Total marks – 84
Attempt Questions 1-7
All questions are of equal value

Abbotsleigh Ext 1 Trial 2004

Answer each question in a SEPARATE writing booklet. Extra booklets are available.

Question 1 (12 marks) — Marks

(a) Use the table of standard integrals to evaluate $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}}$ **2**

(b) A is the point $(3,-2)$. B is the point $(1,4)$. Find the co-ordinates of the point P which divides AB externally in the ratio $5:2$. **2**

(c) Express $\frac{X^3Y^3}{Z^2}$ in the form $2^a \times 3^b$ if $X=\left(\frac{4}{3}\right)^3$, $Y=\left(\frac{9}{2}\right)^4$ and $Z=\left(\frac{3}{8}\right)^2$ **3**

(d) Sketch the graph of $y=3\tan^{-1}\left(\frac{x}{2}\right)$ clearly showing the domain and range. **2**

(e) Evaluate $\int_0^1 x(2-x)^3\,dx$ using the substitution $u=2-x$. **3**

Question 2 (12 marks)
Start a new booklet

(a) The equation $x^3+2x-8=0$ has a root close to $x=1.6$. Use one application of Newton's method to find a better approximation to the root. **3**

(b) Find the coefficient of x^3 in the expansion of $\left(x+\frac{5}{x^2}\right)^9$ **3**

(c) Find $\int \sin^2 6x\,dx$ **2**

(d) (i) Express $6\cos x+8\sin x$ in the form $R\cos(x-\alpha)$, where α is in radians. **2**

(ii) Hence, or otherwise, solve the equation $6\cos x+8\sin x=5$ for $0\le x\le 2\pi$ **2**

Marks

Question 3 (12 marks)
Start a new booklet

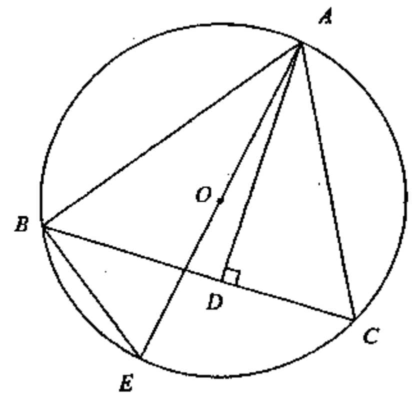
(a) (i) Write out the expansion for $\tan(\alpha+\beta)$ **1**

(ii) Hence evaluate exactly $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{3}{5}\right)$ **2**

(b) ABC is a triangle inscribed in a circle, centre O, and AD is drawn perpendicular to BC.

Copy or trace the diagram into your writing booklet.

Prove $\angle BAE=\angle DAC$ **4**



(c) A sphere is expanding so that its surface area is increasing at the rate of $0.01\text{ cm}^2\text{s}^{-1}$. Calculate the exact rate of change of its volume at the instant when the radius is 5 cm. **3**

(d) Prove $\frac{\sin 2x}{1+\cos 2x}=\tan x$ **2**

Question 4 (12 marks) **Marks**
Start a new booklet

(a) A function f has the following properties for all positive real numbers a and b.

$f(ab) = f(a) + f(b)$
$f(a^c) = cf(a)$
$f(2) = 5$
$f(5) = 12$

Evaluate (i) $f(10)$ **1**
(ii) $f(\sqrt{5})$ **1**
(iii) $f(1)$ **1**

(b) Solve for x $\log_2 x + \log_2(x-2) = 3$ **3**

(c) (i) Sketch $y = \log_e x$ and $y = 1 - x$ on the same diagram. **2**

(ii) Hence or otherwise write down all values of x for which

$\log_e x \le 1 - x$ **1**

(d) Differentiate $x\tan^{-1} x$ and hence show that $\int_0^1 \tan^{-1} x\,dx = \frac{\pi}{4} - \frac{1}{2}\log_e 2$ **3**

Question 5 (12 marks) **Marks**
Start a new booklet

(a) If α, β and γ are the roots of $2x^3 - 6x^2 - 4x + 1 = 0$ find

(i) $\alpha\beta + \beta\gamma + \alpha\gamma$ **1**

(ii) $\alpha^2 + \beta^2 + \gamma^2$ **3**

(b) The population of sheep on a farm is given by the equation $\frac{dN}{dt} = k(N - 2000)$ where N is the number of sheep at any time t and k is a constant. Initially there are 5000 sheep and after 2 years there are 6000 sheep.

(i) Show that $N = 2000 + Ae^{kt}$ is a solution of the differential equation. **1**

(ii) Find the values of A and k. **2**

(iii) How many sheep are on the farm after 5 years? (Answer to the nearest sheep.) **1**

(c) A number of electrical components are wired together in an alarm so that it will operate if at least one of the components works. The probability that each one of the components will work is 0.6.

(i) If an alarm had 3 of these components wired together, find the probability that at least one of the components will work. **2**

(ii) Find the minimum number of components that must be wired together to be 99% certain that the alarm will operate. **2**

Question 6 (12 marks) **Marks**
Start a new booklet

(a) When asked to find $\int \frac{1}{2x}\,dx$ Mary did the following working:

$$\int \frac{1}{2x}\,dx = \frac{1}{2}\int \frac{1}{x}\,dx$$
$$= \frac{1}{2}\log_e x + c$$

Louise attempted the same question with working shown below:

$$\int \frac{1}{2x}\,dx = \frac{1}{2}\int \frac{2}{2x}\,dx$$
$$= \frac{1}{2}\log_e 2x + k$$

Can they both be correct or is only one correct? Justify your answer. **2**

(b) Use mathematical induction to prove that $13\times 6^n + 2$ is divisible by 5 for all n, where n is a positive integer. **3**

(c) Two points $P(6p, 3p^2)$ and $Q(6q, 3q^2)$ lie on the parabola $x^2 = 12y$. The equation of the chord PQ is given by $y - \frac{1}{2}(p+q)x + 3pq = 0$. This chord passes through the fixed point $(4, -3)$.

(i) Show that $3pq = 3 + 2(p+q)$ **1**

(ii) Show that the equation of the tangent to the parabola at P is $y = px - 3p^2$. **2**

(iii) Find the co-ordinates of the point of intersection T of the tangents to the parabola at P and Q. **2**

(iv) Find the locus of T and describe it geometrically. **2**

Question 7 (12 marks) **Marks**
Start a new booklet

(a) Evaluate $\lim_{x\to 0} \frac{\sin 3x}{7x}$ **1**

(b) A particle moves with acceleration $\ddot{x} = 4x + 2$ where x metres is the distance measured from the origin 0. Initially the particle is at the origin with velocity -1 ms^{-1}.

(i) Show that $v^2 = 4x^2 + 4x + 1$ **2**

(ii) Show that $x = \frac{1}{2}\left(e^{-2t} - 1\right)$ **3**

(iii) Describe the position of the particle as t increases indefinitely. **1**

(c) Consider the geometric series

$$S = 1 + (1+x) + (1+x)^2 + \dots + (1+x)^n$$

(i) Write down the expansion of $(1+x)^n$ **1**

(ii) Show that $S = \frac{(1+x)^{n+1} - 1}{x}$ **2**

(iii) Hence, show that

$$S = \binom{n+1}{1} + \binom{n+1}{2}x + \dots + \binom{n+1}{r+1}x^r + \dots + \binom{n+1}{n+1}x^n$$ **2**

END OF PAPER