2007 TRIAL HIGHER SCHOOL CERTIFICATE   EXAMINATION

Mathematics Extension 1

- **Solutions including marking scale**
- **Mapping grid**

Marking Guidelines: Mathematics Extension I – Solutions

Question 1 (a)(b)(c)(d)(i)

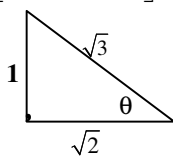| Criteria	Marks
(a) One mark for the diagram and one for the solution.	2
(b)(i) One mark for the answer. **(ii)** One mark for the diagram (or explanation) and one mark for the answer.	3
(c) One mark for derivation.	1
(d) One mark for each step.	2
(e) One mark for finding the domain and range and one mark for the sketch.	2
(f) One mark for showing it's quarter of the area of circle and one for the answer.	2

Answers:

1(a)
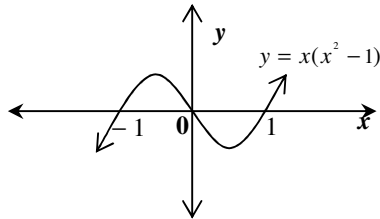
From the graph, the solutions to

$x(x^2 - 1) > 0$ are $-1 < x < 0$ and $x > 1$

1(b)(i)

$$\sin \frac{5\pi}{4} = \sin\left(\pi + \frac{\pi}{4}\right)$$

$$= -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

(ii) $\sin\left[2\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\right)\right] = \sin 2\theta$ from diagram

Now $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 2 \times \frac{1}{\sqrt{3}} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{2}}{3}$$

$$\therefore \sin\left[2\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)\right] = \frac{2\sqrt{2}}{3}$$

(c) $x = \sin\theta + \cos\theta$; $y = \sin\theta - \cos\theta$

$x^2 = (\sin\theta + \cos\theta)^2$

$\quad = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$\quad = 1 + \sin 2\theta$

$y^2 = \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$

$\quad = 1 - \sin 2\theta$

1(c) continued $\therefore$ $x^2 + y^2 = 2$

(d) $\displaystyle\lim_{x\to 0} \frac{3x}{\sin 2x} = \lim_{x\to 0}\left(\frac{3}{2} \times \frac{2x}{\sin 2x}\right)$

$$= \frac{3}{2}\lim_{x\to 0}\left(\frac{2x}{\sin 2x}\right) = \frac{3}{2}$$

as $\displaystyle\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right) = 1$

(e) $y = 4\cos^{-1} 2x$

Domain: $-1 \le 2x \le 1$ i.e. $-\dfrac{1}{2} \le x \le \dfrac{1}{2}$

Range: $0 \le y \le 4\pi$
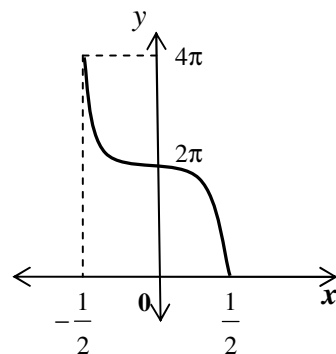
(g) $\displaystyle\int_0^3 \sqrt{\left(9 - x^2\right)}\,dx$ = shaded area

$\quad$ = quarter of circle of radius 3 units
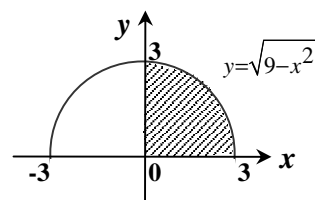
$$= \frac{9}{4}\pi \text{ unit}^2$$

Question2

Criteria	Marks
(a) (i) One mark for $\tan(A+B)$.**(ii)**One mark each for deriving the quadratic equation and the solution.	**3**
(b) One mark for writing expression for the perpendicular distance and one for the coordinates.	**2**
(c) One mark for T_8 and one for the coefficient of a^5b^7.	**2**
(d) One mark each for x and y coordinate	**2**
(e) One mark for the table, one of writing Simpson's rule and one for application.	**3**

Answers

(a) (i) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\tan(A+B) = \dfrac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$

Dividing by $\cos A \cos B$ throughout we get,

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(ii) Given $\tan^{-1} x + \tan^{-1} 2x = \dfrac{\pi}{4}$

$\therefore \tan\left(\tan^{-1} x + \tan^{-1} 2x\right) = \tan\dfrac{\pi}{4}$

$\Rightarrow \dfrac{\tan\left(\tan^{-1} x\right) + \tan\left(\tan^{-1} 2x\right)}{1 - \tan\left(\tan^{-1} x\right) \times \tan\left(\tan^{-1} 2x\right)} = \tan\dfrac{\pi}{4}$

$\Rightarrow \dfrac{x + 2x}{1 - 2x^2} = 1 \Rightarrow 2x^2 + 3x - 1 = 0$

$\Rightarrow x = \dfrac{-3 \pm \sqrt{9+8}}{4} = 0 \cdot 28$ (2 dec.pl.) taking the

positive value as required.

