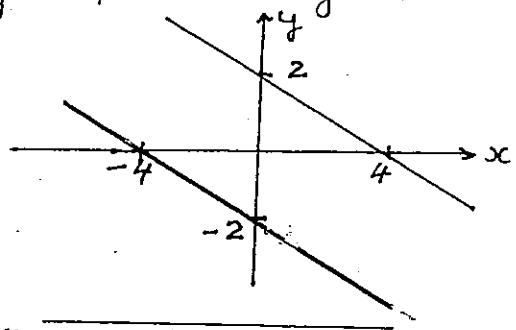
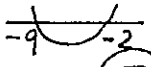
Extension One Mathematics

Trial HSC Examination 2006

- Markers Comments + Worked Solutions
- Dot plots

Mathematics Extension 1: Question 1

Suggested Solutions	Marks Awarded	Marker's Comments
(a) $x^3+27=(x+3)(x^2-3x+9)$ (1)		Well done
(b) $\lvert x+2y\rvert = 4$ $x+2y=-4$ or $x+2y=4$  (2)		poorly answered. All sorts of graphs were presented. 1 Mark awarded for each.
(c) $(2,-1)$ $(5,3)$ $3:-1$ $x_1\ y_1$ $x_2\ y_2$ $m_1\ m_2$ $\left(\frac{3\times5+(-1)\times2}{3+(-1)}, \frac{3\times3+(-1)\times(-1)}{3+(-1)}\right)$ $\left(6\frac{1}{2}, 5\right)$ (2)		1 Mark for correct Substitution into formula 1 Mark for answer
(d) $\frac{3x-1}{x+2} > 4$ $\times(x+2)^2$: $(3x-1)(x+2) > 4(x+2)^2$ $4(x+2)^2-(3x-1)(x+2)<0$ $(x+2)[4(x+2)-(3x-1)]<0$ $(x+2)(x+9)<0$  $-9<x<-2$ (3)		1 Mark multiplying through by $(x+2)^2$ 1 Mark factorisation 1 Mark answer
(e) $y=8-2x$: grad $=-2$ $y=x^2$ $\frac{dy}{dx}=2x$. grad $=4$ at $(2,4)$. $\tan\theta = \left\lvert\frac{m_1-m_2}{1+m_1m_2}\right\rvert$ $= \left\lvert\frac{-2-4}{1+(-2)\times4}\right\rvert$ $= \frac{6}{7}$ $\theta = 40^\circ36'$ OR 41° (nearest degree) (4)		Generally well done. Some used an incorrect formula. 1 Mark Correct gradients 2 Marks Substitution and Simplification in formula 1 Mark answer.

Mathematics Extension 1: Question 2.

Suggested Solutions	Marks Awarded	Marker's Comments

(a) $\int \frac{x}{\sqrt{16-x^4}}\,dx = \frac{1}{2}\int \frac{2x\,dx}{\sqrt{16-x^4}}$ $\quad u = x^2$, $\frac{du}{dx} = 2x$, $du = 2x\,dx$

$= \frac{1}{2}\int \frac{1}{\sqrt{16-u^2}}\,du$

$= \frac{1}{2}\sin^{-1}\frac{u}{4} + C$

$= \frac{1}{2}\sin^{-1}\frac{x^2}{4} + C$ (3)

Well done

(b) $\int_0^{\frac{\pi}{4}} \sin^2 2x\,dx = \int_0^{\frac{\pi}{4}} \frac{1}{2}(1-\cos 4x)\,dx$

$= \frac{1}{2}\left[x - \frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{4}}$

$= \frac{1}{2}\left[\left(\frac{\pi}{4} - \frac{1}{4}\times 0\right) - (0-0)\right]$

$= \frac{\pi}{8}$ (3)

Those that started with the result $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ did better than those that went straight into the expression integrated.

(c) $\sin\left(2\tan^{-1}\frac{2}{3}\right)$

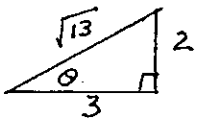
$= \sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2 \times \frac{2}{\sqrt{13}} \times \frac{3}{\sqrt{13}}$

$= \frac{12}{13}$ (3)

Let $\theta = \tan^{-1}\frac{2}{3}$, $\tan\theta = \frac{2}{3}$



Well done

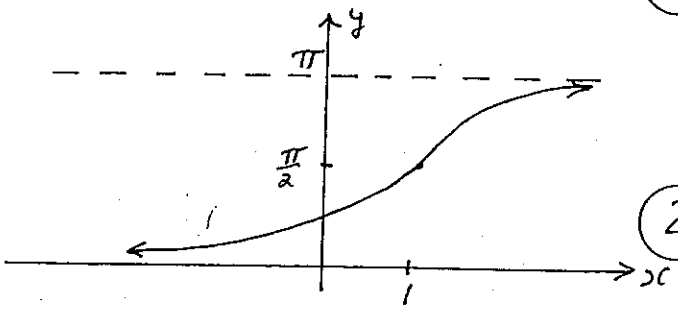
(d) $f(x) = \frac{\pi}{2} + \tan^{-1}(x-1)$

