Question 1 (12 marks) Use a separate page/booklet **Marks**

(a) Simplify:

(i) $e^{\ln a}$ **1**

(ii) $e^{x \ln a}$ **1**

(b) If ${}^{n}c_5 = {}^{n}c_6$, *find n.* **1**

(c) Solve: $\dfrac{x}{x-2} \geq 4, \quad x \neq 2$ **3**

(d) Prove: ${}^{n}c_{k-1} + {}^{n}c_k = {}^{n+1}c_k$ **2**

(e) Differentiate with respect to x: $\log_7 x^2$ **2**

(f) Find the coordinates of the point P that divides the interval $(2, -6)$ and $(7, 9)$ internally in the ratio $2:3$. **2**

Question 2 (12 marks) Use a separate page/booklet **Marks**

(a) (i) How many arrangements can be made of the letters taken altogether of the word POLLUTION ? **2**

(ii) How many will start with T and end with P ? **1**

(b) Find the term independent of x in $\left(3x^3 - \frac{2}{x}\right)^8$ **3**

(c) Find the value of k if the roots of the equation $x^3 - 3x^2 - 6x + k = 0$ are in arithmetic progression. **3**

(d) An archer, finds that in the long run, he scores a bull's eye on 3 out of 5 occasions. He fires 8 rounds at a target. Assuming that each trial is an independent event, find the probability of

(i) exactly 5 bull's eyes. **1**

(ii) at least 7 bull's eyes. **2**

Question 3 (12 marks) Use a separate page/booklet **Marks**

(a) The point P($2ap, ap^2$) lies on the parabola $x^2 = 4ay$ whose focus is at S. The tangent at P meets the Y-axis at Q.

(i) Find the coordinates of Q. **1**

(ii) Show that $\angle SPQ = \angle SQP$ **2**

(b) Find $\int \frac{2x-1}{x^2+4}\, dx$ **2**

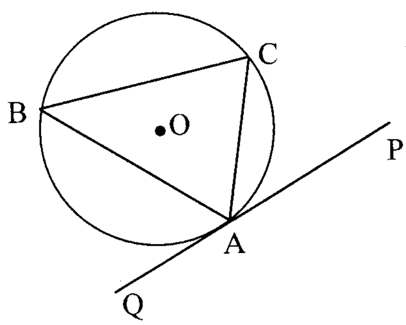
(c) A particle moves with a simple harmonic motion. It starts from rest at a point 6 cm from the centre of motion O. The particle has a speed of 10 *cm/s,* when it passes through O.

(i) Find the period of motion. **3**

(ii) Find the acceleration after 3 seconds. **1**

(d) PAQ is a tangent to the circle centre O of which AC, CB and BA are chords.

Prove that $\angle PAC = \angle ABC$ **3**



NOT TO SCALE

Question 4 (12 marks) Use a separate page/booklet. **Marks**

(a) A particle is projected under gravity with speed v *m/s* at an angle of projection θ.

(i) Obtain expressions for the horizontal and vertical displacements x and y at any time t seconds after projection. Let gravity = g m/s^2. **2**

(ii) Show that the equation of the path of the particle is given by

$$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$ **2**

(iii) A particle has an initial speed of $2\sqrt{70}$ m/s and just clears a pole. The pole is 5m high and its base is 20m from the point of projection. Find two possible angles of projection to the nearest degree. (Take $g = 9{\cdot}8$ m/s^2) **2**

(b) (i) Differentiate: $x\sin^{-1}x + \sqrt{1-x^2}$ **2**

(ii) Hence evaluate $\int_0^{\frac{1}{2}} \sin^{-1} x\, dx$ **2**

(c) The acceleration of a particle is given by $a = -e^{-x}$. Initially $v = \sqrt{2}, x = 0$. Find the velocity as a function of x. **2**

Question 5 (12 marks) Use a separate page/booklet. **Marks**

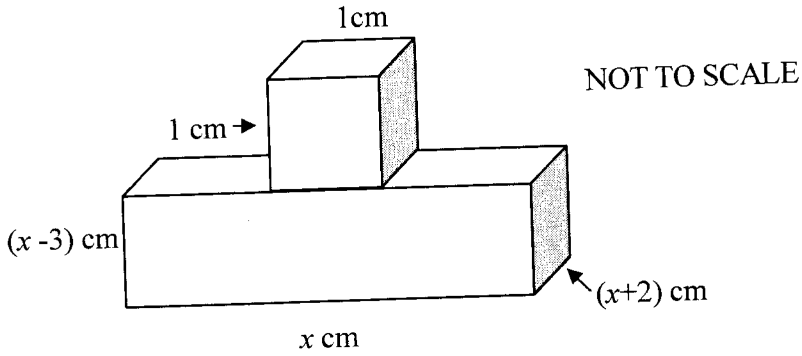
(a) A biologist is performing experiments with certain type of mosquitoes that reproduce at the rate of $(2t+1)e^{t^2+t}$ mosquitoes per month (where t is in months). The biologist starts the experiments with only 50 mosquitoes.

