

Question 1 (4 + 3 + 2 + 3 = 12 Marks)

[4] a) Evaluate the following integrals correct to 2 Decimal places .

$$(i) \int_0^2 \frac{x^3}{x^4 + 1} dx$$

$$(ii) \int_{-2}^4 \left(1 + \frac{x}{2}\right)^5 dx$$

[3] b) Solve the following for x : $\frac{4}{5-x} \geq 1$

[2] c) Find the co - ordinates of the point which divides the interval AB with A(1,4) and B(5,2) externally in the ratio 1 : 3

[3] d) On the same diagram make a neat sketch of the graphs

$$y = |x - 2|$$

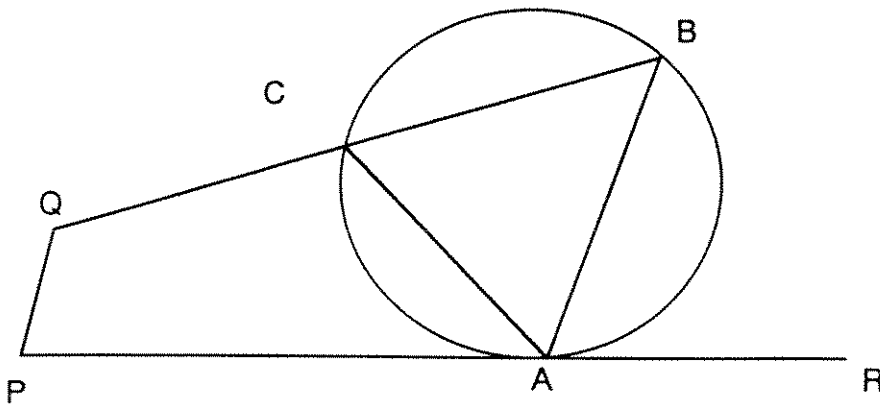
$$y = \frac{3}{x}$$

For what values of x is $|x - 2| < \frac{3}{x}$.

Question 2 (5 + 5 + 2 = 12 Marks)

[5] a) ABC is a triangle inscribed in a circle. PA is a tangent to the circle. PQ is drawn parallel to AB and meets BC produced to Q. Copy the diagram and prove APQC is a cyclic quadrilateral.

Diagram not to scale



[5] b) (i) Expand $(x - a)^3$

(ii) Given $P(x) = 0$ has a triple root at $x = a$,

that is $P(x) = (x - a)^3 Q(x)$ where $Q(x)$ is a another polynomial.

Show $P'(a) = 0$.

(iii) Hence or otherwise find the value of a and b if $x = 1$ is a triple root of

$$x^5 + x^4 + ax^3 + bx^2 - 5x + 1 = 0.$$

[2] c) Use Newton's method to find a second approximation to a root of

$$x - e^{-x} = 0,$$

given that $x = 0.5$ is the first approximation. Give the answer correct to three decimal places.

Question 3 (4 + 4 + 4 = 12 Marks)

[4] a) If the displacement x centimetres from 0 at time t seconds is given by

$$x = 5 + 4 \sin^2 t$$

- (i) Show that the motion is Simple Harmonic Motion,
- (ii) Find the Centre and Amplitude of the motion,
- (iii) Find the value of x for which the speed is maximum and determine this speed.

[4] b) Evaluate, leaving your answer in exact form

$$\int_{\frac{1}{8}}^{\frac{\sqrt{3}}{8}} \frac{dx}{\sqrt{1-16x^2}}$$

[4] c) From a group of seven men and five women a team of six is to be formed

- (i) How many possible teams are there,
- (ii) What is the probability that the team will have at least four men.

Question 4 (7 + 5 = 12 Marks)

[7] a) Molten plastic at a temperature of 250°C is poured into moulds to form car parts. After 20 minutes the plastic has cooled to 150°C . If the temperature after t minutes is $T^\circ\text{C}$, and if the temperature of the surroundings is 30°C , then the rate of cooling is approximately given by

$$\frac{dT}{dt} = -k(T - 30), \text{ where } k \text{ is a positive constant.}$$

- (i) Show that a solution of this equation is $T = 30 + Ae^{-kt}$, where A is a constant.
- (ii) Show that the value of the constant A is 220.
- (iii) Find the value of k correct to 3 significant figures.
- (iv) The plastic can be taken out of the moulds when the temperature has dropped to 80°C . How long after the plastic has been poured will this temperature be reached. Give the answer to the nearest minute.

