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2013 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

Morning Session Monday, 5 August 2013

General Instructions

- Reading time 5 minutes
- Working time 3 hours
- Write using black or blue pen Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided on a separate sheet
- In Questions 11–16, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks - 100

Section I

Pages 2-6

10 marks

- Attempt Questions 1–10
- Allow 15 minutes for this section

Section II

Pages 7-14

90 marks

- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

Disclaimer

Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, Principles for Setting HSC Examinations in a Standards-Referenced Framework (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

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Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

1 If z = 2 + 3i and w = -5 - 2i, what is the value of zw?

- (A) -4
- (B) -3+i
- (C) -4-19i
- (D) -16-19i

What is the gradient of the tangent to the curve $-2x^2 + y^2 + y = 0$ at the point (1,1)?

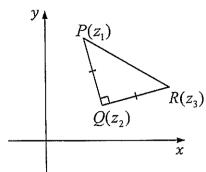
- (A) 1
- (B) $\frac{4}{3}$
- (C) $\frac{3}{2}$
- (D) 2

3 The point $P\left(ct, \frac{c}{t}\right)$ lies on the hyperbola $xy = c^2$.

What is the x-intercept of the tangent to the hyperbola at P?

- (A) (2ct, 0)
- (B) $\left(\frac{2c}{t}, 0\right)$
- (C) $\left(ct-\frac{c}{t^3},0\right)$
- (D) $\left(\frac{c}{t}-ct^3,0\right)$

The vertices of the triangle PQR are represented by the complex numbers z_1 , z_2 and z_3 respectively. The triangle PQR is isosceles and right-angled at Q, as shown in the diagram.



Which of the following statements is true?

(A)
$$z_2 - z_1 = i(z_3 - z_2)$$

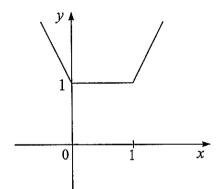
(B)
$$z_1 - z_2 = i(z_3 - z_2)$$

(C)
$$z_2 - z_1 = i(z_1 - z_3)$$

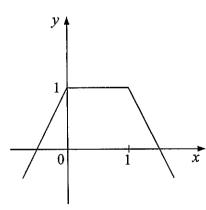
(D)
$$z_1 - z_2 = i(z_1 - z_3)$$

5 Which of the following graphs could represent the graph of y = |x| + |x-1|?

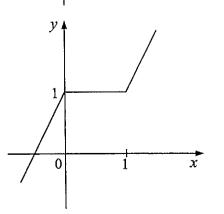
(A)



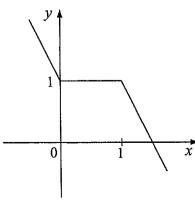
(B)



(C)



(D)



6 Which of the following, for x > 0, is an expression for $\int \frac{1}{x^3 + x} dx$?

(A)
$$\log_e \left(x \sqrt{x^2 + 1} \right) + C$$

(B)
$$\log_e\left(x(x^2+1)\right) + C$$

(C)
$$\log_e \left(\frac{x}{\sqrt{x^2+1}}\right) + C$$

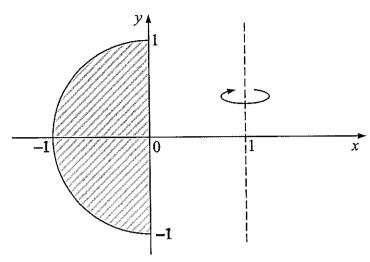
(D)
$$\log_e \left(\frac{x}{x^2+1}\right) + C$$

A stone of mass m is dropped from rest and falls in a medium in which the resistance is directly proportional to the square of the velocity v. Suppose mk is the constant of proportionality and that the displacement downwards from the initial position is x at time t. The acceleration due to gravity is g.

Which of the following is true?

- (A) The terminal velocity is $\frac{g}{k}$.
- (B) As $t \to \infty$, $x \to L$ where L is a positive constant.
- (C) The equation of motion is given by $v \frac{dv}{dx} = g kv^2$.
- (D) The time for the stone to reach velocity V is given by $\int_0^V g kv^2 dv$.

8 The diagram shows the graph $x^2 + y^2 = 1$ for $-1 \le x \le 0$. The region bounded by the graph and the y-axis is rotated about the line x = 1 to form a solid.



Which integral represents the volume of the solid?

(A)
$$2\pi \int_{-1}^{0} (1+x)\sqrt{1-x^2} dx$$

(B)
$$2\pi \int_{-1}^{0} (1-x)\sqrt{1-x^2} dx$$

(C)
$$4\pi \int_{-1}^{0} (1+x)\sqrt{1-x^2} dx$$

(D)
$$4\pi \int_{-1}^{0} (1-x)\sqrt{1-x^2} dx$$

- The polynomial p(x) of degree 4 has real coefficients.
 - p(x) has roots α , β , γ and δ and it is known that $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = -8$.

Which of the following must be true?

- (A) p(x) has all its roots real.
- (B) p(x) has one real and three imaginary roots.
- (C) p(x) has two real and two imaginary roots.
- (D) p(x) has at least two imaginary roots.
- 10 If f(x) is a non-zero odd function with period π , which of the following statements is false?

(A)
$$\int_0^{2\pi} f(x) dx = 0$$

(B)
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x) dx = 2 \int_{0}^{\frac{\pi}{2}} f(x) dx$$

(C)
$$\int_{0}^{\pi} f(x) \, dx = -\int_{0}^{\pi} f(-x) \, dx$$

(D)
$$\int_{\alpha}^{\alpha+\pi} f(x) dx = \int_{0}^{\pi} f(x) dx \text{ for any real number } \alpha.$$

Section II

90 marks

Attempt Questions 11–16

Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

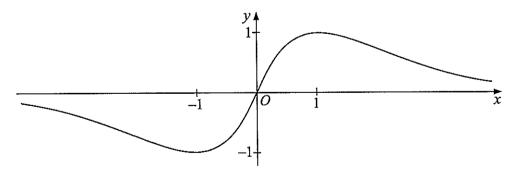
In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Let z = 2 + 3i and w = -3 - 4i.

