FORM 6 MATHEMATICS (2 UNIT) TRIAL 2003

QUESTION 1

a) $\dfrac{x^3+y^4}{y^2} = \dfrac{\frac{3}{5}+\left(\frac{2}{5}\right)^2}{\frac{2}{5}}$

$= \dfrac{19}{10}$

b) $\displaystyle\int \frac{1}{\sqrt{x^2+16}}\,dx = \ln\left(x+\sqrt{x^2+16}\right)+c$

c) $|x-3| \geq 5$

$x-3 \geq 5$ or $x-3 \leq -5$

$x \geq 8$ or $x \leq -2$

d) $\sqrt{p^4-2p^2} = \sqrt{16\times 25 - 2\times 4\times 5}$

$= \sqrt{360}$

$= 6\sqrt{10}$

e) $2x^2+x-6 = (2x-3)(x+2)$

f) $\dfrac{\sqrt5-\sqrt3}{\sqrt5+\sqrt3} = a+b\sqrt{15}$

$lhs = \dfrac{(\sqrt5-\sqrt3)(\sqrt5-\sqrt3)}{(\sqrt5+\sqrt3)(\sqrt5-\sqrt3)}$

$= \dfrac{5-2\sqrt{15}+3}{5-3}$

$= \dfrac{8-2\sqrt{15}}{2}$

$= 4-\sqrt{15}$

$\therefore a = 4$

$b = -1$

QUESTION 2

a) i) $\dfrac{d}{dx}(5-3x^2)^6 = -36x(5-3x^2)^5$

ii) $\dfrac{d}{dx}\,3\tan\left(4x-\frac{\pi}{4}\right) = 12\sec^2\left(4x-\frac{\pi}{4}\right)$

iii) $\dfrac{d}{dx}\,\dfrac{\log_e 3x}{x} = \dfrac{x\cdot\frac{3}{3x}-\log 3x}{x^2}$

$= \dfrac{1-\log 3x}{x^2}$

b) $\displaystyle\int_0^1 5x+\sin 5x\,dx = \left[\frac{5x^2}{2}-\frac{\cos 5x}{5}\right]_0^1$

$= \frac{5}{2}-\frac{\cos 5}{5}-\left(0-\frac{\cos 0}{5}\right)$

$= 2\frac{1}{2}-\frac{1}{5}\cos 5+\frac{1}{5}$

$= 2.64$ (2dp)

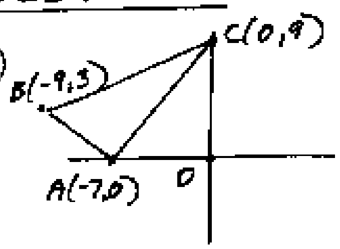
c) $\displaystyle\int_1^e \frac{2}{x}+\frac{x}{2}\,dx = \left[2\ln x+\frac{x^2}{4}\right]_1^e$

$= 2\ln e+\frac{e^2}{4}-2\ln 1-\frac{1}{4}$

$= 1\frac{3}{4}+\frac{e^2}{4}$ or $\dfrac{7+e^2}{4}$

QUESTION 3

a) i)



ii) $m_{AB} = \dfrac{3-0}{-9+7} = -\dfrac{3}{2}$

$m_{BC} = \dfrac{9-3}{0+9} = \dfrac{6}{9} = \dfrac{2}{3}$

$m_{AB}\times m_{BC} = -3/2\times\frac{2}{3} = -1$

$\therefore AB \perp BC$

iii) $y-0 = -3/2(x+7)$

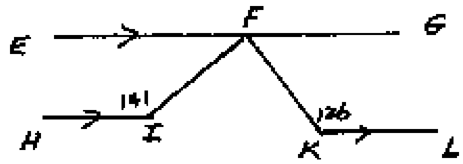
$y = -\frac{3}{2}x-\frac{21}{2}$

$2y = -3x-21$

$3x+2y+21=0$

iv) $d_{AB} = \sqrt{3^2 + (-2)^2}$
$= \sqrt{13}$

v) $pd = \left| \dfrac{0+0+21}{\sqrt{9+4}} \right|$
$= \dfrac{21}{\sqrt{13}}$
$= \dfrac{21\sqrt{13}}{13}$