[5] b) You are walking along a straight level fire track heading south at 6km/hr . The hut you are heading for is 12km along this track and 8km east into the bush. It is getting late and you wish to minimise the time to get to the hut. If you can travel through the bush at only 3km/hr , how far should you continue down this track (to the nearest metre) before you turn into the bush towards the hut ?

Question 5 (8 + 4 = 12 Marks)

[8] a) Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

(i) Derive the equation of the tangent to the parabola at P,

(ii) Find the coordinates of the point of intersection T of the tangents to the parabola at P and Q,

(iii) You are given that the tangents at P and Q in Part (ii) intersect at an angle of 60°

$$\text{Show that } p - q = \sqrt{3}(1 + pq)$$

(iv) By evaluating the expression $x^2 - 4ay$ at T, find the locus of T when the tangents at P and Q intersect as given in Part (iii).

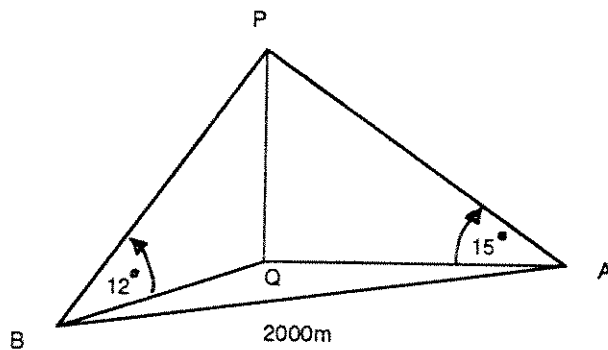
[4] b) Prove by Mathematical Induction that for $n \geq 1$

$$1^2 + 3^2 + \dots + (2n-1)^2 = \frac{1}{3}(2n-1)n(2n+1)$$

Question 6 (2 + 4 + 6 = 12 Marks)

[2] a) Solve $\sin 2x + \sin x = 0$, $0 \leq x \leq 2\pi$

[4] b)



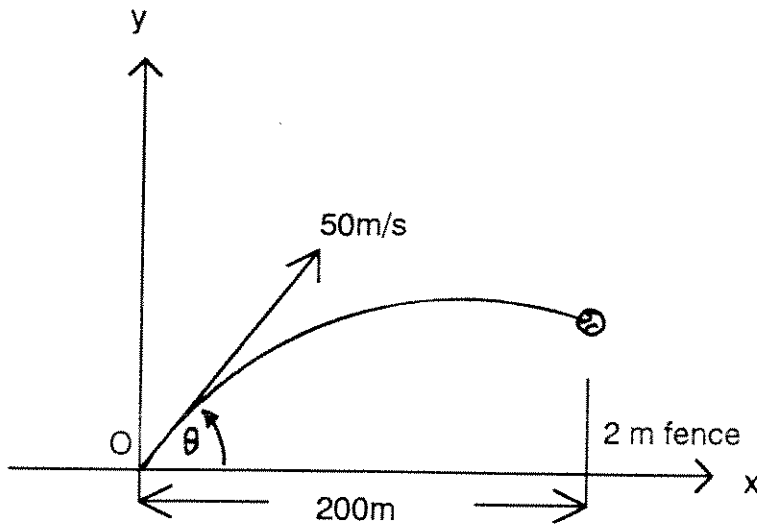
The angle of elevation of a tower PQ of height h metres at a point A due East of it is 15° . From another point B, due South of the tower the angle of elevation is 12° . The point A and B are 2000 metres apart on level ground. Find the height h of the tower to the nearest metre.

[6] c) (i) Draw a sketch of $y = \sin^{-1} x$. State the Domain and Range.

(ii) A region R is bounded by the curve $y = \sin^{-1} x$, the x axis and the line $x=1$. Use Simpson's rule with five function values to give an approximation for the area R. Give your answer correct to two decimal places.

Question 7 (4 + 2 + 2 + 4 = 12 Marks)

One method to score a home run in a baseball game is to hit the ball over the boundary fence on the full.



