Part A

Hand in questions 1 to 4 as a single bundle.
Ensure your name is on the bundle.

Question 1: (12 marks)

a) Differentiate:

(i) $\sin(3x + 1)$ (1)

(ii) $\cos^2 x$. (2)

(iii) $\dfrac{\log_e x}{x}$ (in simplest form) (3)

b)

(i) Show $\cos(2x) = 1 - 2\sin^2(x)$ (1)

(ii) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$ as an exact value (4)

c) Find a primitive of $\dfrac{2x}{x^2+1}$ (1)

Question 2: (7 marks) (*Start a new page*)

a) Given that $\log_a b = 2.75$ and $\log_a c = 0.25$, find the value of:

(i) $\log_a \left(\dfrac{b}{c}\right)$ (1)

(ii) $\log_a (bc)^2$. (2)

b) Solve the equation $3^{2x} + 2.3^x - 15 = 0$. (4)

Question 3: (11 marks) (*Start a new page*)

a) Find the acute angle between the lines $2x + y = 4$ and $x - y = 2$, to the nearest degree. (3)

b) Let $A(-1, 5)$ and $B(3, 2)$ be points in the plane. Find the coordinates of the point C which divides the interval AB externally in the ratio 3 : 1. (2)

c) Solve the inequality $\dfrac{x^2-9}{x} > 0$. (3)

d) (i) State the domain and range of $y = 3\cos^{-1} 2x$

(ii) Hence make a neat sketch of the graph of $y = 3\cos^{-1} 2x$ (3)

Question 4: (14 marks) (*Start a new page*)

a) Consider the function f(x) , where its derivative $f'(x)$ is given by (3)

$$f'(x) = x^2(x-1)(x+2)^3$$

Determine the nature of the stationary points at x = 0, x = 1 and x = -2

b) A Geiger counter is taken into a region after a nuclear accident and gives a reading of 10 000. One year later, the same Geiger counter gives a reading of 9000. It is known that the reading T is given by the formula $T = T_0e^{-kt}$, where T_0 and k are constants and t is the time measured in years from the date of the accident. (5)

(i) Evaluate the constants T_0 and k.

(ii) What is the expected Geiger counter reading 10 years after the accident.

c) The displacement (x metres) from a fixed point, O, at time t seconds is given by $x = \frac{2t^3}{3} - \frac{7t^2}{2} - 4t + 1$. Find (6)

(i) the initial acceleration

(ii) when is the particle at rest

(iii) the distance travelled during the 3rd second.

PART B

Hand in questions 5 to 7 as a single bundle.
Ensure your name is on the bundle.

Question 5: (14 marks) (*Start a new page*)

a) Write down the exact value of 135° in radians. (1)

b) Solve the equation $2\sin^2\theta = \sin\theta\cos\theta$ for $0 \le \theta \le 2\pi$. (4)

c)

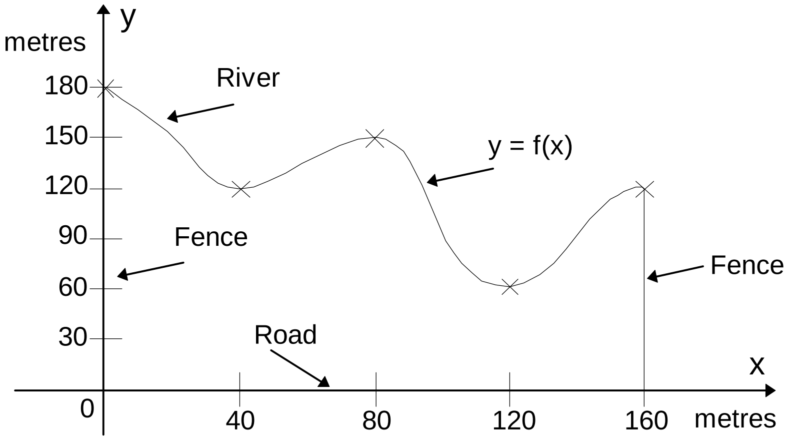
(i) If $\sqrt{3}\sin\theta - \cos\theta = R\sin(\theta - \alpha)$, $R > 0, \alpha$ acute, find *R* and α . (3)

