The Scots College
2002

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 1

General Instructions
- Reading time – 5 minutes.
- Working time – 2 hours.
- Write using black or blue pen.
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown in every question.
- Answer each question in a SEPARATE writing booklet.
- Extra writing booklets are available.
- Question Papers are to be handed in.

Total marks – 84
- Attempt Questions 1–7.
- All questions are of equal value.

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Question 1 (12 marks)

Marks

a. Differentiate $4x\sin^{-1}x$ **2**

b. If $\int_{-a}^{a}\frac{dx}{1+x^2}=\frac{\pi}{2}$ Find the value of a. **2**

c. Find the coordinates of the point P that divides the interval joining (-4, 3) and (2, -7) externally in the ratio 4:3. **2**

d. If $\log_a b = 2.8$ and $\log_a c = 4.1$, find $\log_a bc$. **1**

e. Solve for x: $\frac{x+2}{x}\leq 3,\ x\neq 0$. **2**

f. Evaluate $\int_{0.5}^{1}4x(2x-1)^5\,dx$ by making the substitution $u=2x-1$. **3**

End of Question 1

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

a. Taking $x = 0.5$ as a first approximation for the root of $\log_e x = -x$, use Newton's method to find a second approximation. (Answer correct to 3 significant figures.) **3**

b. Prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ **4**

c. Find the size of α and β in the following diagram (giving reasons). **2**

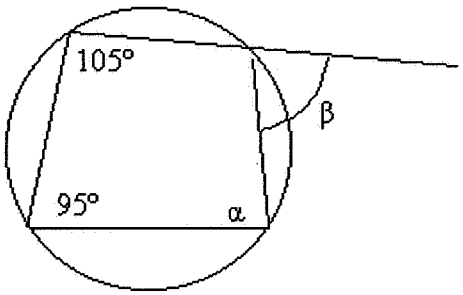


Diagram NOT to scale.

d.

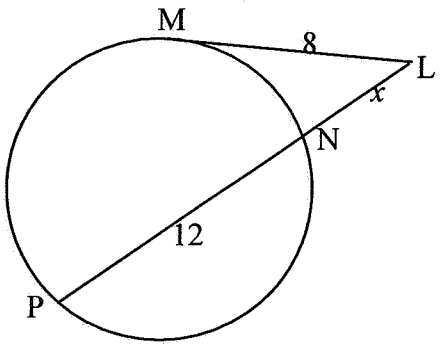


Diagram NOT to scale.

LM is a tangent to the circle, while LNP is a secant intersecting the circle at N and P. Given that LM = 8, NP = 12 and LN = x, find x. **3**

End of Question 2

Question 3 (12 marks) Use a SEPARATE writing booklet.

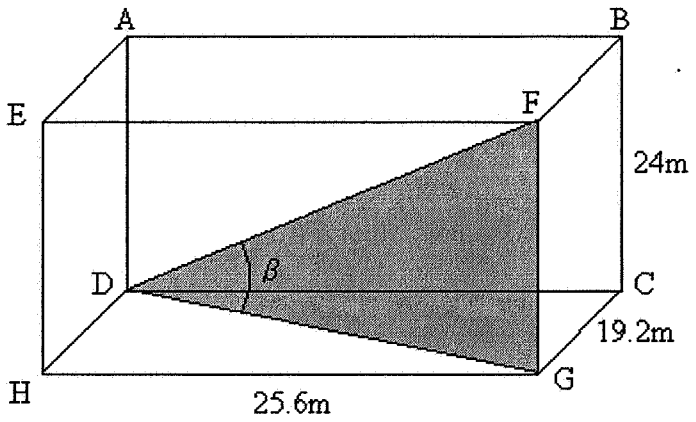
Marks

a. Find, for $0 \le x \le 2\pi$, all solutions for the equation $\cos x = \sin 2x$. **3**

b. If α, β and γ are the roots of the cubic polynomial equation $x^3 + 8x^2 - 4x - 6 = 0$ **3**
Find the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma}$.

c. Find the term independent of x in the expansion of $(3x^4 + \frac{1}{x})^{10}$ **3**

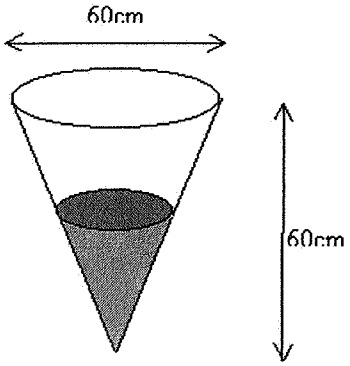
d. Calculate the value of β in the following rectangular prism. **3**
(Answer to the nearest minute.)



