

# ENGG1000

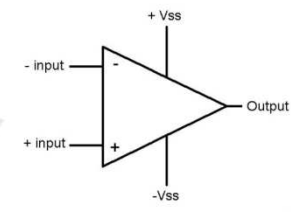
Electrical Stream 2018

Week 5 – Op-Amps and Filters

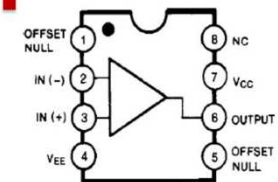
Never Stand Still

Faculty of Engineering

School of Electrical Engineering and Telecommunications

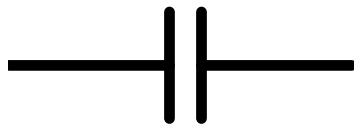


**OP-AMP**



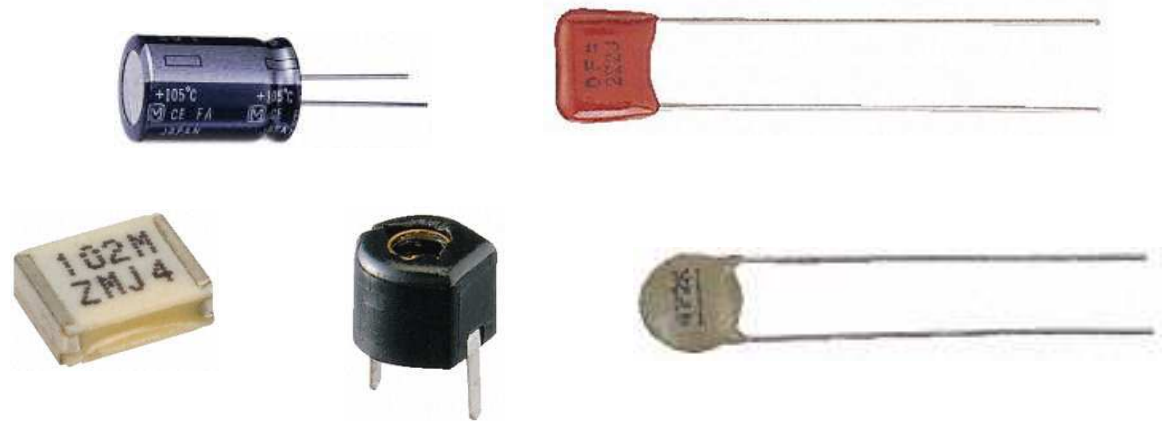
# The Capacitor

- Two conducting plates close to each other
  - Separated by an insulator (dielectric)
  - Stores charge when a voltage is applied
  - Releases charge when voltage taken away



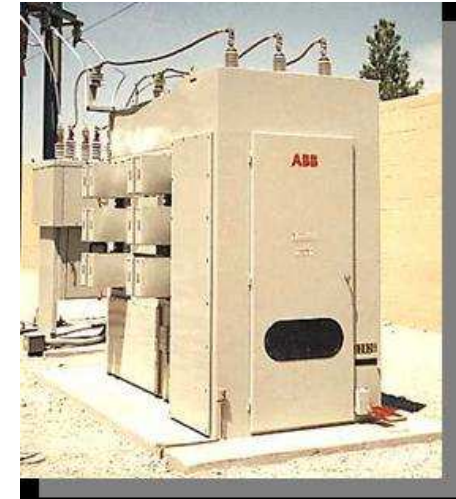
$$q = CV$$

source: <http://australia.rs-online.com>



# The Capacitor

- Capacitance is denoted as  $C$ 
  - In Farads
  - Typical values:  $\sim 1$  pF to  $\sim 10\mu\text{F}$
  - Only certain values available
    - Like resistors
- Values shown on package
- Or use LCR bridge
- Tend to very imprecise
  - 10%-20% typical



# The Capacitor

- Since current is the flow of charge, i.e.

$$I = \frac{dq}{dt}$$

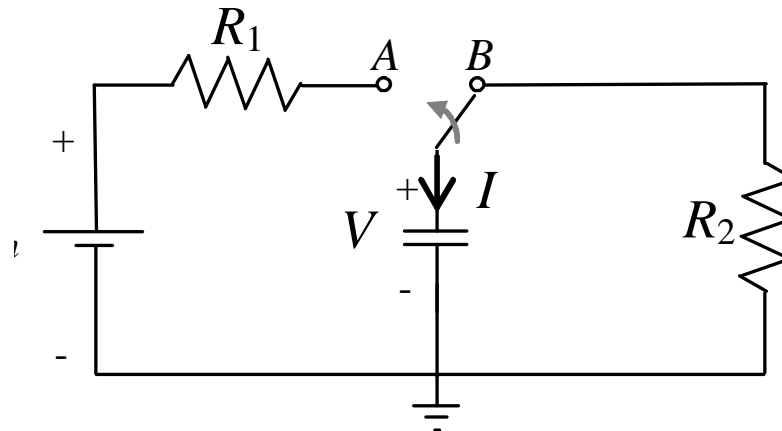
we have the following linear relationship between voltage and current:

$$I(t) = C \frac{dV(t)}{dt}$$

- $I(t)$ ,  $V(t)$  : capacitor current, voltage as a function of time  $t$
- Capacitor voltage constant  $\Rightarrow$  no current
- Voltage changing linearly  $\Rightarrow$  current constant

# First-order Capacitive Circuits

- Example of capacitive behaviour
- Suppose switch  $B \rightarrow A$  in circuit below
  - Before this, no voltage across capacitor  $C$   
 $\therefore V = 0$
  - After switch, capacitor will charge up to  $V_{in}$



# First-order Capacitive Circuits

- Voltage around loop:

$$V_{in} = R_1 I(t) + V(t) = R_1 C \frac{dV(t)}{dt} + V(t)$$

- Differential equation, solution

$$V(t) = V_{in} \left( 1 - e^{-\frac{t}{R_1 C}} \right)$$

$$- \quad I(t) = C \frac{dV(t)}{dt} \rightarrow I(t) = \frac{V_{in}}{R_1} e^{-\frac{t}{R_1 C}}$$

- So at time of switching ( $t=0$ ) and after ( $t \rightarrow \infty$ )

$$t = 0 : V(0) = 0$$

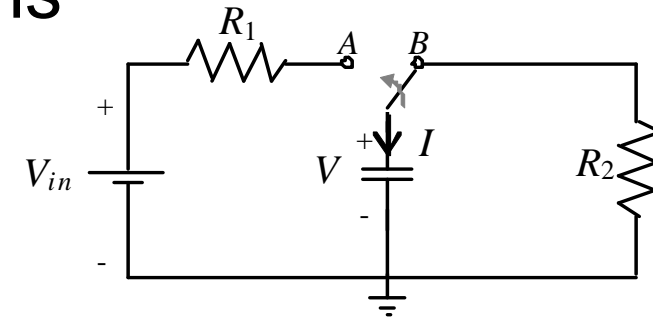
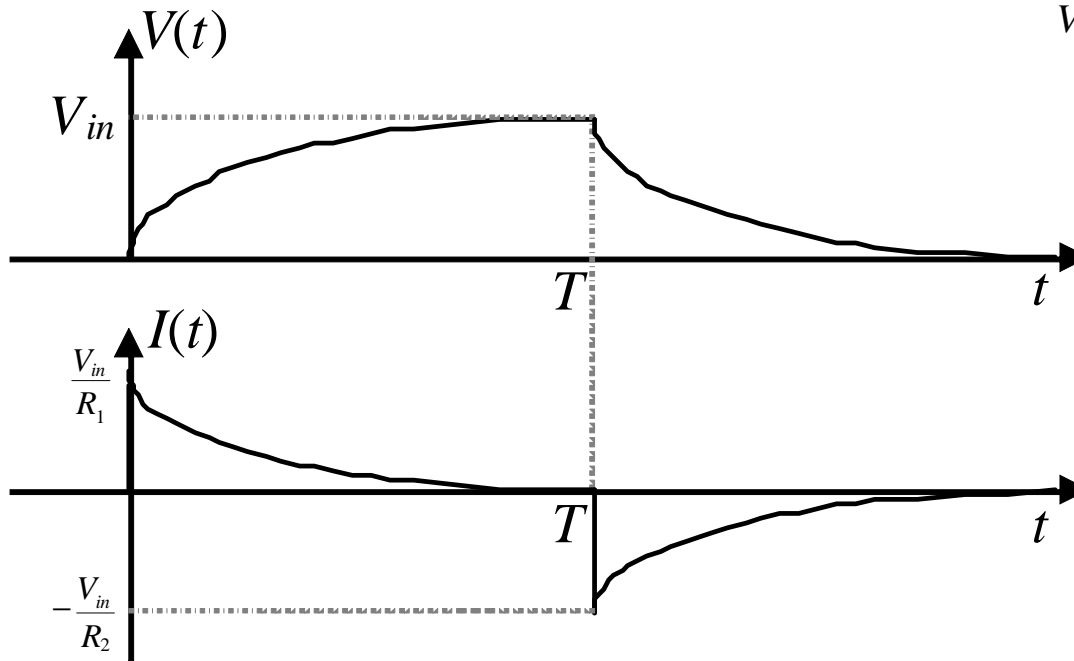
$$t \rightarrow \infty : V(\infty) = V_{in}$$

