Centre Number

[redacted]

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2007
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Afternoon Session
Tuesday 14 August 2007

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1-7
- All questions are of equal value

Disclaimer

Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, *Principles for Setting HSC Examinations in a Standards-Referenced Framework* (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and *Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework* (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

Marks

Question 1 (12 Marks) Use a SEPARATE writing booklet

(a) If $y = \sin^2 3x$ find $\dfrac{dy}{dx}$. **2**

(b) $A(-3, 1)$, $B(8, -2)$ and $C(4, 16)$ are the vertices of a triangle. AM is a median of the triangle.

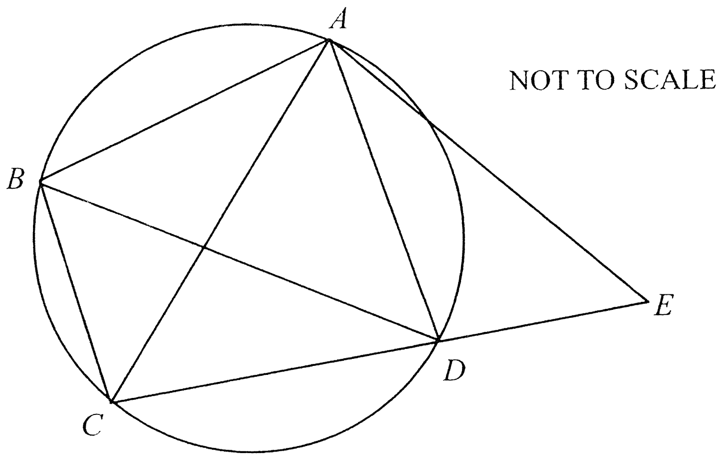
(i) Show that M has coordinates $(6, 7)$. **1**

(ii) Hence find the coordinates of the point which divides the interval AM internally in the ratio 2:1. **2**

(c)(i) Express $x^3 - 3x^2 + 4$ as a product of three linear factors by first showing that $(x-2)$ is a factor of this polynomial. **2**

(ii) Hence solve the inequality $x^3 - 3x^2 + 4 \geq 0$. **1**

(d)



In the diagram, $ABCD$ is a cyclic quadrilateral. E is a point on CD produced.

(i) Give a reason why $\angle ADE = \angle ABC$. **1**

(ii) If $\Delta ADE \parallel\!| \Delta CBA$, show that $AE \parallel BD$. **3**

Marks

Question 2 (12 Marks) Use a SEPARATE writing booklet

(a) If $y = \log_{\frac{1}{a}}\left(\frac{1}{N}\right)$, where $a > 0$ and $N > 0$, show that $y = \log_a N$. **2**

(b) The lines $y = mx$ and $y = 2mx$, where $m > 0$, are inclined to each other at an angle θ such that $\tan\theta = \frac{1}{3}$.

(i) Show that $2m^2 - 3m + 1 = 0$. **1**

(ii) Hence find the possible values of m. **2**

(c)(i) Show that $\tan 2x + \tan x = \dfrac{\sin 3x}{\cos 2x \cos x}$. **2**

(ii) Hence solve the equation $\tan 2x + \tan x = 0$ for $0 < x < \frac{\pi}{2}$. **1**

(d) $P(2ap, ap^2)$, $Q(2aq, aq^2)$ and $R(2ar, ar^2)$, where $p < q < r$, are three points on the parabola $x^2 = 4ay$.

(i) Use differentiation to show that the tangent to the parabola at Q has gradient q. **1**

(ii) If the chord PR is parallel to the tangent at Q, show that p, q and r are consecutive terms in an arithmetic sequence. **3**

Marks

Question 3 (12 Marks) Use a SEPARATE writing booklet

(a) Find the number of ways in which the letters of the word EPSILON can be arranged in a straight line so that the three vowels are all next to each other. **2**

(b) Use Mathematical induction to show that $5^n > 3^n + 4^n$ for all positive integers $n \geq 3$. **4**

(c) Consider the function $f(x) = x + e^{-x}$, $x \geq 0$.

