Chapter One

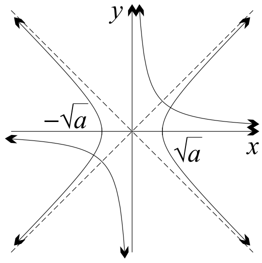
Exercise **1A** (Page 8)

1(a) -1 **(b)** 1 **(c)** $-i$ **(d)** i
(e) i **(f)** -1 **(g)** 1 **(h)** 0
2(a) $-2i$ **(b)** $3-i$ **(c)** $1+i$ **(d)** $5+3i$ **(e)** $-3-2i$
3(a) $12-2i$ **(b)** $-6+2i$ **(c)** $1+5i$ **(d)** $7-11i$
4(a) $-5+4i$ **(b)** $5+5i$ **(c)** $14+5i$ **(d)** $-26+82i$
(e) $24+10i$ **(f)** $-5-12i$ **(g)** $2+11i$ **(h)** -4
(i) $28-96i$
5(a) 5 **(b)** 17 **(c)** 29 **(d)** 65
6(a) $-i$ **(b)** $1-2i$ **(c)** $3+2i$ **(d)** $1-2i$ **(e)** $-1+3i$
(f) $-\frac{1}{5}+\frac{3}{5}i$
7(a) $-2-i$ **(b)** $4-3i$ **(c)** $3+7i$ **(d)** 3 **(e)** $-3+4i$
8(a) $6+2i$ **(b)** 18 **(c)** $19-22i$ **(d)** $8-i$ **(e)** $1+2i$
9(a) $22+19i$ **(b)** $6+15i$ **(c)** $4-2i$ **(d)** $2-3i$
(e) 6
10(a) $x=3$ and $y=-2$ **(b)** $x=2$ and $y=-1$
(c) $x=6$ and $y=2$ **(d)** $x=\frac{14}{5}$ and $y=\frac{3}{5}$
(e) $x=\frac{35}{2}$ and $y=-\frac{39}{2}$
11(a) $\frac{9}{10}-\frac{13}{10}i$ **(b)** 1 **(c)** $-\frac{8}{29}$ **(d)** $-4-\frac{5}{2}i$
16(a) $\frac{x-iy}{x^2+y^2}$ **(b)** $\frac{x^2-y^2-2ixy}{(x^2+y^2)^2}$ **(c)** $\frac{x^2+y^2-1+2iy}{(x+1)^2+y^2}$

Exercise **1B** (Page 15)

1(a) $z=\pm 3i$ **(b)** $z=2\pm 4i$ **(c)** $z=-1\pm 2i$
(d) $z=3\pm i$ **(e)** $z=\frac{1}{2}\pm\frac{1}{4}i$ **(f)** $z=-\frac{3}{2}\pm 2i$
2(a) $(z-6i)(z+6i)$ **(b)** $(z-2\sqrt{2}\,i)(z+2\sqrt{2}\,i)$
(c) $(z-1-3i)(z-1+3i)$ **(d)** $(z+2-i)(z+2+i)$
(e) $(z-3+\sqrt{5}\,i)(z-3-\sqrt{5}\,i)$ **(f)** $(z+\frac{1}{2}-\frac{\sqrt{3}}{2}i)(z+\frac{1}{2}+\frac{\sqrt{3}}{2}i)$
3(a) $z^2+2=0$ **(b)** $z^2-2z+2=0$ **(c)** $z^2+2z+5=0$ **(d)** $z^2-4z+7=0$
4(a) $\pm(1+i)$ **(b)** $\pm(2+i)$ **(c)** $\pm(-1+3i)$ **(d)** $\pm(6+i)$ **(e)** $\pm(2+3i)$ **(f)** $\pm(5-i)$ **(g)** $\pm(1-4i)$
(h) $\pm(5-4i)$
5(a) $\pm(1-2i)$ **(b)** $z=2-i$ or $1+i$
6(a) $\pm(1+3i)$ **(b)** $z=4+i$ or $3-2i$
7(a) $z=1-i$ or i **(b)** $z=-3+2i$ or $-2i$ **(c)** $z=4+i$ or $2-i$ **(d)** $z=-2+i$ or $\frac{1}{2}(3-i)$ **(e)** $z=-5+i$ or $3-2i$ **(f)** $z=3+i$ or $-1-3i$
8(a) $w=-1$ **(b)** $a=-6$ and $b=13$ **(c)** $k=8-i$ and the other root is $2+3i$.
9 $z=\pm(2+i)$
10(a) $\cos\theta+i\sin\theta$ or $\cos\theta-i\sin\theta$
11(a) $z=-1$ or $\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$ **(b)** $z=i$ or $\pm\frac{\sqrt{3}}{2}-\frac{1}{2}i$
12(a) $x=\omega$ satisfies the equation. **(c)** They are complex conjugates.

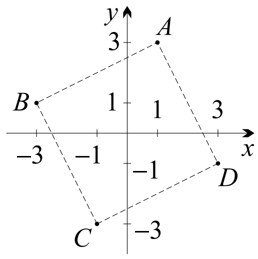
13(a) $\overline{\alpha}$
14(a)(i)



15(a) $\pm\frac{1}{\sqrt{2}}(1-i)$ **(b)** $\pm\sqrt{2}(1+2i)$ **(c)** $\pm(\sqrt{3}+i)$
(d) $\pm\sqrt{2}(3-2i)$
(e) $\pm\left(\sqrt{\sqrt{5}+1}-i\sqrt{\sqrt{5}-1}\right)$
16(a) $-2-i\pm\left(\sqrt{\sqrt{2}+1}+i\sqrt{\sqrt{2}-1}\right)$
(b) $1+i\pm\left(\sqrt{\sqrt{5}-1}-i\sqrt{\sqrt{5}+1}\right)$
(c) $-1+i\sqrt{3}\pm\left(\sqrt{2}-i\sqrt{6}\right)$
(d) $\frac{1}{2}\left(-1+i\pm\left(\sqrt{\sqrt{13}+2}-i\sqrt{\sqrt{13}-2}\right)\right)$

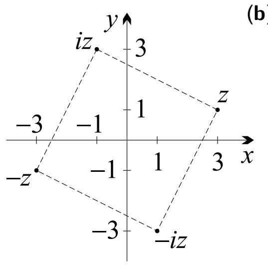
Exercise **1C** (Page 20)

1(a) $(2,0)$ **(b)** $(0,1)$ **(c)** $(-3,5)$ **(d)** $(2,-2)$
(e) $(-5,-5)$ **(f)** $(-1,2)$
2(a) $-3+0i=-3$ **(b)** $0+3i=3i$ **(c)** $7-5i$
(d) $a+bi$
3(a)



(b) A square. **(c)** An anticlockwise rotation of 90° about the origin.

4(a)



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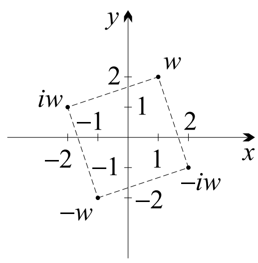

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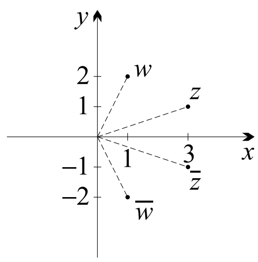
(c)


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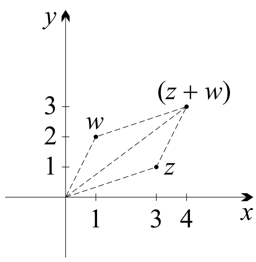

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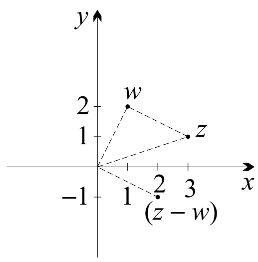

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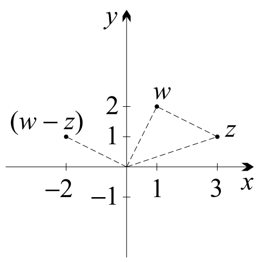

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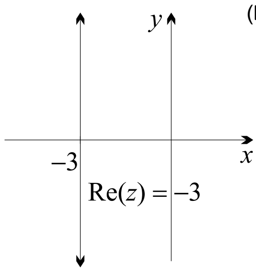

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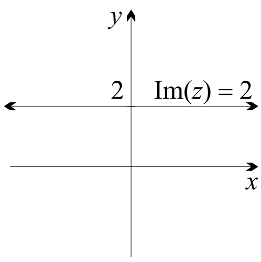

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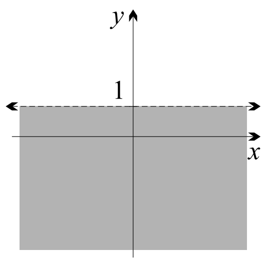

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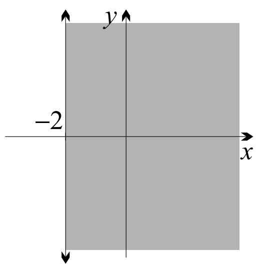

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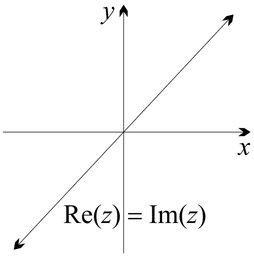

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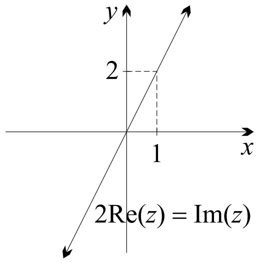

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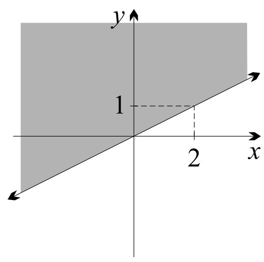

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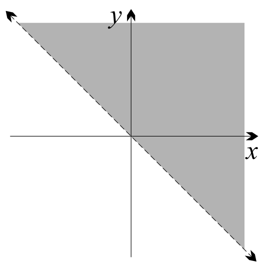

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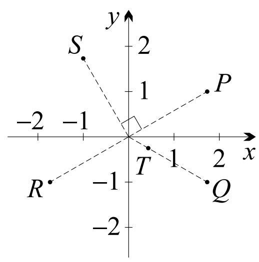
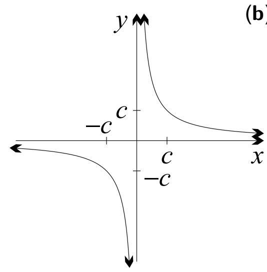


10(c) right-isosceles

11 It is the circle centre $(0, -1)$ with radius 1, omitting the origin.

12 It is the circle centre $(3, 0)$ with radius, omitting the origin.

14 It is a parabola with focus the origin and directrix $x = 1$.

15(a)


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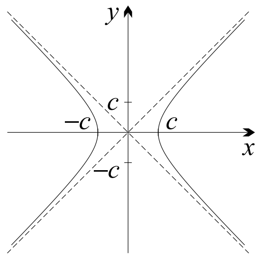


Exercise **1D** (Page 26)

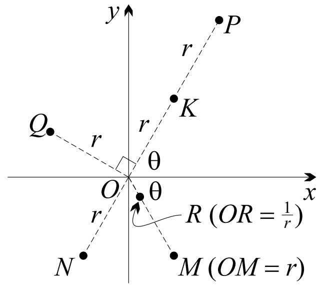
1(a) $2\operatorname{cis}\frac{\pi}{2}$ **(b)** $4\operatorname{cis}\pi$ **(c)** $\sqrt{2}\operatorname{cis}\frac{\pi}{4}$ **(d)** $2\operatorname{cis}\left(-\frac{\pi}{6}\right)$
(e) $2\operatorname{cis}\frac{2\pi}{3}$ **(f)** $\operatorname{cis}\left(-\frac{3\pi}{4}\right)$

2(a) $5\operatorname{cis}(0{\cdot}93)$ **(b)** $13\operatorname{cis}(-0{\cdot}39)$
(c) $\sqrt{5}\operatorname{cis}(2{\cdot}68)$ **(d)** $\sqrt{10}\operatorname{cis}(-1{\cdot}89)$

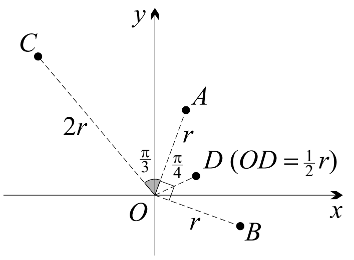
3(a) 3 **(b)** $-5i$ **(c)** $2\sqrt{2} + 2\sqrt{2}\,i$ **(d)** $3\sqrt{3} - 3i$
(e) $-\sqrt{2} + \sqrt{2}\,i$ **(f)** $-1 - \sqrt{3}\,i$

4(a) $\sqrt{2}\operatorname{cis}\left(-\frac{\pi}{4}\right)$ **(b)** $\sqrt{2}\operatorname{cis}\frac{\pi}{4}$ **(c)** $\sqrt{2}\operatorname{cis}\frac{3\pi}{4}$
(d) $\sqrt{2}\operatorname{cis}\frac{\pi}{4}$ **(e)** $2\operatorname{cis}\left(-\frac{\pi}{2}\right)$ **(f)** $\frac{1}{\sqrt{2}}\operatorname{cis}\left(-\frac{\pi}{4}\right)$

5(a) $10\operatorname{cis}\frac{\pi}{3}$ **(b)** $9\operatorname{cis}3\theta$ **(c)** $2\operatorname{cis}\frac{\pi}{3}$ **(d)** $\frac{3}{2}\operatorname{cis}\alpha$
(e) $16\operatorname{cis}\frac{2\pi}{5}$ **(f)** $8\operatorname{cis}\frac{6\pi}{7}$

6


7



9(a) $z_1 = 2\operatorname{cis}\frac{\pi}{6}$ and $z_2 = 4\operatorname{cis}\frac{\pi}{4}$ **(b)** $z_1 z_2 = 8\operatorname{cis}\frac{5\pi}{12}$ and $\frac{z_2}{z_1} = 2\operatorname{cis}\frac{\pi}{12}$
10 $z_1 = 2\operatorname{cis}\frac{5\pi}{6}$, $z_2 = \sqrt{2}\operatorname{cis}(-\frac{3\pi}{4})$, $z_1 z_2 = 2\sqrt{2}\operatorname{cis}\frac{\pi}{12}$ and $\frac{z_2}{z_1} = \frac{\sqrt{2}}{2}\operatorname{cis}\frac{5\pi}{12}$
11(a) $\frac{1}{2}\Big((\sqrt{3}+1) + i(\sqrt{3}-1)\Big)$ **(b)** $\sqrt{2}\operatorname{cis}\frac{\pi}{12}$
(c) $\frac{1}{2\sqrt{2}}(\sqrt{3}+1)$
12(b) $2\sqrt{2}\operatorname{cis}\frac{\pi}{12}$
13(a) $\sqrt{2}$ **(b)** $\frac{\pi}{4}$ **(c)** $1+i$
20 $z + w = 2\cos\left(\frac{\theta-\phi}{2}\right)\operatorname{cis}\left(\frac{\theta+\phi}{2}\right)$
21(c) The tangents at z_0 and z_1 to the circle with centre the origin meet at z_2.
22(a) When $\operatorname{Im}(z) = 0$.

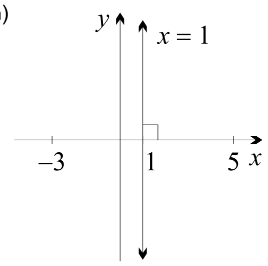
Exercise **1E** (Page 32)

1(a) $7+4i$ **(b)** $-3+2i$ **(c)** $3-2i$
2(a) $-3+4i$ **(b)** $1+7i$ **(c)** $-4-3i$ **(d)** $-7+i$
3 $-3+6i$
4(a) B represents $1+3i$, C represents $-1+2i$
(b) $-\sqrt{2}+2\sqrt{2}\,i$
5(a) $4+3i$ **(b)** $-3+4i$ **(c)** $2+7i$
6(a) $-5+12i$ **(b)** $-3-4i$
8 E represents $w_2 - w_1$, F represents $i(w_2 - w_1)$, C represents $w_2 + i(w_2 - w_1)$ and D represents $w_1 + i(w_2 - w_1)$.
9(a) Vectors BA and BC represent $z_1 - z_2$ and $z_3 - z_2$ respectively, and BA is the anticlockwise rotation of BC through 90° about B. So $z_1 - z_2 = i(z_3 - z_2)$. Squaring both sides gives the result.
(b) $z_1 - z_2 + z_3$
10(a) $2\omega i$ **(b)** $\frac{1}{2}\omega(1+2i)$
11 -2 and $1-\sqrt{3}\,i$
12(a) $w = -4+3i$ or $4-3i$ **(b)** $w = -1+7i$ or $7+i$ **(c)** $w = \frac{1}{2}(7+i)$ or $\frac{1}{2}(-1+7i)$
13(a) $1-5i$, $7+3i$ **(b)** $3+6i$, $-3-2i$ **(c)** $\frac{7}{2}+\frac{5}{2}i$, $\frac{1}{2}-\frac{3}{2}i$
14 $-2+2i$, $12i$, 4
19(a) $z_1 = 2\operatorname{cis}\frac{\pi}{2}$, $z_2 = 2\operatorname{cis}\frac{\pi}{3}$ **(c)(i)** $\frac{5\pi}{12}$ **(ii)** $\frac{11\pi}{12}$

22(c) The sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.
23(c) parallelogram **(d)** $\arg\frac{w}{z} = \frac{\pi}{2}$, so $\frac{w}{z}$ is purely imaginary.
26 Use the converse of the opposite angles of a cyclic quadrilateral.

Exercise **1F** (Page 41)

1(a)



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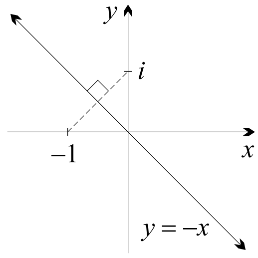

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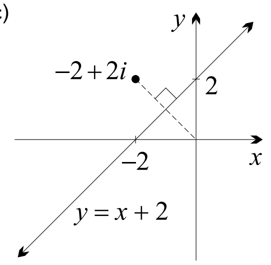

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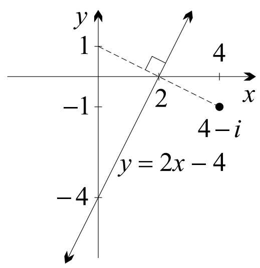

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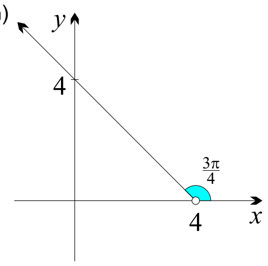

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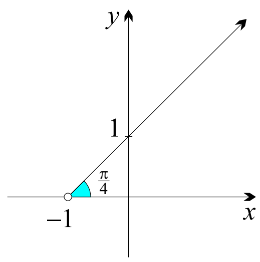

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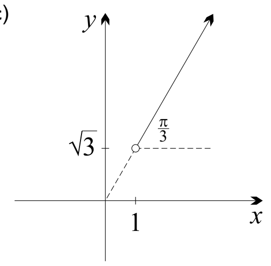

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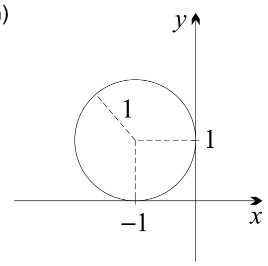

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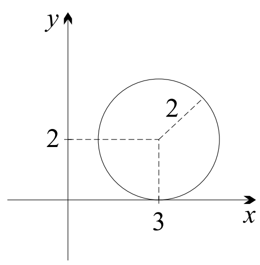

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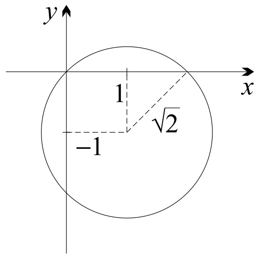
(c)


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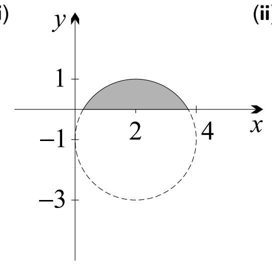

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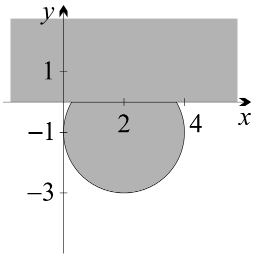

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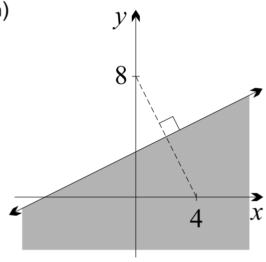

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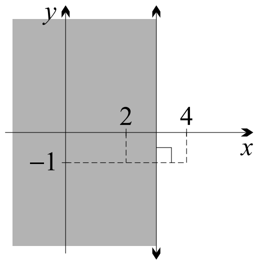

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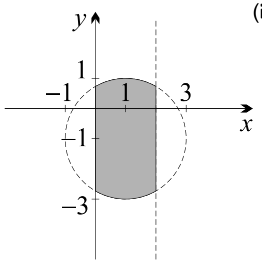

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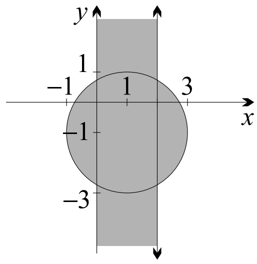

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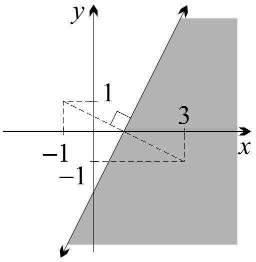

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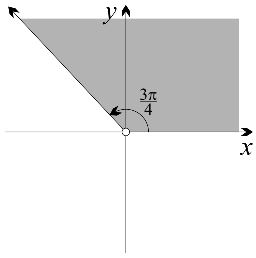

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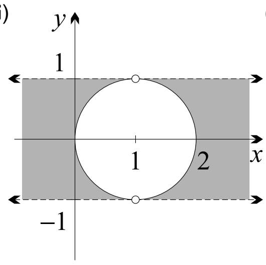

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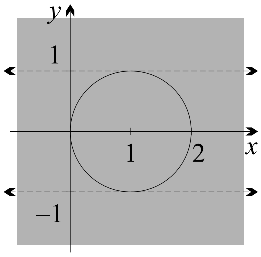

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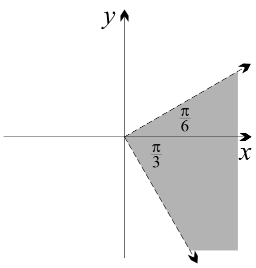

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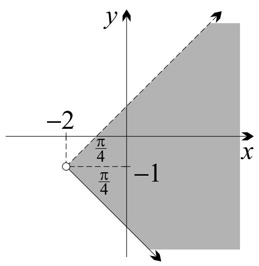

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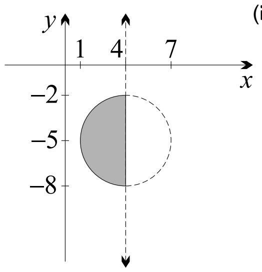

