Summer 2016-2017 Final Exam

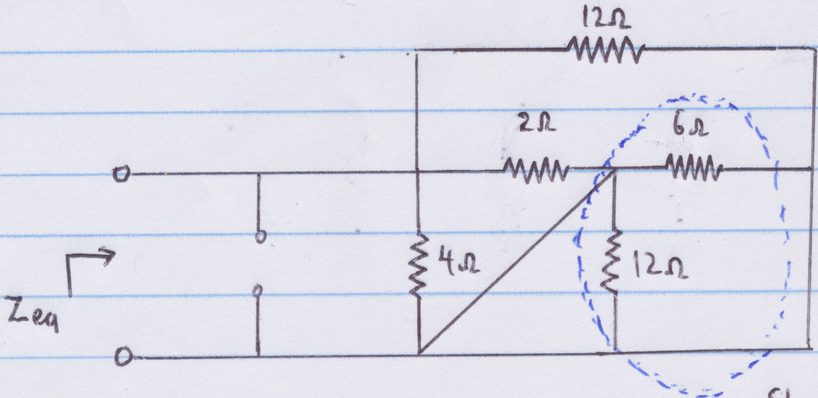
Question 1

i)

a) Under DC conditions

0.5F capacitor ⇒ Open circuit

4H inductor ⇒ Short circuit



Short circuit

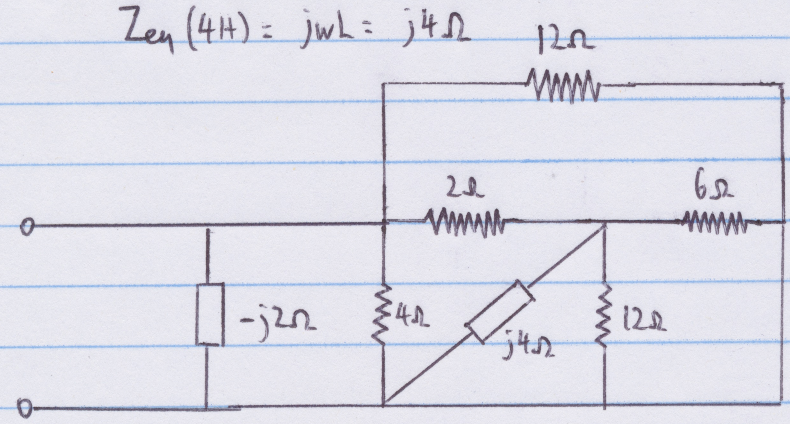
$$Z_{eq} = 12 // 2 // 4$$

$$= \left(\frac{1}{12} + \frac{1}{2} + \frac{1}{4}\right)^{-1} = 1.2\,\Omega$$

b) $w = 1$ rad/s

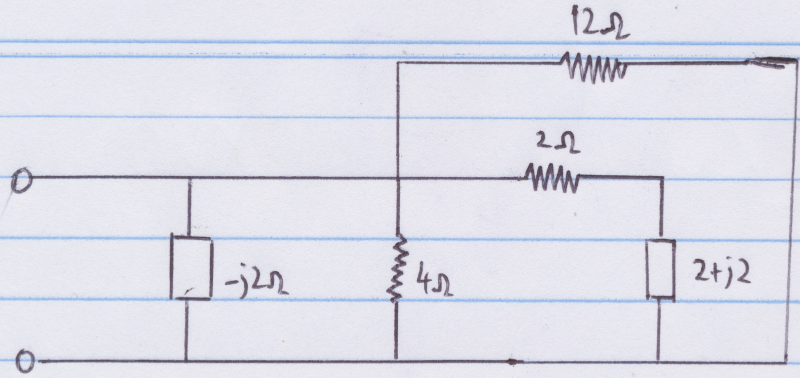
$$Z_{eq}(0.5F) = \frac{-j}{wc} = \frac{-j}{0.5} = -j2\,\Omega$$

