JAMES RUSE AHS
GESTA NON VERBA

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2004

MATHEMATICS

Time Allowed – 3 Hours
(Plus 5 minutes Reading Time)

All questions may be attempted

All questions are of equal value

Department of Education approved calculators are permitted

In every question, show all necessary working

Marks may not be awarded for careless or badly arranged work

No grid paper is to be used unless provided with the examination paper

The answers to all questions are to be returned in separate bundles clearly labeled Question 1, Question 2, etc. Each question must show your Candidate Number.

Year 12 Mathematics - Trial HSC 2004

QUESTION 1

MARKS

(a) Find the value of $e^{-2.5}$ correct to 3 significant figures. **2**

(b) Factorise fully: $16x^2 - 36y^2$. **2**

(c) Solve for t: $\dfrac{4}{2t-3} = \dfrac{5}{t}$. **3**

(d) If $\dfrac{12}{2+\sqrt{10}}$ is written in the form $m + n\sqrt{10}$, where m and n are rational numbers, find the values of m and n. **3**

(f) A customer is given a 6% discount on the purchase of a radio. If the customer paid \$42.30, find the price of the radio before the discount. **2**

QUESTION 2: (START A NEW PAGE)

(a) Differentiate the following with respect to x, leaving your answer in simplest form.

(i) $(3-4x)^7$. **2**

(ii) $\dfrac{2x}{3x+1}$. **2**

(b) (i) Find: $\displaystyle\int \frac{6}{1-2x}\,dx$. **2**

(ii) Evaluate: $\displaystyle\int_0^{\frac{\pi}{4}} \sec^2 3x\,dx$. **3**

(c) Find the equation of the curve $y = f(x)$, if $f'(x) = \dfrac{\sqrt{x}-4}{x}$ and the curve passes through the point $(1, 5)$. **3**

QUESTION 3: (START A NEW PAGE)

MARKS

(a) (i) Sketch the graph of $y = 3\cos 2\theta$ for $0 \le \theta \le \pi$. **2**

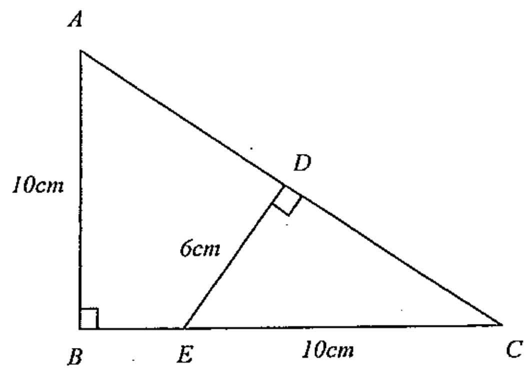
(ii) Solve $3\cos 2\theta = 1$ for $0 \le \theta \le \pi$. Give your answer correct to 2 decimal places. **2**

(b) (i) On the same set of coordinate axes, sketch the functions $y = 6x - x^2$ and $y = 2x$, clearly showing the coordinates of their intersection points. **4**

(ii) Find the area bounded by the above curves and the x-axis. **4**

QUESTION 4: (START A NEW PAGE)

(a) Triangles ABC and CDE are right angled at B and D respectively (as shown in the diagram).



(i) Copy the diagram onto your examination answer sheet and prove that ΔABC and ΔCDE are similar. **2**

(ii) If $AB = EC = 10$cm and $DE = 6$cm, find the length of AC. **2**

(b) $A(5, 20)$, $B(30, 15)$, $C(20, -10)$ and D are the vertices of a quadrilateral $ABCD$.

(i) Given that the diagonals AC and BD are perpendicular, prove that the point D lies on the line $y = \frac{1}{2}x$. **2**

(ii) If also $AB = AD$, prove that the coordinates of D are $(\overset{-6}{\cancel{6}}, -3)$. **3**

(iii) Prove that AC bisects BD. **3**

QUESTION 5: **(START A NEW PAGE)**

MARKS

(a) A balloon drifts 100km from point A to point B on a bearing of 028°T. At point B the balloon changes direction and drifts 160km to point C on a bearing of 114°T.

(i) Draw a neat diagram showing the above information. **1**

(ii) Find the distance from point A to point C. Give your answer correct to the nearest kilometre. **2**

(iii) Find the true bearing of point C from point A. Give your answer correct to the nearest degree. **3**

(b) Water flows into then out of a container at a rate (R litres/minute) given by $R = t(10 - t)$.

