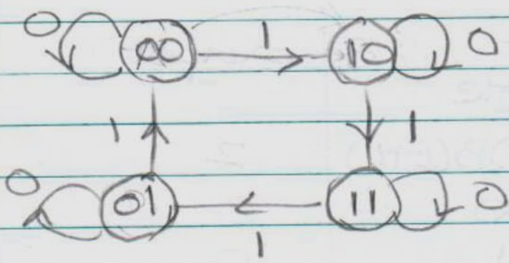
The University of New South Wales

ELEC2141: Digital Circuit Design

Tutorial Week 7 – Sequential Circuit Design

1.



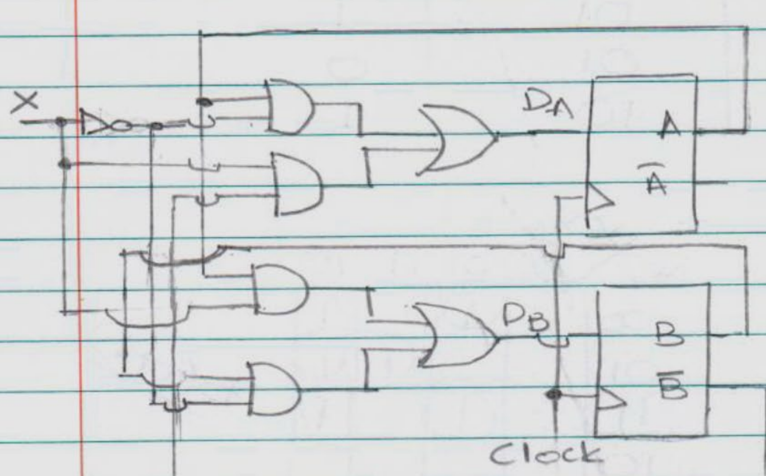
Present State AB	Input X	Next State A(t+1) B(t+1)
00	0	00
00	1	10
01	0	01
01	1	00
10	0	10
10	1	11
11	0	11
11	1	01

A \ BX	00	01	11	10
0		1		
1	1	1		1

$D_A = A\bar{X} + \bar{B}X$

A \ BX	00	01	11	10
0				1
1		1	1	1

$D_B = AX + B\bar{X}$



2.

2. a)

Present state AB	Next state A(t+1)B(t+1)				Output Y			
	X_1X_2=00	01	10	11	X_1X_2=00	01	10	11
S_1	S_1	S_2	S_2	S_1	0	0	1	0
S_2	S_1	S_1	S_4	S_4	0	0	1	1
S_3	S_1	S_1	S_3	S_3	1	0	1	0
S_4	S_4	S_2	S_2	S_3	1	1	0	1

S_1=00 S_3=10
S_2=01 S_4=11

b)

Present state AB	Inputs X_1 X_2	Next state A(t+1)B(t+1)	Output Z
00	00	00	0
00	01	01	0
00	10	01	1
00	11	00	0
01	00	00	0
01	01	00	0
01	10	11	1
01	11	11	1
10	00	00	1
10	01	00	0
10	10	10	1
10	11	10	0
11	00	10	1
11	01	01	1
11	10	01	0
11	11	10	1

c)

Z =

AB \ X_1X_2	00	01	11	10
00				1
01			1	1
11	1	1	1	
10	1			1

B(t+1)

