MATHEMATICS - TRIAL REVISION BOOKLET 1

1) CARRINGBAH 2005

2) GOSFORD 2005

3) GIRRAWEEN 2007

4) SYDNEY TECH 2007.

5) FORT ST. 2008

Student Name: ....

2005
TRIAL HIGHER SCHOOL CERTIFICATE
Sample Examination paper

MATHEMATICS

CARINGBAH HIGH

OMNIA VINCIT DILIGENTIA

General Instructions

Reading Time: 5 minutes
Working Time: 3 hours

- Attempt all questions
- Start each question on a new page
- Each question is of equal value
- Show all necessary working.
- Marks may not be awarded for careless work or incomplete solutions
- Standard integrals are printed on the last page
- Board-approved calculators may be used

Question 1 (12 marks) **MARKS**

(a) Express $\frac{1}{\sqrt{5}-2}$ with a rational denominator **2**

(b) The thickness of a cat's whisker is 0.0000598m. Write this in scientific notation correct to 2 significant figures. **2**

(c) Simplify: $\frac{3}{x-1}-\frac{2}{x+1}$ **2**

(d) Solve the pair of simultaneous equations: **2**

$$x-2y=9$$
$$2x+y=8$$

(e) Find $\frac{dy}{dx}$ given $y=(5-2x)^3$ **2**

(f) Solve: $x^3=4x$ **2**

Question 2 (12 marks) ***Start a new page*** **MARKS**

(a) (i) Find: $\int \frac{\cos 2x}{\sin 2x}\,dx$ **2**

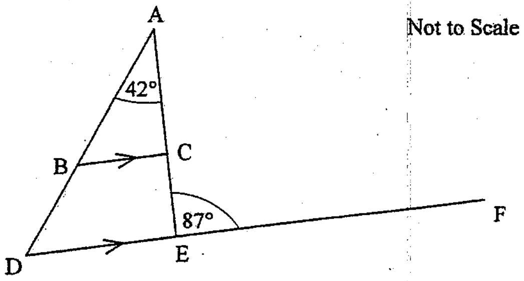
(ii) Evaluate: $\int_0^{\frac{\pi}{3}} \cos 3x\;dx$ **2**

(b) Differentiate with respect to x:

(i) $\frac{x^2}{x+1}$ **2**

(ii) $x^3 \cos x$ **2**

(c) In the diagram below, ADE is a triangle. B and C lie on AD and AE respectively such that BC is parallel to DE. Line DE is produced to F. $\angle AEF=87^\circ$ and $\angle DAE=42^\circ$ **3**
Find the size of $\angle ABC$, giving reasons for your answer.



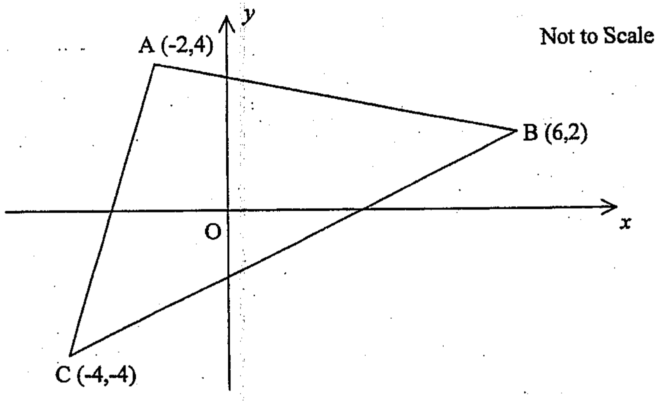
Not to Scale

(d) Evaluate: $\sum_{r=1}^{4} 2^{1-r}$ **1**

Marks

Question 3 (12 marks) ***Start a new page***

(a) The diagram below shows the points A (-2, 4), B (6, 2) and C (-4, -4). Copy or trace the diagram onto your worksheet.



(i) Calculate the length of the interval BC. 1

(ii) Find the gradient of BC. 1

(iii) Find the coordinates of M, the midpoint of BC. 1

(iv) Show that the equation of *l*, the perpendicular bisector of BC, is $5x+3y-2=0$. 2

(v) Show that *l* passes through A 1

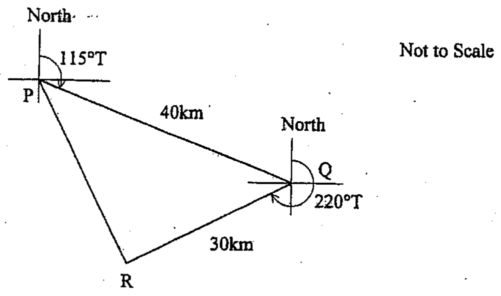
(vi) Hence or otherwise find the area of triangle ABC. 2

(b) Solve: $\sqrt{3}\tan x = -1$ for $0 \le x \le 2\pi$ 2

(c) Solve: $|3-2x| \le 5$ 2

Question 4 (12 marks) ***Start a new page*** **MARKS**

(a) From P the bearing of a Lighthouse Q, 40 kilometres distant from P, is 115°T. From Q the bearing of a headland R, 30 kilometres from Q, is 220°T. This is illustrated in the diagram below.



(i) Find the size of $\angle PQR$ 1

(ii) Use the Cosine Rule to find the length of PR. Give your answer correct to 2 decimal places. 1

(iii) Find the bearing of R from P. Give your answer to the nearest whole degree. 2

(b) For the parabola: $4x = 8y - y^2$

(i) Find the co-ordinates of the vertex. 2

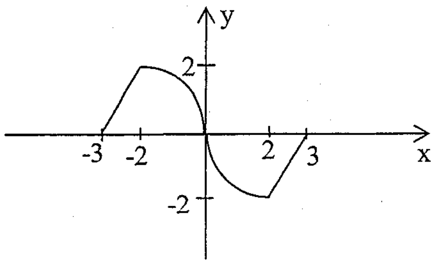
(ii) Find the co-ordinates of the focus. 1

(iii) Sketch the curve, labeling the focus and vertex 2

(c) Find the value of 'k' if the sum of the roots of $x^2 - (k-1)x + 2k = 0$ is equal to the product of the roots. 2

(d) The graph of $y = f(x)$ is shown below. It consists of quadrants of a circle and line segments.

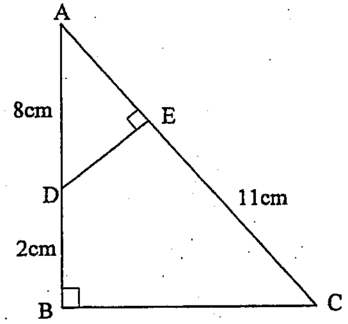
Find $\int_{-3}^{3} f(x)\,dx$ 1



Marks

Question 5 (12 marks) ***Start a new page***

(a) ABC is a right-angled triangle in which $\angle ABC = 90°$. Points D and E lie on AB and AC respectively such that AC is perpendicular to DE.
AD = 8cm, EC = 11cm and DB = 2cm.



Not to Scale

(i) Prove that ΔABC is similar to ΔAED. **3**

(ii) Find the length of AE. **1**

(b) Tom is an enthusiastic gardener. He planted a silky oak tree three years ago when it was 80 centimetres tall. At the end of the first year after planting, it was 130 centimetres tall, that is it grew 50 centimetres. Each years growth was then 90% of the previous years.

(i) What was the growth of the silky oak in the second year? **1**

(ii) How tall was the silky oak after three years? **1**

(iii) Assuming that it maintains the present growth pattern, explain why it will never reach a height of 10 metres. **2**

(iv) In which year will the silky oak reach a height of 5 metres? **2**

(c) For what values of k does $x^2 - (2+k)x + 4 = 0$ have real roots? **2**

Question 6 (12 marks) ***Start a new page*** **MARKS**

(a) For the function: $f(x) = 8x^3 - 8x^2$

(i) Find the stationary point(s) and determine their nature. **3**

(ii) Find the co-ordinates of any points of inflexion.
Confirm that your answer does provide a point of inflexion. **2**

(iii) Sketch the graph of the function $y = f(x)$, showing any stationary Points, points of inflexion and intercepts with the x- and y- axes. **3**

(iv) For what values of x is the curve concave down <u>and</u> decreasing? **2**

(b) For what values of x does the geometric series **2**

$$1 + \ln x + (\ln x)^2 + \dots$$

have a limiting sum?

Question 7 (12 marks) *Start a new page* **MARKS**

(a) A normal is drawn to the curve $y = \sin x$ at the point $P\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$.

The normal cuts the x-axis at Q.

(i) Show that the equation of the normal is: **2**

$$2x + y = \frac{\sqrt{3}}{2} + \frac{2\pi}{3}$$

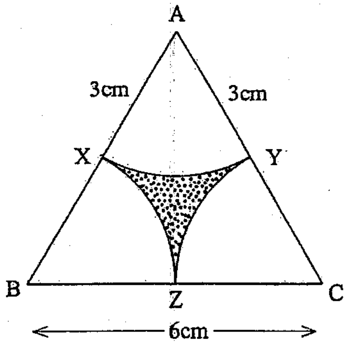
(ii) Find the co-ordinates of Q **1**

(iii) On a diagram, shade the region bounded by the curve $y = \sin x$, the normal at P and the x-axis. **2**

Your diagram should be at least $\frac{1}{3}$ *page and show all of the above information.*

(iv) Find the area of the shaded region. **3**

(b) ABC is an equilateral triangle with sides of length 6cm. An arc, centre A, and radius 3 cm cuts AB and AC at X and Y respectively.
This is repeated at B and C, as shown in the diagram.



Not to Scale

(i) Explain why $\angle ABC = \frac{\pi}{3}$ radians. **1**

(ii) Find the shaded area enclosed by the arcs XY, YZ and ZX **3**

Question 8 (12 marks) ***Start a new page*** **MARKS**

(a) (i) Copy and complete the following table of values: 1

x	-2	-1	0	1	2
3^x					

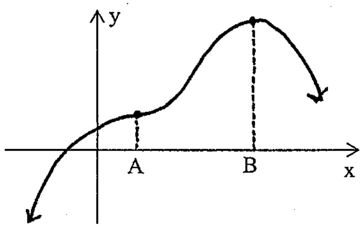
(ii) Use Simpson's Rule with 5 function values to estimate the area enclosed by the curve $y = 3^x$, the x-axis and the ordinates $x = 2$ and $x = -2$ 2

(b) Find the volume of the solid of revolution formed by rotating the curve $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ about the x-axis from $x = 1$ to $x = 9$. 3

(c) The graph of $y = f(x)$ is drawn below.

(i) Copy the diagram onto your answer page

(ii) On the same axes, sketch the graph of its gradient function, $y = f^1(x)$ 2



(d) (i) Sketch the graph of $y = 1 - 2\cos x$ for $0 \le x \le 2\pi$
Clearly mark on your sketch the endpoints of the curve in the given domain as well as its turning points. 3

(ii) Use your graph to solve: $1 - 2\cos x > 0$ in the given domain. 1

Marks

Question 9 (12 marks) ***Start a new page***

(a) Ella borrowed $180 000 to finance an extension on her home. She agreed to pay off the loan in equal monthly instalments of $P, paid at the end of each month, at an interest rate of 6% per annum, compounded monthly.

(i) Show that after the first instalment is paid, the amount owing on the loan is: **1**

$$\$[180\,000(1.005)-P]$$

(ii) Show that after three months she owes: **2**

$$\$\left[180\,000(1.005)^3-P\left((1.005)^2+(1.005)+1\right)\right]$$

(iii) If the loan is repaid after 8 years, find the value of P, the monthly instalment. **3**

(b) A particle moves in a straight line so that its distance x in metres from a fixed point O is given by:

$$x=2t+e^{-2t} \text{ where } t \text{ is measured in seconds}$$

(i) What is the velocity of the particle when $t=\frac{1}{2}$ sec ? **2**

(ii) Show that initially the particle is at rest. **1**

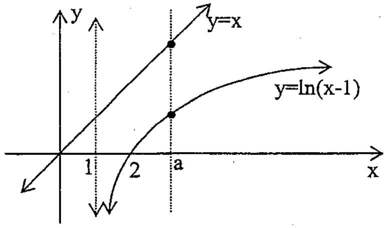
(iii) As t increases, find the limiting velocity of the particle. **1**

(iv) Draw a neat sketch of the graph of the velocity as a function of time. **1**

(v) Using v as the velocity and a as the acceleration, show that $a=4-2v$ **1**

Question 10 (12 marks) ***Start a new page*** **MARKS**

(a) The diagram shows the graphs of $y=\ln(x-1)$ and $y=x$ for $x>0$.



(i) Find an expression for M, the vertical distance between these two curves at any point x = a. **1**

(ii) For what value of 'a' is this vertical distance a minimum? Justify your answer. **3**

(iii) Find this minimum distance. **1**

(b) At the beginning of a drought, the number of sheep on a property was 285 000. Six months after the drought commenced this number had reduced to 202 000. Sheep numbers have continued to decreased so that at any time t, the number of sheep, S, is given by the formula:

$$S=A\,e^{-kt}$$

where A and k are constants and t is the number of months since the drought commenced.

(i) Find the values of A and k. **2**

(ii) Show that $\frac{dS}{dt}=-kS$ **1**

(iii) How many sheep will there be 1 year after the drought started? **1**

(iv) When will the flock reach one-third of its original size? **2**

(v) Find the rate of decrease of the number of sheep at this time. **1**

End of Paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Solutions: 2005 Mathematics TRIAL H.S.C.

①a) $\frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2$

b) $0.0000598 = 6.0 \times 10^{-5}$ m

c) $\frac{3}{x-1} - \frac{2}{x+1} = \frac{3(x+1)-2(x-1)}{(x-1)(x+1)}$

$= \frac{3x+3-2x+2}{x^2-1}$

$= \frac{x+5}{x^2-1}$

d) $x-2y = 9$ — ①

$2x+y = 8$ — ②

①×2 $2x-4y = 18$ —

$5y = -10$

$y = -2$

$x-2(-2) = 9$

$x = 5$ $\therefore (x,y) = (5,-2)$

e) $y = (5-2x)^3$

$\frac{dy}{dx} = 3(5-2x)^2 \times -2 = -6(5-2x)^2$

f) $x^3 = 4x$

$x^3-4x = 0$

$x(x^2-4) = 0$

$x(x+2)(x-2) = 0$

$x = 0, \pm 2$

②a) i) $\frac{1}{2}\int 2\left(\frac{\cos 2x}{\sin 2x}\right) dx = \frac{1}{2}\ln(\sin 2x) + C$

ii) $\int_0^{\pi/3} \cos 3x \, dx = \left[\frac{\sin 3x}{3}\right]_0^{\pi/3}$

$= \frac{1}{3}\left[\sin \pi - \sin 0\right] = 0$

b) i) $\frac{d}{dx} \frac{x^2}{x+1} = \frac{(x+1).2x - x^2.1}{(x+1)^2} = \frac{x^2+2x}{(x+1)^2}$

ii) $\frac{d}{dx} x^3.\cos x = x^3.{-\sin x} + \cos x.3x^2$

$= x^2(3\cos x - x\sin x)$

c) $\angle ECB = 87^\circ$ (alt $\angle$'s, $BC \parallel AF$)

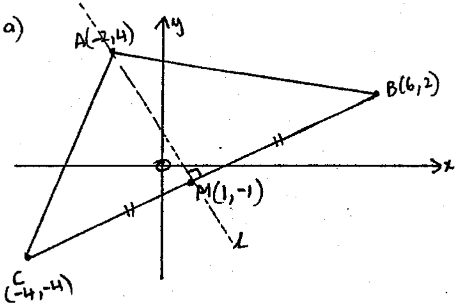
$\angle ABC = 87 - 42$ (ext $\angle$ ΔABC)

$= 45^\circ$

d) $\sum_{r=1}^{4} 2^{1-r} = 2^0 + 2^{-1} + 2^{-2} + 2^{-3}$

$= 1\frac{7}{8}$

③a)



i) $BC = \sqrt{(6+4)^2+(2+4)^2} = \sqrt{100+36} = \sqrt{136}$

ii) $m(BC) = \frac{2+4}{6+4} = \frac{6}{10} = \frac{3}{5}$

iii) $M = \left(\frac{6-4}{2}, \frac{-4+2}{2}\right) = (1,-1)$

iv) $m(l) = \frac{-5}{3}$

Eqn l: $y+1 = -\frac{5}{3}(x-1)$

$3y+3 = -5x+5$

$5x+3y-2 = 0$

v) $A(-2,4)$ LHS $= 5x+3y-2$

$= 5(-2)+3(4)-2$

$= -10+12-2$

$= 0 =$ RHS

$\therefore$ A lies on line l.

vi) $MA = \sqrt{(1+2)^2+(-1-4)^2} = \sqrt{9+25} = \sqrt{34}$

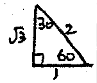
$\therefore$ Area $\Delta ABC = \frac{1}{2} \times \sqrt{136} \times \sqrt{34}$

$= 34$ units^2

b) $\sqrt{3}\tan x = -1$ $0 \le x \le 2\pi$

$\tan x = -\frac{1}{\sqrt{3}}$ Q 2, 4

$\pi/6$



$\therefore x = \frac{5\pi}{6}, \frac{11\pi}{6}$

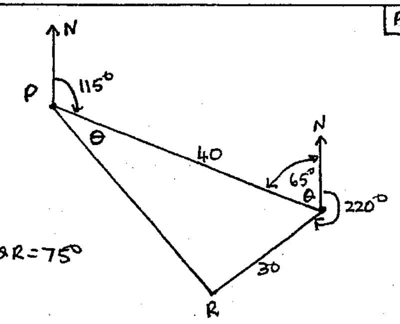
c) $|3-2x| \le 5$

$-5 \le 3-2x \le 5$

$-8 \le -2x \le 2$

$4 \ge x \ge -1$ $\therefore -1 \le x \le 4$

④a)



i) $\angle PQR = 75^\circ$

ii) $PR^2 = 40^2+30^2-2.40.30\cos 75$

$= 1878.83$

$PR = 43.35$

iii) $\frac{\sin\theta}{30} = \frac{\sin 75}{43.35}$

$\sin\theta = \frac{30\sin 75}{43.35} = 0.668$

$\theta \doteqdot 42^\circ$

$\therefore$ Bearing $= 115+42 = 157^\circ$T

b) $4x = 8y - y^2$

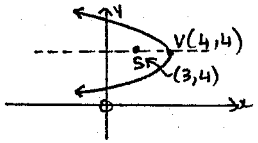
i) $y^2-8y+16 = -4x+16$

$(y-4)^2 = -4(x-4)$ $V = (4,4)$

$a = 1$

ii) $S = (3,4)$

iii)



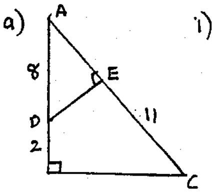
c) $\alpha+\beta = k-1$ $\alpha\beta = 2k$

$k-1 = 2k$

$k = -1$

d) $\int_{-3}^{3} f(x)\,dx = 0$

⑤a)



i) In Δ's ADE, ABC

$\angle A$ is common

$\angle AED = \angle ABC$ (given as 90°)

$\therefore \Delta ADE \equiv \Delta ABC$ (equiangular)

ii) $\frac{AE}{10} = \frac{8}{AE+11}$

$AE^2 + 11AE = 80$

$AE^2 + 11AE - 80 = 0$

$(AE+16)(AE-5) = 0$

$AE = -16, 5$

But $AE > 0$ $\therefore AE = 5$

b) $80 + \underbrace{50 + 45 + 40.5 + \cdots}_{GP \; a=50 \; r=0.9}$

i) 45cm

ii) 3 yrs $= 80+50+45+40.5 = 215.5$cm

iii) max height :- $S_\infty = \frac{a}{1-r} = \frac{50}{1-0.9} = 500$cm

$\therefore$ max height $= 80+500 = 580$cm

$\therefore$ tree never reaches 10m

iv) 5m → $500 = 80 + S_n$

$420 = \frac{50(1-0.9^n)}{1-0.9}$

$420 = \frac{50(1-0.9^n)}{0.1}$

$\frac{42}{50} = 1-0.9^n$

$0.9^n = 1-\frac{42}{50} = 0.16$

$\ln(0.9)^n = \ln 0.16$

$n\ln 0.9 = \ln 0.16$

$n = \frac{\ln 0.16}{\ln 0.9} = 17.393..$

$\therefore$ during the 17th year

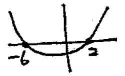
c) Real roots if $b^2-4ac \ge 0$

$[-(2+k)]^2 - 4(1)(4) \ge 0$

$4+4k+k^2-16 \ge 0$

$k^2+4k-12 \ge 0$

$(k+6)(k-2) \ge 0$



$\therefore k \le -6$ or $k \ge 2$

⑥ a) $f(x) = 8x^3 - 8x^2$

i) $f'(x) = 24x^2 - 16x$

$f''(x) = 48x - 16$

Stat. pts. when $f'(x) = 0$

$24x^2 - 16x = 0$

$8x(3x-2) = 0$

$x = 0, 2/3$

at $x=0$ $y=0$ $f''(0) = -16 < 0$ $\therefore$ MAX $(0,0)$

at $x=\frac{2}{3}$ $y = -1\frac{5}{27}$ $f''(\frac{2}{3}) = 16 > 0$ $\therefore$ MIN $(\frac{2}{3}, -1\frac{5}{27})$

ii) Pt of inflexion when $f''(x) = 0$

$48x - 16 = 0$

)ii) cont

$x = \frac{16}{48} = \frac{1}{3}$

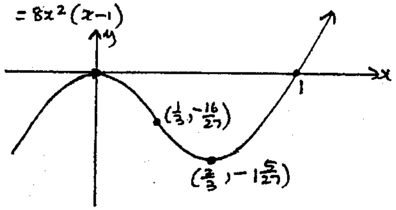
at $x = \frac{1}{3}$, $y = -\frac{16}{27}$

x	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$
y''	$-$	0	$+$

Concavity changes $\therefore$ pt inflexion at $\left(\frac{1}{3}, -\frac{16}{27}\right)$

i) $f(x) = 8x^3 - 8x^2$

$= 8x^2(x-1)$



v) concave down and decreasing:

$0 < x < \frac{1}{3}$

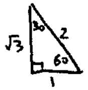
) Limiting sum if $|r| < 1$ $\quad r = \ln x$

$-1 < \ln x < 1$

$\log_e e^{-1} < \log_e x < \log_e e$

$\therefore \frac{1}{e} < x < e$

7a) $y = \sin x \qquad P\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$



i) $\frac{dy}{dx} = \cos x$

at $x = \frac{\pi}{3}$ $\quad m = \cos\frac{\pi}{3} = \frac{1}{2} \quad y = \frac{\sqrt{3}}{2}$

$\therefore m_1 = -2$

Eqn normal: $y - \frac{\sqrt{3}}{2} = -2\left(x - \frac{\pi}{3}\right)$

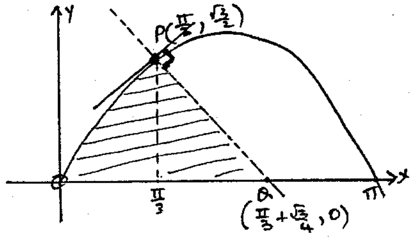
$y - \frac{\sqrt{3}}{2} = -2x + \frac{2\pi}{3}$

$2x + y = \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$

) $y = 0 \qquad 2x = \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3} + \frac{\sqrt{3}}{4} \qquad Q\left[\frac{\pi}{3} + \frac{\sqrt{3}}{4}, 0\right]$

i)



iv) Area $= \int_0^{\pi/3} \sin x \, dx + \frac{1}{2}bh$

$= \left[-\cos x\right]_0^{\pi/3} + \frac{1}{2}\left(\frac{\sqrt{3}}{4}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \left(-\cos\frac{\pi}{3}\right) - (-\cos 0) + \frac{3}{16}$

$= -\frac{1}{2} + 1 + \frac{3}{16}$

$= \frac{11}{16}$ units^2

b) i) As ΔABC is equilateral, all angles are 60°.

