"Final Exam, S2 2017 Solutions"

QUESTION 1 [20 marks]

(i) [10 marks] For the circuit shown in Figure 1,

a. **(8 marks)** Apply mesh analysis and show that the mesh equation are given as below,

$$\begin{cases} i_1 + 3i_2 - 2i_3 = 8 \\ i_1 + i_2 - 4i_3 = -1 \\ i_1 - i_2 = -4 \end{cases}$$

b. **(2 marks)** Given the values of mesh currents as $i_1 = -0.5$ A, $i_2 = 3.5$ A, and $i_3 = 1$ A, find the voltage v across 4-A current source.

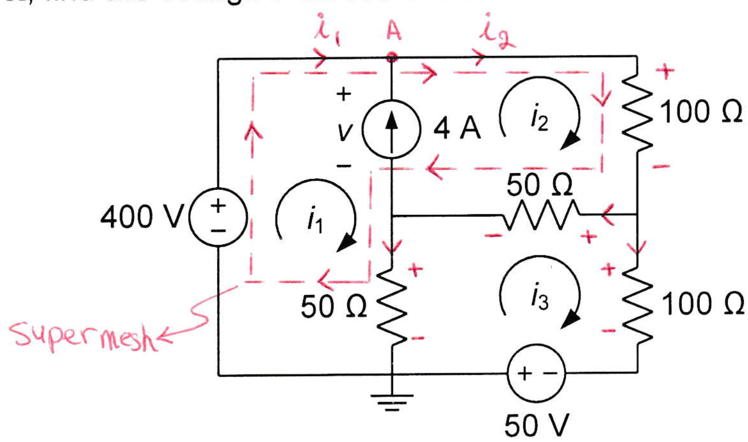


Figure 1

(i)_a:

* KVL in supermesh:

$$-400 + 100i_2 + 50(i_2 - i_3) + 50(i_1 - i_3) = 0 \xrightarrow{\times 1/50}$$

$$\longrightarrow -8 + 2i_2 + i_2 - i_3 + i_1 - i_3 = 0 \implies i_1 + 3i_2 - 2i_3 = 8$$

* KVL in mesh 3:

$$-50 - 50(i_1 - i_3) - 50(i_2 - i_3) + 100i_3 = 0 \xrightarrow{\times 1/50}$$

$$\longrightarrow -1 - i_1 + i_3 - i_2 + i_3 + 2i_3 = 0 \implies i_1 + i_2 - 4i_3 = -1$$

* KCL at node A:

$$i_1 + 4 = i_2 \implies i_1 - i_2 = -4$$

(i)_b: * KVL in mesh 1:

$$v + 50(i_1 - i_3) - 400 = 0 \xrightarrow[i_3 = 1\text{ A}]{i_1 = -0.5\text{A}} v = 50(i_3 - i_1) + 400$$

$$\implies v = 50(1 + 0.5) + 400 = 475\text{V}$$

Question 1 continued

(ii) [10 marks] For the circuit shown in Figure 2,

a. **(8 marks)** Apply nodal analysis and show that the nodal equations are given as below,

$$\begin{cases} 11v_1 - 6v_2 = 108 \\ 4v_1 - 3v_2 = -54 \end{cases}$$

b. **(2 marks)** Given the values of node voltages as $v_1 = 72$ V and $v_2 = 114$ V, Calculate the power of the dependent current source.

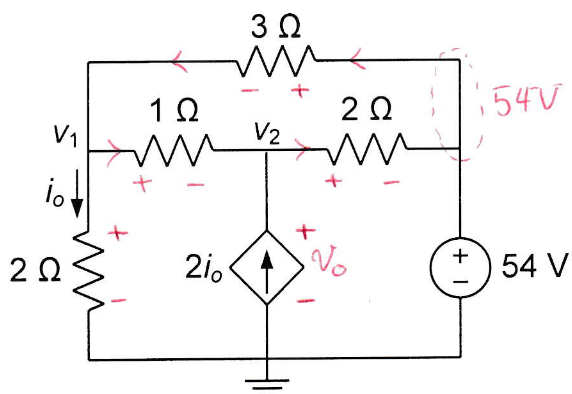


Figure 2

(ii)-a:

* KCL at node 1:

$$\frac{54 - V_1}{3} = \frac{V_1 - 0}{2} + \frac{V_1 - V_2}{1} \xrightarrow{\times 6} 108 - 2V_1 = 3V_1 + 6V_1 - 6V_2$$

$$\Rightarrow 11V_1 - 6V_2 = 108$$

* KCL at node 2:

$$\frac{V_1 - V_2}{1} + 2i_o = \frac{V_2 - 54}{2} \xrightarrow{\boxed{i_o = \frac{V_1}{2}}} V_1 - V_2 + V_1 = \frac{V_2 - 54}{2} \xrightarrow{\times 2}$$

$$\Rightarrow 4V_1 - 2V_2 = V_2 - 54 \Rightarrow 4V_1 - 3V_2 = -54$$

(ii)-b:

$P_o = 2i_o \times V_o \longrightarrow$ passive sign convention for sources (Active element)

$2i_o = 2 \times \frac{V_1}{2} = V_1 = \boxed{72\text{ A}}$

$V_o = V_2 = \boxed{114\text{V}}$

$\Rightarrow P_o = 72 \times 114 = 8208\text{ W} = 8.208\text{ KW}$ Supplied

QUESTION 2 [20 marks]

(i) [10 marks] For the circuit shown in Figure 3,

a. **(4 marks)** Calculate the open-circuit voltage v_{oc} and short-circuit current i_{sc} at the terminals *a-b*.

b. **(4 marks)** Obtain the Norton equivalent circuit with respect to the terminals *a-b* and draw the equivalent circuit.

c. **(2 marks)** Determine the value of the load resistance R_L for maximum power transfer, and then calculate the maximum power that can be delivered to R_L.