$$t = 0 : I(0) = \frac{V_{in}}{R_1}$$

$$t \rightarrow \infty : I(\infty) = 0$$

# First-order Capacitive Circuits

- Voltage and current waveforms



**B  $\rightarrow$  A at  $t = 0$**   
**A  $\rightarrow$  B at  $t = T$**

# First-order Capacitive Circuits

- Capacitor voltage for circuits undergoing a change in voltage from  $V_{initial}$  to  $V_{final}$  at time  $t = 0$ :

$$V(t) = V_{final} - (V_{final} - V_{initial})e^{-\frac{t}{RC}}$$

- $R$  is the charging/discharging resistance
  - Needs to be determined based on the circuit surrounding the capacitor

- In our example

- During charging  $R = R_1$

$$V(t) = V_{in} \left( 1 - e^{-\frac{t}{R_1 C}} \right)$$

- During discharging  $R = R_2$

$$V(t) = V_{in} e^{-\frac{t-T}{R_2 C}}$$



# Capacitors: Peak Detector

Diode output follows positive part of waveform only

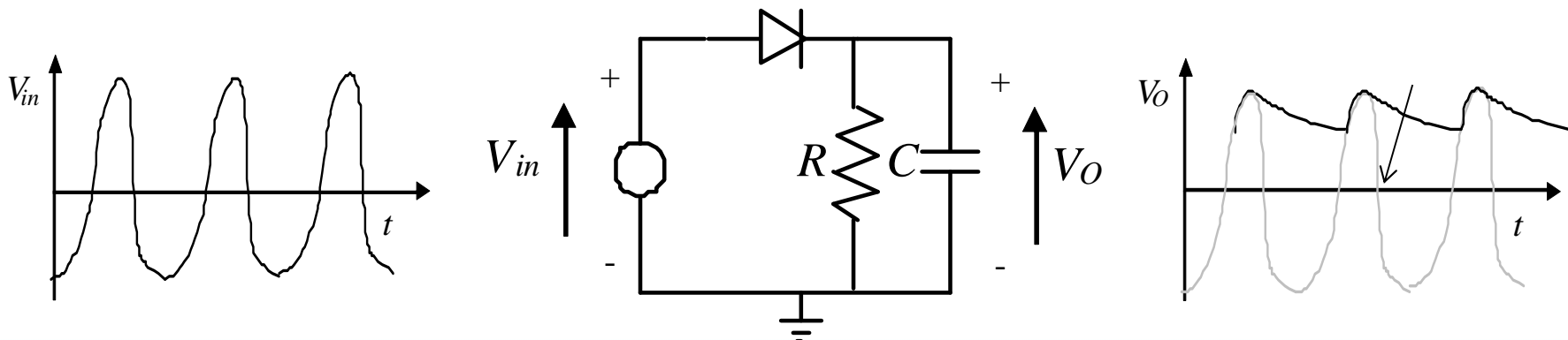
Increasing  $V_{in}$ :

- $C$  charges rapidly through diode

Increasing  $V_{in}$ :

- $C$  discharges slowly through  $R$
- Choose  $R$  to control rate of decay

approximately  
constant if  $R$  is large

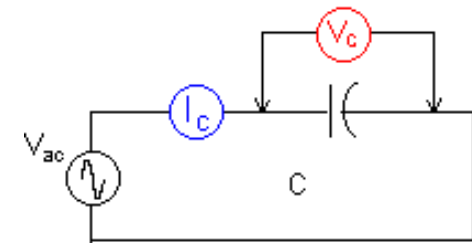


# Capacitors: Frequency Behaviour

- Consider a capacitor with a constant frequency sinusoid input
- How are the voltage and current through the capacitor related?

$$V_{ac} = V_c = V_0 \sin(2\pi ft)$$

$$I_c = C \frac{dV_c}{dt} = 2\pi f C V_0 \cos(2\pi ft)$$



- Effective resistance of the capacitor (called the ‘impedance’)

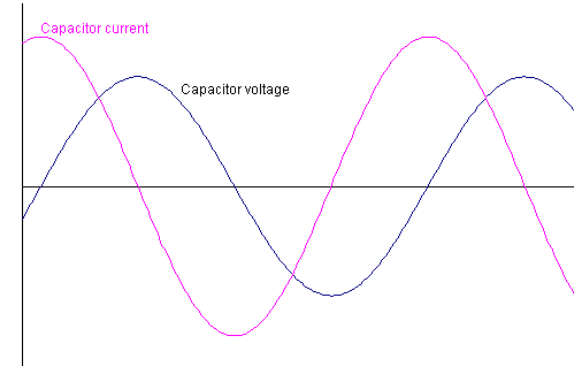
$$X_c = \frac{|V_c|}{|I_c|} = \frac{1}{2\pi f C}$$

- Ratio of voltage amplitude to current amplitude

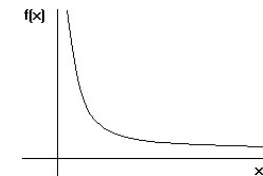
# Capacitors: Frequency Behaviour

- Note that the current and voltage are out of phase
  - One peaks while the other is zero

$$X_C = \frac{|V_C|}{|I_C|} = \frac{1}{2\pi fC}$$



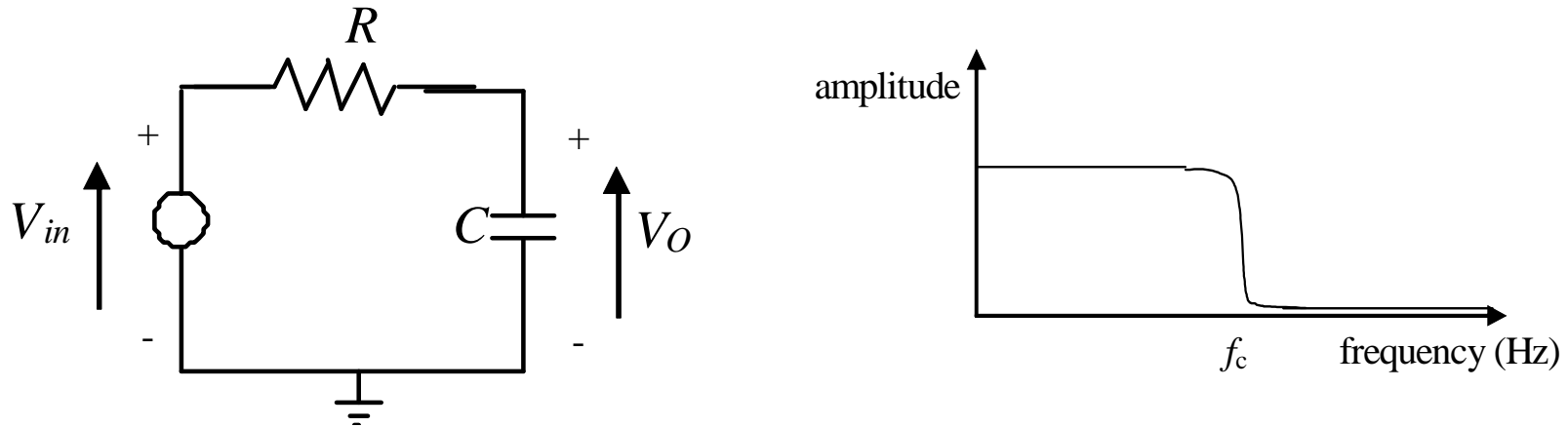
- Impedance as function of frequency
- Note that the capacitor offers a large 'resistance' at low frequencies, but a small resistance at high frequencies
- Forms the basis of 'FILTERS'
- Circuits to separate frequency components in an input signal...