(i) Find the maximum flow rate. **2**

(ii) Find an expression for the volume, V litres, of water in the container at time t minutes assuming that the container is initially empty. **2**

(iii) Find the total time for the container to fill and then empty. **2**

QUESTION 6: **(START A NEW PAGE)**

(a) (i) Sketch the region bounded by the curve $y = \sqrt[3]{x}$, the y-axis and the line $y = 2$. **1**

(ii) Find the *exact* volume of the solid formed when the area in part (i) is rotated one revolution about the x-axis. **4**

(b) The velocity v m/s of an object at time t seconds is given by $v = 3t^2 - 14t + 8$. The object is initially 30m to the right of the origin.

(i) Find the initial acceleration of the object. **1**

(ii) Find when the object is at rest. **2**

(iii) Find the minimum distance between the origin and the object during its motion. **4**

QUESTION 7: (START A NEW PAGE)

MARKS

(a) On an interval $x_1 \le x \le x_2$, a curve $y = f(x)$ has the following three properties: **3**
$f(x_1) < 0$, $f'(x) > 0$ and $f''(x) < 0$.
Draw a section of the curve $y = f(x)$ that illustrates all of above information.

(b) The mass M grams of a radioactive isotope of Carbon (called Carbon 14 and written as C_{14}) found in a rock sample at time t years is given by the formula $M = Ae^{-kt}$, where A and k are constants.

(i) Prove that the rate of decay of the mass of C_{14} is proportional to the mass present at any time t. **2**

(ii) If there is initially 100 grams of C_{14} and this mass decays to 75 grams in 2500 years, find the values of the A and k. Give your value of k correct to three significant figures. **3**

(iii) Find the amount of C_{14} present at the end of 4000 years. Give your answer correct to the nearest gram. **2**

(iv) Find the time required for the mass of C_{14} to decay to 5 grams. Give your answer correct to the nearest 100 years. **2**

QUESTION 8: (START A NEW PAGE)

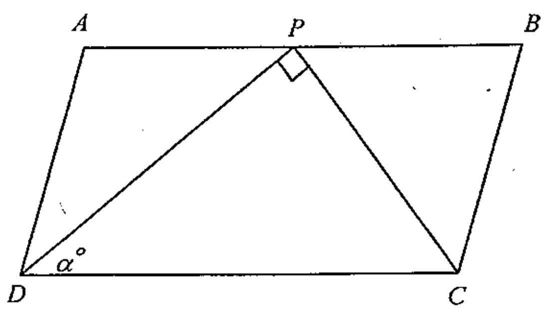
(a) As wire is unwound from a cylinder, the mass of wire remaining on the cylinder decreases. It is given that the mass, M kg, of wire remaining after t minutes can be calculated by the formula $M = 240 - 40\sqrt{t+1}$.

(i) Find the initial mass of wire on the cylinder. **1**

(ii) Find the time taken to remove all the wire from the cylinder. **2**

(iii) Find the rate at which the wire is being removed from the cylinder when half the wire has been removed. **3**

(b) $ABCD$ is a parallelogram. P is a point chosen on side AB so that PD bisects $\angle ADC$ and $\angle DPC = 90°$. (as shown in the diagram)



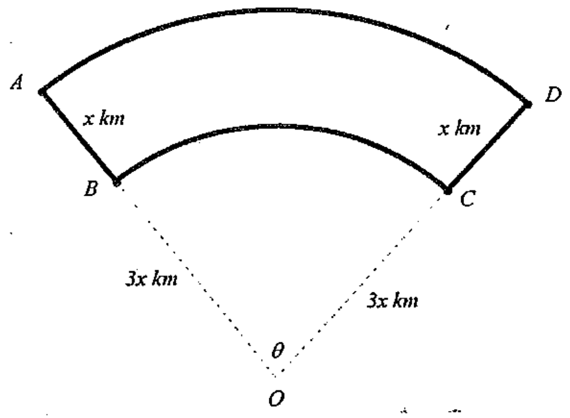
(i) If $\angle PDC = \alpha°$, prove that $\angle BPC = (90 - \alpha)°$. **3**

(ii) Prove that ΔBPC is isosceles. **3**

QUESTION 9: **(START A NEW PAGE)**

MARKS

(a) Four towns A, B, C and D are joined by roads that are either straight or arcs of concentric circles with centre at O. Towns B and C are distance $3x$ km from O and towns A and D are both distance x km from B and C respectively and $\angle AOD = \theta$ radians. (see diagram)



