ST. CATHERINE'S SCHOOL

YEAR 12 TRIAL EXAMINATION

3/4 UNIT MATHEMATICS

TIME ALLOWED: 2 HOURS *(PLUS 5 MINUTES READING TIME)*

DATE: AUGUST 1999

STUDENT NUMBER:______________________________

DIRECTIONS TO CANDIDATES:

- This paper consists of seven questions.
- All questions are to be attempted.
- All questions are of equal value.
- In every question, all necessary working should be shown.
- Marks may be deducted for careless or badly arranged work.
- Approved calculators and geometrical instruments are required.
- Begin a NEW PAGE for every question.
- Attach your question paper to the front of Section A.
- Hand your work in three bundles:

  Section A - Questions 1, 2 and 3

  Section B - Questions 4, 5, 6 and 7

- This sheet will form the cover page for Section A. You will need to write a cover sheet for Section B, which clearly states your Student Number.

Securely staple or tie questions together in sections.

TEACHERS USE ONLY TOTAL MARKS
A
B

3 Unit Trial Mathematics Examination Paper 1999
Section A
Question 1

Marks

a) Solve the inequality $\frac{x^2 - 1}{x} > 0$ — 3

b) Evaluate $\int_0^{\pi} \sin^2 x \, dx$ — 3

c) Integrate $\int \frac{t}{\sqrt{1+t}} \, dt$ by using the substitution $t = u^2 - 1$ — 3

d) A particle moves from rest from the origin in a straight line in such a way that its velocity $v \, m/s$ is given by $v = 20t - 5t^2$, (where t is in seconds). — 3

Find (i) when the particle comes to rest

(ii) the greatest velocity of the particle.

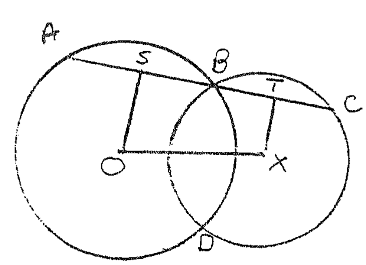
Question 2 (Start a new page)

a) If $A(x)$ is a factor of $P(x)$, find a when $A(x) = x - 4$ and $P(x) = x^3 + 2x^2 + ax - 20$ — 2

b) Express $12\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$ and use it to solve $12\cos\theta + 5\sin\theta = 13$ for $0^0 \le \theta \le 360^0$. — 5

c) The equation $e^x = x + 2$ has a root close to $x = 1.2$. Use Newton's method once to find a better approximation to this root (correct to 2 decimal places). — 2

d) ABC is a straight line
S and T are midpoints of AB and BC respectively
O is centre of cirlce ABD
X is centre of circle BCD — 3



Prove <SOX is the supplement of <OXT.

Question 3 (Start a new page)

Marks

a) Consider the function $f(x) = \dfrac{x}{x^2+1}$ 7

(i) Show that it is an odd function

(ii) Find any stationary points and given that $f''(x) = \dfrac{2x(x^2-3)}{(x^2+1)^3}$, find any points of inflexion.

(iii) Describe the behaviour of $f(x)$ for very large positive and very large negative values of x i.e. when $x \to \infty$ and $x \to -\infty$.

(iv) Sketch the curve.

b) Prove by mathematical induction that 5

$\sum_{r=1}^{n} r(r+2) = \frac{1}{6}n(n+1)(2n+7)$ where n is a positive integer.

SECTION B (Start a new page)

Question 4

Marks

a) (i) How many odd 4 digit numbers can be made from the digits 2, 3, 4, 5, 6 if none of the digits are repeated? 3

(ii) What is the probability of an odd number being selected if the digits may be repeated?

b) Without using your calculator, evaluate 3

(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(i) $\cos\left(\sin^{-1}\left(-\frac{3}{5}\right)\right)$

c) Using the t-results, show that $\dfrac{\cot\frac{\theta}{2}+\tan\frac{\theta}{2}}{\cot\frac{\theta}{2}-\tan\frac{\theta}{2}}=\sec\theta$ 3

d) Evaluate $\int_0^{\frac{1}{4}} \frac{dx}{1+16x^2}$ 3

Question 5 (Start a new page)

a) The sum of three acute angles is 45^0 and the tangent ratios of two of them are $\frac{1}{2}$ and $\frac{1}{4}$ respectively. Without using your calculator, find the tangent ratio of the third angle. 4

b) The normals to the parabola $x^2 = 4ay$ at the points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ intersect at R. 8

(i) Derive the equation of the normal at P

(ii) Find R, the point of intersection of the normals at P and Q

(iii) Derive the equation of the chord PQ.

(iv) If the chord PQ varies in such a way that it always passes through $(0, 2a)$ find the locus of R.

Question 6 (Start a new page)

Marks

a) Find the co-ordinates of the point P which divides the interval AB with end points $A(2,3)$ and $B(5,-7)$ internally in the ratio 4:9. 2

b) A sphere is expanding such that its surface area is increasing at the rate of $0.01 cm / \sec^2$. 5
Calculate the rate of change of

(i) its radius
(ii) its volume

at an instant when the radius is 5 cm.

c) Find $\frac{d}{dx}\sin^{-1} e^{2x}$ and hence evaluate $\int_{-\ln\sqrt{2}}^{0} \frac{2e^{2x}}{\sqrt{1-e^{4x}}}\,dx$ 5

Question 7 (Start a new page)

Marks

a) Brine, containing 1 kg of salt per 10 litres, runs into a tank, initially filled with 500 litres of fresh water, at a rate of 25 litres per minute. 6

The mixture runs out of the tank at the same rate of 25L/min.

(i) If A is the amount of salt in the tank at time t, by calculating the concentration of salt flowing in and out of the tank,

show that $\frac{dA}{dt} = -\frac{1}{20}(A - 50)$

NOTE: 1 L of water weighs 1kg.

(ii) Find the amount of salt in the tank at the end of 100 minutes, assuming that the mixture is kept uniform by stirring.

b) A particle moves with an acceleration which varies linearly as the distance travelled such that $\ddot{x} = mx + b$. It starts at the origin from rest with an accleleration of $3m/s^2$ and reaches maximum speed in a distance of $160m$. 6

Find (i) the maximum speed

(ii) the speed when the particle has moved $80m$.

END OF EXAMINATION

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$