(i) Find $z + \overline{w}$

- (ii) Express $\frac{w}{z}$ in the form a+ib, where a and b are real numbers.
- (b) Sketch the region in the complex plane which satisfies $\frac{\pi}{6} < \arg(z) < \frac{5\pi}{6}$ and $\frac{1}{2} \le \operatorname{Im}(z) \le 2$.
- (c) Find $\int \frac{dx}{(9-x^2)^{\frac{3}{2}}}$ using the substitution $x = 3\sin\theta$. Give your answer in terms of x.
- (d) The following diagram shows the graph of $f(x) = \frac{2x}{x^2 + 1}$.



Draw separate one-third page diagrams of the graphs of each of the following.

(i)
$$y = \left[f(x) \right]^2$$

(ii)
$$y = \sqrt{f(x)}$$

(iii)
$$y = \frac{1}{f(x)}$$

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Find the square roots of -24-10i.

2

(ii) Hence, or otherwise, solve $x^2 - (1-i)x + 6 + 2i = 0$.

2

(b) Use integration by parts to find $\int \frac{\ln x}{x^2} dx$.

2

(c) Using the substitution $t = \tan \frac{x}{2}$, or otherwise, evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{2 + \cos x}$.

4

(d) An ellipse is defined by the parametric equations:

$$x = 2\cos\theta$$

$$y = 3\sin\theta$$

for $0 \le \theta < 2\pi$.

(i) Find the Cartesian equation of the ellipse.

1

(ii) Find the eccentricity of the ellipse.

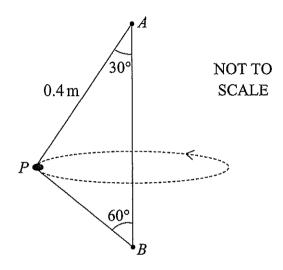
1

(iii) Sketch the ellipse showing the intercepts, foci and directrices.

3

Question 13 (15 marks) Use a SEPARATE writing booklet.

- (a) A group of 30 students is to be divided into three groups consisting of 7, 8 and 15 students. In how many ways can this be done?
- (b) (i) Find a and b such that x = 2 is a double root of $p(x) = x^4 + ax^3 + x^2 + b$.
 - (ii) For the values of a and b above, factorise p(x) over the real numbers.
- (c)



A particle P of mass 0.3 kg is attached to one end of each of two light inextensible strings of different lengths. The longer string is also attached to a fixed point A and the shorter string is also attached to a fixed point B, which is vertically below A.

AP makes an angle of 30° with the vertical and is 0.4 m long. PB makes an angle of 60° with the vertical. The particle moves in a horizontal circle with constant angular speed and with both strings taut. The tension in the string AP is 5 N. Assume the acceleration due to gravity is $10\,\mathrm{ms}^{-2}$.

- (i) Find the tension in the string PB.
 - orrect to 1 decimal place.

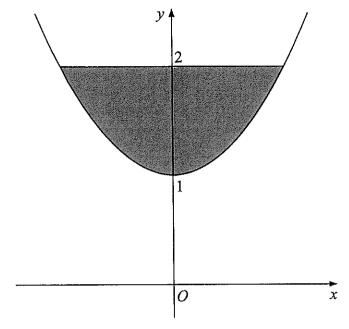
2

(ii) Calculate the angular velocity of the particle P correct to 1 decimal place.

Question 13 continues of page 10

Question 13 (continued)

(d) The base of a solid, S, is the region enclosed by the parabola $y = x^2 + 1$ and the line y = 2.



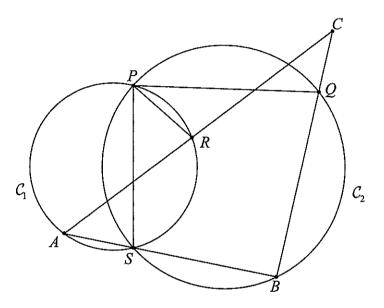
Each cross section of S perpendicular to the y-axis is a rectangle. The height of the rectangle that is y units from the origin is $\frac{1}{2}y$ units.

- (i) Show that the area of the rectangle y units from the origin is given by $y\sqrt{y-1}$ square units.
- (ii) Hence find the volume of the solid S. 3

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

- (a) Consider the polynomial $p(x) = x^3 x^2 21x + 45$ with roots α , β and γ .
 - (i) Find the monic polynomial with roots $\alpha 3$, $\beta 3$, $\gamma 3$.
 - (ii) Hence solve p(x) = 0.
- (b) Two circles C_1 and C_2 meet at P and S. Points A and R lie on C_1 and points B and Q lie on C_2 . AB passes through S and AR produced meets BQ produced at C, as shown in the diagram.

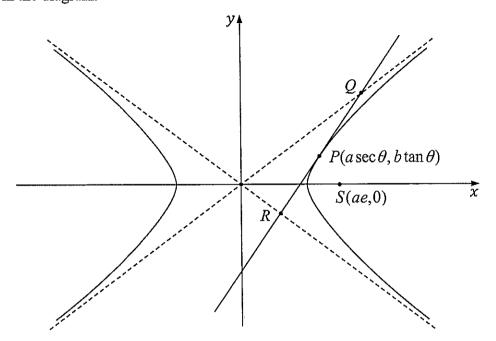


- (i) Prove that $\angle PRA = \angle PQB$.
- (ii) Prove that the points P, R, Q and C are concyclic.

Question 14 continues on page 12

Question 14 (continued)

(c) The point $P(a \sec \theta, b \tan \theta)$ lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with focus S(ae, 0). The tangent to the hyperbola at P meets the asymptotes of the hyperbola at Q and R, as shown in the diagram.



- (i) Show that the equation of the tangent to the hyperbola at P is given by $\frac{x}{a}\sec\theta \frac{y}{b}\tan\theta = 1.$
- (ii) Show that Q has coordinates $\left(\frac{a}{\sec\theta \tan\theta}, \frac{b}{\sec\theta \tan\theta}\right)$.

