2009 TRIAL MATHEMATICS SOLUTION.

Solutions	Marks/Comments

Question 1

(a)Let $x = 0{\cdot}53131..$

$10x = 5{\cdot}3131$

$100x = 53{\cdot}1313$

$1000x = 531{\cdot}3131$

$990x = 526$

$x = \frac{526}{990} = \frac{263}{495}$

$\therefore\ 3{\cdot}5\dot{3}\dot{1} = 3\frac{263}{495}$

or $3{\cdot}5\dot{3}\dot{1} = 5 + \frac{5}{10} + \frac{31}{1000} + \frac{31}{100000}....$

Limiting Sum $\frac{31}{1000} + \frac{31}{100000} + \frac{31}{10000000}....$

$a = \frac{31}{1000}$ $\qquad S_\infty = \frac{a}{1-r}$

$r = \frac{1}{100}$ $\qquad = \dfrac{\frac{31}{1000}}{1-\frac{1}{100}}$

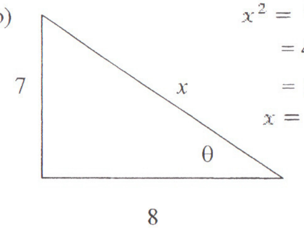
$= \frac{31}{1000} \div \frac{99}{100}$

$= \frac{31}{990}$

$\therefore\ 3{\cdot}5\dot{3}\dot{1} = 3 + \frac{5}{10} + \frac{31}{990} = 3\frac{263}{495}$

2 marks – 1 for correct method
1 correct answer

(b)



$x^2 = 7^2 + 8^2$

$= 49 + 64$

$= 113$

$x = \sqrt{113}$

Since

$\tan\theta = \frac{7}{8}$ and $\cos\theta < 0$

3rd Quadrant $\therefore$ cosec $\theta < 0$

$\therefore\ cosec\ \theta = -\frac{\sqrt{113}}{7}$

1 Mark attempt to use pythag

1 Mark – Correct Answer

(c) $|15 - 4x| \le 3$

$15 - 4x \le 3$ or $15 - 4x \ge -3$

$-4x \le -12 \qquad -4x \ge -18$

$x \ge 3$ or $\quad x \le 4\frac{1}{2}$

2 Marks – 1 for each solution

(d) $k = \frac{1}{3}m(v^2 - u^2)$

$724 = \frac{1}{3}m(14\cdot 2^2 - 7.4^2)$

$2172 = m(146{\cdot}88)$

$m = \frac{2172}{146{\cdot}88} = 14\cdot 7875817$

$= 14{\cdot}8 \quad (3sf)$

2 Marks – 1 for substitution
- 1 for answer

(e) $3y = sin\left(2x - \frac{\pi}{4}\right)$

$y = \frac{1}{3}sin\left(2x - \frac{\pi}{4}\right)$

$\therefore$ amplitude $= \frac{1}{3}$ $\qquad$ period $= \frac{2\pi}{2} = \pi$

2 marks - 1 for peri
1 for a

(f) $130\% = \$67{\cdot}50$

$1\% = \frac{67{\cdot}50}{130} = 0{\cdot}51923......$

Cost Price $= \frac{67{\cdot}50}{130} \times 100 = \$51{\cdot}92$

2 Marks - 1 for 1%
1 for Cos

Solutions	Marks/Comm

Question 2

(a) $x - 2y + 9 = 0$ -----(1)

$4x - y - 20 = 0$ -----(2)

From (2)

$y = 4x - 20$ ------(3)

Sub (3) into (1)

$x - 2(4x - 20) + 9 = 0$

$x - 8x + 40 + 9 = 0$

$-7x = -49$

$x = 7$

Hence $y = 8$

B is the point (7, 8)