AB \ X_1X_2	00	01	11	10
00		1		1
01			1	1
11		1		1
10				

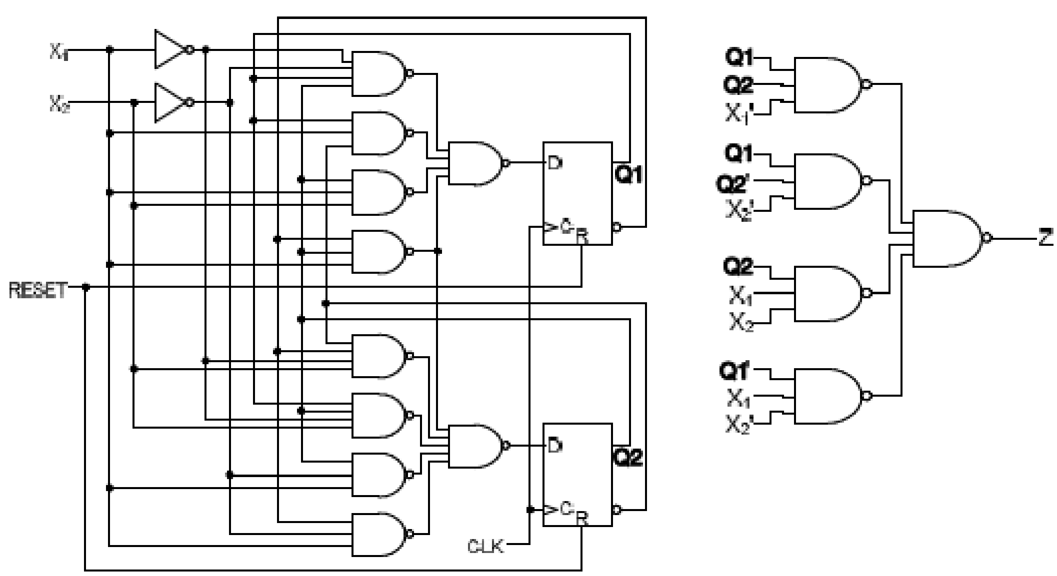
A(t+1)

AB \ X_1X_2	00	01	11	10
00				
01			1	1
11	1		1	
10			1	1

$Z = AB\bar{X}_1 + A\bar{B}\bar{X}_2 + BX_1X_2 + \bar{A}X_1\bar{X}_2$

$A(t+1) = D_A = AB\bar{X}_1\bar{X}_2 + BX_1X_2 + \bar{A}BX_1 + A\bar{B}X_1$

$B(t+1) = D_B = \bar{A}\bar{B}\bar{X}_1X_2 + AB\bar{X}_1X_2 + \bar{A}BX_1$
$+ BX_1\bar{X}_2 + \bar{A}X_1X_2$



Where referenced, questions are taken from the textbook:
M. Mano, C. R. Kime and T. Martin, Logic and Computer Design Fundamentals, 5th Edition (Global Edition), Pearson, 2016

d) $S_1 = 0001$; $S_2 = 0010$; $S_3 = 0100$; $S_4 = 1000$

Present State ABCD	Inputs X_1X_2	Next State $A(t+1)B(t+1)C(t+1)D(t+1)$	Output Z
0001	00	0001	0
0001	01	0010	0
0001	10	0010	1
0001	11	0001	0
0010	00	0001	0
0010	01	0001	0
0010	10	1000	1
0010	11	1000	1
0100	00	0001	1
0100	01	0001	0
0100	10	0100	1
0100	11	0100	0
1000	00	0100	1
1000	01	0010	1
1000	10	0010	0
1000	11	0100	1

$$A(t+1) = CX_1$$

$$B(t+1) = BX_1 + AX_1X_2 + A\overline{X_1}\,\overline{X_2}$$

$$C(t+1) = D\overline{X_1}X_2 + DX_1\overline{X_2} + A\overline{X_1}X_2 + AX_1\overline{X_2}$$

$$D(t+1) = D\overline{X_1}\,\overline{X_2} + DX_1X_2 + C\overline{X_1} + B\overline{X_1}$$

$$Z = DX_1\overline{X_2} + CX_1 + B\overline{X_2} + A\overline{X_1} + AX_1X_2$$

Circuit not shown – but can be drawn based on equations

3.

