ARC Academic Resources Centre

Student Number: …………..……………………..….

2007

HIGHER SCHOOL CERTIFICATE

Sample Examination Paper

MATHEMATICS Extension 1

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using blue or black pen
- Write your student number at the top of this page
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1–7
- All questions are of equal value

Directions to school or college

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Total marks – 84
Attempt Questions 1–7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Find $\displaystyle\int \frac{dx}{\sqrt{49 - x^2}}$. **2**

(b) State the domain and range of $y = 2\sin^{-1}\left(\frac{x}{3}\right)$. **2**

(c) Solve the inequality $\dfrac{3x+5}{x-4} \geq 2$. **3**

(d) Sketch the region in the plane defined by $y \leq |2x+3|$. **2**

(e) Use the substitution $u = e^{-x} + 1$ to find the indefinite integral $\displaystyle\int \frac{e^{-2x}\,dx}{e^{-x}+1}$. **3**

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) A series is given by $1+\frac{1-p}{p}+\left(\frac{1-p}{p}\right)^2+\ldots$, where p is positive.

(i) Find the domain of p such that the series has a sum to infinity. **2**

(ii) Find this sum to infinity in terms of p. **1**

(b) The curves $y=x^3$ and $y=\frac{x^2+3}{4}$ meet at the point (1, 1).

Find the angle between the tangents to the curves at this point. **4**

(c) A jug of water at a temperature of 20°C is placed in a refrigerator. The temperature inside the refrigerator is maintained at 4°C. When the jug has been in the refrigerator for t minutes the temperature of the water in the jug is T °C. The rate at which the water temperature decreases is proportional to the excess of its temperature over the temperature inside the refrigerator. That is, T satisfies the equation $\frac{dT}{dt}=-k(T-4)$, where k is a positive constant.

(i) Show that $T = 4 + Ae^{-kt}$ satisfies the equation. **1**

(ii) The temperature of the water is 10°C after 15 minutes. Find the value of A and k. **2**

(iii) How long will it take the temperature of the water to fall to 6°C? **2**

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Marks

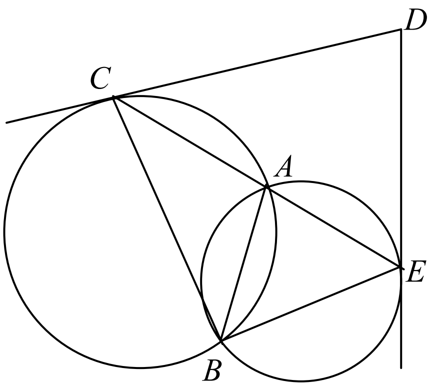
Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) (i) Show that $\tan^{-1} x - x^2 + \frac{\pi}{4} = 0$ has a root in the interval $1 < x < \sqrt{3}$. **2**

(ii) Use one application of Newton's method to find an approximation to this root. Take $x_0 = 1$. **2**

(b) Use mathematical induction to prove that $\displaystyle\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$. **3**

(c)



Two circles intersect at A and B. CAE is a straight line where C is a point on the first circle and E is a point on the second circle. The tangent at C to the first circle and the tangent at E to the second circle meet at D.

Prove that $BCDE$ is a cyclic quadrilateral. **3**

(d) In the expansion of $(2x + k)^6$ the coefficients of x and x^2 are equal. Find the value of k. **2**

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) (i) Given that $f(x) = \tan^{-1}x + x\tan^{-1}x$, show that $f\,'(x) = \tan^{-1}x + \dfrac{x+1}{1+x^2}$ **2**

(ii) The graph of $y = f(x)$ has only a single point of inflection. Find the exact coordinates of this point of inflection. **3**

(b) A bag contains six balls which are identical, apart from colour. Three are red, two are blue and one is black. Two balls are drawn at random.

(i) What is the probability that they are both red? **1**

(ii) After replacing the two balls the drawing is repeated several times.

