SGS Trial 2003 2U

1\. (a) (i) $-2-(-5)=3$ ✓

(ii) $-8-4+2=-10$ ✓

(b) (i) $(2x+3)(x-4)=0$

$x=-\frac{3}{2}$ or 4 ✓✓

(ii) $2x=-7(500-x)$

$2x=-3500+7x$ ✓

$5x=3500$

$x=700$ ✓

(c) $3m^2-12=3(m^2-4)$

$=3(m+2)(m-2)$ ✓

(d) (i) $\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$ ✓

(ii) $\tan x=1$

$x=45°,\ 225°$ ✓✓

(e) (i) $y=5x^2$

$\frac{dy}{dx}=10x$ ✓

(ii) $y=\sin 2x$

$\frac{dy}{dx}=2\cos 2x$ ✓

2 (a) $\int\frac{2x}{x^2+1}\,dx=\log_e(x^2+1)+c$ ✓✓

(give ✓ if they have $\log_e$ (anything)
(don't worry about c)

(b) (i) $y+8=-2(x-6)$ ✓

$y+8=-2x+12$

$2x+y-4=0$

(ii)

Can solve eqns simultaneously (✓)

or Can show that $(-1,6)$ satisfies both eqns (✓)

$2x+y-4=0$ ①

$2x-y+8=0$ ②

add ①+② $4x+4=0$

$x=-1$

$y=2x+8$

$=6$ ✓

so $(-1,6)$ lies on both.

(iii) $QR=\sqrt{(2--1)^2+(12-6)^2}$

$=\sqrt{9+36}$ ✓

$=\sqrt{45}$

$=3\sqrt{5}$

(iv) dist $PL=\left|\frac{2\times 6+(-1)\times(-8)+8}{\sqrt{4+1}}\right|$ ✓

$=\frac{12+8+8}{\sqrt{5}}$

$=\frac{28}{\sqrt{5}}$ ✓

(v) Area of $\Delta PQR = \frac{1}{2} \times$ base $\times$ ht

$= \frac{1}{2} \times 3\sqrt{5} \times \frac{28}{\sqrt{5}}$

$= 3 \times 14$

$= 42 \ u^2$

(c) (i) $\cos C = \frac{a^2+b^2-c^2}{2ab}$

$= \frac{7^2+6^2-4^2}{2 \times 7 \times 6}$

$= \frac{49+36-16}{84}$

$= \frac{69}{84}$

$= \frac{23}{28}$

(ii) 35°

(iii) Area $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 7 \times 6 \times \sin 35^\circ$

$= 12 \ cm^2$

Q3. (a) (i) $y = \frac{\log_e x}{x}$

$\frac{dy}{dx} = \frac{x \cdot \frac{1}{x} - \log_e x}{x^2}$

$= \frac{1 - \log_e x}{x^2}$

(ii) $y = e^x \cos x$

$\frac{dy}{dx} = -\sin x \times e^x + e^x \cos x$

$= e^x(-\sin x + \cos x)$

(b) $a = 13$

$a + 5d = -7$

(i) $13 + 5d = -7$

$5d = -20$

$d = -4$

(ii) $T_3 = a + 2d$

$= 13 + 2(-4)$

$= 5$

(c) i) If two transversals cross 3 parallel lines, then the ratio of the intercepts on one transversal is the same as the ratio of the intercepts on the other transversals

