Question 1

(a) Find the acute angle between the lines $y = x$ and $\sqrt{3}y = x$ (2 marks)

(b) Find the indefinite integral of $\int \dfrac{dx}{\sqrt{1-9x^2}}$ (1 mark)

(c) If α, β and λ are the roots of the cubic equation $2x^3 + x^2 - x - 2 = 0$ find the value of

(i) $\alpha + \beta + \lambda$

(ii) $\alpha\beta\lambda$

(iii) $\alpha\beta + \alpha\lambda + \beta\lambda$

(iv) Hence, or otherwise, find the value of $(\alpha - 1)(\beta - 1)(\lambda - 1)$ (4 marks)

(d) Given $x = 12t$ and $y = 6t^2$ write down

(i) the cartesian equation of the parabola

(ii) the coordinates of the focus

(iii) the equation of the directrix (3 marks)

Question 2 (Start a new page)

(a) (i) Solve the equation $\sin\theta - \sqrt{3}\cos\theta = 1$ for $0 \le \theta \le 2\pi$ (3 marks)

(ii) Solve $\sin 2x = \sin x$ for $0 \le x \le \pi$

(iii) On the same number plane, sketch $y = \sin 2x$ and $y = \sin x$ for $0 \le \theta \le \pi$ showing all important features.

(iii) Hence, or otherwise, find the area bounded by the curves $y = \sin 2x$ and $y = \sin x$ for $0 \le \theta \le \dfrac{\pi}{3}$ (7 marks)

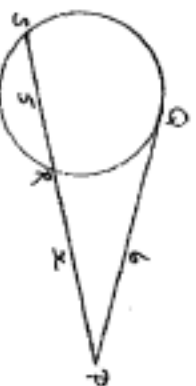**Question 3**   (Start a new page)

-5-

(a)

DIAGRAM
NOT TO SCALE

PQ is a tangent to the circle QRS.
PRS is a secant intersecting the circle in R and S.
Given that $PQ = 6$, $RS = 5$ and $PR = x$, find x, giving reasons.

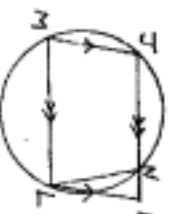*tip of tangents & product of whole segments* (3 marks)

(b)

DIAGRAM
NOT TO SCALE

The circle passes through the points J, N, L and M.
$JKLM$ is a parallelogram.
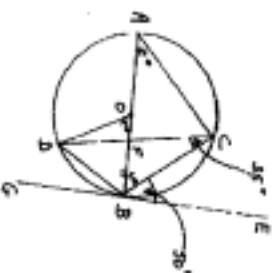Prove that $NL = LK$, giving reasons.

(3 marks)

(c)

DIAGRAM
NOT TO SCALE

O is the centre of the circle. AB is a diameter.
BG is a tangent at B. If $\angle CBE = 50°$ and $\angle DCB = 35°$, find the
values of x, y and z, giving reasons.

(4 marks)

Question 4   (Start a new page)

-6-

(a) Find the general solution for the equation $\tan\theta = \dfrac{1}{\sqrt{2}}$

(1 mark)

(b) For the function $f(x) = \sqrt{x} + 3$ find

(i) the inverse function

(ii) the domain of the inverse function.

(3 marks)

(c) (i) Differentiate $y = \tan^{-1}\dfrac{1}{x}$, $x \neq 0$

(ii) Hence show that

$$\frac{d}{dx}\left[\tan^{-1} x + \tan^{-1}\frac{1}{x}\right] = 0$$

(iii) Then sketch the curve $y = \tan^{-1} x + \tan^{-1}\dfrac{1}{x}$

(5 marks)

Question 5 (Start a new page)

(a) The function $f(x) = x^3 - x^2 - x - 1$ has a zero between 1 and 2

  (i) Taking $x = 2$ as a first approximation to this zero, use Newton's method to calculate a second approximation.

  (ii) Would $x = 1$ have been a suitable first approximation to use? Explain your answer fully.

(4 marks)

(b) The function $f(x) = ax^3 + bx^2 + cx + d$ has a double zero at $x = 1$ and a minimum value of -4 when $x = -1$.

Find the values of a, b, c and d.

(6 marks)

Question 6 (Start a new page)

Consider the parabola $x^2 = 4ay$

(i) Find the equation of the tangent at $P(2ap, ap^2)$

(ii) If the tangent at P cuts the y-axis at T, show that $T = (0, -ap^2)$

(iii) Find the equation of the normal at P.

(iv) If the normal at P cuts the y-axis at N, show that $N = (0, 2a + ap^2)$

(v) Explain why NT is the diameter of the circle passing through PTN. Hence find the equation of the circle.

(vi) If the tangent at P cuts the x-axis at R and M is the midpoint of RN, show that $R = (ap, 0)$ and find the co-ordinates of M.

(vii) Determine the equation of the locus of M and describe this locus geometrically.

(10 marks)

END OF PAPER