Independent.

2004

Higher School Certificate

Trial Examination

Mathematics
Extension 1

General Instructions

- Reading time – 5minutes
- Working time – 2 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided with this paper
- All necessary working should be shown in every question

Total marks - 84

Attempt Questions 1 – 7

All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: ... ...

Student name / number .....

Question 1 **Begin a new page** **Marks**

a. Find the domain of the function $f(x) = \log_e\left(\frac{5-x}{3-x}\right)$ **3**

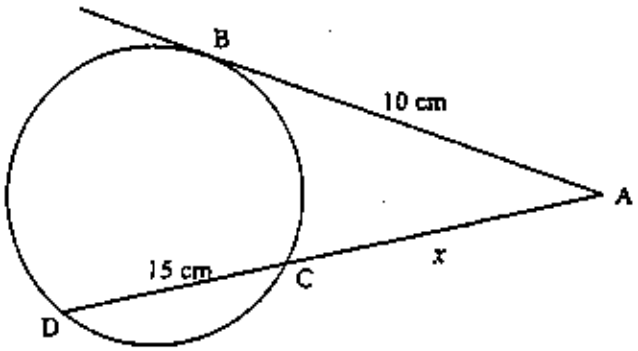
b. Consider the curves $y = x^3$ and $y = x^2 - x$.

i. Show that the curves intersect at the point where $x = 0$. **1**

ii. Find the acute angle between the curves at this point. **2**

c. If $P(x) = x^4 - 3x^3 + ax^2 - ax - 12$ is divisible by $(x - 3)$, find the value of a. **2**

d. In the diagram, AB = 10 cm, CD = 15 cm and AC = x. Find the value of x. **2**



e. Six people are to be placed around a circular table. Two of them want to sit together. **2**

In how many ways can the table be arranged?

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Question 2 **Begin a new page** **Mark**

a. Find the exact value of $\int_{-1}^{1}\sqrt{4-x^2}\ dx$, using the substitution $x = 2\sin\theta$. **3**

b. For the expansion of the expression $\left(x-\frac{3}{x}\right)^8$, find the term independent of x **3**

c. i. Sketch the graph of $y = 2\tan^{-1}3x$. **2**

ii. State the domain and range of the function. **1**

d. Solve the equation $3\cos\theta - 4\sin\theta = 5$, for $-\pi \le \theta \le \pi$. **3**

Express your answers to 2 decimal places.

Question 3 **Begin a new page**

a. In how many ways can 8 prefects be chosen from a group of 20 nominees? **1**

b. Find the exact value of $\int_0^{\pi/12}\sin^2 2x\,dx$. **3**

c. Use Mathematical Induction to show that $\sum_{r=1}^{n}4r - 3 = 2n^2 - n$. **3**

d. Jaime wants to use Newton's Method to obtain the zero of $\sqrt[3]{x} = 0$.

i. Using Newton's Method once with a first approximation of $x = 1$, obtain a second approximation. **2**

ii. Using Newton's Method with a first approximation $x_1 \neq 0$, show that the second approximation, x_2, is such that $|x_2| > |x_1|$. **3**

Explain the significance of this result.

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Question 4 **Begin a new page** **Marks**

a. A particle is moving so that its displacement, x cm, at any time, t seconds, is given by the equation $x = 2\cos\left(3t + \frac{\pi}{6}\right)$.

i. Show that the particle moves in Simple Harmonic Motion. **2**

ii. State the period of the motion. **1**

iii. When does the particle first come to rest after $t = 0$? **1**

b. $P(2ap, ap^2)$ is a point on the parabola $x^2 = 4ay$.

The normal at P cuts the x axis at S and the y axis at T.

i. Draw a half page sketch to show this information. **1**

ii. State the equation of the normal to the parabola at P and hence show that S is the point $(ap(2 + p^2), 0)$ and that T is the point $(0, a(2 + p^2))$. **3**

iii. Find the value(s) of p such that P is the midpoint of ST. **1**

c. i. Explain why the probability of obtaining 2 heads and a tail when three coins are tossed is $\frac{3}{8}$. **1**

ii. Sian tosses three coins 10 times in a row. Calculate the probability of obtaining 2 heads and a tail at least 2 times. Give your answer correct to 3 significant figures. **2**

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Question 5 **Begin a new page** **Marks**

a. The rate at which a body cools in air is proportional to the difference between the temperature, T, of the body and the constant surrounding temperature, S. This can be expressed as

