2011 Year 11 - Ext I - Task One.

Question 1 (14 Marks)

a) $y = \sqrt{x-5}$

D: $x - 5 \geq 0$,
$x \geq 5$,

R: $y \geq 0$. ,

/2.

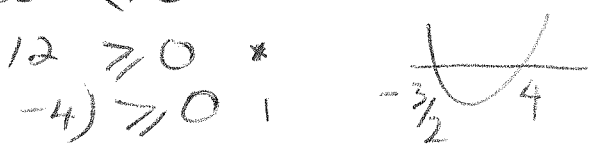
b) $\dfrac{x^2+2x-15}{(x+5)^2} \div \dfrac{x^2-9}{x^3+27}$,

$= \dfrac{(x+5)(x-3)}{(x+5)^2} \times \dfrac{(x+3)(x^2-3x+9)}{(x-3)(x+3)}$, factorising , cancel operation

$= \dfrac{x^2-3x+9}{x+5}$,

/3

c) $12 + 5x - 2x^2 \leq 0$
$2x^2 - 5x - 12 \geq 0$ *
$(2x+3)(x-4) \geq 0$,



,

$x \leq -1\frac{1}{2}$, $x \geq 4$,

/3

d) $x - \dfrac{1}{x} = \dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} \times \dfrac{(\sqrt{7}+\sqrt{5})}{(\sqrt{7}+\sqrt{5})} - \dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} \times \dfrac{(\sqrt{7}-\sqrt{5})}{(\sqrt{7}-\sqrt{5})}$,

rationalise denominators $= \dfrac{7+2\sqrt{35}+5}{7-5} - \dfrac{7-2\sqrt{35}+5}{7-5}$,

$= \dfrac{4\sqrt{35}}{2}$.

$= 2\sqrt{35}$,

/3

Q1 e) $\frac{4x-3}{2x+1} \geq 3(2x+1)^2$ $\quad x \neq -\frac{1}{2}$ 1

$(4x-3)(2x+1) \geq 3(2x+1)^2$ 1

$3(2x+1)^2 - (4x-3)(2x+1) \leq 0$

$(2x+1)[6x+3-(4x-3)] \leq 0$

$(2x+1)(2x+6) \leq 0$

$-3 \leq x < -\frac{1}{2}$ 1 /3

Question 2: (14 Marks)

a) $y^4 - 50y^2 + 49$

$= (y^2-49)(y^2-1)$ 1

$= (y-7)(y+7)(y-1)(y+1)$ 1 /2

b) $\frac{a+1}{a^2-a} - \frac{a+1}{a^2-1}$

$= \frac{a+1}{a(a-1)} - \frac{a+1}{(a-1)(a+1)}$ 1

$= \frac{(a+1)^2 - (a+1)a}{a(a-1)(a+1)}$ 1

$= \frac{a^2+2a+1-a^2-a}{a(a-1)(a+1)}$

$= \frac{a+1}{a(a-1)(a+1)}$

$= \frac{1}{a(a-1)}$ 1 /3

Q2(c) $2 \leq |2x+3| \leq 11$

Solve

$2 \leq 2x+3$ + $-2 \geq 2x+3$

$-1 \leq 2x$ $\quad -5 \geq 2x$

$x \geq -\frac{1}{2}$ $\quad x \leq -2\frac{1}{2}$ .1

$2x+3 \leq 11$ $\quad 2x+3 \geq -11$

$2x \leq 8$ $\quad 2x \geq -14$

$x \leq 4$ $\quad x \geq -7$ .1

3

Solution $-7 \leq x \leq -2\frac{1}{2}$

$-\frac{1}{2} \leq x \leq 4$ } .1

Award 2 for 1 soln.

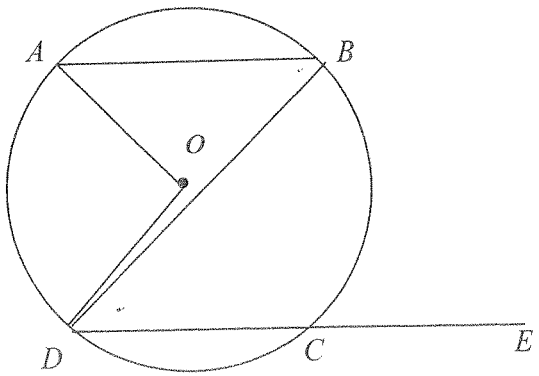
d) $\angle ABD = 65°$ ($\angle$ in centre is double $\angle$ at circumf.) .1

$\angle ABD = \angle BDC$

$= 65°$ (Alternate $\angle$'s) .1

$\therefore \angle BCE = 85°$ (Ext. $\angle$ = sum of opp. interior $\angle$'s)

