SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2007

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time – 5 Minutes
- Working time – 2 Hours
- Write using black or blue pen. Pencil may be used for diagrams.
- Board-approved calculators maybe used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown in every question.
- Each Question is to be returned in a separate bundle.

Total Marks – 84

- Attempt Questions 1 – 7.
- All questions are of equal value.

Examiner: *A. Fuller*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

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Total marks - 84
Attempt Questions 1-7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\lim_{x \to 0} \dfrac{\sin 4x}{5x}$. **1**

(b) Calculate the acute angle (to the nearest minute) between the lines $2x + y = 4$ and $x - 3y = 6$. **2**

(c) (i) Show that $x + 1$ is a factor of $x^3 - 4x^2 + x + 6$. **1**

(ii) Hence, or otherwise factorise $x^3 - 4x^2 + x + 6$ fully. **2**

(d) The point $P(5,7)$ divides the interval joining the points $A(-1,1)$ and $B(3,5)$ externally in the ratio $k:1$. Find the value of k. **2**

(e) Find the horizontal asymptote of the function $y = \dfrac{3x^2 - 4x + 1}{2x^2 - 1}$. **1**

(f) Find a primitive of $\dfrac{1}{\sqrt{4 - x^2}}$. **1**

(g) Solve the equation $|x+1|^2 - 4|x+1| - 5 = 0$. **2**

Marks

Question 2 (12 marks)

(a) Let $f(x)=\frac{1}{2}\cos^{-1}\left(\frac{x}{3}\right)$.

(i) State the domain and range of the function $f(x)$. **2**

(ii) Show that $y=f(x)$ is a decreasing function. **2**

(iii) Find the equation of the tangent to the curve $y=f(x)$ at the point where $x=0$. **2**

(b) Find the derivative of $y=\ln\left(\sin^3 x\right)$. **2**

(c) (i) Write $\cos x-\sqrt{3}\sin x$ in the form $A\cos(x+\alpha)$, where $A>0$ and $0<\alpha<\frac{\pi}{2}$. **2**

(ii) Hence, or otherwise, solve $\cos x-\sqrt{3}\sin x+1=0$ for $0\leq x\leq 2\pi$. **2**

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) (i) Show that the equation $e^x - x - 2 = 0$ has a solution in the interval $1 < x < 2$. **1**

(ii) Taking an initial approximation of $x = 1.5$ use one application of Newton's method to approximate the solution, correct to three decimal places. **2**

(b) The normal at $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$ cuts the y-axis at Q and is produced to a point R such that $PQ = QR$.

(i) Show that the equation of the normal at P is $x + py = 2ap + ap^3$. **2**

(ii) Find the coordinates of Q. **1**

(iii) Show that R has coordinates $(-2ap, ap^2 + 4a)$. **1**

(iv) Show that the locus of R is a parabola, and find its vertex. **3**

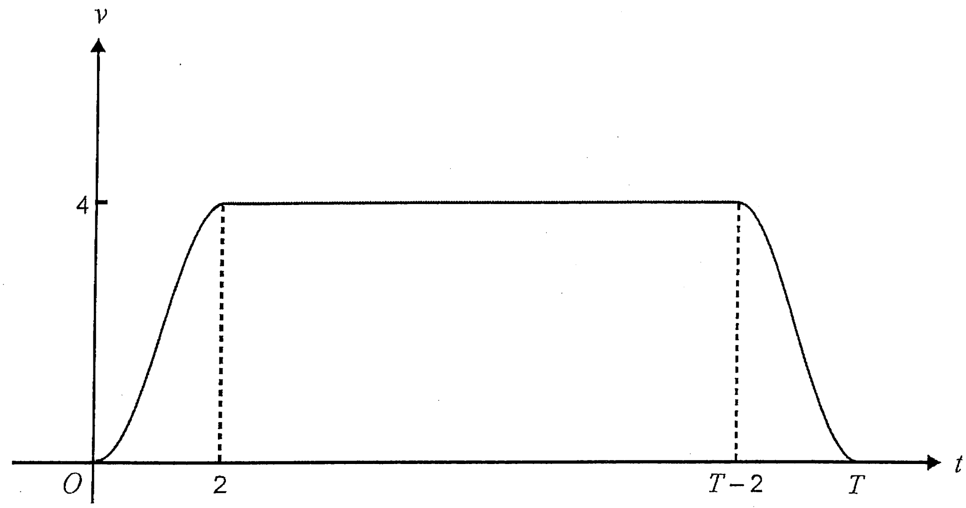
(c) If $\int_1^5 f(x)dx = 3$, find $\int_1^5 (2f(x) + 1)dx$. **2**

