ELEC2141

Final Exam, Session 1, 2011

Question 1

i)

$$F = \cancel{AB} + AC + \cancel{\bar{A}B} + \underline{\underline{B}} + \cancel{BC} + \bar{A}D + \cancel{BD} + CD$$
$$+ A\cancel{B}\cancel{\bar{C}} + A\cancel{B}D + A\bar{B}D + A\bar{C}D + AD$$

$$= B + AC + D(\underline{\underline{\bar{A}}} + C + A\bar{B} + A\bar{C} + \underline{\underline{A}})$$

$$= AC + B + D.$$

(2) a)

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

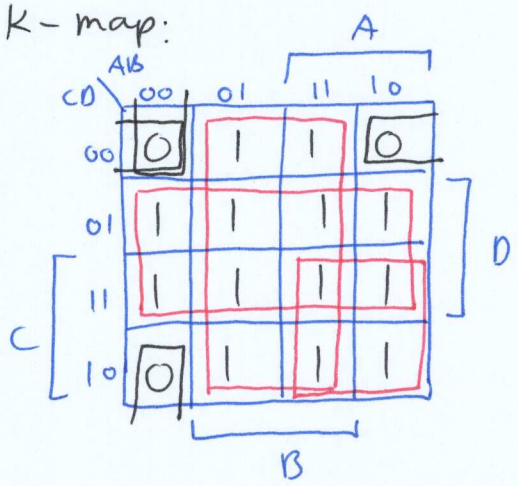
(1) b)

$$F = \Sigma m(1, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15)$$

(1) c)

$$F = \Pi M(0, 2, 8)$$

K-map:



(1) d) B, D, AC

(2) e) B, D, AC

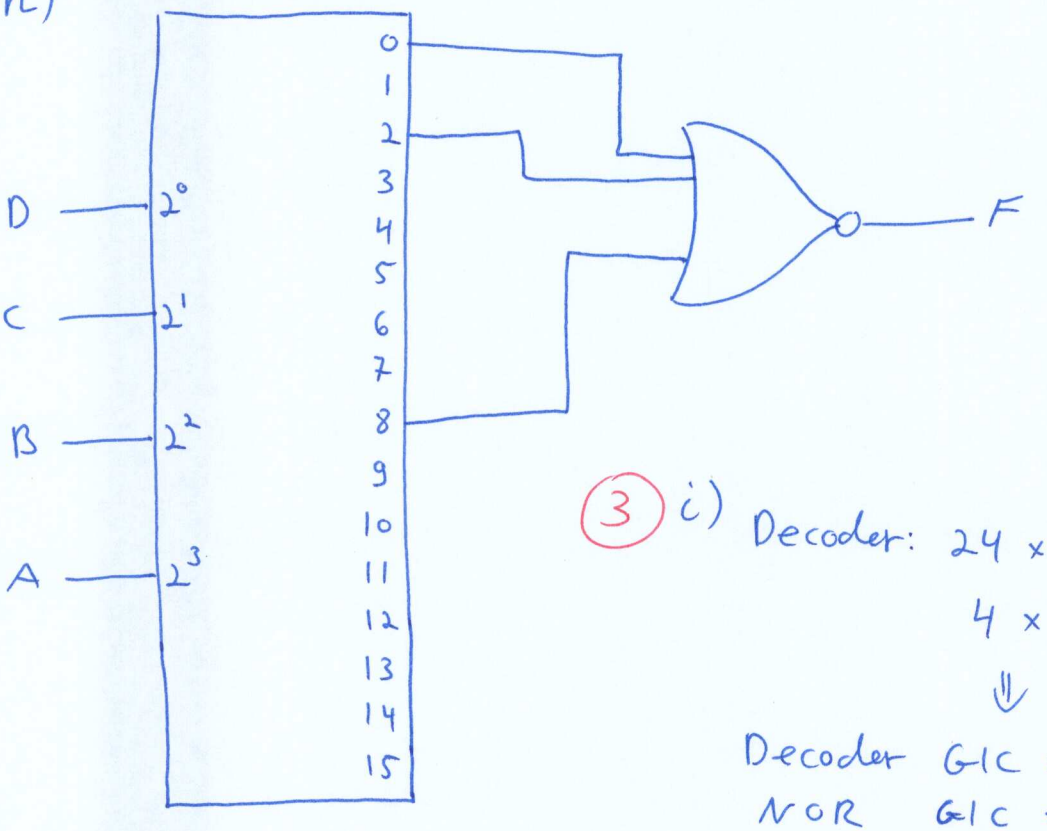
(1) f) $F = AC + B + D$

(2) g) $\bar{F} = \bar{A}\bar{B}\bar{D} + \bar{B}\bar{C}\bar{D}$

$\Downarrow$

$F = (A + B + D)(B + C + D)$

(3) h)



(3) i) Decoder: 24 x 2-input AND
4 x inverter

$\Downarrow$

Decoder GIC = 52
NOR GIC = 3
Total GIC = 55

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(4) ii) $BC + \bar{A}\bar{B} + \bar{A}\bar{C} = ABC + \bar{A}BC + \bar{A}\bar{B} + \bar{A}\bar{C}$

$= ABC + \bar{A}(BC + \bar{B} + \bar{C})$

$= ABC + \bar{A}(\bar{B} + C + \bar{C})$

$= ABC + \bar{A}$ ✓

Question 2

i)

(2) a) min = 1 max = 2^L

(2) b) min = 2^M max = 2^M

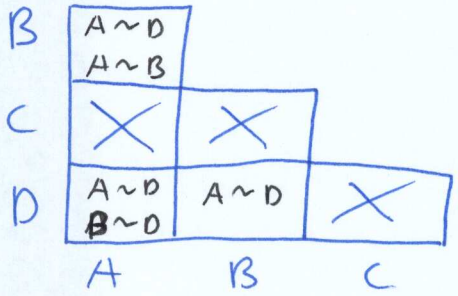
(2) c) min = 0 max = $2^L \times 2^M = 2^{L+M}$

(2) d) min = 1

$$\max = \begin{cases} 2^N & \text{if } N \leq (L+M) \\ 2^{L+M} & \text{if } N > (L+M) \end{cases}$$

ii) (Easier to do (b) and then (a)):

(4) b)



$\Rightarrow$ A ~ B ~ D (state A)

C (state C)

$\Downarrow$

Current	next, output XY=00	01	10	11
A	A,0	C,0	A,0	A,1
C	A,1	A,0	C,0	A,0

(4) a) split state A to: A/0 and B/1.

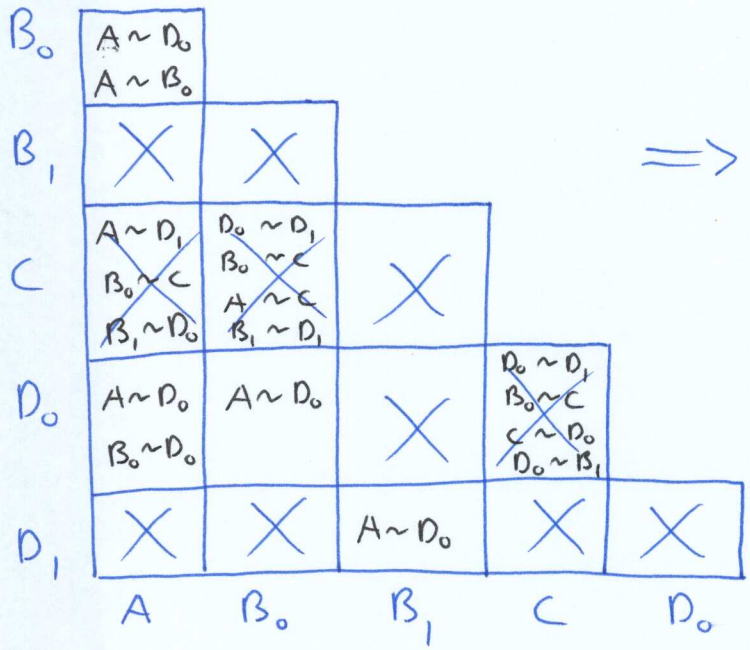
Current	next XY=00	01	10	11	output z
A	A	C	A	B	0
B	A	C	A	B	1
C	B	A	C	A	0

