Q1. Find (i) the eccentricities, (ii) the coordinates of the foci, and (iii) the equations of the directrices of the following ellipses:

1. $\frac{x^2}{2} + y^2 = 1.$ 2. $\frac{x^2}{6} + \frac{y^2}{4} = 1.$ 3. $2x^2 + y^2 = 8.$

4. $4x^2 + 9y^2 = 16.$ 5. $x^2 + 16y^2 = 25.$

Q2. Sketch the following loci for varying values of θ:

1. $(5 \cos\theta, 3 \sin\theta)$. 2. $(2 \cos\theta, \sin\theta)$.

3. $(3 \sin\theta, 4 \cos\theta)$. 4. $(1 + 3 \cos\theta, 2 \sin\theta)$.

5. $(2 \cos\theta - 2, \sin\theta + 1)$.

Find (i) the eccentricity, (ii) the coordinates of the foci, and (iii) the equations of the directrices of the following ellipses:

6. $x = 6 \cos\theta, y = 2 \sin\theta$. 7. $x = 4 \cos\theta, y = 3 \sin\theta$.

8. $x = \sqrt{(2)} \cos\theta, y = \sin\theta$. 9. $x = 2 \sin\theta, y = 3 \cos\theta$.

10. Prove that the locus of the point $(2 + 3 \cos\theta, 2 \sin\theta - 3)$ is an ellipse with centre $(2, -3)$. What is the area of the ellipse?

11. Find the distance between the foci of the ellipse

$x = 1 + 4 \cos\theta, y = 1 + \sin\theta$.

Obtain the parametric equations of the following ellipses:

12. $\frac{x^2}{9} + \frac{y^2}{4} = 1.$ 13. $x^2 + y^2 = 4.$

14. $4x^2 + 9y^2 = 9.$ 15. $\frac{(x-1)^2}{4} + y^2 = 1.$

16. $\frac{(x+2)^2}{16} + \frac{(y-1)^2}{9} = 1.$

17. PQ is a diameter of the ellipse $x^2 + 4y^2 = 9$. If the eccentric angle of P is $\frac{\pi}{3}$, what is the eccentric angle of Q? Find the gradients of the tangents P and Q.

18. Sketch the ellipse $4x^2 + 9y^2 = 36$. By using the auxillary circle, find the points on the curve with eccentric angles: $\frac{\pi}{6}, \frac{2\pi}{3}, \frac{\pi}{3}, -\frac{3\pi}{4}$.

19. If the ratio of the areas of an ellipse and its auxilliary circle is 5:9, find the eccentricity of the ellipse.

20. The distance between the foci of an ellipse of eccentricity $\frac{3}{4}$ is 8 cm. Find the area of the ellipse.

21. The coordinates of a point on the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ are $(a \cos\phi, b \sin\phi)$. Find the coordinates of the corresponding point on the auxilliary circle.

Q3. 1. Find the equations of the tangents to the ellipse $x^2 + 4y^2 = 9$ which are parallel to the line $2x + 3y = 0$.

2. Find the equations of the tangent and normal to the ellipse $16x^2 + y^2 = 169$ at the point $(3,5)$. Find also the equation of that diameter of the ellipse which passes through this point.

3. Show that the diameter $x + 2y = 0$ of the ellipse $x^2 + 3y^2 = 1$ bisect the chord which lies along the line $2x - 3y + 1 = 0$.

4. Show that $3x \cos\theta + 2y \sin\theta = 6$ is the tangent to the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ at the point $(2\cos\theta, 3\sin\theta)$.

Show that the tangents at the points where the line $y = 2x$ meets the ellipse are $9x + 8y = \pm 30$.

5. A segment of length 4 units has its two ends lying on two perpendicular lines. Find the locus of a point on the segment distant 1 unit from one end.

6. What are the lengths of the semi-axes of the ellipse $x^2 + 16y^2 = 25$?

Find the equation of the tangent and normal to the ellipse at the point $(3,1)$.

7. Find the equations of the tangents to the ellipse $9x^2 + 16y^2 = 36$ which are perpendicular to the line $x + y = 4$.

