Faculty of Engineering

School of Electrical Engineering and Telecommunications

ELEC 1111 – Topic 1

Circuit Basics

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Topic 1 Content

This lecture covers:

- Systems of Units.
- Charge, Current and Voltage.
- Power and Energy.
- Independent and Dependent Sources.
- Resistors and Ohm's law.
- Diodes.

Corresponds to Chapter 1 and parts of Chapter 2 of your textbook

Systems of Units

- We need a consistent system of units to describe electrical quantities (e.g. voltage, current) in a circuit.
- The International System of Units (SI Units), is built upon the seven base or fundamental units below.

Quantity	Unit Name	Unit Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Temperature	Kelvin	K
Electric current	Ampere	A
Amount of substance	mole	mol
Luminous intensity	Candela	Cd

Further reading: http://physics.nist.gov/cuu/Units/units.html
http://physics.nist.gov/cuu/Units/current.html (historical context)

Systems of Units

SI derived units

- Derived quantities are defined in terms of the seven base quantities.
- SI derived units are obtained from the seven SI base units according to the expression below.

Quantity	Unit Name	Expression	Symbol
Frequency	Hertz	s^{-1}	Hz
Force	Newton	$kg.m/s^2$	N
Energy (work)	Joule	N.m	J
Power	Watt	J/s	W
Electric Charge	Coulomb	A.s	C
Electric potential	Volt	W/A	V
Resistance	Ohm	V/A	Ω
Conductance	Siemens	A/V	S
Capacitance	Farad	C/V	F
Magnetic flux	Weber	V.s	Wb
Inductance	Henry	Wb/A	H

Systems of Units

Multiples and Prefixes

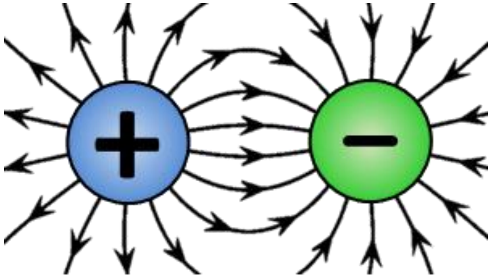
Factor	Name	Symbol
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

Factor	Name	Symbol
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deka	da

600,000,000 mm = 600,000 m = 600 km

Electric Charge

- Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).
- One coulomb is the total charge possessed by 6.25×10^{18} electrons, so a single electron has a charge $e = -1.6\times10^{-19}$ C.
  - This value is also called electronic charge. The charges that occur in nature are integral multiples of the electronic charge.
- Protons carry positive charge of the same magnitude.
- Charge cannot be created or destroyed, only transferred (*law of conservation of charge*).
  - This is an important law as it will help us define many of the laws we use in the analysis of electric circuits.



Notation

- Q : Constant charge
- $q(t)$ or q : Instantaneous or time-varying charge

Exercise

1. How many Coulombs do 93.75×10^{16} electrons represent?

2. How many electrons does it take to have 2 C of charge?

One Coulomb is the total charge possessed by 6.25×10^{18} electrons, so a single electron has a charge of -1.6×10^{-19} C.

Current

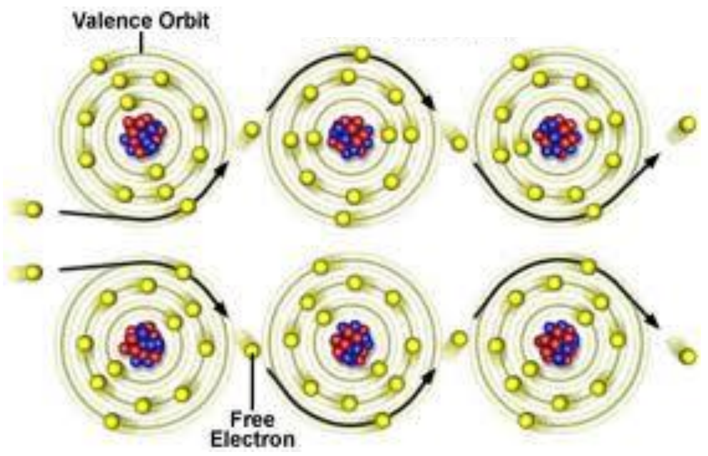
- Electric charge moves, and movement of charge from one place to another results in a transfer of energy.

- Electric current is defined as the time rate of change of charge past a given point (or rate of flow of electrons in a conductor).

$$\boxed{i \triangleq \frac{dq}{dt}}$$

- Current is measured in amperes (A), which are coulombs per second.

$$\boxed{1\ \mathrm{A} = 1\,\frac{\mathrm{C}}{\mathrm{s}}}$$



http://ajitvadakayil.blogspot.com.au/2010/06/basics-for-chemical-tankerman-capt-ajit.html

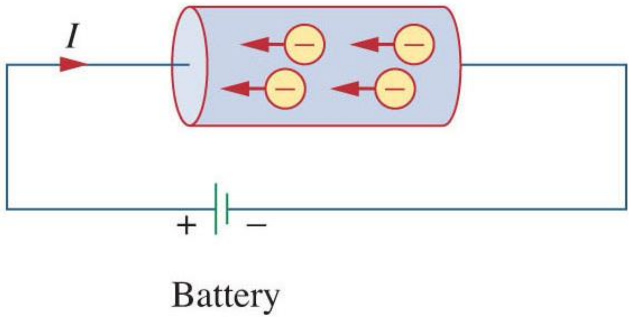
Notation

- I : Constant current
- $i(t)$ or i : Instantaneous or time-varying current

Remark: Since we use "i" as current, complex numbers in electrical engineering are represented with "j". This will come up again in Topic 7!

Current

- Historically, moving charges were thought to be positive.
- Practically, both negative and positive charges are moving when compelled by an electromotive force.
- In metallic conductors, current is created by negatively-charged electrons.
- By convention, the direction of current is the net flow of positive charge.



Convention

- Standard way of describing something so that others in the profession understand what we mean.
- IEEE conventions will be used, e.g., for notations like Q as constant charge.

Current

Charge calculation given a current

- Since $i \triangleq \frac{dq}{dt}$, then charge can be calculated from current as:

$$q = \int_{-\infty}^{t} i \, d\tau = \int_{0}^{t} i \, d\tau + q(0)$$

where $q(0)$ is the charge at $t = 0$ and $i(t) = 0$ for $t \leq 0$.

- For a given time interval $(t_0 \leq t \leq t_1)$:

$$\boxed{q = \int_{t_0}^{t_1} i \, dt + q(t_0)}$$

If nothing is mentioned about initial values (integral constant), we assume zero. This is known as zero initial conditions.

Exercise

Determine the total charge transferred over the time interval $0 \le t \le 10s$ when the current is $i(t) = \frac{1}{2} t \; A$.

$$q = \int_{t_0}^{t_1} i \, dt + q(t_0)$$

$$\int a t^n dt = a \frac{t^{n+1}}{n+1}$$

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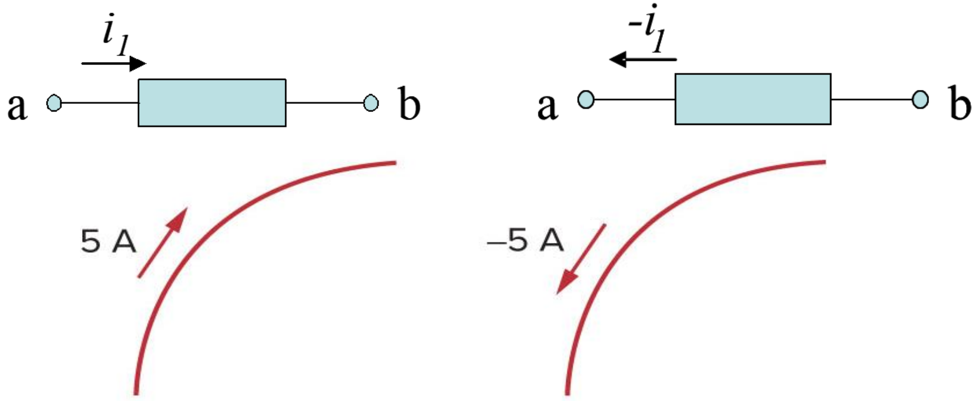
Current

