Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2006
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Morning Session
Monday 7 August 2006

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- All necessary working should be shown in every question
- Write your Centre Number and Student Number at the top of this page

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value

2602-1

Total marks – 120
Attempt Questions 1-10
All questions are of equal value.

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\sqrt[3]{\dfrac{\left|-38.67\times 7.2\right|}{(11.7)^2-(1.83)^2}}$ correct to 3 decimal places. **2**

(b) By rationalising the denominator, simplify $\dfrac{1-\sqrt{2}}{2-\sqrt{8}}$. **2**

(c) Solve $\cos q = -\dfrac{1}{2}$, for $0 \le q \le 2p$. **2**

(d) Completely factorise $4xy + xb + 8ay + 2ab$. **2**

(e) For what values of k does the quadratic equation $4x^2 + kx + 9 = 0$ have equal roots? **2**

(f) Solve the equation $|x-2| = 3$. **2**

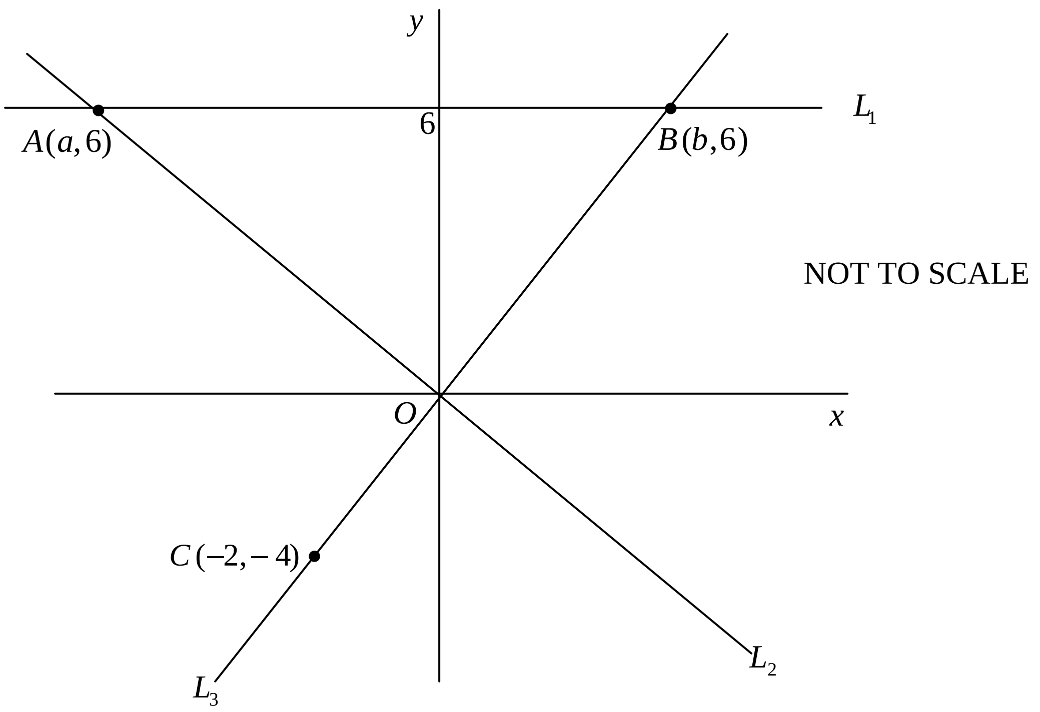
Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Sketch the region represented by $x^2 + y^2 < 4$. **2**

(b) The function $f(x)$ is given by $f(x) = \begin{cases} x+1 & \text{for } x \le 3 \\ x^2 - 9 & \text{for } x > 3 \end{cases}$ **2**

Find $f(3)$ - $f(6)$.

(c)



On the diagram above line L_1 is parallel with the x-axis and crosses the y-axis at 6. Lines L_2 and L_3 each pass through the origin, O, and intersect with the line L_1 at the points $A(a,6)$ and $B(b,6)$ respectively. The point $C(-2,-4)$ lies on the line L_3.

(i) Show that the equation of the line L_3 is $2x - y = 0$. **2**

(ii) Show that the x-ordinate of point B is 3. **1**

(iii) Find the coordinates of point A such that $\angle AOB$ is a right angle. **3**

(iv) Write down the distance between points A and B. Hence, or other calculate the area of ΔAOB. **2**

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) For the parabola $(x-2)^2 = 16y$, find the coordinates of the:

(i) Vertex. **1**

(ii) Focus. **1**

(b) Differentiate with respect to x the following expressions:

(i) $3x \log_e x$. **2**

(ii) $\sin^2 x$. **2**

(c) Find:

(i) $\int \cos 2006x \, dx$. **2**

(ii) $\int_0^1 e^{2x} \, dx$. (Leave your answer in exact form). **2**

(d) Show that the equation of the normal to the curve $y = x^3 - 5x$ at the point $(1, -4)$ is given by $x - 2y - 9 = 0$. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the curve $f(x) = -\frac{1}{3}x^3 - x^2 + 3x + 1$.

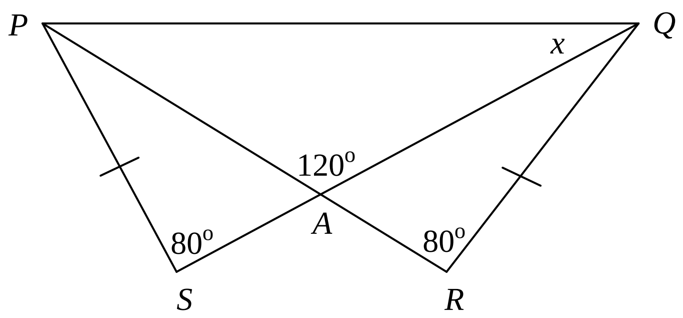
(i) Find the coordinates of any stationary points and determine their nature. **3**

(ii) Find any point(s) of inflexion. **2**

(iii) Sketch the curve in the domain, $-6 \le x \le 3$. **2**

(iv) What is the maximum value of $f(x)$ in the given domain? **1**

(b)



NOT TO SCALE

PR and QS are straight lines intersecting at point A. Also $PS = QR$, $\angle PSA = \angle QRA = 80^\circ$, $\angle PAQ = 120^\circ$ and $\angle PQA = x$.

(i) Copy the diagram into your writing booklet.

(ii) Prove that ΔPSA is congruent to ΔQRA. **2**

(iii) Hence, show that $x = 30^\circ$. **2**

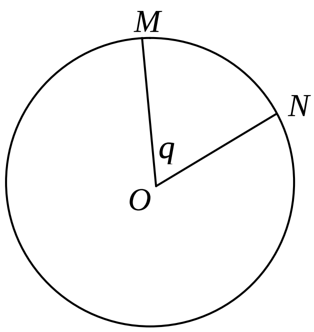
Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Write $0.7\dot{5}$ as the sum of an infinite geometric series. **1**

(ii) Hence, express $0.7\dot{5}$ as a fraction. **2**

(b) Simplify $\dfrac{1-\sin^2 x}{\cot x}$. **2**

(c) The circle below has centre O, radius $\dfrac{6}{p}$ cm and arc length MN = 1 cm.



NOT TO SCALE

(i) Find the size of q in radians. **1**

(ii) Hence, or otherwise, find the exact area of the sector MON. **2**

(d) During qualification for the 2006 World Cup, the Socceroos goalkeeper, Mark, defended many penalty shots at goal. In fact, the probability that he can stop a penalty shot at goal is $\dfrac{3}{5}$. During a particular match, the opposing team had three penalty shots at goal.

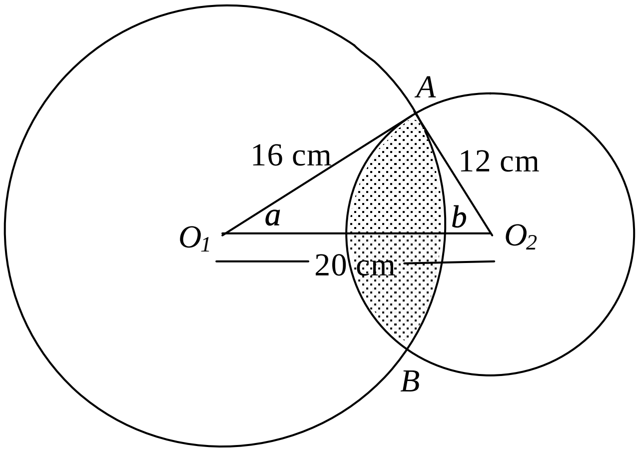
Using a tree diagram, find the probability that:

(i) the goalkeeper will stop all shots at goal. **2**

(ii) the goalkeeper will stop at least 1 shot at goal. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



NOT TO SCALE

A circle with centre at O_1 and radius 16 cm intersects with another circle with centre at O_2 and radius 12 cm. Their points of intersection are A and B and the distance between their centres, O_1O_2, is 20 cm. $\angle AO_1O_2 = a$ and $\angle AO_2O_1 = b$.

(i) Show that ΔO_1AO_2 is a right angled triangle. **1**

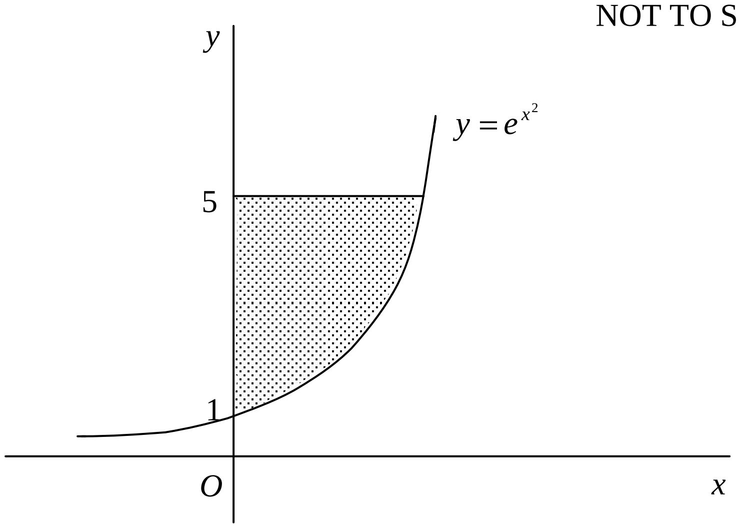
(ii) Find the size of the angles a and b . **2**

(iii) Find the shaded area enclosed by these circles. (Give your answer correct to 2 decimal places). **3**

Question 6 continues on page 8

(b)

NOT TO SCALE



The shaded region bounded by the graph $y = e^{x^2}$, the line $y = 5$ and the y-axis is rotated about the y-axis to form a solid of revolution.

(i) Show that the volume of the solid is given by $V_y = p\int_1^5 \log_e y\, dy$. **2**

(ii) Copy and complete the following table into your writing booklet. Give all answers correct to three decimal places. **1**

y	1	2	3	4	5
$\log_e y$	0	0.693	1.099		1.609

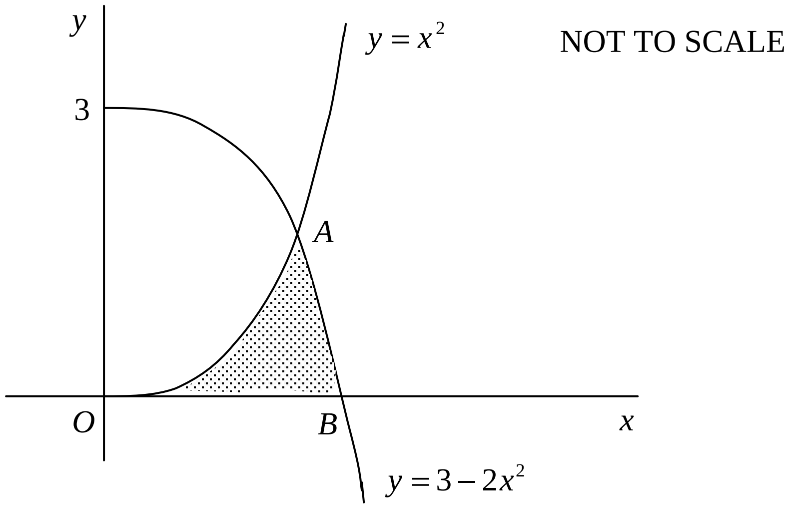
(iii) Use Simpson's Rule with five function values to approximate the volume of the solid of revolution V_y, correct to three decimal places. **3**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve the following equation: $\log_2 x + \log_2 (x+7) = 3$, for $x > 0$. **3**

(b)



The shaded region OAB is bounded by the parabolas $y = x^2$, $y = 3 - 2x^2$ and the x-axis. Point A is the intersection of the two parabolas and point B is the x-intercept of the parabola $y = 3 - 2x^2$.

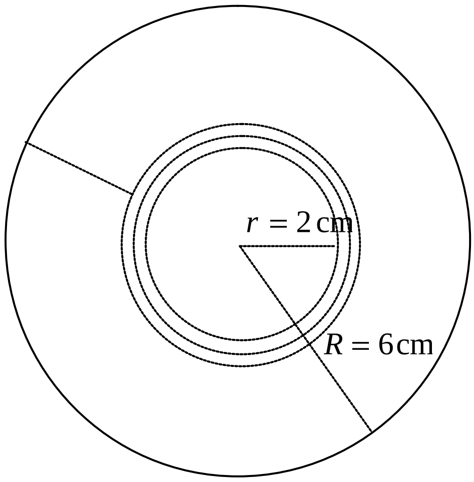
(i) Find the x-ordinates of points A and B. **2**

(ii) By considering the sum of two areas, show that the exact area of the shaded region OAB is given by $2\sqrt{\dfrac{3}{2}} - 2$ square units. **3**

Question 7 continues on page 10

Question 7 (continued) **Marks**

(c)



NOT TO SCALE

The playing track of a *CD* is made out of a number of concentric circles with the inner circle having a radius of 2 cm and the outer circle having a radius of 6 cm. The *CD* is rotating at 5 revolutions per second and takes 25 minutes to completely play.

(i) Find the total number of revolutions for this *CD*. **1**

(ii) Find the total length of the playing track in km, correct to one decimal place. **3**

End of Question 7

Question 8 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) If $A^m = 3,$ find the value of $A^{4m} - 5$. **2**

(b) A 100 mg tablet is dissolved in a glass of water. After t minutes the amount of *undissolved* tablet, U in mg, is given by the formula:

$$U = 100\ e^{-kt}\ , \text{ where } k \text{ is a constant.}$$

(i) Calculate the value of k, correct to 4 decimal places, given that 2 mg of the tablet remain after 10 minutes. **2**

(ii) Find the rate at which the tablet is dissolving in the glass of water after 12 minutes. Give your answer correct to two decimal places. **2**

(c) Mr. Egan borrows $\$P$ from a bank to fund his house extensions. The term of the loan is 20 years with an annual interest rate of 9%. Each month, interest is calculated on the balance at the beginning of the month and added to the balance owing. Mr. Egan repays the loan in equal monthly instalments of \$1 050.

(i) Write an expression for the amount, A_1, Mr. Egan owes immediately at the end of the first month. **1**

(ii) Show that at the end of n months the amount owing, A_n, is given by: **3**

$$A_n = P(1.0075)^n - 140000(1.0075)^n + 140\,000$$

(iii) If at the end of 20 years the loan has been repaid, calculate the amount Mr. Egan originally borrowed, correct to the nearest dollar. **2**

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Two particles, A and B, move along a straight line so that their displacements, x_A and x_B, in metres, from the origin at time t seconds are given by the following equations respectively:

$$x_A = 12t + 5 \qquad\qquad x_B = 6t^2 - t^3$$

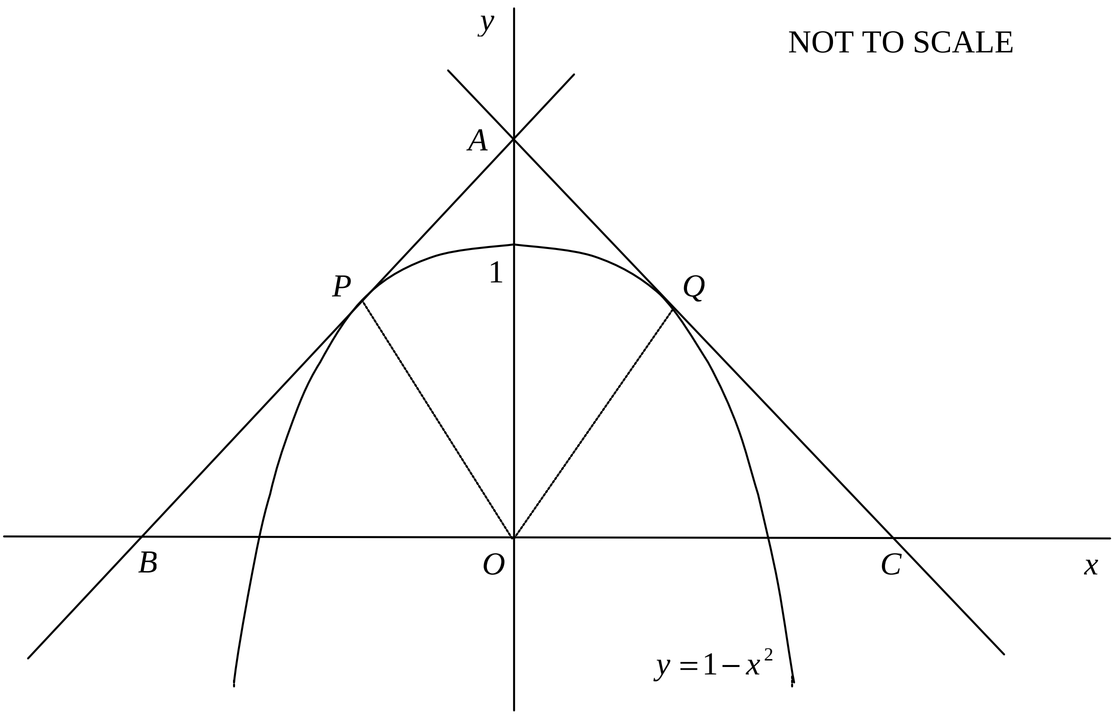
(i) Find two expressions for the velocities of particles A and B. **2**

(ii) Which of the two particles is travelling faster at $t = 1$ second? **1**

(iii) At what time does particle B come to rest? **1**

(iv) Find the maximum positive displacement of particle B. **2**

(b)



In the diagram above P and Q are two points on the parabola $y = 1 - x^2$. The tangents to the parabola at P and Q intersect each other on the y-axis at A and the x-axis at B and C respectively. Triangle ABC is an equilateral triangle.

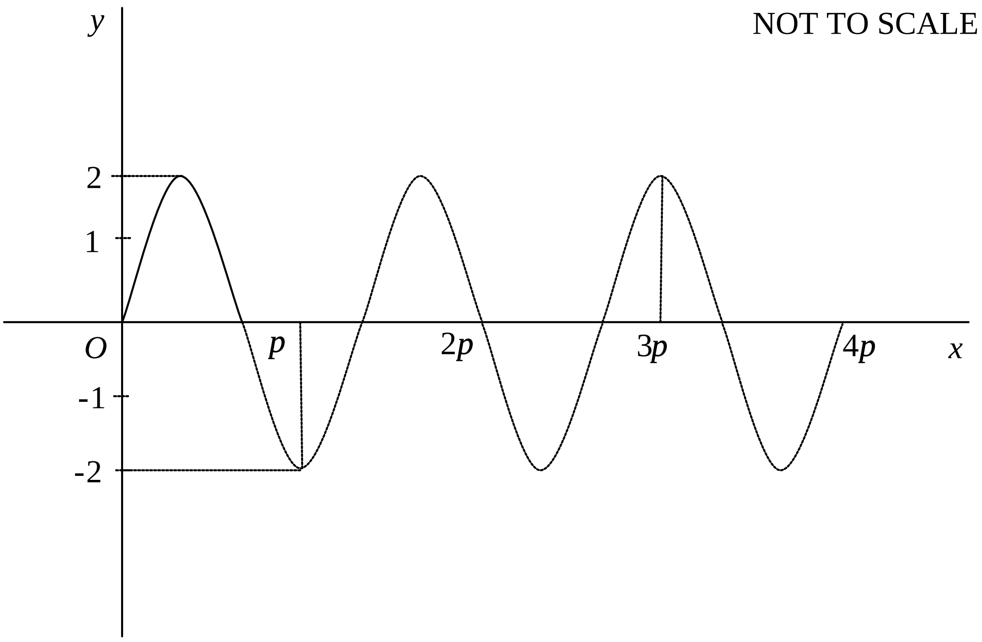
(i) Show that the gradient of the tangent at point P is equal to $\sqrt{3}$. **1**

(ii) Show that the coordinates of point P are $\left(-\frac{\sqrt{3}}{2}, \frac{1}{4}\right)$. **2**

(iii) Find the value of $\angle POQ$, in radians, correct to one decimal place. **3**

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

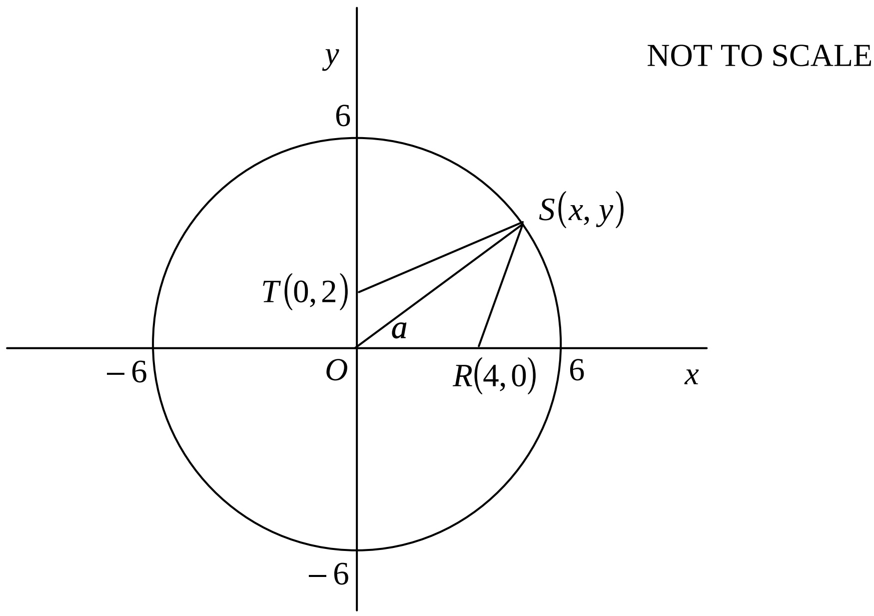
(a) Find the trigonometric equation for the graph below: **2**



Question 10 continues on page 14

Question 10 (continued) **Marks**

(b)



The diagram above shows the circle $x^2 + y^2 = 36$. The point $S(x, y)$ lies on the circle in the first quadrant. O is the origin, $R(4, 0)$ lies on the x-axis and $T(0, 2)$ lies on the y-axis. The size of $\angle ROS$ is α radians, where $0 < \alpha < \frac{\pi}{2}$.

(i) Show that the area of triangle SOR is $12\sin\alpha$. **1**

(ii) Hence show that the area, A, of the quadrilateral $ORST$ is given by: **3**

$$A = 6\cos\alpha\,(2\tan\alpha + 1)$$

(iii) Find the value of $\tan\alpha$ for which the area A is a maximum. **3**

(iv) Hence, show that for this maximum area, the coordinates of point S are $\left(\frac{6}{5}\sqrt{5}, \frac{12}{5}\sqrt{5}\right)$ **3**

End of Paper

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EXAMINERS

Liviu Spiridon (convenor)	LaSalle Catholic College, Bankstown
Magdi Farag	LaSalle Catholic College, Bankstown
Kimon Kousparis	Trinity Catholic College, Auburn / Regents Park
Timothy Dimech	Concord High School, Concord