# Other Inequalities

3U93-3a)!

Solve the inequality $\dfrac{2x+3}{x-4} > 1$.†

«→ $x < -7$ or $x > 4$ »

3U92-2c)!

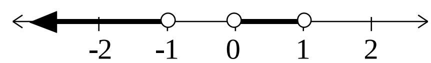
Solve the inequality $\dfrac{x}{x^2-1} > 0$.†

«→ $-1 < x < 0$ or $x > 1$ »

3U90-1c)!

Solve the following inequality for x: $x - \dfrac{1}{x} < 0$ and graph the solution on a number line.†

«→ $x < -1$ or $0 < x < 1$




»

3U89-1c)!

Solve the inequality: $\dfrac{x-3}{x^2-x} \geq -2$.†

«→ $x \leq -1$, $0 < x < 1$, $x \geq \dfrac{3}{2}$ »

3U88-1a)!

Solve for x: $\dfrac{x+4}{x-2} \geq 3$.†

«→ $2 < x \leq 5$ »

3U86-1ii)!

Solve the inequality $\dfrac{1}{x} < \dfrac{1}{x+1}$.†

«→ $-1 < x < 0$ »

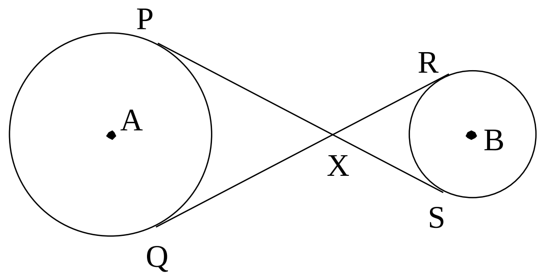
3U85-1i)!

Solve $\dfrac{3x+2}{x-1} > 2$.†

«→ $x < -4$ or $x > 1$ »

# Circle Geometry

3U96-5a)!



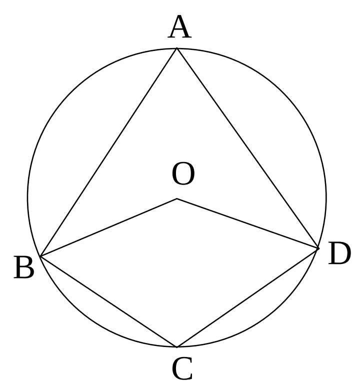

In the diagram PS and QR are tangents to each of the circles with centres A and B. The tangents intersect at X and A, X, B are collinear.

i. Copy the diagram and show that ΔAPX ||| ΔBSX.

ii. Suppose that the diagram represents two circles of radii 5cm and 3cm that are placed in the same plane with their centres 16cm apart. A taut string surrounds the circles and crosses itself between them. Find the exact length of the string.†

«→ i) Proof ii) $\left(\frac{32\pi}{3}+16\sqrt{3}\right)$ cm »

3U95-2c)!



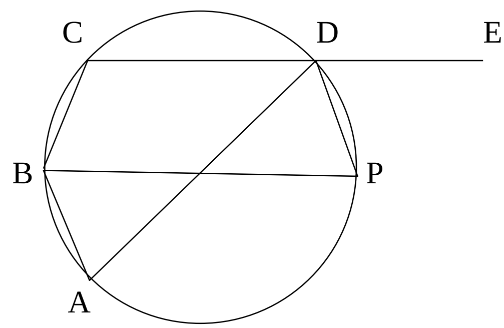

In the diagram A, B, C, and D are points on a circle with centre O. ∠BAD = x° and ∠BOD = ∠BCD.

i. Copy the diagram.

ii. Find the value of x.†

«→ x = 60° »

3U94-3a)!




In the diagram above ABCD is a cyclic quadrilateral. CD is produced to E. P is a point on the circle through A, B, C, D such that ∠ABP = ∠PBC.

i. Copy the diagram showing the above information.

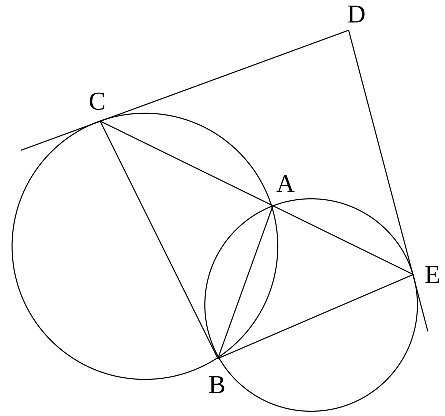
ii. Explain why ∠ABP = ∠ADP.

iii. Show that PD bisects ∠ADE.

iv. If, in addition, ∠BAP = 90° and ∠APD = 90°, explain where the centre of the circle is located.†

«→ ii) ∠ABP and ∠ADP are angles in the same segment standing on the arc AP. iii) Proof iv) The centre of the circle is at the point of intersection of the diametres BP and AD. »

3U93-4a)!

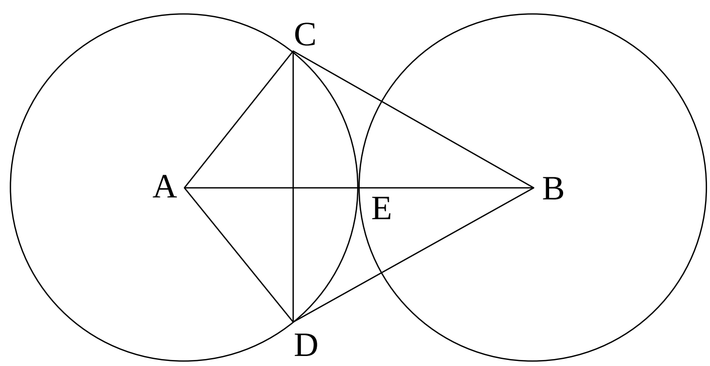



Two circles intersect at A and B. CAE is a straight line where C is a point on the first circle and E is a point on the second circle. The tangent at C to the first circle and the tangent at E to the second circle meet at D.

i. Copy the diagram.
ii. Prove that BCDE is a cyclic quadrilateral.†

«→ Proof »

3U92-4a)!

