DOONSIDE TECHNOLOGY HIGH SCHOOL

MATHEMATICS FACULTY

Trial HSC
Examination
2001

2U Mathematics

Time Allowed : Three Hours

Directions to candidates:

* Attempt ALL questions.
* All questions are of equal value.
* Show all necessary working.
* Marks may be deducted for careless or badly arranged work.
* Only Board-approved calculators may be used.
* Standard Integrals are printed on the last page. These may be removed for your convenience.
* Start each question on a new sheet of paper.

QUESTION 1 Use a SEPARATE Sheet of Paper **Marks**

(a) Express 0.304304304...... in the form $\frac{a}{b}$ where a and b have no common factor. **2**

(b) Factorise $3x^2 - 2x - 1$. **2**

(c) Solve and graph the solution of $|2x+1| < 2$ on a number line. **2**

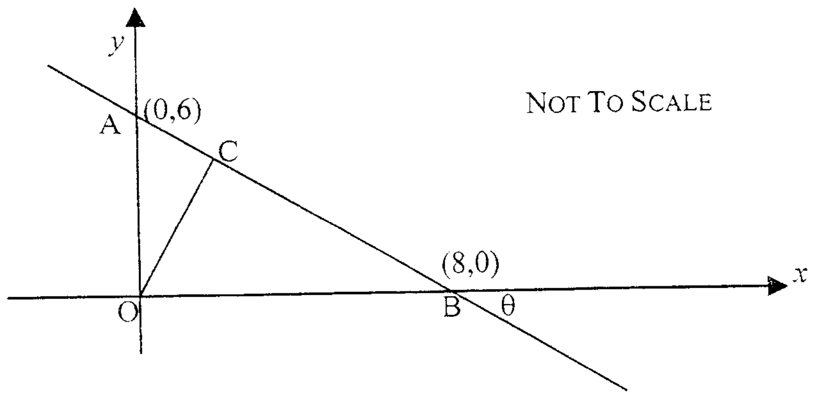
(d) Find the value of $8^{\frac{1}{2}}$ correct to 3 decimal places. **2**

(e) Find a primitive function for **2**

$$x^{-2} + 6.$$

(f) Find the exact value of $\tan 60^0 + \tan 150^0$. **2**

QUESTION 2 Use a SEPARATE Sheet of Paper **Marks**



Find the gradient of the line AB. 1

Show that the equation of AB is $3x + 4y - 24 = 0$ 2

Calculate the acute angle θ to the nearest whole degree. 2

Given that OC meets AB at right angles,
calculate the distance OC. 2

(e) (i) Show OC has equation $4x - 3y = 0$ 2

(ii) Find the distance BC. 2

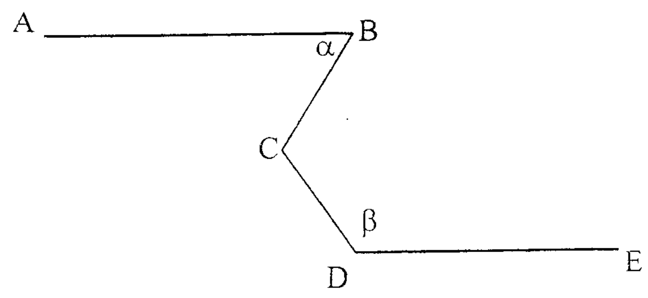
(iii) Show that $\dfrac{OC}{BC} = \dfrac{OA}{OB}$ 1

QUESTION 3 Use a SEPARATE Sheet of Paper **Marks**

(a) Obtain all solutions to **2**

$$9^x - 28 \times 3^x + 27 = 0$$

(b)



In the diagram, AB ∥ DE. $\angle ABC = \alpha$. $\angle CDE = \beta$. **2**

Explain why the reflex angle DCB = $180 + \beta - \alpha$.

