QUESTION 1 [20 marks]

a. (10 marks) i. (8 marks) Apply mesh analysis and find the mesh equations using the labels provided (DO NOT solve the equations).

ii. (2 marks) Find the power dissipated in the 20Ω resistor in the circuit shown in Figure 1, given $i_1 = 64.8$ A, $i_2 = 68.4$ and $i_3 = 39$ A.

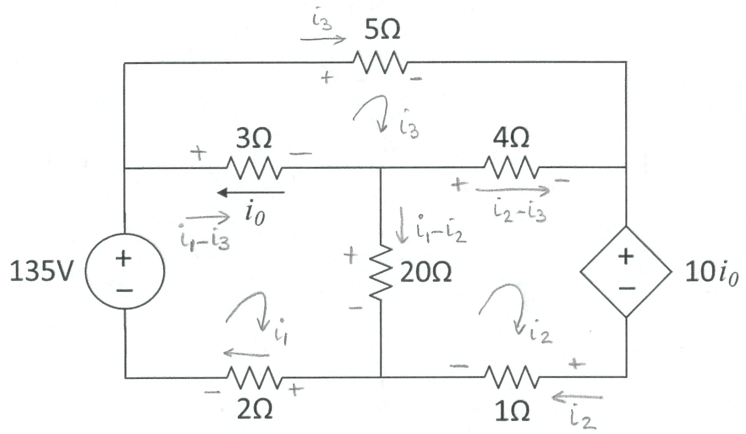


Figure 1

a) Using mesh analysis (KVL @ each mesh):

$$-135 + 3(i_1 - i_3) + 20(i_1 - i_2) + 2i_1 = 0$$
$$-20(i_1 - i_2) + 4(i_2 - i_3) + 10i_o + i_2 = 0$$
$$5i_3 - 4(i_2 - i_3) - 3(i_1 - i_3) = 0$$
$$i_o = -(i_1 - i_3) = i_3 - i_1$$

$\Rightarrow$

$$25i_1 - 20i_2 - 3i_3 = 135$$
$$-30i_1 + 25i_2 + 6i_3 = 0$$
$$-3i_1 - 4i_2 + 12i_3 = 0$$

b)

$i_{20\Omega} = i_1 - i_2 = 64.8 - 68.4 = -3.6$ A

$\boxed{P_{20\Omega} = i_{20\Omega}^2 \times 20 = (-3.6)^2 \times 20 = 259.2 \text{ W}}$

b. **(10 marks)** The variable resistor R_0 in the circuit shown in Figure 2 is adjusted for maximum power transfer.

i. **(2 marks)** Find the value of R_0.

ii. **(8 marks)** Find the maximum power that can be delivered to R_0.

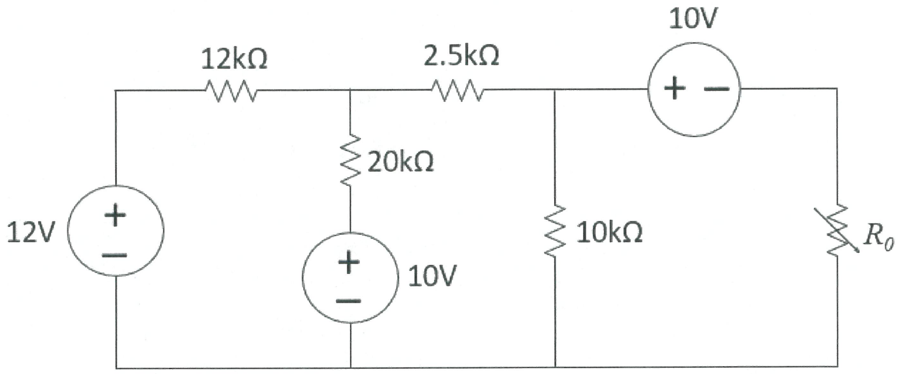
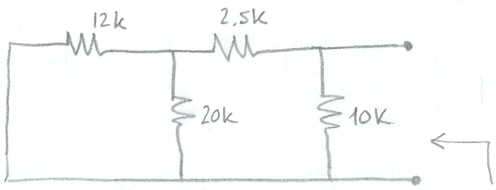