(b) Let $(x, 2x)$ be the point on $y = 2x$

The distance of this point from $x + y - 4 = 0$ is given by

$\dfrac{x + 2x - 4}{\sqrt{2}}$ and this is given equal to $\pm\sqrt{2}$

$\therefore x + 2x - 4 = \pm 2 \Rightarrow 3x = 4 \pm 2$

or $x = 6/3$ or $2/3 \therefore x = 2$ and $\dfrac{2}{3}$

$\therefore$ the points are $\underline{(2,4)}$ and $\underline{\left(\dfrac{2}{3}, \dfrac{4}{3}\right)}$

Note: A point and its image on a line will be of equal length from the line but opposite in sign and hence the sign $\pm$.

(c) For the expansion $(2a-b)^{12}$,

$T_{r+1} = {}^{12}c_r (2a)^{12-r} (-b)^r$

The term $a^5 b^7$ occurs when $r = 7$

Hence $T_8 = {}^{12}c_7 (2a)^5 (-b)^7$

$\qquad = \dfrac{12!}{5!\,7!}(2a)^5 (-b)^7 = 792 \times 32a^5 \times -b^7$

$\qquad = -25344\, a^5 b^7$

Coeff. of $a^5 b^7$ is $\underline{-25344}$

(d)

$x = \dfrac{-2(-4)+3(3)}{-2+3}$, $y = \dfrac{-2(2)+3(5)}{-2+3}$

$\quad = \dfrac{8+9}{1}$ $\qquad = \dfrac{-4+15}{1}$

$\quad = 17$ $\qquad\qquad = 11$ ie $(17,11)$

(e)

x	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$
cos x	0	$\dfrac{1}{\sqrt{2}}$	1	$\dfrac{1}{\sqrt{2}}$	0
	(y_0)	(y_1)	(y_2)	(y_3)	(y_4)

Simpson's Rule:

$\displaystyle\int_a^b f(x)\,dx \approx \dfrac{h}{3}\left\{(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2)\right\}$

where h = width of interval = $\pi/4$

$\therefore \displaystyle\int_a^b \cos x\,dx \approx \dfrac{\pi}{12}\left\{0 + 4\times\dfrac{2}{\sqrt{2}} + 2\right\}$

$\qquad\qquad \approx \dfrac{\pi}{12}\left\{\dfrac{8}{\sqrt{2}} + 2\right\} \approx \underline{2\cdot 00}$ (2dec.pl.)

Question 3 & 4(a)

Criteria	Marks
(a) One mark for the explanation and one for the answer.	2
(b)(i) One mark for showing it's SMH **(ii)** one mark each for α and a **(iii)** one mark for the position	4
(c)(i) &(ii) One mark for each answer **(iii)** One mark for showing blue marbles as a group $= 4!$ and one for $4!4!$	4
(d) One mark for writing the equation and one for finding the value of a	2
4(a) One mark for the solution.	1

Answers

3(a) Out of the 11 letters of MATHEMATICS, three letters – A, T and M are repeated. Hence total no. of words with 11 letter

$$= \frac{11!}{2!2!2!} = 4989600$$

(b) (i) $x = a\cos(4t + \alpha) \quad \Rightarrow \dot{x} = -4a\sin(4t + \alpha)$

$\Rightarrow \ddot{x} = -16a\cos(4t + \alpha)$

$= -16 \times a\cos(4t + \alpha)$

$= -16x$

As the particle's motion can be described in the form $\ddot{x} = -n^2 x$, where $n = 4$, it is undergoing simple harmonic motion

