CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW

2012 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS EXTENSION 1

Section I
10 marks

Questions 1–10 (1 mark each)

Question 1 (1 mark)
Outcomes Assessed: PE2
Targeted Performance Bands: E2

Solution	Answer	Mark
$P(-1) = -1$	A	1

Question 2 (1 mark)
Outcomes Assessed: H5, HE7
Targeted Performance Bands: E2

Solution	Answer	Mark
$\left\|\frac{2-m}{1+2m}\right\| = \tan 45°, m > o$ $m = \frac{1}{3}$	A	1

Question 3 (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: E2

Solution	Answer	Mark
$\sin\theta = \frac{2t}{1+t^2}$ and $\cos\theta = \frac{1-t^2}{1+t^2}$ $\sin\theta + \cos\theta = \frac{1+2t-t^2}{1+t^2}$	C	1

Question 4 (1 mark)
Outcomes Assessed: P4, PE2
Targeted Performance Bands: E2

Solution	Answer	Mark
$(x,y)=\left(\frac{mx_2+nx_1}{m+n},\ \frac{my_2+ny_1}{m+n}\right)$ $=\left(\frac{3\times3+(-2)\times(-2)}{3+(-2)},\ \frac{3\times(-4)+(-2)\times3}{3+(-2)}\right)$ $=(13,-18)$	D	1

Question 5 (1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E2

Solution	Answer	Mark
$\binom{6}{3}\times\binom{4}{2}=120$	B	1

Question 6 (1 mark)
Outcomes Assessed: HE4
Targeted Performance Bands: E3

Solution	Answer	Mark
$f^{-1}(3)=x$ $f(x)=3$ $\frac{2x}{x+1}=3$ $x=-3$	A	1

Question 7 (1 mark)
Outcomes assessed: PE3, PE4
Targeted Performance Bands: E2

Solution	Answer	Mark
$x + py = 2ap + ap^3$ When $x = 0, y = a$: $ap = 2ap + ap^3$ $ap(1 + p^2) = 0$ $p = 0$ is the only solution. Therefore there exists only one solution.	B	1

Question 8 (1 mark)
Outcomes Assessed: HE3, HE7
Targeted Performance Bands: E3

Solution	Answer	Mark
Let $p = 0.8$ and $q = 0.2$ $P(X \geq 5) = P(X = 5) + P(X = 6)$ $= \binom{6}{5}(0.2)(0.8)^5 + (0.8)^6$ $= (0.8)^5(6 \times 0.2 + 0.8)$ $= 2(0.8)^5$	B	1

Question 9 (1 mark)
Outcomes Assessed: HE3
Targeted Performance Bands: E3

Solution	Answer	Mark
$T_{k+1} = \binom{7}{k}(x^2)^{7-k}\left(\frac{2}{x}\right)^k$ $= \binom{7}{k}2^k x^{14-3k}$ When $14 - 3k = 2, k = 4$. The coefficient of $x^2 = \binom{7}{4}2^4$ $= 560$.	D	1

Question 10 (1 mark)
Outcomes Assessed: HE3
Targeted Performance Bands: E3, E4

Solution	Answer	Mark
The FALSE statement is C	C	1

Section II
60 marks

Question 11 (15 marks)
(a) (1 mark)
Outcomes assessed: H5
Targeted Performance Bands: E2

Criteria	Mark
• Correct answer	1

Sample answer:

$$\lim_{x\to 0}\frac{\sin 2x}{x} = 2\lim_{x\to 0}\frac{\sin 2x}{2x}$$
$$=2$$

(b) (2 marks)
Outcomes assessed: HE6
Targeted Performance Bands: E2

Criteria	Marks
• Correct answer	2
• Recognising the correct primitive	1

Sample answer:

$$\int_0^4 \frac{dx}{\sqrt{x^2+9}} = \left[\ln\left(x+\sqrt{x^2+9}\right)\right]_0^4$$
$$= \ln 9 - \ln 3$$
$$= \ln 3$$

(c) (3 marks)

Outcomes assessed: HE4, HE6

Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct answer	3
• Significant progress towards solution	2
• Correct integral with correct limits, after applying given substitution	1

Sample answer:

Let $u = x - 8$

$du = dx$

$x = 8.5 \Rightarrow u = 0.5$ and $x = 8 \Rightarrow u = 0.$

Upon substituting: $\int_0^{0.5} \frac{du}{\sqrt{(7-(u+8))(u+8-9)}}$

$= \int_0^{0.5} \frac{du}{\sqrt{1-u^2}}$

$= [\sin^{-1} u]_0^{0.5}$

$= \frac{\pi}{6}$

(d) (3 marks)

Outcomes assessed: PE3

Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct range of solutions given	3
• Significant progress towards solution (e.g. inequality if using solution presented below or points of intersection if using graphical techniques)	2
• Some progress towards answer (considering technique used e.g. graphical or algebraic)	1

Sample answer:

$\frac{2t}{1-t} \geq t \qquad t \neq 1$

$2t(1-t) \geq t(1-t)^2$

$t(1-t)(1+t) \geq 0$

$\therefore t \leq -1, 0 \leq t < 1$

(e) (i) (2 marks)
Outcomes assessed: PE2, PE3
Targeted Performance Bands: E2

Criteria	Marks
• Correct answer	2
• Recognises the relationship between the tangent and the intercepts from an external point	1

Sample answer:

$$PC^2 = AP.BP$$
$$12^2 = (x+7).x$$
$$x = 9 \quad \text{(since } x > 0\text{)}$$

(e) (ii) (1 mark)
Outcomes assessed: PE3
Targeted Performance Bands: E2

Criteria	Mark
• Correct answer	1

Sample answer:
$\angle BPC = 90°$ since BC is the diameter of the circle passing through P, B and C.
∴ ΔBPC is a right-angled triangle.

$$BC^2 = BP^2 + PC^2$$
$$BC^2 = 9^2 + 12^2$$
$$BC = 15$$

(f) (3 marks)
Outcomes assessed: PE2, PE6, HE7
Targeted Performance Bands: E3

Criteria	Marks
• Correct proof logically presented	3
• Progress towards solution	2
• Correctly identifying a relevant trigonometric relationship in one of the triangles	1

Sample answer:
Let $CD = h$

$$\text{In } \Delta ADC, \tan\alpha = \frac{h}{AC}$$
$$\text{In } \Delta BDC, \tan\beta = \frac{h}{BC}$$
$$\therefore \frac{\tan\alpha}{\tan\beta} = \frac{BC}{AC}$$

ΔABC is right-angled at C and $\angle CAB = 90° - 60° = 30°$ (from the bearings given), hence

$$\frac{BC}{AC} = \tan 30° = \frac{1}{\sqrt{3}}$$
$$\therefore \frac{\tan\alpha}{\tan\beta} = \frac{1}{\sqrt{3}}.$$

Question 12 (15 marks)
(a) (i) (1 mark)
Outcomes assessed: PE3
Targeted Performance Bands: E2

Criteria	Mark
• Correct answer	1

Sample answer:

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

(a) (ii) (2 marks)
Outcomes assessed: H5, HE7
Targeted Performance Bands: E3–E4

Criteria	Marks
• Correctly using the result in part (i) to obtain the result	2
• Solves $f''(x) = 0$	1

Sample answer:

$$f'(x) = 3ax^2 + 2bx + c$$
$$f''(x) = 6ax + 2b$$

Solving $f''(x) = 0$

$$6ax + 2b = 0$$
$$x = \frac{-b}{3a}$$
$$= \frac{\alpha+\beta+\gamma}{3}$$

(a) (iii) (1 mark)
Outcomes assessed: HE7
Targeted Performance Bands: E3

Criteria	Mark
• Correct answer	1

Sample answer:

$$x = \frac{\alpha+\beta+\gamma}{3}$$
$$= \frac{-1+3+4}{3}$$
$$= 2$$

(b) (i) (2 marks)