Find also the coordinates of the points of contact of these two tangents to the ellipse.

8. The tangent at a point P of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ cuts the x-axis at T, and the perpendicular PN is drawn to the x-axis. If O is the origin, prove that $ON.OT = a^2$.

9. Find the equation of the tangent to the ellipse $36x^2 + 100y^2 = 225$ at the point $(\frac{3}{2}, \frac{6}{5})$. Show that the product of its perpendicular distances from the points $(2,0)$, $(-2,0)$ is $\frac{9}{4}$.

10. Show that the product of the perpendiculars from the points $(c,0)$ $(-c,0)$ on the tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to b^2, if $c^2 = a^2 - b^2$.

11. What are the lengths of the semi axes of the ellipse $9x^2 + 25y^2 = 225$?

    Find the equation of the normal to this ellipse at the point $P(3, 2\frac{2}{5})$.

    This normal intersects the x-axis at G, and N is the foot of the perpendicular drawn from P to the x-axis. Calculate the length of GN.

12. Find the equation of the tangent to the ellipse $5x^2 + 9y^2 = 45$ at the point $(2,5/3)$.

    Prove that the feet of the perpendiculars drawn from the points $(2,0)$, $(-2,0)$ to this tangent both lie on the circle $x^2 + y^2 = 9$.

13. Find the equations of the tangents to the ellipse $4x^2 + 9y^2 = 1$ which are perpendicular to the line $y = 2x + 3$.

14. Find the equations of the normals at the points $P(1,4)$ and $Q(7,2)$ on the conic $x^2 + 4y^2 = 65$. Also find the equation of the line through the intersection point of the normals at P and a which passes through the origin.

15. The straight line $x - 2y + 3 = 0$ cuts the ellipse $x^2 + 2y^2 = 9$ at two points A, B. The tangents to the ellipse at A,B meet at C. Find the coordinates of C.

16. Find the equation of the tangent at $(4,-1)$ to the ellipse $9x^2 + 25y^2 = 169$. Prove that the circle $x^2 + y^2 + 28x - 23y = 152$ touches the ellipse at this point.

17. P is the point $(6,4)$ Q the point $(-8,3)$ on the ellipse $x^2 + 4y^2 = 100$.

    The tangents at P, Q meet at T, the normals meet at G. Find the coordinates of T,G, and show that the diameter through G is perpendicular to PQ.

18. Find the equation of the normal to the ellipse $x^2 + 4y^2 = 100$ at the point $P(8,3)$.

    If the normal at P meets the major axis in G, and OY is the perpendicular from the centre O to the tangent at P, prove that PG.OY is equal to the square on the minor semi-axis.

Q4. 1. The moon's mean distance from the earth is 384 000 km. The eccentricity of the moon's orbit is 0.055. Show that the nearest and farthest distances of the moon from the earth is approximately 363 000 k m and 405 000 km respectively. Note: the orbit of the moon is an ellipse with the earth at one of the foci. (See fig. 13.a)

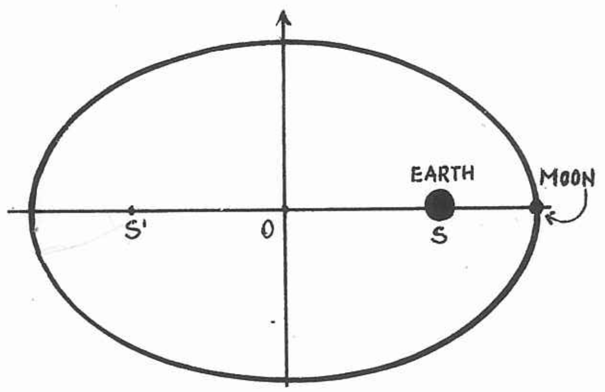


fig. 13a.

2. Write the equations of the circles which are internally and externally touching the ellipse $4x^2 + 25y^2 = 100$ provided the 3 conics are concentric.

3. Show the volume of the solid formed when the ellipse $\frac{x^2}{64} + \frac{y^2}{16} = 1$ is rotated about the
   (i) x axis is $^{512\pi}/_3$ cu. units.
   (ii) y axis is $^{1024\pi}/_3$ cu. units.