(a) What is the probability of getting two red balls on at least one occasion from five drawings of two balls? **2**

(b) What is the probability of getting two red balls on exactly three occasions from five drawings of two balls? **1**

(c) (i) Show that $t^2 + p^2 = (t + p)^2 - 2tp$. **1**

(ii) Given that R has coordinates $(-atp(t + p),\ a(t^2 + tp + p^2 + 2))$, show that when $tp = 2$, the locus of R is the parabola $x^2 = 4ay$. **2**

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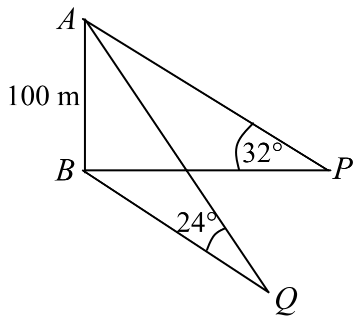
Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a)



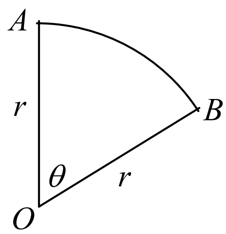
A is 100 metres above the horizontal plane BPQ. AB is vertical. The angle of elevation of A from P is 32° and the angle of elevation of A from Q is 24°. P is due East of B and Q is South 42° East from B.

Calculate the distance from P to Q, to the nearest 10 metres. **3**

(b) (i) Prove that $\cot\theta - 2\cot 2\theta = \tan\theta$. **1**

(ii) Hence deduce that $\tan x + 2\tan 2x + 4\tan 4x = \cot x - 8\cot 8x$. **2**

(c)



AOB is a sector of a circle with centre O and radius r cm, as shown in the diagram. $\angle AOB = \theta$ radians

(i) The area of the sector AOB is 100 cm^2. Show that $\theta = \dfrac{200}{r^2}$. **1**

(ii) If the radius is increasing at a constant rate of 0.5 cms^{-1}, find the rate at which $\angle AOB$ is decreasing when $r = 10$ cm. **2**

(d) Using the expansion of $(1+x)^n = (1+x)^2(1+x)^{n-2}$, or otherwise, prove that
$\binom{n}{r} = \binom{n-2}{r} + 2\binom{n-2}{r-1} + \binom{n-2}{r-2}$. **3**

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) A function is defined as $f(x) = 1 - \cos\frac{x}{2}$ for $0 \le x \le b$.

(i) Find the largest value of b for which the inverse function $f^{-1}(x)$ exists. **2**

(ii) Find $f^{-1}(x)$. **2**

(iii) Sketch the graph of $y = f^{-1}(x)$. **1**

(iv) Calculate the area enclosed between the curve $y = f^{-1}(x)$, the x axis and the line $x = 2$. **1**

(b) An object falling from rest in air is subjected to an acceleration $\ddot{x} = g - \frac{v}{k}$, where g and k are constants and v is the velocity at time t.

(i) Show that $v = gk\left(1 - e^{\frac{-t}{k}}\right)$. **3**

(ii) What is the greatest speed obtained? **1**

(iii) What is the distance travelled in the time $t = k\ln 4$? **2**

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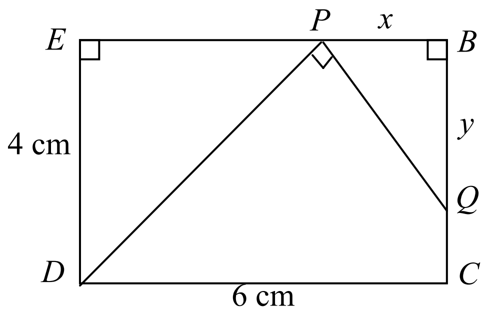
Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) A triangle contains ten points within it. No three of the ten points are collinear and no two points within the triangle are collinear with a vertex of the original triangle. How many triangles may be formed using vertices of the original triangle and, at most, two of the points within the triangle. **2**

(b)



EBCD is a rectangle in which the sides *DC* and *ED* are of length 6 cm and 4 cm respectively. *P* is a point on *EB* and *Q* is a point on *BC* (or *BC* produced) such that $\angle DPQ = 90°$.
Let $PB = x$ cm and $BQ = y$ cm.

