2007

Higher School Certificate

Trial Examination

Mathematics

Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board approved calculators may be used
- Write using black or blue pen
- Write your student number and/or name at the top of every page
- All necessary working should be shown in every question
- A table of standard integrals is provided separately

Total marks – 84

Attempt Questions 1 – 7

All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME..................................

Student name / number ....

Marks

Question 1 **Begin a new booklet**

(a) Solve the inequality $\dfrac{1}{|x-1|}<1$ **2**

(b) Find the acute angle between the lines $2x-y=0$ and $x-2y=0$, giving the answer correct to the nearest degree. **2**

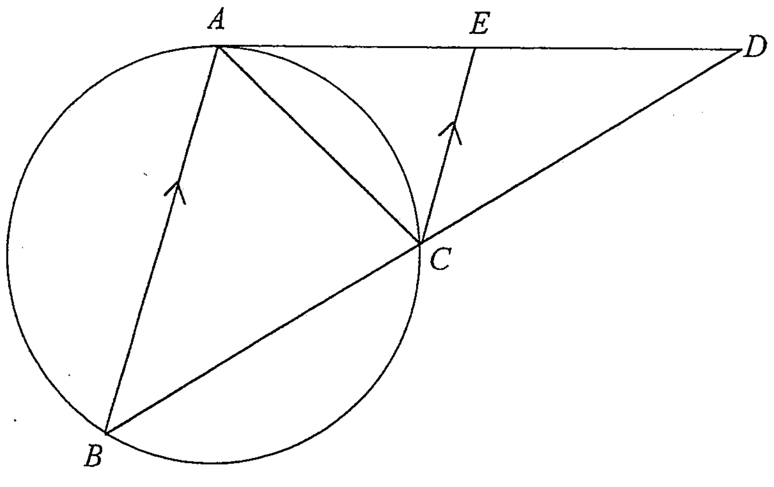
(c) The equation $x^3+px^2+qx+r=0$ has roots 1, α and α^2.

(i) Write down expressions in terms of p and q for $1+\alpha+\alpha^2$ and $\alpha+\alpha^2+\alpha^3$. **2**

Hence show that $\alpha=-\dfrac{q}{p}$.

(ii) Show that $q^3=rp^3$. **2**

(d)



Triangle ABC is inscribed in a circle. The tangent to the circle at A meets BC produced at D. The line through C parallel to BA meets AD at E.

(i) Show that $\Delta ACD \,|||\, \Delta CED$. **3**

(ii) Hence show that $AD=\dfrac{CA\times CD}{CE}$. **1**

Student name / number ....

Marks

Question 2 **Begin a new booklet**

(a) Solve the equation $(n+2)!=72n!$. **2**

(b) $A(-3,2)$ and $B(9,-6)$ are two points. Find the coordinates of the point $P(x,y)$ which divides the interval AB internally in the ratio $3:1$. **2**

(c)(i) Show that $\tan\left(\dfrac{\pi}{4}+A\right)=\dfrac{\cos A+\sin A}{\cos A-\sin A}$. **2**

(ii) Hence show that $\tan\left(\dfrac{\pi}{4}+A\right)=\dfrac{1+\sin 2A}{\cos 2A}$. **2**

(d)(i) $T(2at,at^2)$ is a point on the parabola $x^2=4ay$. Show that the normal to the parabola at T has equation $x+ty-2at-at^3=0$. **2**

(ii) P and Q are points on the parabola $x^2=4ay$ with parameter values $t=1$ and $t=2$ respectively. Show that the normals to the parabola at P and Q intersect at a point R on the parabola. **2**

Student name / number ....

