2007 HSC Mathematics Extension 1 Scripts of Andrew Harvey

Preamble:
The following pages of this document are my original exam responses (scripts) from the 2007 NSW HSC Mathematics Extension 1 examination. I have provided them for research and/or study purposes. The scripts were obtained from the Office of the Board of Studies NSW, under the *Freedom of Information Act 1989* (NSW). The author of the scripts can be contacted at andrew.harey4@gmail.com.

Examination Mark: 47/50

Summary of Estimated Marks Awarded – Andrew Harvey 2007 HSC Mathematics Extension 1 Exam

From my exam script I have derived the raw mark that I would have been awarded. Most questions I am fairly certain that I have received the mark that I estimated however I cannot always be sure.

Key:
2 = Available Marks
2 = Marks Awarded

Got correct final answer so I have assumed I received full marks
Got an incorrect final answer however I have determine the appropriate mark by forensic and logical analysis of the censored information
There is doubt in my mind of what mark was actually awarded

Q1		**12**	**11**
	a)	2	2
	b)	2	2
	c)	2	2
	d)	3	3
	e)	3	2
Q2		**12**	**9**
	a)	2	0
	b) i)	2	2
	b) ii)	1	1
	c)	3	3
	d) i)	2	2
	d) ii)	2	1
Q3		**12**	**12**
	a)	3	3
	b) i)	3	3
	b) ii)	2	2
	c) i)	2	2
	c) ii)	2	2
Q4		**12**	**9-12**
	a) i)	1	1
	a) ii)	1	1
	a) iii)	2	2
	b)	3	3
	c) i)	3	0-3
	c) ii)	2	2
Q5		**12**	**9**
	a) i)	2	1
	a) ii)	2	2
	b)	2	1
	c)	3	3
	d) i)	1	0
	d) ii)	2	2
Q6		**12**	**11**

	a) i)	2	2
	a) ii)	1	1
	a) iii)	2	2
	a) iv)	2	1
	b) i)	1	1
	b) ii)	3	3
	b) iii)	1	1
Q7		**12**	**6-7**
	a) i)	1	1
	a) ii)	2	2
	b) i)	2	1
	b) ii)	2	0
	b) iii)	2	1
	b) iv)	3	1-2
		84	**67-71**

Total Raw Mark: 67-71/84 = 39.9-42.3/50
Which Equates to an Aligned Exam Mark: 47/50

Remember also that a raw mark of 84/84 equates to an aligned exam mark of 50/50 and that the relationship between the two marks for each band is linear. However my reported exam mark has been rounded and that incurs an error margin.

	5	2		

Centre Number

1	7	8	0	7	7	0	6	

Student Number

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Maths Ext 1

4

WRITING BOOKLET

Section	Part	Question Number
		1

Date

31/10/07

Number of booklets used for this question: 1

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Q1

a) $(1+\sqrt{5})(1+\sqrt{5})(1+\sqrt{5})$

$=(1+2\sqrt{5}+5)(1+\sqrt{5})$

$=1+\sqrt{5}+2\sqrt{5}+2\times5+5+5\sqrt{5}$

$=16+8\sqrt{5}$

$=16+8\sqrt{5}$

$\therefore a=16$

$b=8$

[redacted]

b) 4, 5 19, -5

2 : 3

$x=\dfrac{2\times19+3\times4}{5}=10$

$y=\dfrac{2\times-5+3\times5}{5}=1$

[redacted]

$P(10,1)$

c) $\dfrac{1}{1+x^8}\times4x^3=\dfrac{4x^3}{1+x^8}$

[redacted]

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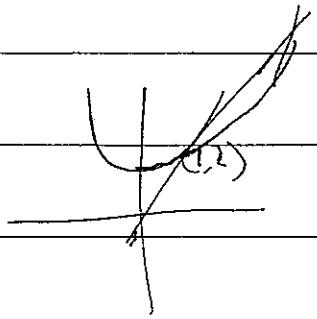
d)



$\frac{dy}{dx} = 3x^2$

at $x = 1$

$\frac{dy}{dx} = 3$

$y - 2 = 3(x-1)$

$y = 3x - 1$

$2y = x + 3$

$y = \frac{1}{2}x + \frac{3}{2}$

$\theta = \tan^{-1}(3) - \tan^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{4}$ radians.