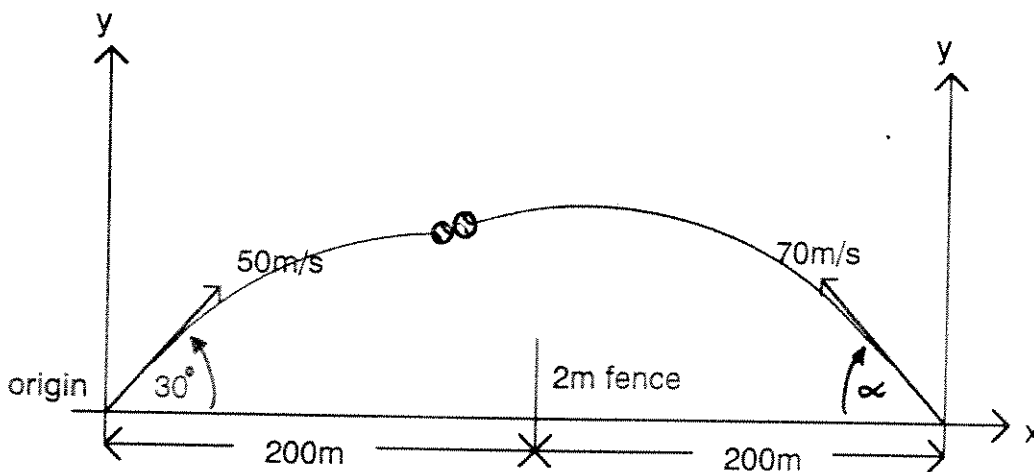
A ball is hit at 50 metres per second. The fence 200 metres away is 2 metres high.

[4] (i) Derive the equations of motion of the ball in the x and y directions. (neglect air resistance and acceleration due to gravity can be taken as 10 ms^{-2}).

[2] (ii) Show the ball would just clear the 2 metre boundary fence when $80 \tan^2 \theta - 200 \tan \theta + 82 = 0$, where θ is the angle of projection.

[2] (iii) In what range must θ lie to score a home run by this method.

[4] (iv) In an adjacent field another ball is hit at the same instant at 70 ms^{-1} and the balls collide. Assuming $\theta = 30^\circ$, Find the angle α the angle of projection of the second ball and the time and position where the balls collide.



T4SC 1995 3 UNIT Solution

Question 1 (4+3+2+3=12 Marks)

a) i) $\int_0^2 \frac{x^3}{x^4+1} dx = \frac{1}{4} \int_0^2 \frac{4x^3}{x^4+1} dx = \left[\frac{1}{4} \ln(x^4+1) \right]_0^2$

$= \frac{1}{4} \ln 17 \div$

ii) $\int_{-2}^4 \left[1 + \frac{x}{2} \right] dx$ Let $u = 1 + \frac{x}{2}$ $\frac{du}{dx} = \frac{1}{2} \therefore 2du = dx$
 $\therefore \int_0^3 u^2 du = \left[\frac{u^3}{3} \right]_0^3 = \frac{27}{3} = 9$
 If $x = -2$ $u = 0$
 $x = 4$ $u = 3$

b) Solve $\frac{4}{5-x} \geq 1$ $\times \ln(5-x)^2$

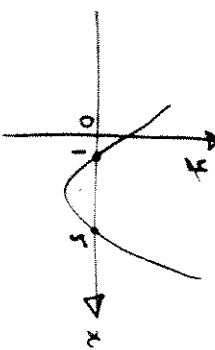
$\therefore \frac{4(5-x)^2}{5-x} \geq (5-x)^2$

$\therefore 4(5-x) \geq 25-10x+x^2$

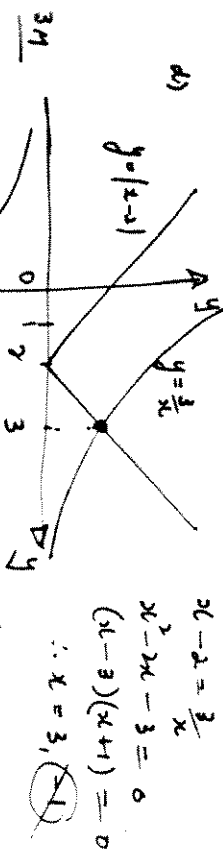
$20-4x \geq 25-10x+x^2$

$0 \geq 5-6x+x^2$

$\therefore x^2-6x+5 \leq 0$



c) External ratio $\left(\frac{mx_2+ny_1}{m+n}, \frac{my_2+ny_1}{m+n} \right)$
 Let $m: n$
 $\left(\frac{1.5+(3) \cdot 1}{1+3}, \frac{1.2+(3) \cdot 4}{1+3} \right)$
 $A(1, 4)$
 $B(5, 2)$
 $m: n$
 $1: 3$

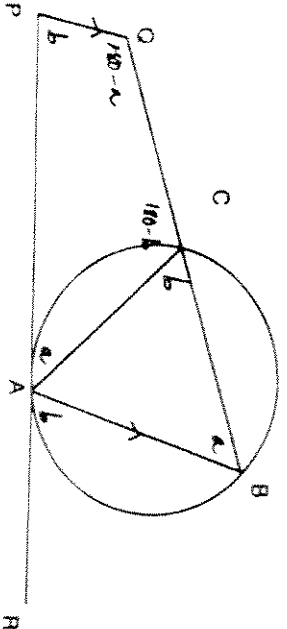


Solve $\{x: x < 3\}$

Question 2 (5+5+2=12 Marks)

a)

Diagram not to scale



Let $\angle PAC = a$, $\angle BAR = b$

Now Aim: To Prove $\triangle POC$ is a cyclic quadrilateral

Proof: 1. $\angle PAC = \angle ABC$ angle in alternate segments

2. $\angle BAR = \angle ACB$

3. $\angle AOC = 180 - \angle BAR$ as $\angle OCB$ is a straight line

4. $\angle POC = 180 - \angle ABC$ as $\angle OCB$ is a straight line

$\therefore \angle POC$ is a cyclic quadrilateral as opposite angles are supplementary

b) i) $(x-a)^3 = (x-a)(x^2-2ax+a^2) = x^3-2ax^2+ax^2-2a^2x+a^3$

ii) $P(x) = (x-a)^3 Q(x)$ $P'(x) = 3(x-a)^2 Q(x) + (x-a)^3 Q'(x)$

$\therefore P'(x) = (x-a)^2 [3Q(x) + (x-a)Q'(x)]$

$\therefore P'(a) = 0$

iii) We know $P(1) = 0$ and $P'(1) = 0$

$P(1) = 0 \rightarrow 1+1+a+b-5+1 = 0$ $a+b = 2 \rightarrow a = 2-b$

$P'(x) = 5x^4 + 4x^3 + 3ax^2 + 2bx - 5$

$P'(1) = 0 \rightarrow 5+4+3a+2b-5 = 0$ $3a+2b = -4$

\therefore sub $a = 2-b$ into $3a+2b = -4$ gives $-b = -10 \therefore b = 10$

$\therefore a = 2-10 = -8$ Soln $a = -8$ $b = 10$

c) Newton's Method $x - e^{-x} = 0$ Let $f(x) = x - e^{-x}$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ with $x_0 = 0.5$ $f'(x) = 1 + e^{-x}$

$\therefore x_{n+1} = 0.5 - \frac{f(0.5)}{f'(0.5)} \div 0.566$ (to 3DP) $f(0.5) \div -0.10653$ $f'(0.5) \div 1.60653$

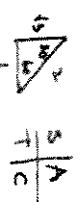
Question 6 (1+4+6 = 12 Marks)

a) Solve $\sin 2x + \sin x = 0 \quad \therefore 2 \sin x \cos x + \sin x = 0$

$\therefore \sin x (2 \cos x + 1) = 0 \quad \therefore \sin x = 0$ or $2 \cos x + 1 = 0$

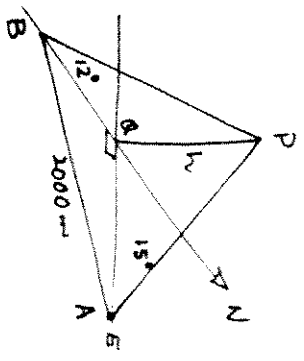
for $0 \leq x < 2\pi \quad \sin x = 0, x = 0, \pi, 2\pi$

