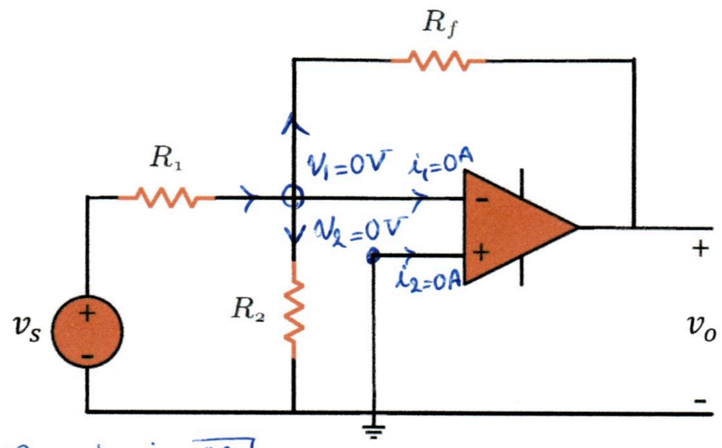
School of Electrical Engineering & Telecommunications

ELEC1111

Topic 6: Operational Amplifiers

1. Find the voltage gain $\frac{v_o}{v_s}$ in the following operational amplifier circuit.



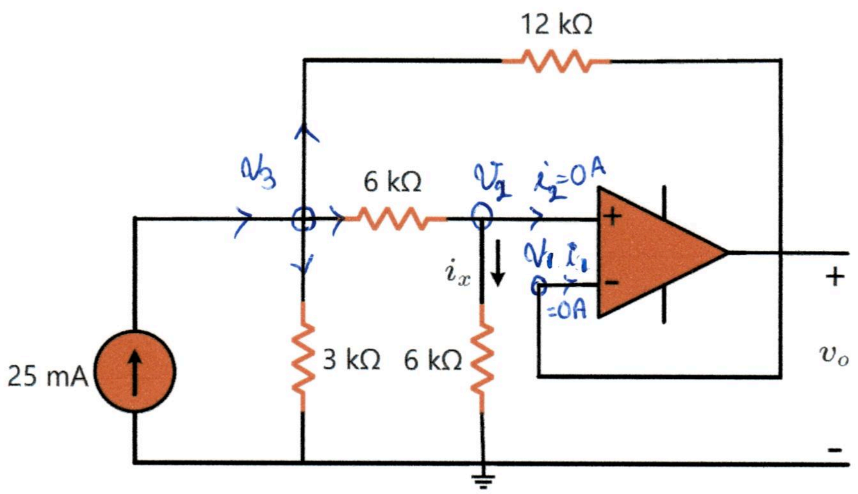
Ideal Op Amp → $i_1 = i_2 = \boxed{0A}$
V_2 is grounded → $V_1 = V_2 = \boxed{0V}$

Answer: $\frac{v_o}{v_s} = \frac{-R_f}{R1}$

Solution: KCL @ V_1: $\frac{V_S - V_1}{R_1} = \frac{V_1}{R_2} + \frac{V_1 - V_0}{R_f} \xrightarrow{V_1 = 0} \frac{V_S}{R_1} = -\frac{V_0}{R_f} \Longrightarrow \frac{V_0}{V_S} = -\frac{R_f}{R_1}$

Another form of inverting Op Amp.

2. Find the current i_x in the following operational amplifier circuit.



Answer: $i_x = 4.545$ mA,

Solution: Ideal opAmp $\rightarrow i_1 = i_2 = 0A$, $V_1 = V_0 \Rightarrow \boxed{V_2 = V_1 = V_0}$

KCL @ V_2: $\frac{V_2}{6K} = \frac{V_3 - V_2}{6K} \xrightarrow[\times 6K]{V_2 = V_0} V_0 = V_3 - V_0 \longrightarrow \boxed{V_3 = 2V_0}$

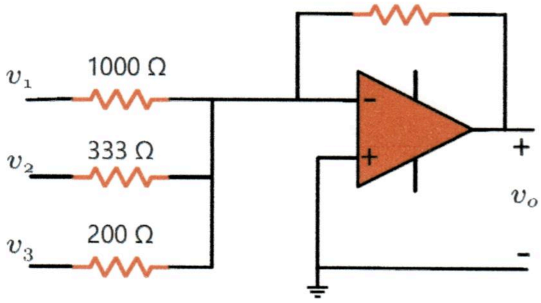
KCL @ V_3: $25mA = \frac{V_3}{3K} + \frac{V_3 - V_2}{6K} + \frac{V_3 - V_0}{12K} \xrightarrow[\times 12]{V_2 = V_0} 300 = 4V_3 + 2V_3 - 2V_0 + V_3 - V_0$

