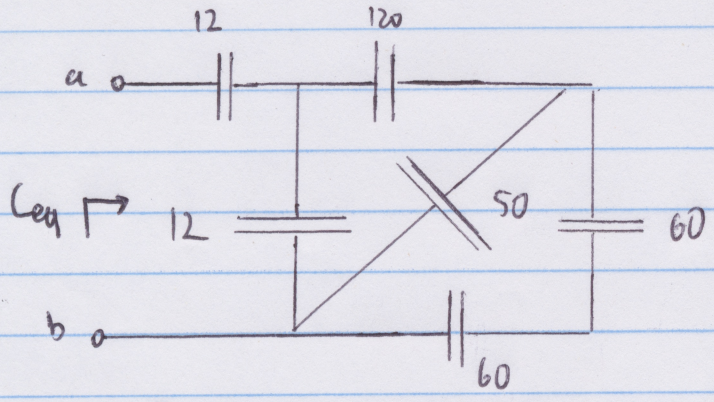
# 2015 Semester 2 Final Exam

Q.1 i) $80 // 40 = 120\,\mu F$

$10 // 20 // 30 = 60\,\mu F$

Note: // ⇒ Parallel

\+ ⇒ In Series




$[(60+60)//50 + 120] // 12 + 12$

$(30//50 + 120) // 12 + 12$

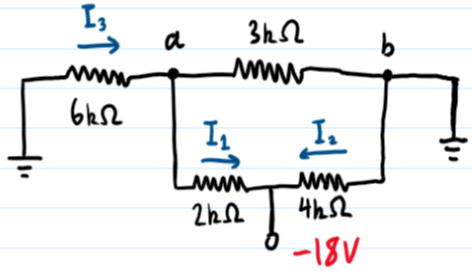
$(80+120) // 12 + 12$

$= 48 // 12 + 12$

$= 60 + 12$

$= 10\,\mu F$

ii) a)




KCL at node a:


$\frac{0-V_a}{6} + \frac{0-V_a}{3} = \frac{V_a-(-18)}{2}$

$-V_A - 2V_A = 3V_A + 54$

$-6V_A = 54$

$V_A = -9V$

$I_1 = \frac{V_A-(-18)}{2000} = \frac{-9+18}{2000} = 4.5\,mA$

$I_2 = \frac{0-(-18)}{4000} = 4.5\,mA$

$I_3 = \frac{0-V_a}{6000} = \frac{0-(-9)}{6000} = 1.5\,mA$

b) $V_{ab} = V_a - V_b = -9V$

c) Power dissipated

$$P_{6k\Omega} = I_3^2(6000) = 13.5\ mW$$

$$P_{2k\Omega} = I_1^2(2000) = 40.5\ mW$$

$$P_{4k\Omega} = I_2^2(4000) = 81\ mW$$

$$P_{3k\Omega} = \frac{V_{ab}^2}{3000} = 27\ mW$$

iii) KCL at node a:

$$\frac{V_a}{2} + \frac{V_a - V_b}{5} + 3 + 2 = 0$$

$$5V_a + 2V_a - 2V_b + 50 = 0$$

$$7V_a - 2V_b = -50 \quad (1)$$

KCL at node b:

$$\frac{V_b - V_a}{5} + \frac{V_b}{4} = 2$$

$$4V_b - 4V_a + 5V_b = 40$$

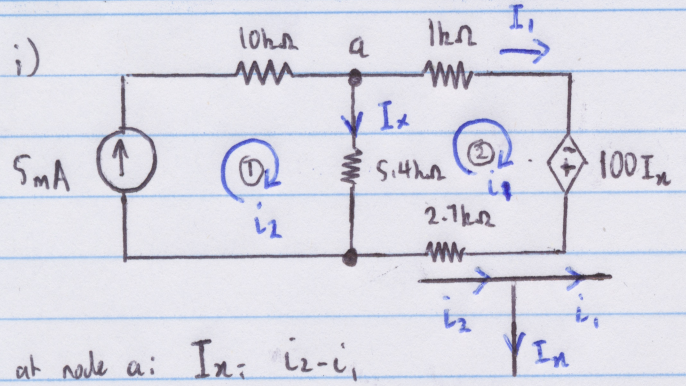
$$9V_b - 4V_a = 40 \quad (2)$$

$$\left(\begin{array}{cc|c} V_a & V_b & \\ -4 & 9 & 40 \\ 7 & -2 & -50 \end{array}\right) \quad R_2 = 4R_2 + 7R_1$$

