2001
Higher School Certificate
Trial Examination

Mathematics
Extension 1

General Instructions

- Reading time – 5minutes
- Working time – 2 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided on the last page
- All necessary working should be shown in every question

Total marks (84)

Attempt Questions 1 – 7

All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NAME/NUMBER: ....

Question 1 (Start a new work book) **Marks**

a. Determine the ratio in which the point C(6, 9) divides the interval AB, where A is the point $(-1, -5)$ and B the point $(3, 3)$. **3**

b. Solve the inequality $x - 1 \leq \dfrac{1}{x - 1}$. **3**

c. For the polynomial $P(x) = x^3 - 2x^2 - x + 2$

i. show that $x - 1$ is a factor. **1**
ii. Hence, or otherwise, find all the factors of $P(x)$. **1**

d. i. If $t = \tan\dfrac{\theta}{2}$, show that $\sin\theta = \dfrac{2t}{1 + t^2}$ and $\cos\theta = \dfrac{1 - t^2}{1 + t^2}$. **2**

ii. Using these results, show that $\dfrac{1 - \cos\theta}{\sin\theta} = \tan\dfrac{\theta}{2}$. **1**

iii. Hence find the exact value of tan 15°. **1**

Question 2 (Start a new work book)

a. For the parabola defined by the parametric equations $x = 4t$, $y = 2t^2$

i. by differentiation, show that the gradient of the tangent at the point, P, where $t = 3$, is 3. **1**

ii. find the gradient of the focal chord through P. **1**

iii. calculate the acute angle between the tangent at P and the focal chord through P. **2**

b. Use one iteration of Newton's method to find an approximation to the root of the equation $x\log_e x - 2x = 0$ near $x = 7$. Give your answer to 1 decimal place. **3**

c. Six people attend a dinner party.

i. In how many different ways can they be arranged around a round table? **1**

ii. In how many different ways can they be arranged if a particular couple must sit together? **1**

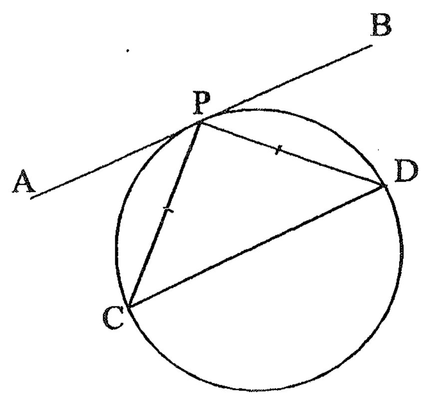
iii. What is the probability that, if the people are seated at random, the couple are sitting apart from each other? **1**

Question 2 (continued)

Marks

d. PC and PD are equal chords of a circle. A tangent, AB, is drawn at P. **2**

Prove that AB is parallel to CD



Question 3 (Start a new work book)

a. Jane, a netball goal shooter, has a 70% probability of scoring a goal at any attempt. In her next 10 attempts at scoring, what is the probability that she scores at least 8 times? Give your answer as a decimal to 2 significant figures. **3**

b. Show that the equation of the circle whose diameter is the join of the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ **2**

c. Use the Principle of Mathematical Induction to prove that $2^{3n} - 3^n$ is divisible by 5 for all positive integers n. **4**

d. The arc of the curve $y = \cos 2x$ between $x = 0$ and $x = \frac{\pi}{6}$ is rotated through $360°$ about the x-axis. **3**

Find the exact volume of the solid formed.

Question 4 (Start a new work book)

a. If $\binom{n}{r} = \binom{n}{r+1}$, where n and r are positive integers, show that n is odd. **3**

b. i. Express $x^2 + 6x + 13$ in the form $(x + a)^2 + b^2$ **1**

ii. Hence, using the substitution $u = x + 3$, find $\displaystyle\int \frac{dx}{x^2 + 6x + 13}$ **2**

STUDENT NAME/NUMBER: ....

Question 4 (continued) **Marks**

c. Show that $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}$ 3

d. If $y = \frac{1}{2}(e^{x} - e^{-x})$, show that $x = \log_e\left(y + \sqrt{y^2 + 1}\right)$ 3

Question 5 (Start a new work book)

a. A particle's motion is defined by the equation $v^2 = 12 + 4x - x^2$, where x is its displacement from the origin in metres and v its velocity in ms^{-1}. Initially, the particle is 6 metres to the right of the origin.

i. Show that the particle is moving in Simple Harmonic Motion 1

ii. Find the centre, the period and the amplitude of the motion 3

iii. The displacement of the particle at any time t is given by the equation $x = a\sin(nt + \theta) + b$.

