CATHOLIC SECONDARY SCHOOLS ASSOCIATION

2006 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS – SUGGESTED SOLUTIONS

These marking guidelines show the criteria to be applied to responses along with the marks to be awarded in line with the quality of responses. These guidelines are suggested and not prescriptive. This is not intended to be an exhaustive list but rather an indication of the considerations that students could include in their responses.

Question 1 (12 marks)

(a) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Gives correct answer.	1
• Correctly rounds to THREE decimal places.	1

Sample Answer

$$\sqrt{\frac{|-38.67\times 7.2|}{(11.7)^2-(1.83)^2}} = 1.277508818$$

$$= 1.278 \quad \text{(3 decimal places)}$$

(b) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly write the conjugate and expands to get the denominator OR the numerator correct	1
• Gives the correct answer	1

Sample Answer

$$\frac{1-\sqrt{2}}{2-\sqrt{8}} = \frac{1-\sqrt{2}}{2-\sqrt{8}}\times\frac{2+\sqrt{8}}{2+\sqrt{8}}$$

$$= \frac{2-2\sqrt{2}+\sqrt{8}-\sqrt{16}}{4-8}$$

$$= \frac{-2}{-4}$$

$$= \frac{1}{2}$$

(c) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Gives ONE correct answer in radians OR TWO correct answers in degrees.	1
• Gives TWO correct answers in radians.	1

Sample Answer

$\cos\theta = -\dfrac{1}{2}$

Basic angle is $\dfrac{\pi}{3}$ (First Quadrant).

$\therefore\ \theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3} \qquad 0 \le \theta \le 2\pi$

(d) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Factorises in pairs, e.g. $x(4y+b)+2a(4y+b)$.	1
• Completes the factorisation into TWO brackets.	1

Sample Answer

$$\begin{aligned} 4xy+xb+8ay+2ab &= x(4y+b)+2a(4y+b) \\ &= (x+2a)(4y+b) \end{aligned}$$

(e) (2 marks)

Outcomes Assessed: P2, P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly writes down the discriminant and equates to zero.	1
• Solves the equation to give correct values of k.	1

Sample Answer

$4x^2+kx+9=0$ equal roots when $\Delta = 0$

$\Delta = k^2 - 144$

$0 = k^2 - 144$

$k = \pm 12$

(f) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Gives ONE correct answer.	1
• Gives the second correct answer.	1

Sample Answer

$|x-2| = 3$

$x-2=3 \quad \text{or} \quad -(x-2)=3$

$\therefore\ x=5 \quad \text{or} \quad x=-1$

Question 2 (12 marks)

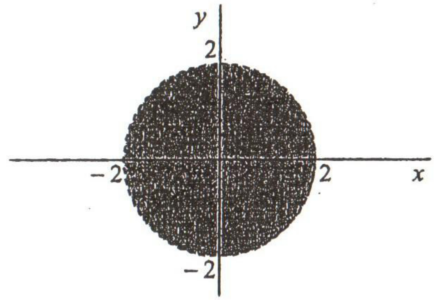
(a) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly draws the circle using a dotted line.	1
• Correctly shades the inside of the circle.	1

Sample Answer



Question 2

(b) (2 marks)

Outcomes Assessed: P3, P4, P5

Targeted Performance Bands: 2-4

Criteria	Mark
• Gives correct answer ONE partial answer (e.g. $f(3)=4$ or $f(6)=27$)	1
• Gives correct answer	1

Sample Answer

$$f(x)=\begin{cases} x+1, & x\le 3 \\ x^2-9, & x>3 \end{cases}$$

$f(3)=(3)+1=4$, $f(6)=6^2-9=27$

$\therefore f(3)-f(6)=4-27=-23$

(c) (i) (2 marks)

Outcomes Assessed: P4, P5

Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly finds the gradient of line L_3.	1
• Correctly finds the equation of line L_3.	1

Sample Answer

$$m_{L_3}=\frac{0+4}{0+2}=\frac{4}{2}=2$$

$y-0=2(x-0)$

$\therefore 2x-y=0$ (as required)

(c) (ii) (1 mark)

Outcomes Assessed: P4, P5

Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly shows that the x-coordinate of point B is 3.	1