2 marks – 1 for meth
1 for corr

b) $m(AC) = \dfrac{4\frac{1}{2} - 0}{0 - 5}$

$= -\frac{9}{10}$

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{9}{10}(x - 5)$

$10y = -9x + 45$

$9x + 10y - 45 = 0$

2 marks – 1 for gradi
1 for equa

c) $AC = \sqrt{(0-5)^2 + \left(4\frac{1}{2} - 0\right)^2}$

$= \sqrt{(-5)^2 + \left(4\frac{1}{2}\right)^2}$

$= \sqrt{25 + \frac{81}{4}}$

$= \frac{\sqrt{181}}{2}$

2 marks – 1 for subst
1 for answ

d) $m(BC) = \frac{8-0}{7-5} = \frac{8}{2} = 4$ $\qquad \therefore\ m\ (line) = -\frac{1}{4}$

$y - y_1 = m(x - x_1)$

$y - 4\frac{1}{2} = -\frac{1}{4}(x - 0)$

$4y - 18 = -x$

$x + 4y - 18 = 0$

2 marks – 1 for gradi
1 for equat

(e) $d = \dfrac{|9(7)+10(8)-45|}{\sqrt{9^2+10^2}} = \dfrac{|63+80-45|}{\sqrt{81+100}} = \dfrac{98}{\sqrt{181}}$

2 marks – 1 for substitution
1 for answer

Area $= \tfrac{1}{2}bh = \frac{1}{2} \times \frac{\sqrt{181}}{2} \times \frac{98}{\sqrt{181}} = 24\frac{1}{2}$ square units.

(f)
$$x - 2y + 9 \geq 0$$
$$4x - y - 20 \leq 0$$
$$9x - 10y - 45 \geq 0$$

2 marks - lose 1 mark for each incorrect

Solutions	Marks/Comments

Question 3

(a) i. $\dfrac{d}{dx}\left[3x\sqrt[5]{x}\right] = vu' + uv'$

$$= 3 \times x^{\frac{1}{5}} + 3x \times \frac{1}{5}x^{-\frac{4}{5}}$$
$$= 4x^{\frac{1}{5}}$$
$$= 4\sqrt[5]{x}$$

OR By using indices

$$3x\sqrt[5]{x} = 3x \times x^{\frac{1}{5}}$$
$$= 3x^{\frac{6}{5}}$$

Derivative $= 4\sqrt[5]{x}$

2 marks – 1 for method
1 for answer

ii. $$\frac{d}{dx}\left[\frac{\sin 2x}{e^{2x}}\right] = \frac{(e^{2x})(2\cos 2x) - (\sin 2x)(2e^{2x})}{(e^{2x})^2}$$
$$= \frac{2e^{2x}\left[\cos 2x - \sin 2x\right]}{(e^{2x})^2}$$
$$= \frac{2\left[\cos 2x - \sin 2x\right]}{e^{2x}}$$

2 marks – 1 for method
1 for answer

(b) i. $$\int \frac{dx}{e^{3x}} = \int e^{-3x}dx = -\frac{1}{3}e^{-3x} + C$$

2 marks – 1 for method
1 for answer

ii. $$\int_0^{\pi} \sec^2\frac{x}{4}\,dx = 4\left[\tan\frac{x}{4}\right]_0^{\pi}$$
$$= 4\left[\tan\frac{\pi}{4} - \tan 0\right] = 4$$

2 marks – 1 for integral
1 for answer

(c) i. $\alpha + \beta = -\frac{b}{a} = -\frac{-4}{3} = \frac{4}{3}$

$$2\alpha^2 + 2\beta^2 = 2\left(\alpha^2 + \beta^2\right)$$
$$= 2\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$$

1 mark

ii. $$= 2\left[\left(\frac{4}{3}\right)^2 - 2\left(\frac{-7}{3}\right)\right]$$
$$= 2\left[\left(\frac{16}{9}\right) + \left(\frac{14}{3}\right)\right]$$
$$= \frac{116}{9}$$

