Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2004
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics
Extension 1

Afternoon Session
Tuesday 10 August 2004

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1-7
- All questions are of equal value

Question 1 **Begin a new page** **Marks**

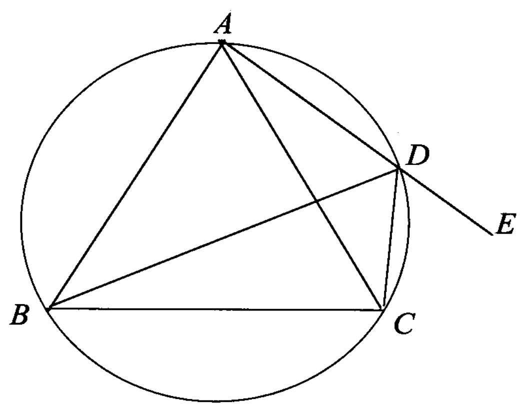
(a) Find $\frac{d}{dx}\left(1+x^2\right)\tan^{-1}x$. **2**

(b) The polynomial $P(x)$ is given by $P(x)=x^3+ax+1$ for some real number a. The remainder when $P(x)$ is divided by $(x-1)$ is equal to the remainder when $P(x)$ is divided by $(x-2)$. Find the value of a. **2**

(c) (i) The line $y=mx$ makes an angle of $45°$ with the line $y=2x$. Show that $|m-2|=|1+2m|$. **2**

(ii) Hence find the equations of the lines $y=mx$ which make an angle of $45°$ with the line $y=2x$. **2**

(d)



ABC is a triangle in which $BC=AC$. D is a point on the minor arc AC of the circle passing through A, B and C. AD is produced to E.

(i) Copy the diagram.

(ii) Give a reason why $\angle CDE=\angle ABC$. **1**

(iii) Hence show that DC bisects $\angle BDE$. **3**

Marks

Question 2 **Begin a new page**

(a) $A(-5,6)$ and $B(1,3)$ are two points. Find the coordinates of the point P which divides the interval AB externally in the ratio $5:2$. **2**

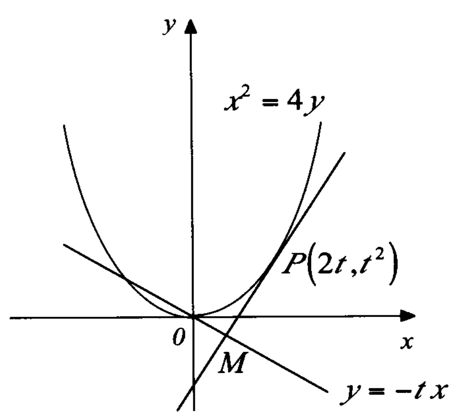
(b) The equation $2x^3+2x^2+4x+1=0$ has roots α, β and γ. Find the value of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$. **2**

(c) Consider the geometric series $\sin 2x+\sin 2x\cos 2x+\sin 2x\cos^2 2x+\ \ldots$ for $0<x<\frac{\pi}{2}$.

(i) Show that the limiting sum S of the series exists. **2**

(ii) Show that $S=\cot x$. **2**

(d)



$P(2t,t^2)$ is a point on the parabola $x^2=4y$. The tangent to the parabola at P and the line $y=-tx$ intersect at the point M.

(i) Show that the tangent to the parabola at P has gradient t and equation $tx-y-t^2=0$. **2**

(ii) Find the Cartesian equation of the locus of M as t varies. **2**

Marks

Question 3

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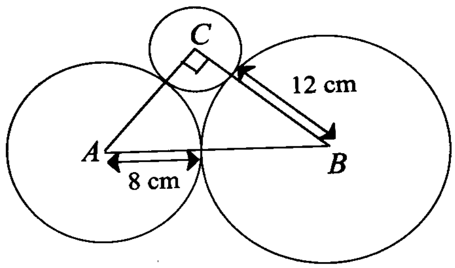
(a)



Three circles with centres A, B and C touch externally in pairs with $\angle BCA = 90°$, as shown in the diagram. The circles with centres A and B have radii 8cm and 12cm respectively.

(i) If the circle with centre C has radius x cm, show that $x^2 + 20x - 96 = 0$. **2**

(ii) Hence find the radius of the circle with centre C. **2**

(b) (i) Show that $\dfrac{u}{u+1} = 1 - \dfrac{1}{u+1}$. **1**

(ii) Hence find $\displaystyle\int \frac{1}{1+\sqrt{x}}\,dx$ using the substitution $x = u^2$, $u \geq 0$. **3**

(c) Use Mathematical Induction to show that $5^n > 4^n + 3^n$ for all integers $n \geq 3$. **4**

Question 4

Begin a new page

(a) Find the term independent of x in the binomial expansion of $\left(x - \dfrac{2}{x^2}\right)^{15}$. **3**

(b) After t years, $t \geq 0$, the number N of individuals in a population is given by $N = A + Be^{-0.5t}$ for some constants $A > 0$ and $B > 0$. The initial population size is 500 individuals and the limiting population size is 100 individuals.

(i) Find the values of A and B. **2**

(ii) Find the time taken for the population size to fall within 10 of its limiting value, giving the answer correct to the nearest month. **2**

(c) Consider the function $f(x) = x - \cos x$.

(i) Show that the equation $f(x) = 0$ has a root α such that $0 < \alpha < 1$. **2**

(ii) Use one application of Newton's Method with an initial approximation of $0 \cdot 7$ to approximate α, giving the answer correct to 2 decimal places. **3**

Marks

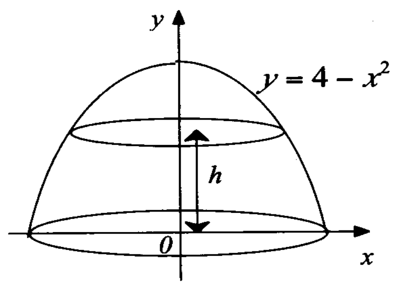
Question 5 **Begin a new page**

(a) In a certain street, 40% of the households have at least 2 cars. If 4 households are chosen at random, find the probability that

(i) exactly 3 of these households have at least 2 cars. **1**

(ii) at most 3 of these households have at least 2 cars. **2**

(b)



A mould for a container is made by rotating the part of the curve $y = 4 - x^2$ which lies in the first quadrant through one complete revolution about the y axis. After sealing the base of the container, water is poured through a hole in the top. When the depth of water in the container is h cm, the depth is changing at a rate $\dfrac{10}{\pi(4-h)}$ cms^{-1}.

(i) Show that when the depth is h cm, the surface area $S\text{cm}^2$ of the water is given by $S = \pi(4 - h)$. **1**

(ii) Find the rate at which the surface area of the water is changing when the depth of the water is 2cm. **3**

(c) Consider the function $f(x) = e^x - x$.

(i) Show that the curve $y = f(x)$ is concave up for all values of x. **1**

(ii) Find the coordinates and nature of the stationary point on the curve $y = f(x)$. **2**

(iii) Hence show that $e^x \geq x + 1$ for all values of x. **2**

Question 6 **Begin a new page**

(a) Consider the function $f(x) = \cos^{-1}(x - 1)$.

(i) Find the domain of the function. **1**

(ii) Sketch the graph of the curve $y = f(x)$ showing clearly the coordinates of the endpoints. **2**

(iii) The region in the first quadrant bounded by the curve $y = f(x)$ and the coordinate axes is rotated through one complete revolution about the y axis. Find the exact value of the volume of the solid of revolution. **3**

Marks

(b) A particle moving in a straight line is performing Simple Harmonic Motion. At time t seconds it has displacement x metres to the right of a fixed point O on the line, where $x = 4\cos^2 t - 2\sin^2 t$.

(i) Show that $x = 1 + 3\cos 2t$. Hence express the acceleration $\ddot{x}\,\text{ms}^{-2}$ of the particle in the form $\ddot{x} = -n^2(x - b)$, where the values of the constants n and b are to be determined. **2**

(ii) Find the set of possible values of x and the period of the motion. **2**

(iii) Find the distance travelled and the time taken (to the nearest tenth of a second) for the particle to first pass through O. **2**

Question 7 **Begin a new page**

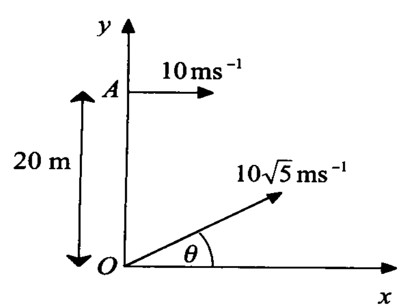
(a) A particle is moving in a straight line. At time t seconds it has displacement x metres to the right of a fixed point O on the line and velocity $v\,\text{ms}^{-1}$ given by $v = \sin x \cos x$. The particle starts $\frac{\pi}{4}$ metres to the right of O.

(i) Show that $\dfrac{d}{dx}\ln(\tan x) = \dfrac{1}{\sin x \cos x}$. **1**

(ii) Hence show that the displacement of the particle is given by $x = \tan^{-1}(e^t)$. **3**

(iii) Find the limiting position of the particle and sketch the graph of x against t. **2**

(b)



OA is a vertical building of height 20 metres. A particle is projected horizontally from A with speed $10\,\text{ms}^{-1}$. At the same instant, a second particle is projected from O with speed $10\sqrt{5}\,\text{ms}^{-1}$ at an angle θ above the horizontal. The two particles travel in the same plane of motion. Take $g = 10\,\text{ms}^{-2}$.

(i) Write down expressions for the horizontal and vertical displacements relative to O of each particle after time t seconds. **2**

(ii) Show that if the two particles collide, then they do so after 1 second. **2**

(iii) Show that if the two particles collide, when they do so their paths of motion are perpendicular to each other. **2**