



Student Number:

2004

HIGHER SCHOOL CERTIFICATE
Sample Examination Paper

MATHEMATICS

Extension 1

General Instructions

Reading time – 5 minutes
Working time – 2 hours

- Attempt ALL questions.
- Show all necessary working, marks may be deducted for careless or untidy work.
- Board-approved calculators may be used.
- Additional Answer Booklets are available.

Directions to School or College

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QUESTION 1 (Start a new booklet)**MARKS**

- (a) Use the substitution $u = \log x$ to find the exact value of $\int_1^2 \frac{1}{x \log x} dx$. 3
- (b) Solve $\frac{5}{(2-x)(x+2)} > 1$. 2
- (c) Show that the point $A(\frac{1}{2}, 4)$ lies on the line joining the points $P(-3, -3)$ and $Q(1, 5)$ and find the ratio in which it divides the line segment PQ . 3
- (d) Show that $\tan^{-1} x = \cos^{-1} \frac{1}{\sqrt{1+x^2}}$. 3
- (e) Find the remainder when the function $P(x) = x^3 - 4x + 2$ is divided by $x + 4$. 1

QUESTION 2 (Start a new booklet)

MARKS

- (a) The word EQUATIONS contains all five vowels. How many 7-letter 'words' consisting of all five vowels can be formed from the letters of EQUATIONS? 2
- (b) Determine the coefficient of x^5 in the expansion of $(1 - 3x + 2x^3)(1 - 2x)^6$. 3
- (c) Find the general solution of the equation $\cos 54^\circ \cos \alpha + \sin 54^\circ \sin \alpha = \sin 2\alpha$. 3
- (d) Find $\frac{d}{dx} \left(\frac{\tan^2 x}{x} \right)$. 2
- (d) If $f(x) = 2x^2 + x$, use the definition $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ to find the derivative of $f(x)$ at the point where $x = a$. 2

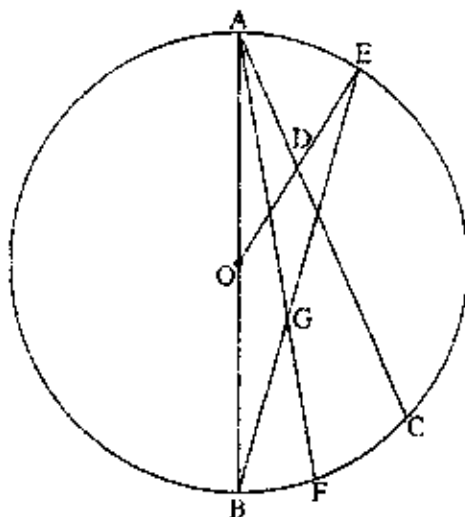
QUESTION 3 (Start a new booklet)

MARKS

- (a) For the parabola $y = x^2 - 4x - 1$, find the coordinates of the focus and the equation of the directrix. 3

- (b) How many numbers smaller than 500 can be formed from the digits 2, 3, 4, 5, 6 and 7 if no repetitions are allowed? 2

- (c) In the figure, AOB is the diameter of a circle centre O . D is a point on chord AC such that $DA = DO$ and OD is produced to E . AF is the bisector of $\angle BAC$ and cuts BE in G .



Prove that

- (i) $GA = GB$ 3
- (ii) $AOGE$ is a cyclic quadrilateral 2
- (iii) If CE is joined, then $CE \parallel AF$. 2

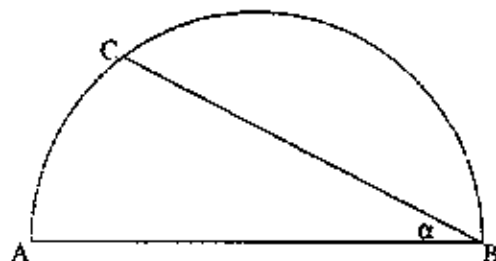
QUESTION 4 (Start a new booklet)**MARKS**

- (a) Use mathematical induction to prove that, for all positive integers
- n
- ,

5

$$\sum_{r=1}^n \frac{r^2}{(2r-1)(2r+1)} = \frac{n(n+1)}{2(2n+1)}$$

- (b) AB is the diameter of a semi-circle of unit radius and BC is a chord which makes an angle α with AB such that the area of the semi-circle is bisected by the chord.