(ii) $\frac{x+2}{11} = \frac{3x}{x+18}$

$(x+2)(x+18) = 33x$

$x^2 + 20x + 36 = 33x$

Q4 (a) $\cot x = -\sqrt{3}$

$\tan x = -\frac{1}{\sqrt{3}}$

related angle is $\frac{\pi}{6}$

$x = \frac{5\pi}{6}$ or $\frac{11\pi}{6}$

(b) (i) $\int_0^1 e^{3x}\,dx$

$= \left[\frac{1}{3}e^{3x}\right]_0^1$

$= \frac{1}{3}e^3 - \frac{1}{3}e^0$

$= \frac{1}{3}(e^3-1)$ or $\frac{1}{3}e^3 - \frac{1}{3}$

(ii) $\int_0^{\frac{\pi}{6}} \sin x\,dx$

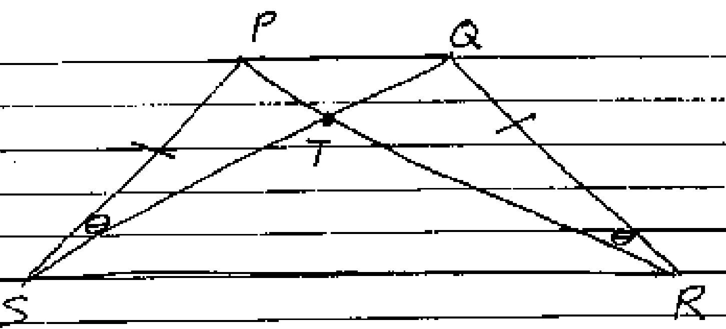
$= \left[-\cos x\right]_0^{\frac{\pi}{6}}$

$= -\cos\frac{\pi}{6} - -\cos 0$

$= -\frac{\sqrt{3}}{2} + 1$

$= 1 - \frac{\sqrt{3}}{2}$

c)



(i) In ΔPTS and ΔQTR

$\angle PSQ = \angle QRP$ (given)

$\angle PTS = \angle QTR$ (vertically opposite angles)

$PS = QR$ (given)

$\therefore \Delta PTS \equiv \Delta QTR$ (AAS)

(ii) $TS = TR$ because the corresponding sides of congruent triangles are equal

(iii) ΔTSR is isosceles ($TS = TR$)

The angles opposite equal sides are equal

so $\angle TSR = \angle TRS$

15.

(a) $\log_2 64 = 3\log_2 x$

$\log 64 = \log x^3$

$64 = x^3$

$x = 4$

(b) (i) 12% p.a $= 1\%$ per month.

Amount owed $= \$3000(1.01)^{24}$

$= \$3809.20$

(ii) find n if $5000 = 3000(1.01)^n$

$1.01^n = \frac{5}{3}$

$n \log 1.01 = \log \frac{5}{3}$

$n = \frac{\log 5/3}{\log 1.01}$

$\doteqdot 51$ months.

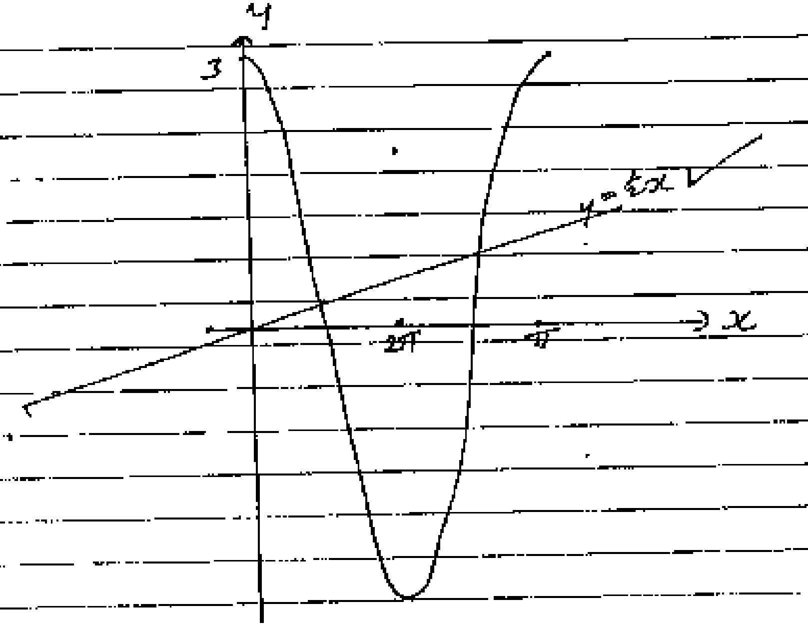
(c) (i) 0

(ii) area $= 4\int_0^{\frac{\pi}{4}} 3\cos 2x \, dx$ (or equivalent entry)

$= 12\left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$

$= 6\left[\sin\frac{\pi}{2} - \sin 0\right]$

$= 6 \quad u^2$



number of solutions is 2.

