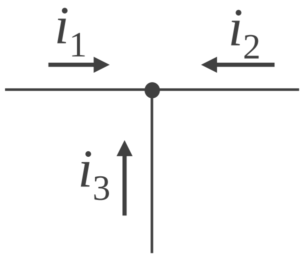
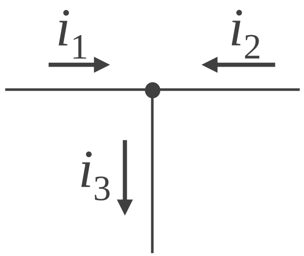
Topic 2 recap

- Kirchhoff’s Current Law (KCL):
  - The sum of all current entering and leaving a node is zero $\sum_{n=1}^{N} i_n = 0$.

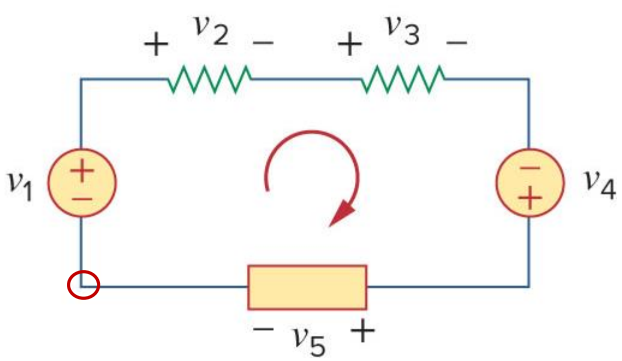




$$i_1 + i_2 + i_3 = 0$$
$$i_1 + i_2 = -i_3$$

$$i_1 + i_2 - i_3 = 0$$
$$i_1 + i_2 = i_3$$

- Kirchhoff’s Voltage Law (KVL):
  - The sum of all voltages in a loop is zero $\sum_{m=1}^{M} v_m = 0$.



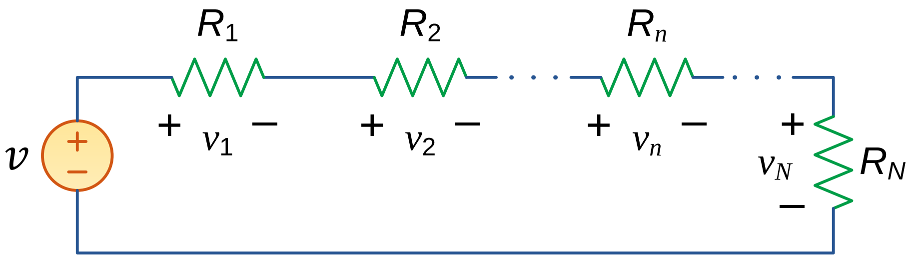
$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

Topic 2 recap

Circuits can be simplified by finding the equivalent resistance from a given terminal.

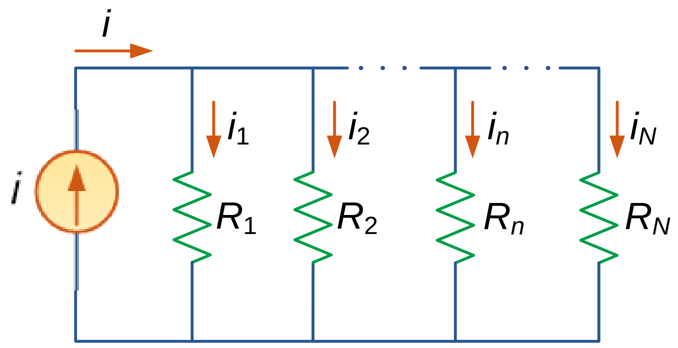
- Resistors in series: $R_{eq} = R_1 + R_2 + \cdots + R_N$
- Resistors in parallel: $\frac{1}{R_{\mathrm{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \cdots + \frac{1}{R_N}$
- Voltage division:

$$v_n = v \frac{R_n}{R_1 + R_2 \cdots + R_N} = v \frac{R_n}{R_{eq}}$$



- Current division:

$$i_n = i \frac{R_1 \| R_2 \| \cdots \| R_N}{R_n} = i \frac{R_{eq}}{R_n}$$



UNSW SYDNEY

Topic 2 recap – Nodal analysis

- Formal circuit analysis aims to derive the **smallest set of simultaneous equations** that completely define the operating characteristics of a circuit.
- Nodal and mesh analysis are based on the systematic application of Kirchhoff's laws.

Nodal analysis

- Five steps:

0. Simplify the circuit (if appropriate).
1. Select a node as the **reference node**.
2. Assign voltages $\boldsymbol{v_1}, \boldsymbol{v_2}, \ldots \boldsymbol{v_n}$ to the remaining $n - 1$ nodes. These voltages are relative to the reference node.
3. Apply **KCL** to each of the $n - 1$ non-reference nodes.
   – For resistors, use Ohm's law to express the currents in terms of node voltages. Keep in mind the passive sign convention.
4. Solve the resulting $n - 1$ simultaneous equations to obtain the unknown node voltages.