$\longrightarrow \boxed{7V_3 - 3V_0 = 300}$

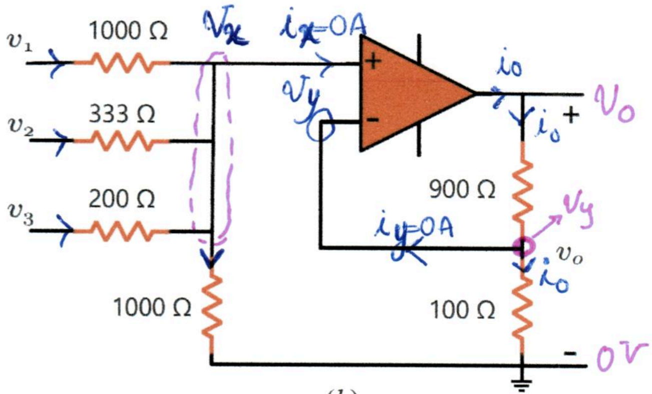
$\Rightarrow \begin{cases} V_3 = 2V_0 \\ 7V_3 - 3V_0 = 300 \end{cases} \longrightarrow 14V_0 - 3V_0 = 300 \Rightarrow V_0 = \frac{300}{11} = \boxed{27.27V}$

$\Rightarrow i_x = \frac{V_2}{6K} = \frac{V_0}{6K} = \frac{27.27}{6K} = 4.54\,m$

3. Calculate v_o in the following two Op Amp circuits in terms of three input voltages v_1, v_2, and v_3.



(a)



(b)

Answer:

a) $v_o = -(v_1 + 3v_2 + 5v_3)$

b) $v_o = v_1 + 3v_2 + 5v_3$

or $-\left(\frac{1000}{1000}V_1 + \frac{1000}{333}V_2 + \frac{1000}{200}V_3\right)$

Solution: a) Typical Summing OpAmp $\rightarrow V_0 = -\left(\frac{1K}{1K}V_1 + \frac{1K}{0.333K}V_2 + \frac{1K}{0.2K}V_3\right)$

$\boxed{1000/333 \simeq 3}$

$\Rightarrow V_0 = -(V_1 + 3V_2 + 5V_3)$

Ideal opAmp $\rightarrow i_1 = i_2 = 0$, $\boxed{V_x = V_y}$ (I)

b) KCL @ V_x: $\frac{V_1 - V_x}{1000} + \frac{V_2 - V_x}{333} + \frac{V_3 - V_x}{200} = \frac{V_x}{1000} \xrightarrow{\times 1000} V_1 - V_x + 3V_2 - 3V_x + 5V_3 - 5V_x = V_x$

$\Rightarrow \boxed{V_x = \frac{V_1 + 3V_2 + 5V_3}{10}}$ (II)

or ($i_o = \frac{V_0}{100 + 900} = \frac{V_y}{100} \rightarrow \frac{V_0}{1000} = \frac{V_y}{100} \xrightarrow{\times 1000} \boxed{V_0 = 10V_y}$

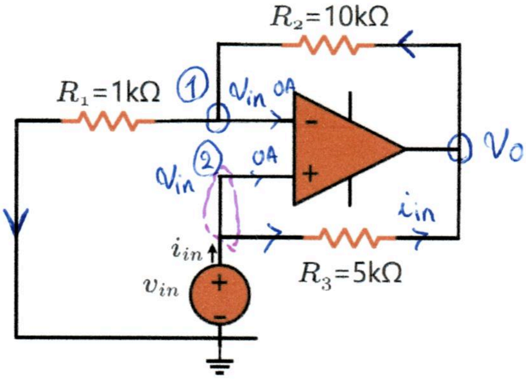
KCL @ V_y: $\frac{V_0 - V_y}{900} = \frac{V_y}{100} \xrightarrow{\times 900} V_0 - V_y = 9V_y \rightarrow \boxed{V_0 = 10V_y}$ (III)

(I) $V_x = V_y$ $\longrightarrow$ (II), (III) $\longrightarrow V_x = \frac{V_0}{10} \longrightarrow \frac{V_0}{10} = \frac{V_1 + 3V_2 + 5V_3}{10} \xrightarrow{\times 10} V_0 = V_1 + 3V_2 + 5V_3$

