N.S.W. DEPARTMENT OF EDUCATION

HIGHER SCHOOL CERTIFICATE EXAMINATION 1981

MATHEMATICS 3 UNIT (AND 4 UNIT - FIRST PAPER)

Instructions: Time allowed 3 hours. All questions may be attempted. All questions are of equal value. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work. Mathematical tables will be supplied. Approved slide rules or calculators may be used.

QUESTION 1 (10 MARKS)

(i) Differentiate with respect to x: (a) $(5 - 3x)^{10}$; (b) $x^3 \log_e 2x$.

(ii) Find the gradient of the tangent to the curve $y = x^{\frac{1}{2}}$ at the point (9, 3).

(iii) The function $f(x)$ is defined by the rule $f(x) = 9x(x - 2)^2$ in the domain $-1 \le x \le 3$. Draw a sketch of the graph of $y = f(x)$, showing clearly the turning points, the intercepts with the x and y axes, and the values at the end-points of the domain. What is the range of $f(x)$?

QUESTION 2 (10 MARKS)

(i) Express $\frac{\sqrt{7} - 2\sqrt{3}}{\sqrt{7} + \sqrt{3}}$ as the sum of two numbers with rational denominators.

(ii) For what values of x is $x(x - 2) \ge 3$?

(iii) O is the point (0, 0, 0), P the point (5, -6, 5) and Q the point (2, 0, 8).

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(a) Write down the equations of the line PQ.

(b) PQ meets the plane $4x - 3y - 2z = 4$ at R. Find the co-ordinates of R.

(c) Show that OR is perpendicular to PQ.

QUESTION 3 (10 MARKS)

(i) Given that $y = \tan^{-1}(1/x)$, find $\frac{dy}{dx}$ in its simplest form.

(ii) (a) Find the derivative of $f(x) = \tan(x^2)$.

(b) Hence, or otherwise, evaluate $\int_{-1}^{1} x \sec^2(x^2)\,dx$.

(iii) Write down the transformed equation of the curve defined by $2x + 3y = xy$ when the origin is moved to (3, 2) with new axes parallel to the original axes. Sketch the curve, showing both sets of axes.

QUESTION 4 (10 MARKS)

(i) Use the binomial theorem to find the term independent of x in the expansion of $(2x + \frac{1}{x^2})^6$.

(ii) Solve the equation $2(2^{2x}) - 3(2^x) + 1 = 0$.

(iii) Find the term of the arithmetic series $1 + 2 + 3 + \ldots + 49$ which is such that the sum of the terms preceding it is equal to the sum of the terms following it.

QUESTION 5 (10 MARKS)

(i) (a) Show that the function $f(x) = x^3 - x^2 - x - 1$ has a zero for x between 1 and 2.

(b) Taking $x = 2$ as a first approximation to this zero, use Newton's method to calculate a second approximation.

(c) Give a geometrical interpretation of the process used in (b). Why is $x = 1$ unsuitable as a first approximation to this zero?

(ii) The polynomial $(x - a)^3 + b$ is zero at $x = 1$ and, when divided by x, the remainder is -7. Find all possible values of the pair a, b.

QUESTION 6 (10 MARKS)

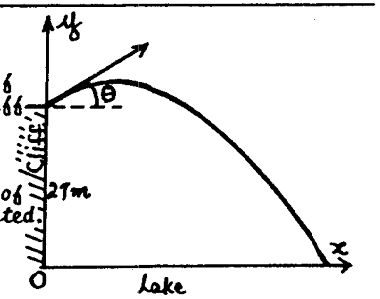
(i) (a) Find the points of intersection of the line $y = x + 1$ with the parabola $y = 2x^2$.

(b) Also find the area enclosed between this line and the given parabola.

(ii) For what values of m does the line $y = m(x + 1)$ have no intersection with the parabola $y = 2x^2$?

(iii) Find the equations of the two tangents to the parabola $y = 2x^2$ which pass through the point (-1, 0).

