ARC Academic Resources Centre

Harcourt Education

Student Number: …………..…………………….….

2006

HIGHER SCHOOL CERTIFICATE

Sample Examination Paper

MATHEMATICS

General Instructions

- Reading Time – 5 minutes.
- Working Time – 3 hours.
- Write using blue or black pen.
- Write your student number at the top of this page.

Total marks – 120

- Attempt Questions 1–10.
- All questions are of equal value.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown in every question.

Directions to school or college

To ensure maximum confidentiality and security, examination papers used for trial examinations must NOT be removed from the examination room or used with students for revision purposes until Monday 4 September 2006.

All care has been taken to ensure that this sample examination paper is error free and that it follows the style, format and material content of the current NSW syllabus. Candidates are advised that the authors of this examination paper cannot in any way guarantee that the actual Board Of Studies Examination will have a similar content or format.

Product code: 734811

Marks

Question 1 (12 marks) *(Start a new booklet)*

(a) Find the value of cos 1.5, correct to three significant figures. **2**

(b) Solve $x^2 + 3x + 2 \geq 0$. **2**

(c) Find the primitive of $x^3 + 2$. **2**

(d) Solve $\dfrac{2x}{5} - \dfrac{5x}{4} = 1$. **2**

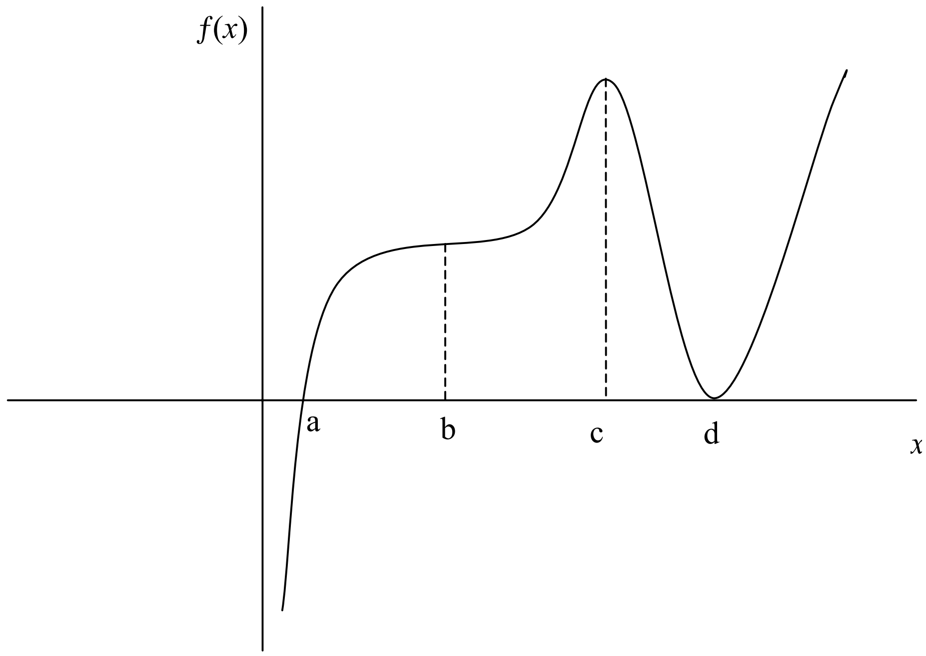
(e) Express $\dfrac{1}{2+\sqrt{3}}$ with a rational denominator. **2**

(f) Solve $|x+2| > 5$. **2**

Marks

Question 2 (12 Marks) *(Start a new booklet)*

(a) The following is a sketch of $f(x)$.



In your answer booklet sketch the graph of $f'(x)$. **2**

(b) For the parabola $y^2 = 12x$

Find (i) the coordinates of the vertex. **1**

(ii) the coordinates of the focus. **1**

(iii) the equation of the directrix. **1**

(c) Solve for x if $4^x = 32$. **1**

(d) Find the equation of a line with slope of 3 which passes through the point (3, 5). **2**

(e) Find the perpendicular distance from the point (-1, 4) to the line whose equation is $2x - 3y + 1 = 0$. **2**

(f) Solve for x
$\ln(x + 2) - \ln x = \ln 4$. **2**

Marks

Question 3 (12 Marks) *(Start a new booklet)*

(a) Differentiate with respect to x:

(i) $\dfrac{1}{x}$ **1**

(ii) $x\cos x$ **2**

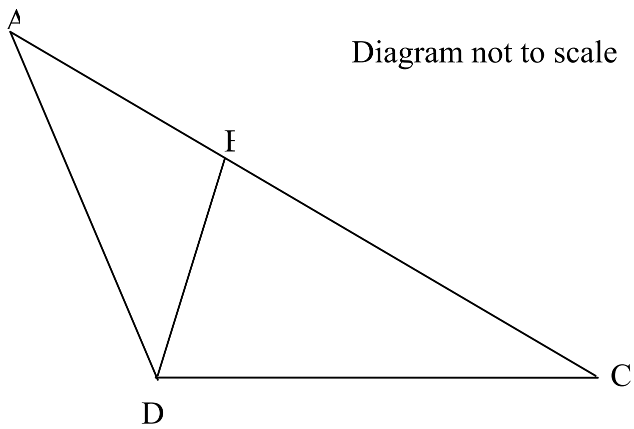
(iii) $\ln\left(\dfrac{x+1}{x-1}\right)$. **2**

(b) (i) Find $\int (4x+3)^2\,dx$. **1**

(ii) Evaluate $\int_0^1 (e^{2x}+1)\,dx$ in exact form. **2**

(c) If each interior angle of regular polygon is 150°, calculate the number of sides. **2**

(d)



In the diagram $AB = BD = DC$ and $\angle DAB = 20°$

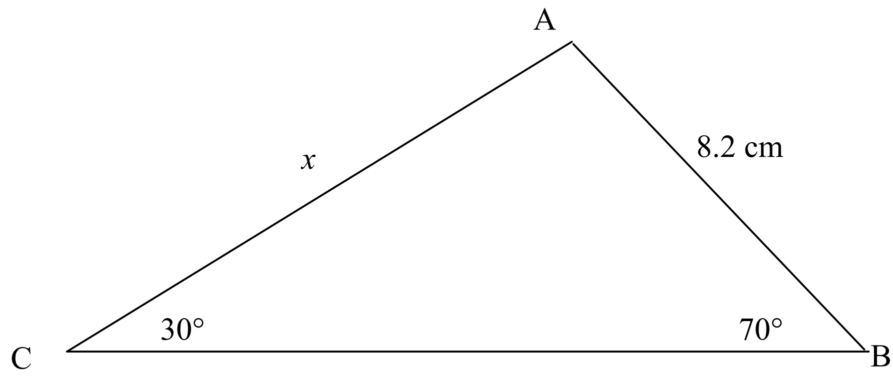
(i) Copy the diagram into your booklet, showing the given information.

(ii) Find $\angle BDC$ giving reasons. **2**

Marks

Question 4 (12 marks) *(Start a new booklet)*

(a) Diagram not to scale



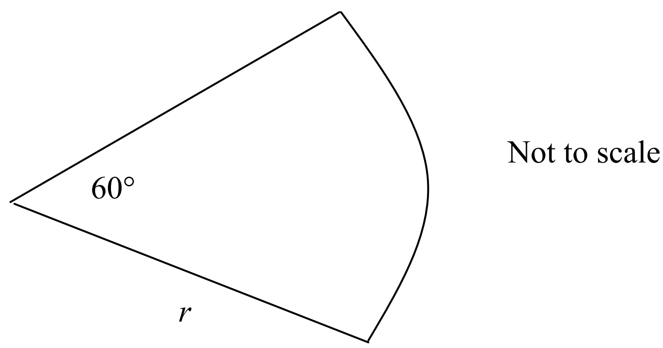
Find (i) the length of AC to the nearest mm. **2**

(ii) the area of the triangle ABC, to the nearest cm². **2**

(b) For the curve $f(x) = 2x^3 + x^2$ find the values of x for which the curve is concave down. **2**

(c) Solve $\cos 2x = \dfrac{1}{2}$ in the domain $-\pi \leq x \leq \pi$ **2**

(d)



If the area of this sector is 10π units squared, find the value of r to 2 decimal places. **2**

(e) Evaluate $\int (\sin x \cos x)\, dx$ **2**

Marks

Question 5 (12 marks) *(Start a new booklet)*

(a) Write $0.\dot{7}$ as an infinite series and hence express it as a rational number. **2**

(b) The semi-circle $y = \sqrt{9 - x^2}$ is rotated about the x-axis. Find the volume of the sphere so formed, in terms of π. **3**

(c) Use Simpson's rule to find an approximation to the definite integral $\int_3^5 (x+1)^{-2}\,dx$ using 4 strips. (Answer correct to 4 decimal places.) **3**

(d) The third and seventh terms of a geometric series are $1\frac{1}{4}$ and 20 respectively.

