Neap:

HSC Trial Examination 2009

Mathematics

This paper must be kept under strict security and may only be used on or after the morning of Monday 10 August, 2009 as specified in the Neap Examination Timetable.

General Instructions

Reading time – 5 minutes

Working time – 3 hours

Write using black or blue pen

Board-approved calculators may be used

A table of standard integrals is provided at the back of this paper

All necessary working should be shown in every question

Total marks – 120

Attempt questions 1–10

All questions are of equal value

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2009 HSC Mathematics Examination.

Total marks 120
Attempt Questions 1–10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

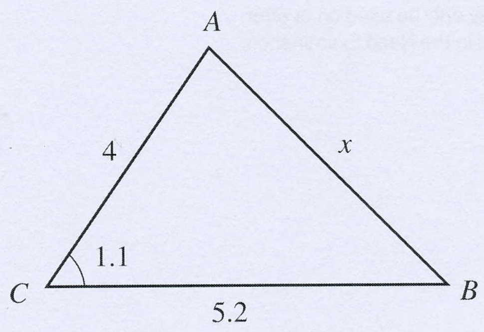
Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Calculate the exact value of $t^2 - t$ when $t = 2\sqrt{3} - 1$. **2**

(b) Simplify $\dfrac{25a^2 - 1}{2 - 10a}$. **2**

(c) Simplify $\log_{10}20A - \log_{10}2A$. **2**

(d) In ΔABC, $AB = 4$ cm, $BC = 5.2$ cm and $\angle ACB = 1.1$ radians. **3**



Calculate the length of AB correct to 1 decimal place.

(e) Jane is going to invest \$20 000 for three years. **3**

Which option will produce the greater interest? Use calculations to justify your answer.

Option 1: Monthly compounding interest at 6% p.a.

Option 2: Annually compounding interest at 6.2% p.a.

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Marks

Question 2 (12 Marks) Use a SEPARATE writing booklet.

(a) Determine an expression for each indefinite integral.

(i) $\int 6e^{2x}dx$ **1**

(ii) $\int \sec^2 \pi x dx$ **1**

(iii) $\int (x^2+1)^2 dx$ **2**

(b) Find the equation of the tangent to the curve $y = \log_e x$ at the point where $x = e^3$. **3**

(c) (i) Show that $6y = x^2 - 10x + 13$ can be expressed as $6y + 12 = (x-5)^2$. **1**

(ii) What is the focal length of the parabola $6y + 12 = (x-5)^2$? **1**

(iii) Determine the coordinates of the vertex and focus of the parabola $6y = x^2 - 10x + 13$. **2**

(iv) Hence determine the range of the function $6y = x^2 - 10x + 13$. **1**

Marks

Question 3 (12 Marks) Use a SEPARATE writing booklet.

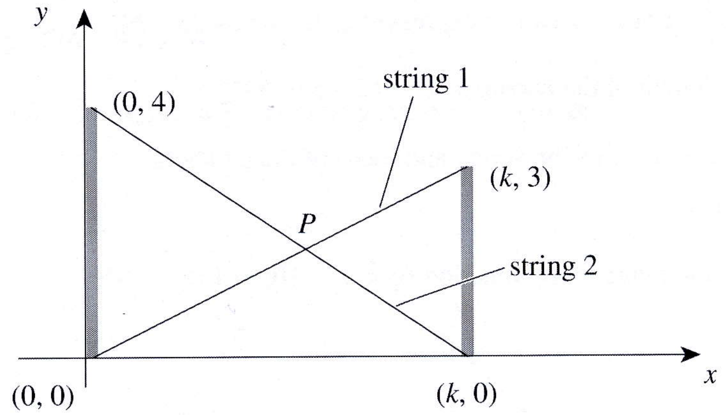
(a) Differentiate the following with respect to x.

(i) $y = 8x - x^2$ **1**

(ii) $y = \cos 2x$ **1**

(iii) $y = \dfrac{2}{x}$ **1**

(b) Aidan has two poles. One is 4 metres long and the other 3 metres long. He positions the poles vertically, k metres apart, on level ground. Aidan joins the top of each pole to the bottom of the other pole with string.



