QUESTION 1

(a) (i) *Find* $\int \frac{dx}{x^2+4}$

(ii) *Find* $\int \frac{x^2dx}{x^3-8}$

(b) *Evaluate:* (i) $\int_2^7 \frac{xdx}{\sqrt{x+2}}$ using the substitution $u=x+2$

(ii) $\int_2^7 \frac{xdx}{\sqrt{x+2}}$ using the substitution $u=\sqrt{x+2}$

(c) (i) *Show* that $\tan x \equiv \frac{\sin 2x}{1+\cos 2x}$

(ii) *Hence* evaluate $\tan\frac{\pi}{12}$.

QUESTION 2:

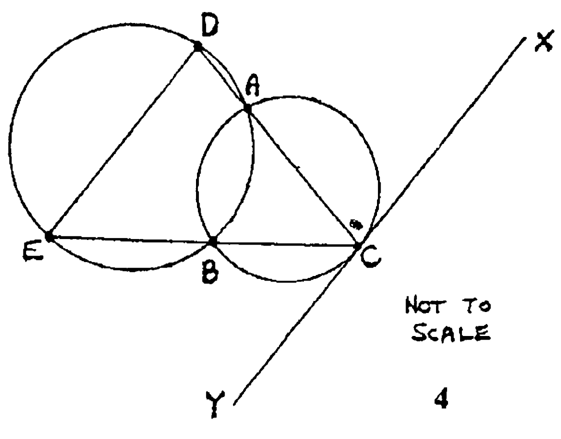
(a) A is the point $(3,2)$ and B is the point (x_B, y_B) .
The point $P(-4,2)$ divides AB internally in the ratio $2:5$
(i.e. $AP : PB = 2:5$). *Find* the values of x_B and y_B .

(b) (i) If $f(x)=\sin^{-1}\frac{x}{2}$ *find* $f^{-1}(x)$.

(ii) *State* the domain and range of $f^{-1}(x)$.

(iii) *Sketch* the graph of $3y=\sin^{-1}\frac{x}{2}$ stating clearly its domain and range.

(c) Two circles intersect at A and B.
From any point C on the smaller circle lines CAD and CBE are drawn cutting the larger circle at D and E respectively.
XY is the tangent at C.
Prove formally that DE is parallel to XY.



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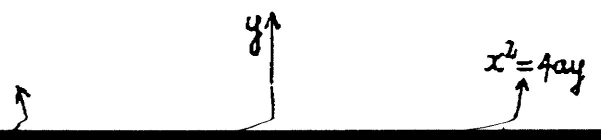
QUESTION 3:

(a) In the diagram P and Q are two points on the parabola $x^2 = 4ay$ having coordinates respectively of $(2ap, ap^2)$ and $(2aq, aq^2)$.



(i) $\alpha+\beta+\gamma$

(ii) $\alpha\beta+\beta\gamma+\gamma\alpha$

(iii) $\alpha\beta\gamma$

(iv) Hence calculate the value of $(\alpha-1)(\beta-1)(\gamma-1)$.

(c) *Solve* for x given that $\dfrac{2x+3}{x-4} > 1$.

Sketch your solution on a number line.

(d) *Differentiate* with respect to x :

(i) $y = x\sin^{-1}\dfrac{x}{2}$

(ii) $y = \tan(x^3)$

(iii) $y = \dfrac{e^{2x}}{1+\cos x}$

QUESTION 5:

(a) (i) *Show* that $\dfrac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta}=\dfrac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$.

(ii) *Find* the most general solution for θ satisfying the equation

$$4\sin^2\theta-1=0$$

(b) A body is heated to a temperature of 120 °C and left to cool in a room whose room temperature is 20 °C . After 10 minutes the temperature of the body cools to 80 °C .

