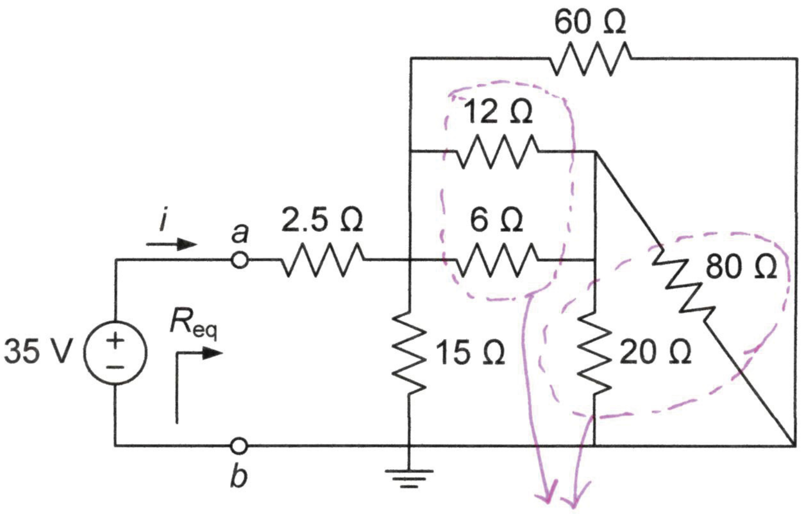
QUESTION 1 [20 marks]

(i) For the circuit shown in Figure 1,

a. **(15 marks)** Calculate the equivalent resistance R_{eq} as seen from terminals *a-b*.

b. **(5 marks)** Find the current i through the network using the result of part (a).



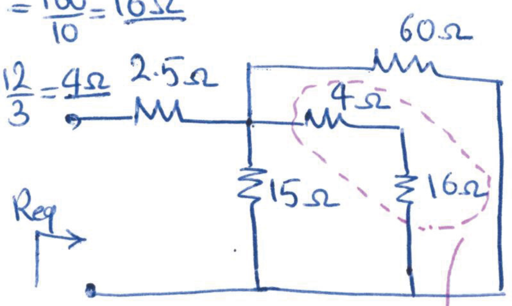
Parallel

Figure 1

a)

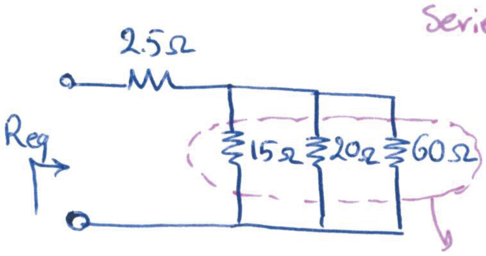
$20 \parallel 80 = \frac{20 \times 80}{100} = \frac{160}{10} = \underline{16\Omega}$

$6 \parallel 12 = \frac{6 \times 12}{18} = \frac{12}{3} = \underline{4\Omega}$



Series

$4 + 16 = \underline{20\Omega}$



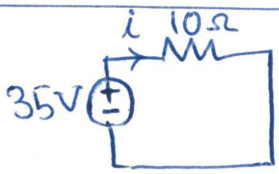
Parallel

$60 \parallel 20 \parallel 15 = \left(\frac{60 \times 20}{80}\right) \parallel 15 = \frac{60}{4} \parallel 15 = 15 \parallel 15 = \frac{15 \times 15}{30} = \frac{15}{2} = \underline{7.5\Omega}$

$\Rightarrow R_{eq} = 2.5 + 7.5 = 10\Omega$

Series

b)



$i = \frac{V}{R} = \frac{35}{10} = 3.5\,A$

Ohm's Law

QUESTION 2 [40 marks]

(i) **[24 marks]** For the circuit shown in Figure 2,

a. **(10 marks)** Apply nodal analysis to obtain the node voltage at nodes v_1 and v_2, and show that $v_1 = 20\ \mathrm{V}$ and $v_2 = 12\ \mathrm{V}$

b. **(12 marks)** Calculate all the powers absorbed/supplied by resistors and sources and specify which element supplies power and which element absorbs power.

c. **(2 marks)** Verify the law of conservation of energy for this circuit.

