MIDTERM EXAM II 2019 SOLUTION

1a) $F(A,B,C,D) = \underbrace{\bar{C}D}_{1} + \underbrace{AB\bar{C}}_{1} + \underbrace{AB\bar{D}}_{1} + \underbrace{\bar{A}\bar{B}D}_{1}$ → terms.

(i) GIC = 11 (literals) + 4 (terms) + 4 (complements) = 19 (1) (1) (1)

(ii) Can be done algebraically or with truth table

Algebraic Method

$\bar{C}D = \bar{C}D(A+\bar{A})(B+\bar{B}) = \bar{C}D(AB+\bar{A}B+A\bar{B}+\bar{A}\bar{B})$

$= \underset{1}{\bar{A}\bar{B}\bar{C}D} + \underset{5}{\bar{A}B\bar{C}D} + \underset{9}{A\bar{B}\bar{C}D} + \underset{1}{\bar{A}\bar{B}\bar{C}D}$. (minterms)

$AB\bar{C}(D+\bar{D}) = \underset{13}{AB\bar{C}D} + \underset{12}{AB\bar{C}\bar{D}}$ (2)

$AB\bar{D}(C+\bar{C}) = \underset{14}{ABC\bar{D}} + \underset{12}{AB\bar{C}\bar{D}}$ ⇒ $F = \Sigma m(1,3,5,9,12,13,14)$

$\bar{A}\bar{B}D(C+\bar{C}) = \underset{3}{\bar{A}\bar{B}CD} + \underset{1}{\bar{A}\bar{B}\bar{C}D}$

Truth table

A	B	C	D	$\bar{C}D$	$AB\bar{C}$	$AB\bar{D}$	$\bar{A}\bar{B}D$	F
0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	1	1
0	0	1	0	0	0	0	0	0
0	0	1	1	0	0	0	1	1
0	1	0	0	0	0	0	0	0
0	1	0	1	1	0	0	0	1
0	1	1	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0
1	0	0	0	0	0	0	0	0
1	0	0	1	1	0	0	0	1
1	0	1	0	0	0	0	0	0
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	0	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	0	1
1	1	1	1	0	0	0	0	0

(2) - Method

most will have 0's in empty spaces here

(iii)

AB \ CD	00	01	11	10
00		1	1	0
01		1		0
11	1	1		1
10		1		

OR ②

CD \ AB	00	01	11	10
00			1	
01	1	1	1	1
11	1			
10			1	

Prime implicants: $\bar{C}D$, $AB\bar{D}$, $AB\bar{C}$, $\bar{A}\bar{B}D$ ③

Essential PIs: $\bar{C}D$, $AB\bar{D}$, $\bar{A}\bar{B}D$ (1.5)

$F = \bar{C}D + AB\bar{D} + \bar{A}\bar{B}D$ (1.5)

(iv)

AB \ CD	00	01	11	10
00	0			0
01	0		0	0
11			0	
10	0		0	0

OR

CD \ AB	00	01	11	10
00	0	0		0
01				
11		0	0	0
10	0	0		0

Prime implicants: ④

$A+D$, $B+D$, $A+\bar{B}+\bar{C}$, $\bar{B}+\bar{C}+\bar{D}$, $\bar{A}+\bar{C}+\bar{D}$, $\bar{A}+B+\bar{C}$

Essential PIs

$A+D$, $B+D$ ②

$F = (A+D)(B+D)(\bar{B}+\bar{C}+\bar{D})(\bar{A}+\bar{C}+\bar{D})$

OR

$F = (A+D)(B+D)(A+\bar{B}+\bar{C})(\bar{A}+\bar{C}+\bar{D})$ ②

OR

$F = (A+D)(B+D)(\bar{B}+\bar{C}+\bar{D})(\bar{A}+B+\bar{C})$

(v)

new GIC = 8 (literals) ① + 3 (terms) ① + 4 (comp.) ①

= 15

Reduction = 19 − 15 = 4

1b)

Solution 2:

$X = Y \Rightarrow EQ = 1$; $X > Y \Rightarrow GT = 1$

$X < Y \Rightarrow LE = 1$; $NULL = 0$

X_1	X_0	Y_1	Y_1	EQ	GT	LE
0	0	0	0	1	0	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	0	1
0	1	0	0	0	1	0
0	1	0	1	1	0	0
0	1	1	0	0	0	1
0	1	1	1	0	0	1
1	0	0	0	0	1	0
1	0	0	1	0	1	0
1	0	1	0	1	0	0
1	0	1	1	0	0	1
1	1	0	0	0	1	0
1	1	0	1	0	1	0
1	1	1	0	0	1	0
1	1	1	1	1	0	0

2

S_1	S_0	F
0	0	NULL
0	1	LE
1	0	EQ
1	1	GT

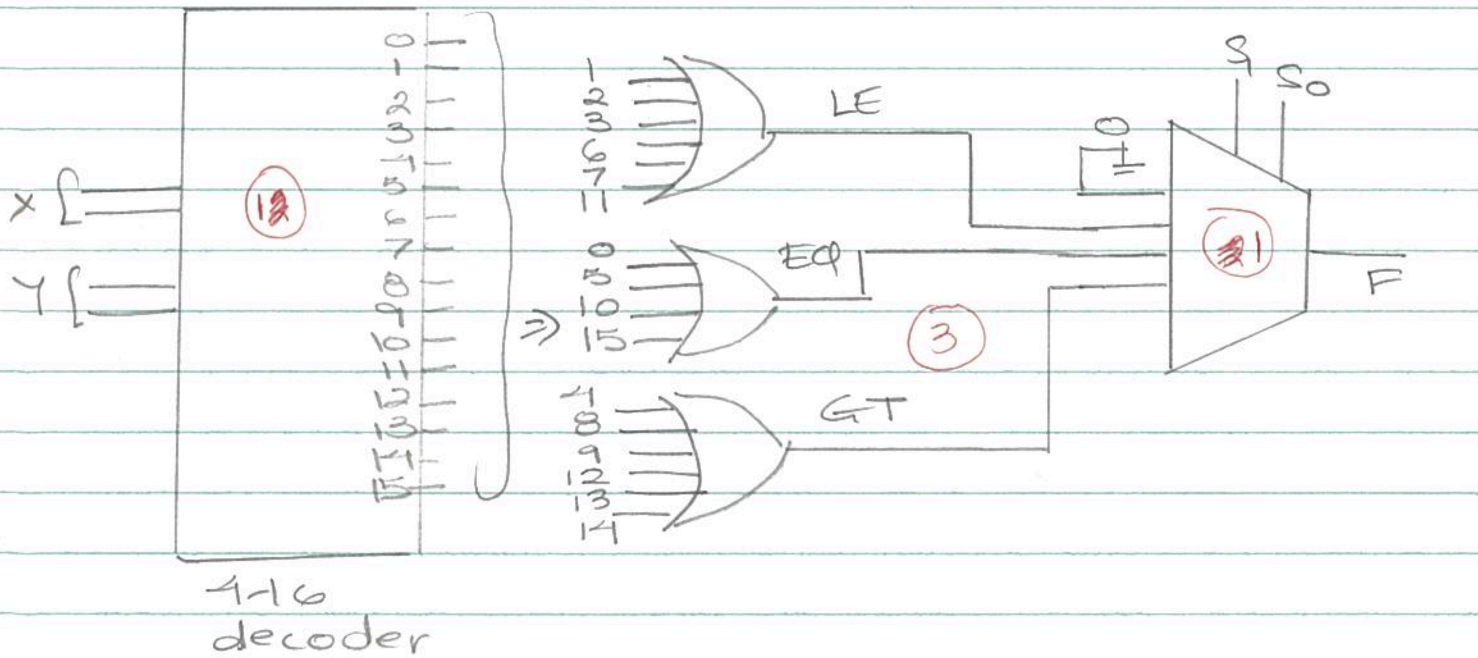
2

$EQ = \Sigma m(0, 5, 10, 15)$

$GT = \Sigma m(4, 8, 9, 12, 13, 14$

$LE = \Sigma m(1, 2, 3, 6, 7, 11)$

6



1b) 2-bit comparator

Solution 1:

$X = Y \Rightarrow EQ = 1$; $X > Y \Rightarrow GT = 1$

$X < Y \Rightarrow LE = 1$; $X - Y = 0 \Rightarrow NULL = 1$

X_1	X_0	Y_1	Y_0	EQ	GT	LE	NULL
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	1	0	0	0
0	1	1	0	0	0	1	0
0	1	1	1	0	0	1	0
1	0	0	0	0	1	0	0
1	0	0	1	0	1	0	0
1	0	1	0	1	0	0	0
1	0	1	1	0	0	1	0
1	1	0	0	0	1	0	0
1	1	0	1	0	1	0	0
1	1	1	0	0	1	0	0
1	1	1	1	1	0	0	0

2

S_1	S_0	F
0	0	NULL
0	1	LE
1	0	EQ
1	1	GT

2

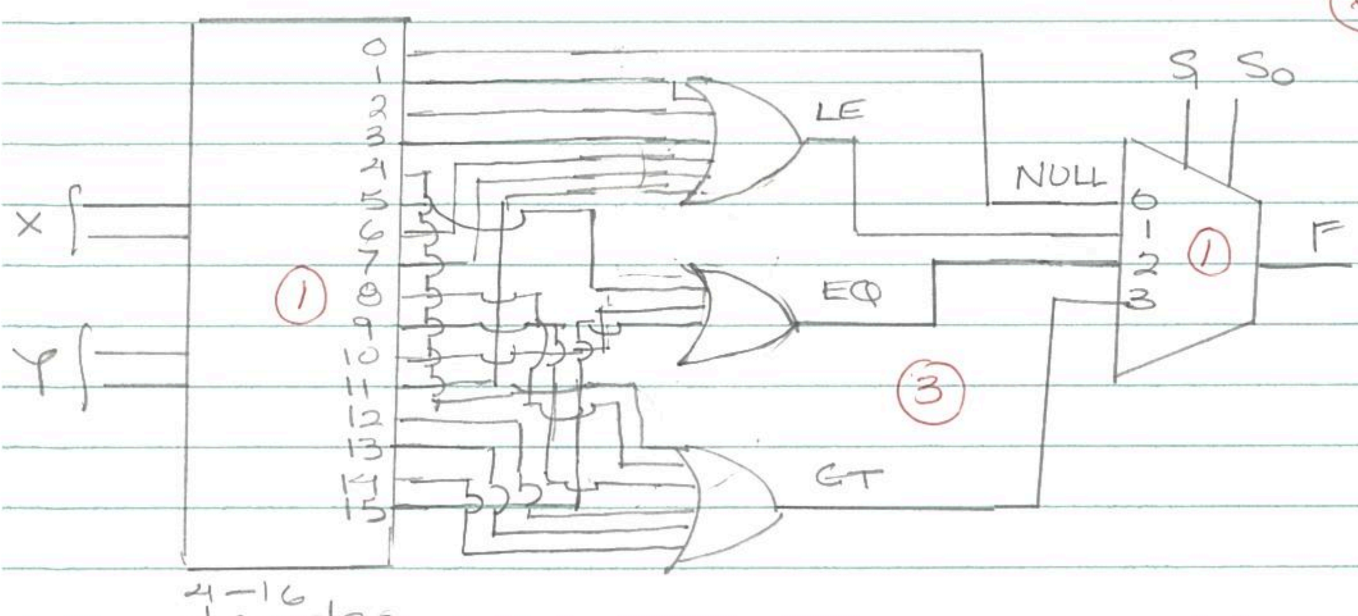
$NULL = \Sigma m(1)$

$EQ = \Sigma m(5, 10, 15)$

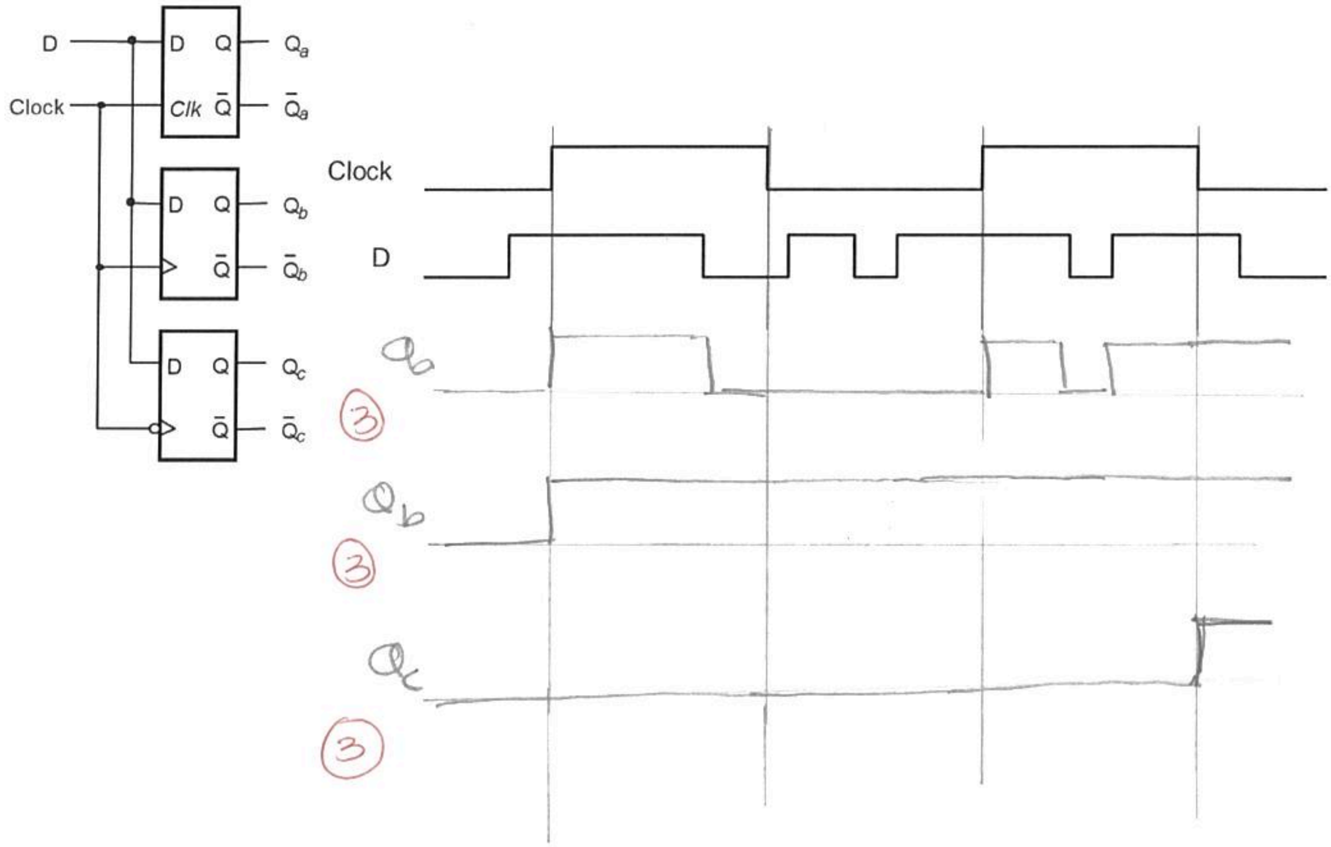
$GT = \Sigma m(4, 8, 9, 12, 13, 14)$

$LE = \Sigma m(1, 2, 3, 6, 7, 11)$

6



4-16 decoder


D
Clock
D
Q
Q_a
Clk
$\bar{Q}$
$\bar{Q}_a$
D
Q
Q_b
$\bar{Q}$
$\bar{Q}_b$
D
Q
Q_c
$\bar{Q}$
$\bar{Q}_c$
Clock
D
Qa
3
Qb
3
Qc
3

2a)

$453.25_8 \longrightarrow$ base 10 (1)

$4\times 8^2 + 5\times 8^1 + 3\times 8^0 + 2\times 8^{-1} + 5\times 8^{-2}$

(5) $= 299.3281_{10}$

base 10 $\rightarrow$ base 7

7	299	
	42	5
	6	0

$0.3281\times 7 = 2.2967$ 2
$0.2967\times 7 = 2.0769$ 2
$0.0769\times 7 = 0.5383$ 0
$0.5383\times 7 = 3.7681$ 3

(5)

605.2203_7

453.25_8

$\downarrow$ Replacing each number with 3 bit binary

$4 - 100,\ 5 - 101,\ 3 - 011,\ 2 - 010,\ 5 - 101$

(5)

$\Rightarrow \overbrace{100}^{4}\overbrace{101}^{5}\overbrace{011}^{3}.\underbrace{010}_{2}\underbrace{1_2}_{5}$

b)

(i) From left side – moving right

AND gate : $W\bar{X}Z$

OR gates : $Y + W\bar{X}Z$ and $W + X$

AND gate : $(W+X)(Y + W\bar{X}Z)$ (4)

OR gate: $F = \bar{Z} + (W+X)(Y+W\bar{X}Z)$

(ii)

$F = \bar{Z} + [(W+X)(Y+W\bar{X}Z)]$

$= \bar{Z} + [WY + XY + WW\bar{X}Z + XW\bar{X}Z]$ $\boxed{WW = 1 \quad X\bar{X} = 0}$

$= \bar{Z} + WY + XY + W\bar{X}Z$ (7)

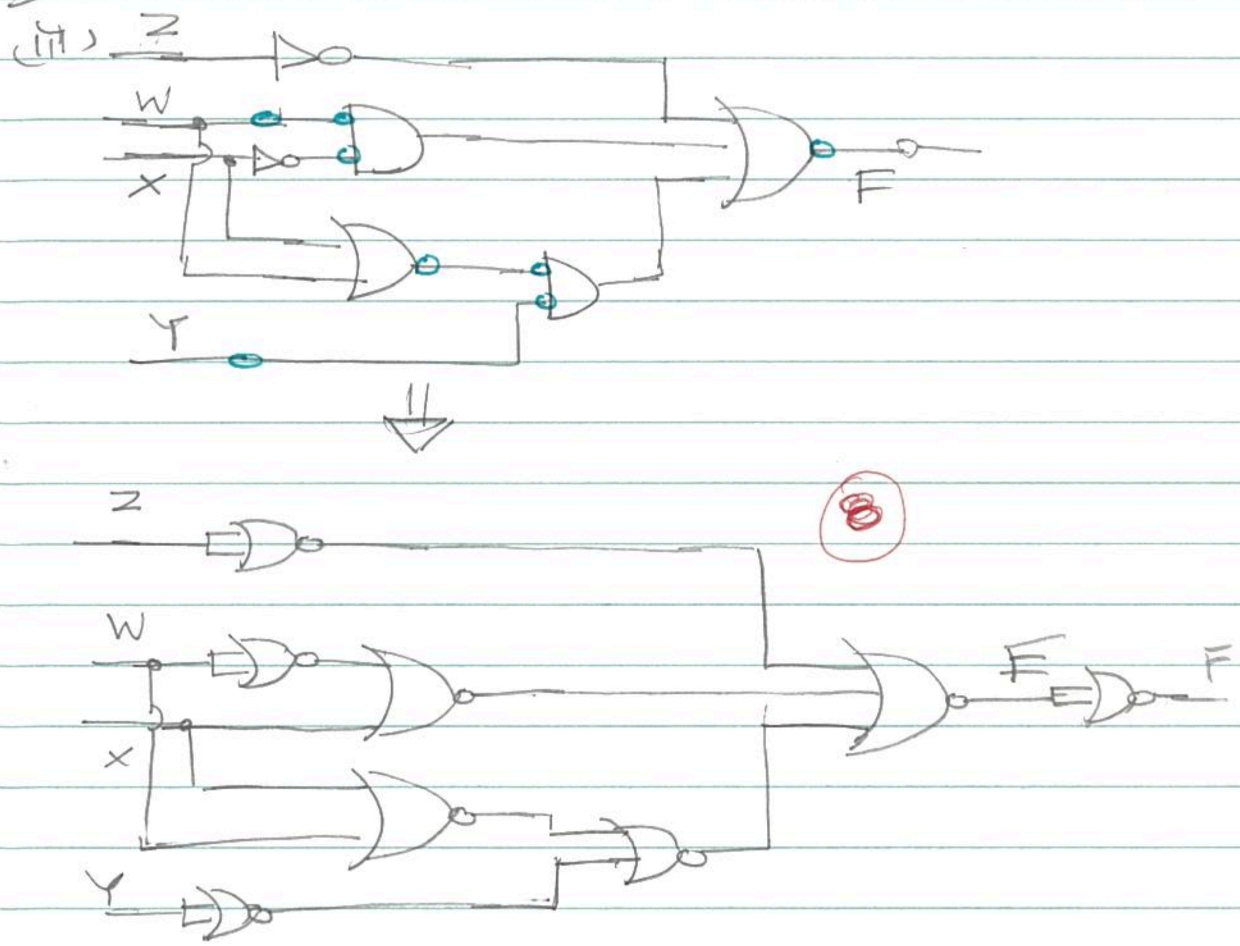
$= WY + XY + (\bar{Z} + W\bar{X}Z)$ $A + \bar{A}B = A + B$

$= Y(W+X) + \bar{Z} + W\bar{X}$

or $= XY + \bar{X}W + \bar{Z}$ consensus: $XY + \bar{X}W + YW = XY + \bar{X}W$

2b

(iii)



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2c

Truth table

4

A	B	BR	D	BL
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$BL = \Sigma m(1,2,3,7)$

$D = \Sigma m(1,2,4,7)$

$= \bar{A}\bar{B}BR + \bar{A}B\overline{BR} + A\bar{B}\overline{BR} + ABBR$

$= BR(\bar{A}\bar{B} + AB) + \overline{BR}(\bar{A}B + A\bar{B})$

$\bar{A}B + A\bar{B} = A \oplus B$

$\bar{A}\bar{B} + AB = \overline{A \oplus B}$

$\rightarrow D = BR(\overline{A \oplus B}) + \overline{BR}(A \oplus B)$

$= BR \oplus A \oplus B$ 2

$BL = \bar{A}\bar{B}BR + \bar{A}B\overline{BR} + \bar{A}BBR + ABBR$

$= (\bar{A}\bar{B} + AB)BR + \bar{A}B(\overline{BR} + BR)$

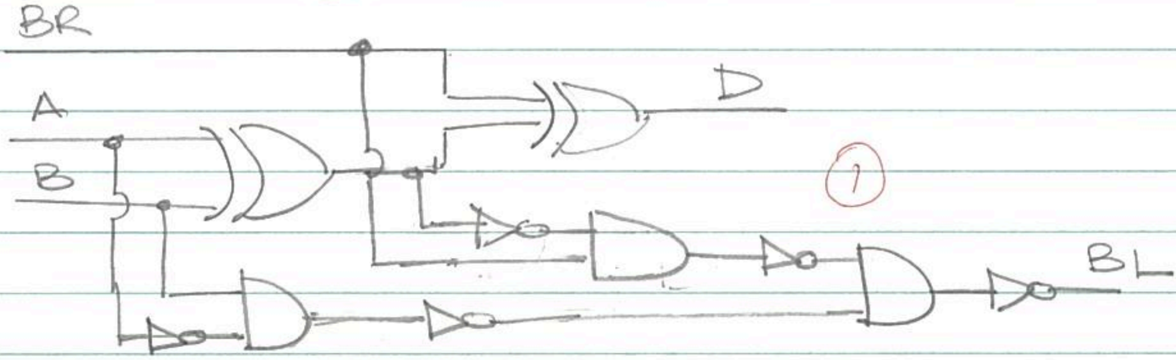
$= (\overline{A \oplus B})BR + \bar{A}B$ $\quad \overline{BR} + BR = 1$

2c)

To get rid of OR need to do double complement

$$BL = \overline{\overline{(A \oplus B)BR + \bar{A}B}} = \overline{\left[\overline{(A \oplus B)BR}\right]\left[\overline{\bar{A}B}\right]}$$

3



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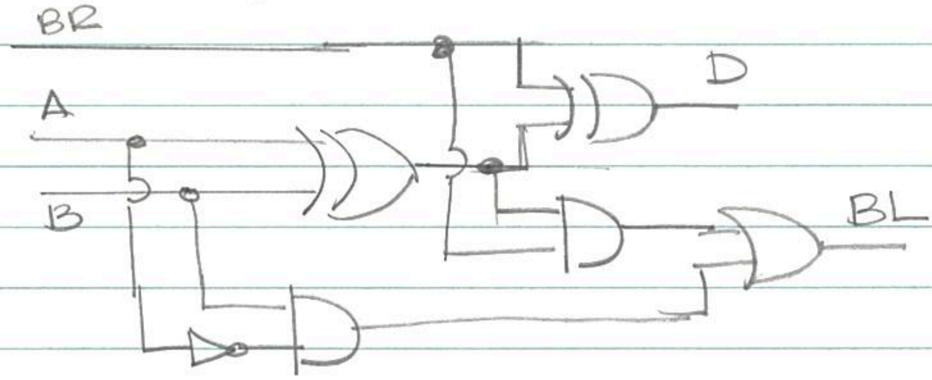
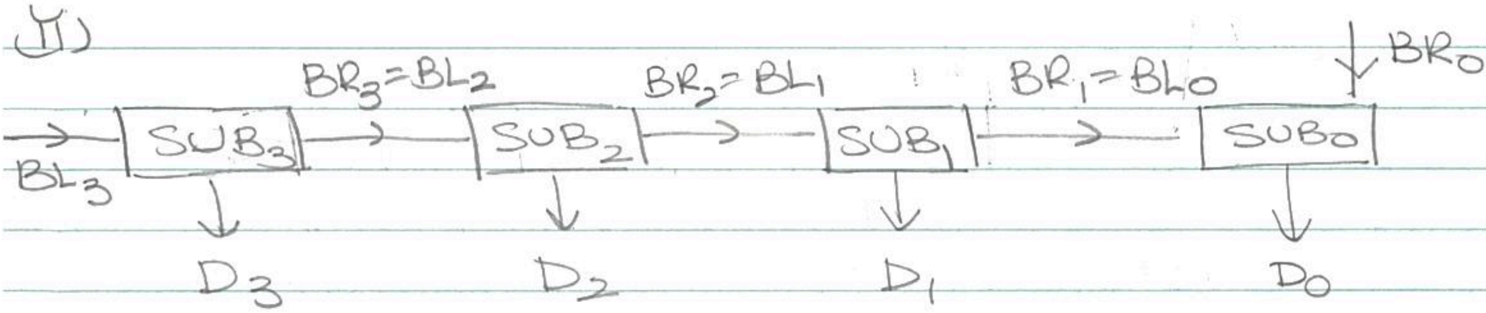
if OR gate kept — circuit will be



(ii)



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