Student Name: ________________________ **Teacher Name:** ________________

Saint Mark's Coptic Orthodox College

Mathematics Department
Preliminary Task One
February, 2007

Year 11 Mathematics Extension 1

Time Allowed: TWO PERIODS

Examiner Mr. W. Micheal

DIRECTIONS TO CANDIDATE:

- Attempt all questions.
- Show all necessary working. Marks may be deducted for careless or badly arranged work.
- Only approved calculators may be used.
- This paper contains **6** questions in 3 pages.

Question	1	2	3	4	5	6
Mark	/	/	/	/	/	/

Question One

1) Graph on the number line the solution set of: $\frac{x-1}{2} - \frac{2x-3}{3} < 1$.† 3marks

2) Using completing the squares, solve the quadratic equation $3x^2 - 4x - 4 = 0$. 3marks

3) Solve the simultaneous equations
$4x - y = 3$
$10x + 3y = 2$. 2marks

4) Mark on a number line the values of x for which $|x-2| < 3$. 2marks

5) Solve inequality $\frac{2}{x} > x - 1$. 2marks

Question Two

6) Find the values of x and y if: $x - y\sqrt{3} = (5 - 2\sqrt{3})(2 - \sqrt{3})$. 2marks

7) Using the Quadratic Formula, Solve the equation $2a^2 - 3a - 1 = 0$. 2marks

8) Solve the equation $\frac{3x-1}{5x+1} = \frac{3x-2}{5x+2}$. 3marks

9) Graph the solution of $4x \leq 15 \leq -9x$ on a number line. 3marks

10) Rationalize the denominator and simplify: $\frac{\sqrt{3-L}}{\sqrt{3+L}}$. 2marks

Question Three

11) For what values of x is $|x| + |x - 1| = 1$? 2marks

12) Find the value of x, If $\dfrac{3}{2 - \dfrac{x}{2}} = 2$ 2marks

13) If $a + b = 7$, $b + c = 9$, $a + c = 8$, find the value of abc. 2marks

14) $A(-1, 5)$ and $B(5, -4)$ are two points. Find the coordinates of the point P which divides the interval AB internally in the ratio $2 : 1$. 3marks

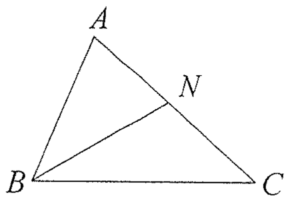
15) A is the point $(-2, 1)$ and B is the point (x, y). The point $P(13, -9)$ divides AB externally in the ratio $5 : 3$. Find the values of x and y. 3marks

Question Four

16) Solve the inequality $\dfrac{1}{|x - 3|} \geq \dfrac{1}{2}$. 3marks

17) Solve the inequality $\dfrac{2x + 3}{x - 4} > 1$. 3marks

18)



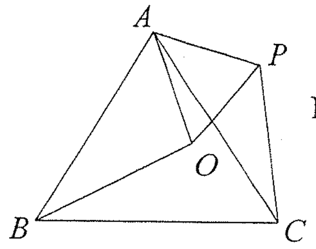
ABC is a triangle and N is a point on AC. $\angle ABN = \angle CBN = \angle BCN$. $BC = 2a$, $CA = b$, $AB = c$. $BN = CN = d$.

i. Given that $\Delta ABN \parallel\!| \Delta ACB$, show that $c^2 = b^2 - 2ac$. 3marks

ii. Hence show that $(a + c)^2 = a^2 + b^2$. 3marks

Question Five

19) In the figure triangles ACB and APO are equilateral.

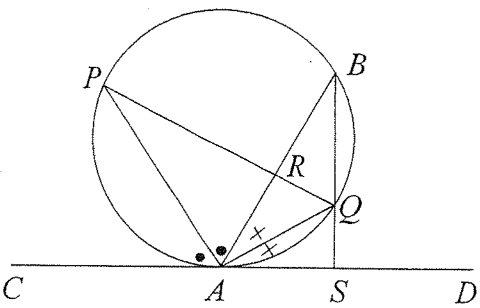


i. Copy this diagram onto your answer sheet and include all the given information. 1mark
ii. Explain why $\angle BAO = \angle PAC$. 1mark
iii. Prove $\Delta AOB \equiv \Delta APC$. 3marks
iv. Hence prove $OB = CP$. 2marks

20) Two points A and B are taken on a circle, and C is the other end of the diameter through A. AE is the line from A perpendicular to the tangent at B.
a. Draw a careful diagram showing this information. 2marks
b. Prove that AB bisects $\angle CAE$. 3marks

