Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2004
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Morning Session
Monday 9 August 2004

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- All necessary working should be shown in every question
- Write your Centre Number and Student Number at the top of this page

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\quad x \neq 0, \text{ if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\quad -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x>0$

Total marks – 120
Attempt Questions 1-10
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) If $x^5 = 5000$, find x correct to 3 significant figures. **2**

(b) Express $0.3 + 0.\dot{3}$ in the form $\frac{a}{b}$, where a and b are integers. **2**

(c) Solve $\tan\alpha = 3$, for $0^\circ \le \alpha \le 360^\circ$, giving the answers to the nearest degree. **2**

(d) Simplify: $1 - \dfrac{a-b}{a+b}$ **2**

(e) Solve: $8^x = 32$, leaving the answer as a fraction. **2**

(f) Find the integers a and b such that $\dfrac{1}{2-\sqrt{3}} = a + b\sqrt{3}$ **2**

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Write down the derivatives of:

(i) $(3x+4)^7$ **2**

(ii) x^3e^x **2**

(iii) $\dfrac{\tan 5x}{5x}$ **2**

(b) (i) Write down the primitive function of $e^{3x} + \sqrt{x}$ **2**

(ii) Find the exact value of $\displaystyle\int_1^2 \frac{x^4+1}{x}\,dx$ **2**

(iii) Given that $\dfrac{dy}{dx} = 2x - \sin x$ and $y = 2$ when $x = 0$, find y in terms of x. **2**

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) For what values of a, will $ax^2 + 5x + a$ be positive definite? **3**

(b) Find the values of k if $\int_1^k (x+1)dx = 6$ **2**

(c) The points A, B and C have co-ordinates $(1,5)$, $(6,0)$ and $(5,7)$ respectively. Plot these points on a number plane. Hence:

(i) Show that the length of AB is $5\sqrt{2}$. **1**

(ii) Show that the triangle ABC is isosceles by finding the length of BC. **1**

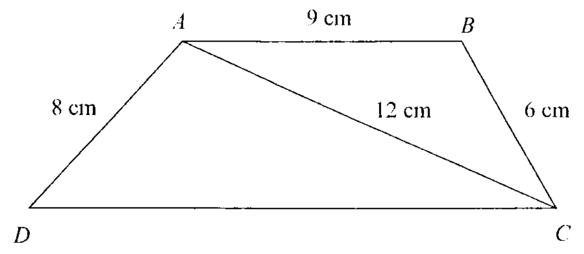
(iii) Find the equation of the line AB. **2**

(iv) BA is produced to meet the line $y = 7$ at P; show that P has co-ordinates $(-1,7)$. **1**

(v) Find the area of triangle PAC. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) In the diagram below, $AB = 9$ cm, $BC = 6$ cm, $AD = 8$ cm, $AC = 12$ cm and $\angle ABC = \angle DAC$



NOT TO SCALE

(i) Prove $\Delta ABC \parallel\!| \Delta CAD$, giving clear reasons. **3**

(ii) Hence, find the value of side CD. **2**

(b) A parabola whose equation is $y = ax^2$, where a is a constant, has the line $y = 12x + 3$ as a tangent.

(i) By equating the two given equations, find a quadratic equation in terms of x and a. **1**

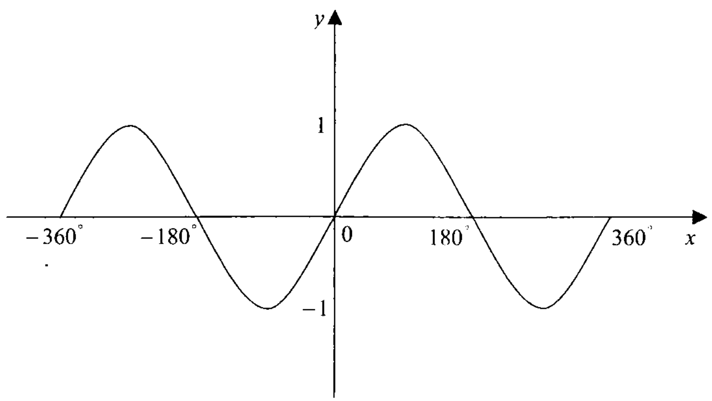
(ii) By using the discriminant of the quadratic equation found, find the value of a. **2**

(iii) Find the coordinates of the point of contact between the tangent and the parabola. **2**

(iv) Sketch the parabola and the tangent line, showing the co-ordinates of the point of contact. **2**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Below is the graph of $y = \sin x$, for $-360^\circ \le x \le 360^\circ$.



