



Name: _____

Teacher: _____

Class: _____

FORT STREET HIGH SCHOOL

2011

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics

**TIME ALLOWED: 3 HOURS
(PLUS 5 MINUTES READING TIME)**

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1	
Manipulates algebraic expressions to solve problems from topic areas such as geometry, co-ordinate geometry, quadratics, trigonometry, probability and logarithms	3,4,7	
Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	2,5,6,8	
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	9,10	

Question	1	2	3	4	5	6	7	8	9	10	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/12	/12	/12	/120	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started in a new booklet
- A table of standard integrals is supplied

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

Note $\ln x = \log_e x, \quad x > 0$

QUESTION 1 (12 marks) Start a NEW booklet.**Marks**(a) Evaluate $2e^2$ correct to three significant figures**2**(b) Factorise $2x^2 - x - 6$ **2**

(c) Simplify

$$\frac{2}{x+1} - \frac{3}{x}$$

2(d) Solve $|2x - 1| = 9$ **2**

(e) Expand and simplify

$$(\sqrt{5} - 1)(2\sqrt{5} + 3)$$

2

(f) Find the sum of the first 12 terms of the arithmetic sequence

2

$$3+6+9+\dots\dots$$

QUESTION 2 (12 marks) Start a NEW booklet.

(a) Differentiate

(i) $(x^3 + 1)^7$

2

(ii) $x^4 \log_e x$

2

(iii) $\frac{\sin x}{x+1}$

2(b) Find the perpendicular distance from (6, -2) to the line
 $4x - 3y + 7 = 0$ **2**

(c) (i) Find

$$\int \frac{1}{x+7} dx$$

1

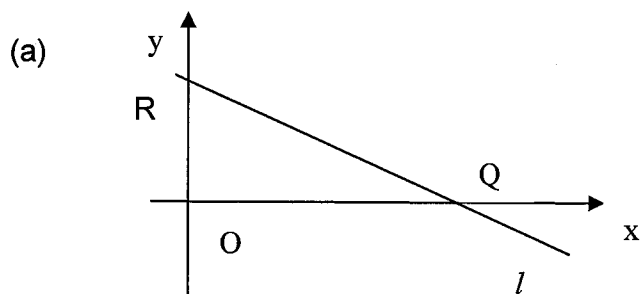
(ii) Evaluate

$$\int_0^{\frac{\pi}{8}} \sec^2 2x dx$$

3

QUESTION 3 (12 marks) Start a NEW booklet.

Marks



The diagram shows the straight line l which cuts the x -axis and y -axis at $Q(6,0)$ and $R(0,2)$ respectively.

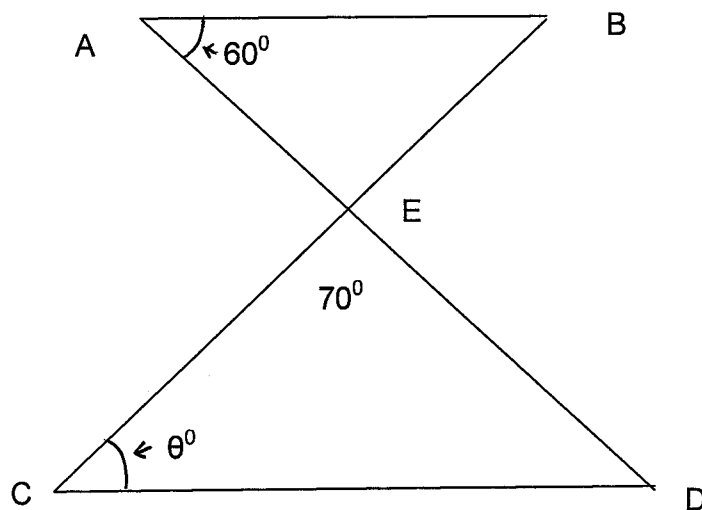
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|---|---|
| (i) Copy the diagram and find the equation of the line l . | 2 |
| (ii) Draw the line k , passing through Q perpendicular to l , and find the equation of the line k . | 2 |
| (iii) Find the coordinates of T , where k cuts the y -axis. | 1 |
| (iv) Find the equation of the circle which has TR as a diameter. Verify that Q lies on this circle. | 2 |
| (v) $(8,m)$ also lies on the circle. Find two possible values of m . | 2 |
- (b) Find the equation of the straight line that passes through $(1,-2)$ and the intersection point of the lines $4x + y - 5 = 0$ and $3x - 2y - 12 = 0$
- 3

QUESTION 4 (12 marks) Start a NEW booklet.

