YR 12 : 2 UNIT

MATHEMATICS TRIAL 2003

SOLUTIONS + MARKING SCALE

QUESTION 1 : MF

(a) $7.26366\ldots$ ✓

$\doteq 7.3$ correct to **2** s.f. ✓

Rounding off to 2 s.f. not well done. Does not mean 2 decimal places.

(b) $\frac{y}{4} - \frac{y-6}{8} = 2$

$\therefore 2y - y + 6 = 16$ ✓

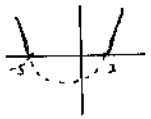
$\therefore y = 10$ ✓

this sign was a problem.

(c) $x^2 + 3x - 10 > 0$

$(x+5)(x-2) > 0$ ✓

$\therefore x < -5$ and $x > 2$ ✓



You must factorise, sketch and solve the quadratic inequality from your sketch.
It is incorrect to solve this way
$x+5 > 0$ $\quad x-2 > 0$
$x > -5$ $\quad x > 2$
Please don't do it.

(d) $y = (x-1)^2 + 4$

Range: $y \geq 4$ ✓

Students who sketched the parabola were the most successful in finding the range.

(e) $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ✓

$\frac{3}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{3\sqrt{2}+3}{1}$ ✓

$\therefore \frac{1}{\sqrt{2}} + \frac{3}{\sqrt{2}-1} = \frac{\sqrt{2}}{2} + \frac{2(3\sqrt{2}+3)}{2}$

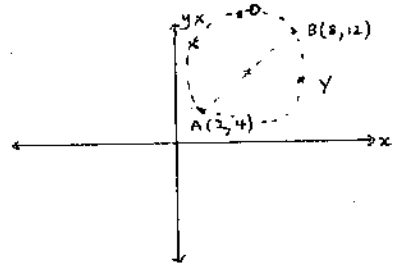
$= \frac{7\sqrt{2}+6}{2}$ ✓

(f) $\log_a a^2 - \log_a a^{-1} = 2 - -1$ ✓

$= 3$ ✓

1 mark for correct use of a log rule.

QUESTION 2 : KB



(a) Centre : $\left(\frac{3+8}{2}, \frac{4+12}{2}\right) = (5, 8)$ ✓

Good

(b) $d = \sqrt{(12-8)^2 + (8-5)^2}$

$= \sqrt{16+9}$

$= 5$ ✓

Good

(c) Equation :

$(x-5)^2 + (y-8)^2 = 5^2$ ✓

Good

(d) When $x = 5$:

$(5-5)^2 + (y-8)^2 = 5^2$

$0 + (y-8)^2 = 25$

$y - 8 = \pm 5$ ✓

$\therefore y = 13$ or 3

$\therefore (5, 13)$ does lie on the circle

Good

(e) $m_{AD} = \frac{13-4}{5-2} = \frac{9}{3} = 3$

$m_{BD} = \frac{13-12}{5-8} = -\frac{1}{3}$ ✓

$\therefore m_{AD} \times m_{BD} = 3 \times -\frac{1}{3} = -1$

$\therefore AD \perp BD$ ✓

Both gradients correct for 1 mark.

(f) $m_{AC} = \frac{12-4}{5-2} = \frac{8}{6} = \frac{4}{3}$

$\therefore$ Grad $\times$: $-\frac{3}{4}$ ✓

$\therefore y - 8 = -\frac{3}{4}(x-5)$

$\therefore 3x + 4y - 47 = 0$ ✓

not well done.
A diagram would assist in the next sections

must be in general form.

Question 2 (cont.)

