# 2000 CSSA TRIAL HSC EXAMINATION
## 2 UNIT MATHEMATICS
## SUGGESTED SOLUTIONS

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### QUESTION 1

(a) 0·30

(b) $\frac{x-6}{3} = \frac{4x}{5}$

$5(x-6) = 12x$

$5x - 30 = 12x$

$-7x = 30$

$x = -4\frac{2}{7}$

(c) $\angle YUV = 53°$ (alt $\angle$s; $XY \parallel UW$)

$\therefore \theta = 108 + 53$ (ext $\angle$ of $\Delta$)

$= 161°$

(d) $\frac{x^4}{4} + 4x + C$

(e) $g(-4) = 2(-4) - 1 = -9$

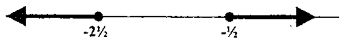
$g(2) = -3$

$g(-4) + g(2) = -12$

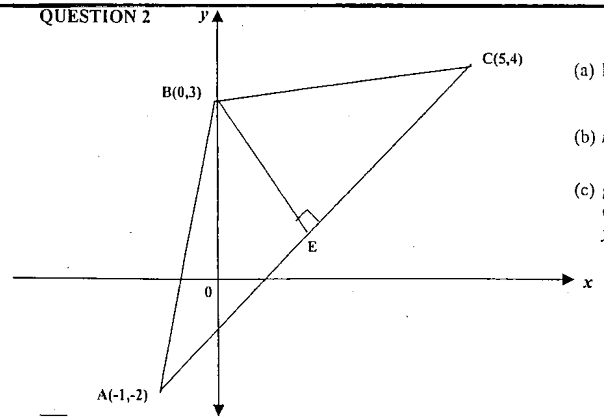
(f) $2x + 3 \geq 2$ or $-(2x+3) \geq 2$

$x \geq -½$ $\quad$ $2x + 3 \leq -2$

$x \leq -2½$




### QUESTION 2




(a) $E(\frac{-1+5}{2}, \frac{-2+4}{2}) = E(2,1)$

(b) $m_{AC} = \frac{4+2}{5+1} = 1$

(c) grad of perp. line is $-1$

eqn of L is

$y - 3 = -1(x - 0)$

$\therefore y = 3 - x$

(d) $1 = 3 - 2$ true

$\therefore$ E lies on L.

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(e)

$3\sqrt{2}$ units In $\Delta BAE$ and $\Delta BCE$

centre E and radius

BE is common

AE = CE (E is the midpoint of AC)

$\angle BEA = \angle BEC = 90°$ ($BE \perp AC$)

$\therefore \Delta BAE \equiv \Delta BCE$ (SAS)

(f) (i) $AC = \sqrt{(5+1)^2 + (4+2)^2} = \sqrt{}$

$\therefore$ radius of the circle is $½ \times 6\sqrt{2} =$

(ii) Equation of the circle

$3\sqrt{2}$ is $(x-2)^2 + (y-1)^2 = 18$

### QUESTION 3

(a)

(i) $\frac{d}{dx}(3x^2+2)^3$

$= 3(3x^2+2)^2 \times 6x$

$= 18x(3x^2+2)^2$

(ii) $\frac{d}{dx}(3x \cos 2x)$

$= 3x \times -2 \sin 2x + 3 \cos 2x$

$= 3 \cos 2x - 6x \sin 2x$

(iii) $\frac{d}{dx}(\frac{e^{3x}}{x})$

$= \frac{3xe^{3x} - e^{3x}}{x^2}$

(b)

(i) $\int_0^{\pi/4} \sin 3x \, dx = \left[-\frac{1}{3} \cos 3x\right]_0^{\pi/4}$

$= -\frac{1}{3}\left[\cos\frac{3\pi}{4} - \cos 0\right]$

$= -\frac{1}{3}\left[-\frac{1}{\sqrt{2}} - 1\right] = \frac{1}{3}\left(\frac{1+\sqrt{2}}{\sqrt{2}}\right)$

(ii) $\int_0^3 e^{2x+3} \, dx = \left[½ e^{2x+3}\right]_0^3$

$= ½ [e^9 - e^3]$

(c) $\int \frac{x}{x^2+1} \, dx = ½ \ln(x^2+1) + C$

### QUESTION 4

(a)

(i) $a + 2d = 7$

$a + 9d = 42$

$\therefore 7d = 35;\ d = 5$

$\therefore a = -3$

(ii) $S_{10} = 5(-3 + 42) = 5 \times 39$

$= 195$

(b)

(i) $P(BB) = \frac{5}{16} \times \frac{4}{15}$

$= \frac{1}{12}$

(ii) P(at most 1 B) = 1 – P(BB)

$= 1 - \frac{1}{12} = \frac{11}{12}$

(c)

(i) $y = 9 - x^2$

$\frac{dy}{dx} = -2x$

At (1,8) grad of the tangent is –2.