(iii) ...

Q6.

(a) $x = t^3 - 21t^2$

(i) $v = \dot{x} = 3t^2 - 42t$

(ii) $a = \ddot{x} = 6t - 42$

(iii) $t = 2$ $x = 2^3 - 21 \times 2^2$
$= -76$ m
76 m to the left of the origin

(iv) find t when $x = 0$
$t^3 - 21t^2 = 0$
$t^2(t-21) = 0$
$t = 0$ or 21 seconds

(v) stationary when $v = 0$
$3t^2 - 42t = 0$
$3t(t-14) = 0$
$t = 0$ or 14 seconds

(vi) $6t - 42 = 0$
$t = 7$ seconds when $a = 0$

$t = 7$, $x = 7^3 - 21 \times 49$
$= -686$ m

(vii) $t^3 - 21t^2 > 0$
$t^2(t-21) > 0$
$t - 21 > 0$ since $t^2 > 0$
$t > 21$
$\therefore$ It is to the right when t is greater than 21 seconds

(b) $\int_0^3 (kx^2 - 6x - 8)\,dx = -15$
area is below axis

$\left[\frac{k}{3}x^3 - 3x^2 - 8x\right]_0^3 = -15$

$(9k - 27 - 24) - (0) = -15$

$9k = -15 + 51$

$9k = 36$

$k = 4$

12

(a) (i) $y = \frac{x^6}{64}$

$64y = x^6$

$x^2 = \sqrt[3]{64y}$

$= 4y^{\frac{1}{3}}$

(ii) $V = \pi \int_0^{64} x^2 \, dy$

$= \pi \int_0^{64} 4y^{\frac{1}{3}} \, dy$

$= 4\pi \left[\frac{3y^{4/3}}{4} \right]_0^{64}$

$= 3\pi \left[y^{4/3} \right]_0^{64}$

$= 3\pi \times 256$

$= 768\pi \ u^3$

(b) (i) $t=0$, $R = 15 + \frac{10}{1}$

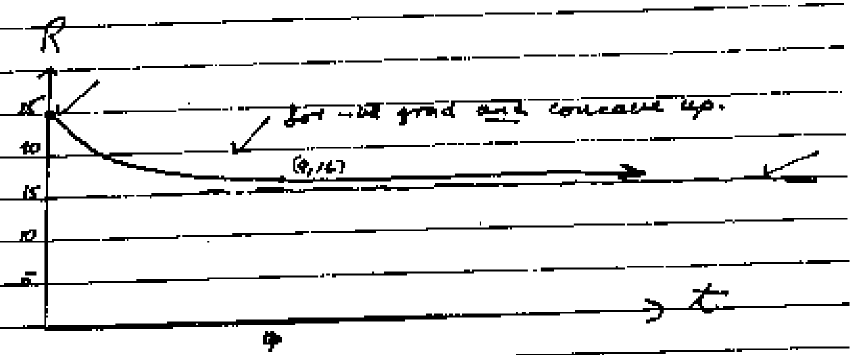
$= 25$ l/min

(ii) $t=9$, $R = 15 + \frac{10}{10}$

$= 16$ l/min

(iii) as $t \to \infty$, $\frac{10}{1+t} \to 0$ and $R \to 15$

(iv)



v) Fuel burned $= \int_0^9 \left(15 + \frac{10}{1+t}\right) dt$ (or equiv integral)

$= \left[15t + 10\log_e(1+t)\right]_0^9$

$= \left(135 + 10\log 10\right) - \left(0 + 10\log 1\right)$

$= 135 + 23.025$

$\doteqdot 158$ L

8.

