CRANBROOK 2UNIT TRIAL 2003.

① (a) $\dfrac{432.5}{18.9 \times 4.6} = 4.974695\ldots$

$= 4.97 \ (2 \text{ d.p.})$

(b) $3x^2 - x - 10 = (3x+5)(x-2)$

(c) $3 - \dfrac{2x}{3} < 4$

$\therefore -\dfrac{2x}{3} < 1$

$\therefore x > -1\frac{1}{2}$

(d) $(2\sqrt{3}-1)(\sqrt{3}+2) = a + \sqrt{b}$

$\therefore 6 + 4\sqrt{3} - \sqrt{3} - 2 = a + \sqrt{b}$

$\therefore 4 + 3\sqrt{3} = a + \sqrt{b}$

$\therefore 4 + \sqrt{27} = a + \sqrt{b}$

$\therefore a = 4, \ b = 27$

(e) $|x-2| \le 8$

$\therefore -8 \le x - 2 \le 8$

$\therefore -6 \le x \le 10$

(f) $\dfrac{3^{-1} a^2}{2a^{-3}} = \dfrac{a^5}{6}$

② $A = (-3, 2)$, $B = (5, 8)$.

(a) Eq'n of AB is: $\dfrac{y-2}{x+3} = \dfrac{8-2}{5+3}$

$8(y-2) = 6(x+3)$

$8y - 16 = 6x + 18$

$6x - 8y + 34 = 0$

i.e. $3x - 4y + 17 = 0$.

(b) $M_{AB} = \left(\dfrac{-3+5}{2}, \dfrac{2+8}{2}\right)$

$= (1, 5)$

(c) length $AB = \sqrt{(5--3)^2 + (8-2)^2}$

$= \sqrt{64+36}$

$= 10$ units.

(d) If AB is the diameter of the circle $\Rightarrow$ centre $= (1, 5)$

radius $= 5$ units

$\therefore$ Eq'n of circle is: $(x-1)^2 + (y-5)^2 = 5^2$

$x^2 - 2x + 1 + y^2 - 10y + 25 = 25$

i.e. $x^2 + y^2 - 2x - 10y + 1 = 0$

(e) Sub $(1, 10)$ into $x^2 + y^2 - 2x - 10y + 1 = 0$

$\therefore LHS = 1 + 100 - 2 - 100 + 1$

$= 0$

$= RHS$

$\Rightarrow C(1, 10)$ lies on the given circle

(f) $m_{AC} = \dfrac{10-2}{1--3} = 2$

$m_{CB} = \dfrac{10-8}{1-5} = -\dfrac{1}{2}$

As $m_{AC} \cdot m_{CB} = -1$

$\Rightarrow AC \perp CB$.

(g) For $12x - 5y + 3 = 0$ $\quad m_1 = \dfrac{-12}{-5}$

$= \dfrac{12}{5}$

For $24x - 10y - 7 = 0$ $\quad m_2 = \dfrac{-24}{-10}$

$= \dfrac{12}{5}$

As $m_1 = m_2 \Rightarrow$ lines are parallel

(h) Sub $(1, k)$ into $12x - 5y + 3 = 0$

$\therefore 12 - 5k + 3 = 0$

$\therefore 5k = 15$

$\therefore k = 3$

(i) perp. distance $= \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}}$

where $(x_1, y_1) = (1, 3)$; $(A, B, C) = (24, -$

$\therefore$ perp. distance $= \dfrac{|24 - 30 - 7|}{\sqrt{24^2 + (-10)^2}} = \dfrac{|-13|}{26} = \dfrac{1}{2}$.

3 (a)(i) Let $y = (7x^2-2)^5$

$\therefore \frac{dy}{dx} = 5(7x^2-2)^4 \cdot 14x$

$= 70x(7x^2-2)^4$

(ii) Let $y = \frac{3x}{2x+5}$

$\therefore \frac{dy}{dx} = \frac{(2x+5).3 - 3x.2}{(2x+5)^2}$

$= \frac{6x+15-6x}{(2x+5)^2}$

$= \frac{15}{(2x+5)^2}$

(b) $y = 3x^3 - 8x^2$

(i) $\frac{dy}{dx} = 9x^2 - 16x$

At $P(2,-8)$ $\frac{dy}{dx} = 4 = m_{tangent}$

$\therefore$ Eq'n of tangent at P is:

$y - -8 = 4(x-2)$

$\therefore y + 8 = 4x - 8$

$\therefore 4x - y - 16 = 0$.

