Total marks – 120
Attempt Questions 1-10
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Write down the value of $|-6|-|-12|$. **2**

(b) If $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$, find the value of f when $u=-5$ and $v=7.5$. **2**

(c) Solve the equation $(x-3)^2=9$. **2**

(d) Differentiate x^5+4x^{-2}. **2**

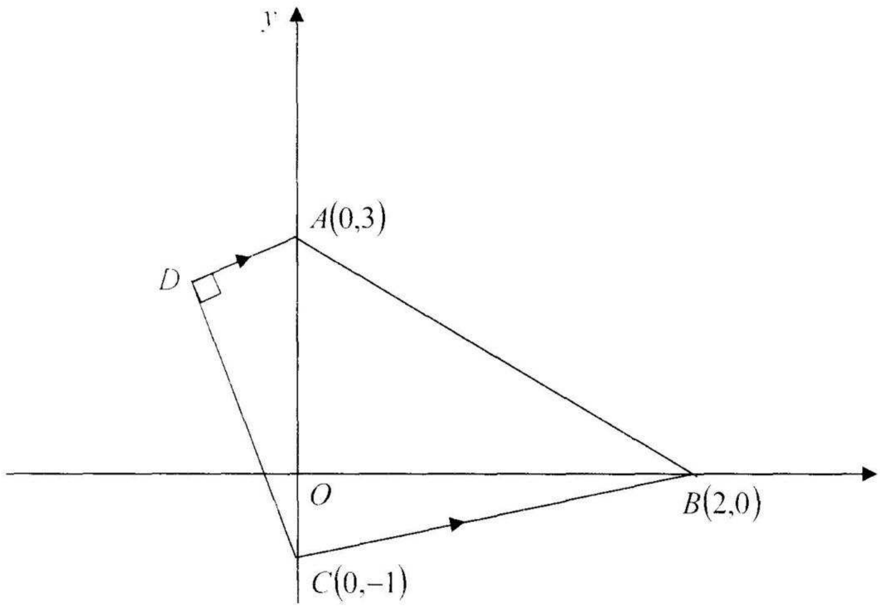
(e) Sketch the curve $y=e^x$. State its range. **2**

(f) If $\frac{1}{a}=\sqrt{10}-3$, show that $a=\sqrt{10}+3$. **2**

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The definition of an odd function $f(x)$ is given by the rule $f(-x) = -f(x)$. Show that the function $f(x) = x^5 - x^3$ is an odd function. **2**

(b)



NOT TO SCALE

In the diagram above, points A, B and C have coordinates $(0,3)$, $(2,0)$ and $(0,-1)$ respectively. Also $AD \parallel BC$ and $AD \perp CD$.

Copy this diagram into your answer sheet.

(i) Show that the gradient of the line BC is equal to $\frac{1}{2}$. **1**

(ii) Show that the equation of the line AD is $x - 2y + 6 = 0$. **2**

(iii) Find the equation of line CD. **2**

(iv) By solving simultaneously the equations from (ii) and (iii), find the coordinates of point D. **2**

(v) Find the area of the quadrilateral $ABCD$. **3**

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) In a right angled triangle $\tan\theta = \frac{3}{4}$. Find $\sin\theta$, for $0 \le \theta \le \frac{\pi}{2}$. 1

(b) Differentiate the following functions:

(i) $\sin x \log_e x$ 2

(ii) $3\tan\frac{\pi x}{3}$ 2

(c) Find: 2

(i) $\int \sin(e - x)\,dx$ 2

(ii) $\int_0^1 \frac{2x}{x^2+1}\,dx$, leaving answer in exact form. 2

(d) Find the equation of the normal to the curve $y = e^{4x} - 1$ at the point on the curve where $x = 0$. 3

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A quadratic equation with roots α and β has the form: **2**

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0.$$

Hence, or otherwise, form a quadratic equation whose roots are $2+\sqrt{3}$ and $2-\sqrt{3}$.

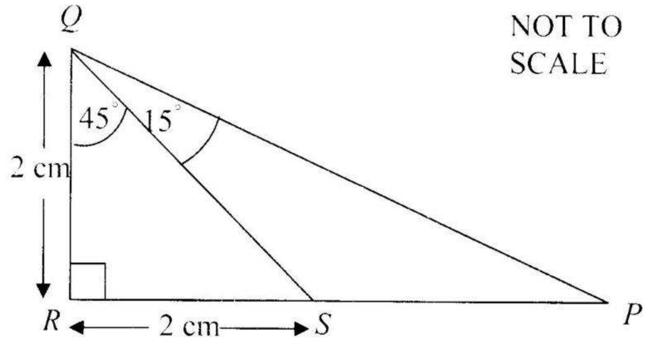
(b) The first and the thirteenth terms of an arithmetic progression are 7 and 1 respectively. Calculate:

(i) the common difference, **2**

(ii) the number of terms which have a sum of zero. **2**

(c) In the diagram below triangle QRP has a right angle at R. Also $\angle RQS = 45^\circ$, $\angle SQP = 15^\circ$ and $QR = RS = 2$ cm.

