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PYMBLE LADIES' COLLEGE

YEAR 12

MATHEMATICS EXTENSION I

ASSESSMENT

Time Allowed: 1 hour 30 minutes + 5 minutes reading time

Test date: 7th May, 2001

INSTRUCTIONS:

- **All questions should be attempted**
- **Start each question on a new page**
- **Approved calculators may be used**
- **A standard integral sheet is attached**
- **Write your name and your teacher's name on each page.**
- **DO NOT staple the questions together**
- **Hand this question paper in with your answers**
- **ALL rough working paper must be attached to the back of the last question**
- **A coloured sheet of paper must be attached, by stapling, to the end of each question**
- **This assessment task has a value of 15%**
- **There are 6 questions in this paper**
- **There are 9 pages in this paper**

ASSESSMENT CRITERIA

- Provide answers which are complete, accurate and comprehensive
- Leave your answers in exact form unless otherwise stated
- Include all necessary working. Correct answers will not necessarily gain full marks unless necessary working is shown. Relevant working might gain marks even if your answer is wrong.
- Take care with mathematical notation
- Show relevant information clearly and unambiguously on sketches if required
- Present well set out solutions using a logical set of steps in which justification is included where necessary.

Question 1

(a) Find the acute angle between the lines $y = -x$ and $\sqrt{3}y = x$ (2 marks)

(b) Find the indefinite integral of $\int \frac{dx}{\sqrt{1 - 9x^2}}$ (1 mark)

(c) If α, β and λ are the roots of the cubic equation $2x^3 + x^2 - x - 2 = 0$ find the value of

(i) $\alpha + \beta + \lambda$

(ii) $\alpha\beta\lambda$

(iii) $\alpha\beta + \alpha\lambda + \beta\lambda$

(iv) Hence, or otherwise, find the value of $(\alpha - 1)(\beta - 1)(\lambda - 1)$

(4 marks)

(d) Given $x = 12t$ and $y = 6t^2$ write down

(i) the cartesian equation of the parabola

(ii) the coordinates of the focus

(iii) the equation of the directrix (3 marks)

Question 2 **(Start a new page)**

(a) Solve the equation $\sin\theta - \sqrt{3}\cos\theta = 1$ for $0 \leq \theta \leq 2\pi$ (3 marks)

(b) (i) Solve $\sin 2x = \sin x$ for $0 \leq \theta \leq \pi$

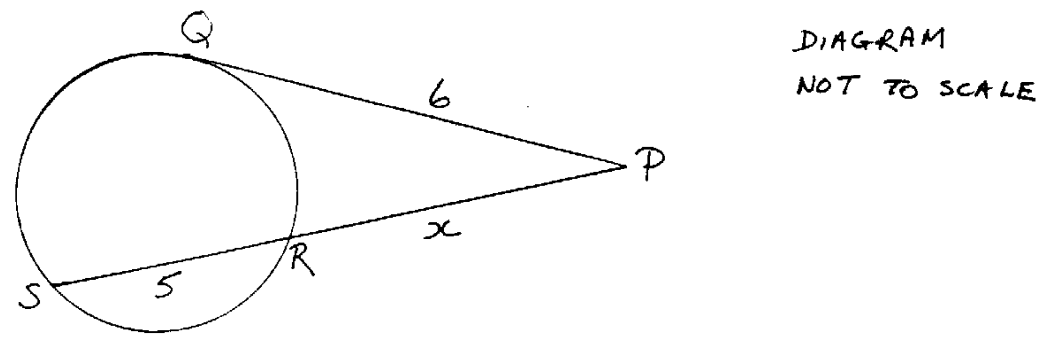
(ii) On the same number plane, sketch $y = \sin 2x$ and $y = \sin x$ for $0 \leq \theta \leq \pi$ showing all important features.

(iii) Hence, or otherwise, find the area bounded by the curves $y = \sin 2x$ and $y = \sin x$ for $0 \leq \theta \leq \dfrac{\pi}{3}$

(7 marks)

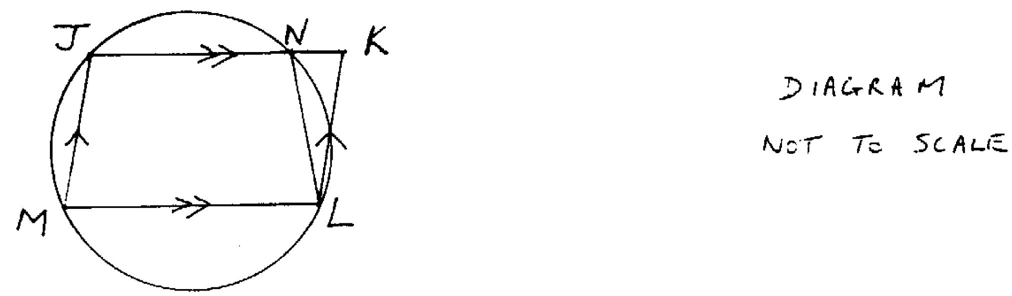
Question 3 **(Start a new page)**

(a)



