ELEC 2141
Final Exam, Session 1, 2009.

Question 1

i)

(3) a) $F = (A+B)(A+C+D) + (\bar{A}+B)(B+C+\bar{D})A$

$= A + \cancel{AC} + \cancel{AD} + \cancel{AB} + BC + BD + \cancel{AB} + \cancel{ABC} + \cancel{AB\bar{D}}$

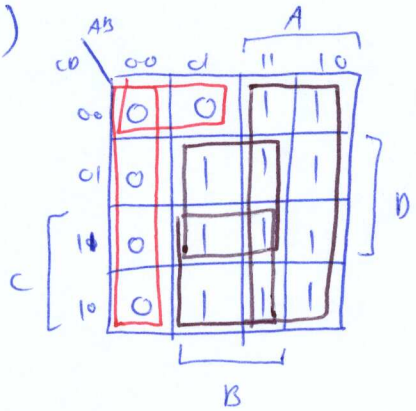
$= A + B(C+D)$

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

(1) b) $F = \Sigma m(5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$

(1) c) $F = \Pi M(0, 1, 2, 3, 4)$

(1) d)



$F = A + BC + BD$

(2) e) $\bar{F} = \bar{A}\bar{B} + \bar{A}\bar{C}\bar{D}$

$F = \overline{\bar{A}\bar{B} + \bar{A}\bar{C}\bar{D}}$

$= (A+B)(A+C+D)$

(2) f) GIC = 19.

ii)

(3) a) $X + Y = X \oplus Y + XY$

$$\Downarrow$$

$$X \oplus Y + XY = X\bar{Y} + \bar{X}Y + XY$$

$$= X\bar{Y} + XY + XY + \bar{X}Y$$

$$= X(\bar{Y} + Y) + Y(X + \bar{X})$$

$$= X + Y.$$

(4) b)

$$G = A\bar{B} + AB\bar{C} + \bar{A}B$$

$$= \underbrace{A \oplus B}_{X} + \underbrace{AB\bar{C}}_{Y}$$

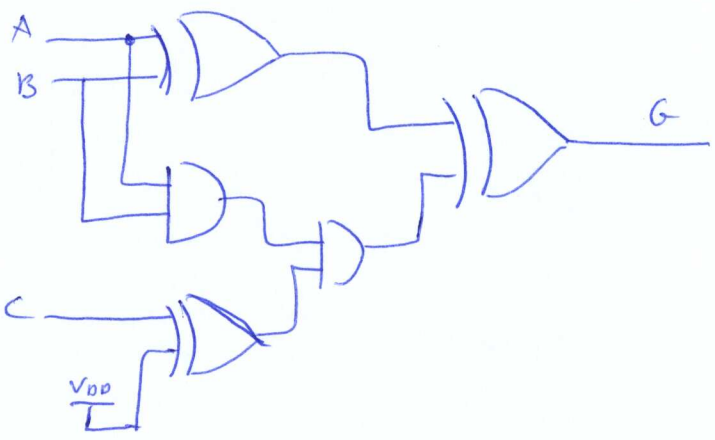
$$= A \oplus B \oplus AB\bar{C} + (A \oplus B)(AB\bar{C})$$

$$= A \oplus B \oplus AB\bar{C} + (A\bar{B} + \bar{A}B)(AB\bar{C})$$

$$= A \oplus B \oplus AB\bar{C}$$

$$= A \oplus B \oplus AB(C \oplus 1)$$

(3) c)

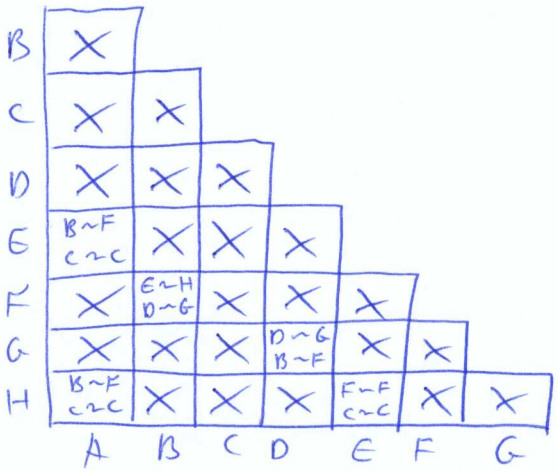


Question 2

(3) i)

state	next state, output x=0	x=1
A	B, 1	C, 1
B	E, 0	G, 0
C	C, 1	A, 0
D	D, 0	B, 1
E	F, 1	C, 1
F	H, 0	D, 0
G	G, 0	F, 1
H	F, 1	C, 1

