Student Name: ....

2003
TRIAL HIGHER SCHOOL CERTIFICATE

MATHEMATICS
Extension 1

General Instructions

Reading Time: 5 minutes
Working Time: 2 hours

- Attempt all questions
- Start each question on a new page
- Each question is of equal value
- Show all necessary working.
- Marks may be deducted for careless work or incomplete solutions
- Standard integrals are printed on the last page
- Board-approved calculators may be used
- This examination paper must not be removed from the examination room

QUESTION 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\frac{d}{dx}(x^2\sin^2 x)$. 2

(b) Write down the Cartesian equation of the locus of a point $P(x, y)$ where $x = 2\cos\theta$ and $y = \frac{1}{2}\sin\theta$. 2

(c) Find the general solution, in terms of π, to $2\sin x + 1 = 0$. 2

(d) The interval AB has end points $A\ (2, 4)$ and $B\ (x, y)$. The point $P\ (-1, 1)$ divides AB internally in the ratio 3 : 4. Find the coordinates of B. 2

(e) If $P(x) = 5x^3 - 3x + k$ has a remainder of 7 when $P(x)$ is divided by $(x + 2)$, find the value of k. 2

(f) Use the table of standard integrals to find the exact value of $\int_0^{\frac{\pi}{6}} \sec 4x \tan 4x\, dx$. 2

QUESTION 2. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Sketch $y = \frac{\pi}{2} + \cos^{-1}\frac{x}{2}$. **2**

(b) Solve $\frac{x^2 - 2}{x} \leq 1$. **4**

(c) Find, correct to the nearest degree, the acute angle between the lines $x + y - 3 = 0$ and $2x - y + 2 = 0$. **2**

(d) Use the substitution $u = x - 2$ to find the exact value of $\int_1^3 x(x-2)^5 dx$. **4**

QUESTION 3. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Write down the expansion of $\tan(A+B)$. 1

(ii) Hence, find the value of $\tan\left(\frac{7\pi}{12}\right)$ as a simplified surd with a rational denominator. 2

(b) Use one application of Newton's method to find an approximate root to the equation $x - \tan^{-1} 2x = 0$ that lies close to $x = 1$. Write your answer correct to two significant figures. 3

(c) (i) Show that the equation of the normal to the parabola $x^2 = 4ay$ at the point $(2ap, ap^2)$ is $x + py = 2ap + ap^3$. 2

(ii) Derive the equation of the line that passes through the focus $S(0, a)$ and is perpendicular to the normal. 1

(iii) If the line in (c) (ii) meets the normal at N, find the coordinates of N. 2

(iv) Find a Cartesian equation for the locus of N. 1

QUESTION 4. (12 marks) Use a SEPARATE writing booklet. Marks

(a) Determine the exact value of $\cos\left(\sin^{-1}\left(-\frac{12}{13}\right)\right)$. 2

(b) A golf ball is hit towards a tree 60 metres away and standing in the same horizontal plane as the ball. The tree is 20 metres high. The initial velocity of the ball is 30 *m/s* and the angle of projection θ.

(i) Show that the vertical distance travelled by the ball is $y = 30t \sin\theta - 5t^2$. (take $g = 10m/s^2$). 1

(ii) Show that the horizontal distance travelled by the ball is $x = 30t \cos\theta$. 1

(iii) Find the range of angles (θ) in which the ball must be hit to clear the tree. 3

(c) Newton's law of cooling states that the rate of cooling of a body is proportional to the excess of the temperature of a body above the surrounding temperature.

The temperature of a cup of chocolate drink satisfies an equation of the form $T = B + Ae^{-kt}$ where T is the temperature of the drink, t is time in minutes, A and k are constants and B is the temperature of the surroundings.

The drink cools from 90°C to 80°C in two minutes in a room of temperature 25°C.