Outcomes assessed: HE4, HE7

Targeted Performance Bands: E3

Criteria	Marks
• Correct value of x	2
• Correct value of α	1

Sample answer:

$$\sin^{-1}x = \alpha,\ \cos^{-1}x = \frac{\pi}{2} - \alpha$$

$$\text{Now,}\quad \sin^{-1}x = \cos^{-1}x$$

$$\text{i.e.}\quad \alpha = \frac{\pi}{2} - \alpha$$

$$\alpha = \frac{\pi}{4}$$

$$\therefore \sin^{-1}x = \frac{\pi}{4}$$

$$x = \sin\frac{\pi}{4}$$

$$\therefore \text{The solution is } x = \frac{\sqrt{2}}{2}.$$

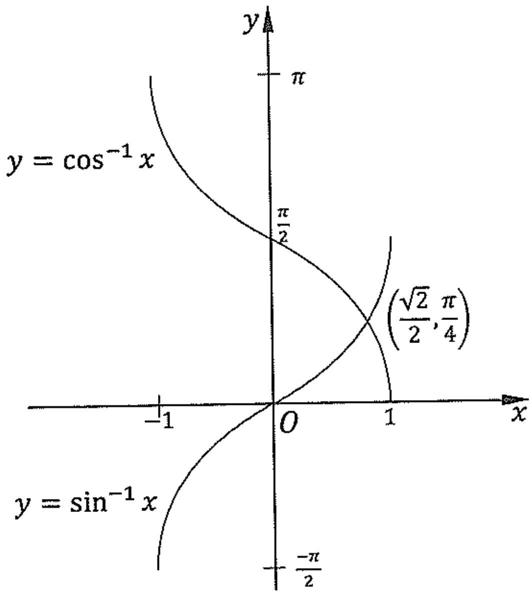
(b) (ii) (1 marks)

Outcomes assessed: HE4, HE7

Targeted Performance Bands: E2

Criteria	Marks
• Sketches curves correctly, showing the point of intersection	1

Sample answer:



(b) (iii) (2 marks)
Outcomes assessed: HE2
Targeted Performance Bands: E3

Criteria	Mark
• Correct explanation	2
• Reference to symmetry of region	1

Sample answer:
The region is symmetrical about the line $y = \frac{\pi}{4}$.
Therefore the volume of the rotated region bounded by $y = \sin^{-1} x$ between $y = 0$ and $y = \frac{\pi}{4}$ (given by $\pi \int_0^{\frac{\pi}{4}} \sin^2 y \, dy$) is equal to the volume of the rotated region bounded by $y = \cos^{-1} x$ between $y = \frac{\pi}{4}$ and $y = \frac{\pi}{2}$ (given by $\pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 y \, dy$).

$$\therefore V = \pi \int_0^{\frac{\pi}{4}} \sin^2 y \, dy + \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 y \, dy$$
$$= 2\pi \int_0^{\frac{\pi}{4}} \sin^2 y \, dy$$

(b) (iv) (2 marks)
Outcomes assessed: H8, HE6
Targeted Performance Bands: E3

Criteria	Marks
• Correct answer	2
• Substantial progress towards integrating $\sin^2 y$	1

Sample answer:

$$\begin{aligned} V &= 2\pi \int_0^{\frac{\pi}{4}} \sin^2 y \, dy \\ &= \pi \int_0^{\frac{\pi}{4}} (1 - \cos 2y) dy \\ &= \pi \left[y - \frac{\sin 2y}{2} \right]_0^{\frac{\pi}{4}} \\ &= \frac{\pi}{4}(\pi - 2) \end{aligned}$$

$\therefore$ The volume is $\frac{\pi}{4}(\pi - 2)$ units^3.

(c) (i) (3 marks)
Outcomes assessed: HE2, HE7
Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	3
• Establishes the induction step	2
• Verifies the result for n=2	1

Sample answer:

(1) When $n = 2$:

$$\begin{aligned} \text{LHS} &= \left(1 - \frac{1}{2^2}\right) \\ &= \frac{3}{4} \\ \text{RHS} &= \frac{2+1}{2\times 2} \\ &= \frac{3}{4} \\ &= \text{LHS} \end{aligned}$$

Assume true for $n = k$: $\left(1-\frac{1}{2^2}\right)\times\left(1-\frac{1}{3^2}\right)\times\left(1-\frac{1}{4^2}\right)\times\ldots\times\left(1-\frac{1}{k^2}\right)=\frac{k+1}{2k}$

