SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2002

TRIAL HIGHER SCHOOL
CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes.
- Working time – 2 hours
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may not be awarded for messy or badly arranged work.

Total Marks - 84 marks

- All questions are of equal value.

Examiner: *E. Choy*

NOTE: This is a trial paper only and does not necessarily reflect the content or format of the final Higher School Certificate Examination Paper for this subject.

Question 1: [12 Marks] **Marks**

(a) Evaluate $\int_{-2}^{2} \frac{dx}{\sqrt{16-x^2}}$, giving your answer in exact form. **2**

(b) If $f(x)=e^{x+1}$ find the inverse function $f^{-1}(x)$ and hence show that **3**

$$f[f^{-1}(x)]=f^{-1}[f(x)]=x$$

(c) Solve the inequality **2**

$$\frac{4-x}{x} \le 1$$

(d) Find the acute angle between the lines $y=\frac{1}{2}x$ and $x+\sqrt{3}y+1=0$. Give your answer in radians correct to two decimal places. **2**

(e) $A(10,1)$, $P(8,5)$ and B are points on the number plane. **3**

Point P divides the interval AB externally in the ratio 2: 3.

Find the coordinates of B.

Question 2: [12 Marks] **Marks**

(a) Differentiate $y = \tan^{-1}(\cot x)$ with respect to x. **2**

(b) Show that $\tan^{-1}(x) = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$ **2**

(c) The polynomial $p(x) = ax^3 + bx^2 - 8x + 3$ has a factor $(x-1)$. When divided by $(x+2)$ the remainder is 15. **2**

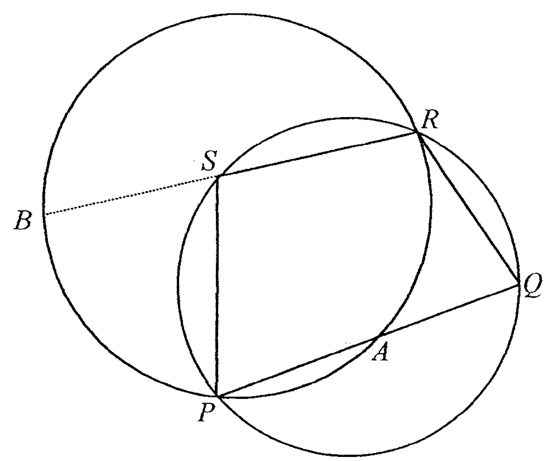
Find the values of a and b.

(d) Find $\frac{d}{dx}\left(\frac{\ln x}{x}\right)$ and hence find the primitive function of $\frac{2-\ln x}{x^2}$ **2**

(e) The word EQUATION contains all five vowels. How many 3 letter "words" consisting of at least 1 vowel and 1 consonant can be made from the letters of EQUATION? **2**

[NB a "word" is ANY arrangement of the letters without any necessary meaning]

(f) **2**



$PQRS$ is a cyclic quadrilateral and A is any point on PQ.

A circle through the points P, A and R cuts RS produced at B.

Prove that $AB \parallel SQ$

Question 3: [12 Marks] **Marks**

(a) Use mathematical induction to show that for all positive integers n **4**

$$\sum_{r=1}^{n} a^{-r} = \frac{a^n - 1}{(a-1)a^n}$$

(b) The tangent at the point $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$ cuts the y - axis at T. **4**

The line through the focus S parallel to this tangent cuts the directrix at V.

M is the midpoint of TV.

Find the locus of M as P moves on the parabola.

(c) Show that $f(x) = x - 3 + \ln x$ has a root between $x = 1$ and $x = 3$. **4**
If x_1 is this root, using Newton's method, prove that the second approximation is given by

$$x_2 = \frac{x_1(4 - \ln x_1)}{1 + x_1}$$

If $x_1 = 2$, find the value of x_2 giving your answer correct to two decimal places.

Question 4: [12 Marks]

Marks

(a) Tidal flow in a harbour is assumed to be simple harmonic motion and water depth x metres at time t hours is given by

$$x = 20 + A\cos(nt + \alpha)$$

where A, n and α are positive constants.

The depth of water is 12 m at low tide and 28 m at high tide which occurs 7 hours later.

(i) Evaluate A and n. **3**

(ii) On a day when low tide occurs at 2.00 am, find the first time period during which the water level is greater than 22 m. **3**

(b) The acceleration of a body moving along a straight line is given by

$$\frac{d^2x}{dt^2} = -\frac{24}{x^2}$$

where x is the displacement from the origin after t seconds.
When $t = 0$, the body is 3 metres to the right of the origin with a velocity of 4 m/s.

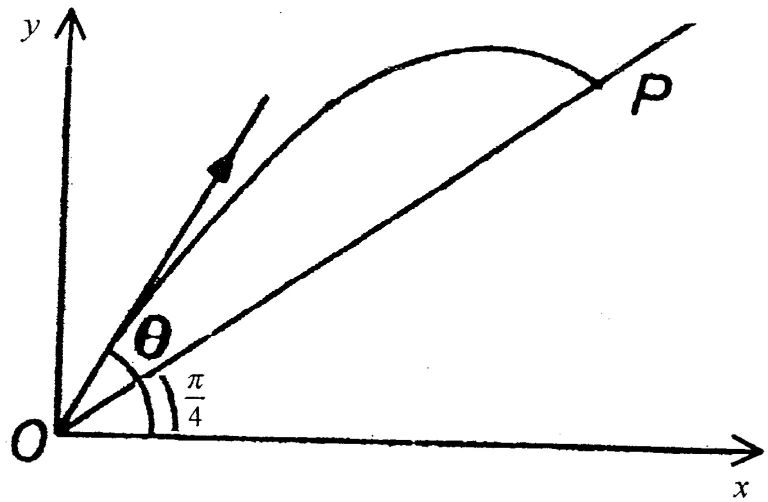
(i) Show that the velocity, v, of the body in terms of x is given by **2**

$$v = \frac{4\sqrt{3}}{\sqrt{x}}$$

(ii) Find an expression for t in terms of x. **2**

(iii) How long does it take for the body to reach a point 10 m to the right of the origin? **2**

Question 5: [12 Marks] **Marks**



A golf ball is hit with a velocity of 5 m/s. It is projected at O, at the bottom of the slope inclined at $\frac{\pi}{4}$ to the horizontal.

The ball is projected at an angle θ to the horizontal, where $\frac{\pi}{4} < \theta < \frac{\pi}{2}$.

The equations of motion are $\ddot{x} = 0$ and $\ddot{y} = -10$

(i) Use calculus to show that the coordinates of the ball's position at time t seconds are given by **3**

$$x = 5t\cos\theta \text{ and } y = -5t^2 + 5t\sin\theta$$

(ii) The ball lands at P, where the length of $OP = R$ metres. **2**

Show that $x = y = \dfrac{R}{\sqrt{2}}$.

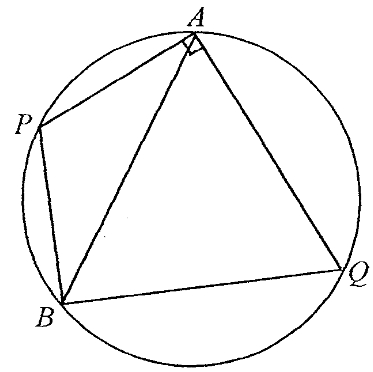
(iii) Show that $R = 5\sqrt{2}\left(\cos\theta\sin\theta - \cos^2\theta\right)$ **3**

(iv) By differentiation, find the exact value of θ (in radians) for the ball to achieve the maximum distance R. **2**

(v) Find the maximum value of R. **2**

Question 6: [12 Marks] **Marks**

(a)



A, P, B, Q are four points on a circle in a horizontal plane.

$\angle AQB = \theta$ and $\angle PAQ = \dfrac{\pi}{2}$

(i) Express $\sin\angle ABQ$ in terms of AB, AQ and θ **2**

(ii) Hence find PQ in terms of AB and θ **3**

(iii) Show that **2**

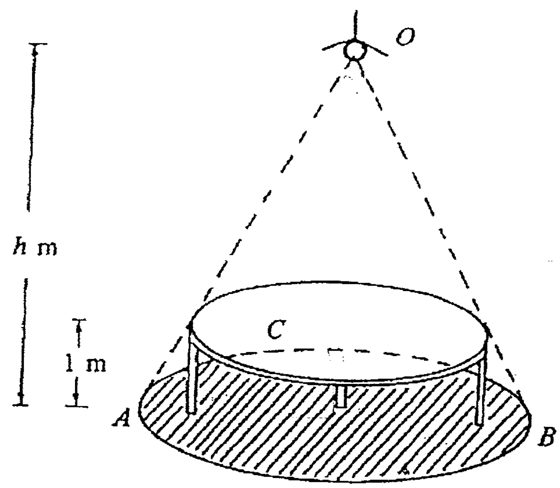
$$PQ = \frac{\sqrt{AP^2 + BP^2 + 2AP \times BP\cos\theta}}{\sin\theta}$$

(b) (i) Prove that $\dfrac{\sin 2x}{1-\cos 2x} = \cot x$ **3**

(ii) Hence, or otherwise, obtain a value for $\cot 67\frac{1}{2}^\circ$ **2**

Question 7: [12 Marks]

Marks



A small lamp O is placed h m above the ground, where $1 < h \leq 5$.

Vertically below the lamp is the centre of a round table of radius 2 m and height 1 m.

The lamp casts a shadow ABC of the table on the ground.

Let S m^2 be the area of the shadow.

(i) Show that $S = \dfrac{4\pi h^2}{(h-1)^2}$ **3**

(ii) If the lamp is lowered vertically at a constant rate of $\dfrac{1}{8}$ m/s, find the rate of change of S with respect to time when $h = 2$. **4**

Let V m^3 be the volume of the cone $OABC$.

(iii) Show that $V = \dfrac{4\pi h^3}{3(h-1)^2}$ **1**

(iv) Find the minimum value of V as h varies. **4**

Does S attain a minimum when V attains its minimum? Explain your answer.

THIS IS THE END OF THE EXAMINATION