Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2002

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Morning Session
Monday 12 August 2002

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on page 15
- All necessary working should be shown in every question

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

2602 – 1

Total marks (120)
Attempt Questions 1 – 10
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\dfrac{x^3 + y^4}{y^2}$ if $x = \left(\dfrac{2}{5}\right)^{\frac{1}{3}}$ and $y = \left(\dfrac{3}{5}\right)^{\frac{1}{2}}$ **2**

Give your answer in fractional form.

(b) Express $0.2\dot{3}$ as a fraction in simplest form. **2**

(c) Factorise $40 - 5y^3$ **2**

(d) Solve $x^2 + 4x - 1 = 0$ leaving your answer in simplest surd form. **3**

(e) (i) Solve $4^x = 32$. **2**

(ii) Hence, or otherwise, write down the value of $\log_4 32$. **1**

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

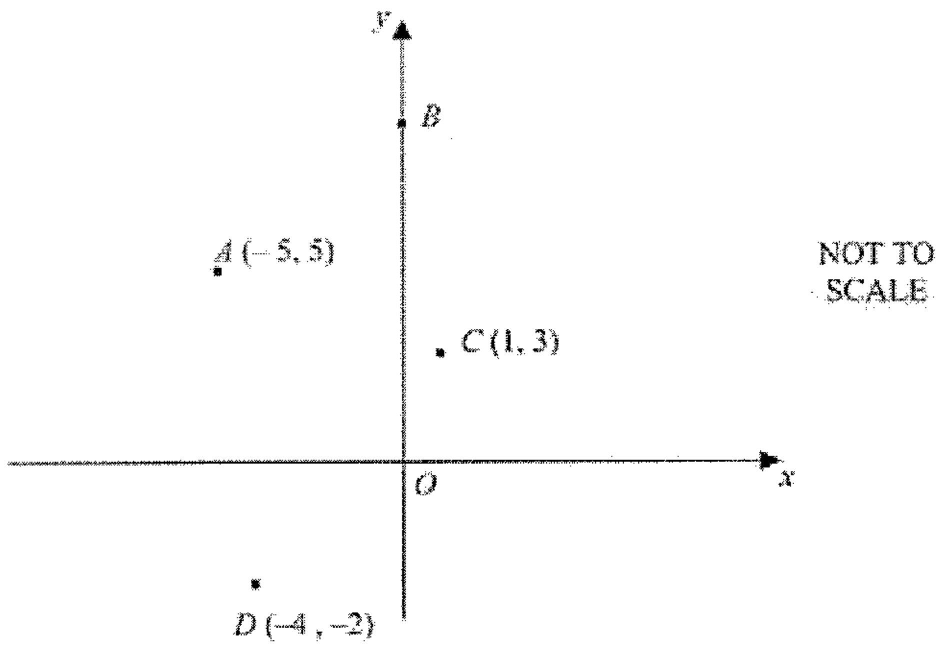
(a) Differentiate with respect to x:

(i) $5x + \dfrac{3}{x^2}$ 1

(ii) e^{2x^2+3} 1

(iii) $\dfrac{3x}{\sin x}$ 2

(b)



The diagram shows the points $A\ (-5, 5)$ and $C\ (1, 3)$ and $D\ (-4, -2)$.
B is a point on the y axis.

(i) Find the gradient of AC. 1

(ii) Find the midpoint of AC. 1

(iii) Show that the equation of the perpendicular bisector of AC is $3x - y + 10 = 0$.

(iv) Find the coordinates of B given that B lies on $3x - y + 10 = 0$. 1

(v) Show that the point $D\ (-4, -2)$ lies on $3x - y + 10 = 0$. 1

(vi) Show that $ABCD$ is a rhombus. 2

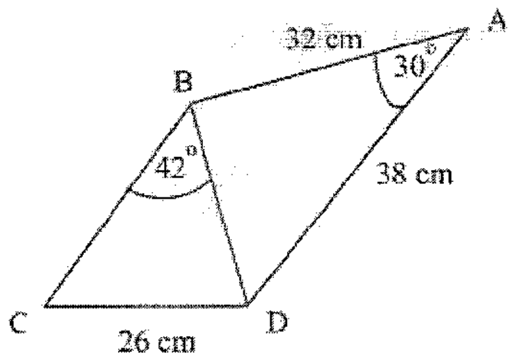
Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\int \frac{x}{x^2+5}\,dx$ **2**

(b) Evaluate $\int_0^{\frac{\pi}{8}} \sec^2 2x\,dx$ **2**

(c) Find the equation of the normal to the curve $y = x\log_e x$ at the point (e, e). **4**

(d)



NOT TO SCALE

In the diagram AB is 32 cm, AD is 38 cm and CD is 26 cm.
$\angle BAD$ is 30° and $\angle CBD$ is 42°.

