Question 1

	Sample answer	Syllabus outcomes and marking guide
(a)	$\frac{d}{dx}(x^2\sin^{-1}x) = 2x\sin^{-1}x + \frac{x^2}{\sqrt{1-x^2}}$.	P7, PE5, HE4 • Gives the correct answer .... 2 • Demonstrates the correct use of the product rule 1
(b)	If $(x+3)$ is a factor of $P(x)$ then $P(-3)=0$. $P(x) = 2x^3 - 5kx + 9$ $P(-3) = 2(-3)^3 - 5k(-3) + 9$ $= -54 + 15k + 9 = 0$ $15k = 45$ $k = 3$.	PE3 • Gives the correct answer .... 2 • Demonstrates the correct use of the factor theorem. OR • Correctly uses long division........... 1
(c)	$x = \frac{(-3)(5)+(3)(-2)}{5-2}$, $y = \frac{5(5)+2(-2)}{5-2}$ $= \frac{-15-6}{3}$ $= \frac{25-4}{3}$ $x = -7$. $y = 7$. $\therefore$ point P is $(-7, 7)$.	P4, PE6 • Gives the correct answer .... 2 • Demonstrates a correct method of finding an external ratio 1
(d)	$x + y = 5 \Rightarrow y = -x + 5$ $m_1 = -1$ $2y = 3x + 5 \Rightarrow y = \frac{3}{2}x + \frac{5}{2}$ $m_2 = \frac{3}{2}$ $\tan\theta = \left\|\frac{m_1 - m_2}{1 + m_1 m_2}\right\|$ $= \left\|\frac{-1-\frac{3}{2}}{1-\frac{3}{2}}\right\|$ ✓ $= 5$. $\therefore \theta = 79°$ (nearest degree). ✓	H5, PE6 • One mark for substituting their values of m into the correct equation. • One mark for finding the correct answer to the nearest degree.
(e)	Number of arrangements $= \frac{11!}{2!2!}$ $= 9\,979\,200$.	PE3 • Gives the correct answer 1
(f)	$\int_0^4 \frac{dx}{\sqrt{x^2+9}} = \left[\ln(x + \sqrt{x^2+9})\right]_0^4$ ✓ $= [\ln(4 + \sqrt{4^2+9})] - [\ln(0 + \sqrt{9})]$ ✓ $= [\ln(4 + \sqrt{25})] - \ln 3$ $= \ln(4+5) - \ln 3$ $= \ln 9 - \ln 3$ ✓ $= \ln\frac{9}{3}$ $= \ln 3$	H5 • Gives the correct answer or correct numerical expression .... 3 • Gives the correct substitution of limits into the correct integral .... 2 • Gives the correct integral from table of standard integrals.................... 1

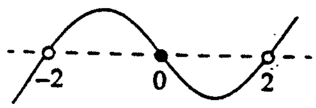
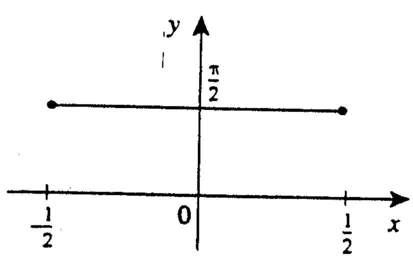
Question 2