(i) Show that $y = \dfrac{x(6-x)}{4}$ **2**

(ii) Find the position of *P* on *EB* so that the area of ΔPBQ is a maximum and find this maximum area. **3**

(iii) Show that the equation $3x^2 - 24x + 52 = 0$ has no real roots. **1**

(iv) Show that the area of $\Delta DPQ = \dfrac{x\left(x^2 - 12x + 52\right)}{8}$ and find when this area is a maximum. Comment on your answer. **4**

End of paper

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STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2} \right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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Mapping grid

Question	Mark	Content	Outcome	Band
1(a)	2	Integration, inverse trigonometry	HE4	E2
1(b)	2	Inverse functions	HE4	E2
1(c)	3	Inequalities	PE3	E2
1(d)	2	Real functions and their graphs	PE6	E2
1(e)	3	Integration by substitution	HE6	E2
2(a)	3	Series and applications	HE1	E3
2(b)	4	Angle between two curves	PE1	E2
2(c)	5	Application of calculus to the physical world	HE3	E3
3(a)	4	Polynomials	HE7	E4
3(b)	3	Mathematical induction	HE2	E4
3(c)	3	Circle geometry	PE3	E3
3(d)	2	Binomial theorem	HE3, HE7	E4
4(a)	5	Inverse functions	PE5, HE4	E3
4(b)	4	Permutations, combinations and further probability	HE3	E3
4(c)	3	Quadratic polynomial and the parabola	PE3	E4
5(a)	3	Harder trigonometry	PE6	E3
5(b)	3	Trigonometric identities	PE3	E4
5(c)	3	Application of calculus to the physical world	HE5	E3
5(d)	3	Binomial theorem	HE3	E4
6(a)	6	Inverse functions	HE4	E4
6(b)	6	Application of calculus to the physical world	HE5	E4
7(a)	2	Permutations and combinations	PE3	E3
7(b)	10	Application of calculus to the physical world	HE7	E4

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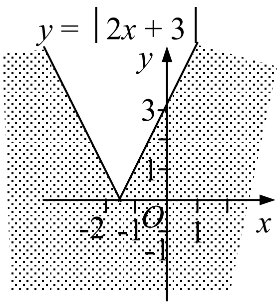
Marking guidelines

Question 1
Marking guidelines

Criteria	Marks
(a) $\int \frac{dx}{\sqrt{49-x^2}} = \sin^{-1}\frac{x}{7} + (C)$	2 Correct answer 1 Any of ($b \neq 0$) : $\frac{1}{b}\sin^{-1}\frac{x}{7}$, $f^{-1}\left(\frac{x}{7}\right)$
(b) Domain: $-3 \le x \le 3$ Range: $-\pi \le y \le \pi$	2 both correct 1 one part correct
(c) $\frac{3x+5}{x-4} \ge 2$: $\frac{(3x+5)(x-4)^2}{x-4} \ge 2(x-4)^2$, $x \neq 4$ * $(3x+5)(x-4) \ge 2(x-4)^2$ $(x-4)(3x+5-2x+8) \ge 0$ $(x-4)(x+13) \ge 0$ * $x = 0$, LHS $= (-4)(13) < 0$ Solution is $x \le -13$, $x > 4$ *	3 Correct solution 1 for each * or equivalent. Award marks if result is correct from previous error (CPE)
(d) $y \le \lvert 2x+3 \rvert$ 	2 Correct region 1 correct boundary 1 correct region for incorrect boundary Need evidence of correct intercepts for boundary.
(e) $u = e^{-x} + 1$: $du = -e^{-x}\,dx$ and $e^{-x} = u - 1$ * $\int \frac{e^{-2x}\,dx}{e^{-x}+1} = \int \frac{e^{-x} \times e^{-x}\,dx}{e^{-x}+1} = \int \frac{-(u-1)\,du}{u}$ * $= \int \left(\frac{1}{u} - 1\right) du = \ln u - u + (C)$ $= \ln(e^{-x}+1) - (e^{-x}+1) + C = \ln(e^{-x}+1) - e^{-x} + (K)$ *	3 Correct solution 1 for each * or equivalent. Be aware of variable constant of integration in last step. (K real)

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Question 2
Marking guidelines