Figure 2

i) $R_0 = R_{th}$

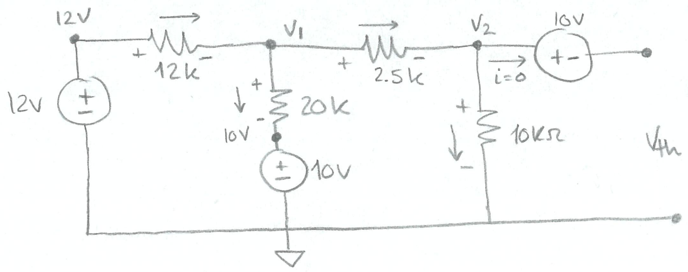


$\boxed{R_0 = R_{th} = 10 \| (2.5 + (12 \| 20))\ k\Omega = 5\ k\Omega}$

$12 \| 20 = \frac{240}{32} = 7.5\Omega$

ii) $P_{max} = \frac{V_{th}^2}{4R_{th}}$

We calculate V_{th}. We can use e.g. nodal analysis:



KCL @ node 1: $\frac{12-V_1}{12\cancel{k}} - \frac{V_1-10}{20\cancel{k}} - \frac{V_1-V_2}{2.5\cancel{k}} = 0 \Rightarrow 1 - \frac{V_1}{12} - \frac{V_1}{20} + \frac{1}{2} - \frac{V_1}{2.5} + \frac{V_2}{2.5} = 0$ ① (×120)

KCL @ node 2: $\frac{V_1-V_2}{2.5\cancel{k}} - \frac{V_2}{10\cancel{k}} = 0 \Rightarrow \frac{V_1}{2.5} - \frac{V_2}{2.5} - \frac{V_2}{10} = 0$ ② (×10)

① $120 - 10V_1 - 6V_1 + 60 - 48V_1 + 48V_2 = 0$
② $4V_1 - 4V_2 - V_2 = 0$

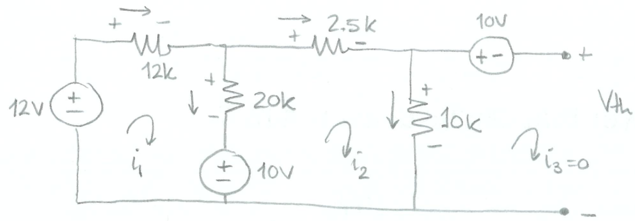
$-64V_1 + 48V_2 = -180$
$4V_1 - 5V_2 = 0$ (×16)

$-\cancel{64V_1} + 48V_2 = -180$
$\cancel{64V_1} - 80V_2 = 0$
$-32V_2 = -180$
$V_2 = 180/32 = 5.625\ V$

KVL @ mesh 3: $-V_2 + 10 + V_{th} = 0 \Rightarrow V_{th} = V_2 - 10 = -4.375V$

$\boxed{P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{(-4.375)^2}{4 \times 5000} = 957\ \mu W}$

* V_{th} calculation using <u>mesh analysis</u>



$$\left.\begin{array}{l} -12 + 12000\, i_1 + 20000\,(i_1 - i_2) + 10 = 0 \\ -10 - 20000\,(i_1 - i_2) + 2500\, i_2 + 10000\, i_2 = 0 \end{array}\right\} \quad \left.\begin{array}{ll} 32000\, i_1 - 20000\, i_2 = 2 & (\times 2) \\ -20000\, i_1 + 32500\, i_2 = 10 & (\times 3.2) \end{array}\right\}$$

$$\left.\begin{array}{l} \cancel{64000}\, i_1 - 40000\, i_2 = 4 \\ -\cancel{64000}\, i_1 + 104000\, i_2 = 32 \end{array}\right\}$$

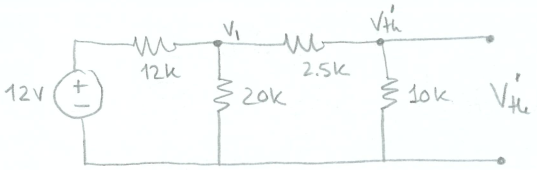
$$64000\, i_2 = 36 \implies i_2 = 36/64000 = 562.5\,\mu A$$

Use KVL in mesh 3 to find V_{th}:

$$-10000\, i_2 + 10 + V_{th} = 0 \implies V_{th} = 10000\, i_2 - 10 = 5.625 - 10 = \underline{-4.375V}$$

