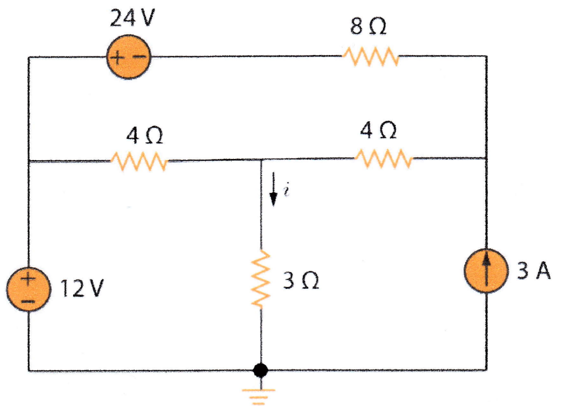
School of Electrical Engineering & Telecommunications

ELEC1111

Topic 3: Circuit Theorems

1. Find the current i in the following circuit by using superposition principle.

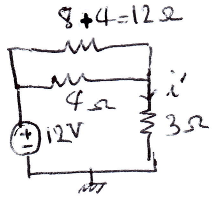


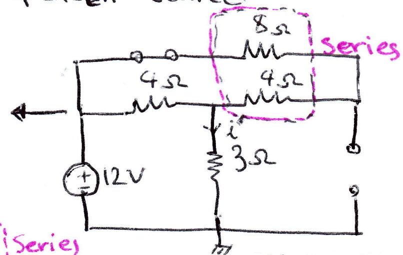
Answer: $i = i' + i'' + i''' = 2$ A, where $i' = 2$ A, $i'' = -1$ A, and $i''' = 1$ A are the responses due to 12-V, 24-V, and 3-A sources.

Solution: Let's find the response to each source individually by setting the other independent sources to zero. Note that there is NO dependent source.

* i' due to 12-V source

Set 24-V and 3-A sources to zero



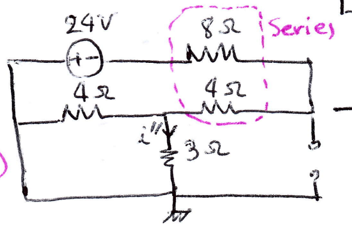


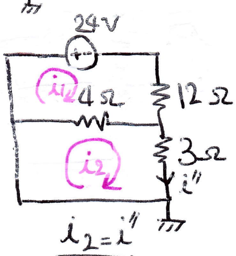
KVL: $-12 \times (3 + \underbrace{4 \| 12}_{3}) i' = 0$

$i' = \frac{12}{6} = \boxed{2A}$

* i'' due to 24-V Source

Set 12-V and 3-A sources to zero





$\underline{i_2 = i''}$

Use Mesh analysis:

KVL @ m1: $24 + 12i_1 + 4(i_1 - i_2) = 0$ (I)

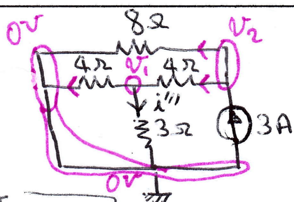
KVL @ m2: $-4(i_1 - i_2) + 3i_2 = 0$ (II)

(II) $\rightarrow$ $\boxed{i_1 = \frac{7}{4} i_2}$

(I) $\rightarrow$ $16i_1 - 4i_2 = -24$ or $4i_1 - i_2 = -6$ $\longrightarrow$ $4 \times \frac{7i_2}{4} - i_2 = -6 \rightarrow \boxed{i_2 = i'' = -1A}$

* i''' due to 3-A Source

Set 12-V and 24V sources to zero



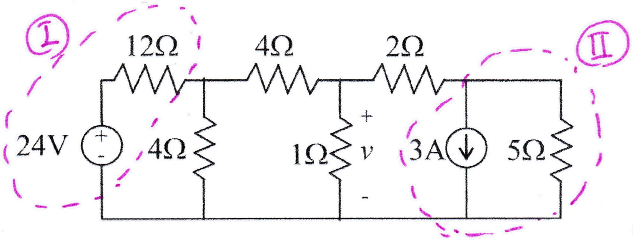
Using Nodal Analysis $\rightarrow$ KCL @ V_1: $\frac{V_1}{4} + \frac{V_1}{3} = \frac{V_2 - V_1}{4}$

