School of Electrical Engineering & Telecommunications

ELEC1111 Tutorial

Topic 10: Digital Logic Circuits

1. Convert the following unsigned binary numbers to decimal:

   1. 101
   2. 10111
   3. 1101

Solution:

1. $(101)_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 4 + 1 = 5$
2. $(10111)_2 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 16 + 4 + 2 + 1 = 23$
3. $(1101)_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 8 + 4 + 1 = 13$

2. Convert the following decimal numbers to binary:

   1. 9
   2. 45
   3. 255

Solution:

1. $9 = (1001)_2$
2. $45 = (101101)_2$
3. $255 = (11111111)_2$

9 | 2
1 4 | 2
0 2 | 2
0 1

45 | 2
05 22 | 2
1 0 11 | 2
1 5 | 2
1 2 | 2
0 1

3. How many binary digits are required to allow a variable to range between 0 and 1000?

Solution: $2^{10} = 1024$ so we need 10 binary digits to represent numbers from 0 to 1023 ie 1024 different numbers (counting 0). 2^9 only allows us to represent half as many so the answer is 10 binary digits.

4. Write a Boolean expression for the following statement: "Z is TRUE if either A or B is FALSE, otherwise Z is FALSE". Write a truth table for this expression.

Solution:

Converting the logic statement to an equation we get:

$$Z = \bar{A} + \bar{B}$$

The truth table for the above expression will be:

A	B	Z
0	0	1
0	1	1
1	0	1
1	1	0

5. Consider the functions $X(A,B,C)$ and $Y(A,B,C)$ specified in the truth table

A	B	C	$X(A,B,C)$	$Y(A,B,C)$
0	0	0	1	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	1	1
1	1	0	0	1
1	1	1	1	1

1. Write a logic expression corresponding to the functions $X(A,B,C)$ and $Y(A,B,C)$.
2. Implement $X(A,B,C)$ using logic gates.
3. Implement $Y(A,B,C)$ using logic gates.
4. Using DeMorgan's Theorem, implement $X(A,B,C)$ using only two-input NAND gates.

Solution:

1. The logic expressions for $X(A,B,C)$ and $Y(A,B,C)$ are:

$$X = \bar{A} \cdot \bar{B} \cdot \bar{C} + A \cdot \bar{B} \cdot \bar{C} + A \cdot \bar{B} \cdot C + A \cdot B \cdot C$$

Simplifying, we get: $X = \bar{B}\bar{C}(\underbrace{\bar{A}+A}_{1}) + AC(\underbrace{\bar{B}+B}_{1}) = \bar{B}\bar{C} + AC$

$$X = \bar{B} \cdot \bar{C} + A \cdot C$$

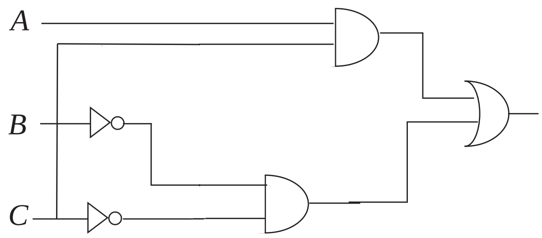
For $Y(A,B,C)$, we have:

$$Y = \bar{A} \cdot B \cdot C + A \cdot \bar{B} \cdot C + A \cdot B \cdot \bar{C} + A \cdot B \cdot C$$

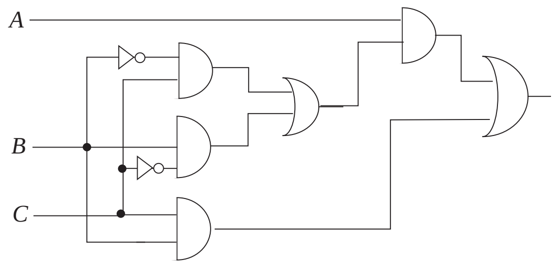
Simplifying the first and the last terms, we get: $Y = BC(\underbrace{\bar{A}+A}_{1}) + A\bar{B}C + AB\bar{C}$

$$Y = A \cdot \bar{B} \cdot C + A \cdot B \cdot \bar{C} + B \cdot C$$

2. From the simplified version of $X(A, B, C)$ we can draw the following logic circuit



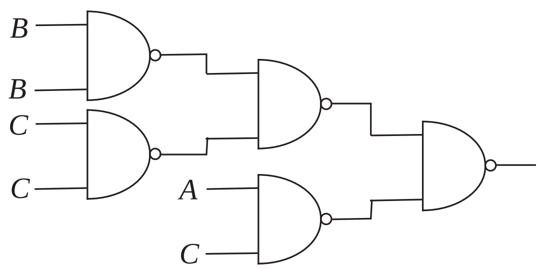
3. From the simplified version of $Y(A, B, C)$ we can draw the following logic circuit