(ii) $x = a\cos(4t + \alpha)$

when $t = 0$, $x = 0$, $\therefore 0 = a\cos(\alpha)$

i.e. $\cos\alpha = 0$ $\therefore \alpha = \pi/2$

$\therefore x = a\cos\left(4t + \dfrac{\pi}{2}\right)$

and $v = \dot{x} = -4a\sin\left(4t + \dfrac{\pi}{2}\right)$

when $t = 0$, $v = -6$, $\therefore -6 = -4a\sin\left(\dfrac{\pi}{2}\right)$

i.e. $4a = 6$ or $a = 3/2$ m

(iii) $x = \dfrac{3}{2}\cos\left(4t + \dfrac{\pi}{2}\right)$

when $t = 4$; $x = \dfrac{3}{2}\cos\left(16 + \dfrac{\pi}{2}\right) = 0.43$m (2dec.pl)

the particle is about 0.43m to the right of origin.

(c) (i) $7! = 5040$ arrangements

(ii) $7! -$ number of arrangements in which blue and white marbles alternate $= 7! - 3!4!$

$= 5040 - 144$

$= 4896$

(iii) blue marbles arranged as a group $= 4!$

blue marbles as a group and white marbles $= 4!$

$\therefore 4!4! = 576$

(d) Since it is a cubic with -1 as a repeated root and 2 as a root it can be written as

$$y = a(x+1)^2(x-2)$$

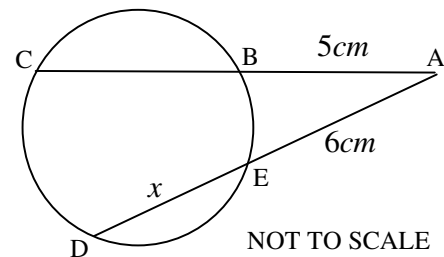It passes through $(0, -4)$,

$-4 = a(0+1)^2(0-2) \quad \Rightarrow -4 = -2a$

$\therefore a = 2$ and the equation of the curve is

$$\underline{y = 2(x+1)^2(x-2)}$$

4(a)

NOT TO SCALE

Since the chords of the circle, DE and CB intersect at A,

$DA \times EA = CA \times BA$

$\Rightarrow (6 + x) \times 6 = 10 \times 5$

$(6 + x) = 8\dfrac{1}{3}$ or $\underline{x = 2\dfrac{1}{3}}$

Question 4(b) –

Criteria	Marks
(b)(i) One mark for showing that the root lies between the two values indicated. **(ii)** One mark for the value of $f'(x)$ and one for applying Newton's method. **(c)** One mark for finding the product of roots and one mark for finding the value of k. **(d)** One mark for the tangent, one mark for $\angle$ APX, one for $\angle$ QPX and one for showing that AP bisects $\angle$BPQ. **(e)** One mark for subtracting 1 and one for the solution	3 2 4 2

Answers

4(b) Let $f(x) = x + \sin x - \dfrac{\pi}{3}$

then $f\left(\dfrac{\pi}{6}\right) = \dfrac{\pi}{6} + \dfrac{1}{2} - \dfrac{\pi}{3} \approx -0 \cdot 0236$

and $f\left(\dfrac{\pi}{4}\right) = \dfrac{\pi}{4} + \dfrac{1}{\sqrt{2}} - \dfrac{\pi}{3} \approx 0 \cdot 445$

Thus over the interval $\dfrac{\pi}{6} \le x \le \dfrac{\pi}{4}$, $f(x)$ changes

from $-$ ive to $+$ ive. Hence it has at least one root

between $\dfrac{\pi}{6}$ and $\dfrac{\pi}{4}$.

Let $x_1 = \dfrac{\pi}{6}$ then $f'(x) = 1 + \cos x$

and $f'\left(\dfrac{\pi}{6}\right) = 1 + \cos\dfrac{\pi}{6} = 1 + \dfrac{\sqrt{3}}{2}$

By Newton's law, $x_2 = \dfrac{\pi}{6} - \dfrac{f\left(\dfrac{\pi}{6}\right)}{f'\left(\dfrac{\pi}{6}\right)}$

$\qquad = \dfrac{\pi}{6} - \dfrac{-0 \cdot 0236}{1 + \dfrac{\sqrt{3}}{2}} \approx 0.54\,(2\,\mathrm{dp})$

(c) Let α, 1/α and β be the roots.

