Extension I

Higher School Certificate
TRIAL EXAMINATION
2006

General Instructions

- **Reading time – 5 minutes**
- **Working time – 2 hours**
- **Use Board approved calculators**
- **All necessary working should be shown in every question if full marks are to be awarded.**
- **Marks may NOT be awarded for messy or badly arranged work**
- **Use writing booklets provided**
- **ALL questions are NOT of equal Value.**

Total Marks – 74 Marks

Examiner: ***Patrick Loi***

Disclaimer: **This does not necessarily reflect the content or format of the Higher School Certificate.**

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Question 1 *(12 Marks)* **Marks**

(a) **Evaluate,**

$$\int_1^4 \frac{1+2\sqrt{x}}{\sqrt{x}}\,dx \qquad \text{using } u=1+2\sqrt{x}$$ **2**

in simplest form.

(b) **Let A (-3, 6) to B (1, 10) be points on the number plane. Find the coordinates of the point C, which divides the interval AB externally in the ratio 5 : 3.** **2**

(c) **Find the size of the acute angle between the line $y = -x$ and $\sqrt{3}y = 2x$.** **2**

(answer to the nearest minute)

(d) **Evaluate** $\lim_{x\to 0}\frac{\sin 2x}{\sin x}$ **2**

(e) **Given that $\log_b pq = 1.544$ and $\log_b qr = 1.113$, find the value of $\log_b(\frac{p}{r})$** **2**

(f) **How many different arrangements can be made using the thirteen letters of the word** **2**

O T E C E D U C A T I O N

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Question 2 (12 Marks)

Marks

(a) **(i)** **Prove that $e^{2x} - e^{x} = 56$ has a root between 2 and 3.** **2**

(ii) **Determine whether this root lies closer to 2 or 3.** **1**

(iii) **Take $x = 2$ as an approximation to this root and use Newton's method to find this root correct to three significant figures** **2**

(b) **Prove that**

$$\frac{\cos 2\theta}{\cos\theta} + \frac{\sin 2\theta}{\sin\theta} = \frac{4\cos^2\theta - 1}{\cos\theta}$$ **3**

(c) **Consider the function $f(x) = \cos^{-1}(2x) - \frac{\pi}{2}$**

(i) **State the domain and range of $f(x)$** **2**

(ii) **Sketch the graph of $f(x)$** **2**

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Question 3 *(12 Marks)*

Marks

(a) **A cup of hot coffee at temperature T degrees Celsius loses heat when placed in cooler environment. It cools according to the law $\frac{dT}{dt} = k(T - T_0)$ where time, t is the time elapsed in minutes and T_o is the temperature of the environment in degrees Celsius**

(i) **A cup of coffee at 100^0C is placed in an environment at -20^oC for 4 minutes and than cools to 70^oC. Find k.** **2**

(ii) **The same cup of coffee at 70^oC is then placed in an environment at 20^oC assuming k stays the same, find the temperature of the coffee after a further 15 minutes.** **3**

(b) **The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. The equation of chord PQ is given by $y - ap^2 = \frac{p+q}{2}(x - 2ap)$.**

(i) **If PQ is a focal chord show that $pq = -1$** **1**

(ii) **Find M, the midpoint of PQ** **1**

(iii) **Find the equation of the locus of M** **2**

(c) **If $y = \frac{\log_e x}{x}$**

(i) **find $\frac{dy}{dx}$** **1**

(ii) **Hence, show that $\int_e^{e^2} \frac{1 - \log_e x}{x \log_e x} = \log_e 2 - 1$** **2**

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Question 4 *(12 Marks)*

Marks

(a)

(i) **Show that** $\int_0^{\pi/4} \cos^2 x\,dx = \frac{\pi + 2}{8}$ **2**

(ii) **The region under the curve** $y = \cos x + \sec x$ **, above the x-axis and between** $x = 0$ **and** $x = \frac{\pi}{4}$ **, makes a revolution about the x-axis.**

Show that the volume of the solid traced out is $\frac{5\pi(\pi + 2)}{8}$ **units**^{3} **3**

(b) **The velocity of a point moving along x-axis is given by** $v^2 = 16x - 4x^2 + 20$ **.**