Current value and direction

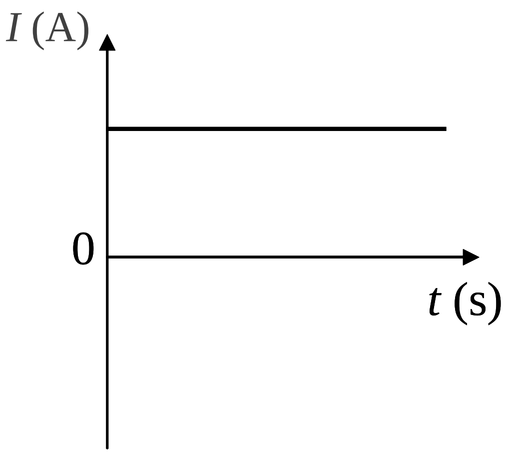
- Current is described by its value and direction.
- By convention, the direction can be chosen arbitrarily in circuits.
- The algebraic sign of the current and the relevant circuit laws will ultimately determine the actual direction in which the current is flowing.
- A positive current through an element is equivalent to a negative current flowing in the opposite direction. E.g. the following two currents are the same:



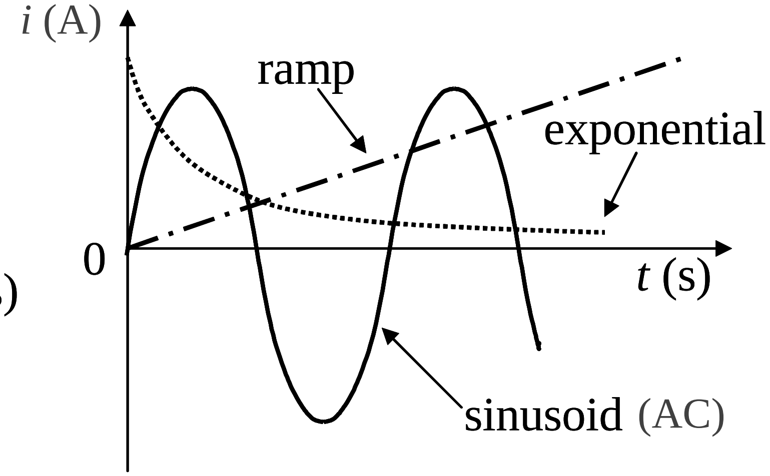
Current

DC vs AC current

- Direct Current (DC): A current whose direction remains constant with time.
- Alternating Current (AC): A time-varying current that changes direction at regular intervals.



DC current



Time-varying current

Voltage

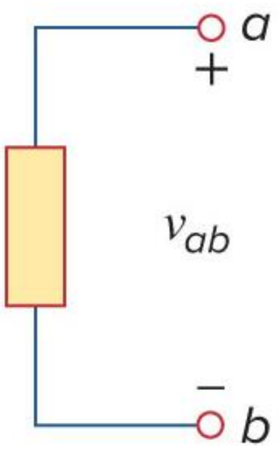
- Electrons move when there is a difference in charge between two locations. This difference is expressed as a *potential difference* or voltage.

- Voltage is defined as the energy (w) required to move a unit of charge (q) from a reference point to another point.

$$\boxed{v \triangleq \frac{dw}{dq}}$$

- Voltage is measured in volts (V) and energy in joules (J). Volts are joules per coulombs:

$$\boxed{1\ \mathrm{V} = 1\,\frac{\mathrm{J}}{\mathrm{C}}}$$



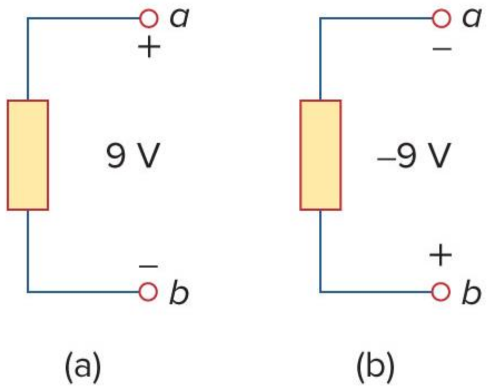
Notation

- V : Constant voltage
- $v(t)$ or v : Instantaneous or time-varying voltage

Voltage

Voltage value and polarity

- Voltage is described by its value and polarity (+ and − signs).
- By convention, the polarity can be chosen arbitrarily in circuits.
- The algebraic sign of the voltage and the relevant circuit laws will ultimately determine the actual polarity.
- A positive voltage across a component is equivalent to a negative voltage with reverse polarity. E.g. the following two voltages are the same:

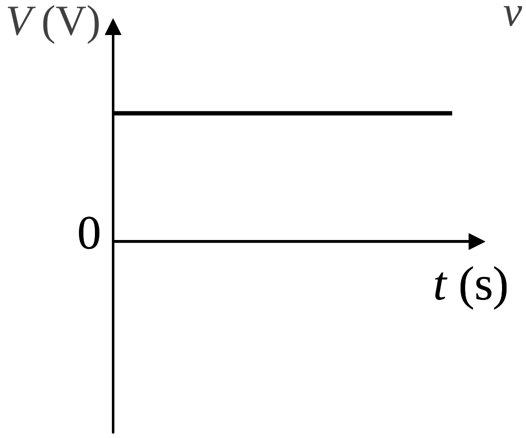


Remark: Adding the signs of the voltage in all of your figures can be extremely helpful in avoiding mistakes.

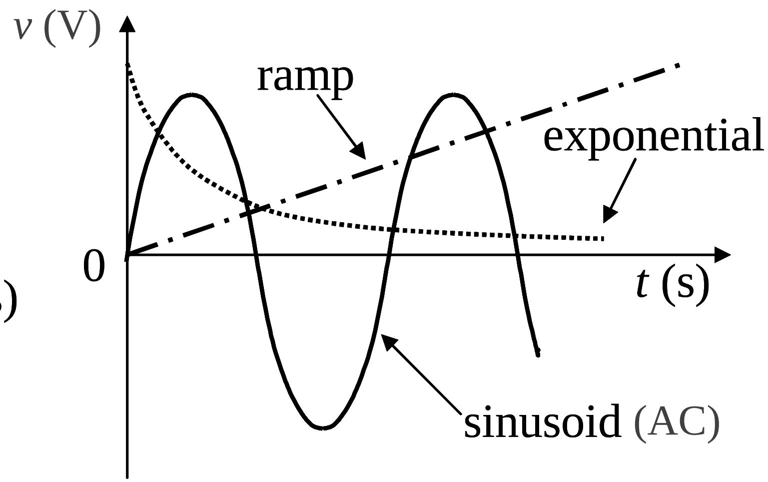
Voltage

DC vs AC voltage

- DC voltage: A voltage with fixed polarity.
- AC voltage: A time-varying voltage with alternating polarity over time.



DC voltage



Time-varying voltage

Exercise

- If on average 10 C of electric charge pass a given point in a wire in 1000 s, what is the mean current in that wire?