Find:

(i) the common ratio. **2**

(ii) the first term. **1**

(iii) the 14th term. **1**

Marks

Question 6 (12 Marks) *(Start a new booklet)*

(a) The chance of a fisherperson catching a legal length fish is 4 in 5. If three fish are caught at random, what is the probability that:

(i) none are of legal length. **1**

(ii) all are of legal length. **1**

(iii) exactly one is of legal length. **1**

(iv) exactly two are of legal length. **1**

(b) A particle moves along the x-axis with acceleration $3t - 2$. Initially it is 4 units to the right of the origin, with velocity 2 units per second. Calculate the position of the particle after 5 seconds. **4**

(c) A circular metal disc is being heated so that the rate of increase of the area, A cm², after t hours is given by $\frac{dA}{dt} = \frac{1}{10}\pi t$. If initially the disc had a radius of 12 cm, find the area of the disc after 6 hours. Leave your answer in terms of π. **4**

Marks

Question 7 (12 Marks) *(Start a new booklet)*

(a) For the function $y = 4 - 2\cos x$,

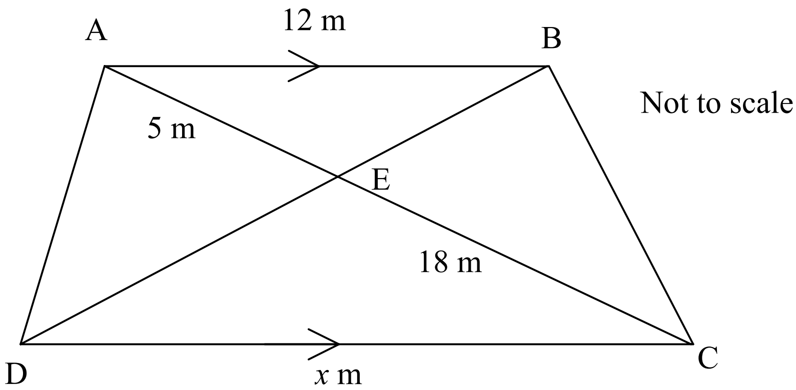
(i) find the greatest value. **1**

(ii) find the period. **1**

(iii) sketch the function for $0 \le x \le \pi$ **1**

(b) A closed cylinder of volume $24\pi\,\text{m}^3$ is to be cut from a single sheet of metal. What is the minimum amount of metal needed to make the cylinder? (answer to the nearest m^2) **4**

(c)



(i) Copy the diagram into your book.

(ii) Prove ΔABE is similar to ΔCDE. **3**

(iii) Find the length of x, giving reasons. **2**

Marks

Question 8 (12 Marks) *(Start a new booklet)*

(a) Find all values of x for $0 \le x \le 2\pi$ which satisfy $(\sin x + 2)(2\sin x + 1) = 0$. **2**

(b) Find the area enclosed between the curves $y = x^2 + 1$ and $y = 3x + 1$ **3**

(c) The population of a colony of microbes is increasing continuously at a rate proportional to the existing population. The present population is 20 000 and the population 3 months ago was 8 000

(i) Show the value of k, the growth constant, to be 0.305, correct to 3 significant figures. **2**

(ii) Find the population at the end of 6 months (nearest thousand). **1**

(iii) Find the rate at which the population is increasing after 6 months to the nearest microbe. **2**

(iv) In which month does the population pass 30 000 microbes? **2**

Marks

Question 9 (12 Marks) *(Start a new booklet)*

(a) Find the derivative of $\log_2 x$. **2**

(b) Find the value(s) of k for which the equation $x^2 - (k+2)x + 3k = 0$ has roots which are equal in value but of opposite sign. **2**

(c) At what points on the curve $y = x^3 - 4x^2 + 2x$ are the tangents parallel to the line $2x + y = 3$ **4**

(d) A and B are the points (3, -2) and (-4, 3) respectively. The point $P(x,y)$ moves so that $\angle APB = 90°$. Show that the equation of the locus is a circle and find its centre and radius. **4**

Marks

Question 10 (12 Marks) *(Start a new booklet)*

(a) Solve for real x: $x^2 - 2x + \dfrac{12}{x^2 - 2x} = 8$ leave your answer as a surd **4**

(b) A sum of \$26 000 is borrowed now at 6% per annum reducible interest. Payment is made by n equal annual instalments of \$4 200 beginning at the end of the first year. Find the value of n. **8**

