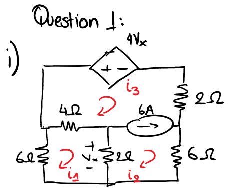
Final Exam Solutions

Thursday, 15 June 2017 16:12

Question 1:

i)



Current source:

$i_2 - i_3 = 6A$ (1)

Supermesh:

$4V_x + 2i_3 + 6i_2 - 2(i_1 - i_2) - 4(i_1 - i_3) = 0$

$= 8i_1 - 8i_2 + 2i_3 + 6i_2 - 2i_1 + 2i_2 - 4i_1 + 4i_3 = 0$

$= 2i_1 + 6i_3 = 0$

Mesh 1:

$6i_1 + 4i_1 - 4i_3 + 2i_1 - 2i_2 = 0$

$\Rightarrow 12i_1 - 2i_2 - 4i_3 = 0$

$i_1 = 6/7$ A

$i_2 = 40/7$ A

$i_3 = -2/7$ A

b) $i_{4\Omega} = i_1 - i_3 = 8/7$ A.

$P_{4\Omega} = i^2 \cdot R = \frac{64}{49} \cdot 4 = 5.224$ W

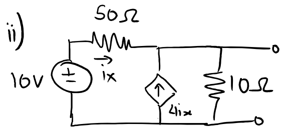
a) $v_x = 2 \cdot (i_1 - i_2) = 2\left(\frac{6}{7} - \frac{40}{7}\right)$

$= 2 \cdot \left(-\frac{34}{7}\right) = -\frac{68}{7}$ V

c) $P = +v \cdot i = +4\left(-\frac{68}{7}\right)\left(-\frac{2}{7}\right) = 11.1$ W

consumes power as power is positive or because of current direction and voltage polarity.

ii)



V_{th}: KCL at node a

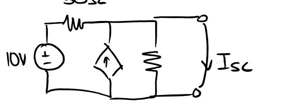
$\frac{10 - V_a}{50} + 4i_x = \frac{V_a}{10} \Rightarrow$

$\Rightarrow \frac{10 - V_a}{50} + 4\frac{10 - V_a}{50} = \frac{V_a}{10}$

$\Rightarrow 50 - 5V_a = 5V_a \Rightarrow V_a = 5V$

$V_a = V_{Th} = 5V$

Isc short-circuit the nodes a–b

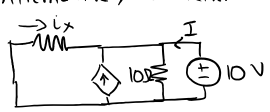


KCL

$\frac{10}{50} + 4 \cdot \frac{10}{50} = I_{sc} \Rightarrow I_{sc} = 1A$

$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{5}{1} = 5\Omega$

Alternatively for R_{Th}.



Ohm's Law

$I_x = -\frac{10}{50} = -0.2A$

$I = \frac{10}{10} - i_x - 4i_x = 1 + 0.2 + 0.8$

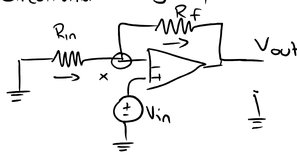
$= 2A$

$R_{Th} = \frac{10}{I} = \frac{10}{2} = 5\Omega$

iii) $P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{5^2}{20} = 1.25W$

Question 2:

Circuit of non-inverting amplifier:



KCL at node x, $V_x = V_{in}$

$\frac{0 - V_{in}}{R_{in}} = \frac{V_{in} - V_{out}}{R_f}$

$\Rightarrow -R_f V_{in} = R_{in} V_{in} - R_{in} V_{out}$

$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{R_{in} + R_f}{R_{in}} \Rightarrow \frac{V_{out}}{R_{in}}$

ii) For a gain of 12, $\frac{R_f}{R_{in}} = 11$ → Resistors 10Ω–110Ω

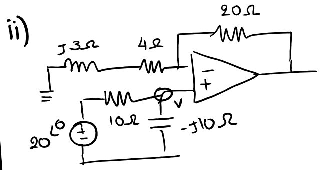
20Ω–220Ω

iii) $\frac{R_f}{R_{in}} = 12$ Resistors 10Ω–120Ω

50Ω–600Ω

20Ω–240Ω

ii)