$$Z_{eq}(4H) = jwL = j4\,\Omega$$



$$6 // 12 // j4 = \left(\frac{1}{6} + \frac{1}{12} + \frac{1}{j4}\right)^{-1}$$

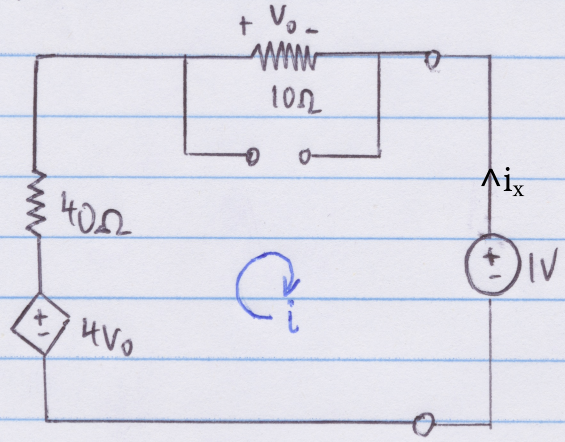
$$= (2 + j2)\,\Omega$$



$$Z_{eq} = -j2 \,//\, 4 \,//\, (4+j2) \,//\, 12$$

$$= \left(\frac{1}{-j2} + \frac{1}{4} + \frac{1}{4+j2} + \frac{1}{12} \right)^{-1} = (1.2 - j0.9)\,\Omega$$

ii) Finding R_{Th}



$$R_{Th} = \frac{1}{i_x}$$

KVL in mesh:

$$-4V_o + 40i + V_o + 1 = 0$$

But $V_o = 10i$

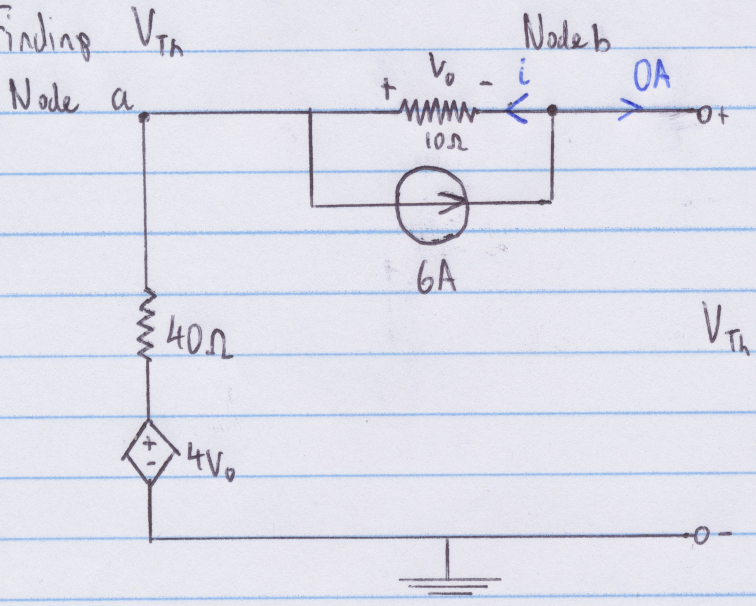
$$40i - 3(10i) = -1$$

$$10i = -1$$

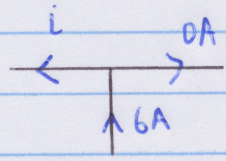
$$i = -0.1\,A \quad \Rightarrow i_x = 0.1\,A$$

$$\therefore R_{Th} = 10\,\Omega$$

Finding V_{Th}



KCL at node b:



$$i + 0 = 6$$
$$\therefore i = 6A$$

Also: $6 = \dfrac{V_b - V_a}{10} \Rightarrow V_a - V_b = -60$

KCL at node a:

$$\frac{V_a - (4x - 60)}{40} + 6 + \frac{V_a - V_b}{10} = 0$$

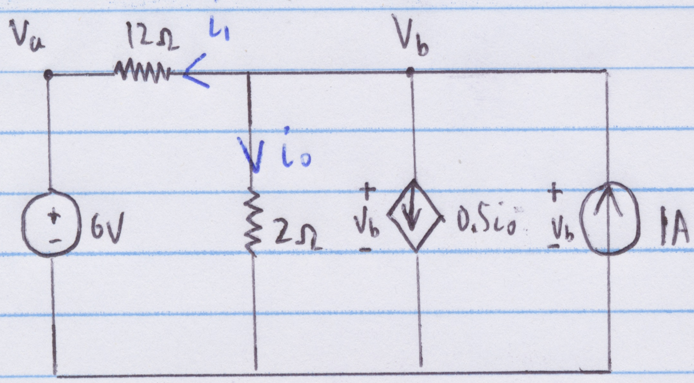
$$V_a + 240 + 240 + 4V_a - 4V_b = 0$$
$$5V_a - 4V_b = -480$$

$$\begin{array}{cc} V_a & V_b \\ \end{array}$$
$$\left(\begin{array}{cc|c} 1 & -1 & -60 \\ 5 & -4 & -480 \end{array}\right) \quad R_2 = R_2 - 5R_1$$

$$\left(\begin{array}{cc|c} 1 & -1 & -60 \\ 0 & 1 & -180 \end{array}\right) \quad \therefore V_b = -180V$$

$$\therefore V_{Th} = V_b - 0$$
$$= -180V$$

iii)



