Name: ______________________________

Teacher: ____________________________

Class: ______________________________

FORT STREET HIGH SCHOOL

2010

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics

TIME ALLOWED: 3 HOURS
(PLUS 5 MINUTES READING TIME)

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1, 2, 3	
Manipulates algebraic expressions to solve problems from topic areas such as functions, quadratics, trigonometry, probability and logarithms	4, 5, 7, 8	
Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	6, 9	
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	10	

Question	1	2	3	4	5	6	7	8	9	10	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/12	/12	/12	/120	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started in a new booklet

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

<u>Question One</u>: (Start a NEW BOOKLET)

a) Evaluate $\dfrac{\pi}{\sqrt{e^2-1}}$ correct to 3 decimal places. [2]

b) Solve $\dfrac{3-2x}{x}=4$. [2]

c) Rationalise the denominator of $\dfrac{2}{1+\sqrt{3}}$. [2]

d) Factorise $4+11x-3x^2$. [2]

e) Sketch the graph of $x+2y-6=0$, showing the intercepts on both axes. [2]

f) Find the equation (in General Form) of the line perpendicular to $4x-3y-1=0$ that passes through the point $(2,-3)$. [2]

<u>Question Two</u>: (Start a NEW BOOKLET)

a) Differentiate with respect to *x*:

i. x^2e^x [2]

ii. $(1+\tan x)^2$ [2]

b) Find

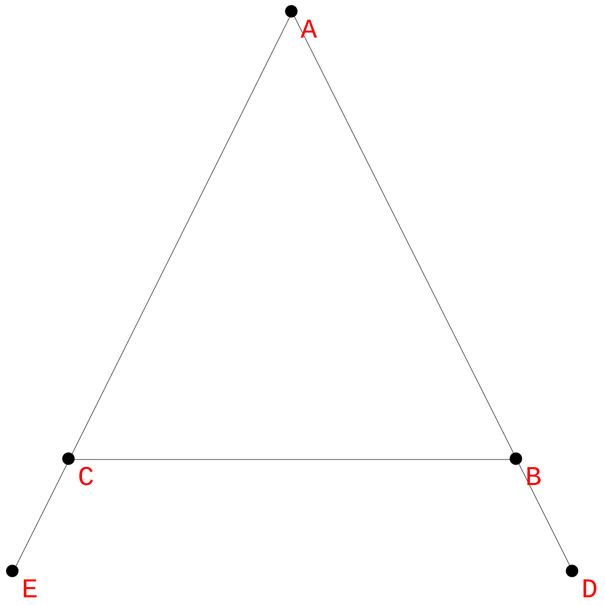
i. $\int 4x-\sin x\,dx$ [2]

ii. $\int_1^3 \dfrac{1}{x^2}\,dx$ [3]

c) Find the equation of the tangent to the curve $y=x-\dfrac{1}{x}$ at the point $(-1,0)$. [3]

<u>Question Three</u>: (Start a NEW BOOKLET)

a) Triangle ABC is isosceles with AB=AC. AB and AC are extended to D and E respectively, with BD=CE, as shown in the diagram below.



i. Copy the diagram into you answer booklet showing the information given. [1]

ii. Prove that $\Delta ABE \equiv \Delta ACD$. [4]

b) Using Simpson's Rule with five function values to find and approximate value for the integral $\int_0^2 e^{x^2}\,dx$, to 2 decimal places. [3]

c) A geometric series has a 3rd term of $\frac{1}{12}$ and an eighth term of $\frac{-1}{384}$. For this series:

i. Find an expression for T_n. [2]

ii. Find the sum of the first 8 terms. [1]

iii. Find the limiting sum. [1]

<u>Question Four</u>: (Start a NEW BOOKLET)

a) Solve $2\cos x = \sqrt{3}$ for $-\pi \le x \le \pi$. [2]

b) For the parabola $x^2 = 6(y+1)$:

i. Write down the coordinates of the vertex. [1]

ii. Find the coordinates of the focus. [1]

iii. Draw a neat sketch of the parabola. [1]

iv. Calculate the area bounded by the parabola and the line $y = 5$. [3]

c) Two ordinary dice are rolled and the score is the sum of the numbers on the top faces.