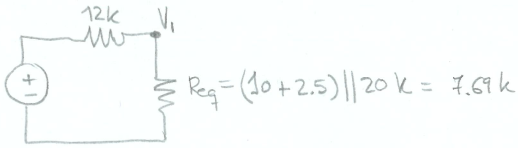
* V_{th} calculation using <u>superposition</u>

- V_{th}' due to 12V source:



$\equiv$

Use voltage division (twice) to find V_{th}':

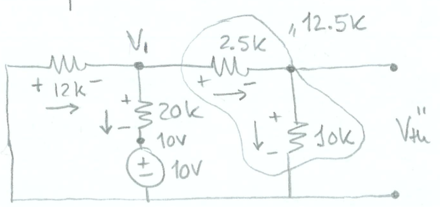


$$V_1 = 12\,\frac{R_{eq}}{R_{eq} + 12k} = 12 \times \frac{7.69\,\cancel{k}}{19.69\,\cancel{k}} = 4.687\text{ V}$$

voltage divider

$$V_{th}' = V_1\,\frac{10\cancel{k}}{2.5\cancel{k} + 10\cancel{k}} = 4.687 \times \frac{10}{12.5} = 3.75\text{ V}$$

- V_{th}'' due to first 10V source:



Use nodal analysis (e.g.) to find V_1 and V_{th}'':

$$\frac{-V_1}{12\cancel{k}} = \frac{V_1 - 10}{20\cancel{k}} + \frac{V_1}{12.5\,\cancel{k}}$$

$$-\frac{V_1}{12} - \frac{V_1}{20} + 0.5 - \frac{V_1}{12.5} = 0$$

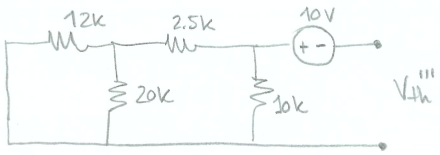
$$\left(-\frac{1}{12} - \frac{1}{20} - \frac{1}{12.5}\right) V_1 = -0.5\,; \quad -0.213\, V_1 = -0.5$$

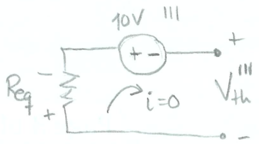
$$V_1 = 2.34V$$

voltage divider

$$V_{th}'' = 2.344\,\frac{10\cancel{k}}{10\cancel{k} + 2.5\cancel{k}} = 1.875\text{ V}$$

* V_{th}''' due to second 10V source:





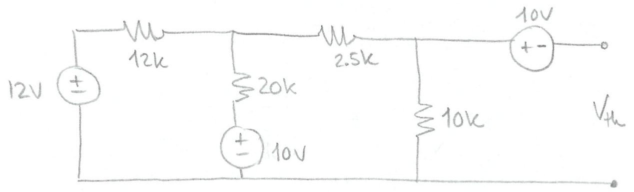
Use KVL to find V_{th}''':

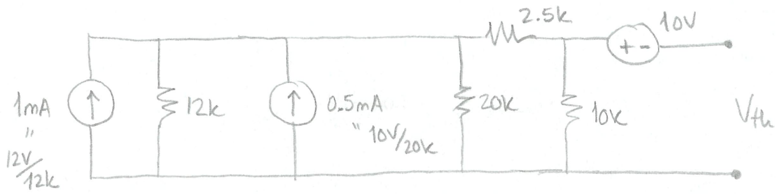
$$\cancel{R_{eq} \times i}^{\,0} + 10V + V_{th}''' = 0 \implies V_{th}''' = -10V$$

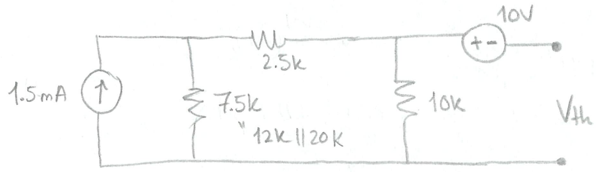
* Calculate final result:

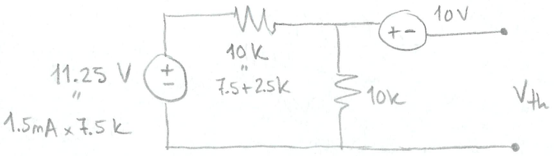
$$V_{th} = V_{th}' + V_{th}'' + V_{th}''' = 3.75 + 1.875 - 10 = \underline{-4.375V}$$