$$\left(\begin{array}{cc|c} -4 & 9 & 40 \\ 0 & 55 & 80 \end{array}\right) \quad \therefore V_b = \frac{80}{55} = \frac{16}{11} V$$

$$V_a = \frac{40 - 9\left(\frac{16}{11}\right)}{-4} = -\frac{74}{11} V$$

Q.2 i)




$i_2 = 5mA$ (due to source)

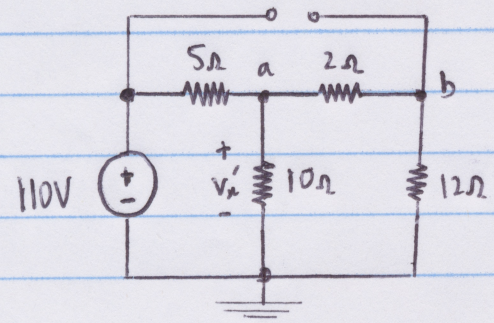
KCL at node a: $I_x = i_2 - i_1$

KVL in mesh ②: $3.7\times10^3 i_1 - 100(i_2 - i_1) + 5.4\times10^3(i_1 - i_2) = 0$

$9.2\times10^3 i_1 - 5.5\times10^3(5\times10^{-3}) = 0$

$i_1 = \frac{27.5}{9.2\times10^3} = \frac{11}{3680}\ A = 2.989\ mA$

ii) Turning off 4A:




KCL at node a:

$\frac{V_a - 110}{5} + \frac{V_a}{10} + \frac{V_a - V_b}{2} = 0$

$2V_a - 220 + V_a + 5V_a - 5V_b = 0$

$8V_a - 5V_b = 220$ ①

KCL at node b:

$\frac{V_b - V_a}{2} + \frac{V_b}{12} = 0$

$-6V_a + 7V_b = 0$ ②

$$\begin{array}{cc|c} V_a & V_b & \\ \hline -6 & 7 & 0 \\ 8 & -5 & 220 \end{array}$$

$R_2 = 3R_2 + 4R_1$

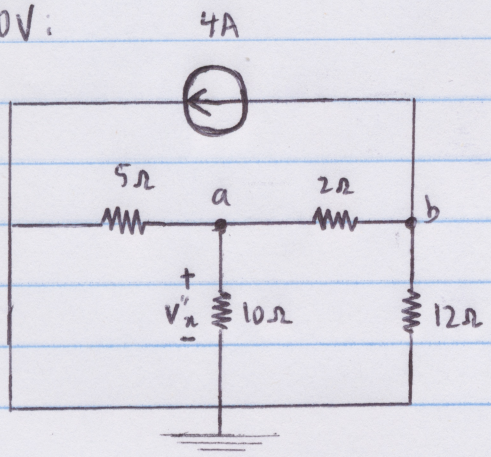
$$\left(\begin{array}{cc|c} -6 & 7 & 0 \\ 0 & 13 & 660 \end{array}\right)$$

$\therefore V_b = \frac{660}{13}\ V = 50.77V$

$\therefore V_a = \frac{-7\left(\frac{660}{13}\right)}{-6} = \frac{770}{13}\ V = 59.231V$

$\therefore V_x' = V_a = \frac{770}{13}\ V$

Turning off 100V:




KCL at node a:

$$\frac{V_a}{5} + \frac{V_a}{10} + \frac{V_a - V_b}{2} = 0$$

$$8V_a - 5V_b = 0 \quad (1)$$

KCL at node b:

$$\frac{V_b - V_a}{2} + \frac{V_b}{12} + 4 = 0$$

$$-6V_a + 7V_b = -48 \quad (2)$$

$$\begin{matrix} V_a & V_b \end{matrix}$$

$$\left(\begin{array}{cc|c} 8 & -5 & 0 \\ -6 & 7 & -48 \end{array}\right) \qquad R_2 = 4R_2 + 3R_1$$

$$\left(\begin{array}{cc|c} 8 & -5 & 0 \\ 0 & 13 & -192 \end{array}\right) \qquad \therefore V_b = \frac{-192}{13} V$$

$$V_a = \frac{5\left(\frac{-192}{13}\right)}{8} = \frac{-120}{13} V$$

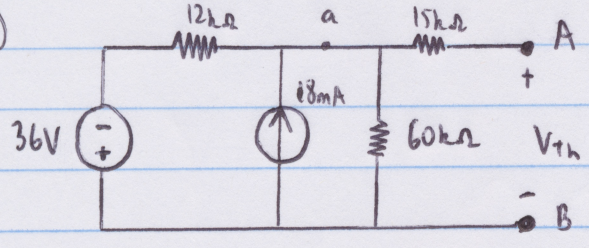
$$\therefore V''_x = V_a = \frac{-120}{13} V$$

By superposition:

$$V_x = V'_x + V''_x$$

$$= \frac{770}{13} - \frac{120}{13} = 50V$$

iii)

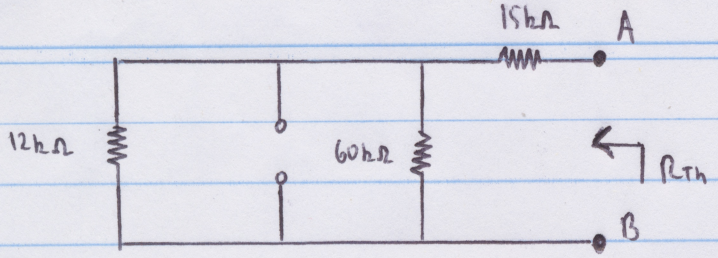



KCL at node a:

$$\frac{V_a + 36}{12 \times 10^3} + \frac{V_a}{60 \times 10^3} = 18 \times 10^{-3}$$

$$6V_a + 180 = 1080$$

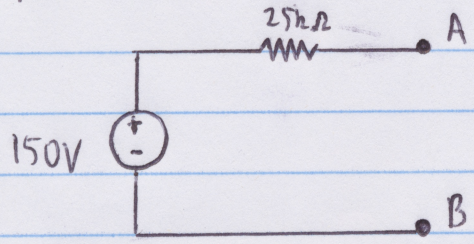
$$\therefore V_{Th} = V_a = 150V$$




$R_{Th} = 12 // 60 + 15$

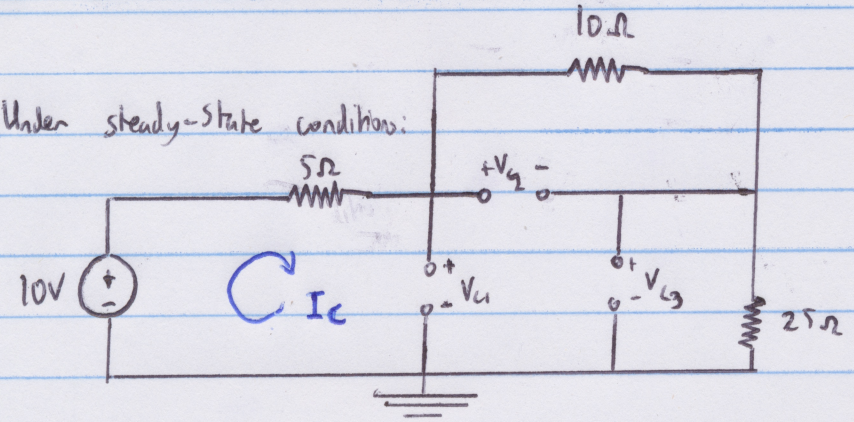
$= 25k\Omega$

Thevenin equivalent circuit



Q.3 i)

a) Under steady-state condition:




KVL in mesh:

$$-10 + (5+10+25)I_c = 0$$

$$I_c = 0.25A$$

b) Current across 5Ω:

$$\frac{10 - V_{c1}}{5} = 0.25$$

$$10 - V_{c1} = 1.25$$

$$\therefore V_{c1} = 8.75V$$

Current across 10Ω:

$$\frac{8.75 - V_{c3}}{10} = 0.25$$

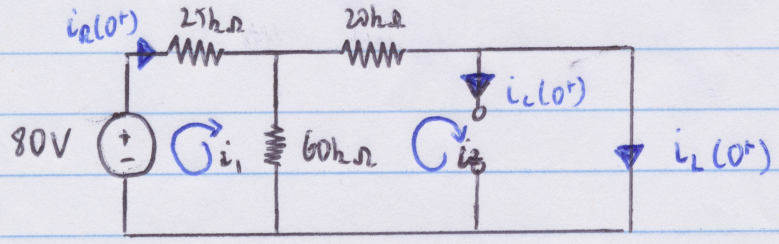
$$8.75 - V_{c3} = 2.5$$

$$\therefore V_{c3} = 6.25V$$

$$V_{c2} = V_{c1} - V_{c3}$$

$$= 8.75 - 6.25 = 2.5V$$

ii) At $t = 0^-$




KVL in mesh ①:

$$0 = -80 + 25\times10^3 i_1 + 60\times10^3(i_1 - i_2)$$

$$85\times10^3 i_1 - 60\times10^3 i_2 = 80 \quad ①$$

KVL in mesh ②:

$$0 = 20\times10^3 i_2 + 60\times10^3(i_2 - i_1)$$

$$-60\times10^3 i_1 + 80\times10^3 i_2 = 0 \quad ②$$

$i_1(10^3)$ $i_2(10^3)$

$$\left(\begin{array}{cc|c} -60 & 80 & 0 \\ 85 & -60 & 80 \end{array}\right) \quad \begin{array}{l} R_1 = R_1 \div 20 \\ R_2 = R_2 \div 5 \end{array} \Rightarrow \left(\begin{array}{cc|c} -3 & 4 & 0 \\ 17 & -12 & 16 \end{array}\right) \quad R_2 = 3R_2 + 17R_1$$

$$\left(\begin{array}{cc|c} -3 & 4 & 0 \\ 0 & 32 & 48 \end{array}\right) \quad \therefore \; i_2 = 1.5A \times 10^{-3} = 1.5mA$$

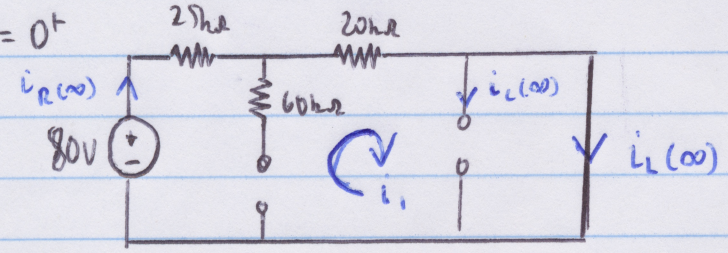
$$i_1 = \frac{-4 \times 1.5}{-3} = 2A \times 10^{-3} = 2mA$$

$\therefore$ $i_R(0^+) = 2mA$

$i_L(0^+) = 1.5mA$

$i_C(0^+) = 0A$

At $t = 0^+$




KVL in mesh:

$-80 + 45 \times 10^3 i_1 = 0$

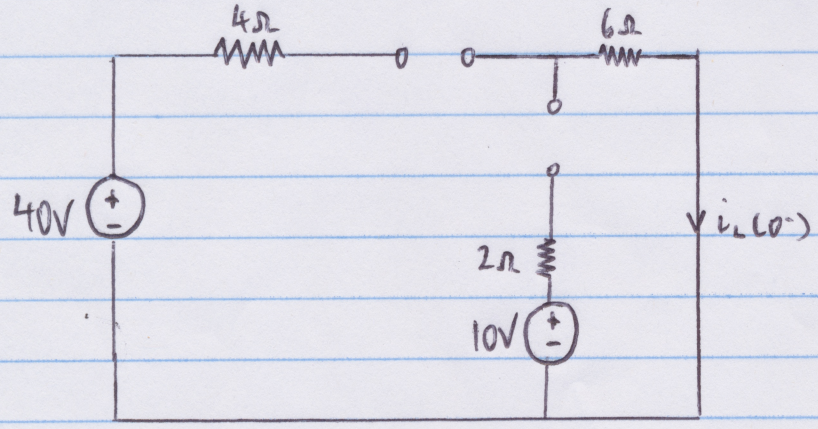
$i_1 = 1.778mA$

$\therefore$ $i_R(\infty) = 1.778mA$

$i_L(\infty) = 1.778mA$

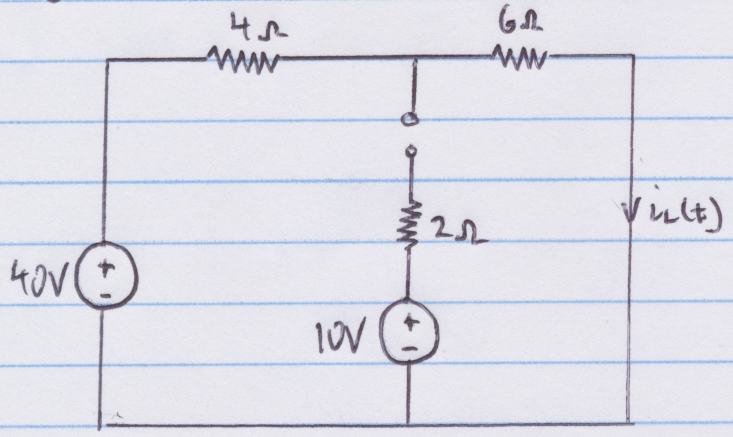
$i_C(\infty) = 0A$

iii) At $t = 0^-$




$i_L(0^-) = 0A$ due to no source connected

At $0 < t < 4$




KVL:

$10\, i_L(t) = 40$

$i_L(t) = 4A$

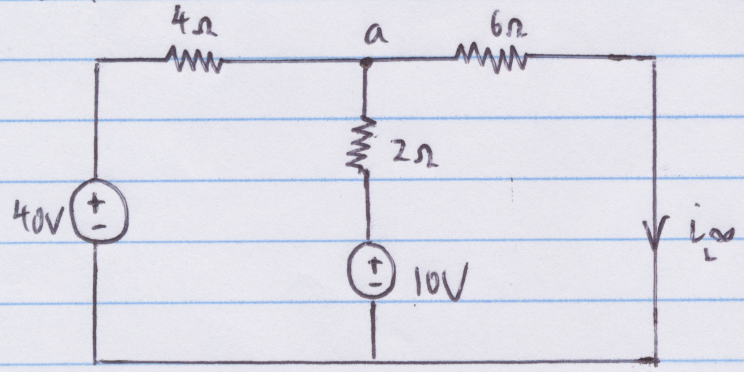
$R_{Th} = 4 + 6 = 10\Omega$

$\tau = \frac{L}{R_{Th}} = \frac{5}{10} = \frac{1}{2}\, s$

$\therefore \quad i_L(t) = 4 + (0-4)e^{-2t} \; A$

$= 4(1 - e^{-2t})\; A \quad$ for $\quad 0 < t < 4$

At $t > 4$




KCL at node a:

$\frac{V_a - 40}{4} + \frac{V_a - 10}{2} + \frac{V_a}{6} = 0$

$11V_a - 180 = 0$

$V_a = \frac{180}{11} = 16.364\, V$

Current across 6Ω

$\frac{V_a}{6} = i_{L\infty}$

$\therefore i_{L\infty} = \frac{30}{11} A \quad$ and $\quad i_L 0 = 4(1 - e^{-8})$ at $t = 4s$

$R_{Th} = 4 // 2 + 6 = \frac{22}{3}\Omega$

$\tau = \frac{L}{R} = \frac{15}{22}\, s$

$\therefore i_L(t) = \frac{30}{11} + \left(4 - 4e^{-8} - \frac{30}{11}\right)e^{-\frac{22}{15}t} \Rightarrow = \frac{30}{11} + \left(\frac{14}{11} - 4e^{-8}\right)e^{-\frac{22}{15}t}, \; t > 4$

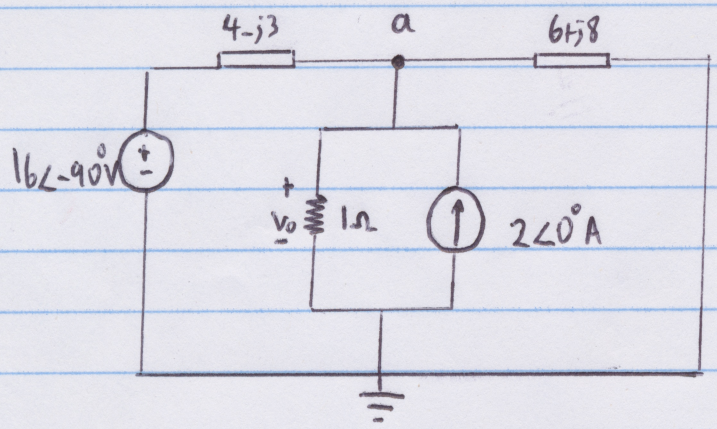
Q.4 i) $Z_{eq}(\frac{1}{12}F) = \frac{-j}{4(\frac{1}{12})} = -j3\Omega$

$Z_{eq}(2H) = j(4)(2) = j8\Omega$

Sources:

$16\sin 4t \Rightarrow 16\cos(4t-90^\circ)V = 16\angle -90^\circ V$

$2\cos 4t \Rightarrow 2\angle 0^\circ A$




KCL at node a:

$$\frac{V_o - 16\angle -90^\circ}{4-j3} + \frac{V_o}{1} + \frac{V_o}{6+j8} = 2$$

$$V_o\left(\frac{1}{4-j3} + 1 + \frac{1}{6+j8}\right) = 2 + \frac{16\angle -90}{4-j3}$$

$\therefore V_o = 3.836\cos(4t-35^\circ)V$

ii) $Z_{eq}(10mH) = j(10\times10^{-3})(100) = j1\Omega$

$Z_{eq}(20\times10^{-6}F) = \frac{-j}{20\times10^{-6}\times 100} = -j500\Omega$

For the circuit:

$Z_{eq} = (40+j)\,//\,60\,//\,30 - j500$

$= \left(\frac{1}{40+j} + \frac{1}{60} + \frac{1}{30}\right)^{-1} - j500 = 500\angle -88.5^\circ\Omega$

$Y_{in} = \frac{1}{Z_{eq}} = 2\angle 88.5^\circ mS$

iii) $i_1 = 100\sin(50t+100^\circ) = 100\cos(50t+10^\circ) = 100\angle 10^\circ A$

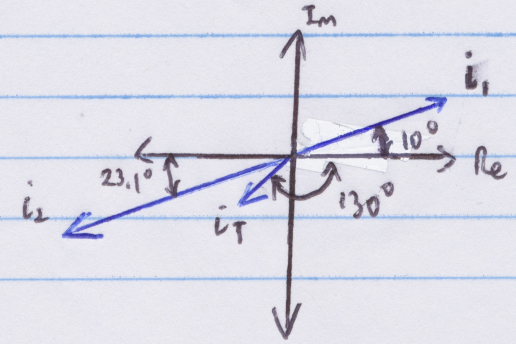
$i_T = 50\sin(50t-40^\circ) = 50\cos(50t-130^\circ) = 50\angle -130^\circ A$