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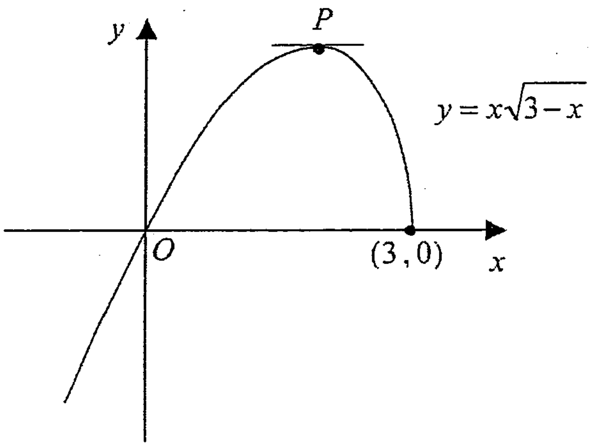
Question 3 **Begin a new booklet**

(a) Find $\int \sin^2 2x \, dx$. **2**

(b) Use Mathematical Induction to show that for all positive integers n, **4**

$$1\times 2^0 + 2\times 2^1 + 3\times 2^2 + ... + n\times 2^{n-1} = 1 + (n-1)2^n.$$

(c)(i) **2**



The diagram shows the graph of the curve $y = x\sqrt{3-x}$. Find the coordinates of the stationary point P on the curve.

(ii) The function $f(x)$ is defined by $f(x) = x\sqrt{3-x}$, $x \le 2$. The inverse function is denoted by $f^{-1}(x)$. On the same diagram, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ and shade the region where both $y \le f(x)$ and $y \ge f^{-1}(x)$. **2**

(iii) Explain why the area A of the shaded region is given by $A = 2\int_0^2 \left(x\sqrt{3-x} - x\right) dx$. **2**
(Do NOT attempt to evaluate this integral).

Student name / number ....

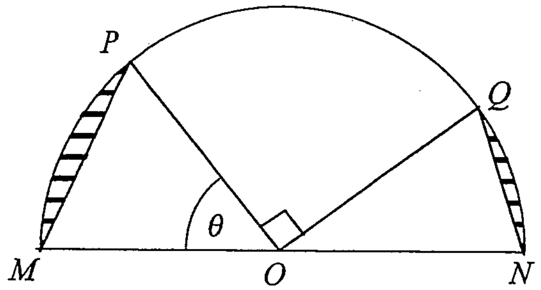
Marks

Question 5 **Begin a new booklet**

(a)(i) Find the domain and range of the function $f(x) = 2\cos^{-1}(1-x)$. **2**

(ii) Sketch the graph of the curve $y = 2\cos^{-1}(1-x)$. **2**

(b)



In the diagram, MN is a diameter of a semicircle with centre O and radius 1 metre. P and Q are variable points which move on the semicircle so that $\angle MOP = \theta$ and $\angle POQ = \frac{\pi}{2}$.

(i) Show that the area A m^2 of the shaded region is given by **2**

$$A = \frac{\pi}{4} - \frac{1}{2}(\sin\theta + \cos\theta).$$

(ii) If θ is increasing at a rate of $0 \cdot 1$ radians/s, find the rate at which the shaded area is changing when $\theta = 1$ radian. **2**

(c) Use the substitution $u = x + 1$ to evaluate $\displaystyle\int_0^3 \frac{x-2}{\sqrt{x+1}}\,dx$. **4**

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Marks

Question 4 **Begin a new booklet**

(a) Use one application of Newton's method with an initial approximation of $x = 1$ to find the next approximation to the root of the equation $\ln x - \frac{1}{x} = 0$. **2**

(b) A fair die is thrown five times.

(i) Find the probability that all of the five scores are different. **2**

(ii) Find the probability that exactly two of the five scores are 1's or 6's. **2**

(c) A particle is moving in a straight line. Initially the particle is at a fixed point O on the line. At time t seconds it has displacement x metres from O, velocity v ms^{-1} given by $v = 10 - x$ and acceleration a ms^{-2}.

(i) Find an expression for a in terms of x. **1**

(ii) Use integration to show that $x = 10 - 10e^{-t}$. **3**

(iii) Find the limiting position of the particle and the time it takes to move within 1cm of this limiting position. **2**

Student name / number ....

