TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2005

MATHEMATICS EXTENSION 1

Time Allowed – 2 Hours
(Plus 5 minutes Reading Time)

All questions may be attempted

All questions are of equal value

Department of Education approved calculators are permitted

In every question, show all necessary working

Marks may not be awarded for careless or badly arranged work

No grid paper is to be used unless provided with the examination paper

The answers to all questions are to be returned in separate bundles clearly labeled Question 1, Question 2, etc. Each question must show your Candidate Number.

Question 1.   [Start a new page]   **Marks**

a) If $P(x) = x^3 - 2x^2 + ax + 4$ is divisible by $(x+2)$, what is the value of a ?   **1**

b) i)   Find $\dfrac{d}{dx} \ln(\cos 2x)$ **1**

　　ii) Hence evaluate exactly $\displaystyle\int_0^{\cdot} \tan 2x \; dx$ **2**

c) Find i) $\displaystyle\int \dfrac{e^{3x} dx}{2 + e^{3x}}$ **1**

　　ii) $\displaystyle\int \dfrac{dx}{\sqrt{9 - 4x^2}}$ **2**

d) Find the acute angle between the straight lines $y = \sqrt{3}x + 2$ and $x = 2$.   **2**

e) Solve : $x + 2 < \dfrac{4}{x-1}$ $(x \neq 1)$ **3**

Question 2.   [Start a new page]   **Marks**

a) By making the substitution $u = \sqrt{x}$, evaluate exactly $\displaystyle\int_0^{\frac{\pi^2}{4}} \dfrac{\sin\sqrt{x}}{\sqrt{x}} \, dx$ **3**

b) i)   Sketch the graph of the curve $y = 3\sin^{-1}(x/2)$, clearly indicating the domain and range.   **2**

　　ii) Find the area enclosed between the curve $y = 3\sin^{-1}(x/2)$, the line $y = (3\pi/2)$ and the positive y axis.   **2**

c) The polynomial equation $3x^3 - 2x^2 + 3x - 4 = 0$ has roots α, β and γ .   **2**
Find the exact value of $\dfrac{1}{\alpha\beta} + \dfrac{1}{\beta\gamma} + \dfrac{1}{\gamma\alpha}$.

d) The letters of the word *MOUSE* are to be rearranged.

　　i)   How many arrangements are there which start with the letter *M* and end with the letter *E*?   **1**

　　ii) How many arrangements are there in which the vowels are grouped together? (A vowel is one of the letters *A, E, I, O, U*)   **1**

　　iii) How would your answers to parts (i) and (ii) change if the given word had been *MOOSE* instead of *MOUSE* ?   **1**

Question 3.   [Start a new page]   **Marks**

a)   Find the general solution (in radian form) of the equation $\cos 2x = \cos x$ **3**

b)   i)   At the distinct points $P(2at, at^2)$ and $Q(2au, au^2)$ on the parabola $4ay = x^2$,
             the tangents are drawn. You may assume, without proof, that the equation
             of the tangent at P is $y = tx - at^2$. Show that the tangents from P and
             Q intersect at the point $(a(u+t) , aut)$.   **2**

      ii)   From the point R $(a, -6a)$, two tangents are drawn to the parabola $4ay = x^2$.
            If the points of contact of these tangents are P and Q, show that the
            triangle PQR is isosceles.   **3**

c)   Suppose that $(5 + 2x)^{12} = \sum_{k=0}^{12} a_k x^k$.

      i)   Using the Binomial Theorem, write an expression for a_k.   **2**

      ii)   Show that $\dfrac{a_{k+1}}{a_k} = \dfrac{24 - 2k}{5k + 5}$ **2**

Question 4.   [Start a new page]   **Marks**

a)   i)   Sketch the function $y = f(x)$ where $f(x) = (x - 2)^2 - 4$, clearly   **2**
            showing all intercepts on the axes. (Use the same scale on both axes)

      ii)   What is the largest positive domain of f for which $f(x)$ has a   **1**
            continuous inverse $f^{-1}(x)$?

      iii) Sketch the graph of $f^{-1}(x)$ on the same axes as (i).   **1**

b)   A particle moves along the x axis according to the equation
            $x = 6\sin 2t - 2\sqrt{3}\cos 2t$.

      i)   Express x in the form $R\sin(2t - \alpha)$ where $R > 0$ and $0 \le \alpha \le \pi/2$.   **2**

      ii)   Prove that the particle moves in simple harmonic motion   **1**

c) A, B and C are three sequential points on a straight line on horizontal ground.   **5**
      A vertical flagpole PQ is situated close by the line (but its base P is not on the line).
      The angles of elevation of the top of the flagpole from A, B and C are $\tan^{-1}\frac{1}{4}, \tan^{-1}\frac{1}{2}$
      and $\tan^{-1}\frac{1}{3}$ respectively. If $AB = 90$m and $BC = 30$m, find the height of the flagpole.

