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Teacher:

Saint Mark's Coptic Orthodox College

Mathematics Department

Yearly\Yr11- EXTENSION I

2003 Yearly Exam

Time Allowed: TWO HOURS

EXAMINER Mr. W. MICHEAL

DIRECTIONS TO CANDIDATE:

- Attempt all questions.
- Show all necessary working. Marks may be deducted for careless or badly arranged work.
- Only approved calculators may be used.

Question	1	2	3	4	5
Mark	/	/	/	/	/

4/100

QUESTION ONE

1) Solve for x: $3^{2x} - 10(3^x) + 9 = 0$.†

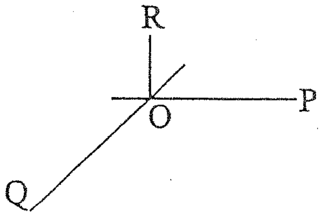
2) The graphs of $y = x$ and $y = x^3$ intersect at $x = 1$. Find the size of the acute angle between these curves at $x = 1$.¤

3) A parabola has equation $y = 2x^2 - 4x + 1$. Find:
   - i. the co-ordinates of its vertex;
   - ii. its focal length;
   - iii. the equation of its directrix.†

4)

In the diagram, which is not to scale, the points P, Q and O are in the same plane. R is a point vertically above O. P and Q are 750 metres apart and ∠POQ is 120°.

If ∠QRO is 30° and ∠PRO is 60°, find the height of R above O



QUESTION TWO

5)
   - a. Draw on a sketch diagram the lines $y = x$ and $y = x + 1$.
   - b. Indicate on your diagram, by shading, the region of the (x, y) plane determined by those points which satisfy all the inequalities $|x| \le 1$ and $y \ge x$ and $y \le x + 1$.¤

6) Solve the inequality $\frac{1}{x} < \frac{1}{x+1}$.†

7) Find all angles θ for which $\sin 2\theta = \frac{1}{2}\cos\theta$.†

8) The point $P(x, y)$ moves in the XY-plane such that its distance from the point $R(-1, 0)$ is always twice its distance from the point $S(2, 0)$. Find the locus of P and describe its geometrical features.

9) Consider the polynomial $P(x) = 6x^3 - 5x^2 - 2x + 1$
   - i. Show that 1 is a zero of $P(x)$.
   - ii. Express $P(x)$ as a product of 3 linear factors.
   - iii. Sketch the polynomial showing all the features on the diagram.
   - iv. Solve the inequality $P(x) \le 0$.

QUESTION THREE

10) If $6x^2 - 11 \equiv A(x+2)^2 + Bx + C$, find the values of A, B and C.†

11) Find the gradient of the normal to the curve $y = \dfrac{1}{\sqrt{x^2 - 3}}$ at the point (2, 1)

12) Use the t results to show that $\dfrac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = t$

13) Two points $P(2Ap, Ap^2)$ and $Q(2Aq, Aq^2)$ lie on the parabola $x^2 = 4Ay$, where $A > 0$. The chord PQ passes through the focus.
   - a. Show that $pq = -1$.
   - b. Show that the point of intersection T of the tangents to the parabola at P and Q lies on the line $y = -A$.
   - c. Show that the chord PQ has length $A\left(p + \dfrac{1}{p}\right)^2$.¤

QUESTION FOUR

14) If $y = (x-1)^3$, find the value(s) of x for which $\dfrac{dy}{dx} = 12$.

15) Given that $0 < x < 45$, prove that $\tan(45 + x) = \dfrac{\cos x + \sin x}{\cos x - \sin x}$.¤

16) The tangents to the curve $y = x^2 - 4x + 5$ at the points $P(3, 2)$ and $Q(1, 2)$ meet at the point R.
   - a. Find the coordinates of R.
   - b. What type of triangle is ΔPQR?
   - c. Find the area of ΔPQR.

