2U Mathematics TRIAL, 2003, SOLUTIONS

1)(a) On calculator:

$\sqrt{(4\pi) \div (3.6x^2 - 9.8)} = 1.1218\ldots$ (4 sig. fig's)

$\therefore$ Answer $= 1.122$

(b) $\frac{6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{6(\sqrt{3}+1)}{(\sqrt{3})^2 - (1)^2}$ $\frac{1}{2}$

$= \frac{6(\sqrt{3}+1)}{3-1}$ $\frac{1}{2}$

$= \frac{^3\cancel{6}(\sqrt{3}+1)}{\cancel{2}_1}$ $\frac{1}{2}$

$= 3(\sqrt{3}+1)$ (2)

c) $\frac{d}{dx}(6 - x^3) = 0 - 3x^2$

$= -3x^2$ (1)

d) $\frac{x}{2} + \frac{x}{3} = 1$ $\frac{1}{2}$ ($\times 6$)

$\therefore 3x + 2x = 6$ $\frac{1}{2}$

$\therefore 5x = 6$ $\frac{1}{2}$

$x = \frac{6}{5}$ or $1\frac{1}{5}$ or 1.2 $\frac{1}{2}$ (2)

e) $\int \frac{4}{x}\,dx = 4\int \frac{1}{x}\,dx$

$= 4\log_e x + c$ (2)

f) $9 - 16t^2 = (3)^2 - (4t)^2$ 1

$\{a^2 - b^2 = (a+b)(a-b)\}$

$= (3+4t)(3-4t)$ (2)

$\frac{1}{2}$ $\frac{1}{2}$

(2)(a)(i) $y = e^{\sin x} + \frac{1}{2}x^4$

$\therefore \frac{dy}{dx} = \cos x \times e^{\sin x} + \frac{1}{2} \times 4x^3$

$= \cos x \cdot e^{\sin x} + 2x^3$ (2)

(ii) $y = \frac{\log_e x}{x}$ $\rightarrow u = \log_e x,\ v = x$; $u' = \frac{1}{x},\ v' = 1$ (1) $\frac{1}{2}$

$\therefore y' = \frac{vu' - uv'}{v^2}$ ($\frac{1}{2}$)

$= \frac{x \times \frac{1}{x} - \log_e x \times 1}{x^2}$ (1)

$= \frac{1 - \log_e x}{x^2}$ ($\frac{1}{2}$) (3)

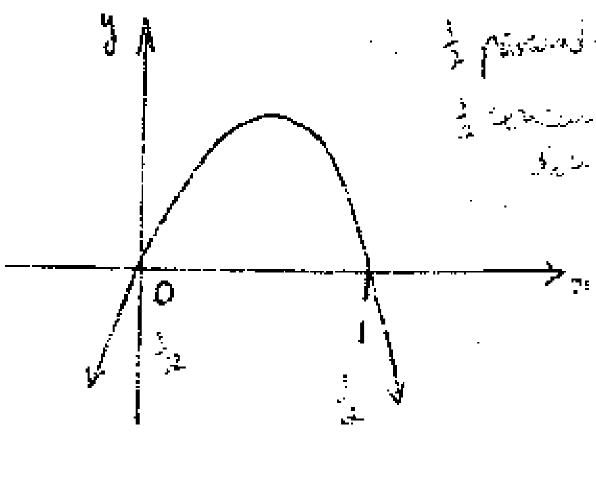
(b) $y = -x^2 + x + 0$ ← y-intercept

and for $x - x^2 = 0$

$x(1-x) = 0$

$\therefore x = 0$ or 1 ← x-intercepts

and parabola; concave down



$\frac{1}{2}$ parabola

$\frac{1}{2}$ concave down

(2)

② continued:

(c) $|x+4| = 1$

$\therefore x+4 = 1$ ½ or $-(x+4) = 1$ ½

$\;\;\; -4 \;\; -4$ $\qquad \therefore x+4 = -1$

$\qquad\qquad\qquad\qquad -4 \;\; -4$

$\therefore x = -3$ ½ or $\therefore x = -5$ ½ 2

(d) $\sec 210° = \dfrac{1}{\cos 210°}$ ½

and for $\cos 210°$:

3rd quad : $\cos(180°+30°)$ } ½

$= -\cos 30°$

$= -\sqrt{3}/2$ ½

$\therefore \sec 210° = \dfrac{1}{-\sqrt{3}/2}$ } ½ 2

{ Answer can be checked on calculator } $= -2/\sqrt{3}$

③(a) Arithmetic, $a=6$, $d=9$.

