

Independent

2007
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Board approved calculators may be used Write using black or blue pen
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in every question
- Write your student number and/or name at the top of every page

Total marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME:

Question 1 (12 marks)

- (a) Evaluate $\frac{2.6^2 - 3.9^3}{2 \times 2.6 \times 3.9}$ correct to 4 significant figures. 2
- (b) Factorise: $1 + 8x^3$ 2
- (c) Find the values of x for which $|2x - 1| \leq 3$ 2
- (d) Differentiate $4x - \cos 3x$, with respect to x . 2
- (e) Solve: $\frac{3x-2}{3} - \frac{2-x}{2} = 1$. 2
- (f) In a group of 50 students, 12 study Geography, 9 study History and 6 of these students study both Geography and History. What is the probability that a student selected at random from the group, studies neither History nor Geography? 2

Question 2 (12 marks)*Start a new page*(a) Differentiate with respect to x .

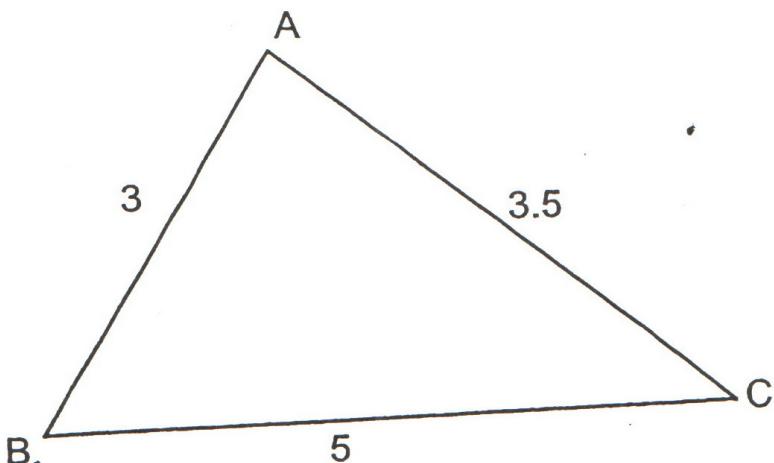
(i) $\frac{x}{\ln x}$.

2

(ii) $(1 + \tan x)^5$.

2

(b)

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In the diagram above $AB = 3\text{m}$, $BC = 5\text{m}$ and $AC = 3.5\text{m}$. Find the size of the largest angle, correct to the nearest minute.

3

(c) (i) Evaluate: $\int_0^2 \frac{6x^2}{1+x^3} dx$.

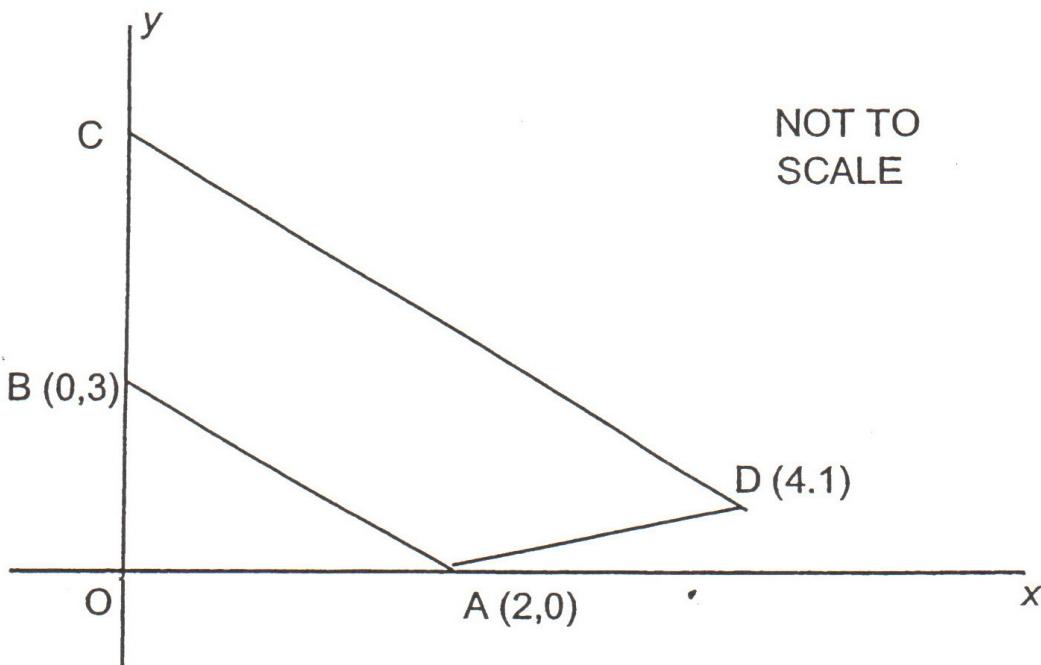
3

(ii) Find: $\int (1+e^{3x}) dx$.

