2016 Semester 2 Final Exam

Question 1

a)

i) KCL at node a:

$$\frac{V_a}{12} + \frac{V_a - V_b}{2} + 2i_x = 0$$

But $i_x = \frac{V_c}{2}$

$$V_a + 6V_a - 6V_b + 12V_c = 0$$

$$7V_a - 6V_b + 12V_c = 0 \quad ①$$

KCL at node b:

$$\frac{V_b - V_a}{2} + 1 + \frac{V_b - V_c}{5} = 0$$

$$5V_b - 5V_a + 10 + 2V_b - 2V_c = 0$$

$$-5V_a + 7V_b - 2V_c = -10 \quad ②$$

KCL at node c:

$$\frac{V_c}{2} + \frac{V_c - V_b}{5} = 2i_x = 2\left(\frac{V_c}{2}\right) = V_c$$

$$5V_c + 2V_c - 2V_b = 10V_c$$

$$2V_b + 3V_c = 0 \quad ③$$

ii)
$$\left(\begin{array}{ccc|c} 7 & -6 & 12 & 0 \\ -5 & 7 & -2 & -10 \\ 0 & 2 & 3 & 0 \end{array}\right)$$

$R_2 = 7R_2 + 5R_1$

$$\left(\begin{array}{ccc|c} 7 & -6 & 12 & 0 \\ 0 & 19 & 46 & -70 \\ 0 & 2 & 3 & 0 \end{array}\right) \qquad R_3 = 19R_3 - 2R_2$$

$$\left(\begin{array}{ccc|c} 7 & -6 & 12 & 0 \\ 0 & 19 & 46 & -70 \\ 0 & 0 & -35 & 140 \end{array}\right)$$

$\therefore\ V_c = -4V$

$$V_b = \frac{-70 - 46(-4)}{19} = 6V$$

$$V_a = \frac{6 \times 6 - 12(-4)}{7} = 12V$$

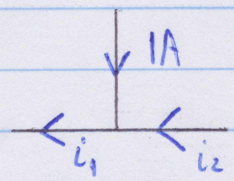
iii) $i_3 = 2i_x = 2i_2$

$\therefore\ 2i_2 - i_3 = 0$ ①

KCL at supermesh:

$i_2 + 1 = i_1$

$i_1 - i_2 = 1$ ②



KVL in supermesh:

$$12i_1 + 2(i_1 - i_3) + 5(i_2 - i_3) + 2i_2 = 0$$

$$14i_1 + 7i_2 - 7i_3 = 0$$

$$2i_1 + i_2 - i_3 = 0 \quad ③$$

iv)

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 2 & -1 & 0 \\ 2 & 1 & -1 & 0 \end{array}\right) \qquad R_3 = R_3 - 2R_1$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 2 & -1 & 0 \\ 0 & 3 & -1 & -2 \end{array}\right) \qquad R_3 = 2R_3 - 3R_2$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & -4 \end{array}\right)$$

$\therefore\ i_3 = -4A$

$i_2 = \dfrac{-4}{2} = -2A$

$i_1 = -1A$

v) Power absorbed:

$P_{12\Omega} = \dfrac{V_a^2}{R} = \dfrac{12^2}{12} = 12W$ absorbed

$P_{2\Omega} = \dfrac{(V_a - V_b)^2}{R} = \dfrac{(12-6)^2}{2} = 18W$ absorbed

$P_{5\Omega} = \dfrac{(V_b - V_c)^2}{R} = \dfrac{(6+4)^2}{5} = 20W$ absorbed

$P_{2\Omega} = \dfrac{(V_c)^2}{R} = \dfrac{(-4)^2}{2} = 8W$ absorbed

$P_{1A} = 1 \times V_b = 1 \times 6 = 6W$ absorbed

Power supplied:

$2i_x = 2i_2 = -4A$

$V_a - V_c = 12+4 = 16V$

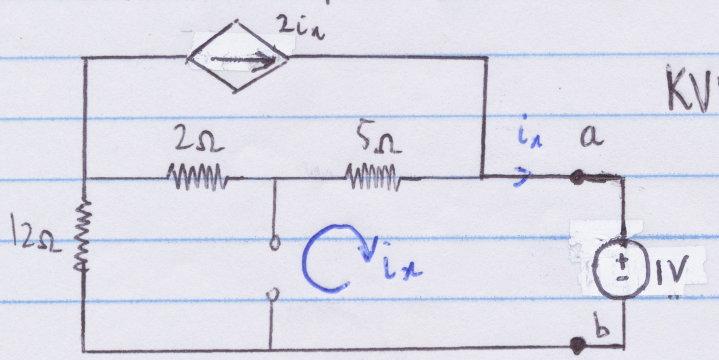
$P_{2i_x} = -4 \times 16 = 64W$ supplied

$\sum P = 64 - 6 - 8 - 20 - 18 - 12$

$= 0W$ $\therefore$ Power is conserved

b) Finding R_{Th}

$i_x = 0A \Rightarrow$ Open circuit



KVL in mesh

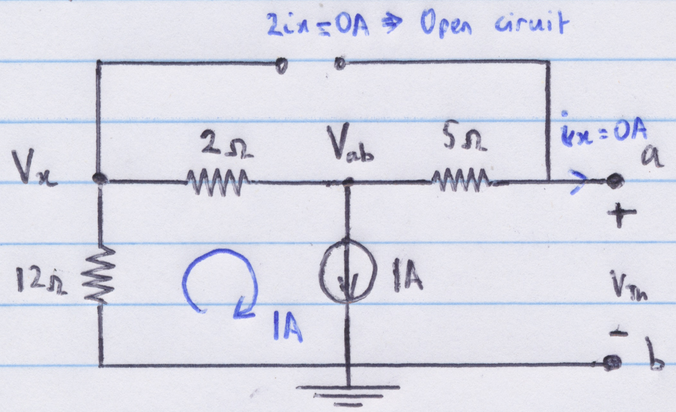
$12i_x + 7(i_x - 2i_x) + 1 = 0$

$5i_x = -1$

$i_x = -0.2A$

$\therefore R_{Th} = \dfrac{1}{i_x} = -5\Omega$

Finding V_{Th}



Current across 5Ω:

$$\frac{V_{ab} - V_{Th}}{5} = 0$$

$$\therefore V_{ab} = V_{Th}$$

Current across 12Ω:

$$\frac{0 - V_x}{12} = 1$$

$$V_x = -12V$$

KCL:

$$\frac{V_{ab} - (-12)}{2} + 1 + 0 = 0$$

$$V_{ab} + 12 + 2 = 0$$

$$\therefore V_{ab} = -14V$$

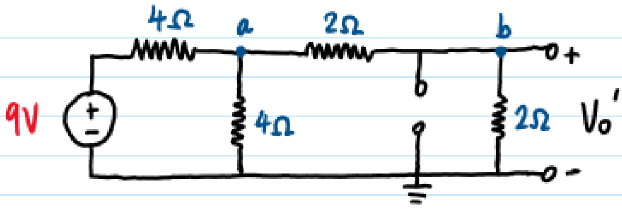
Hence $V_{Th} = -14V$

$$P_{max} = \left| \frac{V_{Th}^2}{4R_{Th}} \right| = \left| \frac{(-14)^2}{4 \times -5} \right| = 9.8W \text{ when } R_L = -5\Omega$$

c)

By Superposition $V_o = V_o' + V_o'' + V_o'''$

Only leaving 9V source on (V_o')



KCL at node a:

$$\frac{V_a - 9}{4} + \frac{V_a}{4} + \frac{V_a - V_b}{2} = 0$$

$$V_a - 9 + V_a + 2V_a - 2V_b = 0$$

$$4V_a - 2V_b = 9$$

$$V_a = \frac{9 + 2V_b}{4}$$

KCL at node b:

$$\frac{V_b - V_a}{2} + \frac{V_b}{2} = 0$$

$$2V_b - V_a = 0$$

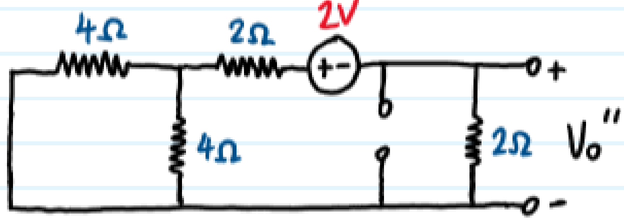
$$2V_b - \left(\frac{9 + 2V_b}{4}\right) = 0$$

