



CATHOLIC SECONDARY SCHOOLS  
ASSOCIATION OF NEW SOUTH WALES

Student Number \_\_\_\_\_

**2008**  
**TRIAL HIGHER SCHOOL CERTIFICATE**  
**EXAMINATION**

# **Mathematics**

## **Extension 1**

Afternoon Session  
Thursday, 14 August 2008

### **General Instructions**

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

### **Total marks – 84**

- Attempt Questions 1-7
- All questions are of equal value

### **Disclaimer**

Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, *Principles for Setting HSC Examinations in a Standards-Referenced Framework* (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and *Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework* (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

**3201 - 1**

**Total marks – 84**

**Attempt Questions 1–7**

**All questions are of equal value**

Answer each question in a SEPARATE writing booklet.

**Marks**

**Question 1** (12 marks) Use a SEPARATE writing booklet.

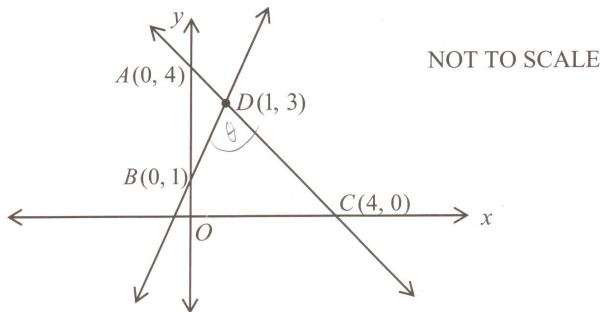
- (a) Find the exact value of  $\int_0^{\frac{\pi}{8}} \sec^2 2x dx$ . 2

- (b) (i) Sketch the graph of  $y = |2 - x|$ . 1

- (ii) Using this graph, or otherwise, find the solution to  $|2 - x| < x$ . 2

- (c) Find the value of  $k$  if  $x + 2$  is a factor of  $P(x) = x^2 + kx + 6$ . 2

- (d)  $A(0, 4)$ ,  $B(0, 1)$ ,  $C(4, 0)$  and  $D(1, 3)$  are points in the plane where  $D$  is the point of intersection of the two lines shown. Find, correct to the nearest minute, the size of the acute angle,  $\angle BDC$ , between the two lines. 3



- (e) Jasi was trying to find the solution to the inequality  $\frac{3}{x+1} < 2$ . 2

He stated that the solution is all values of  $x$  greater than  $\frac{1}{2}$ .

Solve the inequality  $\frac{3}{x+1} < 2$  to determine if Jasi's solution is correct.

**Marks**

**Question 2** (12 marks) Use a SEPARATE writing booklet.

(a) If  $\alpha$ ,  $\beta$  and  $\gamma$  are the roots of  $2x^3 - 5x^2 + 3x - 5 = 0$  find the value of 2

$$\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2.$$

(b) Let  $f(x) = \frac{2x}{\sqrt{1-x^2}}$ .

(i) For what values of  $x$  is  $f(x)$  undefined? 1

(ii) Find  $\int_0^{\frac{1}{2}} \frac{2x}{\sqrt{1-x^2}} dx$ , using the substitution  $x = \sin u$ . 3

(c) (i) Find the derivative of  $\sin^{-1} x + \cos^{-1} x$ . 1

(ii) Explain why  $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ . 2

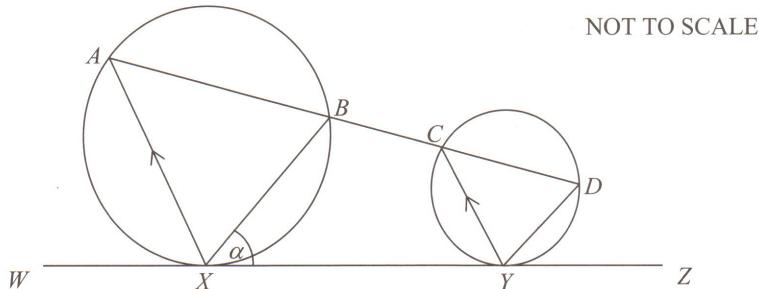
(d) How many different arrangements can be made from the letters of the word EXERCISE if:

(i) there are no restrictions? 1

(ii) the letters C and R are at the ends? 2

**Question 3** (12 marks) Use a SEPARATE writing booklet.

- (a) In the diagram below,  $WZ$  is a common tangent to the two circles and  $AX$  is parallel to  $CY$ .  $AD$  is a straight line through  $B$  and  $C$  on the circles as shown. Let  $\angle BXY = \alpha$ .



Copy or trace this diagram into your writing booklet.

- (i) Explain why  $BX$  is parallel to  $DY$ . 3
- (ii) Show that  $BCYX$  is a cyclic quadrilateral. 1
- (b) If  $A$  and  $B$  are both reflex angles, and given  $\cos A = \frac{3}{5}$  and  $\tan B = \frac{12}{5}$ ,  
find the exact value of  $\sin(A - B)$ . 3
- (c) In the expansion of  $(1 - kx)^9$  the coefficient of  $x^6$  is half that of the coefficient of  $x^5$ .  
Find the value of the constant  $k$ . 3
- (d) Taking  $x = 2$  as the first approximation, use one application of Newton's method to  
obtain a closer approximation to the solution to  $x = \sqrt[3]{9}$ . 2

**Marks**

**Question 4** (12 marks) Use a SEPARATE writing booklet.

- (a) Prove by mathematical induction that  $\sum_{r=1}^n r \times r! = (n+1)! - 1$ . 3

- (b) The acceleration of a particle  $P$ , moving in a straight line, is given by  $\ddot{x} = 2x - 3$  where  $x$  metres is the displacement from the origin  $O$ . Initially the particle is at  $O$  and its velocity  $v$  is 2 metres per second.

- (i) Show that the velocity  $v$  of the particle is  $v^2 = 2x^2 - 6x + 4$ . 2

- (ii) Calculate the velocity and acceleration of  $P$  at  $x = 1$  and briefly describe the motion of  $P$  after it moves from  $x = 1$ . 2

- (c) The rate of change of the number of bees infected by a disease is given by the equation  $\frac{dN}{dt} = N(200 - N)$ , where  $N$  is the number of infected bees in the hive at time  $t$  years. There are 200 bees in the hive.

- (i) If  $k$  is a constant, show that  $N = \frac{200}{1 + ke^{-200t}}$  satisfies the above equation. 2

- (ii) If at time  $t = 0$  one bee was infected, after how many days will half the colony be infected? 2

- (iii) Show that eventually all the bees will be infected. 1

**Marks**

**Question 5** (12 marks) Use a SEPARATE writing booklet

- (a) Let  $P(x) = -2x^3 + px^2 - qx + 5$ .