The diagram shows the poles on a coordinate plane.

(i) Determine an expression for the total length of string required to join the top of each pole to the base of the other pole. Ignore the length of any string required to join the string to the poles. **2**

(ii) Show that the equation of the line representing string 1 is $ky = 3x$. **1**

(iii) Determine the equation of the line representing string 2. **2**

(iv) Show that the coordinates of point P, the point where the two pieces of string cross, are $\left(\dfrac{4k}{7}, \dfrac{12}{7}\right)$. **2**

(v) If Aidan places the poles further apart,

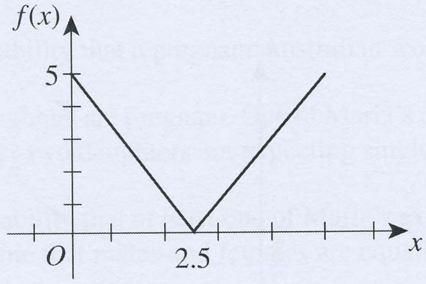
(α) by how much will the height of P above the ground change? Justify your answer. **1**

(β) will the horizontal distance from point P to the 3 m pole increase or decrease? Justify your answer. **1**

Marks

Question 4 (12 Marks) Use a SEPARATE writing booklet.

(a)



The diagram shows the graph of $f(x) = |5 - 2x|$.

(i) Use the graph to determine the value of $f'(4)$. **1**

(ii) Calculate the value of $\int_0^4 |5 - 2x|\,dx$. **2**

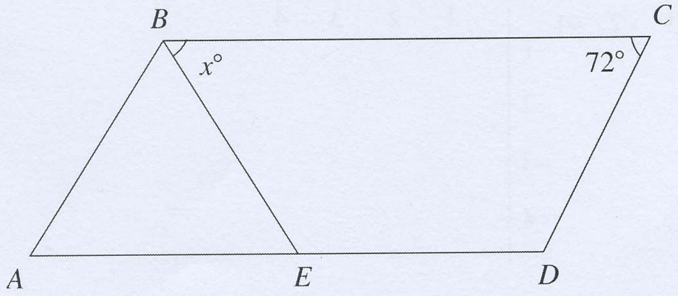
(b) Consider the line $y = 6x - k$ and the parabola $y = x^2$.

(i) For what value of k is the line $y = 6x - k$ a tangent to the parabola $y = x^2$? **2**

(ii) The line $y = 6x - k$ intersects the parabola in two distinct places. What is the largest **integer** value that k can take? **2**

(c) Solve the inequality $x^2 \leq 6 - x$. **2**

(d)



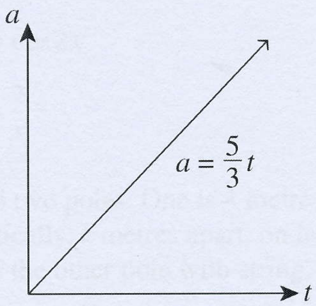
$ABCD$ is a parallelogram and $\angle BCD = 72°$. ΔABE is isosceles with $AB = BE$.

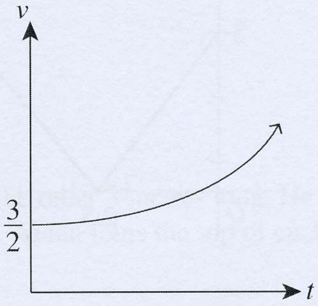
Calculate the size of $\angle CBE$, the angle marked $x°$. **3**

Marks

Question 5 (12 Marks) Use a SEPARATE writing booklet.

(a) The graphs below describe the flight of a sugar glider after it jumps from a tall tree. Assume that the displacement at time $t = 0$ is 0 m.