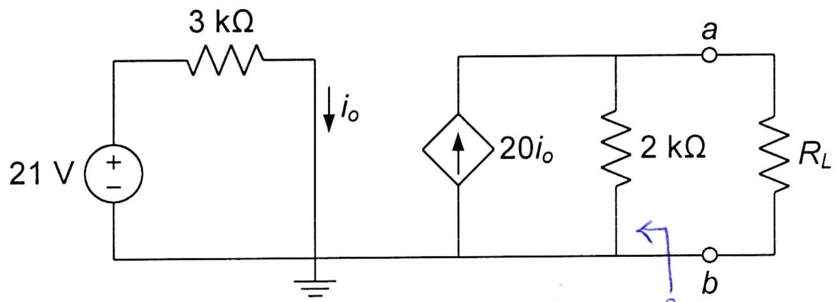
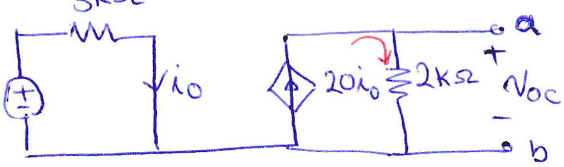


Figure 3

(i)a:

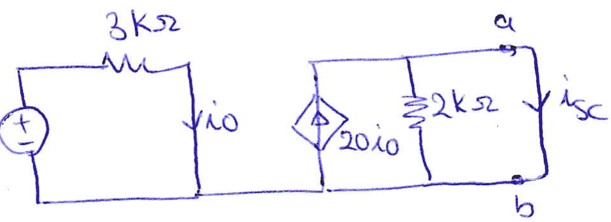


* Open-Circuit Votage Voc:

$i_o = \frac{21^V}{3K} = \boxed{7mA}$ (1)

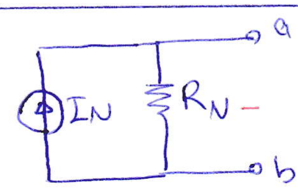
$V_{oc} = V_{2k\Omega} = 2^K \times 20\, i_o = 280^V$

* Short-Circuit Current i_{sc}:



$i_{sc} = 20\, i_o \xrightarrow{(1)} i_{sc} = 140mA$

(i)-b: The Norton equivent circuit



$R_N = R_{eq}$

$I_N = i_{sc}$

Since there exists a dependent source:

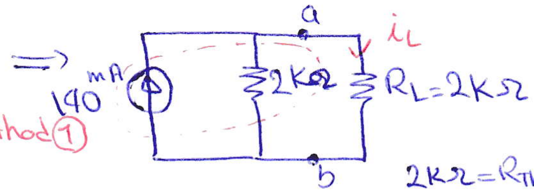
$R_{eq} = \frac{V_{oc}}{i_{sc}} = \frac{280^V}{140mA} = 2k\Omega = R_N$

$I_N = i_{sc} = 140mA$

(i)-c: for maximumpower transfer $R_L = R_{Th} = R_N = 2k\Omega$

⇒

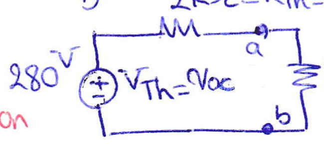
*method(1)



$i_L = \frac{2^K}{2^K + 2^K} \times 140^{mA} = 70^{mA} \Rightarrow P_{max} = R_L i_L^2 = 2^K \times 70^{mA\,2} = 9800^{mW} = 9.8\,W$

*Method(2) Source transformation

$2k\Omega = R_{Th} = R_N$



$\Rightarrow P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{280^2}{4 \times 2^K} = 9800^{mW} = 9.8\,W$

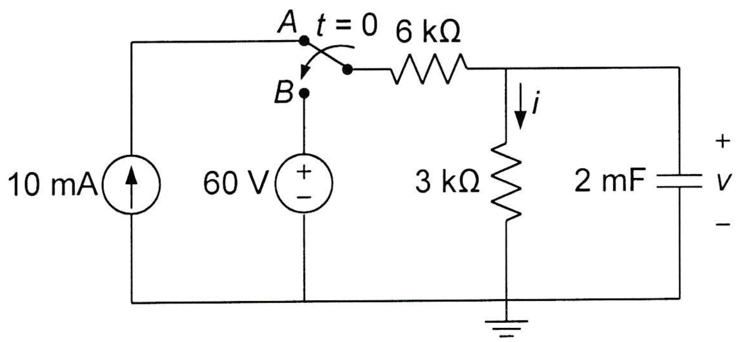
Question 2 continued

(ii) [10 marks] In circuit shown in Figure 4, the switch has been in position *A* for a long time. At $t = 0$, the switch moves to position *B*.

a. **(4 marks)** Find the voltage $v(t)$ across the capacitor immediately after the switch changes to position *B*, $v(0^+)$, and its final voltage when $t \to \infty$, $v(\infty)$.

b. **(4 marks)** Derive an expression for the capacitor voltage $v(t)$ for all time (i.e., for both $t < 0$ and $t > 0$).

c. **(2 marks)** Find the current $i(t)$ through 3-kΩ resistor for $t > 0$.