Topic 2 recap – Nodal analysis

1. Choose **ground** as **reference node** ($v_0 = 0$ V).
2. Assign voltages v_1 and v_2 to nodes 1 and 2.
3. Apply KCL to nodes 1 and 2.

   node 1: $I_1 = I_2 + i_1 + i_2$

   node 2: $I_2 + i_2 = i_3$

   – For resistors, use Ohm's law to express the branch currents in terms of node voltages. Keep in mind the passive sign convention.

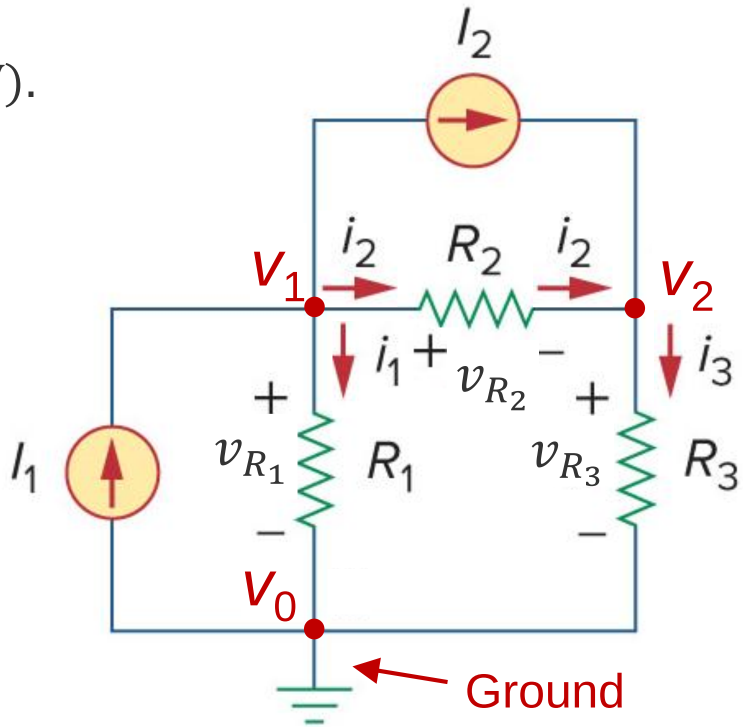
$$\boxed{i_n = \frac{v_{R_n}}{R_n} = \frac{v_{\text{higher}} - v_{\text{lower}}}{R_n}}$$

$$i_1 = \frac{v_{R_1}}{R_1} = \frac{v_1 - 0}{R_1}; \quad i_2 = \frac{v_{R_2}}{R_2} = \frac{v_1 - v_2}{R_2}; \quad i_3 = \frac{v_{R_3}}{R_3} = \frac{v_2 - 0}{R_3}$$

   – Substitute back i_1, i_2, and i_3 into the node equations

$$\begin{cases} I_1 = I_2 + \dfrac{v_1}{R_1} + \dfrac{v_1 - v_2}{R_2} \\ I_2 + \dfrac{v_1 - v_2}{R_2} = \dfrac{v_2}{R_3} \end{cases}$$