1 mark

iii. Equation with roots $2\alpha^2$ and $2\beta^2$ has equation

$$x^2 - (2\alpha^2 + 2\beta^2)x + (2\alpha^2 \times 2\beta^2) = 0$$

i.e. $$x^2 - 2[(\alpha+\beta)^2 - 2\alpha\beta]x + 4(\alpha\beta)^2 = 0$$
$$x^2 - 2\left[\left(\frac{4}{3}\right)^2 - 2\left(\frac{-7}{3}\right)\right]x + 4\left(\frac{-7}{3}\right)^2 = 0$$
$$x^2 - 2\left[\frac{16}{9} + \frac{14}{3}\right]x + \frac{196}{9} = 0$$
$$x^2 - 2\left[\frac{58}{9}\right]x + \frac{196}{9} = 0$$
$$x^2 - \frac{116}{9}x + \frac{196}{9} = 0$$
$$9x^2 - 116x + 196 = 0$$

2 marks – 1 for m
1 for an

Solutions	Marks/Comme

Question 4

(a)

i. $\angle FBD = \theta$ (Given)
$\angle DBC = 90° - \theta$
$\angle BCD = 180° - 90° - (90 - \theta)$ (angle sum of ΔBCD)
$= \theta$
$\therefore \angle FEA = \theta$ (Corresponding angles to $\angle BCD$, FE ∥ BC)

2 marks – 1 for proo
1 for reaso

ii. $\angle AFE = 90°$ (Corresponding angles FE ∥ BC)

$$\tan\theta = \frac{AF}{y}$$
$$\therefore AF = y\tan\theta$$

1 mark

iii. In ΔABD, $\cos\theta = \dfrac{z}{AF+FB}$

$$z = (AF + FB)\cos\theta$$
$$= (x + y\tan\theta)\cos\theta$$

1 mark

iv. $$z = (x + y\tan\theta)\cos\theta$$
$$= x\cos\theta + y\tan\theta\cos\theta$$
$$= x\cos\theta + y\frac{\sin\theta}{\cancel{\cos\theta}}\cancel{\cos\theta}$$
$$= x\cos\theta + y\sin\theta$$

1 mark

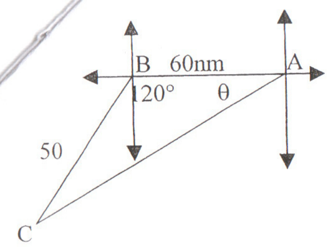
(b) $a = 500\,000000 \times 0.8$

$r = \frac{4}{5}$

$$S_\infty = \frac{a}{1-r}$$
$$= \frac{500\,000\,000 \times 0.8}{1-\frac{4}{5}}$$
$$= \frac{400\,000\,000}{\frac{1}{5}}$$

\$2 000 000 000

2 marks – 1 for subs formula
1 for an



1 for correct diagram

ii.

$C^2 = 50^2 + 60^2 - 2 \times 50 \times 60 \cos 120°$

$C = 95{\cdot}3939$.......... $= 95$ nm

2 marks – 1 for substitution
- 1 for answer

$\frac{\sin\theta}{50} = \frac{\sin 120}{95.3939..}$

$\sin\theta = \frac{50 \sin 120}{95.3939...}$

$\theta = 27°$ (nearest degree)

Bearing $= 270 - 27$
$= 243°$

2 marks – 1 for 27°
- 1 for bearing

ii. $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(\log_a 3) + (10-1)\log_a 3]$
$= 5[2\log_a 3 + 9\log_a 3]$
$= 5[11\log_a 3]$
$= 55\log_a 3$
$= \log_a 3^{55}$

1 for S_{10} in either for

(d) $4x^2 - 4x + 4y^2 + 24y + 21 = 0$

$x^2 - x + y^2 + 6y = -\frac{21}{4}$

$\left(x^2 - x + \frac{1}{4}\right) + (y^2 + 6y + 9) = -\frac{21}{4} + \frac{1}{4} + 9$

$\left(x - \frac{1}{2}\right)^2 + (y+3)^2 = 4$ Centre $\left(\frac{1}{2}, -3\right)$, Radius $= 2$