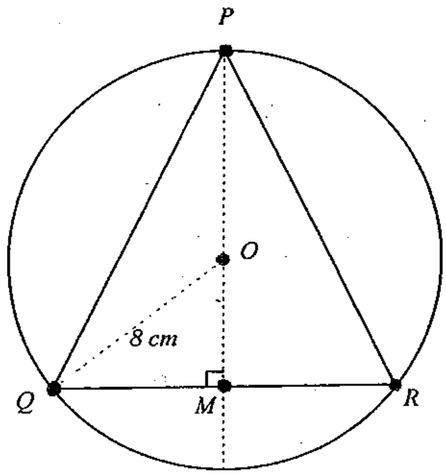
(i) Write an expression, in terms of x and θ, for the length of the journey from town A to town D along the arc AD. **1**

(ii) A salesperson wants to travel from town A to town D but must visit towns B and C on the way. Write an expression, in terms of x and θ, for the length of this journey from town A to town D. **1**

(iii) Find the value of θ for which the journeys described in parts (i) and (ii) are the same distance. **2**

(b) An isosceles triangle PQR with $PQ = PR$ is inscribed in a circle of radius 8cm (as shown in the diagram).

Given that O is the centre of the circle and M is the midpoint of the base QR of the triangle, you may assume that P, O and M are collinear and PM is perpendicular to QR.



(i) If the height, PM cm, of ΔPQR is h cm, prove that its area, A cm^2, is given by $A = h\sqrt{16h - h^2}$. **3**

(ii) Write down the restriction on the values for h. **1**

(iii) Find the maximum area of ΔPQR. **4**

QUESTION 10: (START A NEW PAGE)

MARKS

A fund is established to provide prizes for a basketball team's annual Awards night. \$10 000 is placed in the fund one year before the first Awards night. It is decided that \$450 will be withdrawn from the fund each year to purchase the annual prizes. The money in the fund is invested at 3% p.a. compounded annually with the interest paid into the fund before each annual Awards night.

(i) Show that the fund contains \$9695.50 after the second Awards night. **2**

(ii) If A_n is the amount in dollars remaining in the fund after the n^{th} Awards night, prove that $A_n = 5000(3 - 1.03^n)$. **3**

(iii) Find the amount of money in the fund after the 25^{th} Awards night. Give your answer correct to the nearest dollar. **1**

(iv) Find the maximum number of Awards nights that can be financed using this fund. **2**

(v) For the fund described above it is decided to increase the amount of money withdrawn for each Awards night by 2% each year.

(α) Show that the amount remaining in the fund after the 2^{nd} Awards night is \$9686.50. **2**

(β) Find the amount remaining in the fund after the 25^{th} Awards night. Give your answer correct to the nearest dollar. **2**

THE END

QUESTION 1

(a) $e^{-2.5} = 0.0821$ (to 3 significant figures) 2

(b) $16x^2 - 36y^2 = 4(4x^2 - 9y^2)$ 2

$= 4(2x-3y)(2x+3y)$

(c) $4t = 5(2t-3)$ 3

$= 10t - 15$

$6t = 15$

$t = 2\frac{1}{2}$

(d) $\frac{12}{2+\sqrt{10}} = \frac{12(2-\sqrt{10})}{(2+\sqrt{10})(2-\sqrt{10})}$ 3

$= \frac{12(2-\sqrt{10})}{4-10}$

$= \frac{12(2-\sqrt{10})}{-6}$

$= -2(2-\sqrt{10})$

$= -4 + 2\sqrt{10}$

$\therefore\ m = -4,\ n = 2$

(f) 94% of radio price = \$42.30 2

1% of radio price = $\frac{\$42.30}{94}$

100% of radio price = $\frac{\$42.30}{94} \times 100$

= \$45.00

QUESTION 2: (STAR A NEW PAGE)

(a) (i) Let $f(x) = (3-4x)^7$ 2

$f'(x) = 7(3-4x)^6 \times (-4)$

$= -28(3-4x)^6$

(ii) Let $f(x) = \frac{2x}{3x+1}$ 2

$f'(x) = \frac{(3x+1)(2)-(2x)(3)}{(3x+1)^2}$

$= \frac{6x+2-6x}{(3x+1)^2}$

$= \frac{2}{(3x+1)^2}$

(b) (i) $\int \frac{6}{1-2x}\,dx = -3\ln(1-2x) + c$ 2

(ii) $\int_0^{\frac{\pi}{4}} \sec^2 3x\,dx = \frac{1}{3}[\tan 3x]_0^{\frac{\pi}{4}}$ 3

