SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2001
HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK # 1

Mathematics Extension 1

General Instructions
- Reading time – 5 minutes.
- Working time – 1 hour
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question.

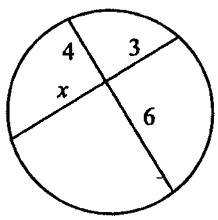
Total Marks - 62 marks
- Attempt questions 1 – 4
- All questions are **NOT** of equal value.

Examiner: *P. R. Bigelow*

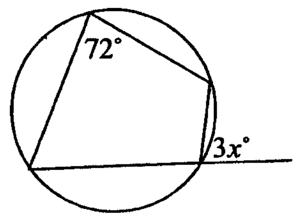
Question 1 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the value of the pronumeral in the following (give a brief reason) [Diagrams are NOT to scale] **4**

(i)



(ii)

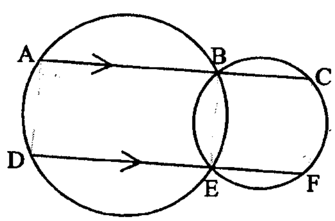


(b) Write down the value of n in the following: **2**

$$\binom{n}{2} = 78.$$

(c) Write down the equation of the chord of contact from the point (5,-2) to the parabola $x^2 = 20y$. **2**

(d) **3**



Two circles intersect at B and E. AC is drawn through B parallel to DF, through E.

Prove that ACFD is a parallelogram.

(e) Find the second derivative for each of:

(i) $\dfrac{x}{x+1}$ **2**

(ii) $(x^2+4)^3$ **2**

Question 2 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) For the function $y = x^3 - 9x + 3$

(i) Find the coordinates of the stationary points and determine their nature **2**

(ii) Find the coordinates of any points of inflexion. **2**

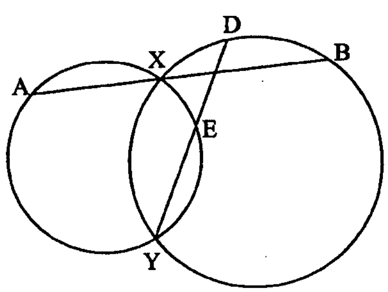
(iii) Sketch the curve in the domain $-4 \leq x \leq 4$. **2**

(iv) What is the greatest value of $x^3 - 9x + 3$ in the domain $-4 \leq x \leq 4$? **1**

(b) (i) Show that the normal to the parabola $x^2 = 4ay$ at the point $T(2at, at^2)$ has equation $x + ty = 2at + at^3$. **3**

(ii) Hence show that there is only one normal to the parabola which passes through its focus. **2**

(c) **3**



The two circles intersect at X and Y.
AXB and DEY are straight lines.

Copy the diagram into your booklet and prove that AE is parallel to DB.

Question 3 (16 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $f''(2)$ if **2**

$$f(x) = 1 - \frac{1}{x^2}$$

(b) Use Mathematical Induction to prove that $7^n + 5$ is divisible by 6 for all positive integers n. **4**

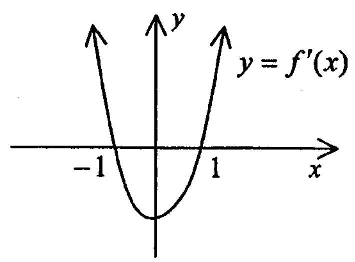
(c) In how many ways can 6 people enter a room? (Assuming they enter one at a time). **2**

(d) From a group consisting of 6 women and 5 men, how many different committees can be formed consisting of 2 women and 3 men? **2**

Question 3 (continued) **Marks**

(e) In how many ways may 9 people be seated around two circular tables if there are 5 at one table and 4 at the other? **2**

(f)



The diagram shows the graph of the *gradient function* of the function $y = f(x)$.

(i) What type of point occurs on $y = f(x)$ at $x = -1$? Justify your answer. **2**

(ii) If $f(-1) = 4$ and $f(x) > 0$ sketch $y = f(x)$. **2**

Marks

Question 4 (16 marks) Use a SEPARATE writing booklet

(a) (i) Express 21 000 as a product of prime numbers (in index notation) **2**

(ii) Hence or otherwise find the number of factors of 21 000. **2**

(b) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

(i) Find the gradient of PQ. **1**

(ii) Hence or otherwise show that the equation of PQ is **2**
$y - \left(\frac{p+q}{2}\right)x + apq = 0$.

(iii) If PQ passes through $(2a, 0)$ show that $pq = p + q$. **1**

(iv) Hence find the locus of M, the mid-point of PQ (subject to the condition in (iii)). **3**

(c) (i) By considering the sum of the terms of an arithmetic series, show that: **2**
$(1 + 2 + 3 + \dots\dots\dots + n)^2 = \frac{1}{4}n^2(n+1)^2$.

(ii) By using the Principle of Mathematical Induction. prove that: **3**
$1^3 + 2^3 + 3^3 + \dots\dots\dots + n^3 = (1 + 2 + 3 + \dots\dots\dots + n)^2$.
for all $n \geq 1$.

THIS IS THE END OF THE PAPER