Year 12 Trial Question No.1	Mathematics Solutions and Marking Guidelines	Examination 2009

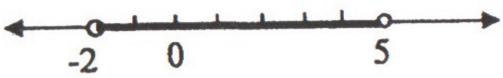
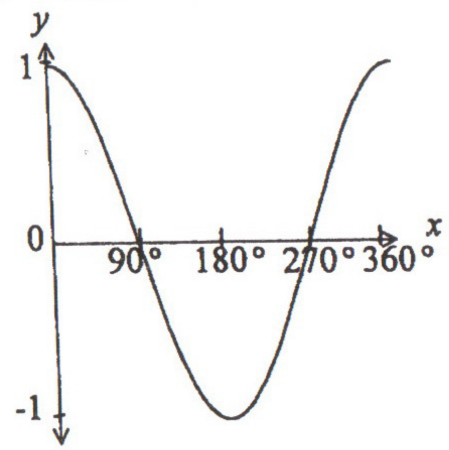
Outcomes Addressed in this Question

P3 Performs routine arithmetic & algebraic manipulation involving surds & simple rational expressions

P4 Chooses and applies appropriate arithmetic, algebraic & trigonometric techniques

P5 Understands the relationship between a function and its graph

H3 Manipulates algebraic expressions involving logarithmic & exponential functions

Outcome	**Solutions**	**Marking Guidelines**
P3	a) $3x^3+24=3\left(x^3+8\right)$ $=3(x+2)\left(x^2-2x+4\right)$	2 marks : factorises correctly twice 1 mark: factorises correctly once
P3	b) $\dfrac{2}{5-\sqrt{3}}=\dfrac{2}{5-\sqrt{3}}\times\dfrac{5+\sqrt{3}}{5+\sqrt{3}}$ $=\dfrac{2\left(5+\sqrt{3}\right)}{22-3}$ $=\dfrac{5+\sqrt{3}}{11}$	2 mark: finds common denominator and correctly simplifies 1 mark : significant progress towards correct answer
P4	c) $\dfrac{3x+2}{2}-\dfrac{x-1}{5}=\dfrac{5(3x+2)-2(x-1)}{10}$ $=\dfrac{15x+10-2x+2}{10}$ $=\dfrac{13x+12}{10}$	2 marks : correct answer 1 mark : significant progress towards correct answer
P4	d) $\lvert 2x-3\rvert<7$ $\therefore -7<2x-3<7$ $\therefore -4<2x<10$ $\therefore -2<x<5$  	2 marks: correct answer 1 mark : partially correct answer
H3	e) $\dfrac{e^{-3.5}}{4}=0.007549\ldots$ $=7.5\times10^{-3}$ to 2 significant figures	2 mark : correct answer 1 mark: correct calculation, incorrect rounding
P5	f) $y=\cos x$  	2 marks: correct answer 1 mark : partially correct answer

Outcomes Addressed in this Question H2

Outcome	Sample Solution	Marking Guidelines
H2	a) $BD^2+20^2=24^2$ (*Pythagoras' Theorem*) $BD=\sqrt{176}$ $x^2+(\sqrt{176})^2=14^2$ (*Pythagoras' Theorem*) $\therefore\ x=\sqrt{20}$ $=2\sqrt{5}$	**2 mark** ~ Correct answer with reasons **1 mark** ~ Correct answer without reasons
H2	b) i) $\dfrac{-4+x}{2}=0$ $\quad\dfrac{0+y}{2}=3$ $x=4$ $\quad y=6$ $\therefore N(4,6)$	**2 marks** ~ Correct solution **1 mark** ~ Substantial progress towards correct solution.
H2	ii) $m_{NP}=\dfrac{8-6}{0-4}=-\dfrac{1}{2}$ $\quad m_{PL}=\dfrac{8-0}{0+4}=2$ $m_{NP}\times m_{PL}=-\dfrac{1}{2}\times 2$ $=-1$ $\therefore NP\perp PL$ $\therefore \angle NPL=90^\circ$	**2 marks** ~ Correct solution **1 mark** ~ Substantial progress towards correct solution.
H2	iii) *Circle centre* $M(0,3)$, *radius* 5 $x^2+(y-3)^2=25$	**2 marks** ~ Correct solution **1 mark** ~ Substantial progress towards correct solution.
H2	c) i) $\angle ADX=\angle CBY$ (*opposite* $\angle$'s *of parallelogram*) ii) $AD=BC$ (*opposite sides of parallelogram*) $AX=BC$ (*given*) $\therefore AD=AX$	**1 mark** ~ correct reasons given **1 mark** ~ correct reasons given
H2	iii) $AD=AX$ $\therefore \Delta ADX$ *is isosceles* $BC=YC$ $\therefore \Delta CBY$ *is isosceles* $\therefore \angle ADX=\angle AXD$ *and* $\angle CBY=\angle CYB$ (*equal angles in isosceles* Δ's) $\angle ADX=\angle CBY$(*shown in* (*i*)) $AX=BC$ (*given*) $\angle AXD=\angle CYB$ (*equals of* $\angle ADX$ *and* $\angle CBY$) $\therefore \Delta ADX\equiv\Delta CBY$ (AAS)	**2 marks** ~ Correct solution **1 mark** ~ Substantial progress towards correct solution.

