Marking Guidelines Mathematics Extension 1 CSSA Trial HSC 2006

Question 1

a. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• writes primitive function	1
• evaluates by substitution of limits	1

Answer

$$\int_0^{\frac{\pi}{6}} \sec 2x \tan 2x \quad dx = \tfrac{1}{2}\left[\sec 2x\right]_0^{\frac{\pi}{6}} = \tfrac{1}{2}\left(\sec\tfrac{\pi}{3} - \sec 0\right) = \tfrac{1}{2}(2-1) = \tfrac{1}{2}$$

b. Outcomes assessed : P4

Marking Guidelines

Criteria	Marks
• substitutes gradients into expression for $\tan\theta$	1
• calculates θ to required accuracy	1

Answer

$3x - y - 2 = 0$

$y = 3x - 2$

Gradient is 3

$x + 2y - 3 = 0$

$y = -\tfrac{1}{2}x + \tfrac{3}{2}$

Gradient is $-\tfrac{1}{2}$

Acute angle θ between the lines is given by

$$\tan\theta = \left|\frac{3 - \left(-\frac{1}{2}\right)}{1 + 3\left(-\frac{1}{2}\right)}\right| = 7.$$

$\therefore \theta \approx 82°$ (to the nearest degree)

c. Outcomes assessed : PE3, P4

Marking Guidelines

Criteria	Marks
i • shows $P(1) = 0$ by substitution	1
ii • deduces that equation $P(x) = 0$ has 3 real roots provided $x^2 + kx + 1 = 0$ has real roots.	1
• finds discriminant of this quadratic in terms of k and realizes $\Delta \geq 0$ for real roots	1
• states values of k	1

Answer

i. $P(1) = 1 + (k-1) + (1-k) - 1 = 0$.

ii. Equation $P(x) = 0$ has 3 real roots if equation $x^2 + kx + 1 = 0$ has two real roots.

For this quadratic equation, $\Delta = k^2 - 4 \geq 0$ for $k^2 \geq 4$.

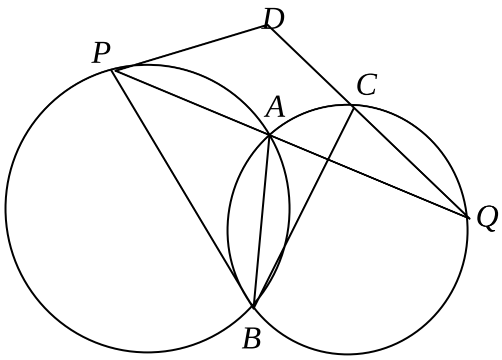
Hence $P(x) = 0$ has 3 real roots for $k \leq -2$ or $k \geq 2$.

d. Outcomes assessed : PE2, PE3

Marking Guidelines

Criteria	Marks
ii • quotes the alternate segment theorem in circle *APB*	1
iii • quotes theorem about angles standing on the same arc (or chord) in circle *AQB*	1
iv • writes a sequence of deductions leading to a test for *BCDP* to be cyclic	1
• supports these deductions with reasons	1

Answer



ii. In circle *APB*, angle between tangent *DP* and chord *PA* is equal to the angle subtended by *PA* in the alternate segment at *B*.
Hence $\angle DPA = \angle PBA$.

iii. In circle *AQB*, angles subtended by the same arc *CA* at points *B* and *Q* on the circumference are equal.
Hence $\angle CQA = \angle CBA$.

iv. $\angle QDP + \angle DPQ + \angle DQP = 180°$ (*Angle sum of* ΔQPD *is* $180°$)
But $\angle QDP = \angle CDP$, $\angle DPQ = \angle DPA$, $\angle DQP = \angle CQA$ (*Q, C, D collinear*; *P, A, Q collinear*)
Hence $\angle CDP + \angle DPA + \angle CQA = 180°$.
$\therefore \angle CDP + \angle PBA + \angle CBA = 180°$ ($\angle DPA = \angle PBA$, $\angle CQA = \angle CBA$ *shown above*)
But $\angle PBA + \angle CBA = \angle PBC$ (*by addition of adjacent angles*)
$\therefore \angle CDP + \angle PBC = 180°$
Hence *BCDP* is a cyclic quadrilateral (*one pair of opposite angles supplementary*)

