Student Name

Student Number

BURWOOD GIRLS HIGH SCHOOL

2008
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

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Marks

Begin each question on a SEPARATE sheet of paper. Extra paper is available.

Question 1 (12 marks) Use a SEPARATE sheet of paper or booklet.

a) Evaluate $\left(\frac{1}{e^{2.5}}-1\right)^2$ correct to 3 significant figures. **2**

b) Solve $|2x-4| \leq 2$ **2**

c) If $\frac{4}{2-\sqrt{3}} = a + b\sqrt{3}$ find the values of a and b. **2**

d) Find the sum of the first ten terms of the series $4\frac{1}{2} + 3 + 1\frac{1}{2} + \ldots$ **2**

e) Factorise $2z^2 + 6zy + xz + 3xy$ **2**

f) Find the perpendicular distance from the point (1, 3) to the line $6x - 8y + 5 = 0$ **2**

End of Question 1

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) Differentiate with respect to x

(i) $2x^3 + x^{-3}$ **2**

(ii) $\dfrac{1}{e^{2x}} - \sin x$ **2**

b) (i) Find $\int \sec^2 x - e^{4x} \; dx$ **2**

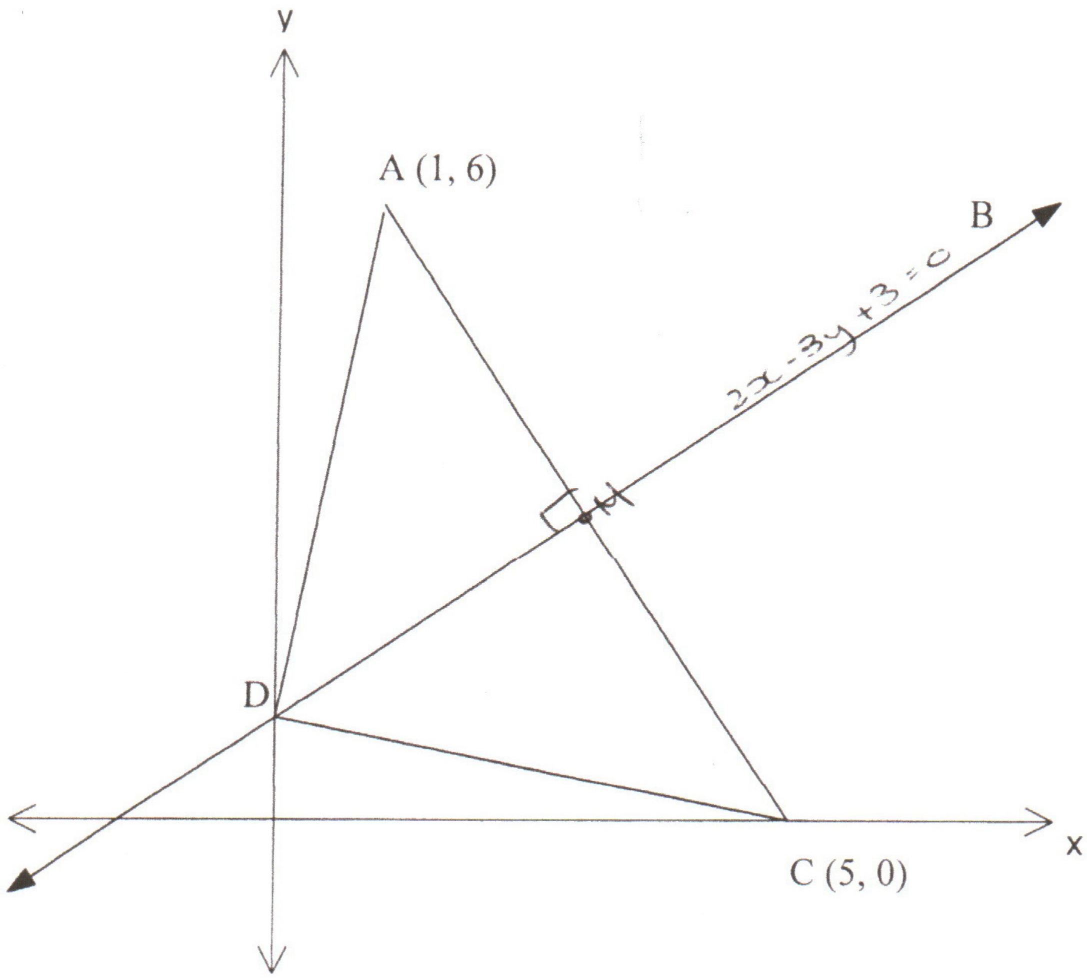
(ii) Evaluate $\int_1^e x^2 + \dfrac{2}{x} dx$ **3**

c) Find the area enclosed by the curve $y = 2\cos 3x$, the line $x = \dfrac{\pi}{12}$ and the x and y axes. **3**