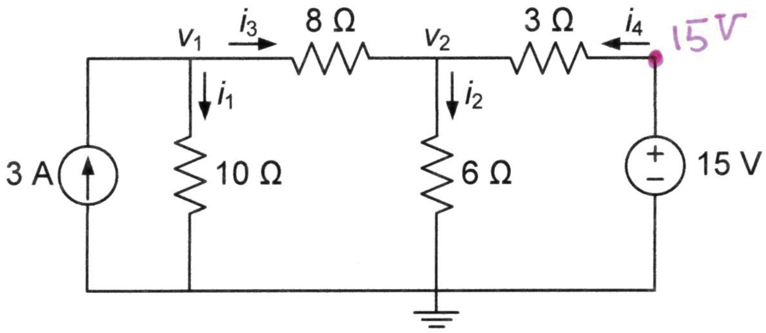


Figure 2

a) KCL @ v_1: $3 = \frac{v_1}{10} + \frac{v_1 - v_2}{8}$ (based on directions of i_1 and i_3)

$\xrightarrow{\times 40} 120 = 4v_1 + 5v_1 - 5v_2 \rightarrow \boxed{9v_1 - 5v_2 = 120}$

KCL @ v_2: $\frac{v_1 - v_2}{8} + \frac{15 - v_2}{3} = \frac{v_2}{6}$ (based on directions of i_2, i_3, i_4)

$\xrightarrow{\times 24} 3v_1 - 3v_2 + 8\times 15 - 8v_2 = 4v_2 \rightarrow 3v_1 - 15v_2 = -120 \xrightarrow{\times 1/3}$

$\rightarrow \boxed{v_1 - 5v_2 = -40}$

$\Rightarrow \begin{cases} \text{I} & 9v_1 - 5v_2 = 120 \\ \text{II} & v_1 - 5v_2 = -40 \end{cases}$

II $\rightarrow v_1 = 5v_2 - 40 \xrightarrow{\text{II}} 9(5v_2 - 40) - 5v_2 = 120 \rightarrow 45v_2 - 360 - 5v_2 = 120$

$\rightarrow 40v_2 = 480 \rightarrow v_2 = \frac{480}{40} \rightarrow v_2 = 12\text{V}$

II $\Rightarrow v_1 = 5\times 12 - 40 = 60 - 40 = 20\text{V}$

$\rightarrow v_1 = 20\text{V}$

b) $P = \frac{v^2}{R}$

$10\Omega \rightarrow P_{10} = \frac{v_1^2}{10} = \frac{20^2}{10} = \boxed{40\text{W absorbed}}$

$8\Omega \rightarrow P_8 = \frac{(v_1 - v_2)^2}{8} = \frac{(20-12)^2}{8} = \frac{8^2}{8} = \boxed{8\text{W absorbed}}$

$6\Omega \rightarrow P_6 = \frac{v_2^2}{6} = \frac{12^2}{6} = \boxed{24\text{W absorbed}}$

$3\Omega \rightarrow P_3 = \frac{(15 - v_2)^2}{3} = \frac{(15-12)^2}{3} = \frac{3^2}{3} = \boxed{3\text{W absorbed}}$

$P = v.i$

3-A source: $P_{3A} = v_1 \times 3 = 20 \times 3 = \boxed{60\text{W supplied}}$ (or -60W)

15-V source: $P_{15V} = 15 \times i_4 = 15 \times \frac{(15 - v_2)}{3} = 15 \times \frac{(15-12)}{3} = \boxed{15\text{W supplied}}$