17) Prove that the equation $3kx^2 - (2k+3a)x + 2a = 0$ has rational roots if k and a are rational.

18) Use the Principle of Mathematical Induction to prove that $5^n + 2(11^n)$ is a multiple of 3 for all positive integers n.¤

EXAMINER MR. W. MICHEAL

QUESTION FIVE

19) Solve $2\cos^2 2\theta - \cos 2\theta - 1 = 0$ for $0° \le \theta \le 360°$

20) Find the coordinates of the point P which divides the interval AB internally in the ratio $2 : 3$ where A and B have coordinates $(1, -3)$ and $(6, 7)$ respectively.¤

21) Differentiate $y = x^5(1 + x)^5$ with respect to x.

22)

i Factorise $3x^3 + 3x^2 - x - 1$.

ii. Solve the equation $3\tan^3\theta + 3\tan^2\theta - \tan\theta - 1 = 0$ for $0 \le \theta \le$ 180°†

23) R is the point $(2ar, ar^2)$ on the parabola $x^2 = 4ay$. From R, perpendiculars are drawn to the x and y axes meeting them at M and N respectively. T is the midpoint of RN and V is the midpoint of TM.

a. Write down the coordinates of T and M.

b. Find the coordinates of V.

c. Show that as R moves along the given parabola, the locus of V is another parabola and find its equation.†

[[End Of Qns]]

Question One ~~13~~ 17

Student`s Name: ______________________ **Class:**

① $3^{2x} - 10(3^x) + 9 = 0$

1 $m^2 - 10m + 9 = 0$

$(m-9)(m-1) = 0$

1 $m = 9$ and $m = 1$

1 $3^x = 9$ $\quad 3^x = 1$

1 $x = 2$ $\quad x = 0$ $-\frac{1}{2}$ for wrong answ.

② $y = x$ $\quad y = x^3$

$m_1 = \frac{dy}{dx} = 1$ $\quad m_2 = \frac{dy}{dx} = 3x^2$ at $x = 1$

$m_1 = 1$ and $m_2 = 3$

$\tan\theta = \left|\frac{1-3}{1+1\cdot 3}\right|$

$= \left|\frac{-2}{4}\right|$

1 $= \frac{1}{2}$

$\theta = 26^\circ 34'$

③ $y = 2x^2 - 4x + 1$

$y - 1 = 2x^2 - 4x$

1 $\frac{1}{2}(y-1) = x^2 - 2x$

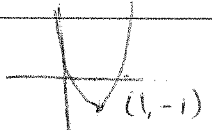
1 for $\boxed{x = 1}$

$\frac{1}{2}y - \frac{1}{2} + 1 = x^2 - 2x + 1$

$\frac{1}{2}(y+1) = (x-1)^2$

$(x-1)^2 = \frac{1}{2}(y+1)$

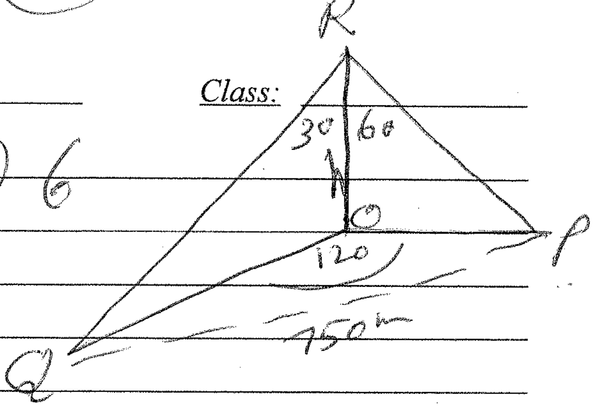
vertex $(1, -1)$



1 focal length $= \frac{1}{8}$

1 $y = -1\frac{1}{8}$

④ 6



$\tan 60 = \frac{OP}{h}$ ~~OP~~

$OP = h\tan 60 \longrightarrow$

$\tan 30 = \frac{OQ}{h}$

$OQ = h\tan 30 \longrightarrow$

$750^2 = h^2\tan^2 60 + h^2\tan^2 30 - 2 \times h\tan 60 \times h\tan 30 \cos 120^\circ \longrightarrow$ up to here only 3

