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Centre Number

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Student Number



**CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES**

2004
**TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION**

Mathematics

Morning Session
Monday 9 August 2004

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- All necessary working should be shown in every question
- Write your Centre Number and Student Number at the top of this page

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value

Disclaimer

Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, *Principles for Setting HSC Examinations in a Standards-Referenced Framework* (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and *Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework* (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

Total marks – 120

Attempt Questions 1-10

All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet.

Marks

- | | | |
|-----|--|---|
| (a) | If $x^5 = 5000$, find x correct to 3 significant figures. | 2 |
| (b) | Express $0.3 + 0.\dot{3}$ in the form $\frac{a}{b}$, where a and b are integers. | 2 |
| (c) | Solve $\tan \alpha = 3$, for $0^\circ \leq \alpha \leq 360^\circ$, giving the answers to the nearest degree. | 2 |
| (d) | Simplify: $1 - \frac{a-b}{a+b}$ | 2 |
| (e) | Solve: $8^x = 32$, leaving the answer as a fraction. | 2 |
| (f) | Find the integers a and b such that $\frac{1}{2-\sqrt{3}} = a + b\sqrt{3}$ | 2 |

Question 2 (12 marks) Use a SEPARATE writing booklet.

Marks

(a) Write down the derivatives of:

(i) $(3x + 4)^7$ 2

(ii) $x^3 e^x$ 2

(iii) $\frac{\tan 5x}{5x}$ 2

(b) (i) Write down the primitive function of $e^{3x} + \sqrt{x}$ 2

(ii) Find the exact value of $\int_1^2 \frac{x^4 + 1}{x} dx$ 2

(iii) Given that $\frac{dy}{dx} = 2x - \sin x$ and $y = 2$ when $x = 0$,
find y in terms of x . 2

Question 3 (12 marks) Use a SEPARATE writing booklet.

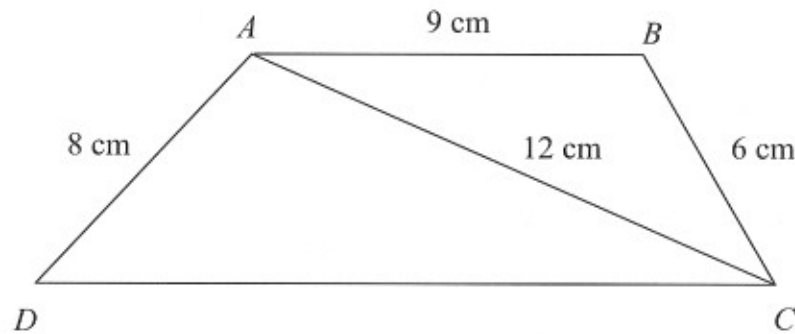
Marks

- (a) For what values of a , will $ax^2 + 5x + a$ be positive definite? 3
- (b) Find the values of k if $\int_1^k (x+1)dx = 6$ 2
- (c) The points A , B and C have co-ordinates $(1,5)$, $(6,0)$ and $(5,7)$ respectively. Plot these points on a number plane. Hence:
- (i) Show that the length of AB is $5\sqrt{2}$. 1
- (ii) Show that the triangle ABC is isosceles by finding the length of BC . 1
- (iii) Find the equation of the line AB . 2
- (iv) BA is produced to meet the line $y = 7$ at P ; show that P has co-ordinates $(-1,7)$. 1
- (v) Find the area of triangle PAC . 2

Question 4 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) In the diagram below, $AB = 9$ cm, $BC = 6$ cm, $AD = 8$ cm, $AC = 12$ cm and $\angle ABC = \angle DAC$



