2/3U (1996) CSSA Trials

Question 1:

(a) $5x^2 - 2x - 3 = (5x+3)(x-1)$.

$5x^{+3}_{-1}$

(b) 1.3.

(c) $\frac{d}{dx}(4x^2+2) = 8x$.

(d) $|x-5| = 6$.

Case 1: $x-5>0$

$x-5 = 6$

$x = 11$

Case 2: $x-5 = -6$.

$x = -1$

or $(x-5)^2 = 36$.

$x-5 = \pm 6$.

$x = 11, -1$.

(e) $\frac{2x-3}{5} + \frac{3x+4}{10} = \frac{2(2x-3)}{10} + \frac{3x+4}{10}$

$= \frac{4x-6-3x-4}{10}$

$= \frac{x-10}{10}$.

(f) ~~$\angle CAE = 180 - 30 - 35$ ($\angle$ sum of $\triangle ABE$)~~

~~$= 115°$.~~

~~$x = 115° + 50°$ (opp. ext. $\angle$ = sum of 2 opp int. $\angle$s)~~

~~$= 165°$~~

(f) $\angle CAE = 180 - 35 - 80 - 50$ ($\angle$ sum of $\triangle ABC$)

$= 65°$

$\angle BDE = 180 - 65$ (supp. $\angle$)

$= 115°$.

(g) $3p > -5$

$p > -5/3$

Question 2:

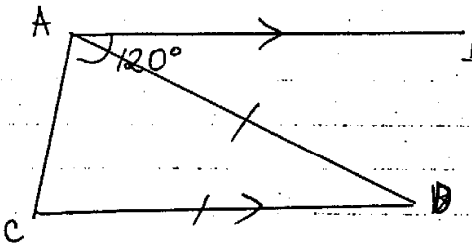
(a) $\frac{d}{dx}[\sin(3x-2)] = \cos(3x-2) \cdot 3$

$= 3\cos(3x-2)$ #

(b) $\frac{d}{dx}\left(\frac{e^x}{x}\right) = \frac{xe^x - e^x}{x^2}$

$= \frac{(x-1)e^x}{x^2}$ #

(b)



(i) $\angle ADC = 60°$ ($AB \parallel CD$, cont. $\angle$'s)

(ii) $\angle DAC = \angle ACD = 60°$ (given $AD = CD$, $\therefore \triangle ACD$ is an isos. $\triangle$ $\therefore$ base $\angle$'s =)

$\angle ADC = 180 - 60 - 60$ ($\angle$ sum of $\triangle$)

$= 60°$.

Since $\angle DAC = \angle ACD = \angle ADC = 60°$ $\therefore$ all $\angle$s equal $\therefore$ equilateral.

(c)(i) $\int \sec^2 3x\, dx = \frac{\tan 3x}{3} + C$.

(ii) $\int (5x-3)^5\, dx = \frac{(5x-3)^6}{6 \times 5} + C$

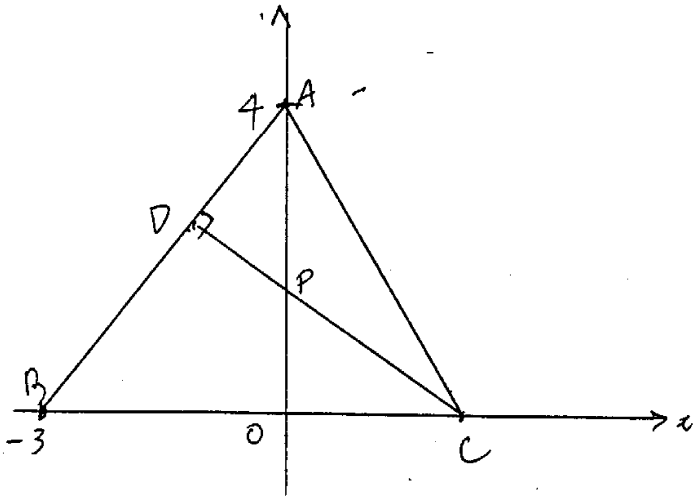
$= \frac{(5x-3)^6}{30} + C$.

(iii) $\int_{-1}^{0} \frac{dx}{2x+3} = \left[\frac{\ln(2x+3)}{2}\right]_{-1}^{0}$

$= (\ln 3 - \ln 1) \times \frac{1}{2}$

$= \ln 3 = 1.10$ (2 dp.)