$= h^2(\tan^2 60 + \tan^2 30 - 2\tan 60\tan 30 \cos 120)$

$= h^2\left(3 + \frac{1}{3} - 2 \times \cancel{\sqrt{3}} \times \frac{1}{\cancel{\sqrt{3}}} \times -\frac{1}{2}\right)$

$750^2 = h^2\left(4 + \frac{1}{3}\right)$ Not (+) lose 2 marks

$750^2 = 4\frac{1}{3}h^2$

$750^2 = \frac{13}{3}h^2$

$\frac{3 \times 750^2}{13} = h^2$

$h = 360.29\text{ m}$

$h \doteqdot 360\text{ m}$ (to whole number)

Please read the instructions carefully

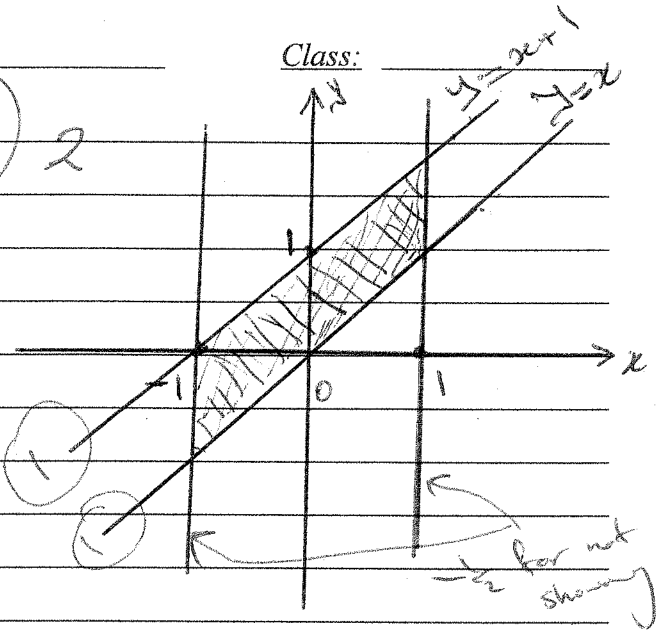
$2 \quad 30 \quad \sqrt{3}$

Question Two ~~19~~ 22

Student`s Name: ______________________ *Class:* __________

ⓐ 2

ⓑ 2



-½ for not showing

ⓒ $\frac{1}{x} < \frac{1}{x+1}$

$x < \frac{1}{x+1} \cdot x^2$

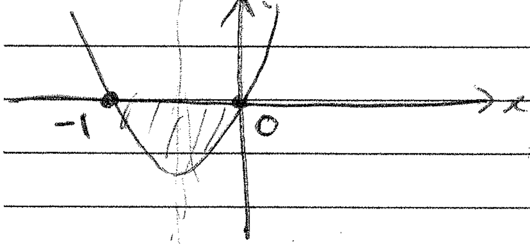
3 $x(x+1)^2 < x^2(x+1)$

$x(x^2+2x+1) < x^3+x^2$

$x^3+2x^2+x < x^3+x^2$

$x^2+x < 0$

$x(x+1) < 0$



$-1 < x < 0$

-1 for having ⊜

Question Three ⑱ ⑳

Student`s Name: ______________________ Class: ______________

⑩ $6x^2 - 11 \equiv A(x+2)^2 + Bx + C$

3 $\equiv A(x^2+4x+4) + Bx + C$

$\equiv Ax^2 + 4Ax + 4A + Bx + C$

$6x^2 - 11 \equiv Ax^2 + (4A+B)x + (4A+C)$

By equating coff^s of both sides.