(i) Show that if  $P(x)$  is to have any stationary points, then  $p^2 - 6q \geq 0$ .

**2**

(ii) Discuss the situation when  $p^2 - 6q = 0$ .

**1**

- (b) A camera, one kilometre away in the horizontal direction from where the Space Shuttle is being launched, is tracking the ascent of the Shuttle. Assume the Shuttle ascends vertically.

Thirty seconds after the launch the Shuttle reaches a height,  $h$ , of 3240 metres and it is travelling at a speed of 230 metres per second.

The angle  $\theta$  is the angle of elevation of the camera as it tracks the Shuttle. At what rate is  $\theta$  increasing 30 seconds after the Shuttle is launched?

- (c) Diana loves to play basketball. From the free throw line she makes 2 out of every 5 baskets that she throws. For every basket that she makes she scores one point.

(i) In her game last week she had 6 free throws.

**1**

What is the probability that she scored 2 points?

- (ii) How many free throws would she need in one game so that the probability that she scores at least one point is 0.9978?

**2**

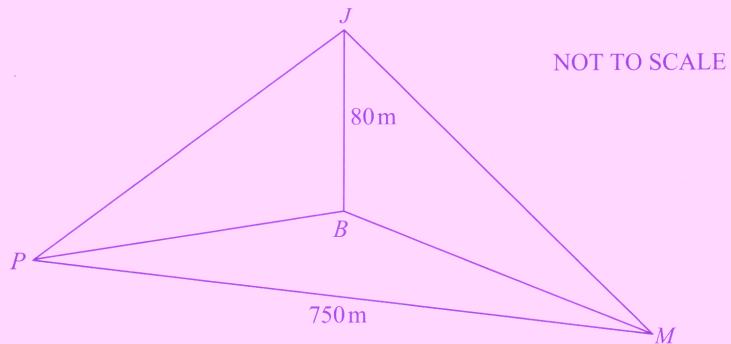
**Question 5 continues on page 7**

**Marks**

Question 5 (continued)

- (d) Janus,  $J$ , is on top of an 80 metre cliff, watching the Sydney to Hobart yacht race. 3

From the base of the cliff,  $B$ , directly below Janus, Poseidon,  $P$ , is on a bearing  
of  $202^\circ$  and Majorca,  $M$ , is on a bearing of  $144^\circ$ . Majorca is  $750$  m from Poseidon  
on a bearing of  $110^\circ$ .



Copy or trace this diagram into your writing booklet.

Find the angle of depression of Poseidon,  $P$ , from Janus,  $J$ .

**End of Question 5**

**Marks**

**Question 6** (12 marks) Use a SEPARATE writing booklet

- (a) Consider the function given by  $f(x) = \frac{e^x}{x-1}$ .

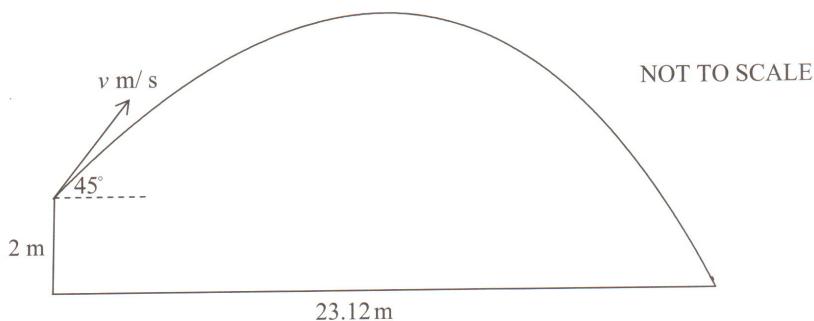
(i) Determine all vertical and horizontal asymptotes of the graph of  $y = f(x)$ . 2

(ii) Find any stationary point(s) and sketch the graph of  $y = f(x)$  including any intercepts with the coordinate axes. 3

(iii) State the largest positive domain for which  $f(x)$  has an inverse. 1

- (b) The world record for men's shot-put is 23.12 metres.

You may assume that the shot-put is projected at an initial velocity of  $v$  m/s from a height of 2 metres at an angle of projection of  $45^\circ$ , there is no air resistance and that the acceleration due to gravity is  $10\text{ m/s}^2$ .



- (i) Use integration to show that the equations of motion are 2

$$x = \frac{vt}{\sqrt{2}} \text{ and } y = -5t^2 + \frac{vt}{\sqrt{2}} + 2.$$

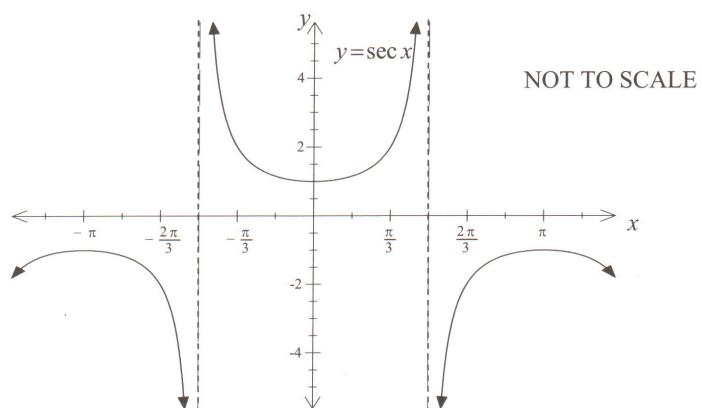
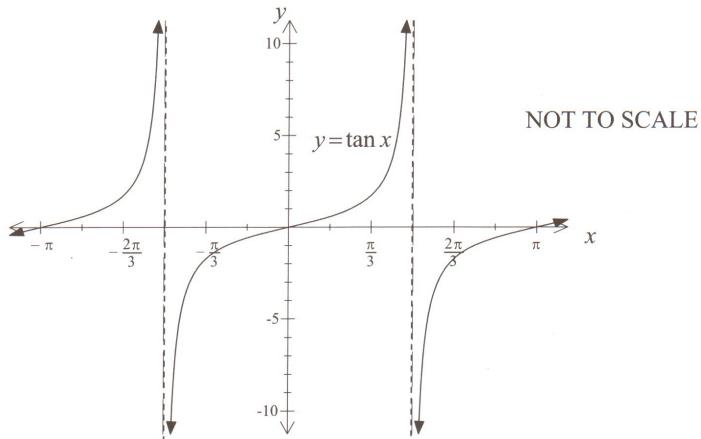
- (ii) Find the minimum velocity  $v$  m/s at which the shot-put must be projected to achieve the world record distance. 2

- (iii) What is the maximum height that the shot-put reaches in its path if it is projected with this velocity? 2

**Marks**

**Question 7** (12 marks) Use a SEPARATE writing booklet

- (a) The graphs shown are of  $y = \tan x$  and  $y = \sec x$  respectively.