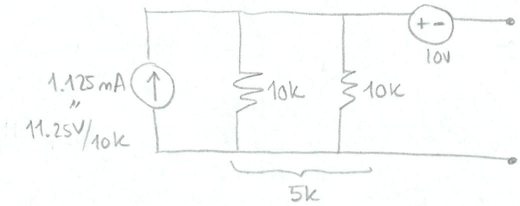
* V_{th} and R_{th} calculation using source transformation :

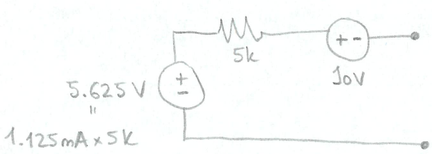




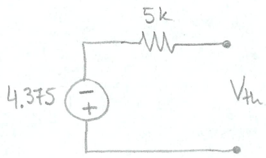








$\Rightarrow$



$\Rightarrow$ $V_{th} = -4.375\,V$

$R_{th} = 5k$

QUESTION 2 [20 marks]

The switch in the circuit shown in Figure 3 has been in position a for a long time. At $t = 0$, the switch moves instantaneously to position b.

a. **(4 marks)** Find the initial current $i(0^-)$ though the inductor under steady-state condition.

b. **(2 marks)** Calculate the initial energy $w_L(0)$ stored in the inductor.

c. **(4 marks)** Find the final current $i(\infty)$ through the inductor under steady-state condition.

d. **(5 marks)** Derive an expression for the current of the inductor $i(t)$ for all time (i.e., for both $t < 0$ and $t \geq 0$).

e. **(5 marks)** Derive an expression for the voltage $v(t)$ across the $40\,\Omega$ resistor for $t \geq 0$.

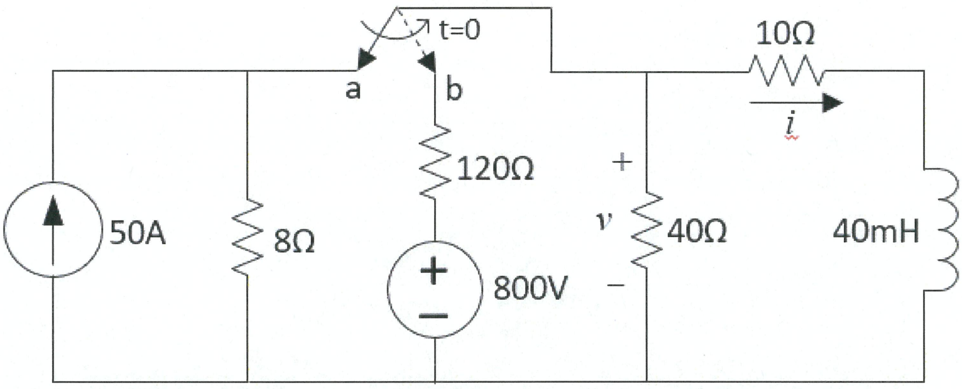
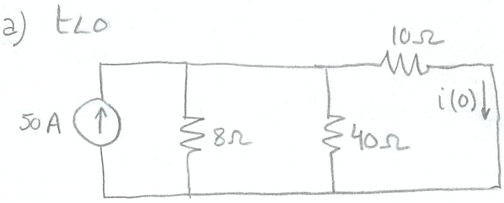


Figure 3

a) $t < 0$



Inductor behaves as short circuit in DC steady state

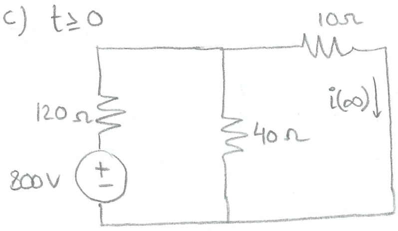
$\boxed{i(0) = 50\,\frac{8\|40\|10}{10} = \frac{50 \times 4}{10} = 20\text{ A}}$

current divider

$8\|40\|10 = \frac{1}{\frac{1}{8}+\frac{1}{40}+\frac{1}{10}} = \frac{1}{\frac{5+1+4}{40}} = \frac{40}{10} = 4\,\Omega$