$$8V_b - 9 - 2V_b = 0$$

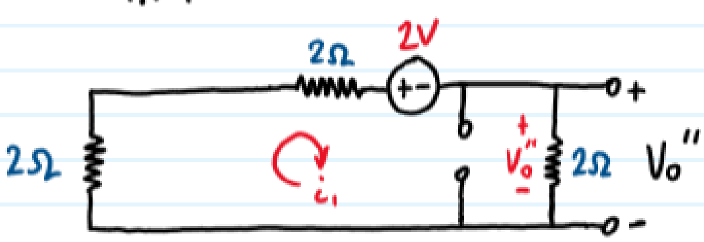
$$6V_b = 9$$

$$V_b = \frac{3}{2} = 1.5V = V_o'$$

Only leaving 2V source on (V_o'')



$4 // 4 = 2\Omega$



KCL in mesh

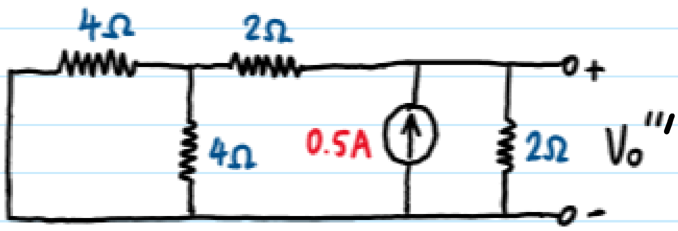
$$2i_1 + 2i_1 + 2 + 2i_1 = 0$$

$$6i_1 = -2$$

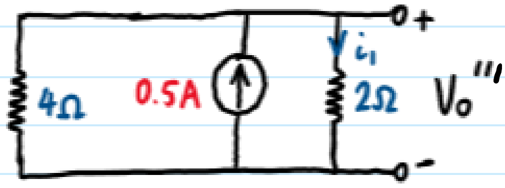
$$i_1 = -\frac{1}{3}$$

$$\therefore V_o'' = 2i_1 = -\frac{2}{3}\,V$$

Only leaving 0.5A source on (V_o''')



$$4//4 + 2 = 4\Omega$$



Current division

$$i_1 = \frac{4}{4+2} \times 0.5 = \frac{1}{3}\,A$$

$$\therefore V_o''' = 2i_1 = \frac{2}{3}\,V$$

$$\therefore V_o = V_o' + V_o'' + V_o'''$$

$$= 1.5 - \frac{2}{3} + \frac{2}{3}$$

$$= 1.5V$$

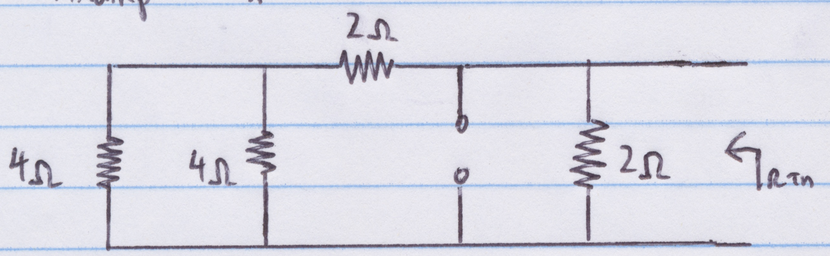
$$\therefore V_o = V_o' + V_o'' + V_o'''$$

