

Name : _____

Number : _____



HURLSTONE AGRICULTURAL HIGH SCHOOL

YEAR 12 2009

MATHEMATICS

TRIAL HIGHER SCHOOL CERTIFICATE

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General Instructions

- Reading time : 5 minutes
- Working time : 3 hours
- Attempt all questions
- Start a new answer booklet for each question
- All necessary working should be shown
- This paper contains 10 questions worth 12 marks each. Total Marks: 120 marks
- Marks may not be awarded for careless or badly arranged work
- Board approved calculators and mathematical templates may be used
- This examination paper must **not** be removed from the examination room

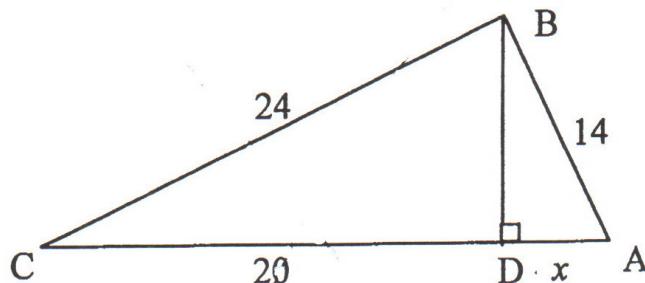
QUESTION 1. Start a new answer booklet.

- a) Factorise $3x^3 + 24$ 2
- b) Rationalise the denominator and simplify: $\frac{2}{5-\sqrt{3}}$ 2
- c) Express $\frac{3x+2}{2} - \frac{x-1}{5}$ as a single fraction, in simplest form 2
- d) Solve the inequality, graphing your solution on a number line: $|2x-3| < 7$ 2
- e) Write, in scientific notation, correct to 2 significant figures: $\frac{e^{-3.5}}{4}$ 2
- f) Sketch the graph of $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$ 2

QUESTION 2. Start a new answer booklet.

Marks

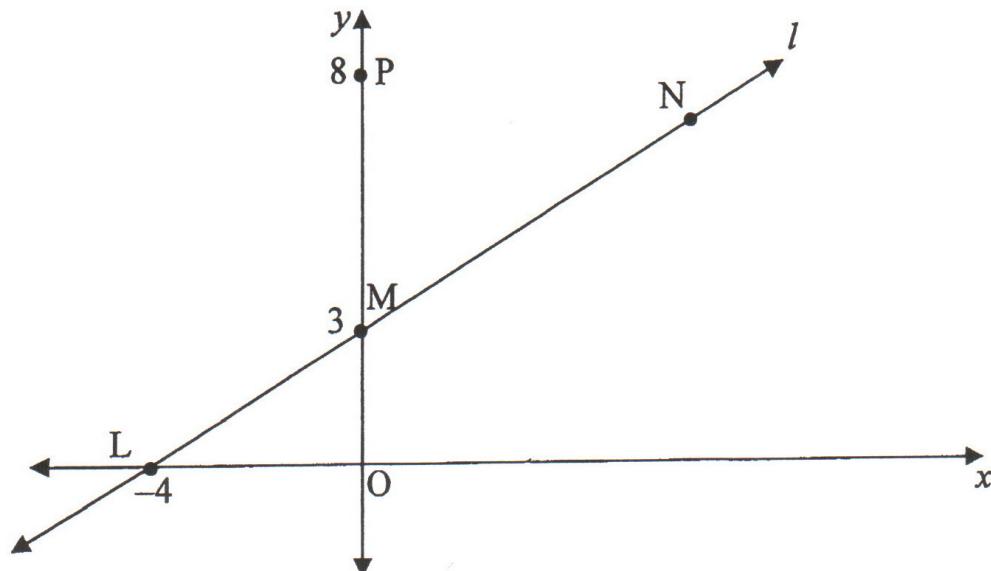
a)



Find x , the length of AD , as an exact value. Justify your answer.

