Question 1 (15 Marks) **Marks**

(a) Draw a neat sketch of $xy = 8$, **clearly** indicating on the sketch, the coordinates of the foci, vertices, and the equations of the directrices. **4**

(b) A raindrop falls so that its velocity v m/s at time t seconds is given by

$$\frac{dv}{dt} = \frac{1}{3}(3g - 2v)$$

where g is the acceleration due to gravity.

(i) Show that $v = \dfrac{3g}{2}\left(1 - e^{-\frac{2}{3}t}\right)$. **3**

(ii) Find the limiting velocity of the raindrop in terms of g. **1**

(iii) Find the time when the velocity reaches $\dfrac{1}{2}g$ m/s. **2**

(c) The rate of increase of the population, $P(t)$, of a particular bird species at time t years is given by the equation:

$$\frac{dP}{dt} = kP(Q - P)$$

where k and Q are positive constants and $P(0) < Q$.

(i) Verify that the expression $P(t) = \dfrac{QC}{C + e^{-kQt}}$, where C is a constant, is a solution of the equation. **3**

(ii) Describe the behaviour of P as $t \to \infty$. **1**

(iii) Describe what happens to the rate of increase of the population as $t \to \infty$. **1**

Question 2 (15 Marks) **(Start a new page)**

(a) A particle of unit mass is projected vertically upwards form the ground with initial speed u m/s. If air resistance at any time t seconds is proportional to the velocity at that instant, and assuming air resistance is $-kv$,

(i) Prove that if the highest point is reached by the particle in time T seconds then **4**

$$kT = \log\left(1 + \frac{ku}{g}\right)$$

where g is the acceleration due to gravity.

(ii) If the highest point reached is at a height h metres above the ground, prove that $hk = u - gT$. **5**

Question 2 continues on the next page

Question 2 continued **Marks**

(b) The normal at a variable point $P\left(2p, \dfrac{2}{p}\right)$ on $xy = 4$, given by $y = p^2x - 2p^3 + \dfrac{2}{p}$,

meets the x – axis at Q.

(i) Find the coordinates of Q. **1**
(ii) Find the coordinates of the midpoint, M, of PQ. **2**
(iii) Hence, find the locus of M. **3**

Question 3 (15 Marks)

(a) (i) Prove that the area bounded by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is πab square units. **3**

(ii) Hence, by the **method of cylindrical shells**, find the volume of the solid formed when the area is rotated through 1 complete revolution about the line $y = b$. **3**

(b) The area enclosed by the graph of the function $y = e^{2x}$, the y – axis and the horizontal line $y = e^2$ is rotated about the y – axis.

(i) Show that the volume is given by $\Delta V = \sum_{0}^{1} 2\pi x(e^2 - e^{2x})\Delta x$. **2**

(ii) Hence, find the exact volume of the solid of revolution formed. **3**

(c) If the gradients of the tangents drawn to the curves $xy = c^2$ and $y^2 = 4ax$ at the point of intersection are m and M respectively. Prove that $m = -2M$. **4**

Question 4 (15 Marks) **(Start a new page)** **Marks**

(c) A particle moves in a straight line so that it's acceleration is inversely proportional to the square of its distance from a point O in the line and is directed towards O. It starts from rest at a distance a units from O.

(i) What is its velocity when it first reaches a distance, $\dfrac{a}{2}$ units, from O? **3**

(ii) Show that the time taken to first reach this distance in part (i) is given by **3**

$$t = \frac{(\pi + 2)a^{\frac{3}{2}}}{4\sqrt{2k}}, \text{ where } k \text{ is a constant,}$$

given that $\dfrac{d}{dx}\left[\sqrt{x(a-x)} + \dfrac{a}{2}\sin^{-1}\left(\dfrac{a-2x}{a}\right)\right] = -\sqrt{\dfrac{x}{a-x}}$

Question 4 is on the next page.

Question 4 continued **Marks**

(b) A is the area of the region R bounded by the upper branch of the hyperbola $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, the x – axis and the lines $x = \pm a$.

(i) Show that $A = \frac{Lb}{2}\left[\sqrt{2} + \ln\left(1 + \sqrt{2}\right)\right]$ square units, where L the length of the base of R is $2a$ units. **5**

(ii) S is the solid whose base is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Cross sections perpendicular to the base and to the minor axis, are plane figures similar to region R where the line of intersection of the planes is the base length of R. Find the volume of S. **4**

~ END OF TEST ~