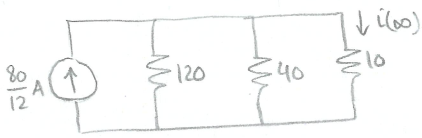
b) $\boxed{W_L = \frac{1}{2}Li^2 = \frac{1}{2} \times 0.04 \times 20^2 = 8\text{ J}}$

c) $t \geq 0$



Inductor behaves as short circuit in DC steady state

source transformation



$\boxed{i(\infty) = \frac{80}{12}\,\frac{120\|40\|10}{10} = \frac{80}{12} \times \frac{7.5}{10} = 5\text{ A}}$

current divider

$120\|40\|10 = \frac{1}{\frac{1}{120}+\frac{1}{40}+\frac{1}{10}} = \frac{1}{\frac{1+3+12}{120}} = \frac{120}{16} = 7.5$

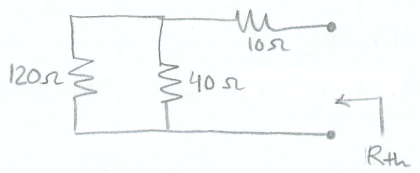
d)

$t<0 \Rightarrow i = i(0) = 20\ A$

$t \geq 0 \Rightarrow$ step response

$$i(t) = i(\infty) + (i(0) - i(\infty))\, e^{-t/\tau} \quad \text{where } \tau = \frac{L}{R_{th}} \text{ and } i(\infty) = 5A$$

We calculate R_{th}:



$$R_{th} = 10 + (40 \| 120) = 10 + 30 = 40\Omega$$

$$40\|120 = \frac{1}{\frac{1}{40}+\frac{1}{120}} = \frac{1}{\frac{3+1}{120}} = \frac{120}{4} = 30$$

So $\tau = \frac{L}{R_{th}} = \frac{40mH}{40\Omega} = 1ms = 0.001\,s$

$$i(t) = 5 + (20-5)\,e^{-t/0.001} = 5 + 15\,e^{-1000t}$$

$$\boxed{i(t) = \begin{cases} 20\ A, & t<0 \\ 5+15e^{-1000t}\ A, & t\geq 0 \end{cases}}$$

e) KVL in inductor mesh

$$-v + 10i + v_L = 0$$

$$v = 10i + L\frac{di}{dt} = 10\left(5+15e^{-1000t}\right) + 0.04 \times 15 \times (-1000)\,e^{-1000t}$$

$$v = 50 + 150e^{-1000t} - 600e^{-1000t}$$

$$\boxed{v = 50 - 450\,e^{-1000t}\ V}$$

QUESTION 3 [18 marks]

The circuit shown in Figure 4 is a simple fire alarm system. When the alarm is triggered, 1 kHz sinusoidal "beeps" are sent to the speakers.

a. **(2 marks)** Express the voltage at the amplifier input v_{in} as a function of time and then as a phasor.

b. **(10 marks)** Find the voltage v_{out} across each speaker as a phasor.

c. **(6 marks)** Find the average power consumed by each speaker.

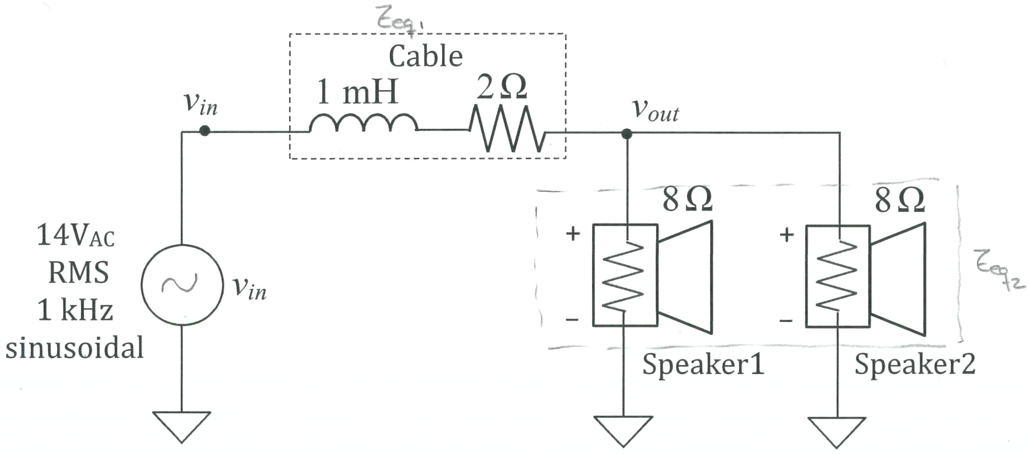
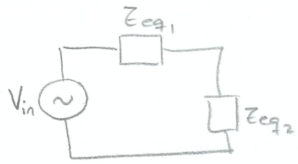