$$= 1.5 - \frac{2}{3} + \frac{2}{3}$$

$$= 1.5V$$

d) $V_{Th} = V_o = 3V$

Finding R_{Th}

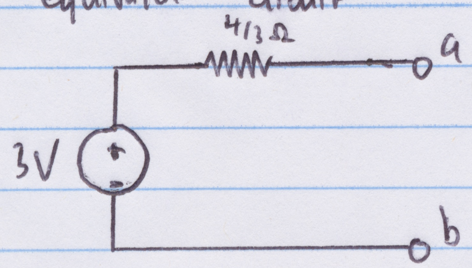


$$R_{Th} = (4 // 4 + 2) // 2$$

$$= 4 // 2$$

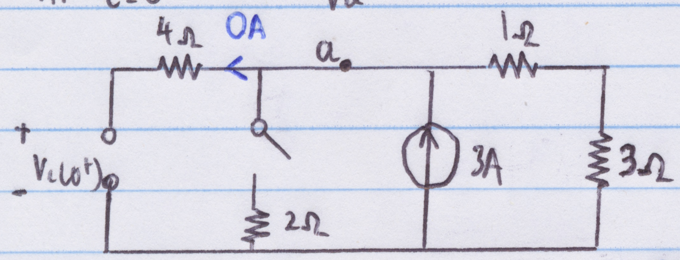
$$= \frac{8}{6} = \frac{4}{3}\,\Omega$$

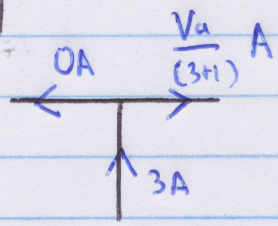
∴ Thevenin equivalent circuit



Question 2

a) At $t=0^-$





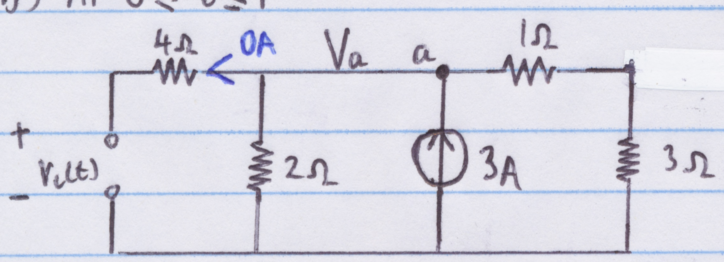
KCL at node a:

$$0 + \frac{V_a}{4} = 3$$

$V_a = 12V \Rightarrow$ Current across 4Ω: $0 = \frac{V_a - V_c(0^+)}{4}$ $\therefore V_a = V_c(0^+)$

$\therefore \quad V_c(0^+) = 12V$

b) At $0 \le t \le 1$



KCL at node a:

$$0 + \frac{V_a}{2} + \frac{V_a}{4} = 3$$

$$3V_a = 12$$

$V_a = 4V \Rightarrow V_a = V_c(t)$

$\therefore \quad V_c(t) = 4V$

$R_{Th} = 4 + (3+1) // 2 \Rightarrow \quad \tau = R_{Th}C = \left(\frac{16}{3}\right)(1) = \frac{16}{3}$ s

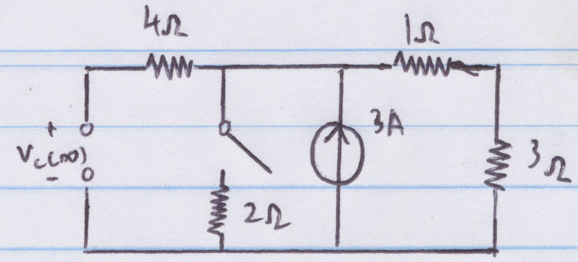
$= \frac{16}{3}\Omega$

$V_c(t) = 4 + (12-4)e^{-\frac{3}{16}t}$ V

$= 4(1 + 2e^{-\frac{3}{16}t})$ V for $0 \le t \le 1$

c) For $t \geq 1$

$$V_c(1) = 4\left(1 + 2e^{-\frac{3}{16}}\right)$$



At $t \geq 1$ (Same figure as a))

$\therefore V_c(\infty) = 12V$

$R_{Th} = 4 + 1 + 3$
$= 8\Omega$

$T = R_{Th}C = 8(1) = 8s$

$$\therefore V_c(t) = 12 + \left(4 + 8e^{-\frac{3}{16}} - 12\right) e^{-\frac{(t-1)}{8}} \, V$$

$$= 12 + 8\left(-1 + e^{-\frac{3}{16}}\right) e^{-\frac{(t-1)}{8}} \, V \quad \text{for } t \geq 1$$

d) From c)

$V_c(\infty) = 12V$ (Similarly solve for $\lim_{t \to \infty}(V_c(t)) = 12V$)

e) At $t = 5$

$$W_c = \frac{1}{2} C(V(5))^2 = \frac{1}{2}(1)\left[12 + 8\left(-1 + e^{-\frac{3}{16}}\right) e^{-\frac{1}{2}}\right]^2 = 62.39 J$$