∴ $A = 6$	$4A + B = 0$	$4A + C = -11$
	$4(6) + B = 0$	$4(6) + C = -11$
	$B = -24$	$C = -35$

⑪ $y = \frac{1}{\sqrt{x^2-3}}$, pt $(2, 1)$

$y = (x^2-3)^{-\frac{1}{2}}$

3 $\frac{dy}{dx} = -\frac{1}{2}(x^2-3)^{-\frac{3}{2}} \cdot 2x$

$= -\frac{x}{\sqrt{(x^2-3)^3}}$ at $(2, 1)$

$M_{Tangt} = -\frac{2}{\sqrt{(4-3)^3}}$

$= -\frac{2}{1}$

$= -2$ ($2\frac{1}{2}$)

∴ $M_{normal} = \frac{1}{2}$ #

⑫

3

$$\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}=t \qquad \sin\theta=\frac{2t}{1+t^2},\ \cos\theta=\frac{1-t^2}{1+t^2}$$

$$\frac{1\ \ +\sin\theta-\cos\theta}{1\ \ +\sin\theta+\cos\theta}=\frac{\frac{1+t^2}{1+t^2}+\frac{2t}{1+t^2}-\frac{1-t^2}{1+t^2}}{\frac{1+t^2}{1+t^2}+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}$$

$$=\frac{\frac{\cancel{1}+t^2+2t\cancel{-1}+t^2}{\cancel{1+t^2}}}{\frac{1+\cancel{t^2}+2t+1\cancel{-t^2}}{\cancel{1+t^2}}}$$

↖ $1\frac{1}{2}$

$$=\frac{2t^2+2t}{2t+2}$$

$$=\frac{\cancel{2}t(\cancel{t+1})}{\cancel{2}\,(\cancel{t+1})}$$

$$=t$$

$$=RHS$$

13) a)

$$m_{PQ} = \frac{Ap^2 - Aq^2}{2Ap - 2Aq}$$

3

$$= \frac{A\cancel{(p-q)}(p+q)}{2\cancel{A}\cancel{(p-q)}}$$

$$m_{PQ} = \frac{(p+q)}{2}$$

$P(2Ap, Ap^2)$

$$y - Ap^2 = \frac{(p+q)}{2}(x - 2Ap)$$

$$2y - 2Ap^2 = (p+q)x - 2Ap(p+q) \qquad f(0, A)$$

$$2A - \cancel{2Ap^2} = (p+q)\cdot 0 - \cancel{2Ap^2} - 2Apq$$

$$2A = -2Apq$$

$$-2Apq = 2A$$

$$\boxed{pq = -1}$$

b) Tangent at $P(2Ap, Ap^2)$

4

$x^2 = 4Ay$

$y = \frac{x^2}{4A}$

$m_{\text{Tangt}} = \frac{2x}{4A}$

$= \frac{x}{2A}$

$= \frac{2Ap}{2A}$

$= p$

eqn. of tangent at P

$y - Ap^2 = p(x - 2Ap)$

$y - Ap^2 = px - 2Ap^2$

$y = px - Ap^2$

similarly $y = qx - Aq^2$ at Q

$y = px - Ap^2$

$y = qx - Aq^2$

$0 = (p-q)x - A(p^2 - q^2)$

$\cancel{(p-q)}x = A\cancel{(p-q)}(p+q)$

$\boxed{x = A(p+q)}$

$y = px - Ap^2$

$= pA(p+q) - Ap^2$

$= Ap^2 + Apq - Ap^2$

$y = Apq$

Since $pq = -1$ from part (a)