(i) Prove that  $\sec \theta - \tan \theta = \frac{1}{\sec \theta + \tan \theta}$ .

**2**

(ii) Explain why  $0 < \sec \theta - \tan \theta \leq 1$  for  $0 \leq \theta < \frac{\pi}{2}$ .

**2**

(iii) Solve the equation  $\sec \theta - \tan \theta = \frac{1}{2}$  for  $0 \leq \theta < \frac{\pi}{2}$ .

**3**

**Question 7 continues on page 10**

**Marks**

Question 7 (continued)

- (b) (i) From a point  $A(p, q)$  perpendiculars  $AP$  and  $AQ$  are drawn to  
meet the  $x$  and  $y$  axes at  $P(p, 0)$  and  $Q(0, q)$  respectively.

Find the equation of  $PQ$ .

- (ii) Show that the condition for the line  $PQ$  to be a tangent to the parabola  $x^2 = 4ay$  is  $aq + p^2 = 0$ .

- (iii) If the points  $P(p, 0)$  and  $Q(0, q)$  move on the  $x$  and  $y$  axes respectively  
such that  $PQ$  is a tangent to the parabola  $x^2 = 4ay$  then the point  $A(p, q)$   
traces out a curve as  $P$  and  $Q$  move.

Find the locus of  $A$ .

**End of paper**

**Examiners**

Carolyn Gavel (convenor)	Kambala, Rose Bay
Cynthia Athayde	St John Bosco College, Engadine
Joe Grabowski	Freeman Catholic College, Bonnyrigg
Anne Hastings	Kambala, Rose Bay
Br Domenic Xuereb fsp	Patrician Brothers' College, Fairfield

## STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \quad \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left( x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left( x + \sqrt{x^2 + a^2} \right)$$

**NOTE:**  $\ln x = \log_e x, \quad x > 0$



CATHOLIC SECONDARY SCHOOLS ASSOCIATION

2008 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS EXTENSION 1

**Question 1** (12 marks)

(a) (2 marks)

*Outcomes assessed: H5*

*Targeted Performance Bands: E2-E3*

Criteria	Marks
• finds the correct primitive	1
• evaluates the integral correctly	1

*Sample Answer:*

$$\begin{aligned} \int_0^{\frac{\pi}{8}} \sec^2 2x \, dx &= \frac{1}{2} [\tan 2x]_0^{\frac{\pi}{8}} \\ &= \frac{1}{2} (\tan \frac{\pi}{4} - \tan 0) \\ &= \frac{1}{2} (1 - 0) \\ &= \frac{1}{2} \end{aligned}$$

(b) (i) (1 mark)

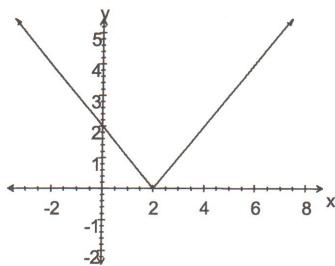
*Outcomes assessed: P4*

*Targeted Performance Bands: E2-E3*

Criteria	Mark
• draws the correct graph of $y =  2 - x $ , including intercepts	1

*Sample Answer:*

$$y = |2 - x|$$



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(b)(ii) (2 marks)

**Outcomes assessed: PE2**

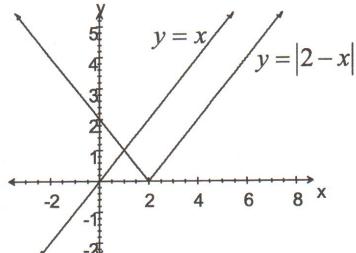
**Targeted Performance Bands: E2-E3**

Criteria	Marks
• finds the point of intersection or other significant step towards solution	1
• writes the correct solution	1

**Sample Answer:**

point of intersection of  $y = x$  and  $y = |2 - x|$  is  $(1,1)$

$\therefore$  from the graph  $|2 - x| < x$  when  $x > 1$



(c) (2 marks)

**Outcomes assessed: PE3**

**Targeted Performance Bands: E2-E3**

Criteria	Marks
• uses the factor theorem with substitution of $x = -2$	1
• solves the equation to find $k$	1

**Sample Answer:**

$$P(x) = x^2 + kx + 6$$

$$P(-2) = 4 - 2k + 6 = 0 \text{ as } (x + 2) \text{ is a factor}$$

$$k = +5$$

(d) (3 marks)

**Outcomes assessed: PE2**

**Targeted Performance Bands: E2-E3**

Criteria	Marks
• finds the TWO gradients	1
• uses the correct formula for $\tan \theta$	1
• finds the correct angle	1

**Sample Answer:**

From the diagram the gradients of the lines are 2 and -1.

$$\begin{aligned}\tan \theta &= \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \\ &= \left| \frac{2 - (-1)}{1 + 2(-1)} \right| \\ &= 3 \\ \therefore \theta &= 71^\circ 34'\end{aligned}$$

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(e) (2 marks)

*Outcomes assessed: PE3*

*Targeted Performance Bands: E2-E3*

Criteria	Marks
• establishes correct quadratic or other correct significant step towards solution	1
• finds full solution	1

*Sample Answer:*

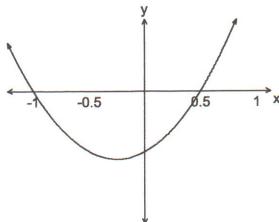
$$\frac{3}{x+1} < 2 \quad \text{multiply by } (x+1)^2$$

$$3(x+1) < 2(x+1)^2$$

$$2(x+1)^2 - 3(x+1) > 0$$

$$(x+1)(2x+2-3) > 0$$

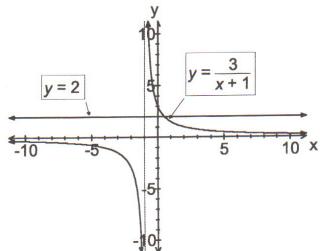
$$(x+1)(2x-1) > 0$$



$$\text{Solution is } x < -1 \text{ or } x > \frac{1}{2}$$

∴ Jasi's solution is only partially correct.

*or graphically:*



ordinate of intersection  $\frac{3}{x+1} = 2$

$$3 = 2x + 2$$

$$x = \frac{1}{2}$$

From the graph  $x > \frac{1}{2}$  satisfies the inequality

BUT the other branch of the hyperbola for  $x < -1$  also satisfies the inequality.

