THE KING'S SCHOOL

2003
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value

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Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

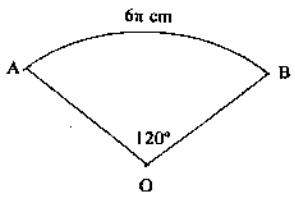
Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Find, correct to two decimal places, $\log_{12} 2003$ **2**

(b) Find the derivative of $12-\cos 12x$ **2**

(c) In sector OAB, $\angle AOB = 120°$ and arc $AB = 6\pi$ cm. Find the radius of the sector. **2**



(d) Solve $\sqrt{2}x = 2\sqrt{3}$, expressing your solution in simplest form. **2**

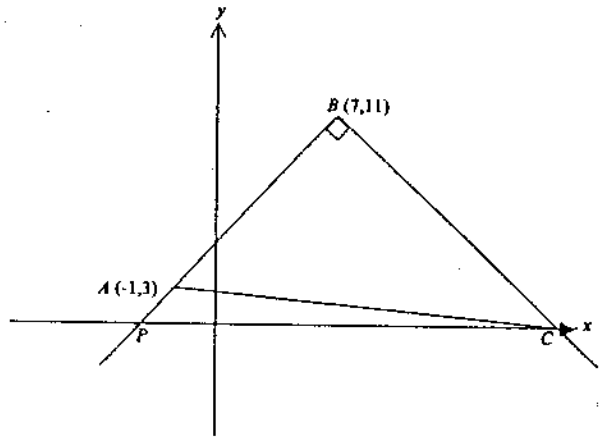
(e) Sketch the region in the number plane where $0 \le y \le e^{-x}$ **2**

(f) Simplify $\dfrac{x^2}{x+\dfrac{x}{x-1}}$ **2**

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a)



In the diagram, PAB is a straight line where P is on the x axis.

ΔABC has vertices $A(-1,3)$, $B(7,11)$ and C, which is on the x axis. $\angle ABC = 90°$.

(i) Find the size of $\angle BPC$ **2**

(ii) Find the equation of BC **2**

(iii) State the coordinates of point C **1**

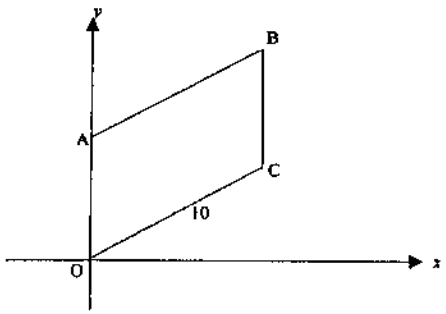
(iv) Find the area of ΔABC **2**

(v) Find the size of $\angle BAC$, nearest degree **2**

Marks

Question 2 (continued)

(b)



OABC is a parallelogram, O is the origin and A is on the y axis.
The equations of OC and AB are $y = 2x$ and $y = 2x + 5$, respectively.
The length of OC is 10 units.

Find the area of the parallelogram. **3**

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) Find, correct to 1 decimal place,

$$\int_0^{0.1} \sec^2(x+1)\,dx$$ **2**

(b) The probability that Max can correctly integrate a function is 0.7. Max is given 7 functions to integrate.

Find the probability that Max gets at least one integration wrong. Give your answer correct to 1 decimal place. **2**

(c) Find the sum of the arithmetic series

$$-7 \;+\; (-2) \;+\; 3 \;+\; \ldots \;+\; 2003$$ **3**

(d) (i) Sketch on the same diagram

$y = |x - 2|$ and $y = 2x$,

showing the x and y intercepts. **2**

(ii) Hence, or otherwise,

solve $|x - 2| = 2x$ **2**

(iii) Using (i), or otherwise,

find $\int_0^4 |x - 2|\,dx$ **1**

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) Find the equation of the tangent to the curve $y=(x+1)e^x$ at the point where $x=0$ — 3

(b) Prove that $\sec^2 A+\operatorname{cosec}^2 A \equiv \sec^2 A\operatorname{cosec}^2 A$ — 3

(c) Find the centre and radius of the circle $x^2+y^2=4y$ — 3

(d) The line $y=2x+c$ is a tangent to the parabola $y=x^2+x$.

Find the value of c. — 3

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) Consider the curve $y=x^3(x-4)$

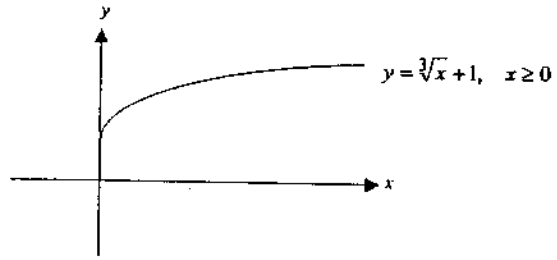
(i) State the x intercepts. — 1

(ii) Show that there are stationary points at $x=0$ and $x=3$ — 2

(iii) Show that the stationary point at $x=0$ is a point of inflection. — 2

(iv) Sketch the curve. — 2

(b)



The diagram shows the sketch of $y=\sqrt[3]{x}+1$ for $x\geq 0$

(i) Show that the line $y=\frac{1}{4}x+1$ meets the curve $y=\sqrt[3]{x}+1, \quad x\geq 0$, at $x=0, \quad 8$ — 1

(ii) Copy the diagram into your booklet and include on it the line $y=\frac{1}{4}x+1$ — 1

(iii) Find the area enclosed between the line and the curve on your diagram. — 3

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) A quantity Q is decreasing at the rate $\frac{dQ}{dt} = kQ$, k a constant.