Year 12 Mathematics Extension 1 Task 4 2009		
Question No. 3	Solutions and Marking Guidelines	
	Outcomes Addressed in this Question	

P3 performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities
P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques
P5 understands the concept of a function and the relationship between a function and its graph
P6 relates the derivative of a function to the slope of its graph

Outcome	Solutions	Marking Guidelines
P4	**3. a)** $a^2 = b^2 + c^2 - 2ab\cos A$ $22^2 = 12^2 + 13^2 - 2(12)(13)\cos A$ $2(12)(13)\cos A = 12^2 + 13^2 - 22^2$ $\cos A = \dfrac{12^2 + 13^2 - 22^2}{2(12)(13)}$ $\therefore A = 123.2351...^\circ$ (by calculator) $\therefore A = 123^\circ$ to nearest degree	1 mark awarded for partial correct solution 2 marks awarded for complete correct solution
P4	**b)** **(i)** $DP = 2cm$ (ΔDAP is equilateral) **Or** $DP = 2cm$ (using cosine rule)	1 mark for partial correct solution. 2 marks for complete correct solution

(ii)

Now $\angle PBC = 180° - 60° = 120°$ (co-interior to $\angle DAP; DA \parallel CB$)

and $BC = AD = 2cm$ (opposite sides in parallelogram)

In ΔPBC:

$$PC^2 = PB^2 + BC^2 - 2(PB)(BC)\cos\angle PBC$$
$$PC^2 = 4^2 + 2^2 - 2(4)(2)\cos 120°$$
$$PC^2 = 16 + 4 - 16\left(-\frac{1}{2}\right)$$
$$PC^2 = 28$$
$$PC = \sqrt{28}$$
$$PC = 2\sqrt{7}$$

1 mark awarded for partial correct solution leading to the correct value of PC.

Now $DC = AB = 6cm$ (opposite sides of parallelogram)

In ΔPCD

$$\cos x° = \frac{DP^2 + PC^2 - DC^2}{2 \times DP \times PC}$$
$$\cos x° = \frac{2^2 + \left(2\sqrt{7}\right)^2 - \left(6^2\right)}{2 \times 2 \times 2\sqrt{7}}$$
$$\cos x° = \frac{4 + 28 - 36}{8\sqrt{7}}$$
$$\cos x° = \frac{-4}{8\sqrt{7}}$$
$$\cos x° = \frac{-1}{2\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$$
$$\cos x° = \frac{-\sqrt{7}}{14}$$

2 marks awarded for a further correct partial solution using the cosine rule and giving any correct value for $\cos x°$