$= \frac{1}{3}\{\tan\frac{3\pi}{4} - \tan 0\}$

$= -\frac{1}{3}$

(c) $\frac{dy}{dx} = \frac{\sqrt{x}-4}{x}$ 3

$= x^{-\frac{1}{2}} - \frac{4}{x}$

$y = 2x^{\frac{1}{2}} - 4\ln x + c$

at point $(1,5)$

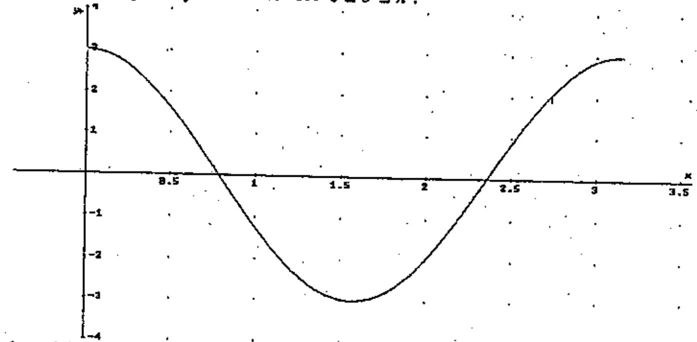
$5 = 2\sqrt{1} - 4\ln 1 + c$

$\therefore c = 3$

$y = 2\sqrt{x} - 4\ln x + 3$

QUESTION 3: (STAR A NEW PAGE)

(a) (i) Sketch the graph of $y = 3\cos 2\theta$ for $0 \le \theta \le \pi$. 2



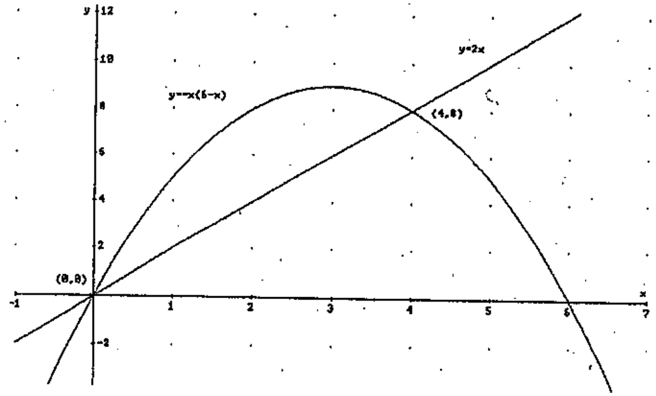
(ii) $3\cos 2\theta = 1$ 2

$\cos 2\theta = \frac{1}{3}$

$2\theta = 1.230959$ or 5.052226

$\theta = 0.62$ or 2.53 (to 2 decimal places)

(b) (i) 4



(ii) 4

$$A = \int_0^4 2x\,dx + \int_4^6 \left(6x - x^2\right)dx$$

$$= \left[x^2\right]_0^4 + \left[3x^2 - \tfrac{1}{3}x^3\right]_4^6$$

$$= \left\{4^2 - 0\right\} + \left\{\left(3\times 6^2 - \tfrac{1}{3}\times 6^3\right) - \left(3\times 4^2 - \tfrac{1}{3}\times 4^3\right)\right\}$$

$$= 25\tfrac{1}{3}$$

Area $= 25\frac{1}{3}$ u^2

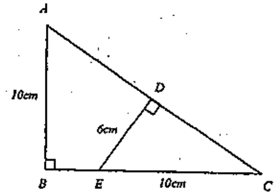
QUESTION 4: (STAR A NEW PAGE)

(a) (i) In ΔACB and ΔCDE 2

$A\hat{C}B = D\hat{C}E$ (common)

$A\hat{B}C = C\hat{D}E$ (both 90°)

$\Delta ABC \equiv \Delta EDC$ (equiangular)



(ii) $\dfrac{AC}{10} = \dfrac{10}{6}$ (ratio of corresponding sides in similar triangles) 2

$AC = 16\frac{2}{3}$

length of $AC = 16\frac{2}{3}$ cm

(b) (i) slope $AC = \dfrac{20+10}{5-20}$ 2

$= -2$

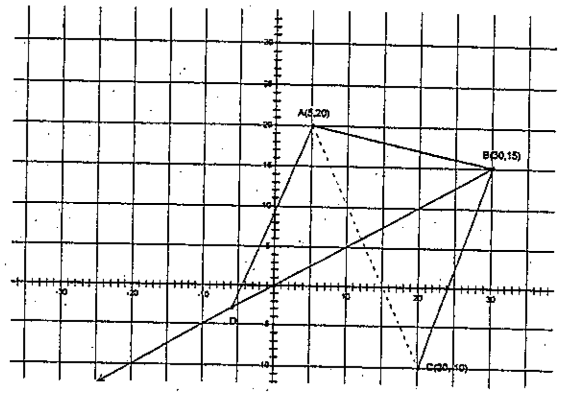
$\therefore$ slope $DB = \frac{1}{2}$

equation DB is

$y - 15 = \frac{1}{2}(x - 30)$

$y - 15 = \frac{1}{2}x - 15$

$y = \frac{1}{2}x$



(ii) Let D be the point $(2a, a)$ 3

$AD^2 = AB^2$

$(2a-5)^2 + (a-20)^2 = (5-30)^2 + (20-15)^2$

$5a^2 - 60a - 225 = 0$

$a^2 - 12a - 45 = 0$

$(a+3)(a-15) = 0$

$a = -3$ or 15

at point D, $a = -3$

$\therefore D$ is $(2a, a) = (-6, -3)$

(iii) Using coordinate geometry
Midpoint of BD is $P(12,6)$
Equation of line AC is
$y-20=-2(x-5)$
$y=-2x+30$
at point $P(12,6)$
$LHS=y$
$=6$
$RHS=-2x+30$
$=-2\times(-12)+30$
$=6$
$\therefore LHS=RHS$
$\therefore$ midpoint P lies on line AC
i.e. line AC bisects DB

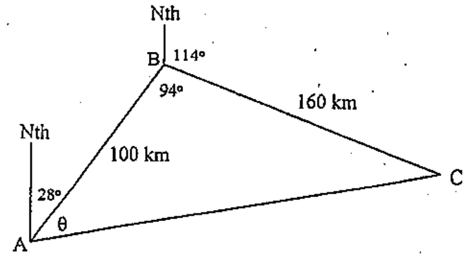
or

Using congruent triangles
Let AC meet BD at P
In ΔADP and ΔABP
$AD=AB$ (given)
$AP=AP$ (common)
$A\hat{P}D=A\hat{P}B$ (both 90°, $AC\perp BD$)
$\therefore \Delta ADP\equiv\Delta ABP$ (RHS)
$\therefore DP=BP$ (corresponding sides in congruent triangles)
$\therefore$ line AC bisects DB

3

QUESTION 5: (STAR A NEW PAGE)

(a) (i) 1



(ii) $AC^2=100^2+160^2-2(100)(160)\cos 94°$ 2
$AC=195\text{km}$ (to nearest km)

(iii) $\cos\theta=\dfrac{100^2+AC^2-160^2}{2(100)(AC)}$ 3
≈ 0.5715
$\theta=55°8'$
bearing $=(28°+55°)T$
$=083°T$ (to nearest degree)

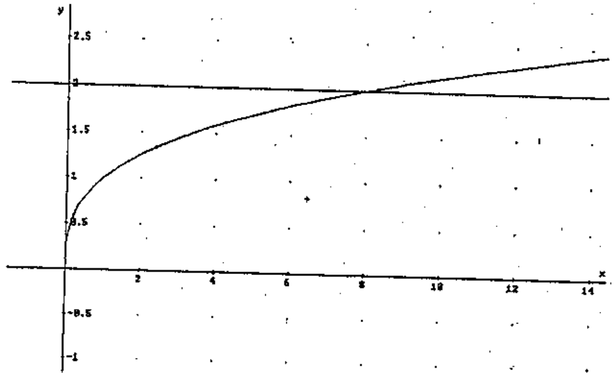
(b) (i) when $t=5$, $R=5(10-5)$ 2
$=25$
max. flow rate $=25$ L/min

(ii) $V=\int(10t-t^2)\,dt$ 2
$=5t^2-\frac{1}{3}t^3+c$
when $t=0, V=0\Rightarrow c=0$
$V=5t^2-\frac{1}{3}t^3$