(i) If $y(t)$ denotes the mosquito population at any time t, find $y(t)$. **2**

(ii) How many mosquitoes will the biologist have after 2 months? **1**

(b) In how many ways can a jury of 6 people reach a majority decision? **2**

(c)



A block is to be made with dimensions as shown above.

(i) Show that the volume is given by the expression $(x^3 - x^2 - 5x + 2)\ cm^3$ **1**

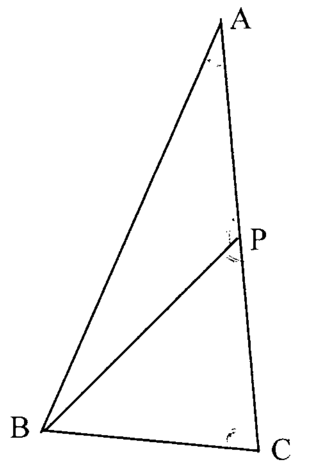
(ii) To make such a solid of volume 100 cm^3, show that the value of x is to be between 5·2 and 5·5 cm. **2**

(iii) Taking $x_1 = 5.2$ as the first approximation, use Newton's method to find a second approximation of x, correct to 3 significant figures. **2**

(d) Using the substitution $x = u^2 - 2$, find $\int \frac{x}{\sqrt{x+2}}\,dx$ **2**

Question 6 (12 marks) Use a separate page/booklet **Marks**

(a) In the triangle ABC, BC = BP = AP and $\angle BAC = 36^0$.



NOT TO SCALE

(i) Prove that $BC^2 = CP \times CA$ 2

(ii) If BC = 1 unit deduce that $\cos 36^0 = \frac{1}{4}(\sqrt{5}+1)$ 2

(b) (i) Sketch the curve $y = \ln(x-2)$ 1

(ii) The inner surface of a bowl is of the shape formed by rotating about the y axis, the curve $y = \ln(x-2)$ between y = 0 and y = 2

The bowl is placed with its axis vertical and water is poured in.

Show that the volume of water in the bowl when it is filled to a depth *h*, where h < 2, is given by $\pi(4h - 4\frac{1}{2} + 4e^h + \frac{1}{2}e^{2h})\ unit^3$. 3

(iii) If the bowl is filled at the rate of 60 $unit^3$ / s, find the rate at which the water level is rising when the depth of water is 1·25 *units*. Give your answer correct to 2 decimal places. 2

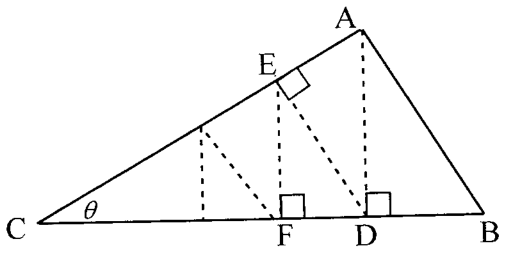
(c) (i) Sketch the curve $y = 4\cos^{-1}\frac{x}{3}$ 1

(ii) State its domain. 1

Question 7 (12 marks) Use a separate page/booklet **Marks**

(a) In the triangle ABC, $\angle ACB = \theta$, where $0 < \theta < \frac{\pi}{2}$ and AC is of length d.

A fly starts at A, flies directly to the line CB, i.e. to the point D. It then flies directly to the line CA, ie to the point E. It then flies directly to the line CB and so on until it ultimately reaches C.



(i) Show that the distance travelled by the fly when it reaches the point E is $d\sin\theta(1+\cos\theta)$ **1**

(ii) Show that the total distance travelled is given by $s = \dfrac{d\sin\theta}{1-\cos\theta}$ **2**

(b) (i) Use a calculator to find the smallest integer N for which **1**

$$ln\, N! > N$$

(ii) Prove by mathematical induction that $ln\, N! > N, for\ N \geq 6$ **3**

(c) John has baked a chocolate cake. At 2 pm he takes it out of a 180 ^{0}C hot oven and places it on a cooling rack in the kitchen where the temperature is 20 ^{0}C. According to Newton's Law of Cooling, the temperature, T, of John's cake t minutes after it comes out of the oven satisfies the equation

$$\frac{dT}{dt} = -k(T-20) \text{ where } k \text{ is a constant.}$$

(i) Show that $T = 20 + 160e^{-kt}$ is a solution of the equation. **2**

(ii) At 2.15 pm the cake's temperature is 100 ^{0}C. Find the value of k, correct to 3 significant figures. **1**

(iii) The cake must cool to 35 ^{0}C before John can ice it. What is the earliest time that the cake can be iced? **2**

ANSWERS QUESTION 1

Question 1 (a) (i)

Criteria	Marks
One mark for the correct answer	1

Answer:

Let $y = e^{\ln a}$

Taking logs on both sides,

$\ln y = \ln e^{\ln a}$

$= \ln a \ \ln e$

$= \ln a \quad since \quad \ln e = 1$

$\therefore y = a \quad or \ e^{\ln a} = a$

Question 1 (a) (ii)

Criteria	Marks
One mark for the correct answer	1

Answer:

Let $y = e^{x \ln a}$

Taking logs *on both sides,*

$\ln y = \ln(e^{x \ln a})$

$= x \ln a \ \ln e$

$= \ln a^x$

$\therefore y = a^x \qquad or \qquad e^{x \ln a} = a^x$

Question 1 (b)

Criteria	Marks
One mark for the correct answer	1

Answer:

$^n c_5 = {}^n c_6$

$$\frac{n(n-1)(n-2)(n-3)(n-4)}{5\times4\times3\times2\times1} = \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{6\times5\times4\times3\times2\times1}$$

Cancelling we have

$$1 = \frac{n-5}{6}$$

$$\therefore \quad n = 11$$

Question 1 (c)

Criteria	Marks
One mark for multiplying correctly both sides by $(x-2)^2$ one for correct factorization and one for choosing the correct values from the parabola	3

Answer:

$$\frac{x}{x-2} \geq 4$$

mutiplying both sides by $(x-2)^2$,

$$\frac{x}{x-2} \times (x-2)^2 \geq 4 \times (x-2)^2$$

$$x(x-2) \quad \geq 4x^2 - 16x + 16$$

$$x^2 - 2x \quad \geq 4x^2 - 16x + 16$$

$$0 \geq 3x^2 - 14x + 16$$

or $3x^2 - 14x + 16 \leq 0$

$(x-2)(3x-8) \leq 0$

$\therefore$ *Solution is* $2 < x \leq \dfrac{8}{3}$

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Question 1(d)

Criteria	Marks
One mark for writing RHS and LHS using the definition of ${}^nC_r$ and one for simplifying and showing LHS = RHS	2

Answer:

$$RHS = {}^{n+1}c_k$$
$$= \frac{(n+1)!}{k!(n+1-k)!}$$
$$LHS = \frac{n!}{(k-1)!\,(n-k+1)!} + \frac{n!}{k!(n-k)!}$$
$$= \frac{n!}{(k-1)!(n-k)!}\left[\frac{1}{n-k+1} + \frac{1}{k}\right]$$
$$= \frac{n!}{(k-1)!(n-k)!}\left[\frac{k+n-k+1}{k\ (n-k+1)}\right]$$
$$= \frac{n!}{(k-1)!(n-k)!}\left[\frac{n+1}{k(n-k+1)}\right]$$
$$= \frac{(n+1)!}{k!(n+1-k)!}$$
$$= RHS$$

Question 1 (e)

Criteria	Marks
One mark for the logarithmic transformation and one for differentiation	2

Answer:

$$y = \log_7 x^2$$
$$= \frac{\log_e x^2}{\log_e 7}$$
$$= \frac{2\log_e x}{\log_e 7} \quad = \frac{2}{\log_e 7} \times \log_e x$$
$$\therefore \frac{dy}{dx} = \frac{2}{\log_e 7} \times \frac{1}{x}$$
$$= \frac{2}{x \ln 7}$$

Question 1 (f)

Criteria	Marks
One mark for the correct substitution of the formula, one for simplification	2

Answer :

$$\left(\frac{2\times 7 + 3\times 2}{2+3}, \frac{2\times 9 + 3\times -6}{2+3}\right)$$
$$\therefore (4,0)$$

ANSWERS QUESTION 2

Question 2 (a) (i)

Criteria	Marks
One mark for writing $\frac{9!}{2!\,2!}$ and one for simplification	2

Answer:

There are nine letters, and out of these two letters O and L are repeated.

$$Hence\ number\ of\ arrangements = \frac{9!}{2!\ 2!}$$
$$= 90\,720$$

Question 2 (a) (ii)

Criteria	Marks
One mark for the correct answer	1

Answer:

When T and P are fixed, there are seven letters left and out of these two letters are repeated.

Hence the no. of arrangements $= \dfrac{7!}{2!2!} = 1260$

Question 2 (b)

Criteria	Marks
One mark for the general term, one for r and one for simplification.	3

Answer:

$$(3x^3 - \frac{2}{x})^8$$

$$\begin{aligned} T_{r+1} &= {}^8c_r\,(3x^3)^{8-r}\left(-\frac{2}{x}\right)^r \\ &= {}^8c_r 3^{8-r} x^{3(8-r)} (-2)^r (x)^{-r} \\ &= {}^8c_r\ 3^{8-r}\ x^{24-3r-r}(-2)^r \\ &= {}^8c_r\ 3^{8-r}\ x^{24-4r}\ (-2)^r \end{aligned}$$

For the term independent of x

$24 - 4r = 0$ *ie* $r = 6$

$$\begin{aligned} \therefore T_7 &= {}^8C_6 3^2 (-2)^6 \\ &= 16128 \end{aligned}$$

Question 2 (c)

Criteria	Marks
One mark for the value of α, one for the value of d^2 and one for the value of k	3

Answer:

Let the roots of the equation be $\alpha - d, \alpha$ *and* $\alpha + d$

Sum of the roots : $\alpha - d + \alpha + \alpha + d = 3\alpha = 3$

$$\therefore \alpha = 1$$

Sum of roots taken two at a time :

$$\alpha(\alpha - d) + (\alpha - d)(\alpha + d) + \alpha(\alpha + d) = -6$$

$$\alpha^2 - \alpha d + \alpha^2 - d^2 + \alpha^2 + \alpha d = -6$$

$$3\alpha^2 - d^2 = -6$$

$$3 - d^2 = -6 \quad \text{as } \alpha = 1$$

$$\therefore d^2 = 9$$

Product of roots:

$$\alpha(\alpha^2 - d^2) = -k$$

$$-8 = -k \quad \text{or} \quad k = 8$$

Question 2 (d) (i)

Criteria	Marks
One mark for the correct answer	1

Answer:

$${}^8C_5\left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^3 = \frac{108864}{390625} \text{ or } 0.279\ (3dp)$$

Question 2 (d) (ii)

Criteria	Marks
One mark for the correct expression and one for simplification	2

Answer:

$${}^8C_7\left(\frac{3}{5}\right)^7\left(\frac{2}{5}\right)^1 + {}^8C_8\left(\frac{3}{5}\right)^8 = \frac{41553}{390625} \text{ or } 0.106(3dp)$$

ANSWERS QUESTION 3

Question 3 (a) (i)

Criteria	Marks
One mark for correct answer.	1

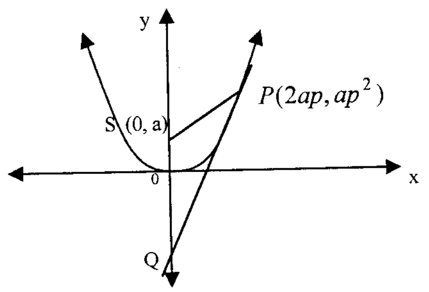
Answer:

Equation of tangent at P is given by
$y - ap^2 = p(x - 2ap)$

PQ meets the y-axis at x = 0
i.e. $y - ap^2 = -2ap^2$

$y = -ap^2$

$\therefore$ *Coordinates of* Q *are* $(0, -ap^2)$



Question 3 (a) (ii)

Criteria	Marks
One mark for $SP^2 = a^2(p^2+1)^2$ and one for simplification	2

Answer:

$$SP^2 = (2ap)^2 + (ap^2 - a)^2$$
$$= 4a^2p^2 + (a^2p^4 - 2a^2p^2 + a^2)$$
$$= a^2p^4 + 2a^2p^2 + a^2$$
$$= a^2(p^2+1)^2$$

$\therefore SP = a(p^2+1)$

$SQ = SO + OQ = a + ap^2 = a(1+p^2)$

$SP = SQ, \quad \angle SPQ = \angle SQP$ (*Base angles of isosceles triangle*)

Question 3 (b)

Criteria	Marks
One mark for the log answer and one for the inverse tan answer along with constant..	2

Answer:

$$\int \frac{2x-1}{x^2+4}dx = \int \frac{2x\,dx}{x^2+4} - \int \frac{dx}{x^2+4}$$
$$= \ln(x^2+4) - \frac{1}{2}\tan^{-1}\frac{x}{2} + C$$

Question 3 (c) (i)

Criteria	Marks
One mark for finding α , one for finding time, one for the period..	3

Answer:

Since the motion is simple harmonic

$$x = a\cos(nt + \alpha)$$

when $t = 0, x = 6$

$$6 = a\ \cos\alpha$$

$$\dot{x} = v = -an\ \sin(nt + \alpha)$$

when $t = 0, v = 0$

$$\therefore 0 = -an\sin\alpha$$

$\therefore\ \sin\alpha = 0\ \ or\ \ \alpha = 0$

$\therefore \cos\alpha\ = 1$

$since\ 6 = a\ \cos\alpha\ ,\ \ a = 6\ and\ x = 6\cos nt$

$$v = -6n\ \sin nt$$

$when\ x = 0,\ v = 10\ and\ 6\cos nt = 0$

$$\therefore\ nt = \frac{\pi}{2}, \frac{3\pi}{2}, -----$$

$$t = \frac{\pi}{2n}, \frac{3\pi}{2n}, ----$$

$$10 = |-6n\ \sin nt|$$

$$= 6n\ \sin\frac{\pi}{2}$$

$$= 6n$$

$$or\ n = \frac{10}{6} = \frac{5}{3}$$

$$Period\ of\ motion = \frac{2\pi}{n}$$

$$= \frac{2\pi}{\left(\frac{5}{3}\right)}$$

$$= \frac{6\pi}{5}\sec s$$

Question 3 (c) (ii)

Criteria	Marks
One for the correct answer	1

Answer:

$$Now\ x = 6\ \cos\frac{5t}{3}$$

$$\therefore\ v = -10\sin\frac{5t}{3}$$

$$And\ \frac{dv}{dt} = -\frac{50}{3}\cos\frac{5t}{3}$$

when $t = 3$

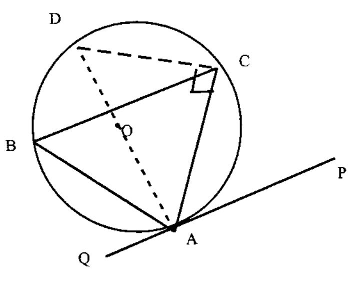
$$\frac{dv}{dt} = -\frac{50}{3}\cos\frac{5\times 3}{3}$$

$$= -4\cdot 73\,cm/s^2\ \text{(2 d.p.)}$$

Question 3 (d)

