The University of New South Wales

ELEC2141: Digital Circuit Design

Tutorial Week 3 – Combinational Logic Circuits Analysis

1. **(2-10)** Obtain the truth table of the following functions, and express each function in sum-of-minterms and product-of-maxterms form:

   a. $(XY + Z)(Y + XZ)$
   b. $(\bar{A} + B)(\bar{B} + C)$
   c. $WX\bar{Y} + WX\bar{Z} + WXZ + Y\bar{Z}$

2. **(2-11)** For the Boolean functions E and F, as given in the following truth table:

X	Y	Z	E	F
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

   a. List the minterms and maxterms of each function.
   b. List the minterms of $\bar{E}$ and $\bar{F}$.
   c. List the minterms of $E + F$ and $E \cdot F$.
   d. Express E and F in sum-of-minterms algebraic form.
   e. Simplify E and F to expressions with a minimum of literals.

3. **(2-12)** Convert the following expressions into sum-of-products and product-of-sums forms:

   a. $(AB + C)(B + \bar{C}D)$
   b. $\bar{X} + X(X + \bar{Y})(Y + \bar{Z})$
   c. $(A + B\bar{C} + CD)(\bar{B} + EF)$

4. **(2-13)** Draw the logic diagram for the following Boolean expressions. The diagram should correspond exactly to the equation. Assume that the complements of the inputs are not available.

   a. $\bar{A}\bar{B}\bar{C} + AB + AC$
   b. $X(Y\bar{Z} + \bar{Y}Z) + \bar{W}(\bar{Y} + \bar{X}Z)$
   c. $AC(\bar{B} + D) + \bar{A}C(\bar{B} + \bar{D}) + BC(\bar{A} + \bar{D})$

Where referenced, questions are taken from the textbook:

M. Mano, C. R. Kime and T. Martin, Logic and Computer Design Fundamentals, 5th Edition (Global Edition), Pearson, 2016

5. **(2-14)** Optimize the following Boolean functions by means of a three-variable Karnaugh map:

    a. $F(A,B,C) = \sum m(3,4,5,6,7)$
    b. $F(A,B,C) = \sum m(1,3,6,7)$
    c. $F(A,B,C) = \sum m(3,6,7)$
    d. $F(A,B,C) = \sum m(1,3,4,5,6,7)$

6. **(2-15)** Optimize the following Boolean expressions using a Karnaugh map:

    a. $\bar{X}\bar{Z} + Y\bar{Z} + XYZ$
    b. $\bar{A}B + \bar{B}C + \bar{A}\bar{B}\bar{C}$
    c. $\bar{A}\bar{B} + A\bar{C} + \bar{B}C + \bar{A}B\bar{C}$

7. **(2-16)** Optimize the following Boolean functions by means of a four-variable Karnaugh map:

    a. $F(A,B,C,D) = \sum m(3,4,5,6,7,12,13)$
    b. $F(A,B,C,D) = \sum m(4,6,7,12,13)$
    c. $F(A,B,C,D) = \sum m(0,1,4,5,6,7,12,13)$
    d. $F(A,B,C,D) = \sum m(1,3,4,5,6,7)$

8. **(2-17)** Optimize the following Boolean functions using a Karnaugh map:

    a. $F(A,B,C,D) = \sum m(0,1,4,5,8,9,12,13,15)$
    b. $F(A,B,C,D) = \sum m(4,6,7,12,13)$

9. **(2-18)** Find the minterms of the following expressions by first plotting each expression on a Karnaugh map:

    a. $XY + XZ + \bar{X}YZ$
    b. $XZ + \bar{W}X\bar{Y} + WXY + \bar{W}YZ + W\bar{Y}Z$
    c. $\bar{B}\bar{D} + ABD + \bar{A}BC$

10. **(2-19)** Find all the prime implicants for the following Boolean functions, and determine which are essential:

    a. $F(W,X,Y,Z) = \sum m(0,2,5,7,8,10,12,13,14,15)$
    b. $F(A,B,C,D) = \sum m(0,2,3,5,7,8,10,11,14,15)$
    c. $F(A,B,C,D) = \sum m(1,3,4,5,9,10,11,12,13,14,15)$

11. **(2-20)** Optimize the following Boolean functions by finding all prime implicants and essential prime implicants and applying the selection rule:

    a. $F(A,B,C,D) = \sum m(0,1,5,6,7,11,12,13,15)$
    b. $F(A,B,C,D) = \sum m(1,3,5,7,13,15)$
    c. $F(A,B,C,D) = \sum m(0,2,4,8,10,12,13,15)$

12. **(2-21)** Optimize the following Boolean functions in product-of-sums form:

    a. $F(A,B,C,D) = \sum m(1,2,3,5,6,7,13,14,15)$

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b. $F(A,B,C,D) = \sum m(0,1,2,3,6,8,9,10,11,14)$

13. **(2-22)** Optimize the following expressions in (1) sum-of-products and (2) product-of-sums forms:

a. $A\bar{C} + \bar{B}D + \bar{A}CD + ABCD$
b. $(\bar{A} + \bar{B} + \bar{D})(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{D})(B + \bar{C} + \bar{D})$
c. $(\bar{A} + \bar{B} + D)(\bar{A} + \bar{D})(A + B + \bar{D})(A + \bar{B} + C + D)$

14. **(2-23)** Optimize the following functions into (1) sum-of-products and (2) product-of-sums forms:

a. $F(A,B,C,D) = \prod M(2,5,6,7,8,9,10,11,14)$
b. $F(A,B,C,D) = \prod M(5,7,9,11)$

15. **(2-24)** Optimize the following Boolean functions F together with the don't-care conditions d:

a. $F(A,B,C,D) = \sum m(2,3,4,6,8,10,12,13,14) + d(0,1,15)$
b. $F(A,B,C,D) = \prod M(5,7,13,15)\,, d(A,B,C,D) = \sum m(9,11)$
c. $F(A,B,C,D) = \prod M(10,13,14,15)\,, d(A,B,C,D) = \sum m(0,3,4,7,12)$

16. **(2-25)** Optimize the following Boolean functions F together with the don't-care conditions d. Find all prime implicants and essential prime implicants, and apply the selection rule.

a. $F(A,B,C) = \sum m(3,5,6) + d(0,7)$
b. $F(W,X,Y,Z) = \sum m(0,2,4,5,8,14,15) + d(7,10,13)$
c. $F(A,B,C,D) = \sum m(4,6,7,8,12,15) + d(2,3,5,10,11,14)$

17. **(2-26)** Optimize the following Boolean functions F together with the don't-care conditions d in (1) sum-of-products and (2) product-of-sums form:

a. $F(A,B,C,D) = \sum m(0,1,4,5,6,11,15) + d(7,8,9,12)$
b. $F(A,B,C,D) = \prod M(2,3,7,8,10,12)\,, d(A,B,C,D) = \sum m(0,9,13,14)$

18. Use decomposition to find minimum gate-input cost, multiple-level implementations for the functions given, using AND and OR gates and inverters.

a. $F(A,B,C,D) = A\bar{B}C + \bar{A}BC + A\bar{B}D + \bar{A}BD$
b. $F(W,X,Y,Z) = WY + XY + \bar{W}XZ + W\bar{X}Z$