You may assume that the rate of cooling can be expressed in the differential equation

$$\frac{dT}{dt}=-k(T-20)$$

(i) *Show* by integration that the temperature T can be expressed in the form

$$T=20+100e^{-kt} \quad \text{where } k=-\frac{1}{10}\ln\frac{3}{5} .$$

(ii) What will be the temperature to the nearest degree of the body after a further 25 minutes?

QUESTION 6:

(a) The speed $|v|$ of a particle moving along the x-axis is given by the equation

$$v^2 = 12 + 8x - 4x^2$$

where x is the displacement of the point from the origin.

(i) *Prove* that the motion is simple harmonic.

(ii) *Find* its centre of motion.

(iii) *Calculate* its period.

(iv) *Show* that its amplitude is 2 units.

(b) (i) *Write down* an expression for $\sin^2\theta$ in terms of $\cos 2\theta$

(ii) *Hence* evaluate $\int_0^{\frac{\pi}{2}} \sin^2\theta d\theta$

(c) (i) *Sketch* the curve $y = 1 + \sin x$ for the domain $-\pi \leq x \leq \pi$.

(ii) *Hence* sketch the shape of the solid of revolution formed by rotation of this curve about the x-axis .

(iii) *Show* that the volume of this solid formed by rotation about the x-axis is $3\pi^2$ units^3.

QUESTION 7:

(a) A projectile **P** is projected with initial velocity U at angle α to the horizontal.

Show by using $\ddot{x} = 0$ and $\ddot{y} = -g$ and *without* assuming a numerical value for g that :

(i) The time taken to reach maximum height is given by

$$t = \frac{U \sin\alpha}{g} \quad .$$

(ii) *Find* this maximum height reached by the projectile.

(iii) *Show* that to obtain a maximum range, the angle of projection must be 45° .

(b) A missile is projected with a speed of ***100 m/s*** at an elevation of 45° aimed at a tall building which is a horizontal distance of ***400 m*** from the point of projection.

(i) *Find* the time of flight until the missile strikes the building.

(ii) *Find* how high on the building the missile strikes. (You may use the approximation $g \approx 10 m/s^2$ for this part, ie part (ii) .)

Question No. 1

(a)

(i) $\int \frac{dx}{x^2+4} = \frac{1}{2}\tan^{-1}\frac{x}{2} + C$ ①

(ii) $\int \frac{x^2\,dx}{x^3-8} = \frac{1}{3}\log_e(x^3+8) + C$ ①

(b) (i) $I = \int_2^7 \frac{x\,dx}{\sqrt{x+2}}$

$u = x+2$, $x = u-2$

$\therefore du = dx$

when $x = 2$, $u = 4$; when $x = 7$, $u = 9$

$$\therefore I = \int_4^9 \frac{(u-2)\,du}{u^{1/2}}$$

$$= \int_4^9 (u^{1/2} - 2u^{-1/2})\,du$$ ③

$$= \left[\frac{2}{3}u^{3/2} - 4u^{1/2}\right]_4^9$$

$$= \left(\frac{2}{3}\times 27 - 4\times 3\right) - \left(\frac{2}{3}\times 8 - 4\times 2\right)$$