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**Question 2** (12 marks)

(a) (2 marks)

**Outcomes assessed: PE3****Targeted Performance Bands: E2-E3**

Criteria	Mark
• finds sum and product of roots	1
• evaluates the expression correctly	1

**Sample Answer:**

$$\begin{aligned}
 2x^3 - 5x^2 + 3x - 5 &= 0 \\
 \alpha + \beta + \gamma &= \frac{5}{2} \quad \alpha\beta + \alpha\gamma + \beta\gamma = \frac{3}{2} \quad \alpha\beta\gamma = \frac{5}{2} \\
 \alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2 &= \alpha\beta\gamma(\alpha + \beta + \gamma) \\
 &= \frac{5}{2} \times \frac{5}{2} \\
 &= \frac{25}{4}
 \end{aligned}$$

(b) (i) (1 mark)

**Outcomes assessed: P4****Targeted Performance Bands: E2-E3**

Criteria	Mark
• gives the correct solutions	1

**Sample Answer:**

Given  $f(x) = \frac{2x}{\sqrt{1-x^2}}$

$f(x)$  is undefined when  $1-x^2 \leq 0$

i.e. when  $x \leq -1$  or  $x \geq 1$

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(b) (ii) (3 marks)

*Outcomes assessed: HE6*

*Targeted Performance Bands: E2-E3*

Criteria	Marks
• rewrites the integral in term of $u$	1
• finds the new limits	1
• evaluates the integral correctly to at least $-2\left[\frac{\sqrt{3}}{2} - 1\right]$ (correct numerical equivalence)	1

*Sample Answer:*

$$\begin{aligned}\int_0^{\frac{1}{2}} \frac{2x}{\sqrt{1-x^2}} dx &= \int_0^{\frac{\pi}{6}} \frac{2 \sin u}{\cos u} \cos u du && x = \sin u \\ &= 2 \int_0^{\frac{\pi}{6}} \sin u du && dx = \cos u du \\ &= -2 \left[ \cos u \right]_0^{\frac{\pi}{6}} && x = 0 \Rightarrow u = 0 \\ &= -2 \left( \cos \frac{\pi}{6} - \cos 0 \right) && x = \frac{1}{2} \Rightarrow u = \frac{\pi}{6} \\ &= -2 \left[ \frac{\sqrt{3}}{2} - 1 \right] \\ &= 2 - \sqrt{3}\end{aligned}$$

(c) (i) (1 mark)

*Outcomes assessed: HE4*

*Targeted Performance Bands: E2-E3*

Criteria	Mark
• differentiates correctly	1

*Sample Answer:*

$$\begin{aligned}\frac{d}{dx} (\sin^{-1} x + \cos^{-1} x) &= \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \\ &= 0\end{aligned}$$

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(c) (ii) (2 marks)

*Outcomes assessed: HE4*

*Targeted Performance Bands: E2-E3*

Criteria	Marks
• identifies that the primitive is a constant	1
• uses a suitable substitution, or otherwise, to show that the constant is $\frac{\pi}{2}$	1

*Sample Answer:*

Since the derivative is zero,  $\sin^{-1} x + \cos^{-1} x = C$  ( $C$  is a constant)

$$\begin{aligned} \text{Let } x = 0 \Rightarrow \sin^{-1} 0 + \cos^{-1} 0 &= 0 + \frac{\pi}{2} \\ &= \frac{\pi}{2} \end{aligned}$$

(d) (i) (1 mark)

*Outcomes assessed: PE3*

*Targeted Performance Bands: E2-E3*

Criteria	Mark
• correct numerical expression for the answer	1

*Sample Answer:*

EXERCISE  $\Rightarrow$  8 letters with 3 Es

$$\text{Number of arrangements} = \frac{8!}{3!} = 6720$$

(d) (ii) (2 marks)

*Outcomes assessed: PE3*

*Targeted Performance Bands: E2-E3*

Criteria	Marks
• significant progress towards solution	1
• correct numerical expression for answer	1

*Sample Answer:*

EXERCISE with C and R at the ends

$$\text{Number of arrangements} = \frac{2!6!}{3!} = 240$$

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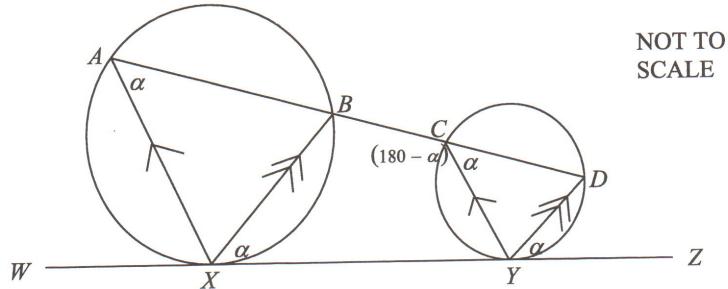
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**Question 3 (12 marks)**

(a) (i) (3 marks)

**Outcomes assessed: PE2, PE3****Targeted Performance Bands: E2-E3**

Criteria	Marks
• correct application of alternate segment theorem or other correct step in explanation	1
• identifying corresponding angles or other significant progress in reasoning	1
• conclusion with reason	1

**Sample Answer:**NOT TO  
SCALEGiven  $\angle BXY = \alpha$  $\therefore \angle BAX = \alpha$  (angle between tangent and chord at the point of contact is equal to the angle in the alternate segment) $\angle DCY = \alpha$  (corresponding to  $\angle BAX$ ,  $AX \parallel CY$ ) $\angle DYZ = \alpha$  (angle between tangent and chord at the point of contact is equal to the angle in the alternate segment) $\therefore \angle DYZ = \angle BXY$  $\therefore BX \parallel DY$  (corresponding angles are equal)

(a) (ii) (1 mark)

**Outcomes assessed: PE2, PE3****Targeted Performance Bands: E2-E3**

Criteria	Mark
• states the correct reason	1

**Sample Answer:** $\angle BCY = 180 - \alpha$  ( $BCD$  is a straight line) $\therefore \angle BCY + \angle BXY = 180^\circ$  $\therefore BCYX$  is a cyclic quadrilateral as one pair of opposite angles are supplementary**DISCLAIMER**

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(b) (3 marks)

*Outcomes assessed: PE2*

*Targeted Performance Bands: E2-E3*

Criteria	Marks
• determines the correct value of $\sin A$ , including the sign	1
• determines the correct value of $\sin B$ and $\cos B$ , including the sign	1
• expands $\sin(A - B)$ and gives a correct numerical expression	1

