EPPING BOYS' HIGH SCHOOL

3 UNIT

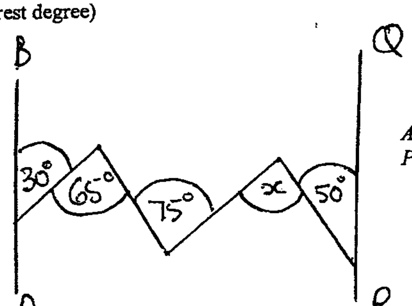
HSC TRIAL EXAMINATION 1999

Start a new page.

a. Evaluate $\int_1^2 \frac{dx}{2x+5}$ to 3 decimal places — 2 marks

b. Show that $\sin 75^0 = \frac{\sqrt{6}+\sqrt{2}}{4}$ — 3 marks

c. Find the acute angle at the intersection of $2x+3y=4$ and $2x-3y=2$ (nearest degree) — 3 marks

d. 

AB is parallel to *PQ*. Find x — 2 marks

e. Find the value of "a" if $(x+2)$ is a factor of $P(x)=x^4+ax^3+7x-10$ — 2 marks

Start a new page.

a. A point moves in the x-y number plane so that its co-ordinates at time t seconds, are given by $x=\cos t \quad y=\cos 2t$ — 4 marks

Show that its path is part of a parabola and sketch the path.

b. A number is drawn at random from the set of numbers 0, 1, 2, 3, ... 35, 36 and is then replaced.

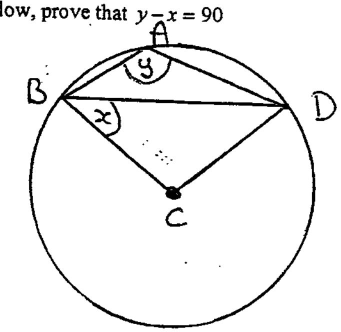
(i) What is the probability that the number drawn will be odd? — 1 mark

(ii) What is the probability that in 2 consecutive draws the number drawn will be the same? — 1 mark

Q2 – continued

(iii) What is the probability that in 100 consecutive draws the number 0" will not be drawn. (correct to 3 decimal places). — 1 mark

c. Using the diagram below, prove that $y-x=90$ — 4 marks



d. Differentiate $\cot x$ — 1 mark

Q3. Start a new page

a. Find the greatest coefficient in the expansion $(2x+3x^{-1})^{12}$ — 4 marks

b. (i) Write an expression for $\cos(A+B)$ — 1 mark

Hence, prove the following:

(ii) $\cos 2\theta = 1-2\sin^2\theta$ — 2 marks

(iii) $\frac{\cos\phi-\cos(2\theta+\phi)}{2\sin\theta} = \sin(\theta+\phi)$ — 3 marks

c. $\int \cos x \sin^2 x$, using the substitution $u=\sin x$ — 2 marks

(10)

Q4. Start a new page

a. (i) Find the number of ways of arranging 10 ladies and 10 gentlemen around a table. 1 mark

(ii) Given that Sue, John and Roy are amongst these 20 people, find the probability that Sue will sit between John and Roy. 2 marks

b. I throw a coin "p" times. Find an expression to describe the probability of throwing:

(i) At least 1 tail 1 mark

(ii) (p-3) heads 1 mark

(iii) 9 tails 1 mark

c. <u>Derive</u> an answer for the sum of the first 20 terms of the series
$3+5+9+17+33+\ldots\ldots\ldots$ 2 marks
(Hint: Consider 3=2+1, 5=4+1, etc)

d. Prove by induction that n^2+n is divisible by 2 for any integer $n \geq 1$ 2 marks

Q5. Start a new page

a. (i) Sketch $y=\sin^{-1} 2x$ 2 marks

b. Newton's Law of Cooling states that the rate of cooling of a body is proportional to the excess of the temperature of the body above the surrounding temperature. This can be expressed as:

$\frac{dT}{dt}=-k(T-T_0)$, where:

T = temperature of the body
T_o = temperature of the surroundings
t = time in seconds
k = a constant

(i) Show that $T=T_o+Ae^{-kt}$, where A is a constant, is a solution of the differential equation. 2 marks

c. A pot of water cools from 95^0C to 55^0C after 5 minutes. The room temperature is a constant 23^0C.

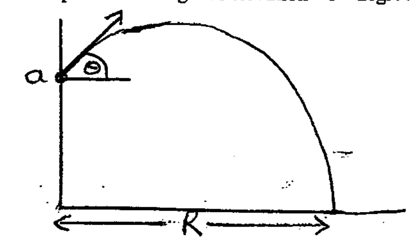
(i) Prove that $T=23+72e^{-162t}$ 2 marks

(ii) Find the temperature of the water after 30 minutes. 2 marks

(iii) As "t" approaches infinity, what does the temperature of the water approach? 2 marks

Q6. Start a new page

a. A particle is projected with speed "V" m/sec from a height "a" metres above a horizontal plane at an angle of elevation "θ" degrees to the horizontal. 7 marks



If the range on the horizontal is "R" metres, prove that:

$$R^2\sec^2\theta-2R\frac{V^2}{g}\tan\theta-\frac{2aV^2}{g}=0$$

b. The equation $x^3+3x^2-\frac{12}{\pi}=0$ has a root close to 0.9. Use 1 application of Newton's method to give a better approximation (correct to 4 dec. places) 3 marks

Q7. Start a new page

a. Prove that: $\frac{d}{dx}\left[\frac{2x}{4+x^2}+\tan^{-1}\frac{x}{2}\right]=\frac{16}{((4+x)^2)^2}$ 2 marks

b. <u>Hence</u>, evaluate $\int_0^2 \frac{dx}{(4+x^2)^2}$ 2 marks

c. Find the turning point and the point of inflexion of $y=\frac{\log_e x}{x^2}$

Hence, sketch the curve. 6 points