

Question One:

a) 1.242886646
 $= 1.243$ (to 3dp)

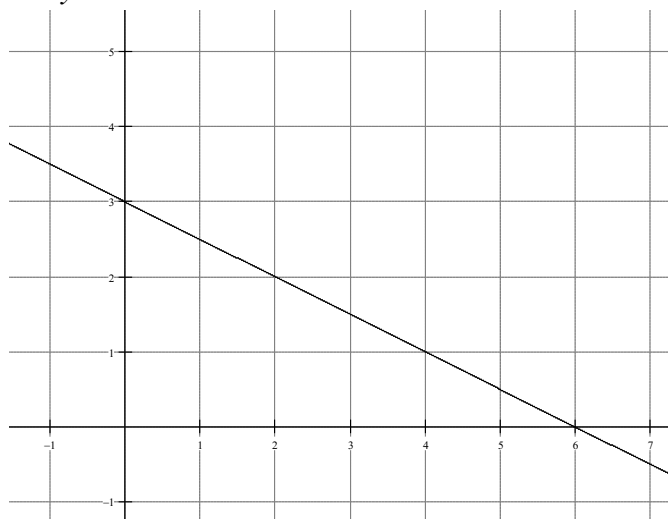
b) $3 - 2x = 4x$
 $3 = 6x$
 $x = \frac{1}{2}$

c) $\frac{2}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$
 $= \frac{2(1-\sqrt{3})}{1-3}$
 $= \frac{2(1-\sqrt{3})}{-2}$
 $= \sqrt{3} - 1$

d) $= 4 + 12x - x - 3x^2$
 $= 4(1+3x) - x(1+3x)$
 $= (1+3x)(4-x)$

e) $2y = 6 - x$
 $y = 3 - \frac{1}{2}x$

f) $x = 0: y = 0:$
 $y = 3 \quad x = 6$



g) $3y = 4x - 1$
 $y = \frac{4}{3}x - \frac{1}{3} \Rightarrow m_1 = \frac{4}{3}$, so perp. gives $m_2 = -\frac{3}{4}$

So through $(2, -3)$:

$$y - (-3) = -\frac{3}{4}(x - 2)$$

$$4y + 12 = -3x + 6$$

$$3x + 4y + 6 = 0$$

Marking

- 1 answer
- 1 dp's
- 1 algebra

Comments

- 1 answer

- 1 \times conjugate

- 1 answer
- 1 resolves pairs

- 1 answer

- 1 intercepts

Some had trouble finding these!

- 1 graph

- 1 perp. gradient

- 1 eqn (in GF) Again, simplify!

Question Two:

a) i) $u = x^2 \quad v = e^x$
 $du = 2x \quad dv = e^x$

$$\therefore \frac{d(x^2 e^x)}{dx}$$

$$= x^2 e^x + 2x e^x$$

$$= x e^x (x + 2)$$

ii) $\frac{d(1 + \tan x)^2}{dx}$

$$= 2(1 + \tan x)^1 \times \frac{d(\tan x)}{dx}$$

$$= 2 \sec^2 x (1 + \tan x)$$

b) i) $\int 4x - \sin x \, dx$

$$= 2x^2 + \cos x + c$$

ii) $\int_1^3 \frac{1}{x^2} \, dx$

$$= \left[-\frac{1}{x} \right]_1^3$$

$$= \frac{-1}{3} - \frac{-1}{1}$$

$$= \frac{2}{3}$$

c) $\frac{dy}{dx} = 1 + \frac{1}{x^2},$

so when $x = -1 \quad \frac{dy}{dx} = 2$

Hence $y - 0 = 2(x + 1)$

or $y = 2x + 2$

Marking**Comments**

❶ product rule Mostly good

❶ answer

❶ chain rule

Some struggled with structure of the Chain Rule.

Some did not know the derivative of $\tan x$.

❶ answer

❶ answer

Quite a few integration errors, and some forgot “+c”

❶ for ‘+c’

❶ int & limits

Many 2 Unit candidates got a log or differentiated.

❶ subst

Many also made sign errors in substitution.

❶ answer

❶ derivative

Some could not find derivative.

