SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2004
HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK # 2

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes.
- Working time – 2 hours.
- Write using black or blue pen.
- Board approved calculators may be used.
- All *necessary* working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- Hand in your answer booklets in **3** sections.
  Section A (Questions 1 - 3), Section B (Questions 4 - 5) and Section C (Questions 6 - 7).
- Start each **NEW** section in a separate answer booklet.

Total Marks - 75 Marks

- Attempt Sections A - C
- All questions are NOT of equal value.

Examiner: *E. Choy*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

Total marks – 78
Attempt Questions 1 – 7
All questions are NOT of equal value

Answer each section in a SEPARATE writing booklet. Extra writing booklets are available.

SECTION A (Use a SEPARATE writing booklet)

Question 1 (15 marks) **Marks**

(a) Evaluate **3**

$$\int_0^3 \frac{x\,dx}{\sqrt{16+x^2}}$$

(b) By completing the square first, find **2**

$$\int \frac{dx}{x^2+6x+13}$$

(c) Use integration by parts to find **2**

$$\int xe^{-x}dx$$

(d) Find **3**

$$\int \cos^3\theta \; d\theta$$

(e) (i) Find real numbers A, B, and C such that **3**

$$\frac{x^2-4x-1}{(1+2x)(1+x^2)} = \frac{A}{1+2x} + \frac{Bx+C}{1+x^2}$$

(ii) Hence find $\int \frac{x^2-4x-1}{(1+2x)(1+x^2)}dx$ **2**

Question 2 (10 marks) **Marks**

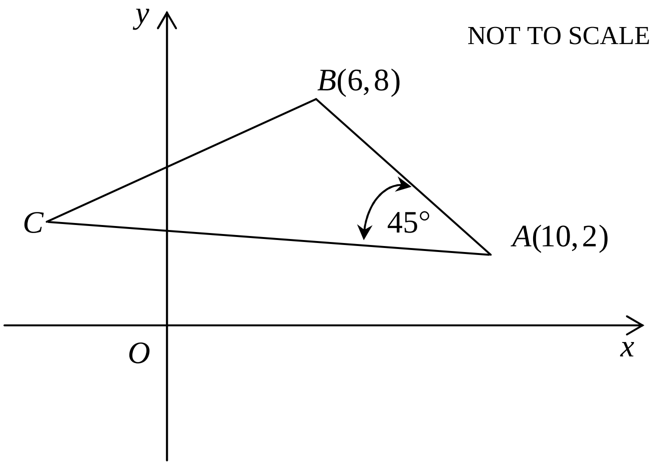
(a) On separate Argand diagrams, sketch the locus defined by:

(i) $2|z| = z + \overline{z} + 2$ **2**

(ii) $\left|z^2 - (\overline{z})^2\right| \geq 4$ **2**

(iii) $\arg(z-1) - \arg(z+1) = -\pi/3$ **2**

(b)



ΔABC is drawn in the Argand diagram above where $\angle BAC = 45°$, A and B are the points $(10, 2)$ and $(6, 8)$ respectively.
The length of side AC is twice the length of side AB.

Find:

(i) the complex number that the vector $\overrightarrow{AB}$ represents the complex number $-4+6i$; **2**

(ii) the complex number that the point C represents. **2**

Question 3 (12 marks)

(a) The quadratic equation $x^2 - x + K = 0$, where K is a real number, has two distinct positive real roots α and β.

(i) Show that $0 < K < \dfrac{1}{4}$ **1**

(ii) Show that $\alpha^2 + \beta^2 = 1 - 2K$ and deduce that $\alpha^2 + \beta^2 > \dfrac{1}{2}$ **2**

(iii) Show that $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} > 8$ **2**

Question 3 continued **Marks**

(b) (i) Show, using De Moivre's Theorem, that $z = \omega$, where $\omega = \sqrt{2} + i\sqrt{2}$ satisfies $z^4 = -16$. **3**
Hence write down, in the form $x + iy$ where x and y are real, all the other solutions of $z^4 = -16$.

(ii) Hence write $z^4 + 16$ as a product of two quadratic factors with real coefficients. **2**

(iii) Show that $\omega + \dfrac{\omega^3}{4} + \dfrac{\omega^5}{16} + \dfrac{\omega^7}{64} = 0$ **2**

SECTION B (Use a SEPARATE writing booklet)

Question 4 (8 marks) **Marks**

(a) Evaluate

(i) $\displaystyle\int_0^a x\sqrt{a-x}\,dx$ **2**

(ii) $\displaystyle\int_0^1 \frac{\sin^{-1} x}{\sqrt{1+x}}\,dx$ **2**

(b) (i) Using the substitution $t=\tan\dfrac{x}{2}$ show that **2**

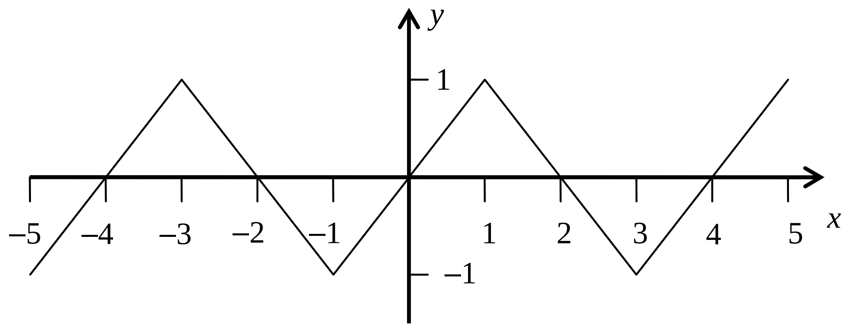
$$\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos x+\sin x}=\ln 2$$

(ii) Hence, by substituting $u=\dfrac{\pi}{2}-x$ evaluate **2**

$$\int_0^{\frac{\pi}{2}} \frac{x\,dx}{1+\cos x+\sin x}$$

Question 5 (11 marks)

(a)



The diagram is a sketch of the function $y=h(x)$ for $-5\le x\le 5$.
On separate diagrams sketch each of the following:

(i) $y=h(x+1)$ **1**

(ii) $y=\dfrac{1}{h(x)}$ **2**

(iii) $y=h\left(|x|\right)$ **1**

(iv) $y=\sqrt{h(x)}$ **2**

(v) $y=h\left(\sqrt{x}\right)$ **2**

(b) Sketch the curve $9y^2=x(x-3)^2$ showing clearly the coordinates of any turning point. **3**

SECTION C (Use a SEPARATE writing booklet)

Question 6 (9 marks) **Marks**

A firework missile of mass $0 \cdot 2$ kg is projected vertically upwards from rest by means of a force that decreases uniformly in 2 seconds from $2g$ newtons to zero and thereafter ceases. Assume no air resistance and that g is the acceleration due to gravity.

(i) If the missile has an acceleration of a m/s^2 at time t seconds, show that **3**

$$a = \begin{cases} g(9-5t) & t \le 2 \\ -g & t > 2 \end{cases}$$

[Hint: Draw a diagram showing the forces on the missile.]

(ii) Hence find:

(α) the maximum speed of the missile; **3**

(β) the maximum height reached by the missile. **3**

Question 7 (10 marks) **Marks**

A particle of mass 1 kg is projected from a point O with a velocity u m/s along a smooth horizontal table in a medium whose resistance is Rv^2 newtons when the particle has velocity v m/s. R is a constant, with $R > 0$.

(i) Show that the equation of motion governing the particle is given by **1**

$$\ddot{x} = -Rv^2$$

where x is the horizontal distance travelled from O.

(ii) Hence show that the velocity, v m/s, after t seconds is given by **3**

$$t = \frac{1}{R}\left(\frac{1}{v} - \frac{1}{u}\right)$$

An equal particle is projected from O simultaneously with the first particle, but vertically upwards with velocity u m/s in the SAME medium.

(iii) Show that the equation of motion governing the second particle is given by **1**

$$\ddot{y} = -(g + Rv^2)$$

where g m/s^2 is the acceleration due to gravity and y represents the vertical distance from O where the particle has a velocity of v m/s.

(iv) Hence show that the velocity V m/s of the first particle when the second one is momentarily at rest is given by **5**

$$\frac{1}{V} = \frac{1}{u} + \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right), \text{ where } Ra^2 = g$$

THIS IS THE END OF THE PAPER

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax,$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$