4. Show the equations of the tangents to the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ which are parallel to the diameter $y = 2x$ are $y = 2x \pm \sqrt{(109)}$.

5. S is the focus of the **ellipse** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with coordinates $(ae,0)$. If $P(x_1,y_1)$ is any point on the curve. Show that $PS = a - ex_1$.

6. The tangent and normal to the ellipse $x^2 + 3y^2 = 2$ at the point $(1, \frac{1}{\sqrt{(3)}})$ meet the x axis at P and the y axis at Q respectively. Prove that PQ touches the ellipse.

7. P is a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ with centre 0. A line drawn through 0, parallel to the tangent to the ellipse at P meets the ellipse at R and R'. Show that the area of triangle RPR' is 6 square units.

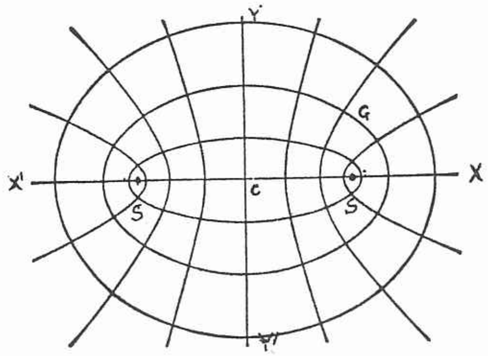


fig. 30

Case 5: $-a^2 < p < -b^2$. When p is less than $-b^2$ the semi-minor axis $\surd(b^2 + p)$ becomes imaginary $(b^2 + p < 0)$ and (3) will represent a HYPERBOLA. The angle between the asymptotes of the hyperbola will become greater and greater as p approaches $-a^2$. As p becomes a quantity nearly equal to $-a^2$ the hyperbola gradually approaches C and widens out as in fig.30.

Case 6: $p \leqq -a^2$. When $p = -a^2$ the conic flattens out into the two coincidental lines $x^2 = 0$. When $p < -a^2$ both semi axes of the conic become imaginary and therefore the confocal becomes wholly imaginary..

EXERCISES - UNIT TWO

Q1. Find (i) the eccentricities, (ii) the coordinates of the foci, (iii) the equations of the directrices, and (iv) the equations to the asymptotes of the following hyperbolas:

1. $\frac{x^2}{4} - y^2 = 1$. 2. $x^2 - y^2 = 4$. 3. $\frac{x^2}{12} - \frac{y^2}{4} = 1$.

4. $\frac{x^2}{144} - \frac{y^2}{25} = 1$. 5. $x^2 - 4y^2 = 36$. 6. $4x^2 - 4y^2 = 9$

7. Sketch the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ and find the acute angle included between its asymptotes.

* 8. Find the asymptotes of the hyperbola $(x - 3)^2 - (y + 1)^2 = 8$.

* 9. Show that $x^2 - y^2 + 2x + 4y = 0$ is the equation of a hyperbola. Find the centre of the hyperbola and prove that the asymptotes are perpendicular.

10. The foci of a hyperbola are the points $(\pm 5,0)$. Find the equation of the curve if $e = \frac{5}{4}$.

Q2. 1. Find the equation of the tangent at the point (2,1) on the hyperbola $x^2 - 2y^2 = 2$. Prove that the product of the perpendiculars from the foci to this tangent is 1.

2. Find the coordinates of the midpoint of the chord $2y = x + 1$ of the hyperbola $4x^2 - 9y^2 = 36$.

3. Find the equations of the tangents to the curve $x^2 - 2y^2 = 1$ parallel to the diameter $4y = 3x$.

4. Trace accurately, the hyperbola with foci (-2, 0), (3,0) and semi-transverse axis of lengths 2 units.

5. Prove that the triangle formed by the asymptotes and any tangent to the curve $\frac{x^2}{4} - \frac{y^2}{2} = 1$ is of constant area.