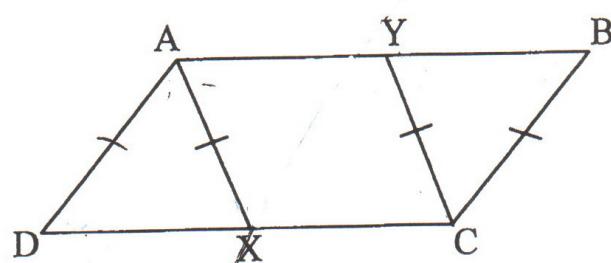
2

b)



For the diagram shown, M is the midpoint of the interval LN

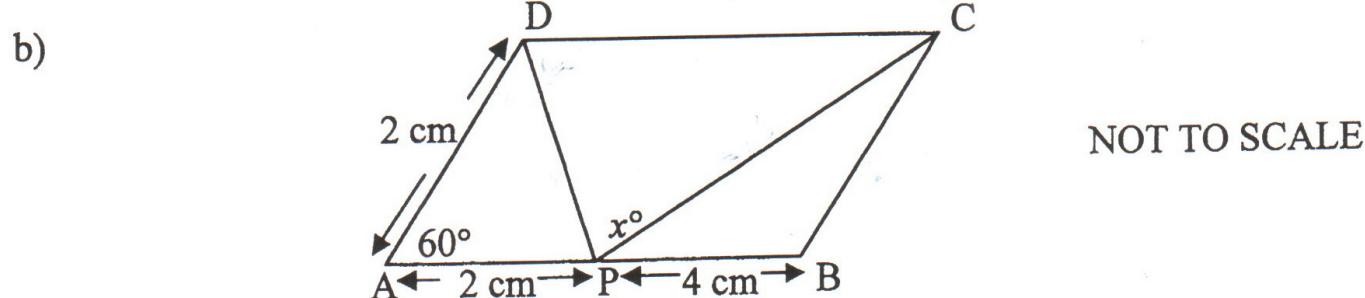
- (i) Find the coordinates of the point N . 2
 - (ii) Show that $\angle NPL$ is a right angle. 2
 - (iii) Find the equation of the circle that passes through the points N , P and L . 2
- c) ABCD is a parallelogram. The point X lies on CD, the point Y lies on AB, and $AX = CY = BC$, as shown in the diagram.



- (i) Explain why $\angle ADX = \angle CBY$. 1
- (ii) Show that $AD = AX$. 1
- (iii) Show that triangles ADX and CBY are congruent. 2

QUESTION 3. Start a new answer booklet.

- a) Use the formula $a^2 = b^2 + c^2 - 2bc \cos A$ to find A , correct to the nearest degree, when $a = 22$, $b = 12$, $c = 13$, and A lies between 0° and 180° . 2



In the figure (not to scale), ABCD is a parallelogram in which $AB = 6\text{cm}$, $AD = 2\text{cm}$, and $\angle DAB = 60^\circ$. The point P on AB is such that $AP = 2\text{cm}$, and $\angle DPC = x^\circ$.

- (i) Find the length of DP, giving reasons. 2

- (ii) Use the cosine rule for each of the triangles PBC, PCD to show that $\cos x^\circ = \frac{-\sqrt{7}}{14}$. 3

- c) Find all the values of θ , for $0^\circ \leq \theta \leq 180^\circ$, such that $\cos 2\theta = \frac{1}{2}$. 2

- (d) Sketch the graph of:

$$f(x) = \begin{cases} 5 & \text{if } x > 2 \\ x^2 & \text{if } 0 \leq x \leq 2 \\ 2x - 1 & \text{if } x < 0 \end{cases}$$
3

QUESTION 4. Start a new answer booklet.

a) Given that $\log_k 5 = 0.627$ and $\log_k 2 = 0.270$, find the value of:

(i) $\log_k 10$

1

(ii) $\log_k 25$

1

b) Differentiate the following with respect to x :

(i) $(e^{2x} + 1)^3$

2

(ii) $\frac{\log_e x}{x+1}$

2

c) Find $\int \frac{x}{x^2 + 2} dx$

2

d) Evaluate $\int_0^1 (e^{2x} - x) dx$

2

e) Consider the equation $\log_e y = x \log_e \left(\frac{1}{2}\right)$.

2

Write an expression for $y = f(x)$.

QUESTION 5. Start a new answer booklet.

- a) The first three terms of a geometric series are:

$$4a^2b^2, x, a^2 + 2ab + b^2$$

Find x in terms of a and b .

