NSW INDEPENDENT TRIAL EXAMS –2004
MATHEMATICS (2 unit) –HSC TRIAL
Suggested Solutions

Q1(a) -1.01

(b) $\dfrac{x-2(x+2)}{x^2-4}$

$\dfrac{-x-4}{x^2-4}$

(c) $3+3e^{3x}$

(d) $7\frac{1}{2}\% = 6$

$92\frac{1}{2}\% = x$

$\dfrac{x}{6} = \dfrac{92\frac{1}{2}}{7\frac{1}{2}}$

$x = \$74$

(e) $6x-2y = 10$

$x+2y = -3$

$7x = 7$

$x = 1$

$y = -2$

(f) $3\ln x + \frac{1}{3}\cos x + c.$

Q2 (a) (i) $m_{BO} = \dfrac{0-3}{0--2} = -3/2$

(ii) $m_{AO} = \dfrac{2}{3}$

$\therefore m_{BO}\cdot m_{AO} = -1$

$AO \perp OB$

(iii) $m = -3/2.$

$y-2 = -3/2\,(x-3)$

$2y-4 = -3x+9$

$3x+2y-13 = 0$

(iv) $x=-1$: $-3+2y-13=0$

$y = 8$

$C(-1,8)$

(v) $AC = \sqrt{(3+1)^2+(2-8)^2}$

$= \sqrt{16+36}$

$= \sqrt{52} = 2\sqrt{13}$

(vi) $AO = OB = \sqrt{13}$

$A = \frac{1}{2}\cdot OA\cdot(OB+AC)$

$= \frac{1}{2}\cdot\sqrt{13}\,(\sqrt{13}+2\sqrt{13})$

$= 39/2$ units^2.

(b) $y' = -\cos x$ $\quad m = -\cos\pi/2 = -1$

$y-0 = -1(x-\pi/2)$

$x+y-\pi/2 = 0$

(c) $V_1 = 85\%$ of 19600

$= \$16\,660$

$V_8 = P(1-\frac{r}{100})^n$

$= 1666 \times 0.875^7$

$= \$6542.31$

$= \$6540$

Q3 (a) (i) $e^{\sin x}\cdot 1 + x e^{\sin x}\cdot\cos x$

$= e^{\sin x}(1+x\cos x)$

(ii) $-5(5-3x)^{-6}\cdot -3$

$= \dfrac{15}{(5-3x)^6}$

(b) $\angle ACB = 65° = \angle CAE$

(alt $\angle$'s $AE \parallel BC$)

$\angle ABC = \angle BDC + \angle BCD$

(Ext $\angle$ of $\triangle$ = ...)

$= 40° + 25° = 65°$

$\therefore \triangle ABC$ is isosceles

(Pair of $\angle$'s =)

(c) (i) $\frac{1}{3}e^{3x} + c$

(ii) $\left[\tan x - \dfrac{x^2}{2}\right]_0^{\pi/4}$

$= \left(1-\dfrac{\pi^2}{16\times 2}\right) - (0-0)$

$= 1 - \pi^2/32$

Q3 (c) $15 - 2 + 3x = -3$

$3x = -16$

$x = -5\frac{1}{3}$

Q4 (a) (i) $AD \parallel BC$ (opp sides rect)

$\therefore \angle ADX = \angle CBY$ (alt $\angle$'s $AD \parallel BC$)

(ii) $\angle ADX = \angle CBY$

$AD = BC$ (opp sides rect)

$A\hat{X}D = 90°$ ($AX \perp DB$)

sim $B\hat{Y}C = 90°$

$\therefore \triangle ADX \equiv \triangle CBY$ (AAS)

(iii) $AX = CY$ (corr sides cong $\triangle$'s)

(iv) $AX = CY$

$\angle AXB = \angle CYD = 90°$

$\therefore AX \parallel CY$ (alt $\angle$'s $=$)

$\therefore AXCY$ is a parm

Pair of opp sides $=$ and $\parallel$

(b) $B\hat{A}C = 120°$ ($\angle$ sum $\triangle = 180°$)

$$\frac{12}{\sin 45} = \frac{BC}{\sin 120°}$$

$$BC = \frac{12 \cdot \sqrt{3}/2}{1/\sqrt{2}}$$

$= 6\sqrt{6}$ metres

(c) $ABCD$ Area $= \frac{1}{2}4^2\theta - \frac{1}{2}2^2\theta$

$\pi = 6\theta$

$\therefore \theta = \pi/6$

$\theta = 30°$

Q5 (a) (i) $T_5 = 20 + (5-1)10$

$= 60m$

Q5(a) (ii) $T_n = 20 + (n-1)10$

$= 10n + 10$

(iii) $S_n = \frac{n}{2}\left(2 \times 20 + (n-1)10\right)$

$= n(15 + 5n)$

$= 15n + 5n^2$

(iv) $900 = 15n + 5n^2$

$n^2 + 3n - 180 = 0$

$(n-12)(n+15) = 0$

$n = 12$ $(n \neq -15)$

ie 12 eggs.