❶ gradient

Many errors to find “m=2”

❶ answer

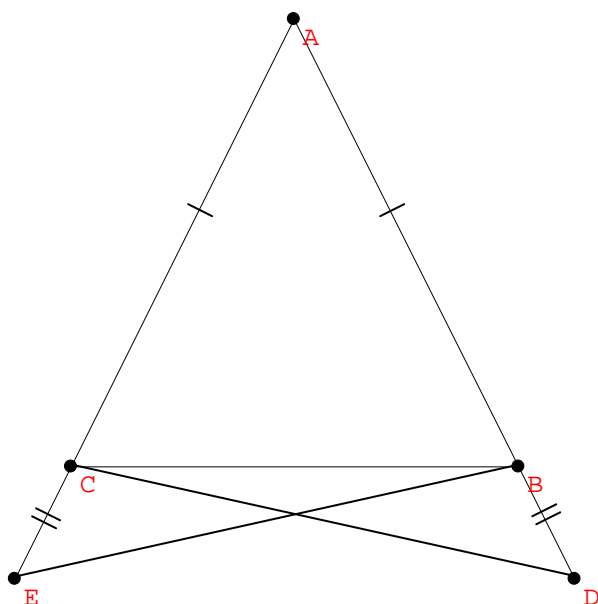
(“m=0” was popular). More seriously, not finding an “m” value but using algebra:

$$y = \left(1 + \frac{1}{x^2}\right)(x + 1)$$

is meaningless!

Question Three:**Marking****Comments**

a) i)



ii) $AB=AC$ and $BE=CD$ (given)
 $AD=AB+BD$, $AE=AC+CE$ (st. lns), so
 $AD=AC+CE$, hence
 $AD=AE$

In Δ 's ABE, ACD ←

i) $AB = AC$ (given)

ii) $\angle EAB = \angle DAC$ (common angle)

iii) $AD=AE$ (shown above)

$\therefore \Delta ABE \equiv \Delta ACE$ (SAS)

b) For $y = e^{x^2}$, with $h = 0.5$:

x	0	0.5	1	1.5	2
y_i	1	$e^{0.25}$	e	$e^{2.25}$	e^4

Simpsons Rule:

$$A = \frac{h}{3} [(y_0 + y_4) + 2(y_1 + y_3) + 4y_2]$$

$$= \frac{0.5}{3} [(1 + e^4) + 4(e + e^{2.25}) + 2e]$$

$$\doteq 17.35362645$$

$$\doteq 17.35$$

c) With $T_3 = \frac{1}{12}$, $T_8 = \frac{-1}{384}$:

i. $ar^2 = \frac{1}{12}$ and $ar^7 = \frac{-1}{384}$, hence

1 markings

1 reason

1 reason

1 reason

1 conclusion

1 values

1 subst

1 ans to 2dp

Some students failed to give all the information required.

Some did not show this (or equivalent);
Need to state the triangles;

Some did not state the test used.

Some students had the 4 and the 2 the wrong way around in the formula.

$$\frac{T_8}{T_3} = \frac{-1}{384} \div \frac{1}{12}$$

$$\frac{ar^7}{ar^2} = \frac{-1}{384} \times \frac{12}{1}$$

$$r^5 = \frac{-1}{32}$$

$$r = \frac{-1}{2}$$

$$a \cdot \left(\frac{-1}{2}\right)^2 = \frac{1}{12}$$

$$a = \frac{1}{3}$$

$$T_n = ar^n$$

$$= \frac{1}{3} \cdot \left(\frac{-1}{2}\right)^n$$

$$\text{ii. } S_n = \frac{a(1-r^n)}{1-r}$$

$$S_8 = \frac{\frac{1}{3} \left(1 - \left(\frac{-1}{2}\right)^8\right)}{1 - \frac{-1}{2}}$$

$$= \frac{\frac{1}{3} \left(1 - \frac{1}{256}\right)}{\frac{3}{2}}$$

$$= \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{255}{256}$$

$$= \frac{85}{384}$$

$$\text{iii. } S_\infty = \frac{a}{1-r}$$

$$= \frac{\frac{1}{3}}{1 - \frac{-1}{2}}$$

$$= \frac{1}{3} \cdot \frac{2}{3}$$

$$= \frac{2}{9}$$

This part done well provided the correct ratio of

$\frac{-1}{2}$ found (some

used $\frac{\pm 1}{2}$ or $\frac{1}{2}$).

❶ a, r values

❶ T_n correct

❶ S_8 correct

This answer best left as a fraction.

❶ S_∞ correct

Question Four:

a) $\cos x = \frac{\sqrt{3}}{2}$

$$x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\pi}{6}$$

$\cos x$ positive in Q1 & Q4, but with $-\pi \leq x \leq \pi$,

$$x = \frac{\pi}{6}, \frac{-\pi}{6}$$

b)

