Q1. (a) $\cos(A-B) = \cos A\cos B + \sin A \sin B$

$\cos(45-30) = \cos 45 \cos 30 + \sin 45 \sin 30$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)$

$\cos 15 = \frac{\sqrt{3}+1}{2\sqrt{2}}$

(b) $|r| < 1$

$\left|\frac{2x}{x+1}\right| < 1$

either $\frac{2x}{x+1} < 1$

Critical points at $x=-1$ and

$\frac{2x}{x+1} = 1$

$2x = x+1 \Rightarrow x = 1$

$x<-1 \{ \ -1<x<1 \ \{ \ x>1$

$-1 \quad 0 \quad 1$

Test $x=0$: true $\therefore -1 < x < 1$

or $\frac{2x}{x+1} > -1$

Critical points at $x=-1$ and

$\frac{2x}{x+1} = -1$

$2x = -x-1 \Rightarrow x = -\frac{1}{3}$

$x<-1 \{ \quad \{ \ x > -\frac{1}{3}$

$-1 \quad -\frac{1}{3} \quad 0$

Test $x=0$: ~~true~~ $\therefore x<-1$ & $x > -\frac{1}{3}$

$\therefore$ solution is: $-\frac{1}{3} < x < 1,\ x \neq 0$

(c) $\int_0^4 \frac{1}{\sqrt{9+x^2}}\,dx$

$= \left[\ln\left(x+\sqrt{9+x^2}\right)\right]_0^4$

$= \ln\left(4+\sqrt{9+4^2}\right) - \ln\left(0+\sqrt{9+0}\right)$

$= \ln 9 - \ln 3$

$= \ln 3$

(d) (i) ${}^6P_6 \times {}^5P_5 = 86400$

(ii) Ignore the women:

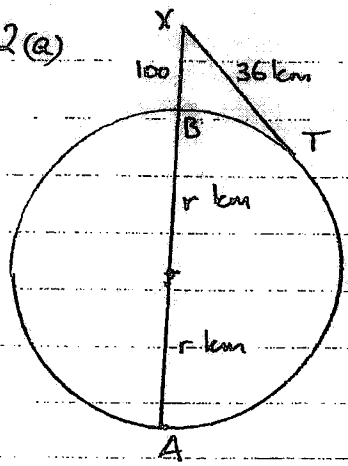
Number of permutations with A & B at the ends is ${}^2P_2 \times {}^4P_4 = 48$

$\therefore P(A \,\&\, B \text{ are at the ends})$

$= \frac{48}{720}$

$= \frac{1}{15}$

12 (a)



Now

$BX.AX = TX^2$

$\therefore .1 \times (2r + .1) = 36^2$

$2r + .1 = \frac{36^2}{.1}$

$r = \frac{1}{2}\left(\frac{36^2}{.1} - .1\right)$

$= 6479.95$ km

~ 6480 km

(b) $\frac{dV}{dt} = 50$; $r = 8$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 . \frac{dr}{dt}$

$\therefore \frac{dr}{dt} = \frac{dV/dt}{4\pi r^2} = \frac{50}{4\pi r^2}$

and $S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r . \frac{dr}{dt}$

$= 8\pi r . \frac{50}{4\pi r^2}$

$= \frac{100}{r}$

$\therefore$ if $r = 8$, $\frac{dS}{dt} = 12.5$ mm^2/s

c) let $\theta = \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right)$

$\tan\theta = \tan\left(\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right)\right)$

$= \frac{\tan\tan^{-1}\left(\frac{1}{4}\right) + \tan\tan^{-1}\left(\frac{3}{5}\right)}{1 - \tan\tan^{-1}\left(\frac{1}{4}\right) \times \tan\tan^{-1}\left(\frac{3}{5}\right)}$

$= \frac{1/4 + 3/5}{1 - 1/4 \times 3/5}$

$\tan\theta = 1$

$\therefore \theta = \pi/4$

(d) $\left(x^{1/5} + x^{1/3}\right)^9 = \sum_{r=0}^{9} {}^9C_r \left(x^{1/5}\right)^{9-r}\left(x^{1/3}\right)^r$

$= \sum_{r=0}^{9} {}^9C_r \, x^{\frac{9-r}{5}} . x^{\frac{r}{3}}$

$= \sum_{r=0}^{9} {}^9C_r \, x^{\frac{27+2r}{15}}$

Integer powers occur when $\frac{27+2r}{15}$ is an integer

This occurs when $r = 9$

$\Rightarrow \frac{27 + 2\times 9}{15} = 3$

$\therefore$ the term is ${}^9C_9 \, x^3$

$= x^3$

Q3 a(i) LHS $= \dfrac{\sec^2 x}{\tan x}$

$= \dfrac{1/\cos^2 x}{\sin x/\cos x}$

$= \dfrac{1}{\cos x \sin x} = $ RHS //

(ii) $I = \displaystyle\int_{\pi/6}^{\pi/3} \frac{\sec^2 x \, dx}{\tan x}$

if $u = \tan x$, $du = \sec^2 x \, dx$

if $x = \pi/3$, $u = \tan \pi/3 = \sqrt{3}$

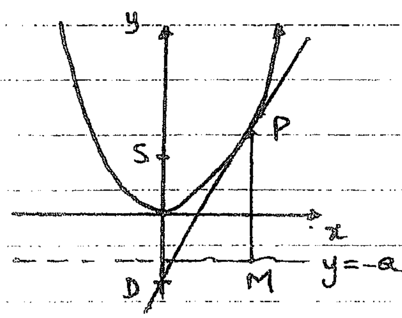
$x = \pi/6$, $u = \tan \pi/6 = 1/\sqrt{3}$

$\therefore I = \displaystyle\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{du}{u}$

$= \Big[\ln u\Big]_{1/\sqrt{3}}^{\sqrt{3}}$

$= \ln\sqrt{3} - \ln 1/\sqrt{3} = \ln 3$

b)



(i) Parabola is locus of points equidistant from focus, S, and directrix, $y=-a$

$\therefore PS = PM$

ii) Tangent at P: $y = px - ap^2$

At $x=0$, $y = -ap^2$

$\therefore D(0, -ap^2)$

iii) $d_{PM} = ap^2 - -a = a(p^2+1)$

$d_{SP} = a - -ap^2 = a(1+p^2)$

so SPMD is a rhombus.

(c) $S(n)$: $\displaystyle\sum_{r=1}^{n} \frac{1}{(4r-3)(4r+1)} = \frac{n}{4n+1}$

$S(1)$: LHS $= \dfrac{1}{(4-3)(4+1)} = 1/5$

RHS $= \dfrac{1}{4+1} = 1/5 =$ LHS

$\therefore S(1)$ is true

Assume $n=k$:

i.e. $S(k)$: $\displaystyle\sum_{r=1}^{k} \frac{1}{(4k-3)(4k+1)} = \frac{k}{4k+1}$

Prove $n = k+1$

i.e. $S(k+1)$: $\displaystyle\sum_{r=1}^{k+1} \frac{1}{(4r-3)(4r+1)} = \frac{k+1}{4k+5}$

LHS $= \dfrac{k}{4k+1} + \dfrac{1}{(4k+1)(4k+5)}$

$= \dfrac{k(4k+5)+1}{(4k+1)(4k+5)}$

$= \dfrac{4k^2+5k+1}{(4k+1)(4k+5)}$

$= \dfrac{(4k+1)(k+1)}{(4k+1)(4k+5)}$

$= \dfrac{k+1}{4k+5} =$ RHS

$\therefore$ If $S(k)$ is true, then $S(k+1)$ is t

But $S(1)$ is true, so $S(2)$ is true, whence $S(3)$ is true and so on for all positive integer values of n.

Q4. (a) $A(0,4)$ + $B(x_2, y_2)$ & $k:l = -3:1$

$x = \frac{kx_2 + lx_1}{k+l} \Rightarrow -6 = \frac{-3x_2 + 1\times 0}{-3+1}$

$12 = -3x_2 \Rightarrow x_2 = -4$

$y = \frac{ky_2 + ly_1}{k+l} \Rightarrow 1 = \frac{-3y_2 + 1\times 4}{-3+1}$

$-2 = -3y_2 + 4 \Rightarrow y_2 = 2$

$\therefore B(-4, 2)$

(b)(i) $A\sin(\theta - \alpha) = A\sin\theta\cos\alpha - A\cos\theta\sin\alpha$

$\therefore A\cos\alpha = 4$

$A\sin\alpha = 3$

whence $\alpha = \tan^{-1}(3/4)$ + $A = 5$

$\therefore 4\sin\theta - 3\cos\theta = 5\sin(\theta - \alpha)$

where $\alpha = \tan^{-1}(3/4)$

(ii) $5\sin(\theta - \alpha) = 1$

$\sin(\theta - \alpha) = 1/5$

$\theta - \alpha = 11°32', 168°28'$

$\therefore \theta = 11°32' + 36°52' = 48°24'$

and $\theta = 168°28' + 36°52' = 205°20'$

(c) Landing area = 3000×2000

Area where hoop does not protrude

$= 2800 \times 1800$

$\therefore P(\text{wins prize}) = \frac{2800 \times 1800}{3000 \times 2000}$

$= 0.84$

(d) Sum of roots is -6

Product of roots is c

(i) Assume the roots are α, $\alpha +$

Then $\alpha + \alpha + 2n = -6$

$\Rightarrow \alpha = -n - 3$

But $\alpha \times (\alpha + 2n) = c$

$(-n-3) \times (-n-3+2n) = c$

$-n^2 + 9 = c$

so $n^2 = 9 - c$

(ii) Since the roots are opposite in sign, the product must be nega

$\therefore c < 0$

but $c = 9 - n^2$ (above)

$\therefore 9 - n^2 < 0$

$n^2 > 9$

+ $n < -3$, $n > 3$

5. (a) Let p = probability of correctly machined part $= 0.98$

q = prob. of incorrectly machined part $= 0.02$

X = no. of correctly machined parts:

$P(X=r) = {}^{40}C_r\,(0.98)^r\,(0.02)^{40-r}$

(i) $P(X=38) = {}^{40}C_{38}\,(0.98)^{38}\,(0.02)^2$

$= 0.145$

(ii) $P(X \geq 38) = P(X=38) + P(X=39) + P(X=40)$

$= 0.1448 + {}^{40}C_{39}(.98)^{39}.02 + {}^{40}C_{40}(.98)^{40}$

$= 0.954$

b) let $f(x) = \log_e x + x^2 - 4x$

(i) $f(3) = \ln 3 + 9 - 12 < 0$

$f(4) = \ln 4 + 16 - 16 > 0$

$\therefore$ root exists between $x = 3$, $x = 4$

(ii) $f'(x) = \frac{1}{x} + 2x - 4$

Then $x_1 = x - \dfrac{\ln x + x^2 - 4x}{1/x + 2x - 4}$

$= 3.5 - \dfrac{\ln 3.5 + 3.5^2 - 4\times 3.5}{1/3.5 + 2\times 3.5 - 4}$

$= 3.6513$

$\therefore$ Better approximation is $x = 3.65$

(c)(i) $\cos 4x = \cos 2\times 2x$

$= 2\cos^2 2x - 1$

$= 2\left[2\cos^2 x - 1\right]^2 - 1$

$= 2\left(4\cos^4 x - 4\cos^2 x + 1\right) - 1$

$= 8\cos^4 x - 8\cos^2 x + 2 - 1$

(ii) $\cos^2 x - \cos^4 x = \frac{1}{16}$

$\therefore \cos 4x = 8 \cdot -\frac{1}{16} + 1$

$= \frac{1}{2}$

$4x = \frac{\pi}{3}$ or $4x = \frac{5\pi}{3}$

$\therefore x = \frac{\pi}{12}$ $\quad\therefore x = \frac{5\pi}{12}$

Q6.(a) $\ddot{x} = 0$ $\quad \ddot{y} = -10$

$\dot{x} = V\cos\alpha$ $\quad \dot{y} = -10t + V\sin\alpha$

$x = Vt\cos\alpha$; $y = -5t^2 + Vt\sin\alpha + 600$

Also 504 km/hr $= 140$ m/s

$\therefore \dot{x} = 70\sqrt{3}$ $\quad \dot{y} = -10t + 70$

$x = 70\sqrt{3}.t$ $\quad y = -5t^2 + 70t + 600$

(i) $y = 0 \Rightarrow -5t^2 + 70t + 600 = 0$

$-5(t^2 - 14t - 120) = 0$

$-5(t-20)(t+6) = 0$

$\Rightarrow t = 20$ seconds

(ii) $\dot{y} = 0 \Rightarrow -10t + 70 = 0$

$t = 7$

At $t=7$, $y = -5\times7^2 + 70\times7 + 600$

$= 845$ metres

iii) At $t = 20$, $x = 70\sqrt{3}\times 20$

$= 2424.87$

$\doteqdot 2.425$ kilometres

(b) (i) $v^2 = 28 + 24x - 4x^2$

if $v = 0 \Rightarrow 28 + 24x - 4x^2 = 0$

$4(7 + 6x - x^2) = 0$

$-4(x^2 - 6x - 7) = 0$

$-4(x-7)(x+1) = 0$

$x = -1$ and $x = 7$

$\therefore$ Oscillates between $x = -1$, $x = 7$

(ii) midpoint of motion is $x = 3$

$\therefore$ amplitude is 4m

(iii) $v^2 = 28 + 24x - 4x^2$

$\frac{1}{2}v^2 = 14 + 12x - 2x^2$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 12 - 4x$

$\therefore a = 12 - 4x$

(iv) $a = -4(x-3)$

$= -2^2(x-3)$

$\therefore n = 2$

Period, $T = \frac{2\pi}{n}$

$= \pi$ seconds

(v)