EXT I 2006 TERM 1

QUESTION ONE - (Start a new page) **marks**

(a) Differentiate $y = \ln(\cos^2 x)$ **2**

(b) The sides of a cube are decreasing at a constant rate of 2.5cms^{-1}. Find the rate at which the volume of the cube is changing when the sides are 15cm. **2**

(c) Show that $k(4k+1)^{-1} + (4k+1)^{-1}(4k+5)^{-1} = (k+1)(4k+5)^{-1}$ **2**

(d) Find the exact sum of the first twenty terms of the series: **3**

$$\log_a 4 + \log_a 16 + \log_a 64 + \ldots$$

QUESTION TWO - (Start a new page)

(a) Solve:

(i) $\cos^2 x - \sin 2x = 0$ for $0 \le x \le 2\pi$ **3**

(ii) $1 = 2\log_{10} x - \log_{10}(\frac{x}{10} + 24)$ **3**

(b) Differentiate $3xe^x$ with respect to x, and hence or otherwise evaluate $\int_0^2 xe^x \, dx$. **3**

QUESTION THREE - (Start a new page)

(a) Water is pouring into a cone shaped funnel at a constant rate of $36\text{cm}^3\text{s}^{-1}$. If the diameter of the funnel is ¾ of its height, find the rate at which the depth of water is increasing when the height is 12cm. Give your answer correct to three sig. figures. **3**

(b) A dinghy is being pulled towards a wharf at a constant rate of 15m per minute. The rope is tied to the dinghy and the dinghy is 5m below the wharf. Find the rate at which the:

(i) rope is being drawn in when the dinghy is 12m from the wharf. **3**

(ii) the angle between the rope and wharf is changing when the dinghy is 12m from the wharf. **3**

QUESTION FOUR - (Start a new page)

(a) If $n! > 2^n$ for all integer values of n greater than 3, prove that $(n+1)! > 2^{n+1}$ **3**

(b) Given that $\int_0^k \frac{3x^2}{x^3+3}\,dx = \ln 10$, find the exact value of k. **2**

(c) Prove by Mathematical Induction, that $n^3 + 2n$ is divisible by 3, for all positive integers n. **4**

QUESTION FIVE - (Start a new page)

Dwayne borrows \$200 000 which is to be repaid in equal monthly repayments of $\$x$ over 20 years. If interest is charged at 6% p.a. calculated monthly on the balance outstanding, find:

(a) The amount owing after the first repayment. **1**

(b) The amount of each monthly repayment to the nearest dollar. **3**

(c) How long it would take to repay the same loan if Dwayne pays an extra \$100 every month from the very start? **3**

(d) Assuming Dwayne makes the extra repayments of \$100, and after 5 years he wins \$50 000, can he pay out the balance of the loan? If not, how much more does he owe? **2**

QUESTION SIX - (Start a new page)

(a) Express $0.\dot{5}\dot{0}$ as a geometric series and hence convert $0.\dot{5}\dot{0}$ to a rational number in its simplest form. **2**

(b) Use Simpson's Rule with 5 functional values, to find the approximate area under the curve $y = \sin(e^{2x})$, the x-axis and the lines $x = 1$ to $x = 3$. Give your answer correct to two decimal places. **4**

QUESTION SIX - continued

(c) Find the exact volume of the solid of revolution when the area under the curve $y = \cos 3x$, from $x = 0$ to $x = \frac{\pi}{6}$ is rotated about the x-axis. **3**

QUESTION SEVEN - (Start a new page)

(a) Given that the sum of the infinite geometric series $1 + 2^n + 2^{2n} + \ldots\ldots$ is 2. Find the exact value of n. **2**

(b) Find $\int_0^1 e^{\ln 4x}\,dx$. **2**

(c) MON is a quadrant of a circle centre O and radius 20cm. P is a point on the arc MN rotating about O at a constant rate, moving from M to N in 15 minutes. A is the total area of ΔOMP and ΔONP in cm^2.

(i) Show that $A = 200(\sin\theta + \cos\theta)$, where θ is the angle MOP. **1**

(ii) Find the exact rate at which A is changing when $\theta = \frac{\pi}{6}$. **4**

END OF PAPER