Sample answer	Syllabus outcomes and marking guide
(a) $$\begin{array}{r|l} & 2x^2 \quad -7x \quad +15 \\ \hline x^2+2x-1 & 2x^4 \quad -3x^3 \quad -x^2 \quad +2x \quad +1 \\ & 2x^4 \quad +4x^3 \quad -2x^2 \\ \hline & \quad -7x^3 \quad +x^2 \quad +2x \\ & \quad -7x^3 \quad -14x^2 \quad +7x \\ \hline & \quad 15x^2 \quad -5x \quad +1 \\ & \quad 15x^2 \quad +30x \quad -15 \\ \hline & \quad -35x+16 \end{array}$$ $Q(x) = 2x^2 - 7x + 15$ and $R(x) = -35x + 16$.	PE3 • Gives the correct answer for $Q(x)$ and $R(x)$ .... 3 • Performs the correct long division. OR • Gives $R(x)$ and $Q(x)$ correctly from a long division containing a minor error....... 2 • Demonstrates a significant understanding of long division...................... 1
(b) $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \dfrac{1-\tan^2 A}{\sec^2 A}$ $= \dfrac{1}{\sec^2 A} - \dfrac{\tan^2 A}{\sec^2 A}$ $= \cos^2 A - \sin^2 A$ $= \cos 2A$.	H5 • Gives a correct proof .... 2 • Makes significant progress towards a correct proof............................ 1
(c) $\dfrac{d}{dx}\left(\dfrac{x+2}{\sqrt{x-1}}\right) = 0, x \neq 1$ $\dfrac{(\sqrt{x-1})(1) - (x+2) \times \frac{1}{2}(x-1)^{-\frac{1}{2}}}{x-1} = 0$ $\sqrt{x-1} - \dfrac{x+2}{2\sqrt{x-1}} = 0$ $2(x-1) - (x+2) = 0$ $x - 2 - 2 = 0$ $x = 4$.	H5 • Gives the correct answer .... 3 • Attempts to solve the correct equation. OR • Correctly solves their equation arising from a minor error in differentiation .... 2 • Gives the correct derivative 1

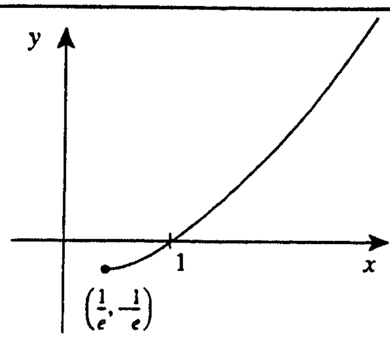
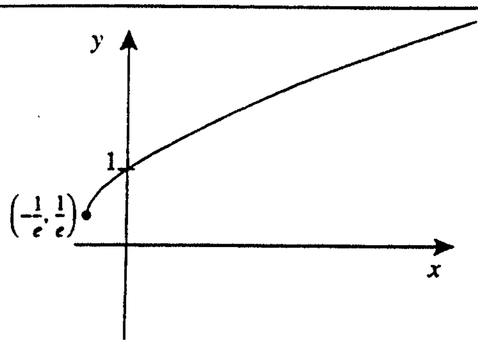
Question 2 (Continued)

Sample answer	Syllabus outcomes and marking guide
(d) $\displaystyle\int_2^5 \frac{x+1}{\sqrt{x-1}}\,dx$ $u = x - 1$ $x + 1 = u + 2$ $du = dx$ $x = 2,\ u = 1$ $x = 5,\ u = 4$ $\displaystyle = \int_1^4 \frac{u+2}{u^{\frac{1}{2}}}\,du$ $\displaystyle = \int_1^4 \left(u^{\frac{1}{2}} + 2u^{-\frac{1}{2}}\right)du$ $\displaystyle = \left[\frac{2}{3}u^{\frac{3}{2}} + 4u^{\frac{1}{2}}\right]_1^4$ $\displaystyle = \left[\frac{2}{3}(4)^{\frac{3}{2}} + 4(4)^{\frac{1}{2}}\right] - \left[\frac{2}{3}(1)^{\frac{3}{2}} + 4(1)^{\frac{1}{2}}\right]$ $\displaystyle = \frac{16}{3} + 8 - 4\frac{2}{3}$ $\displaystyle = 8\frac{2}{3}.$	HE6 • Gives the correct answer .... 4 • Gives the correct integral and correct substitution of correct limits of integration .... 3 • Gives the correct expression to be integrated. OR • Gives the correct integration with incorrect limits .... 2 • Gives the correct expressions and values to be substituted....................... 1

