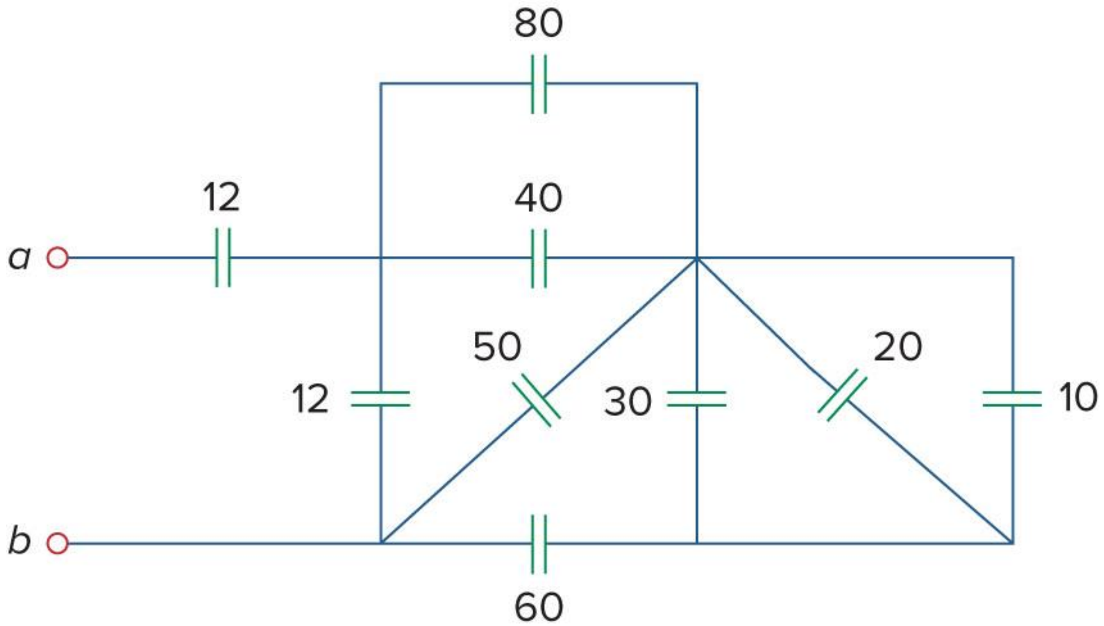
School of Electrical Engineering & Telecommunications

ELEC1111

Topic 4: Capacitors and RC Circuits

1. (**Final Exam – S2, 2015**) The circuit below shows a network of supercapacitors in a hybrid energy storage system where all capacitances are in $\mu\mathrm{F}$
   a) Find the equivalent capacitance at terminals a-b.
   b) What is the energy stored if the voltage of terminals a-b is equal to 100 V.

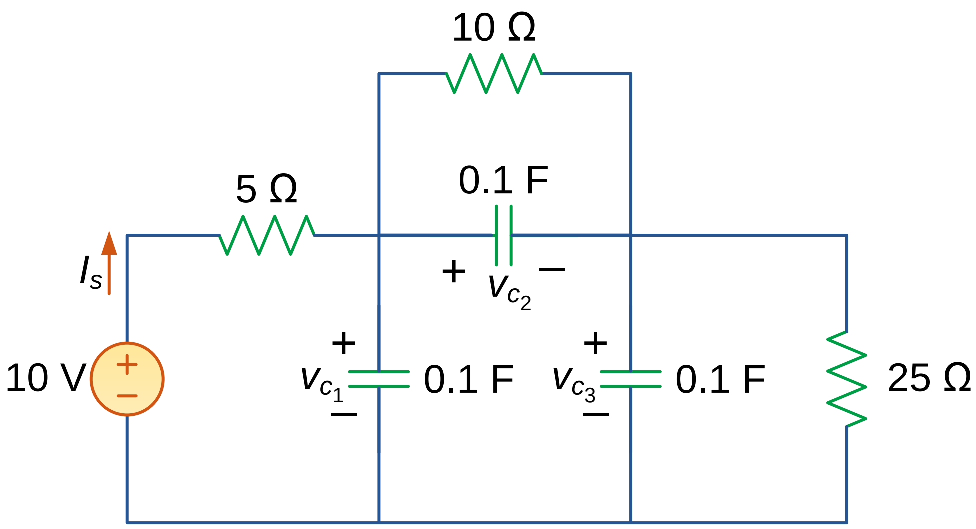


Answer:
a) $C_{\mathrm{eq}} = 10\ \mu\mathrm{F}$.
b) $w = 50\ \mathrm{mJ}$
Hint: since nothing is mentioned about initial conditions, they are assumed to be zero.

2. (**Final Exam – S2, 2015**) For the circuit below,

a) Determine the current I_s after the circuit has reached steady state.

b) Determine the capacitor voltages V_{c_1}, V_{c_2} and V_{c_3} after the circuit has reached steady state.