- 0.1 C of electric charge has 100 J of energy. What is the voltage of that charge?

$$i \triangleq \frac{dq}{dt} \; ; \; v \triangleq \frac{dw}{dq}$$

Power

- Current and voltage alone are not sufficient to describe the amount of energy consumed by an electric element.
- Power is defined as the rate of change of energy (w).

$$\boxed{p = \frac{dw}{dt}}$$

- Also, power is the product of the voltage across an element and the current flowing through it.

$$p = \frac{dw}{dt} = \frac{dw}{dq}.\frac{dq}{dt} = v.i \quad \rightarrow \quad \boxed{p = vi}$$

- Power is measured in watts (W), which are joules per second.

$$\boxed{1\ \mathrm{W} = 1\ \frac{\mathrm{J}}{\mathrm{s}}}$$

Notation

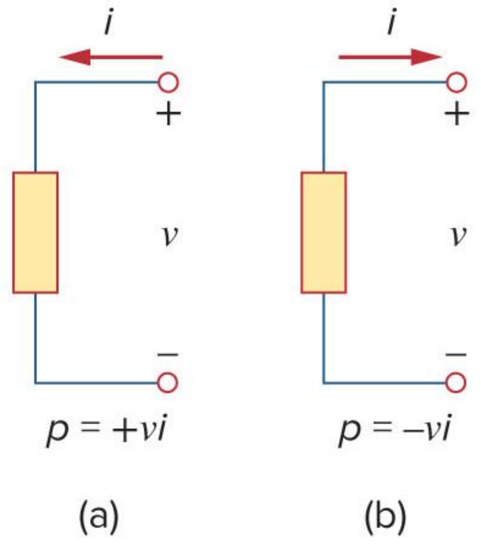
- P : Constant power
- $p(t)$ or p : Instantaneous or time-varying power

Power

Passive Sign Convention

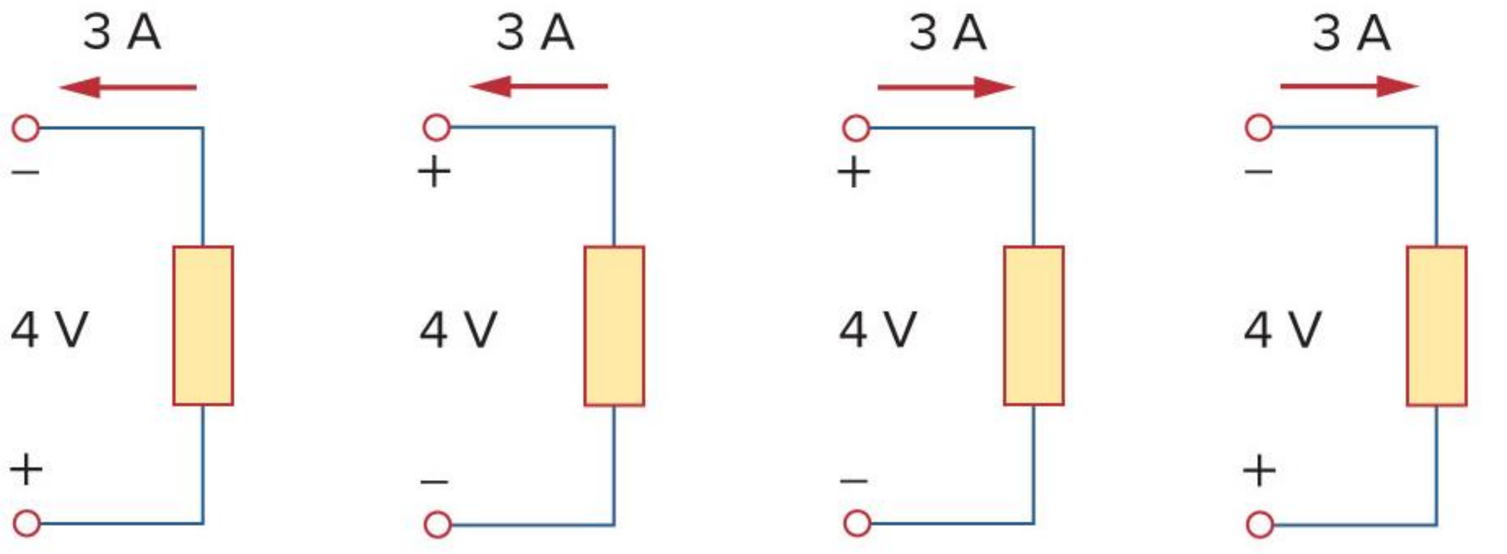
- The direction of current flowing through an element and the polarity of voltage across that element define the algebraic sign of the power.
- **Positive** power is power **absorbed or dissipated** by an element.
  Negative power is power **supplied or generated** by an element.
- Power is positive when the current enters through the positive terminal of an element ($p = +vi$, Fig. (a)).

  Power is negative when the current enters through the negative terminal of the element ($p = -vi$, Fig. (b))



Exercise

Absorbed or Supplied?

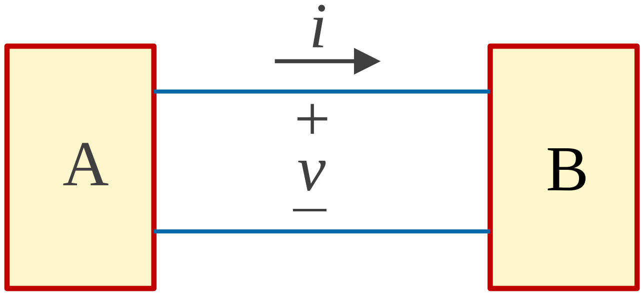


(a) (b) (c) (d)

Power

Power transfer

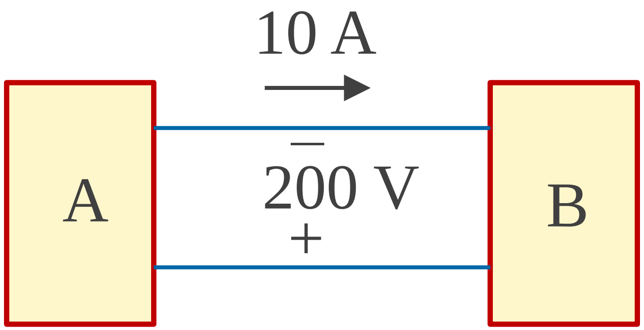
- When elements are connected together in a circuit, power can be transferred from some to others.
- In the following case, element A is supplying and element B is absorbing power. Therefore, power is transferred from A to B.



Exercise

- How much power is being transferred between elements A and B?

- Is the power transfer going from A to B, or from B to A? (take sign convention into account).



$p = vi$

Power

Conservation of Energy

- There can be multiple elements in a circuit. Some elements can supply or generate power and others may absorb or dissipate power.
- Also, in a circuit, energy cannot be created or destroyed (*law of conservation of energy*), so the net power balance should be zero.
- This can be examined by calculating the power supplied or absorbed by each element.

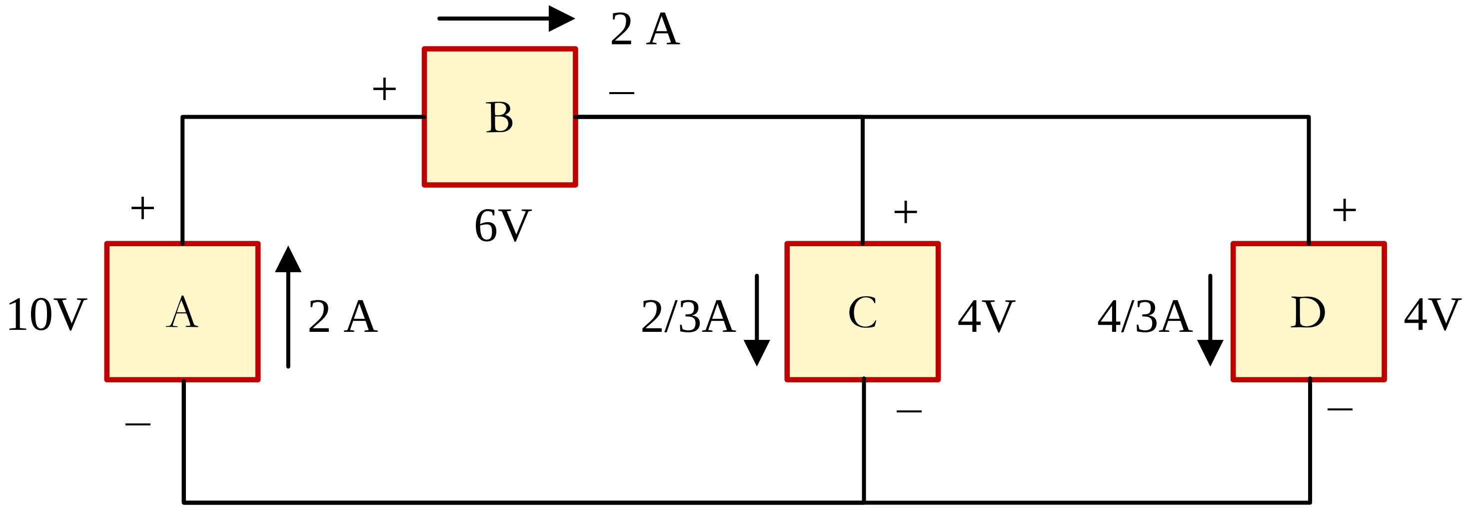
Total power supplied = Total power absorbed

$$\boxed{\sum p_s = \sum p_a}$$

$$\boxed{\sum p = 0}$$

Exercise

Check the net power balance in the following circuit:



Calculate the power ($p=vi$) produced or consumed by each element and add all values.

