

1. a)

X	Y	Z	XY	XY+Z	XZ	Y+XZ	(XY+Z)(Y+XZ)
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	0	0	0	1	0
0	1	1	0	1	0	1	1
1	0	0	0	0	0	0	0
1	0	1	0	1	1	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

$$SOM = \bar{X}YZ + X\bar{Y}Z + X\bar{Y}\bar{Z} + XYZ$$

$$POM = (X+Y+Z)(X+\bar{Y}+\bar{Z})(X+\bar{Y}+Z)(\bar{X}+Y+Z)$$

b)

A	B	C	( $\bar{A}+B$ )	( $\bar{B}+C$ )	( $\bar{A}+B$ )( $\bar{B}+C$ )
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	1	0
1	0	1	0	1	0
1	1	0	1	0	0
1	1	1	1	1	1

$$SOM = \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + \bar{A}BC + ABC$$

$$POM = (A+\bar{B}+C)(\bar{A}+B+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$$

c)

W	X	Y	Z	WX $\bar{Y}$	WX $\bar{Z}$	WXZ	$\bar{Y}\bar{Z}$	WX $\bar{Y}$ +WX $\bar{Z}$ +WXZ+ $\bar{Y}\bar{Z}$
0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	1	1
0	0	1	1	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0
0	1	1	0	0	0	0	1	1
0	1	1	1	0	0	0	0	0
1	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0
1	0	1	0	0	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	1	1	0	0	1
1	1	0	1	1	0	1	0	1
1	1	1	0	0	1	0	1	1
1	1	1	1	1	0	1	0	1

$$SOM = \bar{W}\bar{X}Y\bar{Z} + \bar{W}XY\bar{Z} + W\bar{X}Y\bar{Z} + WX\bar{Y}\bar{Z} + WX\bar{Y}Z + WX\bar{Y}\bar{Z} + WX\bar{Y}Z$$

$$POM = (W+X+Y+Z)(W+X+Y+\bar{Z})(W+\bar{X}+Y+Z)(W+\bar{X}+Y+\bar{Z}) \cdot (W+\bar{X}+\bar{Y}+\bar{Z})(\bar{W}+X+Y+Z)(\bar{W}+X+Y+Z)(\bar{W}+X+\bar{Y}+\bar{Z})$$

$$a) \quad E = \sum m(1, 3, 6, 7) = \prod M(0, 2, 4, 5) \\ F = \sum m(3, 4, 5, 6, 7) = \prod M(0, 1, 2)$$

$$b) \quad \bar{E} = \sum m(0, 2, 4, 5) \\ F = \sum m(0, 1, 2)$$

$$c) \quad E + F = \sum m(1, 3, 4, 5, 6, 7) \\ E \cdot F = \sum m(3, 6, 7)$$

$$d) \quad E = \bar{x}\bar{y}z + \bar{x}yz + x\bar{y}\bar{z} + xy\bar{z} \\ F = xyz$$

$$a) (AB + C)(B + \bar{C}D)$$

$$= \cancel{AB} + \cancel{AB} \bar{C} D + BC + \cancel{C \bar{C}} D$$

$$= AB(1 + \bar{C}D) + BC$$

$$= AB + BC \quad \text{SOP}$$

$$= B(A + C) \quad \text{POS}$$

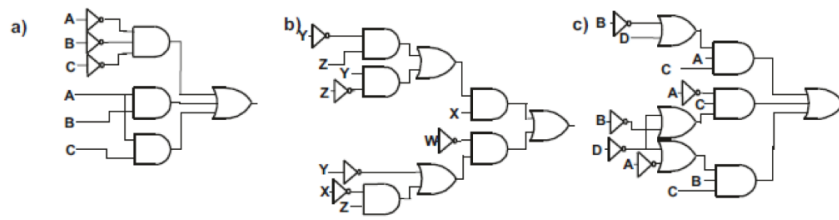
$$b) \bar{x} + x(x + \bar{y})(y + \bar{z})$$

