2011 2U Trial HSC

QUESTION ONE

(a) $2e^2 = 14.7781122\ldots$ ✓
$= 14.8$ (to 3 sig fig) ✓

(b) $2x^2 - x - 6$
$= 2x^2 - 4x + 3x - 6$ ✓
$= 2x(x-2) + 3(x-2)$
$= (2x+3)(x-2)$ ✓

(c) $\dfrac{2}{x+1} - \dfrac{3}{x}$

$= \dfrac{2x}{x(x+1)} - \dfrac{3(x+1)}{x(x+1)}$ ✓

$= \dfrac{2x - 3x - 3}{x(x+1)}$

$= \dfrac{-x-3}{x(x+1)}$ ✓

(d) $|2x-1| = 9$

either $2x - 1 = 9$ or $2x - 1 = -9$
$2x = 10$ $\quad 2x = -8$
$x = 5$ ✓ $\quad x = -4$ ✓

(e) $(\sqrt{5}-1)(2\sqrt{5}+3)$ ✓
$= \sqrt{5}(2\sqrt{5}+3) - 1(2\sqrt{5}+3)$
$= 10 + 3\sqrt{5} - 2\sqrt{5} - 3$
$= 7 - \sqrt{5}$ ✓

(f) $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{12}{6}\left[2 \times 3 + (12-1) \times 3\right]$ ✓

$= 6 \times 39$

$= 354$ ✓

QUESTION TWO

(a) (i) $y = (x^3+1)^7$

$y' = 3x^2 \times 7 \times (x^3+1)^6$ ✓

$= 21x^2(x^3+1)^6$ ✓

(ii) $y = x^4 \log_e x$

$y' = \frac{x^4}{x} + 4x^3 \log_e x$ ✓

$= x^3 + 4x^3 \log_e x$ ✓

(iii) $y = \frac{\sin x}{x+1}$

$y' = \frac{(x+1)\cos x - 1 \times \sin x}{(x+1)^2}$

$= \frac{(x+1)\cos x - \sin x}{(x+1)^2}$ ✓ ✓

(b) perp dist $= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$ ✓

$= \frac{|4 \times 6 - 3 \times -2 + 7|}{\sqrt{4^2 + (-3)^2}}$

$= \frac{37}{5}$ ✓

(c) (i) $\int \frac{1}{x+7}\,dx$

$= \log_e (x+7) + C$

(ii) $\int_0^{\frac{\pi}{8}} \sec^2 2x\,dx$

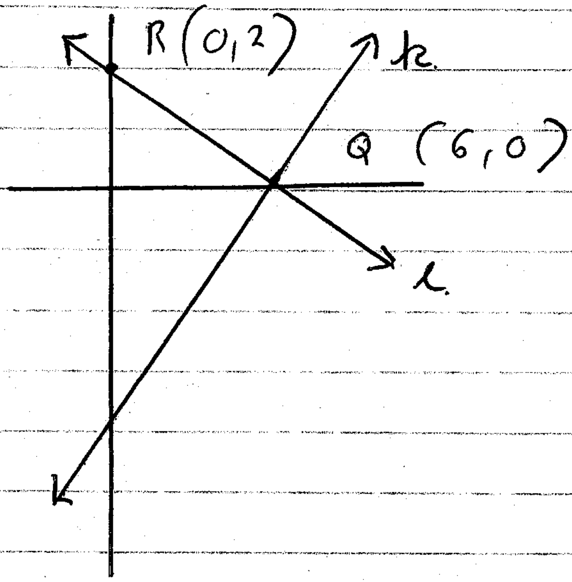
$= \left[\frac{1}{2} \tan 2x\right]_0^{\frac{\pi}{8}}$

$= \frac{1}{2} \tan \frac{\pi}{4}$

$= \frac{1}{2}$

QUESTION THREE

(a) (i)



gradient of $l = \frac{\text{rise}}{\text{run}} = -\frac{2}{6} = -\frac{1}{3}$ ✓

equation of l : $y = mx + b$

$y = -\frac{1}{3}x + 2$ ✓

(b = y-intercept = 2)