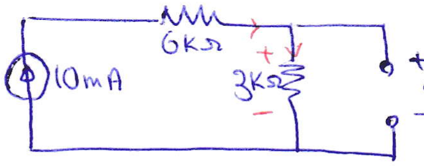


(ii)-a:

Figure 4

$V(0^+) = V(0^-)$ No sudden change in capacitor voltage

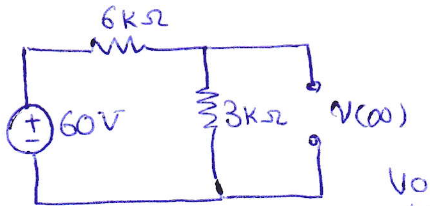
Circuit when $t < 0$ → Switch is at A



$\Rightarrow V(0^-) = V_{3K\Omega} = 3^K \times 10^m = 30V$

$\Rightarrow V(0^+) = V(0^-) = 30V$

Circuit when $t > 0$ and $t \to \infty$ → Switch is at B



$V(\infty) = \dfrac{3^K}{6^K + 3^K} \times 60 = 20V$

Voltage division

(ii)-b: After switch changes to B → $V(t) = V(\infty) + (V(0^+) - V(\infty))e^{-\frac{t}{R_{Th}C}}$

$\tau = R_{Th}C$ →

$R_{Th} = 3^K \| 6^K = \dfrac{3 \times 6}{3+6} = \dfrac{18}{9} = 2K\Omega \Rightarrow \tau = 2^{K\Omega} \times 2^{mF} = 4s$

$\Rightarrow V(t) = \begin{cases} 30V & t < 0 \\ 20 + 10e^{-\frac{t}{4}} V & t \geq 0 \end{cases}$

$\Rightarrow V(t) = 20 + (30 - 20)e^{-\frac{t}{4}}$
$= 20 + 10e^{-\frac{t}{4}} V$

(ii)-c: $i(t) = \dfrac{V_{3K\Omega}}{3K\Omega} = \boxed{\dfrac{V(t)}{3K\Omega}}$ 3KΩ is in parallel with capacitor

$t < 0 \rightarrow i(t) = \dfrac{30}{3K\Omega} = 10mA$

$t \geq 0 \rightarrow i(t) = \dfrac{20}{3} + \dfrac{10}{3}e^{-\frac{t}{4}} =$

$\Rightarrow i(t) = \begin{cases} 10mA & t \geq 0 \\ \dfrac{20}{3} + \dfrac{10}{3}e^{-\frac{t}{4}} mA & t \geq 0 \end{cases}$

QUESTION 3 [20 marks]

(i) [8 marks] For the Op Amp circuit shown in Figure 5 show that the output v_0 can be given by the following equation,

$$v_o = a_1 v_1 - a_2 v_2$$

and determine the parameters a_1 and a_2 in terms of resistors R_1 to R_5.

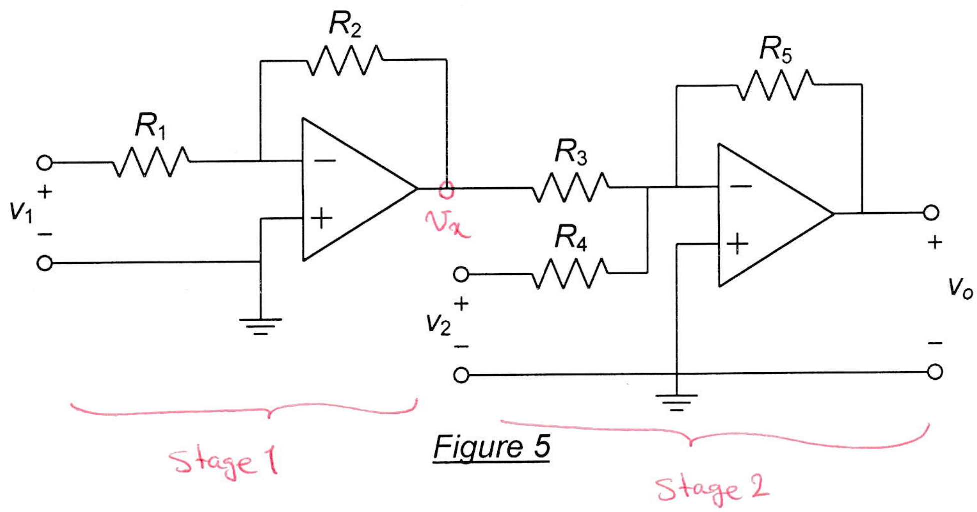


Figure 5

(i):

* Stage 1: Inverting amplifier:

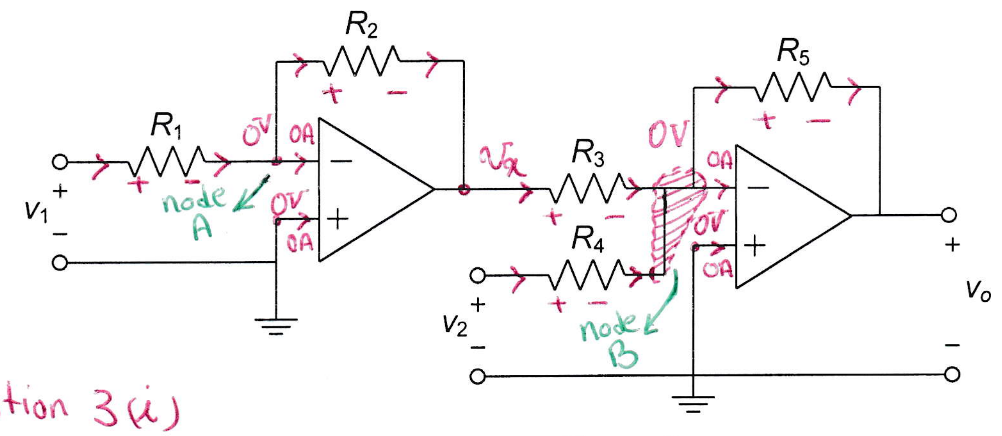
$$\boxed{\frac{V_x}{V_1} = -\frac{R_2}{R_1}} \quad \text{(I)}$$

* Stage 2: Summing amplifier:

$$V_o = -\left(\frac{R_5}{R_3} V_x + \frac{R_5}{R_4} V_2\right) \xrightarrow{\text{(I)}} V_o = -\left(\frac{R_5}{R_3} \times \frac{-R_2}{R_1} V_1 + \frac{R_5}{R_4} V_2\right)$$

$$\Rightarrow V_o = \frac{R_5 R_2}{R_3 R_1} V_1 - \frac{R_5}{R_4} V_2 \xrightarrow{\text{choose}} a_1 = \frac{R_5 R_2}{R_3 R_1}, \quad a_2 = \frac{R_5}{R_4}$$

$$\Rightarrow V_o = a_1 V_1 - a_2 V_2$$



Question 3(i)

Figure 5

Nodal analysis approach:

* KCL at node A: $\dfrac{V_1 - 0}{R_1} = \dfrac{0 - V_x}{R_2} \xrightarrow{\times -R_2} \boxed{V_x = -\dfrac{R_2}{R_1} V_1}$ (I)

* KCL at node B: $\dfrac{V_x - 0}{R_3} + \dfrac{V_2 - 0}{R_4} = \dfrac{0 - V_o}{R_5} \xrightarrow{\times -R_5} \boxed{V_o = -\dfrac{R_5}{R_3} V_x - \dfrac{R_5}{R_4} V_2}$ (II)

$\xrightarrow{\text{(I)} \to \text{(II)}}$ Substitute V_x

$$V_o = +\frac{R_5 R_2}{R_3 R_1} V_1 - \frac{R_5}{R_4} V_2 \xrightarrow{\text{Choose}} a_1 = \frac{R_5 R_2}{R_3 R_1}, \quad a_2 = \frac{R_5}{R_4}$$

$$\Longrightarrow \boxed{V_o = a_1 V_1 - a_2 V_2}$$

Question 3 continued

(ii) [12 marks] In the circuit of Figure 6, the switch has been in position *A* for a long time before changing its position to *B* at $t = 0$.

a. **(10 marks)** Derive an expression for the inductor current $i_L(t)$ for all time (i.e., for both $t < 0$ and $t > 0$) and sketch $i_L(t)$ as a function of time showing all critical points in the sketch.

b. **(2 marks)** Find the voltage $v(t)$ across the 2-Ω resistor for $t > 0$.

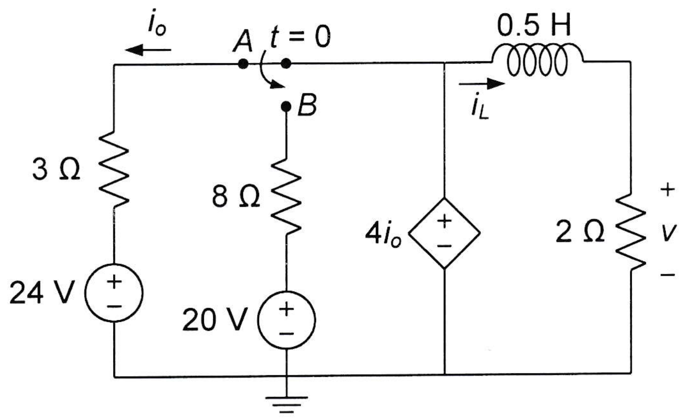
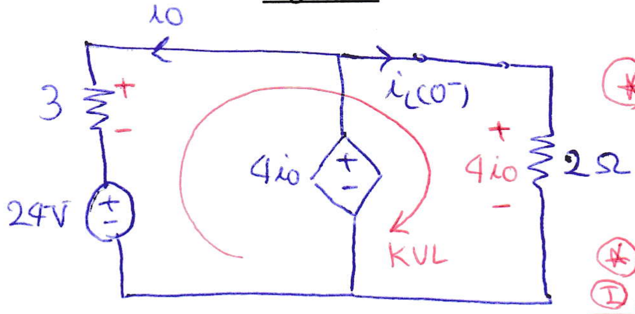