6. Find the coordinates of points on the hyperbola $x^2 - 9y^2 = 9$, the normals at which are parallel to an asymptote.

7. Any straight line cuts the hyperbola $4x^2 - 8y^2 = 64$ at P,Q and the asymptotes at P', Q'. Show that P'P = QQ'.

8. Find the acute angle between the tangents which can be drawn from the point (2,3) to the hyperbola $x^2 - y^2 = 2$.

9. An asymptote of the hyperbola $2x^2 - 8y^2 = 3$ meets a directrix in M and the tangent at a vertex in N. Prove that the lines joining the vertex to M and the corresponding focus to N are parallel.

10. Prove that the tangent at the point $P(x_1,y_1)$ of the curve $\frac{x^2}{16} - \frac{y^2}{9} = 1$ meets the asymptotes at the points $A\left(\frac{48}{3x_1 - 4y_1}, \frac{36}{3x_1 - 4y_1}\right)$ $B\left(\frac{48}{3x_1 + 4y_1}, \frac{-36}{3x_1 + 4y_1}\right)$ respectively show that P is the midpoint of AB.

11. Show that any point on the hyperbola $b^2x^2 - a^2y^2 = a^2b^2$ can be written as $(a \sec \emptyset, b \tan \emptyset)$. Find the equation of the tangent at this point.

12. Show that the locus of the point $\{\frac{a}{2}(t + \frac{1}{t}), \frac{b}{2}(t - \frac{1}{t})\}$ for varying values of t is the hyperbola $x^2/a^2 - y^2/b^2 = 1$. Derive the equation of the tangent at the point parameter t.

Q3. 1. Find the equations of the tangents to the hyperbola $x2 - 4y^2 = 9$ which are parallel to the line $2x + 3y = 0$.

2. Find the equations of the tangent and normal to the hyperbola $16x^2 - y^2 = 12$ at the point $P(1,-2)$. Find also the equation of the diameter through P.

3. Find the equations of the normals to the hyperbola $9x^2 - y^2 = 32$ at the points $(2,-2)$, $(-3,7)$.
Find also the point of intersection of these normals, and the equation of the chord of contact of the tangents from that point to the hyperbola.

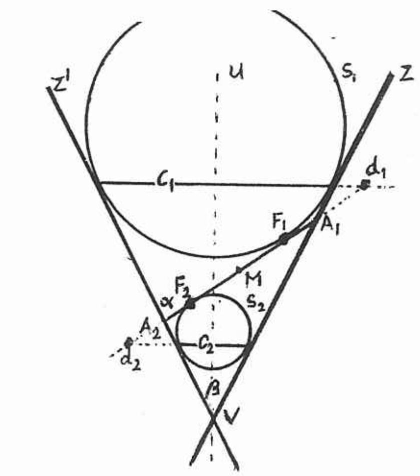
*4. A perpendicular OY is drawn from the centre of the hyperbola $4x^2 - y^2 = 1$ to any tangent. Prove that the equation of the locus of Y is $4(x^2 + y^2)^2 = x^2 - 4y^2$.

5. Write down the equations of the asymptotes of the hyperbola $9x^2 - 4y^2 = 20$. Prove that, if the tangent at $P(-2,2)$ meets the asymptotes in L, M respectively, then P is the midpoint of LM.

6. Prove that the point $P\ (3\frac{1}{2},7)$ lies on one of the asymptotes of the hyperbola $4x^2 - y^2 = 35$. Find the equation of the tangent (other than the asymptote itself) from P to the hyperbola.

7. Find the equation of the normal to the hyperbola $9x^2 - y^2 = 5$ at the point $P(1,2)$.
The normal at P meets the axis $y = 0$ in G, and OY is the perpendicular from the centre O to the tangent at P. Prove that $PG.OY = 5$.

8. Draw a neat sketch for the parabola as a conic section by clearly showing every significant detail.

9. 