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) Using the substitution $u = e^x$, or otherwise, find $\int e^{\left(e^x + x\right)} dx$ 3

(b) The velocity-time graph below shows the velocity of a lift as it travels from the first floor to the twentieth floor of a tall building during the T seconds of its motion.



The velocity v m/s at time t s for $0 \le t \le 2$ is given by $v = t^2(3-t)$. After the First two seconds, the lift moves with a constant velocity of 4 m/s for a time, and then decelerates to rest in the final two seconds.

The velocity-time graph is symmetrical about $t = \frac{1}{2}T$.

(i) Express the acceleration in terms of t for the first two seconds of the motion of the lift. 1

(ii) Hence, find the maximum acceleration of the lift during the first two seconds of its motion. 2

(iii) Given that the total distance travelled by the lift during its journey is 41 metres, find the exact value of T. 2

(c) A solid is formed by rotating about the y-axis the region bounded by the curve $y = \cos^{-1} x$, the x-axis and the y-axis.

(i) Show that the volume of the solid is given by $V = \pi \int_0^{\frac{\pi}{2}} \cos^2 y \, dy$. **1**

(ii) Calculate the volume of this solid. **3**

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) Use mathematical induction to prove that $\sum_{r=1}^{n} r \times r! = (n+1)! - 1$. **3**

(b) In the expansion of $\left(2x + \frac{1}{x^2}\right)^{15}$, determine the coefficient of the term that is independent of x. **3**

(c) The acceleration of a particle P is given by the equation $a = 8x\left(x^2+1\right)$, where x is the displacement of P from the origin in metres after t seconds, with movement being in a straight line.
Initially, the particle is projected from the origin with a velocity of 2 metres per second in the negative direction.

(i) Show that the velocity of the particle can be expressed as $v = 2\left(x^2+1\right)$. **2**

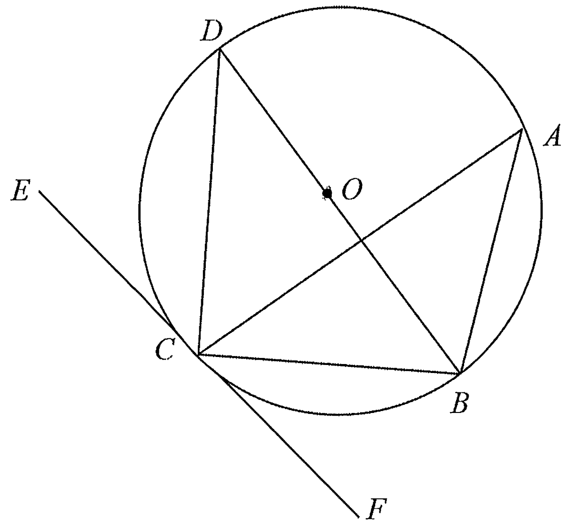
(ii) Hence, show that the equation describing the displacement of the particle at time t is given by $x = \tan 2t$. **2**

(iii) Determine the velocity of the particle after $\frac{\pi}{8}$ seconds. **2**

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a)



A, B, C and D are points on the circumference of a circle with centre O. **3**

EF is a tangent to the circle at C and the angle ECD is $60°$.

Find the value of $\angle BAC$ giving reasons.

