Total marks – 120
Attempt Questions 1–10
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate correct to one decimal place $\dfrac{2.1^2 \times 4.5^2}{2.1^2 + 4.5^2}$. **2**

(b) Factorise fully $128x - 16x^4$. **2**

(c) Graph on a number line the solution to $|2x+1| \leq 5$. **2**

(d) Rationalise the denominator of $\dfrac{\sqrt{5}}{3\sqrt{2}-1}$. **2**

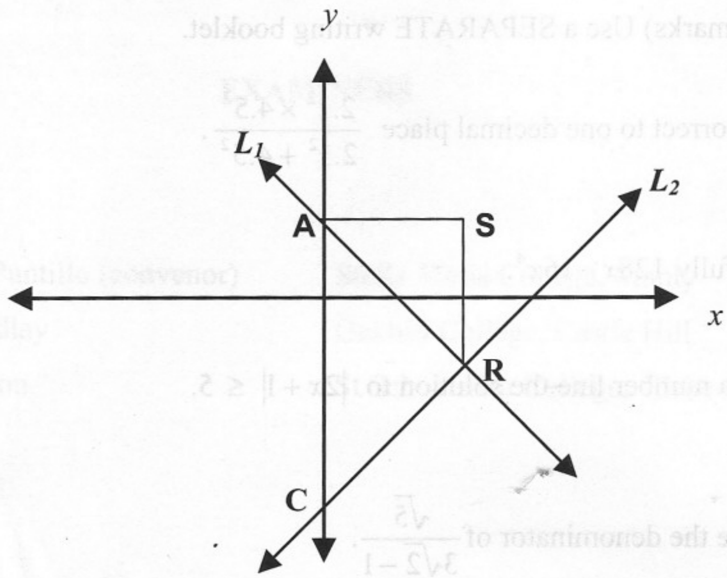
(e) Find the exact value of $\tan\dfrac{\pi}{3} + \operatorname{cosec}\dfrac{\pi}{4}$. **2**

(f) Sketch the curve $y = 2e^{-x}$, clearly showing where the curve cuts the y-axis. **2**

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the quadratic function $x^2 - (k+2)x + 4 = 0$.
For what value of k does the quadratic function have real roots? **2**

(b)



NOT TO SCALE

Line L_1 has equation $x + y = 2$ and intersects the y-axis at point A.
Line L_2 has equation $x - y = 4$ and intersects the y-axis at point C.
Line L_1 and line L_2 intersect at point R.

The horizontal line through A intersects the vertical line through R, at S.

(i) Find the coordinates of point A and C. **2**

(ii) Show that R has coordinates (3, -1). **1**

(iii) State the equation of the line SR. **1**

(iv) Find the gradient of line L_1. **1**

(v) Find the distance AR. **1**

(vi) Show that triangle ARC is a right-angled isosceles triangle. **2**

(vii) Find the equation of the circle with centre R, passing through the points A and C. **2**

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate with respect to x :

