



YEAR 12  
COMMON TEST 4  
TRIAL HSC EXAMS  
2007

# EXTENSION 1 MATHEMATICS

*Time allowed : 2 Hours*

## DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board approved calculators may be used.
- Each question is to be started on a **new page** and you are to write your name, student number and teacher's name on each page.
- The marks allocated for each question are indicated.
- Standard integrals are listed on the last page.

Name : \_\_\_\_\_ Student Number: \_\_\_\_\_

Teacher: \_\_\_\_\_

Q	1	2	3	4	5	6	7	Total
Mark	/12	/12	/12	/12	/12	/12	/12	/84



## STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left( x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left( x + \sqrt{x^2 + a^2} \right)$$

NOTE :  $\ln x = \log_e x, \quad x > 0$

**Question 1 ( 12 marks )**

(a) If  $P(x) = x^3 - bx^2 - bx + 4$  is divisible by  $x - 2$ , find the value of  $b$ . 2

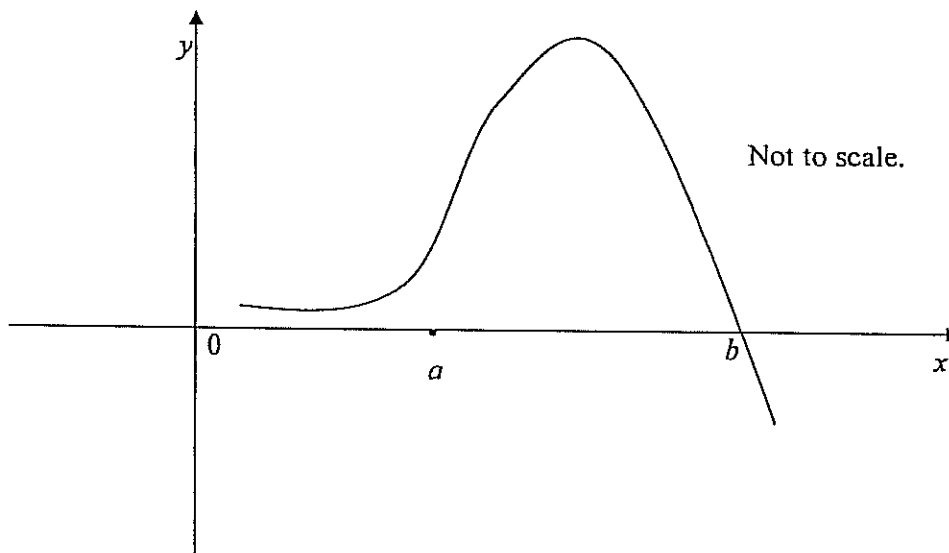
(b) Find  $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$  2

(c) Write down the domain and range of  $3 \sin^{-1}\left(\frac{x}{2}\right)$ . 2

(d) Use the substitution  $u = 2x + 1$  to find the exact value of 4

$$\int_0^4 \frac{x}{\sqrt{2x+1}} dx$$

(e) Consider the graph of  $y = f(x)$ . The value  $a$ , shown on the graph is taken as the first approximation to the solution  $b$  of  $y = f(x)$ . 2



Is the second approximation obtained by Newton's method a better approximation than  $a$ ? Justify your answer.

**Question 2** ( 12 marks ) Start a new page.

(a) Find all values of  $x$  which satisfy the inequality

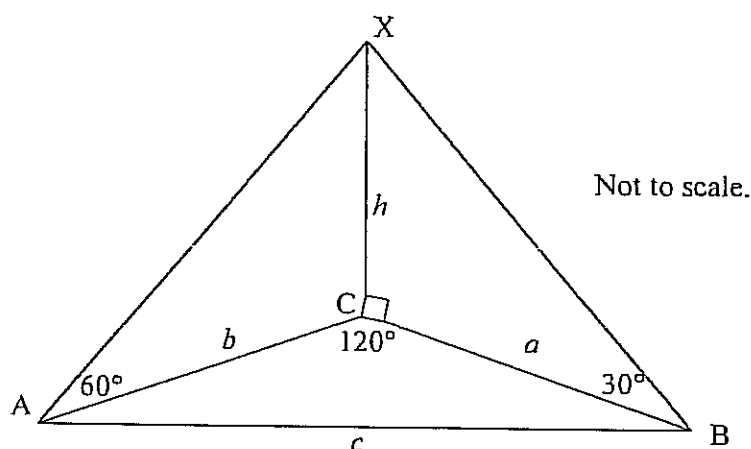
$$\frac{x+1}{x-1} > 2$$

3

(b) Find  $\int \sin^2 2x \, dx$

2

(c)



