N.S.W. DEPARTMENT OF EDUCATION

HIGHER SCHOOL CERTIFICATE EXAMINATION 1975

MATHEMATICS - PAPER B (2F) - EQUIVALENT TO 3U (AND 4U - 1ST PAPER)

Instructions: Time allowed 3 hours. All questions may be attempted. (Questions are not of equal value.) In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.
Mathematical tables will be supplied. Approved slide rules or calculators may be used.

QUESTION 1 (12 Marks)

(i) Find the vertex and the axis of symmetry of $y = 2x^2 + 4x + 3$

(ii) Find the derivative of $e^x \cos x$

(iii) Find the equation of the tangent to $y = \frac{1}{1 + x^2}$ at the point where $x = 1$.

(iv) Sketch (not on graph paper) the curve $y = 2 + x - x^2$ and find the area above the x- axis and under the curve.

QUESTION 2 (9 Marks)

(i) The first two terms of an arithmetic sequence are -17, -14. Write down the sum of the first n terms. What is the least value of n for which the sum of the first n terms is positive?

(ii) Evaluate $\int_0^{e-1} \frac{dx}{1 + x}$

(iii) In how many ways can a team of 4 be chosen from 7 players?

QUESTION 3 (9 Marks)

(i) Write down the equation of the line through $(-4, -3)$ perpendicular to $2x + y = 5$.

(ii) Use one application of Newton's Method to find approximately the root of $\cos x + \sin x = x$ nearest to $x = 1.2$ ($\sin 1.2 = 0.932$, $\cos 1.2 = 0.362$).

(iii) The position at time t of a particle moving along the x- axis is given by $x = 2t^3 - 9t^2 + 12t$.
When and where does the particle first come to rest?

QUESTION 4 (10 Marks)

(i) Find the sum to infinity of the geometric progression $1 + (\sqrt{3} - 1) + (\sqrt{3} - 1)^2 + \ldots$ Give your answer to six significant figures ($\sqrt{3} = 1.73205$ to six significant figures)

(ii) Prove by induction that $\frac{1}{x^n(x-1)} = \frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2} - \ldots - \frac{1}{x^n}$ for all positive integers n and $x \neq 0, 1$

QUESTION 5 (10 Marks)

P is the point $(2at, at^2)$ on the parabola $4ay = x^2$ and ℓ is the tangent at P.

(i) Prove that the equation of ℓ is $y = tx - at^2$

(ii) If ℓ cuts the x- axis at A and the y- axis at B, find the co-ordinates of A and B.

(iii) In what ratio does P divide AB?

(iv) What is the slope of the line joining P to the focus S?

(v) Show that ℓ makes equal angles with the y- axis and with PS.

QUESTION 6 (10 Marks)

(i) Sketch (not on graph paper) $y = \sin x$, $y = \frac{2}{\pi}x$ with the same coordinate axes for $0 \leq x \leq \pi$. Find the area enclosed between the two curves on this sketch.

(ii) Differentiate $\tan^{-1}\sqrt{x^2 - 1}$

(iii) Show that $\cot\theta + \tan\frac{1}{2}\theta = \operatorname{cosec}\theta$

QUESTION 7 (10 Marks)

(i) Indicate by shading the region $a \geq y \geq \frac{x^2}{4a}$. (Do not use graph paper.) Find the volume of the solid of revolution obtained by rotating this region about the y- axis.

(ii) Two dice (each with faces labelled 1, 2, 3, 4, 5, 6) are thrown together.

(a) What is the probability of a double six in a throw?

(b) What is the probability of no double six in n throws?

(c) For what values of n will the probability of (at least) one double six in n throws be greater than $\frac{1}{2}$?

QUESTION 8 (10 Marks)

(i) Use the remainder theorem to find one factor of $x(x + 1) - a(a + 1)$. By division, or otherwise, find the other factor.

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(ii) (a) Find the distance r from the origin O to the plane α whose equation is $2x + y + 2z = 9$.

(b) The sphere of radius r, centre O, touches α at A. Find the co-ordinates of A.

(c) Describe geometrically the sphere passing through O which touches α at A and write down its equation.

QUESTION 9 (10 Marks)

A particle moves on a line so that its distance from the origin at time t is x.

(i) Prove that $\frac{d^2x}{dt^2} = \frac{d}{dx}(\frac{1}{2}v^2)$ where v denotes velocity.

(ii) If $\frac{d^2x}{dt^2} = 10x - 2x^3$ and $v = 0$ at $x = 1$, find v in terms of x.

(iii) Describe the motion. Is it simple harmonic?

QUESTION 10 (10 Marks)

(i) Sketch (not on graph paper) the graph of $\log\{x(1 - x)\}$

(ii) When the temperature of a body is T degrees above the temperature (assumed constant) of its surroundings, its temperature falls at a rate proportional to T.

(a) Write down the equation satisfied by T and, by integration, show that $T = Ae^{-kt}$ where A, k are constants and t denotes time.

(b) If the body is initially at 95° C, the surroundings at 25° C, and the body cools to 75° C after 15 minutes, what will its temperature be after a further 20 minutes?