[redacted] [redacted] [redacted]

Section A 2unit

2 1. (a) 0.272

2 (b) $x^2 - 10x = 0$

$x(x-10) = 0$

$x = 0$ or 10

2 c) i) $-6x$

2 ii) $xe^x + e^x$

$e^x(x+1)$

2 iii) $\dfrac{x\cos x - \sin x}{x^2}$

2 d) $\cancel{(x+3+\sqrt{2})}(x-3+\sqrt{2})$

$\cancel{x^2 - 3x + \sqrt{2}x + 3x\ldots}$

$x^2 - 6x + 7 = 0$

Moltchanski Kotsch

Q2 .a) $\dfrac{x(x-4)}{x-4} = x.$

1

1 b) $\dfrac{3 \times 180}{5} = 108^\circ$

c) $5\sqrt{3} + 4\sqrt{5} - 2\sqrt{3}$

$3\sqrt{3} + 4\sqrt{5} = 4\sqrt{c} + a\sqrt{3}$

2

$a = 3, \quad c = 5$

1 d) i) $\log(1+x) + c$

ii) $\left[\int 4e^{-2x}\right]_0^1$

$= \left[-2e^{-2x}\right]_0^1$

2 $= [-2e^{-2}] - [-2]$

$= -2/e^2 + 2.$

(e) $y = (3x+4)^3$

$\dfrac{dy}{dx} = 9(3x+4)^2$

at $x = -1$ $\dfrac{dy}{dx} = 9$

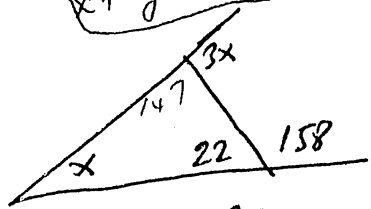
2 $(-1, 1)$ $-\frac{1}{9}$ $y - 1 = -\frac{1}{9}(x+1)$

$9y - 9 = \cancel{9x+9}$ $-x-1$

$\boxed{\cancel{9x - 9y + 18 = 0}}$

$x + 9y - 8 = 0$

(f)



2 $x + 22 = 3x$

$\boxed{x = 11.}$

Q3 (i) domain $-8 \le x \le 8$

[a] (i) range $0 \le y \le 8$

[b] [2]

$x + 3y = -7$

$4x - y = -2$

$13y = -26$

$y = -2$

$\cancel{x = \ldots}$ -1

$\boxed{x = -1 \text{ and } y = -2}$

c) $\dfrac{9}{\sin 30} = \dfrac{y}{\sin 45}$

$9 \times \dfrac{1}{\sqrt{2}} = \dfrac{1}{2} \times y$

[2] $y = 18/\sqrt{2}$

exact $y = 9\sqrt{2}.$

d) $\int_0^a (x-3)\,dx = -4$

(2) $\left[\frac{1}{2}x^2 - 3x\right]_0^a = -4$

e) i) $(4-a)(4+a)($

ii) $(4c-1)(c+4)($

Q4 a) [2] $3^x - 3^{x-1} = 54$

$3^x\left[1 - \frac{1}{3}\right] = 54$

$3^x = 81$

$x = 4$

b] (2) $\dfrac{\sin\theta}{-\sin\theta} = -1$

c] $a = 3$ $d = 2$

$81 = 3 + (n-1) \times 2$

(2) $81 = 3 + 2n - 2$

$n = 40$

[d] $\alpha + \beta = 4$

$\alpha\beta = 2$

(1) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha+\beta}{\alpha\beta} = \dfrac{4}{2} =$