(5) ii)



A ~ E ~ H

B ~ F

D ~ G

state	next, output x=0	x=1
A	B, 1	C, 1
B	A, 0	D, 0
C	C, 1	A, 0
D	D, 0	B, 1

(5) iii)

state	next state x=0	x=1	output
A	B	C	0
B	A	D	1
C	C	A	1
D	D	B	0

(1)

iv) One possibility:

	0	1
0	A	C
1	B	D

A ~ 00

B ~ 01

C ~ 10

D ~ 11

(3)

v)

$q_1\ q_0$ state	$Q_1 Q_0$ next state $x=0$	$Q_1 Q_0$ next state $x=1$	z output
0 0	0 1	1 0	0
0 1	0 0	1 1	1
1 0	1 0	0 0	1
1 1	1 1	0 1	0

D_1:

x \ $q_1 q_0$	00	01	11	10
0	0	0	1	1
1	1	1	0	0

D_0:

x \ $q_1 q_0$	00	01	11	10
0	1	0	1	0
1	0	1	1	0

z:

q_0 \ q_1	0	1
0	0	1
1	1	0

$D_1 = \bar{x} q_1 + x \bar{q}_1$

$= x \oplus q_1$

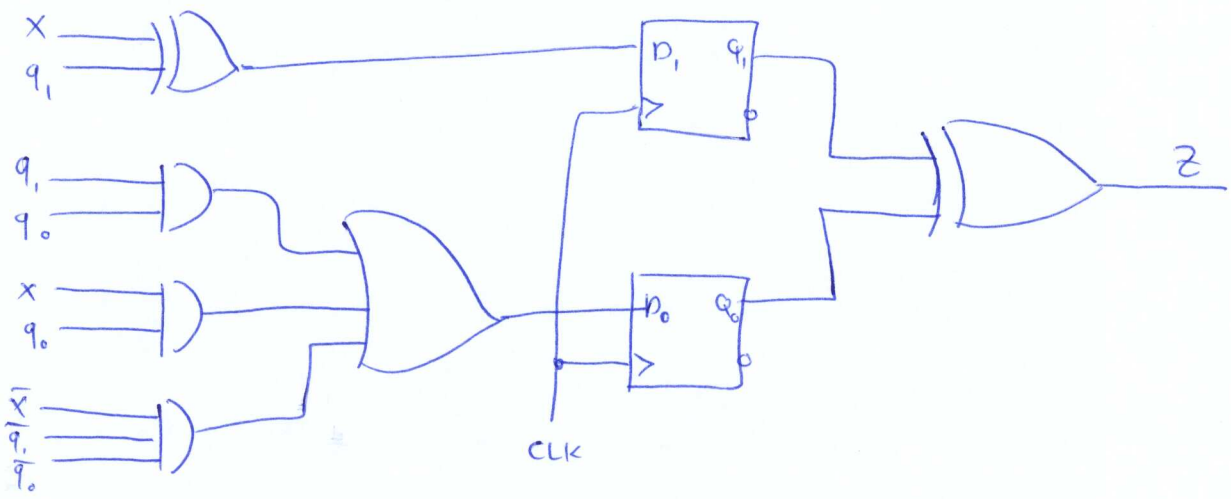
$D_0 = q_1 q_0 + x q_0 + \bar{x} \bar{q}_1 \bar{q}_0$

$z = q_1 \bar{q}_0 + \bar{q}_1 q_0$

$= q_1 \oplus q_0$

(3)

vi)



Question 3

(3) i)