4. In the following circuit,

a) Calculate the ratio $\frac{v_{in}}{i_{in}}$ in terms of R_1, R_2, and R_3, and then use the numerical values of resistances given in the circuit to determine the ratio. What does this ratio represent?

b) If $R_1 = R_2 = R$, determine the value of R_3 for the Op Amp circuit such that $\frac{v_{in}}{i_{in}} = -33\ \text{k}\Omega$



Answer:

a) $\frac{v_{in}}{i_{in}} = -\frac{R_1R_3}{R2}$

b) $R_3 = 33\ \text{k}\Omega$

Solution: Ideal Op Amp $V^- = V^+ = V_{in}$ $\quad i^- = i^+ = 0A$

a) KCL @ node ①: $\frac{V_o - V_{in}}{R_2} = \frac{V_{in} - 0}{R_1} \xrightarrow{\times R_2} V_o - V_{in} = \frac{R_2}{R_1}V_{in} \to \boxed{V_o = (1+\frac{R_2}{R_1})V_{in}}$ (I)

KCL @ node ②: $i_{in} = \frac{V_{in} - V_o}{R_3} \xrightarrow{(I)} i_{in} = \frac{V_{in}}{R_3} - \frac{1}{R_3}(1+\frac{R_2}{R_1})V_{in}$

$\to i_{in} = \frac{V_{in}}{R_3}\left[\cancel{1} - \cancel{1} - \frac{R_2}{R_1}\right] = -V_{in}\frac{R_2}{R_1R_3} \Rightarrow \frac{V_{in}}{i_{in}} = -\frac{R_1R_3}{R_2}$

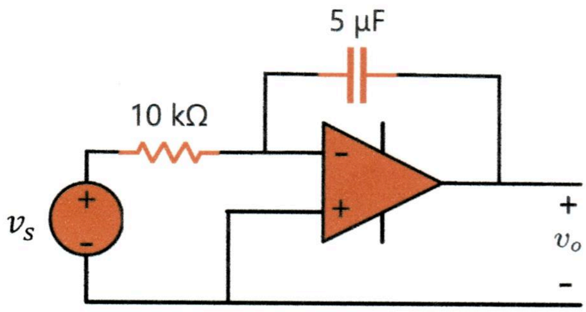
$R_1 = 1K\Omega$, $R_2 = 10K\Omega$, $R_3 = 5K\Omega \to \frac{V_{in}}{i_{in}} = -\frac{1K \times 5K}{10K} = -\frac{5\times10^{6}}{10\times10^{3}} = \boxed{-500\Omega}$

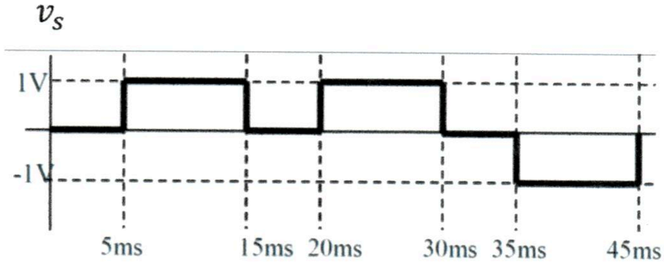
According to Ohm's law: $V = Ri \to R = V/i$, Thus $\frac{V_{in}}{i_{in}} = -500\Omega$ is a negative resistance circuit

This circuit acts like a negative resistance.

b) if $R_1 = R_2 = R \to \frac{V_{in}}{i_{in}} = -\frac{RR_3}{R} = -R_3 \Rightarrow$ Choose $R_3 = 33K\Omega$

5. Calculate and draw the output voltage v_o in the following circuit for input voltage signal v_i given in the graph from $0 \le t \le 45$ ms .





Answer: $v_o(t) = -20\int_0^t v_s(\tau)d\tau$, You must complete the output graph

Solution: Typical Integrator → $V_o = -\frac{1}{RC}\int_0^t V_s(\tau)\,d\tau$, $RC = 10\times10^3\times5\times10^{-6}$

→ $RC = 5\times10^{-2}$ → $\frac{1}{RC} = \frac{1}{5\times10^{-2}} = \frac{100}{5} = \boxed{20}$ ⟹ $V_o = -20\int_0^t V_s(\tau)\,d\tau$

$0 \le t \le 5^{ms}$ → $V_o = -20\int_0^t 0\,d\tau = \boxed{0V}$ initial voltage (integral constant