Question 3

a)

i) $Z_{eq}(C) = \frac{-j}{wC}$

$Z_{eq}(L) = jwL$

$Z_{eq_{AB}} = R // \left(\frac{-j}{wC} + jwL\right)$

$= R // \left(\frac{jw^2CL - j}{wC}\right)$

$= \left(\frac{1}{R} + \frac{wC}{jw^2CL - j}\right)^{-1}$

$= \left(\frac{jw^2CL - j + wCR}{R(jw^2CL - j)}\right)^{-1} = \frac{jR(w^2CL - 1)}{wCR + j(w^2CL - 1)}$

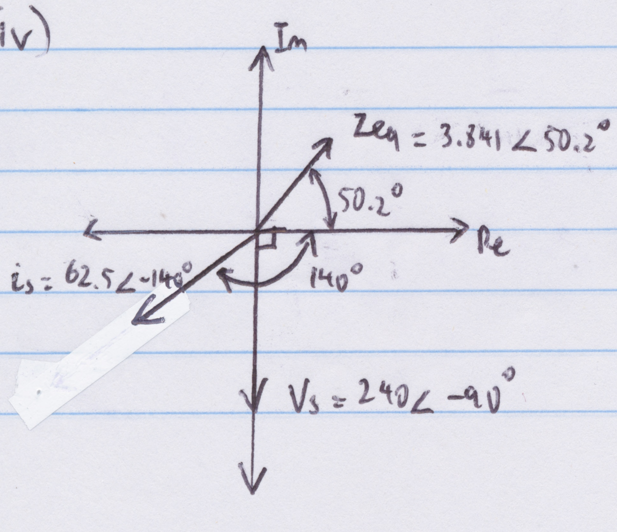
ii) $Z_{eq} = \frac{j6(100^2 \times 2\times10^{-3} \times 0.1 - 1)}{100 \times 2\times10^{-3} \times 6 + j(100^2 \times 2\times10^{-3} \times 0.1 - 1)} = 3.841\cos(100t + 50.2^\circ)\ \Omega$

or $3.841\angle 50.2^\circ\ \Omega$

iii) $V_s = 240\sin(100t) = 240\cos(100t - 90^\circ)\ V = 240\angle -90^\circ$

$i_s = \frac{V_s}{Z_{eq}} = \frac{240\angle -90^\circ}{3.841\angle 50.2^\circ} = -48 - j40 = 62.482\cos(100t - 140^\circ)\ A$

iv)

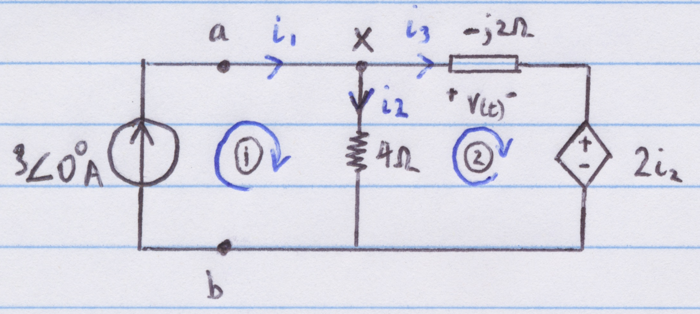


V_s leads i_s by 50°

b)

i) $w = 4\,rad/s$

$$Z_{eq}(0.125F) = \frac{-j}{4(0.125)} = -j2\,\Omega$$



$i_1 = 3\angle 0^\circ A$

KCL at node X:

$$i_1 = i_2 + i_3$$

$$i_2 + i_3 = 3 \quad ①$$

KVL in mesh ②:

$$(-j2)\,i_3 + 2i_2 - 4i_2 = 0$$

$$-2i_2 - (j2)\,i_3 = 0 \quad ②$$

$$① \times 2 \Rightarrow 2i_2 + 2i_3 = 6 \quad ③$$

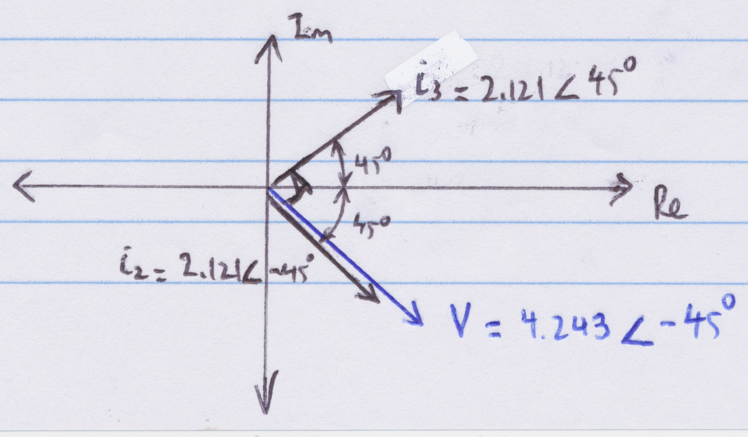
$$② + ③ \Rightarrow (2-j2)\,i_3 = 6$$

$$i_3 = \frac{6}{2-j2} = 2.121\cos(4t+45^\circ)\ A$$

$$\therefore\ i_2 = 3 - 2.121\cos(4t+45^\circ) = 2.121\cos(4t-45^\circ)\,A$$

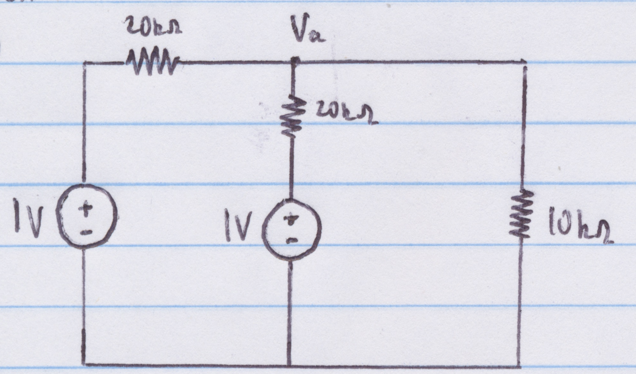
$$\therefore\ v(t) = (-j2)\left(\frac{6}{2-j2}\right) = 4.243\cos(4t-45^\circ)\,V$$

ii)



Question 4

a)



KCL at node a:

$$\frac{V_a-1}{20\times10^3}+\frac{V_a-1}{20\times10^3}+\frac{V_a}{10\times10^3}=0$$

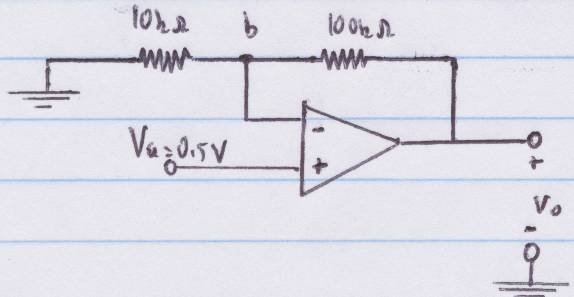
$$4V_a=2$$

$$V_a=0.5V$$

b)

i) $\frac{V_o}{V_a}=\frac{5.5}{0.5}=11$

ii)



$V_a = V_b = 0.5V$ (Op amp principle)

KCL at node b:

$$\frac{0.5}{10{,}000}+\frac{0.5-V_o}{100{,}000}=0$$

$$5.5=V_o$$

$$\therefore V_o=5.5V$$

iii) The output ratio is greater than zero (11>0) so the circuit cannot be an inverting amplifier $\Rightarrow \frac{V_o}{V_a}=-\frac{R_f}{R_{in}}$ (Inverting amplifier formula)

Furthermore supply voltage is supplied through the positive terminal of the op amp.