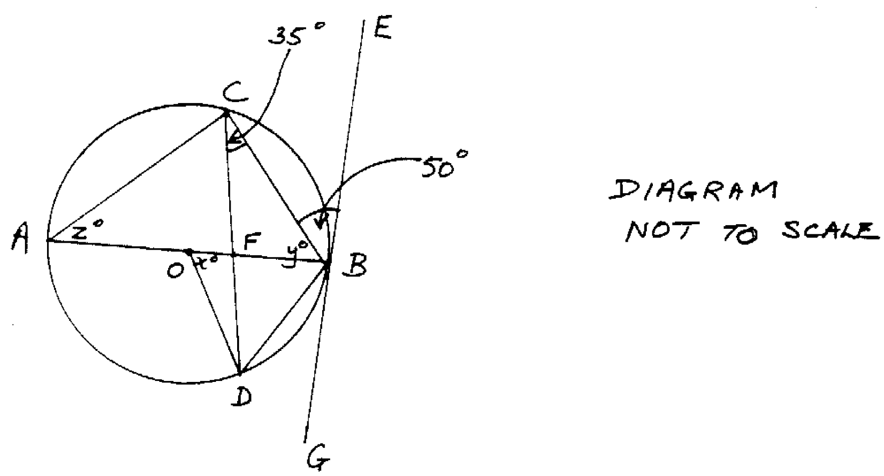
PQ is a tangent to the circle *QRS.*
PRS is a secant intersecting the circle in *R* and *S*
Given that *PQ* = *6, RS* = *5* and *PR* = *x*, find *x*, giving reasons. (3 marks)

(b)



The circle passes through the points *J, N, L,* and *M.*
JKLM is a parallelogram
Prove that *NL* = *LK*, giving reasons. (3 marks)

(c)



0 is the centre of the circle. *AB* is a diameter.
EBG is a tangent at *B*. If $\angle CBE = 50°$ and $\angle DCB = 35°$, find the values of *x*, *y* and *z*, giving reasons. (4 marks)

Question 4 **(Start a new page)**

(a) Find the general solution for the equation $\tan\theta = \dfrac{1}{\sqrt{2}}$ (1 mark)

(b) For the function $f(x) = \sqrt{x} + 3$ find

(i) the inverse function

(ii) the domain of the inverse function. (3 marks)

(c) (i) Differentiate $y = \tan^{-1}\dfrac{1}{x}$, $x \neq 0$

(ii) Hence show that

$$\frac{d}{dx}\left[\tan^{-1}x + \tan^{-1}\frac{1}{x}\right] = 0$$

(iii) Then sketch the curve $y = \tan^{-1}x + \tan^{-1}\dfrac{1}{x}$ (6 marks)

Question 5 **(Start a new page)**

(a) The function $f(x) = x^3 - x^2 - x - 1$ has a zero between 1 and 2

(i) Taking $x = 2$ as a first approximation to this zero, use Newton's method to calculate a second approximation.

(ii) Would $x = 1$ have been a suitable first approximation to use? Explain your answer fully.

(4 marks)

(b) The function $f(x) = ax^3 + bx^2 + cx + d$ has a double zero at $x = 1$ and a minimum value of -4 when $x = -1$.

Find the values of a, b, c and d.

(6 marks)

Question 6 **(Start a new page)**

Consider the parabola $x^2 = 4ay$

(i) Find the equation of the tangent at $P(2ap, ap^2)$

(ii) If the tangent at P cuts the *y-axis* at T, show that $T = (0, -ap^2)$

(iii) Find the equation of the normal at P.

(iv) If the normal at P cuts the *y-axis* at N, show that $N = (0, 2a + ap^2)$

(v) Explain why NT is the diameter of the circle passing through PTN.
Hence find the equation of the circle.

(vi) If the tangent at P cuts the x-axis at R and M is the midpoint of RN,
show that $R = (ap, 0)$ and find the co-ordinates of M.

(vii) Determine the equation of the locus of M and describe this locus geometrically.

(10 marks)

END OF PAPER