Neap:

HSC Trial Examination 2009

Mathematics

Solutions and marking guidelines

Abbreviations:

CFPA = correct from previous answer

IU = ignore units

IR = ignore rounding

CNE = correct numerical expression, i.e. correct substitution into a correct formula

ISE = ignore subsequent errors

Question 1

	Sample answer	Syllabus outcomes and marking guide
(a)	$(2\sqrt{3}-1)^2-(2\sqrt{3}-1)=12-4\sqrt{3}+1-2\sqrt{3}+1$ $=14-6\sqrt{3}$	P3, H9 • Gives correct answer . . . . . 2 • Makes one minor error. 1
(b)	$\frac{(5a-1)(5a+1)}{2(1-5a)}=\frac{-(5a+1)}{2}$ $=\frac{-5a-1}{2}$	P4 • Gives correct answer . . . . . 2 • Factorises one expression correctly 1
(c)	$\log_{10}\left(\frac{20A}{2A}\right)=\log_{10}10$ $=1$	H3, H9 • Gives correct answer . . . . . 2 • Uses one log law correctly. 1
(d)	$(AB)^2=4^2+5.2^2-2\times4\times5.2\times\cos(1.1)$ $=24.1704$ (to four decimal places) $AB=4.9$ (to one decimal place)	P4 • Gives correct answer . . . . . 3 • Obtains $(AB)^2=24.17$. OR • Rounds 4.9 incorrectly. . . . . 2 • Substitutes into the cosine rule. OR • Demonstrates correct rounding to one decimal place 1
(e)	Option 1: Interest $=\$20\,000(1.005)^{36}-\$20\,000$ $=\$3933.61$ Option 2: Interest $=\$20\,000(1.062)^3-\$20\,000$ $=\$3955.41$ Option 2 gives more interest.	H4 • Gives correct answer . . . . . 3 • Calculates the interest for both options correctly . . . . . 2 • Calculates the final value of the investment correctly for either option 1

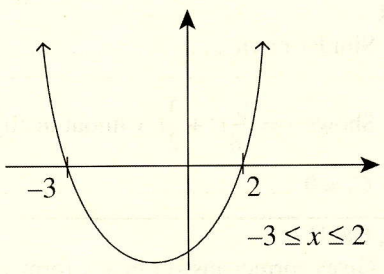
Question 2

Sample answer	Syllabus outcomes and marking guide
(a) (i) $\int 6e^{2x}dx = 3e^{2x} + C$	H5, H9 • Gives correct answer (ignore + C) 1
(ii) $\int \sec^2 \pi x dx = \frac{1}{\pi}\tan \pi x + C$	H5, H9 • Gives correct answer (ignore + C) 1
(iii) $\int (x^2+1)^2 dx = \int (x^4 + 2x^2 + 1)dx$ $= \frac{1}{5}x^5 + \frac{2}{3}x^3 + x + C$ $= \frac{x^5}{5} + \frac{2x^3}{3} + x + C$	H5, H9 • Gives correct answer (ignore + C) . . . . . 2 • Multiplies out brackets before integrating 1
(b) $y = \log_e x$ $\quad x = e^3,\ y = 3$ $y' = \frac{1}{x}$ $\quad m_T = \frac{1}{e^3}$ $y - 3 = \frac{1}{e^3}(x - e^3)$ $y = \frac{1}{e^3}x - 1 + 3$ $y = \frac{1}{e^3}x + 2$	P5, H3, H9 • Gives correct answer in any form . . . . . 3 • Correctly determines gradient and $y = 3$ . . . . . 2 • Correctly differentiations or determines $y = 3$ 1
(c) (i) $6y = x^2 - 10x + 13$ $6y + 12 = x^2 - 10x + 25$ $6y + 12 = (x-5)^2$ $6(y+2) = (x-5)^2$	P3 • Correct demonstration by any method. . . 1
(ii) $4a = 6$ $a = 1\frac{1}{2}$	H5 • Gives correct answer 1
(iii) $6(y+2) = (x-5)^2$ vertex $= (5, -2)$ focus $= \left(5, -\frac{1}{2}\right)$	H5 • Both points correct (or CFPA) . . . . . 2 • Either point correct (CFPA) 1
(iv) Range: $y \geq -2$	P5 • Gives correct answer (CFPA) 1

