Barker College

2003
TRIAL
HIGHER SCHOOL
CERTIFICATE

Mathematics Extension 1

Staff Involved:

- CFR*
- HG*
- DOK
- RMH
- MRB
- BJR
- VAB

90 copies

PM THURSDAY 14 AUGUST

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Make sure your Barker Student Number is on ALL pages
- Board-approved calculators may be used
- A table of standard integrals is provided on page 9
- ALL necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged working

Total marks – 84

- Attempt Questions 1 – 7
- All questions are of equal value

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Attempt Questions 1 – 7
ALL questions are of equal value

Answer each question on a SEPARATE sheet of paper

Marks

Question 1 (12 marks) **[BEGIN A NEW PAGE]**

(a) If $f(x) = x^2$ and $g(x) = -\sqrt{x}$, what is the value of $f(g(9)) - g(f(9))$? **2**

(b) $y = f(x)$ is a linear function with slope $\frac{1}{2}$

(i) Find an expression for the inverse function of $y = f(x)$ **2**

(ii) Hence find the slope of $y = f^{-1}(x)$ **1**

(c) Find $\displaystyle\int \frac{2}{3\sqrt{16 - x^2}}\,dx$ **1**

(d) Find the coordinates of the point that divides the interval AB, where A is $(-1, 3)$ and B is $(2, 8)$, externally in the ratio of $3 : 2$. **2**

(e) If $\sin 2A = \frac{1}{2}$, what is the value of $\dfrac{1}{\sin A \cos A}$? **2**

(f) If $0 \le t \le 1$, find the Cartesian equation of the curve whose parametric equations are $y = t^2$ and $x = \sqrt{t}$ **2**

Marks

Question 2 (12 marks) **[BEGIN A NEW PAGE]**

(a) Consider the function $y = 2\sin^{-1}\frac{x}{3}$

(i) State the domain and range of $y = f(x)$ **2**

(ii) Hence sketch the graph of $y = f(x)$ **1**

(b) From the top, C, of a vertical cliff, 200 m high, two ships P and Q are observed at sea level. A is the foot of the cliff at sea level. P is due south of A and the angle of elevation of C from P is $45°$. Q is S$50°$W of A and the angle of elevation of C from Q is $60°$.

(i) Draw a diagram showing this information. **1**

(ii) Find the distance PQ (to nearest metre). **3**

(c) Consider the curve whose equation is $y = \dfrac{x^2}{1 - x^2}$

(i) Find any vertical asymptotes. **1**

(ii) Find $\lim\limits_{x \to \pm\infty} y$ **1**

(iii) Show that the curve is an even function. **1**

(iv) Hence (without using calculus), sketch the curve, showing all main features. **2**

Marks

Question 3 (12 marks) **[BEGIN A NEW PAGE]**

(a) Differentiate $x\cos^{-1}x$ 2

(b) Find $\int_0^{\pi} \sin^3 x\,dx$ using the result $\sin 3x = 3\sin x - 4\sin^3 x$ 3

(c) A boat is attached by a rope to a jetty 2 m above the bow of the boat.
The rope is being pulled in at the rate of 1 m s^{-1}.
At what rate is the boat approaching the jetty when 3 m of rope still remains to be pulled in? (Answer correct to 1 decimal place) 4

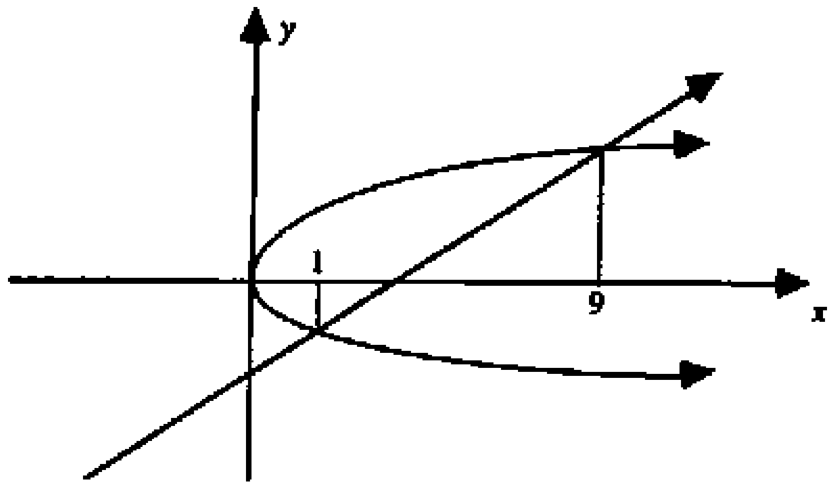
(d) (i) Express $x^2 + x + 1$ in the form $(x - A)^2 + B$ where A, B are constants. 1

(ii) Hence find $\int \frac{dx}{x^2 + x + 1}$ 2

Marks

Question 4 (12 marks) **[BEGIN A NEW PAGE]**

(a) The curves $y^2 = 16x$ and $y = 2x - 6$ intersect at the points where $x = 1$ and $x = 9$.