A tetrahedron has base  $\triangle ABC$  in the horizontal plane with  $\angle ACB = 120^\circ$ .  $XC$  is perpendicular to  $BC$  and  $AC$ .  $\angle XBC = 30^\circ$  and  $\angle XAC = 60^\circ$ .

(i) Show that  $a = h\sqrt{3}$ .

1

(ii) Hence show that  $a = 3b$ .

1

(iii) Show that  $c = b\sqrt{13}$ .

2

(d) Prove by induction that for all positive integers  $n$

3

$$\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{(n+1)(n+2)} = \frac{n}{2(n+2)}$$

**Question 3 ( 12 marks )** Start a new page.

(a) P  $(2ap, ap^2)$  and Q  $(2aq, aq^2)$  are two distinct points on the parabola  $x^2 = 4ay$ .  
The chord PQ passes through  $(0, -a)$ .

(i) Show that  $pq = 1$ . 3

(ii) The normals at P and Q meet at N where N has co-ordinates  $(-apq(p+q), a(p^2+q^2+pq+2))$ . ( Do NOT prove this. ) 2

Show that the equation of the locus of N is  
 $x^2 = a(y-a)$ .

(b) Find all values of  $x$ , where  $0 \leq x \leq 2\pi$ , which satisfy the equation 3

$$\cos x + \sqrt{3} \sin x = 1.$$

(c ) Assume that the rate at which a body warms in air is proportional to the difference between its temperature  $T$  and the constant temperature  $A$  of the surrounding air.

This can be expressed by  $\frac{dT}{dt} = k(T - A)$ .

i) Show that  $T = A + Ce^{kt}$ , where  $C$  is a constant, is a solution of  $\frac{dT}{dt} = k(T - A)$ . 1

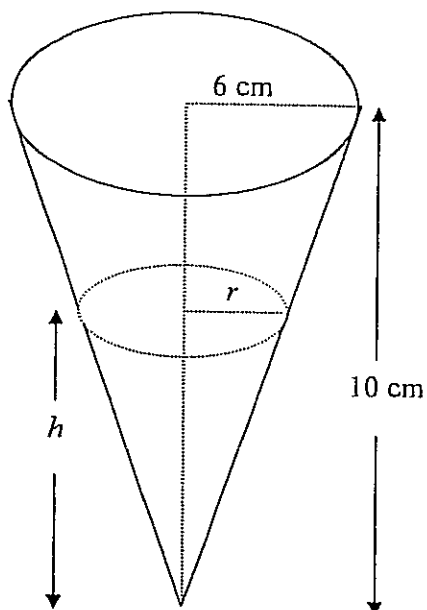
ii) A body warms from  $5^\circ$  to  $10^\circ$  in 20 minutes. The air temperature is  $30^\circ$ . 3  
Find the temperature of the body after 35 minutes.

**Question 4 ( 12 marks )** Start a new page.

(a) Find a general solution to the equation  $\sin 2x = 2 \sin^2 x$

3

(b) The diagram shows a conical drinking cup of height 10 cm and radius 6 cm. The cup is being filled with water at the rate of  $k \text{ cm}^3$  per second. The height of the water at time  $t$  seconds is  $h$  cm and the radius of the water's surface is  $r$  cm.



Not to scale.

i) Show that  $r = \frac{3h}{5}$ .

1

ii) Find the rate at which the height is increasing ie  $\frac{dh}{dt}$ .

3

Question 4 is continued on page 6.

**Question 4 (cont)**

c) The  $\triangle ABC$  has vertices  $A ( 2, 5 )$ ,  $B ( -1 , 2 )$  and  $C ( 3 , -1 )$ . The point

$Y ( \frac{3m+2n}{m+n}, \frac{5n-m}{m+n} )$  divides  $AC$  in the ratio  $m : n$ .