$T_2 = \bar{q}_1$

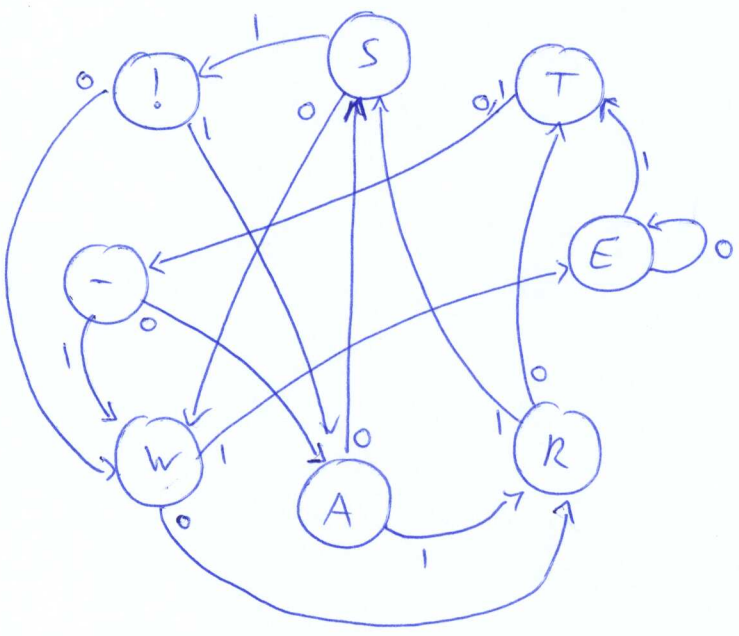
$T_1 = x + q_0 + q_2 q_1$

$T_0 = x + \bar{q}_2 \bar{q}_1$

(10) ii)

	$q_2 q_1 q_0$	$x=0$ $T_2 T_1 T_0$	$x=1$ $T_2 T_1 T_0$	$x=0$ $Q_2 Q_1 Q_0$	$x=1$ $Q_2 Q_1 Q_0$
S	0 0 0	1 0 1	1 1 ϕ	1 0 1	1 1 1
T	0 0 1	1 1 1	1 1 ϕ	1 1 0	1 1 0
E	0 1 0	0 0 0	0 1 ϕ	0 1 0	0 0 1
R	0 1 1	0 1 0	0 1 ϕ	0 0 1	0 0 0
A	1 0 0	1 0 0	1 1 ϕ	0 0 0	0 1 1
W	1 0 1	1 1 0	1 1 1	0 1 1	0 1 0
–	1 1 0	0 1 0	0 1 1	1 0 0	1 0 1
!	1 1 1	0 1 0	0 1 1	1 0 1	1 0 0

(4) iii)



(3) iv)

state	S	W	E	E	T	–	A	S	!
x	0	ϕ	0	1	0	0	0	1	

Question 4

(5) i)

$27/2 = 13$	$R = 1$	↑
$13/2 = 6$	1	
$6/2 = 3$	0	
$3/2 = 1$	1	
$1/2 = 0$ //	1	

$27_{10} = 11011_2$

$0.40625 \times 2 = 0.8125$	$I = 0$	↓
$0.8125 \times 2 = 1.625$	$= 1$	
$0.625 \times 2 = 1.25$	1	
$0.25 \times 2 = 0.5$	0	
$0.5 \times 2 = 1.0$ //	1	

$0.40625_{10} = 0.01101_2$

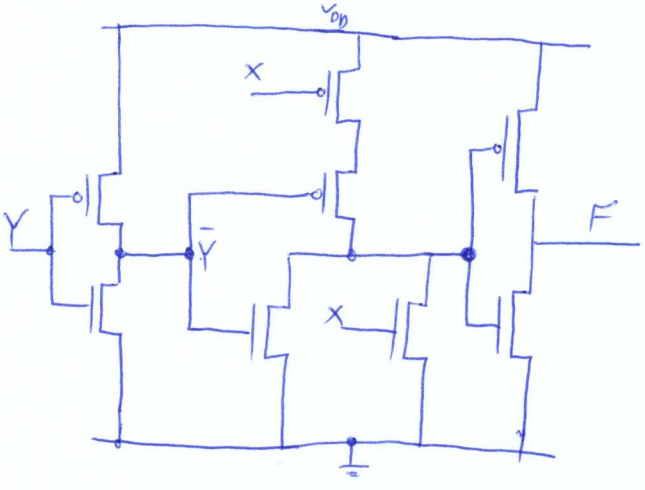
$$27.40625 = 11011.01101_2$$

(8) ii)