$\xrightarrow{\times 12}$ $3V_1 + 4V_1 = 3V_2 - 3V_1 \rightarrow \boxed{V_2 = \frac{10}{3} V_1}$

KCL @ V_2: $3 = \frac{V_2}{8} + \frac{V_2 - V_1}{4}$ $\longrightarrow$ $3V_2 - 2V_1 = 24$ $\longrightarrow$ $3 \times \frac{10}{3} V_1 - 2V_1 = 24 \Rightarrow \boxed{V_1 = 3V}$

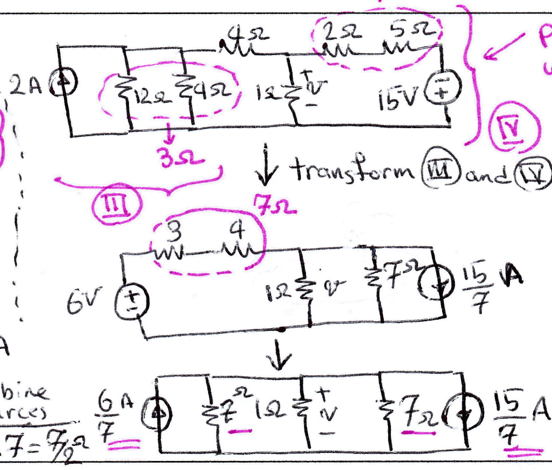
$i''' = \boxed{\frac{V_1}{3} = 1A}$ $\Rightarrow$ Thus $i = i' + i'' + i''' = 2 - 1 + 1 = 2A$ Contribution of 3 Sources via superposition

2. (**Final Exam, S1, 2014**) Use source transformations to determine the voltage v shown in the following circuit.



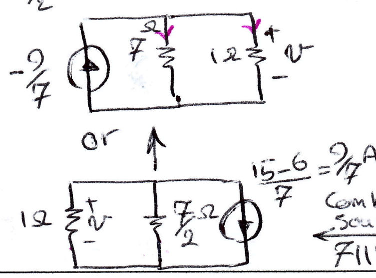
Answer: $v = -1\ \mathrm{V}$

Solution: First transform (I) and (II)



$$V = 1 \times \frac{7/2}{1+7/2} \times \frac{-9}{7} = \frac{-9}{9} = -1\,V$$

Current division

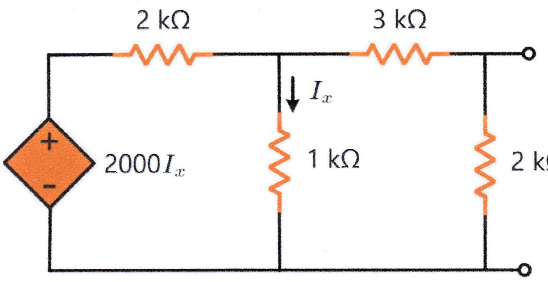


$\frac{15-6}{7} = 9/7\,A$

Combine sources

$7 \| 7 = 7/2\,\Omega$

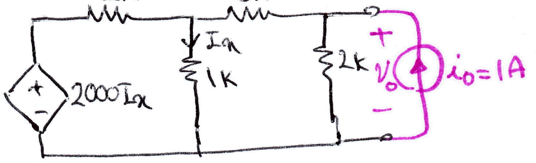
3. Determine R_{Th} for the following circuit. What is the value of the Thevenin equivalent voltage? Why can we find this value without any calculations?



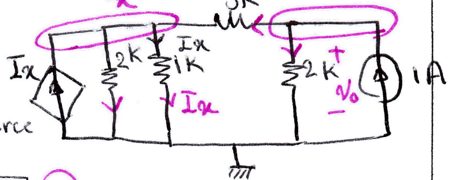
Answer: $V_{\mathrm{Th}} = 0\ \mathrm{V}$, $R_{\mathrm{Th}} = \frac{10}{7} = 1.428\ \mathrm{k\Omega}$

Solution: Since there is no independent source in the circuit $V_{Th} = 0V$

for R_{Th}, attach a current source $i_o = 1A$ and find the voltage V_o across i_o or $2k\Omega$.



