REDDAM
House

Reddam House
Mathematics Department

2004
Trial HSC Examination

Mathematics (Ext I)

GENERAL INSTRUCTIONS:

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided
- All necessary working should be shown in every question

- Attempt Questions 1 – 7
- All questions are of equal value
- Use a SEPARATE writing booklet for each question

TOTAL MARKS (84)

QUESTION 1 *(Start a new booklet)* **MARKS**

(a) Use the substitution $u = \log x$ to find the exact value of $\int_e^{e^2} \frac{1}{x \log x} dx$. **3**

(b) Solve $\frac{5}{(2-x)(x+2)} > 1$. **2**

(c) Show that the point $A\left(\frac{1}{2}, 4\right)$ lies on the line joining the points $P(-3, -3)$ and $Q(1, 5)$ and find the ratio in which it divides the line segment PQ. **3**

(d) Show that $\tan^{-1} x = \cos^{-1} \frac{1}{\sqrt{1+x^2}}$. **3**

(e) Find the remainder when the function $P(x) = x^3 - 4x + 2$ is divided by $x + 4$. **1**

QUESTION 2 *(Start a new booklet)* **MARKS**

(a) The word EQUATIONS contains all five vowels. How many 7-letter 'words' consisting of all fives vowels can be formed from the letters of EQUATIONS? **2**

(b) Determine the coefficient of x^5 in the expansion of $(1-3x+2x^3)(1-2x)^6$. **3**

(c) Find the general solution of the equation $\cos 54° \cos \alpha + \sin 54° \sin \alpha = \sin 2\alpha$. **3**

(d) Find $\frac{d}{dx}\left(\frac{\tan^2 x}{x}\right)$. **2**

(d) If $f(x) = 2x^2 + x$, use the definition **2**

$$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

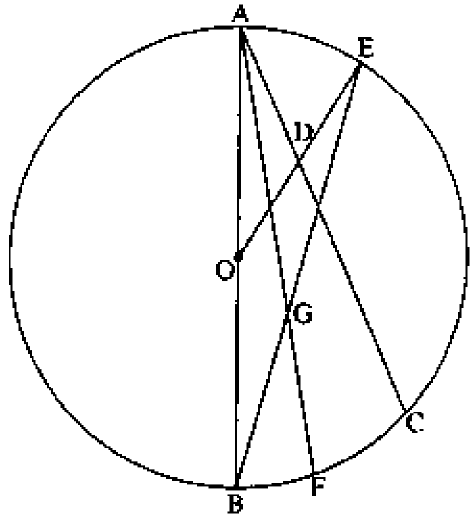
to find the derivative of $f(x)$ at the point where $x = a$.

QUESTION 3 *(Start a new booklet)* **MARKS**

(a) For the parabola $y = x^2 - 4x - 1$, find the coordinates of the focus and the equation of the directrix. **3**

(b) How many numbers smaller than 500 can be formed from the digits 2, 3, 4, 5, 6 and 7 if no repetitions are allowed? **2**

(c) In the figure, AOB is the diameter of a circle centre O. D is a point on chord AC such that $DA = DO$ and OD is produced to E. AF is the bisector of $\angle BAC$ and cuts BE in G.



Prove that

(i) $GA = GB$ **3**

(ii) $AOGE$ is a cyclic quadrilateral **2**

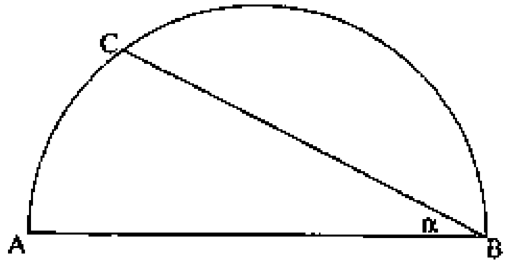
(iii) If CE is joined, then $CE \parallel AF$. **2**

QUESTION 4 *(Start a new booklet)* **MARKS**

(a) Use mathematical induction to prove that, for all positive integers n, 5

$$\sum_{r=1}^{n} \frac{r^2}{(2r-1)(2r+1)} = \frac{n(n+1)}{2(2n+1)}$$

(b) AB is the diameter of a semi-circle of unit radius and BC is a chord which makes an angle α with AB such that the area of the semi-circle is bisected by the chord.