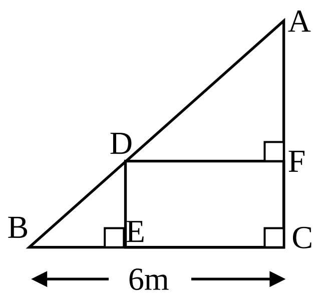
i. What is the probability that the score is 5? [2]

ii. What is the probability that the score is not 5? [1]

iii. What is the probability that the dice show "doubles" (i.e. that both numbers on the top faces are the same)? [1]

<u>Question Five</u>: (Start a NEW BOOKLET)

a) A 10m long ladder (AB) rests against a wall, with its foot (B) 6m from the base (C) of the wall (AC) as shown in the diagram below (Not drawn to Scale). D is a point on the ladder AB.



i. How far up the wall does the ladder reach? [1]

ii. Explain why DF||BC. [1]

iii. Prove $\Delta ADF \,|||\, \Delta ABC$. [2]

iv. Felix climbs the ladder to point D so that he is 3m directly above the ground (E). How far along the ladder (BD) has he climbed? [2]

b) Find the equation of the normal to the curve $y = x(x-2)$ when $x = 2$. [2]

c) Given $g(x) = ax^2 + bx + c$ and that $g(0) = 4, g(1) = 23, g(-1) = 1$, determine the values of a, b and c. [2]

<u>Question Six</u>: (Start a NEW BOOKLET)

a) For the function $y = x^3 - 3x^2 - 9x + 6$:

i. At what point does this curve cut the y-axis? [1]

ii. Find the coordinates of any stationary points and determine their nature. [3]

iii. Find the coordinates of any points of inflection. [1]

iv. For what values of x is the curve concave up? [1]

v. Sketch the curve, showing the information above. [2]

b) If $\sin\theta = -\frac{2}{3}$ and $\cos\theta > 0$, find the value of $\tan\theta$ (in surd form). [2]

c) Show that the derivative of xe^x is $e^x + xe^x$, and hence find $\int xe^x \, dx$. [2]

<u>Question Seven</u>: (Start a NEW BOOKLET)

a) If α and β are the roots of $4x^2 + 8x - 1 = 0$, find the value of

i. $\alpha + \beta$ [1]

ii. $\alpha\beta$ [1]

iii. $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ [2]

b) For the curve $y = 3\sin 2x$:

i. State the amplitude. [1]

ii. Find the period. [1]

iii. Draw a neat sketch of the curve for $0 \le x \le \pi$. [2]

c) Find the value of:

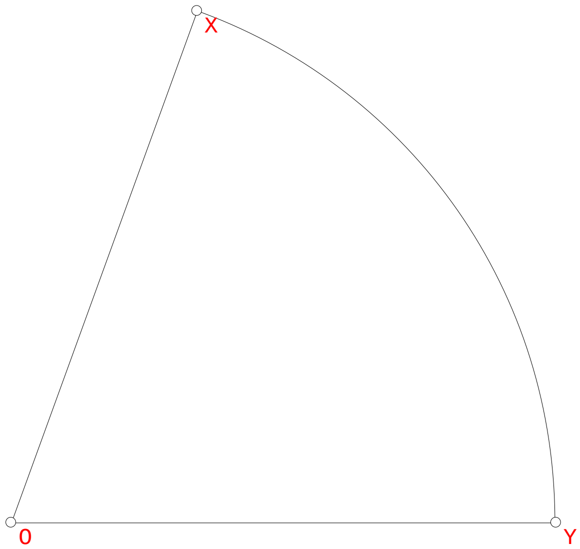
i. $\log_2 45$, given that $\log_2 3 = 1.585$ and $\log_2 5 = 2.322$, without using the change of base law. [1]

ii. $\log_7 0.3$, using change of base law. [1]

d) Find p so that $9x^2 - 3x + p = 0$ has only one root. [2]

<u>Question Eight</u>: (Start a NEW BOOKLET)

a) In the diagram, XY is an arc of a circle with centre O and radius 12cm. The length of the arc XY is 4π cm.

