2009

Higher School Certificate

Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in every question
- Write your student number and/or name at the top of every page

Total marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: .....

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Total marks – 120
Attempt Questions 1 - 10
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Answer the questions on your own paper or writing booklet, if provided.
Start each question on a new page.

Question 1 (12 marks) **Marks**

(a) Evaluate $\sqrt{\dfrac{627}{42+29}}$ correct to two significant figures. **2**

(b) Find integers a and b such that $\left(2\sqrt{3}-1\right)^2 = a\sqrt{3}+b$. **2**

(c) Solve $\dfrac{2x-1}{3}-\dfrac{1-3x}{5}=2$. **2**

(d) Find a primitive function of $3+\sin 2x$. **2**

(e) Find the values of x for which $|2-3x|>11$. **2**

(f) Factorise x^4-16. **2**

STUDENT NUMBER/NAME: .....

Marks

Question 2 (12 marks) Start a new writing booklet.

(a) Differentiate with respect to x:

(i) $e^{2x}\tan x$, **2**

(ii) $\dfrac{\sin x}{4-x}$. **2**

(b) Find the equation of the normal to the curve $y=\log_e x-1$ at the point $(e,\ 0)$. **2**

(c) Find:

(i) $\displaystyle\int \frac{3}{\sqrt{2x-1}}\,dx$. **2**

(ii) $\displaystyle\int_{\frac{\pi}{12}}^{\frac{\pi}{9}} \sec^2 3x\,dx$. Give your answer in exact form. **2**

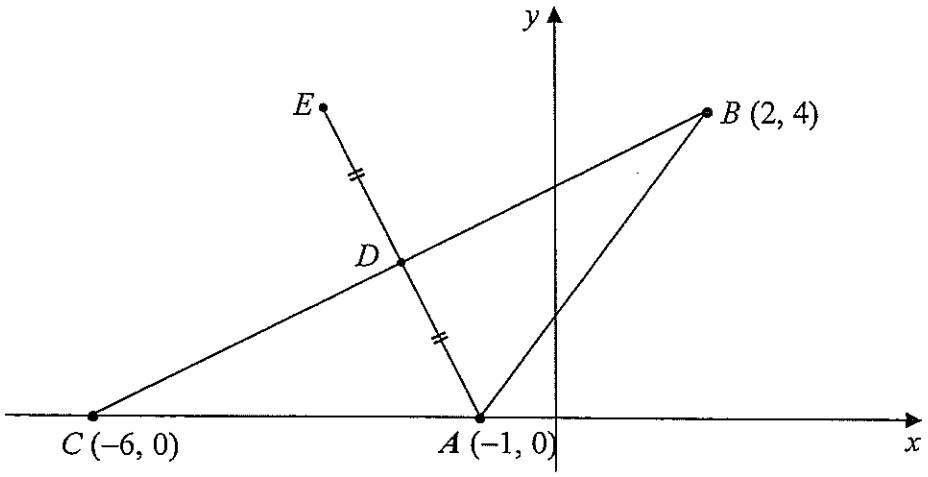
(d) Solve $2\sin\theta+1=0$ for $0\le\theta\le 2\pi$. **2**

STUDENT NUMBER/NAME: .....

Marks

Question 3 (12 marks) Start a new writing booklet.

(a)



NOT TO SCALE

In the diagram A, B and C are $(-1,\ 0)$, $(2,\ 4)$ and $(-6,\ 0)$ respectively.
D has coordinates (–2, 2) and is the midpoint or AE.

(i) Find the length of the interval AB. **1**

(ii) Find the equation of the circle with centre at B which passes through the point A. **1**

(iii) Find the size of $\angle CAB$ (to the nearest degree). **2**

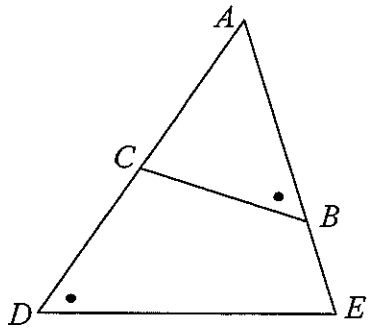
(iv) Find the midpoint of BC. **1**

(v) Show the equation of the line BC is $x-2y+6=0$. **1**

(vi) Find the perpendicular distance of A from the line BC in simplest exact form. **2**

(vii) What type of quadrilateral is $ABEC$? Give reasons for your answer. **2**

(b)



NOT TO SCALE

$\Delta ABC \,|||\, \Delta ADE$. $AC = 4\ cm$, $AB = 6\ cm$ and $BE = 2\ cm$.
Find the length CD. **2**

STUDENT NUMBER/NAME: .....