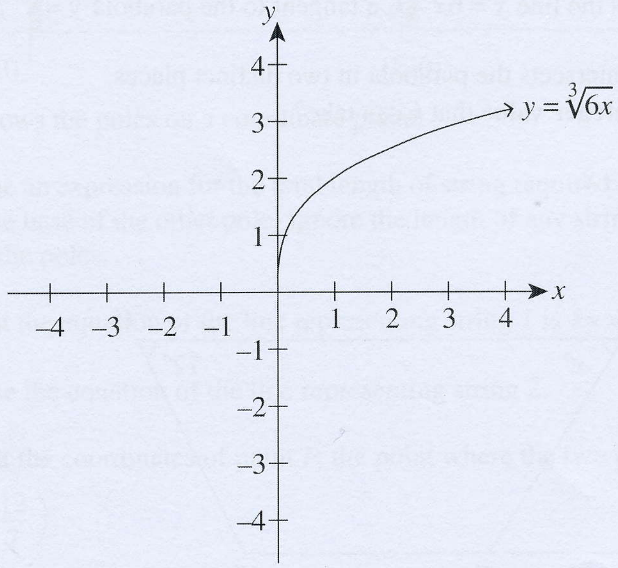




(i) Show that the velocity of the sugar glider for $t \geq 0$ is given by $v = \frac{1}{6}(5t^2 + 9)$. **2**

(ii) Determine the expression for the displacement (s) for $t \geq 0$ and hence calculate the displacement of the sugar glider at $t = 2$. **3**

(b) The diagram shows the graph of $y = \sqrt[3]{6x}$ for $x \geq 0$.



Calculate the volume of the solid formed when the section of the curve $y = \sqrt[3]{6x}$ between $y = 1$ and $y = 3$ is rotated about the y-axis. **3**

(c) (i) Show that $\frac{d}{dx}(x\log_e x) = \log_e x + 1$. **1**

(ii) Hence evaluate $\int_1^e (\log_e x)\,dx$. **3**

Marks

Question 6 (12 Marks) Use a SEPARATE writing booklet.

(a) The probability that any pregnant Australian woman will have twins is $\frac{1}{60}$.

(i) What is the probability that a pregnant Australian woman will not have twins? **1**

(ii) Maria's three daughters are pregnant. One of Maria's daughters is pregnant with twin boys and the other two daughters are expecting single babies of unknown sex.

What is the probability that at least one of Maria's expected four grandchildren will be female? Assume that males and females are equally likely. **2**

(b) Consider the curve $y = x^3 - 6x^2 + 5$ in the domain $-1 \le x \le 7$.

(i) Determine the coordinates of any stationary points and determine their nature. **4**

(ii) Sketch the curve $y = x^3 - 6x^2 + 5$ in $-1 \le x \le 7$. **2**

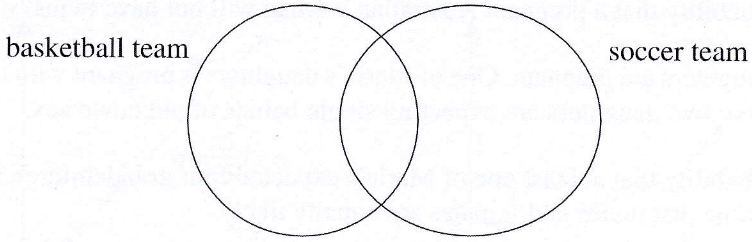
(iii) For what values of x is the curve $y = x^3 - 6x^2 + 5$ concave down? **1**

(iv) What is the maximum and minimum value of $x^3 - 6x^2 + 5$ when $-1 \le x \le 7$? **2**

Marks

Question 7 (12 Marks) Use a SEPARATE writing booklet.

(a) The Wolves are a basketball team and the Hammers are a soccer team. A total of 18 people are members of one or the other or both teams. The soccer team has 14 members and the basketball team has 10 members. The diagram represents the players in the teams.



(i) Copy the diagram and place appropriate numbers in each section. **1**

(ii) A player is selected at random from the basketball team.

