SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

2003
TRIAL
HIGHER SCHOOL CERTIFICATE

Mathematics

General Instructions

- Reading time — 5 minutes
- Working time — 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided on the back of this
- All necessary working should be shown in every question

Total marks — 120

- Attempt questions 1—10
- All questions are of equal value, the mark value is shown beside each part.
- Hand up your paper in three parts: Section A, Questions 1, 2, 3, & 4; Section B, Questions 5, 6, and 7; Section C, Questions 8, 9, and 10.

Examiner: P.Bigelow

Note: This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a1>0, \quad -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x>0$

Total marks – 120
Attempt Questions 1–10
All questions are of equal value

Answer each Section in a SEPARATE writing booklet. Extra writing booklets are available.

Section A **Marks**

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate, correct to three significant figures: **2**

$$\frac{4{\cdot}73 + 3{\cdot}1^2}{5{\cdot}6 \times 9{\cdot}4}$$

(b) Solve $x^2 = 10x$ **2**

(c) Differentiate:

(i) $4 - 3x^2$ **2**

(ii) xe^x **2**

(iii) $\dfrac{\sin x}{x}$ **2**

(d) Write down a quadratic equation with roots $3 + \sqrt{2}$ and $3 - \sqrt{2}$. **2**

Question 2 (12 marks)

(a) Simplify $\dfrac{x^2-4x}{x-4}$. **2**

(b) Convert $\dfrac{3\pi}{5}$ to degrees. **1**

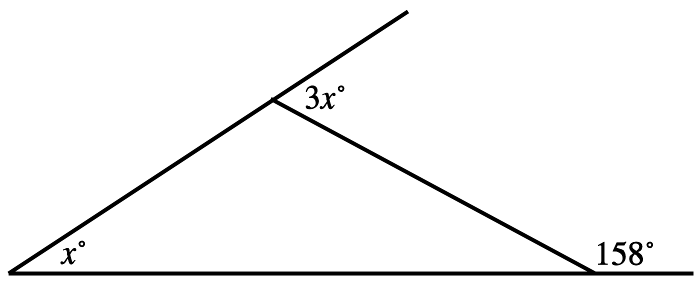
(c) If $\sqrt{75}+\sqrt{80}-\sqrt{12}=4\sqrt{c}+a\sqrt{3}$, find a and c. **2**

(d) Find (i) $\displaystyle\int \frac{dx}{1+x}$ **1**

(ii) $\displaystyle\int_0^1 \frac{4}{e^{2x}}\,dx$ **2**

(e) Find the equation of the normal to $y=(3x+4)^3$ at the point where $x=-1$. **2**

(f) **2**



In the diagram above, find the value of x.

Question 3 (12 marks)

(a) Given the function $f(x) = \sqrt{64 - x^2}$

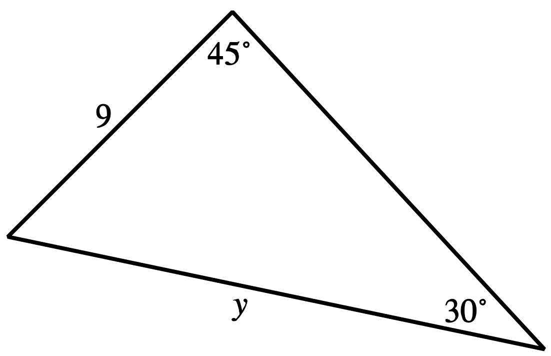
state the (i) domain **1**

and (ii) range of the function. **1**

(b) Solve this pair of equations simultaneously. **2**

$$x + 3y = -7$$
$$4x - y = -2$$

(c) Find the exact value of y. **2**



(d) If $\int_0^a (x-3)\,dx = -4$, find the value(s) of a. **3**

(e) Factorise (i) $16 - a^2$ **1**

(ii) $4c^2 + 15c - 4$. **2**

Question 4 (12 marks)

(a) Solve for x: **2**

$$3^x - 3^{x-1} = 54.$$

(b) Simplify $\dfrac{\cos(90^\circ - \theta)}{\sin(180^\circ + \theta)}$. **2**

(c) Evaluate. **2**

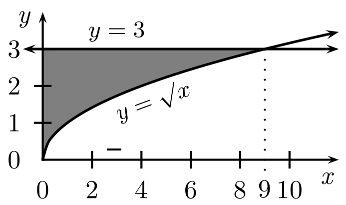
$$3 + 5 + 7 + 9 + \cdots + 81$$

(d) If α and β are the roots of $x^2 - 4x + 2 = 0$, find the value of :

(i) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **1**

(ii) $\alpha^2 + \beta^2$. **2**

(e) **3**



The diagram shows the area bounded by the y-axis, the curve $y = \sqrt{x}$, and the line $y = 3$. Find the area of the shaded region.