Prove true for $n = k + 1$:

i.e. $\left(1-\frac{1}{2^2}\right)\times\left(1-\frac{1}{3^2}\right)\times\left(1-\frac{1}{4^2}\right)\times\ldots\times\left(1-\frac{1}{k^2}\right)\times\left(1-\frac{1}{(k+1)^2}\right)=\frac{k+2}{2(k+1)}$

Now $\text{LHS} = \frac{k+1}{2k}\times\left(1-\frac{1}{(k+1)^2}\right)$ (using assumption)

$$\begin{aligned} &= \frac{k+1}{2k}\times\frac{(k+1)^2-1}{(k+1)^2} \\ &= \frac{k^2+2k}{2k(k+1)} \\ &= \frac{k+2}{2(k+1)} \\ &= \text{RHS} \end{aligned}$$

Hence, by the Principle of Mathematical Induction, the result holds true for all integers $n \geq 2$.

(c) (ii) (1 mark)
Outcomes assessed: HE2, HE7
Targeted Performance Bands: E2

Criteria	Mark
• Correct answer	1

Sample answer:

$n = 100$, hence the product equals $\frac{101}{200}$.

Question 13 (15 marks)
(a) (i) (1 mark)
Outcomes assessed: HE3, HE7
Targeted Performance Bands: E2–E3

Criteria	Mark
• Correct proof	1

Sample answer:

$$T = 100 - Ae^{-0.2t}$$
$$\frac{dT}{dt} = 0.2Ae^{-0.2t}$$
$$= 0.2(100 - T) \qquad \text{as } Ae^{-0.2t} = 100 - T.$$

(a) (ii) (1 mark)
Outcomes assessed: HE3, HE7
Targeted Performance Bands: E2–E3

Criteria	Mark
• Correct value for A	1

Sample answer:

When $t = 0, T = 4$.
$4 = 100 - Ae^{-0.2\times 0}$
$A = 96$

(a) (iii) (1 mark)
Outcomes assessed: HE3, HE7
Targeted Performance Bands: E2–E3

Criteria	Mark
• Correct value for temperature	1

Sample answer:

$T = 100 - 96e^{-0.2t}$
Substitute $t = 4.5$
$T = 100 - 96e^{-0.2\times 4.5}$
$T = 60.9693 \ldots$

Therefore, the temperature correct to three significant figures is 61.0°C.

(a) (iv) (2 marks)
Outcomes assessed: HE3, HE7
Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct answer	2
• Correct value of t for an egg boiled from room temperature	1

Sample answer:

$$T = 100 - 79e^{-0.2t}$$
Substituting $T = 60.9693\ldots$
$$60.9693\ldots = 100 - 79e^{-0.2t}$$
$$t = \ln\left(\frac{100-60.9693\ldots}{79}\right) \div -0.2$$
$$= 3.525498\ldots \approx 3.5 \text{ minutes}$$

Therefore, the egg at room temperature will cook approximately one minute faster than an egg taken from the refrigerator.

(b) (i) (3 marks)
Outcomes assessed: HE3, HE7
Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct answers for A and α	3
• Correct value for either A or α	2
• Significant progress towards correct equations involving A and α	1

Sample answer:

$$x = A\cos(2t+\alpha)$$

When $t = 0, x = 4$: $A\cos\alpha = 4$ (i)

Now $\frac{dx}{dt} = -2A\sin(2t+\alpha)$

When $t = 0, \frac{dx}{dt} = -8\sqrt{3}$ $A\sin\alpha = 4\sqrt{3}$ (ii)

From (i) and (ii) above: $\frac{A\sin\alpha}{A\cos\alpha} = \frac{4\sqrt{3}}{4}$

$$\tan\alpha = \sqrt{3}$$
$$\alpha = \frac{\pi}{3}$$

Substitute $\alpha = \frac{\pi}{3}$ into (i): $A\cos\frac{\pi}{3} = 4$

$$A = 8$$

Therefore, $A = 8$ and $\alpha = \frac{\pi}{3}$.

(b) (ii) (2 marks)
Outcomes assessed: HE3, HE7
Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct answer	2
• Correct equation with ONE unknown, t	1

Sample answer:
Particle is at the centre of motion when $x = 0$:

$$8\cos\left(2t+\frac{\pi}{3}\right)=0$$

$$2t+\frac{\pi}{3}=\frac{\pi}{2}, \quad \text{for the first time.}$$

$$t=\frac{\pi}{12}$$

Therefore, the particle arrives at the origin after $\frac{\pi}{12}$ seconds.