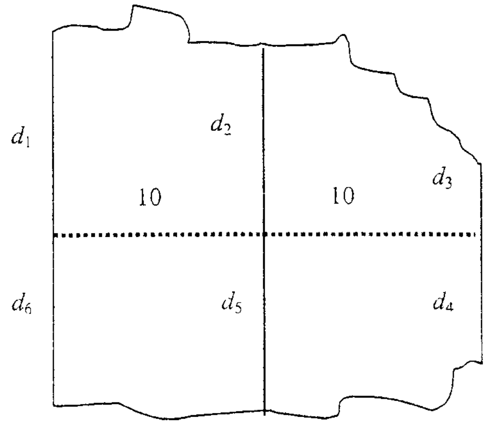
(c) Differentiate the following with respect to x.

(i) $\dfrac{\tan x}{x}$ **2**

(ii) $\sin^3 x$ **2**

QUESTION 3 CONTINUED **MARKS**

(d) The diagram shows the face of a 20m wide vertical cliff. The distances d_1 - d_6 are given in the table.



d_1	d_2	d_3	d_4	d_5	d_6
15	14	5.4	8.8	15	14.4

(i) Find an estimate for the area of the cliff face using the trapezoidal rule. Give your answer correct to the nearest square metre. **2**

(ii) Is the estimate greater than or less than the actual area of the cliff ? Justify your answer. **2**

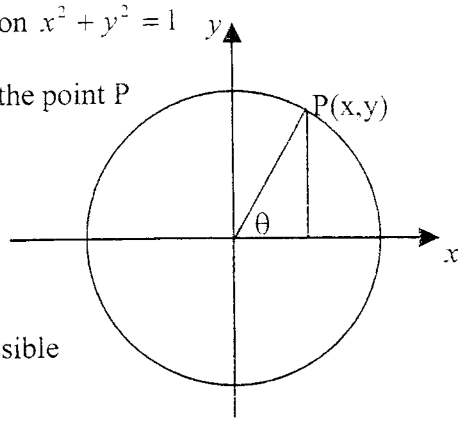
QUESTION 4 Use a SEPARATE Sheet of Paper **Marks**

(a) For the quadratic function $f(x) = Ax^2 - 7x + 3$, $f(2) = -3$.

(i) Find the value of A. **1**

(ii) If the two roots of the equation $f(x) = 0$ are α and β, find the value of $\alpha^2 + \beta^2$. **2**

(b) The unit circle shown has equation $x^2 + y^2 = 1$



(i) Write the coordinates of the point P in terms of angle θ. **1**

(ii) Explain why $\sin^2\theta + \cos^2\theta = 1$ **1**

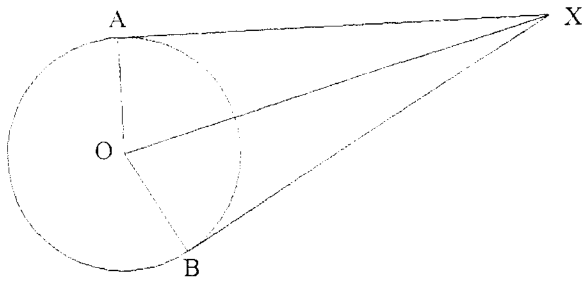
(iii) If $\sin\theta = \frac{8}{17}$ find 2 possible values for $\cos\theta$. **2**

(c) The figure shows a circle, centre O.

AX and BX are tangents to the circle from the external point X.

OA and OB are the radii at the points of contact of the tangents.

AX $\perp$ OA and BX $\perp$ OB



(i) By considering the triangles AOX and BOX prove that AX = BX. **3**

(ii) If AO = r and $\angle AOX = \theta$, show that the area of OAXB = $r^2 \tan\theta$. **2**