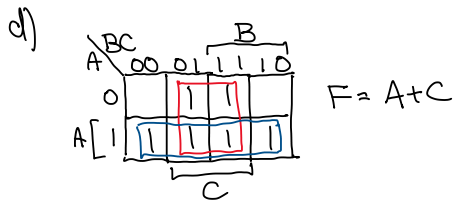
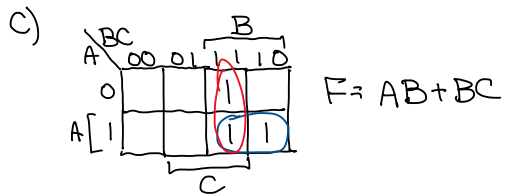
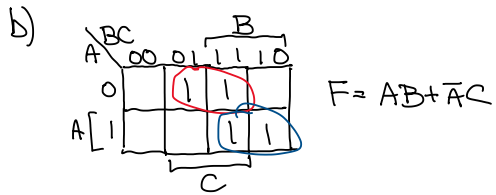
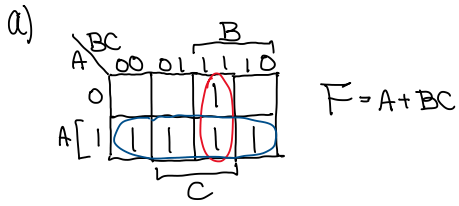
$$= \bar{x} + \cancel{x} x y + \cancel{x} x \bar{z} + x \cancel{\bar{y}} y + x \bar{y} \bar{z}$$

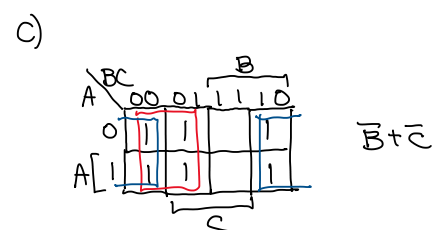
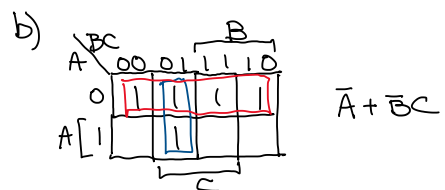
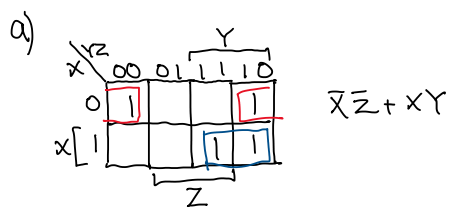
$$= \bar{x} + xy + x\bar{z} + x\bar{y}\bar{z}$$

$$= \bar{x} + xy + x\bar{z}(1 + \bar{y})$$

$$= \bar{x} + xy + x\bar{z}$$







		C			
		00	01	11	10
A	00	$m_0$	$m_1$	$m_3$	$m_2$
	01	$m_4$	$m_5$	$m_7$	$m_6$
	11	$m_{12}$	$m_{13}$	$m_9$	$m_{14}$
	10	$m_8$	$m_9$	$m_{11}$	$m_{10}$

a)

		C			
		00	01	11	10
AB	00			1	
	01	1	1	1	1
	11	1	1		
	10				

$$\bar{A}B + B\bar{C} + \bar{A}CD$$

b)

		C			
		00	01	11	10
AB	00				
	01	1		1	1
	11	1	1		
	10				

$$A\bar{B}\bar{C} + \bar{A}BC + B\bar{C}\bar{D}$$

c)

		C			
		00	01	11	10
AB	00	1	1		
	01	1	1	1	1
	11	1	1		
	10				

$$\bar{A}B + \bar{A}\bar{C} + B\bar{C}$$

d)

		C			
		00	01	11	10
AB	00		1	1	
	01	1	1	1	1
	11				
	10				

$$\bar{A}B + \bar{A}D$$

		CD			
		00	01	11	10
AB	00	$m_0$	$m_1$	$m_3$	$m_2$
	01	$m_4$	$m_5$	$m_7$	$m_6$
	11	$m_{12}$	$m_{13}$	$m_6$	$m_{14}$
	10	$m_8$	$m_9$	$m_{11}$	$m_{10}$

$A = \begin{bmatrix} 11 \\ 10 \end{bmatrix}$  (rows 11, 10)  
 $B = \begin{bmatrix} 01 \\ 11 \\ 10 \end{bmatrix}$  (rows 01, 11, 10)  
 $D = \begin{bmatrix} 11 \\ 10 \end{bmatrix}$  (columns 11, 10)

a)

		CD			
		00	01	11	10
AB	00				
	01				
	11				
	10				

$\bar{C} + ABD$

b)

		CD			
		00	01	11	10
AB	00				
	01				
	11				
	10				

$B\bar{C}\bar{D} + AB\bar{C} + \bar{A}BC$



a)

yz		Y			
x		00	01	11	10
0				1	
1			1	1	1

Z

$$\Sigma m(3, 5, 6, 7)$$

b)

wx		W			
yz		00	01	11	10
00			1		
01			1	1	1
11		1	1	1	
10				1	

X

Z

$$\Sigma m(3, 4, 5, 7, 9, 13, 14, 15)$$

c)