Marks

Question 4 (12 marks) Start a new writing booklet.

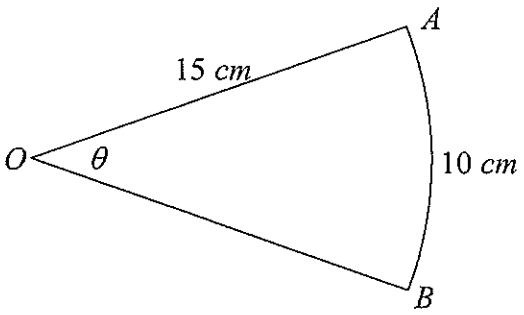
(a) Evaluate $\sum_{x=2}^{20}(3x-5)$. **3**

(b) Consider the parabola $4y = x^2 - 2x + 5$.

(i) Show the vertex is $(1, 1)$, **2**

(ii) Find the coordinates of the focus. **2**

(c)

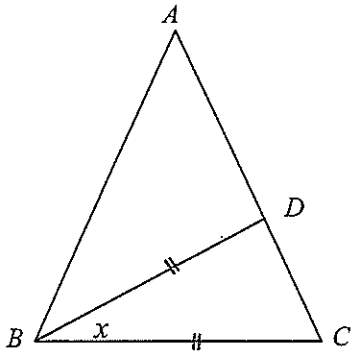


In the diagram, AB is an arc of a circle with centre O.
The arc AB is 10 cm and the radius OA is 15 cm. Find:

(i) the exact size of $\angle AOB$ in radians, **1**

(ii) the exact area of the sector AOB. **1**

(d)



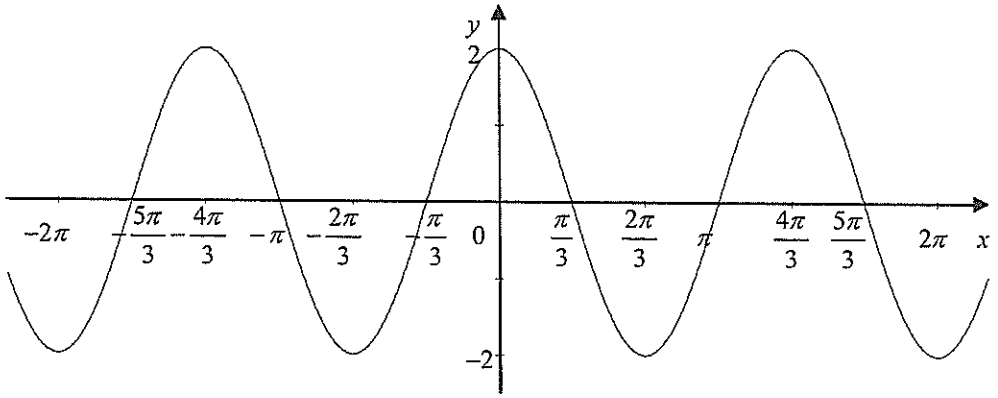
NOT TO SCALE

ΔABC is isosceles with $AB = AC$. **3**
D lies on AC such that $\angle ABD = 3\angle DBC$ and $BD = BC$, as shown above.
Find the value of x.

Marks

Question 5 (12 marks) Start a new writing booklet.

(a)



The graph above can be represented by an equation in the form $y = a\cos nx$. **2**
Find the values of a and n.

(b) A scientist measured the instantaneous velocity, in ms^{-1}, of a particle as it moved through its motion. He made 5 measurements at $t =$ 0, 1, 2, 3 & 4 seconds.
His results, for $v = f(t)$, are tabulated below.