(i) Use the cosine rule to find the length of BD. **2**

(ii) Hence, find the size of $\angle BCD$ to the nearest degree. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The second term of a geometric series is 120 and the fifth term is 50.625.

(i) Find the common ratio and the first term of the series. **2**

(ii) Find the limiting sum of the series. **1**

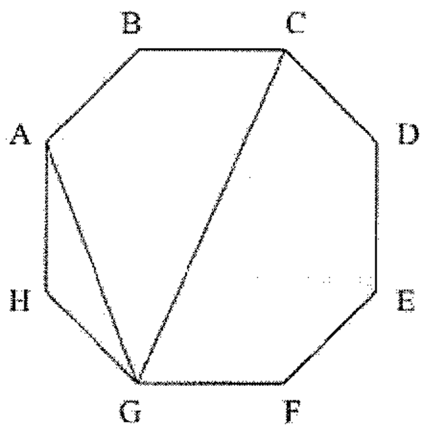
(iii) Hence, find the difference between the limiting sum and the sum of the first 40 terms giving your answer in scientific notation correct to 2 significant figures. **2**

(b) For the quadratic equation $x^2 + kx - 3x + 2 - k = 0$,

(i) find the value of the discriminant in terms of k, **1**

(ii) explain why the roots of this quadratic equation are real for all values of k. **2**

(c)



NOT TO SCALE

ABCDEFGH is a regular octagon.

(i) Explain clearly why $\angle ABC$ is 135^0. **1**

(ii) Calculate the size of $\angle GAH$. **1**

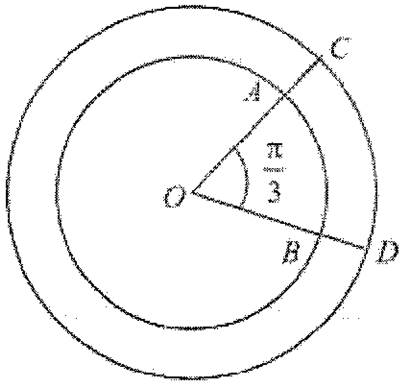
(iii) Using (i), or otherwise, calculate the size of $\angle CGF$. **1**

(iv) Hence, calculate the size of $\angle AGC$. **1**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)

NOT TO SCALE



The diagram shows two concentric circles with centre O.
The radius of the larger circle is 8.2 cm.

(i) Calculate the area of sector COD. **1**

(ii) The area of the sector AOB is 18.4 cm^2. Calculate the radius of this sector AOB. **2**

(iii) Calculate the area of triangle COB. **2**

(b) Let $f(x) = 3x^2 + 1$.

(i) Copy the following table and supply the missing values. **1**

x	0	0.2	0.4	0.6	0.8	1
$f(x)$	1					4

(ii) Use these six values of the function and the trapezoidal rule to find the approximate value of **2**

$$\int_0^1 (3x^2 + 1)\,dx.$$

Question 5 continues on page 8

Question 5 (continued) **Marks**

(c) The population P of a town is growing at a rate proportional to the town's current population. The population at time t years is given by $P = A\,e^{kt}$, where A and k are constants.

The population 20 years ago was 100 000 people and today the population of the town is 150 000 people.

(i) Find the value of A. **1**

(ii) Find the value of k. **1**

(iii) Find the population that will be present 20 years from now. **2**

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the curve given by $y = x^3 + 3x^2 - 9x - 5$.

(i) Find $\dfrac{dy}{dx}$. **1**

(ii) Find the coordinates of the two stationary points. **2**

(iii) Determine the nature of the stationary points. **2**

(iv) Sketch the curve for the domain $-5 \le x \le 3$. **2**

(v) By drawing an appropriate line on your graph, or otherwise, solve **2**

$$x^3 + 3x^2 - 9x + 5 = 0.$$

(b) Calculate the exact volume generated when the region enclosed by the curve **3**

$$y = 1 + 2e^{x} \text{ for } 0 \le x \le 1,$$

is rotated about the x axis.