(g) XY = 10 units (diameter of circle). ✓

⊥ height is dist of O from XY:

$$d = \frac{|3\times5 + 4\times13 - 47|}{\sqrt{9+16}}$$

$$= \frac{20}{5} = 4 \quad ✓$$

$$\therefore \text{Area} = \frac{1}{2}\times10\times4$$

$$= 20\,u^2$$

(h) ⊥ distance of line from centre:

Centre: (5, 8)

$$d = \frac{|3\times5 + 4\times8 - 22|}{\sqrt{3^2+4^2}} \quad ✓$$

$$= \frac{25}{5}$$

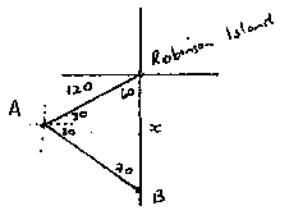
$$= 5$$

∴ Since the line is 5 units from the centre of the circle of radius 5, it is a tangent to the circle. ✓

Com 1
Question 3: Calc 3 CB

Comments

(a)



✓

Com 1 – diagram only

$\angle RBA = 70°$

$$\frac{x}{\sin 50} = \frac{120}{\sin 70} \quad ✓$$

$$\therefore x = 97.824\ldots$$

$$\doteq 98 \text{ km} \quad ✓$$

must round to nearest km.

(b) (i) $y = \ln(x^2+1)$

$$\frac{dy}{dx} = \frac{2x}{x^2+1} \quad ✓$$

(b) Calc /3

(ii) $y = \dfrac{e^{2x}}{\sin 3x}$ $\quad u = e^{2x} \quad v = \sin 3x$

$u' = 2e^{2x} \quad v' = 3\cos 3x$ ✓

$$\frac{dy}{dx} = \frac{2e^{2x}\sin 3x - 3e^{2x}\cos 3x}{(\sin 3x)^2} \quad ✓$$

$$= \frac{e^{2x}(2\sin 3x - 3\cos 3x)}{(\sin 3x)^2}$$

Several students failed to put v^2 in the denominator

(further simplification is not necessary)

ignore subsequent errors

(c) $\lim\limits_{x\to3} \dfrac{(x-3)(x+3)}{(x-3)(x+2)} = \dfrac{6}{5}$ ✓

(d) $\sin^2\theta - \sin\theta - 1 = 0$

$(\sin\theta - 2)(\sin\theta + 1) = 0$

$\therefore \sin\theta = 2$ no solution ✓ $\quad$ or $\quad \sin\theta = -1$ ✓

$\theta = -\frac{\pi}{2}$ ✓

(must not just ignore no sol^n case).

(e) $\alpha+\beta = 2$ ✓

$\alpha\beta = \frac{2}{3}$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$$

$$= 2 \div \frac{2}{3}$$

$$= 3 \quad ✓$$

Well done

Question 4: HG

Com /2 Calc /5 Reas /3

(a) $\sum_{r=1}^{4} r^2 - 1 = 0 + 3 + 8 + 15 = 26$ ✓

(b) (i) $\int 3\sqrt{x} + x^{-4}\,dx$

$= \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{-3}}{-3} + C$ ✓✓✓C

$= 2\sqrt{x^3} - \frac{1}{3x^3} + C$

(ii) $\int_0^2 \left(e^{5x} + e^{-5x}\right)^2 dx$

$= \int_0^2 e^{10x} + 2 + e^{-10x}\,dx$ ✓

$= \left[\frac{1}{10}e^{10x} + 2x + \frac{1}{10}e^{-10x}\right]_0^2$ ✓

$= \left(\frac{1}{10}e^{20} + 4 - \frac{1}{10}e^{-20}\right) - \left(\frac{1}{10} + 0 - \frac{1}{10}\right)$ ← must show evidence of substituting $x=0$!

$= \frac{1}{10}e^{20} + 4 - \frac{1}{10}e^{-20}$ ✓

(c) LHS: $\frac{\cos\theta\left(2\cos^2\theta - 1\right)}{\sin\theta\left(\cos^2\theta - \sin^2\theta\right)}$ ✓

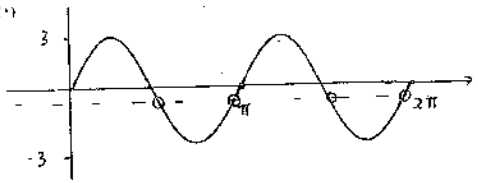
$= \frac{\cos\theta\left(2\cos^2\theta - 1\right)}{\sin\theta\left(\cos^2\theta - (1 - \cos^2\theta)\right)}$ ✓

$= \frac{\cos\theta\left(2\cos^2\theta - 1\right)}{\sin\theta\left(2\cos^2\theta - 1\right)}$

$= \cot\theta$ ✓

$= RHS$

(d) (i)



(ii) 4 solutions to the equation. ✓

Comments

- Some students forget to add!
- Learn index rules carefully.

