Faculty of Engineering

School of Electrical Engineering and Telecommunications

ELEC 1111 – Topic 6

Operational Amplifiers

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Topic 6 Content

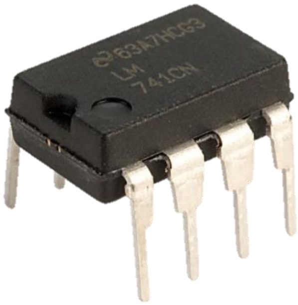
This lecture covers:

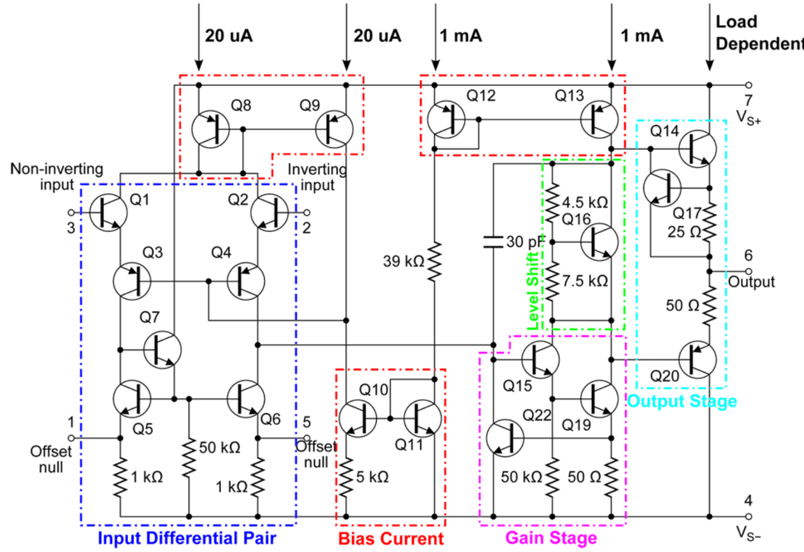
- Operational amplifiers (Op Amp) as active elements
- Ideal and non-ideal Op Amp
- Mathematical operations using Op Amp
  - Inverting Op Amp
  - Non-inverting Op Amp
  - Summing Op Amp
  - Difference Op Amp
  - Integrator
  - Differentiator

Corresponds to Chapter 5 and last part of Chapter 6 of your textbook

Operational Amplifiers

- Operational amplifiers or Op Amp are **active elements.**
- The Op Amp is an electronic device consisting of a complex arrangement of resistors and transistors.
- They are commercially available in integrated circuit packages (e.g. LM 741).
- An Op Amp circuit is designed to perform **mathematical operations** in **analog circuits** such as addition, subtraction, multiplication, division, differentiation, and integration when **resistors** and **capacitors** are connected to its terminals.





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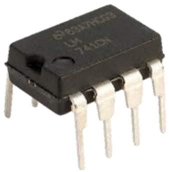
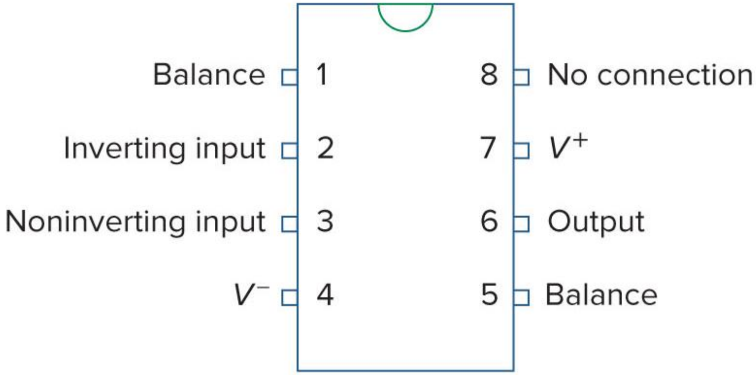
Operational Amplifiers

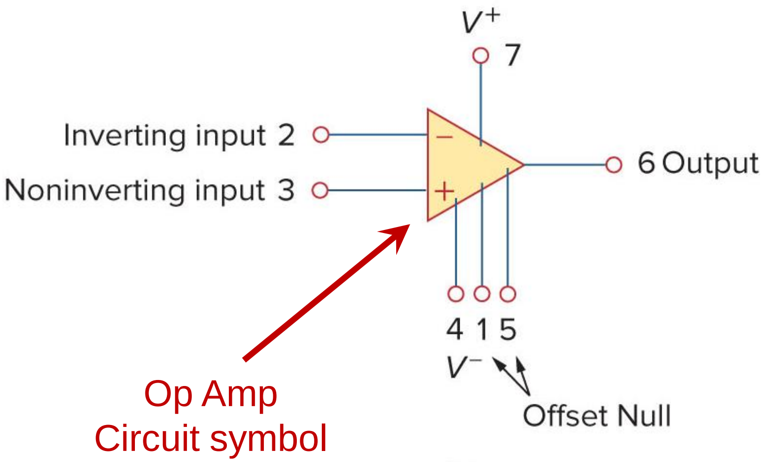


- A typical Op Amp has **eight/8 pins.** We are only interested in **five/5** of them:
  - **Inverting input** (pin 2)
  - **Non-inverting input** (pin 3)
  - **Output** (pin 6)
  - **Positive** power supply V^+ (pin 7)
  - **Negative** power supply V^- (pin 4)



(a)

> - A signal applied to the **non-inverting** input with appear with the **same polarity** in the output.
> - A signal applied to the **inverting** input will appear with the **opposite polarity** in the output.