Question 3

Sample answer	Syllabus outcomes and marking guide
(a) $\dfrac{x}{x^2-4} \le 0$ $x^2-4 \ne 0$ $(x-2)(x+2) \ne 0$ $x \ne 2, x \ne -2.$ $\therefore \dfrac{(x^2-4)^2 x}{x^2-4} \le 0$ $(x^2-4)x \le 0$ $x(x-2)(x+2) \le 0$  $x < -2$ or $0 \le x < 2.$	PE3 • Gives the correct answer .... 3 • Gives a correct expression for the cubic, i.e. $x(x-2)(x+2) \le 0.$ OR • Makes significant progress in solving $\dfrac{x}{x^2-4} \le 0$ by considering when $x^2-4>0$ and $x^2-4<0$ .... 2 • Multiplies by $(x^2-4)^2$. OR • Considers x^2-4 as positive and negative 1
(b) (i) $f(x) = \cos^{-1}2x + \sin^{-1}2x$ $f'(x) = \dfrac{-2}{\sqrt{1-(2x)^2}} + \dfrac{2}{\sqrt{1-(2x)^2}}$ $= 0.$	HE4 • Gives a correct derivative of $\cos^{-1}2x$ and $\sin^{-1}2x$ 1
(ii) Since $f'(x) = 0$, $\therefore$ $y = f(x)$ is a horizontal straight line. $f(0) = \cos^{-1}(0) + \sin^{-1}(0)$ $= \dfrac{\pi}{2}.$ Domain $-1 \le 2x \le 1.$ $\therefore y = \dfrac{\pi}{2},\ -\dfrac{1}{2} \le x \le \dfrac{1}{2}.$ 	HE4 • Draws a horizontal line at $y = \dfrac{\pi}{2}$ for $-\dfrac{1}{2} \le x \le \dfrac{1}{2}$ .... 2 • Draws a horizontal line at $y = \dfrac{\pi}{2}$, incorrect domain. OR • Draws a horizontal line with correct domain 1

Question 3 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) (i) $f(x) = x\ln x$ $f'(x) = x\left(\frac{1}{x}\right) + 1\ln x$ $= 1 + \ln x.$ To find the stationary point, let $f'x = 0$. $\ln x + 1 = 0$ $\ln x = -1$ $x = e^{-1}$ $= \frac{1}{e}.$ $y = e^{-1}\ln e^{-1}$ $= e^{-1}(-1)$ $y = -\frac{1}{e}$ $\therefore$ the stationary point is $\left(\frac{1}{e}, -\frac{1}{e}\right)$. $f''(x) = \frac{1}{x} > 0$ for $x = \frac{1}{e}$. $\therefore \left(\frac{1}{e}, -\frac{1}{e}\right)$ is a minimum turning point.	**H3, H6** • One mark for showing the stationary point is $\left(\frac{1}{e}, -\frac{1}{e}\right)$. • One mark for showing the stationary point is a minimum turning point.
(ii) 	**H6** • Gives the correct sketch 1
(iii) 	**HE3** • Gives the correct sketch 1
(d) If the roots are $\alpha - d$, α and $\alpha + d$, then (i) $(\alpha - d) + \alpha + (\alpha + d) = -\frac{b}{a}$ $3\alpha = -\frac{b}{a}$ $3\alpha = -\frac{12}{2}$ $\therefore \alpha = -2.$	**PE3** • Gives the correct answer 1

Question 3 (Continued)

Sample answer	Syllabus outcomes and marking guide
(ii) $(\alpha - d)(\alpha)(\alpha + d) = -\dfrac{d}{a}$ $\alpha(\alpha^2 - d^2) = \dfrac{20}{2}$ $-2(4 - d^2) = 10$ $4 - d^2 = -5$ $d^2 = 9$ $d = 3$ or $d = -3$.	PE3 • Gives a correct answer of either 3 or –3 1

Question 4

Sample answer | **Syllabus outcomes and marking guide**

(a) $\left(5x^2-\dfrac{1}{2x}\right)^{12}$

$$T_{r+1}={}^{12}C_r\times(5x^2)^{12-r}\times(-1)^r\times\left(\frac{1}{2}\right)^r\times x^{-r}$$

$$=(-1)^r\times{}^{12}C_r\times\frac{5^{12-r}}{2^r}\times x^{24-2r}\times x^{-r}.$$