(b) (i) By considering the expansion of $(1+x)^n$ in ascending powers of x, where n is a positive integer, and differentiating, show that **1**

$$\binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+\ldots\ldots+n\binom{n}{n}=n\left(2^{n-1}\right).$$

(ii) Hence, find an expression for $2\binom{n}{1}+3\binom{n}{2}+4\binom{n}{3}+\ldots\ldots+(n+1)\binom{n}{n}$. **2**

(c) If $f(x+2)=x^2+2$, find $f(x)$. **2**

(d) At a particular dinner, each rectangular table has nine seats, five facing the stage and four with their backs to the stage.

In how many ways can 9 people be seated at the table if

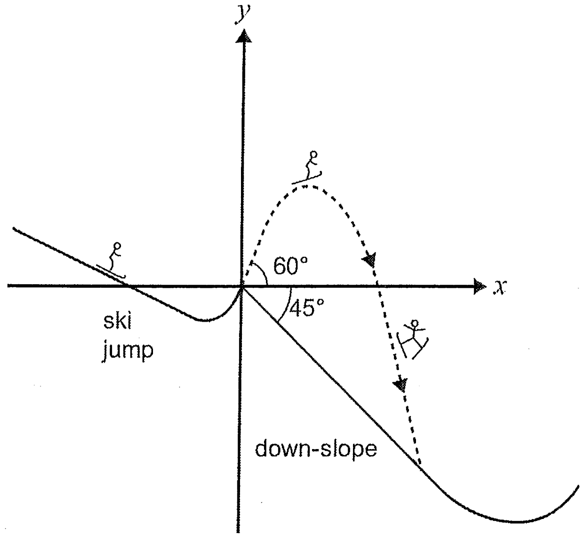
(i) John and Mary sit on the same side? **2**

(ii) John and Mary sit on opposite sides? **2**

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) A skier accelerates down a slope and then skis up a short jump (see diagram). The skier leaves the jump at a speed of 12 m/s and at an angle of 60° to the horizontal. The skier performs various gymnastic twists and lands on a straight line section of the 45° down-slope T seconds after leaving the jump.
Let the origin O of a Cartesian coordinate system be at the point where the skier leaves the jump. Displacements are measured in metres and time in seconds.
Let $g = 10ms^{-2}$ and neglect air resistance.

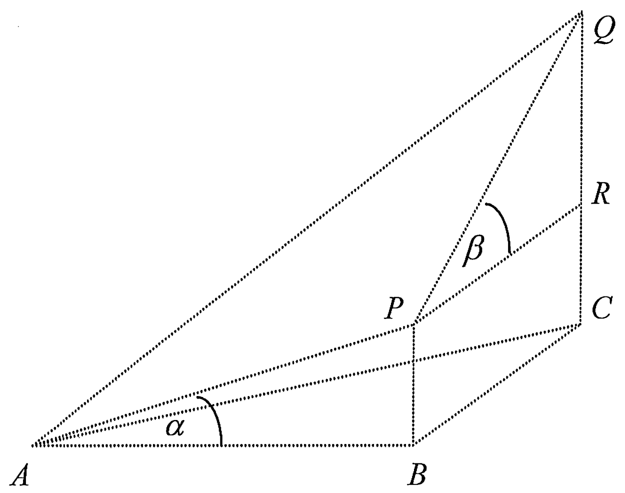


(i) Derive the cartesian equation of the skiers flight as a function of y in terms of x. **3**

(ii) Show that $T = \frac{6}{5}\left(\sqrt{3}+1\right)$. **3**

(iii) At what speed, in metres per second does the skier land on the down-slope? Give your answer correct to one decimal place. **2**

(b)



ABC is a horizontal, right-angled, isosceles triangle where $AB = BC$ and $\angle ABC = 90°$. P is vertically above B; Q is vertically above C. The angle of elevation of P from A, and Q from P are α and β respectively.

(i) If the angle of elevation of Q from A is θ, prove that **2**

$$\tan\theta = \frac{\tan\alpha + \tan\beta}{\sqrt{2}}.$$

(ii) If $\angle APQ = \phi$, prove that $\cos\phi = -\sin\alpha\sin\beta$. **2**

End of Paper

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax,$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$