

CRANBROOK SCHOOL

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(6)

Year 12 MATHEMATICS 3 Unit (Second Paper), 4 Unit (First Paper)

Term 3 1999

Time: 2h

(GJB, MJB, LD, WMF, CGH, KMR, BES)

All questions may be attempted. All questions are of equal value.
All necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.
Standard integrals are provided at the end of the paper.
Approved silent calculators may be used.
Begin each question in a new booklet.

1. (new booklet please)

- a Solve $x^2 + x - 6 < 0$. 2
- b Differentiate $y = \log_e \left(\frac{x-1}{(x^2+1)} \right)$. 3
- c Find the exact value of $\int_0^{\frac{\pi}{2}} \cos 2x dx$. 3
- d Find a and b if $a + \sqrt{b} = (2 + 3\sqrt{2})^2$ 2
- e Sketch the graph of $y = \sqrt{3}x$. If the straight line $y = mx$ is added to the graph axes in such a way that the angle between $y = \sqrt{3}x$ and $y = mx$ is 60° find all possible values for m. 2

2. (new booklet please)

- a Show that $\log\left(\frac{2}{3}\right) + \log\left(\frac{3}{4}\right) + \log\left(\frac{4}{5}\right) + \dots + \log\left(\frac{n}{n-1}\right) = \log\left(\frac{n}{2}\right)$ 3
- b James invests \$1 500 at the end of each financial year in a superannuation fund for a period of 15 years. If the fund returns 5.5% pa over the period of investment, how much is in James' account at the end of 15 years? 4

2 continued

- c** Triangle ABC has a right angle at A and angle C equal to 60° . The side AC is 2 cm long. A circular arc is drawn with centre A, radius 2 cm cutting BC at D and AB at E.

[ED is an arc and DC a straight line.]

- i** Show that the area of the portion BED of the triangle is $\left(\sqrt{3} - \frac{\pi}{3}\right) \text{ cm}^2$. **2**

- ii** Find the area of the remaining portion ACDE of the triangle. **3**

3. (new booklet please)

- a** Using Induction, prove the following true for all positive integers, n

$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2 \quad \text{3}$$

- b** Write $5 \sin x - 3 \cos x$ in the form $r \sin(x - \alpha)$.

Hence solve the equation $5 \sin x - 3 \cos x = 2$ in the domain $-180^\circ \leq x \leq 180^\circ$ **3**

- c** When x metres from an origin on a straight line, the velocity $v \text{ ms}^{-1}$ of a particle is given by $v^2 = 9(3 + 2x - x^2)$

- i** Prove that the particle is in simple harmonic motion.

- ii** Find the centre of motion and the period. **6**

4. (new booklet please)

- a** A perfectly spherical balloon with radius r has a slow leak whereby the balloon remains spherical while slowly decreasing in size.

- i** If the gas is escaping at a rate of $2 \text{ m}^3/\text{min}$ at a particular instant and the radius at that instant is 10 m, what is the rate of decrease in the surface area at that instant? **2**

- ii** Find the radius of the balloon at the instant when the rate of decrease of the surface area is numerically identical to the rate of decrease of the volume. **2**

- b** Consider the function $f(x) = 1 + \frac{2}{x-3}$ for $x > 3$.

- i** Give the equations of the horizontal and vertical asymptotes for $y = f(x)$. **2**