Criteria	Marks
(a) $1+\frac{1-p}{p}+\left(\frac{1-p}{p}\right)^2+\ldots,\ p>0.$ (i) $-1<\frac{1-p}{p}<1$ * $-p^2<p-p^2<p^2$ $0<p<2p^2$ $\quad 2p^2-p>0$ $p(2p-1)>0$ $\quad p>\frac{1}{2}$ (ii) $S_\infty=\frac{1}{1-\frac{1-p}{p}}=\frac{p}{2p-1}$	(i) 2 Correct solution 1 correct answer without working. 1 line 1 1 solution correct from their incorrect line 1 (ii) 1 Correct answer
(b) $y=x^3$: $\quad \frac{dy}{dx}=3x^2$. $\quad$ At (1, 1), $\quad m_1=3$ * $y=\frac{x^2+3}{4}$: $\quad \frac{dy}{dx}=\frac{x}{2}$. $\quad$ At (1, 1), $\quad m_2=\frac{1}{2}$ * $\tan\theta=\left\lvert\frac{3-\frac{1}{2}}{1+3\times\frac{1}{2}}\right\rvert$ * $\quad \tan\theta=1$ so $\theta=45°$ *	4 Correct solution 1 for each * or equivalent. Award marks if CPE
(c)(i) If $T=4+Ae^{-kt}$ then $\frac{dT}{dt}=-kAe^{-kt}$ But $Ae^{-kt}=T-4$ so $\frac{dT}{dt}=-k(T-4)$ (ii) $t=0,\ T=20$: $\quad 20=4+A$ $\quad A=16$ $t=15,\ T=10$: $\quad 10=4+16e^{-15k}$ $e^{-15k}=\frac{3}{8}$ $\quad k=\frac{1}{15}\ln\left(\frac{8}{3}\right)$ (≈ 0.06539) (iii) $T=6,\ t=?$: $\quad 6=4+16e^{\frac{-t}{15}\ln\left(\frac{8}{3}\right)}$ $\frac{1}{8}=e^{\frac{-t}{15}\ln\left(\frac{8}{3}\right)}$ OR $\frac{1}{8}=e^{\ln\left(\frac{3}{8}\right)^{\frac{t}{15}}}$ * $\frac{-t}{15}\ln\left(\frac{8}{3}\right)=-\ln 8$ OR $\frac{1}{8}=\left(\frac{3}{8}\right)^{\frac{t}{15}}$ etc $t=\frac{15\ln 8}{\ln\left(\frac{8}{3}\right)}\approx 31.8\text{ min}$	(i) 1 Correct working shown (ii) 1 correct A 1 correct expression for k (iii) 2 Correct solution 1 line * 1 correct t from their line *

Question 3
Marking guidelines

Criteria	Marks
(a)(i) $f(x) = \tan^{-1} x - x^2 + \frac{\pi}{4}$ $f(1) = \tan^{-1} 1 - 1 + \frac{\pi}{4} = \frac{\pi}{4} - 1 + \frac{\pi}{4} = \frac{\pi}{2} - 1 > 0$ * $f(\sqrt{3}) = \tan^{-1}\sqrt{3} - 3 + \frac{\pi}{4} = \frac{\pi}{3} - 3 + \frac{\pi}{4} < 0$ * $f(x)$ changes sign between $x = 1$ and $x = \sqrt{3}$; hence equation has a root in the interval $1 < x < \sqrt{3}$. *	(i) 2 all three * 1 two *
(ii) $f'(x) = \frac{1}{1+x^2} - 2x$ # $f'(1) = \frac{1}{1+1} - 2 = -1.5$, $f(1) = \frac{\pi}{2} - 1$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{\frac{\pi}{2} - 1}{-1.5} (= 1.3805)$ #	(ii) 2 correct numerical expression for x_1 1 either of #, i.e. correct derivative or correct numerical expression for their derivative
(b) $\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$ $n = 1$: LHS $= \frac{1}{1\times 3} = \frac{1}{3}$ RHS $= \frac{1}{2+1} = \frac{1}{3} =$ LHS Assume true for $n = k$, ie. assume $\sum_{r=1}^{k} \frac{1}{(2r-1)(2r+1)} = \frac{k}{2k+1}$ Prove true for $n = k + 1$, ie. prove $\sum_{r=1}^{k+1} \frac{1}{(2r-1)(2r+1)} = \frac{k+1}{2k+3}$ LHS $= \sum_{r=1}^{k} \frac{1}{(2r-1)(2r+1)} + \frac{1}{(2k+1)(2k+3)}$ $= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$ $= \frac{k(2k+3)+1}{(2k+1)(2k+3)} = \frac{2k^2+3k+1}{(2k+1)(2k+3)} = \frac{(2k+1)(k+1)}{(2k+1)(2k+3)} = \frac{(k+1)}{(2k+3)}$ = RHS Result is true for $n = 1$, so it is true for $n = 1 + 1 = 2$ and, by the principle of mathematical induction, is true for all $n > 0$.	3 Correct solution 1 prove for $n = 1$ and evidence of correct assumption for $n = k$ 1 correct substitution into result for $n = k+1$ 1 correct simplification

