2007 2 unit Maths Trial

/14

1 (a) 4.58 2DP ①

(b) $ax+3ay-x-3y$

$a(x+3y)-1(x+3y)$

$(x+3y)(a-1)$ ①

(c) $\frac{d}{dx}(\ln 5x) = \frac{1}{5x}\times 5 = \frac{1}{x}$ ①

(d) $\frac{2}{(4+\sqrt{3})}\times\frac{(4-\sqrt{3})}{(4-\sqrt{3})} = \frac{2(4-\sqrt{3})}{16-3} = \frac{8-2\sqrt{3}}{13}$ ①

(e) $\left.\begin{array}{l} a+9d=20 \\ \cancel{20}a+\cancel{90}d=12\cancel{0} \end{array}\right\}$ $\begin{array}{r} a+9d=20 \\ 2a+9d=12 \\ \hline -a = 8 \\ a=-8 \end{array}$ −

So $-8+9d=20$ ①

$9d=28$

$d=\frac{28}{9}=3\frac{1}{9}$

(f) $\frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = 1$ ①

(g) $\int\frac{1\,dx}{\sqrt{x^2+a^2}} = \ln(x+\sqrt{x^2+a^2})$

$\int_0^1\frac{1\,dx}{\sqrt{x^2+2^2}} = \Big[\ln(x+\sqrt{x^2+4})\Big]_0^1$

$=\ln(1+\sqrt{5})-\ln(0+2) = \ln\left(\frac{1+\sqrt{5}}{2}\right)$ ②

(h) $\cos x = -\frac{1}{2}$.

quad. 2, 3.

$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$. (1)

$\pi + \frac{\pi}{3} = \frac{4\pi}{3}$ (1)

(i) $\log_x 6 = \log_x (2\times 3)$

$= \log_x 2 + \log_x 3$

$\Rightarrow 0.6 + 0.8$

$= 1.4$ (2)

If $x^{0.6} = 2$ then $\log_x 2 = 0.6$

If $x^{0.8} = 3$ then $\log_x 3 = 0.8$

(j) $3x - 1 = 5$, $3x - 1 = -5$

$3x = 6$, $3x = -4$

$x = 2$ (1), $x = -\frac{4}{3}$. (1)

2 (a) (i) $\frac{d}{dx}(\tan x) = \sec^2 x$ ① $\quad$ 1/12

(ii) $\frac{d}{dx}(x^4+1)^{-2} = -2(x^4+1)^{-3} \times 4x^3$

$= \frac{-8x^3}{(x^4+1)^3}$ ①

(b) $x^2+y^2-8x+6y+9=0$

$\left(-\frac{8}{2}\right)^2$ $\quad$ $x^2-8x+16+y^2+6y+9 = -9+16+9$

$\left(+\frac{6}{2}\right)^2$ $\quad$ $(x-4)^2+(y+3)^2 = 16$

centre $(4,-3)$ ① $\quad r=4$ ①

(c) (i) $y = \frac{k}{(x+2)} = k(x+2)^{-1}$

$\frac{dy}{dx} = -k(x+2)^{-2} \times 1 = \frac{-k}{(x+2)^2}$ ①

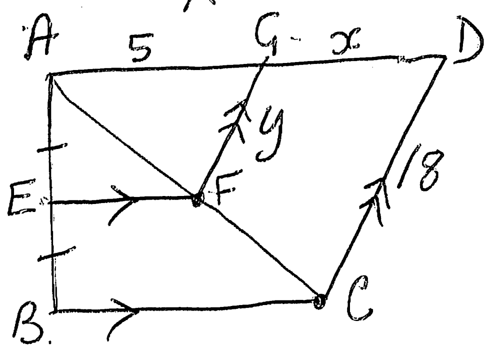
(ii) At $x=2$, $\frac{dy}{dx} = \frac{1}{4}$

$\frac{-k}{16} = \frac{1}{4}$

$-4k = 16$ ①

$k = -4$ ①

(d)



"needs" "reasons"

$x=5$ lines cut off by equal intercepts are equal etc ①

$y=9$ line through the centre of 1 part of a $\triangle$ and $\parallel$ to a 3rd side is half its length etc

2 (e) (i) $-\frac{1}{2}\int -2\sin(2t-1)\,dt$

$= -\frac{1}{2}\cos(2t-1) + c$ (1)

(ii) $\frac{1}{2}\int_0^1 2e^{2x}\,dx$

$= \frac{1}{2}e^{2x}\Big]_0^1 = \frac{1}{2}(e^2-1)$ (1)