Question 3

Sample answer	Syllabus outcomes and marking guide
(a) (i) $y' = 8 - 2x$	P7, P8, H9 • Gives correct answer 1
(ii) $y' = -2\sin 2x$	P7, P8, H9 • Gives correct answer 1
(iii) $y = 2x^{-1}$ $y' = -2x^{-2}$ $= -\dfrac{2}{x^2}$	P7, P8, H9 • Gives correct answer 1
(b) (i) length $= \sqrt{4^2 + k^2} + \sqrt{3^2 + k^2}$ $= \sqrt{k^2 + 16} + \sqrt{k^2 + 9}$	P4 • Gives correct answer (do not ignore subsequent error) . . . . . 2 • Gives correct expression for either length 1
(ii) gradient $= \dfrac{3}{k}$ y-intercept $= 0$ $y = \dfrac{3}{k}x$ $ky = 3x$	P4, H9 • Gives correct demonstration by any method 1
(iii) gradient $= \dfrac{-4}{k}$ y-intercept $= 4$ $\therefore y = \dfrac{-4}{k}x + 4$	P4 • Gives correct answer in any form . . . . . 2 • Gives correct gradient. OR • Gives an answer of equivalent merit 1
(iv) $\dfrac{3}{k}x = \dfrac{-4x}{k} + 4$ $3x + 4x = 4k$ $x = \dfrac{4k}{7}$ $y = \dfrac{3}{k} \times \dfrac{4k}{7}$ $= \dfrac{12}{7}$	P4 • Gives correct demonstration . . . . . 2 • Shows either x or y value correctly. OR • Uses simultaneous equations 1
(v) (α) The height will not change. It will always be $\dfrac{12}{7}$.	P4, H5 • Gives correct answer with justification . . 1
(β) The distance is $\dfrac{3k}{7}$. As k gets bigger $\dfrac{3k}{7}$ gets bigger. P will be farther from the pole.	P4, H5 • Gives correct answer with justification . . 1

Question 4

Sample answer	Syllabus outcomes and marking guide
(a) (i) $f'(4)$ = the gradient at $x = 4$ $= 2$	P5, H6 • Gives correct answer 1
(ii) $\int_0^4 \lvert 5 - 2x \rvert \, dx$ = area under curve $= \frac{1}{2} \times 5 \times 2\frac{1}{2} + \frac{1}{2} \times 1\frac{1}{2} \times 3$ $= 8\frac{1}{2}$	H6, H9 • Gives correct answer . . . . . 2 • Attempts to use the area of the triangles 1
(b) (i) Solving simultaneously: $x^2 = 6x - k$ $x^2 - 6x + k = 0$ Need $\Delta = 0$ $36 - 4 \times 1 \times k = 0$ $k = 9$	P4, H9 • Gives correct answer by any method . . . . 2 • Uses $\Delta = 0$. OR • Solves simultaneously. OR • Equivalent merit. 1
(ii) $\Delta > 0$ $36 - 4k > 0$ $k < 9$ $\therefore$ The greatest integer for k is $k = 8$.	P4, H9 • Gives correct answer by any method . . . . 2 • Obtains $k < 9$ 1
(c) $x^2 + x - 6 \leq 0$ $(x + 3)(x - 2) \leq 0$  $-3 \leq x \leq 2$	H6 • Gives correct answer . . . . . 2 • Determines –3 and 2. OR • Equivalent merit. 1
(d) $\angle BAE = 72°$ (opposite angles in a parallelogram are equal) $\angle BEA = 72°$ (base angles of an isosceles Δ are equal, $AB = BE$) $\angle CBE = 72°$ (alternate angles in parallel lines are equal and $BC \parallel AD$)	H2, H9 • Gives correct answer with reasons. . . . . 3 • Uses two correct, relevant geometrical facts with reasons . . . . . 2 • Uses one correct, relevant geometrical fact with reason. OR • Obtains correct answer without reasons . 1

