Year 11 Extension one, Task One

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Teacher:________________

ST MARK'S COPTIC ORTHODOX COLLEGE

Mathematics Department

2010

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Year 11 Extension 1

Assessment Task One

GENERAL INSTRUCTION

- Reading time 5 minutes
- Working Time – 1 hour
- Write in black or blue pen only
- Approved calculators may be used
- All necessary working must be shown
- Begin each question on a different bo kl
- Attempt all questions

Question 1 (15 marks) *Start work on a new page* **Mark**

a) Solve for x; $|2x-3|+|x+1|=9$ 3

b) Simplify $\frac{x^2+5x+6}{x^2-16} \div \frac{x+3}{x-4}$ 3

c) Solve for x *and* y: $x = 2y-1$ *and* $3x^2 = x+2y^2$ 3

d) Solve for x: $2x^2-3x+1 \geq 0$ 3

e) Write $\frac{1+\sqrt{7}}{3-\sqrt{7}}$ in the form $a+b\sqrt{7}$, where a and b are rational. 3

Question 2 (14 marks) *Start work on a new page*

a) Solve $2 < |x+4| < 6$ 3

b) Simplify $\frac{x}{x^2-y^2} - \frac{x}{x^2+xy}$ 3

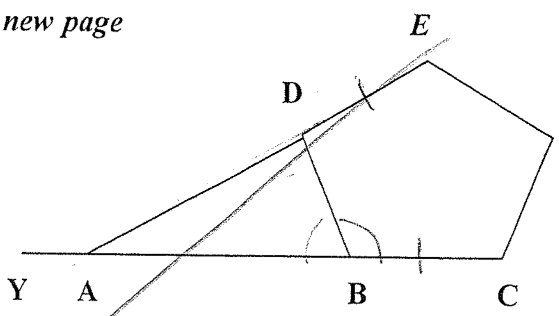
c) Factorise fully a^6-7a^3-8 3

d) Express $1.\dot{4}\dot{5}$ as a fraction in simplest form (All Working Out Must Be Shown). 2

e) Solve for x: $\frac{4}{5-x} \geq 1$ 3

Question 3 (13 marks) *Start work on a new page* Marks

a)



The diagram above shows a regular pentagon BCFED. The sides ED and CB are produced to meet at A. The point Y lies on CBA produced.

i. Find the size of angle DBC. 2

ii. Find the size of angle YAD, giving reasons. 3

b) Solve by completing the square $2x^2 + 8x + 3 = 0$ 4

c) Factorise fully: $zv(4x - 2)^4 - vm(3 - 6x)^3$. 3

d) Find the value of $\frac{x^3}{y^3 z^2}$ in index form if $x = \left(\frac{2}{5}\right)^5, y = \left(\frac{-2}{3}\right)^3, z = \left(\frac{3}{5}\right)^3$ 4

(ALL WORKING OUT MUST BE SHOWN, IN ORDER TO BE AWARDED THE MARKS)

Question 4 (~~17~~ 12 marks) *Start work on a new page* Marks

a) Solve simultaneously: $3x - 2y - z = -8$ 4

$$5x + y + 3z = 23$$

$$4x + y - 5z = -18$$

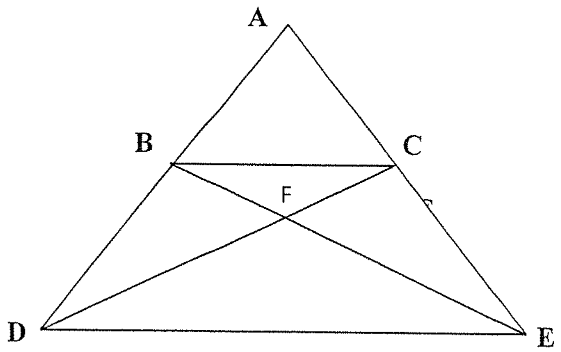
Marks

b) i) Show algebraically that: $a^2 + b^2 = (a+b)^2 - 2ab$ 1

ii) Hence, find the value of:

$\alpha)\ x^2 + y^2 \quad \beta)\ x^4 + y^4, if\ x + y = 3\ and\ xy = -1$ 4

c) In the figure below, $BC \parallel DE\ and\ AB : BD = 3 : 5$, Show that:



i) Δ ABC /// Δ ADE 3

ii) Δ BFC /// ΔEFD 3

iii) DF : FC = 8 : 3 2

END OF EXAM

Yr 11 EXT1 – 2010 Ass. Task One.

Question One

(1) $|2x-3|+|x+1|=9$

$+(2x-3) + +(x+1) = 9$

$3x - 2 = 9$

$x = \frac{11}{3} = 3\frac{2}{3}$

Test $|2(\frac{11}{3})-3| + |\frac{11}{3}+1| = 9$ True.