Question 5

a)

Binary value	16	8	4	2	1
Binary digit	1	1	0	0	1

$11001 \Rightarrow 16+8+1 = 25$

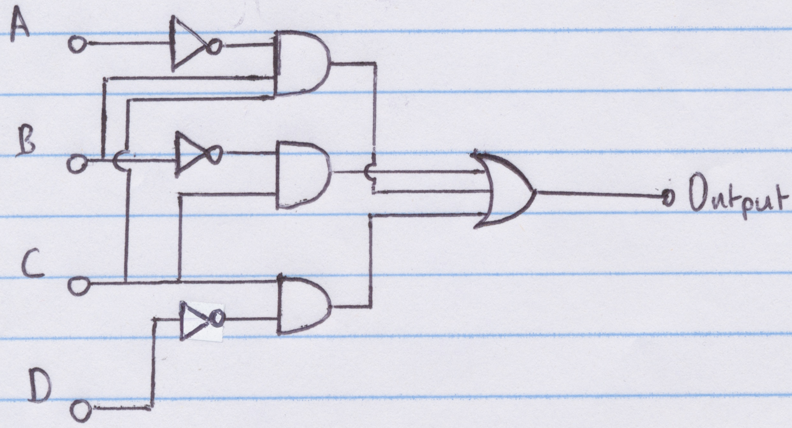
b)

Binary value	32	16	8	4	2	1
Binary digit	1	0	0	0	1	1

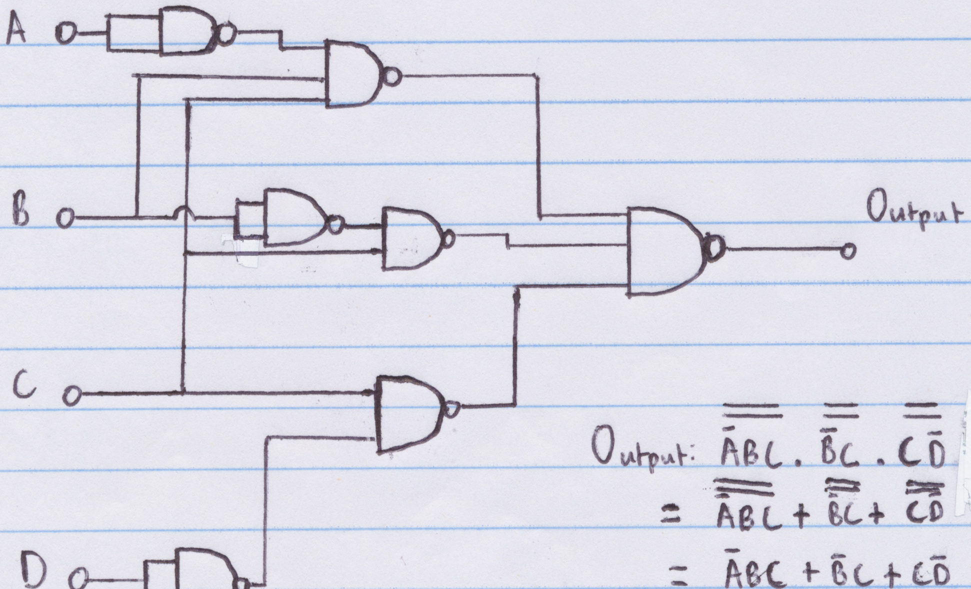
$35 = 32+2+1$

$100011 \Rightarrow 35$

c)



d)



Output: $\overline{\overline{\bar{A}BC} \cdot \overline{\bar{B}C} \cdot \overline{C\bar{D}}}$

$= \overline{\overline{\bar{A}BC}} + \overline{\overline{\bar{B}C}} + \overline{\overline{C\bar{D}}}$

$= \bar{A}BC + \bar{B}C + C\bar{D}$

e)

i) $f(A,B,C) = \overline{(A.B)+(\bar{B}+C)}$

ii)

A	B	C	A.B	$\bar{B}+C$	f(A,B,C)
0	0	0	0	1	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	0	1	0
1	0	0	0	1	0
1	0	1	0	1	0
1	1	0	1	0	0
1	1	1	1	1	0

iii) $\overline{(A.B)+(\bar{B}+C)}$

$= (\overline{A.B}).(\overline{\bar{B}+C})$

$= (\bar{A}+\bar{B}).(\bar{\bar{B}}.\bar{C}) = (\bar{A}+\bar{B}).(B.\bar{C})$

$= \bar{A}.B.\bar{C} + \cancelto{0}{\bar{B}.B}.\bar{C}$

$= \bar{A}.B.\bar{C}$