(iii) when $V=0$ 2
$5t^2-\frac{1}{3}t^3=0$
$\frac{1}{3}t^2(15-t)=0$
$t=0$ or 15
$\therefore$ time taken $=15$ minutes

QUESTION 6: (STAR A NEW PAGE)

(a) (i)



1

(ii) $V = \pi \int_0^8 \left(2^2 - x^{\frac{2}{3}}\right) dx$ 4

$= \pi\left[4x - \frac{3}{5}t^{\frac{5}{3}}\right]_0^8$

$= \pi\left\{\left(4\times 8 - \frac{3}{5}\times 8^{\frac{5}{3}}\right) - (0)\right\}$

$= \frac{64\pi}{5}$

$\therefore$ volume $= \frac{64\pi}{5}$ u^3

(b) (i) $v = 3t^2 - 14t + 8$ 1

$a = 6t - 14$

when $t = 0$, $a = -14$

initial acceleration $= -14\text{ms}^{-2}$

(ii) at rest when $v = 0$ 2

$3t^2 - 14t + 8 = 0$

$(3t-2)(t-4) = 0$

$t = \frac{2}{3}$ or 4

at rest after $\frac{2}{3}$ seconds or 4 seconds

(iii) $x = t^3 - 7t^2 + 8t + c$ 4

$t = 0, x = 30 \Rightarrow c = 30$

$\therefore x = t^3 - 7t^2 + 8t + 30$

when $t = 0$, $x = 30$

when $t = \frac{2}{3}$, $\ddot{x} = -10 < 0$, $\therefore$ concave down $\Rightarrow$ local max.tp

when $t = 4$, $\ddot{x} = 10 > 0$, $\therefore$ concave up $\Rightarrow$ local min.tp.

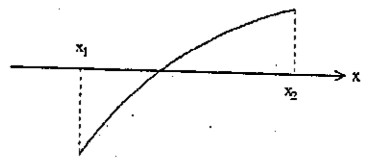
$x = 4^3 - 7\times 4^2 + 8\times 4 + 30$

$= 14$

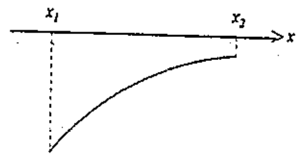
$\therefore$ minimum distance $= 14$km

QUESTION 7: (STAR A NEW PAGE)

(a)



or



3

(b) (i) $\frac{dM}{dt} = -kAe^{-kt}$ 2

$= -kM$ since $M = Ae^{-kt}$

$\therefore \frac{dM}{dt} \propto M$

(ii) when $t = 0$, $M = 100$ 3

$100 = Ae^0$

$A = 100$

when $t = 2500$, $M = 75$

$75 = 100e^{-2500k}$

$e^{-2500k} = 0.75$

$-2500k = \ln 0.75$

$k = \frac{\ln 0.75}{-2500}$

$= 1.15\times 10^{-4}$ (to 3 significant figures)

(iii) when $t = 4000$ 2

$M = 100e^{-4000\times 1.15\times 10^{-4}}$

≈ 63

mass ≈ 63 grams

(iv) when $M = 5$ 2

$5 = 100e^{-kt}$

$e^{-kt} = 0.05$

$-kt = \ln 0.05$

$t = \frac{\ln 0.05}{-k}$

≈ 26049

time $= 26000$years (to nearest 100 years)

QUESTION 8: **(STAR A NEW PAGE)**

(a) (i) when $t = 0$ 1

$M = 240 - 40\sqrt{1}$

$= 200$

$\therefore$ initial mass = 200kg

(ii) when $M = 0$ 2

$0 = 240 - 40\sqrt{t+1}$

$40\sqrt{t+1} = 240$

$\sqrt{t+1} = 6$

$t + 1 = 36$

$t = 35$

$\therefore$ time = 35 minutes

(iii) $\frac{dM}{dt} = 0 - 40 \times \frac{1}{2}(t+1)^{-\frac{1}{2}}$ 3

$= \frac{-20}{\sqrt{t+1}}$

when $M = 100$

$100 = 240 - 40\sqrt{t+1}$

$40\sqrt{t+1} = 140$

$\sqrt{t+1} = 3.5$

$\therefore \frac{dM}{dt} = \frac{-20}{\sqrt{t+1}}$

$= -\frac{20}{3.5}$

$= -\frac{40}{7}$

$\therefore$ rate $= -\frac{40}{7}$ kg/min

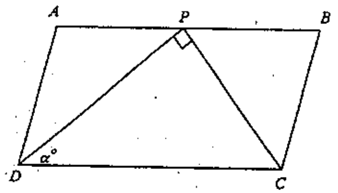
(b) (i) 3

$AB \parallel DC$ (opposite sides of parallelogram are parallel)