Energy

- Energy is the capacity to do work.
- Energy can be calculated from power:

$$p = \frac{dw}{dt} \rightarrow w = \int p \, d\tau$$

For a given time interval $(t_0 \leq t \leq t_1)$:

$$\boxed{w = \int_{t_0}^{t_1} p \, dt + w(t_0) = \int_{t_0}^{t_1} vi \, dt + w(t_0)}$$

- Energy is measured in joules (J).
- Another common unit for energy in electrical engineering is the watt-hour (Wh) or kilowatt-hour (kWh).

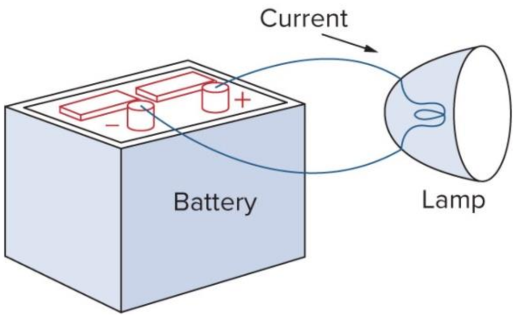
1 Wh = 3,600 J and 1 kWh = 3,600,000 J

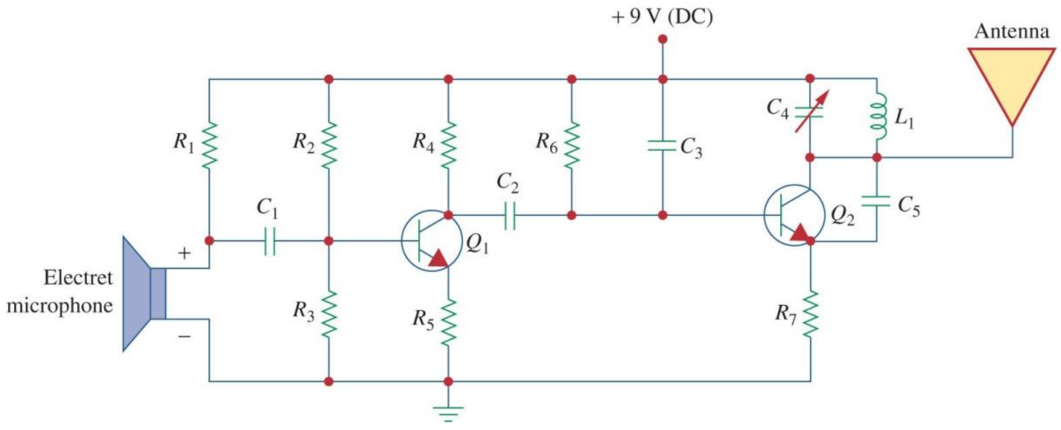
Notation

- W : Constant energy
- $w(t)$ or w : Instantaneous or time-varying energy

Electric circuits

- An electric circuit is an interconnection of electrical elements.
- It may consist of only two elements or many more.





Independent sources

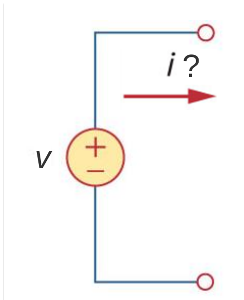
- An ideal independent source is a circuit element that provides a specified voltage or current, and that is completely independent of any other voltage or current in the circuit.

Ideal voltage source

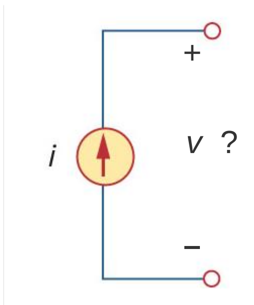
- Generates or dissipates power at a specified voltage with whatever current is required.
- The voltage is known at the terminals, but the current is not.

Ideal current source

- Generates or dissipates power at a specified current with whatever voltage is required.
- The current is known through the terminal, but the voltage is not.



Independent **voltage source**

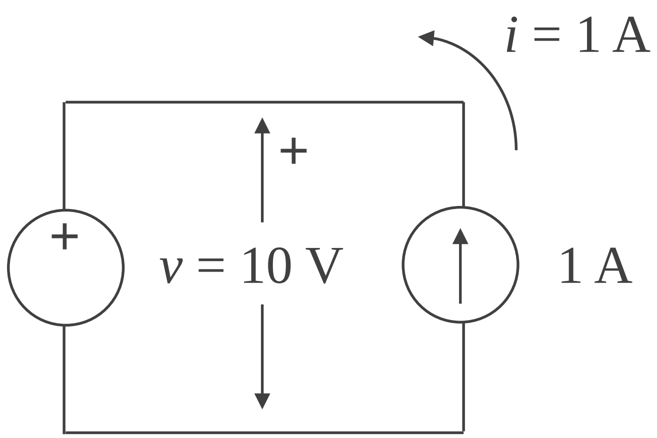


Independent **current source**

Independent sources

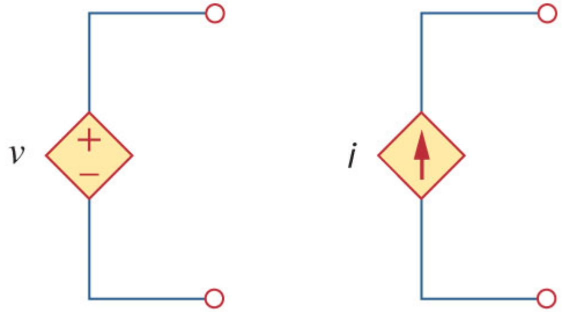
Connected sources

- Since sources can generate and dissipate power, they can be connected.
- In the following example:
  - The voltage source sets the voltage v to 10V, with the upper wire being positive.
  - The current source sets the current i to 1 A flowing anticlockwise.
  - 10 W of power is generated by the current source and dissipated by the voltage source.



Dependent sources

- An ideal dependent (controlled) source is a circuit element whose output value is controlled by another voltage or current in a circuit.
- There are four types:
  - Voltage-controlled voltage source (VCVS).
  - Current-controlled voltage source (CCVS).
  - Voltage-controlled current source (VCCS).
  - Current-controlled current source (CCCS).