Table of Standard Integrals

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\ x > 0$

Mapping grid

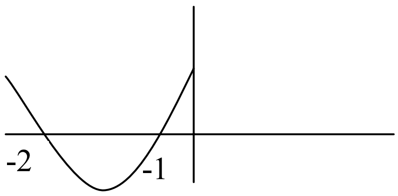
Question	Mark	Content	Outcome	Band
1a	2	Trigonometric Functions	H5	2
1b	2	Quadratic Polynomial	P4	3
1c	2	Integration	H5	2
1d	2	Basic Arithmetic and Algebra	P3	2
1e	2	Basic Arithmetic and Algebra	P3	2
1f	2	Basic Arithmetic and Algebra	P3	2
2a	2	Integration	H7	3
2b	3	The Parabola	P4	2
2c	1	Logarithmic and exponential functions	H3	3
2d	2	Linear Functions	P4	3
2e	2	Linear Functions	P4	3
2f	2	Logarithmic and exponential functions	H3	3
3a	5	Derivative of a Function	P7, H3	3
3b	3	Integration	H5	3
3c	2	Plane Geometry	P4	3
3d	2	Applications of geometric properties	H5	3
4a	4	Trigonometric ratios	P4	3
4b	2	Geometric Applications of differentiation	H6	3
4c	2	Trigonometric Functions	H5	4
4d	2	Trigonometric Functions	H5	3
4e	2	Trigonometric Functions	H5	4/5
5a	2	Series and Series Applications	H5	3
5b	3	Integration	H8	4
5c	3	Integration	H8	4
5d	4	Series and Series Applications	H5	4
6a	4	Probability	H5	3
6b	4	Applications of Calculus	H5	3
6c	4	Applications of Calculus	H5	5
7a	3	Trigonometric Functions	H5	3
7b	4	Applications of Calculus	H5	5
7c	5	Applications of geometric properties	H5	4
8a	2	Trigonometric Functions	H5	4

Question	Mark	Content	Outcome	Band
8b	3	Integration	H5	5
8c	7	Applications of calculus	H3,H5	5
9a	2	Logarithmic and exponential functions	H3	5
9b	2	Quadratic polynomial	P4	5
9c	4	Geometrical applications of differentiation	H6	6
9d	4	Locus	P4	6
10a	4	Quadratic Polynomial	P4	6
10b	8	Series Applications	H3, H5	6

Marking guidelines

Question 1

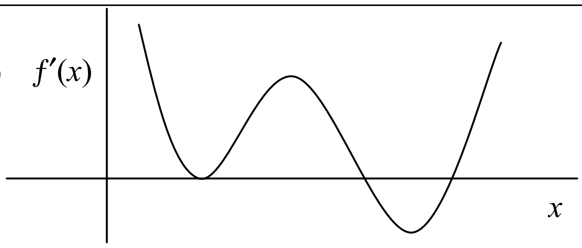
Marking guidelines

Criteria	Marks
• a) 0.070737201 0.0707	1 1
• b) $(x+1)(x+2)\geq 0$  $x\geq -1$, and $x\leq -2$	1 1
• c) $\dfrac{x^4}{4}+2x+c$	2 -1 no c
• d) $8x-25x=20$ $-17x=20$ $x=\dfrac{-20}{17}$	1 1
• e) $\dfrac{1}{2+\sqrt{3}}\times\dfrac{2-\sqrt{3}}{2-\sqrt{3}}$ $=\dfrac{2-\sqrt{3}}{4-3}$ $=2-\sqrt{3}$	1 1

Criteria	Marks
• f) $\lvert x+2\rvert > 5$ $x+2 < -5$ or $x+2 > 5$ or $x+2 > 5$ or $-x-2 > 5$ $x < -7$ or $x > 3$ $\quad$ $x > 3$ $\quad$ $-x > 7$ $x < -7$	 1 1

Question 2

Marking guidelines

Criteria	Marks
• a) 	2 1 for less than 2 errors
• b) $y^2 = 12x$ $\quad$ $y^2 = 4\text{a}x$ $\quad$ a = 3 (i) (0,0) (ii) (3, 0) (iii) $x = -3$	 1 1 1
• c) $(2^2)^x = 2^5$ $2x = 5$ $x = 2\frac{1}{2}$	1 for correct answer
• d) $y = mx + b$ $y = 3x + b$ $5 = 3 \times 3 + b$ $\quad$ OR $\quad$ $y - 5 = 3(x - 3)$ $-4 = b$ $y = 3x - 4$	 1 1
• e) $d = \dfrac{\lvert 2(-1) - 3(4) + 1\rvert}{\sqrt{2^2 + (-3^2)}}$ $d = \dfrac{\lvert -13\rvert}{\sqrt{13}}$ or $d = \sqrt{13}$	 1 1