Source transformation for dependent source



Use Nodal Analysis

$$KCL @ V_o: \quad 1 = \frac{V_o}{2k} + \frac{V_o - V_x}{3k} \xrightarrow{\times 6} 5V_o - 2V_x = 6 \quad (\mathrm{I})$$

$$KCL @ V_x: \quad I_x + \frac{V_o - V_x}{3k} = \frac{V_x}{2k} + \frac{V_x}{1k} \xrightarrow[I_x = \frac{V_x}{1k}]{\times 6} 2V_o = 5V_x \longrightarrow V_x = \frac{2}{5}V_o \quad (\mathrm{II})$$

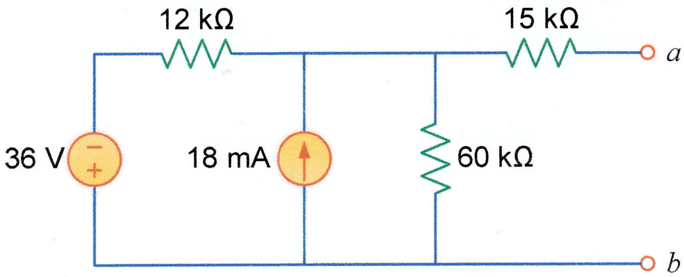
$$\xrightarrow{\text{Sub (II) into (I)}} 5V_o - \frac{4}{5}V_o = 6 \rightarrow V_o = \frac{30}{21} = \frac{10}{7}kV$$

Careful that all restors are in $k\Omega$ and with $i_o = 1A$, the voltage will be in KV

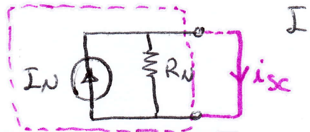
$$\Rightarrow R_{Th} = \frac{V_o}{i_o} = \frac{\frac{10}{7}kV}{1A} = \frac{10}{7}k\Omega$$

pay attention

4. (**Final Exam - S2, 2015**) Find the Norton equivalent of the following circuit.

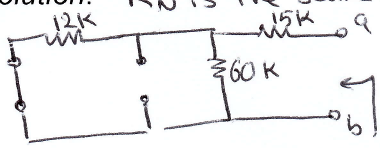


Norton equivalent circuit



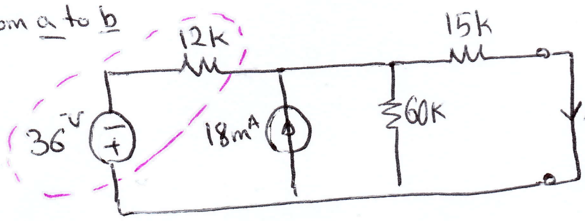
Answer: $I_N =$ 6mA, $R_{Th} = 25\ \text{k}\Omega = R_N$

Solution: R_N is the same as R_{Th}. Set all independent sources to zero (No dependent source)

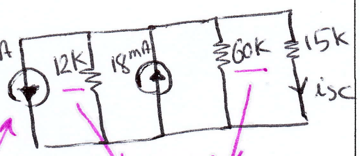


$R_{in} = R_N = R_{Th} = 15^K + (12 \| 60) = 15 + \frac{12 \times 60}{72} = 15 + 10 = 25\,K\Omega$ $R_N = R_{Th}$

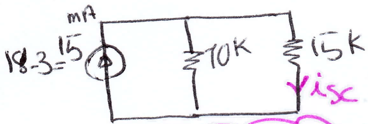
To find I_N, short circuit the terminals and find the short-circuit current flowing from a to b



Source transformation → Careful with direction

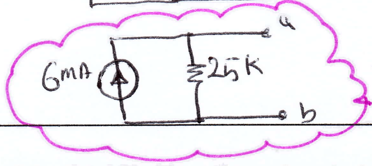


Combine current sources and $\underbrace{12^K \| 60^K}_{10K}$ →



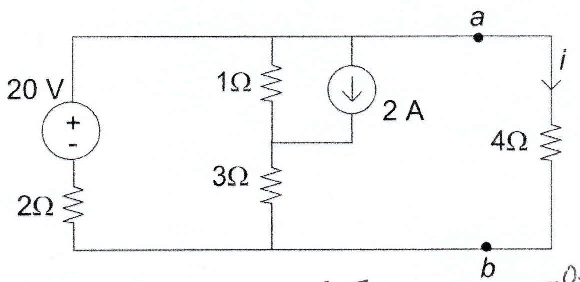
current division → $I_N = i_{sc} = \frac{10^K}{10^K + 15^K} \times 15^{mA} = 6\,mA$