$\therefore \boxed{y = -A}$

$T\left[A(p+q), -A\right]$

(c) Since the y-value at T is $-A$

$\therefore$ pt of intersection lies on the line

$y = -A$

4

(c) $P(2Ap, Ap^2)$, $Q(2Aq, Aq^2)$

$PQ = \sqrt{(2Ap - 2Aq)^2 + (Ap^2 - Aq^2)^2}^{\frac{1}{2}}$

$= \sqrt{4A^2(p-q)^2 + A^2(p-q)^2(p+q)^2}$

$= \sqrt{A^2(p-q)^2\left[4 + (p+q)^2\right]}$

$= \sqrt{A^2(p-q)^2\left[4 + p^2 + 2pq + q^2\right]}$

$= \sqrt{A^2(p-q)^2\left[p^2 + 4 - 2 + q^2\right]}$

$= \sqrt{A^2(p-q)^2\left[p^2 + 2 + q^2\right]}$

$= \sqrt{A^2\left(p - -\frac{1}{p}\right)^2\left(p^2 + 2 + \left(-\frac{1}{p}\right)^2\right)}$

Since $pq = -1$

$\therefore q = -\frac{1}{p}$

$= \sqrt{A^2\left(p + \frac{1}{p}\right)^2\left(p^2 + 2 + \frac{1}{p^2}\right)}$

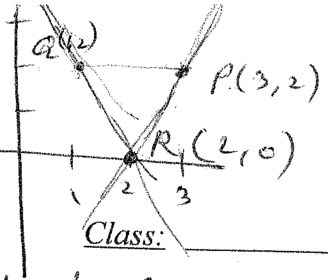
$= \sqrt{A^2\left(p + \frac{1}{p}\right)^2\left(p + \frac{1}{p}\right)^2}$

$= A\left(p + \frac{1}{p}\right)\left(p + \frac{1}{p}\right)$

$= A\left(p + \frac{1}{p}\right)^2$

Q.E.D

Question 4 21



Student`s Name:

Class:

⑭ $y=(x-1)^3$

$\frac{dy}{dx}=3(x-1)^2$ ½

$12=3(x-1)^2$ ½

3

$(x-1)^2=4$ ½

$(x-1)=\pm 2$ ½

$x=1\pm 2$

$x=3$ ½ or $x=-1$ ½

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⑮ $\tan(45+x)=\frac{\cos x+\sin x}{\cos x-\sin x}$

$LHS=\tan(45+x)$

$=\frac{\tan 45+\tan x}{1-\tan 45\cdot\tan x}$

3

$=\frac{1+\frac{\sin x}{\cos x}}{1-1\cdot\frac{\sin x}{\cos x}}$

$=\frac{\frac{\cos x+\sin x}{\cancel{\cos x}}}{\frac{\cos x-\sin x}{\cancel{\cos x}}}$

$=\frac{\cos x+\sin x}{\cos x-\sin x}$

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⑯ $y=x^2-4x+5$

ⓐ $\frac{dy}{dx}=2x-4$

$P(3,2)$, $m_{\text{Tangt}}=2(3)-4$
$=2$ ½

$Q(1,2)$, $m_{\text{Tangt}}=2(1)-4$
$=-2$ ½

equ. of tangent at P

$y-2=2(x-3)$

$y-2=2x-6$

$\boxed{y=2x-4}$ * ½

equ. of tangent at Q

$y-2=-2(x-1)$

$y-2=-2x+2$

$\boxed{y=-2x+4}$ ½

To find pt R (pt of intersec

$y=2\cancel{x}-4$

$y=\cancel{-2x}+4$

$2y=0$

$\boxed{y=0}$ (1 for solving any equat.)