(i) $-\frac{\pi}{2} < \tan^{-1}(x-1) < \frac{\pi}{2}$

$\therefore\ 0 < \frac{\pi}{2} + \tan^{-1}(x-1) < \pi$. (1)

i) well done

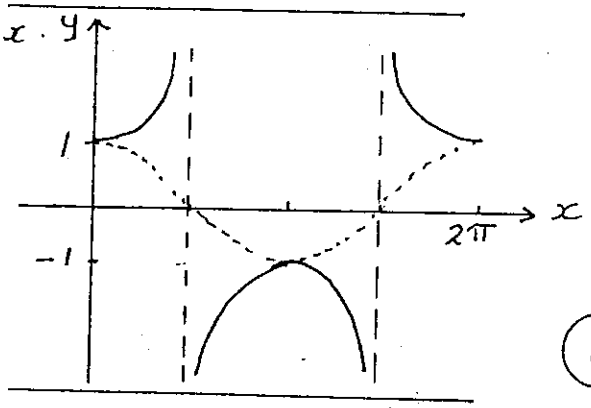
(ii)



(2)

ii) Some forgot to move the curve across 1 unit ∴ had the y-intercept as $\frac{\pi}{2}$.

Mathematics Extension 1: Question 3

Suggested Solutions	Marks Awarded	Marker's Comments
(a) $\cos \pi x = \frac{\sqrt{3}}{2}$ $\pi x = 2n\pi \pm \frac{\pi}{6}$ $x = 2n \pm \frac{1}{6}$ (2)		a) Poorly answered. 1 mark for each part to answer.
(b) $\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\sin 105° = \sin(60°+45°)$ $= \sin 60° \cos 45° + \cos 60° \sin 45°$ $= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$ $= \frac{\sqrt{3}+1}{2\sqrt{2}}$ OR $\frac{\sqrt{6}+\sqrt{2}}{4}$ (2)		b) Well answered. 1 Mark for rule 1 Mark for simplification
(c) $y = \sec x$  (2)		Many sketches were messy. Some lost mark for not showing asymptotes
(d) (i) $6\sin\theta - 8\cos\theta \equiv R\sin(\theta-\alpha)$ $\equiv R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$ $R\cos\alpha = 6$ $R\sin\alpha = 8$ $\tan\alpha = \frac{8}{6}$ $\alpha = 53°08'$ (or $53°$) $R^2(\cos^2\alpha + \sin^2\alpha) = 6^2 + 8^2$ $R^2 = 100$ $R = 10$ $(R>0)$ $\therefore 6\sin\theta - 8\cos\theta \equiv 10\sin(\theta - 53°08')$ (2)		a) Generally well done 1 Mark for R in correct form 1 Mark for correct angle
(ii) $10\sin(\theta - 53°08') = 4$ $\sin(\theta - 53°08') = \frac{4}{10}$ $\theta - 53°08' = 23°35'$ or $156°25'$ $\theta = 76°43'$ or $209°33'$ (2)		Poorly answered. Many couldn't give both answers
(iii) Minimum value of -10 when $\theta - 53°08' = 270°$ i.e. $\theta = 323°08'$ (2)		Poorly answered A large number couldn't give the min. value and value for θ when this occured.

Mathematics Extension 1: Question 4

Suggested Solutions	Marks Awarded	Marker's Comments

(a) $8ay = x^2 - 4ax - 20a^2$

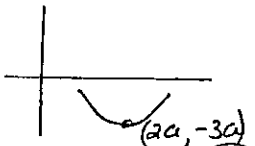
(i) $x^2 - 4ax + 4a^2 = 8ay + 20a^2 + 4a^2$

$(x-2a)^2 = 8a(y+3a)$.

Vertex is $(2a, -3a)$. (2)

(ii) $A = 2a$

Directrix: $y = -5a$



(1)

(b) $x^2 = 8y$ i.e. $y = \frac{x^2}{8}$

(i) $\frac{dy}{dx} = \frac{2x}{8} = \frac{2 \times 4p}{8} = p$ at $(4p, 2p^2)$

Tangent at P: $y - 2p^2 = p(x - 4p)$

$y - 2p^2 = px - 4p^2$

$px - y - 2p^2 = 0$ (2)

(ii) Tangent at P: $px - y = 2p^2$ (1)

Tangent at Q: $qx - y = 2q^2$ (2)

(1)-(2) $(p-q)x = 2p^2 - 2q^2$

$= 2(p-q)(p+q)$

$x = 2(p+q)$

Subst. into (1): $y = p \times 2(p+q) - 2p^2$

$= 2p^2 + 2pq - 2p^2$

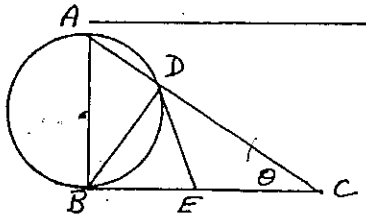
$= 2pq$

T is $[2(p+q), 2pq]$ (3)

(iii) x-coord of $M = \frac{4p+4q}{2} = 2(p+q)$

Since T, M have the same x-coord., then $TM \parallel$ y-axis (axis of parabola) (1)

(c)



$\angle ADB = 90°$ (angle in a semi-circle)

$\therefore \angle BDC = 90°$

$\therefore \angle DBE = 90° - \theta$ (angle sum of $\triangle BDC$)

$\therefore \angle BDE = 90° - \theta$ (base angles of isos. $\triangle BDE$ since $EB = ED$ (tangents from external point are equal))

$\therefore \angle BED = 180° - 2(90° - \theta)$ (angle sum of $\triangle BDE$)

$= 2\theta$

(3)

Many students didn't handle the algebra well. or were confused with working with the 'a'.

