Name:____________________

Teacher:____________________

Saint Mark's Coptic Orthodox College

Mathematics Department

Year 11- Extension I

Assessment Task III
June 2005

Time Allowed: 2 Periods

Topics: Inequalities, Angle Between Lines, Dividing a line in a given ratio, Circle Geometry, 3D Trigonometry & Inductions

DIRECTIONS TO CANDIDATE:

- Attempt all questions.
- Show all necessary working. Marks may be deducted for careless or badly arranged work.
- Only approved calculators may be used.
- This paper contains **10** questions in **3** pages.

Office Use Only						
Section	**A**	**B**	**C**	**D**	**E**	**Total**
Mark	**/10**	**/13**	**/11**	**/6**	**/15**	**/55**

Mrs. S. Gerges

Section A (10Marks)

1) Solve for x: $|x^2 - 5| = 5x + 9$.† **4 Marks**

2) Solve for x: $\dfrac{x+4}{x-2} \geq 3$.† **3 Marks**

3) A is the point $(-2, 1)$ and B is the point (x, y). The point $P(13, -9)$ divides AB internally in the ratio $5 : 3$. Find the values of x and y.† **3 Marks**

Section B (13 Marks)

4) The acute angle between the line $x - 2y + 3 = 0$ and the line $y = mx$ is $45°$.

   i. Show that $\left|\dfrac{2m-1}{m+2}\right| = 1$. **4 Marks**

   ii. Find the possible values of m.† **2 Marks**

5) A and B are the points $(-5, 12)$ and $(4, 9)$ respectively. P is the point which divides AB externally in the ratio $5 : 2$.

   i. Find the co-ordinates of P. **2 Marks**

   ii. Show that if Q is the point $(0, 2)$, then triangle APQ is both right-angled and isosceles.† **5 Marks**

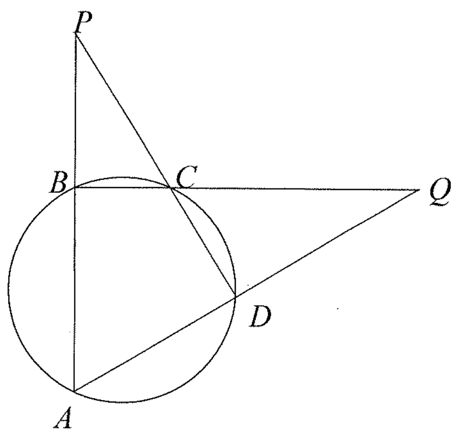
Section C (11Marks)

6) Two points A and B are taken on a circle, and C is the other end of the diameter through A. AE is the line from A perpendicular to the tangent at B.

   i. Draw a careful diagram showing this information. **2 Marks**

   ii. Prove that AB bisects $\angle CAE$.† **4 Marks**

7)



In the diagram above ABP, DCP, BCQ, and ADQ are all straight lines and $\angle APD = \angle BQA$.

   i. Show that $\angle ABC = \angle ADC$. **2 Marks**

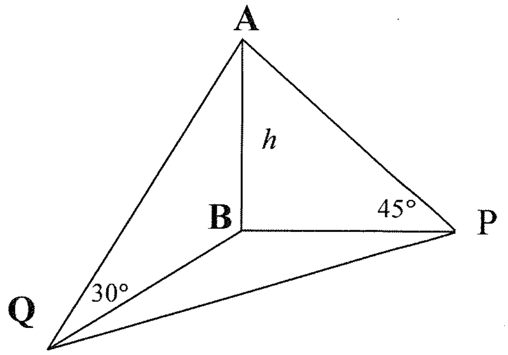
   ii. Prove that AC is a diameter of the circle.† **3 Marks**

Section D (6 Marks)

8) Let $S_n = 1 \times 2 + 2 \times 3 + \ldots + (n-1) \times n$. Use mathematical induction to prove that, for all integers n with $n \geq 2$, $S_n = \frac{1}{3}(n-1)\,n\,(n+1)$. **6 Marks**

Section E (15 Marks)

9)



A vertical tower AB of height h metres stands on horizontal ground. From a point P on the ground due east of the tower the angle of elevation of the top of the tower is $45°$. From a point Q on the ground due south of the tower the angle of elevation of the top of the tower is $30°$.
If the distance PQ is 40 metres, find the exact height of the tower.† **5 Marks**