$\cos x = -\frac{1}{2} \quad x = \frac{2\pi}{3}, \frac{4\pi}{3}$



\therefore Solns $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi$

b)

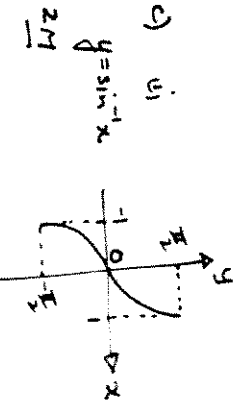


In $\triangle ABC \quad AB = 2000m$
 $BA = h \cot 12^\circ$
 $AB = h \cot 15^\circ$

Using Pythagoras Theorem here
 $2000^2 = h^2 \cot^2 12^\circ + h^2 \cot^2 15^\circ$
 $\therefore h^2 = \frac{2000^2}{(\cot^2 12^\circ + \cot^2 15^\circ)} \quad h > 0$
 $\therefore h \approx 333 \text{ m (to nearest)}$

Domain: $\{x: -1 \leq x \leq 1\}$

Range: $\{y: -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\}$



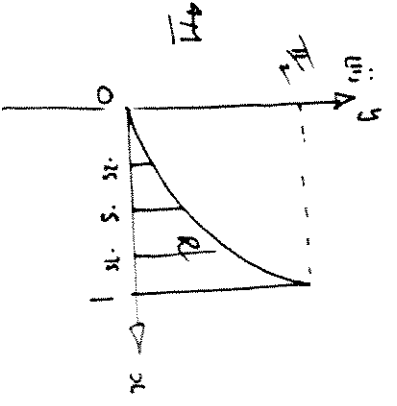
Simpson's Rule 5 function values

$\int_0^1 \sin^{-1} x \, dx \approx \frac{h}{3} [y_0 + y_n + 4(\text{odds}) + 2(\text{evens})]$

$\approx \frac{1}{12} [\sin^{-1} 0 + \sin^{-1} 1 + 4[\sin^{-1}(0.25) + \sin^{-1}(0.75)] + 2[\sin^{-1}(\frac{1}{2})]]$

$\approx \frac{1}{12} [0 + \frac{\pi}{2} + 4(0.25 + 1.107) + 2(0.785)]$

$\approx \frac{7.0209632}{12} \approx 0.5849 (2DP)$



a) $P(2ap, ap^2) Q(2aq, aq^2)$ on $x^2 = 4ay$

i) $y = \frac{x^2}{4a} \quad \frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$ at P $\frac{dy}{dx} = p$, at Q $\frac{dy}{dx} = q$

\therefore eqⁿ of Tangents at P $y - ap^2 = p(x - 2ap) \quad \therefore y = px - ap^2 \quad \text{--- (1)}$

ii) Eqⁿ of tangents at Q $y = qx - aq^2 \quad \text{--- (2)}$

Solve (1) & (2) $\therefore px - ap^2 = qx - aq^2 \quad \therefore x(p - q) = a(p^2 - q^2)$

$\therefore x = a(p + q) \quad \therefore y = p(a(p + q)) - ap^2 = apq$

$\therefore T(a(p + q), apq)$ is point of intersection of tangents at P & Q

iii) $\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2} \quad \therefore \sqrt{3} = \frac{p - q}{1 + pq} \quad m_1 = p, m_2 = q$

$\therefore p - q = \sqrt{3}(1 + pq)$

iv) $x^2 - 4ay = [a(p + q)]^2 - 4a(apq) = ap^2 - 2a^2 pq + a^2 q^2$
 $= a^2 (p - q)^2 = a^2 ((1 + pq)\sqrt{3})^2$
 $= 3a^2 [1 + 2pq + p^2 q^2] \quad apq = y \quad \therefore pq = \frac{y}{a}$
 $\therefore x^2 - 4ay = 3a^2 [1 + 2\frac{y}{a} + \frac{y^2}{a^2}] = 3a^2 + 3a \cdot 2y + 3a^2 \frac{y^2}{a^2}$

\therefore locus $x^2 - 3ay^2 - 10ay - 3a^2 = 0$

b) Induction $1^2 + 3^2 + \dots + (2n-1)^2 = \frac{1}{3}(2n-1)n(2n+1)$

Step 1 Show $n=1$ LHS $1^2 = 1$ RHS $\frac{1}{3} \cdot 1 \cdot 1 \cdot 3 = 1 \quad \therefore$ True for $n=1$