Hence $\angle ABC = 60° = \frac{60\pi}{180}$ rads

$= \pi/3$ rads.

ii) Area $\Delta ABC = \frac{1}{2} \times 6 \times 6 \times \sin 60$

$= 18 \times \frac{\sqrt{3}}{2} = 9\sqrt{3}$ cm^2

Area sector $AXY = \frac{1}{2} \times 3^2 \times \frac{\pi}{3} = \frac{3\pi}{2}$ cm^2

$\therefore$ Shaded area = area Δ $-$ 3 × area sector

$= 9\sqrt{3} - 3\left(\frac{3\pi}{2}\right)$

$= 9\sqrt{3} - \frac{9\pi}{2}$ cm^2

⑧ a) i)

x	-2	-1	0	1	2
3^x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

ii) $A = \int_{-2}^{2} 3^x \, dx \doteqdot \frac{1}{3}\left[\frac{1}{9} + 1 + 4 \times \frac{1}{3}\right] + \frac{1}{3}\left[1 + 9 + 4 \times 3\right]$

$\doteqdot 8\frac{4}{27}$

b) $V_x = \pi \int_1^9 y^2 \, dx \qquad y = \sqrt{x} + \frac{1}{\sqrt{x}}$

$y^2 = x + 2 + \frac{1}{x}$

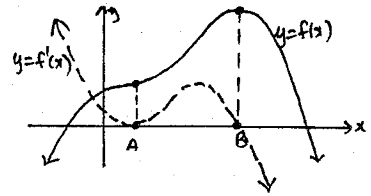
$= \pi \int_1^9 \left(x + 2 + \frac{1}{x}\right) dx$

$= \pi \left[\frac{x^2}{2} + 2x + \ln x\right]_1^9$

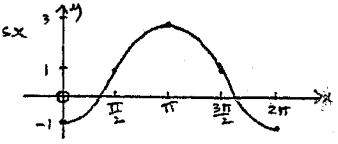
$= \pi \left[\frac{81}{2} + 18 + \ln 9 - \left(\frac{1}{2} + 2 + \ln 1\right)\right]$

$= \pi \left[56 + \ln 9\right]$

c) i)
ii)



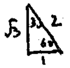
d) i) $y = 1 - 2\cos x$



ii) $1 - 2\cos x = 0$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$



$1 - 2\cos x > 0$ for $\frac{\pi}{3} < x < \frac{5\pi}{3}$

⑨ a) \$180 000 $\quad$ \$P/month $\quad$ 6% pa = 0.5% per month

i) $A_1 = 180\,000 + 180000 \times \frac{0.5}{100} - P$

$= 180000\,(1 + 0.005) - P$

$= 180\,000(1.005) - P$

ii) $A_2 = A_1 \times 1.005 - P$

$= \left[180000 \times 1.005 - P\right] \times 1.005 - P$

$= 180000 \times 1.005^2 - P \times 1.005 - P$

$= 180\,000 \times 1.005^2 - P(1.005 + 1)$

$A_3 = A_2 \times 1.005 - P$

$= \left[180\,000 \times 1.005^2 - P(1.005 +)\right] \times 1.005 - P$

$= 180\,000 \times 1.005^3 - P(1.005^2 + 1.005) - P$

$= 180\,000 \times 1.005^3 - P(1 + 1.005 + 1.005^2)$

iii) $n = 8$ years $= 96$ months

$\therefore A_{96} = 0$

$180\,000 \times 1.005^{96} - P(1 + 1.005 + 1.005^2 + \ldots + 1.005^{95}) = 0$

$$P = \frac{180\,000 \times 1.005^{96}}{1 + 1.005 + \ldots + 1.005^{95}}$$

$\uparrow$ GP $a = 1$, $r = 1.005$ $n = 96$

$$P = \frac{180\,000 \times 1.005^{96}}{\frac{1(1.005^{96} - 1)}{1.005 - 1}} = \frac{180000 \times 1.005^{96} \times 0.005}{1.005^{96} - 1}$$

$P = \$2365.46$

b) $x = 2t + e^{-2t}$

i) $v = \frac{dx}{dt} = 2 - 2e^{-2t}$

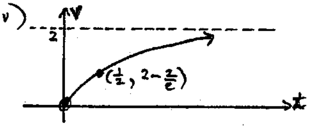
$t = \frac{1}{2} \quad v = 2 - 2e^{-1} = 2 - \frac{2}{e}$ m/sec

ii) $t = 0 \quad V = 2 - 2e^0 = 2 - 2 = 0 \quad \therefore$ at rest

iii) $v = 2 - \frac{2}{e^{2t}}$ $\quad$ as $t \to \infty$ $\quad e^{2t} \to \infty \quad \frac{2}{e^{2t}} \to 0$

$\therefore$ velocity $\to$ 2 m/sec

iv)



v) $a = \frac{dv}{dt} = 4e^{-2t} \qquad V = 2 - 2e^{-2t}$

$2e^{-2t} = 2 - V$

$e^{-2t} = \frac{2 - V}{2}$

$\therefore a = 4e^{-2t} = 4\left(\frac{2 - V}{2}\right) = 2(2 - V)$

$= 4 - 2V$

⑩ a) i) $M = a - \ln(a - 1)$

ii) $\frac{dM}{da} = 1 - \frac{1}{a-1} = 1 - (a-1)^{-1}$

$\frac{d^2M}{da^2} = (a-1)^{-2}$

Stat pts when $\frac{dM}{da} = 0$

$1 - \frac{1}{a-1} = 0$

$a - 1 = 1$

$a = 2$

at $a = 2$, $\frac{d^2M}{da^2} = (2-1)^{-2} > 0 \quad \therefore$ min

iii) $M = 2 - \ln(2-1) = 2 - \ln 1 = 2$

b) $S = 285\,000 \quad t = 0$

$S = 202\,000 \quad t = 6$

i) $S = Ae^{-kt}$

$\left.\begin{array}{l} t = 0 \\ S = 285000 \end{array}\right\} \quad 285\,000 = Ae^0$

$\therefore A = 285000$

$S = 285000e^{-kt}$

$\left.\begin{array}{l} S = 202000 \\ t = 6 \end{array}\right\} \quad 202000 = 285000e^{-6k}$

$\frac{202000}{285000} = e^{-6k}$

$\ln\left(\frac{202}{285}\right) = \ln e^{-6k} = -6k$

$k = \frac{\ln\frac{202}{285}}{-6}$

$\doteqdot 0.05737$

ii) $S = 285\,000\,e^{-kt}$

$\frac{dS}{dt} = -k \cdot 285\,000\,e^{-kt}$

$= -k(S)$

iii) $t = 1$ year $= 12$ months

$S = 285\,000\,e^{-k \times 12}$

$= 143\,171$

iv) $\frac{1}{3}$ original size $= 95000$

$95000 = 285000\,e^{-kt}$

$\frac{1}{3} = e^{-kt}$

$\ln(\frac{1}{3}) = \ln e^{-kt} = -kt$

$t = \frac{\ln\frac{1}{3}}{-k} = 19.15$ mths

v) $\frac{dS}{dt} = -kS = -k \times 95000 = -5450$

$\therefore$ decreasing at 5450 sheep/month.

Student Name: ....

2005

YEAR 12 TRIAL HSC EXAMINATION

MATHEMATICS

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown in every question

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value.

Question 1. (Start this question on a new page) **Marks**

(a) Express 0.031997 correct to three significant figures. 1

(b) Find a primitive of $\frac{2}{x}$ 1

(c) Solve $(v-2)^2=16$ 2

(d) Simplify $\frac{3x-2}{3}-\frac{3x-5}{4}$ 2

(e) If $\sqrt{27}-\frac{1}{\sqrt{3}}=a\sqrt{3}$, find the value of a 2

(f) Find the exact value of $\cos\frac{\pi}{6}+\sin\frac{3\pi}{4}$ 2

(g) Find the values of x for which $x+1=|4-2x|$ 2

Question 2. (Start this question on a new page) **Marks**

(a) On the number plane mark the origin O and the points $A(5,4)$, $B(-1,2)$, $C(-3,-7)$ and $D(3,-5)$, and then:

(i) Show that AB is parallel to DC 1

(ii) Show that the length of AB is the same as DC. 1

(iii) Show that the midpoint M of AC is also the midpoint of BD. 1

(iv) Show that $ABCD$ is a parallelogram. 2

(v) Show that the equation of DC is $x-3y-18=0$ 2

(vi) Find the perpendicular distance from B to $x-3y-18=0$ 2

(vii) Find the area of the parallelogram $ABCD$ 1

(b) Find the length of the longer diagonal of a parallelogram with sides 7 cm and 9 cm and an acute angle of 50°. 2

Question 3. (Start this question on a new page) **Marks**

(a) Draw a neat sketch of $y = 1 - |x|$ **1**

(b) Find the domain of $y = \sqrt{3 - 2x}$ **1**

(c) Differentiate with respect to x:

(i) $\frac{e^{2x}}{x}$ **2**

(ii) $\sin^2 3x$ **2**

(iii) $\ln\left(x^3 - 5\right)^7$ **2**

(d) Find $\int \frac{4}{1+3x} dx$ **2**

(e) Draw a neat sketch of the parabola $y^2 = 8x$ and write down the coordinates of the focus. **2**

Question 4. (Start this question on a new page) **Marks**

(a) Evaluate $\sum_{2}^{4}(2r - 3)$ **1**

(b) Differentiate $\frac{1}{x\sqrt{x}}$ **1**

(c) Given that $f(x) = \begin{cases} -1 & \text{if } x \leq -1 \\ 3x+2 & \text{if } |x| < 1 \\ 7-2x & \text{if } \ x \geq 1 \end{cases}$, **2**

find the value of $f(-3) + f\left(-\frac{1}{3}\right) + f\left(3\frac{1}{2}\right)$

(d) Prove that $\frac{1}{1-\sin A} + \frac{1}{1+\sin A} = 2\sec^2 A$ **2**

(e) Evaluate $\int_0^{\frac{\pi}{2}} \sin 2x \, dx$ **3**

(f) Find the geometric series whose second term is 6 and the sum to infinity is 49. **3**

Question 5. (Start this question on a new page) **Marks**

(a) A bag contains five red and five black balls. A ball is chosen at random from the bag. If it is red it is put to one side, and if it is black it is returned to the bag. A second drawing is then made from the bag.

(i) What is the probability that both balls are red? 1

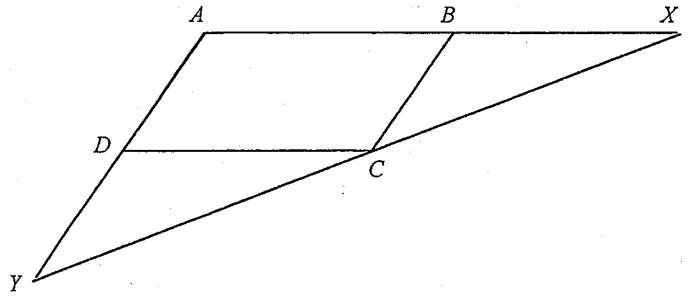
(ii) What is the probability of one ball of each colour? 2

(b) Find the value of a if $\int_2^a (2x+1)\,dx = 14$ 2

(c) ABCD is a parallelogram. Through C a straight line is drawn cutting AB, AD (both produced) at X, Y respectively.

(i) Show that $\angle CBX = \angle YDC$ 1

(ii) Prove that ΔDCY is similar to ΔBXC and hence show that $\frac{XB}{AB} = \frac{AD}{DY}$. 3



(d) Sketch the curve $y = 1 - \sin 2x$ for $0 \le x \le \pi$ 3

Question 6. (Start this question on a new page) **Marks**

(a) The line $y = 2x + 9$ meets the parabola $y = x^2 + 2x$ at two points A and B. Find :

(i) The coordinates of A and B. 1

(ii) the area between the curves $y = 2x + 9$ and $y = x^2 + 2x$ 3

(b) A, B, C and D are respectively the points $(0,2)$, $(0,8)$, $(4,0)$ and $(6,0)$. Find the locus of the point $P(x,y)$ which moves so that the areas of the triangles PAB and PCD are equal in magnitude. 3

(c) A closed tin rectangular box is to have a square base and a volume of 8 cubic metres. The length of the edge of the base is x metres.

(i) Express the height h m, of the box in terms of x. 1

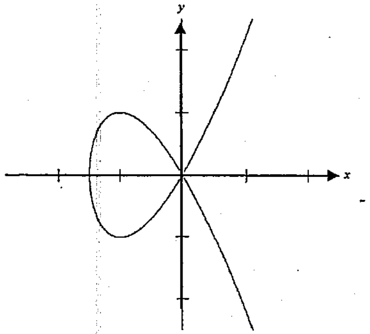
(ii) Show that the total surface area A square metres, is given by $A = \frac{32}{x} + 2x^2$ 1

(iii) Find the value of x for which A is a minimum. Hence find the smallest area of tin sheet necessary to fulfil these specifications. 3

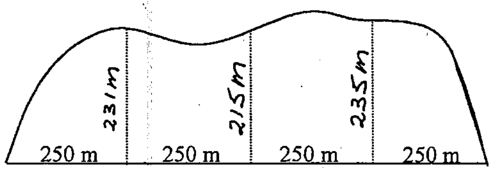
Question 7. (Start this question on a new page) **Marks**

(a) The curve with equation $y = \pm x\sqrt{(x+3)}$ is called **Tschirnhausen's cubic.** 4

Find the volume of the solid generated when the area enclosed by the loop is rotated about the x-axis



(b) The diagram below represents an area of land bounded by a river and a straight fence which is 1 kilometre in length. Four subdivisions are made at equal distances along the straight fence as shown in the diagram. The distance from the fence to the river is indicated. Use the Trapezoidal rule with 5 function values to find the approximate area of the land. 3



(c) Find the coordinates of the point on the curve $y = \frac{1}{2}x^2 - 3x + 2$ at which the tangent is parallel to the line $4x - 2y - 7 = 0$. 2

Question 7 part (d) is on the next page

(d) The curve $y = f(x)$ has a second derivative given by $\frac{d^2y}{dx^2} = (x-2)^2(x-3)$, find the x coordinate of any *possible* points of inflection and show that there is only one inflection. 3

Question 8. (Start this question on a new page) **Marks**

(a) If water drains from a cylindrical tank according to the formula $V = 5000\left(1 - \frac{t}{40}\right)^2$, where V is the volume of water in the tank at any time t. V is in litres and t in minutes.

(i) How much water is initially in the tank? 1

(ii) How long will it take to empty the tank. 1

(iii) Find the rate at which the water is flowing out of the tank after 10 minutes 2

(b) The position of a particle moving along the x-axis is given by $x = 8e^{-2t} - 8 + 16t$, where t is the time in seconds and x is measured in cm.

(i) Show that the particle is at rest when $t = 0$ 2

(ii) What is the limiting velocity which the particle approaches as t increases? 1

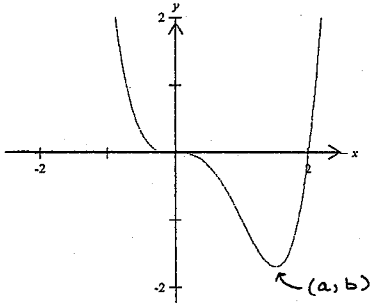
(iii) Show that the acceleration is $32 - 2v$ 2

(c) A disease is spreading through the community. Let N be the number of people with the disease after t days. Let D be the rate at which the number of people who have the disease is increasing. It is known that $D = 5 + \left(\frac{40}{4+t}\right)^2$. 3

Initially 20 people had the disease. How many would you expect to have the disease after 10 days?

Question 9. (Start this question on a new page) **Marks**

(a) The graph below is $y = f(x)$ 2



On your answer sheet draw a neat sketch of the derivative $y = f'(x)$

Show clearly what happens at $x = 0$ and at $x = a$.

(b) Find the equation of the straight line k, such that the x axis is the bisector of the angle between the line with equation $5x + 4y = 1$ and the line, k. 2

(c) The sum of the three middle terms of a nineteen term arithmetic series is 57 and the sum of the last three is 105, find the second term. 3

(d) Xing Borrows \$240 000 in order to buy a house. Interest of 6% per annum on the loan is calculated monthly on the balance owing.

The equal repayments of \$$M$, are made monthly and the loan is to be repaid over 20 years.

(i) Show that A_2 the amount owing at the end of 2 months is given by $A_2 = 240000 \times 1.005^2 - M(1 + 1.005)$. 2

(ii) Show that M is given by $M = \dfrac{1200 \times 1.005^{240}}{1.005^{240} - 1}$ 2

(iii) Find the value of M correct to the nearest \$. 1

Question 10. (Start this question on a new page) **Marks**

(a) The equation $x^2 + 3x - 2 = 0$ has roots α, β.

(i) Find $\alpha + \beta$ and $\alpha\beta$. 2

(ii) Hence or otherwise find the equation with roots α^2, β^2. 2

(b) Find expressions for the perpendicular distances from (x_1, y_1) to $7x - y + 9 = 0$ and to $x + y - 1 = 0$ and hence find the locus of the two lines bisecting the angles between the lines $7x - y + 9 = 0$ and $x + y - 1 = 0$. 4

(c) Two circles have radii 4 cm. and 7 cm. respectively. Their centres are 8 cm. apart. 4

Find the length of the arc of the smaller circle cut off by the larger circle.

End of Examination

MATHEMATICS

GOSFORD HIGH SCHOOL TRIAL HSC

Question 1 a) $\underline{0 \cdot 0320}$

b) $\int \frac{2}{x}\, dx$

$= 2\int \frac{1}{x}\, dx$

$= \underline{2 \ln x + c}.$

c) $(v-2)^2 = 16$

$v - 2 = \pm 4$

$v = 2 \pm 4$

$\underline{v = 6 \text{ OR } -2}.$

d) $\frac{3x-2}{3} - \frac{3x-5}{4}$

$= \frac{4(3x-2) - 3(3x-5)}{12}$

$= \frac{12x - 8 \quad -9x+15}{12}$

$= \frac{3x+7}{12}$

e) $\sqrt{27} - \frac{1}{\sqrt{3}} = \sqrt{9\times 3} - \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 3\sqrt{3} - \frac{\sqrt{3}}{3}$

$= \frac{9\sqrt{3} - \sqrt{3}}{3}$

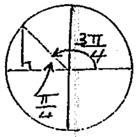
$= \frac{8\sqrt{3}}{3}$

$\therefore \underline{a = \frac{8}{3}}$

f) $\cos\frac{\pi}{6} + \sin\frac{3\pi}{4}$

$= \frac{\sqrt{3}}{2} + \sin\frac{\pi}{4}$

$= \underline{\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}}$



g) $x+1 = |4-2x|$

$x+1 = 4-2x$ OR $x+1 = -(4-2x)$

$3x = 3$ OR $x+1 = -4+2x$

$x = 1$ $\quad 5 = x$

Checking

If $x=1$ $\quad$ If $x=5$

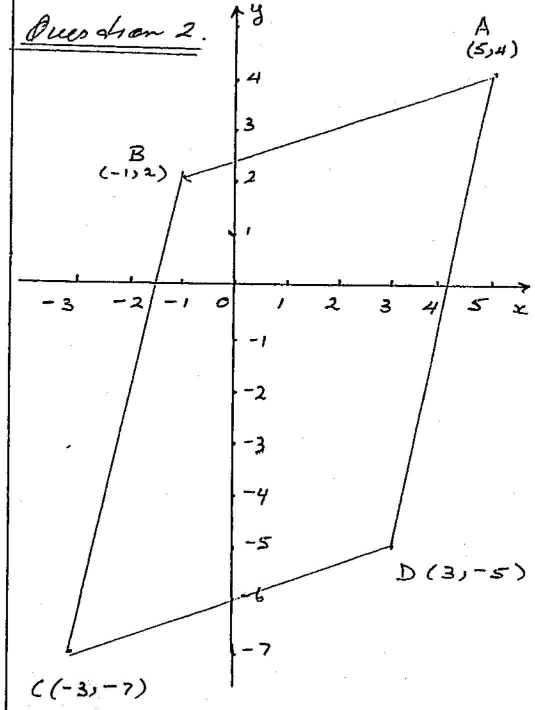
$1+1 = |4-2\times 1|$ $\quad$ $5+1 = |4-2\times 5|$

$2 = |4-2|$ $\quad$ $6 = |-6|$

True $\quad$ True

$\therefore \underline{x = 1 \text{ OR } x = 5}$

Question 2.