3.

a)

A	000
B	001
C	010
D	011
E	101
F	110

Present State	Next State X=0	Next State X=1	Output Z
A	C	E	0
B	E	D	1
C	C	E	0
D	F	A	1
E	B	D	1
F	C	E	0

b)

B	X				
C	✓	X			
D	X	~~E–F A–D~~	X		
E	X	~~B–C D–D~~	X	~~B–F A–D~~	
F	✓	X	✓	X	X
	A	B	C	D	E

A ~ C ~ F
B ~ E

Present State	Next State X=0	Next State X=1	Output
A	A	B	0
B	B	D	1
D	A	A	1

c) Choosing assignment so one variable matches output

A = YZ = 00 ; B = 01 ; D = 11

⇒ Z = output

Present State YZ	Input X	Next State Y(t+1)Z(t+1)	Output Z
00	0	00	0
00	1	01	0
01	0	01	1
01	1	11	1
11	0	00	1
11	1	00	1

$$Y(t+1) = D_Y = X\bar{Y}Z$$

$$Z(t+1) = D_Z = X\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + X\bar{Y}Z$$

$$= \bar{X}\bar{Y}\bar{Z} + \bar{Y}Z = \bar{Y}(\bar{X}\bar{Z} + Z) = \bar{Y}(\bar{X} + Z)$$

$$= \bar{X}\bar{Y} + \bar{Y}Z$$

4.

state assignment: binary coding

A ~ 00 B ~ 01 C ~ 10

Write the state table and the flip-flop inputs:

	Present state $q_1\ q_0$	next state, output Q_1Q_0, z $x=0$	$x=1$	D_1D_0 $x=0$	D_1D_0 $x=1$	T_1T_0 $x=0$	T_1T_0 $x=1$
A	0 0	01, 0	00, 1	01	00	01	00
B	0 1	00, 1	10, 0	00	10	01	11
C	1 0	00, 1	10, 1	00	10	10	00
	1 1	XX, X	XX, X	XX	XX	XX	XX

a)

D_1

$x \backslash q_1q_0$				
	0	0	X	0
x	0	1	X	1

$D_1 = q_0 x + q_1 x$

D_0

$x \backslash q_1q_0$				
	1	0	X	0
x	0	0	X	0

$D_0 = \bar{q}_1 \bar{q}_0 \bar{x}$

Where referenced, questions are taken from the textbook:

M. Mano, C. R. Kime and T. Martin, Logic and Computer Design Fundamentals, 5th Edition (Global Edition), Pearson, 2016

b) T_1

$x \backslash q_1 q_0$				
	0	0	X	1
x	0	1	X	0

$$T_1 = x q_0 + \bar{x} q_1$$

T_0:

$x \backslash q_1 q_0$				
	1	1	X	0
x	0	1	X	0

$$T_0 = \bar{x}\,\bar{q}_1 + q_0$$

For both (a) and (b), the output functions

Z

$x \backslash q_1 q_0$				
	0	1	X	1
x	1	0	X	1

$$Z = q_1 + \bar{x} q_0 + \bar{q}_0 x$$
$$= q_1 + q_0 \oplus x$$

5.

State assignment: Use "good" Coding

	00	01	11	10
0	A	B	C	
1	E	D		

~~place~~ ~~Assign state~~
Place next State and present State ~~in~~ in adjacent squares ~~of the k-map~~ in the k-map

A ~ 000
B ~ 010
C ~ 110
D ~ 011
E ~ 001

Write the state table and the flop-flop inputs:

	$q_2 q_1 q_0$	$x=0$: $q_2 q_1 q_0, Z$	$x=1$: $q_2 q_1 q_0, Z$	$x=0$: $J_2 K_2$	$J_1 K_1$	$J_0 K_0$	$x=1$: $J_2 K_2$	$J_1 K_1$	$J_0 K_0$
A	000	000,0	010,0	0X	0X	0X	0X	1X	0X
E	001	011,0	000,0	0X	1X	X0	0X	0X	X1
B	010	011,1	110,1	0X	X0	1X	1X	X0	0X
D	011	010,1	001,1	0X	X0	X1	0X	X1	X0
	100	XXX,X	XXX,X	XX	XX	XX	XX	XX	XX
	101	XXX,X	XXX,X	XX	XX	XX	XX	XX	XX
C	110	010,1	011,0	X1	X0	0X	X1	X0	1X
	111	XXX,X	XXX,X	XX	XX	XX	XX	XX	XX