$\boxed{I_N = 6\,mA}$



→ Norton equivalent circuit

5. (**Mid-session Exam – S2, 2016**) For the circuit below, find the current i in the 4-Ω resistor using,
   (a) The superposition principle.
   (b) The Thevenin equivalent of the circuit to the left of terminal pair *a*-*b*.



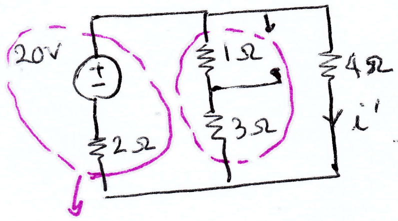
Answer: a) $i = i' + i'' = 2.375$ A, where $i' =$ 2.5 A and $i'' =$ -0.125 A are the responses due to 20-V and 2-A sources.

b) $V_{Th} = 12.66$ V, $R_{Th} = 4/3\ \Omega = 1.33\ \Omega$

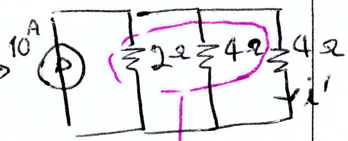
Solution: Let's find the response to each source individ by setting the other independent sources to zero

5.a) $i = i' + i''$

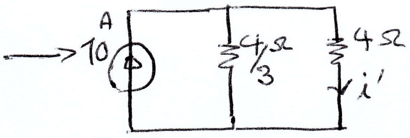
* i' due to 20-V source
  Set 2-A source to zero



Source transformation

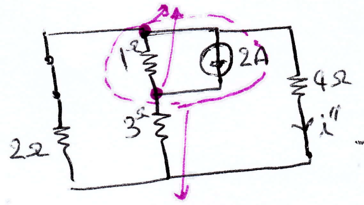


$2 \| 4 = 1.33 = \frac{4}{3}\,\Omega$

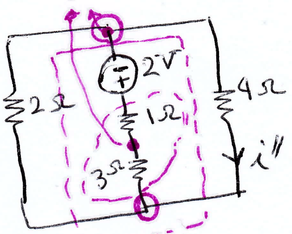


current division: $i' = \dfrac{\frac{4}{3}}{\frac{4}{3}+4} \times 10 = \dfrac{40}{16} = \boxed{2.5\text{ A}}$

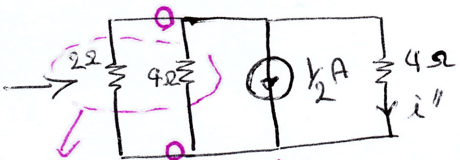
* i'' due to 2-A source
  Set 20V source to zero



Source transformation



Source transformation



$2 \| 4 = \frac{4}{3}\,\Omega$

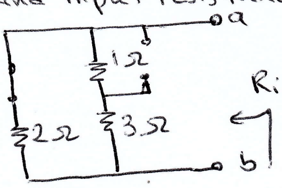
Note the direction

current division: $i'' = \dfrac{\frac{4}{3}}{\frac{4}{3}+4} \times -\frac{1}{2} = \dfrac{-4}{16 \times 2} = \boxed{-0.125\text{ A}}$

superposition

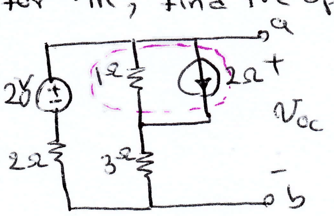
Thus $i = i' + i'' = 2.5 - 0.125 = 2.375\text{ A}$

5.b) First detach the load (4Ω) from the circuit.
Then check for dependent sources, if not, set all independent sources to zero and find input resistance, R_{Th}