For x^9, $24-3r=9$

$3r=15$

$r=5.$

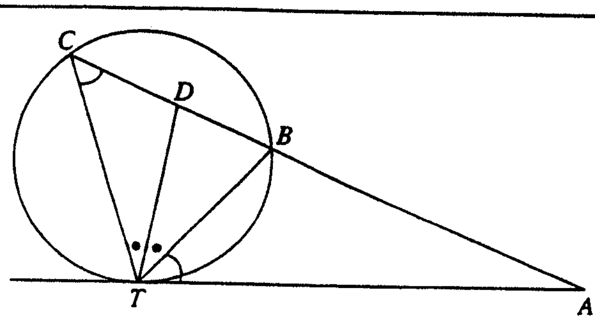
$\therefore$ coefficient of x^9 is

$$(-1)^5\times{}^{12}C_5\times\frac{5^7}{2^5}=-\frac{792\times78\,125}{32}$$

$$=-1\,933\,593\tfrac{3}{4}.$$

HE7
- Gives the correct answer in any form 3
- Gives the correct value for r 2
- Gives the correct expression for T_{r+1} ...1

(b)



$\angle BTD=\angle DTC$ (given).

$\angle TCD=\angle BTA$ (alternate segment theorem).

$\angle TDA=\angle DTC+\angle TCD$ (exterior angle of ΔDCT= sum of interior opposite angles).

$\therefore\ \angle TDA=\angle DTA.$

$\therefore\ \Delta ATD$ is isosceles.

$\therefore\ AT=AD.$

HE2
- Demonstrates a correct proof with reasons............................3
- Demonstrates significant progress towards a correct proof which includes reasons.
  OR
- Demonstrates a 'correct' proof without adequate reasons 2
- Demonstrates partial progress towards a correct proof 1

(c) (i) $v=2+Ae^{-kt}$

$$\frac{dv}{dt}=-k(Ae^{-kt})$$

$$=-k(v-2)$$

$$=k(2-v).$$

HE3
- Shows the correct working 1

(ii) When $t=0$, $v=50$

$v=2+Ae^{-kt}$

$50=2+Ae^0.$

$\therefore A=48.$

H3, HE3
- Gives the correct answer..............1

Question 4 (Continued)

Sample answer	Syllabus outcomes and marking guide
(iii) When $t = 1$, $v = 35$ $v = 2 + 48e^{-kt}$ $35 = 2 + 48e^{-k}$ $e^{-k} = \frac{33}{48}$ $-k = \ln\frac{33}{48}$ $k = 0.374693\ ...$ $\therefore\ k = 0.3747$ (4 decimal places).	H3, HE3 • Gives the correct answer . . . . 2 • Gives a correct value for e^{-k}, e.g. $\frac{33}{48}$ . . . 1
(iv) If t is large, e^{-kt} becomes very small and approaches 0. $\therefore\ v = 2 + 0$. $\therefore$ terminal speed = 2 m sec^{-1}.	H3, HE3 • Gives the correct answer 1
(v) 5% more than terminal speed is 2.1 m sec^{-1}. $\therefore\ 2.1 = 2 + 48e^{-0.3747t}$ $-0.3747t = \ln\frac{1}{480}$ $t = 16.4766\ ...$ $\therefore$ time taken is 16.48 seconds (2 decimal places).	PE6, H3, HE3 • Gives a correct answer 1