Find the acute angle between the two curves at the point where $x = 1$ **3**

(b) If $\tan\dfrac{\theta}{2} = t$ and θ is acute, express $\sqrt{\dfrac{1 - \cos 2\theta}{1 + \cos 2\theta}}$ in terms of t **3**

(c) Evaluate in exact form $\cos 105^\circ$ **2**

(d) Solve $\sqrt{2}\cos x - \sin x = \dfrac{3}{2}$ for $0^\circ \le x \le 360^\circ$ **4**

Marks

Question 5 (12 marks) **[BEGIN A NEW PAGE]**

(a) A body is cooling in a room of constant temperature $15°C$.
At time t minutes its temperature, T, decreases according to the equation

$$\frac{dT}{dt} = -k(T - 15)$$

where k is a positive constant.
The initial temperature of the body is $75°C$, and it cools to $55°C$ after 10 minutes.
What is the temperature of the body after a further 5 minutes?
(Answer correct to 1 decimal place) **4**

(b) (i) Show that the relation $v^2 = -kx^2 + c$, where k and c are constants, is satisfied by the equation $\frac{d}{dx}\left(\frac{v^2}{2}\right) = -kx$ **1**

(ii) A pendulum, P, swings so that it oscillates about its centre of motion according to the equation $\frac{d^2x}{dt^2} = \frac{-x}{9}$, where x is the distance of P from its centre of oscillation at any time t seconds.
Show that $v^2 = \frac{1}{9}(4 - x^2)$, given that its maximum displacement is 2 cm.
Hence find the maximum speed of P. **4**

(c) Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin x \cos^2 x \, dx$ using $u = \cos x$ **3**

Marks

Question 6 (12 marks) **[BEGIN A NEW PAGE]**

(a) Write down the value of ${}^{n}C_{j} - {}^{n}C_{n-j}$ **1**

(b) Find the term independent of x in the expansion of $\left(x^{3} + \frac{5}{x}\right)^{8}$ **3**

(c) By considering the identity $(1 + x)^{2n} = \sum_{k=0}^{2n} \binom{2n}{k} x^{k}$, show that

$$\sum_{k=1}^{2n} k \binom{2n}{k} = n4^{n}$$ **4**

(d) What is the greatest coefficient in the expansion of $(2 + 3x)^{20}$? **4**

Marks

Question 7 (12 marks) **[BEGIN A NEW PAGE]**

(a) Given that $y = \sin x$, and using the result $\cos x = \sin\left(x + \frac{\pi}{2}\right)$, it can be shown that :

$$\frac{dy}{dx} = \cos x$$
$$= \sin\left(x + \frac{\pi}{2}\right)$$
$$\frac{d^2y}{dx^2} = \cos\left(x + \frac{\pi}{2}\right)$$
$$= \sin\left[\left(x + \frac{\pi}{2}\right) + \frac{\pi}{2}\right]$$
$$= \sin\left[x + \frac{2\pi}{2}\right]$$

Similarly :

$$\frac{d^3y}{dx^3} = \sin\left(x + \frac{3\pi}{2}\right)$$

Therefore :

$$\frac{d^ny}{dx^n} = \sin\left(x + \frac{n\pi}{2}\right)$$

Prove, by induction, that the generalisation given above,
i.e. $\frac{d^ny}{dx^n} = \sin\left(x + \frac{n\pi}{2}\right)$, is correct for all positive integers n
when $y = \sin x$ 5

(b) A particle is projected under gravity with speed $u\,\text{m s}^{-1}$ and at an angle $\frac{\pi}{4}$, from a point O on horizontal ground. It strikes the ground at P, where $OP = R$.

(i) Taking the x and y axes through 0, show that the equation of the trajectory is given by $y = x - g\frac{x^2}{u^2}$ 2

(ii) Hence, or otherwise, show that $R = \frac{u^2}{g}$ 2

(iii) A ball is fired from O with velocity $30\,\text{m s}^{-1}$ at an angle $\frac{\pi}{4}$ to the horizontal. Find the speed of the ball when it has travelled a horizontal distance of 15 m from its starting point. (Take $g = 10\,\text{m s}^{-2}$)
(Answer correct to 1 decimal place) 3

End of Paper