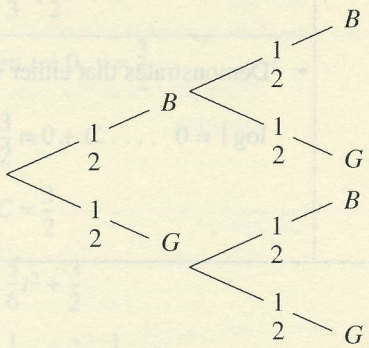
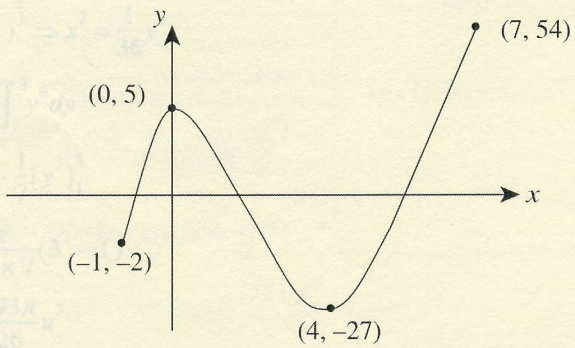
Question 5

Sample answer	Syllabus outcomes and marking guide
(a) (i) $a = \frac{5}{3}t$ $v = \frac{5}{3} \times \frac{1}{2}t^2 + C$ when $t = 0$, $v = \frac{3}{2}$ $\frac{3}{2} = 0 + C$ $\therefore C = \frac{3}{2}$ $v = \frac{5}{6}t^2 + \frac{3}{2}$ $= \frac{1}{6} \times 5t^2 + \frac{1}{6} \times 9$ $= \frac{1}{6}(5t^2 + 9)$	H4 • Correct demonstration . . . . . 2 • Shows $v = \frac{5}{6}t^2 + C$ 1
(ii) $v = \frac{5}{6}t^2 + \frac{3}{2}$ $x = \frac{5}{6} \cdot \frac{1}{3}t^3 + \frac{3}{2}t + C_1$ $C_1 = 0$ as starts at 0 $x = \frac{5}{18}t^3 + \frac{3}{2}t$ when $t = 2$ $x = \frac{5}{18} \times 8 + \frac{3}{2} \times 2$ $= 5\frac{2}{9}m$	H5 • Gives correct answer . . . . . 3 • Shows $x = \frac{5}{18}t^3 + \frac{3}{2}t + C$ and shows $C = 0$. OR • Determines x when $t = 2$ in an incorrect non-trivial equation for x. OR • Similar merit . . . . . 2 • Shows $x = \frac{5}{18}t^3 + \frac{3}{2}t$ without justifying $C_1 = 0$ 1
(b) $V = \pi \int x^2 dy$ $y^6 = (6x)^2$ $y = (6x)^{\frac{1}{3}} \Rightarrow x^2 = \frac{1}{36}y^6$ $V = \frac{\pi}{36}\int_1^3 y^6 dy$ $= \frac{\pi}{36} \cdot \frac{1}{7}[y^7]_1^3$ $= \frac{\pi}{36 \times 7}(3^7 - 1)$ $= \frac{1093\pi}{126}u^3$	P8, H8, H9 • Gives correct answer in any form . . . . . 3 • Shows $V = \frac{\pi}{36}\int_1^3 y^6 dy$. OR • Gives correct answer but makes a minor error . . . . . 2 • Shows $x^2 = \frac{1}{36}y^6$. OR • Similar merit 1
(c) (i) $\frac{d}{dx}(x\log_e x) = 1 \times \log_e x + x + \frac{1}{x}$ $= \log_e x + 1$	P7, H9 • Correct demonstration 1