(i) Show that the area of the segment is 2

$$\frac{1}{2}(\pi - 2\alpha - \sin 2\alpha)$$

(ii) Hence show that $2 \sin 2\alpha + 4\alpha = \pi$ 2

(iii) Prove that a root of this equation lies between $\alpha = 0.4$, and $\alpha = 0.5$. 2

(iv) By using the 'halving the interval' method, determine whether the root lies closer to 0.4 or 0.5. 1

QUESTION 5 *(Start a new booklet)* **MARKS**

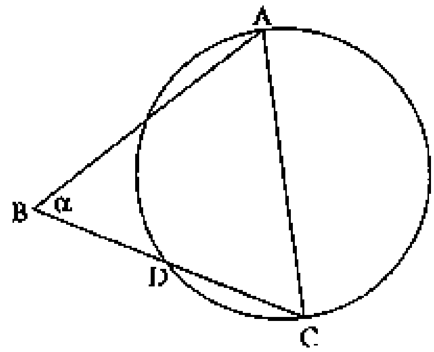
(a) Corn cobs are cooked by immersing them in boiling water. On being removed, a corn cob cools in the air according to the equation $\frac{dT}{dt} = -k(T - T_O)$ where t is time in minutes, T is temperature in °C and T_O is the temperature of the air, while k is a positive constant.

(i) Verify that $T = T_O + Ae^{-kt}$ is a solution of the above equation where A is a constant. 2

(ii) If the temperature of the boiling water is 100°C and that of the air is a constant 25°C, find the values of A and k if a corn cob cools to 70°C in 3 minutes. 2

(iii) How long should a person wait to enjoy the food at a temperature of 50°C? 2

(b) AC is a diameter of a circle
$AC = BC = y$ and $\angle ABC = \alpha$.



(i) Show that $\cos\alpha = \frac{x}{2y}$. 2

(ii) Determine DC in terms of x and y. 4

QUESTION 6 *(Start a new booklet)* **MARKS**

(a) The rise and fall of tides can be approximated to simple harmonic motion. On October 18 the depth of water in a tidal lagoon at low tide is 2 m at 11.00 a.m. At the following high tide at 5.20 p.m. the depth is 6 m. Calculate between what times a yacht could safely cross the lagoon if a minimum depth of 3.5 m of water is needed. **7**

(b) At a dinner party, host and hostess and seven guests sit at a round table.

(i) If there are no restrictions, in how many ways can they be arranged? **1**

(ii) In how may ways can they be arranged if host and hostess are separated? **2**

(c) At a football club a team of 13 players is to be chosen from a pool of 32 players consisting of 20 Australian-born players and 12 players born elsewhere. What is the probability that the team will consist of all Australian-born players? **2**

QUESTION 7 *(Start a new booklet)* **MARKS**

(a) A particle is moving along the x-axis. Its velocity v at position x is given by **2**

$$v = \sqrt{8x - x^2}$$

Find its acceleration when $x = 3$.

(b) A football is kicked at an angle of α to the horizontal. The position of the ball at time t seconds is given by

$x = vt \cos \alpha$

$y = vt \sin \alpha - \frac{1}{2}gt^2$

where g m/s^2 is the acceleration due to gravity and v m/s is the initial velocity of projection.

(i) Show that the equation of the path of the ball is **1**

$$y = x \tan \alpha - \frac{gx^2}{2v^2}\sec^2 \alpha .$$

(ii) Show that the maximum height h reached is given by **2**

$$h = \frac{v^2 \sin^2 \alpha}{2g}.$$

(iii) Hence show that $y = x \tan \alpha(1 - \frac{x \tan \alpha}{4h})$. **2**

(iv) If $g = 10$ m/s^2, $\alpha = 30°$ and the ball just clears the head of a player 1.6 m tall and 10 m away, calculate the maximum height reached by the ball. **1**

(c) Write down the expansion of $(1 + x)^{2n}$ and hence prove that **4**

$$\sum_{r=0}^{n} {}^{2n}C_r = 2^{2n-1} + \frac{(2n)!}{2(n!)^2}.$$

End of paper