(f) $\log_{27}16 = x\log_3 2$

$\frac{\log_{10}16}{\log_{10}27} = \frac{\log_{10}2^x}{\log_{10}3}$

$\log_{10}2^x = \frac{\log_{10}16 \times \log_{10}3}{\log_{10}27}$

$= \frac{4\log_{10}2 \times \cancel{\log_{10}3}}{3\cancel{\log_{10}3}}$

$x\log_{10}2 = \frac{4}{3}\log_{10}2$

$x = \frac{4}{3}$ (2)

Section B

Question 3

(a) i) P has coordinates $(5,5)$

C has coordinates $(5,10)$

$\therefore PC = 5$ units

ii) $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$PE = \dfrac{|4(5) + 3(5) - 10|}{\sqrt{4^2+3^2}}$

$PE = \dfrac{|25|}{5}$

$PE = 5$ units

iii) In Δ's BCP & BEP

PB is common

$PC = PE$ (from (i) and (ii))

$\angle BCP = \angle BEP = 90°$ (given $PE \perp BE$ and $BC \perp PC$)

$\therefore \Delta BCP \equiv \Delta BEP$ (RHS)

iv) B lies on both $y=10$ and $4x+3y-10=0$

clearly $y=10$

when $y=10$

$4x + 3(10) - 10 = 0$

$4x = -20$

$x = -5$

$\therefore B$ has coordinates $(-5, 10)$

v) $BCPE$ is a kite

$BCPE$ is also a cyclic quadrilateral

vi) since $\Delta BCP \equiv \Delta BEP$

area $BCPE = 2 \times$ area ΔBCP

$= \cancel{2} \times \frac{1}{\cancel{2}} \times BC \times CP$

$= 10 \times 5$

$= 50$ units^2

vii) consider a point $Q(x, y)$ that is equidistant from BC and BE.

$$|y - 10| = \frac{|4x + 3y - 10|}{\sqrt{4^2 + 3^2}}$$

$$|y - 10| = \frac{|4x + 3y - 10|}{5}$$

$$5|y - 10| = |4x + 3y - 10|$$

$5y - 50 = 4x + 3y - 10$ or $-5y + 50 = 4x + 3y - 10$

$4x - 2y + 40 = 0$ $\quad$ $4x + 8y - 60 = 0$

$2x - y + 20 = 0$ $\quad$ $x + 2y - 15 = 0$

$\therefore x + 2y - 15 = 0$ is a locus which is equidistant from BC and BE.

(b) $\sum_{n=3}^{8} (2 \times 3^n - 2n) = 2 \times 3^3 - 2(3) + 2 \times 3^4 - 2(4) + 2 \times 3^5 - 2(5) + 2 \times 3^6 - 2(6)$