Criteria	Marks
(c) Let $\angle DCE = x°$ and $\angle DEC = y°$. Since CAE is a straight line, then $\angle CDE = 180° - (x + y)°$ (angle sum of ΔDCE) $\angle CBA = x°$ (angle between the tangent DC and the chord CA equals the angle in the alternate segment) Similarly $\angle EBA = y°$ (tangent is DE) $\therefore\ \angle CBA + \angle EBA = (x + y)°$ $\therefore\ \angle CDE + \angle CBE = 180°$ $\therefore$ $BCDE$ is a cyclic quadrilateral (opposite angles supplementary)	3 Correct solution following with reasons: 1 finding $\angle CDE$ 1 finding $\angle CBA$ or $\angle EBA$ 1 tying results together Maximum 1 if correct and no reasons given
(d) $(2x + k)^6$ Term in x: $\binom{6}{1} 2x \times k^5 = 12k^5 x$ Term in x^2: $\binom{6}{2} (2x)^2 \times k^4 = 60k^4 x^2$ $12k^5 = 60k^4$ so $k = 5$	2 Correct solution 1 either coefficient (or term) correct

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Question 4
Marking guidelines

Criteria	Marks
(a)(i) $f(x) = \tan^{-1}x + x\tan^{-1}x$ $f'(x) = \frac{1}{1+x^2} + \tan^{-1}x + x \times \frac{1}{1+x^2}$ $f'(x) = \tan^{-1}x + \frac{x+1}{1+x^2}$ (ii) $f''(x) = \frac{1}{1+x^2} + \frac{(1+x^2)-(x+1)\times 2x}{(1+x^2)^2}$ $f''(x) = \frac{1+x^2+1+x^2-2x^2-2x}{(1+x^2)^2} = \frac{2-2x}{(1+x^2)^2} = \frac{2(1-x)}{(1+x^2)^2}$ $f''(x) = 0$ when $x = 1$. $f(1) = \tan^{-1}1 + \tan^{-1}1 = \frac{\pi}{2}$. $x < 1$: $f''(x) = \frac{2(+)}{(+)^2} > 0$ $x > 1$: $f''(x) = \frac{2(-)}{(+)^2} < 0$ $\therefore$ Concavity changes at $x = 1$ so $\left(1, \frac{\pi}{2}\right)$ is the point of inflection.	(i) 2 Correct solution 1 correct derivative of $\tan^{-1}x$ 1 correct use of product rule with derivative of $\tan^{-1}x$ consistent. (ii) 3 Correct solution 1 correct $f''(x)$ 1 finding coordinates 1 justifying the change in concavity
(b)(i) $P(RR) = \frac{1}{2} \times \frac{2}{5} = \frac{1}{5}$ (ii)(a) $n = 5$, P(two red at least once) = 1 – P(no reds 5 times) $= 1 - \left(\frac{4}{5}\right)^5 \approx (0.67232)$ (ii)(b) P(two red exactly 3 times) $= \binom{5}{3}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2 \approx (0.0512)$	(i) 1 Correct answer (ii)(a) 2 Correct solution 1 use of complementary events or $\left(\frac{4}{5}\right)^5$ (ii)(b) 1 Correct answer
(c)(i) $(t+p)^2 - 2tp = t^2 + 2tp + p^2 - 2tp$ $= t^2 + p^2$ (ii) $x = -atp(t+p)$, $y = a(t^2 + tp + p^2 + 2)$ $y = a((t+p)^2 - 2tp + tp + 2)$ $y = a((t+p)^2 - tp + 2)$ $tp = 2$: $x = -2a(t+p)$, $y = a(t+p)^2$ $y = a\left(\frac{x}{-2a}\right)^2$ $\therefore$ $x^2 = 4ay$ is the locus of R.	(i) 1 Correct expansion (ii) 2 Correct solution 1 uses part (i) in (ii) 1 correctly simplifies their substitution