Step 2 Assume true for $n=k$, prove true for $n=k+1$

\therefore LHS $1^2 + 3^2 + \dots + (2k-1)^2 + (2k+1)^2$ using assumption
 $= \frac{1}{3} k(2k-1)(2k+1) + (2k+1)^2$
 $= \frac{1}{3} (2k+1) [k(2k-1) + 3(2k+1)] = \frac{1}{3} (2k+1) [2k^2 + 5k + 3]$
 $= \frac{1}{3} (2k+1)(2k+3)(k+1) = RH \text{S}$

Step 3 Since it is true for $n=1$ and we have established

the link between $n=k$ and $n=k+1$ we can induce $n=1+1=2$ and so on for all positive integers n

Question 7 (4+2+2+4 = 12 Marks)

(i) \vec{A} \vec{y} 50 ms^{-1}

$$\ddot{x} = 0$$

$$\dot{y} = -10$$

$$\dot{x} = C_1$$

$$\dot{y} = -10t + C_2$$

$$t=0 \quad C_1 = 50 \cos \theta, \quad C_2 = 50 \sin \theta$$

$$\therefore \dot{x} = 50 \cos \theta, \quad \dot{y} = -10t + 50 \sin \theta$$

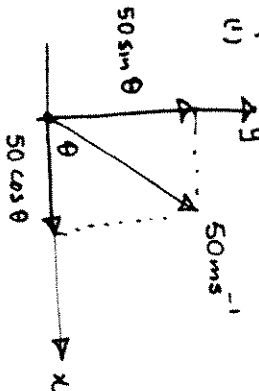
$$x = 50t \cos \theta + C_3, \quad y = -5t^2 + 50t \sin \theta + C_4$$

$$t=0 \quad x=0, y=0$$

$$\therefore x = 50t \cos \theta$$

$$y = -5t^2 + 50t \sin \theta$$

4M



(ii) Boundary fence $x=200$ $y=2$ $\therefore 200 = 50t \cos \theta$

$$\therefore t = \frac{4}{\cos \theta}$$

$$\text{Now } 2 = -5 \left(\frac{4}{\cos \theta} \right)^2 + 50 \left(\frac{4}{\cos \theta} \right) \sin \theta$$

$$2 = \frac{-80}{\cos^2 \theta} + 200 \tan \theta$$

$$\text{using } \frac{1}{\cos^2 \theta} = \sec^2 \theta$$

$$2 = -80 \sec^2 \theta + 200 \tan \theta$$

$$\text{and } 1 + \tan^2 \theta = \sec^2 \theta$$

$$\therefore 2 = -80(1 + \tan^2 \theta) + 200 \tan \theta$$

$$2 = -80 - 80 \tan^2 \theta + 200 \tan \theta$$

$$\text{ie } 80 \tan^2 \theta - 200 \tan \theta + 82 = 0 \text{ as required.}$$

(iii) Now solve for θ $\tan \theta = \frac{200 \pm \sqrt{200^2 - 4 \cdot 80 \cdot 82}}{160}$

2M $\tan \theta = \frac{200 \pm 117.802}{160} \therefore 27.20^\circ < \theta < 63.14^\circ$ θ to nearest minute.

range of θ

Ball 1 $x_1 = 50t \cos 30 = 25\sqrt{3}t$

$$y_1 = -5t^2 + 25t$$



Ball 2 $\dot{x} = 0$ $\dot{y} = -10$

$$x_2 = -70t \cos \alpha$$

$$y_2 = -5t^2 + 70t \sin \alpha$$

$$x_3 = -70t \cos \alpha + 400$$

$$y_3 = -5t^2 + 70t \sin \alpha$$

$$y_4 = -5t^2 + 70t \sin \alpha$$

$$y_5 = -70t \cos \alpha + 400$$

$$y_6 = -5t^2 + 70t \sin \alpha$$

$$y_7 = -5t^2 + 70t \sin \alpha$$

$$y_8 = -5t^2 + 70t \sin \alpha$$

$$y_9 = -5t^2 + 70t \sin \alpha$$

$$y_{10} = -5t^2 + 70t \sin \alpha$$

$$y_{11} = -5t^2 + 70t \sin \alpha$$

$$y_{12} = -5t^2 + 70t \sin \alpha$$

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$$y_{102} = -5t^2 + 70t \sin \alpha$$

