



**2003
YEAR 12
EXTENSION ONE MATHEMATICS**

HALF YEARLY EXAMINATION
Weight: 25%

Total Marks: 70 marks

Time Allowed: 1.5 Hours + 5 minutes reading time

DIRECTIONS TO CANDIDATES

- There are FIVE (5) questions
- Attempt all questions
- Board Approved calculators may be used
- Start a NEW BOOKLET for each question
- All necessary working should be shown in every question. Marks may be deducted for carelessly or badly arranged work.
- An Integral sheet is provided with this paper

(a) If $\sin A = \frac{1}{\sqrt{2}}$, and $\cos A > 0$, find the exact value of $\sin 2A$. 3

(b) Solve the inequality $\frac{4x+3}{x-4} \geq 1$ 3

(c) Deduce, to the nearest minute, the obtuse angle between the lines $\frac{x}{7} + \frac{y}{5} = 1$ and $2x - 3y + 4 = 0$. 3

(d) Use the substitution $u = 1 + t$, to find $\int \frac{t}{\sqrt{1+t}} dt$ 3

Question Two (13 Marks) START A NEW BOOKLET

Marks

(a) Integrate the following:

(i) $\int_{-1}^1 \frac{x^2 - 4x}{x} dx$ 2

(ii) $\int (4-y)^3 dy$ 1.5

(b) (i) Express $7\cos\theta - \sin\theta$, in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$ 2

(ii) Hence, solve $7\cos\theta - \sin\theta = 5$ for $0^\circ \leq \theta \leq 360^\circ$, giving answers to the nearest minute. 2.5

(c) Given $\int_1^e x\sqrt{x} = \frac{62}{5}$, deduce the value of K . 2

(d) Prove $\frac{\sin 2\beta + \sin \beta}{1 + \cos 2\beta + \cos \beta} = \tan \beta$ 3

Question Three (19 Marks)

START A NEW BOOKLET

Marks

- (a) Given $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are points on the parabola $x^2 = 4ay$,

(i) Derive the equation of the tangent to the parabola at P.

2

(ii) The tangent at P and the line through Q parallel to the y-axis, intersect at T. Determine the co-ordinates of T.

2

(iii) Calculate the co-ordinates of M, the midpoint of PT.

2

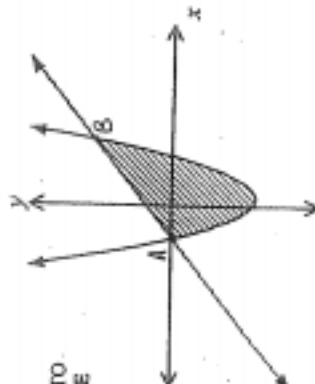
(iv) Given $pq = -1$, find the equation of the locus of M.

3

(b) Prove by Mathematical Induction, that $7^n + 11^n$ is divisible by 9, if n is odd.

4

(c) The diagram below shows the curves $y = x^3 - x - 6$ and $y = x + 2$.



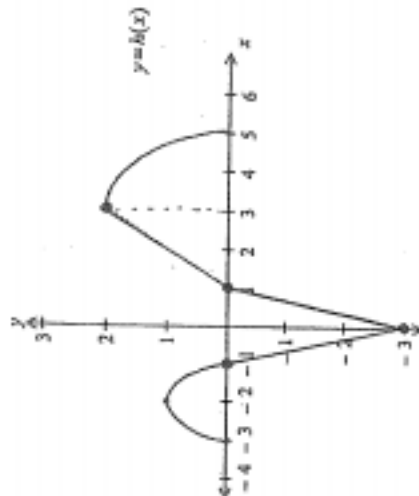
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(i) Find the x-coordinates of points A and B.

3

(ii) Hence, calculate the area enclosed by the two curves.

3



NOT TO SCALE

(i) Evaluate $\int_{-3}^3 h(x) dx$

2

(ii) Calculate the area enclosed by the curve $y = h(x)$, and the x-axis.

2

(b) (i) Differentiate $y = \frac{2x^2 - 1}{3x^3 + 4}$

(ii) Hence integrate $\int_0^1 \frac{x dx}{(3x^3 + 4)^2}$

2

(c) $P(x)$ is a monic polynomial of degree 4 and has exactly 2 real zeros, at 1 and -1.

(i) If $P(x)$ is an even function find a general equation to represent this information.

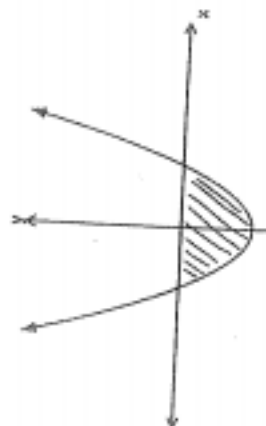
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(ii) Hence, if $P(x) = 33$ when $x = -2$, find the unique polynomial $P(x)$.

2

(d) The region bounded between $y = x^2 - 1$ and the x-axis is rotated about the x-axis. Determine the exact volume of the solid of revolution formed.

3



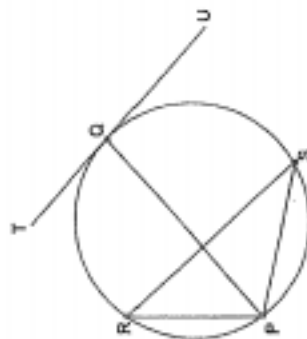
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Question Five (11 Marks) START A NEW BOOKLET

Marks

- (a) The diagram below, shows a circle with a chord PQ and another chord RS, which is parallel to the tangent at Q.

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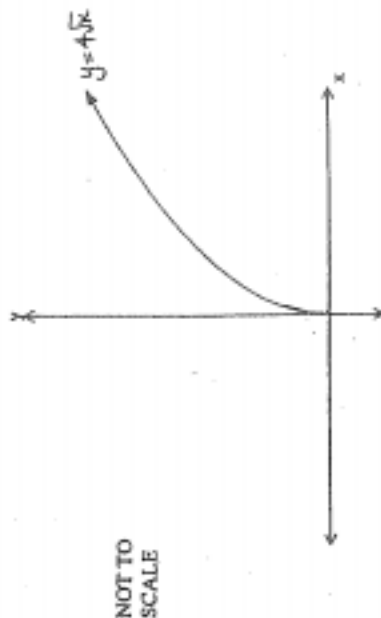
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Copy or trace the diagram onto your page

Prove that chord PQ bisects $\angle RPS$. [HINT: construction lines may be required]

- (b) A vase is formed by the rotation of the curve $y = 4\sqrt{x}$, about the y -axis. Calculate the amount of water needed to fill the vase to a depth of 8 cm.

3

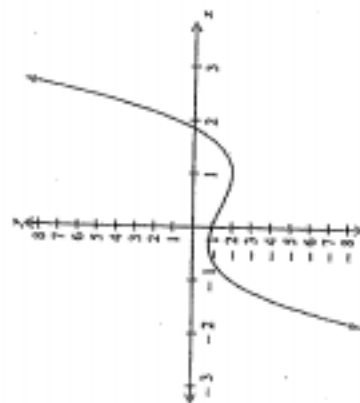


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Question Five continued...

- (c) The function $f(x) = x^3 - x^2 - x - 1$ is shown below.



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- (i) Using $x = 2$ as a first approximation for $f(x) = 0$, use one application of Newton's method to find a better approximation to 1 decimal place.

2

- (ii) Copy or trace the diagram. Describe, in words using your diagram, why $x = 1$ is an unsuitable first approximation to this $f(x) = 0$.

2