$Z = 10 - j10\Omega$

$V = \frac{-j10}{10 - j10} \cdot 20\angle 0 =$

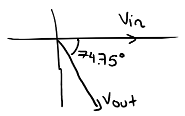
$= 10 - j10$ V $= 10\sqrt{2}\angle{-45^\circ}$ V

$\frac{V_{out}}{V_{in}} = 1 + \frac{Z_f}{3j + 4} = 1 + \frac{16}{5} - \frac{12}{5}j$

$V_{out} = \left(\frac{21}{5} - \frac{12}{5}j\right)V_{in} = \left(\frac{21}{5} - \frac{12}{5}j\right)(10 - j10)$

$= 18 - j66 = 68.4\angle{-74.75^\circ}$ V

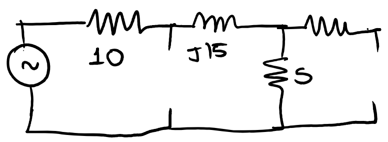
phasor diagram



Question 3

Must be superposition:

for 60cos15t voltage source



$Z = 15 + j15$

$I = \frac{60}{15 + j15} = \frac{4}{1 + j} = 2 - 2j$

or voltage divider

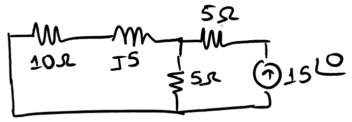
$V = \frac{5}{15 + j15} \cdot 60$

$V = 5 \cdot (2 - 2j) = 10 - j10V$

$= 14.14\angle{-45^\circ}$ V

$= 14.14\cos(15t - 45^\circ)$

b) Current source 15cos5t



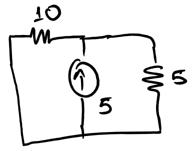
$I = \frac{10 + j5}{15 + j5} \cdot 15$

$= \left(\frac{7}{10} + j\frac{1}{10}\right) \cdot 15 = 10.6\angle{8.13^\circ}$ A

$V = 53.03\angle{8.13^\circ}$ V

$v_2(t) = 53.03\cos(5t + 8.13^\circ)$ V

c) dc current source



$I = \frac{10}{15} \cdot 5 = 3.33$ A

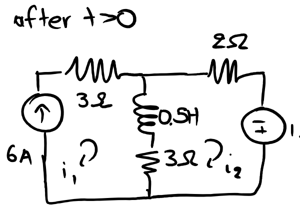
$V = 5 \cdot 3.33 = 16.65V$

So $v_x(t) = 16.65 + 53.03\cos(5t + 8.13^\circ) + 14.14\cos(15t - 45^\circ)$

ii) Initial current of inductor $i_L(0) = 3A$ $e_L = \frac{1}{2} 0.5 \cdot 3^2$

$= 9/4$ J

after t>0



1) Mesh analysis

$i_1 = 6A$

$-12 - 3(i_1 - i_2) + 2i_2 = 0$

$\Rightarrow -12 - 3i_1 + 5i_2 = 0$

$\Rightarrow 3i_2 = 18 \Rightarrow i_2 = 6A$

$i_L = i_1 - i_2 = 0A$

or 2) $6 = \frac{V}{3} + \frac{V + 12}{2}$

$\Rightarrow 36 = 2V + 3V + 36$

$\Rightarrow V = 0$

$i_L = V/3 = 0A$

Any method correct

$i_L(\infty) = 0A$

$\tau = \frac{L}{R} = \frac{0.5}{5}$

$= 0.1s$

$i_L(t) = 3e^{-t/0.1}$ A, t→sec $= 3e^{-10t}$ A, t→sec

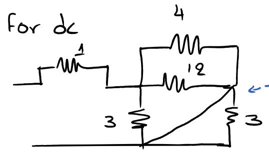
c)

d) $V_{R_2} = i_{R_2} \cdot R = 3 \cdot i_L = 9e^{-10t}$ V, t→sec. 1mark

$V_{R_1} = i_{R_1} \cdot R = 6 \cdot 3 = 18V$ ← 1mark.