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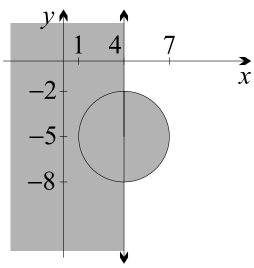

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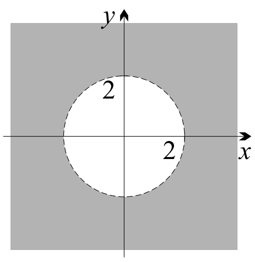

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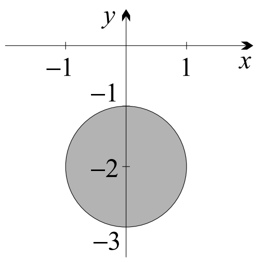

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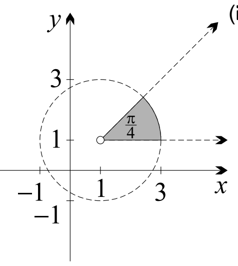

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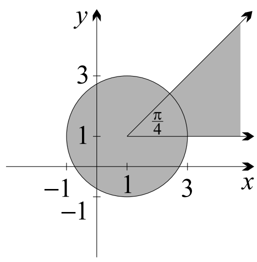

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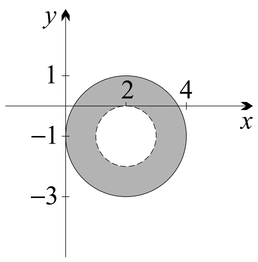

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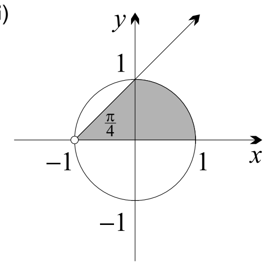

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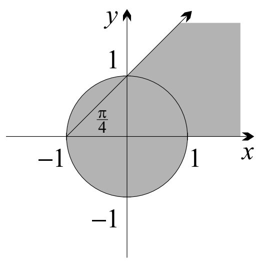

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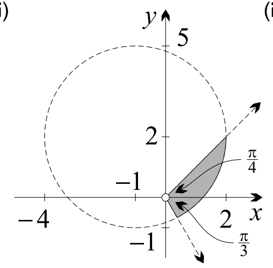

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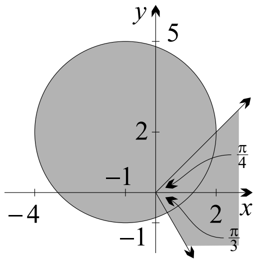

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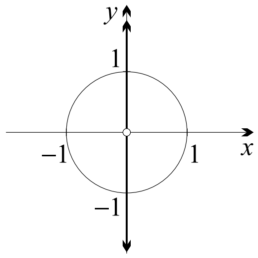

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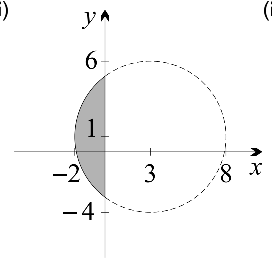

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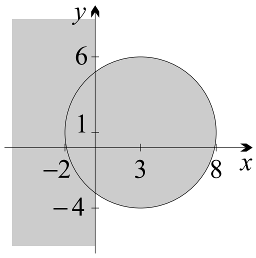

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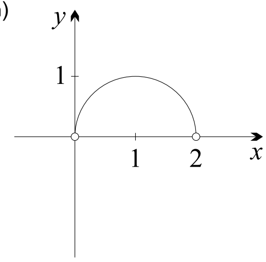

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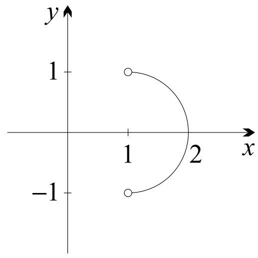

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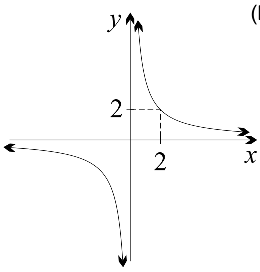

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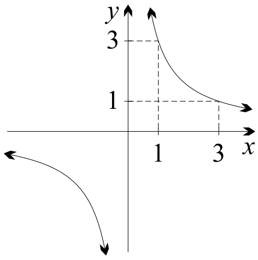

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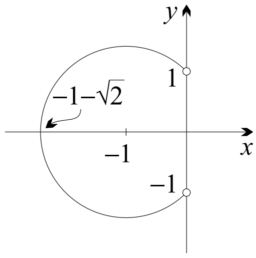

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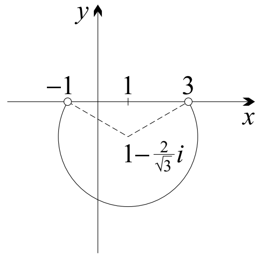

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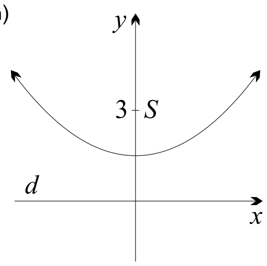

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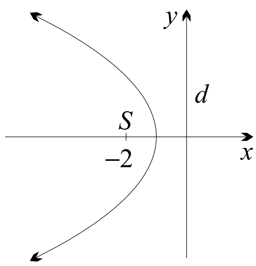

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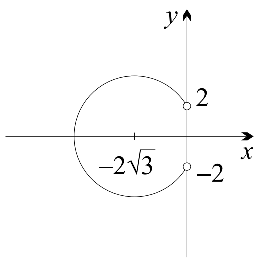

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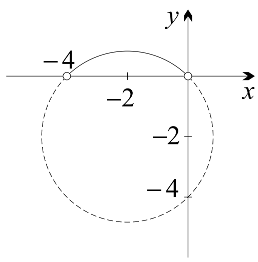

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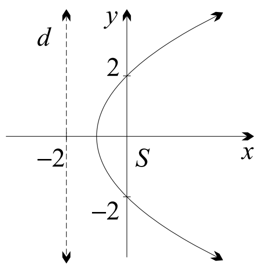

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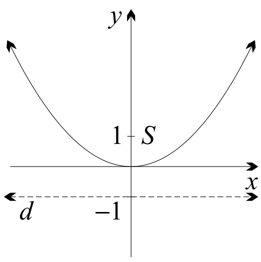

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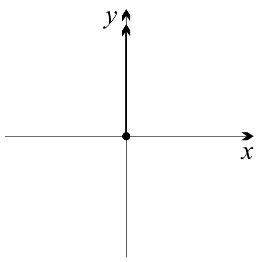

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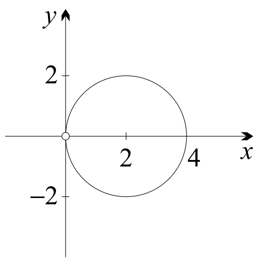
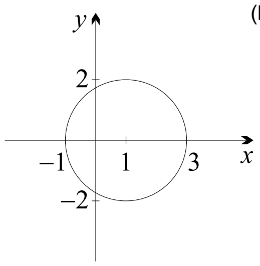


10(b) $\sqrt{3}\operatorname{cis}\frac{\pi}{3} = \frac{\sqrt{3}}{2}(1 + i\sqrt{3})$

11(a) $\arg(z + 3) = \frac{\pi}{3}$ **(b)** $|z| = \frac{3\sqrt{3}}{2}$, $\arg z = \frac{5\pi}{6}$

(c) $-\frac{9}{4} + \frac{3\sqrt{3}}{4}i$

12(a)



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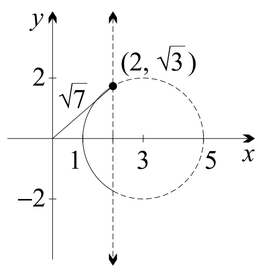

3 and 1

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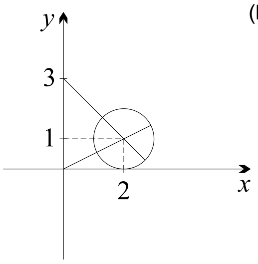
13(a)



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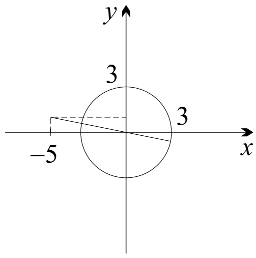


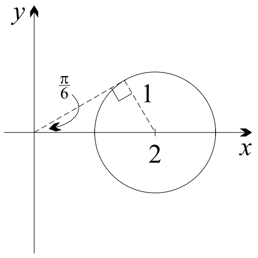
(i) $\sqrt{5}+1$ and $\sqrt{5}-1$ $\quad\sqrt{26}+3$ and $\sqrt{26}-3$

(ii) $2\sqrt{2}+1$ and $2\sqrt{2}-1$

(c)(i) $\Big||z_0|-r\Big| \le |z| \le |z_0|+r$

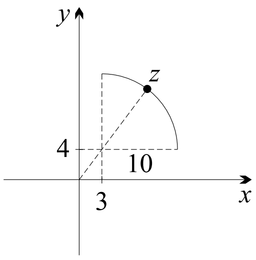
(ii) $\Big||z_0-z_1|-r\Big| \le |z-z_1| \le |z_0-z_1|+r$

14(a)(i)



(b) This is simply part (a) shifted left by 2.

15(a)



(b) 15 **(c)** $9+12i$

16(b)(i) $|z+2|=2$, centre -2, radius 2

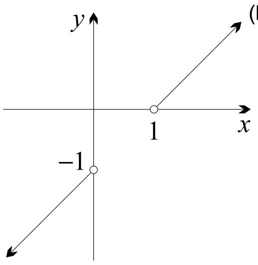
(ii) $|z-(1+i)|=1$, centre $1+i$, radius 1

(iii) $|z-1|=1$, centre 1, radius 1

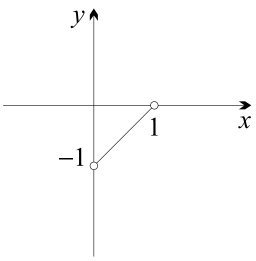
17(a) The line through 1 and i, omitting i.

(b) The circle with diameter joining 1 and i, omitting these two points.

18(a)

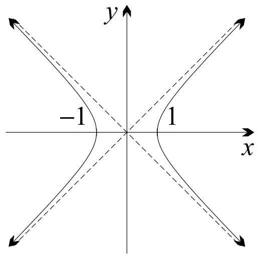


(b)



20(a) straight line external to z_1 and z_2 **(b)** major arc **(c)** semi-circle **(d)** minor arc **(e)** straight line between z_1 and z_2

21



22(a) Angle in the alternate segment theorem: it is the arc taken anticlockwise from z_2 to z_1 of the circle tangent to $y=\mathrm{Im}(z_1)$ and through z_2.

23 The ellipse with eccentricity e, semi-major axis a and semi-minor axis b, where $b^2=a^2(1-e^2)$.

24(b) The locus is the perpendicular bisector of the line joining z_1 and z_2.

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Chapter Seven

Exercise **7A** (Page 45)

1(a) $\operatorname{cis} 5\theta$ **(b)** $\operatorname{cis}(-3\theta)$ **(c)** $\operatorname{cis} 8\theta$ **(d)** $\operatorname{cis}(-\theta)$
(e) $\operatorname{cis} 7\theta$ **(f)** $\operatorname{cis}(-6\theta)$
2(a) $\operatorname{cis} 7\theta$ **(b)** $\operatorname{cis}(-5\theta)$
3(a) -1 **(b)** $-i$ **(c)** $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$ **(d)** $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$
(e) $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ **(f)** $\frac{1}{2} + \frac{\sqrt{3}}{2}i$
4(a) $\sqrt{2} \operatorname{cis} \frac{\pi}{4}$ **(b)** $256 + 256i$
5(a) $2 \operatorname{cis} \frac{\pi}{3}$ **(b)** $1024 - 1024\sqrt{3}\, i$
6(a) $2,\ \frac{5\pi}{6}$
7(a) $2 \operatorname{cis}\left(-\frac{\pi}{6}\right)$ **(b)** $128 \operatorname{cis} \frac{5\pi}{6}$ **(c)** $-64\sqrt{3} + 64i$
8(a) $2 \operatorname{cis}\left(-\frac{2\pi}{3}\right)$ **(b)** $32 \operatorname{cis} \frac{2\pi}{3}$ **(c)** $-16 + 16\sqrt{3}\, i$
9(a) $2 \operatorname{cis}\left(-\frac{\pi}{4}\right)$ **(b)** $2^{22}i$
12(a)(i) 6 **(ii)** 3 **(b)** $-64,\ 8i$
13(b) $n = 2,\ 6,\ 10,\ \ldots$
15(b) -2^{2n}

Exercise **7B** (Page 47)

6(b) $\frac{8}{15}$
7(c) $b = 2,\ c = -1$
(d) No, since $\sin \frac{\pi}{10} = \sin \frac{9\pi}{10}$ and $\sin \frac{13\pi}{10} = \sin \frac{17\pi}{10}$
(e) $\sin \frac{\pi}{10} = \frac{\sqrt{5}-1}{4}$, $\sin \frac{3\pi}{10} = \frac{\sqrt{5}+1}{4}$
8(b) $\theta = 0,\ \frac{\pi}{6},\ \frac{5\pi}{6},\ \pi,\ \frac{7\pi}{6},\ \frac{11\pi}{6}$
11(b) $z = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ or $\frac{3}{5} \pm \frac{4}{5}i$
12(a) $8(1 - 10s^2 + 24s^4 - 16s^6)$
(b) $x = 2 \sin \frac{n\pi}{8}$ for $n = 1,\ 2,\ 3,\ 5,\ 6,\ 7$

Exercise **7C** (Page 52)

1(a) $1,\ \operatorname{cis} \frac{2\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}\, i,\ \operatorname{cis} \frac{4\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}\, i$
(d)(i) 1 **(ii)** 0
2(a) $z = \pm 1,\ \frac{1}{2} + \frac{\sqrt{3}}{2}\, i,\ \frac{1}{2} - \frac{\sqrt{3}}{2}\, i,\ -\frac{1}{2} + \frac{\sqrt{3}}{2}\, i,$
$-\frac{1}{2} - \frac{\sqrt{3}}{2}\, i$ **(e)** $(z^2 - z + 1)(z^2 + z + 1)$
3(a) $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\, i,\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\, i,\ -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\, i,\ -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\, i$
(b) $(z^2 - \sqrt{2}\, z + 1)(z^2 + \sqrt{2}\, z + 1)$
4(a) $i,\ -i,\ \frac{\sqrt{3}}{2} + \frac{1}{2}\, i,\ \frac{\sqrt{3}}{2} - \frac{1}{2}\, i,\ -\frac{\sqrt{3}}{2} + \frac{1}{2}\, i,\ -\frac{\sqrt{3}}{2} - \frac{1}{2}\, i$
5(a) $\operatorname{cis}\left(-\frac{7\pi}{10}\right),\ \operatorname{cis}\left(-\frac{3\pi}{10}\right),\ \operatorname{cis} \frac{\pi}{10},\ \operatorname{cis} \frac{\pi}{2} = i,\ \operatorname{cis} \frac{9\pi}{10}$
(b) $\operatorname{cis}\left(-\frac{5\pi}{8}\right),\ \operatorname{cis}\left(-\frac{\pi}{8}\right),\ \operatorname{cis} \frac{3\pi}{8},\ \operatorname{cis} \frac{7\pi}{8}$
(c) $1 + \sqrt{3}\, i,\ -1 - \sqrt{3}\, i,\ \sqrt{3} - i,\ -\sqrt{3} + i$
(d) $2 \operatorname{cis}\left(-\frac{17\pi}{20}\right),\ 2 \operatorname{cis}\left(-\frac{9\pi}{20}\right),\ 2 \operatorname{cis}\left(-\frac{\pi}{20}\right),\ 2 \operatorname{cis} \frac{7\pi}{20},$
$2 \operatorname{cis} \frac{3\pi}{4}$
6(a) $-1,\ \operatorname{cis} \frac{\pi}{5},\ \operatorname{cis}\left(-\frac{\pi}{5}\right),\ \operatorname{cis} \frac{3\pi}{5},\ \operatorname{cis}\left(-\frac{3\pi}{5}\right)$
7(a) $1,\ \operatorname{cis}\left(\pm\frac{2\pi}{7}\right),\ \operatorname{cis}\left(\pm\frac{4\pi}{7}\right),\ \operatorname{cis}\left(\pm\frac{6\pi}{7}\right)$
(c) $(z - 1) \times (z^2 - 2\cos \frac{2\pi}{7}\, z + 1)\times$
$(z^2 - 2\cos \frac{4\pi}{7}\, z + 1) \times (z^2 - 2\cos \frac{6\pi}{7}\, z + 1)$
8(a)(i) $1,\ \operatorname{cis} \frac{2\pi}{5},\ \operatorname{cis}\left(-\frac{2\pi}{5}\right),\ \operatorname{cis} \frac{4\pi}{5},\ \operatorname{cis}\left(-\frac{4\pi}{5}\right)$
9(a) $\operatorname{cis} \frac{2k\pi}{9}$ for $k = -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4$
13(a) 3, when k is a multiple of 3, 0 otherwise.
(b) $(1 + \omega)^n = \sum_{r=0}^{n} \binom{n}{r} \omega^r$ and

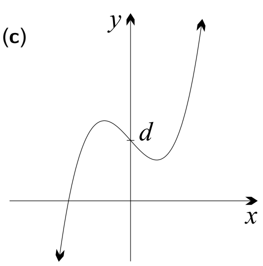
$$(1 + \omega^2)^n = \sum_{r=0}^{n} \binom{n}{r} \omega^{2r}$$

14(a) The roots are $-i \cot \frac{(2k-1)\pi}{4n}$
for $k = 1,\ 2,\ 3,\ \ldots,\ 2n$.

Chapter Five

Exercise 5A (Page 4)

1(a) $(x-2)(x+1-\sqrt{3})(x+1+\sqrt{3})$
(b) $(x-1)(x+2-\sqrt{2})(x+2+\sqrt{2})$
(c) $(x-1)(x-1-\sqrt{5})(x-1+\sqrt{5})$
2(a) The coefficients of $P(x)$ are real, so complex zeroes occur in conjugate pairs. **(b)** 6
3(a) $1+2i$; the coefficients of $P(x)$ are real, so complex zeroes occur in conjugate pairs.
(c) $P(x)=(x+2)(x^2-2x+5)$
4(a) $3i$; the coefficients of $P(z)$ are real, so complex zeroes occur in conjugate pairs. **(b)** z^2+9
(c) $P(z)=(2z+3)(z^2+9)$
5(b) 0; the coefficients of $P(z)$ are real, so complex zeroes occur in conjugate pairs.
(c)(i) $P(z)=(2z-1)(z-3-i)(z-3+i)$
(ii) $P(z)=(2z-1)(z^2-6z+10)$
6(a) The coefficients of $Q(x)$ are real, so complex zeroes occur in conjugate pairs. **(b)** $3+\sqrt{5}$, $3-\sqrt{5}$
(c)(i) $(x-2i)(x+2i)(x-3-\sqrt{5})(x-3+\sqrt{5})$
(ii) $(x^2+4)(x-3-\sqrt{5})(x-3+\sqrt{5})$
(iii) $(x^2+4)(x^2-6x+4)$
7(a) $x=1\pm3i$, 3 or -2 **(b)** $x=1\pm i$ or $2\pm i$
8(a) $a=3$ **(b)** $b=1$
(c) $(x^2-6x+10)(x^2-6x+13)$
9(b) $k=3$
10(b) $m=7$, $n=-4$
11(a) $-7-4i$ **(b)(i)** $-7+4i$ **(ii)** $2x-7$
12(b) $P(z)=\frac{1}{2}(z^4-2)(2z^4-1)$ so one root is $z=\sqrt[4]{2}$. **(c)** $\sqrt[4]{2}$, $\frac{1}{\sqrt[4]{2}}$, $-\sqrt[4]{2}$, $-\frac{1}{\sqrt[4]{2}}$, and $i\sqrt[4]{2}$, $\frac{1}{\sqrt[4]{2}}i$, $-i\sqrt[4]{2}$, $-\frac{1}{\sqrt[4]{2}}i$
13(a) $P(x)$ has minimum value B, when $x=0$. Since $B>0$, it follows that $P(x)>0$ for all real values of x. **(b)** $-ic$, $-id$; the coefficients of $P(x)$ are real, so complex zeroes occur in conjugate pairs.
14(a) They form a conjugate pair, since $P(x)$ has real coefficients. **(c)**



15(a) The minimum stationary point is at $x=1$. $f(1)=k-2>0$. Hence the graph of $f(x)$ has only one x-intercept which lies to the left of the maximum stationary point at $x=-1$.
(b) $f(x)$ has real coefficients **(d)** -14, $7\pm12i$

Exercise 5B (Page 10)

1(a)(ii) 3 is a double zero of $P(x)$ **(b)** 3, 3, -2
(c) $P(x)=(x-3)^2(x+2)$
2(a)(ii) -1 is a triple zero of $P(x)$
(b) -1, -1, -1, -5 **(c)** $P(x)=(x+1)^3(x+5)$
3(a) -3 and 3 **(b)** 3 **(c)** -6
4(a) $\frac{5}{2}$ and -5 **(b)** -5 **(c)** 10
5(a) -2 **(b)** $\frac{3}{2}$, $P(x)=(x+2)^2(2x-3)$
6(a) $\frac{1}{2}$ **(b)** 2, $P(x)=(2x-1)^3(x-2)$
7(b) $x=3$, $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$, $-\frac{1}{2}-\frac{\sqrt{3}}{2}i$
8(a) $k=27$ or -5
(b) When $k=27$, $P(x)=(x-3)^2(x+3)$ and when $k=-5$, $P(x)=(x+1)^2(x-5)$.
9 $a=1$, $b=-3$, $c=2$
10(a) -3 **(b)** $c=-54$ **(c)** $P(x)=(x+3)^3(x-2)$
11(a) $b=-5$ and $c=8$
(b) $x=\frac{1}{2}(3-\sqrt{5})$ or $\frac{1}{2}(3+\sqrt{5})$
12 The Fundamental Theroem of Algebra only applies to polynomials of degree ≥ 1.
15 HINT: consider $P(x)-P'(x)$
16(b)(ii) $m<0$ **(iii)** $x=-\sqrt{-\frac{m}{2}}$ or $\sqrt{-\frac{m}{2}}$
19(a) HINT: $x^2=-(2px+q)$
(b) HINT: $P'(\alpha)=0$.
20(b) $(z-\alpha)^2(z-\overline{\alpha})^2$ is a factor. **(c)** HINT: Begin by writing: $P(z)=\left(z-2\operatorname{Re}(\alpha)+|\alpha|^2\right)^2\times Q(z)$

Exercise 5C (Page 17)

1(a) $\pm3i$ **(b)** $2\pm3i$ **(c)** $Q(x)=x^2-4x+13$
2(a) $-4\pm2i$ **(b)** $-2\pm i$ **(c)** $Q(x)=4x^2+16x+20$
3 $8P(\frac{x}{2})=x^3-20x+24$
4 $x^3-2x^2-7x+1=0$
5(a) 4 **(b)** 4 **(c)** 16 **(d)** 72 **(e)** 224
6(a) $x^3-3x^2+1=0$ **(b)** $x^3-6x^2+9x-1=0$
7(a)(i) $27x^4+18x^3-3x^2+4x-1=0$
(ii) $x^4-4x^3+3x^2-18x-27=0$
(b)(i) $x^4+14x^3+71x^2+160x+135=0$
(ii) $x^4-14x^3+71x^2-160x+135=0$
8(a) $x^3+2mx^2+m^2x-n^2=0$
(b) $n^2x^3-m^2x^2-2mx-1=0$
9(a) $4+2\sqrt{3}$ **(b)** $x^4-8x^2+4=0$
(c) $x=\sqrt{3}$, $-\sqrt{3}$, $-2+\sqrt{3}$ or $-2-\sqrt{3}$
10(a) $x^4-5x^2+6=0$ **(b)** $x=-3\pm\sqrt{2}$ or $-3\pm\sqrt{3}$
11(a)(i) -4 **(ii)** 3 **(b)(i)** $2x^3+32x^2+163x+262=0$

(ii) $2x^3 + 24x^2 + 27x - 162 = 0$

12(a) Use the sum of roots.

