Section A
Question 1.

a) Solve $\frac{2t}{5} + 14 = 8$

$\frac{2t}{5} = -6$ ①

$2t = -30$

$\boxed{t = -15}$ ①

b) $\left(\frac{34-7}{53+34+7}\right) \times 9.8$

$= 2.814893617$

$\boxed{= 2.815 \quad \text{4 sig fig}}$ ①

c) $3k - 2 \times -1 = 23$

$3k + 2 = 23$ ①

$3k = 21$

$\boxed{k = 7}$ ①

d) $\frac{x}{4} + \frac{3x-1}{3}$

$= \frac{3x + 4(3x-1)}{12}$ ①

$= \frac{3x + 12x - 4}{12}$

$\boxed{= \frac{15x - 4}{12}}$ ①

e) Factorise

-36

$3x^2 + 5x - 12$

$= \frac{(3x+9)(3x-4)}{3}$ ①

$= \frac{\cancel{3}(x+9)(3x-4)}{\cancel{3}}$

$= \boxed{(x+9)(3x-4)}$ ①

f) $7 - 4x > 12$

$-4x > 5$ ①

$\boxed{x < -5/4}$ ①

g) $\operatorname{cosec} \pi/4$

$= \frac{1}{\sin \pi/4}$

$= \frac{1}{1/\sqrt{2}}$

$= \boxed{\sqrt{2}}$ ①

Question 2

a) $\tan x° = 1 \quad 0° \leq x \leq 360°$

S | A
T | C

$x = 45°, 225°$ (2)

b) i) $m = -2 \quad (6, -8)$

$y + 8 = -2(x - 6)$

$y + 8 = -2x + 12$

$\boxed{2x + y - 4 = 0}$ (1)

ii) $k: 2x + y - 4 = 0$

$l: 2x - y + 8 = 0$

$k + l \quad 4x + 4 = 0$

$4x = -4$

$\boxed{x = -1}$ (2)

sub into k, $-2 + y - 4 = 0$

$\boxed{y = 6}$

$\therefore \boxed{R = (-1, 6)}$ (1)

iii) $D =$ $Q(2, 12) \quad R(-1, 6)$

$D = \sqrt{(2+1)^2 + (12-6)^2}$

$= \sqrt{3^2 + 6^2}$

$= \sqrt{45} = 3\sqrt{5}$ (1)

iv) $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

line: $2x - y + 8 = 0$

point: $(6, -8)$

$d = \dfrac{|2 \times 6 + (-1) \times (-8) + 8|}{\sqrt{2^2 + (-1)^2}}$

$= \dfrac{|12 + 8 + 8|}{\sqrt{5}}$

$= \dfrac{28}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}}$

$= \boxed{\dfrac{28\sqrt{5}}{5}}$ (2)

v) $A = \frac{1}{2} \times 3\sqrt{5} \times \dfrac{28\sqrt{5}}{5}$

$= 42\ u^2$ (1)

c) i $\cos C = \dfrac{6^2 + 7^2 - 4^2}{2 \times 6 \times 7}$

$= \dfrac{69}{84}$

$= \boxed{\dfrac{23}{28}}$ (1)

ii) $C = 34.77194403$

$= \boxed{35°}$ nearest degree. (1)

iii) $A = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 7 \times 6 \times \sin 35$

$= 12.04510516$

$= \boxed{12\ cm^2}$ (2)

3. HSC Trial 2 unit maths 2009

(a) (i) $\frac{d}{dx}(3-x^2)^3 = 3(3-x^2)^2 \times -2x$

$= -6x(3-x^2)^2$ (2)

(ii) $\frac{d}{dx}(\log_e(x^2+3)) = \frac{2x}{x^2+3}$ (2)

(iii) $\frac{d}{dx}(x\cos x) = x \times -\sin x + \cos x \times 1$

$= -x\sin x + \cos x$ (2)

