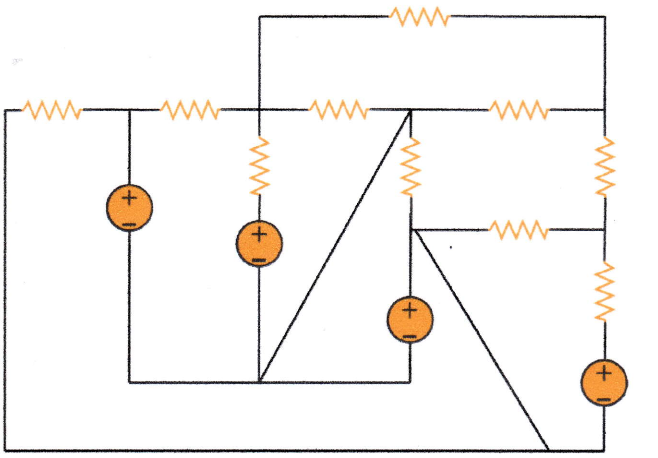
UNSW SYDNEY | Australia's Global University

School of Electrical Engineering & Telecommunications

ELEC1111

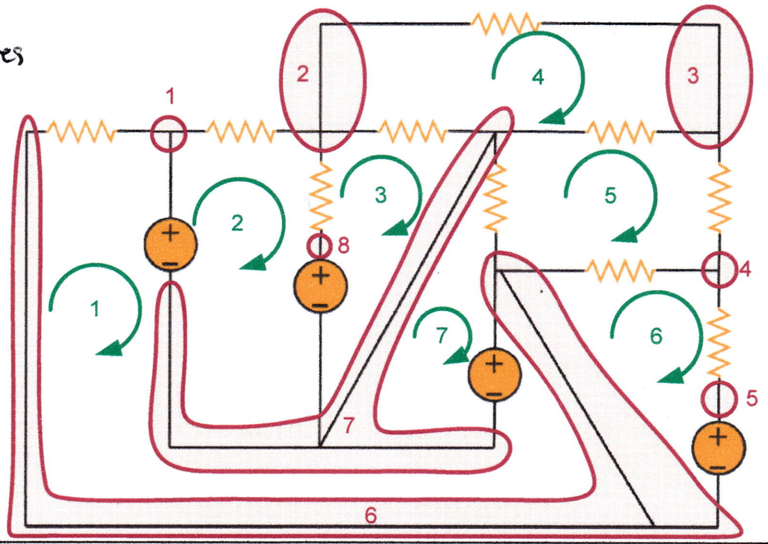
Topic 2: Kirchhoff's Laws, Nodal & Mesh Analysis

1. Identify all the nodes, branches, and meshes in the following figure.

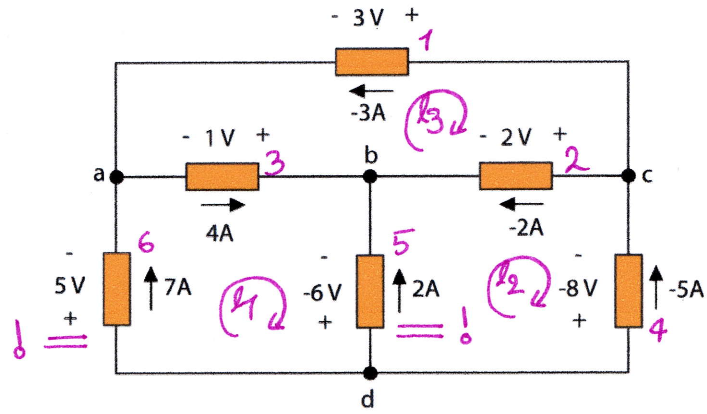


Solution:

14 branches
8 nodes
7 meshes



2. In the following circuit (Question 3 of the Online Tutorial 1) the total sum of powers being consumed and generated is not equal to zero, violating the conservation of energy law.
   (a) Identify the error in the circuit using Kirchhoff's Current and Voltage Laws.
   (b) Confirm your answer by calculating the powers and their total sum.



Solution: First check KCLs and KVLs

KCL: @ node a: $7-3-4=0$ ✓

@ node b: $4+2-2=4\neq 0$ ✗. error

@ node c: $-5+2+3=0$ ✓

@ node d: $7+2-5=4\neq 0$ ✗. error

This suggests branch 5 has the wrong direction; ↓2A ✓ or ↑-2A

KVL: in l_1: $+5-1-(-6)=10\neq 0$ ✗. error

in l_2: $+(-6)-2-(-8)=0$ ✓

in l_3: $+(-3)+2+1=0$ ✓

This suggests branch 6 has the wrong polarity: $^{+}5V_{-}$ or $^{-}{-5V}_{+}$

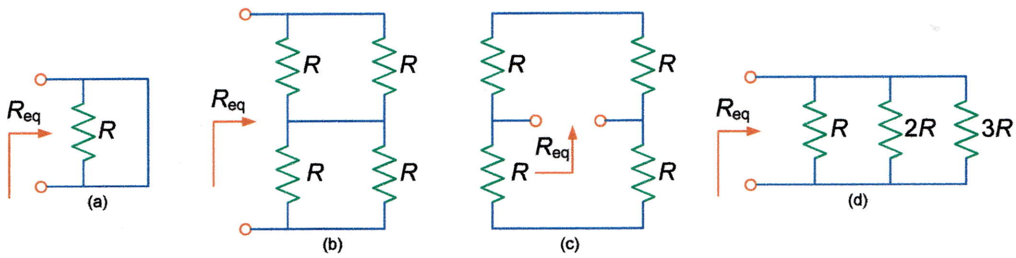
Summation of powers with corrected direction and polarity:

$P_1=-9W$ $P_4=40W$

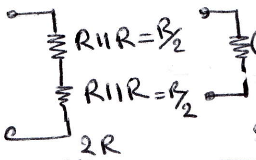
$P_2=-4W$ $P_5=12W$ $\Rightarrow \sum_{n=1}^{6} P_n = 40+12-35-4-4-9=0$ ✓

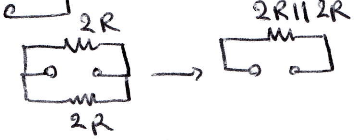
$P_3=-4W$ $P_6=-35W$ Confirmed

3. Find the equivalent resistance of the following networks.



Solution: a) Extreme case of short circuit being parallel with R $\Rightarrow R_{eq}=0\Omega$

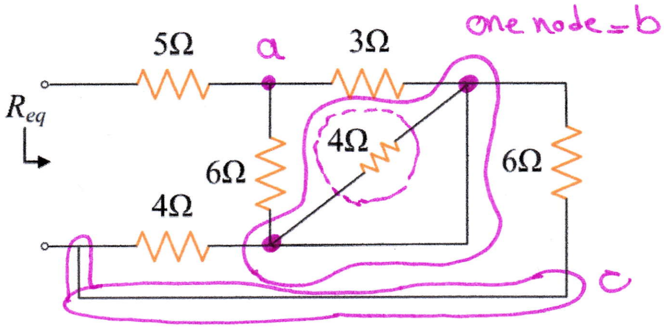
b)  $R\|R=\frac{R}{2}$, $R\|R=\frac{R}{2}$; $(R\|R)+(R\|R) \Rightarrow R_{eq}=\frac{R^2}{2R}+\frac{R^2}{2R}=\boxed{R}$

c)  2R, 2R → 2R||2R $\Rightarrow R_{eq}=\frac{2R\times 2R}{4R}=\boxed{R}$

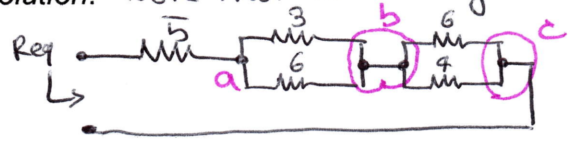
d) $R_{eq}= R\|2R\|3R \Rightarrow \frac{1}{R_{eq}}=\frac{1}{R}+\frac{1}{2R}+\frac{1}{3R}=\frac{6R^2+3R^2+2R^2}{6R^3}=\frac{11R^2}{6R^3}$

$\Rightarrow R_{eq}=\boxed{\frac{6}{11}R}$

4. (**Midterm Exam - S1, 2016**) For the circuit below, calculate the equivalent resistance R_{eq} of the network as seen from the terminals.

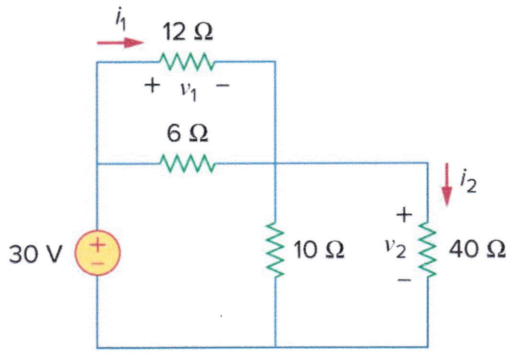


Solution: Note that 4Ω is being short circuited.



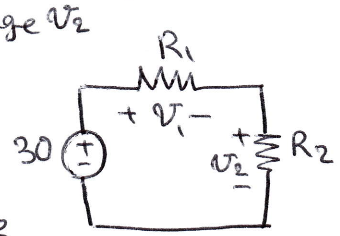
$$R_{eq} = 5 + 3 \| 6 + 6 \| 4 = 5 + \frac{3\times6}{3+6} + \frac{6\times4}{6+4} = 5 + 2 + 2.4 = \boxed{9.4\,\Omega}$$

5. Find v_1 and v_2 in the circuit shown below. Also the currents i_1 and i_2 .



Solution: Note that 12Ω ∥ 6Ω with equal voltage v_1
and 10Ω ∥ 40Ω with equal voltage v_2

$$R_1 = \frac{12\times6}{12+6} = 4\,\Omega \,, \quad R_2 = \frac{10\times40}{10+40} = 8\,\Omega$$



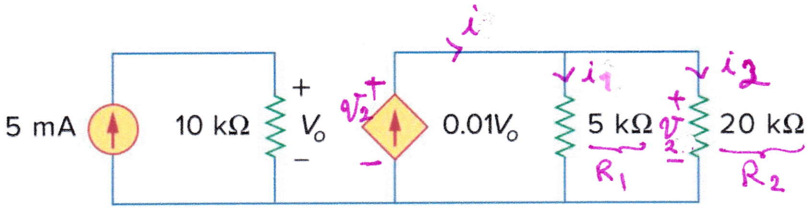
Voltage Divisor Circuit

Voltage division

$$\begin{cases} v_1 = \frac{R_1}{R_1+R_2}\times30 = \frac{4\times30}{4+8} = \boxed{10V} \\ v_2 = \frac{R_2}{R_1+R_2}\times30 = \frac{8\times30}{4+8} = \boxed{20V} \end{cases}$$

Ohm's law: $i_1 = \frac{v_1}{12\,\Omega} = \frac{10}{12} = \frac{5}{6}\,A \simeq \boxed{833.3\,mA}$, $i_2 = \frac{v_2}{40} = \frac{20}{40} = 0.5A = \boxed{500\,mA}$

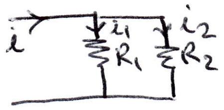
6. In the following circuit, calculate the current and voltage of the 20 kΩ resistor. What is the power generated/supplied by the dependent current source?



Solution: Note that 5mA only flows through 10KΩ

→ $V_o = 10K \times 5mA = \boxed{50V}$

Voltage-Controlled current Source → $i = 0.01V_o = 0.01 \times 50 = 0.5A$



→ $i_2 = \frac{R_1}{R_1+R_2} \times i = \frac{5K}{5K+20k} \times 0.5 = \boxed{0.1A}$

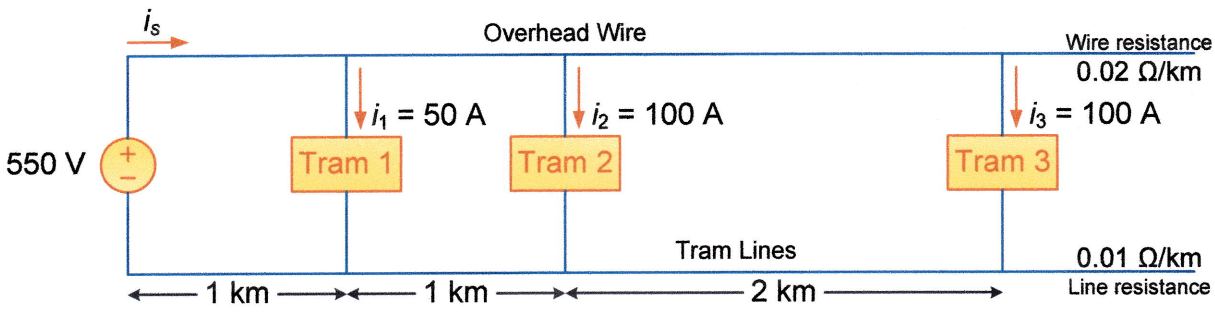
Current divisor circuit

$v_2 = 20K \times i_2 = 20k \cdot 0.1A = 2KV = \boxed{2000V}$

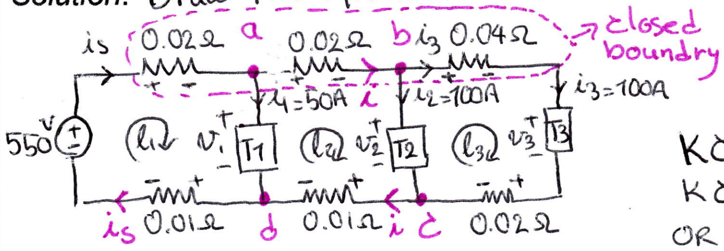
VCCS has the same voltage as v_2 → $P_{VCCS} = v_2 \times i = 2K \times 0.5A = \boxed{1KW}$

7. The DC *traction power supply system* (TPSS) of a light rail network, as shown in the following figure, consists of a DC Voltage source, overhead wires and an earth return. At a given moment, three trams travel within one *section post* and absorb 50 A, 100 A, and 100 A, respectively. Considering the overhead wires electric resistance given in units of Ω/km, calculate the power that is absorbed by each trams. What is the overall **efficiency** of this TPSS?

Efficiency η is defined as the ratio of the total **useful power** consumed by a system (to perform a task) to the **total power supplied** to it. Efficiency is an indication of how much of the supplied power is being lost like the heat dissipation by wires resistances.



Solution: Draw the equivalent Circuit. Note that wires and lines can be modelled with their equivalent resistance according to their length in the circuit diagram.



KCL @ b: $i = i_2 + i_3 = 200\text{ A} \triangleq$ @ c

KCL @ a: $i_S = i + i_1 = \underline{250\text{ A}} \triangleq$ @ d

OR you can consider a closed boundry with resistors inside

Continue on next page →

KVL in l_1: $-550 + 0.02 i_s + V_1 + 0.01 i_s = 0 \Rightarrow V_1 = 550 - 0.03\times 250 = 542.5\,V$

KVL in l_2: $-V_1 + 0.02 i + V_2 + 0.01 i = 0 \Rightarrow V_2 = 542.5 - 0.03\times 200 = 536.5\,V$

KVL in l_3: $-V_2 + 0.04 i_3 + V_3 + 0.02 i_3 = 0 \Rightarrow V_3 = 536.5 - 0.06\times 100 = 530.5\,V$

$P_1 = V_1 . i_1 = 542.5\times 50 = 27.125\,kW$

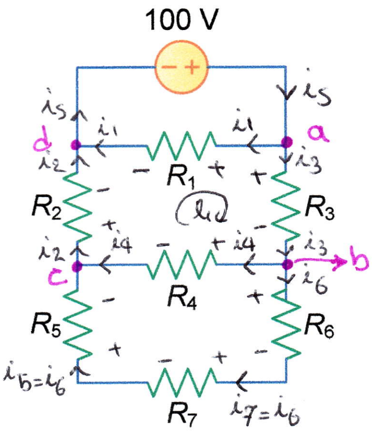
$P_2 = V_2 . i_2 = 536.5\times 100 = 53.65\,kW$

$P_3 = V_3 . i_3 = 530.5\times 100 = 53.05\,kW$

$$\Rightarrow \eta = \frac{P_1+P_2+P_3}{P_s} = \frac{133.825\,kW}{\underset{550\times 250}{137.5\,kW}} = 0.973 = 97.3\%$$

8. An industrial heating application can be modelled by using a connection of resistive elements as shown in the figure below. The two requirements for this industrial heating application are stated as below,
1) The total power consumed is equal to 3500 W.
2) The elements generate heat uniformly (this means that they dissipate equal power).

Based on the above requirements, Calculate the value of resistances R_1 to R_7.



Solution: Since each resistor consumes the same amount of power, thus the power of each resistor is $\frac{3500}{7} = \boxed{500\,W} = P_{R_1} = P_{R_2} = P_{R_3} = P_{R_4} = P_{R_5} = P_{R_6} = P_{R_7} = \boxed{P}$

R_1 is in parallel with 100 V $\Rightarrow \boxed{V_{R_1} = 100\,V} \rightarrow P = \frac{V_{R_1}^2}{R_1} \Rightarrow R_1 = \frac{V_{R_1}^2}{P} = \frac{10000}{500} = \boxed{20\,\Omega}$

Also $i_1 = \frac{V_{R_1}}{R_1} = \frac{100}{20} = \boxed{5\,A}$

Note that Total power absorbed = total power supplied. Thus the 100 V source supplies 3500 W

$\Rightarrow i_s = \frac{3500\,W}{100\,V} = \boxed{35\,A}$

KCL @ b & c: $\begin{cases} i_3 = i_4 + i_6 \;\text{(I)} \\ i_2 = i_4 + i_6 \end{cases} \Rightarrow i_2 = i_3$. Also $\boxed{R_5 = R_6 = R_7}$ Since they have the same current and consume the same power

$\hookrightarrow \boxed{R_2 = R_3}$ as they also consume the same power

KCL @ a $\quad i_s = i_1 + i_3 \Rightarrow i_3 = 35 - 5 = \boxed{30\,A}$, $\boxed{i_2 = 30\,A}$ $\rightarrow P = R_2 i_2^2 \Rightarrow R_2 = \frac{500}{900} = \frac{5}{9} = 0.55\,\Omega$

$\boxed{R_3 = R_2 = 0.55\,\Omega}$

Now $V_{R_3} = V_{R_2}$ Since $R_2 = R_3$ and $i_2 = i_3 \Rightarrow V_{R_2} = \frac{5}{9}\times 30 = \frac{50}{3}\,V = V_{R_3}$

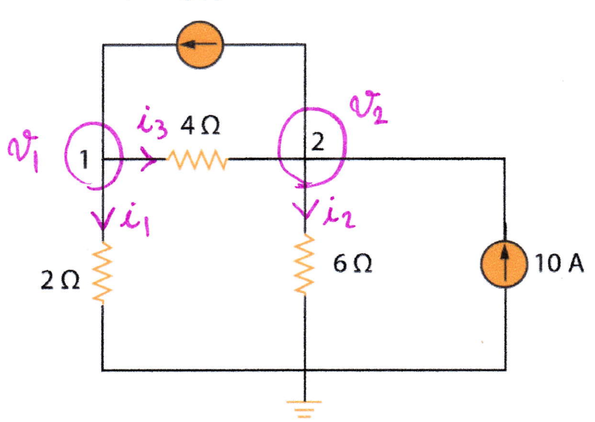
KVL l_1: $-V_{R_1} + V_{R_2} + V_{R_4} + V_{R_3} = 0 \Rightarrow V_{R_4} = 100 - 2\times\frac{50}{3} = \boxed{\frac{200}{3}\,V} \Rightarrow P = \frac{V_{R_4}^2}{R_4} \rightarrow R_4 = \frac{\left(\frac{200}{3}\right)^2}{500} = \frac{400}{45}$

$\boxed{R_4 = 8.88}$

(I) $i_6 = i_3 - i_4 = 30 - \frac{V_{R_4}}{R_4} = 30 - \frac{\frac{200}{3}}{\frac{400}{45}} = 30 - \frac{15}{2} = \boxed{\frac{45}{2}\,A}$

$P = R_6 i_6^2 \Rightarrow \boxed{R_6 = \frac{500}{\left(\frac{45}{2}\right)^2} = \frac{2000}{2025} = 0.987\,\Omega = R_5 = R_7}$

9. Find the node voltages in the circuit given below.



Answer: $v_1 = \frac{40}{3} = 13.333$ V and $v_2 = 20$ V.

Solution: Assign v_1 and v_2 to nodes 1 and 2 and choose arbitrary direction for currents.

KCL @ n1: $5 = \frac{v_1}{2} + \frac{v_1 - v_2}{4} \xrightarrow{\times 4} 20 = 3v_1 - v_2$

KCL @ n2: $10 + \frac{v_1 - v_2}{4} = 5 + \frac{v_2}{6} \xrightarrow{\times 12} 60 = 5v_2 - 3v_1$

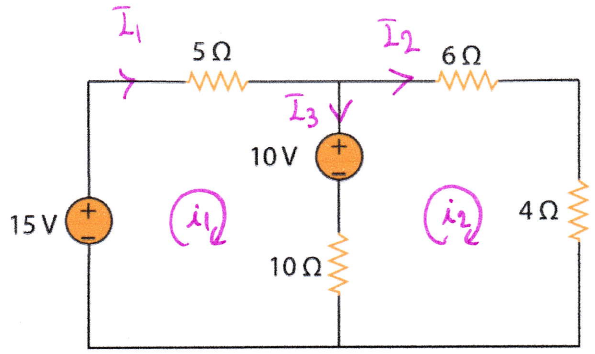
$\Rightarrow \begin{cases} 3v_1 - v_2 = 20 \\ 3v_1 - 5v_2 = -60 \end{cases}$

$\Rightarrow v_1 = \frac{40}{3} = 13.333\text{V}$

$v_2 = 20\text{V}$

Note that in the case of $i_3 = \frac{v_1 - v_2}{4} = -1.66$A showing that the actual direction of current is opposite.

10. In the following circuit, find all the branch currents using mesh analysis. (Assign mesh currents and branch currents individually)



Answer: $I_1 = 1\ \mathrm{A}$ (current in 15-V voltage source and 5-Ω resistor), $I_2 = 1\ \mathrm{A}$ (current in 6-Ω and 4-Ω resistors), and $I_3 = 0\ \mathrm{A}$ (current in 10-V voltage source and 10-Ω resistor).

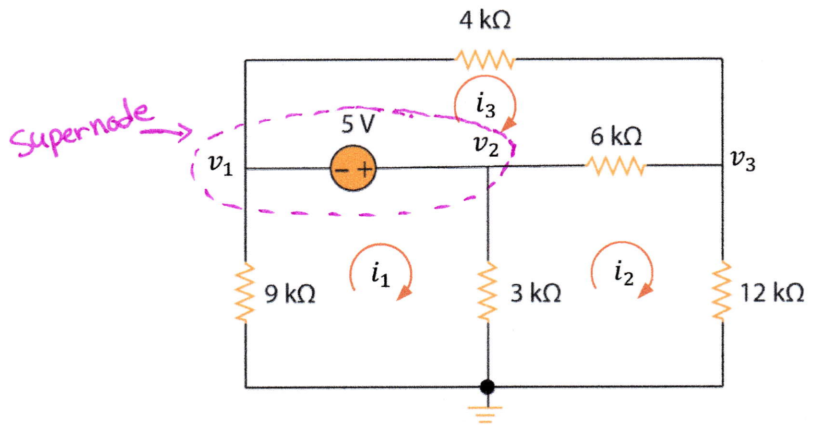
Solution: KVL in m1: $-15 + 5i_1 + 10 + 10(i_1 - i_2) = 0$

KVL in m2: $6i_2 + 4i_2 - 10(i_1 - i_2) - 10 = 0$

$$\Rightarrow \begin{cases} 15i_1 - 10i_2 = 5 \\ -10i_1 + 20i_2 = 10 \end{cases} \begin{array}{c} \xrightarrow{\times 1/5} \\ \xrightarrow{\times -1/10} \end{array} \begin{cases} 3i_1 - 2i_2 = 1 \\ i_1 - 2i_2 = -1 \end{cases} \Longrightarrow \begin{cases} i_1 = 1A \\ i_2 = 1A \end{cases}$$

Thus: $\begin{cases} I_1 = i_1 = 1A \\ I_2 = i_2 = 1A \\ I_3 = i_1 - i_2 = 0A \end{cases}$

11. For the circuit below,
    a) Write nodal equations using nodal analysis
    b) Write mesh equations using mesh analysis



Answer:

a) $\begin{cases} v_1 - v_2 = -5 \\ 13v_1 + 18v_2 - 15v_3 = 0 \\ 3v_1 + 2v_2 - 6v_3 = 0 \end{cases}$

b) $\begin{cases} 12i_1 - 3i_2 = 5 \\ 3i_1 - 21i_2 + 6i_3 = 0 \\ 6i_2 - 10i_3 = 5 \end{cases}$

Solution: Nodal analysis

*KCL@ Supernode: $\frac{V_1}{9}+\frac{V_2}{3}+\frac{V_2-V_3}{6}+\frac{V_1-V_3}{4}=0$

$\xrightarrow{\times 36} \boxed{13V_1+18V_2-15V_3=0}$

*KCL@ n3: $\frac{V_3}{12}=\frac{V_2-V_3}{6}+\frac{V_1-V_3}{4}$

$\xrightarrow{\times 12} \boxed{3V_1+2V_2-6V_3=0}$

* Supernode Constraint (KVL) $\boxed{V_2-V_1=5}$

$\Longrightarrow \begin{cases} V_1-V_2=-5 \\ 13V_1+18V_2-15V_3=0 \\ 3V_1+2V_2-6V_3=0 \end{cases}$ Nodal equation

Mesh analysis

*KVL in m1: $9i_1-5+3(i_1-i_2)=0$

$\longrightarrow \boxed{12i_1-3i_2=5}$

*KVL in m2: $12i_2-3(i_1-i_2)+6(i_2-i_3)=0$

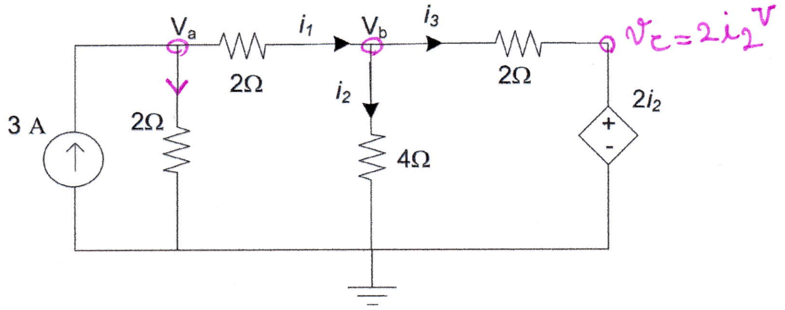
$\longrightarrow \boxed{-3i_1+21i_2-6i_3=0} \xrightarrow{\times -1}$

*KVL in m3: $4i_3-6(i_2-i_3)+5=0$

$\longrightarrow \boxed{10i_3-6i_2=-5} \xrightarrow{\times -1}$

$\Longrightarrow \begin{cases} 12i_1-3i_2=5 \\ 3i_1-21i_2-6i_3=0 \\ 6i_2-10i_3=5 \end{cases}$ Mesh equation

12. **(Mid-session Exam – S2, 2016)** For the circuit below,
   a) Apply nodal analysis to write down the node voltage equations at nodes V_a and V_b.
   b) Solve the voltage equations in part (a) to find the voltages V_a and V_b.
   c) Find the currents i_1, i_2 and i_3 based on your results in part (b)



Answer:

a) $\begin{cases} 2v_a - v_b = 6 \\ 2v_a - 4v_b = 0 \end{cases}$

b) $V_a = 4$ V and $V_b = 2$ V.

c) $i_1 = 1$ A, $i_2 = 0.5$ A and $i_3 = 0.5$ A

Solution: Note that V_c is directly Connected to dependent voltage Source

a) *KCL@ V_a: $3=\frac{V_a}{2}+\frac{V_a-V_b}{2} \xrightarrow{\times 2} \boxed{2V_a-V_b=6}$

*KCL@ V_b: $\frac{V_a-V_b}{2}=\frac{V_b}{4}+\frac{V_b-V_c}{2}$ Also $V_c=2i_2$ and $i_2=\frac{V_b}{4} \Rightarrow \boxed{V_c=\frac{V_b}{2}}$ I

$\xrightarrow{\times 4 \,\&\, \text{I}} 2(V_a-V_b)=V_b+2\left(V_b-\frac{V_b}{2}\right) \longrightarrow \boxed{2V_a-4V_b=0}$

$\Longrightarrow$ Nodal equation $\begin{cases} 2V_a-V_b=6 \\ 2V_a-4V_b=0 \end{cases}$

b) Solve the equations $\longrightarrow V_a=4V$ and $V_b=2V$

c) $i_1=\frac{V_a-V_b}{2}=\frac{4-2}{2}=\boxed{1A}$

$i_2=\frac{V_b}{4}=\frac{2}{4}=\boxed{0.5A}$

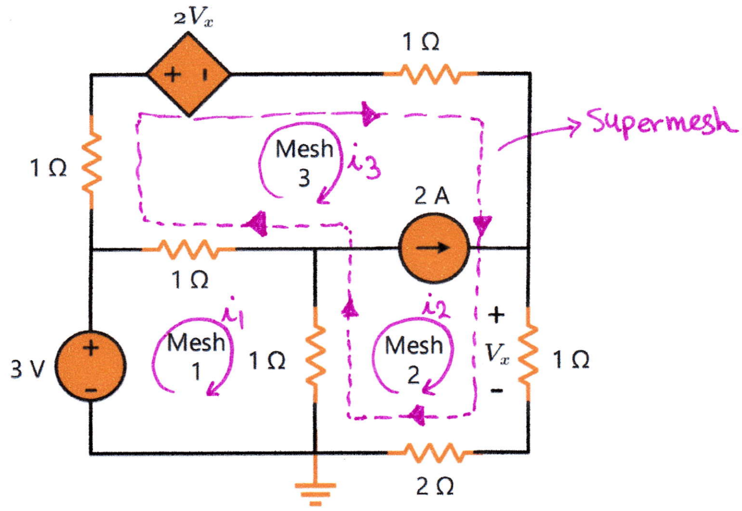
$i_3=\frac{V_b-V_c}{2} \overset{\text{I}}{=} \frac{V_b-\frac{V_b}{2}}{2}=\frac{V_b}{4}=\frac{2}{4}=\frac{1}{2}A$ or $\boxed{0.5A}$

13. (**Mid-session Exam – Summer, 2017**) For the circuit below,

a) Apply mesh analysis to write down the mesh current equations in Mesh 1, Mesh 2, and Mesh 3 (this part was only given in the actual exam).

b) Solve the current equations in part (a) to find the mesh current.

c) Find the power generated by the dependent voltage source based on your results in part (b).



Answer:

a) $\begin{cases} 2i_1 - i_2 - i_3 = 3 \\ 2i_1 - 6i_2 - 3i_3 = 0 \\ i_2 - i_3 = 2 \end{cases}$

b) $i_1 = 1.5$ A, $i_2 = 1$ A and $i_3 = -1$ A

c) $P = 2$ W supplied

Solution: Note that 2 A current source is a common branch between Mesh 2 and Mesh 3.

a) * KVL in m1: $-3+1(i_1-i_3)+1(i_1-i_2)=0 \longrightarrow \boxed{2i_1-i_2-i_3=3}$

* KVL in Supermesh: $1i_3+2V_x+1i_3+1i_2+2i_2-1(i_1-i_2)-1(i_1-i_3)=0$

$\longrightarrow 2V_x-2i_1+4i_2+3i_3=0$ Also $\boxed{V_x=1i_2}$ (I) Note the polarity of V_x an direction of i_2

$\Longrightarrow \boxed{-2i_1+6i_2+3i_3=0} \xrightarrow{\times -1}$

* Supermesh Constraint (KCL): $\boxed{i_2-i_3=2}$

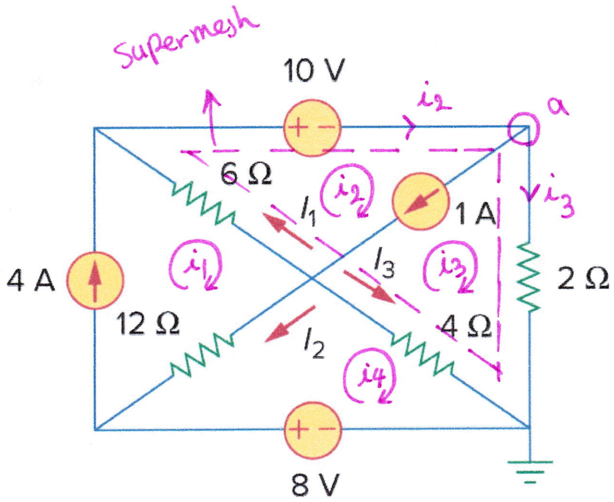
Thus: $\begin{cases} 2i_1-i_2-i_3=3 \\ 2i_1-6i_2-3i_3=0 \\ i_2-i_3=2 \end{cases}$

b) Solve the equations $\rightarrow \begin{cases} i_1=1.5\text{A} \\ i_2=1\text{A} \\ i_3=-1\text{A} \end{cases}$

c) $P=-(-1)\times 2V_x \overset{\text{(I)}}{=} 1\times 2\times 1 = 2\text{W}$ supplied or -2W absorbed

Note that i_3 enters the positive terminal of the dependent voltage source.

14. In the circuit below, find the branch currents I_1, I_2, and I_3.



Answer: $I_1 = -1\text{ A}$, $I_2 = 0\text{ A}$, and $I_3 = 2\text{ A}$

Solution: Note that 1-A current source is a common branch between mesh 2 and mesh 3.
Also, it is clear that $\boxed{i_1 = 4A}$ (I) in mesh 1

* KVL in m4: $-8 - 12(i_1 - i_4) + 4(i_4 - i_3) = 0 \xrightarrow{\times -1/4} \boxed{3i_1 + i_3 - 4i_4 = -2}$
* KVL in Supermesh: $10 + 2i_3 - 4(i_4 - i_3) + 6(i_2 - i_1) = 0$

$\xrightarrow{\times 1/2} \boxed{-3i_1 + 3i_2 + 3i_3 - 2i_4 = -5}$

* Supermesh Constraint (KCL @ node a): $\boxed{i_2 = i_3 + 1}$

$$\xRightarrow{\text{(I)}} \begin{cases} i_3 - 4i_4 = -14 \\ 3i_2 + 3i_3 - 2i_4 = 7 \\ i_2 - i_3 = 1 \end{cases} \xrightarrow{\text{Solve}} \boxed{\begin{cases} i_2 = 3A \\ i_3 = 2A \\ i_4 = 4A \end{cases}} \xRightarrow[\text{(I)}]{\text{Thus}} \begin{cases} I_1 = i_2 - i_1 = -1A \\ I_2 = i_1 - i_4 = 0A \\ I_3 = i_4 - i_3 = 2A \end{cases}$$