13(a)(i) 8 **(ii)** 6 **(c)** $3,\ 1+i,\ 1-i$

14(a)(i) $-p$ **(ii)** $-r$ **(c)(ii)** $P(0) = -8,\ P(1) = 2$

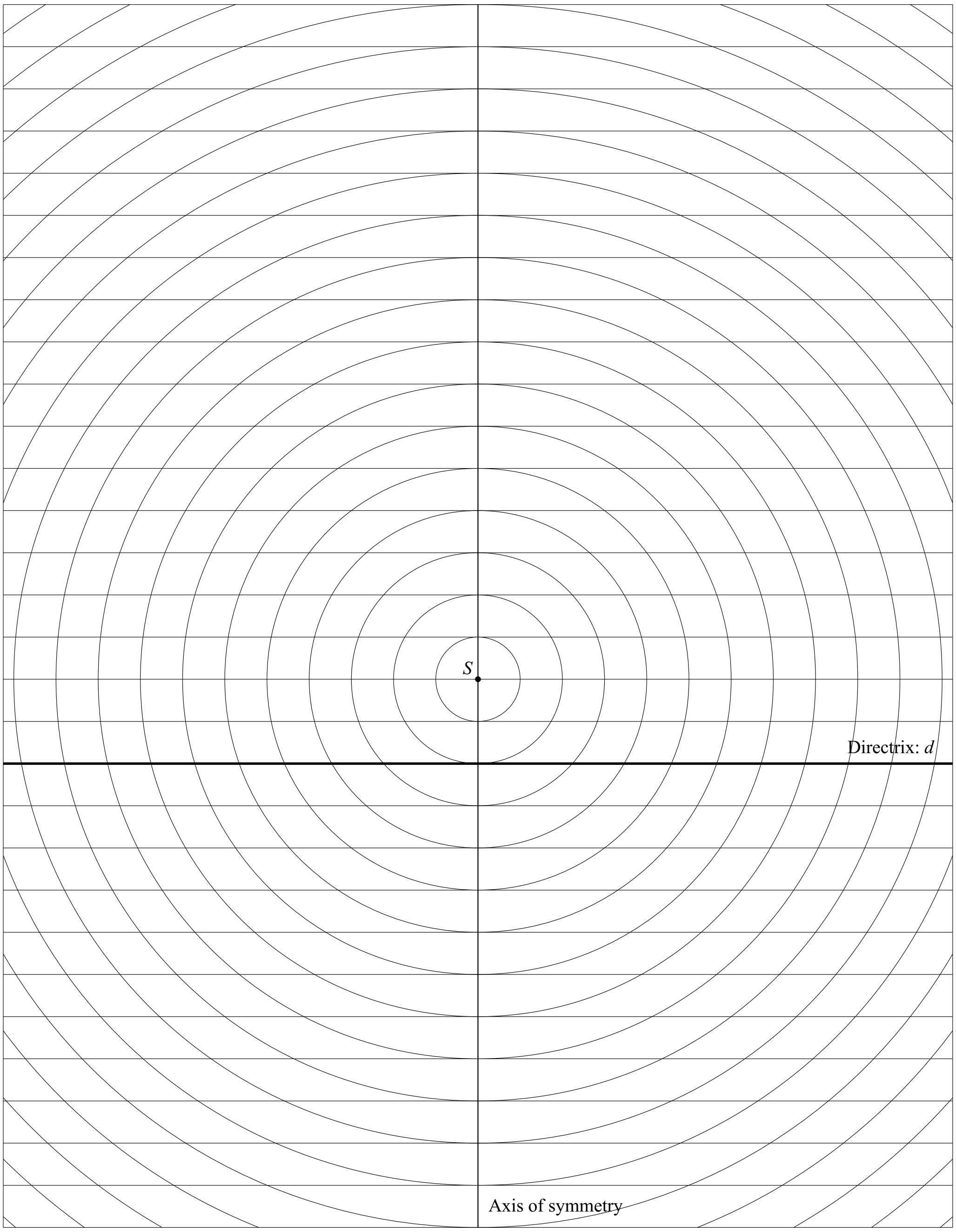
19(a) Replace x with $-\frac{p}{x+1}$ to get

$rx^3 + (3r - pq)x^2 + (p^3 - 2pq + 3r)x + (r - pq) = 0$

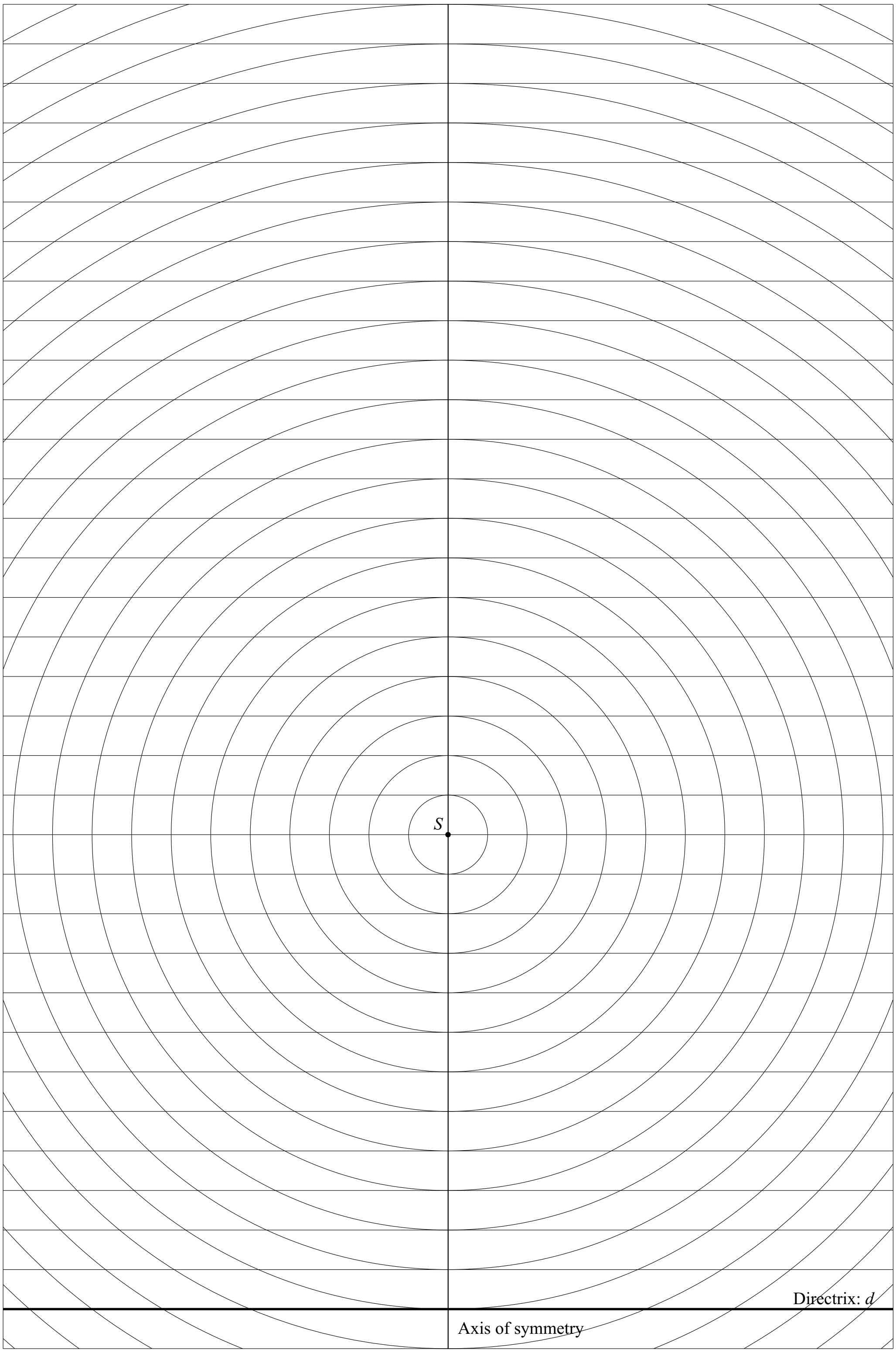
20(a)(i) $-\frac{1}{2}n(n+1)$ **(ii)** $(-1)^n\, n!$

Appendix — Conic Grids

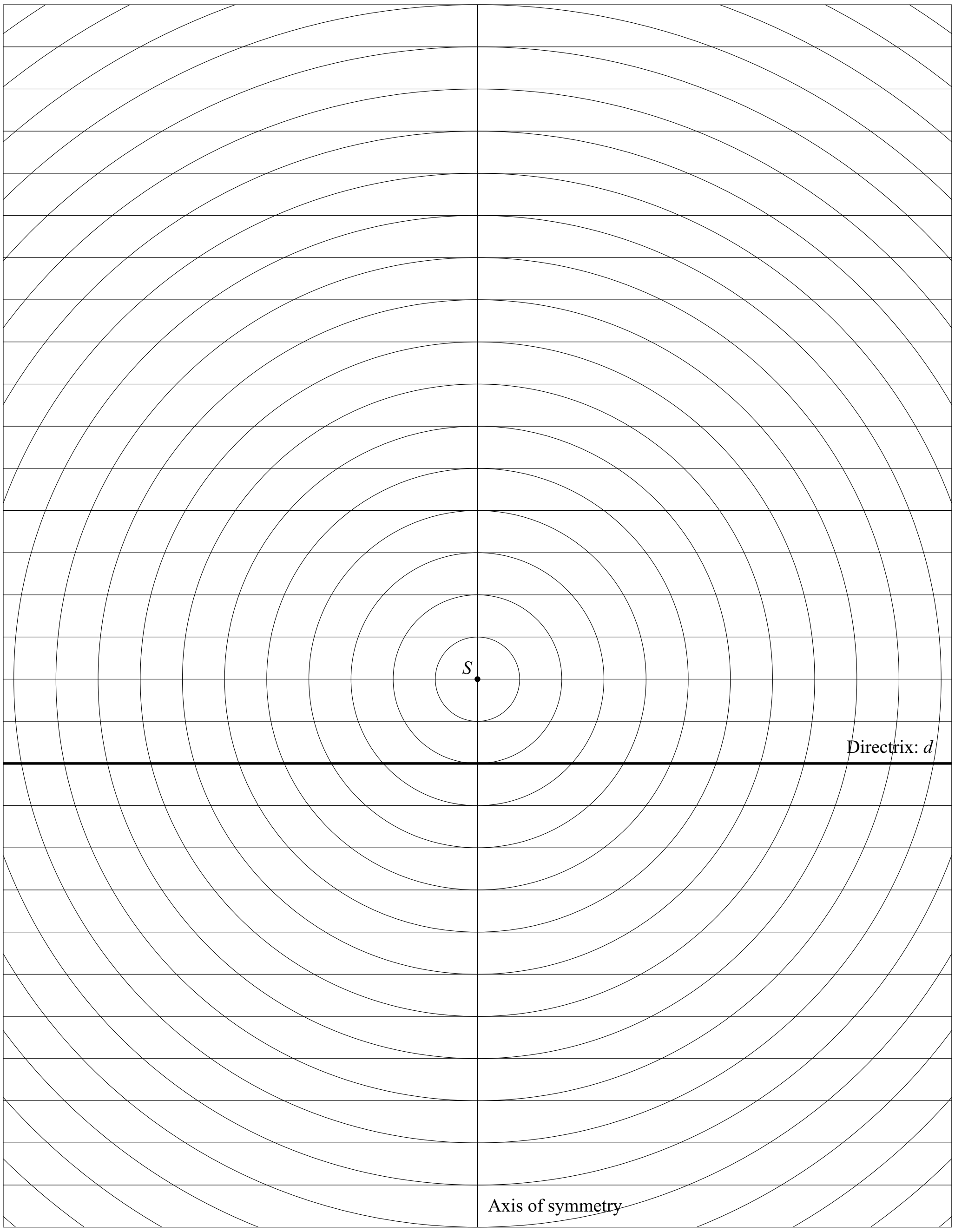
Parabola:



Ellipse:



Hyperbola:



Chapter Three

Exercise **3A** (Page 3)

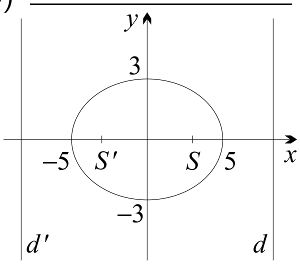
1(b) Both points are equidistant from d and S.
2(b) In both cases $\frac{PS}{PQ} = \frac{1}{2}$.
3(b) In both cases $\frac{PS}{PQ} = 2$.
5(c) They are the same: $PS + PS' = 16$ units.
(d) $\frac{PS'}{PQ'} = \frac{PS}{PQ} = \frac{1}{2}$, always.

Exercise **3B** (Page 7)

1(b) $x_1(y - b) = y_1(x - a)$
2(a) $3x - y = 4$ **(b)** $x + 2y = 6$ **(c)** $x + 4y = -9$
(d) $2y - 2x = 5$
3(a) $C = (-1, 2)$, $r = \sqrt{5}$ **(b)** $C = (3, 0)$, $r = 2\sqrt{2}$
(c) $C = (-3, -2)$, $r = 4$ **(d)** $C = (\frac{1}{2}, -\frac{3}{2})$, $r = \frac{1}{\sqrt{2}}$
4(a) $(x + 1)^2 + (y - 1)^2 = 2$
(b) $(x - 2)^2 + (y + 1)^2 = 10$
(c) $(x - 3)^2 + (y - 4)^2 = 13$
(d) $(x + 3)^2 + (y + 3)^2 = 20$
5(a) $x(x + 2) + y(y - 2) = 0$
(b) $(x - 5)(x + 1) + y(y + 2) = 0$
(c) $(x - 1)(x - 5) + (y - 7)(y + 1) = 0$
(d) $(x + 5)(x + 1) + (y + 7)(y - 1) = 0$
6(a) $x \times r\cos\theta + y \times r\sin\theta = r^2$ **(b)** $x_1x + y_1y = r^2$.
7(a) $3x + 4y = 25$ **(b)** $x + y = 2$ **(c)** $y\sqrt{3} - x = 4$
(d) $x + 7y = -50$
8 $x^2 + (y - 2)^2 = 10$ or $(x - 4)^2 + (y + 2)^2 = 10$
9 $(x + 1)^2 + (y - 3)^2 = 20$
10(b)(i) $(b + mh - k)^2 = r^2(m^2 + 1)$
12(b) $\frac{b^2 - r^2}{m^2 + 1}$ **(c)** $b^2 - r^2$ **(d)** The product of intercepts of intersecting secants is constant.
13(a) $\frac{\sin\theta - \sin\phi}{\cos\theta - \cos\phi}$ **(c)** Angles in the same segment are equal.
14(a) $PA^2 = {x_0}^2 + {y_0}^2 - r^2$ **(b)** Tangents to a circle from an external point are equal.
16 $(x - 5)^2 + (y + 5)^2 = 37$

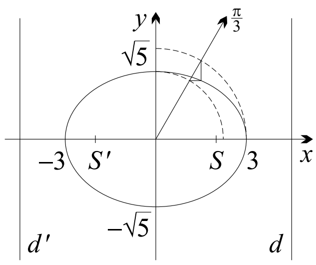
Exercise **3C** (Page 16)

1(a) $\frac{3}{5}$
(b) $(3, 0)$ and $(-3, 0)$
(c) $x = \frac{25}{3}$ and $x = -\frac{25}{3}$
(f) $(\frac{5}{2}, 2\sqrt{3})$

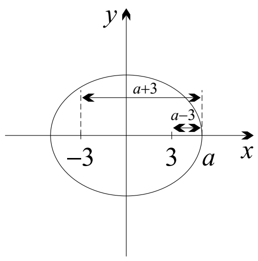


2(b) $x = 2\cos\theta$, $y = \frac{4}{3}\sin\theta$

3(a) $(2, 0)$, $(-2, 0)$,
$x = \frac{9}{2}$, $x = -\frac{9}{2}$



4(b) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ **(c)** $\frac{4}{5}$
5(a) $SB = a$ **(b)(i)** $\frac{x^2}{10} + y^2 = 1$ **(ii)** $\frac{x^2}{25} + \frac{y^2}{9} = 1$
6 $\frac{x^2}{36} + \frac{y^2}{20} = 1$
7



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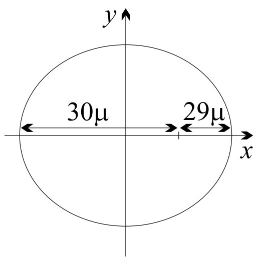
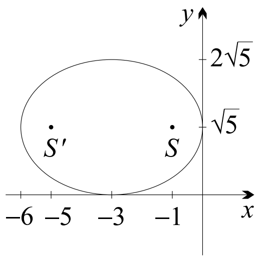


9(a) $3x - 4y = 12$ **(b)** $5x + 8y + 4 = 0$
15(b) $S'P = 2(5 + 2\sqrt{3})$
17(a) $\frac{b}{a} \to 1^-$ and the ellipse becomes more circular. **(b)** $\frac{b}{a} \to 0^+$ and the ellipse becomes long and slender.
18(a) $-3 + i\sqrt{5}$
(c) $\frac{\pi}{2} \le \arg(z) \le \pi$



19(a) $\lambda < 2$ **(b)** The length of the major axis increases from $2\sqrt{3}$ to $2\sqrt{2}$, while the length of the minor axis starts at 2 and approaches zero.
(c) When $\lambda = 2$, $b = 0$, so the ellipse has collapsed onto the interval joining $(-\sqrt{2}, 0)$ and $(\sqrt{2}, 0)$.
21 All three coalesce at $x = a$.
22(b)(ii) In the limit, $y^2 = -4x$ is obtained. This is a parabola with focal length 1.
(c) A parabola with focal length f is obtained.

Exercise **3D** (Page 20)

4(b) $A = \left(\frac{a^2 - b^2}{a}\cos\theta, 0\right)$, $B = \left(0, \frac{b^2 - a^2}{b}\sin\theta\right)$
7(a) $\frac{a}{b} = \frac{1}{e^2} = \frac{1}{2}(\sqrt{5} + 1)$ — the golden ratio.
(b) $e = \frac{1}{\sqrt{2}}$
8(a) $\frac{b^2(ae - x_1)}{aey_1}$
9(a) $\frac{x_1x}{a^2} + \frac{y_1y}{b^2} = 1$

10(b) $R = \left(\frac{a}{e}, \frac{b\sin\theta}{e\cos\theta}\right)$

11(d) Only if $y_1 = 0$, since $|e^2x_1| < ae$.

12(b) $\dfrac{ab}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}$

(c) $R = (a\sin\theta, -b\cos\theta)$, $R' = (-a\sin\theta, b\cos\theta)$

(d) $|\triangle RPR'| = ab$

15(a) Q is outside the ellipse except when $Q = P$.

(b) $\triangle SUP \equiv \triangle S^*UP$ so $S'P + PS^* = S'P + PS$. If the line passed through any other point Q, the distance would be greater than this.

16(a) Matching sides, $\triangle STP \equiv \triangle RTP$

(b) $S'P + PR = S'P + PS = 2a$, since $\triangle STP \equiv \triangle RTP$.

(c) $\triangle S'RS \parallel\!| \triangle OTS$ so $OT = a$.

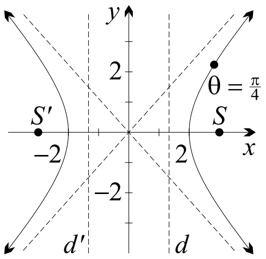
17 Let TS intersect the auxiliary circle again at T^*. By symmetry in the circle, $ST^* = S'T'$. So $ST \times S'T' = ST \times ST^* = AS \times SA'$ (intersecting chords)

Exercise **3E** (Page 31)

1(a) $e = \frac{3}{2}$, $S = (3, 0)$, $S' = (-3, 0)$, $d : x = \frac{4}{3}$, $d' : x = \frac{4}{3}$

(b) $y = \frac{\sqrt{5}}{2}x$, $y = -\frac{\sqrt{5}}{2}x$

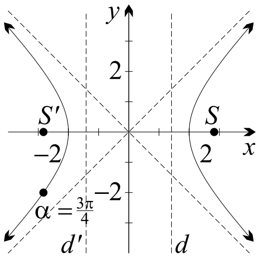
(e) $(2\sqrt{2}, \sqrt{5})$ $\doteqdot (2{\cdot}83, 2{\cdot}24)$



2(a) $S = (2\sqrt{2}, 0)$, $S = (-2\sqrt{2}, 0)$, $d : x = \sqrt{2}$, $d : x = -\sqrt{2}$

(b) $y = x$, $y = -x$

(d) $(2\sqrt{2}, -2)$ $\doteqdot (2{\cdot}28, -2)$

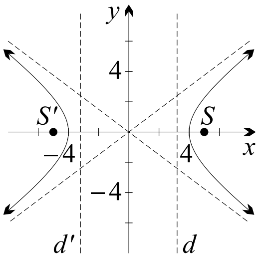


3(a) $e = \frac{5}{4}$, $S = (5, 0)$, $S' = (-5, 0)$, $d : x = \frac{16}{5}$, $d' : x = -\frac{16}{5}$

(b) $y = \frac{3}{4}x$, $y = -\frac{3}{4}x$

(d) $x = 4\sec\theta$, and $y = 3\tan\theta$

(e) $(8, -3\sqrt{3})$



4(a) $(-\frac{2}{\sqrt{3}}, 1)$ **(b)** $x^2 - \frac{y^2}{3} = 1$ **(c)** 2

5(a) $\frac{x^2}{4} - \frac{y^2}{12} = 1$ **(b)** $\frac{x^2}{25} - \frac{y^2}{200} = 1$

7(a) $4x - 9y = 36$

15(b) It collapses onto the asymptotes: $\frac{y^2}{x^2} = e^2 - 1$.

16(b) Since z is closer to 2 than -2, the left branch is omitted. Thus $\arg(z)$ is in the fourth and first quadrants between the asymptotes.

17(b)(ii) In the limit, $y^2 = 4x$ is obtained. This is a parabola with focal length 1.

(c) A parabola with focal length f is obtained.

Exercise **3F** (Page 36)

2(b) $e = \sqrt{2}$ **(c)** $e = \frac{1}{2}(1 + \sqrt{5})$, the golden ratio.

