CATHOLIC SECONDARY SCHOOLS' ASSOCIATION OF NEW SOUTH WALES

YEAR TWELVE FINAL TESTS 1994

MATHEMATICS

3/4 UNIT COMMON PAPER

(i.e. 3 UNIT COURSE – ADDITIONAL PAPER: 4 UNIT COURSE – FIRST PAPER)

ternoon session

Friday 12th August 1994.

Time Allowed — Two Hours

EXAMINERS

Glenn Abrahams, Patrician Brothers' College, Fairfield
Graham Arnold, John Paul II Senior High, Marayong
Sandra Hayes, All Saints Catholic Senior High, Casula.

RECTIONS TO CANDIDATES :

L questions may be attempted.

I. questions are of equal value.

necessary working should be shown in every question.

l marks may not be awarded for careless or badly arranged work.

proved calculators may be used.

ndard integrals are printed on a separate page.

QUESTION 7

(a) An employer wishes to choose two people for a job. There are eight applicants, three of whom are women and five of whom are men.

(i) If each applicant is interviewed separately and all of the women are interviewed before any of the men, find how many ways there are of carrying out the interviews.

(ii) If the employer chooses two of the applicants at random, fir the probability that at least one of those chosen is a woman

(b) A particle moving in a straight line is performing Simple Harmonic Motion. At time t seconds its displacement x metres from a fixed point O on the line is given by $x = 2\cos^2 t$.

(i) Show that its velocity v ms^{-1} and its acceleration $\ddot{x}$ ms^{-2} are given by $v^2 = 4(2x - x^2)$ and $\ddot{x} = -4(x-1)$ respectively.

(ii) Find the centre, amplitude and period of the motion.

QUESTION 1

(a) If the positive numbers a, b, c are three consecutive terms in a geometric sequence show that $\log_e a$, $\log_e b$, $\log_e c$ are three consecutive terms in an arithmetic sequence.

(b) (i) Write down the expansion of $\cos(\alpha + \beta)$.

(ii) Write down the exact values of $\cos 30^\circ$ and $\cos 45^\circ$.

(iii) Hence find the exact value of $\cos 75^\circ$.

(c) The equation $x^3 - 2x^2 + 4x - 5 = 0$ has roots α, β, γ.

(i) Write down the values of $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$.

(ii) Hence find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$.

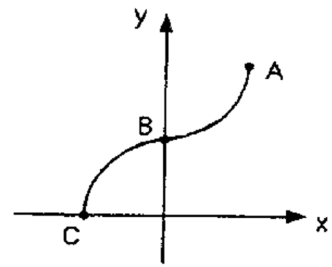
QUESTION 2

(a) (i) Find $\dfrac{d}{dx} e^{3x^2}$ $= 6x e^{3x^2}$

(ii) Hence find $\int x e^{3x^2} \, dx$ $\frac{1}{6}\{e^{3x^2}\} + c$

(b) Use the substitution $u = \log_e x$ to evaluate $\displaystyle\int_1^e \frac{(\log_e x)}{x} \, dx$

(c) The diagram below shows the graph of $y = \pi + 2\sin^{-1} 3x$



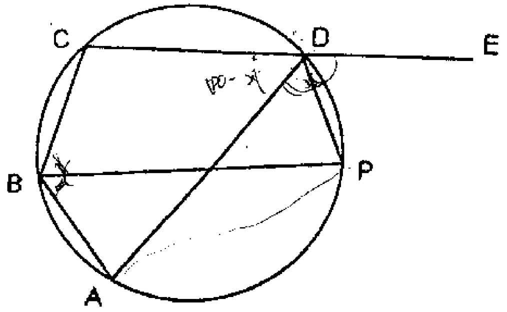
(i) Write down the coordinates of the endpoints A and C.

(ii) Write down the coordinates of the point B.

(iii) Find the equation of the tangent to the curve $y = \pi + 2\sin^{-1} 3x$ at the point B.

QUESTION 3

(a)



In the diagram above $ABCD$ is a cyclic quadrilateral. CD is produced to E. P is a point on the circle through A, B, C, D such that $\angle ABP = \angle PBC$.

(i) Copy the diagram showing the above information.