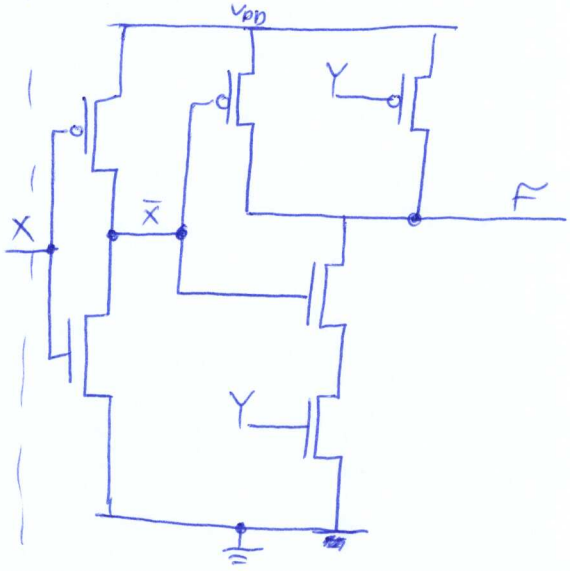
$F = X + \bar{X}\bar{Y} + \bar{Y}\bar{Z}$

$= X + \bar{Y} + \bar{Y}Z$

$= X + \bar{Y}$



OR: $\bar{F} = \overline{X + \bar{Y}} = \bar{X}Y$



(7) (iii)

$$G = \overline{C(AB + D)} = \overline{ABC + CD} = (\bar{A} + \bar{B} + \bar{C})(\bar{C} + \bar{D})$$

$$= \bar{A}\bar{C} + \bar{A}\bar{D} + \bar{B}\bar{C} + \bar{B}\bar{D} + \bar{C} + \bar{C}\bar{D}$$

$$= \bar{C} + \bar{D}(\bar{A} + \bar{B})$$

A	B	C	D	G
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

Question 5

(6)

i)

q	Q	D	J	K
0	0	0	0	X
0	1	1	1	X
1	0	0	X	1
1	1	1	X	0

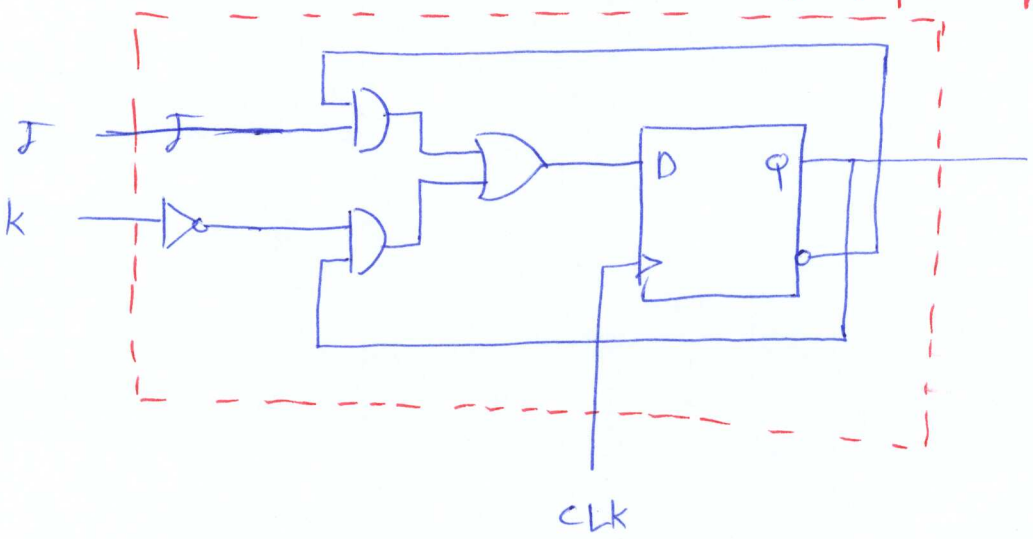
D(q, J, K):