(or -15W)

c) Conservation of energy $\Sigma P = 0$

$P_{10} + P_8 + P_6 + P_3 = P_{3\text{-}A} + P_{15}$

$40 + 8 + 24 + 3 = 60 + 15$

$75\text{W} = 75\text{W}$

absorbed supplied

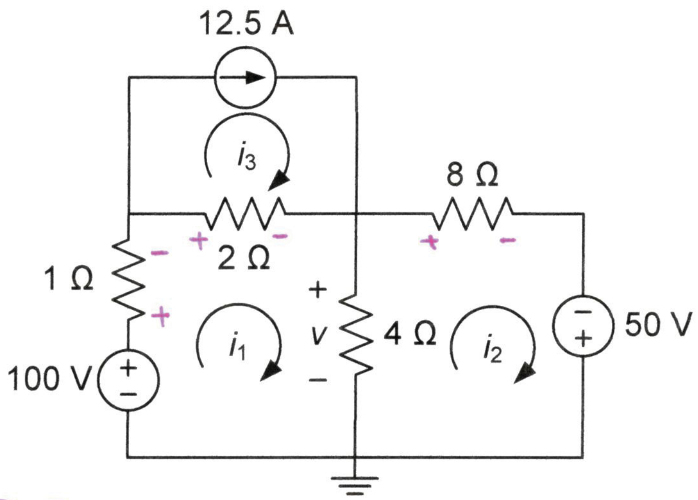
QUESTION 2

(ii) [16 marks] For the circuit shown in Figure 3,

a. **(10 marks)** Apply mesh analysis to obtain the mesh currents i_1, i_2 and i_3, and show that $i_1 = 25\ \mathrm{A}$ and $i_2 = 12.5\ \mathrm{A}$

b. **(2 marks)** Find the voltage v across 4-Ω resistor.

c. **(4 marks)** Calculate the power of the 50-V voltage source and explain whether it supplies or absorbs power.



a) mesh 3: $i_3 = 12.5\,A$ (I)

Figure 3

KVL in mesh 1: $-100 + 1 \times i_1 + 2(i_1 - i_3) + 4(i_1 - i_2) = 0$

$\xrightarrow{\text{I}}$ $i_1 + 2i_1 \underbrace{- 2\times 12.5}_{-25} + 4i_1 - 4i_2 = 100 \longrightarrow \boxed{7i_1 - 4i_2 = 125}$

KVL in mesh 2: $-50 - 4(i_1 - i_2) + 8i_2 = 0 \longrightarrow -4i_1 + 4i_2 + 8i_2 = 50$

$\longrightarrow -4i_1 + 12i_2 = 50 \xrightarrow{\times -\frac{1}{2}} \boxed{2i_1 - 6i_2 = -25}$

$\Longrightarrow \begin{cases} 7i_1 - 4i_2 = 125 & \text{(I)} \\ 2i_1 - 6i_2 = -25 & \text{(II)} \end{cases}$

$\xrightarrow{\text{II}} i_1 = \frac{1}{2}(6i_2 - 25) = 3i_2 - 12.5 \xrightarrow{\text{I}} 7(3i_2 - 12.5) - 4i_2 = 125 \longrightarrow 21i_2 - 87.5 - 4i_2 = 12$

$\longrightarrow 17i_2 = 212.5 \longrightarrow i_2 = \frac{212.5}{17} \longrightarrow i_2 = 12.5A \xRightarrow{\text{II}} i_1 = \frac{1}{2}(6\times 12.5 - 25) = \frac{1}{2}(75-2$