D_2: ($q_1 q_0$ / $x q_2$; q_1, q_2, x, q_0)

0	0	0	0
X	X	X	0
X	X	X	0
0	0	0	1

$$D_2 = x \bar{q}_2 q_1 \bar{q}_0$$

D_1: ($q_1 q_0$ / $x q_2$; q_1, q_2, x, q_0)

0	1	1	1
X	X	X	1
X	X	X	1
[illegible]	0	0	1

$$D_1 = \bar{x} q_0 + x \bar{q}_0 + q_1 \bar{q}_0$$
$$= x \oplus q_0 + q_1 \bar{q}_0$$

D_0: ($q_1 q_0$ / $x q_2$; q_1, q_2, x, q_0)

0	1	0	1
X	X	X	0
X	X	X	1
0	0	1	0

$$D_0 = \bar{x} \bar{q}_2 \bar{q}_1 \bar{q}_0 + \bar{x} \bar{q}_1 q_0 + x q_2 + x q_1 q_0$$

b.

J_2:

$xq_2 \backslash q_1q_0$	00	01	11	10
00	0	0	0	0
01	X	X	X	X
11	X	X	X	X
10	0	0	0	1

$J_2 = x\bar{q}_0 q_1$

K_2:

$xq_2 \backslash q_1q_0$	00	01	11	10
00	X	X	X	X
01	X	X	X	1
11	X	X	X	1
10	X	X	X	X

$K_2 = q_2$

J_1:

$xq_2 \backslash q_1q_0$	00	01	11	10
00	0	1	X	X
01	X	X	X	X
11	X	X	X	X
10	1	0	X	X

$J_1 = \bar{x}q_0 + x\bar{q}_0 = x \oplus q_0$

K_1:

$xq_2 \backslash q_1q_0$	00	01	11	10
00	X	X	0	0
01	X	X	X	0
11	X	X	X	0
10	X	X	1	0

$K_1 = xq_0$

J_0:

$xq_2 \backslash q_1q_0$	00	01	11	10
00	0	X	X	1
01	X	X	X	0
11	X	X	X	1
10	0	X	X	0

$J_0 = xq_2 + \bar{x}q_1\bar{q}_2$

K_0:

$xq_2 \backslash q_1q_0$	00	01	11	10
00	X	0	1	X
01	X	X	X	X
11	X	X	X	X
10	X	1	0	X

$K_0 = x\bar{q}_1 + \bar{x}q_1$
$= x \oplus q_1$

For both (a) and (b), the output Z,

Z:

$xq_2 \backslash q_1q_0$	00	01	11	10
00	0	0	1	1
01	X	X	X	1
11	X	X	X	0
10	0	0	1	1

$Z = \bar{x}q_1 + \bar{q}_2 q_1$

or

$= \bar{x}q_2 + \bar{q}_2 q_1$

6.

use implication Chart

B	X						
C	X	H~D					
D	X	X	X				
E	X	X	X	X			
F	H~D A~F	X	X	X	X		
G	X	✓	D~H	X	X	X	
H	X	X	X	A~F G~B	X	X	X
	A	B	C	D	E	F	G

Read off equivalent States

A ~ F
B ~ C ~ G
D ~ H
E

New State table :

Current State	next state, output x=0	x=1
A	D, 1	A, 0
B	E, 1	D, 1
D	A, 0	B, 0
E	B, 0	E, 1

7.

use implication Chart

B	X			
C	X	~~A~B C~E~~		
D	~~C~D~~	X	X	
E	~~E~D C~B~~	X	X	~~D~B~~
	A	B	C	D

the state machine can not be minimised further.

