**TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2002**

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# MATHEMATICS

## EXTENSION I

*Time Allowed – 2 Hours*
*( Plus 5 minutes reading time )*

All questions may be attempted

All questions are of equal value

In every question, show all necessary working

Marks may not be awarded for careless or badly arranged work

**Standard integral tables are included with the examination paper.**
**Approved silent calculators may be used.**

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**The answers to all questions are to be returned in separate bundles clearly labelled Question 1, Question 2 , etc. Each bundle must show your candidate number.**

## Question 1:

(a) Find the acute angle between the lines

$$2x + y = 17 \quad \text{and} \quad 3x - y = 3$$ **2**

(b) Differentiate $y = \tan^{-1}\sqrt{2x^2 - 1}$ **3**

(c) Evaluate $\displaystyle\int_0^3 \frac{y}{\sqrt{y+1}}\,dy$, using the substitution $y = u^2 - 1$ **3**

(d) Eight identical coins show 3 heads and 5 tails.

(i) In how many ways can they be arranged in a straight line? **1**

(ii) What is the probability that all the tails will be together? **1**

(e) Solve for $x$ : $\dfrac{2x-3}{x-2} \geq 1$ **2**

## Question 2: (START A NEW PAGE)

(a)

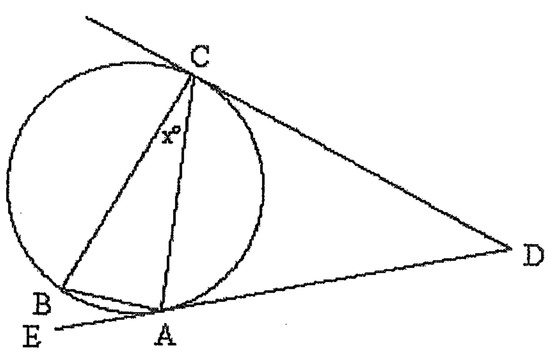


Diagram not to scale **4**

$AD$ and $CD$ are tangents to a circle. $B$ is a point on the circle such that $\angle CBA$ and $\angle CDA$ are equal and are each both double $\angle BCA$. Prove that $b$ is a diameter of the circle.

(b) The roots of the equation $9x^2 + 6x + 1 = 4kx$ where $k$ is a real constant, are $\alpha$ and $\beta$. Show that the equation with roots $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$ is **4**

$$x^2 + 6x + 9 = 4kx$$

(c) Prove by Mathematical Induction that

$$1\times 2^0 + 2\times 2^1 + 3\times 2^2 + \ldots + n\times 2^{n-1} = 1 + (n-1)2^n$$ **4**

for all integers $n \geq 1$.

**Question 3:** **(START A NEW PAGE)**

(a) The angle of elevation of a tower $PQ$ of height $h$ metres ~~at~~ from a point $A$ due east of it is $15^0$. From another point $B$, the bearing of the tower is $032^0$T and the angle of elevation is $13^0$. The points $A$ and $B$ are 500 metres apart and on the same level as the base $Q$ of the tower.

(i) Draw a neat sketch showing all the information on your diagram **1**

(ii) Show that $\angle AQB = 122^0$. **1**

(iii) Calculate the height of the tower $PQ$ to the nearest metre. **2**

(b) The speed $v$ m/s of a particle moving in a straight line is given by

$$v^2 = 64 - 16x - 8x^2$$

where the displacement from a fixed point O is $x$ metres.