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A bag contains 5 blue balls, 4 red balls, 2 yellow balls and 1 green ball. Three balls are selected at random without replacement from the bag. Calculate the probability that

(i) the three balls drawn are blue, **1**

(ii) the three balls drawn are of the same colour, **2**

(iii) exactly two of the balls drawn are blue. **2**

(b) A particle is projected vertically upwards from a point 2 metres above horizontal ground. The displacement at time t seconds is given by

$$x = 24.5t - 4.9t^2, \qquad t \geq 0.$$

(i) Find an expression for the velocity of the particle. **1**

(ii) Find when the particle comes to rest. **2**

(iii) Hence, find the greatest height of the particle above the ground. **2**

(iv) Find the length of time for which the particle is at least 21.6 metres above the ground. **2**

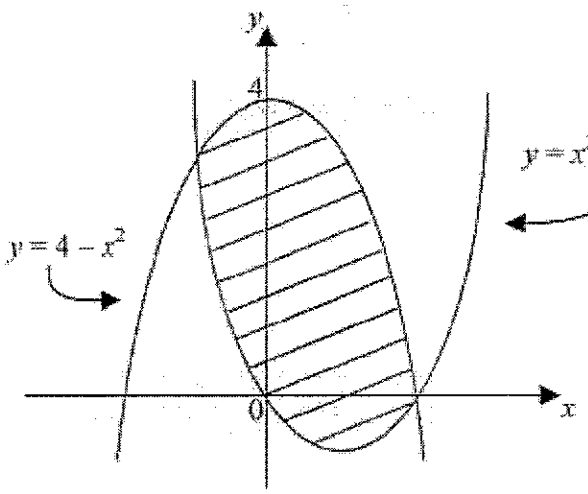
Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A function $y = f(x)$ is continuous for all values. After finding the first and second derivatives a student discovers the following, for all values of x. **2**

When	$x < 2$,	$f'(x) < 0$	and	$f''(x) > 0$
When	$x = 2$,	$f'(x) = 0$	and	$f''(x) = 0$
When	$x > 2$,	$f'(x) < 0$	and	$f''(x) < 0$.

Draw a neat sketch of $y = f(x)$, showing all the important characteristics of the function given that $f(2) = 0$.

(b) The graphs of the functions $y = 4 - x^2$ and $y = x^2 - 2x$.



NOT TO SCALE

(i) Describe, using inequalities, the shaded region. **1**

(ii) By solving simultaneously, show that the points of intersection are at $x = -1$ and $x = 2$. **2**

(iii) Calculate the area of the shaded region. **2**

Question 8 continues on page 12

Question 8 (continued) **Marks**

(c) On a factory production line a tap opens and closes to fill containers with liquid. As the tap opens, the rate of flow increases for the first 10 seconds according to the relation $R = \frac{6t}{50}$, where R is measured in L/sec. The rate of flow then remains constant until the tap begins to close. As the tap closes, the rate of flow decreases at a constant rate for 10 seconds, after which time the tap is fully closed.

(i) Show that, while the tap is fully open, the volume in the container at any time is given by **3**

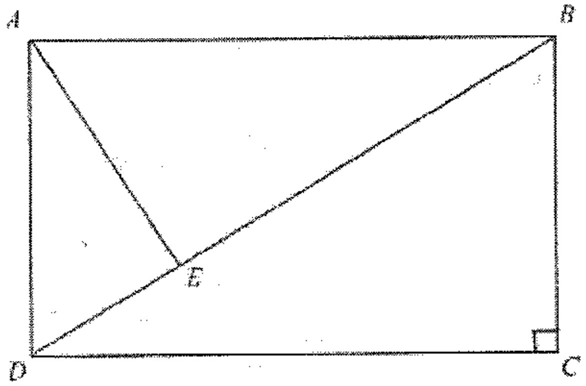
$$V = \frac{6}{5}(t-5).$$

(ii) For how many seconds must the tap remain fully open in order to exactly fill a 120L container with no spillage.

End of Question 8

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) $ABCD$ is a rectangle and $AE \perp BD$. $AE = 5$ cm and $DE = 2$ cm.



(i) Copy the diagram and prove that triangles AED and BCD are similar. **2**

(ii) Hence, show that $AD^2 = BD.DE$. **1**

(iii) Find the area of $ABCD$. **3**

(b) A closed water tank in the shape of a right cylinder is to be constructed with a surface area of 54π cm^2. The height of the cylinder is h cm and the base radius is r cm.

(i) Show that the height of the water tank in terms of r is given by **2**

$$h = \frac{27}{r} - r$$

(ii) Show that the volume V that can be contained in the tank is given by **1**

$$V = 27\pi r - \pi r^3$$

(iii) Find the radius r cm which will give the cylinder its greatest possible volume. Justify your answer. **3**

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Show that $x = \frac{\pi}{8}$ is a solution of $\sin 2x = \cos 2x$. **1**

(ii) On the same set of axes, sketch the graphs of the functions $y = \sin 2x$ and $y = \cos 2x$ for $-\pi \le x \le \pi$. **2**

(iii) Hence, find graphically the number of solutions of $\tan 2x = 1$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. **1**

(iv) Use your graphs to solve $\tan 2x \le 1$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. **1**

(b) Mr and Mrs Matthews decide to borrow \$250 000 to buy a house. Interest is calculated monthly on the balance still owing, at a rate of 6.06% per annum. The loan is to be repaid at the end of 15 years with equal monthly repayments of \$$M$.