Alternatively, D_0 (a) first and then (b):

(4) a) split B to $B_0/0$ and $B_1/1$
split D to $D_0/0$ and $D_1/1$

current	$XY=00$	next 01	10	11	output Z
A	A	C	B_0	B_1	0
B_0	D_0	C	A	B_1	0
B_1	D_0	C	A	B_1	1
C	D_1	B_0	C	D_0	0
D_0	D_0	C	D_0	B_1	0
D_1	D_0	C	D_0	B_1	1

(4) b)



$\Rightarrow$

$A \sim B_0 \sim D_0$ [A]

$B_1 \sim D_1$ [B]

C [C]

current	$XY=00$	next 01	10	11	output Z
A	A	C	A	B	0
B	A	C	A	B	1
C	B	A	C	A	0

(1) c) Assign:

$A \sim 00 \quad B \sim 01 \quad C \sim 10$

(3) d)

current $q_1 q_0$	next $(Q_1 Q_0)$ $XY=00$	01	10	11	output z
0 0	0 0	1 0	0 0	0 1	0
0 1	0 0	1 0	0 0	0 1	1
1 0	0 1	0 0	1 0	0 0	0
1 1	x x	x x	x x	x x	x

D_1:

$q_1 q_0$ \ XY	00	01	11	10
00	0	1	0	0
01	0	1	0	0
11	X	X	X	X
10	0	0	0	1

q_1: 11, 10 ; q_0: 01, 11 ; Y: 01, 11 ; X: 11, 10

D_0:

$q_1 q_0$ \ XY	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	X	X	X	X
10	1	0	0	0

q_1: 11, 10 ; q_0: 00, 01 ; Y: 01, 11 ; X: 11, 10

Flip-flop equations:

$$D_1 = \bar{X} Y \bar{q}_1 + X \bar{Y} q_1$$

$$D_0 = \bar{X} \bar{Y} q_1 + X Y \bar{q}_1$$

output equation:

$$z = q_0$$

Question 3

(3) i)

$$J_2 = X \oplus q_1 \qquad J_1 = X(q_2 + q_0) \qquad J_0 = 1$$

$$K_2 = X \oplus q_1 \qquad K_1 = X\bar{q}_0 \qquad K_0 = 1$$

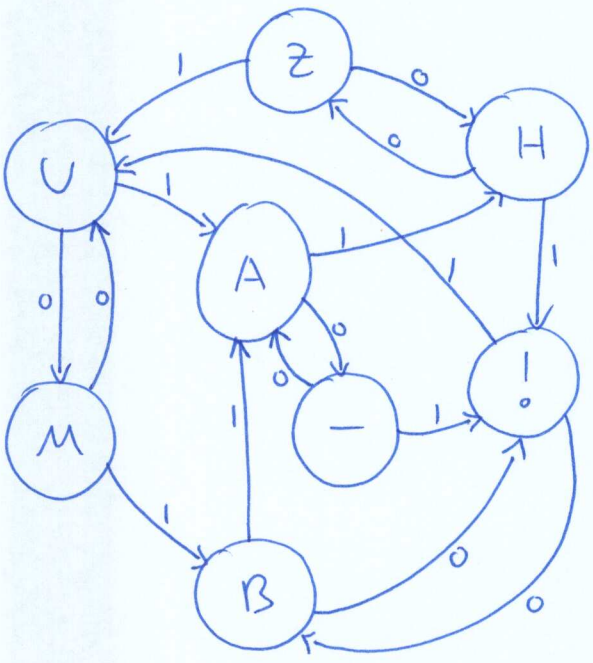
ii)