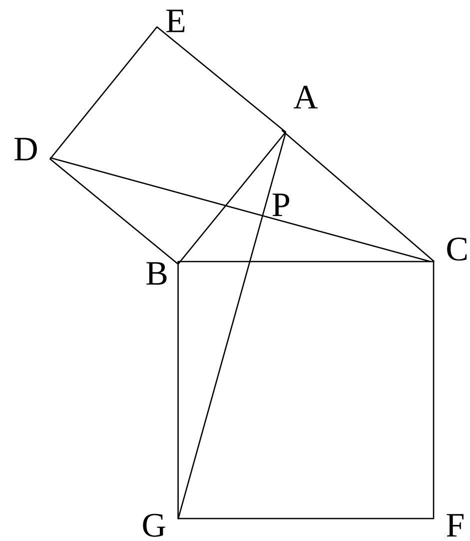



Two circles of equal radius and with centres at A and B respectively touch each other externally at E. BC and BD are tangents from B to the circle with centre A.

i. Copy the diagram.
ii. Show that BCAD is a cyclic quadrilateral.
iii. Show that E is the centre of the circle which passes through B, C, A and D.
iv. Show that ∠CBA = ∠DBA = 30°.
v. Show that triangle BCD is equilateral.†

«→ Proof »

3U91-4b)!



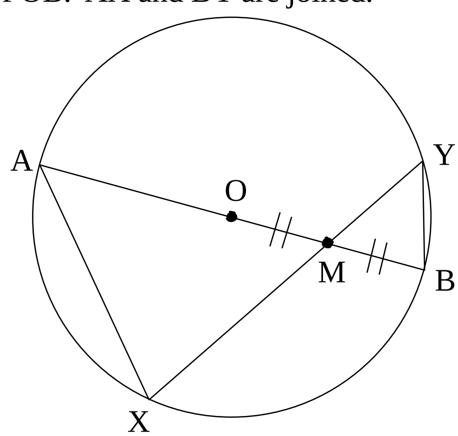

In the above diagram, ABC is an acute angled triangle and ABDE and BCFG are squares constructed on AB and BC, respectively, as sides and lying wholly outside the triangle. AG meets CD at the point P. Prove that:

i. triangles ABG and DBC are congruent;
ii. the points B, P, C, G are concyclic;
iii. AG and DC are perpendicular to each other;
iv. BP bisects angle DPG.†

«→ Proof »

3U90-2b)!

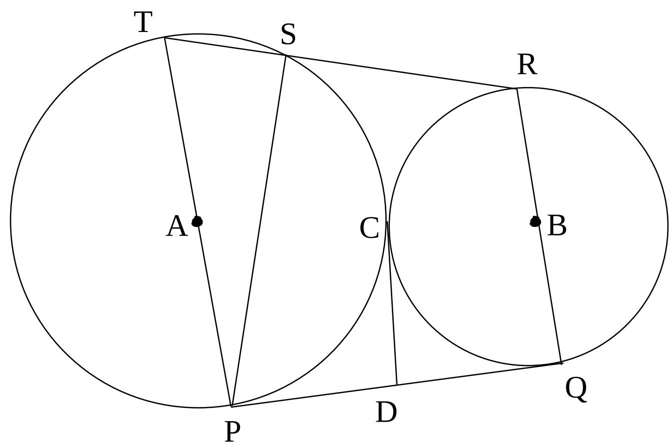
In the diagram below, AB is a diameter of a circle, whose centre is the point O. The chord XY passes through M, the mid-point of OB. AX and BY are joined.




i. Copy the above diagram carefully into your writing booklet.
ii. Prove the two triangles formed (triangles AXM and MYB) are similar.
iii. If XM = 8cm and YM = 6cm, find the length of the radius of the circle.†

«→ ii) Proof iii) 8 »

3U90-7c)!

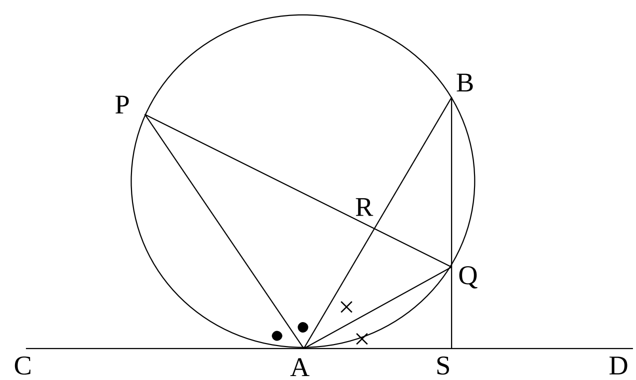



A circle, centre A, touches a smaller circle, centre B, externally at a point C. PQ is a direct tangent to the two circles, touching them at points P and Q. The common tangent to both circles passing through C meets PQ at the point D. PA and QB, when produced, meet the circumferences of the two circles at T and R respectively. TR meets the larger circle at S.

i. Copy the above diagram carefully into your writing booklet.
ii. Show the the points P, C and R are collinear.
iii. Show that BD is parallel to the line RCP.
iv. Show that the points P, Q, R, S are concyclic.†

«→ Proof »

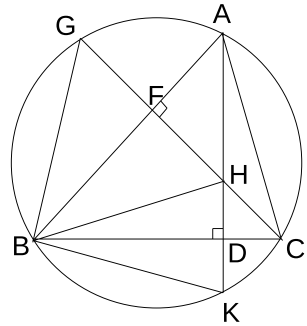
3U89-4b)!




i. AB is a chord of a circle and CAD is a tangent to the circle at the point A. The bisector of angle BAC meets the circle again at P and the bisector of angle BAD meets the circle again at Q. Show that:
   α. PQ is a diameter of the circle;
   β. PQ is perpendicular to the chord AB.
ii. PQ meets AB at R and BQ produced meets CD at S. If BS is perpendicular to CD, prove that:
   α. $\angle BAD = 60°$;
   β. $QR = QS$;
   γ. $AB = AP$.†

«→ Proof »

3U88-6)!

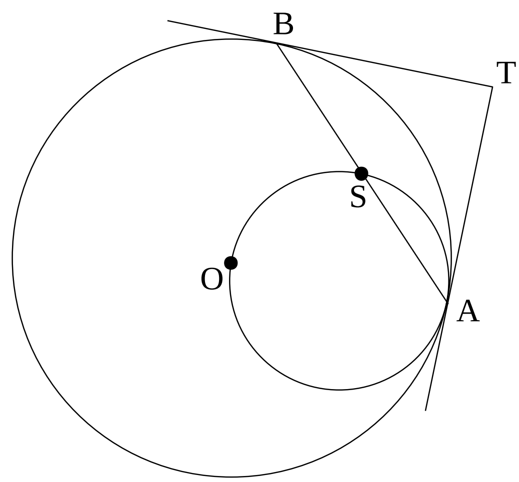



In the above diagram, ABC is a triangle inscribed in a circle. The perpendicular from A onto BC meets it at D and is then produced to meet the circumference at K. The perpendicular from C onto AB meets it at F and is then produced to meet the circumference at G. The two perpendiculars AD and CF meet at the point H.

i. Show that the quadrilaterals AFDC and BFHD are both cyclic.
ii. Prove that AB bisects the angle GBH.
iii. Prove that GB = BK.†

«→ Proof »

3U87-5b)!




