ABBOTSLEIGH

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS

3 UNIT

1999

Time allowed: Two hours
(Plus 5 minutes reading time)

Directions to candidates:

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied.
- Board-approved calculators may be used.
- Answer each question in a SEPARATE Writing Booklet.
- You may ask for extra Writing Booklets if you need them.

Marks

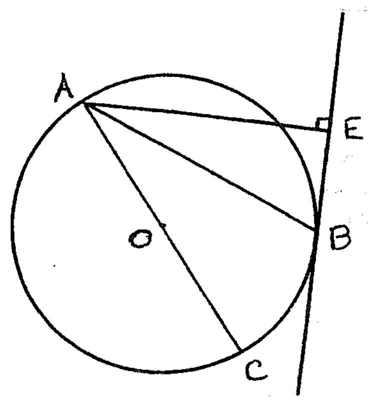
Q1. (a) Let A(-5,12) and B(4,9) be two points in the number plane. Find the coordinates of P which divides the interval AB externally in the ratio 5 : 2. **2**

(b) Find the size of the acute angle between the lines $y = 2x + 3$ and $y = 4x + 1$. (Answer to the nearest minute). **2**

(c) Express $f(x) = x^3 + 3x^2 - 10x - 24$ as a product of three linear factors. **3**

(d) Evaluate $\displaystyle\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ **3**

(e) Two points A and B are placed on a circle and AC is a diameter. AE is perpendicular to the tangent at B. **2**



(i) Draw the diagram on your paper.

(ii) Prove AB bisects $\angle$CAE.

Q2. **Start a new booklet**

(a) Solve for x : $x \geq \dfrac{4}{x}$ **3**

(b) For $y = -3\sin^{-1}\dfrac{x}{2}$

(i) State the domain and range.
(ii) Sketch the curve. **3**

(c) Using the substitution $u = 9 - x^2$, evaluate $\displaystyle\int_0^3 x\sqrt{9-x^2}\,dx$ **3**

(d) The area bounded by the curve $y = \sin x$ between $x = 0$ and $x = \dfrac{\pi}{2}$ is rotated about the x-axis. Find the volume of the solid of revolution. **3**

Marks

Q3. **Start a new booklet**

(a) Express $3\cos x + 4\sin x$ in the form $A\cos(x-\alpha)$ where $A > 0$. Hence, or otherwise, solve $3\cos x + 4\sin x = -3$ for $0 \leq x \leq 360°$. **4**

(b) Find the greatest coefficient in the expansion $(3 + 4x)^{16}$ (leave in index form) **4**

(c) A point P moves on the curve $y = x^3$ in such a way that its x coordinate is changing at a constant rate of 2 units/sec. When $x = 1$, at what rate is

(i) the y coordinate changing?
(ii) the gradient changing? **4**

Q4. **Start a new booklet**

(a) Find x and y if $\frac{4^x}{16} = 8^{x+y}$ and $2^{2x+y} = 128$. **3**

(b) If $x = 2 - \cos t$ and $y = 2t + 2\sin t$, **4**

(i) find $\frac{dx}{dt}$ and $\frac{dy}{dt}$

(ii) Hence or otherwise, find $\frac{dy}{dx}$ in terms of $\frac{t}{2}$.

(c) A particle is oscillating in simple harmonic motion such that its displacement x metres from the origin is given by the equation $\frac{d^2x}{dt^2} = -9x$ where t is time in seconds. **5**

(i) Show that $x = a\cos(3t+\alpha)$ is a solution of motion for this particle (a and α are constants).

(ii) When $t = 0$, $v = 3$ m/s and $x = 5$ m. Show that the amplitude of the oscillation is $\sqrt{26}$ metres.

(iii) What is the maximum speed of the particle?

Marks

Q5. **Start a new booklet**

(a) α, β, γ are the roots of the equation $2x^3 + 3x^2 - 4 = 0$ **3**

Find

(i) $\alpha + \beta + \gamma$

(ii) $\alpha \beta \gamma$

(iii) $\alpha^2 + \beta^2 + \gamma^2$

(b) For the function $y = x^2 - 2x + 1$, find the largest possible domain such that this function has an inverse. Find the equation of this inverse and state its range. **3**

(c) For the parabola $x^2 = 12y$, find **6**

(i) the equation of the tangent at the point $P\ (6p, 3p^2)$ on the parabola.

(ii) the coordinates of the point T where the tangent meets the x axis.

(iii) Show that N, the midpoint of PT, has coordinates $(\frac{9p}{2}, \frac{3p^2}{2})$.

(iv) Find the equation of the locus of N.

Q6. **Start a new booklet**

(a) Find $\lim\limits_{x \to 0} \dfrac{\sin 3x}{5x}$ **2**

(b) The daily growth of a colony of insects is 10% of the excess of the population over 1.2×10^6. **4**
ie $\dfrac{dN}{dt} = 0.1\ (N - 1.2 \times 10^6)$.

Initially, the population is 2.7×10^6,

(i) Determine the population after $3\frac{1}{2}$ days.

(ii) If a scientist checks the population each day, which is the first day on which she should notice that the original population has tripled?

Marks

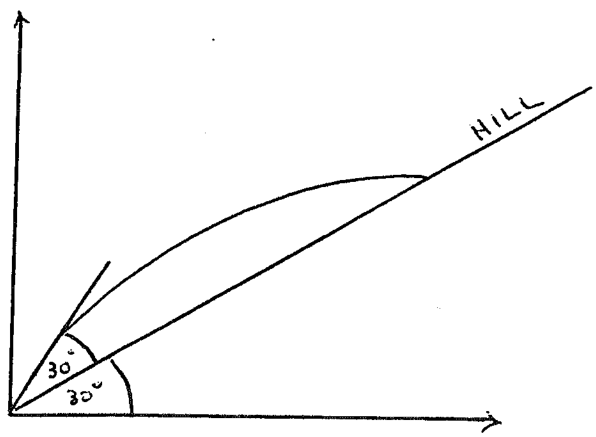
Q6. (continued).....

(c) A ball is thrown with a velocity of $30\sqrt{3}$ m/s at an angle of 60° to the horizontal. **6**

(i) Assuming negligible air resistance and letting $g = 10\ ms^{-2}$, derive the equations of motion.

(ii) Find the time of flight and the range.

(iii) If the ball had been thrown with velocity $30\sqrt{3}$ m/s at an angle of 30° to a hill which is itself inclined at 30° to the horizontal (see diagram), determine the time of flight.



Q 7. **Start a new booklet**

(a) Prove by mathematical induction that for all values of n **5**

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots\ldots \frac{n}{(n+1)!} = \frac{(n+1)! - 1}{(n+1)!}$$

where n is a positive integer.

(b) (i) Show that $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ has no stationary points. **7**

(ii) Prove that the lines $y = \pm 1$ are asymptotes.

(iii) Sketch the curve.

(iv) If k is a positive constant, find the area in the first quadrant enclosed by the above curve and the three lines $y = 1$, $x = 0$ and $x = k$.

(v) Prove that for all values of k, this area is always less than $\log_e 2$.