Question 2

a. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• uses the equivalence of expressions 3^x and $e^{x\ln 3}$	1
• derives the equivalent exponential function with base *e*.	1

Answer

$$3^x = e^{\ln 3^x} = e^{x\ln 3}\ . \quad \text{Hence} \quad \frac{d}{dx}3^x = \frac{d}{dx}e^{x\ln 3} = \ln 3\, e^{x\ln 3} = 3^x \ln 3$$

b. Outcomes assessed : P4

Marking Guidelines

Criteria	Marks
• applies an appropriate process to determine the coordinates	1
• calculates both coordinates correctly	1

Answer

$A(-3\,,7)$ $\qquad\qquad$ $B(4\,,-2)$

$3 \quad : \quad 2$

$$P\left(\frac{3\times 4\ +\ 2\times(-3)}{3+2}\ ,\ \frac{3\times(-2)\ +2\times 7}{3+2}\right)$$

Hence the point of internal division is $P\left(\frac{6}{5}\,,\frac{8}{5}\right)$.

c. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• uses double angle identities for sine and cosine	1
• rearranges and factorises resulting equation	1
• solves $\cos x = 0$ in required domain	1
• solves $\tan x = 1$ in required domain	1

Answer

$$1+\cos 2x = \sin 2x \quad , \quad 0 \le x \le 2p$$

$$2\cos^2 x = 2\sin x\cos x$$

$$\cos x\left(\cos x - \sin x\right) = 0$$

$\therefore \cos x = 0 \quad or \quad \cos x = \sin x$

$\qquad\qquad\qquad\qquad\qquad 1 = \tan x$

$\therefore\ x = \frac{p}{2}, \frac{3p}{2} \quad or \quad x = \frac{p}{4}, \frac{5p}{4}$

$\therefore\ x = \frac{p}{4}\,, \frac{p}{2}\,, \frac{5p}{4}\,, \frac{3p}{2}$

d. Outcomes assessed : P4, PE3

Marking Guidelines

Criteria	Marks
i • finds gradients of OP and OQ and sets product equal to -1	1
ii • uses appropriate rectangle property to find the coordinates of R.	1
iii • writes y coordinate of R in terms of sum and product of p and q.	1
• substitutes for sum and product of p and q to find Cartesian equation.	1

Answer

i Gradient $OP = \dfrac{ap^2}{2ap} = \frac{1}{2}p$. Similarly gradient $OQ = \frac{1}{2}q$.

$\therefore OP \perp OQ \ \Rightarrow\ \frac{1}{2}p \cdot \frac{1}{2}q = -1 \qquad \therefore\ pq = -4$

ii The diagonals of a rectangle bisect each other. Hence M is the midpoint of OR.

Hence at R, $\frac{1}{2}\left(x+0\right) = a\left(p+q\right)$ and $\frac{1}{2}\left(y+0\right) = \frac{1}{2}a\left(p^2+q^2\right)$.

$$\therefore\ x = 2a\left(p+q\right) \quad \text{and} \quad y = a\left(p^2+q^2\right)$$

iii At R, $y = a\left\{\left(p+q\right)^2 - 2pq\right\} = a\left\{\left(\dfrac{x}{2a}\right)^2 + 8\right\}$

Hence locus of R has equation $x^2 = 4a\left(y - 8a\right)$.