.1



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Question 3 : (15 Marks)

a) $$\frac{4^{3-x} \times 12^{2x-1}}{8^{x} \times 15^{-2x}} = \frac{2^{6-2x} \times (3 \times 2^{2})^{2x-1}}{2^{3x} \times (3 \times 5)^{-2x}}$$

$$= \frac{2^{6-5x} \times 3^{2x-1} \times 2^{4x-2}}{3^{-2x} \times 5^{-2x}}$$

$$= 2^{4-x} \times 3^{4x-1} \times 5^{2x}$$

/3

b) $$2x - y - z = 11 \quad \text{---} (1)$$
$$x + 3y + z = -2 \quad \text{---} (2)$$
$$3x - 2y + z = 23 \quad \text{---} (3)$$

$(1) + (2)$ $\quad 3x + 2y = 9 \quad \text{---} (4)$

$(1) + (3)$ $\quad 5x - 3y = 34 \quad \text{---} (5)$

$(4) \times 3$ $\quad 9x + 6y = 27 \quad \text{---} (6)$

$(5) \times 2$ $\quad 10x - 6y = 68 \quad \text{---} (7)$

$(6) + (7)$ $\quad 19x = 95$

$x = 5$

$\rightarrow (4)$ $\quad 15 + 2y = 9$

$2y = -6$

$y = -3$

$\rightarrow (2)$ $\quad 5 - 9 + z = -2$

$z = 2$

$\therefore \quad x = 5, \quad y = -3, \quad z = 2$

/4

Q3(c)(i) $y = \sqrt{25 - x^2}$

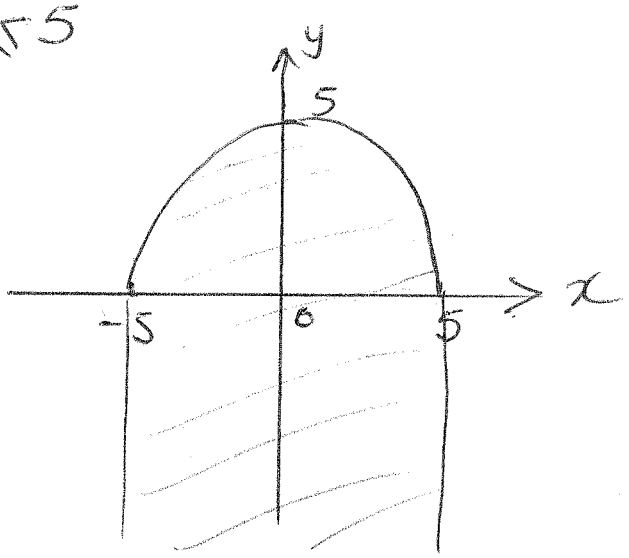
$25 - x^2 \geq 0$

$(5 - x)(x + 5) \geq 0$

D: $-5 \leq x \leq 5$

R $0 \leq y \leq 5$ ½

(ii)



Test (0,0)

$0 \leq \sqrt{25}$

True. ½

d) Prove $TP = TD$

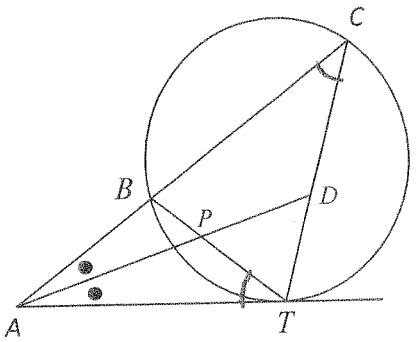
In ΔAPT & ΔACD

$\angle ATB = \angle ACT$

($\angle$ bet. tangent & chord = $\angle$ is alternate segment)

$\angle CAD = \angle DAT$

(AD is a bisector $\angle CAT$)



$\therefore \Delta APT \,|||\, \Delta ACD$ (equiangular)

$\therefore \angle ADC = \angle APT$

$\therefore \angle ADT = \angle DPT$ (exterior $\angle$'s of 2 Δ's)

$\because$ 2 $\angle$'s are equal $\therefore \Delta PTD$ is isos

$\therefore TP = TD$.

