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SYDNEY

2008
YEAR 11, TASK THREE

Mathematics Extension 1

EXAMINER: MR. WAGDY MICHEAL

General Instructions

- Working Time – 2 periods
- Write using a black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.
- Attempt ALL Questions

QUESTION ONE: (10 marks)

1. Solve for : $\dfrac{4}{5-x} \geq 1$. 3

2. Find the coordinates of the point which divides the interval AB with $A(1,4)$ and $B(5,2)$ externally in the ratio 1: 3. 3

3. Given that $x^2 + 4x + 5 = (x+a)^2 + b^2$, find a and b . 3

4. The degree of two polynomials, P(x) and Q(x), are n and m respectively.

   What is the degree of P(x) × Q(x)? 1

QUESTION TWO : (12 marks)

1. The Parabolas $y = x^2$ and $y = (x-1)^2$ intersect at a point A.
   - (i) Find the coordinates of A. 1
   - (ii) Find the angle between the curves at A, giving your answer to the nearest degree. 3

2. If α, β and γ are the roots of $2x^3 + x^2 - 5x + 7 = 0$, find
   - (i) $\alpha + \beta + \gamma$. 1
   - (ii) $\alpha\beta\gamma$ 1
   - (iii) $\alpha^2 + \beta^2 + \gamma^2$ 3

3. A polynomial is given by $P(x) = x^3 + ax^2 + bx - 18$. Find the values of a and b if $(x+2)$ is a factor of $P(x)$ and if -24 is the remainder when $P(x)$ is divided by $(x-1)$. 3

QUSTION THREE: (15 marks)

1. Solve for $x : \dfrac{2x+3}{x-4} \geq 1$ 3

2. (i) If $P(x) = x^6 - 2x^4 - 6x^2 + k$, find the value of k if $(x-2)$ is a factor. 2

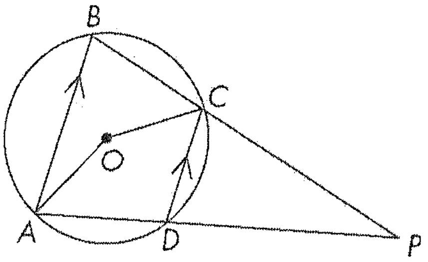
   (ii) Find another linear factor of $P(x)$ in part (i). 2

3. ABC is a triangle inscribed in a circle. The tangent at A meets BC produced at D. Angle DAC =40 degrees and angle CDA = 10.

   i) Draw a neat diagram, showing the above information. 2
   ii) Show that BC is a diameter of the circle. 3
   iii) Prove $AD^2 = DC \times DB$ 3

QUESTION FOUR: (12 marks)

1. In the diagram below, O is the centre of the circle and $AB \parallel DC$. AD and BC meet at P.



Prove: (i) $CP = DP$. 2

(ii) ΔABP is isosceles. 2

(iii) $OAPC$ is a cyclic quadrilateral. 2

2. If α, β, γ are the roots of the quadratic equation $x^3 + 2x^2 - 5x - 4 = 0$, find:
   i) $\alpha + \beta + \gamma$ 1
   ii) $\alpha\beta + \beta\gamma + \gamma\alpha$ 1
   iii) $\alpha\beta\gamma$ 1
   iv) $4\alpha + 4\beta + 4\gamma + 10$ 1
   v) $\dfrac{1}{\alpha\beta} + \dfrac{1}{\beta\alpha} + \dfrac{1}{\gamma\alpha}$ 2

QUESTION FIVE: (11 marks)

1. Find the acute angle between the lines $4x + y = 6$ and $x - 7y = 3$ 2

2. A is the point (-2, 1) and B is the point (x, y). The point P(13,-9) divides AB externally in the ratio 5 : 3. Find the values of x and y. 3

3. A radio tower stands on level ground. The angle of elevation to the top of the tower from a house due east is 30°. From another house due north of the tower, the angle of elevation to the top is 45°. the houses are 100 metres apart.

   (i) Draw a neat diagram and clearly indicate the above information on it. 1

   (ii) Show that $\frac{h^2}{\tan^2 30} + \frac{h^2}{\tan^2 45} = 100^2$, where h is the height of the tower. 3

   (iii) Hence, find the height of the tower. 2

Question ONE

① $x^2+4x+5=(x+a)^2+b^2$

$x^2+4x+4+1=(x+a)^2+b^2$

$(x+2)^2+1=(x+a)^2+b^2$

$\therefore a=2,\ b^2=1$

$\therefore b=\pm 1$

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② $A(\overset{x_1,y_1}{1,4}),\ B(\overset{x_2,y_2}{5,2})$

$-\overset{m}{1}:\overset{n}{3}$

$x=\dfrac{x_2m+x_1n}{m+n},\ y=\dfrac{y_2m+y_1n}{m+n}$

$=\dfrac{5\times -1+1\times 3}{-1+3}$	$=\dfrac{2\times -1+4\times 3}{-1+3}$
$=\dfrac{-2}{2}$	$=\dfrac{10}{2}$
$=-1$	$=5$

$(-1,5)$

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③ $\dfrac{4}{5-x}\geqslant 1$

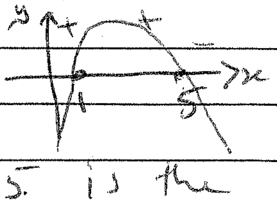
$4(5-x)\geqslant(5-x)^2$

$4(5-x)-(5-x)^2\geqslant 0$

$(5-x)[4-(5-x)]\geqslant 0$

$(5-x)(x-1)\geqslant 0$

$1\leq x\leq 5$



$\therefore 1\leq x<5$ is the Solution to $\dfrac{4}{5-x}\geqslant 1$

④ Degree is "mn" or "nm"