Product of roots $= \alpha \cdot 1/\alpha \cdot \beta = \dfrac{-4}{2} = -2$

i.e. $\beta = -2$

Since -2 is a root, $2(-2)^3 - (-2)^2 + k(-2) + 4 = 0$

$\Rightarrow -2k = 16$ or $\underline{\boldsymbol{k = -8}}$

(d) Construction : Draw through P a common tangent to the two circles to meet AR in X.

Proof: $\angle$XPA = $\angle$ABP (alt. seg) $= x^\circ$ say
$\qquad \angle$XPQ = $\angle$PRQ (alt.seg) $= y^\circ$ say
$\qquad \angle$APQ = $\angle$XPA + $\angle$XPQ = $x^\circ + y^\circ$
Also $\angle$XAP = $\angle$ABP (alt.seg)
$\therefore \angle$XAP = $\angle$XPA $= x^\circ$
Now in $\triangle$ APR, Ext. $\angle$APB = $\angle$PAR + $\angle$PRA
$\qquad\qquad\qquad\qquad\qquad = x^\circ + y^\circ$
$\qquad \therefore \ \angle$APQ = $\angle$APB

(e) $\quad \dfrac{2x-3}{x} - 1 \le 0 \qquad \Rightarrow \dfrac{2x-3-x}{x} \le 0$

$\qquad \Rightarrow \quad \dfrac{x-3}{x} \le 0$

The critical points are 0 and 3.

When $x < 0$, $\quad \dfrac{x-3}{x} > 0$

When $0 < x \le 3$, $\quad \dfrac{x-3}{x} \le 0$

When $x > 3$, $\quad \dfrac{x-3}{x} > 0$

$\therefore$ The solution is $\underline{\boldsymbol{0 < x \le 3}}$

Question 5

Criteria	Marks
(a) One mark for conversion into cos 6*x* and one mark for integration.	2
(b) (i) One mark writing it's not a one to one function. **(ii)**One mark for the sketch.**(iii)** One mark for stating domain **(iv)** One mark for interchange of variables for finding *y*.	4
(c) (i) One mark for showing that $0 < y < 1$. **(ii)** one mark for the differentiation and one for simplifying.	3
(iii) One mark for substitution for *P,q* one for *y* and one mark for *t*.	3

Answers

5 (a) $\cos 2(3x) = 1 - 2\sin^2(3x)$

$\therefore \sin^2(3x) = \dfrac{1 - \cos 2(3x)}{2}$

$\displaystyle\int \sin^2 3x\, dx = \int \dfrac{1 - \cos 2(3x)}{2}\, dx$

$\qquad = \dfrac{1}{2}\displaystyle\int [1 - \cos 6x]\, dx$

$\qquad = \dfrac{1}{2}\left(x - \dfrac{\sin 6x}{6}\right) + c$

(b) (i) *f(x)* does not have an inverse because it is not a one to one function. That is, for at least one *y*-value there is more than one *x*-value.

(ii)
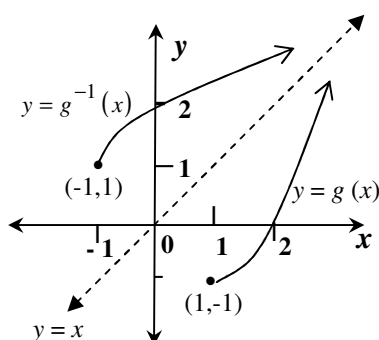
(iii) Domain of $y = g^{-1}(x)$ is $\underline{\mathbf{x \geq -1}}$

(iv) Interchanging *x* and y , we get

$\qquad x = (y-1)^2 - 1$

or $y = \sqrt{x+1} + 1$ i.e $g^{-1}(x) = \sqrt{x+1} + 1$

(c) (i) $y = \dfrac{Pe^{qt}}{1 + Pe^{qt}}, \quad P > 0, q > 0$

Since $Pe^{qt} > 0$ for all *t*

then $1 + Pe^{qt} > Pe^{qt}$ $\therefore$ $\dfrac{Pe^{qt}}{1 + Pe^{qt}} < 1$

or $y > 0$ also since $Pe^{qt} > 0$ **&** $1 + Pe^{qt} > 0$

$y = \dfrac{Pe^{qt}}{1 + Pe^{qt}} > 0$ Hence $\underline{\mathbf{0 < y < 1}}$

(ii) $\dfrac{dy}{dt} = \dfrac{Pe^{qt} \cdot q(1 + Pe^{qt}) - Pe^{qt}(Pe^{qt} \cdot q)}{\left(1 + Pe^{qt}\right)^2}$

$\qquad = q\left[\dfrac{Pe^{qt}\left(1 + Pe^{qt}\right)}{\left(1 + Pe^{qt}\right)^2} - \dfrac{\left(Pe^{qt}\right)^2}{\left(1 + Pe^{qt}\right)^2}\right]$

$\qquad = q\left[\dfrac{Pe^{qt}}{1 + Pe^{qt}} - \left(\dfrac{Pe^{qt}}{1 + Pe^{qt}}\right)^2\right]$

$\qquad = q(y - y^2) = q\, y(1 - y)$

(iii)

Given P=0·01 and q=0·7, to find *t* when *y* =0·5

Substituting these values in $y = \dfrac{Pe^{qt}}{1 + Pe^{qt}} > 0$ we get

$0 \cdot 5 = \dfrac{0 \cdot 01 e^{0 \cdot 7t}}{1 + 0 \cdot 01 e^{0 \cdot 7t}} \Rightarrow 0 \cdot 5 \times \left(1 + 0 \cdot 01 e^{0 \cdot 7t}\right) = 0 \cdot 01 e^{0 \cdot 7t}$

$\Rightarrow 0 \cdot 5 = 0 \cdot 005 e^{0 \cdot 7t} \Rightarrow e^{0 \cdot 7t} = 100$

$\therefore t = \dfrac{\ln 100}{0 \cdot 7} \cong 6 \cdot 58$ or $\underline{t = 7}$ hours to nearest hour

Question 6

Criteria	Marks
(a)(i) One mark for integrating to get v^2, one for finding v and one for expressing x in terms of t. **(ii)** One mark for finding the displacement of P.	4
(b)(i) One mark for the expression showing the volume and one for simplifying it **(ii)** One mark for finding an expression for h, one for finding $d\theta/dV$, one for finding the value of θ when $x = 0.3$ and one mark for the carrying out the calculations.	6
(c) One mark for finding the derivative and one for the value of x for which f(x) decreases.	2

Answers

6(a)(i) $\ddot{x} = 8x\left(x^2 + 1\right)$

$\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = 8x\left(x^2 + 1\right) \Rightarrow \dfrac{1}{2}v^2 = \dfrac{8x^4}{4} + \dfrac{8x^2}{2} + c_1$

When $x = 0$; $v = -2$ $\therefore$ $\dfrac{4}{2} = 0 + 0 + c_1$ or $c_1 = 2$

$\dfrac{1}{2}v^2 = 2x^4 + 4x^2 + 2 = 2\left(x^2 + 1\right)^2$

$\therefore$ $v = \pm 2\left(x^2 + 1\right)$ or speed = $\underline{v = 2\left(x^2 + 1\right) cm/s}$

$v = \dfrac{dx}{dt} = \pm 2\left(x^2 + 1\right) \Rightarrow \pm dt = \dfrac{dx}{2\left(1 + x^2\right)}$

$\therefore$ $\pm t = \dfrac{1}{2}\displaystyle\int \dfrac{dx}{1 + x^2} = \dfrac{1}{2}\tan^{-1} x + c_2$

when $t = 0$, $x = 0$ $\therefore$ $c_2 = 0$

$\therefore$ $\pm t = \dfrac{1}{2}\tan^{-1} x \Rightarrow x = \tan(\pm 2t) = \pm \tan 2t$

(ii) When $t = \pi/8$, $x = \pm \tan \pi/4 = \pm 1$
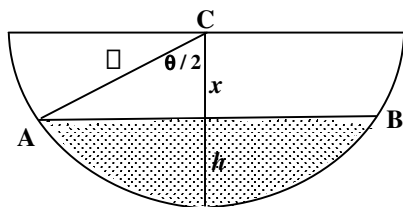
The particle is 1 cm from O.

(b) (i) Volume of water(V)

= (area of sector ACB − Area of $\triangle$ ACB) $\times$ length

= $\left(\dfrac{1}{2}r^2\theta - \dfrac{1}{2}r^2 \sin\theta\right) \times 2$

= $\left(\dfrac{1}{8}\theta - \dfrac{1}{8}\sin\theta\right) \times 2 = \dfrac{1}{4}\left(\theta - \sin\theta\right)$

(ii)

(b)(ii) continued.