Q is in grams and t is time measured in hours.

Initially, $Q = 30$ and 3 hours later, $Q=9$

(i) Show that $Q = 30e^{kt}$ satisfies both the initial condition and the equation $\frac{dQ}{dt} = kQ$ — 2

(ii) Find the one significant figure value for k. — 2

(iii) How much of the quantity, correct to one significant figure, will be left after a further one hour has elapsed? — 2

(b) From P, a ship sails on a bearing of 070° to A, a distance of 150 km. Also, from P, another ship sails on a bearing of 330° to B, a distance of 300 km.

(i) Draw a diagram to show the above information. — 1

(ii) Find the distance from A to B, correct to the nearest kilometre. — 2

(iii) Find the bearing of B from A, correct to the nearest degree. — 3

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) Solve the equation $(3x-1)^4 - 2(3x-1)^2 - 8 = 0$ — 4

(b) A bag contains six discs. Two of the discs have the number 0 on them and the other four discs have the number 1 on them.

Three discs are withdrawn at random.

(i) Find the probability that all of the three discs drawn have the number 1 on them. — 2

(ii) Find the probability that the product of the numbers on the three discs drawn is 0. — 1

(c) Maggie borrows \$10 000 from a bank. This loan plus interest and charges are to be repaid at the end of each month in equal monthly instalments, \$$M$, over five years. Interest of 12% p.a. on the balance owing at the start of each month is added to the account at the end of each month. Additionally, at the end of each month a management charge of \$10 is added to the account.

Let \$$A_n$ be the amount owing after n months.

(i) Show that $A_1 = 10\,000 \times 1.01 - (M - 10)$ — 1

(ii) Show that $A_2 = 10\,000 \times 1.01^2 - (M - 10)(1 + 1.01)$ — 1

(iii) Find \$$M$, correct to the nearest cent. — 3

Marks

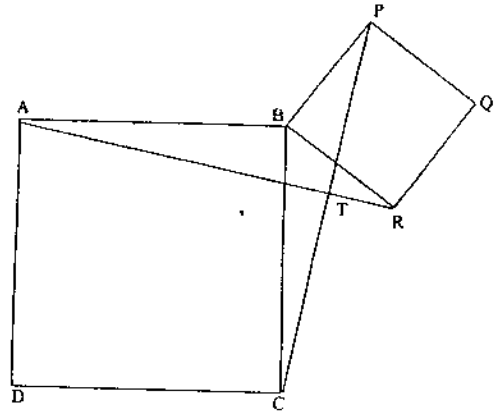
Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) Find

(i) $\int \frac{4x^4}{4x^5+1}\,dx$ **2**

(ii) $\int \frac{4x^5+1}{4x^4}\,dx$ **3**

(b) In the diagram, ABCD and BPQR are squares. AR intersects PC at T.



(i) Copy the diagram into your booklet.

(ii) Prove $\Delta ABR \equiv \Delta CBP$ **3**

(iii) Why does $\angle BAR = \angle PCB$? **1**

(iv) Prove that $AR \perp PC$ **3**

Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a) (i) Sketch the curve $y = 2\sin\left(\frac{x}{2}\right)+1, \quad 0 \le x \le 3\pi$ **2**

(ii) The region bounded by the curve in (i) and the x axis from $x=0$ to $x=\pi$ is revolved about the x axis.

Write down a definite integral which would give the volume of the solid of revolution. **1**

(iii) Use Simpson's Rule with 3 function values to give a one decimal place approximation to the volume in (ii). **3**

(b) A particle moves on the x axis with its velocity, v m/s, given at any time, t seconds, $t \ge 0$, by $v = \frac{1}{\sqrt{2t+1}}$

Initially the particle is at the origin.

(i) Find the initial velocity and the velocity after 12 seconds. **1**

(ii) Sketch the velocity-time graph. **1**

(iii) Find the acceleration of the particle after 12 seconds. **2**

(iv) Find the displacement of the particle as a function of time. **2**

Marks

Question 10 (12 marks) Use a SEPARATE writing booklet.

(a) Find the equation of the directrix of the parabola $(x+1)^2 = 4y+2$ **2**

(b) A circle and two equal squares are to have a total perimeter of 200 cm.

Let the radius of the circle be $4x$ cm.

(i) Show that each side of the squares is $25 - \pi x$ cm. **2**

(ii) Deduce that $0 \le x \le \frac{25}{\pi}$ **1**

(iii) Show that the total area, A cm^2, of the circle and the two squares is given by

$A = 2\pi(8+\pi)x^2 - 100\pi x + 1250$ **2**

(iv) Find the exact value for x for which A is a minimum. **3**

(v) Find the exact value of the minimum area in simplest form. **1**

(vi) Find the exact value for x for which A is a maximum. Give reasons. **1**

End of Paper