(i) $m_{AB} = \frac{4-2}{5-(-1)}$ $\quad$ $m_{DC} = \frac{-5-(-7)}{3-(-3)}$

$= \frac{2}{6}$ $\quad$ $= \frac{2}{6}$

$= \frac{1}{3}$ $\quad$ $= \frac{1}{3}$

$\therefore m_{AB} = m_{DC}$

$\therefore \underline{AB \parallel DC}$

(ii) $d_{AB} = \sqrt{(5-(-1))^2 + (4-2)^2}$

$= \sqrt{6^2+2^2}$

$= \sqrt{40}$

$= \underline{2\sqrt{10}}$

$d_{DC} = \sqrt{(3-(-3))^2 + (-5-(-7))^2}$

$= \sqrt{6^2+2^2}$

$= \sqrt{40}$

$= \underline{2\sqrt{10}}$

$\therefore d_{AB} = d_{DC}$

$\therefore AB = DC$

(iii) Mid pt of AC

$= \frac{5+(-3)}{2}, \frac{4+(-7)}{2}$

$= \underline{\left(1, -\frac{3}{2}\right)}$

Mid pt of BD

$= \left(\frac{-1+3}{2}, \frac{2+-5}{2}\right)$

$= \underline{\left(1, -\frac{3}{2}\right)}$

∴ AC and BD have the same mid-pt.

(iv) ABCD is a //ogram

(α) because from (iii) the diagonals bisect each other

OR

(β) from (i) & (ii)

AB // DC and AB = DC

i.e. one pair of opp. sides is both equal and parallel.

(v) Equ of DC is

$y - y_1 = m(x - x_1)$

$y - (-5) = \frac{1}{3}(x-3)$

$3y + 15 = x - 3$

$\underline{x - 3y - 18 = 0}$

(vi) $d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right|$

$= \left|\frac{(-1) - 3(2) - 18}{\sqrt{1^2+(-3)^2}}\right|$

$= \left|\frac{-1-6-18}{\sqrt{10}}\right|$

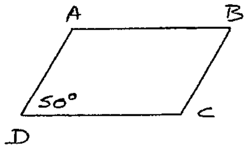
$= \underline{\frac{25}{\sqrt{10}}}$

(vii) Area = base × ⊥ height

$= 2\sqrt{10} \times \frac{25}{\sqrt{10}}$

$= \underline{50 \text{ unit}^2}.$

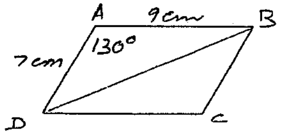
b)



$\angle D = 50°$

$\therefore \angle A = 130°$ since $\angle A + \angle D = 18$

co-int ∠'s AB // DC

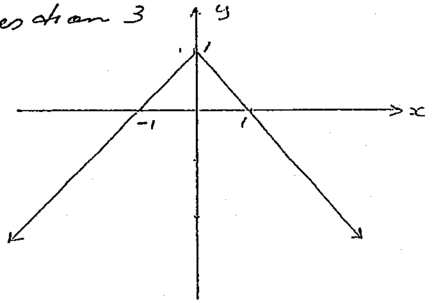


$DB^2 = 9^2 + 7^2 - 2\times 9 \times 7 \cos 130$

$\underline{DB = (14.5 \text{ to } 1 \text{ dec pl.})}$

Question 3

a)



b) Domain $3-2x \geq 0$

$3 \geq 2x$

$\frac{3}{2} \geq x$

Domain $x \leq \frac{3}{2}$

c) (i) $\frac{d}{dx}\left(\frac{e^{2x}}{x}\right) = \frac{x\,2e^{2x} - e^{2x}.1}{x^2}$

$= \frac{e^{2x}(2x-1)}{x^2}$

(ii) $\frac{d}{dx}(\sin 3x)^2$

$= 2(\sin 3x)^1 \cdot 3\cos 3x$

$= 6\sin 3x \cos 3x.$

(iii) $\frac{d}{dx}\left(\ln(x^3-5)^7\right)$

$= \frac{d}{dx}\left(7\ln(x^3-5)\right)$

$= 7.\ \frac{3x^2}{x^3-5}$

$= \frac{21x^2}{x^3-5}$

d) $\int \frac{4}{1+3x}\,dx = \frac{4}{3}\int \frac{3\,dx}{1+3x}$

$= \frac{4}{3}\ln(1+3x) + C$

e)

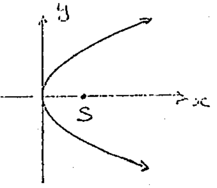
$y^2 = 8x$

$y^2 = 4ay$

$4a = 8$

$a = 2$

Focus $(2,0)$



Question 4

a) $\sum_{2}^{4}(2r-3)$

$= (2\times2-3) + (2\times3-3) + (2\times4-3)$

$= 1 + 3 + 5$

$= 9.$

b) $\frac{d}{dx}\left(\frac{1}{x\sqrt{x}}\right) = \frac{d}{dx}\left(x^{-\frac{3}{2}}\right)$

$= -\frac{3}{2}x^{-\frac{5}{2}}$

$= \frac{-3}{2x^2\sqrt{x}}$

c) $f(-3) + f(-\frac{1}{3}) + f(3\frac{1}{2})$

$= -1 + 3(-\frac{1}{3}) + 2. + 7 - 2\times3\frac{1}{2}$

$= -1 + 1 + 0$

$= 0$

d) $\frac{1}{1-\sin A} + \frac{1}{1+\sin A}$

$= \frac{1+\sin A + 1 - \sin A}{(1-\sin A)(1+\sin A)}$

$= \frac{2}{1-\sin^2 A}$

$= \frac{2}{\cos^2 A}$

$= 2\sec^2 A.$

e) $\int_0^{\pi/2} \sin 2x\,dx$

$= -\frac{1}{2}\left[\cos 2x\right]_0^{\pi/2}$

$= -\frac{1}{2}\{\cos\pi - \cos 0\}$

$= -\frac{1}{2}\{-1-1\}$

$= -\frac{1}{2}\times(-2)$

$= 1.$

f) $ar = 6$ $\frac{a}{1-r} = 49$

$a = \frac{6}{r}$

$a = 49(1-r)$

Substitute $\frac{6}{r}$ for

a in $a = 49(1-r)$

$\frac{6}{r} = 49(1-r)$

$6 = 49r - 49r^2$

$49r^2 - 49r + 6 = 0$

$(7r-1)(7r-6) = 0$

$\therefore r = \frac{1}{7}$ OR $r = \frac{6}{7}$

If $r = \frac{1}{7}$, $a\times\frac{1}{7} = 6$

$a = 42$

$\therefore$ Series is

$42, 6, \frac{6}{7}, \ldots$

If $r = \frac{6}{7}$, $a\times\frac{6}{7} = 6$

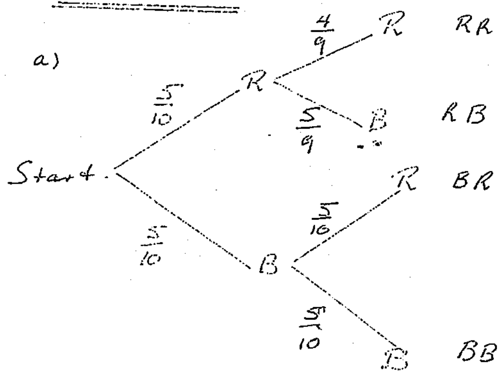
$a = 7.$

$\therefore$ Series is

$7, 6, \frac{36}{7}, \ldots$

Question 5

a)



(i) $P(RR) = \frac{5}{10}\times\frac{4}{9}$

$= \frac{2}{9}$

(ii) P(one of each colour)

$= P(RB) + P(BR)$

$= \frac{5}{10}\times\frac{}{9} + \frac{5}{10}\times\frac{5}{10}$

$= \frac{25}{90} + \frac{25}{100}$

$= \frac{5}{18} + \frac{1}{4}$

$= \frac{10+9}{36}$

$= \frac{19}{36}$

b) $\int_2^a (2x+1)\,dx = 14$

$\left[x^2+x\right]_2^a = 14$

$a^2 + a - (2^2+2) = 14$

$a^2 + a - 6 = 14$

$a^2 + a - 20 = 0$

$(a+5)(a-4) = 0$

$a = -5$ OR $a = 4.$

c) (i) $\angle A = \angle CBX$ (corr. $\angle$'s $DA \parallel CB$)

$\angle A = \angle YDC$ (corr. $\angle$'s $DC \parallel AB$)

$\therefore \angle CBX = \angle YDC$

(ii)

In $\triangle DCY$ and $\triangle BXC$

(i) $\angle YDC = \angle CBX$ (proven above)

(ii) $\angle DCY = \angle BXC$ (corr. $\angle$'s $DC \parallel AX$)

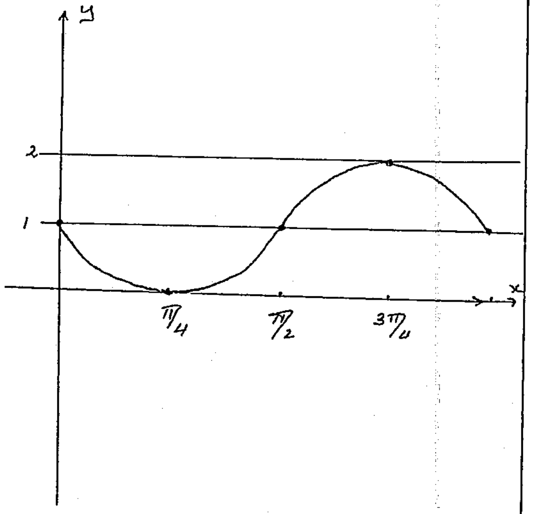
$\therefore \triangle DCY \ ||| \ \triangle BXC$ (equiangular)

$\therefore \frac{XB}{DC} = \frac{BC}{DY}$ (corr sides of $|||\triangle$'s)

But $AB = DC$ and $BC = AD$

opp. side of //ogram

$\therefore \frac{XB}{AB} = \frac{AD}{DY}$



x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$\sin 2x$	0	1	0	−1	0
$-\sin 2x$	0	−1	0	1	0
$-\sin 2x+1$	1	0	1	2	1

Question 6.

a) $y = 2x+9$

$y = x^2+2x$

$\therefore\ x^2+2x = 2x+9$

$x^2 = 9$

$x = \pm 3$

$\therefore$ A is $(3, 15)$

B is $(-3, 3)$

b) $A = \left| \int_{-3}^{3} \{x^2+2x-(2x+9)\} dx \right|$

$= \left| \int_{-3}^{3} (x^2-9)\, dx \right|$

$= \left[\frac{x^3}{3} - 9x \right]_{-3}^{3}$

$= \left| \left(\frac{27}{3} - 27\right) - \left(\frac{-27}{3} + 27\right) \right|$

$= |9-27-(-9+27)|$

$= |-18-18|$

$= |-36|$

$= \underline{\underline{36 \text{ unit}^2}}$.

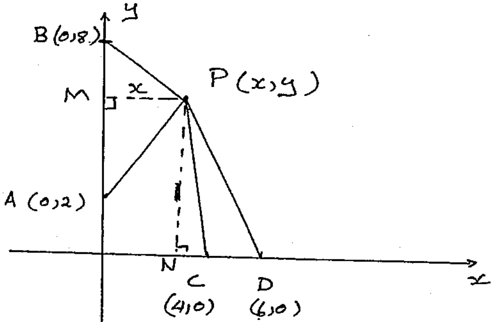
OR since x^2-9 is an even function

$\left| \int_{-3}^{3} (x^2-9) dx \right|$

$= 2 \left| \int_{0}^{3} (x^2-9)\, dx \right|$

$= 2 \times 18$

$= \underline{\underline{36 \text{ square units.}}}$



Area △ PAB = Area △PCD

$\frac{1}{2}$ AB × MP = $\frac{1}{2}$ CD × PN

$\frac{1}{2} \times 6 \times |x| = \frac{1}{2} \times 2 \times |y|$

$3|x| = |y|$

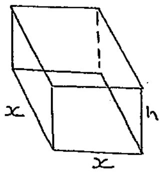
$\therefore\ \underline{y = \pm 3x}$

Note P could be in any of the 4 quadrants

c)

(i) $x x h = 8$

$h = \frac{8}{x^2}$



(ii)

$A = 2x^2 + 4xh$

$= 2x^2 + 4x \times \frac{8}{x^2}$

$= 2x^2 + \frac{32}{x}$

$= 2x^2 + 32x^{-1}$

$\frac{dA}{dx} = 4x - 32x^{-2}$

$= 4x - \frac{32}{x^2}$

Stationary points occur when $\frac{dA}{dx} = 0$

i.e. $4x - \frac{32}{x^2} = 0$

$4x = \frac{32}{x^2}$

$4x^3 = 32$

$x^3 = 8$

$\underline{\underline{x = 2}}$

$\frac{d^2A}{dx^2} = 4. + 64x^{-3}$

$= 4 + \frac{64}{x^3}$

at $x = 2$, $\frac{d^2A}{dx^2} = 4 + \frac{64}{8}$

> 0

$\therefore$ $\underline{\underline{A \text{ is a minimum when } x = 2}}$

If $x = 2$, $A = 2 \times 2^2 + \frac{32}{2}$

$= 8+16$

$= 24$

$\underline{\underline{\text{Minimum Area} = 24 \text{ m}^2}}$

Question 7.

a) $V = \pi \int_{-3}^{0} y^2 dx$

$= \pi \int_{-3}^{0} x^2(x+3)\, dx$

$= \pi \int_{-3}^{0} (x^3+3x^2)\, dx$

$= \pi \left[\frac{x^4}{4} + x^3 \right]_{-3}^{0}$

$= \pi \left\{ (0+0) - \left(\frac{(-3)^4}{4} + (-3)^3 \right) \right.$

$= \pi \left\{ -\left(\frac{81}{4} - 27 \right) \right\}$

$= \pi \left\{ -\frac{81}{4} + 27 \right\}$

$= \pi \left(\frac{-81+108}{4} \right)$

$= \frac{27\pi}{4}$ cubic units

b) Area $\doteqdot \frac{h}{2} \{ y_0 + y_n + 2y_{\text{res}}$

Area $\doteqdot \frac{h}{2} \{ y_0 + y_4 + 2(y_1+y_2+y_3$

$= \frac{250}{2} \{ 0+0+2(231+215+235$

$= 170250 \text{ m}^2$

$= \underline{\underline{17.025 \text{ ha}}}$.

c) $4x - 2y - 7 = 0$

$4x - 7 = 2y$

$y = 2x - \frac{7}{2}$

gradient of line $= 2$

$y = \frac{1}{2}x^2 - 3x + 2$

$\frac{dy}{dx} = x - 3$

We want $x - 3 = 2$

$x = 5$

If $x = 5$, $y = -\frac{1}{2}$

$\therefore$ Point is $\underline{\underline{\left(5, -\frac{1}{2}\right)}}$

d) Possible inflexion when $\frac{d^2y}{dx^2} = 0$

$\therefore (x-2)^2(x-3) = 0$

$\therefore x-2 = 0$ OR $x-3=0$

$x = 2$ OR $x = 3$

at $x = 1\cdot9$, $\frac{d^2y}{dx^2} = (+)(-) < 0$

at $x = 2\cdot1$, $\frac{d^2y}{dx^2} = (+)(-) < 0$

No change in concavity

∴ No inflexion

at $x = 2\cdot9$, $\frac{d^2y}{dx^2} = (+)(-) < 0$

at $x = 3\cdot1$, $\frac{d^2y}{dx^2} = (+)(+) > 0$

Change in concavity

∴ inflexion at $x=3$

∴ Only one inflexion at $x = 3$.

Question 8.

a)
(i)
at $t=0$

$V = 5000\left(1-\frac{0}{40}\right)^2$

$= 5000$ litres

(ii) If the tank is empty $V=0$

$\therefore 0 = 5000\left(1-\frac{t}{40}\right)^2$

$1 - \frac{t}{40} = 0$

$40 = t$

∴ the tank will be empty when $t = 40$

(iii) $V = 5000\left(1-\frac{t}{40}\right)^2$

$\frac{dV}{dt} = 5000 \times 2\left(1-\frac{t}{40}\right)\times\left(\frac{-1}{40}\right)$

$= -250\left(1-\frac{t}{40}\right)$

at $t = 10$

$\frac{dV}{dt} = -250\left(1-\frac{10}{40}\right)$

$= -250 \times \frac{3}{4}$

$= 187\cdot5$ litres/minute

b) $x = 8e^{-2t} - 8 + 16t$

(i) $v = \frac{dx}{dt}$

$= 8(-2e^{-2t}) + 16$

$= -16e^{-2t} + 16$

at $t=0$

$v = -16e^0 + 16$

$= -16 + 16$

$= 0$

∴ particle is at rest when $t=0$

(ii) $\lim\limits_{t\to\infty}(-16e^{-2t} + 16)$

$= -16\times0+16$

$= 16$ cm/sec

(iii) $a = \frac{dv}{dt}$

$= -16(-2e^{-2t})$

$= 32e^{-2t}$

But $v = -16e^{-2t} + 16$

$e^{-2t} = \frac{16-v}{16}$

$\therefore a = 32\left(\frac{16-v}{16}\right) \Rightarrow a = 32 - 2v$

c) $D = 5 + \left(\frac{40}{4+t}\right)^2$

$\frac{dN}{dt} = 5 + 1600(4+t)^{-2}$

$N = \int\{5 + 1600(4+t)^{-2}\}dt$

$N = 5t + 1600\frac{(4+t)^{-1}}{-1} + c$

$N = 5t$ [illegible] $\frac{1600}{(4+t)} + c$

at $t=0$, $N=20$

$20 = 0 - \frac{1600}{4} + c$

$20 + 400 = c$

$c = 420$

$N = 5t - \frac{1600}{(4+t)} + 420$

If $t = 10$

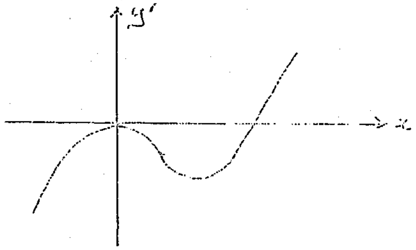
$N = 5\times10 - \frac{1600}{14} + 420$

$\doteqdot 356$

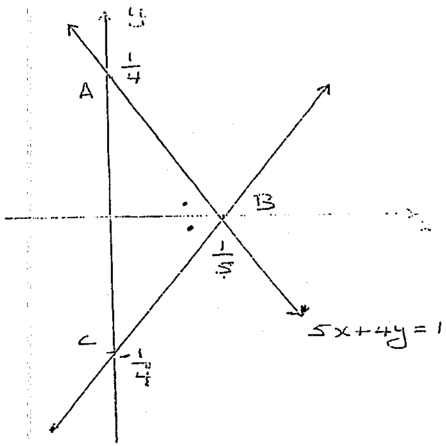
Approximately 356 people will have the disease after 10 days.

