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BAULKHAM HILLS HIGH SCHOOL

HIGHER SCHOOL CERTIFICATE

TRIAL EXAMINATION

2008

MATHEMATICS EXTENSION 1

Time allowed: Two Hours (Plus 5 mins reading time)

GENERAL INSTRUCTIONS

- Attempt all questions
- There are seven questions start each question on a new page
- All necessary working should be shown
- Write, using black or blue pen
- Write your student number at the top of each page of the answer sheets
- At the end of the exam, staple your answers in order, behind the cover sheet.

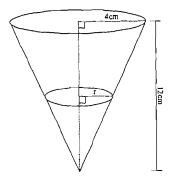
Que	stion 1	Marks
a)	A and C have co-ordinates $(-1,2)$ and $(6,10)$ respectively. Find the point B which divides AC internally in the ratio $2:3$.	2
b)	Find $\int \frac{4x}{2x+1} dx$ using the substitution $u = 2x + 1$.	3
c)	State the domain of the function $y = log_e\left(\frac{3x-1}{x+2}\right)$.	3
d)	i) Show that the curves $y = e^{x-1}$ and $y = e^{-x}$ intersect at $x = \frac{1}{2}$.	1
	ii) Find the acute angle between the curves at this point.	3
Ques	stion 2 (start a new page)	
a)	Find the constant term in the expansion $\left(3x^2 + \frac{5}{x^3}\right)^{10}$.	3
b)	Solve $\sin 4x = \cos 2x$ for $0^{\circ} \le x \le 360^{\circ}$	3
c)	Evaluate $\int_{0}^{\frac{3}{4}} \frac{dx}{\sqrt{9-4x^2}}$.	3
d)	Taking $x = 2$ as the first approximation for the root of $\sin x - \frac{x}{3} = 0$	3
	find a closer approximation of the root using one application of Newton's method.	
Ques	stion 3 (start a new page)	
a)	Find the volume when the area between $y = 2\sin x$, the x and y axes and	4
	$x = \frac{\pi}{4}$ is rotated about the x axis.	
b)	i) State the domain and range of $y = 2\cos^{-1}(x-1)$	2
	ii) Hence sketch the curve	2
c)	If α, β and γ are the roots of the cubic $2x^3 - 5x^2 - 3x + 1 = 0$, find	
	i) $\alpha + \beta + \gamma$	1
	ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$	1
	iii) $\alpha^2 + \beta^2 + \gamma^2$	2

Marks

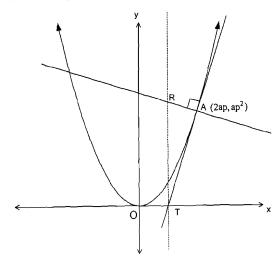
2

Question 4 (start a new page)

The diagram shows a conical drinking cup of height 12cm and radius 4cm. The cup is filled with water at a rate of 3cm³ per second. The height of water at time t seconds is h cm and the radius of the water's surface is r cm.



- Show that $r = \frac{1}{3}h$.
- Find the rate at which the height is increasing when the height ii) of the water is 9cm. ($V = \frac{1}{3}\pi r^2 h$ is the volume of a cone.)
- x = 2at and $y = at^2$ are parametric equations for the parabola below.



Question 4 (cont.)		
i)	By finding the Cartesian equation of the parabola, find the equation of the tangent at the point A.	2
ii)	The tangent cuts the x axis at T . Find the coordinates of T .	1
iii)	Find the equation of the normal at A .	1
iv)	A line through T parallel to the axis of the parabola cuts the normal at R. Show that the coordinates of R are $(ap, ap^2 + a)$.	1

Marks

3

3

2

Question 5 (start a new page)

(v)

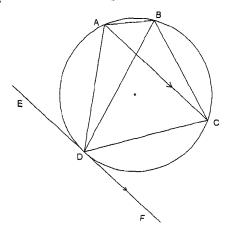
Marks

1

3

ABCD is a cyclic quadrilateral. EF is a tangent to the circle and AC \parallel EF.

Show that the locus of R is a parabola and state the equation of it's directrix.



Prove BD bisects ∠ABC.

The velocity of a particle as it moves along the X axis is given by

$$v^2 = -9x^2 + 18x + 27$$

- Show that the particle undergoes Simple Harmonic Motion. 2 i) 1 What is the period of the motion? ii)
- What is the amplitude of the motion?

Marks

1

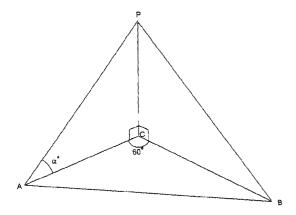
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3

Question 5 (cont.)

c) The position of two yachts, A and B out at sea subtend an angle of 60° at the base C of a cliff. The distance AC is 3 times the height of the cliff and the distance BC is 4 times the height of the cliff.



- i) Show that the angle of elevation α° of the cliff from Point A is 18°26'
- ii) The distance AB is 300 metres greater than the height of the cliff.
 Find the height of the cliff.