- i) Find the point  $X$  which divides  $AB$  in the ratio  $m : n$ . 2
- ii) Show that  $XY$  is parallel to  $BC$ . 1
- iii) Determine  $\frac{XY}{BC}$ . 2

**Question 5 is on page 7.**



**Question 5 ( 12 marks )** Start a new page.

- (a) (i) How many different arrangements of the word REDEEMED are possible? 1
- (ii) An arrangement of the letters of the word REDEEMED is chosen at random  
What is the probability that the Es are together ? 2

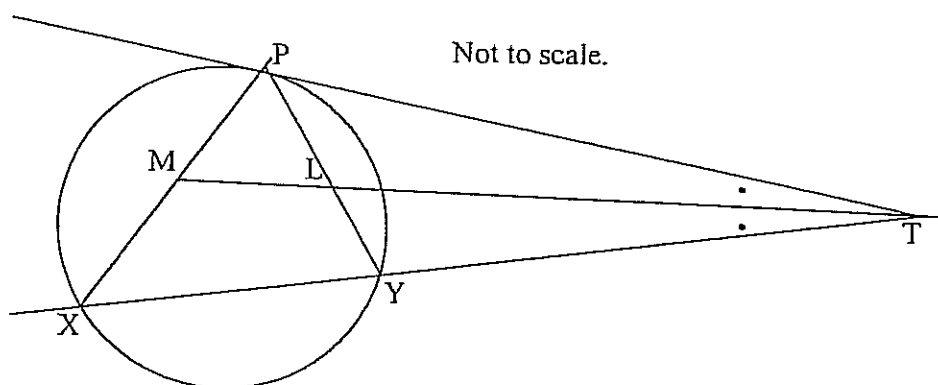
- (b) i) Show that the curve  $y = x^3 - 12x$  has stationary points at  $(-2, 16)$  and  $(2, -16)$ . 2

- ii) Find the largest domain including zero such that the function  
 $f(x) = x^3 - 12x$  has an inverse  $f^{-1}(x)$ . 1

- iii) On the same set of axes sketch the graphs of  $y = f(x)$  and  $y = f^{-1}(x)$  1

- iv) Find the gradient of the tangent to the curve  $y = f^{-1}(x)$  at the point  $(11, -1)$ . 1

- (c) TP is a tangent to the circle at P. XY is a chord of the same circle produced to T. MT bisects  $\angle PTX$ .



- i) Show that  $\angle MPT = \angle LYT$  2
- ii) Hence, or otherwise show that  $\triangle MPL$  is isosceles. 2

**Question 6 ( 12 marks )** Start a new page.

(a) A projectile is launched on a level surface with initial speed  $V$  ( $\text{ms}^{-1}$ ) and at an angle  $\theta$  to the horizontal. The equations of horizontal and vertical motion are given by

$$y = -\frac{1}{2}gt^2 + Vt \sin \theta \text{ and } x = Vt \cos \theta, \text{ } g \text{ is the acceleration due to gravity.}$$

( Do NOT prove this. )

(i) Show that the maximum height  $H$  reached by the projectile is  $\frac{V^2 \sin^2 \theta}{2g}$ . 2

(ii) Show that the range  $R$  of the projectile (i.e. the horizontal distance from its starting position to where it lands) is  $\frac{V^2 \sin 2\theta}{g}$ . 2

A large indoor stadium is 300 metres long. A golf ball is to be hit from floor level from one end of the stadium to the other without hitting the roof. The roof is just high enough for the ball to reach a maximum height of 25 metres.

(iii) Show that if the ball is just going to miss the roof then  $\theta = \tan^{-1}\left(\frac{1}{3}\right)$ . 2

(iv) Using this value of  $\theta$ , and using  $g = 9.8$ , find the value of  $V$  so that the ball just lands at the other end of the stadium. 2

(b) In a raffle 100 tickets are sold. Five tickets are drawn out which will each win a prize.

(i) How many different combinations of five winning tickets are there? 1  
(Leave your answer in unsimplified form)

(ii) Tom buys three tickets. What is the probability that all of Tom's tickets win prizes? 1  
(Leave your answer in unsimplified form)

(iii) What is the probability that at least one of Tom's tickets win prizes? 2  
(Leave your answer in unsimplified form)

**Question 7** ( 12 marks ) Start a new page.

a) The line  $y = mx$  is a tangent to the curve  $y = e^{kx}$ ,  $k > 0$ .

i) Show that  $m = ke$ . 2

ii) Determine the values of  $m$  such that the equation 2

$$e^{kx} = mx$$

will have 2 solutions, 1 solution or 0 solutions.