Two circles touch internally at a point A and the smaller of the two circles passes through O, the centre of the larger circle. AB is any chord of the larger circle, cutting the smaller circle at S. The tangents to the larger circle at A and B meet at a point T.

Prove:

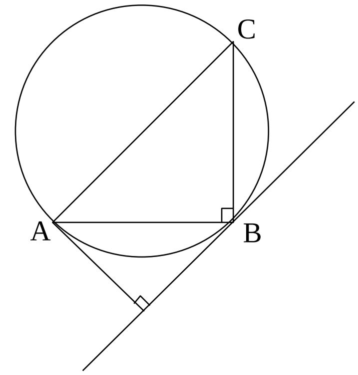
i. AB is bisected at S.
ii. O, S, and T are collinear.†

«→ Proof »

3U86-3i)!

Two points A and B are taken on a circle, and C is the other end of the diameter through A. AE is the line from A perpendicular to the tangent at B.

a. Draw a careful diagram showing this information.
b. Prove that AB bisects ∠CAE.†



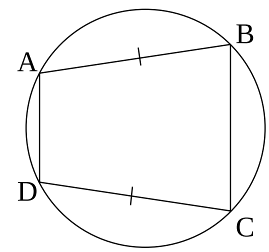

«→ a) b) Proof »

3U86-3ii)!

ABCD is a cyclic quadrilateral in which the opposite sides AB and DC are equal.

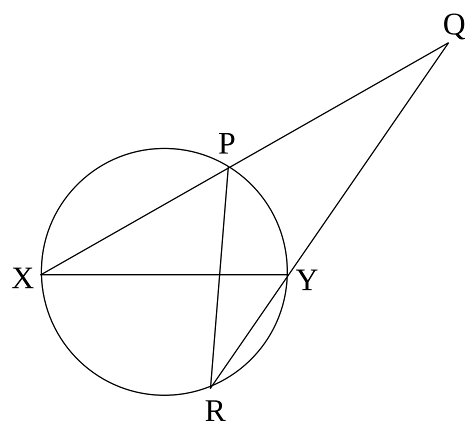
a. Draw a diagram.

b. Prove that the diagonals AC and BD are equal.†




«→ a) b) Proof »

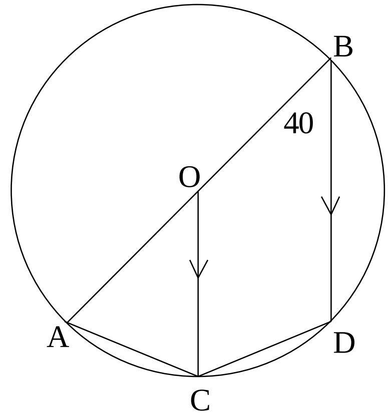
3U85-2i)!




XY is the diameter of the circle XPYR. XPQ and RYQ are straight lines. PR, XY and PY are joined. Given that $\angle PXY = 35°$ and $\angle PQY = 25°$, find the size of $\angle YPR$, giving reasons.†

«→ 30° »

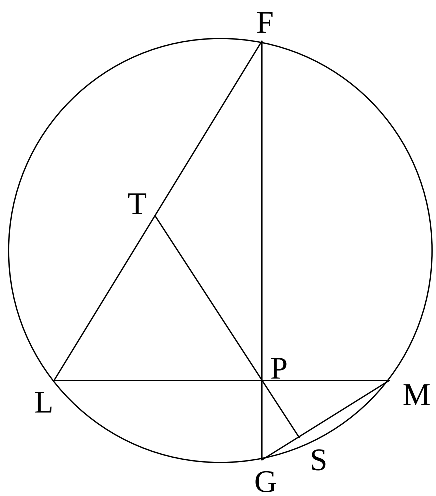
3U85-2ii)!




AB is the diameter of the circle with centre O. BD and OC are parallel, and ∠OBD = 40°. If C and D are joined, find the size of ∠OCD, giving reasons.†

«→ 70° »

3U85-2iii)!

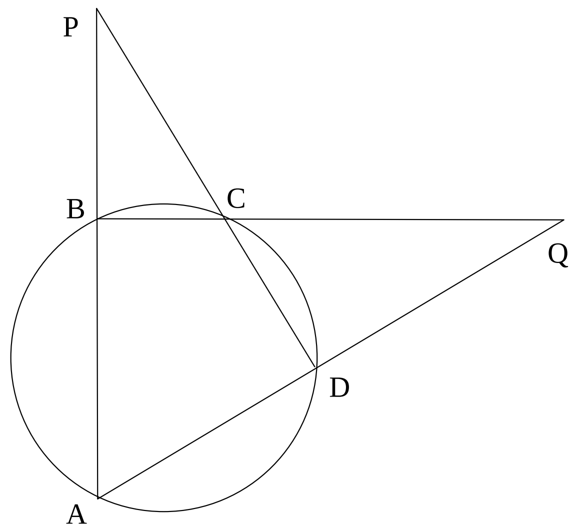



LM and FG are two chords of the circle at right angles to each other, and they intersect at P. From P, a line perpendicular to GM is drawn, meeting it at S. SP is produced to meet LF at T.

a. Show that ΔLTP is isosceles.
b. Show that T is the midpoint of LF.†

«→ Proof »

3U84-4i)!




In the diagram above ABP, DCP, BCQ, and ADQ are all straight lines and ∠APD = ∠BQA..

a. Show that ∠ABC = ∠ADC.

b. Prove that AC is a diameter of the circle.†

«→ Proof »

# Further Trigonometry
# (sums and differences, t formulae, identities and equations)

3U96-3b)!

