N.S.W. DEPARTMENT OF EDUCATION

HIGHER SCHOOL CERTIFICATE EXAMINATION 1970

MATHEMATICS - PAPER B (2F) - EQUIVALENT TO 3U and 4U - 1st PAPER

Instructions: Time allowed 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work. Mathematical tables will be supplied.

QUESTION 1 (12 Marks)

(i) Find the second derivative of e^{3x^2}.

(ii) Find the area above the x-axis and under the curve $y = \sec^2 x$ between $x = 0$ and $x = \pi/4$.

(iii) Find a primitive (indefinite integral) of $\dfrac{\cos x}{1 + \sin x}$

(iv) Write down the formula for $\tan(A + B)$ and hence show that $\tan 2x = 2 \tan x/(1 - \tan^2 x)$.

QUESTION 2 (9 Marks)

(i) The height x at time t of a particle projected vertically upwards from the ground is given by $x = 32t - 16t^2$. Find:

(a) the initial velocity; (b) the greatest height reached.

(ii) The coordinate axes are translated to a new origin at the point (1, 2). What is the new form of the curve $xy = 2$?

(iii) Find the sum $e^x + e^{-x} + e^{-3x} + e^{-5x} + \ldots + e^{-(2k+1)x}$.

QUESTION 3 (9 Marks)

(i) Find the stationary point of the curve $y = x^6 + 1$ and determine its nature.

(ii) Five cards are labelled with the numbers 2, 4, 6, 8 and 10 respectively. Two cards are drawn at random from those five (without replacement). Find the probability that the sum of the numbers on those two cards exceeds 15.

(iii) Use one step of Newton's method to find an improved value of that root of $f(x) = e^{4x} - \sqrt{x + 0.81}$ which lies close to $x = 0$ (express your answer to one significant figure).

QUESTION 4 (10 Marks)

(i) Show that there exists one value of the constant a for which the polynomial $P(x) = x^4 + 2x^3 - x^2 - 8x - a$ is divisible by $Q(x) = x^2 - 4$; and find this value of a.

(ii) Hence, or otherwise, find all real roots of the polynomial $P(x)$ with that particular value of a.

QUESTION 5 (10 Marks)

(i) The segment of the curve $y = \frac{1}{2}x^2$ lying between the abscissae $x = -1$ and $x = +1$ is rotated about the line $y = \frac{1}{2}$ in the xy-plane. Find the volume of the resulting figure of revolution.

(ii) A point source of light rays is placed at the point $P = (0, \frac{1}{2})$ in the xy-plane. By using known properties of parabolas, or otherwise, show that all light rays emitted by that source return again to the point P after a suitable number of reflections from the surface of revolution in (i), considered as a perfectly reflecting mirror surface.

(OMIT)
QUESTION 6 (10 Marks)

Given the two planes $x + y - z = 1$ and $2x - y + z = 0$.

(i) Find the angle between them.

(ii) Find the equation of the plane which passes through the origin and is perpendicular to both planes.

(iii) Find the coordinates of that point P on the line of intersection of the two given planes, which lies nearest the origin.

QUESTION 7 (10 Marks)

(i) Define $\tan^{-1}(x)$ and show that its derivative is $1/(1 + x^2)$.

(ii) Differentiate $\tan^{-1}(x/\sqrt{1 - x^2})$.

QUESTION 8 (10 Marks)

(i) Use mathematical induction to prove the identity

$$1.2.3.4 + 2.3.4.5 + 3.4.5.6 + \dots + n(n + 1)(n + 2)(n + 3)$$
$$= \frac{1}{5}n(n + 1)(n + 2)(n + 3)(n + 4)$$

(ii) Hence state the limit of $S_n = \frac{1}{n^5}\sum_{k=1}^{n} k(k + 1)(k + 2)(k + 3)$ as n increases indefinitely.

QUESTION 9 (10 Marks)

One of six spark plugs in a car is known to be faulty. The plugs are tested one by one.

(i) What is the probability that the faulty spark plug is the first one tested?

(ii) What is the probability that the faulty plug is the very last one tested?

(iii) Find the expected value of the number of good plugs tested before the faulty plug is reached.

(iv) Find the probability that, in five cars with the same condition, the faulty plug should be the very last one tested in exactly three of the cars (express your answer as a fraction).

QUESTION 10 (10 Marks)

An object passes through the point $x = 2$ at time $t = 0$, and has a velocity v at time t given by $v = \frac{1}{2}(1 + 3t)^{-\frac{1}{4}}$.

(i) Find the position x of the object at time $t = 5$.

(ii) How long does this object take to move a distance of 14 units from its starting point?

(iii) How long would that object have taken to move the same distance, if its velocity v had not altered from its initial value?

(iv) Find the acceleration of the object with the velocity given at the beginning of this question. Show that this acceleration is proportional to a power of the velocity, and give the precise relationship between the acceleration and the velocity.