<u>Question 1</u> **Start a new page**

Marks

(a) (i) The base of a certain solid S_1 is the region bounded by the parabola $y^2 = 4ax$ and the line $x = a$, where $a > 0$. **3**

By taking slices parallel to the y-axis in this base where each cross-section is an equilateral triangle, find the volume of S_1.

(ii) The area bounded by $y^2 = 4ax$ and the line $x = a$ is rotated about the line $x = a$ to form a solid of revolution.

By considering slices parallel to the x–axis:

(α) Show that the cross-sectional area A is given by: **2**

$$A = \pi\left(a^2 - \frac{1}{2}y^2 + \frac{1}{16a^2}y^4\right).$$

(β) Hence, find the volume of the solid of revolution. **2**

(b) A particle P of mass m kg, is attached to the end of a light wire 5 cm long which rotates as a conical pendulum with uniform speed in a horizontal plane below a fixed point O to which the wire is attached. The particle rotates so that the angular velocity is ω rads/sec.

(i) Show that the angular velocity is $\dfrac{26\pi}{5}$ rads/sec when the particle is rotating at 156 rpm. **1**

(ii) Find the semi-vertical angle θ of the conical pendulum (answer to the nearest degree and take $g = 9.8$ m/s^2). **2**

(c) A particle moves in a straight line. It is placed at the origin O on the x-axis and is then released from rest.

When it is at position x, the acceleration $\ddot{x}$, of the particle is given by:

$$\ddot{x} = -9x + \frac{5}{(2-x)^2}.$$

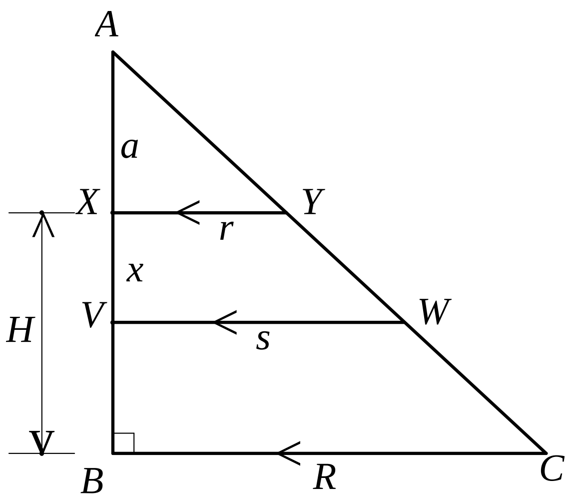
(i) Show that: $v^2 = \dfrac{x(3x-5)(3x-1)}{2-x}$ for $x \neq 2$. **3**

(ii) Prove that the particle moves between two points on the x-axis, and find these points. **2**

<u>Question 2</u> **Start a new page**

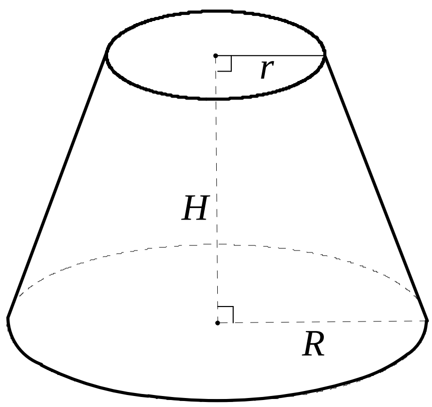
Marks

(a) Consider the right-triangle ABC, where XY and BC are the lengths r and R respectively. Given VW is parallel to XY and BC. The distance between XY and BC is H, the length $VW = s$, $XV = x$ and length $AX = a$, as shown.



(i) Show using similar triangles: $s = \dfrac{R-r}{H}\left(x + \dfrac{Hr}{R-r}\right)$. **3**

(ii) The frustum of a cone where the radius of the top and bottom faces are r and R respectively and the height is H, is shown below.



By considering a cross-sectional slice of the frustum parallel to the top face at a distance x units from the top, and using **integration,** show that the volume V of the frustum is given by : **3**

$$V = \frac{\pi H}{3}\left(R^2 + Rr + r^2\right) .$$

Question 2 Continued

Marks

(b) An object of unit mass falls under gravity through a resistive medium.
The object falls from rest from a height of 50 metres above the ground.
The resistive force, in Newtons, is of magnitude $\frac{1}{100}$ the square of the objects speed v ms^{-1} when it has travelled a distance x metres. Let g be the acceleration due to gravity in ms^{-2}.