(ii) At $P(2,-8)$ $m_{normal} = -\frac{1}{4}$

$\therefore$ Eq'n of normal at P is:

$y - -8 = -\frac{1}{4}(x-2)$

$\therefore 4y + 32 = -x + 2$

$\therefore x + 4y + 30 = 0$.

(iii) At A tangent cuts x-axis

$\therefore y = 0$ $\therefore 4x - 16 = 0$

$\therefore x = 4$

$\Rightarrow A = (4,0)$.

At B normal cuts y-axis

$\therefore x = 0$ $\therefore 4y + 30 = 0$

$\therefore y = -7\frac{1}{2}$

$\Rightarrow B = (0, -7\frac{1}{2})$.

(c) $\lim_{x \to 1} \frac{[illegible]}{x-1}$

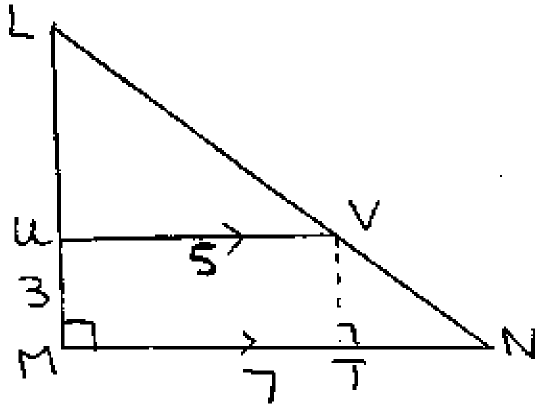
$= \lim_{x \to 1} \frac{(3x+4)\cancel{(x-1)}}{\cancel{(x-1)}}$ $(x \neq 1)$

$= \lim_{x \to 1} (3x+4)$

$= 7$

4 (a)

(i)



(ii) (A) In Δs LUV and LMN

$\angle LUV = \angle LMN$ (corr. $\angle$s in $\parallel$ lines are equal)

Similarly $\angle LVU = \angle LNM$

$\angle L$ is common

$\therefore \Delta LUV \parallel\!\mid \Delta LMN$ (Δs are equiangular)

$\therefore \frac{LU}{LU+3} = \frac{5}{7}$ (corr. sides of similar Δs are in the same ratio.)

$\therefore 7LU = 5LU + 15$

$\therefore 2LU = 15$

$\therefore LU = 7.5$ m

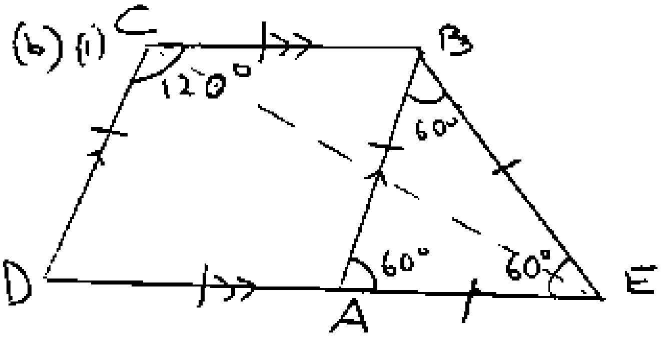
(B) Construct $VT \perp MN$

$\therefore VT = 3$, $TN = 2$

$\therefore VN = \sqrt{3^2 + 2^2}$

$= \sqrt{13}$ m

4 (b) (i)



(ii) $DC \parallel BA$ (property of rhombus)

$\angle CBA = 60°$ (coint. $\angle$s in $\parallel$ lines are supp.)

$\angle ABE = 60°$ (property of equil. Δ.)

