THE KING'S SCHOOL

2003
Higher School Certificate
Trial Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1-7
- All questions are of equal value

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Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) $P(5,7)$ divides the interval AB externally in the ratio $m:n$

If $A=(-1,-5)$ and $B=(0,-3)$, find $m:n$. **2**

(b) Find the derivative of $\tan^{-1}\left(1+x^2\right)$ **2**

(c) In how many ways can 8 rowers be divided into two groups of 5 and 3 rowers? **2**

(d) Find the acute angle between the lines $y=2x+1$ and $y=7-3x$. **2**

(e) $x+1$ is a factor of the polynomial

$$P(x)=x^{2n+1}-x^{2n}+b, \quad n \text{ a positive integer}$$

Find the value of b. **2**

(f) Evaluate $\displaystyle\sum_{n=1}^{9}\left(\frac{1}{n}-\frac{1}{n+1}\right)$ **2**

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Show that the function $f(x)=5x-\sin 4x-12$

increases for all values of x **3**

(b) (i) Find $R>0$ and α, $0<\alpha<\dfrac{\pi}{2}$, so that

$$R\sin(x-\alpha)=\sin x-\sqrt{3}\cos x$$ **2**

(ii) Solve $\sin x-\sqrt{3}\cos x=\sqrt{2}$, $0<x<2\pi$, exactly **2**

(c) (i) Use the substitution $u=4-x^2$ to evaluate

$$\int_0^{\sqrt{3}}\frac{x}{\sqrt{4-x^2}}\,dx$$ **3**

(ii) Evaluate $\displaystyle\int_0^{\sqrt{3}}\frac{4-x}{\sqrt{4-x^2}}\,dx$ **2**

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) The function $f(x) = e^{x^2} - x - 3$ has a zero near $x = 1.2$.

Use Newton's Method once to find a two decimal place approximation to this zero. **3**

(b) (i) Write $\sin 2A$ in terms of t, where $t = \tan A$. **1**

(ii) Prove the identity $\operatorname{cosec} 2A - 3\cot 2A = 2\tan A - \cot A$ **3**

(c) A, B, C are three points in a horizontal plane and M is the mid-point of AB. CD is at right-angles to the horizontal plane ABC.
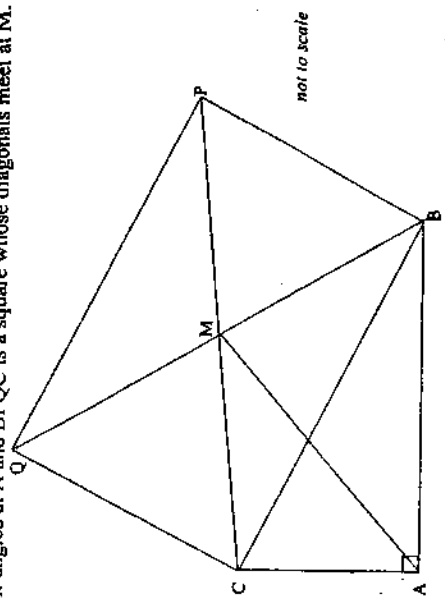
The length of AB = 20 metres.
The angles of elevation from A, M, B to D are 30°, 45°, 30°, respectively.
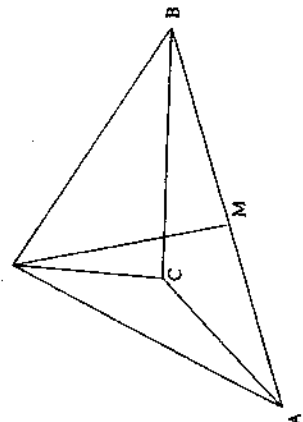
(i) Copy the diagram into your booklet and include on it the information given. **1**

(ii) If $CD = x$, show that $AC = \sqrt{3}x$. **1**

(iii) Find the exact value of x. **3**

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) $\triangle ABC$ is right-angled at A and BPQC is a square whose diagonals meet at M.

not to scale

(i) Copy the diagram into your booklet.

(ii) Prove that ABMC is a cyclic quadrilateral. **2**

(iii) Prove that MA bisects $\angle BAC$ **2**

(b) A particle is moving in simple harmonic motion on the x-axis. Its displacement, x metres, at any time t seconds, where $t \geq 0$, is given by $x = 10\cos nt$, n a constant.

(i) Show that the particle is initially at rest at $x = 10$. **2**

(ii) The period of the motion is T seconds and after $\dfrac{T}{3}$ seconds the particle is at position $x = b$.

Find the value of b. **2**

(iii) The speed at $x = b$ is $20\sqrt{3}$ m/s. Find the period of the motion. **2**

(c) Simplify $\dbinom{n}{3} + \dbinom{n-1}{2}$, $n \geq 3$ **2**

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) $P\left(2ap, ap^2\right)$, $Q\left(2aq, aq^2\right)$ are the end points of a focal chord of the parabola $x^2 = 4ay$.

(i) Show that the equation of the chord PQ is

$$y - \frac{1}{2}(p+q)x + apq = 0$$ [2]

(ii) Deduce that $q = -\dfrac{1}{p}$ [1]

(iii) Show that $PQ = 2a + a\left(p^2 + \dfrac{1}{p^2}\right)$ [2]

(iv) A circle is drawn with PQ as its diameter.

Prove that the directrix is a tangent to this circle. [3]

(b) A cube is expanding in such a manner that it maintains its cubic shape. Initially, each edge is 10cm and the surface area is expanding at a constant rate of 12.6 cm²/s.

(i) Find an expression in terms of t for the surface area of the cube after t seconds. [1]

(ii) Hence, or otherwise, find the rate at which the volume of the cube is increasing after 10 seconds. [3]

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) Prove by mathematical induction for $n \geq 0$ that $E(n) = 9^{n+2} - 4^n$ is a multiple of 5. [3]

(b) (i) With the aid of a diagram, or otherwise,

solve $\left(x^2 - 1\right)\left(x^2 - 4\right) \leq 0$ [2]

(ii) A particle is moving along the x-axis with its acceleration at position x given by

$$\ddot{x} = 10x - 4x^3$$

When $x = \sqrt{2}$ its velocity $v = 2$

(α) Prove that the expression $\dfrac{1}{2}v^2 + x^4 - 5x^2$ is a constant for the motion and find this constant.

(v is its velocity at position x) [2]

(β) Describe the motion. [2]

(c) A random sample of 10 people is made. Assuming that either sex is equally likely, find the probability that

(i) there is an equal number of each sex [1]

(ii) there are more females [2]

Give your answers correct to three decimal places.

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) Solve $\dfrac{-2x}{x+1} > 0$

2

(b) Consider the curve $y = \ln\left(\dfrac{-2x}{x+1}\right)$

(i) Use (a) to explain why

$$\ln\left(\dfrac{-2x}{x+1}\right) = \ln(-2x) - \ln(x+1)$$

1

(ii) Show that the curve has no stationary points.

2

(iii) Sketch the curve, showing the x-intercept.

2

(iv) Find the inverse function of $y = \ln\left(\dfrac{-2x}{x+1}\right)$, expressing your answer with y as subject.

3

(v) Find the area of the region bounded by $y = \ln\left(\dfrac{-2x}{x+1}\right)$ and the y-axis and the lines $y = 0$ and $y = 2$

2

End of Paper