Question 3

a. Outcomes assessed : P5, P8, H6, HE4

Marking Guidelines

Criteria	Marks
i • shows $f(-x) = f(x)$ to deduce function f is even	1
ii • shows formally that required limit is 1	1
iii • finds the first derivative, showing it is zero at the origin	1
• shows the origin is a maximum turning point by applying first or second derivative test	1
iv • shows the two vertical asymptotes and the central branch of the curve	1
• shows the horizontal asymptote and the remaining branches of the curve	1
v • makes x the subject, interchanges x and y to obtain equation for the inverse g^{-1}	1
• writes the domain of the inverse function	1

Answer

i. $f(-x) = \dfrac{(-x)^2}{(-x)^2 - 1} = \dfrac{x^2}{x^2-1} = f(x)$,

$x \neq \pm 1$. Hence f is an even function.

ii. $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{1}{1-\frac{1}{x^2}} = \dfrac{1}{1-0} = 1$

Curve has horizontal asymptote $y = 1$ as $x \to \pm\infty$

iii. $$\frac{dy}{dx} = \frac{2x\left(x^2-1\right) - x^2 . 2x}{\left(x^2-1\right)^2}$$

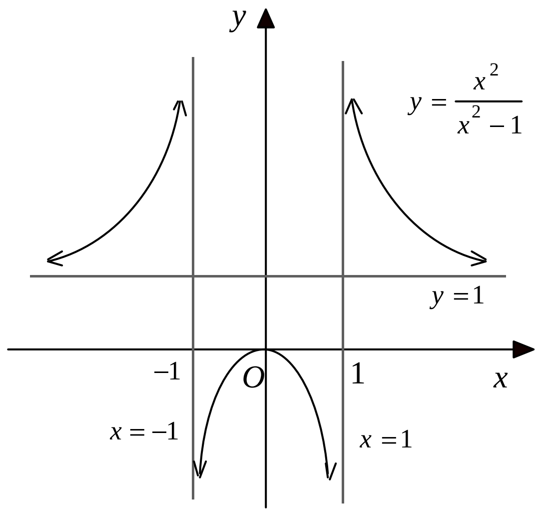
$$= \frac{-2x}{\left(x^2-1\right)^2}$$

$\therefore \dfrac{dy}{dx} = 0$ when $x = 0$

x		-1		0		1	
Sign of $\dfrac{dy}{dx}$	$+$		$+$	0	$-$		$-$
Curve	/	|	/	—	\	|	\

Hence $(0, 0)$ is a maximum turning point.

iv.



v. $y = \dfrac{x^2}{x^2-1}$, $x \geq 0$

$y\left(x^2-1\right) = x^2$

$yx^2 - y = x^2$

$yx^2 - x^2 = y$

$x^2(y-1) = y$

$x^2 = \dfrac{y}{y-1}$

$\therefore$ for the function g, $x = \sqrt{\dfrac{y}{y-1}}$, since $x \geq 0$.

Interchanging x and y, $g^{-1}(x) = \sqrt{\dfrac{x}{x-1}}$.

Inspection of the graph of $y = f(x)$ shows that the range of the function g is $\{y : y \leq 0 \text{ or } y > 1\}$.

Hence the domain of the inverse function g^{-1} is $\{x : x \leq 0 \text{ or } x > 1\}$.

b. Outcomes assessed : HE2

Marking Guidelines

Criteria	Marks
• verifies that statement true for $n = 1$	1
• writes LHS of $(k+1)^{\text{th}}$ statement in terms of RHS of k^{th} statement (assumed true)	1
• rearranges resulting expression into form of RHS of $(k+1)^{\text{th}}$ statement	1
• deduces the required result, showing understanding of the process of mathematical induction	1

Answer

Let $S(n)$, $n = 1, 2, 3, \ldots$ be the sequence of statements $\sum_{r=1}^{n} \frac{r+2}{r(r+1)2^r} = 1 - \frac{1}{(n+1)2^n}$, $n = 1, 2, 3, \ldots$

Consider $S(1)$: $LHS = \frac{3}{1 \times 2 \times 2} = \frac{4-1}{2 \times 2^1} = 1 - \frac{1}{2 \times 2^1} = RHS$. $\therefore S(1)$ is true.

