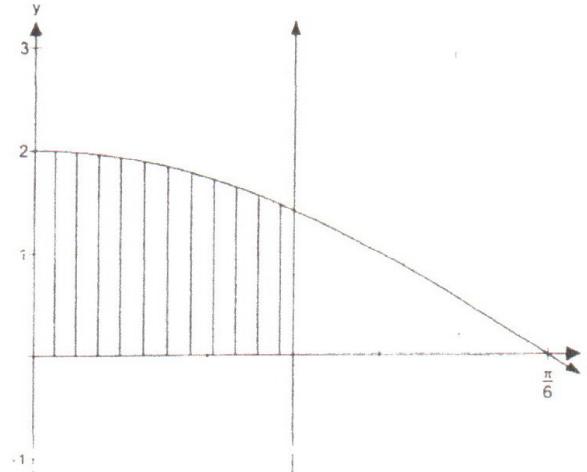


Question 1		Trial HSC Examination- Mathematics	2008
Part	Solution	Marks	Comment
(a)	$\left(\frac{1}{e^{2.5}} - 1\right)^2 = 0.84256 = 0.843 \text{ (3 sig fig)}$	2	1 for answer 1 for rounding
(b)	$ 2x - 4  \leq 2$ $-2 \leq 2x - 4 \leq 2$ $2 \leq 2x \leq 6$ $1 \leq x \leq 3$	2	1 for each part of the inequality
(c)	$\frac{4}{2-\sqrt{3}} = a + b\sqrt{3}$ $\frac{4}{2-\sqrt{3}} = \frac{4}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$ $= \frac{8+4\sqrt{3}}{4-3}$ $a+b\sqrt{3} = 8+4\sqrt{3}$ $a = 8 \text{ and } b = 4$	2	1 for correct method 1 for values of a and b
(d)	$4\frac{1}{2} + 3 + 1\frac{1}{2} + \dots$ Series is arithmetic with $a = 4\frac{1}{2}$ and $d = -1\frac{1}{2}$ $S_n = \frac{n}{2}(2a + (n-1)d)$ $S_{10} = \frac{10}{2}(9 + (9)\left(1\frac{1}{2}\right))$ $= 5\left(22\frac{1}{2}\right)$ $= 112\frac{1}{2}$	2	1 for formula 1 for answer
(e)	$2z^2 + 6zy + xz + 3xy = 2z(z + 3y) + x(z + 3y)$ $= (2z + x)(z + 3y)$	2	1 for partial factorisation or simple mistake leading to incorrect factorisation 2 for correct factorisation.
(f)	$d = \frac{6(1) - 8(3) + 5}{\sqrt{6^2 + (-8)^2}}$ $= \frac{-13}{\sqrt{100}}$ $= 1.3$	2	1 for use of formula 1 for answer

## Question 2

## Trial HSC Examination- Mathematics

2008

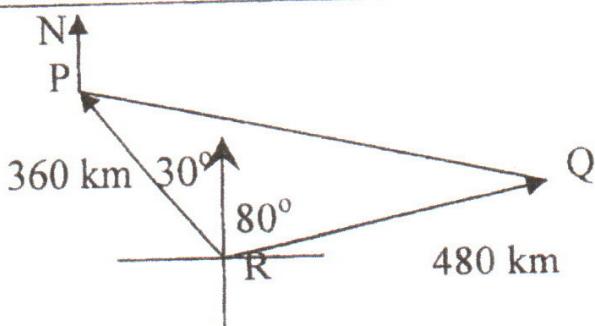
Part	Solution	Marks	Comment
(a) i)	$\frac{d}{dx}(2x^3 + x^{-3}) = 6x^2 - 3x^{-4}$	2	1 for each part of derivative.
ii)	$\begin{aligned} \frac{d}{dx}\left(\frac{1}{e^{2x}} - \sin x\right) &= \frac{d}{dx}(e^{-2x} - \sin x) \\ &= -2e^{-2x} - \cos x \end{aligned}$	2	1 for each part of derivative.
(b) i)	$\int \sec^2 x - e^{4x} dx = \tan x - \frac{e^{4x}}{4} + c$	2	1 for each part of integral.
ii)	$\begin{aligned} \int x^2 + \frac{2}{x} dx &= \left[ \frac{x^3}{3} + 2 \ln x \right]_1^e \\ &= \frac{e^3}{3} + 2 \ln e - \frac{1}{3} - 2 \ln 1 \\ &= \frac{e^3}{3} + 2 - \frac{1}{3} - 0 \\ &= \frac{e^3 - 1}{3} + 2 \end{aligned}$	3	1 for each part of the integral.  1 for substitution.
(c)	$y = 2 \cos 3x$  $\text{Area} = \int_0^{\pi/6} 2 \cos 3x dx$ $= \left[ \frac{2 \sin 3x}{3} \right]_0^{\pi/6}$ $= \frac{2 \sin \frac{\pi}{4}}{3} - \frac{2 \sin 0}{3}$ $= \frac{2}{3\sqrt{2}} - 0$ $= \frac{\sqrt{2}}{3} \text{ square units.}$	3	1 for using correct integral including units.  1 for integration  1 for evaluation