Figure 4

a)

$$\boxed{\begin{aligned} v_{in} &= 14\sqrt{2}\cos(2000\pi t)\ \text{V} \\ V_{in} &= 14\sqrt{2}\angle 0^\circ\ \text{V} \end{aligned}}$$

b)



$$Z_{eq_1} = 2 + j\omega L = 2 + j\,2000\pi \times 0.001 = 2 + j2\pi\ \Omega$$

$$Z_{eq_2} = 8 \| 8 = 4\ \Omega$$

$$\boxed{V_{out}} = V_{in}\frac{Z_{eq_2}}{Z_{eq_2} + Z_{eq_1}} = 14\sqrt{2}\angle 0^\circ \frac{4}{4+2+j2\pi} = 14\sqrt{2}\angle 0^\circ \frac{4}{6+j2\pi} = \frac{56\sqrt{2}\angle 0^\circ}{8.688\angle 46.32^\circ} = \boxed{9.115\angle -46.32^\circ}$$

voltage divider

c) For resistors, $P_{avg} = V_{RMS} \times I_{RMS} = \frac{V_{RMS}^2}{R}$ or $P_{avg} = \frac{1}{2}V_m \times I_m \underbrace{\cos(\theta_v - \theta_i)}_{"1}$

$$\boxed{P_{8\Omega}} = \frac{V_{out_{RMS}}^2}{8} = \frac{(9.115/\sqrt{2})^2}{8} = \frac{41.54}{8} = \boxed{5.19\ \text{W}}$$

QUESTION 4 [30 marks]

a. (15 marks) A 120Ω strain gauge bridge consists of four identical 120Ω resistors arranged in a Wheatstone bridge. When the gauge experiences a positive strain of 0.5%, the resistances of the individual resistor elements change as shown in Figure 4. The op-amp circuit shown has been designed to amplify these small changes in resistance into a larger change in voltage for measurement.

i. (3 marks) Calculate the voltages v_a and v_b at the output of the strain gauge.

ii. (10 marks) Derive an expression for the op-amp output voltage v_{out} in terms of the input voltages v_a and v_b to demonstrate that it is a difference amplifier.

iii. (2 marks) Substitute the values of v_a and v_b from part (a) to calculate the voltage value at the output of the op-amp.

→ NOTE: DO NOT substitute the values of V_a and V_b from part (i) so that you can find a general expression rather than a voltage value.

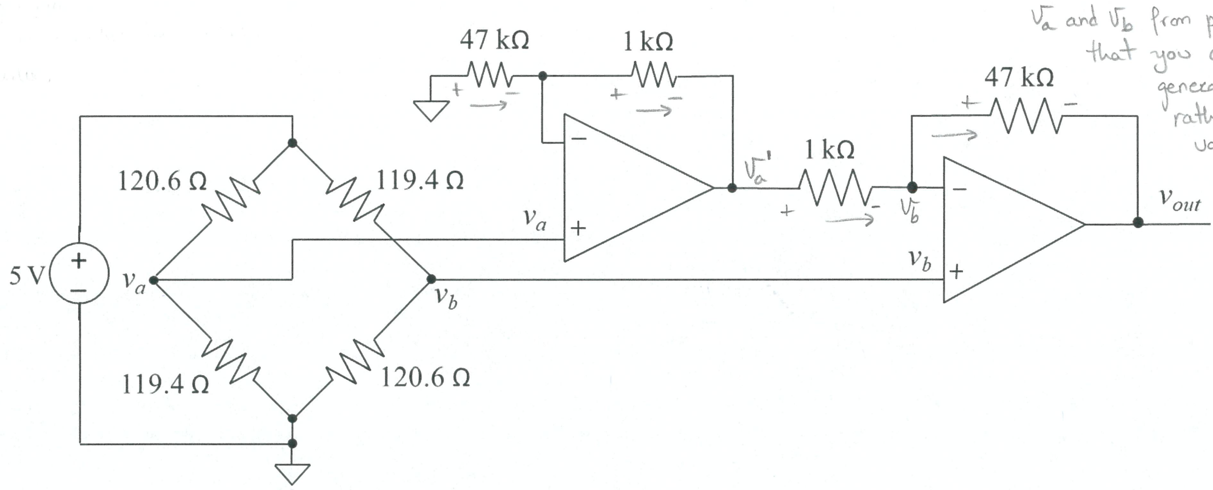
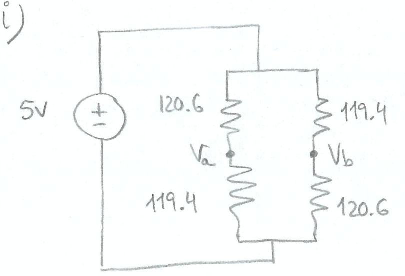