10) A person walking along a straight road observes a tower bearing 065°T, the angle of elevation being 15°. After travelling a distance of 1000m, the tower now bears 305°T and the angle of elevation of 20°.

i. Draw a diagram and mark on it all the information given. **3 Marks**

ii. Show that the height of the tower is given by the expression

$$h^2 = \frac{1000^2}{\cot^2 15^\circ + \cot^2 20^\circ + \cot 15^\circ \cot 20^\circ}$$

Hence, find the height of the tower to the nearest metre. **4 Marks**

iii. Find the bearing of the person now from his starting point. **3 Marks**

[End Of Qns]

Year 11 Ext I Task 3 - 2005

Section A (10 Marks)

1/ $|x^2 - 5| = 5x + 9$.

$x^2 - 5 = 5x + 9$ or $x^2 - 5 = -5x - 9$

$x^2 - 5x - 14 = 0$ $\quad$ $x^2 + 5x + 4 = 0$

$(x-7)(x+2) = 0$ $\quad$ $(x+4)(x+1) = 0$

$x = 7$ or -2 $\quad$ $x = -4, -1$

Test $x = 7$ $\quad |7^2 - 5| = 35 + 9$ ✓ $\quad$ $x = -4, \; |16 - 5| = -20 + 9$ ✗

$x = -2$ $\quad |4 - 5| = -10 + 9$ ✗ $\quad$ $x = -1$ $\quad |1 - 5| = -5 + 9$ ✓

$\therefore x = 7, -1$ only.

/4

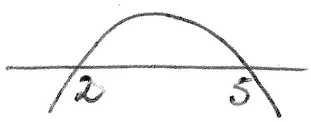
2/ $\dfrac{x+4}{x-2} \geqslant 3$ $\qquad x \neq 2$.

$(x-2)(x+4) \geqslant 3(x-2)^2$

$(x-2)(x+4) - 3(x-2)^2 \geqslant 0$.

$(x-2)[x + 4 - 3x + 6] \geqslant 0$

$(x-2)(10 - 2x) \geqslant 0$



$2 < x \leqslant 5$

/3

3/ $A(-2, 1)$ $\quad B(x, y)$ $\quad P(13, -9)$ Ratio $5:3$.

$x = \dfrac{mx_2 + nx_1}{m+n}$ $\qquad$ $y = \dfrac{my_2 + ny_1}{m+n}$

$13 = \dfrac{5x + 3 \times -2}{8}$ $\qquad$ $-9 = \dfrac{5y + 3 \times 1}{8}$

$104 = 5x - 6$ $\qquad$ $-72 = 5y + 3$

$5x = 110$ $\qquad$ $5y = -75$

$x = 22$ $\qquad$ $y = -15$ $\quad \therefore B(22, -15)$

/3

Section B (13 Marks)

4/ $x - 2y + 3 = 0$ $\qquad$ $y = mx$

$\therefore$ grad $= \frac{1}{2}$ $\qquad$ grad $= m$.

(i) $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ $\qquad$ where m_1 + m_2 are grad of lines

$\tan 45 = \left|\frac{m - \frac{1}{2}}{1 + \frac{m}{2}}\right|$

$1 = \left|\frac{2m-1}{2} \div \frac{2+m}{2}\right|$

$1 = \left|\frac{2m-1}{m+2}\right|$ $\therefore$ Shown.

/4

(ii) $\frac{2m-1}{m+2} = 1$ $\qquad$ or $\qquad$ $\frac{2m-1}{m+2} = -1$

$2m - 1 = m + 2$ $\qquad$ $2m - 1 = -m - 2$

$m = 3$ $\qquad$ $3m = -1$

$m = -\frac{1}{3}$.

