NSW INDEPENDENT TRIAL EXAMS – 2002
Maths 2Unit –HSC Trial Exam
Suggested Answers & Mapping Grid

Q1 (a) $2.2035 \Rightarrow \underline{2.20}$

(b) $x-2 \geq 1 \quad -x+2 \geq 1$

$\underline{x \geq 3} \quad -x \geq -1$

$\underline{\text{OR } x \leq 1}$

(c) $\frac{(x+2)(x-2)}{xy} \times \frac{2x}{x(x-2)}$

$= \underline{\frac{x+2}{y}}$

(d) $\frac{1(2+\sqrt{3}) + 1(2-\sqrt{3})}{2^2 - (\sqrt{3})^2}$

$= \underline{4}$

(e) $x(x-4) = 0$

$\underline{x = 0, 4}$

(f) $\frac{2^n(2-1)}{2^{2n}(2-1)}$

$= 2^{n-2n}$

$= \underline{2^{-n}}$

Q2 (a) $\left[\frac{1}{5} e^{5x}\right]_0^2$

$= \frac{1}{5}(e^{10} - e^0)$

$= \underline{\frac{1}{5}(e^{10} - 1)}$

(b) $\underline{\frac{1}{2} \ln(2x-3) + C}$

(c) $\frac{\sin \hat{R}}{18.6} = \frac{\sin 37}{12.5}$

$\sin \hat{R} = 0.895\ldots$

$\hat{R} = 180° - 63°34'$

$= \underline{116°26'}$

(d) (i) $\underline{\frac{x \sec^2 x - \tan x}{x^2}}$

(ii) $e^x(-\sin x) + \cos x \cdot e^x$

$= \underline{e^x(\cos x - \sin x)}$

(iii) $\underline{\frac{2}{2x-5}}$

Q3 (a) (i) $AB = \sqrt{(5-1)^2 + (2-4)^2}$

$= \sqrt{40}$

$= \underline{2\sqrt{10}}$

(ii) $M = \left(\frac{-1+5}{2}, \frac{4+2}{2}\right)$

$= \underline{(2, 3)}$

(iii) $m_{AB} = \frac{2-4}{5-1}$

$= \underline{\frac{-1}{3}}$

(iv) $y - 4 = -\frac{1}{3}(x+1)$

$3y - 12 = -x - 1$

$\underline{x + 3y - 11 = 0}$

(v) $L: \ y = 3x - 3$

$\therefore m_L = 3$

$m_{AB} \cdot m_L = -\frac{1}{3} \cdot 3$

$= -1$

$\therefore L \perp AB$

Also $3(2) - 3 - 3 = 0$

$\therefore L$ passes through M

$\therefore \underline{L \text{ is } \perp \text{ bisector } AB}$

(vi) At $C \quad y = 0$

$\therefore 3x - 0 - 3 = 0$

$x = 1$

$\underline{C \text{ is } (1, 0)}$

(vii) $\underline{(x-2)^2 + (y-3)^2 = 10}$

(b) (i) $\alpha + \beta = -\frac{b}{a} = 6$

(ii) $\alpha\beta = c/a = 10$

(iii) $(\alpha+1)(\beta+1) = \alpha\beta + \alpha + \beta + 1$

$= 10 + 6 + 1 = 17$

Q4 (a) (i) $\log_a\left(\frac{c}{b}\right) = \log_a c - \log_a b$

$= 0.35 - 3.5$

$= \underline{-3.15}$

(ii) $\log_a (bc)^2 = 2(\log_a b + \log_a c)$

$= 2(3.5 + 0.35)$

$= \underline{7.7}$

(b) (i) In Δ's ABC, AED

$\hat{A}$ is common

$A\hat{B}C = A\hat{E}D$ (data)

$\therefore \Delta ABC \,///\, \Delta AED$

(equiangular)

(ii) $\frac{AB}{AE} = \frac{AC}{AD}$ (corr sides in Simi Δ's)

$\therefore \frac{2}{x+3} = \frac{x}{5}$

$x^2 + 5x - 10 = 0$

$x = \frac{-5 \pm \sqrt{25 + 40}}{2}$

But $x > 0$

$\underline{x = \frac{-5 + \sqrt{65}}{2}}$

(c) (i) $P(FF) = .54 \times .54$

$= \underline{0.29}$

(ii) $P(MF) = 0.46 \times 0.54$

$+ 0.54 \times 0.46$

$= \underline{0.50}$

(iii) $P(\text{Neither F})$

$= 1 - (P(FF) + P(MF))$

$= 1 - (0.29 + 0.50)$

$= \underline{0.21}$

(OR $P(MM)$)