Source: <http://people.sinclair.edu/nickreeder/eet155/mod04.htm>

# Capacitors: Low Pass Filter

Passes low frequencies, blocks high frequencies

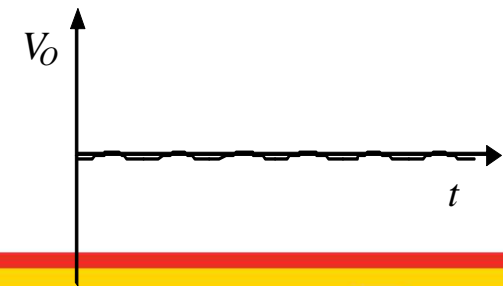
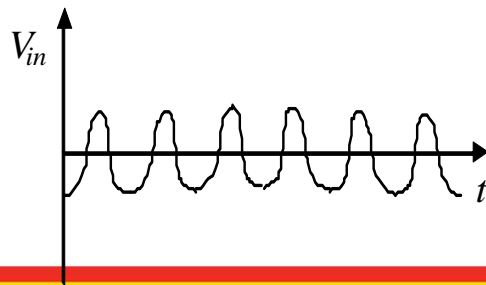
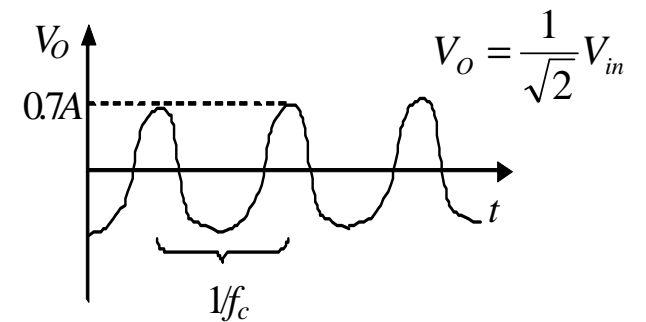
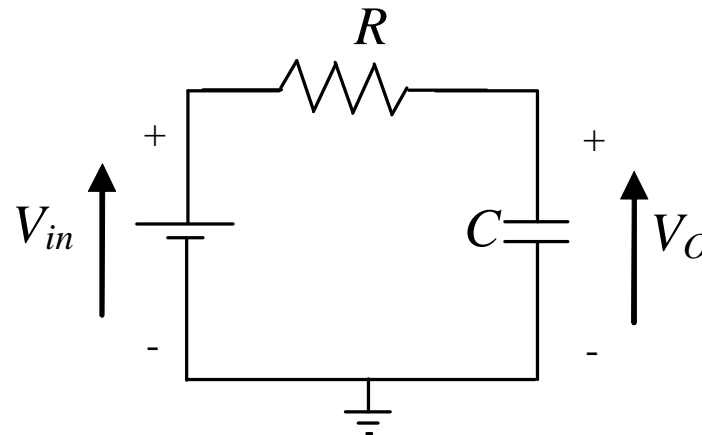
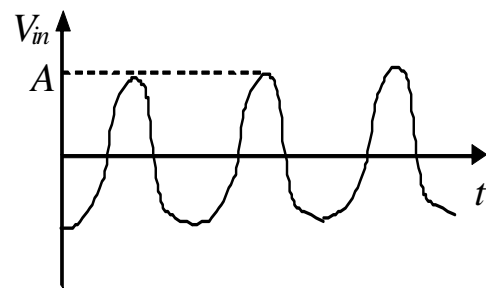
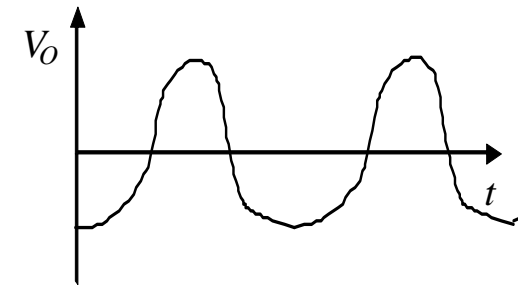
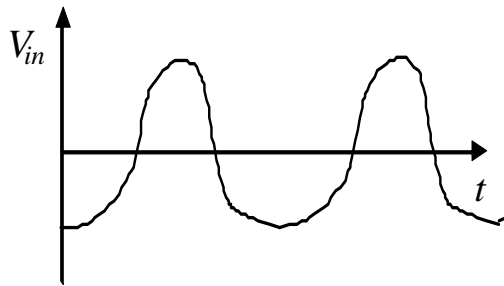


Choose

$$V_o = \frac{1}{\sqrt{1 + (2\pi fRC)^2}} V_{in}$$

$$f_c = \frac{1}{2\pi RC}$$

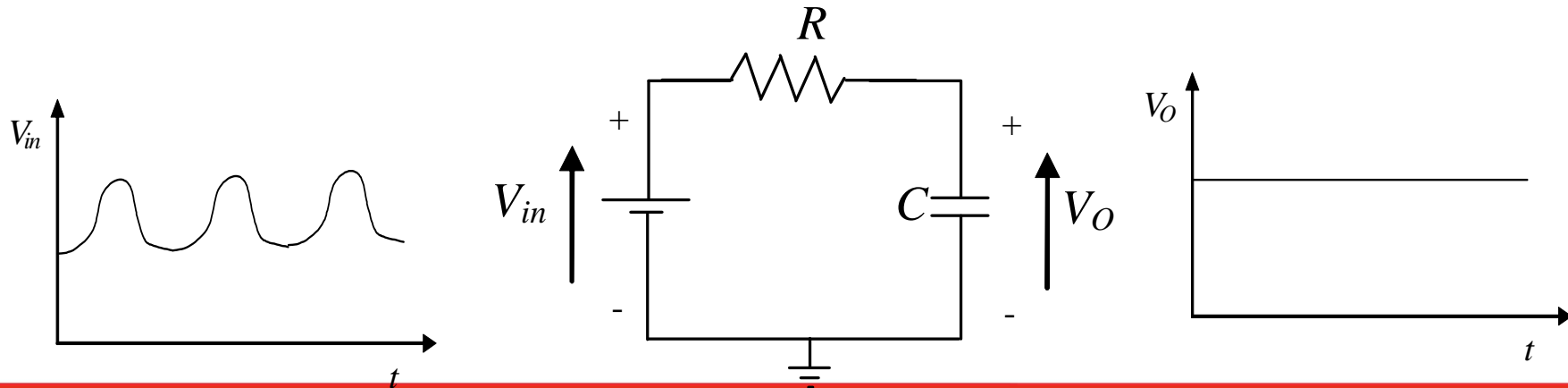
# Capacitors: Low Pass Filter



# Capacitors: Low Pass Filter

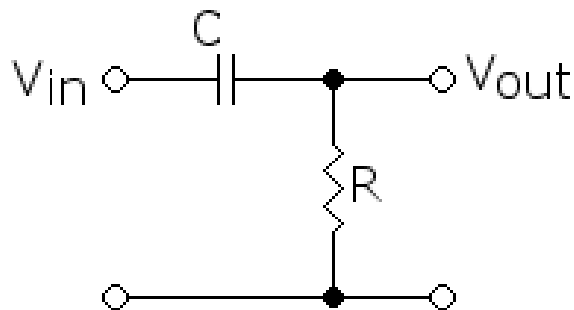
Suppose you have a voltage that is oscillating