Section B Use a SEPARATE writing booklet. **Marks**

Question 5 (12 marks)

(a) Given the parabola $(x+2)^2 = 8(y-1)$, Write down

(i) the co-ordinates of the focus, **1**

(ii) the equation of the directrix. **1**

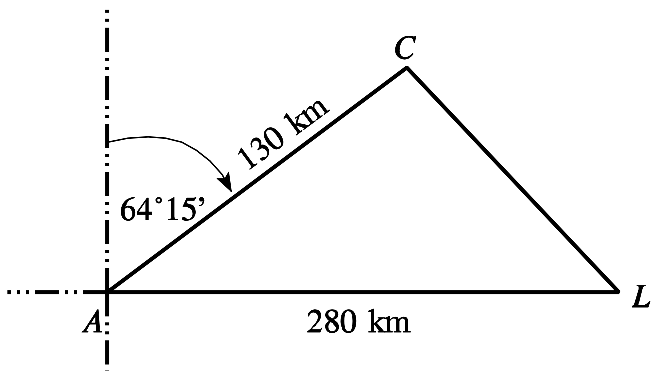
(b) (i) Draw a number plane and mark on it the points A (4, 3), B (12, −3), and C (10, 7). **1**

(ii) Find the equation of the line AB. **1**

(iii) Find the distance of C from the line AB. **2**

(iv) Find the area of the triangle ABC. **2**

(c)



A ship A is 280 km west of a lighthouse L. It travels a distance of 130 km on a bearing of N64°15'E to a position C.

(i) Calculate the distance from the lighthouse to the ship's position at C. **2**

(ii) Find the bearing of C from the lighthouse L. **2**

Question 6 (12 marks)

(a) (i) On the same set of axes, carefully sketch the graphs of $y = \cos x$ and $y = \sqrt{3}\sin x$ where $0 \le x \le 2\pi$. **3**

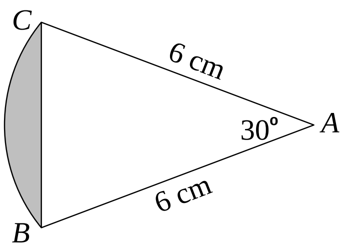
(ii) Find the x-values of the two points of intersection. **2**

(iii) Hence solve $\cos x < \sqrt{3}\sin x$ for $0 \le x \le 2\pi$. **1**

(b) The table below shows the values of $f(x)$ for $0 \le t \le 2$. **3**

t	0	0·5	1	1·5	2
$f(t)$	0	0·32	0·39	0·35	0·26

Use the Trapezoidal Rule with 5 function values to approximate $\int_0^2 f(t)\,dt$ correct to 1 decimal place.

(c) 

In the diagram $\angle CAB = 30^\circ$.
CB is a circular arc of radius 6 cm.

(i) Find the area of ΔABC. **1**

(ii) Calculate the exact area of the shaded region. **2**

Section B continued **Marks**

Question 7 (12 marks)

(a) Consider the curve $y = 2x^3 + 3x^2 - 12x - 9$.

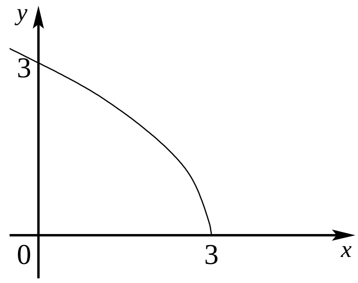
(i) Find all stationary points and determine their nature. **2**

(ii) Find any points of inflexion. **2**

(iii) Sketch the curve for $-3 \le x \le 3$, showing the y-intercept. **2**

(iv) For what values of x is the curve increasing and concave down? **2**

(b) A solid is formed by rotating the part of the curve $y = \sqrt{9 - 3x}$ between the points (3, 0) and (0, 3) about the y-axis, as shown in the diagram below. Find the volume of the solid. **2**



(c) The volume V cm^3 of a balloon is increasing such that its volume at any time t seconds is given by $V = \dfrac{\pi t^3}{3} - \dfrac{\pi t^2}{6} + \dfrac{1}{2}$. Find the rate at which the volume is increasing when $t = 3$. **2**

Section C Use a SEPARATE writing booklet. **Marks**

Question 8 (12 marks)

(a) A football club held a raffle to raise money for the end-of-season trip, 100 tickets were sold and two prizes were offered. Two tickets were drawn without replacement to determine the prize-winners.

Frank bought some of the tickets. The probability that he won both prizes was $\frac{2}{275}$. Find:

(i) The number of tickets bought by Frank. **2**

(ii) The probability of his winning at least one prize. **2**

(b) An Electrical Goods store has a special deal on digital wide-screen TVs. It is offering a loan of \$12 000 with an interest free period of 12 months. From then on, interest is charged at the rate of 12% p.a. monthly reducible.

Patrick takes out the loan and agrees to repay it over four years by making 48 equal monthly repayments of $\$M$.

Let $\$A_n$ be the amount owing after n repayments.

(i) Find an expression for A_{12}. **1**

(ii) Show that $A_{14} = (12\,000 - 12M) \times 1{\cdot}01^2 - M(1 + 1{\cdot}01)$. **2**

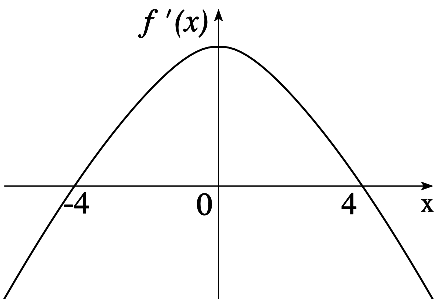
(iii) Find an expression for A_{48}. **2**

(iv) Find the value of M. **3**

Section C continued **Marks**

Question 9 (12 marks)

(a) Solve $\log_3 x - \log_3(x-2) = \frac{2}{3}\log_3 27$. **3**

(b) 

The diagram shows the graph of the gradient function for the curve $y = f(x)$.

(i) What type of point occurs on $y = f(x)$ at $x = 4$? Justify your answer. **2**

(ii) If $f(4) = 6$ and $f(-4) > 0$, sketch $y = f(x)$. **2**

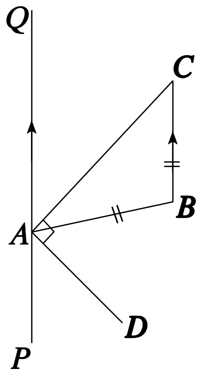
(c) A particle moves with an acceleration given by $f = \sqrt{t} - \dfrac{1}{\sqrt{t}}$. Initially the particle is moving at $\frac{4}{3}$ m.s $^{-1}$ and is $\frac{4}{3}$ m to the right of O.

(i) Express the velocity v in terms of t. **2**

(ii) Find the displacement x when $t = 1$. **3**

Question 10 (12 marks)

(a)

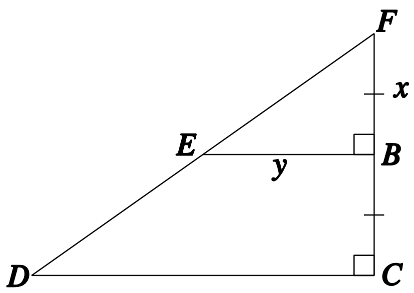


In the diagram ABC is a triangle with $AB = BC$. The line PQ passes through A parallel to BC, and the line AD is perpendicular to AC.

(i) Prove that AC bisects $\angle QAB$. **2**

(ii) Deduce that AD bisects $\angle PAB$. **2**

(b)



Farmer George wishes to establish two separate paddocks and sets up his field FCD so that there are fences at FC, DC, and EB as shown on the diagram. The side FD is an existing fence, so no fencing will be required for that side. B is the middle of FC.
FB is x metres and EB is y metres.

(i) Write down expressions in terms of x and y for:

(α) BC and DC. **2**

(β) The area A of the field FCD. **1**

(γ) The amount of new fencing that the farmer would need. **1**

(ii) If the area of the field is 1200 m^2, show that the length of fencing required is given by: **2**

$$L = 2x + \frac{1800}{x} \text{ metres.}$$

(iii) Hence find the values of x and y so that the farmer uses the minimum amount of fencing. **2**

END OF THE PAPER