[redacted]

d)

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e)
$$\int_3^4 \frac{2x}{\sqrt{25-x^2}}\,dx$$

$u = 25 - x^2$ $\quad x = 3,\ u = 16$

$\frac{du}{dx} = -2x$ $\quad x = 4,\ u = 9$

$dx = \frac{du}{-2x}$

$$-\int_{16}^{9} \frac{-2x}{\sqrt{u}} \frac{du}{-2x}$$

$$= -\int_{16}^{9} u^{-\frac{1}{2}}\,du$$

$$= -\left[\frac{1}{-\frac{1}{2}+1}\,u^{-\frac{1}{2}+1}\right]_{16}^{9}$$

$$= -\left\{2u^{0.5}\right\}_{16}^{9} = -\left(2\times 9^{0.5} - 2\times 16^{0.5}\right)$$

$$= -1$$

[redacted]

	5	2		

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		2

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$t = \tan\frac{\theta}{2}$

$$\frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$$

RHS = ∈

LHS =

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\therefore \tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

$$\therefore \tan\frac{\theta}{2} = \frac{2\tan\frac{\theta}{4}}{1-\tan^2\frac{\theta}{4}}$$

[redacted]

$$= 2\,\frac{\sin\frac{\theta}{4}}{\cos\frac{\theta}{4}}$$

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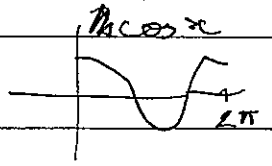
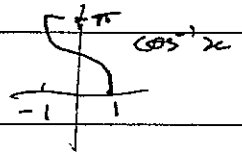
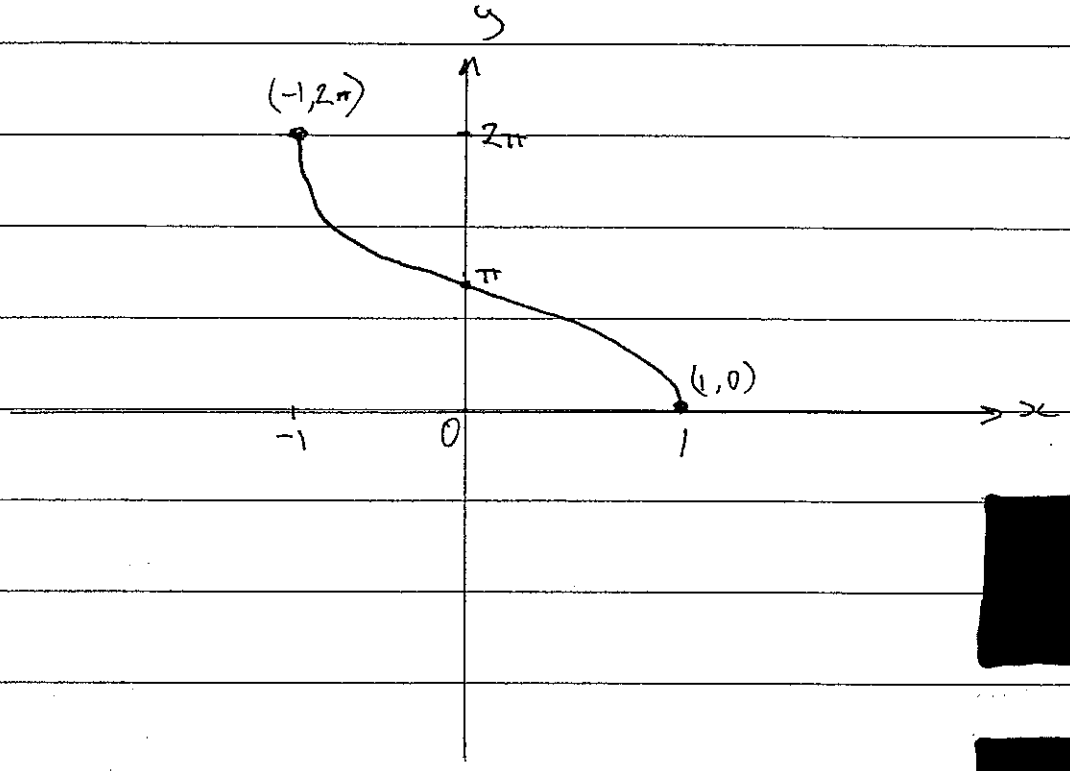
$f(x) = 2\cos^{-1} x$

i)







[redacted]

[redacted]

ii) $0 \leq f(x) \leq 2\pi$

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c) $P(x) = x^2 + ax + b$

$(x-2)$ is a factor

$P(2) = 0$

$2^2 + 2a + b = 0$

$4 + 2a + b = 0$

$2a + b = -4$

$ax - x$

$x+1 \;\big|\; x^2 + ax + b$

$x^2 + x$

$x(a-1) + b$

$x(a-1) + (a-1)$

$(b - a + 1)$

ie $b - a + 1 = 18$ also $2a + b = -4$

$2a$

$2a + 4 = -b$

$b = -2a - 4$

$-2a - 4 - a + 1 = 18$

$-3a = 18 + 3$

$a = -7$

$b = -2x - 7 - 4$

$= 10$

[redacted]

	5	2		

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WRITING BOOKLET

Section	Part	Question Number
		2

Date

31/10/07

Number of booklets used for this question 2/2

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Q2 cont.

d)