From the diagram, $x = \dfrac{1}{2}\cos\dfrac{\theta}{2}$

$h = \dfrac{1}{2} - x = \dfrac{1}{2} - \dfrac{1}{2}\cos\dfrac{\theta}{2}$

$\therefore$ $\dfrac{dh}{d\theta} = \dfrac{1}{4}\sin\dfrac{\theta}{2}$

V $= \dfrac{1}{4}\left(\theta - \sin\theta\right) \Rightarrow \dfrac{dV}{d\theta} = \dfrac{1}{4}\left(1 - \cos\theta\right)$

and $\dfrac{d\theta}{dV} = \dfrac{4}{1 - \cos\theta}$

To find dh/dt when $h = 0.2$ m and $dV/dt = 0.1$

Now $\dfrac{dh}{dt} = \dfrac{dh}{d\theta} \times \dfrac{d\theta}{dV} \times \dfrac{dV}{dt}$

$= \dfrac{1}{4}\sin\dfrac{\theta}{2} \times \dfrac{4}{1 - \cos\theta} \times 0.1$

When $h = 0.2$, $x = 0.5 - 0.2 = 0.3$

substituting $x = 0.3$ in $x = \dfrac{1}{2}\cos\dfrac{\theta}{2}$ we get

$\cos\dfrac{\theta}{2} = 0.6 \Rightarrow \dfrac{\theta}{2} = \cos^{-1} 0.6 \therefore \theta = 1.855$

$\dfrac{dh}{dt} = \dfrac{1}{4}\sin\dfrac{1.855}{2} \times \dfrac{4}{1 - \cos 1.855} \times 0.1$

= 0.0625 m / min (3sig.fig)

(c) $f\left(x\right) = xe^{-2x}$ Hence $f'\left(x\right) = e^{-2x} - 2xe^{-2x}$

i.e. $f'\left(x\right) = e^{-2x}\left(1 - 2x\right)$

$f(x)$ decreases when $f'\left(x\right) < 0$

i.e. $e^{-2x}\left(1 - 2x\right) < 0$

Now $e^{-2x} > 0$ for all values of x and

$1 - 2x < 0$ when $x > \dfrac{1}{2}$

$\therefore f(x)$ decreases for $\underline{x > \dfrac{1}{2}}$

Question 7

Criteria	Marks
(a)(i) One mark for showing no.of ways of selecting 3f and 1m **(ii)** One mark for showing no. ways with 3f. **(iii)** One mark for the explanation and answer.**(b)** One mark for substitution, one for integration.	**5**
(c)(i) One mark for deriving an expression for time for max. height and one mark for showing V sin α in terms of g and h **(ii)** One mark for the quadratic equation, one mark for its solution (t) and one for finding the horizontal distance	**5**
(d)(i) One mark for showing that it is the binomial expansion of $[x+(1-x)]^n$ that equals to 1 **(ii)** One mark for the substitution for x and simplification.	**2**

Answers

7(i) The number of ways of selecting the 2 females from n females is $^nc_2$ and the number of selecting 1 male for n males is $^nc_1$.

$\therefore$ No. of ways of selecting 1 male and 2 females =

$^nc_1 \times {}^nc_2 = n\left({}^nc_2\right)$ since $^nc_1 = n$

(ii) No. of ways of selecting 3 females from n females is $^nc_3$

(iii) Total no. of ways of selecting a committee of 3 people from $2n$ people is $^{2n}c_3$.

This total can be divided into 4 categories:-
3 males, 0 females; 2 males,1 female;
1 male, 2 females;   0 males, 3 females

$\therefore$

$^{2n}c_3 = \left({}^nc_3 \times {}^nc_0\right) + \left({}^nc_2 \times {}^nc_1\right) + \left({}^nc_1 \times {}^nc_2\right) + \left({}^nc_0 \times {}^nc_3\right)$

$= 2\left({}^nc_3\right) + \left({}^nc_2 \times n\right) + \left(n \times {}^nc_2\right)$

$= 2\left({}^nc_3 + n\,{}^nc_2\right)$

or $n({}^nc_2) + {}^nc_3 = \frac{1}{2}\left({}^{2n}c_3\right)$

(b)

$\int x\sqrt{x^2+9}\,dx$ $let\ u = x^2+9$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int u^{\frac{1}{2}}\,du$
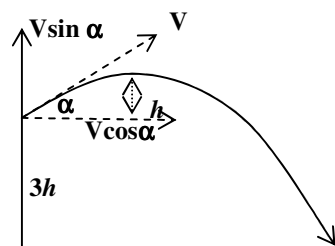
$= \frac{1}{2} \times \frac{2u^{\frac{3}{2}}}{3} + c$