(c) (i) (3 marks)
Outcomes assessed: HE5, HE7
Targeted Performance Bands: E3–E4

Criteria	Marks
• Correctly shows the result	3
• Correct expression for v^2, including the value of the constant	2
• Progress towards expressing the result in terms of v and x, e.g. using the correction version of acceleration $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$	1

Sample answer:

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right)=2x^3+18x$$

$$\frac{1}{2}v^2=\frac{1}{2}x^4+9x^2+c$$

When $x = 0, v = 9$: $\quad c=\frac{81}{2}$

Therefore, $\quad \frac{1}{2}v^2=\frac{1}{2}x^4+9x^2+\frac{81}{2}$

$$v^2=x^4+18x^2+81$$

$$v^2=(x^2+9)^2$$

$$v=\pm(x^2+9)$$

However, since initially the particle has a positive velocity and there are no solutions to $v = 0$, the particle must always travel in a positive direction.
Hence, $v = x^2 + 9$.

(c) (ii) (2 marks)
Outcomes assessed: HE5, HE7
Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct equation for x in terms of t	2
• Progress towards an expression for t in terms of x	1

Sample answer:

$\frac{dx}{dt} = x^2 + 9$
$\frac{dt}{dx} = \frac{1}{x^2+9}$
$t = \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + c$
When $x = 0$, $t = 0$, therefore $c = 0$.
$t = \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)$
$x = 3\tan(3t)$

Question 14 (15 marks)
(a) (3 marks)
Outcomes assessed: H5, HE7
Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct answer	3
• Correct substitution into the formula of Newton's Method	2
• Establishing an appropriate function and determining its derivative	1

Sample answer:

$$3\sin x = \ln x$$

$$\text{Let } f(x) = 3\sin x - \ln x$$

$$f'(x) = 3\cos x - \frac{1}{x}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 3 - \frac{3\sin 3 - \ln 3}{3\cos 3 - \frac{1}{3}}$$

$$= 2.79558\ldots$$

$$= 2.80 \text{ (2 d.p.)}$$

(b) (i) (1 mark)
Outcomes assessed: H5
Targeted Performance Bands: E3, E4

Criteria	Mark
• Correct justification for result	1

Sample answer:
For the gradient of MB: applying Pythagoras' Theorem, rise $= \sqrt{L^2 - x^2}$; run $= x$.
The gradient of $MB = \frac{-\text{rise}}{\text{run}} = \frac{-\sqrt{L^2-x^2}}{x}$.
Since the rope is always tangent to the curve, the line MB is a tangent to $y = f(x)$

$$\therefore \frac{dy}{dx} = \frac{-\sqrt{L^2 - x^2}}{x}.$$

(b) (ii) (2 marks)
Outcomes assessed: HE5, HE7
<u>Targeted Performance Bands: E3–E4</u>

Criteria	Marks
• Correct answer	2
• Substantial progress towards expressing $\frac{dx}{dt}$ as a function of x	1

Sample answer:

$$\begin{aligned}\frac{dx}{dt} &= \frac{dx}{dy} \times \frac{dy}{dt}\\ &= \frac{-x}{\sqrt{L^2-x^2}} \times 3\\ &= \frac{\frac{-L}{2}}{\sqrt{L^2-\frac{L^2}{4}}} \times 3 \text{ when } x = \frac{L}{2}\\ &= -\sqrt{3}\end{aligned}$$

Therefore, the boat approaches the pier at a rate of $\sqrt{3}$ ms^{-1}.

(c) (3 marks)
Outcomes assessed: PE3, PE4, HE2
<u>Targeted Performance Bands: E3–E4</u>

Criteria	Marks
• Correct proof	3
• Substantial progress towards determining both required distances or both required angles	2
• Recognising the gradient at P is given by $\frac{1}{p}$ and progress towards determining a relevant distance or a relevant angle	1

Sample answer:

The gradient of the tangent at $P = \frac{2a}{2ap} = \frac{1}{p}$.

The equation of the tangent at P is given by $y - 2ap = \frac{1}{p}(x - ap^2)$

Let the tangent at P intersect the x-axis at G.