$v = 50(1 - e^{-0.2t})$

i) $\frac{dx}{dt} = v = 50 - 50e^{-0.2t}$

$a = \frac{dv}{dt} = -50 \times -0.2\, e^{-0.2t}$

$= 10e^{-0.2t}$

at $t = 10$

$a = 10e^{-0.2 \times 10}$

$= 1.35\ldots$

$= 1.4\ ms^{-2}$ (1 dp)

ii) $x = \int 50 - 50e^{-0.2t}\ dt$

$= 50t - 50\frac{1}{-0.2}e^{-0.2t} + C$

at $t = 0$, $x = 0$ $\Rightarrow$ $= 50t + 250e^{-0.2t}$

at $t = 0$, $x = 0$ $\Rightarrow$ $C = 0$

at $t = 10$

$x = 500 + 250e^{-0.2 \times 10}$

$= 533.83...$

$\doteq 534$ m

$\therefore$ she has fallen 534 m. (1 dp).

[redacted]

[redacted] ✓

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WRITING BOOKLET

Section	Part	Question Number
		3

Date

31/10/07

Number of booklets used for this question: 1

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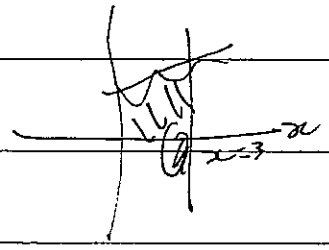
Q3

a) $y = \dfrac{1}{\sqrt{9+x^2}}$



$$V = \pi \int_0^3 \frac{1}{9+x^2}\, dx$$

$$= \pi \left[\frac{1}{3} \tan^{-1} \frac{x}{3} \right]_0^3$$

$$= \frac{\pi}{3} \left(\tan^{-1} 1 - \tan^{-1} 0 \right)$$

$$= \frac{\pi}{3} \times \frac{\pi}{4} = \frac{\pi^2}{12} \ u^3$$

b) i) $x \neq 4$

$\therefore$ vertical asymptote at $x = 4$

test:
$x - 2 = x - 4$

$0 = -2$

which is not true,

$\therefore y \neq 1$

$\therefore$ horizonal asymptote at $y = 1$

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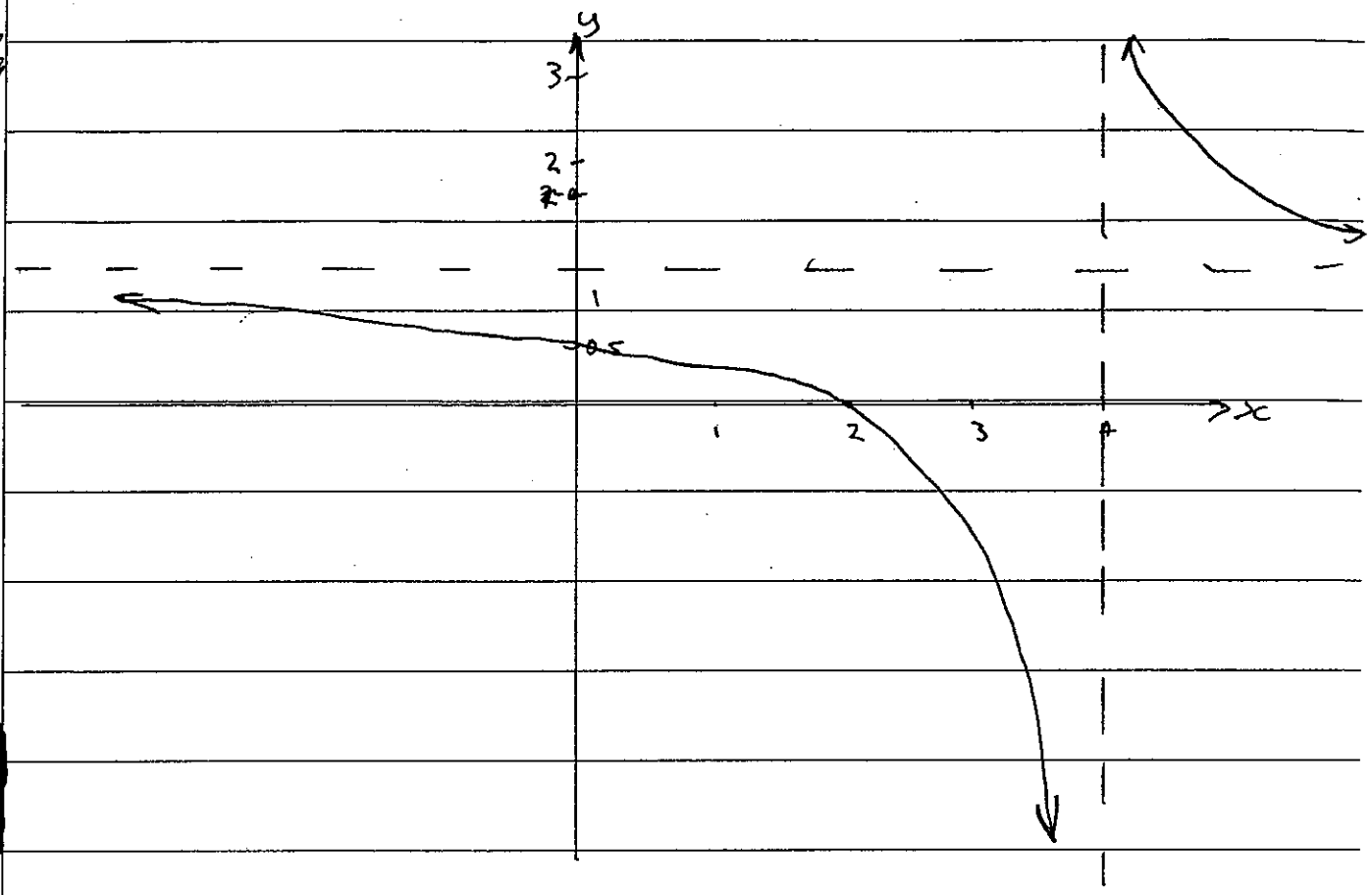