KCL at node b:

$$\frac{V_b - 6}{12} + \frac{V_b}{2} + \frac{1}{2}\left(\frac{V_b}{2}\right) = 1$$

$$V_b - 6 + 6V_b + 3V_b = 12$$
$$10V_b = 18$$
$$V_b = 1.8V$$

Finding i_1:

$$i_1 = \frac{V_b - V_a}{12} = \frac{1.8 - 6}{12} = -0.35A$$

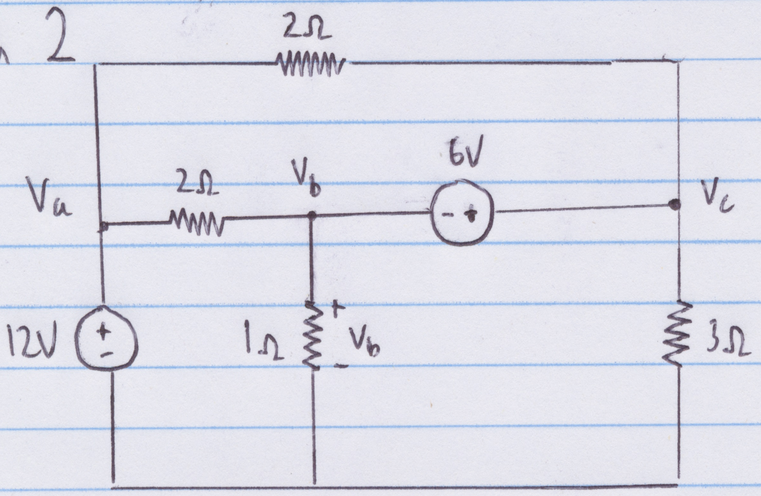
$P_{6V} = -0.35 \times 6 = 2.1\ W$ supplied
$P_{1A} = 1.8 \times -1 = 1.8W$ supplied
$P_{0.5i_o} = \frac{1}{2}\left(\frac{1.8}{2}\right) \times 1.8 = 0.81W$ absorbed

Explanation:

By sign convention, positive power absorbs and negative power supplies.

Question 2

i)



$V_a = 12V$ due to source

Supernode:

$$V_c - V_b = 6V \quad (1)$$

KCL at supernode:

$$\frac{V_b - 12}{2} + \frac{V_b}{1} + \frac{V_c - 12}{2} + \frac{V_c}{3} = 0$$

$$3V_b - 36 + 6V_b + 3V_c - 36 + 2V_c = 0$$

$$9V_b + 5V_c = 72 \quad (2)$$

$$\begin{array}{cc} V_b & V_c \end{array}$$

$$\left(\begin{array}{cc|c} -1 & 1 & 6 \\ 9 & 5 & 72 \end{array}\right) \quad R_2 = R_2 + 9R_1$$

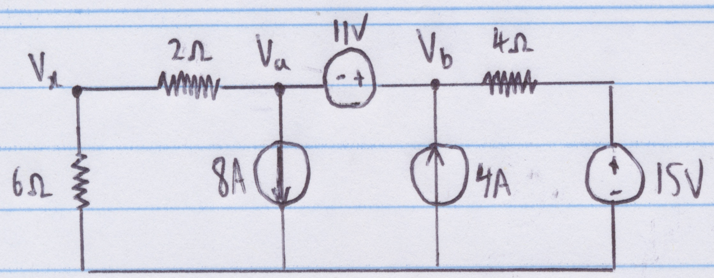
$$\left(\begin{array}{cc|c} -1 & 1 & 6 \\ 0 & 14 & 126 \end{array}\right)$$

$$\therefore \; V_c = \frac{126}{14} = 9V$$

$$V_b = -(6-9) = 3V$$

$$\therefore \; V_{1\Omega} = 3V$$

$$P_{3\Omega} = \frac{V_c^2}{R} = \frac{9^2}{3} = 27W \text{ absorbed}$$

ii) 