Question 3



(a) Using pythagoras' $AB^2 = 3^2 + 4^2$

$AB = 5$ #

(b) Given $|AB| = |BC|$

$5 = |-3| + OC.$

$\therefore OC = 2.$

$\therefore$ C has co-ordinates $(2,0)$. #

(c) Gradient of $AB = \frac{4}{3}$

Since $CD \perp AB$ $\therefore m_{CD} = \frac{-3}{4}$ rqd pt. $(2,0)$

$\therefore$ Using $y - y_1 = m(x - x_1)$

$y - 0 = \frac{-3}{4}(x - 2)$

$4y = -3x + 6.$

$\therefore 3x + 4y = 6$ #

(d) for P, when $x = 0$, then $3x + 4y = 6.$ $\therefore P(0, 1\frac{1}{2})$ #

$4y = 6$

$y = 1\frac{1}{2}.$

(e) $CP^2 = OC^2 + OP^2$

$= 2^2 + \left(\frac{3}{2}\right)^2$

$= 4 + \frac{9}{4}$

$= \frac{25}{4}$

$\therefore$ CP $= 2\frac{1}{2}$ units #

(f) $\angle ADP = 90°$ (given)
$\angle POC = 90°$ (given)
$AP = 4 - 1\frac{1}{2}$
$= 2\frac{1}{2}$ units.
$PC = 2\frac{1}{2}$ units
$= AP$
$\angle APD = \angle CPO$ (vert. opp. $\angle$'s)
$\therefore \triangle ADP \equiv \triangle COP$ (AAS)

(g) Lets consider. $\triangle ABC$ & $\triangle BDC$.

$\angle B$ is common
$\angle A = \angle C$ (corresp. $\angle$'s of $\triangle ADP \equiv \triangle COP$
$\angle O = \angle D = 90°$ (given)
$\therefore \triangle ABC \equiv \triangle BDC$ (AAS)

$\triangle BDPO = 2 \times \text{Area}_{BOA} - 2 \times \triangle COP.$

$= 2 \times \frac{1}{2} \times 4 \times 3 - 2 \times \frac{1}{2} \times \frac{3}{2} \times 2$

$= 12 - 3$
$= 9u^2.$ ##

Question 4:

(a) $y = x \ln x$. ← use product rule.

$$\frac{dy}{dx} = x \cdot x\frac{1}{x} + \ln x.$$
$$= 1 + \ln x.$$
$$= 1 + \ln 1 \quad \text{at } x = 1$$
$$= 1$$

eqn. of tangent: $y - y_1 = \frac{dy}{dx}(x - x_1)$

$$y - 0 = 1(x - 1)$$
$$y = x - 1 \ \#$$

b) i) $\frac{AD}{40} = \sin 60°$

$$AD = 40 \times \frac{\sqrt{3}}{2}$$
$$= 20\sqrt{3} \text{ m.} \ \#.$$

(ii) $AB^2 = 5^2 + 40^2 - 2 \times 40 \times 5 \cos 60°$
$= 25 + 1600 - 400 \times \frac{1}{2}$
$= 1425$
$= \sqrt{25} \times \sqrt{57}$
$AB = 25\sqrt{57}$

(c) $\log 3^{2m} = \log 3 - \log 3^{1/2}$

$2m \log 3 = \log 3 - \frac{1}{2}\log 3$

$2m \log 3 = \frac{1}{2}\log 3$

$2m = \frac{1}{2}$

$m = 1/4.$

(d) $0.4^x \geq 5$ $\quad x \log \frac{2}{5} \geq \log 5$

$\left(\frac{2}{5}\right)^x \geq 5$

~~$x \log \frac{2}{5} \geq \log 5$~~ $\quad x \geq \frac{\log 5}{\log \frac{2}{5}}$

Question 5:

(a) $h = 24/4 = 6$.

$A \doteq \frac{h}{3}[0 + 13 + 4(13+9) + 2 \times 10]$

$\doteq \frac{6}{3}[13 + 76 + 20]$

$\doteq 242$.

(b) i) $\frac{dL}{dt} = 120 \times 2(40-t) \times (-1)$

$= -240(40-t)$

$= -240(40-6)$

$= -8160$.

$\frac{dL}{dt} = 8160$ L/min.

(ii)