(a) Solve $\frac{x+1}{x} \geqslant 2$ **3**

(b) Find the acute angle between the lines $x + 3y = 4$ and $2x - 5y = 0$. Give your answer correct to the nearest degree. **3**

(c) If $\sqrt{3}\cos x - \sin x = R\cos(x + \theta)$, find the values of R and θ. **2**

(d) Evaluate $\int_0^1 \frac{2x\,dx}{(2x+1)^2}$, using the substitution $u = 2x + 1$. **4**

Question 2. (Start a New Page)

(a) It is given that $x^2 + x - 2$ is a factor of $x^3 + rx^2 - 4x + s$, where r and s are constants. **4**

(i) Show that $r + s = 3$.

(ii) Evaluate r and s.

(b) (i) What is the condition for the geometric series $a + ar + ar^2 + \dots$ to have a limiting sum? **4**

(ii) Consider the geometric series $1 - \tan^2 x + \tan^4 x + \dots$, where $0 < x < \frac{\pi}{2}$.

For what values of x does this series have a limiting sum?

(iii) Find the limiting sum in terms of $\cos x$.

(c) Find the exact value of $\int_0^{\frac{\pi}{4}} \cos^2(\tfrac{1}{2}x)\,dx$. **4**

Question 3 over the page

Question 3. (Start a New Page)

Marks

(a) (i) Sketch $y = 3\sin x$ and $y = x$, for $0 \leqslant x \leqslant 2\pi$. **4**

(ii) By substitution show that a solution for $3\sin x - x = 0$ lies between $x = 2.2$ and $x = 2.4$.

(iii) Taking $x = 2.3$ as an approximation to a solution of $3\sin x - x = 0$, apply Newton's Method once to find a better approximation. Give your answer correct to 3 decimal places.

(b) (i) Find $\frac{d}{dx}(2x\tan^{-1}x)$. **4**

(ii) Hence, find the exact value of $\int_0^1 \tan^{-1}x\,dx$.

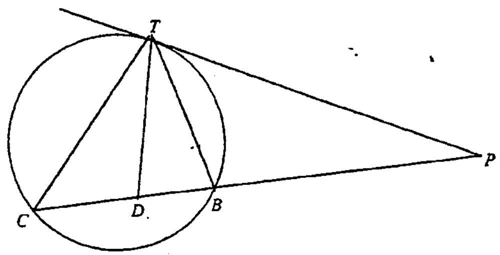
(c) Use Mathematical Induction to show that, for all $n \geq 1$ **4**

$$1\times 2 + 2\times 2^2 + 3\times 2^3 + \dots + n\times 2^n = (n-1)\times 2^{(n+1)} + 2$$

Question 4 over the page

Question 4. (Start a New Page) **Marks**

(a) **4**



PT is a tangent to the circle and PBC is a secant. D is a point on PBC such that $TD = TB$.
Prove that $\angle CTD = \angle P$.

(b) Consider the function $f(x) = \dfrac{1}{1+x^2}$ for $x \leqslant 0$. **4**

(i) Sketch $y = f(x)$. It is not necessary to show working.

(ii) Find the inverse function, $f^{-1}(x)$.

(iii) State the domain of $f^{-1}(x)$.

(c) (i) On the same set of axes sketch $y = \sin^{-1}x$ and $y = \cos^{-1}x$, showing all essential information. **4**

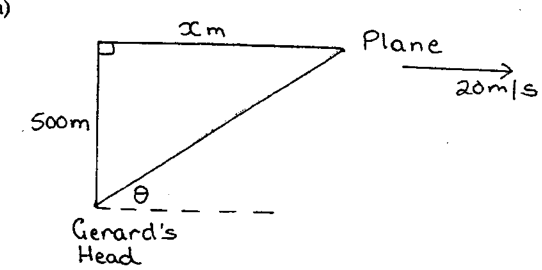
(ii) Let $f(x) = \sin^{-1}x + \cos^{-1}x$.
By referring to the graph in part (i), or otherwise, explain why $f(x)$ is a constant function.

iii) Hence, evaluate $\int_0^1 f(x)\,dx$.

Question 5 over the page

Question 5. (Start a New Page) **Marks**

(a) **5**



At 9 am an ultralight aircraft flies directly over Gerard's head, at a height of 500 metres. It maintains a constant speed of $20 m/s$, and a constant altitude.
If x is the horizontal distance travelled by the plane, and θ is the angle of elevation from Gerard's head to the plane,

(i) show that $\dfrac{dx}{d\theta} = -\dfrac{500}{\sin^2\theta}$.

