MORIAH COLLEGE

Year 12

MATHEMATICS

Extension 1 and 2 Common

Date: Tuesday 13th March, 2001

Time Allowed: 45 minutes

Examiners: N. Franks, D. Steel

Instructions:

- **Start each question on a separate page.**

- **Show all necessary working.**

- **Mark values are shown but are subject to change.**

Extension 1 and 2 Common March Assessment

1)

a)   i)   Find the differential of: $\sin^{-1}(\sqrt{1-4x^2})$ (2 marks)

ii)   Find $\int \dfrac{dx}{\sqrt{25-4x^2}}$ (2 marks)

iii)   Evaluate $\displaystyle\int_0^1 \dfrac{x\,dx}{1+x^4}$ using the substitution $u = x^2$ (3 marks)

b) Sketch the graphs of   (6 marks)

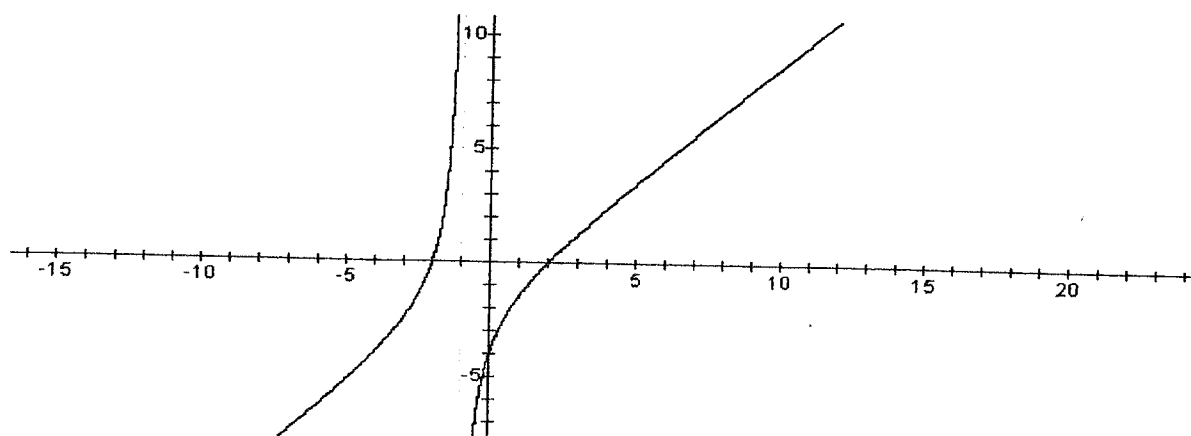i) $y = 4\cos^{-1}(3x-2)$

ii) $y = \cos^{-1}(\sin x)$

2. a) Without the use of a calculator give your answer in exact form:
       [marks will be allocated for working]

    i) Evaluate $\cos(2\sin^{-1}\dfrac{-3}{5})$ (3 marks)

    ii) Find x if $\tan^{-1}x = \tan^{-1}(\dfrac{1}{3}) + \tan^{-1}(-2)$ (3 marks)

    b) The graph of $f(x) = \dfrac{x^2 - 4}{x+1}$ is below.   (7 marks)

    i) Explain why f(x) does not have an inverse function, $f^{-1}(x)$.

    ii) State the largest domain containing x = 0 for which f(x) has an
         inverse function.

    iii) State the domain and range of this inverse function, $f^{-1}(x)$.

    iv) On the graph at the back of the question paper, draw the line $y = x$
         and draw $f^{-1}(x)$.

3.

a) $P(\frac{\pi}{4},1)$ is a point on $y = tan\ x$ and O is the origin. Find to the nearest

minute, the acute angle between OP and the tangent to $y = tan\ x$ at P.

(5 marks)

b)

i) Using a series of diagrams, write down the maximum number of regions into which a circle can be divided, by

α) 1 line

β) 2 lines

γ) 3 lines

δ) 4 lines.

ii) Using the diagrams in part (i), show that the nth line adds n regions and the $(n+1)^{th}$ line adds $(n+1)$ regions.

iii) Hence, use the method of Mathematical Induction to prove that

the greatest number of regions that n straight lines can divide a circle

is $\frac{1}{2}(n^2 + n) + 1, \quad n \geq 1$

(9 marks)