KCL at node x:

$$\frac{V_x}{6} + \frac{V_x - V_a}{2} = 0$$

$$4V_x - 3V_a = 0 \quad (1)$$

Supernode between nodes a and b:

$$V_b - V_a = 11V \Rightarrow V_a - V_b = -11 \quad (2)$$

KCL at supernode:

$$\frac{V_a - V_x}{2} + 8 + \frac{V_b - 15}{4} = 4$$

$$2V_a - 2V_x + 32 + V_b - 15 = 16$$

$$-2V_x + 2V_a + V_b = -1 \quad (3)$$

$$\begin{array}{ccc} V_x & V_a & V_b \end{array}$$

$$\left(\begin{array}{ccc|c} 4 & -3 & 0 & 0 \\ 0 & 1 & -1 & -11 \\ -2 & 2 & 1 & -1 \end{array}\right) \quad R_3 = 2R_3 + R_1$$

$$\left(\begin{array}{ccc|c} 4 & -3 & 0 & 0 \\ 0 & 1 & -1 & -11 \\ 0 & 1 & 2 & -2 \end{array}\right) \quad R_3 = R_3 - R_2$$

$$\left(\begin{array}{ccc|c} 4 & -3 & 0 & 0 \\ 0 & 1 & -1 & -11 \\ 0 & 0 & 3 & 9 \end{array}\right)$$

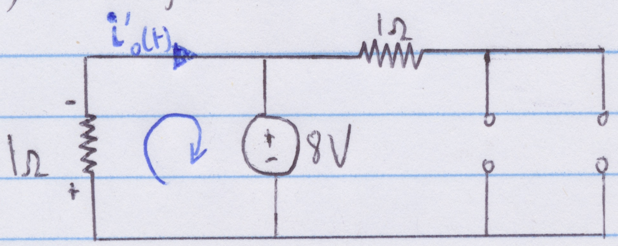
$\therefore V_b = 3V$

$V_a = -8V$

$\underline{\underline{V_x = -6V}}$

iii) Must use superposition

Only turning on 8V source on



Inductor ⇒ Closed circuit

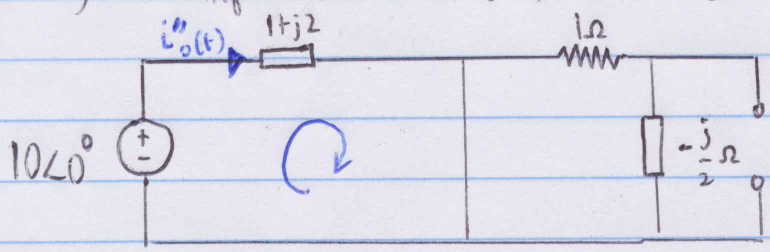
Capacitor ⇒ Open circuit

KVL:

$$i'_o + 8 = 0$$

$$i'_o = -8A$$

Only turning on $10\cos(2t)$ V source on



$W = 2$ rad/s

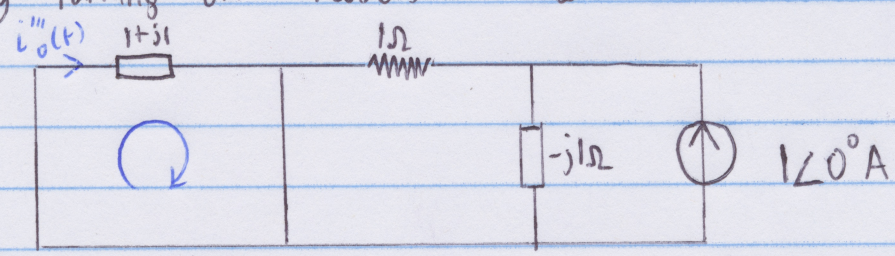
Inductor ⇒ $jWL = j2\Omega$

Capacitor ⇒ $\frac{-j}{WC} = \frac{-j}{2}\Omega$

KVL: $(1+j2)\,i''_o - 10 = 0$

$$i''_o = \frac{10}{1+j2} = 4.472\cos(2t - 63.4^\circ)\ A$$

Only turning on $1\cos(t)$ A source on



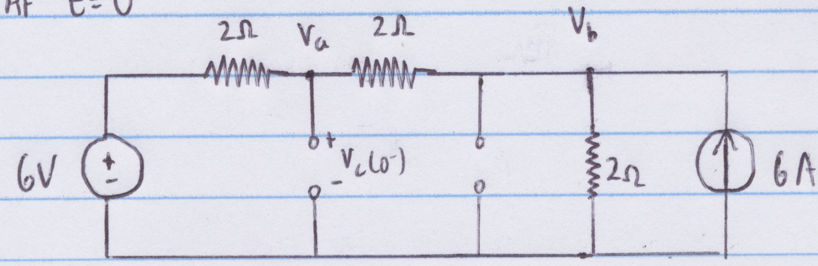
$i'''_o(t) = 0A$ due to short circuit

∴ By superposition:

$$i_o(t) = [-8 + 4.472\cos(2t - 63.4^\circ)]\ A$$

Question 3

i) At $t = 0^-$



KCL at node a:

$$\frac{V_a - 6}{2} + \frac{V_a - V_b}{2} = 0$$

$$2V_a - V_b = 6 \quad (1)$$

KCL at node b:

$$\frac{V_b}{2} + \frac{V_b - V_a}{2} = 6$$

$$-V_a + 2V_b = 12 \quad (2)$$

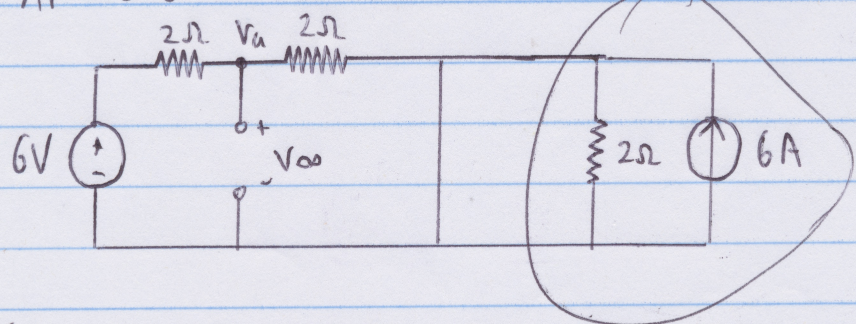
$$(2) \times 2 \Rightarrow \quad -2V_a + 4V_b = 24 \quad (3)$$

$$(1) + (3) \Rightarrow 3V_b = 30$$

$$V_b = 10V$$

$$\therefore V_a = 8V = V_c(0^-)$$

At $t = 0^+$



Short circuit

KCL at node a:

$$\frac{V_a - 6}{2} + \frac{V_a}{2} = 0$$

$$\therefore V_a = 3V = V_\infty$$

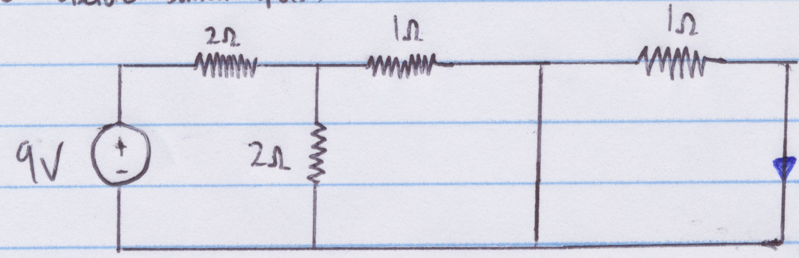
$R_{Th} = 2 // 2$ $\Rightarrow$ $\tau = RC$

$= 1\Omega$ $= 10 ms = 10 \times 10^{-3}$

$V_c(t) = V_\infty + (V_c(0^-) - V_\infty) e^{-\frac{t}{\tau}}$ V

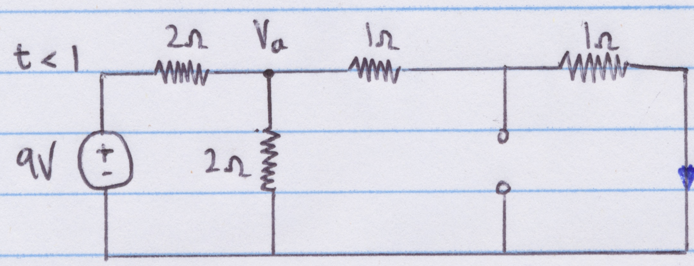
$= 3 + 5e^{-100t}$ V for $t > 0$

ii) At $t = 0^-$ (Before switch opens)



$i_L(0^-) = i_L(0^+) = 0$ A

At $0 < t < 1$



$i_L(t)$

KCL at node a:

$$\frac{V_a - 9}{2} + \frac{V_a}{2} + \frac{V_a}{2} = 0$$

$$3V_a = 9$$

$$V_a = 3V$$

$\therefore$ $i_L(t) = \frac{3}{2}$ A $= i_L(\text{infinity})$

Assuming that the circuit remains in this condition for a long time

$R_{Th} = 2 + 2 // 2 = 3\Omega$

$\tau = \frac{L}{R_{th}} = \frac{4}{3}$ s

$i_L(t) = 1.5 - 1.5e^{-\frac{3}{4}t}$ A for $0 < t < 1$

At $t > 1$ (Circuit is the same as first one in question)

