N.S.W. DEPARTMENT OF EDUCATION

HIGHER SCHOOL CERTIFICATE EXAMINATION 1983

MATHEMATICS - 3 UNIT/4 UNIT COMMON PAPER

Time 2 hours

Instructions: All questions may be attempted. All questions are of equal value. In every question, all necessary working should be shown. Marks may not be awarded for careless or badly arranged work. Mathematical tables will be supplied; approved slide-rules or calculators may be used.

QUESTION 1

(i) (a) Given that $y = x \sin^{-1}x$, find $\frac{d^2y}{dx^2}$. (b) Given that $\int_0^1 \frac{dx}{x^2 + 3} = a\pi$, find a.

(ii) Express $\sin\theta$ and $\cos\theta$ in terms of $t = \tan(\theta/2)$.

Hence, or otherwise, prove that, for $0 < \theta < \frac{1}{2}\pi$, $\frac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = \tan(\theta/2)$.

QUESTION 2

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(i) Sketch and describe the following surfaces or regions for $0 \le z \le 4$:

(a) $x^2 + y^2 \le 9$, (b) $x^2 + y^2 = 9z$.

(ii) O, P and Q are the points $(0,0,0)$, $(5,-6,5)$ and $(2,0,8)$ respectively.

(a) Write down equations for the line PQ in parametric form.

(b) Hence find the co-ordinates of R, the point of intersection of PQ with the plane $4x - 3y - 2z = 4$.

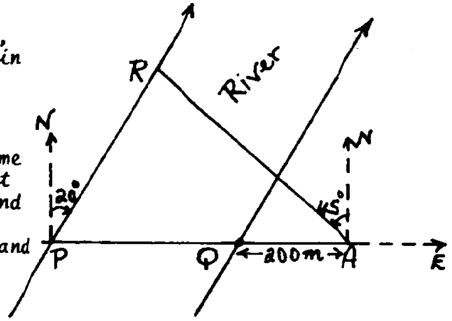
(c) Show that $OR \perp PQ$.

QUESTION 3

(i) Find, to the nearest degree, all solutions of $3 \cos 2A - \sin A + 2 = 0$, such that $0^0 < A < 360^0$.

(ii) In the figure (not drawn to scale), a river with parallel banks flows in a direction N 20° E, and P,R,Q are points on the banks such that Q is due East of P.

A surveyor standing at A at the same horizontal level as Q, and due east of Q, observes PQ to be in line, and that R bears N 45^0W. He measures the distance QA to be 200 metres, and finds that the bearing of R from Q is N 28^0W. Find the distance PQ.



QUESTION 4

(i) Given that $f(x) = ax^3 + bx^2 + cx + d$ is a function with a double zero at $x = 1$, and with a minimum value of -4 when $x = -1$, find the values of a,b,c and d.

(ii) The nth term of a series is given by $u_n = \frac{1}{(2n - 1)(2n + 1)}$.

(a) Find u_5 and u_{k+1}.

(b) Assuming that the sum S_k of the first k terms of this series is given by the formula $S_k = \frac{k}{2k + 1}$, prove that $S_{k+1} = \frac{k + 1}{2k + 3}$.

(c) Explain why the sum of the first n terms of the series is $n/(2n + 1)$.

QUESTION 5

A steady wind is blowing with speed 36 km per hour. From clouds moving horizontally with the wind, heavy raindrops fall to the ground 200 metres below.

(a) Find the time taken for a drop to reach the ground.

(b) Find the speed and angle at which a drop hits the ground (assumed horizontal).

(c) At what angle does a drop hit the ground when the wind speed is doubled?

(Air resistance may be neglected, and the approximate value $g = 10$ m s^{-2} may be assumed.)

QUESTION 6

(i) In the expansion of $(x^2 + \frac{2}{x})^{10}$, find the coefficient of x^2.

(ii) Anna plays a game in which she throws alternately with each hand at a mark. She has a probability of 0.7 of hitting the mark with a right-handed throw, and of 0.5 of hitting the mark using the left hand.

A completed game consists of 4 throws with each hand (i.e. a total of 8 throws). Find:

(a) Anna's expected score over a series of games;

(b) Anna's most likely score in one completed game.

(iii) A firm makes a gross profit of \$10,000 in 1980. In every subsequent year the gross profit decreases by 20% of that in the preceding year. Each year there is a tax of 50% on all the gross profit over \$2000 in that year, and all the remaining (net) profit is then distributed to shareholders.

Show that there is a limit to the total net profit, accumulated from 1980, that is distributed to shareholders, and find the limiting profit.

QUESTION 7

The distinct points P,Q correspond respectively to the values $t = t_1$, $t = t_2$ on the parabola $x = 2t$, $y = t^2$.

(i) (a) Write down the equation of the tangent to the parabola at P.

(b) Show that the equation of the chord PQ is $2y - (t_1 + t_2)x + 2t_1t_2 = 0$.

(c) Show that M, the point of intersection of the tangents to the parabola at P and at Q, has co-ordinates $(t_1 + t_2,\ t_1t_2)$.

(ii) Prove that for any value of t_1, except $t_1 = 0$, there are exactly two values of t_2 for which M lies on the parabola $x^2 = -4y$ and find these two values in terms of t_1. Find also the co-ordinates of the corresponding points M.

(iii) Show that, for these values of t_2, the chord PQ is a tangent line to the parabola $x^2 = -4y$.