(ii) Explain why $\angle ABP = \angle ADP$. angles in the same segment

(iii) Show that PD bisects $\angle ADE$.

(iv) If, in addition, $\angle BAP = 90^\circ$ and $\angle APD = 90^\circ$, explain where the centre of the circle is located.

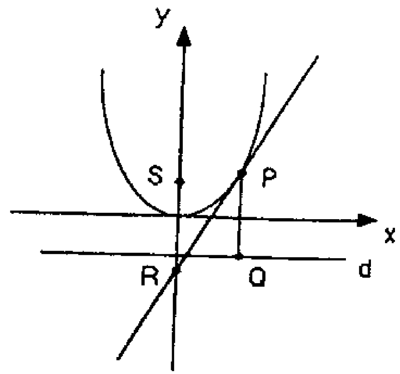
(b) For the function $y = x + e^{-x}$

(i) find the coordinates and the nature of any stationary points on the graph of $y = f(x)$ and show that the graph is concave upwards for all values of x.

(ii) sketch the graph of $y = f(x)$ showing clearly the coordinates of any turning points and the equations of any asymptotes

QUESTION 4

(a)



$P(2at, at^2)$ is a point on the parabola $x^2 = 4ay$. S is the focus of the parabola. PQ is the perpendicular from P to the directrix d the parabola. The tangent at P to the parabola cuts the axis of th parabola at the point R.

(i) Show that the tangent at P to the parabola has equation $tx - y - at^2 = 0$.

(ii) Show that PR and QS bisect each other.

(iii) Show that PR and QS are perpendicular to each other.

(iv) State with reason what type of quadrilateral $PQRS$ is.

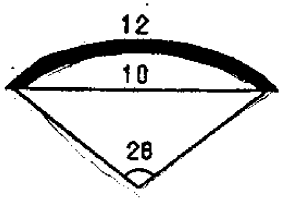
(b) In the expansion of $(1-2x)(1+ax)^{10}$ the coefficient of x^6 is 0. Find the value of a.

QUESTION 5

(a) A body is moving in a straight line. At time t seconds its displacement is x metres from a fixed point O on the line and its velocity is v ms^{-1}. If $v = \frac{1}{x}$ find its acceleration when $x = 0.5$.

$v = \frac{1}{x}$ $a = \int \frac{1}{x}\,dx$ $= \log_e x + c$ when $x = 0.5$ $a = \log_e 0.5 + c$

(b) A pipe which is 12 metres long is bent into a circular arc which subtends an angle of 2θ radians at the centre of the circle. The chord of the circle joining the ends of the arc is 10 metres long.



(i) Show that $6 \sin\theta - 5\theta = 0$.

(ii) Show that $\theta_0 = 1$ radian is a good first approximation to the value of θ.

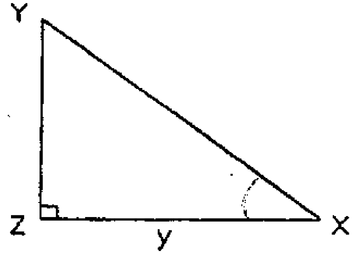
(iii) Use one application of Newton's method to find a better approximation θ_1 to the value of θ.
Use this value of θ_1 to find an approximation to the length of the radius of the arc, rounding off this approximation correct to two decimal places.

QUESTION 6

(a) (i) Write down the expression for $\tan 2a$ in terms of $\tan a$.

(ii) If $f(a) = a \cot a$ show that $f(2a) = (1 - \tan^2 a)\, f(a)$.

(b)



In ΔXYZ, $ZX = y$ and $\angle YZX = 90°$.

(i) Show that the area A and perimeter P of the triangle are given by $A = \frac{1}{2} y^2 \tan X$ and $P = y(1 + \tan X + \sec X)$ respectively.

(ii)
(α) If $X = \frac{\pi}{4}$ radians and y is increasing at a constant rate of 0.1 cm s^{-1} find the rate at which the area of the triangle is increasing at the instant when $y = 20$ cm.

(β) If $y = 10$ cm and X is increasing at a constant rate of 0.2 radians s^{-1} find the rate at which the perimeter of the triangle is increasing when $X = \frac{\pi}{6}$ radians.