

QUESTION ONE (Start a new page)

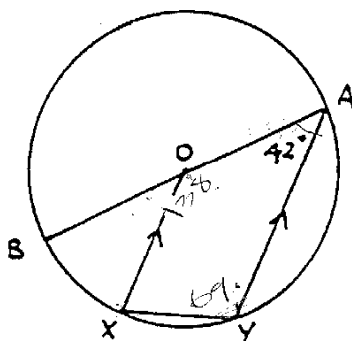
Marks

- A. In group A there are 5 men and 3 women. In group B there are 6 women and 4 men.
- (i) If one person is chosen at random from each group what is the probability that the people chosen are of opposite sexes ? 2
- (ii) If a group, and then one person from that group is chosen at random, what is the probability that the person chosen was a man ? 2
- B. Show that $\tan x = \frac{\sin x}{\cos x}$ Hence, find k to 3 decimal places, given that $\int_0^k \tan x \, dx = 1$ 5
- (C) Using the substitution $u = \log_e x$, evaluate $\int_1^e \frac{\log_e x}{x} \, dx$ 3

QUESTION TWO (start a new page)

- (A) The area between the curve $y = \sin x$, the x axis and the ordinates $x = 0$ and $x = \frac{3\pi}{4}$ is revolved about the x axis. 5
- Find the volume of the solid so formed, leaving your answer in exact form.

(B)



O is the centre of the circle. AY is parallel to OX. Angle OAY measures 42° . Find the measure of angle XYA, giving reasons for your answer.

- (C) A student can borrow 4 books from the library. he wants to read 3 history books, 2 romances, 4 science fiction and 3 murder mysteries. 4
- (i) How many different selections of 4 books can be made ?
- (ii) How many selections can be made if he takes both romances ?
- (iii) How many selections can be made if he decides to take 2 history books ?

QUESTION THREE (Start a new page)**Marks**

(A) Consider the function $f(x) = 3\sin^{-1}\left(\frac{x}{2}\right)$

(i) Find the value of $f(2)$

(ii) Draw the graph of $y = f(x)$

(iii) State the domain and range of this function

(iv) Find the slope of the curve $y = f(x)$ at $x = 0$

6

(B) $\int_0^{1/3} \frac{4}{\sqrt{4-9x^2}} dx$

3

(C) Express $\sqrt{3} \sin x - \cos x$ in the form $n \sin(x - B)$ and hence solve the equation $n \sin(x - B) = 1$ in the region $0 \leq x \leq \pi$

3

QUESTION FOUR (Start a new page)

(A) If a , b and c are the roots of $x^3 - 3x + 2 = 0$, find $a^2 + b^2 + c^2$

2

(B) The position at time t of a particle moving along the x -axis is given by $x = 2t^3 - 9t^2 + 12t$. When and where does the particle first come to rest?

3

(C) What is the coefficient of x^{-3} in the expansion of $(2x - \frac{1}{2x^2})^9$

4

(D) Prove the differentiation of $y = \sin^{-1} x$

2

(E) Find $\lim_{x \rightarrow 0} \frac{\sin 3x}{\frac{1}{\tan 2x}}$

2

QUESTION FIVE (Start a new page)**Marks**

- (A) Prove by mathematical induction that :

4

$$\frac{1}{1(4)} + \frac{1}{4(7)} + \frac{1}{7(10)} + \frac{1}{1(3n-2)(3n+1)} = \frac{n}{3n+1}$$

- (B) (i) Write down the value of
- ${}^nC_k - {}^nC_{n-k}$

3

- (ii) By comparing the coefficients of
- x^6
- on both sides of the identity
- $(1+x)^6 (1+x)^6 = (1+x)^{12}$
- or otherwise, show that :

3

$$\sum_{k=0}^6 ({}^6C_k)^2 = {}^{12}C_6$$

- (C) Find the equation of the normal to the curve
- $y = \operatorname{cosec} 3x$
- at the point P with x coordinate
- $\frac{\pi}{4}$

3

QUESTION SIX (Start a new page)

- (A) For the curve
- $y = \frac{x+2}{(x+1)(x-3)}$

- (i) Find the x and y intercepts

2

- (ii) Find any asymptotes

2

- (iii) Find any stationary points and determine their nature

3

- (iv) find any points of inflexion

3

- (v) Sketch the curve

2

QUESTION SEVEN (Start a new page)**Marks**

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|-----|-------|--|---|
| (A) | (i) | Derive the equation of the tangent to the parabola whose parametric equation is $x = 2at$ $y = at^2$. | |
| | (ii) | P and Q are two points on the parabola above with parameters p and q respectively. If the tangents at P and Q meet at T find the co-ordinates of T in terms of p and q. | 6 |
| | (iii) | If T lies on the line $x + y + 5a = 0$ find the relationship between p and q. Hence or otherwise find the locus of the Midpoint M of PQ as P moves around the parabola. | |
| (B) | | A spherical balloon is being inflated and its radius is increasing at the constant rate of 3 cm/min. At what rate is its volume increasing when the radius of the balloon is 5 cm ? | 2 |
| (C) | | For the time interval $0 \leq t \leq \frac{1}{2}$ the velocity of a particle is given by :
$v = \frac{10}{\sqrt{1-t^2}} + \frac{1}{(1-t)^2} \quad (\text{the units are metres, seconds})$ | 4 |

During the given time interval find :

- (i) The distance travelled by the particle.
- (ii) The maximum velocity attained.