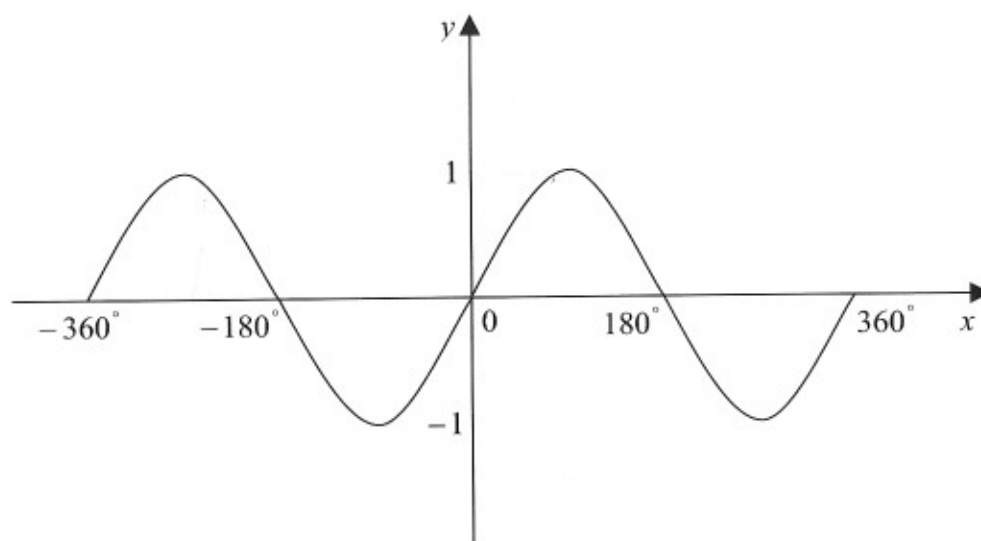
NOT TO SCALE

- (i) Prove $\triangle ABC \parallel \triangle CAD$, giving clear reasons. 3
- (ii) Hence, find the value of side CD . 2
- (b) A parabola whose equation is $y = ax^2$, where a is a constant, has the line $y = 12x + 3$ as a tangent.
- (i) By equating the two given equations, find a quadratic equation in terms of x and a . 1
- (ii) By using the discriminant of the quadratic equation found, find the value of a . 2
- (iii) Find the coordinates of the point of contact between the tangent and the parabola. 2
- (iv) Sketch the parabola and the tangent line, showing the co-ordinates of the point of contact. 2

Question 5 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) Below is the graph of $y = \sin x$, for $-360^\circ \leq x \leq 360^\circ$.



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One solution of the equation $\sin x = 0.5$ is $x = -210^\circ$.

3

Find the other solutions of this equation for $-360^\circ \leq x \leq 360^\circ$.

- (b) (i) On the same graph, sketch the curves $y_1 = 2 \sin x$ and $y_2 = -\sin 2x$, for $0 \leq x \leq 2\pi$.

2

- (ii) Give three solutions to the equation $2 \sin x + \sin 2x = 0$, for $0 \leq x \leq 2\pi$

1

- (c) A radioactive substance decays at a rate proportional to the mass present.

The rate of change is given by $\frac{dM}{dt} = -kM$, where k is a positive constant and M grams is the mass present at any time, t hours.

- (i) Show that $M = M_0 e^{-kt}$ is a solution to this equation.

1

- (ii) If 100 grams of this substance decays to 80 grams in 20 hours, find:

(α) The value of k , correct to 2 decimal places.

2

(β) The mass present after further 10 hours, to the nearest gram.

1

- (iii) The half-life time is the time taken for 100 grams of this substance to decay to 50 grams. What is the half-life time of this substance? Give your answer to the nearest hour.

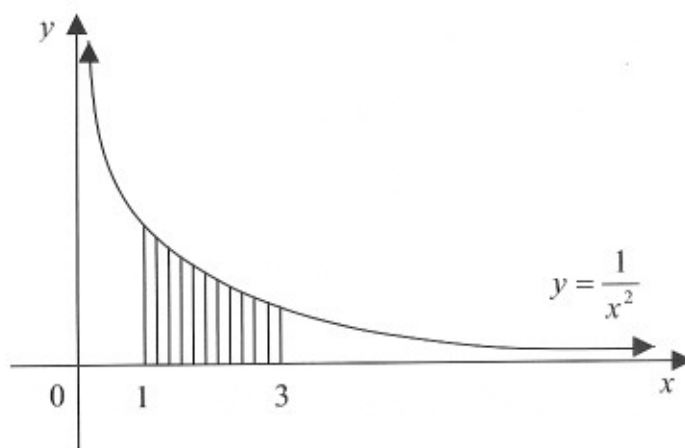
2

Question 6 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) (i) The curve $y = x^3 + ax^2 + 7x - 5$ has a stationary point at $x = 1$.
Find the value of a , and hence: 2
- (ii) Find the coordinates of all stationary points. 2
- (iii) Determine the nature of the stationary points. 2
- (iv) Sketch the curve and then determine for what values of x
the curve is increasing. 2
- (b) If the K^{th} term of an arithmetic series is L , and the L^{th} term is K :
- (i) Show that $L = a + (K - 1)d$. 1
- (ii) Find another expression for K . 1
- (iii) By solving the two equations founded, show that $d = -1$. 1
- (iv) Hence, find the first term of this series in terms of L and K . 1

