SAINT IGNATIUS' COLLEGE

Trial Higher School Certificate

2004

MATHEMATICS EXTENSION 1

1:25pm – 3:30 pm
Thursday 19th August 2004

Directions to Students

- Reading Time : 5 minutes
- Working Time : 2 hours
- Write using blue or black pen. (sketches in pencil).
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown in every question.
- **Answer each question in the booklets provided and clearly label your name and teacher's name.**

- Total Marks 84
- Attempt Question 1 – 7
- All questions are of equal value

Total marks (84)
Attempt Questions 1 – 7
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

QUESTION 1 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $\dfrac{5}{2x-1} < 3$. **3**

(b) Find the acute angle between the lines $2x - y + 1 = 0$ and $x + 3y - 4 = 0$. **3**
Give answer to the nearest degree.

(c) Find the coordinates of the point that divides the interval joining $(-2, 5)$ and $(8, -9)$ internally in the ratio $2 : 3$. **2**

(d) If α, β, γ are the roots of the equation $x^3 - 5x^2 - 3x + 2 = 0$, **2**
find the value of $\dfrac{1}{\alpha\beta} + \dfrac{1}{\beta\gamma} + \dfrac{1}{\gamma\alpha}$.

(e) Write down the general solution, in terms of π, of the equation **2**

$$\cos\theta = -\frac{1}{2}.$$

QUESTION 2 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Use the substitution $x = u^2 + 1$, for u > 0, to evaluate **4**

$$\int_1^5 (x+1)\sqrt{x-1}\,dx.$$

(b) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sin^2(\tfrac{1}{2}x)\,dx$. **3**

(c) Prove, using the principle of mathematical induction, that $9^{n+2} - 4^n$ is divisible by 5, for n a positive integer. **5**

QUESTION 3 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the exact value of $\tan\left(2\sin^{-1}\dfrac{3}{4}\right)$. **3**

(b) Consider the function $f(x) = \sin^{-1}(x+1) + \dfrac{\pi}{2}$.

(i) What is the domain of $f(x)$? **1**

(ii) Sketch the graph of $y = f(x)$. **2**

(c) Consider the function $f(x) = \log_e(2x+1)$.

(i) Write down the domain of $f(x)$. **1**

(ii) Find the inverse function of $f(x)$, and write it in the form $f^{-1}(x) = \ldots\ldots$ **2**

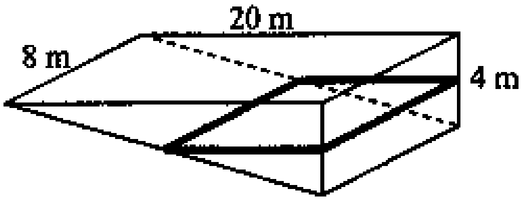
(iii) Find the gradients of the graphs of $y = f(x)$ and $y = f^{-1}(x)$ at the origin. **1**

(iv) On the same diagrams, draw the graphs of $y = f(x)$ and $y = f^{-1}(x)$. **2**

QUESTION 4 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the coefficient of x^3 in the expansion of $(2-x)(1+x)^5$. **3**

(b)



NOT TO SCALE

A swimming pool is 20 metres long, 8 metres wide, 4 metres deep at one end, and zero depth at the other end. The floor of the pool is a plane rectangular surface.

(i) When the depth of water at the deeper end is h metres, show that the volume (V m^3) of water in the pool is given by $V = 20h^2$. **2**

(ii) If water is being poured into the pool at the rate of 2 m^3/minute, find the rate at which the depth of the water is increasing at the deepest end, when the depth is 1 metre. **2**

(c) The value of a home business, $\$V$, is increasing at a rate proportional to the amount by which the value is less that \$4000.

$$\text{i.e. } \frac{dV}{dt} = k(4000 - V)$$

Initially, the value of the business was \$2000 and after 5 years it was \$3000.