(a) $\displaystyle\int_{\frac{1}{3}}^{\frac{1}{2}} \sec^2 \frac{\pi x}{2}\, dx$

$= \frac{2}{\pi}\left[\tan \frac{\pi x}{2}\right]_{\frac{1}{3}}^{\frac{1}{2}}$

$= \frac{2}{\pi}\left(\tan \frac{\pi}{4} - \tan \frac{\pi}{6}\right)$

$= \frac{2}{\pi}\left(1 - \frac{1}{\sqrt{3}}\right)$

(b) $S_n = 2n + 3n^2$

$S_{n-1} = 2(n-1) + 3(n-1)^2$

$= 2n - 2 + 3n^2 - 6n + 3$

$= 3n^2 - 4n + 1$

$T_n = S_n - S_{n-1}$

$= 2n + 3n^2 - (3n^2 - 4n + 1)$

$= 2n + 3n^2 - 3n^2 + 4n - 1$

$= 6n - 1$

(c) (i) Number left $= 256(0.75)^4$

$= 81$

(ii) 1st lot of P plants becomes $P(0.75)^3$ Jan 2001

2nd lot of P plants becomes $P(0.75)^2$ Jan 2002

3rd lot of P plants become $P(0.75)^1$ Jan 2003

So, number of plants $= 81 + P(0.75^3 + 0.75^2 + 0.75)$

$= 81 + P$ (sum of GP, $a = 0.75$, $r = 0.75$, $n = 3$)

$= 81 + P\left(\frac{0.75(1 - 0.75^3)}{1 - 0.75}\right)$

$= 81 + P\left(\frac{\cancel{0.75}^{\,3}(1 - 0.75^3)}{\cancel{0.25}}\right)$

$= 81 + 3P(1 - 0.75^3)$

(iii) $81 + 3P(1 - 0.75^3) \geq 200$

$3P(1 - 0.75^3) \geq 119$

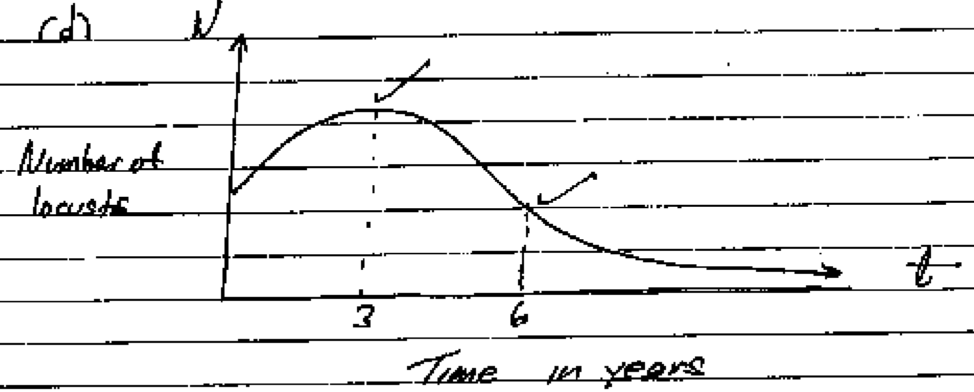
$P \geq \frac{119}{3(1 - 0.75^3)}$

$P \geq \frac{119}{1.73437}$

$P \geq 68.6$

so P is 69.

(d)



9.