- You want just the DC part, or some smooth version of the voltage
- e.g. removing fluorescent light noise from an IR signal



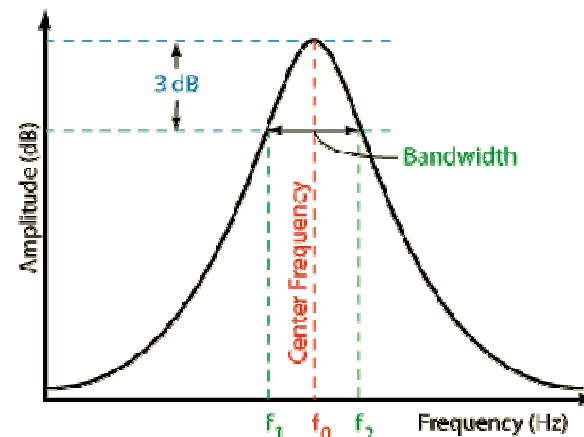
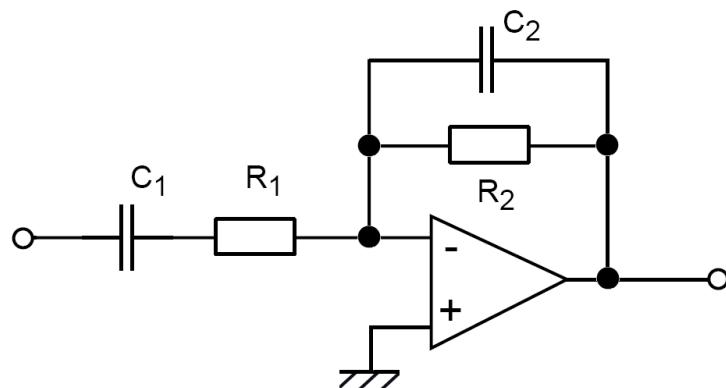
# Capacitors: High Pass Filter

- On the other hand, you could reverse the location of the capacitor and the resistor and you would get a 'high pass filter'
  - A filter that removes low frequency parts of the signal, but passes the high frequencies



# Capacitors: Band Pass Filter

- Cascade a low pass filter with a high pass filter to pass only a range of frequencies
  - Choose the LPF cut-off and HPF cut-off to obtain the desired response

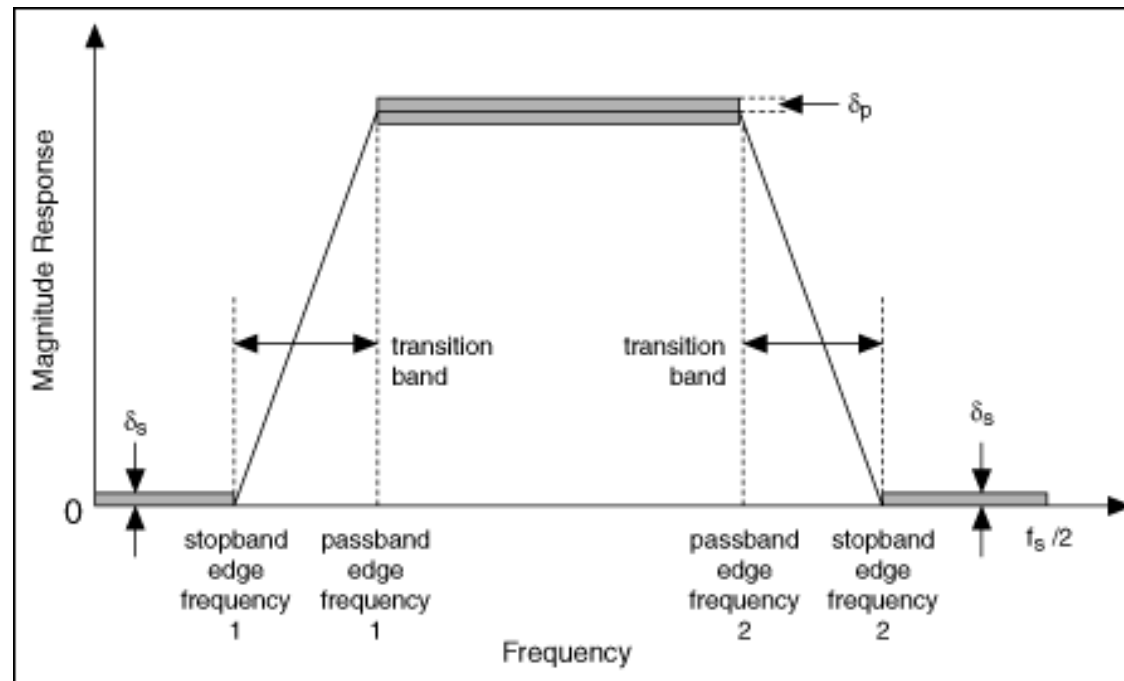


Sources: [http://www.changpuak.ch/electronics/Simple\\_Bandpass.php](http://www.changpuak.ch/electronics/Simple_Bandpass.php); <http://www.rane.com/note170.html>



# Filters - Characteristics

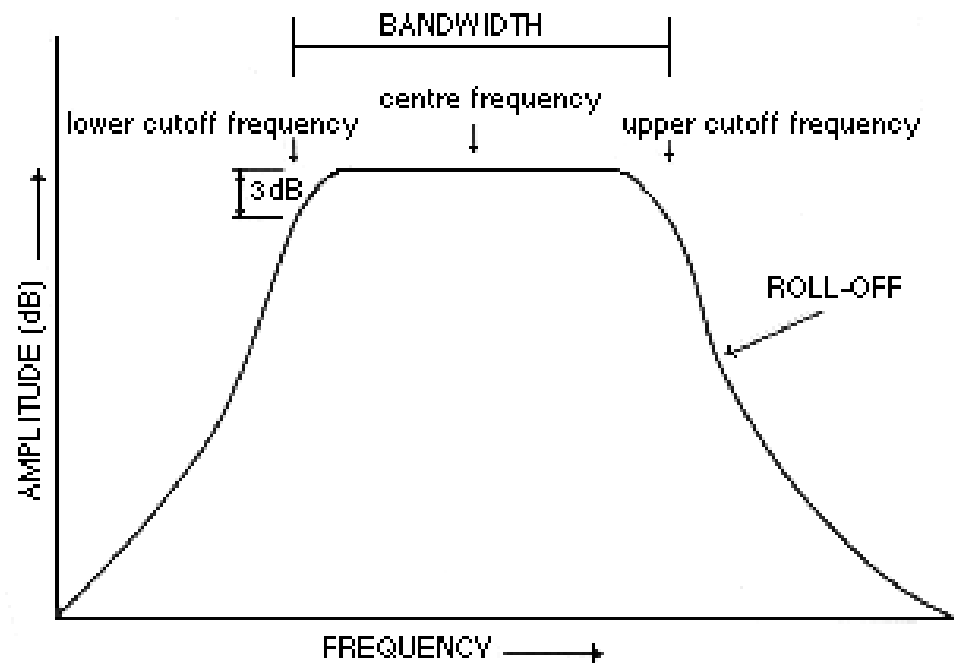
- Examples: Bandpass Filter



Source: [http://zone.ni.com/reference/en-XX/help/371988B-01/lvdfdtconcepts/dfd\\_filter\\_spec/](http://zone.ni.com/reference/en-XX/help/371988B-01/lvdfdtconcepts/dfd_filter_spec/)

# Filters - Characteristics

- Examples:



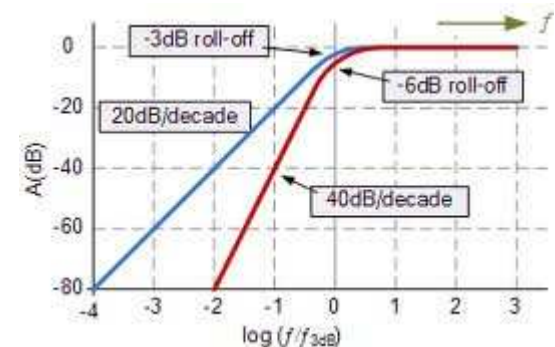
Source: <http://www.sfu.ca/sonic-studio/handbook/Filter.html>

