NAME:____________________

WESTERN REGION

2000

MATHEMATICS

3 UNIT (Additional)
and
3/4 UNIT (Common)

TRIAL HSC EXAMINATION

TIME ALLOWED: 2 hours plus 5 minutes reading time

DIRECTIONS TO CANDIDATES

* Attempt ALL questions.
* ALL questions are of equal value.
* All necessary working should be shown in every question.
* Marks may be deducted for careless or badly arranged work.
* Start each question on a new page.
* Approved calculators may be used.
* Standard Integrals are supplied at the end of this examination paper.

QUESTION	MARK
1	
2	
3	
4	
5	
6	
7	
TOTAL	

QUESTION 1. ***Start a new page.***

Marks

(a) Simplify $\dfrac{1+a^{-1}}{1+a^{-3}}$. **2**

(b) If $y = \sec x$ **4**

(i) prove $\dfrac{dy}{dx} = \sec x \tan x$

(ii) find $\dfrac{d^2y}{dx^2}$

(c) Find $\displaystyle\lim_{x\to 0} \frac{1-\cos 2x}{x^2}$ **2**

(d) Use the substitution $u = 1 + x^3$ to evaluate $\displaystyle\int_0^1 x^2\left(1+x^3\right)^3 dx$ **2**

(e) Find the exact value of $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{1-\cos x}\, dx$ **2**

QUESTION 2. *Start a new page.* **Marks**

(a) If $\frac{dy}{dx} = 1 + y$, and when $x = 0,\ y = 2$; show that $y = 3e^{x} - 1$ 2

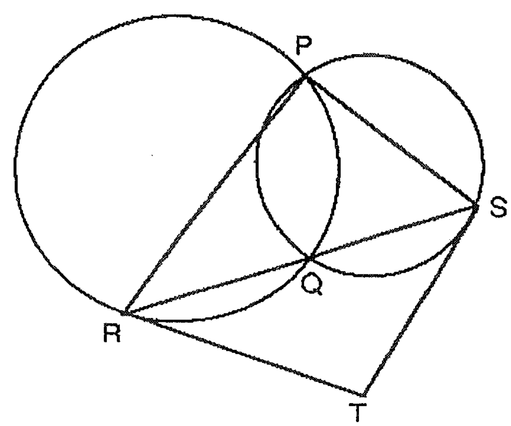
(Hint examine $\frac{dx}{dy}$ *)*

(b) In the expansion of $\left(t^{3} + \frac{1}{t}\right)^{7}$, does the expression contain a constant term? Justify your answer. 2

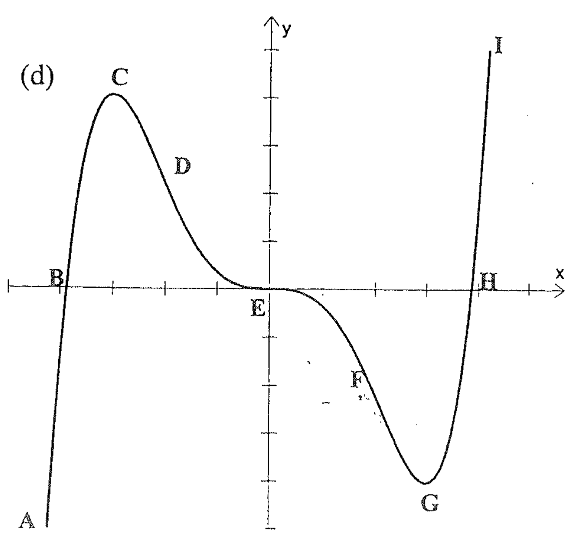
(c) The circles intersect at P and Q 3

RQS is a straight line. TR and TS are tangents.

Prove that $TSPR$ is a cyclic quadrilateral



(d)



The graph of $y = f(x)$ is shown. 5

i. Is the function *odd* , *even* or *neither* ? Justify for your answer.

ii. Using the points indicated on the graph, state between which points ;

a. $f(x)$ is decreasing and the curve is concave up.

b. $f(x) < 0$ and $f''(x) < 0$

iii. Sketch the graph of $y = f'(x)$.

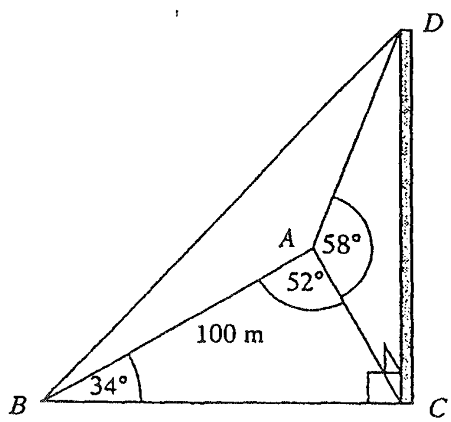
Label the corresponding points A to I on your graph

QUESTION 3. *Start a new page.* **Marks**

(a) (i) In how many ways can the letters of the word **MONSTERS** be arranged if the S, S and **T** occur together. **4**

(ii) A jury of seven is to be formed from 6 males and 4 females. If the jury is chosen at random, find the ***probability*** that it will contain a majority of females.