Looking "side on" at right circular cone vertex V, the plane of the ellipse with vertices A_1A_2 (looks like a line) is touching two spheres S_1 and S_2 (they look like 2 circles) at the foci F_1 and F_2. C_1 and C_2 are the circles of contact between the spheres and the cone. The plane of the ellipse intersects the planes of the circles of contact in the lines d_1, d_2 (they look like two points) which are the directrixes corresponding to F_1 and F_2 respectively. UV is the axis of the cone, M is the centre of the ellipse. (Note that the centre of the ellipse M is not lying on the axis UV!) VZ and VZ' are generators of the cone. α is the angle $Z'A_2A_1$ what the plane of the ellipse makes with generator Z'V. β is the angle ZVZ' i.e. the "vertex angle".

Suppose that S_2 (the sphere closer to the vertex V) remains fixed and the plane of the conic (ellipse) is being tilted in such a manner that meanwhile A_1 moves "up" on generator VZ (thus α decreases) the plane of the ellipse remains in contact with both of the spheres. S_1 also remains in contact with the cone, however its size increases.

As α approaches the value of β describe:

(a) How the size and position of C_1 change?

(b) How the positions of the two directrixes d_1 and d_2 and the foci F_1, F_2 change?

(c) How does it affect the size of the ellipse and the position of M, the centre of the ellipse?

(d) What is the "limiting shape" of the conic as β is approached?

(e) What is the "limiting position" of the directrix d_1, the focus F_1 and of the centre M as β is approached?

10. (a) Determine the real values of p for which the equation $C \equiv \frac{x^2}{9+p} + \frac{y^2}{4+p} = 1$ defines respectively an ellipse and an hyperbola.

(b) Show that for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a^2 = b^2 + a^2e^2$ and illustrate it on a freehand sketch. Take S(ae,0).

(c) If S_1 and S_2 are the foci of the confocal C with its centre at O, show that OS_1 is independent of p.

(d) Sketch the conic C for $p = 0$ and $p = -3$ and describe how the shape of the conic changes as p decreases from 0 towards -4.

(e) What are the limiting lengths of the semi minor and semi major axes as -4 is approached?

(f) Describe the shape and position and equation of C when $p = -4$.

(g) Describe the shape and position of C when p is just less than -4.

(h) Describe the semi minor axis when $p < -4$.

(i) Describe the change in the shape and in the position of C as p approchaes -9.

(j) What is the shape, position and equation of C when $p = -9$?

Q1. Find the parametric coordinates of a point on each of the following curves:

1. $xy = 9$. 2. $xy = 16$. 3. $4xy = 25$.
4. $9xy = 1$. 5. $xy = 2$. 6. $xy = -4$.

For the following loci, find (i) the Cartesian equation, (ii) the coordinates of the vertices:

7. $(5t, \frac{5}{t})$ 8. $(3t, \frac{3}{t})$ 9. $(6t, \frac{6}{t})$ 10. $(t, -\frac{1}{t})$

Sketch the following rectangular hyperbolas:

11. $2x = \frac{25}{y}$ 12. $x = 3t,\ y = \frac{3}{t}$ 13. $x = 4t,\ y = \frac{4}{t}$.

14. $xy = -2$. 15. $y = \frac{2}{x - 2}$ 16. $y = \frac{-2}{x + 1}$

17. $(y - 1)(2x + 1) = 16$

Find (i) the lengths of the transverse axes, (ii) the coordinates of the foci of the following curves:

18. $xy = 18$ 19. $xy = 4$. 20. $x = 8t,\ y = 8/t$.

21. $2xy = 25$.

22. Find the equation of the tangents at the vertices of the rectangular hyperbola $xy = 8$.

23. Find the equation of the tangent and normal to the curve $xy = 16$ at the point $(4t, \frac{4}{t})$.

24. Find the length of the diameter of the curve $xy = 4$ drawn through the point $(4,1)$.

25. Sketch the locus $(\frac{t}{2}, \frac{1}{2t})$ for varying values of t and show the points with parameters $t = 1, -2$ and $\frac{1}{2}$.

26. Find the equations of the tangents to the curve $xy = 3$ which are parallel to the line $y + 3x = 0$. What is the distance between the tangents?