$= \frac{(x^2+9)^{\frac{3}{2}}}{3} + c$

(c) (i) Horizontal motion: $\ddot{x} = 0$, Integrating we get

$\dot{x} = c_1$ when $t = 0$, $\dot{x} = V\cos\alpha$ $\therefore c_1 = V\cos\alpha$

$\dot{x} = V\cos\alpha$

(c) (i) continued

Integrating again we get,

$x = Vt\cos\alpha + c_2$

when $t = 0$, $x = 0$ $\therefore c_2 = 0$

or $x = Vt\cos\alpha$

Vertical motion:

$\ddot{y} = -g$, Integrating we get,

$\dot{y} = -gt + c_3$

when $t = 0$, $\dot{y} = V\sin\alpha$ and so $c_3 = V\sin\alpha$

$\therefore \dot{y} = -gt + V\sin\alpha$, Integrating again we get

$y = -\frac{1}{2}gt^2 + Vt\sin\alpha + c_3$

when $t = 0$, $y = 3h$ $\therefore c_3 = 3h$

$\therefore y = -\frac{1}{2}gt^2 + Vt\sin\alpha + 3h$

Greatest height is reached when $\dot{y} = 0$

$-gt + V\sin\alpha = 0$ or $t = \frac{V\sin\alpha}{g}$

When $t = \frac{V\sin\alpha}{g}$, $y = 4h$

$\therefore 4h = -\frac{g}{2}\left[\frac{V\sin\alpha}{g}\right]^2 + V\left[\frac{V\sin\alpha}{g}\right]\sin\alpha + 3h$

$h = -\frac{1}{2}\frac{V^2\sin^2\alpha}{g} + \frac{V^2\sin^2\alpha}{g}$

$= \frac{1}{2g}V^2\sin^2\alpha \Rightarrow V\sin\alpha = \sqrt{2gh}$

7(c)(ii).

We substitute $y = 0$, in $y = -\dfrac{1}{2}gt^2 + Vt\sin\alpha + 3h$

to get the time taken for the journey.

$$-\frac{1}{2}gt^2 + Vt\sin\alpha + 3h = 0$$

$$\Rightarrow\ gt^2 - 2Vt\sin\alpha - 6h = 0$$

$$\therefore\ t = \frac{2\sqrt{2gh} \pm \sqrt{8gh + 24gh}}{2g}$$

$$= \frac{2\sqrt{2gh} \pm 4\sqrt{2gh}}{2g}$$

or $\quad t = \dfrac{6\sqrt{2gh}}{2g} = 3\sqrt{\dfrac{2h}{g}}$

Horizontal distance (or the time $t = 3\sqrt{\dfrac{2h}{g}}$)

$$x = Vt\cos\alpha = V\sin\alpha\left(\frac{\cos\alpha}{\sin\alpha}\right) \times 3\sqrt{\frac{2h}{g}}$$

$$= \sqrt{2gh} \times \cot\alpha \times 3\sqrt{\frac{2h}{g}} = \underline{6h\cot\alpha}$$

(d)(i)

$$\left[x + (1-x)\right]^n = {}^n c_0 x^n + {}^n c_1 x^{n-1}(1-x)$$

$$+ {}^n c_2 x^{n-2}(1-x)^2 + \cdots + {}^n c_n (1-x)^n$$

Now $\left[x + (1-x)\right]^n = \left[x + 1 - x\right]^n = 1^n = 1$

$$\therefore\ {}^n c_0 x^n + {}^n c_1 x^{n-1}(1-x) + {}^n c_2 x^{n-2}(1-x)^2$$

$$+ {}^n c_2 x^{n-2}(1-x)^2 + \cdots + {}^n c_n (1-x)^n = 1$$

(ii) Substituting $x = \dfrac{1}{2}$ in the above equation,

we get

$${}^n c_0\left(\frac{1}{2}\right)^n + {}^n c_1\left(\frac{1}{2}\right)^{n-1}\left(\frac{1}{2}\right) + {}^n c_2\left(\frac{1}{2}\right)^{n-2}\left(\frac{1}{2}\right)^2$$

$$+ \cdots + {}^n c_n\left(\frac{1}{2}\right)^n = 1$$

$$\Rightarrow {}^n c_0 2^{-n} + {}^n c_1 2^{-n} + {}^n c_2 2^{-n} + \cdots + {}^n c_n 2^{-n} = 1$$

$$\Rightarrow {}^n c_0 + {}^n c_1 + {}^n c_2 + \cdots + {}^n c_n = \underline{2^n}$$