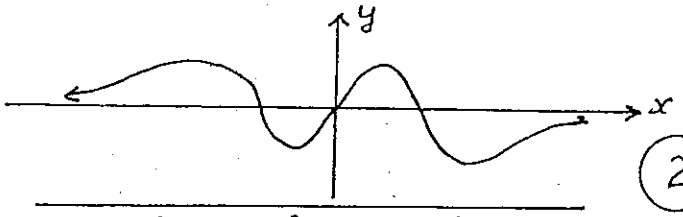
i) well done

ii) didn't have to derive tangent at Q. a number of students couldn't solve the simultaneous eqns or didn't see that $2p^2 - 2q^2 = 2(p+q)(p-q)$.

iii) many students found gradient of TM and showed that it was undefined $\therefore$ TM is a vertical line. Correct but more working than given explanation.

c) well done by those that worked through it.

Mathematics Extension 1: Question 5

Suggested Solutions	Marks Awarded	Marker's Comments
(a)  (2)		Generally well done
(b). $P(x) = x^3 + 4x^2 + ax + b$ (i) $P(1)=0$: $1+4+a+b=0$ $\therefore a+b=-5$ (1) $P(-2)=0$: $-8+16-2a+b=0$ $\therefore -2a+b=-8$ (2) (1) $-$ (2): $3a=3 \therefore a=1,\ b=-6$ (2)		Well done.
(ii) $(x-1)(x+2)(x+c) \equiv x^3+4x^2+x-6$ $\therefore -2c=-6 \therefore c=3$ Third factor is $(x+3)$ OR Use "sum of roots" (1)		Well done
(c) $P(x) = x^3 - 5x + 3$ (i) $P(1) = -1$; $P(2) = 1$ $P(1.5) = 1.5^3 - 5\times 1.5 + 3 = -1.125$ $\therefore$ Root lies between 1.5 and 2 (1)		For mark needed to show $P(1.5)<0$ $P(2)>0$ $\therefore$ lies between 1.5 and 2.
(ii) $x_2 = x_1 - \frac{P(x_1)}{P'(x_1)}$ $\quad P'(x) = 3x^2 - 5$ $= 1.5 - \frac{-1.125}{1.75}$ $\quad P'(1.5) = 3\times 1.5^2 - 5$ $= 2.14$ (2dp) $\quad = 1.75$ (2)		Well done
(d) Prove $5^n + 2\times 11^n$ is a multiple of 3 When $n=1$, $5^n + 2\times 11^n = 5 + 2\times 11 = 27 = 3\times 9$ $\therefore$ it is true for $n=1$ Assume it is true for $n=k$. i.e. assume $5^k + 2\times 11^k = 3M$ (M integer) When $n=k+1$, $5^n + 2\times 11^n = 5^{k+1} + 2\times 11^{k+1}$ $= 5\times 5^k + 2\times 11\times 11^k$ $= 5(3M - 2\times 11^k) + 22\times 11^k$ by assumption $= 15M - 10\times 11^k + 22\times 11^k$ $= 15M + 12\times 11^k$ $= 3(5M + 4\times 11^k)$ $\therefore$ if it is true for $n=k$, then it is true for $n=k+1$. Since it is true for $n=1$, it is true for all positive integers n. (4)		Poorly answered. 1 Mark for proving true for $n=1$ 1 Mark for assuming $5^k + 2\times 11^k = 3M$ and substitution into T_{k+1}. 1 Mark for showing T_{k+1} is a multiple of 3 1 Mark for Conclusion. Those that could not prove it true for $n=k+1$ were only awarded 1 Mark

Mathematics Extension 1: Question 6

Suggested Solutions	Marks Awarded	Marker's Comments
(a) $PV = 3000$ $P = 3000V^{-1}$ $\frac{dP}{dV} = -3000V^{-2} = -\frac{3000}{V^2}$ ✓ $\frac{dP}{dt} = \frac{dP}{dV} \times \frac{dV}{dt}$ ✓ When $V=100$, $\frac{dV}{dt} = 30$ $\frac{dP}{dt} = -\frac{3000}{(100)^2} \times 30$ $= -9$ ✓ Pressure is decreasing at 9 kpa/minute.	3	a) Well done by those that attempted. Disappointing to see that a number of students didn't know where to start.
(b) $\frac{dM}{dt} = -0.01(M-50)$ (i) $M = 50 + Ae^{-0.01t}$ $\frac{dM}{dt} = -0.01Ae^{-0.01t}$ $= -0.01(M-50)$	1	i) well done ii) well done
(ii) When $t=0$, $M=80$ $\therefore 80 = 50 + Ae^0$ $A = 30$	1	iii) well done
(iii) When $t=60$, $M = 50 + 30 \times e^{-0.01 \times 60}$ Mass of salt = 66.5 kg	1	iv) Needed to know that as $t \to \infty$, $e^{-0.01t} \to 0$.
(iv) Least amount of salt = 50 kg.	1	
(c) (i) $x = A \sin nt$ Period $= \frac{2\pi}{n} = 6 \therefore n = \frac{2\pi}{6} = \frac{\pi}{3}$ $\therefore x = 20 \sin \frac{\pi}{3}t$ ✓	2	i) Some students incorrectly let $n = 6$ instead of $\frac{2\pi}{n} = 6$.
(ii) $v = 20 \times \frac{\pi}{3} \cos \frac{\pi}{3}t$ Maximum velocity $= \frac{20\pi}{3}$ cm/sec	1	ii) Many students found max. velocity by letting acceleration = 0. Easier & less working to consider amplitude of velocity function.
(iii) $v^2 = n^2(a^2 - x^2)$ $\left(\frac{10\pi}{3}\right)^2 = \left(\frac{\pi}{3}\right)^2(400 - x^2)$ $100 = 400 - x^2$ $x^2 = 300$ Distance $= 10\sqrt{3}$ cm.	2	iii) Most students used second method. First method produced less errors.
OR When $v = \frac{10\pi}{3}$, $\frac{10\pi}{3} = \frac{20\pi}{3}\cos\frac{\pi}{3}t$ $\cos\frac{\pi}{3}t = \frac{1}{2} \therefore \frac{\pi}{3}t = \frac{\pi}{3} \therefore t = 1$ When $t=1$, $x = 20\sin\frac{\pi}{3}$ $= 20 \times \frac{\sqrt{3}}{2}$ $= 10\sqrt{3}$ ✓		