$+(2x-3) + -(x+1) = 9$

$x - 4 = 9$

$x = 13$

Test $|2(13)-3| + |13+1| = 9$. False

$-(2x-3) + +(x+1) = 9$

$-x + 4 = 9$

$x = -5$

Test $|2(-5)-3| + |-5+1| = 9$ False

13

$-(2x-3) + -(x+1) = 9$

$-3x + 2 = 9$

$-3x = 7 \Rightarrow x = -2\frac{1}{3}$

Test $|2(-2\frac{1}{3})-3| + |-2\frac{1}{3}+1| = 9$ True

(b) $\frac{x^2+5x+6}{x^2-16} \div \frac{x+3}{x-4}$

$= \frac{(x+2)\cancel{(x+3)}}{\cancel{(x-4)}(x+4)} \times \frac{\cancel{(x-4)}}{\cancel{(x+3)}}$

$= \frac{x+2}{x+4}$

(c) $x = 2y-1, \quad 3x^2 = x+2y^2$

$3(2y-1)^2 = 2y-1+\cancel{2}(2y^2\cancel{-1})$

$3(4y^2-4y+1) = 2y-1+2y^2$

$12y^2-12y+3-2y+1-2y^2=0$

$10y^2-14y+4=0$

$5y^2-7y+2=0$

$(5y-2)(y-1)=0$

$y=\frac{2}{5}$ OR $y=1$

$y=\frac{2}{5} \Rightarrow x = 2y-1$

$= 2(\frac{2}{5})-1$

$y=\frac{2}{5}, \quad x = -\frac{1}{5}$

$y=1 \Rightarrow x = 2y-1$

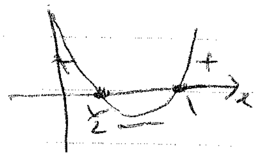
$= 2(1)-1$

$y=1, \quad x=1$

d) $2x^2-3x+1 \geq 0$

$(2x-1)(x-1) \geq 0$

$x \leq \frac{1}{2}, \quad x \geq 1$



e) $\frac{1+\sqrt{7}}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}}$

$= \frac{3+\sqrt{7}+3\sqrt{7}+7}{9-7}$

$= \frac{10+4\sqrt{7}}{2}$

$= 5+2\sqrt{7} = a+b\sqrt{7}$

Question 2

a) $2<|x+4|<6$

$2<+(x+4)<6$ or $2<-(x+4)<6$

$-2<x<2$ $\quad 2<-x-4<6$

$6<-x<10$

$-6>x>-10$

$-10<x<-6$

b) $\frac{x}{x^2-y^2} - \frac{x}{x^2+xy}$

$= \frac{x}{(x-y)(x+y)} - \frac{\cancel{x}\,1}{\cancel{x}(x+y)}$

$= \frac{x-(x-y)}{(x-y)(x+y)}$

$= \frac{\cancel{x}-\cancel{x}+y}{(x-y)(x+y)}$

$= \frac{y}{(x-y)(x+y)}$

Q2 -

c) $a^6 - 7a^3 - 8$

M $= (a^3 - 8)(a^3 + 1)$

$= (a-2)(a^2+2a+4)(a+1)(a^2-a+1)$

d) $1.\dot{4}\dot{5}$

$x = 1.45\ 45\ 45\ldots$

√ $100x = 145.45\ 45\ldots$

$x = 1.45\ 45\ldots$

$99x = 144$

$x = \frac{144}{99} = \frac{48}{33}$

$x = \frac{16}{11} = 1\frac{5}{11}$

e) $\frac{4}{\cancel{5-x}} \times (5-x)^{\cancel{2}} \geq 1 \times (5-x)^2$

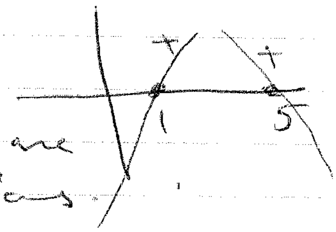
$4(5-x) - (5-x)^2 \geq 0$

$(5-x)[4-(5-x)] \geq 0$

M $(5-x)(x-1) \geq 0$

$1 \leq x \leq 5$

∴ $1 \leq x < 5$ are the solutions.



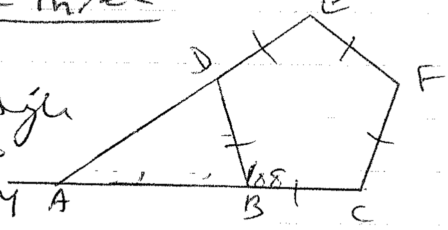
Question Three

a) i) Interior Angle

$= \frac{(n-2)\times 180}{n}$

$= \frac{(5-2)\times 180}{5}$

$= 108°$ ∴ ∠DBC = 108°



ii) ∠DBA = 72° (adj. supp. angles)

and ∠EDB = 72° (adj. supp. angles)

∴ ∠YAD = ∠ADB + ∠ABD

= 72 + 72

= 144° (Exterior angle of a triangle equal the sum of the interior opp. angles.

b) $2x^2 + 8x + 3 = 0$

$x^2 + 4x + (2)^2 - (2)^2 + \frac{3}{2} = 0$

$(x+2)^2 = \frac{5}{2}$

$(x+2) = \pm\sqrt{\frac{5}{2}}$

$x = -2 \pm \sqrt{\frac{5}{2}}$

c) $ZV(4x-2)^4 - Vm(3-6x)^3$

$= ZV(2)^4(2x-1)^4 - Vm(-3)^3(2x-1)^3$

$= 16ZV(2x-1)^4 + 27Vm(2x-1)^3$

$= V(2x-1)^3[16Z(2x-1) + 27m]$

$= V(2x-1)^3(32xZ - 16Z + 27m)$

d) $\frac{x^3}{y^3 z^2}$, $x = (\frac{2}{5})^5$, $y = (\frac{-2}{3})^3$, $z = (\frac{3}{5})^3$

$= \frac{((\frac{2}{5})^5)^3}{((\frac{-2}{3})^3)^3((\frac{3}{5})^3)^2}$

$= \frac{\frac{2^{15}}{5^{15}}}{(\frac{(-2)^9}{3^9})(\frac{3^6}{5^6})}$

$= \frac{2^{15}}{5^{15}} \div \left(-\frac{2^9}{\cancel{3^9}\,3^3} \times \frac{\cancel{3^6}}{5^6}\right)$

$= \frac{2^{15}}{5^{15}} \div \frac{-2^9}{3^3 \times 5^6}$

$= \frac{\cancel{2^{15}}\,2^6}{\cancel{5^{15}}\,5^9} \times \frac{3^3 \times \cancel{5^6}}{-\cancel{2^9}}$

$= \frac{2^6 \times 3^3}{-5^9}$

= ______

Question four

(a) $3x - 2y - z = -8$ ①
$5x + y + 3z = 23$ ②
$4x + y - 5z = -18$ ③

① $\rightarrow$ $3x - \cancel{2y} - z = -8$
② $\xrightarrow{\times 2}$ $10x + \cancel{2y} + 6z = 46$ (+)

$13x + 5z = 38$ ④

② $\rightarrow$ $5x + \cancel{y} + 3z = 23$
③ $\rightarrow$ $4x + \cancel{y} - 5z = -18$ (−)

$x + 8z = 41$ ⑤

④ $\rightarrow$ $13\cancel{x} + 5z = 38$
⑤ $\xrightarrow{\times 13}$ $13\cancel{x} + 104z = 533$ (−)

$-99z = -495$
$\boxed{z = 5}$

$\therefore x + 8z = 41$
$x + 8(5) = 41$
$\boxed{x = 1}$

$\therefore 3x - 2y - z = -8$
$3(1) - 2y - 5 = -8$
$-2y - 2 = -8$
$-2y = -6$
$\boxed{y = 3}$

c(i) $a^2 + b^2 = (a+b)^2 - 2ab$

$R.HS = a^2 + 2ab + b^2 - 2ab$
$= a^2 + b^2$

$= L.HS$

(ii) α) $x^2 + y^2 = (x+y)^2 - 2xy$
$= (3)^2 - (-1)$
$= 10$

B) $x^4 + y^4 = (x^2+y^2)^2 - 2x^2y^2$
$= (10)^2 - 2(-1)^2$
$= 100 - 2$
$= 98$

d) (i) In Δ's ABC, ADE

$\angle A$ is a common angle

$\angle ABC = \angle ADE$ (corresponding angles are equal, $BC \parallel DE$)

$\angle ACB = \angle AED$ (corresponding angles are equal, $BC \parallel DE$)

$\therefore \Delta ABC \parallel\!| \Delta ADE$

(ii) In Δ's BFC, EFD

$\angle BFC = \angle DFE$ (vertically oppos. angles)

$\angle CBF = \angle FED$ (alternate angles are equal, $BC \parallel DE$)

$\angle BCF = \angle FDE$ (alternate angles are equal, $BC \parallel DE$)

$\therefore \Delta BFC \parallel\!| \Delta EFD$

(iii) $\because \Delta ABC \parallel\!| \Delta ADE \therefore \dfrac{AB}{AD} = \dfrac{BC}{DE}$ ①

and $\because \Delta BFC \parallel\!| \Delta EFD \therefore \dfrac{CF}{FD} = \dfrac{BC}{DE}$ ②

From ① & ② $\dfrac{AB}{AD} = \dfrac{CF}{FD}$, Since $\dfrac{AB}{BD} = \dfrac{3}{5}$

$\therefore \dfrac{3}{8} = \dfrac{CF}{FD}$

$\therefore \dfrac{DF}{FC} = \dfrac{8}{3}$

$\therefore DF : FC = 8 : 3$