$\therefore \angle EBC = 120°$ ($\angle$s sum of adj. $\angle$s)

(iii) As $CB = AB$ (property of rhombus)

and $AB = BE$ (property of equil. Δ)

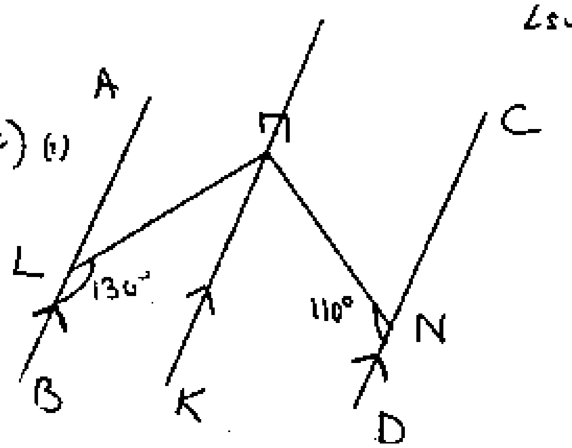
$\therefore CB = BE$

$\Rightarrow \Delta CBE$ is isosceles

$\therefore \angle ECB = \dfrac{180° - 120°}{2}$

$= 30°$ (base $\angle$s of isos. Δ are equal; $\angle$ sum of $\Delta = 180°$)

(c) (i)



(ii) (A) Construct $MK \parallel BA \parallel DC$

$\therefore \angle LMK = 50°$ (coint. $\angle$s in $\parallel$ lines are supp.)

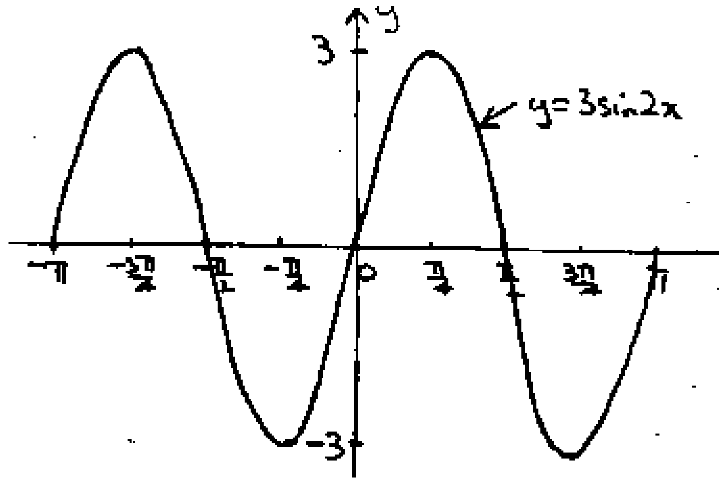
similarly $\angle KMN = 70°$

$\therefore \angle LMN = 120°$ ($\angle$ sum of adj. $\angle$s)

(B) $\angle MNC = 70°$ ($\angle$ sum on st. line $= 180°$)

5. (a) $y = 3\sin 2x$, $|x| \le \pi$

Period $= \frac{2\pi}{2} = \pi$; Amplitude $= 3$ units



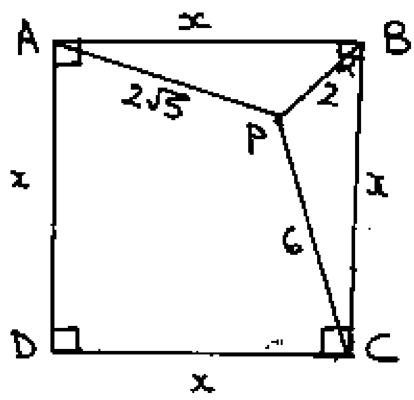
Required Area $= 4\int_0^{\frac{\pi}{2}} 3\sin 2x \, dx$

$= 12\left[\dfrac{-\cos 2x}{2}\right]_0^{\frac{\pi}{2}}$

$= 6\left[-\cos\pi + \cos 0\right]$

$= 12$ units^2

(b)



(i) In ΔPBC : $\cos\alpha = \dfrac{2^2 + x^2 - 6^2}{2 \times 2 \times x}$

$= \dfrac{x^2 - 32}{4x}$ —(1)

(ii) In ΔPBA $\angle ABP = \left(\frac{\pi}{2} - \alpha\right)$

$\therefore \cos\left(\frac{\pi}{2} - \alpha\right) = \dfrac{x^2 + 2^2 - (2\sqrt{5})^2}{2 \times 2 \times x}$