If $S(k)$ is true : $\sum_{r=1}^{k} \frac{r+2}{r(r+1)2^r} = 1 - \frac{1}{(k+1)2^k}$ $**$

Consider $S(k+1)$: $LHS = \sum_{r=1}^{k+1} \frac{r+2}{r(r+1)2^r}$

$$= \sum_{r=1}^{k} \frac{r+2}{r(r+1)2^r} + \frac{(k+1)+2}{(k+1)(k+2)2^{k+1}}$$

$$= 1 - \frac{1}{(k+1)2^k} + \frac{(k+1)+2}{(k+1)(k+2)2^{k+1}} \quad \textit{if } S(k) \textit{ is true, using } **$$

$$= 1 - \frac{2(k+2) - (k+3)}{(k+1)(k+2)2^{k+1}}$$

$$= 1 - \frac{k+1}{(k+1)(k+2)2^{k+1}}$$

$$= 1 - \frac{1}{(k+2)2^{k+1}}$$

$$= RHS$$

Hence if $S(k)$ is true, then $S(k+1)$ is true. But $S(1)$ is true, hence $S(2)$ is true, and then $S(3)$ is true and so on. Hence by mathematical induction $S(n)$ is true for all positive integers $n \geq 1$.

Question 4

a. Outcomes assessed : HE4

Marking Guidelines

Criteria	Marks
• makes x the subject of the equation of the curve	1
• expresses the volume as a definite integral with respect to y with integrand $\tan^2\left(\frac{1}{2}y\right)$	1
• uses an appropriate trig. identity to find the primitive function	1
• substitutes the limits to evaluate the exact volume	1

Answer

$$y = 2\tan^{-1} x$$
$$\tfrac{1}{2} y = \tan^{-1} x$$
$$\tan\left(\tfrac{1}{2} y\right) = x$$

Hence volume is V cubic units where

$$V = p \int_0^{\frac{p}{2}} \tan^2\left(\tfrac{1}{2} y\right) \ dy$$

$$V = p \int_0^{\frac{p}{2}} \left\{\sec^2\left(\tfrac{1}{2} y\right) - 1\right\} \ dy$$
$$= p \ \left[2\tan\left(\tfrac{1}{2} y\right) - y\right]_0^{\frac{p}{2}}$$
$$= p \ \left\{2\left(\tan \tfrac{p}{4} - \tan 0\right) - \left(\tfrac{p}{2} - 0\right)\right\}$$
$$= p \ \left\{2 - \tfrac{p}{2}\right\}$$

Hence volume is $\tfrac{1}{2} p \ \left(4 - p\right)$ cubic units.

b. Outcomes assessed : H5, PE3

Marking Guidelines

Criteria	Marks
i • writes equation using expressions for areas of segment and sector	1
• simplifies to obtain required equation	1
ii • writes second approximation in terms of $f(2)$, $f'(2)$ where $f(q) = q - 2\sin q$	1
• evaluates expression for second approximation correct to 2 decimal places	1

Answer

i. *area segment* $= \tfrac{1}{2}$ *area sector*

$$\tfrac{1}{2} r^2 q - \tfrac{1}{2} r^2 \sin q = \tfrac{1}{4} r^2 q$$
$$\tfrac{1}{4} r^2 q - \tfrac{1}{2} r^2 \sin q = 0$$
$$r^2 \left(q - 2\sin q\right) = 0$$
$$\therefore r \neq 0 \ \Rightarrow \ q - 2\sin q = 0$$

ii. Let $f(q) = q - 2\sin q$

Then $f'(q) = 1 - 2\cos q$

Using Newton's method with $q_1 = 2$,

$$q_2 = 2 - \frac{f(2)}{f'(2)} \approx 2 - \frac{0 \cdot 1814}{1 \cdot 8323}$$

Hence second approximation is $1 \cdot 90$ (to 2 dec. pl.)

c. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
i • writes numerical expression for required probability	1
• evaluates probability as a fraction	1
ii • writes numerical expression for required probability	1
• evaluates probability as a fraction	1

Answer

Probability distribution is Binomial with $n = 6$, $p = \tfrac{1}{3}$, $q = \tfrac{2}{3}$.

i. $P\left(exactly\ 2\ correct\right) = {}^6C_2 \left(\tfrac{1}{3}\right)^2 \left(\tfrac{2}{3}\right)^4 = 15 \times \tfrac{16}{729} = \tfrac{80}{243}$

ii. $P\left(exactly\ 1\ correct\ out\ of\ first\ 5,\ then\ 6^{th}\ correct\right) = {}^5C_1 \left(\tfrac{1}{3}\right)^1 \left(\tfrac{2}{3}\right)^4 \times \tfrac{1}{3} = 5 \times \tfrac{16}{243} \times \tfrac{1}{3} = \tfrac{80}{729}$

Question 5

a. Outcomes assessed : HE6

Marking Guidelines

Criteria	Marks
• writes dx in terms of du and converts x limits to u limits	1
• writes integrand in terms of u	1
• finds primitive function in terms of u	1
• evaluates integral in simplest exact form by substitution of limits	1

Answer

$$u = x - 1$$
$$du = dx$$

$$x = 0\cdot 5 \;\Rightarrow\; u = -0\cdot 5$$
$$x = 1\cdot 5 \;\Rightarrow\; u = 0\cdot 5$$

$$2x - x^2 = 2(u+1) - (u^2 + 2u + 1)$$
$$= 1 - u^2$$

$$\int_{0\cdot 5}^{1\cdot 5} \frac{1}{\sqrt{2x - x^2}}\, dx = \int_{-0\cdot 5}^{0\cdot 5} \frac{1}{\sqrt{1 - u^2}}\, du$$
$$= \left[\sin^{-1} u\right]_{-0\cdot 5}^{0\cdot 5}$$
$$= \tfrac{\pi}{6} - \left(-\tfrac{\pi}{6}\right)$$
$$= \tfrac{\pi}{3}$$

b. Outcomes assessed : P4, HE5, HE7

Marking Guidelines

Criteria	Marks
i • uses similar triangles or tangent ratio to write r in terms of h	1
ii • writes $\dfrac{dr}{dt}$ in terms of $\dfrac{dh}{dt}$	1
• substitutes values of h and $\dfrac{dh}{dt}$	1
• finds required rate	1

Answer

i. The ray of light from P makes equal angles with the horizontal in both right triangles. Corresponding sides in these similar triangles are in proportion.

$$\therefore\ \frac{r}{6} = \frac{10}{h} \text{ and hence } r = \frac{60}{h}$$

ii. $$\frac{dr}{dt} = \frac{dr}{dh}\cdot\frac{dh}{dt} = -\frac{60}{h^2} \times \frac{dh}{dt}$$

But $\dfrac{dh}{dt} = -0\cdot 1$. Hence when $h = 5$,

$$\frac{dr}{dt} = \frac{60}{25} \times 0\cdot 1 = 0\cdot 24$$

Hence r is increasing at a rate of $0\cdot 24\,\text{cm}\,\text{s}^{-1}$.

c. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
i • differentiates $\frac{1}{2}v^2$ to find a in terms of x	1
ii • states the centre of the motion	1
• states the amplitude of the motion	1
iii • finds the maximum speed	1

Answer

i. $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}\left(16 + 4x - 2x^2\right)$ $\therefore a = 4 - 4x$

ii. $v^2 = 4\left(-x^2 + 2x + 8\right)$

$\therefore v^2 = 4\left(x+2\right)\left(4-x\right)$

$v^2 \geq 0 \;\Rightarrow\; -2 \leq x \leq 4$

The midpoint of this interval is $x = 1$.
Hence centre of motion is 1 m to the right of O
and the amplitude is 3 m.