(a) $\ddot{x} = e^{-3t}$

(i) $v = \dot{x} = \int e^{-3t}\,dt$

$\dot{x} = -\frac{1}{3}e^{-3t} + c$

$t = 0,\quad 0 = -\frac{1}{3}e^{0} + c$

so $c = \frac{1}{3}$

and $v = -\frac{1}{3}e^{-3t} + \frac{1}{3}$

(ii) It is stationary when $-\frac{1}{3}e^{-3t} + \frac{1}{3} = 0$

so solve $\frac{1}{3}e^{-3t} = \frac{1}{3}$

$e^{-3t} = 1$

this has only one solution, $t = 0$

(iii) Distance $= \int_0^3 \left(-\frac{1}{3}e^{-3t} + \frac{1}{3}\right) dt$

$= \left[\frac{1}{9}e^{-3t} + \frac{1}{3}t\right]_0^3$

$= \left(\frac{1}{9}e^{-9} + 1\right) - \left(\frac{1}{9}e^{0} + 0\right)$

$= \frac{1}{9}e^{-9} + \frac{8}{9}$

(b) $\frac{dP}{dt} = -kP$

(i) $P = P_0 e^{-kt}$

$\frac{dP}{dt} = -kP_0 e^{-kt}$

$= -kP$

(ii) when $t=0$, $P = 3060$

so $P_0 = 3060$

(iii) $t = 2,\quad 1530 = 3060e^{-2k}$

$e^{-2k} = \frac{1}{2}$

$-2k = \log_e \frac{1}{2}$

$k = -\frac{1}{2}\log_e \frac{1}{2}$

$= -\frac{1}{2}\log_e 2^{-1}$

$= \frac{1}{2}\log 2$

(iv) $t = 3,\quad P = 3060e^{-3/2 \log 2}$

$= 3060 \times 2^{-3/2}$

$= 1082$

(v) $50 = 3060 e^{(-\frac{1}{2}\ln 2)t}$

$\frac{5}{306} = e^{(-\frac{1}{2}\ln 2)t}$

$\log_e \frac{5}{306} = (-\frac{1}{2}\ln 2)\, t$

$t = \frac{\log \frac{5}{306}}{-\frac{1}{2}\log 2}$

$= 11.8704$ years

So light out ~~during~~ 11.8 years after August 2000
ie during 2012, during May 2012

Q 10.

(a) (i) $P = 2r + r\theta$

$= r(2+\theta)$

(ii) $36 = r(2+\theta)$

so $r = \frac{36}{2+\theta}$

$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2}\left(\frac{36}{2+\theta}\right)^2 \theta$

$= \frac{648\,\theta}{(2+\theta)^2}$

(iii) $\frac{dA}{d\theta} = \frac{(2+\theta)^2 648 - 648\,\theta \times 2 \times (2+\theta)}{(2+\theta)^4}$

$= \frac{(2+\theta)648 - 1296\,\theta}{(2+\theta)^3}$

$= \frac{1296 - 648\,\theta}{(2+\theta)^3}$

At min/max, $\frac{dA}{d\theta} = 0$

solve $1296 - 648\,\theta = 0$

$\theta = \frac{1296}{648}$

$= 2$

check for max

θ	1	2	3
$\frac{dA}{d\theta}$	$\frac{2\times 648 - 2\times 144}{27}$	0	$\frac{6\times 648 - 6\times 64}{53}$
	+ve	—	−ve

So we have maximum area at $\theta = 2$.

$$A = \frac{648 \times 2}{4\pi}$$
$$= 81 \text{ cm}^2$$

(b)

(i) (s,t) satisfies both equations

so $t^2 = 4as$ ①

and $ls + mt = 1$ ②

from ① $s = \frac{t^2}{4a}$

from ② $s = \frac{1-mt}{l}$

so $\frac{t^2}{4a} = \frac{1-mt}{l}$

$4a - 4amt = lt^2$

$lt^2 + 4amt - 4a = 0$

(ii) If the line is a tangent then there is only 1 value of t that satisfies $lt^2 + 4amt - 4a = 0$

so we want $\Delta = 0$

say something about 1 value, or explain

$(4am)^2 + 4(4a)(l) = 0$

$16a^2m^2 + 16al = 0$

$a^2m^2 + al = 0$

$am^2 + l = 0$ we can divide by a since $a \neq 0$ because $y^2 = 4ax$ is a parabola

(iii) For the line to be a tangent we want $l = -am^2$

so we have $-am^2x + my = 1$

$my = 1 + am^2x$

$y = amx + \frac{1}{m}$