End of Question 2

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

a)



The points A and C have coordinates (1, 6) and (5, 0) respectively.
The line BD has an equation of $2x - 3y + 3 = 0$ and meets the y axis in D.

i) The point M is the midpoint of AC. Show that M has coordinates (3, 3). **1**

ii) Show that M lies on BD. **1**

iii) Find the gradient of the line AC. **1**

iv) Show that BD is perpendicular to AC. **2**

v) Find the distance AC. **1**

vi) Explain why the quadrilateral ABCD is a kite regardless of the position of B. **1**

Question 3 continues on page 5

Question 3 continued **Marks**

b) A pile driver is hitting the top of a large pole. The first hit drives the pole 60 cm into the ground. The second hit drives the pole another $60 \times 0.75 = 45$ cm into the ground. The additional distance the pile goes into the ground with each drive is 75% of the previous distance.

i) How far will the pole be driven into the ground on the 6^{th} drive? **2**

ii) Determine the total distance the pile will be driven on the 6^{th} drive. **2**

iii) Calculate the maximum distance the driver can drive the pole into the ground. **1**

End of Question 3

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

a) Show that: **2**

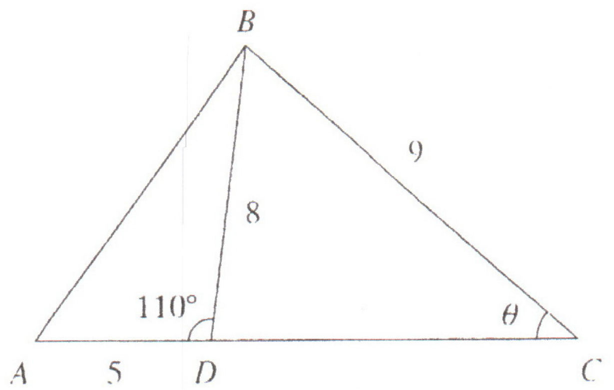
$$\sqrt{\frac{\operatorname{cosec}^2 x - \cot^2 x - \cos^2 x}{\cos^2 x}} = \tan x$$

b) Two dice are painted so that the first has four blue and two red faces and the second has one blue and five red faces. The two dice are rolled together.

i) What is the probability that the dice both show blue faces uppermost? **1**

ii) What is the probability that different colours show uppermost? **2**

c)



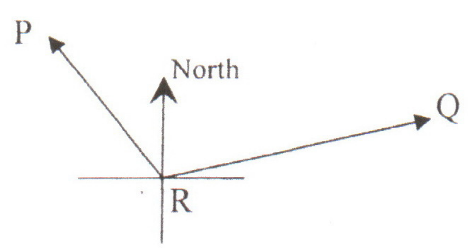
NOT TO SCALE

In the diagram AD=5 cm, BC=9 cm, DB=8 cm and $\angle ADB = 110°$.

i) Calculate the size of $\angle DCB\ (\theta)$ correct to the nearest minute. **2**

ii) Calculate the area of ΔABD in cm^2 correct to one decimal place. **1**

d) Peta and Quentin are pilots of two light planes which leave Resthaven station at the same time. Peta flies on a bearing of 330° at a speed of 180 km/h and Quentin flies on a bearing of 080° at a speed of 240 km/h. Copy the diagram below onto your answer page and mark the information on the diagram.