(ii) See diagram for drawing of k

$m_l = -\frac{1}{3} \Rightarrow m_k = 3$ ✓

(neg reciprocal)

equation of k : $y - y_1 = m(x - x_1)$

$y - 0 = 3(x - 6)$ ✓

$\therefore \quad y = 3x - 18$

(iii) k cuts the y-axis

when $x = 0$

ie when $y = 3(0) - 18$

$= -18$

$\therefore T\ (0, -18)$ ✓

(iv) midpoint $TR = \left(\frac{0+0}{2}, \frac{2-18}{2}\right)$

$= (0, -8)$

$\therefore$ circle has centre $(0, -8)$

radius 10

✓

$\therefore$ eqn is $x^2 + (y+8)^2 = 100$

sub $Q\ (6, 0)$ into

$$x^2 + (y+8)^2 = 100$$

$LHS = 6^2 + (0+8)^2$

$= 36 + 64$

$= 100$ ✓

$= RHS$ $\therefore Q\ (6, 0)$ lies on circle

(v) $x^2 + (y+8)^2 = 100$

sub $(8, m) \Rightarrow$

$8^2 + (m+8)^2 = 100$

$(m+8)^2 = 36$

$m + 8 = \pm 6$ ✓

$m = \pm 6 - 8$

$\therefore m = -2$ or $m = -14$ ✓

(b) Using "k" method

$4x + y - 5 + k(3x - 2y - 12) = 0$

sub $(1, -2)$

$4 - 2 - 5 + k(3 + 4 - 12) = 0$

$-3 + -5k = 0$

$k = -\frac{3}{5}$

$\therefore 4x + y - 5 - \frac{3}{5}(3x - 2y - 12) = 0$

$20x + 5y - 25 - 9x + 6y + 36 = 0$

$11x + 11y + 11 = 0$

$x + y + 1 = 0$

$y = -x - 1$

OR

$4x + y - 5 = 0$ (1)

$3x - 2y - 12 = 0$ (2)

$8x + 2y - 10 = 0$ (3) = (1) $\times 2$

$11x - 22 = 0$ (2) + (3)

$x = 2 \quad y = -3$

$\therefore$ Pt of intersection of (1) & (2) is $(2, -3)$

grad from $(2, -3)$ to $(1, -2)$

$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 + 2}{2 - 1} = -1$

$\therefore$ required eqn : $y - y_1 = m(x - x_1)$

$y + 2 = -1(x - 1)$

$y + 2 = -x + 1$

$y = -x - 1$

QUESTION FOUR

(a) $\angle EDC = \angle BAD$
(alternate angles $AB \parallel CD$) ✓
$= 60^\circ$
$\therefore \angle ECD = \theta$
$= 180^\circ - 70^\circ - 60^\circ$
(angle sum of $\triangle ECD$ is 180°)
$= 50^\circ$ ✓

(b)
$\frac{x}{7} = \frac{5}{9}$ (ratio of intercepts) ✓
$x = \frac{7 \times 5}{9}$
$= \frac{35}{9}$
$= 3\frac{8}{9}$ ✓

(c) (i) $\angle ADE = \angle CBE$ } $AD \parallel BC$
$= x^\circ$ } alternate $\angle s$
Similarly $\angle DAE = x^\circ$ ✓
$AE = DE$ (sides opposite equal $\angle s$ ADE, DAE) ✓
(ii) In $\triangle BEC$
$EB = EC$ (opposite equal $\angle s$ EBC, ECB)
But $AC = AE + EC$
and $DB = DE + EB$
$\therefore AC = DB$ ✓
In $\triangle s$ ABC, DCB ✓
$AC = DB$ (as above)
BC is common

$\angle ACB = \angle DBC$ (given)
$\therefore \triangle ABC \equiv \triangle DCB$ (SAS) ✓

(iii) $\because \angle ABC = \angle DCB$
(Corresponding $\angle$s in congruent $\triangle$s) ✓

$\therefore \angle ABD + \angle DBC = \angle DCA + \angle ACB$ ✓
But $\angle DBC = \angle ACB$ (given)
$\therefore \angle ABD = \angle DCA$ ✓

