Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2010
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Afternoon Session
Thursday 12 August 2010

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided as a separate page
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1-7
- All questions are of equal value

Disclaimer
Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, *Principles for Setting HSC Examinations in a Standards-Referenced Framework* (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and *Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework* (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

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Total marks – 84
Attempt Questions 1–7
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) A is the point $(-2, -1)$ and B is the point $(1, 5)$. Find the coordinates of the point Q which divides AB externally in the ratio $5:2$. **2**

(b) Show that $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$. **2**

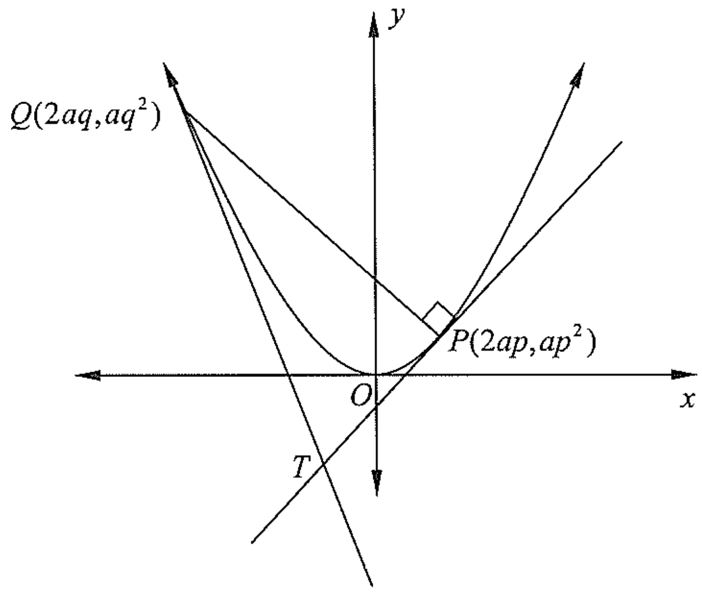
(c) Solve the inequality $\dfrac{2x}{x-1} \geq 1$. **3**

(d) Evaluate $\sin^{-1}\left(\sin \dfrac{7\pi}{6}\right)$ exactly. **2**

(e) Using the substitution $u = \ln 3x$, find $\displaystyle\int \frac{dx}{x(\ln 3x)^2}$. **3**

Question 3 (continued)

(c) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. The equation of the tangent at the point P is $y = px - ap^2$ and the gradient of the chord PQ is $\frac{p+q}{2}$. The point T is the intersection of the tangents at P and Q.



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(i) Show that the coordinates of T are $\left(a(p+q), apq\right)$. **2**

(ii) The chord PQ is also the normal at P. Show that $p + q + \frac{2}{p} = 0$. **2**

(iii) Hence, or otherwise, show that the equation of the locus of T as P moves on the parabola is $y = \frac{-4a^3}{x^2} - 2a$. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) Harry and Bill are in a competition. Harry is the more skilful, having a probability of $\frac{2}{3}$ of winning any game against Bill. **3**

Find the probability that Harry wins 6 games to 4, taking into account that Harry wins the last game.

(b) (i) By considering the graph of $y = \sin^{-1} x$, or otherwise, show that the equation $\sin^{-1} x + x - \frac{\pi}{2} = 0$ has only one real and positive root. **2**

(ii) Taking $x = 0.7$ as the first approximation to this root, use one application of Newtown's method to find another approximation. Give your answer correct to 2 decimal places. **3**

(c) A series is given as $S_n = \tan^2 x - \tan^4 x + \tan^6 x - \ldots$ where $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(i) Find the values of x for which this series has a limiting sum. **2**

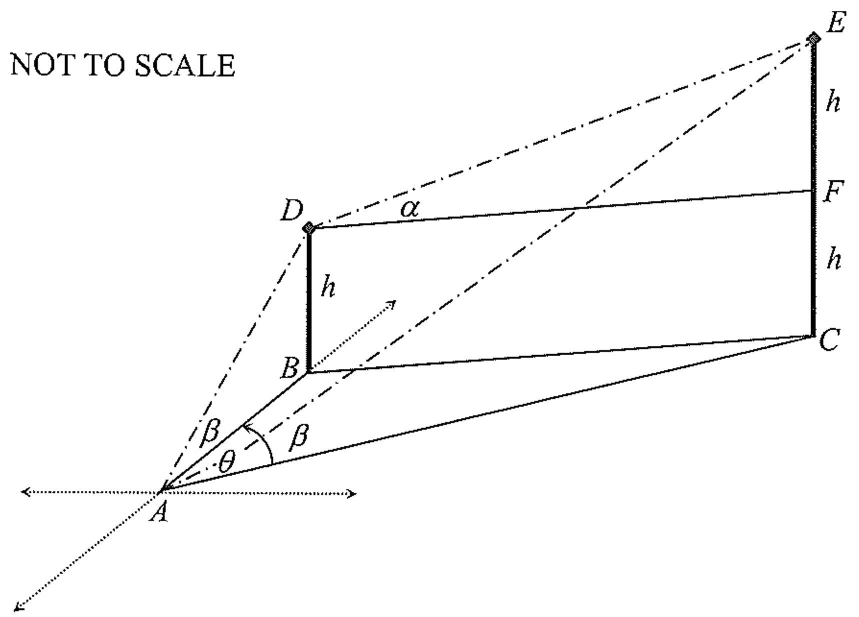
(ii) Express the limiting sum in simplest form. **2**

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) A man, standing on level ground at A, notices two vertical towers, BD and CE. The foot of tower BD, B, is due North of A and the foot of tower CE, C, is on a bearing of θ from A.