$= \dfrac{x^2 - 16}{4x} = \sin\alpha$ —(2)

(iii) Now as $\sin^2\alpha + \cos^2\alpha = 1$

sub. ① and ②: $\left(\frac{x^2-16}{4x}\right)^2 + \left(\frac{x^2-32}{4x}\right)^2 = 1$

$\therefore x^4 - 32x^2 + 256 + x^4 - 64x^2 + 1024 = 16x^2$

$\therefore 2x^4 - 112x^2 + 1280 = 0$

$\therefore x^4 - 56x^2 + 640 = 0$

(iv) $x^2 = \frac{56 \pm \sqrt{(-56)^2 - 4.1.640}}{2}$

$= \frac{56 \pm 24}{2}$

$= 40$ or 16

$\therefore x = 2\sqrt{10}$ ($x^2 > 32$ otherwise ① would not yield an acute value for α)

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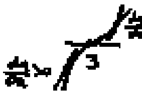
6 (a) $y = (x-3)^3$

$\frac{dy}{dx} = 3(x-3)^2$, $\frac{d^2y}{dx^2} = 6(x-3)$

For a stat. pt $\frac{dy}{dx} = 0$ $\therefore x = 3$

x	3^-	3	3^+
$\frac{dy}{dx}$	+	0	+

$\rightarrow$ 

$\Rightarrow$ horiz. pt. of inflexion on a rising curve at $(3,0)$

ie the only stationary point is a horizontal point of inflexion at $(3,0)$.

(b) $y = x^3 - 9x$

(i) For x-intercepts $y=0$

$\therefore 0 = x(x^2-9)$ $\therefore x = 0, \pm 3$.

(ii) $\frac{dy}{dx} = 3x^2 - 9$, $\frac{d^2y}{dx^2} = 6x$

(iii) Let $y = f(x) = x^3 - 9x$

Now $f(-x) = -x^3 + 9x = -f(x)$

$\Rightarrow y = x^3 - 9x$ is an odd function

(iv) For a stat. pt $\frac{dy}{dx} = 0$

$\therefore 3x^2 - 9 = 0$ $\therefore x = \pm\sqrt{3}$

when $x = \sqrt{3}$ $\frac{d^2y}{dx^2} > 0$ $\Rightarrow$ min. turning pt at $(\sqrt{3}, -6\sqrt{3})$

as the function is odd $\Rightarrow$ max. turning pt at $(-\sqrt{3}, 6\sqrt{3})$

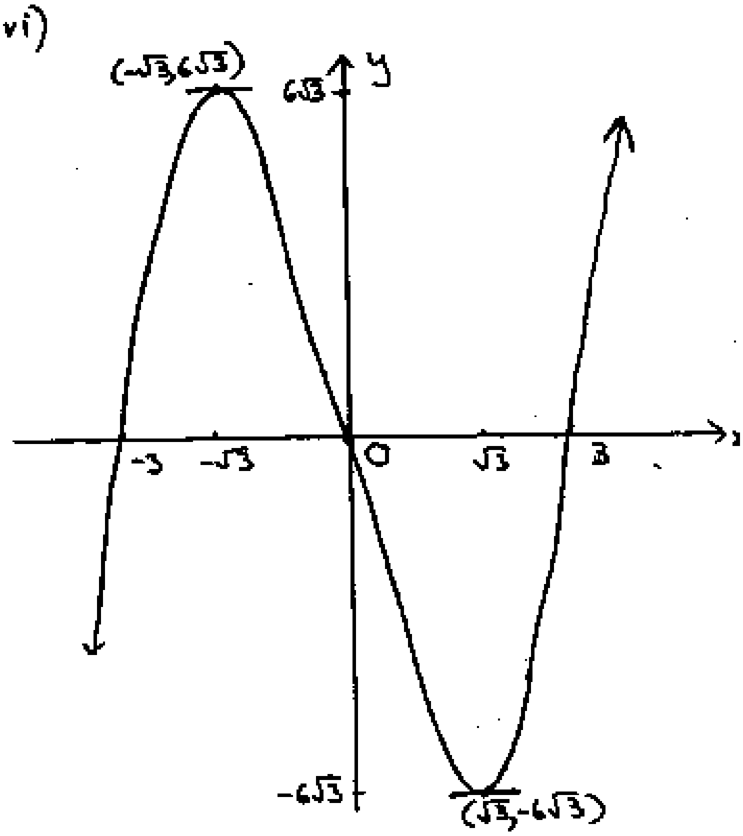
(v) For a possible pt of inflexion $\frac{d^2y}{dx^2} = 0$

