
Centre Number

1	9	9	0	4	8	6	5	

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Morning Session
Monday 17 August 2009

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on the back page
- All necessary working should be shown in every question
- Write your Centre Number and Student Number at the top of this page AND on the separate Answer Booklets provided for each question

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value

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Total marks – 120
Attempt Questions 1-10
All questions are of equal value.

Answer each question in a SEPARATE writing booklet.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\dfrac{2+\sqrt{2}}{7(e^2-4)}$ correct to three significant figures. **2**

(b) Solve $(y-2)^2=9$. ? **2**

(c) Find a primitive of $\dfrac{x}{3}+\dfrac{1}{x^2}$. **2**

(d) Solve $|\,5a+3\,|\le 13$. **2**

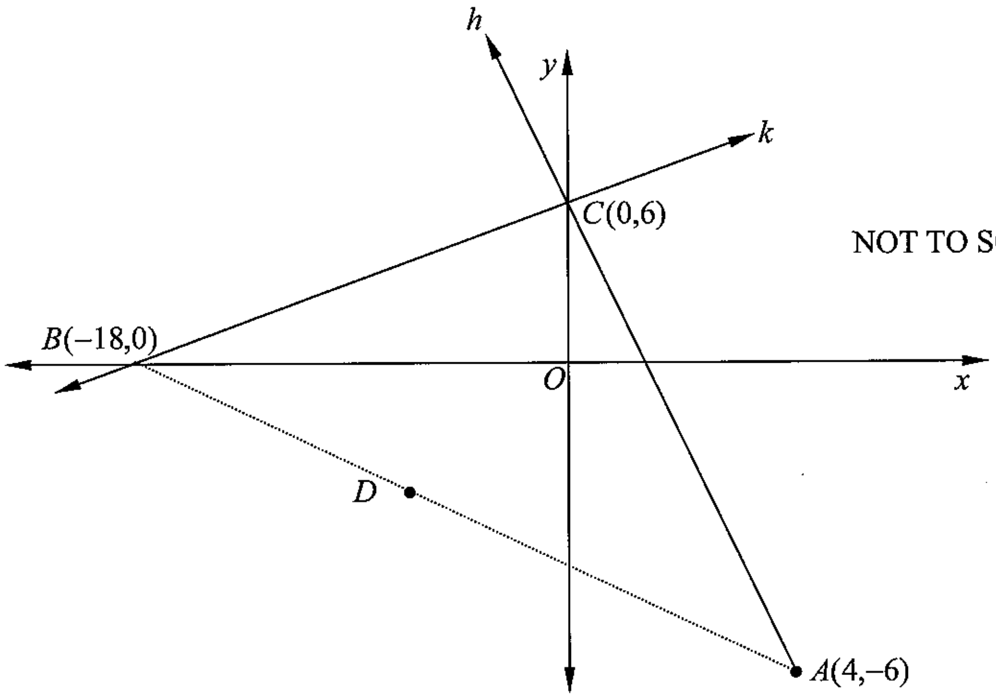
(e) Find the limiting sum of the series $20+4+\dfrac{4}{5}+\ldots$ **2**

(f) If $g(x)=7x^3-3x+1$ evaluate $g'(2)$. **2**

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) In the diagram, the lines h and k are drawn. The coordinates of A, B and C are $(4,-6)$, $(-18,0)$ & $(0,6)$ respectively. D is the midpoint of AB.



NOT TO SCALE

(i) Show that D has coordinates $(-7,-3)$. **1**

(ii) Calculate the length of DC. Leave your answer in exact form. **1**

(iii) Show that the equation of the line h is given by $3x+y-6=0$. **2**

(iv) Show that the line h is perpendicular to the line k. **2**

(v) AB is the diameter of a circle which passes through the points A, B and C. Show that the equation of the circle is given by $(x+7)^2+(y+3)^2=130$. **2**

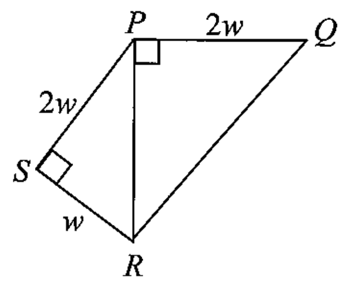
(vi) Find the area of the circle which passes through A, B and C. Leave your answer in terms of π. **1**

(b) Find the values of q if $3qx^2-5x+3q=0$ is negative definite. Leave your answer in exact form. **3**

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) The diagram shows two right angled triangles *PQR* and *PRS*.



