CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2004
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1
Marking guidelines/ solutions

Please note: Mapping grid for this examination is on the last page of these Marking guidelines/solutions

2603 - 2

Question 1

a. **Outcomes Assessed: PE5 , HE4**

Marking Guidelines

Criteria	Marks
• Applies the product rule with correct derivative of $\tan^{-1}x$	1
• Simplifies resulting expression	1

Answer

$$\frac{d}{dx}(1+x^2)\tan^{-1}x = 2x\tan^{-1}x + (1+x^2)\frac{1}{1+x^2} = 1 + 2x\tan^{-1}x$$

b. **Outcomes Assessed: PE3**

Marking Guidelines

Criteria	Marks
• uses the remainder theorem to obtain an equation for a	1
• solves the equation to evaluate a.	1

Answer

$P(1) = P(2) \Rightarrow a + 2 = 2a + 9 \quad \therefore a = -7$

c. **Outcomes Assessed: (i) H5 (ii) P4**

Marking Guidelines

Criteria	Marks
i. • writes the expression for $\tan 45°$ in terms of the gradients of the lines	1
• obtains the required equation by putting $\tan 45° = 1$ and rearranging	1
ii. • finds one of the values of m with the corresponding line	1
• finds the second value of m and the equation of the second line	1

Answer

i. $\left|\frac{m-2}{1+2m}\right| = \tan 45° = 1$

$\therefore |m-2| = |1+2m|$

ii. $m - 2 = 1 + 2m$ or $m - 2 = -(1+2m)$

$-3 = m$ $\qquad m - 2 = -1 - 2m$

$3m = 1$

$\therefore m = -3$ or $m = \frac{1}{3}$

The required lines are $y = -3x$ and $y = \frac{1}{3}x$

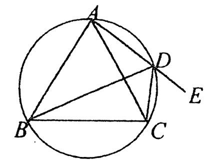
d. **Outcomes Assessed: (i) PE3 (ii) PE2 , PE3**

Marking Guidelines

Criteria	Marks
i. •	0
ii. • gives suitable reason referring to appropriate property of cyclic quadrilateral	1
iii. • explains why $\angle BDC = \angle BAC$	1
• explains why $\angle BAC = \angle ABC$	1
• uses these facts to make final deduction about DC	1

Answer

i.



ii. $\angle CDE = \angle ABC$ (*exterior angle of cyclic quadrilateral ABCD is equal to the opposite interior angle*).

iii. $\angle BDC = \angle BAC$ (*$\angle$ s subtended at circumference by same arc BC are equal*)

$\angle BAC = \angle ABC$ (*$\angle$ s opposite equal sides BC and AC in ΔABC are equal*)

$\therefore \angle BDC = \angle ABC$

$\therefore \angle BDC = \angle CDE$ (*both equal to $\angle ABC$*)

$\therefore DC$ bisects $\angle BDE$.

Question 2

a. **Outcomes Assessed: P4**

Marking Guidelines

Criteria	Marks
• applies an appropriate formula or pattern for external division	1
• evaluates the coordinates of P.	1

Answer

$A \quad B$

$(-5,6) \quad (1,3)$

$5 : -2$

$\left(\frac{5+10}{5-2}, \frac{15-12}{5-2}\right) \quad \therefore P(5,1)$

b. **Outcomes Assessed: PE3**

Marking Guidelines

Criteria	Marks
• expresses $\sum\frac{1}{\alpha}$ in terms of $\sum\alpha\beta$ and $\alpha\beta\gamma$.	1
• reads correct values of $\sum\alpha\beta$ and $\alpha\beta\gamma$ from coefficients to evaluate $\sum\frac{1}{\alpha}$	1

DISCLAIMER
The information contained in this document is intended for the professional assistance of teaching staff. It does not constitute advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible Trial HSC answers. Rather the purpose is to provide teachers with information so that they can better explore, understand and apply HSC marking requirements, as established by the NSW Board of Studies.
No guarantee or warranty is made or implied with respect to the application or use of CSSA Marking Guidelines in relation to any specific trial exam question or answer. The CSSA assumes no liability or responsibility for the accuracy, completeness or usefulness of any Marking Guidelines provided for the Trial HSC papers.