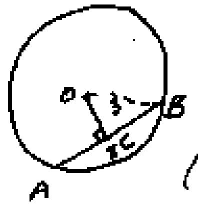
b) 

$\angle EFI = 39°$ (co-int ∠s $EF \parallel HI$)
$\angle GFK = 54°$ (co-int ∠s $FG \parallel KL$)
$\therefore \angle IFK = 87°$ (st line)

QUESTION 4

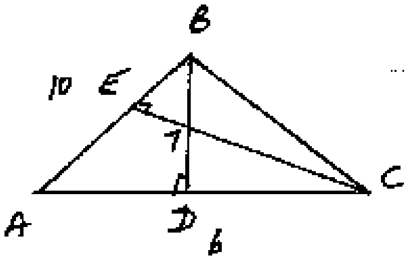
a) $0 = x^2 - 6y + 4x + 16$
i) $x^2 + 4x = 6y - 16$
$x^2 + 4x + 4 = 6y - 12$
$(x+2)^2 = 6(y-2)$
$(x+2)^2 = 4 \times \frac{3}{2}(y-2)$

ii) Vertex $= (-2, 2)$
Directrix: $y = \frac{1}{2}$

b) 

$OB = 5$ (Pythag)
b/c $CB = 4$
(line fr centre ⊥ chord)

To find $\angle COB$: $\tan \angle COB = \frac{4}{3}$
$\angle COB \doteq 0.927\ldots^c$
$l = r\theta$
$= 5 \times 0.927\ldots \times 2$
$= 9.27$ (2dp) cm

c) 

i) In △s ECA, DBA
$\angle CEA = \angle BDA$ (rt ∠s given)
$\angle A$ is common
$\therefore \triangle ECA \parallel\!\mid \triangle DBA$ (equiangular)

ii) $\dfrac{EC}{DB} = \dfrac{CA}{BA} = \dfrac{EA}{DA}$ (corr sides sim △s)
$\dfrac{EC}{7} = \dfrac{6}{10}$
$CE = 4.2$ cm

QUESTION 5

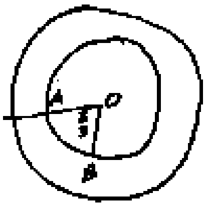
a) i) Area $\doteq \frac{h}{2}(y_0 + y_2 + 2y_1)$
$\doteq \frac{60}{2}(62 + 67 + 2 \times 50)$
$\doteq 6870 \text{ m}^2$

ii) Estimate is greater because the two trapezia formed are greater than the actual area.

b) $2x^2 - 5x + 1 = 0$
$\alpha + \beta = \frac{5}{2}$, $\alpha\beta = \frac{1}{2}$
i) $\dfrac{2}{\alpha} + \dfrac{2}{\beta} = \dfrac{2\beta + 2\alpha}{\alpha\beta}$
$= \dfrac{2(5/2)}{1/2}$
$= 10$

ii) $\beta^2 + \alpha^2 = (\alpha+\beta)^2 - 2\alpha\beta$
$= \frac{25}{4} - 1$
$= 5\frac{1}{4}$

c) b) 9·4 → C



i) $\text{Area} = \frac{1}{2}r^2\theta$

$18\cdot4 = \frac{1}{2}r^2\,\pi/3$

$r = \sqrt{\frac{18\cdot4\times6}{\pi}}$

$= 5\cdot93\text{cm (2dp)}$

ii) $\text{Area}\ \Delta COB = \frac{1}{2}\,9\cdot4\times5\cdot9\ldots\sin\frac{\pi}{3}$

