QUESTION 1 [12 Marks] **Marks**

(a) Differentiate the following:

(i) $f(x) = \cos^{-1} 2x$ 1

(ii) $y = \ln(\tan^{-1} x)$ 2

(b) Find $\int \cos^2 2x dx$ 3

(c) Find $\lim_{x \to 0} \dfrac{x^2}{2 - 2\cos 2x}$ 3

(d) $\int_0^1 \dfrac{dx}{x^2 + 3} = ap$ Find the exact value of a 3

QUESTION 2 [12 Marks] **Marks**

(a) (i) Graph accurately the curve $y = \dfrac{2}{x-1}$ 3

(ii) Hence, solve $\dfrac{2}{x-1} \geq -1$ 2

(b) The interval PQ has endpoints P(2,3) and Q(-3,5). Find the coordinates of the point T, which divides the interval PQ externally in the ration 3:1. 2

(c) Find the general solution of $\tan 3J = 1$ 2

(d) A particle is moving in simple harmonic motion. Its displacement x at time t is given by $x = 3\sin(2t - \dfrac{p}{4})$.

(i) Find the period of the motion. 1

(ii) Find the velocity of the particle when t=0 2

QUESTION 3 [12 Marks] **Marks**

(a) A particle is moving along the x-axis. Its velocity, v m/s at position x metres is given by 2

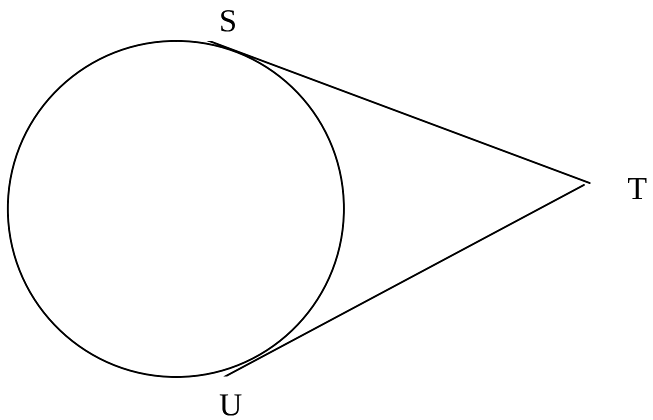
$$v = \sqrt{3x - x^2}$$

Find the acceleration of the particle when x = 5

(b) $Q(x) = x^3 + ax^2 + 2x + b$. Given that Q($x$) has a factor of ($x$+3) and when Q($x$) is divided by ($x$-1) the remainder is 4. 3

Find the values of a and b.

(c)



S and U are points on a circle. The tangents to the circle at S and U meet at T. R are a point on the circle so that the chord SR is parallel to UT.

(i) Draw a neat sketch showing the given information. 1

(ii) Prove that SU=UR 3

(d) Find the ratio of the 5th term to the 8th term in the expansion $(2x+3)^{10}$ when $x=\frac{1}{2}$ 3

QUESTION 4 [12 Marks]

Marks

(a) Using the substitution $x=1-u^2$, find $\int \frac{xdx}{\sqrt{1-x}}$ 3

(b) Consider the function $f(x)=\frac{1}{2}\sin^{-1}x$.

(i) State the domain and range of the function. 2

(ii) Find the area of the region bounded by the curve, the *x*-axis and the line *x*=1. 3

(c) Show that the constant term in the expansion $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$ is $\frac{{}^9C_6}{6^3}$ 4

QUESTION 5 [12Marks]

Marks

(a) Solve for $0 \le q \le p$, $\cos q + 3\sin\frac{q}{2} - 2 = 0$ 3

(b) Homer Simpson borrows \$15 000 at 11.95% per annum reducible interest, calculated monthly. The loan is to be repaid in 60 monthly instalments of \$333.30 at the end of the month.

The amount A_n, of the loan remaining after *n* months is given by

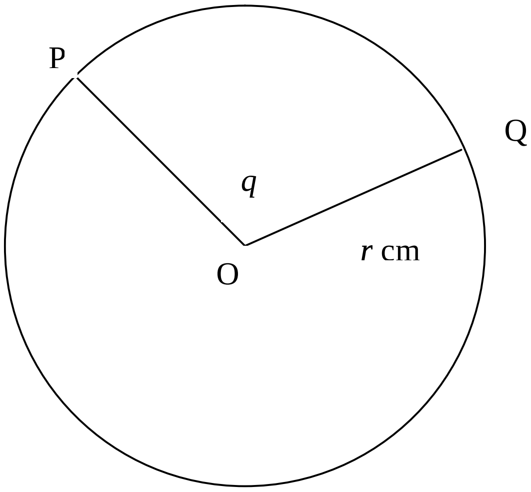
$A_n = MR^n - \$333.30(\frac{R^n-1}{R-1})$, where *M* is the principle amount borrowed.