$$= 18 - 12 - 5\tfrac{1}{3} + 8$$

$$= 8\tfrac{2}{3} \text{ Unit.}$$

(ii) $I = \int_2^7 \frac{x\,dx}{\sqrt{x+2}}$

$u = \sqrt{x+2}$

$\therefore u^2 = x+2$

$\therefore 2u\,du = dx$

when $x = 2$, $u = 2$; when $x = 7$, $u = 3$

$$= \int_2^3 \frac{(u^2-2)\cdot 2u\,du}{u}$$

$$= 2\int_2^3 (u^2-2)\,du$$ ③

$$= 2\left[\frac{u^3}{3} - 2u\right]_2^3$$

$$= 2\left[(9-6) - \left(\frac{8}{3} - 4\right)\right]$$

$$= 8\tfrac{2}{3} \text{ units}$$

(c) To show $\tan x = \frac{\sin 2x}{1+\cos 2x}$

$$RHS = \frac{\sin 2x}{1+\cos 2x}$$

$$= \frac{2\sin x\cos x}{1+2\cos^2 x - 1}$$ ②

$$= \frac{2\sin x\cos x}{2\cos^2 x}$$

$$= \frac{\sin x}{\cos x}\cdot\frac{\cos x}{\cos x}$$

$$= \tan x$$

$$= LHS$$

Using the above: (Replace x by $\frac{\pi}{12}$) $\tan\frac{\pi}{12} = \frac{\sin 2\times\frac{\pi}{12}}{1+\cos 2\times\frac{\pi}{12}}$

$$= \frac{\sin\frac{\pi}{6}}{1+\cos\frac{\pi}{6}}$$ ②

$$= \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}$$

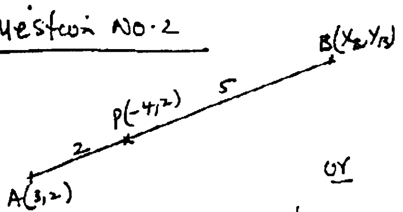
$$= \frac{1}{2+\sqrt{3}}$$

$(\text{or } = 2-\sqrt{3})$

Total 12 Marks

Question No. 2

(a)



$\frac{2x_B + 15}{7} = -4$

$\therefore 2x_B + 15 = -28$

$\therefore 2x_B = -43$

$\therefore x_B = -21\tfrac{1}{2}$ ①

$\frac{2y_B + 10}{7} = 2$

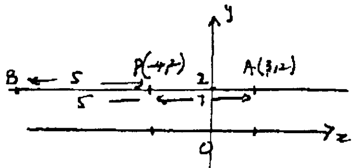
$\therefore 2y_B + 10 = 14$

$\therefore 2y_B = 4$

$\therefore y_B = 2$ ①

$\therefore$ B is the point $(-21\tfrac{1}{2}, 2)$

OR



AP = 7 units which is equivalent to

1 part = $3\tfrac{1}{2}$ units.

PB corresponds to 5 parts

$\therefore$ length PB $= 5\times 2\tfrac{1}{2}$ units $= 17\tfrac{1}{2}$ units

$\therefore$ Co-ords of P are: $(-21\tfrac{1}{2}, 2)$

(b) (i) $f(x) = \sin^{-1}\frac{x}{2}$

f: $y = \sin^{-1}\frac{x}{2}$ ②

f^{-1}: $x = \sin^{-1}\frac{y}{2}$

$\therefore \sin x = \frac{y}{2}$

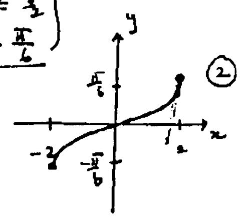
$\therefore y = 2\sin x$

(ii) $\boxed{D: -\frac{\pi}{2} \le x \le \frac{\pi}{2}}$
$R: -1 \le \frac{y}{2} \le 1$ ①

(iii) $3y = \sin^{-1}\frac{x}{2}$

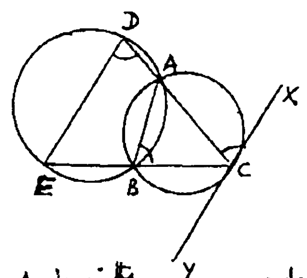
$D: -1 \le \frac{x}{2} \le 1$
ie $-2 \le x \le 2$ ①

$R: -\frac{\pi}{2} \le 3y \le \frac{\pi}{2}$
ie $-\frac{\pi}{6} \le y \le \frac{\pi}{6}$



②

(c) ④



Data: XY is a tangent at C
CAD, CBE and st line.