(2) ii) $(\alpha+\beta)^2 - 2\alpha\beta$

$16 - 4 = 12$

e] $A = \int_0^3 x^2\,dy$

$A = \int_0^3 y^2\,dy$

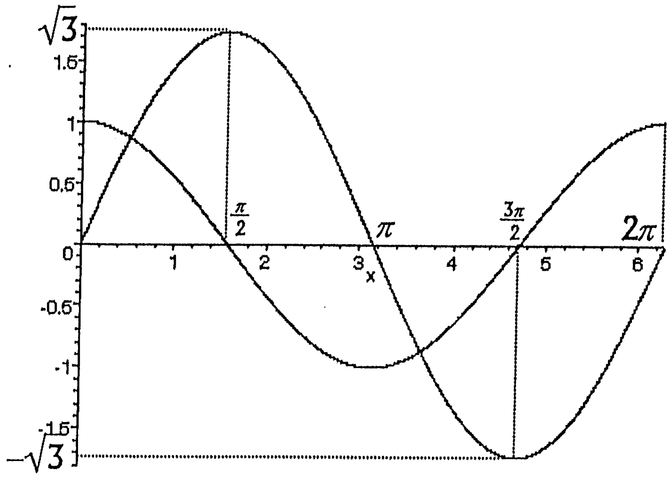
$A = \left[\frac{1}{3}y^3\right]_0^3$

$A = [9] - [0]$

$A = 9$ sq units.

Question 6

(a) (i)



(ii) $\sqrt{3}\sin x = \cos x \Rightarrow \dfrac{\sin x}{\cos x} = \dfrac{1}{\sqrt{3}} \Rightarrow \tan x = \dfrac{1}{\sqrt{3}}$

$x = \dfrac{\pi}{6}, \dfrac{7\pi}{6}$

(iii) $\dfrac{\pi}{6} < x < \dfrac{7\pi}{6}$ (We need $y = \cos x$ to be 'below' $y = \sqrt{3}\sin x$)

(b)

x	y	w	$y \times w$
0	0	1	0
$0\cdot5$	$0\cdot32$	2	$0\cdot64$
1	$0\cdot39$	2	$0\cdot78$
$1\cdot5$	$0\cdot35$	2	$0\cdot7$
2	$0\cdot26$	1	$0\cdot26$
			$\sum(y \times w) = 2\cdot38$

$h = 0\cdot5$

$$\int_0^2 f(t)\,dt \cong \frac{h}{2} \times 2\cdot38 = 0\cdot6$$

(c) (i) Area $\Delta ABC = \tfrac{1}{2} \times 6^2 \times \sin 30^\circ = 9$

(ii) $30^\circ = \dfrac{\pi}{6}^{c}$

Sector $ABC = \tfrac{1}{2} \times 6^2 \times \dfrac{\pi}{6} = 3\pi$

Shaded area = $3\pi - 9$ cm^2

Question 5

(a) $(x-p)^2 = 4a(y-q)$ is the parabola with vertex (p,q) and focal length $|a|$

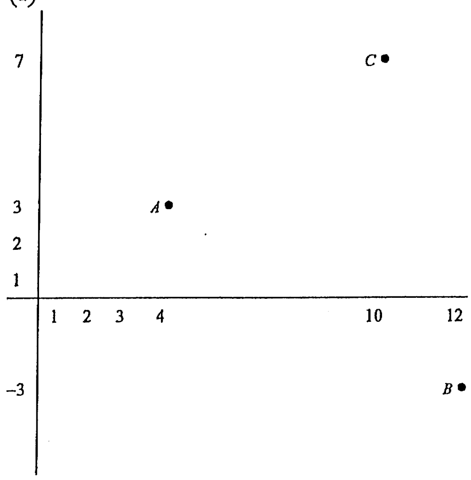
$(x+2)^2 = 8(y-1) \Rightarrow$ vertex $(-2,1)$ & $a=2$

(i) focus: $(-2, 1+a) = (-2,3)$

(ii) directrix: $y = 1 - a = -1$

(b)

(i)



(ii) $m_{AB} = \dfrac{-3-3}{12-4} = -\frac{3}{4}$

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{3}{4}(x-4) \Rightarrow y = -\frac{3}{4}x + 3 + 3$

$y = -\frac{3}{4}x + 6 \Leftrightarrow 3x + 4y - 24 = 0$

(iii) $d = \dfrac{|Ax_c + By_c + C|}{\sqrt{A^2+B^2}}$

$3x + 4y - 24 = 0 \Rightarrow A = 3, B = 4, C = -24$

$C(10,7) = (x_c, y_c)$

$d = \dfrac{|3\times 10 + 4\times 7 - 24|}{\sqrt{3^2+4^2}} = \dfrac{34}{5}$

(iv) $AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} = \sqrt{(4-12)^2 + (3-(-3))^2} = \sqrt{100} = 10$

