THE HILLS GRAMMAR SCHOOL

STRIVE FOR EXCELLENCE

THE HILLS GRAMMAR SCHOOL

TRIAL HSC
1999

MATHEMATICS

3 UNIT

TIME ALLOWED: 2 Hours (plus 5 minutes reading time)

Teacher Responsible: Mrs B Spencer
Mrs S Maxton

INSTRUCTIONS:

- Attempt all questions.
- In every question show all necessary working.
- Silent calculators and approved templates may be used.
- Start each question on a NEW page and hand up your paper in ONE bundle with your name marked on EVERY page.
- All questions are of equal value.

Question 1

Marks

a) Express $\dfrac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\dfrac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$ in the form of $a\sqrt{5}+b\sqrt{6}$ 2

b) Show that $\dfrac{{}^nC_r}{{}^nC_{r-1}}=\dfrac{n-r+1}{r}$ 2

c) Find the acute angle between the lines 2

$y = 4x - 2$ and $2x + 3y - 9 = 0$

to the nearest minute.

d) Find the co-ordinates of the point P which divides the interval AB externally in the ratio 3:1. A is (–4,2) and B is (6,5). 2

e) (i) What is the maximum value of $5\sin\theta+12\cos\theta$; and 2

(ii) What is the first positive value of θ for which this maximum occurs. 2

Question 2

a) Find $\int x^2\sqrt{(x^3-9)}\,dx$ using $u=x^3-9$ 3

b) Evaluate $\int_0^{\frac{\pi}{12}}\cos^2 x\;dx$. Answer in exact value terms. 3

c) Find the term independent of x in the expansion of $(x^2-\frac{1}{x})^{12}$. 3

d) The remainder when x^3+ax+b is divided by $(x-2)(x+3)$ is $2x+1$. Find a and b. 3

Question 3

Marks

a) The points P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. The angle POQ is 90°. 0 is (0,0). Show that $pq = -4$ and hence find the equation of the locus of M, the midpoint of PQ. 2 3

b) A projectile is fired from O with a velocity of 20m/s at an angle of 60°. The projectile just clears a wall 25m from the point of projection. How high is the wall? Take $g = 10\text{m/sec}^2$. 3

c) $f(x) = x^3 - x^2 - x - 1$.

(i) Show that the equation $f(x) = 0$ has a root in the interval $1 < x < 2$. 1

(ii) Use Newton's method once to find a better approximation for the root, taking x to be 1.5. 3

Question 4

a) A stone is thrown into a pond and creates a circular ripple which expands so that $\frac{dr}{dt} = 1.5m/s$. Find the rate at which the area of the circle is increasing when the radius is 2m. Answer to 4 significant figures. 2

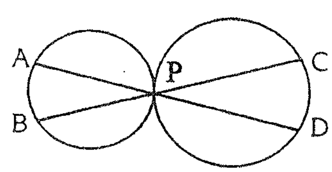
b) A particle moving along the x-axis has a velocity given by $v^2 = 15 - 2x - x^2$.

(i) Show that the centre of the motion is at $x = -1$. 2

(ii) Show that $a = -n^2 x$ and thus the motion is simple harmonic. 2

(iii) Find the amplitude and period of the motion. 2

c)



Two unequal circles touch at P. APD and BPC are straight lines.

(i) Copy the diagram in a larger scale. 4

(ii) Draw the common tangent XPY.

(iii) Prove that AB is parallel to CD.

Question 5

Marks

a) Use the expansion $(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$ to prove that

$1 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots\dots\dots + (-1)^n C_n = 0$. 2

b) In how many ways can a committee of 4 people be selected from a group of 10 people if;

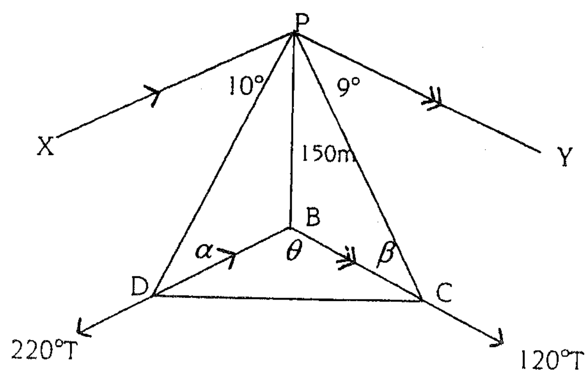
(i) 2 particular members P and Q are included; 3

(ii) 2 particular members P and Q are excluded. 3

c) I participate in a raffle where my chances of winning a prize are one in ten. If I buy a book of 12 tickets, what is the probability of my winning exactly two prizes. Leave answer in index form. 4

Question 6

a)



A ship is observed from the top of a 150m cliff BP with an angle of depression 9° when the ship is at the point C. Ten minutes later it is seen at D with an angle of depression of 10°. $\angle PDB = \alpha$, $\angle PCB = \beta$ and $\angle DBC = \theta$. BC bears 120°T. BD bears 220°T.

(i) Show that $\alpha = 10°$, $\beta = 9°$ and $\theta = 100°$. 3

(ii) Show that BD = 150 cot α and BC = 150 cot β and hence that $CD^2 = 150^2\ (\cot^2\alpha + \cot^2\beta - 2\cot\alpha\cot\beta\cos\theta)$. 4

(iii) Find the speed of the ship in km/h to three significant figures. 2

c) Solve for all x, $2\sin^2 x = \sin 2x$. Answer in radian measure. 3

Question 7

Marks

a) My loungeroom is kept at a constant temperature of 25°c. A cup of tea left standing in the room cools at a rate proportional to the difference in temperature between the tea and its surroundings so that $\frac{dT}{dt} = k(T - 25)$. After 20 minutes the temperature of the tea has dropped from 95°c to 65°c.

(i) Show that $T = 25 + Ae^{kt}$ is a solution of $\frac{dT}{dt} = k(T - 25)$. 1

(ii) Find the values of A and k. 3

(iii) Find the temperature of the tea after a further 10 minutes. 2

b) Consider the function $f(x) = x\sin^{-1}x$.

(i) Show that $f(x)$ is an even function. 1

(ii) Find $f'(x)$ and hence the co-ordinates of the only turning point. 2

(iii) Determine the domain and range of $f(x)$. 2

(iv) Sketch $f(x)$. 1