$= 24\cdot1\ \text{cm}^2\ (1dp)$

c) $y = 2\sqrt{x}$

$y/2 = \sqrt{x}$

$x^2 = \left(\frac{y^2}{4}\right)^2$

$= \frac{y^4}{16}$

$Vol = \pi\int_1^3 \frac{y^4}{16}\,dy = \pi\left[\frac{y^5}{16\times5}\right]_1^3$

$= \pi\left[\frac{243}{80} - \frac{1}{80}\right]$

$= \frac{121\pi}{40}\ u^3$

QUESTION 6

a) i) $\int_0^4 p(x)\,dx = B + C + D$

ii) $\int_1^5 p(x) - h(x)\,dx = D + E + F$

iii) $\left|\int_4^5 p(x)\,dx\right| + \left|\int_5^6 h(x)\,dx\right|$

b) i) $y = x^2 - 2x - 3$ & $y = 3x - 3$

$x^2 - 2x - 3 = 3x - 3$

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0$ or 5

$\therefore$ pts of int $(0,-3)$ & $(5, 12)$

ii) $\text{Area} = \int_0^5 3x - 3 - x^2 + 2x + 3\,dx$

$= \int_0^5 5x - x^2\,dx$

$= \left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_0^5$

$= \frac{125}{2} - \frac{125}{3} - 0$

$= 20\frac{5}{6}$ sq units

QUESTION 7

a) $y = x^3 + 3x^2 - 9x - 5$

i) $y' = 3x^2 + 6x - 9$

ii) for st pts $y' = 0$

$3(x^2 + 2x - 3) = 0$

$(x+3)(x-1) = 0$

$x = -3, 1$

$y = 22, -10$

$\therefore$ st pts are $(-3, 22)$ & $(1, -10)$

iii) $y'' = 6x + 6$

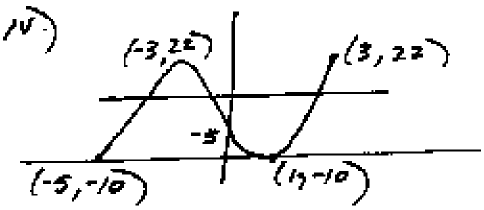
If $x = -3$ $y'' = -18 + 6 < 0$

$\therefore (-3, 22)$ max t/pt

If $x = 1$, $y'' = 6 + 6$
> 0

$\therefore (1, -10)$ min t/pt

iv)



v) $x^3 + 3x^2 - 9x + 5 = 0$

$x^3 + 3x^2 - 9x - 5 = -10$

$y = -10$ drawn on graph

$\therefore x = -5$ or 1

b) $\frac{9}{8}, \frac{3}{4}, \frac{1}{2}$ $\quad r = \frac{3/4}{9/8} = \frac{2}{3}$

$$S_\infty = \frac{a}{1-r}$$
$$= \frac{\frac{9}{8}}{1-2/3}$$
$$= \frac{27}{8}$$

QUESTION 8

a) let no. be x

$2+x,\ 5+x,\ 9+x$

$$\frac{5+x}{2+x} = \frac{9+x}{5+x}$$

$$25+10x+x^2 = 18+11x+x^2$$
$$-x = -7$$
$$x = 7$$

$\therefore$ 7 must be added

b) i) $A_1 = 350000(1\cdot006) - M$

ii) $A_2 = 350000(1\cdot006)^2 - M(1+1\cdot006)$

$$A_{240} = 350000(1\cdot006)^{240} - M(1+1\cdot006+\cdots 1\cdot006^{239})$$
$$= 350000(1\cdot006)^{240} - M\left(\frac{1\cdot006^{240}-1}{0\cdot006}\right)$$