Note: You can also use three-input AND and OR gates to make the logic circuit simpler

4. From the simplified version of the expression for X:

$$X = \overline{\overline{(\overline{B} \cdot \overline{C}) \cdot (\overline{AC})}}$$



$X = \bar{B}\bar{C} + AC =$

$= \overline{\overline{\bar{B}\bar{C} + AC}} = \overline{\overline{\bar{B}\bar{C}} \cdot \overline{AC}}$

DeMorgan's

$= \overline{\overline{\overline{B.B} . \overline{CC}} . \overline{AC}}$

$Z = ZZ \Rightarrow \bar{Z} = \overline{ZZ}$

6. Complete the truth tables of the following logic equations:

   1. Output $= A \cdot \bar{B}$
   2. Output $= A \cdot \bar{B} \cdot C$
   3. Output $= \bar{A} + B$
   4. Output $= A \cdot \bar{B} + C$

Solution:

1. Output $= A \cdot \bar{B}$

A	B	$A \cdot \bar{B}$
0	0	0
0	1	0
1	0	1
1	1	0

2. Output $= A \cdot \bar{B} \cdot C$

A	B	C	$A \cdot \bar{B} \cdot C$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

3. Output $= \bar{A} + B$

A	B	$A + \bar{B}$
0	0	1
0	1	1
1	0	0
1	1	1

4. Output $= A \cdot \bar{B} + C$

A	B	C	$A \cdot \bar{B} + C$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

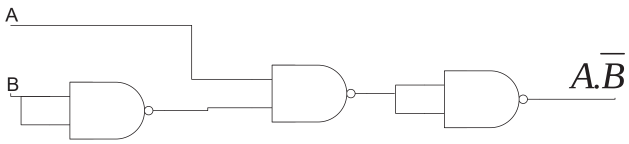
7. Draw the logic diagrams which represent the function of these logic equations using NAND Gates only:

$Z = Z.Z$

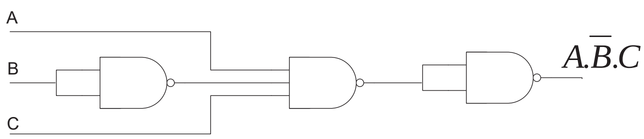
1. Output $= A \cdot \bar{B}$ $= \overline{\overline{A.\bar{B}}} = \overline{\overline{A.\bar{B}}.\overline{A\bar{B}}}$
2. Output $= A \cdot \bar{B} \cdot C$ $= \overline{\overline{A\bar{B}C}} = \overline{\overline{A\bar{B}C}.\overline{A\bar{B}C}}$
3. Output $= \bar{A} + B$ $= \overline{\overline{\bar{A}+B}} = \overline{\bar{\bar{A}}.\bar{B}} = \overline{A.\bar{B}}$
4. Output $= A \cdot \bar{B} + C$ $= \overline{\overline{A.\bar{B}+C}} = \overline{\overline{A\bar{B}}.\bar{C}}$

Solution:

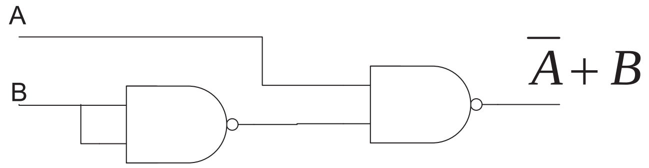
1. Output $= A \cdot \bar{B}$



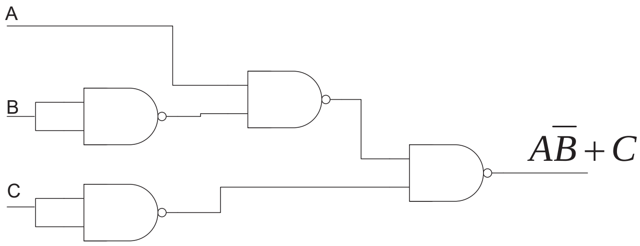
2. Output $= A \cdot \bar{B} \cdot C$



3. Output $= \bar{A} + B$



4. Output $= A \cdot \bar{B} + C$



8. Draw the logic diagram which represent the function of this logic equation
$X = A \cdot \bar{B} \cdot C + A \cdot B \cdot \bar{C}$

Solution: