Sydney Boys High School

4 unit mathematics

Trial HSC Examination 1986

1. **(a)** Find the moduli and arguments of the roots of the equation $z^2 - 2iz - 2 = 0$.
(b) Determine the locus on the Argand diagram specified by $z\overline{z} = z + \overline{z}$.
(c) Illustrate on separate Argand diagrams
(i) $|z - 2 - i| = i$
(ii) $\arg(z - i) = \frac{\pi}{3}$
(iii) $\Im(z^2) = 2$

2. **(a)** Find y correct to 2 decimal places if $2^{3y+1} = 5^{y+1}$.
(b) Solve for x: $\tan^{-1} 3x - \tan^{-1} 2x = \tan^{-1} \frac{1}{5}$.
(c) For the function $y = \frac{x}{(x-1)^2}$
(i) Find any turning points or points of inflexion.
(ii) Sketch the graph of the function.

3. **(a)** Given $f(x) = \frac{7x}{(x^2+3)(x+2)}$
(i) Express $f(x)$ as a sum of partial fractions.
(ii) Evaluate $\int_0^3 f(x)\, dx$.
(b) Find the co-ordinates of the point on the graph of $x^2y + xy^2 = 16$ at which the tangent is parallel to the x-axis.
(c) Find the general solution of the equation $\cos x + \cos 2x + \cos 3x = 0$.

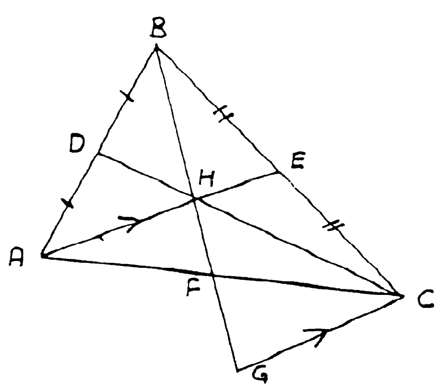
4. **(a)** **(i)** Show that if the polynomial $P(x)$ has a zero b of multiplicity m, then $P'(x)$ has the zero b with multiplicity $m - 1$.
(ii) Given that the polynomial $P(x) = x^4 + x^3 - 3x^2 - 5x - 2$ has a three-fold zero, find all the zeros of $P(x)$.
(b) Show that the volume of the largest cylinder that can be cut from a solid sphere of radius r cm is $\frac{4\pi r^3}{3\sqrt{3}}$ cm^3.

5. **(a)** **(i)** Sketch $\frac{x^2}{9} + \frac{y^2}{4} = 2$ indicating the centre, foci and directrices.
(ii) P and Q are the end-points of a focal chord of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If P and Q have parameters α and β, show that $e^{-1} = \frac{\sin\alpha - \sin\beta}{\sin(\alpha-\beta)}$
(b) **(i)** Prove that the tangent at a point $(ct, \frac{c}{t})$ to $xy = c^2$ is $x + t^2y = 2ct$.
(ii) The point P is a point of intersection of the rectangular hyperbolas $x^2 - y^2 = 2$ and $xy = 1$. The tangent at P to the first hyperbola meets its asymptotes in A, C and the tangent at P to the second hyperbola meets its asymptotes in B, D. Prove that $ABCD$ is a square.

6. (a) A particle of unit mass moves in a straight line. It is placed at the origin on the x-axis and is then released from rest. When at position x, its acceleration is given by $-9x + \frac{5}{(2-x)^2}$. Prove that the particle moves between two points on the x-axis and find these points.
(b) If two sources of light are 10m apart, and one is 8 times as bright as the other, where will be the darkest point on the interval joining them? (The intensity of light varies inversely as the square of the distance from the source of light.)

7. (a) The equation $x^3 + 2x - 1 = 0$ has roots $x = \alpha, \beta, \gamma$. In each of the following cases find an equation with numerical coefficients having the roots stated.
(i) $-\alpha, -\beta, -\gamma$.
(ii) $\alpha, -\alpha, \beta, -\beta, \gamma, -\gamma$.
(iii) $\alpha^2, \beta^2, \gamma^2$.
(b) Write down in mod-arg form, the five roots of $z^5 = 1$. Show that when these five roots are plotted on an Argand diagram they form the vertices of a regular pentagon of area $\frac{5}{2}\sin\frac{2\pi}{5}$. By combining appropriate pairs of these roots show that for $z \neq 1$. $\frac{z^5-1}{z-1} = (z^2 - 2z\cos\frac{2\pi}{5} + 1)(z^2 - 2z\cos\frac{4\pi}{5} + 1)$. Deduce that $x = \cos\frac{2\pi}{5}, \cos\frac{4\pi}{5}$ are the roots of the equation $4x^2 + 2x - 1 = 0$.

8. (a)



In the triangle ABC, AE bisects BC and CD bisects AB. Through the point H, the intersection of AE and CD, BH is drawn to G cutting AC in F and making CG parallel to AE.
(i) Prove that $AF = FC$.
(ii) What property of a triangle is proven?
(iii) Prove that $BH : HF = 2 : 1$.
(b) A sequence of wholw numbers $\{u_0, u_1, u_2, \ldots, u_n, \ldots\}$ is generated by $u_{r+2} = 3u_{r+1} - 2u_r$ where $r = 0, 1, 2, \ldots$. Defining $u_{r+1} - u_r = d_r$, $(r \geq 0)$ prove that
(i) $d_{r+1} = 2d_r$
(ii) $d_{n-1} = 2^{n-1}d_0$.