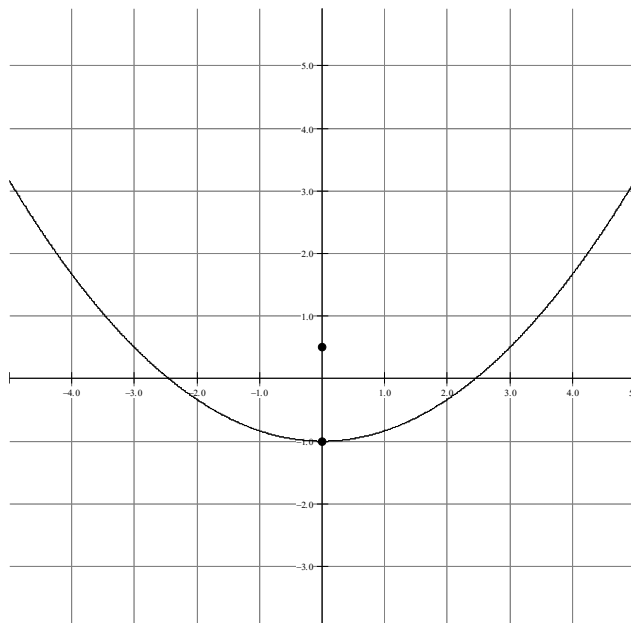
i. Vertex is $(0, -1)$

ii. $4a = 6$

Hence $a = \frac{3}{2}$, so focus is $(0, -1 + a)$ or

$$\left(0, \frac{1}{2}\right)$$

iii.



iv. $y = 5$ and $x^2 = 6(y + 1)$

$$\begin{aligned} x^2 &= 6(5 + 1) \\ &= 36 \end{aligned}$$

$$y + 1 = \frac{x^2}{6}$$

$$y = \frac{x^2}{6} - 1$$

$x = \pm 6$, hence

Marking**Comments**

❶ base angle

❶ answers

❶ answer

❶ answer

❶ graph

❶ eqn for Int

❶ boundaries

Many did not read the requirements of the question: $-\pi \leq x \leq \pi$ also implies radians!

Sketch was poorly done by many – must show vertex, focus, intercepts.

Use a ruler to draw axes!

Poorly done. Students need to review area between two curves.

$$\begin{aligned}
 A &= \int_{-6}^6 5 - \left(\frac{x^2}{6} - 1 \right) dx \\
 &= 2 \int_0^6 5 - \left(\frac{x^2}{6} - 1 \right) dx \\
 &= 2 \left[6x - \frac{x^3}{18} \right]_0^6 \\
 &= 2 \left[\left(6^2 - \frac{6^3}{18} \right) - 0 \right] \\
 &= 48
 \end{aligned}$$

1 answer

c)

- i. First die 1 to 4 must correspond to 2nd die 4 to 1, so 4 outcomes give a total of 5

1 reason for 4

$$\begin{aligned}
 P(5) &= \frac{4}{36} \\
 &= \frac{1}{9}
 \end{aligned}$$

1 answer

- ii. $P(\text{not } 5) = 1 - P(5)$

$$\begin{aligned}
 &= 1 - \frac{1}{9} \\
 &= \frac{8}{9}
 \end{aligned}$$

1 answer

- iii. $P(\text{doubles}) = \frac{1}{6}$

1 answer

Question Five:**Marking****Comments**

a)

i. By Pythagoras: $AC^2 = 10^2 - 6^2$
 $= 64$

$$AC = 8$$

ii. $\angle DFA = \angle BCA = 90^\circ$
 For BC and DF , $\angle DFA$ and $\angle BCA$ are
 in a corresponding position and equal.
 Hence $BC \parallel DF$.

iii. In Δ 's ADF, ABC
 $\angle DFA = \angle BCA = 90^\circ$ (given in diagram)
 $\angle A$ is common.
 Hence all angles are equal so
 $\Delta AFD \parallel \Delta ABC$

iv. Similarly, $\Delta BDE \parallel \Delta ABC$, hence

$$\frac{DE}{AC} = \frac{DB}{AB}$$

$$\frac{3}{8} = \frac{DB}{10}$$

$$DB = 3.75$$

b) $y = x^2 - 2x$; $x = 2, y = 0$

$$\frac{dy}{dx} = 2x - 2; x = 2, \frac{dy}{dx} = 2, \text{ so normal gradient is}$$

$$\frac{-1}{2}$$

$$y - 0 = \frac{-1}{2}(x - 2)$$

$$y = 1 - \frac{x}{2}$$

$$0 = x + 2y - 2$$

c) $g(0) = 4:4 = a.0^2 + b.0 + c$

$$g(1) = 23:23 = a.1^2 + b.1 + c$$

$$g(-1) = 1:1 = a.(-1)^2 + b.(-1) + c, \text{ giving the eqns}$$

$$4 = c \quad \langle 1 \rangle$$

$$23 = a + b + c \quad \langle 2 \rangle$$

$$1 = a - b + c \quad \langle 3 \rangle$$

$$\langle 1 \rangle \text{ in } \langle 2 \rangle \text{ and } \langle 3 \rangle \text{ gives:}$$

$$a + b = 19$$

$$a - b = -3, \text{ then adding gives:}$$

$$2a = 16; \text{ back-substitution gives } 23 = 8 + b + 4$$

$$a = 8$$

$$b = 11$$

$$\text{Hence } a = 8, b = 11, c = 4$$

1 answer

1 reasoning

1 reasons

1 conclusion

1 subst

1 answer

1 gradient

1 answer

1 set-up

1 answers

Some students
 failed to recognise
 corresponding
 angles, or failed to
 write that fact
 down!