The coordinates of R are given by $\left(\frac{a}{\sec\theta + \tan\theta}, \frac{-b}{\sec\theta + \tan\theta}\right)$. (Do NOT prove this).

(iii) Prove that $\left|\tan \angle QSR\right| = \frac{b}{a}$.

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Evaluate
$$\int_0^{\frac{\pi}{4}} \tan x \, dx.$$
 2

(ii) Suppose
$$I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$$
.

Given that $I_n = \frac{1}{n-1} - I_{n-2}$ for any integer $n \ge 2$, find the value of
$$\int_0^{\frac{\pi}{4}} \tan^5 x \, dx$$
.

- (b) (i) Find the five fifth roots of $z^5 = 1$ and plot these on an Argand diagram.
 - (ii) Express $z^5 1$ as the product of real linear and quadratic factors.

(iii) Prove that
$$\cos \frac{2\pi}{5} + \cos \frac{4\pi}{5} = -\frac{1}{2}$$
.

(c) A projectile starting from the origin has acceleration given by

$$\frac{d^2x}{dt^2} = -6\frac{dx}{dt} - 9x$$

where x is the displacement from the origin at time t.

The solution to this equation is known to be of the form, $x(t) = f(t)e^{-3t}$, for some function f(t).

- (i) Show that f(t) = At + B, where A and B are constants.
- (ii) Find the value of B.
- (iii) Assuming that A is positive, at what time is the displacement a maximum? 2 You do not need to prove a maximum is attained.

Examiners

Gerry Sozio (Convenor) Catholic Education Office, Wollongong

Peter Brown University of New South Wales, Kensington

Robert Muscatello Mount Carmel Catholic High School, Varroville

Frank Reid University of New South Wales, Australian Catholic University

Thanom Shaw SCEGGS, Darlinghurst

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CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW 2013 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION **MATHEMATICS EXTENSION 2**

Section I 10 marks

Questions 1-10 (1 mark each)

Question 1 (1 mark) Outcomes Assessed: E3

Targeted Performance Bands: E2

Solution	Answer	Mark
(2+3i)(-5-2i)		
$= -10 - 4i - 15i - 6i^2$	C	1
=-4-19i		7/2114

Question 2 (1 mark) Outcomes Assessed: E6

Targeted Performance Bands: E3

Solution	Answer	Mark
Using implicit differentiation		
$-4x + 2y\frac{dy}{dx} + \frac{dy}{dx} = 0$		
$\frac{dy}{dx} = \frac{4x}{2y+1}$	В	1
Substituting (1, 1), $\frac{dy}{dx} = \frac{4}{3}$		

Outcomes Assessed: E4 Question 3 (1 mark)

Targeted Performance Bands: E2

Solution	$=\frac{-c^2}{x^2}.$	At $P\left(ct, \frac{c}{t}\right)$, $\frac{dy}{dx} = \frac{-c^2}{c^2t^2} = \frac{-1}{t^2}$.
	$\frac{dy}{dx} = \frac{1}{x}$	$ At P(\epsilon) $

Equation of the tangent at
$$P$$
:
$$y - \frac{c}{t} = \frac{-1}{t^2}(x - ct)$$

$$x + t^2 y = 2ct$$

4

The x-intercept is $(2\alpha, 0)$.

Outcomes Assessed: E3 Question 4 (1 mark)

Targeted Performance Bands: E3

Solution	Answer	Mark
$\overline{QP} = i \times \overline{QR}$	ä	
$\begin{vmatrix} z_1 - z_2 = i(z_3 - z_2) \end{vmatrix}$	ì	1

Outcomes Assessed: E6 Question 5 (1 mark)

Targeted Performance Bands: E2

Mark	-	
Answer	¥	
Solution		. t

Outcomes Assessed: E8 Question 6 (1 mark)

Targeted Performance Bands: E3

Mark

Answer

Solution	Answer	Mark
$\int \frac{1}{x^3 + x} dx = \int \left(\frac{1}{x} + \frac{-x}{x^2 + 1} \right) dx$		
$= \log_e x - \frac{1}{2} \log_e \left(x^2 + 1 \right) + C$	U	H
$= \log_e \frac{x}{\sqrt{x^2 + 1}} + C$		

Question 7 (1 mark)

Targeted Performance Bands: E3 Outcomes Assessed: E5

Solution	Answer	Mark
The equation of motion is		
$m\ddot{x} = mg - mk\alpha^2$		
$\ddot{x} = g - kv^2$	Ü	-
Since $\ddot{x} = v \frac{dv}{dx}$, $v \frac{dv}{dx} = g - kv^2$.		

Question 8 (1 mark)

Outcomes Assessed: E7

Targeted Performance Bands: E3

Solution	Answer	Mark
$V = 2\pi {\binom{n}{2} (1-x) \times 2\sqrt{1-x^2} dx}$		
\ \	Д	_
$=4\pi \left[(1-x)\sqrt{1-x^2} dx \right]$		
\ \		

Outcomes Assessed: E4 Question 9 (1 mark)

Targeted Performance Bands: E4

Solution	Answer	Mark
p(x) has at least 2 imaginary roots.	D	

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Question 10 (1 mark) Outcomes Assessed: E8

Targeted Performance Bands: E4

Solution	Answer	Mark
$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x) dx \neq 2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x) dx$	В	·
- <u></u>		

Section II 60 marks Question 11 (15 marks)

(a) (i) (1 mark)
Outcomes assessed: E3

Targeted Performance Bands: E2

1	0			
		Criteria	Mark	
L.,,,,,,	Evaluates	₩ correctly	1	
	Contraction of the Contraction o			

Sample answer:

$$z + i\overline{v} = 2 + 3i + (-3 + 4i)$$

= -1 + 7i

(a) (ii) (2 marks)

Targeted Performance Bands: E3 Outcomes assessed: E3

	Criteria	
an Scient & city or manner war and		
The Court of the C		

Mark

•	Criteria • Correct answer
•	Attempts to realize the denominator
Sa	Sample answer:
g	3-4i $-3-4i$ $2-3i$
7	$\frac{2+3i}{2+3i} = \frac{x}{2+3i} \times \frac{x}{2-3i}$
	-18+i
	13
	-18 i
	13 13