$+ 2 \times 3^7 - 2(7) + 2 \times 3^8 - 2(8)$

$= 48 + 154 + 476 + 1446 + 4360 + 13106$

$= 19590$

Question 4

(a) i) $\sin 60 = \frac{AP}{8}$

$AP = 8 \sin 60$

$AP = 8\left(\frac{\sqrt{3}}{2}\right)$

$AP = 4\sqrt{3}$ cm

ii) $BQ = AP = 4\sqrt{3}$

$\tan 30 = \frac{4\sqrt{3}}{QC}$

$QC = \frac{4\sqrt{3}}{\tan 30}$

$QC = \frac{4\sqrt{3}}{\left(\frac{1}{\sqrt{3}}\right)}$

$QC = 4\sqrt{3} \times \sqrt{3}$

$QC = 12$ cm

(b) i) $P = \frac{4}{52}$

$= \frac{1}{13}$

ii) $P = \frac{26}{52}$

$= \frac{1}{2}$

red Kings are counted twice

iii) $P = \frac{28}{52}$ OR $P = \frac{4}{52} + \frac{26}{52} - \frac{2}{52}$

$= \frac{7}{13}$

c) let $y = 0$

$3x^2 - 4x + 5 = 0$

$\Delta = b^2 - 4ac$

$\Delta = (-4)^2 - 4(3)(5)$

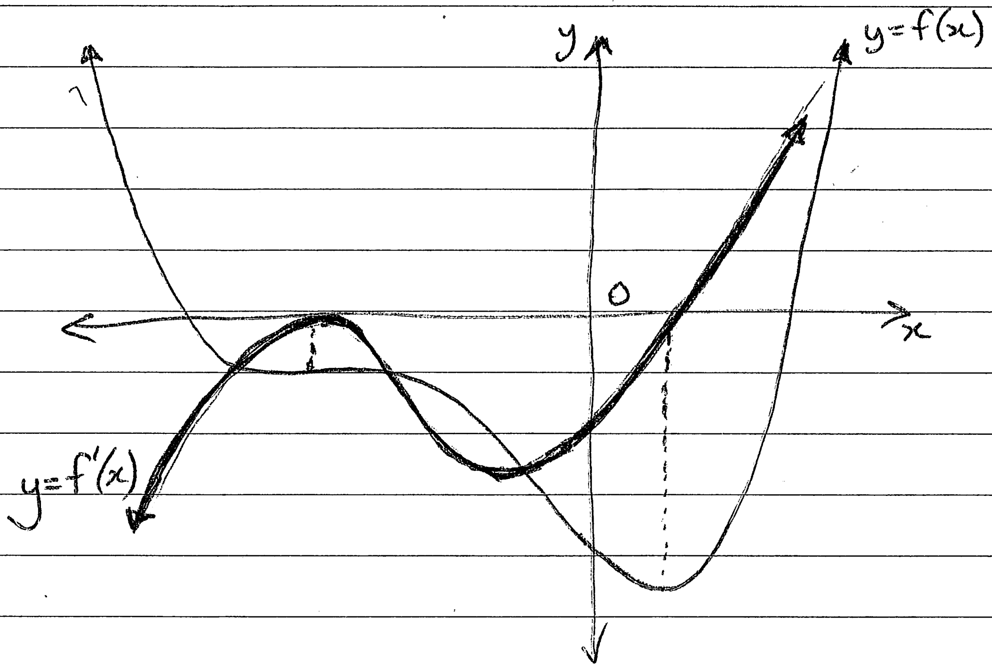
$\Delta = -44$ since $\Delta < 0$ the curve $y = 3x^2 - 4x + 5$ does not cut the x-axis.

(d) $\lim_{x\to 3} \dfrac{x^2-5x+6}{x^2-9} = \lim_{x\to 3} \dfrac{\cancel{(x-3)}(x-2)}{\cancel{(x-3)}(x+3)}$

$= \dfrac{3-2}{3+3}$

$= \dfrac{1}{6}$

(e)



2007 THSC Mathematics: **Solutions— Section C**

5. (a) For the series $486 + 324 + 216 + 144 + \ldots$

(i) Which term is $\dfrac{2048}{243}$? (Working must be shown.) $\boxed{2}$

Solution:

$$\begin{aligned} r &= \tfrac{324}{486} = {}^{2}\!/_{3}, \quad a = 486. \\ u_n = 486 \times ({}^{2}\!/_{3})^{n-1} &= \frac{2048}{243}, \\ ({}^{2}\!/_{3})^{n-1} &= \frac{1024}{59049}, \\ &= ({}^{2}\!/_{3})^{10}. \\ n - 1 &= 10, \\ n &= 11. \end{aligned}$$

$\therefore$ It is the 11^{th} term.

(ii) Does this series have a limiting sum? Give a reason for your answer. $\boxed{1}$

Solution: Yes, $|r| < 1$.

(b) A polygon has 25 sides, the lengths of which form an arithmetic sequence.

(i) Find an expression for the perimeter in terms of the shortest side and the common difference. $\boxed{1}$

Solution:

$$\begin{aligned} \text{Put } a &= \text{shortest side}, \\ d &= \text{common difference}, \\ \text{then perimeter, } P &= \tfrac{25}{2}(2a + 24d), \\ &= 25a + 300d. \end{aligned}$$

(ii) The perimeter of the polygon is 1100 cm and the longest side is 10 times the length of the shortest side. Find the length of the shortest side of the polygon and the common difference of the sequence. $\boxed{3}$

Solution:

$$\begin{aligned} 10a &= a + 24d, \\ 9a &= 24d, \\ a &= \frac{8d}{3} \quad \boxed{1} \\ 1100 &= 25a + 300d \quad \boxed{2} \\ \text{Sub } \boxed{1} \text{ in } \boxed{2}: \quad 1100 &= \frac{200d}{3} + 300d, \\ 3300 &= 1100d, \\ d &= 3, \\ a &= 8. \end{aligned}$$

$\therefore$ The shortest side is 8 cm and the common difference is 3 cm.