(b) Calc /5 (i) Com /2

further simplification not required

+C = 2nd mark.

(ii) Poorly done

- Expansion poor
- Many integration errors eg $\int e^{10x}\,dx \neq \frac{1}{10}e^{10x} + C$

(c) Reas /3

factorise

- Many students gave up after factoring!

substituting expression.

(Ext students may use $\cos 2\theta$)

simplify.

✓ period
✓ amplitude

(i) Com /2

- Well done!

(or cfge).

Question 5: MF

Calc /5 Reas /5

(a) (i) $T = Ce^{-kt}$

$\frac{dT}{dt} = -k \times Ce^{-kt}$

$= -kT$ ✓

(ii) when $t=0$, $T=40$

$\therefore 40 = Ce^0$

$\therefore C = 40$ ✓

(iii) when $t=17$, $T=24$

$\therefore 24 = 40e^{-k\times 17}$

$\frac{3}{5} = e^{-17k}$

$\therefore \ln\left(\frac{3}{5}\right) = -17k$ ✓

$\therefore k = -\frac{1}{17}\ln\left(\frac{3}{5}\right)$

$= \frac{1}{17}\ln\left(\frac{3}{5}\right)^{-1}$ ← The log-law used here should really be shown in working. No students did this so the mark was awarded for having the required answer.

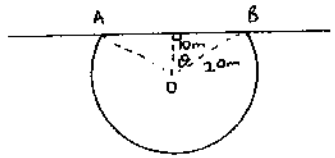
$= \frac{1}{17}\ln\left(\frac{5}{3}\right)$ $(\doteq 0.03)$ ✓

(ii) $T = 40 \times e^{-\frac{1}{17}\ln\left(\frac{5}{3}\right)\times 45}$

$= 10.987\ldots$

$= 11°$. ✓

(b)



$\cos\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{3}$

$\therefore \angle AOB = \frac{2\pi}{3}$ ✓

$\therefore$ Area minor seg $= \frac{1}{2} \times 20^2 \times \left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$

$= 200\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$ $(\doteq 245.7)$ ✓

$\therefore$ Area of major seg $= \pi \times 20^2 - 200\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= \frac{800\pi}{3} + 100\sqrt{3}$

$\left(\doteq 1010.96\right)$ ✓

Comments

(a) Calc /5

First mark given for knowing to take logs both sides and using a log-law.

(b) Reas /3

Note Very important

Angle must be in radians.

Required Area

= Circle − Minor Segment

= Circle − (Sector − triangle)

Question 5 (cont.)

(c) (i) $P(4, H) = \frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$ ✓

(ii) $P(\text{odd, vowel}) = \frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$ ✓

(iii) $P(<3 \text{ or } L) = \frac{2}{5}$ ✓ $+ \frac{3}{5} \times \frac{2}{5}$

$= \frac{16}{25}$ ✓

COMMENTS

Some students added the fractions. You must multiply the successive events probabilities together.

(iii) Reas ✓2

These events in iii) are not mutually exclusive. They have something in common

$P(<3 \text{ or } L) = P(<3) + P(L) - P(<3 \text{ and } L)$

$= \frac{2}{5} + \frac{2}{5} - \frac{2}{5} \times \frac{2}{5}$

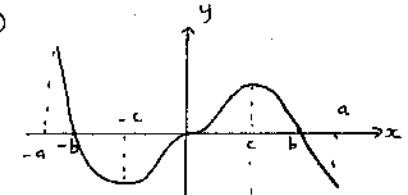
$= \frac{10}{25} + \frac{10}{25} - \frac{4}{25}$

$= \frac{16}{25}$

QUESTION 6

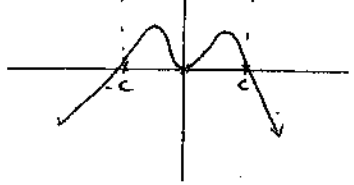
Com ✓3
Calc ✓4
Reas ✓3 MF

(a) (i)



✓

(ii)



✓ut. at $x=c$
✓t.p at $(0,a)$

(b)

(i) 6000, 14000, 22000, ...