(i) Find the exact value of *R*. 1

(ii) After 2 years, Homer inherits \$1500 and wishes to pay this towards his loan. By how many months is the term of his loan reduced, by paying this extra amount? 3

QUESTION 5 continued

(c)



A sector with centre O and radius r cm, is bounded by radii OP and OQ and arc PQ. $\angle$POQ is q radians.

(i) Given that r and q vary in such a way that the area of the sector POQ is always equal to 50 cm^2, show that $q = \dfrac{100}{r^2}$. 2

(ii) Given also that the radius is increasing at a constant rate of 0.5cm/s, finds the rate at which the angle POQ is decreasing when r=10 cm. 3

QUESTION 6 [12 Marks]

Marks

(a) (i) Find the equation of the tangent to the parabola $x = 2at$, $y = at^2$ at the point P where $t = p$. 1

(ii) If the point Q is the point where $t = q$, and O is the origin, show that if OQ is parallel to the tangent, then $q = 2p$. 1

(iii) If M is the midpoint of PQ, find the equation of the locus of M as P and Q vary long the parabola such that OQ remains parallel to the tangent at P. 4

(b) Using the principles of mathematical induction, prove that 4
$\ln[(n+2)!] > n+2$, for $n \geq 4$

(c) At a referendum, 30% of parents were in favour of a new uniform logo. An SRC member approached 8 parents chosen at random. 2

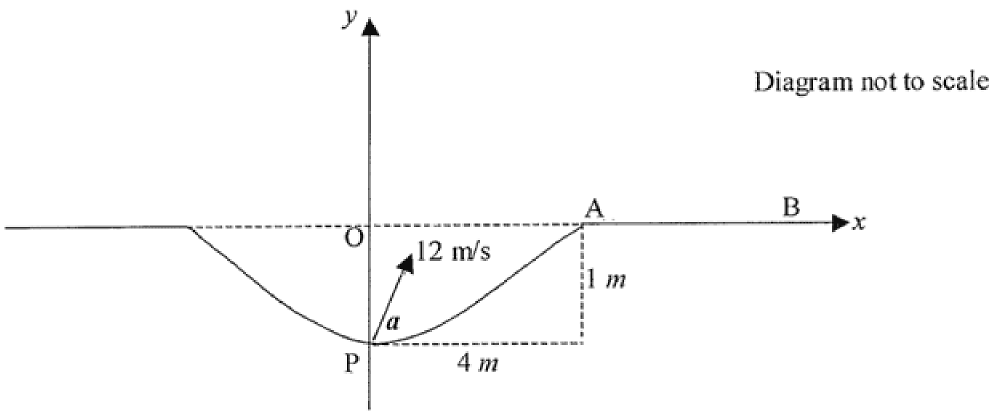
Find the probability that from this group, exactly 3 parents voted in favour of the logo.

Question 7 [12 Marks]

Marks

(a) All the letters of the word **ENCUMBRANCE** are arranged in a line. Find the total number of arrangements, which contain all the vowels in alphabetical order but separated by at least one consonant. 4

(b) A golf ball is lying at point P, at the middle of a sand bunker, which is surrounded by level ground. The point A is at the edge of the bunker and the line AB lies on the level ground. The bunker is 8 metres wide and 1 metre deep.



The ball is hit towards A with an initial speed of 12m/s and angle of elevation α. (You may assume that the acceleration due to gravity is 10m/s^2)

The golf ball's trajectory at time t seconds after being hit may be defined by the equations $x = (12\cos a)t$ and $y = -5t^2 + (12\sin a)t - 1$ where x and y are the horizontal and vertical displacements, in metres, of the ball from the origin O, shown in the diagram.

(i) If $a = 30^o$, how far to the right of A will the ball land? (Give your answer correct to 0.1m) 4

(ii) Find the range of values of α, to the nearest degree, at which the ball must hit so that it will land to the right of A 4