Sample Answer

B lies on the line $y=6$

$y=6 \therefore 6=2x \therefore x=3$

$\therefore B(3,6)$

(c) (iii) (3 marks)

Outcomes Assessed: P2, P4, P5

Targeted Performance Bands: 2-4

Criteria	Mark
• Correctly finds the gradient of L_2 which gives $\angle AOB=90^\circ$.	1
• Correctly finds the equation of L_2.	1
• Correctly finds the value of the x-ordinate of A.	1

Sample Answer

$$m_{L_2}=\frac{-1}{m_{L_3}}=\frac{-1}{2}=-\frac{1}{2}$$

$$\therefore y-0=-\frac{1}{2}(x-0)$$

$$y=-\frac{1}{2}x$$

For $y=6 \therefore 6=-\frac{1}{2}x \therefore x=-12$

$A(-12,6)$

(c) (iv) (2 marks)

Outcomes Assessed: P2, P4, P5

Targeted Performance Bands: 2-3

Criteria	Mark
• Writes down the correct distance between points A and B.	1
• Calculates the correct area of ΔAOB.	1

Sample Answer

Distance between points A and B is 15 units.

$$\text{Area of } \Delta AOB=\frac{1}{2}\times 15\times 6$$
$$=45 \text{ units}^2$$

Question 3 (12 marks)
(a) (i) (1 mark)
Outcomes Assessed: P3, P4
Targeted Performance Band: 2-4

Criteria	Mark
• Finds the correct vertex	1

Sample Answer

$Vertex \equiv (2,0)$

(a) (ii) (1 mark)
Outcomes Assessed: P3, P4
Targeted Performance Band: 2-4

Criteria	Mark
• Finds the correct focus	1

Sample Answer

$Focus \equiv (2,4)$

(b) (i) (2 marks)
Outcomes Assessed: P7, H5
Targeted Performance Band: 2-3

Criteria	Mark
• Correctly uses the product rule of differentiation but has ONE mistake in calculation	1
• Correctly finds the answer	1

Sample Answer

$$\frac{d}{dx}(3x\log_e x) = 3 + 3\log_e x$$

(b) (ii) (2 marks)
Outcomes Assessed: P7, H5
Targeted Performance Band: 2-4

Criteria	Mark
• Correctly uses the chain rule of differentiation but has ONE mistake in calculation	1
• Correctly finds the answer	1

Sample Answer

$$\frac{d}{dx}(\sin^2 x) = 2(\sin x)^1.\cos x = 2\sin x\cos x$$

(c) (i) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 2-4

Criteria	Mark
• Gives an answer of $\sin 2006x + c$	1
• Correctly finds the answer	1

Sample Answer

$$\int \cos 2006x\ dx = \frac{1}{2006}\sin 2006x + c$$

(c) (ii) (2 marks)
Outcomes Assessed: H3, H5
Targeted Performance Band: 3-4

Criteria	Mark
• Finds the primitive $\frac{1}{2}e^{2x}$ but has an error in calculating the integral	1
• Correctly applies the Newton-Leibnitz formula to obtain the correct answer in exact form	1

Sample Answer

$$\int_0^1 e^{2x}\ dx = \left[\frac{1}{2}e^{2x}\right]_0^1 = \frac{1}{2}e^2 - \frac{1}{2}e^0 = \frac{1}{2}e^2 - \frac{1}{2} \text{ or } \frac{1}{2}(e^2 - 1)$$

(d) (2 marks)
Outcomes Assessed: P6, H5
Targeted Performance Band: 2-4

Criteria	Mark
• Correctly finds the gradient of the normal	1
• Correctly substitutes the values for x and y into the point/gradient formula to find the equation of the normal	1

Sample Answer

$$y = x^3 - 5x \quad \therefore \frac{dy}{dx} = 3x^2 - 5 \text{ ? } \quad \text{At } x = 1,\ m_T = -2 \therefore m_N = \frac{1}{2}$$

$$\therefore \text{ equation of normal is given by } y - -4 = \frac{1}{2}(x-1)$$

$$\therefore\ x - 2y - 9 = 0 \ \left(\text{or } y = \frac{1}{2}x - \frac{9}{2}\right)$$

Question 4 (12 marks)
(a) (i) (3 marks)
Outcomes Assessed: P7, H6
Targeted Performance Band: 3-5

Criteria	Mark
• Finds the stationary points	1
• Finds the nature of ONE stationary point	1
• Finds the nature of the other stationary point	1

Sample Answer

$f(x)=-\frac{1}{3}x^3-x^2+3x+1 \quad \therefore f'(x)=-x^2-2x+3$

For stationary points $f'(x)=0 \quad \therefore (1-x)(x+3)=0 \quad \therefore x=1$ or $x=-3$

$\therefore$ the stationary points are $\left(1,2\frac{2}{3}\right)$ & $(-3,-8)$

Also for the nature of the stationary points, $f''(x)=-2x-2$

At $x=1$, $f''(1)=-4<0 \qquad \therefore \left(1,2\frac{2}{3}\right)$ is a MAXIMUM turning point

At $x=-3$, $f''(-3)=4>0 \qquad \therefore (-3,-8)$ is a MINIMUM turning point

(a) (ii) (2 marks)
Outcomes Assessed: P7, H6
Targeted Performance Band: 2-4

Criteria	Mark
• Correctly solves the equation $f''(x)=0$	1
• Analyse the sign of the second derivative and gives correct answer	1

Sample Answer

$f''(x)=-2x-2 \quad \therefore -2x-2=0 \rightarrow x=-1 \quad (y=-2\frac{2}{3})$

x	-1^-	-1	-1^+
$f''(x)$	+	0	-

$\therefore$ a change in the sign of the second derivative has occurred

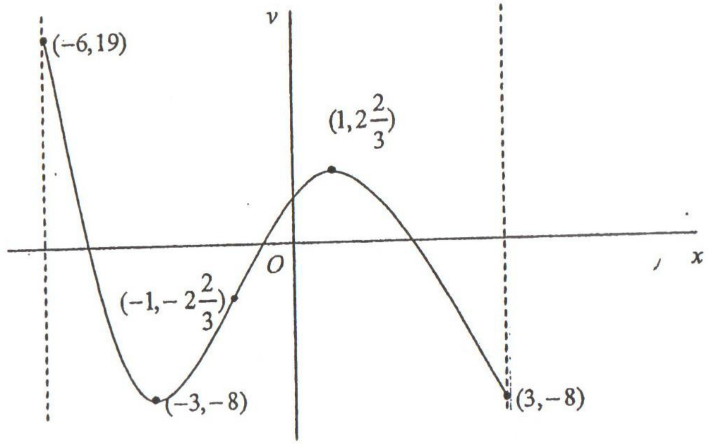
$\therefore \left(-1,-2\frac{2}{3}\right)$ is a point of inflection

(a) (iii) (2 marks)
Outcomes Assessed: P6, H6, H7, H9
Targeted Performance Band: 3-5

Criteria	Mark
• Draws the correct cubic curve	1
• Plots all important points	1

Sample Answer

$f(x)=-\frac{1}{3}x^3-x^2+3x+1$



(a) (iv) (1 mark)
Outcomes Assessed: H7
Targeted Performance Band: 3-4

Criteria	Mark
• Gives the correct answer	1

Sample Answer

$y=19$

i) (0 marks)
ii) (2 marks)
comes Assessed: P2, H2
geted Performance Band: 2-4

Criteria	Mark
Shows that $\angle PAS = \angle QAR$ (vertically opposite $\angle's$) or equivalent	1
Correctly completes proof using (AAS)	1

ple Answer

$SA = \angle QRA$ (given)
$AS = \angle QAR$ (vertically opposite $\angle's$)
$= QR$ (given)
$PSA \equiv \Delta QRA$ (AAS)

iii) (2 marks)
comes Assessed: P2, H2
geted Performance Band: 3-4

Criteria	Mark
Realises that ΔPAQ is isosceles & base $\angle's$ are equal	1
Gives the correct answer	1

ple Answer

$= QA$ (corresponding sides are equal in congruent ?'s)
PAQ is isosceles $\therefore \angle PQA = \angle QPA = x$

$- 120 = 180°$ ($\angle$ sum of a ? is 180°)

$30°$

estion 5 (12 marks)
i) (1 mark)
comes Assessed: H5
geted Performance Band: 2-3

Criteria	Mark
Writes the correct sum of an infinite geometric series	1

ple Answer

$$\dot{5} = \frac{7}{10}+\frac{5}{100}+\frac{5}{1000}+\frac{5}{10000}\ldots$$

(a) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 3-4

Criteria	Mark
• Writes the limiting sum formula with correct a and r	1
• Gives the correct answer	1

Sample Answer

$$S_\infty = \frac{a}{1-r}$$
$$= \frac{7}{10} + \frac{5/100}{1-1/10}$$
$$= \frac{7}{10} + \frac{1}{18}$$
$$= \frac{34}{45}$$

(b) (2 marks)
Outcomes Assessed: P3, H5
Targeted Performance Band: 2-4

Criteria	Mark
• Realises that $\cos^2\theta$ is a substitute or equivalent	1
• Simplifies to give the correct answer	1

Sample Answer

$$\frac{1-\sin^2 x}{\cot x}$$
$$= \frac{\cos^2\theta}{\frac{\cos\theta}{\sin\theta}} = \cos^2\theta \times \frac{\sin\theta}{\cos\theta}$$

$$= \sin\theta\cos\theta$$

(c) (i) (1 mark)

Outcomes Assessed: H5

Targeted Performance Band: 2-4

Criteria	Mark
• Gives the correct answer	1

Sample Answer

$$l = r\theta \quad \therefore \quad \theta = \frac{l}{r}$$

$$\theta = \frac{1}{6/\pi} = \frac{\pi}{6}$$

(c) (ii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Band: 2-4

Criteria	Mark
• Writes the correct formula with necessary substitutions	1
• Gives the correct answer in exact form	1

Sample Answer

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2} \times \left(\frac{6}{\pi}\right)^2 \times \frac{\pi}{6}$$

$\therefore$ Area of sector $MON \Rightarrow A = \dfrac{3}{\pi}\text{ cm}^2$

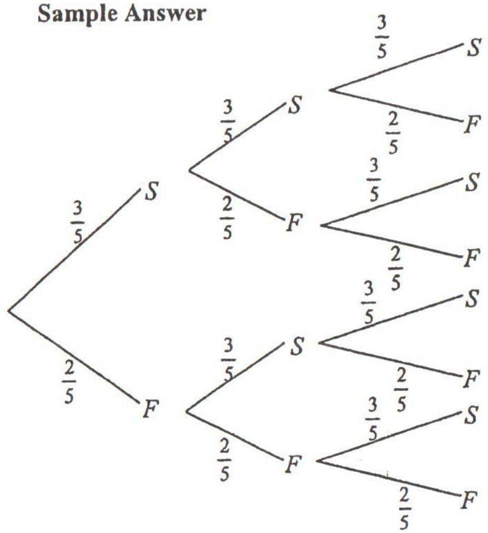
(d) (i) (2 marks)

Outcomes Assessed: H5

Targeted Performance Band: 3-4

Criteria	Mark
• Correctly draws a tree diagram	1
• Gives the correct answer	1

Sample Answer



$$\therefore P(S) = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \left(\frac{3}{5}\right)^3 = \frac{27}{125}$$

(d) (ii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Band: 3-4

Criteria	Mark
• Uses the complementary events method (or otherwise)	1
• Gives the correct answer with required working	1

Sample Answer

$$P(S) = 1 - \left[\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}\right] = 1 - \left(\frac{2}{5}\right)^3 = \frac{117}{125}$$

Question 6 (12 marks)

(a) (i) (1 mark)

Outcomes Assessed: P2

Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly verifies Pythagoras' Theorem	1

Sample Answer

Triangle with sides 12 cm, 16 cm and 20 cm is right angled at A ($12^2 + 16^2 = 20^2$, converse of Pythagoras' Theorem)

(a) (ii) (2 marks)

Outcomes Assessed: P4

Targeted Performance Bands: 2-3

Criteria	Marks
• Correctly gives answer for α in either degrees or radians	1
• Correctly gives answer for β in either degrees or radians	1

Sample Answer

$\sin\alpha = \frac{12}{20}$ $\therefore$ $\alpha = 36^\circ 52'$ or $\alpha = 0.644^c$

$\beta = 90^\circ - 36^\circ 52' = 53^\circ 8'$ or $\beta = 0.927^c$ (using complementary angles in ΔAO_1O_2)

(a) (iii) (3 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-5

Criteria	Marks
• Correctly substitutes in the segment formula for the circle O_1	1
• Correctly substitutes in the segment formula for the circle O_2	1
• Correctly gives answer (to 2 d.p.)	1

Sample Answer

Area enclosed between the two circles is the sum of the two segments, with angles 2α and 2β subtended respectively at the centre.

$$\text{Area} = \frac{1}{2}\times 16^2(2\times 0.644 - \sin 2\times 0.644) + \frac{1}{2}\times 12^2(2\times 0.927 - \sin 2\times 0.927)$$

$\therefore$ Area = 106.2667952 cm^2 $\therefore$ Area = 106.27 cm^2 (2 d.p.).