# Filters - Notes

- Be wary of the input and output impedance of your filters
  - Op-amp based circuits are a good choice here
- Cut-off frequency – usually defined as the 3dB point
  - At this point output amplitude drops to  $1/\sqrt{2}$  (and power by  $1/2$ )
  - If you're looking to remove a component, you'll need to cut-off well before this point
- Be wary of variations in component tolerances
  - Actual cut-off frequencies will be different from what you calculate
  - Consider variable resistors to allow 'in the field' tuning

# Filters - Notes

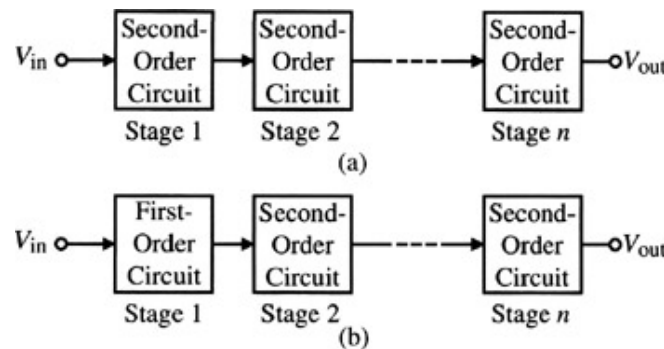
- Roll-off issues
  - The rate at which it attenuates out of band signals
  - First order filter rolls off at 20dB/dec
  - Recall:  $X_c \propto \frac{1}{|f - f_0|} \rightarrow P_c \propto \frac{1}{|f - f_0|^2}$
  - For every power of 10 increase in frequency, the output power will drop by 1/100 (we say '20dB/dec')
- Faster roll-off – Higher-order filters
  - Second order filter rolls-off at 40dB/dec
  - Here,  $X_c \propto \frac{1}{|f - f_0|^2} \rightarrow P_c \propto \frac{1}{|f - f_0|^4}$
  - Can form a by cascading first order filters together



Source: <http://www.electronics-tutorials.ws/blog/second-order-filter.html>

# Filters - Notes

- Cascaded Filters
  - The attenuation of each stage multiplies



- Can be problematic
  - Leads to many large number of components, wires, stages -> lots of places it can go wrong...
  - Tolerances of components reducing performance
  - Impedance matching between stages (buffers...)
  - Consider ICs (Integrated Circuits)

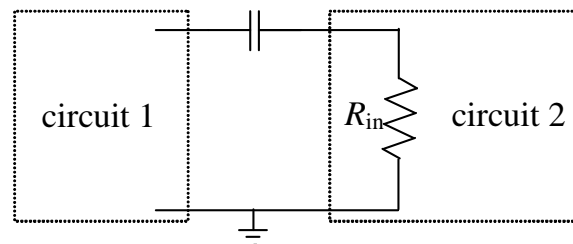
Source: <http://www.globalspec.com/reference/56055/203279/9-6-second-order-switched-capacitor-circuits>

# Capacitors: Other Uses

Lots of other uses

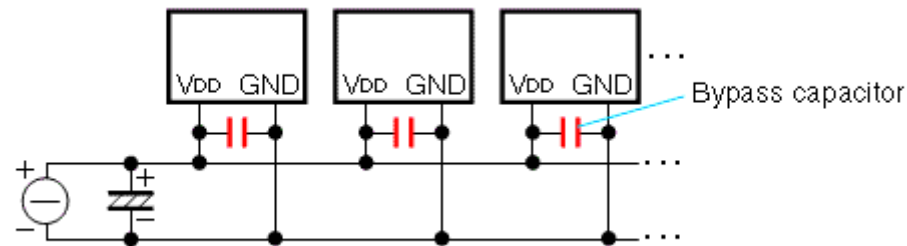
You may come across:

- Coupling capacitor or DC Blocking capacitor
  - Only passes AC signals (DC blocked)
  - Might see in amplifier circuits
  - Together with input impedance of next stage, creates high pass filter



# Capacitors: Other Uses

- Bypass or decoupling capacitor
  - Placed across supply rails of an integrated circuit
  - Helps to improve stability of supply voltage (removes fluctuations)
  - Used in voltage regulators
    - Research for yourself!!!



Source: [http://www2.renesas.com/faq/en/f\\_tech.html](http://www2.renesas.com/faq/en/f_tech.html)

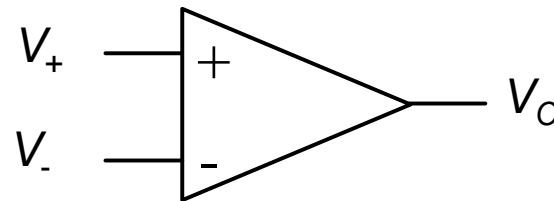
# The Ideal Amplifier

- Earlier: transistor amplifier
- Amplifiers important
  - $\therefore$  refined over many years
- Characteristics of an ideal amplifier
  - Very large / infinite gain
  - Infinite input resistance
    - Does not load previous circuit stages
  - Negligible output resistance
    - Want output to act as ideal voltage source



# The Operational Amplifier

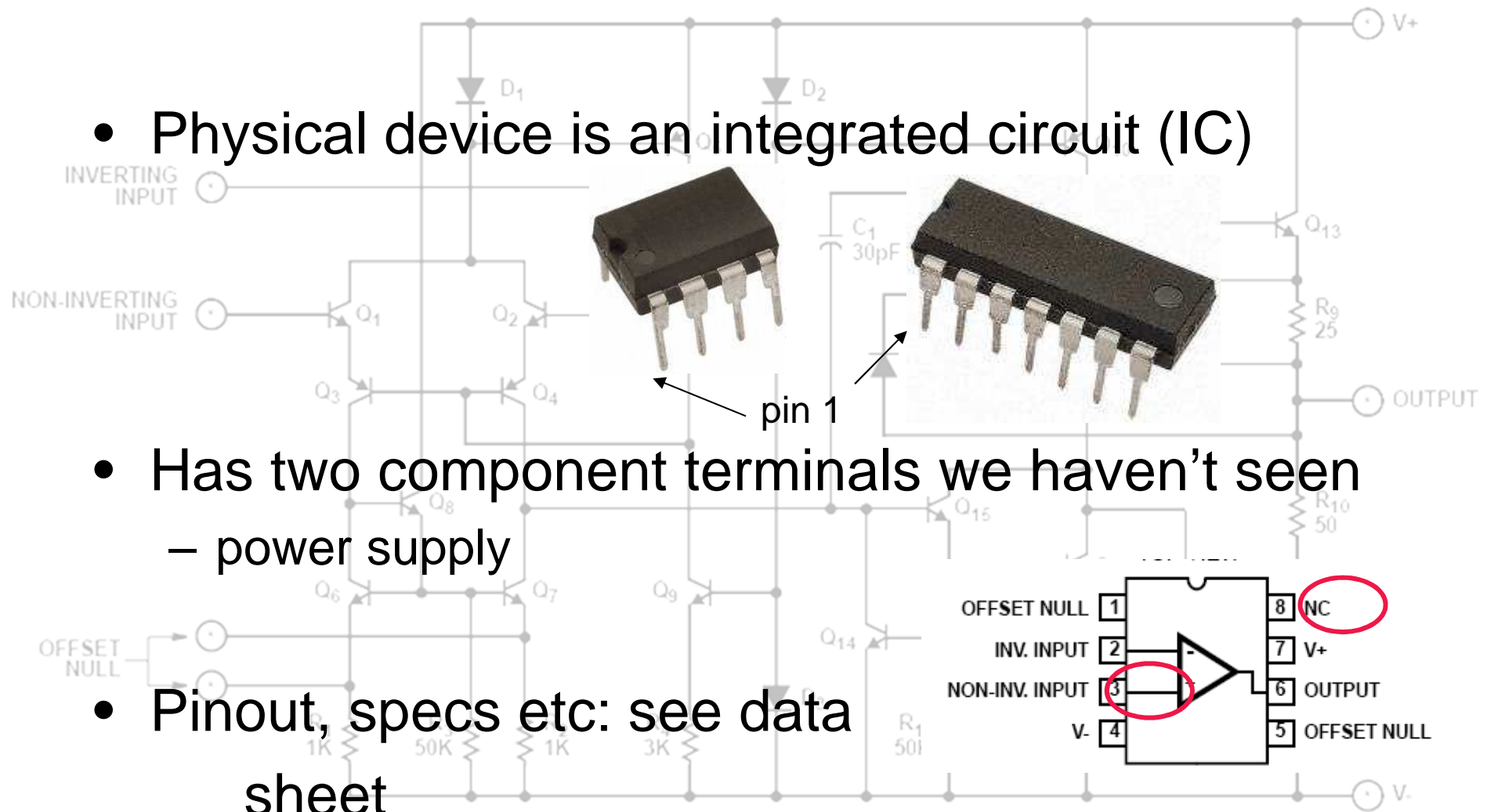
- An approximation to the ideal amplifier