$i_\infty = 0A$

$R_{Th} = 1\Omega$

$\tau = \frac{L}{R} = 4s$

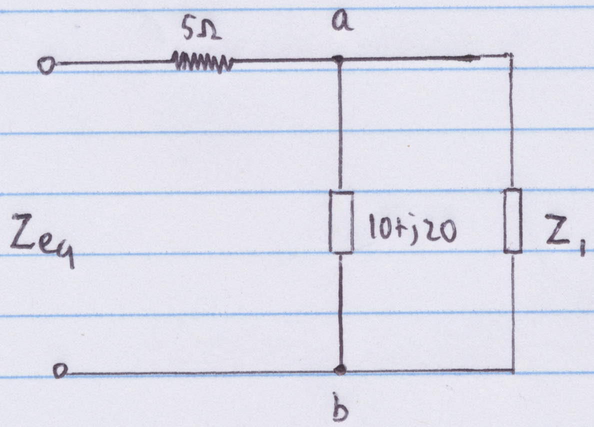
i(1) = 1.5(1-e$^{-0.75}$) = 0.79 A

i(1) is used as initial value in the equation instead of i(0$^+$) since the switch changes at t = 1 s.

$\therefore \; i_L(t) = 0.79\, e^{-\frac{(t-1)}{4}}$ A for $t > 1$

$$i_L(t) = \begin{cases} 1.5\left(1 - e^{-\frac{3}{4}t}\right) \text{ A} & \text{for } 0 < t < 1 \\ 0.79\, e^{-\frac{(t-1)}{4}} \text{ A} & \text{for } t > 1 \end{cases}$$

iii)



$Z_1 = a + jb$

$Z_{eq}(2H) = j(2)(10) = j20\,\Omega$

$Z_{eq} = 5 + Z_1 // 10 + j20$

For Z_{eq} to be purely resistive

$$I_m\left(Z_1 // 10 + j20\right) = 0$$

$$Z_1 // 10 + j20 = \frac{(a+jb)(10+j20)}{(a+10) + j(b+20)} = \frac{(10a - 20b) + j(20a + 10b)}{(a+10) + j(b+20)} = \frac{r_1 \angle \theta_1}{r_2 \angle \theta_2}$$

If $\theta_1 = \theta_2$ the circuit will be purely resistive:

$$\frac{20a + 10b}{10a - 20b} = \frac{b + 20}{a + 10}$$

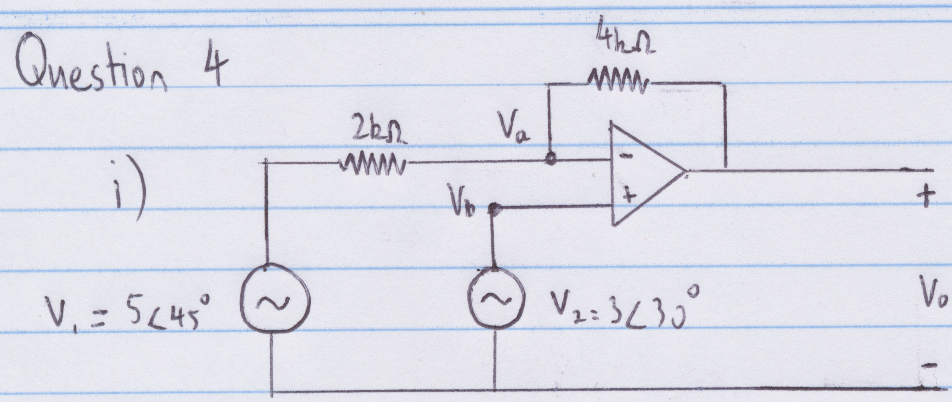
$$20a^2 + 20b^2 + 500b = 0 \Rightarrow a^2 + b^2 + 25b = 0 \Rightarrow a^2 + b(b+25) = 0$$

$$b = -25 \qquad a = 0$$

$$\therefore Z_1 = -j25 = \frac{-j}{\omega C} = \frac{-j}{10C} \Rightarrow C = \frac{1}{250} = 4mF$$

Question 4

i)