$$\begin{aligned}
 &2 \left[ \ln 2 - \ln 1 \right] \\
 &= 2 \left( \ln 2 - 0 \right)
 \end{aligned}$$

Question 3		Trial HSC Examination- Mathematics	2008
Part	Solution	Marks	Comment
(a) i)	Midpoint of (1, 6) and (5, 0). $MP = \left( \frac{1+5}{2}, \frac{6+0}{2} \right) = \left( \frac{6}{2}, \frac{6}{2} \right) = (3, 3)$	1	1 for answer
ii)	Show that (3,3) lies on $2x - 3y + 3 = 0$ $\begin{aligned} LHS &= 2(3) - 3(3) + 3 \\ &= 6 - 9 + 3 \\ &= 0 = RHS \end{aligned}$ So M lies on BD.	1	1 for answer
iii)	Gradient AC = $m_1 = \frac{6-0}{1-5} = \frac{6}{-4} = -\frac{3}{2}$	1	1 for answer
iv)	Find gradient $m_2$ of BD $2x - 3y + 3 = 0$ $2x - 3y + 3 = 0$ $3y = 2x + 3$ $y = \frac{2}{3}x + 1$ $\therefore m_2 = \frac{2}{3}$ $m_1 \cdot m_2 = -\frac{3}{2} \cdot \frac{2}{3} = -1$ $\therefore$ BD is perpendicular to AC	2	1 for gradient of BD  1 for showing product of gradients = -1
v)	$\begin{aligned} AC &= \sqrt{(5-1)^2 + (0-6)^2} \\ &= \sqrt{16+36} \\ &= \sqrt{52} \\ &= 2\sqrt{13} \end{aligned}$	1	1 for answer
(vi)	The lines AC and BD would form the diagonals of the quadrilateral ABCD. BD is the perpendicular bisector of AC from ii and iv above.. The diagonals of a kite meet at right angles and one diagonal bisects the other, so ABCD meets the criteria for a kite.	1	Need to mention perpendicular (or meet at right angles) and midpoint (or bisect) for point for mark.

(a)	(i) $T_6 = 60 \times (0.75)^5$ = 14.24 cm	P4 • Gives correct answer (IU, IR) ..... 2 • Indicates a geometric sequence, e.g. $a = 60$ , $r = 0.75$ ..... 1
	(ii) $S_6 = \frac{60(0.75^6 - 1)}{0.75 - 1}$ = 197.29 cm	H4, H5 • Gives correct answer (IU, IR) ..... 2 • Correct substitution into the formula ..... 1
	(iii) $S_\infty = \frac{60}{1 - 0.75}$ = 240 cm	H4, H5 • Gives correct answer (IU) ..... 1

Question 4		Trial HSC Examination- Mathematics	2008
Part	Solution	Marks	Comment
(a)	$\sqrt{\frac{\operatorname{cosec}^2 x - \cot^2 x - \cos^2 x}{\cos^2 x}} = \sqrt{\frac{1 - \cos^2 x}{\cos^2 x}}$ $= \sqrt{\frac{\sin^2 x}{\cos^2 x}}$ $= \frac{\sin x}{\cos x}$ $= \tan x$	2	2 marks for required result. 1 mark for partial work toward required result.
(b) i)	$P(\text{Blue on first die}) = \frac{4}{6} = \frac{2}{3}$ $P(\text{Blue on 2nd die}) = \frac{1}{6}$ $P(\text{2 Blue showing}) = \frac{2}{3} \times \frac{1}{6} = \frac{1}{9}$	1	1 mark for answer.
(b) ii)	$P(\text{Different}) = P(\text{BR}) + P(\text{RB})$ $= \frac{2}{3} \times \frac{5}{6} + \frac{1}{3} \times \frac{1}{6}$ $= \frac{5}{9} + \frac{1}{18}$ $= \frac{11}{18}$	$OR = 1 - P(\text{same})$ $= 1 - (P(\text{RR}) + P(\text{BB}))$ $= 1 - \left( \frac{5}{6} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{6} \right)$ $= 1 - \frac{7}{18}$ $= \frac{11}{18}$	2 for correct answer.  1 mark if correct strategy used, but mistake made in the process.