3 marks awarded for complete correct solution

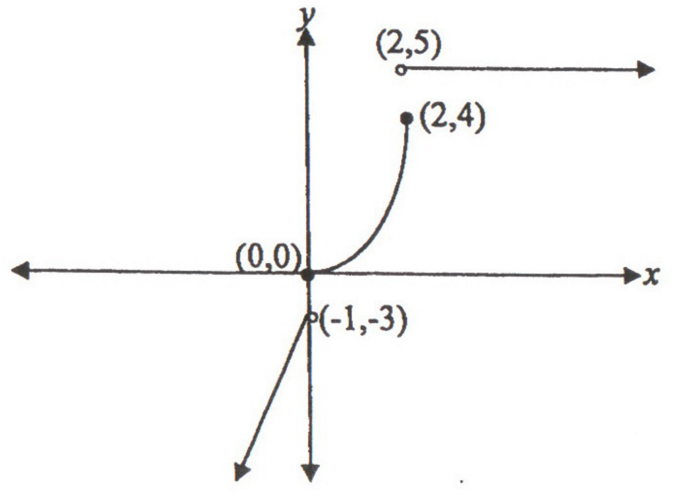
P3 (c)

$$\cos 2\theta = \frac{1}{2} \text{ for } 0° \le \theta \le 180°$$
$$\therefore 2\theta = 60° \text{ or } (360° - 60°) \text{ for } 0° \le 2\theta \le 360°$$
$$\therefore 2\theta = 60° \text{ or } 300°$$
$$\therefore \theta = 30° \text{ or } 150° \text{ for } 0° \le \theta \le 180°$$

1 mark awarded for partial correct solution

2 marks awarded for complete correct solution

P5 (d)



1 marked awarded for sketching $f(x) = 5$ for $x > 2$

1 marked awarded for sketching $f(x) = x^2$ for $0 \le x \le 2$

ear 12 Mathematics Trial HSC Examination 2009

uestion No. 5 Solutions and Marking Guidelines

Outcomes Addressed in this Question

5 applies appropriate techniques from the study of series to solve problems

utcome	Solutions	Marking Guidelines
H5	**(a)** Since the series is geometric: $\frac{T_3}{T_2}=\frac{T_2}{T_1}$ ie. $\frac{a^2+2ab+b^2}{x}=\frac{x}{4a^2b^2}$ $x^2=4a^2b^2(a^2+2ab+b^2)$ $=4a^2b^2(a+b)^2$ $x=\pm 2ab(a+b)$	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution
H5	**(b) (i)** $T_1=50$ $T_2=100$ $T_3=150$	**1 mark** Correct terms shown
H5	**(ii)** Since the above series is arithmetic: $T_n=1200 \quad a=50 \quad d=50$ $T_n=a+(n-1)d$ $1200=50+(n-1).50$ $=50+50n-50$ $=50n$ $n=\frac{1200}{50}$ $=24$ $\therefore$ The water had been leaking for 24 hours when the leak was detected.	**2 marks** Correct solution **1 mark** Correctly identifies series as arithmetic, giving first term, common difference and demonstrating substantial knowledge or required formula.
H5	**(iii)** Total volume of water lost $=S_{24}$ $S_n=\frac{n}{2}(a+l)$ where $l=T_{24}$ $S_{24}=\frac{24}{2}(50+1200)$ $=12.1250$ $=15000$ $\therefore$ 15000L of water had been lost when the leak was detected.	**1 mark** Correct solution
H5	**(c) (i)** $1+\sin A+\sin^2 A+\ldots$ is a geometric series where $r=\sin A$ Since $-1\le \sin A\le 1$ and provided $\sin A\ne \pm 1$ series will have a limiting sum as limiting sum exists where $-1<r<1$	**1 mark** Correct justification

H5

(ii)

$$
\begin{aligned}
r &= \sin\frac{4\pi}{3} \\
&= -\frac{\sqrt{3}}{2} \\
S_\infty &= \frac{a}{1-r} \\
&= \frac{1}{1+\frac{\sqrt{3}}{2}} \\
&= \frac{2}{2+\sqrt{3}} \\
&= 4-2\sqrt{3}
\end{aligned}
$$

2 marks
Correct solution
1 mark
Gives correct value for sine ratio **OR** uses incorrect value in correct formula for limiting sum.

H5

(d) (i)

$$
\begin{aligned}
A_1 &= P(1+r)^n \\
&= 250(1.03)^{20} \\
&= 451.53
\end{aligned}
\qquad \textbf{OR} \qquad
\begin{aligned}
A_1 &= P(1+r)^n \\
&= 250(1.06)^{10} \\
&= 447.71
\end{aligned}
$$

\$447.71 will be available for withdrawal in 10 years.

1 mark
Correct answer. Accept both interest compounded 6 monthly and compounded yearly.

