(Western Region)

2003

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a blue or black pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (84)

- Attempt Questions 1-7
- All questions are of equal value

Question 1 12 Marks Start a fresh sheet of paper. **Marks**

a) Find the horizontal asymptote for $y = \dfrac{3x^2+4x+5}{x^2}$ **2**

b) Solve the inequality $\dfrac{x}{2-x} \leq 4$ **3**

c) Find $\sum_{n=2}^{20} 3n-4$ **2**

d) Use the substitution $u = x^2 - 3$ to evaluate **3**

$$\int_2^6 \frac{x}{\sqrt{x^2-3}}\,dx$$

e) A parabola is defined by the parametric equations

$$x = 3t$$
$$y = 6t^2$$

i) What point is defined when $t = 5$? **1**

ii) What is the Cartesian equation of the parabola? **1**

Question 2 12 Marks Start a fresh sheet of paper. **Marks**

a) Consider the points A (-1, -1) B (2, 4) C (8, 14)

i) Find the ratio AB:BC **1**

ii) Complete the statement "C divides AB externally in the ratio …" **1**

b) i) If $\log_3 12 = 2.26186$, find $\log_3 4$ **1**

ii) Find $\log_e e^{1.09}$ **1**

c) Find the quotient and the remainder when **2**

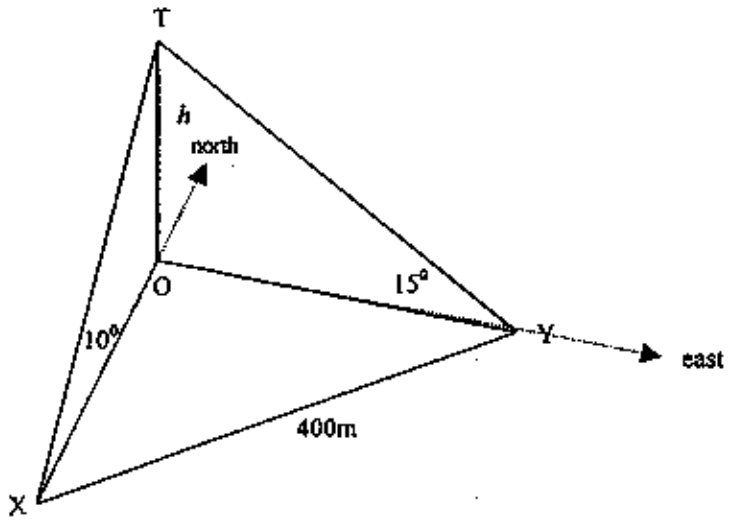
$x^3 + 4x^2 - 2x + 3$ is divided by $x^2 - 1$.

d) A surveyor at X observes a tower due north.

The angle of elevation to the top of the tower is 10^0.

He then walks 400m to a position Y which is due east of the tower.

The angle of elevation from Y to the top of the tower is 15^0.



i) Write an expression for OY in terms of h. **1**

ii) Calculate h to the nearest metre. **3**

iii) Find the bearing of Y from X. **2**

Question 3 12 Marks Start a fresh sheet of paper. **Marks**

a) Find $\int_0^{\pi/16} \cos^2 4x\,dx$ **3**

b) i) How many arrangements can be made from the letters of the word EXCESSIVE ? **1**

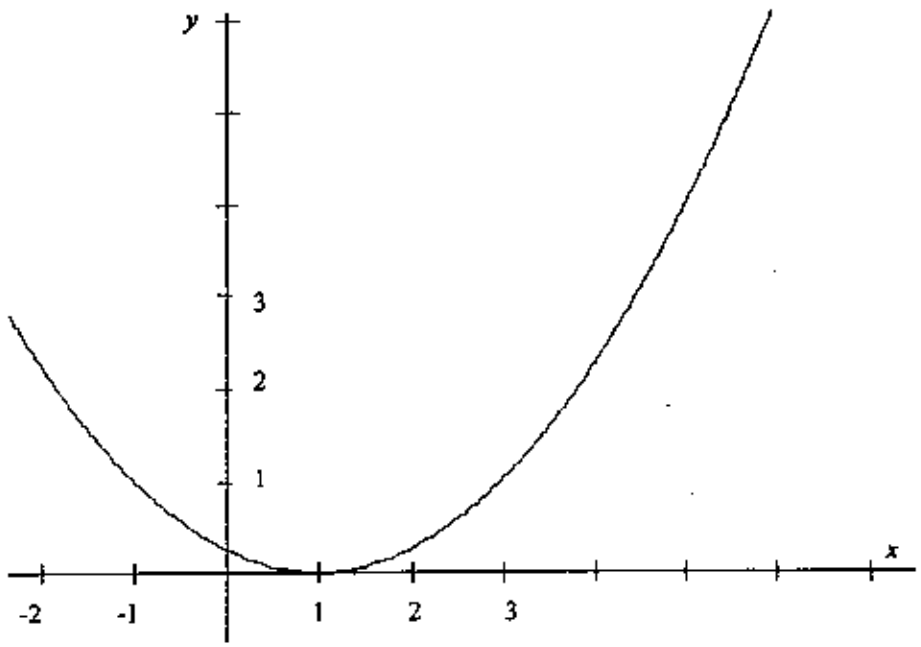
ii) Find the probability that such an arrangement has the consonants and vowels in alternating positions. **2**

c) Calculate the solutions to $4\cos\theta + 3\sin\theta = 2$ in the range $0 \le \theta \le 2\pi$ **4**
Express your answers to the nearest hundredth of a radian.

d) The graph below shows a function $y = f(x)$.