Question 5 **(Continued)**

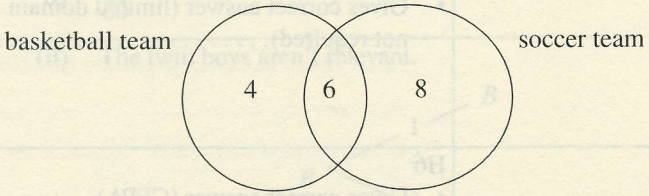
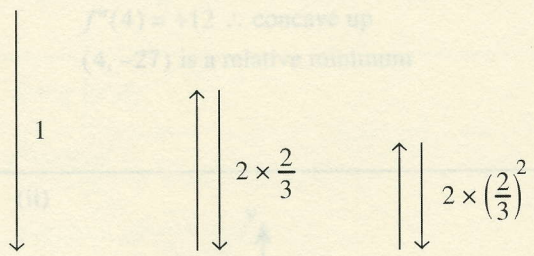
Sample answer	Syllabus outcomes and marking guide
(ii) $\log_e x = \frac{d}{dx}(x\log_e x) - 1$	H5
$\int \log_e x\,dx = x\log_e x - \int 1\,dx$	• Gives correct answer 3
$= x\log_e x - x + c$	• Determines a correct primitive function . 2
$\int_1^e \log_e x\,dx = e\log_e e - e - [\log_e 1 - 1]$	• Demonstrates that either $\log_e e = 1$ or $\log 1 = 0$ 1
$= e \times 1 - e - 0 + 1$	
$= 1$	

Question 6

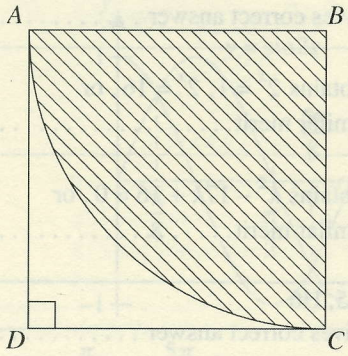
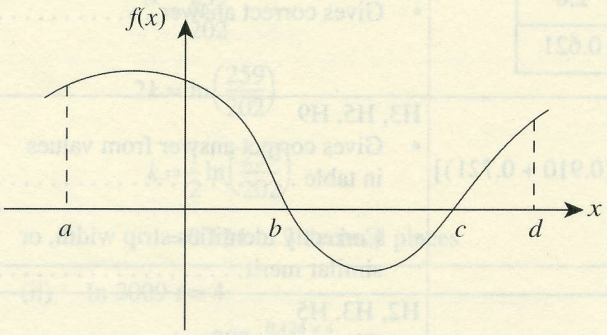
Sample answer	Syllabus outcomes and marking guide
(a) (i) $\frac{59}{60}$	H5 • Gives correct answer 1
(ii) The twin boys aren't relevant.  1 – probability that both (unknown sex) babies are boys $= 1 - \left(\frac{1}{2}\right)^2$ $= \frac{3}{4}$	H5, H9 • Gives correct answer . . . . . 2 • Any indication that the twin boys are not involved in the calculation (the indication could be implied). OR • Uses a tree diagram 1
(b) (i) $y' = 3x^2 - 12x$ For stationary points, $y' = 0$. $3x(x-4) = 0$ $x = 0$ or 4 (0, 5) and (4, –27) $y'' = 6x - 12$ $f''(0) = -12$ $\therefore$ concave down (0, 5) is a relative maximum $f''(4) = +12$ $\therefore$ concave up (4, –27) is a relative minimum	H6 • Gives correct answer . . . . . 4 • Correct points without showing justifications for the nature. OR • Gives correct answer but makes a minor error . . . . . 3 • Obtains $x = 0$ and $x = 4$. OR • Similar merit . . . . . 2 • Obtains $3x^2 - 12x = 0$. OR • Similar merit 1
(ii)  	H6 • Gives correct answer (must indicate turning points and end points (CFPA)). . . . . 2 • Draws 'correct' graph (CFPA) without end points. 1

