FORM 12 EXTENSION MATHEMATICS (3UNIT)

ASSESSMENT TASK NUMBER 1

Time allowed : 60 minutes plus 5 minutes reading time

Instructions : Begin each question on a new sheet of green paper.
Show all necessary working.
All questions are of equal value.
Approved calculators may be used.

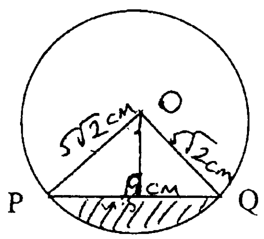
QUESTION 1:(12 marks) Marks

(a) Differentiate with respect to x:

(i) $\tan^2 4x$ 2

(ii) $x^2 \sin 2x$ 2

(b) In a circle with centre O and radius $5\sqrt{2}$ cm, a chord PQ of length 9 cm is drawn.



(i) Find $\angle$ POQ in radians, correct to 2 decimal places. 2

(ii) Hence find correct to the nearest cm^2, the area of the minor segment cut off by the chord. 3

(c) Show that the derivative of sec x is sec x tan x 3

Marks

QUESTION 2: (12 marks)

(a) Calculate the area enclosed between the curves $y = 4x - x^2$ and $y = x^2$. 3

(b) (i) Sketch the graph of the curve $\mathbf{y = 3\cos 2x}$ for $0 \le x \le \pi$, showing all essential features. 2

(ii) Use your graph to find the number of solutions to the equation $3\cos 2x = x$ in the domain $0 \le x \le \pi$. 1

(iii) Calculate the area enclosed between the curve $y = 3\cos 2x$, the x-axis and the ordinates at $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$. 3

(c) The section of the curve $y = e^x$ between $x = -2$ and $x = 2$ is rotated about the x-axis. Calculate the volume of the solid generated. Leave your answer in exact form. 3

QUESTION 3 : (12 marks)

(a) Given that $y = e^{-kx}$, where k is a constant, find the values of k such that 4

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$$

(b) Find the gradient of the tangent to the curve $y = e^{\tan x}$ at the point on the curve where $x = \frac{\pi}{4}$. 3

(c) (i) Find the derivative of xe^x 2

(ii) Hence, evaluate $\displaystyle\int_{\ln 2}^{\ln 3} e^x(x+1)\,dx$ 3

Q1 (a) (i) $\frac{d}{dx}\{\tan^2 4x\} = 2\tan 4x . 4\sec^2 4x$

$= 8\tan 4x \sec^2 4x$ 2

(ii) $\frac{d}{dx}\{x^2 \sin 2x\} = x^2 . 2\sin 2x \cos 2x + \sin 2x . 2x$

$= 2x\sin 2x(x\cos 2x + 1)$ 2

(b) (i) use cosine rule

$\cos\angle POQ = \frac{(5\sqrt{2})^2 + (5\sqrt{2})^2 - 9^2}{2\times 5\sqrt{2}\times 5\sqrt{2}}$

$\angle POQ = 79° \Rightarrow 1{\cdot}38$ radians 2

(ii) area of minor segment

$= \frac{1}{2}\times(5\sqrt{2})^2 \times 1{\cdot}38 - \frac{1}{2}\times(5\sqrt{2})^2 \times \sin 1{\cdot}38$ 3

$= 10 \text{ cm}^2$

(c) $f(x) = \sec x = (\cos x)^{-1}$

$f'(x) = -1.(\cos x)^{-2} . -\sin x$

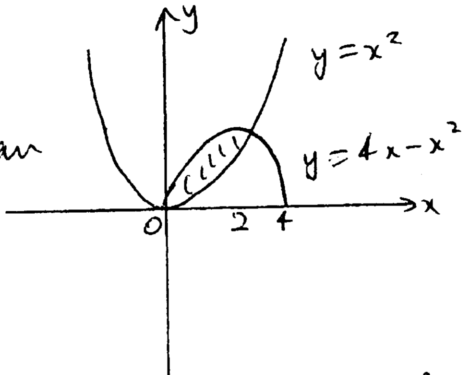
$= \frac{\sin x}{\cos^2 x}$

$= \frac{1}{\cos x} . \frac{\sin x}{\cos x}$

$= \sec x . \tan x$ 3

Q2 (a)

No diagram (-1)



graphs intersect when $x^2 = 4x - x^2$

i.e. $2x^2 = 4x$

$x^2 = 2x$

$x(x-2) = 0$ (1)

