Question 1

$\int 6\cos x\, e^{3\sin x}\, dx = 2e^{3\sin x} + c$ (2)

$\lim_{x\to 0} \frac{\sin 2x}{12x} = \frac{1}{6}$ (2)

$\frac{5x-7}{x} \leq 4$ (3)

Case 1 $x > 0$

$5x - 7 \leq 4x$

$x - 7 \leq 0$

$x \leq 7$

$\therefore \quad 0 < x \leq 7$

Case 2 $x < 0$

$5x - 7 \geqslant 4x$

$x \geqslant 7$

No Solution

$\therefore \quad 0 < x \leq 7$

Note inequality

d) $LHS = \frac{\cos x + \sin x}{\cos x - \sin x} \times \frac{\cos x + \sin x}{\cos x + \sin x}$ (2)

$= \frac{\cos^2 x + 2\sin x\cos x + \sin^2 x}{\cos^2 x - \sin^2 x}$

$= \frac{1 + \sin 2x}{\cos 2x} = RHS$

e) $\int_0^{\ln 4} \frac{e^x\, dx}{e^x + 2} = \left[\ln(e^x + 2)\right]_0^{\ln 4}$ (1) (3)

$= \ln(4+2) - \ln(1+2)$ (1)

$= \ln 2$ (1)

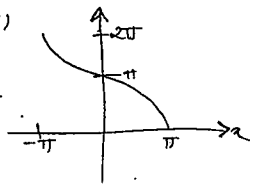
Question 2

a) i) $D: \quad -1 \leq \frac{x}{\pi} \leq 1$ (1)

$-\pi \leq x \leq \pi$ (1)

$R: \quad 0 \leq y \leq 2\pi$

ii)



(1)

b) Let $\tan\alpha = 4$, $\tan\beta = \frac{3}{5}$ (2)

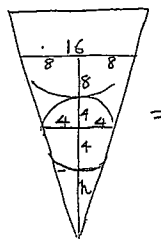
Consider $\tan(\alpha + \beta)$.

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

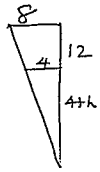
$\frac{4 + \frac{3}{5}}{1 - 4\times\frac{3}{5}} = \frac{17}{17} = 1 = \tan\frac{\pi}{4}$

$\therefore \quad LHS = RHS$

c) (3)



$\Rightarrow$



(1)

$\frac{4+h}{4} = \frac{16+h}{8}$ (1)

$8 + 2h = 16 + h$

$h = 8$ (1)

Question 2 d

i) Let $2\cos x + 2\sqrt{3}\sin x = R\cos x\cos\theta + R\sin x\sin\theta$

$\Rightarrow R\cos\theta = 2$ (2)

$R\sin\theta = 2\sqrt{3}$

$\therefore \tan\theta = \sqrt{3} \quad \therefore \theta = \frac{\pi}{3}$ (1)

$R = \sqrt{4 + 12}$

$= 4$ (1)

$\Rightarrow 4\cos\left(x - \frac{\pi}{3}\right)$

ii) $4\cos\left(x - \frac{\pi}{3}\right) = 2$ (2)

$\cos\left(x - \frac{\pi}{3}\right) = \frac{1}{2}$

$x - \frac{\pi}{3} = \frac{\pi}{3}$ or $\frac{5\pi}{3}$

$\therefore x = \frac{2\pi}{3}$ or 2π (1)(1)

Question 3

$x = B\cos(4t + \alpha)$ (1)

$\dot{x} = -4B\sin(4t + \alpha)$ (2)

$\ddot{x} = -16B\cos(4t + \alpha)$ (3)

$\ddot{x} = -16x$ (4)

$\therefore$ x is moving S.H.M. (2)

i) $t = 0, \; x = 0, \; v = -6$ (2)

(1) $\Rightarrow 0 = B\cos\alpha$

$\therefore \alpha = \frac{\pi}{2}$

(2) $\Rightarrow -6 = -4B\sin\left(0 + \frac{\pi}{2}\right)$

$\therefore -6 = -4B \times 1$

iii) $x = \frac{3}{2}\cos\left(4t + \frac{\pi}{2}\right)$ (2)

$x(4) = \frac{3}{2}\cos\left(16 + \frac{\pi}{2}\right)$

$= 0.432$ (3DP)

Question 3

b) (3)

x	-2	0	2
$f(x)$	$-\frac{7}{4}$	-1	2

(1)

S.R $\Rightarrow \int_{-2}^{2}(2x-2)\,dx \doteq \frac{2-(-2)}{6}\left[-\frac{7}{4} + 4x - 1 + 2\right]$ (1)

$= -\frac{5}{2}$ (1)

c) $y = \frac{x^2}{4}$ (2)

i) $\frac{dy}{dx} = \frac{2x}{4} = \frac{x}{2}$

at $x = 2p$

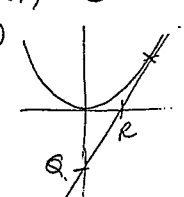
$m = p$

$\therefore \; y - p^2 = p(x - 2p)$

$y - p^2 = px - 2p^2$

$y - px + p^2 = 0$

ii)



R $\Rightarrow y = 0 \therefore x = p$.