(b) (i) (2 marks)

Outcomes Assessed: H3, H8

Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly works out x^2 as a subject ($x^2 = \log_e y$)	1
• Correctly substitutes $x^2 = \log_e y$ in the volume formula and deduce the answer	1

Sample Answer

$V_y = \pi\int_1^5 x^2\,dy$; Since $y = e^{x^2}$ $\therefore$ $\log_e y = x^2$ $\therefore$ $V_y = \pi\int_1^5 \log_e y\,dy$ (as required)

(b) (ii) (1 mark)

Outcomes Assessed: H3

Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly completes the required value ($\log_e y = 1.386$)	1

Sample Answer

y	1	2	3	4	5
$\log_e y$	0	0.693	1.099	1.386	1.609

(b) (iii) (3 marks)

Outcomes Assessed: H3, H5

Targeted Performance Bands: 2-4

Criteria	Mark
• Substitutes the correct values in the correct Simpson's formula	1
• Correctly calculates the answer in decimal form (e.g. 4.041)	1
• Gives the correct answer	1

Sample Answer

Using Simpson's Formula: $\int_a^b y\,dx \approx \frac{h}{3}[y_0 + 4(y_1 + y_3 + \ldots) + 2(y_2 + y_4 + \ldots) + y_n]$

$V_y = \pi\times\frac{1}{3}[0 + 4(0.693 + 1.386) + 2\times 1.099 + 1.609]$ $\therefore$ $V_y = 4.041\pi$

$\therefore$ $V_y = 12.695$ (3 d.p.)

Question 7 (12 marks)

(a) (3 marks)

Outcomes Assessed: H3

Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly uses the properties of logarithms	1
• Correctly uses the definition of logarithms to obtain a quadratic equation	1
• Solves the quadratic equation, indicating the correct answer	1

Sample Answer

$\log_2 x + \log_2 (x+7) = 3 \therefore \log_2 x(x+7) = 3 \therefore x(x+7) = 2^3 \therefore x^2 + 7x - 8 = 0$

$\therefore (x+8)(x-1) = 0 \therefore x = -8$ or $x = 1 \therefore$ solution is $x = 1$ $(x > 0)$ or checking the solution by substitution.

(b) (i) (2 marks)

Outcomes Assessed: P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly finds x-ordinate of point A	1
• Correctly finds x-ordinate of point B	1

Sample Answer

At point A: $x^2 = 3 - 2x^2 \therefore 3x^2 = 3 \therefore x = \pm 1 \therefore x_A = 1$ (first quadrant)

At point B: $3 - 2x^2 = 0 \therefore x = \pm\sqrt{\frac{3}{2}} \therefore x_B = \sqrt{\frac{3}{2}}$ (first quadrant)

(b) (ii) (3 marks)

Outcomes Assessed: H8

Targeted Performance Bands: 2-4

Criteria	Mark
• Correctly uses the sum of integrals to find area under curves	1
• Correctly finds the primitives	1
• Correctly finds the area using Leibnitz – Newton Formula	1

Sample Answer

$$A_{AOB} = \int_0^1 x^2\,dx + \int_1^{\sqrt{\frac{3}{2}}} (3 - 2x^2)\,dx = \frac{1}{3}[x^3]_0^1 + \left[3x - \frac{2}{3}x^3\right]_1^{\sqrt{\frac{3}{2}}}$$

$$\therefore A_{AOB} = \frac{1}{3}(1-0) + \left[\left(3\sqrt{\frac{3}{2}} - \frac{2}{3}\sqrt{\frac{3}{2}}^3\right) - \left(3 - \frac{2}{3}\right)\right] = 2\sqrt{\frac{3}{2}} - 2 \text{ square units}$$

(c) (i) (1 mark)

Outcomes Assessed: P4

Targeted Performance Bands: 2

Criteria	Mark
• Correctly shows the number of revolutions the track will rotate	1

Sample Answer

Record is played at 5 revolutions per second for 25 minutes therefore will rotate

$5 \times 60 \times 25 = 7500$ revolutions

(c) (ii) (3 marks)

Outcomes Assessed: H1, H5

Targeted Performance Bands: 3-5

Criteria	Mark
• Correctly realises that circles' radii are in a arithmetic sequence	1
• Correctly uses the formula for arithmetic series	1
• Finds the correct answer	1

Therefore there will be 7500 concentric circles whose radii increases in an arithmetic sequence from 2 cm to 6 cm. The length of the track record can be found using the sum of the arithmetic sequence with 7500 terms:

$$S_{7500} = \frac{7500}{2}(2\pi \times 2 + 2\pi \times 6) = 188495.5595 \text{ cm} = 1.9 \text{ km (1 d.p.)}$$

Question 8 (12 marks)
(a) (2 marks)
Outcomes Assessed*: *P3, P4
Targeted Performance Bands*: *2 – 4

Criteria	Mark
• Correctly uses index law	1
• Find the correct answer	1

Sample Answer

If $A^m = 3 \therefore (A^m)^4 = 3^4 = 81 \therefore A^{4m} - 5 = 81 - 5 = 76$

(b) (i) (2 marks)
Outcomes Assessed*: *H3, H4, H5
Targeted Performance Bands*: *3 – 4

Criteria	Mark
• Correctly substitutes in the formula and writes $-10k$ as a subject	1
• Gives the correct answer	1

Sample Answer

$U = 100e^{-kt} \quad \therefore 2 = 100e^{-10k} \quad \therefore \frac{1}{50} = e^{-10k} \quad \therefore \ln\frac{1}{50} = \ln e^{-10k} \quad \therefore k = \frac{-\ln 50}{-10}$

$\therefore k = 0.3912$

(b) (ii) (2 marks)
Outcomes Assessed*: *H3, H4, H5
Targeted Performance Bands*: *3-5

Criteria	Mark
• Correctly differentiates U	1
• Gives the correct answer	1

Sample Answer

$\frac{dU}{dt} = -k\left(100\,e^{-kt}\right)$

For $t = 12$ and $k = 0.3912$: $\frac{dU}{dt} = -0.36$ mg/minute

(c) (i) (1 mark)
Outcomes Assessed*: *H4, H5
Targeted Performance Bands*: *2-3

Criteria	Mark
• Correctly gives answer for A_1	1

Sample Answer

$A_1 = P\left(1 + \frac{0.75}{100}\right) - 1050 \quad \therefore A_1 = P(1.0075) - 1050$

(c) (ii) (3 marks)
Outcomes Assessed*: *H4, H5
Targeted Performance Bands*: *3-6

Criteria	Mark
• Correctly gives answer for A_2	1
• Write the correct sum of the geometric series	1
• Gives the correct answer	1

Sample Answer

$A_2 = [P(1.0075) - 1050](1.0075) - 1050$

$= P(1.0075)^2 - 1050(10075) - 1050$

$= P(1.0075)^2 - 1050(1 + 1.0075)$

$A_3 = P(1.0075)^3 - 1050(1 + 1.0075 + 1.0075^2)$

$\vdots$

$A_n = P(1.0075)^n - 1050(1 + 1.0075 + \ldots + 1.0075^{n-1})$

$= P(1.0075)^n - 1050\left[\frac{1(1.0075^n - 1)}{0.0075}\right]$

$= P(1.0075)^n - 140\,000\,[1.0075^n - 1]$

$= P(1.0075)^n - 140\,000\,(1.0075)^n + 140000$

(c) (iii) (2 marks)
Outcomes Assessed*: *H4, H5
Targeted Performance Bands*: *3-4

Criteria	Mark
• Correctly substitutes n=240 and makes $A_n = 0$	1
• Gives the correct answer	1

Sample Answer

When $n = 240$

$$\therefore 0 = P(1.0075)^{240} - 140000(1.0075)^{240} + 140\,000$$

$$\therefore P(1.0075)^{240} = 140\,000(1.0075)^{240} - 140\,000$$

$$\therefore P = \frac{140\,000(1.0075)^{240} - 140\,000}{(1.0075)^{240}}$$

$\therefore P =$ \$116 702 (to the nearest dollar)

Question 9 (12 marks)
(a) (i) (2 marks)
Outcomes Assessed*: *H4, H5
Targeted Performance Bands*: *2-3

Criteria	Mark
• Correctly differentiate to find the velocity v_A	1
• Correctly differentiate to find the velocity v_B	1

Sample Answer

$x_A = 12t + 5 \quad \therefore v_A = 12$

$x_B = 6t^2 - t^3 \quad \therefore v_B = 12t - 3t^2$

(a) (ii) (1 mark)
Outcomes Assessed*: *H4, H5
Targeted Performance Bands*: *2-3

Criteria	Mark
• Gives the correct answer	1

Sample Answer

At $t = 1$ second $\therefore v_A = 12$ m/s, $v_B = 12 - 3 = 9$ m/s

$\therefore$ particle A is faster

(a) (iii) (1 mark)
Outcomes Assessed*: *H4, H5
Targeted Performance Bands*: *3 – 4

Criteria	Mark
• Gives the correct answer	1

Sample Answer

$v_B = 0 \therefore 12t - 3t^2 = 0 \therefore 3t(4-t) = 0 \therefore t = 0$ or $t = 4$ seconds

$\therefore$ Particle comes at rest at $t = 4$ seconds

(a) (iv) (2 marks)
Outcomes Assessed*: *H4, H5
Targeted Performance Bands*: *3 – 5

Criteria	Mark
• Correctly substitutes the value of $t = 4$ into the x_B equation	1
• Gives the correct answer	1

Sample Answer

For maximum displacement, $v_B = 0 \therefore t(12 - 3t) = 0 \therefore t = 4$ seconds

$\therefore x_B = 6 \times 4^2 - 4^3 = 32$ metres

(b) (i) (1 mark)
Outcomes Assessed*: *P6
Targeted Performance Bands*: *2 – 4

Criteria	Mark
• Gives the correct answer	1

Sample Answer

The gradient of the tangent at point P is $m_P = \tan 60^\circ = \sqrt{3}$

(b) (ii) (2 marks)
Outcomes Assessed*: *P3, P4
Targeted Performance Bands*: *3 – 5

Criteria	Mark
• Correctly finds the gradient of the tangent to the curve and equates to the gradient obtained from (i)	1
• Correctly finds the coordinates of point P	1

Sample Answer

$y = 1 - x^2 \therefore \dfrac{dy}{dx} = -2x$ and since the gradient of the tangent is $\sqrt{3}$

$\therefore \sqrt{3} = -2x \therefore x = \dfrac{-\sqrt{3}}{2} \therefore y = \dfrac{1}{4} \therefore$ coordinates of point P are $\left(-\dfrac{\sqrt{3}}{2}, \dfrac{1}{4}\right)$

(b) (iii) (3 marks)
Outcomes Assessed*: *P4, H5
Targeted Performance Bands*: *3-6

Criteria	Mark
• Correctly find the length of PQ	1
• Correctly find the length of PO^2 (or QO^2)	1
• Gives the correct answer	1

Sample Answer

Coordinates of point Q are $\left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{4}\right)$ (By symmetry)

$\therefore PQ = \sqrt{3}$

$PO^2 = \dfrac{3}{4} + \dfrac{1}{16} = \dfrac{13}{16}$

Using the Cosine Rule in ΔPOQ: $\cos \angle POQ = \dfrac{\dfrac{13}{16} + \dfrac{13}{16} - (\sqrt{3})^2}{2 \times \sqrt{\dfrac{13}{16} \times \dfrac{13}{16}}}$

$\angle POQ = 2.6$ radians

Question 10 (12 marks)

(a) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 2-5

Criteria	Mark
• Writes the correct value of the amplitude A	1
• Find the correct value of n	1

Sample Answer

Since $y = A\sin nx \therefore A = 2$

Period $(T) = \dfrac{4\pi}{3}$ (from graph) and since Period $(T) = \dfrac{2\pi}{n}$ $\therefore n = \dfrac{2\pi}{T}$