$a = 6000$
$d = 8000$ ✓
$n = 10$

$T_{10} = 6000 + 9 \times 8000$
$= \$78000$ ✓

(ii) $S_n = \frac{n}{2}(12000 + (n-1)8000)$

$= 6000n + 4000n^2 - 4000n$

$= 4000n^2 + 2000n$ ✓

But $S_n \geq 1000000$

$4000n^2 + 2000n - 1000000 \geq 0$

$2n^2 + n - 500 \geq 0$ ✓

$\therefore n \leq -16.1$ or $n \geq 15.6$

$\therefore$ Must answer 16 questions ✓ correctly to exceed one million.

COMMENTS

(a) Com ✓3

✓✓✓ odd fn + correct derivative showing horiz. P.O.I.

✓✓ odd fn + correct derivative with no indication of horiz. P.O.I.

✓ max. for even fn. + correct derivative.

Show clear formulae, setting out + working.

(ii) Reas ✓3

Question 6 (cont.)

(c) (i) Volume = 32

$\therefore x^2 h = 32$

$\therefore h = \frac{32}{x^2}$

$S.A = x^2 + 4xh$

$= x^2 + 4.x.\frac{32}{x^2}$ ✓

$= x^2 + \frac{128}{x}$

(ii) $S = x^2 + 128x^{-1}$

$\frac{dS}{dx} = 2x - 128x^{-2}$ ✓

min SA $\Rightarrow \frac{dS}{dx} = 0$

$2x - \frac{128}{x^2} = 0$

$2x = \frac{128}{x^2}$

$\therefore x^3 = 64$

$\therefore x = 4$ ✓

$\therefore$ Dimensions are $4 \times 4 \times 2$ ✓

COMMENTS

(c) Calc 4

Substitution of h must be clearly shown.

Students found it difficult to solve this sort of equation.

Must give both x + h values.

Question 7: Com 3, Lea 2, HG

(a) RHS: $(x+1)(x-1)^2$

$= (x^2-1)(x-1)$

$= x^3 - x^2 - x + 1$ ✓

$=$ LHS

{or by factorising LHS}

(b) (i) $f(x) = \frac{x-1}{x^2}$

$f'(x) = \frac{x^2.1 - 2x.(x-1)}{x^4}$ ✓

$= \frac{x^2 - 2x^2 + 2x}{x^4}$

$= \frac{2x - x^2}{x^4}$ ✓

$= \frac{2-x}{x^3}$

(ii) Stat pt $\Rightarrow f'(x) = 0$

$\frac{2-x}{x^3} = 0$

$2 - x = 0$

$\therefore x = 2$ ✓

x	2^-	2	2^+
$f'(x)$	+ve	0	-ve

$\therefore (2, \frac{1}{4})$ is a max t.p. ✓

(iii) $f(x) = 0$

$\frac{x-1}{x^2} = 0$

$\therefore x - 1 = 0$

$\therefore x = 1$ ✓

$\therefore (1, 0)$

COMMENTS

. Many students tried to fudge it!

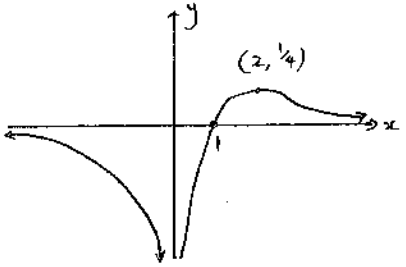
. (i), (ii), (iii) well done by most student.

(or by 2nd derivative)

$f''(x) = \frac{2x-6}{x^4}$

QUESTION 7 (cont.)