Question Six

21)



i. AB is a chord of a circle and CAD is a tangent to the circle at the point A. The bisector of angle BAC meets the circle again at P and the bisector of angle BAD meets the circle again at Q. Show that:
α. PQ is a diameter of the circle; 3marks
β. PQ is perpendicular to the chord AB. 3marks

[[End Of Qns]]

Question One

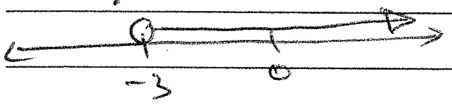
① $\frac{x-1}{\cancel{2}} \times \overset{3}{\cancel{6}} - \frac{2x-3}{\cancel{3}} \times \overset{2}{\cancel{6}} < 1 \times 6$

$3x - 3 - 4x + 6 < 6$

$-x + 3 < 6$

$-x < 3$

$x > -3$



② $3x^2 - 4x - 4 = 0$

$x^2 - \frac{4}{3}x = \frac{4}{3}$

$x^2 - \frac{4}{3}x + \left(\frac{2}{3}\right)^2 = \frac{4}{3} + \left(\frac{2}{3}\right)^2$

$\left(x - \frac{2}{3}\right)^2 = \frac{16}{9}$

$x - \frac{2}{3} = \pm \frac{4}{3}$

$x = \frac{2}{3} \pm \frac{4}{3}$

$x = 2$ or $x = -\frac{2}{3}$

③ $4x - y = 3 \longrightarrow$ ①

$10x + 3y = 2 \longrightarrow$ ②

①$\times 3 \longrightarrow$ $12x - \cancel{3y} = 9$

$10x \cancel{+3y} = 2$

$22x = 11$

$\boxed{x = \frac{1}{2}} \longrightarrow$ ③

Sub. ③ into ①

$4\left(\frac{1}{2}\right) - y = 3$

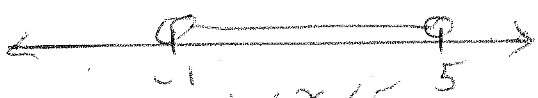
$2 - y = 3$

$\boxed{y = -1}$

④ $|x - 2| < 3$

$x - 2 < 3$ or $-x + 2 < 3$

$x < 5$ $\quad x > -1$



$-1 < x < 5$

5) $\frac{2}{\cancel{x}} \times x^2 > x - 1 \times x^2$

$2x > (x-1)x^2$

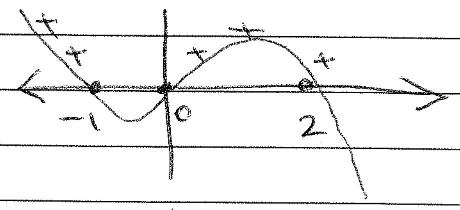
$2x - (x-1)x^2 > 0$

$x[2 - (x-1)x] > 0$

$x[2 - x^2 + x] > 0$

$x[2 + x - x^2] > 0$

$x(2 - x)(1 + x) > 0$



$x < -1, \ 0 < x < 2$

Question Two

⑥ $x - y\sqrt{3} = (5 - 2\sqrt{3})(2 - \sqrt{3})$

$= 10 - 5\sqrt{3} - 4\sqrt{3} + 6$

$x - y\sqrt{3} = 16 - 9\sqrt{3}$

$\therefore x = 16 \ ; \ y = 9$

⑦ $2a^2 - 3a - 1 = 0$

$a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{3 \pm \sqrt{9 - 4 \times 2 \times -1}}{2 \times 2}$

$a = \frac{3 \pm \sqrt{17}}{4}$

⑧ $\frac{3x - 1}{5x + 1} = \frac{3x - 2}{5x + 2}$

$(3x - 1)(5x + 2) = (3x - 2)(5x + 1)$

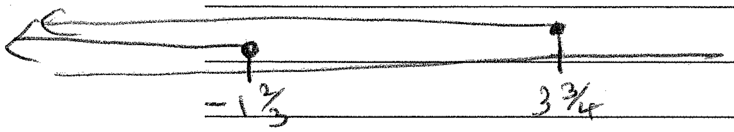
$\cancel{15x^2} + x - 2 = \cancel{15x^2} - 7x - 2$

$-8x = 0$

$\therefore \boxed{x = 0}$

(9) $4x \le 15 \le -9x$

$4x \le 15$	$15 \le -9x$
$x \le \frac{15}{4}$	$\frac{15}{-9} \ge x$
$x \le 3\frac{3}{4}$	$-1\frac{2}{3} \ge x$