QUESTION 5 Use a SEPARATE Sheet of Paper **Marks**

(a) In a geometric sequence $T_1 = 27$ and $T_4 = 1$

(i) Find the common ratio, r. **1**

(ii) Find the limiting sum **2**

b) Consider the series $97 + 91 + 85 + 79 + \ldots\ldots$

(i) Find the common difference, d **1**

(ii) Find the largest n such that $S_n > 0$ **2**

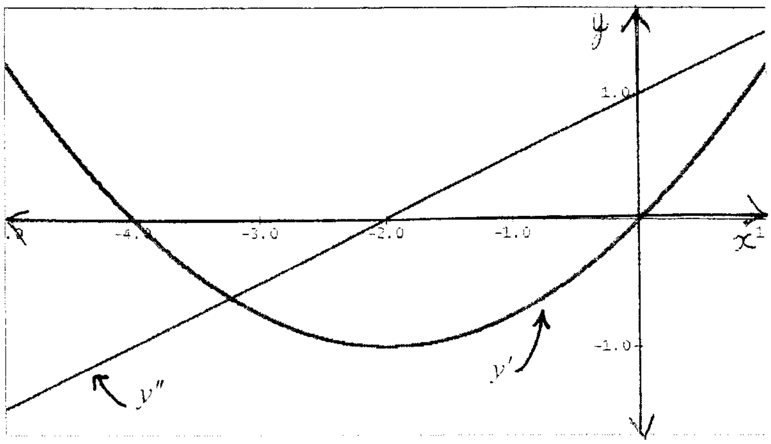
(c Find the following Integrals

(i) $\int_0^{\frac{\pi}{4}} \sec^2 x.dx$ **1**

(ii) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x} dx$ **2**

QUESTION 5 CONTINUED **MARKS**

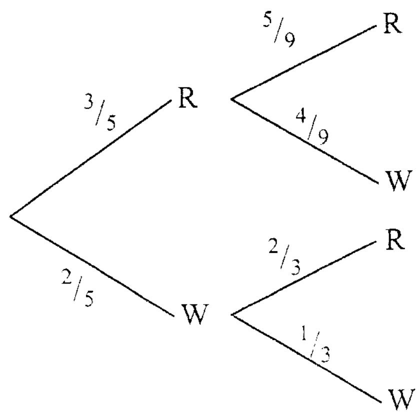
(d) The graph shows y' and y'' for a function $y = f(x)$. **3**



Sketch the graph of $y = f(x)$ clearly showing the x values of any turning points and points of inflexion.

QUESTION 6 Use a SEPARATE Sheet of Paper **Marks**

(a)



Some red and white balls are placed in a bag.

The tree diagram shows the probabilities relating to the selection of two balls from the bag, without replacement.

Find (i) The probability that the two balls are different colours. **2**

(ii) The number of red balls and white balls in the bag. **1**

b) Due to adverse weather conditions the population P of a certain invertebrate is falling according to the rule :

$$P = 2000000e^{kt}$$

where t is the time in years after January 1st 2002.

At the end of 4 years the population is half the initial population.

(i) What is the initial population? **1**

(ii) Calculate the value of k. **2**

(iii) How many invertebrates will there be after 16 years? **1**

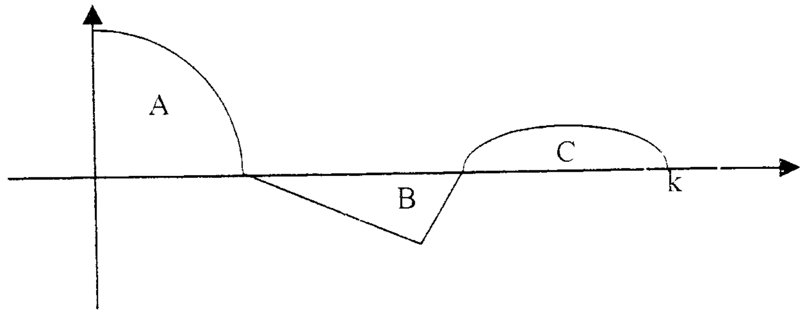
(iv) When will the species be extinct? ($P < 1$) **1**

(c) (i) Sketch the curve $y = 2\sin 2x + 1$ in the domain $0 \leq x \leq \pi$ showing the main features of the graph. **3**

(ii) State a solution to $0 = 2\sin 2x + 1$ in the domain $0 \leq x \leq \pi$ **1**