(10)

current		x=0			x=1			next x=0		next x=1	
	$q_2 q_1 q_0$	$J_2 K_2$	$J_1 K_1$	$J_0 K_0$	$J_2 K_2$	$J_1 K_1$	$J_0 K_0$	$Q_2 Q_1 Q_0$		$Q_2 Q_1 Q_0$	
Z	0 0 0	0 0	0 0	1 1	1 1	0 1	1 1	H	0 0 1	U	1 0 1
H	0 0 1	0 0	0 0	1 1	1 1	1 0	1 1	Z	0 0 0	!	1 1 0
A	0 1 0	1 1	0 0	1 1	0 0	0 1	1 1	-	1 1 1	H	0 0 1
B	0 1 1	1 1	0 0	1 1	0 0	1 0	1 1	!	1 1 0	A	0 1 0
M	1 0 0	0 0	0 0	1 1	1 1	1 1	1 1	U	1 0 1	B	0 1 1
U	1 0 1	0 0	0 0	1 1	1 1	1 0	1 1	M	1 0 0	A	0 1 0
!	1 1 0	1 1	0 0	1 1	0 0	1 1	1 1	B	0 1 1	U	1 0 1
-	1 1 1	1 1	0 0	1 1	0 0	1 0	1 1	A	0 1 0	!	1 1 0

iii)

(4)



(3) iv)

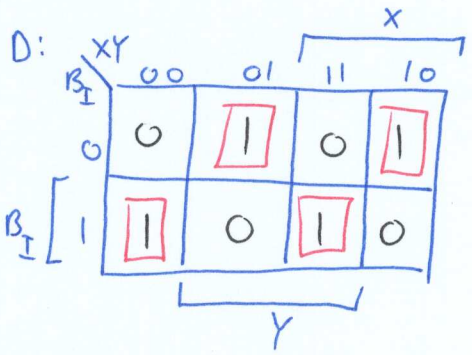
State	Z	U	M	B	A	-	A	H	!
X	1	0	1	1	0	0	1	1	

Question 4

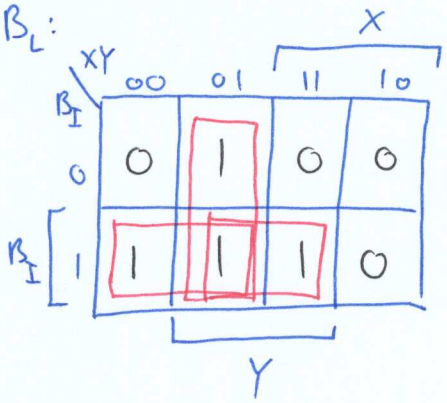
i) (8) a) Truth table for the subtractor:

	B_I	X	Y	D	B_L
	0	0	0	0	0
*	0	0	1	1	1
	0	1	0	1	0
	0	1	1	0	0
	1	0	0	1	1
	1	0	1	0	1
*	1	1	0	0	0
	1	1	1	1	1

D:



B_L:



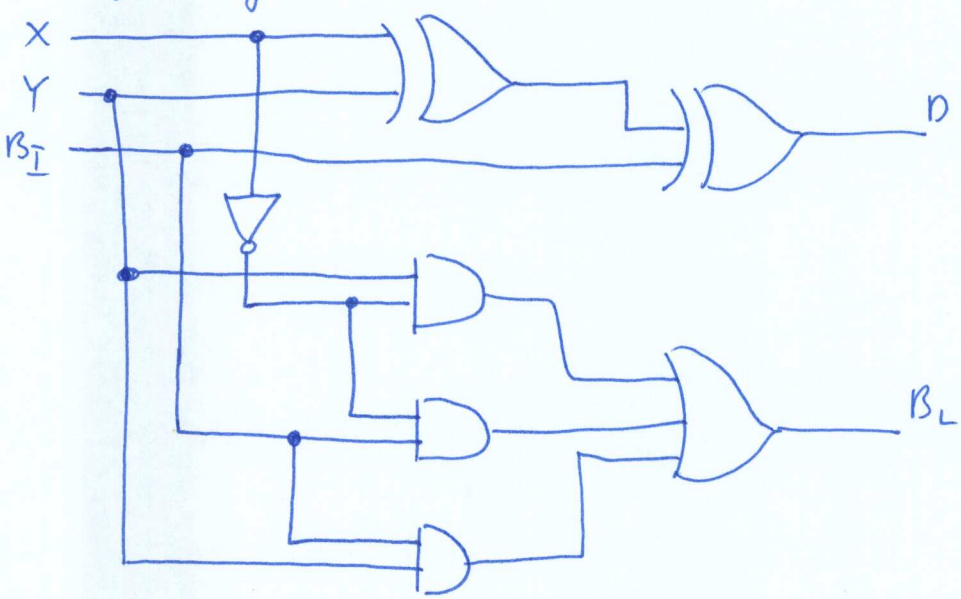
From k-maps:

$$D = \bar{X}\bar{Y}B_I + \bar{X}Y\bar{B}_I + XYB_I + X\bar{Y}\bar{B}_I$$

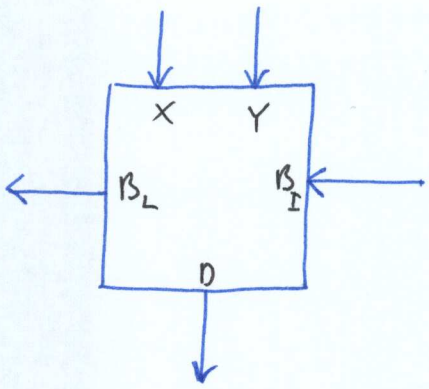
$$= X \oplus Y \oplus B_I$$

$$B_L = \bar{X}Y + \bar{X}B_I + YB_I$$

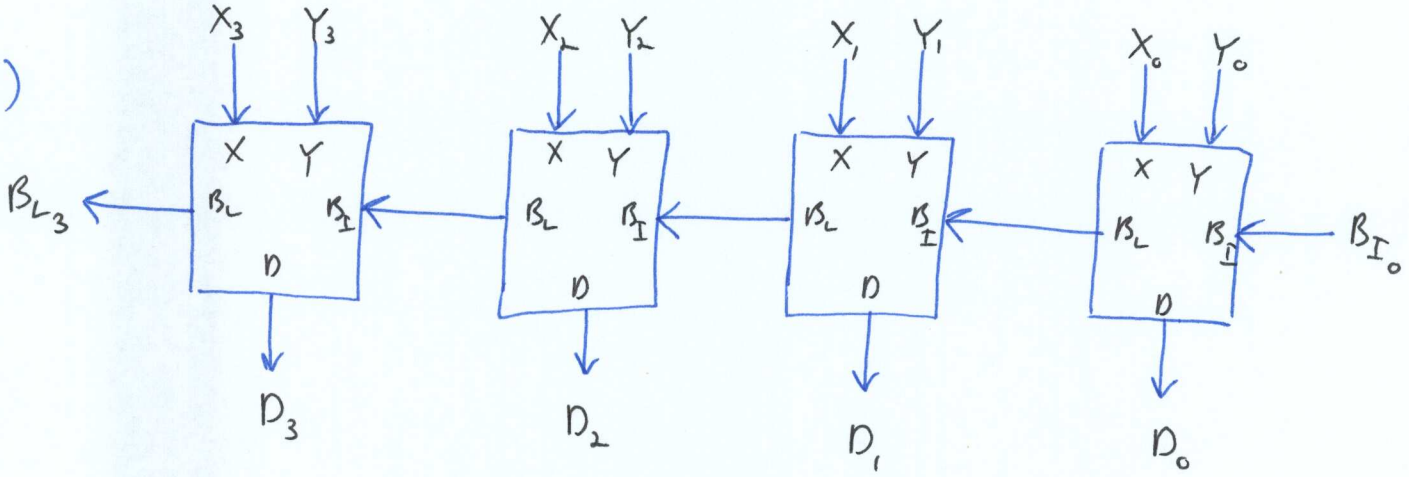
Logic Diagram:



(2) b)



(3) c)



(3) d) Underflow will occur when subtracting a positive number from a negative one and the result is positive or when subtracting a negative number from a positive one and the result is negative. Looking at the MSB, this is one of two cases:

$$\begin{array}{r} X_3 \\ -\,Y_3 \\ \hline D_3 \end{array} \Rightarrow \begin{array}{r} 1 \\ -\,0 \\ \hline 0 \end{array} \quad \text{or} \quad \begin{array}{r} 0 \\ -\,1 \\ \hline 1 \end{array}$$