4(b) $x = \frac{4}{3}$, $x = -\frac{4}{3}$, $y = \frac{\sqrt{5}}{2}x$, $y = -\frac{\sqrt{5}}{2}x$

11(a) $2b|\sec\theta|$ **(b)** $OT = \frac{a}{|\sec\theta|}$

17(c) $(-b, 0)$

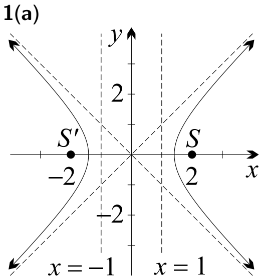
18(a) $\frac{a^2}{x_0}$

21(a) It is the hyperbola $\dfrac{x^2}{c^2\cos^2\theta} - \dfrac{y^2}{c^2\sin^2\theta} = 1$ with eccentricity $e = \sec\theta$ and the same foci as the ellipse.

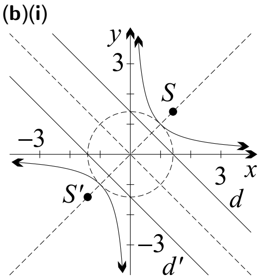
(b) It is the ellipse $\dfrac{x^2}{c^2\sec^2\theta} + \dfrac{y^2}{c^2\tan^2\theta} = 1$ with eccentricity $e = \cos\theta$ and the same foci as the hyperbola.

Exercise **3G** (Page 44)

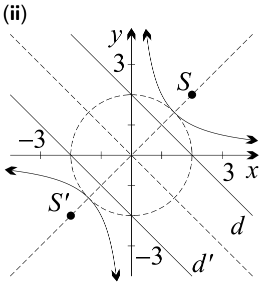
1(a)



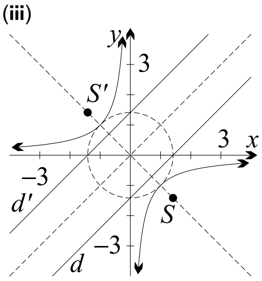
(b)(i)



(ii)



(iii)



4(b) $Q\left(-\frac{c}{t^3}, -ct^3\right)$ **(d)** $Q \equiv R$

5(a) p and q have opposite sign.

6(c) Q is the result of reflecting P in $y = x$.

9(b) $Q = \left(3ct, -\frac{c}{t}\right)$, $Q' = \left(-ct, \frac{3c}{t}\right)$

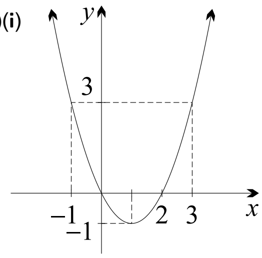
10(a) $x + pqy = c(p + q)$

13(b) $\left(\frac{c}{2t^3}(t^4 - 1), \frac{c}{t}\right)$

Chapter Eight

Exercise **8A** (Page 60)

1(a)(i)



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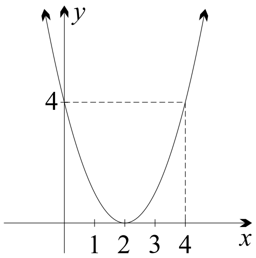

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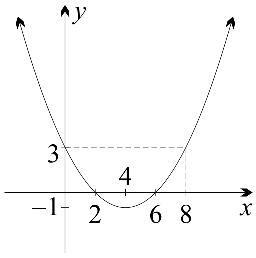
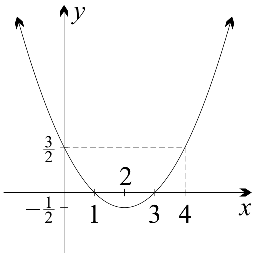
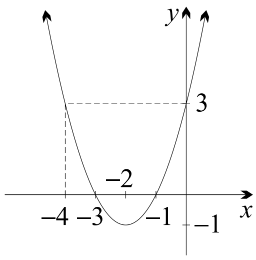


(iv)



(v)



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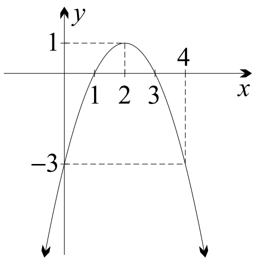

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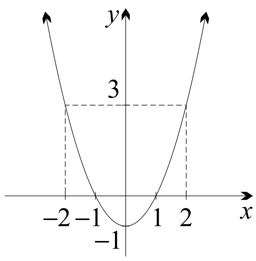

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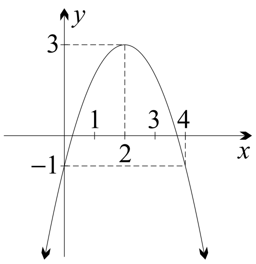
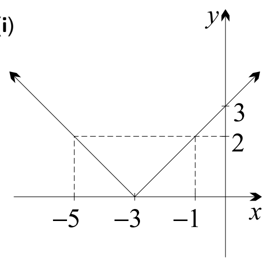


(b)(i)



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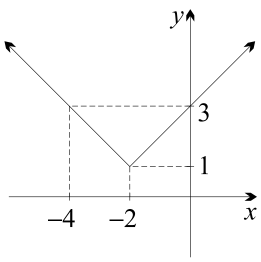
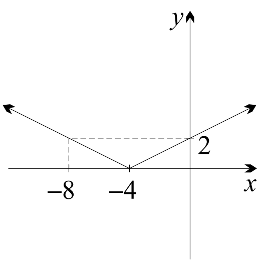


(iii)



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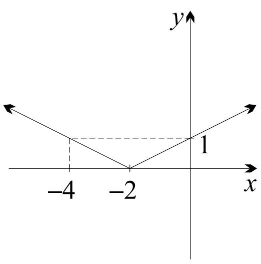

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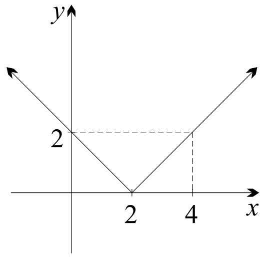

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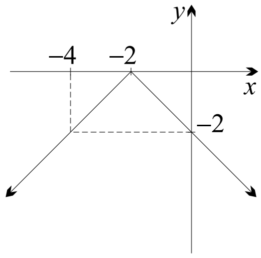

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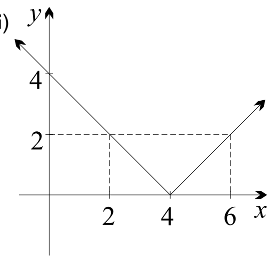

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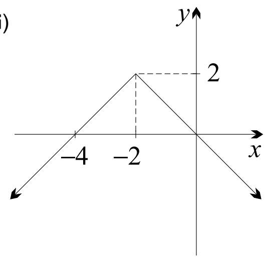
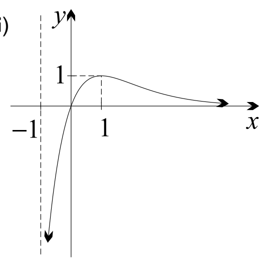
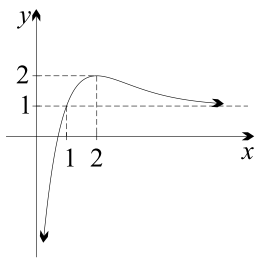


(c)(i)



(ii)



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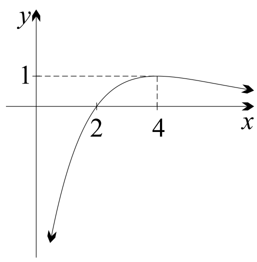

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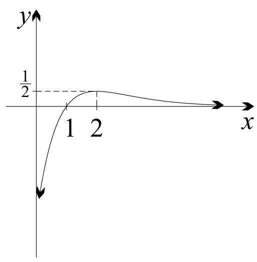

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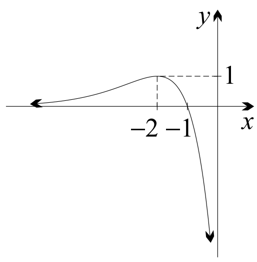

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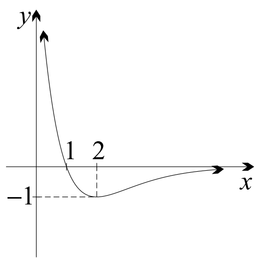
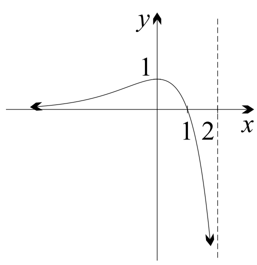
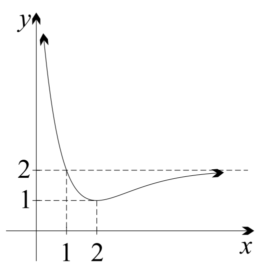


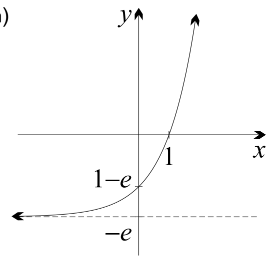
(vii)



(viii)

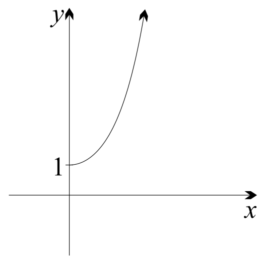


3(a)



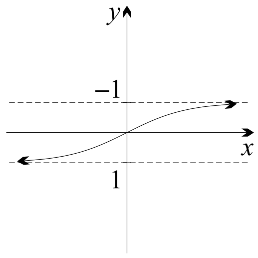
$f^{-1}(x) = e^x - e$

(b)



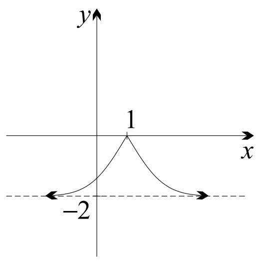
$f^{-1}(x) = \frac{1}{2}(e^x + e^{-x})$

(c)

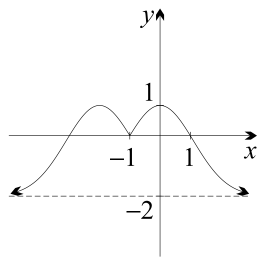


$$f^{-1}(x) = \frac{e^x - 1}{e^x + 1}$$

4(a)



(b)



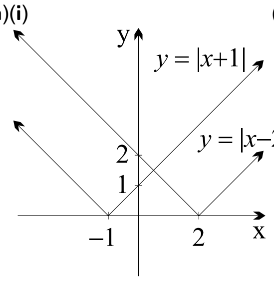
5(a) It could be a vertical shift of two down or a reflection in the x-axis. **(b)** It could be a shift left by $\frac{\pi}{2}$ or a reflection in the x-axis or a reflection in the y-axis.

7(a)(i) $x = n\pi$ for integer n.

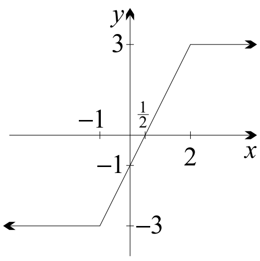
9(b) The converse is not true. For example, a primitive of $3x^2$ is $x^3 + 1$ which is neither even nor odd. **(c)** The converse is true in this case.

Exercise **8B** (Page 64)

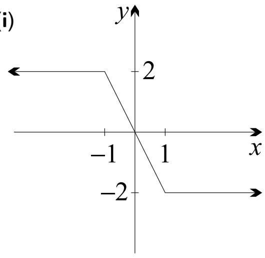
1(a)(i)



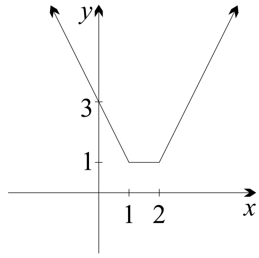
(ii)



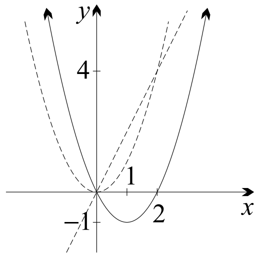
(b)(i)

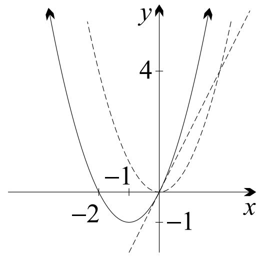


(ii)

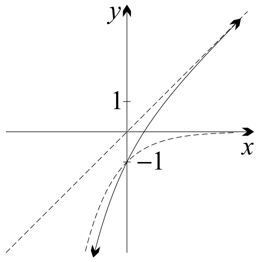


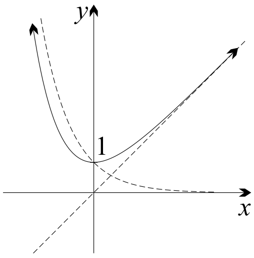
2(a)



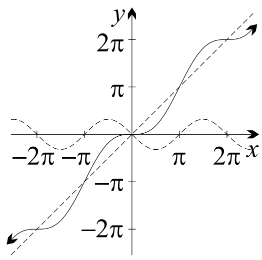


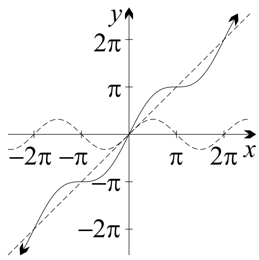
(b)



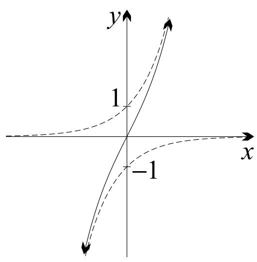


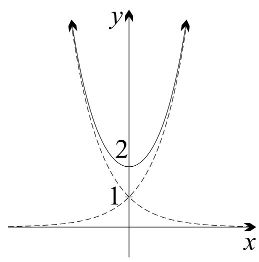
(c)



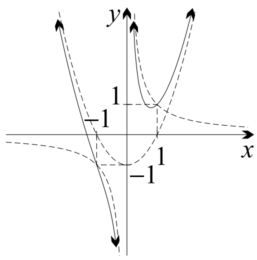


(d)

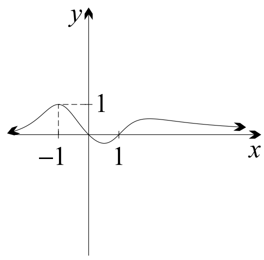




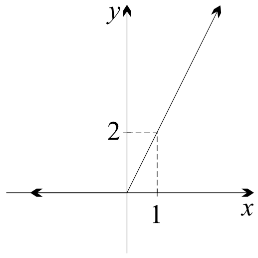
3(a)



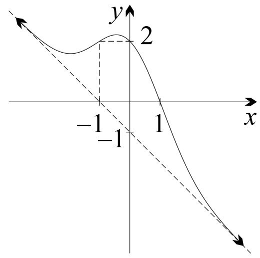
(b)



(c)



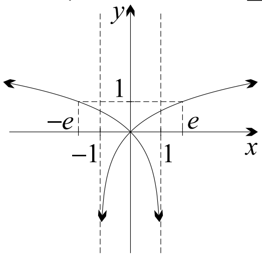
(d)



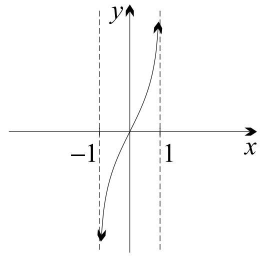
4(b)(i) $y = \begin{cases} 2 & \text{for } x < -1 \\ -2x & \text{for } -1 \le x < 1 \\ -2 & \text{for } x \ge 1 \end{cases}$

(ii) $y = \begin{cases} 3 - 2x & \text{for } x < 1 \\ 1 & \text{for } 1 \le x < 2 \\ 2x - 3 & \text{for } x \ge 2 \end{cases}$

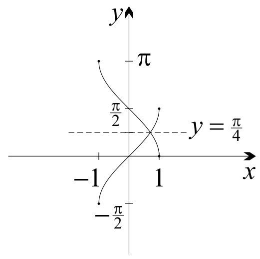
5(a)



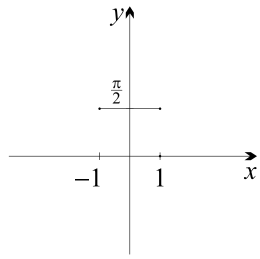
(b)



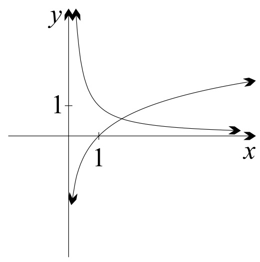
6(a)



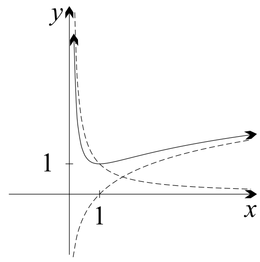
(b)



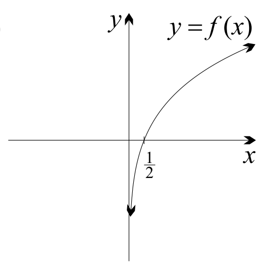
7(a)

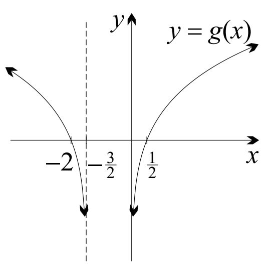


(c)



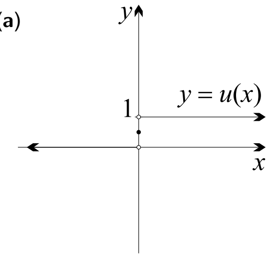
8(b)

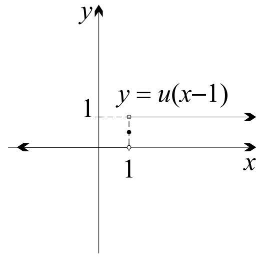




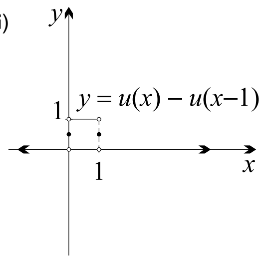
9(a) One possible selection is as follows.
Both odd: $f(x) = x$, $g(x) = \sin x$.
Both even: $f(x) = x^2$, $g(x) = \cos x$.
Odd and even: $f(x) = x$, $g(x) = \cos x$.
(c) When $g(x) = -f(x)$, $h(x) \equiv 0$ which is even.

10(a)

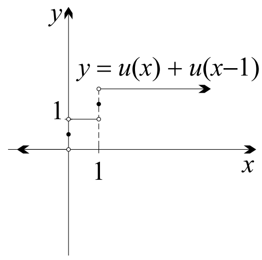




(b)(i)



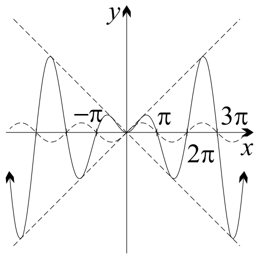
(ii)



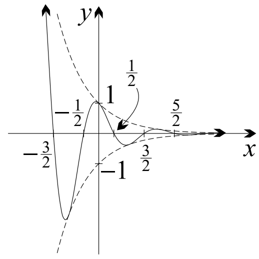
11 2

Exercise **8C** (Page 68)

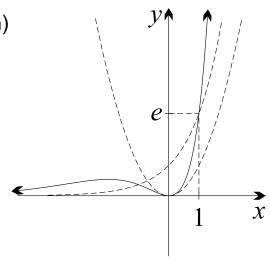
1(a)



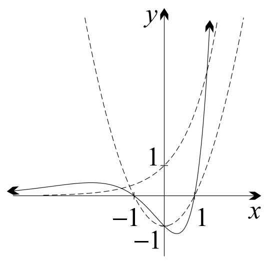
(b)



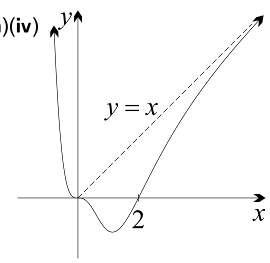
2(a)



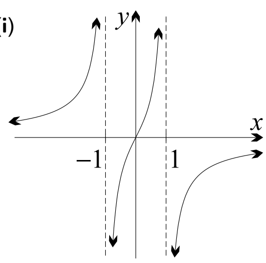
(b)



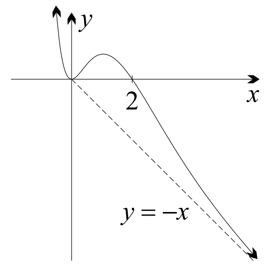
3(a)(iv)



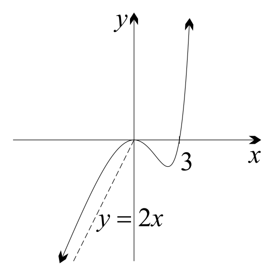
(b)(i)



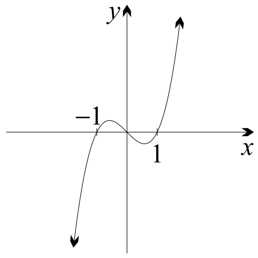
(ii)



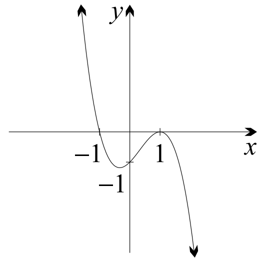
(iii)



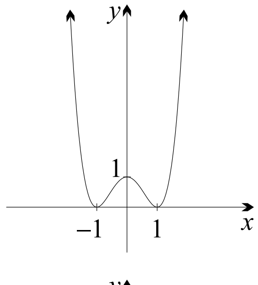
4(a)



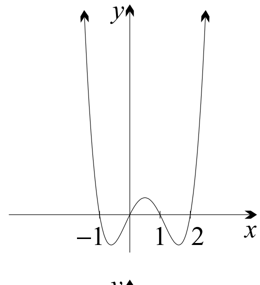
(b)



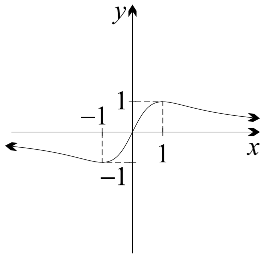
(c)



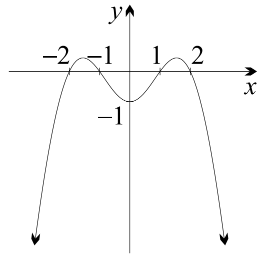
(d)



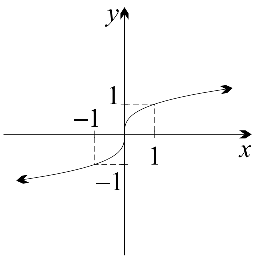
(e)



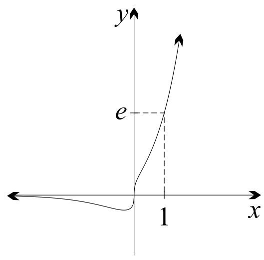
(f)



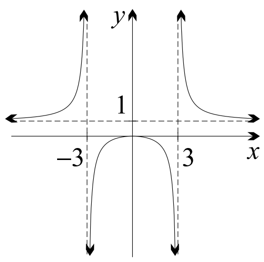
5(a)



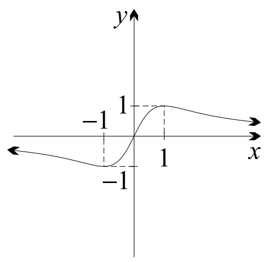
(b)



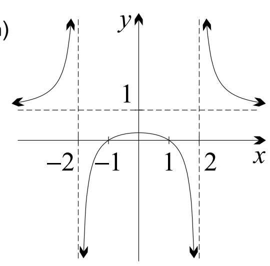
6



7



8(a)



(b) $x < -2$ or $x > 2$

9(a) $x = -2, -1, 1, 2$

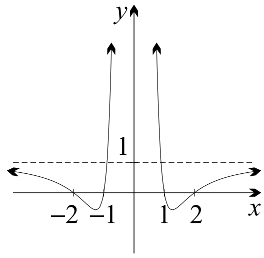
(b) $x = 0$

(c)(i) $y = 1$

(ii) $\left(\frac{\pm 4}{\sqrt{10}}, -\frac{9}{16}\right)$

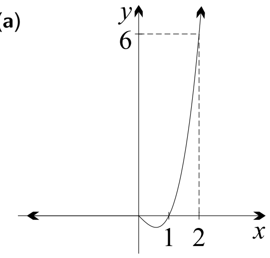
(d) $-\frac{9}{16} < b < 1$

(d)

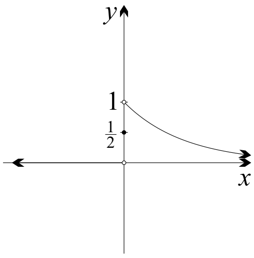


11 $y = h(x)$ has a hole at the origin.