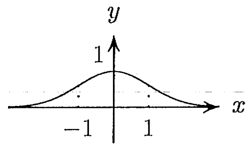
(c) For the function $y = e^{-\frac{x^2}{2}}$,

(i) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. $\boxed{2}$

Solution:
$$\begin{aligned}\frac{dy}{dx} &= -xe^{-x^2/2}.\\ \frac{d^2y}{dx^2} &= -e^{-x^2/2} - x\left(-xe^{-x^2/2}\right),\\ &= e^{-x^2/2}\left(x^2-1\right).\end{aligned}$$

(ii) For what values of x is the curve of $y = e^{-\frac{x^2}{2}}$ concave down? $\boxed{1}$

Solution:



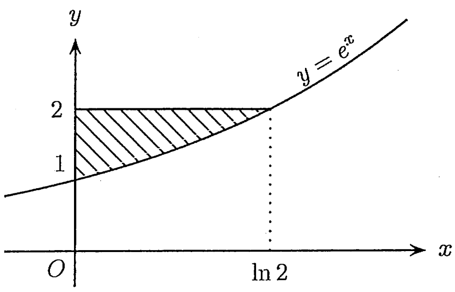
$-1 < x < 1$

(d) (i) Find the point of intersection of $y = 2$ and $y = e^x$. $\boxed{1}$

Solution: Equating ys, $e^x = 2 \quad \Rightarrow \quad x = \ln 2$.
$\therefore$ Intersection is at $(\ln 2,\, 2)$.

(ii) Indicate, by shading on a diagram, the region in the first quadrant bounded by the y-axis, the line $y = 2$, and the curve $y = e^x$. $\boxed{1}$

Solution:

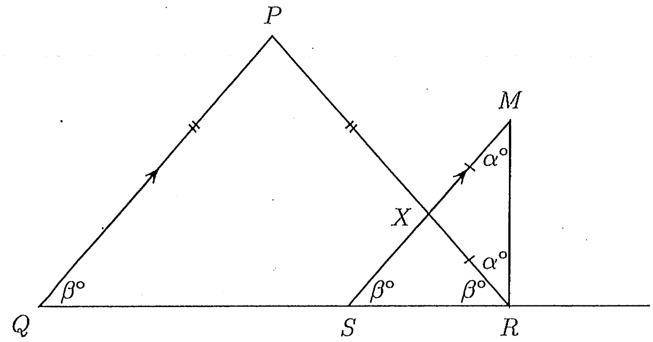


(iii) Use Simpson's Rule with 3 function values, to calculate the volume of the solid generated when the region in part (ii) is rotated about the y-axis (answer to 2 d.p.) $\boxed{2}$

Solution: Volume
$$\begin{aligned} &= \pi\int_1^2 x^2\,dy,\\ &= \pi\int_1^2 (\ln y)^2\,dy,\\ &\approx \pi\times\tfrac{1}{6}(0\times 1 + 0.1644\times 4 + 0.4805\times 1),\\ &\approx 0.60 \text{ (2 dec. pl.)}\end{aligned}$$

y	1	1.5	2
$(\ln y)^2$	0	0.1644	0.4805

6. (a) $\boxed{3}$

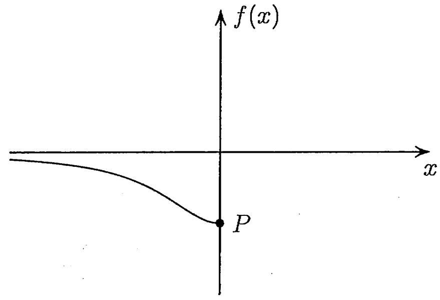


In the diagram $PQ = PR$, $XM = XR$ and $PQ \parallel MS$. By letting $\angle MRX = \alpha°$, show that $MR \perp QR$, giving full reasons.

Solution:

$$\begin{aligned} \Delta XMR &\text{ is isosceles} && (XM = XR) \\ X\widehat{M}R &= M\widehat{R}X = \alpha° && (\text{base } \angle\text{s of isosceles } \Delta XMR) \\ \Delta PQR &\text{ is isosceles} && (PQ = PR) \\ P\widehat{Q}R &= Q\widehat{R}P = \beta° && (\text{base } \angle\text{s of isosceles } \Delta PQR) \\ M\widehat{S}R &= P\widehat{Q}R = \beta° && (\text{corresponding } \angle\text{s, } PQ \parallel MS) \\ 180° &= 2\alpha° + 2\beta° && (\angle \text{ sum of } \Delta MSR) \\ \therefore\ \alpha° + \beta° &= 90°, \\ i.e.\ M\widehat{R}S &= 90°. \\ \therefore\ MR &\perp QR. \end{aligned}$$

(b) The diagram below shows part of the function $f(x) = \dfrac{-2}{1+x^2}$.