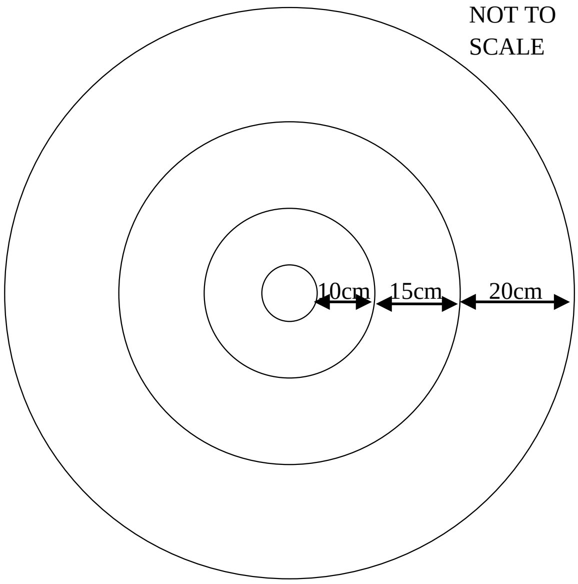


i. Find the exact size of θ in radians. [1]

ii. Find the area of the sector OXY [1]

b) The region bounded by the curve $y = e^{x} + e^{-x}$, the *x*-axis and the lines $x = 0$ and $x = 2$ is rotated about the *x*-axis. Find the volume of the solid formed. (Answer in terms of *e*). [3]

c) Beginning with a circular piece of fabric of radius 5cm, Lynn sewed together circular strips of different coloured fabrics which increased in width to make a circular tablecloth. The finished width of the first strip was 10cm, the second was 15cm, the third 20cm, and so on, as shown opposite.



i. Show that the width of the 10^{th} strip was 55cm. [2]

ii. The radius of the Tablecloth was 455cm. How many strips were sewn onto the edge of the first circular piece? [3]

d) Solve for *x*: $4e^{2x} - e^{x} = 0$. [2]

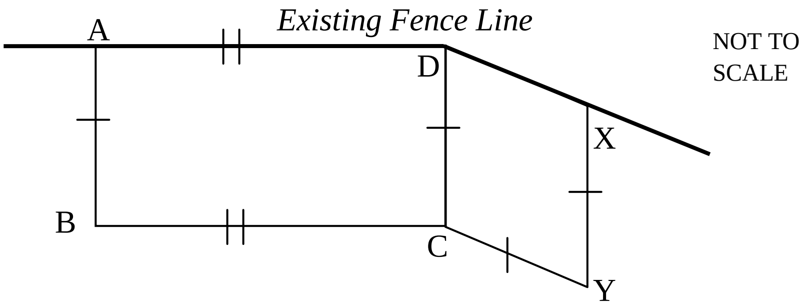
<u>Question Nine</u>: (Start a NEW BOOKLET)

a) For the inequality $y \le 4 - x^2$:

i. Shade the region bounded simultaneously by the inequality above, and the inequalities $x \ge 0$ and $y \ge 3x$. [2]

ii. Find the volume of the solid of revolution formed when the region defined in (i) above is rotated about the *y*-axis. [4]

b) A farmer needs to construct two holding paddocks, one rectangular (ABCD) and one a rhombus (CDXY) for horses and cattle respectively. The diagram below shows an aerial view of the paddocks, including the use of an existing fence as part of the boundary.



The farmer has only 700m of fencing. We also know that $\angle CDX = 30^\circ$.

i. By letting $AB = x$, show that the area A of the two paddocks is given by $A = 700x - \dfrac{7x^2}{2}$. [2]

ii. Hence find the maximum area that can be enclosed. [3]

iii. Calculate the dimensions for the rectangular paddock, when the overall area is a maximum. [1]

<u>Question Ten</u>: (Start a NEW BOOKLET)

a) Jerry joins a Superannuation fund, investing \$P at the beginning of every year at 9% p.a. compounding annually.

i. Write an expression for the value of his investment A_1 at the end of the first year. [1]

ii. Write an expression for the value of his investment A_2 at the end of the second year. [1]

iii. Show that, after n years, the value of his investment A_n is given by
$A_n = \frac{109P}{9}\left(1.09^n - 1\right)$ [2]

iv. If, after 30 year, he wishes to collect \$1,000,000, calculate the value of \$P to the nearest dollar. [1]

b) A triangle is right-angled at *B*. *D* is the point on *AC* such that *BD* is perpendicular to *AC*. Let $\angle BAC = \theta$.

i. Draw a diagram showing this information. [1]

You are given that $6AD + BC = 5AC$:

ii. Show that $6\cos\theta + \tan\theta = 5\sec\theta$ [2]

iii. Deduce that $6\sin^2\theta - \sin\theta - 1 = 0$ [2]

iv. Find θ. [2]