Criteria	Marks
• f) $\ln\left(\dfrac{x+2}{x}\right) = \ln 4$	1
$\left(\dfrac{x+2}{x}\right) = 4$ $x+2 = 4x$ $3x = 2$	
$x = \dfrac{2}{3}$	1

Question 3

Marking guidelines

Criteria	Marks
• a) (i) $\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = \dfrac{d}{dx}x^{-1}$ $= -x^{-2}$	1
• (ii) $\dfrac{d}{dx}x\cos x = -x\sin x + \cos x$ $u = x \quad v = \cos x$, $u' = 1 \quad v' = -\sin x$	2 1 for each term
• (iii) $\dfrac{d}{dx}\ln\dfrac{(x+1)}{(x-1)} = \dfrac{(x-1)}{(x+1)} \times \dfrac{(x-1)(1)-(x+1)(1)}{(x-1)^2}$	1
$= \dfrac{x-1-x-1}{(x+1)(x-1)}$	1
$= \dfrac{-2}{x^2-1}$	
OR	OR
$\dfrac{d}{dx}\ln\dfrac{(x+1)}{(x-1)} = \dfrac{d}{dx}\left(\ln(x+1)-\ln(x-1)\right)$	1
$= \dfrac{1}{x+1} - \dfrac{1}{x-1}$	
$= \dfrac{x-1-x-1}{(x+1)(x-1)}$	1
$= \dfrac{-2}{x^2-1}$	

Criteria	Marks
• b) (i) $\int (4x+3)^2\,dx = \frac{(4x+3)^3}{12} + c$	1
• (ii) $\int_0^1 (e^{2x}+1)\,dx = \left[\frac{1}{2}e^{2x} + x\right]_0^1$	1
$= (\frac{1}{2}e^2 + 1) - (\frac{1}{2})$	1
$= \frac{1}{2}(e^2+1)$	
• c) $\frac{(n-2)180}{n} = 150$ OR $\frac{360}{n} = 30$	1
$180n - 360 = 150n$ OR $360 = 30n$	
$30n = 360$; $n = 12$	
$n = 12$	1
• d) (i) diagram no marks	
(ii) In ΔABD: $AB = BD$ given	
$\angle BAD = \angle ADB = 20°$ ΔABD isosceles	
$\angle DBC = 40°$ Exterior angle	1
In ΔBDC $BD = DC$ given	
$\angle DCB = \angle DBC = 40°$ ΔBDC isosceles	
$\angle BDC = 180 - 2\times 40°$ angle sum of triangle	
$\angle BDC = 100°$	1

Question 4

Marking guidelines

Criteria	Marks
• a) (i) $\frac{AC}{\sin 70^\circ} = \frac{8.2}{\sin 30^\circ}$	1
$AC = \frac{8.2\sin 70^\circ}{\sin 30^\circ}$	
$AC = 15.41095898$	1
$AC = 15.4$ (1 d.p.)	
(ii) Area $= \frac{1}{2}\times 8.2\times 15.4\times \sin 80°$	1
$= 62.18$ cm^2	

Criteria	Marks
$= 62\text{ cm}^2$ nearest whole number	1
• b) Concave down for $f''(x) < 0$ $f(x) = 2x^3 + x^2$ $f'(x) = 6x^2 + 2x$ $f''(x) = 12x + 2$ For $f''(x) < 0$ need $12x + 2 < 0$ $12x < -2$ $x < \frac{-1}{6}$	 1 1
• c) $\cos 2x = \frac{1}{2}$ $-\pi \le x \le \pi$ the basic angle whose cos is $\frac{1}{2}$ is $\frac{\pi}{3}$ $2x = \frac{\pi}{3}$ $-2\pi \le 2x \le 2\pi$ $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{-5\pi}{6}, \frac{-\pi}{6}$	 1 1
• d) $10\pi = \frac{1}{2} \times r^2 \times \frac{\pi}{3}$ $60\pi = r^2\pi$ $60 = r^2$ $r = 7.7459$ $r = 7.75$ (2 d.p.)	1 1
• e) $\int \sin x \cos x dx = \frac{-1}{2} \cos^2 x + k$	1 for $\cos^2 x$ 1 for the answer

Question 5

Marking guidelines

Criteria	Marks
• a) $0.\dot{7} = \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \ldots..$ G.P. with $a = \frac{7}{10}$ and $r = \frac{1}{10}$	1