Area $= \frac{1}{2} \times 10 \times \frac{34}{5} = 34$

(c) (i) $CL^2 = AC^2 + AL^2 - 2\times AL \times AC \times \cos 25°45'$

$CL^2 = 130^2 + 280^2 - 2\times 130 \times 280 \times \cos 25°45'$

$CL \cong 172\cdot 4\,\text{km}$

(ii) Let $\theta = \angle CLA$, $\angle CAL = 25°45'$

$\dfrac{\sin\angle CLA}{AC} = \dfrac{\sin\angle CAL}{CL} \Rightarrow \sin\theta = \dfrac{\sin 25°45'}{172\cdot 4} \times 130$

$\therefore \theta = 19°7'$

Bearing $= 270° + \theta = 289°7'\text{T} = N70°53'W$

(b) $y=\sqrt{9-3x} \Rightarrow y^2=9-3x \Rightarrow 3x=9-y^2 \Rightarrow 9x^2=\left(9-y^2\right)^2$

$$V=\pi\int_{y=a}^{y=b} x^2 dy$$

$$=\tfrac{1}{9}\times\pi\int_0^3 9x^2 dy$$

$$=\tfrac{\pi}{9}\int_0^3 \left(9-y^2\right)^2 dy$$

$$=\tfrac{\pi}{9}\int_0^3 \left(81-18y^2+y^4\right) dy$$

$$=\tfrac{\pi}{9}\left[81y-6y^3+\tfrac{1}{5}y^5\right]_0^3$$

$$=\tfrac{\pi}{9}\left(\frac{648}{5}\right)=\frac{72\pi}{5}\text{ c.u.}$$

(c) $V=\dfrac{\pi t^3}{3}-\dfrac{\pi t^2}{6}+\dfrac{1}{2} \Rightarrow \dfrac{dV}{dt}=\pi t^2-\dfrac{\pi t}{3}$

$t=3, \dfrac{dV}{dt}=\pi\times 9-\pi=8\pi \text{ cm}^3/\text{s}$

Question 7

(a) (i) $y = 2x^3 + 3x^2 - 12x - 9$

$y' = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6(x+2)(x-1)$

$y'' = 12x + 6 = 6(2x+1)$

Stationary points when $y' = 0 \Rightarrow x = -2, 1$

$x = -2 \Rightarrow y = 11, y'' = -18 \Rightarrow (-2, 11)$ is a rel. max.

$x = 1 \Rightarrow y = -16, y'' = 18 \Rightarrow (1, -16)$ is a rel. min.

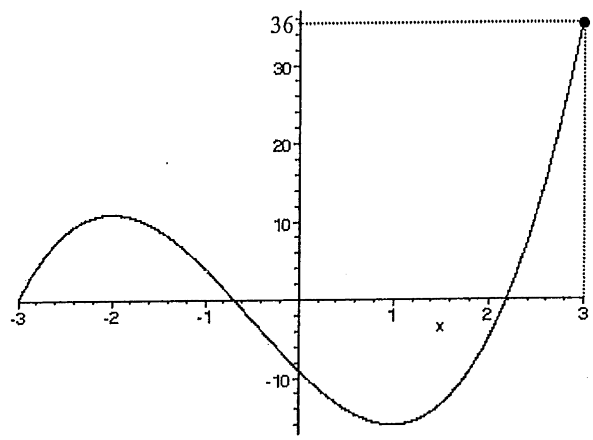
(ii) P.O.I. if $y'' = 0 \Rightarrow x = -\frac{1}{2} \Rightarrow y = -2\frac{1}{2}$ **AND** a change of concavity

x	-1	$-\frac{1}{2}$	0
y''	-6	0	6

So $(-\frac{1}{2}, -2\frac{1}{2})$ is a P.O.I

(iii) $x = -3, y = 0$ & $x = 3, y = 36$

$y-$intercept $(0, -9)$



(iv) From the graph, it is increasing and concave down for $x < -2$

In the domain for (iii) it would be $-3 \le x < -2$

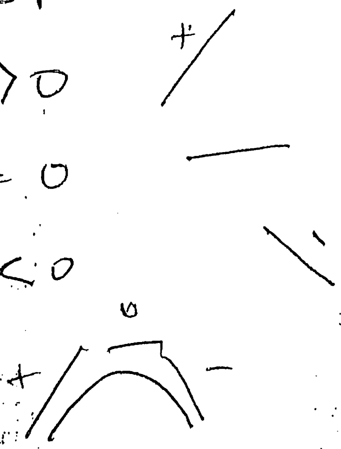
Question 9 (contd)

b) (i) Relative Maximum Turning point.