27. Find the points of contact of the two tangents which can be drawn from the point $(-5,1)$ to the curve $xy = 4$.

28. The normal at the point P $(8,2)$ on the locus $(4t, 4/t)$ meets the curve again at Q. Find the length PQ.

29. Find the equations of the tangents to the curve $xy = 9$ which pass through the point $(-9,3)$.

Q2. 1. Find the equation of the tangent at the point $P(6,2)$ of rectangular hyperbola $xy = 12$; and, if it meets the axes of coordinates in Q,R, show that $QP = PR$.

2. Find the coordinates of the foot of the perpendicular drawn from the origin to the tangent to the hyperbola $xy = 1$ at the point $(t, 1/t.)$.
Show that the locus of the foot of the perpendicular is the curve $(x^2 + y^2)^2 = 4xy$.

3. A is the point $(m,0)$ and B is the point $(0,4/m)$. Find the equation of the straight line AB and show that, for all values of m, AB touches the rectangular hyperbola $xy = 1$. Find the coordinates of the point of contact in terms of m.

4. Find the equation of the tangent at the point $(2t, 1/t)$ to the rectangular hyperbola $xy = 2$.
Show that the product of the perpendiculars from the points $(2,2)$, $(-2,-2)$ to this tangent is equal to -4.

5. A,B are two points on OX,OY respectively. If $OA = a$, $OB = b$, what are the coordinates of P, the mid-point of AB? If the line AB moves so that the area of the triangle OAB is always $2c^2$, find the locus of P. Show that the line AB touches the locus of P.

6. $P(ct, \frac{c}{t})$ is any point on the rectangular hyperbola $xy = c^2$. U and V are the points where the tangent at P intersects the asymptotes. O is the centre of the hyperbola. Prove that U, V and O are concyclic points of the circle centre P.

7. Find the equation of the normal to the rectangular hyperbola $xy = 6$ at the point $A(2,3)$.
If B is the point $(1,6)$, find the coordinates of the point C where a line through A at right angles to AB cuts the hyperbola again.
Show that BC is parallel to the normal at A.

8. Find the equations of the normal to the curve $xy = 4$ which are parallel to the line $4x - y = 2$.

Q3. The following examples all refer to the rectangular hyperbola $(cp, c/p)$.

1. Prove that the length of the segment of the tangent at P intercepted between the asymptotes is equal to 2OP.
2. The tangent at a variable point P meets the asymptotes in U,V. Prove that the area of the triangle OUV is constant.
3. The line $2x + y = 3c$ meets the asymptotes in U,V. Prove that the hyperbola passes through the two points of trisection of UV.
4. P,Q are two variable points on the hyperbola, $xy = c^2$, such that the tangent at Q passes through the foot of the ordinate P. Show that the locus of the mid-point of the chord PQ is a hyperbola with the same asymptotes as the given hyperbola.

Q4. 1. Prove that the locus of the midpoints of parallel chords of the rectangular hyperbola

(a) $xy = c^2$

(b) $x^2 - y^2 = a^2$

is a diameter. (i.e. a line through O).

2. P is a variable point on the rectangular hyperbola. The tangent at P cuts the x axis and the y axis at A and B respectively. Q is the fourth vertex of the rectangle whose other vertices are the origin, A and B. Show that P moves along the curve

(a) $xy = c^2$, Q also describes a hyperbola with the same asymptotes.

(b) $x^2 - y^2 = a^2$ then the equation of the locus of Q is $Y^2 - X^2 = \frac{X^2Y^2}{a^2}$

3. The tangents at P and Q on the rectangular hyperbola meet at R. Show that the locus of R is a straight line through O, if

   (a) P and Q moves on the curve with the equation $xy = c^2$, with parameters p and q respectively and if $pq = k$ (constant)

   (b) $P(x_1,y_1)$ and $Q(x_2,y_2)$ moves on the curve with equation $x^2 - y^2 = a^2$ and if the gradient of PQ is k (constant).