Let \$$A_n$ be the amount owing after the nth repayment.

(i) Derive an expression for A_{60}. **1**

(ii) Find the value of M. **2**

(iii) Hence, calculate the amount still owing after 5 years of payment at this rate. **2**

(iv) At the end of five years, the interest rate is increased to 7.2% per annum and Mr and Mrs Matthews change their payments to \$1800 per month. How many more months are needed to pay off the remainder of the loan? **2**

End of paper

Question 1

(a) $\dfrac{x^3+y^4}{y^2}$

$= \dfrac{\dfrac{2}{5}+\dfrac{9}{25}}{\dfrac{3}{5}} = 1\dfrac{4}{15}$

(b) Let $x = 0.23333\ldots$ ①
$10x = 2.33333\ldots$ ②

② − ①:
$9x = 2.1$
$x = \dfrac{21}{90} = \dfrac{7}{30}$
i.e. $0.2\dot{3} = \dfrac{7}{30}$

OR $0.2\dot{3} = 0.233333\ldots$
$= 0.2 + (0.03 + 0.003 + 0.0003 + 0.00003 + \ldots)$
∴ Infinite sum of a geometric progression,
where $a = 0.03$, $r = \dfrac{0.003}{0.03} = 0.1$

∴ $S = \dfrac{a}{1-r} = \dfrac{0.03}{1-0.1} = \dfrac{0.03}{0.9} = \dfrac{1}{30}$

∴ $0.2\dot{3} = 0.2 + \dfrac{1}{30} = \dfrac{6}{30} + \dfrac{1}{30} = \dfrac{7}{30}$

(c) $40 - 5y^3$
$= 5(8 - y^3)$
$= 5(2 - y)(4 + 2y + y^2)$

(d) $x^2 + 4x - 1 = 0$

$x = \dfrac{-4 \pm \sqrt{20}}{2}$

$x = \dfrac{-4 \pm 2\sqrt{5}}{2}$

$x = -2 \pm \sqrt{5}$

(e) (i) $4^x = 32$
$2^{2x} = 2^5$
$2x = 5$
$x = 2.5$

(ii) ∴ $\log_4 32 = 2.5$

Question 2

(a) (i) $\dfrac{d}{dx}\left(5x + \dfrac{3}{x^2}\right)$

$= \dfrac{d}{dx}\left(5x + 3x^{-2}\right)$

$= 5 - 6x^{-3}$

$= 5 - \dfrac{6}{x^3}$

(ii) $\dfrac{d}{dx}\left(e^{2x^2+3}\right)$

$= 4xe^{2x^2+3}$

(iii) $\dfrac{d}{dx}\left(\dfrac{3x}{\sin x}\right) = \dfrac{\sin x \times 3 - 3x \times \cos x}{\sin^2 x}$

$= \dfrac{3\sin x - 3x\cos x}{\sin^2 x}$

$= \dfrac{3(\sin x - x\cos x)}{\sin^2 x}$

(b) (i) Gradient of $AC = \dfrac{3-5}{1+5} = \dfrac{-2}{6} = -\dfrac{1}{3}$.

(ii) Midpoint of $AC = \left(\dfrac{-5+1}{2}, \dfrac{5+3}{2}\right) = (-2, 4)$.

(iii) Use $y - y_1 = m(x - x_1)$, with $(x_1, y_1) = (-2, 4)$ and $m = 3$.

∴ $y - 4 = 3(x + 2)$

∴ $y = 3x + 10$

∴ $3x - y + 10 = 0$.

(iv) Substitute $x = 0$ in equation $3x - y + 10 = 0$.
$3(0) - y + 10 = 0$
$y = 10$.
∴ B has coordinates $(0, 10)$.

(v) Substitute $D(-4, -2)$ into $3x - y + 10 = 0$.
LHS $= 3(-4) - (-2) + 10$
$= -12 + 2 + 10$
$= 0$
$=$ RHS.
∴ D lies on the line.

(vi) Midpoint of $BD = \left(\dfrac{0+(-4)}{2}, \dfrac{10+(-2)}{2}\right) = (-2, 4)$.

Since B and D both lie on the perpendicular bisector of AC and the midpoint of BD is equal to the midpoint of AC, then the diagonals AC and BD bisect each other at right angles.

∴ $ABCD$ is a rhombus.

Question 3

(a) $\displaystyle\int \frac{x}{x^2+5}\,dx = \frac{1}{2}\int \frac{2x}{x^2+5}\,dx$

$= \dfrac{1}{2}\ln\left(x^2+5\right) + C$

(or $\ln\sqrt{x^2+5} + C$).