$f'(4^-) > 0$

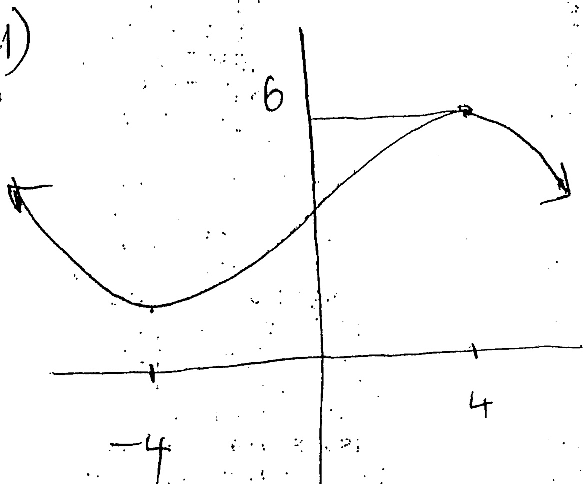
$f'(4) = 0$

$f'(4^+) < 0$



[2]

(ii)



[2]

c) $f = \sqrt{t} - \frac{1}{\sqrt{t}}$

When $t=0$, $v = \frac{4}{3}$, $x = \frac{4}{3}$

(i) $\ddot{x} = t^{\frac{1}{2}} - t^{-\frac{1}{2}}$

$\dot{x} = \int (t^{\frac{1}{2}} - t^{-\frac{1}{2}})\,dt + C$

$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$

$\dot{x} = \frac{2}{3}t^{\frac{3}{2}} - 2\sqrt{t} + C$

From initial conditions:

$\frac{4}{3} = 0 + 0 + C$ [2]

$\therefore v = \dot{x} = \frac{2}{3}t\sqrt{t} - 2\sqrt{t} + \frac{4}{3}$

(ii) $x = \int \left(\frac{2}{3}t^{\frac{3}{2}} - 2t^{\frac{1}{2}} + \frac{4}{3}\right) dt + D$

$= \frac{2}{3}\frac{t^{\frac{5}{2}}}{\frac{5}{2}} - \frac{2t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{4}{3}t + D$

$= \frac{4}{15}t^2\sqrt{t} - \frac{4}{3}t\sqrt{t} + \frac{4}{3}t + D$

By initial conditions $D = \frac{4}{3}$

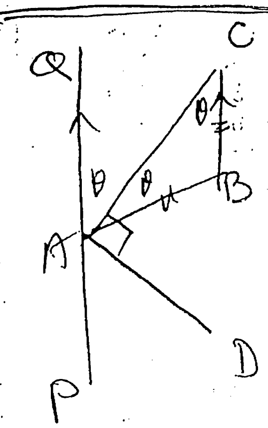
$\therefore x = \frac{4}{15}t^2\sqrt{t} - \frac{4}{3}t\sqrt{t} + \frac{4}{3}t + \frac{4}{3}$

When $t = 1$

$x = \frac{4}{15} - \frac{4}{3} + \frac{4}{3} + \frac{4}{3}$

$= \underline{\frac{8}{5}}$ m [3]

Question 10



(i) Let $\angle ACB = \theta$.

$\therefore \angle BAC = \theta$ (Isosceles Δ)

SBHS THSC Maths 03

Section C

Question 8

(i) Let n be the no. of tickets

$p(\text{both}) = \frac{n}{100} \times \frac{n-1}{99} = \frac{2}{275}$

$\therefore \frac{n(n-1)}{9900} = \frac{2}{275}$

$n^2 - n = 72$

$n^2 - n - 72 = 0$

$(n-9)(n+8) = 0$

$\therefore n = 9$ or -8

(-8 is extraneous)

2] $\therefore n = 9$

$\therefore$ Frank bought 9 tickets.