Figure 6

(ii)_a:

Circuit at $t<0$ → Switch is at A



(*) 2-Ω resistor is in parallel with dependent voltage source thus $\boxed{V_{2\Omega} = 4i_o}$ (I)

(*) Also $V_{2\Omega} = 2i_L(0^-)$

(I) → Thus $4i_o = 2i_L(0^-)$

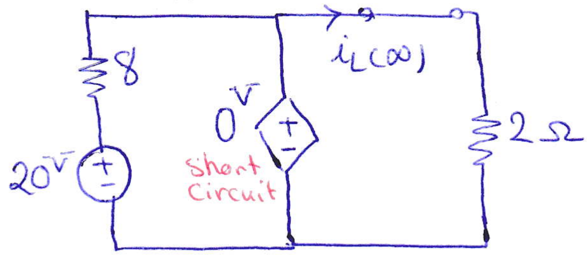
$\Rightarrow \boxed{i_L(0^-) = 2i_o}$ (II)

* KUL in big loop to find i_o

$$-24 - 3i_o + 4i_o = 0 \rightarrow \boxed{i_o = 24A}$$

Using (II) $i_L(0^-) = 2i_o = 48\,A$ $t<0$

Circuit at $t>0$ and $t \to \infty$ → Switch is at B



when switch changes to B, $i_o = 0\,A$ which means any dependent source relying on i_o will be zero.

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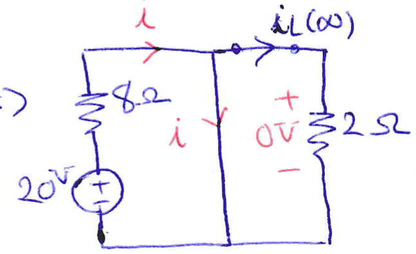

$$\Rightarrow i_L(\infty) = \frac{0}{2} = 0A \quad t \to \infty$$

$$i_L(t) = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}, \quad \tau = \frac{L}{R_{Th}}$$

$\boxed{i_L(0^+) = i_L(0^-) = 48A}$

No sudden change in inductor current

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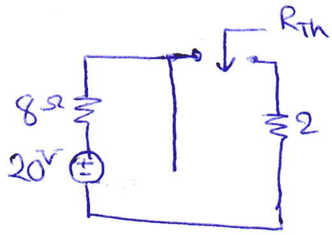
Q3 part (ii)-a Continued

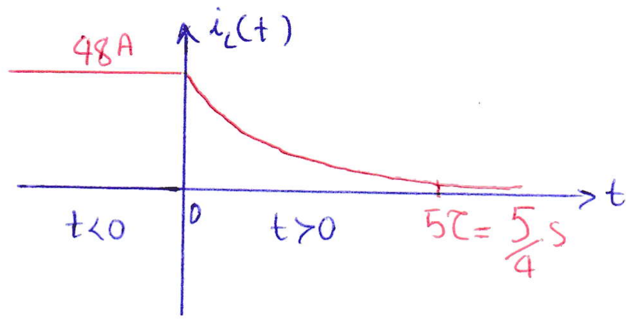
Finding R_{Th}:



$R_{Th} = 2\Omega \Rightarrow \tau = \frac{L}{R_{Th}} = \frac{0.5}{2} = \boxed{\frac{1}{4}\,s}$

$\Rightarrow i_L(t) = 0 + [48-0]e^{-t/\frac{1}{4}} = 48e^{-4t}\text{ A} \quad t \geq 0$

Thus: $i_L(t) = \begin{cases} 48\text{ A} & t \leq 0 \\ 48e^{-4t}\text{ A} & t \geq 0 \end{cases}$



(ii)-b: $v(t) = 2i_L(t)$ for all time

$\Rightarrow v(t) = 2\times 48 = 96\text{V} \quad t \leq 0$

$v(t) = \underbrace{2\times 48}_{96} e^{4t}\text{ V} \quad t \geq 0$

$\Rightarrow v(t) = \begin{cases} 96\text{ V} & t \leq 0 \\ 96e^{-4t}\text{ V} & t \geq 0 \end{cases}$

QUESTION 4 [20 marks]

(i) [10 marks] For the circuit shown in Figure 7,

a. **(4 marks)** Find the equivalent impedance Z_{eq} seen from terminals *a-b*

b. **(6 marks)** Apply phasor analysis and source transformation to calculate the voltage v_x across the 80-Ω resistor.