(b) Let  $f(x) = \tan^{-1}(\tan x)$

(i) Show that  $f'(x) = 1$  for all  $x$  where  $f'(x)$  is defined. 1

ii) Sketch  $y = f(x)$  for  $-\pi \leq x \leq \pi$  indicating any discontinuities. 2

iii) For what values of  $x$  is  $\tan^{-1}(\tan x) = x$ . 1

iv) Show that if  $-1 < x < 1$  then  $2 \tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ . 2

v) Suppose  $x > 1$ . Write down a relationship between 2

$$2 \tan^{-1} x \text{ and } \tan^{-1}\left(\frac{2x}{1-x^2}\right).$$

**End of paper.**



Question 1.

a)  $P(2) = 8 - 4b - 2b + 4 = 0$

$4b + 2b = +12$

$6b = 12$

$b = 2$

b)  $\lim_{x \rightarrow 0} \frac{\sin 3x}{x} = 3 \lim_{x \rightarrow 0} \frac{\sin 3x}{3x}$

$= 3$

c)  $-1 \leq \frac{x}{2} \leq 1$

$-2 \leq x \leq 2$  : domain

Range :  $-\frac{3\pi}{2} \leq y \leq \frac{3\pi}{2}$

d)  $\int_0^4 \frac{x}{\sqrt{2x+1}} dx$

$u = 2x+1 \Rightarrow \frac{u-1}{2} = x$   
 $\frac{du}{dx} = 2$

$\int_0^4 \frac{(u-1)}{2 \cdot 2} du$   
 $x=0, u=1$   
 $x=4, u=9$

$= \frac{1}{4} \int_1^9 u^{1/2} - u^{-1/2} du$

$= \frac{1}{4} \left[ \frac{2}{3} u^{3/2} - 2u^{1/2} \right]_1^9$

$= \frac{1}{4} \left[ \left( \frac{2}{3} \cdot 9 - 2 \cdot 3 \right) - \left( \frac{2}{3} - 2 \right) \right]$

$= \frac{3^2}{3}$

e) No, the x intercept of the tangent at a is further from b than a is.

Question 2

a)  $\frac{x+1}{x-1} = 2, x \neq 1$

$x+1 = 2x-2$

$3 = x$

test  $x=0, LHS = -1$

$LHS \neq RHS$

$\therefore \text{No soln } 1 < x < 3$

b)  $\int \sin^2 2x dx$

$= \int \frac{1}{2} - \frac{1}{2} \cos 4x dx$

$= \frac{x}{2} - \frac{1}{8} \sin 4x + C$

$= \frac{x}{2} - \frac{1}{8} \sin 4x + C$

c) i)  $\tan 60 = \frac{h}{a}$

$a = h \cot 60$

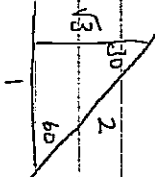
$a = h\sqrt{3}$

ii)  $\tan 60 = \frac{h}{b}$

$b\sqrt{3} = h$

$a = b\sqrt{3} \cdot \sqrt{3}$

$a = 3b$



$\cos 2x = \cos^2 x - \sin^2 x$   
 $= 1 - \sin^2 x - \sin^2 x$   
 $= 1 - 2\sin^2 x$

(3)

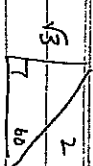
$$\text{iii)} \quad c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = (3b)^2 + b^2 - 2 \cdot 3b \cdot b \cos 120^\circ$$

$$= 10b^2 + 6b^2 \cdot \frac{1}{2}$$

$$c^2 = 13b^2$$

$$c = b\sqrt{13}$$



$$\text{d)} \quad n=1, \quad LHS = \frac{1}{2 \cdot 3} = \frac{1}{6}$$

$$LHS = RHS$$

$$RHS = \frac{1}{2(1+2)} = \frac{1}{6}$$

$$\text{Assume true for } n=k \text{ i.e. } \frac{1}{2 \cdot 3} + \dots + \frac{1}{(k+1)(k+2)} = \frac{k}{2(k+2)}$$