Question TWO

①(i) $y=x^2,\ y=(x-1)^2$

$(x-1)^2=x^2$

$\cancel{x^2}-2x+1=\cancel{x^2}$

$-2x=-1$

$\boxed{x=\frac{1}{2}}$

$\therefore y=\left(\frac{1}{2}\right)^2=\frac{1}{4}$

$\therefore A\left(\frac{1}{2},\frac{1}{4}\right)$

(ii) $y=x^2$

$m_1=y'=2x$ at $\left(\frac{1}{2},\frac{1}{4}\right)$

$m_1=1$

$y=(x-1)^2$

$m_2=y'=2(x-1)$ at $\left(\frac{1}{2},\frac{1}{4}\right)$

$m_2=-1$

$\tan\theta=\left|\dfrac{m_1-m_2}{1+m_1m_2}\right|$

$\tan\theta=\left|\dfrac{2}{1-1}\right|=\dfrac{2}{0}$

$\therefore\theta=90^\circ$

② (i) $2x^3 + x^2 - 5x + 7 = 0$

(i) $\alpha + \beta + \gamma = -b/a$

$= \frac{-1}{2}$

(ii) $\alpha\beta\gamma = -d/a$

$= -7/2$

(iii) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$

$= (-1/2)^2 - 2\left(\frac{-5}{2}\right)$

$= 1/4 + 5 = 5\frac{1}{4}$

③ $p(x) = x^3 + ax^2 + bx - 18$

Since $(x+2)$ is a factor

$\therefore p(-2) = 0$

$\therefore 0 = (-2)^3 + a(-2)^2 + b(-2) - 18$

$0 = -8 + 4a - 2b - 18$

$13 = 2a - b \longrightarrow ①$

$-24 = 1^3 + a(1)^2 + b(1) - 18$

$-7 = a + b \longrightarrow ②$

①+② $\Rightarrow 6 = 3a$

$\boxed{a = 2}$

$a + b = -7$

$2 + b = -7$

$\boxed{b = -9}$

③ Question Three

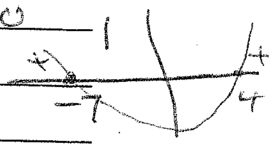
(i) $\frac{2x+3}{x-4} \geq 1$

$(2x+3)(x-4) \geq (x-4)^2$

$(2x+3)(x-4) - (x-4)^2 \geq 0$

$(x-4)\left[(2x+3) - (x-4)\right] \geq 0$

$(x-4)(x+7) \geq 0$



$x \leq -7 \ , \ x \geq 4$

$\therefore x \leq -7 \ , \ x > 4$ are the solution to $\frac{2x+3}{x-4} \geq 1$

② (i) $p(x) = x^6 - 2x^4 - 6x^2 + k$

Since $(x-2)$ is a factor

$\therefore p(2) = 0$

$0 = 2^6 - 2(2)^4 - 6(2)^2 + k$

$\therefore$ [illegible] $= 64 - 32 - 24 + k$

$k = -8$

Test $x = -2$

$p(-2) = (-2)^6 - 2(-2)^4 - 6(-2)^2 - 8$

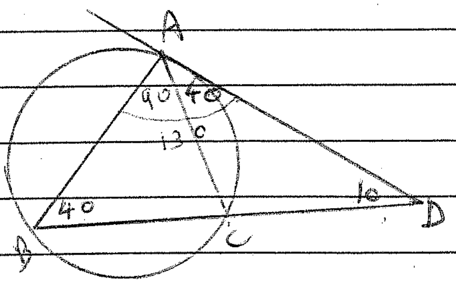
$= 0$

$\therefore x = -2$ is a root

$\therefore (x+2)$ is another factor.

Question Three (Cont.)

③ⓘ



ii) $\angle DAC = \angle ABC = 40^\circ$
(angle between tangent and a chord equal to the angle in the alternate segment)

$\angle BAD = 180^\circ - (40 + 10)$
$= 130^\circ$
(angle sum of a Δ is 180°)

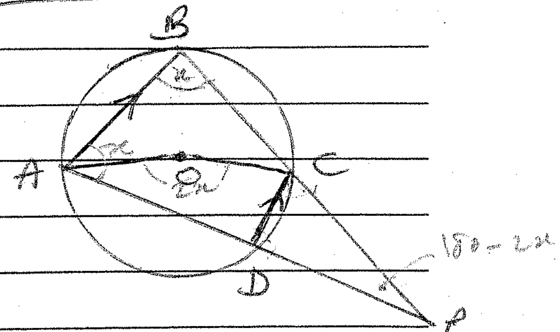
$\therefore \angle BAC = 130^\circ - 40^\circ$
$= 90^\circ$

$\therefore$ BC is a diameter Since angle in a semi-circle is a right angle.

iii) In Δ's ADC, ADB
$\angle D$ is a common angle.
$\angle DAC = \angle ABC$ proven in (i)
$\therefore \Delta ADC \,|||\, \Delta ADB$
$\therefore$ corresponding sides in the same ratio
$\therefore \frac{AD}{BD} = \frac{DC}{AD}$
$\therefore AD^2 = DC \times DB$.

Question Four

①



ⓘ $\angle PCD = \angle CBA \rightarrow$ ①
(Corresponding angles are equal and $DC \parallel AB$)

$\angle PDC = \angle CBA \rightarrow$ ② (Exterior angle of a cyclic Quad. equal to the interior opp. angle).

from 1, 2
$\therefore \angle PCD = \angle PDC \rightarrow$ ③
$\therefore \Delta PDC$ is an Isos. Δ
(Base angles of Isos. Δ are equal)
$\therefore CP = DP$

(ii) Since $DC \parallel AB$ (given)
$\therefore \angle PCD = \angle PBA$ and $\angle PDC = \angle PAB$
Corresponding angles on Parallel lines are equal.
But $\angle PCD = \angle PDC$ from ③

$\therefore \angle PBA = \angle PAB \rightarrow$ ④
$\therefore \Delta PAB$ is an Isos. Δ
(Base angles of Isos. Δ are equal)

(iii) $\angle PBA = \angle PAB = x$ (say) from ④
$\therefore \angle APB = 180 - 2x$ (angle sum of a Δ)
$\angle AOC = 2\angle ABC = 2x$
(angle at the centre is twice the size
Now, $\angle APB + \angle AOC = 180 - 2x + 2x$
$= 180^\circ$
$\therefore$ OAPC is a cyclic Quad.
Since (Sum of opposite angles are supplementary).