(i) Find the coördinates of point P on the diagram. $\boxed{1}$

Solution: $f(0) = -2$.
$\therefore\ P(0, -2)$.

(ii) Prove that $f(x)$ is an even function. $\boxed{1}$

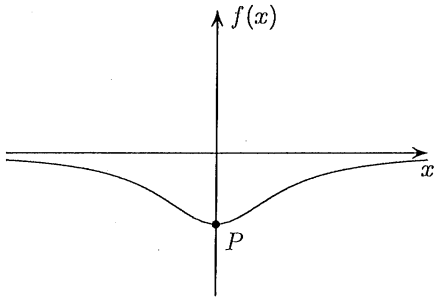
Solution:

$$\begin{aligned} f(-x) &= \frac{-2}{1+(-x)^2}, \\ &= \frac{-2}{1+x^2}, \\ &= f(x). \end{aligned}$$

$\therefore\ f(x)$ is even.

(iii) Copy the diagram into your answer booklet and complete the curve. $\boxed{1}$

Solution:



(iv) State the range of this function. $\boxed{1}$

Solution: $-2 \leq f(x) < 0$.

(c) The volume V litres, of a tank after t minutes is given by $V = 60 + 4t - t^2$.

(i) At what time(s) will the tank be empty? $\boxed{1}$

Solution:
$$\begin{aligned} V &= 60 + 4t - t^2, \\ &= 60 + 10t - 6t - t^2, \\ &= 10(6+t) - t(6+t), \\ &= (10-t)(6+t), \\ &= 0 \text{ when } t = -6,\ 10. \end{aligned}$$

P	-60
S	4
F	10, -6

$\therefore$ The tank *will* be empty after 10 minutes (and was empty 6 minutes before observation started, although this is not required by the question).

(ii) Find an expression for the rate at which the volume is changing. $\boxed{1}$

Solution: $\dfrac{dV}{dt} = 4 - 2t$.

(iii) Find the maximum volume in the tank. $\boxed{1}$

Solution:
$$\begin{aligned} \frac{dV}{dt} &= 0 \text{ when } t = 2. \\ \frac{d^2V}{dt^2} &= -2. \end{aligned}$$
$$\begin{aligned} \therefore \text{Max. volume (when } t = 2) &= 60 + 8 - 4, \\ &= 64 \text{ litres.} \end{aligned}$$

(iv) Find the rate when $t = 4$ and comment on your answer. $\boxed{2}$

Solution: When $t = 4$, $\frac{dV}{dt} = 4 - 8 = -4$.
This means the tank is emptying at $4\,\text{L/min}$.

Question 7

(a) $P = 5000e^{kt}$

(i) $\frac{dP}{dt} = 5000ke^{kt}$

$= k\left[5000e^{kt}\right]$

$\frac{dP}{dt} = kP$ 1

(ii)

$\left.\begin{array}{l} t=1 \\ P=6500 \end{array}\right\}$ $P = 5000e^{kt}$

$6500 = 5000e^{k}$

$k = \ln\left(\frac{13}{10}\right)$ 1

$k \doteqdot 0.26$

(iii) $P = 5000e^{0.26t}$

$15000 = 5000e^{0.26t}$

$3 = e^{0.26t}$

$t = \frac{1}{0.26}\ln 3$ 1

$\Rightarrow t \doteqdot 4.23$

During the 5th day

(b)

(i)

$\sin 2x = 2\cos x$

$2\sin x\cos x = 2\cos x$

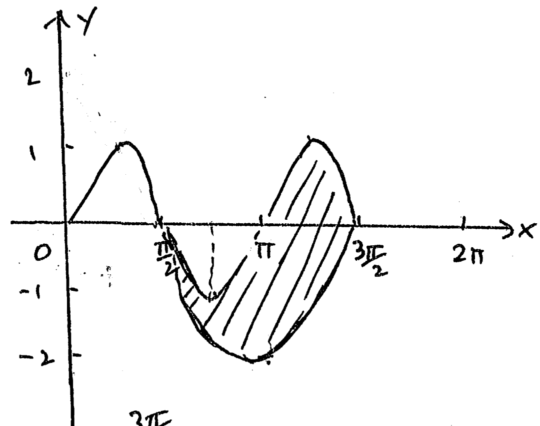
$2\cos x\left[\sin x - 1\right] = 0$ 2

$\cos x = 0$ or $\sin x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$, $x = \frac{\pi}{2}$

