UNSW SYDNEY Australia's Global University

School of Electrical Engineering & Telecommunications

ELEC1111 Tutorial

Topic 7: AC Circuits I

1. For the following pairs of sinusoids, find the phasors corresponding to each pair and then determine which one leads and by how much. Also, sketch each pair in phasor diagram using rectangular form of the phasors.

   a) $v(t) = 50\cos(4t - 60°)\ \mathrm{V}$ and $i(t) = 17\sin(4t + 50°)\ \mathrm{A}$
   b) $v_1(t) = 4\cos(314t + 10°)\ \mathrm{V}$ and $v_2(t) = -20\cos(314t)\ \mathrm{V}$
   c) $i_1(t) = 13\cos(2t) + 5\sin(2t)\ \mathrm{A}$ and $i_2(t) = 15\cos(2t - 11.8°)\ \mathrm{A}$
   d) $v(t) = 50\cos(20t + 70°)\ \mathrm{V}$ and $i(t) = 40\cos(25t + 20°)\ \mathrm{A}$

   Answer:
   a) $\mathbf{V} = 50\angle(-60°)\ \mathrm{V}$, $\mathbf{I} = 17\angle(-40°)\ \mathrm{A}$. i leads v by $20°$
   b) $\mathbf{V}_1 = 4\angle(10°)\ \mathrm{V}$, $\mathbf{V}_2 = 20\angle(\pm 180°)\ \mathrm{V}$. v_2 leads v_1 by $170°$ <u>or</u> v_1 leads v_2 by $190°$
   c) $\mathbf{I}_1 = 13.92\angle(-21.03°)\ \mathrm{A}$, $\mathbf{I}_2 = 15\angle(-11.8°)\ \mathrm{A}$. i_2 leads i_1 by $9.23°$
   d) Not comparable! Why?

2. Evaluate the following complex numbers and express your results in both polar and rectangular forms.

   a) $\frac{(5\angle 10°)(10\angle(-40°))}{(4\angle(-80°))(-6\angle 50°)}$
   b) $\frac{2+j3}{1-j6} + \frac{7-j8}{-5+j11}$
   c) $\begin{vmatrix} 2+j3 & -j2 \\ 2e^{-j\pi/2} & 8-j5 \end{vmatrix}$
   d) $\frac{60e^{j45°}}{7.5-j10} + j2$

   Answer:
   a) Polar form: $2.083\angle \pm 180°$, rectangular forms: -2.083
   b) Polar form: $1.284\angle 173.2°$, rectangular forms: $-1.275 + j0.152$
   c) Polar form: $37.696\angle 21.8°$, rectangular forms: $35 + j14$
   d) Polar form: $6.786\angle 95.741°$, rectangular forms: $-0.678 + j6.752$

3. Obtain the sinusoids corresponding to each of the following phasors.

a) $\mathbf{V}_1 = 60\angle 15°\ \mathrm{V},\ \ \omega = 1\ \mathrm{rad/s}$
b) $\mathbf{V}_2 = 6 + j8\ \mathrm{V},\ \ \omega = 40\ \mathrm{rad/s}$
c) $\mathbf{I}_1 = 2.8e^{-j\pi/3}\ \mathrm{A},\ \ f = 50\ \mathrm{Hz}$

Answer:
a) $v_1(t) = 60\cos(t + 15°)\ \mathrm{V}$
b) $v_2(t) = 10\cos(40t + 53.13°)\ \mathrm{V}$
c) $i_1(t) = 2.8\cos(314.16t - 60°)\ \mathrm{A}$

4. Find $v(t)$ and $i(t)$ in the following integrodifferential equations using phasor approach.

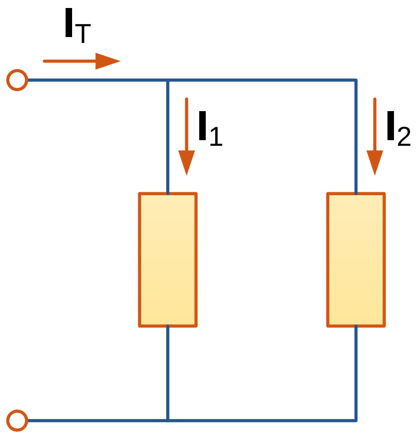
a) $v(t) + \int v(t)\,dt = 10\cos(t)$

b) $10\int i(t)\,dt + \frac{di(t)}{dt} + 6i(t) = 5\cos(5t + 22°)$

Answer:
a) $v(t) = 7.071\cos(t + 45°)\ \ \mathrm{V}$
b) $i(t) = \ 745\cos(5t - 4.56°)\ \ \mathrm{mA}$