Mathematics Extension 1: Question 7.

Suggested Solutions	Marks Awarded	Marker's Comments
(a) $\left(x - \frac{2}{x^2}\right)^9$ By inspection, term in x^3 is $\binom{9}{2} x^7 \left(-\frac{2}{x^2}\right)^2$ Coefficient of $x^3 = \binom{9}{2}(-2)^2$ $= 144$	2	Well done. Some needed to expand completely as they couldn't find the coefficient by using the general term.
(b) $v = \frac{5}{x}$ for $x > 0$. (i) $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ $= \frac{d}{dx}\left(\frac{1}{2} \times \frac{25}{x^2}\right)$ $= \frac{d}{dx}\left(\frac{25}{2} x^{-2}\right)$ $= -25x^{-3}$ When $x = 2$, $a = -\frac{25}{2^3}$ Acceleration $= -3\frac{1}{8}$ m/s^2	3	(b)(i) Generally well done. 1 Mark $\frac{d}{dx}\left(\frac{25}{2}x^{-2}\right)$ 1 Mark Correct differentiation 1 Mark answer.
(ii) $\frac{dx}{dt} = \frac{5}{x}$ $\frac{dt}{dx} = \frac{x}{5}$ $t = \frac{x^2}{10} + c$ When $t = 0$, $x = 10$: $0 = \frac{10^2}{10} + c \therefore c = -10$ $t = \frac{x^2}{10} - 10$ $x^2 = 10(t + 10)$ $x = \sqrt{10(t+10)}$ $(x > 0)$	3	(ii) Poorly answered. Many thought this was a log problem and tried to integrate 1 Mark integrating correctly 1 Mark Finding the constant 1 Mark Making x subject.
(c) $x = Vt\cos\theta$ $y = Vt\sin\theta - \frac{1}{2}gt^2$ (i) $\dot{y} = V\sin\theta - gt$. Max. height when $\dot{y} = 0$. $\therefore t = \frac{V\sin\theta}{g}$ Max. ht. $= V\left(\frac{V\sin\theta}{g}\right)\sin\theta - \frac{g}{2}\left(\frac{V\sin\theta}{g}\right)^2$ $= \frac{V^2\sin^2\theta}{g} - \frac{V^2\sin^2\theta}{2g}$ $= \frac{V^2\sin^2\theta}{2g}$	3	(c) Many got 2/3 Marks as they couldn't simplify to achieve the answer. 1 Mark for finding $\dot{y}$ 1 Mark for finding time when at max ht. 1 Mark for max ht.
(ii) $\dot{x} = V\cos\theta$ At maximum height, direction is horizontal. $\therefore$ Speed at maximum height is $V\cos\theta$.	1	(ii) Poorly answered. A wide variety of answers.

Year 12 Extension 1 Mathematics Q1 Trial HSC Examination

BLC mean = 10.1 Std Dev = 1.4

									x	x		
									x	x		x
									x	x	x	x
									x	x	x	x
							x	x	x	x	x	x
0	1	2	3	4	5	6	7	8	9	10	11	12

