1-3

Monday, 24 February 2020 1:52 am

1. a) 11 bits $\rightarrow$ $2^{11}-1 = 2047$
   b) 25 bits $\rightarrow$ $2^{25}-1 = 33554431$

2. a) $1011001_2 = 2^6 + 2^4 + 2^3 + 2^0$
   $= 64 + 16 + 8 + 1$
   $= 89_{10}$

   b) $1100111.001_2 = 2^6 + 2^5 + 2^2 + 2^1 + 2^0 + 2^{-3}$
   $= 64 + 32 + 4 + 2 + 1 + 0.125$
   $= 103.125_{10}$

   c) $10110010.10101 = 2^7 + 2^5 + 2^4 + 2^1 + 2^{-1} + 2^{-3} + 2^{-5}$
   $= 128 + 32 + 16 + 2 + 0.5 + 0.125 +$
   $= 178.65625_{10}$

3. a)
   $255/2 = 127$ r1
   $127/2 = 63$ r1
   $63/2 = 31$ r1
   $31/2 = 15$ r1
   $15/2 = 7$ r1
   $7/2 = 3$ r1
   $3/2 = 1$ r1

   $255_{10} = 11111111_2$

   b)
   $452/2 = 226$ r0
   $226/2 = 113$ r0
   $113/2 = 56$ r1
   $56/2 = 28$ r0
   $28/2 = 14$ r0
   $14/2 = 7$ r0
   $7/2 = 3$ r1
   $3/2 = 1$ r1

   $452_{10} = 111000100_2$

   c)
   $124/2 = 62$ r0
   $62/2 = 31$ r0
   $31/2 = 15$ r1
   $15/2 = 7$ r1
   $7/2 = 3$ r1
   $3/2 = 1$ r1

   $0.5 \times 2 = 1$

   $124.5_{10} = 1111100.1_2$

   d)
   $587/2 = 293$ r1
   $293/2 = 146$ r1
   $146/2 = 73$ r0
   $73/2 =$

4.

$369/2 = 184$ r1
$184/2 = 92$ r0
$92/2 = 46$ r0
$46/2 = 23$ r0
$23/2 = 11$ r1
$11/2 = 5$ r1
$5/2 = 2$ r1
$2/2 = 1$ r0

$0.3125 \times 2 = 0.625$
$0.625 \times 2 = 1.25$
$0.25 \times 2 = 0.5$
$0.5 \times 2 = 1.0$

$369.3125_{10} = 101110001.0101$

Binary

$\underbrace{101}_{5}\,\underbrace{110}_{6}\,\underbrace{001}_{1}.\underbrace{010}_{2}\,\underbrace{100}_{4}$ Octal

$\underbrace{0001}_{1}\,\underbrace{0111}_{7}\,\underbrace{0001}_{1}.\underbrace{0101}_{5}$ Hexadecimal

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$\overset{2^7}{1}\,0\,\overset{2^5}{1}\,\overset{2^4}{1}\,\overset{2^3}{1}\,\overset{2^2}{1}\,0\,\overset{2^0}{1}.\overset{2^{-1}}{1}\,0\,\overset{2^{-3}}{1} = 2^7 + 2^5 + 2^4 + 2^3 + 2^2 + 2^0$
$+ 2^{-1} + 2^{-3}$
$= 189.625$

$\underbrace{010}_{2}\,\underbrace{111}_{7}\,\underbrace{101}_{5}.\underbrace{101}_{5}$ Octal

$\underbrace{1011}_{B}\,\underbrace{1101}_{D}.\underbrace{1010}_{A}$ Hexadecimal

---

326.5

$011\,010\,110.101$ Binary

$2^7 + 2^6 + 2^4 + 2^2 + 2^1 + 2^{-1} + 2^{-3} = (214.625)_{10}$

Decimal

$0\underbrace{1101}_{D}\,\underbrace{0110}_{6}.\underbrace{1010}_{A}$ Hexadecimal

---

F 3 C 7 . A

$1111\,0011\,1100\,0111.1010$

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5. a) $7562/8 = 945 \quad r\,2$
$945/8 = 118 \quad r\,1$
$118/8 = 14 \quad r\,6$
$14/8 = 1 \quad r\,6$

$0.45 \times 8 = 3.6$
$0.6 \times 8 = 4.8$
$0.8 \times 8 = 6.4$
$0.4 \times 8 = 3.2$
$0.2 \times 8 = 1.6$
⋮

$7562.45_{10} = 16612.34631\ldots$

b) $1938/16 = 121 \quad r\,2$
$121/16 = 7 \quad r\,9$

$1938.257_{10} = 792.41CAC$

$0.257 \times 16 = 4.112$
$0.112 \times 16 = 1.792$
$0.792 \times 16 = 12.672$ C
$0.672 \times 16 = 10.752$ A
$0.752 \times 16 = 12.032$ C

c) $175/2 = 87 \quad r\,1$
$87/2 = 43 \quad r\,1$
$43/2 = 21 \quad r\,1$
$21/2 = 10 \quad r\,1$
$10/2 = 5 \quad r\,0$
$5/2 = 2 \quad r\,1$
$2/2 = 1 \quad r\,0$

$0.175 \times 2 = 0.35$ 0
$0.35 \times 2 = 0.7$ 0
$0.7 \times 2 = 1.4$ 1
$0.4 \times 2 = 0.8$ 0
$0.8 \times 2 = 1.6$ 1
$0.6 \times 2 = 1.2$ 1

$(175.175)_{10} = (10101111.001011)_2$

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6. a) $56180/20 = 2809 \quad r0$
$2809/20 = 140 \quad r9$
$140/20 = 7 \quad r0$ $\uparrow$

$56180_{10} = 7090_{20}$

b) $A \rightarrow 10$
$B \rightarrow 11$
$F \rightarrow 15$

$$9ABF_{20} = 9 \times 20^3 + 10 \times 20^2 + 11 \times 20^1 + 15 \times 20^0$$
$$= 72000 + 4000 + 220 + 15$$
$$= 76235$$

c) $D \rightarrow 13$
$H \rightarrow 17$
$A \rightarrow 10$

$$D5HA.5 = 13 \times 20^3 + 5 \times 20^2 + 17 \times 20^1 + 10 \times 20^0 + 5 \times 20^{-1}$$
$$= 106350.25$$

7

Tuesday, 25 February 2020 12:00 pm

7 a) $\underbrace{0010}_{2}\ \underbrace{1001}_{9}\ \underbrace{0111}_{7}\ \underbrace{0101}_{5}$

2975/2 = 1487 r 1
1487/2 = 743 r 1
743/2 = 371 r 1
371/2 = 185 r 1
185/2 = 92 r 1
92/2 = 46 r 0
46/2 = 23 r 0
23/2 = 11 r 1
11/2 = 5 r 1
5/2 = 2 r 1
2/2 = 1 r 0

$2975_{10} = 101110011111$

b) $\underbrace{0001}_{1}\ \underbrace{1001}_{9}\ \underbrace{0010}_{2} . \underbrace{0101}_{5}\ \underbrace{0100}_{4}$

$192.54_{10} = 11000000.100010 1$

192/2 = 96 r 0	0.54 × 2 = 1.08
96/2 = 48 r 0	0.08 × 2 = 0.16
48/2 = 24 r 0	0.16 × 2 = 0.32
24/2 = 12 r 0	0.32 × 2 = 0.64
12/2 = 6 r 0	0.64 × 2 = 1.28
6/2 = 3 r 0	0.28 × 2 = 0.56
3/2 = 1 r 1	0.56 × 2 = 1.12

8

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8. a) $7_{10} = 0111_{BCD}$

$1_{10} = 0001_{BCD}$

$5 = 0101_{BCD}$

011100010101_{BCD}

b) $3_{10} = 0011_{BCD}$

$5_{10} = 0101_{BCD}$

$4_{10} = 0100_{BCD}$

001101010100_{BCD}

9

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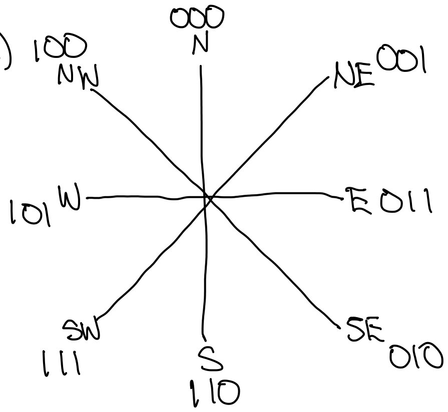
9a)



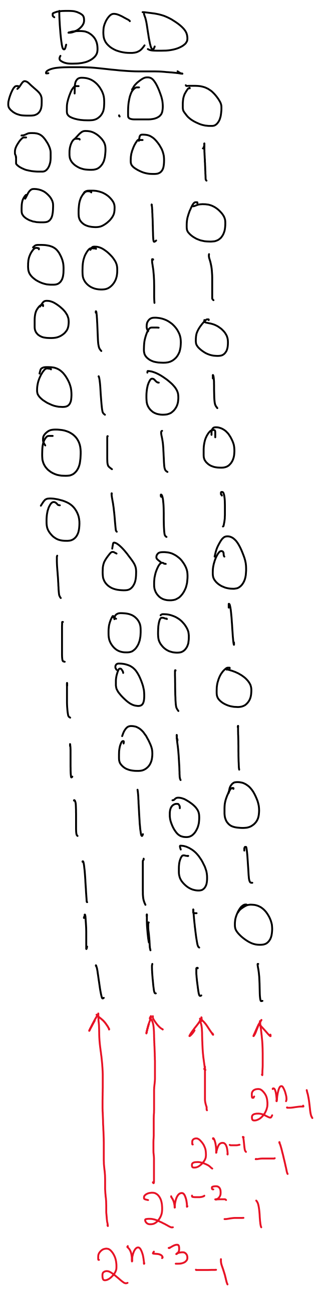
b) See lecture notes

10

Monday, 24 February 2020 4:27 am

10. In Gray code, 1-bit change per cycle.
∴ total bit change for 2^n word
$= 2^n$ bit changes

Decimal	BCD	Gray Code
0	0000	0000
1	0001	0100
2	0010	0101
3	0011	0111
4	0100	0110
5	0101	0010
6	0110	0011
7	0111	0001
8	1000	1001
9	1001	1000



In BCD, LSB changes every cycle ⇒ $2^n - 1$ changes
Second LSB changes every 2^{nd} cycle ⇒ $2^{n-1} - 1$ changes
Third LSB changes every 4^{th} cycle ⇒ $2^{n-2} - 1$ changes
∴ total bit change for n-bits

$$\sum_{i=0}^{n} (2^{n-i} - 1)$$

Percentage power of Gray code to BCD:

$$\boxed{\% \text{ Power} = \frac{2^n}{\sum_{i=0}^{n} (2^{n-i} - 1)}}$$

11

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11a) $\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$

X	Y	Z	$\overline{XYZ}$	$\overline{X}+\overline{Y}+\overline{Z}$
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

b) $X + YZ = (X+Y)(X+Z)$

X	Y	Z	YZ	$X+Y$	$X+Z$	$(X+Y)(X+Z)$	$X+YZ$
0	0	0	0	0	0	0	0
0	0	1	0	0	1	0	0
0	1	0	0	1	0	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

c) $\overline{X}Y + \overline{Y}Z + X\overline{Z} = X\overline{Y} + Y\overline{Z} + \overline{X}Z$

X	Y	Z	$\overline{X}Y$	$\overline{Y}Z$	$X\overline{Z}$	$\overline{X}Y+\overline{Y}Z+X\overline{Z}$	$X\overline{Y}$	$Y\overline{Z}$	$\overline{X}Z$	$X\overline{Y}+Y\overline{Z}+\overline{X}Z$
0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0	0	1	1
0	1	0	1	0	0	1	0	1	0	1
0	1	1	1	0	0	1	0	0	1	1
1	0	0	0	0	1	1	1	0	0	1
1	0	1	0	1	0	1	1	0	0	1
1	1	0	0	0	1	1	0	1	0	1
1	1	1	0	0	0	0	0	0	0	0

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12\. a) $\overline{X}\,\overline{Y} + \overline{X}Y + XY = \overline{X} + Y$

$= \overline{X}\,\overline{Y} + \overline{X}Y + XY + \overline{X}Y$

$= \overline{X}(\cancel{\overline{Y}+Y}) + Y(\cancel{\overline{X}+X})$

$= \overline{X} + Y$

b) $\overline{A}B + \overline{B}\,\overline{C} + AB + \overline{B}C = 1$

$\Rightarrow B(\cancel{\overline{A}+A}) + \overline{B}(\cancel{\overline{C}+C})$

$= B + \overline{B}$

$= 1$

Distributive

$\boxed{A + BC = (A+B)(A+C)}$

c) $Y + \overline{X}Z + X\overline{Y} = X + Y + Z$

$(Y+X)(\cancel{Y+\overline{Y}}) + \overline{X}Z$

$= Y + X + \overline{X}Z$

$= Y + X + Z$

Simplification

$\boxed{A + \overline{A}B = A + B}$

d) $\overline{X}\,\overline{Y} + \overline{Y}Z + XZ + XY + Y\overline{Z} = \overline{X}\,\overline{Y} + XZ + Y\overline{Z}$

$= \overline{X}\,\overline{Y} + \overline{Y}Z(X+\overline{X}) + XZ + XY(Z+\overline{Z}) + Y\overline{Z}$

$= \overline{X}\,\overline{Y} + X\overline{Y}Z + \overline{X}\,\overline{Y}Z + XZ + XYZ + XY\overline{Z} + Y\overline{Z}$

$= \overline{X}\,\overline{Y}(\cancel{1+Z}) + XZ(\cancel{\overline{Y}+1+Y}) + Y\overline{Z}(\cancel{X+1})$

$= \overline{X}\,\overline{Y} + XZ + Y\overline{Z}$

13

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13. a) $A B \bar{C} + B \bar{C} \bar{D} + BC + \bar{C} D = \underline{B + \bar{C} D}$

$= A B \bar{C} + BC + B \bar{C} \bar{D} + \bar{C} D$

$= A B \bar{C} + BC(A+1) + B \bar{C} \bar{D} + \bar{C} D (B+1)$

$= A B \bar{C} + ABC + BC + B \bar{C} \bar{D} + B \bar{C} D + \bar{C} D$

$= AB(\bar{C}+C) + B\bar{C}(\bar{D}+D) + BC + \bar{C} D$

$= AB + B\bar{C} + BC + \bar{C} D$

$= B(\bar{C}+C) + AB + \bar{C} D$

$= B + AB + \bar{C} D$

$= B(1+A) + \bar{C} D$

$= \underline{B + \bar{C} D}$

b) $WY + \bar{W} Y \bar{Z} + WXZ + \bar{W} X \bar{Y} = \underline{WY + \bar{W} X \bar{Z} + \bar{X} Y \bar{Z} + X \bar{Y} Z}$

$= WY + \bar{W} Y \bar{Z}(X+\bar{X}) + WXZ(Y+\bar{Y}) + \bar{W} X \bar{Y}(Z+\bar{Z})$

$= WY + \bar{W} X Y \bar{Z} + \bar{W} \bar{X} Y \bar{Z} + WXYZ + WX\bar{Y}Z + \bar{W} X \bar{Y} Z + \bar{W} X \bar{Y} \bar{Z}$

$= WY(1+XZ) + \bar{W} X \bar{Z}(Y+\bar{Y}) + \bar{X} Y \bar{Z}(W+\bar{W}) + X \bar{Y} Z (W+\bar{W})$

$= \underline{WY + \bar{W} X \bar{Z} + \bar{X} Y \bar{Z} + X \bar{Y} Z}$

c) $A\bar{D} + \bar{A}B + \bar{C}D + \bar{B}C = \underline{(\bar{A}+\bar{B}+\bar{C}+\bar{D})(A+B+C+D)}$

$= \overline{\overline{A\bar{D} + \bar{A}B + \bar{C}D + \bar{B}C}}$

$= \overline{\overline{A\bar{D}} \cdot \overline{\bar{A}B} \cdot \overline{\bar{C}D} + \overline{\bar{B}C}}$

$= \overline{(\bar{A}+D)(A+\bar{B})(C+\bar{D})(B+\bar{C})}$

$= \overline{\bar{A}ACB + \bar{A}AC\bar{C} + \bar{A}A\bar{D}B + \bar{A}A\bar{D}\bar{C}}$
$\overline{+ \bar{A}\bar{B}CB + \bar{A}\bar{B}C\bar{C} + \bar{A}\bar{B}\bar{D}B + \bar{A}\bar{B}\bar{D}\bar{C}}$
$\overline{+ DACB + DAC\bar{C} + DA\bar{D}B + DA\bar{D}\bar{C}}$
$\overline{+ D\bar{B}CB + D\bar{B}C\bar{C} + D\bar{B}\bar{D}B + D\bar{B}\bar{D}\bar{C}}$

$= \overline{\bar{A}\bar{B}\bar{C}\bar{D} + ABCD}$

$= \underline{(A+B+C+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})}$

14

Tuesday, 25 February 2020 12:35 pm

Given $A \cdot B = 0$ $\quad A + B = 1$

$$(A+C)\cdot(\bar{A}+B)\cdot(B+C) = BC$$

$$= A\bar{A}B + A\bar{A}C + C\bar{A}B + C\bar{A}C +$$
$$ABB + ABC + CBB + CBC$$

$$= C\bar{A}B + C\bar{A} + AB + ABC + CB + CB$$

$$= \bar{A}BC + \bar{A}C + AB + ABC + BC$$

$$= \bar{A}C(B+1) + AB(1+C) + BC$$

$$= \bar{A}C + \underbrace{\cancel{AB}}_{0,\ \text{given}} + BC$$

$$= C(\bar{A}+B)\cdot(A+B) = 1,\ \text{given}$$

$$= C\bar{A}A + C\bar{A}B + CBA + CBB$$

$$= CB(\bar{A}+A+1)$$

$$= BC$$

15

Tuesday, 25 February 2020 12:48 pm

a) $\bar{A}\bar{C} + \bar{A}BC + \bar{B}C$

$= \bar{A}\bar{C} + \bar{A}BC + \bar{B}C\cdot 1$

$= \bar{A}\bar{C} + \bar{A}BC + \bar{B}C(1+\bar{A})$

$= \bar{A}\bar{C} + \bar{A}BC + \bar{B}C + \bar{A}\bar{B}C$

$= \bar{A}\bar{C} + \bar{A}C(B+\bar{B}) + \bar{B}C$

$= \bar{A}\bar{C} + \bar{A}C + \bar{B}C$

$= \bar{A}(\bar{C}+C) + \bar{B}C$

$= \bar{A} + \bar{B}C$

b) $\overline{(A+B+C)} \cdot \overline{ABC}$

$= \bar{A}\bar{B}\bar{C} \cdot (\bar{A}+\bar{B}+\bar{C})$

$= \bar{A}\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}\bar{C}$

$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

$= \bar{A}\bar{B}\bar{C}$

c) $AB\bar{C} + AC$

$= A(B\bar{C} + C)$

$= A(C+B)$

d) $\bar{A}\bar{B}D + \bar{A}\bar{C}D + BD$

$= D(\bar{A}\bar{B} + \bar{A}\bar{C} + B)$

$= D(B + \bar{A} + \bar{A}\bar{C})$

$= D(B + \bar{A}(1+\bar{C}))$

$= D(B+\bar{A})$

e) $(\overline{\bar{A}+B})(\overline{\bar{A}+\bar{C}})(\overline{A\bar{B}C})$

$= (A\bar{B})(AC)(\bar{A}+B+\bar{C})$

$= A\bar{B}C(\bar{A}+B+\bar{C})$

$= \underbrace{\cancel{A\bar{A}}\bar{B}C}_{0} + \underbrace{A\cancel{B\bar{B}}C}_{0} + \underbrace{A\bar{B}\cancel{C\bar{C}}}_{0}$

$= 0$

16.

a) $\bar{X}\bar{Y} + XYZ + \bar{X}Y$

$= \bar{X}(\bar{Y}+Y) + XYZ$

$= \bar{X} + XYZ$

$= \bar{X} + YZ$

Simplification

$\boxed{A + \bar{A}B = A + B}$

b) $X + Y(Z + \overline{X+Z})$

$= X + Y(Z + \bar{X}\cdot\bar{Z})$

$= X + Y(Z + \bar{X})$

$= X + YZ + \bar{X}Y$

$= X + Y + YZ$

$= X + Y(1+Z)$

$= X + Y$

c) $\bar{W}X(\bar{Z} + \bar{Y}Z) + X(W + \bar{W}YZ)$

$= \bar{W}X\bar{Z} + \bar{W}X\bar{Y}Z + WX + \bar{W}XYZ$

$= X(\bar{W}\bar{Z} + \bar{W}\bar{Y}Z + W + \bar{W}YZ)$

$= X(W + \bar{W}(\bar{Z} + \bar{Y}Z + YZ))$

$= X(W + \bar{W}(\bar{Z} + Z))$

$= X(W + \bar{W})$

$= X$

d) $(AB + \bar{A}\bar{B})(\bar{C}\bar{D} + CD) + \overline{AC}$ De Morgan's

$= AB\bar{C}\bar{D} + ABCD + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A} + \bar{C}$

$= \bar{A}(\bar{B}\bar{C}\bar{D} + \bar{B}CD + 1) + \bar{C}(1 + AB\bar{D}) + ABCD$

$= \bar{A} + \bar{C} + ABCD$

$= \bar{A} + \bar{C} + BCD$

$= \bar{A} + \bar{C} + BD$

Simplification

$\boxed{A + \bar{A}B = A + B}$

17

Wednesday, 26 February 2020 1:05 am

17 a) $F = A\bar{B}C + \bar{A}\bar{C} + AB$

$= \overline{\overline{A\bar{B}C}} + \overline{\overline{\bar{A}\bar{C}}} + \overline{\overline{AB}}$

$= (\overline{\bar{A} + B + \bar{C}}) + (\overline{A + C}) + (\overline{\bar{A} + \bar{B}})$

<u>OR only</u>

b) $F = A\bar{B}C + \bar{A}\bar{C} + AB$

$= \overline{\overline{A\bar{B}C + \bar{A}\bar{C} + AB}}$

$= \overline{(\overline{A\bar{B}C})(\overline{\bar{A}\bar{C}})(\overline{AB})}$

<u>AND only</u>

18

Wednesday, 26 February 2020 1:10 am

18 a) $\overline{A\overline{B}+\overline{A}B} = (\overline{A}+B)(A+\overline{B})$

b) $\overline{(\overline{V}W+X)Y+\overline{Z}} = \overline{(\overline{V}W+X)Y}\cdot\overline{Z}$

$= ((\overline{\overline{V}W+X})+\overline{Y})\cdot\overline{Z}$

$= (\overline{\overline{V}W}\cdot\overline{X}+\overline{Y})Z$

$= ((V+\overline{W})\overline{X}+\overline{Y})\cdot Z$

c) $\overline{WX(\overline{Y}Z+Y\overline{Z})+\overline{W}\,\overline{X}(\overline{Y}+Z)(Y+\overline{Z})}$

$= \overline{WX(\overline{Y}Z+Y\overline{Z})}\cdot\overline{\overline{W}\,\overline{X}(\overline{Y}+Z)(Y+\overline{Z})}$

$= (\overline{WX}+(\overline{\overline{Y}Z+Y\overline{Z}}))\cdot(\overline{\overline{W}\,\overline{X}}+\overline{(\overline{Y}+Z)}+\overline{(Y+\overline{Z})})$

$= (\overline{W}+\overline{X}+\overline{\overline{Y}Z}\cdot\overline{Y\overline{Z}})\cdot(W+X+Y\cdot\overline{Z}+\overline{Y}Z)$

$= (\overline{W}+\overline{X}+(Y+\overline{Z})(\overline{Y}+Z))(W+X+Y\overline{Z}+\overline{Y}Z)$

d) $\overline{(A+\overline{B}+C)(\overline{A}\,\overline{B}+C)(A+\overline{B}\,\overline{C})}$

$= (\overline{A+\overline{B}+C})+(\overline{\overline{A}\,\overline{B}+C})+(\overline{A+\overline{B}\,\overline{C}})$

$= (\overline{A}B\overline{C})+(\overline{\overline{A}\,\overline{B}}\cdot\overline{C})+(\overline{A}\cdot\overline{\overline{B}\,\overline{C}})$

$= \overline{A}B\overline{C}+(A+B)\cdot\overline{C}+\overline{A}(B+C)$