Question Four (Cont.)

② (i) $x^3+2x^2-5x-4=0$

$\alpha+\beta+\gamma=-2$

(ii) $\alpha\beta+\beta\gamma+\alpha\gamma=\frac{c}{a}$

$=-5$

(iii) $\alpha\beta\gamma=-d/a=4$

(iv) $4(\alpha+\beta+\gamma)+10$

$=4(-2)+10$

$=2$

(v) $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}$

$=\frac{\gamma+\alpha+\beta}{\alpha\beta\gamma}$

$=\frac{-2}{4}$

$=-\frac{1}{2}$

Question five

① $4x+y=6$ | $x-7y=3$

$y=-4x+6$

$m_1=-4$ | $m_2=\frac{1}{7}$

$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$=\left|\frac{-4-\frac{1}{7}}{1+-4\times\frac{1}{7}}\right|$

$=\left|\frac{-4\frac{1}{7}}{3/7}\right|$

$=\left|\frac{-29}{3}\right|$

$\therefore\ \theta=84^\circ 6'$

② $A(\overset{x_1,\ y_1}{-2,1})$ $B(\overset{x_2\ y_2}{x,y})$ $p(\overset{x\ y}{13,-9})$

$m\ n$

$5:-3$

$x=\frac{x_2m+x_1n}{m+n}$

$13=\frac{x\cdot5+-2\cdot-3}{5+-3}$

$13=\frac{5x+6}{2}$

$26=5x+6$

$20=5x$

$\boxed{x=4}$

$y=\frac{y_2m+y_1n}{m+n}$

$-9=\frac{y\cdot5+1\cdot-3}{5+-3}$

$-9=\frac{5y-3}{2}$

$-18=5y-3$

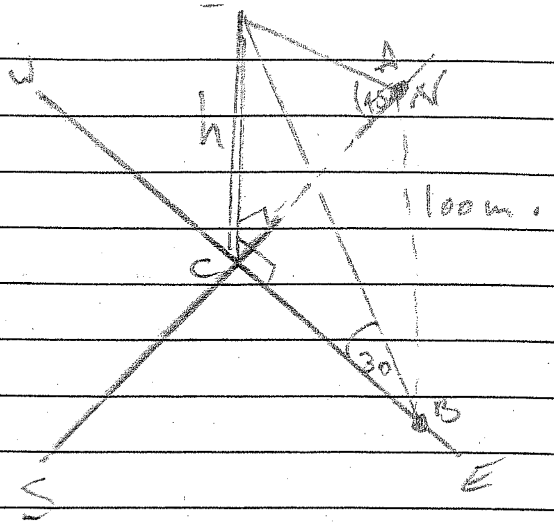
$-15=5y$

$\boxed{y=-3}$

$\therefore B(4,-3)$

③

(i)



(ii) In ΔBCD, $\tan 30 = \frac{h}{BC}$

$BC = \frac{h}{\tan 30}$

In ΔACD, $\tan 45 = \frac{h}{AC}$

$\therefore AC = \frac{h}{\tan 45}$

In ΔBCA,

$AB^2 = AC^2 + BC^2$

Since $\angle ACB = 90$

$\frac{h^2}{\tan^2 45} + \frac{h^2}{\tan^2 30} = 100^2$

(iii) $h^2\left(\frac{1}{\tan^2 45} + \frac{1}{\tan^2 30}\right) = 100^2$

$h^2\left(\frac{1}{1} + \frac{1}{\left(\frac{1}{\sqrt{3}}\right)^2}\right) = 100^2$

$h^2(1 + 3) = 100^2$

$h^2 = \frac{10000}{4}$

$h^2 = 2500$

$\therefore h = 50m$.