$$y_{103} = -5t^2 + 70t \sin \alpha$$

$$y_{104} = -5t^2 + 70t \sin \alpha$$

$$y_{105} = -5t^2 + 70t \sin \alpha$$

$$y_{106} = -5t^2 + 70t \sin \alpha$$

$$y_{107} = -5t^2 + 70t \sin \alpha$$

$$y_{108} = -5t^2 + 70t \sin \alpha$$

$$y_{109} = -5t^2 + 70t \sin \alpha$$

$$y_{110} = -5t^2 + 70t \sin \alpha$$

$$y_{111} = -5t^2 + 70t \sin \alpha$$

$$y_{112} = -5t^2 + 70t \sin \alpha$$

$$y_{113} = -5t^2 + 70t \sin \alpha$$

$$y_{114} = -5t^2 + 70t \sin \alpha$$

$$y_{115} = -5t^2 + 70t \sin \alpha$$

$$y_{116} = -5t^2 + 70t \sin \alpha$$

$$y_{117} = -5t^2 + 70t \sin \alpha$$

$$y_{118} = -5t^2 + 70t \sin \alpha$$

$$y_{119} = -5t^2 + 70t \sin \alpha$$

$$y_{120} = -5t^2 + 70t \sin \alpha$$

$$y_{121} = -5t^2 + 70t \sin \alpha$$

$$y_{122} = -5t^2 + 70t \sin \alpha$$

$$y_{123} = -5t^2 + 70t \sin \alpha$$

$$y_{124} = -5t^2 + 70t \sin \alpha$$

$$y_{125} = -5t^2 + 70t \sin \alpha$$

$$y_{126} = -5t^2 + 70t \sin \alpha$$

$$y_{127} = -5t^2 + 70t \sin \alpha$$

$$y_{128} = -5t^2 + 70t \sin \alpha$$

$$y_{129} = -5t^2 + 70t \sin \alpha$$

$$y_{130} = -5t^2 + 70t \sin \alpha$$

$$y_{131} = -5t^2 + 70t \sin \alpha$$

$$y_{132} = -5t^2 + 70t \sin \alpha$$

$$y_{133} = -5t^2 + 70t \sin \alpha$$

$$y_{134} = -5t^2 + 70t \sin \alpha$$

$$y_{135} = -5t^2 + 70t \sin \alpha$$

$$y_{136} = -5t^2 + 70t \sin \alpha$$

$$y_{137} = -5t^2 + 70t \sin \alpha$$

$$y_{138} = -5t^2 + 70t \sin \alpha$$

$$y_{139} = -5t^2 + 70t \sin \alpha$$

$$y_{140} = -5t^2 + 70t \sin \alpha$$

$$y_{141} = -5t^2 + 70t \sin \alpha$$

$$y_{142} = -5t^2 + 70t \sin \alpha$$

$$y_{143} = -5t^2 + 70t \sin \alpha$$

$$y_{144} = -5t^2 + 70t \sin \alpha$$

$$y_{145} = -5t^2 + 70t \sin \alpha$$

$$y_{146} = -5t^2 + 70t \sin \alpha$$

$$y_{147} = -5t^2 + 70t \sin \alpha$$

$$y_{148} = -5t^2 + 70t \sin \alpha$$

$$y_{149} = -5t^2 + 70t \sin \alpha$$

$$y_{150} = -5t^2 + 70t \sin \alpha$$

$$y_{151} = -5t^2 + 70t \sin \alpha$$

$$y_{152} = -5t^2 + 70t \sin \alpha$$

$$y_{153} = -5t^2 + 70t \sin \alpha$$

$$y_{154} = -5t^2 + 70t \sin \alpha$$

$$y_{155} = -5t^2 + 70t \sin \alpha$$

$$y_{156} = -5t^2 + 70t \sin \alpha$$

$$y_{157} = -5t^2 + 70t \sin \alpha$$

$$y_{158} = -5t^2 + 70t \sin \alpha$$