$i_1 = \frac{50}{2} \longrightarrow i_1 = 25A$

---

b) $v = 4(i_1 - i_2) = 4(25 - 12.5) = 4\times 12.5 \Rightarrow v = 50\ V$

---

c) $P = v.i = 50 \times i_2 = 50 \times 12.5 = 625\ W$ supplied

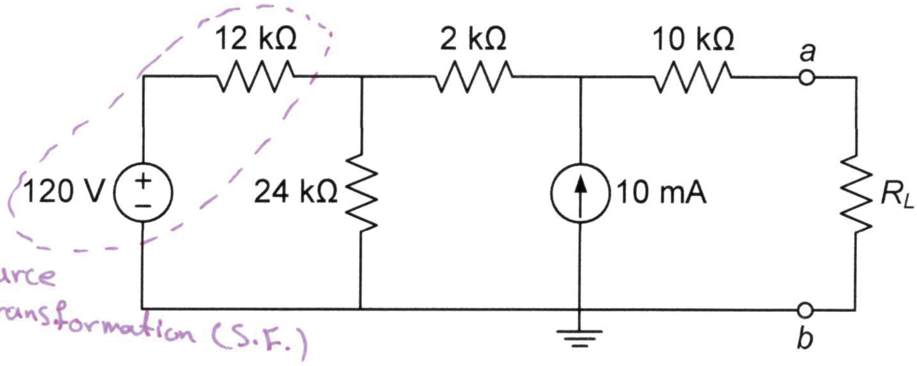
(or -625W)

QUESTION 3 [20 marks]

(i) [10 marks] For the circuit shown in Figure 4,

a. **(6 marks)** Use source transformation to obtain Thevenin equivalent circuit from terminals *a-b* and draw the Thevenin equivalent circuit.

b. **(4 marks)** Determine the value of load resistance R_L for maximum power transfer, and then calculate the maximum power that can be delivered to R_L.

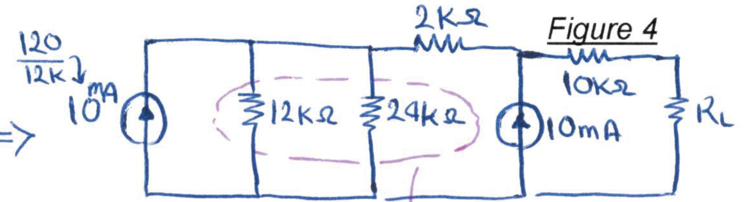


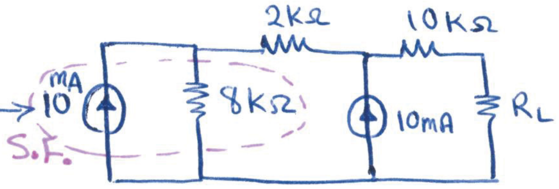
Source transformation (S.F.)

Figure 4

a)