Parts b) and c)
 generally well
 done.

Question Six:

a)

i. y-intercept at (0,6).

ii. $y' = 3x^2 - 6x - 9$

$y'' = 6x - 6$

Stat Pts when $y' = 0$:

$0 = 3x^2 - 6x - 9$

$= x^2 - 2x - 3$

$= (x-3)(x+1)$

$\therefore x = -1, 3$

$x = -1$

$x = 3$

$y = (-1)^3 - 3(-1)^2 - 9(-1) + 6$

$= 11$

$= -21$

Pts are (-1,11) and (3,-21)

$x = -1$

$x = 3$

$y'' = -12$

$y'' = 12$

$\Rightarrow ccu$

$\Rightarrow ccu$

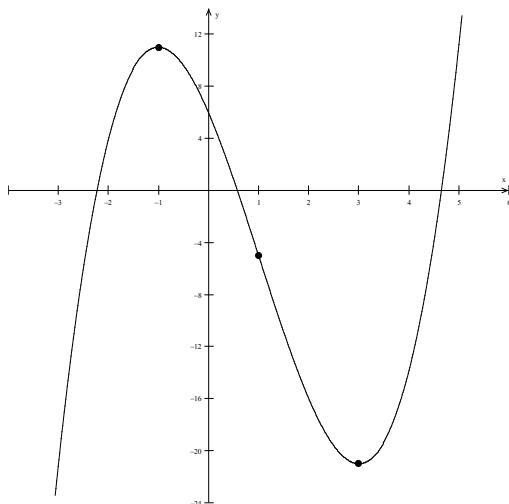
 $\therefore (-1,11)$ is a max t.p.and $(3,-21)$ is a min t.p.iii. $y'' = 0: 0 = 6x - 6$, hence possibleinflexion pt when $x = 1$.when $x < 1, y'' < 0 \Rightarrow ccu$ when $x > 1, y'' > 0 \Rightarrow ccu$ hence concavity changes, so $(1,-5)$ is a point of inflexion.iv. Concave up when $y'' > 0$

$6x - 6 > 0$

$6x > 6$

i.e. when $x > 1$

v.

**Marking****Comments**

1 answer

1 x values

1 test

1 points

1 point & test

1 answer

1 t.p's

1 intercepts

Make sure co-ordinates are stated when the question asks for them

Must test the nature of Stat. Pts and Inflexion Pts!

Sketch very poorly done.

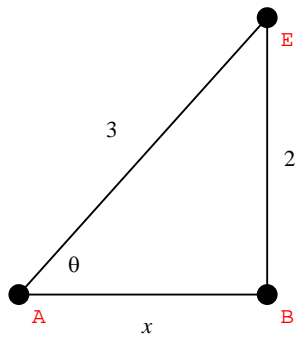
Use:

- a ruler for axes
- a suitable scale

Show the information requested:

- turning points
- inflexion point
- intercepts (x and y)

b) $\sin \theta = \frac{-2}{3}$



Hence $x^2 = 3^2 - 2^2$, or $x = \sqrt{5}$

$\sin \theta < 0 \Rightarrow Q_3, Q_4$

$\cos \theta > 0 \Rightarrow Q_1, Q_4 \Rightarrow \tan \theta \text{ in } Q_4; \tan \theta < 0$

Hence $\tan \theta = \frac{-2}{\sqrt{5}}$

c) $\frac{d(xe^x)}{dx} \quad \begin{matrix} u = x & v = e^x \\ u' = 1 & v' = e^x \end{matrix}$

$= xe^x + 1 \cdot e^x$

$= e^x + xe^x$ as reqd.

Hence $\frac{d(xe^x)}{dx} = e^x + xe^x$, so integrating gives:

$xe^x = \int (e^x + xe^x) dx$

$= \int e^x dx + \int xe^x dx$

$\therefore \int xe^x dx = xe^x - \int e^x dx$

$= xe^x - e^x + c$

$= e^x(x-1) + c$

Some students did not know the ASTC results.

❶ quadrant

❶ answer

❶ product rule Showing a result – you must clearly demonstrate the link for each step.