(i) $T_n = a + (n-1)d$ ½

$= 6 + (n-1)\times 9$ } ½

$= 6 + 9n - 9$

ie. $T_n = 9n - 3$ ½ 1½

(ii) Let $9n - 3 = 4623$ ½

$\qquad +3 \qquad\quad +3$

$9n = 4626$ } ½

$\div 9 \qquad \div 9$

$n = 514$ 1½

ie. 514th term ½

(b)(i) $\text{Gradient}_{AB} = \dfrac{-6-3}{11-(-1)}$ } ½ $\left(\dfrac{y_2-y_1}{x_2-x_1}\right)$ ②

$= \dfrac{-9}{12}$

$= -3/4$ } ½ 1

(ii) $y - y_1 = m(x - x_1)$ ½

$\times 4 \;\; \times 4$

$y - 3 = -3/4\,(x+1)$ $\times 4$ ½

$\therefore 4y - 12 = -3x - 3$ ½

$+3x \;\; +3 \qquad +3x \;\; +3$

$\therefore 3x + 4y - 9 = 0$ ½ (QED!) 2

(or : can be done by showing - by substitution - that A and B satisfy equation). (OR 1, 1)

(iii) // to AB $\therefore m_L = -3/4$ ½

and through origin : $y = mx + b$ ½ $\qquad b \to 0$

i.e. $y = mx$

$\therefore y = -3/4\,x$ } 1 2

(iv) $(4, k)$ on L $\therefore k = -3/4 \times 4$ 1

$k = -3$ 1 2

(v) $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$ ½

$= \dfrac{|3\times 4 + 4\times -3 + -9|}{\sqrt{3^2+4^2}}$ ½

$= \dfrac{|12-12-9|}{\sqrt{25}}$ ½

$= |-9|$ ½ [illegible]

4 (a)(i) $\int \cos ax \, dx = \frac{1}{a}\sin ax + c$

(from Standard Integrals Sheet)

$\therefore \int \cos 2x \, dx = \frac{1}{2}\sin 2x + c$

$\frac{1}{2}$ 1 $\frac{1}{2}$ (2)

(ii) $\int \frac{dx}{2x+3} = \frac{1}{2}\int \frac{2}{2x+3}\,dx$

$= \frac{1}{2}\log_e(2x+3) + c$

$\frac{1}{2}$ 1 $\frac{1}{2}$ (2)

(iii) $\int e^{3x}\,dx = \frac{1}{3}\int 3e^{3x}\,dx$

$= \frac{1}{3}e^{3x} + c$

$\frac{1}{2}$ 1 $\frac{1}{2}$ (2)

(b) X: $A = P(1+r/100)^n$

$= 5000\,(1+9/100)^6$ } 1

$= 5000 \times 1.09^6$

$= \$8385.50$ (rounded)

$\therefore$ C.I. $= 8385.50 - 5000$

$= \$3385.50$ .... (1) $\frac{1}{2}$

Y: $SI = \frac{Prn}{100}$ $\frac{1}{2}$

$= \frac{5000 \times 9 \times 6}{100}$

$= \$2700$ ...... (2) $\frac{1}{2}$

$\therefore$ Difference $=$ (1) $-$ (2)

$= 3385.50 - 2700$

$= \$685.50$ $\frac{1}{2}$ 5

(c) Pos. definite if:

$a > 0$ AND $\Delta < 0$ $\frac{1}{2}$

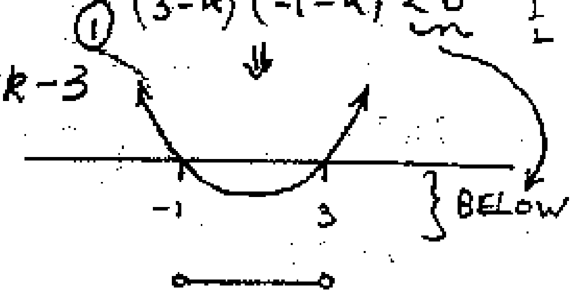
ie $3-k>0$ $\frac{1}{2}$ $\quad b^2 - 4ac < 0$ $\frac{1}{2}$

$3>k$ $\quad (3-k)^2 - 4(3-k) < 0$

or $k<3$ ....(1) $\quad (3-k)[(3-k)-4] < 0$

$(3-k)(-1-k) < 0$ $\frac{1}{2}$

$\Delta = k^2 - 2k - 3$



ie $-1 < k < 3$ .... $\frac{1}{2}$ (2)

from (1) AND (2) we have:

$-1 < k < 3$ $\frac{1}{2}$ 3

5 (a)(i) $\sin\alpha = AB/AD$ (from ΔABD) $\frac{1}{2}$

$= 1x/2x$ $\frac{1}{2}$

$= \frac{1}{2}$

$\therefore \alpha = \sin^{-1}(\frac{1}{2})$

$\alpha = 30°$ $\frac{1}{2}$ (QED) $1\frac{1}{2}$

(ii) $\angle ADC = 30°$ (above)

$\therefore \angle BAD = 60°$ ($\angle$ sum of ΔABD) $\frac{1}{2}$

$\therefore \angle DAC = 30°$ $\frac{1}{2}$ 1

(iii) ΔACD is isosceles $\frac{1}{2}$

$\therefore AC = DC = 2$ $\frac{1}{2}$

$\therefore$ In ΔABC:

$\cos\alpha = AB/2$ $\frac{1}{2}$ $2\frac{1}{2}$

ie $\cos 30° = AB/2 = \sqrt{3}/2$ $\frac{1}{2}$

5 continued:

(b) $9^x + 6 \times 3^x - 27 = 0$

$9^x = (3^2)^x = (3^x)^2$

∴ Letting $A = 3^x$ gives: $\frac{1}{2}$

$$\left.\begin{array}{l} A^2 + 6A - 27 = 0 \\ (A+9)(A-3) = 0 \end{array}\right\} \frac{1}{2}$$

∴ $A = -9$ or $A = 3$

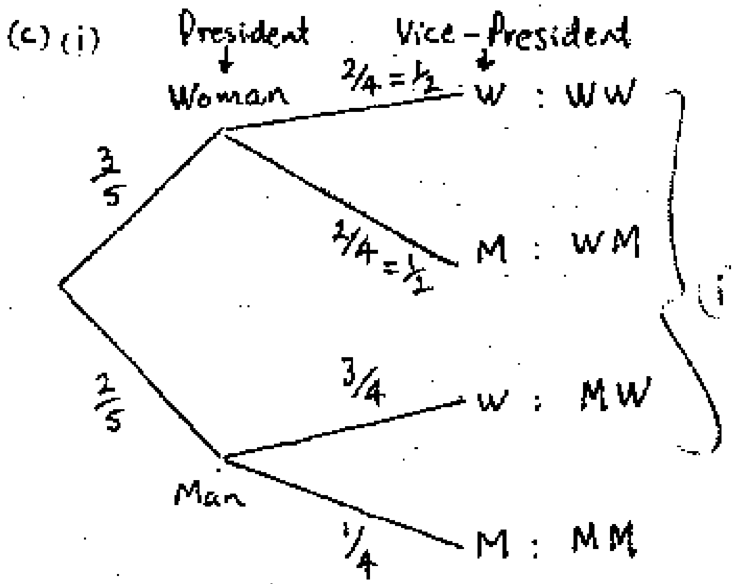
i.e. $3^x = -9$ $\frac{1}{2}$ $\quad 3^x = 3$ $\frac{1}{2}$

no solution $\frac{1}{2}$ $\quad 3^x = 3^1$ $\frac{1}{2}$

∴ $x = 1$

3

(c) (i)



Note President can not be Vice-Pres.