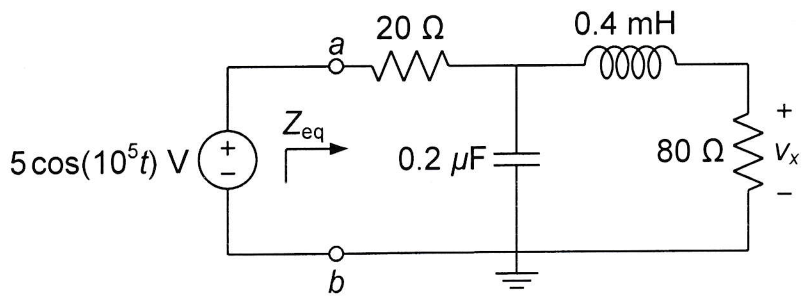


Figure 7

(i)-a:

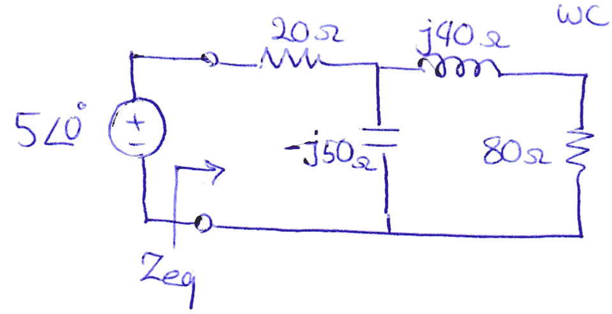
* Draw the circuit in phasor domain

$5\cos(10^5 t) \longrightarrow 5\angle 0^\circ$

$\omega = 10^5 \frac{rad}{s} \Rightarrow 0.4\,mH \longrightarrow j\omega L = j\times(10^5 \times 0.4\times 10^{-3}) = \boxed{j40\,\Omega}$

$0.2\,\mu F \longrightarrow \frac{-j}{\omega C} = \frac{-j}{10^5 \times 0.2\times 10^{-6}} = \boxed{-j50\,\Omega}$

$\Rightarrow$

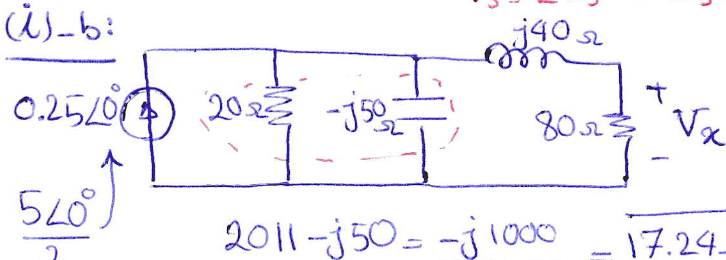


$Z_{eq} = (80+j40)\,||\,(-j50) + 20$

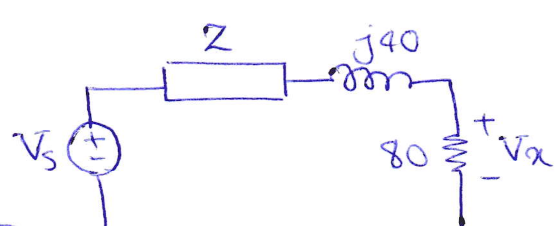
$\Rightarrow Z_{eq} = \frac{(80+j40)\times(-j50)}{80+j40-j50} + 20 = 50.769 - j46.153\,\Omega = 68.61\angle -42.27^\circ\,\Omega$

Source transformation $V_S = Z I_S$ or $I_S = \frac{V_S}{Z}$

(i)-b:



$\longrightarrow$



$20\,||\,-j50 = \frac{-j1000}{20-j50} = \boxed{17.24 - j6.89\,\Omega = Z}$

$V_S = Z\times 0.25\angle 0^\circ$

$= \underline{4.642\angle -21.8^\circ V}$

$\Rightarrow$ Voltage division $V_x = \frac{80}{Z + j40 + 80} \times V_S = 3.615\angle -40.6^\circ V$

$\Rightarrow v_x(t) = 3.615\cos(10^5 t - 40.6^\circ)\,V$

Question 4 continued

(ii) [10 marks] For the circuit shown in Figure 8,

a. (6 marks) Find the Thevenin equivalent circuit as seen from the terminals *a-b*, and draw the equivalent circuit.

b. (2 marks) Determine the value of the load impedance Z_L for maximum average power transfer.

c. (2 marks) Calculate the maximum average power that can be delivered to the load Z_L from this circuit.

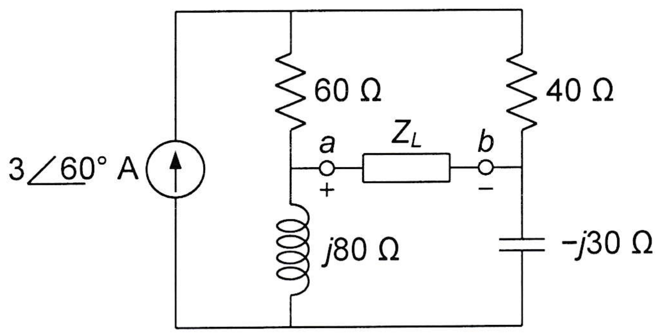
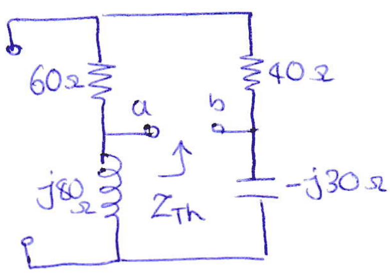
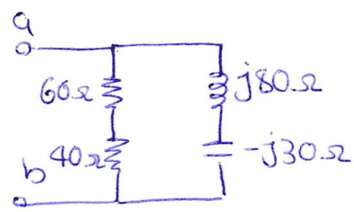