Question 6 (Continued) Sample answer		Syllabus outcomes and marking guide
(iii)	$y'' = 6x - 12$ $6x - 12 = 0$ $x = 2$ The curve is concave down for $x < 2$.	H6 • Gives correct answer (limited domain not required). 1
(iv)	maximum = 54 minimum = –27	H6 • Gives correct answer (CFPA). . . . . 2 • Includes an endpoint of the domain. 1

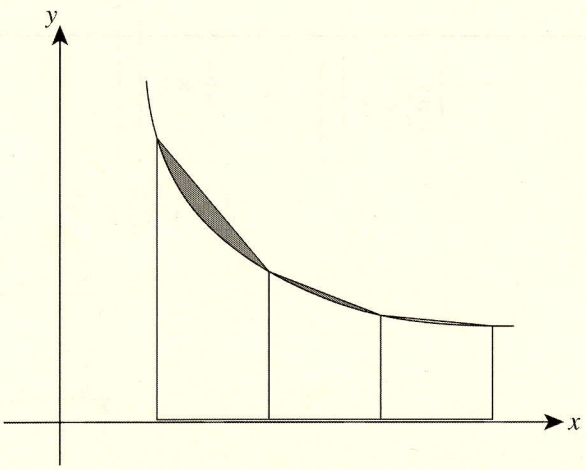
Question 7

Sample answer	Syllabus outcomes and marking guide
(a) (i)  	H5 • Gives correct answer 1
(ii) $\frac{4}{10}$	H5 • Gives correct answer (CFPA, non trivial) 1
(b) (i) $\sqrt{7}+\sqrt{28}+\sqrt{63}\ldots=\sqrt{7}+2\sqrt{7}+3\sqrt{7}\ldots$ This is an arithmetic series as the common difference is $\sqrt{7}$.	P3, H5, H9 • Correct demonstration 1
(ii) $a=\sqrt{7}$ $\quad d=\sqrt{7}$ $\quad S_n=300\sqrt{7}$ $S_n=\frac{n}{2}\{2a+(n-1)d\}$ $300\sqrt{7}=\frac{n}{2}\{2\sqrt{7}+(n-1)\sqrt{7}\}$ $300\sqrt{7}=\frac{n}{2}(\sqrt{7}+n\sqrt{7})$ $300=\frac{n}{2}(1+n)$ $600=n+n^2$ $n^2+n-600=0$ $(n+25)(n-24)=0$ $\therefore n=24$ as $n>0$	H5, H9 • Gives correct answer . . . . . 2 • Correctly substitutes $a=\sqrt{7}$, $d=\sqrt{7}$ and $S_n=300\sqrt{7}$ into the formula for the sum of an arithmetic sequence. 1
(c)  $1+2\times\left[\frac{2}{3}+\left(\frac{2}{3}\right)^2+\ldots\right]$ $=1+2\times\frac{a}{1-r}$ $=1+2\times\frac{\frac{2}{3}}{1-\frac{2}{3}}$ $=1+2\times\frac{2}{3}\times\frac{3}{1}$ $=5$ m	H4, H5, H9 • Gives correct answer . . . . . 3 • Makes substantial progress, e.g. uses twice the limiting sum of a geometric sequence . . . . . 2 • Identifies a correct geometric sequence. . 1

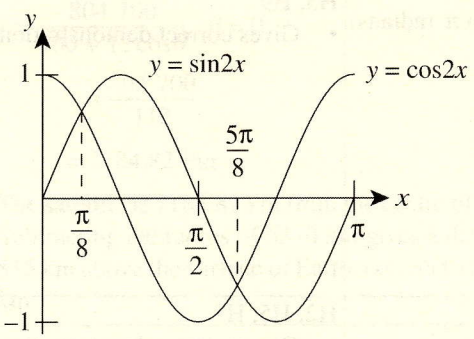
Question 7 (Continued)