$$\text{Prove true for } n=k+1 \text{ i.e. } \frac{1}{2 \cdot 3} + \dots + \frac{1}{(k+2)(k+3)} = \frac{k+1}{2(k+3)}$$

$$S_{k+1} = S_k + T_{k+1}$$

$$= \frac{k}{2(k+2)} + \frac{1}{(k+2)(k+3)}$$

$$= \frac{k(k+3) + 2}{2(k+2)(k+3)}$$

$$= \frac{k^2 + 3k + 2}{2(k+2)(k+3)}$$

$$= \frac{(k+2)(k+1)}{2(k+2)(k+3)}$$

$$= \frac{k+1}{2(k+3)}$$

Q3

a) i)

$$\frac{y - ay^2}{x - 2ay} = \frac{ay^2 - ay^2}{2ay - 2ay}$$

$$= \frac{a(p-q)(p+q)}{2a(p-q)}$$

$$= \frac{p+q}{2}$$

$$2y - 2aq^2 = (2 - 2aq)(p+q)$$

passed through (0, -a)

$$-2y - 2aq^2 = -2aq(p+q)$$

$$1 + q^2 = qp + q^2$$

$$pq = 1$$

ii)

$$x = -apq(p+q) \quad ; \quad y = a(p^2 + q^2 + pq + 2)$$

$$x = -a(p+q) \quad ; \quad y = a(p^2 + q^2 + 3)$$

$$x^2 = a^2(p+q)^2$$

$$\frac{x^2}{a^2} = p^2 + 2pq + q^2$$

$$\frac{x^2}{a^2} - 2 = p^2 + q^2$$

$$\text{Sub into } y : \quad y = a \left( \frac{x^2}{a^2} - 2 + 3 \right)$$

$$y = a \left( \frac{x^2}{a^2} + 1 \right)$$

$$y = \frac{x^2}{a} + a$$

$$ay = x^2 + a^2$$

$$x^2 = ay - a^2$$

$$x^2 = a(y - a)$$

(5)

$$b) \cos x + \sqrt{3} \sin x = 1$$

$$\cos x + \sqrt{3} \sin x = r \cos(x - \bar{\phi})$$

$$r \cos \alpha = 1, \quad r \sin \alpha = \sqrt{3}$$

$$\frac{r \sin \alpha}{r \cos \alpha} = \tan \alpha = \sqrt{3} \quad r^2 = 4$$

$$r = 2$$

$$\alpha = \frac{\pi}{3}$$

$$2 \cos(x - \frac{\pi}{3}) = 1$$

$$\cos(x - \frac{\pi}{3}) = \frac{1}{2}$$

$$x - \frac{\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$x = \frac{2\pi}{3}, \frac{8\pi}{3}, 0$$

$$c) i) T = A + Ce^{kt}$$

$$\frac{dT}{dt} = Cke^{kt}$$

$$= kA + Cke^{kt} - kA$$

$$= kT - kA$$

$$= k(T - A)$$

$$ii) \text{ when } t=0, T=5^\circ, A=30^\circ$$

$$t=20, T=10^\circ$$

$$\text{when } t=35, T=?$$

$$\text{when } t=0, A=30^\circ, T=5$$

$$5 = 30 + Ce^0$$

$$C = -25$$

$$T = 30 - 25e^{kt}$$

(6)