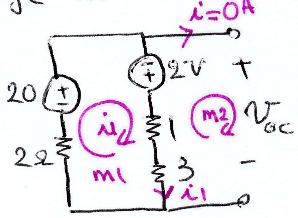


$R_{in} = R_{Th} = 2 \| (1+3) = \frac{4}{3}\,\Omega$

For V_{Th}, find the open-circuit voltage after detaching the load, $V_{Th} = V_{oc}$



Source transformation

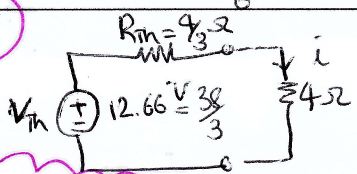


KVL$_{m1}$: $-20 - 2 + (1+3+2)\,i_1 = 0$

$\rightarrow i_1 = \boxed{\frac{22}{6}\text{ A}}$

KVL in m_2: $V_{oc} - (1+3)\,i_1 + 2 = 0 \rightarrow V_{oc} = 4 \times \frac{22}{6} - 2 = \frac{38}{3} = 12.66\text{ V}$

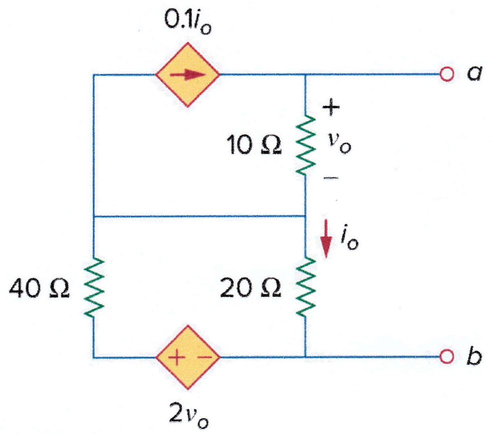
$\Rightarrow V_{Th} = V_{oc} = 12.66\text{ V}$



using Thevenin $\Rightarrow$ equivalent circuit

$i = \dfrac{V_{Th}}{R_{Th}+4} = \dfrac{\frac{38}{3}}{\frac{4}{3}+4} = \dfrac{38}{16} = 2.375\text{ A}$

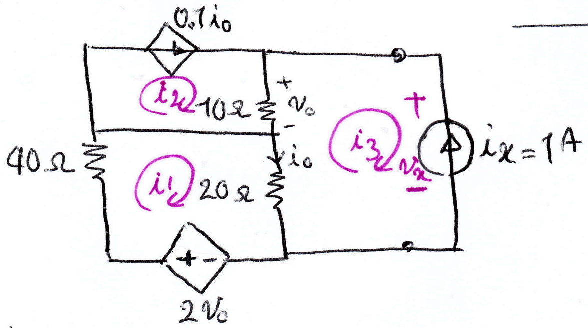
6. Find the Thevenin equivalent of the circuit given below.



Answer: $V_{Th} = 0$ V, $R_{Th} = 31.73$ kΩ

Solution: Since there is no independent source in the circuit, $\boxed{V_{Th} = 0\,V}$

For R_{Th}, attach a current source $i_x = 1A$ and then find the voltage V_x across the new current source. Then $\boxed{R_{Th} = \frac{V_x}{i_x} = \frac{V_x}{1}}$



$$\begin{cases} i_3 = -i_x = -1 \quad \text{(I)} \\ V_o = 10(i_2 - i_3) \quad \text{(II)} = \boxed{10(i_2+1)} \\ i_o = i_1 - i_3 \quad \text{(III)} \\ \quad = i_1 + 1 \end{cases}$$

Also from m2 $\rightarrow 0.1\,i_o = i_2 \xrightarrow{\text{(III)}} \boxed{0.1(i_1+1) = i_2}$ (*)

Using Mesh Analysis : *KVL in m1 : $40i_1 + 20i_o - 2V_o = 0$

$$\xrightarrow[\text{(I)}]{\text{(II)},\text{(III)}} 40i_1 + 20(i_1+1) - 2\times 10(i_2+1) = 0$$

$$\rightarrow 60i_1 - 20i_2 = 0 \rightarrow \boxed{3i_1 = i_2} \text{ (**) } \xrightarrow{(*)} 3i_1 = 0.1(i_1+1)$$

$$\Rightarrow 30i_1 = i_1 + 1 \Rightarrow \boxed{i_1 = \frac{1}{29}A} \Rightarrow \boxed{i_2 = \frac{3}{29}A}$$

