SYDNEY GIRLS HIGH SCHOOL
TRIAL HIGHER SCHOOL CERTIFICATE

2000

MATHEMATICS

3 UNIT (Additional)
and
3/4 UNIT (Common)

Time Allowed – 2 hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES   NAME ______________________________

- Attempt ALL questions
- ALL questions are of equal value
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used
- Each question attempted should be started on a new sheet. Write on one side of the paper only

This is a trial paper ONLY. It does not necessarily reflect the format or the contents of the 2000 HSC Examination Paper in this subject

Question 1

(a) Find $\int_0^{0.4} \frac{3dx}{4+25x^2}$ **2**

(b) At the Sydney 2000 Olympic Games the semi-finals of the mens 100m freestyle consists of 9 swimmers wearing full body wetsuits and 7 swimmers wearing normal swimwear. How many groups of 8 swimmers, containing exactly 5 swimmers wearing full-bodied wetsuits, can be in the final? **2**

(c) If $\sin\alpha = \frac{3}{4}$ $\quad 0 < \alpha < \frac{\pi}{2}$

and $\sin\beta = \frac{2}{3}$ $\quad \frac{\pi}{2} < \beta < \pi$

Find the exact value of:

(i) $\tan 2\alpha$

(ii) $\cos(\alpha - \beta)$ **4**

(d) Solve the equation

$2\ln(3x+1) - \ln(x+1) = \ln(7x+4)$ **4**

Question 2

(a) Use the substitution $u = 2 - x$ to evaluate $\int_{-1}^{2} x\sqrt{2-x}\,dx$ **4**

(b) (i) Find the value of x such that $\sin^{-1}x = \cos^{-1}x$

(ii) On the same axes sketch the graph of $y = \sin^{-1}x$ and $y = \cos^{-1}x$

(iii) On the same diagram as the graphs in (ii) draw the graph of $y = \sin^{-1}x + \cos^{-1}x$ **4**

(c) Solve $\frac{2}{3-x} \geq x$ **4**

Question 3

(a) (i) Show that the equation $\log_e x - \cos x = 0$ has a root between $x = 1$ and $x = 2$

(ii) By taking 1.2 as the first approximation, use 1 step of Newton's method to find a better approximation to this root correct to 2 decimal places **3**

(b) Prove by mathematical induction that:

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots+\frac{1}{n(n+1)}=1-\frac{1}{n+1}$$ **4**

(c) Consider the binominal expansion of $(3 + 2x)^{11}$

(i) Let T_k be the kth term in the expansion (where the terms are written out in increasing powers of x) Show that

$$\frac{T_{k+1}}{T_k} = \frac{2x(12-k)}{3k}$$

(ii) Find the greatest coefficient in the expansion. **5**

Question 4

(a) A spherical metal ball is being heated such that the volume increases at a rate of 2π mm^3/min. At what rate is the surface area increasing when the radius is 3mm? **3**

(b) A is the point (-4,1) and B is the point (2,4). Q is the point which divides AB internally in the ratio 2:1 and R is the point which divides AB externally in the ratio 2:1.
P (x,y) is a variable point which moves so that PA = 2PB.
(i) find the co-ordinates of Q and R
(ii) show that the locus of P is a circle on QR as diameter. **5**

(c) At any time t the rate of cooling of the temperature T of a body when the surrounding temperature is P, is given by the equation.

$$\frac{dT}{dt} = -k\,(T-P) \text{ for some constant } k$$

(i) Show that the solution
$T = P + Ae^{-kt}$ for some constant A satisfies this equation

(ii) A metal bar has a temperature of 1340° and cools to 1010° in 12 minutes when the surrounding temperature is 25°C. Find how much longer it will take the bar to cool to 60°C, giving your answer correct to the nearest minute **4**

Question 5

(a) (i) Prove $\dfrac{d^2x}{dt^2} = \dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right)$ where v denotes velocity **6**

(ii) The acceleration of a particle moving in a straight line is given by $\ddot{x} = -2e^{-x}$ where x is the displacement from O. The initial velocity of the particle is 2m/s at O

a) Show that $v^2 = 4e^{-x}$

b) Describe the subsequent motion of the particle making reference to its speed and direction.

(b) Consider the binominal expansion

$(1+x)^n = \binom{n}{o} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \ldots + \binom{n}{n}x^n$ **3**

(i) Use a suitable substitution to find the value of

$\binom{n}{o} + 2\binom{n}{1} + 2^2\binom{n}{2} + \ldots + 2^n\binom{n}{n}$

(ii) Differentiate both sides of the identity and then use a suitable substitution to find the value of

$\binom{n}{1} - 2\binom{n}{2} + 3\binom{n}{3} - \ldots + (-1)^{n-1}n\binom{n}{n}$

(c) Write $2\cos\theta + \sin\theta$ in the form

$A\cos(\theta - \alpha)$. Hence solve $2\cos\theta + \sin\theta = \sqrt{5}$ $O \le \theta \le 2\pi$: **3**

Question 6

(a) (i) Using long division divide the polynomial $f(x) = x^4 - x^3 + x^2 - x + 1$ by the polynomial $d(x) = x^2 + 4$.
Express your answer in the form $f(x) = d(x).q(x) + r(x)$

(ii) Hence find the values of the constants a and b so that $x^4 - x^3 + x^2 + ax + b$ is Divisible by $x^2 + 4$ **3**

(b) Find the volume of revolution formed when the area bounded by the x axis and the curve $y = \cos x$ between $x = \frac{-\pi}{2}$ and $x = \frac{\pi}{2}$ is rotated about the x axis **4**

(c) A competitor shoots an arrow with velocity 20m/s^{-1} to hit a target at a horizontal distance 20m from the point of projection and a height of 10m above the ground

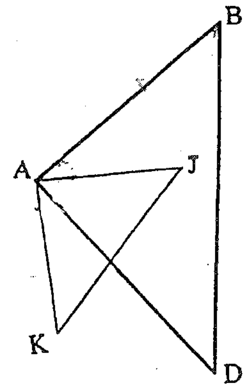
(i) Using calculus prove that the co-ordinates of the arrow at time t are given by

$$x = 20t\cos\alpha$$
$$y = -5t^2 + 20t\sin\alpha$$

(ii) Find two possible angles of projection (g = 10m/s) **5**

Question 7.

(a) ABD and AJK are two isosceles triangles both right angled at A



Copy the diagram onto your answer sheet 6

(i) Show that $B\hat{J}A = D\hat{K}A$

(ii) BJ is produced to meet DK at X. Show that $BX \perp DK$

(ii) The square ABCD is completed. Show that $B\hat{X}C = 45^{\circ}$

(b) A ship needs 7.5m of water to pass down a channel safely. At high tide the channel is 9m deep and at low tide the channel is 3m deep.
High tide is at 4:00am
Low tide is at 10:20 am.
Assume that the tide rises and falls in Simple Harmonic Motion

(i) What is the latest time before noon, to the nearest minute, that the ship can safely proceed through the channel?

(ii) In the 12 hours starting from 9:00 am between what times will the ship be able to proceed safely down the channel? 6

END OF PAPER