End of Question 3

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

a. Sand is being poured into a conical container at a constant rate of $36\text{cm}^3\text{s}^{-1}$. The height and diameter of the container are both 60cm. After t seconds the depth of the sand in the container is h cm.



i. Show that the depth of sand in the container after 5 seconds is 8.826cm correct to 2d.p. **2**

ii. Find the rate at which the depth of sand is changing after after 5 seconds. (answer to 3d.p.) **2**

iii. Find the rate at which the surface area, S, of the sand in the container is changing when the depth of the sand is 20cm. **2**

b. i. Find the domain and range of the function $y = 4\cos^{-1}\left(\frac{x}{3}\right)$. **1**

ii. Sketch the graph of the function $y = 4\cos^{-1}\left(\frac{x}{3}\right)$ showing clearly the intercepts on the coordinate axes and the coordinates of any endpoints. **2**

iii. Find the area of the region in the first quadrant bounded by the curve $y = 4\cos^{-1}\left(\frac{x}{3}\right)$ and the coordinate axes **3**

End of Question 4

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

a. The parabola given by $x = 2at$ and $y = at^2$ has points P and Q where $t = p$ and $t = q$ respectively.
Prove:

i. The equation of the chord PQ is given by $y - \frac{(p+q)x}{2} + apq = 0$ **2**

ii. The equation of the tangent at P is given by $y - px + ap^2 = 0$ **2**

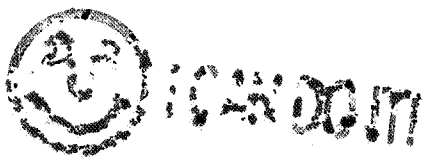
iii. The tangents at the ends of any focal chord meet on the directrix and are perpendicular to each other. **3**

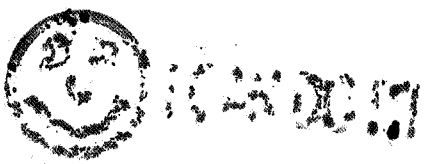
b. At time t the temperature $T°$ Celsius of a piece of iron in a room of constant temperature 30° Celsius, is decreasing according to the equation $\frac{dT}{dt} = -k(T - 30)$ for some constant k > 0.

i. Verify that $T = 30 + Ae^{-kt}$, A constant, is a solution of the equation. **2**

ii. The initial temperature, T, of the piece of iron is 100°C and it falls to 60°C after 20 minutes. Find the temperature of the body after a further 10 minutes. (Answer to nearest degree.) **3**

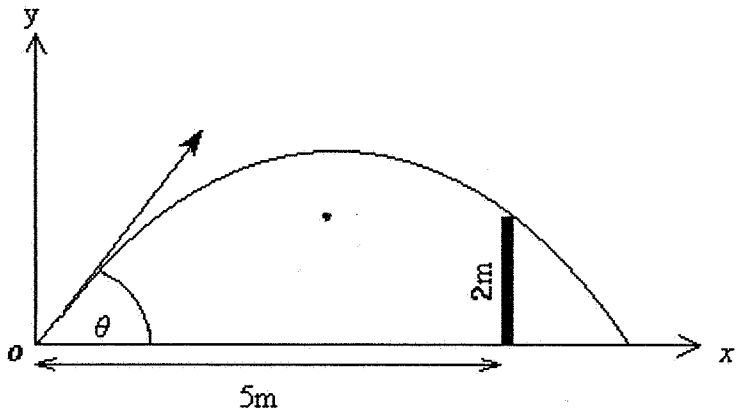
End of Question 5





Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

a. A ball is shot out of a small cannon on the ground at point O with velocity 15ms^{-1}, fired at an angle of θ with the horizontal.



i. Taking acceleration due to gravity to be a constant 10ms^{-2}, show that the equation for the horizontal (x) and vertical (y) components of the particles displacement from the origin O are given by $x = 15t\cos\theta$ and $y = -5t^2 + 15t\sin\theta$ **2**

ii. Show that the Cartesian equation for displacement is given by **2**

$$y = \frac{-x^2}{45}\sec^2\theta + x\tan\theta$$

iii. The ball just clears a 2 metre high fence that is 5 metres from the origin . Find two values of θ (measured in degrees) for this to happen. (Give your answer to the nearest minute) **3**

b. A particle is moving such that its acceleration is given by $\ddot{x} = -16x$.
The particle has an initial displacement of 3m and an initial velocity of 12ms^{-1}.

i. Show the velocity is given by $\dot{x} = \left|4\sqrt{18 - x^2}\right|$ **3**

ii. Find the equation for the particle displacement, x, over time t. **2**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

a. Use the principal of mathematical induction to prove that, for every positive integer n, $13 \times 6^n + 2$ is divisible by 5. **4**

b. A particle moves in a straight line. Its displacement x metres from the origin, after t seconds is given by $x = \sin^2 5t + 2$, $t > 0$.

i. Find the time when the particle is first at $x = \frac{5}{2}$. **2**

ii. In what direction is the particle travelling when it is first at $x = \frac{5}{2}$? **1**

iii. Express the acceleration of the particle in terms of x. **2**

iv. Hence, or otherwise, show that the particle is undergoing simple harmonic motion. **2**

v. State the period of the motion. **1**

End of paper