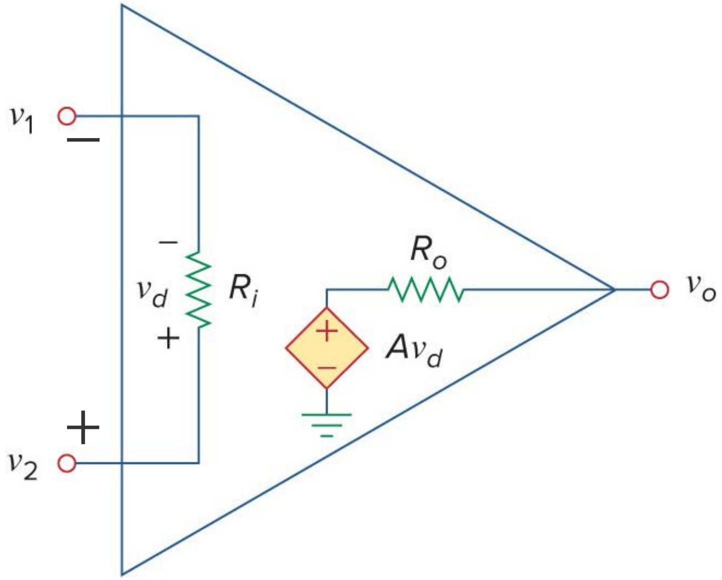


(b)

Equivalent circuit model

- An Op Amp is modelled from its input terminals by an **input resistor** R_i, and from its output terminal by a **voltage-controlled voltage source** in series with an **output resistor** R_o.
- The output voltage v_o is given by:

$$\boxed{v_o = Av_d = A(v_2 - v_1)}$$



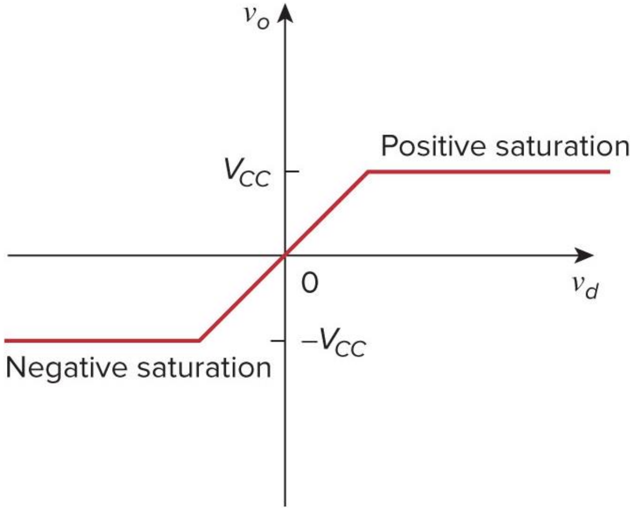
Typical range for Op Amp parameters

Parameter	Typical range	Ideal value
Open-loop gain, A	10^5 to 10^8	∞
Input resistance R_i	10^5 to 10^{13} Ω	∞ Ω
Output resistance R_o	10 to 100 Ω	0 Ω
Supply voltage, V_{cc}	5 to 24 V	

- A: **Open-loop** voltage **gain**
- v_d: **Differential** input voltage
- v_1: **Inverting** terminal voltage to the **ground**
- v_2: **Non-inverting** terminal voltage to the **ground**
- R_o: Thevenin equivalent resistance seen at the **output terminals**
- R_i: Thevenin equivalent resistance seen at the **input terminals**

Ideal vs non-ideal Op Amp

- In practice, the magnitude of output voltage v_o cannot exceed its power supply voltage $|V_{cc}|$.
- Depending on the differential input voltage in open-loop form, the Op Amp can operate in three modes:
  1. Positive saturation: $v_o = V_{cc}$
  2. Linear region: $-V_{cc} \leq v_o \leq V_{cc}$
  3. Negative saturation: $v_o = -V_{cc}$



We always operate Op Amp in its **linear region** throughout this course

$$-V_{cc} \leq v_o \leq V_{cc}$$

- Assuming **linear operation**, an **ideal Op Amp** has the following characteristics:
  - **Infinite** open-loop gain, $A \simeq \infty$
  - **Infinite** input resistance, $R_i \simeq \infty$
  - **Zero** output resistance, $R_o \simeq 0$

Ideal Op Amp in circuit analysis

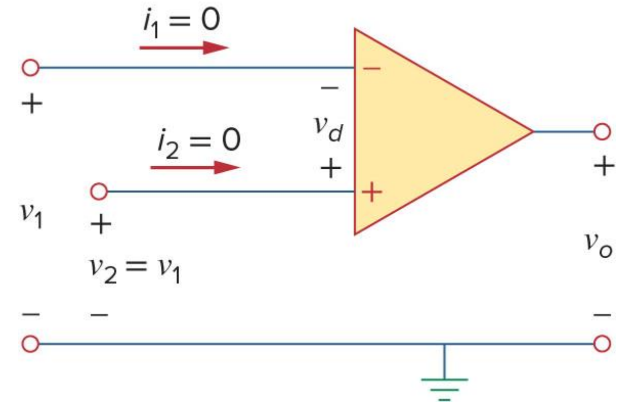
There are **two important properties** of Ideal Op Amp that are needed for analysing them in electric circuits.