Find the values of θ and b, given $0 \leq \theta \leq 2\pi$ 2

b. Newton's Law of Cooling states that the rate of change in the temperature, $T°$, of a body is proportional to the difference between the temperature of the body and the surrounding temperature, $P°$.

i. If A and k are constants, show that the equation $T = P + Ae^{kt}$ satisfies Newton's Law of Cooling. 2

ii. A cup of tea with a temperature of 100°C is too hot to drink. Two minutes later, the temperature has dropped to 93°C. If the surrounding temperature is 23°C, calculate A and k. 2

iii. The tea will be drinkable when the temperature has dropped to 80°C. How long, to the nearest minute, will this take? 2

STUDENT NAME/NUMBER: ....

Question 6 (Start a new work book) **Marks**

a. A particle is projected horizontally with velocity, V ms^{-1}, from a point h metres above the ground. Take g ms^{-2} as the acceleration due to gravity.

i. Taking the origin at the point on ground immediately below the projection point, find expressions for x and y, the horizontal and vertical displacements respectively of the particle at time t seconds. **2**

ii. Hence show that the equation of the path of the particle is given by the equation $y = \dfrac{2hV^2 - gx^2}{2V^2}$. **2**

iii. Find how far the particle travels horizontally from its point of projection before it hits the ground. **2**

b. A particle moves in a straight line so that its velocity after t seconds is v ms^{-1} and its displacement is x.

i. Given that $\dfrac{d^2x}{dt^2} = 10x - 2x^3$ and that $v = 0$ when $x = -1$, find v in terms of x. **3**

ii. Explain why the motion cannot exist between $x = -1$ and $x = 1$. **2**

iii. Describe briefly what would have happened if the motion had commenced at $x = 0$ with $v = 0$. **1**

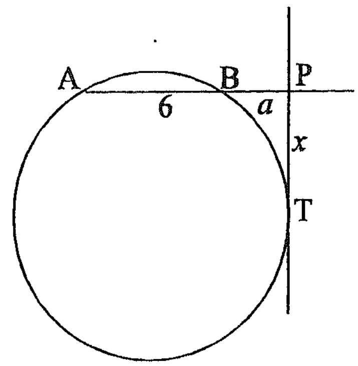
STUDENT NAME/NUMBER: ....

Question 7 (Start a new work book) **Marks**

a. In the circle, the chord AB is 6 metres long. The chord is produced to the point P and BP is a metres.

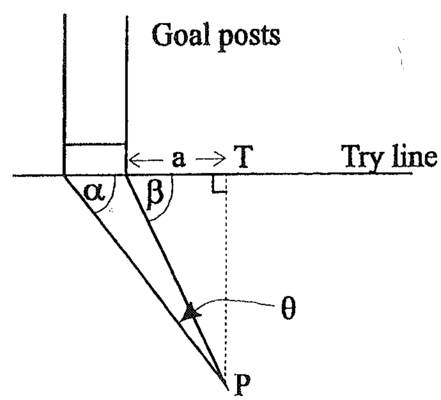
A tangent to the circle cuts the chord at P. PT is x metres.

Show that $x = \sqrt{a(a+6)}$. 2



b. In a rugby game, teams score points by placing the ball over the try line at the end of the field. A kicker may then take the ball back at right angles from the try line and attempt to kick the ball between the goal posts.

In the diagram, a try has been scored a metres to the right of the goal posts. The kicker has brought the ball back to the point P to attempt his kick. The kicker wants to maximise θ, his angle of view of the goalposts.



Let PT be x metres and assume that the goal posts are 6 metres wide.

i. Show that $\tan\theta = \dfrac{6x}{a^2 + 6a + x^2}$. 3

ii. Letting $T = \tan\theta$, find the value of x for which T is a maximum. 2

iii. Hence show that the maximum angle, θ, is given by $\theta = \tan^{-1}\left(\dfrac{3}{\sqrt{a^2 + 6a}}\right)$ 2

iv. If a try is scored 10 metres to the right of the goal posts, find the maximum value of θ (to the nearest minute) and the corresponding value of x (to the nearest centimetre) . 2

v. Explain why the goal kicker, to maximise his angle of view of the goal posts, should imagine himself at the point of contact of a tangent to the circle passing through the goal posts. 1

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$