❶ answer

Question Seven:

a) $4x^2 + 8x - 1 = 0$:

i. $\alpha + \beta = \frac{-8}{4}$
 $= -2$

ii. $\alpha\beta = \frac{-1}{4}$

iii. $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$
 $= \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2}$
 $= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$
 $= \frac{(-2)^2 - 2 \cdot \frac{-1}{4}}{\left(\frac{-1}{4}\right)^2}$
 $= 16 \left(4 + \frac{1}{2}\right)$
 $= 72$

b) $y = 3 \sin 2x$

i. Amplitude is 3

ii. Period is $\frac{2\pi}{2}$, or π

iii. ① amplitude, ① period/shape

Marking**Comments**

① answer

① answer

① resolves

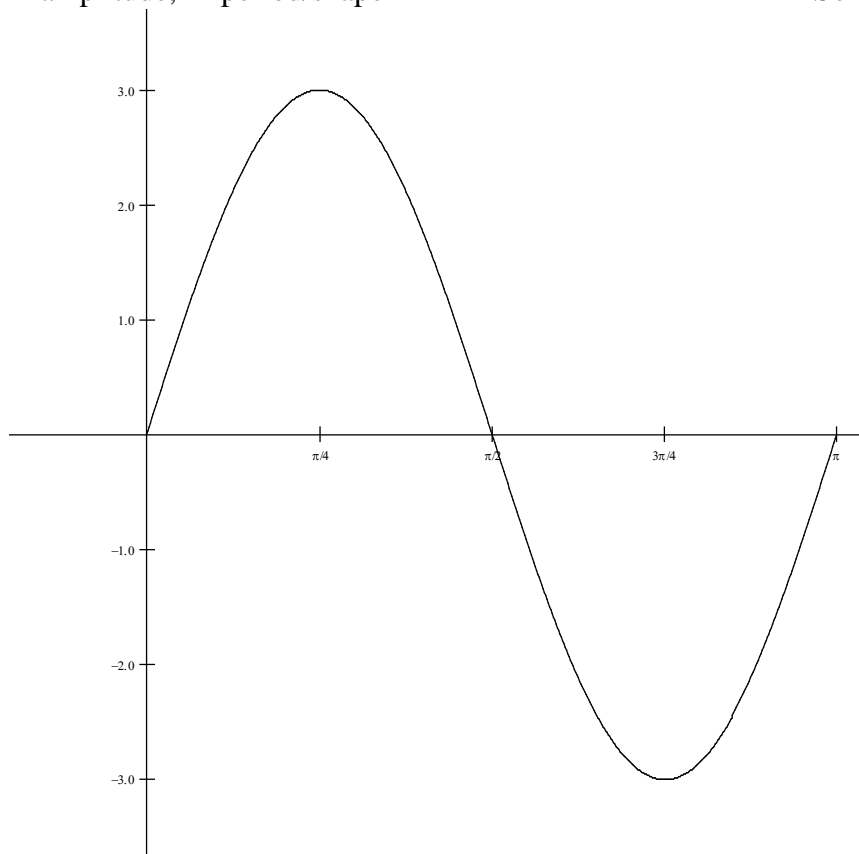
Many had problems with this identity

① answer

① answer

① answer

Some drew Cos!



c)

$$\begin{aligned}
 \text{i. } \log_2 45 &= \log_2 (5 \times 9) \\
 &= \log_2 5 + \log_2 3^2 \\
 &= \log_2 5 + 2 \log_2 3 \\
 &= 2.322 + 2 \times 1.585 \\
 &= 5.492
 \end{aligned}$$

1 answer

Many multiplied the logs, instead of adding them.

$$\begin{aligned}
 \text{ii. } \log_7 0.3 &= \frac{\ln 0.3}{\ln 7} \\
 &= -0.6187196284 \\
 &\approx -0.619
 \end{aligned}$$

1 answer

$$\text{d) } 9x^2 - 3x + p = 0$$

For only one root, $\Delta = 0$

Too many had $\Delta > 0$ for one real root!

$$\begin{aligned}
 \therefore b^2 - 4ac &= 0 \\
 0 &= 9 - 4 \cdot 9 \cdot p \\
 36p &= 9 \\
 p &= \frac{1}{4}
 \end{aligned}$$

1 set-up

1 answer

This was also successfully resolved by sums and products of roots by many:

Alternate marking for (d):

$$\alpha + \beta = -\frac{-3}{9}, \alpha\beta = \frac{p}{9} \quad \alpha = \beta, \text{ giving:}$$

$$2\alpha = \frac{1}{3}$$

$$\alpha = \frac{1}{6} (= \beta)$$

1 uses = roots

$$\left(\frac{1}{6}\right)^2 = \frac{p}{9}$$

$$p = \frac{9}{36}$$

$$= \frac{1}{4}$$

1 answer

Question Eight:

a)

$$\text{i. } l = r\theta$$

$$4\pi = 12\theta$$

$$\therefore \theta = \frac{\pi}{3}$$

$$\text{ii. } A = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2} \cdot 12^2 \cdot \frac{\pi}{3}$$

$$= 24\pi \text{ cm}^2$$

$$\text{b) } y = e^x + e^{-x}, \text{ so}$$

$$y^2 = (e^x + e^{-x})^2$$

$$= e^{2x} + 2e^x e^{-x} + e^{-2x}$$

$$= e^{2x} + e^{-2x} + 2$$

Volume is given by:

$$v = \int_0^2 \pi y^2 dx$$

$$= \pi \int_0^2 (e^{2x} + e^{-2x} + 2) dx$$

$$= \pi \left[\frac{1}{2} e^{2x} - \frac{1}{2} e^{-2x} + 2x \right]_0^2$$

$$= \pi \left[\left(\frac{1}{2} e^4 - \frac{1}{2} e^{-4} + 4 \right) - \left(\frac{1}{2} e^0 - \frac{1}{2} e^0 + 0 \right) \right]$$

$$= \frac{\pi}{2} (e^4 - e^{-4} + 8) \text{ cu. units}$$

c)

$$\text{i. } T_1 = 10, T_2 = 15, T_3 = 20 \dots$$

$$T_2 - T_1 = 5; T_3 - T_2 = 5, \text{ so this is an AP}$$

$$\text{with } a = 10, d = 5, \text{ hence}$$

$$T_n = 10 + 5(n-1)$$

$$= 5 + 5n$$

When $n = 10$

$$T_n = 5 + 5 \cdot 10$$

$$= 55$$

$$\text{ii. } S_n = \frac{n}{2}(2a + (n-1)d), \text{ so radius will be}$$

$$r = 5 + S_n, \text{ so}$$

$$S_n = 455 - 5$$

$$= 450$$

Marking**Comments**

Well done.

1 answer

Mostly good. A few tried to find the area of a segment or triangle.

1 answer

Students who could expand y^2 correctly generally got full marks.1 y^2 correct

A few made horrible integration attempts ☹:

$$\int e^{2x} dx = \left[\frac{e^{x^2}}{x^2} \right]$$

1 int & limits

Some mistakes substituting:
 $e^0 - e^0$

1 answer

i) Was poorly set out – when a question says “Show that...” you must show your thinking, theory used and calculations!

1 justifies AP then shows subst

1 to get answer

For full marks, students needed to show that they recognised an AP and then substituted.

1 radius correct

A good answer would have proved an AP (as shown in solns).

$$450 = \frac{n}{2}(2 \times 10 + 5(n-1))$$

$$900 = n(15 + 5n)$$

$$0 = 5n^2 + 15n - 900$$

$$= n^2 + 3n - 180$$

$$0 = (n+15)(n-12)$$

$$n = -15, 12$$

As $n > 0$, there are 12 strips needed.

d) $4e^{2x} - e^x = 0$, let $u = e^x$, then

$$0 = 4u^2 - u$$

$$= u(4u - 1)$$

$$u = 0, \frac{1}{4}, \text{ hence}$$

$$e^x = 0, e^x = \frac{1}{4}$$

$e^x = 0$ has no solution.

$$e^x = \frac{1}{4}$$

$$x = \ln\left(\frac{1}{4}\right)$$

$$= -2 \ln 2$$

$$(= -1.386294361)$$

① forms quadratic

① answer justified

① resolves quad

① answer justified

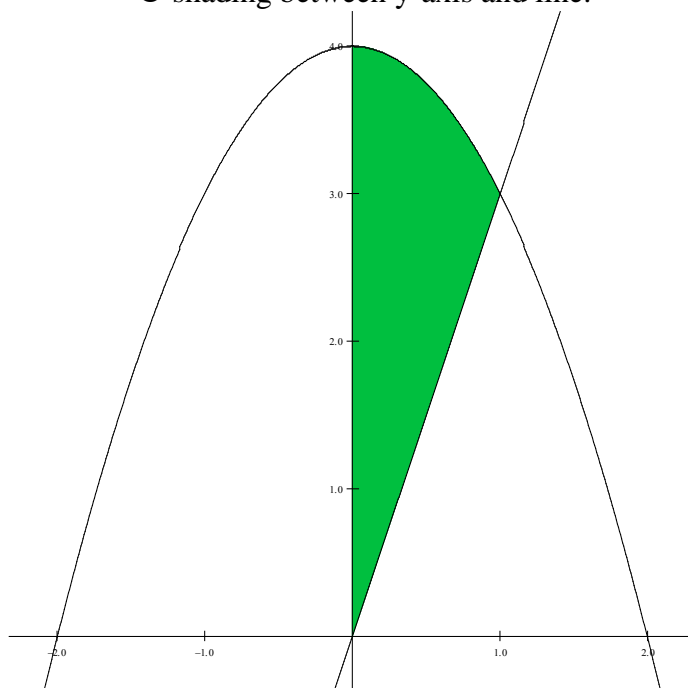
Students needed to recognise the extra 5cm centre and deduct it; for full marks, justification for the positive solution was needed.