(ii) $p(\text{at least 1 prize}) = 1 - p(\text{no prize})$

$= 1 - \frac{91}{100} \times \frac{90}{99}$

2] $= \frac{19}{110}$

b) (i) $A_{12} = 12000 - 12M$

1]

(ii) $A_{13} = A_{12} \times 1.01 - M$

(Monthly interest = 1%)

$= (12000 - 12M)1.01 - M$

$A_{14} = A_{13} \times 1.01 - M$

$= (12000 - 12M)1.01^2 - M \times 1.01 - M$

[2] $= (12000 - 12M)1.01^2 - M(1 + 1.01)$

(iii)

$A_{48} = (12000 - 12M)(1.01)^{36} - M\underbrace{(1 + 1.01 + \dots + 1.01^{35})}$

AS: $a = 1, r = 1.01, n =$

$= (12000 - 12M)1.01^{36} - \frac{M(1.01^{36} - 1)}{1.01 - 1}$ [

(iv)

But $A_{48} = 0$

$\therefore (12000 - 12M)1.01^{36} = \frac{M(1.01^{36} - 1)}{1.01 - 1}$

$12000 \times 1.01^{36} = 12M \times 1.01^{36} + \frac{M(1.01^{36} - 1)}{1.01 - 1}$

$12000 \times 1.01^{36} \times 0.01 = 12M \times 1.01^{36} \times 0.01 + M(1.01^{36} - 1)$

$= M(12 \times 1.01^{36} \times 0.01 + 1.01^{36} -$

$M = \frac{12000 \times 1.01^{36} \times 0.01}{12 \times 1.01^{36} \times 0.01 + 1.01^{36} - 1}$

$= \frac{171.692254}{0.60246}$

$\doteqdot \$284.98$ [3

Question 9

(a) $\log_3 x - \log_3 (x-2) = \frac{2}{3} \log_3 27$

$\therefore \log_3 \left(\frac{x}{x-2}\right) = \log_3 27^{\frac{2}{3}}$

$\therefore \frac{x}{x-2} = 9 \qquad x \neq 2$

$x = 9x - 18$

$8x = 18$

$x = \frac{18}{8} = \frac{9}{4}$ [3

Question 10 (Contd)

But $\angle QAC = \angle ACB$
$= \theta$ (alternate $\angle$'s)

$\therefore \angle QAC = \angle CAB$

[2] $\therefore$ AC bisects $\angle QAB$.

(ii) Now $\angle QAP = 180°$ (str. $\angle$)

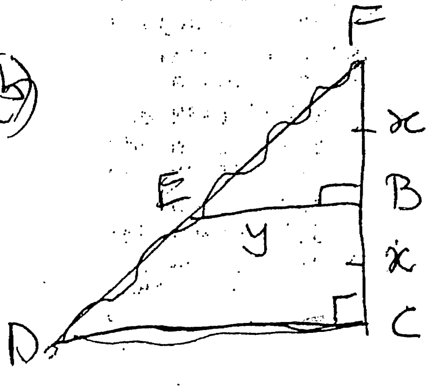
$\therefore \angle PAD = 180 - 90 - \theta$
$= 90 - \theta$

But $\angle CAD = \angle CAB + \angle BAD$
ie $90° = \theta + \angle BAD$

$\therefore \angle BAD = 90 - \theta$
$= \angle PAD$
(See above)

[2] $\therefore$ AD bisects $\angle PAB$

(b)



(i)(α) BC = x

$\frac{DC}{EB} = \frac{FC}{BC}$ (||| Δ's)

$\frac{DC}{y} = \frac{2x}{x}$

$\therefore DC = 2y$ [2]

(β) $A = \frac{1}{2}(2x)(2y)$
$= 2xy$ [1]

(γ) New Fencing
$L = 2x + 3y$ [1]

(ii) $1200 = 2xy$
$\therefore y = \frac{600}{x}$

$\therefore L = 2x + 3 \times \frac{600}{x}$
$= 2x + \frac{1800}{x}$ [2]

(iii) $\frac{dL}{dx} = 2 - \frac{1800}{x^2}$

$\frac{d^2L}{dx^2} = \frac{3600}{x^3}$

Now $\frac{dL}{dx} = 0$ for $\frac{2x^2 - 1800}{x^2} = 0$
ie $2x^2 - 1800 = 0$
$x^2 - 900 = 0$
$(x-30)(x+30) = 0$
$x = 30$ or -30
(-30 is extraneous)
2nd derivative > 0 for $x > 0$.
$\therefore$ Minimum L for $x = 30$
$y = 20$ I