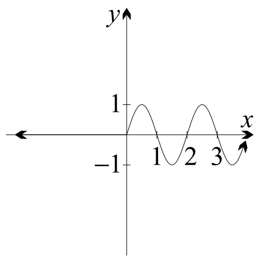
12(a)



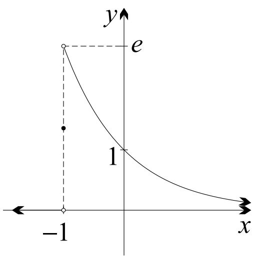
(b)



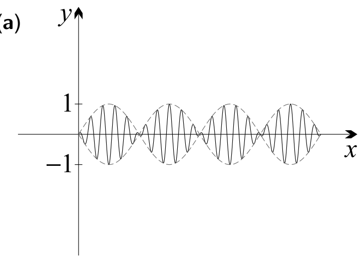
(c)



(d)



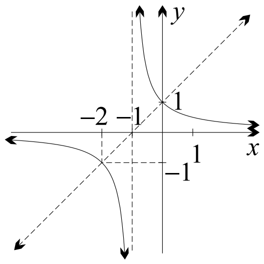
13(a)



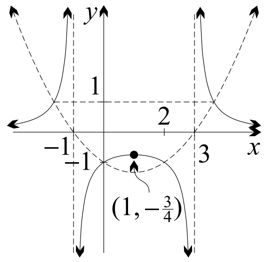
(b) $y = \cos 5x$

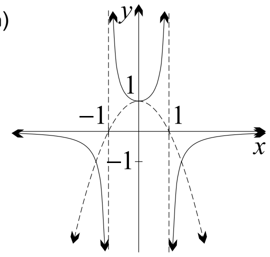
Exercise **8D** (Page 72)

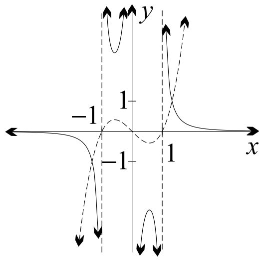
1

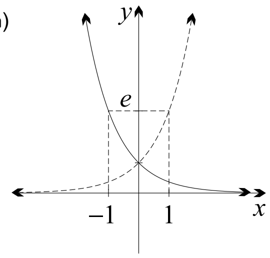


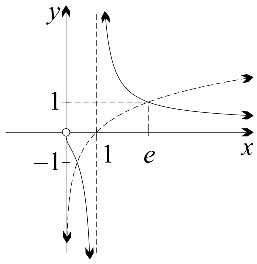
2

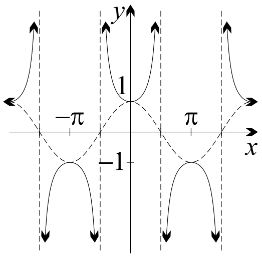


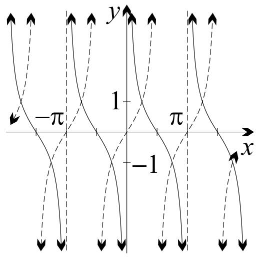
3(a)


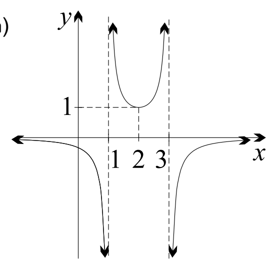
(b)


4(a)


(b)


(c)


(d)


5(a)


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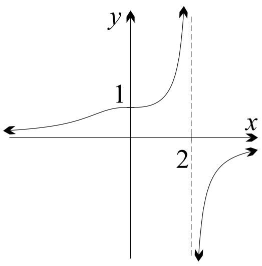

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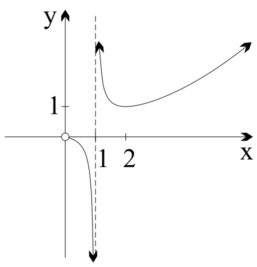

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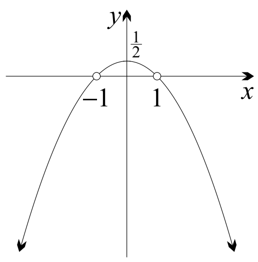

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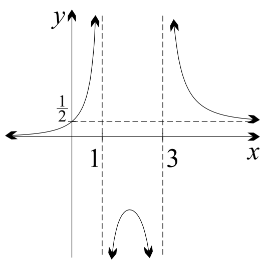

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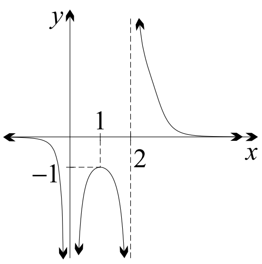
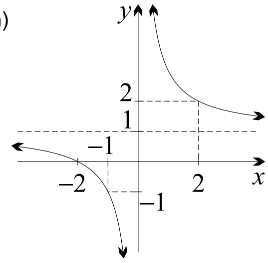


6(a)


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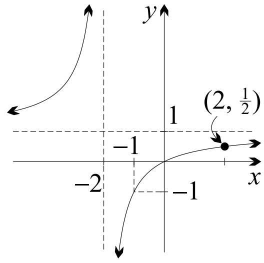
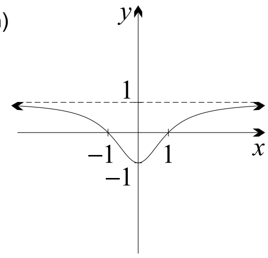


7(a)


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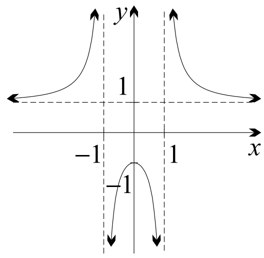
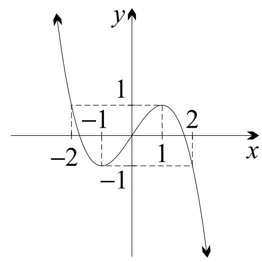


8(a)(ii) $x = -\sqrt{3}, 0, \sqrt{3}$ **(iii)** $x = 1, 1, -2$

(iv) $f(x) = 1$ has a double root at $x = 1$.

(v)


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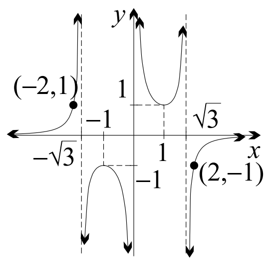

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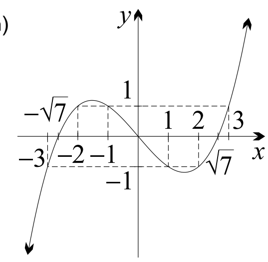

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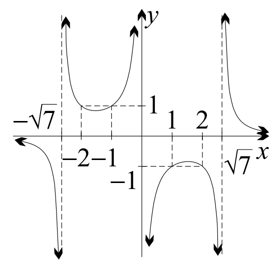
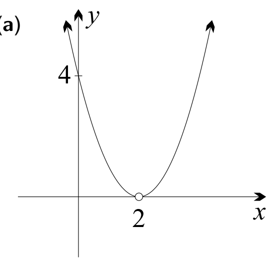


10(a)


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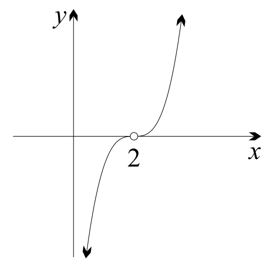
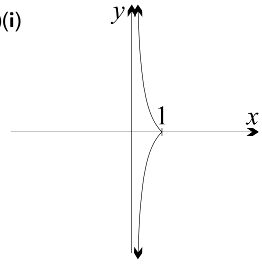


Exercise **8E** (Page 77)

1(a)(i)



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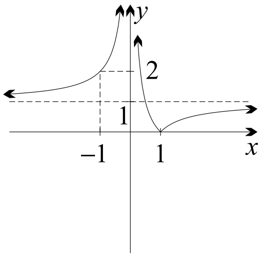

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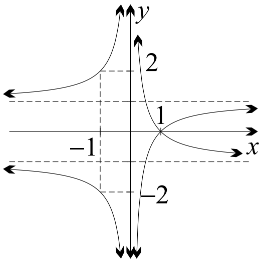
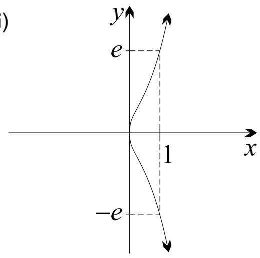


(b)(i)



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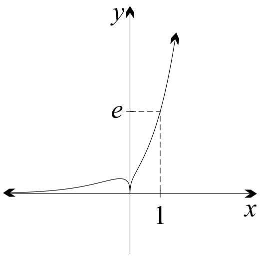

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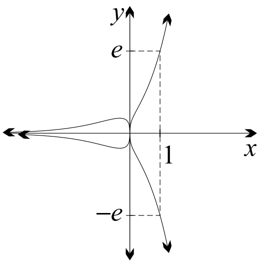

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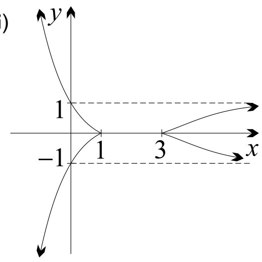

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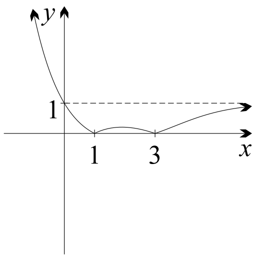

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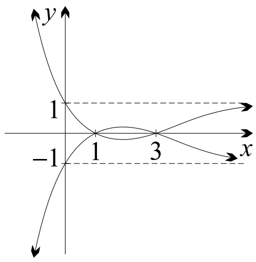

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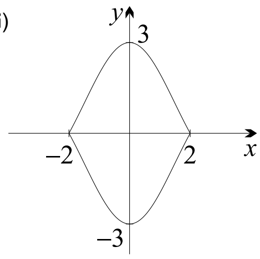

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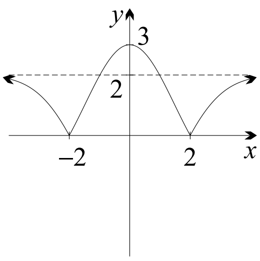

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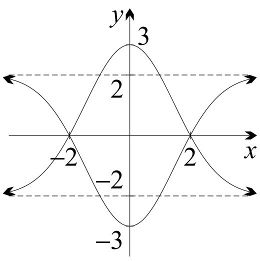
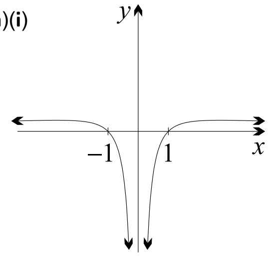


2(a)(i)



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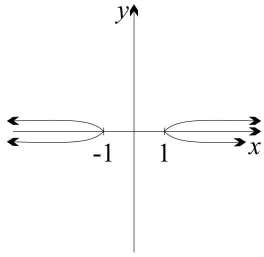

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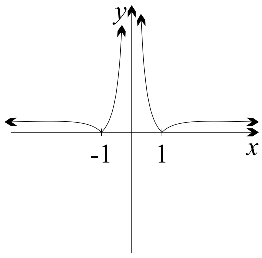

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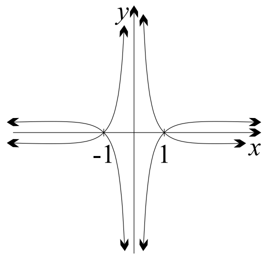

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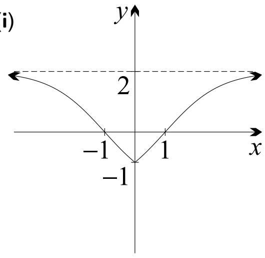

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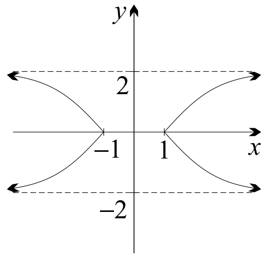

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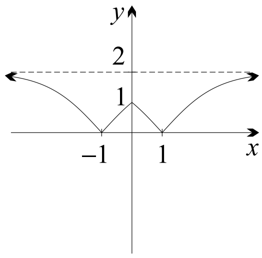

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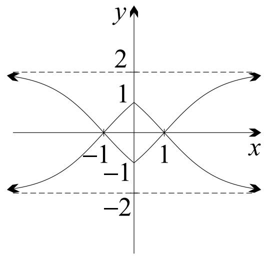

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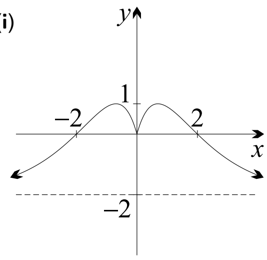

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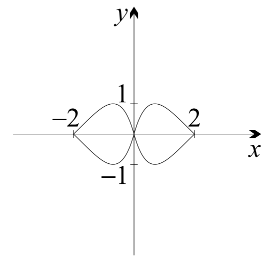

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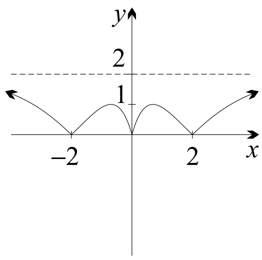

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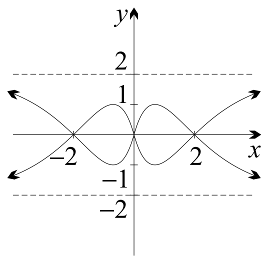
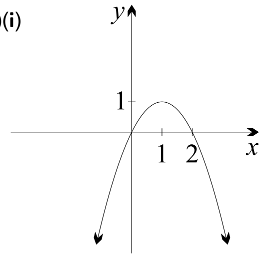


3(a)(i)



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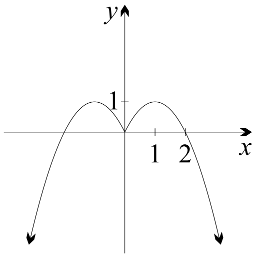

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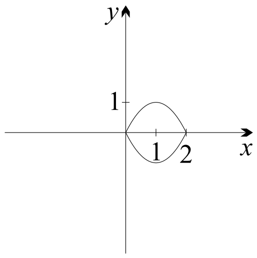

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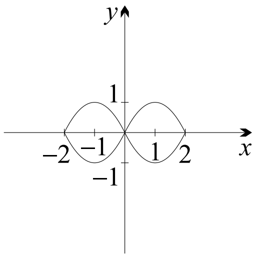
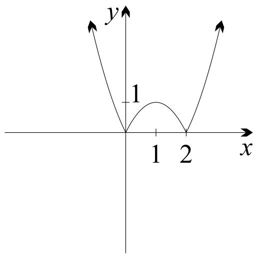


(v)



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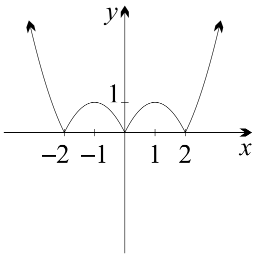

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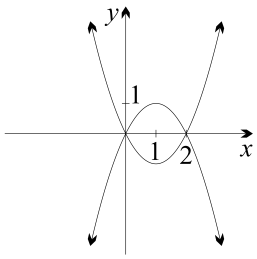

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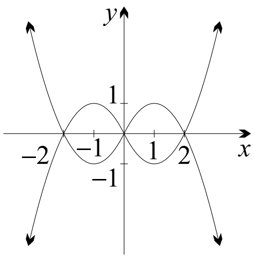

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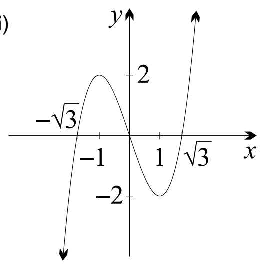

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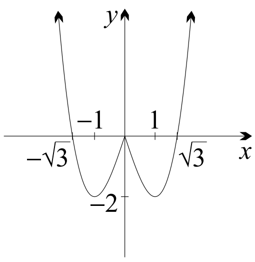

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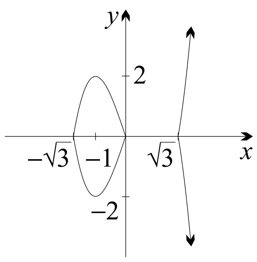

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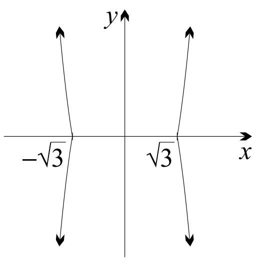

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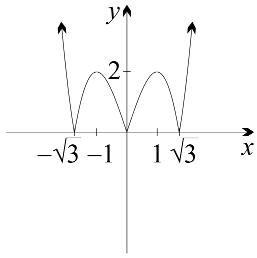

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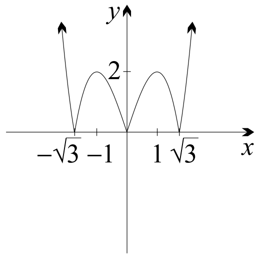

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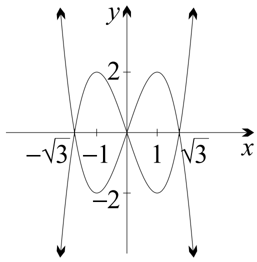

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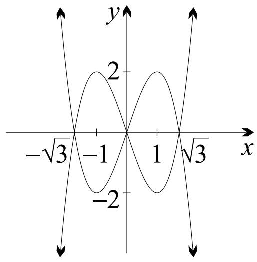
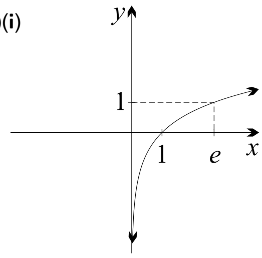


4(a)(i)



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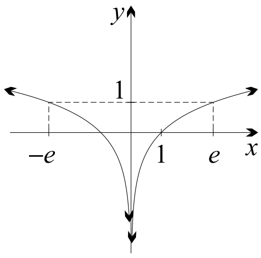

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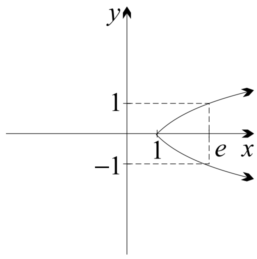

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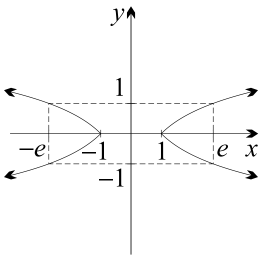

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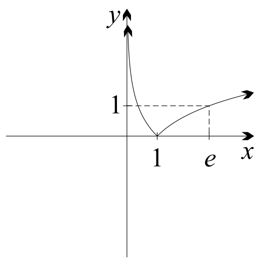

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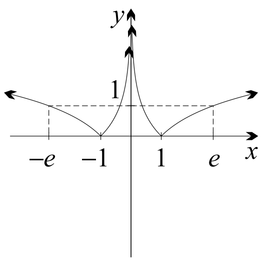

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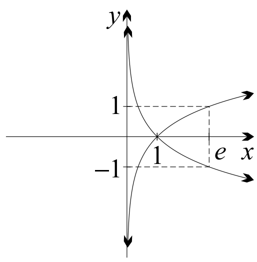

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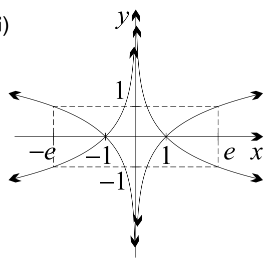

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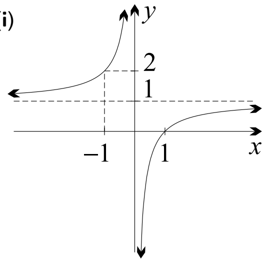

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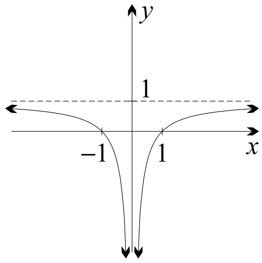

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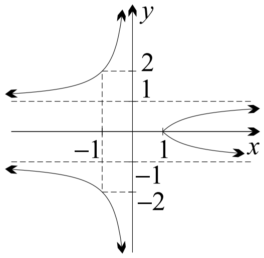

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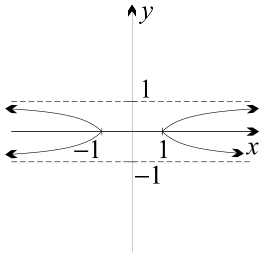

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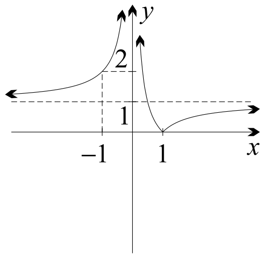

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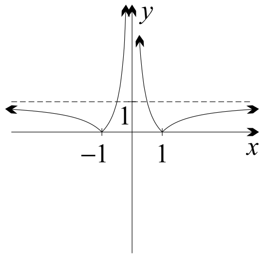

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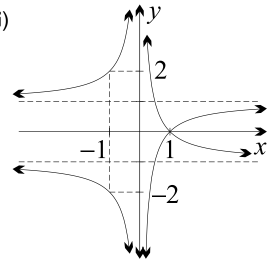
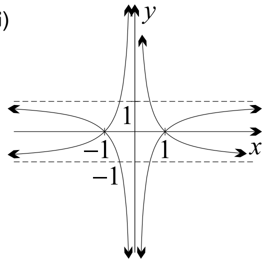
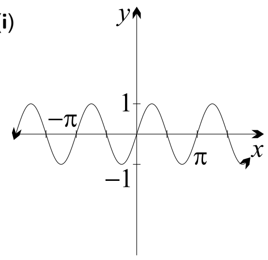


