PETRUS KY COLLEGE
NEW SOUTH WALES

in partnership with

VIETNAMESE COMMUNITY IN AUSTRALIA
NSW CHAPTER

JULY 2007

MATHEMATICS EXTENSION 2

PRE-TRIAL TEST

HIGHER SCHOOL CERTIFICATE (HSC)

Student Number: | | | | |

Student Name: ______________________________

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Write your Student Number and your Name on all working booklets

Total marks – 84

- Attempt Questions 1–7
- All questions are of equal value

Marks

Question 1 **12**

(i) $\int \frac{3x^2-6x+1}{(x-3)(x^2+1)}dx$ 2

(ii) $\int_0^1 x.\tan^{-1} x.dx$ 2

(iii) $\int_0^{\pi/2} \sqrt{1+\sin 2x}.dx$ 2

(iv) If $I_n = \int_0^{\pi/2} \frac{\cos(2n+1)\theta}{\cos\theta}d\theta$, show that 4

$I_n + I_{n-1} = 0$ for $n \geq 1$. Hence find the value of I_n for $n \geq 0$

(v) $\int_{\sqrt{2}}^{2} \frac{1}{x\sqrt{x^2-1}}dx$ 2

Question 2 **12**

(A) Express $Z=\sqrt{3}+i$ and $W=1+i$ in the MOD-ARG forms and hence evaluate $\frac{Z^{20}}{Z^{30}}$ in the form $a+bi$ 2

(B) If $Z=i-1$, show clearly on an Argand diagrams all the points representing the complex numbers. 2

$$Z, Z^2, Z^3, Z^{-1}, \sqrt{2}.Z, -Z, \bar{Z}, iZ, Z^2-Z, \sqrt{Z}$$

(C) Simplify $Z=\dfrac{1+cos2\theta+i\sin 2\theta}{1+cos2\theta-i\sin 2\theta}$ 2

Hence show that $Z^n = cos2n\theta + i\sin 2n\theta$

(D) Express $cos6\theta$ as a polynomial in terms of $cos\theta$ hence show that 4

$\cos\frac{\pi}{12}$, $\cos\frac{3\pi}{12}$, $\cos\frac{5\pi}{12}$, $\cos\frac{7\pi}{12}$, $\cos\frac{9\pi}{12}$ and $\cos\frac{11\pi}{12}$ are the roots of the equation $32x^6-48x^4+18x^2-1=0$

(E) If w is the complex cube root of unity, $z^3=1$ then simplify 2

$$\frac{1}{3+5w+3w^2}+\frac{1}{7+7w+9w^2}$$

Question 3 **12**

(A)

(i) If $P(x) = x^3 - 6x^2 + 9x + c$ for some real number c, find the value of x for which $P'(x) = 0$. 3

Hence find the values of c for which the equation $P(x)$ has a repeated root.

(ii) Sketch the graphs of $y = P(x)$ with this values of c, hence find the set of values of c for which the equation $P(x) = 0$ has only one real root. 3

(B) Show that the equation $\dfrac{x^2}{36-k} + \dfrac{y^2}{20-k} = 1$, where k is a real number , represents:

(i) an ellipse if $k < 20$ 2

(ii) a hyperbola if $20 < k < 36$ 2

(iii) Show that the foci of the ellipse in (i) or hyperbola in (ii) are independent of the value of k. 2

Question 4 **12**

(A)

(i) The normal at point $P\left(ct, \frac{c}{t}\right)$ on the hyperbola $xy = c^2$ cuts the line $y = x$ at Q. Find the co-ordinates of Q. 2

(ii) Show that OP = PQ and hence show that there is no point on the parabola for which the length of PQ is less than $c\sqrt{2}$ 4

(B) Two points P and Q lie on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Their parameters are given as θ and $\theta + \frac{\pi}{2}$.

(i) Show that Q has co-ordinates $(-a\sin\theta, b\cos\theta)$. Hence prove: 2

$$OP^2 + OQ^2 = a^2 + b^2$$

(ii) Find the locus of midpoint M of PQ. 2

(iii) If α is the acute angle between the 2 tangents at P and at Q, show that 2

$$\tan\alpha = \frac{2\sqrt{1-e^2}}{e^2.\sin 2\theta}$$

Question 5 **12**

(A) By using the division of two graphs, or otherwise, sketch the curve 2

$$y = \frac{3x}{x^2 - 4}$$

(B) Find the domain and range of curve $y = \cos^{-1}(e^x)$ and hence sketch the graph of $y = \cos^{-1}(e^x)$ 2

(C) Let $f(x) = (\sin x - \cos x)^2$, find the period and range of $f(x)$, hence sketch the curve of $f(x)$ with $-\pi \le x \le \pi$. 4

From the separated graph, sketch the following curve.

(i) $y = \dfrac{1}{f(x)}$ 1

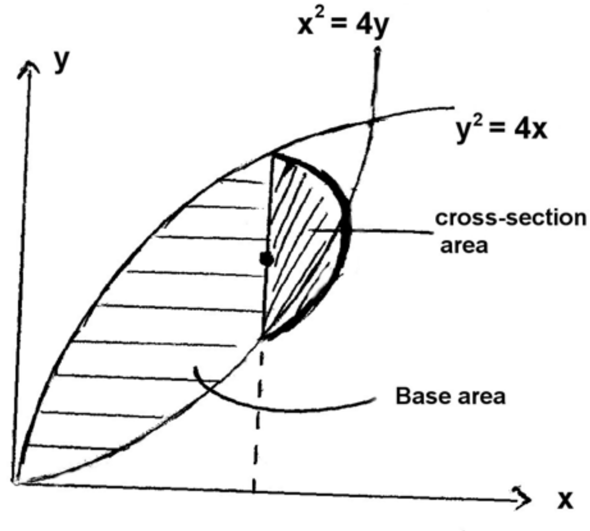
(ii) $y = \sqrt{f(x)}$ 1

(iii) $y = \ell n\left[f(x) \right]$ 1

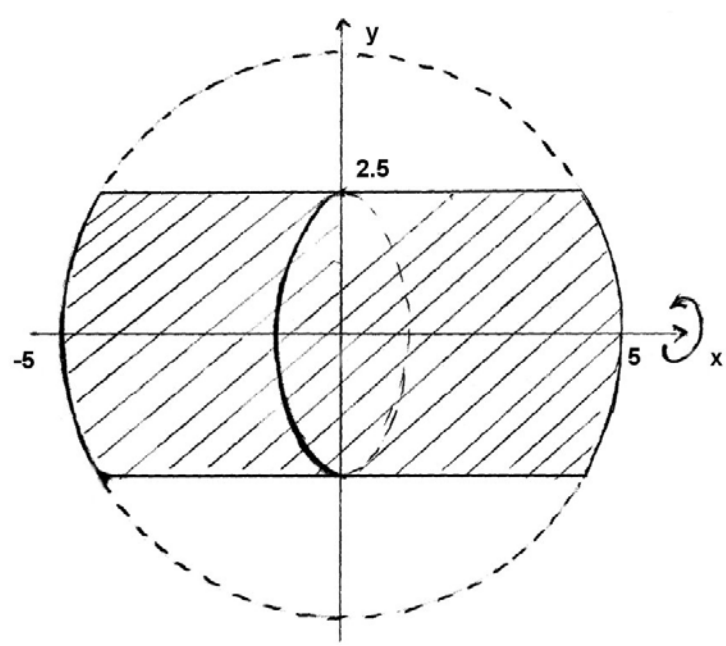
(iv) $y = f\left(|x| \right)$ 1

Question 6 **12**

(A) The base of a certain solid is the region bounded by the curves $y^2 = 4x$ and $x^2 = 4y$, and its cross-sections by planes perpendicular to the x-axis are semi circles. Find the volume of the solid. 6



(B) The area bounded by 2 arcs and 2 chords of a circle as shown in the figure below, is let to rotate about the x-axis. Find the volume of the solid shape. 6



Question 7 **12**

(A) Mice are placed in the centre of a maze which has 5 exits. Each mouse is equally likely to leave the maze through any one of the 5 exits. Four mice A, B, C, D are put into the maze and behave independently.

(i) Find the probability that A, B, C, D all come out the same exit. 1

(ii) What is the probability that A, B and C come out the same exit and D comes out a different exit. 1

(iii) What is the probability that any 3 of 4 mice come out the same exit and the other comes out a different exit. 1

(iv) What is the probability that no more than 2 mice come out the same exit. 1

(B) If $\mu_1 = 1$ and $\mu_n = \sqrt{3 + 2\mu_{n-1}}$ for $n \geq 2$

(i) show that $\mu_n < 3$ for $n \geq 1$ 2

(ii) deduce that $\mu_{n+1} > \mu_n$ for $n \geq 1$ 2

(C) By using induction method , prove that $3^{4n+2} + 2.4^{3n+1}$ is divisible by 17 for $n \geq 1$ 4

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$