2 marks – 1 for centr
1 for radiu

Question 5

a) i. α. P(Wins first Prize) $= \frac{1}{1000}$

1 mark

β. P(At least \$500) $= \frac{2}{1000}$ or $0 \cdot 002$

1 mark

γ. P(no prizes) $= 1 - \frac{3}{1000} = \frac{997}{1000}$ or $0 \cdot 997$

1 mark

ii. P(At least \$500) $= 1 - \left(\frac{998}{1000} \times \frac{997}{999}\right)$
$= 0{\cdot}003997997$

1 mark

int
tin

b) ABCD is a parallelogram and BP = DQ
Then
AP = AB − PB
= DC − DQ
= QC
In Δs APD, BCQ
AP = QC (proven above)
AD = BC (opposite sides of parallelogram)
∠PAD = ∠QCB (opposites angles of a parallelogram)

∴ $\Delta APD \equiv \Delta BCQ$ (SAS)

∴ DP = BQ (corresponding side in congruent triangles)

3 marks – 1 for showing AP = QC
1 for proving triangles congruent
1 for DP = BQ

flex
tin

(c) i. $\log 3 + \log 9 + \log 27 + \ldots\ldots\ldots$
$\log 3 + \log 3^2 + \log 3^3 + \ldots\ldots\ldots$
$\log 3 + 2\log 3 + 3\log 3 + \ldots\ldots\ldots\ldots$

Series is Arithmetic with a common difference of log 3

2 marks – 1 for type of series
1 for reason
i.e the value of d

Question 6

(a) i. $\frac{dy}{dx} = 6(x-1)(x-2)$ $\qquad \frac{d^2y}{dx^2} = 12x - 18$
$= 6(x^2 - 3x + 2)$
$= 6x^2 - 18x + 12$
$y = \int(6x^2 - 18x + 12)\,dx$
$= 2x^3 - 9x^2 + 12x + C$
When $x = 1, y = 2$
$2 = 2(1)^3 - 9(1)^2 + 12(1) + C$
$2 = 2 - 9 + 12 + C$
0 = 3 + C
C = − 3
Equation of curve is $y = 2x^3 - 9x^2 + 12x - 3$

2 marks – 1 for integr
1 for equat
correct value of "c"

ii. $\frac{dy}{dx} = 6(x-1)(x-2)$ but $\frac{dy}{dx} = 0$ for Stationary Points
i.e. $6(x-1)(x-2) = 0$

$x = 1$ or $x = 2$
i.e. (1, 2), (2,1)

At (1, 2), $\frac{d^2y}{dx^2} < 0$ Maximum at (1, 2)

At (2, 1), $\frac{d^2y}{dx^2} > 0$ Mimimum at (2,1)

2 marks – 1 for po
1 for tes

iii. $\frac{d^2y}{dx^2} = 12x - 18 = 0$ for inflexion point
$12x = 18$
$x = 1½$ i.e. (1½, 1½)

At (1½, 1½)

x	1	1½	2
$\frac{d^2y}{dx^2}$	−	0	+

Concavity changes at $x = 1½$

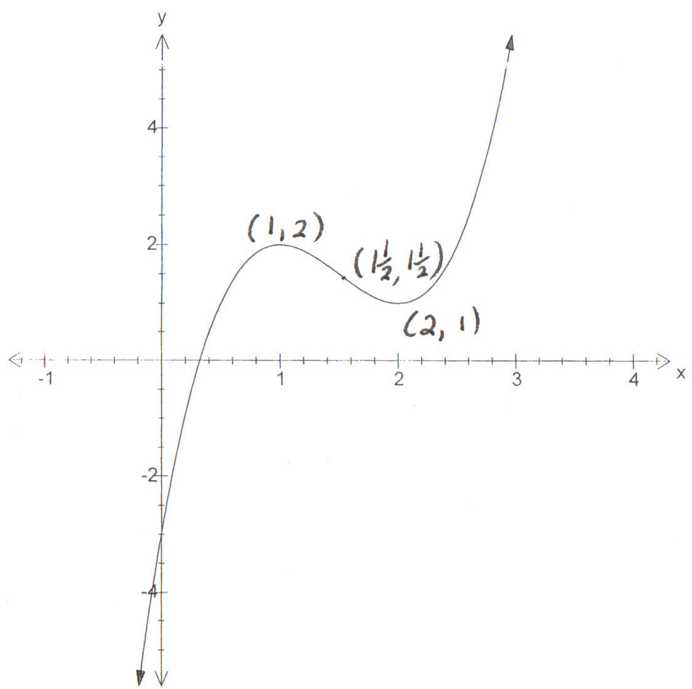
Point of inflexion at (1½, 1½)