*Sample Answer:*

$$\cos A = \frac{3}{5} \quad \therefore \sin A = -\frac{4}{5} \quad (A \text{ is reflex and in the 4th quad})$$

$$\tan B = \frac{12}{5} \quad \therefore \sin B = -\frac{12}{13} \quad \text{and} \quad \cos B = -\frac{5}{13} \quad (B \text{ is reflex and in the 3rd quad})$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$= \left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right) - \left(\frac{3}{5}\right)\left(-\frac{12}{13}\right)$$

$$= \frac{56}{65}$$

(c) (3 marks)

*Outcomes assessed: HE7*

*Targeted Performance Bands: E3-E4*

Criteria	Marks
• writes down at least one correct term in $x^6$ or $x^5$	1
• writes down the correct equivalence	1
• finds the correct value of $k$ (correct numerical equivalence)	1

*Sample Answer:*

Consider terms of the expansion of  $(1 - kx)^9$

$$\begin{array}{ll} \text{Term in } x^5 = {}^9C_5(-kx)^5 & \text{Term in } x^6 = {}^9C_6(-kx)^6 \\ = {}^9C_5 k^5 x^5 & = {}^9C_6 k^6 x^6 \end{array}$$

$$\therefore -{}^9C_5 k^5 = 2 {}^9C_6 k^6$$

$$k = -\frac{{}^9C_5}{2 {}^9C_6}$$

$$= -\frac{1}{2} \times \frac{9!}{5!4!} \times \frac{3!6!}{9!}$$

$$= -\frac{3}{4}$$

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(d) (2 marks)

*Outcomes assessed: PE3, HE7*

*Targeted Performance Bands: E2-E3*

Criteria	Marks
• identifies the function	1
• uses Newton's Method to find correct approximation (correct numerical equivalence)	1

*Sample Answer:*

$$x = \sqrt[3]{9} \\ \therefore x^3 = 9 \Rightarrow f(x) = x^3 - 9 \therefore f'(x) = 3x^2 \\ \text{Let } x_1 = 2 \quad \therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow x_2 = 2 - \frac{-1}{12} \\ x_2 = 2 \frac{1}{12}$$

**Question 4 (12 marks)**

(a) (3 marks)

*Outcomes assessed: HE2*

*Targeted Performance Bands: E3-E4*

Criteria	Marks
• establishes the truth of $S(1)$	1
• establishes the correct relationship between $S(k)$ and $S(k+1)$	1
• deduces the required result	1

*Sample Answer:*

Let  $S(n)$  be the statement  $\sum_{r=1}^n r \times r! = (n+1)! - 1$   
 Consider  $S(1)$ :  $LHS = 1 \times 1! ; RHS = (1+1)! - 1 = 1$ .  
 Hence  $S(1)$  is true

If  $S(k)$  is true:  $\sum_{r=1}^k r \times r! = (k+1)! - 1 \quad *$

RTP  $S(k+1)$  is true i.e. to prove  $\sum_{r=1}^{k+1} r \times r! = (k+2)! - 1$

$$\begin{aligned} LHS &= \sum_{r=1}^k r \times r! + (k+1)(k+1)! \\ &= (k+1)! - 1 + (k+1)(k+1)! \quad \text{if } S(k), \text{ using } * \\ &= (k+1)! (1+k+1) - 1 \\ &= (k+1)! (k+2) - 1 \\ &= (k+2)! - 1 \end{aligned}$$

Hence if  $S(k)$  then  $S(k+1)$  is true. Thus since  $S(1)$  is true it follows by induction that  $S(n)$  is true for positive integral  $n$ .

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(b) (i) (2 marks)

*Outcomes assessed: HE5*

*Targeted Performance Bands: E2-E3*

Criteria	Marks
• sets up the correct differential equation or significant progress towards result	1
• finds the desired equation	1

*Sample Answer:*

$$\begin{aligned}\ddot{x} &= 2x - 3 \\ \therefore \frac{d}{dx} \left( \frac{1}{2} v^2 \right) &= 2x - 3 \\ \frac{1}{2} v^2 &= x^2 - 3x + c \\ \text{when } x = 0, v = 2 &\Rightarrow c = 2 \\ \therefore \frac{1}{2} v^2 &= x^2 - 3x + 2 \\ \therefore v^2 &= 2x^2 - 6x + 4\end{aligned}$$

(b) (ii) (2 marks)

*Outcomes assessed: HE5*

*Targeted Performance Bands: E3-E4*

Criteria	Marks
• calculates the correct velocity and acceleration	1
• describes the motion	1

*Sample Answer:*

at  $x = 1$ ,  $v = 0$  and  $\ddot{x} = -1 \text{ m/s}^2$

After the object comes to rest at  $x = 1$  it then moves towards the origin, and will continue moving in a negative direction.

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(c) (i) (2 marks)

**Outcomes assessed: HE3**

**Targeted Performance Bands: E3-E4**

Criteria	Marks
• differentiates correctly	1
• proves correct result	1

**Sample Answer:**

$$\begin{aligned}
 N &= \frac{200}{1+ke^{-200t}} = 200\left(1+ke^{-200t}\right)^{-1} \\
 \frac{dN}{dt} &= -200\left(1+ke^{-200t}\right)^{-2}(-200ke^{-200t}) \\
 &= \frac{200}{1+ke^{-200t}} \left( \frac{200ke^{-200t}}{1+ke^{-200t}} \right) \\
 &= N \left( \frac{200 + 200ke^{-200t} - 200}{1+ke^{-200t}} \right) \\
 &= N \left( \frac{200(1+ke^{-200t})}{(1+ke^{-200t})} - \frac{200}{1+ke^{-200t}} \right) \\
 &= N(200 - N)
 \end{aligned}$$

(c) (ii) (2 marks)

**Outcomes assessed: HE3**

**Targeted Performance Bands: E3-E4**

Criteria	Marks
• finds the value of $k$	1
• finds $t$ (correct numerical equivalence)	1

**Sample Answer:**

$$\begin{aligned}
 \text{when } t = 0, N = 1 &\quad \text{i.e. } 1 = \frac{200}{1+k} \quad \therefore k = 199 \\
 N &= \frac{200}{1+199e^{-200t}} \quad \text{half the colony infected i.e. } N = 100 \\
 100 &= \frac{200}{1+199e^{-200t}} \\
 1+199e^{-200t} &= 2 \\
 e^{-200t} &= \frac{1}{199} \\
 -200t &= \log_e \frac{1}{199} \\
 \therefore t &= -\frac{1}{200} \log_e \frac{1}{199} \\
 t &= 0.0265 \text{ years or } 9.66 \text{ days}
 \end{aligned}$$