- ii** Find the inverse function $f^{-1}(x)$ **2**

- iii** State the domain of the inverse function. **1**

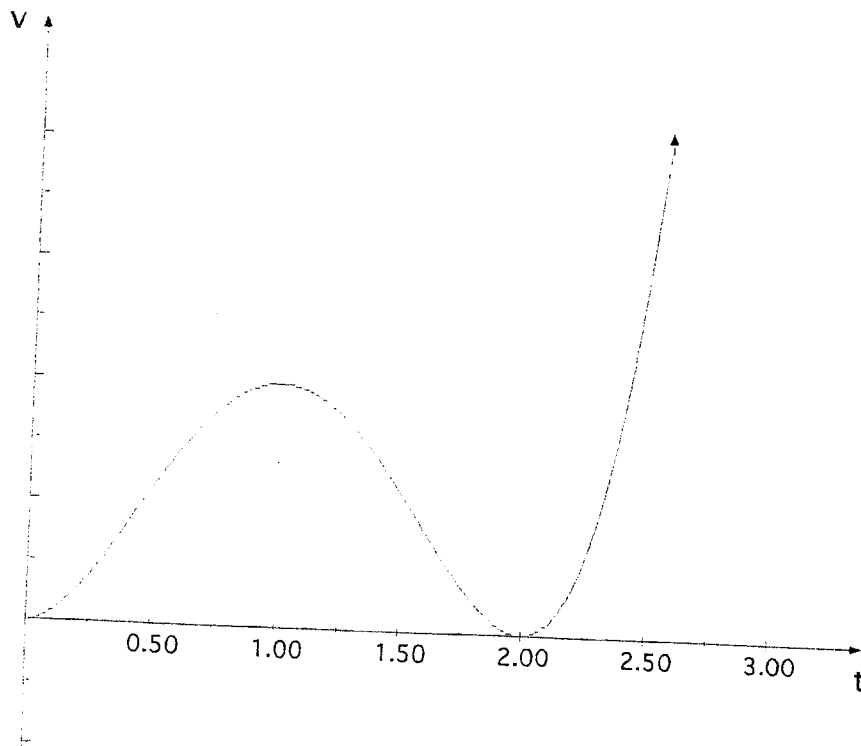
5. (new booklet please)

- a Write $13 + 21 + 31 + \dots + (n^2 + n + 1)$ using \sum notation. 1
- b The polynomial $P(x) = 4x^3 + 3x^2 + 2$ has one real zero in the interval $-2 < x < -1$. 6
- i Show that this is the only real zero and sketch the graph of $P(x)$.
- ii Using the approximation $x = -1.2$, find a better approximation using one application of Newton's method.
- iii Explain what would happen if the approximation $x = -0.25$ had been used.
- c Four men and four women are seated in a line. 2
- i In how many different ways can the people be seated?
- ii What is the probability that if one arrangement is chosen at random that the men and women are seated alternately?
- d A biased coin has a probability of 0.6 of coming up Heads. If the coin is tossed 10 times what is the probability getting: 3
- i 5 Heads?
- ii 5 Heads, then 5 Tails?
- iii At least two Heads, giving your answer to 4 significant figures?

6 (new booklet please)

- a The graph below represents the velocity function of a particle moving in a straight line.
- Copy the diagram into your answer booklet.
- i State, with reason, whether the particle changes direction. 1
- ii Indicate appropriately, on the diagram, the total distance travelled by the particle in the first two seconds. 1
- iii On the same diagram, draw a curve that could be the displacement function of the particle. 2

6 continued



- b A projectile is launched from the top of an 80 metre tower that stands on level ground. The particle's initial speed is 20 ms^{-1} and its angle of projection is 60° to the horizontal.

Assuming acceleration due to gravity is 10 ms^{-2}

- | | | |
|-----|---|---|
| i | Derive the functions for the position of the projectile showing that $x = 10t$ and $y = 10\sqrt{3}t - 5t^2$ | 2 |
| ii | Find the maximum height reached above ground level. | 2 |
| iii | Find how long it takes for the projectile to hit the ground. | 2 |
| iv | When the projectile is half way through its flight to the ground, calculate the direction of its flight. | 2 |

7. (new booklet please)

- | | | |
|----|---|---|
| a | Given the polynomial $P(x) = x^3 - (k+1)x^2 + kx + 12$, | 3 |
| i | Find the remainder when $P(x)$ is divided by $A(x) = x - 3$. | |
| ii | Find k if $P(x)$ is divisible by $A(x)$. | |

7 continued

- b i Using the given substitution, evaluate $\int_0^3 x \sqrt{(x^2 + 1)^3} dx$, $u = x^2 + 1$ 4
- ii Find $\int \frac{x dx}{\sqrt{x^2 + 1}}$ 2
- c Find all real x such that $\frac{2}{|x-2|} > x+1$ 3

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1} \quad (n \neq -1; x \neq 0 \text{ if } n < 0)$$

$$\int \frac{1}{x} dx = \log_e x \quad (x > 0)$$

$$\int \cos ax dx = \frac{1}{a} \sin ax \quad (a \neq 0)$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax \quad (a \neq 0)$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} \quad (a \neq 0)$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} \quad (a > 0, -a < x < a)$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log_e \left\{ x + \sqrt{x^2 - a^2} \right\} \quad (|x| > |a|)$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \log_e \left\{ x + \sqrt{x^2 + a^2} \right\}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} \quad (a \neq 0)$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax \quad (a \neq 0)$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax \quad (a \neq 0)$$