(i) Find an expression for the acceleration and show the motion is simple harmonic. **2**

(ii) Find the period of the motion **1**

(iii) Find the amplitude of the motion **1**

(c) (i) Find the largest possible domain for which $f(x) = \sin^{-1}(2x+1)$ defines a function **1**

(ii) Hence find and sketch $f^{-1}(x)$, stating its domain and range. **3**

## Question 4: (START A NEW PAGE)

(a) $N$ is the number of kangaroos in a certain population at time $t$ years. The population size $N$ satisfies the equation

$$\frac{dN}{dt} = -k(N-500), \text{ for some constant } k.$$

(i) Verify that $N = 500 + Ae^{-kt}$ with $A$ constant, is a solution of the equation **1**

(ii) Initially, there are 3500 kangaroos but after 3 years there are only 3300 left. Find the values of $A$ and $k$. **2**

(iii) Find when the number of kangaroos begins to fall below 2300 **2**

(iv) Sketch the graph of the population size against time **2**

(b) An urn contains 6 cards numbered 1, 2, 3, 4, 5, 6. One card is drawn at random and a second card is drawn without the first card being replaced. Find the probability that: -

(i) the second number is 3 **1**

(ii) the larger number is 5 **2**

(iii) the larger number is even **2**

## Question 5: (START A NEW PAGE)

(a) At an air show, a Harrier Jump Jet leaves the ground 200 metres from an observer and rises vertically at the rate of 25 $m$/sec. At what rate is the observer's angle of elevation of the aircraft changing when the jet is 500 metres above the ground? **3**

Question 5 continued over page.....

(b) A chord joining the points $P(2p, p^2)$ and $Q(2q, q^2)$ on the parabola $x^2 = 4y$ passes through the point (0,-1)

(i) Find the coordinates of $M$, the midpoint of $PQ$, as a function of $m$, the gradient of the chord **3**

(ii) Show that the cartesian equation of the locus of $M$ is

$$x^2 = 2(y+1) \text{ for } |x| \geq 2.$$ **2**

(c) (i) Express $\sin x + \sqrt{3}\cos x$ in the form $A\cos(x+\alpha)$. **2**

(iii) Hence solve $\sin x + \sqrt{3}\cos x = 1$ for $0 \leq x \leq 2\pi$. **2**

**Question 6: (START A NEW PAGE)**

(a) The deck of a ship was $1\cdot4\,m$ below the level of a wharf at low tide and $0\cdot6\,m$ above wharf level at high tide. Low tide was at 8:24 am and hightide at 2:40pm. If tide's motion is simple harmonic, find the first time after low tide that the deck was level with the wharf. **4**

(b) Steven borrows \$50 000 to pay for a new car. He plans to repay the loan by making 60 equal monthly instalments. Interest is charged at the rate of $0\cdot6$ % per month on the balance owing.

(i) Show that immediately after making two monthly instalments of \$$P$, the balance owing is given by

$$\$(50\,601\cdot80 - 2\cdot006P)$$ **2**

(ii) Calculate the value of each monthly instalment **2**

(c) A particle is projected with an initial velocity of 60 $m/s$ at an angle of 45° to the horizontal. (use $g = 10ms^{-2}$)

(i) Calculate the greatest height reached by the particle. **3**

(ii) What is the speed of the particle at the greatest height? **1**

**Question 7:** **(START A NEW PAGE)**

(a) In a box, there are 10 black counters (each marked with the digit "2") and 5 white counters (each marked with digits "3") 4 counters are withdrawn one at a time, the first being replaced before the second is drawn. Find the probability that

(i) 2 blacks and 2 white counters are drawn in any order **2**

(ii) The sum of digits on the counters drawn is greater than 9 **3**

(b) (i) Show that $(1+x)^m(1-\frac{1}{x})^m = (x-\frac{1}{x})^m$ **1**

(ii) By considering the term(s) independent of $x$ in the expansion of the result from part (b) (i), justify the result: **3**

$$\binom{2002}{0}^2 - \binom{2002}{1}^2 + \binom{2002}{2}^2 - \dots + \binom{2002}{2002}^2 = -1\binom{2002}{1001}$$

(iii) Hence, or otherwise, show that: **3**

$$\sum_{k=0}^{1001}(-1)^k\binom{2002}{k}^2 = -\frac{1}{2}\binom{2002}{1001}\left[1+\binom{2002}{1001}\right].$$

**END OF PAPER**

## STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$