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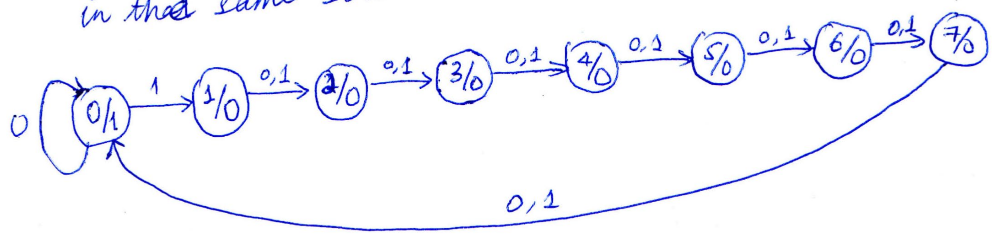
M. Mano, C. R. Kime and T. Martin, Logic and Computer Design Fundamentals, 5th Edition (Global Edition), Pearson, 2016

8.

1. (a) When E=1, the circuit will output the required sequence regardless of input E
When E=0, the circuit outputs constant 1 and remains in that same state



(b) use binary coding assignment. The state table is

	$q_2\ q_1\ q_0$	E=0 $Q_2\ Q_1\ Q_0$	E=1 $Q_2\ Q_1\ Q_0$	Z
	0 0 0	0 0 0	0 0 1	1
	0 0 1	0 1 0	0 1 0	0
2	0 1 0	0 1 1	0 1 1	0
3	0 1 1	1 0 0	1 0 0	0
4	1 0 0	1 0 1	1 0 1	0
5	1 0 1	1 1 0	1 1 0	0
6	1 1 0	1 1 1	1 1 1	0
7	1 1 1	0 0 0	0 0 0	0

$Z = \bar{q}_2 \cdot \bar{q}_1 \bar{q}_0$

(c) D_2

$Eq_2 \backslash q_1q_0$				
	0	0	1	0
q_2	1	1	0	1
E, q_2	1	1	0	1
E	0	0	1	0

(q_1 over the last two columns, q_0 over the middle two columns)