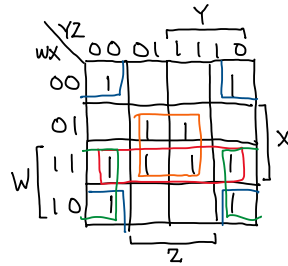
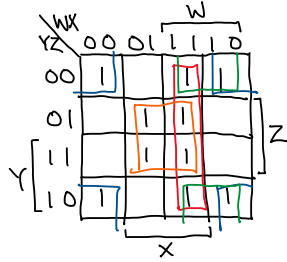
AB		A			
CD		00	01	11	10
00		1			1
01				1	
11			1	1	
10		1	1		1

B

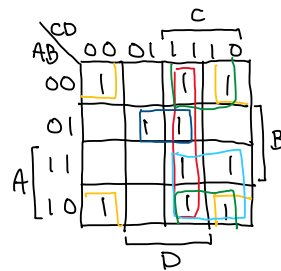
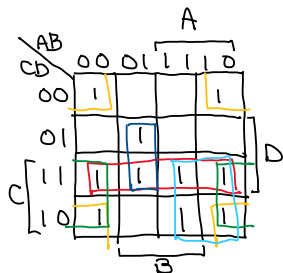
D

$$\Sigma m(0, 2, 6, 7, 8, 10, 13, 15)$$

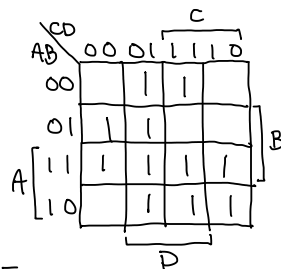
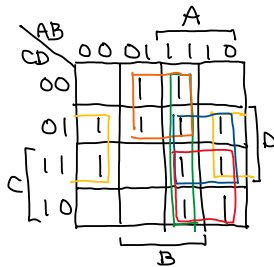
a)

Prime:  $\bar{x}z, xz, w\bar{z}, wx$ Essential:  $\bar{x}z, xz$ 

b)

Prime:  $\bar{B}D, CD, AC, \bar{B}C, \bar{A}BD$ Essential:  $\bar{B}D, AC, \bar{A}BD$ 

c)

Prime:  $AB, \bar{C}D, \bar{B}D, AD, AC, \bar{B}C$ Essential:  $\bar{B}D, \bar{B}C, AC$

		C			
A \ B	B	00	01	11	10
	00	$m_0$	$m_1$	$m_3$	$m_2$
A	01	$m_4$	$m_5$	$m_7$	$m_6$
	11	$m_{12}$	$m_{13}$	$m_{15}$	$m_{14}$
	10	$m_8$	$m_9$	$m_{11}$	$m_{10}$

a)

		C			
A \ B	B	00	01	11	10
	00	1	1		
A	01		1	1	1
	11	1	1	1	
	10			1	

Prime:

$$\bar{A}\bar{B}\bar{C}, BD, AB\bar{C}, \bar{A}BC, ACD, \bar{A}ED$$

Essential:

$$\bar{A}\bar{B}\bar{C}, BD, AB\bar{C}, \bar{A}BC, ACD$$

$$F = \bar{A}\bar{B}\bar{C} + BD + AB\bar{C} + \bar{A}BC + ACD$$

b)

		C			
A \ B	B	00	01	11	10
	00		1	1	
A	01		1	1	
	11		1	1	
	10				

Prime:

$$\bar{A}D, BD$$

Essential:

$$\bar{A}D, BD$$

$$F = \bar{A}D + BD$$

a)

		CD			
		00	01	11	10
AB	00	0	1	1	1
	01	0	1	1	1
	11	0	1	1	1
	10	0	0	0	0

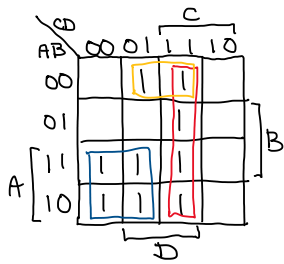
$\bar{F} = \bar{C}\bar{D} + AB$   
 $F = (C+D)(\bar{A}+B)$

b)

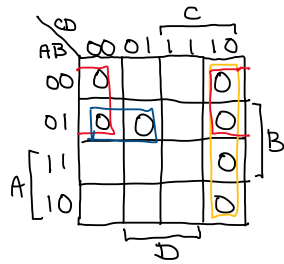
		CD			
		00	01	11	10
AB	00	1	1	1	1
	01	0	0	0	1
	11	0	0	0	1
	10	1	1	1	1