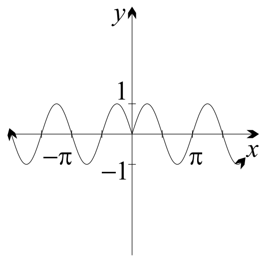
(viii)



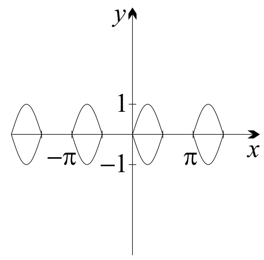
(c)(i)



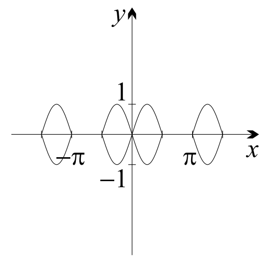
(ii)



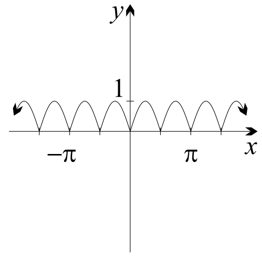
(iii)



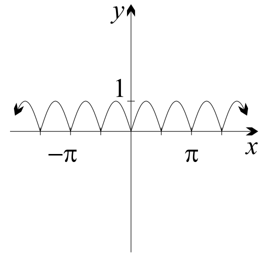
(iv)



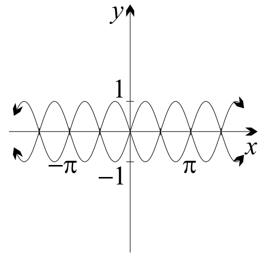
(v)



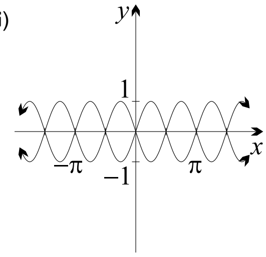
(vi)



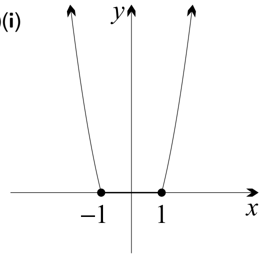
(vii)



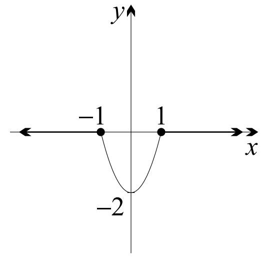
(viii)



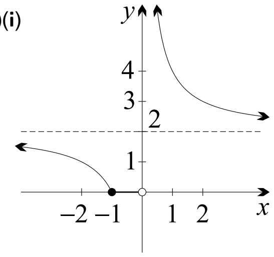
5(a)(i)



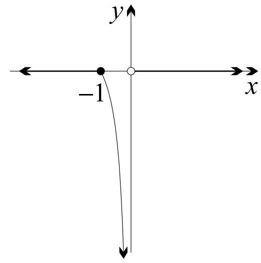
(ii)



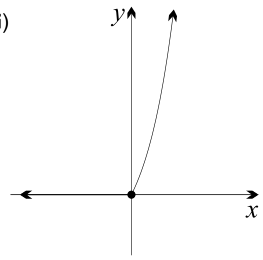
(b)(i)



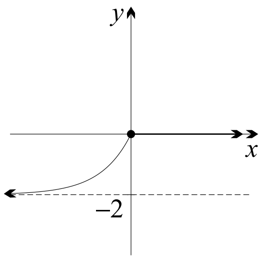
(ii)



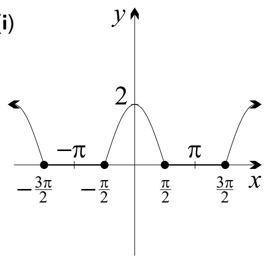
(c)(i)



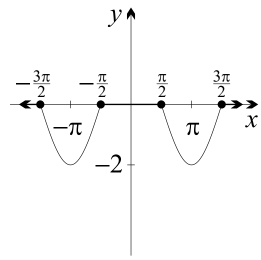
(ii)



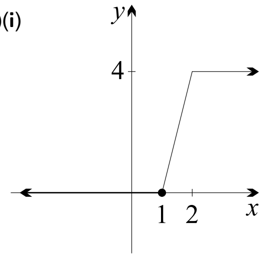
(d)(i)



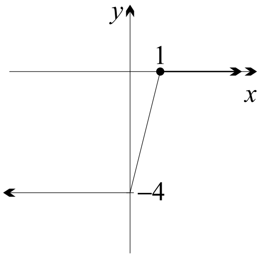
(ii)



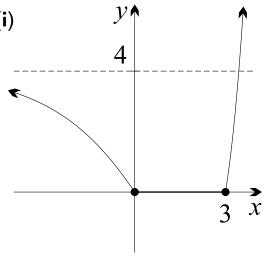
6(a)(i)



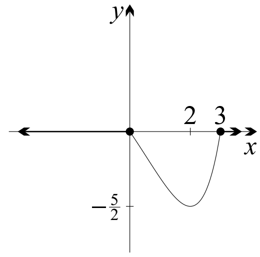
(ii)



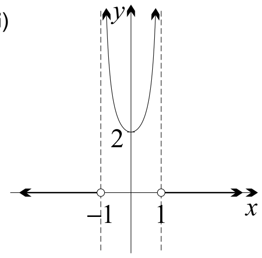
(b)(i)



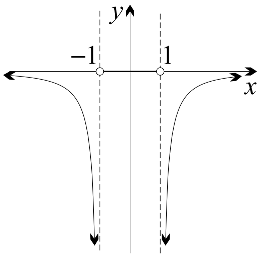
(ii)



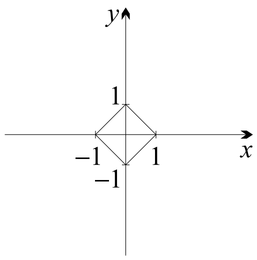
(c)(i)



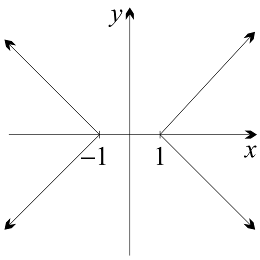
(ii)



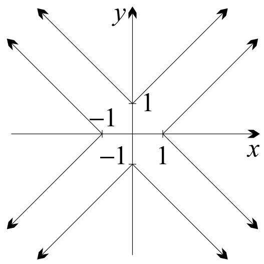
7(a)



(b)



(c)

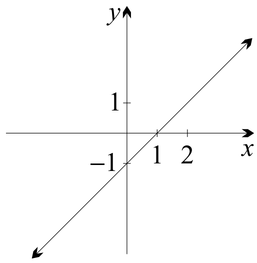


8(b) $|y| = |f(x)|$ and $|y| = \left|f(|x|)\right|$

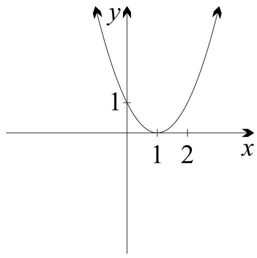
(e) Yes: $|y| = f(|x|)$ and $|y| = \left|f(|x|)\right|$ are the same if $f(|x|) \ge 0$, for example $f(x) = e^x - 1$.

Exercise **8F** (Page 81)

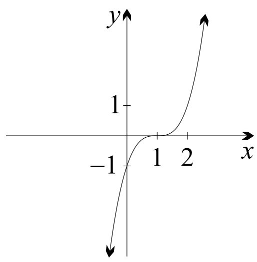
1(a)



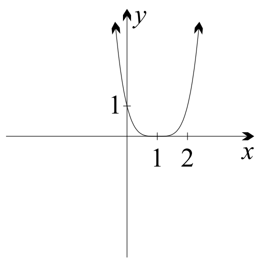
(b)



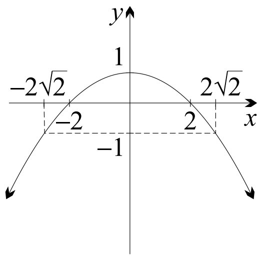
(c)



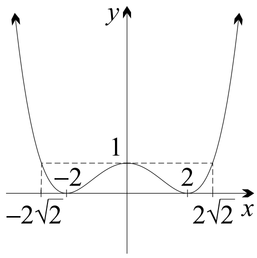
(d)



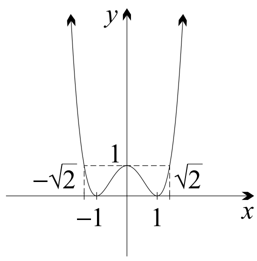
2(a)



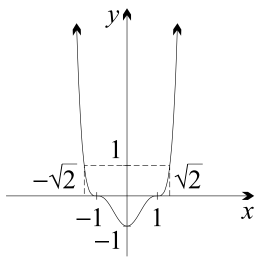
(b)



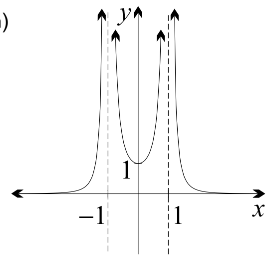
3(a)



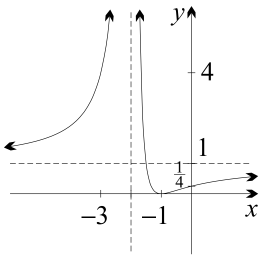
(b)



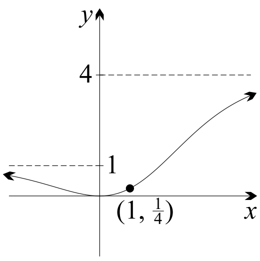
4(a)



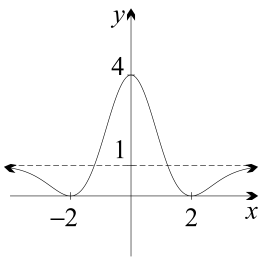
(b)



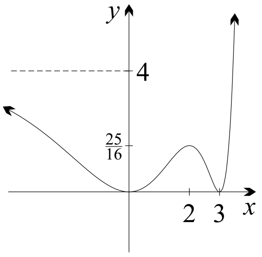
(c)



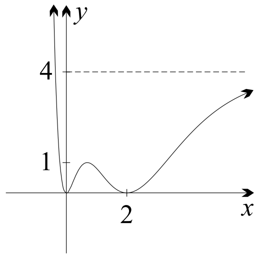
(d)



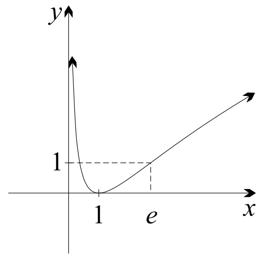
(e)



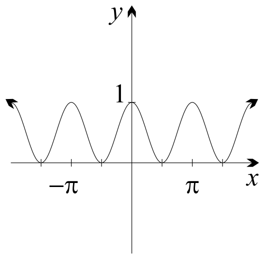
(f)



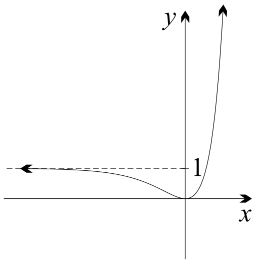
5(a)



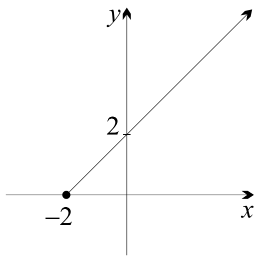
(b)



(c)

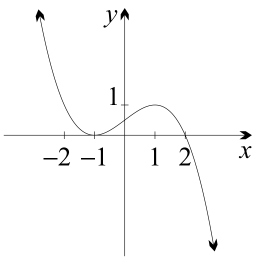


(d)

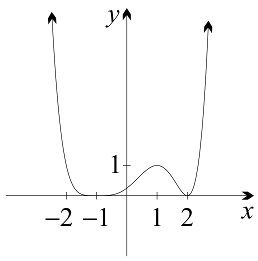


6(a) $(-2, 1)$ **(b)** $(-1, 0)$

(c)

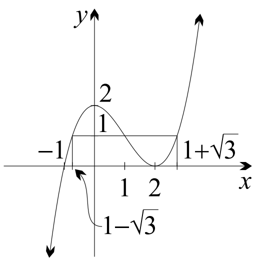


(d)

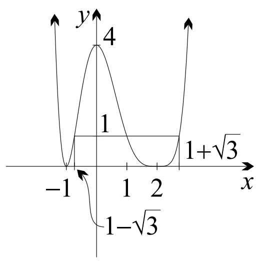


7(a) $(1-\sqrt{3}, 1)$ and $(1+\sqrt{3}, 1)$ **(b)** $(2, 0)$

(c)



(d)



(e) 4

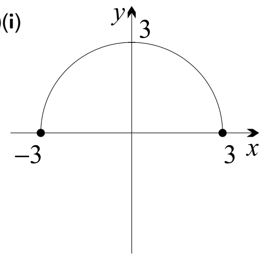
10(a) Either $(2, 0)$ is a minimum of $f(x)$, or n is even and $f(x)$ changes sign across $x = 2$.

(b) n is odd and $f(x)$ has a maximum at $(2, 0)$.

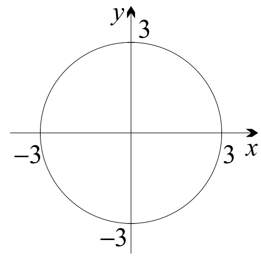
(c) n is odd and $f(x)$ changes sign across $x = 2$.

Exercise **8G** (Page 85)

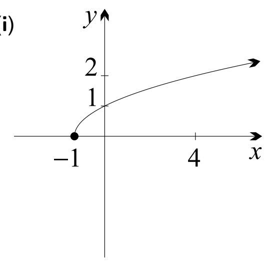
1(a)(i)



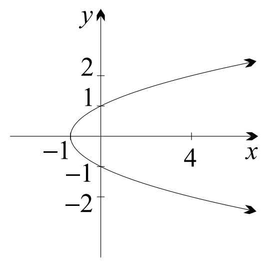
(ii)



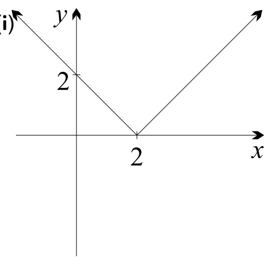
(b)(i)



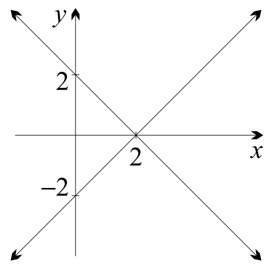
(ii)



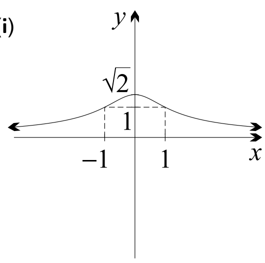
(c)(i)



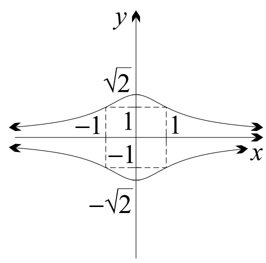
(ii)



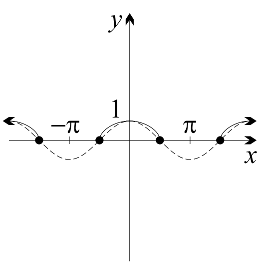
(d)(i)



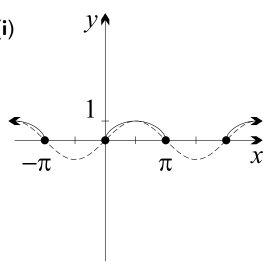
(ii)



2(a)

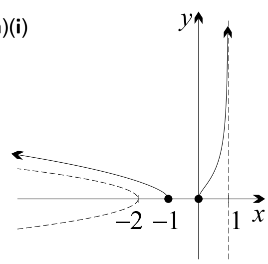


(b)(i)

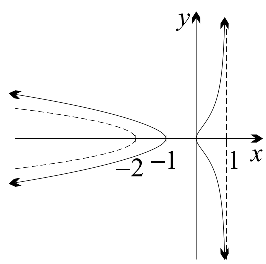


(ii) There is symmetry in $x = \frac{\pi}{2}$

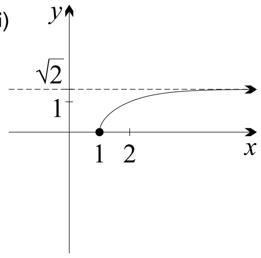
3(a)(i)



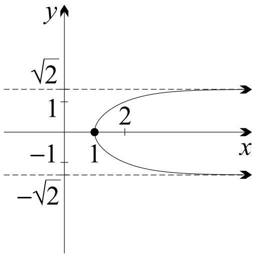
(ii)



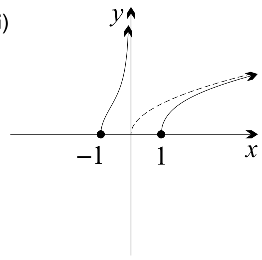
(b)(i)



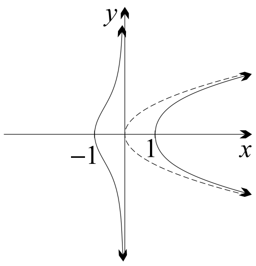
(ii)



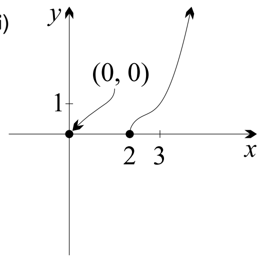
(c)(i)



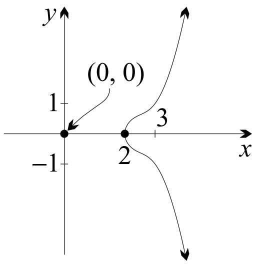
(ii)



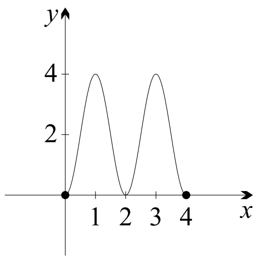
(d)(i)



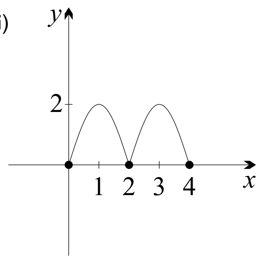
(ii)



4(a)



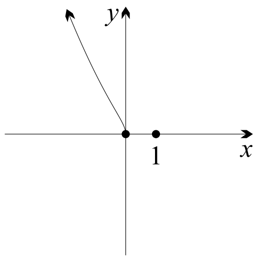
(b)(i)



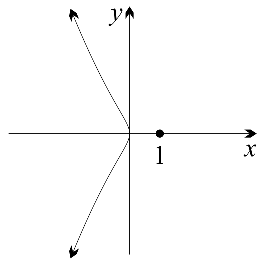
$f(x) = 4\sin^2\left(\frac{\pi}{2}x\right)$

(ii) $0,\ \frac{1}{3},\ \frac{5}{3},\ 2,\ \frac{7}{3},\ \frac{11}{3},\ 4$

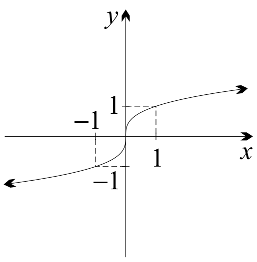
5(a)



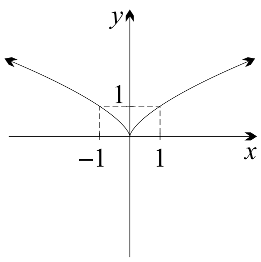
(b)



6(a)



(b)



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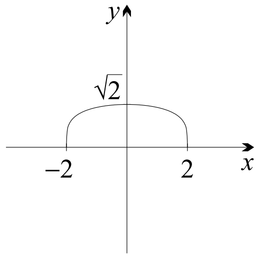
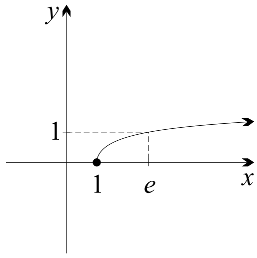
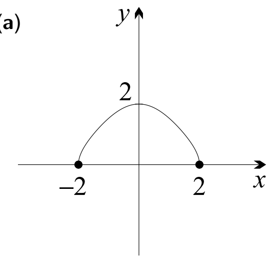


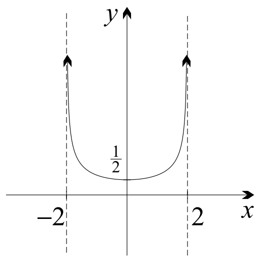
10



11(a)

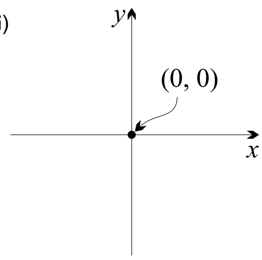


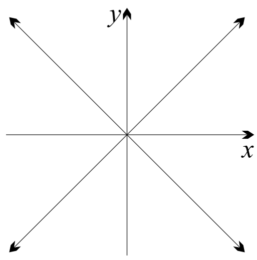
(b)



12(b) The upper half of the hyperbola in part (a).

(c)(i)



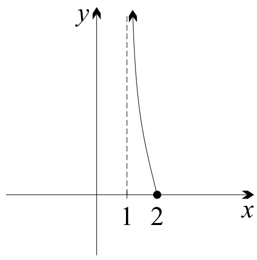


(ii) A horizontal plane through the apex yields a solitary point at the origin. A vertical plane through the apex yields a pair of perpendicular lines through the origin.

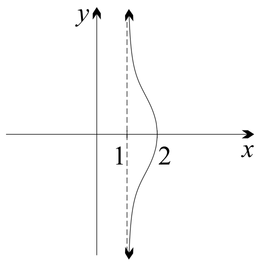
13 When $f(x) \le 0$ for all x in the natural domain.

