SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2002
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may not be awarded for messy or badly arranged work.

Total Marks - 120 marks

- All questions are of equal value.

Examiners: *P. Bigelow, P. Parker*

NOTE: This is a trial paper only and does not necessarily reflect the content or format of the final Higher School Certificate Examination Paper for this subject.

Question 1: (12 Marks) **Marks**

(a) Solve $\frac{3}{x}=\frac{4}{5}$ **2**

(b) If $a=2\cdot673$ and $b=1\cdot049$, evaluate $\frac{a^2+b^2}{ab}$ correct to 2 decimal places. **2**

(c) Factorise $cd-c-dy+y$ **2**

(d) The line $kx-y=29$ passes through the point $(4,-1)$, find the value of k. **2**

(e) Graph the solution to $|x+3|\leq 1$ on a number line. **2**

(f) Find integers a and b such that **2**

$$\frac{1}{\sqrt{3}+2}=a\sqrt{3}+b$$

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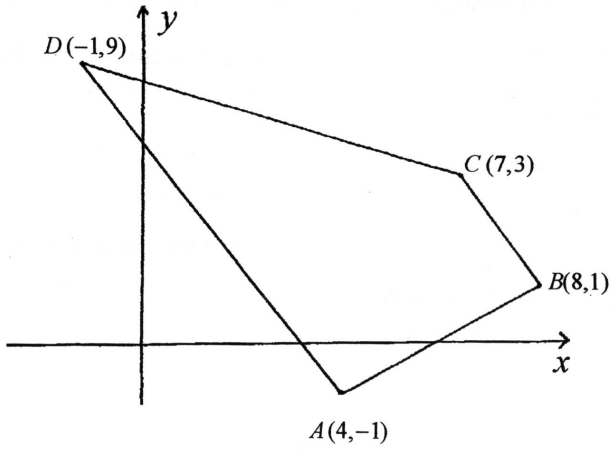
Question 2: (12 Marks) **Marks**



A, B, C and D are the points $(4,-1)$, $(8,1)$, $(7,3)$ and $(-1,9)$ respectively.

(a) Show that the equation of AC is $4x-3y-19=0$ **2**

(b) Show $BC \parallel AD$ **2**

(c) Show $\angle ACD = 90^\circ$ **2**

(d) Show the length of AC is 5 units **2**

(e) Find the perpendicular distance of B from AC **2**

(f) Find the area of the trapezium $ABCD$. **2**

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Question 3: (12 Marks) **Marks**

(a) Differentiate

(i) $(2x-1)^7$ **1**

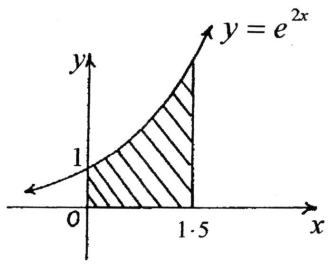
(ii) $\log_e(4+5x)$ **2**

(iii) $x^2 \sin x$ **2**

(b) Find $\int \sec^2 3x \, dx$ **1**

(c) Evaluate $\int_1^{e^2} \frac{3}{x} \, dx$ **3**

(d) **3**



The diagram above shows the region bounded by the curve $y = e^{2x}$, the line $x = 1 \cdot 5$ and the coordinate axes.
Find the EXACT area of this region.

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Question 4: (12 Marks) **Marks**

(a) A parabola has equation $x^2 - 2x + 25 = 8y$.

(i) By completing the square, express this in the general form $(x-p)^2 = 4a(y-q)$ **2**

(ii) State the coordinates of the vertex. **1**

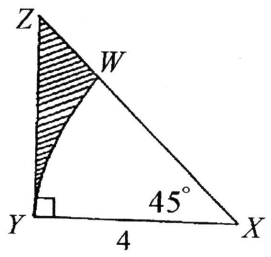
(iii) State the coordinates of the focus. **1**

(iv) State the equation of the directrix. **1**

(b) Find the equation of the line through the intersection of the lines $x - y = 0$ and $x + 2y - 6 = 0$ and which is parallel to the line $3x - 2y + 7 = 0$. **4**

Leave your answer in general form.

(c) **3**



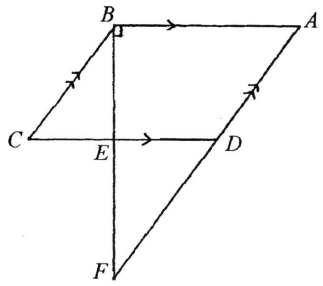
In the diagram above, XYZ is a right-angled triangle, $\angle ZXY = 45^\circ$ and $XY = 4$ units. A circular arc, centre X and radius YX cuts the side XZ at W.
Find the EXACT area of the shaded region WZY.

START A NEW BOOKLET

Question 5: (12 Marks) **Marks**

(a) Find the equation of the normal to $y = x^2 - 3x + 5$ at the point $(3,5)$. Leave your answer in general form. **2**

(b)

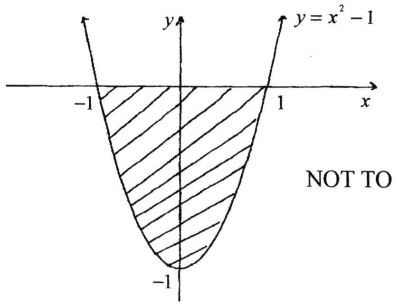


$ABCD$ is a parallelogram and $FB \perp AB$

(i) Prove $\Delta CBE \;|||\; \Delta AFB$ **3**

(ii) If $CE = 3$ cm, $BC = 7$ cm and $AF = 15$ cm, find AB. **2**

(c) **3**



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The area bounded by the curve $y = x^2 - 1$ and the x-axis is rotated about the x-axis.
Find the volume of the solid of rotation. Leave your answer in EXACT form.

