MATHEMATICS (2 unit) – HSC Trial Exam

Suggested Answers & Mapping Grid

Q1 (a) $1.488437\ldots$

$\doteqdot 1.49$

(b) $x^2(x-4) = 0$

$x = 0, 4$

(c) $-\frac{1}{3e^x} + C$

(d) $\frac{1}{(m-3)(m-1)} - \frac{1}{(m-1)(m+1)}$

$= \frac{(m+1) - (m-3)}{(m-3)(m-1)(m+1)}$

$= \frac{4}{(m-3)(m-1)(m+1)}$

(e) $6x - 2y = 30$
$x + 2y = -2$
$7x = 28$
$x = 4$
$4 + 2y = -2$
$2y = -6$
$y = -3$
$(4, -3)$ Solution

(f) $115\% = \$19.55$
$125\% = x$
$x = \frac{125 \times 19.55}{115}$
$= \$21.25$

Q2 (a)(i) $-2x(-\sin(2-x^2))$

$= 2x \sin(2-x^2)$

(ii) $\frac{e^{2x} \cdot 1 - x \cdot 2e^{2x}}{(e^{2x})^2}$

$= \frac{e^{2x}(1-2x)}{e^{4x}}$

$= \frac{1-2x}{e^{2x}}$

(b) $BC = CD$ (Rhombus sides)
$\therefore \triangle BCD$ is isos. (2 sides $=$)
$\therefore B\hat{D}C = D\hat{B}C = 30^\circ$
$D\hat{B}C = B\hat{C}E + B\hat{E}C$ (ext $\angle$ of $\triangle BEC$)
$\therefore 30^\circ = B\hat{C}E + 10^\circ$
$B\hat{C}E = 20^\circ$

(c)(i) $\frac{1}{2}\tan 2x \Big]_0^{\pi/8}$

$= \frac{1}{2}\left(\tan\frac{2\pi}{8} - \tan 0\right)$

$= \frac{1}{2}(1-0)$

$= \frac{1}{2}$

(ii) $-3\ln(1-x) + C$

(d) $128 = \frac{24}{1-x}$

$1 - x = \frac{24}{128} = \frac{3}{16}$

$x = 13/16$

3(a)(i) $m = \frac{7-0}{-2-5} = -1$

(ii) $y - 0 = -1(x-5)$

$y = -x + 5$

$x + y = 5$

(iii) $\tan B\hat{A}x = -1$

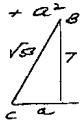
$B\hat{A}x = 135^\circ$

$B\hat{A}C = 45^\circ$

(iv) $\sqrt{53}^2 = 7^2 + a^2$

$a = 2\sqrt{3}$

$\therefore OC = 4$



$C = (4, 0)$

(v) $A = \frac{1}{2} \cdot b \cdot h$

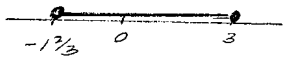
$= \frac{1}{2} \cdot 9 \cdot 7$

$= 31.5$ units^2

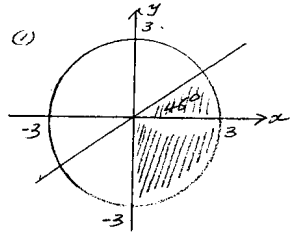
(b) $2 - 3x \leq 7$
$-3x \leq 5$
$x \geqslant -1\frac{2}{3}$

$-2 + 3x \leq 7$
$3x \leq 9$
$x \leq 3$

$-1\frac{2}{3} \leq x \leq 3$



(c)(i)



(ii) $A = \frac{3}{8}\pi \times 3^2$

$= \frac{27\pi}{8}$ units^2

4(a) $\cos 120^\circ = -\frac{1}{2}$

$2^2 = x^2 + (x-1)^2 - 2x(x-1)\cos(120^\circ)$

$4 = x^2 + x^2 - 2x + 1 + x^2 - x$

$3x^2 - 3x - 3 = 0$

$x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{1+4}}{2}$

$= \frac{1+\sqrt{5}}{2}$ as $x > 0$.

(b) $\frac{dV}{dt} = 0.5t^2$

$V = \frac{0.5t^3}{3} + c$

$t = 0,\ V = 0 \Rightarrow C = 0$

$t = 6,\ V = \frac{0.5 \times 6^3}{3}$

$= 36\ m^3$

(c)

x	-2	-1	0	1	2
y	3.25	3.5	4	5	7

$A = \frac{1}{2}\left(3.25 + 7 + 2(3.5 + 4 + 5)\right)$

$= 17.625$ units^2

(d) $y' = xe^x + 1 \cdot e^x$

$x = 1,\ y' = 2e = m$

$y = e$

$\therefore y - e = 2e(x-1)$

$y = 2ex - e$

Q5(a)

(i) $4y - 4 = x^2 - 4x + 4$

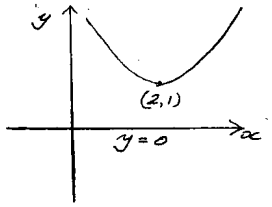
$4(y-1) = (x-2)^2$

$V = (2, 1)$

(ii) $a = 1$

directrix $y = 0$

(iii)



(b) $a = 1 \quad x = -\sqrt{2}$

$S_{10} = \frac{1\left(1 - (-\sqrt{2})^{10}\right)}{1 - (-\sqrt{2})}$

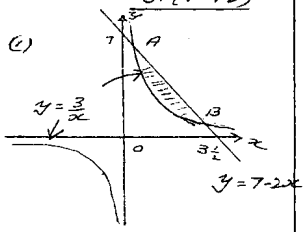
$(\sqrt{2})^{10} = 2^5 = 32$

$= \frac{-31}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$

$= \frac{-31(1-\sqrt{2})}{1-2}$

$= 31(1-\sqrt{2})$

(c) (i)



(ii) $\frac{3}{x} = 7 - 2x$

$2x^2 - 7x + 3 = 0$

$(2x-1)(x-3) = 0$

$x = \frac{1}{2}, 3$

$A(\frac{1}{2}, 6) \quad B(3, 1)$

(iii) $A = \int_{\frac{1}{2}}^{3} \left\{(7 - 2x) - \frac{3}{x}\right\} dx$

$= 7x - x^2 - 3\ln x \Big]_{\frac{1}{2}}^{3}$

$= (21 - 9 - 3\ln 3) - (3\frac{1}{2} - \frac{1}{4} - 3\ln\frac{1}{2})$

$= \frac{35}{4} - 3\ln 6$

Q6(a)

Coeff of $x^2 \Rightarrow A^2 = 9$

$A = \pm 3$

Coeff of $x \Rightarrow 2A + B = 4$

$B = -2, 10$

$x = 0 \Rightarrow 1 - 2B + C = 16$

$C = 11, 35$

Solutions $3, -2, 11$

$-3, 10, 35$

(b) $V = \pi \int_0^2 (y^2+3)^2 dy$

$= \pi \left[\frac{y^5}{5} + y^3 + 9y\right]_0^2$

$= 162\pi/5$ units^3

(c) (i) $P(SS) = \cdot 3 \times \cdot 4 = 12\%$

(ii) $P(SSS) = \cdot 3 \times \cdot 4 \times \cdot 05$

$= 0.6\%$

(iii) $P(SSF) = \cdot 3 \times \cdot 4 \times \cdot 95$

$= 11.4\%$

(iv) $P(F) = 1 - P(SSS)$

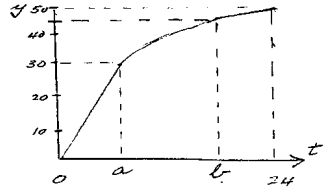
$= 99.4\%$

OR $P(F) + P(SF) + P(SSF)$

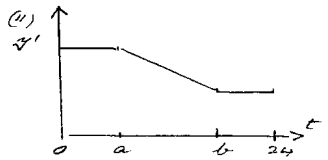
$= \cdot 7 + \cdot 18 + \cdot 114$

$= 99.4\%$

Q7(a)(i)



(ii)



(b) $m = 4 \quad (y = 4x - 3)$

$y' = 2 - 2x = 4$

$x = -1$

$y = 2(-1) - (-1)^2 = -3$

$(-1, -3)$

(c) $\sec^2 x = 4/3$

$\cos^2 x = 3/4$

$\cos x = \pm \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(d) $S_{70} = \frac{70}{2}(2 \times 2 + 69 \times \frac{1}{3})$

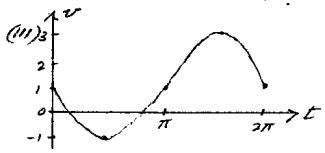
$= 945$

Q8(a)(i) $v = 1 - 2\sin t$

(ii) $v = 0 \quad \sin t = \frac{1}{2}$

second is $t = \frac{5\pi}{6}$.

(iii)



$t = 0 \quad v = 1$

Reduces to 0 at $t = \pi/6$

Becomes negative

reduces to $v = -1, \ t = \pi/2$

increases to $v = 0, \ t = 5\pi/6$

Becomes positive

increases to $v = 1 \quad t = \pi$

$v = 3 \quad t = 3\pi/2$

reduces to $v = 1 \quad t = 2\pi$

(b) (i) $75 = M_0 e^0$

$M_0 = 75$.

$69 = 75 e^{-0.5k}$

$e^{-0.5k} = 0.92$

$-0.5k = \ln(.92)$

$k \doteq 0.1668$

(ii) $M = 75 e^{-0.1668 \times 4}$

$\doteq 75 \times 0.513$

$\doteq 0.513 \times M_0$

(c) $y' = 3e^{3x} \quad y'' = 9e^{3x}$

$9e^{3x} + 6e^{3x} + 5e^{3x} = ae^{3x}$

$\therefore a = 20 \ (e^{3x} \neq 0)$

Q9(a)

(i) $y' = 4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

$x = 0, 3$

$y'' = 12x^2 - 24x$.

$x = 0, \ y = 0 \quad y'' = 0$

x	0^-	0	0^+
y'	$-$	0	$-$

Horizontal

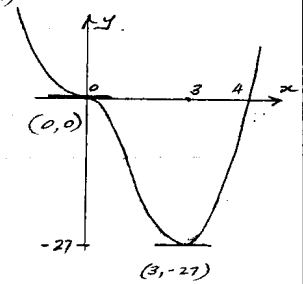
(0,0) Pt of Infl.

$x = 3, \ y = -27$

$y'' > 0$

$\therefore$ (3, -27) Min TP.

(iii)



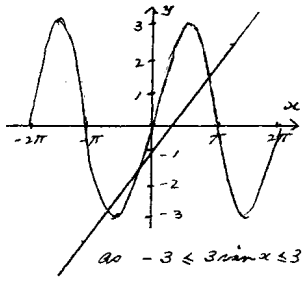
(iv) $y'' = 12x^2 - 24x = 0$

$x = 0, 2$.

ie $x = 2 \Rightarrow$ 2nd Pof I.

Concave up

$x < 0$ and $x > 2$



as $-3 \leq 3\sin x \leq 3$

$\therefore -3 \leq x - 1 \leq 3$

$\therefore -2 \leq x \leq 4$

(c) $(2A - 4) - (A + 4)$

$= (A - 8)$

Q10(a)(i) 6% p.a $= 0.5\%$ p month

$\therefore$ In 1st mth Int $= 0.5\%$ of 1 200 000

$= 0.005 \times 1\,200\,000$

$\therefore A_1 = 1\,200\,000 - 0.005 \times 1\,200\,000 - 8000$

$= 1\,200\,000(1.005) - 8000$

(ii) $A_2 = A_1 + 0.5\% A_1 - 8000$

$= 1.005 A_1 - 8000$

$= 1\,200\,000(1.005)^2 - 8000(1.005) - 8000$

$= 1\,200\,000(1.005)^2 - 8000(1.005 + 1)$

$A_3 = A_2 + 0.5\% A_2 - 8000$.

$= 1.005 A_2 - 8000$

$= 1\,200\,000(1.005)^3 - 8000(1.005 - 1)(1.005) - 8000$

$= 1\,200\,000(1.005)^3 - 8000((1.005)^2 + 1.005 + 1)$

(iii) $A_n = 1\,200\,000(1.005)^n - 8000\left((1.005)^{n-1} + \ldots + 1.005 + 1\right)$

$A_n = 0$ GP $a = 1, \ r = 1.005, \ n$ terms.

$\therefore 1\,200\,000(1.005)^n = 8000\left\{\frac{1((1.005)^n - 1)}{1.005 - 1}\right\}$

$(1.005)^n = 4$

$n \log(1.005) = \log 4$

$n = 277.95$ months

ie 23 years to nearest year

(ii) (i) $300 = \pi r^2 h$

$h = 300/\pi r^2$

(ii) $A = 2\pi r^2 + 2\pi r h$

$= 2\pi r^2 + 2\pi r \cdot 300/\pi r^2$

$= 2\pi r^2 + 600/r$

$= \frac{2\pi r^3 + 600}{r}$

(iii) $A = 2\pi r^2 + 600 r^{-1}$

$A' = 4\pi r - 600 r^{-2} = 0$

$4\pi r^3 = 600$

$r = \sqrt[3]{\frac{150}{\pi}}$

$A'' = 4\pi + 1200 r^{-3} > 0$

$\therefore A$ is min

Min Value of A

$= \frac{2\pi \cdot \frac{150}{\pi} + 600}{\sqrt[3]{\frac{150}{\pi}}}$

$= 900\sqrt[3]{\frac{\pi}{150}}$ m^2