The height of BD is h metres and the height of CE is twice the height of BD.

The angle of elevation from A to the top of both towers is β. The angle of elevation to the top of CE from the top of BD is α.



(i) Show that $AC = 2h\cot\beta$. **1**

(ii) Find similar expressions for AB and BC. **2**

(iii) Use the cosine rule, or otherwise, to show that **2**

$$\cos\theta = \frac{5\cot^2\beta - \cot^2\alpha}{4\cot^2\beta}.$$

Question 5 continues on page 8

Question 5 (continued)

(b) Find the range of values of b for which the seventh term in the expansion of $(2+bx)^{11}$ has the largest coefficient. **3**

(c) A particle is moving in simple harmonic motion in a straight line between $x=a$ and $x=-a$. Its acceleration is given by $\ddot{x}=-n^2x$, where x cm is its displacement from the origin at time $t \geq 0$ seconds and n is a constant.

(i) Show that the velocity of the particle, v, is given by $v^2=n^2(a^2-x^2)$. **2**

(ii) Find the extremities of the motion given that the particle has a velocity of 6cms^{-1} when $x=4$ cm and its maximum velocity is 10cms^{-1}. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) (i) Show that ${}^nC_k = {}^nC_{n-k}$. **1**

(ii) Use the identity $(1+x)^n(1+x)^n \equiv (1+x)^{2n}$ to show that **2**

$$\sum_{k=0}^{n}\left({}^nC_k\right)^2 = \frac{(2n)!}{(n!)^2}$$

(b) A function is defined as $f(x) = x^3 + x + 1$.

(i) Show that $f(x)$ has an inverse function, $f^{-1}(x)$, for all x. **1**

(ii) Find the point of intersection of $f(x)$ and $f^{-1}(x)$. **2**

Question 6 continues on page 10

Question 6 (continued)

(c) The position coordinates of any point on the path of a projectile at time $t \geq 0$, in seconds, with initial velocity v ms^{-1} at an angle of projection θ, and acceleration downwards due to gravity, g, are:

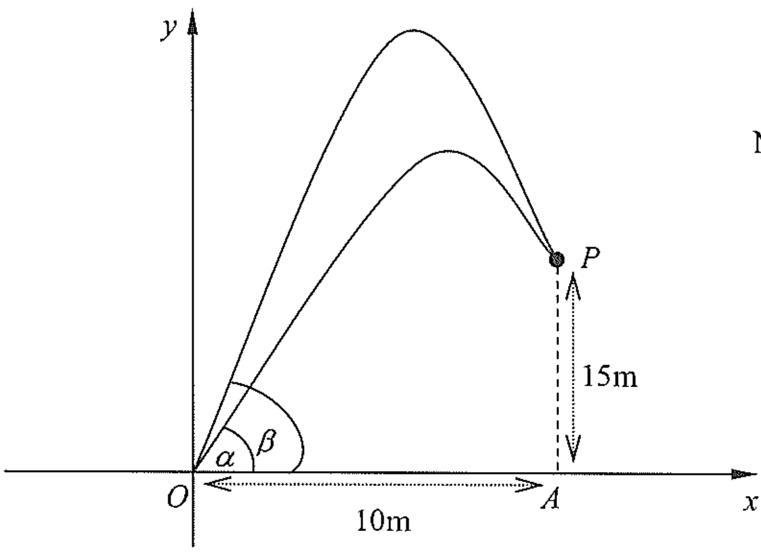
$$x = vt\cos\theta \text{ and } y = vt\sin\theta - \frac{1}{2}gt^2$$

(i) Show that the equation of the path of a projectile is given by **2**

$y = x\tan\theta - \dfrac{gx^2}{2v^2}\sec^2\theta$.

Nicholas throws a small pebble from a fixed point O on level ground, with a velocity $v = 7\sqrt{10}$ ms^{-1} at an angle β with the horizontal. Shortly afterwards he throws another small pebble from the same point at the same speed but at a different angle to the horizontal, α, where $\alpha < \beta$ as shown.

The pebbles collide at a point P, vertically above the point A on the ground, where $OA = 10$ m and $AP = 15$ m. The acceleration downwards due to gravity is $g = 9.8$ ms^{-2}.



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(ii) Show that $\tan\alpha = 2$ and $\tan\beta = 8$. **2**

(iii) Show that the time elapsed between when the pebbles were thrown **2**

was $\dfrac{\sqrt{650} - \sqrt{50}}{7}$ seconds.

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) Use Mathematical Induction to prove that $2n^2 > n^2 + n + 1$ for positive integers $n > 1$. **3**

(b) By definition $\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$ and $\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$.

(i) Show that $2\sinh x \cosh x = \sinh(2x)$. **2**

(ii) Show that the equation $p\cosh x + q\sinh x = r$ can be written as **2**

$(p+q)e^{2x} - 2re^x + (p-q) = 0$ where p, q and r are constants.

(iii) The constants p, q and r are all positive and $p^2 = q^2 + r^2$. **3**

Show that the equation $p\cosh x + q\sinh x = r$ has only one solution.

(iv) Solve the equation $13\cosh x + 5\sinh x = 12$. **2**

Give your answer in the form $\ln k$, where k is rational.

End of paper

Examiners

Carolyn Gavel (convenor)	Kambala, Rose Bay
Cynthia Athayde	St John Bosco College, Engadine
Margaret Clemson	Kambala, Rose Bay
Joe Grabowski	Freeman Catholic College, Bonnyrigg
Br Domenic Xuereb fsp	Patrician Brothers' College, Fairfield