Criteria	Marks
Sum $= \dfrac{\frac{7}{10}}{1-\frac{1}{10}}$ $= \dfrac{7}{9}$	1
• b) $V = \pi \int y^2 dx$ $V = 2\pi \int_0^3 \left(\sqrt{9-x^2}\right)^2 dx$ $= 2\pi \left[9x - \dfrac{x^3}{3} \right]_0^3$ $= 2\pi[(27-9)-0]$ $= 36\pi$ units^3	 1 1 1
• c) $f(x) = \int_0^5 (x+1)^{-2} dx$	
| x | 3 | 3.5 | 4 | 4.5 | 5 | | $f(x)$ | 0.0625 | 0.0494 | 0.04 | 0.0331 | 0.0278 |	1
$A \approx \dfrac{1}{6}(0.0625 + 4\times 0.0494 + 2\times 0.04 + 4\times 0.0331 + 0.0278)$	1
$A \approx 0.083383333$ ≈ 0.0834 (4 d.p.)	 1

x	3	3.5	4	4.5	5
$f(x)$	0.0625	0.0494	0.04	0.0331	0.0278

Criteria	Marks
• d) (i) $T_3 = ar^2$ and $T_7 = ar^6$	1
i.e. $\frac{5}{4} = ar^2$	
and $20 = ar^6$	
so $r^4 = 16$	1
$r = \pm 2$	
(ii) $a(\pm 2)^6 = 20$	
$a = \frac{20}{64}$	
$a = \frac{5}{16}$	1
(iii) $T_{14} = ar^{13}$	Because of the similarity to Part (ii) need the (±) for the mark
$= \frac{5}{16}(\pm 2)^{13}$	
$= \pm 2560$	1

Question 6

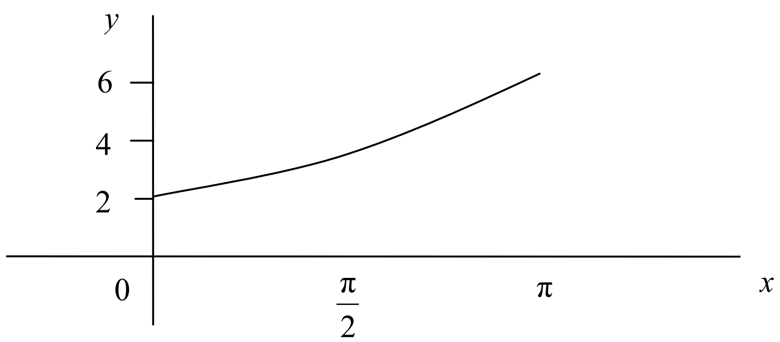
Marking guidelines

Criteria	Marks
• a) (i) $\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}=\frac{1}{125}$	1
(ii) $\frac{4}{5}\times\frac{4}{5}\times\frac{4}{5}=\frac{64}{125}$	1
(iii) $\frac{4}{5}\times\frac{1}{5}\times\frac{1}{5}\times 3=\frac{12}{125}$	1
(iv) $\frac{4}{5}\times\frac{4}{5}\times\frac{1}{5}\times 3=\frac{48}{125}$	1

Criteria			Marks
• b)	Acceleration	$\ddot{x} = 3t - 2$	
	Velocity	$v = \dfrac{3t^2}{2} - 2t + c$	1
	When $t = 0$, $v = 2$	$2 = 0 - 0 + c$ so $c = 2$	1 for c
		$v = \dfrac{3t^2}{2} - 2t + 2$	
	Distance	$x = \dfrac{t^3}{2} - t^2 + 2t + k$	1
	When $t = 0$, $x = 4$	$4 = 0 - 0 + 0 + k$ so $k = 4$	
		$x = \dfrac{t^3}{2} - t^2 + 2t + 4$	
	Find x when $t = 5$	$x = \dfrac{5^3}{2} - 5^2 + 2 \times 5 + 4$	
		$x = 51.5$ units to the right.	1
• c)	$\dfrac{dA}{dt} = \dfrac{1}{10}\pi t$		
	$A = \dfrac{1}{20}\pi t^2 + c$		1
	at $t = 0$, $r = 12$, using $A = \pi r^2$. i.e. $A = 144\pi$		
	$144\pi = 0 + c$	$c = 144\pi$	1
	$A = \dfrac{1}{20}\pi t^2 + 144\pi$		
	After 6 hours, $A = \dfrac{1}{20}\pi \times 6^2 + 144\pi$		1
	$A = \dfrac{729\pi}{5}\text{ cm}^2$		1