Figure 4

i)



Use voltage division:

$$\boxed{V_a = V_{119.4\Omega}} = 5\,\frac{119.4}{119.4+120.6} = \boxed{2.4875\text{ V}}$$

$$\boxed{V_b = V_{120.6\Omega}} = 5\,\frac{120.6}{119.4+120.6} = \boxed{2.5125\text{ V}}$$

ii) First stage is a non-inverting amplifier → Or using nodal analysis: $\frac{0-V_a}{47k} = \frac{V_a - V_a'}{1k}$

$$V_a' = \left(1+\frac{R_2}{R_1}\right)V_a = \left(1+\frac{1}{47}\right)V_a = \frac{48}{47}V_a$$

We use nodal analysis to find the transfer function of the second stage. We also know that $V^+ = V^- = V_b$

$$\frac{V_a'-V_b}{1k} - \frac{V_b - V_{out}}{47k} = 0$$

$$V_a' - V_b - \frac{V_b}{47} + \frac{V_{out}}{47} = 0 \xrightarrow{\times 47} 47V_a' - 47V_b - V_b + V_{out} = 0$$

$$V_{out} = -47V_a' + 48V_b \xrightarrow[V_a' = \frac{48}{47}V_a]{} \boxed{V_{out} = 48V_b - 48V_a = 48(V_b - V_a)}$$

We substitute V_a and V_b:

$$\boxed{V_{out} = 48(V_a - V_b) = 48(2.5125 - 2.4875) = 1.2V}$$

b. **(15 marks)** Calculate the steady-state value for v_{out} in the circuit shown in Figure 5 if $v_g = 25\cos 50000t\ V$.

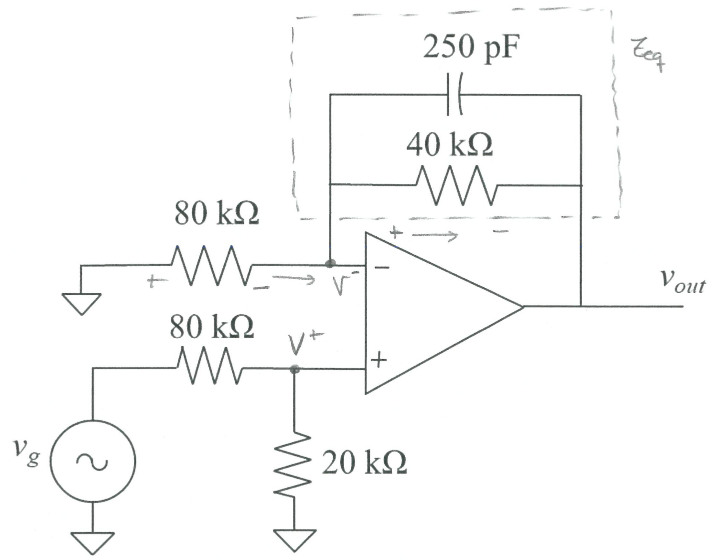


Figure 5

$v_g = 25\cos(50000t)$

$V_g = 25\angle 0^\circ$

First, we calculate V^+ using voltage division:

$$V^+ = V_g \frac{20\,k}{(20+80)\,k} = 0.2V_g = 0.2\times 25\angle 0^\circ = 5\angle 0^\circ$$

Option A

Then, we use nodal analysis to calculate V_{out}:

$$\frac{0-V^-}{80k} - \frac{V^- - V_{out}}{Z_{eq}} = 0 \;;$$

Also: $\dfrac{0-V^-}{80\,k} - \dfrac{V^- - V_{out}}{40\,k} - \dfrac{V^- - V_{out}}{Z_C} = 0$ where $Z_C = -j80\,k$ (see below)

$$\frac{-V^-}{80000} - \frac{V^-}{Z_{eq}} + \frac{V_{out}}{Z_{eq}} = 0 \implies V_{out} = Z_{eq}\left(\frac{1}{80000} + \frac{1}{Z_{eq}}\right)V^- = \left(\frac{Z_{eq}}{80000}+1\right)5\angle 0^\circ$$