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(c) (iii) (1 mark)

*Outcomes assessed: HE7*

*Targeted Performance Bands: E3-E4*

Criteria	Mark
• shows that the limiting value of $N$ is 200	1

*Sample Answer:*

$$\text{as } t \rightarrow \infty \quad e^{-200t} \rightarrow 0$$

$$\therefore N \rightarrow \frac{200}{1+0} = 200 \quad \text{i.e. eventually all the bees will be infected.}$$

**Question 5** (12 marks)

(a) (i) (2 marks)

*Outcomes assessed: H5, HE7*

*Targeted Performance Bands: E2-E3*

Criteria	Marks
• differentiates and determines the correct quadratic	1
• identifies that $\Delta \geq 0$ and sets up the correct inequality	1

*Sample Answer:*

$$P(x) = -2x^3 + px^2 - qx + 5$$

$$P'(x) = -6x^2 + 2px - q = 0 \text{ for stationary points}$$

For this quadratic to have real solutions  $\Delta \geq 0$

$$\text{i.e. } 4p^2 - 24q \geq 0$$

$$p^2 - 6q \geq 0$$

(a) (ii) (1 mark)

*Outcomes assessed: H5, HE7*

*Targeted Performance Bands: E3-E4*

Criteria	Mark
• gives the correct conclusion	1

*Sample Answer:*

When  $p^2 - 6q = 0$   $P'(x)$  has a double root and there is only one stationary point which would be a horizontal point of inflection.

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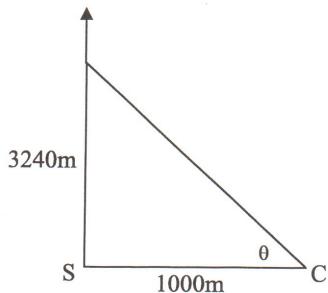
(b) (3 marks)

*Outcomes assessed: HE5, HE7*

*Targeted Performance Bands: E3-E4*

Criteria	Marks
• establishes $\frac{dh}{d\theta}$	1
• evaluating $\theta$ at $t = 30$ s, or other significant progress towards the result such as correct use of chain rule	1
• correct answer	1

*Sample Answer:*



$$\begin{aligned}\frac{dh}{dt} &= 230 \text{ m/s} & \tan \theta &= \frac{h}{1000} \\ h &= 1000 \tan \theta & \frac{dh}{d\theta} &= 1000 \sec^2 \theta\end{aligned}$$

$$\text{at } t = 30 \text{ seconds, } h = 3240 \text{ m} \Rightarrow \tan \theta = \frac{3240}{1000} \Rightarrow \theta = 1.271 \text{ radians}$$

$$\begin{aligned}\frac{d\theta}{dt} &= \frac{d\theta}{dh} \times \frac{dh}{dt} \\ &= \frac{\cos^2 1.271}{1000} \times 230 \\ &= 0.02 \text{ rads/sec}\end{aligned}$$

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(c) (i) (1 mark)

*Outcomes assessed: HE3*

*Targeted Performance Bands: E2-E3*

Criteria	Mark
• gives the correct answer (correct numerical equivalence)	1

*Sample Answer:*

$$P(\text{a basket}) = \frac{2}{5}$$

$$\begin{aligned} P(\text{2 points}) &= {}^6C_2 \left(\frac{2}{5}\right)^2 \left(\frac{3}{5}\right)^4 \\ &= \frac{972}{3125} \quad (\text{or } 0.31104) \end{aligned}$$

(c) (ii) (2 marks)

*Outcomes assessed: HE3, HE7*

*Targeted Performance Bands: E3-E4*

Criteria	Marks
• sets up correct inequality or other significant progress	1
• gives the correct solution, rounding to the nearest whole number	1

*Sample Answer:*

$$\begin{aligned} P(\text{at least one}) &= 1 - P(\text{none}) \\ &= 1 - 0.6^n \end{aligned}$$

$$\therefore 1 - 0.6^n \geq 0.9978$$

$$0.6^n \leq 0.0022$$

take logs of both sides

$$\text{i.e. } n \ln 0.6 \leq \ln 0.0022$$

$$n \geq \frac{\ln 0.0022}{\ln 0.6}$$

$$n \geq 11.979$$

$$n \geq 12$$

Diana would need 12 free throws.

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(d) (3 marks)

*Outcomes assessed: PE6, HE7*

*Targeted Performance Bands: E2-E3*

Criteria	Marks
• some progress towards result, e.g. finds the missing angles	1
• significant progress towards result, e.g. finding $BP$ (correct numerical equivalence)	1
• finds the angle of depression	1

*Sample Answer:*

$$\angle BPM = 110^\circ - 22^\circ = 88^\circ$$

$$\angle PBM = 202^\circ - 140^\circ = 62^\circ$$

$$\therefore \angle PMB = 30^\circ$$

$$\therefore \frac{BP}{\sin 30^\circ} = \frac{750}{\sin 62^\circ}$$

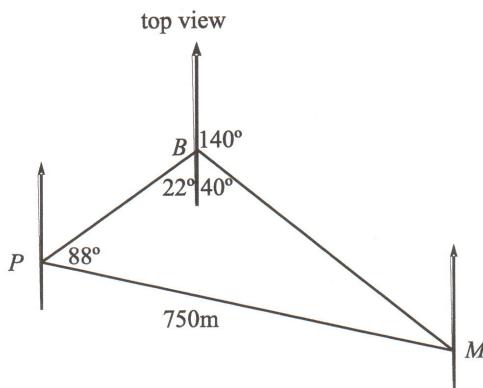
$$\therefore BP = \frac{750 \sin 30^\circ}{\sin 62^\circ}$$

$$= 424.71 \quad (2 \text{ d.p.})$$

$$\text{angle of elevation} = \tan^{-1}\left(\frac{80}{BP}\right)$$

$$= 10^\circ 40'$$

$$\therefore \text{angle of depression is } 10^\circ 40'$$



**Question 6 (12 marks)**

(a) (i) (2 marks)

*Outcomes assessed: H5*

*Targeted Performance Bands: E3-E4*

Criteria	Marks
• finds the vertical asymptote	1
• finds the horizontal asymptote	1

*Sample Answer:*

$$f(x) = \frac{e^x}{x-1}$$

vertical asymptote at  $x = 1$

horizontal asymptote :  $x \rightarrow \infty, f(x) \rightarrow \infty; x \rightarrow -\infty, f(x) \rightarrow 0^-$

$\therefore y = 0$  is a horizontal asymptote as  $x \rightarrow -\infty$

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(a) (ii) (3 marks)