4. The tangent at $P(x_1,y_1)$, a variable point on the hyperbola $x^2 - y^2 = a^2$ meets the asymptote $y = x$ at Q. Show that the locus of $M(X,Y)$, the midpoint of PQ, has equation $X^2 - Y^2 = \frac{3a^2}{4}$

5. PN is the perpendicular to an asymptote from a point P on a rectangular hyperbola. Prove that the locus of the midpoint of PN is also a rectangular hyperbola with the same asymptotes. Consider both:

   (a) $xy = c^2$ and

   (b) $x^2-y^2 = a^2$

6. The normal to the rectangular hyperbola $x^2 - y^2 = 1$ at P meets the asymptotes at G and H. Show that the locus of $M(X,Y)$, the midpoint of GH is $4X^2Y^2 = (Y^2 - X^2)^3$.

7. The perpendiculars drawn from the origin to the tangent and normal at any point on the hyperbola $x^2 - y^2 = 1$ meet them in T and U. Show that the locus of T is $X^2 - y^2 = (X^2 + Y^2)^2$, and of U is $(X^2-Y^2)(X^2+Y^2)^2 = 4X^2Y^2$.

1. $ON.OT = a^2$. 2. $On.OT' = b^2$. 3. $SG = eSP$.

4. $S'G = eS'P$. 5. $SG = e^2PM$. *6. $G\hat{P}S' = P\hat{M}'S'$

7. $P\hat{S}'R = 90^o$ 8. $SY \cdot S'Y' = b^2$

9. Any tangent to an ellipse meets the tangents at the ends of the major axis in M,M'. Prove that MM' subtends a right angle at either focus.

10. Show that the line $y = mx - c$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ when $c = \pm \; (a^2m^2 + b^2)$.

*11. Any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the circle $x^2 + y^2 = a^2$ at L and M, prove that SL and S'M are each perpendicular to the tangent, S,S' being the foci nearest to L,M respectively.

12. A latus rectum of an ellipse is defined as a focal chord which is perpendicular to the major axis. Show that the semi-latus rectum of the ellipse $(a \cos \phi , b \sin \phi)$ is length $\frac{b^2}{a}$.

13. The tangent and normal at any point P on an ellipse meet the major axis at T and G respectively. If PN is the ordinate of P and C the centre, prove that $CT.NG = b^2$.

14. The tangents to an ellipse at P,P' meet at T. If PP' meets a directrix at K, prove that $T\hat{S}K$ is a right angle where S is the focus corresponding to the directrix.

*15 P is any point on ellipse with major axis AA'. If PA,A'P meet the directrix corresponding to focus S at K,K' respectively, prove

(i) $K\hat{S}K' = 90^o$; (ii) $K'K = SX^2$; (iii) $PN:NA' = XK':XA'$.

X is the point of intersection of the directrix and the x axis and PN the ordinate of P.

Q2. 1. Show that the product of the focal distances of a point on a rectangular hyperbola is equal to the square of the distance from the centre for (i) $xy=c^2$ and (ii) $x^2-y^2=a^2$.

*2. The point p on a hyperbola with focus S is such that the tangent at P, the latus rectum through S, and one asymptote are concurrent. Prove that SP is parallel to the other asymptote.

*3. Defining a hyperbola as the locus of a point which moves so that the difference of its distances from two fixed points is constant, obtain its equation in the form $x^2/a^2 - y^2/b^2 = 1$.

*4. Prove that the tangent at a point $(ct, c/t)$ to $xy = c^2$ is $x + t^2y = 2ct$. P is a point of intersection of the rectangular hyperbolas $x^2 - y^2 = a^2$, $xy = c^2$. The tangent at p to the first hyperbola meets its asymptotes in A,C, and the tangent at P to the second hyperbola meets its asymptotes in B,D. Prove that ABCD is a square.

*5. A variable tangent to a hyperbola meets the asymptotes in Q,R. Show that QR subtends a constant angle at a focus.

Q3. 1. Any tangent to an ellipse is cut by the tangents at the ends of the major axis in the points T,T'. Prove that the circle whose diameter is TT' will pass through the foci.

2. $P(x_1, y_1)$ is any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. S and S' are the foci. Prove that the tangent at P bisects $S\hat{P}S'$.

3. $P(a\cos\theta, b\sin\theta)$ is any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. S and S' are the foci. Prove that the normal at P bisects $S\hat{P}S'$.

