Trial HSC Mathematics Extension 1 Solutions 2008

Question 1

(a) (i) $$\int \frac{1}{\sqrt{9-x^2}}\,dx = \sin^{-1}\left(\frac{x}{3}\right) + C$$

(ii) $$\int \frac{e^x}{e^x+2}\,dx = \ln\left(e^x+2\right) + C$$

(b) $$\frac{d}{dx}\left(\cos^{-1}\left(\tfrac{2}{x}\right)\right) = \frac{-1}{\sqrt{1-\left(\tfrac{2}{x}\right)^2}}\cdot\left(-2x^{-2}\right)$$

$$= \frac{2}{x^2\sqrt{1-\tfrac{4}{x^2}}}$$

$$= \frac{2}{x\sqrt{x^2-4}}$$

(c) $A(5,\, a)$ $\qquad$ $B(b,\, -1)$

$-2:3$

$$\left(7,\ 2\right) = \left(\frac{5\times3-2b}{-2+3},\ \frac{3a+(-2)(-1)}{-2+3}\right)$$

$$= \left(15-2b,\ 3a+2\right)$$

$\therefore 15-2b=7$ and $3a+2=2$

i.e. $b=4$ and $a=0$

(d) A general term of $\left(2x-\dfrac{1}{x^2}\right)^{11}$ has the form

$$\binom{11}{k}(2x)^{11-k}\left(-\frac{1}{x^2}\right)^k = \binom{11}{k}2^{11-k}(-1)^k\,x^{11-3k}$$

For the term in x^5: $\quad 11-3k=5$

$$3k=6$$

$$k=2$$

$\therefore$ the required coefficient is $\dbinom{11}{2}2^{11-2}(-1)^2 = 28160$

(e) $$\int_6^{11} x\sqrt{x-2}\,dx$$

let $u^2 = x-2$

$x = u^2+2$

$\dfrac{dx}{du} = 2u$

$dx = 2u\,du$

If $x=11$, $u^2=9$

$u=3$ taking $u>0$

If $x=6$, $u^2=4$

$u=2$ taking $u>0$

$$\text{Now } \int_6^{11} x\sqrt{x-2}\,dx = \int_2^3 \left(u^2+2\right)\sqrt{u^2}\,(2u)\,du$$

$$= \int_2^3 \left(2u^4+4u^2\right) du$$

$$= \left[\frac{2u^5}{5}+\frac{4u^3}{3}\right]_2^3$$

$$= \frac{2(3)^5}{5}+\frac{4(3)^3}{3}-\left[\frac{2(2)^5}{5}+\frac{4(2)^3}{3}\right]$$

$$= 109\tfrac{11}{15} \qquad \left(\tfrac{1646}{15} = 109.7\dot{3}\right)$$

Question 2

(a)

$$\frac{8}{x-3} \ge 1 \qquad x \ne 3$$

$$8(x-3) \ge (x-3)^2$$

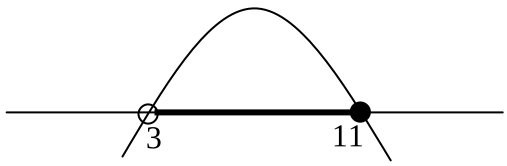
$$8(x-3)-(x-3)^2 \ge 0$$

$$(x-3)\left(8-(x-3)\right) \ge 0$$

$$(x-3)(8-x+3) \ge 0$$

$$(x-3)(11-x) \ge 0$$

$$\therefore \quad 3 < x \le 11$$



(b)

$$\int_{\sqrt{3}}^{3} \frac{2}{9+x^2}\,dx = \left[\frac{2}{3}\tan^{-1}\left(\frac{x}{3}\right)\right]_{\sqrt{3}}^{3}$$

$$= \frac{2}{3}\tan^{-1}\left(\frac{3}{3}\right)-\frac{2}{3}\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$$

$$= \frac{2}{3}\tan^{-1}1-\frac{2}{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{2}{3}\left(\frac{\pi}{4}\right)-\frac{2}{3}\left(\frac{\pi}{6}\right)$$