- Gain  $A \geq 10^5$
  - Input resistance  $\geq 10\text{M}\Omega$
  - Output resistance  $< 500\Omega$
- Differential amplifier  $V_o = A(V_+ - V_-)$

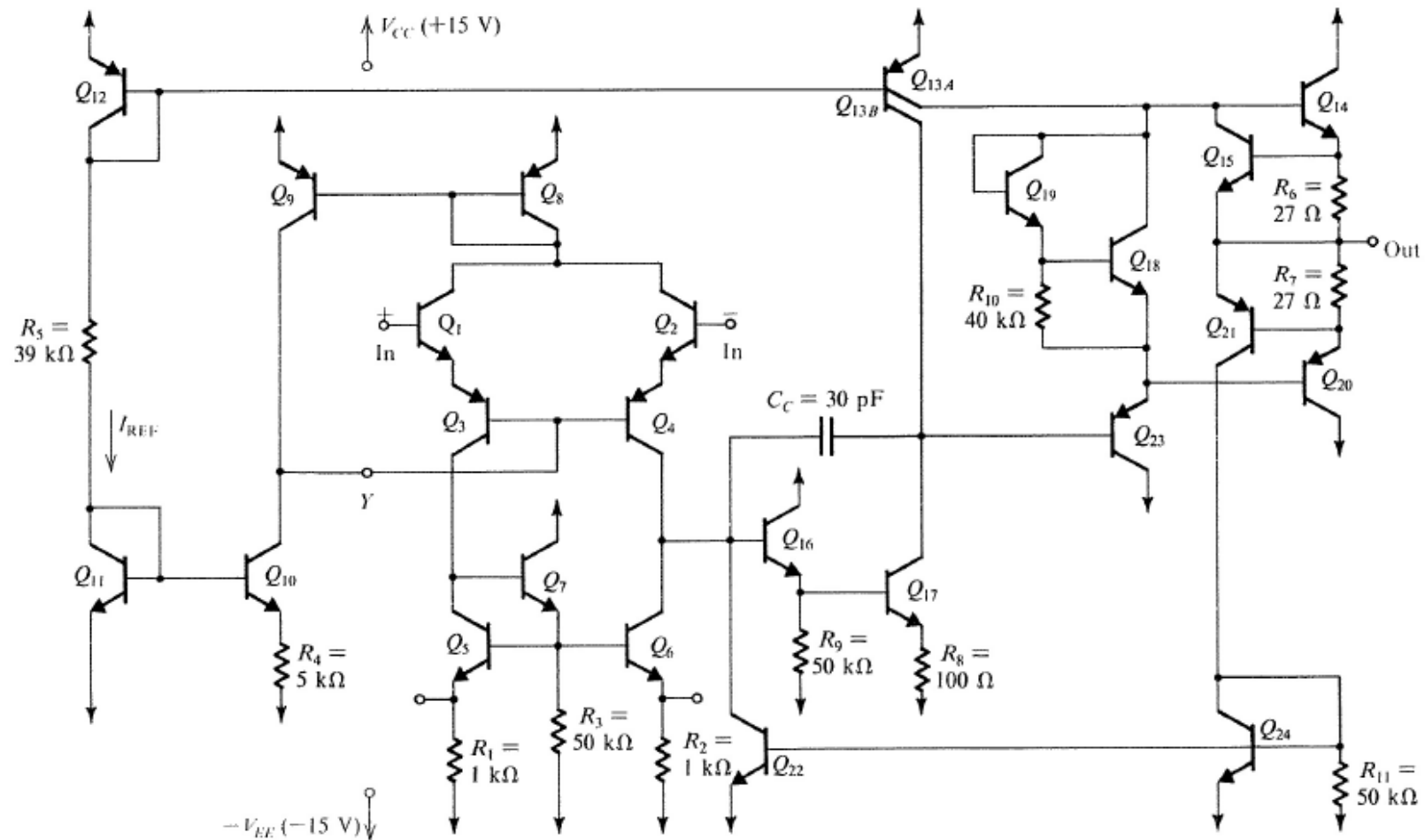
# The Operational Amplifier

- Physical device is an integrated circuit (IC)
- Has two component terminals we haven't seen
  - power supply
- Pinout, specs etc: see data sheet



sources: <http://australia.rs-online.com>, <http://engg1000.ee.unsw.edu.au/datasheets/Analog-ICs/LM741.pdf>

# LM741 Circuit



# The Operational Amplifier

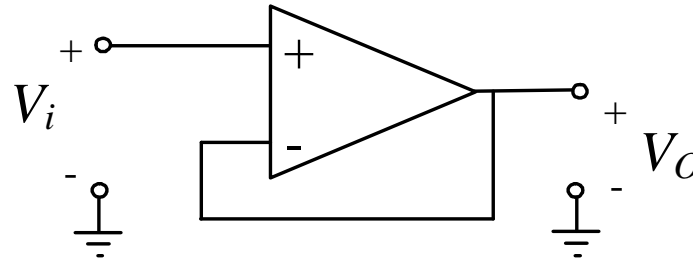
- The op-amp is assumed ideal

⇒ Two rules for analysis:

- Because the input resistance is infinite, *no current enters either input terminal*
- Because the gain  $A = \frac{V_o}{V_+ - V_-}$  is infinite, if the output voltage is **finite**, then  $V_+ - V_- = 0$ , i.e. *both input terminals have identical voltage*

# Negative Feedback

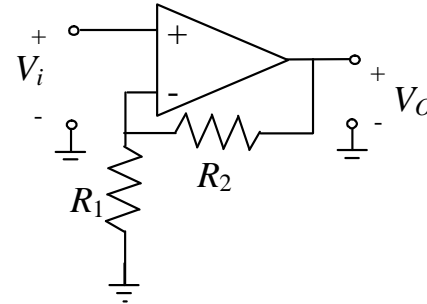
- Path from output to inverting input
  - Provides stable *closed-loop* gain
    - See notes for more detail
- Op-amp buffer



- $V_- = V_+$ , so  $V_o = V_- = V_+ = V_i$
- Gain of 1
- Output is isolated from input (input resistance is  $\infty$ )
- Input current small ( $\mu\text{A}$  or even  $\text{pA}$ ), output large ( $\text{mA}$  or even 100s of  $\text{mA}$ )

# Negative Feedback Applications

- Non-inverting amplifier



- No current flows into input terminals, so

$$V_- = \frac{R_1}{R_1 + R_2} V_o$$

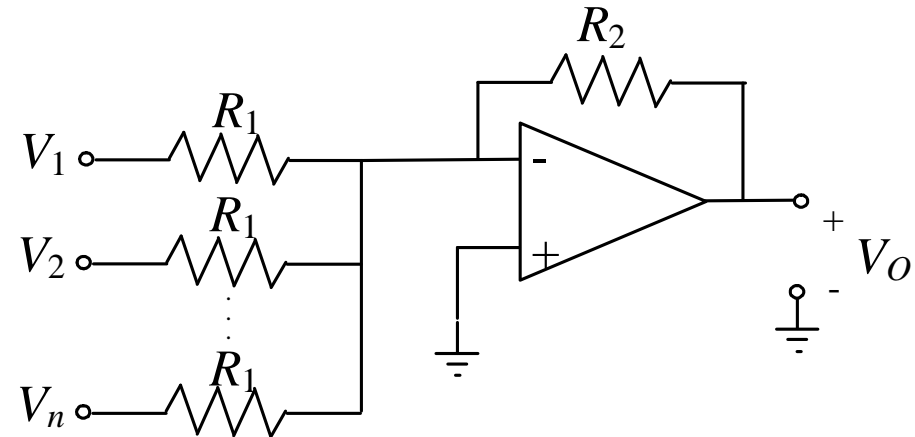
- So  $V_o = \frac{R_1 + R_2}{R_1} V_i$  and closed-loop gain is  $1 + \frac{R_2}{R_1}$