Criteria	Marks
One mark for drawing the diagram showing construction line and marking the right angles. One for showing that angles PAC and CAD are complimentary and one for the remainder.	3

Answer:



<u>To prove:</u> $\angle PAC = \angle ABC$

<u>Construction:</u> *Draw AD, the diameter. Join DC*

<u>Proof:</u>

$\angle PAC + \angle CAD = 90^0$ ($\angle PAD$ *is the angle between the tangent and diameter*)

$\angle ACD = 90^0$ (*angle in a semi-circle*)

$\angle CAD + \angle ADC = 90^0$ (*ADC is a right angled triangle*)

$\therefore\ \angle PAC = \angle ADC$

But $\angle ADC = \angle ABC$ (*angles in the same segment*)

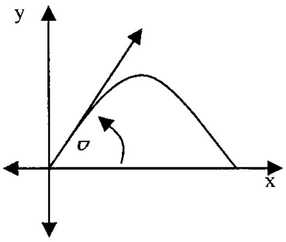
$\therefore\ \angle PAC = \angle ABC$

ANSWERS QUESTION 4

Question 4 (a) (i)

Criteria	Marks
One each for parametric equation involving x and y	2

Answer:



We take the origin to be the point of projection and x and y axes to be horizontal and vertical directions. The two components of initial speed are

$v\cos\theta$ and $v\sin\theta$

Hence the initial conditions are $t = 0$,

$x = 0, y = 0, \dfrac{dx}{dt} = v\cos\theta$ and

$\dfrac{dy}{dt} = v\sin\theta$

Horizontal motion

$$\frac{d^2x}{dt^2} = 0$$

$$\therefore \frac{dx}{dt} = c_1$$

when $t = 0$, $\dfrac{dx}{dt} = v\cos\theta$,

$$\therefore c_1 = v\cos\theta$$

$$\frac{dx}{dt} = v\cos\theta$$

$$\therefore x = vt\cos\theta + c_2$$

when $t = 0$, $x = 0$ and hence $c_2 = 0$

or $\therefore x = vt\cos\theta$ ---------------(1)

Vertical motion

$$\frac{d^2y}{dt^2} = -g$$

$$\therefore \frac{dy}{dt} = -gt + c_3$$

when $t = 0$, $\dfrac{dy}{dt} = v\sin\theta$

and hence $c_3 = v\sin\theta$

or $\dfrac{dy}{dt} = -gt + v\sin\theta$

$$y = -\frac{gt^2}{2} + vt\sin\theta + c_4$$

when $t = 0, y = 0$ *and hence* $c_4 = 0$

or $y = -\dfrac{gt^2}{2} + vt\sin\theta$ --------(2)

Question 4(a) (ii)

Criteria	Marks
One mark for $t = \dfrac{x}{v\cos\theta}$ and one for simplification	2

Answer:

From equation (1) we have $t = \dfrac{x}{v\cos\theta}$

Substituting the value of t in equation (2), we have $y = -\dfrac{g}{2}\left(\dfrac{x}{v\cos\theta}\right)^2 + v\left(\dfrac{x}{v\cos\theta}\right)\sin\theta$

$$= x\tan\theta - \frac{g}{2}\left(\frac{x^2}{v^2\cos^2\theta}\right) \quad \text{-------------------(3)}$$

$$= x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$

Question 4(a) (iii)

Criteria	Marks
One mark for obtaining the quadratic equation involving $\tan\theta$ and one for simplification	2

Answer:

Here we are given the values of v, x, y and g, we have to find the values of θ.

Substituting the given values in equation (3), we have

$$5 = 20\tan\theta - \frac{9.8}{2}\left(\frac{20^2}{\left(2\sqrt{70}\right)^2\cos^2\theta}\right)$$

$$5 = 20\tan\theta - (9.8\times 400)\times\frac{\sec^2\theta}{2\times 280}$$

$$= 20\tan\theta - 7\sec^2\theta$$

$$= 20\tan\theta - 7(1+\tan^2\theta)$$

$$7\tan^2\theta - 20\tan\theta + 12 = 0$$

$$or\ (7\tan\theta - 6)(\tan\theta - 2) = 0$$

$$\tan\theta = \frac{6}{7} \quad and \quad \tan\theta = 2$$

$$\therefore \theta = \tan^{-1}\frac{6}{7} \approx 41^0 \ and\ \theta = \tan^{-1}2 = 63^0 (to\ the\ nearest\ \deg ree)$$

Question 4(b) (i)

Criteria	Marks
One mark for differentiation of $\sin^{-1}x$ and one for simplification	2

Answer:

$$Let\ \ y = x\sin^{-1}x + \sqrt{1-x^2}$$

Then $\dfrac{dy}{dx} = \sin^{-1}x + x \times \dfrac{1}{\sqrt{1-x^2}} - \dfrac{x}{\sqrt{1-x^2}}$

$$= \sin^{-1}x$$

Question 4 (b) (ii)

