SYDNEY BOYS' HIGH SCHOOL

MOORE PARK, SURRY HILLS

VERITATE ET VIRTUTE

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 1998

MATHEMATICS

3 UNIT ADDITIONAL
(3/4 UNIT COMMON)

Time allowed: *3 Hours*
(plus five minutes reading time)

Examiners: P.R. Bigelow & P.S. Parker

DIRECTIONS TO CANDIDATES

- *ALL* questions may be attempted.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- Start each question in a new answer booklet. Indicate your name, class and teacher on each new booklet
- Additional answer booklets may be obtained from the supervisor upon request.

NOTE: This is a trial paper only and does not necessarily reflect the content or format of the final Higher School Certificate Examination Paper for this subject.

Question 1 (Start a new page) **Marks**

(a) Find the value of a such that $P(x) = x^3 - 2x^2 - ax + 6$ is divisible by $x + 2$ **2**

(b) For a given series $T_{n+1} - T_n = 7$, $T_1 = 3$, find the value of S_{100}, where $S_n = T_1 + T_2 + \cdots + T_n$. **2**

(c) The interval joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is trisected by the points $P(-2, 3)$ and $Q(1, 0)$.
Write down the coordinates of A and B. **3**

(d) Find the acute angle (to the nearest degree) between the lines $x - y = 2$ and $2x + y = 1$. **2**

(e) Solve $|2x - 1| - |x| \leq 0$ **3**

Question 2 (Start a new page) **Marks**

(a) Find:

(i) $\displaystyle\int \frac{dx}{4+x^2}$ **1**

(ii) $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2\frac{t}{2}\,dt$ **3**

(b) Given $f(x)=\sin^{-1}2x$ **3**

(i) Write down the domain and range of $y=f(x)$

(ii) Sketch the curve.

(iii) Find the exact value of $f'(0\cdot 25)$

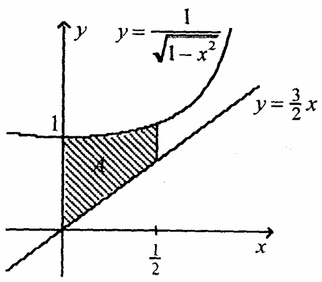
(c) Solve $1+\cos 2x=\sqrt{3}\sin 2x$ where $-\pi < x < \frac{\pi}{4}$ **3**

(d) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sec^2 x e^{\tan x}\,dx$ using the substitution $u=\tan x$ **2**

Question 3 **(Start a new page)** **Marks**

(a) (i) Show algebraically that the line $y=\frac{3}{2}x$ does not meet the curve with equation $y=\frac{1}{\sqrt{1-x^2}}$ **3**

(ii) Find the area of the region A, bounded by the curve $y=\frac{1}{\sqrt{1-x^2}}$ and the lines $x=0$, $x=\frac{1}{2}$ and $y=\frac{3}{2}x$. **3**



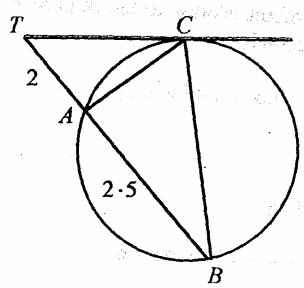
(b) A spherical balloon is expanding so that its volume V mm^3 increases at a constant rate of 72 mm^3 per second. What is the rate of increase of its surface area A mm^2, when the radius is 12 mm. **2**

(c) Given $(3x-2)^{100}=a_{100}x^{100}+a_{99}x^{99}+\cdots+a_1x+a_0$, where a_i $(i=0,\cdots,100)$ is a real number.
Evaluate $a_{100}+a_{99}+\cdots+a_1+a_0$ **2**

(d) Differentiate $x^2\cos^{-1}x$ **2**

Question 4 (Start a new page) **Marks**

(a)



TC is a tangent. $TA = 2$ units, $AB = 2\cdot 5$ units. **2**

Find the length of TC.

(b) Find the coefficient of y^{10} in the expansion $(1+y)(3y^2-2)^7$ **3**

(c) (i) Show that the equation of the tangent at $T(-2t, t^2)$ on the parabola $y = \frac{1}{4}x^2$ is given by $y + tx - t^2 = 0$ **2**

(ii) If the point $M(x, y)$ is the midpoint of the interval TA where A is the x intercept of the tangent at T. Find the equation of the locus of M as T moves on the parabola. **3**

(d) Evaluate $\lim\limits_{x\to 0} \dfrac{5x\cos 2x}{\sin x}$ **2**

Question 5 (Start a new page) **Marks**

(a) A particle is moving in a straight line with Simple Harmonic Motion. If the amplitude of the motion is 8 cm and the period of the motion is 6 seconds. **4**

(i) Express the displacement, x, of the particle as a function of time, t.

(ii) Calculate the maximum velocity of the particle.

(iii) Calculate the maximum acceleration of the particle.

(iv) Calculate the speed when it is 4 cm from the centre of the motion.

(b) Twelve students sit around a circular table. **4**

(i) How many ways can they be arranged?

(ii) If 4 students wish to sit together, how many seating arrangements can be made?

(iii) Let three of the students be A, B and C. Find the probability that A does not sit next to either B or C.

(c) A particle is projected from a point O. After 5 seconds its horizontal and vertical displacements from O are 60 m and $57 \cdot 5$ m respectively. **4**
If the particle is still rising, find its initial velocity.
(You may take $g = 10$ m/s^2)

Question 6 (Start a new page) **Marks**

(a) How many times should a die be thrown so that the probability of obtaining at least one multiple of 3 exceeds $0 \cdot 95$? **2**

(b) At any time t, the rate of cooling of the temperature T of a body when the surrounding temperature is S is given by the equation **5**

$\frac{dT}{dt} = -k(T - S)$, for some constant k

(i) Show that $T = S + Ae^{-kt}$, for some constant A, satisfies this equation.

(ii) A metal rod has a temperature of 1390°C and cools to 1060°C in 10 minutes when the surrounding temperature is 30°C.
Find how much longer it will take the rod to cool to 110°C, giving your answer correct to the nearest minute.

(c) A particle is moving along the x axis with velocity v m s^{-1}, and acceleration $\ddot{x}$ m s^{-2}. **5**

(i) Show that $\ddot{x} = \frac{d}{dx}(\frac{1}{2}v^2)$

(ii) If $v^2 = 24 - 6x - 3x^2$ find the acceleration of the particle at the particle's greatest displacement from the origin O.

Question 7

(a) (i) Show that $P(x) = 8x^3 - 12x^2 + 6x + 13$ has only one zero x_1, and that this zero is negative. **4**

(ii) Find the least value of c, where c is a positive integer, such that $-c < x_1 < 0$

(iii) With $-\frac{c}{2}$ as a first approximation, find a better approximation to x_1, using Newton's Method once. Express your answer correct to two decimal places.

(b) Consider $\tan^{-1} y = 2\tan^{-1} x$ **4**

(i) Express y as a function of x.

(ii) Show that the function has no turning points.

(iii) State the domain of the function.

(iv) Sketch the graph of the function.

(c) If $\tan\alpha$ and $\tan\beta$ are the two values of $\tan\theta$ which satisfy the quadratic equation: **4**

$a\tan^2\theta + b\tan\theta + c = 0$

(i) Find $\tan(\alpha+\beta)$

(ii) Show that $\tan^2(\alpha-\beta) = \dfrac{b^2-4ac}{(a+c)^2}$

END OF THE PAPER