$y=2x-4$

$0=2x-4$ 2

$2x=4$ $R(2,0)$

$\boxed{x=2}$

ⓑ $PQ=2$ units 2

$RP=\sqrt{(2-3)^2+(0-2)^2}=\sqrt{5}$ units

$RQ=\sqrt{(2-1)^2+(0-2)^2}=\sqrt{5}$ units

$\therefore \triangle RPQ$ is an Isosc. $\triangle$

ⓒ Area of $\triangle RPQ=\frac{1}{2}\times b\times h$

1 $=\frac{1}{2}\times 2\times 2$

$=2$ units^2

1) $3kx^2 - (2k+3a)x + 2a = 0$

$\Delta = b^2 - 4ac$

$= [-(2k+3a)]^2 - 4 \times 3k \times 2a$

2

$= 4k^2 + 12ka + 9a^2 - 24ka$

$= 4k^2 - 12ka + 9a^2$

$= (2k - 3a)^2 \geqslant 0$ ✓

∴ eqn. has rational roots

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(18) Step 1 when $n=1$

6

$5^n + 2(11^n) = 5^1 + 2(11^1)$

$= 27$ which is a multiple of 3.

∴ $5^n + 2(11^n)$ is a multiple of 3 for $n = 1$

Step 2

since $5^n + 2(11^n)$ is a multiple of 3 for $n = k$

i.e assume $5^k + 2(11^k) = 3M$

∴ $5^k = 3M - 2 \cdot 11^k \longrightarrow$ Ⓐ

Prove $5^n + 2(11^n)$ is a multiple ~~for~~ of 3 for $n = k+1$

i.e Prove $5^{k+1} + 2(11^{k+1})$ is a multiple of 3

$5^{k+1} + 2(11^{k+1})$

$= 5^k \cdot 5^1 + 2 \cdot 11^{k+1}$

$= (3M - 2 \cdot 11^k) \cdot 5 + 2 \cdot 11^{k+1}$

from A

$= 15M - 10 \cdot 11^k + 2 \cdot 11^{k+1}$

$= 15M - 10 \cdot 11^k + 2 \cdot 11^1 \cdot 11^k$

$= 15M - 10 \cdot 11^k + 22 \cdot 11^k$

$= 15M + 12 \cdot 11^k$

$= 3(5M + 4 \cdot 11^k)$

which is ~~does~~ a multiple of 3.

∴ If ~~st~~ $5^n + 2(11^n)$ is a multiple of 3 for $n = k$ thus it's also a multiple of 3 for $n = k+1$.

Step 3.

Question Solve #7 (20)

Student`s Name: ____________________ *Class:* ____________

(19) $2\cos^2 2\theta - \cos 2\theta - 1 = 0$

$(2\cos 2\theta + 1)(\cos 2\theta - 1) = 0$

3

$2\cos 2\theta = -1$	$\cos 2\theta = 1$
$\cos 2\theta = -\frac{1}{2}$	
$\theta = 120, 240, 480, 600$	$2\theta = 0^\circ, 360, 720$
$= 60, 120, 240, 300$	$\theta = 0^\circ, 180, 360$
1	1

(20) $A(\overset{x_1}{1}, \overset{y_1}{-3})$, $B(\overset{x_2}{6}, \overset{y_2}{7})$, $m:n = 2:3$

$x = \frac{mx_2 + nx_1}{m+n}$	$y = \frac{my_2 + ny_1}{m+n}$
$= \frac{2\cdot 6 + 3\cdot 1}{2+3}$	$= \frac{2\cdot 7 + 3\cdot -3}{2+3}$
$= \frac{15}{5}$	$= \frac{5}{5}$
$= 3$	$= 1$

1

$\therefore P(3, 1)$

(21) $y = x^5 (1+x)^5$

$y' = x^5 \cdot 5(1+x)^4 \cdot 1 + 5x^4 (1+x)^5$

$= 5x^5 (1+x)^4 + 5x^4 (1+x)^5$

$= 5x^4 (1+x)^4 (x + 1 + x)$

$= 5x^4 (1+x)^4 (2x + 1)$

3

(22) (i) $3x^3 + 3x^2 - x - 1$

$= 3x^2(x+1) - (x+1)$ 2

$= (x+1)(3x^2 - 1)$

$= (x+1)(\sqrt{3}x - 1)(\sqrt{3}x + 1)$

(ii) $3\tan^3\theta + 3\tan^2\theta - \tan\theta - 1 = 0$

Let $\tan\theta = x$ and from (i)