Question 7

Marking guidelines

Criteria	Marks
• a) (i) Max value at $\cos x = -1$ i.e. when $x = \pi$ Max value is $y = 6$	1
• (ii) Period $= 2\pi$	1
• (iii)  	1 Must show a curve in the domain
• b) $V = \pi r^2 h$ $24\pi = \pi r^2 h \qquad h = \dfrac{24}{r^2}$ Surface Area $= 2\pi r^2 + 2\pi rh$ i.e. $SA = 2\pi r^2 + 2\pi r\dfrac{24}{r^2}$ $SA = 2\pi r^2 + 48\pi r^{-1}$ Min Area when $\dfrac{dA}{dr} = 0$ and $\dfrac{d^2A}{dr^2} > 0$ $\dfrac{dA}{dr} = 4\pi r - 48\pi r^{-2}$ $\dfrac{d^2A}{dr^2} = 4\pi + 96\pi r^{-3}$ this will be > 0 for all $r > 0$ So $0 = 4\pi r - 48\pi r^{-2}$ $48\pi r^{-2} = 4\pi r$ $r^3 = 12$ $r = \sqrt[3]{12}$ or 2.289 Min Area $= 2\pi(\sqrt[3]{12})^2 + 48\pi(\sqrt[3]{12})^{-1}$ $= 98.79962293$ $= 99 \text{ m}^2$	1 1 1 1

Criteria	Marks
• c) (i) diagram no marks	
• (ii) In ΔABE and ΔDCE $\angle BAC = \angle ECD$ (Alternate angles $AB \parallel DC$ given) $\angle ABE = \angle BDC$ (Alternate angles $AB \parallel DC$ given) $\angle AEB = \angle DEC$ (vertically opposite angles) $\Delta ABE \mid\mid\mid \Delta CDE$ equiangular	1 1 1
• (iii) $\Delta ABE \mid\mid\mid \Delta CDE$ $\frac{x}{12} = \frac{18}{5}$ corresponding sides in similar triangles $x = 43.2$ m	1 1

Question 8

Marking guidelines

Criteria	Marks
• a) $2\sin x + 1 = 0$ OR $\sin x + 2 = 0$ $2\sin x = -1$ OR $\sin x = -2$ (has no solution) so $\sin x = \frac{-1}{2}$ basic angle is $\frac{\pi}{6}$ solutions are $x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$	1 1

Criteria	Marks
• b) $y = x^2 + 1$ $y = 3x + 1$ Points of intersection $x^2 + 1 = 3x + 1$ $x^2 - 3x = 0$ $x(x-3) = 0 \quad x = 0 \text{ or } 3$	1
$A = \int_0^3 (3x+1) - (x^2+1)dx$ $= \int_0^3 (3x - x^2)dx$ $= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$	1
$= (\frac{3\times 3^2}{2} - \frac{3^3}{3}) - (0)$ $= \frac{9}{2}\ u^2$	1
• c) (i) $P = P_0 e^{kt}$ $t = 0, \quad P = 8\ 000 \qquad \text{So } P_0 = 8\ 000$ $t = 3 \quad P = 20\ 000$ $20\ 000 = 8\ 000e^{3k}$ OR $P = 8\ 000e^{3k}$	1
$2.5 = e^{3k}$ $3k = \ln 2.5$ $k = \frac{\ln 2.5}{3}$ $k = 0.305430244$ $k = 0.305$ correct to 3 significant figures	1
• (ii) $t = 6$ $P = 8\ 000e^{6(0.305)}$ $P = 49\ 871.09327$ $= 49\ 871$ $= 50\ 000$ to the nearest thousand	1

Criteria	Marks
• (iii) Rate = $\frac{dP}{dt}$ $\frac{dP}{dt} = kP_0e^{kt}$ $t = 6$ $\quad \frac{dP}{dt} = 0.305 \times 8000e^{6(0.305)}$ Rate = 15210.68345 = 15 211 nearest whole number OR (for 2nd Mark) using P from part (ii) $\frac{dP}{dt} = 0.305 \times 50000$ = 15 250	 1 1
• (iv) $P = 30\,000,\ t = ?$ $30\,000 = 8\,000e^{t(0.305)}$ $3.75 = e^{t(0.305)}$ $t = \frac{1}{0.305}\ln 3.75$ $t = 4.33$ months Passes 30 000 in the 5th month	 1 1

Question 9

Marking guidelines

Criteria	Marks
• a) $\log_2 x = \frac{\ln x}{\ln 2}$ $\frac{d}{dx}(\log_2 x) = \frac{d}{dx}\left(\frac{\ln x}{\ln 2}\right)$ $= \frac{1}{\ln 2} \times \frac{1}{x}$ $= \frac{1}{x\ln 2}$	1 1