(i) Find the values of A and k. 3

(ii) Find the temperature of the cup of chocolate, to the nearest degree, after a further five minutes have passed. 2

QUESTION 5. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Given the polynomial $P(x) = 2x^3 - 9x^2 + kx + 6$,

(i) find the value of k if $(x - 3)$ is a factor of $P(x)$. 1

(ii) Hence, or otherwise, determine all the roots of the equation $P(x) = 0$. 3

(b) A particle is moving in simple harmonic motion. Its velocity v m s^{-1} is given by $v^2 = 15 + 4x - 4x^2$.

(i) Find an expression for the acceleration, $\ddot{x}$, of the particle in terms of x. 1

(ii) Find the centre, amplitude and period of the motion. 3

(c) Use mathematical induction to show that $\cos(x + n\pi) = (-1)^n \cos x$ for all positive integers $n \geq 1$ 4

Marks

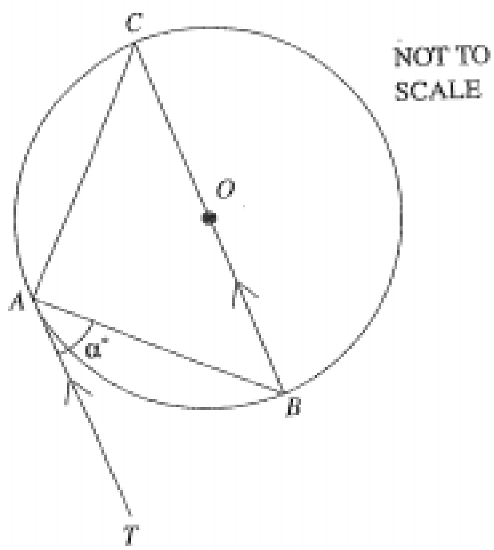
QUESTION 6. (12 marks) Use a SEPARATE writing booklet.

(a) Consider the function $f(x) = \dfrac{1}{1 + x^2}$.

(i) What is the largest domain containing $x = 1$ for which $f(x)$ has an inverse function? 1

(ii) Find an expression for $f^{-1}(x)$. 2

(b) 4



In the diagram, A, B and C are points on the circle with centre O. The line AT is a tangent to the circle at A and is parallel to the diameter CB. Angle $TAB = \alpha^\circ$.

Find the value of α° giving reasons.

(c) A surveyor observes two towers, one due north of height $80m$ and the other on a bearing of θ ($<90^\circ$) of height $120m$. The angles of elevation of the two towers are 40° and 36° respectively. The towers are $150m$ apart on a horizontal plane.

(i) Find an expression in terms of $\tan 50^\circ$ for the distance of the surveyor from the base of the first tower. 1

(ii) Find an expression in terms of $\tan 54^\circ$ for the distance of the surveyor from the base of the second tower. 1

(iii) Calculate the value of θ to the nearest minute. 3

QUESTION 7. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $3\cos\theta - \sqrt{3}\sin\theta + 3 = 0$ for $0 \le \theta \le 2\pi$. **4**

(b) A car leasing company provides finance to customers. Clients can borrow $\$P$ at $r\%$ per month reducible interest, calculated monthly. The loan is to be repaid in equal monthly payments of $\$M$.

Let $R = \left(1 + \frac{r}{100}\right)$ and let $\$A_n$ be the amount owing after n monthly repayments have been made.

(i) Write an expression for the amount owing after two repayments, A_2, in terms of P, R and M. **1**

(ii) Show that the amount owing after the nth repayment is given by **2**

$$A_n = PR^n - \frac{M(R^n - 1)}{R - 1}.$$

(iii) If the amount owing after the nth repayment is $K\%$ of the amount borrowed, show that **3**

$$R^n = \frac{PK(R-1) - 100M}{100[P(R-1) - M]}.$$

(iv) Hence, find the minimum number of years required for the amount owing to fall to 20% of the amount borrowed, if a client borrows \$40 000 and undertakes to make equal monthly payments of \$800. Interest is charged at 9% per annum compounding monthly. **2**

End of paper

Standard integrals

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right).$$

Note: $\ln x = \log_e x, \quad x > 0$