Sample answer	Syllabus outcomes and marking guide
(d)  Shaded area $= \frac{1}{4} \times \pi \times 1^2$ $= \frac{\pi}{4}$ Area of square $ABCD = 1$ $\therefore$ Area $ABCQ = 1 - \frac{\pi}{4}$ $= \frac{4-\pi}{4} u^2$	H4, H5 • Gives correct demonstration . . . . . 2 • Shows shaded area is $\frac{\pi}{4} u^2$ 1
(e) There are many correct solutions. e.g.  	H4, H8, H9 • Draws any correct graph with x-axis labels . . . . . 2 • Draws a correct graph with minor error or omission. 1

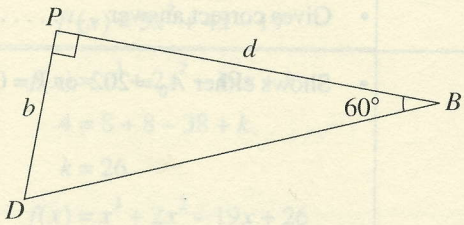
Question 8

Sample answer	Syllabus outcomes and marking guide
(a) Let $k = 2^x$ $k + \frac{16}{k} = 17$ $k^2 - 17k + 16 = 0$ $(k-1)(k-16) = 0$ $\therefore 2^x = 1$ or $2^x = 16$ $\therefore x = 0$ or $x = 4$	P4, H3, H9 • Gives correct answer . . . . . 3 • Obtains $2^x = 1$, $2^x = 16$, or similar merit . . . . . 2 • Obtains $k^2 - 17k + 16 = 0$, or similar merit 1
(b) (i) $f''(x) = 6x + 4$ $f'(x) = 3x^2 + 4x + C$ When $x = 2$, $f'(x) = 1$ $1 = 12 + 8 + C$ $C = -19$ $f'(x) = 3x^2 + 4x - 19$	P8, H5, H6 • Gives correct answer . . . . . 3 • Gives correct answer but makes a minor error . . . . . 2 • Obtains $f'(x) = 3x^2 + 4x$ 1
(ii) $f(x) = x^3 + 2x^2 - 19x + k$ $4 = 8 + 8 - 38 + k$ $k = 26$ $f(x) = x^3 + 2x^2 - 19x + 26$	H5, H6 • Answer correct from answer to (b)(i) . . . 2 • Obtains $f(x) = x^3 + 2x^2 - 19x$ omitting k, or evaluates k incorrectly 1
(c) (i) x: 0.5, 1.0, 1.5, 2.0 y: 1.443, 0.910, 0.721, 0.621	H3, H9 • Gives correct answer 1
(ii) $\int_{0.5}^{2} \frac{1}{\log_e(2x+1)}\,dx \approx \frac{0.5}{2}[1.443 + 0.621 + 2 \times (0.910 + 0.721)]$ $= 1.33$ to 2 decimal places	H3, H5, H9 • Gives correct answer from values in table . . . . . 2 • Correctly identifies strip width, or similar merit 1
(iii)  The shaded area above the curve is not included in the integral, however it is included in the trapezoidal approximation. $\therefore$ Trapezoidal approximation > exact value of integral	H2, H3, H5 • Gives correct answer 1

