Mrs Choong
Mr Keanan-Brown
Mrs Leslie
Mrs Stock
Mrs Williams

Name : ____________

Teacher's Name : ____________

Pymble Ladies' College

Year 12

Extension I Mathematics Trial

11th August 2003

Time allowed : 2 hours plus 5 minutes reading time

Marking guidelines : The marks for each part are indicated beside the question

Instructions :

- **All questions should be attempted**
- **All necessary working must be shown**
- **Start each question on a new page**
- **Put your name and your teacher's name on each page**
- **Marks may be deducted for careless or untidy work**
- **Only approved calculators may be used**
- **All questions are of equal value**
- **Diagrams are not drawn to scale**
- **A standard integral sheet is attached**
- **DO NOT staple different questions together**
- **All rough working paper must be attached to the end of the last question**
- **Staple a coloured sheet of paper to the back of each question**
- **Hand in this question paper with your answers**
- **There are seven (7) questions and eight (8) pages in this paper**

Question 1

a) If P is the point (-3, 5) and Q is the point (1, -2), find the coordinates of the point R which divides the interval PQ externally in the ratio of 3 : 2. **2**

b) When $(x+3)(x-2)+2$ is divided by $x-k$, the remainder is k^2. Find the value of k. **2**

c) Solve $\dfrac{x}{x-3} \ge 1$. **3**

d) Find the general solution of $\sin\theta = \cos\theta$. **2**

e) Find the exact value of $\int_0^{\frac{\pi}{8}} 2\sin^2 x \, dx$. **3**

Question 2 (Start a new page)

a) i) Show that $x^2+4x+13=(x+2)^2+9$. **1**

ii) Hence find $\int \frac{1}{x^2+4x+13}\,dx$. **2**

b) A stone is projected from the ground with a velocity of $20\,ms^{-1}$ at an angle of 30°. Assume that $\ddot{x}=0$ and $\ddot{y}=-10$.

i) Prove that :

(1) $x=10\sqrt{3}\,t$ **2**

(2) $y=-5t^2+10t$ **2**

ii) Hence find the :

(1) time of flight **1**

(2) horizontal range **1**

(3) greatest height reached **1**

(4) velocity of the particle after $1\frac{1}{2}$ seconds **2**

Question 3 (Start a new page)

a) Evaluate $\int_0^{\sqrt{3}} x\sqrt{x^2+1}\,dx$ using the substitution that $u=x^2+1$. **3**

b) i) Express $\cos\theta+\sqrt{3}\sin\theta$ in the form $r\cos(\theta-\alpha)$ where $r>0$ and $0<\alpha<\frac{\pi}{2}$. **2**

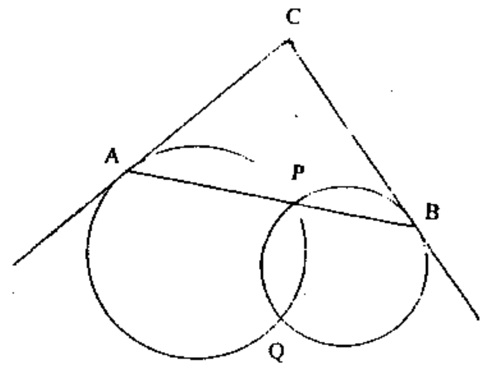
ii) Hence solve $\cos\theta+\sqrt{3}\sin\theta=1$ for $-2\pi\le\theta\le2\pi$. **2**

c) Given $f(x)=\frac{x-1}{x+2}$.

i) Write an expression for the inverse function $f^{-1}(x)$. **1**

ii) Write down the domain and range of $f^{-1}(x)$. **1**

d) Two circles meet at P and Q. A line APB is drawn through P and the tangents at A and B meet at C. Prove that ACBQ is a cyclic quadrilateral. **3**



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Question 4 (Start a new page)

a) Assume that the rate at which a body warms in air is proportional to the difference between its temperature T and the constant temperature A of the surrounding air. This rate can be expressed by the differential equation $\frac{dT}{dt} = -k(T - A)$ where t is the time in minutes and k is a constant.

i) Show that $T = A - Ce^{-kt}$ is a solution of the differential equation where C is a constant. **1**

ii) A body warms from 3°C to 10°C in 15 minutes. The air temperature around the body is 30°C. Find the temperature of this body after a further 15 minutes have elapsed. Answer correct to the nearest $^\circ$C. **4**

iii) With the aid of the graph of T against t, explain the behaviour of T as t becomes large. **1**

b) The acceleration of a particle moving in a straight line is given by $\ddot{x} = -4x + 8$ where x is the displacement, in metres, from the origin O and t is the time in seconds.

i) Show that the particle is moving in simple harmonic motion. **1**

ii) Write down the centre of motion. **1**

iii) Show that $v^2 = 20 + 16x - 4x^2$ given, that the particle is initially at rest at $x = 5$. **2**

iv) Write down the amplitude of the motion. **1**

v) Find the maximum speed of the particle. **1**

Question 5 (Start a new page)

a) Consider the curve $f(x) = \ln(x+1)$. Find the gradient(s) of the possible tangent(s) to $f(x)$ which makes an angle of 45° with the tangent to $f(x)$ at the point where $x = 1$. **3**

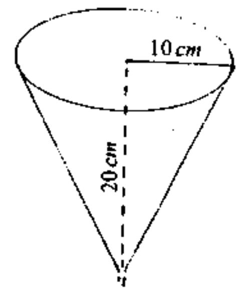
b) i) Use the table of standard integrals given to find $\frac{d}{dx}\left\{\ln\left(x+\sqrt{x^2+9}\right)\right\}$. **1**

ii) Hence use Newton's method to find a second approximation to the root of $x = \ln\left(x+\sqrt{x^2+9}\right)$. Take the first approximation as $x = -4.5$. **2**

c) Water is running out of a filled conical funnel at the rate of $5\,cm^3s^{-1}$. The radius of the funnel is $10\,cm$ and the height is $20\,cm$.

i) How fast is the water level dropping when the water is $10\,cm$ deep? **4**

ii) How long does it take for the water to drop to $10\,cm$ deep? **2**



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Question 6 (Start a new page)

a) Given θ is acute.

i) Write $\sin\frac{\theta}{2}$ in terms of $\cos\theta$. **1**

ii) Prove that $\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$. **2**

iii) If $\sin\theta = \frac{4}{5}$, find the value of $\tan\frac{\theta}{2}$. **2**

b) Find $\frac{d}{dx}\cos^{-1}(\sin x)$ **3**

c) Suppose the roots of the equation $x^3 + px^2 + qx + r = 0$ are real. Show that the roots are in a geometric progression if $q^3 = p^3 r$. **4**

Hint : let the roots be $\frac{a}{b}$, a and ab.

Question 7 (Start a new page)

a)i) Prove by mathematical induction that **4**

$$\frac{12}{1\cdot3\cdot4}+\frac{18}{2\cdot4\cdot5}+\frac{24}{3\cdot5\cdot6}+\ldots+\frac{6(n+1)}{n(n+2)(n+3)} = \frac{17}{6}-\frac{1}{n+1}-\frac{1}{n+2}-\frac{4}{n+3}.$$

ii) Hence find $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{6(r+1)}{r(r+2)(r+3)}$. **1**

b) Consider the variable point $P(x, y)$ on the parabola $x^2 = 2y$.
The x value of P is given by $x = t$;

i) write its y value in terms of t **1**

ii) write an expression, in terms of t, for the square of the distance, m, from P to the point $(6, 0)$ **1**

iii) hence find the coordinates of P such that P is the closest to the point $(6, 0)$. **5**

*** **End of Paper** ***