(iv)



(v) $f'(x) = \frac{2-x}{x^3}$

At $P(1,0)$, $f'(x) = 1$

$\therefore y - 0 = 1(x-1)$ ✓

$\therefore y = x-1$

(vi) Tangent meets curve:

$$\begin{cases} y = x-1 \\ y = \frac{x-1}{x^2} \end{cases}$$

$\therefore x-1 = \frac{x-1}{x^2}$

$\therefore x^3 - x^2 = x-1$ ✓

$x^3 - x^2 - x + 1 = 0$

using (a):

$(x+1)(x-1)^2 = 0$

$\therefore x = \pm 1$

$\therefore$ The tangent also meets the curve at $(-1,-2)$ ✓

COMMENTS

✓ – max
✓ – asymptote $x=0$
✓ $\lim x \to \pm\infty$

Com 1/3

- Many students didn't sketch the curve for $x<0$.

Reas 1/2

(vi) Poorly done. many students did not use the hint to use part (a).

QUESTION 8

Calc 6
Com 1, CB

(a) $\int_1^5 \frac{2}{x(x+1)}\,dx$

$\doteq \frac{1}{3}\left(1 + 4\left(\frac{1}{2}+\frac{1}{10}\right) + 2\left(\frac{1}{6}\right) + \frac{1}{15}\right)$ ✓

$\doteq \frac{1}{3} \times 3\frac{2}{15}$

$\doteq 1.044$ ✓

(ii) $\int_1^5 \frac{2}{x(x+1)}\,dx$

$= \int_1^5 \frac{2}{x} - \frac{2}{x+1}\,dx$

$= \left[2\ln x - 2\ln(x+1)\right]_1^5$ ✓

$= 2\ln 5 - 2\ln 6 - 2\ln 1 + 2\ln 2$ ✓

$= 2\ln\frac{5}{3}$

$\doteq 1.022$ ✓

(iii) Simpson's rule is an approximation ✓ for the integral (using parabolic arcs) whereas (ii) calculated the exact value of the integral.

COMMENTS

Several students chose $n=5$ rather than $n=4$ and several confused the formula.

Generally well done by those who used the appropriate substitution.

Com 1

Mostly well understood.

Question 8 (cont.)

(b) $v = 4 - 2t$

(i) $x = 4t - t^2 + C$ ✓

when $t = 0$, $x = 1$

$\therefore 1 = 0 - 0 + C$

$\therefore C = 1$

$\therefore x = 4t - t^2 + 1$ ✓

(ii) Rest $\Rightarrow v = 0$

$0 = 4 - 2t$

$\therefore t = 2$ ✓

$a = -2$ at all time, t. ✓

(iii) At $t = 0$, $x = 1$

$t = 2$, $x = 5$ ✓

$t = 4$, $x = 1$

$\therefore$ Distance travelled $= 4 + 4$

$= 8$ metres ✓

COMMENTS

(b) Calc 6

Only a few students were able to manage this question.

QUESTION 9 Reas 8 CB

(a) If $a = 4$ and $S_\infty = \frac{2}{3}$

then $\frac{2}{3} = \frac{4}{1-r}$

$2 - 2r = 12$

$\therefore r = -5$ ✓

But $|r| < 1$ for S_∞ to exist.

$\therefore$ No series exists. ✓

(b) (i) The parabola must pass thro' $(2,2)$

$\therefore 2 = a \times 2(4-2)$

$2 = 4a$ ✓

$\therefore a = \frac{1}{2}$

(ii) A is the amplitude

$\therefore A = 2$ ✓

(iii) Parabola: $A = \int_0^4 \frac{1}{2}x(4-x)\,dx$

$= \int_0^4 2x - \frac{1}{2}x^2\,dx$

$= \left[x^2 - \frac{1}{6}x^3 \right]_0^4$ ✓

$= 16 - \frac{32}{3}$

$= \frac{16}{3}$ unit^2 ✓

Sine Curve $A = \int_0^4 2\sin\frac{\pi x}{4}\,dx$

$= \left[-\frac{8}{\pi}\cos\frac{\pi x}{4} \right]_0^4$ ✓

$= -\frac{8}{\pi}\cos\pi + \frac{8}{\pi}\cos 0$

$= \frac{16}{\pi}$ unit^2 ✓

$\therefore$ Sine curve is smaller & hence cheaper to build

COMMENTS

Students did not generally know the condition for a limiting sum with several incorrectly stating that $|r| \leq 1$.