MISCELLANEOUS EXAMINATION TYPE QUESTIONS

1. P is a point on the ellipse $x^2/a^2 + y^2/b^2 = 1$ with centre O. A line drawn through O, parallel to the tangent at P meets the ellipse at R and R'. Another line is drawn from P also through O meets the ellipse at P'.

   (a) Prove that the area of triangle RPR' is independent of the position of P.

   (b) Prove that the quadrilateral RPR'P' is a parallelogram and find its area.

2. With respect to axes Ox, Oy, the line $x = 1$ is a directrix of the hyperbola H. The equation of the asymptotes are $y = \pm x$.

   (a) Deduce that H is a rectangular hyperbola and find its equation.

   (b) Sketch the curve, indicating its asymptotes, directrixes and the numerical values of the foci.

(c) Show that the equation of the normal to the rectangular hyperbola $x^2 - y^2 = a^2$ at $P(x_1,y_1)$ is $y_1x + x_1y = 0$.

(d) The normal to $x^2 - y^2 = a^2$ at P intersects Ox in $(X,0)$ and Oy in $(0,Y)$. If T is the point (X,Y) show that as P moves on the hyperbola $x^2 - y^2 = a^2$, the locus of T is the rectangular hyperbola $X^2 - Y^2 = 4a^2$.

(e) Hence or otherwise find the locus of T if T moves on H.

3. The normal at a variable point P on the hyperbola $xy = c^2$ meets the x axis at Q. Show that the locus of the midpoint of PQ has equation $8y^4 = c^2(c^2 - 2xy)$.

4. If the coordinates of a point on the hyperbola $xy = c^2$ are represented by $x = ct$, $y = {}^{c}/_{t}$ prove that the normals at the four points t_1, t_2, t_3, t_4 will be concurrent if $\Sigma\, t_1t_2 = 0$ and $t_1t_2t_3t_4 = -1$.

5. A circle touches the x axis and cuts off a constant length 2t from the y axis. Prove that the locus of the centre of the circle is $y^2 - x^2 = t^2$.

6. (a) Show that the equation of the tangent to the hyperbola $xy = c^2$ at the point $P(cp, {}^{c}/_{p})$ is $x + p^2y = 2cp$.

(b) If the tangents at the points p and q meet at the point $R(x_o,y_o)$ prove that $pq = \frac{x_o}{y_o}$ and $p + q = \frac{2c}{y_o}$.

(c) If the length of chord PQ is d units, show that

$$d^2 = c^2(p-q)^2 \left\{1 + \frac{1}{p^2q^2}\right\}$$

(d) If d remains fixed, deduce that the locus of R has equation $4c^2(x^2 + y^2)(c^2 - xy) = x^2y^2d^2$.

7. (a) Given the ellipse ${x^2}/_{225} + {y^2}/_{144} = 1$. Prove that the section of the tangent between the point of contact and its point of intersection with the directrix subtends a right angle at the corresponding focus.

(b) The normal to the ellipse ${x^2}/_{a^2} + {y^2}/_{b^2} = 1$ at $P(x_1,y_1)$ meets Ox at N and Oy at G. Prove that

(i) ${PN}/_{NG} = \frac{1 - e^2}{e^2}$ (ii) ${PN}/_{PG} = 1 - e^2$

8. (a) Find the slope of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point P $(a \cos \theta, b \sin \theta)$ and hence show that the equation of this tangent is $\frac{x \cos \theta}{a} + \frac{y \sin \theta}{b} = 1$

(b) If the point P$(a \cos \theta, b \sin \theta)$ is on the ellipse in quadrant one, find the minimum area of the triangle made by this tangent and the coordinate axes.
Also find the coordinates of P in terms of a and b for this case.

9. (a) Prove that the condition that the line $ax + by + c = 0$ is a tangent to the circle $x^2 + y^2 = R^2$ is that $R^2(a^2 + b^2) = c^2$.

(b) A straight line moves so that the sum of the perpendicular distances from the points (2,0), (-2,0) is always equal to 6 units; prove that it always touches a circle and find the equation of this circle.