TDS mean = 9.1 Std Dev = 2.1

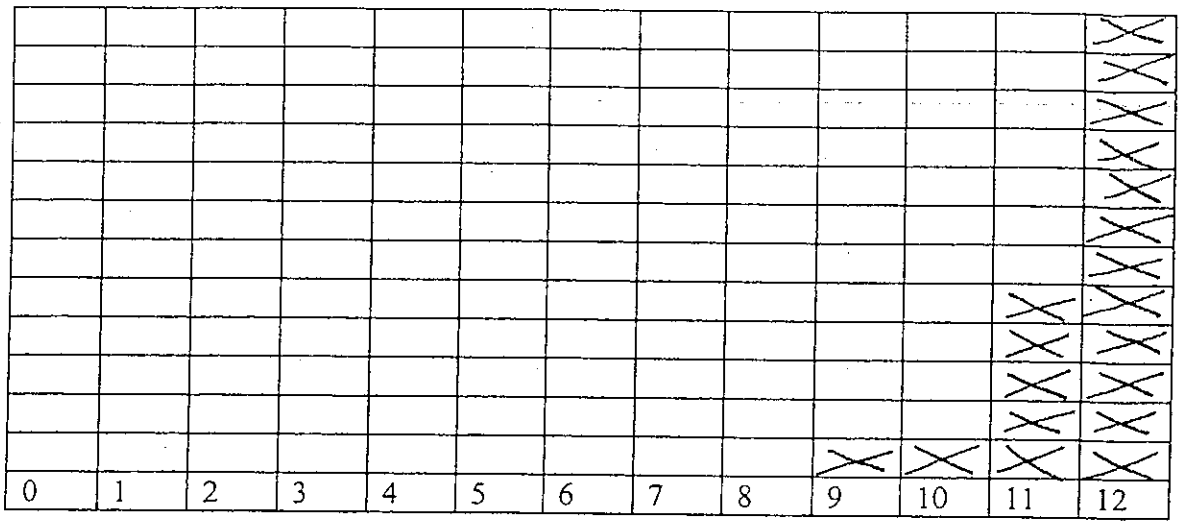
								x				
								x	x			
								x	x	x	x	x
							x	x	x	x	x	x
			x				x	x	x	x	x	x
0	1	2	3	4	5	6	7	8	9	10	11	12

BDD mean = 8.1 Std Dev = 1.8

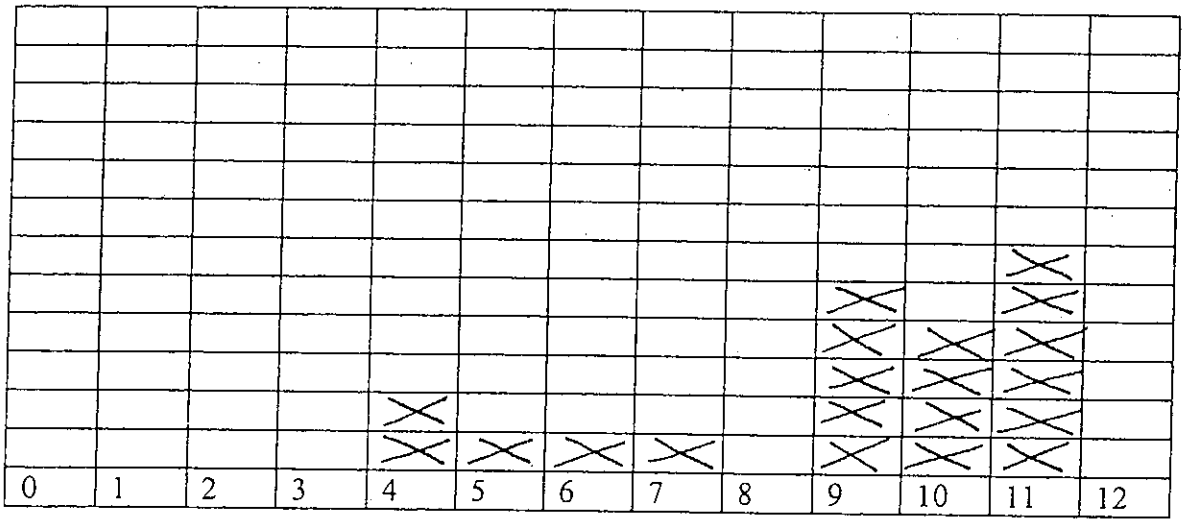
								x	x			
					x			x	x	x		
					x	x	x	x	x	x	x	
0	1	2	3	4	5	6	7	8	9	10	11	12

Extension 1 Trial

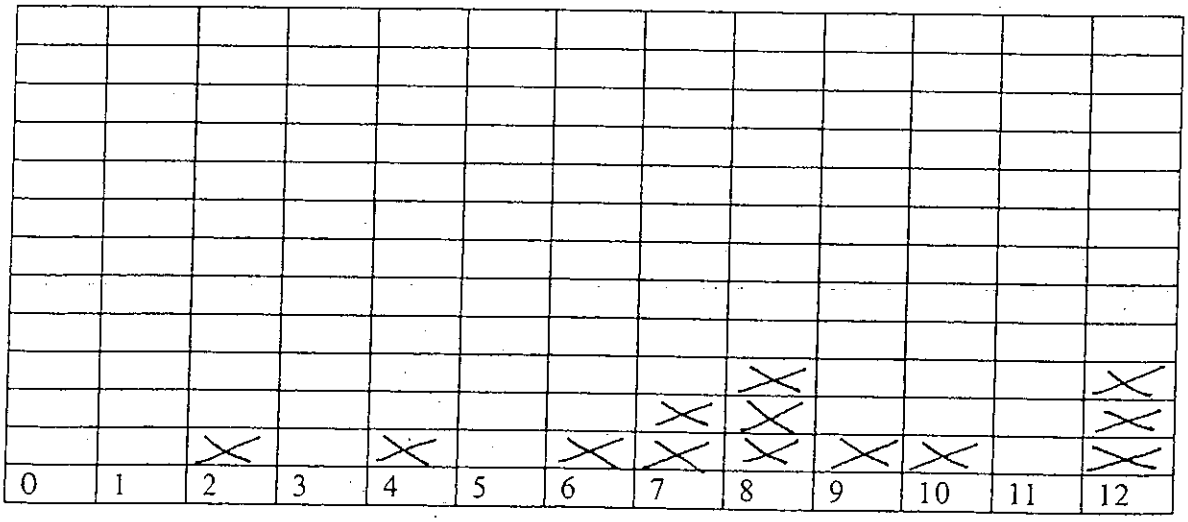
Question 2



BLC $\bar{x} = 11 \cdot 47$



TDS $\bar{x} = 8 \cdot 85$



BDD $\bar{x} = 8 \cdot 08$

Year 12 Extension 1 Mathematics Q3 Trial HSC Examination

BLC mean = 10.5 Std Dev = 1.1

										x	x	
										x	x	
										x	x	x
										x	x	x
									x	x	x	x
								x	x	x	x	x
0	1	2	3	4	5	6	7	8	9	10	11	12