QUESTION 7 (10 MARKS)



A stone is projected with velocity 10 m/s at an angle of elevation $\theta = \tan^{-1}(3/4)$ from the top of a vertical cliff 27 m high overlooking a lake.

(i) Write down the equations of motion of the stone, assuming the origin to be a point O at the base of the cliff, and that air resistance may be neglected. Hence derive expressions for the horizontal and vertical components of the stone's displacement from the origin after t seconds.

(ii) Calculate the time which elapses before the stone hits the lake and find the horizontal distance of the point of contact from the base of the cliff. (You may assume the approximate value 10 m/s^2 for g.)

(iii) What is the maximum height reached by the stone?

(iv) The path of the stone in the air is a parabolic arc. Write down its equation in cartesian form.

QUESTION 8 (10 MARKS)

(i) "In a certain State, of population 4.8 million, about 1,200 persons are killed on the roads each year. Therefore, there is an approximately 1 in 4,000 chance of a particular person being killed in a road accident each year."

Is this statement valid? Give brief reasons for your answer.

(ii) In a game of chess between two players X, Y, of about equal ability, the player with the White pieces, having the first move, has a probability of 0.5 of winning, and the probability that the player with the Black pieces wins is 0.3. What is the probability that the game ends in a draw (i.e. neither player wins)?

The two players X, Y, play each other twice in a chess competition, each player having the White pieces once. In the competition a player who wins a game scores 1 point, a player who loses a game scores 0 points, and a draw scores as ½ point to each player.

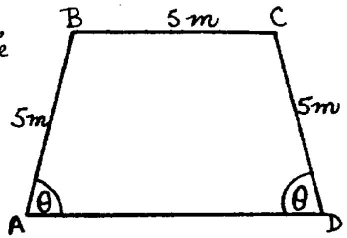
What is the probability that, as a result of these two games,

(a) X scores 2 points? (b) X scores less than 1½ points?

QUESTION 9 (10 MARKS)

(i) In a quadrilateral ABCD, BC is parallel to AD, the sides AB, BC, CD are each 5 m long and the angles BAD, ADC each have size θ, as shown in the diagram. Find an expression for the area of the quadrilateral in terms of θ.

Find the value of θ for which this area is a maximum.

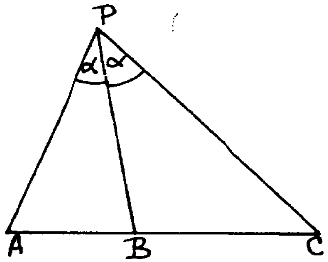


(ii) In the triangle APC, B is the point on AC such that $\angle APB = \angle BPC = \alpha$.

(a) Using the sine rule, or otherwise, show that AB:BC = AP:PC.

(b) Express the area of the triangle BPA in terms of α.

By considering area of triangles, or otherwise, show that if $\alpha = 60^0$ then $\frac{1}{PA} + \frac{1}{PC} = \frac{1}{PB}$.



QUESTION 10 (10 MARKS)

(i) Under ideal conditions, it is estimated that the rate of increase in the population $P(t)$ of a particular species of bird is given by the equation $\frac{d}{dt}P(t) = kP(t)$; $(t \geq 0)$, where k is a positive constant.

(a) Given that the population trebles in two years, write down an expressio for $P(t)$ in terms of t and $P(0)$.

(b) The initial population of a colony is 10 birds. Estimate the colony's population after three years.

(ii) In practice, population growth is restricted by limitations on food, space and other natural resources. The equation $\frac{d}{dt}P(t) = kP(t)(L - P(t))$; $(t \geq 0$ where k and L are positive constants and $P(0) < L$, is found to be more

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appropriate to describe some situations.

(a) Verify that, for any positive constant C, the expression $P(t) = \frac{LC}{C + e^{-kLt}}$ satisfies the equation.

(b) What can be deduced about P as t increases?

(c) What can be deduced about $\frac{dP}{dt}$ as t increases?