Question 5

Sample answer	Syllabus outcomes and marking guide
(a) (i) $R\cos(2t+\alpha) = R\cos 2t\cos\alpha - R\sin 2t\sin\alpha$ $= \sqrt{3}\cos 2t - \sin 2t.$ $\therefore R\cos\alpha = \sqrt{3},\ R\sin\alpha = 1.$ $\tan\alpha = \frac{1}{\sqrt{3}}.$ $\therefore \alpha = \frac{\pi}{6} \quad \left(0 < \alpha < \frac{\pi}{2}\right).$ $\sin\alpha = \frac{1}{2}.$ $\therefore R = 2.$ $\therefore \sqrt{3}\cos 2t - \sin 2t = 2\cos\left(2t + \frac{\pi}{6}\right).$	PE6 • Gives the correct expression .... 2 • Gives the correct value for either R or α, and makes substantial progress towards finding the other value. 1
(ii) $\sqrt{3}\cos 2t - \sin 2t = 0$ $2\cos\left(2t + \frac{\pi}{6}\right) = 0$ $\cos\left(2t + \frac{\pi}{6}\right) = 0$ $2t + \frac{\pi}{6} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$ $t = \left(-\frac{\pi}{12} + \frac{\pi}{4}\right), \left(-\frac{\pi}{12} + \frac{3\pi}{4}\right), \left(-\frac{\pi}{12} + \frac{5\pi}{4}\right) \ldots$ $= \frac{\pi}{6}, \frac{4\pi}{6}, \frac{7\pi}{6} \ldots$ $t = \frac{3n+1}{6}\pi$, where $n = 0, 1, 2, 3 \ldots$	PE2, PE6 • States a correct expression which generates all the positive values of t .... 2 • Gives at least 2 correct positive values for t. OR • Makes substantial progress towards finding the correct multiple values of t 1
(b) (i) $x = 5 + \sqrt{3}\cos 2t - \sin 2t$ $\dot{x} = -2\sqrt{3}\sin 2t - 2\cos 2t$ $\ddot{x} = -4\sqrt{3}\cos 2t + 4\sin 2t$ $= -4(\sqrt{3}\cos 2t - \sin 2t)$ $-4(x-5) = -4(5 + \sqrt{3}\cos 2t - \sin 2t - 5)$ $= -4(\sqrt{3}\cos 2t - \sin 2t)$ $= \ddot{x}.$ $\therefore$ acceleration $= -4(x-5).$	HE3 • One mark for using differentiation to obtain the correct expression for $\ddot{x}$. • One mark for using substitution to prove the required result. 2
(ii) $\ddot{x} = -4(x-5)$ $x = 5 + \sqrt{3}\cos 2t - \sin 2t$ $= 5 + 2\cos\left(2t + \frac{\pi}{6}\right)$ from part (a). $\therefore$ the motion is simple harmonic, about the position $x = 5$, with amplitude $= 2$. End-points: $5 + 2 = 7,\ 5 - 2 = 3.$	HE3 • States the correct end-points of the motion 1

Question 5 (Continued)

Sample answer | **Syllabus outcomes and marking guide**

(iii) Let $x = 5$

$$5 + \sqrt{3}\cos 2t - \sin 2t = 5$$

$$\sqrt{3}\cos 2t - \sin 2t = 0$$

$$2\cos\left(2t + \frac{\pi}{6}\right) = 0 \quad \text{from (a) (i).}$$

$$\therefore t = \frac{\pi}{6} \quad \text{from (a) (ii).}$$

The particle first passes through $x = 5$ at time $t = \frac{\pi}{6}$ seconds.

HE3

- Gives the correct answer 1

(c) Consider the statement S_n:

$$1^2 + 3^2 + \ldots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$$

where $n = 1, 2, 3 \ldots$

When $n = 1$,

LHS of $S_1 = 1^2 = 1$,

RHS of $S_1 = \frac{1}{3} \times 1 \times 1 \times 3 = 1$.

$\therefore S_1$ is true.

Assume S_n is true for some positive integer, k.

LHS of S_{k+1}

$$= 1^2 + 3^2 + 5^2 + \ldots + (2k-1)^2 + (2k+1)^2$$

$$= \frac{1}{3}k(2k-1)(2k+1) + (2k+1)^2 \quad \text{(by assumption)}$$

$$= \frac{1}{3}(2k+1)[k(2k-1) + 3(2k+1)]$$

$$= \frac{1}{3}(2k+1)(2k^2 + 5k + 3)$$

$$= \frac{1}{3}(2k+1)(2k+3)(k+1)$$

$$= \frac{1}{3}(k+1)(2k+1)(2k+3)$$

= RHS of S_{k+1}.

Hence, if S_n is true for a particular positive integer, k, it is also true for $k + 1$. But S_n is true for $n = 1$. Therefore, S_n is true for all positive integers, n.