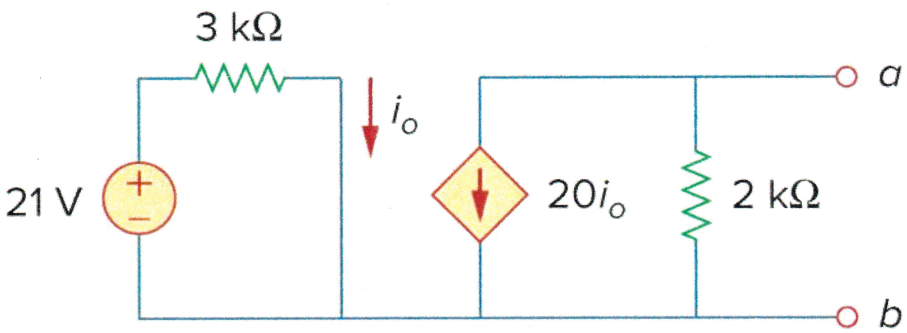
KVL in m3 $V_x - 20i_o - V_o = 0 \xrightarrow[\text{(II)}]{\text{(III)}} V_x = 20(i_1+1) + 10(i_2+1) \xrightarrow{(**)} V_x = 20i_1 + 20 + 30i_1 + 10$

$$\Rightarrow V_x = 50i_1 + 30 = \frac{50}{29} + 30 = \boxed{31.73\,V}$$ (All resistor ar in Ω)

$$\Rightarrow R_{Th} = \frac{V_x}{i_x} = 31.73\,\Omega$$

7. For the transistor model given below, do the following,

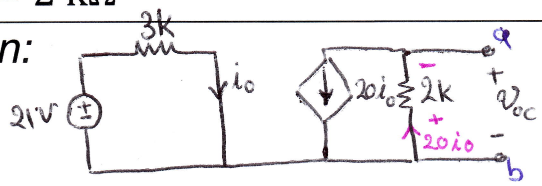
a) Obtain the open-circuit voltage v_{oc} across terminals a-b.
b) Calculate the short-circuit current i_{sc} at terminals a-b.
c) Find the equivalent resistance R_{eq} seen from the terminals a-b (use the results from previous parts)



Answer:

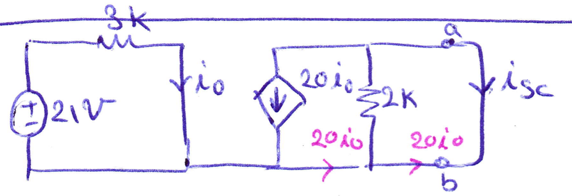
a) $v_{oc} = -280\text{ V}$
b) $i_{sc} = -140\text{ mA}$
c) $R_{\text{eq}} = 2\text{ k}\Omega$

Solution:



a) $i_o = \frac{21^V}{3k} = 7mA$ Note the Polarity

$V_{oc} = -2^k \times 20i_o = \boxed{-280^V}$

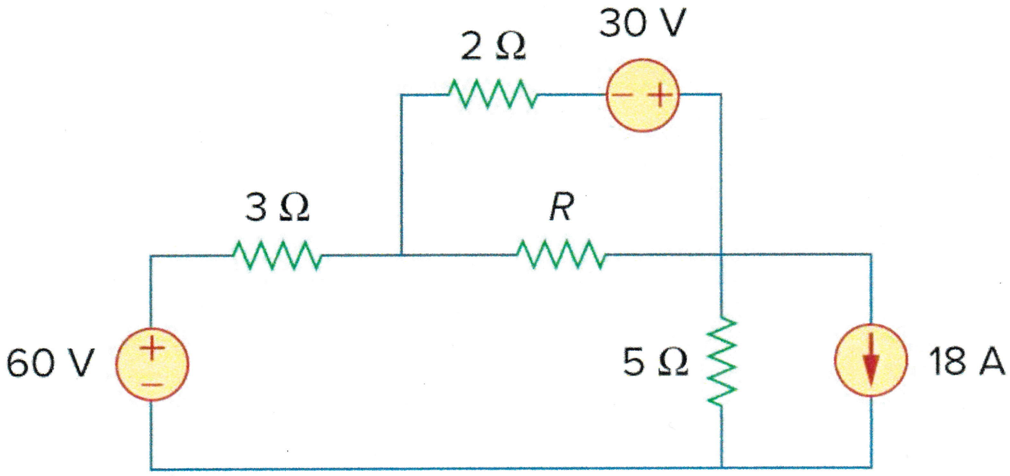


b) $i_{sc} = -20i_o \xrightarrow[\text{form part (a)}]{i_o = 7mA} \boxed{i_{sc} = -140mA}$