q \ JK	00	01	11	10
0	0	0	1	1
1	1	0	0	1

J

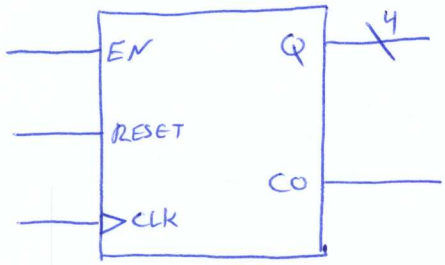
K

$D = J\bar{q} + \bar{K}q$



ii) (3)

a)



(2)

b) Four-bit counter with an asynchronous Reset.

(5)

c) Inputs:

CLK - A clock signal to synchronize the counter

EN - Enable signal - the counter increments when EN = 1 and keep steady when EN = 0.

RESET - Asynchronous signal to reset the counter to 0, only resets on the positive edge of the signal.

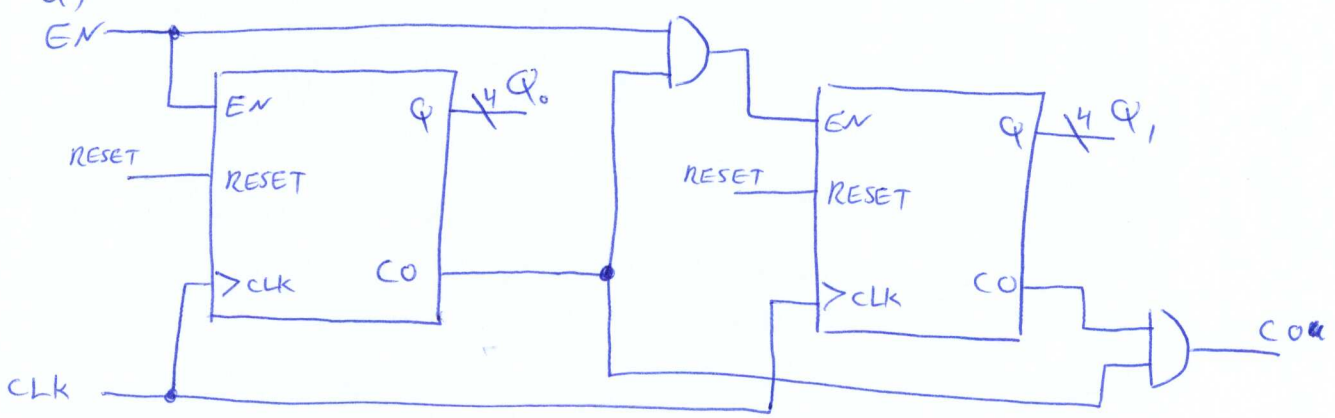
Outputs:

Q - The 4-bit counter value.

CO - Carry-out signal. Goes HIGH when the counter is at its highest value (1111) and LOW otherwise. Can be used to cascade two counters.

(4)

d)



Question 6

(8)

i)

```
always @(posedge S) begin
    Q <= 1;
end

always @(posedge R) begin
    Q <= 0;
end

always @(negedge CLK) begin
    Q <= D;
end
```

ii) (2) a)

```
  0 1 1 1
  0110 1010
+ 0010 1100
-----------
  1001 0110
```

(2) b)

N = 1 C = 0

Z = 0 V = 1

(2) c) No overflow!

iii) (2) a)

```
  1001 0001
- 0010 0100
```

=>

```
  1 0 1
  1001 0001
+ 1101 1100
-----------
  0110 1101
```

(2) b) N = 0 C = 1

Z = 0 V = 1

(2) c) Overflow!