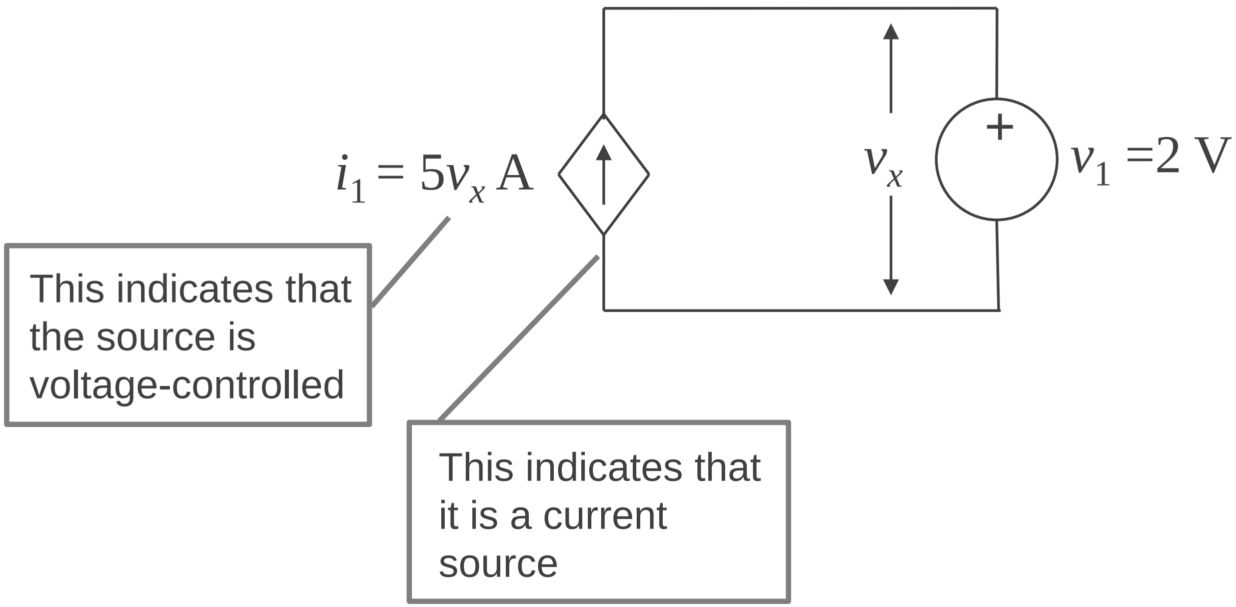


Dependent voltage source

Dependent current source

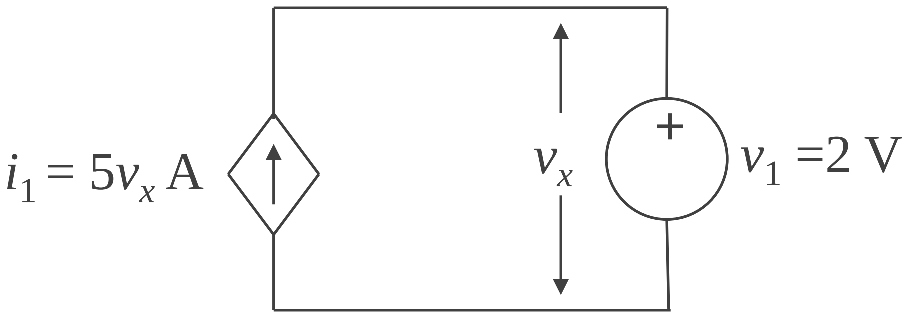
Dependent sources

- The symbol indicates the type of source (voltage or current).
- The expression next to the symbol indicates whether it is voltage-controlled or current-controlled.
  - The example below shows a voltage-controlled current source.



Exercise

Find i_1 in the following circuit.



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Ideal vs Real sources

- Ideal voltage and current sources can generate/dissipate infinite power, while actual voltage and current sources have limited output power:
  - Real voltage sources have an upper current limit.
  - Real current sources have an upper voltage limit.

- Ideal voltage sources have no internal resistance (series), while real voltage sources have a very low, but non-zero internal resistance.

- Ideal current sources have infinite internal resistance (parallel), while real current sources have a very high, but non-infinite internal resistance.

Resistivity

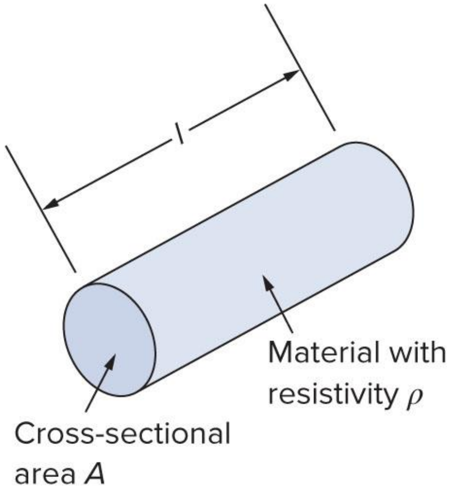
- **Resistivity** (ρ) is a measure of the ability to resist the flow of charge by a certain material.

Material	Resistivity	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

Resistance

- **Resistance** is a physical property of a circuit element that impedes the flow of charge. It is measured in Ohms, Ω.
- The resistance *R* of a conducting wire of uniform cross-section is a function of its length, l, cross sectional area, A, and material's resistivity, ρ.

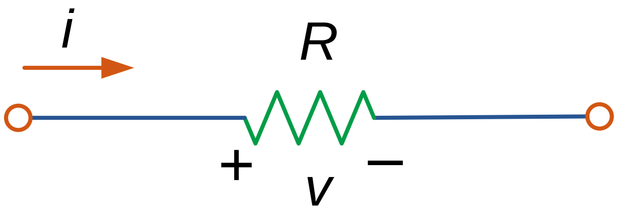
$$\boxed{R = \rho \frac{l}{A}}$$

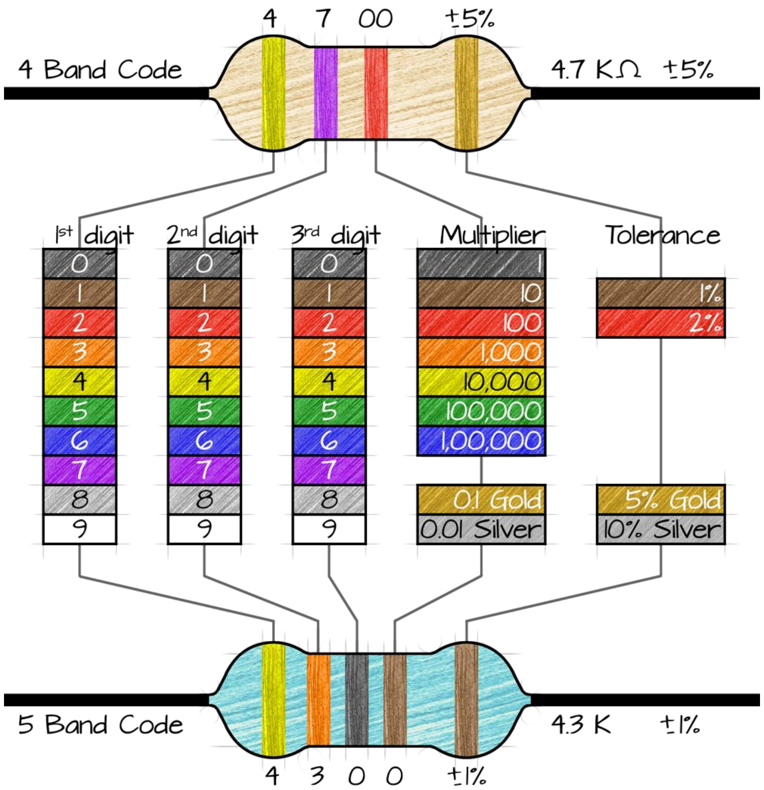


Resistance

Resistor

- **Resistor** is the circuit element which models the current-resisting behaviour of a material.