1. The **currents** into both terminals are **zero**

$$\boxed{i_1 = 0 \quad \& \quad i_2 = 0}$$

2. The **voltage** across the input terminals is **zero**

$$\boxed{v_d = v_2 - v_1 = 0} \quad \text{or} \quad \boxed{v_1 = v_2}$$



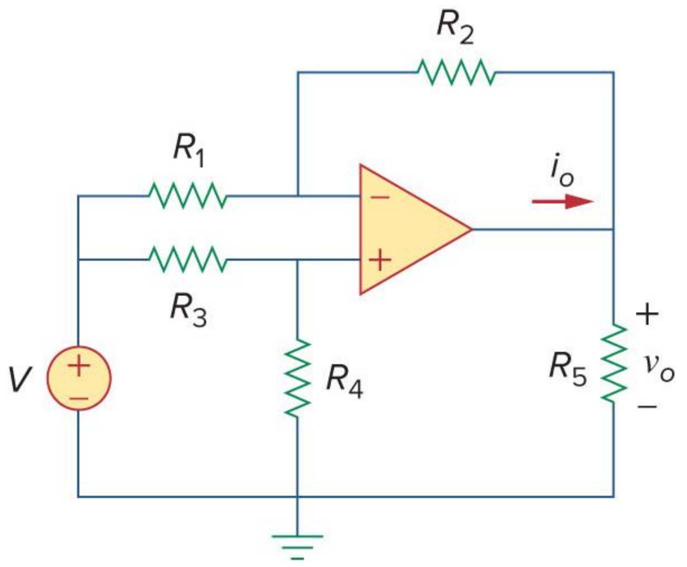
- Having $R_i \simeq \infty$ implies **open circuit** at the **input** terminals. Thus, **input currents** i_1 and i_2 should be **zero.**
- If $v_d \neq 0$, the output voltage $v_o = Av_d$ may become so high that it would exceed V_{cc} due to the large value of the open-loop gain A.

Note: $v_2 = v_1$ only happens when connecting the **output** to the **inverting** terminal (**negative feedback**).

Problem Solving in Op Amp Circuits

- Follow these three steps when solving Op Amp circuits:

1. Use **Ideal** Op Amp model:
   - $i_1 = i_2 = 0$
   - $v_1 = v_2$
   - $R_i \simeq \infty$
   - $R_o \simeq 0$
2. Apply **nodal** analysis.
3. Solve nodal equations to express the **output voltage** in terms of **input signals** (voltage or current) or just solve for the **unknown** voltage or current in the circuit.

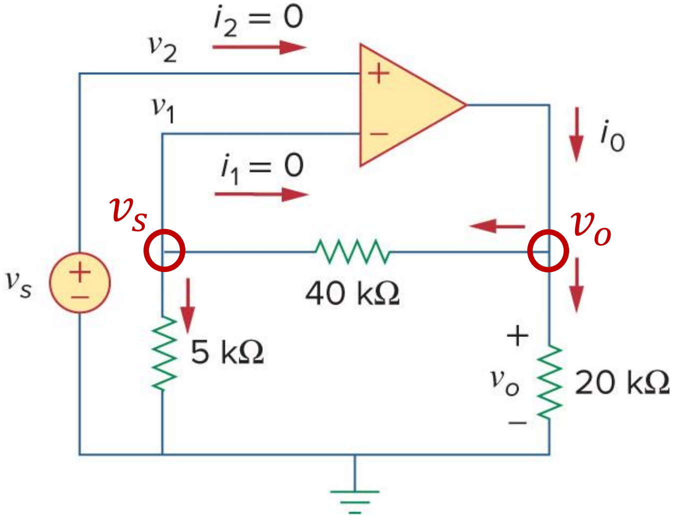


- Having $R_0 \simeq 0$ implies **short circuit** at the **output** terminals, thus **output current** i_o can attain **any value** and **direction** depending on the circuit

Exercise

a) Find the ratio of output voltage v_0 to input voltage v_s, known as *closed-loop gain* $\frac{v_0}{v_s}$.

b) Calculate i_o if $v_s = 1$ V.



Inverting amplifier

- An **inverting amplifier reverses** the polarity of the input signal while **amplifying** it.
  - The **noninverting** input is **grounded.**
  - The **inverting** input is connected to the output via a **feedback resistor**, R_f.
  - The input voltage v_i is also connected to the inverting input via another resistor, R_1.