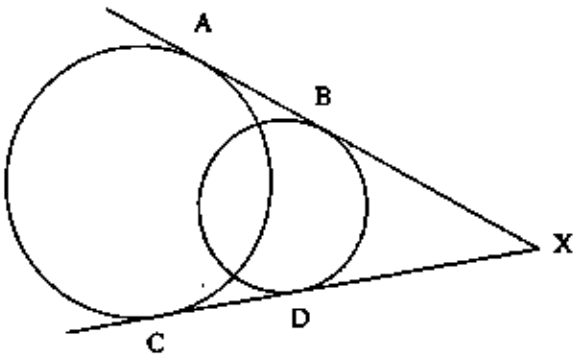
i) Specify a portion of the domain for which $f(x)$ has an inverse **1**

ii) Copy the graph of the curve onto your answer sheet and neatly draw $y = f^{-1}(x)$ for the domain you specified in i) **1**



Question 4 12 Marks Start a fresh sheet of paper. **Marks**

a) **3**



In the diagram AB is common tangent to the two circles.

Likewise CD is also a common tangent.

The two tangents meet externally at X.

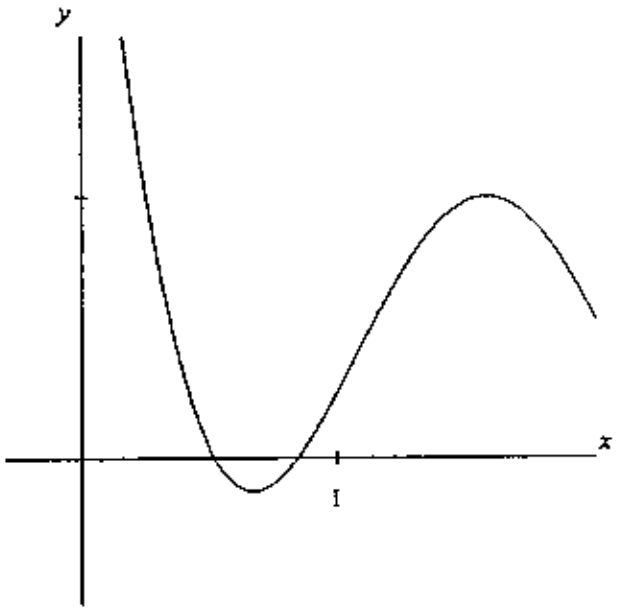
Explain why AC || BD.

b) Given that $\cos 3\theta = \cos(\theta + 2\theta)$, use the double angle formulae to express $\cos 3\theta$ in terms of $\cos\theta$. **2**

c) **5**

The graph represents a part of the curve $y = 8\sin^2 x - 10\sin x + 3$.

Calculate the two roots shown in the diagram and evaluate the minimum value shown in the graph.

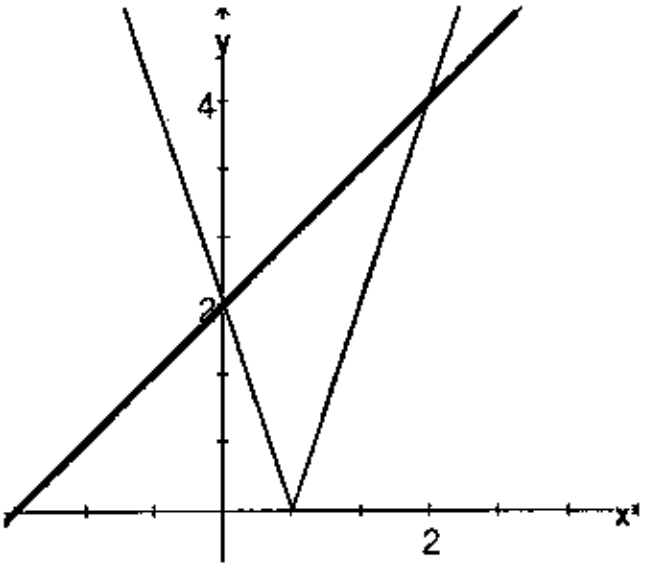


Question 4 is continued on page 6

Question 4 **continued** Marks

d) i) Specify the equation graphed by the thinner of the two lines. **1**

ii) What values of x are defined by $x+2 \geq |3x-2|$? **1**



Question 5 12 Marks Start a fresh sheet of paper. **Marks**

a) i) Use the method of proof by induction to show that **3**

$1+7+19+.....+(3n^2-3n+1)=n^3$

ii) Show that the rule $T_n = S_n - S_{n-1}$ holds true in part (i). **1**

b) i) Use the Chain Rule to show that $\frac{dv}{dt}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ **1**

ii) The acceleration due to gravity is inversely proportional to the square of the distance x from the centre of the earth. **1**

This can be written as $a=\frac{-k}{x^2}$.