HE2

- One mark for proving the result is true for $n = 1$.
- One mark for correctly substituting $\frac{1}{2}k(2k-1)(2k+1)$ into the correct form of the result where $n = k + 1$.
- One mark for using correct algebraic manipulation to obtain $\frac{1}{3}(k+1)(2k+1)(2k+3)$.
- One mark for giving a correct conclusion statement about proof by induction.

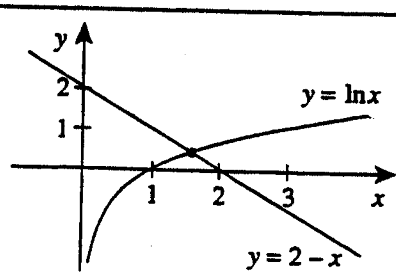
Question 6

		Sample answer	Syllabus outcomes and marking guide
(a)	(i)	$\ddot{x} = \sqrt{3x+4}$, but $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$. $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = (3x+4)^{\frac{1}{2}}$ $\frac{1}{2}v^2 = \frac{2}{3}(3x+4)^{\frac{3}{2}} \times \frac{1}{3} + c.$ $v^2 = \frac{4}{9}(3x+4)^{\frac{3}{2}} + c.$	HE5 • Correctly applies the formula $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ to obtain the requested result. 1
	(ii)	At $x = 0$, $v = 0$ (given) $0 = \frac{4}{9}(3 \times 0 + 4)^{\frac{3}{2}} + c.$ $\therefore c = -\frac{32}{9}.$	HE7 • Gives the correct answer 1
	(iii)	At $x = 0$, $\dot{x} = v = 0$ $\ddot{x} = \sqrt{3 \times 0 + 4}$ $= 2.$ Also $\ddot{x} = \sqrt{3x+4}$ > 0 (for all $x > 0$). The particle starts from rest at 0 with an acceleration of 2 m s^{-2} in a positive direction. The acceleration remains always positive. Hence the motion is always in a positive direction.	HE7 • Gives a correct explanation based on the acceleration of the particle 1
(b)	(i)	Probability of both long hair and grey eyes $= 0.2 \times 0.45$ $= 0.09.$	H5 • Gives the correct answer 1
	(ii)	P(3 with long hair and grey eyes) $= {}^{10}C_3 \times (0.09)^3 \times (0.91)^7$ $= 0.0452 \ldots$ $= 0.045$ (correct to 3 decimal places).	HE3 • Gives the correct answer based on their answer to (i). . . . . 2 • Correctly substitutes their answer to (i) into the binomial probability result 1

Question 6 (Continued)

Sample answer | **Syllabus outcomes and marking guide**

(c) (i)



From the graphs, the curves intersect close to $x = 1.5$.

OR

$$f(x) = \ln x - (2 - x).$$

Consider $f(1.5) = -0.09... < 0.$

$$f(1.6) = +0.07... > 0.$$

$\therefore$ curves intersect near $x = 1.5$.

HE7
- Correctly uses graphs or some other method to show x is close to 1.5 1

(ii) Let

$$f(x) = \ln x - (2 - x)$$
$$f(x) = \ln x - 2 + x$$
$$f'(x) = \frac{1}{x} + 1$$
$$f(1.5) = \ln 1.5 - 0.5$$
$$f'(1.5) = 1\tfrac{2}{3}$$
$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \quad \text{(Newton's method)}$$
$$= 1.5 - \frac{\ln 1.5 - 0.5}{1\tfrac{2}{3}}$$
$$= 1.5567...$$
$$= 1.56 \quad \text{(correct to 2 decimal places).}$$

HE7
- Give the correct answer................ 2

- States correct expressions for $f(x)$, $f'(x)$ and correctly evaluates $f(1.5)$, $f'(1.5)$.