Question 6 (start a new page)

a) Solve $|x^2 - 9| < 8$

b) By integrating both sides of the expansion

$$(I+x)^n = {}^nC_0 + {}^nC_1x + \dots + {}^nC_nx^n \quad \text{prove}$$

$$1 - \frac{1}{2}{}^nC_1 + \frac{1}{3}{}^nC_2 + \dots + \frac{(-1)^n}{n+1}{}^nC_n = \frac{1}{n+1}$$

Question 6 (cont.)

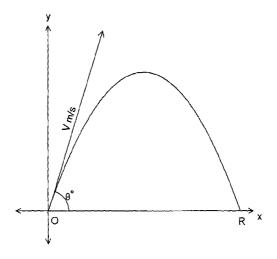
A projectile is fired with velocity V m/s from a point 0 at an angle θ with the horizontal and hits the ground at a horizontal distance R from 0. Taking g = 10m/s² you may assume the equations of motion for the projectile.

i.e.
$$\dot{x} = 0$$
 $\dot{y} = -10$
 $\dot{x} = V \cos \theta$ $\dot{y} = -10t + V \sin \theta$
 $x = Vt \cos \theta$ $y = -5t^2 + Vt \sin \theta$

Marks

3

3



Show that the range $R = \frac{V^2 \sin 2\theta}{10}$ and that the maximum range is given by $\frac{V^2}{10}$.

ii) The maximum range of a certain rifle is 2000 metres.

How much is the range increased when the rifle is mounted on a car travelling at 30 m/s towards the target, the angle of elevation being unaltered.

Marks

Question 7 (start a new page)

a) i) Show that
$$\frac{d}{dx}(\tan^3 x) = 3\tan^2 x + 3\tan^4 x$$

ii) Hence find
$$\int \tan^4 x \ dx$$
 3

b) Prove by Mathematical Induction that

$$3 \times 2^2 + 3^2 \times 2^5 + \dots + 3^n \times 2^{n+1} = \frac{12}{5} (6^n - 1)$$
 for all positive integers n .

c) If the 3^{rd} and 4^{th} terms of the binomial $(1 + ax)^n$ are $264x^2$ and $1760x^3$ and n > a, find the values of a and n.

End of Exam

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \ n \neq -1; \ x \neq 0, \ if \ n < 0$$

$$\int_{-x}^{1} dx = \ln x, \qquad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \ a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \ a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \ a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \ a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \ a > 0, \ -a < x < a$$

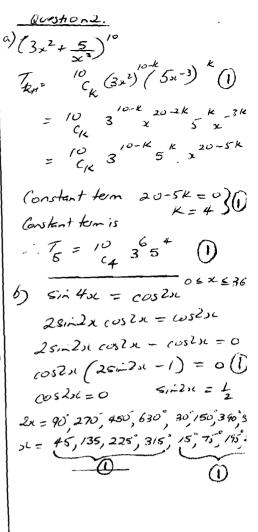
$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}), \ x > a > 0$$

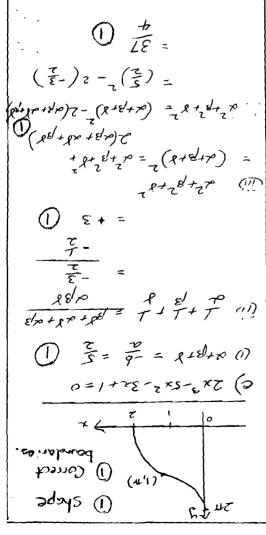
$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

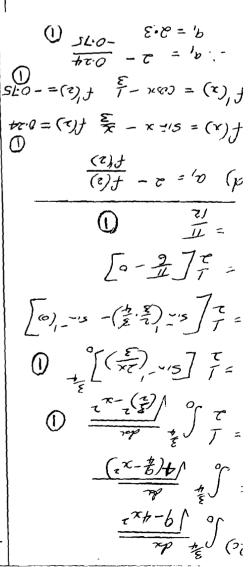
NOTE: $\ln x = \log_e x$, x > 0

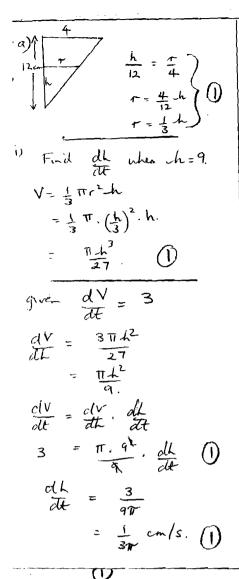
$$\begin{array}{lll}
(a) & (-1,2) & (6,10) \\
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(1,0) & (2,2)$$

$$\frac{2008}{2008} = \frac{200}{200} = \frac{200}{200}$$



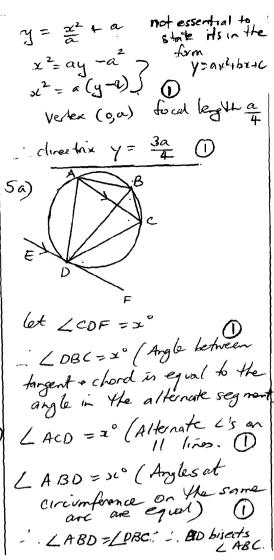


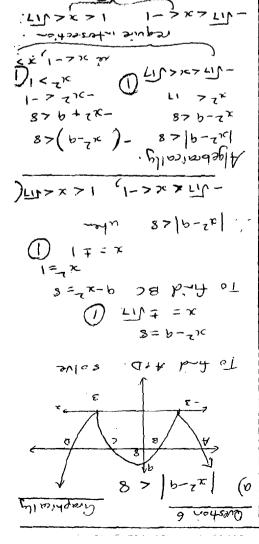


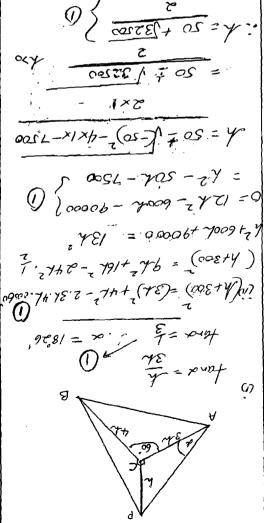


b)
$$x^{2} = 4ay$$

(i) $y = \frac{x^{2}}{4a}$
 $y' = \frac{2x}{4a}$
 $x' =$







12=12 : True for n=1 Issume true for n=k $a^{2} + \dots + 3^{k} \times 2^{k+1} = \frac{12}{5} \left(6^{k} - 1 \right)$ Done true for n= k+1 $2^{2} + \cdots + 3 + 2^{k+1} + 3 + 2^{k+2} = \frac{12}{5} (\zeta^{|2+1|})$ $\frac{12}{5} \left(6^{k} - 1 \right) + 3^{k+1} \times 2^{k+2} = \frac{12}{5} \left(6^{k+1} \right)$ $\frac{12}{5}(6^{k}) - \frac{12}{5} + 3.3^{k} \times 2^{k}.2^{2}$ $\frac{12}{5}(6^{k}) - \frac{12}{5} + 12.6^{k}$

 $=\frac{12}{5}\left(6^{k+1}-1\right)$ Proved true for n=1 + assumed

true for n=k. Proven true for n= |(+1 : tre for n=1, n=2 -- . for all n by M. I.

70) $n_{2a}^{2} = 264$ $\frac{(n(n-1)a^2-264-1)}{2}$ $0 = \frac{1760}{1000}$ $\frac{1760}{1000}$ $\frac{1760}{1000}$ $\frac{1760}{1000}$

w009=

i. a = 1, 2, 4, 5, 10, \$0 p n = 22, 12, 7, 6, \$, 1. but n > a :. trial a=1, 2, 4, 5. sch a=1 n=22 into (A) $\frac{22(21)}{3}$ + 264 a=2 n=12 $\frac{12(11)}{2}.2^2 = 264$ -1.4 = 2 n = 12.0

There are other nethods of doing this ...

(1) 25,20 care (0) =

(2+ x + 100pt - xenat = 0+ [x8+xub+8-x2vot] == 2 = K-14) - 1 = K-14) - 3 to. 1) 36 (Link) 36 (1)) (x2n2+1) 12n2+8 = 3 (Fenz.) , Sec. 22 (i) (i) de (tenz) = =

E TOX3D only make S = 0C = 学·少001=7 . Sh= & Shoot = 1 +09 10-51 = 7 0-7 4 (-56 Wills) = 2 0= 44571 1275horizon velocity unchesod $\bigoplus_{\substack{01\\01}} \frac{2\sqrt{15}}{\sqrt{15}} = 3.$ Max rays when a= Hs

x = 1 + cos & (a451+79-)x =0 (c) Hange occurs when yes 2 (1) 1 2 1 - 0 4 = 1+14 1-= x +07 $\frac{1}{1} \frac{1}{1} \frac{1}$ D 144 = 37-17. 14. (x+1) = (x+1) = (x+1) = (x+1) = (x+1) = (x+1)

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st to	$\chi < -\gamma$
N.	

$(1+cm)^{1} = \dots \qquad T_{1} = 2cv n^{2} \qquad T_{v} = 17cc n^{2}$ $(1_{1} = 2^{2} + cv - C) \qquad (l_{1} = 2^{2} + cv - 2^{2})$ $(1_{1} = 2^{2} + cv - C) \qquad (l_{1} = 2^{2} + cv - 2^{2})$ $(1_{1} = 2^{2} + cv - 2^{2} + cv - 2^{2})$ $(1_{1} = 2^{2} + cv - 2^{2} + cv - 2^{2})$ $(1_{1} = 2^{2} + cv - 2^{2} + cv - 2^{2})$ $(1_{1} = 2^{2} + cv - 2^{2} + cv - 2^{2})$ $(1_{1} = 2^{2} + cv - 2^{2} + cv - 2^{2} + cv - 2^{2})$ $(1_{1} = 2^{2} + cv - 2^{$	$(\alpha \lambda)^{1/2} \qquad \qquad T_{3} = 264 \text{m}^{2} \qquad T_{4} = 0$ $\alpha^{2} = 264 \qquad \qquad -(3 \text{M}_{3} \text{m}^{2}) = 1760 (1)$
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