NOT TO SCALE

Let $SR = w$. The lengths *SP* and *PQ* are each twice the length of *SR*.

(i) Prove that $QR = 3w$. **2**

(ii) Find the area of quadrilateral *PQRS* in terms of w. **2**

(b) Differentiate with respect to x and leave your answers in simplest form:

(i) $\dfrac{\ln x}{x}$. **2**

(ii) $(x-5)^2 e^x$. **2**

(c) (i) Find $\displaystyle\int \frac{3x}{x^2-9}\,dx$. **2**

(ii) Evaluate $\displaystyle\int_0^3 \sqrt{x}\,dx$. **2**

Marks

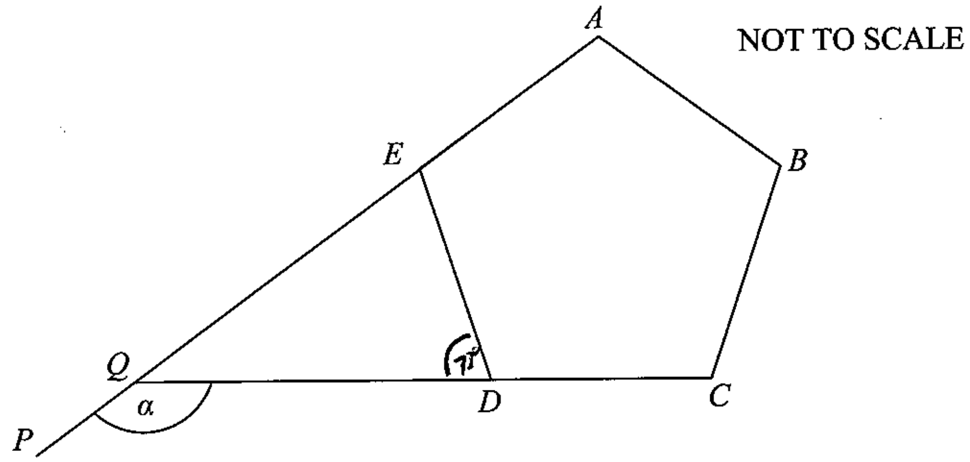
Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\sum_{n=2}^{16}(13-5n)$. **2**

(b) Prove that **3**

$$\frac{\sin\theta}{1-\cos\theta}+\frac{\sin\theta}{1+\cos\theta}=2\operatorname{cosec}\theta$$

(c) $ABCDE$ is a regular pentagon. The points P, Q, E and A are collinear. The points Q, D and C are also collinear.



Find the size of angle α giving reasons. **2**

(d) A six-sided die is biased so that the number 3 occurs twice as often as any other number.

(i) The die is rolled once. Show that the probability that an odd number occurs is $\frac{4}{7}$. **1**

(ii) If the biased die is rolled twice, find the probability of the sum of the uppermost numbers being six. **2**

This biased die is now rolled together with TWO fair six-sided dice.

(iii) What is the probability that at least two odd numbers are uppermost? **2**

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) Solve for x: $\log_{10} x^6 - 8 = 3\log_{10} x$. **2**

Give your answer correct to 1 decimal place.

(b) Consider the curve given by $y = 2x^3 - 9x^2 + 12x$.