A vertical tower of height h metres stands on horizontal ground. From a point P on the ground due east of the tower the angle of elevation of the top of the tower is 45°. From a point Q on the ground due south of the tower the angle of elevation of the top of the tower is 30°. If the distance PQ is 40 metres, find the exact height of the tower.†

«→ 20 metres »

3U94-1b)!

i. Write down the expansion of $\cos(\alpha + \beta)$.
ii. Write down the exact values of cos 30° and cos 45°.
iii. Hence find the exact value of cos 75°.†

«→ i) $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ ii) $\cos 30° = \frac{\sqrt{3}}{2}$, $\cos 45° = \frac{1}{\sqrt{2}}$ iii) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ »

3U94-6a)!

i. Write down the expression for tan 2a in terms of tan a.
ii. If f(a) = a cot a show that $f(2a) = (1 - \tan^2 a)f(a)$.†

«→ i) $\tan 2a = \frac{2\tan a}{1-\tan^2 a}$ ii) Proof »

3U93-1b)!

If $\sin\alpha = \frac{3}{4}$, $0 < \alpha < \frac{\pi}{2}$ and $\sin\beta = \frac{2}{3}$, $\frac{\pi}{2} < \beta < \pi$ find the exact values of:

i. $\tan 2\alpha$;
ii. $\cos(\alpha - \beta)$.†

«→ i) $-3\sqrt{7}$ ii) $\frac{1}{12}(6-\sqrt{35})$ »

3U93-4b)!

i. Solve the equation $\sqrt{3}\cos x - \sin x = 1$ for $0 \le x \le 2\pi$
ii. What is the general solution of the equation?†

«→ i) $\frac{\pi}{6}$ or $\frac{3\pi}{2}$ ii) $\frac{\pi}{6} \pm 2n\pi$ or $\frac{3\pi}{2} \pm 2n\pi$ »

3U93-7a)!

Use the result $\lim_{x\to 0}\frac{\sin x}{x} = 1$ to find $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$.†

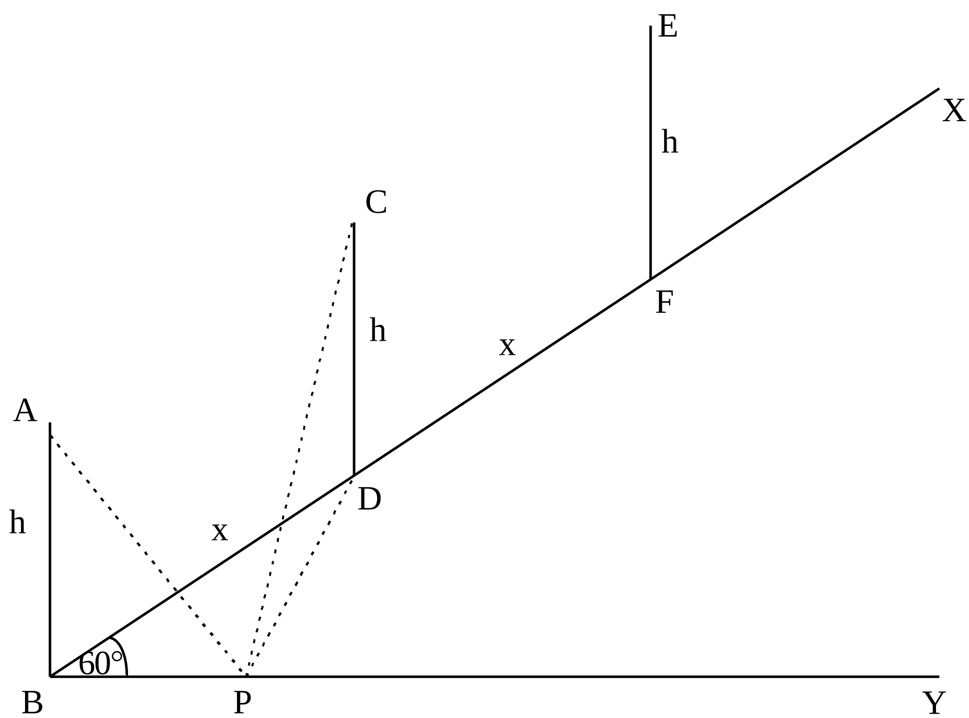
«→ 2 »

3U92-1a)!

Solve the equation $\cos 2A = \cos A$ where $0 \le A \le 360°$.†

«→ A = 0°, 120°, 240° or 360° »

3U91-7b)!




In the above diagram, BX and BY represent two roads intersecting at an angle of 60°. On the road BX are situated three telegraph poles AB, CD and EF, all of equal height, the same distance, x metres, apart (i.e. BD = DF = x). P is a point on the road BY and the angles of elevation of A and C from P are 45° and 30° respectively.

i. Show that BP = h and DP = $h\sqrt{3}$.

ii. By the use of the Sine Rule in triangle BDP, show that angle BDP = 30° and hence that triangle BDP is right angled at P.

iii. Prove that x = 2h.

iv. By the use of the Cosine Rule in triangle PDF, show that PF = $h\sqrt{13}$ and hence show that the angle of elevation of E from P is approximately 15.5°.†

«→ Proof »

3U90-1b)!

i. Show that: $\cos\theta - \sqrt{3}\sin\theta = 2\cos\left(\theta + \frac{\pi}{3}\right)$.

ii. Hence solve the equation $\cos\theta - \sqrt{3}\sin\theta = 1$ for θ in the interval $0 \le \theta \le 2\pi$.†

«→ i) Proof ii) $\theta = 0, \frac{4\pi}{3}$ or $2\pi$ »

3U90-4b)!

If $2\sin\left(\theta - \frac{\pi}{3}\right) = \cos\left(\theta - \frac{\pi}{3}\right)$, express tan θ in the form $a + b\sqrt{3}$, where a and b are integers.†

«→ $-4 + 3\sqrt{3}$ »

3U90-5a)!

If tan A and tan B are the roots of the equation $3x^2 - 5x - 1 = 0$, find the value of tan(A + B).†

«→ $\frac{5}{4}$ »

3U89-2b)!

i. Write down the expansion for sin(A + B).

ii. By letting A = B = x in the above expansion, derive an expression for sin 2x in terms of both sin x and cos x.

iii. Find a general solution for the equation: $\sin 2x = 2\cos^2 x$.†

«→ i) sin(A + B) = sinA cosB + cosA sinB ii) sin2x = 2sinx cosx iii) $x = n\pi + \frac{\pi}{4}$ or $x = 2n\pi \pm \frac{\pi}{2}$ »

3U89-7b)!

A man 1.8 metres tall, standing due east of a street light, notices that his shadow is 6.5 metres long. He walks on a bearing of 150°T from his first position for a distance of 12 metres and now finds his shadow is 9.1 metres long. Find the height of the street light above ground level (in metres correct to 1 decimal place).†

«→ 7.3 metres »

3U88-2b)!