(d) A continuous curve $y = f(x)$ has the following properties for the closed interval $x_1 \le x \le x_2$: **2**

$$f(x) > 0, \quad f'(x) < 0, \quad f''(x) > 0$$

Sketch a curve satisfying these conditions.

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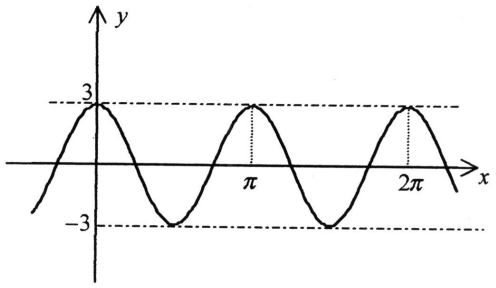
Question 6: (12 Marks) **Marks**

(a) The curve $y = x^3 + mx + n$ has a stationary point at $P(1,5)$. Find the values of the constants m and n. **3**

(b)



The graph above represents $y = a\cos mx$. **2**

Write down the values of a and m.

(c) Express as a single logarithm in simplest form **2**

$$\ln 2 + 2\ln 18 - \tfrac{3}{2}\ln 36$$

(d) Simplify $\sqrt{\dfrac{1-\cos^2\theta}{1+\tan^2\theta}}$ **2**

(e) The table shows the value of a function $f(x)$ for five values of x. **3**

x	-1	1	3	5	7
$f(x)$	5	9	2	-1	-6

Use Simpson's rule with these five values to estimate $\int_{-1}^{7} f(x)dx$

START A NEW BOOKLET

Question 7: (12 Marks) **Marks**

(a) The value $\$V$ of a particular make of car can be calculated using the equation

$$V = 50000e^{-0.25t}, \text{ where } t \text{ years is the age of the car}$$

(i) State the value of the car when it is new **1**

(ii) What is the value of the car after 3 years? (to the nearest \$100). **2**

(iii) How long will it take for the car to be worth \$15 000? (Answer to the nearest tenth of a year) **2**

(b) A total of \$15 600 is to be shared among three people A, B and C. If A is to get the smallest share of \$1600, find the value of the remaining shares when

(i) the values of the three shares form an arithmetic series **2**

(ii) the values of the three shares form a geometric series **2**

(c) Two cards are chosen at random, without replacement from the seven cards below.

2	2	4	6	6	6	6

What is the probability that

(i) both cards are 2? **1**

(ii) the sum of the two numbers on the cards chosen, is greater than 8? **2**

START A NEW BOOKLET

Question 8: (12 Marks) **Marks**

(a) Two artillery guns are situated 3 kilometres apart at positions X and Y respectively. They are both aiming at a target T. **3**

The angles XYT and YXT are respectively 72° and 78°.

Find the distance between the target and the gun nearer to it

(b) Given that $x = -\frac{1}{2}$ is one root of the following quadratic equation **3**

$$mx^2 - 20x + m = 0,$$

find the exact value of the other root.

(c) Consider the function $y = x^3 + 3x^2 - 9x$

(i) Find the coordinates of the stationary points. **2**

(ii) Find the domain in which the curve is concave upwards. **1**

(iii) Sketch the curve for $-5 \leq x \leq 3$. **3**

START A NEW BOOKLET

Question 9: (12 Marks) **Marks**

(a) Solve $|x+1|=2x+7$ **3**

(b) Comment on the following reasoning ie JUSTIFY whether the following statement is TRUE or FALSE **2**

"When two coins are tossed, they can either fall as two heads or two tails or as a tail and a head.
As there are three possibilities the probability of 2 heads is $\frac{1}{3}$."

(c) The acceleration a metres per second per second of a moving object is given at time t seconds $(t \geq 0)$ by

$$a = 2\pi^2 \cos \pi t$$

At time $t = 0$, the object is at the point $x = 0$, and travelling with velocity $v = \pi$ metres per second.

(i) Find the velocity v and the displacement x as a function of t, for $t \geq 0$ **3**

(ii) Find, for t in the range $0 \leq t \leq 4$, the values of t for which the object is stationary. **2**

(iii) Show that, for t in the range $2 \leq t \leq 4$, the largest value of x is **2**

$$2 + \sqrt{3} + \frac{19\pi}{6}.$$

START A NEW BOOKLET

Question 10: (12 Marks) **Marks**

(a) $C(n) = 1000(0\cdot 8)^n + 10\,000[1 + 0\cdot 8 + (0\cdot 8)^2 + \ldots + (0\cdot 8)^{n-1}]$ **3**

Find the limiting sum of C as the number of terms increases indefinitely

(b) A scientist has found that the amount, $Q(t)$, of a substance present in a mineral at time $t \geq 0$ satisfies

$$4\frac{d^2Q}{dt} + 4\frac{dQ}{dt} + Q = 0$$

(i) Verify that $Q(t) = A(1+t)e^{-0.5t}$ satisfies this equation for any constant $A > 0$ **4**

(ii) If $Q(0) = 10$ mg, find the maximum value of $Q(t)$ and the time at which this occurs. **4**

(iii) Describe what happens to $Q(t)$ as $t \to \infty$ **1**

THIS IS THE END OF THE PAPER