(b) $\displaystyle\int_0^{\frac{\pi}{8}} \sec^2 2x\,dx = \left[\frac{1}{2}\tan 2x\right]_0^{\frac{\pi}{8}}$

$= \dfrac{1}{2}\tan\dfrac{\pi}{4} - \dfrac{1}{2}\tan 0$

$= \dfrac{1}{2}$

(c) $y = x\log_e x$

∴ $\dfrac{dy}{dx} = x \times \dfrac{1}{x} + \log_e x \times 1 = 1 + \log_e x$.

Let m_1 = gradient of tangent at (e, e)
and m_2 = gradient of normal at (e, e).

∴ $m_1 = 1 + \log_e e = 1 + 1 = 2$.

$m_2 = -\dfrac{1}{m_1} = -\dfrac{1}{2}$

∴ Equation of normal is
$y - e = -\dfrac{1}{2}(x - e)$

$2y - 2e = -x + e$

∴ $x + 2y - 3e = 0$

3 continued

$$a^2 = b^2 + d^2 - 2bd\cos A$$
$$a^2 = 38^2 + 32^2 - 2(38)(32)\cos 30^\circ$$
$$\therefore \quad a = 19.02173\ldots$$

$BD = 19$ cm (2 s.f.)

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin C}{19.02173\ldots} = \frac{\sin 42^\circ}{26}$$

$\angle BCD = 29.31\ldots^\circ$

$\therefore \angle BCD = 29^\circ$ (nearest degree).

Question 4

(a) (i) $ar = 120$
$ar^4 = 50.625$

$$\frac{ar^4}{ar} = \frac{50.625}{120}$$

$$r^3 = \frac{27}{64}$$

$\therefore$ the common ratio, $r = \frac{3}{4} = 0.75$

$$a = \frac{120}{r} = \frac{120}{0.75} = 160$$

$\therefore$ the first term, $a = 160$.

(ii) $$S_\infty = \frac{a}{1-r}$$

$$= \frac{160}{1-0.75}$$

$$= \frac{160}{0.25} = 640$$

$\therefore$ the limiting sum, $S_\infty = 640$

(iii) $$S_{40} = \frac{160\left(1-0.75^{40}\right)}{1-0.75} = 639.9935\ldots$$

$S_\infty - S_{40} = 0.0064362\ldots$

$= 0.0064$ (2 s.f.)

$= 6.4 \times 10^{-3}$ (2 s.f.)

(b) (i) $x^2 + kx - 3x + 2 - k = 0$

$x^2 + x(k-3) + (2-k) = 0$

$a = 1,\ b = k-3,\ c = 2-k$

$\Delta = b^2 - 4ac$

$= (k-3)^2 - 4(1)(2-k)$

$= k^2 - 6k + 9 - 8 + 4k$

$= k^2 - 2k + 1$

$\therefore \Delta = k^2 - 2k + 1$

(ii) For real roots, $b^2 - 4ac \geq 0$ for all k

i.e. $k^2 - 2k + 1 \geq 0$ for all k

Now $k^2 - 2k + 1 = (k-1)^2$

and $(k-1)^2 \geq 0$ for all k

$\therefore$ the roots are real for all values of k.

(c) (i) For a regular polygon, interior angle = $180^\circ -$

$\therefore$ for a regular octagon, interior angle = 180°

$= 180^\circ$

$= 135^\circ$

$\therefore \angle ABC = 135^\circ$.

(ii) $$\angle GAH = \frac{180^\circ - 135^\circ}{2}$$

$\therefore \angle GAH = 22.5^\circ$.

Question 4 continued

(c) (iii) $$\angle CGF = \frac{135^\circ}{2}$$

$\therefore \angle CGF = 67.5^\circ$.

(iv) $\angle AGC = 135^\circ - (67.5^\circ + 22.5^\circ)$

$\therefore \angle AGC = 45^\circ$.

Question 5

(a) (i) Area of a sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 8.2^2 \times \frac{\pi}{3}$

$= 35.2067\ldots$ cm^2

$\therefore$ Area of sector $COD = 35$ cm^2 (nearest cm^2)

(ii) Area $AOB = \frac{1}{2}r^2\theta = 18.4$

$$r^2 = \frac{18.4 \times 2}{\theta} = 36.8 \div \frac{\pi}{3} = 35.1414\ldots$$

$\therefore r = 5.928\ldots$ cm

$\therefore$ radius of sector $AOB = 5.9$ cm (2 s.f.)