(i) Find the stationary points and determine their nature. **3**

(ii) Show that a point of inflexion occurs at $x = \frac{3}{2}$. **1**

(iii) Sketch the graph of $y = f(x)$ indicating clearly the stationary points and point of inflexion. **2**

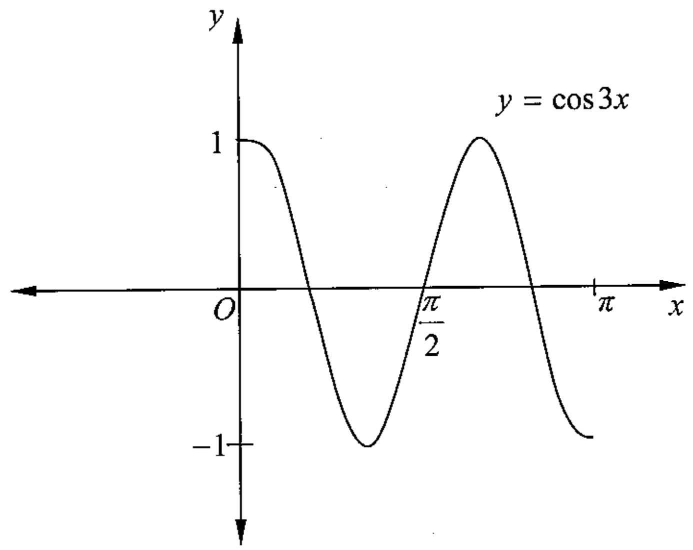
(iv) For what values of x is the curve concave up? **1**

(c) Solve $2(x^2+1)^2 - 19(x^2+1) - 10 = 0$. **3**

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet

(a) The graph of $y = \cos 3x$ is shown below.



NOT TO SCALE

(i) Solve $\cos 3x = 0$ for $0 \le x \le \pi$. **2**

(ii) State the amplitude and period of $y = \cos 3x$. **2**

(iii) Copy this diagram into your answer booklet showing the x-intercepts. **2**

Hence sketch the graph of $y = \sec 3x$ in the domain $0 \le x \le \pi$ showing any asymptotes.

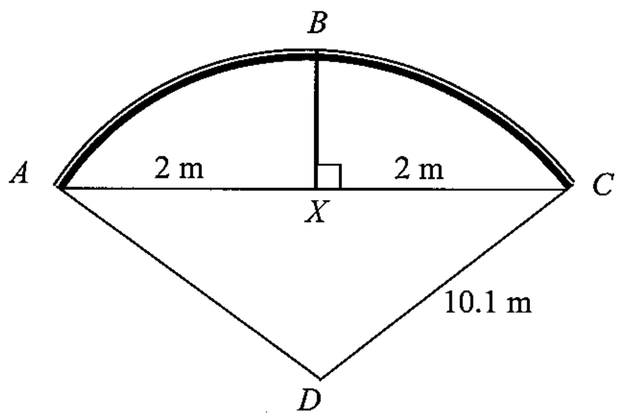
(iv) Using (iii), find the number of solutions to $\sec 3x = x$ in the domain $0 \le x \le \pi$. **2**

Question 6 continues on page 8

Marks

Question 6 (continued)

(b) A bridge's steel arch ABC is part of a circle of radius 10.1 metres. BX bisects the chord AC which is 4 metres long.



NOT TO SCALE

(i) Find the size of angle ADC correct to the nearest degree. **2**

(ii) Find the length of steel needed to make the arch ABC. **2**

End of Question 6

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet

(a) Consider the parabola $x^2 = 8(y+2)$.

(i) Find the coordinates of the vertex. **1**

(ii) Find the coordinates of the focus. **1**

(iii) Find the equation of the tangent to the parabola at the point $\left(2, -\frac{3}{2}\right)$. **2**

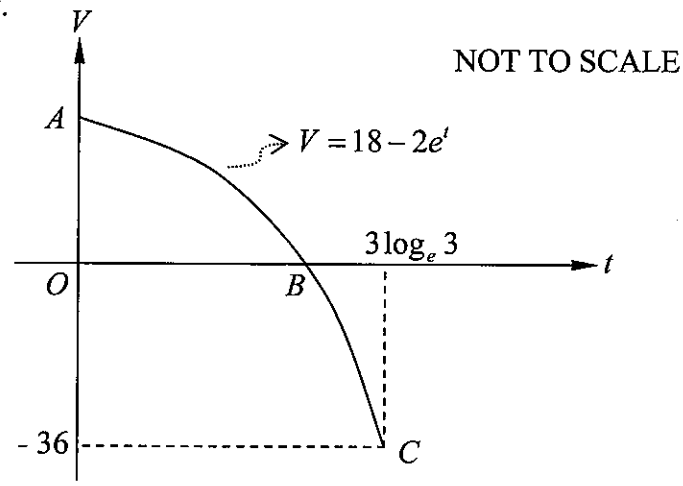
(iv) Find the coordinates of the point where the tangent meets the directrix. **1**

(b) The velocity, V, in m/s of a particle moving in a straight line is given by $V = 18 - 2e^t$, where t is the time in seconds.