(a)



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The diagram above shows the area bounded by the graph $y = \frac{1}{x^2}$, (for $x > 0$), the x -axis and the lines $x = 1$ and $x = 3$.

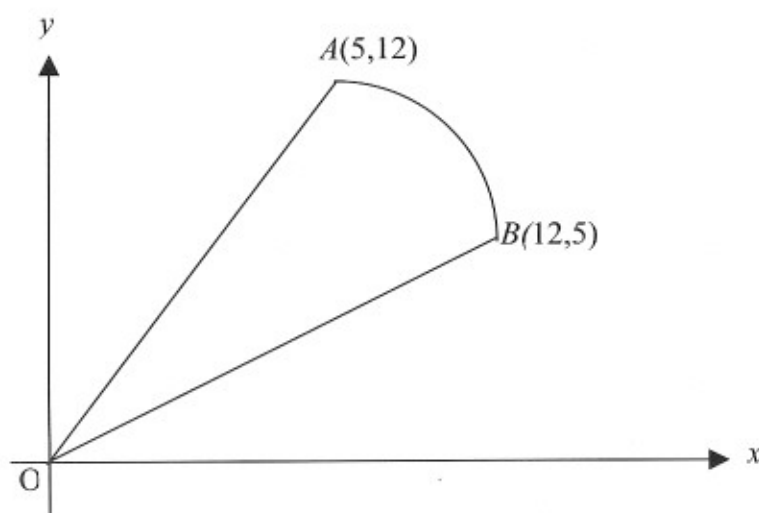
- | | | |
|---------|--|---|
| (i) | Find the shaded area. Leave your answer as a fraction. | 2 |
| (ii) | Find the volume of the solid formed when the shaded area is rotated about x -axis. Leave your answer in exact form. | 3 |
| (b) (i) | Sketch the graph of $f(x) = e^x$ for all values of x in the domain and state its range. | 2 |
| (ii) | The curve $f(x) = e^x$ is rotated about the y -axis to give a solid. Show that the volume V_y of the solid formed, from $y = 3$ to $y = 5$, is given by $V_y = \pi \int_3^5 (\ln y)^2 dy$. | 2 |
| (iii) | Use Simpson's rule with 5 function values to find the volume of this solid, correct to 2 significant figures. | 3 |

Question 8 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) A certain soccer team has a probability of 0.6 of winning a match and a probability of 0.3 of drawing a match.
- (i) If this soccer team plays two matches, draw a tree diagram to show all possible outcomes. 2
- (ii) Find the probability of this soccer team winning at least one match out of the two matches. 2
- (iii) Find the probability of this soccer team not winning either of the two matches. 2

(b)



NOT TO SCALE

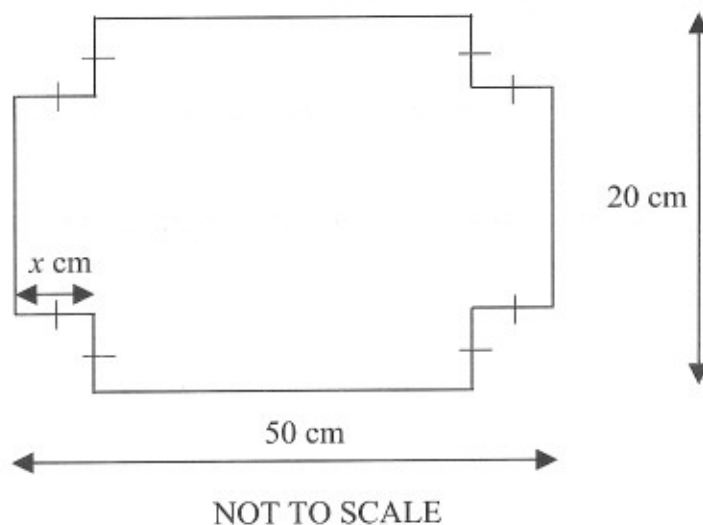
The figure above shows a sector of a circle OAB , centre O , with its arc joining the points $A(5,12)$ and $B(12,5)$. Copy this figure into your answer booklet.

- (i) Find the value, in degrees, of one radian, correct to the nearest minute. 1
- (ii) Show that the size of $\angle AOB$ is 0.78 radians, correct to 2 decimal places. 3
- (iii) Calculate the perimeter of sector OAB , correct to 2 decimal places. 2

Question 9 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) A box is made from a 50 cm by 20 cm rectangle of cardboard by cutting out four equal squares of side x cm from each corner as shown below:



The edges are turned up to make an open box.

- (i) Show that the volume V of this box is given by the equation: 2

$$V = 4x^3 - 140x^2 + 1000x \text{ (cm}^3\text{)}$$

- (ii) Find the value of x , correct to one decimal place, that gives this box its greatest volume. 3

- (iii) Hence, find the maximum volume of this box, correct to 2 decimal places. 1

- (b) Jordan has to pay annual instalments for his superannuation at the beginning of each year according to the formula:

$$M_n = \left(1 + \frac{r}{100}\right) M_{n-1}, \quad n \geq 2$$

where $r(\%)$ is the annual rate of interest paid by the fund and M_n is the instalment at the beginning of the n^{th} year.

If the interest rate is 12 % p.a., compounded yearly, and Jordan's first instalment is \$500, find:

- (i) How much is his second instalment? 2
- (ii) Find the amount Jordan has to pay into the fund at the beginning of the 20th year. 2
- (iii) Find the total value of his investment after 20 years. 2

Question 10 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) The acceleration of a particle at any time t seconds is given by $\frac{dv}{dt} = k$, where k is a constant.

- (i) Show that $v = kt + c_1$, for some constant c_1 .

1

- (ii) The displacement x metres, at any time t seconds, is shown in the table below:

3

$t \text{ (sec)}$	0	1	2
$x \text{ (metres)}$	1	2	9

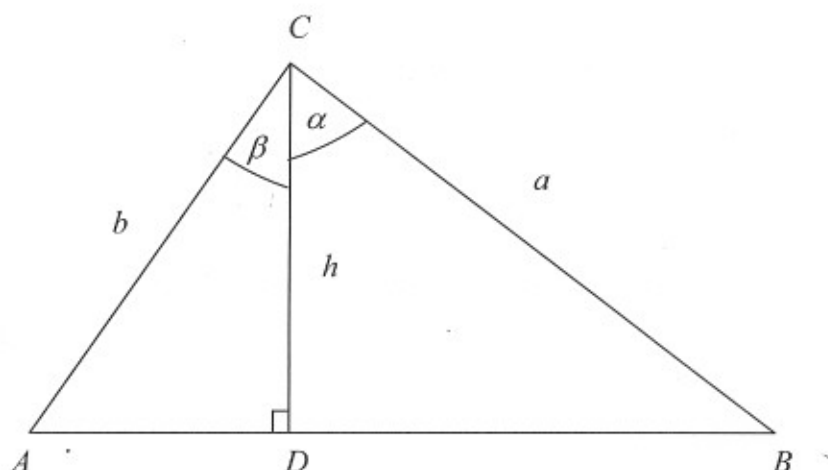
Show that $x = 3t^2 - 2t + 1$

- (iii) Find when the particle comes to rest.

2

Question 10 continues on page 12

(b)



The diagram above shows a triangle ABC , and CD is perpendicular to AB . It is given that $BC = a$, $AC = b$, $\angle ACD = \beta$ and $\angle BCD = \alpha$.

- (i) By using triangles ACD and BCD , show that $h = b \cos \beta = a \cos \alpha$. 1
- (ii) Show that the area of triangle ACD is equal to $\frac{1}{2}ab \sin \beta \cos \alpha$ 1
- (iii) Find another expression for the area of triangle BCD in terms of a , b , α and β . 1
- (iv) Show that the area of triangle ABC is equal to $\frac{1}{2}ab \sin(\alpha + \beta)$ 1
- (v) Hence, but not otherwise, deduce that: 2

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

End of paper

EXAMINERS

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