Question 9.

a)



b)



Equ of BC is $\frac{x}{a} + \frac{y}{b} = 1$

i.e. $\frac{x}{\left(\frac{1}{5}\right)} + \frac{y}{\left(-\frac{1}{4}\right)} = 1$

$5x - 4y = 1.$

c) $T_9 + T_{10} + T_{11} = 57$

$a+8d+a+9d+a+10d = 57$

$3a + 27d = 57$

$a + 9d = 19$

$T_{17} + T_{18} + T_{19} = 105$

$a+16d + a+17d + a+18d = 105$

$3a + 51d = 105$

$a + 17d = 35$

Solve $a+17d = 35$ and $a + 9d = 19$

$8d = 16$

$d = 2$

$\therefore a + 9\times2 = 19$

$a = 1$

$T_2 = a+d$

$= 1+2$

$= 3.$

d) $r = \frac{6}{12}\%$

$= 0.5\%$

$A_1 = 240\,000 + 0.5\% \text{ of } 240000 - M$

$= 240000(1+0.005) - M$

$= 240000 \times 1.005 - M$

$A_2 = A_1 + 0.5\% \text{ of } A_1 - M$

$= A_1(1+0.005) - M$

$= A_1 \times 1.005 - M$

$= (240000 \times 1.005 - M) \times 1.005 - M$

$= 240000 \times 1.005^2 - 1.005M - M$

$= 240000 \times 1.005^2 - M(1+1.005)$

$A_3 = A_2 \times 1.005 - M$

$= 240000 \times 1.005^3 - M(1+1.005+1.005^2)$

$A_n = 240000 \times 1.005^n - M(1+1.005+1.005^2+\ldots \ldots 1.005^{n-1})$

$= 240000 \times 1.005^n - M \times 1\frac{(1.005^{240}-1)}{(1.005-1)}$

$= 240000 \times 1.005^n - M \times \frac{(1.005^{240}-1)}{0.005}$

$= 240000 \times 1.005^n - M \times 200(1.005^{240}-1)$

Note $0.005 = \frac{5}{1000} = \frac{1}{200}$

If $A_n = 0$

Then

$200M(1.005^{240}-1) = 240000 \times 1.005^{240}$

$M = \frac{240\,000 \times 1.005^{240}}{200 \times (1.005^{240}-1)}$

$= \frac{1200 \times 1.005^{240}}{(1.005^{240}-1)}$

(iii) $\underline{M = \$1719.43}$

Question 10

a) $\alpha+\beta = -\frac{b}{a}$ $\quad$ $\alpha\beta = \frac{c}{a}$

$= -3$ $\quad$ $= -2$

b) Equation with roots α^2 & β^2 is

$x^2 - (\alpha^2+\beta^2)x + \alpha^2\beta^2 = 0$

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (-3)^2 - 2(-2)$

$= 9+4$

$= 13$

$\alpha^2\beta^2 = (\alpha\beta)^2$

$= (-2)^2$

$= 4$

∴ Required equation is

$\underline{x^2 - 13x + 4 = 0}$

b) from (x_1, y_1) to $7x - y + 9 = 0$

$D_1 = \left|\frac{7x_1 - y_1 + 9}{\sqrt{7^2+(-1)^2}}\right|$

$D_1 = \left|\frac{7x_1 - y_1 + 9}{\sqrt{50}}\right|$

from (x_1, y_1) to $x + y - 1 = 0$

$D_2 = \left|\frac{x_1 + y_1 - 1}{\sqrt{1^2+1^2}}\right|$

$= \left|\frac{x_1 + y_1 - 1}{\sqrt{2}}\right|$

Required locus is such that the perpendicular distance from (x, y) to both lines is equal

$\therefore \left|\frac{7x - y + 9}{5\sqrt{2}}\right| = \left|\frac{x + y - 1}{\sqrt{2}}\right|$

$\therefore \frac{7x - y + 9}{5\sqrt{2}} = \frac{x + y - 1}{\sqrt{2}}$

$7x - y + 9 = 5x + 5y - 5$

$2x - 6y + 14 = 0$

$\underline{x - 3y + 7 = 0}$

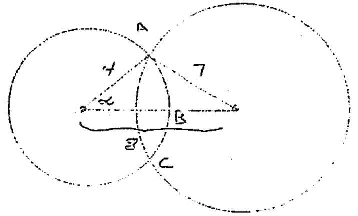
OR $\frac{7x - y + 9}{5\sqrt{2}} = -\frac{(x + y - 1)}{\sqrt{2}}$

$7x - y + 9 = -5x - 5y + 5$

$12x + 4y + 4 = 0$

$\underline{3x + y + 1 = 0}$

c)



$\cos\alpha = \frac{4^2 + 8^2 - 7^2}{2 \times 4 \times 8}$

$\alpha \doteqdot 1.065^c$ $\quad$ $\boxed{\doteqdot 61°2'}$

$\therefore \text{Arc } ABC = r\theta$

$= 4 \times 2\alpha$

$= 4 \times (2 \times 1.065^c)$

$= 8.52 \text{ cm}$

GIRRAWEEN HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

2007

MATHEMATICS

Time allowed - Three hours
(Plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are on sheet provided.
- Board-approved calculators may be used.
- Each question attempted is to be returned on a *separate* piece of paper clearly marked Question 1 , Question 2 , etc. Each piece of paper must show your name.
- You may ask for extra pieces of paper if you need them.

Question 1 (12 marks) **Marks**

(a) Evaluate $\sqrt{\dfrac{762.8}{2.7\times3.5}}$ correct to 3 significant figures. **2**

(b) Factorise $5x^2-16x-3$ **2**

(c) Find the primitive for e^{2x} . **2**

(d) Find the values of x for which $|2x-3|<7$ **2**

(e) Express $\dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}$ in the form $a+b\sqrt{6}$ where a and b are integers. **2**

(f) Karan pays $\$153.00$ for a DVD player which has been discounted by 15%. What was the original price of the DVD player? **2**

Question 2 (12 marks)

(a) Differentiate with respect to x:

(i) x^2e^x **2**

(ii) $\dfrac{3x}{\cos x}$ **2**

(b) Find:

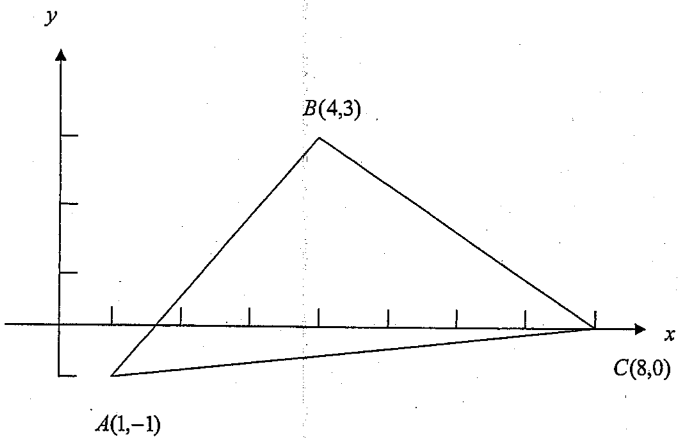
(i) $\displaystyle\int\frac{10x}{x^2+5}\,dx$ **2**

(ii) $\displaystyle\int_0^{\frac{\pi}{8}}5\sec^2 2x\,dx$ **3**

(c) Find the equation of the tangent to $f(x)=e^{2x-4}$ at the point where $x=2$. **3**

Question 3 (12 marks)

(a)



In the diagram above, $A, B,$ and C are the points $(1,-1), (4,3)$ and $(8,0)$ respectively.

Copy the diagram on to your own paper and answer the following questions:

(i) Find the gradient of the line AC. **2**

(ii) Find D, the midpoint of AC. **1**

(iii) Show that the equation of the line through B which is perpendicular to AC is $7x+y-31=0$. **3**

(iv) Show that D lies on the line in part (iii). **1**

(v) Show that ΔABC is isosceles. **3**

Question 3 (continued)

(b) Find the angle θ in the diagram below: **2**

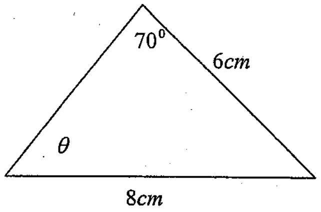


Diagram not to scale

Question 4 (12 marks)

(a) Evaluate $\sum_{n=4}^{6} \frac{1}{n-2}$ **1**

(b) Use the change of base rule to find $\log_3 7$. **1**

(c) A sector of a circle has an area of $8\pi\ cm^2$. The arc at the circumference of this sector is 2π cm long. Find

(i) The radius of the circle. **2**

(ii) The angle subtended by the arc at the centre of the circle. **1**

(d) (i) Find the focus of the parabola $x^2 = 12(y-1)$ **2**

(ii) Find the volume of the solid of revolution formed when the area between $x^2 = 12(y-1)$ and the y axis is rotated about the y axis between $y=1$ and $y=3$. **3**

(e) For what values of k does the equation $4x^2 - 4x + k = 0$ have real roots? **2**

Question 5 (12 marks)

(a) For the function $f(x) = 4x^2(2x+3)$

(i) Find the stationary points and determine their nature. **3**

(ii) Find the point of inflexion. **2**

(iii) Sketch the graph of f(x) showing all stationary points, points of inflexion and intercepts with the co-ordinate axes. **2**

(b) The probability that Rusty will beat Danielle in a set of tennis is $\frac{3}{5}$. On a particular day they play 3 sets of tennis.

(i) What is the probability that Rusty will win all 3 sets? **1**

(ii) Draw a probability tree to illustrate the possible results of the 3 sets. **2**

(iii) What is the probability that Danielle will win exactly 2 sets? **1**

(iv) What is the probability that Danielle will win at least 1 set? **1**

Question 6 (12 marks)

(a) A farmer is delivering loads of cement from a pile at the end of an irrigation ditch 3 kilometres long to points 120 metres apart along the ditch. After delivering each load, the farmer must return to the pile at the end of the ditch to collect the next load. He starts at the pile and delivers his first load to the first point (120 metres away) then after returning to the pile delivers his second load to the second point (240 metres away) and so on.

(i) How far along the ditch is the 12th load delivered? **2**

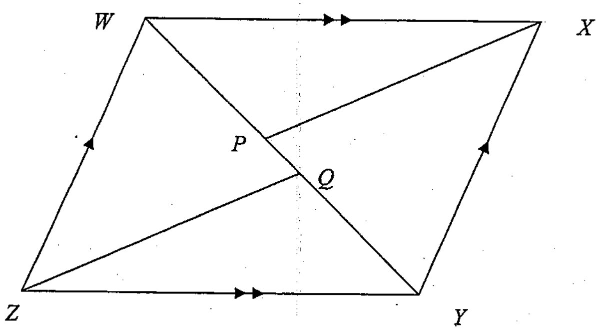
(ii) How many loads are delivered along the entire length of the 3km ditch? (The last load is delivered to the very end of the ditch.) **2**

(iii) How many km has the farmer travelled in order to deliver all of the loads, then return to the end of the ditch where the pile was? **2**

Question 6 (continued)

(b) $WXYZ$ is a parallelogram. XP bisects $\angle WXY$ and ZQ bisects $\angle WZY$.

Copy the diagram on to your answer sheet and answer the following questions:



(i) Explain why $\angle WXY = \angle WZY$. **2**

(ii) Prove $\Delta WXP \equiv \Delta YZQ$ **3**

(iii) Hence find the length of PQ given $WY = 20cm$ and $QY = 8cm$. **1**

Question 7 (12 marks)

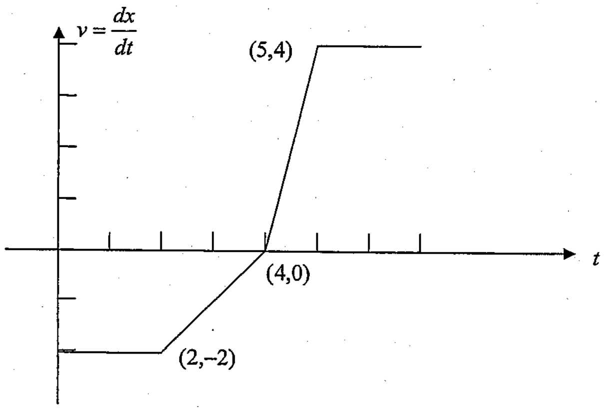
(a) If α and β are the roots of the quadratic equation $2x^2 - 3x + 7 = 0$ find:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **1**

(b) Below is the graph of the velocity of a particle in metres per second. Initially the particle is at the origin.



(i) When is the particle furthest from the origin? **1**

(ii) How far, and in what direction, is the particle from the origin after 7 seconds? **2**

(iii) Sketch the acceleration of the particle from time $t = 0$ to time $t = 7$. **2**

Question 7 (continued)

(c) (i) Differentiate $f(x)=\cos^3 x$. 2

(ii) Hence find $\int_0^{\frac{\pi}{3}} \cos^2 x \sin x.dx$ 2

Question 8 (12 marks)

(a) Use Simpson's Rule with 5 function values to find an approximation 3

for $\int_0^1 \ln(x+1).dx$

(b) The population of a town is growing according to the formula

$\frac{dP}{dt}=kP$.

(i) Show that $P=Ae^{kt}$ is a solution to this differential equation. 1

(ii) If the town's population was 3000 in 1980 and 5000 in 1990 2

find values for A and k given 1980 is when $t=0$.

(iii) Find the town's population in 2007. 1

(c) An arithmetic series has $T_3=60$ and $T_7=95$. Find the sum of the 3

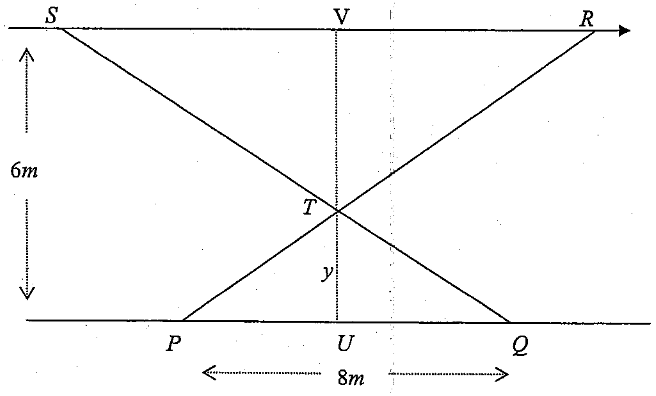
first 10 terms.

(d) The limiting sum of the series $1+3^x+3^{2x}+3^{3x}+...$ is equal to $\frac{9}{8}$. 2

Find the value of x.

Question 9 (12 marks)

(a) In the diagram below PQ and SR are parallel railings which are $6m$ apart. The points P and Q are fixed $8m$ apart on the lower railing. Two crossbars PR and QS intersect at T as shown in the diagram. The line through T perpendicular to PQ intersects PQ at U and SR at V. The length of UT is y metres.



(i) By using similar triangles or otherwise show that $\frac{SR}{PQ}=\frac{VT}{UT}$. 3

(ii) Show that $SR=\frac{48}{y}-8$. 1

(iii) Hence show that the total area of ΔPTQ and ΔRTS is 2

given by $\frac{144}{y}+8y-48$.

(iv) Find the value of y that minimises A. Justify your answer. 3

Girraween High School p.1
Trial Exam
Mathematics — 2007
Solutions:

Q.(1)(a) $\sqrt{\dfrac{762.8}{2.7 \times 3.5}} = 8.9844\ldots$
$\doteqdot 8.98$ (to 3SF). ②

(b) $5x^2 - 16x - 3 = 5\left(x - \dfrac{8-\sqrt{79}}{5}\right)\left(x - \dfrac{8+\sqrt{79}}{5}\right)$ ②

using quadratic formula $x = \dfrac{16 \pm \sqrt{16^2 - 4 \times 5 \times -3}}{2 \times 5}$.

(c) $\int e^{2x}.dx = \frac{1}{2}e^{2x} + C$. ②

(d) $|2x-3| < 7$
$-7 < 2x-3 < 7$
$-4 < 2x < 10$
$-2 < x < 5$. ②

(e) $\dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{2}} \times \dfrac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$ ②

$= \dfrac{3+\sqrt{6}}{1}$

$= 3+\sqrt{6}$.

(f) \$153.00 = 85 % of original price.
\$1.80 = 1 %
\$180.00 = Original price ②

GHS 2007 Mathematics Trial
Solutions p.2

Q.(2)(a)(i) $\dfrac{d}{dx}\left(x^2 e^x\right)$

$= 2xe^x + x^2e^x$

$= e^x(2x + x^2)$ ②

or $= xe^x(2+x)$.

(ii) $\dfrac{d}{dx}\left(\dfrac{3x}{\cos x}\right)$ ②

$= \dfrac{\cos x \times 3 - 3x \times -\sin x}{\cos^2 x}$

$= \dfrac{3\cos x + 3x\sin x}{\cos^2 x}$.

or $= \dfrac{3(\cos x + x\sin x)}{\cos^2 x}$.

(b)(i) $\int \dfrac{10x}{x^2+5}.dx$

$= 5\int \dfrac{2x}{x^2+5}.dx$ ②

$= 5\ln(x^2+5) + C$.

(ii) $\int_0^{\frac{\pi}{8}} 5\sec^2 2x.dx$ ③

$= \left[\dfrac{5}{2}\tan 2x\right]_0^{\frac{\pi}{8}}$

$= \dfrac{5}{2}\tan\dfrac{\pi}{4} - \dfrac{5}{2}\tan 0$

$= \dfrac{5}{2}$.

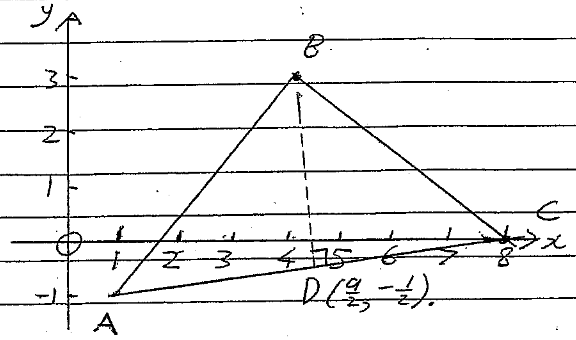
(c) $f(x) = e^{2x-4}$
$f'(x) = 2e^{2x-4}$
Where $x = 2$,
$f(x) = e^{2\times2-4}$
$= e^0$
$= 1$.
$f'(x) = 2e^{2x-4}$ ③
$= 2e^0$
$= 2$.

So tangent is line passing through $(2,1)$ with $m=2$.

By $y - y_1 = m(x - x_1)$
$y - 1 = 2(x-2)$
$y - 1 = 2x - 4$
$y = 2x - 3$

Or in general form:
$2x - y - 3 = 0$.

Q.(3)(a)



(i) $m\ AC = \frac{y_2-y_1}{x_2-x_1}$ ②

$= \frac{0+1}{8-1}$

$= \frac{1}{7}$.

(ii) D, midpoint of AC

$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{1+8}{2}, \frac{-1+0}{2}\right)$ ①

$= \left(\frac{9}{2}, -\frac{1}{2}\right)$

(iii) Line through B perpendicular to AC:

$m = -7$.

By $y - y_1 = m(x - x_1)$ ③

$y - 3 = -7(x-4)$

$y - 3 = -7x + 28$ $\scriptstyle +7x-28$ $\scriptstyle +7x-28$

$7x + y - 31 = 0$.

(iv) Substituting co-ordinates of D into $7x+y-31 = 0$

$7\left(\frac{9}{2}\right) - \frac{1}{2} - 31 = 0$. ①

$\therefore$ D is on line $7x+y-31=0$

(v) Showing $\triangle ABC$ is isosceles:

BD common

AD = DC [as D is midpoint of AC]

$\angle ADB = \angle BDC$ [proven in (iii) and (iv)].

$\therefore \triangle ADB \equiv \triangle CDB$ [SAS].

AB = BC [matching sides in $\equiv\triangle$]

Hence $\triangle ABC$ is isosceles. ③

Alternatively,

distance $AB = \sqrt{25} = 5$.

distance $BC = \sqrt{25} = 5$

$AB = BC$

$\triangle ABC$ is isosceles.

(b) By $\frac{\sin A^\circ}{a} = \frac{\sin B^\circ}{b}$ ②

$\frac{\sin\theta}{6} = \frac{\sin 70^\circ}{8}$

$\sin\theta = \left(\frac{\sin 70^\circ \times 6}{8}\right)$

$\theta = 44^\circ 49'$ [to nearest minute].

Q.(4)(a) $\sum_{n=4}^{6} \frac{1}{n-2}$

$= \frac{1}{4-2} + \frac{1}{5-2} + \frac{1}{6-2}$

$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

$= 1\frac{1}{2}$ or $\frac{3}{2}$ ①

(b) $\log_3 7 = \frac{\ln 7}{\ln 3}$ ①

$\doteqdot 1.77$ (2DP).

(c)(i) Sector area:

$\frac{1}{2}r^2\theta = 8\pi$.

Radius:

$r\theta = 2\pi$.

$\frac{\frac{1}{2}r^2\theta}{r\theta} = \frac{8\pi}{2\pi}$

$\frac{1}{2}r = 4$

$r = 8cm$. ②

(ii) Angle θ:

$r = 8$, $r\theta = 2\pi$

$\theta = \frac{2\pi}{8}$

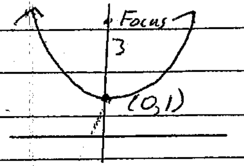
Angle subtended $= \frac{\pi}{4}$. ①

(d)(i) $x^2 = 12(y-1)$

Vertex = (0,1)

Focal length: $4a = 12$

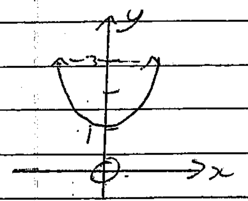
$a = 3$.



②

Focus = (0,4)

(ii)



$V = \pi\int x^2 . dy$

$= \pi\int_{y=1}^{y=3} 12(y-1) . dy$

$= \pi\int_1^3 12y - 12 . dy$ ③

$= \pi\left[6y^2 - 12y\right]_1^3$

$= \pi\left[(6\times3^2 - 12\times3) - (6\times1^2 - 12\times1)\right]$

$= 24\pi$ cubic units.

$4x^2 - 4x + k = 0$ has real roots:

$\therefore \Delta = b^2 - 4ac \geq 0$

$(-4)^2 - 4\times4\times k \geq 0$ ②

$16 - 16k \geq 0$

$16 \geq 16k$

$1 \geq k$

$\therefore 4x^2 - 4x + k = 0$ will have real roots where $k \leq 1$.

Q.(5)(a)(i) $f(x)=4x^2(2x+3)$
$=8x^3+12x^2$

$f'(x)=24x^2+24x.$

Stationary points are where
$f'(x)=0$
$24x^2+24x=0$
$24x(x+1)=0$ ③
$x=0$ or $x=-1.$
If $x=0, y=4(0)^2[2(0)+3]=0.$

If $x=-1, y=4(-1)^2[2(-1)+3]=4$

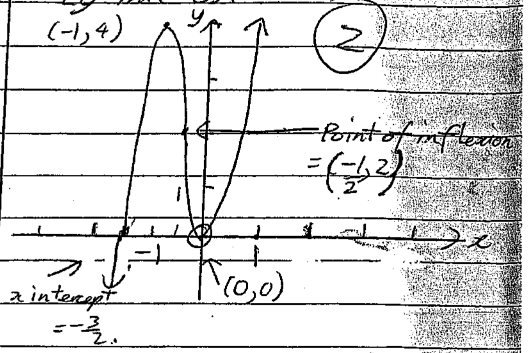
Stationary points at $(0,0)$
& $(-1,4)$

Nature of stationary points: $f''(x)=48x+24.$
At $x=-1, f''(x)=48(-1)+24=-24.$
$(-1,4)$ is a LOCAL MAXIMUM.
At $x=0, f''(x)=48(0)+24=24.$
$(0,0)$ is a LOCAL MINIMUM.