2 marks – 1 for in
1 for tes

At $x = -1$, $y = 2(-1)^3 - 9(-1)^2 + 12(-1) - 3$
$= -26$

At $x = 3$, $y = 2(3)^3 - 9(3)^2 + 12(3) - 3$
$= 6$

At $x = 0$, $y = -3$

Solutions	Marks/Comments
	2 marks – 1 for graph 1 for labels

b) $\dfrac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta} = \tan\theta$ — **(2 marks)**

LHS $= \dfrac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta} = \dfrac{\sec^2\theta . \cot\theta}{\operatorname{cosec}^2\theta}$ — 1 mark

$= \dfrac{1}{\cos^2\theta} \cdot \dfrac{\cos\theta}{\sin\theta} \div \dfrac{1}{\sin^2\theta}$ — 0.5 mark

$= \dfrac{1}{\cos\theta \sin\theta} \times \dfrac{\sin^2\theta}{1}$

$= \dfrac{\sin\theta}{\cos\theta}$ — 0.5 mark

$= \tan\theta$ $=$ RHS

(c) $\dfrac{3 \cdot 24^2 - 2 \cdot 1^2}{\sqrt{36 + 2 \cdot 1}} = 0 \cdot 986242288$ — 1 mark attempt solut

$= 0 \cdot 986$ — 1 mark sig figs

Solutions	Marks/Comm

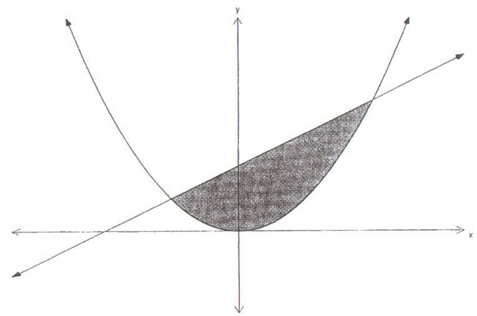
Question 7

(a) $y = x^2$ --- (1)
$y = x + 2$ --- (2)

(1) In (2) — **4 marks – 1 for each**
1 for inte
1 for ansv

$x^2 = x + 2$
$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1$ or $x = 2$

i.e. $A(-1, 1)$ $B(2, 4)$



$A = \left|\int_a^b (f(x) - g(x))\, dx\right|$
$= \left|\int_{-1}^{2} (x + 2 - x^2)\, dx\right|$
$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$
$= \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$
$= 3\frac{1}{3} - -1\frac{1}{6}$
$= 4\frac{1}{2}$ *sq units*

(b) *i.* *Angle* $= \frac{40}{60} \times 360$
$= 240 \times \frac{\pi}{180}$
$= \frac{4\pi}{3}$ — **1 mark**

ii. $l = r\theta$
$= 12\left(\frac{4\pi}{3}\right)$
$= 16\pi$ cm. — **1 mark**

$= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(12)^2\left(\frac{4\pi}{3}\right)$

$= 96\pi \ cm^2$

1 mark

) $A = \frac{h}{3}[ends + 2odds + 4\ evens]$

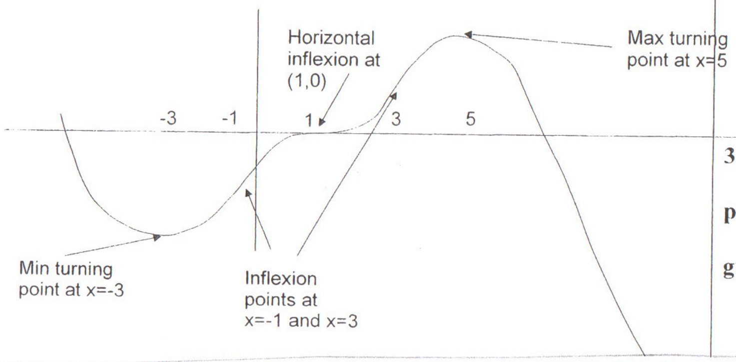
$= \frac{0.25}{3}[(3\cdot43 + 1\cdot97) + 2(0\cdot38 + 2.65) + 4(2\cdot17 + 1\cdot87 + 2\cdot31)]$