Question 5
Marking guidelines

Criteria	Marks
(a) $BQ = 100\tan 66°$ $BP = 100\tan 58$ * $\angle QBP = 90° - 42° = 48°$ $PQ^2 = BQ^2 + BP^2 - 2BQ \times BP\cos 48°$ $PQ^2 = (100\tan 66°)^2 + (100\tan 58°)^2 - \times 100^2 \times \tan 66° \times \tan 58° \times \cos 48°$ * $PQ^2 = 10000(\tan^2 66° + \tan^2 58° - 2\tan 66° \tan 58° \cos 48°)$ $PQ^2 = 27955$ $PQ = 167.2$ (≈ 170) metres *	3 Correct solution 1 each * Pay if CPE
(b)(i) $\cot\theta - 2\cot 2\theta = \frac{1}{\tan\theta} - 2 \times \frac{1-\tan^2\theta}{2\tan\theta}$ $= \frac{1}{\tan\theta} - \frac{1}{\tan\theta} + \tan\theta = \tan\theta$ (ii) $\tan x = \cot x - 2\cot 2x$ } * $\tan 2x = \cot 2x - 2\cot 4x$ } $\tan 4x = \cot 4x - 2\cot 8x$ } $\tan x + 2\tan 2x + 4\tan 4x$ $= \cot x - 2\cot 2x + 2(\cot 2x - 2\cot 4x) + 4(\cot 4x - 2\cot 8x)$ $= \cot x - 2\cot 2x + 2\cot 2x - 4\cot 4x + 4\cot 4x - 8\cot 8x$ ** $= \cot x - 8\cot 8x$	(i) 1 Correct solution (ii) 2 Correct solution 1 3 results at * 1 for **
(c)(i) $\frac{1}{2}r^2\theta = 100$ so $\theta = \frac{200}{r^2}$ (ii) $\frac{dr}{dt} = 0.5$, find $\frac{d\theta}{dt}$ when $r = 10$ $\frac{d\theta}{dr} = \frac{-400}{r^3}$ * $\frac{d\theta}{dt} = \frac{d\theta}{dr} \times \frac{dr}{dt} = \frac{-400}{r^3} \times \frac{1}{2} = \frac{-200}{r^3}$ $r = 10$: $\frac{d\theta}{dt} = \frac{-200}{1000} = -0.2$ * $\angle AOB$ is decreasing at 0.2 radians/second.	(i) 1 Correct solution (ii) 2 Correct solution 1 each *, CPE

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<table>
<tr><th>Criteria</th><th>Marks</th></tr>
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(d) $(1+x)^n$: $\binom{n}{r}x^r$

$(1+x)^2 = 1 + 2x + x^2$

$(1+x)^{n-2}$: $\binom{n-2}{r-2}x^{r-2} + \binom{n-2}{r-1}x^{r-1} + \binom{n-2}{r}x^r$ *

For the term in x^r in the expansion of $(1+x)^2(1+x)^{n-2}$ you have to consider the appropriate terms from:

$(1+2x+x^2)\left[\binom{n-2}{r-2}x^{r-2} + \binom{n-2}{r-1}x^{r-1} + \binom{n-2}{r}x^r\right]$ *

ie.: $1\times\binom{n-2}{r}x^r + 2x\times\binom{n-2}{r-1}x^{r-1} + x^2\times\binom{n-2}{r-2}x^{r-2}$

$\binom{n-2}{r}x^r + 2\binom{n-2}{r-1}x^r + \binom{n-2}{r-2}x^r$ *

Equating the coefficient of x^r on each side gives

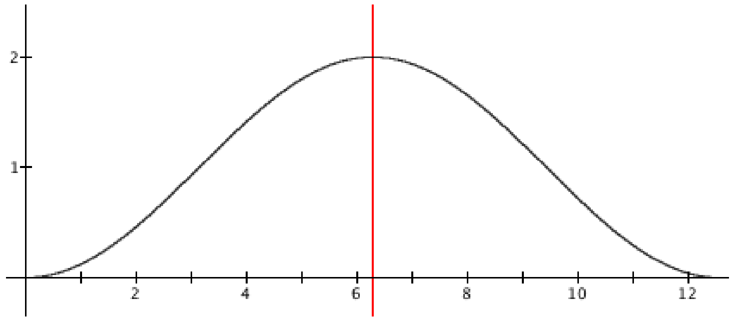
$\binom{n}{r} = \binom{n-2}{r} + 2\binom{n-2}{r-1} + \binom{n-2}{r-2}$

</td><td>3 Correct solution
1 for each *, CPE</td></tr>
</table>

Question 6
Marking guidelines

<table>
<tr><th>Criteria</th><th>Marks</th></tr>
<tr><td>

(a)(i) $f(x) = 1 - \cos\dfrac{x}{2}$, $0 \le x \le b$.