∴ names selected but not replaced

(ii) $P(WM \text{ or } MW)$

$= \frac{3}{5} \times \frac{1}{2}$ 1 $+ \frac{2}{5} \times \frac{3}{4}$ 1

$= \frac{3}{10} + \frac{3}{10}$ $\quad \frac{1}{2}$

$= \frac{3}{5}$ (or 60%) $\frac{1}{2}$

3

6 (a) $\int_0^3 f(x)\,dx$ is positive, say $+9$: $\frac{1}{2}$ .....(1)

$\int_3^4 f(x)\,dx$ is negative, say -2 $\frac{1}{2}$

(NOTE Area from 0 to 3 > Area from 3 to 4 } $\frac{1}{2}$

or $|+9| > |-2|$)

Now: $\int_0^4 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^4 f(x)\,dx$

$$\left.\begin{array}{l} = 9 + (-2) \\ = +7 \;.....(2) \end{array}\right\} \frac{1}{2}$$

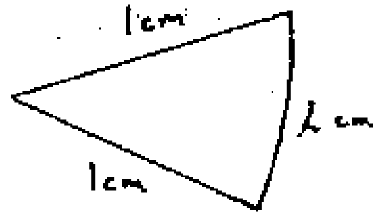
∴ Comparing (1) and (2):

$7 < 9$ } $\frac{1}{2}$

i.e. $\int_0^4 f(x)\,dx < \int_0^3 f(x)\,dx$ (QED)

3

(b)



(i) $l + 1 + 1 = 4$cm (perimeter) $\frac{1}{2}$

∴ $l + 2 = 2$

$l = 2$cm

$l = r\theta$ ∴ $\theta = l/r$ 1

$= 2/1$

i.e. $\theta = 2$ rad QED $\frac{1}{2}$

2

(ii) $A = \frac{1}{2} r^2 \theta$ 1

$= \frac{1}{2} \times 1^2 \times 2$ $\frac{1}{2}$

∴ $A = 1\text{cm}^2$ $\frac{1}{2}$

2

(6) continued:

(c) $h = \frac{b-a}{n}$ $\frac{1}{2}$ — 5 function values = 4 strips

$= \frac{5-1}{4}$ $\qquad \frac{b-a}{n} = 2$

$= 1$ (strip width) $\frac{1}{2}$

x	y ($\log_e x = \ln x$)	$\times$	$=$
1	$\ln 1 = 0$	1	0
2	$\ln 2 = 0.6931$	4	2.7724
3	$\ln 3 = 1.0986$	2	2.1972
4	$\ln 4 = 1.3863$	4	5.5452
5	$\ln 5 = 1.6094$	1	1.6094
$\frac{1}{2}$	$\frac{1}{2} + \frac{1}{2}$	$\frac{1}{2}$ TOTAL =	12.1242

$\therefore \int_1^5 \log_e x \, dx \doteq \frac{h}{3} \times \text{TOTAL}$ $\frac{1}{2}$ $\frac{1}{2}$ (or $\frac{1}{3}$)

$= \frac{1}{3} \times 12.1242$ $\frac{1}{2}$

$= 4.0414$

$= 4.041$ (3dp) $\frac{1}{2}$

($-\frac{1}{2}$ for each "error") $\qquad$ 5

(NOTE: Graph of $y = \log_e x$ above x-axis for $x = 1$ to $x = 5$

$\therefore \int_1^5 \log_e x \, dx = \text{Area}$)