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One solution of the equation $\sin x = 0.5$ is $x = -210^\circ$. **3**
Find the other solutions of this equation for $-360^\circ \le x \le 360^\circ$.

(b) (i) On the same graph, sketch the curves $y_1 = 2\sin x$ and $y_2 = -\sin 2x$, for $0 \le x \le 2\pi$. **2**

(ii) Give three solutions to the equation $2\sin x + \sin 2x = 0$, for $0 \le x \le 2\pi$ **1**

(c) A radioactive substance decays at a rate proportional to the mass present. The rate of change is given by $\frac{dM}{dt} = -kM$, where k is a positive constant and M grams is the mass present at any time, t hours.

(i) Show that $M = M_0 e^{-kt}$ is a solution to this equation. **1**

(ii) If 100 grams of this substance decays to 80 grams in 20 hours, find:

(α) The value of k, correct to 2 decimal places. **2**
(β) The mass present after further 10 hours, to the nearest gram. **1**

(iii) The half-life time is the time taken for 100 grams of this substance to decay to 50 grams. What is the half-life time of this substance? Give your answer to the nearest hour. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) The curve $y = x^3 + ax^2 + 7x - 5$ has a stationary point at $x = 1$. Find the value of a, and hence: **2**

(ii) Find the coordinates of all stationary points. **2**

(iii) Determine the nature of the stationary points. **2**

(iv) Sketch the curve and then determine for what values of x the curve is increasing. **2**

(b) If the K^{th} term of an arithmetic series is L, and the L^{th} term is K:

(i) Show that $L = a + (K-1)d$. **1**

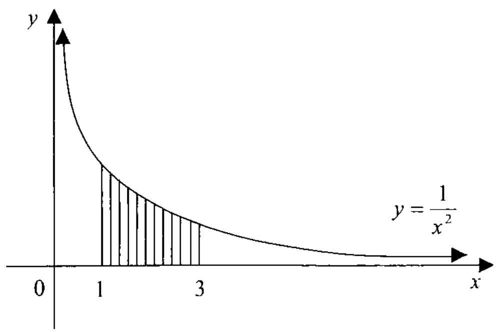
(ii) Find another expression for K. **1**

(iii) By solving the two equations founded, show that $d = -1$. **1**

(iv) Hence, find the first term of this series in terms of L and K. **1**

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



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The diagram above shows the area bounded by the graph $y = \frac{1}{x^2}$, (for $x > 0$), the x – axis and the lines $x = 1$ and $x = 3$.

(i) Find the shaded area. Leave your answer as a fraction. **2**

(ii) Find the volume of the solid formed when the shaded area is rotated about x – axis. Leave your answer in exact form. **3**

(b) (i) Sketch the graph of $f(x) = e^x$ for all values of x in the domain and state its range. **2**

(ii) The curve $f(x) = e^x$ is rotated about the y – axis to give a solid. Show that the volume V_y of the solid formed, from $y = 3$ to $y = 5$, is given by $V_y = \pi \int_3^5 (\ln y)^2 \, dy$. **2**

(iii) Use Simpson's rule with 5 function values to find the volume of this solid, correct to 2 significant figures. **3**

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

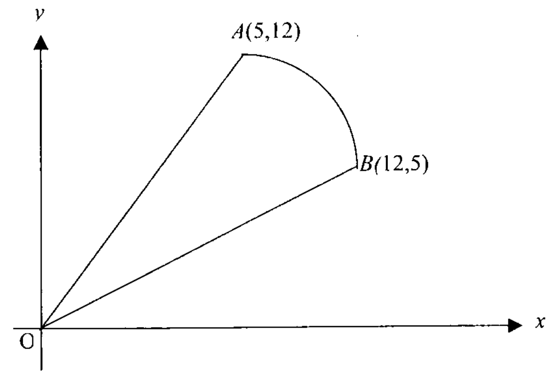
(a) A certain soccer team has a probability of 0.6 of winning a match and a probability of 0.3 of drawing a match.