$$0 = \frac{x-2}{x-4}$$

$x - 2 = 0$

$x = 2$

$x < 4 \quad , \quad x \geq 5$

$$\frac{x-2}{x-4} = 3$$

$x - 2 = 3x - 12$

$10 = 2x$

$x = 5$

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c) $\dot{x} = -e^{-2x}$

at $t = 0$

$x = 0$

$\frac{dx}{dt} = 1$

i) ~~$t = \int$~~ $a = -e^{-2x}$

$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$\frac{1}{2}v^2 = -\int e^{-2x}\,dx$

$= -\frac{1}{-2}e^{-2x} + C$

$\frac{1}{2} = \frac{1}{2} + C$

$\therefore C = 0$

$v^2 = \frac{2}{2}e^{-2x}$

$= e^{-2x}$

$v = \left(e^{-2x}\right)^{\frac{1}{2}} = e^{-x}$

$\therefore \dot{x} = e^{-x}$

ii)

$$\frac{dx}{dt} = e^{-x}$$

$$\frac{dt}{dx} = \frac{1}{e^{-x}} = (e^{-x})^{-1} = e^{x}$$

$$t = \int e^{x}\, dx$$

$$= e^{x} + C$$

as $t = 0$

$x = 0$

$$0 = 1 + C$$

$$\therefore C = -1$$

$$t = e^{x} - 1$$

$$t + 1 = e^{x}$$

$$\therefore x = \log_e (t+1)$$

[redacted]

[redacted]

		5	2		

Centre Number

1	7	8	0	7	7	0	6	

Student Number

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Maths Ext 1

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WRITING BOOKLET

Section	Part	Question Number
		4

Date

31/10/07

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a) $\frac{1}{10}$ has green eyes

i) $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$

ii) ${}^{20}C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^{18} = 0.285$ (3dp)

iii) ie. 3 or 4 or 5 ...

$= 1 - (0 \text{ or } 1 \text{ or } 2)$

$= 1 - \left[{}^{20}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{20} + {}^{20}C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{19} + {}^{20}C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^{18} \right]$

$= 0.32$ (2dp)

b) $7^{2n-1} + 5$ div 12 $n \geq 1$ $n \in \mathbb{Z}$

For $n = 1$

$7^{2\times1-1} + 5 = 7^{2-1} + 5 = 7^1 + 5 = 7 + 5 = 12$ ✓

as $\frac{12}{12} = 1$, which is an integer true for $n = 1$

Assume true for $n = k$

$7^{2k-1} + 5 = 12n$ where $n \in \mathbb{Z}$

$7^{2k} + 5 = 12n$

$7^{2k} = (12n - 5)7$

$7^{2k} = 84n - 35$

For $n=k+1$

$$LHS = 7^{2(k+1)-1} + 5$$

$$= 7^{2k+2-1} + 5$$

$$= 7^{2k+1} + 5$$

$$= 7 \cdot 7^{2k} + 5$$

$$= 7(84m - 35) + 5$$

$$= 588m - 245 + 5$$

$$= 588m - 240$$

$$= 12(49m - 20)$$

[redacted]

now as $m \times 49$ is integer

then $(m \times 49) - 20$ is still an integer

$\therefore 12 \times$ integer is divisible by 12.

Hence is true for $n=1$, etc.