$\therefore x \le -1\frac{2}{3}$

(10) $\frac{\sqrt{3-L}}{\sqrt{3+L}} \times \frac{\sqrt{3+L}}{\sqrt{3+L}}$

$= \frac{\sqrt{9-L^2}}{(3+L)}$

Question Three

(11) $|x| + |x-1| = 1$

$+(x) + (x-1) = 1$

$\boxed{x = 1}$

$-(x) + (x-1) = 1 \times$

$(x) + -(x-1) = 1 \times$

$-x - (x-1) = 1$

$-2x + 1 = 1$

$\boxed{x = 0}$

$\therefore 0 \le x \le 1$

(12) $\frac{3}{2 - \frac{x}{2}} - 2$

$3 = 4 - x$

$\boxed{x = 1}$

(13) $a + b = 7 \rightarrow (1)$

$b + c = 9 \rightarrow (2)$

$a + c = 8 \rightarrow (3)$

$(1) - (2)$ $a - c = -2$

$(3) \rightarrow$ $a + c = 8$

$2a = 6$

$\boxed{a = 3}$

$\therefore \boxed{b = 4}$ and $\boxed{c = 5}$ By inspection

$\therefore abc = 3 \times 4 \times 5 = 60$

(14) $A(\overset{x_1}{-1}, \overset{y_1}{5})$, $B(\overset{x_2}{5}, \overset{y_2}{-4})$, $\overset{m}{2} : \overset{n}{1}$

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

$= \frac{2 \times 5 + 1 \times -1}{2+1}$	$= \frac{2 \times -4 + 1 \times 5}{2+1}$
$= \frac{9}{3}$	$= \frac{-3}{3}$
$= 3$	$= -1$

$\therefore P(3, -1)$

(15) $A(\overset{x_1}{-2}, \overset{y_1}{1})$, $B(\overset{x_2}{x}, \overset{y_2}{y})$ $P(\overset{x}{13}, \overset{y}{-9})$

$5 : -3$

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

$13 = \frac{5 \times x + -3 \times -2}{5-3}$	$-3 = \frac{5 \times y + -3 \times 1}{5 + -3}$
$26 = 5x + 6$	$-6 = 5y - 3$
$20 = 5x$	$-3 = 5y$
$x = 4$	$y = -\frac{3}{5}$

$\therefore B(4, -\frac{3}{5})$

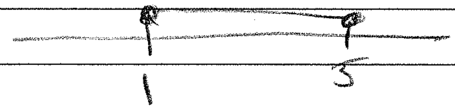
Question four

(16) $\frac{1}{|x-3|} \geq \frac{1}{2}$

$|x-3| \leq 2$

$x-3 \leq 2$ or $-x+3 \leq 2$

$x \leq 5$ $\quad x \geq 1$



$\therefore 1 \leq x \leq 5$

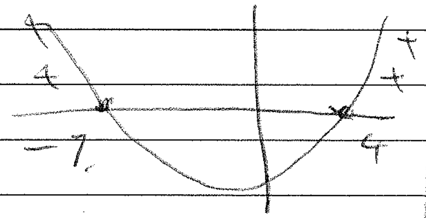
(17) $\frac{2x+3}{\cancel{(x-4)}} \times (x-4)^{\cancel{2}} > 1 \times (x-4)^2$

$(2x+3)(x-4) - (x-4)^2 > 0$

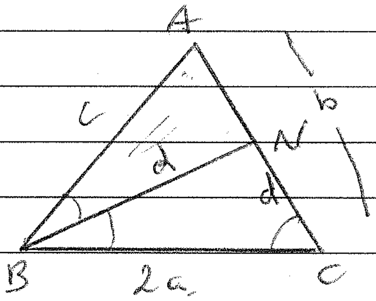
$(x-4)[(2x+3)-(x-4)] > 0$

$(x-4)(x+7) > 0$

$\therefore x < -7,\ x > 4$



(18)



(i) Since $\triangle ABN \parallel\!| \triangle ACB$

$\therefore$ corresp. sides in the same ratio

$\frac{AN}{AB} = \frac{BN}{BC} = \frac{AB}{AC}$

$\frac{b-d}{c} = \frac{d}{2a} = \frac{c}{b}$

$\therefore \frac{b-d}{c} = \frac{c}{b}$ | $\frac{d}{2a} = \frac{c}{b}$

$c^2 = b^2 - bd$ (1) | $2ac = bd$ (2)

from (1) and (2)

$\therefore c^2 = b^2 - 2ac$

(ii) Since $c^2 = b^2 - 2ac$ proven above.