$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}$

(ii)



Area $= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left[\sin 2x - 2\cos x\right] dx$ 3

$= \left[-\frac{1}{2}\cos 2x - 2\sin x\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$

$= \left[\left(-\frac{1}{2}\cos 3\pi - 2\sin\frac{3\pi}{2}\right) - \left(-\frac{1}{2}\cos\pi - 2\sin\frac{\pi}{2}\right)\right]$

$= \left[-\frac{1}{2}(-1) - 2(-1)\right] + \frac{1}{2}(-1) + 2$

$= 4$ units^2

Question 8

(a) $$y = x^3(2-x)$$

(i) $$y = 2x^3 - x^4$$ 3

$$\frac{dy}{dx} = 6x^2 - 4x^3 = 0$$

when $x = 0$

or $x = 3/2$

$[0,0]$ $[3/2, 27/16]$

(ii) $$\frac{d^2y}{dx^2} = 12x - 12x^2$$

When $x = 0$, $y'' = 0$ 3

$\Rightarrow$ possible P.O.I.

x	<	0	>
y''	−	0	+

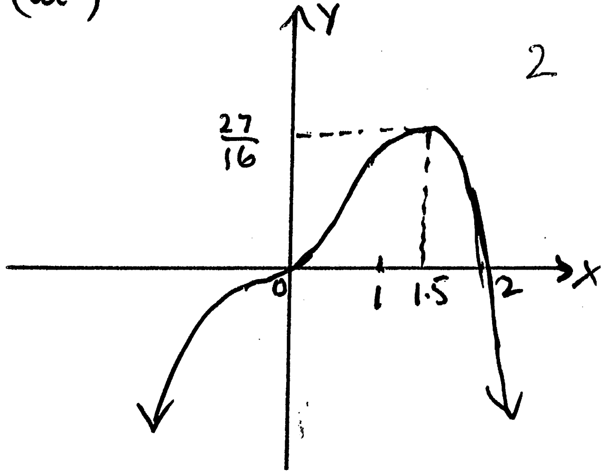
Change of concavity

$\Rightarrow$ Horiz. P.O.I at $(0,0)$

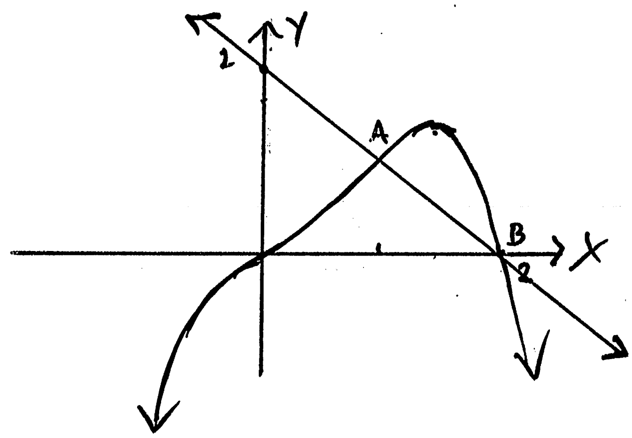
When $x = \frac{3}{2}$, $y'' = -9 < 0$

$\Rightarrow$ Max T.P. at $(\frac{3}{2}, \frac{27}{16})$

(iii) 2



(b)



(i) Solve simultaneously (or simply substitute pts) 2

$\Rightarrow 2 - x = x^3(2-x)$

$(2-x)[1 - x^3] = 0$

$\therefore \begin{pmatrix} x = 2 \\ y = 0 \end{pmatrix}$ or $\begin{pmatrix} x = 1 \\ y = 1 \end{pmatrix}$