$R(p, 0)$

Q $\Rightarrow x = 0 \quad y = -p^2$

$(0, -p^2)$

Mid Pt $\left(\frac{p}{2}, -\frac{p^2}{2}\right)$

(2)

$\Rightarrow x = \frac{p}{2} \Rightarrow p = 2x$

$y = -\frac{(2x)^2}{2}$

$y = -2x^2$ (1)

Question 4

c)

i) $\frac{d(\frac{1}{2}v^2)}{dx} = 3-4x$

$\frac{1}{2}v^2 = \int (3-4x)\,dx$

$= 3x - 2x^2 + c$

$x = 1 \quad v = 0$

$\Rightarrow 3 - 2 + c = 0$

$1 + c = 0$

$c = -1$

$\therefore \frac{1}{2}v^2 = 3x - 2x^2 - 1$

$v^2 = 6x - 4x^2 + 2$

$v = \pm\sqrt{-2(2x^2 - 3x + 1)}$

ii) $2x^2 - 3x + 1 = 0$

$(x-1)(2x-1) = 0$

$x = 1$ or $x = \frac{1}{2}$

d)

$P(x) = x^n(2^m - 1) + x^m(1 - 2^n) + (2^n - 2^m)$

$(x^2 - 3x + 2) = (x-1)(x-2)$

$P(1) = 1(2^m - 1) + 1(1 - 2^n) + 2^n - 2^m = 0$

$P(2) = 2^n(2^m - 1) + 2^m(1 - 2^n) + 2^n - 2^m$

$= 2^{m+n} - 2^n + 2^m - 2^{m+n} + 2^n - 2^m$

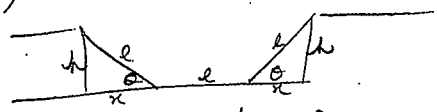
$= 0$

$\therefore (x-1)$ & $(x-2)$ are factors

$\therefore (x^2 - 3x + 2)$ is a factor of $P(x)$

Question 6

b)



$\frac{x}{\ell} = \cos\theta$ $\quad \frac{h}{\ell} = \sin\theta$; (3)

$x = \ell\cos\theta$ $\quad h = \ell\sin\theta$

$A = (\ell + 2x + \ell) \times \frac{h}{2}$;

$= (x+\ell)h$

$= (\ell\cos\theta + \ell) \times \ell\sin\theta$

$= \ell^2(\sin\theta + \sin\theta\cos\theta)$; $\quad \sin\theta = \frac{\sin 2\theta}{2}$

(3)

i) $\frac{dA}{d\theta} = \ell^2(\cos\theta + 2\cos 2\theta)$;

$A' = 0 \Rightarrow \cos\theta = \sin^2\theta - \cos^2\theta$

$= 1 - \cos^2\theta - \cos^2\theta$.

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1)$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$. ← out of the domain.

$\theta = \frac{\pi}{3}$;

$\frac{d^2A}{d\theta^2} = \ell^2(-\sin\theta - 2\sin 2\theta)$

at $x = \frac{\pi}{3}$

$= \ell^2(-\sin\frac{\pi}{3} - 2\sin 2\frac{\pi}{3})$

< 0 ∴ Max at $x = \frac{\pi}{3}$.

Question 7

a) i) $\ddot{x} = 0$ $\quad \ddot{y} = -10$

$\dot{x} = 35\cos\alpha$ $\quad \dot{y} = -10t + 35\sin\alpha$ (2)

$x = 35t\cos\alpha$ $\quad y = -5t^2 + 35t\sin\alpha + 15$;

ii) $x = 105 = 35t\cos\alpha$

$t = \frac{3}{\cos\alpha} = 3\sec\alpha$; (1)

iii) at $t = 3\sec\alpha$ $\quad y = 0$;

$\Rightarrow -5(3\sec\alpha)^2 + 35(3\sec\alpha)\sin\alpha + 15 = 0$ (3)

$-45(1 + \tan^2\alpha) + 105\tan\alpha + 15 = 0$.

$-45 - 45\tan^2\alpha + 105\tan\alpha + 15 = 0$

$-45(3\tan^2\alpha - 7\tan\alpha + 2) = 0$;

$(3\tan\alpha - 1)(\tan\alpha - 2) = 0$

$\therefore \tan\alpha = \frac{1}{3}$ $\quad \tan\alpha = 2$.

OR

i) $\ddot{x} = 0$ $\quad \ddot{y} = -10$

$\dot{x} = 35\cos\alpha$ $\quad \dot{y} = -10t + 35\sin\alpha$

$x = 35t\cos\alpha$ $\quad y = -5t^2 + 35t\sin\alpha$

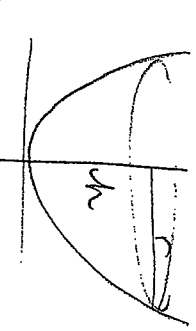
iii) $t = 3\sec\alpha$ $\quad y = -15$

hence

eqn $= -5(3\sec\alpha)^2 + 35(3\sec\alpha)\sin\alpha = -15$ etc.

Question 7 b

$y = \frac{16x^2}{9}$ $\quad x^2 = \frac{9y}{16}$ (3)



i) $V = \pi\int_0^h x^2\,dy = \pi\int_0^h \frac{9y}{16}\,dy$;

$= \pi\left[\frac{9y^2}{32}\right]_0^h$; (1)

$= \pi \times \frac{9h^2}{32}$ (1)

ii) $y = h$ $\quad x = r$

$h = \frac{16r^2}{9}$ (2)

$\frac{9h}{16} = r^2$ (2)

(2) into (1)

$V = \frac{8}{9}\pi r^4$;

$\pi \times \frac{9}{16} \times 2\frac{5}{8}r^4$

$\frac{3}{4} \times \frac{21}{9}$

$\frac{dV}{dr} = \frac{32\pi r^3}{9}$;

iii) $h = 10$

$r = \sqrt{\frac{90}{16}}$ (3)

$= 2.371$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

@ $h = 10$

$\frac{dV}{dt} = 15$

$15 = \frac{32\pi r^3}{9} \times \frac{dr}{dt}$;

$15 = 148.9\ \frac{dr}{dt}$

$\frac{15}{148.9} = \frac{dr}{dt}$

$\cdot 1 = \frac{dr}{dt}$