$\therefore$ the eqn of the tangent at (1,8) is $y - 8 = -2(x-1)$.

(ii) Tangent crosses the $x$ axis when $y = 0$.

$\therefore 2x + 0 = 10$

$x = 5$.

$\therefore$ the point is (5,0).

(iii) Area = area of $\Delta$ - curve

$= ½ \times 4 \times 8 - \int_1^3 (9 - x^2)dx$

$= 16 - \left[9x - \frac{x^3}{3}\right]_1^3$

$= 16 - [(27-9) - (9 - \frac{1}{3})]$

$= 16 - 9\frac{1}{3} = 6\frac{2}{3}$ units$^2$

### QUESTION 5

(a)

(i) $\frac{BD}{\sin 41°29'} = \frac{9{\cdot}8}{\sin 103°16'}$

$BD = \frac{9{\cdot}8}{\sin 103°16'} \times \sin 41°29'$

$= 6.669534032$ cm

$= 6.7$ cm or 67 mm

(ii) $\angle ABC = 180 - (41°\,29' + 61°19')$

$= 77°12'$

Area of $\Delta ABC = ½ \times 9.8 \times 7.4 \times \sin 77°12'$

$= 35.35891559$

$= 35$ cm$^2$

(b)

(i) $y = 4x^3 + 6x^2$

$\frac{dy}{dx} = 12x^2 + 12x$

$\frac{d^2y}{dx^2} = 24x + 12$

Stationary points $\frac{dy}{dx} = 0 \therefore 12x(x+1) = 0$

Hence $x = 0$ or $x = -1$

The coordinates of the stationary points are (0,0) or (-1,2)

Nature of the stationary points:

At (0,0), $\frac{d^2y}{dx^2} = 24(0) + 12 > 0 \therefore$ Min at (0,0)

At (-1,2), $\frac{d^2y}{dx^2} = 24(-1) + 12 < 0 \therefore$ Max at (-1,2)

(ii) Points of inflexion $\frac{d^2y}{dx^2} = 0$

$24x + 12 = 0$

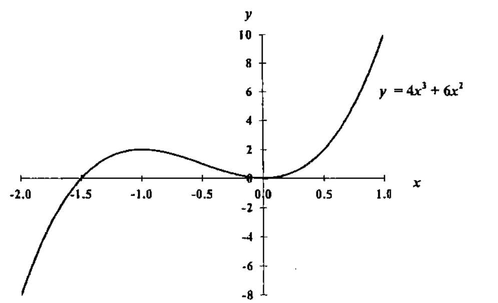
$x = -½$

When $x < -½$, $\frac{d^2y}{dx^2} < 0$

When $x > -½$, $\frac{d^2y}{dx^2} > 0$

$\therefore$ Concavity changes. Point of inflexion at the point where $x = -½$.

(iii)




(iv) The maximum value of $4x^3 + 6x^2$ is 10

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**QUESTION 6**

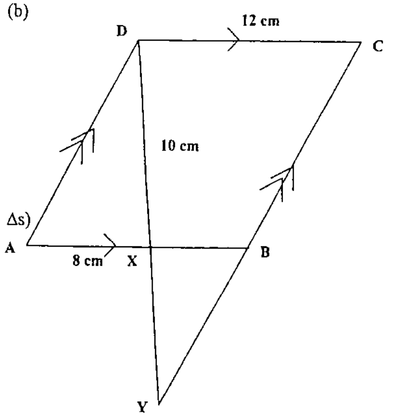
(a) $\ln x = \ln x^2$

$\therefore x = x^2$

$\therefore x = 0$ or $1$

But $x \neq 0$ since $\ln x$ is defined for $x > 0$ $\therefore x = 1$

(b)




(ii) In $\Delta ADX$ and $\Delta CYD$

$\angle ADX = \angle CYD$ (alt. $\angle$s; AD||YC)

$\angle DAX = \angle YCD$ (opp $\angle$s of parm)

$\therefore \Delta ADX ||| \Delta CYD$ (AA)

(iii) $\dfrac{DX}{YD} = \dfrac{AX}{CD}$ (corr. sides of similar Δs)

$\dfrac{10}{YD} = \dfrac{8}{12}$

$YD = \dfrac{120}{8}$ $\quad \therefore YD = 15$ cm

Hence XY = 15 – 10 = 5 cm

(c) $V = \pi \int_0^5 (5y+5)^2\, dy \; - \pi \int_0^5 (y^2+5)^2\, dy$

$= \pi \left[ \dfrac{(5y+5)^3}{3 \times 5} - \left(\dfrac{y^5}{5} + \dfrac{10y^3}{3} + 25y\right) \right]_0^5$

$= \pi \left[ \dfrac{30^3}{15} - \left(625 + \dfrac{1250}{3} + 125 - 0\right) - \dfrac{25}{3} \right]$

$= 625\pi$ units$^3$.

