Marks

Question 1 **Begin a new page.**

(a) Evaluate $\displaystyle\int_0^{\frac{\pi}{6}} \sec 2x \tan 2x \; dx$. **2**

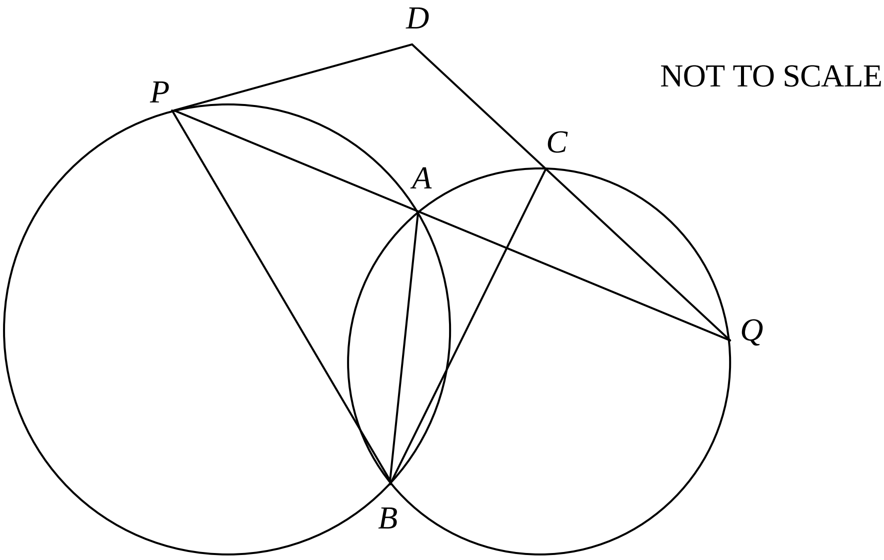
(b) Find the acute angle between the lines $3x - y - 2 = 0$ and $x + 2y - 3 = 0$. **2**
Give the answer correct to the nearest degree.

(c) The polynomial $P(x)$ is given by $P(x) = x^3 + (k-1)x^2 + (1-k)x - 1$ for some real number k.

(i) Show that $x = 1$ is a root of the equation $P(x) = 0$. **1**

(ii) Given that $P(x) = (x-1)(x^2 + kx + 1)$, find the set of values of k such that the equation $P(x) = 0$ has 3 real roots. **3**

(d)



Two circles intersect at *A* and *B*. P is a point on the first circle and Q is a point on the second circle such that PAQ is a straight line. C is a point on the second circle. The line QC produced and the tangent to the first circle at P meet at *D*.

(i) Copy the diagram.

(ii) Give a reason why $\angle DPA = \angle PBA$. **1**

(iii) Give a reason why $\angle CQA = \angle CBA$. **1**

(iv) Hence show that $BCDP$ is a cyclic quadrilateral. **2**

Marks

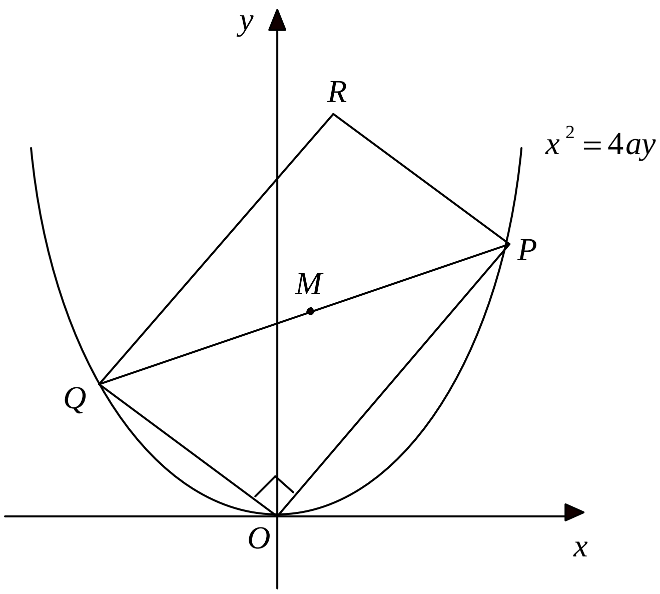
Question 2 **Begin a new page.**

(a) Show that $\dfrac{d}{dx}3^x = 3^x \ln 3$. **2**

(b) $A(-3, 7)$ and $B(4, -2)$ are two points. Find the coordinates of the point P which divides the interval AB internally in the ratio $3:2$. **2**

(c) Solve the equation $1 + \cos 2x = \sin 2x$ for $0 \le x \le 2p$. **4**

(d)



$P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points which move on the parabola $x^2 = 4ay$ such that $\angle POQ = 90°$, where $O(0, 0)$ is the origin. $M\left(a(p+q), \frac{1}{2}a(p^2+q^2)\right)$ is the midpoint of PQ. R is the point such that $OPRQ$ is a rectangle.