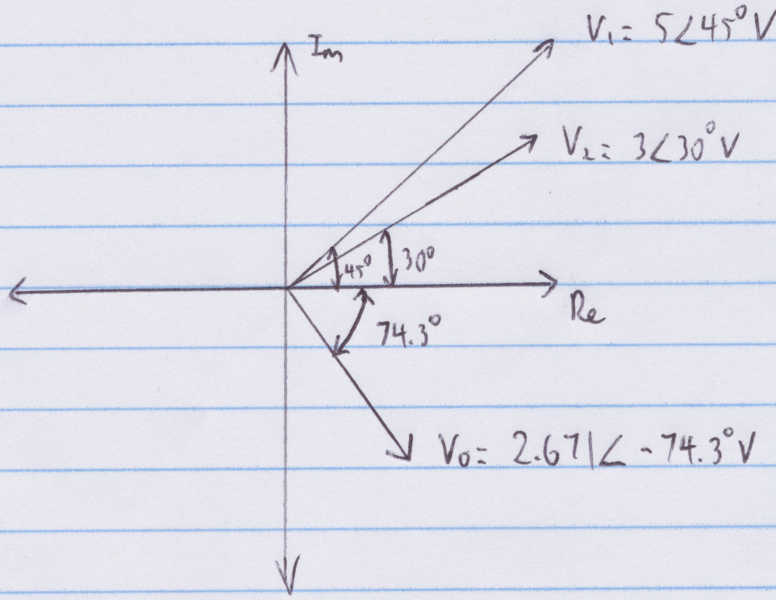
$$V_a = V_b = 3\angle 30°$$

KCL at node a:

$$\frac{3\angle 30° - 5\angle 45°}{2{,}000} + \frac{3\angle 30° - V_o}{4{,}000} = 0$$

$$\therefore V_o = 4{,}000\left(\frac{3\angle 30° - 5\angle 45°}{2{,}000} + \frac{3\angle 30°}{4{,}000}\right)$$

$$= 2.671\angle -74.3° \text{ V}$$

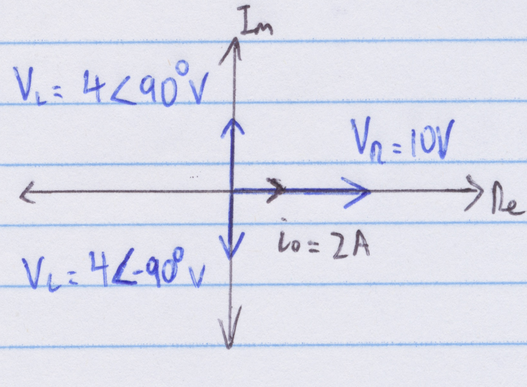


ii) $w = 1$ rad/s

$Z_{eq}(2H) = j2\,\Omega$

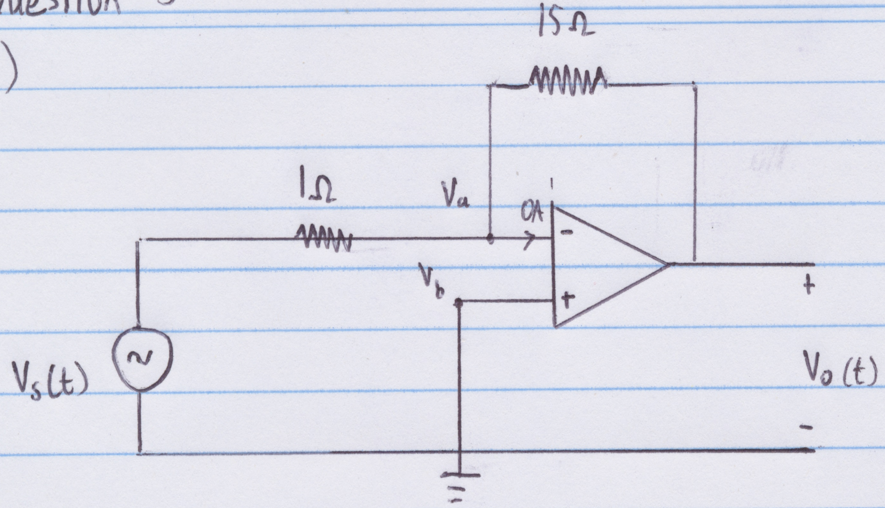
$Z_{eq}(0.5F) = -j2\,\Omega$

$$i_o = \frac{10\angle 0°}{5} = 2A$$



Question 5

i)



$V_b = 0V$

$V_b = V_a$ (Principle of Op amp)