Show that $\frac{\sin 3x - \sin x}{\cos x - \cos 3x} = \cot 2x$.†

«→ Proof »

3U87-1b)!

Solve for x in the interval $0 \le x \le 2\pi$, the equation $\sin 2x = \tan x$.†

«→ $x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$ »

3U87-4b)!

Two ships, A and B, sail from a port simultaneously with A sailing twice as fast as B. Ship A is on a course of 170°T whilst Ship B's course is 290°T. After 1 hour, the two ships are found to be 70 nautical miles apart. Find the speed of each ship, in knots, correct to two decimal places and determine also the bearing of B from A at this time, expressed as a true bearing, correct to the nearest degree. (NOTE: a knot is defined as a speed of one nautical mile per hour.)†

«→ The speed of A is 52.92 knots and the speed of B is 26.46 knots. The bearing of B from A is 331°. »

3U86-2iii)!

Given that $\cos 2\theta = 1 - 2\sin^2\theta$, show that $\frac{\cos x - \cos(x + 2\theta)}{2\sin\theta} = \sin(x + \theta)$.†

«→ Proof »

3U86-2iv)!

If $\cos A = \frac{7}{9}$ and $\sin B = \frac{1}{3}$ where $0 \le A \le \frac{\pi}{2}$, and $0 \le B \le \frac{\pi}{2}$.

a. Show that A = 2B.

b. Find the value of tan(A + B) in simplest surd form.†

«→ a) Proof b) $\frac{23\sqrt{2}}{20}$ »

3U85-4ii)!

Find all angles θ for which $\sin 2\theta = \frac{1}{2}\cos\theta$.†

«→ $\theta = \frac{\pi}{2} \pm n\pi$ or $\theta = n\pi + (-1)^n \sin^{-1}\frac{1}{4}$ »

# Angles between 2 Lines.
# Internal and External Division of Lines into Given Ratios

3U96-1a)!

Find the acute angle between the lines $2x - y = 0$ and $x + 3y = 0$, giving the answer correct to the nearest minute.†

«→ 81° 52′ »

3U95-2a)!

A is the point (-2, 1) and B is the point (x, y). The point P(13, -9) divides AB externally in the ratio 5:3. Find the values of x and y.†

«→ x = 4, y = -3 »

3U93-5a)!

A is the point (-4, 1) and B is the point (2, 4). Q is the point which divides AB internally in the ratio 2:1 and R is the point which divides AB externally in the ratio 2:1. P(x, y) is a variable point which moves so that PA = 2PB.

i. Find the co-ordinates of Q and R.
ii. Show that the locus of P is a circle on QR as diameter.†

«→ i) Q(0, 3), R(8, 7) ii) Proof »

3U92-4b)!

A is the point (-2, -1), B is the point (1, 5). Find the co-ordinates of the point Q which divides AB externally in the ratio 5 : 2.†

«→ Q(3, 9) »

3U90-4a)!

A is the point (-2, -1). B is the point (1, 5). Find the co-ordinates of the point Q, which divides AB externally in the ratio 5:3.†

«→ Q($5\frac{1}{2}$, 14) »

3U88-3a)!

A and B are the points (-5, 12) and (4, 9) respectively. P is the point which divides AB externally in the ratio 5:2. Find the co-ordinates of P and show that if Q is the point (0, 2), then triangle APQ is both right-angled and isosceles.†

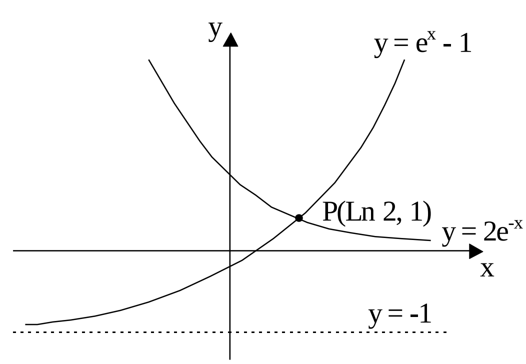
«→ (10, 7) »

3U87-6b)!

The two curves $y = e^x - 1$ and $y = 2e^{-x}$ intersect at the point P.

i. Sketch the curve and find the co-ordinates of P.
ii. Find the acute angle (to the nearest degree) between the two curves at P.
iii. Find the size of the area bounded by the two curves and the y-axis, leaving your answer in exact form.†

«→ i)




ii) 72° iii) Ln 2 »

3U85-1iii)!

Find the size of the acute angle between the tangents drawn to $y = \log_e x$ at the points where $x = 1$ and $x = 2$.†

«→ 18°26′ (to nearest minute) »

# Parametric Representation

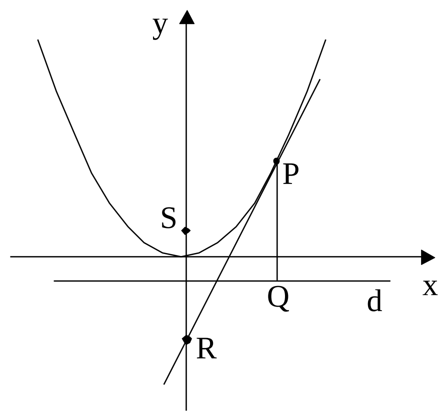
3U95-1c)!

i. Show that the normal to the parabola $x^2 = 4ay$ at the point $T(2at, at^2)$ has equation $x + ty = 2at + at^3$.

ii. Hence show that there is only one normal to the parabola which passes through its focus.†

«→ Proof »

3U94-4a)!




$P(2at, at^2)$ is a point on the parabola $x^2 = 4ay$. S is the focus of the parabola. PQ is the perpendicular from P to the directrix d of the parabola. The tangent at P to the parabola cuts the axis of the parabola at the point R.

i. Show that the tangent at P to the parabola has equation $tx - y - at^2 = 0$.

ii. Show that PR and QS bisect each other.

iii. Show that PR and QS are perpendicular to each other.

iv. State with reason what type of quadrilateral PQRS is.†

«→ i) ii) iii) Proof iv) PQRS is a rhombus since the diagonals QS and PR bisect each other at right angles. »

3U92-3c)!