$V^- = V^+ = 5\angle 0^\circ$

where

$$Z_{eq} = 40000 \,\|\, Z_C = 40000 \,\|\, -j80000$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j\times 50000\times 250\,pF} = \frac{1}{j\,1.25\times 10^{-5}} = -j\,80000\,\Omega = -j\,80\,k\Omega$$

$$Z_{eq} = 40000 \,\|\, -j\,80000 = \frac{1}{\frac{1}{40000}+j\omega C} = \frac{1}{2.5\times 10^{-5} + j\,1.25\times 10^{-5}} = \frac{10^5}{2.5+j1.25} = \frac{10^5}{2.795\angle 26.56^\circ} =$$

$$= 35778.17\angle -26.56^\circ$$

$$V_{out} = \left(\frac{Z_{eq}}{80000}+1\right)5\angle 0^\circ = \left(\frac{35778.17\angle -26.56^\circ}{80000}+1\right)5\angle 0^\circ = (0.4472\angle -26.56^\circ + 1)\times 5\angle 0^\circ =$$

$$= \left[(0.4472\cos\angle -26.56 + 1) + j\,0.4472\sin(-26.56^\circ)\right]\times 5\angle 0^\circ = (1.4 - j0.2)\times 5\angle 0^\circ = 1.414\angle -8.1^\circ \times 5\angle 0^\circ$$

$$\boxed{V_{out} = 7.07\angle -8.1^\circ = 7.07\cos(50000t - 8.1^\circ)}$$

Option B

We can also use directly the formula of the non-inverting amplifier to calculate V_{out}:

$$V_{out} = \left(1 + \frac{Z_{eq}}{80k}\right)\times V^+$$

QUESTION 5 [12 marks]

a. **(6 marks)** Draw the logic diagram which represents the function of the following logic equation:

$$Z = (\bar{A} + \bar{B} + C)(A + B + \bar{C})(A + \bar{B} + C)$$

b. **(6 marks)** Consider the logical diagram in Figure 7.

i. **(3 marks)** Derive the logical expression for Z.

ii. **(3 marks)** Write the truth table of the circuit.

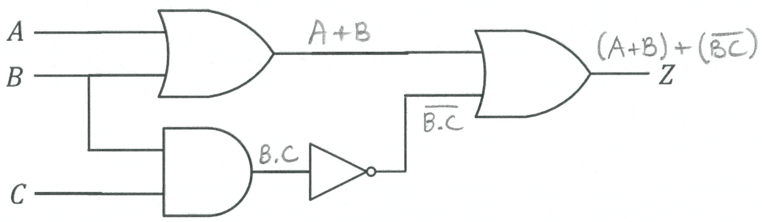


Figure 7

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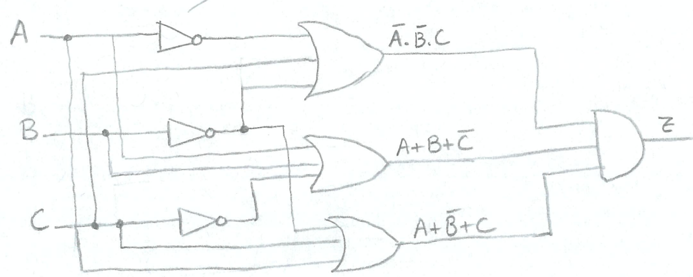
Note: NOTs can also be represented with a circle at the input of the gates. E.g.

a)



b)

i) $Z = (A+B) + \overline{BC}$ (as seen in Figure 7) → This can be simplified to make the truth table easier

$Z = (A+B) + \overline{BC} = A + \underbrace{B + \bar{B}}_{1} + \bar{C} =$

↑ DeMorgan's

$= A + 1 + \bar{C}$

↳ Output of truth table will be all 1s.

ii)

A	B	C	A+B	BC	$\overline{BC}$	$(A+B)+\overline{BC}$
0	0	0	0	0	1	1
0	0	1	0	0	1	1
0	1	0	1	0	1	1
0	1	1	1	1	0	1
1	0	0	1	0	1	1
1	0	1	1	0	1	1
1	1	0	1	0	1	1
1	1	1	1	1	0	1