(ii) Point of inflexion: $f''(x)=0.$
$48x+24=0$
$48x=-24$ ②
$x=-\frac{1}{2}$
$y=4(-\frac{1}{2})^2[2\times-\frac{1}{2}+3]$
$=(-\frac{1}{2},2).$

Testing point of inflexion:
At $x=-1, f''(x)=48(-1)+24$
$=-24.$
~~At $x=0, f''(x)=48(0)+24$~~
$=24.$
$\therefore f''(x)$ changes sign $\rightarrow$
$(-\frac{1}{2},2)$ is a point of inflexion.

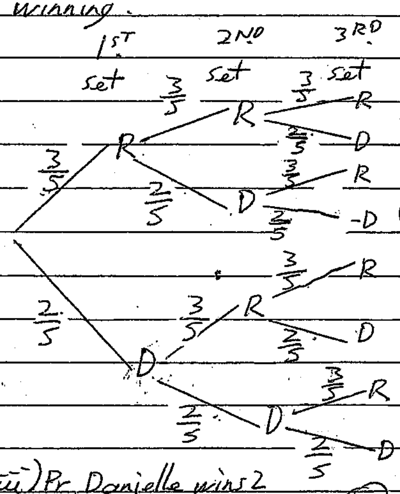
(iii) Note: x intercepts $=0$ & where $2x+3=0$
i.e. $x=-\frac{3}{2}.$
[y int. $=0$].



②

(b)(i) Pr[Rusty winning]
$=0.6\times0.6\times0.6$ ①
$=0.216.$

(ii) Probability Tree:
R = Rusty winning, D = Danielle winning.

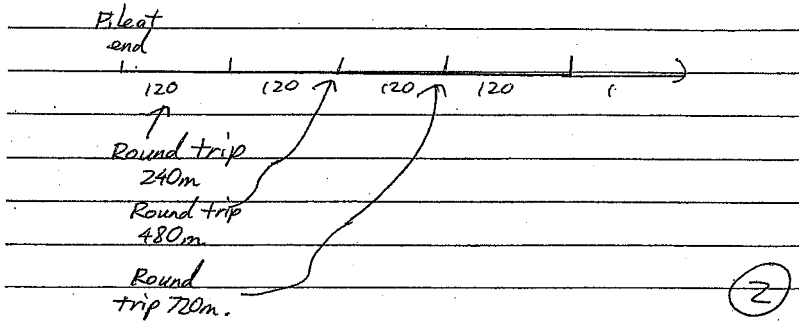


②

(iii) Pr Danielle wins 2
$=0.4\times0.4\times0.6\times3$ ①
$=0.288.$

Pr Danielle wins at least 1 set
$=1-$ Pr[Rusty wins all 3] ①
$=1-0.216=0.784$

Q.(6)(a)



②

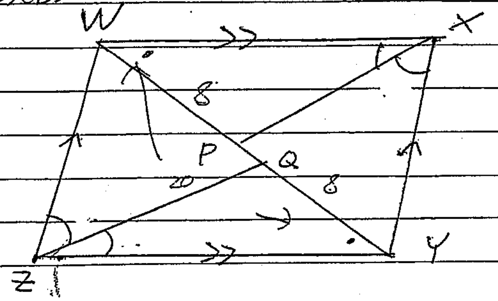
(i) 12TH load delivered $12\times120\text{m}=1440\text{m}$ along ditch.

(ii) Total no. of loads $=3000\div120$
$=25$ loads. ②

(iii) Farmer travels $240+480+720+\ldots+6000\text{m}.$
By $S_n=\frac{n}{2}(a+L)$ ②
Total distance $=\frac{25}{2}(240+6000)$
$=78000\text{m}$ or
$78\text{km}.$

Note: Could also do $S_n=\frac{n}{2}(2a+(n-1)d)$
with $n=25, a=240, d=240$

Q.(6)(b)



(i) $\angle WXY = \angle WZY$ [opposite angles of parallelogram =]. ②

(ii) Hence $\angle WXP = \angle PXY = \angle YZQ = \angle PZQ$
[as XP bisects $\angle WXY$ & ZQ bisects $\angle WZY$].
$\angle XWY = \angle ZYW$ [alternate $\angle$'s in $\parallel$ lines =].
$WX = ZY$ (opposite sides of parallelogram =].
$\therefore \triangle WXP \equiv \triangle YZQ$ (AAS). ③

(iii) $WP = 8$cm [matching sides in $\equiv \triangle WXP$ and $\triangle YZQ$ =].
Hence $PQ = \underline{4\text{cm}}$. ①

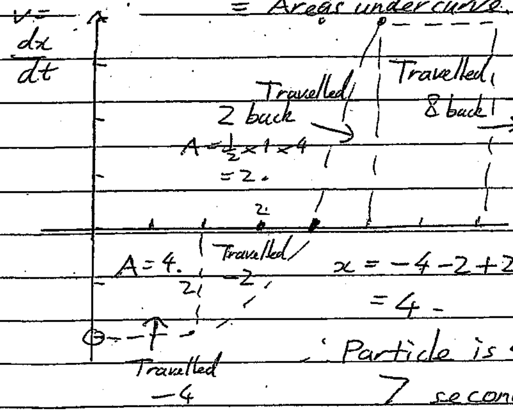
Q.(7)(a)(i) $\alpha+\beta = \frac{-b}{a}$ ①
$= \frac{3}{2}$

(ii) $\alpha\beta = \frac{c}{a}$ ①
$= \frac{7}{2}$.

(iii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
$= \left(\frac{3}{2}\right)^2 - 2\times\frac{7}{2}$ ①
$= -4\frac{3}{4}$.

(b)(i) Particle furthest from origin when $v=0$ ①
[$t = 4$ seconds].

(ii) x will be $\int v.dt$
$\doteqdot$ Areas under curve.

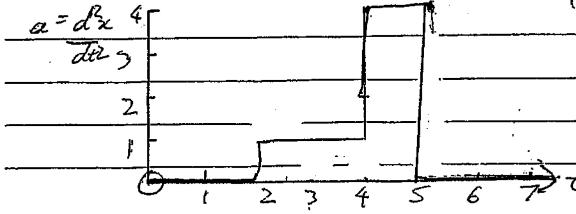


→ $A = 4\times2 = 8$. ②

$x = -4-2+2+8$
$= 4$.

$\therefore$ Particle is 4m to RIGHT of origin after 7 seconds.

(iii) Acceleration $= \frac{dv}{dt}$ = GRADIENT of velocity: ②



Q.(7)(c) (i) $f(x) = \cos^3 x.$

$$f'(x) = 3\cos^2 x \times -\sin x \quad ②$$

$$= -3\cos^2 x \sin x.$$

(ii) Hence $\int_0^{\frac{\pi}{3}} \cos^2 x \sin x . dx$

$$= -\frac{1}{3}\int_0^{\frac{\pi}{3}} -3\cos^2 x \sin x . dx$$

$$= -\frac{1}{3}\left[\cos^3 x\right]_0^{\frac{\pi}{3}}$$

$$= -\frac{1}{3}\left[\cos^3\left(\frac{\pi}{3}\right) - \cos^3(0)\right] \quad ②$$

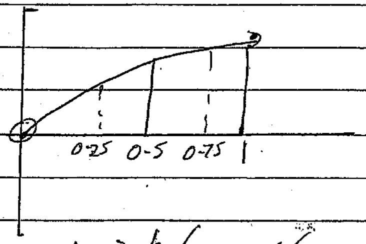
$$= -\frac{1}{3}\left[\left(\frac{1}{2}\right)^3 - 1^3\right]$$

$$= \frac{7}{24}.$$

Q.(8)(a) $\int_0^1 \ln(x+1) . dx$

$h = \frac{1}{4} = 0.25.$

$y_0 = \ln 1 = 0.$



$$A \doteqdot \frac{h}{3}\left(y_0 + 4(y_1 + y_3) + 2y_2 + y_5\right)$$

$$= \frac{0.25}{3}\left(\ln 1 + 4\left(\ln 1.25 + \ln 1.75\right) + 2\ln 1.5 + \ln 2\right)$$

$$= 0.386\ldots$$

$$\therefore \int_0^1 \ln(x+1) . dx \doteqdot 0.386 \text{ [3SF]}. \quad ③$$

(b)

(i) If $P = Ae^{kt}$

$$\frac{dP}{dt} = kAe^{kt} \;\&\; kP = kAe^{kt} \quad ①$$

Hence $\frac{dP}{dt} = kP$

(ii) ② $P = 3000$ when $t = 0$

$3000 = Ae^0$

$\therefore 3000 = A.$

$P = 3000e^{kt}$

$P = 5000$ when $t = 10$

$5000 = 3000e^{10k}$

$\frac{5}{3} = e^{10k}$

$\ln\left(\frac{5}{3}\right) = 10k.$

$\frac{1}{10}\ln\left(\frac{5}{3}\right) = k.$

$k \doteqdot 0.05108\ldots$

(iii) ① Population in '07 [$t = 27$]

$= 3000e^{0.05108\ldots \times 27}$

$= 11\,915.51\ldots$

Population $\doteqdot 11\,900$

[to nearest 100 people].

Q.(8)(c) $T_7 = a+6d = 95$ (1)

$T_3 = a+2d = 60$ (2) (1)−(2)

$4d = 35.$

$d = 8{\cdot}75.$

From $T_3 = a+2d = 60$

$a+2\times 8{\cdot}75 = 60$

$a = 42{\cdot}5.$ ③

Sum of first 10 terms:

$S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{10}{2}[2\times 42{\cdot}5 + 9\times 8{\cdot}75]$

$= 818{\cdot}75.$

(d) Limiting sum $= \frac{a}{1-r} = \frac{9}{8}$

$\frac{1}{1-3^x} = \frac{9}{8}$

$\times 8(1-3x)$

$8 = 9(1-3^x)$

$8 = 9 - 9\times 3^x \quad +9\times 3^x - 8$

$9\times 3^x = 1.$

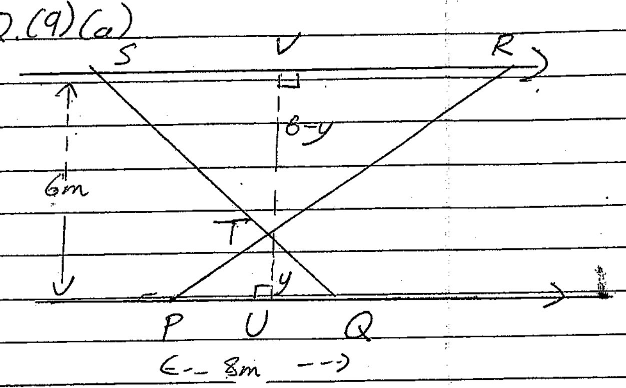
As $9 = 3^2$, & $1 = 3^0$

$3^{x+2} = 3^0$

$x+2 = 0$ ②

$x = -2.$

Q.(9)(a)



③

(i) $\angle PTQ = \angle STR$ [Vertically opposite $\angle$'s =].

$\angle TSR = \angle TQP$ [alternate $\angle$'s in // lines =].

$\angle TRS = \angle TPQ$ [" " "]. Any 2 of

$\therefore \triangle TSR \,|||\, \triangle TQP$ [2 pairs of matching $\angle$'s =].

$\therefore \frac{SR}{PQ} = \frac{ST}{TQ}$ [ratio of matching sides in ||| $\triangle$].

$SV \perp VT$ [data]

$QU \perp UT$ ["].

$\angle STV = \angle QTU$ [vertically opposite $\angle$'s =].

$\angle TSR = \angle TQ.U$ [proven earlier] This is 1 Any 2 of

$\therefore \triangle STV \equiv \triangle QTU$ [2 pairs of matching $\angle$'s =].

$\therefore \frac{ST}{TQ} = \frac{VT}{UT}$ [ratio of matching sides in ||| $\triangle$'s].

As $\frac{ST}{TQ} = \frac{SR}{PQ}$ [proven earlier].

$\therefore \frac{SR}{PQ} = \frac{VT}{UT}.$

(ii) As $\frac{SR}{PQ} = \frac{VT}{UT}$ ①

$\frac{SR}{8} = \frac{6-y}{y}$

$SR = \frac{48-8y}{y}$

$= \frac{48}{y} - 8$

Q. (9) (a) (iii) Total area of $\triangle PTQ$ & $\triangle RTS$

$$= \frac{1}{2} \times 8xy \quad + \frac{1}{2} \times \left(\frac{48}{y} - 8\right) \times (6-y)$$

$$= 4y \quad + \frac{144}{y} + 4y - 48$$ ②

$$= \frac{144}{y} + 8y - 48$$

(iv) Value of y that minimises A:

Find where $\frac{dA}{dy} = 0$.

$$-\frac{144}{y^2} + 8 = 0$$

$$8y^2 - 144 = 0$$ ③

$$y^2 - 18 = 0$$

$$y = \pm\sqrt{18}$$

$$= \pm 3\sqrt{2}$$

→ As y is a measurement, $y = 3\sqrt{2}$.

Justifying: This is a minimum if $\frac{d^2A}{dy^2} > 0$.

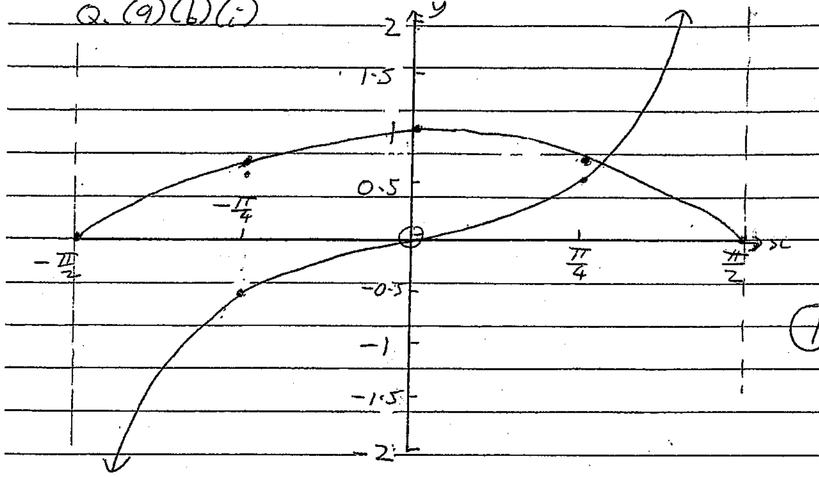
$$\frac{d^2A}{dy^2} = \frac{288}{y^3}$$

Where $y = 3\sqrt{2}$, $\frac{d^2A}{dy^2} = \frac{288}{(3\sqrt{2})^3}$

$$\doteqdot 33 \cdot 9.$$

→ As $\frac{d^2A}{dy^2} > 0$ where $y = 3\sqrt{2}$, area is a MINIMUM.

Q. (9) (b) (i)



①

(ii) $\cos x = \frac{1}{2} \tan x$

$\cos x = \frac{\sin x}{2\cos x}$ [as $\tan x = \frac{\sin x}{\cos x}$].

$2\cos^2 x = \sin x$ ②

$2(1 - \sin^2 x) = \sin x$ [as $\cos^2 x + \sin^2 x = 1$].

$2 - 2\sin^2 x - \sin x = 0$

$2\sin^2 x + \sin x - 2 = 0.$

This is a quadratic equation in $\sin x$:

$$\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4 \times 2 \times -2}}{2 \times 2}$$

$$= \frac{-1 \pm \sqrt{17}}{4}.$$

→ Only possible solution (from graph) is in 1st quadrant.

So $\sin x = \frac{-1+\sqrt{17}}{4}$	$\cos x \doteqdot 0 \cdot 624..$	Co-ordinates
$\doteqdot 0 \cdot 78..$	$\frac{1}{2} \tan x \doteqdot 0 \cdot 624..$	$= (0 \cdot 895, 0 \cdot 624)$
$x \doteqdot 0 \cdot 895.$		

Q. (10)(a) Limiting sum $= \frac{a}{1-r}$

$= \frac{1}{1-\sin^2 x}$

$= \frac{1}{\cos^2 x}$ [as $\cos^2 x + \sin^2 x = 1$].

$= \sec^2 x$ ③

$= 1 + \tan^2 x$. [as $1+\tan^2 x = \sec^2 x$].

(b)(i) Amount left to be repaid: 6 % P.A. = 0·5 % per month

Month:	Start of month:	End:
1	\$400 000	$\$400\,000 \times 1{\cdot}005 - P$
2		$(\$400\,000 \times 1{\cdot}005 - P) \times 1{\cdot}005 - P$ ② $= \$400\,000 \times 1{\cdot}005^2 - P \times 1{\cdot}005 - P$
3		$(\$400\,000 \times 1{\cdot}005^2 - P \times 1{\cdot}005 - P) \times 1{\cdot}005 - P$ $= \$400\,000 \times 1{\cdot}005^3 - P \times 1{\cdot}005^2 - P \times 1{\cdot}005 - P$ $= \$400\,000 \times 1{\cdot}005^3 - P(1 + 1{\cdot}005 + 1{\cdot}005^2)$

ii) After 20 years [240 months]

Amount left to be repaid $= 0$

$400000 \times 1{\cdot}005^{240} - P(1 + 1{\cdot}005 + 1{\cdot}005^2 + \ldots + 1{\cdot}005^{239}) = 0$ (1) ③

$1 + 1{\cdot}005 + 1{\cdot}005^2 + \ldots$ $\doteq$ [By $S_n = \frac{a(r^n - 1)}{r-1}$

$= \frac{1(1{\cdot}005^{240} - 1)}{1{\cdot}005 - 1}$

$\doteqdot 462{\cdot}04\ldots$ [Keep in calculator].

ub in (1): $400\,000 \times 1{\cdot}005^{240} - 462{\cdot}04\ldots P = 0$

$\$400\,000 \times 1{\cdot}005^{240} = \$462{\cdot}04\ldots P$

$\$2\,865{\cdot}72 = P$

They repay \$2 865·72 [pay \$2 865·70] per month.

Q. (10)(b)(iii) Repaying loan at \$4000 per month:

→ Time = n months. P = \$4000

Amount left to be repaid:

$\$400\,000 \times 1{\cdot}005^n - \$4000(1 + 1{\cdot}005 + \ldots + 1{\cdot}005^{n-1}) = 0$ (1)

By $S_n = \frac{a(r^n - 1)}{r-1}$

$1 + 1{\cdot}005 + \ldots + 1{\cdot}005^{n-1} = \frac{1(1{\cdot}005^n - 1)}{0{\cdot}005}$

$= 200(1{\cdot}005^n - 1)$ ④

$= 200 \times 1{\cdot}005^n - 200.$

Sub. in (1):

$\$400\,000 \times 1{\cdot}005^n - 4000 \times (200 \times 1{\cdot}005^n - 200) = 0$

$400\,000 \times 1{\cdot}005^n - 800\,000 \times 1{\cdot}005^n + 800\,000 = 0$

$400\,000 \times 1{\cdot}005^n = 800\,000.$

$1{\cdot}005^n = 2$

$\ln(1{\cdot}005^n) = \ln 2$

$n \ln(1{\cdot}005) = \ln 2$

$n = \frac{\ln 2}{\ln(1{\cdot}005)}$

$n \doteq 138{\cdot}97\ldots$

→ The loan will be paid off in 139 months [11 years 7 months].

Name:__________________________ Maths Class:___________________

SYDNEY TECHNICAL HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE

2007

MATHEMATICS

Time Allowed: 3 hours plus 5 mins reading time

Instructions:

- Write your name and class at the top of this page, and at the top of each answer sheet
- At the end of the examination this examination paper must be attached to the front of your answers
- All questions are of equal vale and may be attempted
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.