(i) $\sqrt{x}$ **1**

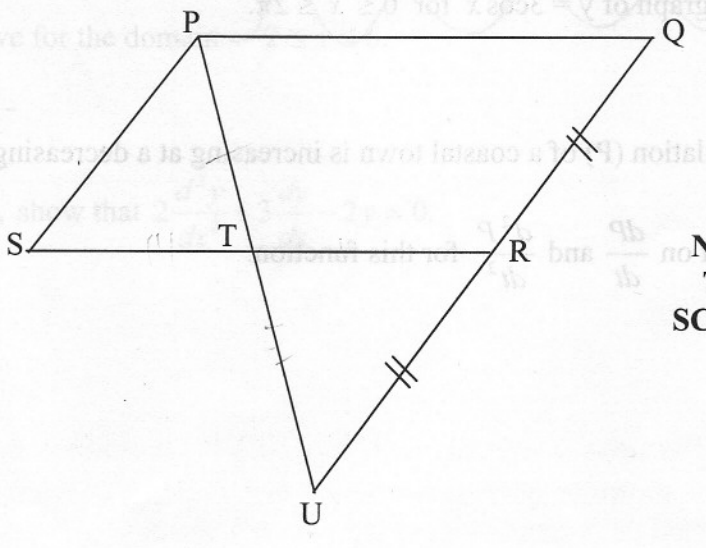
(ii) $x^3 e^{-3x}$ **2**

(iii) $\dfrac{\tan x}{2x+1}$ **2**

(b) Find $\displaystyle\int \frac{e^{2x}}{e^{2x}+4}\,dx$. **2**

(c) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \left(\frac{1}{2}x + \cos 2x\right)\,dx$. **2**

(d) In the diagram, PQRS is a parallelogram. QR is produced to U so that QR = RU.
Copy this diagram into your answer booklet.



NOT TO SCALE

(i) Giving clear reasons, show that the triangles PST and URT are congruent. **2**

(ii) Hence, or otherwise, show that T is the midpoint of SR. **1**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\sum_{k=4}^{20} 2k-5$. **2**

(b) The third term of a geometric series is $\frac{3}{4}$ and the seventh term is 12.

Find the 14th term of this series. **2**

(c) Consider the function $f(x)=|4-x|$.

(i) Sketch the function $f(x)$. **1**

(ii) Evaluate $\int_0^6 f(x)\ dx$. **1**

(d) Solve for x the equation $\sqrt[x]{m}=n^3$. **2**

(e) Sketch a graph of $y=3\cos x$ for $0\le x\le 2\pi$. **2**

(f) The population (P) of a coastal town is increasing at a decreasing rate.

Comment on $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$ for this function. **2**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) In a bag there are 20 marbles. The bag consists of 7 red marbles, 9 gold marbles and 4 blue marbles. One marble is drawn from the bag and not replaced, and then a second marble is drawn.

With the aid of a tree diagram, or otherwise, find the probability of choosing:

(i) two gold marbles **1**

(ii) marbles of different colour **2**

(b) Consider the curve given by $y = 6x^2 - x^3$.

(i) Find the coordinates of the two stationary points. **2**

(ii) Determine the nature of the stationary points. **2**

(iii) Show that there exists a point of inflexion when $x = 2$. **1**

(iv) Sketch the curve for the domain $-2 \le x \le 6$. **2**

(c) Given that $y = 3e^{-2x}$, show that $2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 2y = 0$. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

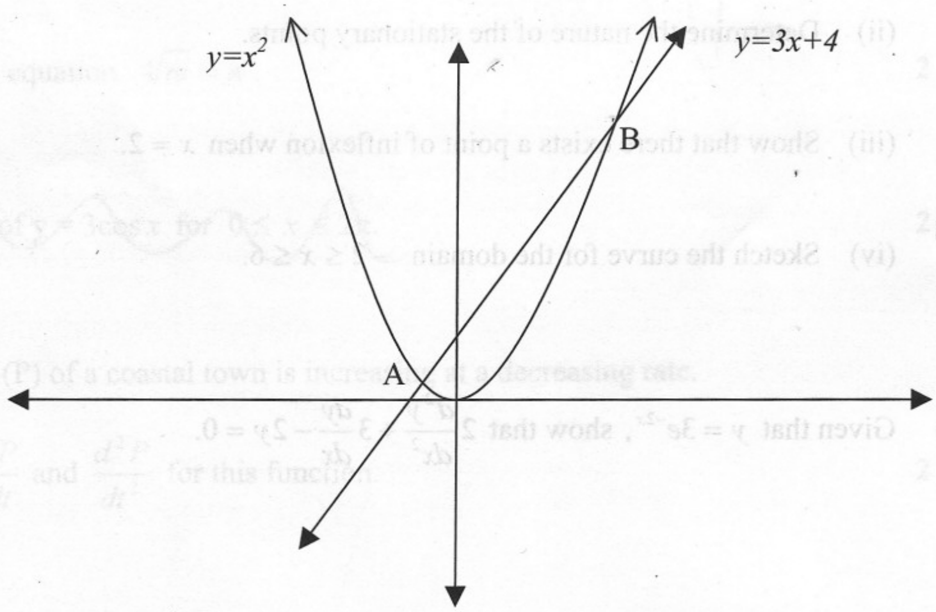
(a) Find the equation of the normal to the curve $y = x \sin x$ at the point where $x = \dfrac{\pi}{2}$. **3**