TDS mean = 7.5 Std Dev = 2.5

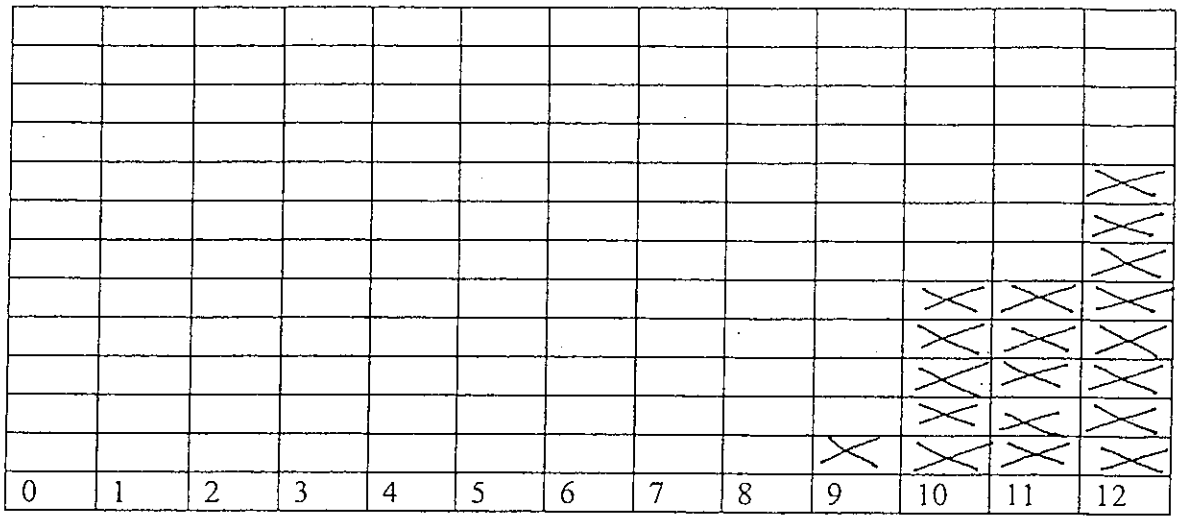
				x				x	x	x		
				x				x	x	x		
				x			x	x	x	x		
		x		x		x	x	x	x	x	x	
0	1	2	3	4	5	6	7	8	9	10	11	12

BDD mean = 5.5 Std Dev = 1.8

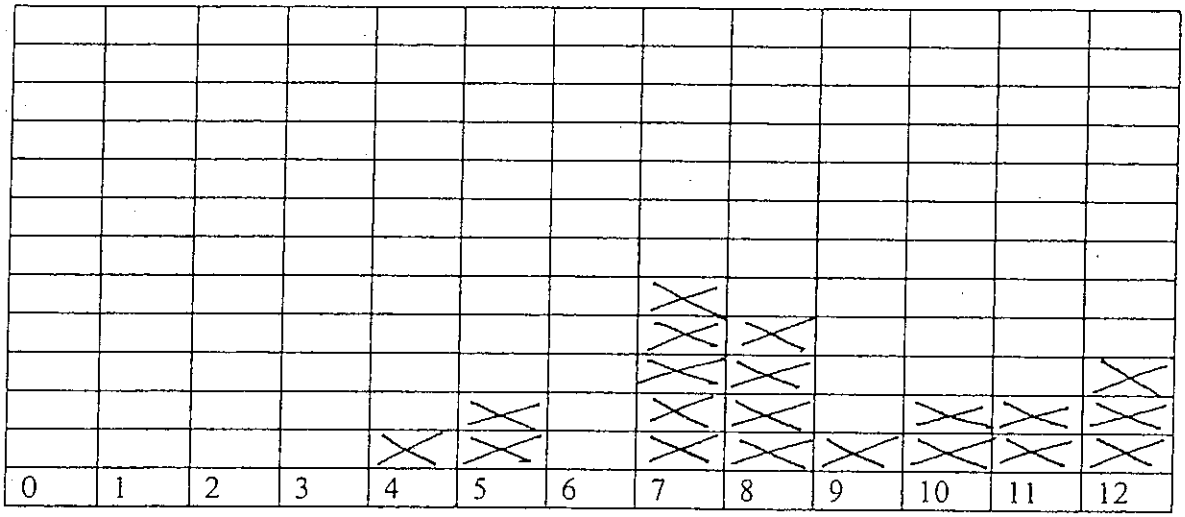
					x							
					x							
					x							
			x		x	x		x				
			x	x	x	x		x	x			
0	1	2	3	4	5	6	7	8	9	10	11	12