$= 3\cdot07\dot{1}\dot{6}$

$= 3\cdot1\ (1dp)$

2 marks – 1 for use of formula
1 for answer

)



3 marks – 1 for inflexions
1 for stationary points
1 for point on graph (1, 0)

Solutions	Marks/Comments

uestion 8

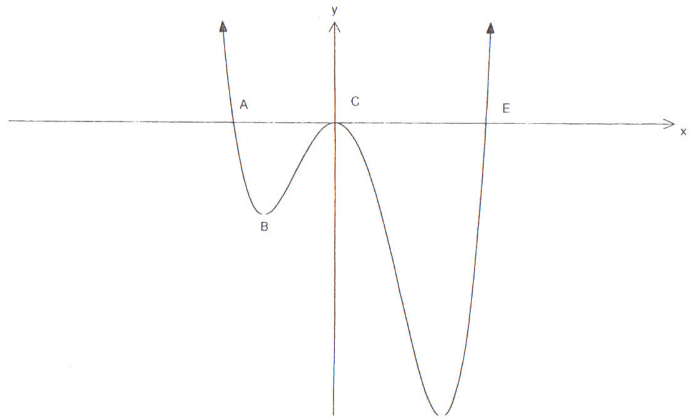
) i. B, C, D

1 mark

ii. From A to C and then from C to E

1 mark

iii.



1 mark

(b) i. Let $P be the amount repaid each month

$$A_n$ - Amount owing after n repayments

$A_1 = 15\,000 \times 1\cdot005 - P$

$A_2 = A_1 \times 1\cdot005 - P$

$= (15\,000 \times 1\cdot005 - P) \times 1\cdot005 - P$

$= 15\,000 \times 1\cdot005^2 - P(1 + 1\cdot005)$

$A_3 = A_2 \times 1\cdot005 - P$

$= [15\,000 \times 1\cdot005^2 - P(1 + 1\cdot005)] \times 1.005 - P$

$= 15\,000 \times 1\cdot005^3 - P(1\cdot005 + 1\cdot005^2) - P$

$= 15\,000 \times 1\cdot005^3 - P(1 + 1\cdot005 + 1\cdot005^2)$

$= 15226\cdot13 - P(3\cdot015025)$

2 marks - 1 for wor
1 for pro

ii. $A_{24} = 15\,000 \times 1\cdot005^{24} - P(1 + 1\cdot005 + \ldots\ldots\ldots + 1\cdot005^{23})$

but $A_{24} = 10\,000$

$\therefore 10\,000 = 15\,000 \times 1\cdot005^{24} - P(1 + 1\cdot005 \ldots\ldots + 1\cdot005^{23})$

$P(1 + 1\cdot005 \ldots\ldots + 1\cdot005^{23}) = 15\,000 \times 1\cdot005^{24} - 10\,000$

$P = \frac{15\,000 \times 1\cdot005^{24} - 10\,000}{(1 + 1\cdot005 \ldots\ldots + 1\cdot005^{23})}$

GS with a = 1, r = 1·005, n = 24

$S = \frac{1(1\cdot005^{24} - 1)}{1\cdot005 - 1} = \frac{1\cdot005^{24} - 1}{0\cdot005}$

$= 15\,000 \times 1\cdot005^{24} - 10\,000 \div \frac{1\cdot005^{24} - 1}{0\cdot005}$

$= \frac{(15\,000 \times 1\cdot005^{24} - 10\,000) \times 0\cdot005}{1\cdot005^{24} - 1}$