(b) The table shows the values of a function $f(x)$ for five values of x.

x	0	1	2	3	4
$f(x)$	2	3	12	35	80

Use Simpson's rule with these five values to find an approximation to $\int_0^4 f(x)\, dx$. **2**

(c)



NOT TO SCALE

(i) The curve $y = x^2$ and the line $y = 3x + 4$ intersect at the points A and B as shown in the diagram above.
Find the x coordinates of the points A and B. **2**

(ii) Find the area bounded by the curve $y = x^2$ and the line $y = 3x + 4$. **2**

(d) Find the volume generated when the curve $y = \sqrt{\cot x}$ is rotated about the $x - axis$ between $x = \dfrac{\pi}{3}$ and $x = \dfrac{\pi}{4}$. Leave your answer in exact form. **3**

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\frac{dy}{dx}$ given that $y = \log_e\left(\frac{2x+1}{3x-7}\right)$. **2**

(b) A particle moves in a straight line so that its velocity, v metres per second, at time t is given by $v = 3 - \frac{2}{1+t}$

The particle is initially 1 metre to the right of the origin.

(i) Find an expression for the position x, of the particle at time t. **2**

(ii) Explain why the velocity of the particle is never 3 metres per second. **1**

(iii) Find the acceleration of the particle when $t = 2$ seconds. **2**

(c) (i) Show that $(\text{cosec}^2 A - 1)\sin^2 A = \cos^2 A$. **2**

(ii) Hence, or otherwise, solve $(\text{cosec}^2 A - 1)\sin^2 A = \frac{3}{4}$ for $-\pi \le A \le \pi$. **3**

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Cristina borrows \$480 000 from a finance company to buy a house. She pays interest at 6% per annum, calculated quarterly on the balance still owing. The loan is to be repaid at the end of 20 years with equal quarterly repayments of \$P.

Let A_n = the amount owing after the n th repayment.

(i) Show that after the first quarterly repayment of \$P Cristina owes an amount equivalent to A_1 = \$ 487 200 - \$P. **1**

(ii) Find an expression for the amount still owing after 3 repayments of \$P. **2**

(iii) Find the value of \$P to the nearest cent. **2**

(b) Water is draining from a storage tank at a rate which is proportional to the volume of water contained in the tank. When full, the storage tank holds 1 000 litres of water. On inspection the tank was found to be full but 40 minutes later it was found to contain only 800 litres of water.

(i) How much water (to the nearest litre) will the tank hold after 1 hour? **2**

(ii) How long, in hours and minutes, will it take to reach 1 litre of water in the tank? **2**

(c) Consider the series $\sin^2 x + \sin^4 x + \sin^6 x + \ldots.$ $0 < x < \dfrac{\pi}{2}$.

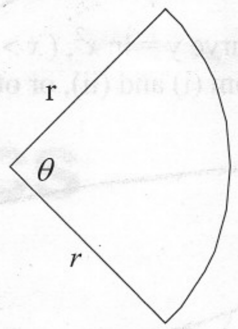
(i) Show that a limiting sum exists. **1**

(ii) Find the limiting sum expressing the answer in simplest form. **2**

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Arcsec Landscaping Company are designing a garden bed for a local park in the shape of a sector with radius r and sector angle θ.

They have a total of 375 metres of garden edging materials to use as the perimeter of the garden bed.