Find k if $a = -g$ when $x = R$.

iii) If the initial velocity of a rocket is u ms^{-1}, show that **2**

$v^2=\frac{2R^2g}{x}+u^2-2gR$ where g is the acceleration due to gravity and R is the radius of the earth.

iv) Find the maximum distance that the rocket will travel from the centre of the earth. **1**

(Answer in terms of g, R and u)

v) Taking $g = 9.8$, $R = 6400$ km find the value of u in ms^{-1} for which the rocket will escape the gravity of the earth. **1**

c) Given that $f(x)=ax^3+bx^2+cx+d$ is a function with a double root at $x = -1$ and with a minimum value of -4 when $x = 1$, find the values of a, b, c and d. **2**

Question 6 12 Marks Start a fresh sheet of paper. **Marks**

a) A body is moving in a straight line and its position x is given by $x = 2\sin^2 t$.

i) What are the extremities of its position? **1**

ii) Express the acceleration of the particle in terms of x. **2**

iii) Show the particle is undergoing SHM. **1**

iv) Find its maximum speed. **1**

b) The binomial theorem states that $(1+x)^n = \sum_{k=0}^{n} {}^nC_k x^k$ **2**

Show that ${}^nC_1 + 2{}^nC_2 + 3{}^nC_3 + \ldots\ldots n{}^nC_n = n \times 2^{n-1}$

c) $\left(\frac{1}{2}+\frac{1}{2}\right)^7$ represents the outcomes in terms of gender of children **1**

for a family with 7 children.

Calculate the probability of 5 boys and 2 girls.

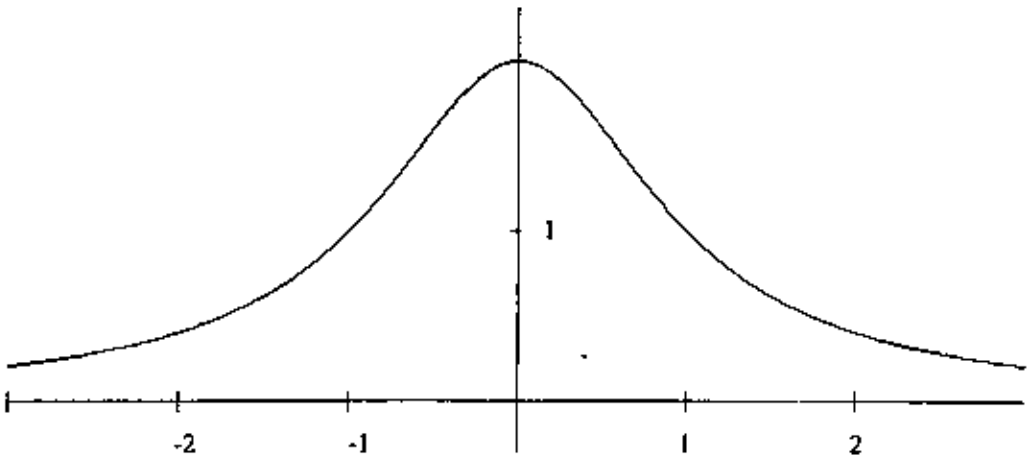
d) The graph below shows the derivative of $y = 2\tan^{-1} x$.

i) Where does $y = 2\tan^{-1} x$ have its greatest slope and what is this slope? **1**

ii) What x values correspond with $\frac{dy}{dx} = \frac{1}{3}$ **1**

iii) What is the total area bounded by this curve and the x axis? **2**

(Note: Domain of the function is $-\infty \le x \le \infty$)



Question 7 12 Marks Start a fresh sheet of paper. **Marks**

a) A projectile has an initial velocity V and an angle of projection θ.

i) Assuming $\frac{d^2y}{dt^2} = -10$, $\frac{d^2x}{dt^2} = 0$ and the initial point of projection is 10m above the origin, find expressions for x and y in terms of t. **3**

ii) If $V = 13\text{ms}^{-1}$ and $\theta = \tan^{-1}\frac{5}{12}$ find the range of the projectile. **2**

b) P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ are extremities of a focal chord for the parabola $x^2 = 4ay$.

i) Form the equation of the chord PQ and deduce the constraint on p and q. **2**

ii) Find where the tangents at P and Q meet. **2**

iii) Show that the chord PQ has length $a\left(p + \frac{1}{p}\right)^2$. **3**

End of Paper