Marks

(a)

Figure not
to scale

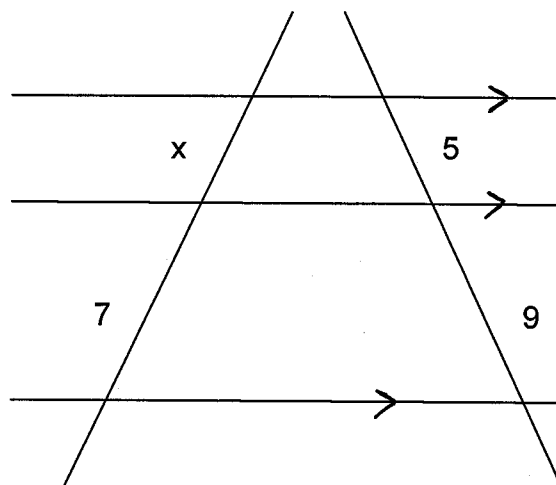


In the diagram AB is parallel to CD .
Find the value of θ . Give reasons.

2

(b)

Figure not
to scale



Find the value of x . Give reasons. All measurements are in cm.

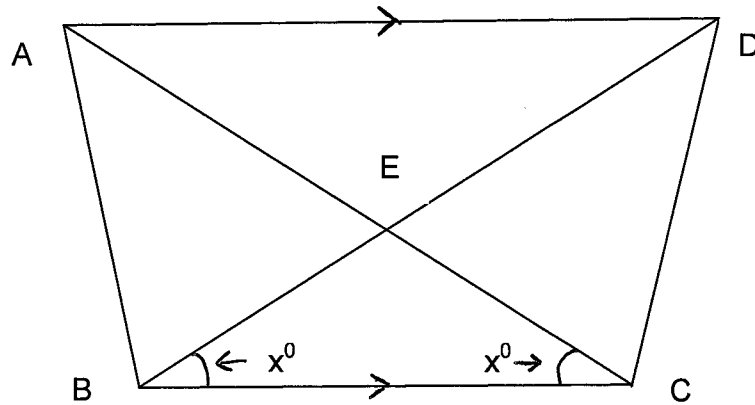
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QUESTION 4 (continued)

Marks

(c)

Figure not
to scale



In the diagram AD is parallel to BC and
angle DBC = angle ACB = x° .

(i) Show that $AE = DE$.

2

(ii) Prove that the triangles ABC and DCB are congruent.

3

(iii) Deduce angle ABD = angle DCA.

3

QUESTION 5 (12 marks) Start a NEW booklet.

Marks

- (a) The gradient of a curve is given by

$$\frac{dy}{dx} = \frac{1}{\sqrt{2x+1}}$$

3

The curve passes through (4, 5). What is the equation of the curve? (Answer in surd form)

- (b) Consider the geometric series

$$1 + 3x + 9x^2 + 27x^3 + \dots$$

- (i) For what values of x does this series have a limiting sum?

2

- (ii) The limiting sum of this series is 100.
Find the value of x

2

- (c) A 5kg mass of metal is placed in an acid and it starts to Dissolve. After t hours, the mass M kg of undissolved metal is given by

$$M = 5e^{-kt}.$$

- (i) Find k , given $M = 4.2$ when $t = 2$
(Answer in exact form)

2

- (ii) After how many hours will half of the initial mass remain undissolved? Note: This known as the half-life of the metal in this acid. (Answer to 2 decimal places)

3

QUESTION 6 (12 marks) Start a NEW booklet.

Marks

- (a) A particle moves along a straight line so that its distance x , in metres, from a fixed point O is given by

$$x = 1 - 2\sin 2t,$$

where the time t is measured in seconds from $t = 0$.

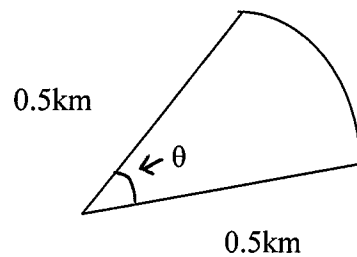
- | | |
|--|---|
| (i) Where is the particle initially? | 1 |
| (ii) When and where, does the particle come to rest? | 3 |
| (iii) Where does the particle next come to rest? | 2 |
| (iv) What is the acceleration of the particle when
$t = \frac{\pi}{12}$? | 2 |
| (b) (i) Graph $y = \cos x$, for $0 \leq x \leq 2\pi$ | 1 |
| (ii) On your diagram shade the regions bounded by the curve $y = \cos x$, the x -axis and the lines $x = 0$ and $x = \pi$.
Calculate the total area of these regions. | 2 |
| (iii) Solve $\cos x = \frac{\sqrt{3}}{2}$ for $0 \leq x \leq 2\pi$
(Answer in terms of π) | 1 |

QUESTION 7 (12 marks) Start a NEW booklet**Marks****(a) Solve**

$$\log_e x - \frac{3}{\log_e x} = 2$$

3

(Answer in exact form)

(b)

A car travels at 45 km/h on a circular curve whose radius is 0.5km.