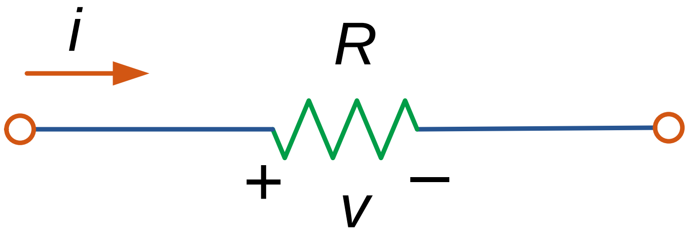




Resistor

- The voltage across a resistor is directly **proportional** to the current flowing through it.
- The constant of proportionality is the **resistance, R.**
- **Ohm's law** states:

$$\boxed{v = Ri}$$

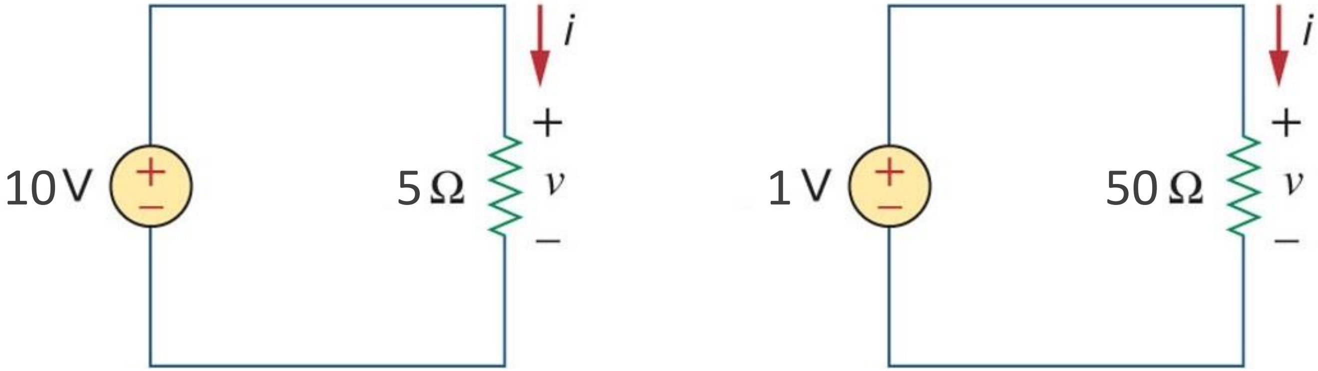


- Resistance is measured in Ohms, Ω, which are volts per ampere.

$$R = \frac{v}{i} = \frac{1\,\mathrm{V}}{1\,\mathrm{A}} = 1\,\Omega$$

Exercise

- Calculate the current flowing through the resistor in the following circuits:



$v = Ri$

Conductance

- **Conductance** is the ability of an element to conduct electric current.
- It is the reciprocal of resistance.
- Conductance *G* is measured in mho (℧) or siemens (S). Siemens are amperes per volt.

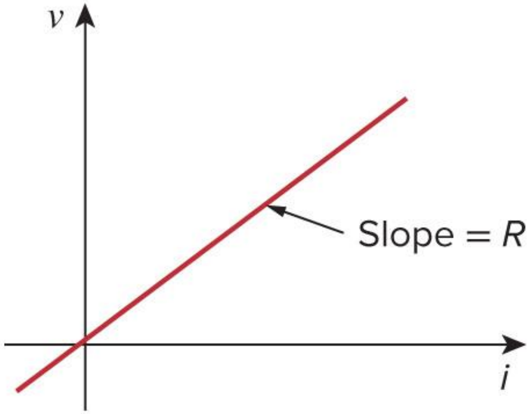
$$\boxed{G = \frac{1}{R} = \frac{i}{v}}$$

$$1S = 1℧ = 1\frac{A}{V}$$

Linear vs nonlinear resistance

Linear

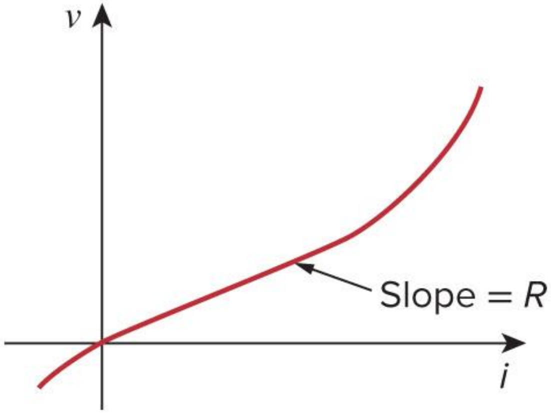
- Resistors have linear resistance. They obey Ohm's law.
- Their current-voltage (i-v) graph or v-i characteristic is a straight line passing through the origin with positive slope (Fig.(a)).



(a)

Nonlinear

- Some circuit elements have nonlinear resistance, so they **do not** obey Ohm's law.
- Resistance varies with current.
- A sample i-v graph of an element with nonlinear resistance is shown in Fig.(b).
- Examples of this are incandescent light bulbs.

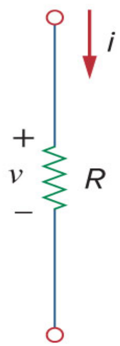


(b)

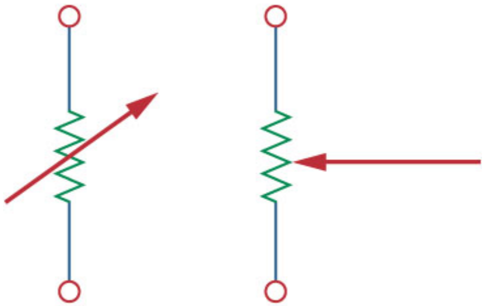
Fixed and variable resistors

- Resistors can be fixed or variable.
- Most resistors are of the fixed type, meaning their resistance remains constant.

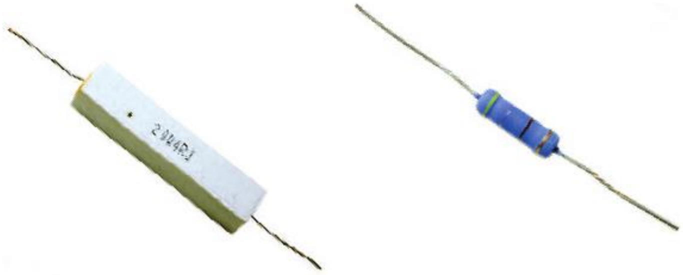
Fixed-valued resistors

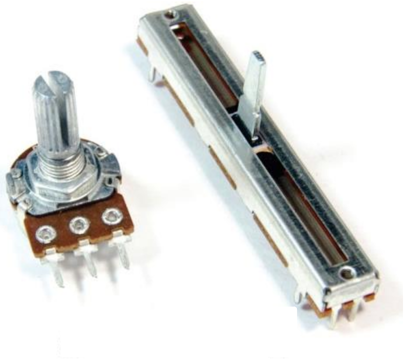


Variable resistors



Potentiometer

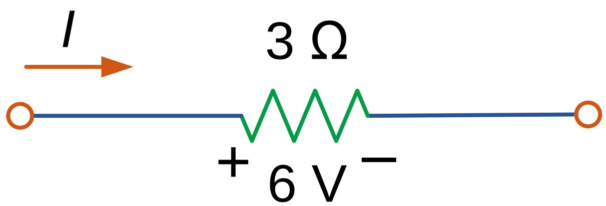


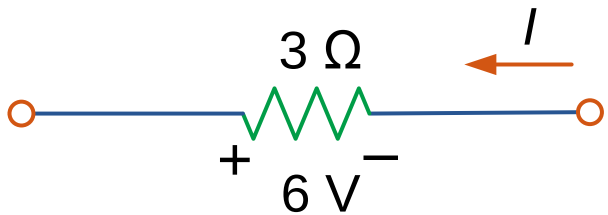


For further reading: http://www.resistorguide.com/

Polarity convention

- Resistors are passive elements which **always dissipate power.**
- Ohm's law requires conforming to the **passive sign convention.**
  - Current flows from the higher potential (**positive terminal**) to the lower potential (**negative terminal**).





$$i = \frac{v}{R} = \frac{6\ \text{V}}{3\ \Omega} = 2\ \text{A}$$

$$i = -\frac{v}{R} = -\frac{6\ \text{V}}{3\ \Omega} = -2\ \text{A}$$

Power dissipation

- Using Ohm's law, we can derive **two forms** of the power equation for resistors:

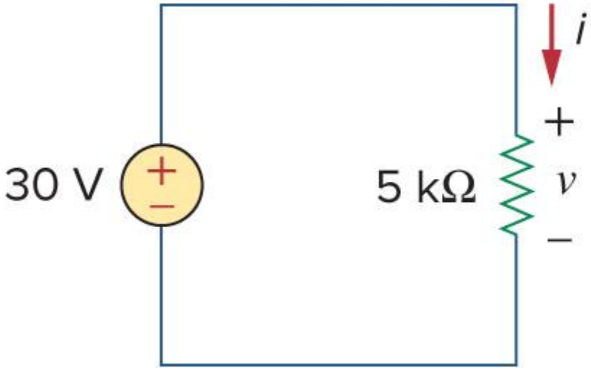
$$1.\ \begin{cases} p = vi \\ v = Ri \end{cases} \quad \Rightarrow \quad \boxed{p = Ri^2}$$

$$2.\ \begin{cases} p = vi \\ i = \dfrac{v}{R} \end{cases} \quad \Rightarrow \quad \boxed{p = \dfrac{v^2}{R}}$$

- **Power** in a resistor is **always positive**, since resistors **always dissipate** power (they can never generate power).

