ZU THSC 08

(1)

Question 1

(a) $1^c = \frac{180^\circ}{\pi}$

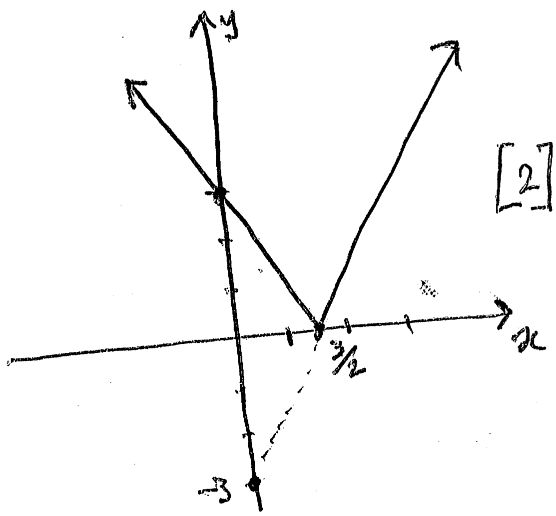
$\doteqdot 57^\circ 18'$ [2]

(b) $\frac{2\sqrt{2}}{\sqrt{7}-\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{7}-\sqrt{3}} \times \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

$= \frac{2\sqrt{14}+2\sqrt{6}}{7-3}$

$= \frac{1}{2}(\sqrt{14}+\sqrt{6})$ [2]

(c) $y = |2x-3|$

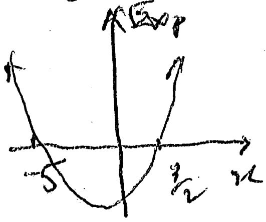


[2]

(d) $2x^2+7x-15 \geq 0$

$(2x-3)(x+5) \geq 0$

$x \leq -5, \; x \geq \frac{3}{2}$



[2]

(e) $\sum_{k=0}^{19} (3k-1) = -1+2+5+\dots+56$

This is an A.S.; $a=-1$, $l=56$

$n=20$

$S_{20} = \frac{20}{2}(-1+56)$

$= 550$ [2]

(f) $\ln 5x - \ln 2 = 2\ln x$

$\ln \frac{5x}{2} = \ln x^2$

$\therefore \frac{5x}{2} = x^2, \; x>0$

$2x^2-5x=0$

$x(2x-5)=0$

$x=0$ or $5/2$

But $x>0$

$\therefore x = 5/2$ [2]

Question 2

a) i $y = \tan(x^2)$

$\frac{dy}{dx} = 2x\sec^2(x^2)$ (1) (1)

ii $y = 2x\sin(2x)$

$\frac{dy}{dx} = 2x \times \cos(2x) \times 2 + \sin(2x) \times 2$

$= 4x\cos(2x) + 2\sin(2x)$ (1) (1)

b) i $\int \frac{x^2}{x^3-1}\,dx = \frac{1}{3}\int \frac{3x^2}{x^3-1}$

$= \frac{1}{3}\log_e(x^3-1) + C$ (1) (1)

ii $\int_{\pi/2}^{\pi} \cos\left(\frac{1}{2}x\right)dx = \left[2\sin\left(\frac{1}{2}x\right)\right]_{\pi/2}^{\pi}$ (1)

$= 2\sin\frac{\pi}{2} - 2\sin\frac{\pi}{4}$ (1)

$= 2 - \sqrt{2}$ (1)

c) $y = \sin(x + \pi/3)$

$\frac{dy}{dx} = \cos\left(x + \frac{\pi}{3}\right)$

at $x = \pi$

$\frac{dy}{dx} = \cos\left(\frac{4\pi}{3}\right)$

$= -1/2$ (1)

when $x = \pi$

$y = \sin\left(\frac{4\pi}{3}\right)$

$= \frac{-\sqrt{3}}{2}$ (1)

$\therefore$ pt$\left(\pi, -\frac{\sqrt{3}}{2}\right)$ $m = -1/2$

$y + \frac{\sqrt{3}}{2} = \frac{-1}{2}(x - \pi)$

$2y + \sqrt{3} = -1(x - \pi)$

$2y + \sqrt{3} = -x + \pi$

$x + 2y + \sqrt{3} - \pi = 0$ (1)

Question (3) [12 marks]

(a)

(i) $\dfrac{y-y_1}{x-x_1} = \dfrac{y_2-y_1}{x_2-x_1}$

$\dfrac{y-3}{x-2} = \dfrac{1}{3}$ (2)

$\therefore$ $\boxed{x - 3y + 7 = 0}$

(ii) $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$.

$\left(\dfrac{2+5}{2}, \dfrac{3+4}{2}\right)$

$\boxed{\left(\tfrac{7}{2}, \tfrac{7}{2}\right)}$ (1)

(iii) $m_{AB} = \tfrac{1}{3}$.