2

Question 3 (12 marks)*Start a new page*

(a)



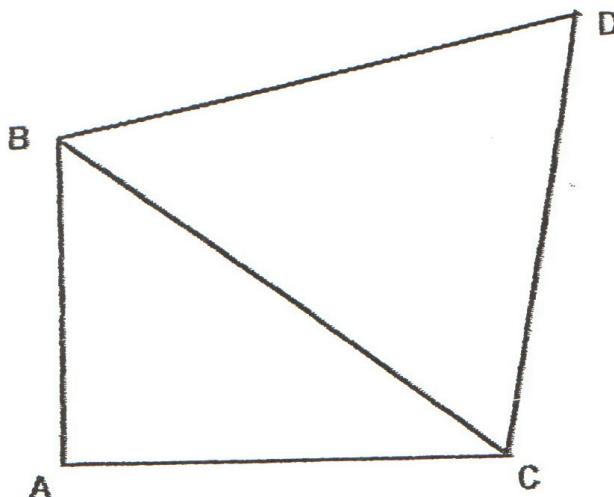
In the diagram above, the coordinates of A, B and D are (2,0), (0,3) and (4,1) respectively. Point C lies on the y-axis such that AB is parallel to DC.
Copy or trace the diagram onto your worksheet.

- (i) What type of quadrilateral is ABCD? 1
- (ii) Write down the gradient of AB. 1
- (iii) Show that the equation of DC is $3x + 2y - 14 = 0$. 2
- (iv) Find the coordinates of point C. 1
- (v) Using Pythagoras' Theorem or otherwise, show that the length of AB = $\sqrt{13}$ units. 1
- (vi) Find the length of CD. 1
- (vii) Find the perpendicular distance from A to DC. 1
- (viii) Hence or otherwise, find the area of quadrilateral ABDC. 1

- (b) Find the equation of the tangent to the curve $y = e^{2x} + x$ at the point with x-coordinate 0. 3

Question 4 (12 marks)*Start a new page*

(a)

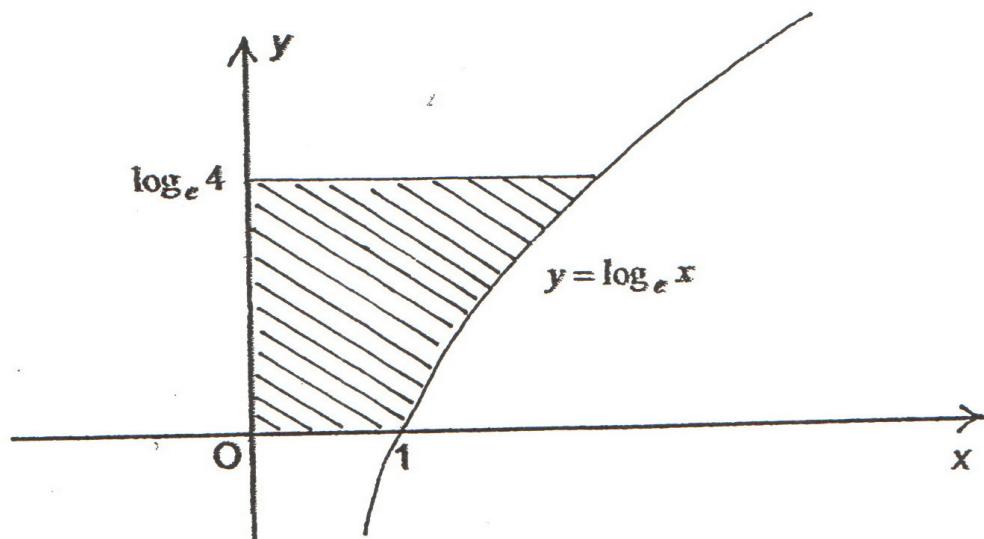


In the diagram above, ABC is an isosceles triangle in which $\angle BAC = 90^\circ$.
BCD is an equilateral triangle.