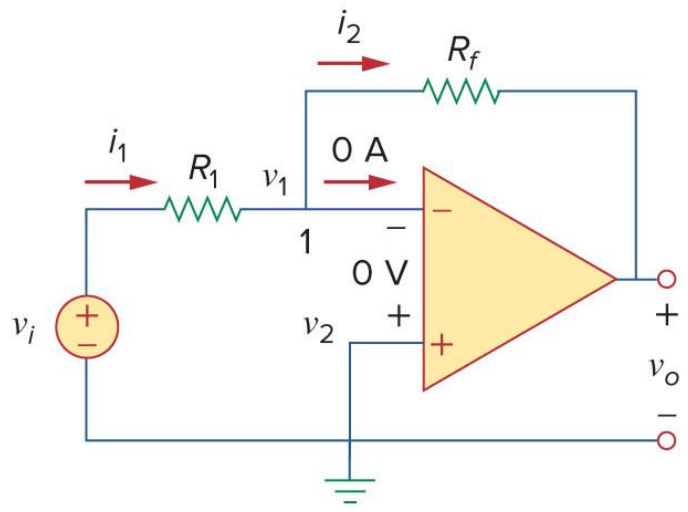
Apply KCL at node 1: $i_1 = i_2 \Rightarrow \dfrac{v_i - v_1}{R_1} = \dfrac{v_1 - v_0}{R_f}$

Ideal Op Amp with negative feedback: $v_1 = v_2 = 0\ \mathrm{V}$

Thus: $\dfrac{v_i}{R_1} = -\dfrac{v_0}{R_f}$

$A_v = \dfrac{v_o}{v_i} = -\dfrac{R_f}{R_1}$ (voltage gain)

$$\boxed{v_0 = -\frac{R_f}{R_1} v_i}$$



Non-inverting amplifier

- A **non-inverting amplifier** is designed to provide **positive** voltage gain amplification
  - The voltage input v_i is directly connected to **noninverting** input
  - The **inverting** input is **grounded** via R_1 and also connected to the output via the **feedback resistor**, R_f

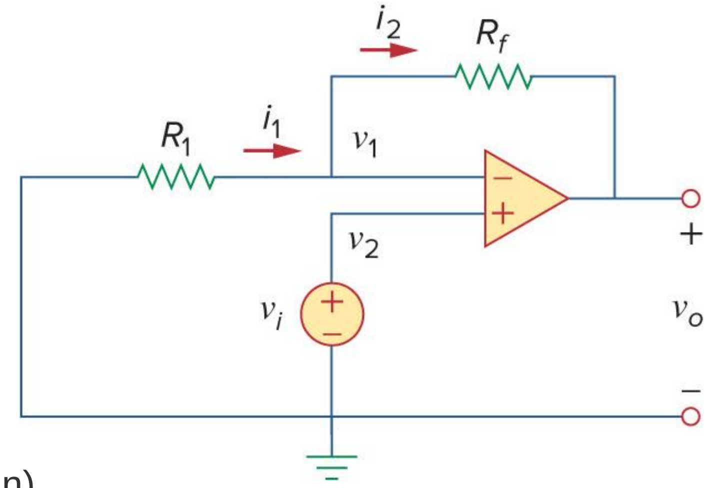
Apply KCL at node 1: $$i_1 = i_2 \Rightarrow \frac{0 - v_1}{R_1} = \frac{v_1 - v_0}{R_f}$$

Ideal Op Amp with negative feedback: $$v_1 = v_2 = v_i$$

Thus: $$\frac{-v_i}{R_1} = \frac{v_i - v_0}{R_f}$$

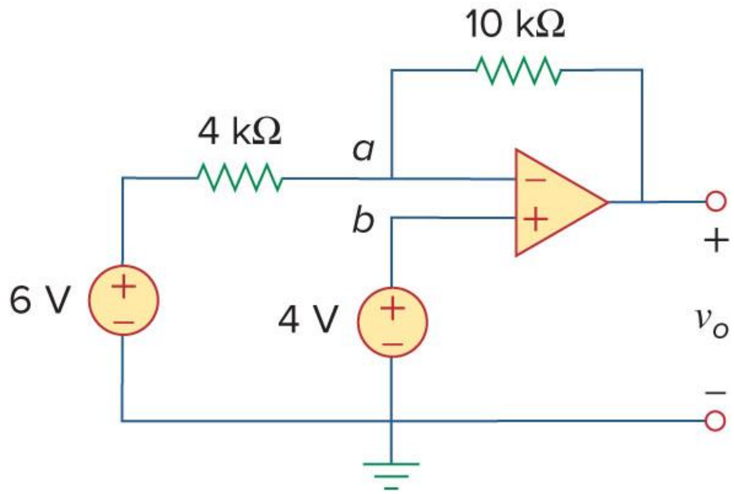
$$A_v = \frac{v_o}{v_i} = 1 + \frac{R_f}{R_1}$$ (voltage gain)