$$= \frac{\pi}{18}$$

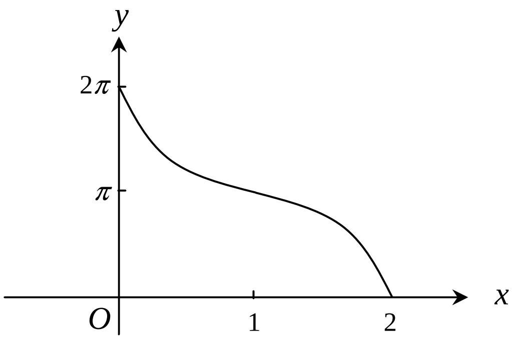
(c) $y = 2\cos^{-1}(x-1)$

(i) $\dfrac{y}{2} = \cos^{-1}(x-1)$

Domain: $-1 \le x-1 \le 1$ Range: $0 \le \dfrac{y}{2} \le \pi$

$0 \le x \le 2$ $0 \le y \le 2\pi$

(ii)



(iii) $y = 2\cos^{-1}(x-1)$

$$\frac{dy}{dx} = \frac{-2}{\sqrt{1-(x-1)^2}}$$

At $x=1$: $\dfrac{dy}{dx} = \dfrac{-2}{\sqrt{1-(1-1)^2}} = -2$

$\therefore$ the gradient of the tangent at x = 1 is –2.

(d) $y = 6 - 2x$ has $m_1 = -2$

For $y = 2x^2 + x - 8$: $\dfrac{dy}{dx} = 4x+1$

At $x = 2$: $\dfrac{dy}{dx} = 4(2)+1 = 9$ $\therefore m_2 = 9$

Let θ be the acute angle between curves where x = 2, then

$$\tan\theta = \left|\frac{-2-9}{1+(-2)(9)}\right|$$

$$= \frac{11}{17}$$

$$\theta = 32^\circ 54'$$

$$= 33^\circ \quad \text{correct to the nearest degree}$$

<u>Question 3</u>

(a) (i) Let $f(x) = x^2 - 4x + \log_e x$

Now $f(3) = 3^2 - 4(3) + \log_e 3 = -1\cdot 901...$

and $f(4) = 4^2 - 4(4) + \log_e 4 = 1\cdot 386...$

$\therefore$ as the sign of the function changes over the interval $3 \le x \le 4$, and the function is continuous over this domain, there is a root between x = 3 and x = 4.

(ii) Now $f\left(\dfrac{3+4}{2}\right) = f(3\cdot 5) = 3\cdot 5^2 - 4(3\cdot 5) + \log_e 3\cdot 5 = -0\cdot 497...$

$\therefore$ the root lies in the interval $3\cdot 5 < x < 4$

$f\left(\dfrac{3\cdot 5+4}{2}\right) = f(3\cdot 75) = 3\cdot 75^2 - 4(3\cdot 75) + \log_e 3\cdot 75 = 0\cdot 384...$

$\therefore$ the root lies in the interval $3\cdot 5 < x < 3\cdot 75$

(b) (i) Let $P(x)=(x^2-x)Q(x)+R(x)$

Now $P(1)=(1^2-1)Q(1)+R(1)$

i.e. $P(1)=R(1)$ but $P(1)=3$ $\therefore R(1)=3$

(ii) Now $P(x)=(x^2-x)Q(x)+ax+b$ as $R(x)=ax+b$

When $P(x)$ is divided by x, the remainder is -4

i.e. $P(0)=-4=R(0)$

Now $R(0)=-4$: $a(0)+b=-4$ $\therefore$ $b=-4$

But $R(1)=3$: $a(1)+b=3$

Substituting $b=-4$: $a-4=3$ $\therefore$ $a=7$

$\therefore$ $R(x)=7x-4$

(c)
$$\int_{\frac{\pi}{2}}^{\pi}\cos^2 2x\,dx=\frac{1}{2}\int_{\frac{\pi}{2}}^{\pi}(\cos 4x+1)\,dx \quad \text{as } \cos 4x=2\cos^2 2x-1$$
$$=\frac{1}{2}\left[\frac{1}{4}\sin 4x+x\right]_{\frac{\pi}{2}}^{\pi}$$
$$=\frac{1}{2}\left[\frac{1}{4}\sin 4\pi+\pi\right]-\frac{1}{2}\left[\frac{1}{4}\sin 4\left(\frac{\pi}{2}\right)+\frac{\pi}{2}\right]$$
$$=\frac{\pi}{2}-\frac{\pi}{4}$$
$$=\frac{\pi}{4}$$

(d) Aim: Prove that $5^n+2(11)^n$ is divisible by 3 for all positive integer values of n.