Answer

$2x^3+2x^2+4x+1=0$ has roots α,β,γ. $\quad \therefore \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\beta\gamma+\gamma\alpha+\alpha\beta}{\alpha\beta\gamma}=\frac{\left(\frac{4}{2}\right)}{\left(-\frac{1}{2}\right)}=-4$

c. **Outcomes Assessed: (i) H5 (ii) H5**

Marking Guidelines

Criteria	Marks
i. • identifies common ratio as $\cos 2x$	1
• applies condition for existence of limiting sum	1
ii. • writes expression for S in terms of $\sin 2x$ and $\cos 2x$	1
• uses appropriate trig. identities to simplify expression for S.	1

Answer

i. $r=\cos 2x,\ 0<x<\frac{\pi}{2}\Rightarrow |r|<1$.
Hence limiting sum S exists.

ii. $S=\dfrac{\sin 2x}{1-\cos 2x}$
$=\dfrac{2\sin x\cos x}{2\sin^2 x}$
$\therefore S=\dfrac{\cos x}{\sin x}=\cot x$

d. **Outcomes Assessed: (i) PE3 , PE4 (ii) PE3**

Marking Guidelines

Criteria	Marks
i. • finds $\frac{dy}{dx}$ to show that the tangent has gradient t	1
• finds the equation of the tangent	1
ii. • finds x and y coordinates of M in terms of t	1
• finds Cartesian equation of locus of M	1

Answer

i. $x=2t\Rightarrow\dfrac{dx}{dt}=2$

$y=t^2\Rightarrow\dfrac{dy}{dt}=2t$

$\therefore\dfrac{dy}{dx}=\dfrac{2t}{2}=t$

Tangent has gradient t and equation

$y-t^2=t(x-2t)$

$y-t^2=tx-2t^2$

$tx-y-t^2=0$

ii. at M, $\quad tx-y-t^2=0 \quad and \quad y=-tx$

$\therefore 2tx-t^2=0$

$2t\left(x-\frac{1}{2}t\right)=0$

If $t=0$, P and M both lie at the origin.
Otherwise at M $\quad x=\frac{1}{2}t,\ y=-\frac{1}{2}t^2$,
giving $y=-\frac{1}{2}(2x)^2$.
$\therefore$ locus of M has equation $y=-2x^2$.

Question 3

a. **Outcomes Assessed: (i) P4 (ii) P4**

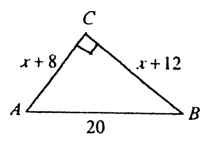
Marking Guidelines

Criteria	Marks
i. • uses Pythagoras to obtain an equation for x	1
• simplifies this equation by expanding squares and collecting like terms	1
ii. • factors this quadratic (or applies an alternative method)	1
• finds the radius of the circle with centre C.	1

Answer

When circles touch, the line joining centres passes through the point of contact, giving the sides of right triangle ABC as shown below

i.



$(x+8)^2+(x+12)^2=20^2$

$2x^2+40x+64+144=400$

$2x^2+40x-192=0$

$x^2+20x-96=0$

ii.

$(x+24)(x-4)=0$

$\therefore x>0\Rightarrow x=4$

Circle with centre C has radius 4 cm.

b. **Outcomes Assessed: (i) P3 (ii) HE6**

Marking Guidelines

Criteria	Marks
i. • rearranges either LHS or RHS to establish result	1
ii. • transforms integral into form $2\int\frac{u}{1+u}\,du$	1
• finds primitive in terms of u	1
• finds primitive in terms of x	1

Answer

i. $\dfrac{u}{1+u}=\dfrac{(1+u)-1}{1+u}$
$=1-\dfrac{1}{1+u}$

ii. $u\ge 0$
$x=u^2$
$dx=2u\,du$

$\int\frac{1}{1+\sqrt{x}}\,dx=\int\frac{1}{1+u}2u\,du$

$=2\int\frac{u}{1+u}\,du$

$=2\int\left(1-\frac{1}{1+u}\right)du$

$=2\{u-\ln(1+u)\}+c$

$=2\sqrt{x}-2\ln\left(1+\sqrt{x}\right)+c$

c. **Outcomes Assessed:** HE2

Marking Guidelines

Criteria	Marks
• shows the statement is true for $n=3$	1
• shows that $5^{k+1} > 5(4^k+3^k)$ if $S(k)$ is true	1
• completes the explanation that $S(k)$ true implies $S(k+1)$ true	1
• makes final statements to complete the Mathematical Induction	1

Answer

Let $S(n)$ be the statement $5^n > 4^n + 3^n$, $n = 3, 4, 5, \ldots$

Consider $S(3)$: $5^3 = 125$, $4^3 + 3^3 = 64 + 27 = 91$. Hence $S(3)$ is true.