The two cases are marked with a "*" on the subtractor truth table, and can be flagged by the function:

$$UF = B_{I_3} \oplus B_{L_3}$$

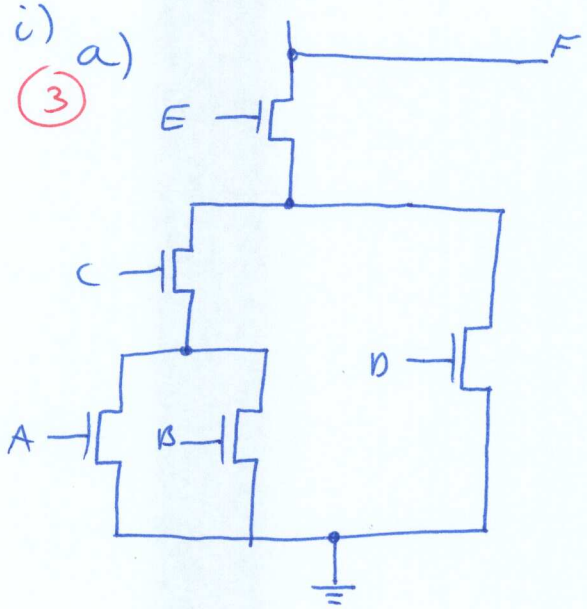
ii) (4)

A datapath can be modeled as a state machine as it has memory elements to store the current state (the registers). It has some number of inputs (such as the control inputs, data input, constant input, etc.) and produces outputs that depend on the inputs and the current machine state (Address output, data output, NZCV flags).

The registers are clocked elements such that a clock trigger will cause a state transition.

Question 5

i) a) (3)



b) (3)

$$F = \overline{\left[(A+B)C + D\right] \cdot E}$$

$$= (\bar{A}\,\bar{B} + \bar{C})\,\bar{D} + \bar{E}$$

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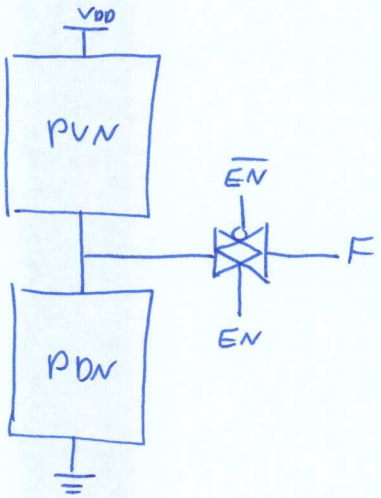
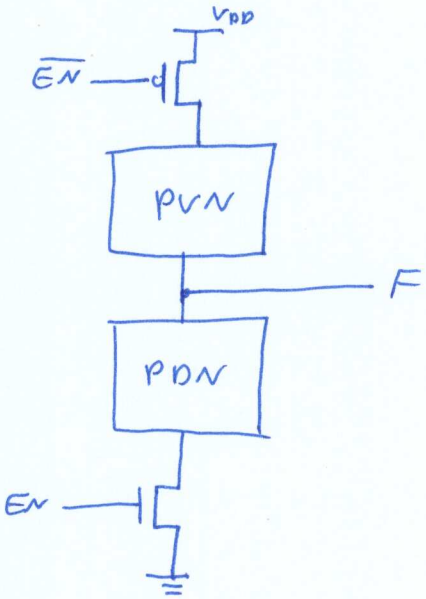
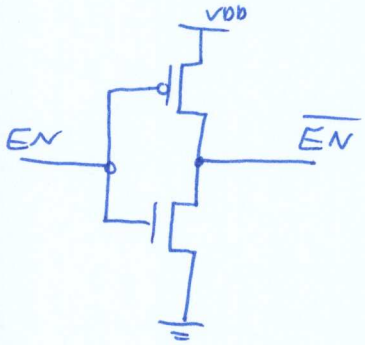
(4) c) One of two possibilities:



Or



where $\overline{EN}$ is derived from:

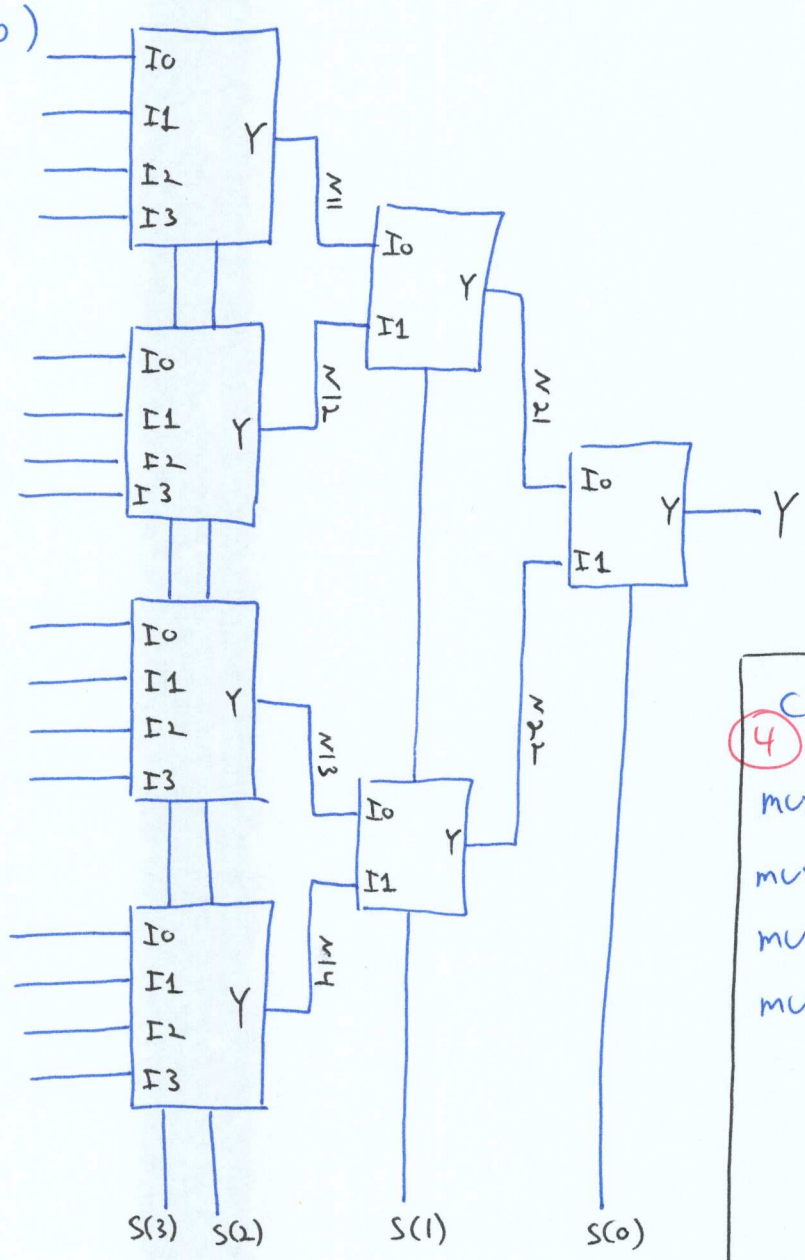


ii)

(3) a) Both multiplexer implementations are missing the 'I' inputs in the always sensitivity list. Currently, if a certain input is selected, the output will take the input's initial value but will not track it in case the input chages. The bug can be fixed by adding the 'I' inputs to the sensitivity list. Example for mux2:

always @(s or I0 or I1) begin

(3) b)



(4) c)

```
mux4 mx11 (S[3:2], I0, I4, I8, I12, n11);
mux4 mx12 (S[3:2], I2, I6, I10, I14, n12);
mux4 mx13 (S[3:2], I1, I5, I9, I13, n13);
mux4 mx14 (S[3:2], I3, I7, I11, I15, n14);
```