Question 4		Trial HSC Examination- Mathematics	2008
Part	Solution	Marks	Comment
(d) i)	 $PQ^2 = 360^2 + 480^2 - 2 \times 360 \times 480 \cos 110^\circ$ $PQ^2 = 478202$ $PQ = 692 \text{ km (nearest km)}$	2	2 for complete answer. 1 if started using the cos rule correctly but simple mistake made.
(d) ii)	<p>First find <math>\angle QPR</math></p> $\frac{\sin \angle QPR}{480} = \frac{\sin 110^\circ}{692}$ $\sin \angle QPR = \frac{480 \times \sin 110^\circ}{692}$ $\sin \angle QPR = 0.652$ $\angle QPR = 41^\circ$ $\angle NPR = 150^\circ$ $\text{Bearing}(\angle NPQ) = 150^\circ - 41^\circ$ $= 109^\circ$	2	2 for complete answer. 1 if started using the sin or cos rule correctly but simple mistake made.

(c)

$$(i) \quad \angle BDC = 70^\circ$$

The angles making a straight line add to  $180^\circ$ .

---


$$(ii) \quad \frac{\sin \theta}{8} = \frac{\sin 70^\circ}{9}$$

$$\sin \theta = \frac{8 \sin 70^\circ}{9}$$

$$\theta = 56^\circ 39'$$

---


$$(iii) \quad A = \frac{1}{2} \times 5 \times 8 \times \sin 110^\circ$$

$$\approx 18.8 \text{ cm}^2$$

Question 5		Trial HSC Examination- Mathematics	2008
Part	Solution	Mark s	Comment
(a) i)	$\angle EAD = 60^\circ$ (equilateral $\Delta$ ) $\angle DAC = 45^\circ$ (isosceles right $\Delta$ ) $\therefore \angle EAB = \angle EAD + \angle DAC + \angle CAB$ $= 60^\circ + 45^\circ + 51^\circ$ $= 156^\circ$	3	1 for equilateral $\Delta$ 1 for isosceles $\Delta$ 1 for answer
ii)	$\angle ABC = 180^\circ - 156^\circ$ (cointerior $\angle$ on $\parallel$ lines AE and BC) $= 24^\circ$	1	1 for answer
(b) i)	$x = \frac{4t^2 + t + 8}{4t + 1}$ $x = \frac{4(0)^2 + (0) + 8}{4(0) + 1}$ when $t = 0$ $= 8$	2	1 for answer 1 for working
(b) ii)	$x = \frac{4t^2 + t + 8}{4t + 1}$ $\dot{x} = \frac{(4t + 1)(8t + 1) - (4t^2 + t + 8)(4)}{(4t + 1)^2}$ $= \frac{32t^2 + 12t + 1 - 16t^2 - 4t - 32}{(4t + 1)^2}$ $= \frac{16t^2 + 8t - 31}{(4t + 1)^2}$	1	1 for answer

Question 5		Trial HSC Examination- Mathematics	2008
Part	Solution	Mark s	Comment
(b) iii)	$\dot{x} = 0$ $\frac{16t^2 + 8t - 31}{(4t+1)^2} = 0$ $16t^2 + 8t - 31 = 0$ $t = \frac{-8 \pm \sqrt{8^2 - 4(16)(-31)}}{2(16)}$ $= \frac{-8 \pm \sqrt{2048}}{32}$ $= \frac{-8 \pm 32\sqrt{2}}{32}$ $= \frac{-1 \pm 4\sqrt{2}}{4}$ So it is stationary when $t = \frac{-1 \pm 4\sqrt{2}}{4}$ Only us positive value so $t = \frac{-1 + 4\sqrt{2}}{4}$	2	1 for quadratic
(b) iv)	$t = \frac{-1 + 4\sqrt{2}}{4} \approx 1.2 \text{ sec}$ $v = 0, x \approx 2.6$ When $t = 0$ $x = 8$ and $v = -3$ When $t = 2$ $x = 2.9$ and $v = 0.6$ Particle starts 8 units to the right of the origin, moving toward the origin, it decelerates and stops after 1.2 sec at 2.6 m to right of origin, then begins to move away from the origin, being 2.9 units to the right of the origin after 2 sec.	2	1 for mention of direction of travel before and after turning. 1 for mention of position at at least one point.

### Sample answer

Let the roots be  $\alpha$  and  $2\alpha$ .

$$\alpha + 2\alpha = -\frac{b}{a}$$

$$3\alpha = \frac{24}{4}$$

$$= 6$$

$$\alpha = 2$$

The roots are 2 and 4.

$$\alpha \times 2\alpha = \frac{c}{a}$$

$$2 \times 4 = \frac{k}{4}$$

$$k = 32$$

### Syllabus outcomes and marking guide

- P4
- Gives correct answer ..... 2
  - Makes significant progress, e.g. determines that the roots are 2 and 4 ..... 1

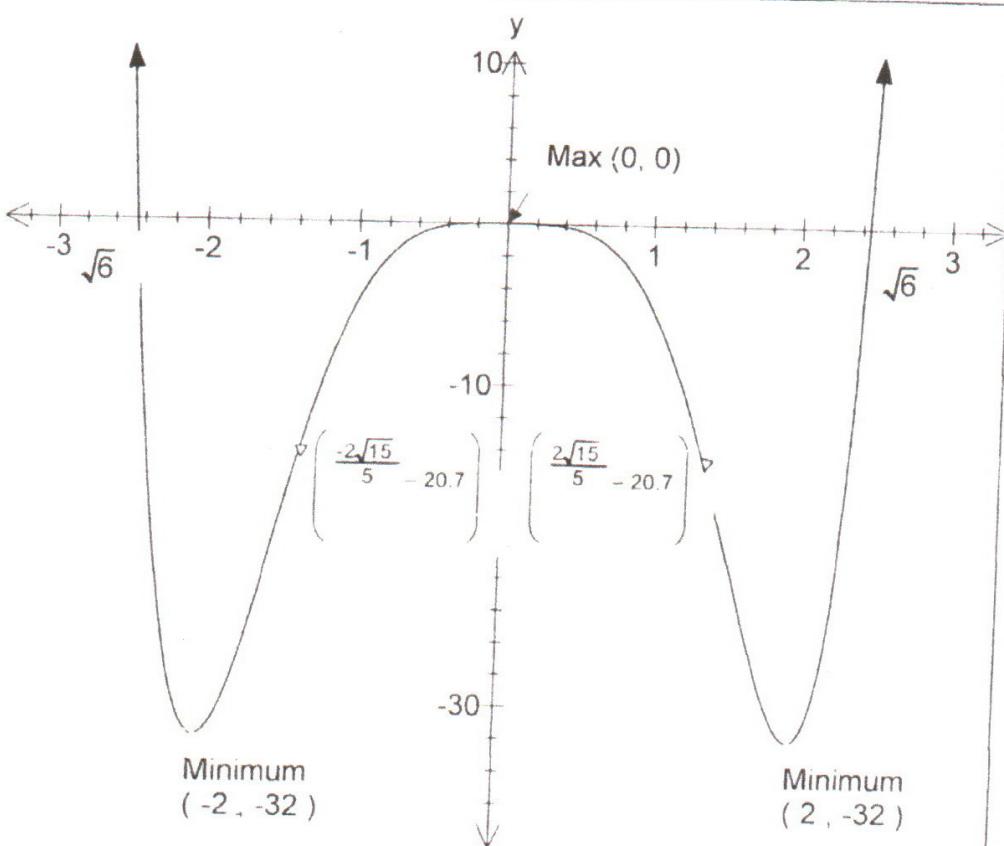
Question 6		Trial HSC Examination- Mathematics	2008
Part	Solution	Mark s	Comment
(a) (i)	$y = x^6 - 6x^4$ Crosses axis where $x^6 - 6x^4 = 0$ $x^4(x^2 - 6) = 0$ $x^4(x - \sqrt{6})(x + \sqrt{6}) = 0$ Crosses axis where $x = 0$ and $x = \pm\sqrt{6}$	2	2 if all given.  1 if only one or two given
(a) (ii)	$y = x^6 - 6x^4$ $y' = 6x^5 - 24x^3$ $= 6x^3(x^2 - 4)$ $= 6x^3(x - 2)(x + 2)$ $y'' = 30x^4 - 72x^2$ Stationary points where $x = 0, y = 0, y'' = 0$ $x = 2, y = -32, y'' = 192$ $x = -2, y = -32, y'' = 192$  Stationary points $(-2, -32), (0, 0), (2, -32)$ $y'' = 30x^4 - 72x^2$ At $x = 0$ $y'' = 0$ so test either side At $x = 1$ $y'' = -42$ $\therefore$ concave down. At $x = -1$ $y'' = -42$ $\therefore$ concave down $\therefore$ maximum at $(0, 0)$ At $x = 2$ $y'' = 192$ $\therefore$ minimum at $(2, -32)$ . At $x = -2$ $y'' = 192$ $\therefore$ minimum at $(-2, -32)$ .	4	1 for derivative  1 for solving for x values  1 for y values  1 for nature

## Question 6

## Trial HSC Examination- Mathematics

2008

Part	Solution	Mark s	Comment
(a) (iii)	$y'' = 30x^4 - 72x^2$ $= 6x^2(5x^2 - 12)$ $= 6x^2(\sqrt{5}x - 2\sqrt{3})(\sqrt{5}x + 2\sqrt{3})$ $x = 0 \quad y = 0$ $x = \frac{2\sqrt{3}}{\sqrt{5}} = \frac{2\sqrt{15}}{5} \quad y = -20.736$ $x = -\frac{2\sqrt{3}}{\sqrt{5}} = -\frac{2\sqrt{15}}{5} \quad y = -20.736$ <p>Check for changes of concavity From above, no change at (0, 0) but there is a change at <math>\left(\pm \frac{2\sqrt{15}}{5}, -20.736\right)</math> Inflexions at <math>\left(\pm \frac{2\sqrt{15}}{5}, -20.736\right)</math></p>	2	1 for Coordinate s 1 for testing to determine only 2 inflexions 



Part	Solution	Mark s	Comment
(b)	<p><math>f''(x)</math> is positive where <math>f(x)</math> is increasing</p> <p><math>f'(x)</math> is zero where <math>f(x)</math> is stationary</p> <p><math>f'(x)</math> is negative where <math>f(x)</math> is decreasing</p>	2	1 mark for x coordinates and 1 mark for nature of both.

Stationary points on  $y$  occur where  $f'(x) = 0$  i.e. at

$$x = -4$$

here  $f''(x)$  is positive  $\therefore$  min turning point at  $x = -4$

$$\text{and } x = 2$$

here  $f''(x)$  is negative  $\therefore$  max turning point at  $x = 2$

7

$$(a) \quad (i) \quad x^2 = y + 2$$

$$x^2 = 4a(y + 2)$$

$$\text{vertex} = (0, -2) \text{ and } a = \frac{1}{4}$$

$$(ii) \quad \text{directrix: } y = -2\frac{1}{4}$$

$$(iii) \quad \text{when } x = 3, \text{ and } y = 7$$

$$y' = 2x$$

$$m_T = 2 \times 3$$

$$= 6$$

$$m_N = -\frac{1}{6}$$

$$y - 7 = -\frac{1}{6}(x - 3)$$

$$y = -\frac{1}{6}x + 7.5$$

P4, P6, P7, H6

- Gives correct answer in any form . . . . . 3

- Gives the gradient of the normal as  $-\frac{1}{6}$

OR

- Numerical error in gradient calculation but equation otherwise correct

OR

- Determines the equation of the tangent . . . 2

- Gives  $y' = 2x$  or the point  $(3, 7)$  . . . . . 1

7

(b)  
i)Substitute  $y = x^2 - 3x$  into  $y = 5x - x^2$ 

$$5x - x^2 = x^2 - 3x$$

$$2x^2 - 8x = 0$$

$$2x(x - 4) = 0$$

$$x = 0, \quad y = 0$$

$$x = 4, \quad y = 4$$

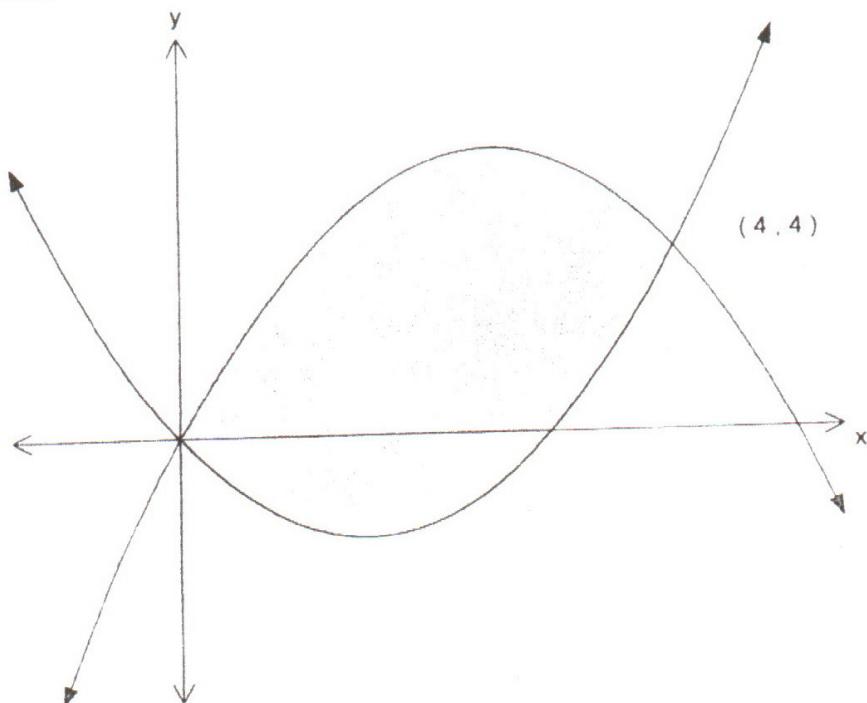
Intersect at  $(0, 0)$  and  $(4, 4)$ .

2

1 for x  
substitution

1 for points

ii)



3

$$\text{Area} = \int_0^4 5x - x^2 dx - \int_0^4 x^2 - 3x dx$$

$$= \int_0^4 8x - 2x^2 dx$$

$$= \left[ 4x^2 - \frac{2x^3}{3} \right]_0^4$$

$$= \left( 64 - \frac{128}{3} \right) - 0$$

$$= \frac{64}{3} = 21\frac{1}{3}$$

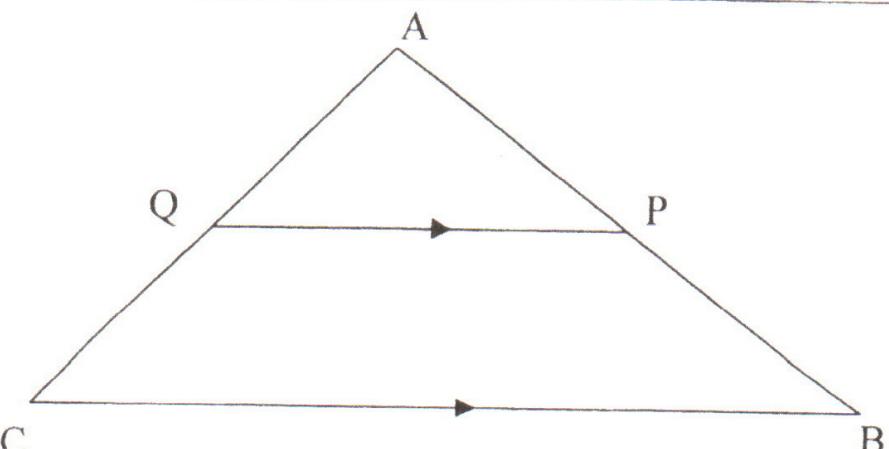
1 for correct  
integral stated1 for  
integration1 for sub and  
correct  
answer

## Question 8

## Trial HSC Examination- Mathematics

2008

Part	Solution	Marks	Comment
(a) i)	$P = Ae^{kt}$ $\frac{dP}{dt} = Ae^{kt} \cdot k$ $= kAe^{kt}$ $= kP$	1	1 for answer
ii)	$t = 1$ was 147 200 $P = Ae^{kt}$ $147200 = Ae^k$ (i) $t = 2$ was 154 800 $154800 = Ae^{2k}$ (ii) $\frac{154800}{147200} = \frac{Ae^{2k}}{Ae^k}$ (ii) $\div$ (i) $1.0516 = e^k$ $k = \ln(1.0516)$ $k \approx 0.05$ $147200 = Ae^{0.05(1)}$ $A = \frac{147200}{e^{0.05}} = 139973$	2	1 for k  1 for A
iii)	When $t = 4$ $P = Ae^{kt}$ $P = 139973e^{0.05(4)}$ $= 171197$	1	1 for answer
iv)	$P = Ae^{kt}$ $200000 = 139973e^{0.05t}$ $\frac{200000}{139973} = e^{0.05t}$ $1.429 = e^{0.05t}$ $\ln(1.429) = \ln(e^{0.05t})$ $0.05t = \ln(1.429)$ $t = \frac{\ln(1.429)}{0.05}$ $= 7.1$ $t = 7$ is start of 2012 Population will reach 200 000 in 2012	1	1 for answer

Question 8		Trial HSC Examination- Mathematics	2008
Part	Solution	Marks	Comment
(b) i)	 <p>In <math>\triangle APQ</math> and <math>\triangle ABC</math>  <math>\angle A</math> is common  <math>\angle AQP = \angle ACB</math> (Corresp <math>\angle</math> on <math>\parallel</math> lines)  <math>\angle APQ = \angle ABC</math> (Corresp <math>\angle</math> on <math>\parallel</math> lines) (1)  <math>\therefore \triangle APQ \sim \triangle ABC</math> (Corresponding angles equal) (1)</p>	2	notes on • $\angle AQP = \angle ACB$ (Corresp $\angle$ on $\parallel$ lines) • $\angle APQ = \angle ABC$ (Corresp $\angle$ on $\parallel$ lines) • $\therefore \triangle APQ \sim \triangle ABC$ (Corresponding angles equal)  AAA not acceptable reason must use words
ii)	$\frac{AP}{AB} = \frac{1}{2}$ (P is midpoint of AB) $\frac{AP}{AB} = \frac{AQ}{AC}$ (sides of similar triangle in same ratio) $\frac{AQ}{AC} = \frac{1}{2}$ (from above) $\therefore Q$ is midpoint of AC.	2	2 for any reasonable explanation using ratio of correspond sides 1 for partial explanation
(c)	$\int_1^3 g(x)dx \approx \frac{1}{6} \{12 + 4(8) + 2(0) + 4(3) + 5\}$ $\approx \frac{61}{6}$ $\approx 10\frac{1}{6}$	2	1 for sub in formula correctly 1 evaluate correctly
(d)	$\sum_{n=2}^5 n^2 - 1 = (2^2 - 1) + (3^2 - 1) + (4^2 - 1) + (5^2 - 1)$ $= 3 + 8 + 15 + 24$ $= 50$	1	1 for answer

Question 9		Trial HSC Examination- Mathematics	2008
Part	Solution	Marks	Comment
(a)	$y = 2x^2 - 2$ $V = \pi \int_0^6 x^2 dy$ $= \pi \int_0^6 \frac{y+2}{2} dy$ $= \pi \left[ \frac{y^2}{4} + y \right]_0^6$ $= \pi \left[ \left( \frac{36}{4} + 6 \right) - (0) \right]$ $= 15\pi \text{ u}^3$	3	1 for formula used correctly 1 for integration 1 for evaluation
(b) i)	$P(WWW) = 0.8 \times 0.8 \times 0.6 = 0.384$	1	1 answer
ii)	$P(LLL) = 0.2 \times 0.2 \times 0.4 = 0.016$	1	1 answer
iii)	$P(\text{at least 1 win}) = 1 - P(LLL)$ $= 1 - 0.016$ $= 0.984$	1	1 answer
(c) i)	$A_6 = 20000 - 6M$	1	1 answer
ii)	$A_7 = (20000 - 6M)1.01 - M$ $A_8 = [(20000 - 6M)1.01 - M]1.01 - M$ $= (20000 - 6M)1.01^2 - 1.01M - M$ $= (20000 - 6M)1.01^2 - M(1 + 1.01)$	1	1 answer
iii)	$A_9 = (20000 - 6M)1.01^3 - M(1 + 1.01 + 1.01^2)$ $A_n = (20000 - 6M)1.01^{n-6} - M(1 + 1.01 + 1.01^2 + \dots + 1.01^{n-7})$ $A_{36} = (20000 - 6M)1.01^{30} - M(1 + 1.01 + 1.01^2 + \dots + 1.01^{29})$	2	1 for developing series further 1 for result for $A_{36}$

Question 9		Trial HSC Examination- Mathematics	2008
Part	Solution	Marks	Comment
iv)	<p>Since repaid after 36 months <math>A_{36} = 0</math></p> $(20000 - 6M)1.01^{30} - M(1 + 1.01 + 1.01^2 + \dots + 1.01^{29}) = 0$ $M(1 + 1.01 + 1.01^2 + \dots + 1.01^{29}) = (20000 - 6M)1.01^{30}$ <p>Need to evaluate <math>1 + 1.01 + 1.01^2 + \dots + 1.01^{29}</math></p> <p>Geometric series with <math>a = 1</math>, <math>r = 1.01</math>, <math>n = 30</math></p> $S_{30} = \frac{a(r^n - 1)}{r - 1}$ $= \frac{1(1.01^{30} - 1)}{1.01 - 1}$ $= 34.785$ $34.785M = (20000 - 6M)1.01^{30}$ $\frac{34.785M}{1.01^{30}} = 20000 - 6M$ $6M + \frac{34.785M}{1.01^{30}} = 20000$ $M\left(6 + \frac{34.785}{1.01^{30}}\right) = 20000$ $31.8M = 20000$ $M = \frac{20000}{31.8}$ $= \$629 \quad (\text{nearest dollar})$	2	