If $S(k)$ *is true* : $5^k > 4^k + 3^k$ **

Consider $S(k+1)$: $5^{k+1} = 5.5^k$

$$\begin{aligned} &> 5(4^k + 3^k) \quad \textit{if } S(k) \textit{ is true, using}** \\ &= 5.4^k + 5.3^k \\ &> 4.4^k + 3.3^k \\ &= 4^{k+1} + 3^{k+1} \end{aligned}$$

Hence if $S(k)$ is true, then $S(k+1)$ is true. But $S(3)$ is true, hence $S(4)$ is true and then $S(5)$ is true and so on. Hence by Mathematical Induction $5^n > 4^n + 3^n$ for all integers $n \geq 3$.

Question 4

a. **Outcomes Assessed:** PE3

Marking Guidelines

Criteria	Marks
• writes an expression for the general term in the expansion	1
• identifies the term independent of x	1
• calculates the term independent of x	1

Answer

General term is ${}^{15}C_r\left(-\frac{2}{x^2}\right)^r x^{15-r} = {}^{15}C_r(-2)^r x^{15-3r}$, $r = 0, 1, 2, \ldots, 15$

Constant term has $15 - 3r = 0 \Rightarrow r = 5$

$\therefore$ term independent of x is ${}^{15}C_5(-2)^5 = -96\,096$.

b. **Outcomes Assessed:** (i) HE3 (ii) H3

Marking Guidelines

Criteria	Marks
i. • uses given information to show one of $A=100$ or $A+B=500$	1
• shows the second result about A, B and deduces the values of A and B	1
ii. • obtains $t \geq 2\ln 40$	1
• calculates the time to nearest month	1

Answer

i. $N = A + Be^{-0.5t}$

$t = 0$, $N = 500 \Rightarrow A + B = 500$

$t \to \infty$, $N = 100 \Rightarrow A + 0 = 100$

$\therefore A = 100$, $B = 400$

ii. $N \leq 110 \Rightarrow 100 + 400e^{-0.5t} \leq 110$

$$\begin{aligned} 400e^{-0.5t} &\leq 10 \\ e^{-0.5t} &\leq \tfrac{1}{40} \\ e^{0.5t} &\geq 40 \\ \tfrac{1}{2}t &\geq \ln 40 \\ t &\geq 2\ln 40 \end{aligned}$$

Population falls within 10 of limiting size after $7 \cdot 38$ yrs ≈ 7 yrs 5 months.

c. **Outcomes Assessed:** (i) PE3 (ii) PE3

Marking Guidelines

Criteria	Marks
i. • shows $f(0)$, $f(1)$ have opposite signs	1
• notes continuity of f to justify deduction.	1
ii. • obtains expression for α by substitution into Newton's formula	1
• calculates at least one of $f(0 \cdot 7)$, $f'(0 \cdot 7)$ correctly	1
• approximates α to 2 decimal places	1

Answer

i. $f(x) = x - \cos x$

f is a continuous function and

$f(0) = 0 - 1 < 0$

$f(1) = 1 - \cos 1 > 0$

$\therefore f(\alpha) = 0$ for some α such that $0 < \alpha < 1$.

ii. $f'(x) = 1 + \sin x$

$$\begin{aligned} \alpha &\approx 0 \cdot 7 - \frac{0 \cdot 7 - \cos 0 \cdot 7}{1 + \sin 0 \cdot 7} \\ &\approx 0 \cdot 7 - \frac{-0 \cdot 065}{1 \cdot 644} \\ &\approx 0 \cdot 74 \end{aligned}$$

Question 5

a. Outcomes Assessed (i) HE3 (ii) HE3:

Marking Guidelines

Criteria	Marks
i. • writes appropriate expression for binomial probability	1
ii. • interprets *at most* as either sum or complement of appropriate binomial probabilities	1
• calculates the probability in fraction or decimal form	1

Answer

Binomial distribution: $n=4,\ p=\frac{2}{5},\ q=\frac{3}{5}$

i. ${}^4C_3\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)=\frac{96}{625}$ ii. $1-{}^4C_4\left(\frac{2}{5}\right)^4=1-\frac{16}{625}=\frac{609}{625}$

b. Outcomes Assessed: (i) P3 (ii) HE 5

Marking Guidelines

Criteria	Marks
i. • obtains required expression for S in terms of h	1
ii. • writes expression for $\frac{dS}{dt}$ in terms of $\frac{dh}{dt}$	1
• evaluates $\frac{dS}{dt}$ when $h=2$	1
• interprets negative value and provides appropriate units	1

Answer

i. The surface of the water is a circle with radius x when the depth is y, where $x^2=4-y$.
When the depth is h, $S=\pi x^2=\pi(4-h)$

ii. $\therefore \frac{dS}{dt}=-\pi\frac{dh}{dt}=-\pi\frac{10}{\pi(4-h)}=-5$ when $h=2$

When depth is 2 cm, surface area of the water is decreasing at a rate of $5\text{cm}^2\text{s}^{-1}$.

c. Outcomes Assessed: (i) H5 (ii) H5 (iii) PE3

Marking Guidelines

Criteria	Marks
i. • finds $f''(x)$ and notes $f''(x)>0$ for all x	1
ii. • finds coordinates of stationary point	1
• states nature of stationary point	1
iii. • deduces that $f(x)\ge 1$ for all x	1
• uses this result to deduce $e^x\ge x+1$ for all x	1

Answer

i. $f(x)=e^x-x$
$f'(x)=e^x-1$
$f''(x)=e^x$
$f''(x)>0$ for all x, hence curve is concave up for all x.

ii. $f'(x)=0\Rightarrow e^x=1$ $\therefore$ stationary point is $(0,1)$
Since curve is concave up, $(0,1)$ is a minimum turning point

iii. $f(x)\ge 1$ for all $x\Rightarrow e^x-x\ge 1$ for all x
$\therefore e^x\ge x+1$ for all x.

Question 6

a. Outcomes Assessed: (i) HE4 (ii) HE4 (iii) H8

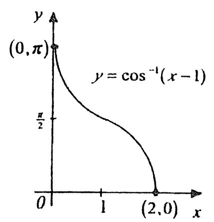
Marking Guidelines

Criteria	Marks
i. • states domain of function	1
ii. • sketches curve with correct shape and position	1
• shows endpoints with correct coordinates	1
iii. • writes integral for V in terms of y	1
• finds primitive function	1
• evaluates V by substitution of correct limits	1

Answer

i. $f(x)=\cos^{-1}(x-1)\ \Rightarrow -1\le x-1\le 1$
$\therefore$ Domain is $\{x: 0\le x\le 2\}$

ii.



iii. $V=\pi\int_0^{\pi} x^2\ dy$, where $\cos y=x-1\ \Rightarrow\ x=1+\cos y$.

$$\therefore V=\pi\int_0^{\pi}(1+\cos y)^2\ dy$$
$$=\pi\int_0^{\pi}\left(1+2\cos y+\cos^2 y\right)\ dy$$
$$=\pi\int_0^{\pi}\left(1+2\cos y+\tfrac{1}{2}(1+\cos 2y)\right)\ dy$$
$$=\pi\int_0^{\pi}\left(\tfrac{3}{2}+2\cos y+\tfrac{1}{2}\cos 2y\right)\ dy$$
$$=\pi\left[\tfrac{3}{2}y+2\sin y+\tfrac{1}{4}\sin 2y\right]_0^{\pi}$$
$$=\pi\left(\tfrac{3}{2}\pi+0+0\right)$$

Volume is $\frac{3}{2}\pi^2$ cubic units.

b. Outcomes Assessed: (i) HE3 (ii) HE3 (iii) HE3

Marking Guidelines

Criteria	Marks
i. • expresses x in terms of $\cos 2t$	1
• expresses $\ddot{x}$ in required form	1
ii. • finds possible values for x	1
• states period of motion	1
iii. • finds smallest t for which $x=0$	1
• finds initial x and deduces distance travelled	1

DISCLAIMER
The information contained in this document is intended for the professional assistance of teaching staff. It does not constitute advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible Trial HSC answers. Rather the purpose is to provide teachers with information so that they can better explore, understand and apply HSC marking requirements, as established by the NSW Board of Studies.
No guarantee or warranty is made or implied with respect to the application or use of CSSA Marking Guidelines in relation to any specific trial exam question or answer. The CSSA assumes no liability or responsibility for the accuracy, completeness or usefulness of any Marking Guidelines provided for the Trial HSC papers.