(For Markers Use Only)

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Total

Question 1 (12 Marks) **Marks**

a) Find the value of $\frac{16.2^2}{14.7-8.1}$ correct to 3 significant figures 2

b) Simplify $4\sqrt{32}-2\sqrt{8}$ 2

c) Write down the exact value of $\sin\frac{5\pi}{4}$ 2

d) Simplify $4(2x+1)-(x^2+2x-3)$ 2

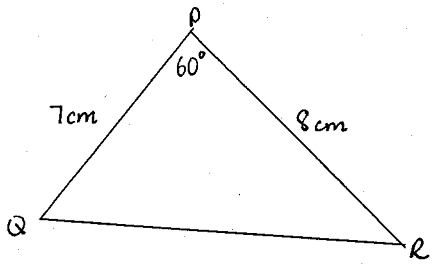
e) Fully factorise $2x^3-2y^3$ 2

f) Find the primitive of $x^2-2x+\frac{1}{x}$ 2

Question 2 **(12 marks)** **Start a new page** **Marks**

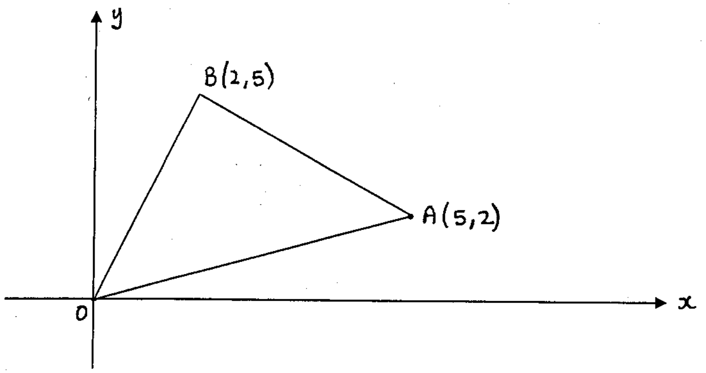
a) Solve $|1-2x|>7$ 2

b) Find the exact area of ΔPQR 2



Not to scale

c)



Not to scale

The points **0 (0,0)** ***A (5, 2)*** and ***B (2, 5)*** are the vertices of a triangle *ABO.*

(i) Find the distance ***OA*** and the distance ***OB*** 2

(ii) Show that the equation AB is $x+y-7=0$ 2

(iii) Calculate the perpendicular distance from O to AB 2

(iv) Find the midpoint, M, of AB 1

(v) Without any more calculations what is the distance of OM, give a reason for answer. 1

Question 3 **(12 marks)** **Start a new page** **Marks**

a) Differentiate with respect to x:

i) $y=x^2-4x+1$ 1

ii) $y=(e^{2x}+1)^2$ 2

iii) $y=x^2\cos 2x$ 2

b) i) Find $\int \frac{4}{4x+1}\,dx$ 1

ii) Evaluate $\int_0^{\frac{\pi}{4}} 2\sec^2 x \quad dx$ 2

c) The roots of the equation $x^2+5x=7$ are α *and* β

Find the value of

i) $\alpha+\beta$ 1

ii) $\alpha\beta$ 1

iii) $\alpha^2+\beta^2$ 2

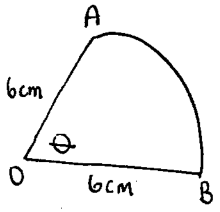
Question 4 **(12 marks)** **Start a new page** **Marks**

a) A ship sails from Port A 70 nautical miles due west to Port B. It then proceeds 40 nautical miles on a bearing of 120°T to Port C.

i) Find the distance of Port C from Port A (correct to 2 decimal places) 2

ii) Find the bearing of Port C from Port A (correct to the nearest degree). 2

b)

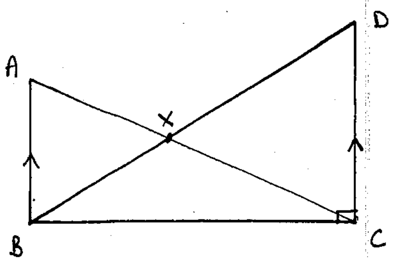


The perimeter of sector AOB is 13.5cm

i) Find the size of $\angle AOB$, correct to the nearest minute 2

ii) Find the area of sector AOB 2

c)



In the diagram AB is parallel to CD and $CD \perp BC$ 2

i) Show that triangle AXB is similar to triangle CXD 2

ii) Given $AB:DC=2:3$ Show that $9(BX)^2 = 4(XD)^2$ 2

Question 5 **(12 marks)** **Start a new page** **Marks**

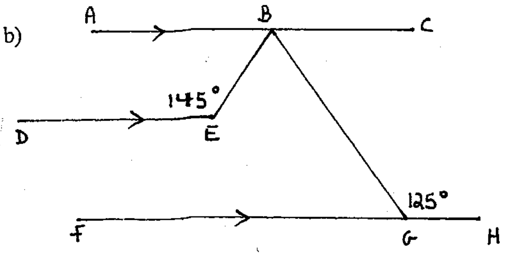
a) For the sequence 95, 91, 87 …………, find,

i) An expression for the *nth* term, ***Tn***, in its simplest form 2

ii) Which term is the first term less than zero 2

iii) What is the sum of all the terms greater than zero 2

b)



In the diagram given

AC // DE and AC // FH

$\angle DEB = 145°$ and $\angle BGH = 125°$

Find the size of $\angle EBG$, giving reasons 2

c) i) For what values of x will a limiting sum exist for the geometric series, $3 - 12x + 48x^2 -$,……………? 2

ii) Find the value of x for which the limiting sum is 9. 2

Question 6 **(12 marks)** **Start a new page** **Marks**

a) Find the equation of the *normal* to the curve $y = \ln(2x+3)$ at the point where $x = -1$. 3

b) The function $f(x)$ is given by $f(x) = 2x(x-3)^2$

i) Find the coordinates of the points where the curve $y = f(x)$ cuts the x-axis 2

ii) Find the coordinates of any turning points on the curve $y = f(x)$, and determine their nature 4

iii) Sketch the curve $y = f(x)$ in the domain $-1 \le x \le 4$ 2

iv) Hence solve $2x^3 - 12x^2 + 18x - 8 = 0$ 1

Question 7 **(12 marks)** **Start a new page** **Marks**

a) What is the value of $\log_2 \sqrt{8}$ 1

b) Given $3x^2 + 4x + 5 \equiv A(x+1)^2 + B(x+1) + C$

Find the value of the constants *A, B* and *C* 3

c) Consider the function $f(x) = x\sin^2 x$

i) Copy and complete the table below in your writing booklet. Values of $f(x)$ are given to 3 decimal places where appropriate.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$f(x)$	0	0.393	1.571		0

1

ii) Using Simpson's Rule with five function values, evaluate

$\int_0^{\pi} x\sin^2 x dx$, correct to 2 decimal places. 3

d) i) Sketch the curve $y = 1 - \cos 2x, \quad 0 \le x \le 2\pi$ 2

ii) Find the area bounded by the curve, $y = 1 - \cos 2x$, the x- axis and the lines $x = 0$ and $x = \pi$ 2

Question 8 **(12 marks)** **Start a new page** **Marks**

a) Given $\log_a x = 0.417$ and $\log_a y = 0.609$ find the value of

i) $\log_a (ax)$ 2

ii) $\log_a \frac{x^2}{y}$ 2

b) The region beneath the curve $y = 3e^{-2x} + 1$ which is above the x – axis and between the lines $x = 0$ and $x = 1$ is rotated about the x – axis

i) Sketch the region 2

ii) Find the volume of the solid revolution 4

c) The price of one gram of gold, \$P, was studied over the period of *t* days.

i) Throughout the period of study $\frac{dP}{dt} > 0$

What does this say about the price of gold? 1

ii) If it was noted over this time that the rate of change in the price of gold increased. What does this statement imply about $\frac{d^2P}{dt^2}$? 1

Question 9 (12 marks) Start a new page

Marks

a) For what values of k does the equation $x^2-(k+2)x+1=0$ have;

i) Equal roots — 2

ii) No real roots — 1

b) The population of a town at the end of t years is given by $P = Ae^{kt}$, where A and k are constants.

After 1 year the population is 1060

i) Find the value of A if the population was initially 1020 — 1

ii) Find the value of k — 2

iii) Calculate the population after 12 years — 2

iv) What is the **rate** of increase in the population after 12 years — 2

v) How many years will it take the population to double? — 2

Question 10 (12 marks) Start a new page

Marks

a) Shrek borrows \$1 000 000 from the Muffin man, at 7.8% p.a. monthly reducible interest to buy a new swamp in Far-Far away land.

He repays the loan in equal monthly repayments of \$8000.

i) Write an expression for the amount Shrek owes immediately **before** the 1st repayment — 1

ii) Show that Shrek owes the Muffin man after n months: — 3

$$An = 1000000(1.0065)^n - 8000\left[\frac{1.0065^n - 1}{0.0065}\right]$$

iii) How many months does Shrek take to repay half the loan to the Muffin man? — 2

b) A new grain silo with a capacity of $4000m^3$ is to be constructed on a farm. The silo is a fully enclosed cylinder and is to be constructed from concrete.

To Save costs, the farmer wants to minimise the surface area of the silo.

i) Write an expression for the volume of the silo in terms of radius (r) and height (h) — 1

ii) Write an expression for the surface area (A) of the concrete silo in terms of r — 2

iii) Show that $\frac{dA}{dr} = \frac{4\pi r^3 - 8000}{r^2}$ — 1

iv) Hence, find the dimensions of the silo to minimise the surface are of the silo. Express your dimensions to 1 decimal place. — 2

END OF EXAM

2007 - Maths Trial

Answers

(24)

Question 1

a) $39.7636\ldots$ ①

$= 39.8$ ①

b) $4\sqrt{32} - 2\sqrt{8} = 16\sqrt{2} - 4\sqrt{2}$ ①

$= 12\sqrt{2}$ ①

c) $\sin\frac{5\pi}{4} = -\sin\pi/4$ ①

$= -1/\sqrt{2}$ ①

d) $4(2x+1) - (x^2+2x-3)$

$= 8x + 4 - x^2 - 2x + 3$ ①

$= 6x - x^2 + 7$ ①

e) $2x^3 - 2y^3$

$= 2(x^3 - y^3)$ ①

$= 2(x-y)(x^2+xy+y^2)$ ①

f) $\int x^2 - 2x + 1/x \, dx$

$= \frac{x^3}{3} - x^2 + \ln x + C$ ②

Question 2

a) $|1-2x| > 7$

$1-2x > 7$ $\quad$ $-1+2x > 7$

$-2x > 6$ $\quad$ $2x > 8$

$x < -3$ ① $\quad$ $x > 4$ ①

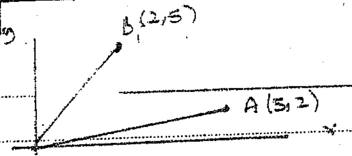
b) $A = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 7 \times 8 \times \sin 60^\circ$

$= \frac{1}{2} \times 7 \times 8 \times \sqrt{3}/2$ ①

$= 14\sqrt{3}$ cm^2 ①

c)



(i) $d_{OA} = \sqrt{29}$ ①

$d_{OB} = \sqrt{29}$ ①

(ii) $m_{AB} = \frac{2-5}{5-2}$

$= -1$

$\therefore y - 5 = -1(x-2)$

$y - 5 = -x + 2$

$x + y - 7 = 0$

(iii) pt $(0,0)$ line $x + y - 7 = 0$

$d_\perp = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$ ①

$= \frac{|0 + 0 - 7|}{\sqrt{1^2+1^2}}$

$= \frac{7}{\sqrt{2}}$ ①

(iv) Midpt $m(3.5, 3.5)$ ①

v) dist $om = \frac{7}{\sqrt{2}}$ as ΔAOB is isosceles $\therefore$ om is $\perp$ bisector of AB.

① → must have a suitable reason.

Question 3

a) i. $\frac{dy}{dx} = 2x - 4$ ①

ii. $\frac{dy}{dx} = 2(e^{2x}+1) \cdot 2e^{2x}$

$= 4e^{2x}(e^{2x}+1)$ ②

iii. $\frac{dy}{dx} = \cos 2x(2x) + x^2(-2\sin 2x)$

$= 2x\cos 2x - 2x^2 \sin 2x$ ②

b) i. $\int \frac{4}{4x+1} dx = \ln(4x+1) + c$ ①

ii. $\int_0^{\pi/4} 2\sec^2 x \, dx = \left[2\tan x\right]_0^{\pi/4}$ ①→

$= 2\left[\tan\frac{\pi}{4} - \tan 0\right]$

$= 2\left[1 - 0\right]$

$= 2$ ①

c) $x^2 + 5x - 7 = 0$ $\quad a=1, b=5$ $c=-7$

(i) $\alpha + \beta = -b/a$ $\quad$ (ii) $\alpha\beta = c/a$

$= -5$ ① $\quad$ $= -7$ ①

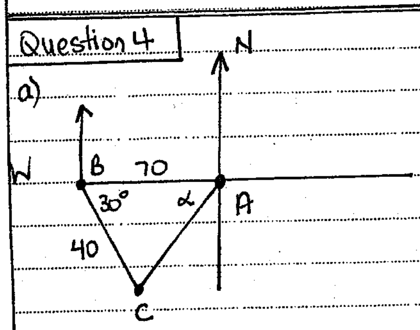
(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ①

$= (-5)^2 - 2(-7)$

$= 39$ ①

Question 4

a)



$AC^2 = 70^2 + 40^2 - 2 \times 70 \times 40 \times \cos 30^\circ$

ii. $\frac{\sin\alpha}{40} = \frac{\sin 30^\circ}{40.62}$

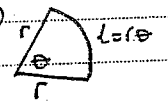
$\sin\alpha = 0.49236\ldots$

$\alpha = 29^\circ 30'$

Bearing $= 270^\circ - 29^\circ 30'$

$= 240^\circ 30'$

b)



$13.5 = 2r + r\theta$

$13.5 = 12 + 6\theta$ ①

r=6 $\quad$ $0.25 = \theta$ (rad)

$\theta = 0.25 \times \frac{180^\circ}{\pi} = \underline{\underline{14^\circ 19'}}$ ①

ii) $A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 6^2 \times 0.25$

$= 4.5$ cm^2

c) In ΔAXB and ΔCXD

① → $\angle BAX = \angle DCX$ (alternate angles AB ∥ CD)

→ $\angle AXB = \angle CXD$ (vertically opposite)

$\therefore \Delta AXB$ ||| ΔCXD (equiangular) ①

ii) $\frac{AB}{CD} = \frac{XB}{XD}$ $\quad$ corresponding sides of ||| Δ's in proportio ①

$\frac{2}{3} = \frac{XB}{XD}$

$2XD = 3BX$

$4(XD)^2 = 9(BX)^2$

both = ①

Question 5

a) 95, 91, 87, ... $a = 95$ $d = -4$ AP

1) $T_n = a + (n-1)d$ ①
$= 95 + (n-1)(-4)$
$= 95 - 4n + 4$ ①
$= 99 - 4n$

ii) $T_n < 0$
$99 - 4n < 0$
$4n > 99$
$n > 24.75$ ①
∴ 25th term is 1st negative. ①

iii) ie 24 terms > 0 $n = 24$ $a = 95$ $d = -4$
$S_n = \frac{n}{2}(2a + (n-1)d)$ ①
$= \frac{24}{2}(2(95) + 23(-4))$ ①
$= 1176$

b) $\angle CBG + 125° = 180°$ (co-interior angles AC ∥ FH)
$\angle CBG = 55°$ ① both
$\angle ABE + 145° = 180°$ (co-interior angles AC ∥ DE)
$\angle ABE = 35°$
$35 + 55 + \angle EBG = 180°$ (straight) ①
$\angle EBG = 90°$

c) $a = 3$ $r = -4x$
1) ∴ $-1 < r < 1$ ①
$-1 < -4x < 1$
$1/4 > x > -1/4$
∴ $-1/4 < x < 1/4$ ①

ii) $S_\infty = \frac{a}{1-r}$ $9 = \frac{3}{1+4x}$ ①
$9(1+4x) = 3$
①

Question 6

a) $\frac{dy}{dx} = \frac{2}{2x+3}$ ① at $x = -1$ $y = 0$
$M_T = 2$
$M_N = -1/2$ ①
eq: $y - 0 = -1/2(x+1)$
$2y = -x - 1$ ①
$x + 2y + 1 = 0$

b) $f(x) = 2x(x-3)^2 = 2x^3 - 12x^2 + 18x$

(i) x-int $y = 0$
(0,0) ① and (3,0) ①

(ii) Stat pts $f'(x) = 0$
$f'(x) = 6x^2 - 24x + 18 = 0$
$6(x-3)(x-1) = 0$
$x = 3$ $x = 1$
$y = 0$ $y = 8$

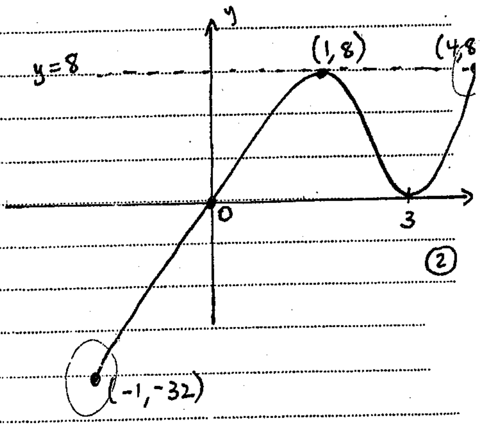
test

x	0	1	2
y'	14 /	0 —	−12 \

x	2	3	4
y'	−12 \	0 —	/

MAX (1,8) 1 MIN (3,0) 1

(iii) End pts (−1,−32) & (4,8)



②

iv) $2x^3 - 12x^2 + 18x = 8$
∴ $x = 1$, $x = 4$ ①

Question 7

a) $\log_2 \sqrt{8} = \frac{1}{2}\log_2 8$
$= \frac{1}{2} \times 3\log_2 2$
$= 1.5$ ①

b) $3x^2 + 4x + 5 \equiv A(x^2 + 2x + 1) + Bx + B + C$

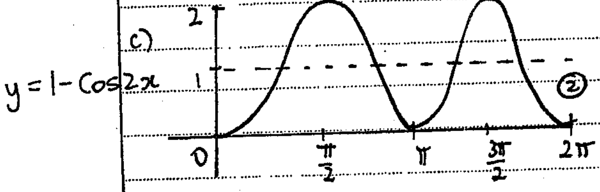
equating
$3 = A$
$4 = 2A + B$
$4 = 6 + B$
$B = -2$
$5 = A + B + C$
$5 = 3 - 2 + C$
$C = 4$
∴ $A = 3$ ①, $B = -2$ ①, $C = 4$ ①

c)

$\frac{3\pi}{4}$
1.178

①

ii. $h/3[f + L + 4m]$ ①
$\frac{\pi/4}{3}[0 + 1.571 + 4 \times 0.393]$ ①
$+ \frac{\pi}{12}[1.571 + 0 + 4 \times 1.178]$
$= 2.46772...$ ①
$= 2.47$ (2dp)

c)


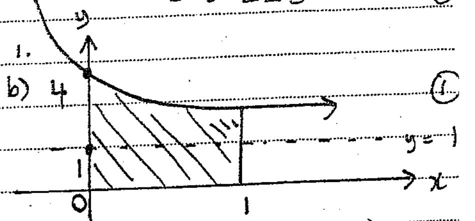
②

ii. $\int_0^\pi 1 - \cos 2x \, dx$ ①
$= \left[x - \frac{1}{2}\sin 2x\right]_0^{\pi/}$

Question 8

a) $\log_a(ax) = \log_a a + \log_a x$ ①
$= 1 + 0.417$
$= 1.417$ ①

ii) $\log_a \frac{x^2}{y} = 2\log_a x - \log_a y$ ①
$= 2(0.417) - 0.609$
$= 0.225$ ①

b) i.


①

ii.
$V_x = \pi \int y^2 \, dx$
$= \pi \int_0^1 (3e^{-2x} + 1)^2 \, dx$
$= \pi \int_0^1 9e^{-4x} + 6e^{-2x} + 1 \, dx$
$= \pi \left[\frac{9}{-4}e^{-4x} + \frac{6e^{-2x}}{-2} + x\right]_0^1$
$= \pi \left[\frac{9}{-4}e^{-4} - 3e^{-2} + 1 - \left(\frac{-9}{4} - 3\right)\right.$
$= \pi \left[-\frac{9}{4}e^{-4} - 3e^{-2} + \frac{25}{4}\right]$

c) $\frac{dP}{dt} > 0$ price of gold increasing ①

ii) $\frac{d^2P}{dt^2} > 0$ ①

Question 9

a) $x^2 - (k+2)x + 1 = 0$

i) Equal roots $\Delta = 0$

$b^2 - 4ac = 0$

$(k+2)^2 - 4(1)(1) = 0$ ①

$k^2 + 4k + 4 - 4 = 0$

$k^2 + 4k = 0$

$k(k+4) = 0$

$k = 0, \quad k = -4$ ①

ii) $\Delta < 0, \quad -4 < k < 0$ ①

b) $t = 0 \quad P = 1020$

$\therefore A = 1020$ ①

ii. $t = 1 \quad P = 1060$

$1060 = 1020e^{k(1)}$

$\frac{1060}{1020} = e^k$ ①

$\ln\left(\frac{106}{102}\right) = k$

$k = \ln\left(\frac{106}{102}\right)$ ①

$\doteqdot 0.038466....$

iii) $t = 12 \quad P = ?$ ①

$P = 1020e^{k.12} \quad k = \ln\left(\frac{106}{102}\right)$

$= 1618.335....$

$\doteqdot 1618$ ①

iv) rate $= \frac{d}{dt}$

①

$\frac{dP}{dt} = k.(1020e^{kt}) \quad k = \ln\left(\frac{106}{102}\right)$

$t = 12$

$= 62.2513....$

$= 62.25$ people/yr. ①

v) $t = ? \quad P = 2A$

$2A = Ae^{kt} \quad k = \ln\left(\frac{106}{102}\right)$

$2 = e^{kt}$ ①

$\ln 2 = \ln e^{kt}$

$\ln 2 = k.t$

$t = \ln 2 \div k$

$= 18.0196....$ ①

$\doteqdot$ 18 years.

Question 10

a) monthly repayment = 8000

Principal = 1 000 000

rate = 7.8% ÷ 12 (monthly)

= 0.0065

(i) $1000\,000(1.0065)$ ①

(ii) $A_1 = 1000\,000(1.0065) - 8000$

$A_2 = A_1(1.0065) - 8000$

$= 1000\,000(1.0065)^2 - 8000(1.0065) - 8000$ ①

$A_n = 1000\,000(1.0065)^n - 8000\left[1.0065^{n-1} + 1.0065^{n-2} ... + 1\right]$ ①

$= 1000\,000(1.0065)^n - 8000\left[\frac{a(r^n - 1)}{r - 1}\right]$ ①

$a = 1 \quad r = 1.0065 \quad n = n$

$= 1000\,000(1.0065)^n - 8000\left[\frac{1.0065^n - 1}{0.0065}\right]$

(iii)

$500\,000 = 100\,000(1.0065)^n - 1230769\left[1.0065^n - 1\right]$

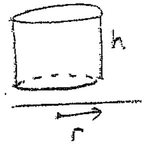
① $500\,000 = 1000000(1.0065)^n - 1230769(1.0065)^n + 1230769$

$230769(1.0065)^n = 730769$

$1.0065^n = 3.1666...$

$\log 1.0065^n = \log 3.1666....$

$n\left[\log 1.0065\right] = \log 3.166..$

b) 

$V = 4000\text{ m}^3$

$V = \pi r^2 h$

(i) $4000 = \pi r^2 h$ ①

(ii) $SA = 2\pi r^2 + 2\pi r h$ ① $\quad h = \frac{4000}{\pi r^2}$

$SA = 2\pi r^2 + 2\pi r\left[\frac{4000}{\pi r^2}\right]$

$= 2\pi r^2 + \frac{8000}{r}$ ①

$= 2\pi r^2 + 8000r^{-1}$

(iii) $\frac{dA}{dr} = 4\pi r - 8000r^{-2}$ ①

$= \frac{4\pi r^3 - 8000}{r^2}$

(iv) Min Surface Area $dA/dr = 0$

$\frac{4\pi r^3 - 8000}{r^2} = 0$

$4\pi r^3 = 8000$

$r^3 = \frac{8000}{4\pi}$

$r = \sqrt[3]{\frac{2000}{\pi}}$ ①

$\doteqdot 8.6025...$

$\doteqdot 8.6$ (1dp)

test

r	8	8.6025...	9
$\frac{dA}{dr}$	\	—	/

$\therefore$ dimensions are ①

$r \doteqdot 8.6\text{m} \quad h = 17.2\text{ m}$

Name: ____________________________

Teacher: ___________________________

Class: ___________________________

FABER EST SUAE QUISQUE FORTUNAE

FORT STREET HIGH SCHOOL

2008

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 4: TRIAL HSC

Mathematics

TIME ALLOWED: 3 HOURS
(PLUS 5 MINUTES READING TIME)

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1,2	
Manipulates algebraic expressions to solve problems from topic areas such as functions, quadratics, trigonometry, probability and series	3,4,5	
Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	6,7,8	
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	9, 10	

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**	**%**
Marks	**/12**	**/12**	**/12**	**/12**	**/12**	**/12**	**/12**	**/12**	**/12**	**/12**	**/120**	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started in a new booklet

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Question 1 **Start a new booklet**

(a) Evaluate $\frac{2.4\times\sqrt{30}}{24.9}$ correct to 3 significant figures. **1**

(b) Solve $x^2-3=3x+1$ **2**

(c) Express $\frac{5}{3-2\sqrt{3}}$ with a rational denominator. **2**

(d) Solve and graph on the number line $|3x-1|<8$. **3**

(e) A patient in hospital is fed intravenously (into the vein) 3.6 litres of fluid per 24 hours. If there are 15 drops of fluid per mL, find how many drops per minute the patient receives. **2**

(f) Simplify $\frac{2}{x(x-3)}-\frac{1}{x}$ **2**

Question 2 **Start a new booklet**

The line L has equation $x+2y=5$ and P is the point $(2,4)$.

