrow V_C(0^-) = V_{3K\Omega} = 3^K \times 10^m = 30V = V_C(0^+)$$

* for $t>0$

switch is at B $\longrightarrow$

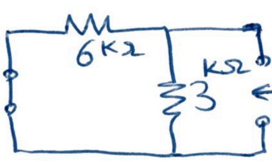
$t \to \infty$



$$V_C(\infty) = \frac{3^K}{6^K + 3^K} \times 60^V = 20V$$

b) $V_C(t) = V_C(\infty) + [V_C(0^+) - V_C(\infty)]e^{-t/\tau}$ $t>0$

$\tau = R_{Th_\infty} C$ $\xrightarrow{t>0}$



$$R_{Th\infty} = 3^K \| 6^K = \frac{3^K \times 6^K}{3^K + 6^K} = 2K\Omega$$

$$\Rightarrow \tau = 2^{K\Omega} \times 2^{mF} = 4s$$

$$\Rightarrow V_C(t) = 20 + [30 - 20]e^{-t/4} V = 20 + 10e^{-t/4} V \quad t>0$$

c) $i(t) = \frac{V_{3K}}{3K\Omega} = \frac{V_C(t)}{3K\Omega} \Longrightarrow i(t) = \frac{20}{3} + \frac{10}{3}e^{-t/4}$ mA $t>0$

3KΩ is parallel with capacitor

QUESTION 4 [20 marks] *Continued*

(ii) [12 marks] In the circuit of Figure 8, the switch has been closed for a long time before being opened at $t = 0$.

a. **(10 marks)** Derive an expression for the inductor current $i_L(t)$ for all time (i.e., for both $t < 0$ and $t > 0$) and sketch $i_L(t)$ as a function of time showing all critical points in the sketch.

b. **(2 marks)** Find the total energy stored or released by the inductor for $t \geq 0$, i.e., from $t = 0$ to $t \to \infty$.

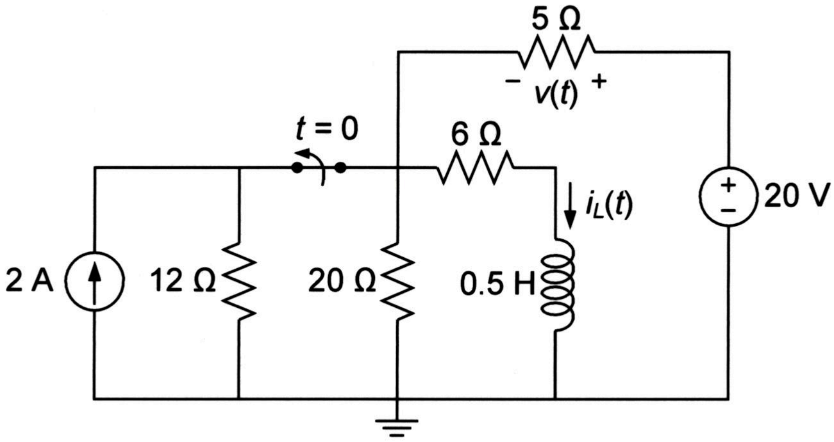
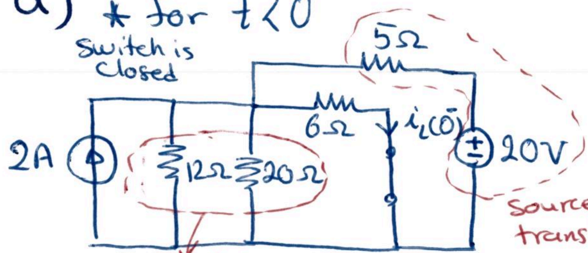


Figure 8

a) * for $t<0$

Switch is closed



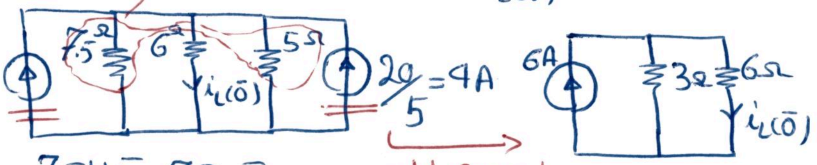
$$\begin{cases} i_L(t) = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau} \\ \tau = L/R_{Th\infty} \text{ and } i_L(0^+) = i_L(0^-) \end{cases}$$

no sudden change in $i_L(t)$

parallel

source transformation

$12 \| 20 = \frac{12\times 20}{12+20} = 7.5\Omega$



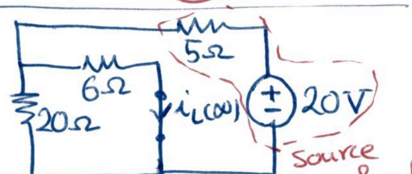
$7.5 \| 5 = \frac{7.5\times 5}{7.5+5} = 3\Omega$