$$\text{when } t=20, T=10^\circ$$

$$10 = 30 - 25e^{20k}$$

$$-20 = -25e^{20k}$$

$$\frac{4}{5} = e^{20k}$$

$$\ln\left(\frac{4}{5}\right) = 20k$$

$$k = \frac{1}{20} \ln\left(\frac{4}{5}\right)$$

$$\text{when } t=35, T=?$$

$$T = 30 - 25e^{kt}$$

$$T = 30 - 25e^{35k}$$

$$T = 29.99^\circ \approx 30^\circ$$

$$T = 13.082059$$

$$\approx 13.1^\circ$$

# Question 4

a)  $\sin 2x = 2\sin^2 x$

$2\sin x \cos x = 2\sin^2 x$

$\sin^2 x - \sin x \cos x = 0$

$\sin x (\sin x - \cos x) = 0$

$\sin x = 0, \sin x - \cos x = 0$

$x = 0, \sin x = \cos x$

$\tan x = 1$

$x = \frac{\pi}{4}$

b) i)  $\frac{b}{10} = \frac{r}{h}$

$r = \frac{3h}{5}$

ii)  $\frac{dV}{dt} = k, V = \frac{1}{8}\pi r^2 h$

$V = \frac{1}{8}\pi \left(\frac{3h}{5}\right)^2 h$

$V = \frac{3\pi h^3}{25}$

$\frac{dV}{dh} = \frac{9\pi h^2}{25}$

$\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$

$= \frac{25}{9\pi h^2} \cdot k$

$\frac{dh}{dt} = \frac{25k}{9\pi h^2}$

$\frac{1}{\pi} = \frac{25k}{25 \cdot 9\pi}$

$k = 9$

c) i)  $A(2,5), B(-1,2)$

$X\left(\frac{-m+2n}{m+n}, \frac{2m+5n}{m+n}\right)$

ii)  $m_{BC} =$

$\frac{2+1}{-1-3}$

$= \frac{-3}{4}$

$m_{xy} = \frac{5n-m-(2m+5n)}{3m+2n-(2n-m)}$

$= \frac{-3m}{4m}$

$= -\frac{3}{4}$

iii)  $XY = \sqrt{(-m+2n-3m-2n)^2 + (2m+5n-5n+m)^2}$

$= \sqrt{16m^2 + 9m^2}$

$= \sqrt{25m^2}$

$= 5m$

$BC = \sqrt{(-1-3)^2 + (2+1)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$

$\frac{XY}{BC} = \frac{5m}{5}$

$= m$



### Question 5

$$\begin{array}{r} 81 \\ 214 \overline{) 17124} \\ \underline{1712} \phantom{4} \\ 0 \end{array}$$

$$\text{ii) } P(\text{E's together}) = \frac{5!}{2! \cdot 8! \cdot 40}$$

$$= \frac{120}{1650}$$

$$\frac{4}{14} = \frac{2}{7}$$

$$b) \quad y = x^3 - 12x$$

$$y' = 3x^2 - 12$$

$$y'' = 6x$$

At  $y' = 0$  pts

$$3x^2 - 12 \geq 0$$

$$x^2 - 4x = 0$$

$$(x-2)(x+2) = 0$$

$$x = \pm 2$$

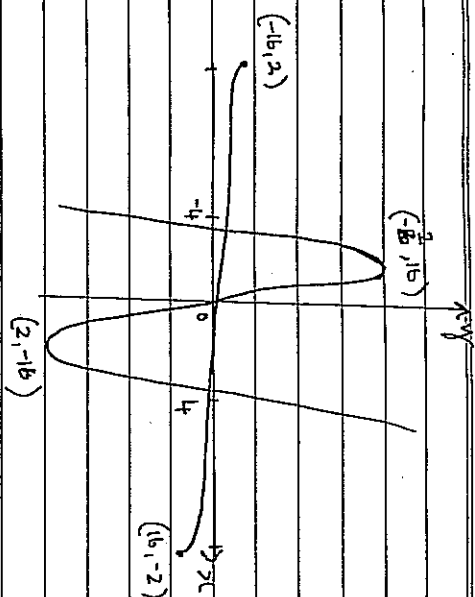
when  $x=2$ ,  $y=8-24=-16$

$$x = -2, y = -8 + 24 = 16 \quad (-2, 16)$$

$$-2 \leq x \leq 2$$

⑨

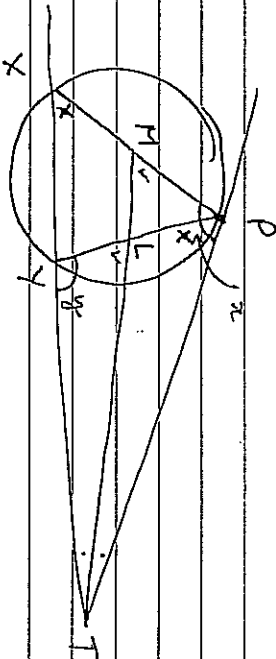
iii)