Marks

Question 6 **Begin a new booklet**

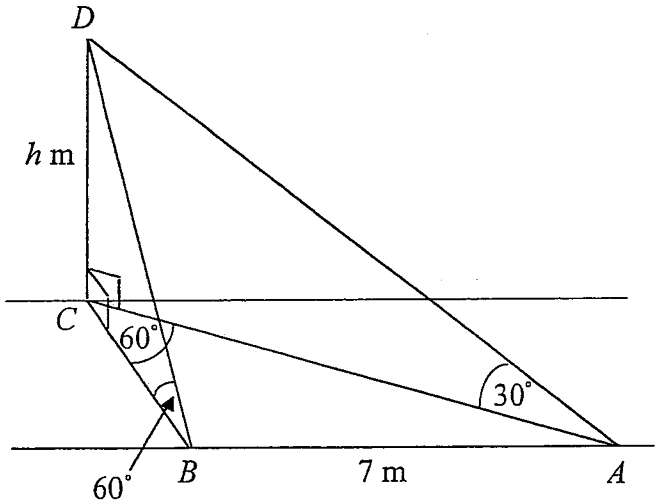
(a) A particle is moving in a straight line with Simple Harmonic Motion. At time t seconds it has displacement x metres from a fixed point O on the line, where $x = A\cos(\frac{\pi}{4}t + \alpha)$, $A > 0$, $0 < \alpha < \frac{\pi}{2}$. After 1 second the particle is 2 metres to the right of O, and after 3 seconds it is 4 metres to the left of O.

(i) Show that $A\cos\alpha - A\sin\alpha = 2\sqrt{2}$ and $A\cos\alpha + A\sin\alpha = 4\sqrt{2}$. **2**

(ii) Solve these equations simultaneously to show that $A = 2\sqrt{5}$ and $\alpha = \tan^{-1}\frac{1}{3}$. **2**

(iii) Show that the particle first passes through O after $\frac{4}{\pi}\tan^{-1}3$ seconds. **2**

(b)



A footpath on horizontal ground has two parallel edges. CD is a vertical flagpole of height h metres which stands with its base C on one edge of the footpath. A and B are two points on the other edge of the footpath such that $AB = 7\,\text{m}$ and $\angle ACB = 60^\circ$. From A and B the angles of elevation of the top D of the flagpole are 30° and 60° respectively.

(i) Find the exact height of the flagpole. **4**

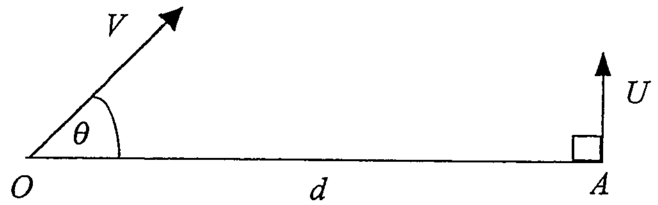
(ii) Find the exact width of the footpath. **2**

Student name / number ....

Marks

Question 7 **Begin a new booklet**

(a)



O and A are two points d metres apart on horizontal ground. A rocket is projected from O with speed V ms^{-1} at an angle θ above the horizontal, where $0<\theta<\frac{\pi}{2}$. At the same instant, another rocket is projected vertically from A with speed U ms^{-1}. The two rockets move in the same vertical plane under gravity where the acceleration due to gravity is g ms^{-2}. After time t seconds, the rocket from O has horizontal and vertical displacements x metres and y metres respectively from O, while the rocket from A has vertical displacement Y metres from A. The two rockets collide after T seconds.

(i) Write down expressions for x, y and Y in terms of V, θ, U, t and g. **2**

(ii) Show that $d = VT\cos\theta$ and $U = V\sin\theta$. **2**

(iii) Show that $V > U$. **1**

(iv) Show that the two rockets are the same distance above ground level at all times. **1**

(v) Show that $T = \dfrac{d}{\sqrt{V^2-U^2}}$. **1**

(vi) If the two rockets collide at the highest points of their flights, show that **1**

$$d = \frac{U\sqrt{V^2-U^2}}{g}.$$

(b)(i) Write down the Binomial expansion of $(1-x)^{2n}$ in ascending powers of x. **1**

(ii) Hence show that **3**

$$^{2n}C_1 + 3\,^{2n}C_3 + \ \dots\ + (2n-1)\,^{2n}C_{2n-1} = 2\,^{2n}C_2 + 4\,^{2n}C_4 + \ \dots\ + 2n\,^{2n}C_{2n}.$$