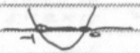
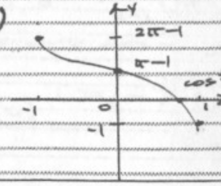
MATHS EXTENSION 1 TRIAL, 2008

MATHEMATICS Extension 1 : Question 1

Suggested Solutions	Marks	Marker's Comments
Q1(a) $\lim_{x\to0}\frac{3x}{\tan 5x} = \lim_{x\to0}\frac{3x(5x)}{\tan(5x)\times5}$ ✓ $=\frac{3}{5}\lim_{x\to0}\frac{5x}{\tan 5x}$ $=\frac{3}{5}\times1$ ✓ $\boxed{2}$ $=\frac{3}{5}$	1 1	
(b) $x-y-1=0$ $m_1=1$ $2x+y-1=0$ $m_2=-2$ $\boxed{2}$ $\tan\theta = \frac{-2-1}{1+-2\times1} = \frac{-3}{1-2} = \frac{-3}{-1}$ $\therefore \tan\theta = 3$ $\therefore$ obtuse angle $= 180^\circ - \tan^{-1}3$ ✓ $= \underline{108^\circ 26'}$	1 1	or $\tan^{-1}(-3)$
(c) $\sin\theta = \frac{\sqrt{3}}{2}$ $\boxed{2}$ $\theta = n\pi + (-1)^n\sin^{-1}\frac{\sqrt{3}}{2}$ ✓ $\theta = n\pi+(-1)^n\frac{\pi}{3}$ where ✓ $n \in \mathbb{Z}$	1 1	or $\theta=\begin{cases}\frac{\pi}{3}+2n\pi\\ \pi-\frac{\pi}{3}+2n\pi\end{cases}$ Acc $\theta = 180^\circ n+(-1)^n.60^\circ$
(d) $f(x) = (x^2-16)Q(x) + 3x-1$ $\boxed{2}$ Rem $= f(4) = 0 + 3\times4-1$ $= \underline{11}$ ✓	1 1	
(e) $\frac{1-2x}{1+x} \geq 1$ $\boxed{3}$ $\frac{1-2x-1(1+x)}{1+x} \geq 0$ $\frac{-3x}{1+x} \geq 0$ $\frac{3x}{1+x} \leq 0$ Now $x \neq -1$ so $3x(1+x) \leq 0$  $\Rightarrow \underline{-1 < x \leq 0}$	1 1+1	
(f) Primitive $\ln[x+\sqrt{x^2-9}]\ (+c)$ ✓ $\boxed{1}$		1 For $\ln[x+\sqrt{x^2-9}]$

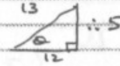
MATHEMATICS Extension 1 : Question 2

Suggested Solutions	Marks	Marker's Comments
Q2(a) $g(x) = \sqrt{x+2}$ $\boxed{2}$ $g^{-1}(5)$ is $g(x) = 5$ $\therefore 5 = \sqrt{x+2}$ $\therefore \underline{x = 23}$	1 1	$g^{-1}(x) = x^2-2$
(b) (i) $\frac{2+\tan x}{1+\tan^2 x} = \frac{2\frac{\sin x}{\cos x}}{\sec^2 x} = \frac{2\sin x}{\cos x}\times\cos^2 x$ ✓ $= 2\sin x\cos x$ $\boxed{1}$ $= \sin 2x$ qed.	1	
(ii) $\int_0^{\pi/4}\frac{\tan x}{1+\tan^2 x}\,dx = \frac{1}{2}\int_0^{\pi/4}\sin 2x\,dx$ $\boxed{2}$ $= -\frac{1}{4}\Big[\cos 2x\Big]_0^{\pi/4}$ $= -\frac{1}{4}\left[\cos\frac{\pi}{2} - \cos 0\right]$ $= -\frac{1}{4}[0-1]$ $= \underline{\frac{1}{4}}$	1 1	
(c) $I = \int_0^3 \frac{5x^2+10x}{\sqrt{1+x}}\,dx$ $\boxed{4}$ $u = \sqrt{1+x}$ $u^2 = 1+x$ $2u\,du = dx$ ✓ $\begin{array}{c\|c} x & u \\ \hline 3 & 2 \\ 0 & 1 \end{array}$ ✓ $\therefore I = 5\int_1^2 \frac{(u^2-1)^2+2(u^2-1)}{\cancel{u}}\times 2\cancel{u}\,du$ $x = u^2-1$ $=10\int_1^2 u^4-2u^2+1+2u^2-2\,du$ ✓ $=10\int_1^2 u^4-1\,du$ $=10\left[\frac{1}{5}u^5-u\right]_1^2 = 10\left[\left(\frac{32}{5}-2\right)-\left(\frac{1}{5}-1\right)\right]$ $=10\left(\frac{31}{5}-1\right) = \underline{52}$ ✓	1 1 1 1	
(d)  NTS $y = 2\cos^{-1}x - 1$ $\boxed{3}$		1 For x-int $\cos\frac{1}{2} \doteqdot 0.88$ 1 For $2\pi-1$ and/or -1 $\frac{1}{2}$ For $\pi-1$ $\frac{1}{2}$ For shape

MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments

Q3(a) $\tan\left(2\cos^{-1}\frac{12}{13}\right)$



Let $\theta = \cos^{-1}\frac{12}{13} \Rightarrow \cos\theta = \frac{12}{13}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\times\frac{5}{12}}{1-\frac{25}{144}}$ ✓ $\boxed{2}$

$= \frac{2\times5\times12}{144-25} = \frac{120}{119}$.

Marker's Comments: $\frac{1}{2}$ For $\cos\theta = \frac{12}{13}$

$\frac{1}{2}$ For $\tan\theta = \frac{5}{12}$

1 For $\frac{2\times\frac{5}{12}}{1-\frac{25}{144}}$ or $\frac{120}{119}$

(b) (i) $x^2 = 8y$

$\therefore y = \frac{x^2}{8}$ $\quad P(4p, 2p^2)$ ①

$\frac{dy}{dx} = \frac{2x}{8} = \frac{x}{4}$

Gradient of tangent at P : $m_T = \frac{4p}{4} = p$

Equ. of tangent at P: $y - 2p^2 = p(x-4p)$ ✓ 1

$y - 2p^2 = px - 4p^2$

$\therefore y = px - 2p^2$.

Marker's Comments: 1 For getting to ✓

(ii) $C = (0, -2p^2)$

For Q $(0, -2p^2)$ 1:3 $P(4p, 2p^2)$

$Q = \left(\frac{4p+0}{4}, \frac{2p^2-6p^2}{4}\right) = (p, -p^2)$. ✓ 1+1

③

Let Q (x, y) be the general point on the required locus

$\therefore x \equiv p$ — (1)

$y \equiv -p^2$ — (2)

(1) ⇒ $p = x$ in (2) $y = -(x)^2$

$\therefore$ locus of Q $x^2 = -y$ ✓ 1

(c) $v = x^3 - x$ $\boxed{2}$

$\ddot{x} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ 1

$\ddot{x} = (x^3-x)(3x^2-1)$ 1

(d) For two 1s $1\ \underline{1}\ \_\ \_$ No of ways $= 3\times9.8 = 216$ 1

For not having two 1s $1\ \underline{X}\ \underline{X}\ \underline{Y}$ No of ways $= 9.3\times8 = 216$ 1

TOTAL 432

or ${}^4C_2\times1\times9.8 = 432$ $\boxed{2}$

(e) $I = \int\cos^2\frac{x}{2}\,dx = \frac{1}{2}\int(1+\cos x)\,dx$ 1

$= \frac{1}{2}[x + \sin x] + c$ 1

$\boxed{2}$

Marker's Comments: 1 For $\cos^2A = \frac{1}{2}(1+\cos 2A)$ or equiv.

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments

Q4(a) $\left(2x^2 - \frac{3}{x}\right)^9$ $\boxed{2}$

General term $T_{r+1} = {}^9C_r(2x^2)^{9-r}\times\left(-\frac{3}{x}\right)^r \equiv Ax^0$

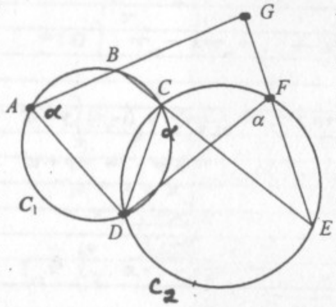
$\therefore {}^9C_r\,2^{9-r}(-3)^r.\,x^{18-2r}\times x^{-r} = Ax^0$. 1

$\Rightarrow 18 - 3r = 0$

$r = 6$ ✓

$\therefore$ Term is the seventh ✓ $T_7 = {}^9C_6\,2^3.3^6$ (= 489888) 1

(b) $\boxed{3}$



1. $\angle DCE = \alpha$ (Angles in same segment standing on arc DE are equal) ✓

2. $\angle DAB = \alpha$ (Exterior angle of cyclic Quad ABCD equals interior opposite angle) ✓

3. As $\angle DFE = \angle DAB = \alpha$

$\therefore$ AGED is a cyclic Quad as ✓
(Exterior angle equals interior opposite angle [converse])

(c) (i)

No of ways = 1 ODD + 3 ODD + 5 ODD

$= {}^6C_1\times{}^5C_5 + {}^6C_3\times{}^5C_3 + {}^6C_5\times{}^5C_1$ 1

$= 6 + 20\times10 + 6\times5$

$= 236$ 1

$\boxed{2}$

$\boxed{3}$

Marker's Comments: Note: O+O = E
E+E = E
Need Odd No of Odd No's for sum to be ODD

(ii) $P(E) = \frac{236}{{}^{11}C_6}$ ✓ $= \frac{236}{462} = \frac{118}{231}$ ① 1

(d) Let $P = 2000$, int rate $= 0\cdot08$, n is...

$R = 1\cdot08$ $M = 200$ ③

(i) After 1st prize:

$B_1 = P\times R - 200$

After 2nd prize awarded:

$B_2 = B_1R - 200 = (PR-200)R - 200$

$= PR^2 - 200(1+R)$ $\frac{1}{2}$

After 3rd :

$B_3 = B_2R - 200 = \left(PR^2 - 200(1+R)\right)R - 200$ $\frac{1}{2}$

$= PR^3 - 200(1+R+R^2)$

$\therefore$ After n^{th} : $B_n = PR^n - 200(1+R+R^2+\ldots+R^{n-1})$

$= PR^n - 200\left(\frac{R^n-1}{R-1}\right)$

$= 2000R^n - \frac{200}{0\cdot08}(R^n-1)$ 1

$= 2000R^n - 2500(R^n-1)$ 1

$= -500R^n + 2500 = 500[5 - 1\cdot08^n]$ req.

$\boxed{4}$

(ii) ①

Set $B_n = 0$

$\Rightarrow 1\cdot08^n = 5$

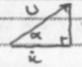
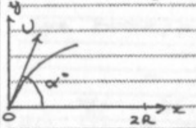
$n = \frac{\log 5}{\log 1\cdot08} = 20\cdot912$

$\therefore$ No of years is 20

MATHEMATICS Extension 1 : Question 5

Suggested Solutions	Marks	Marker's Comments
Q5(a) (i)		
$(9+5x)^{29} = {}^{29}C_0\, 9^{29} + {}^{29}C_1\, 9^{28}(5x) + {}^{29}C_2\, 9^{27}(5x)^2 + \dots$	①	
$\therefore p_k = {}^{29}C_k\, 9^{29-k}.5^k$ ✓ $k = 0, 1, 2, \dots, n$		
(ii) $\frac{p_{k+1}}{p_k} = \frac{{}^{29}C_{k+1}\cdot 9^{29-(k+1)}\cdot 5^{k+1}}{{}^{29}C_k\cdot 9^{29-k}\cdot 5^k}$ ②		
$= \frac{29!}{(k+1)!(28-k)!} \times \frac{k!(29-k)!}{29!} \times 9^{-1} \times 5^1$	1	
$= \frac{(29-k)}{(k+1)} \times \frac{1}{9} \times 5 = \frac{5(29-k)}{9(k+1)}$ ✓	1	1 For showing how to get the result
(iii) Find the least positive integer k such that $\frac{p_{k+1}}{p_k} = \frac{5(29-k)}{9(k+1)} \le 1$ ②		If do $\frac{p_{k+1}}{p_k} \geqslant 1$
so $145 - 5k \le 9k + 9$ and $k > 0$		$k = 9$
$136 \le 14k$		but still
$\therefore k \geqslant \frac{136}{14} = 9.714\dots$		$p_{9+1} = p_{10}$
$\therefore k = 10$ ✓	1	
$\therefore$ Largest coefft. is $p_{10} = {}^{29}C_{10}.9^{19}.5^{10} = {}^{29}C_{10}\, 3^{38}\, 5^{10}$	1	
(b) (i) $\frac{d}{dt}(We^{kt}) = \frac{dW}{dt}.e^{kt} + W.ke^{kt}$	1	
② $= -k(W+15)e^{kt} + kWe^{kt}$	1	
$= -kWe^{kt} - 15ke^{kt} + kWe^{kt}$		
$\therefore \frac{d}{dt}(We^{kt}) = -15ke^{kt}$ q.e.d.		
(ii) As $\frac{d}{dt}(We^{kt}) = -15ke^{kt}$ ②		
$\therefore We^{kt} = -15e^{kt} + C$	1	
when $t = 0$ $W = 20$		
so $20 = -15 + C$		
ie $C = 35$		
$\therefore We^{kt} = -15e^{kt} + 35$	1	
so $W = -15 + 35e^{-kt}$ q.e.d.		
(iii) (α) As $t = 5$ $W = 6$ ②		
$\therefore 6 = -15 + 35e^{-5k}$		
so $e^{-5k} = \frac{21}{35} = \frac{3}{5} = 0.6$		
$-5k = \ln 0.6$; $k = -\frac{\ln 0.6}{5}$ ✓	1	
$\therefore$ Rate $= -\left(-\frac{\ln 0.6}{5}\right)(6+15) = \frac{21 \ln 0.6}{5}$ using (i)		
$= -2.145\dots$		
Rate $= -2.1\,°C/min$ ✓	1	
(β) $-15 + 35e^{-kt} = -10$		
① $e^{-kt} = \frac{5}{35} = \frac{1}{7} \Rightarrow t = \frac{\ln(\frac{1}{7})}{-k} = 19.0467\dots$		

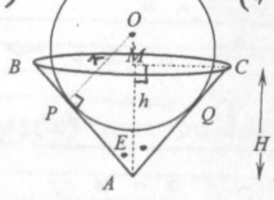
MATHEMATICS Extension 1 : Question 6.

Suggested Solutions	Marks	Marker's Comments
Q6(a) (i) $t = 0 \begin{cases} x = 0 \\ y = 0 \end{cases}$  $\dot{y} = U\sin\alpha$ ②		
 $\ddot{y} = -g$		
$\dot{y} = \int -g\,dt$		
$\dot{y} = -gt + c$		
but $t = 0$ $\dot{y} = U\sin\alpha$		
$\therefore c = U\sin\alpha$ ✓	1	
$\dot{y} = U\sin\alpha - gt$		
$y = \int(U\sin\alpha - gt)\,dt$		
$y = Ut\sin\alpha - \frac{1}{2}gt^2 + D$		
$t = 0$ $y = 0$ so $D = 0$ ✓	1	
$\Rightarrow y = Ut\sin\alpha - \frac{1}{2}gt^2$		
(ii) For the range : $y = 0$		
so $t(U\sin\alpha - \frac{1}{2}gt) = 0$		
② $\therefore t = 0$ or $t = \frac{2U\sin\alpha}{g}$	1	
$\therefore R = x = U.\frac{2U\sin\alpha}{g}.\cos\alpha = U^2\frac{\sin 2\alpha}{g}$ ✓	1	
(iii) (α) Max. height is 3.5m		or $\dot{y} = U\sin\alpha - gt = 0$
when $t = \frac{1}{2} \times \frac{2U\sin\alpha}{g} = \frac{U\sin\alpha}{g}$	1	
so $3.5 = U.\frac{U\sin\alpha}{g}.\sin\alpha - \frac{1}{2}g \times \frac{U^2\sin^2\alpha}{g^2}$		
$= \frac{U^2\sin^2\alpha}{g} - \frac{U^2\sin^2\alpha}{2g}$ ②		
$3.5 = \frac{U^2\sin^2\alpha}{2g}$	1	
so $U^2 = \frac{3.5 \times 2g}{\sin^2\alpha} = 7g\,\mathrm{cosec}^2\alpha$		
(β) Max R will then be $R = U^2\frac{\sin 2\alpha}{g}$		1 For subst (iii)(α) into (ii) and showing how $\frac{14\cos\alpha}{\sin\alpha}$
① $= \frac{7g}{\sin^2\alpha}.\frac{2\sin\alpha\cos\alpha}{g}$	1	
$= 14\frac{\cos\alpha}{\sin\alpha}$		
$\therefore$ max $R = 14\cot\alpha$ q.e.d.		

MATHEMATICS Extension 1 : Question 6

Suggested Solutions	Marks	Marker's Comments
Q6(b) (i) $f_2(x) = 1 + \frac{x}{1!} + \frac{x(x+1)}{2!}$ $= \frac{2 + 2x + x(x+1)}{2} = \frac{2+2x+x^2+x}{2}$		
$= \frac{x^2+3x+2}{2}$ ✓ $= \frac{1}{2!}(x+1)(x+2)$ ②	1	1 For getting to $\frac{x^2+3x+2}{2}$
and the zeros are -1 and -2	1	
(ii) Let $P(n)$ be the proposition that: $f_n(x) = 1 + \frac{x}{1!} + \frac{x(x+1)}{2!} + \ldots + \frac{x(x+1)x\ldots(x+n-1)}{n!} = \frac{1}{n!}(x+1)(x+2)\ldots(x+n)$ • Now $P(1)$ was given $P(2)$ was shown true in part (i) * Assume $P(n)$ is true for some integer $k \geq 1$	1	
i.e. $f_k(x) = 1 + \frac{x}{1!} + \frac{x(x+1)}{2!} + \ldots + \frac{x(x+1)\ldots(x+k-1)}{k!} = \frac{1}{k!}(x+1)(x+2)\ldots(x+k)$ —(*) RTP : $P(k+1)$ is true i.e. $f_{k+1}(x) = \frac{1}{(k+1)!}(x+1)(x+2)\ldots(x+k)$ PROOF : For $P(k+1)$ $f_{k+1}(x) = 1 + \frac{x}{1!} + \frac{x(x+1)}{2!} + \ldots + \frac{x(x+1)\ldots(x+k-1)}{k!} + \frac{x(x+1)(x+2)\ldots(x+k)}{(k+1)!}$		
$= \frac{1}{k!}(x+1)(x+2)\ldots(x+k) + \frac{x(x+1)\ldots(x+k-1)(x+k)}{(k+1)!}$	1	using assumption (*) 1 For using/substituting assumption
$= \frac{(x+1)(x+2)\ldots(x+k)}{k!}\left\{1 + \frac{x}{k+1}\right\}$ $= \frac{1}{k!}(x+1)(x+2)\ldots(x+k)\left\{\frac{k+1+x}{k+1}\right\}$	1	
$= \frac{1}{(k+1)!}(x+1)(x+2)\ldots(x+k+1)$ ③ $\therefore P(k+1)$ is true * $\therefore$ by the PMI $P(n)$ is true for $n = 1, 2, 3, \ldots$		

MATHEMATICS Extension 1 : Question 7

Suggested Solutions	Marks	Marker's Comments
(a) $V = \pi\int_{r-w}^{r}(r^2 - x^2)\,dx$ $\boxed{2}$ $= \pi\left[r^2x - \frac{1}{3}x^3\right]_{r-w}^{r}$ $= \pi\left[\left(r^3 - \frac{1}{3}r^3\right) - \left(r^2(r-w) - \frac{1}{3}(r-w)^3\right]$	1	
$= \pi\left[\frac{2}{3}r^3 - \frac{(r-w)}{3}\left(3r^2 - (r-w)^2\right)\right]$ $= \frac{\pi}{3}\left[2r^3 - (r-w)\left(3r^2 - r^2 + 2rw - w^2\right)\right]$ $= \frac{\pi}{3}\left[2r^3 - (r-w)\left(2r^2 + 2rw - w^2\right)\right]$ $= \frac{\pi}{3}\left[2r^3 - \left(2r^3 + \cancel{2r^2w} - rw^2 - \cancel{2r^2w} - 2rw^2 + w^3\right)\right]$ $= \frac{\pi}{3}\left[3rw^2 - w^3\right] = \frac{\pi}{3}(3r - w)w^2$	1	
(b)  (i) As $\Delta OPA \,\|\|\, \Delta CMA$ (equiangular) $\frac{r}{R} = \frac{OA}{AC}$ ✓ (corresponding sides in similar Δs are in the same ratio)	1	
$\frac{r}{R} = \frac{H + (r-h)}{L}$ $rL = HR + rR - hR$ $r(L-R) = (H-h)R$ ✓ ② $\therefore r = \frac{(H-h)R}{L-R}$	1	
(ii) using (a) where $h = w$, $r = \frac{(H-h)R}{L-R}$ $\therefore V = \frac{\pi}{3}\left(3.\frac{(H-h)R}{L-R} - h\right)h^2$ $= \frac{\pi}{3(L-R)}\left[3HR - \underline{3hR} - hL + \underline{hR}\right]h^2$ ① $= \frac{\pi}{3(L-R)}\left[3RHh^2 - (L+2R)h^3\right]$	1	1 For subst and simplifying to
(iii) $\frac{dV}{dh} = \frac{\pi}{3(L-R)}\left[6RHh - 3(L+2R)h^2\right]$ $= \frac{\pi}{L-R}\left[2RHh - (L+2R)h^2\right]$	½	
For possible max/min values of V to occur $\frac{dV}{dr} = 0$ $\therefore h(2RH - (L+2R)h) = 0$	1	
④ $\therefore h = 0$ or $h = \frac{2HR}{L+2R}$ but $h \neq 0$	1	
TEST : $\frac{d^2V}{dh^2} = \frac{\pi}{L-R}\left[2RH - 2(L+2R)h\right]$ at $h = \frac{2RH}{L+2R}$ $\frac{d^2V}{dh^2} = \frac{\pi}{L-R}\left[2RH - 4RH\right] = -\frac{2RH}{L-R} < 0$ as $L > R$ $\therefore$ a Relative max T.P. at $h = \frac{2RH}{}$. $r = \frac{RHL}{(L-R)(L+2R)}$	1 + 1	

MATHEMATICS Extension 1 : Question 7

Suggested Solutions	Marks	Marker's Comments
(c) (i) ① $(1-x)^{2n} = \binom{2n}{0} - \binom{2n}{1}x + \binom{2n}{2}x^2 - \binom{2n}{3}x^3 + \ldots + \binom{2n}{2n}x^{2n}$	1	
(ii) By differentiating both sides w.r.t x $-2n(1-x)^{2n-1} = -\binom{2n}{1} + 2\binom{2n}{2}x - 3\binom{2n}{3}x^2 + \ldots + 2n\binom{2n}{2n}x^{2n-1}$ ✓ put $x = 1$ ② $0 = -\binom{2n}{1} + 2\binom{2n}{2} - 3\binom{2n}{3} + \ldots - (2n-1)\binom{2n}{2n-1} + 2n\binom{2n}{2n}$ ✓ so $\binom{2n}{1} + 3\binom{2n}{3} + \ldots + (2n-1)\binom{2n}{2n-1} =$ $= 2\binom{2n}{2} + 4\binom{2n}{4} + \ldots + 2n\binom{2n}{2n}$ q.e.d.	1 1	1 For Differe... 1 For subst $x=1$ and