15(b) $1 < x \le 2$

(c)

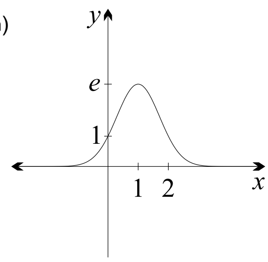


(d)

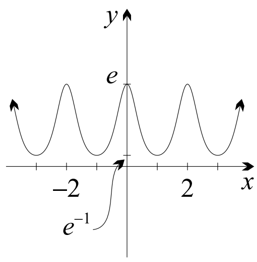


Exercise **8H** (Page 88)

1(a)



(b)



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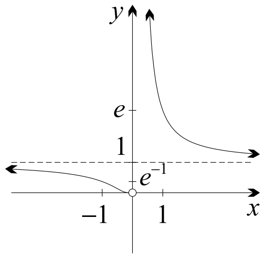
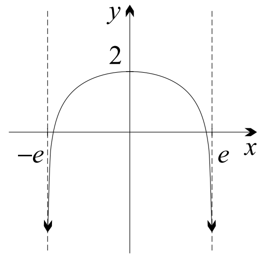
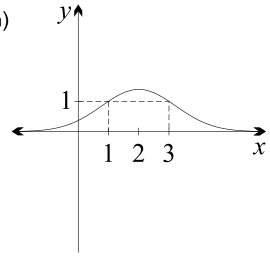
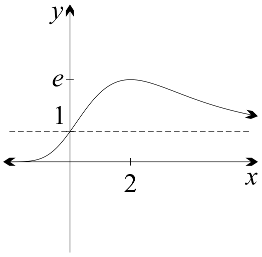
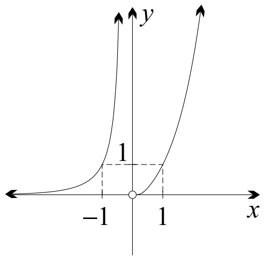
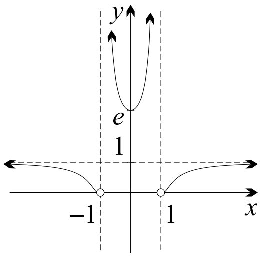
(c)


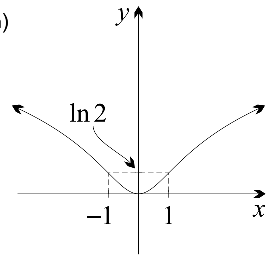
(d)


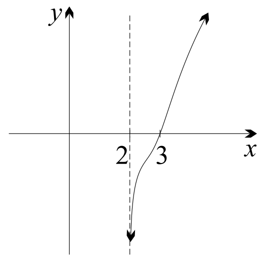
2(a)


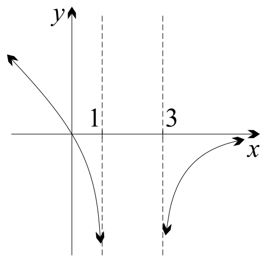
(b)


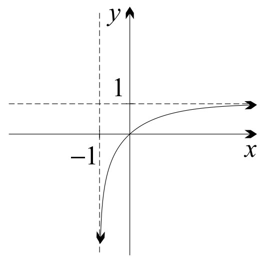
(c)


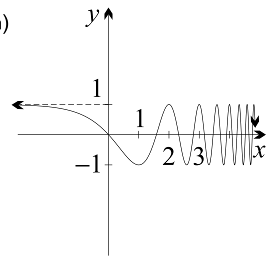
(d)


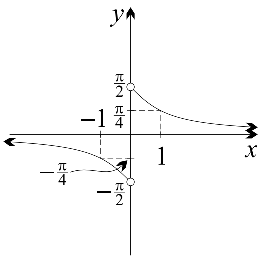
3(a)


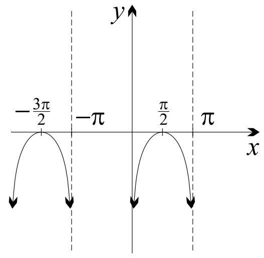
(b)


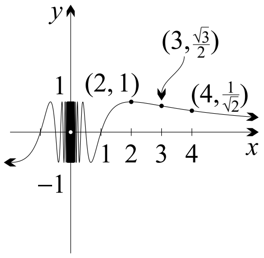
(c)


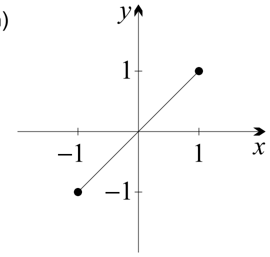
(d)


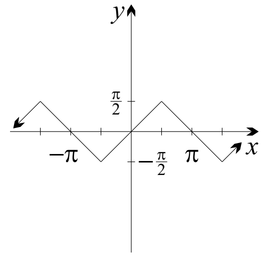
4(a)


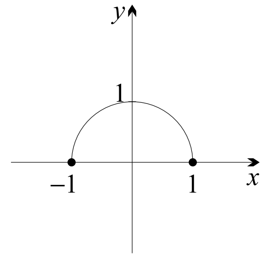
(b)


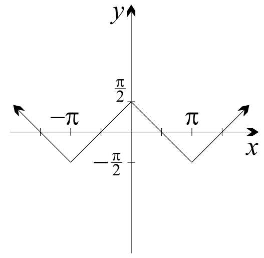
(c)


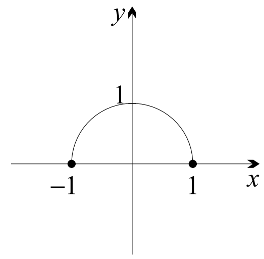
(d)


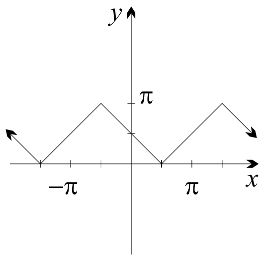
5(a)


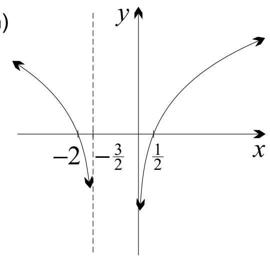
(b)


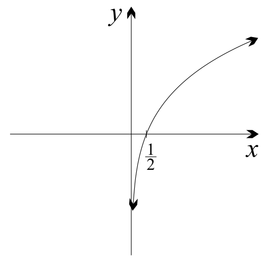
(c)


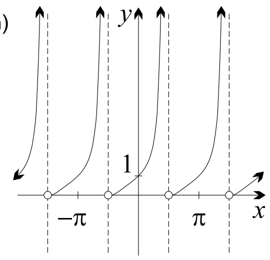
(d)


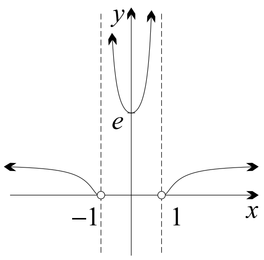
(e)


(f)


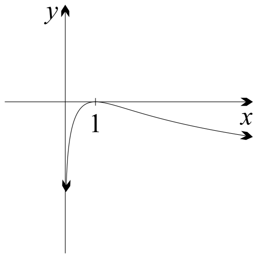
6(a)


(b)


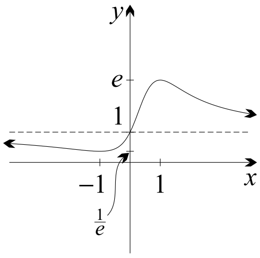
7(a)


(b)


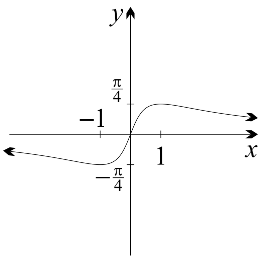
9(a)



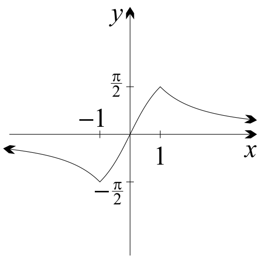
(b)



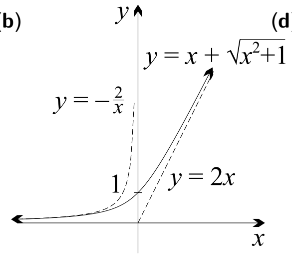
(c)



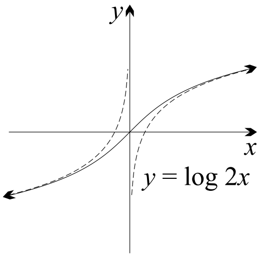
(d)



11(b)



(d)



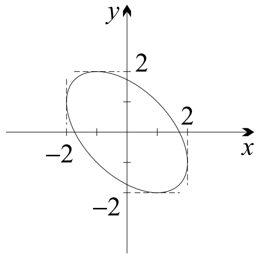
(e) $\sinh x = \dfrac{e^x - e^{-x}}{2}$

Exercise 8I (Page 93)

1(a) $y' + 1$ **(b)** $y + xy'$ **(c)** $2x - 2yy'$
(d) $3y^2y' + 3y + 3xy'$ **(e)** $y^{-1}y'$ **(f)** $e^y y'$
(g) $y'(2x + 3y) + y(2 + 3y')$ **(h)** $3(x + y)^2(1 + y')$
(i) $4(x^2 + y^2)(x + yy')$
2(a) $y = \sqrt{x^2 - 9}$ or $y = -\sqrt{x^2 - 9}$
(b) $y = \sqrt{4 - x^2}$ or $y = -\sqrt{4 - x^2}$
(c) $y = 1 + \sqrt{1 - x^2}$ or $y = 1 - \sqrt{1 - x^2}$
(d) $y = -x + \sqrt{1 - x^2}$ or $y = -x - \sqrt{1 - x^2}$
3(a) $y' = -\frac{x}{y}$, $(-6, 0)$, $(6, 0)$ **(b)** $y' = \frac{x}{y}$, $(-4, 0)$, $(4, 0)$ **(c)** $y' = \frac{x-y}{x-2y}$, $(-2, -1)$, $(2, 1)$ **(d)** $y' = \frac{3x^2+y^2}{2y(2-x)}$, $(0, 0)$
4(a) $(0, -6)$, $(0, 6)$ **(b)** none **(c)** $(-\sqrt{2}, -\sqrt{2})$, $(\sqrt{2}, \sqrt{2})$ **(d)** none
5(a) $\frac{5}{4}$ **(b)** $\frac{1}{4}$ **(c)** 0 **(d)** $-\frac{1}{4}$ **(e)** $\frac{1}{2}$ **(f)** $\frac{13}{48}$
6(a) $y = x + 4$ **(b)** $10x - 7y = 1$ **(c)** $x - 2y - 5 = 0$
(d) $y = 3x + 2$ **(e)** $y = 12x - 23$ **(f)** $y = 2x - 3$
7(a) $4x - 7y + 19 = 0$ **(b)** The denominator of y' is never zero. **(d)** 1
8(b) $y = 1 - x$

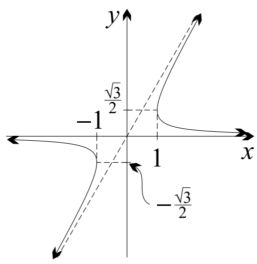
9(a) $-2 \le x \le 2$
(b) $(-\sqrt{3}, 0)$, $(\sqrt{3}, 0)$
(c)(ii) $(-1, -1)$, $(1, 1)$
(e) $(-1, 2)$, $(1, -2)$
(f) $(-2, 1)$, $(2, -1)$

(g)



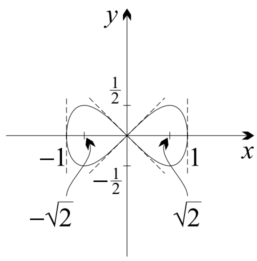
10(a) $x \le -1$ or $x \ge 1$
(b) no
(d) $y = 0$, $y = x\sqrt{3}$
(f) no

(g)



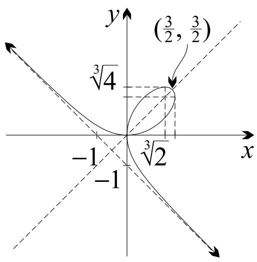
11(a) $-1 \le x \le 1$
(b) $(-1, 0)$, $(0, 0)$, $(1, 0)$
(c) The relations is even in both x and y.
(g) $(\frac{1}{\sqrt{2}}, \frac{1}{2})$

(h)



12(a) $(0, 0)$ **(b)(ii)** $(\frac{3}{2}, \frac{3}{2})$ **(c)(ii)** $(\sqrt[3]{2}, \sqrt[3]{4})$ **(d)(iii)** 0
(e)(i) $(0, 0)$, $(\sqrt[3]{4}, \sqrt[3]{2})$
(ii) The curve crosses itself. **(f)(ii)** $x + y + 1 = 0$

(g)



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Appendix — Table of Standard Integrals

Here is a table of standard integrals. A similar table is supplied on the last page of each examination paper.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Chapter Two

Exercise **2A** (Page 53)

1(a) $\frac{1}{2}\sin 2x + C$ **(b)** $3\tan\frac{x}{3} + C$
(c) $\frac{1}{5}\tan^{-1}(\frac{x}{5}) + C$ **(d)** $\sin^{-1}(\frac{x}{2}) + C$
(e) $\log\left(x + \sqrt{x^2+3}\right) + C$
(f) $\log\left(x + \sqrt{x^2-5}\right) + C$
2(a) $2(e^2-1)$ **(b)** $\frac{1}{2}$ **(c)** $\frac{\pi}{8}$ **(d)** $\frac{\pi}{4}$
(e) $\log\left(\frac{3+\sqrt5}{1+\sqrt5}\right) = \log\left(\frac{1+\sqrt5}{2}\right)$ **(f)** $2\log 3$
3(a) $-\frac{1}{2}\log(1-x^2) + C$ **(b)** $\log(x+\tan x) + C$
(c) $\frac{1}{3}\log(1+\sin 3x) + C$
4(a) $\frac{1}{3}\log 2$ **(b)** $\frac{1}{2}\log(\frac{e^2+1}{2})$ **(c)** $\log 2$
5(a) $\frac{\pi}{3\sqrt3}$ **(b)** $\frac{\pi}{18}$ **(c)** $\frac{1}{2}\log\left(\frac{\sqrt2+1}{\sqrt2-1}\right) = \log(\sqrt2+1)$
(d) $\frac{1}{\sqrt5}\log\left(\frac{15+7\sqrt5}{5+\sqrt5}\right) = \frac{1}{\sqrt5}\log(2+\sqrt5)$
6(a) $x + \log(x-1) + C$ **(b)** $x - 2\log(x+1) + C$
(c) $x + 2\log(x-1) + C$
7(a) $1 - \log 4$ **(b)** $1 - \frac{1}{4}\log 5$ **(c)** $\pi - 1$
8(a) $\frac{\pi}{3} - \frac{1}{2}$ **(b)** $\frac{\pi}{4} + \log 2$ **(c)** $\frac{1}{4}(\pi - \log 4)$
(d) $\frac{\pi}{8} + \frac{1}{2}\log 2$
10(a) $\frac{x^3}{3} - \frac{x^2}{2} + x - \log(x+1) + C$
(b) $\frac{1}{2}\left(x^2 - \log(x^2+1)\right) + C$
(c)(i) $\frac{x^3}{3} + \frac{x^2}{2} + x + \log(x-1) + C$
(ii) $\frac{x^3}{3} - x + \tan^{-1}x + C$ **(iii)** $x - \log(1+e^x) + C$
(iv) $\frac{1}{3}(2x-8)\sqrt{2+x} + C$ **(v)** $-\frac{2}{3}(2+x)\sqrt{1-x} + C$
(vi) $\frac{1}{2}x^2 - 2\log(x^2+4) + C$
11(a) $\log(e+e^{-1})$ **(b)** $\frac{1}{2}\log\left(\frac{e^2+1}{2}\right)$ **(c)** $\frac{\pi}{12} + \log 2$
12(a) $\frac{1}{2}x^2 + \log(x+1) + C$ **(b)** $\frac{1}{3}x^3 + 3\log(x-2) + C$
(c) $x + \log(1+x^2) + C$
13 $2\log(1+\sqrt{x}) + C$

Exercise **2B** (Page 57)

1(a)(i) $-\frac{1}{2}\log(1-x^2) + C$ **(ii)** $\log(1+\sin x) + C$
(iii) $\log(\log x) + C$ **(b)(i)** $\frac{1}{2}\left(\log(e^2+1) - \log 2\right)$
(ii) $\frac{1}{3}\log 2$ **(iii)** $\frac{1}{2}\log 3$
2(a)(i) $2e^{x^3} + C$ **(ii)** $e^{\tan x} + C$ **(iii)** $-e^{\frac{1}{x}} + C$
(b)(i) $\frac{1}{2}(e-1)$ **(ii)** $e - 1$ **(iii)** $2e(e-1)$
3(a) $\frac{1}{5}(x^2+1)^5 + C$ **(b)** $\frac{1}{7}(1+x^3)^7 + C$
(c) $-\frac{2}{1+x^3} + C$ **(d)** $\frac{1}{2(x^2-3)^4} + C$
(e) $\sqrt{x^2-2} + C$ **(f)** $\frac{1}{2}\sqrt{1+x^4} + C$
4(a) $\frac{-1}{2\sin^2 x} + C$ **(b)** $\frac{-1}{1+\tan x} + C$ **(c)** $\frac{1}{3}(\log x)^3 + C$
(d) $2\sin\sqrt{x} + C$ **(e)** $\frac{1}{2}\tan^{-1}x^2 + C$
(f) $\sin^{-1}x^3 + C$
5(a) $\frac{7}{4}$ **(b)** $2 - \sqrt3$ **(c)** $3(\sqrt3 - \sqrt2)$
(d) $\frac{1}{5}$ **(e)** $\frac{1}{3}$ **(f)** 2
6(a) $-\frac{1}{42}$ **(b)** Begin by writing $x = (x-1) + 1$.
7(a) $\frac{2}{15}(3x-2)(1+x)\sqrt{1+x} + C$
(b) $2\left(\sqrt{x} - \log(1+\sqrt{x})\right) + C$
(c) $4\left(x^{\frac14} - \frac{1}{2}\sqrt{x} + \frac{1}{3}x^{\frac34} - \log(1+x^{\frac14})\right) + C$
(d) $\tan^{-1}\sqrt{e^{2x}-1} + C$
8(a) $\frac{1}{9}$ **(b)** $\frac{128}{15}$ **(c)** $4 + 10\log\frac{5}{7}$ **(d)** $\frac{\pi}{12}$
9(a) $2\tan^{-1}\left(\sqrt{x}\right) + C$ **(b)** $\frac{2}{3}(x-2)\sqrt{x+1} + C$
10(a) $\frac{x}{\sqrt{1+x^2}} + C$ **(b)** $2\sin^{-1}\frac{x}{2} - \frac{1}{2}x\sqrt{4-x^2} + C$
(c) $-\frac{\sqrt{25-x^2}}{25x} + C$ **(d)** $-\frac{1}{x}\sqrt{1+x^2} + C$
11(a) $\frac{2}{3}$ **(b)** Begin by writing $x^3 = x(x^2+1) - x$.
12(b) The region is half a segment.
13(b) Begin by writing $x^2 = 1 - (1-x^2)$.
14(a) $\tan^{-1}\sqrt{x^2-1} + C_1$ **(b)** $\tan^{-1}\sqrt{x^2-1} + C_2$
15(a) $\frac{\sqrt3}{8} - \frac{\sqrt{\epsilon(4+\epsilon)}}{4(2+\epsilon)}$ **(b)** $\frac{\sqrt3}{8}$

Exercise **2C** (Page 64)

1(a) $\frac{1}{x-1} - \frac{1}{x+1}$ **(b)** $\frac{1}{3(x-4)} - \frac{1}{3(x-1)}$ **(c)** $\frac{2}{x-3} + \frac{2}{x+3}$
(d) $\frac{2}{x-2} - \frac{1}{x-1}$ **(e)** $\frac{1}{5(x-2)} + \frac{4}{5(x+3)}$ **(f)** $\frac{1}{x-1} + \frac{2-x}{x^2+3}$
2(a) $\ln(x-4) - \ln(x-2) + C$
(b) $2\ln(x+1) - 2\ln(x+3) + C$
(c) $4\log(x-2) - \log(x-1) + C$
(d) $3\log(x-1) - \log(x+3) + C$
(e) $\log(x+1) + \log(2x+3) + C$
(f) $2\log(x+1) + 3\log(2x-3) + C$
3(a) $\frac{1}{4}\log\frac{3}{2}$ **(b)** $\log 2$ **(c)** $\log\frac{14}{3}$ **(d)** $\frac{1}{2}\log 2$
4(a) $\log(x-2) - 2\tan^{-1}x + C$
(b) $\log(2x+1) - \frac{1}{2}\log(x^2+3) + C$
(c) $\tan^{-1}x + 3\log x - \log(x^2+1) + C$
5(a) $\frac{\pi}{4} - \log\frac{3}{2}$ **(b)** $\pi + \log 2$ **(c)** $\log 4 - \frac{3}{2}\log 3$
6(a) $5\log(x-1) + 7\log(x-2) - 12\log(2x-3) + C$
(b) $\frac{3}{2}\log(x) - 5\log(x-2) + \frac{7}{2}\log(x-4) + C$
7(a) $\frac{5}{3}\log 3 - \log 2$ **(b)** $2\log 3 - 8\log 2$
8(a)(i) $A = 2,\ B = 1,\ C = -3$
(ii) $2x + \log(x-1) - 3\log(x+2) + C$
(b)(i) $x + \log(x-2) - 2\log(x+1) + C$
(ii) $3x + 2\log(x+4) + \log(x-5) + C$
9(a)(i) $A = 1,\ B = -1,\ C = 2,\ D = -1$
(ii) $\log 3 + \log 2 - \frac{1}{2}$ **(b)** $12 + \log 2$
10(a)(i) $A = 12,\ B = 2$
(ii) $3x + 12\log(x-2) - \frac{2}{x-2} + C$
(b)(i) $A = 23,\ B = 10,\ C = -23,\ D = 13$
(ii) $23\log\left(\frac{x-1}{x-2}\right) - \frac{10}{x-1} - \frac{13}{x-2} + C$
12(a) $A = 0,\ B = 1,\ C = 0,\ D = 2$
13(a) $x + \log(x-1) - \log(x+1) + C$
(b) $x + 2\log(x-1) - \log x + C$
(c) $x - \tan^{-1}x + \log x - \frac{1}{2}\log(x^2+1) + C$
(d) $x + 9\log(x-3) - 4\log(x-2) + C$

(e) $\frac{1}{2}x^2 - x + 5\log(x) - 4\log(x+1) + C$
(f) $\frac{1}{3}x^3 + \frac{3}{2}x^2 + 7x + 16\log(x-2) - \log(x-1) + C$

Exercise **2D** (Page 68)

1(a) $\frac{1}{3}\tan^{-1}\frac{x}{3} + C$ **(b)** $\log(x + \sqrt{9+x^2}) + C$
(c) $\sin^{-1}\frac{x}{3} + C$ **(d)** $\log(x + \sqrt{x^2-9}) + C$
(e) $\frac{1}{6}\Big(\log(x-3) - \log(x+3)\Big) + C$
(f) $\frac{1}{6}\Big(\log(3+x) - \log(3-x)\Big) + C$
2(a) $\tan^{-1}(x+2) + C$ **(b)** $\frac{1}{4}\tan^{-1}\left(\frac{x-2}{4}\right) + C$
(c) $\log(x - 3 + \sqrt{x^2-6x+13}\,) + C$
(d) $\log(x + 4 + \sqrt{x^2+8x+12}\,) + C$
(e) $\sin^{-1}\frac{x-4}{5} + C$
(f) $\frac{1}{2}\log\left(x + 1 + \sqrt{x^2+2x+\frac{3}{2}}\right) + C$
3(a) $\frac{\pi}{8}$ **(b)** π **(c)** $\frac{\pi}{6}$ **(d)** $\frac{\pi}{2}$ **(e)** $\log 3$ **(f)** $\log 3$
4(a) $\log(x^2+2x+2) - \tan^{-1}(x+1) + C$
(b) $\frac{1}{2}\log(x^2+2x+10) - \frac{1}{3}\tan^{-1}\frac{x+1}{3} + C$
(c) $\sqrt{(x+1)^2+9} - \log\left(x+1+\sqrt{(x+1)^2+9}\right)$
(d) $\sqrt{x^2-2x-4} + 4\log\left(x-1+\sqrt{x^2-2x-4}\right)$
(e) $-\sqrt{6x-x^2} + 3\sin^{-1}\frac{x-3}{3} + C$
(f) $-\sqrt{4-2x-x^2} + 2\sin^{-1}\frac{x+1}{\sqrt{5}} + C$
5(a) $\frac{1}{2}\log 2 + \frac{\pi}{8}$ **(b)** $\frac{1}{4}(3\pi - \log 4)$ **(c)** $\log 2 - \frac{\pi}{4}$
(d) $2 - \sqrt{3} - \frac{\pi}{6}$ **(e)** $3\log(3+2\sqrt{2}\,) - 4\sqrt{2}$
(f) $\log\left(1+\sqrt{\frac{2}{3}}\right) + \sqrt{6} - 1$
6(a) $\sqrt{x^2-1} - \log\left(x+\sqrt{x^2-1}\right) + C$
(b) $\sin^{-1}x - \sqrt{1-x^2} + C$
(c) $\sqrt{6+x-x^2} + \frac{5}{2}\sin^{-1}\frac{2x-1}{5} + C$
7(a) $\frac{\pi}{3} + \sqrt{3} - 2$ **(b)** $3\sin^{-1}\frac{1}{3}$
(c) $2\sqrt{2} - \sqrt{3} + \log\left(\frac{2+\sqrt{3}}{3+2\sqrt{2}}\right)$
8(a) $\frac{x}{\sqrt{4x-x^2}}$ is undefined at $x=0$.

