Topic 5 recap

- An inductor is a circuit element that stores energy in its magnetic field.
- Inductance L is the property of inductors by which they oppose to changes in the current flowing through them. It is measured in henry (H).
- Voltage in an inductor is proportional to the time rate of change of its current.

$$v = L\frac{di}{dt}$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$$

- Energy stored in an inductor is proportional to the square of its current.

$$w_L = \frac{1}{2}Li^2$$

- Inductor acts as a short circuit to DC voltage.
- Series combination of inductors is similar to series resistors.

$$L_{eq} = L_1 + L_2 + \cdots + L_N$$

- Parallel combination of inductors is similar to parallel resistors.

$$\frac{1}{L_{\mathrm{eq}}} = \frac{1}{L_1} + \frac{1}{L_2} \cdots + \frac{1}{L_N}$$

Topic 5 recap

- Natural response of RL circuits
  - Behaviour of the circuit (in terms of voltage or current) due to initial energy stored.
  - If the inductor has an initial current $i(0) = I_0$, the natural response of the RL circuit is:
    $$i(t) = I_0 e^{-\frac{t}{\tau}}$$
  - The natural response decays to zero exponentially.
  - The speed at which the current decays is given by the *time constant*:
    - Time constant is the time required for the response to decay to a factor of 1/e or **36.8%** of its **initial value** or to reach **63.2%** of its **final value.**
    - For an RL circuit $\tau = \frac{L}{R}$.
    - The resistance for the time constant is the **Thevenin equivalent resistance** as seen from the inductor terminals.
    - After **5 time constant**, 5τ, the inductor current is considered to have reached its final value.

Topic 5 recap

- Step response of RL circuits
  - The unit step function can be used in electric circuits to model switching.

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

  - The step response is the response to a sudden change in the input sources.
  - The inductor current over time is obtained as an exponential function:

$$i(t) = \begin{cases} I_0, & t < 0 \\ \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{t}{\tau}}, & t > 0 \end{cases}$$

  - If the initial current $i(0) = 0\ A$, the response is known as **forced response**.
  - The step response with non-zero initial condition is known as **complete response**.
    - It can be described as the sum of **transient** and **steady state responses**.
  - The solution to the step response of RL circuits can be given as follows:

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-\frac{t}{\tau}}, \quad t > 0$$

    - $i(0)$: **Initial current** at $t = 0$.
    - $i(\infty)$: **Final or steady-state value** at $t \to \infty$.
    - $\tau = \frac{L}{R_{\text{Th\_}\infty}}$: Time constant at $t \to \infty$.