Topic 4 recap

- A capacitor is a circuit element that stores energy in its electric field.
- The ratio of voltage to charge across a capacitor is its *capacitance* (F).

$$q = Cv$$

- Current in a capacitor is proportional to the time rate of change of its voltage.

$$i = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

- Energy stored in a capacitor is proportional to the square of its voltage.

$$w_c = \frac{1}{2}Cv^2$$

- Capacitor acts as an open circuit to DC voltage.
- Parallel combination of capacitors is similar to series resistors.

$$C_{eq} = C_1 + C_2 + \cdots + C_N$$

- Series combination of capacitors is similar to parallel resistors.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}\cdots + \frac{1}{C_N}$$

Topic 4 recap

- Natural response of RC circuits
  - Behaviour of the circuit (in terms of voltage or current) due to initial energy stored.
  - If the capacitor has a initial voltage $v(0) = V_0$, the natural response of the RC circuit is:

$$v(t) = V_0 e^{\frac{-t}{RC}}$$

  - The natural response decay to zero exponentially.
  - The speed at which the voltage decays is given by the *time constant*:
    - Time constant is the time required for the response to decay to a factor of 1/e or **36.8%** of it **initial value** or to reach **63.2%** of its **final value.**
    - For an RC circuit $\tau = RC$.
    - After **5 time constant**, 5τ, the capacitor voltage is considered to have reached its final value.
    - The higher the resistance and capacitance, the longer it would take for the capacitor to charge or discharge.
    - The resistance for the time constant is the **Thevenin equivalent resistance** as seen from the capacitor terminals.

Topic 4 recap

- Step response of RC circuits
  - The unit step function can be used in electric circuits to model switching.

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

  - The step response is the response to a sudden change in the input sources.
  - The capacitor voltage over time is obtained as an exponential function:

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-\frac{t}{\tau}}, & t > 0 \end{cases}$$

  - If the initial voltage $v(0) = 0\ \mathrm{V}$, the response is known as **forced response**.
  - The step response with non-zero initial condition is known as **complete response**.
    - It can be described as the sum of **transient** and **steady state responses**.
  - The solution to the step response of RC circuits can be given as follows:

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-\frac{t}{\tau}}, \quad t > 0$$

    - $v(0)$: **Initial voltage** at $t = 0$.
    - $v(\infty)$: **Final or steady-state value** at $t \to \infty$.
    - $\tau = R_{\mathrm{Th\_}\infty}C$: Time constant at $t \to \infty$.