Note the directions

$R_{eq} = \frac{V_{oc}}{i_{sc}} = \frac{-280^V}{-140\,mA} = \boxed{2k\Omega}$

8. In the following circuit, find the maximum power that can be delivered to the resistor R in the circuit below. What should be the value of R for maximum power transfer?

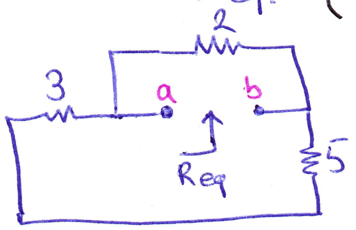


Answer: $R_{\text{Th}} = 1.6\ \Omega$, $V_{\text{Th}} = 6\ \text{V}$, $P_{\max} = 5.625\ \text{W}$, $R = 1.6\ \Omega$

Hint: Find the Thevenin equivalent circuit from the terminals across the load R.

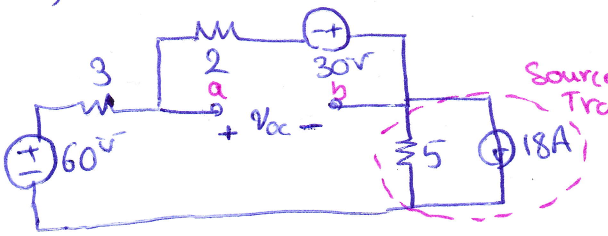
Solution: No dependent Source → Set all Independent Sources to zero and find Req. (The objective is to find Thevenin equivalent circuit)

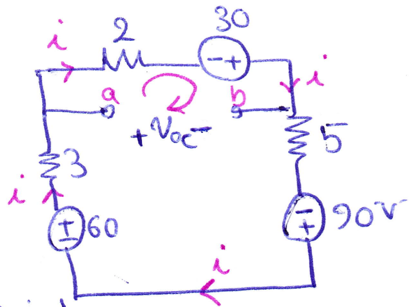
Rmove $\underline{R}$ and find the equivalent resistance first.



$\rightarrow$ $\boxed{R_{eq} = R_{Th} = (3+5) \| 2 = 1.6\,\Omega}$

Now, find Voc across the terminals





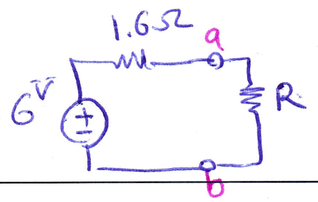
Assign the current i and write KVL in the main loop

* KVL: $-90 - 60 + 3i + 2i - 30 + 5i = 0 \rightarrow 10i - 180 = 0 \rightarrow \boxed{i = 18\,\text{A}}$ (I)

Now write KVL in top mesh or bottom mesh

* KVL in top mesh: $-V_{oc} + 2i - 30 = 0 \xrightarrow{\text{(I)}} \boxed{V_{oc} = V_{Th} = 6\,\text{V}}$

