Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2003
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Afternoon Session
Tuesday 12 August 2003

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- All necessary working should be shown in every question

Total marks **(84)**

- Attempt Questions 1 – 7
- All questions are of equal value

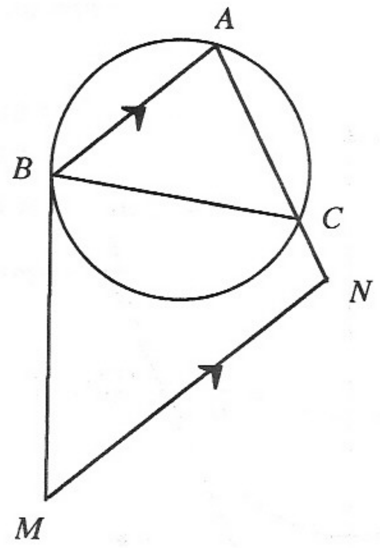
Question 1 **Begin a new page** **Marks**

(a) Find the value of $\lim_{n\to\infty} \dfrac{5(10^n)+3}{2(10^n)+1}$. **2**

(b) $A(-2,5)$ and $B(7,-1)$ are two points. Find the coordinates of the point $M(x,y)$ which divides the line AB internally in the ratio $2:1$. **2**

(c) Solve the inequality $\dfrac{2}{x} > x-1$. **4**

(d)



ABC is a triangle inscribed in a circle. M is a point on the tangent to the circle at B and N is a point on AC produced so that MN is parallel to BA.

(i) Copy the diagram.

(ii) Give a reason why $\angle MBC = \angle BAC$. **1**

(iii) Show that $MNCB$ is a cyclic quadrilateral. **3**

Question 2 **Begin a new page** **Marks**

(a) If $y=\left(x^2+1\right)^5$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **2**

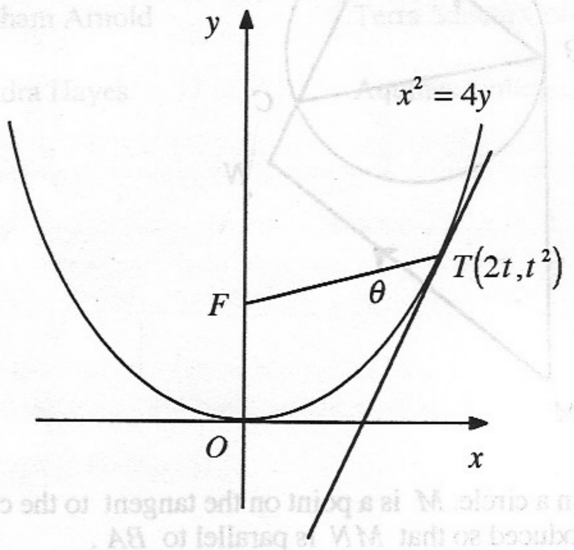
(b) Find the value of $\sum_{n=2}^{5} {}^{n}C_2$. **2**

(c)

(i) Show that $(\sin A-\cos A)^2 = 1-\sin 2A$. **2**

(ii) Hence find the exact value of $\sin 15° - \cos 15°$. **2**

(d)



$T(2t,t^2)$ is a point on the parabola $x^2=4y$ with focus F. The tangent to the parabola at T makes an acute angle θ with the line FT.

(i) Show that the tangent to the parabola at T has gradient t. **1**

(ii) Find $\tan\theta$ in simplest form in terms of t. **3**

Marks

Question 3 **Begin a new page.**

(a) Consider the function $f(x) = x - e^{-x}$.

(i) Show that the function is increasing and its graph is concave down for all values of x in its domain. **2**

(ii) Use one application of Newton's Method with an initial approximation of $x = 0 \cdot 5$ to find the value of the x intercept on the graph of $y = f(x)$, giving the answer correct to one decimal place. **2**

(iii) Sketch the graph of $y = f(x)$ showing clearly the intercepts on the axes and the equations of any asymptotes. **3**

On the same diagram sketch the graph of the inverse function $y = f^{-1}(x)$.