$D_2 = \bar{q}_2 q_1 q_0 + q_2 \bar{q}_0 + q_2 \bar{q}_1$

D_1

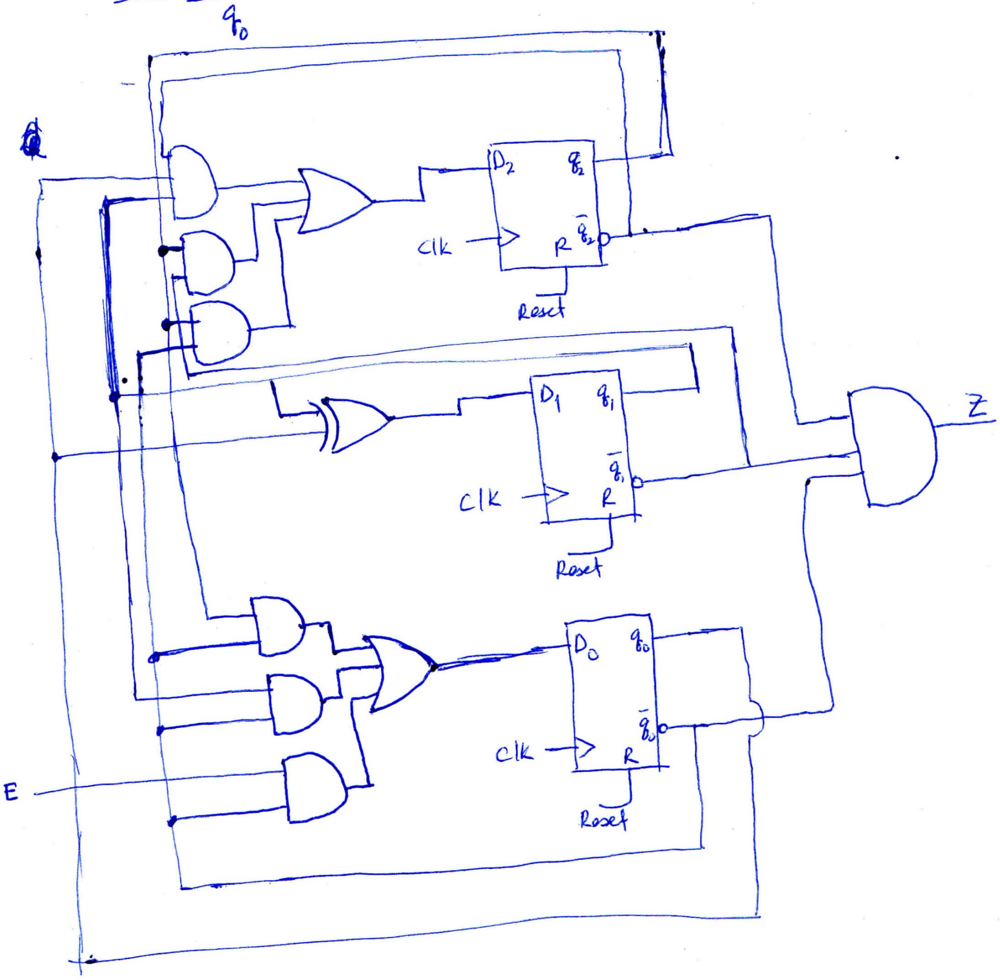
$Eq_2 \backslash q_1q_0$				
	0	1	0	1
q_2	0	1	0	1
E, q_2	0	1	0	1
E	0	1	0	1

$D_1 = \bar{q}_1 q_0 + q_1 \bar{q}_0 = q_0 \oplus q_1$

D_0

$Eq_2 \backslash q_1q_0$			q_1	
	0	0	0	1
q_2	1	0	0	1
E, q_2	1	0	0	1
E	1	0	0	1
		q_0		

$$D_0 = q_1\bar{q}_0 + E\bar{q}_0 + q_2\bar{q}_0$$

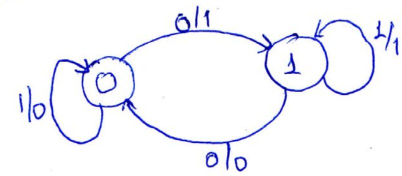


9.

Define the States as the last value at the NRZI message.

2 possible states: 0 and 1

a)



b) State table

q	$X=0$ Q, Z	$X=1$ Q, Z
0	1, 1	0, 0
1	0, 0	1, 1

c) Derive flip-flop input equation and output equation:

D:

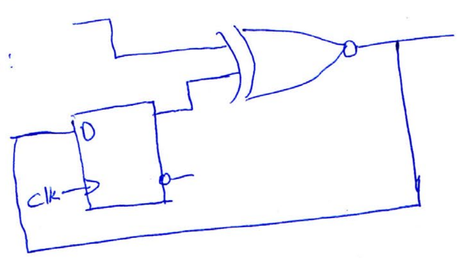
$x \backslash q$		q
	1	0
x	0	1

Z:

$x \backslash q$		q
	1	0
x	0	1

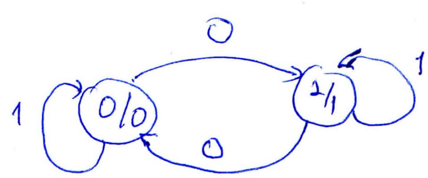
$$Z = D = \overline{q \oplus x}$$

Logic diagram:



d. Simple Moore model:

q	$X=0$ Q	$X=1$ Q	Z
0	1	0	0
1	0	1	1



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