**QUESTION 7**

(a)

(i) $M = e^{-2(0)} + 3$

$= 4$ mL was initially injected into the cat.

(ii) $\dfrac{dM}{dt} = -2e^{-2t}$

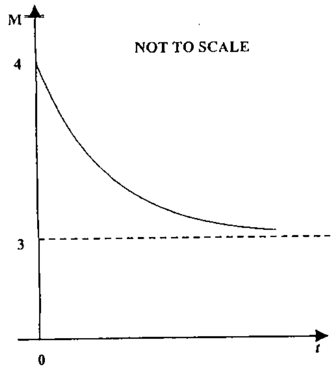
When $t = 3$, $\dfrac{dM}{dt} = -2e^{-2(3)} = -2e^{-6}$

$\therefore$ The amt of vaccine is decreasing at the rate of $2e^{-6}$ mL/h which is approximately 0.005 mL/h.

(iii) As $t \to \infty$, $e^{-2t} \to 0$, $\therefore M \to 3$.

$\therefore$ There will always be more than 3 mL of vaccine present in the cat's bloodstream.

(iv)

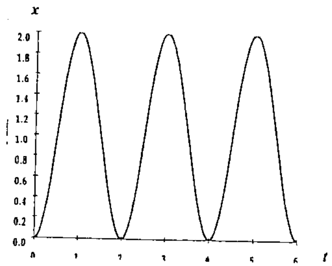



(b)

(i) When $t = 0$, $x = 1 - \cos 0 = 0$

$\therefore$ The particle is initially at $x = 0$.

(ii)




(iii) $v = \pi \sin \pi t$.

(iv) When $t = \dfrac{1}{6}$, $v = \pi \sin \dfrac{\pi}{6} = \dfrac{\pi}{2}$ m/s

(v) Maximum speed when acceleration is zero.

$a = \pi^2 \cos \pi t = 0$

$\therefore \cos \pi t = 0$

$\pi t = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \ldots$

$\therefore t = ½$ second when the particle first reaches its maximum speed.

**QUESTION 8**

(a)

(i) Interest rate 18% pa = 1.5% per month = 0.015 per month.

Interest charged at the end of the first month \$(0.015 × 15 000)

$\therefore$ Total amount owing after making the first instalment is \$(15 000 + 0.015 × 15 000 – M)

= \$[15 000(1.015) – M]

(ii) After making the second instalment the amount owing is:

\$[15 000(1.015) – M](1.015) – M

= \$[15 000$(1.015)^2$ – 1.015M – M]

Immediately after making the third instalment the amount owing is:

\$[15 000$(1.015)^2$ – 1.015M – M](1.015) – M]

= \$[15 000$(1.015)^3$ – $(1.015)^2$M – 1.015M – M]

=\$[15 000$(1.015)^3$ – M(1 + 1.015 + $1.015^2$)]

(iii) $\therefore$ Immediately after making the 60th instalment the amount owing is 0.

$\therefore 15\,000(1.015)^{60} - M(1 + 1.015 + 1.015^2 + \ldots + 1.015^{59}) = 0$

$\therefore M\left[\dfrac{1(1.015^{60} - 1)}{1.015 - 1}\right] = 15\,000(1.015)^{60}$

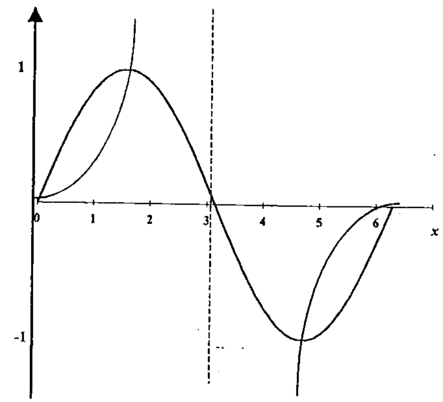
$M = \dfrac{15\,000(1.015)^{60} \times 0.015}{1.015^{60} - 1}$

$M = \$380.90$

(b) $\int_0^8 f(x)\,dx \approx \dfrac{2}{3}[(0.9 + 1.7) + 4(1.4 + 2.1) + 2(1.8)] \approx 13{\cdot}5$

(c)

(i)




(ii) $\sin x = \tan \dfrac{x}{2}$ has 4 solutions in the domain $0 \le x \le 2\pi$.