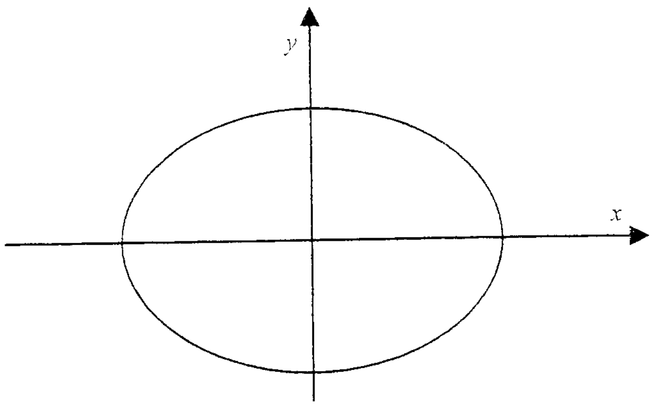
QUESTION 7 Use a SEPARATE Sheet of Paper **Marks**

(a) The graph shows $y = f(x)$ for $0 \le x \le k$ **2**



The value of $\int_0^k f(x).dx$ is known to be 3.5 units.
If A = 5 and B = 4 find the area C.

(b)

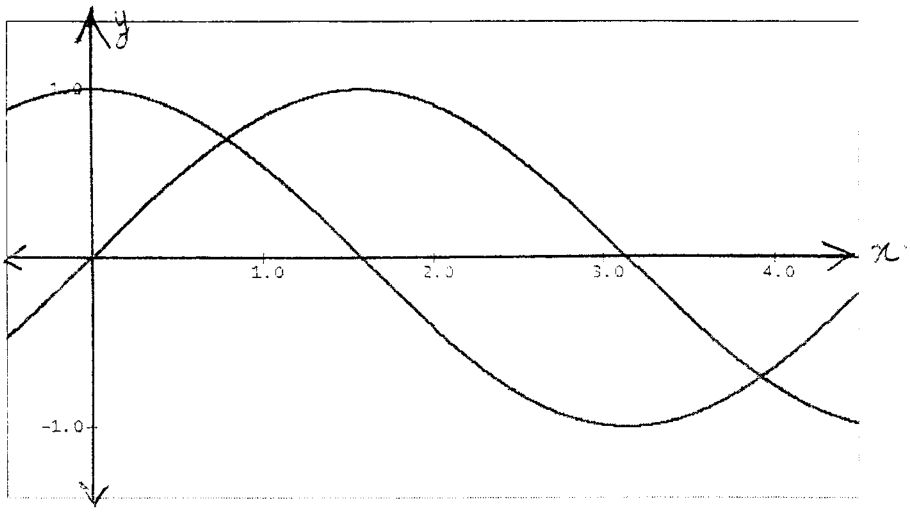


The curve represented on the graph is an ellipse which has equation $4x^2 + 9y^2 = 36$

(i) Show that the curve crosses the x axis at (3,0) and (-3,0). **1**

(ii) Obtain the volume generated when the curve is rotated about the x axis. **3**

QUESTION 7 CONTINUED **Marks**



(c

T graph shows the functions $y = \sin x$ and $y = \cos x$

(i) Show that the x coordinates of the points of intersection of these two curves in the domain shown are $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ **2**

(ii) Calculate the area enclosed between the two curves in the diagram. (Give your answer in exact terms.) **4**

QUESTION 8 Use a SEPARATE Sheet of Paper **Marks**

(a) (i) For what values of k does the quadratic equation 2

$$kx^2 + (k+3)\ x - 1 = 0$$

have real roots ?

(ii) For what values of k is the quadratic expression 2

$$kx^2 + (k+3)\ x - 1$$

positive definite ?

(b) (i) Show that $\frac{d}{dx}(x\ln x - x) = \ln x$ 1

(ii) Hence evaluate $\int_1^{e^2} \ln x.dx$. 2

Leave your answer in exact form.