(b) (3 marks)

Outcomes assessed: E6

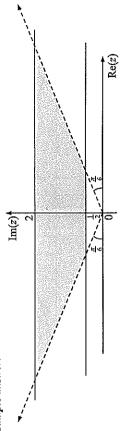
Targeted Performance Bands: E3

Correct solution	3
Both graphs correct 2	2
ONE correct graph	1

Criteria

Mark

Sample answer:



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(c) (4 marks)

Outcomes assessed: E8

Targeted Performance Bands: E3

Criteria	Mark
Correct answer in terms of x	4
Correct answer in terms of θ	3
Significant working towards answer	2
Correctly substitutes $x = 3\sin\theta$	-

Sample answer:

$$\frac{dx}{(9-x^2)^{\frac{1}{2}}} = \int \frac{3\cos\theta \, d\theta}{(9-9\sin^2\theta)^{\frac{1}{2}}}$$
$$= \int \frac{\cos\theta \, d\theta}{9\cos^3\theta}$$
$$= \frac{1}{9}\int \sec^2\theta \, d\theta$$
$$= \frac{1}{9}\int \tan\theta + C$$
$$= \frac{x}{9\sqrt{9-x^2}} + C$$

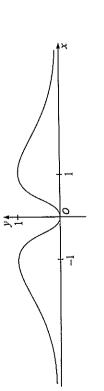
(d) (i) (2 marks)

Outcomes assessed: E6

Targeted Performance Bands: E3

3	largetea i ei formance Dannas. Lo	
L	Criteria	Mark
•	Correct graph, including turning point at origin and (±1,1) and asymptotic features at	2
	6.1	
Ľ	Date	,
•	Dasic stabe	-

Sample answer:



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(d) (ii) (1 mark)

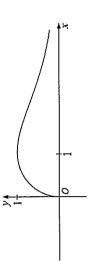
Targeted Performance Bands: E2 Outcomes assessed: E6

• Correct graph, including maximum turning point at (1,1) and vertical tangent at the origin

Criteria

Mark

Sample answer:



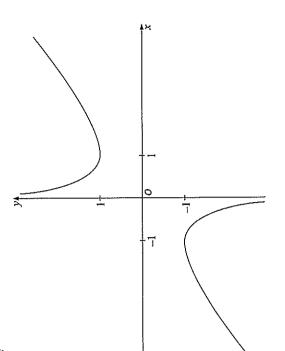
(d) (iii) (2 marks)

Targeted Performance Bands: E2 Outcomes assessed: E6

Sanas: E4	Criteria Mark	iding vertical asymptote $x = 0$ and turning points at (1,1) and	7
largetea rerjormance banas.	- Attachen	Correct graph, including v	(-1,-1).

Basic shape

Sample answer:



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Question 12 (15 marks)

(a) (i) (2 marks)

Outcomes assessed: E3

Targeted Performance Bands: E2

	Criteria	Mark	
	Correct answer	2	
١.	Significant progress solving simultaneously for the real and imaginary parts	1	

Sample answer:

Let $-24-10i = (a+ib)^2$ where a and b are real.

Then,
$$a^2 - b^2 + 2abi = -24 - 10i$$

Equating real and imaginary parts: $a^2 - b^2 = -24$ and 2ab = -10

Substituting
$$b = \frac{-5}{a}$$
 into $a^2 - b^2 = -24$, gives

$$a^4 + 24a^2 - 25 = 0$$
$$(a^2 + 25)(a^2 - 1) = 0$$

 $a = \pm 1$ since a is real.

.. The square roots of -24-10i are 1-5i, -1+5i.

(a) (ii) (2 marks)

Targeted Performance Bands: E2 Outcomes assessed: E3

 Significant progress solving the quadratic equation e.g. applies the quadratic formula Correct answer

Criteria

Sample answer:

$$x = \frac{1 - i \pm \sqrt{(1 - i)^2 - 4(6 + 2i)}}{2}$$

$$= \frac{1 - i \pm \sqrt{-24 - 10i}}{2}$$

$$= \frac{1 - i \pm (1 - 5i)}{2}$$
$$= 1 - 3i, 2i$$

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(b) (2 marks)

Outcomes assessed: E8

Targeted Performance Bands: E2

	CHELLA	LITTER	
-	The property of the second sec	(
•	Correct angiver		
•	COLLOCI ALIS W CL		
	+ - +		
•	Correctly applies the method of integration by parts	-	

Sample answer:

$$\int \frac{\ln x}{x^2} dx = \int \ln x \cdot x^{-2} dx$$
$$= \ln x \left(-x^{-1} \right) - \int \left(-x^{-1} \right) \frac{1}{x} dx$$

$$= -\frac{\ln x}{x} + \int x^{-2} dx$$
$$= -\frac{\ln x}{x} - \frac{1}{x} + C$$

(c) (4 marks)

Outcomes assessed: E8

Targeted Performance Bands: E2

	Critéria	Mark	
•	Correct answer	4	
•	Correct integration and correct treatment of the limits either by substitution or rewriting the integrated expression in terms of x	3	
•	Correct expression for the integrand using the substitution $t = \tan \frac{x}{2}$	2	
•	• Significant progress towards writing the integrand using the substitution $t = \tan \frac{x}{2}$	-	

Sample анswer:

$$\int_{0}^{\frac{\pi}{2}} \frac{dx}{2 + \cos x} = \int_{0}^{\infty} \frac{1}{2 + \frac{1 - t^{2}}{1 + t^{2}}} \times \frac{2dt}{1 + t^{2}}$$

$$= \int_{0}^{1} \frac{2dt}{t^{2} + 3}$$

$$= \left[\frac{2}{\sqrt{3}} \tan^{-1} \frac{t}{\sqrt{3}} \right]_{0}^{1}$$

$$= \frac{\sqrt{3\pi}}{9}$$

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(d) (i) (1 mark)

Outcomes assessed: E4

Targeted Performance Bands: E2

뇓	
Mar	
• Correct answer	

Sample answer:

(d) (ii) (1 mark)
Outcomes assessed: E4

Targeted Performance Bands: E2

Mark	1
a	Correct answer

Sample answer:

$$4 = 9(1 - e^2)$$

$$e = \frac{\sqrt{5}}{3}$$

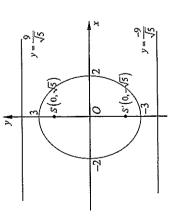
(d) (iii) (3 marks)

Outcomes assessed: E4

Targeted Performance Bands: E3

	Critéria	IVIAI N.
•	Entire correct diagram	3
•	Diagram of an ellipse with TWO of the following shown: intercepts, foci, directrices	2
•	Diagram of an ellinse with ONE of the following shown: intercepts, foci, directrices	I

Sample answer:



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Question 13 (15 marks)

(a) (1 mark)

Targeted Performance Bands: E3 Outcomes assessed: HE3

Criteria Correct answer

Mark

Sample answer:

$$\frac{30!}{7!8!15!}$$
 or $\binom{30}{7}\binom{23}{8}\binom{15}{15}$

(b) (i) (3 marks)

Outcomes assessed: E4

Targeted Performance Bands: E3

Criteria	Mark
Correct answer	m
Correctly formulates simultaneous equations	2
Substitutes $x=2$ into either $n(x)$ or $n'(x)$	-

Sample answer:

$$p(\bar{x}) = x^4 + ax^3 + x^2 + b$$

$$p'(x) = 4x^3 + 3ax^2 + 2x$$

$$p'(2) = 0 \Rightarrow 32 + 12a + 4 = 0 \Rightarrow a = -3$$

$$p(2) = 0 \Rightarrow 16 - 24 + 4 + b = 0 \Rightarrow b = 4$$

 $\therefore a = -3$ and b = 4

(b) (ii) (1 mark)

Outcomes assessed: E4

Targeted Performance Bands: E3

Criteria

Mark

$$p(x) = x^4 - 3x^3 + x^2 + 4$$

$$(x^4 - 3x^3 + x^2 + 4) \div (x^2 - 4x + 4) = x^2 + x + 1$$

\therefore\tau (x) = (x - 2)^2 (x^2 + x + 1)

1

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(c) (i) (2 marks)

Outcomes assessed: E5

Targeted Performance Bands: E3

	Criteria	Mark	
	A STATE OF THE PERSON NAMED IN COLUMN TO THE PERSON NAMED IN COLUM	,	
•	Correct answer	7	
1	The state of the s		
۰	Significant progress towards resolving forces vertically		

Sample answer: Let T be the tension in the string PB.

$$5\cos 30^{\circ} = T\cos 60^{\circ} + mg$$

$$\frac{5\sqrt{3}}{2} = \frac{T}{2} + 0.3 \times 10$$

$$T = 5\sqrt{3} - 6 \text{ Newtons}$$

($\approx 2.66 \text{ Newtons}$)

(c) (ii) (3 marks)

Outcomes assessed: E5

Targeted Performance Bands: E3

	Criteria	IVIALK
•	Correct answer	3
	Resolves forces horizontally	2
	Calculates horizontal forces on P	I

Sample answer: Let P move in a horizontal circle of radius r metres at ω radians/second.

$$r = 0.4 \sin 30^{\circ} = 0.2 \text{ metres}$$

Resolving forces horizontally

$$5\sin 30^{\circ} + T \sin 60^{\circ} = m\omega^{2}r$$

$$\frac{5}{2} + \left(5\sqrt{3} - 6\right)\frac{\sqrt{3}}{2} = 0.3 \times \omega^{2} \times 0.2$$

$$\omega = \sqrt{\frac{50\left(10 - 3\sqrt{3}\right)}{3}}$$

$$\omega = \sqrt{\frac{50\left(10 - 3\sqrt{3}\right)}{3}}$$

$$\approx 8.9 \text{ radians / second (1 d.p.)}$$

(d) (i) (2 marks)

Outcomes assessed: E7

Targeted Performance Bands: E3

	Criteria	Mark
•	Correct answer	2
•	Make progress towards finding the length of the base of the rectangle	1

Sample answer:

Parabola has equation $y = x^2 + 1 \Rightarrow x = \pm \sqrt{y - 1}$

 $\cdot \cdot y$ units from the origin, the base of the rectangular cross-section has length $2\sqrt{y-1}$. The height of the rectangular cross-section is $\frac{1}{2}y$ (given). Hence, the area of the rectangle y units from the origin is $2\sqrt{y-1} \times \frac{1}{2}y = y\sqrt{y-1}$ square units.

(d) (ii) (3 marks)

Outcomes assessed: E8

Targeted Performance Bands: E3

ļ	Correct evaluation of integral	3
<u></u>	Some progress using substitution or integration by parts	2
	Correct integral	1

Sample answer:

$$V = \int_{1}^{2} y \sqrt{y - 1} \, dy$$

Jsing the substitution y = u + 1

$$V = \int_0^1 (n+1) \sqrt{u} \, du$$
$$= \int_0^1 \left(u^{\frac{5}{2}} + u^{\frac{1}{2}} \right) du$$
$$= \left[\frac{2u^{\frac{5}{2}}}{5} + \frac{2u^{\frac{3}{2}}}{3} \right]_0$$

 \therefore The volume of the solid S is $\frac{16}{15}$ cubic units.

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Question 14 (15 marks)

(a) (i) (3 marks)

Targeted Performance Bands: E3 Outcomes assessed: E4

	Criteria	ITAGILA
		cc
•	Correct simplified polynomial	,
Ľ	Correct not mornial connection $(x+3)^3-(x+3)^2-21(x+3)+45=0$ or correctly finds	7
•	Control Post recommendation of the control of the c	
	the sums and products of roots	
_1	The state of the s	-
•	Correct transformation $x \mapsto x + 3$ or attempts to find sums and products of 100ts	7
•		

Sample answer:

$$p(x) = x^3 - x^2 - 21x + 45 = (x - \alpha)(x - \beta)(x - \gamma)$$
 has roots α , β and γ .