(i) On a number plane, mark the origin O, the point P and draw the line L. **1**

(ii) Find the midpoint M, of the interval OP. **1**

(iii) Show M lies on the line L. **1**

(iv) Find the gradients of the line OP and the line L. **2**

(v) Show the line L is the perpendicular bisector of the interval OP. **2**

(vi) Line L meets the $x-$axis at Q. Find the co-ordinates of Q. **1**

(vii) A line is drawn through O parallel to PQ and it meets line L in R. Find the equation of ~~QR~~.OR. **2**

(viii) Explain why PQOR is a rhombus. **2**

Question 3 **Start a new booklet**

(a)

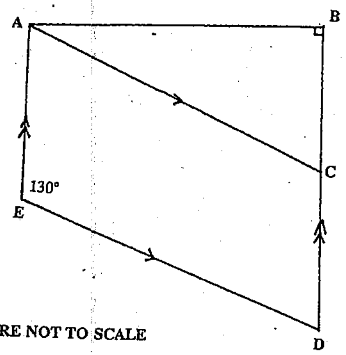


FIGURE NOT TO SCALE

In the diagram AE$\parallel$BD and AC$\parallel$ED, $\angle$AED = 130° and $\angle$ABC = 90°.

(i) Copy this diagram onto your answer sheet.

(ii) Find the size of $\angle$BAC giving reasons. **2**

(b) Differentiate

(i) xe^{3x} **2**

(ii) $\dfrac{2x^4-3}{x^2}$ **2**

(c) **(i)** Find the primitive function of $\dfrac{1}{3x^2}$ **1**

(ii) Find exactly in simplest form $\int_2^3 \dfrac{x}{x^2-1}$ **2**

(d) Find the range of values of k if the equation $4x^2-kx+1=0$ has no real roots. **3**

Question 4 **Start a new booklet**

(a) If α and β are the roots of the equation $(3x-2)^2+4=0$

Find **(i)** $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $3\alpha^2+3\beta^2$ **2**

(b) An arithmetic progression has a first term 1 and last term 14.
The sum of the series is 90.

(i) Find the number of terms in the series. **2**

(ii) Show that the common difference is $\dfrac{13}{11}$. **2**

(c) Two dice are rolled. The score for the roll is given by the difference between the numbers on the uppermost faces (e.g. if the numbers are 2 and 6, the score is 4).

Find the probability that the score will be

(i) 0 **2**

(ii) At least 3. **2**

Question 5 **Start a new booklet**

(a) If $\log_x 128 = \frac{7}{3}$, find x. 1

(b) **(i)** Sketch the graph of $y = 5\cos\frac{x}{2}$ for $-360^0 \le x \le 360^0$. 2

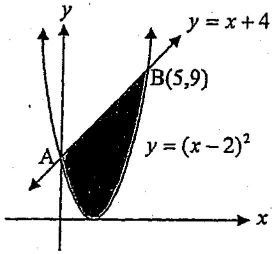
(ii) Mark clearly on your graph the point or points where

$5\cos\frac{x}{2} = -1$. 1

(iii) Calculate the value(s) of x which satisfy the equation

$5\cos\frac{x}{2} = -1$. Express your answer(s) to the nearest minute. 2

(c)



NOT TO SCALE

The graphs of $y = (x-2)^2$ and $y = x + 4$ intersect at the point A and the point B(5,9).

(i) Show that the point A lies on the y-axis. 2

(ii) Write down the two inequalities whose intersection describes the shaded area shown in the diagram above. 1

(iii) Find the area of the shaded regions bounded by the graphs of $y = (x-2)^2$ and $y = x + 4$. 3

Question 6 **Start a new booklet**

(a) For the curve $f(x) = \frac{1}{3}x^3 - x^2 - 8x + 12$,

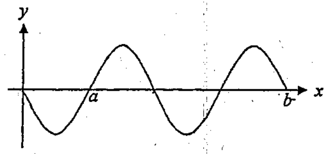
(i) Find any turning points and determine their nature. 6

(ii) Find any points of inflexion. 2

(iii) Sketch the curve clearly labelling points of intersection with the axes and the features you have found in (i) and (ii). 1

(iv) For what value of x is the curve concave upwards? 1

(b)



The graph of $y = 2\cos(2x + \frac{\pi}{2})$ is shown over two complete cycles.

(i) Find the value of b. 1

(ii) Given that $\int_0^a 2\cos(2x + \frac{\pi}{2})dx = -8$, find $\int_0^b 2\cos(2x + \frac{\pi}{2})\,dx$ without using calculus. 1

Question 7 **Start a new booklet**

(a) For the curve $y^2 - 2y - 6x = 0$ find

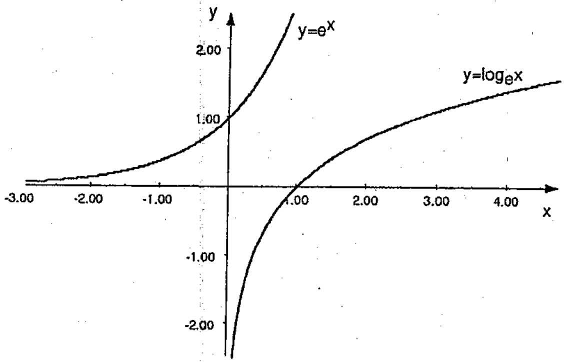
(i) the co-ordinates of the focus 3

(ii) the equation of the directrix. 1

(b) Prove that the line $y = 2x + c$ cuts the curve $y = x^2 + 6x + 7$ at two distinct points if $c > 3$. 2

(c) Evaluate $\sum_{r=1}^{\infty} 3^{-r}$ 2

(d) The graphs show the two functions $y = e^x$ and $y = \log_e x$.



(i) With reference to the graph above, explain how the two graphs $y = e^x$ and $y = \log_e x$ are related to each other. 1

(ii) Show that the equation of the tangent drawn at $x = 2$ on the graph of $y = \log_e x$ is given by the equation

$x - 2y - 2 + \log 4 = 0$ 2

(iii) find the acute angle that the tangent in **(ii)** makes with the x - axis, to the nearest degree. 1

Question 8 **Start a new booklet**

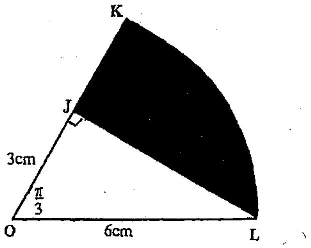


FIGURE NOT TO SCALE

(a) In the diagram KL is an arc of a circle with centre 0 and radius 6cm. OJ =3cm, $\angle KOL = \frac{\pi}{3}$ and $JL \perp OK$.

Calculate the perimeter of the shaded region JKL. Give your answer correct to 1 decimal place. 4

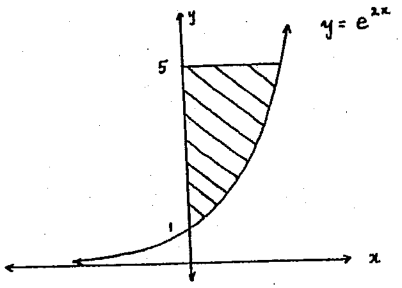
(b) **(i)** Copy and complete the table below for $f(x) = \left(\log_e \sqrt{x}\right)^2$, calculating each value correct to 3 decimal places. 1

x	1	2	3	4	5
$f(x)$	0	0.120			

(ii) Using Simpson's Rule with 5 function values, show that

$\int_1^5 \left(\log_e \sqrt{x}\right)^2 dx \doteqdot 1.22$ 1

(c)



The diagram above shows the region bounded by the curve $y = e^{2x}$, the y-axis and the line $y = 5$.

(i) Show that $x = \log_e \sqrt{y}$ **1**

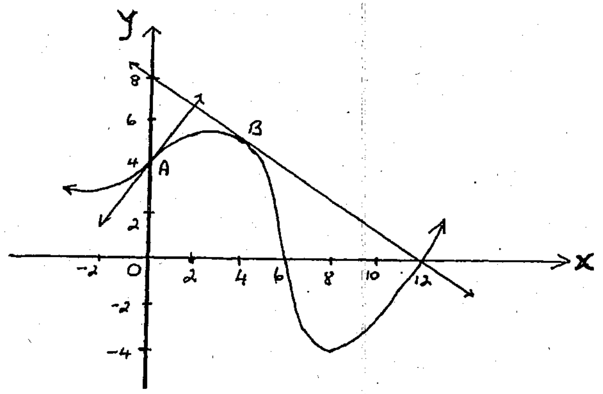
(ii) The shaded area is rotated about the y-axis. Write down the integral equal to the volume formed. **1**

(iii) Evaluate the volume of this solid of revolution using the approximation in **Part (b) (ii)** above, leaving your answer correct to 2 significant figures. **1**

(d) A function $y = f(x)$ has $\frac{d^2y}{dx^2} = 6x - 2$ and a stationary point at $(3,0)$. Find $f(x)$. **3**

Question 9 **Start a new booklet**

(a)



The above is a graph of the function $y = f(x)$. Tangents are drawn at A(0,4) and B(4,5). Use the graph to evaluate:

(i) $f(6)$

(ii) $f'(4)$

(iii) $f'(8)$

(iv) $f''(0)$ **4**

(b) A school softball team has a probability of 0.8 of losing or drawing any match and a probability of 0.2 of winning any match.

(i) Find the probability of the team winning at least one of the three consecutive matches. **2**

(ii) What is the least number of consecutive matches the team must play to be 90% certain it will win at least one match? **2**

(c) Maxamillian's daughter was born on the 1st January. On that day he opened a trust account by depositing \$250. Each year, on her birthday, he deposited \$250 into this trust fund. He continued to do this up to and including her 17th birthday. When she turned eighteen, he collected the total amount including interest from this account and presented to her.

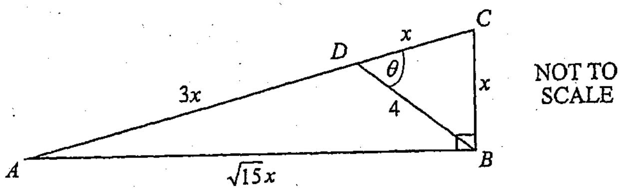
This account paid an interest of 6% p.a. compounded every six months.

(i) Show the initial deposit amounted to approximately \$724.57 after 18 years. **1**

(ii) How much did Maximillian give his daughter on her eighteenth birthday? **3**

Question 10 **Start a new booklet**

(a)



In the diagram, ABC is a right angled triangle where $AB = \sqrt{15}\,x$ cm and $BC = x$ cm. The point D lies on AC and $CD = BC = x$ cm, $AD = 3x$ cm and $BD = 4$ cm. Let $< BDC = \theta$.

(i) Use the cosine rule to show that $\cos\theta = \frac{2}{x}$. **1**

(ii) Use the sine rule in triangle BCD to show that $\sin\theta = \frac{\sqrt{15}\,x}{16}$. **2**

(iii) Hence show that $15x^4 - 256x^2 + 1024 = 0$. **2**

(iv) Explain why one of the solutions to the equation in part (iii), namely $x = 2.53$ (to 2 decimal places), could not be the value of x indicated in the diagram above. **1**

(b) ABCDE is a pentagon of fixed perimeter P cm. Its shape is such that ABE is an equilateral triangle and BCDE is a rectangle. If the length of AB is x cm :

(i) Show that the length BC is $\frac{P-3x}{2}$ cm. **1**

(ii) Show that the area of the pentagon is given by

$A=\frac{1}{4}[2Px-(6-\sqrt{3})x^2]$ **2**

(iii) Find the value of $\frac{P}{x}$ for which the area of the pentagon is a maximum. **3**

END OF EXAMINATION

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2UNIT TRIAL HSC – 2008 – SOLUTIONS

Question 1

a) $\frac{2.4 \times \sqrt{30}}{24.9} = 0.527925\ldots$

$= 0.528$ (3 sig fig) ✓

- mostly well done, but a significant number of students wrote 0.53... clearly not understanding significance.

b) $x^2 - 3 = 3x + 1$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$ ✓

$x = 4, \quad x = -1$ ✓

- mostly well done.

c) $\frac{5}{3-2\sqrt{3}} \times \frac{3+2\sqrt{3}}{3+2\sqrt{3}}$ ✓

$= \frac{15 + 10\sqrt{3}}{9-12}$

$= \frac{15+10\sqrt{3}}{-3}$

$= -5 - \frac{10}{3}\sqrt{3}$ ✓

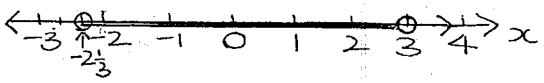
- mostly well done
- many made errors with signs!

d) $|3x - 1| < 8$

$3x - 1 < 8, \quad -3x + 1 < 8$

$3x < 9 \qquad -3x < 7$

$x < 3$ ✓ $\qquad x > -2\frac{1}{3}$ ✓



✓

- mostly well done
- those who didn't do well did not have or do the negative case properly.

e) Rate = 3.6 L / 24 h

$= 3600$ mL/1440 min ✓

$= 2.5$ mL/min

$= 2.5 \times 15$ drops/min

$= 37.5$ drops/min ✓

- usually well done
- check reasonableness of answer – a torrent of 135000 drips/min is quite unreasonable!!

f) $\frac{2}{x(x-3)} - \frac{1}{x}$

$= \frac{2-(x-3)}{x(x-3)}$ ✓

$= \frac{5-x}{x(x-3)}$ ✓

- often had an extra x, which caused problems cancelling (ie $\frac{2x - x(x-3)}{x^2(x-3)}$)
- Signs!! $(x-5)$ or $(x-1)$ was con

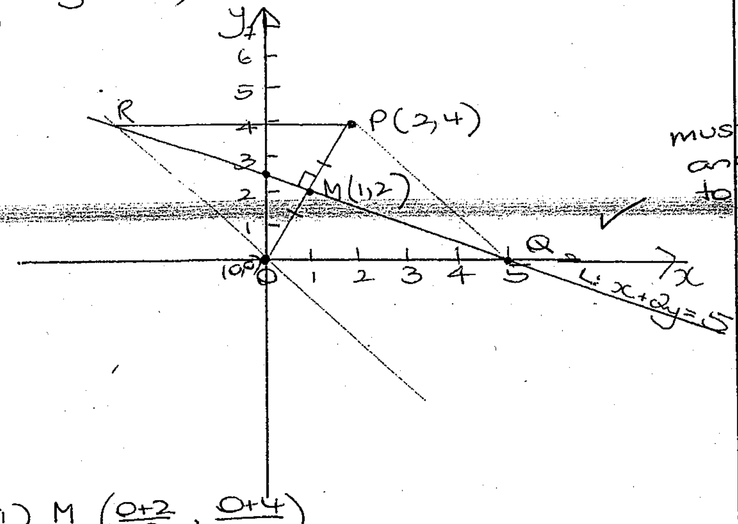
Question 2.

line L: $x + 2y = 5$, $P(2,4)$

$x = 0, \quad y = 2\frac{1}{2}$

$y = 0, \quad x = 5$

i)



✓

The graphing was poorly done. Give the intercepts, use a ruler and indicate interval size on axes

must have O, P, and line L have to be shown.

ii) $M\left(\frac{0+2}{2}, \frac{0+4}{2}\right)$

$= M(1,2)$ ✓

iii) $x + 2y = 5$

L.H.S $= 1 + 2 \times 2$

$= 5$

$=$ R.H.S. ✓

$\therefore$ M lies on $x + 2y = 5$

iv) $m_{OP} = \frac{4-0}{2-0}$

$=$ ✓

$x + 2y = 5$

$2y = -x + 5$

$y = -\frac{1}{2}x + \frac{5}{2}$

$m \quad = -1$ ✓

Q2 cont'd

v) $m_{OP} = 2$

$\perp m = -\frac{1}{2}$ $M(1,2)$ ✓

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{2}(x - 1)$

$2y - 4 = -x + 1$ ✓

$x + 2y - 5 = 0$

which is line L

OR since L passes through ✓ the midpt. of OP and $m_L \times m_{OP} = -1$

i.e. $-\frac{1}{2} \times 2 = -1$ ✓

then line L is the perpendicular bisector of OP.

many students did not explain why L is also the bisector of OP ∴ lost one mark

vi) $Q(5,0)$ ✓

vii) $m_{PQ} = \frac{0-4}{5-2}$

$= \frac{-4}{3}$

$\therefore m_{OR} = \frac{-4}{3}$ ✓

$m = -\frac{4}{3}$, $O(0,0)$

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{4}{3}(x - 0)$

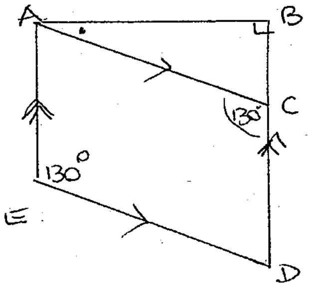
$y = -\frac{4}{3}x$ ✓

ie $4x + 3y = 0$

viii) the diagonals bisect each other at rightangles. ✓✓ (or other suitable reason)

Students need to learn the tests for all types of quadrilaterals

Question 3



$\angle DCA = 130°$ (opp. ∠'s of ‖gm =) ✓

$\angle BAC + 90 = 130$ (ext. ∠ $\triangle ABC$)

$\angle BAC = 40°$ ✓

reasons must be given

poor reasoning with many students

Q3 cont'd

(b) i) Let $y = \overset{}{\underset{u}{x}}\,\underset{v}{e^{3x}}$

$\frac{dy}{dx} = v \cdot \frac{du}{dx} + u \cdot \frac{dv}{dx}$

$= e^{3x} + x \cdot 3e^{3x}$ ✓

$= e^{3x}(1 + 3x)$ ✓

ii) let $y = \frac{2x^4 - 3}{x^2}$

$= 2x^2 - \frac{3}{x^2}$

$= 2x^2 - 3x^{-2}$ ✓

$y' = 4x + 6x^{-3}$

$= 4x + \frac{6}{x^3}$ ✓

many did not simplify

* or use the quotient rule.

(c) i) $\int \frac{1}{3x^2}\,dx = \frac{1}{3}\int x^{-2}\,dx$

$= -\frac{1}{3}x^{-1} + c$

$= -\frac{1}{3x} + c$ ✓

ii) $\int_2^3 \frac{x}{x^2 - 1}\,dx = \frac{1}{2}\left[\ln(x^2 - 1)\right]_2^3$ ✓

$= \frac{1}{2}\left[\ln 8 - \ln 3\right]$

$= \frac{1}{2}\ln\frac{8}{3}$ ✓

(d) $4x^2 - kx + 1 = 0$

$a = 4,\ b = -k\ \ c = 1$

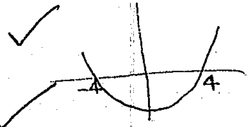
No real roots $\Rightarrow \Delta < 0$ ✓

$b^2 - 4ac < 0$

$k^2 - 16 < 0$ ✓

$(k - 4)(k + 4) < 0$

$-4 \ldots k < 4$ ✓



many had a good idea i.e. $\Delta < 0$ but could not solve properly

Question 4

(a) $(3x-2)^2 + 4 = 0$

$9x^2 - 12x + 4 + 4 = 0$

$9x^2 - 12x + 8 = 0$

$a = 9 \quad b = -12, \quad c = 8$

i) $\alpha + \beta = -\frac{b}{a}$

$= \frac{12}{9}$

$= \frac{4}{3}$ ✓

ii) $\alpha\beta = \frac{c}{a}$

$= \frac{8}{9}$ ✓

iii) $3\alpha^2 + 3\beta^2 = 3(\alpha^2 + \beta^2)$

$= 3\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$ ✓

$= 3 \times \left[\frac{16}{9} - \frac{16}{9}\right]$

$= 0$ ✓

(b) A P: $a = 1 \quad l = 14 \quad S_{14} = 90$

i) $S_n = \frac{n}{2}(a + l)$ ✓

$90 = \frac{n}{2}(1 + 14)$

$180 = 15n$

$n = 12$ ✓

12 terms in the series.

ii) $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$90 = 6\left[2 + (12-1)d\right]$ ✓

$90 = 6(2 + 11d)$

$15 = 2 + 11d$ ✓

$11d = 13$

$d = \frac{13}{11}$ as req'd.

c)

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

✓

i) $P(0) = \frac{6}{36}$

$= \frac{1}{6}$ ✓

ii) P(score at least 3)

$= \frac{12}{36}$ ✓

$= \frac{1}{3}$ ✓

Some student had difficulty in simplifying the equation into $ax^2+bx+c=0$.

