HIGHER SCHOOL CERTIFICATE EXAMINATION 1987

MATHEMATICS - 3/4 UNIT

Directions to Candidates

Time allowed - Two hours (includes reading time)

All questions may be attempted. All questions are of equal value.

All necessary working should be shown in every question. Marks may not be awarded for careless or badly arranged work.

Standard integrals are provided (See page 74); approved slide-rules or silent calculators may be used.

QUESTION 1

(i) Differentiate: (a) $\dfrac{1}{3+x^2}$; (b) $e^x\log_e(2x)$.

(ii) Write down primitive functions of: (a) $(3x+2)^{10}$; (b) $\dfrac{5}{2+x^2}$.

(iii) (a) Find the equation of the normal n to the curve $y=x^4+4x^{3/2}$ at the point A(1,5).

(b) Find, to the nearest degree, the size of the acute angle between the line n and the line ℓ with equation $2x+3y-7=0$.

QUESTION 2

(i) Two circles cut at points B and C as shown in the diagram. A diameter of one circle is AB while BD is a diameter of the other.

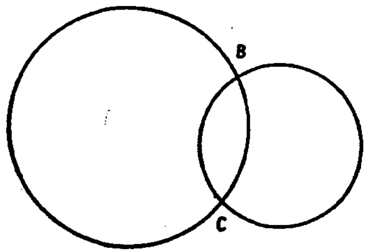

Figure not to scale

(a) Draw a neat sketch showing the given information.

(b) Prove that A, C and D are collinear, giving reasons.

(ii) Find the values of x for which $(x-2)^2\geq 4$.

(iii) (a) Differentiate $x\sin^{-1}x+\sqrt{1-x^2}$.

(b) Hence evaluate $\int_0^1 \sin^{-1}x\,dx$.

(iv) State the domain and range of $y = 2\sin^{-1}(3x)$.

QUESTION 3

(i) Find, for $0\leq x\leq 2\pi$, all solutions of the equation $\sin 2x = \cos x$.

(ii) Find the coefficient of x^3 in the expansion of $\left(2x-\dfrac{1}{x}\right)^{11}$.

(iii) One fifth of all jellybeans are black. A random sample of ten jellybeans is chosen.

(a) What is the probability that this sample contains exactly two black jellybeans? Give your answer correct to 3 decimal places.

(b) What is the probability that the sample contains fewer than two black jellybeans? Give your answer correct to 3 decimal places.

(c) Which is more likely: the sample contains fewer than 2 black jellybeans

or the sample contains more then 2 black jellybeans?

Give reasons for your answer.

QUESTION 4

(i) Find the volume of the solid formed when the region bounded by the x-axis and the curve $y = x(8-x^3)^4$ between x=0 and x=2 is rotated about the x-axis. (You may need to use the substitution $u = 8-x^3$ to evaluate the integral involved.)

(ii)

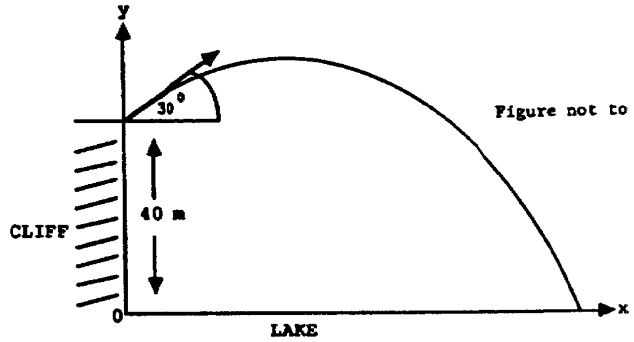

Figure not to scale

A pebble is projected from the top of a vertical cliff with velocity $20ms^{-1}$ at an angle of elevation of 30°. The cliff is 40 metres high and overlooks a lake.

(a) Take the origin 0 to be the point at the base of the cliff immediately below the point of projection. Derive expressions for the horizontal component x(t) and vertical component y(t) of the pebble's displacement from 0 after t seconds. (Air resistance is to be neglected.)

(b) Calculate the time which elapses before the pebble hits the lake and the distance of the point of impact from the foot of the cliff. [Assume the acceleration due to gravity is $10ms^{-2}$.]

QUESTION 5

The polynomial equation $f(x) = 8x^3+12x^2-18x-20 = 0$ has a root at $x=-2$.

(a) Find all roots of $f(x) = 0$.

(b) Draw a sketch of the graph of $y = f(x)$ showing the coordinates of its points of intersection with the axes and all stationary points.

(c) Apply Newton's method once to approximate a root of $f(x) = 0$ beginning with an initial approximation $x_1 = 1$.

(d) Willy chose an initial approximation of $x_1 = 0.49$ and used Newton's method a number of times in order to approximate a root of $f(x) = 0$. State, giving reasons, the root of $f(x) = 0$ to which Willy's approximations are getting closer. (It is not necessary to do additional calculations.)

QUESTION 6

(i) The rate at which a body cools in air is assumed to be proportional to the difference between its temperature T and the constant temperature S of the surrounding air. This can be expressed by the differential equation $\frac{dT}{dt} = k(T-S)$ where t is the time in hours and k is a constant.

(a) Show that $T = S+Be^{kt}$, where B is a constant, is a solution of the differential equation.

(b) A heated body cools from 80°C to 40°C in 2 hours. The air temperature S around the body is 20°C. Find the temperature of the body after one further hour has elapsed. Give your answer correct to the nearest degree.

(ii) Two points $P(2Ap,Ap^2)$ and $Q(2Aq,Aq^2)$ lie on the parabola $x^2 = 4Ay$, where $A>0$. The chord PQ passes through the focus.

(a) Show that $pq = -1$.

(b) Show that the point of intersection T of the tangents to the parabola at P and Q lies on the line $y = -A$.

(c) Show that the chord PQ has length $A(p+\frac{1}{p})^2$.

QUESTION 7

The rectangular piece of paper PQRS shown is folded along a line AB, where A and B lie on edges PQ and PS respectively. This line is so positioned that, after folding, P coincides with a point P' which lies on the edge QR. This fold line AB makes an acute angle θ with the edge PQ. The length of AB is ℓ and that of PQ is w.

(a) Show that $\angle P'AQ = (\pi - 2\theta)$.

(b) Prove that $\ell = \frac{w}{\cos\theta(1-\cos2\theta)}$.

(c) More than one fold line exists such that P coincides with a point on QR after folding. Find the value of θ which corresponds to the fold line of minimum length.

(d) Let CD be the fold line of minimum length, where C lies on PQ and D lies on PS. Calculate the length of CP.

ANSWERS – 1987 3UNIT/4UNIT –