(i) Show that $V = 4000 - Ae^{-kt}$ satisfies this equation. **1**

(ii) Find the value of A and the value of k to 4 decimal places. **2**

(iii) Find the number of years for the value of the business to grow to \$3800. **2**

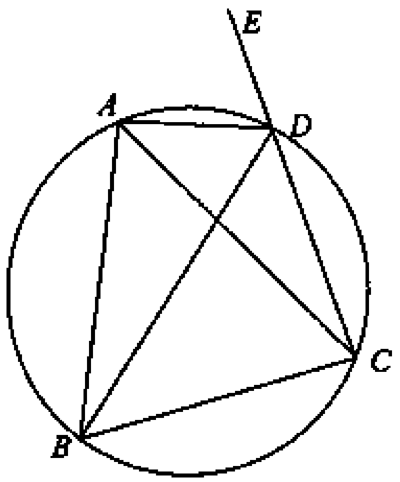
QUESTION 5 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Show that the derivative of x^2e^{-x} is $xe^{-x}(2-x)$. **1**

(ii) Show that $x^2e^{-x} = 0.4$ has a root between $x = 1$ and $x = 2$. **1**

(iii) Use Newton's approximation to find an approximation to the root of $x^2e^{-x} = 0.4$, taking $x = 1$ as a first approximation. **3**

(b)



ABCD is a cyclic quadrilateral in which $AB = AC$, and CD is produced to E.
Prove that AD bisects the angle BDE. **3**

(c) In the expansion of $(3+2x)^8$, c_r is the coefficient of x^r.

(i) Show that $\dfrac{c_r}{c_{r-1}} = \dfrac{18-2r}{3r}$. **2**

(ii) Hence or otherwise find the largest coefficient in the expansion of $(3+2x)^8$. **2**

QUESTION 6 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) The position of a particle at time t is given by:

$$x = 3\sin 2t - 4\cos 2t.$$

(i) Show that this equation satisfies $\ddot{x} = -n^2 x$. **2**

(ii) What is the initial velocity of the particle? **1**

(iii) At what time does the particle first come to rest? **3**

(b) The acceleration of a particle at position x is given by:

$$\ddot{x} = -\frac{1}{4x^3}.$$

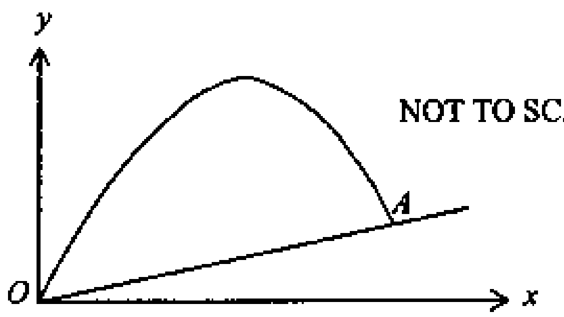
Initially the particle is at $x = 1$ moving with a velocity of ½ unit in the positive direction.

(i) Prove that the velocity of the particle at position x is given by: **3**

$$v = \frac{1}{2x}.$$

(ii) Hence find the position of the particle at time t. **3**

QUESTION 7 (12 Marks) Use a SEPARATE writing booklet. **Marks**



An object is thrown from ground level with a speed of 40 m/s at an angle of 60° to the horizontal.
Assume acceleration due to gravity is 10 m/s² and neglect air resistance.

(a) Find equations for x and y in terms of time t seconds, starting from the acceleration equations $\ddot{x} = 0$ and $\ddot{y} = -10$, and hence show that: **4**

$$y = \sqrt{3}\,x - \frac{x^2}{80}.$$

(b) The object is thrown up a slope with a gradient of ¼. **2**
Show that the horizontal distance travelled by the object when it lands on the slope is given by:

$$x = 80\sqrt{3} - 20.$$

(c) Hence find the distance OA (to the nearest metre) up the slope from the point of projection to the point of landing. **2**

(d) Show that the maximum height reached by the object above the slope is $(61.25 - 10\sqrt{3})$ metres. **4**

End of paper