Mostly well done.

Some students did not rewrite $3\alpha^2+3\beta^2$ correctly

Mostly well done

Many used Tn expression to correctly determine d.

* a diagram need no be shown, but some working has to be shown.

Lack of table or working for some students

Question 5

(a) $\log_x 128 = \frac{7}{3}$

$x^{\frac{7}{3}} = 128$

$x = 128^{\frac{3}{7}}$ ✓

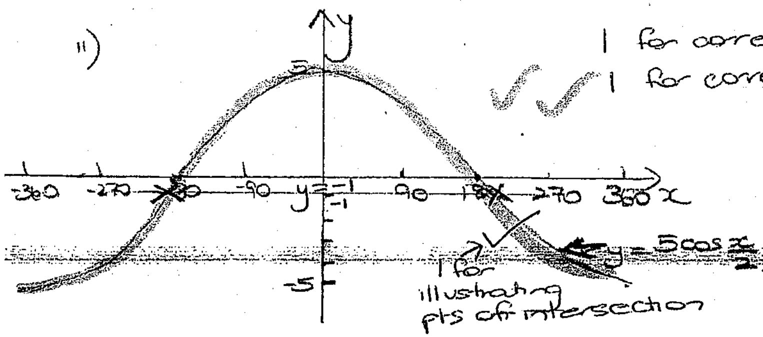
$x = 8$

(b) i) $y = 5\cos\frac{x}{2}$

$A = 5 \quad P = \frac{2\pi}{\frac{1}{2}}$

$= 4\pi \; (720°)$

ii)



1 for correct shape ✓

1 for correct Amplitude/Range. ✓

1 for illustrating pts of intersection ✓

iii) $5\cos\frac{x}{2} = -1$

$\cos\frac{x}{2} = -\frac{1}{5}$ ✓

$\frac{x}{2} = 101°32', -101°32'$

$x = 203°4', -203°4'$ ✓

(c) $y = x + 4$

$y = (x-2)^2$

$(x-2)^2 = x + 4$

$x^2 - 4x + 4 = x + 4$

$x^2 - 5x = 0$

$x(x-5) = 0$ ✓

$x = 0, \quad x = 5$

when $x = 0$, $y = 4 \Rightarrow (0,4)$

when $x = 5$, $y = 9 \Rightarrow (5,9)$ ✓

A is (0,4) which lies on

(a)

* Many 24 students coul not do. This using the log defn

(b)

* Graph poor drawn in ma instances

- axes must labelled
- divisions o axes need t be shown

* y=-1 was o not in a rele position compared to y=-5

* $\cos\frac{x}{2} = -\frac{1}{5}$ is not equiva to cosx = -

* need to observe gra & realise the was a ± valu

(c) i) some stud didn't actuall show that A lies on the y axis MUST explain that the x co- is zero. Some used the substitution method, but d not sub x=0 i both eq^ns to s

Q5 (cont'd)

ii) $y < x+4 \cap y > (x-2)^2$ ✓

iii) $A = \int_0^5 (x+4)\,dx - \int_0^5 (x^2-4x+4)\,dx$

$= \int_0^5 (x+\cancel{4} - x^2 + 4x - \cancel{4})\,dx$ ✓

$= \int_0^5 5x - x^2 \, dx$

$= \left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_0^5$ ✓

$= \left(\frac{125}{2} - \frac{125}{3}\right) - 0$ ✓

$A = 20\frac{5}{6}\,u^2$

(ii) accepted
← $y<x+4 \cap y > (x-$
also.

iii) generally well done but there were still some careless error with the actual integration and/or substituting

---

Question 6.

(a) $f(x) = \frac{1}{3}x^3 - x^2 - 8x + 12$

$f'(x) = x^2 - 2x - 8$

$f''(x) = 2x - 2$

i) for statny pts $f'(x) = 0$ ✓

i.e. $x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, \ x = -2$ ✓

when $x = 4$, $y = -14\frac{2}{3} \Rightarrow (4, -14\frac{2}{3})$ ✓

when $x = -2$ $y = 21\frac{1}{3} \Rightarrow (-2, 21\frac{1}{3})$ ✓

$x = 4$: $f''(4) > 0 \Rightarrow$ <u>min^m</u> at $(4, -14\frac{2}{3})$ ✓

$x = -2$: $f''(-2) < 0 \Rightarrow$ max^m at $(-3, 21\frac{1}{3})$ ✓

Many students did not state why f'(x)= others did not let it equal zero but still found 2 x values fr an expression

} y values

The test for max/min was often not shown

} test for max/min.

ii) for possible inflexions $f''(x) = 0$

i.e. $2x - 2 = 0$

$2x = 2$

$x = 1$ ✓

test $x = 1$

x	1^-	1	1^+
$f''(x)$	−	0	+

∴ concavity changes.

when $x = 1$

$y = \frac{1}{3} - 1 - 8 + 12$

$= 3\frac{1}{3}$

∴ inflexion at $(1, 3\frac{1}{3})$ ✓

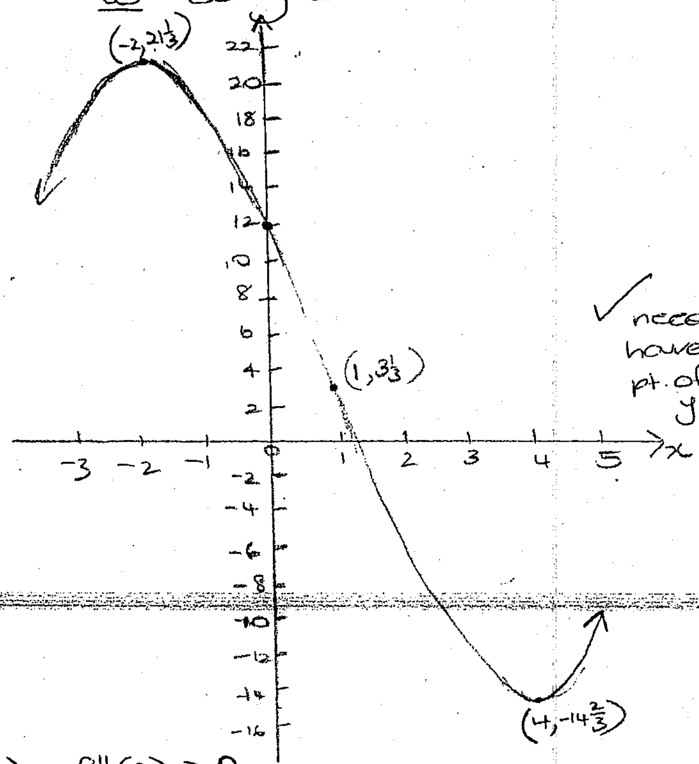
The test for the change in concavity was often ignored

*1 only if did not test for change in concavity.

Question 6 (cont'd)

* cuts y axis ⇒ $x = 0$

i.e. at $y = 12$



✓ need to have max^m/min^m/ pt. of inflexion + y int. labelled.

Sketch was mostly well done but lack of detail in some cases.

iv) $f''(x) > 0$

i.e. $x > 1$ ✓

students included $x=1$, others could not use their graph to answer this.

(b) i) $P = \frac{2\pi}{2}$

$= \pi$

2 cycles

$\therefore b = 2\pi$ ✓

Some students did not recognise that there was 2 cycles for the curve.

ii) $\int_0^b 2\cos\left(2x + \frac{\pi}{2}\right) dx$

$= -8 + 8 + -8 + 8$

$= 0$ ✓

Some people did not realise that -8 was appropriate & tried to use absolute value.

Question 7

(a) $y^2-2y-6x=0$

$y^2-2y=6x$

$y^2-2y+1=6x+1$ ✓

$(y-1)^2=6(x+\frac{1}{6})$

$V(-\frac{1}{6},1)$ ✓

$4a=6$

$a=\frac{3}{2}$

i) $F(\frac{4}{3},1)$ ✓

ii) $x=-1\frac{2}{3}$ ✓

- students got the completing the square wrong, adding 4 instead of 1.
- students did not realise that the parabola was in the form $x=y^2$ not $y=x^2$.

(b) $y=2x+c$

$y=x^2+6x+7$

$\therefore x^2+6x+7=2x+c$

$x^2+4x+7-c=0$

$a=1 \quad b=4 \quad c=7-c$ ✓

2 distinct roots $\Rightarrow \Delta>0$

ie $b^2-4ac>0$

$16-4(7-c)>0$ ✓

$16-28+4c>0$

$4c>12$

$c>3$ as req'd.

- students tried to sub in a value, eg. let $c=4$, and then show 2 distinct roots.

(c) $\sum_{r=1}^{\infty} 3^{-r}=\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\dots$

$a=\frac{1}{3},\ r=\frac{1}{3}$ ✓

$S_\infty=\frac{a}{1-r}$

$=\frac{\frac{1}{3}}{\frac{2}{3}}$

$=\frac{1}{3}\times\frac{3}{2}$

$=\frac{1}{2}$ ✓

- some students let $r=3$ or at times $a=3$

Question 7 (cont'd)

(d) i) $y=e^x$ and $y=\log_e x$ are reflections of each other in the line $y=x$ ✓

students used ~~the~~ words such as opposite & reciprocal instead of inverse.

ii) $y=\log_e x$

$y'=\frac{1}{x}$

at $x=2$: $y'=\frac{1}{2}$ ✓

when $x=2$ $y=\log_e 2$

$y-y_1=m(x-x_1)$

$y-\log_e 2=\frac{1}{2}(x-2)$

$2y-2\log_e 2=x-2$

$x-2y-2+2\log_e 2=0$ ✓

~~$x-2y-2+\log_e 2^2=0$~~

$x-2y-2+\log_e 4=0$ (as req'd)

done well

iii) $m=\tan\theta$

$\therefore \tan\theta=\frac{1}{2}$

$\theta=\tan^{-1}\frac{1}{2}$

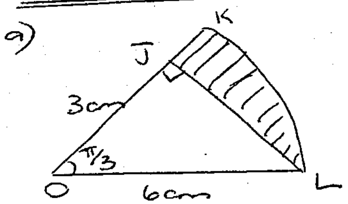
$\theta=26°33'54''$ ✓

$\theta=27°$ (nearest degree)

done well

Question 8:

a)



$6^2 = JL^2 + 3^2$

$36 = JL^2 + 9$

$JL^2 = 27$

$JL = \sqrt{27}$

$\underline{JL = 3\sqrt{3}}$ ✓

$\underline{JK = 3\text{ cm}}$ ✓

$l = r\theta$

$= \frac{\pi}{3} \times 6$

$= 2\pi$ ✓

$\therefore KL = 2\pi$

Perimeter $= 3\sqrt{3} + 3 + 2\pi$

$= 14.5$ cm (1dp) ✓

b) i) $f(x) = (\log_e \sqrt{x})^2$

x	1	2	3	4	5
$f(x)$	0	0.125	0.302	0.480	0.648

✓

$\int_1^5 (\log_e \sqrt{x})^2 dx$

$= \frac{h}{3}\left\{f(x_1) + f(x_5) + 2f(x_3) + 4f(x_2) + 4f(x_4)\right\}$

$= \frac{1}{3}\left\{0 + 0.648 + 2 \times 0.302 + 4 \times 0.125 + 4 \times 0.48\right\}$ ✓

$\doteqdot 1.22$ (as req'd.)

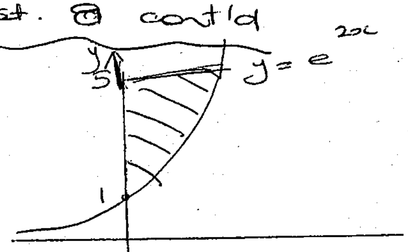
care

eg. $\sqrt{36-9} \neq 5$

perimeter not area

Quest. ⑧ cont'd

c)



(i) $y = e^{2x}$

$\log_e y = \log_e e^{2x}$

$\log_e y = 2x$

$x = \frac{1}{2} \log_e y$

$= \log_e y^{\frac{1}{2}}$

$\underline{x = \log_e \sqrt{y}}$ as req'd ✓

ii) $V = \pi \int_1^5 (\log_e \sqrt{y})^2 \, dy$ ✓

iii) $V = \pi \left[(1.22)\right]$ ✓

$\underline{= 3.8 \text{ u}^3}$ (2 sig fig)

(d) $\frac{d^2y}{dx^2} = 6x - 2$

$\frac{dy}{dx} = 3x^2 - 2x + c$ ✓

$\frac{dy}{dx} = 0, x = 3$: $0 = 3(3)^2 - 2(3) + c$

$= 27 - 6 + c$

$c = -21$

$\therefore \frac{dy}{dx} = 3x^2 - 2x - 21$ ✓

$y = x^3 - x^2 - 21x + k$

$x = 3, y = 0$: $0 = 27 - 9 - 63 + k$

$= -45 + k$ ✓

$k = 45$

$\therefore \underline{y = x^3 - x^2 - 21x + 45}$

more steps required in many solutions

missed connection with this Q and Q b) ii)

many careless errors

+

many did not find last constant

Question 9

(a) i) $f(6) = 0$ ✓

ii) $(0,8)\ (12,0)$

$m = \frac{-8}{12}$

$\therefore f'(4) = -\frac{2}{3}$ ✓

iii) $f'(8) = 0.$ ✓

iv) $f''(0) = 0.$ ✓

b) $P(W) = 0.2 \quad P(\tilde{W}) = 0.8$

$P(\text{Win at least one}) = 1 - P(\text{win none})$ ✓ Done well.

$= 1 - (0.8)^3$

$= 1 - 0.512$

$= 0.488$ ✓

$\frac{61}{125}$

ii) $P(\text{winning at least one in } n \text{ matches}) = 1 - (0.8)^n$

Now $1 - (0.8)^n = 0.9$ ✓

$0.8^n = 0.1$

$\log_e 0.8^n = \log_e 0.1$

Done well.

$n = \frac{\log_e 0.1}{\log_e 0.8}$

$= 10.3\ldots$ ✓

$\therefore$ 11 matches need to be played.

(c) $P = \$250 \quad R = 0.06/\text{pa}$

$n = 18$ years $= 36\ \frac{1}{2}$yrs

$R = 0.03 / 6$ months

Done well.

i) $A_1 = P(1+R)^n$

$= 250(1.03)^{36}$ ✓

$A_1 = \$724.57$

$\therefore$ initial deposit amounts to \$724.57.

Q9 (cont'd)

ii) A_1 amounts to: $25(1.03)^{36}$

A_2 amounts to: $250(1.03)^{34}$

A_3 amounts to: $250(1.03)^{32}$

⋮

A_{16} amounts to: $250(1.03)^6$

A_{17} amounts to: $250(1.03)^4$

A_{18} amounts to: $250(1.03)^2$

- students got r and a incorrect
- Also did not get the powers corre ✓

$$\text{Total amount} = 250\left(\underbrace{1.03^2 + 1.03^4 + \ldots + 1.03^{36}}\right)$$

G.S. $a = 1.03^2$ $r = 1.03^2$, $n = 18$ ✓

$S_n = a(r^n - 1)$

$$= \frac{1.03^2(1.03^{36} - 1)}{1.03^2 - 1}$$

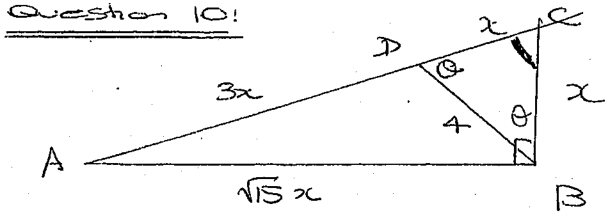
$\text{Total} = 250 \times S_{18}$

$= 250 \times 33.06869\ldots$

$= 8267.173556$ ✓

Total $= \$8267.17$ (nearest ¢)

Question 10:



1) In △DCB.

$$\cos\theta = \frac{x^2 + 4^2 - x^2}{2 \times 4 \times x}$$

$= \frac{16}{8x}$ ✓

$\cos\theta = \frac{2}{x}$ (as req'd)

generally well done

Question 10 (cont'd)

ii) In △ ABC:

$\sin C = \frac{\sqrt{15}x}{4x}$

$= \frac{\sqrt{15}}{4}$ ✓

Now in △BCD

$\frac{\sin\theta}{x} = \frac{\frac{\sqrt{15}}{4}}{4}$

$\sin\theta = \frac{\frac{\sqrt{15}}{4}x}{4}$ ✓

$= \frac{\sqrt{15}x}{16}$

- this preperatory link was often missed.
- otherwise well done

iii) $\sin^2\theta + \cos^2\theta = 1$

$\frac{15x^2}{256} + \frac{4}{x^2} = 1$ ✓

$\frac{15x^4}{256} + 4 = x^2$

$15x^4 + 1024 = 256x^2$ ✓

$15x^4 - 256x^2 + 1024 = 0$
(as req'd)

- asked to form this equation, not solve it! Many students wasted a lot of time here because they didn't READ the quest.

iv) $\sin\angle C = \frac{\sqrt{15}}{4}$

$= 79°31'$

if $x = 2.53$

$\cos\theta = \frac{2}{x}$

$= \frac{2}{2.53}$

$\therefore \theta = 37°46'$

Now in △ BCD

$37°46' \times 2 + 79°31'$ ✓

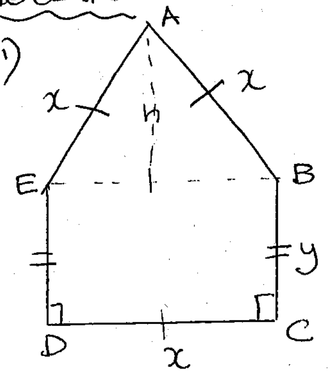
$\neq 180°$.

∴ x cannot equal 2.53.

- not well done - many could not interpret the requirements correctly
- many statements (with no supporting evidence) gained no marks.

Q10 cont'd

(b) i)



Let AB = x cm

BC = y cm.

Perimeter (P) $= 3x + 2y$

$\therefore 2y = P - 3x$

$y = \frac{P - 3x}{2}$ cm

as req'd

demonstrate this ✓

- usually well done

ii) $h^2 + \left(\frac{1}{2}x\right)^2 = x^2$

$h^2 + \frac{1}{4}x^2 = x^2$

$h^2 = \frac{3}{4}x^2$

$h = \frac{\sqrt{3}}{2}x$

- $A_T = \frac{1}{2}x.x.\sin 60$ also used well for Area △.

Area = Area △ + Area rect.

$= \frac{1}{2}bh + xy$

$= \frac{1}{2} \times x \times \frac{\sqrt{3}}{2}x + x\left(\frac{P - 3x}{2}\right)$ ✓

$= \frac{\sqrt{3}x^2}{4} + \frac{Px - 3x^2}{2}$

$= \frac{\sqrt{3}x^2 + 2Px - 6x^2}{4}$ ✓

$= \frac{1}{4}\left[2Px - x^2(6 \mp \sqrt{3})\right]$

$A = \frac{1}{4}\left[2Px - (6-\sqrt{3})x^2\right]$ as req'd

- usually well done

iii) $A' = \frac{1}{2}P - \frac{1}{2}(6-\sqrt{3})x$

$A'' = -\frac{1}{2}(6-\sqrt{3}) < 0 \Rightarrow \max^m$ ✓

for statny values $A' = 0$

ie $\frac{1}{2}P - \frac{1}{2}(6-\sqrt{3})x = 0$ ✓

$\cancel{\tfrac{1}{2}}P = \cancel{\tfrac{1}{2}}(6-\sqrt{3})x$

$x = \frac{P}{6-\sqrt{3}}$

$\therefore \frac{P}{x} = 6 - \sqrt{3}$. ✓

the value of $\frac{P}{x}$ that makes the area a max is $(6-\sqrt{3})$ cm.

$A'' < 0 \Rightarrow$ maxm

- many differentiation issues!
- signs (as always!)
- $0 = \frac{P}{2} - 3x + \frac{\sqrt{3}}{2}x$ often became $\frac{P}{2} = -3x + \frac{\sqrt{3}}{2}x$ resulting in $\frac{P}{x} = \sqrt{3} - 6$!
- Check signs!!!
- many did not demonstrate a max value!

Question 9 (continued)

(b) (i) Sketch the graphs of $y = \cos x$ and $y = \frac{1}{2}\tan x$ from $x = \frac{-\pi}{2}$ to $x = \frac{\pi}{2}$ on the same set of axes. **1**

(ii) By solving the equation $\cos x = \frac{1}{2}\tan x$ find the point of intersection of the two graphs that lies between $x = 0$ and $x = \frac{\pi}{2}$. **2**

Question 10 (12 marks)

(a) Prove that the limiting sum of the series $1 + \sin^2 x + \sin^4 x + \sin^6 x + ...$ is equal to $1 + \tan^2 x$ where $\tan x$ is defined. **3**

(b) Fred and Wilma take out a home loan of $400 000 to be repaid over 20 years at an interest rate of 6% per annum compounding monthly. They repay the loan in instalments of $\$P$ at the end of each month *after* the monthly interest has been calculated.

(i) Show that the amount left to be repaid after 3 months (just after Fred and Wilma have paid their third instalment) is given by **2**

$$\$400\,000 \times 1.005^3 - P(1 + 1.005 + 1.005^2)$$

(ii) Given that the home loan is completely repaid in 20 years find the value of P. **3**

(iii) Fred and Wilma decide to pay off the loan at $4000 per month instead. After how many months will the loan be repaid in this case? **4**