$5^{ms} \le t \le 15^{ms}$ → $V_o = -20\int_{5^m}^t 1\,d\tau + V_o(5^m) = -20\tau\Big]_{5^m}^t + 0 = -20(t - 5\times10^{-3}) = \boxed{-20t + 0.1V}$

$15^{ms} \le t \le 20^{ms}$ → $V_o = -20\int_{15^m}^t 0\,d\tau + V_o(15^m) = 0 - 20\times15\times10^{-3} + 0.1 = -0.3 + 0.1 = \boxed{-0.2V}$

$20^{ms} \le t \le 30^{ms}$ → $V_o = -20\int_{20^m}^t 1\,d\tau + V_o(20^m) = -20\tau\Big]_{20^m}^t - 0.2 = -20t + 20\times20\times10^{-3} - 0.2$

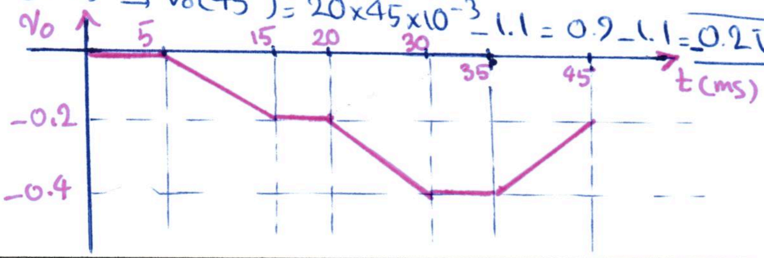
$= \boxed{-20t + 0.2V}$

$30^{ms} \le t \le 35^{ms}$ → $V_o = -20\int_{30^m}^t 0\,d\tau + V_o(30^m) = 0 - 20\times30\times10^{-3} + 0.2 = \boxed{-0.4V}$

$35^{ms} \le t \le 45^{ms}$ → $V_o = -20\int_{35^m}^t -1\,d\tau + V_o(35) = 20\tau\Big]_{35}^t - 0.4 = 20t - \overbrace{20\times35\times10^{-3}}^{-0.7} - 0.4$

$= \boxed{20t - 1.1V}$

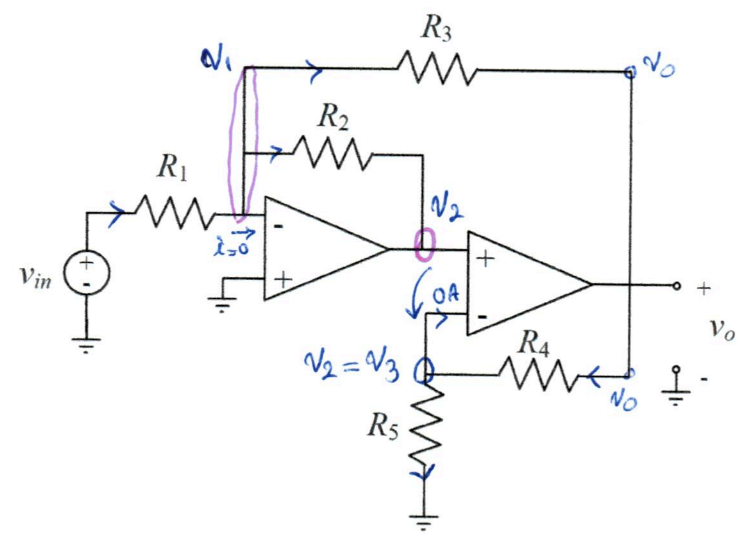
$t = 45^{ms}$ → $V_o(45^m) = 20\times45\times10^{-3} - 1.1 = 0.9 - 1.1 = \boxed{-0.2V}$



6. (**Final Exam – S1, 2014**) Consider the Op Amp circuit below with input v_{in} and output v_o.

a) Derive an expression for the voltage gain $\frac{v_o}{v_{in}}$ in terms of the resistor values R_1, R_2, R_3, R_4, and R_5.

b) If $R_1 = 1\ \text{k}\Omega$, $R_2 = 2\ \text{k}\Omega$, $R_3 = 3\ \text{k}\Omega$, and $R_5 = 4\ \text{k}\Omega$, determine the value of R_4 such that the voltage gain $\frac{v_o}{v_{in}} = -1.8$.