(iii) Area $\Delta COB = \frac{1}{2}(8.2)(5.9)\sin\frac{\pi}{3}$

$= 21.0486\ldots$ cm^2

$\therefore$ Area of $\Delta COB = 21$ cm^2 (nearest cm^2)

(b) (i)

x	0	0.2	0.4	0.6	0.8	1
$f(x)$	1	1.12	1.48	2.08	2.92	4

(ii) $h = \frac{1}{5}$

Trapezoidal rule:

$$\int_0^1 (3x^2+1)\,dx \approx \frac{h}{2}\left[f(0) + 2\{f(0.2) + f(0.4) + f(0.6) + f(0.8)\} + f(1)\right]$$

$$= \frac{1}{10}\left[1 + 2\{1.12 + 1.48 + 2.08 + 2.92\} + 4\right]$$

$= 2.02$

5 continued

$A = 100\,000$

) When $t = 20$, $P = 150\,000$

By substitution into $P = Ae^{kt}$,

$$150000 = 100000e^{20k}$$

$$e^{20k} = 1.5$$

$$20k = \ln 1.5$$

$$k = \frac{\ln 1.5}{20} = 0.02027\ldots$$

$\therefore k = 0.0203$ (3 s.f.)

i When $t = 40$, $P = 100000e^{40k}$

$$= 100000e^{2\ln 1.5}$$

$$= 225\,000$$

$\therefore$ population that will be present 20 years from now is 225 000.

n 6

$= x^3 + 3x^2 - 9x - 5$

$$\frac{dy}{dx} = 3x^2 + 6x - 9$$

$$= 3(x^2 + 2x - 3).$$

Stationary points when $\frac{dy}{dx} = 0$.

That is, $(x^2 + 2x - 3) = 0$

$$(x + 3)(x - 1) = 0.$$

$\therefore x = -3$ or $x = 1$

Stationary points are $(-3, 22)$ and $(1, -10)$.

$$\frac{d^2y}{dx^2} = 6x + 6$$

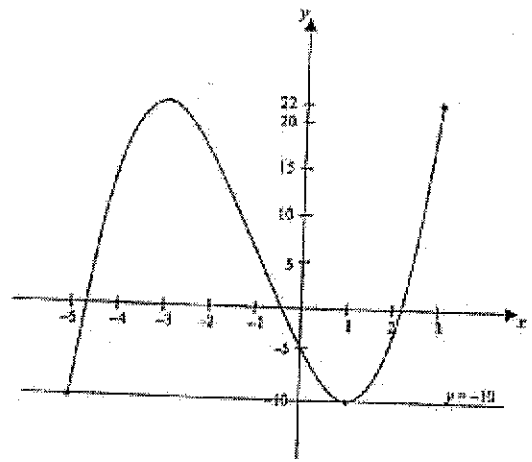
When $x = -3$, $\frac{d^2y}{dx^2} = -18 + 6 < 0$,

$\therefore$ the curve is concave down and $(-3, 22)$ is a relative maximum.

When $x = 1$, $\frac{d^2y}{dx^2} = 6 + 6 > 0$,

$\therefore$ the curve is concave up and $(1, -10)$ is a relative minimum.

(a) (iv)



(v) $x^3 + 3x^2 - 9x + 5 = 0$

when $x^3 + 3x^2 - 9x - 5 = -10$

$\therefore$ by drawing the line $y = -10$ on the graph,

Solutions are $x = -5$ and $x = 1$.

(b) $V = \pi\int_0^1 y^2\,dx \quad = \pi\int_0^1 \left(1 + 2e^{-x}\right)^2 dx$

$$= \pi\int_0^1 \left(1 + 4e^{-x} + 4e^{-2x}\right) dx$$

$$= \pi\left[x - 4e^{-x} - 2e^{-2x}\right]_0^1$$

$$= \pi\left[\left(1 - 4e^{-1} - 2e^{-2}\right) - \left(0 - 4e^{-0} - 2e^{-0}\right)\right]$$

$\therefore$ Volume $= \pi\left(7 - 4e^{-1} - 2e^{-2}\right)$ unit^3.

Question 7

(a) (i) $P(BBB) = \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10} = \frac{1}{22}$

(ii) $P(BBB) + P(RRR) = \frac{1}{22} + \left(\frac{4}{12} \times \frac{3}{11} \times \frac{2}{10}\right) = \frac{1}{22} + \frac{1}{55} = \frac{7}{110}$

(iii) $P(B,B,NB) + P(NB,B,B) + P(B,NB,B) = 3\left(\frac{5}{12} \times \frac{4}{11} \times \frac{7}{10}\right) = \frac{7}{22}$

(b) (i) $x = 24.5t - 4.9t^2$

$$\frac{dx}{dt} = v = 24.5 - 9.8t$$

(ii) $v = 24.5 - 9.8t$

Particle comes to rest when $v = 0$.