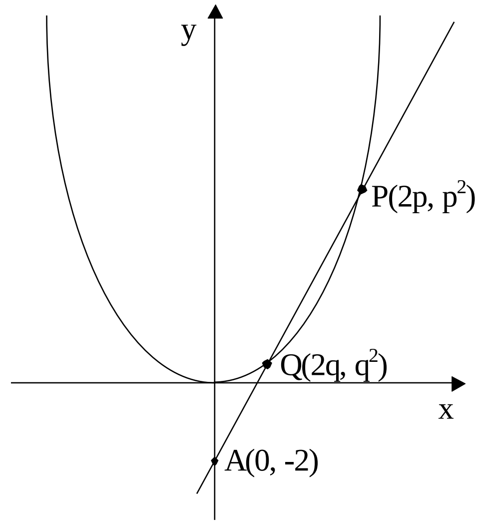
$P(2p, p^2)$ and $Q(2q, q^2)$ are two points of the parabola $x^2 = 4y$. The chord PQ subtends a right angle at the origin O.

i. Show that $pq = -4$.

ii. If M is the mid point of PQ find the locus of M as P and Q move on the parabola.†

«→ i) Proof ii) $y = \frac{1}{2}(x^2 + 8)$ »

3U92-7)!




i. Find the equations of the normals to the parabola $\begin{cases} x = 2t \\ y = t^2 \end{cases}$ at the points $P(2p, p^2)$ and $Q(2q, q^2)$, where $p \neq q$.
Hence show that these normals intersect at the point R(X, Y) where $X = -pq(p + q)$ and $Y = (p + q)^2 - pq + 2$.

ii. If the chord PQ has gradient m and passes through the point A(0, -2) find, in terms of m, the equation of PQ. Hence show that p and q are the roots of the equation $t^2 - 2mt + 2 = 0$.

iii. By considering the sum and the product of the roots of this quadratic equation show that the point R lies on the original parabola.

iv. Find the least value of $m^2$ for which p and q are real. Hence find the set of possible values of the y co-ordinate of R.†

«→ i) $x + py = 2p + p^3$ and $x + qy = 2q + q^3$ ii) $y = mx - 2$ iii) Proof iv) 2, $y \geq 8$ »

3U91-6b)!

P is a point in the first quadrant of the co-ordinate plane, lying on the parabola $x^2 = 4y$. The normal to the parabola at P meets the parabola again at a point Q in the second quadrant. The tangents to the parabola at P and Q meet at a point T. If S is the focus of the parabola and if QS = 2PS, show that:

i. QP subtends a right angle at S;

ii. PQ = PT.†

«→ Proof »

3U89-5a)!

$P(2ap, ap^2)$ is a point on the parabola $x^2 = 4ay$.

i. Show that the equation of the normal to the curve of the parabola at the point, P, is: $x + py = 2ap + ap^3$.

ii. Find the co-ordinates of the point Q where the normal at P meets the y axis.

iii. Determine the co-ordinates of the point R which divides PQ externally in the ratio 2:1.

iv. Find the cartesian equation of the locus of R and describe the locus in geometrical terms.†

«→ i) Proof ii) $Q(0, 2a + ap^2)$ iii) $R(-2ap, 4a + ap^2)$ iv) $x^2 = 4a(y - 4a)$. A parabola (concave up) with focal length 'a' and vertex (0, 4a) »

3U88-4)!

$P(2p, p^2)$ is a point on the parabola $x^2 = 4y$.

i. Show that the normal to the curve of the parabola at P has the equation $x + py = 2p + p^3$.

ii. This normal meets the y-axis at N, and M is the midpoint of PN. Find the co-ordinates of M and N.

iii. Show that as P moves on the parabola $x^2 = 4y$, the locus of M is another parabola. State the cartesian equation (i.e. the x-y equation) of this second parabola and show that its vertex is S, the focus of the original parabola.

iv. Show that SM is parallel to the tangent at P.

v. If the triangle PNS is equilateral, find the co-ordinates of P.†

«→ i) Proof ii) N is $(0, 2 + p^2)$ and M is $(p, 1 + p^2)$ iii) Proof iv) Locus of M is $x^2 = y - 1$. S is (0, 1) v) P is $(\pm 2\sqrt{3}, 3)$ »

3U87-4a)!

The normal at any point $P(2at, at^2)$ on the parabola $x^2 = 4ay$ cuts the y-axis at Q and is produced to a point R such that PQ = QR. Show that R has co-ordinates $(-2at, at^2 + 4a)$ and thus show also that the locus of R is another parabola. State the co-ordinates of the vertex and focus of this second parabola.†

«→ Vertex is (0, 4a) and focus is (0, 5a) »

3U86-6)!

$P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$. M is the midpoint of the chord PQ. The tangents at P and Q meet at the point N.

a. Show that the equation of the tangent at P is $y = px - ap^2$ and write down the equation of the tangent at Q.

b. Find the co-ordinates of M and N.

c. Show that MN is parallel to the y-axis.

d. Find the co-ordinates of T, the midpoint of MN.

e. Show that T lies on the parabola.†

«→ a) $y = qx - aq^2$ b) $M\left(a(p + q), \frac{a}{2}(p^2 + q^2)\right)$, $N(a(p + q), apq)$ c) Proof d) $T\left(a(p + q), \frac{a}{4}(p + q)^2\right)$ e) Proof »

3U84-2ii)!

R is the point $(2ar, ar^2)$ on the parabola $x^2 = 4ay$. From R, perpendiculars are drawn to the x and y axes meeting them at M and N respectively. T is the midpoint of RN and V is the midpoint of TM.

a. Write down the coordinates of T and M.

b. Find the coordinates of V.

c. Show that as R moves along the given parabola, the locus of V is another parabola and find its equation.†

«→ a) $T(ar, ar^2)$, $M(2ar, 0)$ b) $V\left(\frac{3ar}{2}, \frac{ar^2}{2}\right)$ c) $x^2 = \frac{9a}{2}y$. A parabola, focus $\left(0, \frac{9a}{8}\right)$, vertex (0, 0) and directrix $y = -\frac{9a}{8}$ »

# Permutations and Combinations

3U93-5b)!

Twelve people are going to the local swimming pool. Five are to go by car and the rest are to walk to the pool.

i. How many different groups of five of the people can be found to fill the car?

ii. In one of these groups it is found that only one person can drive. In how many ways can the seats be filled in this group under this condition?†

«→ i) 792 ii) 24 »

3U91-1c)!

In a tennis club, there are five married couples available to play a "mixed doubles" match, that is, a match in which a combination of one man and one woman play against a combination of another man and another woman. In how many ways can a group of four persons be chosen for this match if:

i. a man and his wife play in a match but not as partners;

ii. a man and his wife may not play in the match either as partners or as opponents.†

«→ i) 130 ii) 60 »

3U90-3c)!