Criteria	Marks
One mark for $\left[x\sin^{-1}x + \sqrt{1-x^2}\right]_0^{\frac{1}{2}}$ and one for simplification	2

Answer:

From (b) (i)

$$\int \sin^{-1}x\,dx = x\sin^{-1}x + \sqrt{1-x^2} + c$$

$$\therefore \int_0^{\frac{1}{2}} \sin^{-1}x\,dx = \left[x\sin^{-1}x + \sqrt{1-x^2}\right]_0^{\frac{1}{2}}$$

$$= \frac{1}{2}\sin^{-1}\frac{1}{2} + \sqrt{1-\frac{1}{4}} - 1$$

$$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$$

Question 4 (c)

Criteria	Marks
One mark for finding v^2 and one for simplification	2

Answer:

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -e^{-x}$$

$$\frac{1}{2}v^2 = e^{-x} + c$$

$$v^2 = 2e^{-x} + c_1$$

$$v = \sqrt{2}, x = 0$$

$$\therefore v = \sqrt{2}e^{\frac{-x}{2}}$$

ANSWERS QUESTION 5

Question 5 (a) (i)

Criteria	Marks
One mark for explaining the representation of population and reproduction as functions of *x*. One for finding the population function *y(t)*	2

Answer:

Let $y(t)$ denote the mosquito population at any time t, where t is in months

$$Given\ \frac{dy}{dt} = (2t+1)e^{t^2+t}$$

$$\therefore\ y(t) = \int (2t+1)e^{t^2+t}\,dt$$

$$= e^{t^2+t} + c$$

$$when\ t = 0, y = 50$$

$$\therefore 50 = 1 + c$$

$$c = 49$$

$$y(t) = e^{t^2+t} + 49$$

Question 5 (a) (ii)

Criteria	Marks
One mark for the correct answer	1

Answer:

$$y(t) = e^{t^2+t} + 49$$

$$t = 2$$

$$y(2) = e^6 + 49$$

$$\approx 452$$

Question 5(b)

Criteria	Marks
One for $\frac{6!}{4!2!}+\frac{6!}{5!1!}+\frac{6!}{6!4!}$ and one for simplification.	2

Answer:

$$^6C_4+{}^6C_5+{}^6C_6=\frac{6!}{4!2!}+\frac{6!}{5!1!}+\frac{6!}{6!4!}$$

$$=15+6+1$$
$$=22\ ways$$

Question 5 (c) (i)

Criteria	Marks
One mark for the correct answer	1

Answer:

Volume of the solid $= x(x-3)(x+2)+(x+2)$
$= x(x^2-x-6)+(x+2)$
$= x^3-x^2-6x+x+2$
$= x^3-x^2-5x+2$

Question 5 (c) (ii)

Criteria	Marks
One for finding $f(5.2)$ or $f(5.5)$, and one for the conclusion	2

Answer:

$$x^3-x^2-5x+2 = 100$$
or $$x^3-x^2-5x-98 = 0$$
Let $$f(x) = x^3-x^2-5x-98$$

$$f(5\cdot 2)=(5\cdot 2)^3-(5\cdot 2)^2-5\times(5\cdot 2)-98 = -10\cdot 432$$
$$f(5\cdot 5)=(5\cdot 5)^3-(5\cdot 5)^2-5\times 5\cdot 5-98 = 10.625$$

Since f (5·2) and f (5·5) have opposite signs there is root between 5·2 and 5·5.

Question 5 (c) (iii)

Criteria	Marks
One for applying Newton's formula and one for simplification	2

Answer:

If $x_1=5\cdot 2$ *be the first approximation,*

by Newton's law the second approximation is given by $x_2=x_1-\frac{f(x_1)}{f'(x_1)}$

$$f'(x)=3x^2-2x-5$$

$$f'(5\cdot 2)=3(5\cdot 2)^2-2(5\cdot 2)-5= 65\cdot 72$$

$$\therefore x_2=5\cdot 2-\frac{-10\cdot 432}{65\cdot 72}$$
$$=5.358..$$
$$ie\ x_2=5.36(3s.f)$$

Question 5 (d)

Criteria	Marks
One for correct integral involving u and one for simplification	2

Answer:

$$x = u^2 - 2$$
$$dx = 2udu$$
$$\therefore \int \frac{x}{\sqrt{x+2}}dx = 2\int \frac{(u^2-2)udu}{u}$$

$$= 2\int (u^2-2)\,du$$
$$= 2\left[\frac{u^3}{3} - 2u\right] + c$$
$$= \frac{2}{3}(x+2)^{\frac{3}{2}} - 4(x+2)^{\frac{1}{2}} + c$$

ANSWERS QUESTION 6

Question 6 (a) (i)

Criteria	Marks
One for proving ΔABC and ΔBPC are similar, and one for the conclusion	2

Answer:

$BP = BC$ (given)ΔPBC is isosceles

$\therefore \angle BCP = \angle BPC$

$= 72^0$

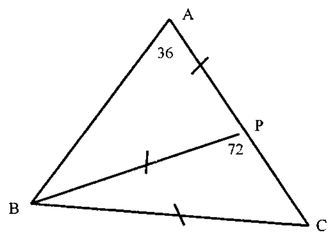
$\angle BAC + \angle ACB + \angle CBA = 180^0$ (Angle sum of a triangle)

In Δ's ABC and BPC $\angle C$ is common

$\angle ABC = \angle BPC = 72^0$

$\therefore$ ΔABC and ΔBPC are similar

$$\therefore \frac{CA}{BC} = \frac{BC}{CP} \quad or \quad BC^2 = CP \times CA$$



Question 6 (a) (ii)

Criteria	Marks
One for showing $AC = 2\,Cos\,36^0$ and one for simplification	2

Answer:

Let $BC = BP = AP = 1$

Using sine rule in ΔBPC $\quad \frac{BC}{Sin\angle BPC} = \frac{PC}{Sin\angle PBC}$

$$\therefore PC = \frac{BC \times Sin\angle PBC}{Sin\angle BPC} = \frac{Sin\,36^0}{Sin\,72^0}$$
$$= \frac{Sin\,36^0}{2Sin\,36^0\,Cos36^0}$$
$$= \frac{1}{2\,Cos\,36^0}$$

From above $BC^2 = AC \times PC$

$$or\ AC = \frac{BC^2}{PC} = \frac{1}{PC} = 2\,Cos\,36^0$$

Now $AC - PC = AP = 1$

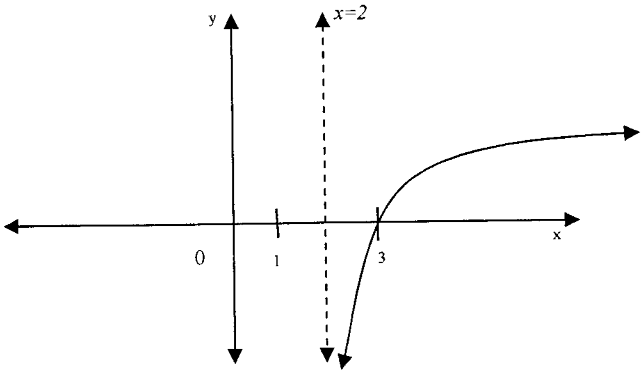
$$2\,Cos36^0 - \frac{1}{2Cos36^0} = 1$$
$$\Rightarrow 4\,Cos^2 36^0 - 2\,Cos\,36^0 - 1 = 0$$
$$\therefore Cos\,36^0 = \frac{2 \pm \sqrt{4+16}}{8}$$
$$= \frac{1}{4}\left(\sqrt{5}+1\right)$$

Since $\cos 36^0 > 0$, $\frac{1}{4}(1-\sqrt{5})$ is rejected

Question 6 (b) (i)

Criteria	Marks
One for the correct answer	1

Answer:



Question 6 (b) (ii)

Criteria	Marks
One for the x coordinate, one for finding x^2 and one for simplification	3

Answer:

$$\ln(x-2) = y$$

$$x - 2 = e^y$$

$$\therefore x = e^y + 2$$

$$or\ x^2 = \left(e^y + 2\right)^2$$

$$V = \pi \int_0^h x^2\,dy$$

$$= \pi \int_0^h \left(e^{2y} + 4e^y + 4\right) dy$$

$$= \pi \left[\frac{1}{2}e^{2y} + 4e^y + 4y\right]_0^h$$

$$= \pi \left[\left(\frac{1}{2}e^{2h} + 4e^h + 4h\right) - \left(\frac{1}{2} + 4\right)\right]$$

$$= \pi \left[\frac{1}{2}e^{2h} + 4e^h + 4h - 4\cdot 5\right]\ unit^3$$

Question 6 (b) (iii)

Criteria	Marks
One for finding an expression for $\frac{dh}{dt}$, and one for simplification	2

Answer:

$$\frac{dV}{dh} = \pi\left(e^{2h} + 4e^{h} + 4\right)$$

To find $\frac{dh}{dt}$ given $\frac{dV}{dt} = 60$ and h = 1·25

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$$60 = \pi\left(e^{2\cdot5} + 4e^{1\cdot25} + 4\right) \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{60}{\pi\left(e^{2\cdot5} + 4e^{1\cdot25} + 4\right)}$$

$$= 0\cdot6335...$$

$$= 0.63$$

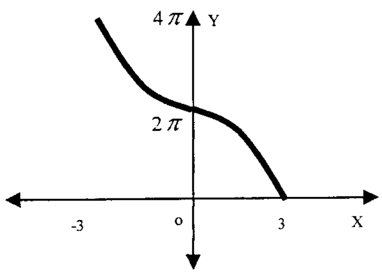
Rate of water level rising = 0· 63 ***unit/s***

Question 6 (c) (i)

Criteria	Marks
One for sketch	1

Answer:

$$y = 4\,Cos^{-1}\frac{x}{3}$$



Question 6 (c) (ii)

