Student Number:..... Teacher's Name:.....

North Sydney Boys High School

YEAR 12 Trial Higher School Certificate Examination

2002

Mathematics Extension 1

Time allowed – 2 hours (plus 5 minutes reading time)

General Instructions

- Attempt all questions on the writing paper supplied
- Write on one side of the paper only
- Start each question on a new page
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is supplied
- All necessary working should be shown in every question

Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Total marks – 84

- Attempt Questions 1–7
- All questions are of equal value

Marks

QUESTION 1

(a) Differentiate: (i) $\sin^2 x$ **2**

(ii) $\sin^{-1}(2x)$ **2**

(b) Find the coordinates of the point P which divides the interval AB internally in the ratio 2 : 3 where A and B have coordinates (1, –3) and (6, 7) respectively. **2**

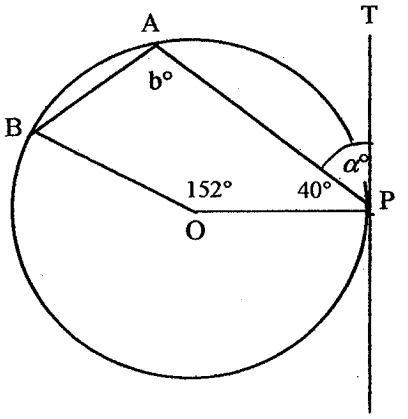
(c) Solve the inequality $\dfrac{2x+3}{x-4} > 1$ **2**

(d) $\int x\sqrt{x+1}\,dx$, using the substitution $u = 1 + x$ **3**

(e) Find $\lim\limits_{x\to 0} \dfrac{2\sin\frac{x}{2}}{x}$ **1**

QUESTION 2

(a) PT is a tangent to the circle centre 0. Find the sizes of the angles marked a and b giving reasons for your answers. **4**



(b) (i) Write down the expansion of $\tan(\alpha + \beta)$. **3**

(ii) Hence find the exact value of $\tan 75°$.

(c) Consider the function $f(x) = 3\cos^{-1}\left(\dfrac{x}{2}\right)$

(i) Evaluate $f(2)$. **1**

(ii) State the domain and range of $y = f(x)$. **2**

(iii) Draw the graph of $y = f(x)$. **2**

Marks

QUESTION 3

(a) Write $9 + 16 + 25 + \ldots\ldots + n^2$ using $\sum$ notation **1**

(b) Solve $\sin 2x = \cos x$ for $0 \leq x \leq 2\pi$ **4**

(c) Find the indefinite integrals: **3**

(i) $\int \frac{dx}{x^2 + 4}$

(ii) $\int \sin^2 2x \, dx$

(d) Evaluate $\int_0^{\ln 3} \frac{e^x dx}{\sqrt{1+e^x}}$ using the substitution $u = e^x$. **4**

QUESTION 4

(a) The polynomial $P(x) = x^3 + ax^2 - 3ax$ has a factor $(x + 2)$. **2**
Find the value of a.

(b) Express $\sqrt{3}\cos\theta + \sin\theta$ in the form $A\cos(\theta - \alpha)$. **4**
Hence solve the equation $\sqrt{3}\cos\theta + \sin\theta = 1$ for $-\pi \leq \theta \leq \pi$.

(c) Differentiate $x \tan^{-1}x$ and hence evaluate $\int_0^1 \tan^{-1}x \, dx$. **4**

(d) Sketch $y = \sin(\cos^{-1}x)$ showing clearly the domain and range. **2**

QUESTION 5 **Marks**

(a) (i) Draw the graph of $y = e^{-x}$. By drawing another graph on the same set of axes, show that $f(x) = e^{-x} - x + 1$ has exactly one root. **2**

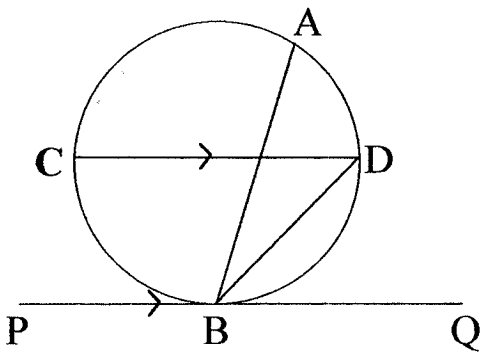
(ii) Let $x = 1$ be a first approximation to the root. Apply Newton's method once to obtain another approximation. Answer to 3 significant figures. **3**

(b) Prove that $\dfrac{cosec\beta - cot\beta}{cosec\beta + cot\beta} = tan^2\dfrac{\beta}{2}$. Hint: Let $tan\dfrac{\beta}{2} = t$ **3**

(c) AB and CD are two intersecting chords of a circle and CD is parallel to the tangent to the circle at B. **4**

Copy the diagram in your booklet.

Prove that AB bisects $\angle CAD$.



QUESTION 6

(a) $P(4p,2p^2)$ is any point on the parabola $x^2 = 8y$. The tangent to the parabola meets the x-axis at M, and the y-axis at N.

(i) Show that the tangent at P is given by $y = px - 2p^2$. **2**

(ii) Find the co-ordinates of M and N. **2**

(iii) Find the equation of the locus of the midpoint of MN as P varies. **2**

(b) A spherical balloon leaks air such that the radius decreases at the rate of 5 mm/sec. Calculate the rate of change of the volume of the balloon when the radius is 100mm. **3**

(c) Evaluate $\sin^{-1}\left(\dfrac{1}{\sqrt{5}}\right) + \sin^{-1}\left(\dfrac{1}{\sqrt{10}}\right)$. **3**

Marks

QUESTION 7

(a) (i) Given that $S_n = 1\times2 + 2\times3 + 3\times4 + ... + n(n+1)$, show by mathematical induction,

$S_n = \frac{n}{3}(n+1)(n+2)$, for all positive integers n. **4**

(ii) Evaluate $\lim_{n\to\infty} \frac{1\times2 + 2\times3 + 3\times4 + ... + n(n+1)}{n^3}$ **1**

(b) $P(x) = x^3 - 6x^2 + ax - 4$ where $a > 0$. Given that all the roots of $P(x) = 0$ are real and positive, and that one of the roots is the product of the other 2 roots, find the value of a. **3**

(c) Given $f(x) = 2\cos^{-1}\left(\frac{x}{\sqrt{2}}\right) - \sin^{-1}\left(1 - x^2\right)$. Show that $f'(x) = 0$. **4**

Hence sketch $f(x)$.

End of paper