The Thevenin equivalent Circuit →

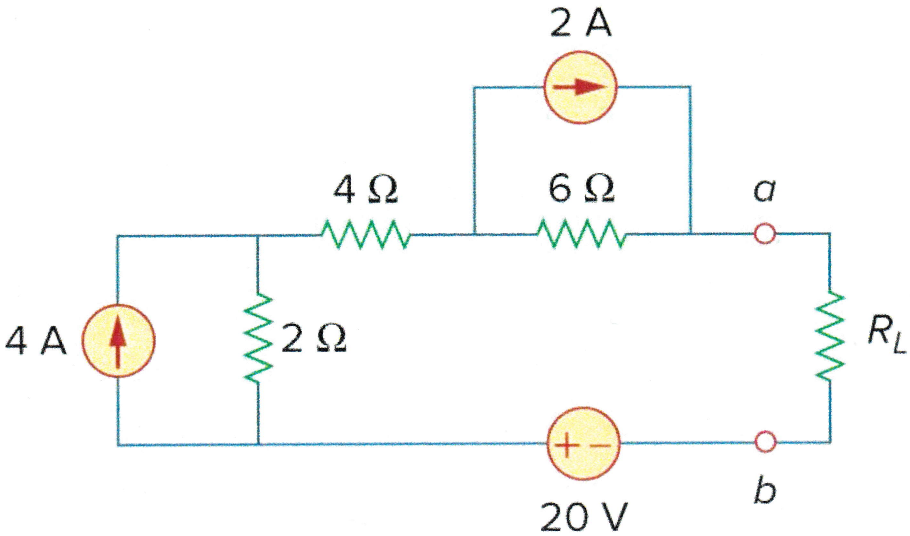


for Max Power Transfer $R = R_{Th} = 1.6\,\Omega$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{6^2}{4 \times 1.6} = 5.625\,\text{W}$$

9. For the circuit below,

a) Obtain the Norton equivalent circuit at terminals a-b.
b) Calculate the current in $R_L = 13\ \Omega$.
c) Find R_L for maximum power deliverable to R_L.
d) Determine that maximum power.



Answer:

a) $R_N = 12\ \Omega$, $I_N = \frac{10}{3} = 3.33\ \mathrm{A}$

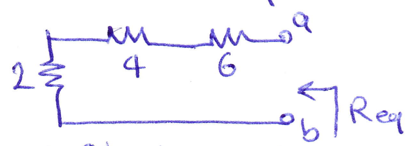
b) $I_L = \frac{8}{5} = 1.6\ \mathrm{A}$

c) $R_L = 12\ \Omega$

d) $P_{\max} = \frac{100}{3} = 33.33\ \mathrm{W}$

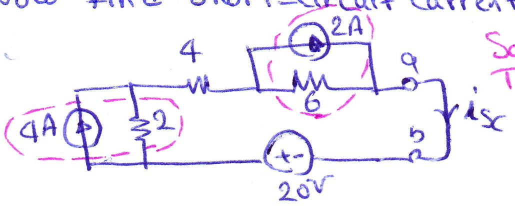
Solution: No dependent source → Set all Independent Sources to zero. Then find $R_N = R_{eq}$ from the terminals a-b

a)

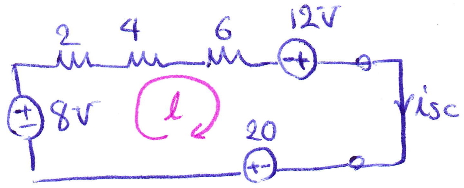


$\boxed{R_{eq} = R_N = 2+4+6 = 12\ \Omega}$

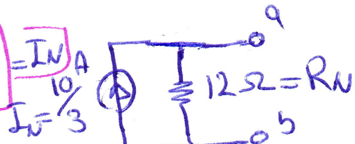
Now find Short-circuit Current



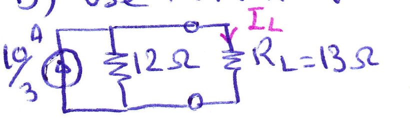
Source Transformation →



*KVL in ℓ: $-12-20-8+i(2+4+6)=0 \rightarrow \boxed{i_{sc} = \frac{10}{3}\ \mathrm{A}} = I_N$



b) Use Norton equivalent circuit

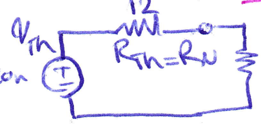


Current division → $\boxed{I_L = \frac{12}{12+13} \times \frac{10}{3} = \frac{8}{5} = 1.6\mathrm{A}}$

c) Max Power Transfer $\boxed{R_L = R_{Th} = R_N = 12\ \Omega}$

$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{40^2}{4 \times 12} = \boxed{\frac{100}{3}\ \mathrm{W}}$

d) Source Transformation



$R_L = R_{Th} = 12 \rightarrow V_{Th} = I_N R_N = 40\mathrm{V}$