What is the probability that the player is **not** in the soccer team? **1**

(b) (i) Show that $\sqrt{7} + \sqrt{28} + \sqrt{63} + \ldots$ is an arithmetic series. **1**

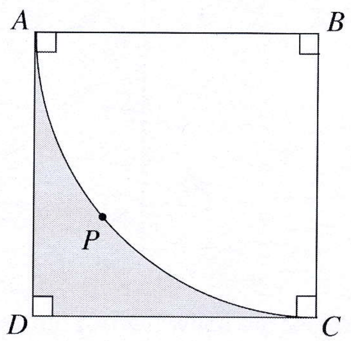
(ii) $\sqrt{7} + \sqrt{28} + \sqrt{63} + \ldots + p = 300\sqrt{7}$.

How many terms are in the sequence? **2**

(c) A golf ball is dropped from a height of one metre. Each time it hits the ground it bounces to two-thirds of its previous height.

Calculate the distance that the golf ball travels before it comes to rest. **3**

(d) $ABCD$ is a square with side length of 1 unit. P is a point on arc AC of the circle centre at B.



Show that the area of the region bounded by AD, DC and arc APC (the shaded region in the diagram) is $\dfrac{4-\pi}{4}$. **2**

(e) Sketch and label a function on the number plane whose area between the curve and the x-axis can be represented by the following: **2**

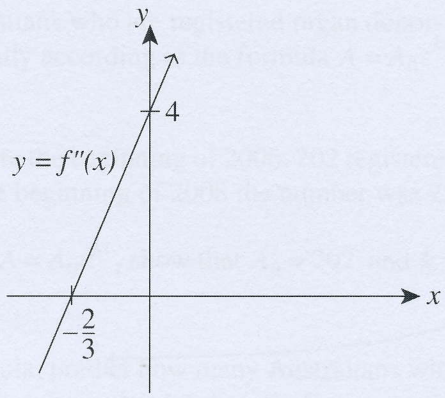
$$\text{area} = \int_a^b f(x)\,dx + \int_c^d f(x)\,dx - \int_b^c f(x)\,dx$$

Marks

Question 8 (12 Marks) Use a SEPARATE writing booklet.

(a) Solve the equation $2^x + 16 \times 2^{-x} = 17$. **3**

(b) The diagram shows the graph of $y = f''(x)$.



The point (2, 4) lies on the curve $y = f(x)$ and at this point on the curve $f'(x) = 1$.

(i) Determine the equation of the line $y = f''(x)$ and hence the equation of the curve $y = f'(x)$. **3**

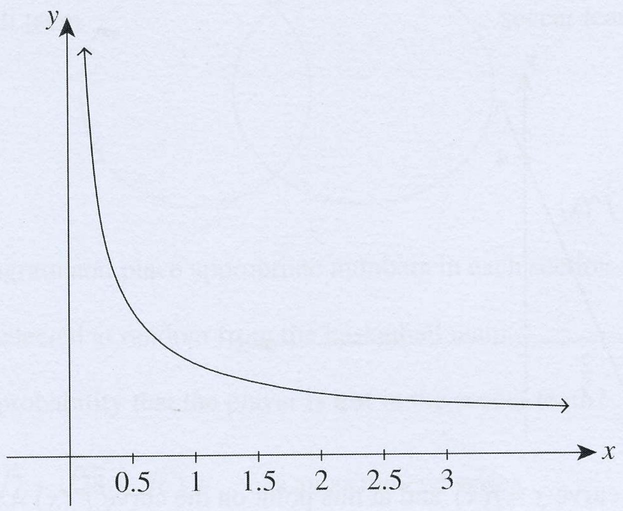
(ii) What is the equation of the curve $y = f(x)$? **2**

Question 8 continues on page 10

Marks

Question 8 (continued)

(c) The diagram shows part of the graph of the function $y = \dfrac{1}{\log_e(2x+1)}$.