(ii) grad. tangent at $A(1,1)$

$$\frac{dy}{dx} = 6x^2 - 4x^3$$ 2

At $x = 1$, grad. $= 2$

$\tan \alpha = 2 \Rightarrow \alpha = 63^\circ$

grad. tangent at $B(2,0)$

$$\frac{dy}{dx} = 6x^2 - 4x^3$$

At $x = 2$, grad $= -8$

$\tan\theta = -8 \Rightarrow \theta = 97^\circ$

(iii) $63^\circ + A\hat{C}B = 97^\circ$ 1

$A\hat{C}B = 34^\circ$

Q9. (a) $x_A = 12t + 5$ $\qquad$ $x_B = 6t^2 - t^3$

$\dot{x}_A = 12$ $\qquad$ $\dot{x}_B = 12t - 3t^2$

(i) when $t = 1$ $\qquad$ $\dot{x}_A = 12$ & $\dot{x}_B = 12 - 3 = 9$

$\therefore$ x_A ✓✓

(ii) let $12 = 12t - 3t^2$

$3t^2 - 12t + 12 = 0$

$3(t^2 - 4t + 4) = 0$

$3(t-2)^2 = 0$

$t = 2$. $\qquad$ $\therefore$ after 2 seconds ✓✓

(iii) $\ddot{x}_B = 12 - 6t$

$\therefore$ when $t = 3$

$\ddot{x}_B = 12 - 18$

$= -6$. $\qquad$ $\therefore$ $-6\ m\,s^{-2}$ ✓

(iv) let $\dot{x}_B = 0$ ie $12t - 3t^2 = 0$

$3t(4 - t) = 0$

$\therefore t = 0, 4$.

$\therefore$ after 4 secs. $x_B = 6 \times 16 - 64$

$= 96 - 64$

$= 32\ m$

✓✓

9 (CONT'D)

(b) (i) $AB = 240\theta.$

$DC = 170\theta.$

(ii) let $70 + 170\theta + 70 < 240\theta.$

$140 < 70\theta.$

$\theta \cancel{>} \frac{140}{70}$

$\boxed{\theta \cancel{>} 2.}$

$\therefore \boxed{2 < \theta \leq \pi.}$

Q10. (a)(i) $f(x) = \sqrt{3}x - 2\sin x \quad , 0 \le x \le \pi.$

$f'(x) = \sqrt{3} - 2\cos x.$

$f''(x) = 2\sin x.$

now for max/min $f'(x) = \sqrt{3} - 2\cos x = 0$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}.$

$\therefore y = \frac{\sqrt{3}.\pi}{6} - 2\sin\frac{\pi}{6}.$

$= \frac{\sqrt{3}\pi}{6} - 2 \times \frac{1}{2}.$

$= \frac{\sqrt{3}\pi}{6} - 1.$

Test $f''(\frac{\pi}{6}) = 2 \times \sin\frac{\pi}{6}$

$= 2 \times \frac{1}{2}$

$= 1.$ ✓✓

$\therefore$ $\frac{\sqrt{3}\pi}{6} - 1$ is a local MIN.

(ii). Consider $f(0) = \sqrt{3}.0 - 2\sin 0$

$= 0.$ ✓

& $f(\pi) = \sqrt{3}\pi - 2\sin\pi.$

$= \sqrt{3}\pi - 0$

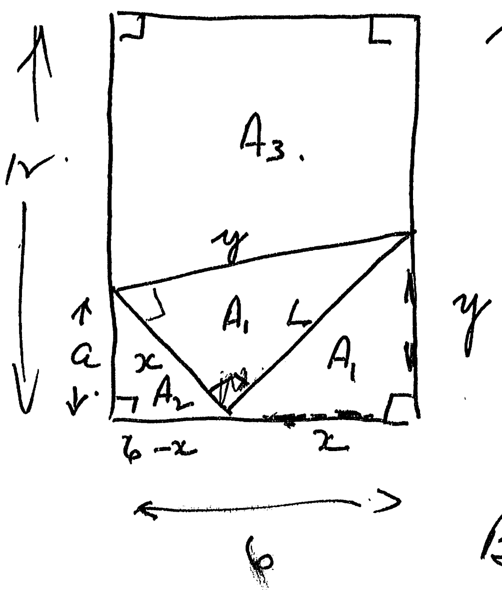
$= \sqrt{3}\pi$ ✓

$\therefore$ GREATEST VALUE is $\boxed{\sqrt{3}\pi.}$ ✓

& LEAST VALUE is $\boxed{\frac{\sqrt{3}\pi}{6} - 1.}$ (NB $\approx 0.093.$)

Q10. (b) (i) $L = \sqrt{x^2 + y^2}$ ✓

(ii)



Using Pythagoras

$$a^2 = x^2 - (6-x)^2$$
$$= x^2 - (36 - 12x + x^2)$$
$$a^2 = 12x - 36.$$
$$a = \sqrt{12x - 36}.$$
$$= 2\sqrt{3x - 9}.$$

By considering areas.