/2

5/ $A(-5, 12)$ $\quad$ $B(4, 9)$ $\quad$ divides externally ie $5 : -2$

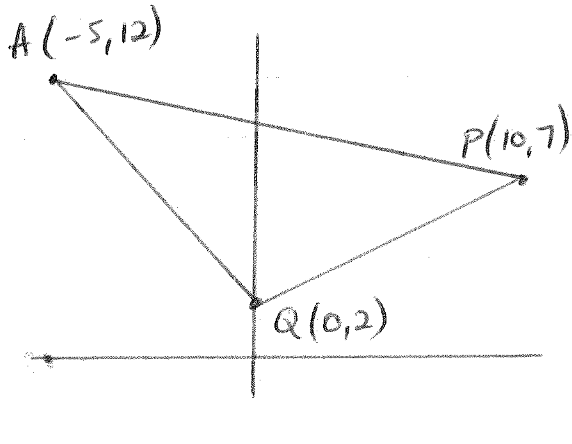
(i) $x = \frac{5 \times 4 - 2 \times -5}{3}$ $\qquad$ $y = \frac{5 \times 9 - 2 \times 12}{3}$

$= 10$ $\qquad$ $= 7$

$\therefore P(10, 7)$

/2

(ii) APQ $\quad$ $Q(0, 2)$



$m_{AQ} = \frac{12-2}{-5}$ $\qquad$ $m_{PQ} = \frac{7-2}{10-0}$

$= -2$ $\qquad$ $= \frac{1}{2}$

$\because m_{AQ} \times m_{PQ} = -1$ $\quad$ $\therefore AQ \perp PQ$.

$d_{AQ} = \sqrt{(-5-0)^2 + (12-2)^2}$
$= \sqrt{125}$ u.

$d_{PQ} = \sqrt{(10-0)^2 + (7-2)^2}$
$= \sqrt{125}$ u.

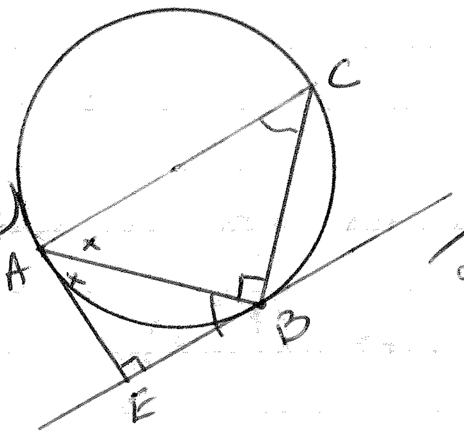
$\therefore d_{AQ} = d_{PQ}$

$\therefore \Delta APQ$ is both right angled + isosc. Δ. /5

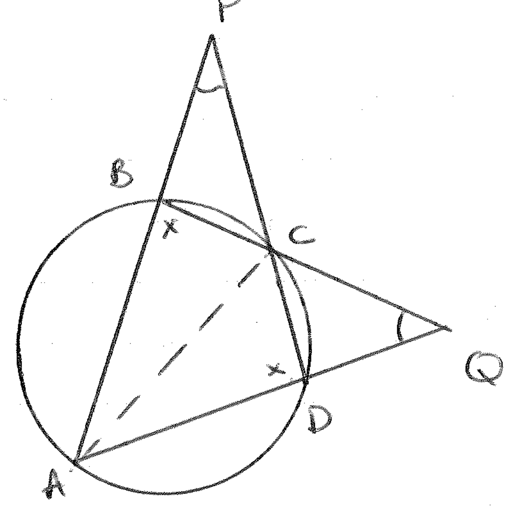
Section C (11 Marks)

6/

(ii) Prove AB bisects $\angle CAE$ (i)



$\because$ AC is a diameter.
$\therefore \angle ABC = 90°$ ($\angle$ in semi-circle) /2
$\angle ABE = \angle ACB$
($\angle$ bet tang + chord = $\angle$ in alt. seg).
$\therefore \angle CAB = 90° - \angle ACB$.
$\because$ AE $\perp$ EB.
$\therefore \angle EAB = 90° - \angle ABE$.
$\therefore \angle CAB = \angle EAB$
$\therefore$ AB bisects $\angle CAE$ ..... /4



7/ given $\angle APD = \angle BQA$

(i) Show $\angle ABC = \angle ADC$
In ΔAPD + ΔABQ
$\angle A$ is common
$\angle APD = \angle BQA$ (given)
$\therefore \angle ABC = \angle ADC$ (3rd $\angle$ of Δ). /2.