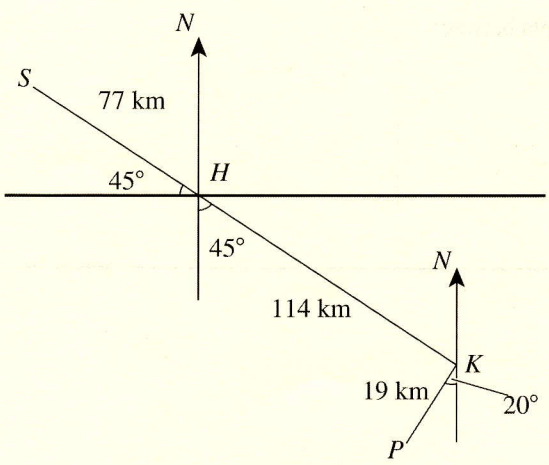
Question 9

Sample answer	Syllabus outcomes and marking guide
(a) (i) 	H5, H9 • Both graphs correct over correct domain, correct x values . . . . . 2 • One graph correct and one intersection correct. OR • Both graphs correct over incorrect domain 1
(ii) $\frac{\pi}{8} < x < \frac{5\pi}{8}$ i.e the portion of the graph where y-values of the sine curve are greater than the y-values of the cos curve.	P4, H5 • Gives correct answer (ignore incorrect inclusion of $\leq$, accept approximate points of intersection correct from graph) 1
(b) (i) $A = A_0 e^{kt}$ when $t = 0,\ A = 202$ $202 = A_0 e^0$ $A_0 = 202$ when $t = 2,\ A = 259$ $259 = 202e^{2k}$ $e^{2k} = \frac{259}{202}$ $2k = \ln\left(\frac{259}{202}\right)$ $k = \frac{1}{2}\ln\left(\frac{259}{202}\right)$ $= 0.124$ to 3 decimal places	H3, H4, H9 • Gives correct answer . . . . . 2 • Shows either $A_0 = 202$ or $k = 0.124$ 1
(ii) In 2009 $t = 4$ $A = 202e^{0.124 \times 4}$ $= 332$ In 2009 there will be 332 donors.	H3, H4, H9 • Gives correct answer (ignore rounding) 1
(iii) $\int_0^R 202e^{0.124t}\,dt = 1388$ $\left[\frac{202}{0.124}e^{0.124t}\right]_0^R = 1388$ $e^{0.124R} - 1 = 1388 \times \frac{0.124}{202}$ $e^{0.124R} = 1.852$ $0.124R = \ln 1.852$ $R = 4.97$ $\therefore$ beginning 2006 2009 2010 2011 $R = 0$ $R = 3$ $R = 4$ $R = 5$ The formula predicts that the waiting list will be empty just before the end of 2010.	H3, H4, H5, H9 • Gives correct answer (ignore rounding, accept end of 2010 or beginning of 2011) . . . . . 2 • Makes some progress, e.g. recognises that a summation or integral is required 1

Question 9 **(Continued)**

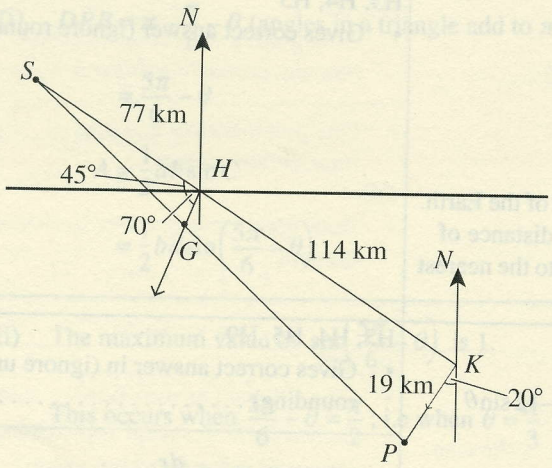
Sample answer	Syllabus outcomes and marking guide
(c) (i) $DPB = \pi - \frac{\pi}{6} - \theta$ (angles in a triangle add to π radians) $= \frac{5\pi}{6} - \theta$ $A = \frac{1}{2}ab\sin C$ $= \frac{1}{2}bd\sin\left(\frac{5\pi}{6} - \theta\right)$	H5, H9 • Gives correct demonstration 1
(ii) The maximum value of $\sin\left(\frac{5\pi}{6} - \theta\right)$ is 1. This occurs when $\frac{5\pi}{6} - \theta = \frac{\pi}{2}$; i.e when $\theta = \frac{\pi}{3}$ The maximum area is when $\angle DPB = \frac{\pi}{2}$  $\therefore \frac{b}{d} = \tan 60°$ $\therefore \frac{b}{d} = \sqrt{3}$ $\therefore b = \sqrt{3}d$	H2, H5, H9 • Gives correct demonstration . . . . . 3 • Determines that the maximum area is when $\angle DPB = \frac{\pi}{2}$ (trig graph or derivative approach) . . . . . 2 • Makes some progress, e.g. differentiates $A = \frac{1}{2}bd\sin\left(\frac{5\pi}{6} - \theta\right)$ or considers a sine graph 1