Test the result for $n=1$:
$$5^1+2(11)^1=5+22$$
$$=27$$
$$=3(9) \quad \text{which is divisible by 3}$$

$\therefore$ the result is true for n = 1

Let n = k be a value of n for which the result is true:

i.e. $5^k+2(11)^k=3M$ where M is an integer (1)

then $5^k=3M-2(11)^k$

Test the result for n = k + 1:
$$5^{k+1}+2(11)^{k+1}=5(5^k)+2(11)(11)^k$$
$$=5(5^k)+22(11)^k$$
$$=5\left[3M-2(11)^k\right]+22(11)^k$$

$$= 5(3M) + 12(11)^k \qquad \text{by (1)}$$

$$= 3\left[5M + 4(11)^k\right]$$

Now as M and k are both integral, $\left[5M + 4(11)^k\right]$ is an integer, say N

$\therefore 3\left[5M + 4(11)^k\right] = 3N$ where N is an integer

and hence $5^{k+1} + 2(11)^{k+1} = 3N$ which is divisible by 3.

$\therefore$ by Mathematical induction, the result is true for all positive integral values of n.

<u>Question 4</u>

(a) $\sin^{-1}\left(\cos\frac{2\pi}{3}\right) = \sin^{-1}\left(-\frac{1}{2}\right)$

$$= -\frac{\pi}{6}$$

(b) $\dfrac{d}{dx}(x\tan x) = x\sec^2 x + \tan x$

Now $\quad x\sec^2 x = \dfrac{d}{dx}(x\tan x) - \tan x$

$$\therefore \int_0^{\frac{\pi}{4}} x\sec^2 x\,dx = \int_0^{\frac{\pi}{4}} \left\{\frac{d}{dx}(x\tan x) - \tan x\right\} dx$$

$$= \left[x\tan x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan x\,dx$$

$$= \frac{\pi}{4}\tan\frac{\pi}{4} - 0\tan 0 - \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x}\,dx$$

$$= \frac{\pi}{4} + \left[\log(\cos x)\right]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{4} + \log\left(\cos\frac{\pi}{4}\right) - \log(\cos 0)$$

$$= \frac{\pi}{4} + \log\left(\frac{1}{\sqrt{2}}\right) - \log 1$$

$$= \frac{\pi}{4} + \log(2)^{-\frac{1}{2}}$$

$$= \frac{\pi}{4} - \frac{1}{2}\ln 2$$

(c) (i) A general term of $(1+2x)^n = \dbinom{n}{k}(2x)^k$

$\therefore$ the coefficient of the term in $x^4 = \dbinom{n}{4}(2)^4$

(ii) The coefficient of the term in $x^6 = \binom{n}{6}(2)^6$

$$\therefore \quad \frac{\text{coefficient of } x^4}{\text{coefficient of } x^6} = \frac{5}{8}$$