$\therefore 6x = 0$ $\therefore x = 0$

x	0^-	0	0^+
$\frac{d^2y}{dx^2}$	−	0	+

concavity change $\Rightarrow$ pt. of inflexion at $(0,0)$.

(vi)



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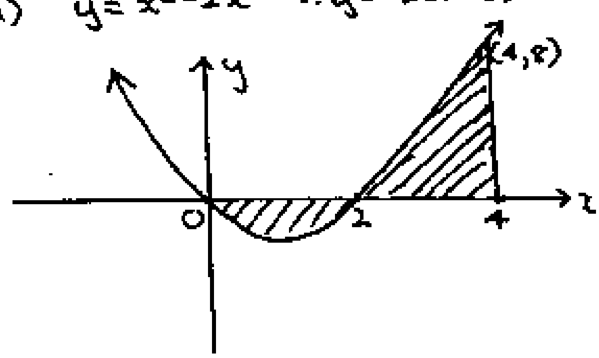
7 (a) $y = x^2 - 2x$ $\therefore y = x(x-2)$

(i)



Ernie's method to find the area was wrong as $\int_0^4 (x^2-2x)\,dx$ did not take into account that part of the area was below the x-axis.

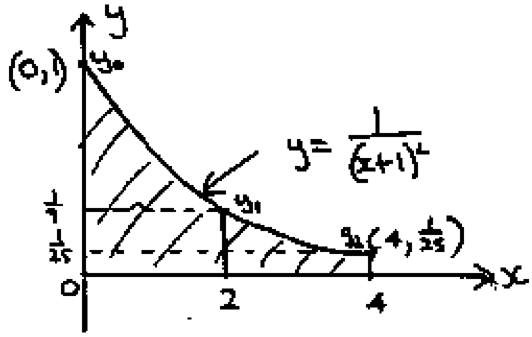
(ii) Req'd area $= \left|\int_0^2 (x^2-2x)\,dx\right| + \int_2^4 (x^2-2x)\,dx$

$= \left|\left[\frac{x^3}{3} - x^2\right]_0^2\right| + \left[\frac{x^3}{3} - x^2\right]_2^4$

$= \left|\left(\frac{8}{3}-4\right)-0\right| + \left[\left(\frac{64}{3}-16\right)-\left(\frac{8}{3}-4\right)\right]$

$= 8 \text{ units}^2$

(b)



By Simpson's Rule, $\int_0^4 \frac{1}{(x+1)^2}\,dx \doteq \frac{h}{3}\left[y_0+y_2+4y_1\right]$

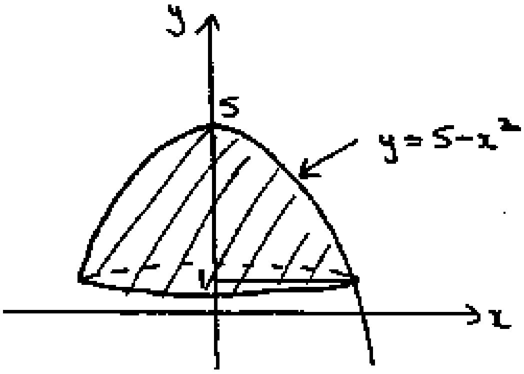
$h = \frac{4-0}{2} = 2$

$\therefore \text{Area} = \frac{2}{3}\left[1+\frac{1}{25}+4\left(\frac{1}{9}\right)\right]$

$= \frac{668}{675}$

$= 0.99 \text{ units}^2$ (2 sig. figs)

(c)