(i) Draw a diagram to show the forces acting on the body. **1**
Hence, show that the equation of motion of the body is:

$$\ddot{x} = g - \frac{v^2}{100}.$$

(ii) Show that the terminal speed, u ms^{-1}, of the body is given by: **1**

$$u = \sqrt{100g}.$$

(iii) Prove that: $\ddot{x} = v\frac{dv}{dx}$. **1**

(iv) Show that: $\frac{v^2}{u^2} = 1 - e^{-\frac{x}{50}}$. **3**

(v) Find the distance fallen when the object has reached a speed equal to 50% of its terminal speed (correct to 1 decimal place). **2**

(vi) Find the speed attained, as a percentage of the terminal speed, when the object hits the ground (correct to 1 decimal place). **1**

Question 3 Start a new page

Marks

(a) The region bounded by the curves $y = \frac{1}{x+1}$ and $y = \frac{1}{x+2}$ and the lines $x = 0$ and $x = 2$, is rotated about the y-axis, forming a solid of revolution with a volume of V units^3.

(i) Show that: $V = 2\pi \int_0^2 \frac{x}{(x+1)(x+2)}\,dx$. **2**

(ii) Find V, correct to three significant figures. **3**

(b) A vehicle is travelling along a horizontal straight road with a speed of 42 ms^{-1}. The engine is stopped as it passes a point marked O on the road and then the car is allowed to come to rest at a point B. The frictional resistance force is $\frac{1}{7}$ of the weight of the car and the air resistive force is $\frac{v}{14}$ per unit mass, where v is the speed of the car.

(i) If x is the distance travelled in metres, explain why $\ddot{x} = -\left(\frac{v+2g}{14}\right)$, **1**

where g is the acceleration due to gravity, in ms^{-2}.

(ii) Find the distance travelled (to the nearest metre) **and** the exact time taken for the car to come to rest once the engine is stopped. **4**

Take g = 10 ms^{-2}.

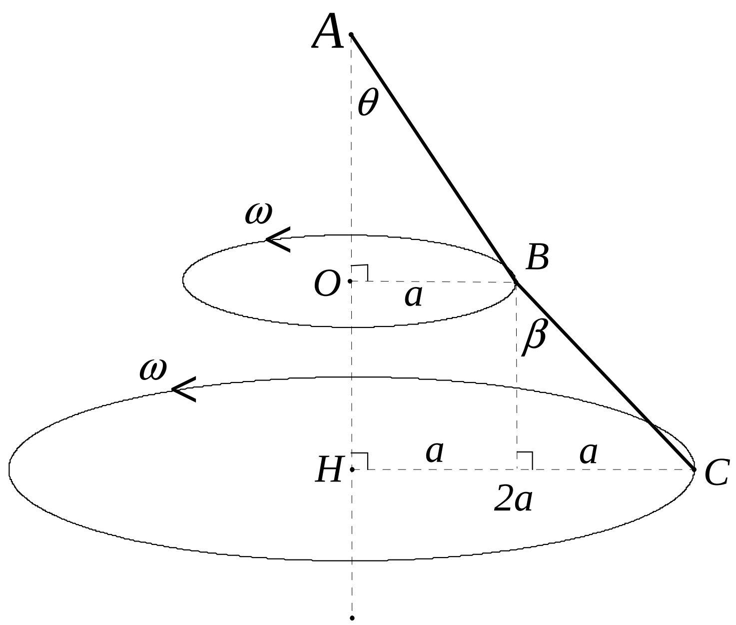
Question3 Continued

(c) A light inextensible string ABC is such that $AB = \frac{5a}{3}$ and $BC = \frac{5a}{4}$.

A particle of mass m kg is attached to the string at C and another particle of mass $7m$ kg is fixed at B. The end A is tied to a fixed point and the whole system rotates steadily about the vertical AH (as shown), in such a way that B and C describe horizontal circles of radii a and $2a$ respectively and each has the same angular velocity ω.

(i) By resolving the forces at C, **2**

show that the tension in the string BC is $\frac{5mg}{3}$ Newtons.



(ii) Hence, find the tension in the part of string AB. **1**

(iii) Find the speed of the particle at B. **2**

<u>Question 4</u> **Start a new page**

Marks

(a) A rectangular hyperbola has the equation $x^2 - y^2 = 8$.

Write down its eccentricity, the coordinates of the foci and the equation of each directrix.

Sketch the curve, indicating on your diagram each focus, directrix and asymptote. **5**

(b) This curve is rotated anti-clockwise through 45^0, where the equation of the curve takes the form $xy = 4$.

(i) Prove that the equation of the normal to the rectangular hyperbola $xy = 4$ at the point $P\left(2p, \dfrac{2}{p}\right)$ is $py - p^3x = 2(1 - p^4)$. **2**

(ii) If this normal meets the hyperbola again at $Q\left(2q, \dfrac{2}{q}\right)$ with parameter q, prove that $q = -\dfrac{1}{p^3}$. **2**

(iii) Hence, or otherwise, explain why there exists only one chord of the hyperbola where the gradients of the normal, at both ends, are equal.

Find the equation of this special chord *PQ*. **3**

(iv) Find the equation of the locus of the midpoint *R* of the chord *PQ*, as p and q vary. **3**

<u>End of Exam Paper</u>