**Outcomes assessed: PE5, PE6, H5**

**Targeted Performance Bands: E3-E4**

Criteria	Marks
• finds the stationary point	1
• identifies intercept	1
• sketches the correct function	1

**Sample Answer:**

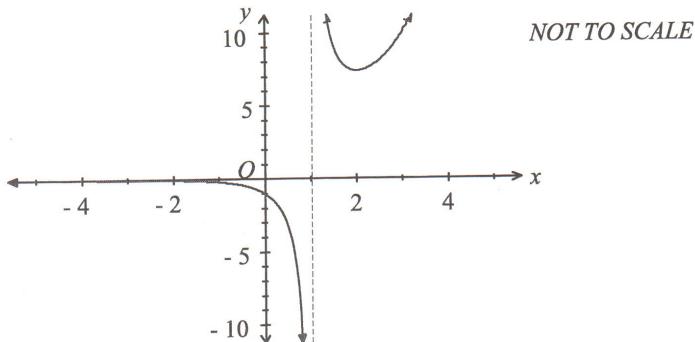
$$\begin{aligned}f''(x) &= \frac{(x-1)e^x - e^x}{(x-1)^2} \\&= \frac{e^x(x-2)}{(x-1)^2} \\f'(x) = 0 &\text{ when } x = 2 \text{ and } y = e^2\end{aligned}$$

testing nature:

$x$	$2^-$	$2$	$2^+$
$f'(x)$	-	0	+

\ \ \ \ /  
∴  $(2, e^2)$  is a minimum point

$$\text{when } x = 0, y = \frac{e^0}{0-1} = -1 \quad \therefore \text{y intercept is } (0, -1)$$



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(a) (iii) (1 mark)

*Outcomes assessed: HE\$*

*Targeted Performance Bands: E2-E3*

Criteria	Marks
• gives correct domain	1

*Sample Answer:*

For an inverse function domain is  $x \geq 2$

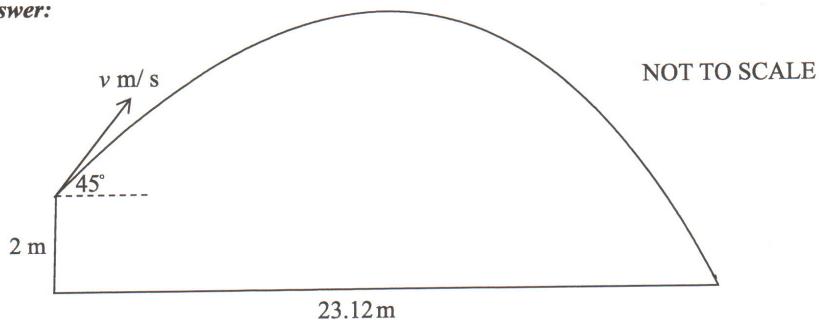
(b) (i) (2 marks)

*Outcomes assessed: HE3*

*Targeted Performance Bands: E2-E3*

Criteria	Marks
• finds the correct differential equations for the horizontal motion	1
• finds solves the correct differential equations for the vertical motion	1

*Sample Answer:*



#### Horizontal

$$\ddot{x} = 0$$

$$\dot{x} = c_1$$

$$\text{at } t = 0, \dot{x} = v \cos 45^\circ \Rightarrow c_1 = v \cos 45^\circ$$

$$\therefore \dot{x} = \frac{v}{\sqrt{2}}$$

$$x = \frac{vt}{\sqrt{2}} + c_2$$

$$\text{at } t = 0, x = 0 \Rightarrow c_2 = 0$$

$$x = \frac{vt}{\sqrt{2}}$$

#### Vertical

$$\ddot{y} = -10$$

$$\dot{y} = -10t + c_3$$

$$\text{at } t = 0, \dot{y} = v \sin 45^\circ \Rightarrow c_3 = v \sin 45^\circ$$

$$\therefore \dot{y} = -10t + \frac{v}{\sqrt{2}}$$

$$y = -5t^2 + \frac{vt}{\sqrt{2}} + c_4$$

$$\text{at } t = 0, y = 2 \Rightarrow c_4 = 2$$

$$y = -5t^2 + \frac{vt}{\sqrt{2}} + 2$$

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(b) (ii) (2 marks)

*Outcomes assessed: HE3*

*Targeted Performance Bands: E3-E4*

Criteria	Marks
• determines the Cartesian equation of motion or other significant progress	1
• substitutes and solves for $v$	1

*Sample Answer:*

$$t = \frac{\sqrt{2}x}{v}$$

$$\therefore y = -5 \times \frac{2x^2}{v^2} + x + 2$$

at world record range  $x = 23.12$  and  $y = 0$

$$\therefore 0 = -\frac{10 \times 23.12^2}{v^2} + 25.12$$

$$\frac{5345.344}{v^2} = 25.12$$

$$v^2 = 212.7923567$$

$$v = 14.59 \text{ m/s (2 decimal places)}$$

(b) (iii) (2 marks)

*Outcomes assessed: HE3*

*Targeted Performance Bands: E3-E4*

Criteria	Marks
• finds the value of $t$ in terms of $v$	1
• substitutes into $y$ and solves	1

*Sample Answer:*

maximum height when  $\dot{y} = 0$

$$\text{i.e. } t = \frac{v}{10\sqrt{2}}$$

$$y = -5 \times \frac{v^2}{10^2 \times 2} + \frac{v^2}{10 \times 2} + 2$$

$$= \frac{v^2}{20} - \frac{v^2}{40} + 2$$

$$= \frac{v^2}{40} + 2$$

$$= \frac{212.7923567}{40} + 2$$

$$= 7.32 \text{ m (2 decimal places)}$$

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**Question 7** (12 marks)

(a) (i) (2 marks)

**Outcomes assessed: H5, PE2****Targeted Performance Bands: E3-E4**

Criteria	Marks
• Showing $\frac{\sec^2 \theta - \tan^2 \theta}{\sec \theta + \tan \theta}$ or other significant progress	1
• completing the proof	1

**Sample Answer:**

$$\text{RTP } \sec \theta - \tan \theta = \frac{1}{\sec \theta + \tan \theta}$$

LHS

$$= \frac{(\sec \theta - \tan \theta)(\sec \theta + \tan \theta)}{(\sec \theta + \tan \theta)}$$

$$= \frac{\sec^2 \theta - \tan^2 \theta}{\sec \theta + \tan \theta}$$

$$= \frac{1}{\sec \theta + \tan \theta} = \text{RHS}$$

(a) (ii) (2 marks)

