

NEW SOUTH WALES

HIGHER SCHOOL CERTIFICATE

Mathematics Extension 2

Exercise 40/67

BY JAMES CORONEOS*

Find the following integrals.

1. $\int \frac{x dx}{x^2+4}$ 2. $\int \frac{x dx}{\sqrt{x^2+4}}$ 3. $\int \frac{5x+2}{x^2-4} dx$ 4. $\int \sin x \cos^3 x dx$ 5. $\int \sin x \sec^3 x dx$
6. $\int \cos^2 \frac{x}{2} dx$ 7. $\int x \sin x dx$ 8. $\int x \sec^2 2x dx$ 9. $\int \tan^{-1} 2x dx$ 10. $\int \frac{x^3 dx}{x^2+1}$
11. $\int \frac{x dx}{(x+2)(x+4)}$ 12. $\int \frac{(x-1)(x+1) dx}{(x-2)(x-3)}$ 13. $\int \frac{(2x-1) dx}{x^2+2x+3}$ 14. $\int \frac{x^3 dx}{2x-1}$ 15. $\int \frac{(1+x) dx}{\sqrt{1-x-x^2}}$
16. $\int \frac{dx}{x^2(1-x^2)^{\frac{1}{2}}}$ 17. $\int \frac{dx}{x\sqrt{a^2+x^2}}$ 18. $\int \frac{dx}{x\sqrt{a^2-x^2}}$ 19. $\int \frac{dx}{x\sqrt{x^2-a^2}}$ 20. $\int \frac{x dx}{\sqrt{x+1}}$
21. $\int \frac{\cos^{-1} x}{\sqrt{1-x^2}} dx$ 22. $\int \sqrt{\frac{x+1}{x-1}} dx$ 23. $\int \frac{dx}{x(\log x)^3}$ 24. $\int \sec^4 3x dx$ 25. $\int \frac{dx}{x^2(1-x)}$
26. $\int \frac{dx}{x^2(1+x^2)}$ 27. $\int \frac{dx}{(1+x^2)^2}$ 28. $\int \tan^3 x dx$ 29. $\int \frac{dx}{5+3 \cos x}$ 30. $\int \frac{dx}{3+5 \cos x}$
31. $\int \frac{\sin x dx}{5+3 \cos x}$ 32. $\int \frac{dx}{1+\cos^2 x}$ 33. $\int \frac{dx}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}$ 34. $\int x^2 \sin x dx$
35. $\int \frac{x^2 dx}{(x-1)(x-2)(x-3)}$ 36. $\int \frac{e^x dx}{e^x-1}$ 37. $\int \frac{dx}{3 \sin^2 x + 5 \cos^2 x}$ 38. $\int x^3 e^{5x^4-7} dx$
39. $\int x^5 \log x dx$ 40. $\int \frac{(3x+2) dx}{x(x+1)^3}$ 41. $\int \log x^3 dx$ 42. $\int \frac{dx}{e^x + e^{-x}}$
43. $\int (5x^3 + 7x - 1)^{\frac{3}{2}} \cdot (15x^2 + 7) dx$ 44. $\int \frac{dx}{(x^2+1)(x^2+4)}$ 45. $\int (x^2 + x - 1)^{-1} dx$
46. $\int e^x \sin 2x dx$ 47. $\int (x^2 + x - 1)^{-1} dx$ 48. $\int (x^2 - x)^{-\frac{1}{2}} dx$ 49. $\int \frac{1-2x}{3+x} dx$
50. $\int x^3(4+x^2)^{-\frac{1}{2}} dx$ 51. $\int \frac{\sin 2x dx}{3 \cos^2 x + 4 \sin^2 x}$ 52. $\int \frac{x^2 dx}{1-x^4}$ 53. $\int \frac{dx}{\sin x \cos x}$
54. $\int \log \sqrt{x-1} dx$ 55. $\int \frac{dx}{e^x-1}$ 56. $\int \frac{\sec^2 x dx}{\tan^2 x - 3 \tan x + 2}$ 57. $\int \frac{(x+1) dx}{(x^2-3x+2)^{\frac{1}{2}}}$
58. $\int \sin 2x \cos x dx$ 59. $\int \frac{x dx}{1+x^3}$ 60. $\int x \tan^{-1} x dx$ 61. $\int (1+3x+2x^2)^{-1} dx$
62. $\int (9-x^2)^{\frac{1}{2}} dx$ 63. $\int (9+x^2)^{\frac{1}{2}} dx$ 64. $\int x(9+x^2)^{\frac{1}{2}} dx$ 65. $\int \sec^2 x \tan^3 x dx$
66. $\int x^2 e^{-x} dx$ 67. $\int x e^{x^2} dx$ 68. $\int \sin x \tan x dx$ 69. $\int \sin^4 x \cos^3 x dx$
70. $\int \frac{(x^3+1) dx}{x^3-x}$ 71. $\int \log(x + \sqrt{x^2-1}) dx$ 72. $\int \frac{dx}{(x+1)^{\frac{1}{2}} + (x+1)}$

Evaluate the following definite integrals, leaving results in irrational form.

73. $\int_0^4 \frac{x dx}{\sqrt{x+4}}$ 74. $\int_1^2 \frac{dx}{x(1+x^2)}$ 75. $\int_1^2 \frac{\log x}{x} dx$ 76. $\int_0^1 \cos^{-1} x dx$ 77. $\int_1^2 \frac{(x+1) dx}{\sqrt{-2+3x-x^2}}$
78. $\int_0^{\frac{\pi}{2}} \frac{dx}{\cos^2 x + 2 \sin^2 x}$ 79. $\int_0^1 x \sqrt{1-x^2} dx$ 80. $\int_2^4 x \log x dx$ 81. $\int_1^2 \frac{dx}{x^2+5x+4}$
82. $\int_0^{\frac{\pi}{2}} (1 + \frac{1}{2} \sin x)^{-1} dx$ 83. $\int_0^1 x^2 e^{-x} dx$ 84. $\int_0^1 \frac{(7+x) dx}{1+x+x^2+x^3}$ 85. $\int_0^1 \frac{e^{-2x} dx}{e^{-x}+1}$

*Other resources by James Coroneos are available. Write to P.O. Box 25, Rose Bay, NSW, 2029, Australia, for a catalogue.

- 86.** $\int_0^{\frac{a}{2}} \frac{y}{a-y} dy$ **87.** $\int_0^a \frac{(a-x)^2 dx}{a^2+x^2}$ **88.** $\int_0^1 \frac{(x+3) dx}{(x+2)(x+1)^2}$ **89.** $\int_0^1 \frac{x^2 dx}{x^6+1}$
90. $\int_0^\pi \cos^2 mx dx$, m integral **91.** $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x \sin 2x dx$ **92.** $\int_0^{\frac{a}{2}} x^2 \sqrt{a^2 - x^2} dx$
93. $\int_0^{\frac{\pi}{4}} \sec^2 x \tan x dx$ **94.** $\int_0^1 (x+2)(x^2+4x+5)^{\frac{1}{2}} dx$ **95.** $\int_1^2 x(\log x)^2 dx$
96. $\int_3^4 \frac{x^2+4}{x^2-1} dx$ **97.** $\int_1^4 \frac{x^2+4}{x(x+2)} dx$ **98.** $\int_0^{\frac{\pi}{2}} \frac{\cos x dx}{5-3 \sin x}$ **99.** $\int_0^1 \frac{dx}{(4-x^2)^{\frac{3}{2}}}$
100. $\int_0^{\frac{\pi}{2}} 2 \sin \theta \cos \theta (3 \sin \theta - 4 \sin^3 \theta) d\theta$