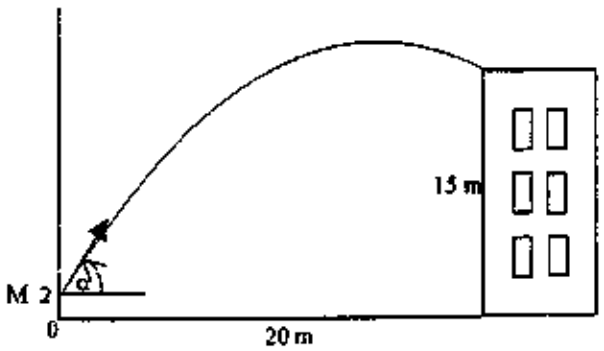
$$\frac{dT}{dt} = k(T - S)$$

where t is time in minutes and k is a constant.

i. Show that $T = S + Be^{kt}$, where B is a constant, is a solution of the above equation. **1**

ii. If a particular body cools from 100° to 80° in 30 minutes, find the temperature of the body after a further 30 minutes, given the surrounding temperature remains constant at 25°. Give your answer to the nearest degree. **3**

b. A man of height 2 metres throws a ball from M to the roof of a 15 metre high building. He throws the ball at an initial velocity of 25m/s, and he is 20 m from the base of the building.



Between which two angles of projection must he throw the ball to ensure that it lands on the roof of the building? **5**

(Assume $\ddot{x} = 0$ and $\ddot{y} = -10$)

c. i. By considering $(1 + x)^{n+3} = (1 + x)^n(1 + x)^3$, show that $\binom{n+3}{k} = \binom{n}{k} + 3\binom{n}{k-1} + 3\binom{n}{k-2} + \binom{n}{k-3}$. **2**

ii. Between what values must k lie? **1**

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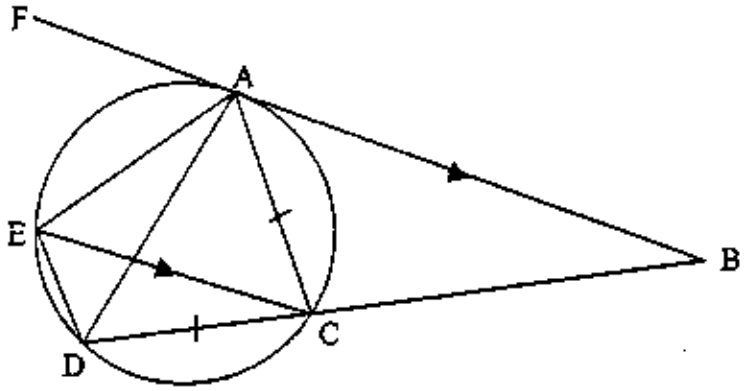
Question 6 **Begin a new page** **Marks**

a. AB is a tangent to the circle. AB ∥ EC and CD = AC. **4**



Copy the diagram into your booklet.

Prove that AC ∥ ED.

b. A particle is moving in a straight line. At time t seconds, it has displacement x metres from a fixed point O on the line, velocity v ms^{-1} and acceleration a ms^{-2}. The particle starts from O and you are given that $v = (2 - x)^2$.

i. Find an expression for a in terms of x. **1**

ii. Find an expression for x in terms of t. **3**

iii. Find the distance from O when the particle has a speed of 1 ms^{-1}. **1**

c. i. Given a function, $y = f(x)$, under what geometrical conditions would $f(x) = f^{-1}(x)$? **2**

ii. Give an example of a function for which $f(x) = f^{-1}(x)$. **1**

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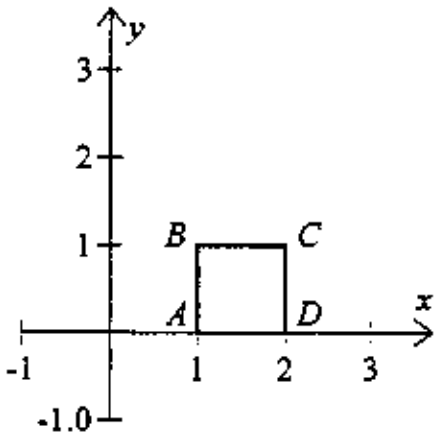
Question 7 **Begin a new page** **Marks**

a. i. Find $\frac{d}{dx}\left(x\cos^{-1}x-\sqrt{1-x^2}\right)$ **2**

ii. Find the area between the curve $y = \cos x$, the y axis and the lines $y=\frac{1}{2}$ and $y=\frac{\sqrt{3}}{2}$. **3**

b. The diagram shows a unit square, $ABCD$, where $A(1, 0)$, $B(1, 1)$, $C(2, 1)$, $D(2, 0)$.



Copy the diagram into your workbook.

i. A line, l, passing through the origin with gradient m, cuts the sides AB and CD at P and Q respectively.

Comment on the possible values of m. **1**

ii. For what value(s) of m does the line, l, divide the area of the square in the ratio 2:1? **3**

iii. Another line, k, passes through the origin with gradient, n, and cuts the square through sides AB and BC at S and T respectively.

Show that it is not possible for k to divide the area of the square in the ratio 2:1. **3**