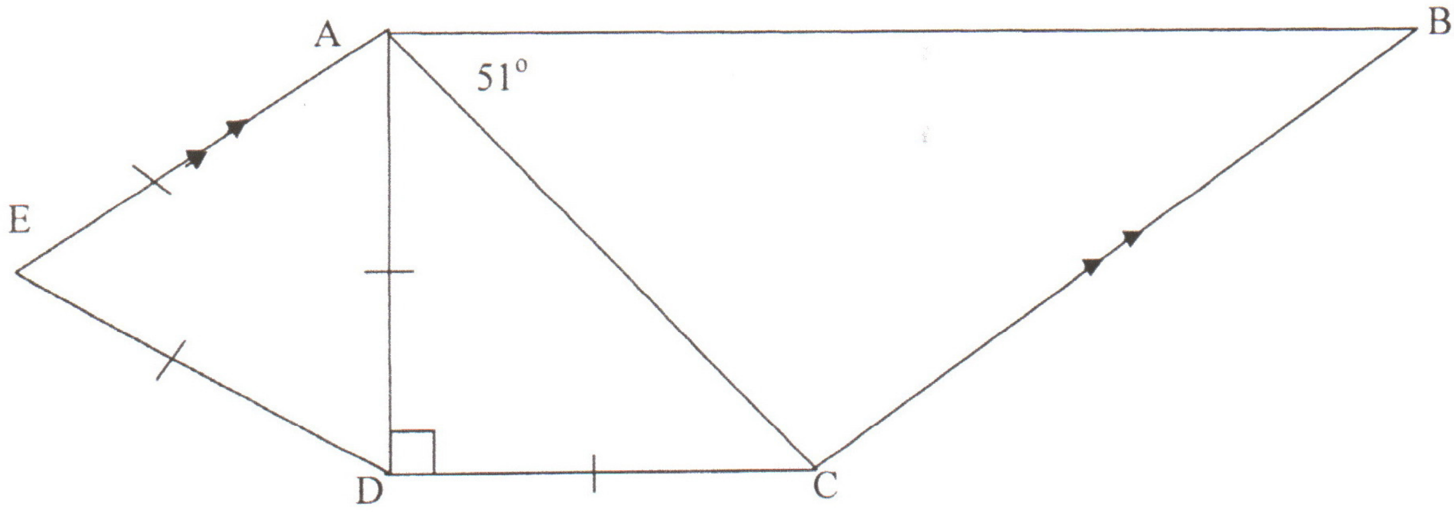
i) How far apart are Peta and Quentin after 2 hours? **2**

ii) What is the bearing of Quentin from Peta after 2 hours. **2**

End of Question 4

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) In the diagram below AE = ED = AD = DC, $\angle ADC = 90^\circ$ and AE || BC.
$\angle BAC = 51^\circ$



i) Find the size of $\angle EAB$. Give reasons for your answer. **3**

ii) Find the size of $\angle ABC$. Give reasons for your answer. **1**

b) A particle moves in a straight line so that its displacement, in metres, is given by

$x = \dfrac{4t^2 + t + 8}{4t + 1}$ where t is measured in seconds.

i) Find the initial position of the particle. **1**

ii) Find an expression for the velocity of the particle. **1**

iii) Show that the particle is stationary when $t = \dfrac{-1 + 4\sqrt{2}}{4}$ seconds. **2**

iv) Describe the motion of the particle in the first two seconds. **2**

c) One root of the quadratic equation $4x^2 - 24x + k = 0$ is twice the other root. Determine the value k. **2**

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) For the function $y = x^6 - 6x^4$

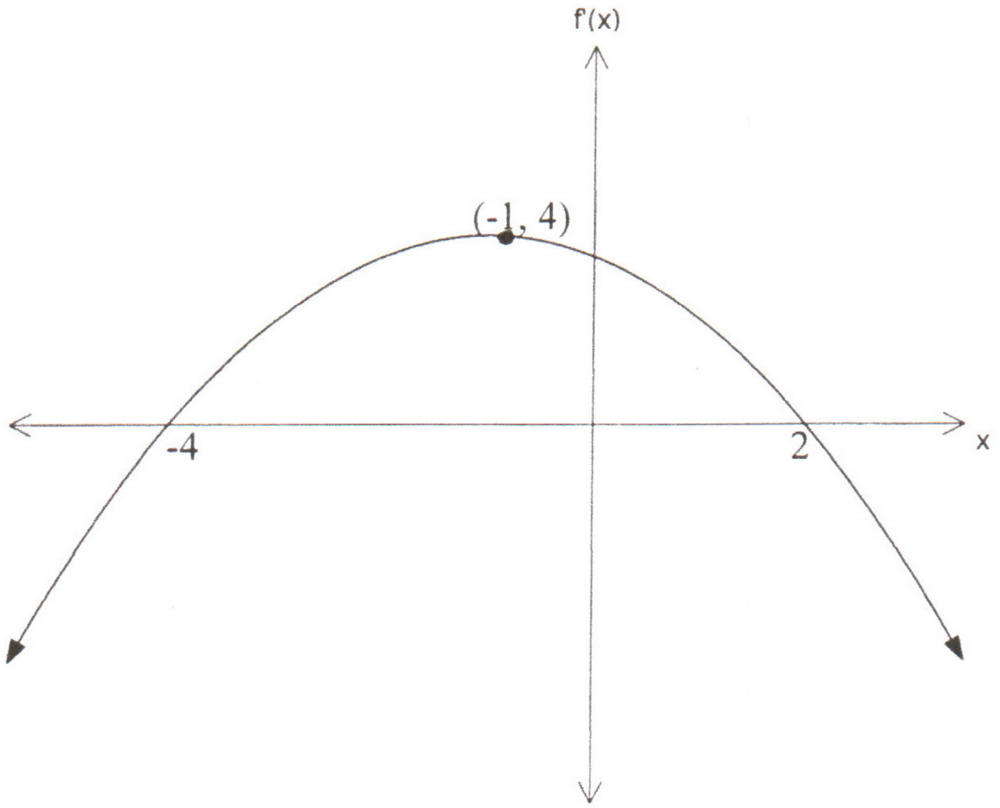
i) Find the x coordinates of the points where the curve crosses the axes. **2**