Copy or trace the diagram onto your worksheet.

- (i) Find the size of $\angle ACD$ giving reasons. 2
- (ii) If $BC = 3$ centimetres, find the perimeter of ABDC, giving reasons. 2

- (b) In the diagram below, the shaded region bounded by the curve $y = \log_e x$, the x and y -axes and the line $y = \log_e 4$ is rotated about the y -axis. Find the exact volume of the solid of revolution formed. 3

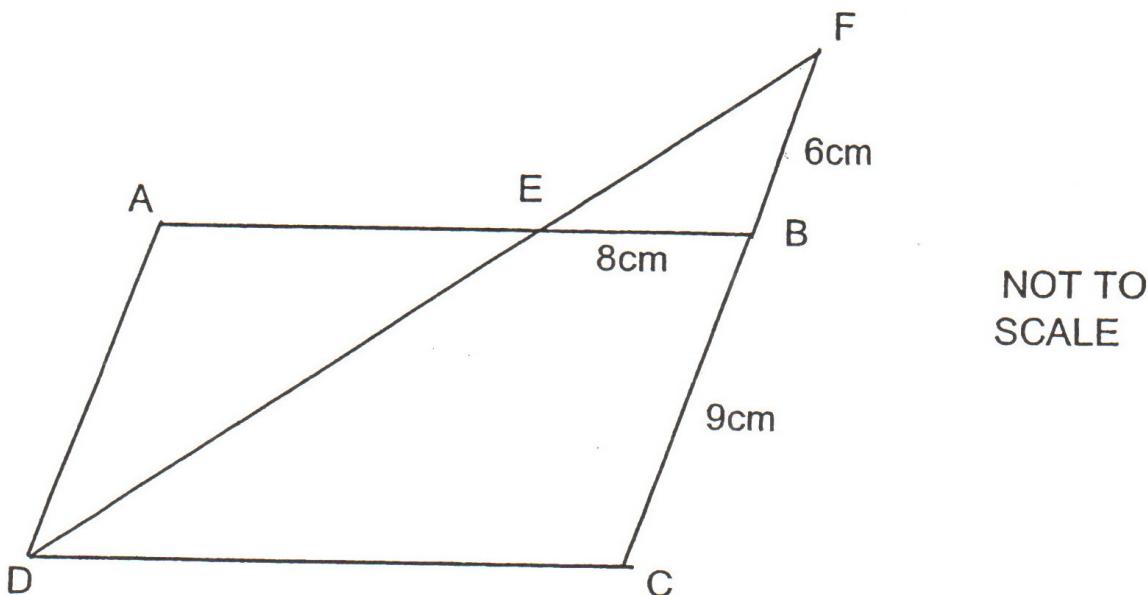


- (c) The geometric series $1 - x + x^2 - \dots$ has a limiting sum of 4.
Find the value of x . 3

- (d) Find the coordinates of the focus of the parabola $6y = x^2 - 2x - 11$. 2

Question 5 (12 marks)*Start a new page*

(a)



In the diagram above ABCD is a parallelogram. F lies on CB produced such that $BF = 6 \text{ cm}$. AB and DF intersect at E. $EB = 8 \text{ cm}$ and $BC = 9 \text{ cm}$. Copy or trace the diagram onto your worksheet.

- (i) Prove that $\triangle AED$ is similar to $\triangle BEF$.

3

- (ii) Find the lengths of AE and DC.

2

(b)

Farmer Brown has hired a driller to drill a borehole to enable her to have access to the underground water on her property. The driller quotes a price of \$260 for the first 3 metres drilled, \$280 for the next 2 metres, \$300 for the next 2 metres and so on. The price increases by the same amount for each successive 2 metres of borehole drilled.

- (i) Show that the cost of drilling the portion from a depth of 25 metres to 27 metres is \$500.

1

- (ii) Calculate the total cost of drilling to a depth of 27 metres.

1

- (iii) The cost of drilling the borehole to reach water was \$12500. Find the total depth drilled to give access to the water.

2

(c)

- Use Simpson's Rule with five function values to find an approximation for the value of $\int_0^1 10^x dx$. Give your answer correct to three decimal places.