$x = 0$ and $x = 2$

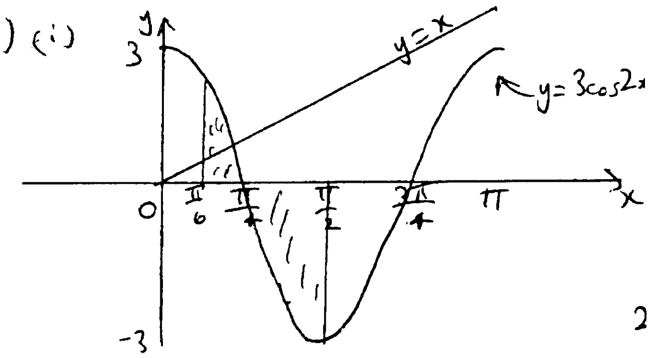
required area $= \int_0^2 (4x - x^2 - x^2)\,dx$

$= \int_0^2 (4x - 2x^2)\,dx$ (1)

$= \left[2x^2 - \frac{2}{3}x^3\right]_0^2$ (1)

$= 2\times 2^2 - \frac{2}{3}\times 2^3 = \frac{8}{3}$ units^2 3

(b) (i)



2

(ii) graphs of $y = 3\cos 2x$ and $y = x$ intersect in 1 place only

$\Rightarrow 3\cos 2x = x$ has 1 solution in given domain 1

(iii) required area $= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 3\cos 2x\,dx + \left|\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 3\cos 2x\,dx\right|$

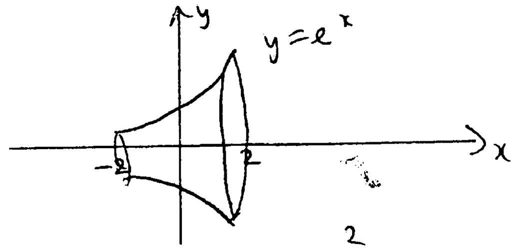
$= \left[\frac{3}{2}\sin 2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}} + \left|\left[\frac{3}{2}\sin 2x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}\right|$

$= \frac{3}{2}\sin\frac{\pi}{2} - \frac{3}{2}\sin\frac{\pi}{3} + \left|\frac{3}{2}\sin\pi - \frac{3}{2}\sin\frac{\pi}{2}\right|$

$= \frac{3}{2} - \frac{3\sqrt{3}}{4} + \left|0 - \frac{3}{2}\right|$

$= \frac{3}{2} - \frac{3\sqrt{3}}{4} + \frac{3}{2} = 3 - \frac{3\sqrt{3}}{4}$ units^2 3

(c)



required volume $= \pi\int_{-2}^{2} y^2\,dx$

$= \pi\int_{-2}^{2} e^{2x}\,dx$ no. $(2\frac{1}{2})$

$= \frac{\pi}{2}\left[e^{2x}\right]_{-2}^{2}$

$= \frac{\pi}{2}(e^4 - e^{-4})$ units^3 why $(1\frac{1}{2})$ 3

Q3 (a) $y = e^{-kx}$

$\frac{dy}{dx} = -k e^{-kx}$

$\frac{d^2y}{dx^2} = k^2 e^{-kx}$

sub. in $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$

$\Rightarrow k^2 e^{-kx} + 7k e^{-kx} + 12 e^{-kx} = 0$

$e^{-kx} \neq 0 \Rightarrow k^2 + 7k + 12 = 0$

$(k+3)(k+4) = 0$

$k = -3$ or $k = -4$ 4

(b) $y = e^{\tan x}$

$\frac{dy}{dx} = \sec^2 x \cdot e^{\tan x}$

At $x = \frac{\pi}{4}$, $\frac{dy}{dx} = \sec^2 \frac{\pi}{4} \cdot e^{\tan \frac{\pi}{4}}$

$= 2 \times e^1$

$= 2e$ 3

(c) (i) $y = xe^x$

$\frac{dy}{dx} = xe^x + e^x \cdot 1$

$= e^x(x+1)$ 2

(ii) $\int_{\ln 2}^{\ln 3} e^x(x+1)\,dx = \left[xe^x\right]_{\ln 2}^{\ln 3}$

$= \ln 3 \times e^{\ln 3} - \ln 2 \times e^{\ln 2}$

$= 3\ln 3 - 2\ln 2$

$= \ln \frac{27}{4}$ 3

$\frac{3}{2} - \frac{3}{2} \times \frac{\sqrt{2}}{2}$

$\frac{3}{2} - \frac{3\sqrt{2}}{4}$

$\sin^2 2x \cdot \cos^2 2x \; dx$

$= \left(\frac{1}{2}\sin 4x\right)^2$

$= \frac{1}{4}\int \sin^2 4x \, dx$

$= \frac{1}{4}\int \frac{1 - \cos 8x}{2}\, dx$

$= \frac{1}{8}\left[x - \frac{\sin 8x}{8}\right] + c$