$\therefore$ G has coordinates $(-ap^2, 0)$. P has coordinates $(ap^2, 2ap)$. S has coordinates $(a, 0)$.

$SG = ap^2 + a$

$$\begin{aligned}SP &= \sqrt{(ap^2-a)^2 + (2ap)^2}\\ &= \sqrt{a^2p^4 + 2a^2p^2 + a^2}\\ &= \sqrt{(ap^2+a)^2}\\ &= ap^2 + a\end{aligned}$$

Since $SG = SP$, ΔSPG is isosceles.

$\therefore \angle SGP = \angle SPG$ (angles opposite equal sides of an isosceles triangle are equal), thus the angle of inclination to the axis of the parabola equals the angle between the tangent and the focal chord.

Hence the tangent to the parabola at P is equally inclined to the axis of the parabola and the focal chord through P.

(d) (i) (1 mark)
Outcomes assessed: HE2, HE7,
Targeted Performance Bands: E3

Criteria	Mark
• Correct proof	1

Sample answer:
The coefficient of x^r in the expansion $(1+x)^n$ is $\binom{n}{r}$.
The coefficient of x^r in the expansion $(x+1)^n$ is $\binom{n}{n-r}$.
Since $(1+x)^n = (x+1)^n$, the expansions are the same. Therefore, $\binom{n}{n-r} = \binom{n}{r}$.

(d) (ii) (3 marks)
Outcomes assessed: HE2, HE7
Targeted Performance Bands: E4

Criteria	Marks
• Correct proof	3
• Correctly identifies and matches coefficients of x^{n-r} in the two binomial expansions	2
• Expansion of ONE relevant binomial expression	1

Sample answer:
$f(r) = \binom{n}{0}\binom{n}{r} + \binom{n}{1}\binom{n}{r+1} + \cdots + \binom{n}{n-r}\binom{n}{n}$.
Therefore, $f(r) = \binom{n}{0}\binom{n}{n-r} + \binom{n}{1}\binom{n}{n-r-1} + \cdots + \binom{n}{n-r}\binom{n}{0}$, using the result in part (i).

Now, consider $(1+x)^n(1+x)^n = (1+x)^{2n}$
$(1+x)^n(1+x)^n = \left(\binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n\right)\left(\binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n\right)$

The coefficient of x^{n-r} in this expansion is:

$$\binom{n}{0}\binom{n}{n-r} + \binom{n}{1}\binom{n}{n-r-1} + \binom{n}{2}\binom{n}{n-r-2} + \cdots + \binom{n}{n-r}\binom{n}{0} = f(r) \text{ from above}$$

But the coefficient of x^{n-r} in the expansion of $(1+x)^{2n}$ is $\binom{2n}{n-r}$.
Therefore, $f(r) = \binom{2n}{n-r}$, for $r = 0, 1, 2, \ldots, n$

(d) (iii) (2 marks)

Outcomes assessed: HE2, HE7

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct proof	2
• Correctly identifies and matches coefficients in relevant binomial expansions	1

Sample answer:

Consider $(1+x)^{3n} = (1+x)^{2n}(1+x)^n$

$$(1+x)^{2n}(1+x)^n = \left(\binom{2n}{0} + \binom{2n}{1}x + \binom{2n}{2}x^2 + \cdots + \binom{2n}{2n}x^{2n}\right)\left(\binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n\right)$$

The coefficient of x^n in this expansion is:

$$\binom{2n}{0}\binom{n}{n} + \binom{2n}{1}\binom{n}{n-1} + \binom{2n}{2}\binom{n}{n-2} + \cdots + \binom{2n}{n}\binom{n}{0}$$
$$= f(n)\binom{n}{n} + f(n-1)\binom{n}{n-1} + f(n-2)\binom{n}{n-2} + \cdots + f(0)\binom{n}{0}, as\ f(r) = \binom{2n}{n-r} \text{ for } r = 0, 1, 2 \ldots, n$$

But the coefficient of x^n in the expansion of $(1+x)^{3n}$ is $\binom{3n}{n}$.

Therefore, $\binom{n}{0}f(0) + \binom{n}{1}f(1) + \cdots + \binom{n}{n}f(n) = \binom{3n}{n}$.