Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics
Extension 1

Afternoon Session
Thursday, 20 August 2009

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 84
- Attempt Questions 1-7
- All questions are of equal value

Disclaimer
Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, *Principles for Setting HSC Examinations in a Standards-Referenced Framework* (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and *Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework* (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

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Total marks – 84
Attempt Questions 1–7
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the remainder when $P(x) = x^3 - 3x^2 + 3x - 5$ is divided by $x - 2$. **2**

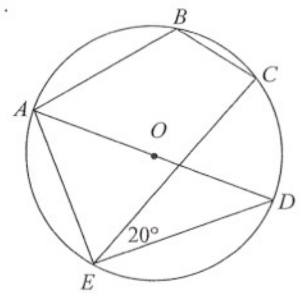
(b) Find $\int \sin^2 6x \, dx$. **2**

(c) Sketch the graph of $y = 3\sin^{-1}(2x)$, clearly indicating the domain and range. **3**

(d) (i) Find the Cartesian equation of the curve with parametric equations $x = \cos t$ and $y = 3 + \sin t$. **2**

(ii) Describe this locus geometrically. **1**

(e) In the diagram AOD and EC are straight lines, O is the centre of the circle, and $\angle CED = 20^\circ$. **2**



NOT TO SCALE

Find $\angle ABC$, giving reasons for your answer.

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\lim_{x \to 0} \dfrac{\sin 3x}{x}$. **1**

(b) Use the substitution $u = 3x - 1$ to evaluate $\int_1^2 \dfrac{x}{3x-1}\,dx$. **3**

(c) Find all real numbers such that $\ln(2x+3) + \ln(x-2) = 2\ln(x+4)$. **4**

(d) (i) From a group of 7 girls and 6 boys, 3 girls and 2 boys are chosen. How many different groups of 5 are possible? **2**

(ii) If the group of 5 stands in a line what is the probability that the boys stand together? **2**

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve the inequality $\dfrac{x^2-4}{x+3} < x - 4$ for x. **3**

(b) Prove by Mathematical Induction that $3^{3n} + 2^{n+2}$ is divisible by 5, for all positive integers n. **4**

(c) A particle, P, moves on the x-axis for time $t \geq 0$, in seconds, with velocity $v = \dfrac{2}{1+3x}$ cms^{-1}, where x, in centimetres, is the displacement from the origin $x = 0$.

(i) Find an expression for the acceleration, a cms^{-2}, and show that a varies directly with v^3. **3**

(ii) If the particle was initially at the origin, describe the motion both initially and as $t \to \infty$. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The function $f(x) = e^x - x - 2$ has a zero near $x = 1.2$. **2**

Use one application of Newton's method to find a second approximation to the zero. Write your answer correct to three significant figures.

(b) A function is defined by $f(x) = e^{3x} - 1$ for all real x.

(i) Draw the graph of $y = f(x)$ and state the range of the function. **2**

(ii) Find the inverse function, $f^{-1}(x)$, clearly indicating any restrictions. **3**

(c) A particle moves in a straight line so that its displacement x cm from the origin at time $t \geq 0$, in seconds, is given by $x = \sqrt{3}\cos 3t - \sin 3t$.

(i) Show that the particle moves in simple harmonic motion. **2**

(ii) Find the velocity when the particle is 1 cm from the origin on its first oscillation. **3**

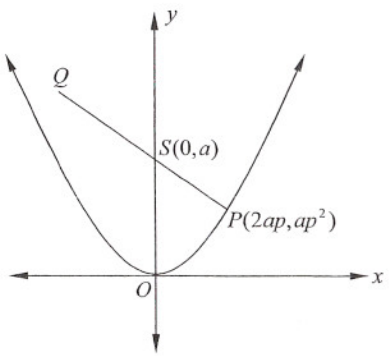
Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) If the roots of $x^3 - 6x^2 + 3x + k = 0$ are consecutive terms of an arithmetic series show that one of the roots is 2. **2**

(ii) Hence find the value of k and the other two roots. **3**

(b) Show that $\dfrac{\tan 2\theta - \tan\theta}{\tan 2\theta + \cot\theta} = \tan^2\theta$. **3**

(c) $P(2ap,\ ap^2)$ is a point on the parabola $x^2 = 4ay$ with focus $S(0,\ a)$. The point Q lies on PS produced and Q divides PS so that $PQ : QS = -4 : 3$.



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(i) Show that Q has coordinates $\left(-6ap,\ a(4-3p^2)\right)$. **2**

(ii) Show that as P varies, the locus of Q is a parabola. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Simplify $\dfrac{2^{4n} \times 3^{2n}}{8^n \times 6^n} + 3^n$. **3**

(b) A balloon in the shape of a cylinder, with height h and radius r, expands so that h is always proportional to r, that is $h = kr$ for some constant k. When $r = 4\,\text{cm}$, the volume is expanding at the rate of $0.2\,\text{cm}^3\,\text{s}^{-1}$.

(i) Show that when $r = 4\,\text{cm}$ the rate of change of the radius is given by $\dfrac{dr}{dt} = \dfrac{1}{240\pi k}$. **2**

(ii) If the surface area of the balloon is expanding at the rate of $0.1\,\text{cm}^2\,\text{s}^{-1}$ when $r = 4\,\text{cm}$, find the constant of proportionality, k. **3**

(c) (i) Differentiate both sides of the expansion $(1+x)^{2n} = \sum_{k=0}^{2n} {}^{2n}C_k\, x^k$. **2**

(ii) Hence show that $\sum_{k=1}^{2n} k\, {}^{2n}C_k = n \times 4^n$. **2**

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

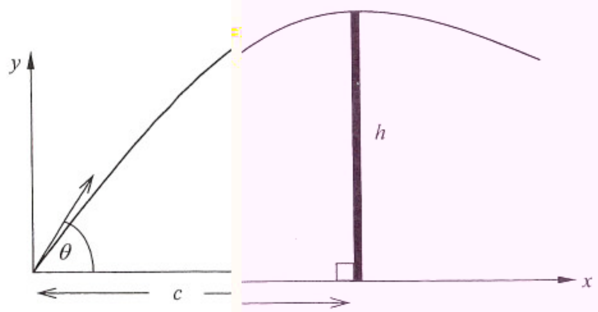
(a) A student is taking a test with 50 multiple-choice questions and guesses the answer to each one. The probability of guessing a question correctly is 0.3.

(i) What is the probability that the student answers 25 questions correctly? **2**

(ii) What is the most likely number of questions answered correctly? **3**

(b) A vertical wall, height h metres, stands on horizontal ground. When a projectile is fired, in a vertical plane which is at right angles to the wall, from a point on the ground c metres from the wall, it just clears the wall at the highest point of its path. The equations of motion for the projectile with angle of projection, θ, are:

$x = Vt\cos\theta \qquad y = Vt\sin\theta - \frac{1}{2}gt^2$ (Do not prove these.)



(i) Show that the particle reaches the highest point on its path when $t = \dfrac{V\sin\theta}{g}$. **2**

(ii) Show that the speed of projection is given by $V^2 = \dfrac{g}{2h}\left(4h^2 + c^2\right)$. **3**

(iii) Find the angle of projection, θ, in terms of h and c. **2**

End of paper