To Prove: DE ∥ XY

Constr: Join AB

Proof:

∠ACX = ∠ABC — Angles in the Alternate Segment - XY is a tangent

But ∠ABC = ∠ADE — Exterior ∠ of Cyclic Quad ABED

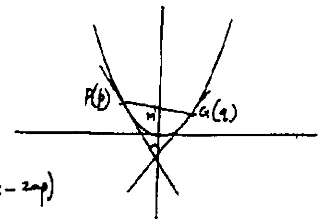
∴ ∠ACX = ∠ADE

But these are alternate angles to lines XY and DE

∴ XY ∥ DE

Total 12 Marks

Question No. 3



$P(2ap, ap^2)$
$Q(2aq, aq^2)$

Equation of PQ is given by:

$$y - ap^2 = \frac{aq^2 - ap^2}{2aq - 2ap}(x - 2ap)$$

$$y - ap^2 = \frac{a(q^2-p^2)}{2a(q-p)}(x-2ap) \qquad p \neq q$$

$$y - ap^2 = \frac{q+p}{2}(x-2ap) \qquad (2)$$

i.e. $2y - \cancel{2ap^2} = (p+q)x - 2apq - \cancel{2ap^2}$

$$\therefore \boxed{y = \frac{p+q}{2}x - apq}$$

To Find the Gradient of tangent at P:

$x^2 = 4ay$

i.e. $2x = 4a\frac{dy}{dx}$

$\therefore \frac{dy}{dx} = \frac{x}{2a}$ (2)

at the pt $(2ap, ap^2)$

$$\boxed{\frac{dy}{dx} = \frac{2ap}{2a} = p}$$

Similarly the Gradient of the tangent at Q is q

$\therefore$ The Condition that they are $\perp^{ar}$ to each other is $\boxed{pq = -1}$ (1)

(iv) Co-ord. of M are $\frac{2ap+2aq}{2}, \frac{ap^2+aq^2}{2}$

i.e. $[a(p+q)], \frac{a}{2}(p^2+q^2)$ (1)

(v). Let $x = a(p+q)$

$\therefore p+q = \frac{x}{a}$ —— (1)

Let $y = \frac{a}{2}(p^2+q^2)$

$\therefore p^2+q^2 = \frac{2y}{a}$ —— (2)

And $pq = -1$

$\therefore (p+q)^2 = p^2+q^2+2pq$ (2)

i.e. $\frac{x^2}{a^2} = \frac{2y}{a} - 2$

i.e. $x^2 = 2ay - 2a^2$

$x^2 = 2a(y-a)$

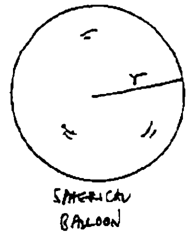
$\therefore$ The Locus of M is $x^2 = 2a(y-a)$.

Co-ords of Vertex of the parabola is $(0, a)$
" " focus " " " is $(0, a+\frac{a}{2})$
i.e. $(0, \frac{3a}{2})$ (1)

(b) $\frac{dV}{dt} = 450.\ cm^3/s.$

$\frac{dr}{dt} = ?$

$$\boxed{\frac{dr}{dt} = \frac{dr}{dV}\cdot\frac{dV}{dt}}$$



$V = \frac{4}{3}\pi r^3$

$\therefore \frac{dV}{dr} = 3\times\frac{4}{3}\pi r^2 = 4\pi r^2$

$\therefore \frac{dr}{dV} = \frac{1}{\frac{dV}{dr}} = \frac{1}{4\pi r^2}$ (3)

$$\therefore \boxed{\frac{dr}{dt} = \frac{1}{4\pi r^2}\times 450}$$

$$\therefore \left(\frac{dr}{dt}\right)_{r=15} = \frac{1}{4\pi\times 15^2}\times 450 \quad cm/s$$

$$= \frac{1}{2\pi}\ cm/s.$$

Question No. 4

(a) $f(x) = x^3 + 3x^2 - 10x - 24$

$f(-2) = (-2)^3 + 3(-2)^2 - 10(-2) - 24$
$= -8 + 12 + 20 - 24$
$= 0$

$\therefore (x+2)$ is a factor

$$
\begin{array}{r}
x^2 + x - 12 \\
x+2 \overline{\smash{\big)} x^3 + 3x^2 - 10x - 24} \\
x^3 + 2x^2 \\
\hline
x^2 - 10x \\
x^2 + 2x \\
\hline
-12x - 24 \\
-12x - 24
\end{array}
$$

$\therefore f(x) = (x+2)(x^2+x-12)$
$= (x+2)(x+4)(x-3)$ (3)

(b). $x^3 + 2x^2 - 3x + 5 = 0$ (α, β, γ)

(i) $\alpha + \beta + \gamma = -2$
(ii) $\alpha\beta + \beta\gamma + \alpha\gamma = -3$
(iii) $\alpha\beta\gamma = -5$ $\Big\}$ $(1\frac{1}{2})$

(iv) $(\alpha-1)(\beta-1)(\gamma-1)$
$= (\alpha-1)[\gamma\beta - \beta - \gamma + 1]$
$= \alpha\beta\gamma - \alpha\beta - \alpha\gamma + \alpha - \gamma\beta + \beta + \gamma - 1$
$= \alpha\beta\gamma - (\alpha\beta + \beta\gamma + \alpha\gamma) + (\alpha + \beta + \gamma) - 1$
$= -5 - (-3) + (-2) - 1$
$= -5 + 3 - 2 - 1 = \underline{-5}$ $\Big\}$ $(1\frac{1}{2})$

(c) $\dfrac{2x+3}{x-4} > 1$ $\quad x \neq 4$

$\therefore \dfrac{2x+3}{x-4} \cdot (x-4)^2 > (x-4)^2$
ie $(2x+3)(x-4) > (x-4)^2$
ie $(x-4)[2x+3-(x-4)] > 0$
ie $(x-4)(x+7) > 0$
ie $\underline{x > 4 \text{ or } x < -7}$ (3)

$-7 \qquad 4$

(d) (i) $y = x\sin^{-1}\left(\frac{x}{2}\right)$
$\frac{dy}{dx} = \sin^{-1}\left(\frac{x}{2}\right) \times 1 + x \times \frac{1}{2} \cdot \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$ (1)
$= \sin^{-1}\frac{x}{2} + \dfrac{x}{\sqrt{4-x^2}}$

(ii) $y = \tan(x^3)$
$\frac{dy}{dx} = 3x^2 \cdot \sec^2(x^3)$ (1)

(iii) $y = \dfrac{e^{2x}}{1+\cos x}$
$\frac{dy}{dx} = \dfrac{(1+\cos x) \cdot 2e^{2x} - e^{2x}(0 - \sin x)}{(1+\cos x)^2}$
$= \dfrac{e^{2x}(2 + 2\cos x + \sin x)}{(1+\cos x)^2}$ (1)

Total: 12 marks

Question No. 5

(a) (i) Show that $\dfrac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \dfrac{\tan\frac{\alpha+\beta}{2}}{\tan\left(\frac{\alpha-\beta}{2}\right)}$

LHS $= \dfrac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta}$
$= \dfrac{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}{2\cos\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2}}$ (3)
$= \tan\frac{\alpha+\beta}{2} \cdot \cot\frac{\alpha-\beta}{2}$
$= \dfrac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

(ii) $4\sin^2\theta = 1$
$\therefore \sin^2\theta = \frac{1}{4}$
$\therefore \sin\theta = \pm\frac{1}{2}$ (3)
$\therefore \underline{\theta = n\pi \pm \frac{\pi}{6}}$

(b) (i) $\dfrac{dT}{dt} = -k(T-20)$
$\therefore \dfrac{dT}{T-20} = -k\,dt$
$\therefore \displaystyle\int \frac{dT}{T-20} = -k\int dt + A$
$\therefore \log_e(T-20) = -kt + A$
$\therefore T-20 = e^{-kt+A} = Be^{-kt}$

$T = 20 + Be^{-kt}$ ——— (1)

when $t=0$, $T = 120°C$
$\therefore 120 = 20 + B$
$\therefore B = 100$
$\therefore \underline{T = 20 + 100e^{-kt}}$

when $t = 10$ min., $T = 80°C$
$\therefore$ Sub in (1):
$80 = 20 + 100e^{-10k}$
$\therefore 60 = 100e^{-10k}$
$\therefore e^{-10k} = \frac{60}{100} = \frac{3}{5}$ (5)
$\therefore -10k = \log_e\frac{3}{5}$
$\therefore \underline{k = -\frac{1}{10}\log_e\frac{3}{5}}$

$\therefore T = 20 + 100e^{\frac{1}{10}\left(\log_e\frac{3}{5}\right)t}$

After a further 25 mts: total time taken to cool will be 35 mts. (1)
$\therefore T = 20 + 100e^{\frac{1}{10}\left(\ln\frac{3}{5}\right)35}$
$= 36.73° \; (37°)$.

Total: 12 marks

Question No. 6

(a) $v^2 = 12 + 8x - 4x^2$

(i) $\frac{1}{2}v^2 = 6 + 4x - 2x^2$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 4 - 4x$

$= -4(x-1)$ (2)

ie $\ddot{x} = -4(x-1)$

This is of the form $\ddot{X} = -n^2 X$, when $X = x-1$

$\therefore$ The motion is Simple Harmonic.

(ii) Centre of motion is when $X = 0$
ie when $x = 1$ (½)

(iii) $T = \frac{2\pi}{n}$. $n = 2$

$\therefore T = \frac{2\pi}{2} = \pi$ sec (½)

(1)

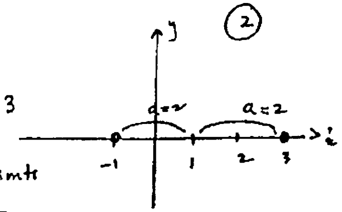
(iv) At the position of maximum displacement $v = 0$

ie $12 + 8x - 4x^2 = 0$

ie $x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\therefore x = -1, 3$



(2)

$\therefore$ The amplitude $= 2$ units

(b). (i) $\cos 2\theta = 1 - 2\sin^2\theta$

$\therefore \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$ (1)

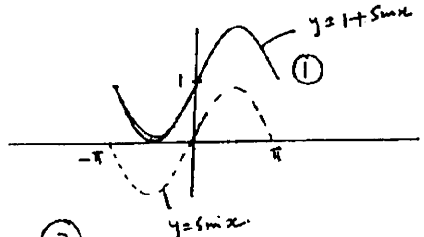
(ii) $\int_0^{\pi/2} \sin^2\theta \, d\theta = \frac{1}{2}\int_0^{\pi/2}(1 - \cos 2\theta)\,d\theta$

$= \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$

$= \frac{1}{2}\left[\left(\frac{\pi}{2} - 0\right) - 0\right]$ (2)

$= \frac{\pi}{4}$ units

(c). $y = 1 + \sin x$ for $-\pi \le x \le \pi$



(1)

$V = \pi \int_{-\pi}^{\pi} y^2\,dx$

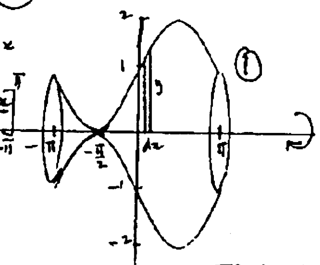
$= \pi \int_{-\pi}^{\pi} (1 + \sin x)^2\,dx$ (2)

$= \pi \int_{-\pi}^{\pi} (1 + 2\sin x + \sin^2 x)\,dx$

$= \pi \left[x - 2\cos x + \frac{x}{2} - \frac{\sin 2x}{4}\right]_{-\pi}^{\pi}$

$= \pi\left[\left(\frac{3\pi}{2} + 2\right) - \left(-\frac{3\pi}{2} + 2\right)\right]$