Question 4.

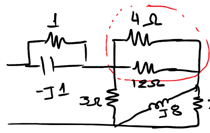
i) For dc



← 1mark if C: open circuit L: short circuit

$R = 1 + 4 \| 12 \| 3 = 2.5\Omega$ ← 2marks for all correct

ii)



$Z_{eq} = (1 \| -j1) + 3 \| (3 + 3 \| j8)$

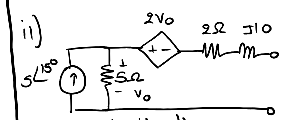
$= 0.5 - j0.5 + 3 \| \left(3 + \frac{j24}{3 + j8}\right)$

$= 0.5 - j0.5 + 3 \| (5.63 + j0.98)$

$= 0.5 - j0.5 + 1.97 + j0.11$

$= 2.47 - j0.39\ \Omega$

ii)



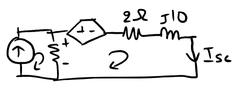
Open circuit voltage:

$V_{Th} = -V_o$

$V_o = 5 \cdot 5\angle 15^\circ = 25\angle 15^\circ$

$V_{Th} = -25\angle 15^\circ$ V

short circuit current



$-5(i_1 - i_2) + 2(5(i_1 - i_2)) + 2i_2 + j10i_2 = 0$

$-5i_1 + 5i_2 + 10i_1 - 10i_2 + 2i_2 + j10i_2 = 0$

$\Rightarrow 5i_1 - 3i_2 + j10i_2 = 0$

$\Rightarrow i_2(-3 + j10) = -5i_1$

$\Rightarrow i_2 = -\frac{25\angle 15}{-3 + j10} = I_{sc}$

$Z_{Th} = \frac{V_{Toc}}{I_{sc}} = \frac{25\angle 15^\circ}{\frac{25\angle 15^\circ}{-3 + j10}} = -3 + j10\Omega$

iii) $Z_L = Z_{Th}^*$ or conjugate of Z_{Th}

2marks

Question 5:

i) Load 1: 20kVA 0.8pf leading → 16 − j12 kVA 0.5marks

Load 2: 6kW pf 0.6 lagging → 6 + j8 kVA 0.5marks

Total complex power = 22 − j4 kVA ← 1mark

Apparent power = $\sqrt{22^2 + 4^2} = 22.36$kVA ← 1mark

1mark

$I^* = \frac{S}{V} = \frac{22 - j4}{230} = 95.65 - j17.39$

↑ 1mark

$I = 95.65 + j17.39$ A

either of the 2

$I_{rms} = 97.21$ A

or

↓ 1mark

1mark

$S = V \cdot I \Rightarrow I = 22.36\text{kVA}/230 = 97.21$ A

$pf = \frac{22}{22.36} = 0.983$ leading

1mark 1mark

ii) $V_p = 500V$

$V_s = 100V$

$I_s = \frac{100}{10 + j10} = 5 - 5j = 7.07\angle{-45^\circ}$ A

$I_p = 1 - 1j = 1.41\angle{-45^\circ}$ A

$S = V \cdot I^* = 100(5 + 5j) = 500 + j500$ VA

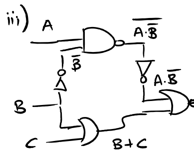
P = 500W, Q = 500VAr

S = 707 VAr

% loading $= \frac{707}{2000} = 35.35\%$ 2marks

if 25% 1mark.

iii)



1mark

$\overline{A \cdot \overline{B} + B + C} = \overline{A \cdot \overline{B}} \cdot \overline{B + C}$

$= (\overline{A} + B) \cdot \overline{B} \cdot \overline{C}$

$= \overline{A}\,\overline{B} \cdot \overline{C} + B\overline{B}\,\overline{C}$

$= \overline{A} \cdot \overline{B} \cdot \overline{C}$

2marks

Truth table

A	B	C	Out
0	0	0	1
All others			0

← 1mark

← 2marks