Question 1:

(a)(i) Find the derivative of $x^2 \cos x$. 2

(ii) Evaluate $\int_1^6 \frac{x}{x^2+4}\,dx$. 2

(b)(i) Sketch $y = |x+1|$. 2

(ii) Hence or otherwise solve $|x+1| = 3x$. 1

(c) If $f(x) = 2\sin^{-1}(3x)$, find

(i) the domain and range of $f(x)$, 2

(ii) $f\left(\frac{1}{6}\right)$, 1

(iii) $f'\left(\frac{1}{6}\right)$. 2

QUESTION 2: **(START A NEW PAGE)**

(a) P(-7,3), Q(9,15) and B(14,0) are three points and A divides the interval PQ in the ratio 3:1. Prove that PQ is perpendicular to AB. 3

(b) By using the substitution $u^2 = x+1$ evaluate $\int_0^3 \frac{x+2}{\sqrt{x+1}}\,dx$. 3

(c) Water flows from a hole in the base of a cylindrical vessel at a rate given by 6

$$\frac{dh}{dt} = -k\sqrt{h}$$

where k is a constant and h mm is the depth of water at time t minutes.
If the depth of water falls from 2500mm to 900mm in 5 minutes, find how much longer it will take to empty the vessel.

QUESTION 3: **(START A NEW PAGE)**

(a) Find the value of the constant term in the expansion of $\left(3x+\dfrac{2}{\sqrt{x}}\right)^6$. 3

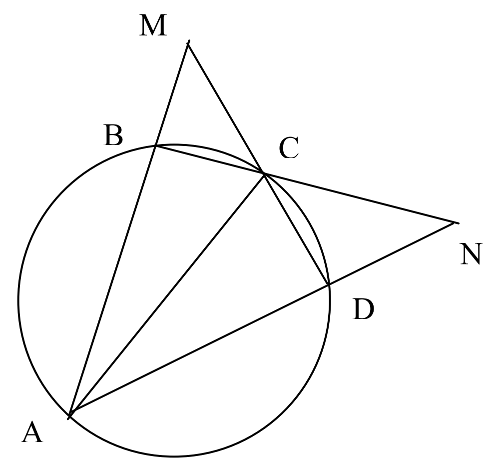
(b) Three boys (Adam, Bruce, Chris) and three girls (Debra, Emma, Fay) form a single queue at random in front of the school canteen window. Find the probability that:

(i) the first two to be served are Emma and Adam in that order, 2

(ii) a boy is at each end of the queue, 1

(iii) no two girls stand next to each other. 1

(c) In the figure ABM, DCM, BCN and ADN are straight lines and $\angle AMD = \angle BNA$.



(i) Copy the diagram onto your answer sheet and prove that $\angle ABC = \angle ADC$. 3

(ii) Hence prove that AC is a diameter. 2

QUESTION 4: **(START A NEW PAGE)**

(a)(i) Given that $\sin^2 A + \cos^2 A = 1$, prove that $\tan^2 A = \sec^2 A - 1$. 2

(ii) Sketch the curve $y = 4\tan^{-1} x$ clearly showing its range. 2

(iii) Find the volume of the solid formed when the area bounded by the curve $y = 4\tan^{-1} x$, the y-axis and the line $y = \pi$ is rotated one revolution about the y-axis. 2

(b)(i) An object has velocity $v\ ms^{-1}$ and acceleration $\ddot{x}\ ms^{-2}$ at position $x\ m$ from the origin, show that $\dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right) = \ddot{x}$. 2

(ii) The acceleration (in ms^{-2}) of an object is given by $\ddot{x} = 2x^3 + 4x$.

(α) If the object is initially 2 m to the right of the origin traveling with velocity $6\ ms^{-1}$, find an expression for v^2 (the square of its velocity) in terms of x. 2

(β) What is the minimum speed of the object? (Give a reason for your answer) 2

<u>QUESTION 5</u>: **(START A NEW PAGE)**

(a) The curves $y = e^{-2x}$ and $y = 3x + 1$ meet on the y-axis. Find the size of the acute angle between these curves at the point where they meet. 3

(b)(i) Sketch the function $y = f(x)$ where $f(x) = (x-1)^2 - 4$ clearly showing all intercepts with the co-ordinate axes. (Use the same scale on both axes) 2

(ii) What is the largest positive domain of f for which *f(x)* has an inverse $f^{-1}(x)$? 1

(iii) Sketch the graph of $y = f^{-1}(x)$ on the same axes as (i). 1

(c) In tennis a player is allowed a maximum of two serves when attempting to win a point. If the first serve is not legal it is called a fault and the server is allowed a second serve. If the second serve is also illegal then it is called a double fault and the server loses the point. The probability that Pat Smash's first serve will be legal is 0.4. If Pat Smash needs to make a second serve then the probability that it will be legal is 0.7.

(i) Find the probability that Pat Smash will serve a double fault when trying to win a point. 2

(ii) If Pat Smash attempts to win six points, what is the probability that he will serve at least two double faults? (Give answer correct to 2 decimal places) 3

<u>QUESTION 6</u>: **(START A NEW PAGE)**

(a) A spherical bubble is expanding so that its volume is increasing at $10\ cm^3 s^{-1}$. 3
Find the rate of increase of its radius when the surface area is $500\ cm^2$.
(Volume = $\frac{4}{3}\pi r^3$, Surface area = $4\pi r^2$)

(b) Prove by Mathematical Induction that: 4

$$2(1!) + 5(2!) + 10(3!) + \ldots + (n^2+1)n! = n(n+1)! \text{ for positive integers } n \geq 1.$$

(c) If $y = \dfrac{\log_e x}{x}$ find $\dfrac{dy}{dx}$ and hence show that $\displaystyle\int_e^{e^2} \frac{1-\log_e x}{x \log_e x}\, dx = \log_e 2 - 1$. 5

QUESTION 7: **(START A NEW PAGE)**

(i) By considering the expansion of $\sin(X+Y)-\sin(X-Y)$ prove that 3

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right).$$

(ii) Also given that $\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$ prove that 2

$$\frac{\sin A - \sin B}{\cos A - \cos B} = -\cot\left(\frac{A+B}{2}\right).$$

(iii) Prove that the position of a projectile t seconds after projection from ground level with initial horizontal and vertical velocity components of $V\cos\alpha$ and $V\sin\alpha$ respectively is given by $x = Vt\cos\alpha$ and $y = -\frac{1}{2}gt^2 + Vt\sin\alpha$. (Assume that there is no air resistance) 2

(iv) Two objects P and Q are projected from the same ground position at the same time with initial speed $V\ ms^{-1}$ at angles α and β respectively (β > α).

(α) If at time t seconds the line joining P and Q makes an acute angle θ with the horizontal prove that $\tan\theta = \left|\frac{\sin\beta - \sin\alpha}{\cos\beta - \cos\alpha}\right|$. 3

(β) Hence show that $\theta = \frac{1}{2}(\pi - \alpha - \beta)$. 2

THIS IS THE END OF THE EXAMINATION PAPER