(i) Copy and complete the table of values for $y = \dfrac{1}{\log_e(2x+1)}$ with each value correct to 3 decimal places. **1**

x	0.5	1.0	1.5	2.0
y				

(ii) Use the trapezoidal rule with the four y-values in the table to approximate $\displaystyle\int_{0.5}^{2} \frac{1}{\log_e(2x+1)}\,dx$. **2**

(iii) Is the value obtained using the trapezoidal rule greater than or less than the exact value of the integral? Use a diagram to justify your answer. **1**

End of Question 8

Marks

Question 9 (12 Marks) Use a SEPARATE writing booklet.

(a) (i) Sketch the graphs of $y = \sin 2x$ and $y = \cos 2x$ for $0 \le x \le \pi$. Give the x values where the curves intersect. **2**

(ii) Hence find all solutions to the inequality $\sin 2x > \cos 2x$ for $0 \le x \le \pi$. **1**

(b) The number of Australians who are registered organ donors when they die (A) is increasing exponentially according to the formula $A = A_0 e^{kt}$, where t is the number of years since 2006.

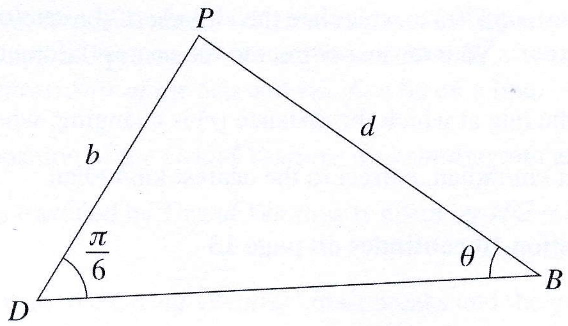
In the 12 months before the beginning of 2006, 202 registered organ donors died, and in the 12 months prior to the beginning of 2008 the number was 259.

(i) In the formula $A = A_0 e^{kt}$, show that $A_0 = 202$ and $k = 0.124$, correct to three decimal places. **2**

(ii) Using the formula, predict how many Australians who die during 2009 will be registered organ donors, that is when $t = 4$. **1**

(iii) At present there are 1388 Australian people waiting for kidney transplants. If the number of registered donor deaths continues to follow the exponential formula, when will there be no one on the waiting list? (Assume no additional people join the waiting list, and that each donor provides kidneys for exactly one person.) **2**

(c)



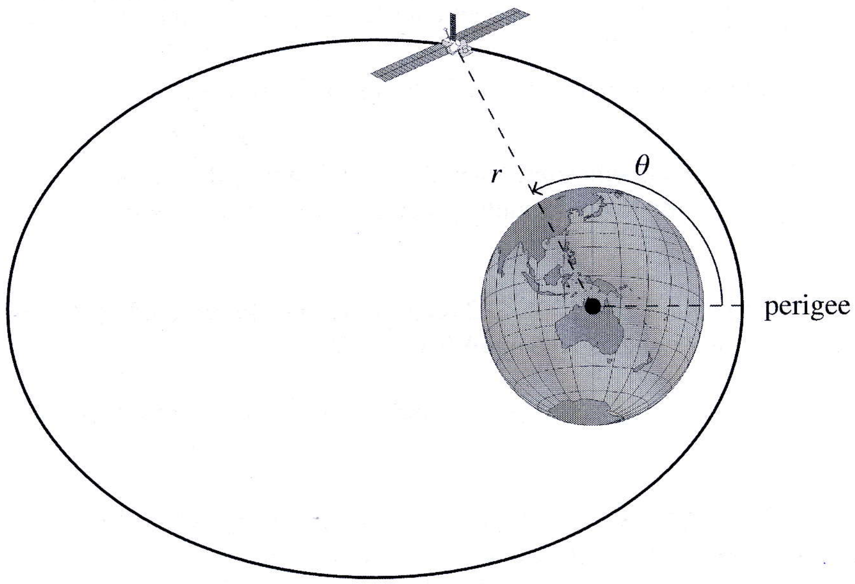
(i) Show that the area, A, of triangle DBP is given by $A = \frac{1}{2} db \sin\left(\frac{5\pi}{6} - \theta\right)$. **1**

(ii) Hence prove that the maximum area of the triangle occurs when $b = \sqrt{3}d$. **3**

Marks

Question 10 (12 Marks) Use a SEPARATE writing booklet.