Question 5.   [Start a new page]   **Marks**
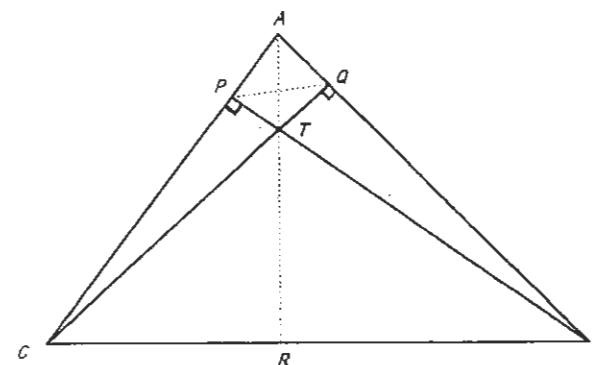
a)   A particle is moving along the x-axis. Its velocity, v m/s at position x metres
      is given by
      $$v = \sqrt{5x - x^2} .$$
      Find the acceleration of the particle when $x = 2$.

b)   Prove by induction that, for any positive integer n, the product $(n+1)(n+2)...(n+n)$
      is always a multiple of 2^n but never a multiple of 2^{n+1}.   **5**

c)

      In the diagram, CQ and BP are altitudes of the triangle ABC. The lines CQ and BP
      intersect at T, and AT is produced to meet CB at R.

      i)   Prove that $\angle TAQ = \angle QCB$.   **3**

      ii)   Prove that $AR \perp CB$.   **2**

Question 6.   [Start a new page]   Marks

Cane sugar, when placed in water, converts into dextrose at a rate which is proportional to the amount of unconverted material remaining. That is, if M grams is the amount of material converted after t minutes, then

$$dM/dt = k(S - M)$$

where S grams is the initial amount of cane sugar and k is a constant.

i)   Show that $M = S + Ae^{-kt}$ satisfies the equation, where A is a constant.   1

ii)   If a certain amount of cane sugar is placed in water at time $t = 0$ and 40% of it has been converted after 10 minutes, show that the value of k is $\frac{1}{10} \log_e(\frac{5}{3})$.   2

iii) How long will it take, to the nearest minute, for 99% of the cane sugar to be converted into dextrose.   2
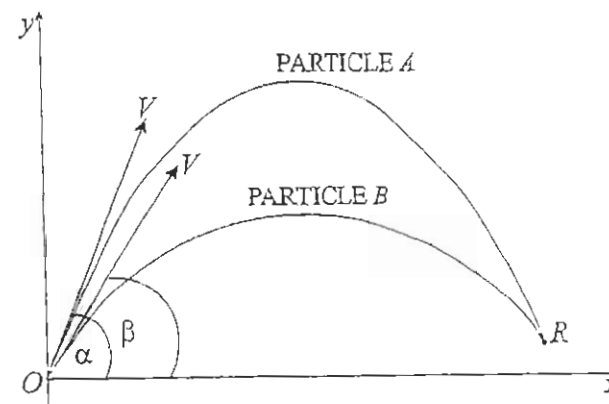
(Question 6 is continued on the next page)

Question 6 (Continued)   Marks

b)

The diagram above shows two particles A and B projected from the origin. Particle A is projected with initial velocity V m/s at an angle α and particle B is projected T seconds later with the same initial velocity V m/s but at an angle of β. The particles collide at the point R.

i)   You may assume that the equation of the path of A is given by

$$y = -\frac{gx^2}{2V^2}\sec^2\alpha + x\tan\alpha$$

Write down the equation of the path of B.   1

Show that the x-coordinate of the collision point R is given by

$$x = \frac{2V^2\cos\alpha\cos\beta}{g\sin(\alpha+\beta)}$$ 3

ii)   You may assume that the horizontal displacement of A after t seconds is given by
$$x = Vt\cos\alpha$$

(α)   Write down the equation for the horizontal displacement of B (Remember that B is projected T seconds after A)   1

(β)   Show that, for the collision to take place, the value of T is given by

$$T = \frac{2V(\cos\beta - \cos\alpha)}{g\sin(\alpha+\beta)}$$ 2