SYDNEY BOYS' HIGH SCHOOL

MOORE PARK, SURRY HILLS

ASSESSMENT TASK - April 2001

MATHEMATICS

EXTENSION 1

Time allowed — One and a half hours
(Plus 5 minutes reading time)

Examiners: Mr A.M.Gainford

DIRECTIONS TO CANDIDATES

- *All* questions may be attempted.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- *Each section* is to be returned in a *separate* booklet, clearly marked Section A (Questions 1 and 2), Section B (Questions 3 and 4), etc. Each booklet must also show your name.
- Start each question on a new page.
- If required, additional booklets may be obtained from the Examination Supervisor upon request.

Question 3. (start a new page)

(a)

(i) Find the equation of the normal to the curve [2]

$y = 3\sin 4x$ at the point $\left(\frac{\pi}{2}, 0\right)$

(ii) Prove that the function [3]

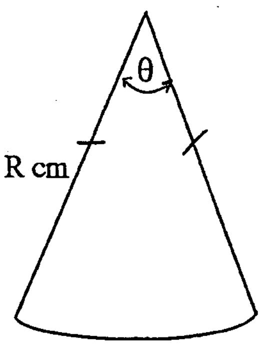
$y = \frac{\sin x}{1+\cos x}$ does not have a stationary point.

(b) Given that $y = \sqrt{12}\ \sin x + 2\cos x$ can be written as [2]

$y = 4\cos(x - \frac{\pi}{3})$ for $\frac{\pi}{3} \le x \le \frac{4\pi}{3}$,

find the equation for the inverse function of y.

(c) The diagram below shows a sector of a circle of radius R cm and angle θ radians. The area of the sector is 25 cm^2



(i) Show that $\theta = \frac{50}{r^2}$ [1]

(ii) If P denotes the perimeter of the sector, show that [1]

$P = 2r + \frac{50}{r}$

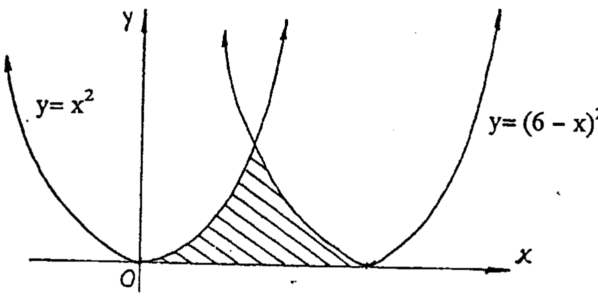
(iii) Determine the value of r which gives the minimum perimeter. [2]

(d) Find

$\lim_{x \to 0} \frac{\sin 2x}{3x}$ [2]

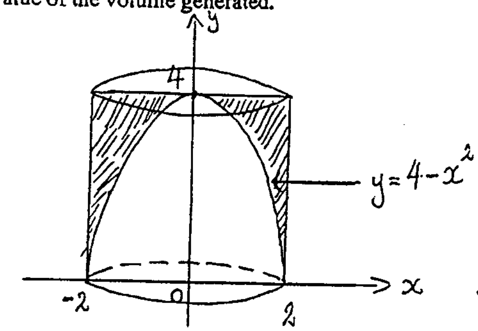
Question 4. (start a new page)

(a) [3



The shaded figure is that enclosed by the branches of the parabolae $y = x^2$ and $y = (6 - x)^2$ and the x axis. Calculate its area.

(b) The area bounded by $y = 4 - x^2$, $x = 2$ and $y = 4$ is revolved about the y axis. Find the exact value of the volume generated. [3]



(c) (i) Sketch $y = 3\cos x$ and $y = x$ for $0 \le x \le 2\pi$ on the same set of axis. [2]

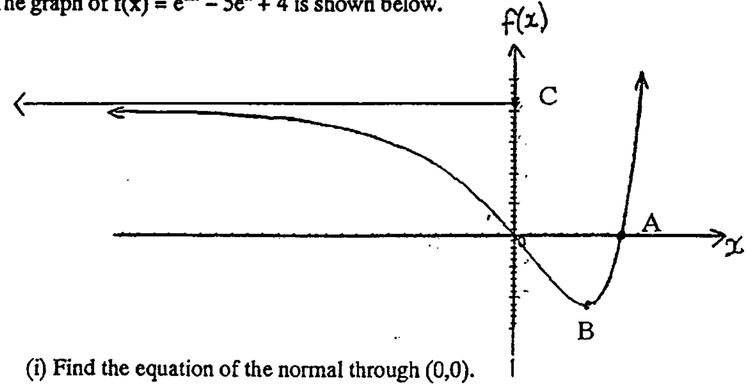
(ii) An approximate solution to the equation $3\cos x - x = 0$ is $x = 1{\cdot}15$. Use one application of Newton's Method to find a better approximation to the solution. [2]

(d) Evaluate [3]

$\frac{\pi}{4}$

Question 5. (start a new page)

(a) The graph of $f(x) = e^{2x} - 5e^{x} + 4$ is shown below.