.. The monic polynomial with roots $\alpha - 3$, $\beta - 3$ and $\gamma - 3$ is given by

$$((x+3)-\alpha)((x+3)-\beta)((x+3)-\gamma)$$

= $(x+3)^3 - (x+3)^2 - 21(x+3) + 45$
= $x^3 + 8x^2$

(a) (ii) (1 mark)

Outcomes assessed: E4

Targeted Performance Bands: E2

Mark	_	4
Criteria	Addition of the state of the st	Correct solution

Sample answer:

$$x^3 + 8x^2 = x^2(x+8)$$
 has roots $\alpha - 3 = 0$, $\beta - 3 = 0$, $\gamma - 3 = -8$

$$\therefore \alpha = 3, \beta = 3, \gamma = -5.$$

 $\therefore x = 3, -5$ are the solutions to p(x) = 0.

(b) (i) (2 marks)

Outcomes assessed: H5

Targeted Performance Bands: E3

Sample answer:

 $\angle PRA = \angle PSA$ (angles in the same segment are equal)

 $\angle PSA = \angle PQB$ (exterior angle of cyclic quadrilateral PSBQ equals the opposite interior angle)

:. ZPR4 = ZPQB

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(b) (ii) (2 marks)

Outcomes assessed: H5, PE3

Targeted Performance Bands: E3

_	Cinetia	MATER
•	Complete answer	2
L	nt progress towards the solution	_

Sample answer:

$$\angle PRA = \angle PQB$$
 (from part i)

Therefore,
$$\angle PRC = \angle PQC$$

 $\therefore P,R,Q$ and C are concyclic as PC subtends equal angles at R and Q (on the same side of PC)

(c) (i) (2 marks)

Outcomes assessed: E2

Targeted Performance Bands: E2

 L	Criteria	Mark
•		2
 •	Correct expression for the gradient of the tangent at P	-

Sample answer:

$$x = a \sec \theta \Rightarrow \frac{dx}{d\theta} = a \sec \theta \tan \theta$$

$$y = b \tan \theta \Rightarrow \frac{dy}{d\theta} = b \sec^2 \theta$$

$$\frac{dy}{dx} = \frac{b\sec\theta}{a\tan\theta}$$

The equation of the tangent at P:

$$y - b \tan \theta = \frac{b \sec \theta}{a \tan \theta} (x - a \sec \theta)$$

$$ay \tan \theta - ab \tan^2 \theta = bx \sec \theta - ab \sec^2 \theta$$

$$bx \sec \theta - ay \tan \theta = ab(\sec^2 \theta - \tan^2 \theta)$$

$$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$$

(c) (ii) (2 marks)

Outcomes assessed: E2

Mark • Significant progress towards solving tangent equation simultaneously with $y = \frac{b}{a}x$ Criteria Targeted Performance Bands: E2 Correct solution

~

Sample answer:

For the coordinates of $\underline{\mathcal{Q}}$ solve $\frac{x}{a} \sec \theta - \frac{y}{b} \tan \theta = 1$ simultaneously with the equation of the

asymptote
$$y = \frac{b}{a}x$$
.

$$\frac{x}{a} \sec \theta - \frac{b}{a}x \tan \theta = 1$$

$$x \left(\frac{\sec \theta}{a} - \frac{\tan \theta}{a} \right) = 1$$

$$x = \frac{a}{\sec \theta - \tan \theta}$$

Substituting into
$$y = \frac{b}{a}x$$
:
 $y = \frac{b}{a} \left(\frac{a}{\sec \theta - \tan \theta} \right)$

Hence, the coordinates of
$$Q$$
 are $\left(\frac{a}{\sec\theta - \tan\theta}, \frac{b}{\sec\theta - \tan\theta}\right)$

 $= \frac{1}{\sec \theta - \tan \theta}$

(c) (iii) (3 marks)

Targeted Performance Bands: E4 Outcomes assessed: E2, E4

	Cineria	MARK
•	 Correct proof 	3
•	Significant progress towards finding $ an(\angle QRS)$	2
•	Significant progress towards finding expressions for the gradients of OS and RS	-

Sample answer:

$$m_{gs} = \frac{\sec \theta - \tan \theta}{\sec \theta - \tan \theta} = \frac{b}{a(1 - e\sec \theta + e\tan \theta)}$$
$$\sec \theta - \tan \theta - ae$$
$$\frac{-b}{-b}$$
$$m_{gs} = \frac{\sec \theta + \tan \theta}{a} = \frac{-b}{a(1 - e\sec \theta - e\tan \theta)}$$

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Question 15 (15 marks)

(a) (i) (2 marks)

Outcomes assessed: E8

Targeted Performance Bands: E3

Mark

z = 1, $\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$, $\cos \frac{-2\pi}{5} + i \sin \frac{-2\pi}{5}$, $\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}$, $\cos \frac{-4\pi}{5} + i \sin \frac{-4\pi}{5}$

Correct roots or correct diagram

Sample answer:

Correct answer

Targeted Performance Bands: E3

Outcomes assessed: E3, E4

(b) (i) (2 marks)

 $\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$

 $\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}$

4		_		
IVA GAR AN	c	1		-
Criteria		- Correct annual	• Collect allower	• Correctly integrates tan x

Sample answer:

$$\int_0^{\frac{\pi}{4}} \tan x \, dx = \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} \, dx$$
$$= \left[-\ln(\cos x) \right]_0^{\frac{\pi}{4}}$$
$$= \ln \sqrt{2}$$

(a) (ii) (2 marks)

Outcomes assessed: E8

Targeted Performance Bands: E3

₹	Criteria	Mark	
Ľ	Comact anginar	5.	
_	Collect answer	-	_
•	Some progress towards the correct answer	1	_,

Sample answer:

$$I_{s} = \frac{1}{4} - I_{s}$$

$$= \frac{1}{4} - \left(\frac{1}{2} - I_{1}\right)$$

$$= \frac{-1}{4} + \ln \sqrt{2}$$

(b) (ii) (2 marks)

 $\frac{-2\pi}{5}$ + $i\sin\frac{-2\pi}{5}$

 $\cos\frac{-4\pi}{5} + i\sin\frac{-4\pi}{5}$

Outcomes assessed: E3, E4

Targeted Performance Bands: E3

	Criteria	Mark
·	Expression written as a product of linear and quadratic factor	2
•	Progress towards answer	1

Sample answer:

The quadratic with roots $\alpha = \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$, $\beta = \cos \frac{-2\pi}{5} + i \sin \frac{-2\pi}{5} = \cos \frac{2\pi}{5} - i \sin \frac{2\pi}{5}$

is
$$z^2 - (\alpha + \beta)z + \alpha\beta = z^2 - \left(2\cos\frac{2\pi}{5}\right)z + 1$$
.