$$\frac{\binom{n}{4}(2^4)}{\binom{n}{6}(2^6)} = \frac{5}{8}$$

$$8\binom{n}{4}(2^4) = 5\binom{n}{6}(2^6)$$

$$\frac{n!(2^7)}{4!(n-4)!} = \frac{5(n!)(2^6)}{6!(n-6)!}$$

$$\frac{2}{(n-4)(n-5)} = \frac{5}{6\times 5}$$

$$12 = (n-4)(n-5)$$

$$n^2 - 9n + 8 = 0$$

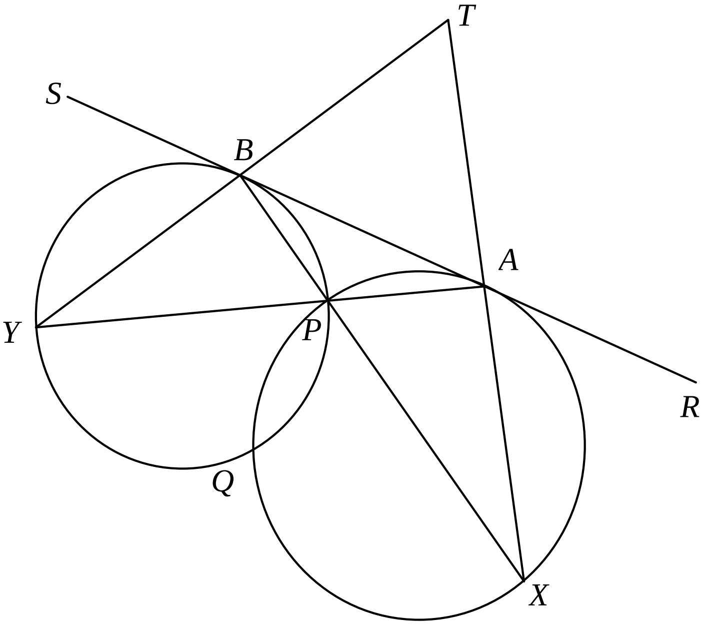
$$(n-8)(n-1) = 0$$

$$n = 1,\ 8$$

But $n \geq 6$ for the x^6 term to exist

$$\therefore \ n = 8$$

(d) (i)



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(ii) $\angle SBY = \angle BPY$ as $\angle SBY$ is the angle between the tangent *SR* and the chord *BY* and $\angle BPY$ is the angle in the alternate segment standing on *BY*.

(iii) $\angle TBA = \angle SBY$ (vertically opposite)

$= \angle BPY$ (angle in the alternate segment)

$= \angle APX$ (vertically opposite)

$= \angle RAX$ (angle between the tangent and the chord AX)

$= \angle TAB$ (vertically opposite)

$\therefore \angle TBA = \angle TAB$

$\therefore AT = TB$ (opposite equal sides in ΔTAB)

Question 5

(a)
$$f(x) = 3e^{-x^2}$$
$$f(-x) = 3e^{-(-x)^2}$$
$$= 3e^{-x^2}$$
$$= f(x)$$
$\therefore$ the function is even

(b)
$$f(x) = 3e^{-x^2}$$
$$f'(x) = -6xe^{-x^2}$$
Stationary points occur when $f'(x) = 0$

$\therefore \quad -6xe^{-x^2} = 0$

$x = 0$ or $e^{-x^2} = 0$

but $e^{-x^2} > 0$ for all values of x

$\therefore$ the only stationary point occurs at $x = 0$

$f(0) = 3e^{-0^2} = 3$

$\therefore (0,3)$ is the only stationary point.

(c)
$$f'(x) = -6xe^{-x^2}$$
$$f''(x) = -6x\left(-2xe^{-x^2}\right) + e^{-x^2}(-6)$$
$$= 6e^{-x^2}\left(2x^2 - 1\right) \text{ as required}$$

Points of inflexion occur when $f''(x) = 0$ and concavity changes sign

$\therefore \quad 6e^{-x^2}\left(2x^2 - 1\right) = 0$

$x^2 = \dfrac{1}{2}$ or $e^{-x^2} = 0$ but $e^{-x^2} > 0$ for all values of x

$\therefore \quad x = \pm\dfrac{1}{\sqrt{2}}$

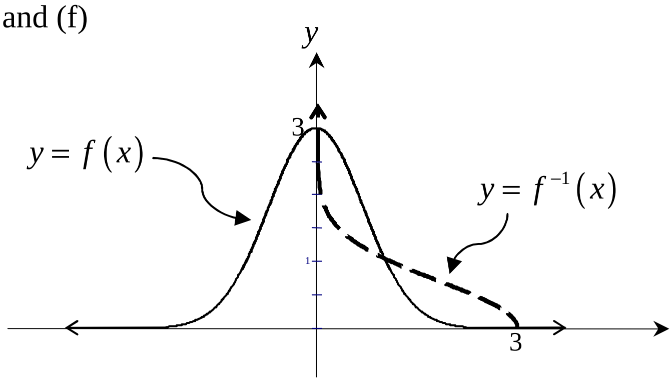
x	$\left(-\frac{1}{\sqrt{2}}\right)^-$	$-\frac{1}{\sqrt{2}}$	$\left(-\frac{1}{\sqrt{2}}\right)^+$	$\left(\frac{1}{\sqrt{2}}\right)^-$	$\frac{1}{\sqrt{2}}$	$\left(\frac{1}{\sqrt{2}}\right)^+$
$f''(x)$	+	0	–	–	0	+

$\therefore$ the concavity changes sign at both values of x.

