ASSESSMENT TASK I EXAMINATION

2006
HSC COURSE
EXTENSION I MATHEMATICS

Time Allowed – One Period

DIRECTIONS TO CANDIDATES

- ***Attempt ALL questions.***
- Write your Student Name on every page of answer sheets.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board approved calculators may be used.
- This question paper must **not** be removed from the examination room.

BUNDLING INSTRUCTIONS:

Please return in two bundles, clearly labeled:
PART A (Questions 1-4) and
PART B (Questions 5-8).
Ensure your name is on each bundle

STUDENT'S NAME:_____

CLASS: 11M_____

*Teacher's Name:*_____ *Mark :* _____%

PART A: Question 1 to Question 4

Question 1

a) Find the acute angle between the lines $y = 2x$ and $y = \frac{-1}{3}x$, giving the answer correct to the nearest minute. (2)

b) Let A(-1, 2) and B(3, 5) be points in the number plane. Find the coordinates of the point C which divides the interval AB externally in the ratio 3 : 1. (2)

Question 2

a) If $\sin\alpha = \frac{3}{4}$, $0° < \alpha < 90°$ and $\sin\beta = \frac{2}{3}$, $0° < \beta < 90°$ find the exact values of:

i. $\tan 2\alpha$; (2)

ii. $\cos(\alpha - \beta)$. (2)

b) i. Solve the equation $\sqrt{3}\cos x - \sin x = 1$ for $0° \le x \le 360°$; (4)

ii. What are the general solutions of the equation? (1)

c) i. Write down the expansion for $\sin(A + B)$; (1)

ii. By letting $A = B = x$ in the above expansion, derive an expression for $\sin 2x$ in terms of both $\sin x$ and $\cos x$; (1)

iii. Find all solutions for the equation: $\sin 2x = 2\cos^2 x$. For $0° \le x \le 360°$. (3)

Question 3

a) Solve the equation $x^2 + 2x - 4 + \frac{3}{x^2 + 2x} = 0$. (4)

b) Solve the inequality $\frac{2x+3}{x-4} > 1$. (3)

Question 4

a) Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

i. Show that the equation of the tangent to the parabola at P is $y = px - ap^2$. (2)

ii. The tangent at P and the line through Q parallel to the y axis intersect at T. Find the coordinates of T. (2)

iii Write down the coordinates of M, the midpoint of PT. (1)

iv Determine the Cartesian equation of the locus of M when $pq = -1$. (2)

PART B: **Question** 5 to Question 8

Question 5

a) In how many ways can the letters of the word GEOMETRY be arranged in a straight line if the vowels must occupy the 2nd, 4th and 6th places? (NOTE: The vowels in the English alphabet are the letters A, E, I, O, U). (1)

b) A school council at a co-ed school consists of 7 girls and 6 boys. In how many ways can a sub-committee of 4 girls and 3 boys be chosen from this council so as to exclude a particular girl, Mary, but include a particular boy, John? (1)

Question 6

a) Consider the polynomial $P(x) = 6x^3 - 5x^2 - 2x + 1$

i. Show that 1 is a zero of P(x). (1)

ii. Express P(x) as a product of 3 linear factors. (2)

iii. Solve the inequality $P(x) \leq 0$. (1)

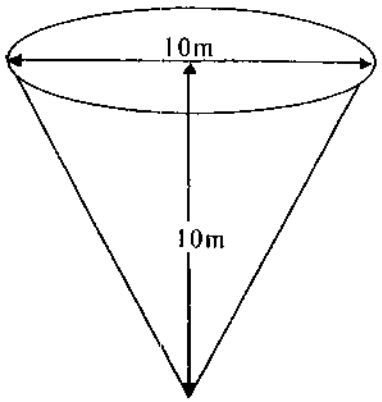
b) The equation $x^3 - 2x^2 + 4x - 5 = 0$ has roots α, β, γ.

i. Write down the values of $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. (2)

ii. Hence find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$. (2)

Question 7

a)

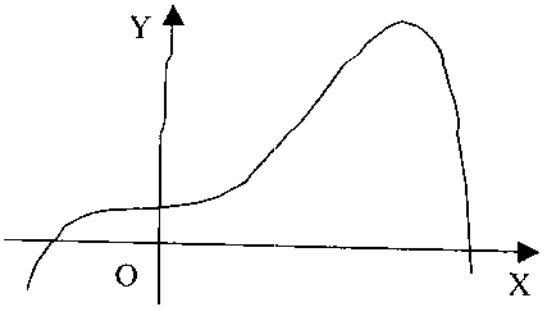


A large grain storage container is in the shape of an inverted cone, in which the diameter of the top is 10 metres and the vertical height is 10 metres.

i. If the height (in metres) of the grain in the container at any given time is denoted by h, show that the volume V (in cubic metres) of grain present at the time is given by $V = \frac{1}{12}\pi h^3$. (1)

ii. If grain runs out of the bottom of the container at the rate of 2 cubic metres per second, find the rate of change of the height of grain in the container at the instant when this height is 5 metres. (Give answer in exact form.). (2)

b) The graph given below shows $y = f(x)$.

