FORT STREET HIGH SCHOOL

YEAR 12
TRIAL HIGHER SCHOOL CERTIFICATE

2001

MATHEMATICS

EXTENSION 1

Time allowed: 2 Hours
(+ 5 Minutes Reading Time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- The marks allocated for each question are indicated.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board approved calculators may be used.
- Each new question is to be started on a new page.
- Standard integrals are included.
- If required additional paper may be obtained from the Examination Supervisor on request.

Name :____________________ Class Teacher: ____________________

Question No	1	2	3	4	5	6	7	Total	Total
Mark	12	12	12	12	12	12	12	84	100

QUESTION 1

(a) (i) Find $\frac{d}{dx}(x\ln x - x)$ **4**

(ii) Hence evaluate $\int_1^e \ln x\,dx$. Leave the answer in exact form.

(b) Solve the inequality $\frac{x}{x-2} \le 3$. **3**

(c) By using the substitution $u = x^3 + 1$, find $\int x^2\sqrt{x^3+1}\,dx$ **3**

(d) The polynomial $x^3 + 2x^2 + ax + b$ has a factor (x+2) and when divided by (x-2) there is a remainder of 12. Find a and b. **2**

QUESTION 2

(a) (i) Write down the expansion of tan(A+B) **4**

(ii) Find the exact value of $\tan\frac{7\pi}{12}$ in simplest form with rational denominator.

(b) Solve $8\cos^2 x - 8\sin^2 x = 5$ for $0° \le x \le 360°$ **3**

(c) Prove by mathematical Induction that $6^n - 1$ is divisible by 5 for $n \ge 1$ **4**

(d) Given that $\lim_{x\to 0}\frac{\sin x}{x} = 1$, show that $\lim_{x\to 0}\frac{\sin 4x}{9x} = \frac{4}{9}$ **1**

QUESTION 3

(a) A particle moves in a straight line so that its displacement x metres from the origin 0 at the time t seconds is given by $x = 10\sin\frac{t}{2}$ **5**

(i) Show that $\frac{d^2x}{dt^2} = -\frac{x}{4}$

(ii) State the amplitude and the period of the motion.

(iii) Find the maximum speed of the particle.

(b) (i) Show that the normal to the parabola $x^2 = 4ay$ at the point $(2at, at^2)$ has the equation $x + ty = 2at + at^3$ **4**

(ii) Hence show that there is only one normal which passes through its focus.

(c) Find $\int_0^{\frac{\pi}{2}} \sin^3 x \cos x\,dx$ **3**

QUESTION 4

(a) Consider the function $f(x) = 3\sin^{-1} 2x$ **7**

(i) Evaluate $f(\frac{1}{4})$.

(ii) Write down the domain and range of f(x).

(iii) Draw the graph of y=f(x) showing any key features.

(iv) Find the derivative of f(x).

$\frac{1}{2}\sin^2 x \sin^2 x$

(b) The roots α, β and δ of the equation $2x^3 + 9x^2 - 27x - 54 = 0$ are in geometric progression. **5**

(i) Show $\beta^2 = \alpha\delta$

(ii) Write down the value of αβδ.

(iii) Find α, β and δ.

QUESTION 5

(a) The acceleration of a particle is given by $\frac{d^2x}{dt^2} = \frac{-72}{x^2}$ where x metres is the displacement from the origin after 10 seconds. When t=0 the particle is 9 metres to the right of the origin with a velocity of $4m/sec$. **6**

(i) Show the velocity, v, of the particle, in terms of x is $v = \frac{12}{\sqrt{x}}$.

(ii) Find t in terms of x.

(iii) How many seconds does it take for the particle to reach a point 35metres to the right of the origin?

(b) Prove $\frac{\operatorname{cosec}^2 A}{\cot^2 A - 1} = \sec 2A$ $\operatorname{cosec}^2 - 1 = \cot^2$ **2**

(c) For the function $y = \frac{\pi}{2} - \cos^{-1}(2x)$ **4**

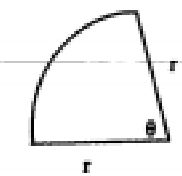
(i) State the domain and range

(ii) Find the value of y when x= 0.25

(iii) Sketch the curve of the function.

QUESTION 6

(a) The diagram below shows the sector of a circle of radius r cm and angle θ radians. The area of the sector is $25\ cm^2$ **7**



(i) show $\theta = \frac{50}{r^2}$

(ii) If P denotes the perimeter of the sector, show that $P = 2r + \frac{50}{r}$

(iii) Determine the value of r which gives the minimum perimeter

(b) Let T be the temperature inside a room at time t and let A be the constant outside air temperature. Newton's law of cooling states the rate of change of the temperature T is proportional to (T-A). **5**

(i) Show that $T = A + Ce^{kt}$ (where C and k are constants) satisfies Newton's law of cooling.

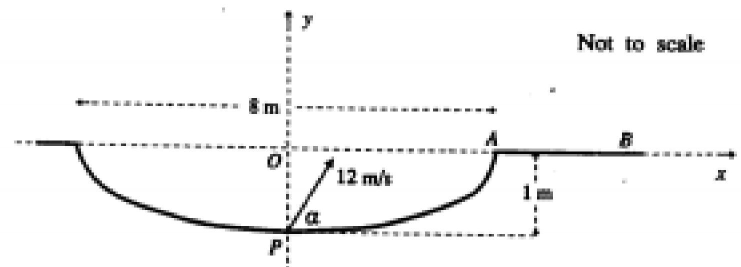
(ii) The out side air temperature is 5°C and a heating system breakdown causes the inside air temperature to drop from 20°C to 17°C in half an hour. After how many hours is the inside room temperature equal to 10°C?

QUESTION 7

(a) Find the maximum value of the function $y = e^{-x}\sin x$, where x is in radians, for the domain $0 \le x \le 2\pi$.(a full explanation is required) **3**

(b) A golf ball is lying at a point P, at the bottom of a bunker, which is surrounded by level ground. The point A is at the edge of the bunker, and the line AB lies on level ground. The bunker is 8 metres wide and 1 metre deep. **9**

The ball is hit towards A with an initial speed of 12 metres per second, and an angle of elevation α. (Have $g = 10\frac{m}{s^2}$)



(i) Show that the golf ball's trajectory at time t seconds after being hit can be defined by the equations.

$$x = (12\cos\alpha)t \quad \text{and} \quad y = -5t^2 + (12\sin\alpha)t - 1$$

Where x and y are the horizontal and vertical displacements, in metres, of the ball from the origin O as shown in the diagram.

(ii) Given $\alpha = 30°$, how far from A will the ball land?

(iii) Find the maximum height the level groung reached by the ball if $\alpha = 30°$.

(iv) Find the range of values of α, to the nearest degree, at which the ball must be hit so it will land to the right of A.