$f\left(\frac{1}{\sqrt{2}}\right) = 3e^{-\frac{1}{2}}$ and $f\left(-\frac{1}{\sqrt{2}}\right) = 3e^{-\frac{1}{2}}$

i.e. inflexions occur at $\left(\frac{1}{\sqrt{2}},\ 3e^{-\frac{1}{\sqrt{2}}}\right)$ and $\left(-\frac{1}{\sqrt{2}},\ 3e^{-\frac{1}{\sqrt{2}}}\right)$

(d) and (f)



(e) **D**: $x \geq 0$

(f) See above

(g) Let $y = 3e^{-x^2}$

Then the inverse is $x = 3e^{-y^2}$

Now
$$\frac{x}{3} = e^{-y^2}$$
$$\ln\left(\frac{x}{3}\right) = -y^2$$
$$\ln\left(\frac{3}{x}\right) = y^2$$
$$y = \pm\sqrt{\ln\left(\frac{3}{x}\right)}$$

but the domain of the function is $x \geq 0$ so the range of the inverse function is $y \geq 0$

$\therefore$ the inverse function is $f'(x) = \sqrt{\ln\left(\frac{3}{x}\right)}$

The domain of the inverse function is **D**: $0 < x \leq 3$

(h) Let $x = N$ where $N < 0$.

Then $f^{-1}\left(f(N)\right) = f^{-1}\left(f(-N)\right)$ as $f(x)$ is an even function

$= -N$

Question 6

(a) (i) $2\cos\theta + 3\sin\theta = A\cos(\theta - \alpha)$ where $A > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$

$= A\cos\theta\sin\alpha + A\sin\theta\cos\alpha$

Equating coefficients gives:

$2 = A\cos\alpha$

$3 = A\sin\alpha$

$\therefore \qquad \dfrac{A\sin\alpha}{A\cos\alpha} = \dfrac{3}{2}$

$$\tan\alpha = \frac{3}{2}$$

$$\alpha = \tan^{-1}\left(\tfrac{3}{2}\right)$$

Also $A^2\sin^2\alpha + A^2\cos^2\alpha = A^2$

$\therefore \qquad 2^2 + 3^2 = A^2$

$$A = \sqrt{13} \quad \text{as } A > 0$$

$\therefore \qquad 2\cos\theta + 3\sin\theta = \sqrt{13}\cos\left(\theta - \tan^{-1}\left(\tfrac{3}{2}\right)\right)$

(ii) Now $2\cos\theta + 3\sin\theta - 3 = \sqrt{13}\cos\left(\theta - \tan^{-1}\left(\tfrac{3}{2}\right)\right) - 3$.

But the maximum value of $\sqrt{13}\cos(x) = \sqrt{13}$

$\therefore$ the maximum value of $\sqrt{13}\cos\left(\theta - \tan^{-1}\left(\tfrac{3}{2}\right)\right) - 3 = \sqrt{13} - 3$

(b) (i) (α) $P(\text{win}) = \tfrac{1}{5}$

$\therefore P(\text{lose then win}) = \tfrac{4}{5} \times \tfrac{1}{5} = \tfrac{4}{25}$

(β) Probabilities are given by the terms of $\left(\tfrac{1}{5} + \tfrac{4}{5}\right)^6$

$$P(\text{win exactly twice}) = P(X = 2)$$

$$= \binom{6}{2}\left(\tfrac{1}{5}\right)^2\left(\tfrac{4}{5}\right)^4$$

$$= 0\cdot 24576$$

(ii) Now probabilities are given by $\left(\tfrac{1}{5} + \tfrac{4}{5}\right)^n$ and we need $P(X \geq 1) = 0\cdot 95$