$$\boxed{v_0 = \left(1 + \frac{R_f}{R_1}\right) v_i}$$



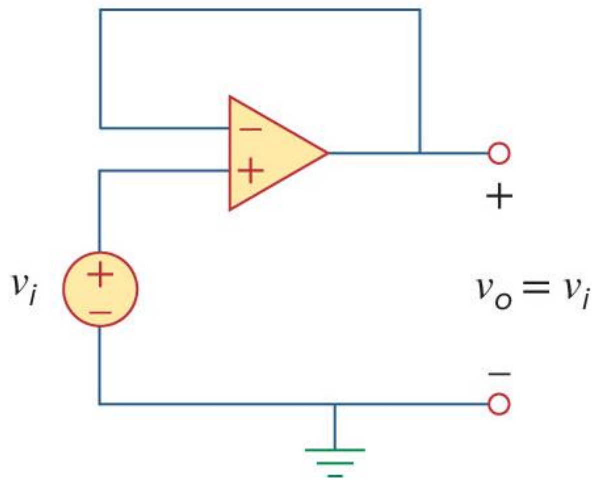
Exercise

Find the output voltage v_0 in the Op Amp circuit shown below.



Voltage follower

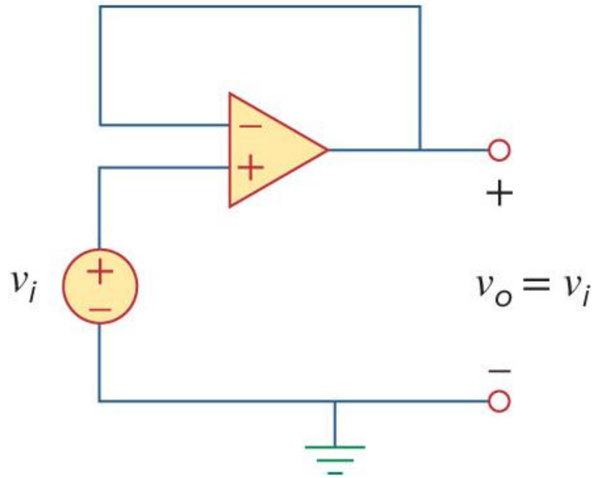
- A **voltage follower** or *unity gain amplifier* is an Op Amp circuit whose **output** is the **same as the input** (i.e. gain equals 1).
  - Using Op Amp with negative feedback makes the ***closed-loop*** **gain** depend only on the **external resistors** rather that open-loop gain *A.*
  - A special case of non-inverting Op Amp is obtained when $R_f = 0$ or $R_1 = \infty$ or both. This makes the gain to become 1, i.e., $v_o = \left(1 + \frac{R_f}{R_1}\right) v_1 = v_1$.



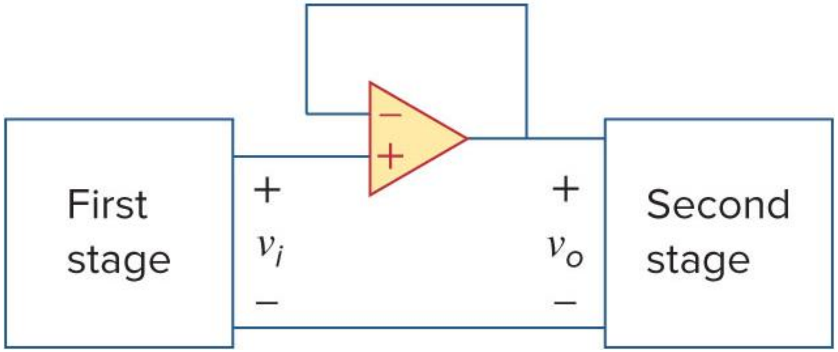
$$\boxed{v_o = v_i}$$

Voltage follower

- Voltage follower has a very **high input resistance** (impedance) and it is useful to **isolate** one circuit (first stage) from another (second stage).
- The second stage circuit **would not draw current** from the first stage, instead **Op Amp provides current** and power while maintaining the same voltage as v_i, **removing** the **loading effect** from the first stage circuit.



$$\boxed{v_o = v_i}$$



Summing amplifier

- A **summing amplifier** is an Op Amp circuit that **combines several inputs** and produces a **weighted sum** of the inputs in its output.
  - Aside from amplification, the Op Amp can be made to **sum** the values of input signals.
  - Using the **inverting Op Amp**, we can connect many inputs at the same time.

KCL at node a: $i = i_1 + i_2 + i_3$

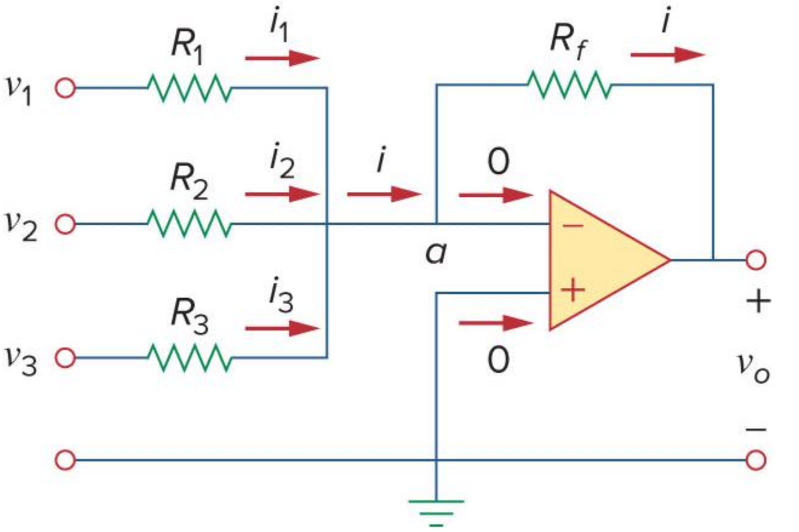
Ohm's Law:

$$\frac{v_a - v_o}{R_f} = \frac{v_1 - v_a}{R_1} + \frac{v_2 - v_a}{R_2} + \frac{v_3 - v_a}{R_3}$$