iv) at  $x = -1$ ,  $f'(x) = 3x^2 - 2$ 

$$f'(-1) = 3 - 12 = -9$$

gradient of tangent in  $f^{-1}(x)$  is  $-\frac{1}{g}$

2


$$i) \quad \angle LPT = \angle PXY \quad (\angle \text{ in alt segment})$$

$$LTYL = LYXM + LYPM \quad (\text{ext } L \text{ of } \Delta)$$

$$L_{MPT} = L_{KPI} + L_{IP} = L_{PY} + L_{IP} = L_{PY} + L_{IP}^H$$

$L_{\theta W} = L_{\theta T}$

ii)  $\Delta MP T \parallel \Delta LYT$  (equiangular)

$$Z_{\text{LT}} = Z_{\text{MP}} \left( \text{corr.}^{0.15} \text{ of } 111 \Delta S \right)$$

$L_{YLT} = L_{PLM}$  (vertically opp 2's)

$\Delta p_{H_2}$  in nacosoles

# Question 6

a)  $y = -\frac{1}{2}gt^2 + vt \sin \theta$ ,  $x = vt \cos \theta$

i) when  $y = 0$

$$0 = -\frac{1}{2}gt^2 + vt \sin \theta$$

$$0 = t \left( v \sin \theta - \frac{1}{2}gt \right)$$

$$t = 0, \quad \frac{1}{2}gt = v \sin \theta$$

$$t = \frac{2v \sin \theta}{g} \leftarrow \text{time of flight}$$

max ht when  $t = \frac{v}{g} \sin \theta$

$$H = y = -\frac{1}{2}g \left( \frac{v \sin \theta}{g} \right)^2 + v \left( \frac{v \sin \theta}{g} \right) \sin \theta$$

$$H = -\frac{1}{2} \frac{v^2 \sin^2 \theta}{g} + \frac{v^2 \sin^2 \theta}{g}$$

$$H = \frac{v^2 \sin^2 \theta}{2g}$$

ii) when  $t = \frac{2v \sin \theta}{g}$ ,  $y = 0$  &  $x = R$ .

$$x = R = v \left( \frac{2v \sin \theta}{g} \right) \cos \theta$$

$$= \frac{v^2 \sin 2\theta}{g}$$

iii) if  $H = 25 = \frac{v^2 \sin^2 \theta}{2g}$

$$50g = v^2 \sin^2 \theta$$

$$\sin^2 \theta = \frac{50g}{v^2}$$

(16)

when  $y = H$ ,

$$x = vt \cos \theta$$

$$150 = v \left( \frac{y}{g} \sin \theta \right) \cos \theta$$

$$150 = \frac{v^2 \sin \theta \cos \theta}{g}$$

$$\frac{150g}{v^2} = \sin \theta \cos \theta$$

$$\frac{\sin^2 \theta}{\sin \theta \cos \theta} = \frac{50g}{v^2} \div \frac{150}{v^2}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{50g}{v^2} \times \frac{v^2}{150g}$$

$$\tan \theta = \frac{1}{3}$$

$$\theta = \tan^{-1} \left( \frac{1}{3} \right)$$

iv)  $R = \frac{v^2 \sin 2\theta}{g}$

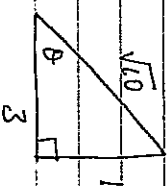
$$300 = \frac{v^2 \sin 2\theta}{g}$$

$$300g = v^2 \cdot 2 \cdot \frac{1}{\sqrt{10}} \cdot \frac{3}{\sqrt{10}}$$

$$90000g = v^2$$

$$49000 = v^2$$

$$v = 70 \text{ m/s}$$



b) i)  $100 C_5$

ii)  $\frac{3 C_0 \cdot {}^{91}C_2}{100 C_5} \left( = \frac{{}^{91}C_2}{100 C_5} \right)$

iii)  $1 - P(\text{winning none})$

$= 1 - \frac{{}^{91}C_5}{100 C_5}$

Question 7

i)  $y' = k e^{kx}$

$y = mx$

at  $x, kx$   
 $m = k e^{kx}$   
 $\frac{m}{k} = e^{kx}$

$\ln\left(\frac{m}{k}\right) = kx$

$x = \frac{1}{k} \ln\left(\frac{m}{k}\right)$

now  $y = \frac{m}{k} \ln\left(\frac{m}{k}\right)$

$\frac{m}{k} \ln\left(\frac{m}{k}\right) = \frac{1}{k} \ln\left(\frac{m}{k}\right)$