(b) $\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$

$\theta = 45°, 135°, 225°, 315°$

(c) $7 - 2x \geq 5$ $\qquad$ $-7 + 2x \geq 5$

$-2x \geq -2$ $\qquad$ $2x \geq 12$

$x \leq 1$ $\qquad$ $x \geq 6$

(d) $\alpha + \beta = {}^{+}5/2$

$\alpha\beta = \frac{12}{2} = 6$

$$\frac{\alpha + \beta}{\alpha\beta} = \frac{5}{12}$$

Q6 (a) (i) $\frac{2}{x} = 3 - x$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 2, 1$

$y = 1, 2$

$A(1,2)$ $\quad$ $B(2,1)$

Q6 (a) (ii) $A = \int_1^2 \left(3 - x - \frac{2}{x}\right) dx$

$= 3x - \frac{x^2}{2} - 2\ln x \Big]_1^2$

$= \left(6 - \frac{4}{2} - 2\ln 2\right) - \left(3 - \frac{1}{2} - 0\right)$

$= \left(\frac{3}{2} - 2\ln 2\right)$ units^2

(b) (i) $4y - 12 = -(x^2 - 4x)$

$4y - 12 - 4 = -(x^2 - 4x + 4)$

$4(y - 4) = (x - 2)^2$

$\therefore$ Vertex is $(2, 4)$

(ii) Max Val of $12 - 4x - x^2$

$= 4 \cdot 4 = 16$

$\therefore$ Min Val of $x^2 + 4x - 12$

$= -16$

(c) (i) $V = x^2 \cdot h$

$4 = x^2 h$

$h = 4/x^2$

(ii) $A = 4xh + x^2$

$= \frac{16}{x} + x^2$

$= x^2 + 16x^{-1}$

(iii) $\frac{dA}{dx} = 2x - 16x^{-2} = 0$

$2x^3 = 16$

$x = 2$

$\frac{d^2A}{dx^2} = 2 + 16x^{-3} > 0$

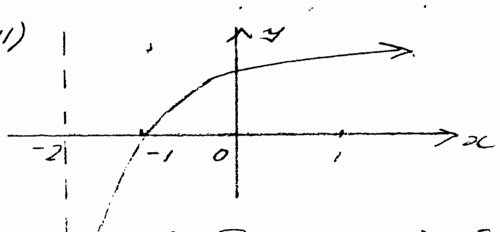
$\therefore$ Min A

Min $A = 2^2 + \frac{16}{2}$

$= 12\ m^2$

Q7 (a) (i) 0, 0·1761, 0·3010, 0·3979, 0·4771

(iii)



(ii) Domain $x > -2$

(iv) $A = 0.5\{0 + 0.4771 + 4(\cdot 1761 + \cdot 3979) + 2(\cdot 3010)\}$

$\doteqdot 1.69$

(b) (i) $P(BB) = \frac{4}{8} \cdot \frac{4}{8} = \frac{1}{4}$

(ii) $P(\bar{G}\bar{G}) = \frac{5}{8} \cdot \frac{5}{8} = \frac{25}{64}$

(iii) $P(\text{Diff}) = 1 - P(\text{same})$

$= 1 - \left(\frac{1}{4} + \frac{3}{8}\cdot\frac{3}{8} + \frac{1}{8}\cdot\frac{1}{8}\right)$

$= 1 - \frac{13}{32} = \frac{19}{32}$

(iv) $P(GG) + P(BB)$

$= \frac{3}{8}\cdot\frac{2}{7} + \frac{4}{8}\cdot\frac{3}{7}$

$= \frac{9}{28}$

Q8 (a) (i) $f'(x) = 6x - 3x^2$

$3x(2 - x) = 0$

$x = 0, 2$

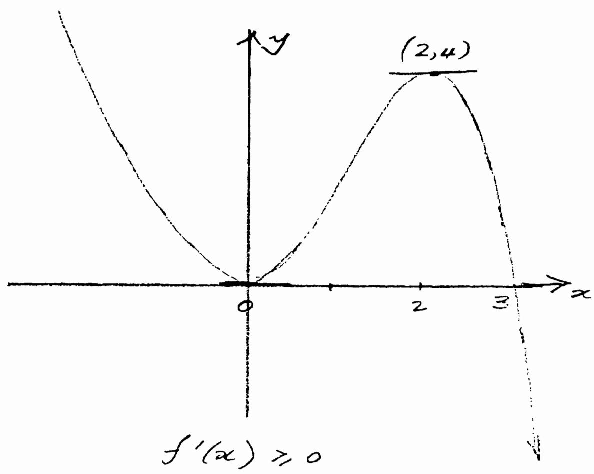
(ii) $x = 0 \quad y = 0 \qquad (0,0)$

$f''(x) = 6 - 6x$

$f''(0) = 6 > 0$ Min TP

$x = 2 \quad y = 4 \qquad (2,4)$

$f''(2) = -6 < 0$ Max TP



$f'(x) \geq 0$

$0 \leq x \leq 2$

Q8(b) (i) $V = -500t + 5t^2 + c$

$12500 = -0 + 0 + c$

$\therefore V = 5t^2 - 500t + 12500$

(ii) $V = 0 \quad \therefore t^2 - 100t + 2500 = 0$

$(t-50)^2 = 0$

Empty when $t = 50$ mins

(iii) $t = 0 \quad \frac{dV}{dt} = -10(50-0)$

$= -500$

ie decreasing at 500 l/min

(iv) $V = 2500 \quad (20\%)$

$t^2 - 100t + 2000 = 0$

$t = \frac{100 \pm \sqrt{2000}}{2}$

$= 50 \pm 10\sqrt{5}$

$\therefore t = 50 - 10\sqrt{5} = 28$ mins

as $50 + 10\sqrt{5} > 50$

Q9 (a) $V = \pi \int_1^3 e^{4x}\,dx$

$= \pi \left[\frac{e^{4x}}{4}\right]_1^3$

$= \frac{\pi}{4}(e^{12} - e^4)$

(b) (i) $W_0 = 800$

$1200 = 800e^{3k}$

$\therefore e^{3k} = 1.5$

$3k = \ln 1.5$

$k \doteqdot 0.135$

(ii) $t = 6 \quad W = 800e^{6k}$

$= 1800$

(iii) $8000 = 800e^{0.135t}$

$0.135t = \ln 10$

$t = 17$ days

(c) $\frac{1 + \cos\theta/\sin\theta}{1/\sin\theta} - \frac{\sec\theta}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$

$= \sin\theta + \cos\theta - \frac{\frac{1}{\cos\theta}}{1/\cos\theta\sin\theta}$

$= \sin\theta + \cos\theta - \sin\theta$

$= \cos\theta$

Q10 (a) (i) $I = 6\%$ of $P = .06 \times P$

$\therefore B = P + .06P = 1.06P$

(ii) Year 2 - Start of Yr

$\therefore$ Bal $= P + 1.06P$

$\therefore I = 6\%(P + 1.06P)$

End of Yr Bal $= P + 1.06P + 6\%$ of $(P + 1.06P)$

$= (P + 1.06P)(1.06)$

$B = P(1.06)(1.06 + 1)$

Sim Yr 3 $\quad B = P(1.06)(1.06^2 + 1.06 + 1)$

(iii) 25 yrs $\quad B = P(1.06)(1.06^{24} + \dots + 1.06 + 1)$

$500000 = P(1.06)\left(\frac{1((1.06)^{25} - 1)}{1.06 - 1}\right)$

$P = \frac{500000 \times .06}{1.06\{1.06^{25} - 1\}}$

$= 8597.51$

$= \$8598$

Q10.

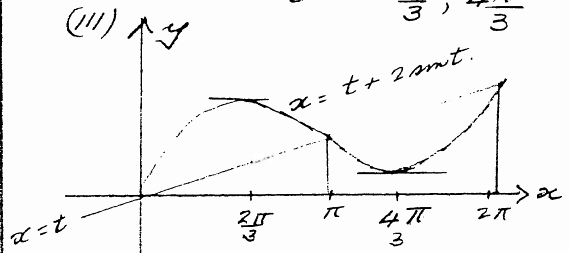
(b) (i) $v = 1 + 2\cos t$

(ii) $1 + 2\cos t = 0$

$\cos t = -\frac{1}{2}$

$t = \frac{2\pi}{3}, \frac{4\pi}{3}$

(iii)



(iv) grad is zero

(v) t from $0 \to \frac{2\pi}{3}$ moves away from O, Stops & reverses direction $t = \frac{2\pi}{3}$ $\left(x = \frac{2\pi}{3} + \sqrt{3}\right)$ moves towards O at $t = \pi$

$x = \pi$