(b) $f'(x) = 3-2x$

$f(x) = \int (3-2x)\,dx$

$= 3x - x^2 + C$

data $(3,5)$ $5 = 9 - 9 + C$

$C = 5$

$f(x) = 3x - x^2 + 5$ (2)

(c) (i) In $\triangle ABC \;|||\; \triangle AXY$

$X\hat{A}Y = B\hat{A}C$ common angle

$\frac{AX}{AB} = \frac{8}{10} = \frac{4}{5}$

$\frac{AY}{AC} = \frac{12}{15} = \frac{4}{5}$

Common angle, sides in same ratio test. (2)

$\triangle ABC \;|||\; \triangle AXY$

(ii) Because $A\hat{X}Y = A\hat{B}C$ angles in corresp position

$\therefore XY \parallel BC$ (1)

3 (d) $\int (x-6)^{\frac{1}{2}}\,dx$

$$= \frac{(x-6)^{1\frac{1}{2}}}{1\frac{1}{2} \times 1} + c$$

$$= \frac{2}{3}(x-6)\sqrt{x-6} + c \quad (1)$$

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4 (a) $(x-3)(x+k) = k(x+2)$

$$x^2 + xk - 3x - 3k = kx + 2k$$

$$x^2 + \cancel{xk} - \cancel{kx} - 3x - 3k - 2k = 0$$

$$x^2 - 3x - 5k = 0$$

equal roots $\Rightarrow \Delta = b^2 - 4ac = 0$

$a = 1$

$b = -3$

$c = -5k$

$$9 - 4 \times 1 \times {}^{-}5k = 0$$

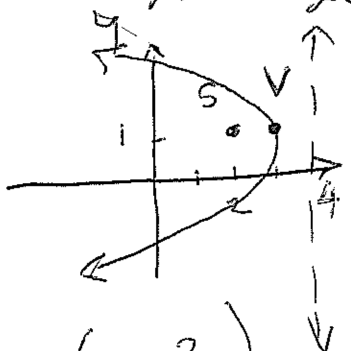
$$9 + 20k = 0$$

$$20k = -9$$

$$k = -\frac{9}{20} \quad (2)$$

(c) (i) $S(2,1)$

$x = 4$



(1)

$(y-k)^2 = {}^{-}4a(x-h)$

(ii) $(y-1)^2 = -4(x-3)$ (2)

$a = 1$

$V = (3,1)$

4 (b) borrows \$130,000

9.75% p.a compounded monthly $\Rightarrow \frac{9.75}{12}\% = 0.008125$

equal monthly instalments \$m.

(i) $\$A_1 = 130{,}000 + 130{,}000 \times 0.008125$

$= 130{,}000(1 + 0.008125)$

$= 130{,}000(1.008125) = \$131056.25$ 1

(ii) $\$130000(1.008125) - m.$ 1

(iii) 13 years = 156 months

$\$A_2 = 130000(1.008125)^2 - m(1 + 0.008125)$

$\$A_{156} = 130000(1.008125)^{156} - m(1 + 1.008125 + \cdots 1.008125^{155})$

$\$A_{156} = 0$

$$m = \frac{130000(1.008125)^{156}}{1 + 1.008125 + \cdots + (1.008125)^{155}}$$

denom: $S_n = \frac{rl - a}{r - 1} = \frac{1.008125 \times 1.008125^{155} - 1}{1.008125 - 1}$

$= \frac{1.008125^{156} - 1}{0.008125} = 311.8543626$

$\frac{a(r^n - 1)}{r - 1}$

$= \frac{1(1.008125^{156} - 1)}{1.008125 - 1}$

$m = \$1473.11$ 3

$\$A_n$:

(iv) $130000(1.008125)^{n} - 1700(1 + 1.008125 + \cdots + 1.008125^{n-1})$.

let $\$A_n = 0$

$1700(1 + 1.008125 + \cdots + 1.008125^{n-1}) = 130000(1.008125)$

sum $1 + 1.008125 + \cdots + 1.008125^{n-1}$

$S_n = \dfrac{a(r^n - 1)}{r - 1} = \dfrac{1.(1.008125^n - 1)}{1.008125 - 1} = \dfrac{1.008125^n - 1}{.008125}$

$\dfrac{1700(1.008125^n - 1)}{0.008125} = 130000(1.008125)^n$

$1700(1.008125^n - 1) = 1056.25(1.008125)^n$

$1700(1.008125)^n - 1700 = 1056.25(1.008125)^n$

$643.75(1.008125)^n = 1700$

$1.008125^n = 2.64077669$

$n \log 1.008125 = \log 2.64077669$

$n = 120$ months. 2

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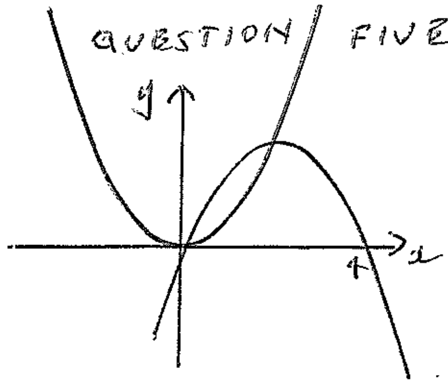
QUESTION FIVE



$y = x^2$

$y = 4x - x^2$

$4x - x^2 = x^2$

$2x^2 - 4x = 0$

$2x(x-2) = 0$

$x = 0, 2$

$y = 0, 4$

Point is $(2, 4)$ (2)

$$A = \int_0^2 (4x - x^2)\,dx - \int_0^2 x^2\,dx$$

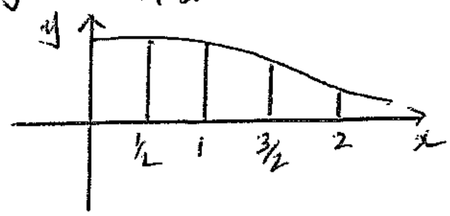
$$= \int_0^2 (4x - 2x^2)\,dx$$

$$= \left[2x^2 - \frac{2}{3}x^3\right]_0^2$$

$$= 8 - \frac{16}{3}$$

$= \frac{8}{3}$ square units (2)

$f(x) = \frac{1}{1+x^2}$



x	0	$1/2$	1	$3/2$	2	
y	1	$4/5$	$1/2$	$4/13$	$1/5$	

(2)

$$A \approx \frac{1}{6}(1-0)\left(1 + 4\times\frac{4}{5} + \frac{1}{2}\right) + \frac{1}{6}(2-1)\left(\frac{1}{2} + 4\times\frac{4}{13} + \frac{1}{5}\right)$$

$$= \frac{1}{6}\left(4\tfrac{7}{10}\right) + \frac{1}{6}\left(1\tfrac{121}{130}\right)$$

$$= \frac{47}{60} + \frac{251}{780}$$

$$= 1\tfrac{41}{390}$$

$= 1.1051$ (4dp)

OR $h = \frac{2-0}{4} = \frac{1}{2}$

$$A \doteq \frac{h}{3}\left[\left(1 + \frac{1}{5}\right) + \left(2\times\frac{1}{2}\right) + 4\left(\frac{4}{5} + \frac{4}{13}\right)\right]$$

$$= \frac{1}{6}\left(\frac{431}{65}\right)$$

$$= 1\tfrac{41}{390}$$

$= 1.1051$ (4dp)

NOTE: EXACT ANSWER IS 1.107148 . . . . .