$f(x)$ is monotonic increasing for $0 \le x \le 2\pi$
Hence $b = 2\pi$.

</td><td>

(i)
2 Correct solution
1 correct sketch of $f(x)$
1 correct b from their sketch

</td></tr>
<tr><td>

(ii) $x = 1 - \cos\dfrac{y}{2}$, $\cos\dfrac{y}{2} = 1 - x$, $y = 2\cos^{-1}(1-x)$

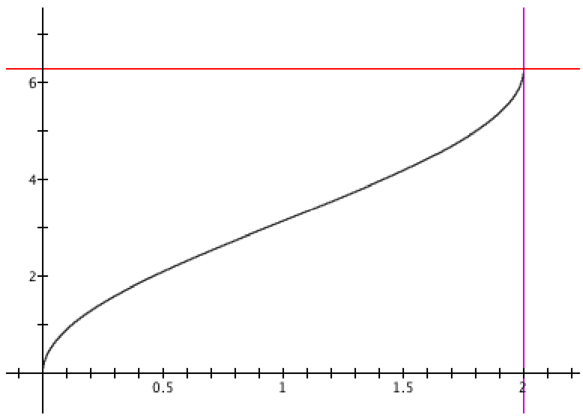
$f^{-1}(x) = 2\cos^{-1}(1-x)$

</td><td>

(ii)
2 correct solution
1 $x = 1 - \cos\dfrac{y}{2}$
1 correct rearrangement of their expression

</td></tr>
<tr><td>

(iii)



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(iii)
1 Correct graph

</td></tr>
<tr><td>

(iv) By symmetry, area is half the area of the rectangle contained by the axes and $x = 2$ and $y = 2\pi$.
Area = 2π square units.
OR

Area $= \displaystyle\int_0^2 2\cos^{-1}(1-x)\,dx$. This can't be evaluated. Use the rectangle and the area bounded by the y-axis.

Area $= 2 \times 2\pi - \displaystyle\int_0^{2\pi}\left(1 - \cos\frac{y}{2}\right)dy = 4\pi - \left[y - 2\sin\frac{y}{2}\right]_0^{2\pi}$

$= 4\pi - (2\pi - 2\sin\pi - 0) = 4\pi - 2\pi = 2\pi$

</td><td>

(iv)
1 Correct answer

</td></tr>
</table>

Criteria	Marks
(b)(i) $\ddot{x} = g - \frac{v}{k}$ $\frac{dv}{dt} = g - \frac{v}{k} = \frac{kg - v}{k}$ $\frac{dt}{dv} = \frac{k}{kg - v}$ * $t = \int \frac{k\,dv}{kg - v} = -k\ln(kg - v) + C$ $t = 0, v = 0$: $0 = -k\ln(kg) + C$ $C = k\ln(kg)$ * $t = k\ln(kg) - k\ln(kg - v)$ $\frac{t}{k} = \ln\frac{kg}{kg - v}$ $e^{\frac{t}{k}} = \frac{kg}{kg - v}$ $kg - v = kge^{-t/k}$ $v = kg - kge^{-t/k}$ * $v = gk(1 - e^{-t/k})$ or $\dot{x} = gk\left(1 - e^{-t/k}\right)$	(i) 3 Correct solution 1 each * , CPE
(ii) Greatest speed when $\ddot{x} = 0$: $g - \frac{v}{k} = 0$, i.e. $v = kg$	(ii) 1 Correct solution
(iii) $\dot{x} = gk\left(1 - e^{-t/k}\right)$, $t = k\ln 4$ $x = \int_0^{k\ln 4} gk\left(1 - e^{-t/k}\right)dt$ $x = gk\left[t + ke^{-t/k}\right]_0^{k\ln 4}$ * $x = gk\left(k\ln 4 + ke^{-\ln 4} - (0 + k)\right)$ $x = gk^2\left(\ln 4 + \frac{1}{4} - 1\right)$ * $x = gk^2\left(\ln 4 - \frac{3}{4}\right)$	(iii) 2 Correct solution 1 each * , CPE