ii) Find the coordinates of the stationary points and determine their nature. **4**

iii) Find the coordinates of the points of inflexion. **2**

iv) Sketch the graph of $y = x^6 - 6x^4$ indicating clearly the intercepts, stationary points and points of inflexion. **2**

b) For a certain function $y = f(x)$, the sketch of $y = f'(x)$ is shown.



Give the x coordinates of the stationary points on $y = f(x)$ and indicate if they are maxima or minima. **2**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) i) Find the co-ordinates of the focus of the parabola $y = x^2 - 2$. **3**

ii) Determine the equation of the directrix of the parabola $y = x^2 - 2$. **1**

iii) Determine the equation of the normal to the parabola $y = x^2 - 2$ at the point on the parabola where $x = 3$. **3**

b) i) Show that the curves $y = x^2 - 3x$ and $y = 5x - x^2$ intersect at the points (0, 0) and (4, 4). **2**

ii) Find the area enclosed between the two curves. **3**

End of Question 7

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) A city has a population which is growing at a rate that is proportional to the current population. The population at time t years is given by

$$P = Ae^{kt}$$

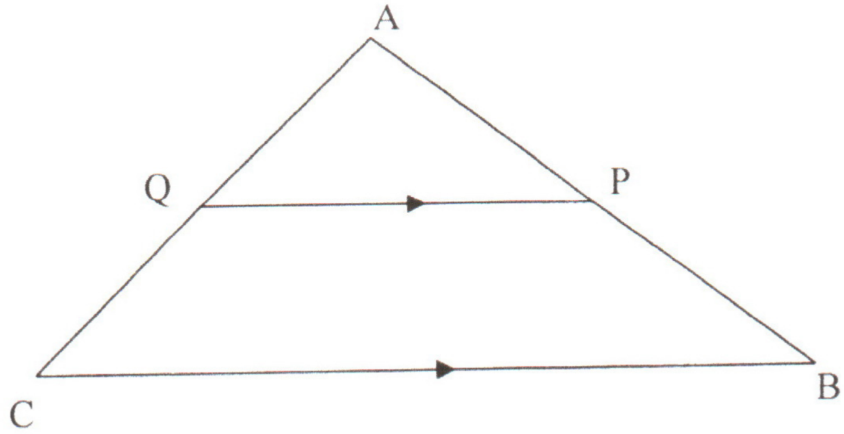
i) Show that $P = Ae^{kt}$ satisfies the equation $\frac{dP}{dt} = kP$. **1**

ii) If the population at the start of 2006 when $t = 1$ was 147 200 and at the start of 2007 when $t = 2$ was 154 800, find the values of A and k. **2**

iii) Find the population at the start of 2009. **1**

iv) Find during which year the population will first exceed 200 000. **1**

b) In the diagram below, P is the midpoint of the side AB of the ΔABC. PQ is drawn parallel to BC.



i) Prove that $\Delta ABC \,|||\, \Delta APQ$. **2**

ii) Explain why Q is the midpoint of AC. **2**

c) Find an approximation for $\int_1^3 g(x)\,dx$ by using Simpson's Rule with the values in the table below. **2**