- (i) Show that the area of the segment is

2

$$\frac{1}{2} (\pi - 2\alpha - \sin 2\alpha)$$

- (ii) Hence show that
- $2 \sin 2\alpha + 4\alpha = \pi$

2

- (iii) Prove that a root of this equation lies between
- $\alpha = 0.4$
- , and
- $\alpha = 0.5$
- .

2

- (iv) By using the 'halving the interval' method, determine whether the root lies closer to 0.4 or 0.5.

1

QUESTION 5 (Start a new booklet)

MARKS

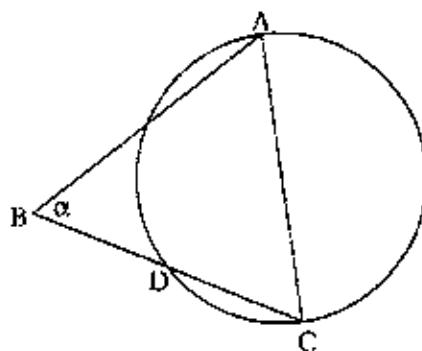
- (a) Corn cobs are cooked by immersing them in boiling water. On being removed, a corn cob cools in the air according to the equation $\frac{dT}{dt} = -k(T - T_0)$ where t is time in minutes, T is temperature in $^{\circ}\text{C}$ and T_0 is the temperature of the air, while k is a positive constant.

(i) Verify that $T = T_0 + Ae^{-kt}$ is a solution of the above equation where A is a constant. 2

(ii) If the temperature of the boiling water is 100°C and that of the air is a constant 25°C , find the values of A and k if a corn cob cools to 70°C in 3 minutes. 2

(iii) How long should a person wait to enjoy the food at a temperature of 50°C ? 2

- (b) AC is a diameter of a circle
 $AC = BC = y$ and $\angle ABC = \alpha$.



(i) Show that $\cos \alpha = \frac{x}{2y}$. 2

(ii) Determine DC in terms of x and y . 4

QUESTION 6 (Start a new booklet)

MARKS

- (a) The rise and fall of tides can be approximated to simple harmonic motion. On October 18 the depth of water in a tidal lagoon at low tide is 2 m at 11.00 a.m. At the following high tide at 5.20 p.m. the depth is 6 m. Calculate between what times a yacht could safely cross the lagoon if a minimum depth of 3.5 m of water is needed. 7
- (b) At a dinner party, host and hostess and seven guests sit at a round table.
- (i) If there are no restrictions, in how many ways can they be arranged? 1
- (ii) In how many ways can they be arranged if host and hostess are separated? 2
- (c) At a football club a team of 13 players is to be chosen from a pool of 32 players consisting of 20 Australian-born players and 12 players born elsewhere. What is the probability that the team will consist of all Australian-born players? 2

QUESTION 7 (Start a new booklet)

MARKS

- (a) A particle is moving along the x -axis. Its velocity v at position x is given by

2

$$v = \sqrt{8x - x^2}$$

Find its acceleration when $x = 3$.

- (b) A football is kicked at an angle of α to the horizontal. The position of the ball at time t seconds is given by

$$x = vt \cos \alpha$$

$$y = vt \sin \alpha - \frac{1}{2}gt^2$$

where $g \text{ m/s}^2$ is the acceleration due to gravity and $v \text{ m/s}$ is the initial velocity of projection.

- (i) Show that the equation of the path of the ball is

1

$$y = x \tan \alpha - \frac{gx^2}{2v^2} \sec^2 \alpha.$$

- (ii) Show that the maximum height h reached is given by

2

$$h = \frac{v^2 \sin^2 \alpha}{2g}.$$

- (iii) Hence show that $y = x \tan \alpha \left(1 - \frac{x \tan \alpha}{4h}\right)$.

2

- (iv) If $g = 10 \text{ m/s}^2$, $\alpha = 30^\circ$ and the ball just clears the head of a player 1.6 m tall and 10 m away, calculate the maximum height reached by the ball.

1

- (c) Write down the expansion of $(1 + x)^{2n}$ and hence prove that

4

$$\sum_{r=0}^n {}^{2n}C_r = 2^{2n-1} + \frac{(2n)!}{2(n!)^2}.$$

End of paper