(a) A satellite is in an elliptical orbit around Earth as shown below.



The distance (r) in kilometres between the satellite and the centre of Earth is given by:

$$r = \frac{804\ 700}{100 + 12\cos\theta}.$$

The angle θ represents the angle measured from the point on the orbit nearest Earth's surface. This point is called the perigee.

(i) Given that Earth's radius is 6370 km, calculate the altitude of the satellite above Earth at the perigee. Express your answer correct to the nearest kilometre. **1**

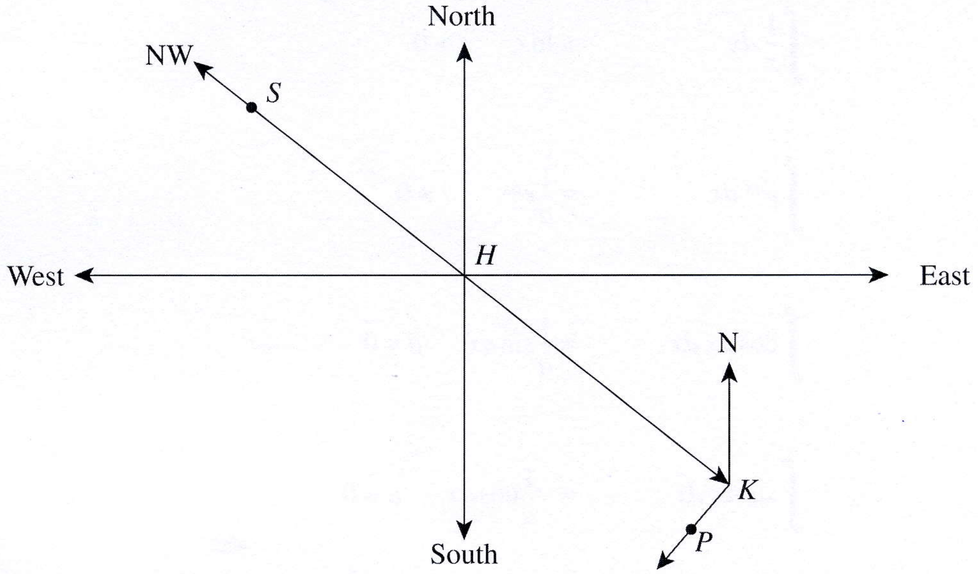
(ii) Calculate $\frac{dr}{d\theta}$, that is, the rate at which the distance (r) is changing, when $\theta = \frac{3\pi}{4}$. Express your answer in km/radian, correct to the nearest km/radian. **3**

Question 10 continues on page 13

Marks

Question 10 (continued)

(b) Two yachts – *Star of the Sea* (S) and *Pacifica* (P) – leave the tiny Holiday Island (H) in opposite directions to each other. *Star of the Sea* sails NW at a speed of 11 km/h for 7 hours. The *Pacifica* sails at a speed of 19 km/h. After six hours, the *Pacifica* changes direction and heads on a bearing of 200° T for a further one hour.



(i) Copy the diagram and show the distances both ships have travelled during the seven hours. **2**

(ii) A third yacht, *Grand Ventures* (G), leaves the same island three hours later than the other two ships and proceeds on a bearing of 200° T. After travelling for four hours, *Grand Ventures*, *Star of the Sea* and *Pacifica* lie on a line.

Label the position of the *Grand Ventures* on your diagram as G and hence show that the distance travelled by *Grand Ventures* is given by $HG = \dfrac{19 \times 77}{191}$ km. **3**

(iii) At this moment, the *Grand Ventures*' mast breaks and the yacht needs help.

Which of the two ships – *Star of the Sea* and *Pacifica* – can get to the *Grand Ventures* first if they maintain their current speeds? Justify your answer with a calculation or otherwise. **3**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0, \ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Note: $\ln x = \log_e x, \quad x > 0$

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