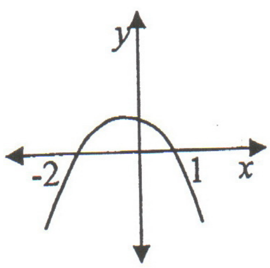
Question No. 1 **Solutions and Marking Guidelines**

Outcomes Addressed in this Question

PE3 **solves problems involving inequalities**
H5 **applies appropriate techniques from the study of calculus & trigonometry**

Outcome	**Solutions**	**Marking Guidelines**
PE3	(a) External point of division $\therefore$ ratio of $3:-1$ Point of division is $\left(\dfrac{lx_1+kx_2}{k+l},\dfrac{ly_1+ky_2}{k+l}\right)=\left(\dfrac{-2\times-1+-5\times3}{3-1},\dfrac{3\times-1-3\times3}{3-1}\right)$ $=\left(\dfrac{-13}{2},-6\right)$	2 marks : correct answer 1 mark : significant progress towards answer
H5	(b) For $2x-3y+1=0,\ y=\dfrac{2x}{3}+\dfrac{1}{3},\ m=\dfrac{2}{3}$ For $y=2x-1,\ m=2$ Using $\tan\theta=\left\|\dfrac{m_1-m_2}{1+m_1m_2}\right\|,\ \tan\theta=\left\|\dfrac{2-\frac{2}{3}}{1+2\times\frac{2}{3}}\right\|$ $\therefore \tan\theta=\dfrac{4}{7},\ \theta=30°$ (to the nearest degree)	2 marks : correct answer 1 mark : significant progress towards answer
PE3	(c) $\dfrac{3}{x+2}\le 1 \qquad x\ne-2$ $\dfrac{3(x+2)^2}{x+2}\le(x+2)^2$ $3(x+2)-(x+2)^2\le 0$ $(x+2)(1-x)\le 0$  $\therefore x\le-2$ or $x\ge1$ But $x\ne-2,\ \therefore x<-2$ or $x\ge1$	3 marks : multiply both sides by denominator squared and correctly factorise and correctly solve inequality 2 marks : 2 of above 1 mark : one of above **or** 3 marks : correct answer with justification 2 marks : correct answer 1 mark : partially correct answer
PE3	(d) $\|x+1\|>\|x-2\|$ $\sqrt{(x+1)^2}>\sqrt{(x-2)^2}$ $(x+1)^2>(x-2)^2$ (squaring both sides as both sides positive) $x^2+2x+1>x^2-4x+4$ $6x>3$ $\therefore x>\dfrac{1}{2}$	3 marks : square both sides and correctly expand and correctly solve inequality 2 marks : 2 of above 1 mark : one of above **or** 3 marks : correct answer with justification 2 marks : correct answer with limited justification 1 mark : solves inequality, incorrect method

Question No. 2	Solutions and Marking Guidelines	
	Outcomes Addressed in this Question	
HE2	uses inductive reasoning in the construction of proofs	
PE3	solves problems involving (permutations and combinations, inequalities, polynomials, circle geometry and) parametric representations	
Outcome	**Solutions**	**Marking Guidelines**
HE2	(a) Show true for $n=1$ $\text{LHS}=\dfrac{1}{1(1+1)}=\dfrac{1}{2} \quad \text{RHS}=\dfrac{1}{1+1}=\dfrac{1}{2}$ $\therefore$ true for $n=1$ Assume for $n=k$ ie $\dfrac{1}{1\times2}+\dfrac{1}{2\times3}+\dfrac{1}{3\times4}+\ldots\ldots\dfrac{1}{k(k+1)}=\dfrac{k}{k+1}$ Prove true for $n=k+1$ $\dfrac{1}{1\times2}+\dfrac{1}{2\times3}+\dfrac{1}{3\times4}+\ldots\ldots\dfrac{1}{k(k+1)}+\dfrac{1}{(k+1)(k+2)}=\dfrac{k+1}{k+2}$ $\text{LHS}=\dfrac{1}{1\times2}+\dfrac{1}{2\times3}+\dfrac{1}{3\times4}+\ldots\ldots\dfrac{1}{k(k+1)}+\dfrac{1}{(k+1)(k+2)}$ $=\dfrac{k}{(k+1)}+\dfrac{1}{(k+1)(k+2)}$ $=\dfrac{k(k+2)+1}{(k+1)(k+2)}$ $=\dfrac{k^2+2k+1}{(k+1)(k+2)}$ $=\dfrac{(k+1)^2}{(k+1)(k+2)}$ $=\dfrac{k+1}{k+2}$ $=\text{RHS}$ $\therefore$ true by mathematical induction	**3 marks** : correct solution **2 marks** : substantially correct solution **1 mark** : partially correct solution
PE3	(b) $x=\sin\theta$ and $y=\cos^2\theta-3$ $y=(1-\sin^2\theta)-3$ $y=(1-x^2)-3$ $y=-2-x^2$	**2 marks** : correct solution **1 mark** : correct approach with no more than one algebraic error.

PE3	(c) (i) $x+py=2ap+ap^3 \quad \ldots(1)$ $x+qy=2aq+aq^3 \quad \ldots(2)$ $(1)-(2)$: $py-qy=2ap+ap^3-2aq-aq^3$ $(p-q)y=2a(p-q)+a(p-q)(p^2+pq+q^2)$ $y=2a+a(p^2+pq+q^2)$ $=a(2+p^2+pq+q^2)$ Sub into (1): $x+py=2ap+ap^3$ $x+pa(2+p^2+pq+q^2)=2ap+ap^3$ $x=-ap^2q-apq^2$ $=-apq(p+q)$ $\therefore R$ is $(-apq[p+q], a[p^2+pq+q^2+2])$	**<u>2 marks</u>** : Correct solution **<u>1 mark</u>** : Applies correct method but obtains incorre result. (substantially correc
	(ii) Substitute $(0,a)$ in $y=\frac{1}{2}(p+q)x-apq$ $a=\frac{1}{2}(p+q).0-apq$ $a=-apq$ $pq=-1 \quad (\text{since } a\neq 0)$	**<u>1 mark</u>** : correct solution
	(iii) $x=-apq(p+q)$, from (i) $=a(p+q)$, using $pq=-1$ so $p+q=\frac{x}{a} \quad \ldots(1)$ $y=a(p^2+pq+q^2+2)$, from (i) $=a(p^2+2pq+q^2-pq+2)$ $=a([p+q]^2+3)$, using $pq=-1$ $=a\left(\left[\frac{x}{a}\right]^2+3\right)$, using (1) $=\frac{1}{a}(x^2+3a^2)$ $x^2+3a^2=ay$ $x^2=a(y-3a)$	**<u>2 marks</u>** : Correct solutio **<u>1 mark</u>** : Applies correct method but obtains inco result. (substantially cor *note: need some sort of in x,y before mark awar*

Outcomes Addressed in this Question

PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

HE3 uses a variety of strategies to investigate mathematical models of situations involving binomial probability, projectiles, simple harmonic motion, or exponential growth and decay

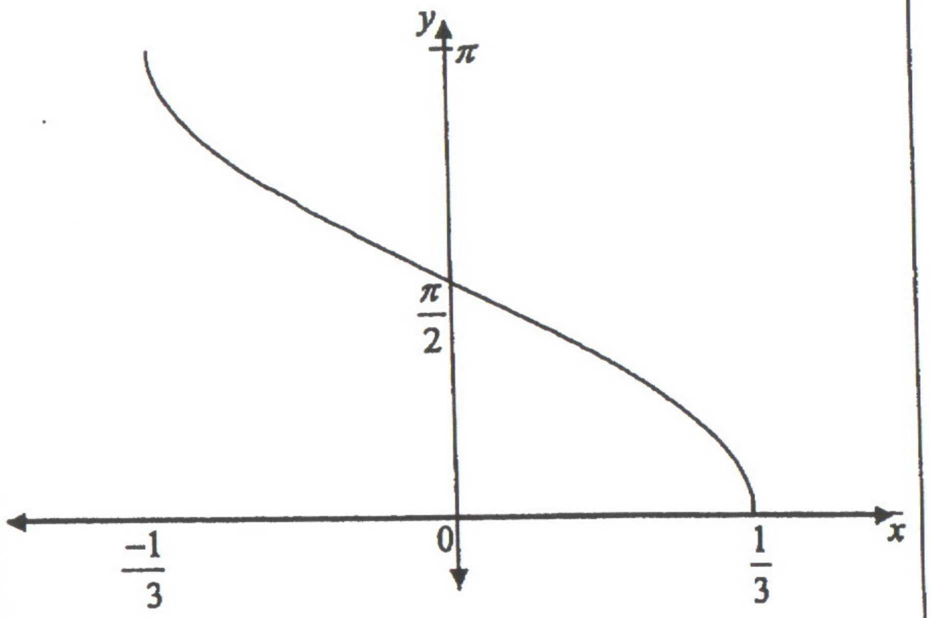
H3 manipulates algebraic expressions involving logarithmic and exponential functions

Part	Solutions	Marking Guidelines
(a) H3	$y=e^{kx},\ \dfrac{dy}{dx}=ke^{kx},\ \dfrac{d^2y}{dx^2}=k^2e^{kx}$ $\dfrac{d^2y}{dx^2}-4\dfrac{dy}{dx}+3y=0 \Rightarrow k^2e^{kx}-4ke^{kx}+3e^{kx}=0$ $\therefore e^{kx}\left(k^2-4k+3\right)=0$ $\therefore k^2-4k+3=0 \quad \left(\because e^{kx}\neq 0\right)$ $\therefore (k-1)(k-3)=0$ $\therefore k=1 \text{ or } 3$	**Award 2** Correct solution. **Award 1** Finds the first and second derivatives only.
(b) PE3	$\angle TAB=\angle ACB$ (Angle in Alternate Segment) $=y^0$ Let $\angle CAD=x^0=\angle BAD$ (AD bisects $\angle BAC$) $\therefore \angle TDA=(x+y)^0$ (Exterior Angle of ΔDCA) But $\angle TAD=\angle TAB+\angle BAD=(x+y)^0$ $\therefore TA=TD$ (Sides opposite equal angles)	**Award 3** Correct solution. **Award 2** Correct approach but with insufficient reasoning. **Award 1** Correct approach but with incorrect (or no) reasoning.
(c) (i) H3	LHS $=\dfrac{dT}{dt}=-k.Ae^{-kt}$ RHS $=-k(T-4)=-k\left(4+Ae^{-kt}-4\right)$ $=-k.Ae^{-kt}$ $=$ LHS $\therefore$ Solution to given differential equation.	**Award 1** Correct solution.
(ii) HE3	$t=0, T=30$ $\therefore 30=4+Ae^{-k.0}$ $\therefore A=26$ $t=20, T=15$ $15=4+26e^{-20k}$ $e^{-20k}=\dfrac{11}{26}$ $k=-\dfrac{1}{20}\ln\left(\dfrac{11}{26}\right)\approx -0.04301006326$	**Award 2** Correct solution. **Award 1** Finds only one of A or k or Attempts to use the correct method

(iii) HE3	$T =$ $8 = 4 + 26e^{\frac{1}{20}\ln\left(\frac{11}{26}\right)t}$ $e^{\frac{1}{20}\ln\left(\frac{11}{26}\right)t} = \frac{4}{26}$ $\frac{1}{20}\ln\left(\frac{11}{26}\right)t = \ln\left(\frac{4}{26}\right)$ $t = \frac{\ln\left(\frac{4}{26}\right)}{\frac{1}{20}\ln\left(\frac{11}{26}\right)} \approx 43.5200982$ minutes	**Award 2** Correct solution. **Award 1** Substantial progress towards solution.

Outcomes Addressed in this Question

HE4 Uses the relationship between trig functions, their inverse functions and their derivatives

Outcome	Solutions	Marking Guidelines
	(a) Domain of $y=\cos^{-1}x$ is $-1\le x\le 1$ $\therefore$ for $y=\cos^{-1}3x$, need $-1\le 3x\le 1$ $\therefore$ domain of $y=\cos^{-1}3x$ is $\frac{-1}{3}\le x\le\frac{1}{3}$ 	2 marks : correct domain and correct graph for given domain 1 mark : one of above
	(b) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $=\frac{\pi}{3}$ (note $\frac{-\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$)	2 marks : correct answer 1 mark : significant progress towards correct answer
	(c) Let $\alpha=\tan^{-1}\frac{1}{2}$ and $\beta=\tan^{-1}\frac{1}{4}$. $\therefore\tan\alpha=\frac{1}{2},\ \tan\beta=\frac{1}{4}$ As $\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$, $\tan(\alpha-\beta)=\frac{\frac{1}{2}-\frac{1}{4}}{1+\frac{1}{2}\times\frac{1}{4}}$ $\therefore\tan(\alpha-\beta)=\frac{2}{9}$ $\therefore\alpha-\beta=\tan^{-1}\left(\frac{2}{9}\right)$ $\therefore\tan^{-1}\left(\frac{1}{2}\right)-\tan^{-1}\left(\frac{1}{4}\right)=\tan^{-1}\left(\frac{2}{9}\right)$	3 marks : writes expansion for $\tan(\alpha-\beta)$ and proves result 2 marks : writes expansion for $\tan(\alpha-\beta)$ and makes significant progress towards proving result 1 mark : writes expansion for $\tan(\alpha-\beta)$

(d) $f(x)=\tan^{-1}\left(\frac{2}{x}\right)$

$$f'(x)=\frac{\frac{d}{dx}\left(2x^{-1}\right)}{1+\left(\frac{2}{x}\right)^2} \text{ since } \frac{d}{dx}\left(\tan^{-1}f(x)\right)=\frac{f'(x)}{1+\left(f(x)\right)^2}$$

$$=\frac{\frac{-2}{x^2}}{1+\frac{4}{x^2}}$$

$$\therefore f'(x)=\frac{-2}{x^2+4} \text{ and } \therefore f'(2)=\frac{-2}{2^2+4}=\frac{-1}{4}$$

When $x=2$, $=\tan^{-1}\left(\frac{2}{2}\right)=\tan^{-1}1=\frac{\pi}{4}$

Equation of tangent with $m=\frac{-1}{4}$, passing through

$\left(2,\frac{\pi}{4}\right)$ is $y-\frac{\pi}{4}=\frac{-1}{4}(x-2)$

$4y-\pi=-x+2$

Tangent is $x+4y-\pi-2=0$

3 marks : corr
derivative; co
finds gradient
and correctly
equation
2 marks : two
1 mark : one

Question No. 5 Solutions and Marking Guidelines

Outcomes Addressed in this Question

HE6: Determines integrals by reduction to a standard form through a given substitution

	Solutions	Marking Guidelines
5(a)	$\cos 2x = 1 - 2\sin^2 x$ $\therefore \sin^2 x = \frac{1}{2}(1-\cos 2x)$ $\int_0^{\frac{\pi}{4}} \sin^2 x\,dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} (1-\cos 2x)\,dx$ $= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$ $= \frac{1}{2}\left\{\left(\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{2}\right) - (0)\right\}$ $= \frac{1}{2}\left(\frac{\pi}{4} - \frac{1}{2}\right)$	**2 marks:** Fully correct solution **1 mark:** Correct integration, but incorrect answer.
(b)	$\int_0^{\frac{1}{2}} 2x\sqrt{1-2x}\,dx$ $= \int_1^0 (1-u)\sqrt{u}\,.\frac{du}{-2}$ $= -\frac{1}{2}\int_1^0 (1-u)\sqrt{u}\,du$ $= \frac{1}{2}\int_0^1 u^{\frac{1}{2}} - u^{\frac{3}{2}}\,du$ $= \frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}} - \frac{2}{5}u^{\frac{5}{2}}\right]_0^1$ $= \frac{1}{2}\left(\frac{2}{3} - \frac{2}{5}\right)$ $= \frac{2}{15}$ $u = 1-2x;\ 2x = 1-u$ $du = -2dx$ $dx = \frac{du}{-2}$ when $x = \frac{1}{2},\ u = 0$ when $x = 0,\ u = 1$	**3 marks:** Fully correct solution **2 marks:** Substantial progress towards correct result **1 mark:** Some progress towards correct result.
(c)	$\int_0^t \frac{1}{1+x^2}\,dx = \left[\tan^{-1} x\right]_0^t$ $= \tan^{-1} t - \tan^{-1} 0$ $= \tan^{-1} t$ *i.e.* $\tan^{-1} t = 0.9$ $\therefore t = 1.26$	**2 marks:** Fully correct **1 mark:** Correct integration, but incorrect answer.
(d)	$\int_0^{\frac{\pi}{4}} \tan x\,dx = \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x}\,dx$ $= [-\ln(\cos x)]_0^{\frac{\pi}{4}}$ $= \left(-\ln(\cos\frac{\pi}{4})\right) - (-\ln(\cos 0))$ $= -\ln\frac{1}{\sqrt{2}} + \ln 1$ $= -\ln 2^{-\frac{1}{2}}$ $= \frac{1}{2}\ln 2$	**3 marks:** Fully correct solution **2 marks:** Substantial progress towards correct result **1 mark:** Some progress towards correct result.

Question No.6 Solutions and Marking Guidelines

Outcomes Addressed in this Question

HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form
H8 uses techniques of integration to calculate areas and volumes

Outcome	Solutions	Marking Guidelines
HE7	a (i) $8\cos^4 x = 2\left(2\cos^2 x\right)^2$ $= 2\left(1+\cos 2x\right)^2$ $= 2\left(1+2\cos 2x+\cos^2 2x\right)$ but $\cos^2 2x = \frac{1}{2}(1+\cos 4x)$ $= 2\left(1+2\cos 2x+\frac{1}{2}(1+\cos 4x)\right)$ $= 2+4\cos 2x+1+\cos 4x$ $= 3+4\cos 2x+\cos 4x$	2 marks correct method leading to correct conclusion 1 mark substantially correct solution
H8	(ii) $V = \pi\int_0^{\frac{\pi}{4}} \cos^4 x\,dx$ $V = \pi\int_0^{\frac{\pi}{4}} \frac{1}{8}(3+4\cos 2x+\cos 4x)\,dx$ from part (i) $V = \frac{\pi}{8}\left[3x+2\sin 2x+\frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{4}}$ $V = \frac{\pi}{8}\left[3\left(\frac{\pi}{4}\right)+2\sin\left(\frac{\pi}{2}\right)+\frac{1}{4}\sin\pi\right]-0$ $V = \frac{\pi}{8}\left(\frac{3\pi^2}{4}+2\right)u^3$	2 marks correct method leading to correct conclusion 1 mark substantially correct solution
HE7	b $LHS = \frac{\sin A}{\cos A+\sin A}+\frac{\sin A}{\cos A-\sin A}$ $= \frac{\sin A(\cos A-\sin A)}{(\cos A+\sin A)}\times\frac{\sin A(\cos A+\sin A)}{(\cos A-\sin A)}$ $= \frac{\sin A\cos A-\sin^2 A+\sin A\cos A+\sin^2 A}{\cos^2 A-\sin^2 A}$ $= \frac{2\sin A\cos A}{\cos^2 A-\sin^2 A}$ $= \frac{\sin 2A}{\cos 2A}$ $= \tan 2A$ $= RHS$	2 marks correct method leading to correct conclusion 1 mark substantially correct solution
HE7	c $2\sin^2\theta-\sin 2\theta = 0$ $2\sin^2\theta-2\sin\theta\cos\theta = 0$ $2\sin\theta(\sin\theta-\cos\theta) = 0$ $\sin\theta = 0, \sin\theta = \cos\theta$ $\tan\theta = 1$ $\theta = n\pi, \theta = n\pi+\frac{\pi}{4}$	2 marks correct method leading to correct conclusion 1 mark substantially correct solution
HE7	d $\sin\theta+\cos\theta = 1$ $\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta\right) = 1$ $\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right) = 1$ $\sin\left(\theta+\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ $\theta+\frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}$ $\theta = 0, \frac{\pi}{2}, 2\pi$	2 marks correct method leading to correct conclusion 1 mark substantially correct solution

Question No.8 Solutions and Marking Guidelines

Outcomes Addressed in this Question

PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

PE6 makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations

HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form

Outcome	Solutions	Marking Guidelines
PE3/6	**a** (i) $\frac{7!}{3!2!} = 420$	1 mark correct answer
PE3 PE6	(ii) D's together $\frac{6!}{3!2!}$ D's separated $\frac{7!}{3!2!} - \frac{6!}{3!2!} = 420 - 120 = 300$	1 mark correct answer
PE3 PE6	**b** $x^2 + 1 \overline{)\, x^4 + 0x^3 - 3x^2 + 4x - 8}$ quotient $x^2 - 4$ $x^4 + x^2$ $-4x^2 + 4x - 8$ $-4x^2 - 4$ $4x - 4$ $R(x) = 4x - 4$	2 marks correct method leading to correct answer 1 mark substantially correct solution
PE3 PE6	**c** $f(x) = x^3 - x^2 - x - 1$ $f(1) = 1^3 - 1^2 - 1 - 1 < 0$ $f(2) = 2^3 - 2^2 - 2 - 2 > 0$ $\therefore$ root between 1 and 2. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ $f(2) = 1$ $f'(x) = 3x^2 - 2x - 1$ $f'(2) = 3(2)^2 - 2(2) - 1 = 7$ $x_2 = 2 - \frac{1}{7}$ $x_2 = 1\frac{6}{7}$	1 mark correct answer 1 mark correct answer
HE7	**iii** Newtons method finds where the tangent to the curve cuts the x axis its value should be closer to the root than the previous approximation. $f'(x) = 3x^2 - 2x - 1$ $f'(1) = 3(1)^2 - 2(1) - 1 = 0$ $x=1$ is a turning point therefore the tangent is parallel to the x axis and does not cut the axis.	2 marks correct reasoning and solution 1 mark substantially progress towards correct solution
PE3	**d** (i) $P(x)$ has $P(2^1) = 0 \therefore x = 2$ is a root $P(x)$ has $P(2^3) = 0 \therefore x = 3$ is a root $\therefore P(x) = a(x-2)(x-8)$ $P(0) = 32$ $a(0-2)(0-8) = 32$ $a = 2$ $\therefore P(x) = 2(x-2)(x-8)$ $P(x) = 2x^2 - 20x + 32$ (ii) $\therefore P(x) = a(x-2)^2(x+2)^2$ $P(3) = 50$ $a(3-2)^2(3+2)^2 = 50$ $a = 2$ $\therefore P(x) = 2(x-2)^2(x+2)^2$ $P(x) = 2x^4 - 16x^2 + 32$	1 mark correct answer 1 mark correct answer