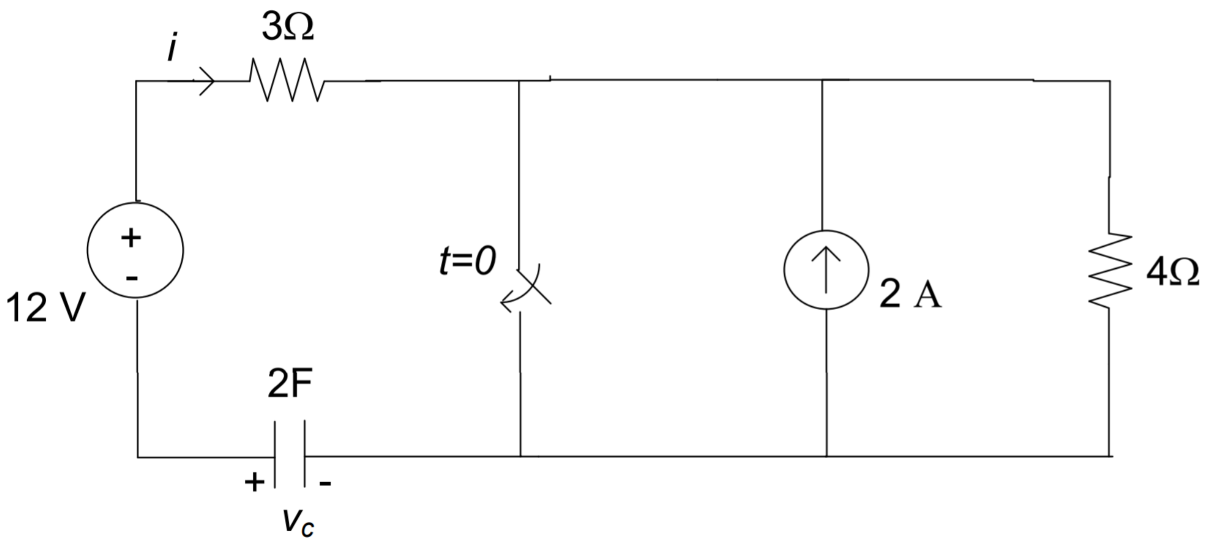
Answer:

a) $I_s = 0.25 \text{ A}$

b) $V_{c_1} = 8.75 \text{ V}$, $V_{c_2} = 2.5 \text{ V}$, $V_{c_3} = 6.25 \text{ V}$

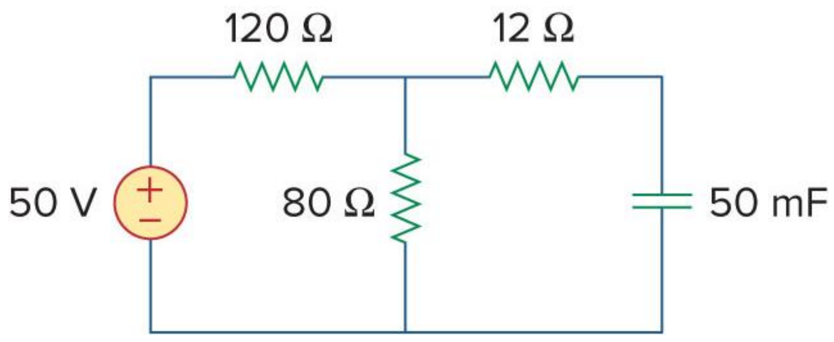
Hint: The steady state means that the capacitors are fully charged and under DC conditions.

3. (**Final Exam – S2, 2015**) For the circuit below, the switch has been in open position for a long time before changing position as shown at time $t = 0$.
   a) Find the voltage v_c and the current i when $t < 0$, i.e., $v_c(0^-)$ and $i(0^-)$.
   b) Find v_c and i when $t > 0$ and in steady state condition, i.e., $v_c(+\infty)$ and $i(+\infty)$.



Answer:
a) $v_c(0^-) = -4\text{ V}$ and $i(0^-) = 0\text{ A}$
b) $v_c(+\infty) = -12\text{ V}$ and $i(+\infty) = 0\text{ A}$

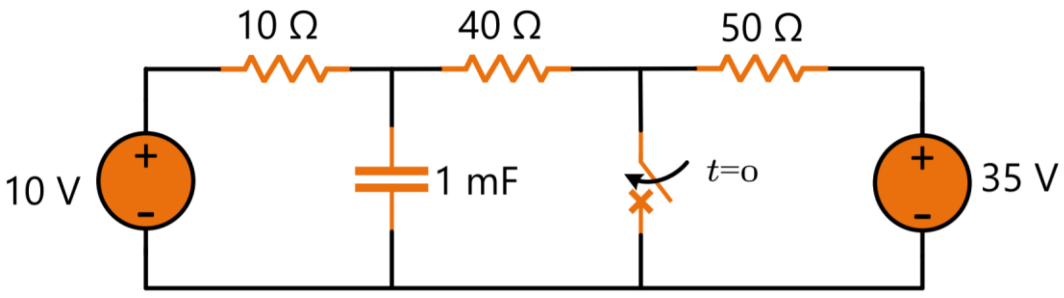
4. Find the time constant for the RC circuit given below.