(c) Solve the pair of simultaneous equations 2

$$\log_{10}\frac{x}{y} = 2$$

$$\log_{10} x + \log_{10} y = 4$$

(d) Find the equation of the tangent to the curve $y = \ln\left(\sqrt{x}\right)$ when $x = e$ 3

QUESTION 9 Use a SEPARATE Sheet of Paper **Marks**

(a) For the parabola $8x = y^2$ find

(i) The Vertex **1**

(ii) The Focus **1**

(iii) The Directrix **1**

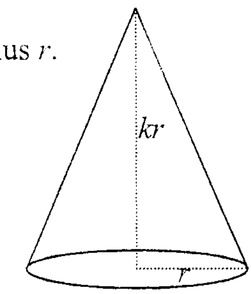
(b) If $\log_x a = 3.6$ and $\log_x b = 2$ find :

(i) $\log_x \sqrt[3]{a}$ **1**

(ii) $\log_x ab$ **1**

(iii) $\log_x \dfrac{b}{a}$ **1**

(c) The diagram represents a right conical container, with radius r. The height of the container = kr. Also the sum of the radius and the height = 1m.



(i) Show that the volume of the cone is given by $V = \dfrac{\pi}{3} \cdot \dfrac{k}{(1+k)^3}$ **2**

(ii) Find the value of k which maximises the volume of the cone. **3**

(iii) Calculate this maximum volume. **1**

QUESTION 10 Use a SEPARATE Sheet of Paper **Marks**

(a) A particle is moving along the x axis and its velocity is given by the equation

$v = 8t - t^2$.

(i) At what time(s) was the particle at rest ? **1**

(ii) Find the displacement (x) as a function of t if the particle was initially at $x = 3$. **1**

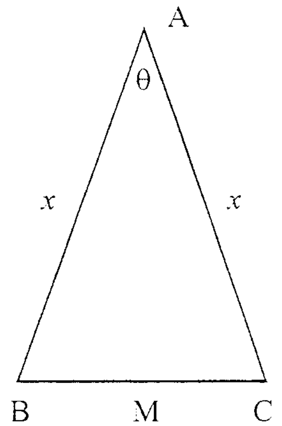
(iii) Find when the particle has zero acceleration. **1**

(iv) What is the total distance travelled by the particle in the first 4 seconds. **2**

b) Consider the isosceles triangle ABC which has perimeter 1m

and two sides of length x as shown in the diagram.

The angle at the apex is θ radians and M is the midpoint of the base BC.



(i) Calculate the length of the altitude of the triangle, AM **2**

(ii) Show $\cos\left(\dfrac{\theta}{2}\right) = \dfrac{\sqrt{x - \frac{1}{4}}}{x}$ **1**

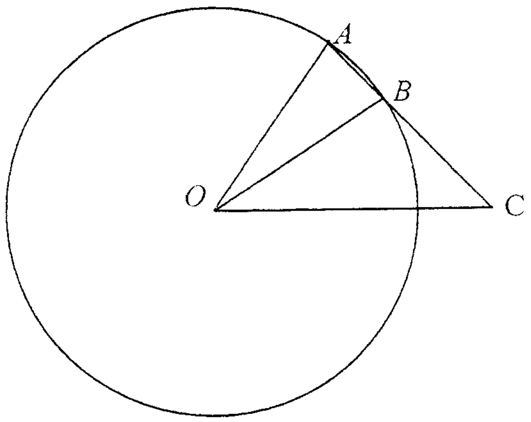
(iii) Show that the area of the triangle is given by **1**

$A = (\frac{1}{2} - x)\sqrt{x - \frac{1}{4}}$

QUESTION 10 CONTINUED **MARKS**

(c) In the diagram OA = OB = 11units. AC is a straight line segment.

OC = 13 units. $\angle OCA = 50^0$.



(i) Use the sine rule to calculate $\angle OAC$ correct to the nearest degree. **2**

(ii) Determine the size of $\angle OBC$. **1**

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$