$\therefore$ grad. of perp. line to AB $= -3$.

$\therefore$ Equation of the perp. bisector of AB.

[ ]

$y - \tfrac{7}{2} = -3\left(x - \tfrac{7}{2}\right)$

$2y - 7 = -6x + 21$

$\therefore$ $\boxed{3x + y - 14 = 0}$

(2)

(iv) $3x + y - 14 = 0$

When $y = 0$, $x = 14/3$.

$\therefore$ $\boxed{C\left(\tfrac{14}{3}, 0\right) \quad 4\tfrac{2}{3}}$ (1)

(v)

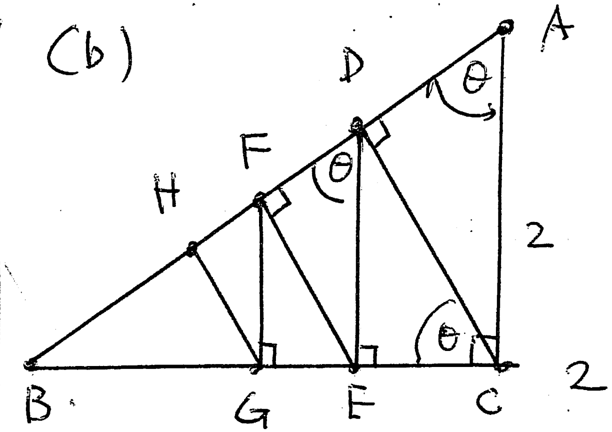
$A = \tfrac{1}{\cancel{2}} \times \tfrac{14}{3} \times \cancel{4}\ 2.$

$= 28/3$

$= 9\tfrac{1}{3}$. (2)

$\boxed{\text{Area} = 9\tfrac{1}{3}}$

(b)



• In $\triangle ADC$, $\angle ACD = 90-\theta$

$\therefore \angle DCE = \theta$.

$\dfrac{|DC|}{2} = \sin\theta \Rightarrow |DC| = 2\sin\theta$ — (1)

• In $\triangle ECD$

$\dfrac{|DE|}{|DC|} = \sin\theta$ [From (1)

$\therefore |DE| = DC\sin\theta$

$= 2\sin^2\theta$ — (2)

• In $\triangle DEF$ 2

$\dfrac{|FE|}{|DE|} = \sin\theta \Rightarrow \therefore FE = 2\sin^3\theta$

i.e sum

$2\sin\theta + 2\sin^2\theta + 2\sin^3\theta + \cdots$

$= 2(\sin\theta + \sin^2\theta + \cdots$

$= \boxed{\dfrac{2\sin\theta}{1-\sin\theta}}$ (4)

Question (3) [12 marks]

(a)

(i) $\dfrac{y-y_1}{x-x_1} = \dfrac{y_2-y_1}{x_2-x_1}$

$\dfrac{y-3}{x-2} = \dfrac{1}{3}$ 2

$\therefore$ $\boxed{x-3y+7=0}$

(ii) $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$.

$\left(\dfrac{2+5}{2}, \dfrac{3+4}{2}\right)$

$\boxed{\left(\tfrac{7}{2}, \tfrac{7}{2}\right)}$ 1

(iii) $m_{AB} = \tfrac{1}{3}$.

$\therefore$ grad. of perp. line to AB $= -3$.

$\therefore$ Equation of the perp. bisector of AB.

$y - \tfrac{7}{2} = -3\left(x - \tfrac{7}{2}\right)$

$2y - 7 = -6x + 21$

$\therefore$ $\boxed{3x + y - 14 = 0}$

2

(iv) $3x + y - 14 = 0$

When $y = 0$, $x = 14/3$.

$\therefore$ $\boxed{C\left(\tfrac{14}{3}, 0\right) \quad 4\tfrac{2}{3}}$ 1

(v)

$A = \dfrac{1}{\cancel{2}} \times \dfrac{14}{3} \times \cancel{4}\ 2.$

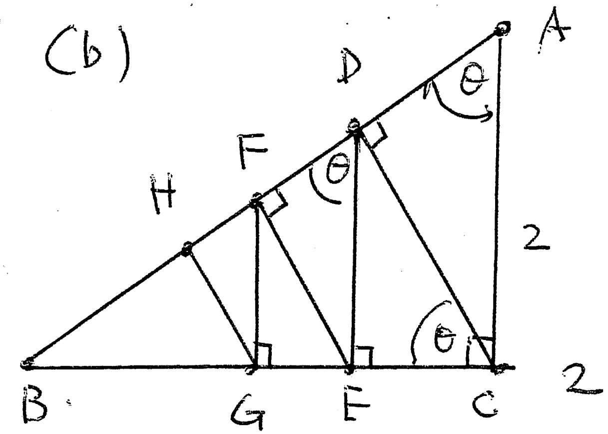
$= 28/3$

$= 9\tfrac{1}{3}$. 2

$\boxed{\text{Area} = 9\tfrac{1}{3}.}$

[7919158121322267]

(b)



• In $\triangle ADC$, $\angle ACD = 90 -$
$\therefore \angle DCE = \theta$.