(i) Show that for all values of $x > 0$, the function $f(x)$ is increasing and the curve $y = f(x)$ is concave up. **2**

(ii) Sketch the graph of $y = f(x)$ showing clearly the coordinates of the endpoint and the equation of the asymptote. **2**

(iii) On the same diagram, sketch the graph of the inverse function $y = f^{-1}(x)$. **1**

(iv) Find the domain of the function $g(x) = f(x) + f^{-1}(x)$. **1**

Marks

Question 4 (12 Marks) Use a SEPARATE writing booklet

(a)(i) Show that the equation $x^3 - 2x - 5 = 0$ has a root α such that $2 < \alpha < 3$. **2**

(ii) Use one application of Newton's method and an initial approximation of 2 to find the next approximation for α. **2**

(b)(i) Sketch the graph of $y = 2\sin^{-1} 2x$ showing clearly the coordinates of the endpoints. **2**

(ii) Find the exact area of the region in the first quadrant bounded by the curve $y = 2\sin^{-1} 2x$, the y-axis and the line $y = \pi$. **2**

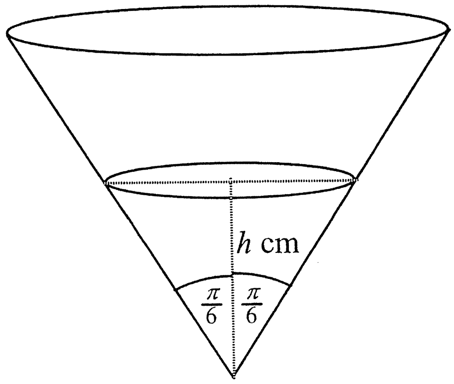
(c)(i) Show that $\dfrac{u^2}{1+u^2} = 1 - \dfrac{1}{1+u^2}$. **1**

(ii) Use the substitution $x = u^2$, $u \geq 0$, to find $\displaystyle\int \frac{\sqrt{x}}{1+x}\,dx$. **3**

Marks

Question 5 (12 Marks) Use a SEPARATE writing booklet

(a)



An egg timer in the shape of an inverted right circular cone of semi vertical angle $\frac{\pi}{6}$ contains sand to a depth of h cm. The sand flows out of the apex of the cone at a constant rate of $0\cdot 5\ \text{cm}^3/\text{s}$.

(i) Show that the volume $V\ \text{cm}^3$ of sand in the cone is given by $V = \frac{1}{9}\pi h^3$. **1**

(ii) Find the value of h when the depth of sand in the egg timer is decreasing at a rate of $0\cdot 05\ \text{cm/s}$, giving your answer correct to 2 decimal places. **3**

(b) A particle is moving in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line, velocity $v\ \text{ms}^{-1}$ given by $v = -\frac{1}{8}x^3$, and acceleration $a\ \text{ms}^{-2}$. The particle is initially 2 metres to the right of O.

(i) Show that $a = \frac{3}{64}x^5$. **1**

(ii) Find an expression for x in terms of t. **2**

(iii) Describe the limiting motion of the particle. **1**

(c) A game is played by throwing three fair coins. The game is considered a failure if all three coins show heads or all three coins show tails. Otherwise the game is considered a success.

(i) Show that the probability of success in any play of the game is $\frac{3}{4}$. **2**

(ii) Find the probability of exactly two successes in four plays of the game. **2**

Marks

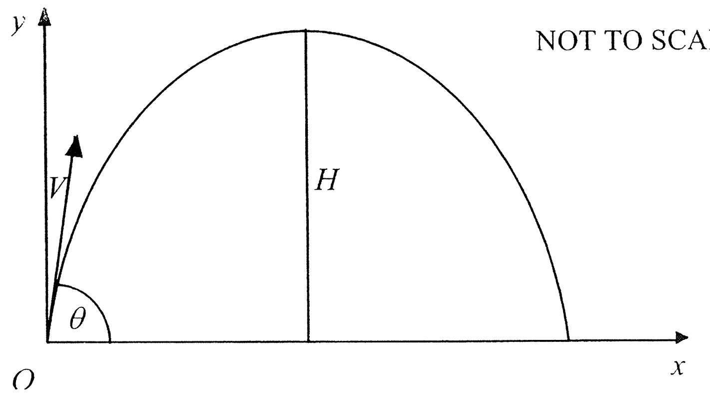
Question 6 (12 Marks) Use a SEPARATE writing booklet

(a) At time t years the area A km^2 controlled by a colony of animals is given by $A = 300 - 200e^{-kt}$ for some $k > 0$.

(i) Sketch the graph of A as a function of t showing clearly the initial and limiting areas controlled by the colony. **2**

(ii) Find the value of k if the area controlled by the colony is increasing at a rate of $10\,\text{km}^2$ per year when this area is twice its initial value. **2**

(b)



A particle is projected from a point O with speed V ms^{-1} at an angle θ above the horizontal, where $\frac{\pi}{4} < \theta < \frac{\pi}{2}$. It moves in a vertical plane under gravity where the acceleration due to gravity is g ms^{-2}. At time t seconds its horizontal and vertical displacements from O are x metres and y metres respectively.

(i) Use integration to show that $x = Vt\cos\theta$ and $y = Vt\sin\theta - \frac{1}{2}gt^2$. **2**

(ii) The particle takes T seconds to reach its greatest height H metres. Find expressions for T and H in terms of g, V and θ. **2**

(iii) The path of the particle is inclined at an angle $\frac{\pi}{4}$ above the horizontal at time $\frac{1}{4}T$. Show that $\theta = \tan^{-1}\frac{4}{3}$. **2**

(iv) What fraction of its maximum height has the particle attained at time $\frac{1}{4}T$? **2**

Marks

Question 7 (12 Marks) Use a SEPARATE writing booklet

(a) A particle is moving in a straight line with Simple Harmonic motion. At time t seconds it has displacement x metres from a fixed point O on the line, where $x=(\cos t+\sin t)^2$, velocity v ms^{-1} and acceleration a ms^{-2}.

(i) Show that $a=-4(x-1)$. **2**

(ii) Find the extreme positions of the particle during its motion. **1**

(iii) Find the time taken by the particle to move from one extreme position of its motion to the other extreme position. **1**

(iv) A second particle moves along a parallel straight line with Simple Harmonic motion so that $x=1-\cos t$, where x metres is its displacement from a fixed point level with O. During the first complete oscillation of the particle with the slower average speed, find the number of times the particles pass each other, and state their relative directions of travel on each occasion. **3**

(b) (i) Considering the identity $(1-t)^n(1+t)^n \equiv (1-t^2)^n$, where n is a positive integer, show that for integer values of r, **2**

$$\sum_{k=0}^{2r}(-1)^k\ {}^nC_k\,{}^nC_{2r-k}=(-1)^r\ {}^nC_r \quad \text{provided} \quad 0\le r\le \tfrac{1}{2}n.$$

(ii) Hence show that $\displaystyle\sum_{k=0}^{r}(-1)^k\ {}^nC_k\,{}^nC_{2r-k}=\tfrac{1}{2}(-1)^r\ {}^nC_r\left\{1+{}^nC_r\right\}$ for $0\le r\le \tfrac{1}{2}n$. **2**

(iii) Hence evaluate $\displaystyle\sum_{k=0}^{10}(-1)^k\left({}^{20}C_k\right)^2$ as a basic numeral. **1**

Examiners

Graham Arnold	Terra Sancta College, Quakers Hill
Denise Arnold	Patrician Brothers' College, Blacktown
Dianne Williams	St Andrews College, Marayong
Franklin Samyia	Holy Cross College, Ryde