Answer: $\tau = 3\ \mathrm{s}$

5. (**Mid-semester Exam - Summer 2017**) In the circuit below, the switch has been in the open position for a long time before closing at time $t = 0$.

a) Give an expression for the capacitor voltage $v_c(t)$ (i.e., as a function of time) for $t > 0$.
b) Give an expression for the capacitor current $i_c(t)$ for $t > 0$
c) Give an expression for the current of the 40-Ω resistor $i_{R40}(t)$ for $t > 0$.



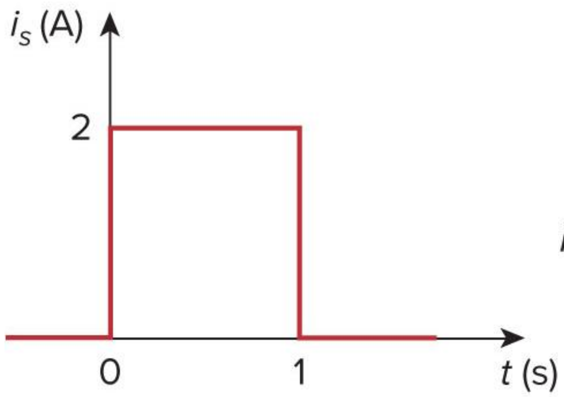
Answer:

a) $v_c(t) = 8 + 4.5e^{\frac{-t}{8\times10^{-3}}}\ \mathrm{V}$

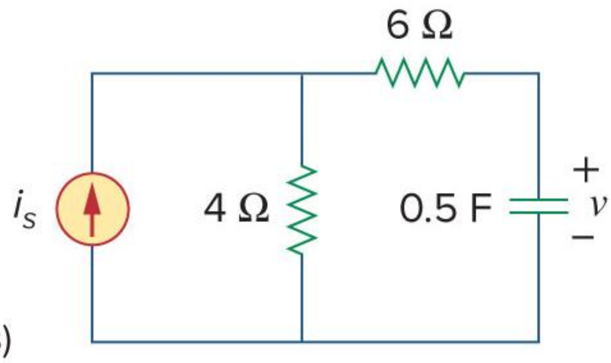
b) $i_{c(t)} = -\frac{4.5}{8}e^{\frac{-t}{8\times10^{-3}}} = -0.5625e^{\frac{-t}{8\times10^{-3}}}\ \mathrm{A}$

c) $i_{R40}(t) = \frac{8}{40} + \frac{4.5}{40}e^{\frac{-t}{8\times10^{-3}}} = 0.2 + 0.1125e^{\frac{-t}{8\times10^{-3}}}\ \mathrm{A}$

6. If the waveform given in Fig. (a) is applied to the circuit of Fig. (b), find the expression for $v(t)$ assuming $v(0) = 0$.



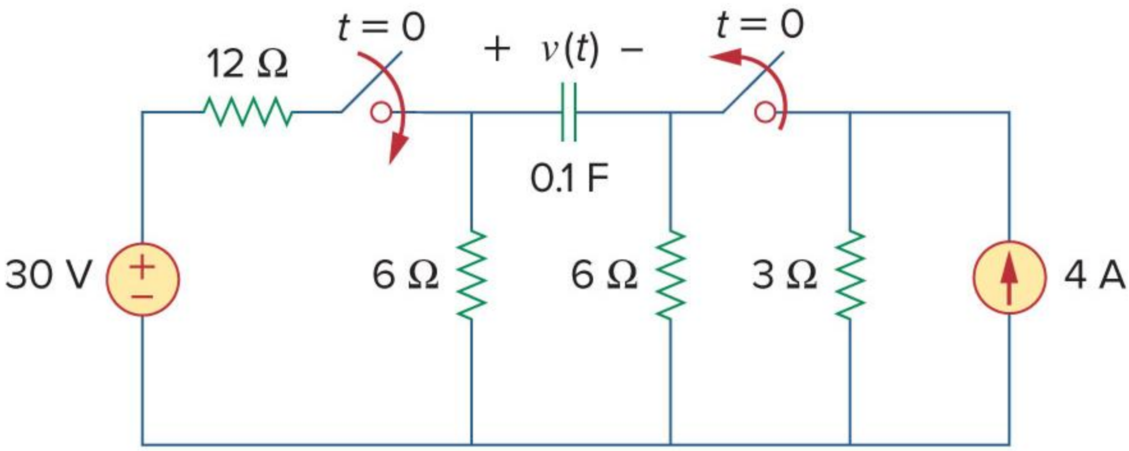
(a)



(b)

Answer: $v(t) = \begin{cases} 8\left(1 - e^{\frac{-t}{5}}\right)\text{V}, & 0 < t < 1 \\ 1.4502e^{\frac{-(t-1)}{5}}\text{V}, & t \geq 1 \end{cases}$

7. In the circuit below, determine the response $v(t)$ for all time (i.e., for both $t < 0$ and $t > 0$), and sketch $v(t)$ waveform showing all critical points in the sketch.



(a)

Answer: $v(t) = \begin{cases} -8\text{ V} & t \leq 0 \\ 10 - 18e^{-t}\text{ V} & t \geq 0 \end{cases}$