$\tan\theta = -1$	$\tan\theta = +\frac{1}{\sqrt{3}}$
$\theta = 180^\circ - 45$	$\theta = 30^\circ, 210^\circ$
$= 135^\circ$	
$\theta = 360 - 45$	
$= 315^\circ$	

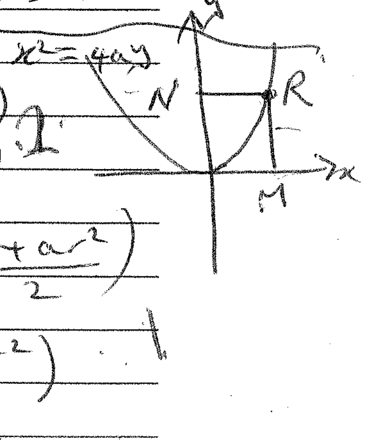
3

$\tan\theta = -\frac{1}{\sqrt{3}}$

$\theta = 150^\circ, 330$

answers $\theta = 30^\circ, 135, 150^\circ$

(23) (a) $M(2ar, 0)$, $T(ar, ar^2)$ 1



(b) $V\left(\frac{2ar + ar}{2}, \frac{0 + ar^2}{2}\right)$

$V\left(\frac{3}{2}ar, \frac{1}{2}ar^2\right)$ 1

(c) $x = \frac{3}{2}ar$, $y = \frac{1}{2}ar^2$

$\boxed{\frac{2x}{3a} = r}$

$y = \frac{1}{2}a\left(\frac{2x}{3a}\right)^2$ 3

$\boxed{x^2 = \frac{9a}{2}y}$ another Parabola

no a

Please read the instructions carefully

Student`s Name: ____________________ Class: __________

(9) $P(x) = 6x^3 - 5x^2 - 2x + 1$

(i)

$P(1) = 6(1)^3 - 5(1)^2 - 2(1) + 1$

1 $= 6 - 5 - 2 + 1$

$= 7 - 7$

$P(1) = 0$

$-\frac{1}{2}$ for not explaining

∴ 1 is a zero of ($P(x)$ since $P(1) = 0$

(ii) Since $x = 1$ is a zero

∴ $x - 1$ is a factor

$6x^2 + x - 1$

$x - 1 \,\big)\, 6x^3 - 5x^2 - 2x + 1$

$6x^3 - 6x^2$

2 $0 \quad x^2 - 2x + 1$

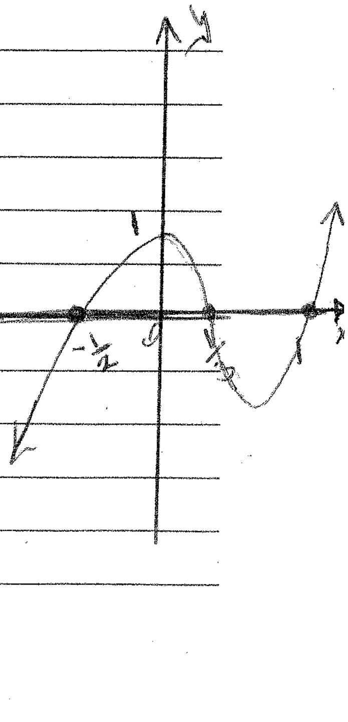
$x^2 - x$

$0 \quad -x + 1$

$\cancel{-x + 1}$

$6x^3 - 5x^2 - 2x + 1 = (x-1)(6x^2 + x - 1)$

$= (x-1)(3x-1)(2x+1)$



(iii) 2 Roots are $x = 1$, $x = \frac{1}{3}$, $x = -\frac{1}{2}$

(iv) 2 $x \le -\frac{1}{2}$, $\frac{1}{3} \le x \le 1$

Please read the instructions carefully