Exercise **2E** (Page 72)

1(a) $e^x(x-1) + C$ **(b)** $-e^{-x}(x+1) + C$
(c) $\frac{1}{9}e^{3x}(3x+2) + C$ **(d)** $x\sin x + \cos x + C$
(e) $-\frac{1}{2}(x-1)\cos 2x + \frac{1}{4}\sin 2x + C$
(f) $(2x-3)\tan x + 2\log(\cos x) + C$
2(a) π **(b)** $\frac{\pi}{2} - 1$ **(c)** $\frac{\pi}{4} - \frac{1}{2}\log 2$ **(d)** $\frac{1}{4}(e^2+1)$
(e) e^{-1} **(f)** $1 + e^{-2}$
3(a) $x(\log x - 1) + C$ **(b)** $2x(\log x - 1) + C$
(c) $x\cos^{-1}x - \sqrt{1-x^2} + C$
4(a) $\frac{\pi}{4} - \frac{1}{2}\log 2$ **(b)** 1 **(c)** $\frac{1}{2}$
5(a) $\frac{1}{4}x^2(2\log x - 1) + C$ **(b)** $\frac{1}{9}x^3(3\log x - 1) + C$
(c) $-\frac{1}{x}(\log x + 1) + C$
6(a) $(2 - 2x + x^2)e^x + C$
(b) $x^2\sin x + 2x\cos x - 2\sin x + C$
(c) $x(\log x)^2 - 2x\log x + 2x + C$
7(a) $-\frac{1}{42}$ **(b)** $\frac{4}{15}(1+\sqrt{2}\,)$ **(c)** $\frac{128}{15}$
8(a) $\frac{1}{2}e^x(\cos x + \sin x) + C$
(b) $-\frac{1}{2}e^{-x}(\cos x + \sin x) + C$
9(a) $\frac{1}{5}(e^{\pi} - 2)$ **(b)** $\frac{1}{5}(e^{\frac{\pi}{4}} + 2)$
10(a) $\frac{1}{2\sqrt{3}}(\pi - \sqrt{3})$ **(b)** $\frac{\sqrt{3}\pi}{2}$ **(c)** $\pi - 2$
12(a) $\frac{1}{2}\left(x\sqrt{a^2-x^2} + a^2\sin^{-1}(\frac{x}{a})\right) + C$
(b) $x\log(x + \sqrt{x^2+a^2}\,) - \sqrt{x^2+a^2} + C$
(c) $x\log(x + \sqrt{x^2-a^2}\,) - \sqrt{x^2-a^2} + C$
13(a) $\frac{1}{4}x^2(2\log x - 1) + C$
(b) $\frac{1}{4}x^2\left(2(\log x)^2 - 2\log x + 1\right) + C$
15(a) $\frac{1}{32}(\sin 4x - 4x\cos 4x + 8x\cos 2x - 4\sin 2x) + C$
(b) $\frac{1}{18}(3x\sin 3x + \cos 3x + 9x\sin x + 9\cos x) + C$
(c) $\frac{1}{4}e^x(\sin 3x - 3\cos 3x + 5\sin x - 5\cos x) + C$
16(a) $\frac{1}{48}(3\sqrt{3} - \pi)$ **(b)** $\frac{1}{12}(\pi + 2\log 2 - 2)$

Exercise **2F** (Page 78)

1(a) $\sin x + C$ **(b)** $-\cos x + C$ **(c)** $-\log(\cos x) + C$
(d) $\log(\sin x) + C$
2(a) $\frac{1}{3}\sin^3 x + C$ **(b)** $-\frac{1}{3}\cos^3 x + C$
(c) $\frac{1}{3}\cos^3 x - \cos x + C$ **(d)** $\sin x - \frac{1}{3}\sin^3 x + C$
(e) $\frac{1}{5}\sin^5 x - \frac{2}{3}\sin^3 x + \sin x + C$
(f) $\frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + C$
3(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{12}$ **(c)** $\frac{\pi}{8}$
4(a) $\tan x + C$ **(b)** $\tan x - x + C$
(c) $\frac{1}{3}\tan^3 x + \tan x + C$ **(d)** $\frac{1}{3}\tan^3 x - \tan x + x + C$
5(a) $\sqrt{2} - 1$ **(b)** $\frac{1}{27}(8\sqrt{3} - 9)$ **(c)** $\frac{1}{2}(1 - \log 2)$
(d) $\frac{4}{3}$ **(e)** $\frac{1}{3}(2 - \sqrt{2}\,)$ **(f)** $\frac{58}{15}$
6(a) $\frac{1}{4}$ **(b)** $\frac{11}{24}$ **(c)** $\frac{9}{64}$ **(d)** $\frac{53}{480}$ **(e)** $\frac{4}{15}$ **(f)** $\frac{7}{60\sqrt{2}}$
7(a) $\frac{1}{32}(\sin 4x + 8\sin 2x + 12x) + C$
(b) $\frac{1}{32}(\sin 4x - 8\sin 2x + 12x) + C$
(c) $\frac{1}{1024}(24x - 8\sin 4x + \sin 8x) + C$
9(a) 1 **(b)** $\frac{1}{3}\log 2$ **(c)** $\frac{\pi}{4}$
10(a) $\frac{\pi}{4}$ **(b)** $\frac{2}{15}(1 + sqrt2)$ **(c)** $\frac{\pi}{16}$
11(a) $\frac{1}{2}\sin^2 x + C_1$ **(b)** $-\frac{1}{4}\cos 2x + C_2$
13(a) $\frac{1}{2}\Big(\sec x\tan x - \log(\sec x + \tan x)\Big) + C$
(b) $\frac{1}{2}\Big(\sec x\tan x + \log(\sec x + \tan x)\Big) + C$
(c) $\sec x\tan x(2\sec^2 x + 3) + 3\log(\sec x + \tan x) + C$
14(a) $\frac{1}{2}$ **(b)** $\frac{4}{3}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{\sqrt{3}}{4}$
15(a) $-\frac{1}{8}\cos 4x - \frac{1}{4}\cos 2x + C$
(b) $-\frac{1}{8}\cos 4x + \frac{1}{4}\cos 2x + C$
(c) $\frac{1}{16}\sin 8x + \frac{1}{8}\sin 4x + C$
16(a) $\frac{1}{4}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{3}{8}$
17(a) $\tan\frac{x}{2} + C$ **(b)** $\log\left(\frac{\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right) + C$
(c) $\frac{1}{5}\log\left(\frac{1+2\tan\frac{x}{2}}{2-\tan\frac{x}{2}}\right) + C$

18 $x \sec x - \log(\sec x + \tan x) + C$

20 $\frac{1}{3}\sin 3\theta + \sin^3\theta + C$

Exercise **2G** (Page 83)

3(b) $\frac{1}{2}(e^2 - 1)$

4(b) $\frac{8}{15}$

6(b) $(\frac{\pi}{2})^6 - 30(\frac{\pi}{2})^4 + 360(\frac{\pi}{2})^2 - 720$

7(b) $I_0 = 1,\ I_4 = \frac{128}{315}$

8(b) $u_4 = \frac{243}{1540}$

9(b) $J_2 = -\frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x + C$

12(d) $\frac{1}{15}(14\sqrt{2} - 16)$

13(d) $\frac{1}{9}\Big((1+x^2)^4 + \frac{8}{7}(1+x^2)^3 + \frac{48}{35}(1+x^2)^2$
$+\frac{192}{105}(1+x^2) + \frac{384}{105}\Big)$

14(d) $J_n = \dfrac{2n}{2n+3}\,J_{n-1}$

15(d) $I_5 = \frac{1}{4}\,(2\ln 2 - 1)$

Exercise **2H** (Page 86)

1(a) $\frac{1}{36}$ **(b)** π **(c)** $\log\frac{12}{5}$ **(d)** $2 - 2\log 3$ **(e)** $2\sqrt{2} - 1$
(f) $\frac{1}{2}\log(2+\sqrt{5}\,)$

2(a) $\sqrt{1+x^2} + C$ **(b)** $\tan^{-1}x + \frac{1}{2}\ln(1+x^2) + C$
(c) $-\frac{1}{5}\cos^5 x + C$ **(d)** $\log\left(\frac{2x+1}{x+1}\right) + C$
(e) $\frac{1}{4}x^4\log x - \frac{1}{16}x^4 + C$ **(f)** $\frac{1}{6}\cos^3 2x - \frac{1}{2}\cos 2x + C$
(g) $\frac{1}{4}\tan^{-1}\frac{x+3}{4} + C$ **(h)** $x\sin 3x + \frac{1}{3}\cos 3x + C$
(i) $\frac{2}{3}(x-8)\sqrt{4+x} + C$

4(a) $A = -\frac{2}{3}\,,\ B = \frac{2}{3}\,,\ C = -\frac{1}{3}$

6 $\frac{1}{\sqrt{2}} + \frac{1}{2}\log(1+\sqrt{2})$

8(a) $A = 0,\ B = -2,\ C = 0,\ D = 2$ **(b)** $\frac{\pi}{2} - 1$

10 $\frac{1}{2}a^2\sin^{-1}\frac{x}{a} + \frac{1}{2}x\sqrt{a^2-x^2} + C$

11(b) $\frac{1}{10}(\pi + \log\frac{27}{16})$

12(a) $P = 2,\ Q = -1$
(b) $2x - \log(3\sin x + 2\cos x - 1) + C$

14(b) $6 - 2e$

Exercise **2I** (Page 93)

1(a) 0 **(b)** 0

2(a) $\frac{1}{132}$ **(b)** $\frac{16}{105}$ **(c)** $\frac{\pi^2}{4}$

6(a) The integrand is undefined at $x = 1$.
(b) $2(1 - \sqrt{1-a})$ **(c)** 2

7(a) The interval is unbounded.
(b) $\frac{1}{2}\tan^{-1}\left(\frac{N}{2}\right)$ **(c)** $\frac{\pi}{4}$

9(a) $\frac{\pi}{4}$ **(b)** 0 **(c)** $1 - \frac{\pi}{4}$

10(b) $\frac{\pi^2}{4}$

11(a) $2\sqrt{2}$ **(b)** 4 **(c)** $\frac{\pi}{2}$ **(d)** $\frac{\pi}{2}$ **(e)** $1 + \frac{\pi}{2}$ **(f)** e

12(a) 1 **(b)** $\frac{\pi}{2}$ **(c)** $\frac{3\pi}{4}$ **(d)** $\frac{1}{2}$ **(e)** 1 **(f)** $\frac{\pi}{2}$

20(b) $u_n = -\frac{n}{2}u_{n-1}$ **(c)** $\frac{3}{4}$

22(d) e

Chapter Ten

Exercise 10A (Page 2)

15(a) Converse of angles in a semi-circle at X, Y and Z.

20 Let D be the foot of the perpendicular from A to BC, then use trigonometry in $\triangle OAD$.

Exercise 10B (Page 8)

1 Begin with LHS − RHS.

2 Begin with LHS − RHS.

4(a) Begin with $(a-b)^2 \geq 0$ and put $a = \sqrt{x}$ and $b = \sqrt{y}$. **(b)** Use part (a) three times.

5(a) Begin with $(p-q)^2 \geq 0$.

(b) Use part (a) three times and add.

(c) Begin with $(p+q+r)^2$ and use part (b).

6(a) Use Question 4(b) with $p = a^2$ and so on.

(b) Use Question 4(b) with $p = ab$ and so on.

(c) Use parts (a) and (b).

7(a) See Question 4(a).

(b) See Question 4(b).

(c) Use part (b).

(d) Use part (c) and put $a^3 = x$ and so on.

(e) Expand and then used part (d). A more sophisticated approach is to use part (a) and in the first bracket put $a^2 = 1$ and $b^2 = x$, and so on.

8(a) See Question 4(a).

(b)(i) Expand the LHS and use part (a).

(c)(i) The triangle inequality: the length of any side is between the sum of the other two and the difference of the other two.

(ii) Begin with LHS − RHS and use part (i).

9(a) Use Question 7(a) and divide by ab.

(b) Expand the LHS and use part (a).

(c)(i) Begin with Question 7(a) and multiply by $(a+b)$.

(ii) Add part (i) and use part (a).

(iii) Begin with part (ii) and replace a^3 with $\frac{a}{b}$.

10(a) Replace a^2 with $a^2 + b^2$ and so on.

(b) Use part (a) and put $a^2 = w$ and so on.

11(a) Begin with Question 7(a) and put $a^2 = \frac{1}{x}$ and so on.

(b) Begin with Question 7(a) and put $a^2 = \frac{1}{x^2}$ and so on.

13(c) When $z = kw$, with $k > 0$, or when either $z = 0$ or $w = 0$.

15(b)(i) $x > 1 + \sqrt{2}$ or $x < 1 - \sqrt{2}$

17(d) $I_5 = \frac{1}{4}(2\ln 2 - 1)$

19(a) $\dfrac{1 + x^{4n+2}}{1 + x^2}$

Exercise 10C (Page 13)

1(a) $|\triangle OAB| = \frac{1}{4}$ sq. units, 3 sq. units

(b)(ii) $|\triangle OGH| = (2 - \sqrt{3})$ sq. units, $12(2-\sqrt{3})$ sq. units

2(a) $(1 - e^{-1})$ sq. units **(b)(i)** $\frac{1}{2}(1 + e^{-1})$ sq. units

(ii) $e^{-\frac{1}{2}}$ sq. units

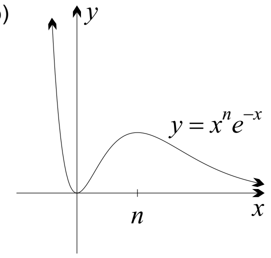
3(a) $\frac{\pi}{36}(4 + \sqrt{3})$

4(a) $\frac{3}{4}$ and $\frac{2}{3}$ square units.

6(b) It diverges to infinity.

7(c) $\left(\frac{a+2b}{3}, \frac{\ln a + 2\ln b}{3}\right)$

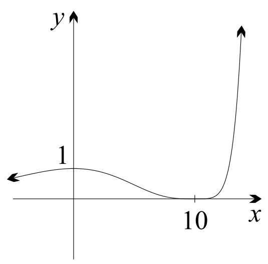
8(b)



10(a) $(0, 1)$ is a maximum turning point, $(10, 0)$ is a minimum turning point.

(b) $y \to \infty$ as $x \to \infty$, and $y \to 0$ as $x \to -\infty$.

(c)



11(c) When $z = kw$, with $k < 0$, or when either $z = 0$ or $w = 0$.

12(a)(i) $6^6 = 46\,656$, $3 \times 5^6 = 46\,875$

(ii) $5 \times 6^6 = 233\,280$, $2 \times 7^6 = 235\,298$

13(f) $0{\cdot}693$

16(c) $n = 9$

18(b) In part (a), put $f(x) = x^{-2}$, $a = (n-1)$ and $b = n$.

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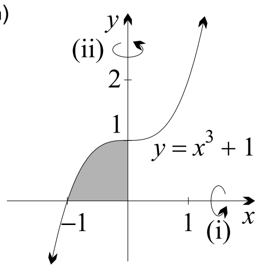
Chapter Six

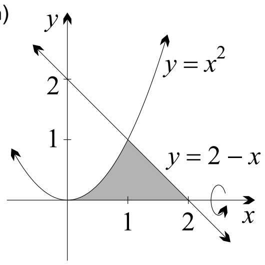
Exercise **6A** (Page 27)

1(a)



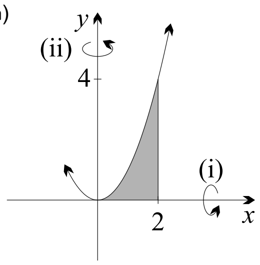
(b)(i) $\frac{9\pi}{14}$ **(ii)** $\frac{3\pi}{5}$

2(a)



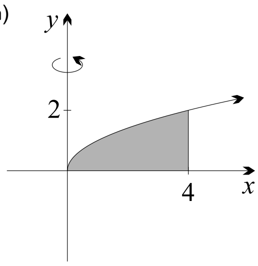
(b) $\frac{8\pi}{15}$

3(a)



(b)(i) $\frac{32\pi}{5}$ **(ii)** 8π

4(a)



(b) $\frac{128\pi}{5}$

5 $\frac{6\pi}{7}$

6(a) 2π **(b)** $\frac{48\pi}{5}$ **(c)** $\frac{7\pi}{10}$

7 4π

8 $\frac{32\pi}{3}$

9 $\frac{256\pi}{15}$

10(a) $\frac{8\pi}{3}$ **(b)** $\frac{224\pi}{15}$ **(c)** 8π

11(a) $\frac{9\pi}{14}$ **(b)** $\frac{15\pi}{7}$

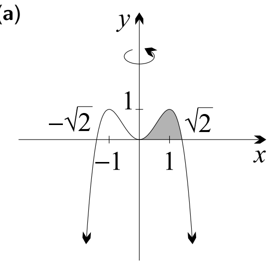
12(a) $\frac{8\pi}{3}$ **(b)** 8π

13(b) $\frac{296\pi}{15}$

14(a) $\frac{32\pi a^3}{15}$ **(b)** $\frac{112\pi a^3}{15}$ **(c)** $\frac{128\pi a^3}{15}$

15(b) $\frac{8\pi}{3}$

16(a)



(b) $\frac{4\pi}{3}$

17(b) $288\pi^2$

19(a) $\frac{4}{3}\pi ab^2$ **(b)** $2\pi^2 ab^2$ **(c)** $2\pi^2 abc$

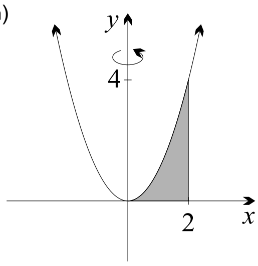
20(a) $AD = \frac{9}{5}$, $CD = \frac{16}{5}$ **(c)** $\frac{92\pi}{15}\text{cm}^3$

23(a) $\frac{1}{\sqrt{2}}(x - x^2)$ **(b)** $\delta V = \frac{\pi}{\sqrt{2}}(x - x^2)^2\,\delta x$

(c) $V' = \frac{\pi}{\sqrt{2}}(x - x^2)^2$

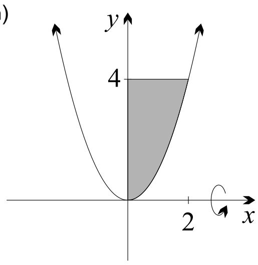
Exercise **6B** (Page 32)

1(a)



(b) 8π

2(a)



(b) $\frac{128\pi}{5}$

3(a) $\frac{128\pi}{3}$ **(b)** $\frac{3888\pi}{5}$ **(c)** $\frac{\pi}{10}$

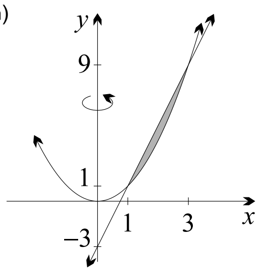
4(a) $\frac{3\pi}{5}$ **(b)** $\frac{9\pi}{14}$

5(a) $\frac{8\pi}{3}$ **(b)** 8π

6(a) $\frac{256\pi}{15}$ **(b)** $\frac{192\pi}{5}$

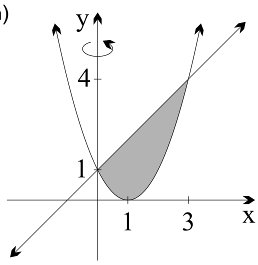
7(a) $\frac{8\pi}{3}$ **(b)** 8π

9(a)



(b) $\frac{16\pi}{3}$

10(a)



(b) $\frac{27\pi}{2}$

11 2π

12 8π

13 $\frac{28\pi}{5}$

14 $\frac{256\pi}{15}$

15(a) $\frac{4}{3}\pi ab^2$ **(b)** $2\pi^2 ab^2$ **(c)** $2\pi^2 abc$

16(a) $\frac{32\pi a^3}{15}$ **(b)** $\frac{112\pi a^3}{15}$ **(c)** $\frac{128\pi a^3}{15}$

21(b) $V = \lim_{\delta x \to 0} \sum_{x=0}^{x=\sqrt{2}} \pi(2x^2 - x^4)(2x + \delta x)\,\delta x$

(c) $\frac{4\pi}{3}$

22(a) $(x - \frac{1}{2}\delta x),\ (x + \frac{1}{2}\delta x)$

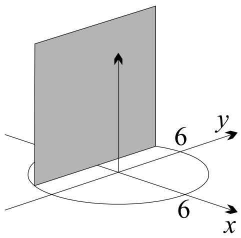
(c) $V = \lim_{\delta x \to 0} \sum_{x=0}^{x=2} 2\pi x(2x - x^2)\,\delta x$ **(d)** $\frac{8\pi}{3}$

Exercise **6C** (Page 36)

1 $126\,\mathrm{m}^3$

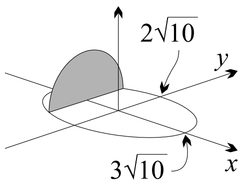
2(a) $\frac{y^2\sqrt{3}}{144}\,\mathrm{m}^2$

4(a)

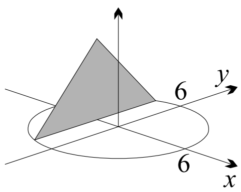


(b) $4(36 - x^2)$ **(c)** 1152

5(a)



6(a)



(b) $36 - x^2$ **(c)** 288

8 $\frac{9}{280}$

11(b) $\frac{1}{3}\pi abh$

13 $128\,\mathrm{ml}$

14(b) $56\,\mathrm{m}^3$

15(b) $45\frac{2}{3}\,\mathrm{m}^3$

16(a) $4(r^2 - y^2)$ **(b)** $\frac{16}{3}r^3$

18(a) $a^2 - x^2$ **(b)** $\frac{4}{3}a^3$

19(a) $\frac{\sqrt{3}}{2}a$ **(b)** $\frac{ax}{b}$ **(e)** $\frac{1}{12}a^2 b(5 + 2\sqrt{3})\,\mathrm{m}^3$

20(a)(ii) $\pi(1 - e^{-R^2})$ **(c)(i)** $R\sqrt{2}$ **(ii)** $\pi(1 - e^{-2R^2})$

(d) $\pi(1 - e^{-R^2}) \le 4I^2 \le \pi(1 - e^{-2R^2})$ **(e)** $\frac{\sqrt{\pi}}{2}$