Centre Number

Student Number

SCEGGS Darlinghurst

2003
Higher School Certificate
Trial Examination

Mathematics

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown in every question.

Total marks - 120

- Attempt Questions 1–10
- All questions are of equal value.

Total marks – 120
Attempt Questions 1–10
All questions are of equal value

Answer each question on a NEW page.

Marks

Question 1 (12 marks)

(a) Evaluate, correct to two significant figures, **2**

$$\frac{195\cdot 32}{4\cdot 6^2 + 5\cdot 73}$$

(b) Solve $\frac{y}{4} - \frac{y-6}{8} = 2$. **2**

(c) Solve $x^2 + 3x > 10$ **2**

(d) State the range of: **1**

$$y = (x-1)^2 + 4$$

(e) Express $\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{2}-1}$ as a single fraction with a rational denominator. **3**

(f) Simplify fully: **2**

$$\log_a a^2 - \log_a \frac{1}{a}$$

Question 2 (12 marks) Start a NEW page. **Marks**

$A(2,4)$ and $B(8,12)$ are the ends of a diameter of a circle.

(a) Find the co-ordinates of the centre, C, of the circle. **1**

(b) Find the radius of the circle. **1**

(c) State the equation of the circle. **1**

(d) Hence show that $D(5,13)$ lies on the circle. **1**

(e) Show that $AD \perp BD$. **2**

(f) The perpendicular bisector of AB meets the circle at X and Y. Find the equation of XY in general form. **2**

(g) Show that the area of ΔXDY is 20m^2 **2**

(h) Show that $3x + 4y - 22 = 0$ is a tangent to the circle. **2**

Question 3 (12 marks) Start a NEW page. **Marks**

(a) A yacht sails from Robinson Island on a bearing of 240° for 120km. It then turns and sails on a bearing of 110° until it reaches a destination due south of its original position. **3**

Calculate the distance of the yacht from Robinson Island to the nearest kilometre.

(b) Differentiate:

(i) $y = \ln\left(x^2 + 1\right)$ **1**

(ii) $y = \dfrac{e^{2x}}{\sin 3x}$ **2**

(c) Evaluate **1**

$$\lim_{x \to 3} \frac{x^2 - 9}{x^2 - x - 6}$$

(d) Solve $\sin^2\theta - \sin\theta - 2 = 0$, $-\pi \le \theta \le \pi$. **3**

(e) α and β are the roots of the equation $3x^2 - 6x + 2 = 0$ **2**

Find the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$

Marks

Question 4 (12 marks) Start a NEW page.

(a) Evaluate $\sum_{r=1}^{4} r^2 - 1$ **1**

(b) Find:

(i) $\int 3\sqrt{x} + \frac{1}{x^4}\,dx$ **2**

(ii) $\int_0^2 \left(e^{3x} + e^{-3x}\right)^2 dx$ **3**

(c) Show that: **3**

$$\frac{2\cos^3\theta - \cos\theta}{\sin\theta\cos^2\theta - \sin^3\theta} = \cot\theta$$

(d) (i) Sketch the curve $y = 3\sin 2x$ in the domain $0 \le x \le 2\pi$ showing the main features of the graph. **2**

(ii) Hence use your graph to find the number of solutions to the equation $3\sin 2x - 1 = 0$ for $0 \le x \le 2\pi$. **1**

Marks

Question 5 (12 marks) Start a NEW page.

(a) A bottle of water is placed in the common room fridge where the temperature is maintained at $0°C$. The rate at which the temperature of the water falls is proportional to its temperature at that time. ($\frac{dT}{dt} = -kT$ where T is its temperature.) When the water is placed in the fridge its temperature is $40°C$ and after 17 minutes its temperature is $24°C$.

(i) Show that the function $T = Ce^{-kt}$ satisfies the equation $\frac{dT}{dt} = -kT$. **1**

(ii) Find the value of the constant C. **1**

(iii) Show the exact value of k is $\frac{1}{17}\ln\left(\frac{5}{3}\right)$. **2**

(iv) Find the temperature of the water after 43 minutes to the nearest degree. **1**

(b) A sheep, grazing in a paddock, is tethered to a stake by a rope 20m long. If the stake is 10m from a long fence, find the area over which the sheep can graze. **3**

(c) One set of cards contains the numbers 1, 2, 3, 4, 5 and another set contains the letters H, O, L, L, Y. One card is selected at random from each set. Find the probability of selecting:

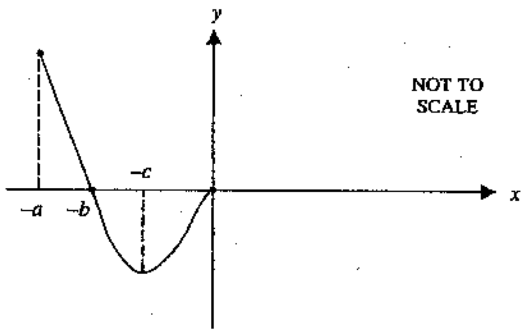
(i) a 4 and an H. **1**

(ii) an odd number and a vowel. **1**

(iii) a number less than 3 or an L. **2**

Question 6 (12 marks) Start a NEW page. **Marks**

(a) The diagram shows the graph of a function $y = f(x)$, for $-a \le x \le 0$.



It is known that $f(x)$ is an odd function and is stationary at $(0, 0)$.

(i) Sketch the graph $y = f(x)$, for $-a \le x \le a$. **1**

(ii) On a separate diagram, sketch $y = f'(x)$ **2**

(b) In a new quiz show called "Dopier than Ever", you win \$6 000 for answering the first question correctly, \$14 000 for answering the second question correctly and \$22 000 for answering the third question correctly and so on for the following questions. You finish when you answer a question incorrectly. Your total winnings for the contest is the sum of money you win on each question.