4. Solve simultaneous equations (for v_1 and v_2).



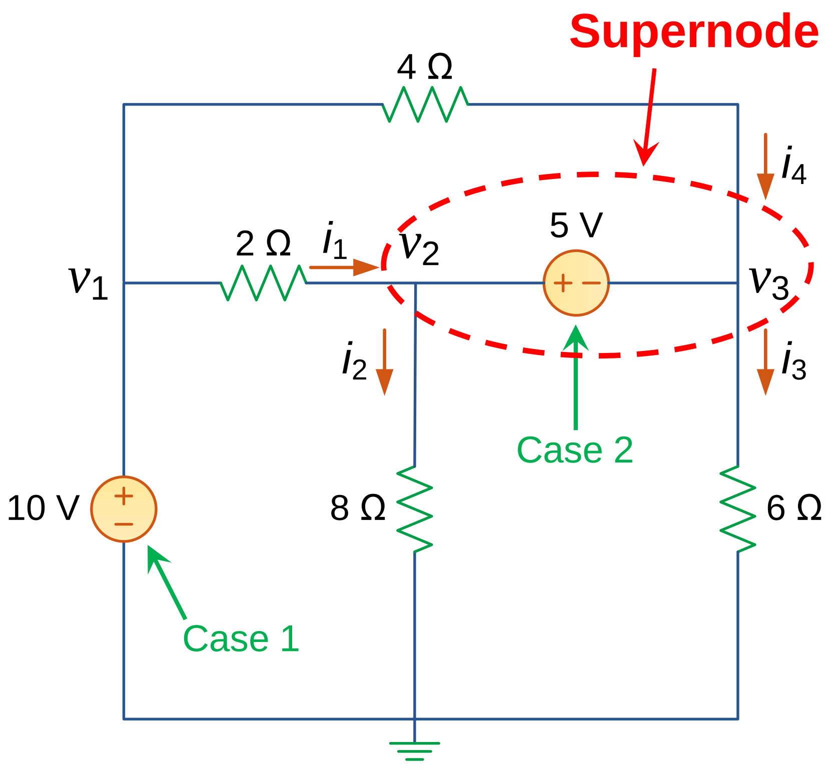
Topic 2 recap – Nodal analysis

- Nodal analysis with voltage source:
  - Case 1: Voltage source is connected to the reference node.

$$v_k = v_{\text{source}}$$

  - Case 2: Voltage source is between two non-reference nodes.
    - Form a supernode by enclosing voltage source and any parallel element with it.
    - Write the constraint equation relating the node voltages inside the supernode.

$$v_a - v_b = v_{\text{source}}$$



Topic 2 recap – Mesh analysis

Mesh analysis

- Four steps:

0. Simplify the circuit (if appropriate).
1. Assign mesh currents $\boldsymbol{i_1}$, $\boldsymbol{i_2}$,…$\boldsymbol{i_n}$ to the n meshes with a direction (generally clockwise).
2. Apply **KVL** to each of the n meshes (following the same direction as mesh currents).
   – For resistors, use Ohm's law to express the voltages in terms of mesh currents.
3. Solve the resulting n simultaneous equations to obtain the unknown mesh currents.

Topic 2 recap – Mesh analysis

1. Assign currents i_1 and i_2 to meshes 1 and 2.

2. Apply KVL to meshes 1 and 2.

   mesh 1: $-V_1 + v_{R_1} + v_{R_3} = 0$

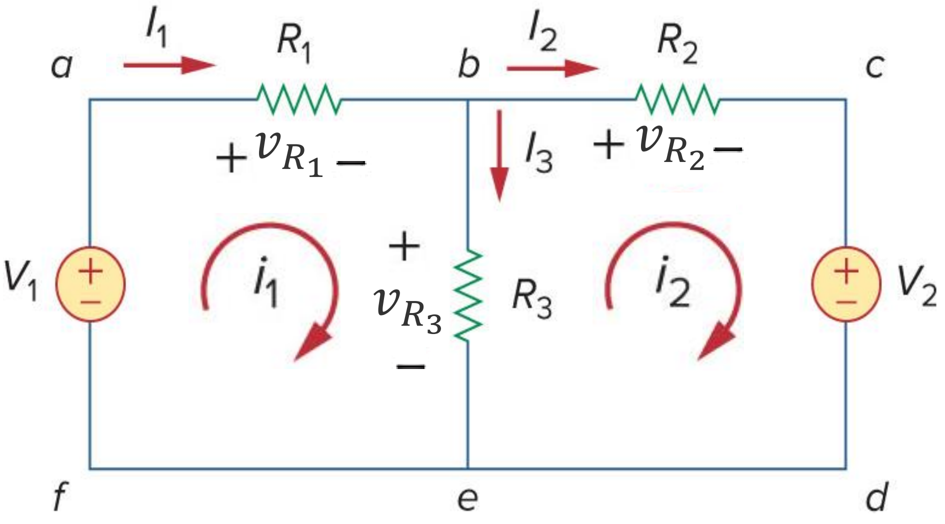
   mesh 2: $-v_{R_3} + v_{R_2} + V_2 = 0$

   – For resistors, use Ohm's law to express the voltages in terms of mesh currents.

   $$v_{R_1} = R_1 I_1 = R_1 i_1$$

   $$v_{R_2} = R_2 I_2 = R_2 i_2$$

   $$v_{R_3} = R_3 I_3 = R_3(i_1 - i_2)$$



   – Substitude back v_{R_1}, v_{R_2}, and v_{R_3} into the mesh equations.

   $$\begin{cases} -V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0 \\ -R_3(i_1 - i_2) + R_2 i_2 + V_2 = 0 \end{cases}$$

3. Solve simultaneous equations (for i_1 and i_2).

Topic 2 recap – Mesh analysis

- Mesh analysis with current source:
  - Case 1: Current source is only in one mesh.

    $$i_k = i_{\text{source}}$$

  - Case 2: Current source is shared between two meshes.
    - Form a supermesh by excluding the current source and any series element with it.
    - Write the constraint equation relating the mesh currents inside the supermesh.

    $$i_a - i_b = i_{\text{source}}$$