Exercise

- Voltage v across the resistor?
- Current i through the resistor?
- Dissipated power p?
- Conductance G?



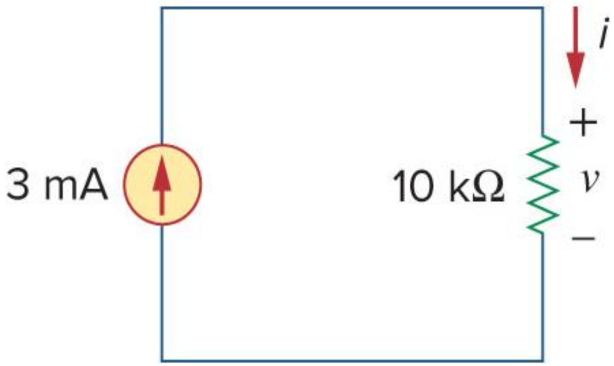
$$v = Ri$$

$$p = vi = Ri^2 = \frac{v^2}{R}$$

$$G = \frac{1}{R}$$

Exercise

- Current i going through the resistor?
- Voltage v across the resistor?
- Dissipated power p?
- Conductance G?



$$v = Ri$$

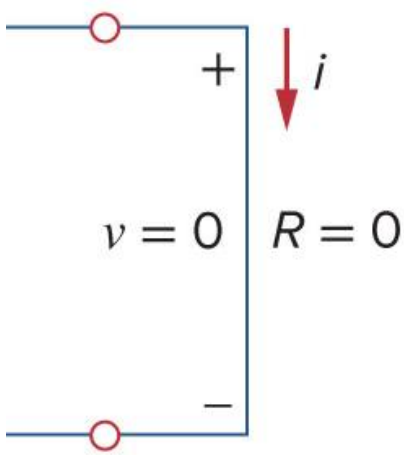
$$p = vi = Ri^2 = \frac{v^2}{R}$$

$$G = \frac{1}{R}$$

Short and open circuits

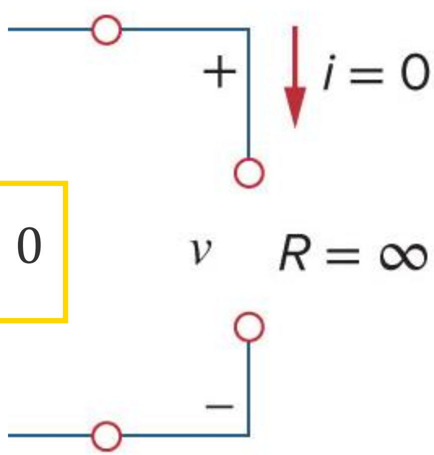
- A circuit element with almost **zero resistance** is called a **short circuit**.
  - Voltage in a short circuit is zero.
  - Ideally, any current may flow through a short circuit.
- A circuit element with **infinite resistance** is called an **open circuit**.
  - Current in an open circuit is zero.
  - Ideally, any voltage may drop across an open circuit.

$$\boxed{v = Ri = 0}$$



Short circuit

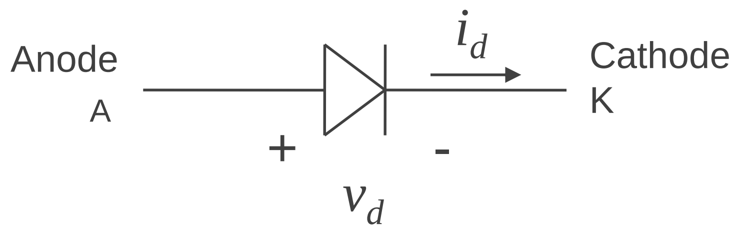
$$\boxed{i = \lim_{R\to\infty} \frac{v}{R} = 0}$$



Open circuit

Diodes

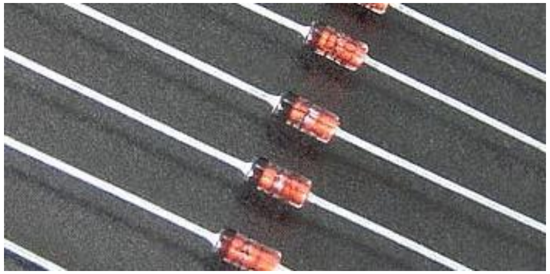
- A diode is a semiconductor electrical component that allows the flow of current in only one direction.
- Current flows in the direction of the arrow, and not against the direction of the arrow.
- Voltage is required to start current flow in the forward direction.
  - When v_d > forward voltage, the diode is **forward biased**, and $i_d > 0$.
  - When v_d < forward voltage, the diode is **reverse biased**, and $i_d \approx 0$.



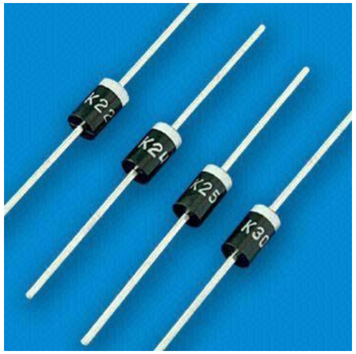
Diodes

- There are many different types of diodes

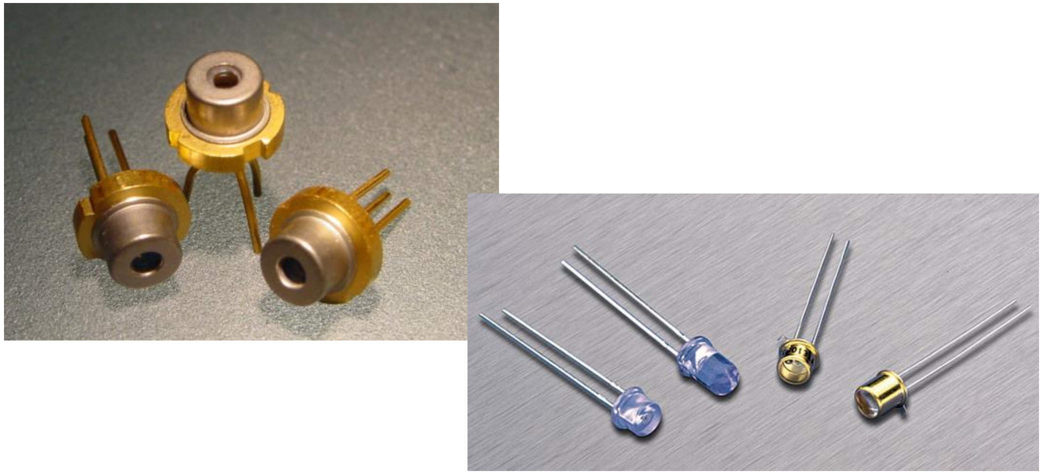
Light Emitting Diodes (LEDs)