If true for $n=k$

then by ~~induction~~ true for $n=k+1$

hence by induction true for all

$n \geq 1 \quad n \in \mathbb{Z}$

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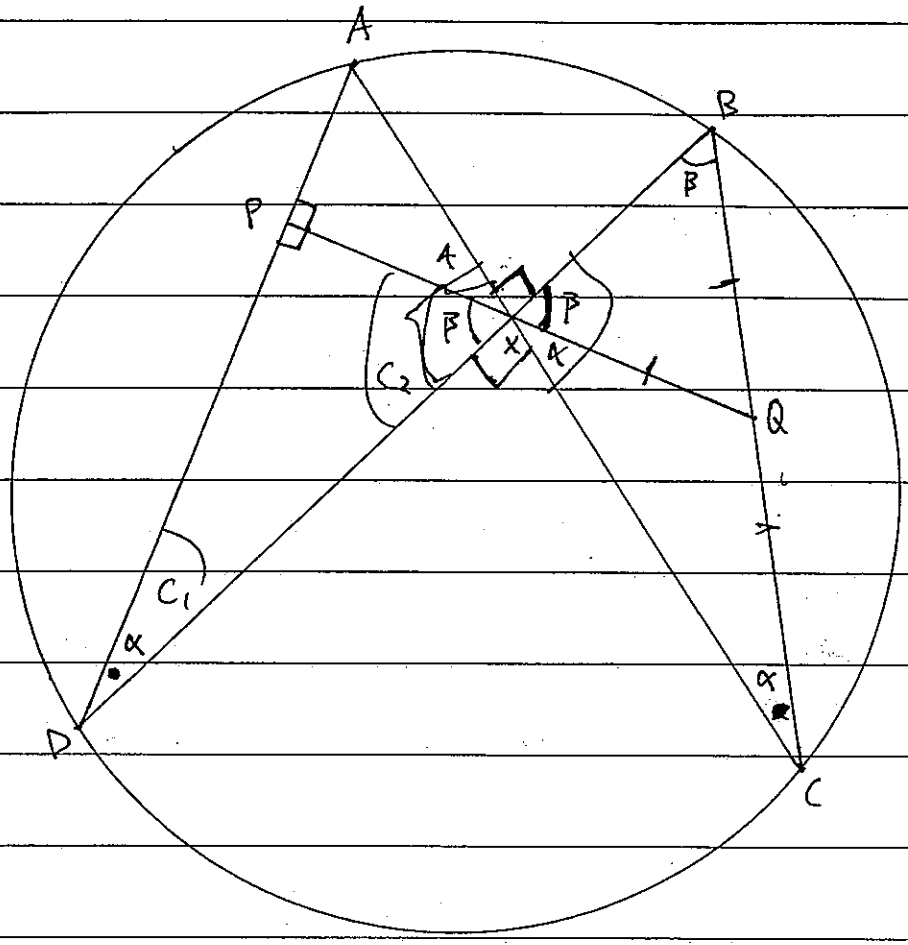
c)



$\alpha + \beta = 90$ (∠ sum of △)

$\beta = 90 - \alpha$

$\angle AXD = 90°$ (∠ sum of line is 180°)

$\angle PXD = 90° - \angle ADX$ (angle sum of △ is 180°)

$\angle PXD = \angle BXQ$ (vertically opposite ∠'s)

$\therefore \angle BXQ = 90° - \angle ADX$

$\angle ADX = \angle XCB$ (angles standing on same arc)

$\therefore \angle BXQ = 90° - \angle XCB$

as $\angle BXC = 90°$ ~~(vertically opposite, or~~ (angle sum of line)

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$\angle XBQ + \angle BCX = 90°$ (cyclic sum of $\triangle$)

$\angle XBQ = 90° - \angle BCX$

$\angle BCX = \angle ADX$ (vertically opp.) which $= \angle BXQ$

$\angle ADX + \angle PXD = 90$

$\angle ADX = 90 - \angle BXQ$

$\therefore \angle BCX = 90 - \angle BXQ$

$\angle XBQ = 90 - 90 + \angle BXQ$

$\therefore \angle XBQ = \angle BXQ$

[redacted]

as $\angle BXQ = \angle XBQ$

$\triangle XBQ$ is isosceles.

$\therefore XQ = QB$

$\beta = \alpha + 90$

$\alpha + \beta = 90$

$\therefore \alpha$

from diagram

$C_1 + C_2 = 90°$

But $\angle AXD = 90°$

so $\angle AXP = 90 - C_2$

$= 90 - 90 + C_1$

$= C_1$

see next page

but $\angle AXP$ also $= \angle QXC$ (Vertically opp)

$\therefore \angle QXC = \angle XCQ$

$\therefore \triangle XQC$ is isosceles

$\therefore XQ = QC$

$\therefore BQ = QC$.