H5

(ii)

At the end of 6 months

$A_1 = 250(1.03)$

At the end of 12 months (ie. 1 year)

$$
\begin{aligned}
A_2 &= 250(1.03)(1.03)+250(1.03) \\
&= 250(1.03)^2+250(1.03) \\
&= 250(1.03)(1+1.03)
\end{aligned}
$$

Similarly, after 3 time periods (18 months)

$$
\begin{aligned}
A_3 &= 250(1.03)^3+250(1.03)^2+250(1.03) \\
&= 250(1.03)(1+1.03+1.03^2)
\end{aligned}
$$

After 20 time periods ie. 10 years

$$
\begin{aligned}
A_{20} &= 250(1.03)^{20}+250(1.03)^{19}+250(1.03)^{18}+\ldots+250(1.03)^2+250(1.03) \\
&= 250(1.03)(1+1.03+1.03^2+\ldots+1.03^{19})
\end{aligned}
$$

Now, $1+1.03+1.03^2+\ldots+1.03^{19}$ is a geometric series with $a=1, r=1.03$ and $n=20$

$$
\begin{aligned}
S_{20} &= \frac{a(r^n-1)}{r-1} \\
&= \frac{1.03^{20}-1}{0.03} \\
&= 26.8704 \\
\therefore A_{20} &= 250(1.03)(26.8704) \\
&= 6919
\end{aligned}
$$

ie. \$6919 is available for withdrawal after 10 years, as required.

2 marks
Correct solution
1 mark
Substantial progress towards correct solution

Outcomes Addressed in this Question

H5 applies appropriate techniques from the study of probability and trigonometry to solve problems

Outcome	Solutions	Marking Guidelines
H5	**(a)** The statement is not valid. It assumes that all Fijians have access to motor vehicles with similar safety levels and the roads being driven on are in a similar state of repair. In fact, many rural dwelling Fijians rarely use motor vehicles, so the probability of them being killed in a road accident is almost zero, whereas a city dwelling Fijian would have a comparably higher probability of being a road fatality. ie. the event of being killed on the road is not equally likely for each Fijian.	**2 marks** Correct assessment of validity of statement **1 mark** Some aspects of assessment are valid.
	(b) Probability of requiring treatment for food poisoning $= P(F)$ $= 0.15$ Probability of requiring treatment for influenza $= P(I)$ $=0.45$	
H5	(i) $P(F \text{ and } I)$ $=P(F)\times P(I)$ $=0.15\times 0.45$ $=0.0675$	**1 mark** Correct answer.
H5	(ii) $P(F \text{ or } I)$ $=P(F)+P(I)-P(F \text{ and } I)$ $=0.15+0.45-0.0675$ $=0.5325$ Note: These events are not mutually exclusive	**2 marks** Correct solution. **1 mark** Solution substantially correct.
H5	**(c)** (i) $P(H)=\frac{2}{3}$	**1 mark** Correct answer.
H5	(ii) $P(\text{at least 1 tail}) = 1 - P(5\text{ heads})$ $= 1-\left(\frac{2}{3}\right)^5$ $= 1-\frac{32}{243}$ $= \frac{211}{243}$	**1 mark** Correct solution.
H5	**(d)** (i) $36^\circ = \frac{36\pi}{180}$ radians $= \frac{\pi}{5}$ radians	**1 mark** Correct answer.
H5	(ii) $A = \frac{1}{2}r^2(\theta - \sin\theta)$ $= \frac{1}{2}\times 4^2\left(\frac{\pi}{5}-\sin\frac{\pi}{5}\right)$ $= 0.32$ $\therefore 0.32\text{m}^2$ of the road is being watered.	**2 marks** Correct solution **1 mark** Correct formula and substitution.

H5	(iii) Water wasted $= \dfrac{\text{area of road watered}}{\text{area of circle}}$ $= \dfrac{0.32}{\pi r^2} \times 3.5\text{kL}$ $= \dfrac{0.32}{\pi \times 4^2} \times 3.5\text{kL}$ $= 22\text{L per hour}$	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution.