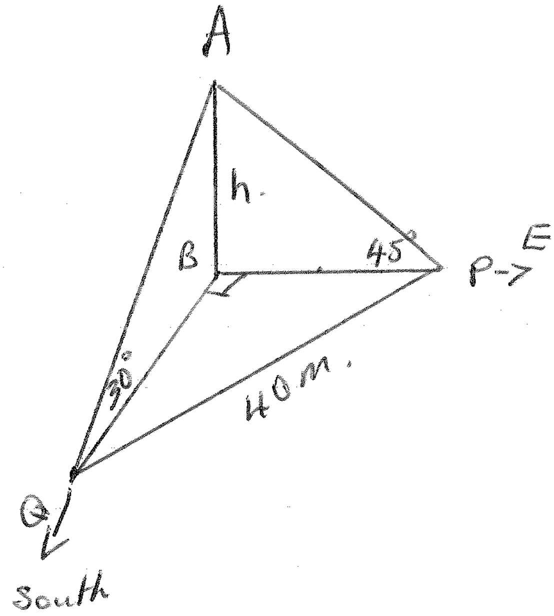
(ii) Prove AC is a diameter.
ABCD is a cyclic quad.
$\therefore \angle ABC + \angle ADC = 180°$.
$\because \angle ABC = \angle ADC$ shown above /3.
$\therefore \angle ABC = \angle ADC = 90°$ $\therefore$ AC is a diameter, forming angles of 90° in semi-circle.

Section E: (15 Marks)

8) In Δ ABP

$$\tan 45^\circ = \frac{h}{BP}$$

$$BP = h.$$



In ΔABQ

$$\tan 30^\circ = \frac{h}{BQ}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{BQ}$$

$$BQ = \sqrt{3}h.$$

In Δ PBQ

$$40^2 = BP^2 + BQ^2$$

$$1600 = h^2 + 3h^2$$

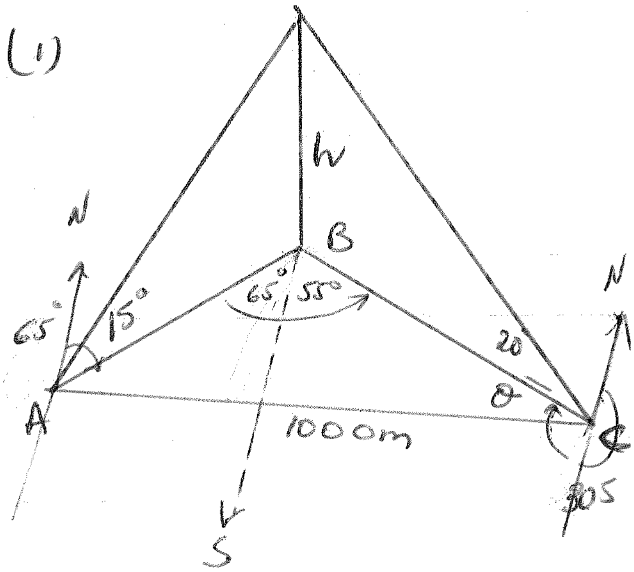
$$1600 = 4h^2$$

$$h = \sqrt{400}$$

$$= 20 \text{ m}.$$

5

(i)



9/(ii) $\tan 15^\circ = \frac{h}{AB}$

$$AB = h\cot 15^\circ$$

$$\tan 20^\circ = \frac{h}{BC}$$

$$BC = h\cot 20^\circ.$$

In ΔABC.

$$1000^2 = AB^2 + BC^2 - 2\times AB\times BC\times\cos 120^\circ$$

$$= h^2\cot^2 15 + h^2\cot^2 20^\circ - 2\times h\cot 15^\circ\times h\cot 20\times\frac{-1}{2}$$

$$1000^2 = h^2(\cot^2 15 + \cot^2 20 + \cot 15\cot 20)$$

$$h = 177.5 \text{ m}.$$

$$\sim 178\text{m}.$$

(iii) $BC = h\cot 20$
$= 178 \cot 20$
$= 489.1 \text{ m}$

$$\frac{\sin\theta}{489.1} = \frac{\sin 120}{1000}$$

$$\sin\theta = \frac{489.1 \sin 120}{1000}$$

$$\theta \sim 25^\circ$$

Bearing $= 65^\circ + 25$
$\sim 90^\circ T$