[redacted]

[redacted]

[redacted]

	5	2		

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WRITING BOOKLET

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		5

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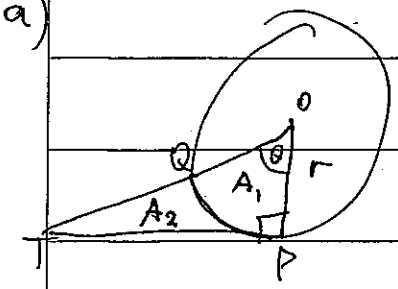
Q5

a)



i) $\tan\theta = \frac{TP}{r}$ $\qquad \frac{\theta}{2\pi} = \frac{A_1}{\pi r^2}$

$$A_1 = \frac{\theta \pi r^2}{2\pi} = \frac{\theta r^2}{2}$$

$$\frac{1}{2} \cdot r \cdot TP - A_1 = A_1$$

$$\frac{1}{2} rTP = 2A_1$$

$$\frac{1}{2} r\pi = \frac{2\theta r^2}{2}$$

$$\pi = \frac{\theta r^2 2}{r}$$

$$= \theta r 2$$

$$r = \frac{\pi}{2\theta}$$

$$\tan\theta = \frac{TP}{1} \times \frac{2\theta}{\pi} = \frac{2\theta \cdot TP}{\pi}$$

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ii) $2\theta - \tan\theta = 0$

$\theta = 1.15$

$f(\theta) = 2\theta - \tan\theta$

$f'(\theta) = 2 - \sec^2\theta$

$$\theta_2 = 1.15 - \frac{f(1.15)}{f'(1.15)} = 1.15 - \frac{2\times 1.15 - \tan 1.15}{2 - \sec(1.15)^2}$$

$$= 1.1664 \text{ rad (4 dp)}$$

$\text{cosec} = \frac{1}{\sin}$

$\sec = \frac{1}{\cos}$

b) R · R - - - -

X X X X X X

$$\frac{\left(\frac{3!}{4!}\right)}{6!} = \frac{1}{2880}$$

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c) $\sin^{-1}x + \frac{1}{2}\cos^{-1}y = \frac{\pi}{3}$

$\sin^{-1}x = \frac{\pi}{3} - \frac{1}{2}\cos^{-1}y$

$3\sin^{-1}x - \frac{1}{2}\cos^{-1}y = \frac{2\pi}{3}$

$\frac{3\pi}{3} - \frac{3}{2}\cos^{-1}y - \frac{1}{2}\cos^{-1}y = \frac{2\pi}{3} - \pi$

$\cos^{-1}y\left(\frac{-3}{2} - \frac{1}{2}\right) = \frac{2\pi}{3} - \pi$

$\cos^{-1}y = \frac{\left(\frac{2\pi}{3} - \pi\right)}{\left(\frac{-3}{2} - \frac{1}{2}\right)} = \frac{\frac{-\pi}{3}}{-2} = \frac{\pi}{6}$

$y = \frac{\sqrt{3}}{2}$

$\sin^{-1}x = \frac{\pi}{3} - \frac{1}{2}\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$x = \frac{\sqrt{2}}{2}$

[redacted]

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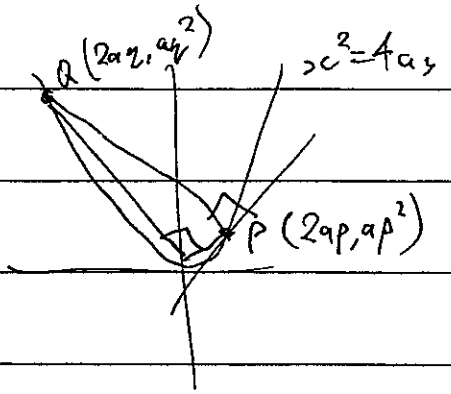
d)



i)

PQ: $x + py - 2ap - ap^3 = 0$

$2ap + p^2 \quad 2ap - ap^3$

$x = 2ap + ap^3 - py$

$x = \frac{x^2}{4a}$

$2ap = 2ap + ap^3 - ap^3$

ii)

$\frac{ap^2}{2ap} \times \frac{aq^2}{2aq} = -1$

$\frac{pq}{4} = -1$

$pq = -4$

$q = \frac{-4}{p}$

$p^2 + p\left(\frac{-4}{p}\right) + 2 = 0$

$p^2 - 4 + 2 = 0$

$p^2 = 2$

[redacted]

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	5	2		

Centre Number

1	7	8	0	7	7	0	6	

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2007

HIGHER SCHOOL CERTIFICATE EXAMINATION

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4

WRITING BOOKLET

Section	Part	Question Number
		6

Date

31/10/07

Number of booklets used for this question: 1

Instructions

- Write your Centre Number and Student Number at the top of this page and of each page that you use.
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Q6

a) $x = \sqrt{3}\sin 2t - \cos 2t + 3$

i) $\frac{dx}{dt} = 2\sqrt{3}\cos 2t + 2\sin 2t$

$\frac{d^2x}{dt^2} = -4\sqrt{3}\sin 2t + 4\cos 2t$

$= -4(\sqrt{3}\sin 2t - \cos 2t)$

$= -4(x-3)$

$\therefore$ in SHM

when $\frac{d^2x}{dt^2} = 0$, it is at the point it oscillates

so, $0 = -4(x-3)$

$x - 3 = 0$

$x = 3$

$\therefore$ in SHM about $x = 3$

ii) period $= \frac{2\pi}{n} =$

$-n^2 x$

$n = 2$

Period $= \frac{2\pi}{2} = \pi$ sec.