(i) Show that $pq = -4$. **1**

(ii) Show that R has coordinates $\left(2a(p+q), a(p^2+q^2)\right)$. **1**

(iii) Find the equation of the locus of R. **2**

Marks

Question 3 **Begin a new page.**

(a) Consider the function $f\left(x\right)=\dfrac{x^2}{x^2-1}$.

(i) Show that $f\left(x\right)$ is an even function. **1**

(ii) Show that $\lim\limits_{x\to\infty} f\left(x\right)=1$. **1**

(iii) Show that the graph $y=f\left(x\right)$ has a maximum turning point at the origin $\left(0\ ,0\right)$. **2**

(iv) Sketch the graph $y=f\left(x\right)$ showing clearly the equations of any asymptotes. **2**

(v) The function $g\left(x\right)$ is defined by $g\left(x\right)=\dfrac{x^2}{x^2-1}$, $x\geq 0$. Find the equation of the inverse function $g^{-1}\left(x\right)$ and state its domain. **2**

(b) Use Mathematical Induction to show that for all positive integers $n\geq 1$ **4**

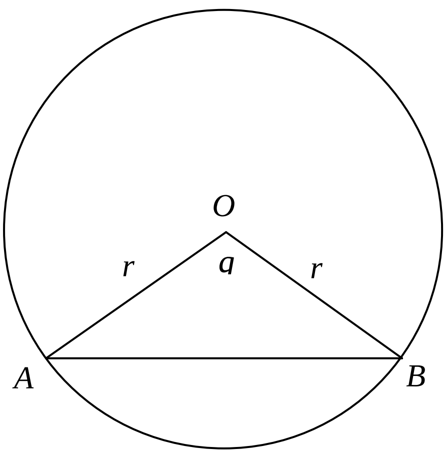
$$\frac{3}{1\times 2\times 2}+\frac{4}{2\times 3\times 2^2}+\ \ldots\ +\frac{n+2}{n\left(n+1\right)2^n}=1-\frac{1}{\left(n+1\right)2^n}\ .$$

Marks

Question 4 **Begin a new page.**

(a) The region in the first quadrant bounded by the curve $y = 2\tan^{-1} x$ and the y axis between $y = 0$ and $y = \frac{p}{2}$ is rotated through one complete revolution about the y axis. Find the exact volume of the solid of revolution so formed. **4**

(b)



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AB is a chord of a circle of radius r which subtends an angle q, $0 < q < p$, at the centre O. The area of the minor segment cut off by chord AB is one half of the area of the sector AOB.

(i) Show that $q - 2\sin q = 0$. **2**

(ii) Use an initial approximation $q_1 = 2$ and one application of Newton's method to find a second approximation to the value of q. Round your answer to 2 decimal places. **2**

(c) Don guesses at random the answers to each of 6 multiple choice questions. In each question there are 3 alternative answers, only one of which is correct.

(i) Find the probability in simplest exact form that Don answers exactly 2 of the 6 questions correctly. **2**

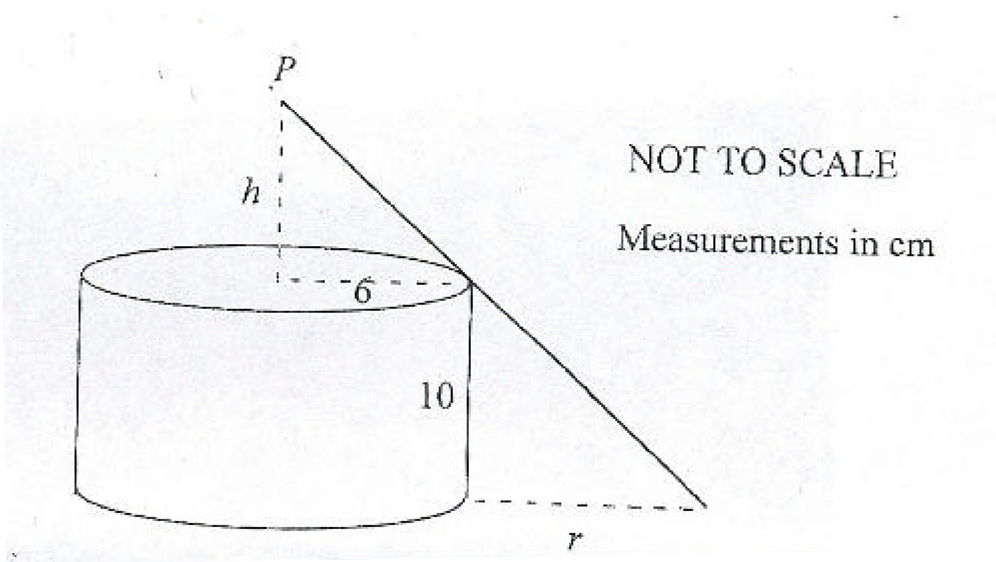
(ii) Find the probability in simplest exact form that the 6th question that Don attempts is only the 2nd question that he answers correctly. **2**

