KAMBALA SCHOOL

YEAR 12

MATHEMATICS

3 UNIT (ADDITIONAL)

HALF-YEARLY EXAMINATION

March 2001

Time Allowed: 2 hours plus 5 minutes reading time

DIRECTIONS TO CANDIDATES

1. This paper contains 6 questions.
2. All questions may be attempted.
3. All questions are of equal value.
4. All necessary working should be shown in every question.
5. Marks may not be awarded for careless or badly arranged work.
6. Board-approved calculators may be used.
7. Start each question on a separate page.

Question 1 (*Start a new page.*)

a) Differentiate with respect to x:

(i) $\tan^2(5x)$

(ii) $\log_{10} x$

b) Find the primitive function of $\frac{7}{1-3x}$.

c) Evaluate

(i) $\lim_{x \to 0} \frac{\sin 3x}{4x}$

(ii) $\lim_{x \to \infty} \frac{x^2 - x - 12}{3x^2 + 2}$

d) Evaluate $\int_0^1 \frac{e^x}{e^x + 1}\,dx$. Leave your answer in exact form.

Question 2 (*Start a new page.*)

a) Solve $\frac{1}{x-5} \leq 1$

b) Simplify $\cos(A+B)\cos B + \sin(A+B)\sin B$.

c) Given that $\frac{dy}{dx} = x - \sin x$ and $y = 2$ when $x = 0$, find y in terms of x.

d) $A(2,4)$ and $B(6,1)$ are two points. Find the co-ordinates of the point $P(x, y)$ that divides the interval AB externally in the ratio $3:5$.

Question 3 (*Start a new page.*)

a) If $f(x) = 8x^3$, find the inverse function $f^{-1}(x)$.

b) Sketch $y = \left|x^2 - 3x - 4\right|$ showing all relevant features.

c) On the same diagram, sketch the curves $y = \sin\theta$ and $y = \dfrac{\theta}{2}$ for $0 \leq \theta \leq \pi$ and hence state the number of solutions to the equation $\sin\theta = \dfrac{\theta}{2}$ in this domain.

d) Solve for x the equation $2\sin 2x = 1$ for $0 \leq x \leq 2\pi$.

Question 4 (*Start a new page.*)

a) An arc AB of a sector of a circle is of length $\dfrac{\pi}{4}$ metres and subtends an angle of 30° at the centre, O, of the circle. Find

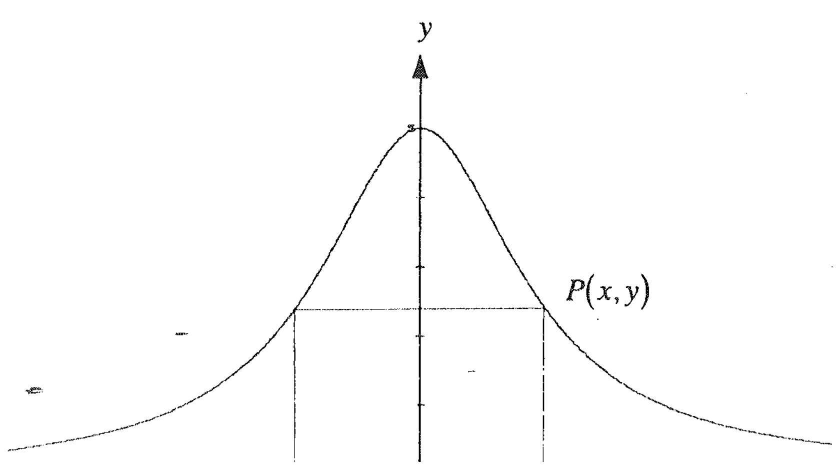
(i) the length of the radius

(ii) the area of sector AOB (in exact form).

b) (i) State the period, amplitude and range of the curve $y = 3 + 3\cos 2x$.

(ii) Sketch the curve in the domain $-\pi \leq x \leq \pi$.

c) A rectangle is inscribed under a curve $y = \dfrac{10}{x^2 + 2}$ such that the rectangle is symmetrical about the y-axis. Find the maximum area of the rectangle.



Question 5 (*Start a new page.*)

a) Find, to the nearest minute, the acute angle between the lines $2x + y + 5 = 0$ and $x - 3y + 6 = 0$.

b) Find the volume (in exact form) of the solid of revolution when the area between the curve $y = \log_e x$ and the y-axis from $y = 1$ to $y = 5$ is rotated about the y-axis.

c) An electrical condenser discharges at a rate proportional to the charge such that $Q = Q_0 e^{-kt}$, where Q is the charge at time t minutes. It takes 8 minutes for the original charge of 1 unit to reduce to half.

(i) State the value of Q_0.

(ii) Find the value of k.

(iii) At what rate is the condenser discharging when the charge has been reduced to a quarter (to 4 significant figures)?

d) An urn contains 3 white balls, 4 red balls and 5 black balls. Two balls are drawn without replacement. Find the probability that both are of a different colour.

Question 6 (*Start a new page.*)

a) Without using a calculator show that $\dfrac{\cos 40^\circ + \sin 50^\circ}{\sqrt{1 - \sin^2 140^\circ}} = 2$

b) Prove by mathematical induction that:

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \ldots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

for all positive integers n.

Hence find $\displaystyle\sum_{r=1}^{\infty} \frac{1}{(3r-2)(3r+1)}$.

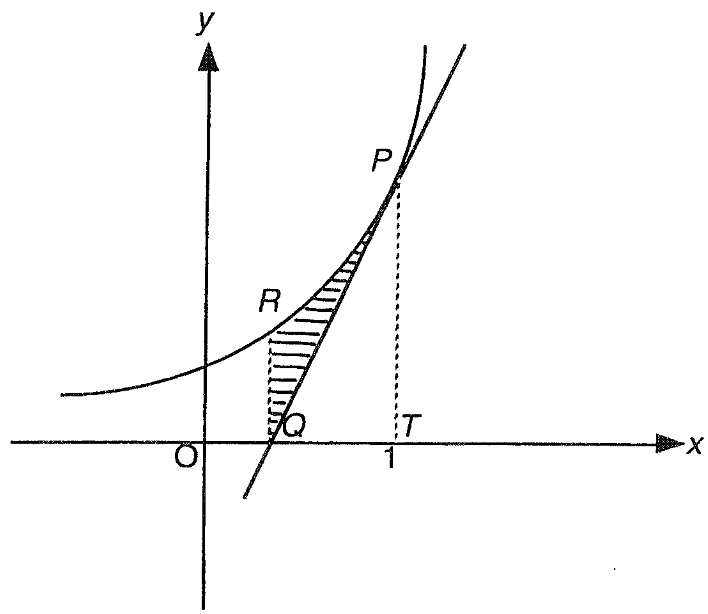
Question 6 continued

c) A tangent is drawn to the curve $y = e^{3x}$ at the point $P\ (1,\ e^3)$. The tangent cuts the x-axis at Q. QR is drawn perpendicular to the x-axis as shown.

(i) Show that the equation of the tangent at P is $y = 3e^3x - 2e^3$.

(ii) Find the co-ordinates of Q.

(iii) Show that the shaded region QRP has area $\dfrac{e^2}{6}(e-2)$ square units.



END OF EXAMINATION