$\therefore \quad 1 - P(X = 0) = 0\cdot 95$

$$P(X = 0) = 0\cdot 05$$

$$\binom{n}{0}\left(\tfrac{1}{5}\right)^0\left(\tfrac{4}{5}\right)^n = 0\cdot 05$$

$$\left(\tfrac{4}{5}\right)^n = 0\cdot 05$$

$$n\ln\left(\tfrac{4}{5}\right) = \ln 0\cdot 05$$

$$n = \frac{\ln 0\cdot 05}{\ln\left(\tfrac{4}{5}\right)}$$

$$= 13\cdot 425\ldots$$

$\therefore$ she would have to open 14 bottles

(c) (i) $2\sin(X + Y)\cos(X - Y)$

$= 2[\sin X\cos Y + \cos X\sin Y][\cos X\cos Y + \sin X\sin Y]$

$= 2\left(\sin X\cos^2 Y\cos X + \sin^2 X\cos Y\sin Y + \cos^2 X\sin Y\cos Y + \sin X\cos X\sin^2 Y\right)$

$$= 2\left[\sin X \cos X\left(\cos^2 Y + \sin^2 Y\right) + \sin Y \cos Y\left(\cos^2 X + \sin^2 X\right)\right]$$
$$= 2\left[\sin X \cos X + \sin Y \cos Y\right]$$
$$= 2\left[\tfrac{1}{2}\sin 2X + \tfrac{1}{2}\sin 2Y\right]$$
$$= \sin 2X + \sin 2Y$$

(ii) Let $\sin\theta + \sin 3\theta = \sin 2X + \sin 2Y$

then $2X = \theta$ and $2Y = 3\theta$ and hence $X + Y = 2\theta$ and $X - Y = -\theta$

$\therefore$ as $\sin 2X + \sin 2Y = 2\sin\left(X+Y\right)\cos\left(X-Y\right)$ from (i) above

i.e. $\sin\theta + \sin 3\theta = 2\sin 2\theta\cos\left(-\theta\right)$ but $\cos\left(-\theta\right) = \cos\theta$

$\therefore \quad \sin\theta + \sin 3\theta = 2\sin 2\theta\cos\theta$

Now $\sin\theta + \sin 3\theta = \cos\theta$

becomes $2\sin 2\theta\cos\theta = \cos\theta$

$\therefore \quad 2\sin 2\theta\cos\theta - \cos\theta = 0$

$\cos\theta\left(2\sin 2\theta - 1\right) = 0$

$\cos\theta = 0$ or $\sin 2\theta = \tfrac{1}{2}$ but $0 \le \theta \le 2\pi$

$\therefore \quad \theta = \tfrac{\pi}{2}, \tfrac{3\pi}{2}$ or $2\theta = \tfrac{\pi}{6}, \tfrac{5\pi}{6}, \tfrac{13\pi}{6}, \tfrac{17\pi}{6}$ as $0 \le 2\theta \le 4\pi$

$\therefore \quad \theta = \tfrac{\pi}{2}, \tfrac{3\pi}{2}, \tfrac{\pi}{12}, \tfrac{5\pi}{12}, \tfrac{13\pi}{12}, \tfrac{17\pi}{12}$

Question 7

(a) (i) $f\left(x\right) = \ln x + \sin 5x$ then $f'\left(x\right) = \dfrac{1}{x} + 5\cos 5x$

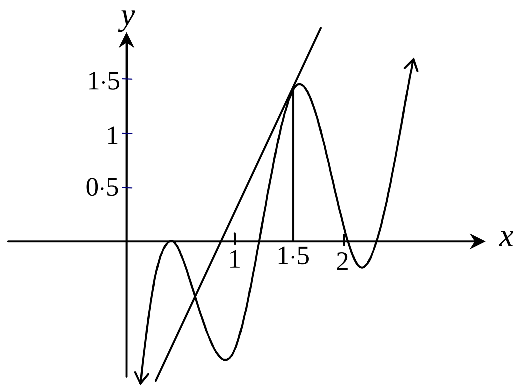
Let $x_0 = 1\cdot 5$

Then $x_1 = 1\cdot 5 - \dfrac{f\left(1\cdot 5\right)}{f'\left(1\cdot 5\right)}$

$$= 1\cdot 5 - \frac{\ln 1\cdot 5 + \sin 5\left(1\cdot 5\right)}{\dfrac{1}{1\cdot 5} + 5\cos 5\left(1\cdot 5\right)}$$

$= 0\cdot 940...$ which is obviously not between 1 and 2

(ii) This attempt fails because a stationary point is very close to x = 1·5 and consequently the tangent to the curve at x = 1·5 has a small gradient. This causes the tangent to intersect the x-axis closer to the root between 0 and 1 than the root between 1 and 2. This argument is illustrated in the diagram below.