$= \$271\cdot60$

3 marks - 1 for wor
- 1 for su
- 1 for an

(c) $2x = y^2 - 8y + 4$

$y^2 - 8y = 2x - 4$

$y^2 - 8y + 16 = 2x - 4 + 16$

$(y-4)^2 = 2x + 12$

$(y-4)^2 = 2(x+6)$

Vertex $= (-6, 4)$

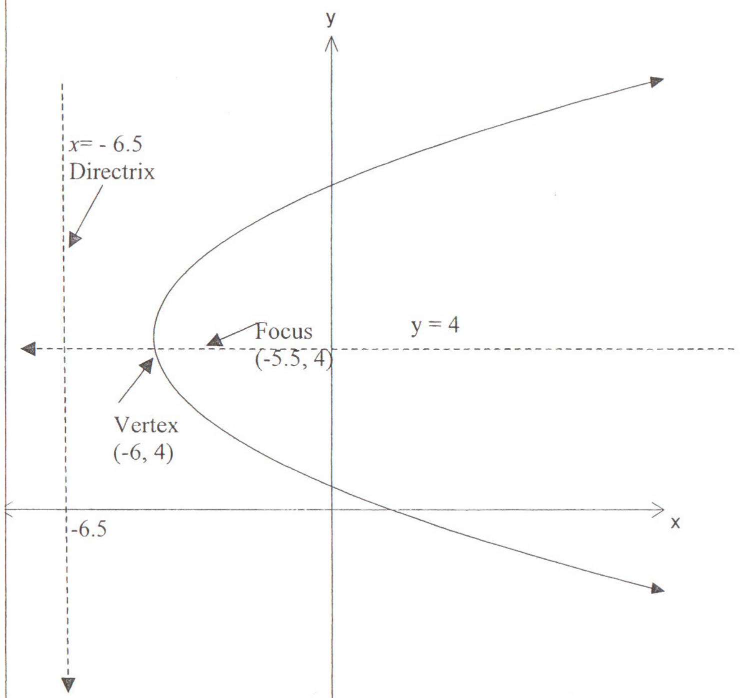
$4a = 2$

$a = \frac{1}{2}$

Focus $= \left(-5\frac{1}{2}, 4\right)$

Directrix : $x = -6\frac{1}{2}$

4 marks – 1 for sket
1 for vert
1 for focu
1 fo dire



Solutions	Marks/Comments

Question 9

(a) $x = 2\sin 2t$

$\dot{x} = 4\cos 2t$

$\ddot{x} = -8\sin 2t$

i. $t = 0 \quad \dot{x} = 4\cos 2(0)$

$= 4 \times 1$

$= 4 \text{ m / s}$

1 mark

ii. $t = \frac{\pi}{12} \quad \ddot{x} = -8\sin 2\left(\frac{\pi}{12}\right)$

$= -8\sin\left(\frac{\pi}{6}\right)$

$= -8 \times \frac{1}{2}$

$= -4 \text{ m / s}^2$

1 mark

iii. $\dot{x} = 0 \quad then\ 4\cos 2t = 0$

i.e.

