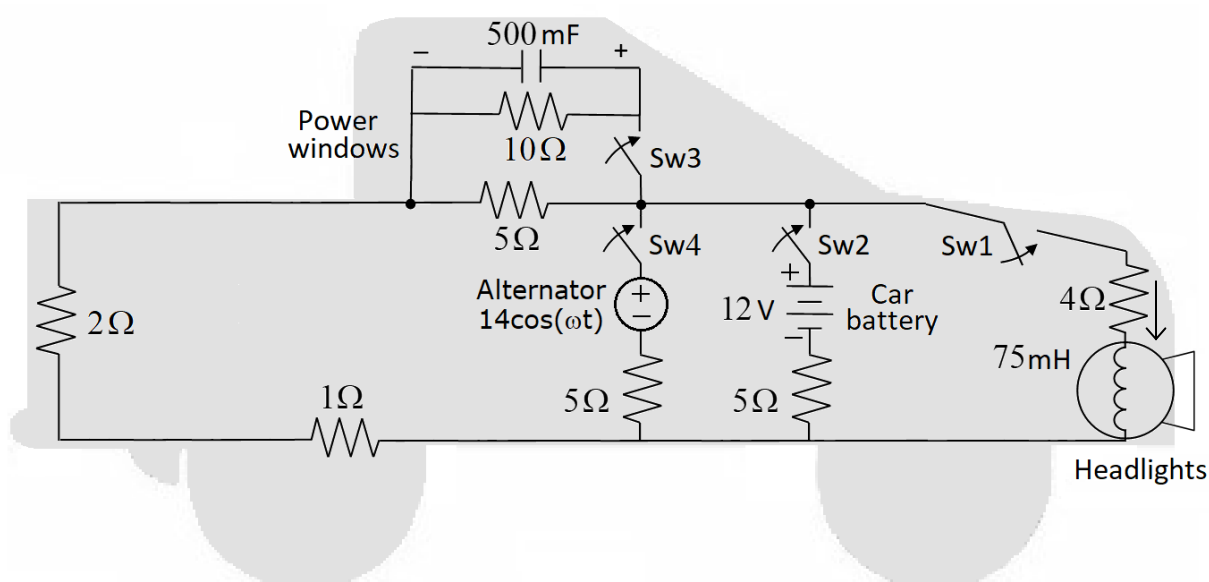


1. Analysis of the headlights and power windows of a car

The circuit below shows the wiring of the headlights and power windows of a car. The power window system has an internal capacitance of 500mF in parallel with a $10\ \Omega$ resistor and the headlamp bulb has an internal resistance of $4\ \Omega$ with an inductance of 75 mH in series. The car also features a 12V car battery which can power the system when the car engine (alternator) is turned off. When the alternator is on, it produces AC power with an amplitude of 14V. The frequency of the AC signal is directly tied to the speed of the car. Due to poor wiring, there are $5\ \Omega$, $2\ \Omega$ and $1\ \Omega$ resistances connected to the power windows circuit.



- Q1.** While driving at 10 km/h, the frequency of the alternator is 50Hz. In this situation, calculate the voltage across the power windows (capacitance and $10\ \Omega$ resistor) and the current through the headlights (inductance and $4\ \Omega$ resistor) when all switches are closed.

Answer: $v_{\text{window}}(t) = 1.57 + 0.008 \cos(100\pi t - 86.58^\circ)\text{ V}$

$i_{\text{headlights}}(t) = 0.745 + 0.158 \cos(100\pi t - 77.17^\circ)\text{ A}$

2. Analysis of an oscillator

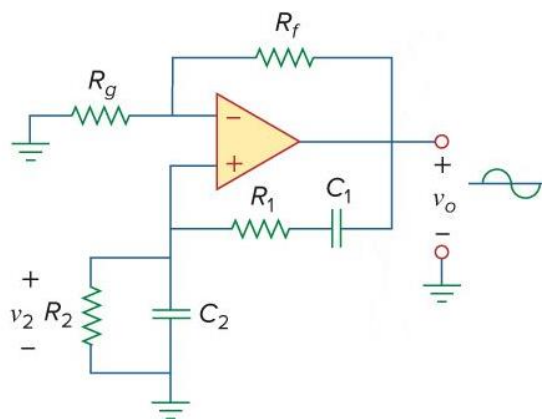
An oscillator is a circuit that produces an AC waveform when powered by a DC input. The only external source an oscillator needs is the DC power supply. Ironically, the DC power supply is usually obtained by converting the AC supplied by the electric utility company to DC. The reason why we need to use the oscillator to convert the DC to AC again is that the AC supplied by the utility company operates at a pre-set frequency of 50 Hz, whereas many applications such as electronic circuits, communication systems, microwave devices, etc. require internally generated frequencies that range from 0 to 10 GHz or higher. Oscillators are used for generating these frequencies.

For sine wave oscillators to sustain oscillations, they must meet the *Barkhausen criteria*:

1. The overall gain of the oscillator must be unity or greater.
2. The overall phase shift must be zero.

The frequency at which the overall phase is zero is known as the oscillation frequency.

The circuit below is a *Wien-bridge oscillator*, which is widely used for generating sinusoids in the frequency range below 1 MHz. It is an RC op amp circuit with only a few components, easily tunable and easy to design. The oscillator essentially consists of a noninverting amplifier with two feedback paths: the positive feedback path to the non-inverting input creates oscillations, while the negative feedback path to the inverting input controls the gain.



If $R_1 = R_2 = 10\text{ k}\Omega$, $C_1 = C_2 = 159\text{ pF}$, $R_g = 10\text{ k}\Omega$ and $R_f = 20\text{ k}\Omega$ in the Wien-bridge oscillator above,

Q2. Demonstrate that the oscillation frequency of the oscillator is 100 KHz by demonstrating that V_2 is in phase with V_0 at this frequency.

Answer: For $f_0 = 100\text{ KHz}$, $V_2 = \frac{Z_p}{Z_p + Z_s} V_0 = \frac{V_0}{3}$, where $Z_p = Z_{R2} \parallel Z_{C2}$ and $Z_s = Z_{R1} + Z_{C1}$ (i.e., V_2 is in phase with V_0).

Q3. Use KCL at the inverting input of the amplifier to demonstrate that the gain of the oscillator is $V_0/V_2 = 3$.

Answer: Using KCL, $V_0/V_2 = 1 + \frac{R_f}{R_g} = 3$ (non-inverting amplifier). Note that this result is consistent with the result obtained in Q2.