$$\begin{cases} v_S = R i_S \\ i_S = \frac{v_S}{R_S} \end{cases} \Rightarrow$$



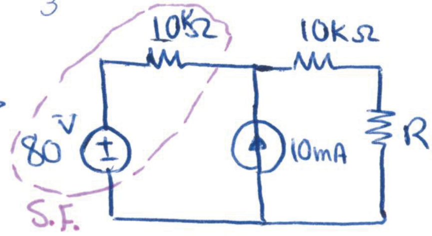


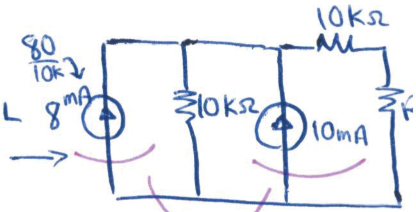
Parallel

$$12 \| 24 = \frac{12\times 24}{36} = \frac{24}{3} = \boxed{8\,k\Omega}$$

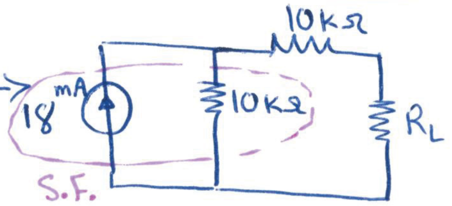
Series

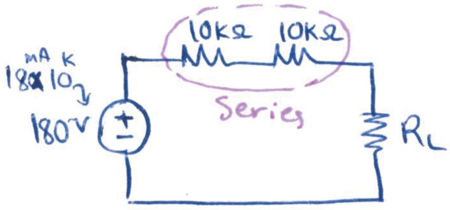
$$\boxed{8+2=10k\Omega}$$



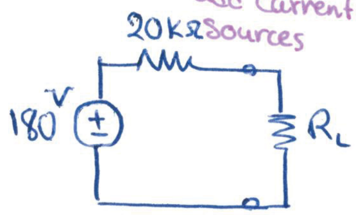


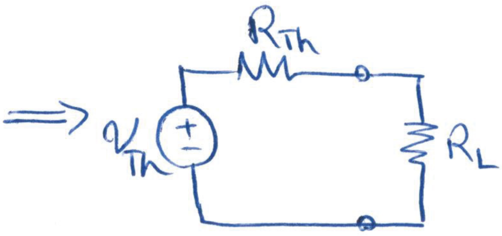
add current sources





$$10+10=20k\Omega$$





$$V_{Th} = 180\,V$$
$$R_{Th} = 20\,k\Omega$$

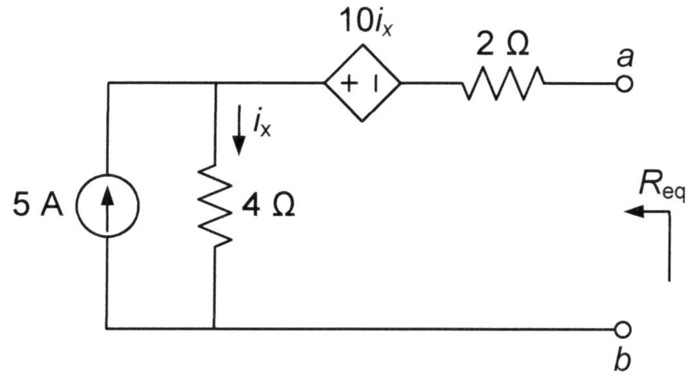
---

b) For max power transfer $\rightarrow \boxed{R_L = R_{Th}} \Rightarrow R_L = 20k\Omega$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(180)^2}{4\times 20\times 10^3} = 405\,mW$$

QUESTION 3

(ii) [10 marks] Find the equivalent resistance R_{eq} in the circuit given in Figure 5.



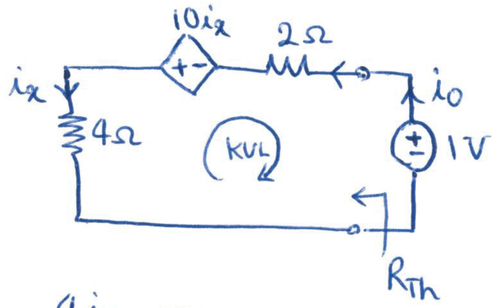
Dependent source → Connect an external voltage or current source

Figure 5

First approach: I-V

* Connect a Voltage Source $V_o = 1V$

disconnect 5-A source ⟹



$$\boxed{R_{Th} = \frac{V_o}{i_o} = \frac{1}{i_o}}$$

$\boxed{i_x = i_o}$ ⟹ KVL: $-4i_x + 10i_x - 2i_x + 1 = 0 \rightarrow 4i_x + 1 = 0$

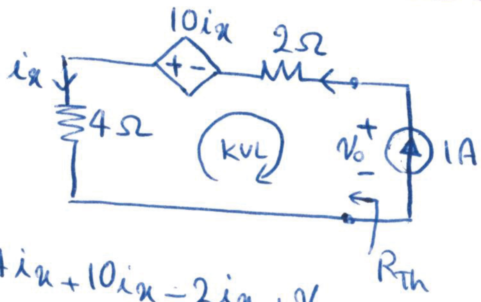
$\rightarrow 4i_x = -1 \rightarrow \boxed{i_x = -\frac{1}{4}A}$ ⟹ $R_{Th} = \frac{1}{-\frac{1}{4}} = -4\Omega$

$i_o = i_x = -\frac{1}{4}A$

Second approach:

* Connect a 1-A current source $i_o = 1A$

disconnect 5-A source ⟹



$$\boxed{R_{Th} = \frac{V_o}{i_o} = \frac{V_o}{1}}$$

$\boxed{i_x = 1A}$ ⟹ KVL: $-4i_x + 10i_x - 2i_x + V_o = 0 \rightarrow 4i_x + V_o = 0$