iii) $A\cos(2t - \alpha)$

$= A\cos 2t \cos\alpha + A\sin 2t \sin\alpha$

$\therefore A\cos\alpha = 2\sqrt{3}$ | $A\sin\alpha = 2$

$A = \frac{2\sqrt{3}}{\cos\alpha}$ | $A = \frac{2}{\sin\alpha}$

07/MATH

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$\frac{2}{\sin\alpha} = \frac{2\sqrt{3}}{\cos\alpha}$

$2\cos\alpha = 2\sqrt{3}\sin\alpha$

$\frac{2}{2\sqrt{3}} = \tan\alpha$

$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{6}$

$A = \frac{2}{\sin\alpha} = \frac{2}{\frac{1}{2}} = 4$

$\therefore \ddot{x} = 4\cos\left(2t - \frac{\pi}{6}\right)$

[redacted]

$\ddot{x} = 2$

$\frac{1}{2} = \cos\left(2t - \frac{\pi}{6}\right)$

$2t - \frac{\pi}{6} = \cos^{-1}\left(\frac{1}{2}\right) + \frac{\pi}{6}$

2

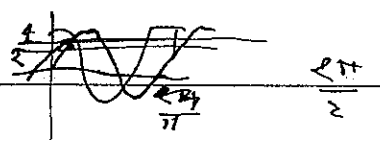
$t = \frac{\pi}{4}, \frac{3\pi}{4}$ sec.

$\ddot{x} = -2$

$t = \frac{\cos^{-1}\left(\frac{1}{2}\right) + \frac{\pi}{6}}{2}$

$=$

$\frac{S \mid A}{T \mid C}$



$\frac{2\pi}{2}$

[redacted]

only 2 sol. —

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6) $f(x) = e^x - e^{-x}$

i) $f'(x) = e^x + e^x$

$= 2e^x$

as $e^x > 0$ for all real x

$f'(x) > 0$ for all real x,

$\therefore f(x)$ is increasing for all real x.

[redacted]

ii) $x = e^y - e^{-y}$

$e^y - (e^y)^{-1}$

$= \frac{(e^y)^2}{e^y} - \frac{1}{e^y}$

$x = \frac{e^{2y} - 1}{e^y}$

$xe^y - e^{2y} + 1 = 0$

$-(e^y)^2 + xe^y + 1 = 0$

$e^y = \frac{-x \pm \sqrt{x^2 - 4 \times -1 \times 1}}{2 \times -1} = \frac{-x \pm \sqrt{x^2 + 4}}{-2}$

$\therefore y = \log_e \left(\frac{x \mp \sqrt{x^2+4}}{2} \right)$

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as $\log_e a$, ~~x~~ $a > 0$.

$\therefore \frac{x + \sqrt{x^2+4}}{2} > 0$

$\therefore \frac{x + \sqrt{x^2+4}}{2}$ as $\frac{x - \sqrt{x^2+4}}{2}$

gives a negative answer.

$\therefore f^{-1}(x) = \log_e \left(\frac{x + \sqrt{x^2+4}}{2} \right)$

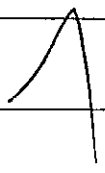
[redacted]

ii) $\log_e \left(\frac{5 + \sqrt{5^2+4}}{2} \right)$

[redacted]

$= 1.65$ (2dp)

[redacted]



	5	2		

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Student Number

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WRITING BOOKLET

Section	Part	Question Number
		7

Number of booklets used for this question: 1

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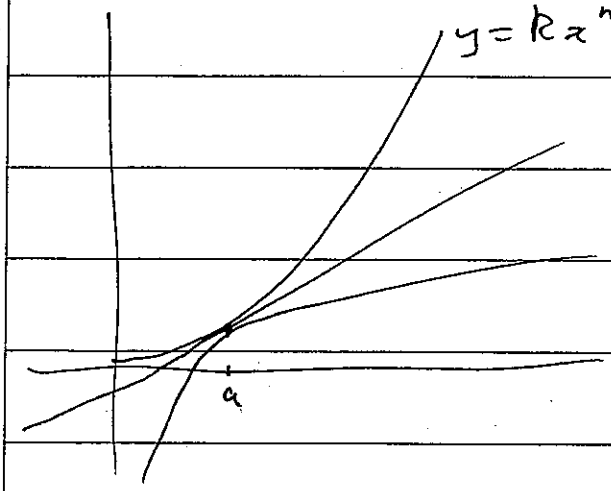
Q7