iii. Maximum speed occurs at the centre of the motion.
$x = 1 \;\Rightarrow\; v^2 = 36$. Hence maximum speed is $6\ \text{ms}^{-1}$

Question 6

a. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
i • states maximum rate of flow	1
ii • expresses total amount of water as a definite integral	1
• uses an appropriate trig. identity to find the primitive	1
• evaluates by substitution of limits, giving answer to nearest litre	1

Answer

i.
$0 \leq \sin^2 t \leq 1$

$\therefore\; 0 \leq R \leq 4$

Maximum rate of flow is 4 kL/min ,
since $R = 4$ when $t = \frac{\pi}{2}$.

ii. $\int_0^{\pi} 4\sin^2 t \; dt = 2\int_0^{\pi} \left(1 - \cos 2t\right) \; dt$

$= \left[2t - \sin 2t\right]_0^{\pi}$

$= 2\left(\pi - 0\right) - \left(\sin 2\pi - \sin 0\right)$

$= 2\pi$

$\therefore 2\pi$ kL $\approx 6 \cdot 283$ kL (to the nearest L) flows into the tank.

b. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
i • substitutes one pair of N, t values to obtain one equation in A and B	1
• similarly obtains a second equation in A and B	1
• solves simultaneously to evaluate A and B	1
ii • states limiting value of N.	1

Answer

i. $N = A + Be^{-t}$

$60 = A + B\,e^{-\ln 2}$ $36 = A + B\,e^{-\ln 5}$

$= A + B\,e^{\ln\frac{1}{2}}$ $= A + B\,e^{\ln\frac{1}{5}}$

$= A + \frac{1}{2}B$ $= A + \frac{1}{5}B$

$\therefore\; 120 = 2A + B$ and $180 = 5A + B$

By subtraction, $3A = 60$
$\therefore A = 20$ and $B = 80$

ii. As $t \to \infty$, $N \to A + B \times 0 = 20$
Hence limiting population size is 20.

c. Outcomes assessed : P4, HE5

Marking Guidelines

Criteria	Marks
i • establishes result algebraically	1
ii • writes $\frac{dt}{dx}$ as sum of two algebraic fractions using (i)	1
• integrates and evaluates constant to find t as a function of x	1
• rearranges to find x as a function of t	1

Answer

i. $\frac{1}{x}+\frac{1}{2-x}=\frac{(2-x)+x}{x(2-x)} = \frac{2}{x(2-x)}$

ii. Initially particle is at $x=1$ moving right with $v=\frac{1}{2}$.

But $v=\frac{x(2-x)}{2}$ and $a=v\frac{dv}{dx}$. Hence if particle reaches $x=2$, $v=a=0$ and particle will remain at rest at this point. Hence $1\le x\le 2$.

$$\frac{dx}{dt}=\frac{x(2-x)}{2}$$

$$\frac{dt}{dx}=\frac{2}{x(2-x)}$$

$$=\frac{1}{x}+\frac{1}{2-x}$$

$$t=\ln x-\ln(2-x)+c$$

$$=\ln\left(\frac{x}{2-x}\right)+c \qquad (c \text{ constant})$$

$$\left.\begin{array}{l} t=0 \\ x=1 \end{array}\right\} \Rightarrow \begin{array}{l} \ln 1 + c = 0 \\ \therefore\ c=0 \end{array}$$

$$\therefore\quad t=\ln\left(\frac{x}{2-x}\right)$$

$$-t=\ln\left(\frac{2-x}{x}\right)$$

$$e^{-t}=\frac{2-x}{x}$$

$$e^{-t}=\frac{2}{x}-1$$

$$1+e^{-t}=\frac{2}{x}$$

$$\therefore\quad x=\frac{2}{1+e^{-t}}$$

Question 7

a. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
i • uses integration to find expression for x	1
• uses integration to find expression for y	1
ii • substitutes given values to write two equations in V and α	1
• finds exact value of V	1
• finds required approximate value of α to required accuracy	1
iii • finds horizontal and vertical components of impact velocity	1
• finds speed of impact to required accuracy	1
• finds angle of impact to required accuracy	1