$\angle APD = \alpha^\circ$ ($AB \parallel DC$, alternate angles are equal)

$\angle BPC + \alpha^\circ + 90^\circ = 180^\circ$ (straight angle $\angle APB = 180^\circ$)

$\angle BPC = (90 - \alpha)^\circ$



(ii) $\angle ADC = 2\alpha^\circ$ (PD bisects $\angle ADC$) 3

$\angle ABC = 2\alpha^\circ$ (opposite angles of parallelogram are equal)

$\angle BCP + 2\alpha^\circ + (90 - \alpha)^\circ = 180^\circ$ (angle sum of $\Delta BPC = 180^\circ$)

$\angle BCP = (90 - \alpha)^\circ$

$\therefore \Delta BPC$ is isosceles ($\angle BPC = \angle BCP = (90 - \alpha)^\circ$)

QUESTION 9: **(STAR A NEW PAGE)**

(a) (i) $AD = 4x\theta$ 1

(ii) $ABCD = 2x + 3x\theta$ 1

(iii) $AD = ABCD$ 2

$4x\theta = 2x + 3x\theta$

$x\theta - 2x = 0$

$x(\theta - 2) = 0$

$\theta = 2 \; (x \neq 0)$

$\therefore$ angle = 2 radians

(b) (i) $Area = \frac{1}{2}QR.PM$ 3

$QR = 2QM$

Case (1) if $h \geq 8$

$QM^2 = 8^2 - (h-8)^2$ (Pythagoras' Theorem)

Case 2 if $h < 8$

$QM^2 = 8^2 - (8-h)^2$ (Pythagoras' Theorem)

$\therefore QM^2 = 16h - h^2$

$QM = \sqrt{16h - h^2}$ $(QM > 0)$

$QR = 2\sqrt{16h - h^2}$

$Area: A = \frac{1}{2} \times 2\sqrt{16h - h^2} \times h$

$\therefore A = h\sqrt{16h - h^2}$

$= 16h - h^2$

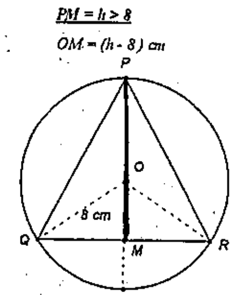
$QM = \sqrt{16h - h^2}$ $(QM > 0)$

$QR = 2\sqrt{16h - h^2}$

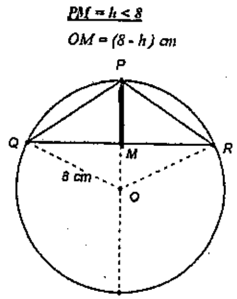
$Area: A = \frac{1}{2} \times 2\sqrt{16h - h^2} \times h$

$\therefore A = h\sqrt{16h - h^2}$

Case (1)



Case (2)



(ii) $0 \leq h \leq 16$ 1

(iii) $A = h(16h - h^2)^{\frac{1}{2}}$ 4

$\frac{dA}{dh} = (1)(16h - h^2)^{\frac{1}{2}} + (h) \times \frac{1}{2}(16h - h^2)^{-\frac{1}{2}} \times (16 - 2h)$

$= \sqrt{16h - h^2} + \frac{8h - h^2}{\sqrt{16h - h^2}}$

$= \frac{2h(12 - h)}{\sqrt{16h - h^2}}$

for stat. pt. $\frac{dA}{dh}=0$

$\frac{2h(12-h)}{\sqrt{16h-h^2}}=0$

$2h(12-h)=0$

$h=0$ or 12

test stat. points

when $h=0$, $A=0$, $\therefore$ min.area

when $h=12$

h	<12 (=11)	12	>12 (=13)
$\frac{dA}{dh}$	$\frac{2(11)(1)}{\sqrt{16\times11-11^2}}$ >0	0	$\frac{2(13)(-1)}{\sqrt{16\times13-13^2}}$ <0

Change in gradient (+ , 0 , -) and curve is continuous for $11\le h\le13$ $\therefore$ stat. pt. is a local max. tp.