(i) Find the initial velocity of the particle. **1**

(ii) Show that the time at which the particle comes to rest is $2\log_e 3$ seconds. **2**

(iii) The graph of the velocity V of the particle as a function of t is given below.



The coordinates of point C are $(3\log_e 3, -36)$.
Write down the coordinates of points A and B. **1**

(iv) Hence, or otherwise, find the distance travelled by the particle between $t = 0$ and $t = 3\log_e 3$. **3**

Marks

Question 8 (12 marks) Use a SEPARATE writing booklet

(a) Kenny begins his retirement with \$500 000 at the beginning of 2009. The annual interest rate is 8% p.a. Interest is calculated annually on the balance at the beginning of the year and added to the remaining balance. Kenny plans to withdraw \$56 000 annually, with the first withdrawal at the end of 2009.

Let A_n be the remaining balance after the nth withdrawal.

(i) Show that $A_2 = (5\times10^5)(1.08)^2 - 5.6\times10^4(1.08+1)$. **2**

(ii) Show that $A_n = 10^5\left[7 - 2(1.08)^n\right]$. **2**

(iii) In which year will Kenny's fund reach zero? **3**

(b) Populations cannot increase indefinitely. Environmental and economic factors such as limited food, weather and space control the size of the population.

Two thousand kangaroos, each aged 2 years old, are released into the wild on an island. After 3 years there are approximately 1800 kangaroos that inhabit the island. The size of the population, N, after t years is predicted by the equation

$$N = N_0\, e^{-kt}.$$

(i) Show that the size of the kangaroo population decreases at a rate proportional to the size of the population. **1**

(ii) Find the value of N_0 and k. **2**

(iii) After how many years will the kangaroo population halve? **2**

Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a) The equation below refers to the filtering cycle of a pump in Helen's garden.

The flow rate of the volume of water that the filter pumps water into and out of a pond in litres per minute, is given by

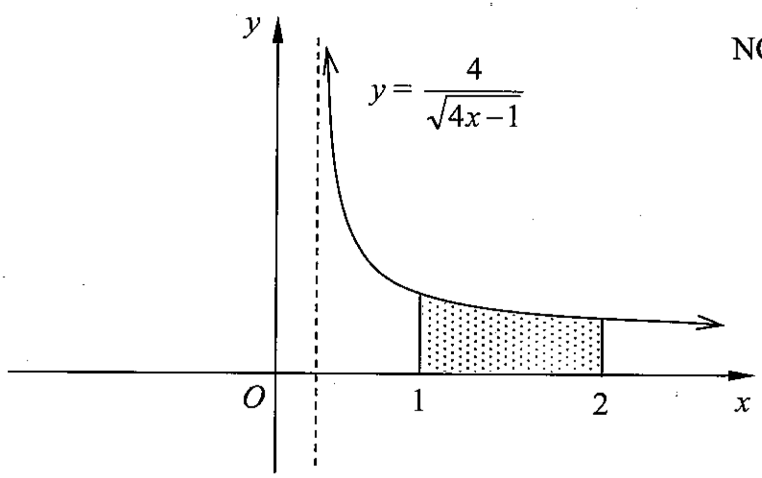
$$\frac{dV}{dt} = 20\sin\frac{\pi}{35}t.$$

(i) If the pump started at 8.55 pm, what is the first time after 8.55 pm at which the flow rate is zero? **2**

(ii) If the pond is initially empty find an expression for the volume, V, of water in the pond after t minutes. **3**

(iii) Find the maximum volume of water in the pond during the filtering cycle. Leave your answer in terms of π. **2**

(b)



The area enclosed by the curve $y = \frac{4}{\sqrt{4x-1}}$ the lines $x = 1$ and $x = 2$ is shaded as shown in the diagram above.

