Topic 7 recap

- A **sinusoid** is a signal (voltage or current) in the form of **sine** or **cosine** function.

$$v(t) = V_m \cos(\omega t + \phi)$$

V_m: Amplitude.

$\omega = 2\pi f$: Angular frequency (rad/s).

$(\omega t + \phi)$: Argument of the sinusoid.

ϕ: Phase (in degrees or radians).

- A **phasor** is a **complex quantity** that represents both the **magnitude** and the **phase** of a **sinusoid**.

$$v(t) = V_m \cos(\omega t + \phi) \quad \Leftrightarrow \quad \mathbf{V} = V_m \angle \phi$$

- An electric circuit is called **AC circuit** mostly when it is operated by **sinusoidal** voltage or current **sources**.
  - They can be analyzed in what is known as **sinusoidal steady-state**.
  - All voltages and currents can be written in phasor domain rather than time domain.

Topic 7 recap

- Circuit elements have a fixed relationship between voltage and current phasors.
- Given $v(t) = V_m \cos(\omega t + \phi_v) \Leftrightarrow \mathbf{V} = V_m \angle \phi_v$ as the voltage across an element and $i(t) = I_m \cos(\omega t + \phi_i) \Leftrightarrow \mathbf{I} = I_m \angle \phi_i$ as the current through the element:
  - For resistor R, voltage and current are **in phase**:

$$v = Ri \;\Rightarrow\; \mathbf{V} = R\mathbf{I} = RI_m \angle \phi_i = V_m \angle \phi_v \;\Rightarrow\; \phi_v = \phi_i$$

  - For inductor L, current **lags** voltage by $90°$:

$$v = L\frac{di}{dt} \;\Rightarrow\; \mathbf{V} = j\omega L\mathbf{I} = \omega L I_m \angle(\phi_i + 90°) = V_m \angle \phi_v \;\Rightarrow\; \phi_v = \phi_i + 90°$$

  - For capacitor C, current **leads** voltage by $90°$:

$$v = \frac{1}{C}\int i\,dt \;\Rightarrow\; \mathbf{V} = \frac{1}{j\omega C}\mathbf{I} = \frac{I_m}{\omega C}\angle(\phi_i - 90°) = V_m \angle \phi_v \;\Rightarrow\; \phi_v = \phi_i - 90°$$

Topic 7 recap

- **Impedance Z** of a circuit is the **ratio** of the **phasor voltage** across it to the **phasor current** through it.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = R(\omega) + jX(\omega)\ \Omega, \quad R\text{: Resistance, } X\text{: Reactance.}$$

- **Admittance Y** is the **reciprocal** of impedance.

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{\mathbf{I}}{\mathbf{V}} = G(\omega) + jB(\omega)\ S, \quad G\text{: Conductance, } \quad B\text{: Susceptance.}$$

- Impedances of circuit elements:
  - For resistor R: $\mathbf{Z}_R = R$
  - For inductor L: $\mathbf{Z}_L = j\omega L$
  - For capacitor C: $\mathbf{Z}_C = 1/j\omega C = -j/\omega C$
- Impedances are **combined** in series and parallel in the **same way** as **resistances** in series and parallel.
- Basic circuit laws (**Ohm's** and **Kirchhoff's**) apply to AC circuits in the same manner as DC circuits, as well as voltage and current **divisions**.
  - $\mathbf{V} = \mathbf{ZI}$ (Ohm's law)
  - $\sum \mathbf{I}_k = 0$ (KCL) and $\sum \mathbf{V}_k = 0$ (KVL)