¼

Question 4 : 15 Marks

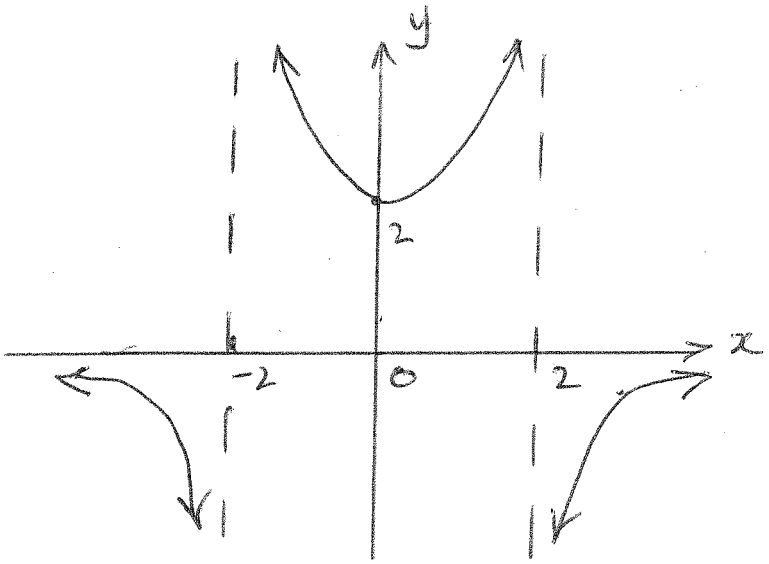
a) (i) $f(x) = \dfrac{2}{1-\frac{1}{2}x^2}$

$f(-x) = \dfrac{2}{1-(-x)^2}$

$= \dfrac{2}{1-x^2} = f(x)$

$\therefore$ Even

(ii)



b) $f(x) = x^2 - 2x - 8$

$= 2x - 8$

$= 2$

$x < 0$

$0 \leq x \leq 5$

$x > 5.$

(i) $f(-1) = (-1)^2 + 2 - 8$

$= -5$

$f(4) = 2 \times 4 - 8$

$= 0$ ✓2

(ii) $x = 5$

$f(5) = 2 \times 5 - 8$

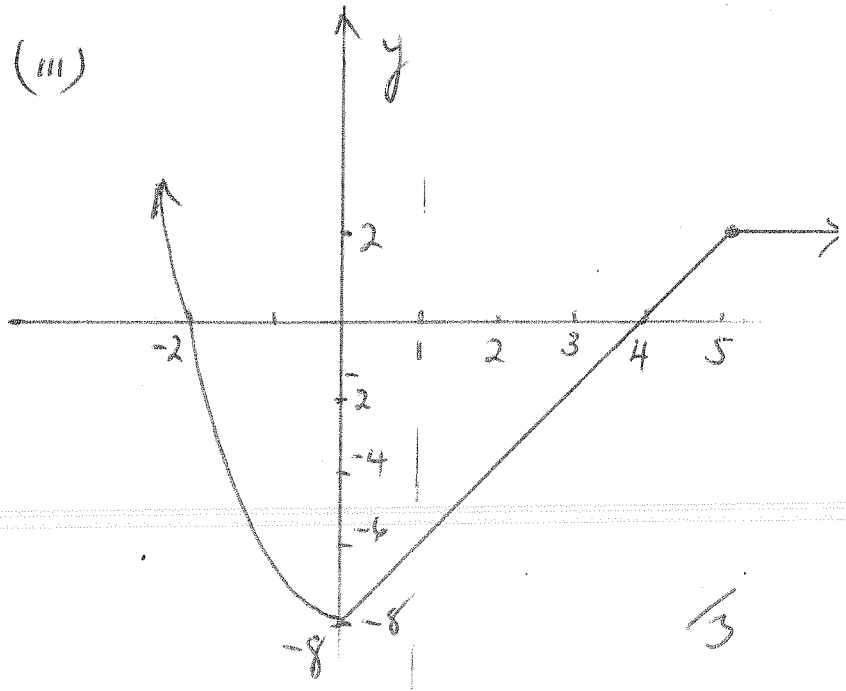
$= 2.$

also

$f(x) = 2 \quad x > 5.$

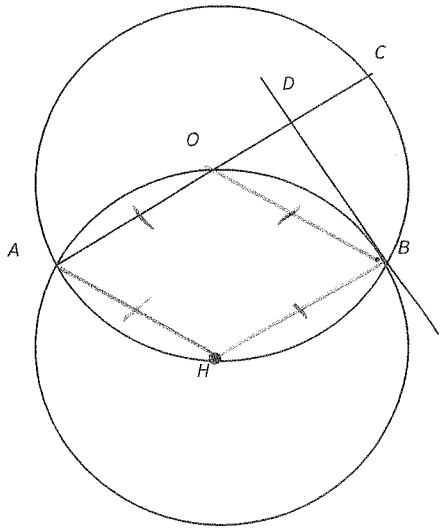
$\therefore$ continuous. ✓1

(iii)



✓3

Q 4(c)



Prove
$AC \perp BD$

Join OB, AH, BH

$OB = OA = HA = HB$ (radii of equal circles)
$\therefore$ $OAHB$ is a rhombus (all sides are equal).
$\therefore$ $AO \parallel BH$ (opp sides)
$\therefore$ $AC \parallel BH$
$BH \perp DB$ ($\angle$ bet. radius and tangent)
$\therefore$ $\angle ADB = 90°$ (Co-interior $\angle$'s, $AC \parallel BH$)
$\therefore$ $AC \perp BD$