$\therefore c^2 + 2ac = b^2$

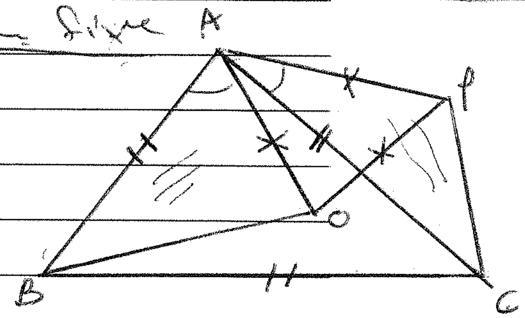
$+a^2 \qquad +a^2$

$\therefore a^2 + 2ac + c^2 = a^2 + b^2$

$\therefore (a+c)^2 = a^2 + b^2$

Question five

(i)



(ii) Since $B\hat{A}C = 60°$ (equilateral $\triangle$)

and $P\hat{A}O = 60°$

$P\hat{A}C = P\hat{A}O - C\hat{A}O$

$= 60 - C\hat{A}O \rightarrow$ (1)

Also $B\hat{A}O = B\hat{A}C - C\hat{A}O$

$= 60 - C\hat{A}O \rightarrow$ (2)

from (1) & (2) $P\hat{A}C = B\hat{A}O$

(iii) In $\triangle$'s AOB, APC

$AP = AO$ (equilateral $\triangle$)

$AC = AB$ (equilateral $\triangle$)

$P\hat{A}C = B\hat{A}O$ (proven above)

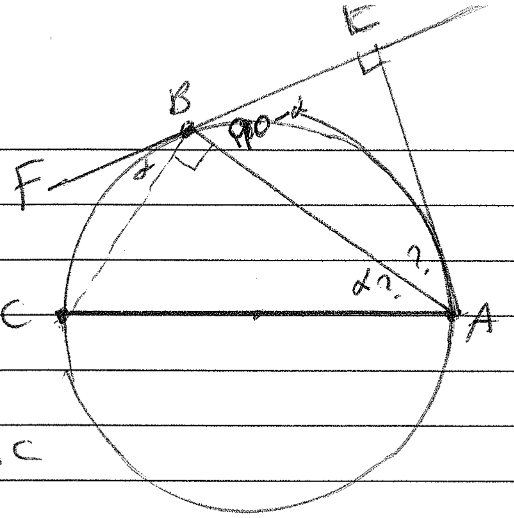
$\therefore \triangle AOB \equiv \triangle APC$ (SAS)

(iv) Since $\triangle AOB \equiv \triangle APC$ proven

$\therefore OB = PC$

Corresponding sides of congruent $\triangle$'s are equal.

(20)



Construct: BC

Let $C\hat{B}F = C\hat{A}B = \alpha$ (1) (angle between the tangent and chord equal to the angle in the alternate segment)

$A\hat{B}C = 90°$ (angle in a semi circle)

$\therefore E\hat{B}A = 180 - (90 + \alpha)$
$= 90 - \alpha$ (adj. suppl. angles)

Since $AE \perp FB$

$\therefore E\hat{A}B = 180 - (90 - \alpha + 90)$

$E\hat{A}B = \alpha°$ — (2)

From (1) and (2)

$\therefore E\hat{A}B = C\hat{A}B = \alpha$

$\therefore$ BA Bisects $C\hat{A}E$.

Question Six (c)

(21) (a)

Let $C\hat{A}P = P\hat{A}R = x°$ (Since PA is the bisector of $B\hat{A}C$

Also let $B\hat{A}Q = Q\hat{A}S = y°$ (Since QA is the bisector of $B\hat{A}D$).

$\therefore P\hat{A}R + Q\hat{A}B = x° + y°$

However $2x + 2y = 180°$ (adj. suppl. $\angle$'s)

$\therefore x + y = 90°$

$\therefore P\hat{A}Q = 90°$

$\therefore$ PQ is the diameter of the circle (angle in a semi circle is a right angle).

(B) $Q\hat{A}S = Q\hat{P}A = y$ and $C\hat{A}P = A\hat{Q}P = x$ Since (angle between tangent and the chord equal to the angle in the alternate segment)

In $\triangle ARQ$,

$A\hat{Q}P + B\hat{A}Q = x + y = 90°$ ~~from above~~ (proven above)

$\therefore Q\hat{R}A = 90°$ (angle sum of a $\triangle$)

(ii)