Extension one HSC trial

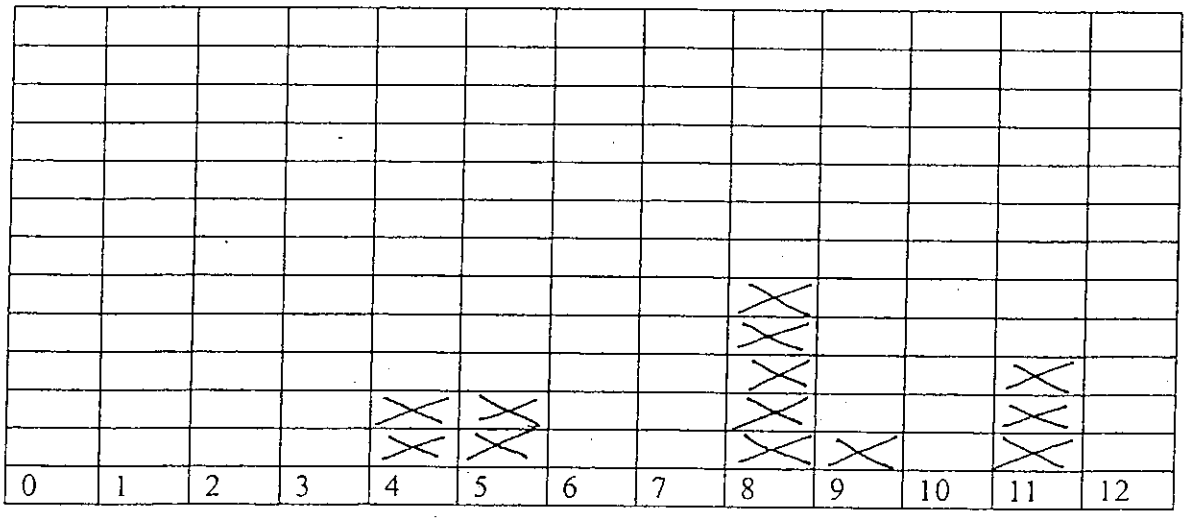
Question 4



BLC $\bar{x} = 11.1$



TDS $\bar{x} = 8.4$



BDD $\bar{x} = 7.7$

Year 12 Extension 1 Mathematics Q5 Trial HSC Examination

BLC mean = 10.9 Std Dev = 1.1

												x
											x	x
											x	x
										x	x	x
										x	x	x
										x	x	x
								x	x	x	x	x
0	1	2	3	4	5	6	7	8	9	10	11	12

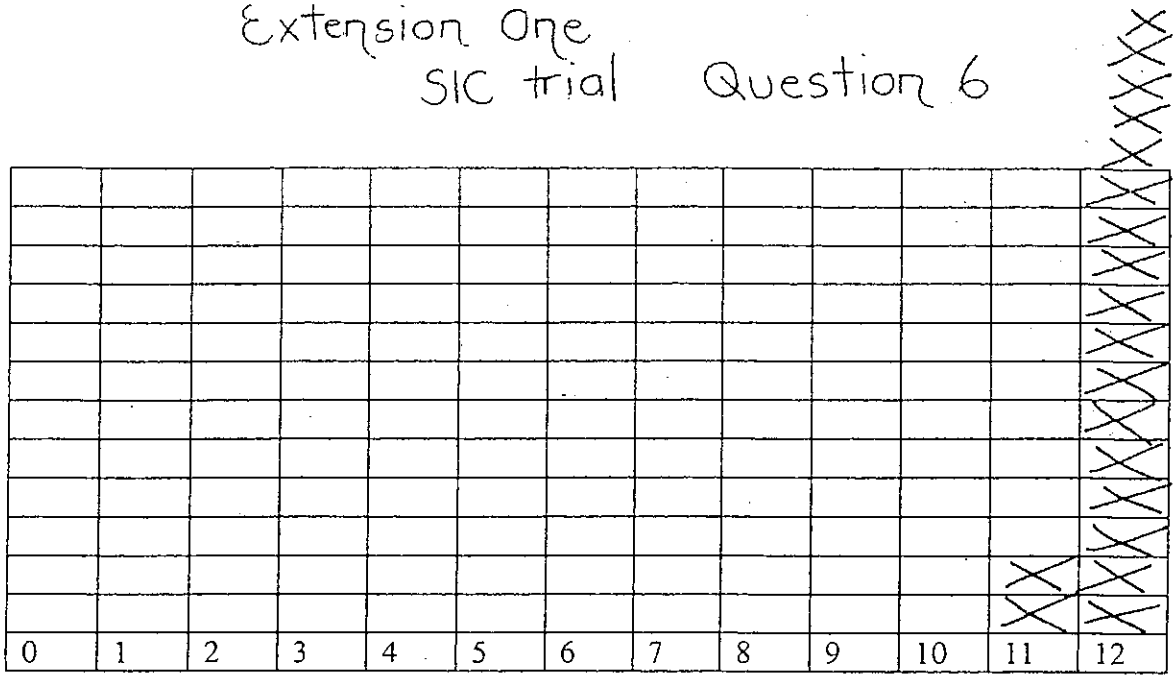
TDS mean = 7.4 Std Dev = 3.0

			x									
			x		x		x		x			x
			x		x		x		x	x		x
			x		x	x	x	x	x	x	x	x
0	1	2	3	4	5	6	7	8	9	10	11	12

BDD mean = 7.6 Std Dev = 2.7

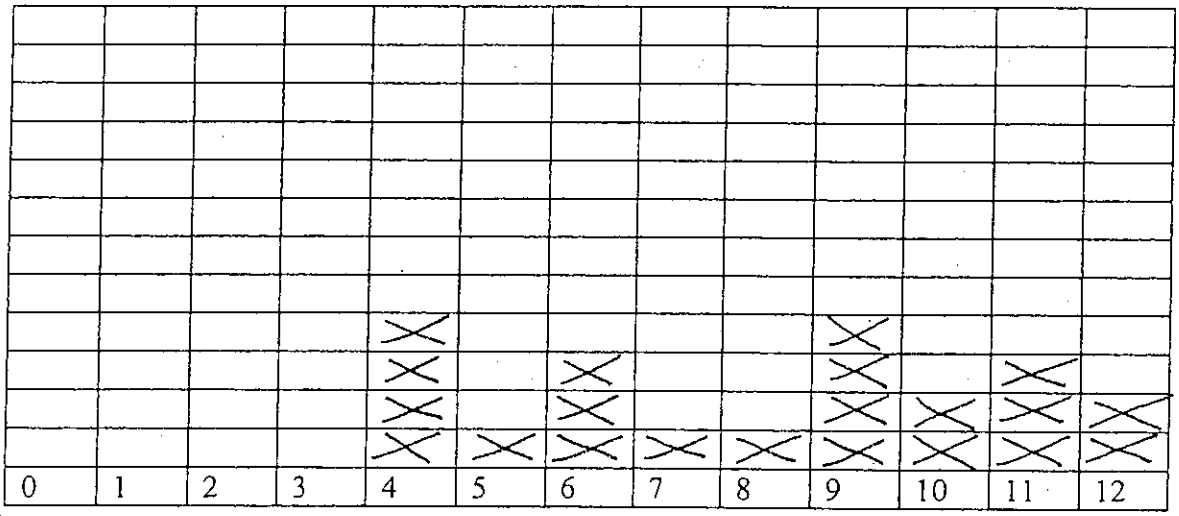
								x				
			x			x	x	x				x
			x			x	x	x	x	x		x
0	1	2	3	4	5	6	7	8	9	10	11	12

Extension One
SIC trial Question 6



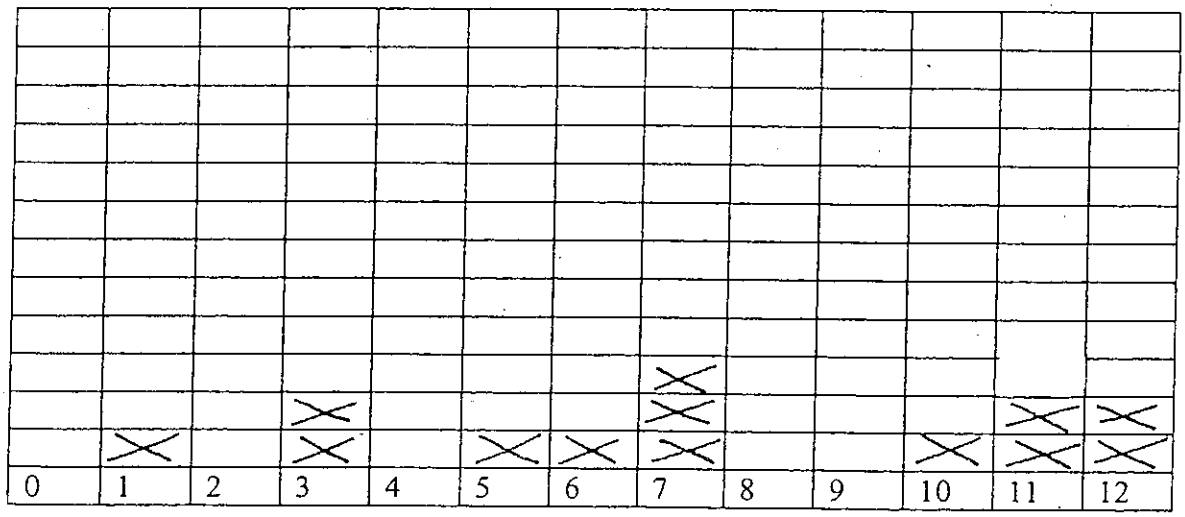
BLC

$\bar{x} = 11.9$



TDS

$\bar{x} = 7.95$



BDD

$\bar{x} = 7.31$

Year 12 Extension 1 Mathematics Q7 Trial HSC Examination

BLC mean = 10.0 Std Dev = 2.0

												x
												x
												x
												x
									x	x		x
								x	x	x	x	x
					x		x	x	x	x	x	x
0	1	2	3	4	5	6	7	8	9	10	11	12

TDS mean = 4.8 Std Dev = 2.5

			x		x							
			x		x	x						
		x	x		x	x	x					
x	x	x	x	x	x	x	x	x	x	x		
0	1	2	3	4	5	6	7	8	9	10	11	12

BDD mean = 5.2 Std Dev = 2.9

				x								
				x		x	x					
x	x		x	x	x	x	x		x		x	
0	1	2	3	4	5	6	7	8	9	10	11	12