Marks

Question 5 **Begin a new page.**

(a) Use the substitution $u = x - 1$ to evaluate $\int_{0\cdot5}^{1\cdot5} \frac{1}{\sqrt{2x - x^2}}\, dx$. Give the answer in simplest exact form. **4**

(b)



A solid wooden cylinder of height 10 cm and radius 6 cm rests with its base on a horizontal table. A light source P is being lowered vertically downwards from a point above the centre of the top of the cylinder at a constant rate of $0\cdot1\ \text{cm s}^{-1}$. When the light source is h cm above the top of the cylinder the shadow cast on the table extends r cm from the side of the cylinder.

(i) Show that $r = \frac{60}{h}$. **1**

(ii) Find the rate at which r is changing when $h = 5$. **3**

(c) A particle is performing Simple Harmonic Motion in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line, velocity v ms^{-1} given by $v^2 = 32 + 8x - 4x^2$ and acceleration a ms^{-2}.

(i) Find an expression for a in terms of x. **1**

(ii) Find the centre and amplitude of the motion. **2**

(iii) Find the maximum speed of the particle. **1**

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Question 6 **Begin a new page.**

(a) At time t minutes the volume flow rate R kilolitres per minute of water into a tank is given by $R = 4\sin^2 t$, $0 \le t \le p$.

(i) Find the maximum rate of flow of water into the tank. **1**

(ii) Find the total amount of water which flows into the tank. Give the answer correct to the nearest litre. **3**

(b) At time t years the number N of individuals in a population is given by $N = A + Be^{-t}$ for some real constants A and B. After $\ln 2$ years there are 60 individuals and after $\ln 5$ years there are 36 individuals.

(i) Show that A and B satisfy the equations $2A + B = 120$ and $5A + B = 180$. Hence find the values of A and B. **3**

(ii) Find the limiting population size. **1**

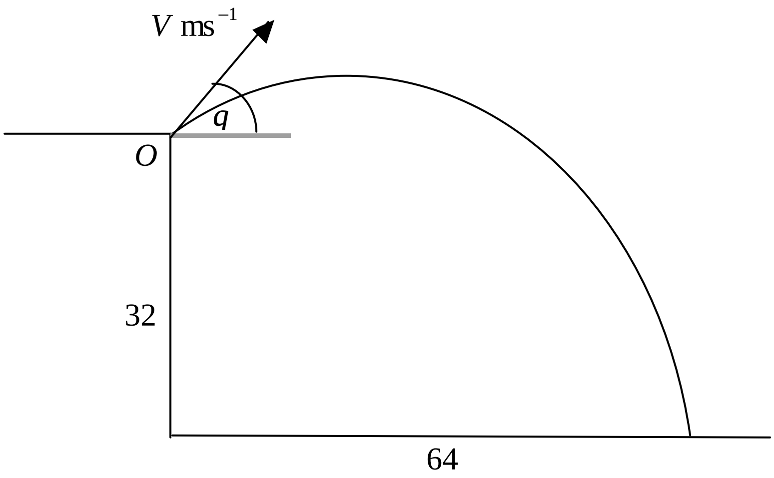
(c) A particle is moving in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line and velocity v ms^{-1} given by $v = \dfrac{x(2-x)}{2}$. The particle starts 1 metre to the right of O.

(i) Show that $\dfrac{2}{x(2-x)} = \dfrac{1}{x} + \dfrac{1}{2-x}$. **1**

(ii) Find an expression for x in terms of t. **3**

Question 7 **Begin a new page.**

(a)



A particle is projected with velocity V ms^{-1} at an angle θ above the horizontal from a point O on the edge of a vertical cliff 32 metres above a horizontal beach. The particle moves in a vertical plane under gravity, and 4 seconds later it hits the beach at a point 64 metres from the foot of the cliff. The acceleration due to gravity is 10 ms^{-2}.

(i) Use integration to show that after t seconds the horizontal displacement x metres and the vertical displacement y metres of the particle from O are given by $x = (V\cos\theta)t$ and $y = (V\sin\theta)t - 5t^2$ respectively. **2**

(ii) Write down two equations in V and θ then solve these equations to find the exact value of V and the value of θ in degrees correct to the nearest minute. **3**

(iii) Find the speed of impact with the beach correct to the nearest whole number and the angle of impact with the beach correct to the nearest minute. **3**

(b)(i) Write down the expansion of $x(1+x)^n$ in ascending powers of x. **1**

(ii) Hence show that $2\,{}^nC_1 + 3\,{}^nC_2 + \ldots + n\,{}^nC_{n-1} = (n+2)(2^{n-1}-1)$. **3**

EXAMINERS

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