In how many ways can the letters of the word GEOMETRY be arranged in a straight line if the vowels must occupy the 2nd, 4th and 6th places. (NOTE: The vowels in the English alphabet are the letters A, E, I, O, U).†

«→ 360 »

3U88-3b)!

A class debating group consists of 12 students, 8 of whom are girls. How many debating teams of three students can be formed, allowing:

i. no girls at all;

ii. exactly one girl;

iii. at least one girl.†

«→ i) 4 ii) 48 iii) 216 »

3U87-6a)!

In the English alphabet of 26 letters, the five letter a, e, i, o, u are known as 'vowels' and the remaining twenty one as 'consonants'.

i. The letters of the word FACETIOUS are arranged in a line in all possible ways. How many ways are there, if there are no restrictions at all?

ii. Out of all the possible arrangements found in (i) above, how many will contain:

  a. all the vowels together in the order AEIOU?

  b. all the vowels separated by consonants?†

«→ i) 96 ii) a) 120 b) 2880 »

3U86-7i)!

A school council at a co-ed school consists of 7 girls and 6 boys. In how many ways can a sub-committee of 4 girls and 3 boys be chosen from this council so as to exclude a particular girl, Mary, but include a particular boy, John?†

«→ 150 »

3U85-7ii)!

Mary is to celebrate her 18th birthday by having a dinner party for herself and 11 other people. Mary is to sit at the head of the table.

a. In how may ways can the people be seated round the table?

b. If there are six men and six women at the party, and Mary decides to seat the men and women alternately, in how many ways can this be done?†

«→ a) $11!$ b) $6! \times 5!$ »

# Polynomials

3U96-1c)!

Consider the polynomial $P(x) = 6x^3 - 5x^2 - 2x + 1$

i. Show that 1 is a zero of P(x).

ii. Express P(x) as a product of 3 linear factors.

iii. Solve the inequality $P(x) \le 0$.†

«→ i) Proof ii) $P(x) = (x - 1)(3x - 1)(2x + 1)$ iii) $x \le -\frac{1}{2}$ or $\frac{1}{3} \le x \le 1$ »

3U94-1c)!

The equation $x^3 - 2x^2 + 4x - 5 = 0$ has roots $\alpha, \beta, \gamma$.

i. Write down the values of $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$.

ii. Hence find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$.†

«→ i) $\alpha\beta + \beta\gamma + \gamma\alpha = 4$, $\alpha\beta\gamma = 5$ ii) $\frac{4}{5}$ »

3U93-2b)!

$\alpha$, $\beta$ and $\gamma$ are the roots of the equation $x^3 + 2x^2 - 3x + 5 = 0$.

i. State the values of $\alpha + \beta + \gamma$, $\alpha\beta + \alpha\gamma + \beta\gamma$ and $\alpha\beta\gamma$.

ii. Find the value of $(\alpha - 1)(\beta - 1)(\gamma - 1)$.†

«→ i) $\alpha + \beta + \gamma = -2$, $\alpha\beta + \alpha\gamma + \beta\gamma = -3$, $\alpha\beta\gamma = -5$ ii) -5 »

3U92-2a)!

If $f(x) = x^3 + 3x^2 - 10x - 24$ calculate f(-2) and express f(x) as the product of three linear factors.†

«→ $f(-2) = 0$, $f(x) = (x + 2)(x + 4)(x - 3)$ »

3U92-2b)!

Two of the roots of the equation $x^3 + px^2 + qx + r = 0$ are equal in magnitude but opposite in sign.

i. Show that $x = -p$ is the other root.

ii. Show that $r = pq$.†

«→ Proof »

3U91-2c)!

P(x) denotes the quadratic polynomial $kx^2 + (k - 1)x - (2k - 1)$, where k is a real, rational number.

i. Show that the equation $P(x) = 0$ always has real, rational roots for all values of k.

ii. Find the value of k for which the roots of $P(x) = 0$ are equal.

iii. Find the value (or values) of k for which one of the roots of $P(x) = 0$ will be double the other root.†

«→ i) Proof ii) $\frac{1}{3}$ iii) $\frac{1}{4}, \frac{2}{5}$ »

3U91-7a)!

Two of the roots of the equation $x^3 + ax^2 + b = 0$ are reciprocals of each other (a, b are both real).

i. Show that the third root is equal to $-b$.

ii. Show that $a = b - \frac{1}{b}$.

iii. Show that the two roots, which are reciprocals, will be real if $-\frac{1}{2} \le b \le \frac{1}{2}$.†

«→ Proof »

3U90-3a)!

If $\alpha, \beta, \gamma$ are the roots of the equation: $x^3 - x^2 + 4x - 1 = 0$. Find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$.†

«→ 7 »

3U90-6a)!

The polynomial $P(x) = x^3 + ax^2 + bx + c$ leaves the same remainder, whether divided by $(x + 1)$, $(x + 2)$ or $(x + 3)$. Find the values of a, b and c if the polynomial has a zero equal to 1 and then show that it has no other real zeros.†

«→ $a = 6$, $b = 11$ and $c = -18$ »

3U88-1c)!

The polynomial $P(x) = x^4 - 3x^3 + ax^2 + bx - 6$ leaves a remainder of 8 when divided by $(x + 1)$. If $(x - 3)$ is a factor of P(x), find the values of a and b.†

«→ 3, –7 »

3U87-2b)!

A monic cubic polynomial when divided by $x^2 + 4$ leaves a remainder of $x + 8$ and when divided by x leaves a remainder of -4. Find the polynomial in the form $ax^3 + bx^2 + cx + d$.†

«→ $P(x) = x^3 - 3x^2 + 5x - 4$ »

3U86-4i)!

The polynomial $P(x) = x^3 - 6x^2 + kx + 14$ has a zero at $x = 1$. Determine the value of the constant k, and for this value of k find:

a. the linear factors P(x);
b. the roots of the equation $P(x) = 0$;
c. the set of values of x for which $P(x) > 0$.†

«→ $k = -9$ a) $P(x) = (x - 1)(x - 7)(x + 2)$ b) $x = -2, 1$ or $7$ c) $-2 < x < 1$ or $x > 7$ »

3U84-7ii)!