  OR
- Correctly substitutes their values for $f(1.5)$, $f'(1.5)$ into Newton's method formula 1

(d) (i)

$$V = \pi\int_a^b y^2\,dx$$
$$= \pi\int_0^{\frac{\pi}{4}} (1 + \sqrt{2}\cos x)^2\,dx$$
$$= \pi\int_0^{\frac{\pi}{4}} (1 + 2\sqrt{2}\cos x + 2\cos^2 x)\,dx.$$

Note: $2\cos^2 x = 1 + \cos 2x$

$$= \pi\int_0^{\frac{\pi}{4}} (1 + 2\sqrt{2}\cos x + 1 + \cos 2x)\,dx$$
$$= \pi\int_0^{\frac{\pi}{4}} (2 + 2\sqrt{2}\cos x + \cos 2x)\,dx.$$

HE6
- One mark for showing the correct integral expression for the volume.............. 1

Question 6 (Continued)

Sample answer	Syllabus outcomes and marking guide
(ii) $= \pi\left[2x + 2\sqrt{2}\sin x + \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$ $= \pi\left[\frac{\pi}{2} + 2\sqrt{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times 1 - 0\right]$ $= \frac{\pi}{2}(\pi + 5).$ $\therefore$ volume of a solid $= \frac{\pi}{2}(\pi + 5)$ unit^3.	HE6 • One mark for writing a correct primitive expression. • One mark for substituting the limits of integration into their primitive to obtain a correct value.

Question 7

Sample answer	Syllabus outcomes and marking guide
(a) $f'(c) = \lim_{x \to c} \dfrac{f(x)-f(c)}{x-c}$ $= \lim_{x \to c} \dfrac{x^3-c^3}{x-c}$ $= \lim_{x \to c} \dfrac{(x-c)(x^2+xc+c^2)}{x-c}$ $= \lim_{x \to c} (x^2+xc+c^2)$ $f'(c) = 3c^2.$ $\therefore f'(a) = 3a^2.$	P6 • Applies the correct working to find $f'(a)$ .... 2 • Partially applies the correct working to find $f'(a)$ 1
(b) (i) For every group of 6 at the large table, there is a corresponding group of 4 at the small table. $\therefore$ number of arrangements $= {}^{10}C_6 \times 5! \times 3!$ $= 151\ 200.$	PE3 • Gives the correct answer .... 2 • Gives the correct number of ways in which people can be grouped at the tables (${}^{10}C_6$ or ${}^{10}C_4$). OR • Using their result for the number of groupings, finds the correct number of arrangements for sitting at the circular tables 1

Question 7 (Continued)

Sample answer	Syllabus outcomes and marking guide
(iii) Note 1: $({}^{2n}C_0)^2 = ({}^{2n}C_n)^2$, $({}^{2n}C_1)^2 = ({}^{2n}C_{2n-1})^2$, etc. Note 2: The left side of the identity in (ii) has an odd number of terms. The middle term occurs where $r = n$ (e.g. 0, 1, 2, 3, 4). $\therefore ({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 + \ldots + (-1)^n({}^{2n}C_n)^2 + \ldots + ({}^{2n}C_{2n-2})^2 - ({}^{2n}C_{2n-1})^2 + ({}^{2n}C_{2n})^2$ $= \left(2({}^{2n}C_0)^2 - 2({}^{2n}C_1)^2 + 2({}^{2n}C_2)^2 - \ldots + 2(-1)^n({}^{2n}C_n)^2\right) - (-1)^n({}^{2n}C_n)^2$ $= (-1)^n({}^{2n}C_n)$ from part (ii). $\therefore \left(2\sum_{r=0}^{n}(-1)^r({}^{2n}C_r)^2\right) - (-1)^n({}^{2n}C_n)^2$ $= (-1)^n({}^{2n}C_n)$. On rearranging, $\sum_{r=0}^{n}(-1)^r({}^{2n}C_r)^2 = \frac{1}{2}\times(-1)^n({}^{2n}C_n) + \frac{1}{2}\times(-1)^n({}^{2n}C_n)^2$ $= \frac{1}{2}\times(-1)^n\times{}^{2n}C_n(1+{}^{2n}C_n)$ $= \frac{1}{2}(-1)^n\,{}^{2n}C_n(1+{}^{2n}C_n)$.	HE7 • One mark for showing that the terms on the left of the identity in (ii) can be written as twice the sum of the first n terms, minus the nth term. • One mark for rearranging the identity to obtain the given result.