KCL:

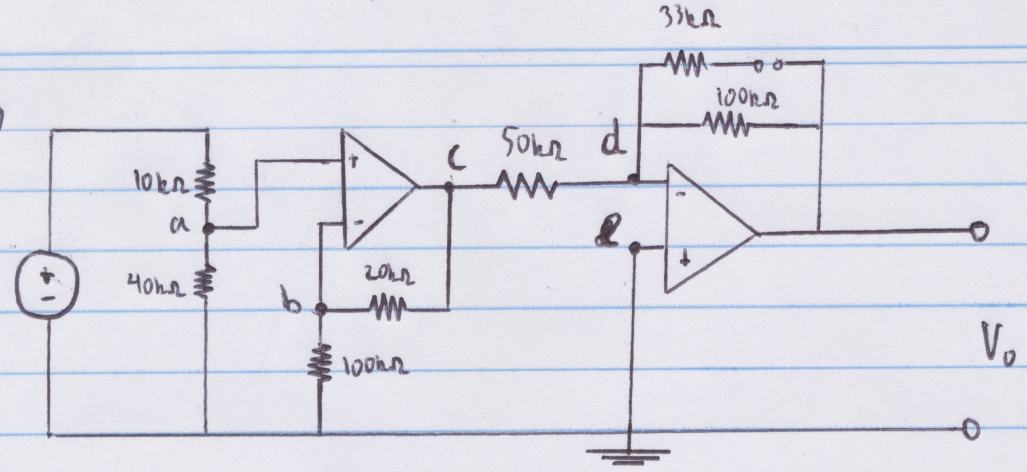
$i_T = i_1 + i_2$

$i_2 = 50\angle -130^\circ - 100\angle 10^\circ$

$= 141.99\angle -156.9^\circ$



Q.5 i)




KCL at node a:

$$\frac{V_a - 10}{10{,}000} + \frac{V_a}{40{,}000} = 0$$

$$5V_a = 40$$

$$V_a = 8V = V_b \text{ (Op amps principle)}$$

KCL at node b:

$$\frac{8}{100{,}000} + \frac{8 - V_c}{20{,}000} = 0$$

$$8 + 40 - 5V_c = 0$$

$$V_c = 9.6V$$

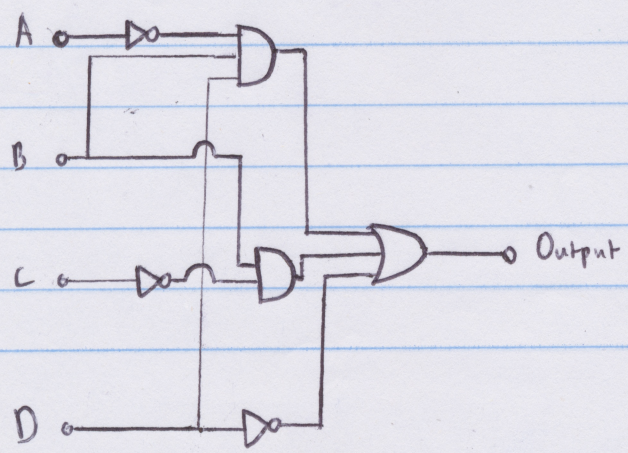
$$V_e = V_d = 0V$$

KCL at node d:

$$\frac{0 - 9.6}{50{,}000} + \frac{0 - V_o}{100{,}000} = 0$$

$$V_o = -19.2V$$

ii)



iii)

| A | B | C | $\bar{A}$ | $\bar{B}$ | $\bar{C}$ | $\bar{A}+B$ | $B+\bar{C}$ | Output |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |