Q1a

i) $$R_{ab} = \left\{ \left[(R_6 \parallel R_7) + R_5 \right] \parallel R_4 \parallel R_3 \right\} + R_1 + R_2$$

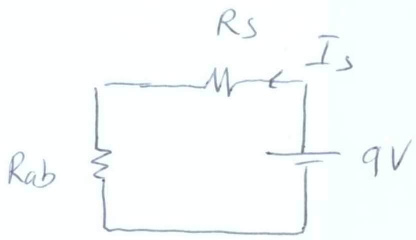
$$= \left(\frac{R_6 R_7}{R_6 + R_7} + R_5 \right) \parallel \left(\frac{R_3 R_4}{R_3 + R_4} \right) + R_1 + R_2$$

$$= \frac{\left(\frac{R_6 R_7}{R_6 + R_7} + R_5 \right)\left(\frac{R_3 R_4}{R_3 + R_4} \right)}{\frac{R_6 R_7}{R_6 + R_7} + R_5 + \frac{R_3 R_4}{R_3 + R_4}} + R_1 + R_2$$

ii) $$R_{ab} = \frac{(10 + 20)(30)}{10 + 20 + 30} + 60 + 60 = 135\,\Omega$$

Q1b

i) $$I_s = \frac{9}{R_{ab} + R_s} = \frac{9}{135 + 15} = 0.06\,A$$



ii) $$P_{resistor} = I_s^2 \times (R_{ab} + R_s) = 0.06^2 \times (135 + 15) = 0.54\,W$$

$$P_{V_s} = -9 \times I_s = -9 \times 0.06 = -0.54\,W$$

$$P_{resistor} + P_{V_s} = 0 \rightarrow \text{energy conservation}$$

Q2

i) Node V_a : $\frac{V_a}{2} + \frac{V_a - V_b}{2} = 3 \rightarrow 2V_a - V_b = 6$

Node V_b : $i_2 = \frac{V_b}{4}$; $\frac{V_b - V_a}{2} + \frac{V_b}{4} + \frac{V_b - 2i_2}{2} = 0$

$2(V_b - V_a) + V_b + 2\left(V_b - 2 \times \frac{V_b}{4}\right) = 0$

$3V_b - 2V_a + 2V_b - V_b = 0 \rightarrow -2V_a + 4V_b = 0$

equation system: $\begin{cases} 2V_a - V_b = 6 \\ -2V_a + 4V_b = 0 \end{cases}$

or: $\begin{cases} \frac{V_a}{2} + \frac{V_a - V_b}{2} = 3 \\ \frac{V_b - V_a}{2} + \frac{V_b}{4} + \frac{V_b - 2\frac{V_b}{4}}{2} = 0 \end{cases}$

ii) $\begin{cases} 2V_a - V_b = 6 \\ -2V_a + 4V_b = 0 \end{cases} \rightarrow \begin{matrix} V_b = 2\ V \\ V_a = 4\ V \end{matrix}$

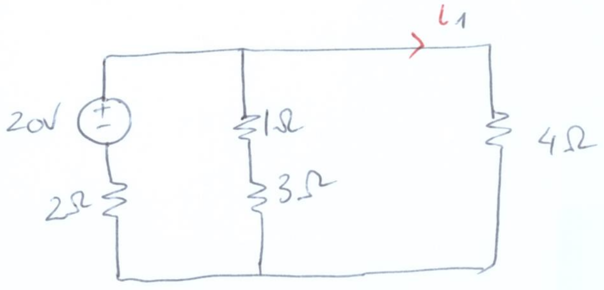
iii) $i_1 = \frac{V_a - V_b}{2} = \frac{4-2}{2} = 1A$

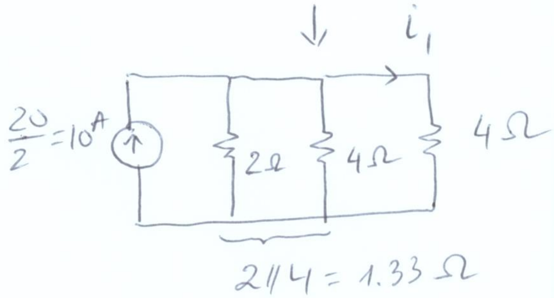
$i_2 = \frac{V_b}{4} = \frac{2}{4} = 0.5A$

$i_3 = i_1 - i_2 = 1 - 0.5 = 0.5A$

(Q3a)

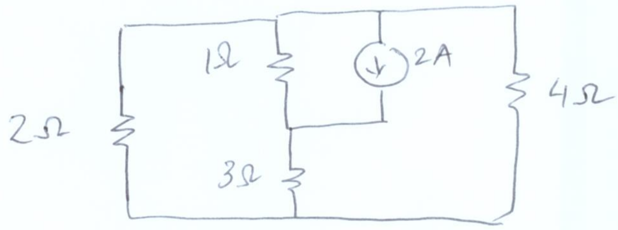
By 20V





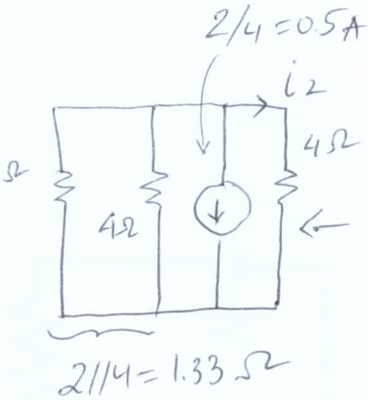
$$i_1 = \frac{1.33}{1.33 + 4} \times 10 = 2.5\,A$$

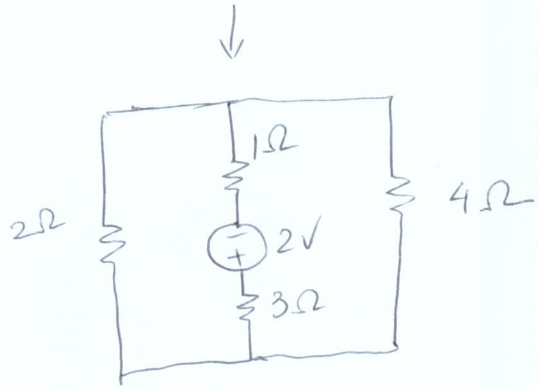
By 2A



$$i_2 = \frac{1.33}{1.33 + 4} \times (-0.5)$$

$$= -0.125\,A$$



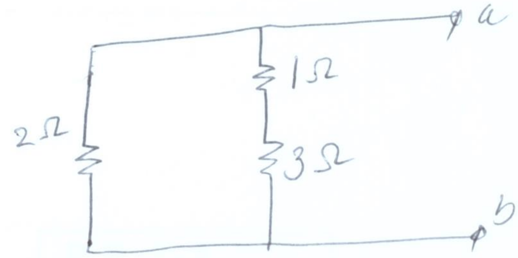


Superposition: $i = i_1 + i_2 = 2.5 - 0.125 = 2.375\,A$

Q3b

* Find R_{th} :

$$R_{th} = 2 \| 4 = 1.33\,\Omega$$



* Find V_{th} :

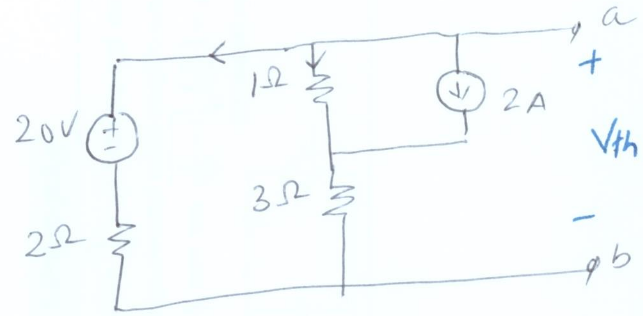
KVL for loop (1)

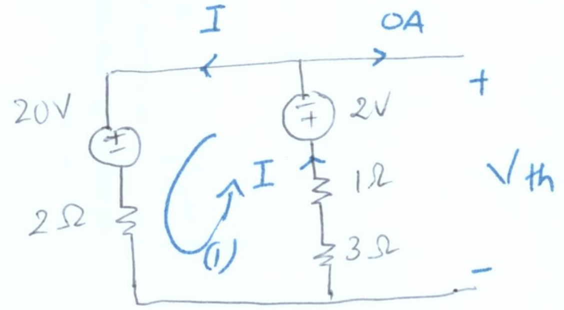
$$(2+1+3)I + 2 + 20 = 0$$

$$I = -\frac{22}{6} = -3.67\,A$$

$$V_{th} = 20 + 2I = 20 + 2(-3.67)$$

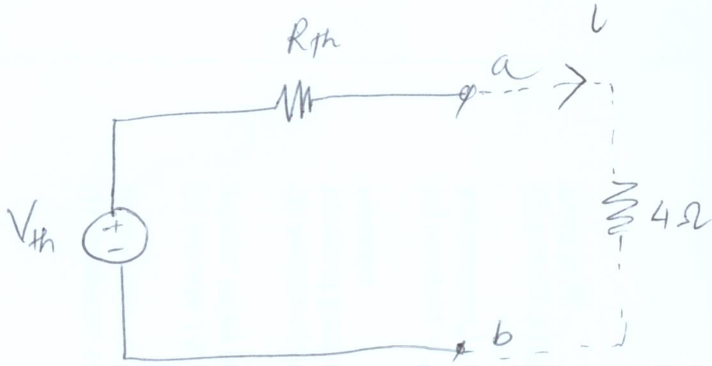
$$V_{th} = 12.66\,V$$





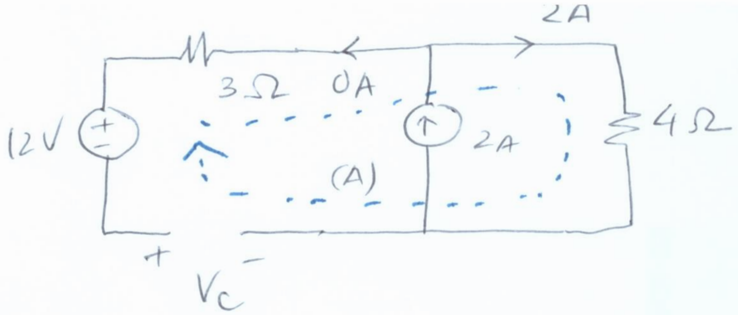
$$i = \frac{V_{th}}{R_{th} + 4} = \frac{12.66}{4 + 1.33}$$

$$i = 2.375\,A$$



Q4a)

a) $t < 0$:



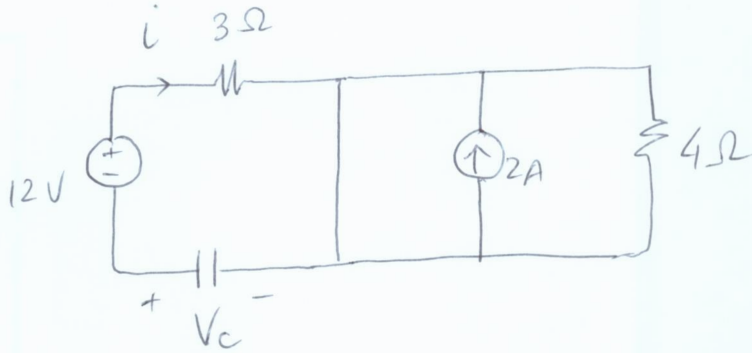
KVL for loop (A)

$-12 - 3\times 0 + 2\times 4 - V_c = 0$

$\rightarrow V_c = -12 + 8 = -4\,V = V_c(0^+)$

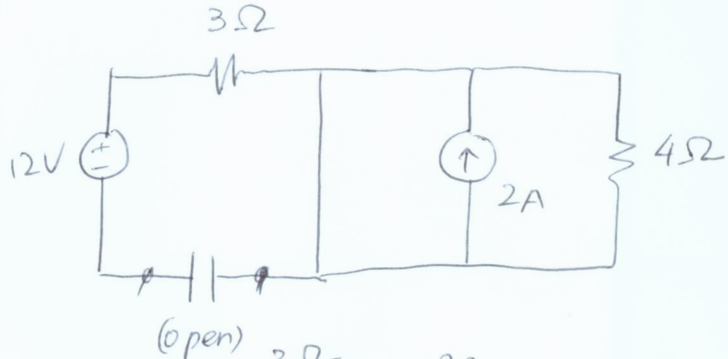
$i = 0\,A$

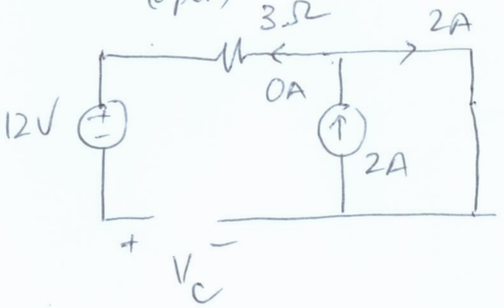
Q4b) $t > 0$ and steady:



$V_c = -12\,V = V_c(\infty)$

$i = 0\,A$





Q4c)

$\tau = RC = 2\times 3 = 6$ (sec)

$V_c(t) = V_c(\infty) + [V_c(0+) - V_c(\infty)] \times e^{-\frac{t}{\tau}} = -12 + (-4 + 12)\, e^{-\frac{t}{6}}$

$V_c(t) = -12 + 8\, e^{-t/6}$ (V)

$i = -C\,\frac{dV_c}{dt} = -2\times 1.33\, e^{-t/6}$ (A) $= -2.66e^{-t/6}$ A