(b) A particle is moving in a straight line. At time t seconds it has displacement x metres (where $0 \leq x < \frac{\pi}{2}$) from a fixed point O on the line, velocity v ms $^{-1}$ given by $v = \cos^2 x$ and acceleration a ms $^{-2}$. The particle starts at O.

(i) Find expressions for a in terms of x and for x in terms of t. **2**

(ii) Sketch the graph of x against t. **1**

(iii) Describe the motion of the particle from its initial position to its limiting position. **2**

Marks

Question 4 **Begin a new page.**

(a) Consider the function $f(x) = \cos^{-1}\sqrt{x}$.

(i) Find the domain and the range of the function and sketch the graph of $y = f(x)$. **3**

(ii) Use Simpson's Rule with three function values to find an approximation to the area bounded by the curve $y = f(x)$ and the coordinate axes. **2**

(iii) Use integration to find the exact area bounded by the curve $y = f(x)$ and the coordinate axes. **2**

(b) A particle moving in a straight line is performing Simple Harmonic Motion. At time t seconds it has displacement x metres from a fixed point O on the line, velocity v ms^{-1} and acceleration $\ddot{x}$ ms^{-2} given by $\ddot{x} = -4(x-2)$. The particle is at rest at the fixed point O.

(i) Show that $v^2 = -4x^2 + 16x$. **2**

(ii) Find the period and amplitude of the motion. **2**

(iii) Find the distance travelled by the particle in the first minute of its motion, giving the answer correct to the nearest metre. **1**

Question 5 **Begin a new page.**

(a) In the expansion of $(1+ax)^9$ the coefficient of x^5 is twice the coefficient of x^6. Find the value of the constant a. **4**

(b) Evaluate $\displaystyle\int_1^{49} \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}}\,dx$ using the substitution $u = 1+\sqrt{x}$, expressing the answer in the form $\sqrt{n}$ for some positive integer n. **4**

(c) A container with capacity A litres is being filled with water. After t minutes the volume V litres of water in the container is given by $V = A\left(1-e^{-kt}\right)$ for some constant $k > 0$.

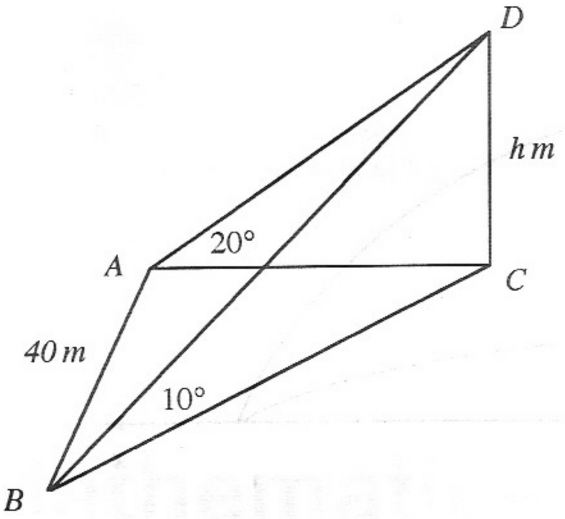
(i) Show that $\dfrac{dV}{dt} = k(A-V)$. **2**

(ii) If one quarter of the container is filled in the first two minutes find what fraction of the container is filled in the next two minutes. **2**

Marks

Question 6 **Begin a new page.**

(a)



