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PYMBLE LADIES' COLLEGE

YEAR 12

MATHEMATICS EXTENSION 1

HSC TRIAL EXAMINATION 2002

Time Allowed: 2 hours + 5 mins reading time

INSTRUCTIONS

- **All questions should be attempted**
- **Write your name and your teacher's name on each page**
- **Start each question on a new page**
- **DO NOT staple the questions together**
- **Only approved calculators may be used**
- **A standard integral sheet is attached**
- **Marks might be deducted for careless or untidy work**
- **Hand this question paper in with your answers**
- **ALL rough working paper must be attached to the back of the last question**
- **Staple a coloured sheet of paper to the back of each question**
- **There are seven (7) questions in this paper**
- **All questions are of equal value**

Question 1 — Marks

(a) Evaluate $\lim_{x \to 0} \frac{\sin 3x}{x}$ — 1

(b) The point P (7, -1) divides the interval AB externally in the ratio 3 : 2. If A is (-2, 5) find the coordinates of B. — 2

(c) Solve for x

$$\frac{x+1}{x-2} < 2$$ — 2

(d) Find the gradient of the tangent to the curve $y = \tan^{-1}(2x)$ at the point where $x = \frac{1}{2}$. — 2

(e) Evaluate $\int_0^3 \frac{1}{\sqrt{9-x^2}}\,dx$ — 2

(f) On the same number plane, sketch the graphs of

(i) $y = |2x-1|$ and $y = |x+1|$ — 2

(ii) Hence, or otherwise, solve $|2x-1| \le |x+1|$ — 1

Question 2 (Start a new sheet of paper)

(a) Prove that $\frac{\sin 2\theta}{\sin\theta} - \sec\theta = \frac{\cos 2\theta}{\cos\theta}$ 2

(b) Evaluate $\int_{\frac{1}{2}}^{1} 4t(2t-1)^5 dt$ by using the substitution $u = 2t-1$ 4

(c) The angle between the lines $y = 3x$ and $y = mx$ is 45°.
Find the value(s) of m. 3

(d) Solve $\tan 2\theta - \cot\theta = 0$ where $0 \le \theta \le \pi$ 3

Question 3 (Start a new sheet of paper) Marks

(a) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^3\left(\frac{x}{2}\right) dx$ 2

(b) Use Mathematical Induction to prove that

(i) $4(1^3 + 2^3 + 3^3 + \cdots + n^3) = n^2(n+1)^2$, for $n = 1,2,3\ldots$
3

(ii) Hence find the value of $\lim_{n\to\infty}\left(\frac{1^3 + 2^3 + 3^3 + \cdots + n^3}{n^4}\right)$ 1

(c) (i) Express $\sin x + \sqrt{3}\cos x$ in the form $R\sin(x+\alpha)$
where $R > 0$ and $0 \le \alpha \le \frac{\pi}{2}$ 2

(ii) Hence sketch $y = \sin x + \sqrt{3}\cos x$ for $-2\pi \le x \le 2\pi$ showing any x and y intercepts. 2

(iii) Find the general solution to $\sin x + \sqrt{3}\cos x = \sqrt{2}$ 2

Question 4 (Start a new sheet of paper) — Marks

(a) α, β and γ are the roots of the equation $x^3 + 2x^2 - 3x + 5 = 0$

(i) State the values of $\alpha + \beta + \gamma$, $\alpha\beta + \alpha\gamma + \beta\gamma$ — 2

(ii) Find the value of $\alpha^2 + \beta^2 + \gamma^2$ — 2

(b) If a polynomial $P(x)$ is divided by $(x+1)$ the remainder is 5 and when $P(x)$ is divided by $(2x+1)$ the remainder is 3. Find the remainder when $P(x)$ is divided by $(x+1)(2x+1)$. — 3

(c) From a point S the bearings of two points P and Q are found to be 331° T and 011°T respectively. From a point F, 7 km due north of S, the bearings of P and Q are 299° T and 020° T respectively.

(i) Show that $PF = \sin 29^\circ \times \dfrac{7}{\sin 32^\circ}$ — 2

(ii) By considering the triangle FPQ, show that if the distance between P and Q is d metres, then

$$d^2 = 49\left(\frac{\sin^2 29^\circ}{\sin^2 32^\circ} + \frac{\sin^2 11^\circ}{\sin^2 9^\circ} - 2\frac{\sin 29^\circ \sin 11^\circ \cos 81^\circ}{\sin 32^\circ \sin 9^\circ}\right)$$ — 3

Question 5 (Start a new sheet of paper) — Marks

(a) Consider the function $f(x) = \dfrac{x-1}{x^2}$

(i) Show that there is only one stationary point and determine its nature — 3

(ii) Determine the point of inflexion. — 1

(iii) What happens to $f(x)$ as $x \to \pm\infty$? — 1

(iv) What happens to $f(x)$ as $x \to 0$? — 1

(v) Sketch the curve showing all its essential features. (Use at least half a page.) — 2

(b) (i) Prove that $\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = \ddot{x}$ — 2

(ii) An object moving in a straight line has an acceleration given by $\ddot{x} = x(8 - 3x)$ where x metres is its position relative to a fixed point 0.

At $x = 0$, it has a speed of 4 m/s. Find its speed when it is 1 m on the positive side of 0. — 2

Question 6 (Start a new sheet of paper) — Marks

(a) A particle is oscillating in simple harmonic motion such that its displacement x metres from the origin is given by the equation $\frac{d^2x}{dt^2} = -16x$ where t is time in seconds.

(i) Show that $x = a\cos(4t + \alpha)$ is a solution of motion for this particle. (a and α are constants). 1

(ii) When $t = 0$, $v = 4$ m/s and $x = 5$ m. Show that the amplitude of the oscillation is $\sqrt{26}$ metres. 2

(iii) What is the maximum speed of the particle? 1

(b) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$. The tangents at P and Q meet at T which is always on the parabola $x^2 = -4ay$.

(i) Derive the equation of the tangent at P. 2

(ii) Hence write down the equation of the tangent at Q. 1

(iii) Show that T is the point $(a(q+p), apq)$. 1

(iv) Show that $p^2 + q^2 = -6pq$ 1

(v) Find M, the midpoint of PQ. 1

(vi) Hence, or otherwise, find the locus of M. 2

Question 7 (Start a new sheet of paper) — Marks

(a) (i) On the same number plane, sketch the graphs of $y = \cos^{-1} x$ and $y = \sin^{-1}(\frac{x}{2})$. Label the important features. 2

(ii) Show $y = \cos^{-1} x$ and $y = \sin^{-1}(\frac{x}{2})$ intersect at $x = \frac{2}{\sqrt{5}}$. 2

(iii) Find the inverse function of $y = \sin^{-1}(\frac{x}{2})$ 1

(iv) Hence or otherwise find the area bounded by the $x-axis$ and the graphs $y = \cos^{-1} x$ and $y = \sin^{-1}(\frac{x}{2})$ (answer correct to 2 decimal places.) 3

(b) Wheat is the only crop grown on Sandy's property in outback NSW. Per hectare the amount of water, W, in kilolitres, used during irrigation times is given by

$$W = Cg^2 + \frac{D}{g}$$

where g is the amount of grain produced in tonnes per hectare and C and D are positive constants. There is a limited amount of water available for irrigation.

(i) Show that, for maximum hectares under irrigation, production of grain per hectare, g, is given by

$$g = \left(\frac{D}{2C}\right)^{\frac{1}{3}}$$ 2

(ii) Show that for maximum grain produced on Sandy's property, grain production per hectare needs to be about 59% more than that given in part (i) above. 2

THE END