**Outcomes assessed: PE2, HE7****Targeted Performance Bands: E3-E4**

Criteria	Marks
• establishing $\sec \theta + \tan \theta \geq 1$ or other significant progress	1
• justifying the inequality	1

**Sample Answer:**From the graphs  $\sec \theta \geq 1$  and  $\tan \theta \geq 0$  for  $0 \leq \theta < \frac{\pi}{2}$ 

$$\therefore \sec \theta + \tan \theta \geq 1$$

$$\therefore 0 < \frac{1}{\sec \theta + \tan \theta} \leq 1$$

i.e.  $0 < \sec \theta - \tan \theta \leq 1$  using (i)**DISCLAIMER**

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(a) (iii) (3 marks)

**Outcomes assessed: HE7**

**Targeted Performance Bands: E3-E4**

Criteria	Marks
• progress towards obtaining a quadratic equation e.g. establishing $2\cos\theta = 1 + \sin\theta$	1
• obtaining the correct quadratic in terms of $\sin\theta$ or equivalent progress	1
• correct solution	1

**Sample Answer:**

$$\sec\theta - \tan\theta = \frac{1}{2}$$

$$\text{i.e. } \frac{1}{\sec\theta + \tan\theta} = \frac{1}{2}$$

$$\therefore \sec\theta + \tan\theta = 2$$

$$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = 2$$

$$\text{i.e. } 2\cos\theta = 1 + \sin\theta$$

Square and solve the quadratic:

$$4\cos^2\theta = 1 + 2\sin\theta + \sin^2\theta$$

$$4(1 - \sin^2\theta) = 1 + 2\sin\theta + \sin^2\theta$$

$$5\sin^2\theta + 2\sin\theta - 3 = 0$$

$$(5\sin\theta - 3)(\sin\theta + 1) = 0$$

since  $\sin\theta$  is positive in the interval,  $0 \leq \theta < \frac{\pi}{2}$

$$\sin\theta = \frac{3}{5}$$

$$\text{i.e. } \theta = 0.644 \text{ radians}$$

**OR**

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• progress towards use of auxiliary angle method e.g. establishing $2\sin\theta + \cos\theta = 2$	1
• obtaining the correct value for $R$ or $\alpha$ using the auxiliary angle method	1
• correct solution	1

$$\begin{aligned}\sec\theta - \tan\theta &= \frac{1}{2} \\ \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} &= \frac{1}{2} \\ 2 - 2\sin\theta &= \cos\theta \\ 2\sin\theta + \cos\theta &= 2\end{aligned}$$

let  $2\sin\theta + \cos\theta \equiv R\sin(\theta + \alpha)$  where  $R$  is positive and  $\alpha$  is acute

i.e.  $2\sin\theta + \cos\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$   
 $\Rightarrow R\cos\alpha = 2$  and  $R\sin\alpha = 1$

i.e.  $\tan\alpha = \frac{1}{2}$  and  $R = \sqrt{5}$

$$\therefore \sqrt{5}\sin\left(\theta + \tan^{-1}\frac{1}{2}\right) = 2$$

$$\sin\left(\theta + \tan^{-1}\frac{1}{2}\right) = \frac{2}{\sqrt{5}}$$

$$\left(\theta + \tan^{-1}\frac{1}{2}\right) = \sin^{-1}\frac{2}{\sqrt{5}}$$

$$\theta = \sin^{-1}\frac{2}{\sqrt{5}} - \tan^{-1}\frac{1}{2}$$

$$\theta = 0.644 \text{ radians}$$

## OR

From simplifying to  $2\cos\theta = 1 + \sin\theta$  i.e.  $2\cos\theta - \sin\theta = 1$

let  $2\cos\theta - \sin\theta \equiv R\cos(\theta + \alpha)$  where  $R$  is positive and  $\alpha$  is acute  
i.e.  $2\cos\theta - \sin\theta \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$   
 $\Rightarrow R\cos\alpha = 2$  and  $R\sin\alpha = 1$

i.e.  $\tan\alpha = \frac{1}{2}$  and  $R = \sqrt{5}$

$$\therefore \sqrt{5}\cos\left(\theta + \tan^{-1}\frac{1}{2}\right) = 1$$

$$\cos\left(\theta + \tan^{-1}\frac{1}{2}\right) = \frac{1}{\sqrt{5}}$$

$$\left(\theta + \tan^{-1}\frac{1}{2}\right) = \cos^{-1}\frac{1}{\sqrt{5}}$$

$$\theta = \cos^{-1}\frac{1}{\sqrt{5}} - \tan^{-1}\frac{1}{2}$$

$$\theta = 0.644 \text{ radians}$$

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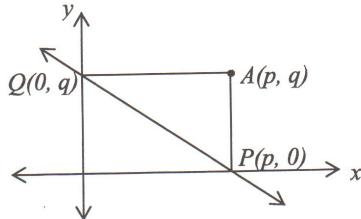
(b) (i) (1 mark)

**Outcomes assessed: HE7**

**Targeted Performance Bands: E2-E3**

Criteria	Mark
• finds the correct equation of $PQ$ in any form	1

**Sample Answer:**



$$y = -\frac{q}{p}x + q$$

$$py = -qx + pq$$

$$\frac{x}{p} + \frac{y}{q} = 1$$

(b) (ii) (3 marks)

**Outcomes assessed: HE7**

**Targeted Performance Bands: E3-E4**

Criteria	Marks
• determines the quadratic or other significant progress towards the solution	1
• uses $\Delta = 0$ when $PQ$ is a tangent to the curve	1
• establishes the relationship	1

**Sample Answer:**

If  $PQ$  is a tangent to the parabola then there is one point of intersection.

solve  $\frac{x}{p} + \frac{y}{q} = 1$  and  $y = \frac{x^2}{4a}$  simultaneously

$$\frac{x}{p} + \frac{x^2}{4aq} = 1$$

$$4aqx + px^2 = 4apq$$

$$px^2 + 4aqx - 4apq = 0$$

for  $PQ$  to be a tangent then  $\Delta = 0$  in this quadratic (i.e. only one root/solution)

$$\text{i.e. } 16a^2q^2 + 16ap^2q = 0$$

$$16aq(aq + p^2) = 0$$

$$\text{i.e. } aq + p^2 = 0$$

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(b) (iii) (1 mark)

**Outcomes assessed: HE7**

**Targeted Performance Bands: E3-E4**

Criteria	Mark
• finds the locus	1

**Sample Answer:**

$$\text{Coordinates of } A \Rightarrow x = p \text{ and } y = q$$

$$\therefore \text{since } aq + p^2 = 0 \text{ from (ii) then } ay + x^2 = 0$$

i.e.  $x^2 = -ay$  is the locus of  $A$

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