(i) **Show that** $\ddot{x} = -4(x - 2)$ **2**

(ii) **State the centre and period of the motion** **1**

(iii) **What is the amplitude of the motion** **2**

(iv) **Find the maximum speed of the particle** **2**

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Question 5 *(12 Marks)*

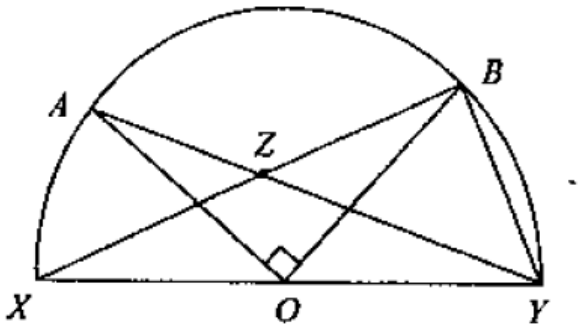
Marks

(a) **Prove, using mathematical induction, that $7^n - 4^n$ is divisible by 3, where n is a positive integer.** **2**

(b) **(i) Express $\sqrt{3}\cos x - \sin x$ in the form $R\cos(x + \alpha)$ where $R > 0$ and α is acute.** **2**

(ii) Hence, solve $\sqrt{3}\cos x - \sin x = \sqrt{2}$ for $0 \le x \le 2\pi$ **2**

(c)



Not to scale

O is the centre of the semicircle, diameter XY.
OA and OB is perpendicular, AY and XB bisect at Z.

(i) **Explain why $\angle AYB = 45^o$** **1**

(i) **Prove that $BY = BZ$** **2**

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Question 6 (12 Marks)

Marks

(a)

(i) Consider the polynomial $P(x) = x^3 - 5x + c$ **2**

(ii) Find the value of c if $x + 2$ is a factor of $P(x)$. **2**

For this value of c, find $Q(x)$ such that $P(x) = (x + 2)Q(x)$

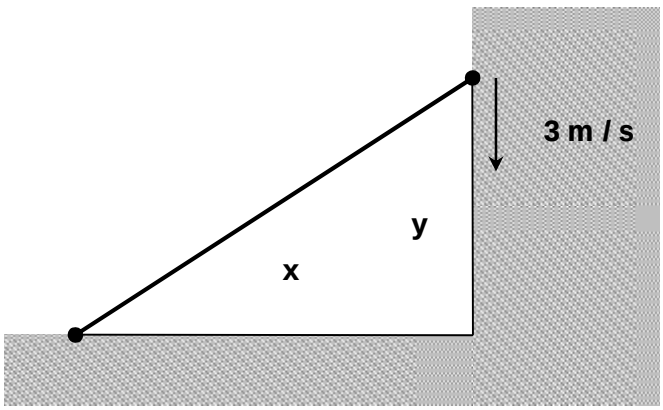
(b) If α, β and γ are the roots of $2x^3 + 3x^2 + x + 5 = 0$.

(i) Find $\alpha^2 + \beta^2 + \gamma^2$ **2**

(ii) Find $\alpha^3 + \beta^3 + \gamma^3$ **2**

(iii) Find the values of $(\alpha + 1)(\beta + 1)(\gamma + 1)$ **1**

(c) A ladder is slipping down a vertical wall. If the ladder is 4 m long and its top is slipping at 3 m/s, how fast is the bottom of the ladder moving along the ground when it is 2 m from the wall. **3**



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Question 7 *(12 Marks)*

Marks

(a) **By considering the expansion of $\sin(X+Y)-\sin(X-Y)$** **3**

Prove that $\sin A-\sin B=2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})$

(b) **Also given that $\cos A-\cos B=2\sin(\frac{A+B}{2})\sin(\frac{B-A}{2})$ prove that**

$$\frac{\sin A-\sin B}{\cos A-\cos B}=-\cot(\frac{A+B}{2})$$ **2**

(c) **Prove that the position of a projectile t seconds after projection from ground level with initial horizontal and vertical velocity components of $V\cos\alpha$ and $V\sin\alpha$ respectively is given by**

$x=Vt\cos\alpha$ and $y=-\frac{1}{2}gt^2+Vt\sin\alpha$. **2**

(d) **Two objects P and Q are projected from the same ground position at the same time with initial speed V m/s at angle α and β respectively ($\beta>\alpha$).**

(i) **If at time t seconds the line join P and Q makes an acute angle of θ with the horizontal**

prove that $\tan\theta=\left|\frac{\sin\beta-\sin\alpha}{\cos\beta-\cos\alpha}\right|$ **3**

(ii) **Hence show that $\theta=\frac{1}{2}(\pi-\alpha-\beta)$** **2**

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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad \boldsymbol{a} \neq 0$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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