Small Signal Diodes



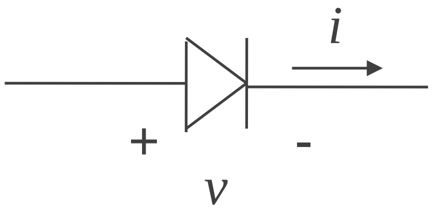
Power Diodes

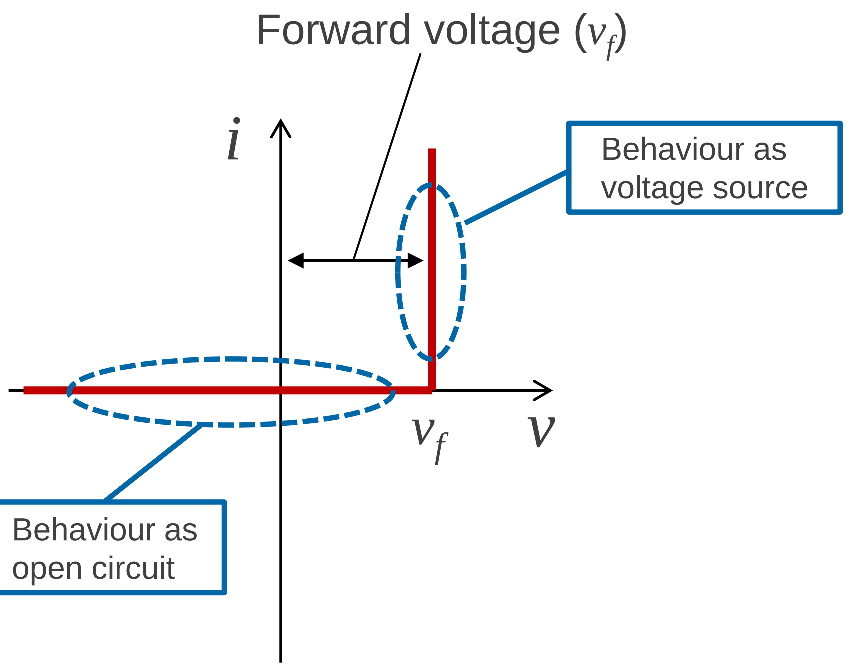


Laser diodes and photodiodes

Simplified diode characteristic

- Diodes have a *non-linear* v-i characteristic.
  - When the diode is forward biased, it behaves as a voltage source of value v_f V.
  - When the diode is reverse biased, it behaves as an open circuit (i = 0).





Analysis of circuits with diodes

Split into two cases:

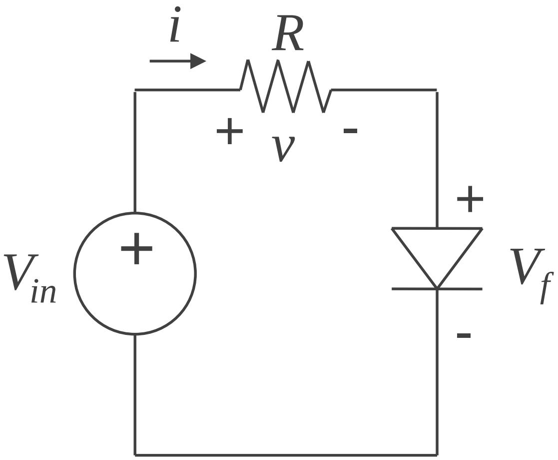
- $V_{in} < V_f$ (reversed biased → 'off')

  No current flows through diode $\Rightarrow$ i = 0 and v = 0

- $V_{in} < V_f$ (forward biased → 'on')

  Diode has voltage $v_d = V_f$, so $v = V_{in} - V_f$ (according to KVL, as we will see in Lecture 2), and

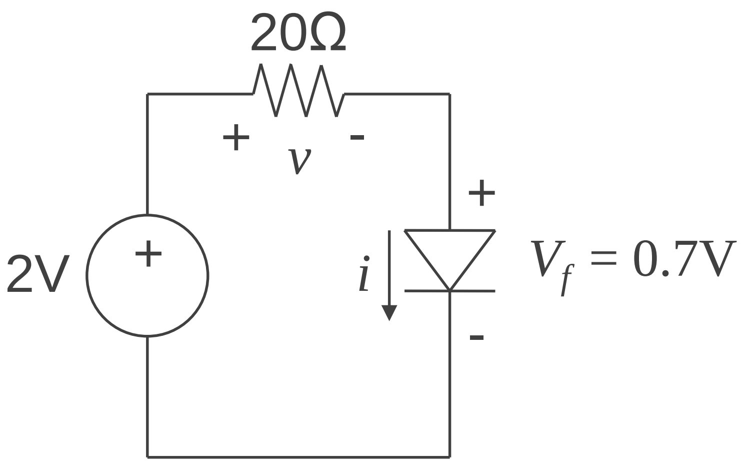
$$i = \frac{v}{R} = \frac{V_{in} - V_f}{R}$$



When a diode is 'on', it **always** has a voltage drop of V_f across it

Exercise

- Calculate the current flowing through the resistor in the following circuit:



$$v = Ri$$
$$v = V_{in} - V_f$$

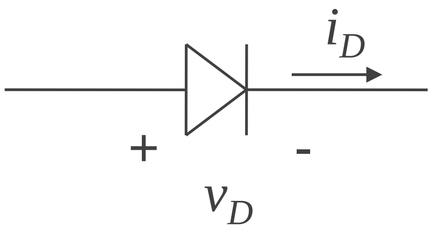
Full diode characteristic

- The diode can be better modelled using Shockley's equation. In its simplest form:

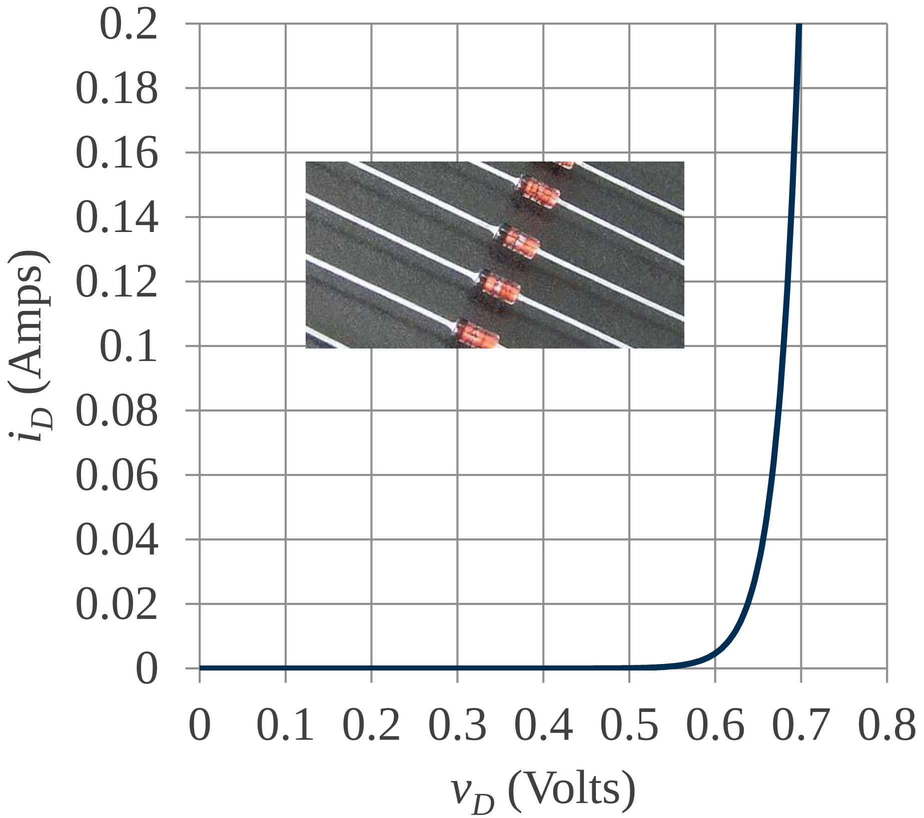
Saturation current

Thermal voltage

$$i_D = I_S \exp\left(\frac{v_D}{0.026}\right)$$



VI Curve for I_s = 4.45 x 10^{-13}



Questions?