



2008
TRIAL HSC EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 84

Attempt Questions 1–7
All questions are of equal value

At the end of the examination, place your solution booklets in order and put this question paper on top. Submit one bundle. The bundle will be separated before marking commences so that anonymity will be maintained.

Student Number: _____

Teacher: _____

Student Name: _____

QUESTION	MARK
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
TOTAL	/84
	%

Total Marks – 84

Attempt Questions 1–7

All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Find

(i) $\int \frac{1}{\sqrt{9-x^2}} dx$ **1**

(ii) $\int \frac{e^x}{e^x + 2} dx$ **1**

(b) Differentiate $\cos^{-1}\left(\frac{2}{x}\right)$. **2**

(c) The interval $A(5, a)$ and $B(b, -1)$ is divided externally in the ratio 2:3 to give the point $(7, 2)$. Find the values of a and b . **2**

(d) Find the coefficient of x^5 in the expansion of $\left(2x - \frac{1}{x^2}\right)^{11}$. **3**

(e) Find the value of $\int_6^{11} x\sqrt{x-2} dx$ using the substitution $u^2 = x - 2$. **3**

Question 2 (12 marks) Use a SEPARATE writing booklet.

- (a) Solve $\frac{8}{x-3} \geq 1$. **2**
- (b) Find the exact value of $\int_{\sqrt{3}}^3 \frac{2}{9+x^2} dx$. **2**
- (c) Consider the curve $y = 2 \cos^{-1}(x-1)$.
- (i) State the domain and range. **2**
- (ii) Sketch the curve. **1**
- (iii) Find the gradient of the tangent to the curve at the point where $x = 1$. **2**
- (d) Find the acute angle between the curves $y = 6 - 2x$ and $y = 2x^2 + x - 8$ at the point where $x = 2$, giving your answer correct to the nearest degree. **3**

Question 3 (12 marks) Use a SEPARATE writing booklet.

- (a) (i) Show that $x^2 - 4x + \log_e x = 0$ has a root between $x = 3$ and $x = 4$. **1**
- (ii) Using two applications of the method of halving the interval, find a smaller interval containing the root. **2**
- (b) When the polynomial $P(x)$ is divided by $x^2 - x$, the quotient is $Q(x)$ and the remainder is $R(x) = ax + b$.
- (i) Given that $P(1) = 3$, show that $R(1) = 3$. **1**
- (ii) Further, when $P(x)$ is divided by x the remainder is -4 . Find $R(x)$. **2**
- (c) Evaluate $\int_{\frac{\pi}{2}}^{\pi} \cos^2 2x \, dx$. **3**
- (d) Prove that $5^n + 2(11)^n$ is divisible by 3 for all positive integer values of n . **3**

Question 4 (12 marks) Use a SEPARATE writing booklet.

- (a) Find the exact value of $\sin^{-1}\left(\cos\frac{2\pi}{3}\right)$. 1

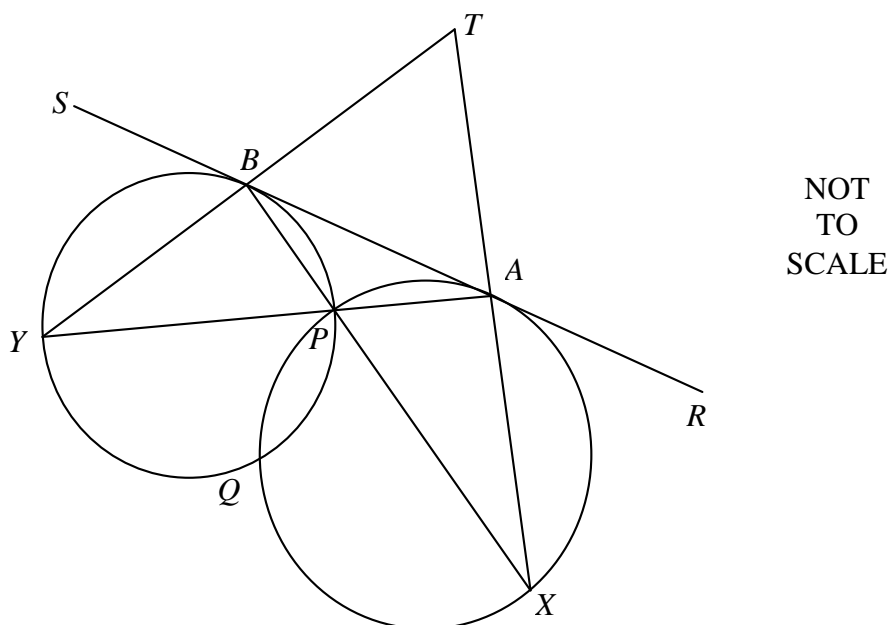
- (b) Find $\frac{d}{dx}(x \tan x)$ and hence show that $\int_0^{\frac{\pi}{4}} x \sec^2 x \, dx = \frac{\pi}{4} - \frac{1}{2} \ln 2$. 3

- (c) Consider $(1 + 2x)^n$:

- (i) Write an expression for the coefficient of the term in x^4 . 1

- (ii) The ratio of the coefficient of x^4 to the coefficient of x^6 is $5:8$.
Find the value of n . 3

- (d) AB is a common tangent to two circles which intersect at P and Q as illustrated in the diagram below.
 XPB and YPA are straight lines. XA and YB intersect at T .