Outcomes Addressed in this Question

H8 uses techniques of integration to calculate areas and volumes

Outcome	Solutions	Marking Guidelines
H8	a) $\int(2x+3)^3dx=\dfrac{(2x+3)^4}{4\times2}+c$ $=\dfrac{(2x+3)^4}{8}+c$	1 mark : correct integral
H8	b) $\int_1^4\dfrac{x^2+8}{x^2}\,dx=\int_1^4\left(\dfrac{x^2}{x^2}+\dfrac{8}{x^2}\right)dx=\int_1^4\left(1+8x^{-2}\right)dx$ $=\left[x-8x^{-1}\right]_1^4$ $=\left[x-\dfrac{8}{x}\right]_1^4$ $=4-2-(1-8)=9$	3 marks: correct solution 2 marks: significant progress towards correct solution 1 mark: progress towards correct solution
H8	c) $f'(x)=3x^2+x$ $\therefore f(x)=\dfrac{3x^3}{3}+\dfrac{x^2}{2}+c$ $\therefore f(x)=x^3+\dfrac{x^2}{2}+c$ $f(-2)=4,\ \therefore -8+2+c=4.\ \therefore c=10$ $\therefore f(x)=x^3+\dfrac{x^2}{2}+10$	2 marks : correct answer with justification 1 mark: one of above
H8	d) (i) $y=x^2$ and $y=12-2x^2$ meet when $x^2=12-2x^2$. $\therefore 3x^2=12$ $\therefore x^2=4\quad x=\pm2$ $\therefore$meet at $(-2,4)$ and $(2,4)$	1 mark : correct answer
H8	(ii) Area = Area under top curve − area under bottom curve $=\int_{-2}^{2}\left(12-2x^2-x^2\right)dx=2\int_0^2\left(12-3x^2\right)dx$ $=2\left[12x-x^3\right]_0^2$ $=2(24-8)=32$ units^2	2 marks : correct answer with justification 1 mark : significant progress towards correct answer
H8	(iii) Volume,in relation to y axis = $\pi\int_a^b x^2dy$ V = sum of the volume when the area between $y=x^2$ and the y axis between $y=0$ and $y=4$ is rotated about the y axis, and the volume when the area between $y=12-2x^2$ and the y axis between $y=4$ and $y=12$ rotated about the y axis	3 marks: correct solution 2 marks: significant progress towards correct solution 1 mark: progress towards correct solution

From $y=x^2$, ie. $x^2=y$

From $y=12-2x^2$, $2x^2=12-y$, $\therefore x^2=6-\frac{y}{2}$.

$$\therefore \text{V}=\pi\int_0^4 y\,dy+\pi\int_4^{12}\left(6-\frac{y}{2}\right)dy$$

$$\therefore \text{V}=\pi\left[\frac{\text{y}^2}{2}\right]_0^4+\pi\left[6y-\frac{\text{y}^2}{4}\right]_4^{12}$$

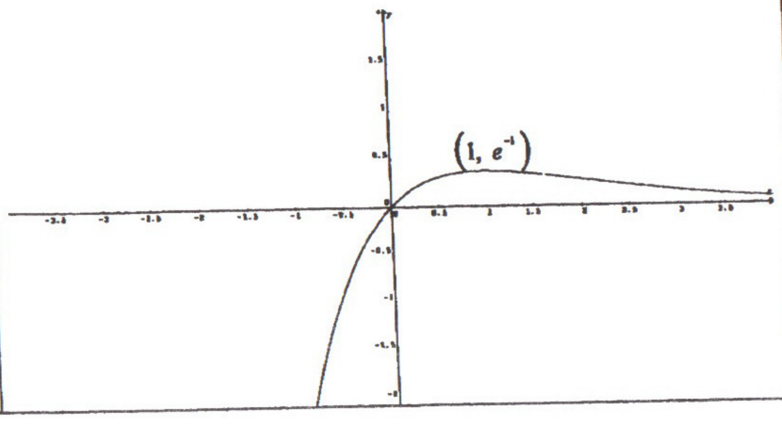
$$\therefore \text{V}=8\pi+\pi\left(72-36-\left(24-4\right)\right)$$

$$\therefore \text{V}=24\pi \text{ units}^3$$

Outcomes Addressed in this Question

H3 manipulates algebraic expressions involving logarithmic and exponential functions