1(i) Find the distance l km, that the car travel in one minute.**2**(ii) Calculate the size of the angle θ through which the car turns in one minute. Give your answer to the nearest degree.**(c) Pat and Chris each threw a die****1**

(i) Find the probability that they throw the same number.

1

(ii) Find the probability that the number thrown by Chris is greater than the number thrown by Pat.

(d) The focus of a parabola is S (1, -2) and the directrix is the line $y = 5$.**1**

(i) Write down the co-ordinates of the vertex.

1

(ii) Find the focal length.

2

(iii) Write down the equation of the parabola in the form

$$(x - h)^2 = 4a(y - k)$$

and sketch the parabola

Marks**QUESTION 8 (12 marks) Start a NEW booklet**

(a) Consider the curve given by $y = 3x^2 - x^3$

4

(i) Find the stationary points and determine their nature.

(ii) Sketch the curve, indicating where it crosses the x-axis and showing stationary points.

2

2

(iii) Find the equation of the tangent to the curve at the point R (-1,4).

1

(b) (i) Differentiate $\log_e(\cos x)$ with respect to x.

(ii) Hence or otherwise, show

3

$$\int_0^{\frac{\pi}{4}} \tan x dx = \frac{1}{2} \log_e 2$$

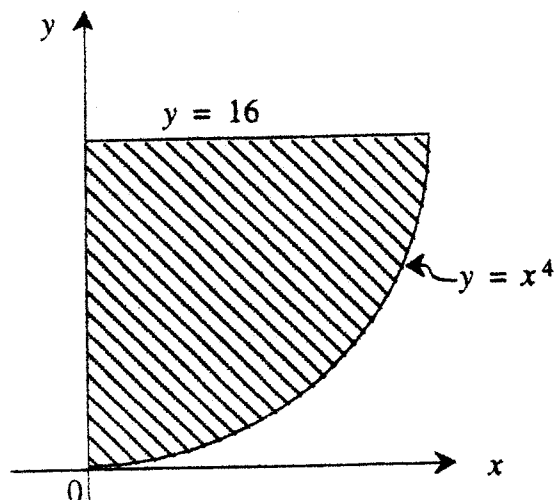
QUESTION 9 (12 marks) Start a NEW booklet.

(a) Solve the equation

$$3^{2x} + 2 \times 3^x - 15 = 0$$

2

(b)



The shaded region in the diagram is bounded by the curve $y = x^4$, the y-axis, and the line $y = 16$

4

Calculate the volume of solid of revolution formed when this region is rotated about the y-axis. (Answer in terms of π)

(c) A woman contributes \$1500 each year into a superannuation fund for the first 25 years of her working life. For the next fifteen years (until retirement), she decides to increase this, and invest a total of \$5000 each year. Each contribution is paid at the beginning of the year.

6

If the investment earns 7% p.a. paid yearly over the whole year period, how much will her investment be upon retiring? (Answer to the nearest dollar)

QUESTION 10 (12 marks) Start a NEW booklet.**Marks**

- (a) The speed of a train was recorded at intervals of one minute. The times, in minute, and the corresponding speeds v , in kilometres per hour, are listed in the following table.

time (min)	0	1	2	3	4
v (km/h)	0	25	34	30	40

- (i) Explain why the distance x , in km, travelled by the train in these four minutes is given by

$$x = \int_0^{\frac{1}{15}} v dt$$

2

- (ii) Estimate x by using Simpson's Rule with five function values. (Answer to 1 decimal place)

3

- (b) A truck is to travel 1000 kilometres at a constant speed of v km/h.

When travelling at v km/h, the truck consumes fuel at the rate of

$$\left(6 + \frac{v^2}{50}\right) \text{ litres per hour.}$$

The truck company pays \$1.50/litre for fuel and pays each of 2 drivers \$36 per hour whilst the truck is travelling.

- (i) Let the total cost of fuel and the drivers' wages for the trip be C dollars. Show that

$$C = 30v + \frac{81000}{v}$$

3

- (ii) The truck must take no longer than 12 hours to complete the trip, and speed limits require that $v \leq 100$. At what speed v should the truck travel to minimize the cost C . (Answer to 2 decimal places)

4**END OF EXAMINATION**