$\bar{F} = B\bar{C} + BD$   
 $F = (\bar{B}+C)(\bar{B}+\bar{D})$

a)



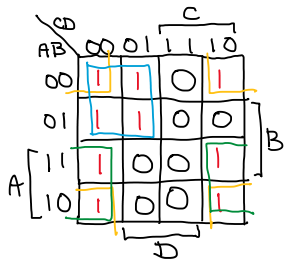
$$\text{SOP: } CD + \bar{B}D + A\bar{C}$$



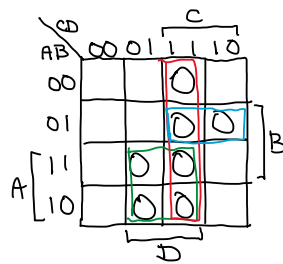
$$\text{POS: } (A + \bar{B} + C)(A + \bar{D})(C + \bar{D})$$

$$\overline{A\bar{B}\bar{C} + A\bar{D} + C\bar{D}}$$

b) Fill in the 0s first from the POS expression, then fill the 1's on the K-map. Group the 1's to get SOP expression

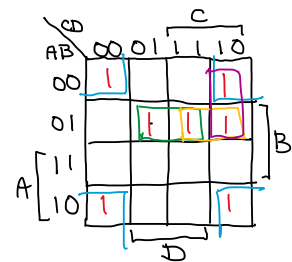


$$\text{SOP: } A\bar{D} + \bar{B}\bar{D} + A\bar{C}$$



$$\text{POS: } (\bar{A} + \bar{D})(\bar{C} + \bar{D})(A + \bar{B} + \bar{C})$$

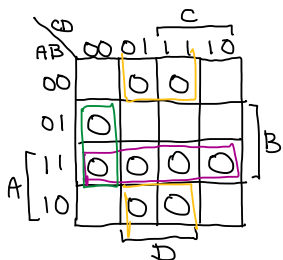
c)



$$\text{SOP: } \bar{B}\bar{D} + \bar{A}BD + \bar{A}BC$$

or

$$\bar{B}\bar{D} + \bar{A}BD + A\bar{C}\bar{D}$$



$$\text{POS: } (\bar{A} + \bar{B})(B + \bar{D})(\bar{B} + C + D)$$

		C			
		00	01	11	10
A \ B	00	$m_0$	$m_1$	$m_3$	$m_2$
	01	$m_4$	$m_5$	$m_7$	$m_6$
	11	$m_{12}$	$m_{13}$	$m_8$	$m_{14}$
	10	$m_9$	$m_{11}$	$m_{10}$	

a)

		C			
		00	01	11	10
A \ B	00	1	1	1	0
	01	1	0	0	0
	11	1	1	1	0
	10	0	0	0	0

SOP:  $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}D + B\bar{C}\bar{D} + A\bar{B}D$

		C			
		00	01	11	10
A \ B	00				0
	01		0	0	0
	11				0
	10	0	0	0	0

POS:  $(A+B)(A+\bar{B}+\bar{D})(\bar{C}+D)$

b)

		C			
		00	01	11	10
A \ B	00	1	1	1	1
	01	1	0	0	1
	11	1	1	1	1
	10	1	0	0	1

SOP:  $\bar{D} + \bar{A}\bar{B} + AB$

		C			
		00	01	11	10
A \ B	00				
	01		0	0	
	11				
	10		0	0	

POS:  $(A+B+\bar{D})(\bar{A}+\bar{B}+\bar{D})$

		C			
AB \ CD		00	01	11	10
A	00	$m_0$	$m_1$	$m_3$	$m_2$
	01	$m_4$	$m_5$	$m_7$	$m_6$
	11	$m_{12}$	$m_{13}$	$m_{10}$	$m_{11}$
	10	$m_8$	$m_9$	$m_{14}$	$m_{15}$
		D			

a)

		C			
AB \ CD		00	01	11	10
A	00	X	X	1	1
	01	1			1
	11	1	1	X	1
	10	1			1

$F = \bar{B} + \bar{A}B + AB$

b)