Q5(a) $y' = 3e^{3x}$

$x=0 \quad y = e^0 + 1 = 2$

$m_1 = 3e^0 = 3$

Normal $m_2 = -\frac{1}{3}$

$y - 2 = -\frac{1}{3}(x-0)$

$3y - 6 = -x$

$\underline{x + 3y - 6 = 0}$

(b) $5\pi = \frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta$

$= \frac{1}{2}\cdot\frac{\pi}{6}(R^2 - 2^2)$

$60 = R^2 - 4$

$R = 8$

$\therefore \underline{AC = 6}$

(c) $y = 2x(x-3)^2$

$y' = 2(x-3)^2 + 2x \cdot 2(x-3)$

$= (x-3)(6x-6)$

$y' = 0 \quad x = 3, 1$

$y' = 6(x^2 - 4x + 1)$

$y'' = 6(2x-4)$

$y'' = 0 \quad x = 2$

(i) $x=1 \quad y=8 \quad y'' < 0$

$\therefore \underline{(1,8)}$ Max TP

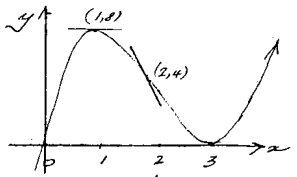
$x=3 \quad y=0 \quad y'' > 0$

$\underline{(3,0)}$ Min TP

(ii) $x=2 \quad y=4$

y'' changes sign

$\therefore \underline{(2,4)}$ P of I



Q6 (a) (i) $t=0 \quad \underline{P = 3000}$

(ii) $\frac{dP}{dt} = 3000 \cdot -k \cdot e^{-kt}$

$= -k(3000e^{-kt})$

$= \underline{-kP}$

(iii) $t=4 \quad P = 2000$

$2000 = 3000e^{-4k}$

$e^{-4k} = \frac{2}{3}$

$4k = \ln(\frac{3}{2})$

$\underline{k = 0.10137}$

(iv) $P = 300$

$300 = 3000e^{-kt}$

$e^{-kt} = \frac{1}{10}$

$kt = \ln(\frac{10}{1})$

$t = \frac{\ln 10}{0.10137}$

$= \underline{22.7 \text{ days}}$

(b)(i) $SA = 2\pi r^2 + 2\pi rh$

$600\pi = 2\pi r^2 + 2\pi rh$

$r^2 + rh = 300$

$h = \frac{300 - r^2}{r}$

$= \underline{\frac{300}{r} - r}$

(ii) $V = \pi r^2 h$

$= \pi r^2 (\frac{300}{r} - r)$

$= \underline{\pi(300r - r^3)}$

(iii) $\frac{dV}{dr} = \pi(300 - 3r^2) = 0$

$r = 10 \quad (r \neq -10)$

$\frac{d^2V}{dr^2} = -6\pi r < 0$

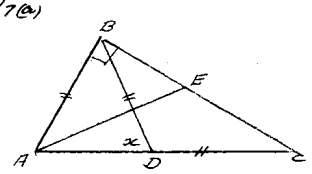
$\therefore$ Max V

$h = \frac{300}{10} - 10$

$\underline{h = 20 \text{ cm}}$

when V is max

Q7(a)



(i)

$x = A\hat{D}B = D\hat{A}B$ (opp $\angle$s = sides) $\triangle ABD$

$\therefore A\hat{B}D = 180 - 2x°$ ($\angle$ sum $\triangle ABD$)

$\therefore D\hat{B}C = 90 - (180 - 2x)$

$= \underline{(2x - 90)°}$

(ii) $D\hat{B}C = B\hat{C}D = (2x - 90°)$

(opp $\angle$s = sides) $\triangle BDC$

$\therefore x = 2(2x - 90)$ (ext $\angle$ of $\triangle$)

$\underline{x = 60°}$

$B\hat{A}D = 60°$ (see (i))

$\therefore E\hat{A}D = 30°$ (AE bisects)

$D\hat{B}C = D\hat{C}B = 2x - 90$

$= 30°$.