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## QUESTION 9

(a)

(i) $A = 90\pi = \frac{1}{2} \times 15^2 \times \theta$

$\therefore\ \theta = \frac{180\pi}{15^2}$ radians

$\theta = \frac{4\pi}{5}$

(ii) Length of arc ABC $= 15 \times \frac{4\pi}{5} = 12\pi$ cm

$AC^2 = 15^2 + 15^2 - 2 \times 15 \times 15 \times \cos\frac{4\pi}{5}$

$\therefore$ AC = 28.53 cm

$\therefore$ Perimeter of segment ABC is $12\pi + 28.5$

$\approx 66$ cm.

(b)

(i) Distance from lighthouse to B is $\sqrt{6^2 + x^2}$ km

Rowing speed is 6 km/h

$\therefore\ 6 = \frac{\sqrt{36 + x^2}}{T}$

$\therefore\ T = \frac{\sqrt{36 + x^2}}{6}$ hours

(ii) Distance from B to C is $(10 - x)$

Jogging speed 10 km/h

Time taken to jog from B to C

$10 = \frac{10 - x}{T}$

$\therefore T = \frac{10 - x}{10}$ hours

Therefore the total time it takes to travel from the lighthouse to the general store is given by

$T = \frac{\sqrt{36 + x^2}}{6} + \frac{10 - x}{10}$ hours.

(iii) $\frac{dT}{dx} = \frac{x}{6\sqrt{36 + x^2}} - \frac{1}{10}$

$\frac{dT}{dx} = 0$ when

$\frac{x}{6\sqrt{36 + x^2}} = \frac{1}{10}$

$100x^2 = 36(36 + x^2)$

minimum

$(100 - 36)x^2 = 36^2$

km.

$\therefore x^2 = \frac{36^2}{64}$

$x = \frac{36}{8} = 4\frac{1}{2}$ km

When $0 < x < 4\frac{1}{2}$, $\frac{dT}{dx} < 0$

When $x > 0$, $\frac{dT}{dx} > 0$

$\therefore$ Minimum time when $x = 4\frac{1}{2}$ km.

Alt. $\frac{d^2T}{dx^2} = \frac{6\sqrt{36 + x^2} - x \times 6(1/2)(36+x^2)^{-1/2} \times 2x}{36(36+x^2)}$

$= \frac{6(36 + x^2) - 6x^2}{36(36 + x^2)^{3/2}}$

$= \frac{6}{(36+x^2)^{3/2}} > 0$ for all values of $x$

$\therefore$ the concavity is always upwards so

time at the stationary point, when $x = 4\frac{1}{2}$

(iv) The quickest time when

$T = \frac{(36 + 81/4)^{1/2}}{6} + \frac{11}{20}$

$\approx$ 1 hour and 48 minutes.

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## QUESTION 10

(a)

(i) AO = OP (radius of circle)

Δ)

$\therefore \Delta APO$ is isosceles (2 sides equal)

Δ)

(ii) $\angle OAP = \angle OPA$ (base $\angle$s of isos

$\angle OAP + \angle OPA = 2x$ (ext. $\angle$ of

$\therefore\ \angle OAP = \angle OPA = x$

(iii) Using $\Delta OPN$, $\sin 2x = \frac{PN}{OP}$

But $OP = \frac{1}{2}AB$ (radius & diameter)

$\therefore \sin 2x = \frac{PN}{1/2AB} = \frac{2PN}{AB}$

(iv) In $\Delta PAB$, $\cos x = \frac{AP}{AB}$

In $\Delta APN$, $\sin x = \frac{PN}{AP}$

$\therefore \sin x \times \cos x = \frac{PN}{AB} = \frac{1}{2}\sin 2x$

$\therefore 2\sin x \cos x = \sin 2x$

(b)

(i) P(Kellie loses and Lachlan wins on 1st throw)

$= \frac{30}{36} \times \frac{6}{36} = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$

(ii) P( Lachlan wins on 1st or 2nd throw)

$= \frac{5}{36} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

$= \frac{5}{36} + \frac{5^3}{6^4}$

(iii) For Lachlan to win the game:

P(Lachlan wins) $= (\frac{5}{6})(\frac{1}{6}) + (\frac{5}{6})^3(\frac{1}{6}) + (\frac{5}{6})^5(\frac{1}{6}) + (\frac{5}{6})^7(\frac{1}{6}) + \ldots$

$= \frac{(\frac{5}{6})(\frac{1}{6})}{1 - (\frac{5}{6})^2} = \frac{5}{36} \times \frac{36}{11} = \frac{5}{11}$