Answer:

a) $\frac{v_o}{v_{in}} = -\frac{R_2R_3(R_4+R_5)}{R_1R_3R_5+R_1R_2(R_4+R_5)}$

b) $R_4 = 5\ \text{k}\Omega$

Solution:

a)

KCL@v_1: $\frac{v_{in}-v_1}{R_1} = \frac{v_1-v_2}{R_2} + \frac{v_1-v_o}{R_3} \xrightarrow[v_1=0V]{\text{Ideal op Amp}} \boxed{\frac{v_{in}}{R_1} = -\frac{v_2}{R_2} - \frac{v_o}{R_3}}$ I

KCL@v_3: $\frac{v_o-v_3}{R_4} = \frac{v_3}{R_5} \xrightarrow[\times R_4]{v_3=v_2} v_o - v_2 = \frac{R_4}{R_5}v_2 \rightarrow \boxed{v_o = \left(1+\frac{R_4}{R_5}\right)v_2}$ II

Non-inverting Op Amp from v_2 to v_o

$\xrightarrow{\text{II}} \boxed{v_2 = \frac{R_5}{R_4+R_5}v_o}$ III

sub III to I $\rightarrow$ $\frac{v_{in}}{R_1} = -\frac{1}{R_2}\left(\frac{R_5}{R_4+R_5}\right)v_o - \frac{v_o}{R_3} =$

$\xrightarrow{\times R_1} v_{in} = -\left(\frac{R_1R_5}{R_2(R_4+R_5)} + \frac{R_1}{R_3}\right)v_o = -\frac{R_1R_3R_5 + R_1R_2(R_4+R_5)}{R_2R_3(R_4+R_5)}v_o$

$\Rightarrow \frac{v_o}{v_{in}} = -\frac{R_2R_3(R_4+R_5)}{R_1R_3R_5 + R_1R_2(R_4+R_5)}$

All resistors in kΩ
factor out kΩs

b) $\frac{v_o}{v_{in}} = -1.8 = -\frac{2k \cdot 3k(R_4+4k)}{1k \times 3k \times 4k + 1k \times 2k(R_4+4k)} = \frac{6(R_4+4)\times\cancel{10^9}}{[12+2(R_4+4)]\times\cancel{10^9}}$

kΩ kΩ

$\Rightarrow 1.8 = \frac{6R_4+24}{12+2R_4+8} \longrightarrow 36+3.6R_4 = 6R_4+24 \longrightarrow 12 = 2.4R_4$

$\longrightarrow R_4 = \frac{12}{2.4} = 5\ \text{k}\Omega$

7. Design a circuit with operational amplifiers that can generate the following output

$$v_o = 9v_1 - 6v_2$$

considering that the range of inputs are $1\text{ V} \le v_1 \le 2\text{ V}$ and $2\text{ V} \le v_2 \le 3\text{ V}$. The operational amplifiers have a supply voltage of ± 12 V.

Hint: Consider the impact that the supply voltage of the operational amplifier has on the output of the device and how you can design around this limitation. This means that you have to break the design in two steps:
Step1: Design an Op Amp circuit for $v_{o1} = 3v_1 - 2v_2$ as first Op amp stage,
Step2: Cascade the Op Amp circuit in Step 1 to another Op Amp stage for $v_o = 3v_{o1}$.

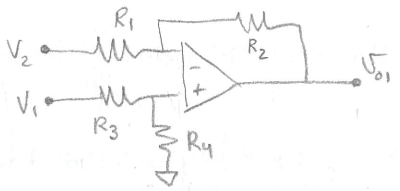
To avoid saturation we use Op Amp circuits in a cascade

$V_o = 9V_1 - 6V_2 = 3(3V_1 - 2V_2)$

* STEP 1 : $V_{o1} = 3V_1 - 2V_2$

Different options:

a) Using difference amplifier



From lecture slides (or doing nodal analysis):

$$V_o = \frac{R_2(1+R_1/R_2)}{R_1(1+R_3/R_4)} V_1 - \frac{R_2}{R_1} V_2$$

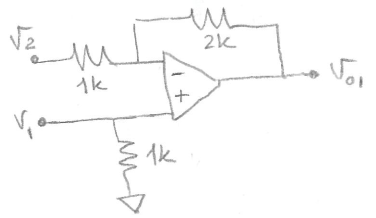
We want $V_{o1} = 3V_1 - 2V_2$, so:

$R_2/R_1 = 2 \Rightarrow$ $R_1 = 1k$, $R_2 = 2k$

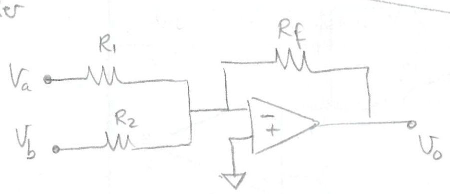
$$\frac{R_2(1+R_1/R_2)}{R_1(1+R_3/R_4)} = 3 \Rightarrow \frac{2(1+1/2)}{1+R_3/R_4} = 3 \Rightarrow \frac{3}{1+R_3/R_4} = 3 \Rightarrow \frac{R_3}{R_4} = 0$$

$R_3 = 0$

$R_4 = 1k$



b) Using summing amplifier



$$V_o = -\frac{R_f}{R_1} V_a - \frac{R_f}{R_2} V_b$$

If we want $V_{o1} = 3V_1 - 2V_2$

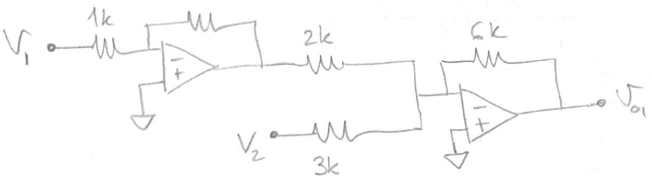
$V_a = -V_1 \Rightarrow$ we need an inverting amplifier with gain 1 first.

$V_b = V_2$

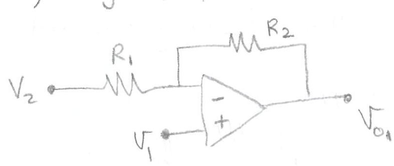
$R_f/R_1 = 3$

$R_f/R_2 = 2$

$\Rightarrow$ If we choose $R_f = 6k\Omega$ Then, $R_1 = 2k\Omega$, $R_2 = 3k\Omega$



c) Using superposition



$$V_{o1} = -\frac{R_2}{R_1} V_2 + \left(1 + \frac{R_2}{R_1}\right) V_1 = -2V_2 + 3V_1$$

$V_1 = 0 \Rightarrow$ inverting amplifier

$V_2 = 0 \Rightarrow$ non-inverting amplifier

↳ We get 2 equations

$-\frac{R_2}{R_1} = -2$

$1 + \frac{R_2}{R_1} = 3$

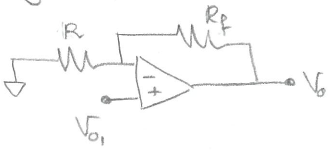
$R_1 = 1k$

$R_2 = 2k$

Note: Options a) and c) might not work for other values of gains (i.e. the simultaneous equations might not have a solution)

* STEP 2 : $V_o = 3V_{o_1} = 3\,(3V_1 - 2V_2) = 9V_1 - 6V_2$

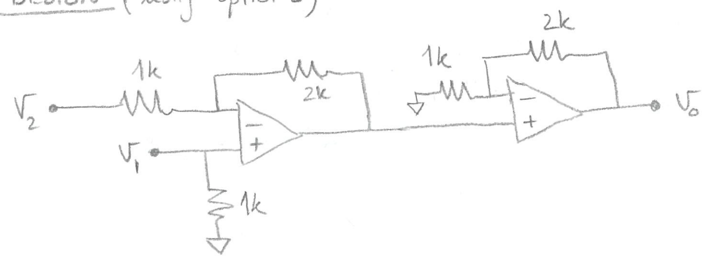
Non-inverting amplifier with gain 3



$$V_o = \left(1 + R_f/R\right) V_{o_1} = 3V_{o_1}$$

$$1 + R_f/R = 3 \quad \Rightarrow \quad \begin{matrix} R_f = 2k \\ R = 1k \end{matrix}$$

* FULL DESIGN (using option 2)



Note: Using Op Amp circuits in a cascade help to make the circuit more robust, since the range of input voltages can be bigger and we would avoid saturation.
In any case, a design without cascading would also work for the given range of input voltages:

$$\begin{cases} 1V \leq V_1 \leq 2V \\ 2V \leq V_2 \leq 3V \end{cases} \xrightarrow[\text{(that would lead to highest } V_o)]{\text{worst case scenario}} \begin{matrix} V_1 = 1V \\ V_2 = 3V \end{matrix} \Rightarrow V_o = 9 - 18 = -9 \;\; \underbrace{< -12}_{\text{we do not reach saturation}}$$

$$\begin{cases} V_{cc}^{+} = 12V \\ V_{cc}^{-} = -12V \end{cases}$$