Since the area function is continuous for $0\le h\le16$ and there is only one max. tp. for $0<h<16$ then the local max. tp. is the absolute max.

maximum $A=12\sqrt{12(16-12)}$

$=48\sqrt{3}$

$\therefore$ maximum area $=48\sqrt{3}$ cm^2

QUESTION 10: (STAR A NEW PAGE)

(i) Let $\$A_n$ = amount in the fund after the n^{th} awards night 2

$A_1=10000\times1.03-450$

$A_2=A_1\times1.03-450$

$=(10000\times1.03-450)\times1.03-450$

$=10000\times1.03^2-(1.03+1)\times450$

$=9695.50$

$\therefore$ amount in fund = \$9695.50

(ii) $A_1=10000\times1.03-450$ 3

$A_2=A_1\times1.03-450$

$=(10000\times1.03-450)\times1.03-450$

$=10000\times1.03^2-(1.03+1)\times450$

$\vdots$

$A_n=10000\times1.03^n-(1.03^{n-1}+1.03^{n-2}+\cdots+1.03+1)\times450$

$=10000\times1.03^n-1\times\left(\frac{1.03^n-1}{1.03-1}\right)\times450$

$=10000\times1.03^n-\left(\frac{1.03^n-1}{0.03}\right)\times450$

$=10000\times1.03^n-(1.03^n-1)\times15000$

$=10000\times1.03^n-1.03^n\times15000+15000$

$=15000-5000\times1.03^n$

$A_n=5000(3-1.03^n)$

(iii) when $n=25$ 1

$A_{25}=5000(3-1.03^{25})$

$=4531.11$

$\therefore$ amount = \$4351 (to nearest dollar)

(iv) $A_n\ge0$ 2

$5000(3-1.03^n)\ge0$

$3-1.03^n\ge0$

$1.03^n\le3$

$n\ln(1.03)\le\ln3$

$n\le\frac{\ln(1.03)}{\ln3}$

$n\le37.16$

$\therefore$ max. number of awards nights = 37

(v) (α) Let $\$B_n$ = amount in the fund after the n^{th} awards night 2

$B_1 = 10000 \times 1.03 - 450$

$B_2 = B_1 \times 1.03 - 450 \times 1.02$

$= (10000 \times 1.03 - 450) \times 1.03 - 450 \times 1.02$

$= 10000 \times 1.03^2 - (1.03 + 1.02) \times 450$

$= 9686.5$

$\therefore$ amount in fund = \$9686.50

(β) Let $\$B_n$ = amount in the fund after the n^{th} awards night 2

$B_1 = 10000 \times 1.03 - 450$

$B_2 = B_1 \times 1.03 - 450 \times 1.02$

$= (10000 \times 1.03 - 450) \times 1.03 - 450 \times 1.02$

$= 10000 \times 1.03^2 - (1.03 + 1.02) \times 450$

$\vdots$

$B_n = 10000 \times 1.03^n - 450 \times \{1.03^{n-1} + 1.03^{n-2}(1.02) + 1.03^{n-3}(1.02^2) + \cdots + 1.03(1.02^{n-2}) + 1.02^{n-1}\}$

B_n series is a GP with $a = 1.02^{n-1}$ and $r = \dfrac{1.03}{1.02}$

$$B_n = 10000 \times 1.03^n - 450 \times 1.02^{n-1} \times \left\{ \frac{\left(\frac{1.03}{1.02}\right)^n - 1}{\left(\frac{1.03}{1.02}\right) - 1} \right\}$$

$$= 10000 \times 1.03^n - \frac{450 \times 1.02^{n-1} \left\{ \frac{1.03^n - 1.02^n}{1.02^n} \right\}}{\left(\frac{1.03 - 1.02}{1.02}\right)}$$

$$= 10000 \times 1.03^n - \frac{450 \times (1.03^n - 1.02^n)}{0.01}$$

$= 10000 \times 1.03^n - 45000 \times (1.03^n - 1.02^n)$

$= 10000 \times 1.03^n - 45000 \times 1.03^n + 45000 \times 1.02^n$

$B_n = 45000 \times 1.02^n - 35000 \times 1.03^n$

$B_{25} = 45000 \times 1.02^{25} - 35000 \times 1.03^{25}$

$\therefore$ amount in fund = \$545

THE END