Criteria	Marks
• b) Need $\alpha + \beta = 0$ Now $\alpha + \beta = \frac{-b}{a}$ $a = 1$ and $b = -(k + 2)$ So g - (k + 2) = 0 and k = - 2	1 1
• c) Line $2x + y = 3$ i.e. $y = -2x + 3$ So $m_T = -2$ Curve $y = x^3 - 4x^2 + 2x$ $\frac{dy}{dx} = 3x^2 - 8x + 2$ Need $3x^2 - 8x + 2 = -2$ $3x^2 - 8x + 4 = 0$ $(3x - 2)(x - 2) = 0$ So $x = \frac{2}{3}$ or $x = 2$ $x = \frac{2}{3}$ $y = \left(\frac{2}{3}\right)^3 - 4\left(\frac{2}{3}\right)^2 + 2\left(\frac{2}{3}\right)$ $y = \frac{-4}{27}$ Point $\left(\frac{2}{3}, \frac{-4}{27}\right)$ $x = 2$ $y = 2^3 - 4(2)^2 + 2(2)$ $= -4$ Point (2 , -4)	1 1 1 1
• d) For $\angle APB = 90°$ need $m_{ap} \times m_{bp} = -1$ So $\frac{y-3}{x+4} \times \frac{y+2}{x-3} = -1$ $y^2 - y - 6 = -1(x^2 + x - 12)$ $y^2 - y - 6 = -x^2 - x + 12$ $x^2 + x - 12 + y^2 - y - 6 = 0$ $x^2 + x + y^2 - y = 18$ $x^2 + x + \left(\frac{1}{2}\right)^2 + y^2 - y + \left(\frac{-1}{2}\right)^2 = 18 + \left(\frac{1}{2}\right)^2 + \left(\frac{-1}{2}\right)^2$ $\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = 18.5$ This is a circle centre $\left(\frac{-1}{2}, \frac{1}{2}\right)$, radius $\sqrt{18.5}$	1 1 1 1

Question 10

Marking guidelines

Criteria	Marks
• a) let $v = x^2 - 2x$ $v + \frac{12}{v} = 8$ $v^2 + 12 = 8v$ $v^2 - 8v + 12 = 0$ $(v - 6)(v - 2) = 0$ $v = 6$ or $v = 2$	1
$x^2 - 2x = 6$ OR $x^2 - 2x = 2$ $x^2 - 2x - 6 = 0$ $\quad$ $x^2 - 2x - 2 = 0$	1
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-6)}}{2(1)}$ $\quad$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$ $x = \frac{2 \pm \sqrt{28}}{2}$ $\quad$ $x = \frac{2 \pm \sqrt{12}}{2}$ $x = 1 \pm \sqrt{7}$ $\quad$ $x = 1 \pm \sqrt{3}$	1 for each

Criteria	Marks
• b) $A_1 = 26\,000(1.06) - 4\,200$	1
$A_2 = A_1(1.06) - 4\,200$	
$A_2 = [26\,000(1.06) - 4\,200](1.06) - 4\,200$	
$= 26\,000(1.06)^2 - 4\,200(1.06) - 4\,200$	
$= 26\,000(1.06)^2 - 4\,200(1.06 + 1)$	1
$A_3 = 26\,000(1.06)^3 - 4\,200(1.06^2 + 1.06 + 1)$	
$A_3 = 26\,000(1.06)^3 - 4\,200(1 + 1.06 + 1.06^2)$	
$A_n = 26\,000(1.06)^n - 4\,200(1 + 1.06 + \ldots\ldots + 1.06^{n-1})$	1
$A_n = 0$	
$4\,200(1 + 1.06 + \ldots\ldots + 1.06^{n-1}) = 26\,000(1.06)^n$	
$(1 + 1.06 + \ldots\ldots + 1.06^{n-1})$ is a GP with $a = 1$, $r = 1.06$, $n = n$	1
$4\,200\left(\dfrac{1(1.06^n - 1}{1.06 - 1}\right) = 26\,000(1.06)^n$	1
$4\,200\left(\dfrac{1.06^n - 1}{0.06}\right) = 26\,000(1.06)^n$	
$4\,200(1.06)^n - 4\,200 = 1560(1.06)^n$	1
$2\,640(1.06)^n = 4\,200$	
$(1.06)^n = \dfrac{4200}{2640}$	
$(1.06)^n = 1.590909$ taking logs	
$n \log(1.06) = \log(1.590909)$	1
$n = \dfrac{\log 1.590909}{\log 1.06}$	
$n = 7.9683$	
Loan repaid in 8 years	1