3

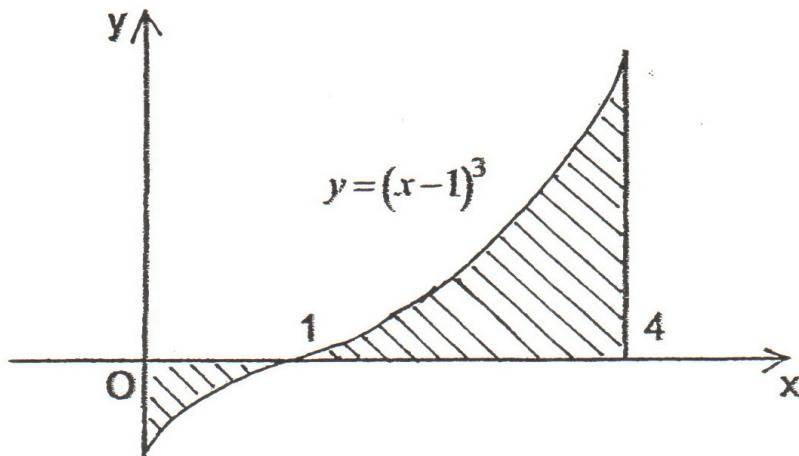
Question 6 (12 marks)*Start a new page*

- (a) A function $f(x)$ is defined by $f(x) = x(x^2 - 3x - 9)$.
- (i) Find the turning points for the curve $y = f(x)$ and determine their nature. 3
- (ii) Find the coordinates of the point of inflexion. 1
- (iii) Sketch the graph of $y = f(x)$ showing the turning points and point of inflexion. 2
- (vi) Find the values of x for which both $f'(x) < 0$ and $f''(x) > 0$. 2
- (b) The quadratic equation $2x^2 - 3x + 6 = 0$ has roots α and β . Find the value of:
- (i) $\alpha + \beta$ 1
- (ii) $\alpha\beta$ 1
- (iii) $\alpha^2 + \beta^2$ 2

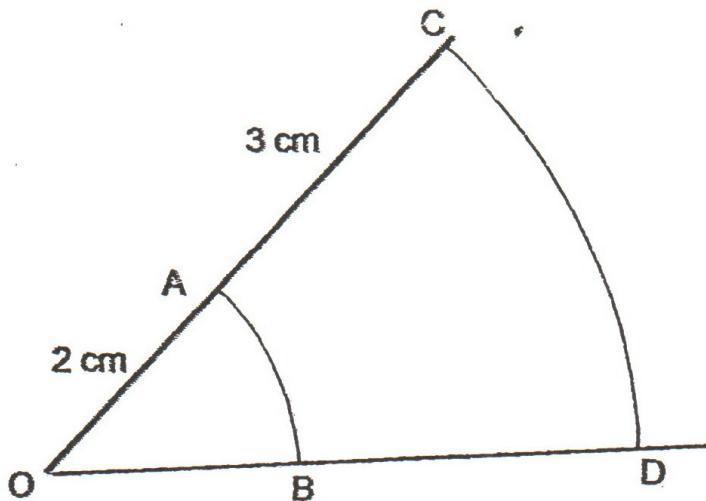
Question 7 (12 marks)*Start a new page*

- (a) The shaded area in the diagram below is the region bounded by the curve $y = (x-1)^3$, the x and y -axes and the line $x = 4$. Calculate the shaded area.

4



(b)



The arcs AB and CD are parts of concentric circles with centre O .
 $OA = 2$ centimetres and $AC = 3$ centimetres.

- (i) Show that the ratio of length of arc AB : length of arc $CD = 2 : 5$.

1

- (ii) Find the ratio of the area of sector AOB : the area of $ABDC$.

2

- (c) The probability that the school bus runs late on any particular day is 1 in 8 . Find the probability that on three successive days, the bus is:

- (i) late on all three days.

1

- (ii) late on exactly two days.

1

- (iii) late on at least two days.

2

- (iv) on time on all three days.

1

Marks**Question 8 (12 marks)***Start a new page*

- (a) A particle is moving in a straight line and its velocity v metres/second at time t seconds is given by;

$$v = \frac{dx}{dt} = 1 - 2\sin 2t, t \geq 0$$

Initially the particle is at the origin.