For full marks, students needed to give the two expressions for e^x and state that $e^x = 0$ has no solution.

Question Nine:**Marking****Comments**

a) $y \leq 4 - x^2$

- i. ❶ shading below parabola,
 ❷ shading between y-axis and line.



- ii. Solving $y = 3x$ and $y = 4 - x^2$ simultaneously:

$$3x = 4 - x^2$$

$$0 = x^2 + 3x - 4$$

$$= (x + 4)(x - 1)$$

$$x = -4, 1$$

Hence intersection at (1, 3)

So volume of solid to y-axis is $V = \pi \int_a^b x^2 dy$

$$y = 4 - x^2 \text{ becomes } x^2 = 4 - y$$

$$y = 3x \text{ becomes } x = \frac{y}{3} \text{ or } x^2 = \frac{y^2}{9}$$

$$V = \pi \int_0^3 \frac{y^2}{9} dy + \pi \int_3^4 4 - y dy$$

Poorly done.
 Most students failed to establish the equation in terms of y, and hence did not find the correct y values for the integration.
 Many also did not recognise the need to split the integral into two parts.

❶ values

❷ both x^2 eqns

❸ int & limits

$$\begin{aligned}
&= \pi \int_0^3 \frac{y^2}{9} dy + \int_3^4 4 - y dy \\
&= \pi \left(\left[\frac{y^3}{27} \right]_0^3 + \left[4y - \frac{y^2}{2} \right]_3^4 \right) \\
&= \pi \left(\left(\frac{27}{27} - 0 \right) + \left(\left(16 - \frac{16}{2} \right) - \left(12 - \frac{9}{2} \right) \right) \right) \\
&= \pi \left(1 + 8 - \frac{15}{2} \right) \\
&= \frac{3\pi}{2} \text{ cu. units}
\end{aligned}$$

① answer

b) $AB = x$, hence CD, CY and XY are all also x .

i. Total length of fencing is given by:

$$700 = AB + CD + CY + XY + BC$$

$$BC = 700 - 4x$$

For rhombus $CDXY$:

$$A = 2 \times \frac{1}{2} CD \cdot DY \cdot \sin 30^\circ$$

$$= \frac{x^2}{2}$$

For rectangle $ABCD$:

$$A = AB \cdot BC$$

$$= x(700 - 4x)$$

$$= 700x - 4x^2$$

Total area is therefore:

$$A = 700x - 4x^2 + \frac{1}{2}x^2$$

$$= 700x - \frac{7x^2}{2} \quad \text{as reqd.}$$

① Perim link

Poorly done. Most students did not use the sine version of the area of a triangle.

① areas & alg

ii. For a possible maximum, $\frac{dA}{dx} = 0$:

$$\therefore \frac{dA}{dx} = 700 - 7x; \frac{dA}{dx} = 0 \text{ gives}$$

$$0 = 700 - 7x$$

$$x = 100$$

$$\frac{d^2A}{dx^2} = -7 \Rightarrow \text{c.c.d.}, \text{ or a max tp.}$$

Hence max area is

$$A = 700 \times 100 - \frac{7 \times 100^2}{2}$$

$$= 35000 \text{ sq.m}$$

iii. Paddock is therefore 100m by 300m.

① x-value

① max shown

① answer

① answer

Question Ten:

a) \$P invested at 9% p.a.

i. First Year: $A_1 = P(1 + 0.09)$

or $A_1 = 1.09P$

Marking**Comments**

1 answer

ii. Second Year: $A_2 = (A_1 + P)(1 + 0.09)$

or $A_2 = (1.09P + P)(1.09)$

$$= 1.09^2 P + 1.09P$$

$$= P(1.09^2 + 1.09)$$

1 answer

iii. After n years: $A_n = (A_{n-1} + P)(1 + 0.09)$

Using the above pattern, this becomes:

$$A_n = P(1.09^n + 1.09^{n-1} + \dots + 1.09)$$

Now, this is a GP with

$$a = 1.09, r = 1.09, n = n, \text{ thus}$$

1 GP & values

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1.09(1.09^n - 1)}{1.09 - 1}$$

$$= \frac{1.09(1.09^n - 1)}{0.09}$$

$$= \frac{1.09}{0.09}(1.09^n - 1)$$

$$= \frac{109(1.09^n - 1)}{9}$$

Need to state this is a GP and write the formula

This step 'fudged' by many.