$\dfrac{|DC|}{2} = \sin\theta \Rightarrow |DC| = 2\sin$ — 1

• In $\triangle ECD$

$\dfrac{|DE|}{|DC|} = \sin\theta$ [From 1

$\therefore |DE| = DC\sin\theta$

$= 2\sin^2\theta$ — 2

• In $\triangle DEF$ 2

$\dfrac{|FE|}{|DE|} = \sin\theta$ $\Rightarrow \therefore FE = 2\sin^3\theta$

i.e sum

$2\sin\theta + 2\sin^2\theta + 2\sin^3\theta + \cdots$

$= 2(\sin\theta + \sin^2\theta + \cdots$

$= \boxed{\dfrac{2\sin\theta}{\cdots}}$ 4

Question(9). [12 marks]

(a) $\frac{dP}{dt} = kP$

2 $P = P_0 e^{kt}$ —— (1)

When $t = 0$, $P = 9000$

i.e $P = P_0 = 9000$

When $t = 10$, $P = 11000$ 1

$\therefore 11000 = 9000\, e^{10k}$

$\therefore e^{10k} = 11/9$ 1

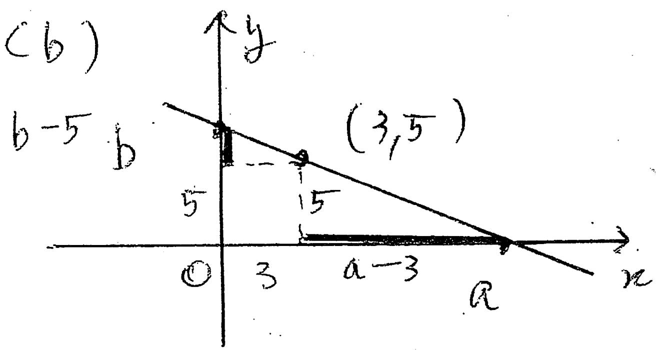
$\boxed{k = \frac{1}{10} \ln(11/9) = 0.02}$

$\therefore P = 9000\, e^{25k}$

1 $= 9000 \times 1.65149$

$\boxed{P = 14863}$

(b)



From similar $\triangle$'s

(i) $\frac{b-5}{3} = \frac{5}{a-3}$ 1

$\therefore a - 3 = \frac{15}{b-5}$

$\Rightarrow a = \frac{15}{b-5} + 3$

$= \frac{15 + 3b - 15}{b-5}$

2 $\therefore \boxed{a = \frac{3b}{b-5}}$ 1

(ii) $A = \frac{1}{2}ab$

$\therefore A = \frac{3b^2}{2(b-5)}$ 1

$\frac{dA}{db} = \frac{12b(b-5) - 6b^2}{(b-5)^2}$

$0 = \frac{6b(b-10)}{(b-5)^2}$ 2

$\Rightarrow b = 0, 10$

When $b = 10$, $a = 6$.

Equation:

$y - 5 = -\frac{5}{3}(x - 3)$ 1

Equation

4 $\boxed{5x + 3y - 30 = 0}$

Test

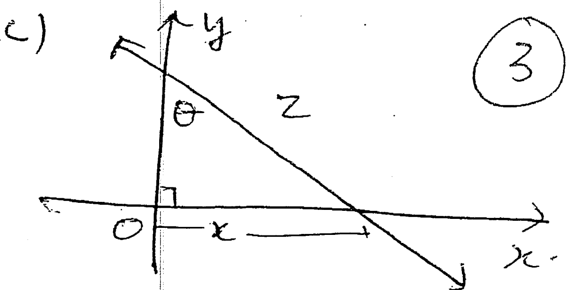
b	9	10	11
$\frac{dA}{db}$	$-27/8$ (−)	0	$11/6$ (+)

$-\ \diagdown\_\_\diagup\ +$

$\therefore 5x + 3y - 30 = 0$

Cuts least Area.

(c)



3

$\frac{x}{2} = \sin\theta$ 1

$x = 2\sin\theta$

$\frac{dx}{d\theta} = 2\cos\theta$ 1

$\boxed{\left.\frac{dx}{d\theta}\right|_{\theta = \pi/3} = 2\cos\frac{\pi}{3} = 1}$

1 m/radian. 1

QUESTION 4

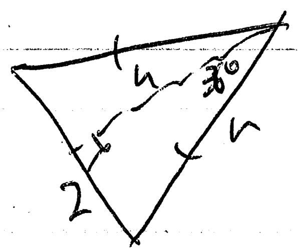
a (i) 0.01

(ii) $0.3 \times 0.7 + 0.7 \times 0.3 = 0.42$

(iii) $0.1 \times 0.9 \times 0.3 \times 0.7 \times 4 = 0.0756$

(iv) $1 - 0.9 \times 0.9 \times 0.7 \times 0.7 = 0.6031$

b (i)



$\tan 36° = \frac{2}{h}$

$h = \frac{2}{\tan 36°}$

$\text{Area} = 5 \times \frac{1}{2} \times 4 \times h$

$= \frac{20}{\tan 36°}$ cm²

(ii) $\sin 36° = \frac{2}{r}$

$r = \frac{2}{\sin 36°}$

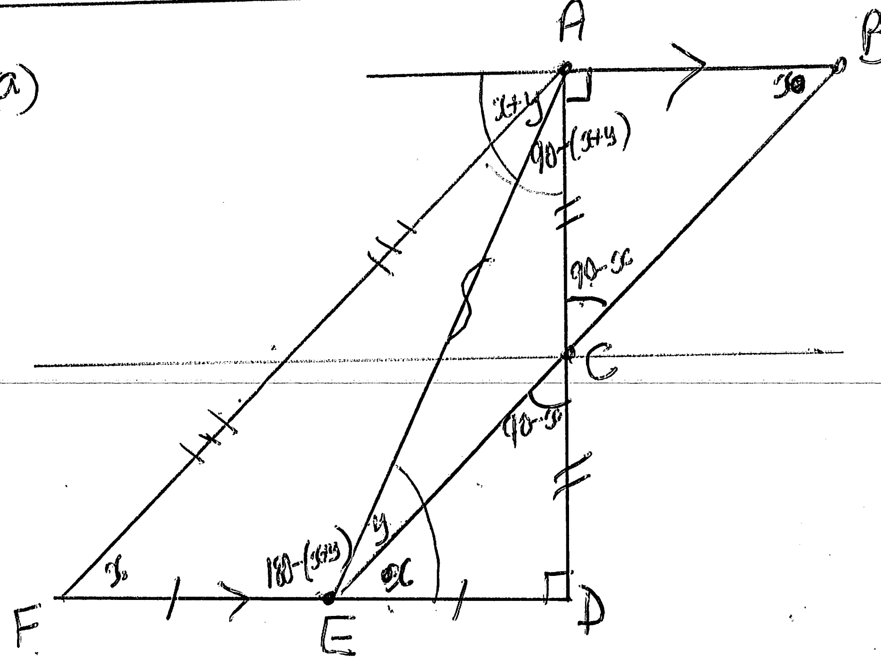
$= 3.40$ cm

(iii) $\dfrac{\frac{4\pi}{\sin^2 36°} - \frac{20}{\tan 36°}}{5} = 1.77$ cm²

Trial HSC 2008 2 unit

(5) (a)

12



(i) alternate angles in // lines AB // FD , EB transversal. (1)

(ii) show $\triangle CDE \equiv \triangle CAB$

$CD = AC$ given

$C\hat{E}D = A\hat{B}C$ alt. angles

$E\hat{C}D = A\hat{C}B$ vert. opp

$\therefore \triangle CDE \equiv \triangle CAB$ AAS. (2)

Cannot use RHS, SSS, SAS.

(iii) show $AF = 2BC$

$\triangle AFE \equiv \triangle EBA$ (AAS).

ABEF is a parm

Line through midpt of 1 side of $\triangle ADF$ // to a 3rd side FD, bisects the other side in proportion, so $AF = BE = BC + CD$, $EC = BC$ from (ii) (2)

$AF = 2BC$.

(iv) Show $\hat{ACB} = \hat{DAF}$

From diagram $\hat{ACB} = 90 - x$ (angle sum $\triangle$)

$\hat{BAC} = 90^\circ$ // lines, transversal

$\hat{DAF} = \hat{DAE} + \hat{EAF}$

$= 90^\circ - (x+y) + y = 90^\circ - x$ (1)

b) $u(x) = f(x)g(x)$

$u'(x) = f(x)g'(x) + g(x)f'(x)$

Given $y = f(x)$ slope $= 2 \quad 0 \le x \le 3$

$= -\frac{1}{4} \quad 3 \le x \le 7$

Given $y = g(x)$ slope $= -3 \quad 0 \le x \le 3$

$= 1 \quad 3 \le x \le 7$

i) $u'(1) = f(1)g'(1) + g(1)f'(1)$

$= 2 \times -3 + 6 \times 2$ (2)

$= 6$

ii) $V(x) = f(g(x))$

$V'(x) = f'(g(x)) \times g'(x)$

So $V'(1) = f'(g(1)) \times g'(1) = f'(6) \times g'(1) = -\frac{1}{4} \times -3 = \frac{3}{4}$ (2)

c) $|4x+13| < 3$

$|4x+13| \le |4x+12| + |1|$

$= 4|x+3| + 1$

$< 4 \times \frac{1}{2} + 1$

$= 3$

or using $|x+3| < \frac{1}{2}$

$-\frac{1}{2} < x+3 < \frac{1}{2}$

$\times 4 \quad -2 < 4x+12 < 2$

$+1 \quad -1 < 4x+13 < 3$

So $-3 < 4x+13 < 3$ is also true

$\therefore |4x+13| < 3$

(2)

QUESTION 6 2U Trial 2008

$y = \frac{x}{x^2+1}$ Let $u = x$ $u' = 1$
$v = x^2+1$ $v' = 2x$.