Other students did not interpret/read the question and substituted $r = \frac{2}{3}$ instead of $S_\infty = \frac{2}{3}$

Several students failed to recognize that the problem could be solved by mere substitution of a point into the equation

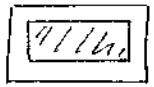
(iii) Reas 4

Product rule does not apply to integration!

Few students were able to handle this integration

QUESTION 9 (cont.)

(c) (i)



Lawn = 1800 ✓

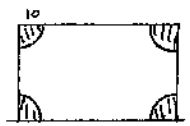
Area = 40×10

$= 400$

$\therefore P = \frac{400}{1800}$

$= \frac{2}{9}$ ✓

(ii)



Lawn = 1800

Area = $\pi . 10^2$

$= 100\pi$ ✓

$\therefore P = \frac{100\pi}{1800}$

$= \frac{\pi}{18}$ ✓

COMMENTS

(c) Reas 4

Well done.

QUESTION 10

Calc 3
Reas 4 HG

(a) $y = xe^{-2x}$

$\frac{dy}{dx} = e^{-2x} \cdot 1 + x.(-2e^{-2x})$

$= e^{-2x} - 2xe^{-2x}$ ✓

$\frac{d^2y}{dx^2} = -2e^{-2x} - 2\left(e^{-2x} - 2xe^{-2x}\right)$

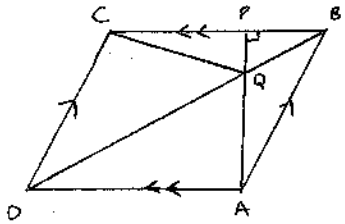
$= -4e^{-2x} + 4xe^{-2x}$ ✓

$\therefore \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y$ ✓

$= -4e^{-2x} + 4xe^{-2x} + 4e^{-2x} - 8xe^{-2x} + 4xe^{-2x}$ ✓ ✓

$= 0$

(b)



(i) $\angle ADB = \angle CDB$ (diagonals of a rhombus bisect the angles they pass through). ✓

(ii) $\angle ADB = \angle CDB$ (as above)

DQ is common ✓

CD = AD (given)

$\therefore \Delta AQD \equiv \Delta CQD$ (SAS) ✓

(iii) $\angle QAD = 90°$ (alt $\angle$ = as CB // AD) ✓

$\therefore \angle QCD = 90°$ (corr. $\angle$ in $\equiv \Delta$ are =)

COMMENTS

(a) Calc 3

- Many students forgot to do the product rule!

(b) Reas 4

- Reasons - poor.

QUESTION 10 (cont.)	COMMENTS
(c) (i) $A_1 = 500000(1.01) - M$	Real 's
$A_2 = A_1 \times 1.01 - M$	• Many attempt at fudging - not very successful!
$= 500,000(1.01)^2 - M(1.01) - M$	
$A_3 = A_2 \times 1.01 - M$	
$= 500000(1.01)^3 - M(1.01)^2 - M(1.01) - M$	
⋮ ✓	
$\therefore A_n = 500000(1.01)^n - M[1.01^{n-1} + 1.01^{n-2} + \ldots + 1]$	
↑ GP	
$a = 1$	
$r = 1.01$	
$n = n$	
$\therefore S_n = \frac{1((1.01)^n - 1)}{1.01 - 1}$ ✓	
$= \frac{1.01^n - 1}{0.01}$	
$\therefore A_n = 500000(1.01)^n - M \times \left(\frac{1.01^n - 1}{0.01}\right)$	
$= 500000(1.01)^n - 100M(1.01^n - 1)$ ✓	
$= 100M - 1.01^n(100M - 500000)$	
(ii) $250000 = 100 \times 5505 - 1.01^n(100 \times 5505 - 500000)$	• Many students still got 2 marks in (ii) despite being stuck in (i).
$250000 = 550500 - 1.01^n(50500)$	
$\therefore 1.01^n = \frac{550500 - 250000}{50500}$ ✓	
$= 5.95$	
$\therefore n = \frac{\ln 5.95}{\ln 1.01}$	
$= 179.23$ months ✓	
$\therefore$ In the 15th year.	