KINCOPPAL-ROSE BAY

Cor Unum

2003

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown on every question

Total marks – 84

- Attempt Questions 1-7
- All questions are of equal value
- Start each question in new writing booklet

Examiners KH,MN June 2003

Question 1 (12 marks)

(a) Evaluate $\lim_{x \to 0} \dfrac{\tan 4x}{x}$. **1**

(b) Find $\dfrac{d}{dx}\left(2x^3 e^{3x}\right)$. **2**

(c) Solve $\dfrac{1}{2-x} > 3$. **3**

(d) State the domain and range of the function **2**

$$f(x) = 2\cos^{-1}\left(\frac{x}{3}\right).$$

(e) Find the acute angle between the lines **2**

$$y = 3x - 5$$
$$2x + y - 7 = 0$$

to the nearest degree.

(f) Evaluate $\displaystyle\int \frac{\cos x}{1 + 2\sin x}\, dx$ **2**

Question 2 (12 marks)

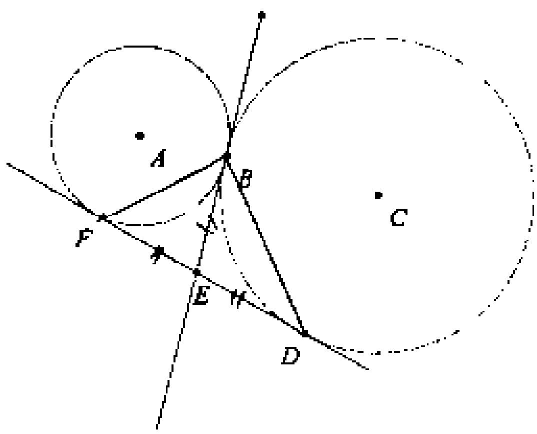
(a) Evaluate $\displaystyle\int_0^2 \frac{8x}{\sqrt{1+2x^2}}\,dx$, using the substitution $u = 1 + 2x^2$. **3**

(b) Find the general solution to $\sqrt{3}\tan x - 1 = 0$. **2**
Express your answer in terms of π.

(c) Prove that $(x-2)$ is a factor of $2x^4 - 4x^3 + 4x^2 - 15x + 14$ **1**

(d) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sin^2 2x \;dx$ **3**

(e)



(e) (i) Explain why $BE = \cancel{BF} = DE$ **1**
EF

(ii) Let $\angle BFE = \alpha$ and $\angle BDE = \beta$. **2**
Prove that $\angle FBD = 90°$

Question 3 (12 marks)

(a) Six people are seated in a straight line.

(i) How many seating arrangements are possible? **1**

(ii) How many arrangements are possible if Tarzan and Jane occupy the seats at either end? **2**

(b) (i) Show that $x^3 + 2x - 17 = 0$ has a root between $x=2$ and $x=3$ **1**

(ii) Using an approximation of $x = 2{\cdot}4$, use one application of Newton's method to find a better approximation for this root. Give your answer to two decimal places. **3**

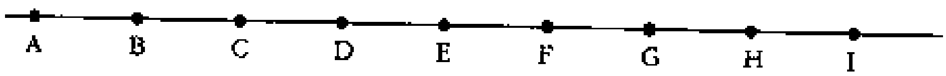
(c) Use a table of standard integral to evaluate **2**

$$\int \frac{1}{\sqrt{x^2+9}}\ dx$$

(d) $$\int_0^{\frac{3}{4}} \frac{1}{9+16x^2}\,dx$$ **3**

Question 4 (12 marks)

(a) (i) In what ratio does I divide AG? **1**



(ii) W(2,3) divides XY internally in the ratio $k:l$ where X(-1,1) and Y(7,9). Find the ratio $k:l$. **2**

(b) The polynomial $P(x)=x^3-3x^2+kx-2$ has roots α,β,γ.