QUESTION FIVE

(a) $$\frac{dy}{dx} = \frac{1}{\sqrt{2x+1}}$$

$$\frac{dy}{dx} = (2x+1)^{-\frac{1}{2}}$$

$$\therefore y = \frac{(2x+1)^{\frac{1}{2}}}{\frac{1}{2}\times 2} + C \checkmark$$

$$y = \sqrt{2x+1} + C$$

sub $(4, 5) \Rightarrow 5 = \sqrt{2(4)+1} + C$

$$5 = 3 + C \checkmark$$

$$C = 2$$

$\therefore$ equation is $y = \sqrt{2x+1} + 2$ ✓

(b) (i) For limiting sum

$$-1 < 3x < 1 \checkmark$$

$$-\frac{1}{3} < x < \frac{1}{3} \checkmark$$

(ii) $$S_\infty = \frac{a}{1-r}$$

$$100 = \frac{1}{1-3x} \checkmark$$

$$1 - 3x = \frac{1}{100}$$

$$-3x = -\frac{99}{100}$$

$$x = \frac{33}{100} \checkmark$$

(c) $M = 5e^{-kt}$

(i) when $M = 4.2$, $t = 2$

$\therefore 4.2 = 5e^{-2k}$

$e^{-2k} = 0.84$

$\therefore \ln(0.84) = -2k$ ✓

$\therefore k = \dfrac{\ln(0.84)}{-2}$

$= 0.0871766693$ ✓

(Calc)

(ii) We want to find t
when $M = \frac{1}{2} \times 5 = 2.5$

$2.5 = 5e^{-0.087176693t}$ ✓

$0.5 = e^{-0.08717 6693t}$

$\ln(0.5) = -0.087176693t$ ✓

$t = \dfrac{\ln(0.5)}{-0.087176693}$

$= 7.95$ hours ✓

(to 2 dec pls)

QUESTION SIX

(a) (i) $x = 1 - 2\sin 2x$

at $t = 0$, $x = 1 - 2\sin 0 = 1$ ✓

ie 1 metre on right-hand side of O

(ii) $v = \frac{dx}{dt} = -4\cos 2t$ ✓

'comes to rest' means '$v = 0$'

$$-4\cos 2t = 0$$
$$\cos 2t = 0$$
$$2t = \frac{\pi}{2} \text{ (1st time)}$$
$$t = \frac{\pi}{4}$$ ✓

$t = \frac{\pi}{4}$

$$x = 1 - 2\sin 2\tfrac{\pi}{4}$$
$$= 1 - 2\sin\tfrac{\pi}{2}$$
$$= 1 - 2$$
$$= -1$$ ✓

ie first comes to rest 1 metre on the left-hand side of O

(iii) $v = 0$ $\cos 2t = 0$

$$2t = \frac{\pi}{2}, \frac{3\pi}{2}$$
$$\therefore t = \frac{\pi}{4}, \frac{3\pi}{4}$$

For $t = \frac{3\pi}{4}$ ✓

$$x = 1 - 2\sin\left(2 \times \tfrac{3\pi}{4}\right)$$
$$= 1 - 2\sin\tfrac{3\pi}{2}$$
$$= 1 - 2 \times -1$$
$$= 1 + 2 = 3$$ ✓

ie 3 metres on right-hand of O

(iii) $a = \frac{dv}{dt} = \frac{d}{dt}(-4\cos 2t)$

$= 8\sin 2t$ ✓

$t = \frac{\pi}{12}$ $\quad a = 8\sin\left(2 \times \frac{\pi}{12}\right)$

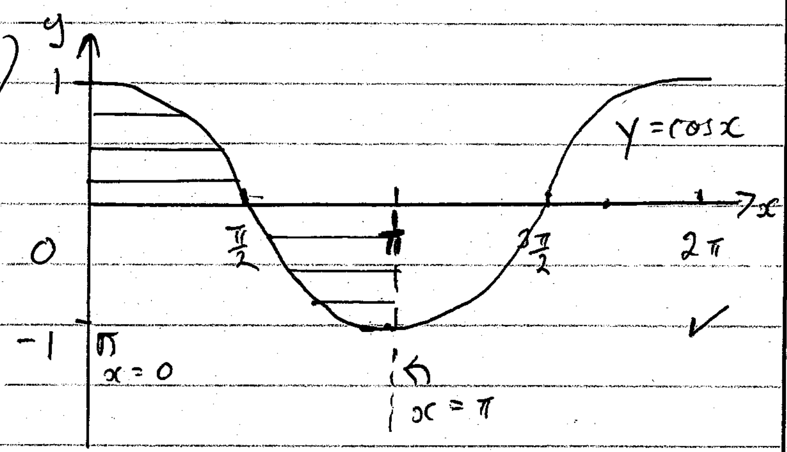
$= 8\sin\frac{\pi}{6}$

$= 8 \times \frac{1}{2}$

$= 4$

ie acceleration is $4\ ms^{-2}$ ✓

(b) (i)