(i) Show that the volume, V, of the solid formed when this shaded region is rotated about the x-axis is given by: **2**

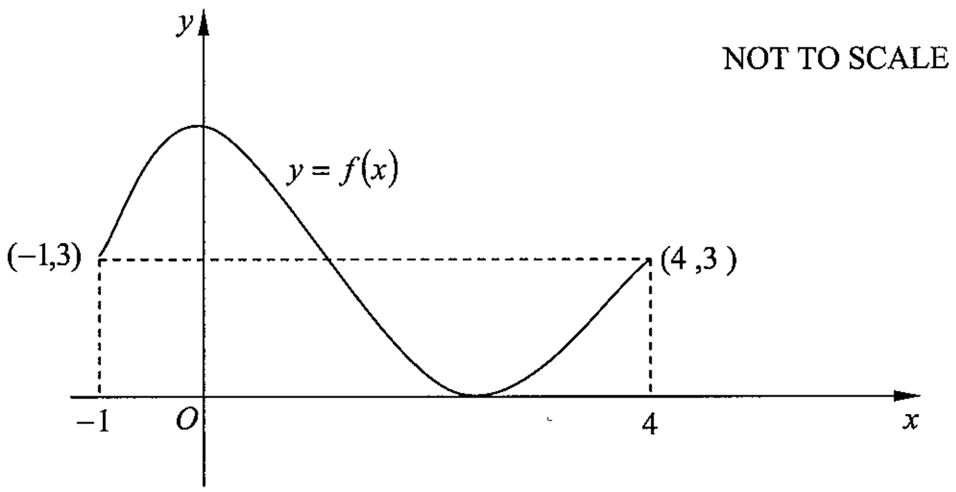
$$V = 4\pi\int_1^2 \frac{4}{4x-1}\,dx.$$

(ii) Hence calculate the volume, V. Leave your answer in exact form. **3**

Marks

Question 10 (12 marks) Use a SEPARATE writing booklet.

(a) The graph below represents the function $y = f(x)$.



If $\int_{-1}^{4} f(x)\, dx = \frac{15}{2}$, find the value of $\int_{-1}^{4} [f(x)+4]\, dx$. **2**

(b) Given $\frac{d}{dx}\left(b^x\right) = b^x \log_e b$, **3**

Evaluate $\int_{0}^{\pi} \pi^x \, dx$ correct to one decimal place.

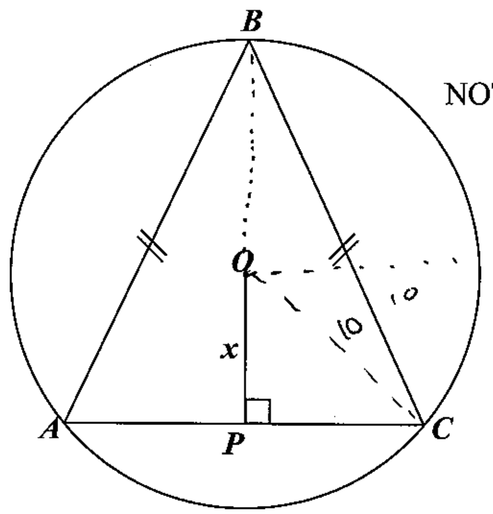
Question 10 continues on page 13

Marks

Question 10 (continued)

(c) An isosceles triangle ABC, where $AB = BC$, is inscribed in a circle of radius 10cm.

$OP = x$ and OP bisects AC, such that $AC \perp OP$.



NOT TO SCALE

(i) Show that the area, A, of ΔABC is given by $A = (10+x)\sqrt{100-x^2}$. **2**

(ii) Show that $\dfrac{dA}{dx} = \dfrac{100-10x-2x^2}{\sqrt{100-x^2}}$. **2**

(iii) Hence prove that the triangle with maximum area is equilateral. **3**

End of Paper

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EXAMINERS

Kimon Kousparis (convenor)	Casimir Catholic College, Marrickville
Magdi Farag	LaSalle Catholic College, Bankstown
Kathleen Roffey	Trinity Catholic College, Auburn / Regents Park
Svetlana Onisczenko	Sydney Grammar School, Darlinghurst

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \quad \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$