Answer

i. $x = 4\cos^2 t - 2\sin^2 t$

$= 2(1+\cos 2t) - (1-\cos 2t)$

$= 1 + 3\cos 2t$

$\dot{x} = -6\sin 2t$

$\ddot{x} = -12\cos 2t$

$= -4(x-1)$

$\ddot{x} = -2^2(x-1)$

ii.

$-1 \le \cos 2t \le 1$

$-3 \le 3\cos 2t \le 3$

$-2 \le 1 + 3\cos 2t \le 4$

$\therefore -2 \le x \le 4$

Period if the motion is $\dfrac{2\pi}{n} = \pi$ s

iii. $x = 0 \Rightarrow \cos 2t = -\frac{1}{3}$

Smallest such t is $\frac{1}{2}\cos^{-1}(-\frac{1}{3}) \approx 1\cdot 0$

Initially particle is at $x = 4$.

Hence particle first passes through O after $1\cdot 0$s when particle has travelled a distance of 4m.

Question 7

a. **Outcomes Assessed: (i) HE5 (ii) HE5 (iii) HE5 , HE7**

Marking Guidelines

Criteria	Marks
i. • uses chain rule then simplifies using trig. identities	1
ii. • writes expression for $\dfrac{dt}{dx}$	1
• finds expression for t in terms of x, evaluating the constant of integration	1
• finds expression for x in terms of t	1
iii. • states limiting position	1
• sketches graph of x against t with correct shape, endpoint and asymptote	1

Answer

i.

$\dfrac{d}{dx}\ln(\tan x) = \dfrac{\sec^2 x}{\tan x}$

$= \dfrac{1}{\cos^2 x}\,\dfrac{\cos x}{\sin x}$

$= \dfrac{1}{\sin x \cos x}$

ii. $v = \sin x \cos x$

$\dfrac{dx}{dt} = \sin x \cos x$

$\dfrac{dt}{dx} = \dfrac{1}{\sin x \cos x}$

$t = \ln(\tan x) + c$

$t = 0,\ x = \frac{\pi}{4} \Rightarrow c = 0$

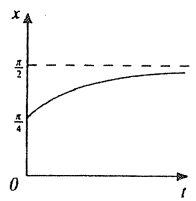
$\therefore t = \ln(\tan x)$

$e^t = \tan x$

$x = \tan^{-1}(e^t)$

iii. as $t \to \infty,\ x \to \frac{\pi}{2}$

$\therefore$ limiting position is $\frac{\pi}{2}$ metres to the right of O.



b. **Outcomes Assessed: (i) HE3 (ii) HE3 (iii) HE3**

Marking Guidelines

Criteria	Marks
i. • writes horizontal and vertical displacements for particle projected from A	1
• writes horizontal and vertical displacements for particle projected from O	1
ii. • equates expressions for x and y to obtain equations (1) and (2) if particles collide	1
• solves simultaneously to find $\cos\theta$, $\sin\theta$ and t if collision occurs	1
iii. • obtains values for $\dot{x}$, $\dot{y}$ for each particle for $t = 1$ and $\theta = \tan^{-1}2$	1
• deduces that if particles collide, their velocities are perpendicular at that time	1

Answer

i. Partical projected from A :

horizontal displacement $x = 10t$

vertical displacement $y = 20 - 5t^2$

Particle projected from O :

horizontal displacement $x = 10\sqrt{5}\, t\cos\theta$

vertical displacement $y = 10\sqrt{5}\, t\sin\theta - 5t^2$

ii. If the particles collide at some time t

$10t = 10\sqrt{5}\, t\cos\theta$ (1) and

$20 - 5t^2 = 10\sqrt{5}\, t\sin\theta - 5t^2$

$20 = 10\sqrt{5}\, t\sin\theta$ (2)

From (1), $\cos\theta = \frac{1}{\sqrt{5}}$. $\therefore \sin\theta = \frac{2}{\sqrt{5}}$

Substituting in (2) gives $t = 1$

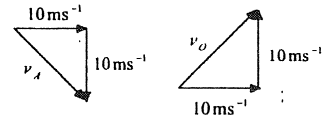
Hence the particles collide if $\theta = \tan^{-1}2$, and in this case they collide after 1 s.

iii. If $\theta = \tan^{-1}2$, when $t = 1$

the particle from A has $\dot{x} = 10$ and $\dot{y} = -10$

the particle from O has $\dot{x} = 10$ and $\dot{y} = 20 - 10 = 10$

Hence the particles have velocities v_A and v_O as shown in the diagrams below :



Hence if the particles collide, when they do so the particle from A is travelling in a direction 45° below the horizontal while the particle from O is travelling in a direction 45° above the horizontal, and their paths of motion are perpendicular to each other.

CSSA Trial HSC 2004 Mathematics Extension 1 Mapping Grid

Question	Marks	Syllabus Content	Syllabus Outcomes	Targeted Performance Bands
1(a)	2	Inverse functions	PE5 , HE4	E2 — E3
1(b)	2	Polynomials	PE3	E2 — E3
1(c)(i)	2	Angles between 2 lines	H5	E2 — E3
(ii)	2	Basic arithmetic and algebra	P4	E2 — E3
1(d)(ii)	1	Circle geometry	PE3	E2 — E3
(iii)	3	Circle geometry	PE2 , PE3	E2 — E3
2(a)	2	Internal and external division of lines	P4	E2 — E3
2(b)	2	Polynomials	PE3	E2 — E3
2(c)(i)	2	Series	H5	E2 — E3
(ii)	2	Further trigonometry	H5	E2 — E3
2(d)(i)	2	Parametric representation	PE3 , PE4	E2 — E3
(ii)	2	Parametric representation	PE3	E2 — E3
3(a)(i)	2	Basic arithmetic and algebra	P4	E2 — E3
(ii)	2	Basic arithmetic and algebra	P4	E2 — E3
3(b)(i)	1	Basic arithmetic and algebra	P3	E2 — E3
(ii)	3	Methods of integration	HE6	E2 — E3
3(c)	4	Induction	HE2	E3 — E4
4(a)	3	Binomial theorem	PE3	E2 — E3
4(b)(i)	2	Equation $\frac{dN}{dt} = k(N-P)$	HE3	E2 — E3
(ii)	2	Logarithmic and exponential functions	H3	E2 — E3
4(c)(i)	2	Polynomials	PE3	E2 — E3
(ii)	3	Iterative methods	PE3	E2 — E3
5(a)(i)	1	Further probability	HE3	E2 — E3
(ii)	2	Further probability	HE3	E2 — E3
5(b) (i)	1	Basic arithmetic and algebra	P3	E2 — E3
(ii)	3	Applications of calculus to the physical world	HE5	E3 — E4
5(c)(i)	1	Logarithmic and exponential functions	H5	E2 — E3
(ii)	2	Logarithmic and exponential functions	H5	E2 — E3
(iii)	2	Inequalities	PE3	E3 — E4
6(a)(i)	1	Inverse functions	HE4	E2 — E3
(ii)	2	Inverse functions	HE4	E2 — E3
(iii)	3	Primitive of $\sin^2 x$	H8	E3 — E4
6(b)(i)	2	Simple harmonic motion	HE3	E2 — E3
(ii)	2	Simple harmonic motion	HE3	E2 — E3
(iii)	2	Simple harmonic motion	HE3	E3 — E4
7(a)(i)	1	Velocity and acceleration as a function of x	HE5	E2 — E3
(ii)	3	Velocity and acceleration as a function of x	HE5	E3 — E4
(iii)	2	Velocity and acceleration as a function of x	HE5 , HE7	E3 — E4
7(b)(i)	2	Projectile motion	HE3	E2 — E3
(ii)	2	Projectile motion	HE3	E3 — E4
(iii)	2	Projectile motion	HE3	E3 — E4