✓

(ii) See above, for shaded regions

$A = \int_0^{\frac{\pi}{2}} \cos x\,dx + \left|\int_{\frac{\pi}{2}}^{\pi} \cos x\,dx\right|$ ✓

$= \left[\sin x\right]_0^{\frac{\pi}{2}} + \left|\left[\sin x\right]_{\frac{\pi}{2}}^{\pi}\right|$

$= \sin\frac{\pi}{2} - \sin 0 + \left|\sin\pi - \sin\frac{\pi}{2}\right|$

$= 1 - 0 + |0 - 1|$

$= 1 + 1$

$\therefore$ Area $= 2$ units^2 ✓

OR $\quad A = 2\int_0^{\frac{\pi}{2}} \cos x \, dx$ ✓

$= 2\left(\sin x\right)_0^{\frac{\pi}{2}}$

$= 2\left(\sin \frac{x}{2} - \sin 0\right)$

$= 2 \ (1 - 0)$ ✓

Area $= 2$ units^2

(iii) $\quad \cos x = \frac{\sqrt{3}}{2} \qquad 0 \leq x \leq 2\pi$

$x = \frac{\pi}{6}, \ 2\pi - \frac{\pi}{6}$

$x = \frac{\pi}{6}, \ \frac{11\pi}{6}$ ✓

QUESTION SEVEN

(a) $\log_e x - \dfrac{3}{\log_e x} = 2$

let $u = \log_e x$

$u - \dfrac{3}{u} = 2$

$u^2 - 3 = 2u$

$u^2 - 2u - 3 = 0$ ✓

$(u-3)(u-1) = 0$

$u = 3$ or $u = -1$ ✓

$\log_e x = 3$ $\quad\quad$ $\log_e x = -1$

$x = e^3$ $\quad\quad$ $x = e^{-1}$ ✓

(b) (i) 45 km in 1 hour

= 45 km in 60 minutes

= $\frac{3}{4}$ km in 1 min ✓

(ii) $l = r\theta$

$\frac{3}{4} = 0.5 \times \theta$ ✓

$\theta = \frac{3}{4} \times 2$

$= 1.5$ radians

$= 1.5 \times \dfrac{180^\circ}{\pi}$

$\theta = 85.94366927$

$= 86^\circ$ (nearest deg) ✓

(c) (i) Probability

$= P[(1,1) \text{ or } (2,2) \text{ or } (3,3) \text{ or } (4,4) \text{ or } (5,5) \text{ or } (6,6)]$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36}$

$= \frac{6}{36} = \frac{1}{6}$ ✓

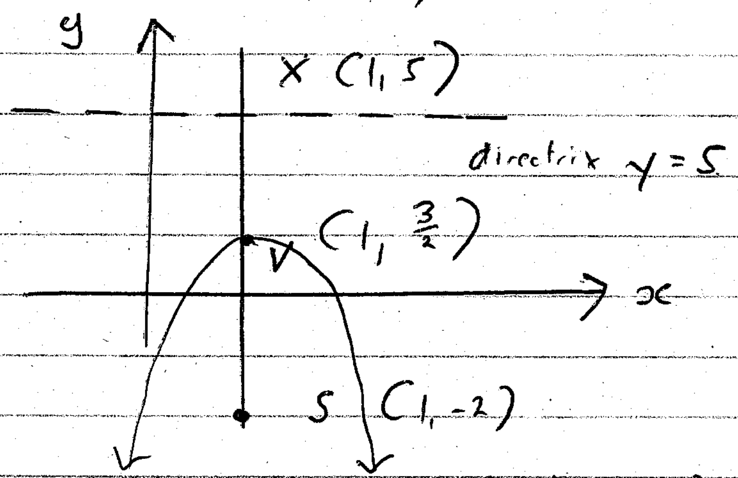
or Probability (Chris's throw same as Pat's)

$= \frac{1}{6}$ ✓

(ii) Let Pat be first, Chris be second relevant pairs are (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 3) (2, 4) (2, 5) (2, 6)
(3, 4) (3, 5) (3, 6) ✓
(4, 5) (4, 6)
(5, 6)

P (Chris' no > Pat's no) $= \frac{15}{36} = \frac{5}{12}$ ✓

(d)