Copy the diagram in your writing booklet.



(i) Using triangle QRS find the exact length of QS. **1**

(ii) Using triangle QRP find the exact length of PR and hence the exact length of PS. **2**

(iii) Use the Sine Rule in triangle QPS to prove that $\sin 15^0 = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$ **3**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the curve given by $y = x^3 - 6x^2 + 9x + 4$.

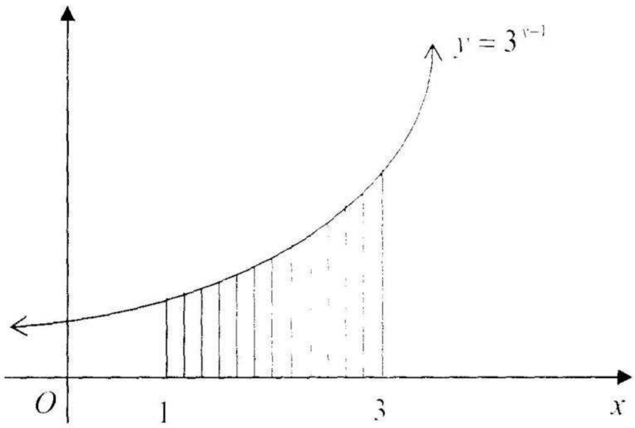
(i) Find the coordinates of the stationary points and determine their nature. **4**

(ii) Find the coordinates of any point of inflexion. **2**

(iii) Sketch the curve, showing all of the above information. **2**

(iv) Determine the values of x for which $\frac{dy}{dx} < 0$ **1**

(b) The diagram below shows the shading of a region bounded by the graph $y = 3^{x-1}$ and the lines $x = 1$ and $x = 3$.



(i) Copy and complete the following table giving your answer correct to three decimal places: **1**

x	1	1.5	2	2.5	3
$y = 3^{x-1}$	1	1.732			

(ii) Use Simpson's Rule with five function values to approximate the shaded area to three decimal places. **2**

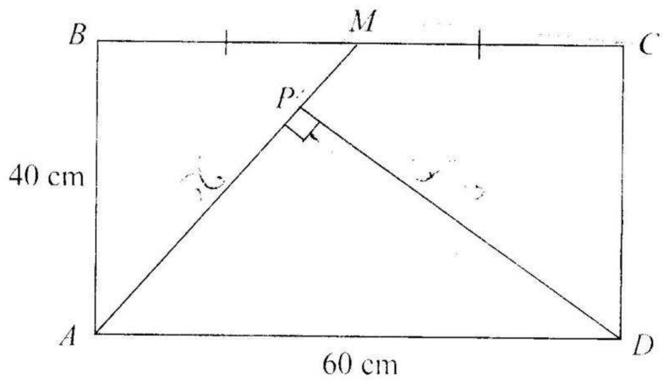
Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Factorise the expression $2a^2 - 7a + 3$. **1**

(ii) Hence, solve the following equation for x: **3**

$$2(\log_2 x)^2 - 7(\log_2 x) + 3 = 0$$

(b)



NOT TO SCALE

$ABCD$ is a rectangle in which $AB = 40$ cm and $AD = 60$ cm. M is the midpoint of BC and DP is perpendicular to AM.

Draw a neat sketch on your answer sheet. Hence:

(i) Prove that triangles ABM and APD are similar. **2**

(ii) Calculate the length of PD. **2**

(iii) Using Pythagoras' Theorem in triangle APD show that $AP = 36$ cm. **1**

(iv) By finding the two areas of the triangles ABM and APD, prove that the area of the quadrilateral $PMCD$ is 936 cm^2. **3**

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Nicole and Mariana play against each other, in the third round of the Australian Open. In this tournament, the first player to win 2 sets wins the match. The probability that Nicole wins any set is 70%.

(i) Find the probability that the game will last two sets only. **2**

(ii) Find the probability that Nicole wins the match. **2**

(iii) Find the probability that Mariana wins the match. **1**

→ see sheet 7b A and 7b B 7b Y

(b) The number N of bacteria in a culture at time t seconds is given by the equation $N = 20000e^{0.003t}$.

(i) What is the number of bacteria initially? **1**

(ii) Determine the number of bacteria after 20 seconds. **2**

(iii) After what period of time will the number of bacteria have doubled? **2**

(iv) At what rate is the number of bacteria increasing when $t = 20$ seconds? **2**

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Sketch the graph of $y = \cos x$, for $0 \le x \le 2\pi$. **1**

(ii) Solve the trigonometric equation $\cos x = \frac{1}{2}$, for $0 \le x \le 2\pi$. **2**

(iii) Hence, find the values of x for which $\frac{1}{2} > \cos x$. **2**

(b) At time t seconds, the position x cm of a point moving in the straight line $X'OX$ is given by $x = at^2 + bt$ cm, where a and b are constants.

The particle passes through the origin O with velocity 16 cm / s in the positive direction at time $t = 0$ seconds, and after 8 seconds, it is again at O.