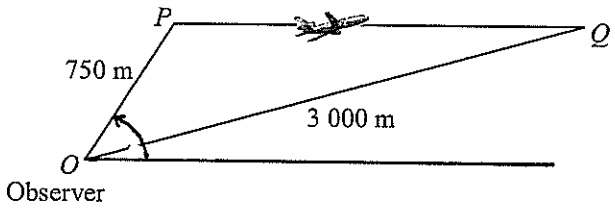
t	0	1	2	3	4
v	6	10	11	8	2

(i) Use Simpson's Rule, with five function values, to approximate the value of **2**

$$\int_0^4 f(t)\,dt$$

(ii) What does $\int_0^4 f(t)\,dt$ represent? **1**

(c)



An observer is standing at point O and sees a plane at P a distance of 750 m from O. **4**
Eight seconds later the plane is sighted at Q, a distance of 3 000 m from O.
The angles of elevation of P and Q from O are 73° and 7° respectively.
Find the speed of the plane to the nearest km/h.

(d) The gradient of a curve is given by $\frac{dy}{dx} = \frac{2x}{x^2 + e}$. The curve passes through **3**
the point $(0,\ 2)$.

What is the equation of the curve?

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Marks

Question 6 (12 marks) Start a new writing booklet.

(a) A function is defined by $f(x) = x^3 - 3x^2 - 9x + 22$.

(i) Find the coordinates of the turning points of the graph $y = f(x)$, and determine their nature. **3**

(ii) Find the coordinates of the point of inflexion. **2**

(iii) Hence sketch the graph of $y = f(x)$, showing the turning points, the point of inflexion and where the curve meets the y–axis. **2**

(iv) For what values of x is the graph of $y = f(x)$ concave up? **1**

(b) Find the values of k for which the quadratic equation $2x^2 - kx + 5 = 0$ has real roots **2**

(c) Fiona is planting exotic flowers in her garden. The probability that an individual flower survives is 0.35. Fiona plants 5 flowers. **2**

What is the probability that at least one flower survives?

Marks

Question 7 (12 marks) Start a new writing booklet.

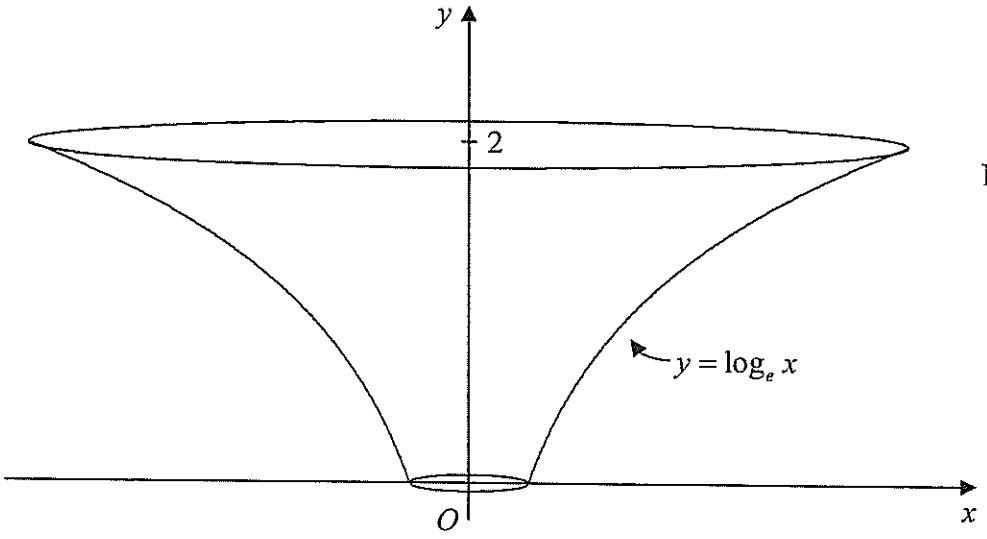
(a) (i) Differentiate $\log_e(\cos x)$ with respect to x. **2**

(ii) Sketch the curve $y = \tan x$ for $0 \le x < \frac{\pi}{2}$. **1**

(iii) Hence, using parts (i) and (ii) or otherwise, find the area bounded by the curve $y = \tan x$, the x axis and the line $x = \frac{\pi}{3}$. **3**
Leave answer in simplest exact form.