DIAGRAM
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(b) (i) A B C D E F and 2 doubles

$$\text{Number of tunes } = {}^6C_2 \times \frac{8!}{2! \times 2!}$$
$$= 151\ 200$$

(ii) Number of choices for the double tones = ${}^6C_2$

The 4 remaining tones can be arranged in 4! ways.

__ tone __ tone__ tone__ tone__

Between these tones, there are 5 'gaps', so the first double tone can be placed in any of these gaps. The second double tone then has only 4 gaps into which it can be placed.

Number of ways of placing the double tones = ${}^6C_2 \times 4! \times 5 \times 4$

= 7200

(c) Consider $(1+x)^{2n} = \binom{2n}{0} + \binom{2n}{1}x + \binom{2n}{2}x^2 + \binom{2n}{3}x^3 + \binom{2n}{4}x^4 + \ldots + \binom{2n}{2n}x^{2n}$

Differentiating both sides with respect to x:

$$2n(1+x)^{2n-1} = \binom{2n}{1} + 2\binom{2n}{2}x + 3\binom{2n}{3}x^2 + 4\binom{2n}{4}x^3 + \ldots + 2n\binom{2n}{2n}x^{2n-1}$$

Differentiating both sides again:

$$2n(2n-1)(1+x)^{2n-2} = 2\binom{2n}{2} + 3(2)\binom{2n}{3}x + 4(3)\binom{2n}{4}x^2 + \ldots + 2n(2n-1)\binom{2n}{2n}x^{2n-2}$$

Substituting $x = 1$:

$$2n(2n-1)(1+1)^{2n-2} = 2\binom{2n}{2} + 3(2)\binom{2n}{3} + 4(3)\binom{2n}{4} + \ldots + 2n(2n-1)\binom{2n}{2n}$$

$$2n(2n-1)(2)^{2n-2} = 2\binom{2n}{2} + 3(2)\binom{2n}{3} + 4(3)\binom{2n}{4} + \ldots + 2n(2n-1)\binom{2n}{2n}$$

Observing the pattern:

$$2n(2n-1)(2)^{2n-2} = 0(-1)\binom{2n}{0} + 1(0)\binom{2n}{1} + 2(1)\binom{2n}{2} + 3(2)\binom{2n}{3} + \ldots + 2n(2n-1)\binom{2n}{2n}$$

$$n(2n-1)(2)^{2n-1} = 0(-1)\binom{2n}{0} + 1(0)\binom{2n}{1} + 2(1)\binom{2n}{2} + 3(2)\binom{2n}{3} + \ldots + 2n(2n-1)\binom{2n}{2n}$$

$$n(2n-1)(2)^{2n-1} = \sum_{k=0}^{2n} k(k-1)\binom{2n}{k}$$

i.e. $\displaystyle\sum_{k=0}^{2n} k(k-1)\binom{2n}{k} = n(2n-1)2^{2n-1}$ as required

(e)

$$2\log_y x + 2\log_x y = 5$$

$$\frac{2\log x}{\log y} + \frac{2\log y}{\log x} = 5$$

$$2(\log x)^2 + 2(\log y)^2 = 5\log x \log y$$

$$2\left(\log x\right)^2 - 5\log x\log y + 2\left(\log y\right)^2 = 0$$

$$\left(2\log x - \log y\right)\left(\log x - 2\log y\right) = 0$$

$$\therefore \quad 2\log x = \log y \quad \text{or} \quad \log x = 2\log y$$

$$\frac{\log x}{\log y} = \frac{1}{2} \quad \text{or} \quad \frac{\log x}{\log y} = 2$$

$$\therefore \quad \log_y x = \tfrac{1}{2} \ \text{ or } \ 2$$

End of Solutions