1 GP resolved

$$\text{Hence } A_n = \frac{109P}{9}(1.09^n - 1), \text{ as reqd.}$$

iv. For $A_n = \$1,000,000$ and $n=30$:

$$1,000,000 = \frac{109P}{9}(1.09^{30} - 1)$$

$$\frac{9000000}{109} = P(1.09^{30} - 1)$$

$$P = \frac{9000000}{109(1.09^{30} - 1)}$$

$$= 6730.597606$$

$$= \$6731$$

(nearest \$)

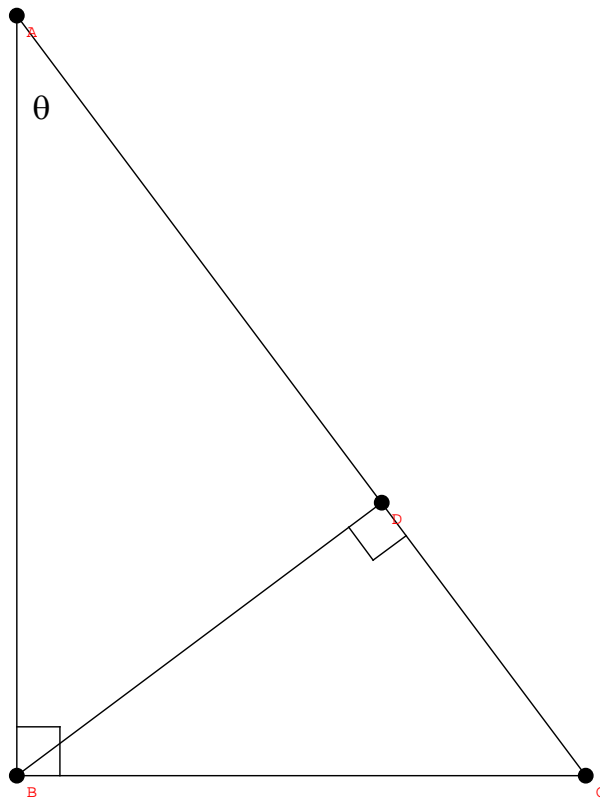
1 answer

Check degree of accuracy required (no marks lost if gave to nearest ϕ).

b)

i.

① diagram



- ii. Given $6AD + BC = 5AC$, expressions for AD, BC and AC in terms of θ :

$$\cos \theta = \frac{AD}{AB} \quad (\text{from } \triangle ADB)$$

$$AD = AB \cos \theta$$

$$\tan \theta = \frac{BC}{AB} \quad \text{and} \quad \cos \theta = \frac{AB}{AC} \quad (\text{from}$$

$$BC = AB \tan \theta$$

$$AC = \frac{AB}{\cos \theta}$$

$\triangle ABC$)

Substituting these into the expression above:

$$6AB \cos \theta + AB \tan \theta = 5 \frac{AB}{\cos \theta}$$

Hence, $6 \cos \theta + \tan \theta = 5 \sec \theta$ as reqd.

- iii. $6 \cos \theta + \tan \theta = 5 \sec \theta$, becomes

$$6 \cos \theta + \frac{\sin \theta}{\cos \theta} = \frac{5}{\cos \theta}$$

$$6 \cos^2 \theta + \sin \theta = 5$$

$$6(1 - \sin^2 \theta) + \sin \theta = 5$$

$$6 - 6 \sin^2 \theta + \sin \theta - 5 = 0$$

Hence $6 \sin^2 \theta - \sin \theta - 1 = 0$ as reqd.

If students saw the common connection of AB , they generally did well with the question.

① exp for each

① correct subst & alg

Many poor with Trig identities!

① sin/cos resolved

① subst & alg

iv. Let $u = \sin \theta$

$$0 = 6u^2 - u - 1$$

$$= 6u^2 - 3u + 2u - 1$$

$$= 3u(2u - 1) + 1(2u - 1)$$

$$= (2u - 1)(3u + 1)$$

❶ quad resolved

Hence

$$0 = 2 \sin \theta - 1$$

$$0 = 3 \sin \theta + 1$$

$$\frac{1}{2} = \sin \theta$$

$$\frac{-1}{3} = \sin \theta$$

$$\theta = 30^\circ \quad \text{or} \quad \theta \approx 199^\circ 28'$$

Reject $199^\circ 28'$, as θ is in a right triangle,
so $\theta = 30^\circ$

❶ θ correct

Need to solve 2 eqns (a negative angle was not acceptable – need the reflex angle).

Need to give both solutions and reject the invalid one with correct reasoning given.