$\xrightarrow{i_x = 1A} 4 + V_o = 0 \rightarrow \boxed{V_o = -4V}$

$R_{Th} = \frac{-4}{1} = -4\Omega$

QUESTION 4 [20 marks]

(i) In the circuit shown in Figure 6, the switch has been closed for a long time before it is opened at $t = 0$. The voltage source is given a step function.

a. (4 marks) Find the initial voltage $v(0^-)$ across the capacitor under steady-state condition.

b. (2 marks) Calculate the initial energy $w_C(0)$ stored in the capacitor.

c. (4 marks) Find the final voltage $v(\infty)$ across the capacitor under steady-state condition.

d. (4 marks) Derive an expression for the voltage of the capacitor $v(t)$ for all time (i.e., for both $t < 0$ and $t > 0$).

e. (2 marks) Sketch the obtained voltage $v(t)$ in part (d) as a function of time.

f. (4 marks) Derive an expression for the current $i(t)$ through the $5\text{-}\Omega$ resistor for all time (i.e., for both $t < 0$ and $t > 0$).

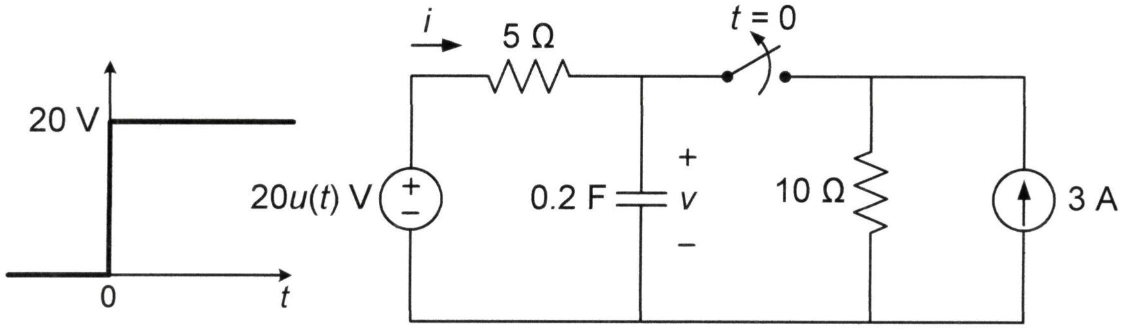
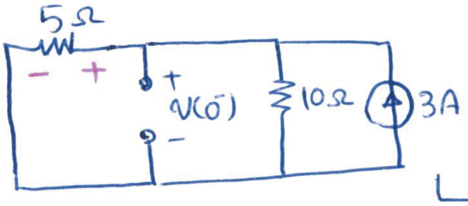


Figure 6

a) $20u(t) = \begin{cases} 0 & t<0 \\ 20 & t>0 \end{cases}$

Step input Voltage

$t<0$ $\xrightarrow{\text{Circuit}}$ before the change

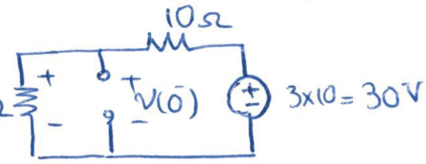


Circuit is in steady-state

Capacitor → open circuit

$v(0^-) = v_{5\Omega}$ (parallel)

$\xrightarrow{S.T}$

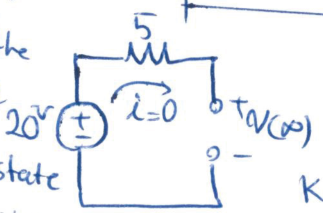


$\xrightarrow{\text{Voltage division}}$ $v(0^-) = \frac{5}{5+10} \times 30 = 10V$

b) $W_c(0) = \frac{1}{2} C v^2(0) \longrightarrow v(0) = v(0^-) \Rightarrow W_c(0) = \frac{1}{2} \times 0.2 \times 10^2 = 10J$