Similarly, the quadratic with roots $\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}$, $\cos \frac{-4\pi}{5} + i \sin \frac{-4\pi}{5}$ is $z^2 - \left(2\cos \frac{4\pi}{5}\right)z + 1$. Hence, $z^5 - 1 = (z - 1)\left(z^2 - \left(2\cos \frac{2\pi}{5}\right)z + 1\right)\left(z^2 - \left(2\cos \frac{4\pi}{5}\right)z + 1\right)$.

Hence,
$$z^5 - 1 = (z - 1) \left(z^2 - \left(2\cos\frac{2\pi}{5} \right) z + 1 \right) \left(z^2 - \left(2\cos\frac{4\pi}{5} \right) z + 1 \right)$$
.

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(b) (iii) (2 marks)

Outcomes assessed: E3, E4

Targeted Performance Bands: E4

Criteria • Correct proof	Mark 2	
Significant progress applying sum of roots		

Sample answer:

sum of roots =
$$\frac{-b}{a}$$
 = 0

$$1 + \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5} + \cos\frac{-2\pi}{5} + i\sin\frac{-2\pi}{5} + \cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5} + \cos\frac{-4\pi}{5} + i\sin\frac{-4\pi}{5} = 0$$

$$1 + \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5} + \cos\frac{2\pi}{5} - i\sin\frac{2\pi}{5} + \cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5} + \cos\frac{4\pi}{5} - i\sin\frac{4\pi}{5} = 0$$

$$1 + 2\cos\frac{2\pi}{5} + 2\cos\frac{4\pi}{5} = 0$$

$$\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = \frac{-1}{2}$$

(c) (i) (2 marks)

Outcomes assessed: E5, E9

Targeted Performance Bands: E3

Criteria	plete proof	inficant progress towards finding $f'(t)$ and $f''(t)$
	Complete pr	Significant p

Sample answer:

$$x(t) = f(t)e^{-3t}$$

$$x'(t) = (-3f(t) + f'(t))e^{-3t}$$

$$x''(t) = (9f(t) - 3f'(t))e^{-3t} + (-3f'(t) + f''(t))e^{-3t}$$

$$= (9f(t) - 6f'(t) + f''(t))e^{-3t}$$

$$\operatorname{Since} \frac{d^2 x}{dt^2} = -6 \frac{dx}{dt} - 9x,$$

$$(9f(t) - 6f'(t) + f''(t))e^{-3t} = (9f(t) - 6f'(t))e^{-3t}$$

Hence,
$$f''(t) = 0$$

 $\therefore f(t)$ is linear, i.e. f(t) = At + B where A and B are constants

Outcomes assessed: HE3 (c) (ii) (1 mark)

Targeted Performance Bands: E2

Mark	1	
Criteria	Correct answer	

Sample answer:

$$x(t) = (At + B)e^{-3t}$$

When t = 0, x = 0 (as the projectile is starting from the origin)

$$(A \times 0 + B)e^{-3x0} = 0$$
$$B = 0$$

(c) (iii) (2 marks)

Outcomes assessed: HE3

Targeted Performance Bands: E3

L	Criteria	Mark
	CA ACCEPTANT	
	Correct answer	7
•	• Correct expression for $x'(t)$ for calculated value of B	-

Sample answer:

$$x(t) = Ate^{-3t}$$

$$x'(t) = Ae^{-3t} - 3Ate^{-3t}$$

$$=Ae^{-3t}\left(1-3t\right)$$

Displacement is a maximum when
$$x'(t) = 0$$
, i.e. $t = \frac{1}{2}$

Therefore, the particle's displacement is at maximum after $\frac{1}{2}$ seconds.

Note: since
$$x''\left(\frac{1}{3}\right) = -3Ae^{-1} < 0$$
 since $A > 0$, a maximum is attained.

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Question 16 (15 marks)

(a) (3 marks)

Targeted Performance Bands: E4 Outcomes assessed: E3, E4

Criena	MINIT
Correct disoram	3
1 = z	2
Attenuts to make z the subject	1

Sample answer:

$$w = \frac{3z+2}{z-1}$$

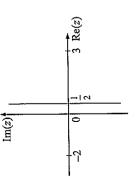
$$wz - w = 3z + 2$$

$$z = \frac{w+2}{w-3}$$

Since
$$|z|=1$$
,

$$\left| \frac{w+2}{w-3} \right| = 1 \Rightarrow \left| w+2 \right| = \left| w-3 \right|$$

Thus w is equidistant from -2 and 3.



(b) (i) (1 mark)

Targeted Performance Bands: E3 Outcomes assessed: E9

Sample answer:

$$\frac{1}{2^{k}+1} + \frac{1}{2^{k}+2} + \frac{1}{2^{k}+3} + \dots + \frac{1}{2^{k}+2^{k}} \ge \frac{1}{2^{k}+2^{k}} + \frac{1}{2^{k}+2^{k}} + \frac{1}{2^{k}+2^{k}} + \dots + \frac{1}{2^{k}+2^{k}}$$

$$= 2^{k} \times \frac{1}{2(2^{k})}$$

$$= \frac{1}{2}$$

(b) (ii) (3 mark)

Outcomes assessed: HE2, E9

Targeted Performance Bands: E3, E4

<u> </u>	Criteria	Mark
•	Complete proof	3
•	Makes use of the assumption or the result in part i	2
•	Proof for $P(1)$	1

Let $\hat{P}(n)$ be the given proposition. P(1) is true since $1 + \frac{1}{2} = \frac{3}{2} \ge \frac{1}{2}(1+1) = 1$.