H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems

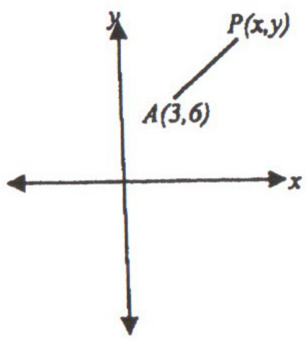
Outcome	Solutions	Marking Guidelines
(a) H3	$A^{4m}-5=\left(A^{m}\right)^{4}-5$ $=3^{4}-5$ $=76$	**Award 2** for correct solution. **Award 1** for attempting to use an appropriate process.
(b) H3, H5	$\int_{3}^{5} x\log_e x\,dx \approx \frac{1}{3}\left(3\log_e 3+5\log_e 5+4\times 4\log_e 4\right)$ $=11.17457874$	**Award 2** for correct solution. **Award 1** for attempting to use Simpson's rule.
(c) (i) H3, H5	$y=xe^{-x}$ $\frac{dy}{dx}=x.-e^{-x}+e^{-x}.1=e^{-x}(1-x)$ $\frac{d^2y}{dx^2}=e^{-x}(-1)+(1-x).-e^{-x}=e^{-x}(x-2)$ Stationary point(s) occur @ $\frac{dy}{dx}=0$ $e^{-x}(1-x)=0$ $\therefore 1-x=0 \quad (\because e^{-x}\neq 0)$ $\therefore x=1$ Test $x=1$ $\frac{d^2y}{dx^2}=e^{-1}(1-2)=-e^{-1}<0$ $\therefore$ Relative maximum turning point @ $(1,\ e^{-1})$	**Award 3** for correct stationary point, with full justification. **Award 2** for correct stationary point, without full justification. **Award 1** for attempting to find the stationary point.
(ii) H3	$\lim\limits_{x\to\infty}\left(xe^{-x}\right)=0$ or function approaches $y=0$. $\lim\limits_{x\to-\infty}\left(xe^{-x}\right)=-\infty$ or function gets very big and negative.	**Award 2** for correct solutions. **Award 1** for only one correct solution.
(iii) H3, H5	When $x=0$, $y=0e^{-0}=0$ $\therefore$ Graph passes through the origin.	**Award 1** for correct solution.
(iv) H3, H5		**Award 2** for correct graph, showing relevant details from (i), (ii) and (iii). **Award 1** for correct graph, but lacking sufficient detail.

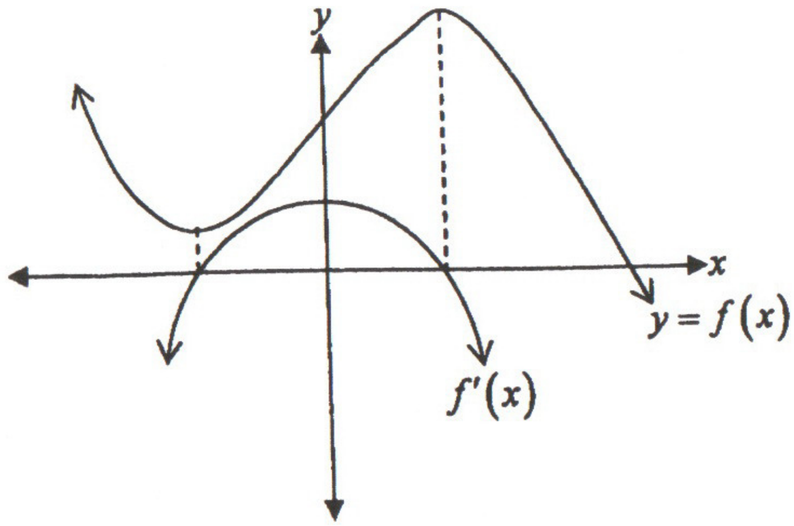
Outcomes Addressed in this Question H5

Outcome	Sample Solution	Marking Guidelines
H5	a) $y=\sqrt{x-1}$ $y=(x-1)^{\frac{1}{2}}$ $y'=\frac{1}{2}(x-1)^{-\frac{1}{2}}.1$ $y'=\frac{1}{2\sqrt{x-1}}$ *when* $x=2$ $y'=\frac{1}{2}$ *eqn of tangent*: $y-1=\frac{1}{2}(x-2)$ $y=\frac{1}{2}x$ *or* $x-2y=0$	**2 mark** ~ Correct equation **1 mark** ~ Substantial progress towards correct solution
H5	b) $x^2-8x+12>0$ $(x-6)(x-2)>0$ $\therefore x>6,\ x<2$	**2 marks** ~ Correct solution **1 mark** ~ Substantial progress towards correct solution.
H5	c) i) $S=\left\|\frac{2x-x^2-5}{\sqrt{2^2+(-1)^2}}\right\|$ $=\left\|\frac{2x-x^2-5}{\sqrt{5}}\right\|$	**2 marks** ~ Correct solution **1 mark** ~ Substantial progress towards correct solution.
H5	ii) $f(x)=2x-x^2-5$ $=-x^2+2x-5$ *For negative definite*, $a<0$ *and* $\Delta<0$ $a=-1$ $\Delta=4-4\times(-1)\times(-5)$ $=4-20$ $=-16$ $\therefore f(x)$ *is negative definite.*	**2 marks** ~ Correct solution **1 mark** ~ Substantial progress towards correct solution.
H5	iii) since $f(x)$ is negative definite, $\|f(x)\|=-f(x)$ $\therefore \|2x-x^2-5\|=-(2x-x^2-5)$ $=x^2-2x+5$ $\therefore S=\frac{x^2-2x+5}{\sqrt{5}}$	**1 mark** ~ correct reasons given
H5	iv) $S=\frac{x^2-2x+5}{\sqrt{5}}$ $S'=\frac{2x-2}{\sqrt{5}}$ $S''=\frac{2}{\sqrt{5}}$ *stat. pt* $S'=0$ $\frac{2x-2}{\sqrt{5}}=0$ $x=1$ *when* $x=1$ $S''=\frac{2}{\sqrt{5}}>0$ $\therefore$ *Minimum at* $x=1$ *when* $x=1$ $S=\frac{1^2-2+5}{\sqrt{5}}=\frac{4}{\sqrt{5}}$ $\therefore$ Shortest possible distance is $\frac{4}{\sqrt{5}}$ units.	**3 marks** ~ Correct solution **2 marks** ~ Substantial progress towards correct solution. **1 mark** ~ Some progress towards correct solution.

Outcomes Addressed in this Question

P3 performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities
P5 understands the concept of a function and the relationship between a function and its graph
P6 relates the derivative of a function to the slope of its graph

Outcome	Solutions	Marking Guidelines
	10.	
P5	**(a)** **(i)** $4a=8$ $\therefore a=2$ $\therefore$ focal length is 2	1 mark awarded for correct answer
	(ii) **Vertex (2,-1)**	1 mark awarded for correct answer
	(iii) **Focus (2,1)**	1 mark awarded for correct answer
P5	**(b)** $y=-4x$	1 mark awarded for correct answer
P5	**(c)**  $PA=\sqrt{(x-3)^2+(y-6)^2}$ comdition is $PA=5$ $5=\sqrt{(x-3)^2+(y-6)^2}$ $\therefore (x-3)^2+(y-6)^2=25$ $\therefore x^2-6x+9+y^2-12y+36=25$ $\therefore x^2-6x+y^2-12y+20=0$ The locus is $x^2-6x+y^2-12y+20=0$	1 mark awarded for partial correct solution 2 marks awarded for a further correct partial solution 3 marks awarded for complete correct solution

P6	**(d)** 	1mark awarded for partial correct graph 2 marks awarded for complete correct graph
P3	**(e)** **(i)** **Roots are** $\alpha, 2\alpha$ **Sum of roots:** $3\alpha = -m$ **Product of roots:** $2\alpha^2 = 10$	1 mark awarded for correct solution
	(ii) $3\alpha = -m \ldots\ldots\ldots\ldots\ldots\ldots (A)$ $2\alpha^2 = 10 \ldots\ldots\ldots\ldots\ldots\ldots (B)$ From (A): $\alpha = \frac{-m}{3} \ldots\ldots\ldots\ldots\ldots\ldots (C)$ substitute (C) into (B): $2\left(\frac{-m}{3}\right)^2 = 10$ $\frac{2m^2}{9} = 10$ $m^2 = 45$ $m = \pm\sqrt{45}$ $m = \pm 3\sqrt{5}$	1 mark for partial correc solution. 2 marks for complete correct solution