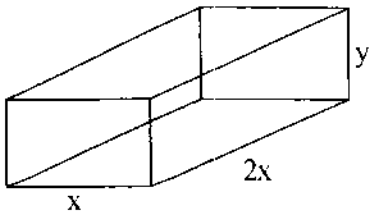


i. Copy the graph onto your answer sheet

ii. On the same axes, draw the graph of its gradient function $y = f'(x)$. (1)

c) If $y = \frac{x^2}{x+1}$, show that $\frac{dy}{dx} = \frac{x^2+2x}{(x+1)^2}$. Hence find the equation of the tangent to the curve $y = \frac{x^2}{x+1}$ at the point $(1, \frac{1}{2})$. (3)

d) Boxes in the shape of rectangular prisms are to be constructed from special materials. The width (x metres) of the base is to be half the length of the base and each box is to hold a volume of 4 cubic metres.



Material that is used to build the base and top costs \$15 per m^2. A cheaper material at \$10 per m^2 is used for the four sides.

i. Show that the total cost (\$C) of building each box is given by: (1)

$$C = 60x^2 + \frac{120}{x}.$$

ii. What is the width of the base of the cheapest box that can be constructed? (3)

e) The function $y = x^3 - 3x^2 - 9x + 1$ is defined in the domain $-4 \leq x \leq 5$.

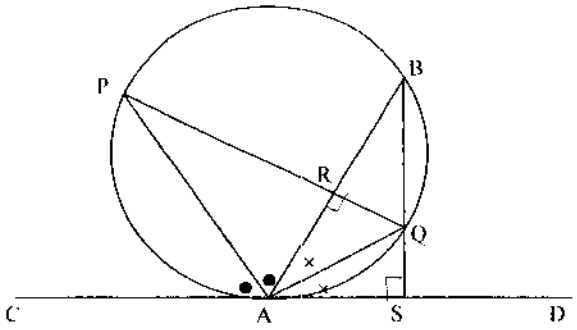
i. Find the co-ordinates of any turning points and determine their nature. (3)

ii. Find the coordinates of any points of inflexion. (1)

iv. Draw a neat sketch of the curve. (1)

v. Determine the minimum value of the function y, in the domain $-4 \leq x \leq 5$. (1)

Question 8



AB is a chord of a circle and CAD is a tangent to the circle at the point A. The bisector of angle BAC meets the circle again at P and the bisector of angle BAD meets the circle again at Q. Show that:

i. α. PQ is a diameter of the circle; (3)

β. PQ is perpendicular to the chord AB. (3)

ii. PQ meets AB at R and BQ produced meets CD at S. If BS is perpendicular to CD, prove that:

α. $\angle BAD = 60°$; (1)

β. QR = QS. (1)

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2005/2006 December Exam 3U Maths
Answers

Q1.

a) 81°52’

b) $\left(5, \frac{13}{2}\right)$

Q2.

a) i) $\frac{-1}{\sqrt{7}}$

ii) $\frac{\sqrt{35}+6}{12}$

b) i) 30°, 270°

ii) 30° ± 360°n, 270° ± 360°n (for integer n)

c) i) $\sin A\cos B + \cos A\sin B$

ii) $2\sin x\cos x$

iii) $x = 45°, 225°, 90°, 270°$

Q3.

a) ?

b) $x < -7$, $x > 4$

Q4.

a) i) Proof

ii) $\mathrm{T}\left(2aq\ ,\ 2apq - ap^2\right)$

iii) $\left(ap + aq\ ,\ apq\right)$

iv)?

Q5.

a) $\frac{3!}{2!} \times 5! = 360$

b) ${}^5\mathrm{C}_2 \times {}^6\mathrm{C}_4 = 150$

Q6.

a) i) Proof

ii) $\mathrm{P}(x) = (x-1)(3x-1)(2x+1)$

iii) $x \le \frac{-1}{2}$, $\frac{1}{3} \le x \le 1$

b) i) 4 , 5

ii) $\frac{4}{5}$

Q7.

a) i) Proof

ii) ?

b) …

c) $4y - 3x + 1 = 0$

d) i) Proof

ii) 1