Kambala Church of England Girls' School

Trial Higher School Certificate Examination , 2000

Year 12

August, 2000

MATHEMATICS

3/4 UNIT

Time Allowed : 2 hours (plus 5 minutes reading time)

16

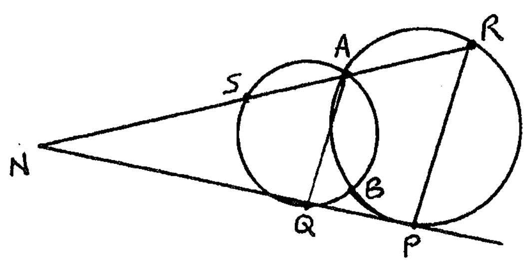
Instructions :

- ALL questions may be attempted.
- ALL questions are of equal value.
- All necessary working should be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Approved scientific calculators and drawing templates may be used.
- A Table of Standard Integrals is contained at the end of the examination paper.
- Start each question in a NEW BOOK.
- This is a Trial Paper only, and does NOT necessarily reflect either the content or format of the final HSC Examination.

Question 1 :

(a) Solve $\dfrac{x^2-1}{x} > 0$.

(b) Two circles intersect at A and B and a common tangent touches them at P and Q.
PR // QA.
RA is produced to cut the other circle at S and the tangent at N.
Prove that PRSQ is a cyclic quadrilateral.



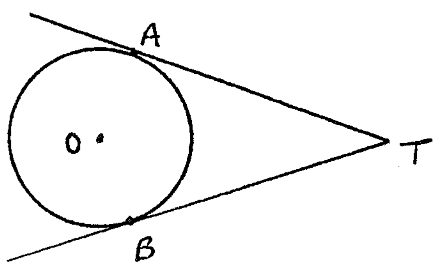
(c) Find $\displaystyle\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$.

(d) Find the equation of the two lines through the point (5,3) which make acute angles of $\dfrac{\pi}{4}^c$ with the line $2x - y + 2 = 0$.

Question 2 : (Start a NEW BOOK)

(a) TA and TB are tangents to the circle drawn below, with centre O.

Prove that : $\dfrac{\angle OAB}{\angle ATB} = \dfrac{1}{2}$.



(b) Find the general solution of the equation $\sin 2\theta = \sin \theta$, θ measured in radians.

(c) Prove by the Principle of Mathematical Induction that $3^{3n} + 2^{n+2}$ is a multiple of 5 for all positive integers n.

Question 3 : (Start a NEW BOOK)

(a) The point P(1,6) divides the interval AB in the ratio m:n.
If A = (7,0) and B = (3,4), find the value of the ratio m:n.

(b) Find $\frac{d}{dx}(x\sin^{-1}2x+\frac{1}{2}\sqrt{1-4x^2})$.

Hence evaluate $\int_0^{\frac{1}{2}} \sin^{-1}2x\,dx$.

(c) (i) If $t=\tan\frac{\theta}{2}$, find $\cos\theta$ and $\sin\theta$ in terms of t .
(ii) Hence solve the equation $3\sin\theta+4\cos\theta=5$ for values of θ in the range $0°\le\theta\le 360°$.

Question 4 : (Start a NEW BOOK)

(a) Evaluate the following definite integrals using the substitutions given :

(i) $\int_0^3 \frac{x}{\sqrt{4-x}}\,dx$ substitute $u=4-x$.

(ii) $\int_0^2 \frac{dx}{(4+x^2)^2}$ substitute $x=2\tan\theta$.

(b) The polynomial $P(x)=ax^3+bx^2-8x+3$ has a factor of $(x-1)$ and leaves a remainder of 15 when divided by $(x+2)$.

(i) Find the values of a and b .
(ii) Hence, factorise $P(x)$ fully and sketch the curve.
(iii) Determine the set of values of x for which $P(x)>0$.

Question 5 : (Start a NEW BOOK)

(a) $P(2ap,ap^2)$ is any point on the parabola $x^2 = 4ay$. S is the focus $(0,a)$. The tangent to the parabola at P meets the Y-axis in M. The perpendicular to the tangent PM from S meets PM in N. Find :

(i) the co-ordinates of M and N.
(ii) the co-ordinates of the midpoint K of MN.
(iii) the equation of the locus of K as P varies.

(b) A circular oil slick lies on the surface of a body of calm water. If its area is increasing at the rate of 1500 m^2/h , at what rate is its circumference increasing when the radius of the slick is 1250 m.

Question 6 : (Start a NEW BOOK)

A stone is thrown horizontally with a velocity of 20 m/s from the top of a tower 100 m high. Assuming no air resistance, and that the acceleration due to gravity, $g \approx 10\ m/s^2$;

(i) express x and y in terms of t.
(ii) find the equation of the trajectory.
(iii) find how long the stone takes to reach the ground .
(iv) find how far from the foot of the tower the stone strike the ground .
(v) find the velocity and direction of the stone on impact with the ground .

Question 7 : (Start a NEW Page)

(a) Define Simple Harmonic Motion.

(b) A particle moves from the origin, O with velocity (2p) m/s, and is subject to a retardation of its motion equal to q times its distance x from the origin ($q>0$).
(Note : retardation means negative acceleration)

(i) Show that the distance it travels, before coming to rest is $\frac{2p}{\sqrt{q}}$ metres.

(ii) Find the time when the particle first comes to rest.

(iii) Find where the particle is after $\frac{\pi}{4\sqrt{q}}$ seconds.

END OF EXAM