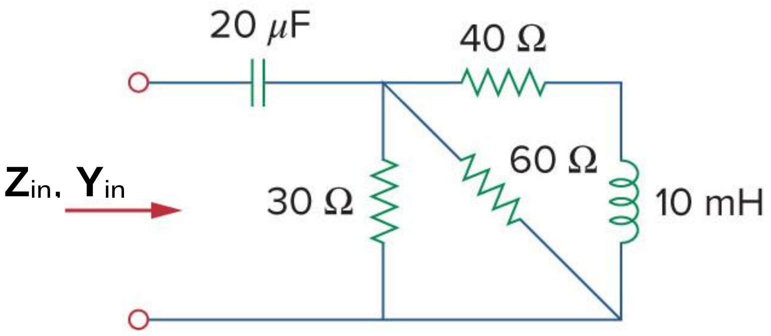
Hint: use the properties of differentiation and integration in time domain and their equivalents in phasor domain ,i.e., $\frac{dv(t)}{dt} \Leftrightarrow j\omega\mathbf{V}$ and $\int v(t)\,dt \Leftrightarrow \frac{\mathbf{V}}{j\omega}$, then solve the equations for the phasor of the main variables by writing the equations in phasor domain assuming that $\mathbf{V}$ is the phasor of $v(t)$ and $\mathbf{I}$ is the phasor of $i(t)$.

5. (**Final Exam – S1, 2015**) In the following figure, $i_1 = 100\sin(50t + 100°)\ \mathrm{A}$ and $i_T = 50\sin(50t - 40°)\ \mathrm{A}$. Determine phasors $\mathbf{I}_1$, $\mathbf{I}_2$ and $\mathbf{I}_\mathrm{T}$ and draw the phasor diagram showing $\mathbf{I}_1$, $\mathbf{I}_2$ and $\mathbf{I}_\mathrm{T}$. Also, determine which one lags and by how much.



Answer: $\mathbf{I}_1 = 100\angle 10°\ \mathrm{A}$, $\mathbf{I}_2 = 141.98\angle(-156.92°)\ \mathrm{A}$ and $\mathbf{I}_\mathrm{T} = 50\angle(-130°)\ \mathrm{A}$. $\mathbf{I}_2$ lags $\mathbf{I}_\mathrm{T}$ by $26.92°$ and lags $\mathbf{I}_1$ by $166.92°$, also $\mathbf{I}_\mathrm{T}$ lags $\mathbf{I}_1$ by $140°$.

6. For the following circuit,
    a) Find the input impedance $\mathbf{Z}_{\text{in}}$ and input admittance $\mathbf{Y}_{\text{in}}$ with their corresponding resistance, reactance, conductance and susceptance if $\omega = 10^3 \text{ rad/s}$.
    b) Is the circuit *inductive* or *capacitive*?



Answer:

a) $\begin{cases} \mathbf{Z}_{\text{in}} = R + jX = 13.51 - j48.92\ \Omega = 50.75\angle(-74.56°)\ \Omega \\ \mathbf{Y}_{\text{in}} = \frac{1}{\mathbf{Z}_{\text{in}}} = G + jB = 5.24 + j18.99 \text{ mS} = 19.704\angle 74.56° \text{ mS} \end{cases}$

b) Capacitive.