(i) Find the velocity of the particle at any time, in terms of a and b. **1**

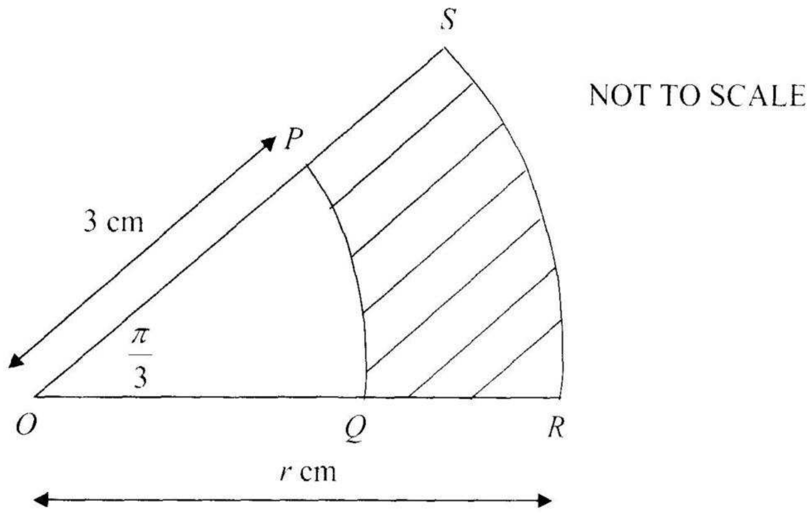
(ii) Find the values of the constants a and b. **3**

(iii) Find the time when the object is at rest. **1**

(iv) Find the position of the particle when it is at rest. **2**

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



In the diagram above PQ and RS are arcs of concentric circles with centre O.
$\angle POQ = \frac{\pi}{3}$ radians and $OP = 3$ cm.

(i) Find the area of the sector OPQ. **1**

(ii) If OR is r cm, find the area of the sector OSR in terms of r. **2**

(iii) If the shaded area is $\frac{27\pi}{6}$ cm^2, find the length of PS. **2**

(b) On 1 July 2005, Nadia invested \$12 000 in a bank account that paid interest at a rate of 6% p.a., compounded annually.

(i) How much would be in the account after the payment of interest on 1 July 2015 if no additional deposits were made? **2**

(ii) In fact Nadia added \$1 000 to her account on 1 July each year, beginning on 1 July 2006. After the payment of interest and her deposit on 1 July 2015, how much was in her account? **4**

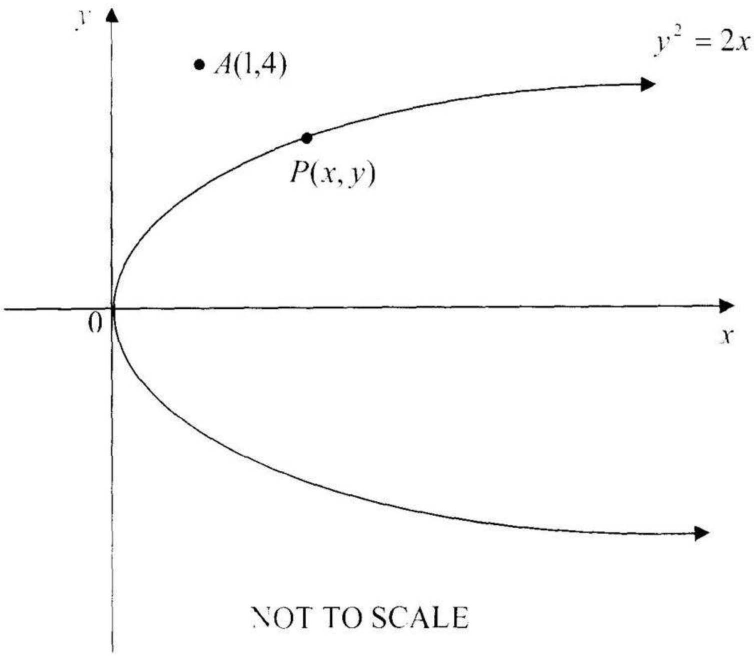
(iii) Nadia's friend Ana deposited \$12 000 in an account at another bank on 1 July 2005 and made no further deposit. On 1 July 2015, the balance of her account was \$35 639.36. What was the annual rate of compound interest paid on Ana's account? **2**

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Simplify $\log_e e^{2ax}$. **1**

(ii) Hence evaluate $\int_0^a \log_e e^{2ax}\,dx$. **2**

(b)



The diagram above shows the graph of the parabola $y^2 = 2x$. The point $A(1,4)$ is outside the parabola while the point $P(x,y)$ is on the parabola as shown in the above diagram.

(i) If D is the distance between the two points A and P, show that **3**

$$D^2 = \left(\frac{1}{2}y^2 - 1\right)^2 + (y-4)^2.$$

(ii) Show that the value of D in the equation in part (i) is a minimum when $y = 2$. **4**

(iii) Show that the minimum distance between A and P is $\sqrt{5}$ units. **2**

End of paper