Mathematics Extension 1
Examination Mapping Grid

Question	Marks	Content	Syllabus outcomes	Targeted Performance bands
1(a)	2	Harder Applications of Maths (2 unit)	H5	1-2
1(b)(i)	1	Harder Applications of Maths (2 unit)	H5	2-3
1(b)(ii)	2	Inverse Functions and the Inverse Trigonometric Functions	HE4	1-2
1(c)	1	Parametric representation	PE4	2-3
1(d)	2	Harder Applications of Maths (2 unit)	H5	1-2
1(e)	2	Harder Applications of Maths (2 unit)	H5	1-2
1(f)	2	Inverse Functions and the Inverse Trigonometric Functions	HE4	1-2
2(a)(i)	1	Inverse Functions and the Inverse Trigonometric Functions	HE4	2-3
2(a)(ii)	2	Inverse Functions and the Inverse Trigonometric Functions	HE4	2-3
2(b)	2	Harder Applications of Maths (2 unit)	H5	2-3
2(c)	2	Binomial Theorem	HE3	2-3
2(d)	2	Internal and external division of lines into a given ratio	PE2	2-3
2(e)	3	Harder Applications of Maths (2 unit)	H5	2-3
3(a)	2	Permutations, Combinations and Further Probability	PE3	2-3
3(b)(i)	1	Applications of Calculus to the Physical World	HE4, HE7	2-3
3(b)(ii)	2	Applications of Calculus to the Physical World)	HE4, HE7	2-3
3(b)(iii)	1	Applications of Calculus to the Physical World	HE4	2-3
3(c)(i)	1	Permutations, Combinations and Further Probability	PE3	2-3
3(c)(ii)	1	Permutations, Combinations and Further Probability	PE3	2-3
3(c)(iii)	2	Permutations, Combinations and Further Probability	PE3	2-3
3(d)	2	Polynomials	PE3	2-3
4(a)	1	Circle geometry	PE2,PE3,PE6	2-3
4(b)(i)	1	Polynomials	PE3	2-3
4(b)(ii)	2	Polynomials	PE3	2-3
4(c)	2	Polynomials	PE3	2-3
4(d)	4	Circle geometry	PE2,PE3,PE6	2-3
4(e)	2	Other inequalities	PE3	2-3
5(a)	2	Integration	HE4	2-3
5(b)(i)	1	Inverse Functions and the Inverse Trigonometric Functions	PE4	2-3
5(b)(ii)	1	Inverse Functions and the Inverse Trigonometric Functions	HE4	2-3
5(b)(iii)	1	Inverse Functions and the Inverse Trigonometric Functions	HE4	2-3
5(b)(iv)	1	Inverse Functions and the Inverse Trigonometric Functions	HE4	2-3

5(c)(i)	1	Applications of Calculus to the Physical World	HE4	2-3
5(c)(ii)	2	Applications of Calculus to the Physical World	HE4	2-3
5(c)(iii)	3	Applications of Calculus to the Physical World	HE4	2-3
6(a)(i)	3	Applications of Calculus to the Physical World	HE4	2-3
6(a)(ii)	1	Applications of Calculus to the Physical World	HE4	2-3
6(b)(i)	2	Applications of Calculus to the Physical World	HE4	2-3
6(b)(ii)	4	Applications of Calculus to the Physical World	HE4	3-4
6(c)	2	Harder Applications of Maths (2 unit)	HE5	2-3
7(a)(i)	1	Permutations, Combinations and Further Probability	HE5	3-4
7(a)(ii)	1	Permutations, Combinations and Further Probability	PE3	3-4
7(a)(iii)	1	Permutations, Combinations and Further Probability	PE3	3-4
7(b)	2	Integration	HE5	3-4
7(b)(i)	2	Applications of Calculus to the Physical World	HE5	3-4
7(b)(ii)	3	Applications of Calculus to the Physical World	HE5	3-4
7(c)(i)	1	Binomial Theorem	HE3	3-4
7(c)(ii)	1	Binomial Theorem	HE3	3-4