7. Calculate $\mathbf{I}_o$ in Fig. 1 and $v_o(t)$ in Fig. 2.

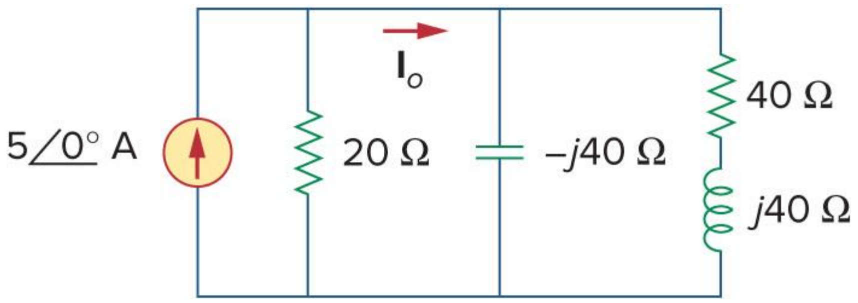


Fig. 1

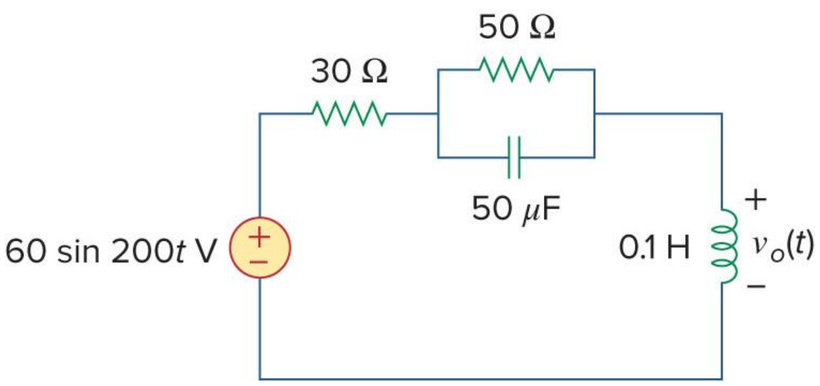
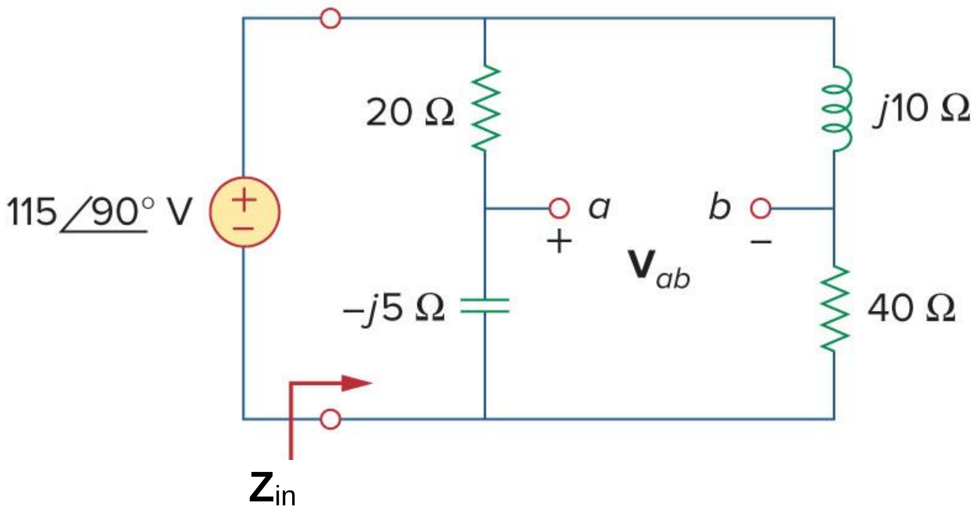


Fig. 2

Answer: Fig. 1: $\mathbf{I}_o = 1.386\angle 33.69° \text{ A}$, and Fig. 2: $v_o(t) = 17.14 \cos(200t) \text{ V}$

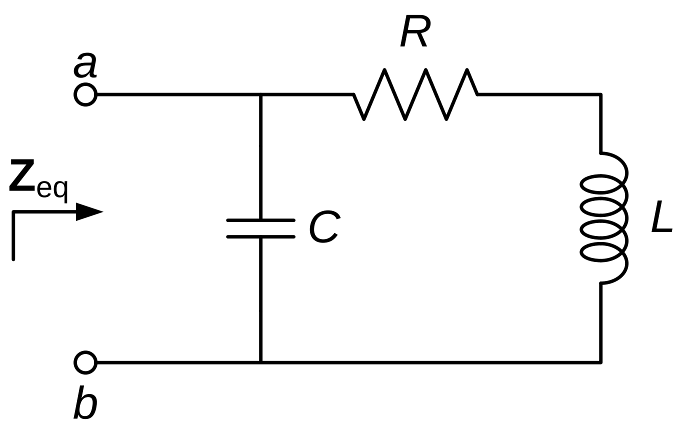
8. For the circuit given below, calculate input impedance $\mathbf{Z}_{\text{in}}$ and $\mathbf{V}_{ab}$.



Answer: $\mathbf{Z}_{\text{in}} = 14.069 - j1.172\ \Omega = 14.117\angle(-4.76°)\ \Omega$, and $\mathbf{V}_{ab} = 101.4\angle(-90°)\ \text{V} \equiv 101.4\angle 270°\ \text{V}$

Challenging Problem

9. (**Final Exam – S1, 2016**) Consider the circuit shown in the figure below, where $L = 27\ \text{mH}$, $C = 22\ \mu\text{F}$ and $R = 6\ \Omega$. Find the angular frequency ω for which the impedance $\mathbf{Z}_{\text{eq}}$ between terminals *a*-*b* is purely resistive.



Answer: $\omega = 1.278 \times 10^3\ \text{rad/s}$
Hint: The reactance of $\mathbf{Z}_{eq}$ is a function of ω and it should be zero for pure resistive impedance, or the phase of $\mathbf{Z}_{eq}$ should be zero for pure resistive impedance.