$$2A_1 + A_2 + A_3 = 72.$$

where $A_1 = \frac{xy}{2}$

$$A_2 = \frac{2\sqrt{3x-9}\,(6-x)}{2}$$
$$= (6-x)\sqrt{3x-9}$$

& $A_3 = \frac{1}{2} \times 6 \times \left[(12-y) + 12 - 2\sqrt{3x-9}\right]$

$$\therefore xy + (6-x)\sqrt{3x-9} + 3\left[(12-y) + 12 - 2\sqrt{3x-9}\right] = 72$$

$$xy + 6\sqrt{3x-9} - x\sqrt{3x-9} + 3\left[24 - y - 2\sqrt{3x-9}\right] = 72$$

$$\therefore xy + 6\sqrt{3x-9} - x\sqrt{3x-9} + 72 - 3y - 6\sqrt{3x-9} = 72.$$ ✓✓✓

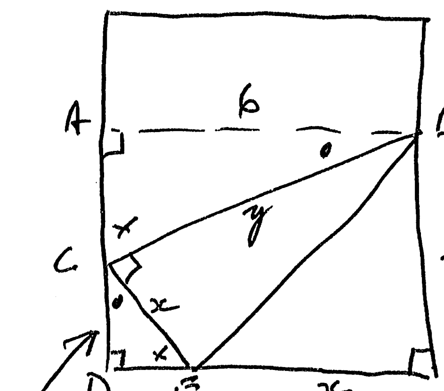
$$\therefore xy - x\sqrt{3x-9} + 72 - 3y = 72.$$

$$y(x-3) = x\sqrt{3x-9}$$

$$y = \frac{x\sqrt{3}\sqrt{(x-3)}}{x-3}$$

$$\therefore y = \frac{\sqrt{3}.\,x}{\sqrt{x-3}}.$$

OR Using Pythagoras



$a = 2\sqrt{3x-9}$

$$y^2 = 36 + \left(y - 2\sqrt{3x-9}\right)^2$$

$$y^2 = 36 + y^2 - 4y\sqrt{3x-9} + 4(3x-9)$$

$$y^2 = 36 + y^2 - 4y\sqrt{3x-9} + 12x - 36.$$

$$4y\sqrt{3x-9} = 12x.$$

$$y\sqrt{3x-9} = 3x.$$

$$y = \frac{3x}{\sqrt{3x-9}}$$

$$= \frac{3x}{\sqrt{3}\sqrt{x-3}}$$

$$\therefore y = \frac{\sqrt{3}x}{\sqrt{x-3}}$$

OR Establish that $\triangle ABC \parallel\!| \triangle DCE$.

then. $$\frac{y}{6} = \frac{x}{2\sqrt{3x-9}}$$

$$y = \frac{3x}{\sqrt{3}\sqrt{x-3}}$$

$$y = \frac{x\sqrt{3}}{\sqrt{x-3}}.$$

Q10 (CONTD)

(iii) $$L^2 = x^2 + y^2$$

$$= x^2 + \left(\frac{\sqrt{3}x}{\sqrt{x-3}}\right)^2$$

$$= x^2 + \frac{3x^2}{x-3}$$

$$= \frac{x^2(x-3)+3x^2}{x-3}$$

$$= \frac{x^3-3x^2+3x^2}{x-3}$$

$$= \frac{x^3}{x-3}.$$

now L will be a min. when L^2 is a min.

$$\frac{d}{dx}(L^2) = \frac{(x-3)3x^2 - x^3}{(x-3)^2}$$

$$= \frac{3x^3-9x^2-x^3}{(x-3)^2}$$

$$= \frac{2x^3-9x^2}{(x-3)^2}$$

Let $\frac{d}{dx}(L^2)=0$ $\therefore$ $2x^3-9x^2=0$

$$x^2(2x-9)=0$$

$$x=0, \frac{9}{2}$$

clearly $x \neq 0$ $\therefore$ at $x=\frac{9}{2}$ $L^2 = \frac{\left(\frac{9}{2}\right)^3}{9/2-3}$

$$= \frac{9^3}{3/2 \times 8}$$

$$= \frac{3 \times 9^2}{4}$$

$$\therefore L = \frac{9\sqrt{3}}{2} \quad (\sim 7.79)$$

Test

x	4	$4\frac{1}{2}$	5
$\frac{d}{dx}L^2$	-16	0	$25/4$
	\	—	/

$\therefore$ $L = \frac{9\sqrt{3}}{2}$ is a <u>MINIMUM.</u>

✓✓✓