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2} = \frac{x^2+1-2x}{(x^2+1)^2}$

Turning points when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{1-x^2}{(x^2+1)} = 0$

$x = \pm 1$.

Turning points at $(+1, \frac{1}{2})(-1, -\frac{1}{2})$

NATURE

x	-2	-1	0	1	2
dy/dx	$-3/25$	0	1	0	$-\frac{3}{25}$
gradient	\	—	/	—	\
		Min		max	

$\therefore$ min at $(-1, -\frac{1}{2})$
max at $(1, \frac{1}{2})$.

b) Points of inflexion occur when $d^2y/dx^2 = 0$.

$\frac{d^2y}{dx^2} = \frac{vu' - uv'}{v^2}$ $u = 1-x^2$
$u' = -2x$.
$v = (x^2+1)$
$v' = 4x(x^2+1)$

$\frac{d^2y}{dx^2} = \frac{(x^2+1)^2(-2x) - (1-x^2)4x(x^2+1)}{(x^2+1)^4}$

$= \frac{-2x(x^2+1) - 4x(1-x^2)}{(x^2+1)^3}$

$\frac{d^2y}{dx^2} = \frac{2x^3 - 6x}{(x^2+1)^3} = 0$.

$\therefore$ $2x(x^2-3) = 0$.
$x = 0$ $x = \pm\sqrt{3}$.

Points of inflexion are

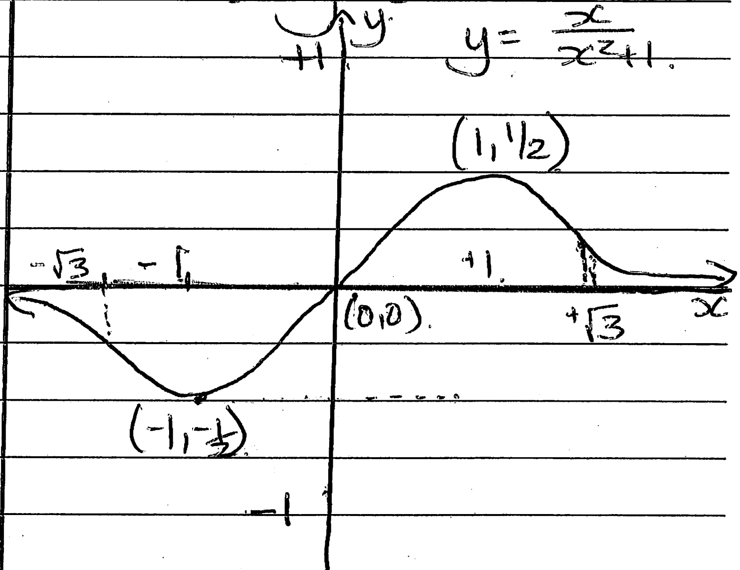
$(0,0), (-\sqrt{3}, \frac{-\sqrt{3}}{4}), (+\sqrt{3}, \frac{+\sqrt{3}}{4})$.

c) $\lim_{x \to \infty} \left(\frac{x}{x^2+1}\right) = \lim_{x \to \infty} \left(\frac{x/x^2}{x^2/x^2 + \frac{1}{x^2}}\right)$

since $\lim_{x \to \infty} \frac{1}{x} = 0$.

$\lim_{x \to \infty} \frac{x}{x^2+1} = 0$.

Horizontal assymptote at $y=0$.



Q6 b)

i) $A = 750(1\cdot09)^{40}$
$= 23557\cdot065$
$= \$23557$
nearest \$.

ii) Yr₂ $750(1\cdot09)^{40} + 750(1\cdot09)^{39}$

Yr_3. $750(1\cdot09)^{40} + 750(1\cdot09)^{39} + 750(1\cdot09)^{38}$.

Y_{40}. $750(1\cdot09^{40} + 1\cdot09^{39} + \ldots + 1\cdot09)$

S_{40} when $a = 1\cdot09$ $n = 40$ $r = 1\cdot09$.
$S_n = \dfrac{a(r^n - 1)}{r-1}$
$S_{40} = \dfrac{1\cdot09(1\cdot09^{40} - 1)}{0\cdot09}$.

$Y_{40} = 750 \times \dfrac{1\cdot09(1\cdot09^{40} - 1)}{0\cdot09}$

$= 276218.898$
$= \$276219$ nearest \$.

Q7 (a) given $3x^2-12x-9=0$ with roots α, β.

$$\alpha+\beta = -\frac{b}{a} = \frac{12}{3} = 4$$

$$\alpha\beta = \frac{c}{a} = \frac{-9}{3} = -3$$

(i) $\frac{1}{\alpha^3\beta^3} = \frac{1}{(\alpha\beta)^3} = \frac{1}{(-3)^3} = -\frac{1}{27}$ (1)

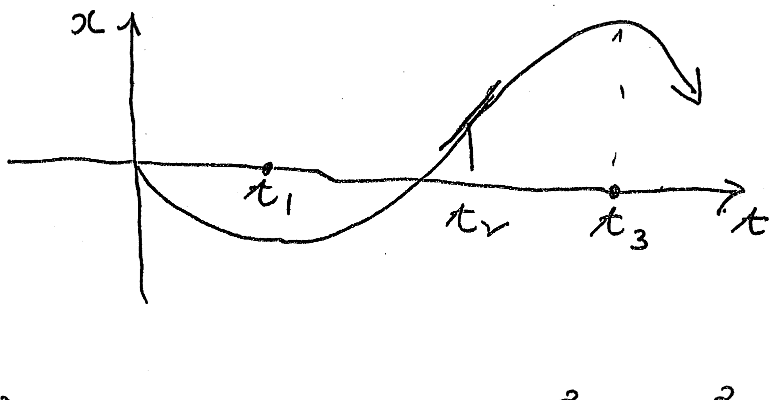
(ii) $\frac{\beta}{\alpha}+\frac{\alpha}{\beta} = \frac{\beta^2+\alpha^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} = \frac{16+6}{-3}$

$= -\frac{22}{3}$ (2)

(b) (i) The particle is moving to the left, then stops at t_1 then moves to the right (1)

(ii) The particle reaches its maximum velocity at t_2, then starts to slow down. (1)

(iii)



(3)

(c) $AP^2 + BP^2 = 118 \Rightarrow (x-2)^2+(y-4)^2+(x-6)^2+(y+8)^2 = 118$

(4)

$\therefore x^2-4x+4+y^2-8y+16+x^2-12x+36+y^2+16y+64 = 118$

$2x^2+2y^2-16x+8y+120 = 118$

$x^2+y^2-8x+4y = -1$

$(x-4)^2 + (y+2)^2 = -1+16+4 = 19$

$\boxed{\therefore \text{centre } (4,-2) \text{ radius } \sqrt{19}}$

2U THSC 08

Question 8

(a) $y = 10^x + 10x$

$\frac{dy}{dx} = 10^x \ln 10 + 10$ [2]

(b) $x = t + \frac{1}{t+1}$

(i) When $t=0$, $x = 1$ [1]

(ii) $\dot{x} = 1 - \frac{1}{(t+1)^2}$

$\dot{x} = 0$ when

$1 - \frac{1}{(t+1)^2} = 0$ $\quad t \neq -1$

$(t+1)^2 - 1 = 0$

$t^2 + 2t + 1 - 1 = 0$

$t^2 + 2t = 0$

$t(t+2) = 0$

$t = 0, -2$

(-2 is extraneous)

$\therefore t = 0$

Particle is initially at rest. [2]

(iii) $\ddot{x} = \frac{2}{(t+1)^3}$

$\ddot{x}(5) = \frac{2}{6^3}$

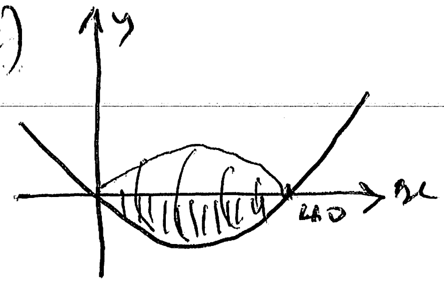
$= \frac{1}{108}$ cm/sec²

$\doteqdot 9.2593 \times 10^{-3}$ cm/s² [2]

(iv) $\ddot{x} \to 0$ as $t \to \infty$,

So $\dot{x} \to$ a limit of 1 cm/sec. [2]

(c)



$V = \pi \int_0^{40} \left[\frac{1}{5}(x^2 - 40x)\right]^2 dx$

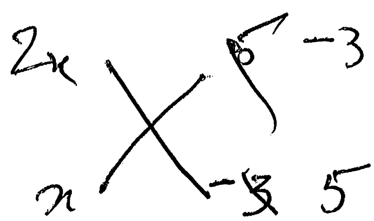
$= \frac{\pi}{25} \int_0^{40} (x^4 - 80x^3 + 1600x^2)\, dx$

$= \frac{\pi}{25} \left[\frac{x^5}{5} - \frac{80x^4}{4} + \frac{1600x^3}{3} \right]_0^{40}$

$= \frac{\pi}{25} (341333\tfrac{1}{3})$

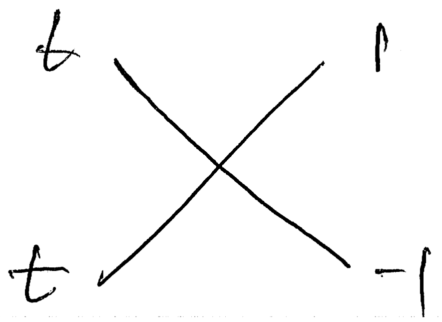
$\doteqdot 428932.117$ cm³

$\doteqdot 429$ L [3]



t^2+2t-1

$\frac{d}{dx}(t+1)^{-1}$



$= \frac{-1}{(t+1)^2}$

$\dot{x} = 1-(t+1)^{-2}$

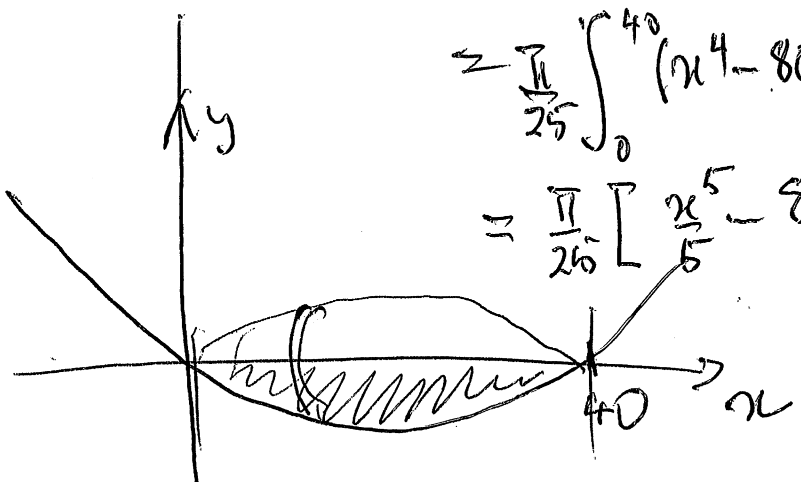
$\ddot{x} = -(-2)(t+1)^{-3}$

$= \frac{2}{(t+1)^3}$

$V = \pi \int_0^{40} \left[\frac{1}{5}(x^2-40x)\right]^2 dx$

$= \frac{\pi}{25}\int_0^{40} (x^4-80x^3+1600x^2)\, dx$

$= \frac{\pi}{25}\left[\frac{x^5}{5}-\frac{80x^4}{4}+\frac{1600x^3}{3}\right]_0^{40}$



$= \frac{\pi}{25}\left(3413333\tfrac{1}{3}\right)$

$\doteq 428932.117 \text{ cm}^3$

$\doteq 429 \text{ L}$

2008 Trial HSC Mathematics:
Solutions— Question 10

10. (a) If $x \sin \pi x = \displaystyle\int_0^{x^2} f(t)\, dt$, find $f(4)$. $\boxed{2}$

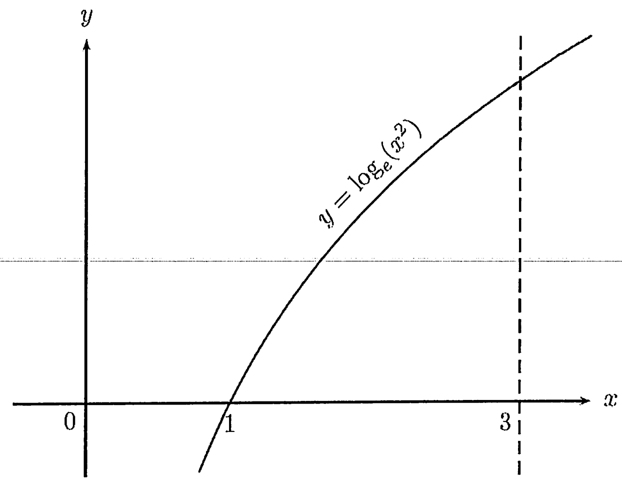
Solution: Let $x^2 = u$,

$$\begin{aligned}
f(u) &= \frac{d}{du}\int_0^u f(t)\,dt,\\
&= \frac{d}{du}\{\pm\sqrt{u}\sin(\pm\pi\sqrt{u})\},\\
&= \pm\sqrt{u}\cos(\pm\pi\sqrt{u}) \times \frac{\pi}{\pm 2\sqrt{u}} + \frac{\sin(\pm\pi\sqrt{u})}{\pm 2\sqrt{u}}.\\
\therefore\ f(4) &= \pm\sqrt{4}\cos(\pm\pi\sqrt{4}) \times \frac{\pi}{\pm 2\sqrt{4}} + \frac{\sin(\pm\pi\sqrt{4})}{\pm 2\sqrt{4}}.\\
&= \frac{2\pi}{4} + 0,\\
&= \frac{\pi}{2}.
\end{aligned}$$

Solution: Alternative method

$$\begin{aligned}
F(x^2) - F(0) &= x\sin(\pi x),\\
2xF'(x^2) - 0F'(0) &= \sin(\pi x) + \pi x\cos(\pi x),\\
F'(x^2) &= \frac{\sin(\pi x) + \pi x\cos(\pi x)}{2x},\\
&= f(x^2).\\
\text{When } x^2 &= 4,\\
x &= \pm 2.\\
\therefore\ f(4) &= \frac{\sin(\pm 2\pi) \pm 2\pi\cos(\pm 2\pi)}{\pm 4},\\
&= \frac{0 + 2\pi}{4},\\
&= \frac{\pi}{2}.
\end{aligned}$$

(b) The graph of the function $y = \log_e(x^2)$ is shown below.



(i) Use the Trapezoidal rule with 5 function values to approximate $\displaystyle\int_1^3 \log_e(x^2)\,dx$ and explain why this approximation underestimates the value of the integral. $\boxed{3}$

Solution: $\displaystyle\int_1^3 \ln(x^2) \approx \frac{0.5}{2}\{0 + 2(0.8109 + 1.3863 + 1.8326) + 2.1972\},$

≈ 2.564 (3 sig. fig.).

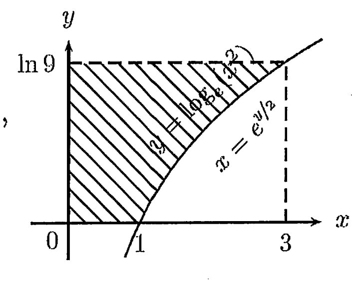
Each trapezium's sloping edge is under the curve as the curve is always concave downwards. The approximation is short by the amounts between the top of the trapezia and the curve.

(ii) Find $\displaystyle\int_0^{\ln 9} e^{\frac{y}{2}}\,dy$ and hence find the exact value of $\displaystyle\int_1^3 \log_e(x^2)\,dx.$ $\boxed{3}$

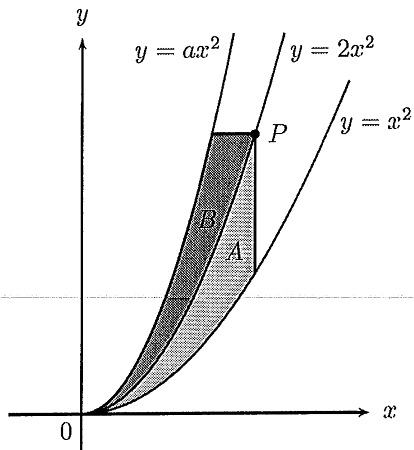
Solution:

$$\begin{aligned}\int_0^{\ln 9} e^{\frac{y}{2}}\,dy &= \left[2e^{\frac{y}{2}}\right]_0^{2\ln 3},\\ &= 2e^{\frac{2\ln 3}{2}} - 2 \times 1,\\ &= 6 - 2,\\ &= 4.\end{aligned}$$

$$\therefore \int_1^3 \ln(x^2)\,dx = 3\ln 9 - 4.$$



(c) $\boxed{4}$



The figure shows a function $y = ax^2$ with the property that, for every point P on the middle function $y = 2x^2$, the areas A and B are equal.
Find the value of a.

Solution:

$$
\begin{aligned}
\text{Let } P &= (p,\, q). \\
\text{Area } A &= \int_0^p \left(2x^2 - x^2\right) dx, \\
&= \int_0^p \left(x^2\right) dx, \\
&= \left[\frac{x^3}{3}\right]_0^p, \\
&= \frac{p^3}{3}. \\
\text{Area } B &= \int_0^q \left(\sqrt{\frac{y}{2}} - \sqrt{\frac{y}{a}}\right) dy, \\
&= \left[\frac{2y^{3/2}}{3\sqrt{2}} - \frac{2y^{3/2}}{3\sqrt{a}}\right]_0^q, \\
&= \frac{2}{3}q^{3/2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{a}}\right). \\
\text{But } q &= 2p^2, \\
\text{so area } B &= \frac{2}{3}.2^{2/3}.p^3\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{a}}\right). \\
\text{Now } A &= B, \\
\textit{i.e. } \frac{p^3}{3} &= \frac{4p^3}{3}\left(1 - \sqrt{\frac{2}{a}}\right), \\
1 &= 4 - 4\sqrt{\frac{2}{a}}, \\
\sqrt{\frac{2}{a}} &= \frac{3}{4}, \\
\frac{2}{a} &= \frac{9}{16}, \\
a &= \frac{32}{9}.
\end{aligned}
$$