2

- b) A hole in a water reservoir wall will allow through it 50L more for each hour that it remains undetected. At the moment that this hole was detected, water was leaking through it at the rate of 1200L/h.

- (i) Write down the first three terms of a series which represents the water lost through the hole for each of the first three hours. 1
- (ii) For how long had the water been leaking when the hole was detected? 2
- (iii) What was the total volume of water lost through the hole, up to the time when it was detected? 1

- c) For the series:

$$1 + \sin A + \sin^2 A + \sin^3 A + \dots$$

- (i) Explain why the series has a limiting sum. 1
- (ii) Find the exact value of this limiting sum when $A = \frac{4\pi}{3}$. 2

- d) An investment fund intends to pay interest at the rate of 6% p.a. every six months.

- (i) If an investment of \$250 is made today, what amount (ie. principal plus interest) will be available for withdrawal in 10 years time? 1
- (ii) If nineteen further investments of \$250 are made every six months, show that the amount available for withdrawal in ten years time will be \$6919 to the nearest dollar. (Assume that no withdrawals are made from the fund during this time.) 2

QUESTION 6. Start a new answer booklet.

- a) Consider the following statement:

"In 2008, Fiji had a population of approximately 906 000. The number of fatalities on Fijian roads was 78. If similar figures were to be maintained into the future, the probability of a Fijian, chosen at random, being killed in a road accident in any particular year is approximately 1 in 12 000."

Comment upon the validity of this statement.

2

- b) A General Practitioner has compiled statistics on patients visiting his practice. He has determined that during the winter months, there is a 45% chance that a patient will require treatment for influenza and a 15% chance that a patient will require treatment for food poisoning.

(i) What is the probability that one of the doctor's patients will require treatment for influenza *and* food poisoning during the same season.

1

(ii) What is the probability that one of the doctor's patients will require treatment for either influenza *or* food poisoning during the same season.

2

- c) A coin is specially weighted so that the probability of tossing a "head" on any single toss is twice that of tossing a "tail".

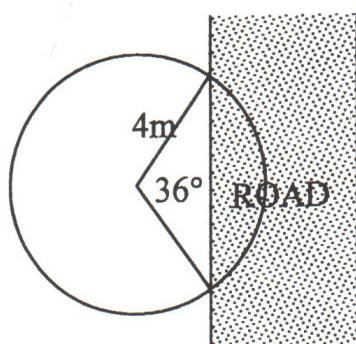
(i) What is the probability of tossing a "head" with this coin.

1

(ii) Calculate the probability of tossing at least one "tail" if the coin is tossed five times.

1

- d)



A sprinkler spraying water in a circular pattern of radius 3m is watering a lawn adjacent to a straight section of road as shown in the diagram. The angle subtended by the road at the sprinkler head is 36° .

(i) Convert 36° to radians. Give your answer in terms of π .

1

(ii) Find the area of road being watered in square metres correct to 2 decimal places.

2

(iii) Calculate the volume of water being wasted each hour if the sprinkler delivers 3.5kL per hour. Give your answer to the nearest litre. (Assume the sprinkler disperses water evenly over its spray area.)

2

QUESTION 7. Start a new answer booklet.

a) $\int (2x+3)^3 dx$

1

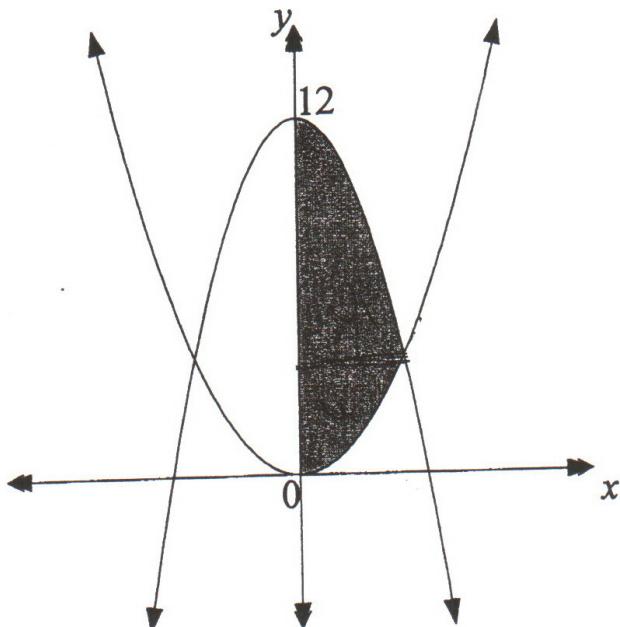
b) Evaluate $\int_1^4 \frac{x^2 + 8}{x^2} dx$

3

c) Given $f'(x) = 3x^2 + x$, find $f(x)$ given $f(-2) = 4$

2

d) The graphs of the curves $y = x^2$ and $y = 12 - 2x^2$ are shown in the diagram.