OR USING DECIMALS

$$A \doteq \frac{1}{6}[1 + 3.2 + 0.5] + \frac{1}{6}[.5 + 1.23077 + 0.2]$$

$$= \frac{1}{6}[6.63077]$$

$$= 1.1051$$

AND

$$A \doteq \frac{1}{6}[1 + .2 + 2\times .5 + 4(.8 + .30769)]$$

$$= \frac{1}{6}[6.63076]$$

$= 1.1051$ (3)

c) $a(1 + r^2) = 13$ — (1)

$ar(1 + r^2) = \frac{39}{2}$ — (2)

From (1)

$$a = \frac{13}{1+r^2}$$

In (2) $\frac{13}{1+r^2} \cdot r(1+r^2) = \frac{39}{2}$

$$13r = \frac{39}{2}$$

$$r = 3/2$$

$a = \frac{13}{1 + 9/4} = 4$ (3)

Series is $4 + 6 + 9 + 13\tfrac{1}{2}$

$T_1 + T_3 = 13, \quad T_2 + T_4 = 19\tfrac{1}{2}$

6) a) $LHS = \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$

$$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$$

$\sin^2\theta + \cos^2\theta = 1.$

$$= \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

$= RHS$

OR

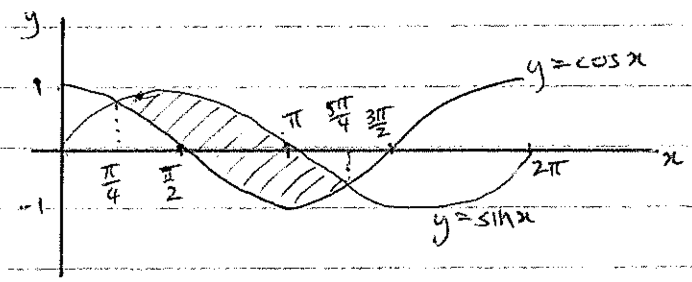
$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = (1-\cos\theta)(1+\cos\theta)$$

$$\frac{\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$$

b) i)



ii) Area $= \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\,dx$

$$= \left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$= -\cos\frac{5\pi}{4} - \sin\frac{5\pi}{4} - \left(-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right)$$

$$= -\left(-\frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}}\right) - \left(-\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)\right)$$

$$= \frac{4}{\sqrt{2}}$$

$= 2\sqrt{2}$ units^2

c) i) Area $= \frac{1}{2} ab \sin C$

$= \frac{1}{2}(12)(12) \sin 60^\circ$

$= 72\left(\frac{\sqrt{3}}{2}\right)$

$= 36\sqrt{3}\ cm^2$

ii) Area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(6)^2 \cdot \frac{\pi}{3}$

$= 6\pi\ cm^2$

iii) Area $= 36\sqrt{3} - 3(6\pi)$

$= 5.81\ cm^2$ (to 3 sig. figures)

SECTION D

QUESTION 7.

(a) $f(0)=10$.

$B(0,10)$ 1

(ii) $f(x)=x^4-8x^2+10$

$f'(x)=4x^3-16x$ 1

(iii) $f'(0)=0$

$f'(2)=4\times8-16\times2$

$=0$

$f'(-2)=-4\times8+16\times2$.

$=0$. 2

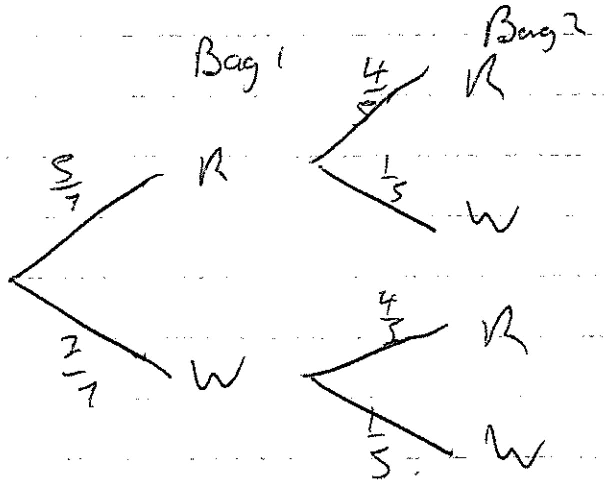
(iv) $f(2)=16-8\times4+10$

$=-6$

$f(-2)=-6$.

$A(-2,-6)$ $C(2,-6)$ 2

(b) (i)



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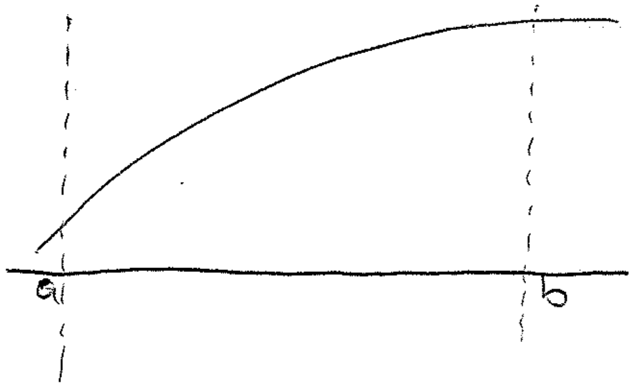
(ii) $P(RW)+P(WR)=\frac{5}{7\times5}+\frac{2\times4}{7\times5}$ 2

$=\frac{13}{35}$

(c)



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QUESTION 8

(a) (i) $\frac{dy}{dx} = -e^{-x}$.

at $x = -1$

$m = -e$.

$y - e = -e(x+1)$.

$y - e = -ex - e$.

$y = -ex$. 2

(ii) $\frac{dy}{dx} = -2x$

$-2x = -e$

$x = \frac{e}{2}$. 2

(iii) $y = -e \times \frac{e}{2}$. from tangent.

$= -\frac{e^2}{2}$. $P\left(\frac{e}{2}, -\frac{e^2}{2}\right)$. 1

$-\frac{e^2}{2} = -\left(\frac{e}{2}\right)^2 - a$.

$-\frac{e^2}{2} = -\frac{e^2}{4} - a$

$a = \frac{e^2}{2} - \frac{e^2}{4}$

$a = \frac{e^2}{4}$. 2.

(b) at $t=0$ $Q = Q_0$.

$Q_0 = Ce^0$.

i.e. $C = Q_0$.

$Q = Q_0 e^{-kt}$.

at $t=20$ $\frac{Q_0}{2}$

$\frac{Q_0}{2} = Q_0 e^{-20k}$.

$e^{-20k} = \frac{1}{2}$

$-20k = \ln \frac{1}{2}$.

$k = \frac{1}{20} \ln 2$. 2

(ii). $\frac{Q_0}{10} = Q_0 e^{-kt}$.

$\frac{1}{10} = e^{-kt}$

$-kt = \ln \frac{1}{10}$

$t = \frac{20 \ln 10}{\ln 2}$

$= 66$ mins 3

Question 9

(a) $R = 15 + \frac{10}{1+t}$

(i) $R = 15 + \frac{10}{1} = 25$

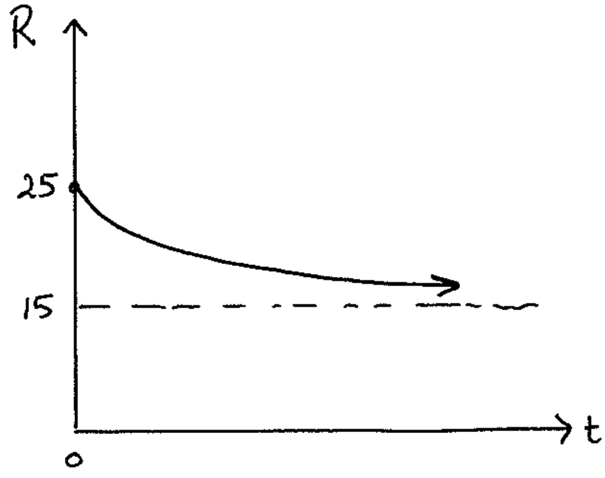
(ii) $R = 15 + \frac{10}{1+9} = 16$

(iii) As $t \to \infty$

$R \to 15$

since $\frac{10}{1+t} \to 0$

(iv)



(v) $\int_0^9 \left(15 + \frac{10}{1+t}\right) dt$

$= \left[15t + 10 \log_e (1+t)\right]_0^9$

$\doteqdot 158\text{ L}$

(b) $x = 3t + e^{-3t}$

(i) When $t = 1$, $x = 3 + e^{-3}$

ie $x \doteqdot 3.05$

(ii) $v = \frac{dx}{dt} = 3 - 3e^{-3t}$

When $t = 0$, $v = 3 - 3e^0$

$= 3 - 3(1)$

$v = 0$

$\therefore$ initially at rest.

(iii) $\ddot{x} = \frac{dv}{dt} = 9e^{-3t}$

(iv) $\lim_{t \to \infty} (3 - 3e^{-3t})$

$= \lim_{t \to 0} \left(3 - \frac{3}{e^{3t}}\right)$

$= 3$

$\left(\text{Since } \frac{3}{e^{3t}} \to 0\right)$

Q10. (a) $S = \frac{BC \times AO}{2} = BO \times AO$

$$= \frac{a}{\cos\theta} \times \frac{a}{\sin\theta}$$

$$= \frac{2a^2}{2\sin\theta\cos\theta}$$

$$= \frac{2a^2}{\sin 2\theta}$$ 3.

(b) $\frac{dS}{d\theta} = -2a^2(\sin 2\theta)^{-2} \times 2\cos 2\theta$.

$$= \frac{-4a^2\cos 2\theta}{\sin^2 2\theta}$$

now ~~increasing~~ decreasing where $\frac{-4a^2\cos 2\theta}{\sin^2 2\theta} > 0$ for $0 < \theta < \frac{\pi}{2}$.

ie $\cos 2\theta > 0$ for $0 < 2\theta < \pi$

$0 < 2\theta < \frac{\pi}{2}$ 2

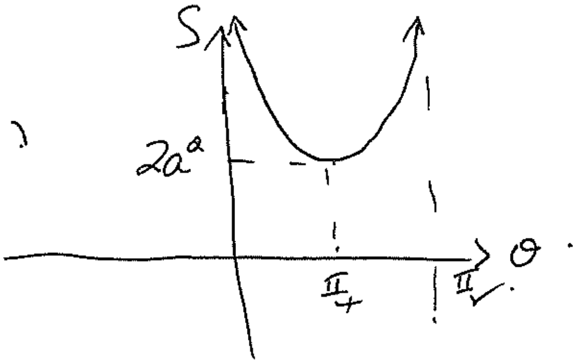
$\boxed{0 < \theta < \frac{\pi}{4}}$

~~decreasing~~ increasing where $\cos 2\theta < 0$ for $0 < 2\theta < \pi$

$\frac{\pi}{2} < 2\theta < \pi$ 2.

$\boxed{\frac{\pi}{4} < \theta < \frac{\pi}{2}}$

(c)



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(d) $2a < \frac{a}{\sin\theta} < 3a$

$2 < \frac{1}{\sin\theta} < 3$

$\frac{1}{2} > \sin\theta > \frac{1}{3}$

where $\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$

$\therefore S = \frac{a^2}{\frac{1}{2} \times \frac{\sqrt{3}}{2}} = \frac{4a^2}{\sqrt{3}}$ 3

where $\sin\theta = \frac{1}{3}$ $\cos\theta = \sqrt{1 - \frac{1}{9}} = \frac{\sqrt{8}}{3}$

$\therefore S = \frac{a^2}{\frac{1}{3} \times \frac{\sqrt{8}}{3}} = \frac{9a^2}{\sqrt{8}}$ $\therefore$ MAX.