(i) If this soccer team plays two matches, draw a tree diagram to show all possible outcomes. **2**

(ii) Find the probability of this soccer team winning at least one match out of the two matches. **2**

(iii) Find the probability of this soccer team not winning either of the two matches. **2**

(b)



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The figure above shows a sector of a circle OAB, centre O, with its arc joining the points A (5,12) and B(12,5). Copy this figure into your answer booklet.

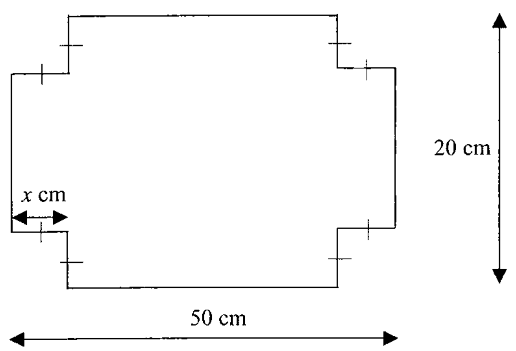
(i) Find the value, in degrees, of one radian, correct to the nearest minute. **1**

(ii) Show that the size of $\angle AOB$ is 0.78 radians, correct to 2 decimal places. **3**

(iii) Calculate the perimeter of sector OAB, correct to 2 decimal places. **2**

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A box is made from a 50 cm by 20 cm rectangle of cardboard by cutting out four equal squares of side x cm from each corner as shown below:



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The edges are turned up to make an open box.

(i) Show that the volume V of this box is given by the equation: **2**

$$V = 4x^3 - 140x^2 + 1000x \ (\text{cm}^3)$$

(ii) Find the value of x, correct to one decimal place, that gives this box its greatest volume. **3**

(iii) Hence, find the maximum volume of this box, correct to 2 decimal places. **1**

(b) Jordan has to pay annual instalments for his superannuation at the beginning of each year according to the formula:

$$M_n = \left(1 + \frac{r}{100}\right) M_{n-1}, \ n \geq 2$$

where $r(\%)$ is the annual rate of interest paid by the fund and M_n is the instalment at the beginning of the n^{th} year.

If the interest rate is 12 % p.a., compounded yearly, and Jordan's first instalment is \$500, find:

(i) How much is his second instalment? **2**

(ii) Find the amount Jordan has to pay into the fund at the beginning of the 20^{th} year. **2**

(iii) Find the total value of his investment after 20 years. **2**

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The acceleration of a particle at any time t seconds is given by $\frac{dv}{dt} = k$, where k is a constant.

(i) Show that $v = kt + c_1$, for some constant c_1. **1**

(ii) The displacement x metres, at any time t seconds, is shown in the table below: **3**

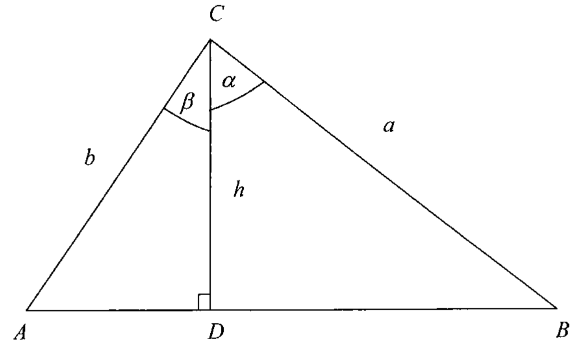
t (sec)	0	1	2
x(metres)	1	2	9

Show that $x = 3t^2 - 2t + 1$

(iii) Find when the particle comes to rest. **2**

Question 10 continues on page 12

(b)



The diagram above shows a triangle ABC, and CD is perpendicular to AB. It is given that $BC = a$, $AC = b$, $\angle ACD = \beta$ and $\angle BCD = \alpha$.

(i) By using triangles ACD and BCD, show that $h = b\cos\beta = a\cos\alpha$. **1**

(ii) Show that the area of triangle ACD is equal to $\frac{1}{2}ab\sin\beta\cos\alpha$ **1**

(iii) Find another expression for the area of triangle BCD in terms of a, b, α and β. **1**

(iv) Show that the area of triangle ABC is equal to $\frac{1}{2}ab\sin(\alpha+\beta)$ **1**

(v) Hence, but not otherwise, deduce that: **2**

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

End of paper

EXAMINERS

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