WESTERN REGION

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

1999

MATHEMATICS

3 Unit (Additional)
and
3/4 Unit (Common)

Time allowed - TWO hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- ALL necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on the last page. These may be removed for your convenience.
- Board-approved calculators may be used.
- Each question should be started on a new page.

Question One. *Start a new page* **Marks**

(a) Find a general solution for x if $\tan x = \frac{1}{\sqrt{3}}$ (in terms of π). **2**

(b) If α, β and χ are the roots of **2**

$$3x^3 + 5x^2 - 7x + 4 = 0.$$

Find the values of $\alpha\beta + \alpha\chi + \beta\chi$ and $\alpha + \beta + \chi$.

(c) Solve $\sec^2 x + \tan x - 7 = 0$ for $0° \le x \le 360°$ (to the nearest minute). **3**

(d) Given that **2**

$$x = \cos\theta + 1 \text{ and } y = \sin\theta - 2$$

By eliminating θ determine a relationship between x and y only.

(e) Solve $\frac{2x}{x-3} \le 1$ if $x \ne 3$. **3**

Question Two. *Start a new page* **Marks**

(a) Given $\dfrac{x^3+3x^2+9x-1}{x^2+9} = Q(x) + \dfrac{R(x)}{x^2+9}$ 4

(i) By performing the division or otherwise find $Q(x)$ and $R(x)$.

(ii) Hence find $\displaystyle\int \frac{x^3+3x^2+9x-1}{x^2+9}\,dx$

(b) Using Mathematical Induction prove that $7^n - 5^n$ is even for all positive integer n. 3

(c) Eight people are seated about a round table. 3

(i) How many different seating arrangements are possible ?

(ii) If the eight people consist of four couples find the probability that each person is seated adjacent to their partner.

(d) Given that the quadrilateral ABCD is cyclic show that the sum of the tangents of the angles is zero. 2
That is, $\tan A + \tan B + \tan C + \tan D = 0$

Question Three. *Start a new page* **Marks**

(a) Evaluate $\displaystyle\int_0^1 \frac{2x}{(2x+1)^2}\,dx$ **3**

using the substitution $u = 2x + 1$.

(b) By making suitable substitutions for A and B in the expansion of $\cos(A + B)$ find the exact value of $\cos 75°$. **2**

(c) If $\sqrt{3}\cos x - \sin x = R\cos(x + \alpha)$ determine the values of R and α. **3**

(d) Find the constant term in the expansion of **2**

$$\left(2x - \frac{1}{x^3}\right)^{20}$$

(Do not evaluate).

(e) If $f(x) = \sin^{-1} x + \cos^{-1} x$ and $1 \geq x \geq 0$, find **2**

(i) $f'(x)$

(ii) $\displaystyle\int_0^1 f(x)\,dx$

Question Four. *Start a new page* **Marks**

(a) A Horticulturalist knows that the probability of grafting a particular rose successfully is 0·4. **5**

(i) If ten grafts are made find the probability of six successful grafts. Give your answer correct to 3 significant figures.

(ii) How many grafts must be made to ensure that the probability of at least one success is at least 99·9 %.

(b) A spherical balloon is being inflated at the rate of $1000 \text{ cm}^3 \text{ s}^{-1}$ and given that **5**

$$V = \frac{4}{3}\pi r^3 \quad \text{and} \quad A = 4\pi r^2$$

find :

(i) An expression for the instantaneous rate of change of the radius in terms of r. $\left(\text{Find } \frac{dr}{dt}\right)$.

(ii) The rate of change of the surface area of the balloon when the radius is 10 cm.

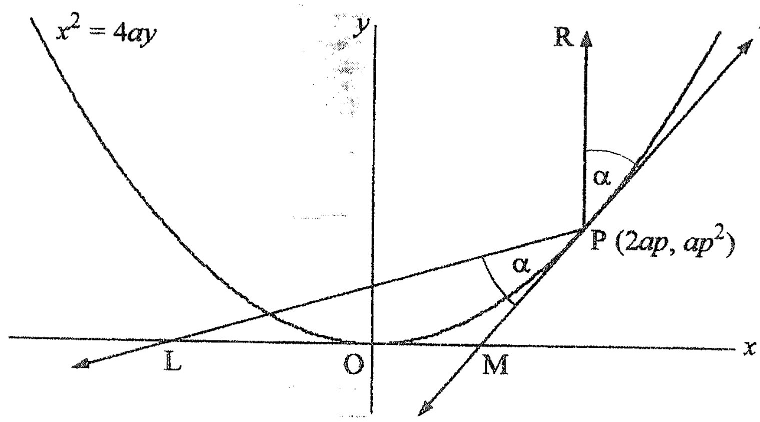
(c) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2 x \; dx$ **2**

Question Five. *Start a new page* **Marks**

(a) It is known that $f(x) = \frac{1}{2}x - \sin x$ has a root of $f(x) = 0$ near $x = 2$. **2**

By using one application of Newton's method find a better approximation to two decimal places.

(b) **7**



The parabola $x^2 = 4ay$ is shown in the sketch above.
The tangent at $P(2ap, ap^2)$ cuts the x axis at M and passes through the point N.
PR is parallel to the axis of the parabola and makes $\angle RPN = \alpha°$.
PL is such that it cuts the x axis at L and $\angle LPM = \alpha°$.

(i) Show that $\tan\alpha = \frac{1}{p}$.

(ii) Show that the gradient of LP is $\dfrac{p^2 - 1}{2p}$.

(iii) Show that the line LP passes through the focus of the parabola.

(c) A particle is projected such that at any time 't', the equation of the trajectory is given by **3**

$$x = 36t \quad \text{and} \quad y = 15t - \tfrac{1}{2}gt^2$$

find the angle of projection to the nearest minute.

Question Six. *Start a new page* **Marks**

(a) (i) State the domain and range of **3**

$$y = 2\sin^{-1}(3x).$$

(ii) Sketch $y = 2\sin^{-1}(3x)$.

(b) The rate at which a body cools is proportional to the difference between its temperature (T) and the constant temperature of the surrounding air (S). That is **7**

$$\frac{dT}{dt} = k\,(T - S)$$

where t is the time elapsed and k is a constant.

(i) Show that $T = S + B\,e^{kt}$, where B is a constant, is a solution of the above differential equation.

(ii) A body cools from 150° C to 90° C in three hours. If the air temperature is 30° C find the value of B and hence the value of k to 3 decimal places.

(iii) Using the values of B and k found in (ii) determine the temperature of the body after a further three hours.

(c) Find the coordinates of the point P which divides the interval joining A (−1 , 2) and B (5 , −3) internally such that AP : PB = 3 : 2. **2**

Question Seven. *Start a new page* **Marks**

(a) Two circles touch internally at P. A line through P cuts the smaller circle at A and the larger circle at B. A second line through P cuts the smaller and larger circles at C and D respectively. **4**

(i) Sketch this information.

(ii) Prove that the line joining A and C is parallel to the line joining B and D.

(b) You are given that x, v, a and t represent displacement, velocity, acceleration and time elapsed respectively. **2**
Show that

$$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

(c) A particle moves in a straight line such that its acceleration is given by **6**

$$a = -e^{-2x}$$

If $v = 1$ and $x = 0$ when $t = 0$

(i) Express v in terms of x. (Use the result proven in (b))

(ii) Express x in terms of t.

(iii) Express v in terms of t.