Student Number: ………….…………………….…

2008
HIGHER SCHOOL CERTIFICATE
Sample Examination Paper

MATHEMATICS

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is supplied at the back of this paper
- All necessary working should be shown in every question
- Write your student number at the top of this page

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

Directions to school or college

ISBN: 978 1 44250 357 1

Total marks – 120
Attempt Questions 1–10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\dfrac{0.1}{\sqrt{e+1}}$ correct to two significant figures. **2**

(b) Factorise $2x^2 - 4x + 2$ completely. **2**

(c) Write down the primitive function of $\dfrac{3}{x} + 5$. **2**

(d) Solve $\dfrac{x}{4} = 3 - \dfrac{x-2}{3}$, leaving your answer as an improper fraction. **3**

(e) Fred invests \$1000 at 4½% p.a. compound interest (compounded annually). Doreen also invests \$1000 but is paid simple interest at 5% p.a. (with interest added at the end of each year).

What is the minimum period of investment for Fred to have earned more interest than Doreen, and how much extra does he have at this time? **3**

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Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Draw a neat sketch of a number plane and plot the points $A(-4, 0)$, $B(4, 0)$ and $C(0, 8)$ on it. **1**

(b) Find the gradient of AC and show that the equation of AC is $2x - y + 8 = 0$. **2**

(c) Find the perpendicular distance of AC from $Z(0, 3)$. **2**

(d) If X the midpoint of AC and Y the midpoint of BC, find the coordinates of X and Y. **1**

(e) Show that XZ is perpendicular to AC. **2**

(f) Show that the lengths $AZ = BZ = CZ = 5$ units. **2**

(g) Find the equation of the circle passing through A, B and C. **2**

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Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) Differentiate

(i) $\left(x^2+3\right)^6$ **1**

(ii) $x^2 \sin 3x$ **2**

(b) Anna sets sail from the port of Newton on a bearing of 230° at a speed of 10 kilometres per hour. At the same time, Bree sets off from Newton in her motor boat, travelling on a bearing of 140° at 24 kilometres per hour. After 2 hours, Bree's motor breaks down and she calls for Anna to come and help her.

(i) Draw a diagram to show Anna and Bree's positions 2 hours after they leave port. **2**

(ii) Determine how far Anna and Bree are from port when Bree's motor breaks down, and calculate the distance Anna and Bree are apart. **2**

(iii) Show that, in order to rescue Bree, Anna must sail on a bearing of $117°23'$ (to the nearest minute). **2**

(iv) When Anna has sailed 40 km towards Bree, she sends a message giving her distance from the port. What is this distance (correct to three significant figures)? **3**

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Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) (i) A circle of radius 5 units, centred at the origin, is rotated about the x-axis. Show that the volume of the solid of revolution is given by **2**

$$V = 2\pi\int_0^5 (25 - x^2)\,dx\,.$$

(ii) Use calculus to show that the volume may be given as **2**

$$V = \frac{4}{3}\pi\times 5^3 \text{ unit}^3.$$

(iii) Explain why Simpson's rule would give the exact value of this volume. **1**

(iv) Use one application of Simpson's rule (two strips) to find the volume. **3**

(b) A bag contains 6 white socks and 4 black socks. When you select a sock, you cannot tell what colour it is until it is removed from the bag. Two socks are selected from the bag.

(i) What is the probability that the first sock is white? **1**

(ii) What is the probability that a pair is selected? **2**

(iii) If four socks are selected, what is the probability that they are all black? **1**

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Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) Rhonda earns \$50 000 in her first year working. Each year, she is given a 5% pay rise at the end of December.

(i) Let the amount Rhonda earns in year n be A_n. Write down A_1, A_2, A_3 and show that $A_n = 50\,000 \times 1.05^{n-1}$. **2**

(ii) By using the formula for the sum of a geometric series, derive an expression for the total amount Rhonda earns in 40 years. **3**

(iii) Calculate this total to the nearest cent. **1**

(iv) Rhonda is paid monthly. How long will she have to keep working (in years and months) to earn a total over \$10 million? **3**

(b) (i) Show that $N = N_0 e^{\frac{t}{5}}$ is a solution to the equation $\dfrac{dN}{dt} = \dfrac{N}{5}$. **1**

(ii) Show that the time t taken for N to double from N_0 to $2N_0$ is $t = \ln 32$. **2**

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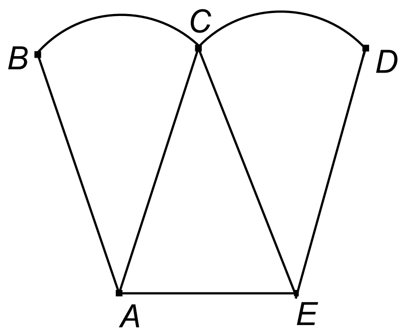
Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

A business logo is designed in the shape below.
ACE is an isosceles triangle with base angles 65°.
ABC and ECD are sectors of a circle of radius 100 cm.
$\angle BAC = \angle ACE = \angle CED$



NOT
TO
SCALE

(a) Copy or trace the diagram into your answer booklet and mark the given information on the diagram. **1**

(b) Prove that $\angle ACE = 50°$. **1**

(c) Explain why $BA = AC = CE = ED = 100$ cm. **1**

(d) Find the size of $\angle ACE$ in radians in terms of π. **1**

(e) Find the area ΔACE of in cm^2 correct to two significant figures. **2**

(f) Find the area of sector ABC to two significant figures. Hence find the area of the whole logo and the cost of painting the logo at a rate of $\$500/\text{m}^2$. **3**

(g) Find the cost of gold chain to cover all of the lines and curves in the diagram of the logo at a cost of \$1000/m. **3**

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Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

Consider the function $A = \dfrac{x}{\ln x}$.

(a) For what values of x is the function differentiable? **2**

(b) Find the gradient of the tangent to the curve when $x = e^2$. **3**

(c) Find where the gradient of a tangent to the curve is -1. **3**

(d) Find the coordinates of all stationary points and determine their nature. **4**

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Marks

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) Sketch a number plane using $-1 \le x \le 8$ and $-1 \le y \le 8$.
Mark the points $A(0, e^2)$, $B(e^2, 0)$ and $C(e^2, e^2)$ clearly on this number plane.
Draw a neat sketch of the graph of $y = e^{2x}$ over the domain $-1 \le x \le 1$. **3**

(b) Change the subject of $y = e^{2x}$ to show that $x = \dfrac{\log_e y}{2}$. **1**

(c) On the same sketch as part (a), draw $y = \dfrac{\log_e x}{2}$ over the domain $0 \le x \le e^2$. **2**

(d) Evaluate $\int_0^1 e^{2x} dx$. **2**

(e) Hence find the area bounded by $y = e^{2x}$, the y-axis and $y = e^2$. **2**

(f) Evaluate the area bounded by $y = e^{2x}$, $y = \dfrac{\log_e x}{2}$, $y = e^2$, $x = e^2$, $y = 0$
and $x = 0$. **2**

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Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

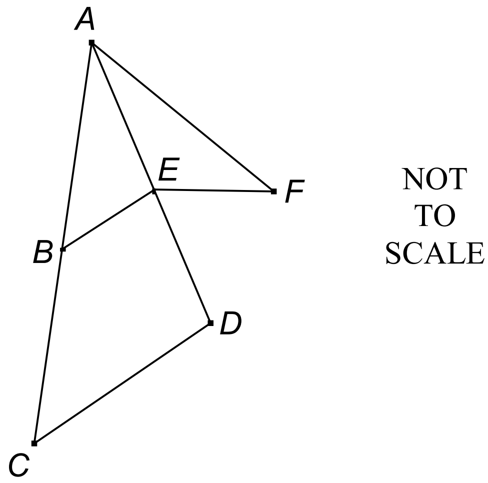
In the diagram:

$BE \parallel CD$

AD bisects $\angle BAF$

$\angle AFE = \angle ABE$

$AF = 6$, $AE = 5$ and $ED = 8$



(a) Copy or trace the diagram into your answer booklet and mark the given information on the diagram. **1**

(b) Prove that: $\Delta ABE \equiv \Delta AFE$ **2**

(c) Prove that: $AB.AD = AC.AE$ **3**

(d) Evaluate the length BC. **2**

(e) Show that: $AB.ED = AE.BC$ **2**

(f) Deduce that: $AC.ED = BC.AD$ **2**

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Marks

Question 10 (12 marks) Use a SEPARATE writing booklet.

Two objects, A and B, are projected vertically from the edge of a cliff of height π^2 metres. The position of object A, x_A, is given by the equation

$$x_A = 2\pi t - t^2 + \pi^2.$$

The velocity of object B, $\dot{x}_B$, is given by the equation

$$\dot{x}_B = \frac{3}{2}\cos\frac{t}{2}.$$

(a) At what time(s) is object A

(i) at height π^2? **1**

(ii) at ground level? (Give your answer in exact form.) **3**

(b) Draw a neat sketch graph showing the position of object A, from the time it is projected until it hits the ground. **2**

(c) Find the equation determining the position of object B. Draw the graph of this equation on the same number plane as part (b). **3**

(d) At what time do the two objects have the same velocity? **1**

(e) At what times are the two objects accelerating in opposite directions? How is this indicated on the graph of x_A and x_B? **2**

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Mapping grid

Question	Mark	Content	Outcome	Band
1(a)	2	Basic arithmetic and algebra	P1	1–2
1(b)	2	Basic arithmetic and algebra	P4	2–3
1(c)	2	Integration	H3	2–3
1(d)	3	Basic arithmetic and algebra	P4	2–4
1(e)	3	Basic arithmetic and algebra	P4	2–5
2(a)	1	Linear functions and straight lines	P4	1–2
2(b)	2	Linear functions and straight lines	P4	2–3
2(c)	2	Linear functions and straight lines	P4	3–5
2(d)	1	Linear functions and straight lines	P4	1–2
2(e)	2	Linear functions and straight lines	P4	2–3
2(f)	2	Linear functions and straight lines	P4	2–3
2(g)	2	Linear functions and straight lines	H5	3–5
3(a)	3	Derivative of a function	P7	1–3
3(b)	9	Use of trig ratios	P4	2–4
4(a)	8	Integration	H8, H9	3–6
4(b)	4	Probability	H5	1–4
5(a)	9	Series and applications	H5	3–5
5(b)	3	Applications of calculus to the physical world	H5	3–5
6(a)	1	Circular measure	H3	1–2
6(b)	1	Circular measure	H4	1–3
6(c)	1	Circular measure	H5	1–2
6(d)	1	Circular measure	H5	2–4
6(e)	2	Circular measure	H5	2–3
6(f)	3	Circular measure	H5	2–4
6(g)	3	Circular measure	H5	2–4
7(a)	2	Geometric applications of calculus	H6	4–6
7(b)	3	Geometric applications of calculus	H3, H7	3–5
7(c)	3	Geometric applications of calculus	H3, H9	3–5
7(d)	4	Geometric applications of calculus	H3, H9	3–6
8(a)	3	Exponential functions	H1	2–4
8(b)	1	Logarithmic and exponential functions	H3	2–3
8(c)	2	Logarithmic functions	H5	2–3

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Question	Mark	Content	Outcome	Band
8(d)	2	Exponential functions	H5	2–3
8(e)	2	Exponential functions	H3	3–4
8(f)	2	Logarithmic and exponential functions	H5	3–4
9(a)	1	Geometry	H2	2
9(b)	2	Geometry	H5	1–2
9(c)	3	Geometry	H9	2–3
9(d)	2	Geometry	H5	3–5
9(e)	2	Geometry	H5	3–4
9(f)	2	Geometry	H6	3–5
10(a)	4	Applications of calculus to the physical world	H5	3–6
10(b)	2	Applications of calculus to the physical world	H5	3–4
10(c)	3	Applications of calculus to the physical world	H6	3–5
10(d)	1	Applications of calculus to the physical world	H5	3
10(e)	2	Applications of calculus to the physical world	H9	3–6

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Marking guidelines

Question 1

Criteria	Marks
(a) $\dfrac{0.1}{\sqrt{e+1}} = 0.0518...$ $= 0.052$ (correct to 2 significant figures)	1 value 1 rounding 2 full answer
(b) $2x^2 - 4x + 2 = 2(x^2 - 2x + 1)$ $= 2(x-1)^2$	1 common factor 2 full
(c) $\int \left(\dfrac{3}{x} + 5 \right) dx = 3\ln x + 5x + c$	1 for each part 2 full
(d) $\dfrac{x}{4} = 3 - \dfrac{x-2}{3}$ $3x = 36 - 4(x-2)$ $3x = 36 - 4x + 8$ $7x = 44$ $x = \dfrac{44}{7}$	1 for 2nd line 2 for +8 3 full
(e) $1000 + 1000 \times 0.05 \times n = 1000(1.045)^n$ Solve by trial and error $n = 1$ $1000 + 50 = 1050$ $1000(1.045) = 1045$ $n = 2$ $1000 + 50 \times 2 = 1100$ $1000(1.045)^2 = 1092.025$ $= 1092.02$ rounded down $n = 3$ $1000 + 50 \times 3 = 1150$ $1000(1.045)^3 = 1141.166125$ $= 1141.16$ rounded down $n = 4$ $1000 + 50 \times 4 = 1200$ $1000(1.045)^4 = 1192.518600$ $= 1192.51$ rounded down $n = 5$ $1000 + 50 \times 5 = 1250$ $1000(1.045)^4 = 1246.18193$ $= 1246.17$ rounded down $n = 6$ $1000 + 50 \times 6 = 1300$ $1000(1.045)^6 = 1302.260$ $= 1302.24$ rounded down Fred has earned more than Doreen after 6 years. Difference = \$2.26 (if full accuracy used) or \$2.24 (if rounded down after each compound interest is added) or \$2.25 (if rounded off).	1 for equation 1 for a reasonable attempt to solve 3 full answer

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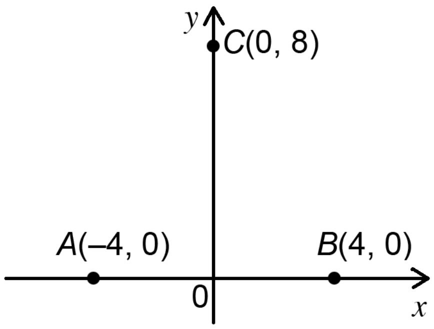
Question 2

Criteria	Marks
(a) 	1
(b) $m = \dfrac{8-0}{0-(-4)} = 2$ C is the y-intercept, so $b = 8$ $y = 2x + 8$ $2x - y + 8 = 0$	1 for mostly correct 2 full answer
(c) $d = \dfrac{\lvert 2x - y + 8 \rvert}{\sqrt{2^2 + (-1)^2}}$ $= \dfrac{\lvert 2\times 0 - 3 + 8 \rvert}{\sqrt{5}}$ $= \dfrac{5}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = \sqrt{5}$	1 for mostly correct 2 full answer
(d) $X\left(\dfrac{-4+0}{2}, \dfrac{0+8}{2}\right)$ and $Y\left(\dfrac{4+0}{2}, \dfrac{0+8}{2}\right)$ $X(-2, 4)$ and $Y(2, 4)$	1
(e) $m_{XZ} = \dfrac{3-4}{0-(-2)} = \dfrac{-1}{2}$ $m_{XZ} \times m_{AC} = 2 \times \dfrac{-1}{2} = -1$, so XZ is perpendicular to AC.	1 for gradient 1 for test 2 full answer
(f) $d_{AZ} = \sqrt{(0 - -4)^2 + (3-0)^2} = 5$ $d_{BZ} = \sqrt{(0-4)^2 + (3-0)^2} = 5$ $d_{CZ} = \sqrt{(0-0)^2 + (3-8)^2} = 5$	1 for mostly correct 2 full answer
(g) From part (f), Z is the centre of the circle with radius 5 passing through A, B and C. $x^2 + (y-3)^2 = 25$	1 for circle equation 2 full answer

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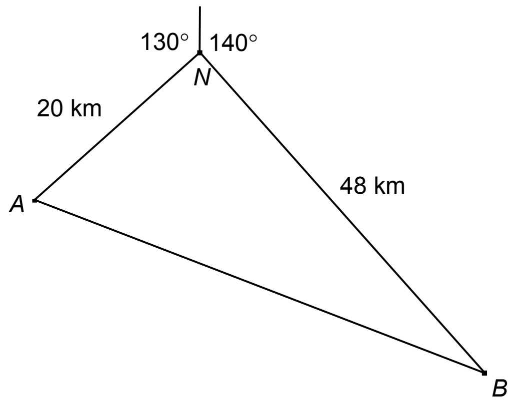
Question 3

Criteria	Marks
(a) (i) $\frac{d}{dx}\left[\left(x^2+3\right)^6\right]=6\left(x^2+3\right)^5\times 2x$ $=12x\left(x^2+3\right)^5$	1
(ii) $\frac{d}{dx}\left[x^2\sin 3x\right]=2x\times\sin 3x+x^2\times 3\cos 3x$ $=x\left(2\sin 3x+3x\cos 3x\right)$	1 for mostly correct 2 full answer
(b) (i) Position of Anna (*A*) and Bree (*B*) from Newton (*N*) after 2 hours:  	1 for mostly correct 2 full answer
(ii) Anna is 20 km from Newton and Bree is 48 km from Newton. $d=\sqrt{20^2+48^2}$ $d=\sqrt{2704}=52$ Anna and Bree are 52 km apart when Bree's motor breaks down.	1 for mostly correct 2 full answer
(iii) $\tan\left(\angle NAB\right)=\frac{48}{20}$ $\angle NAB=67.38013505$ $=67°22'48.49$ $=67°23'$ to the nearest minute Bearing from A to $B = 67°23'+50°=117°23'$ to the nearest minute	1 correct angle 2 full answer
(iv) $d^2=20^2+40^2-2\times 20\times 40\cos 67°23'$ $d=37.2346...$ $=37.2$ (to 3 significant figures)	1 formula correct 2 for mostly correct 3 full answer

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Question 4

Criteria	Marks
(a)(i) $V=\pi\int_{-5}^{5} y^2dx$ $=\pi\int_{-5}^{5}\left(25-x^2\right)dx$ $=2\pi\int_{0}^{5}\left(25-x^2\right)dx$	1 for mostly correct 2 full answer
(ii) $V=2\pi\left[25x-\dfrac{x^3}{3}\right]_0^5$ $=2\pi\left(\left[125-\dfrac{125}{3}\right]-0\right)$ $=2\pi\left(\dfrac{375-125}{3}\right)$ $=\dfrac{500\pi}{3}$ $=\dfrac{4\times125\pi}{3}$ $=\dfrac{4}{3}\pi\times5^3 \text{ unit}^3$	1 for mostly correct 2 full answer
(iii) Since the integral to be evaluated is a quadratic *and* Simpson's rule uses parabolic sections to estimate integrals.	1
(iv) $V=\pi\int_{-5}^{5} y^2dx$ Using $h=5,\ x_1=-5,\ x_2=0,\ x_3=5$ $V=\pi\dfrac{h}{3}\left[y_0+4y_1+y_2\right]$ $=\dfrac{5\pi}{3}\left[\left(25-(-5)^2\right)+4(25-0)+\left(25-(5)^2\right)\right]$ $=\dfrac{5\pi}{3}\left[0+100+0\right]$ $=\dfrac{500\pi}{3} \text{ unit}^3$	1 for using Simpson's rule 2 for calculating area 3 full answer
(b)(i) $\text{P(white)}=\dfrac{6}{10}=\dfrac{3}{5}$	1
(ii) P(pair) = p(ww)+p(bb) $=\dfrac{3}{5}\times\dfrac{5}{9}+\dfrac{2}{5}\times\dfrac{3}{9}=\dfrac{15}{45}+\dfrac{6}{45}$ $=\dfrac{21}{45}=\dfrac{7}{15}$	1 for mostly correct 2 full answer
(iii) $\text{P(bbbb)}=\dfrac{4}{10}\times\dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{1}{7}=\dfrac{1}{210}$	1

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Question 5

Criteria	Marks
(a)(i) $A_1 = 50\,000$ $\quad a = 50\,000 \quad r = 1.05$ $A_2 = 50\,000 \times 1.05$ $\quad A_n = ar^{n-1}$ $A_3 = 50\,000 \times 1.05^2$ $\quad A_n = 50\,000 \times 1.05^{n-1}$	1 for A_1, A_2, A_3 1 use of GP formula 2 full answer
(ii) $S_{40} = A_1 + A_2 + A_3 + \ldots + A_{40}$ $= 50\,000 + 50\,000 \times 1.05 + 50\,000 \times 1.05^2 + \ldots + 50\,000 \times 1.05^{39}$ $= 50\,000(1 + 1.05 + 1.05^2 + \ldots + 1.05^{39})$ $= \dfrac{50\,000((1.05)^{40} - 1)}{0.05}$	1 for line 2 1 for formula 1 for 3rd line 3 full answer
(iii) \$6 039 988.712 = \$6 039 988.71 to the nearest cent	1
(iv) $\dfrac{50\,000\left[(1.05)^n - 1\right]}{0.05} = 10\,000\,000$ $(1.05)^n - 1 = \dfrac{10\,000\,000}{50\,000} \times 0.05$ $(1.05)^n = 10 + 1$ $\ln(1.05)^n = \ln 11$ $n \ln(1.05) = \ln 11$ $n = \dfrac{\ln 11}{\ln(1.05)}$ $= 49.147104$ $= 49$ years and 1.76 months Rhonda must work for 49 years and 2 months to earn over \$10 million.	1 for 3rd line 1 for using logs 3 full answer

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Question 5 (continued)

Criteria	Marks
(b)(i) $N = N_0 e^{\frac{t}{5}}$ $\frac{d}{dt}N = N_0 e^{\frac{t}{5}} \times \frac{1}{5}$ $= \frac{N_0 e^{\frac{t}{5}}}{5}$ $= \frac{N}{5}$	1
(ii) $2N_0 = N_0 e^{\frac{t}{5}}$ $e^{\frac{t}{5}} = 2$ $\frac{t}{5} = \ln 2$ $t = 5\ln 2$ $= \ln 2^5$ $= \ln 32$	1 for 3rd line 2 full answer

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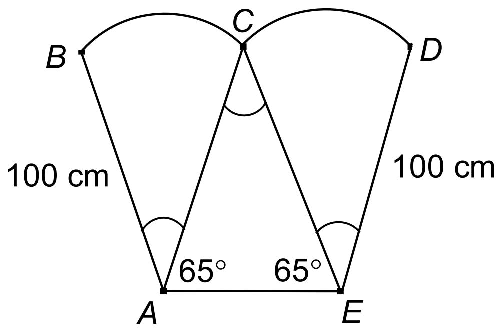
Question 6

Criteria	Marks
(a) 	1
(b) $\angle ACE + \angle CAE + \angle CEA = 180°$ (angle sum of a triangle) $\angle ACE + 65° + 65° = 180°$ $\angle ACE = 50°$	1
(c) BA, AC, CE and ED are all radii of equal circles.	1
(d) $\angle ACE = \frac{50}{180}\pi$ $= \frac{5\pi}{18}$ radians	1
(e) $\Delta ACE = \frac{r^2 \sin\theta}{2}$ $= \frac{10\,000 \sin 50°}{2}$ $= 3830.222216...$ $= 3800 \text{ cm}^2$ (to 2 significant figures)	1 for sig fig 2 full answer
(f) Area of sector $ABC = \frac{\theta r^2}{2}$ $= \frac{5\pi}{18} \times \frac{10\,000}{2}$ $= 4363.3213...$ $= 4400 \text{ cm}^2$ (to 2 significant figures) Area of whole logo = $4400 \times 2 + 3800 = 12\,600 \text{ cm}^2$ Cost to paint logo $= \frac{12\,600}{10\,000} \times \$500 = \$630$	1 for sector 1 for total area 1 for cost 3 full answer
(g) Chain required $= 4 \times 100 + 2 \times 100 \times \frac{5\pi}{18} + AE$ $= 400 + 174.5329... + 200\cos 65°$ $= 659.0565...$ $= 659 \text{ cm}$ (to nearest cm) Cost of chain $= \frac{659}{100} \times \$1000 = \$6590$ $\frac{\frac{AE}{2}}{100} = \cos 65°$ $AE = 200\cos 65°$	1 for AE 1 for most of perimeter 1 for cost 3 full answer

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Question 7

Criteria	Marks
(a) Where the function is defined and continuous, that is, $x > 0$, $x \neq 1$ ($x > 0$ because lnx is defined only for $x > 0$ and $x \neq 1$ because denominator $\ln x \neq 0$)	1 for $x > 0$ 1 for $x \neq 1$ 2 full answer
(b) $\dfrac{dA}{dx} = \dfrac{1.\ln x - x \times \frac{1}{x}}{(\ln x)^2}$ $= \dfrac{\ln x - 1}{(\ln x)^2}$ When $x = e^2$, gradient $= \dfrac{\ln e^2 - 1}{(\ln e^2)^2}$ $= \dfrac{2-1}{2^2} = \dfrac{1}{4}$	1 for derivative mostly correct 2 for derivative 3 full answer
(c) $\dfrac{\ln x - 1}{(\ln x)^2} = -1$ $\ln x - 1 = -(\ln x)^2$ $(\ln x)^2 + \ln x - 1 = 0$ $\ln x = \dfrac{-1 \pm \sqrt{1+4}}{2}$ $= \dfrac{-1+\sqrt{5}}{2}$ as $\ln x > 0$. $= 0.618033988...$ $x = e^{0.618033988...}$ $= 1.855276959...$ ≈ 1.9	1 for 3rd line 2 for mostly correct 3 for full answer
(d) $\dfrac{\ln x - 1}{(\ln x)^2} = 0$ $\ln x - 1 = 0$ $\ln x = 1$ $x = e,\ y = \dfrac{e}{\ln e} = e$ Stationary point at (e, e) From part (c), the gradient just before $x = e$ is –1, i.e. < 0 From part (b), the gradient just after $x = e$ is $\dfrac{1}{4}$, i.e. > 0 The function is continuous and differentiable for $x > 1$ so (e, e) must be a minimum.	2 for SP 2 reasoning for min 3 for mostly correct 4 full answer

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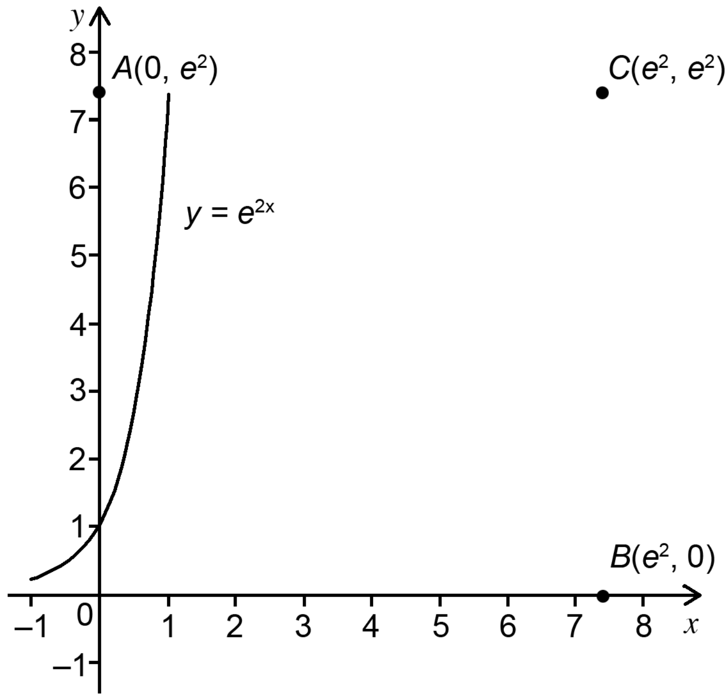
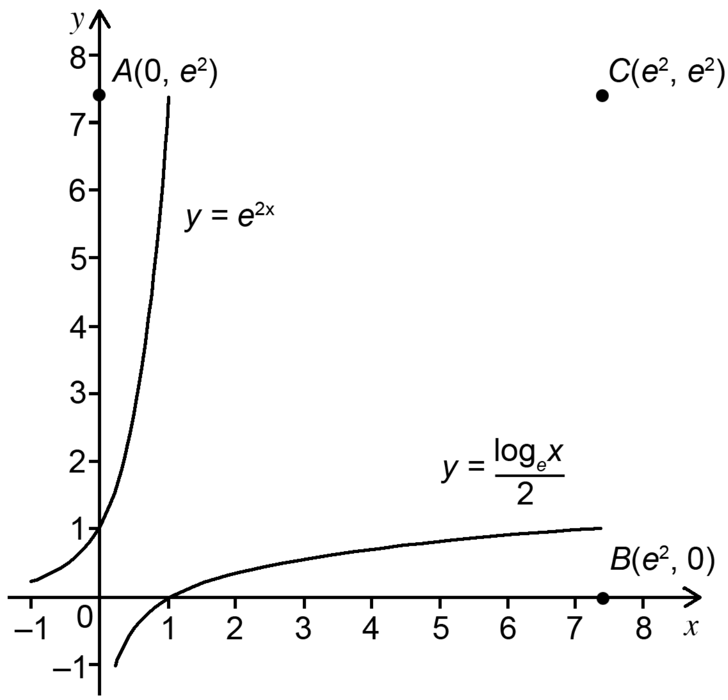
Question 8

Criteria	Marks
(a) 	1 for *A, B, C* 1 for shape of curve 3 full answer
(b) $y = e^{2x}$ $\log_e y = \log_e e^{2x}$ $\log_e y = 2x$ $x = \dfrac{\log_e y}{2}$	1
(c) 	1 for basic shape 2 full answer

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Question 8 (continued)

Criteria	Marks
(d) $\int_0^1 e^{2x}\,dx = \left[\frac{e^{2x}}{2}\right]_0^1$ $= \frac{e^2}{2} - \frac{1}{2}$ $= \frac{e^2 - 1}{2}$	1 for integral 1 for substitution 2 full answer
(e) Area $A = e^2 \times 1 - \int_0^1 e^{2x}\,dx$ $= e^2 - \frac{e^2 - 1}{2}$ $= \frac{2e^2 - e^2 + 1}{2}$ $= \frac{e^2 + 1}{2}$ unit^2	1 for mostly correct 2 full answer
(f) The area bounded by $y = e^2$, $x = e^2$, $y = 0$ and $x = 0$ is $\left(e^2\right)^2 = e^4$ Subtracting $\int_1^{e^2} \log_e x\,dx$ and area A (above) from e^4 gives: Area $= e^4 - 2 \times \frac{e^2 + 1}{2}$ $= e^4 - \left(e^2 + 1\right)$ $= e^4 - e^2 - 1$ unit^2	1 for mostly correct 2 full answer

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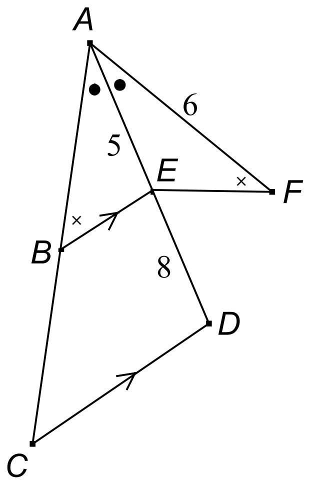
Question 9

<table>
<tr><th>Criteria</th><th>Marks</th></tr>
<tr><td>(a)
</td><td>1</td></tr>
<tr><td>(b) In ΔABE and ΔAFE ,
$\angle ABE = \angle AFE$ (given)
$\angle BAE = \angle FAE$ (as AE bisects $\angle BAF$)
AE is common
$\therefore \Delta ABE \equiv \Delta AFE$ (AAS)</td><td>1 for mostly correct

2 full answer</td></tr>
<tr><td>(c) In ΔABE and ΔACD ,
$\angle ABE = \angle ACD$ (corresponding angles in parallel lines $BE \parallel CD$)
$\angle BAE$ is common
$\therefore \Delta ABE \parallel\!\mid \Delta ACD$ (equiangular)
$\dfrac{AB}{AC} = \dfrac{AE}{AD}$ (ratio of sides in similar Δs)
$\therefore AB.AD = AE.AC$</td><td>1 for some correct
2 for mostly correct
3 full answer</td></tr>
</table>

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Question 9 (continued)

Criteria	Marks
(d) $AB = AF = 6$ (corresponding sides in congruent triangles) $\dfrac{BC}{AB} = \dfrac{ED}{AE}$ (ratios of parallel intercepts) $\dfrac{x}{6} = \dfrac{8}{5}$ $x = \dfrac{48}{5}$ OR $\dfrac{AB}{AC} = \dfrac{AE}{AD}$ (corresponding sides in similar Δs) $\dfrac{6}{6+x} = \dfrac{5}{13}$ $78 = 30 + 5x$ $x = \dfrac{48}{5}$	1 for mostly correct 2 full answer
(e) $\dfrac{BC}{AB} = \dfrac{ED}{AE}$ (ratios of parallel intercepts) $BC.AE = AB.ED$	1 ratio and reason 2 full answer
(f) $\dfrac{AB}{AE} = \dfrac{AC}{AD}$ from part (c) $\dfrac{AB}{AE} = \dfrac{BC}{ED}$ from part (e) Therefore, $\dfrac{AC}{AD} = \dfrac{BC}{ED}$ $AC.ED = BC.AD$	1 for mostly correct 2 full answer

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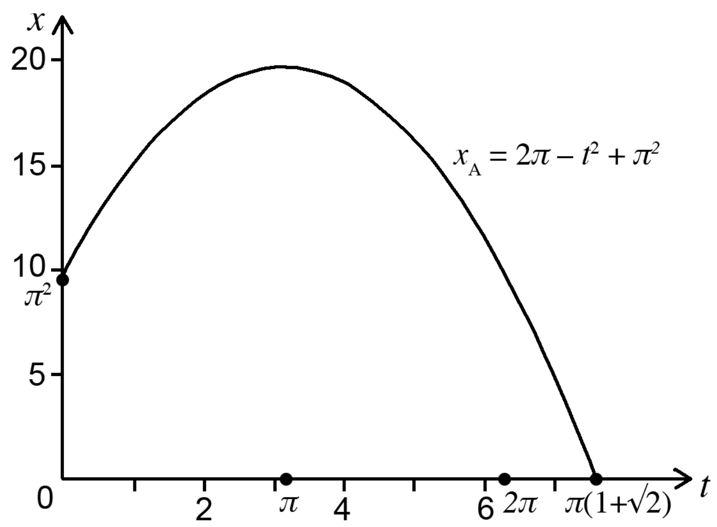
Question 10

Criteria	Marks
(a) (i) $x_A = 2\pi t - t^2 + \pi^2 = \pi^2$ $2\pi t - t^2 = 0$ $t(2\pi - t) = 0$ $t = 0 \;\; or \;\; t = 2\pi$	1
(ii) $x_A = 2\pi t - t^2 + \pi^2 = 0$ $2\pi t - t^2 + \pi^2 = 0$ $t = \dfrac{-2\pi \pm \sqrt{(2\pi)^2 - 4(-1)\pi^2}}{-2}$ $= \dfrac{-2\pi \pm \sqrt{4\pi^2 + 4\pi^2}}{-2}$ $= \dfrac{-2\pi \pm \pi\sqrt{8}}{-2}$ $= \dfrac{-2\pi\left(1 \pm \sqrt{2}\right)}{-2}$ $= \pi\left(1 \pm \sqrt{2}\right)$ $= \pi\left(1 + \sqrt{2}\right) \; as \; t \geq 0$	1 for line 2 1 for quadratic formula 1 for simplifying exact form 2 for mostly correct 3 full answer
(b) 	1 parabolic curve 2 full answer

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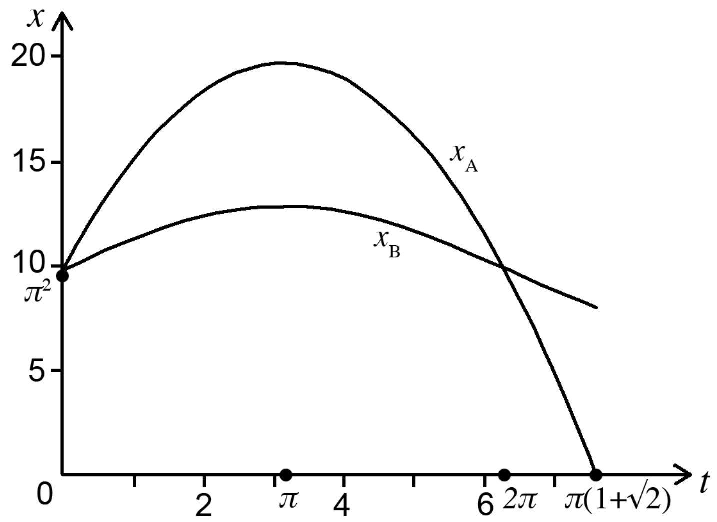
Question 10 (continued)

(c) $\dot{x}_B = \frac{3}{2}\cos\frac{t}{2}$ $x_B = \dfrac{\frac{3}{2}\sin\frac{t}{2}}{\frac{1}{2}} + c$ $= 3\sin\frac{t}{2} + c$ $x_B = \pi^2$ when $t = 0$ $\pi^2 = 3\sin 0 + c$ $\pi^2 = c$ $x_B = 3\sin\frac{t}{2} + \pi^2$ 	1 for integration 1 for constant 2 full equation 1 for graph 3 full answer
(d) The two objects have the same velocity when gradients are the same, that is, at their stationary points when $t = \pi$.	1
(e) $2\pi < t < \pi\left(1+\sqrt{2}\right)$ x_A is concave down x_B is concave up	1 for domain 1 for reason 2 full answer

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