Ideal Op Amp with negative feedback: $v_a = 0\ \mathrm{V}$

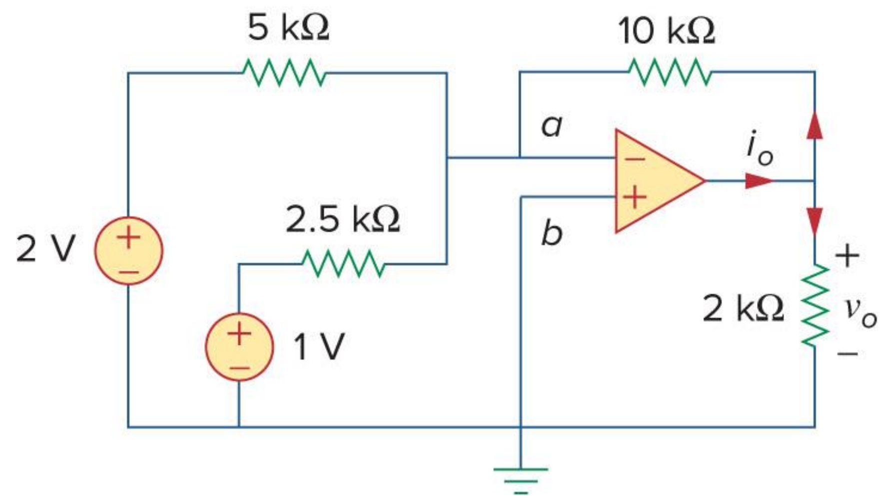
Thus,

$$\boxed{v_0 = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)}$$



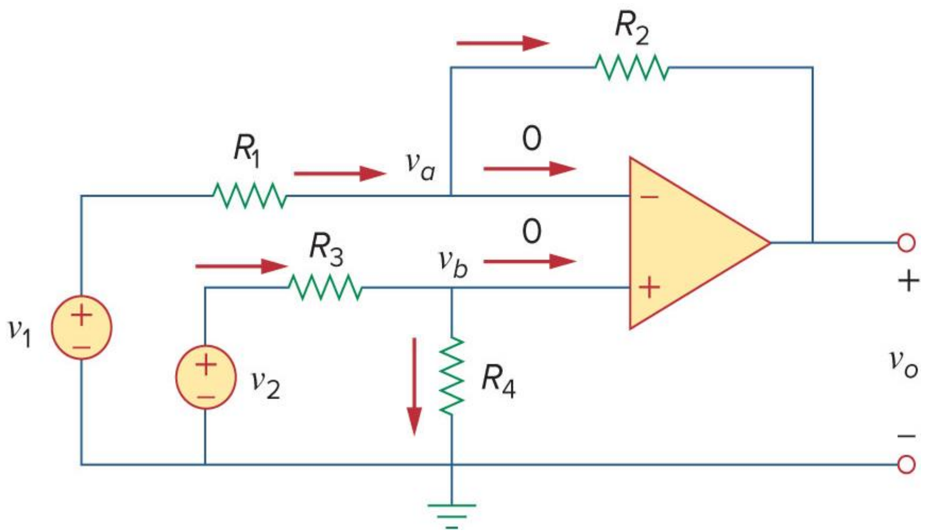
Exercise

Find the output voltage v_0 and output current i_0 in the Op Amp circuit shown.



Difference amplifier

- **Difference amplifiers** are used in various applications where there is a need to **amplify the difference** between two input signals.



KCL at node a: $$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2}$$ or $$v_o = \left(\frac{R_2}{R_1} + 1\right) v_a - \frac{R_2}{R_1} v_1 \quad \text{I}$$

KCL at node b: $$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4}$$ or $$v_b = \frac{R_4}{R_3 + R_4} v_2 \quad \text{II}$$

Ideal Op Amp with negative feedback makes $v_a = v_b$. Substitute Eq. II into Eq I:

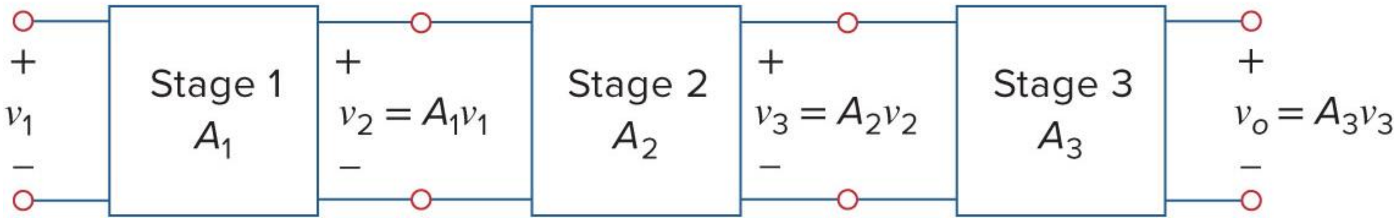
$$v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1$$

A difference amplifier **must reject** a signal **common** to the two inputs. This condition exists if:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow v_0 = \frac{R_2}{R_1}(v_2 - v_1)$$

Cascade of Op Amp stages

- It is common to use multiple Op Amp circuits chained together to **increase** the **overall gain** of an amplifier.
  - Due to input and output impedances of the ideal Op Amps, stages can be connected together without affecting the performance of each other (**no loading effect**).
- This head to tail configuration is called **“cascading”**.
- Each amplifier is then called a **“stage”**.



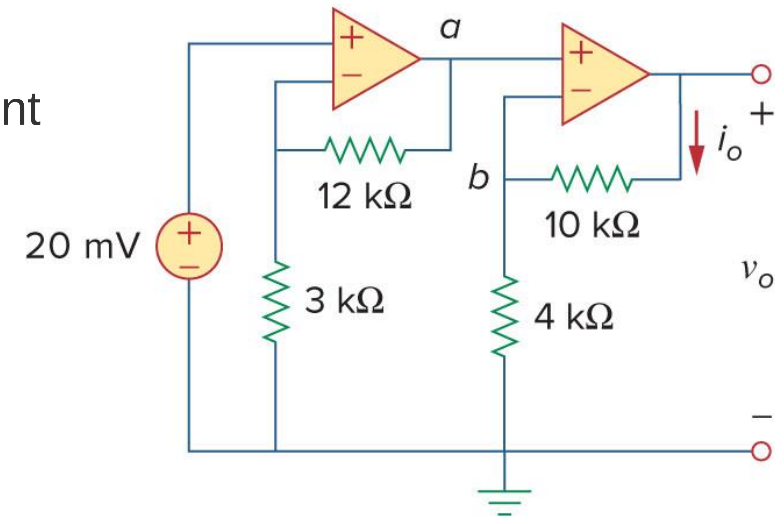
- The gain of a **series of amplifiers connected in cascade** is the **product** of the **individual gains**:

$$\boxed{A = A_1 \times A_2 \times A_3}$$

Note: The range of input voltage must be **small enough** to avoid **saturation** when amplified.

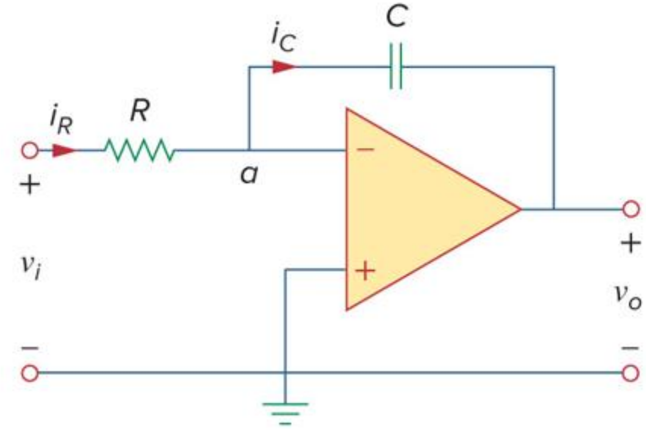
Exercise

Find the output voltage v_0 and output current i_0 in the Op Amp circuit shown.



Integrator

- **Capacitors** in combination with op-amps can be used to perform advanced mathematical functions.
- An **integrator** is an Op Amp circuit used to **integrate** signals.
  - By replacing the **feedback resistor** with a **capacitor**, the output voltage is obtained as follows.



KCL at node a: $i_R = i_C$

Current-voltage relations: $i_R = \dfrac{v_i}{R}, \quad i_C = -C\dfrac{dv_o}{dt}$

Substitute in KCL: $dv_o = -\dfrac{1}{RC}v_i dt$

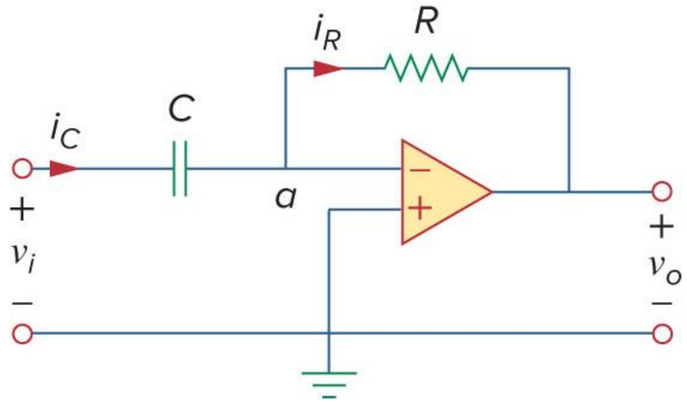
Integrate both sides: $v_o(t) - v_o(0) = -\dfrac{1}{RC}\int_0^t v_i(\tau)d\tau$

Ensuring zero initial conditions $v_o(0) = 0$ V:

$$\boxed{v_o(t) = -\frac{1}{RC}\int_0^t v_i(\tau)d\tau}$$

Differentiator

- A **differentiator** is an Op Amp circuit used to **differentiate** signals.
  - If the **capacitor** is used in place of the **input resistor** instead of the feedback resistor, there will only be current flowing if the capacitor is changing.



