The University of New South Wales

ELEC2141: Digital Circuit Design

Tutorial Week 2 – Number Systems and Boolean Algebra

1. **(1-6)** What is the decimal equivalent of the largest binary integer that can be obtained with:

    a. 11 bits
    b. 25 bits

2. **(1-7)** Convert these binary numbers into decimal numbers

    a. 1011001
    b. 1100111.001
    c. 10110010.10101

3. **(1-8)** Convert the following decimal numbers to binary

    a. 255
    b. 452
    c. 124.5
    d. 587.625

4. **(1-9)** Convert the following numbers from the given base to the other three bases in the table

Decimal	**Binary**	**Octal**	**Hexadecimal**
369.3125			
	10111101.101		
		326.5	
			F3C7.A

5. **(1-10)** Convert the following decimal numbers to the indicated bases

    a. 7562.45 to octal
    b. 1938.257 to hexadecimal
    c. 175.175 to binary

6. **(1-15)** Considerable evidence suggests that base 20 has historically been used for number systems. If the numbers are represented as 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F,G,H,I,J

    a. Convert 56180_{10} to base 20
    b. Convert $9ABF_{20}$ to decimal
    c. Convert $D5HA.5_{20}$ to decimal

Where referenced, questions are taken from the textbook:

M. Mano, C. R. Kime and T. Martin, Logic and Computer Design Fundamentals, 5th Edition (Global Edition), Pearson, 2016

7. **(1-18)** Find the binary representation of the following BCD numbers

    a. 0010 1001 0111 0101
    b. 0001 1001 0010.0101 0100

8. **(1-19)** Represent the following numbers in BCD

    a. 715
    b. 354

9. **(1-28)** The wind direction can be measured by a wireless weather station using a disk encoder like the one presented in lectures.

    a. Assuming that the code 000 corresponds to N, list the Gray code values for each of the directions, S, E, W, NW, NE, SW, and SE.
    b. Explain why the Gray code you have assigned avoids the reporting of major errors in wind direction.

10. **(1-29)** What is the percentage of power consumed for continuous counting (either up or down but not both) at the outputs of a binary Gray code counter (with all 2^n code words used) compared to a binary counter as a function of the number of bits, *n*, in the two counters? Assume each bit change consumes the same amount of power.

11. **(2-1)** Demonstrate by means of truth tables the validity of the following identities:

    a. DeMorgan's theorem for three variables: $\overline{XYZ} = \bar{X} + \bar{Y} + \bar{Z}$
    b. The second distributive law: $X + YZ = (X + Y)(X + Z)$
    c. $\bar{X}Y + \bar{Y}Z + X\bar{Z} = X\bar{Y} + Y\bar{Z} + \bar{X}Z$

12. **(2-2)** Prove the identity of each of the following Boolean equations, using algebraic manipulation:

    a. $\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + Y$
    b. $\bar{A}B + \bar{B}\bar{C} + AB + \bar{B}C = 1$
    c. $Y + \bar{X}Z + X\bar{Y} = X + Y + Z$
    d. $\bar{X}\bar{Y} + \bar{Y}Z + XZ + XY + Y\bar{Z} = \bar{X}\bar{Y} + XZ + Y\bar{Z}$

13. **(2-3)** Prove the identity of each of the following Boolean equations, using algebraic manipulation:

    a. $AB\bar{C} + B\bar{C}\bar{D} + BC + \bar{C}D = B + \bar{C}D$
    b. $WY + \bar{W}Y\bar{Z} + WXZ + \bar{W}X\bar{Y} = WY + \bar{W}XZ + \bar{X}Y\bar{Z} + X\bar{Y}Z$
    c. $A\bar{D} + \bar{A}B + \bar{C}D + \bar{B}C = (\bar{A} + \bar{B} + \bar{C} + \bar{D})(A + B + C + D)$

14. **(2-4)** Given that $A \cdot B = 0$ and $A + B = 1$, use algebraic manipulation to prove that

    $$(A + C) \cdot (\bar{A} + B) \cdot (B + C) = B \cdot C$$

15. Simplify the following Boolean expressions to expressions containing a minimum number of literals:

a. $\bar{A}\bar{C} + \bar{A}BC + \bar{B}C$
b. $(\overline{A + B + C}) \cdot \overline{ABC}$
c. $AB\bar{C} + AC$
d. $\bar{A}\bar{B}D + \bar{A}\bar{C}D + BD$
e. $(\bar{A} + B)(\bar{A} + \bar{C})(\overline{A\bar{B}C})$

16. **(2-7)** Reduce the following Boolean expressions to the indicated number of literals:

a. $\bar{X}\bar{Y} + XYZ + \bar{X}Y$ to three literals
b. $X + Y(Z + \overline{X + Z})$ to two literals
c. $\bar{W}X(\bar{Z} + \bar{Y}Z) + X(W + \bar{W}YZ)$ to one literal
d. $(AB + \bar{A}\bar{B})(\bar{C}\bar{D} + CD) + \overline{AC}$ to four literals

17. **(2-8)** Using DeMorgan's theorem, express the function

$$F = A\bar{B}C + \bar{A}\bar{C} + AB$$

a. with only OR and complement operations.
b. with only AND and complement operations.

18. **(2-9)** Find the complement of the following expressions:

a. $A\bar{B} + \bar{A}B$
b. $(\bar{V}W + X)Y + \bar{Z}$
c. $WX(\bar{Y}Z + Y\bar{Z}) + \bar{W}\bar{X}(\bar{Y} + Z)(Y + \bar{Z})$
d. $(A + \bar{B} + C)(\bar{A}\bar{B} + C)(A + \bar{B}\bar{C})$