(7)(a)(i) $M = M_0 e^{-kt}$

$\therefore \frac{dM}{dt} = M_0 \times -ke^{-kt}$ $\frac{1}{2}$

$= -k\left(M_0 e^{-kt}\right)$ $\frac{1}{2}$

ie. $\frac{dM}{dt} = -kM$ (QED) $\frac{1}{2}$ $\qquad 1\frac{1}{2}$

7(d) $\int_0^4 = \int_0^3 + \int_3^4$ ✓

but $\int_3^4 < 0$ ✓ and $\int_0^3 > 0$ ✓

So $\int_0^4 < \int_0^3$ $\qquad$ 3

(ii) $M = \frac{1}{2}M_0$ at $t = 17600$

ie. $\frac{1}{2}M_0 = M_0 e^{-17600k}$ $\frac{1}{2}$

$\therefore \frac{1}{2} = e^{-17600k}$

$\ln\frac{1}{2} = -17600k$ $\frac{1}{2}$ $\qquad 1\frac{1}{2}$

$\therefore k = \ln\frac{1}{2} \div -17600$

ie. $k = 0.000039$ (6dp) $\frac{1}{2}$

(iii) For $\frac{2}{3}$ decayed, $M = \frac{1}{3}M_0$ $\frac{1}{2}$

ie. $\frac{1}{3} = e^{-0.000039t}$ $\frac{1}{2}$

$\ln\frac{1}{3} = -0.000039t$

$\therefore t = \ln\frac{1}{3} \div -0.000039$ $\frac{1}{2}$

$= 28169.5\ldots$ $\qquad$ 2

ie. 28 200 years $\frac{1}{2}$ (3 sig. figs)

b)(i) $f(x) = 0 \rightarrow$ x-intercept(s) : O

(ii) $f'(x) = 0 \rightarrow$ st. points : D, G

(iii) $f''(x) = 0 \rightarrow$ inf. pts : B, O, I

(iv) $f(x) > 0 \rightarrow$ above x-axis : F, G, H, I, J

(v) $f'(x) > 0 \rightarrow$ increasing : E, O, F

(vi) $f''(x) > 0 \rightarrow$ concave up : C, D, E, J

(v) $\lim_{x\to\infty} f(x) = 0 \rightarrow$ approaches x-axis as $x \uparrow$ in positive direction : J

} EACH

correct $\left(+\frac{1}{2}\right)$

incorrect $\left(-\frac{1}{2}\right)$

(8)(a)(i) $y = \cos^3 x$

$\therefore y = (\cos x)^3$

$\therefore \frac{dy}{dx} = 3(\cos x)^2 \times -\sin x$

$= -3\cos^2 x \sin x$ (2)

(ii) $\int_0^{\frac{\pi}{4}} (\cos^2 x \sin x)\, dx$

$= -\frac{1}{3} \int_0^{\frac{\pi}{4}} (-3\cos^2 x \sin x)\, dx$ ½

$= -\frac{1}{3} \left[\cos^3 x \right]_0^{\frac{\pi}{4}}$ (from (i)) ½

$= -\frac{1}{3} \left[(\cos \pi/4)^3 - (\cos 0)^3 \right]$ ½

½ ½

$= -\frac{1}{3} \left[\left(\frac{1}{\sqrt{2}}\right)^3 - (1)^3 \right]$

$= -\frac{1}{3} [0.3536 - 1]$ (4dp)

$= 0.215$ 3dp ½ (3)

or $-\frac{1}{3}\left(\frac{1}{2\sqrt{2}} - 1\right) = \frac{1}{3}\left(1 - \frac{1}{2\sqrt{2}}\right)$

(b)(i) $x^2 - 8x + 16 = 12y - 28 + 16$ ½

$(-8/2)^2$ (1½)

$\therefore (x-4)^2 = 12y - 12$ ½

ie $(x-4)^2 = 12(y-1)$ ½ (QED)

can also be done by removing brackets and rearranging

Using: $(x-h)^2 = 4a(y-k)$

Vertex at (h, k), focal length $= a$

$\therefore$ **(ii)** Vertex $(4, 1)$ ½ each (1)

(iii) Concave up $\therefore$ Focus at

$(4, 1+a)$ ½

where $4a = 12$ $\therefore a = 3$ ½

Focus at: $(4, 1+3)$ (1½)

(iv) Directrix:

$y = 1 - a$ ½

$= 1 - 3$

ie $y = -2$ ½ (1)