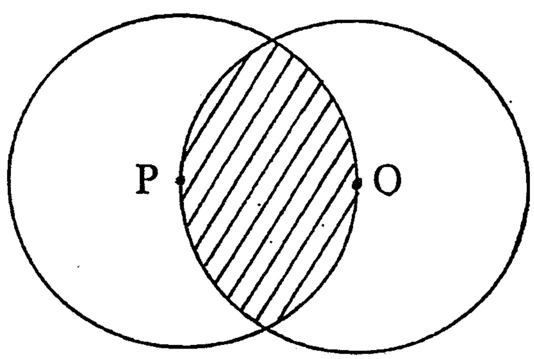
(i) Find the equation of the normal through (0,0). [2]

(ii) Using a suitable substitution, or otherwise, [5]
find the coordinates of A (x-intercept),
and B (stationary point)
Leave your answers in exact form.

(iii) Give the equation of the horizontal asymptote through C. [2]

(b) Given that $\frac{dy}{dx} = \frac{1}{1+x^2}$ and $x = 1$ when $y = 0$, find y when $x = \sqrt{3}$. [2]

(c) In the diagram below, the two circles are of radius 1 metre and pass through centres O and P. Find the area of their intersection correct to two decimal places. [3]



Question 1. (start a new page)

a) Differentiate

(i) $y = \log_e(\cos x)$ expressing your answer in simplest form. [2]

(ii) $y = (x+1)e^{-x}$ [2]

(iii) $y = \tan^{-1} 3x$ [2]

(iv) $y = \tan^3 \theta$ leaving your answer in terms of $\sec\theta$ only. [2]

(b) Show that $\log_4 9 + \log_4 8 - 2\log_4 6 = \frac{1}{2}$ [2]

(c) Show that the derivative of [4]

$$x\tan x - \ln(\sec x) \quad \text{is } x\sec^2 x$$

Hence or otherwise, evaluate.

$$\int_0^{\frac{\pi}{4}} x\sec^2 x\,dx$$ leaving your answer in *exact* form.

Question 2. (start a new page)

(a) Write down primitives (indefinite integrals) of

(i) $\cos 3x$ [2]

(ii) $\frac{e^{2x}}{e^{2x}+1}$ [2]

(iii) $\frac{1}{\sqrt{9-x^2}}$ [2]

(b) Evaluate the following [3]

$$\int_0^{\pi}(2\sin x - \sin 2x)\,dx$$

(c) Sketch the graph of the following function clearly showing the domain and range. [3

$$y = 3\sin^{-1}\frac{x}{2}$$

(d) Solve $\tan\theta = \sin 2\theta$ for $0 \le \theta \le \pi$ [3

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0.$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0.$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{(a^2-x^2)}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{(x^2-a^2)}}\,dx = \ln\{x+\sqrt{(x^2-a^2)}\},\ |x| > |a|.$$

$$\int \frac{1}{\sqrt{(x^2+a^2)}}\,dx = \ln\{x+\sqrt{(x^2+a^2)}\}.$$

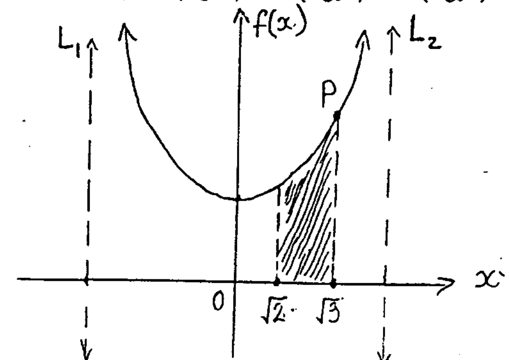
NOTE: $\ln x = \log_e x,\ x > 0.$

Question 6. (start a new page)

(a) If $f(n) = 2(\log_e 2)^n - n \times f(n-1)$ and $f(0) = 2$, Show that [2]

$$f(4) = 2(\log_e 2)^4 - 8(\log_e 2)^3 + 24(\log_e 2)^2 - 48(\log_e 2) + 48$$

(b)

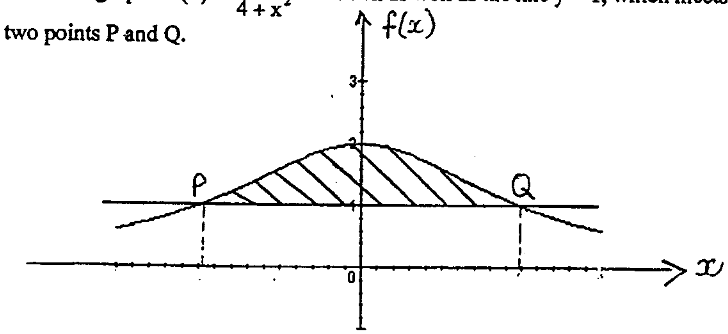


The above diagram shows the graph of the function

$$f(x) = \frac{1}{\sqrt{4-x^2}}$$

(i) Find the equations of the asymptotes L_1 and L_2. [1]
(ii) Find the equation of the tangent to the curve at the point P where $x = \sqrt{3}$ Leave your answer in exact form. [3]
(iii) Find the exact area of the shaded region. [3]

(c) Part of the graph $f(x) = \dfrac{8}{4+x^2}$ is drawn as well as the line $y = 1$, which meets the curve at two points P and Q.



(i) Find the x coordinates of P and Q. [1]
(ii) Show that f(x) is an even function. What is the geometrical significance of this result? [1]
(iii) Calculate the area of the region enclosed by the interval PQ and the arc PQ of the curve. Leave your answer in terms of π. [3]
(iv) The region in (iii) makes a revolution about the y axis. Show that the volume of the solid formed is $4\pi(2\ln 2-1)$ cubic units. [3]