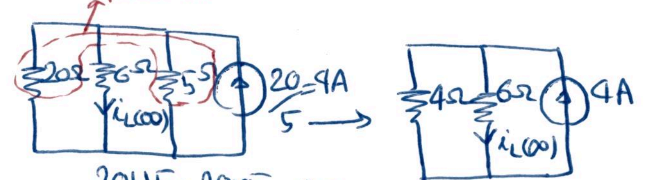
add current sources

$\Rightarrow$ Current division $\to i_L(0^-) = \frac{3}{3+6}\times 6A = 2A$

* for $t>0$

switch is open

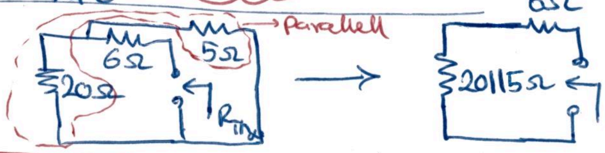




$20 \| 5 = \frac{20\times 5}{20+5} = 4\Omega$

$\Rightarrow$ Current division $\to i_L(\infty) = \frac{4}{4+6}\times 4 = 1.6A$

* $\tau = L/R_{Th\infty}$ $\xrightarrow{t>0}$



$R_{Th\infty} = 6 + 5\|20 = 6 + \frac{5\times 20}{5+20} = 10\Omega$

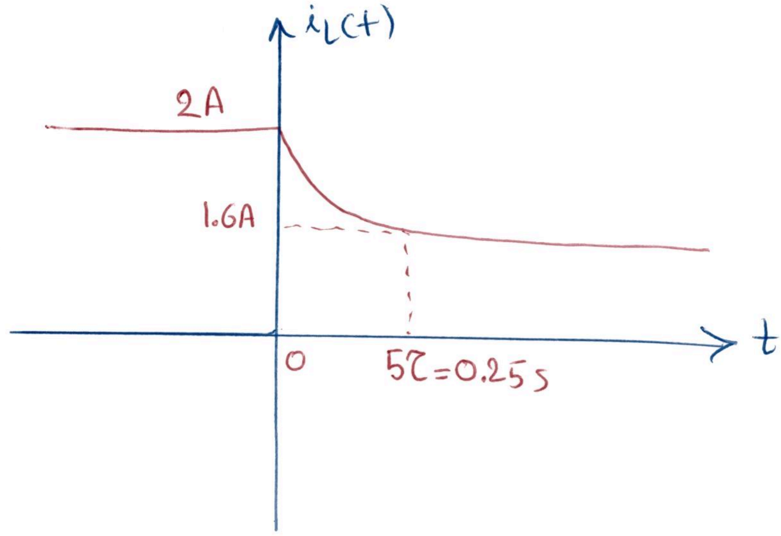
$\Rightarrow \tau = \frac{0.5H}{10\Omega} = \boxed{1/20\,S = 50ms}$

$\Rightarrow i_L(t) = 1.6 + (2-1.6)e^{-t/(1/20)} A = \boxed{1.6 + 0.4e^{-20t} A}$ $t>0$

$$\Rightarrow i_L(t) = \begin{cases} 2\,A & t<0 \\ 1.6 + 0.4e^{-20t}\,A & t\geq 0 \end{cases}$$

→ next page

Part (ii)-a Continued



b) $W_\infty = \frac{1}{2} L\, i_L^2(\infty)$ and $W_0 = \frac{1}{2} L\, i_L^2(0)$ also $W = \int_{-\infty}^{t} p(\tau)\, d\tau = \int_0^\infty L\, i_L\, di_L$

$\Rightarrow W_T = \frac{1}{2} L \left(i_L^2(\infty) - i_L^2(0) \right)$ $\quad = \frac{1}{2} L \left(i_L^2(\infty) - i_L^2(0) \right)$

$= \frac{1}{2} \times 0.5 \left(1.6^2 - 2^2 \right) = -0.36\ J$ $\quad$ assuming $i_L(-\infty) = i_L(0)$

Thus, the inductor releases 0.36 J from $t=0$ to $t \to \infty$