Figure 8

(ii)-a:

* Find Z_{Th}:

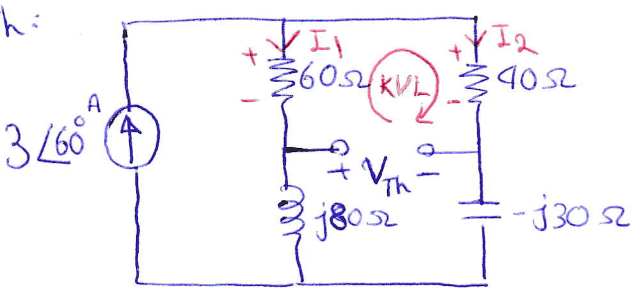




$$\Rightarrow Z_{Th} = (60+40) \parallel (j80 - j30) = \frac{100 \times j50}{100 + j50} = 20 + j40\,\Omega = 44.72\angle 63.43^\circ\,\Omega$$

* Find V_{Th}:

$V_{Th} = V_{oc}$



Current division $\rightarrow$

$$\begin{cases} I_1 = \dfrac{40 - j30}{60 + j80 + 40 - j30} \times 3\angle 60^\circ \\ I_2 = \dfrac{60 + j80}{60 - j80 + 40 - j30} \times 3\angle 60^\circ \end{cases}$$

$$\Rightarrow \begin{aligned} I_1 &= 1.34 - j0.08\text{ A} = 1.34\angle -3.43^\circ\text{ A} \\ I_2 &= 0.16 + j2.67\text{ A} = 2.68\angle 86.56^\circ\text{ A} \end{aligned}$$

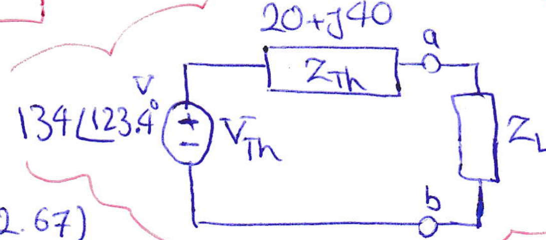
* KVL in upper-right mesh:

$$-V_{Th} - 60 I_1 + 40 I_2 = 0$$

$$\Rightarrow V_{Th} = -60 \times (1.34 - j0.08) + 40 \times (0.16 + j2.67)$$

$$V_{Th} = -74 + j111.6\text{ V} = 134\angle 123.4^\circ\text{ V}$$

$\rightarrow$



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Q4 part (ii) Continued

(ii)-b: For maximum average power transfer $\boxed{Z_L = Z_{Th}^*}$

Thus $Z_L = 20 - j40\ \Omega = 44.72\angle -63.43^\circ\ \Omega$

(ii)-c: $P_{max} = \dfrac{|V_{Th}|^2}{8R_{Th}} = \dfrac{(134)^2}{8\times 20} = 112.225\ W$

$Z_{Th} = 20 + j40\ \Omega$

$\Rightarrow \boxed{R_{Th} = 20\ \Omega}$

QUESTION 5 [20 marks]

(i) [10 marks] For the circuit shown in Figure 9,

a. **(6 marks)** Find the output voltage v_x using phasor analysis and superposition principle.

b. **(4 marks)** Sketch the phasors of voltage source V and current source I along with phasor of voltage V_x on a phasor diagram.

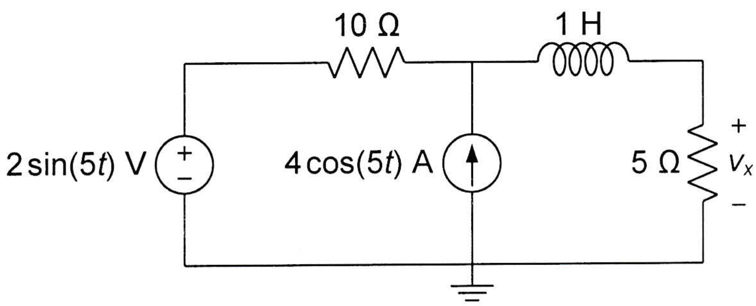


Figure 9

(i)-a:

Both sources have the same frequency

$V = 2\sin(5t) = 2\cos(5t-90) = \boxed{2\angle -90^\circ\ V} = \boxed{-2j}$

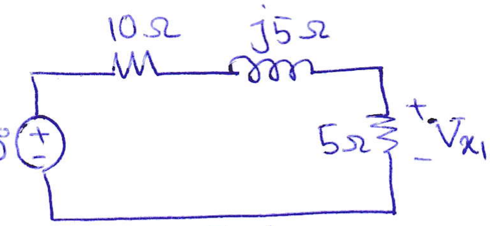
$I = 4\cos(5t) = \boxed{4\angle 0^\circ A}$

$\omega = 5 \longrightarrow 1H \longrightarrow j\omega L = \boxed{j5\Omega}$

$v_x = v_{x1} + v_{x2}$ $\begin{cases} v_{x1} \text{ is due to voltage source} \\ v_{x2} \text{ is due to current source} \end{cases}$