a)



i) $\frac{dy}{dx} = kn x^{n-1}$ • $\frac{dy}{dx} = \frac{1}{x}$ •

as they have same gradient at same $x = a$

$kn\, a^{n-1} = \frac{1}{a}$

$kn \frac{a^n}{a} = \frac{1}{a}$

$kn\, a^n = 1$

$a^n = \frac{1}{kn}$

ii) $n = \log_a \left(\frac{1}{kn}\right)$

$n = \frac{\log\left(\frac{1}{kn}\right)}{\log a}$

at $x = a$
y's are same
so $ka^n = \log_e a$

$e^{ka^n} = a$

$\therefore$ $k\left(\frac{1}{nk}\right) = \log_e \sqrt[n]{\frac{1}{nk}}$

$$\frac{1}{n} = \log_e \sqrt[n]{\frac{1}{nk}}$$

$$e^{\frac{1}{n}} = \sqrt[n]{\frac{1}{nk}}$$

$$\left(e^{\frac{1}{n}}\right)^n = \frac{1}{nk}$$

$$e^1 = \frac{1}{nk}$$

$$ne = \frac{1}{k}$$

$$k = \frac{1}{ne}$$

b) i) $t = \dfrac{x}{14\cos\theta}$

$$y = \frac{14x}{14\cos\theta}\sin\theta - 4.9\left(\frac{x}{14\cos\theta}\right)^2$$

$$= x\tan\theta - 4.9\frac{x^2}{14^2\cos^2\theta}$$

Hence $x\tan\theta$

$$= x\tan\theta - \left(\frac{1}{40\cos^2\theta}\right)x$$

Now, $\dfrac{1}{40\cos^2\theta} \times \dfrac{\frac{1}{\cos^2}}{\frac{1}{\cos^2}}$

$$= \frac{\sec^2\theta}{40}$$

$\cos^2 + \sin^2 = 1$

$\frac{\cos^2}{\sin^2} \times \frac{\cos^2}{1} + \tan^2 = \frac{1}{\tan^2}$

$\frac{\cos^2}{\cos^2} = \frac{1 - \sin^2}{\cos^2} \quad \frac{\tan^2}{\cos^2}$

$\tan^2 = \frac{\sin^2}{\cos^2}$

$\frac{\cos^2}{\tan^2} + \tan^2 = \frac{1}{\tan^2}$

$\tan^2 = \frac{1 - \cos^2}{\tan^2}$

$= \frac{\sin^2 x}{\tan^2 x}$

$= \frac{\sin^2 x}{1} \times \cos^2$

$1 + \tan^2$ $\therefore \cos($

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$\sin^2 + \cos^2 \theta = 1$

$\tan^2 \theta + 1 = \frac{1}{\cos^2}$

$\therefore$

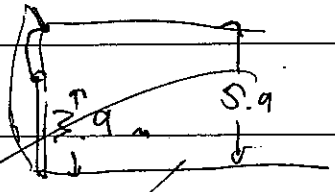
$xm = \left(\frac{1+m^2}{40}\right) x^2$

(ii) $y = \left(2 \pm \sqrt{3-0.4h}\right)10 - 100\left(\frac{1 + \left(2 \pm \sqrt{3-0.4h}\right)^2}{40}\right)$

$10\left(2 \pm \sqrt{3-0.4h}\right) - \frac{100}{10} - \frac{100}{40}\left(2 \pm \sqrt{3-0.4h}\right)^2$

[redacted]

(iii)



$y = 5.9$

$5.9 = 2 \pm \sqrt{3-0.4h}$

$3.9 = 2 \pm \sqrt{3-0.4h}$

$h = 2 \pm \sqrt{3-0.4 \times 5.9}$

$\oplus$ 2.8

$\ominus$ 1.2

$h = 2 \pm \sqrt{3-0.4 \times 3.9}$

$\oplus$ 3.2

$\ominus$ 0.8

[redacted]

$1.2 \leq m \leq 0.8$

$0.8 \leq m \leq 1.2$

iv)

$$0 = x\left(m - \left(\frac{1+m^2}{40}\right)x\right)$$

$x = 0$

$$m - \left(\frac{1+m^2}{40}\right)x = 0$$

$$m = \frac{1+m^2}{40}x$$

$$x = \frac{m}{1} \times \frac{40}{1+m^2} = \frac{40m}{1+m^2}$$

$2.8 \leq m \leq 3.2$ $\qquad$ $0.8 \leq m \leq 1.2$

$x = \frac{2800}{221}, \frac{3200}{281}$ $\qquad$ $x = \frac{800}{41}, \frac{1200}{61}$

$11\frac{109}{281} \leq x \leq 12\frac{148}{221}$ $\qquad$ $19\frac{21}{41} \leq x \leq 19\frac{41}{61}$

[redacted]

[redacted]