(ii) Hence, show that $\dfrac{d\theta}{dt} = -\dfrac{1}{25}\sin^2\theta$.

(iii) find the rate of change of the angle of elevation at 9:01 am.

(b) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x = 2at, y = at^2$. **4**

(i) Find the co-ordinates of M, the midpoint of PQ.

(ii) Show that if the gradient of PQ is constant, the locus of M is a line parallel to the y-axis.

(d) (i) State the angle property of a cyclic quadrilateral. **3**

(ii) Given that the quadrilateral $ABCD$ is cyclic, show that the sum of the tangents of the angles in the quadrilateral is zero.
That is:
$\tan A + \tan B + \tan C + \tan D = 0$.

Question 6. (Start a New Page)

(a) Find a general solution for x if $\tan x = \dfrac{1}{\sqrt{3}}$. **2**
Give your answer in terms of π.

(b) (i) On the same set of axes graph $y = |2x-1|$ and $y = 3x+2$. **3**

(ii) Hence, or otherwise, solve $|2x-1| < 3x+2$.

Question 6 continued over the page

8

Question 6. (Continued) — Marks

(c) The rate at which a body cools is proportional to the difference between its temperature (T), and the constant temperature of the surrounding air (S). — 7

That is $\frac{dT}{dt} = k(T - S)$, where t is the elapsed time and k is a constant.

(i) Show that $T = S + Be^{kt}$, where B is a constant, is a solution of the above differential equation.

(ii) A body cools from 150^0 to 90^0 in three hours. If the air temperature is 30^0C, find the value of B and hence the value of k, correct to 3 decimal places.

(iii) Using the values of B and k found in part (ii), determine the temperature of the body after a further three hours.

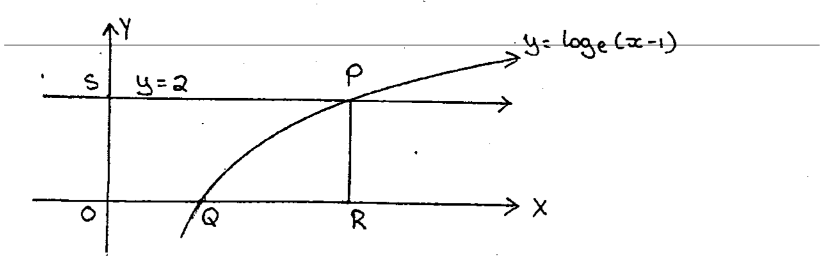
Question 7. **(Start a New Page)**

(a) $P(x)$ is a polynomial of degree 3 with the following properties: $P(0) = 4, P(2) = 0, P(-2) = 0$ and $P(x)$ has a turning point at $x = -2$. — 5

(i) Find $P(x)$.
(You may assume that $P(x) = ax^3 + bx^2 + cx + d$.)

(ii) What is the nature of the turning point at $x = -2$?

(b) The curve $y = \log_e(x - 1)$ meets the line $y = 2$ at the point P and the x-axis at the point Q. From P, perpendiculars are drawn to the x-axis and y-axis, meeting them at R and S, respectively, as shown in the diagram. — 7



(i) Show that the co-ordinates of P are $(e^2 + 1, 2)$.

(ii) Show that the normal to the curve at Q passes through S.

(iii) Show that the arc QP divides the rectangle $OSPR$ into two portions of equal area, where O is the origin.

End of Paper

Cherrybrook 2u 1999

Question One. *Start a new page* — Marks

(a) Evaluate to 4 significant figures — 2

$$\frac{12 \times (1{\cdot}05)^3}{2{\cdot}31 \times 0{\cdot}627}$$

(b) Express in scientific notation, correct to 3 sig fig, — 1

$$\sqrt[4]{\frac{4{\cdot}3 \times 10^{18} - 2{\cdot}9 \times 10^3}{2{\cdot}4^3 + 3{\cdot}31^2}}$$

(c) Find the integers a and b such that — 2

$$\frac{\sqrt{3}}{2 + \sqrt{3}} = a + b\sqrt{3}$$

(d) Factorise $2ax + 4xb - a - 2b$. — 2

(e) The price of tickets to *Future World* has increased 5·5% to \$48. Find the price before the increase. — 2

(f) Solve and graph the solution on the number line — 3

$$|6x - 9| > 21$$

(90)