$\xrightarrow[\text{frequency}]{\text{same}} V_x = V_{x1} + V_{x2}$

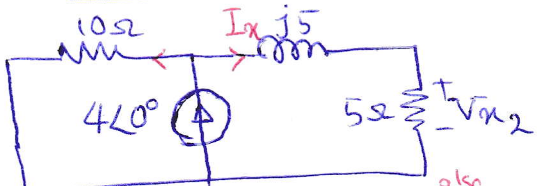
* V_{x1}:



$\xrightarrow[\text{division}]{\text{Voltage}} V_{x1} = \dfrac{5}{10+j5+5} \times 2\angle -90$

$= \boxed{-0.2 - j0.6V = 0.63\angle -108.4^\circ V}$

* V_{x2}



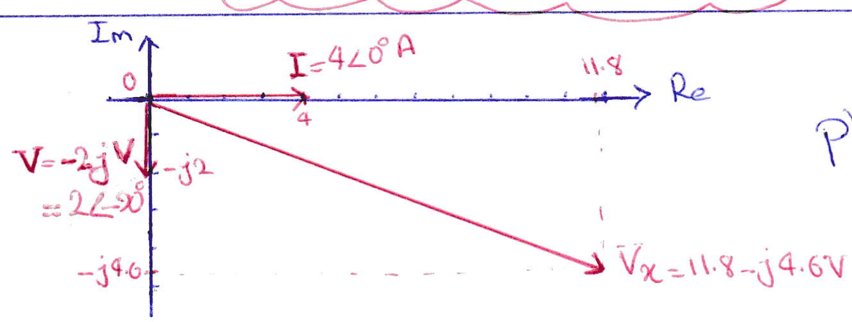
Source transformation is also applicable

$\xrightarrow[\text{division and Ohm's Law}]{\text{Current}} V_{x2} = 5I_x = 5 \times \underbrace{\dfrac{10}{10+j5+5} \times 4\angle 0^\circ}_{I_x}$

$= \boxed{12 - j4\ V = 12.65\angle -18.93^\circ V}$

$\Rightarrow V_x = V_{x1} + V_{x2} = 11.8 - j4.6 = 12.66\angle -21.29^\circ V$

(i)-b:

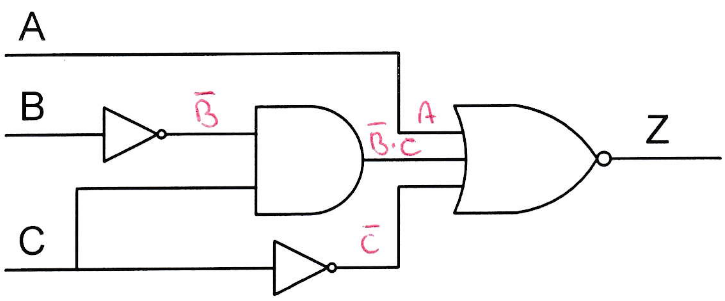


Phasor diagram

Question 5 continued

(ii) [10 marks] Consider the following logical diagram shown in Figure 10.

a. **(4 marks)** Derive and simplify the logical expression for Z.
b. **(4 marks)** Construct the truth table relating Z to inputs A, B, and C.
c. **(2 marks)** Using part a. and/or part b., show that Z can be realised using exactly one 3-input AND gate and one 1-input NOT gate showing all working.



(ii)-a:

Figure 10

$\boxed{Z = \overline{A + \bar{B}.C + \bar{C}}}$

DeMorgan's Theorem → $Z = \bar{A} . \overline{\bar{B}C} . \bar{\bar{C}} = \bar{A}.(\bar{\bar{B}}+\bar{C}).C = \bar{A}.(B.C+\underbrace{\bar{C}.C}_{0})$

$= \bar{A}.(BC+0)$

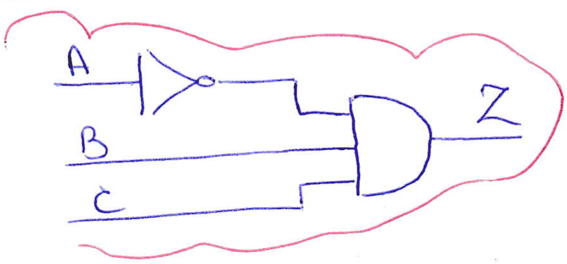
$\Rightarrow Z = \bar{A}.B.C$

(ii)-b: Truth Table:

A	B	C	$\bar{A}$	$\bar{B}$	$\bar{C}$	$\bar{B}.C$	$A+\bar{B}.C+\bar{C}$	$\overline{A+\bar{B}.C+\bar{C}}$	$\bar{A}.B.C$
0	0	0	1	1	1	0	1	0	0
0	0	1	1	1	0	1	1	0	0
0	1	0	1	0	1	0	1	0	0
0	1	1	1	0	0	0	0	1	1
1	0	0	0	1	1	0	1	0	0
1	0	1	0	1	0	1	1	0	0
1	1	0	0	0	1	0	1	0	0
1	1	1	0	0	0	0	1	0	0

(ii)-c:

$Z = \bar{A}.B.C \Rightarrow$



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