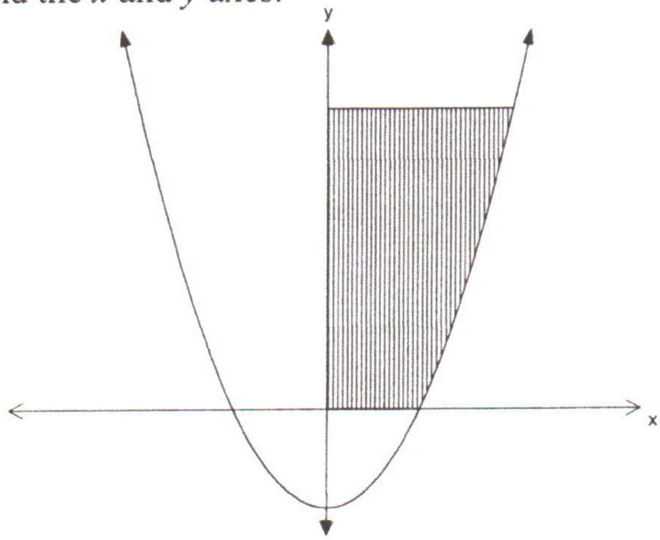
x	1	1.5	2	2.5	3
$g(x)$	12	8	0	3	5

d) Evaluate $\sum_{n=2}^{5} n^2 - 1$ **1**

End of Question 8

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) The diagram shows the region bounded by the curve $y = 2x^2 - 2$ the line $y = 6$ and the x and y axes. 3



Find the volume of the solid of revolution formed when the region is rotated about the y axis.

b) Paul plays computer games competitively. From past experience, Paul has a 0.8 chance of winning a game of *Beastie* and a 0.6 chance of winning a game of *Dragonfire*. In one afternoon of competition he plays two games of *Beastie* and one of *Dragonfire*.

i) What is the probability that he will win all three games? 1

ii) What is the probability that he will win no games? 1

iii) What is the probability that he will win at least one game? 1

c) A car dealership has a car for sale for a cash price of \$20 000. It can also be bought on terms over three years. The first six months are interest free and after that interest is charged at the rate of 1% per month on that months balance. Repayments are to be made in equal monthly instalments beginning at the end of the first month.

A customer buys the car on these terms and agrees to monthly repayments of \$$M$. Let \$$A_n$ be the amount owing at the end of the nth month.

i) Find an expression for A_6. 1

ii) Show that $A_8 = (20\,000 - 6M)1{\cdot}01^2 - M(1 + 1{\cdot}01)$ 1

iii) Find an expression for A_{36}. 2

iv) Find the value of M. 2

End of Question 9

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) A plant nursery has a watering system which repeatedly fills a storage tank then empties it's contents to water different sections of the nursery.
The volume of water (in cubic metres) in the tank at a time t is given by the equation

$V = 2 - \sqrt{3}\cos t - \sin t$ where t is measured in minutes.

i) Give an equation for $\frac{dV}{dt}$, the rate of change of the volume at a time t. **1**

ii) Is the tank initially filling or emptying? **1**

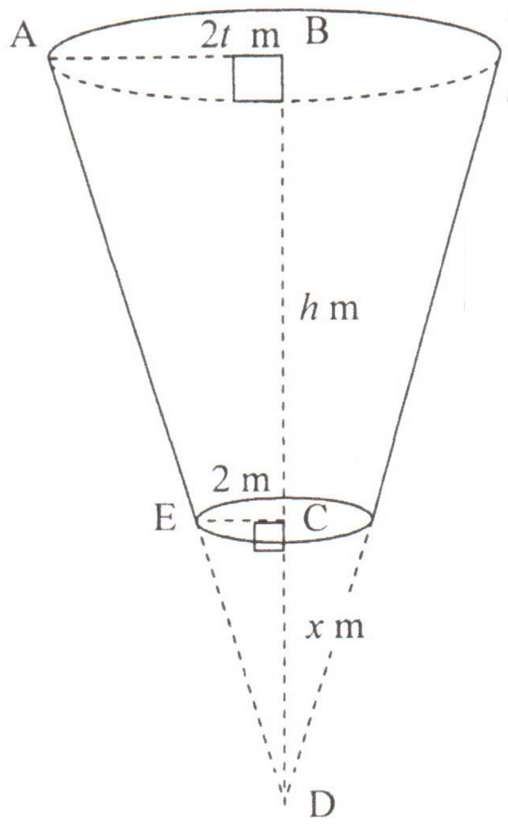
iii) At what time does the tank first become completely full and what is it's capacity when full? **3**

Question 10 continues on page 13

Question 10 continued

Marks

b) A truncated cone is to be used as a part of a hopper for a grain harvester. It has a total height of h metres. The top radius is to be t times greater than the bottom radius which is 2 metres.



AB = $2t$ metres
BC = h metres
EC = 2 metres
CD = x metres

i) If x is the height of the removed section of the original cone, show using similar triangles that $x = \dfrac{h}{t-1}$ **2**

ii) Show that the volume of the truncated cone is given by **2**

$$V = \left(\frac{4\pi h}{3}\right)\left(t^2 + t + 1\right)$$

iii) If the upper radius plus the lower radius plus the height of the truncated cone must total 12 metres, calculate the maximum volume of the hopper. **3**

End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$