P(x) is a monic polynomial of the fourth degree. When P(x) is divided by $x + 1$ and $x - 2$, the remainders are 5 and -4 respectively. Given that P(x) is an even function ie. one where $P(x) = P(-x)$.

a. Express it in the form $p_0 + p_1x^1 + p_2x^2 + p_3x^3 + p_4x^4$.
b. Find all the zeros of P(x).†

«→ a) $12 - 8x^2 + x^4$ b) $\pm\sqrt{6}$ or $\pm\sqrt{2}$ »

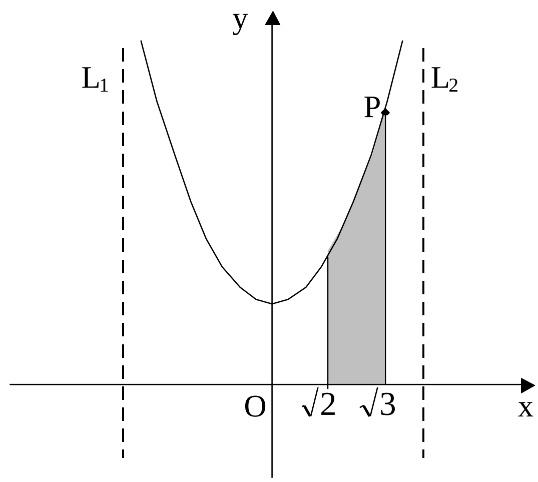
# Harder Applications of the Preliminary 2 Unit Course

3U95-3a)!

Solve the equation $\cos^2 x - \sin^2 x = \frac{1}{2}$ for $0 < x < 2\pi$.†

«→ $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ »

3U95-4b)!




The diagram shows the graph of the function $f(x) = \frac{1}{\sqrt{4 - x^2}}$.

i. Find the equations of the asymptotes $L_1$ and $L_2$.

ii. By comparing the values of f(-x) and f(x) show that f is an even function. What is the geometrical significance of this result?

iii. Find the exact equation of the tangent to the curve at the point P where $x = \sqrt{3}$.

iv. Find the exact area of the shaded region.†

«→ i) x = -2, x = 2 ii) The function is symmetrical about the y-axis iii) $y = x\sqrt{3} - 2$ iv) $\frac{\pi}{12}$ units$^2$ »

3U93-1c)!

i. Find the centre and the radius of the circle C whose equation is

$$x^2 + y^2 - 4x + 6y - 12 = 0.$$

ii. Find, in terms of the constant k, the length of the perpendicular from the centre of C to the line L whose equation is 3x + 4y = k.

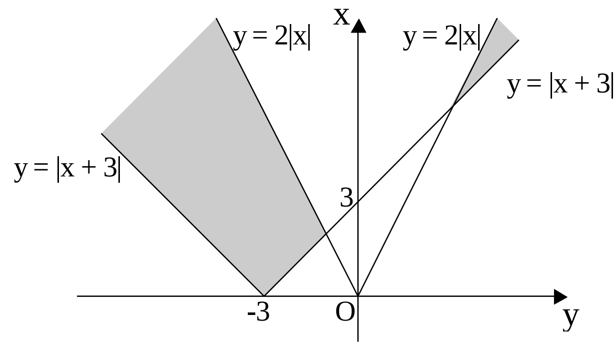
iii. Hence find the values of k for which L is a tangent to C.†

«→ i) centre (2, -3), radius = 5 ii) $\frac{|k+6|}{5}$ units iii) k = -31 or 19 »

3U91-1a)!

i. On the same set of axes, sketch the graphs of y = 2|x| and y = |x + 3|.

ii. On your diagram, shade in the region where $y \le 2|x|$ and $y \ge |x + 3|$ hold simultaneously.†

«→ 

»

3U91-3a)!

Solve for θ in the range $0 \le \theta \le 360°$: $6\cot^2\theta - 4\cos^2\theta = 1$.†

«→ θ = 60°, 120°, 240° or 300° »

3U90-2a)!

Find any values of k, which will make the expression: $(k + 1)x^2 - 2(k - 1)x + (2k - 5)$ a perfect square.†

«→ k = 3 »

3U90-2c)!

The equation $(x - 3y + 5) + k(x + 2y) = 0$ represents a family of straight lines passing through a fixed point P.

i. For what value of k is one of the lines in the family parallel to the straight line $x + y = 2$?

ii. For what value of k does one of the lines in the family pass through the centre of the circle $x^2 + y^2 - 10y + 21 = 0$?

iii. Find the co-ordinates of P.†

«→ i) k = 4 ii) k = 1 iii) P(-2, 1) »

3U90-7a)!

Find: $\lim_{x \to -5} \frac{\sqrt{20 - x} - 5}{5 + x}$.†

«→ $-\frac{1}{10}$ »

3U88-2a)!

For what value (or values) of k, will the quadratic equation $(k + 4)x^2 - 3kx - 4(k - 2) = 0$ have two roots which differ by 1. (HINT: Let the roots of the equation be a and (a + 1).)†

«→ –3 or 2 »

3U87-1a)!

Solve for x: $|x^2 - 5| = 5x + 9$.†

«→ x = -1 and x = 7 »

3U87-2c)!

Find the conditions that must be satisfied by k in order that the expression $2x^2 + 6x + 1 + k(x^2 + 2)$ is positive for all values of x.†

«→ k > 1 »

3U86-1i)!

Solve the equation $x^2 + 2x - 4 + \frac{3}{x^2 + 2x} = 0$.†

«→ x = -3, 1 or $-1 \pm \sqrt{2}$ »

3U86-2i)!

Find all the solutions of the equation $\tan^2\theta = \tan\theta$.†

«→ $\theta = n\pi$ or $n\pi + \frac{\pi}{4}$ »

3U85-3ii)!

a. State the conditions that the quadratic expression $ax^2 + bx + c$ is negative definite.

b. Hence or otherwise show that the expression $(k^2 + k)x^2 - (2k - 6)x + 2$ can never be negative definite.

c. Find the range of values of k for which the expression is positive definite.†

«→ a) $a < 0$ and $\Delta < 0$ b) Proof c) $k < -9$ or $k > 1$ »

3U85-4i)!

Show that $\tan^2 A - \tan^2 B = \dfrac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B}$.†

«→ Proof »

3U84-4ii)!


X and Y are two towns 200 km apart, Y being due east of X. A car A leaves town X travelling in a direction N30°E at 30km/hr and at the same time a car B departs from town Y travelling in a direction S30°W at 45km/hr. How far apart are the two cars after one hour?†

«→ 175 km »