- Choose  $R_1, R_2$  to give desired gain

# Negative Feedback Applications

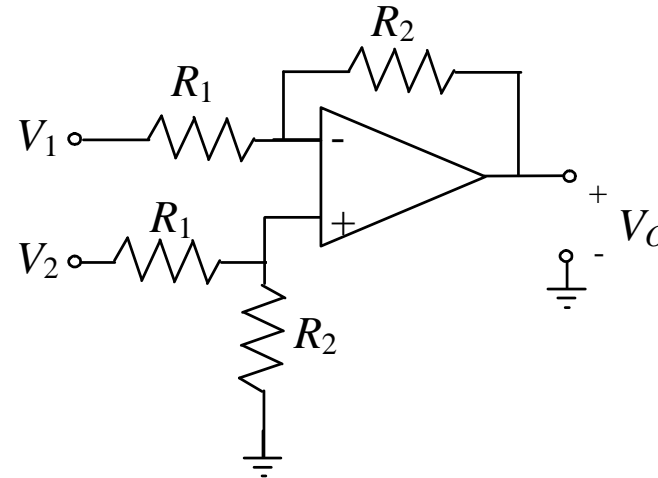
- Adder (inverting)

$$V_O = -\frac{R_2}{R_1}(V_1 + V_2 + \dots + V_n)$$



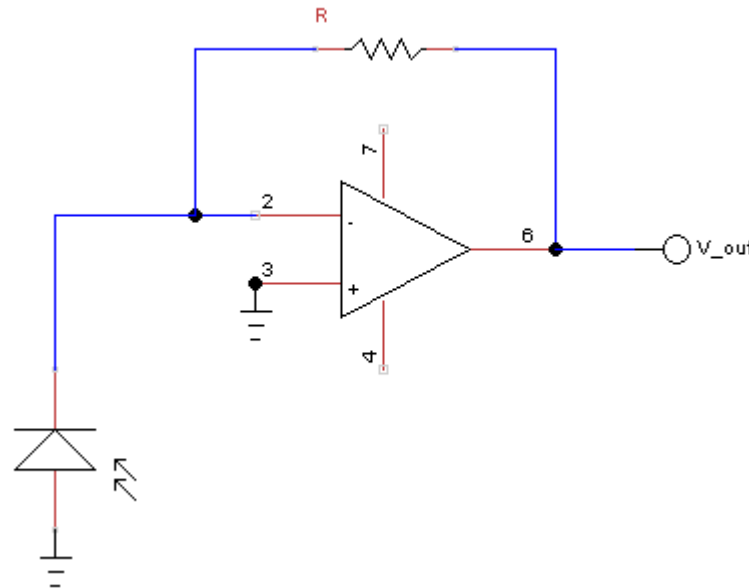
- Subtractor  
– Differential Amp!!!

$$V_O = \frac{R_2}{R_1}(V_2 - V_1)$$



# Current to Voltage Converter

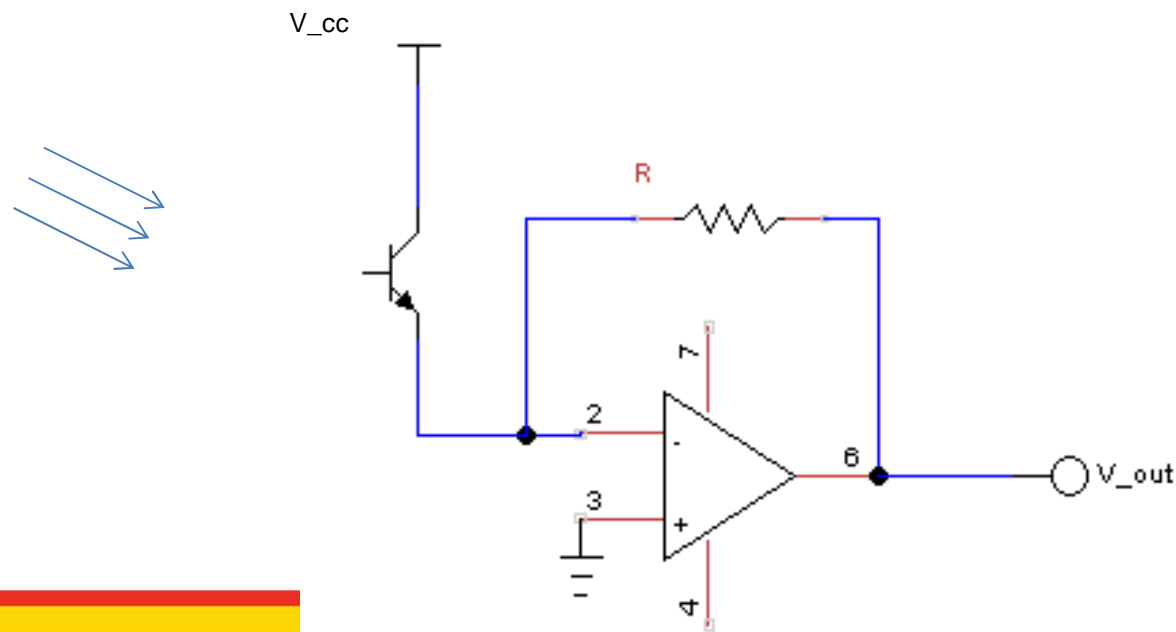
- For example, to convert the current received from a photodiode into a voltage signal
- Photodiode operates in reverse-biased mode
- Output voltage:  $V_{out} = R \times I$  , where  $I$  is the photocurrent (proportional to the incident light intensity)





# Current to Voltage Converter

- An equivalent circuit using a phototransistor
- Note: incident light provides the base current
- In a photodiode, light creates the reverse leakage current



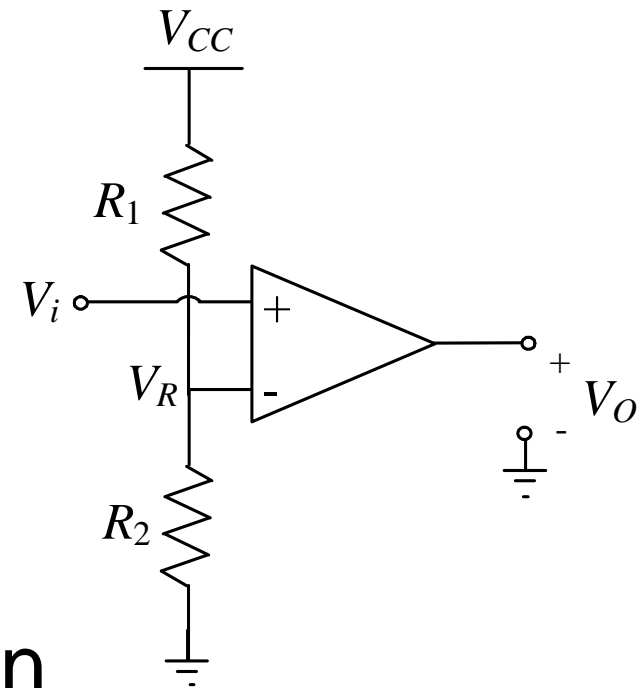
# Positive or No Feedback

- Positive feedback
  - Path from output to non-inverting input
    - Causes output to grow exponentially
- No feedback
  - Small difference in inputs  $\rightarrow$  very large difference in output
    - $V_o = A(V_+ - V_-)$  ,  $A \sim 10^5$
- In practise, output voltage is limited by supply to  $\pm V_{CC}$