(i) Vertex = midpt SX $= \left(\frac{1+1}{2}, \frac{5-2}{2}\right)$

$= \left(1, \frac{3}{2}\right)$ ✓

(ii) Focal length = dist XV

$= 5 - \frac{3}{2} = \frac{7}{2}$ ✓

(iii) $(x-h)^2 = 4a(y-k)$

$(x-1)^2 = -4 \times \frac{7}{2}\left(y - \frac{3}{2}\right)$

$(x-1)^2 = -14\left(y - \frac{3}{2}\right)$

$x^2 - 2x + 1 = -14y + 21$

$x^2 - 2x + 14y - 20 = 0$ ✓

See above for sketch

QUESTION EIGHT

(a) (i) $y = 3x^2 - x^3$ ✓

$\frac{dy}{dx} = 6x - 3x^2 = 0$

for stationary points

$2x - x^2 = 0$

$x(2-x) = 0$

$x = 0$ or 2 ✓

$\frac{d^2y}{dx^2} = 6 - 6x$

at $x = 0$ $\frac{d^2y}{dx^2} > 0$

$\therefore$ minimum at $(0, 0)$ ✓

at $x = 2$ $\frac{d^2y}{dx^2} < 0$

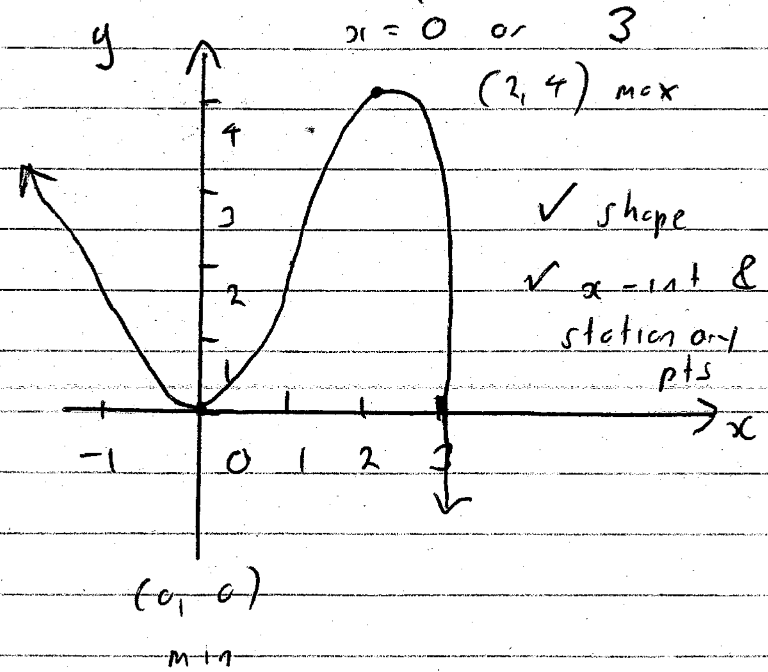
$\therefore$ maximum at $(2, 4)$ ✓

(ii) x intercepts at $y = 0$

$3x^2 - x^3 = 0$

$x^2(3-x) = 0$

$x = 0$ or 3



✓ shape

✓ x-int & stationary pts

(iii) $\frac{dy}{dx} = 6x - 3x^2$

at $x = -1$ $\frac{dy}{dx} = -6 - 3 = -9$ ✓

eqn of tangent.

$$y - 4 = -9(x+1)$$
$$y - 4 = -9x - 9$$
$$y = -9x - 5$$

$9x + y + 5 = 0.$ ✓

(b) (i) $y = \log_e(\cos x)$

$$= -\frac{\sin x}{\cos x}$$
$$= -\tan x$$

(ii) $\int_0^{\frac{\pi}{4}} \tan x = -\left[\log_e \cos x\right]_0^{\frac{\pi}{4}}$ ✓

$$= -\left[\log_e \frac{1}{\sqrt{2}} - \log_e 1\right]$$
$$= -\log_e \frac{1}{\sqrt{2}}$$ ✓
$$= -\log_e 2^{-\frac{1}{2}}$$ ✓
$$= \frac{1}{2}\log_e 2$$

QUESTION NINE

(a) $3^{2x} + 2\times 3^{x} - 15 = 0$

Let $u = 3^x$, $u^2 = 3^{2x}$

Then $u^2 + 2u - 15 = 0$

ie $(u+5)(u-3) = 0$

$\therefore$ $u = -5$ or $u = 3$ ✓

$\therefore$ $3^x = -5$ or $3^x = 3$

ie no soln $\quad x = 1$ ✓

(b) $V = \pi \int_0^{16} x^2 \, dy$

$= \pi \int_0^{16} y^{\frac{1}{2}} \, dy$ ✓

$\left(y = x^4 \text{ so } y^{\frac{1}{2}} = x^2 \right)$

$V = \pi \left[\frac{2y^{\frac{3}{2}}}{3} \right]_0^{16}$ ✓

$= \pi \left[\frac{2\times 16^{\frac{3}{2}}}{3} - \frac{2\times 0^{\frac{3}{2}}}{3} \right]$ ✓