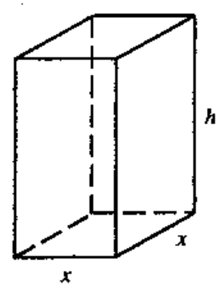
(i) What is the prize money for the 10th question only? **2**

(ii) How many questions must you correctly answer to exceed \$1 000 000 in total winnings? **3**

Question 6 continues on page 8

Question 6 (continued) **Marks**

(c) A box in the shape of a square prism has a volume of 32cm^3 and no lid. The square base has length x cm and the box is h cm high.



(i) Show that the surface area of the box is given by: $SA = x^2 + \dfrac{128}{x}$. **1**

(ii) Find the dimensions of the box that has the least surface area. **3**

Marks

Question 7 (12 marks) Start a NEW page.

(a) Show that $x^3 - x^2 - x + 1 = (x+1)(x-1)^2$. **1**

(b) Consider the function, $f(x) = \frac{x-1}{x^2}$.

(i) Prove that $f'(x) = \frac{2-x}{x^3}$. **2**

(ii) Find the co-ordinates of the stationary point on $y = f(x)$ and determine its nature. **2**

(iii) Find the co-ordinates of P, the only point where $y = f(x)$ meets the x-axis. **1**

(iv) Sketch $y = f(x)$ showing all important features. **3**

(v) Show that the equation of the tangent at P is given by the equation $y = x - 1$. **1**

(vi) Using part (a) or otherwise, find the co-ordinates of the other point where this tangent meets the curve. **2**

Marks

Question 8 (12 marks) Start a NEW page.

(a) The following is a table of values for the function $y = \frac{2}{x(x+1)}$.

x	1	2	3	4	5
y	1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{10}$	$\frac{1}{15}$

(i) Using the information in the table and Simpson's rule with 5 function values, find an approximation for $\int_1^5 \frac{2}{x(x+1)}\,dx$ correct to 3 decimal places. **2**

(ii) It is also true that $\frac{2}{x(x+1)} = \frac{2}{x} - \frac{2}{x+1}$. Use direct integration to find the value of $\int_1^5 \frac{2}{x(x+1)}\,dx$ correct to 3 decimal places. **3**

(iii) Explain the difference between your answers in parts (i) and (ii). **1**

(b) A particle moves in a straight line so that its velocity v in metres per second at time t is given by $v = 4 - 2t$. At time $t = 0$ the particle is at $x = 1$.

(i) Find the displacement x of the particle as a function of t. **2**

(ii) When is the particle at rest and what is its acceleration at that time? **2**

(iii) Find the distance the particle travels in the first 4 seconds. **2**

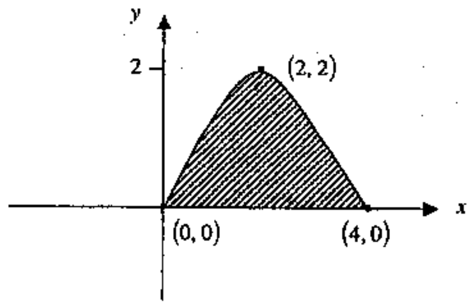
Marks

Question 9 (12 marks) Start a NEW page.

(a) Can there be a geometric series with a limiting sum of $\frac{2}{3}$ and a first term of 4? **2**

Justify your answer with appropriate calculations.

(b) The producers of Play School are replacing the Arched Window. It will still have a base length of 4m and a height of 2m as shown in the diagram below.



NOT TO SCALE

The new arch is to be either an arc of a parabola or a half-cycle of a sine curve.

(i) If the arch is the arc of a parabola, the equation of the curve is of the form: **1**

$$f(x) = ax(4 - x)$$

Show that the value of a is $\frac{1}{2}$.

(ii) If the arch is a sine curve, the equation of the curve is of the form, **1**

$$g(x) = A\sin\frac{\pi x}{4}$$

Find the value of A.

(iii) Calculate the area for each window design and hence decide which would be cheaper to build. **4**

(c) A rectangular lawn is 60 metres long and 30 metres wide. A pigeon wanders randomly around the lawn. Find the probability that the pigeon is:

(i) more than 10 metres from the edge of the lawn. **2**

(ii) not more than 10 metres from a corner of the lawn. **2**

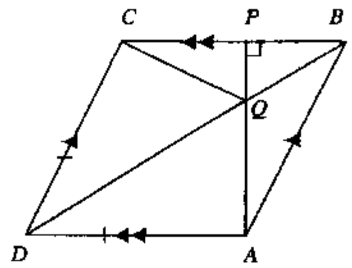
Marks

Question 10 (12 marks) Start a NEW page.

(a) Consider the function $y = xe^{-2x}$. **3**

Prove that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$

(b) In the rhombus $ABCD$, AP is constructed perpendicular to BC and intersects the diagonal BD at Q.



NOT TO SCALE

(i) State why $\angle ADB = \angle CDB$. **1**

(ii) Prove that $\Delta AQD \equiv \Delta CQD$. **2**

(iii) Hence find $\angle QCD$. **1**

(c) After several mornings of horrendous traffic, Chris decides to move closer to work. She takes out a loan for \$500,000 at an interest rate of 12% p.a. compounded monthly for 20 years.

(i) Show that the amount she owes on the loan after n months, A_n, is given by the expression: **3**

$$A_n = 100M - 1.01^n(100M - 500{,}000)$$

where M is the size of her monthly repayments.

(ii) Her repayments are fixed at \$5505 per month. In which year does Chris still owe \$250,000? **2**

END OF PAPER