$0 = 24.5 - 9.8t$

$9.8t = 24.5$

$t = 2.5$ seconds.

$\therefore$ particle comes to rest after 2.5 seconds.

(iii) Greatest height occurs when velocity is zero,

At $t = 2.5$, $\quad x = 24.5(2.5) - 4.9(2.5)^2$

$\quad x = 30.625$

$$\frac{d^2x}{dt^2} = -9.8 < 0 \text{ for all } t$$

$\therefore$ the curve is concave down and $(2.5, 30.625)$ is a absolute maximum.

However, if the particle is projected from 2 metres above the ground then greatest height is 32.625 metres.

on 7 continued

(iv) For particle to be at least 21.6 metres above the ground,

$\therefore x = 21.6 - 2 = 19.6$ metres

and $24.5t - 4.9t^2 \geq 19.6$

$5t - t^2 \geq 4$

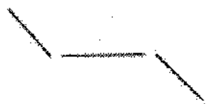
$t^2 - 5t + 4 \leq 0$

$\therefore 1 \leq t \leq 4$ seconds.

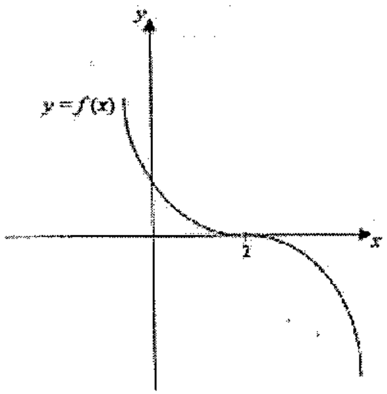
$\therefore$ the particle is at least 21.6 metres above the ground for 3 seconds.

ion 8

x	<2	2	>2
$f'(x)$	Decreasing	Stationary point	Decreasing
$f''(x)$	Concave Up	Point of inflexion	Concave down



Also $f(2) = 0$,



Question 8 continued

(b) (i) $y \leq 4 - x^2, \quad y \geq x^2 - 2x$

(ii) Solving simultaneously,

$x^2 - 2x = 4 - x^2$

$2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1$ or $x = 2$.

(iii) $\text{Area} = \int_{-1}^{2} (4 - x^2 - x^2 + 2x)\, dx$

$= \int_{-1}^{2} (4 - 2x^2 + 2x)\, dx$

$= \left[4x - \frac{2x^3}{3} + x^2\right]_{-1}^{2}$

$= \left(8 - \frac{16}{3} + 4\right) - \left(-4 + \frac{2}{3} + 1\right)$

$= 9$ square units.

(c) (i) For the first 10 seconds, $\frac{dV}{dt} = \frac{6t}{50}$

$\therefore V = \frac{3t^2}{50} + C$

When $t = 0$, $V = 0$

$\therefore V = \frac{3t^2}{50}$, $t \leq 10$.

When $t = 10$ seconds, $V = \frac{3(10)^2}{50} = 6$ Litres

After 10 seconds, rate of flow remains constant

and so, $\frac{dV}{dt} = \frac{6(10)}{50} = \frac{6}{5}$ L/sec

$\therefore V = \frac{6t}{5} + C$

When $t = 10$, $V = 6$

$\therefore 6 = \frac{6(10)}{5} + C$

$C = -6$

$\therefore V = \frac{6t}{5} - 6 = \frac{6t - 30}{5} = \frac{6}{5}(t - 5)$.

(ii) Volume that flows into container while tap is closing is 6 Litres.

$\therefore$ Volume required $= 120 - 6 = 114$ Litres

$\frac{6}{5}(t - 5) = 114$

$t - 5 = 95$

$t = 100$ seconds

$\therefore$ tap must remain fully open for 90 seconds.

Question 9

(a) (i) $\angle AED = \angle BCD = 90^\circ$ ($AE \perp BD$ and $ABCD$ is a rectangle)

$\angle ADE = \angle DBC$ (Alternate angles on parallel lines, $AD \parallel BC$)

$\therefore \Delta AED \parallel\!| \Delta BCD$ (equiangular).

(ii) $\Delta AED \parallel\!| \Delta BCD$.

$\therefore \frac{AD}{BD} = \frac{DE}{BC}$

Now $BC = AD$ (opposite sides of rectangle are equal)

$\therefore \frac{AD}{BD} = \frac{DE}{AD}$

$\therefore AD^2 = BD.DE$.

(iii) $AD = \sqrt{25+4} = \sqrt{29}$ cm

$\therefore BD.DE = 29$

$BD \times 2 = 29$

$BD = 14.5$ cm

$\therefore$ Area $ABCD = 14.5 \times 5 = 72.5$ cm^2.

