Harcourt Education

Student Number: ………….……………………….

2006

HIGHER SCHOOL CERTIFICATE

Sample Examination Paper

MATHEMATICS
Extension 1

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using blue or black pen
- Write your student number at the top of this page

Total marks – 84

- Attempt ALL questions.
- Show all necessary working, marks may be deducted for careless or untidy work.
- Board-approved calculators may be used.
- Additional Answer Booklets are available.

Directions to school or college

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Product code: 734805

QUESTION 1 (*Start a new booklet*) **Marks**

(a) Find the exact value of $\int_0^1 \left(\frac{1}{1+x} + e^{-x} + \frac{1}{\sqrt{1-x^2}} \right) dx$ **4**

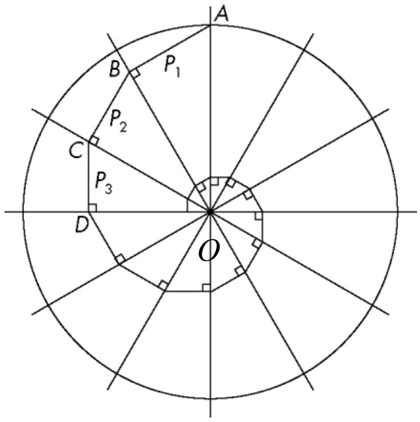
(b) How many numbers greater than 5000 can be formed with the digits 4, 5, 6, 7 and 8 if no digit is used more than once in a number? **3**

(c) Solve for x if $\log_{10}(10^x + 2) = 2x + 1$ **3**

(d) Show that $\frac{1-\cos 2x}{\sin 2x} = \tan x$ and hence express $\tan 15^{\circ}$ in simplest surd form. **2**

QUESTION 2 *(Start a new booklet)* **Marks**

(a) The figure shows a circle centre O and a radius 2 units. It is divided into 12 sectors and perpendiculars have been drawn as indicated. The points A, B, C, etc are shown as well as the distances $P_1, P_2, P_3, \ldots$



(i) Find the value of P_1 by considering ΔOBA. **1**

(ii) Show that the distances $P_1, P_2, P_3, \ldots$ form a geometric series. **3**

(iii) Find the exact length of one spiral. $(P_1 + P_2 + P_3 + \ldots + P_{12})$ **2**

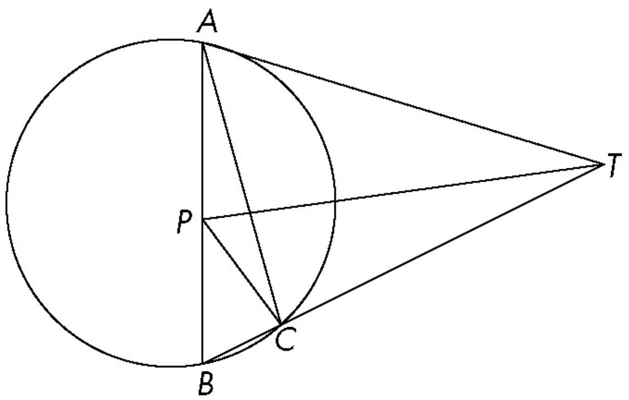
(iv) If the spiral is continued indefinitely, show that its total length will not exceed 7.5 units. **1**

(b) Using the substitution $u = e^x$, find $\int \frac{e^x}{1+e^{2x}}\,dx$. **2**

(c) Use Newton's method once to find a better root of the following equation $\cos x = \log_e x$ given that it has a root near $x = 1$. **3**

QUESTION 3 *(Start a new booklet)* **Marks**

(a) TA is a tangent to the circle at A, and AB is a chord.
P is a point on AB such that $TA = PT$. The secant TB cuts the circle at C and AC and PC are joined.



(i) Prove that $TAPC$ is a cyclic quadrilateral. **3**

(ii) Prove that TP is a tangent to circle PCB at P. **2**

(b) Differentiate with respect to x : $\cos^{-1}\left(\frac{1}{x}\right)$ **2**

(c) A bag contains twice as many white marbles as blue marbles. If a single marble is chosen at random, what is the probability that it is

(i) white? **1**

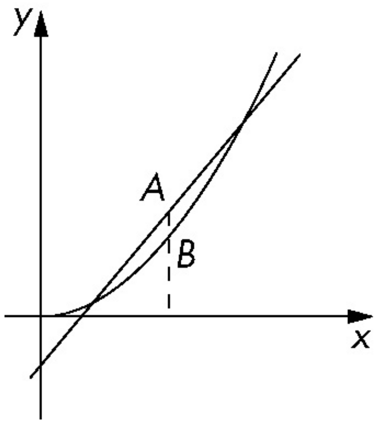
(ii) blue? **1**

(iii) If two marbles are chosen with replacement, find the probability that they will be of different colours. **1**

(d) A particle is moving along the x-axis. Its velocity V at position x is given by $V = \sqrt{8x - x^2}$. Find the acceleration when $x = 3$. **2**

QUESTION 4 *(Start a new booklet)* **Marks**

(a) The diagram represent the graphs of $f(x) = x^2$ and $g(x) = 7x - 6$.



Not to scale

(i) What is the maximum value of AB? **3**

(ii) Show that the line AB at its maximum value divides the area between the curve and the straight line in half. **4**

(b) Given that $f(x) = x^2 + bx + c$ and $g(x) = c + x + bx^2$ and if both $f(x)$ and $g(x)$ are divided by $x - t$, the remainder is 2, prove that $t = \dfrac{2-c}{b+1}$ **2**

(c) (i) Express $\sin x + \sqrt{3}\,\cos x$ in the form $A \sin(x + \alpha)$ where α is in radians and A>0. **2**

(ii) Hence or otherwise, sketch the graph of $y = \sin x + \sqrt{3}\,\cos x$ for $0 \le x \le \pi$. **1**

QUESTION 5 *(Start a new booklet)* **Marks**

(a) (i) Draw graphs of $y = |\sin x|$ and $y = -\cos x$ for $0 \leq x \leq 360°$. **1**

(ii) For what values of x will $|\sin x| + \cos x = 0$? **1**

(iii) For what values of x will $|\sin x| .\cos x < 0$? **1**

(b) A bowl of hot soup at temperature TºC, when placed in a cooler environment, loses heat according to the law

$$\frac{dT}{dt} = k(T - T_0)$$

where t is the time elapsed in minutes and T_0 is the temperature of the environment in degrees Celsius.

(i) A bowl of soup at 96ºC is left to stand in a room at a temperature of 18ºC. After 3 minutes the soup cools down to 75ºC. Calculate the value of k to 4 decimal places. **2**

(ii) Susan wishes to enjoy her soup at a temperature of 60ºC. How long should she wait? **2**

(c) On certain days of constant weather the variation of temperature each day follows a pattern of simple harmonic motion. If it is 13ºC at 5 a.m. and 23ºC at 5 p.m., at what daylight times would the temperature be

(i) 18ºC? **3**

(ii) 15ºC? **1**

(iii) What would be the expected temperature at 1 p.m.? **1**

QUESTION 6 *(Start a new booklet)* **Marks**

(a) Differentiate $(x^2 + 2x + 2)e^{-x}$ and hence evaluate $\int_1^2 x^2 e^{-x}\,dx$ correct to 3 decimal places. **2**

(b) $P\ (2ap, ap^2)$ is a point on the parabola $x^2 = 4ay$.

(i) Show that the equation of the normal at P is $x + py = 2ap + ap^3$. **2**

(ii) This normal cuts the y-axis at R. State the coordinates of R. **1**

(iii) From P a line PT is drawn perpendicular to the directrix meeting it in T. State the coordinates of T. **1**

(iv) If M is the midpoint of RT, find the coordinates of M. **1**

(v) Find the locus of M and show that it is a parabola with vertex at the focus of the original parabola. **3**

(c) A point P is moving on the curve $y = 2x^3$ in such a way that its x-coordinate is changing at a constant rate of 0.5 units/s. At what rate is the gradient changing when $x = 1$? **2**

QUESTION 7 *(Start a new booklet)* **Marks**

(a) A particle is projected with a speed of 20 m/s and passes through a point P whose horizontal distance from the point of projection is 30 m and whose vertical height above the point of projection is $8\frac{3}{4}$ m. Taking $g = 10$ m/s^2 and θ to be the angle of elevation, **8**

(i) Prove that $x = 20t \cos \theta$ and $y = -5t^2 + 20t \sin \theta$.

(ii) Find the angle of elevation of θ and the time taken for the particle to reach P.

(b) In the expansion $(1 + x + kx^2)^9$ in ascending powers of x, the coefficient of x^2 is zero. Find the value of k. **4**

End of paper

STANDARD INTEGRALS

$\int x^n dx$	$= \frac{1}{n+1} x^{n+1}, \ n \neq -1; \ x \neq 0, \text{ if } n < 0$
$\int \frac{1}{x} dx$	$= \ln x, \ x > 0$
$\int e^{ax} dx$	$= \frac{1}{a} e^{ax}, \ a \neq 0$
$\int \cos ax \, dx$	$= \frac{1}{a} \sin ax, \ a \neq 0$
$\int \sin ax \, dx$	$= -\frac{1}{a} coxax, \ a \neq 0$
$\int \sec^2 ax \, dx$	$= \frac{1}{a} \tan ax, \ a \neq 0$
$\int \sec ax \tan ax \, dx$	$= \frac{1}{a} \sec ax, \ a \neq 0$
$\int \frac{1}{a^2 + x^2} dx$	$= \frac{1}{a} \tan^{-1} \frac{x}{a}, \ a \neq 0$
$\int \frac{1}{\sqrt{a^2 - x^2}} dx$	$= \sin^{-1} \frac{x}{a}, \ a > 0, \ -a < x < a$
$\int \frac{1}{\sqrt{x^2 - a^2}} dx$	$= \ln\left(x + \sqrt{x^2 + a^2}\right), \ x > a > 0$
$\int \frac{1}{\sqrt{x^2 - a^2}} dx$	$= \ln\left(x + \sqrt{x^2 + a^2}\right)$
NOTE: $\ln x = \log_e x, \quad x > 0$	

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2006 MATHEMATICS EXTENSION 1 HSC TRIAL
Examination Mapping Grid

Question	Marks	Content	Syllabus Outcomes	Targeted Performance Bands
1(a)	4	Integration of various functions	HE4	E2 – E3
1(b)	3	Permutations and combinations	PE3	E2 – E3
1(c)	3	Logarithmic functions	H3	E2 – E3
1(d)	2	Trigonometric functions	PE2 H5	E2 – E4
2(a)	7	Sequences and series	H5	E2 – E3
2(b)	2	Integration by substitution	HE6	E2 – E4
2(c)	3	Polynomials	PE3	E2 – E4
3(a)	5	Circle Geometry	PE3	E2 – E4
3(b)	2	Differentiation of inverse trigonometric functions	HE4/5	E2 – E3
3(c)	3	Probability	H5	E2 – E3
3(d)	2	Applications of calculus to the physical world	HE5	E2 – E3
4(a)	7	Calculus	HE1 HE4 PE6	E2 – E3
4(b)	2	Real functions of a real variable	PE3	E2 – E4
4(c)	3	Trigonometry	HE1 H9	E2 – E3
5(a)	3	Trigonometry	PE2 H5	E2 – E4
5(b)	4	Applications of calculus to the physical world	HE3	E2 – E3
5(c)	5	Applications of calculus to the physical world	HE3	E2 – E3
6(a)	2	Calculus	HE1	E2 – E4
6(b)	8	Parametric representation	PE3	E2 – E4
6(c)	2	Calculus	HE5	E2 – E4
7(a)	8	Applications of calculus to the physical world	HE3	E2 – E4
7(b)	4	Binomial theorem	HE3	E2 – E4

SOLUTIONS

QUESTION 1

(a) $\int_0^1 \left(\frac{1}{1+x} + e^{-x} + \frac{1}{\sqrt{1-x^2}} \right) dx$

$= \left[\log(1+x) - e^{-x} + \sin^{-1} x \right]_0^1$

$= \log 2 - e^{-1} + \sin^{-1} 1 - (\log 1 - e^0 + \sin^{-1} 0)$

$= \log 2 - \frac{1}{e} + \frac{\pi}{2} + 1$

(b) If all 5 digits are used: number of permutations $= 5!$

$= 120$

If only 4 digits are used

4	4	3	2

Number of permutations $= 4 \times 4 \times 3 \times 2 = 96$

$\therefore$ Total = 216

(c) $\log_{10}(10^x + 2) = 2x + 1$

$10^x + 2 = 10^{2x+1}$

$10^x + 2 = 10(10^{2x})$

Let $10^x = a$

$\therefore\ 10a^2 - a - 2 = 0$

$(5a + 2)(2a - 1) = 0$

$a = -\frac{2}{5},\ a = \frac{1}{2}$

$10^x = -\frac{2}{5};\ 10^x = \frac{1}{2}$

No solution; $x = \log_{10} \frac{1}{2}$

(d) $$\frac{1-\cos 2x}{\sin 2x} = \frac{1-\left(1-2\sin^2 x\right)}{2\sin x\cos x}$$

$$= \frac{2\sin^2 x}{2\sin x\cos x}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

$$\tan 15° = \frac{1-\cos 30°}{\sin 30°}$$

$$= \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= 2-\sqrt{3}$$

QUESTION 2

(a) (i) $OA = 2$ $\angle AOB = 30^\circ$ $\therefore \underline{P_1 = 1}$ and $OB = \sqrt{3}$

(ii) $OB = \sqrt{3}$ $\angle BOC = 30^\circ$ $\therefore P_2 = \sqrt{3}\ \sin 30^\circ = \dfrac{\sqrt{3}}{2}$

and $OC = OB \cos 30^\circ = \sqrt{3} \,.\, \dfrac{\sqrt{3}}{2} = \dfrac{3}{2}$

$OC = \dfrac{3}{2}$, $\angle COD = 30^\circ$, $\therefore P_3 = \dfrac{3}{2}\ \sin 30^\circ = \dfrac{3}{4}$

and $OD = OC \cos 30^\circ = \dfrac{3}{2} \,.\, \dfrac{\sqrt{3}}{2} = \dfrac{3\sqrt{3}}{4}$

$$\frac{P_3}{P_2} = \frac{\frac{3}{4}}{\frac{\sqrt{3}}{2}} = \frac{3}{4} \times \frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\frac{P_2}{P_1} = \frac{\sqrt{3}}{2}$$

$\therefore$ $P_1, P_2, P_3, \ldots$ form a G.P.

(iii) Length of 1 spiral $S_{12} = \dfrac{a(r^{12} - 1)}{r - 1} = \dfrac{a(1 - r^{12})}{1 - r}$

$$= \frac{1\left[1 - \left(\frac{\sqrt{3}}{2}\right)^{12}\right]}{1 - \frac{\sqrt{3}}{2}}$$

$$= \frac{\left[1 - \frac{729}{4096}\right]}{\frac{2 - \sqrt{3}}{2}}$$

$$= \frac{(3367) \times 2}{4096(2 - \sqrt{3})}$$

$$= \frac{3367}{2048(2 - \sqrt{3})}$$

(iv) $S_\infty = \dfrac{a}{1-r}$

$= \dfrac{1}{1-\frac{\sqrt{3}}{2}}$

$= \dfrac{2}{2-\sqrt{3}}$

$\approx 7.464 < 7.5$

(b) $u = e^x$

$du = e^x dx$

$\therefore \int = \int \dfrac{du}{1+u^2} = \tan^{-1} u + c$

$= \tan^{-1}(e^x) + c$

(c) Let $f(x) = \cos x - \log x$

$f'(x) = -\sin x - \dfrac{1}{x}$

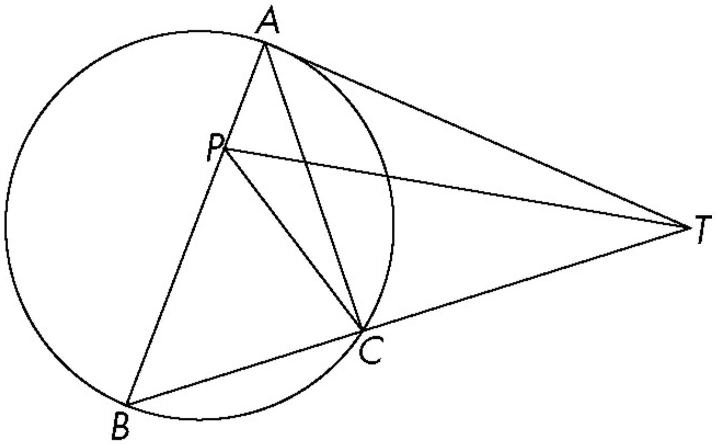
$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$= 1 - \dfrac{\cos 1 - \log 1}{-\sin 1 - 1}$

≈ 1.29

QUESTION 3

(a) (i)



$\angle PAT = \angle TPA$ ($TA = TP$)

$\therefore\ \angle PAC + \angle TAC = \angle TPA$

$\angle TPA = \angle ABT + \angle PTB$ (exterior angle of ΔPBT)

$\therefore\ \angle ABT + \angle PTB = \angle PAC + \angle TAC$

But $\angle TAC = \angle TBA$ (angle between tangent and chord)

$\therefore\ \angle PTB = \angle PAC$

$\therefore$ $TACP$ is a cyclic quadrilateral (angles subtended by arc PC are equal).

(ii) $TAC = \angle TPC$ ($TAPC$ is a cyclic quadrilateral)

But $\angle TAC = \angle ABC$ (angle between tangent and chord)

$\therefore\ \angle TPC = \angle ABC$

$\therefore$ PT is a tangent to circle PCB at P.

(b) Let $u = \dfrac{1}{x}$

$$y = \cos^{-1} u$$

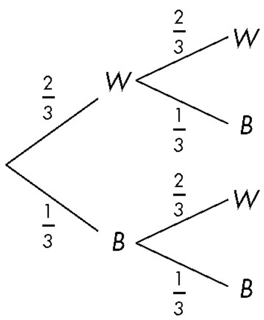
$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = -\frac{1}{\sqrt{1-u^2}}\left(-\frac{1}{x^2}\right)$$

$$= -\frac{1}{\sqrt{1-\frac{1}{x^2}}}\left(-\frac{1}{x^2}\right)$$

$$= \frac{-x}{\sqrt{x^2-1}}\left(-\frac{1}{x^2}\right)$$

$$= \frac{1}{x\sqrt{x^2-1}}$$

(c)



(i) $P(E) = \dfrac{2}{3}$

(ii) $P(E) = \dfrac{1}{3}$

(iii) $P(E) = \dfrac{2}{3}\left(\dfrac{1}{3}\right) + \dfrac{1}{3}\left(\dfrac{2}{3}\right)$

$= \dfrac{4}{9}$

(d) $v = \sqrt{8x - x^2}$

$\therefore v^2 = 8x - x^2$

$\dfrac{1}{2}v^2 = 4x - \dfrac{x^2}{2}$

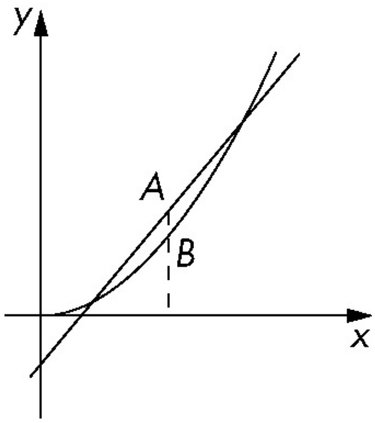
$a = \dfrac{d}{dx}\left(4x - \dfrac{x^2}{2}\right)$

$a = 4 - x$

When $x = 3$, $a = 1$

QUESTION 4

(a) (i)



Let distance $AB = s$

$s = 7x - 6 - x^2$

$\frac{ds}{dx} = 7 - 2x = 0$ for a maximum or minimum

$x = \frac{7}{2}$

$\frac{d^2s}{dx^2} = -2 \quad \therefore$ maximum

s maximum $= 7.\frac{7}{2} - 6 - \left(\frac{7}{2}\right)^2$

$= \frac{49}{2} - 6 - \frac{49}{4}$

$= \frac{25}{4}$

(ii) The parabola and the line meet when $x^2 = 7x - 6$

$x^2 - 7x + 6 = 0$

$(x - 6)(x - 1) = 0$

$x = 6, x = 1$

Area between the parabola and the line is

$\int_1^6 \left(7x - 6 - x^2\right) dx$

$= \left[\frac{7x^2}{2} - 6x - \frac{x^3}{3}\right]_1^6$

$= 126 - 36 - 72 - \left(\frac{7}{2} - 6 - \frac{1}{3}\right)$

$= \frac{125}{6}$

Area between parabola and line and line $x = \dfrac{7}{2}$ is

$$\int_1^{\frac{7}{2}} \left(7x - 6 - x^2\right) dx = \left[\frac{7x^2}{2} - 6x - \frac{x^3}{3}\right]_1^{\frac{7}{2}}$$

$$= \frac{343}{8} - 21 - \frac{343}{24} - \left(\frac{7}{2} - 6 - \frac{1}{3}\right)$$

$$= \frac{125}{12}$$

$\therefore$ Line AB divides the area in half.

(b) $f(x) = x^2 + bx + c$ $\quad$ $f(t) = t^2 + tb + c = 2 \ldots (1)$

$g(x) = c + x + bx^2$ $\quad$ $g(t) = c + t + bt^2 = 2 \ldots (2)$

From (1) $t^2 = 2 - tb - c$

Sub in (2) $c + t + b(2 - tb - c) = 2$

$$c + t + 2b - tb^2 - bc = 2$$

$$t(1 - b^2) = 2 - c - 2b + bc$$

$$= 2 - c - b(2 - c)$$

$$t(1 - b)(1 + b) = (2 - c)(1 - b)$$

$$t(1 + b) = 2 - c$$

$$t = \frac{2 - c}{1 + b}$$

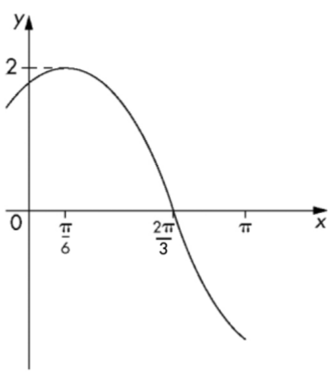
(c) (i) $A = \sqrt{1 + 3} = 2$

$$\tan \alpha = \frac{\sqrt{3}}{1}$$

$$\therefore \alpha = \frac{\pi}{3}$$

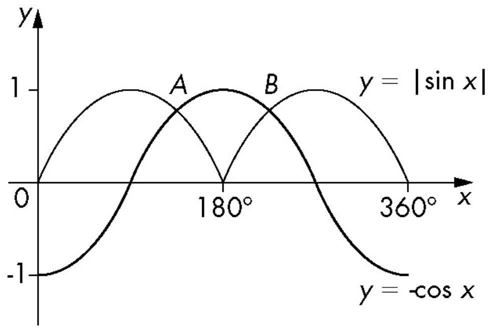
$$\therefore y = 2 \sin\left(x + \frac{\pi}{3}\right)$$

(ii)



QUESTION 5

(a) (i)



(ii) The solutions are found where the graphs intersect (see A and B)

$|\sin x| = -\cos x$

when $x = 135°$ or $225°$

(iii) $\cos x\,|\sin x| < 0$

when $-\cos x\,|\sin x| > 0$

i.e. when $90° < x < 180°$ and

$180° < x < 270°$

(b) (i) The solution of the differential equation is

$T = T_0 + Ae^{-kt}$ where A is a constant

When $t = 0$, $T = 96$, $T_0 = 18$ so that $A = 78$

$T = 18 + 78e^{-kt}$

When $t = 3$, $T = 75°$

$75 = 18 + 78e^{-3k}$

$57 = 78e^{-3k}$

$e^{-3k} = \dfrac{57}{78}$

$-3k = \ln\dfrac{57}{78}$

$k = \dfrac{\ln\frac{57}{78}}{-3}$

≈ 0.1046

(ii) $60 = 18 + 78e^{-0.1046t}$

$42 = 78e^{-0.1046t}$

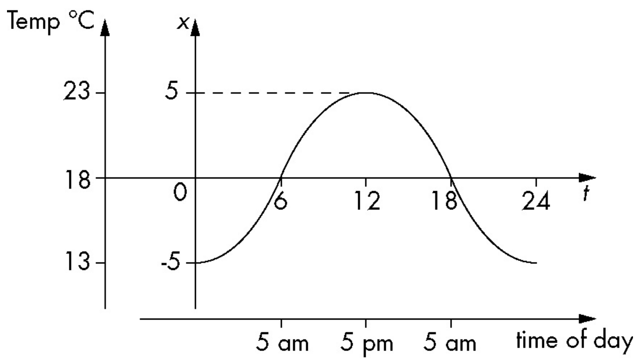
$\frac{42}{78} = e^{-0.1046t}$

$-0.1046t = \ln\frac{42}{78}$

$t = \frac{\ln\frac{42}{78}}{-0.1046}$

≈ 5.92 minutes

(c) Let x be the number of degrees by which the temperature differs from the mean temperature at time t after 5 a.m. (This places the origin of the temperature at mean temperature and the origin of time at 5 a.m.)



The motion is simple harmonic with amplitude 5°C and period 24 hours

$\therefore\ T = \frac{2\pi}{n}$ and $n = \frac{\pi}{12}$

$\therefore\ x = -5\cos\frac{\pi}{12}t$

(i) When temperature = 18°C

$x = 0$

$\therefore\ \cos\frac{\pi t}{12} = 0$

$\frac{\pi t}{12} = \frac{\pi}{2}$

$t = 6$ h

$\therefore$ Time would be 11 a.m.

(ii) $-5\cos\frac{\pi t}{12} = -3$

$\cos\frac{\pi t}{12} = 0.6$

$\frac{\pi t}{12} = 0.927$

t = 3 h 32 mins

$\therefore$ Time would be 8.32 a.m.

(iii) At 1 p.m. $t = 8$

$x = -5\cos\frac{\pi t}{12}$

$x = -5\cos\frac{8\pi}{12}$

$= -5\cos\frac{2\pi}{3}$

$= -5\left(-\frac{1}{2}\right)$

$= \frac{5}{2}$

$\therefore$ Temperature is 18 + 2.5

= 20.5°C

QUESTION 6

(a) $\dfrac{d}{dx}\left[x^2+2x+2\right]e^{-x} = (2x+2)e^{-x} - e^{-x}\left(x^2+2x+2\right)$

$$= e^{-x}\left(2x+2-x^2-2x-2\right)$$

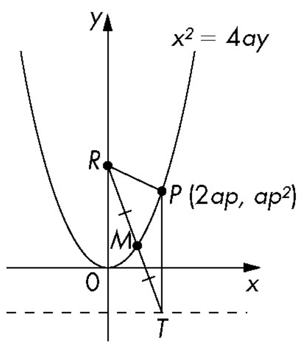
$$= e^{-x}\left(-x^2\right)$$

$$\therefore \int_1^2 x^2 e^{-x}\,dx = -\left[\frac{x^2+2x+2}{e^x}\right]_1^2$$

$$= \frac{5}{e} - \frac{10}{e^2}$$

$$\approx 0.486$$

(b)



(i) $x^2 = 4ay$

$$y = \frac{x^2}{4a}$$

$$\frac{dy}{dx} = \frac{x}{2a}$$

At $x = 2ap$, $\dfrac{dy}{dx} = \dfrac{2ap}{2a} = p$

$\therefore$ Gradient of normal at P is $-\dfrac{1}{p}$

$\therefore$ Equation of normal at P is

$$y - ap^2 = -\frac{1}{p}(x-2ap)$$

$$py - ap^3 = -x + 2ap$$

i.e. $x + py = ap^3 + 2ap$

(ii) Normal cuts y-axis when $x = 0$

i.e. $py = ap^3 + 2ap$

i.e. $y = ap^2 + 2a$

$\therefore$ Coordinates of $R(0, ap^2 + 2a)$

(iii) Coordinates of $T(2ap, -a)$

(iv) Coordinates of $M\left(ap, \dfrac{ap^2 + a}{2}\right)$

(v) $x = ap$

$$y = \frac{ap^2 + a}{2}$$

$$\therefore p = \frac{x}{a}$$

$$y = \frac{a\left(\frac{x}{a}\right)^2 + a}{2}$$

$$2y = \frac{x^2}{a} + a$$

$2ay = x^2 + a^2$

$x^2 = 2ay - a^2$

$x^2 = 2a(y - a)$ which is a parabola with vertex at $(0, a)$, the focus of the original parabola

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(c) Gradient $= m = \dfrac{dy}{dx}$

$$\begin{aligned} &= 6x^2 \\ \frac{dm}{dt} &= \frac{dm}{dx}.\frac{dx}{dt} \\ &= 12x.(0.5) \\ &= 12(1)(0.5) \\ &= 6 \text{ u / s} \end{aligned}$$

QUESTION 7

(a) (i) $\ddot{x} = 0$, $\dot{x} = 20 \cos \theta$, $x = 20t \cos \theta$

$\ddot{y} = -10$, $\dot{y} = -10t + 20 \sin \theta$, $y = -5t^2 + 20t \sin \theta$

(ii) At P $30 = 20t \cos \theta$

$$3 = 2t \cos \theta \qquad \therefore\ t = \frac{3}{2\cos\theta}$$

$$\frac{35}{4} = -5t^2 + 20t \sin \theta$$

$$35 = -20t^2 + 80t \sin \theta$$

$$7 = -4t^2 + 16t \sin \theta$$

$$\therefore\ 7 = -4\left(\frac{3}{2\cos\theta}\right)^2 + 16\left(\frac{3}{2\cos\theta}\right) \sin \theta$$

$$= -9 \sec^2 \theta + 24 \tan \theta$$

$$= -9(\tan^2 \theta + 1) + 24 \tan \theta$$

$$\therefore\ 9 \tan^2 \theta - 24 \tan \theta + 16 = (3 \tan \theta - 4)^2$$

$$= 0$$

$$\theta = \tan^{-1} \frac{4}{3} \qquad \text{and } \cos \theta = \frac{3}{5}$$

$$= 53°08'$$

$$t = \frac{3}{2\left(\frac{3}{5}\right)}$$

$$= 2.5 \text{ s}$$

(b) $(1 + x + kx^2)^9 = 1 + \binom{9}{1}(x + kx^2) + \binom{9}{2}(x + kx^2)^2 + \ldots\ldots$

$$= 1 + 9x + 9kx^2 + 36(x + 2kx^3 + \ldots) + \ldots\ldots$$

$$= 1 + 9x + (9k + 36)x^2 + \ldots\ldots$$

$$\therefore\ 9k + 36 = 0$$

$$k = -4$$

2006 Mathematics Extension 1 HSC Trial
Marking Guidelines

Questions			**Marks**	**Criteria**
1	(a)		3	Integration
			1	Evaluation
1	(b)		1	All digits
			1	Four digits
			1	Total
1	(c)		1	Index form
			1	Quadratic equation
			1	Solution
1	(d)		1	Identity
			1	Substitution
2	(a)	(i)	1	Values of P_1
		(ii)	3	Ratio constant
		(iii)	2	Length of one spiral
		(iv)	1	Evaluation
2	(b)		1	Substitution
			1	Integration
2	(c)		1	First derivative
			1	Correct substitution into formulae
			1	Evaluation
3	(a)	(i)	3	Statements with reasons
		(ii)	2	Statements with reasons
3	(b)		1	Chain rule
			1	Algebra
3	(c)	(i)	1	Correct answer
		(ii)	1	Correct answer
		(iii)	1	Correct answer
3	(d)		1	Acceleration
			1	Evaluation
4	(a)	(i)	1	Expression for AB
			1	Differentiation
			1	Maximum value
		(ii)	1	Coordinates of intersection
			1	First area
			1	Second area
			1	Conclusion
4	(b)		1	$f(t)$ and $g(t)$
			1	Various methods of obtaining result
4	(c)	(i)	1	Value of A
			1	Value of α
		(ii)	1	Graph
5	(a)	(i)	1	Graphs
			1	Answer
			1	Answer
5	(b)	(i)	1	Solution of differential equation
			1	Value of k
		(ii)	1	Correct substitution

Questions			**Marks**	**Criteria**
			1	Answer
5	(c)	(i)	1	Value of n
			1	Trigonometric equation
			1	Time at 18ºC
		(ii)	1	Time at 16ºC
		(iii)	1	Temperature at 1 o'clock
6	(a)		1	Differentiation
			1	Integration
6	(b)	(i)	1	Gradient of normal
			1	Equation of normal
		(ii)	1	Coordinates of R
		(iii)	1	Coordinates of T
		(iv)	1	Coordinates of M
		(v)	3	Locus of M and vertex
	(c)		1	Gradient
			1	Rate
7	(a)	(i)	1	Horizontal motion
			1	Vertical motion
		(ii)	1	T in terms of θ
			1	Quadratic equation of t
			1	Quadratic equation of $\tan \theta$
			1	Factorisation and solution
			1	θ
			1	Time
7	(b)		1	Expansion
			1	Terms containing x^2
			1	Equation
			1	Solution