Criteria	Marks
One for the correct answer	1

Answer:

$$y = 4\,Cos^{-1}\frac{x}{3}$$

Domain : $-1 \le \frac{x}{3} \le 1$

Or $-3 \le x \le 3$

ANSWERS QUESTION 7

Question 7(a)

Criteria	Marks
One mark for the correct answer	1

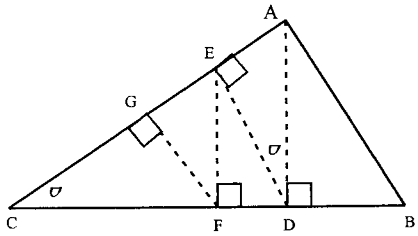
Answer:

Let D, E, F, G be the points the fly touches the inside the triangle. From ΔADC

$$AD = AC\sin\theta$$
$$= d\ \sin\theta$$

From ΔADE

$$DE = AD\cos\theta = d\sin\theta\cos\theta$$
$$\therefore distance = d\sin\theta + d\sin\theta\cos\theta$$
$$= d\sin\theta(1+\cos\theta)$$



Question 7 (a) (ii)

Criteria	Marks
One for $s = d\sin\theta + d\sin\theta\cos\theta + d\sin\theta\cos^2\theta$ and one for simplification	2

Answer:

Let D, E, F, G, be the points where the fly touches inside of the triangle.

From Δ ADC, $AD = AC\sin\theta$

$= d\ \sin\theta$

From Δ ADE, $DE = AD\cos\theta = d\sin\theta\cos\theta$

From Δ DEF, $EF = DE\cos\theta = d\sin\theta\cos^2\theta$

From Δ EFG, $FG = EF\cos\vartheta = d\sin\theta\cos^3\theta$

The total distance travelled is given by

$s = d\sin\theta + d\sin\theta\cos\theta + d\sin\theta\cos^2\theta + \text{-----}$

This is an infinite geometric series where

$a = d\sin\theta$ *and* $r = \cos\theta$ *and* $|\ r\ | \le 1$

$$\therefore\ s = \frac{d\sin\theta}{1-\cos\theta}$$

Question 7 (b) (i)

Criteria	Marks
One mark for the correct answer.	1

Answer:

Using the calculator: ln 5! = 4·79 (2dp)

ln 6! = 6· 58 (2dp)

The smallest integer n for which $\ln N! > N$ *is 6.*

Question 7 (b) (ii)

Criteria	Marks
One for stating step 3, one for proving step 3 and one for the conclusion	3

Answer:

Proposition : $\ln N! > N \quad for \quad N \geq 6$

Step 1: We have shown that it is true for N=6

Step2: Assume the proposition is true for N = k, i.e. $\ln k! > k, \quad for \ k > 6$

Step3: Show that the proposition is true for $N = k+1$ *i.e.* $\ln(k+1)! > k+1$

$$Now \quad \ln(k+1)! = \ln[(k+1) \times k!]$$
$$= \ln(k+1) + \ln k!$$

Since $\ln k! > k$ *and* $\ln(k+1) > 1 \ \ as \ \ k > 6,$

$\therefore \ \ln(k+1)! > k+1$ *i.e. the proposition is true for* $N = k+1$

Step4: If the proposition is true for N = k it is true for N = k +1. But it is true for N = 6.

Hence it is true for N = 6+1 = 7, 8, 9 - - - - so on for all positive integers greater than or equal to 6

Question 7 (c) (i)

Criteria	Marks
One mark for the differentiation and one for substituting for e^{-kt} to get the differential equation.	2

Answer:

$Now \quad T = 20 + 160\, e^{-kt}$

Differentiating wrt t ,

$$\therefore \frac{dT}{dt} = -160\, k\, e^{-kt}$$
$$= -160\, k \left(\frac{T-20}{160} \right)$$
$$= -k\,(T-20)$$

$\therefore \quad T = 20 + 160\, e^{-kt}$ *is a solution of the equation*

Question 7 (c) (ii)

Criteria	Marks
One mark for final answer.	1

Answer:

Given : $T = 100$ when $t = 15$

$\therefore 100 = 20 + 160\, e^{-15k}$

$$e^{-15k} = \frac{80}{160}$$
$$= 0 \cdot 5$$

taking logs on both sides,

$-15k = \ln 0 \cdot 5$

$$k = \frac{\ln 0 \cdot 5}{-15}$$
$$= 0.0462 \ (3sf)$$

Question 7 (c) (iii)

Criteria	Marks
One mark for substituting the values of T and t in $T = 20 + 160\,e^{-kt}$ and one for evaluating t.	2

Answer:

When $T = 35$ to find t

Now $35 = 20 + 160\,e^{-0.0462\,t}$

$$e^{-0\cdot 0462\,t} = \frac{15}{160}$$

$$= \frac{3}{32}$$

taking logs on both sides we have ,

$$-0\cdot 0462\,t = \ln\left(\frac{3}{32}\right)$$

$$\text{or } t = \frac{\ln\left(\frac{3}{32}\right)}{-0\cdot 0462} \approx 51$$

The cake will be ready for icing at about *2 pm + 51 min = 2: 51pm.*