# Application of No Feedback

- Comparator

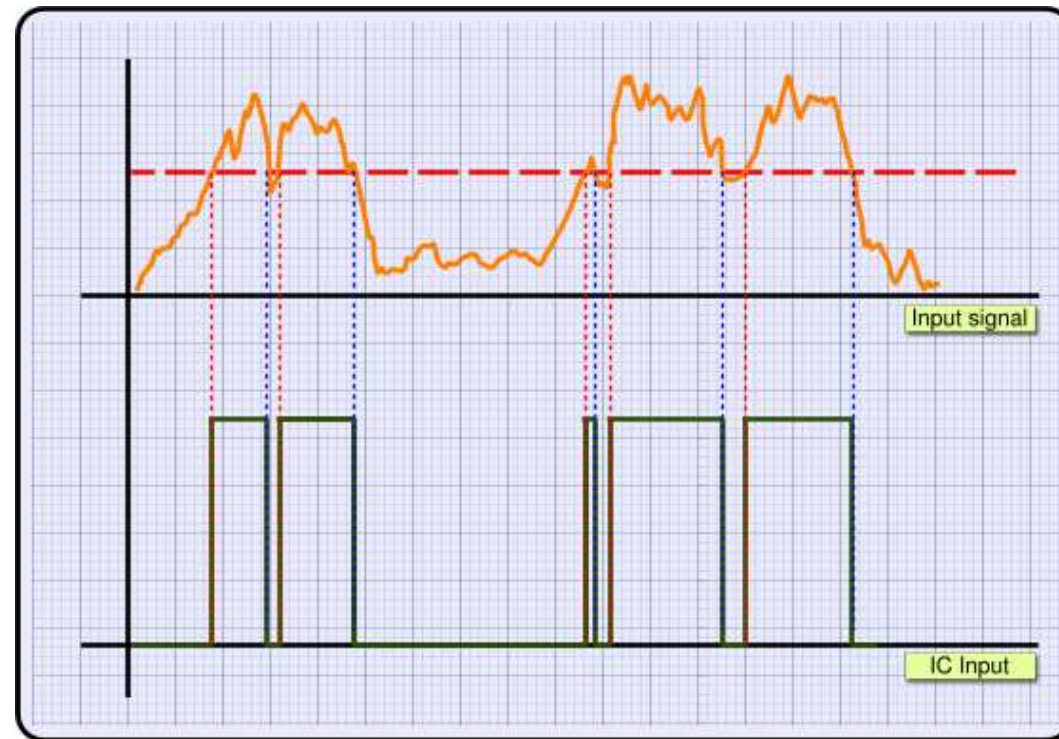
- $V_R$  is constant
  - “reference voltage”
- $V_i > V_R \Rightarrow V_O = +V_{CC}$
- $V_i < V_R \Rightarrow V_O = -V_{CC}$



- Good for threshold detection
- Can reverse terminals to get opposite behaviour

# Comparators under Noise

- Comparators on noisy signals can result in many misdetections



Source: [http://pcbheaven.com/wikipages/The\\_Schmitt\\_Trigger/](http://pcbheaven.com/wikipages/The_Schmitt_Trigger/)

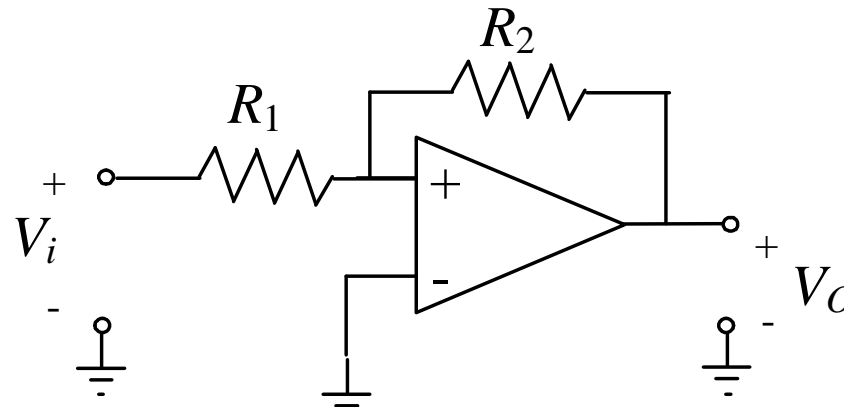
# Schmitt Trigger

- Set-up a lower threshold -> only switch back when the signal falls below the lower threshold:



# Application of Positive Feedback

- Schmitt trigger
  - Comparator not suitable for threshold detection in noise
  - Schmitt trigger uses hysteresis
    - Choose  $R_1$ ,  $R_2$  to determine how much hysteresis



# Schmitt Trigger

- When input goes above 0V, output goes HIGH
- Voltage at +ve terminal will then be:

$$V_+ = V_{CC} - \frac{R_2}{(R_1 + R_2)} (V_{CC} - V_{in})$$

- The output will not swing low until  $V_+ < 0$
- That is, the input must fall to:  $V_{in} = -\frac{R_1}{R_2} V_{CC}$
- In Lab 3, you'll use a slightly different circuit...

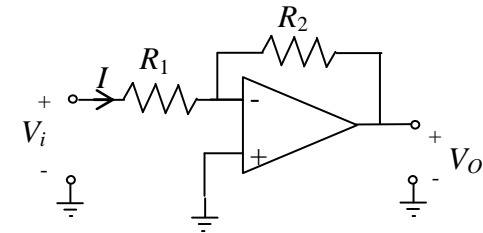
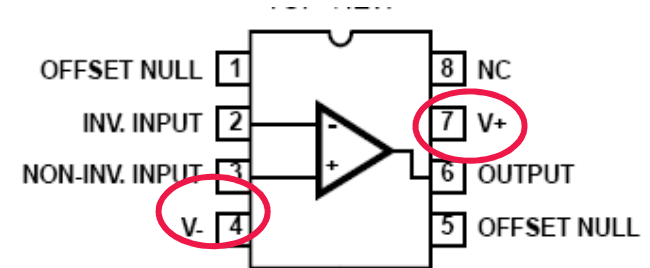
# Practical Considerations

- Op-amps are nearly ideal in many respects
- Cannot produce large output current
  - e.g. max 25 mA for LM741 op-amp
    - Check data sheets
  - An op-amp cannot power a loudspeaker, for example
  - Might need to add a transistor amplifier stage to the output of the op-amp circuit



# Op-amps: Offset

- Recall:
  - Op-amps have a positive and a negative supply
  - “double-sided” supply
- This means
  - Supplying  $-V_{CC}$ , ground and  $+V_{CC}$  to the circuit
  - Input to circuit has average value of zero
    - Can achieve this using???



# Electronics design notes

- Op-amps (negative feedback)
  - Ideal behaviour will only be observed if the inputs and outputs are not saturated
    - i.e. all voltages  $< +V_{CC}$  and  $> -V_{CC}$
  - Ensure that feedback is negative
    - Don't mix up inverting and non-inverting inputs
  - Ensure that there is DC feedback in op-amp circuits
    - e.g. don't put a capacitor in series from the output back to the input (i.e. on the feedback path)

# Electronics design notes

- Op-amps
  - Often the maximum differential input voltage limit is small
    - i.e. the difference between  $V_+$  and  $V_-$
    - e.g. 5V
    - Exceed it and larger input currents will flow, op-amp does not behave as expected
  - Keep leads to power supply short
    - Avoid disturbances from power supply affecting output signal (see Scherz for tips)

Source: Horowitz and Hill, 1989, Scherz, 2000

# Coming Up

- Lab Openings during Mid-session break
  - Thursday 1-5pm (5<sup>th</sup> April)
  - Friday 1-4pm (6<sup>th</sup> April)
  - Lab EEG14
- Week 6 Labs Open (EEG14 & EE214)
  - Monday 2-6pm
  - Thursday 2-6pm
- Circuit Quiz coming up in Week 7
  - Enrolment available on Moodle during Week 6