NOT TO SCALE

(i) Show that the area A of the garden bed is given by $A = \frac{r}{2}(375 - 2r)$. **2**

(ii) Find the greatest area of garden bed which can be made using 375 metres of edging material. **3**

(iii) After inspecting the location for the garden bed the designers calculate that the sector angle for the garden must be less than $110°$. Can they still create the garden bed with maximum area found in (ii)? Justify your answer. **2**

(b) A train is travelling at a constant velocity of 80 kilometres per hour as it passes through the railway station at a town. At the same time, a second train commences its journey from rest at the railway station. The second train accelerates uniformly for 15 minutes until it reaches 100 kilometres per hour and maintains this velocity for a further 5 minutes.

At this time each of the trains then begins to slow down at a constant rate, arriving at the next station at the same time.

(i) By illustrating graphically the relationship between velocity and time, calculate the time taken for the trains to travel between the two stations. **3**

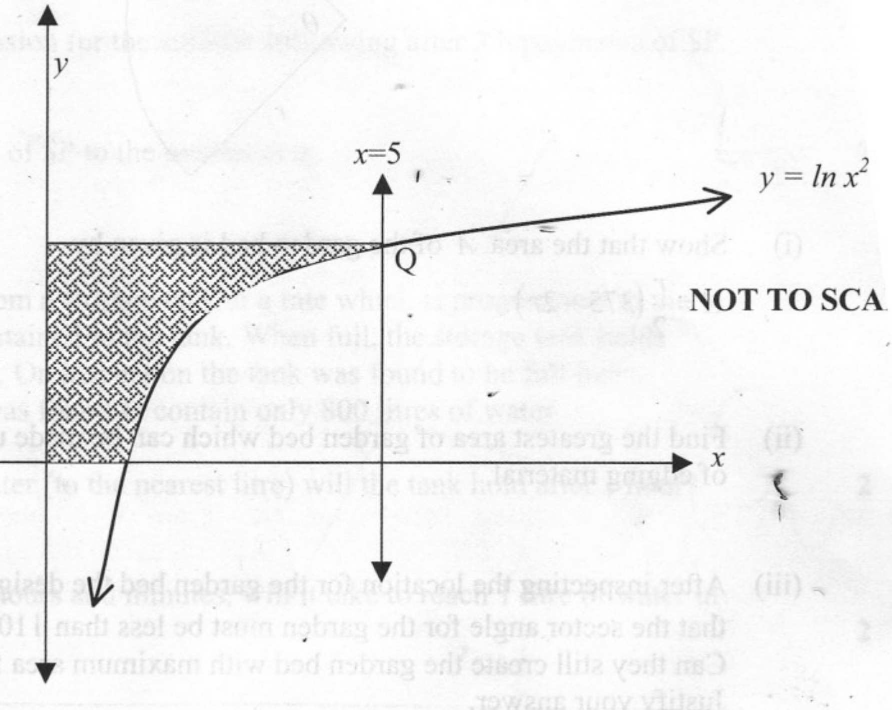
(ii) How far apart are the stations? **2**

Question 10 (12 marks) Use a SEPARATE writing booklet.

(a) (i) Show that $\dfrac{d}{dx}(x\ln x - x) = \ln x$.

(ii) Hence, or otherwise, find $\int \ln x^2 \, dx$.

(iii) The graph shows the curve $y = \ln x^2$, ($x > 0$) which meets the line $x = 5$ at Q. Using your answers from (i) and (ii), or otherwise, find the area of the shaded region.



(b) Consider the function $f(x) = e^{-x}\cos x$ for $0 \le x \le 2\pi$.

(i) Find the x values where the stationary points occur. **2**

(ii) Determine the nature of the stationary points. **2**

(iii) Sketch the curve showing the coordinates of the stationary points in exact form and the intercepts with the axes. **2**

(iv) Find the number of solutions to the equation $e^{-x}\cos x - \dfrac{1}{2}x = 0$.

In the domain $0 \le x \le 2\pi$. Justify your answer. **1**

End of paper