) (i) Surface Area $= 2\pi r^2 + 2\pi rh$.

$54\pi = 2\pi r^2 + 2\pi rh$

$h = \dfrac{54\pi - 2\pi r^2}{2\pi r}$

$\therefore h = \dfrac{27}{r} - r$

i) $V = \pi r^2 h$

$V = \pi r^2\left(\dfrac{27}{r} - r\right)$

$\therefore V = 27\pi r - \pi r^3$.

i) Greatest possible volume V occurs when $\dfrac{dV}{dr} = 0$ and

$V = 27\pi r - \pi r^3$,

$\dfrac{dV}{dr} = 27\pi - 3\pi r^2$

$\dfrac{d^2V}{dr^2} = -6\pi r$

$\dfrac{dV}{dr} = 0$, $\quad\therefore\quad 27\pi - 3\pi r^2 = 0$

$r^2 = \dfrac{27\pi}{3\pi} = 9$

$r = \pm 3$.

But $r > 0$, so $r = 3$ cm.

When $r = 3$, $\dfrac{d^2V}{dr^2} = -6\pi(3) < 0$.

The value of ...

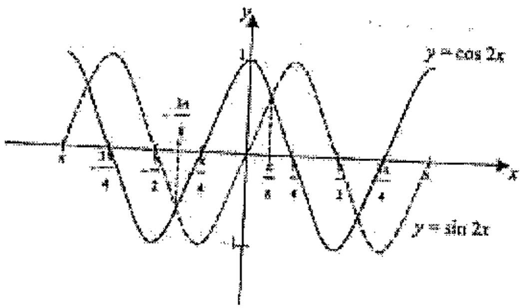
Question 10

(a) (i) LHS: $\sin 2x = \sin\dfrac{2\pi}{8} = \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$.

RHS: $\cos 2x = \cos\dfrac{2\pi}{8} = \cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}} =$ LHS

That is, $\sin 2x = \cos 2x$ when $x = \dfrac{\pi}{8}$.

(ii) Period $= \dfrac{2\pi}{n} = \dfrac{2\pi}{2} = \pi$



(iii) $\tan 2x = 1$ when $\dfrac{\sin 2x}{\cos 2x} = 1$

That is, when $\sin 2x = \cos 2x$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

From the diagram, it can be seen that the curves have two points of intersection for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

Therefore, the equation $\tan 2x = 1$ has two solutions for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

(iv) $\tan 2x \le 1$ when $\sin 2x \le \cos 2x$ for $-\dfrac{3\pi}{8} < x < \dfrac{\pi}{8}$.

(b) (i) $A_1 = (250000 \times 1.00505) - M$

$A_2 = [(250000 \times 1.00505) - M] \times 1.00505 - M$

$= 250000 \times 1.00505^2 - M(1 + 1.00505)$

Continuing the pattern

$A_{60} = 250000 \times 1.00505^{60} - M(1 + 1.00505 + \dots + 1.00505^{59})$

$\therefore A_{60} = 250000 \times 1.00505^{60} - M \times \dfrac{(1.00505^{60} - 1)}{(1.00505 - 1)}$.

(ii) If the loan is to be repaid at the end of 15 years then $A_{180} = 0$.

$\therefore 250000 \times 1.00505^{180} - M \times \dfrac{(1.00505^{180} - 1)}{(1.00505 - 1)} = 0$

$\therefore M = \dfrac{(250000 \times 1.00505^{180}) \times 0.00505}{(1.00505^{180} - 1)}$

$\therefore M = 2117.7545571$

$\therefore$ The monthly repayment is \$2117.75 to the nearest cent.

(iii) Amount still owing after 5 years,

$A_{60} = 250000 \times 1.00505^{60} - 2117.7545571 \times \dfrac{(1.00505^{60} - 1)}{(1.00505 - 1)}$

$\therefore A_{60} = 190236.7605$

$\therefore$ The amount still owing after 5 years is \$190236.76 to the nearest cent.

v) After 5 years, number of months needed to pay off remainder of loan at interest rate of 7.2% per annum with monthly repayments of \$1800,

$190236.7605 \times 1.006^n = 1800 \times \dfrac{(1.006^n - 1)}{0.006}$

$190236.7605 \times 1.006^n = 300000 \times (1.006^n - 1)$

$1.006^n = \dfrac{300000}{300000 - 190236.7605}$

$n = \dfrac{\ln\left(\dfrac{300000}{300000 - 190236.7605}\right)}{\ln 1.006}$

$n = 168.07836$

$\therefore$ Approximately 169 months are needed to pay off the remainder of the loan.