(b) **3**



A pole is seen from the two points A and B. The angle of elevation from A is 58°. If $\angle CAB = 52^\circ$ and $\angle ABC = 34^\circ$, and A and B are **100m** apart, find;

(i) how far A is from the foot of the pole, to the nearest metre.

(ii) the height of the pole, correct to 1 decimal place.

(c) Given the function $f(x) = 1 - \tan x$ for the domain $0 \le x \le \frac{\pi}{4}$ **5**

(i) Sketch the graph of $y = f(x)$

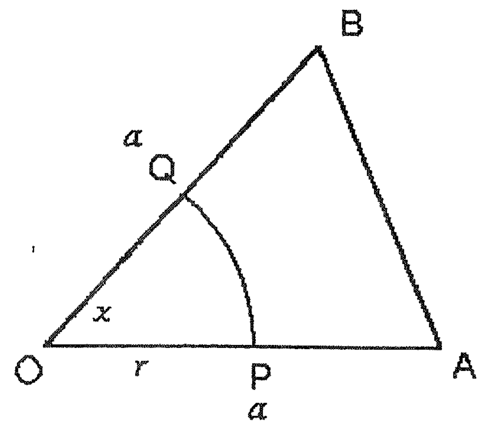
(ii) Prove that the area of the region enclosed by the graph of $y = f(x)$ and the coordinate axes is

$$\frac{\pi - \ln 4}{4} \quad \text{units}^2$$

(iii) The region in (ii) makes a revolution about the x - axis. Find the volume of the solid so formed.

QUESTION 4. *Start a new page.* **Marks**

(a) 3



In ΔOAB , $OA = OB = a$ which is a constant.
$\angle AOB = x$ radians, where x is the variable. PQ is a circular arc, centre O and radius r.

If the area of ΔOAB is twice that of the sector OPQ ,

(i) express r^2 in terms of a and x

(ii) find r when $\angle AOB$ is a right angle, in terms of π and a .

(b) The polynomial $P(x) = 6x^3 - 7x^2 + ax + b$ has a zero at $x = -1$ and the remaining zeros are reciprocals $(\alpha, \frac{1}{\alpha})$ 5

(i) by examining the product of the three roots determine the value of b and hence of a.

(ii) find all the zeros of $P(x)$

(c) Prove by Mathematical Induction, that for all positive integers n 4

$$\sum_{r=1}^{n} r(r+1) = \frac{n(n+1)(n+2)}{3}$$

QUESTION 5. *Start a new page.* **Marks**

(a) If the two lines $y = m_1x + c_1$ and $y = m_2x + c_2$ meet at an angle of 45°, prove that ; **3**

$$m_1m_2 = m_1 - m_2 - 1$$

or

$$m_1m_2 = m_2 - m_1 - 1$$

(b) A metal cube has sides of x cm and volume V cm³. **5**
The cube is cooling so that the length of its sides are *decreasing* at a rate of 0.075 cm/min.

(i) Write an expression for this rate of change.

Find the rate of change in its volume, when

(ii) the sides are 4 cm long.

(iii) the total surface area is 100 cm²

(c) A sky-diver opens his parachute when falling at 30 ms^{-1}. **4**
Thereafter his acceleration is given by

$$\frac{dv}{dt} = k(6 - v) \qquad \text{where } k \text{ is a constant.}$$

(i) Show that this condition is satisfied when $v = 6 + Ae^{-kt}$ and find the value of A.

(ii) One second after opening his chute, his velocity has fallen to 10.7 ms^{-1}. Find the value of k correct to two decimal places.

(iii) Find his velocity, correct to one decimal place, 2 seconds after his chute is opened.

QUESTION 6. *Start a new page.* **Marks**

(a) The equation $\sin x = x^2 - 10$ has a root close to $x = \pi$. **2**

Use one application of *Newton's Method* to give a better approximation, correct to 4 decimal places.

(b) A particle is x cm from an origin on a line after t seconds, where **6**

$$x = a\cos nt + b\sin nt$$

(i) Prove that, at position x, its acceleration is $-n^2x$ ms^{-2}. What does this prove about the nature of the motion?

(ii) If, at position x, its velocity is v ms^{-1}, prove that $v^2 + n^2x^2$ remains constant throughout the motion.

(iii) What is the amplitude of the motion?

(c) By equating the coefficients of x^n in the identical expressions **4**

$$(1+x)^{2n} \quad \text{and} \quad (1+x)^n(1+x)^n$$

prove that

$$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \ldots\ldots\ldots\ldots + \binom{n}{n}^2$$

QUESTION 7. *Start a new page.* **Marks**

(a) A projectile is fired horizontally with speed v ms^{-1} from a point h m above horizontal ground. **5**

(i) Prove that it will reach the ground after $\sqrt{\frac{2h}{g}}$ seconds.

(ii) If it does so at an angle of 60° to the horizontal, prove that

$$3v^2 = 2gh$$

$$hint\ \frac{dy}{dx} = \tan 120^0$$

(b) Given $y = \log_e(e^x \sin^2 x)$ **4**

(i) Show $\frac{dy}{dx} = 1 + 2\cot x$

(ii) Prove that the equation of the normal at $x = \frac{\pi}{2}$ is given by $x + y = \pi$

(c) Let $y = \sin^{-1}x$. **3**

(i) Show $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

(ii) Hence evaluate $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}dx$