1

(i) Find the points of intersection of the curves

2

(ii) Calculate the area between the two curves

3

(iii) The shaded region between the curves and the y axis is rotated about the y axis. By splitting the shaded region into two parts, or otherwise, find the volume of the solid formed.

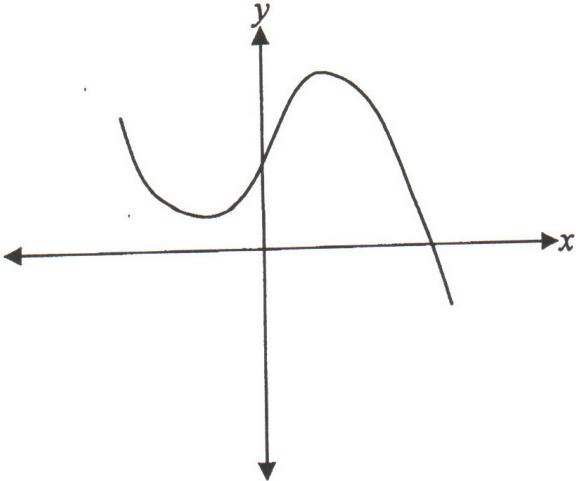
QUESTION 8. Start a new answer booklet.

- a) If $A^m = 3$, find the value of $A^{4m} - 5$ 2
- b) Use Simpsons Rule, with three function values, to approximate $\int_3^5 x \log_e x \, dx$ 2
- c) Consider the function $y = xe^{-x}$.
- (i) Find the coordinates of any stationary point(s) and determine their nature 3
 - (ii) Describe the behaviour of the function as x :
 - increases to negative infinity
 - increases to positive infinity
 - (iii) Does the graph pass through the origin? Justify your answer. 1
 - (iv) Sketch the graph of this function 2

QUESTION 9. Start a new answer booklet.

- a) Find the equation of the tangent to the curve $y = \sqrt{x-1}$ at the point (2,1). 2
- b) Solve for x : $x^2 - 8x + 12 > 0$ 2
- c) Point $P(x, y)$ is a point on the parabola $y = x^2$.
- (i) Show that the distance, S , from the line $y = 2x - 5$ is given by $S = \left| \frac{2x - x^2 - 5}{\sqrt{5}} \right|$. 2
 - (ii) Show that $f(x) = 2x - x^2 - 5$ is negative definite. 2
 - (iii) Hence show that $S = \frac{x^2 - 2x + 5}{\sqrt{5}}$. 1
 - (iv) Hence find the shortest distance possible between the point $P(x, y)$ and the line $y = 2x - 5$. 3

QUESTION 10. Start a new answer booklet.

- a) For the parabola $(x - 2)^2 = 8(y + 1)$, find:
- (i) the focal length 1
 - (ii) the co-ordinates of the vertex 1
 - (iii) the co-ordinates of the focus 1
- b) The parabola $y^2 = 4x$ is reflected in the y axis. What is the equation of the resultant parabola formed? 1
- c) Show that the locus of a point, $P(x, y)$, which is always 5 units from the point $A(3, 6)$ is $x^2 - 6x + y^2 - 12y + 20 = 0$. 3
- d) The diagram shows a graph of a certain function $y = f(x)$.
- 
- (i) Copy this graph into your writing booklet. 2
 - (ii) On the same set of axes, draw a sketch of the derivative $f'(x)$ of the function. 2
- e) The quadratic equation $x^2 + mx + 10 = 0$ has one root twice the other.
- (i) If one of the roots is α , write expressions for the sum and product of the roots. 1
 - (ii) Hence, or otherwise, find the value of m . 2