No sudden change in Capacitor Voltage

c) $t>0$ $\xrightarrow{\text{Circuit}}$ after the change



circuit is in steady-state

Capacitor → open circuit

$\xrightarrow[\text{current}]{\text{No}}$ $v(\infty) = 20V$

KVL: $-20 + 0 + v(\infty) = 0$

d) $v(t) = \begin{cases} v(0^-) & t<0 \\ v(\infty) + (v(0^+) - v(\infty))e^{-t/\tau} & t>0 \end{cases}$

for all time

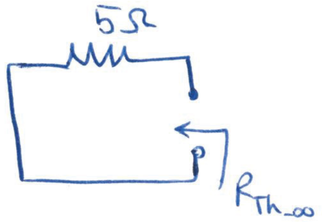
$v(0^+) = v(0) = v(0^-) = \boxed{10V}$

no sudden change After change

$$\Rightarrow \tau = R_{Th\_\infty} \cdot C$$

$R_{Th\infty}$: Thevenin equivalent resistance in the circuit from capacitor terminals after changes.

Q4 Continued



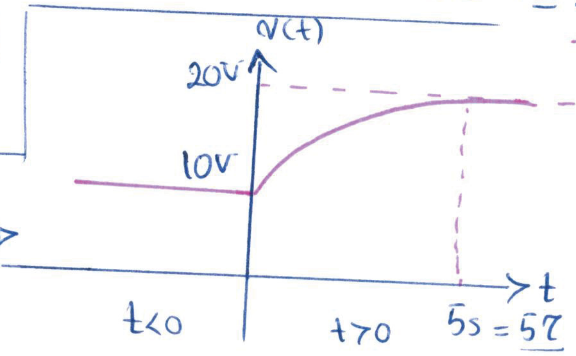
$$R_{Th\_\infty} = 5\Omega \longrightarrow \tau = 5 \times 0.2 = 1\,s$$

$$V(t) = 20 + (10-20)e^{-t/1} = 20 - 10e^{-t}\,V \quad t>0$$

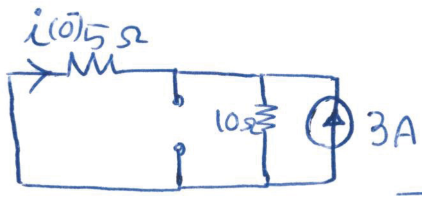
$$\Rightarrow V(t) = \begin{cases} 10\,V & t<0 \\ 20 - 10e^{-t}\,V & t>0 \end{cases}$$

e)

$\longrightarrow$



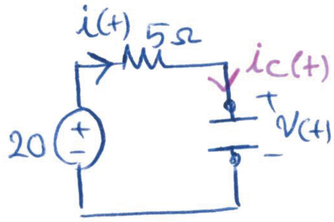
f) $t<0$ $\xrightarrow{\text{Circuit}}$ befor the change (steady-state)



$$i(0^-) = -\frac{10}{5+10} \times 3 = -\frac{10}{5} = \boxed{-2A}$$

$\longrightarrow$ Current division

$t>0$ $\xrightarrow{\text{Circuit}}$ After the change



$\ast$ first approach:

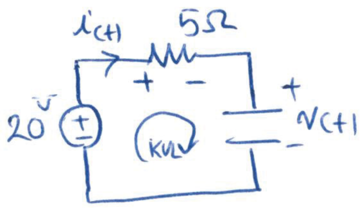
$$i(t) = i_c(t) = C\frac{dV(t)}{dt} = 0.2 \times (-10)(-1)e^{-t}$$

$$\longrightarrow i(t) = 2e^{-t}\,A$$

$$i(t) = \begin{cases} -2A & t<0 \\ 2e^{-t}A & t>0 \end{cases}$$

for all time

$\ast$ Second approach:



$$\text{KVL: } -20 + 5i(t) + V(t) = 0 \quad t>0$$

$$\Rightarrow i(t) = \frac{20 - V(t)}{5} \quad t>0$$

$$\Rightarrow i(t) = \frac{\cancel{20} - \cancel{20} + 10e^{-t}}{5} = 2e^{-t}A \quad t>0$$

$$\Rightarrow i(t) = \begin{cases} -2A & t<0 \\ 2e^{-t}A & t>0 \end{cases}$$