$= 3\pi^2$ units^3

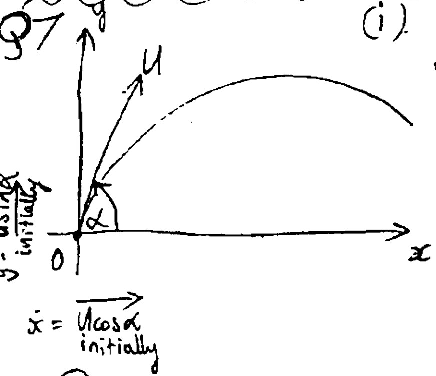


(1)

Total: 12 marks

SOLUTION TO 3U TRIAL HSC - 1998

Q7



(1)

(i)

$\ddot{x} = 0$

$\Rightarrow \dot{x} = C_1$

When $t = 0$

$\dot{x} = U\cos\alpha$

$\Rightarrow C_1 = U\cos\alpha$

$\Rightarrow \dot{x} = U\cos\alpha$

$\Rightarrow x = \int (U\cos\alpha)\,dt$

$x = (U\cos\alpha)t + C_2$

When $t = 0,\ x = 0$

$\Rightarrow C_2 = 0$

$x = (U\cos\alpha)t$

$\ddot{y} = -g$

$\Rightarrow \dot{y} = -gt + C_3$

When $t = 0$

$\dot{y} = U\sin\alpha$

$\Rightarrow C_3 = U\sin\alpha$

$\Rightarrow \dot{y} = U\sin\alpha - gt$

$\Rightarrow y = \int (U\sin\alpha - gt)\,dt$

$= (U\sin\alpha)t - \frac{g}{2}t^2 + C_4$

When $t = 0,\ y = 0$

$\Rightarrow C_4 = 0$

$y = (U\sin\alpha)t - \frac{g}{2}t^2$ (2)

At maximum height, $\dot{y} = 0 \Rightarrow 0 = U\sin\alpha - gt \Rightarrow t = \frac{U\sin\alpha}{g}$ # (1)

(ii) $\Rightarrow y_{MAX} = (U\sin\alpha).\frac{U\sin\alpha}{g} - \frac{g}{2}\left(\frac{U\sin\alpha}{g}\right)^2$

$= \frac{U^2\sin^2\alpha}{g} - \frac{U^2\sin^2\alpha}{2g}$

$= \frac{U^2\sin^2\alpha}{2g}$. This is the maximum height reached by the projectile. (1)

(iii) Range occurs when $y = 0$ the second time $\Rightarrow 0 = (U\sin\alpha)t - \frac{g}{2}t^2$

$\Rightarrow 0 = t\left[U\sin\alpha - \frac{g}{2}t\right] \Rightarrow t = 0$ or $\frac{2U\sin\alpha}{g}$ (1)

Range is reached when $t = \frac{2U}{g}\sin\alpha$ and the range must be $x_R = U\cos\alpha . \frac{2U}{g}\sin\alpha$

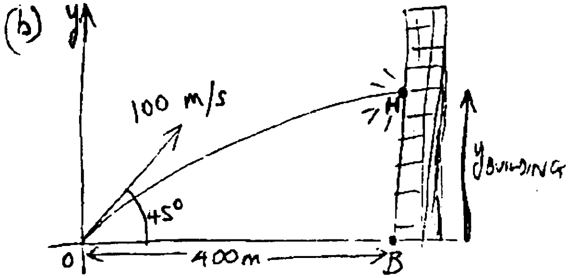
$= \frac{U^2}{g}\sin 2\alpha$ (1)

This is a MAXIMUM when $\sin 2\alpha = 1 \Rightarrow$ when $2\alpha = 90^\circ$

$\alpha = 45^\circ$ (1) #

SOLUTIONS - 3U TRIAL HSC 1998

(b)



(i) Using from (a) $x = (U\cos\alpha)t$

The building is struck when $x = 400$ m

$400 = (100\cos 45^\circ)t$

$t = \frac{4}{\cos 45^\circ} = 4\sqrt{2}$ s. (2)