(b) State the domain and range of the function $y = 2\sqrt{9 - x^2}$. **3**

(c) **3**



NOT TO SCALE

A mould for vase is formed by rotating that part of the curve $y = \log_e x$ between $y = 0$ and $y = 2$ about the y axis.

Find the volume of the mould. Leave your answer in simplest exact form.

STUDENT NUMBER/NAME: .....

Marks

Question 8 (12 marks) Start a new writing booklet.

(a) Consider the geometric series $3-6y+12y^2-24y^3+.....$

(i) For what values of y does this series have a limiting sum? **2**

(ii) The limiting sum of the series is $2\frac{1}{4}$. Find the value of y. **2**

(b) A drawer contains 6 identical blue socks and 6 identical white socks. Two socks are pulled out at random.

(i) What is the probability that they are the same colour? **2**

(ii) How many socks would need to be pulled out to ensure a matching pair? **1**

(c) The acceleration of an object is given by $12t-2$ cm /sec. If the initial velocity is 5 cm/sec and the object is 3 cm to the left of the origin, find its displacement after 4 seconds. **5**

Marks

Question 9 (12 marks) Start a new writing booklet.

(a) A farm hand has to fill a trailer with bales of hay stretched out in a straight line at 8 metre intervals. The trailer is 14 metres from the first bale. The farm hand starts from the trailer and can carry only one bale at a time.

(i) How far has he walked after putting the 4^{th} bale on the trailer? **2**

(ii) Show that, after putting the n^{th} bale on the trailer, the farm hand has walked $8n^2 + 20n$ metres. **2**

(iii) Union regulations require a rest after every 1000 metres of walking. How many bales must he load before he gets a rest? **2**

(b) Jonah is saving for a cruise. He opens an "Incentive Saver Account" which pays interest at the rate of 0.4% per month compounded monthly at the end of each month. Jonah decides to deposit $400 into the account on the first of each month. He makes his first deposit on the 1st January 2007 and his last on the 1st July 2009. He withdraws the entire amount, plus interest, immediately after his final interest payment on 31st July 2009.

(i) How much did Jonah deposit into his "Incentive Saver Account"? **1**

(ii) How much did Jonah withdraw from his account on the 31st July 2009? **3**

(iii) Jonah's cruise is cancelled due to swine flu. He decides to deposit $10000 into a different account which offers interest at the rate of 5% p.a. compounded quarterly for 2 years. How much will Jonah receive at the end of the investment period? **2**

STUDENT NUMBER/NAME: .....

Marks

Question 10 (12 marks) Start a new writing booklet.

(a) A particle moves in a straight line so that its displacement, x metres from a fixed point O on a line, is given by

$$x = t + \frac{16}{t+1}$$

where t is measured in seconds.

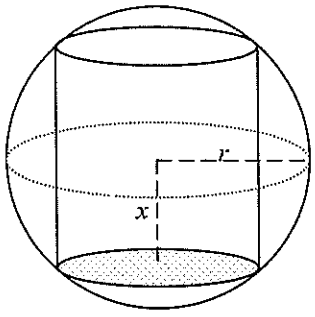
(i) Find the particle's initial position. **1**

(ii) Find expressions for the velocity and acceleration of the particle in terms of t. **2**

(iii) Find when and where the particle is at rest. **2**

(iv) Find the limiting velocity of the particle. **1**

(b) A cylinder is to be made to fit inside a sphere of radius r cm, as shown.



Let x be the distance of the base of the cylinder from the centre of the sphere as shown.

(i) Find an expression for the radius of the base of the cylinder in terms of r and x. **1**

(ii) Show the volume, V, of the cylinder is given by **2**

$$V = 2\pi x\left(r^2 - x^2\right)$$

(iii) Find, in terms of r, the maximum volume of the cylinder. Give answer in exact form. **3**

End of paper

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \, \tan ax \, dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$