(c) $\int_1^k \frac{1}{x}\, dx = 1$

$\therefore [\ln x]_1^k = 1$ ½

$\therefore \ln k - \ln 1 = 1$ ½

$\therefore \ln k - 0 = 1$ ½

$\ln k = 1$ (2)

or $\log_e k = 1$

$\therefore e^1 = k$ ½

ie $k = e$

(9) $x = 3 - 2\cos t$

(i) at $t = 0$, $x = 3 - 2\cos 0$ ½

$= 3 - 2 \times 1$ ½

$= 3 - 2$

$\therefore x = +1\text{m}$ (1)

(ii) $x = 3 - 2\cos t$

$\therefore v = \dot{x} = 0 - 2 \times (-\sin t)$

½ $= 2\sin t$ ½

$\therefore$ at $t = 0$, $v = 2\sin 0$ ½

$= 2 \times 0$

$v = 0 \rightarrow$ stationary

9 continued:

(iii) Rest $\rightarrow v = 0$ $\frac{1}{2}$

$\therefore 2\sin t = 0 \; (\div 2)$
$\sin t = 0$ $\frac{1}{2}$

$\therefore t = 0, \pi, 2\pi, \ldots$ $\frac{1}{2}$

∴ next time (2)

$\therefore \boxed{t = \pi \text{ seconds}}$ $(\doteqdot 3.1 \text{ sec.})$ $\frac{1}{2}$

(iv) When $x = 2$:

$2 = 3 - 2\cos t$
$\therefore 2\cos t = 3 - 2 = 1$ $\frac{1}{2}$ $\frac{1}{2}$

$\therefore \cos t = \frac{1}{2}$ (gap angle = $60°$ or $\pi/3$)

$\therefore t = \frac{\pi}{3}, \; 2\pi - \frac{\pi}{3}, \; \frac{\pi}{3} + 2\pi, \ldots$ $\frac{1}{2}$

1st quad, 4th quad, 1st quad + 1 revolution

second time $\frac{1}{2}$

$\therefore t = 5\pi/3$ sec.

$\therefore v = 2\sin(5\pi/3)$
$= 2 \times -\sqrt{3}/2$ $\frac{1}{2}$

$\therefore \boxed{v = -\sqrt{3} \text{ m/s}}$ $\frac{1}{2}$ (3)

(v) Maximum (greatest) velocity when $\frac{dv}{dt} = 0$ (ie $a = \ddot{x} = 0$) $\frac{1}{2}$

Here: $\ddot{x} = 2\cos t$ $\frac{1}{2}$

$\therefore$ for $2\cos t = 0$
$\cos t = 0$
$t = \pi/2, \; 3\pi/2, \ldots$ $\frac{1}{2}$

$\therefore v_{max} = 2\sin \pi/2$
$= 2 \times 1$
$= \boxed{2 \text{ m/s}}$ $\frac{1}{2}$ (2)

OR $v = 2\sin t$
Amplitude $= 2$ 1
$\therefore \boxed{v_{max} = 2 \text{ m/s}}$ 1

"graphical" approach

(vi) $\ddot{x} = a = 2\cos t$

$\therefore$ for $a = 0$: $2\cos t = 0$ $\frac{1}{2}$
$\cos t = 0$
$t = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$ $\frac{1}{2}$

$\therefore$ at $t = \pi/2$:

$x = 3 - 2\cos \pi/2$
$= 3 - 2 \times 1$ $\frac{1}{2}$
$= 3 - 2$

$\boxed{x = 1 \text{ m}}$ $\frac{1}{2}$

(10)(a)(i) Show: $\frac{1}{x^2-9} = \frac{1}{6}\left(\frac{1}{x-3} - \frac{1}{x+3}\right)$

$$RHS = \frac{1}{6}\left(\frac{1}{x-3}\times\frac{x+3}{x+3} - \frac{1}{x+3}\times\frac{x-3}{x-3}\right) \quad \tfrac{1}{2}$$

$$= \frac{1}{6}\left(\frac{x+3-(x-3)}{x^2-9}\right)$$

$$= \frac{1}{6}\left(\frac{\cancel{x}+3-\cancel{x}+3}{x^2-9}\right) \quad \tfrac{1}{2}$$

$$= \frac{1}{\cancel{6}}\left(\frac{\cancel{6}}{x^2-9}\right)$$

$$= \frac{1}{x^2-9} \quad \tfrac{1}{2} \qquad 1\tfrac{1}{2}$$

$= LHS$ (QED)