$\ln\left(\frac{m}{k}\right) = \frac{1}{m} \cdot \frac{m}{k} \cdot \frac{k}{m}$

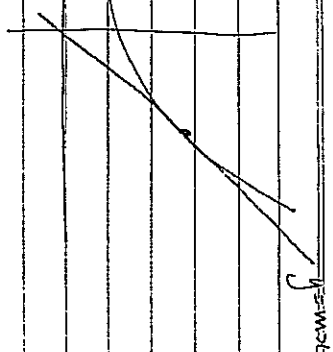
$\ln\left(\frac{m}{k}\right) = 1$

$\frac{m}{k} = e$

$m = ke$

13

ii)



now when  $y = mx$  is a tangent line in 1st qd  
 ie  $mx = k e^{kx}$   
 $\Rightarrow m = ke$

to have 2 roots  $y = mx$  must be a chord

ie  $mx > k e^{kx}$

as the y value on curve

$y = mx >$  than y value

$mx > e^{kx}$   
 ie  $m > ke$

to have 0 roots  $y = mx$

must not touch  $y = e^{kx}$

ie  $m < ke$

b)  $f(x) = \tan^{-1}(\tan x)$

$f'(x) = \frac{1}{1 + \tan^2 x} \cdot \sec^2 x$

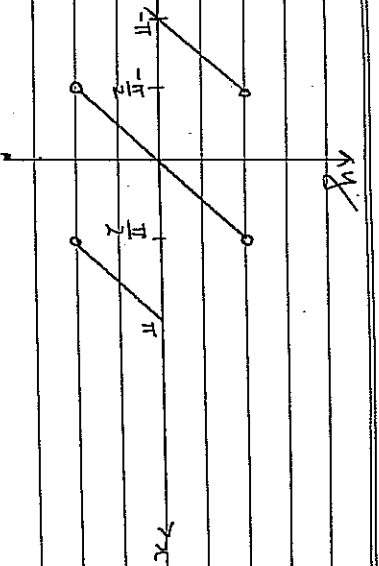
$= \frac{\sec^2 x}{\sec^2 x}$

$= 1$ ,  $\tan \frac{\pi}{2}$  is not defined.

14

(5)

iii)



$$\text{iii) } -\frac{\pi}{2} < 2x < \frac{\pi}{2} \quad \text{from graph}$$

$$\text{iv) if } -1 < x < 1, \text{ let } \tan^{-1} x = \alpha \\ x = \tan \alpha$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$= \frac{2x}{1 - x^2}$$

$$\text{also } -1 < x < 1$$

$$-1 < \tan \alpha < 1$$

$$-\frac{\pi}{4} < \alpha < \frac{\pi}{4}$$

$$-\frac{\pi}{2} < 2\alpha < \frac{\pi}{2}$$

$$\text{and } \tan^{-1}(\tan 2\alpha) = 2\alpha \quad \text{for } -\frac{\pi}{2} < 2\alpha < \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1}(\tan 2x) = 2x \quad \text{as } -\frac{\pi}{2} < 2x < \frac{\pi}{2}$$

$$\text{Now } \tan 2x = \frac{2x}{1-x^2} \quad \therefore \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2x$$

$$\text{and } \alpha = \tan^{-1} x \quad \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2 \tan^{-1} x$$

$$\text{v) if } x > 1 \quad \tan^{-1} \frac{\pi}{4} < \alpha < \frac{\pi}{2} \quad (\alpha = \tan^{-1} x)$$

$$\frac{\pi}{2} < 2\alpha < \pi$$

from the graph in iii) it can be seen that

$$\tan^{-1}(\tan x) = x - \pi$$

$$\text{let } x = 2\alpha, \quad \tan^{-1}(\tan 2\alpha) = 2\alpha - \pi$$

$$\text{Now } \tan 2\alpha = \frac{2x}{1-x^2} \quad \text{as } \alpha = \tan^{-1} x$$

$$\therefore \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2 \tan^{-1} x - \pi$$