Question 10

Sample answer	Syllabus outcomes and marking guide
(a) (i) $r = \dfrac{804\,700}{100 + 12\cos\theta}$, $\theta = 0$ $= \dfrac{804\,700}{112}$ $= 7184.82$ km The satellite is 7184.82 km from the centre of the Earth. Subtracting the radius of 6370 km gives a distance of 815 km above the surface of Earth, correct to the nearest km.	H3, H4, H5 • Gives correct answer (ignore rounding) . 1
(ii) $r = 804\,700(100 + 12\cos\theta)^{-1}$ $\dfrac{dr}{d\theta} = 804\,700 \times (-1)(100 + 12\cos\theta)^{-2} \times -12\sin\theta$ $= \dfrac{9\,656\,400 \times \sin\theta}{(100 + 12\sin\theta)^2}$ When $\theta = \dfrac{3\pi}{4}$ $\dfrac{dr}{d\theta} = \dfrac{9\,656\,400 \times \dfrac{1}{\sqrt{2}}}{\left(100 - 12 \times \dfrac{1}{\sqrt{2}}\right)^2}$ $= \dfrac{6\,828\,105.9}{8374.94}$ $= 815$ km/radian	H3, H4, H5, H9 • Gives correct answer in (ignore units and rounding) . . . . . 3 • Correctly finds $\dfrac{dr}{d\theta}$. OR • Correctly evaluates non-trivial expression for $\dfrac{dr}{d\theta}$ when $\theta = \dfrac{3\pi}{4}$ . . . . . 2 • Correctly finds $\dfrac{dr}{d\theta}$ 1
(b) (i)  	H4, H5, H9 • Draws correct diagram showing 77 km, 114 km and 19 km in correct positions . . 2 • Draws correct diagram with one measurement missing or incorrect 1

Question 10 **(Continued)**

Sample answer	Syllabus outcomes and marking guide
(ii)	H4, H5, H9



$\Delta SHG \text{ III } \Delta SKP \ (HG \parallel KP)$

$\therefore \dfrac{SH}{SK} = \dfrac{HG}{KP}$

$\therefore HG = \dfrac{SH \times KP}{SK}$

$= \dfrac{77 \times 19}{77 + 114}$

$= \dfrac{77 \times 19}{191}$ km

$= 7.66$ km (to two decimal places)

ALTERNATIVE SOLUTION

Because $\dfrac{SG}{GP} = \dfrac{SH}{HK} = \dfrac{77}{144}$ (ratio of intercepts between parallel lines)

$\dfrac{SG}{GP} = \dfrac{77}{144}$

$SG = \dfrac{77}{144} GP$

$\therefore GP > SG$

- Correct demonstration 3
- Recognises and uses similar triangles . . . 2
- Makes some relevant progress towards the answer 1

Question 10 **(Continued)**

Sample answer	Syllabus outcomes and marking guide
(iii) Lengths of SG and GP are required. $\frac{SG}{SP} = \frac{SH}{SK}$ $SG = \frac{77}{191} \times SP$ i.e. $SP = SG \times \frac{191}{77}$ Now $SG^2 = 77^2 + (7.66)^2 - 2 \times 77 \times 7.66 \times \cos 115$ $SG = 80.54$ km (to two decimal places) Time to travel $SG = \frac{80.54}{11}$ = 7 hours, 19 minutes $SP = 80.54 \times \frac{191}{77}$ $PG = 199.78 - 80.54$ = 119.24 (to two decimal places) Time to travel $GP = \frac{119.4}{19}$ = 6 hours, 17 minutes Therefore the *Pacifica* (travelling from P) will arrive first.	H4, H5, H9 • Gives correct answer showing calculations . . . . . 3 • Gives correct lengths for SG and GP. OR • Calculates the time for S to move to G . . 2 • Makes some progress, e.g. determines the length of SG 1