But $A_{240} = 0$

iii) $$M = \frac{350000(1\cdot006)^{240} \times 0\cdot006}{1\cdot006^{240}-1}$$
$$= \frac{2100(1\cdot006)^{240}}{1\cdot006^{240}-1}$$

iv) $M = \$2755\cdot72$

c) i) amp $= 3$

ii) period $= 4\pi$

iii) $y = -3\sin\frac{x}{2}$

QUESTION 9

a) i) $Q = Q_0 e^{-kt}$

$$\tfrac{1}{2}Q_0 = Q_0 e^{-k \times 20}$$
$$\tfrac{1}{2} = e^{-20k}$$
$$\ln\tfrac{1}{2} = -20k$$
$$k = \frac{-\ln 2}{-20}$$
$$= \frac{\ln 2}{20}$$

ii) $$\frac{1}{10} = e^{-\frac{\ln 2}{20}t}$$
$$\ln\frac{1}{10} = -\frac{\ln 2}{20}t$$
$$t = \frac{-20\ln\frac{1}{10}}{\ln 2}$$
$$\doteqdot 66\cdot438\ldots$$
$$= 66 \text{ min (to n min)}$$

b) $x = 5t + \log(1-2t)$ $\quad 0 \le t \le \frac{1}{2}$

i) $$v = 5 + \frac{1}{1-2t} \times -2$$
$$= 5 - \frac{2}{1-2t}$$

$t=0 \quad v = 3$ m/s

$$a = \frac{-4}{(1-2t)^2}$$

$t=0 \quad a = -4$ m/s^2

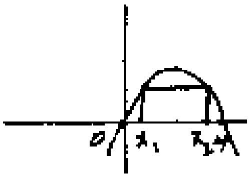
ii) $v=0$

$$5 = \frac{2}{1-2t}$$
$$5-10t = 2$$
$$10t = 3$$
$$t = 3/10$$

At rest after $\frac{3}{10}$ sec

QUESTION 10

a)



$y = 4x - x^2$
$= x(4-x)$

i) $y = c$: $c = 4x - x^2$

$$x^2 - 4x + c = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4c}}{2}$$

$$= 2 \pm \sqrt{4-c}$$

$\therefore x_2 = 2 + \sqrt{4-c}$

$x_1 = 2 - \sqrt{4-c}$

length of rec $= x_2 - x_1$

$= 2\sqrt{4-c}$

Area $= c \times 2\sqrt{4-c}$

$= 2c\sqrt{4-c}$ cm^2

ii) $A' = 2c \cdot \frac{1}{2}(4-c)^{-1/2} \cdot -1 + \sqrt{4-c} \times 2$

$$= \frac{-c}{\sqrt{4-c}} + 2\sqrt{4-c}$$

Max A when $A' = 0$, $A'' < 0$

$$\frac{c}{\sqrt{4-c}} = 2\sqrt{4-c}$$

$c = 8 - 2c$

$3c = 8$

$c = \frac{8}{3}$

Test A'

c	2	$8/3$	3
A'	1·4...	0	−1

$\therefore$ max A when $c = 8/3$

Max Area $= 2 \times \frac{8}{3}\sqrt{4 - 8/3}$

$$= \frac{16}{3}\sqrt{4/3}$$

$$= \frac{16 \times 2}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{32\sqrt{3}}{9} \text{ cm}^2$$

b) $(p^2 + q^2)x^2 + 2q(p+r)x + (q^2 + r^2) = 0$

$\Delta = 4q^2(p+r)^2 - 4(p^2+q^2)(q^2+r^2)$

$= 4q^2(p^2 + 2pr + r^2) - 4(p^2q^2 + p^2r^2 + q^4 + q^2r^2)$

$= \cancel{4p^2q^2} + 8q^2pr + \cancel{4q^2r^2} - \cancel{4p^2q^2} - 4p^2r^2 - 4q^4 - \cancel{4q^2r^2}$

$= 8q^2pr - 4p^2r^2 - 4q^4$

For equal rts $\Delta = 0$

$4(2q^2pr - p^2r^2 - q^4) = 0$

$p^2r^2 - 2q^2pr + q^4 = 0$

$(pr - q^2)^2 = 0$

$pr = q^2$