## Question 10

## Trial HSC Examination- Mathematics

2008

Part	Solution	Marks	Comment
a) i)	$V = 2 - \sqrt{3} \cos t - \sin t$ $\frac{dV}{dt} = \sqrt{3} \sin t - \cos t$	1	1 for answer
ii)	When $t = 0$ $\frac{dV}{dt} = \sqrt{3} \sin 0 - \cos 0$ $= -1$ $\therefore$ the tank is emptying at this time.	1	1 for answer or for correct value of derivative.
iii)	Full (or empty) when $\frac{dV}{dt} = 0$ $\frac{dV}{dt} = 0$ $\sqrt{3} \sin t - \cos t = 0$ $\sqrt{3} \sin t = \cos t$ $\frac{\sin t}{\cos t} = \frac{1}{\sqrt{3}}$ $\tan t = \frac{1}{\sqrt{3}}$ $= \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \dots$ As tank is initially emptying, second value corresponds to when full.  Tank is first full when $t = \frac{7\pi}{6}$ $V = 2 - \sqrt{3} \cos t - \sin t$ $= 2 - \sqrt{3} \cos \frac{7\pi}{6} - \sin \frac{7\pi}{6}$ $= 4$	3	2 if final value of $t$ is given. 1 if equation formed correctly but not solved correctly 1 if an incorrect equation is formed and is solved correctly 1 for finding volume from value of $t$

(b) i)	<p><math>\angle D</math> is common</p> <p><math>\angle C = \angle B = 90^\circ</math></p> <p><math>\angle E = \angle A</math> (corresponding angles)</p> <p><math>\Delta ABD \parallel \Delta ECD</math> (equiangular)</p> $\frac{x+h}{x} = \frac{2t}{2} = t$ $x+h = tx$ $tx - x = h$ $x(t-1) = h$ $x = \frac{h}{t-1}$	2	1 for statement of similarity  1 for finding required expression
(b) ii)	$V = \frac{1}{3}\pi(2t)^2 \cdot (h+x) - \frac{1}{3}\pi(2)^2 \cdot x$ $= \frac{1}{3}\pi(2t)^2 \cdot \left(h + \frac{h}{t-1}\right) - \frac{1}{3}\pi(2)^2 \cdot \left(\frac{h}{t-1}\right)$ $= \frac{1}{3}\pi(2t)^2 \cdot \left(\frac{ht}{t-1}\right) - \frac{1}{3}\pi(2)^2 \cdot \left(\frac{h}{t-1}\right)$ $= \frac{1}{3}\pi(2)^2 \cdot \left(\frac{h}{t-1}\right)(t^3 - 1)$ $= \frac{4}{3}\pi \cdot \left(\frac{h}{t-1}\right)(t-1)(t^2 + t + 1)$ $= \left(\frac{4\pi h}{3}\right)(t^2 + t + 1)$	2	1 for using correct formulae  1 for algebraic manipulation to achieve result

iii)	<p>Sum of radii and height = 12</p> $2 + h + 2t = 12$ $h = 10 - 2t$ $V = \left(\frac{4\pi h}{3}\right)(t^2 + t + 1)$ $= \left(\frac{4\pi}{3}\right)(10 - 2t)(t^2 + t + 1)$ $= \left(\frac{4\pi}{3}\right)(10t^2 + 10t + 10 - 2t^3 - 2t^2 - 2t)$ $= \left(\frac{4\pi}{3}\right)(8t^2 + 8t - 2t^3 + 10)$ $V = \left(\frac{4\pi}{3}\right)(8t^2 + 8t - 2t^3 + 10)$ $\frac{dV}{dt} = \left(\frac{4\pi}{3}\right)(16t + 8 - 6t^2)$ $\frac{dV}{dt} = 0$ $(16t + 8 - 6t^2) = 0$ $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-16 \pm \sqrt{(16)^2 - 4(-6)(8)}}{2(-6)}$ $= \frac{-16 \pm \sqrt{448}}{-12}$ $= -0.43 \text{ or } 3.10 \quad (\text{Ignore negative value})$ $\frac{d^2V}{dt^2} = \left(\frac{4\pi}{3}\right)(16 - 12t)$ <p>When <math>t = 3.10</math></p> $\frac{d^2V}{dt^2} = \left(\frac{4\pi}{3}\right)(16 - 12(3.10)) = \left(\frac{4\pi}{3}\right)(-21.2)$ $= -88.7$ $\therefore \frac{d^2V}{dt^2} < 0$ <p><math>\therefore V</math> is a maximum</p> $V = \left(\frac{4\pi}{3}\right)(8(3.10)^2 + 8(3.10) - 2(3.10)^3 + 10)$ $= 218.2$	3	<p>1 for expressing <math>V</math> in terms of <math>t</math> or <math>h</math></p> <p>1 for derivative</p> <p>1 for finding values for max case. If only found the correct value of <math>t</math>, give the mark.</p> <p>Accept any reasonable rounding of values which indicate correct working has been done. i.e. don't deduct for error in rounding.</p>
------	--	---	--