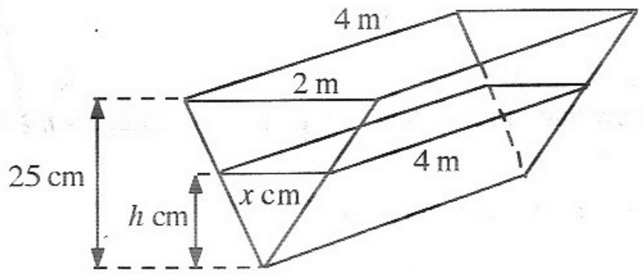
A vertical flagpole CD of height h metres stands with its base C on horizontal ground. A is a point on the ground due West of C and B is a point on the ground 40 metres due South of A. From A and B the angles of elevation of the top D of the flagpole are 20° and 10° respectively. Find the height of the flagpole correct to the nearest metre. **4**

(b) A die is biased so that in any single throw the probability of an odd score is p where p is a constant such that $0 < p < 1$, $p \neq 0\cdot 5$.

(i) Show that in six throws of the die the probability of at most one even score is $6p^5 - 5p^6$. **2**

(ii) Find the probability that in six throws of the die the product of the scores is even. **2**

(c)



An open flat topped water trough in the shape of a triangular prism is being emptied through a hole in its base at a constant rate of 16 litres per second. Its top measures 2 metres by 4 metres and its triangular end has a vertical height of 25 centimetres. When the water depth is h centimetres the water surface measures x centimetres by 4 metres.

(i) Show that when the water depth is h centimetres the volume $V\,\text{cm}^3$ of water in the trough is given by $V = 1600h^2$. **2**

(ii) Find the rate at which the depth of water is changing when $h = 10\,\text{cm}$. **2**

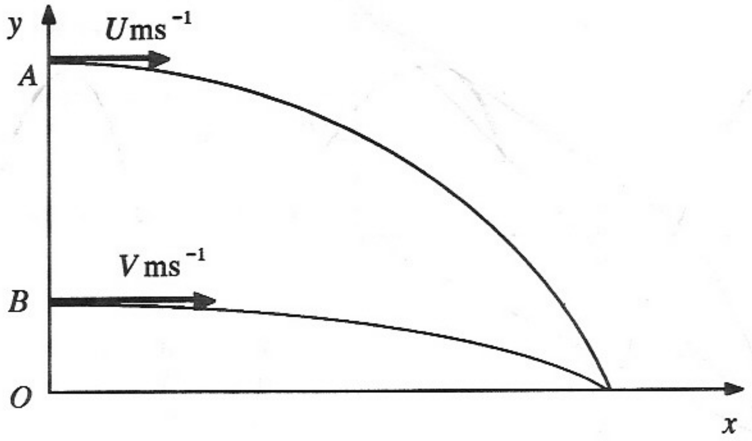
Marks

Question 7 **Begin a new page.**

(a)

$OB = h$ metres

$OA = 4h$ metres



A vertical building stands with its base O on horizontal ground. A and B are two points on the building vertically above each other such that A is $4h$ metres above O and B is h metres above O. A particle is projected horizontally with speed $U\text{ms}^{-1}$ from A and 10 seconds later a second particle is projected horizontally with speed $V\text{ms}^{-1}$ from B. The two particles hit the ground at the same point and at the same time.

(i) Write down expressions for the horizontal and the vertical displacements relative to O of each particle t seconds after the first particle is projected. **2**

(ii) Find the time of flight of each particle. **3**

(iii) Show that $V = 2U$. **1**

(b)

(i) Use Mathematical Induction to show that $\ln(n!) > n$ for all positive integers $n \geq 6$. **3**

(ii) Hence show that $\dfrac{1}{n!} < \dfrac{1}{e^n}$ for all positive integers $n \geq 6$. **1**

(iii) Hence show that $\dfrac{1}{1!}+\dfrac{1}{2!}+\dfrac{1}{3!}+\dfrac{1}{4!}+\dfrac{1}{5!}+\dfrac{1}{6!}+\dfrac{1}{7!}+\dfrac{1}{8!}+\ldots\ldots < \dfrac{103}{60} + \dfrac{1}{e^5(e-1)}$. **2**