$= \pi \times \frac{128}{3}$

Volume $= \frac{128}{3}\pi$ units^3 ✓

(c) Treat the investment in 2 parts

Part I $\quad$ \$1500 invested each year for 40 years

1st $\quad$ \$1500 $= \$1500 \times 1.07^{40}$

2nd $\quad$ \$1500 $= \$1500 \times 1.07^{39}$

$\downarrow$

last $\quad$ \$1500 $= \$1500 \times 1.07^{1}$ ✓

Part 2: For the last 15 years, the extra \$3500 (to make up the \$5000 to be invested each year for 15 years)

1st \$3500 $= \$3500 \times 1.07^{15}$

2nd \$3500 $= \$3500 \times 1.07^{14}$

$\downarrow$

last \$3500 $= \$3500 \times 1.07^{1}$ ✓

$$\text{total} = 1500\left(1.07 + 1.07^2 + \dots + 1.07^{40}\right) \text{ plus}$$
$$3500\left(1.07 + 1.07^2 + \dots + 1.07^{15}\right) ✓$$
$$= 1500 \times 1.07 \frac{\left(1.07^{40} - 1\right)}{1.07 - 1} + 3500 \times 1.07 \frac{\left(1.07^{15} - 1\right)}{1.07 - 1} ✓$$
$$= \$320419.35 + \$94108.19 ✓$$
$$= \$414\,523 \text{ (nearest dollar)} ✓$$

(3 marks for correctly calculating one part only)

QUESTION TEN

(a) (i) $v = \frac{dx}{dt}$

$$\therefore x = \int_a^b v\,dt \checkmark$$

Since the initial time is 0, $a = 0$
The unit for velocity was km/h
The final time was 4 mins or $\frac{1}{15}$ h

$$\therefore b = \frac{1}{15} \checkmark$$

$$\therefore x = \int_0^{\frac{1}{15}} v\,dt$$

(ii) Using the formula:

$$\int_0^{\frac{1}{15}} f(t)\,dt$$

$$= \frac{h}{3}\left\{y_0 + y_4 + 4\times(y_1 + y_2) + 2y_2\right\}$$

where $h = \frac{1}{4}\left(\frac{1}{15} - 0\right)$ ie $\frac{1}{60}$ ✓

$$x = \frac{1}{3} * \frac{1}{60}\left(0 + 40 + 4\times(25+30) \checkmark + 2\times 34\right)$$

$$= \frac{1}{180}(328)$$

$$\approx 1.8 \text{ km (to 1 dp)} \checkmark$$

(b) (i) Using speed $= \frac{\text{distance}}{\text{time}}$

$$\therefore \text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\therefore \text{time} = \frac{1000}{v} \checkmark$$

But cost per hour

= driver cost/h + fuel cost/h

$= 2 \times 36 + \left(6 + \frac{v^2}{50}\right) \times 1.5$

$= 72 + 9 + \frac{1.5v^2}{50}$

$= 81 + \frac{3v^2}{100}$ ✓

$\therefore$ Total Cost $= \left(81 + \frac{3v^2}{100}\right) \times \frac{1000}{v}$

$= 30v + \frac{81000}{v}$ ✓

(ii) $\frac{dc}{dv} = -\frac{81000}{v^2} + 30 = 0$ ✓

$30v^2 = 81000$

$v^2 \; \frac{81000}{30}$

$= 2700$

$v = \sqrt{2700}$

$= 51.96$ km/h ✓

(to 2decpl)

$\frac{d^2c}{dv^2} = \frac{162000}{v^3} > 0$

$\therefore$ min at $v = 51.96$ ✓

however the trip time must be less than 12 hours

$\frac{1000}{v} \leq 12$ ✓

$v \geq \frac{1000}{12} \geq 83\frac{1}{3}$

Hence the soln found to $\frac{dc}{dv} = 0$ does not satisfy the time constraint. Since cost increases the farther you are away from 51.96 km/h, the speed to minimise cost is $83\frac{1}{3}$ km/h. $= 83.33$ km/h (to 1 dp)