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Question 7
Marking guidelines

Criteria	Marks
(a) 3 points from triangle: 1 triangle 2 points from triangle + 1 from inside: $\binom{3}{2} \times 10 = 30$ triangles 1 point from triangle + 2 from inside: $\binom{3}{1} \times \binom{10}{2} = 3 \times 45 = 135$ Total number of triangles = 1 + 30 + 135 = 166	2 Correct solution 1 either 2nd or 3rd lines 1 evidence that 3 possibilities were considered
(b)(i) $EP = 6 - x$, $CQ = 4 - y$ $DQ^2 = 36 + (4 - y)^2$ $DP^2 = 16 + (6 - x)^2$ $PQ^2 = x^2 + y^2$ But $DQ^2 = DP^2 + PQ^2$ $36 + (4 - y)^2 = 16 + (6 - x)^2 + x^2 + y^2$ $20 + 16 - 8y + y^2 = 36 - 12x + x^2 + x^2 + y^2$ $8y = 12x - 2x^2$ * $y = \dfrac{6x - x^2}{4} = \dfrac{x(6 - x)}{4}$ (ii) Area ΔPBQ, $A = \dfrac{xy}{2} = \dfrac{x}{2} \times \dfrac{6x - x^2}{4} = \dfrac{6x^2 - x^3}{8}$ * $\dfrac{dA}{dx} = \dfrac{1}{8}\left(12x - 3x^2\right) = 0$ when $3x(4 - x) = 0$, ie. when $x = 0, 4$ * $\dfrac{d^2A}{dx^2} = \dfrac{1}{8}(12 - 6x)$. When $x = 4$, $\dfrac{d^2A}{dx^2} = \dfrac{-12}{8} < 0$ * Maximum area when $x = 4$. Area $= \dfrac{6 \times 4^2 - 4^3}{8} = 4\ \text{cm}^2$ (iii) $3x^2 - 24x + 52 = 0$ $\Delta = 24^2 - 4 \times 3 \times 52 = -48 < 0$ $\therefore$ equation has no real roots.	(i) 2 Correct solution 1 state 3 Pythagoras results 1 line *, CPE (ii) 3 Correct solution 1 each *, CPE Need to fraction in second derivative for mark and < 0 stated. (iii) 1 Correct solution

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Criteria	Marks
(iv) Area $\Delta DPQ = \dfrac{\sqrt{52-12x+x^2} \times \sqrt{x^2+y^2}}{2}$ * $x^2+y^2 = x^2 + \dfrac{\left(6x-x^2\right)^2}{16} = \dfrac{16x^2+36x^2-12x^3+x^4}{16}$ $\sqrt{x^2+y^2} = \dfrac{x}{4}\sqrt{52-12x+x^2}$ Area $\Delta DPQ = \dfrac{x\left(x^2-12x+52\right)}{8}$ * Call area H, $H = \dfrac{x^3-12x^2+52x}{8}$ $\dfrac{dH}{dx} = \dfrac{1}{8}\left(52-24x+3x^2\right)$ $\dfrac{dH}{dx} \neq 0$ as in part (iii) we showed that $3x^2 - 24x + 52 = 0$ has no real roots. $\therefore$ H has no stationary points. * $x = 0$, Area $\Delta DPQ = 0$ $x = 6$, Area $\Delta DPQ = \dfrac{6\left(36-72+52\right)}{8} = 12$ The greatest area occurs at one of the endpoints of the domain, $x = 6$ when DQ is a diagonal of the rectangle and $\angle DPQ$ coincides with $\angle DEB$. *	(iv) 4 Correct solution 1 for each *, CPE Penalise 1 mark if fraction is left out of first line of derivative Need to show no real roots if (iii) is not mentioned or implied in the working Must check both endpoints of the domain or comment on $x = 0$ for mark. If mistakes make question easier so that any of the above do not occur, do not award marks.

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