(ii) $V = \pi \int_5^6 \left(\frac{1}{\sqrt{x^2-9}}\right)^2 dx$ $\tfrac{1}{2}$

$$= \pi \int_5^6 \frac{1}{x^2-9}\, dx \quad \tfrac{1}{2}$$

$$= \pi \int_5^6 \frac{1}{6}\left(\frac{1}{x-3} - \frac{1}{x+3}\right) dx \quad \text{(from (i))} \quad \tfrac{1}{2}$$

$$= \frac{\pi}{6} \int_5^6 \frac{1}{x-3} - \frac{1}{x+3}\, dx$$

$$= \frac{\pi}{6}\left[\ln(x-3) - \ln(x+3)\right]_5^6 \quad \tfrac{1}{2}$$

$$= \frac{\pi}{6}\left[\ln\left(\frac{x-3}{x+3}\right)\right]_5^6$$

$$= \frac{\pi}{6}\left[\ln\left(\frac{6-3}{6+3}\right) - \ln\left(\frac{5-3}{5+3}\right)\right] \quad \tfrac{1}{2}$$

$$= \frac{\pi}{6}\left[\ln \tfrac{1}{3} - \ln \tfrac{1}{4}\right]$$

$$= \frac{\pi}{6} \ln\left(\frac{1/3}{1/4}\right) \quad \tfrac{1}{2} \qquad 3\tfrac{1}{2}$$

$$= \frac{\pi}{6} \ln \frac{4}{3} \text{ units}^3 \quad (\doteq 0.15 \text{ 2dp})$$

(b)(i) $\angle MOQ = \frac{1}{2} \times \angle ROQ$

$= \theta$

In $\triangle MOQ$: $\sin\theta = \frac{QM}{QO}$ — QO = radius = 1

i.e. $\sin\theta = \frac{QM}{1}$ $\tfrac{1}{2}$

$\therefore QM = \sin\theta$ (QED)

Similarly: $\cos\theta = \frac{OM}{QO}$ $\tfrac{1}{2}$ $\quad$ 1

$\therefore OM = \cos\theta$ (QED)

(ii) Area $\triangle PQR = \frac{1}{2}bh$

$= \frac{1}{2} \times QR \times MP$ $\tfrac{1}{2}$

but: $\frac{1}{2} \times QR = QM = \sin\theta$ ....(1) $\tfrac{1}{2}$

and: $MP = OM + OP$

$= \cos\theta + \text{radius}$

$= \cos\theta + 1$ ....(2) $\tfrac{1}{2}$ $\quad$ 2

$\therefore$ from (1)/(2): Area $= \sin\theta(\cos\theta + 1)$ $\tfrac{1}{2}$ (QED)

(iii) Area$_{max}$ when $\frac{dA}{d\theta} = 0$ ($= A'$) $\tfrac{1}{2}$

Using product rule: $u = \sin\theta$, $v = (\cos\theta + 1)$

$u' = \cos\theta$ $\quad v' = -\sin\theta$

$\therefore A' = (\cos\theta + 1)\cos\theta + \sin\theta \times (-\sin\theta)$ $\tfrac{1}{2}$

$= \cos^2\theta + \cos\theta - \sin^2\theta$

$= \cos^2\theta + \cos\theta - (1 - \cos^2\theta)$

$= 2\cos^2\theta + \cos\theta - 1$ $\tfrac{1}{2}$ $\quad$ 4

$\therefore$ for: $2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$ $\tfrac{1}{2}$

$\therefore \cos\theta = \frac{1}{2}$ or $\cos\theta = -1$

$\therefore \theta = 60°$ (or $270°$: not possible)

and if $\theta = 60°$, $\angle QOR = 120°$

$\therefore \angle PQR = \angle PRQ = 60°$ $\therefore$ equilateral

AND: $\tfrac{1}{2}$

θ	$59°$	$60°$	$61°$
A'	$+0.05$	0	-0.05

easier than A'' test ?

when ...