$\therefore \underline{\triangle AEC}$ isosceles

(b)(i) $(\frac{\sin\alpha}{\cos\alpha} + \sin\alpha)(\frac{\cos\alpha}{\sin\alpha} + \cos\alpha)$

$= 1 + \sin\alpha + \cos\alpha + \cos\alpha\sin\alpha$

$= \underline{(1+\cos\alpha)(1+\sin\alpha)}$

(ii) $\cos\alpha = -1$ or $\sin\alpha = -1$

$\underline{\alpha = \pi \text{ or } 3\pi/2}$

(c)(i)

$A = \frac{1}{2}\cdot 5\cdot(0 + 2 + 2(3.2 + 5.3 + 4.7))$

$= \underline{710 \text{ m}^2}$

(ii) $V = 710 \times 0.21$

$= 149.1 \text{ m}^3$

$= \underline{149100 \text{ litres}}$

Q8 (a) $x=1 \quad y=1$

$x=4 \quad y=2.$

$V = \pi\int_1^2 x^2 \cdot dy$

$= \pi\int_1^2 y^4 \cdot dy$

$= \frac{\pi}{5}\left[y^5\right]_1^2$

$= \frac{\pi}{5} \times (32 - 1)$

$= \underline{\frac{31\pi}{5} \text{ units}^3}$

(b) $x = t + \cos t$

(i) $t = 0 \quad \underline{x = 1}$

(ii) $v = 1 - \sin t = 0$

$\sin t = 1$

$\therefore \underline{t = \pi/2}$ s

$x = \pi/2 + \cos\pi/2$

$\underline{x = \pi/2 \text{ m}}$

(iii) $t = \underline{5\pi/2 \text{ s}}$

(iv) $\ddot{x} = a = -\cos t$

$t = \frac{\pi}{6} \quad a = -\cos\pi/6$

$= \underline{-\sqrt{3}/2 \text{ ms}^{-2}}$

(c) $\Delta = 9 - 8k \geq 0$

$\underline{k \leq 1\frac{1}{8}}$

Q9 (a) $a = 24 \quad d = 5$ AP

(i) $T_8 = 24 + 7\times 5$

$= 59$

(ii) $24 + (n-1)5 > 150$

$19 + 5n > 150$

$n > 26\frac{1}{5}$

$n = 27$

Q10(a)

(i) $A_1 = 67000 + 8\%$ of 67000

$- 12000$

$= 67000 \times 1.08 - 12000$

(ii) $A_2 = (67000 \times 1.08 - 12000) \times 1.08 - 12000$

$= 67000 \times 1.08^2 - 12000(1.08 + 1)$

(iii) $2895 = \frac{n}{2}(48 + (n-1)5)$

$5790 = n(5n + 43)$

$5n^2 + 43n - 5790 = 0$

$(n - 30)(5n + 193) = 0$

$\therefore \underline{n = 30}$ as $n \neq \frac{193}{5}$

(Formula easier)

(b)(i) A $y=0 \quad x=1 \quad \underline{(1,0)}$

B $x=3 \quad y = \log_e 3$

$\underline{(3, \log_e 3)}$

(ii) $\frac{y-0}{x-1} = \frac{3\ln 3 - 0}{3-1}$

$\therefore \underline{y = \frac{3\ln 3}{2}(x-1)}$

(iii) $y = \log_e x$

$x = e^y$

$\therefore A = \int_0^{\log 3} e^y \cdot dy$

$= e^y\Big]_0^{\log 3}$

$= e^{\log 3} - e^0$

$= 3 - 1$

$= \underline{2 \text{ units}^2}$

(iv) $OABC = \frac{1}{2}\cdot\log_e 3(1+3)$

$= 2\log_e 3$

Required A $= \underline{(2\log_e 3 - 2)}$

$A_3 = 67000 \times (1.08)^3 - 12000(1.08^2 + 1.08 + 1)$

$A_n = 67000(1.08)^n - 12000(1.08^{n-1} + \ldots + 1.08 + 1)$ GP

$A_n = 0.$

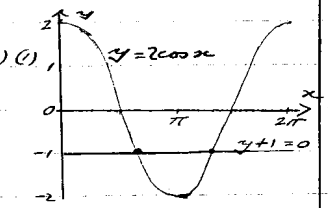
$\therefore 67000(1.08)^n = 12000 \times 1(\frac{1.08^n - 1}{1.08 - 1})$

$6680 \times 1.08^n = 12000$

$n \log 1.08 = \log \frac{12000}{6680}$

$\underline{n \doteq 7.69 \text{ years}}$

(b) (i)



(ii) $2\cos x = -1$

$\cos x = -\frac{1}{2}$

2 solutions

(c)(i) $V = 20t - t^2 + c$

$t=0 \quad V = 500 = c$

$\therefore \underline{V = 20t - t^2 + 500}$

(ii) $t = 10$

$V = 20(10) - 10^2 + 500$

$= \underline{600 \text{ ml}}$

(iii) $t = 0$

$\frac{dV}{dt} = 2(10)$

$= \underline{20 \text{ ml s}^{-1}}$