Volume $= \pi\int_1^5 x^2\,dy$

$= \pi\int_1^5 (5-y)\,dy$

$= \pi\left[\frac{(5-y)^2}{-2}\right]_1^5$

$= \frac{-\pi}{2}\left[0-16\right]$

$= 8\pi \text{ units}^3$

(8) (a) $x^2+(k+6)x-2k=0$

(i) $\Delta = b^2-4ac$

$= (k+6)^2-4.1.(-2k)$

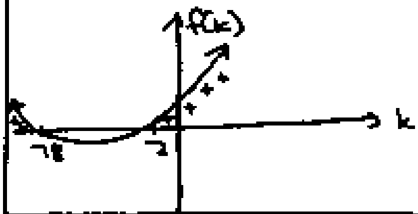
$= k^2+12k+36+8k$

$= k^2+20k+36$

(ii) For real roots $\Delta \geq 0$

$\therefore k^2+20k+36 \geq 0$

$\therefore (k+18)(k+2) \geq 0$



$\Rightarrow k \leq -18$ or $k \geq -2$

(b) $3x^2+4x-3=0$

(i) $\alpha+\beta = \frac{-b}{a} = \frac{-4}{3}$

(ii) $\alpha\beta = \frac{c}{a} = \frac{-3}{3} = -1$

(iii) $2\alpha^2+2\beta^2 = 2(\alpha^2+\beta^2)$

$= 2\left((\alpha+\beta)^2-2\alpha\beta\right)$

$= 2\left(\left(\frac{-4}{3}\right)^2+2\right)$

$= 7\frac{5}{9}$

(c) $x^2+6x-33=12y$

$\therefore (x+3)^2-9-33=12y$

$\therefore (x+3)^2 = 12y+42$

$\therefore (x+3)^2 = 12(y+3\frac{1}{2})$

is of form $(x-h)^2 = 4a(y-k)$

(i) vertex $= (-3,-3\frac{1}{2})$

(ii) focal length $=$ 3 units

(iii) Directrix is: $y=-6\frac{1}{2}$

(iv) Now focus is: $(-3,-\frac{1}{2})$

$\therefore$ sub $(-3,-\frac{1}{2})$ into $x-2y+7=0$

LHS $= -3+1+7$

$= 5$

$\neq 0$

$\neq$ RHS

$\therefore x-2y+7=0$ is not a focal chord of the parabola.

9 (a) (i) let $y = x^3 e^{5x-1}$

$$\therefore \frac{dy}{dx} = x^3 . 5e^{5x-1} + e^{5x-1} . 3x^2$$

(ii) let $y = \log_e \left[\frac{3x^5-4}{6x^3-5}\right]$

$$\therefore y = \log_e[3x^5-4] - \log_e[6x^3-5]$$

$$\therefore \frac{dy}{dx} = \frac{15x^4}{3x^5-4} - \frac{18x^2}{6x^3-5}$$

(b) $$I = \int_0^1 \frac{24x^2-14}{4x^3-7x-5}\,dx$$

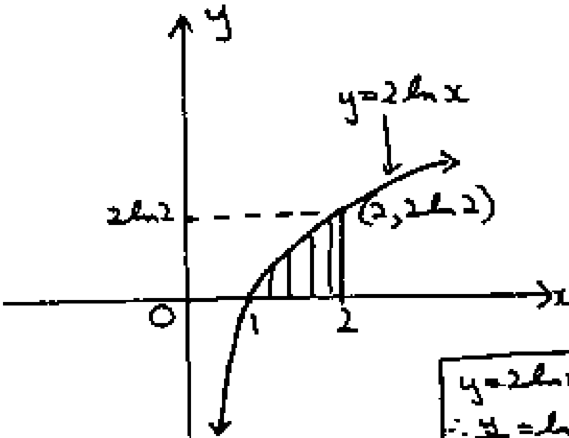
$$= \int_0^1 \frac{2(12x^2-7)}{4x^3-7x-5}\,dx$$

$$= 2\Big[\ln|4x^3-7x-5|\Big]_0^1$$

$$= 2\big[\ln|-8| - \ln|-5|\big]$$

$$= 2\ln\tfrac{8}{5}$$

(c)