(ii) Hence or otherwise, find all angles θ, where $0 \le \theta \le 2\pi$, for which $\sqrt{3}\sin\theta - \cos\theta = 1$ (3)

d) Prove the following identity: $\frac{\sin A}{\cos A + \sin A} + \frac{\sin A}{\cos A - \sin A} = \tan 2A$ (3)

Question 6: (15 marks) (*Start a new page*)

a)



The diagram is a scale drawing of a paddock bounded by a river, a road, and two fences perpendicular to the road. A farmer wishes to calculate the area of this paddock and has measured the perpendicular distances of the river from the road at intervals of 40 metres. These distances can be read off the diagram.

(i) Take the road as the x axis, the fences as the y axis and the line x = 160, and the river as y = f(x). Copy and complete the following table of values in your examination booklet:

x	0	40	80	120	160
y = f(x)					

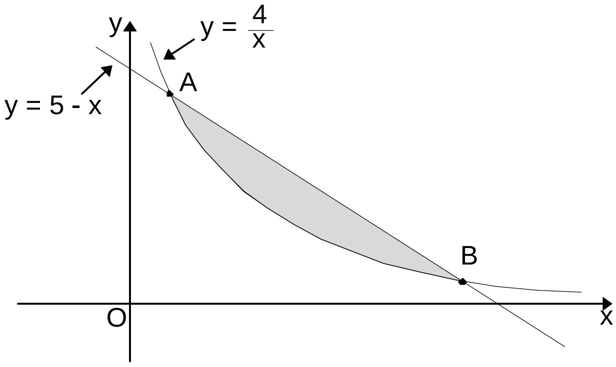
(1)

(ii) Estimate the area of the paddock using Simpson's Rule with five function values. (2)

b) (i) If $y = x\sin^{-1}x + \sqrt{1-x^2}$, show $\dfrac{dy}{dx} = \sin^{-1}x$ (3)

(ii) Hence or otherwise evaluate $\int_0^{\frac{1}{2}} \sin^{-1}x\,dx$ as an exact value. (3)

c)



NOT TO SCALE

The diagram shows the graphs of $y = \frac{4}{x}$ and $y = 5 - x$. The graphs intersect at the points A and B as shown.

(i) Find the x coordinates of the points A and B. (3)

(ii) Find the area of the shaded region between $y = \frac{4}{x}$ and $y = 5 - x$ (leave in exact form) (3)

<u>Question 7</u>: (15 marks) (*Start a new page*)

a) The point $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$.

(i) Show the equation of the chord PQ is given by (3)

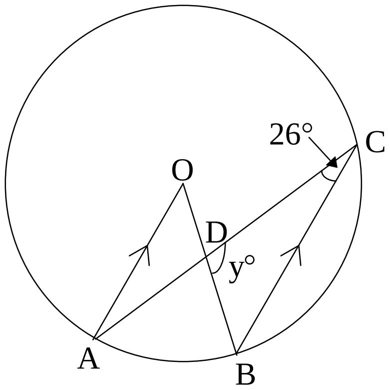
$$y = (\frac{p+q}{2})x - apq$$

(ii) If PQ is a focal chord, show $pq = -1$ (1)

(iii) Given the equation of the tangent at P is $y = px - ap^2$, find the co-ordinates of the point of intersection. (3)

(iv) If PQ is a focal chord, show the tangents intersect on the directrix. (1)

c)



The points A, B and C lie on a circle with centre O. The lines AO and BC are parallel, and OB and AC intersect at D. Also, $\angle ACB = 26°$ and $\angle BDC = y°$, as shown in the diagram. Copy or trace the diagram into your Writing Booklet.

(i) State why $\angle AOB = 52°$. (1)

(ii) Find y. Justify your answer. (2)

d) Let $S_n = 1 \times 2 + 2 \times 3 + . . . + (n - 1) \times n$. Use mathematical induction to prove that, for all integers n with $n \geq 2$, (4)

$S_n = \frac{1}{3}(n-1)n(n+1)$

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \; x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \; a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \; a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \; a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \; a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \; a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \; a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \; a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln(x + \sqrt{x^2 - a^2}), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE: $\ln x = \log_e x, \; x > 0$