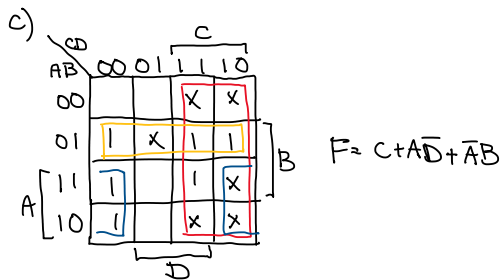
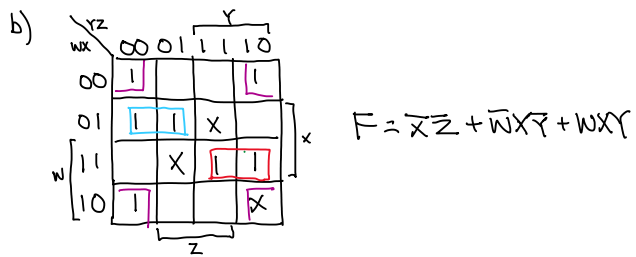
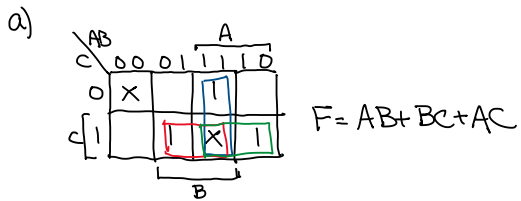
		C			
		00	01	11	10
A	AB	00	01	11	10
	00	1	1	1	1
	01	1	0	0	1
	11	1	0	0	1
B	10	1	X	X	1

$F = \bar{D} + \bar{B}$

c)

		C			
		00	01	11	10
A	00	X	1	X	1
	01	X	1	X	1
	11	X			
	10	1	1	1	

$F = A + \bar{C}\bar{D} + \bar{B}D$





a)

		cd		c			
		00	01	11	11	10	
AB	00	1	1	0	0		
	01	1	1	x	1		
	11	x	0	1	0		
	10	x	x	1	0		

$\overline{A}\overline{C} + \overline{A}B + ACD$   
 $(\overline{A}+C)(A+B+\overline{C})(\overline{A}+D)$

$$\text{SOP: } \overline{A}\overline{C} + \overline{A}B + ACD$$

$$\text{POS: } (\overline{A}+C)(A+B+\overline{C})(\overline{A}+D)$$

b)

		cd		c			
		00	01	11	11	10	
AB	00	x	1	0	0		
	01	1	1	0	1		
	11	0	x	1	x		
	10	0	x	1	0		

$\overline{A}\overline{C} + AD + \overline{A}B\overline{D}$   
 $\overline{A}\overline{C} + AD + BC\overline{D}$   
 $(B+D)(A+\overline{C}+\overline{D})(\overline{A}+D)$

$$\text{SOP: } \overline{A}\overline{C} + AD + \overline{A}B\overline{D}$$

$$\overline{A}\overline{C} + AD + BC\overline{D}$$

$$\text{POS: } (B+D)(A+\overline{C}+\overline{D})(\overline{A}+D)$$

a)  $F = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}D + \bar{A}BD \rightarrow$  literals = 12  
 terms = 4  
 complement = 2  
GIC = 18

Define  
 $X_1 = \bar{A}\bar{B}$  GIC = 3  
 $X_2 = \bar{A}B$  GIC = 3

$$\Rightarrow F = X_1C + X_2C + X_1D + X_2D$$

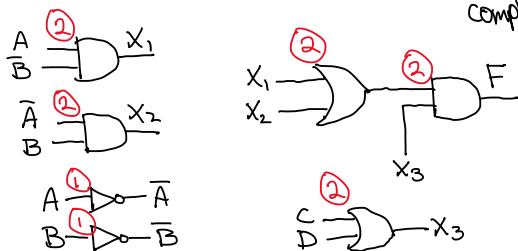
$$= (X_1 + X_2)(C + D)$$

Define  $X_3 = C + D$  GIC = 2

$$\Rightarrow F = (X_1 + X_2)X_3 \rightarrow$$

literals = 3  
 terms = 1  
 complement = 0

Total GIC = 3 + 3 + 2  
 3 + 1  
= 12



b)  $F = WY + XY + \bar{W}XZ + W\bar{X}Z \rightarrow$  literals = 10  
 terms = 4  
 complements = 2  
GIC = 16

$$= Y(W + X) + Z(\bar{W}X + W\bar{X})$$

$$= Y(W + X) + Z(W + X)(\bar{W} + \bar{X})$$

Define  
 $T = W + X$  GIC = 2  
 $S = \bar{W} + \bar{X}$  GIC = 4

$$F = YT + ZST$$

$\rightarrow$  literals = 5  
 terms = 2  
 complement = 0

Total GIC = 5 + 2 + 2 + 4  
= 13