$y = 2\ln x$

$\therefore \frac{y}{2} = \ln x$

$\therefore x = e^{\frac{y}{2}}$

$$\text{Area} = \int_1^2 2\ln x\,dx$$

$$= \text{Area rectangle} - \int_0^{2\ln 2} x\,dy$$

$$= 2\times 2\ln 2 - \int_0^{2\ln 2} e^{y/2}\,dy$$

$$= 4\ln 2 - \left[\frac{e^{y/2}}{1/2}\right]_0^{2\ln 2}$$

$$= 4\ln 2 - 2\left[e^{\ln 2} - e^0\right]$$

$$= 4\ln 2 - 2[2-1]$$

$$= (4\ln 2 - 2)\ \text{units}^2$$

10 (a) If x, y and 1 are in ...

$$\therefore \frac{y}{x} = \frac{9}{y} \qquad \therefore y^2 = 9x \quad \text{— (1)}$$

If y, x and 2 are in A.P.

$$\therefore x - y = 2 - x \qquad \therefore 2x - 2 = y \quad \text{— (2)}$$

sub (2) into (1): $(2x-2)^2 = 9x$

$$\therefore 4x^2 - 8x + 4 = 9x$$

$$\therefore 4x^2 - 17x + 4 = 0$$

$$(4x-1)(x-4) = 0$$

$$\therefore x = \tfrac{1}{4} \text{ or } 4$$

when $x = \frac{1}{4}$ $y = -1\frac{1}{2}$

$x = 4$ $y = 6$

$\Rightarrow (x, y) = (\frac{1}{4}, -1\frac{1}{2})$ or $(4, 6)$.

(b) (i) 6% p.a. $= \frac{6}{12}$ % p. month

$= 0.005$ p. month

Let A_n be the amount owing after n instalments.

Let $P = 1\,000\,000$

After 1 instalment amount owing, $A_1 = P(1.005) - M$

" 2 instalments " ", $A_2 = A_1(1.005) - M$

$= (P(1.005) - M)1.005 - M$

$= P(1.005)^2 - M(1 + 1.005)$

" 3 instalments, " ", $A_3 = A_2(1.005) - M$

$= (P(1.005)^2 - M(1+1.005))1.005 - M$

$\therefore A_3 = P(1.005)^3 - M(1.005^2 + 1.005 + 1)$

$\therefore$ continuing this process the amount owing after n instalments, $A_n = P(1.005)^n - M(1.005^{n-1} + \dots + 1.005 + 1)$

Now after n instalments, $A_n = 0$.

After 10 years $n = 10\times 12 = 120$

$$\therefore 0 = 1000000(1.005)^{120} - M(1 + 1.005 + \dots + 1.005^{119})$$

$$\therefore M = \frac{1\,000\,000(1.005)^{120}}{\underbrace{1 + 1.005 + \dots + 1.005^{119}}_{\text{G.P. } a=1,\ r=1.005,\ n=120}}$$

$$\therefore M = \frac{1000000(1.005)^{120}}{1\left[\frac{1.005^{120}-1}{0.005}\right]}$$

$$\therefore M = \frac{5000(1.005)^{120}}{1.005^{120}-1}$$

(ii) $\therefore$ Monthly instalment, $M = \$11\,102.05$ (to the nearest cent)

(iii) After 5 years, $n = 5 \times 12 = 60$.

$\therefore$ Amount owing, $A_{60} \doteqdot 1000000(1.005)^{60} - 11102.05\left[1 + 1.005 + \ldots + 1.005^{59}\right]$

$$\therefore \; A_{60} = 1000000(1.005)^{60} - 11102.05\left[\frac{1.005^{60}-1}{0.005}\right]$$

$$= \$574259.79 \; \left(\text{to nearest cent}\right)$$

(iv) Interest paid over 10 years

$$= 120 \times \$11102.05 - 1000000$$

$$= \$332246$$

Now Simple Interest @ 4.5% p.a. over 10 years $= \dfrac{P.R.T}{100}$

$$= \frac{1000000 \times 4.5 \times 10}{100}$$

$$= \$450\,000$$

$\therefore$ The investor would have been worse off by some $117754 if he had chosen to borrow from F.B. Knightly Investments Ltd. instead of financial institution X.