ST. MARK'S COPTIC ORTHODOX COLLEGE

END OF SEMESTER ONE EXAMINATIONS
PRELIMINARY
2008

Mathematics Extension 1

EXAMINER : MR. WAGDY MICHEAL

General Instructions

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (**84**)

- Attempt Questions 1-7
- All questions are of equal value

Total Marks – 84
Attempt Questions 1-7
All Questions are of equal value

Begin each question on a NEW SHEET of paper, writing your name and question number at the top of the page. Extra paper is available.

QUESTION 1 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) Divide the interval A (-2, 7) B (12, 0) internally in the ratio 4:3 **2**

b) Simplify $\frac{3ab^2}{5xy} \div \frac{12ab-6a}{x^2y+2xy^2}$ **2**

c) Factorise $27x^6+\frac{1}{8}$ **2**

d) Solve by completing the square $x^2+2x-7=0$, leaving your answer in simplest exact form. **2**

e) Find the horizontal asymptote of the function $y=\frac{2x^2-4x+3}{x^2-5}$ **2**

f) For what values of x is $|6x-3| \leq 5$ **2**

QUESTION 2 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

(a) Solve $\frac{3}{x-2} \leq 1$. **3**

(b) (i) Write down, in surd form, the values of

$\sin 45^\circ, \cos 45^\circ, \sin 30^\circ, \cos 30^\circ$ **2**

(ii) Hence, show that $\cos 75 = \frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$ **(FULL WORKING OUT MUST BE SHOWN, USE OF CALCULATOR WILL RESULT IN A ZERO MARK)** **2**

(c) Micheal who is W metres south of a tower sees the top of it with an angle of elevation of 20°. Gerges is T metres east of the tower. From his position the angle of elevation is 24° to the top of the tower.

The two men are 1400m apart.

(i) Show that $W = h \cot 20^\circ$ **1**

(ii) Find the height of the tower to the nearest metre. **4**

QUESTION 3 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

(a) Show that $\dfrac{\sec^2 x}{\tan x} = \dfrac{1}{\sin x \cos x}$ **2**

(b) (i) Two lines with gradients m_1 and m_2 intersect on the Cartesian Plane. **1**

If the acute angle between the lines is θ, write the formula for $\tan\theta$.

(ii) If $m_1 = 3$, express the exact value(s) of m_2 in the form $a \pm \dfrac{b\sqrt{c}}{d}$

if $\theta = 30^0$. **2**

(c) (i) Express $\cos\theta - 2\sin\theta$ in the form

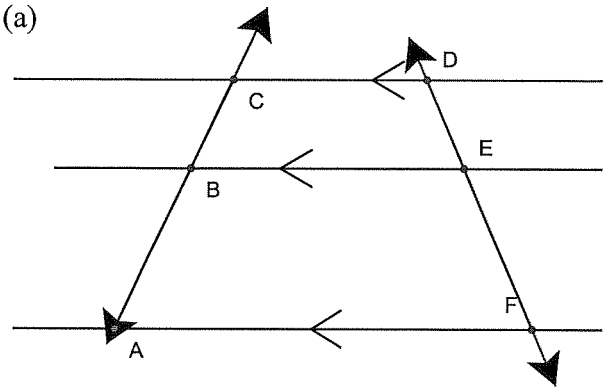
$A\cos(\theta + \alpha)$, $A > 0$, $0 < \alpha < 90^\circ$ **2**

(ii) Hence, solve the equation $\cos\theta - 2\sin\theta = 1$, $0 \le \theta \le 360^\circ$ **2**

(d) Given that $a\sqrt{b} - c = \sqrt{24 - 16\sqrt{2}}$, find the integers a, b and c. **3**

QUESTION 4 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

(a)



The diagram shows 3 parallel lines; CD || BE || AF and 2 transversals AC and DF

(i) Copy the diagram onto your answers and include the parallel to AC through E. **1**

(ii) Prove $\frac{BC}{BA} = \frac{DE}{EF}$ **2**

(You may assume that the opposite sides of a parallelogram are equal)

(b) A (1, -1) B (-3, 1) C (-3, 4) and D (3, 1) are points on the Cartesian Plane. AB||CD

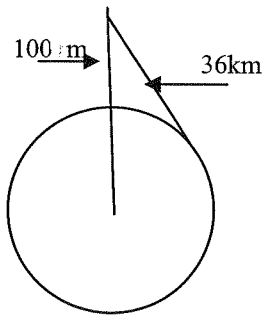
(i) Find the distances AB and DC **2**

(ii) Show that the equation of CD is $x + 2y - 5 = 0$ **2**

(iii) Find the perpendicular distance of A from CD **1**

(iv) Hence or otherwise obtain the area of the trapezium ABCD **1**

(c) From a cliff 100 metres high, the straight line distance to the horizon is 36 kilometres. Calculate the radius of the earth. **3**



QUESTION 5 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

(a) (i) On the same diagram sketch the graphs of $y = \sin x$ and $y = 2\sin x + 1$, where, $0 \le x \le 360^\circ$ **3**

(ii) Hence, or otherwise, solve $2\sin x + 1 \ge \sin x$, where $0 \le x \le 360^\circ$ **2**

(b) (i) Prove $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$ **2**

(ii) Hence, find the exact value of $\tan 15^\circ$ **2**

(c) Boat A sails 15km from port P on a bearing of 055^0. Boat B sails from P for 25 km on a bearing of 135^0.

(i) Show the angle APB = 80^0 **1**

(ii) Calculate their distance apart to 1 decimal place. **2**

QUESTION 6 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

(a) Show that $\dfrac{1-\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} = 2\tan\dfrac{\theta}{2}$ **2**

(b) An Isosceles triangle has base angles of θ° and a base of 12cm. If $\tan\theta = \dfrac{2\sqrt{3}}{3}$, find the exact area of the triangle. **3**

(c) (i) If $t = \tan\dfrac{x}{2}$, show that $3\cos x + 4\sin x + 5 = \dfrac{2t^2+8t+8}{1+t^2}$. **2**

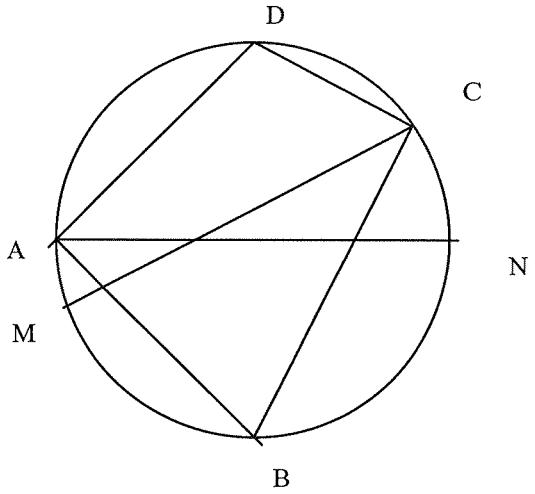
(ii) Hence, solve the equation $3\cos x + 4\sin x + 5 = 0$ for $0 \le x \le 360^\circ$ **2**

(d) Solve $|x-2| + |x+2| > 6$ and graph your solution on a number line. **3**

Question 7 (12 marks) Begin a NEW sheet of writing paper. **Marks**

(a) Find all real solutions for the equation: $\sin 2x = 2\cos^2 x, 0 < x < 360°$. **4**

(b)



In the diagram above , $ABCD$ is a cyclic quadrilateral. M and N are points on the circle through A, B, C and D, such that CM bisects $\angle BCD$ and AN bisects $\angle DAB$.

(i)Copy the diagram. **1**

(ii) Show that MN is a diameter of the circle. **3**

(iii) **Using the fact that MN is a diameter**, prove $\angle NMB = 90° - \angle BCM$. **4**

- End of Paper -