(i) Find the value of $\alpha+\beta+\gamma$. **1**

(ii) Find the value of $\alpha\beta\gamma$. **1**

(iii) It is known that two roots are the reciprocal of each other. Find the value of the third root and hence find the value of k. **2**

(c) Marvin the Martian has a body temperature of $100°$C. When Marvin sleeps his body temperature obeys Newton's Law of Cooling according tho the law $\frac{dT}{dt}=k(T-A)$, where T is Marvin's body temperature and A is the temperature of the surrounding air.

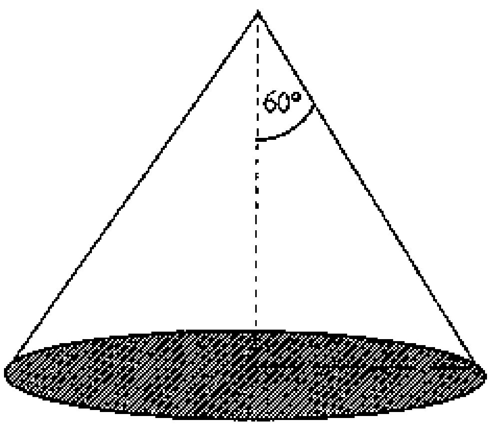
(i) Show that $T = A + Ce^{kt}$, where C and k are constants, satisfies Newton's Law of Cooling. **2**

(ii) Marvin goes to sleep at 10 pm. His temperature at midnight is 95 °C. Marvin's bedroom is air conditioned with the temperature set at 20 °C. Assuming Marvin continues to sleep what will be his body temperature at 8am? **3**

Question 5 (12 marks)

(a) Use the principle of Mathematical Induction to show that $7^n + 13^n$ is divisible by 10 for n odd integers. **3**

(b) Sand pours onto the ground and forms a cone where the semi-vertical angle is $60°$. The height of the cone at time t seconds is h *cm* and the radius of the base is r *cm*. Sand is being poured onto the pile at a rate of $12cm^3/s$.



(i) Show that $r = \sqrt{3}h$ **1**

(ii) Find the rate at which the height is increasing at the instant when the height is 12 *cm*. **3**

[Volume of a cone = $\frac{1}{3}\pi r^2 h$]

(c) Consider the function

$$f(x) = \tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)$$

(i) State any values of x for which $f(x)$ is undefined. **1**

(ii) Show that $f(1) = \frac{\pi}{2}$ **1**

(iii) Show that $f'(x) = 0$ **2**

(iv) Sketch the graph of $y = f(x)$ **1**

Question 6 (12 marks)

A particle moves in Simple Harmonic Motion with amplitude a, in the form $\ddot{x} = -4x$ where x is the displacement, in metres, from the origin O and t is the time in seconds.

(i) Prove that $v^2 = 4(a^2 - x^2)$ **3**

(ii) The particle moves so that $x = 2$, $v = 4$ find the value of a. **1**

(iii) Find an expression for v in terms of displacement. **2**

(iv) By setting $v = \frac{dx}{dt}$ and taking the reciprocal, prove that $x = 2\sqrt{2}\sin 2t$ if when $t = \frac{\pi}{4}$, $x = 2\sqrt{2}$. **3**

(v) Where would you expect the maximum speed to occur? **1**

(vi) Hence, or otherwise, find the maximum speed of the particle. **2**

Question 7 (12 marks)

(a) A particle moves according to the equation $x = 2e^{-t}(\cos t + \sin t)$.

It moves in the interval $0 \le t \le 2\pi$.

(i)	Show that $\dot{x} = -4e^{-t}\sin t$ and find the acceleration function $\ddot{x}$.	2
(ii)	Discuss the displacement as $t \to \infty$.	1
(iii)	Find the times when the particle is at the origin.	2
(iv)	When is the particle moving in the positive direction.	1
(v)	Find the times when the particle will be stationary.	2
(vi)	Find the displacement at the times when the particle is stationary, (give your answers correct to three decimal places).	1
(vii)	Draw a neat, **full-page** sketch of $x = 2e^{-t}(\cos t + \sin t)$, giving endpoints, stationary points and intercepts	3