- (i) Copy or trace this diagram into your writing booklet.
- (ii) Explain why $\angle SBY = \angle BPY$ 1
- (iii) Prove that $AT = TB$. 3

Question 5 (12 marks) Use a SEPARATE writing booklet.

Consider the function $f(x) = 3e^{-x^2}$.

- | | | |
|-----|---|----------|
| (a) | Show that the function is even. | 1 |
| (b) | Find the stationary point of $y = f(x)$. | 2 |
| (c) | Show that $\frac{d^2y}{dx^2} = 6e^{-x^2}(2x^2 - 1)$ and hence find any points of inflexion. | 3 |
| (d) | Sketch the curve $y = f(x)$ using the same scale on both the x and y axis. | 1 |
| (e) | State the greatest positive domain of $y = f(x)$ for which an inverse function exists. | 1 |
| (f) | Sketch $y = f^{-1}(x)$ on the same diagram as (d) above. | 1 |
| (g) | Find the equation of the inverse function and state its domain. | 2 |
| (h) | Let $x = N$ where $N < 0$. Find the value of $f^{-1}(f(N))$. | 1 |

Question 6 (12 marks) Use a SEPARATE writing booklet.

- (a) (i) Express $2 \cos \theta + 3 \sin \theta$ in the form $A \cos(\theta - \alpha)$, 2
 where $A > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$.

- (ii) Hence or otherwise find the maximum value of $2 \cos \theta + 3 \sin \theta - 3$. 1

- (b) In order to promote a new brand of bottled water, the word WIN is printed on the inside of some of the bottle caps.

The advertising slogan claims that ‘one in every five bottles wins a prize’.

- (i) Sandra buys 6 bottles of the new brand of water.

Find the probability that:

- (α) The first bottle she opens does not win her a prize but the next one does. 1

- (β) She will win exactly twice after opening all six of the bottles. 1

- (ii) How many bottles would she have to open to ensure that her chance of winning a prize is at least 95%? 2

- (c) (i) Prove that $\sin 2X + \sin 2Y = 2 \sin(X + Y) \cos(X - Y)$. 2

- (ii) Hence or otherwise, solve $\sin \theta + \sin 3\theta = \cos \theta$ if $0 \leq \theta \leq 2\pi$. 3

Question 7 (12 marks) Use a SEPARATE writing booklet.

- (a) The function $f(x)$, where $f(x) = \ln x + \sin 5x$, has a zero between $x = 1$ and $x = 2$. This is illustrated in the diagram below.

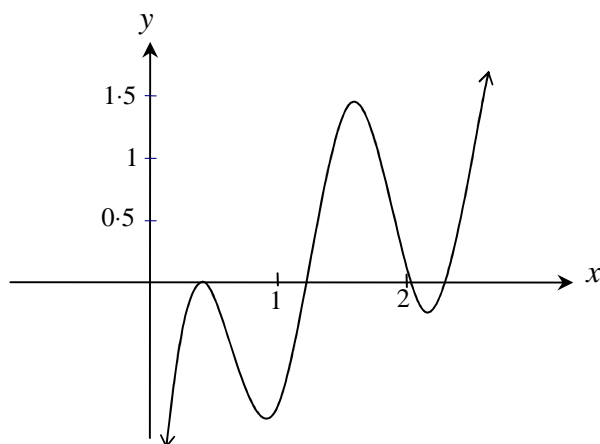


DIAGRAM
TO
SCALE

- (i) Beginning with an approximation of $x = 1.5$, attempt to find an improved value for this root using one application of Newton's method. 2
- (ii) Explain why this attempt fails. 1
- (b) An idle computer programmer decides to develop a 'signature tune' which will play each time she logs onto her computer. She plans to use six tones which she refers to as A, B, C, D, E and F. She plans to repeat two of the tones once only in the tune. For example: A B A C C F D E
- (i) How many such 8 tone signature tunes will she be able to program if there is no restriction on when the repeated tones are played? 2
- (ii) For the sake of a more 'interesting' tune, the repeated tones are played together but the pairs are not to sound immediately after each other. That is, A A B C C... is allowed, but A A B B C... is not. If she applies this condition, how many 8 tone signature tunes will she then be able to program? 2
- (c) By considering the expansion of $(1+x)^{2n}$ prove that 3
- $$\sum_{k=0}^{2n} k(k-1) \binom{2n}{k} = n(2n-1)2^{2n-1}.$$
- (d) Given that $2\log_y x + 2\log_x y = 5$, show that $\log_y x$ is equal to either 2 or $\frac{1}{2}$. 2

End of paper

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STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE : $\ln x = \log_e x$, $x > 0$