$\cos 2t = 0$

$2t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots\ldots$

$t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots\ldots$

2 marks – 1 for working
1 for answer

iv. $x = 2\sin 2t$

$= 2\sin 2\left(\frac{\pi}{4}\right)$

$= 2$

$\therefore x = \pm 2 \text{ m}$

2 marks - 1 for wo
1 for answ

(b) $V = \pi\int_a^b [f(x)]^2\, dx$

$= \pi\int_1^3 \left(\sqrt{\frac{2x}{3x^2-1}}\right)^2 dx$

$= \pi\int_1^3 \frac{2x\, dx}{3x^2-1}$

$= \frac{1}{3}[\ln(3x^2 - 1)]_1^3$

$= \frac{1}{3}[\ln(3\times 3^2 - 1) - \ln(3 \times 1^2 - 1)]$

$= \frac{1}{3}(\ln 26 - \ln 2)$

$= \frac{1}{3}\left(\ln\frac{26}{2}\right)$

$= \frac{\pi}{3}(\ln 13)$ units^3

3 marks – 1 use of fo
1 for Integ
1 for answ

(c) i. $\frac{dV}{dt} = \frac{1}{100}(30t - t^2)$

when $t = 15$

$\frac{dV}{dt} = \frac{1}{100}[30(15) - (15)^2]$

$= \frac{225}{100}$

$= 2\frac{1}{4} \text{ cm}^3 / \text{min}$

1 mark

ii. $V = \int_0^{15} \frac{1}{100}(30t - t^2)$

$= \frac{1}{100}\left[15t^2 - \frac{t^3}{3}\right]_0^{15}$

$= \frac{1}{100}\left\{\left[15(15)^2 - \frac{15^3}{3}\right] - [0]\right\}$

$= \frac{1}{100}[3375 - 1125]$

$= \frac{1}{100}(2250)$

$= 22{\cdot}5 \text{ cm}^3$

2 marks – 1 for integ
1 for answ

Solutions	Marks/Comments
...tion 10	
a) i. $SA = \pi r^2 + 2\pi r h = 300$ $2\pi r h = 300 - \pi r^2$ $h = \frac{300 - \pi r^2}{2\pi r}$ $V = \pi r^2 h$ $= \pi r^2 \left(\frac{300 - \pi r^2}{2\pi r}\right)$ $= 150r - \frac{\pi r^3}{2}$	**2 marks 1 for "h"** **1 for "V"**
ii. $V = 150r - \frac{1}{2}\pi r^3$ $\dot{V} = 150 - \frac{3}{2}\pi r^2$ $\ddot{V} = -3\pi r$ which is less than 0 for positive r Stat Pts when $\dot{V} = 0$ i.e. $150 - \frac{3}{2}\pi r^2 = 0$ $150 = \frac{3}{2}\pi r^2$ $100 = \pi r^2$ $r^2 = \frac{100}{\pi}$ $r^2 = \pm\sqrt{\frac{100}{\pi}}$ Now max Volume when $r > 0$ i.e. $r = \sqrt{\frac{100}{\pi}} = 5{\cdot}641895835$ $V = 150\sqrt{\frac{100}{\pi}} - \frac{\pi}{2}\left(\sqrt{\frac{100}{\pi}}\right)^3 = 564{\cdot}1895835$ $= 564 \text{ m}^3$ (nearest m^3)	**4 marks – 1 for differentials** **1 for value of 'r'** **1 for test** **1 for max volume**
(b) i. $\frac{dP}{dt} = kP \quad \therefore P = P_0 e^{kt}$ When $t = 0$, $P = 20\,000$, $\therefore P_0 = 20\,000$ So $P = 20\,000e^{kt}$ When $t = 2$, $P = 25\,000$ $25\,000 = 20\,000e^{2k}$ $\frac{5}{4} = e^{2k}$ $\ln\left(\frac{5}{4}\right) = 2k$ $k = \ln\left(\frac{5}{4}\right) \div 2$ $k = 0{\cdot}111571775$ $\therefore P = 20\,000e^{0{\cdot}111571775\,t}$	**3 marks – 1 for value of 'k'** **1 for equation** **1 population**

When $t = 10$ $P = 20\,000e^{0{\cdot}111571775\,(10)}$ $= 61\,000$ people (nearest 100)	
ii. $\frac{dP}{dt} = 61\,000 \times 0 \cdot 111571775$ $= 6\,806$ people / year	**1 for rate of change**
(c) $\log_a 2 + 2\log_a x - \log_a 6 = \log_a 3$ $\log_a 2 + \log_a x^2 - \log_3 6 = \log_a 3$ $\text{Log}_a \frac{2x^2}{6} = \log_a 3$ $\therefore \frac{2x^2}{6} = 3$ $2x^2 = 18$ $x^2 = 9$ $x = \pm 3$ going back to original equation, cannot have log (-3) so $x = 3$	**2 marks – 1 manip...** **logs** **1 for an...**