Assume P(k) is true for some positive integer k. i.e. $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2^k} \ge \frac{1}{2} (k+1)$

Prove P(k+1) is true:

$$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2^{k+1}} = \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2^k}\right) + \left(\frac{1}{2^k + 1} + \frac{1}{2^k + 2} + \frac{1}{2^k + 2^k} + \dots + \frac{1}{2^k + 2^k}\right)$$

 $\geq \frac{1}{2}(k+1) + \frac{1}{2}$ using the assumption and the result in part i

$$=\frac{1}{2}\Big(\big(k+1\big)+1\big)$$

 \therefore By the Principle of Mathematical Induction, P(n) is true for integers $n \ge 1$.

DISCLAMER

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(c) (i) (2 marks)

Outcomes assessed: E3

Mark Criteria Attempts to apply DeMoivre's Theorem Targeted Performance Bands: E3 Complete proof

By DeMoivre's theorem,

$$x^{k} = (\cos \theta + i \sin \theta)^{k} = \cos k\theta + i \sin k\theta$$

$$x^{-k} = (\cos \theta + i \sin \theta)^{-k} = \cos(-k\theta) + i \sin(-k\theta) = \cos k\theta - i \sin k\theta$$

 $x^{k} + x^{-k} = \cos k\theta + i \sin k\theta + \cos k\theta - i \sin k\theta$ $=2\cos k\theta$

(c) (ii) (3 marks)

Outcomes assessed: HE3, E4, E9

Targeted Performance Bands: E4

	Criteria	Mark	
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١.	Correct prince	C	
•	CONTROL DATOR	,	
•	Cimificant progress towards the correct proof	7	
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	1,20		
	1 3	_	
•	Correct binomial expansion of $x + \frac{1}{x}$	ı	

$$= \binom{2n}{0} x^{2n} + \binom{2n}{1} x^{2n-2} + \binom{2n}{2} x^{2n-4} + \dots + \binom{2n}{n} + \dots + \binom{2n}{2} \frac{1}{x^{2n-4}} + \binom{2n}{1} \frac{1}{x^{2n-2}} + \binom{2n}{0} \frac{1}{x^{2n}}$$

$$\left(x + \frac{1}{x}\right)^{2n} = \binom{2n}{0} \left(x^{2n} + \frac{1}{x^{2n}}\right) + \binom{2n}{1} \left(x^{2n-2} + \frac{1}{x^{2n-2}}\right) + \binom{2n}{2} \left(x^{2n-4} + \frac{1}{x^{2n-4}}\right) + \dots + \binom{2n}{n-1} \left(x^2 + \frac{1}{x^2}\right) + \binom{2n}{n} \left(x^2 + \frac{1}{x^2}\right) + \binom{2n}{n-1} \left(x^2 +$$

Note
$$\binom{2n}{0} = 1$$

(c) (iii) (1 mark)

Targeted Performance Bands: E4 Outcomes assessed: E9

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Criteria		• Correct proof

Using the result from part i, $x^k + \frac{1}{x^k} = 2\cos k\theta$ in the identity from part ii:

$$(2\cos\theta)^{2n} = \binom{2n}{0} (2\cos 2n\theta) + \binom{2n}{1} (2\cos(2n-2)\theta) + \binom{2n}{2} (2\cos(2n-4)\theta) + \dots + \binom{2n}{n-1} (2\cos 2\theta) + \binom{2n}{n}$$

Dividing both sides by 2:

$$2^{2n-1}\cos^{2n}\theta = \cos 2n\theta + \binom{2n}{1}\cos(2n-2)\theta + \binom{2n}{2}\cos(2n-4)\theta + \dots + \binom{2n}{n-1}\cos 2\theta + \frac{1}{2}\binom{2n}{n}$$

(c) (iv) (2 marks)
Outcomes assessed: E8

fargeted Performance Bands: E4

Mark

Sample answer: $ \begin{bmatrix} 2^{n-1}\cos^{2n}\theta & \theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}\theta & \theta \\ 0 & \cos^{2n}\theta & \theta \end{bmatrix} = \begin{bmatrix} 2^{n}\cos^{2n}$	•	• Comprete correct program	1
= 0	•	Integration of an expression progressing towards the correct proof	-
	San	nple answer:	
	25	$ 2^{2n-1}\cos^{2n}\theta d\theta = \begin{bmatrix} 2^{2n} & \cos 2n\theta + \binom{2n}{2} & \cos (2n-2)\theta + \binom{2n}{2} & \cos (2n-4)\theta + \dots + \binom{2n}{2} & \cos (2n-4)\theta + \dots + \binom{2n}{2} & \cos (2n-4)\theta + \dots \end{bmatrix} $	$2\theta + \frac{1}{2} \binom{2n}{n}$

$$\int_{0}^{2^{n-1}} \cos^{2n}\theta \, d\theta = \int_{0}^{2^{n}} \cos 2n\theta + \binom{2n}{1} \cos(2n-2)\theta + \binom{2n}{2} \cos(2n-4)\theta + \dots + \binom{2n}{n-1} \cos 2\theta + \frac{1}{2} \binom{2n}{n}$$

Since $\int_{a}^{2\pi} \cos k\theta d\theta = 0$ for all even integers k, all the integrals on the right hand side are zero except

for the constant term.

since $\binom{2n}{2n-k} = \binom{2n}{k}$

$$\int_0^{2\pi} 2^{2n-1} \cos^{2n} \theta \, d\theta = \int_0^{2\pi} \frac{1}{2} \binom{2n}{n} d\theta$$
$$= \left[\frac{1}{2} \binom{2n}{n} \theta \right]^{2\pi}$$
$$= \pi \binom{2n}{n}$$

Dividing both sides by 22n-1

 $\int_0^{2\pi} \cos^{2n}\theta \, d\theta = \frac{\pi}{2^{2n-1}} \binom{2n}{n}.$

DISCLAMER.
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