School of Electrical Engineering & Telecommunications

ELEC1111

Topic 1: Circuit Basics

1. Determine the total charge flowing through an element for $0 < t < 2$ seconds when the current entering the positive terminal is $i(t) = e^{-2t}$ mA.

Solution: Recall that electric current is the rate of change of charge, thus to calculate the total being transferred over the interval of 2 seconds, we take integral of the current with respect to time as follows,

$$q = \int_{t_0}^{t_1} i\,dt + q(0) \qquad t_0 \le t \le t_1$$

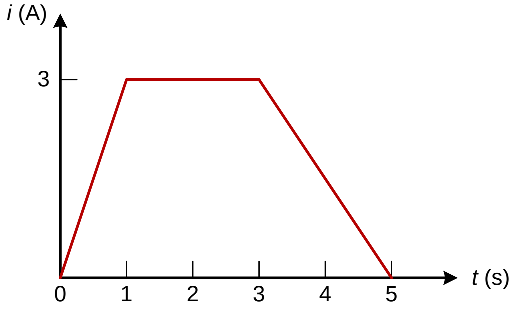
or more accurately when seeking the time function for charge

$$q(t) = \int_{t_0}^{t} i(\tau)\,d\tau + q(0)$$

Note that since nothing is mentioned about initial charge, we assume it is zero. Also, pay attention to unit of current being in mA.

$$q = \int_0^2 e^{-2t}\,dt = -\frac{1}{2}e^{-2t}\Big]_0^2 = -\frac{1}{2}e^{-4} + \frac{1}{2}e^{0} = -\frac{1}{2}(e^{-4} - 1) = 0.4908 \text{ mC or } 490.8\ \mu\text{C}$$

2. The current flowing through an element is shown in the graph below. Assuming charge entering the element before $t = 0$ is zero, i.e., $q(0) = 0$, calculate the total charge that has entered the element at the following times,
   (a) $t = 1$ s
   (b) $t = 3$s
   (c) $t = 5$ s



Solution:
(a) The current between the time interval [0 1] is $i(t) = 3t$ A with no initial charge, thus the charge at $t = 1$ s is

$$q(1) = \int_0^1 3t\,dt = 3\frac{t^2}{2}\Big]_0^1 = 1.5 \text{ C or } 1500 \text{ mC}$$

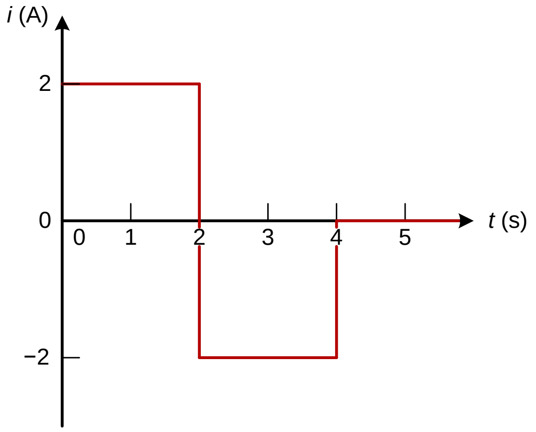
(b) From $t = 1$ s to $t = 3$ s the current is constant and equal to 3 A with a build-up initial charge of 1.5 C at $t = 1$ (initial condition). Thus the charge at $t = 3$ s is

$$q(3) = \int_1^3 3\,dt + q(1) = 3t]_0^1 + 1.5 = 7.5 \text{ C or } 7500 \text{ mC}$$

(c) The current in the last interval is obtained as $i(t) = -\frac{3}{2}(t-5)$ A with the initial charge of 7.5 C at $t = 3$ s, Thus we have

$$q(5) = \int_1^3 \left(-\frac{3}{2}t + \frac{15}{2}\right) dt + q(3) = -\frac{3t^2}{4}t + \frac{15}{2}t\Big]_3^5 + 7.5 = 10.5 \text{ C}$$

3. If the voltage $v(t)$ across an element is 10 V, and the current through the element $i(t)$ is shown in the following figure, calculate the power and energy and sketch their time functions.



Solution: Recall that the power of an element is the product of the voltage across that element to the current through it, so at any given time $p(t) = v(t)i(t)$.

The current is a piecewise function which can be described as below,

$$i = \begin{cases} 2 \text{ A}, & 0 < t < 2 \\ -2 \text{ A}, & 2 \le t < 4 \\ 0 \text{ A}, & t > 4 \end{cases}$$

With constant voltage of 10 V over the whole period, the power in then given as follows,

$$p = vi = 10i = \begin{cases} 20 \text{ W}, & 0 < t < 2 \\ -20 \text{ W}, & 2 \le t < 4 \\ 0 \text{ W}, & t > 4 \end{cases}$$

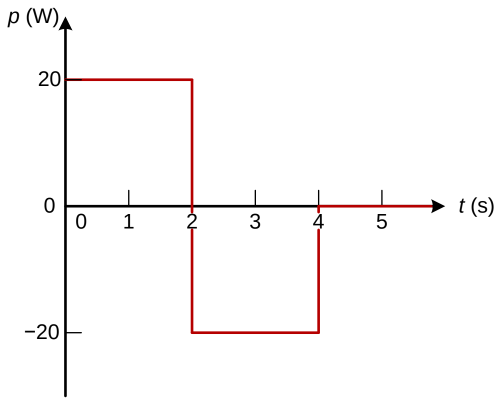
The energy is the integral of power with respect to time, i.e.,

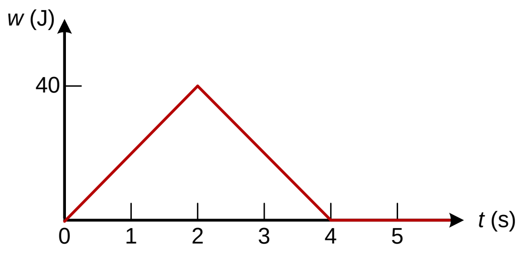
$$w(t) = \int_{t_0}^{t} p(\tau)\,d\tau + w(t_0)$$

And since nothing is mentioned about initial value of energy at $t = 0$, we assume that $w(0) = 0$. Thus the energy function is obtained as follows,

$$w = \begin{cases} \int_0^2 20dt + w(0) = 20t \text{ J}, & 0 < t \leq 2 \\ \int_2^4 -20dt + w(2) = -20t + 80 \text{ J}, & 2 < t \leq 4 \\ \int_0^\infty 0dt + w(4) = 0 \text{ J}, & t > 4 \end{cases}$$

And here are the sketches.





4. How much energy does a 100 W electric bulb consume in one day (both in joules and watt-hours)?

Solution: when the consumed power is constant over a fix period, the result of integration is simply the product of the constant power to the overall period. Thus we have

$$w = p \times t = 100 \text{ W} \cdot 24 \text{ h} = 2400 \text{ Wh} = 2.4 \text{ kWh} \triangleq 2.4 \times 3600 \text{ kJ} = 8.640 \text{ MJ}$$

5. The current entering the positive terminal of a device is $i(t) = 6e^{-2t}$ mA and the voltage across the device is $v(t) = 10\, di/dt$ V.
   (a) Calculate the power absorbed.
   (b) Determine the energy absorbed in 3 s.

Solution: Note that the voltage is a function of the current going through the same element. Thus you have to first find the voltage as a function of time as below,

$$\mathrm{v} = 10\,\frac{\mathrm{di}}{\mathrm{dt}} = -120\mathrm{e}^{-2\mathrm{t}}\,\mathrm{mV} = 0.12\mathrm{e}^{-2\mathrm{t}}\,\mathrm{V}$$

(a) So the power is then given by

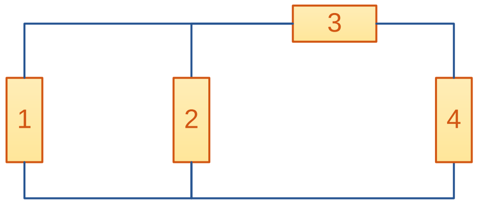
$$\mathrm{p} = \mathrm{vi} = -120\mathrm{e}^{-2\mathrm{t}}\mathrm{mV} \times 6\mathrm{e}^{-2\mathrm{t}}\mathrm{mA} = -\,720\mathrm{e}^{-4\mathrm{t}}\ \mu\mathrm{W}$$

(b) And energy in the interval of 0 to 3 is assuming zero initial energy

$$w = \int_0^3 p\, dt = -\,720 \int_0^3 e^{-4t}\, dt = \frac{720}{4} e^{-4t}\Big]_0^3 = 180(e^{-12} - 1) \cong -180\ \mu\text{J}$$

Note that the power absorbed by the device is negative, which means the device is an active element that supplies $+720e^{-4t}$ μW to the rest of the circuit.

6. The figure below shows a circuit with four elements, $P_1 = 60$ W absorbed, $P_3 = -145$ W absorbed, and $P_4 = 75$ W absorbed.
   (a) How many watts does element 2 absorb?
   (b) Is element 2 an active element or passive element?



Solution: the law of conservation of energy says that the total sum of all powers in a network/circuit must be zero

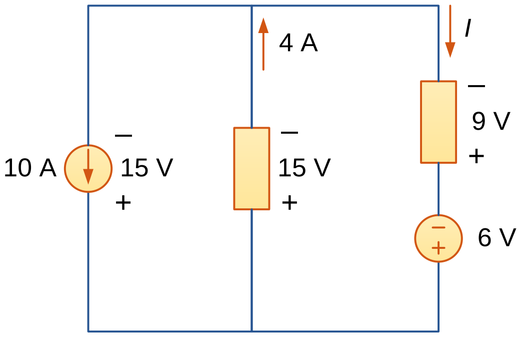
(a) Thus we have

$$\sum p = 0 \quad \rightarrow \quad P_1 + P_2 + P_3 + P_4 = 0$$

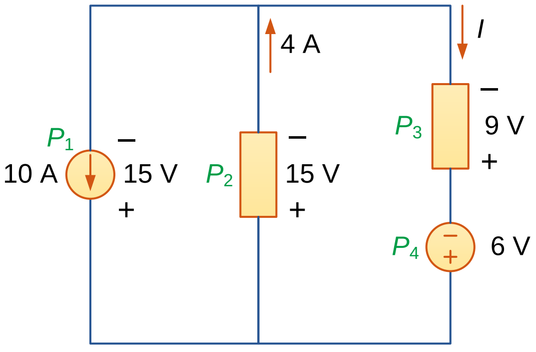
$$P_2 = -P_1 - P_3 - P_4 = -60 + 145 - 75 = 10 \text{ W absorbed}$$

(b) Since the power is positive, the element 2 could be passive, but remember that **ideal sources can absorb infinite power** in the circuit resulting in having positive power. Therefore, the correct answer to this question is that it is **unknown** as we need more information about the element.

7. In the circuit below, find the current I and the power absorbed by each element.



Solution: Note that we can use conservation of energy to find the powers for the elements where the unknown current I is flowing through. Thus we have



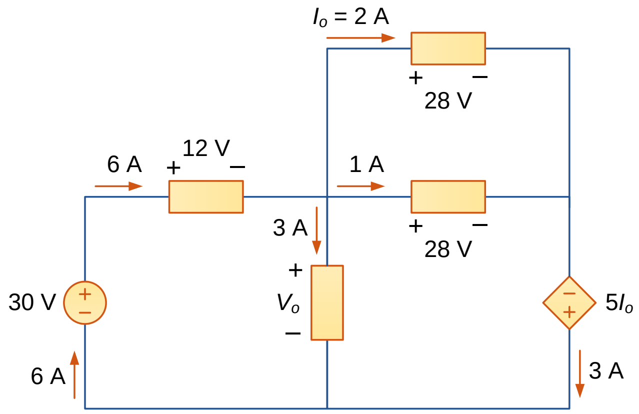
$$P_1 = -15 \times 10 = -150 \text{ W}$$
$$P_2 = 15 \times 4 = 60 \text{ W}$$
$$P_3 = -9 \times I = -9I \text{ W}$$
$$P_4 = -6 \times I = -6I \text{ W}$$

$$\Rightarrow P_1 + P_2 + P_3 + P_4 = 0$$

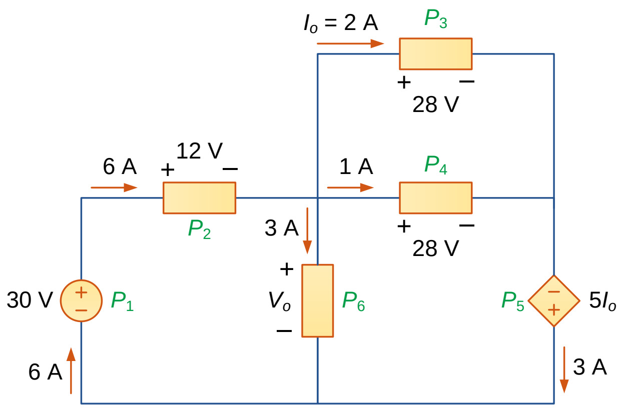
$$-150 + 60 - I(9 + 6) = 0$$
$$\Rightarrow I = \frac{-90}{15} = -6 \text{ A}$$
and thus $P_3 = 54$ W and $P_4 = 36$ W

As you can see, the current I is negative which shows that the assumed direction for I can be reversed if we want to have positive current. Later on, you learn that the current in such circuit can be obtained directly using Kirchhoff's current law (KCL)

8. In the circuit below, find V_o and the power absorbed by each element.



Solution: Similar to the previous question, we can use conservation of energy to find all the powers including the one with voltage V_o, but now we have a dependent voltage source being controlled by current I_o which is given on the top-right of the circuit with 2 A.



$$P_1 = -30 \times 6 = -180 \text{ W}$$
$$P_2 = 12 \times 6 = 72 \text{ W}$$
$$P_3 = 28 \times I_0 = 28 \times 2 = 56 \text{ W}$$
$$P_4 = 28 \times 1 = 28 \text{ W}$$
$$P_5 = -5I_0 \times 3 = -15 \times 2 = -30 \text{ W}$$
$$P_6 = V_0 \times 3 = 3V_0 \text{ W}$$

$$\Rightarrow P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = 0$$

$$-180 + 72 + 56 + 28 - 30 + 3V_0 = 0$$
$$\Rightarrow V_0 = \frac{54}{3} = 18 \text{ V}$$
$$\text{and thus } P_6 = 3 \times 18 = 54 \text{ W}$$