Answer

i. *Horizontal component*

$$\ddot{x} = 0$$

$\dot{x} = c_1$, c_1 *const*.

$$\left.\begin{array}{l} t = 0 \\ \dot{x} = V\cos q \end{array}\right\} \Rightarrow c_1 = V\cos q \qquad \therefore\ \dot{x} = V\cos q$$

$x = (V\cos q)t + c_2$, c_2 *const*

$$\left.\begin{array}{l} t = 0 \\ x = 0 \end{array}\right\} \Rightarrow c_2 = 0 \qquad \therefore\ x = (V\cos q)t$$

Vertical component

$$\ddot{y} = -10$$

$\dot{y} = -10t + c_3$, c_3 *const*.

$$\left.\begin{array}{l} t = 0 \\ \dot{y} = V\sin q \end{array}\right\} \Rightarrow c_3 = V\sin q \qquad \therefore\ \dot{y} = -10t + V\sin q$$

$y = -5t^2 + (V\sin q)t + c_4$, c_4 *const*

$$\left.\begin{array}{l} t = 0 \\ y = 0 \end{array}\right\} \Rightarrow c_4 = 0 \qquad \therefore\ y = (V\sin q)t - 5t^2$$

ii. When $t = 4$, $x = 64$ and $y = -32$

$$\left.\begin{array}{r} 4V\cos q = 64 \\ 4V\sin q - 80 = -32 \end{array}\right\} \qquad \therefore \left.\begin{array}{r} V\cos q = 16 \\ V\sin q = 12 \end{array}\right\}$$

$$\therefore V^2(\cos^2 q + \sin^2 q) = 16^2 + 12^2$$

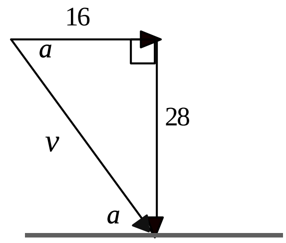
$$\therefore V^2 = 4^2(4^2 + 3^2)$$

Also $\cos q = \frac{4}{5}$ and $\sin q = \frac{3}{5}$

$\therefore V = 20,\quad q \approx 36°52'$

iii. When $t = 4$,

$\dot{x} = V\cos q = 16$ and $\dot{y} = -40 + V\sin q = -28$



$v^2 = 16^2 + 28^2 \Rightarrow v \approx 32 \cdot 2$

$\tan a = \frac{7}{4} \Rightarrow a \approx 60°15'$

Speed of impact is $32\ ms^{-1}$ (to nearest integer)
Angle of impact with beach is $60°15'$ (nearest minute).

b. Outcomes assessed : H9, HE3

Marking Guidelines

Criteria	Marks
i • writes expansion as required	1
ii • differentiates both sides with respect to x	1
• substitutes $x = 1$	1
• rearranges to obtain required identity	1

Answer

i. $x(1+x)^n \equiv x + {}^nC_1x^2 + {}^nC_2x^3 + \ldots + {}^nC_{n-1}x^n + x^{n+1}$

ii. Differentiation with respect to x gives

$$(1+x)^n + nx(1+x)^{n-1} \equiv 1 + 2\ {}^nC_1x + 3\ {}^nC_2x^2 + \ldots + n\ {}^nC_{n-1}x^{n-1} + (n+1)x^n$$

Substituting $x = 1$, $2^n + n \cdot 2^{n-1} = 1 + 2\ {}^nC_1 + 3\ {}^nC_2 + \ldots + n\ {}^nC_{n-1} + (n+1)$

$\therefore\ 2\ {}^nC_1 + 3\ {}^nC_2 + \ldots + n\ {}^nC_{n-1} = (n+2)2^{n-1} - (n+2) = (n+2)(2^{n-1} - 1)$