$\therefore n = \dfrac{3}{2}$ $\therefore$ the equation is $y = 2\sin\dfrac{3}{2}x$

(b) (i) (1 mark)

Outcomes Assessed: P4, H5

Targeted Performance Bands: 3 – 4

Criteria	Mark
• Correctly finds the area of ? SOR	1

Sample Answer

In ? SOR, $OS = 6$ (radius of the circle) and $OR = 4$

$\therefore$ area of ? $SOR = ½ \times 6 \times 4 \times \sin\alpha = 12\sin\alpha$ (as required)

(b) (ii) (3 marks)

Outcomes Assessed: P4, H5

Targeted Performance Bands: 3 – 6

Criteria	Mark
• Correctly finds the area of ? SOT	1
• Correctly finds the area of the quadrilateral $ORST$	1
• Correctly factorise to find the correct answer	1

Sample Answer

$\therefore$area of ? $SOT = ½ \times 6 \times 2 \times \sin\left(\dfrac{\pi}{2} - \alpha\right) = 6\cos\alpha$

$\therefore$area of the quadrilateral $ORST$ is:

A = 12 sina + 6 cosa

A = 6cosa(2tana + 1)

(b) (iii) (3marks)

Outcomes Assessed: P4, H5

Targeted Performance Bands: 3 – 6

Criteria	Mark
• Correctly finds the derivative of A	1
• Correctly finds the value of tan a	1
• Correctly shows that the area is a maximum when tan a = 2	1

Sample Answer

A = 12 sina + 6 cosa $\therefore \dfrac{dA}{d\alpha} = 12\cos\alpha - 6\sin\alpha$

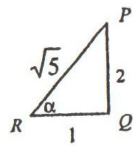
For maximum area $\dfrac{dA}{d\alpha} = 0 \therefore 0 = 12\cos\alpha - 6\sin\alpha$

$\therefore$ 6sina = 12cosa $\therefore \tan\alpha = 2$

To prove the area is maximum when $\tan\alpha = 2$:

$\dfrac{d^2A}{d\alpha^2} = -12\sin\alpha - 6\cos\alpha$ and since $\tan\alpha = 2$ $\therefore$ from the triangle PQR

$\therefore \sin\alpha = \dfrac{2}{\sqrt{5}}$ and $\cos\alpha = \dfrac{1}{\sqrt{5}}$



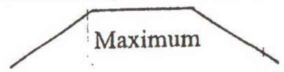
$\therefore \dfrac{d^2A}{d\alpha^2} = -12\times\dfrac{2}{\sqrt{5}} - 6\times\dfrac{1}{\sqrt{5}} = -\dfrac{30}{\sqrt{5}} < 0$

$\therefore$Maximum Area occurs at $\tan\alpha = 2$

__Alternative method:__ To prove the area is maximum when $\tan\alpha = 2$:

Since $\tan\alpha = 2$ $\therefore\alpha = 1.1$ radians

α	1^c	1.1^c	1.2^c
$\dfrac{dA}{d\alpha}$	$1.43 > 0$	0	$-1.24 < 0$



(b) (iv) (3 marks)
Outcomes Assessed: P4, H5
Targeted Performance Bands: 3 – 6

Criteria	Mark
• Correctly finds the gradient of the line OS and equates to 2 to get the equation $y = 2x$	1
• Correctly substitutes into the equation of the circle to get the x-ordinate of point S	1
• Correctly finds the y-ordinate of point S	1

Sample Answer

The gradient of the line $OS = \tan\alpha = \dfrac{y}{x}$ and since $\tan\alpha = 2$

$\therefore \dfrac{y}{x} = 2 \quad \therefore y = 2x \quad [1]$

Substitute into $x^2 + y^2 = 36 \quad \therefore x^2 + 4x^2 = 36$

$\therefore 5x^2 = 36 \quad \therefore x^2 = \dfrac{36}{5} \qquad \therefore x = \dfrac{6}{\sqrt{5}} = \dfrac{6}{5}\sqrt{5}$

$\therefore$ Substitute into $[1] \therefore y = \dfrac{12}{\sqrt{5}} = \dfrac{12}{5}\sqrt{5} \quad \therefore$ point S is $\left(\dfrac{6}{5}\sqrt{5}, \dfrac{12}{5}\sqrt{5}\right)$.