Western Region

Trial Higher School Certificate Examination
1995

MATHEMATICS

3 UNIT (Additional)
and
3/4 UNIT (Common)

Time allowed - Two Hours
(Plus 5 minutes' reading time)

Directions to Candidates:

* Attempt ALL questions
* All questions are of equal value
* Show all necessary working. Marks may be deducted for careless or badly arranged work.
* Standard integrals are printed on the last page which may be removed for your convenience
* Board approved calculators may be used

Question 1.

(3 marks) (a) Solve the equation $\sin 2A = \cos A$ for $0 \leq A \leq 2\pi$.

(3 marks) (b) Solve $\frac{2}{x - 1} < 1$

(2 marks) (c) Evaluate $\int_0^1 \frac{1}{1 + x^2}\, dx$

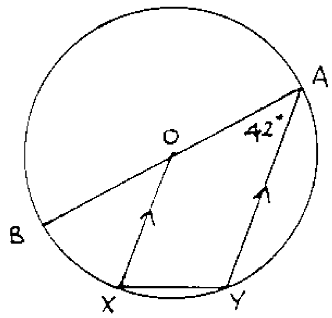
(4 marks) (d) Using the substitution $u = \log_e x$, evaluate $\int_1^e \frac{\log_e x}{x}\, dx$

Question 2.

(5 marks) (a) The area between the curve $y = \sin x$, the x - axis and the ordinates $x = 0$ and $x = {}^{3\pi}/_{4}$ is revolved about the x - axis.

Find the volume of the solid so formed, leaving your answer in exact form.

(3 marks) (b)



O is the centre of the circle. AY is parallel to OX. Angle OAY measures 42^0.

Find the measure of angle XYA, giving reasons for your answer.

(4 marks) (c) At a factory which produces cameras it was found that, on average, 10% of cameras produced have a fault. A batch of 20 cameras is tested.

(i) What is the probability that there are exactly 3 cameras in the batch containing faults? (Give your answer as a percentage correct to the nearest whole number).

(ii) Find the probability that at least one camera in the batch contains a fault.

Question 3.

(6 marks) (a) Consider the function $f(x) = 2\,sin^{-1}\,(x/_2)$.

(i) Find the value of $f(2)$.

(ii) Draw the graph of $y = f(x)$.

(iii) State the domain and range of this function.

(iv) Find the slope of the curve $y = f(x)$ at $x = 0$.

(6 marks) (b) Show that the motion of a particle whose associated velocity is described by $v^2 = 4 + 24x - 4x^2$ is simple harmonic.

For such a particle determine:

(i) the period of the motion.

(ii) the amplitude of the motion.

(iii) the maximum velocity of the particle.

Question 4.

(6 marks) (a) Axco Pty. Ltd makes a gross profit of $20 000 in 1995.
In each following year the gross profit will increase by 10% of that of the preceding year. Each year there is a tax of 50% on all the gross profit over $5 000. The remaining net profit is then distributed to the owners.

The profit, $P, distributed at the end of 1996 will be

$P = ((20 000 x 1.1) - 5 000) x 0.5 + 5 000

Write an expression for the profit distributed :

(i) at the end of 1997

(ii) *n* years after 1995.

(iii) Find the <u>total</u> profit which will be distributed to the owners until the end of the year 2010.

(6 marks) (b) A water tank is treated with a chemical for an outbreak of bacterial contamination. At a time t days after the treatment, the rate of decrease in the number of bacteria is given by

$$\frac{dQ}{dt} = k(1000 - Q)\text{, where Q is the number of bacteria.}$$

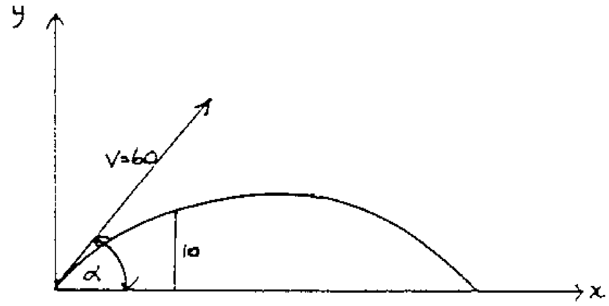
(i) Show that $Q = 1000 + Ae^{-kt}$, where A is a constant, satisfies this equation.

(ii) Initially, the bacteria count is 30 000 and 1 day after the treatment the count is 24 000.
Find the bacteria count after 10 days.

Question 5.

(4 marks) (a) Show that $x = 0.7$ is an approximate solution of the equation $\cos x = x$, and use one application of Newton's method to find a better approximation correct to two decimal places.

(8 marks) (b)



A golf ball is hit with an initial velocity of 60 ms^{-1}. After one second, on rising, it just clears a tree of height 10 metres.
Assuming that the ball follows the path of a projectile and that the acceleration due to gravity $g = 10\ ms^{-2}$;

(i) Let the angle of projection be α. By deriving expressions for the vertical ($\dot{y}$ and y) and horizontal ($\dot{x}$ and x) components of velocity and displacement show that

$$y = -5t^2 + (60 \sin\alpha)\, t$$

and

$$x = 60 (\cos\alpha)\, t$$

where t is the time in seconds.

(ii) Show that the angle of projection is given by $\sin \alpha = {}^1/_4$.

(iii) Find the maximum height reached.

(iv) Find the horizontal distance travelled by the ball.

Question 6.

(4 marks) (a) Prove by using mathematical induction that

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \cdots\cdots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

(3 marks) (b) Find the coefficient of the term in x^2 in the expansion of $(2x + {}^{1}/_{x^2})^{11}$.

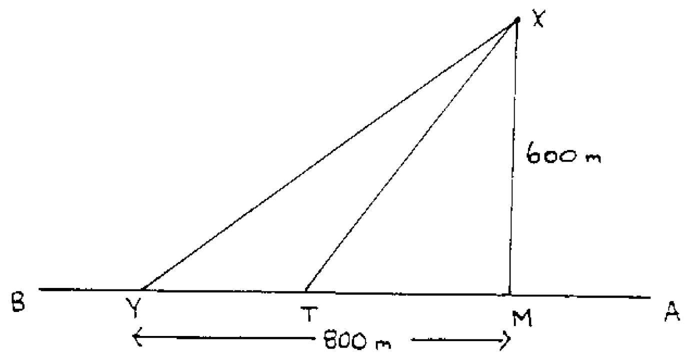
(5 marks) (c) (i) Prove that $\quad {}^{n}C_{k} = {}^{n}C_{n-k}$

(ii) By equating the coefficient of x^n on both sides of the identity

$(1+x)^n(1+x)^n = (1+x)^{2n}$, show that

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \cdots\cdots\cdots + \binom{n}{n-1}^2 + \binom{n}{n}^2 = \frac{2n!}{(n!)^2}$$

Question 7.



A walker is at a point X in a park and has to walk to point Y on a straight road AB. M is the point on the road nearest to X. The distances from X to M and from M to Y are as shown in the diagram. The walker can average 5 minutes per kilometre when walking along the road and 10 minutes per kilometre otherwise.

(3 marks) (a) How long will the walker take to walk

(i) in a straight line from X to Y?

(ii) from X to Y via M?

(3 marks) (b) The walker wishes to minimise the total time taken to walk from X to a point T on the road, between M and Y, and then along the road from T to Y.
If T is x metres from M, show that the time taken to walk from X to T and then T to Y is

$$\frac{2\sqrt{360\,000 + x^2} + 800 - x}{200} \quad \text{minutes}$$

(6 marks) (c) Hence find the minimum time for the walker to walk from X to Y.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$