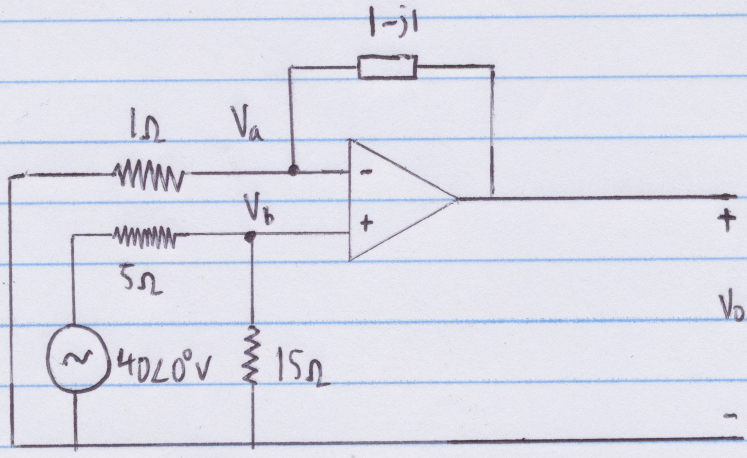
KCL at node a

$$\frac{0 - V_s(t)}{1} + \frac{0 - V_o(t)}{15} = 0$$

$$-V_s(t) = \frac{V_o(t)}{15}$$

$$\therefore V_o(t) = -15\,V_s(t)$$

ii)



KCL at node b:

$$\frac{V_b - 40\angle 0^\circ}{5} + \frac{V_b}{15} = 0$$

$$3V_b - 120 + V_b = 0$$

$$V_b = 30\text{ V} = V_a \quad \text{(Op amp principle)}$$

KCL at node a:

$$\frac{V_a}{1} + \frac{V_a - V_0}{1-j} = 0$$

$$40 + \frac{40}{1-j} = \frac{V_0}{1-j}$$

$$\therefore V_0 = (1-j)\left(40 + \frac{40}{1-j}\right)$$

$$= 80 - j40 = 89.44\angle -26.57^\circ \text{ V}$$

iii) a)

$$Z = \overline{\overline{A+\bar{B}} + \overline{B+C}}$$

$$= \overline{\overline{A+\bar{B}}} \cdot \overline{\overline{B+C}} = (A+\bar{B}).(B+C)$$

$$= A.B + A.C + \cancelto{0}{\bar{B}.B} + \bar{B}.C$$

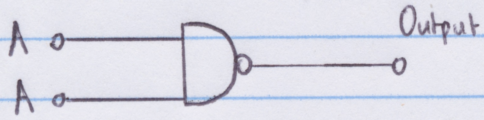
$$= A.B + A.C + \bar{B}.C$$

b)

A	B	C	A.B	A.C	$\bar{B}$.C	Z
0	0	0	0	0	0	0
0	0	1	0	0	1	1
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	0	0	0	0
1	0	1	0	1	1	1
1	1	0	1	0	0	1
1	1	1	1	1	0	1

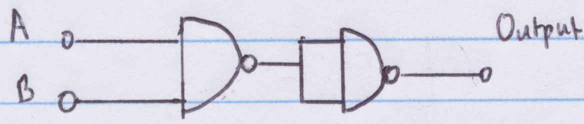
c)

i) NOT gate



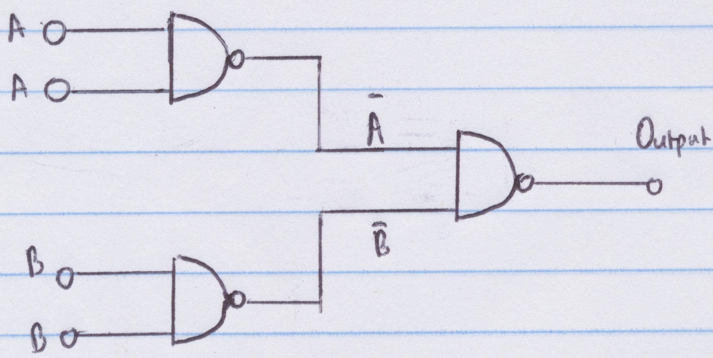
Output = $\overline{A.A} = \bar{A}+\bar{A} = \bar{A}$

ii) AND gate



Output = $\overline{\overline{A.B}} = A.B$

iii) OR gate



Output = $\overline{\bar{A}.\bar{B}} = \bar{\bar{A}} + \bar{\bar{B}} = A+B$