This is the time of flight until the missile strikes the building //

(ii) The height is given by $y = (U\sin\alpha)t - \frac{g}{2}t^2$ from (a)

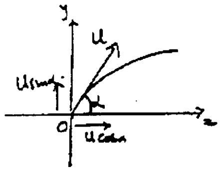
$= 100\sin 45^\circ \times 4\sqrt{2} - \frac{10}{2} \times (4\sqrt{2})^2$

$= \frac{400\sqrt{2}}{\sqrt{2}} - 160$

$= 240$ m (2)

The missile strikes the building 240 m above the ground. //

Question no. 7



$\ddot{y} = -g$

$\therefore \dot{y} = -\int g\,dt + A$

$\therefore \dot{y} = -gt + A$

when $t=0$, $\dot{y} = U\sin\alpha$

$\therefore A = U\sin\alpha$

$\therefore \boxed{\dot{y} = -gt + U\sin\alpha}$ —— (1)

(2)

At Maximum height $\dot{y} = 0$ $\therefore \boxed{t = \frac{U\sin\alpha}{g}}$

Integrating (1) further

$\frac{dy}{dt} = -gt + U\sin\alpha$

$\therefore \int\left(\frac{dy}{dt}\right)dt = \int(-gt + U\sin\alpha)\,dt + B$

$y = (U\sin\alpha).t - \frac{1}{2}gt^2 + B$

when $t=0$, $y=0$ $\therefore B = 0$

$\therefore y = (U\sin\alpha).t - \frac{1}{2}gt^2$ —— (2)

$\therefore$ the maximum height reached is

$y_{max} = U\sin\alpha.\frac{U\sin\alpha}{g} - \frac{1}{2}g.\left(\frac{U\sin\alpha}{g}\right)^2$

$= \frac{u^2\sin^2\alpha}{g} - \frac{u^2\sin^2\alpha}{2g}$

$= \frac{u^2\sin^2\alpha}{2g}$ —— (3)

(3)

$\ddot{x} = 0$

$\therefore \dot{x} = k$

when $t=0$, $\dot{x} = u\cos\alpha$ $\therefore k = u\cos\alpha$

$\therefore \dot{x} = u\cos\alpha$

ie $\frac{dx}{dt} = u\cos\alpha$

$\therefore \int\frac{dx}{dt}.dt = \int u\cos\alpha.dt + C$

$\therefore x = (u.\cos\alpha)\,t + c$

when $t=0$, $x=0$. $\therefore C=0$

$\therefore x = (u\cos\alpha)\,t$ —— (4)

Time taken to cover the range = 2 × time taken to reach maximum height

ie $x = U\cos\alpha.\frac{2U\sin\alpha}{g}$

$= \frac{u^2}{g}.\sin\alpha.\cos\alpha$

$x = \frac{u^2}{g}\sin 2\alpha$

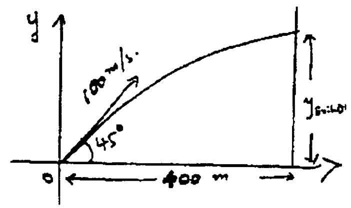
For maximum range $\sin 2\alpha = 1$

$\therefore 2\alpha = 90°$

$\therefore \alpha = 45°$

(3)

(b)



(2)

Using Eq^n (4)

$x = U\cos\alpha.t$

$400 = 100\cos 45°.t$

$t = \frac{400}{100\cos 45}$

$= 4\sqrt{2}$ secs.

Using Eq^n (2): $y_{build} = (U\sin\alpha)t - \frac{1}{2}gt^2$

$= 100\frac{1}{\sqrt{2}} \times 4\sqrt{2} - \frac{1}{2} \times g.(4\sqrt{2})^2$

$= 400 - 160$

$= 240$ m

$\therefore$ The missile strikes the building 240 m above the ground

(2)

12 marks