- (i) Express the displacement x , as a function of t . 2

- (ii) Find the position of the particle when $t = \frac{\pi}{6}$. 1

- (iii) Find an expression for the acceleration $a = \frac{d^2x}{dt^2}$. 1

- (iv) Sketch the graph of the acceleration as a function of time, $0 \leq t \leq \pi$. 2

- (v) What is the maximum acceleration of the particle? 1

- (b) For what values of k does the quadratic equation $x^2 - (k+3)x + 4k = 0$

- (i) have one root equal to -3 ? 1

- (iii) have no real roots? 2

- (c) Solve: $2\cos A + \sqrt{3} = 0, 0 \leq A \leq 2\pi$. 2

Question 9 (12 marks)*Start a new page*

(a) Evaluate: $\sum_{n=2}^4 3^{-n}$ (Give your answer in exact form). 1

- (b) Pauline wishes to invest in a superannuation fund. She decides to invest \$2400 in the fund at the beginning of each year. The fund is paying interest at 9% per annum, compounded annually.

- (i) Show that the value of the first \$2400 invested when she retires after working for 30 years will be $\$2400(1.09)^{30}$ 1
- (ii) Write down similar expressions for the values of the second and third \$2400 amounts invested, at the end of the thirty year period. 2
- (iii) Calculate the total value of her investment when she reaches retirement. 2

- (c) An industrial plant produces vacuum cleaners. The annual production, P cleaners, at time t years, is given by:

$$P = P_0 e^{kt} \text{ where } P_0 \text{ and } k \text{ are constants.}$$

Initially the production of the plant was 2500 cleaners per annum. Five years later it had increased to 4000 cleaners per annum.

- (i) Find the values of P_0 and k . 2
- (ii) What is the predicted production after 10 years? 1
- (iv) How many years will it take for the production to double its original output? 2
- (v) Find the rate of increase in production when the plant has been operating for 5 years. 1

Question 10 (12 marks)*Start a new page*

- (a) Simplify: $\frac{\cosec A \sec A}{\tan A}$, expressing your answer in simplest possible form. 3

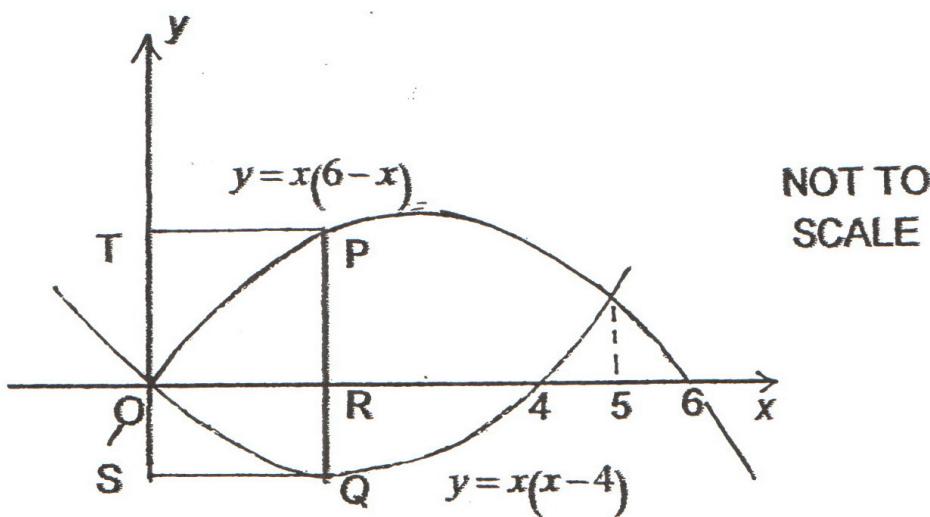
- (b) The rate of fall in the price $\$D$, of shares in a company is given by:

$$\frac{dD}{dt} = -\frac{16}{t^3},$$

where $t > 0$ is the time in months after the shares were put on the market.

One month after going on the market, the shares were selling for \$24 each

- (i) Show that the price of the shares is given by: $D = \frac{8}{t^2} + 16$. 2
- (ii) Find the value of the shares after they have been on the market for 2 months. 1
- (iii) Find the rate at which the share price is falling after 3 months. 1
- (iv) Show that the price of the shares will not fall below a certain amount. Give this amount. 1
- (c) In the diagram below, P is a point on the curve $y = x(6-x)$ and Q is a point on the curve $y = x(x-4)$. PQ cuts the x-axis at right angles at R. S and T are points on the y-axis such that PQST is a rectangle. 1



- (i) Show that the length of PQ is given by $10x - 2x^2$. 1
- (ii) Find an expression for the area of PQST as a function of x . 1
- (iii) Find the value of x which gives the maximum area for PQST ($0 \leq x \leq 5$) 2