Sydney Girls High School

Trial Higher School Certificate

2001

Mathematics

Extension 1

Time Allowed – 2 hours
(Plus 5 minutes reading time)

Directions to Candidates Name ____________

- Attempt ALL questions
- ALL questions are of equal value
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work
- Board-approved calculators may be used
- Each question attempted should be started on a new sheet. Write on one side o the paper only

This is a trial paper ONLY. It does not necessarily reflect the format or the contents of the 2001 HSC Examination Paper in this subject.

Question 1

(a) Solve $\frac{4}{x-1} < 2$

(b) Differentiate $y = \tan^{-1} 4x$

(c) Find the coordinates of the point which divides the interval PQ where $P = (2, 5)$ and $Q = (6, 2)$ externally in the ratio 1:3

(d) Evaluate $\int_{-1}^{0} 2x\sqrt{1+x}\,dx$ using the substitution $u = 1 + x$

(e) Find $\int_{1}^{2} \frac{4}{\sqrt{4-x^2}}\,dx$

Question 2 **Marks**

(a) The polynomial $x^3 + mx^2 + nx - 18$ has $(x + 2)$ as one of its factors. Given that the remainder is -24 when the polynomial is divided by $(x - 1)$, find constants m and n. (3)

(b) A circular disc of radius r cm is heated. The area increases due to expansion at a constant rate of 3.2 cm^2 per minute. Find the rate of increase of the radius when $r = 20$ cm. (3)

(c) Solve the equation $\sin 2\theta = 2 \sin^2\theta$

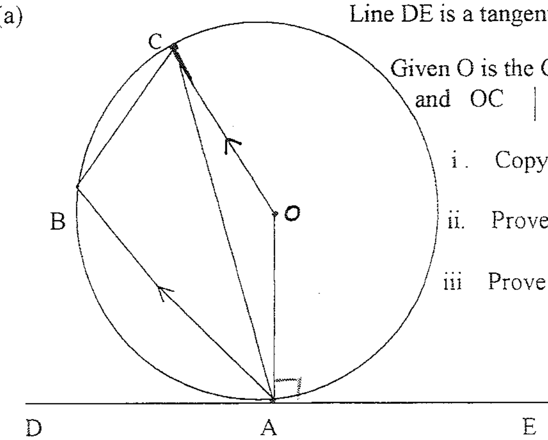
for $0 \le \theta \le 2\pi$ (3)

(d) For the function $y = 3 \sin^{-1} \frac{x}{2}$

(i) State the domain and range
(ii) Sketch the graph of this function (3)

Question 3

(a)



Line DE is a tangent to the Circle at Po

Given O is the Centre of the Circle and OC || AB

i. Copy this diagram

ii. Prove $\angle CAD = \angle E$

iii Prove $\angle CBA = 90^0 +$.

(b) Points P (2 ap, ap^2) and Q (2aq, aq^2) lie on the parabola $x^2 = 4ay$

i. Find the equation of chord PQ

ii. If PQ subtends a right angle at the origin, show that $pq = -4$

iii. Find the equation of the locus of the midpoint of PQ

(c) Taking a first approximation of $x = 0.6$ solve the equation $\tan x = x$ using 1 application of Newton's approximation.

Question 4 **Marks**

(a) For $y = 10^x$, find $\frac{dy}{dx}$ when $x = 1$ (2)

(b) Prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ (2)

(c) Two roots of the polynomial $x^3 + ax^2 + 15x - 7 = 0$ are equal and rational. Find a (3)

(d) For a falling object, the rate of change of its velocity is $\frac{dv}{dt} = -k(v - A)$ where k and A are constants. (5)

i. Show that $v = A + Ce^{-kt}$ is a solution of the above equation, where C = constant.

ii. If $A = 500$ then initial velocity is 0 and velocity when $t = 5$ seconds is 21 m/s. Find C and k

iii. Find the velocity when $t = 20$ seconds

iv. Find the maximum velocity as t approaches infinity.

Question 5

(a) Find the term of the expansion $\left(\frac{2}{x^3} - \frac{x}{3}\right)^8$ which is independent of x

(b) A particle is moving in S.H.M. with acceleration $\frac{d^2x}{dt^2} = -4x$ m/s^2

The particle starts at the origin with a velocity of 3 m/s.

Find i. the period of the motion
ii. the amplitude
iii. the speed as an exact value
when the particle is 1m from the origin

(c) Prove by mathematical induction that the expression $(13 \times 6^n + 2)$ is divisible by 5 for all positive integers $n \geq 1$

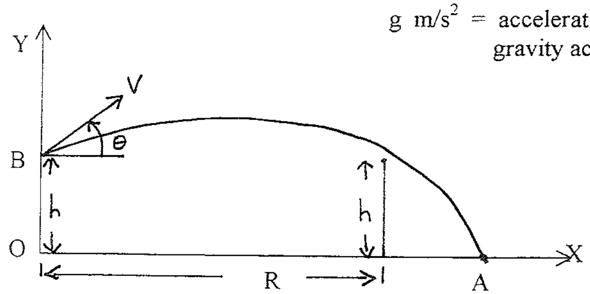
(d) Solve $\sqrt{3}\ \sin\theta - \cos\theta = 1$ for $0 \leq \theta \leq 2\pi$

Question 6 **Marks**

(a) Find the acute angle between the lines $x + y = 0$ and $x - \sqrt{3}\ y = 0$ (3)

(b) Show that $\dfrac{2\sin^3 x + 2\cos^3 x}{\sin x + \cos x} = 2 - \sin 2x$ (3)

if $\sin x + \cos x \neq 0$



OC = R metres

$g\ m/s^2$ = acceleration due to gravity acting downwards

A ball is hit from point B which is h metres above the ground level (OX) at an angle of θ from the horizontal level with initial velocity V m/s
DC represents a fence also of height h metres.

i. Show that the position of the ball at time t secs is given by

$$x = Vt\cos\theta$$
$$y = Vt\sin\theta - \frac{1}{2}gt^2 + h$$ (2)

ii. Hence show that the equation of flight of the ball is given by

$$y = h + 2\tan\theta - \frac{x^2 g}{2V^2\cos^2\theta}$$ (2)

iii. If the ball clears the fence DC, show that $V^2 \geq \dfrac{gR}{2\sin\theta\cos\theta}$ (2)

Question 7

(a) Use the identity $(1 + x)^n = (1 + x)(1 + x)^{n-1}$ to prove that
$^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$

(b) A car rental company rents 200 cars per day when it sets its hiring rat at \$30 per car for each day.
For every \$1 increase in the hiring rate, 5 fewer cars are rented per d
i. What rate will produce the maximum income per day?
ii. Find the maximum possible income per day.

(c) On a building construction site, an object falls from a crane in a vertical straight line. The object passes a 2 metre high window in a time interval of one tenth of 1 second.
Find the height above the top of the window from which the object was dropped
(Take $g = 9.8\ ms^{-2}$)

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$