

2001 Assessment #1

Year 12

Class Test

Integration and Induction

Name: _____

Teacher: _____



Instructions:

- Answer on the paper provided.
- Show all necessary working.
- Marks may be deducted for careless or untidy work.
- Marks for each question shown as: [1]

Question One: (10 marks)

- a) Find $\int \sqrt{4x-3} dx$ [2] b) Evaluate $\int_3^6 (x^2 - 5) dx$ [3]
- c) Find the area between the curve $y = x^2 - 4$ and the x-axis between the values of $x = 1$ and $x = 4$. [5]

Question Two: (9 marks)

- a) Using the Trapezoidal Rule and 2 intervals, find an approximation for the area given by $\int_1^6 (x^3 - 1) dx$. By evaluating $\int_1^6 (x^3 - 1) dx$ using normal integration techniques, calculate the percentage error in the approximation the Trapezoidal Rule makes. [5]
- b) Use Simpson's Rule to evaluate the area under the curve given by $f(x) = \frac{\sqrt{x-1}}{x}$ and bounded by $x=1$ and $x=3$, using four intervals (i.e. 5 ordinates). Draw up a table of values you will use, and give your answers to 4 decimal places. [4]

Question Three: (11 marks)

- a) Sketch the curves $y = x^4$ and $y = \sqrt{x}$ on the same axes. [2]
- b) Determine the area between these two curves. [3]
- c) Find the volume of the solid of revolution formed when this area is rotated about
- i) the x-axis [3]
 - ii) the y-axis [3]

Question Four: (15 marks)

- a) Prove by Mathematical Induction that $\sum_{r=1}^n r^3 = \frac{1}{4} n^2 (n+1)^2$. [5]
- b) Show, by Mathematical Induction, that $3^n + 7^n$ is divisible by 10, if n is odd. [5]
- c) Prove by Mathematical Induction that $9^n > 4^n + 5^n, n \geq 2$. [5]