KCL at node a: $i_R = i_C$

Current-voltage relations: $i_R = -\dfrac{v_o}{R}$, $i_C = C\dfrac{dv_i}{dt}$

Substitute in KCL: $$\boxed{v_o(t) = -RC\frac{dv_i}{dt}}$$

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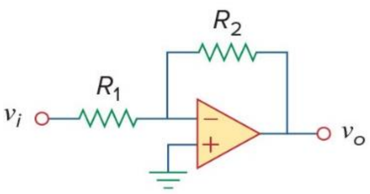
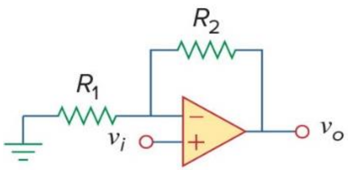
Summary



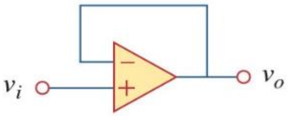
Inverting amplifier

$$v_o = -\frac{R_2}{R_1}v_i$$



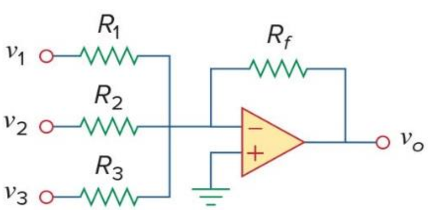
Noninverting amplifier

$$v_0 = \left(1 + \frac{R_2}{R_1}\right)v_i$$



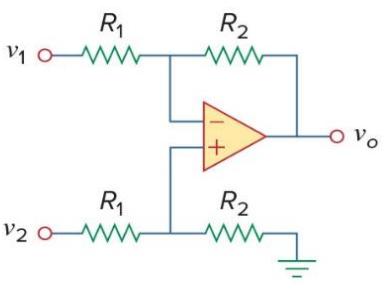
Voltage follower

$$v_o = v_i$$



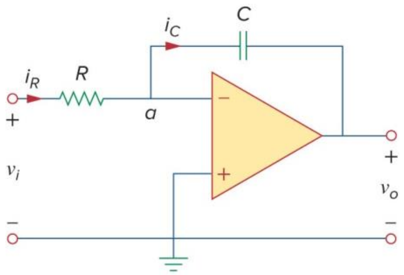
Summer

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$



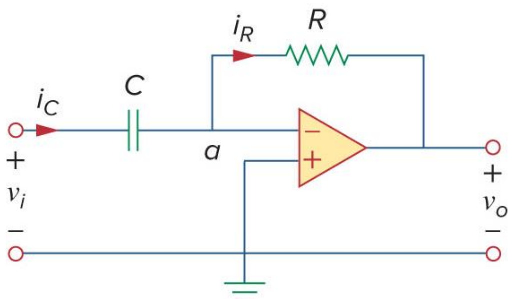
Difference amplifier

$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$



Integrator

$$v_o(t) = -\frac{1}{RC}\int_0^t v_i(\tau)d\tau$$

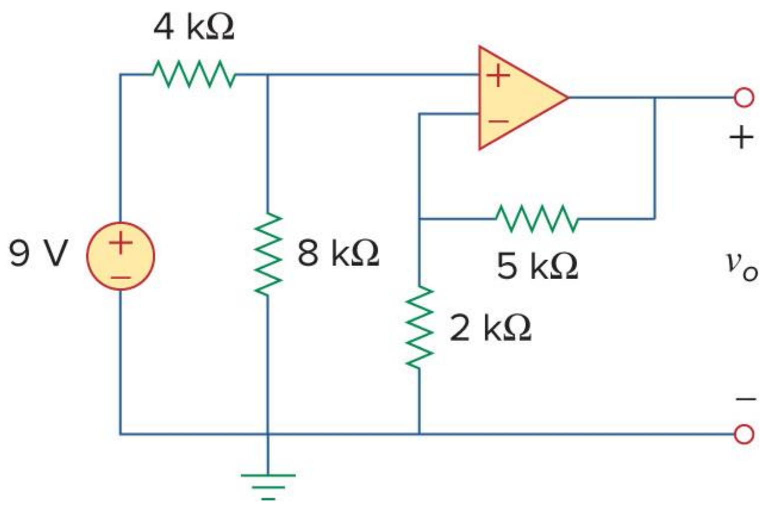


Differentiator

$$v_o(t) = -RC\frac{dv_i}{dt}$$

Exercise

Calculate the output voltage v_o in the Op Amp circuit below.



Questions?