MATHEMATICS - TRIAL REVISION

BOOKLET 3

1) INDEPENDENT 2001
2) CATHOLIC 2002
3) FORT ST. 2003

2001
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5minutes
- Working time – 3 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided on the last page
- All necessary working should be shown in every question

Total marks (120)

Attempt Questions 1 – 10

All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: ... ...

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

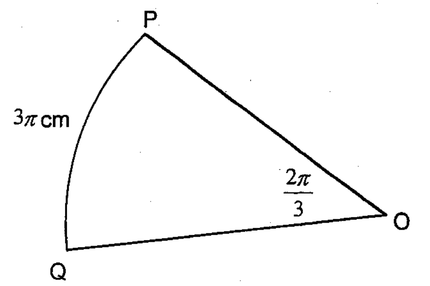
Question 1 **Marks**

(a) Evaluate correct to 3 significant figures : $\frac{\pi}{e^3}$. **2**

(b) Solve $7-3x<5$ and graph your solution set on a number line. **2**

(c) Express $\frac{1}{\sqrt{3}-2}-\frac{1}{\sqrt{3}+2}$ in its simplest form. **2**

(d) **2**



In the diagram above, PQ is an arc of a circle, centre O. The length of the arc is 3π centimetres and angle POQ is $\frac{2\pi}{3}$ radians.
Find the radius of the circle.

(e) The length of the line joining the points A $(a,-2)$ and B $(3,-7)$ is $5\sqrt{2}$ units **2**
Find all possible values of a.

(f) Solve the simultaneous equations; **2**

$$2x-y=7$$
$$x+3y=0$$

Question 2 *Start a new page* **Marks**

(a) Differentiate:

(i) $(2x^3-5)^7$ **2**

(ii) x^2e^{3x} **2**

(iii) $\frac{\sin x}{x}$. **2**

(b) On the same number plane draw the graphs of: **3**

$$y=\sqrt{4-x^2} \text{ and } y=|x|$$

Shade in on your graph, the region where $y\le\sqrt{4-x^2}$ and $y\ge|x|$ hold simultaneously

(c) The curve $y=ax^3+bx$ passes through the point $(1,7)$. The tangent at this point is parallel to the line $y=2x-6$. **3**
Find the values of a and b.

Question 3 *Start a new page* **Marks**

(a) Find primitives of:

(i) $\sec^2 7x$ **1**

(ii) $\dfrac{x^2}{x^3+3}$ **2**

(b) (i) On the same diagram draw graphs of the functions $y = x^2$ and $y = 5 - 4x$ showing all intercepts with the x and y- axes. **2**

(ii) Show that the graphs intersect at $x = 1$ and $x = -5$. **1**

(iii) Hence find the exact area bounded by the two functions. **2**

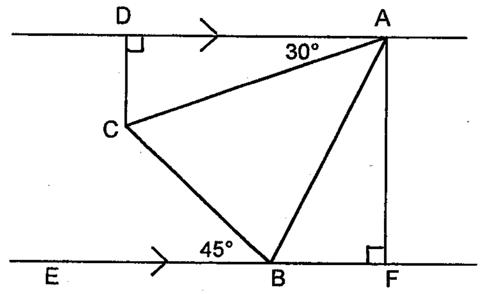
(c) Evaluate :

(i) $\displaystyle\int_{-2}^{-1}\left(\frac{1}{x^2}\right)dx$ **2**

(ii) $\displaystyle\int_{0}^{\frac{\pi}{3}}\cos(2x+\pi)\,dx$ **2**

Question 4 *Start a new page* **Marks**

(a)



Not to Scale

In the diagram above AC = AB and DA is parallel to EF. Angle DAC = 30° and angle CBE = 45°. CD is perpendicular to DA and AF is perpendicular to EF.

Copy or trace the diagram onto your working paper.

(i) Find the size of angle ACB, giving reasons. **2**

(ii) Hence find the size of angle CAB. **1**

(iii) Prove that $\Delta ACD \equiv \Delta ABF$. **2**

(b) A set of packing crates has been designed each in the shape of a rectangular prism. When empty, each crate packs inside the next sized crate. The largest crate is 2 metres long by 2metres wide by 1 metre high. The crate inside this one is 1 metre by 1 metre by 0.5 metres. Each succeeding crate has dimensions which are half those of the preceding one.

(i) Write down the dimensions of the third largest crate. **1**

(ii) Calculate the maximum possible total volume for the complete set. **2**

(c) For the parabola : $(y-1)^2 = 8(x+2)$

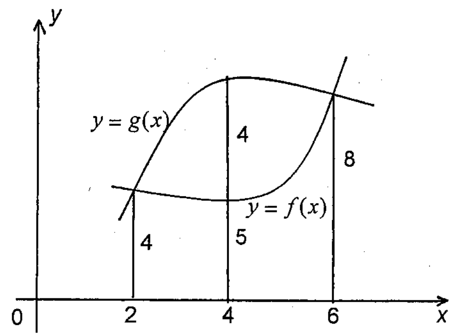
(i) State the coordinates of the vertex and the focus. **2**

(ii) Sketch the graph of the parabola showing the above. **1**

(iii) Write down the equation of the axis of symmetry. **1**

Question 5 *Start a new page* **Marks**

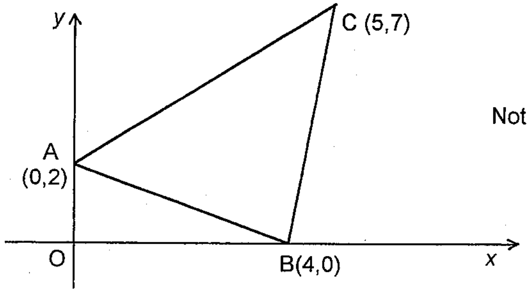
(a) **3**



Not to Scale

In the diagram above, the graphs of the functions $y = f(x)$ and $y = g(x)$ are shown. Using Simpson's Rule, find an approximate value for the area enclosed by the two curves.

(b)



Not to Scale

The diagram shows the points A (0,2), B (4,0) and C (5,7).
Copy the diagram onto your worksheet.

(i) Find the coordinates of M, the midpoint of AB. **1**

(ii) Show that the gradient of AB is $-\frac{1}{2}$ **1**

(iii) Find the equation of the perpendicular bisector of AB. **2**

(iv) Show that the perpendicular bisector of AB passes through C. **1**

(v) What type of triangle is ABC? (Give a reason for your answer) **1**

(c) Solve : $2^{2x} - 15(2^x) - 16 = 0$ **3**

Question 6 *Start a new page* **Marks**

(a) Consider the curve $y = x^3 + 4x^2 - 3x$.

(i) Show that the gradient of the curve at $x = 1$ is 8. **1**

(ii) Hence find the equation of the normal at $x = 1$ **2**

(b) Janice lives in Springwood and is starting a new job in Parramatta. She needs to catch the train to get to work. Her new boss says that she cannot be late on the first two days of her new job or she will lose it.
The probability that her train arrives on time is 0.95.

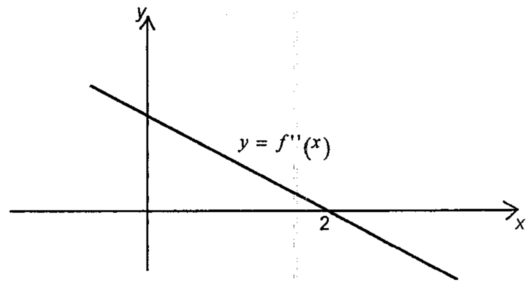
(i) What is the probability that Janice's train is late on the first day? **1**

(ii) What is the probability of the train being late on the first two days? **1**

(iii) What is the probability of Janice keeping her job? **1**

(iv) What is the probability that Janice arrives late on exactly one of the first three days of her new job? **2**

(c) **2**



The diagram above shows the graph of the $y = f''(x)$, the second derivative of the function $y = f(x)$. Given that $f(2) = 0$ and $f'(1) = 0$, draw a possible sketch of the function $y = f(x)$.

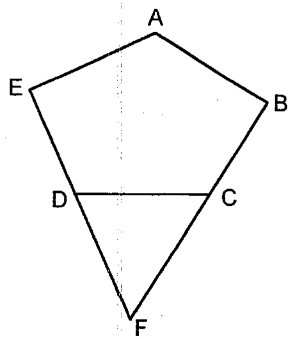
(d) Consider the sequence : $a,\ 3a-1,\ 5a-2, \ldots$

(i) Find the twentieh term **1**

(ii) Find the sum of the first twenty terms. **1**

Question 7 *Start a new page* **Marks**

(a)



Not to Scale

ABCDE is a regular pentagon. BC and ED are produced to meet at F.

Copy or trace the diagram onto your working paper.

(i) Show that the size of each internal angle in the pentagon is $108°$ **1**

(ii) Show that triangle FCD is isosceles. **1**

(iii) Prove that triangle FCD is similar to triangle FBE. **2**

(iv) If the sides of the pentagon are each 5 centimetres and BE = 8 centimetres, determine the length of CF.. **2**

(b) For the curve represented by the equation $y = x^3 + 3x^2 - 1$

(i) Find $\frac{dy}{dx}$. **1**

(ii) Find all stationary points and determine their nature. **3**

(iii) Sketch the curve in the domain $-3 \le x \le 2$, showing the above information. **2**

Question 8 *Start a new page* **Marks**

(a) A particle is moving along the x-axis. The distance of the particle, x metres, from the origin O is given by the equation $x = 6t + e^{-4t}$, where t is the time in seconds.

(i) What is the position of the particle when $t = \frac{1}{2}$? **1**

(ii) Write down an expression for the velocity of the particle and find the initial velocity. **2**

(iii) Show that the initial acceleration of the particle is 16 cm/sec^2. **2**

(iv) Explain why the particle will never come to rest. **1**

(b) Given that $\cos\alpha = -\frac{5}{\sqrt{29}}$ and $\tan\alpha < 0$, find the value of $\operatorname{cosec}\alpha$ **2**

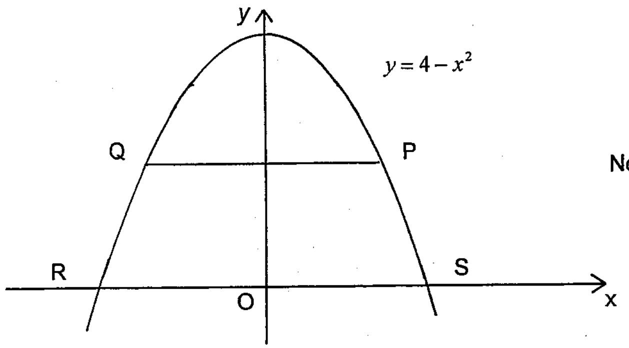
(c) (i) Sketch the graph of $y = \cos 2x$, for $0 \le x \le \pi$ **1**

(iii) Find all values of x for $0 \le x \le \pi$, such that $2\cos 2x = 1$ **2**

(iii) By drawing a straight line on your graph, illustrate these solutions. **1**

Question 9 *Start a new page* **Marks**

(a)



Not to Scale

The parabola $y = 4 - x^2$ cuts the x-axis at R and S. The point P (x,y) lies on the parabola in the first quadrant. Q also lies on the parabola such that PQ is parallel to the x-axis.

(i) Write down the coordinates of R and S. **1**

(ii) Show that the area of trapezium PQRS is given by : **2**

$$A = (2 + x)(4 - x^2)$$

(iii) Hence find the value of x which gives a maximum value of A, justifying your answer **3**

(b) The size of the population, P, of a colony of whiteants after t days is given by the equation $P = 3000e^{kt}$

(i) What was the initial size of the colony? **1**

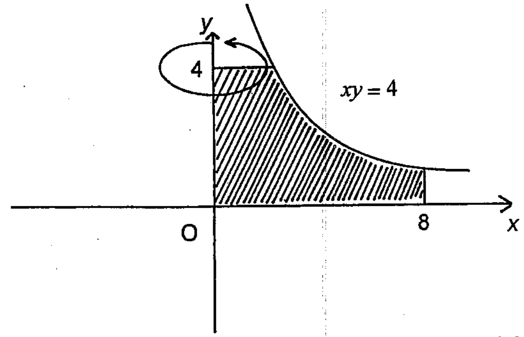
(ii) If there are 4000 whiteants in the colony after 1 day, find the value of k correct to 2 decimal places. **2**

(iii) What is the size of the colony after 2 days? **2**

(iv) When will the colony quadruple in size? (Answer to the nearest day) **1**

Question 10 *Start a new page* **Marks**

(a)



The area enclosed by the curve $xy = 4$, the x and y-axes and the lines $y = 4$ and $x = 8$, is rotated about the y-axis.

(i) Show that the volume of the solid of revolution obtained is given by; **2**

$$V_y = 32\pi + \pi \int_{\frac{1}{2}}^{4} \frac{16}{y^2}\, dy$$

(ii) Hence find the volume of the solid. **2**

(b) Given the quadratic equation $3x^2 - 5x + 6 = 0$.

(i) Find the value of the discriminant. **1**

(ii) Explain the nature of the roots of the equation $3x^2 - 5x + 6 = 0$. **1**

(c) Michael has decided to invest in a superannuation fund. He calculates that he will need \$1 000 000 if he is to retire in 20 years time and maintain his present lifestyle. The superannuation fund pays 12% per annum interest on his investments.

(i) Michael invests \$P at the beginning of each year. Show that at the end of the first year his investment is worth \$P(1.12). **1**

(ii) Show that at the end of the third year the value of his investment is given by the expression \$P$(1.12)(1.12^2 + 1.12 + 1)$. **2**

(iii) Find a similar expression for the value of his investment after 20 years and hence calculate the value of P needed to realise the total of \$1 000 000 required for his retirement. **3**

End of Paper

NSW INDEPENDENT TRIAL EXAMS –2001
MATHEMATICS (2 unit) SUGGESTED ANSWERS

Q1. (a) $0.156410\ldots$

$\doteqdot 0.156$

(b) $7-3x < 5$

$-3x < -2$

$x > 2/3$

$2/3$

(c) $\dfrac{(\sqrt{3}+2)-(\sqrt{3}-2)}{(\sqrt{3})^2-2^2}$

$=\dfrac{4}{-1} = -4$

(d) $3\pi = r \cdot 2\pi/3$

$r = 9\pi/2\pi$

$= 4.5\,cm$

(e) $(a-3)^2+(-2+7)^2 = (5\sqrt{2})^2$

$a^2-6a+34 = 50$

$a^2-6a-16=0$

$(a-8)(a+2)=0$

$a = 8, -2$

(f) $6x-3y = 21$

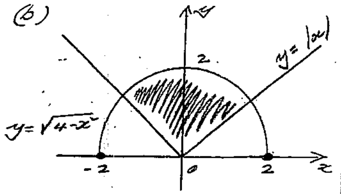
$x+3y = 0$

$7x = 21$

$x = 3$

$y = -1$

(b)



(c) Sub $(1,7)$

$7 = a+b$ — ①

$y' = 3ax^2+b$

$m=2$ (// $y=2x-6$)

$\therefore 2 = 3a+b$ — ②

②−① $-5 = 2a$

$a = -2\tfrac{1}{2}$

$b = 9\tfrac{1}{2}$

Q2. (a)

(i) $7(2x^3-5)^6 \cdot 6x^2$

$= 42x^2(2x^3-5)^6$

(ii) $e^{3x}\cdot 2x + x^2\cdot 3e^{3x}$

$xe^{3x}(2+3x)$

(iii) $\dfrac{x\cos x - \sin x}{x^2}$

(c) (i) $\left[\dfrac{x^{-1}}{-1}\right]_{-2}^{-1} = 1 - \tfrac{1}{2}$

$= \tfrac{1}{2}$

(ii) $\left[\tfrac{1}{2}\sin(2x+\pi)\right]_0^{\pi/3}$

$= \tfrac{1}{2}\left(\sin\tfrac{5\pi}{3} - \sin\pi\right)$

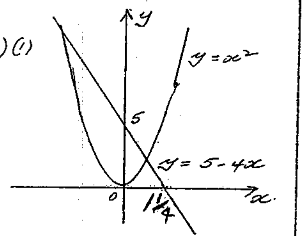
$= \tfrac{1}{2}\left(-\tfrac{\sqrt{3}}{2} - 0\right)$

$= -\dfrac{\sqrt{3}}{4}$

Q3 (a) (i) $\tfrac{1}{7}\tan 7x + c$

(ii) $\tfrac{1}{3}\ln(x^2+3)+c$

(b) (i)



(ii) $x^2 = 5-4x$

$x^2+4x-5=0$

$(x+5)(x-1)=0$

$x = -5, 1$

(iii) $A = \int_{-5}^{1}(5-4x-x^2)\,dx$

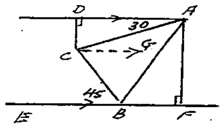
$= \left[5x-2x^2-\dfrac{x^3}{3}\right]_{-5}^{1}$

$= 2\tfrac{2}{3} - (-33\tfrac{1}{3})$

$= 36$ units^2

Q4.

(a)



(i) Draw CG // DA & EF

$\hat{ACG} = 30°$ (alt ∠'s DA//CG)

Similarly $\hat{BCG} = 45°$

$\therefore \hat{ACB} = 75°$

(ii) $\hat{ABC} = 75°$

(Base ∠'s isos △ ACB)

$\therefore \hat{CAB} + 2\times75° = 180°$

(∠ Sum △ ACB)

$\hat{CAB} = 30°$

(iii) $\hat{DAF} = 90°$ ($AF \perp EF$

$\therefore AF \perp DA$ // EF)

$\therefore \hat{BAF} = 30°$

In △'s ACD, ABF

$\hat{BAF} = \hat{CAD} = 30°$

$AC = AB$ (data)

$\hat{ADC} = 90° = \hat{AFB}$

($CD \perp DA$ & $AF \perp EF$)

$\therefore$ △ ACD ≡ △ ABF (AAS)

(b) (i) $0.5\times0.5\times0.25$

(ii) G.P. $a=4$ $r=\tfrac{1}{8}$

$S_\infty = \dfrac{4}{1-1/8}$

$= \tfrac{32}{7}\ m^3$

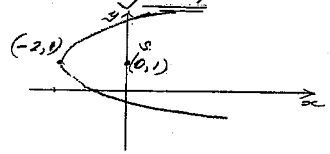
Q4 (c)

$(y-1)^2 = 4\times2(x+2)$

$V = (-2,1)$

$S = (0,1)$

Axis $y=1$



Q5 (a)

$A = \tfrac{4}{6}(4+4\times9+8)$

$-\tfrac{4}{6}(4+4\times5+8)$

$= \tfrac{4}{6}(4\times4)$

$= \tfrac{32}{3}$ units^2.

(b) (i) $M(2,1)$

(ii) $m_1 = \dfrac{2-0}{0-4} = -\tfrac{1}{2}$

(iii) $m_2 = 2$.

$y-1 = 2(x-2)$

$y = 2x-3$

(iv) $7 = 2\times5-3$.

(v) Isos △. as ⊥ bisector of base passes through the vertex.

(c) Let $X = 2^x$

$X^2-15X-16=0$

$(X-16)(X+1)=0$

$2^x = 16, -1$

$x = 4$ $(2^x \neq -1)$

Q6.

(a) (i) $y' = 3x^2+8x-3$

$x=1$ $m_1 = 3+8-3 = 8$.

(ii) $y = 1+4-3 = 2$

$m_2 = -\tfrac{1}{8}$.

$\therefore y-2 = -\tfrac{1}{8}(x-1)$

$8y-16 = -x+1$

$x+8y-17=0$

(b) (i) 0.05

(ii) $(0.05)^2 = 0.0025$

(iii) $1-0.0025$

$= 0.9975$

(iv) Arrive late day 1 only

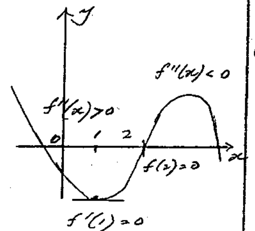
$= 0.05\times0.95\times0.95$

Sim for days 2, 3.

$\therefore 3\times0.05\times0.95\times0.95$

$= 0.135375$

(c)



(d) (i) $T_{20} = a+19\times(2a-1)$

$= 39a-19$

(ii) $S_{20} = \tfrac{20}{2}(a+39a-19)$

$= 10(40a-19)$

Q7 (a) (i) $S = (2n-4)\times90$

$= 540°$

$\therefore$ Each ∠ $= 108°$.

(ii) $\hat{EOC} + \hat{CDF} = 180°$

$\therefore \hat{CDF} = 72°$

Sim $\hat{DCF} = 72°$

△ FCD isosceles

(iii) $FC = FD$ (isos △)

$CB = DE$ (reg pent.)

$\therefore FB = FE$

$\therefore \dfrac{FC}{FB} = \dfrac{FD}{FE}$

$\hat{F}$ is common

$\therefore$ △ FCD ||| △ FBE

(iv) $\dfrac{FC}{FB} = \dfrac{DC}{BE}$

Let $FC = x$ cm

$\therefore \dfrac{x}{x+5} = \dfrac{5}{8}$

$8x = 5x+25$

$CF = x = 8\tfrac{1}{3}$ cm.

(b) (i) $y' = 3x^2+6x = 0$

(ii) $3x(x+2)=0$

$x = 0, -2$.

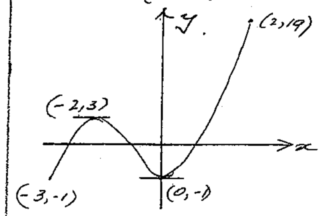
$y'' = 6x+6$

$(0,-1)$ $y'' = 6$. ∪

$\therefore$ Min T.P.

$(-2,3)$ $y'' = -6$ ∩

Max T.P.



Q8

(a) (i) $t = \frac{1}{2}$

$x = 6(\frac{1}{2}) + e^{-4(\frac{1}{2})}$

$= \underline{\underline{3 + e^{-2}}}$

or $\underline{\underline{3 + \frac{1}{e^2}}}$

$(3.135335283..)$

(ii) $v = \underline{\underline{6 - 4e^{-4t}}}$

$t = 0 \quad v = 6 - 4e^0$

$= \underline{\underline{2 \text{ m/sec}}}$

(iii) $a = 16e^{-4t}$

$t = 0 \quad a = 16.$

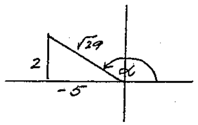
(iv) As $t \to \infty \quad e^{-4t} \to 0$

$\therefore \underline{\underline{v \to 6}}$

$\therefore$ Never stops

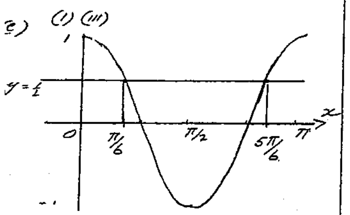
(b) $\cos < 0 \quad \tan < 0$

$\therefore$ Quad 2.



$\operatorname{cosec} \alpha = \underline{\underline{+\frac{\sqrt{29}}{2}}}$

(c) (i) (iii)



(ii) $\cos 2x = \frac{1}{2}$

$0 \le 2x \le 2\pi$

$2x = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

Q9 (a) (i) $\underline{R(-2,0)}$ $\underline{S(2,0)}$

(ii) $A = \frac{h}{2}(a+b)$

$= \frac{y}{2}(4 + 2x)$

$= \left(\frac{4-x^2}{2}\right)(4+2x)$

$= \underline{(2+x)(4-x^2)}$

(iii) $A = 8 + 4x - 2x^2 - x^3$

$A' = 4 - 4x - 3x^2 = 0$

for Max, Min

$(2 - 3x)(2 + x) = 0$

$x = 2/3, -2.$

$A'' = -4 - 6x$

$x = 2/3 \quad A'' = -4 - 6(2/3)$

< 0

$\therefore$ Max value of A

when $\underline{\underline{x = 2/3}}$

(b) (i) $t = 0 \quad \underline{P = 3000}$

(ii) $4000 = 3000e^k$

$e^k = 4/3$

$k = \ln(4/3)$

$\underline{\underline{k \doteqdot 0.29}}$

(iii) $t = 2 \quad P = 3000e^{2(0.29)}$

$= \underline{\underline{5358}}$

(iv) $12000 = 3000e^{0.29t}$

$4 = e^{0.29t}$

$0.29t = \ln 4$

$t = \frac{\ln 4}{0.29}$

$\doteqdot 4.78$

ie after $\underline{\underline{5 \text{ days}}}$

Q10 (a)

(i) $x = 8 \quad y = \frac{1}{2}$

$V = \pi r^2 h + \pi \int_{\frac{1}{2}}^{4} x^2 \, dy$

$= \pi(8)^2 \cdot \frac{1}{2} + \pi \int_{\frac{1}{2}}^{4} 16y^{-2} \, dy$

(ii) $= 32\pi + \pi\left[-16y^{-1}\right]_{\frac{1}{2}}^{4}$

$= 32\pi + \pi[-4 - (-32)]$

$= \underline{\underline{60\pi \text{ units}^3}}$

(b) (i) $\Delta = (-5)^2 - 4(3)(6)$

$= \underline{-47.}$

As $\Delta < 0$

$\therefore$ Unreal roots

(c) (i) Let I_n = value of investment after n yrs.

$I_1 = \$P + 12\%$ of $\$P$

$= \underline{\$P(1.12)}$

(ii) $I_2 = P(1.12) + P$

$+ 12\%(P(1.12) + P)$

$= (1.12)(P(1.12) + P)$

$= P(1.12)(1.12 + 1)$

Sim $I_3 = P(1.12)(1.12+1) + P$

$+ 12\%\{P(1.12)(1.12+1) + P\}$

$= \underline{P(1.12)\{(1.12)^2 + 1.12 + 1\}}$

(iii) $I_{20} = 1\,000\,000$

$= P(1.12)(1.12^{19} + \dots 1.12^2 + 1.12 + 1)$

$1000000 = P(1.12)\frac{(1.12^{20} - 1)}{(1.12 - 1)}$

$P = \frac{1000000 \times 0.12}{1.12(1.12^{20} - 1)}$

$= \underline{\underline{\$12391.77}}$

Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2002

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Morning Session
Monday 12 August 2002

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on page 15
- All necessary working should be shown in every question

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

Total marks (120)
Attempt Questions 1 – 10
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\dfrac{x^3+y^4}{y^2}$ if $x=\left(\dfrac{2}{5}\right)^{\frac{1}{3}}$ and $y=\left(\dfrac{3}{5}\right)^{\frac{1}{2}}$. **2**

Give your answer in fractional form.

(b) Express $0.2\dot{3}$ as a fraction in simplest form. **2**

(c) Factorise $40-5y^3$ **2**

(d) Solve $x^2+4x-1=0$ leaving your answer in simplest surd form. **3**

(e) (i) Solve $4^x=32$. **2**

(ii) Hence, or otherwise, write down the value of $\log_4 32$. **1**

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

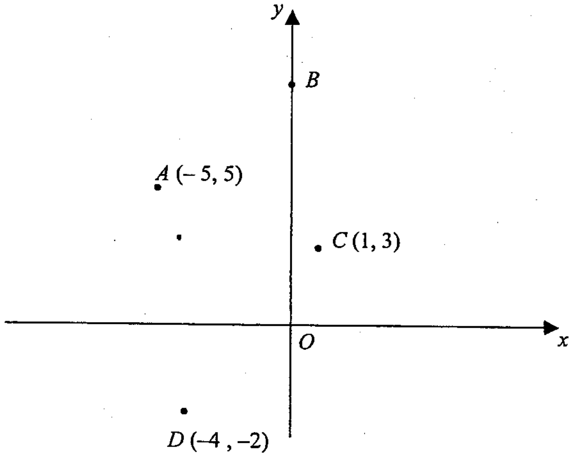
(a) Differentiate with respect to x:

(i) $5x + \dfrac{3}{x^2}$ **1**

(ii) e^{2x^2+3} **1**

(iii) $\dfrac{3x}{\sin x}$ **2**

(b)



NOT TO SCALE

The diagram shows the points A (–5, 5) and C (1, 3) and D (–4, –2).
B is a point on the y axis.

(i) Find the gradient of AC. **1**

(ii) Find the midpoint of AC. **1**

(iii) Show that the equation of the perpendicular bisector of AC is $3x - y + 10 = 0$.

(iv) Find the coordinates of B given that B lies on $3x - y + 10 = 0$. **1**

(v) Show that the point D (– 4, –2) lies on $3x - y + 10 = 0$. **1**

(vi) Show that $ABCD$ is a rhombus. **2**

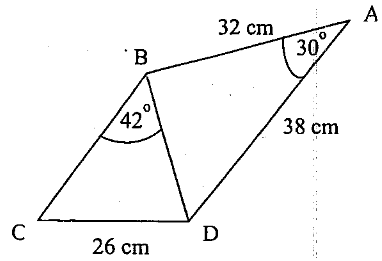
Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\displaystyle\int \frac{x}{x^2+5}\,dx$ **2**

(b) Evaluate $\displaystyle\int_0^{\frac{\pi}{8}} \sec^2 2x\,dx$ **2**

(c) Find the equation of the normal to the curve $y = x\log_e x$ at the point (e, e). **4**

(d)



NOT TO SCALE

In the diagram AB is 32 cm, AD is 38 cm and CD is 26 cm.
∠BAD is 30° and ∠CBD is 42°.

(i) Use the cosine rule to find the length of BD. **2**

(ii) Hence, find the size of ∠BCD to the nearest degree. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The second term of a geometric series is 120 and the fifth term is 50.625.

(i) Find the common ratio and the first term of the series. **2**

(ii) Find the limiting sum of the series. **1**

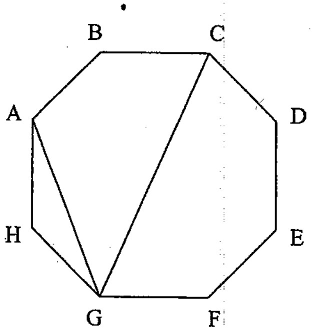
(iii) Hence, find the difference between the limiting sum and the sum of the first 40 terms giving your answer in scientific notation correct to 2 significant figures. **2**

(b) For the quadratic equation $x^2 + kx - 3x + 2 - k = 0$,

(i) find the value of the discriminant in terms of k, **1**

(ii) explain why the roots of this quadratic equation are real for all values of k. **2**

(c)



NOT TO SCALE

ABCDEFGH is a regular octagon.

(i) Explain clearly why $\angle ABC$ is 135°. **1**

(ii) Calculate the size of $\angle GAH$. **1**

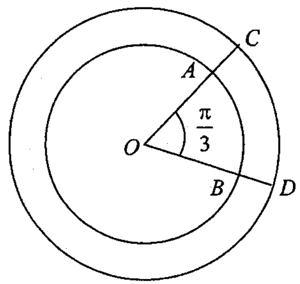
(iii) Using (i), or otherwise, calculate the size of $\angle CGF$. **1**

(iv) Hence, calculate the size of $\angle AGC$. **1**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)

NOT TO SCALE



The diagram shows two concentric circles with centre O.
The radius of the larger circle is 8.2 cm.

(i) Calculate the area of sector COD. **1**

(ii) The area of the sector AOB is 18.4 cm^2. Calculate the radius of this sector AOB. **2**

(iii) Calculate the area of triangle COB. **2**

(b) Let $f(x) = 3x^2 + 1$.

(i) Copy the following table and supply the missing values. **1**

x	0	0.2	0.4	0.6	0.8	1
$f(x)$	1					4

(ii) Use these six values of the function and the trapezoidal rule to find the approximate value of **2**

$$\int_0^1 (3x^2 + 1)\, dx .$$

Question 5 continues on page 8

Question 5 (continued)

Marks

(c) The population P of a town is growing at a rate proportional to the town's current population. The population at time t years is given by $P = A\,e^{kt}$, where A and k are constants.

The population 20 years ago was 100 000 people and today the population of the town is 150 000 people.

(i) Find the value of A. **1**

(ii) Find the value of k. **1**

(iii) Find the population that will be present 20 years from now. **2**

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet.

Marks

(a) Consider the curve given by $y = x^3 + 3x^2 - 9x - 5$.

(i) Find $\frac{dy}{dx}$. **1**

(ii) Find the coordinates of the two stationary points. **2**

(iii) Determine the nature of the stationary points. **2**

(iv) Sketch the curve for the domain $-5 \le x \le 3$. **2**

(v) By drawing an appropriate line on your graph, or otherwise, solve **2**

$$x^3 + 3x^2 - 9x + 5 = 0\,.$$

(b) Calculate the exact volume generated when the region enclosed by the curve **3**

$$y = 1 + 2e^{-x} \text{ for } 0 \le x \le 1,$$

is rotated about the x axis.

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A bag contains 5 blue balls, 4 red balls, 2 yellow balls and 1 green ball. Three balls are selected at random without replacement from the bag. Calculate the probability that

(i) the three balls drawn are blue, **1**

(ii) the three balls drawn are of the same colour, **2**

(iii) exactly two of the balls drawn are blue. **2**

(b) A particle is projected vertically upwards from a point 2 metres above horizontal ground. The displacement at time t seconds is given by

$$x = 24.5t - 4.9t^2, \qquad t \geq 0.$$

(i) Find an expression for the velocity of the particle. **1**

(ii) Find when the particle comes to rest. **2**

(iii) Hence, find the greatest height of the particle above the ground. **2**

(iv) Find the length of time for which the particle is at least 21.6 metres above the ground. **2**

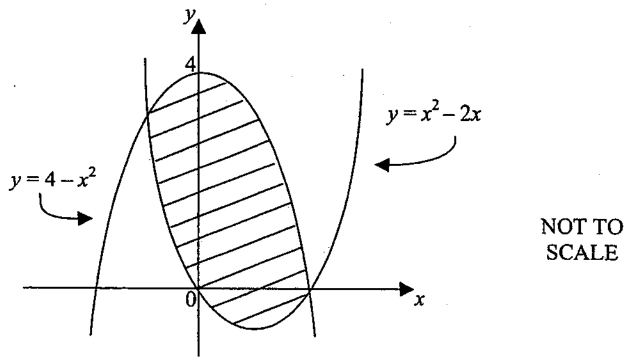
Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A function $y = f(x)$ is continuous for all values. After finding the first and second derivatives a student discovers the following, for all values of x. **2**

When	$x < 2$,	$f'(x) < 0$	and	$f''(x) > 0$
When	$x = 2$,	$f'(x) = 0$	and	$f''(x) = 0$
When	$x > 2$,	$f'(x) < 0$	and	$f''(x) < 0$.

Draw a neat sketch of $y = f(x)$, showing all the important characteristics of the function given that $f(2) = 0$.

(b) The graphs of the functions $y = 4 - x^2$ and $y = x^2 - 2x$.



(i) Describe, using inequalities, the shaded region. **1**

(ii) By solving simultaneously, show that the points of intersection are at $x = -1$ and $x = 2$. **2**

(iii) Calculate the area of the shaded region. **2**

Question 8 continues on page 12

Question 8 (continued) **Marks**

(c) On a factory production line a tap opens and closes to fill containers with liquid. As the tap opens, the rate of flow increases for the first 10 seconds according to the relation $R = \frac{6t}{50}$, where R is measured in L/sec. The rate of flow then remains constant until the tap begins to close. As the tap closes, the rate of flow decreases at a constant rate for 10 seconds, after which time the tap is fully closed.

(i) Show that, while the tap is fully open, the volume in the container at any time is given by **3**

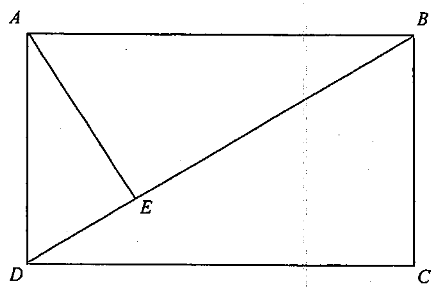
$$V = \frac{6}{5}(t-5).$$

(ii) For how many seconds must the tap remain fully open in order to exactly fill a 120L container with no spillage.

End of Question 8

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) $ABCD$ is a rectangle and $AE \perp BD$. $AE = 5$ cm and $DE = 2$ cm.



(i) Copy the diagram and prove that triangles AED and BCD are similar. **2**

(ii) Hence, show that $AD^2 = BD.DE$. **1**

(iii) Find the area of $ABCD$. **3**

(b) A closed water tank in the shape of a right cylinder is to be constructed with a surface area of 54π cm^2. The height of the cylinder is h cm and the base radius is r cm.

(i) Show that the height of the water tank in terms of r is given by **2**

$$h = \frac{27}{r} - r$$

(ii) Show that the volume V that can be contained in the tank is given by **1**

$$V = 27\pi r - \pi r^3$$

(iii) Find the radius r cm which will give the cylinder its greatest possible volume. Justify your answer. **3**

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Show that $x = \frac{\pi}{8}$ is a solution of $\sin 2x = \cos 2x$. **1**

(ii) On the same set of axes, sketch the graphs of the functions $y = \sin 2x$ and $y = \cos 2x$ for $-\pi \le x \le \pi$. **2**

(iii) Hence, find graphically the number of solutions of $\tan 2x = 1$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. **1**

(iv) Use your graphs to solve $\tan 2x \le 1$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. **1**

(b) Mr and Mrs Matthews decide to borrow \$250 000 to buy a house. Interest is calculated monthly on the balance still owing, at a rate of 6.06% per annum. The loan is to be repaid at the end of 15 years with equal monthly repayments of \$$M$.

Let \$$A_n$ be the amount owing after the nth repayment.

(i) Derive an expression for A_{60}. **1**

(ii) Find the value of M. **2**

(iii) Hence, calculate the amount still owing after 5 years of payment at this rate. **2**

(iv) At the end of five years, the interest rate is increased to 7.2% per annum and Mr and Mrs Matthews change their payments to \$1800 per month. How many more months are needed to pay off the remainder of the loan? **2**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \ne -1; \quad x \ne 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \ne 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \ne 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \ne 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \ne 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \ne 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \ne 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1 Continued

(d) $x^2 + 4x - 1 = 0$

$$x = \frac{-4 \pm \sqrt{20}}{2}$$

$$x = \frac{-4 \pm 2\sqrt{5}}{2}$$

$$x = -2 \pm \sqrt{5}$$

(e) (i) $4^x = 32$

$2^{2x} = 2^5$

$2x = 5$

$x = 2.5$

(ii) $\therefore \log_4 32 = 2.5$

Question 2

(a) (i) $\frac{d}{dx}\left(5x + \frac{3}{x^2}\right)$

$= \frac{d}{dx}\left(5x + 3x^{-2}\right)$

$= 5 - 6x^{-3}$

$= 5 - \frac{6}{x^3}$

(ii) $\frac{d}{dx}\left(e^{2x^2+3}\right)$

$= 4xe^{2x^2+3}$

(iii) $\frac{d}{dx}\left(\frac{3x}{\sin x}\right) = \frac{\sin x \times 3 - 3x \times \cos x}{\sin^2 x}$

$= \frac{3\sin x - 3x\cos x}{\sin^2 x}$

$= \frac{3(\sin x - x\cos x)}{\sin^2 x}$

(b) (i) Gradient of $AC = \frac{3-5}{1+5} = \frac{-2}{6} = -\frac{1}{3}$.

(ii) Midpoint of $AC = \left(\frac{-5+1}{2}, \frac{5+3}{2}\right) = (-2, 4)$.

(iii) Use $y - y_1 = m(x - x_1)$, with $(x_1, y_1) = (-2, 4)$ and $m = 3$.

$\therefore \quad y - 4 = 3(x + 2)$

$\therefore \quad y = 3x + 10$

$\therefore \quad 3x - y + 10 = 0.$

Question 2 continued

Alternative method:

Points on the perpendicular bisector are equidistant from A and C.
So if $P(x, y)$ is on the perpendicular bisector, then

$$PA^2 = PC^2.$$

i.e. $(x+5)^2 + (y-5)^2 = (x-1)^2 + (y-3)^2$

$\cancel{x^2} + 10x + 25 + \cancel{y^2} - 10y + 25 = \cancel{x^2} - 2x + 1 + \cancel{y^2} - 6y + 9$

$10x - 10y + 50 = -2x - 6y + 10$

$12x - 4y + 40 = 0$

$3x - y + 10 = 0.$

(iv) Substitute $x = 0$ in equation $3x - y + 10 = 0$.

$3(0) - y + 10 = 0$

$y = 10.$

$\therefore$ B has coordinates (0,10).

(v) Substitute $D(-4, -2)$ into $3x - y + 10 = 0$.

LHS $= 3(-4) - (-2) + 10$

$= -12 + 2 + 10$

$= 0$

$=$ RHS.

$\therefore$ D lies on the line.

(vi) Midpoint of $BD = \left(\dfrac{0+(-4)}{2}, \dfrac{10+(-2)}{2}\right) = (-2, 4)$.

Since B and D both lie on the perpendicular bisector of AC and the midpoint of BD is equal to the midpoint of AC, then the diagonals AC and BD bisect each other at right angles.

$\therefore$ $ABCD$ is a rhombus.

Question 3

(a) $\displaystyle\int \frac{x}{x^2+5}\,dx = \frac{1}{2}\int \frac{2x}{x^2+5}\,dx$

$\displaystyle = \frac{1}{2}\ln\left(x^2+5\right) + C$

(or $\ln\sqrt{x^2+5} + C$).

(b) $\displaystyle\int_0^{\frac{\pi}{8}} \sec^2 2x\,dx = \left[\frac{1}{2}\tan 2x\right]_0^{\frac{\pi}{8}}$

$\displaystyle = \frac{1}{2}\tan\frac{\pi}{4} - \frac{1}{2}\tan 0$

$\displaystyle = \frac{1}{2}.$

(c) $y = x\log_e x$

$\displaystyle\therefore\ \frac{dy}{dx} = x \times \frac{1}{x} + \log_e x \times 1 = 1 + \log_e x.$

Let m_1 = gradient of tangent at (e, e)
and m_2 = gradient of normal at (e, e).

$\therefore$ $m_1 = 1 + \log_e e = 1 + 1 = 2.$

$\displaystyle m_2 = -\frac{1}{m_1} = -\frac{1}{2}$

$\therefore$ Equation of normal is

$\displaystyle y - e = -\frac{1}{2}(x - e)$

$2y - 2e = -x + e$

$\therefore$ $x + 2y - 3e = 0$

Question 3 continued

(d) (i) $a^2 = b^2 + d^2 - 2bd\cos A$

$a^2 = 38^2 + 32^2 - 2(38)(32)\cos 30°$

$\therefore \quad a = 19.02173\ldots$

$\therefore \quad BD = 19$ cm (2 s.f.)

(ii) $\dfrac{\sin C}{c} = \dfrac{\sin B}{b}$

$\dfrac{\sin C}{19.02173...} = \dfrac{\sin 42°}{26}$

$\angle BCD = 29.31\ldots°$

$\therefore \angle BCD = 29°$ (nearest degree).

Question 4

(a) (i) $ar = 120$

$ar^4 = 50.625$

$\dfrac{ar^4}{ar} = \dfrac{50.625}{120}$

$r^3 = \dfrac{27}{64}$

$\therefore$ the common ratio, $r = \dfrac{3}{4} = 0.75$

$a = \dfrac{120}{r} = \dfrac{120}{0.75} = 160$

$\therefore$ the first term, $a = 160$.

(ii) $S_\infty = \dfrac{a}{1-r}$

$= \dfrac{160}{1-0.75}$

$= \dfrac{160}{0.25} = 640$

$\therefore$ the limiting sum, $S_\infty = 640$

(iii) $S_{40} = \dfrac{160\left(1-0.75^{40}\right)}{1-0.75} = 639.9935\ldots$

$S_\infty - S_{40} = 0.0064362\ldots$

$= 0.0064$ (2 s.f.)

$= 6.4 \times 10^{-3}$ (2 s.f.)

Question 4 continued

(c) (iii) $\angle CGF = \dfrac{135^\circ}{2}$

$\therefore \angle CGF = 67.5^\circ$.

(iv) $\angle AGC = 135^\circ - (67.5^\circ + 22.5^\circ)$

$\therefore \angle AGC = 45^\circ$.

Question 4 continued

(b) (i) $x^2 + kx - 3x + 2 - k = 0$

$x^2 + x(k-3) + (2-k) = 0$

$a = 1,\ b = k-3,\ c = 2-k$

$\Delta = b^2 - 4ac$

$= (k-3)^2 - 4(1)(2-k)$

$= k^2 - 6k + 9 - 8 + 4k$

$= k^2 - 2k + 1$

$\therefore\ \Delta = k^2 - 2k + 1$

(ii) For real roots, $b^2 - 4ac \geq 0$ for all k

i.e. $k^2 - 2k + 1 \geq 0$ for all k

Now $k^2 - 2k + 1 = (k-1)^2$

and $(k-1)^2 \geq 0$ for all k

$\therefore$ the roots are real for all values of k.

(c) (i) For a regular polygon, interior angle $= 180^\circ - \dfrac{360^\circ}{n}$

$\therefore$ for a regular octagon, interior angle $= 180^\circ - \dfrac{360^\circ}{8}$

$= 180^\circ - 45^\circ$

$= 135^\circ$

$\therefore \angle ABC = 135^\circ$.

(ii) $\angle GAH = \dfrac{180^\circ - 135^\circ}{2}$

$\therefore \angle GAH = 22.5^\circ$.

Question 5

(a) (i) Area of a sector = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 8.2^2 \times \frac{\pi}{3}$

$= 35.2067\ldots \text{ cm}^2$

$\therefore$ Area of sector $COD = 35 \text{ cm}^2$ (nearest cm^2)

(ii) Area $AOB = \frac{1}{2}r^2\theta = 18.4$

$r^2 = \frac{18.4 \times 2}{\theta} = 36.8 \div \frac{\pi}{3} = 35.1414\ldots$

$\therefore r = 5.928\ldots$ cm

$\therefore$radius of sector $AOB = 5.9$ cm (2 s.f.)

(iii) Area $\Delta COB = \frac{1}{2}(8.2)(5.9)\sin\frac{\pi}{3}$

$= 21.0486\ldots \text{ cm}^2$

$\therefore$ Area of $\Delta COB = 21 \text{ cm}^2$ (nearest cm^2)

(b) (i)

x	0	0.2	0.4	0.6	0.8	1
$f(x)$	1	**1.12**	**1.48**	**2.08**	**2.92**	4

(ii) $h = \frac{1}{5}$

Trapezoidal rule:

$$\int_0^1 (3x^2+1)\,dx \approx \frac{h}{2}\left[f(0) + 2\{f(0.2) + f(0.4) + f(0.6) + f(0.8)\} + f(1)\right]$$

$$= \frac{1}{10}\left[1 + 2\{1.12 + 1.48 + 2.08 + 2.92\} + 4\right]$$

$= 2.02.$

Question 5 continued

(c) (i) $A = 100\,000$

(ii) When $t = 20$, $P = 150\,000$

By substitution into $P = Ae^{kt}$

$150000 = 100000e^{20k}$

$e^{20k} = 1.5$

$20k = \ln 1.5$

$k = \frac{\ln 1.5}{20} = 0.02027\ldots$

$\therefore k = 0.0203$ (3 s.f.)

(iii) When $t = 40$, $P = 100000e^{40k}$

$= 100000e^{2\ln 1.5}$

$= 225\,000$

$\therefore$population that will be present 20 years from now is 225 000.

Question 6

(a) $y = x^3 + 3x^2 - 9x - 5$

(i) $\frac{dy}{dx} = 3x^2 + 6x - 9$

$= 3(x^2 + 2x - 3).$

(ii) Stationary points when $\frac{dy}{dx} = 0$.

That is, $(x^2 + 2x - 3) = 0$

$(x + 3)(x - 1) = 0.$

$\therefore x = -3$ or $x = 1$

Stationary points are $(-3, 22)$ and $(1, -10)$.

(iii) $\frac{d^2y}{dx^2} = 6x + 6$

When $x = -3$, $\frac{d^2y}{dx^2} = -18 + 6 < 0$,

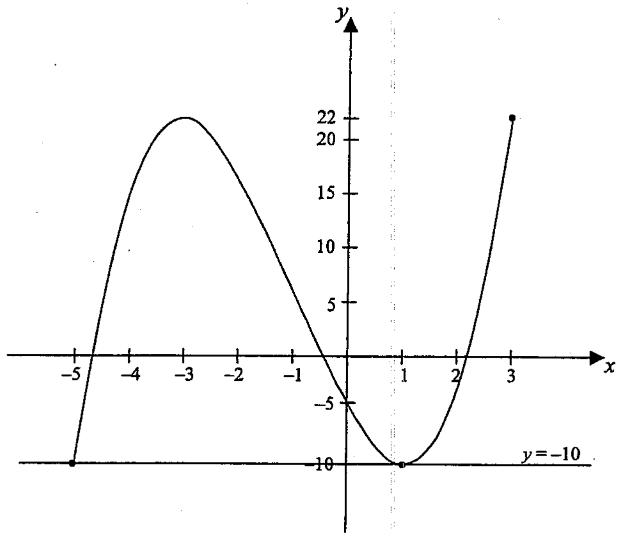
$\therefore$ the curve is concave down and $(-3, 22)$ is a relative maximum.

When $x = 1$, $\frac{d^2y}{dx^2} = 6 + 6 > 0$,

$\therefore$ the curve is concave up and $(1, -10)$ is a relative minimum.

Question 6 continued

(a) (iv)



(v) $x^3 + 3x^2 - 9x + 5 = 0$

when $x^3 + 3x^2 - 9x - 5 = -10$

$\therefore$ by drawing the line $y = -10$ on the graph,

Solutions are $x = -5$ and $x = 1$.

(b) $V = \pi \int_0^1 y^2 \, dx \quad = \pi \int_0^1 \left(1 + 2e^{-x}\right)^2 dx$

$$= \pi \int_0^1 \left(1 + 4e^{-x} + 4e^{-2x}\right) dx$$

$$= \pi \left[x - 4e^{-x} - 2e^{-2x}\right]_0^1$$

$$= \pi\left[\left(1 - 4e^{-1} - 2e^{-2}\right) - \left(0 - 4e^{-0} - 2e^{-0}\right)\right]$$

$\therefore$ Volume $= \pi\left(7 - 4e^{-1} - 2e^{-2}\right)$ unit^3.

Question 7

(a) (i) $P(BBB) = \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10} = \frac{1}{22}$

(ii) $P(BBB) + P(RRR) = \frac{1}{22} + \left(\frac{4}{12} \times \frac{3}{11} \times \frac{2}{10}\right) = \frac{1}{22} + \frac{1}{55} = \frac{7}{110}$

(iii) $P(B,B,NB) + P(NB,B,B) + P(B,NB,B) = 3\left(\frac{5}{12} \times \frac{4}{11} \times \frac{7}{10}\right) = \frac{7}{22}$

(b) (i) $x = 24.5t - 4.9t^2$

$\frac{dx}{dt} = v = 24.5 - 9.8t$

(ii) $v = 24.5 - 9.8t$

Particle comes to rest when $v = 0$,

$0 = 24.5 - 9.8t$

$9.8t = 24.5$

$t = 2.5$ seconds.

$\therefore$ particle comes to rest after 2.5 seconds.

(iii) Greatest height occurs when velocity is zero,

At t = 2.5, $x = 24.5(2.5) - 4.9(2.5)^2$

$x = 30.625$

$\frac{d^2x}{dt^2} = -9.8 < 0$ for all t

$\therefore$ the curve is concave down and (2.5, 30.625) is a absolute maximum.

However, if the particle is projected from 2 metres above the ground then greatest height is 32.625 metres.

Question 7 continued

(iv) For particle to be at least 21.6 metres above the ground,

$\therefore x = 21.6 - 2 = 19.6$ metres

and $24.5t - 4.9t^2 \geq 19.6$

$5t - t^2 \geq 4$

$t^2 - 5t + 4 \leq 0$

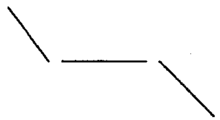
$\therefore 1 \leq t \leq 4$ seconds.

$\therefore$ the particle is at least 21.6 metres above the ground for 3 seconds.

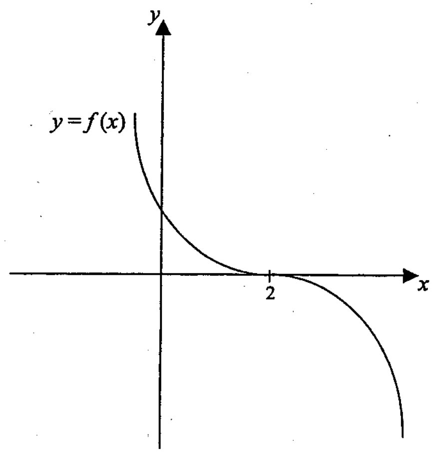
Question 8

(a)

x	<2	2	>2
$f'(x)$	Decreasing	Stationary point	Decreasing
$f''(x)$	Concave Up	Point of inflexion	Concave down



Also $f(2) = 0$,



Question 8 continued

(b) (i) $y \le 4 - x^2, \quad y \ge x^2 - 2x$

(ii) Solving simultaneously,

$x^2 - 2x = 4 - x^2$

$2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1$ or $x = 2$.

(iii) Area $= \int_{-1}^{2} (4 - x^2 - x^2 + 2x)\, dx$

$= \int_{-1}^{2} (4 - 2x^2 + 2x)\, dx$

$= \left[4x - \frac{2x^3}{3} + x^2 \right]_{-1}^{2}$

$= \left(8 - \frac{16}{3} + 4 \right) - \left(-4 + \frac{2}{3} + 1 \right)$

$=$ 9 square units.

(c) (i) For the first 10 seconds, $\frac{dV}{dt} = \frac{6t}{50}$

$\therefore V = \frac{3t^2}{50} + C$

When $t = 0$, $V = 0$

$\therefore V = \frac{3t^2}{50}$, $t \le 10$.

When $t = 10$ seconds, $V = \frac{3(10)^2}{50} = 6$ Litres

Question 8 continued

After 10 seconds, rate of flow remains constant

and so, $\frac{dV}{dt} = \frac{6(10)}{50} = \frac{6}{5}$ L / sec

$\therefore V = \frac{6t}{5} + C$

When $t = 10$, $V = 6$

$\therefore 6 = \frac{6(10)}{5} + C$

$C = -6$

$\therefore V = \frac{6t}{5} - 6 = \frac{6t - 30}{5} = \frac{6}{5}(t - 5).$

(ii) Volume that flows into container while tap is closing is 6 Litres.

$\therefore$ Volume required = 120 – 6 = 114 Litres

$\frac{6}{5}(t - 5) = 114$

$t - 5 = 95$

$t = 100$ seconds

$\therefore$ tap must remain fully open for 90 seconds.

Question 9

(a) (i) $\angle AED = \angle BCD = 90^\circ$ ($AE \perp BD$ and $ABCD$ is a rectangle)

$\angle ADE = \angle DBC$ (Alternate angles on parallel lines, $AD \parallel BC$)

$\therefore \Delta AED \parallel\!| \Delta BCD$ (equiangular).

(ii) $\Delta AED \parallel\!| \Delta BCD$.

$\therefore \frac{AD}{BD} = \frac{DE}{BC}$

Now $BC = AD$ (opposite sides of rectangle are equal)

$\therefore \frac{AD}{BD} = \frac{DE}{AD}$

$\therefore AD^2 = BD.DE.$

(iii) $AD = \sqrt{25 + 4} = \sqrt{29}$ cm

$\therefore BD.DE = 29$

$BD \times 2 = 29$

$BD = 14.5$ cm

$\therefore$ Area $ABCD = 14.5 \times 5 = 72.5$ cm^2.

(b) (i) Surface Area = $2\pi r^2 + 2\pi rh.$

$54\pi = 2\pi r^2 + 2\pi rh$

$h = \frac{54\pi - 2\pi r^2}{2\pi r}$

$\therefore h = \frac{27}{r} - r$

Question 9 continued

(b) (ii) $V = \pi r^2 h$

$$V = \pi r^2\left(\frac{27}{r} - r\right)$$

$\therefore\ V = 27\pi r - \pi r^3.$

(iii) Greatest possible volume V occurs when $\dfrac{dV}{dr} = 0$ and $\dfrac{d^2V}{dr^2} < 0.$

$V = 27\pi r - \pi r^3.$

$$\frac{dV}{dr} = 27\pi - 3\pi r^2$$

$$\frac{d^2V}{dr^2} = -6\pi r$$

$$\frac{dV}{dr} = 0, \qquad \therefore\ 27\pi - 3\pi r^2 = 0$$

$$r^2 = \frac{27\pi}{3\pi} = 9$$

$$r = \pm 3.$$

But $r > 0$, so $r = 3$ cm.

When $r = 3$, $\dfrac{d^2V}{dr^2} = -6\pi(3) < 0.$

$\therefore$ The value of r that maximises the volume V is 3 cm.

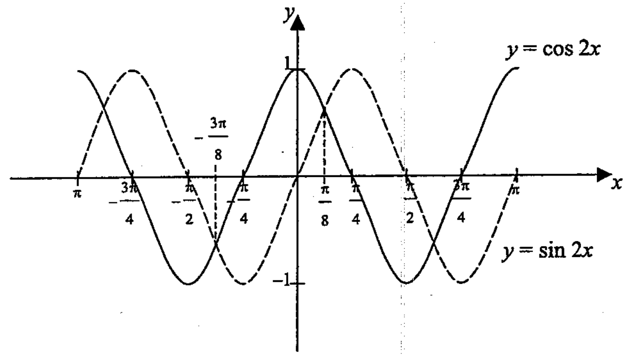
Question 10

(a) (i) LHS: $\sin 2x = \sin\dfrac{2\pi}{8} = \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}.$

RHS: $\cos 2x = \cos\dfrac{2\pi}{8} = \cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}} =$ LHS.

That is, $\sin 2x = \cos 2x$ when $x = \dfrac{\pi}{8}.$

(ii) Period $= \dfrac{2\pi}{n} = \dfrac{2\pi}{2} = \pi$



(iii) $\tan 2x = 1$ when $\dfrac{\sin 2x}{\cos 2x} = 1$

That is, when $\sin 2x = \cos 2x$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}.$

From the diagram, it can be seen that the curves have two points of intersection for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}.$

Therefore, the equation $\tan 2x = 1$ has **two** solutions for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}.$

(iv) $\tan 2x \le 1$ when $\sin 2x \le \cos 2x$ for $-\dfrac{3\pi}{8} < x < \dfrac{\pi}{8}.$

Question 10 continued

(b) (i) $A_1 = (250000 \times 1.00505) - M$

$A_2 = [(250000 \times 1.00505) - M] \times 1.00505 - M$

$= 250000 \times 1.00505^2 - M(1 + 1.00505)$

Continuing the pattern

$A_{60} = 250000 \times 1.00505^{60} - M(1 + 1.00505 + \ldots + 1.00505^{59})$

$\therefore A_{60} = 250000 \times 1.00505^{60} - M \times \dfrac{(1.00505^{60} - 1)}{(1.00505 - 1)}.$

(ii) If the loan is to be repaid at the end of 15 years then $A_{180} = 0$.

$\therefore 250000 \times 1.00505^{180} - M \times \dfrac{(1.00505^{180} - 1)}{(1.00505 - 1)} = 0$

$\therefore M = \dfrac{(250000 \times 1.00505^{180}) \times 0.00505}{(1.00505^{180} - 1)}$

$\therefore M = 2117.7545571$

$\therefore$ The monthly repayment is \$2117.75 to the nearest cent.

(iii) Amount still owing after 5 years,

$A_{60} = 250000 \times 1.00505^{60} - 2117.7545571 \times \dfrac{(1.00505^{60} - 1)}{(1.00505 - 1)}$

$\therefore A_{60} = 190236.7605$

$\therefore$ The amount still owing after 5 years is \$190236.76 to the nearest cent.

Question 10 continued

(iv) After 5 years, number of months needed to pay off remainder of loan at interest rate of 7.2% per annum with monthly repayments of \$1800,

$190236.7605 \times 1.006^n = 1800 \times \dfrac{(1.006^n - 1)}{0.006}$

$190236.7605 \times 1.006^n = 300000 \times (1.006^n - 1)$

$1.006^n = \dfrac{300000}{300000 - 190236.7605}$

$n = \dfrac{\ln\left(\dfrac{300000}{300000 - 190236.7605}\right)}{\ln 1.006}$

$n = 168.07836$

$\therefore$ Approximately 169 months are needed to pay off the remainder of the loan.

FORT STREET HIGH SCHOOL

TRIAL HIGHER SCHOOL EXAMINATION
(ASSESSMENT TASK 4)

2003

MATHEMATICS

Reading Time : Five Minutes
WorkingT ime : Three Hours

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work
- Standard integrals are supplied.
- Board approved calculators may be used.
- Each question is to be started on a **new page**

Name :____________________
Class Teacher :____________________

Q	1	2	3	4	5	6	7	8	9	10	Total	%
Mk	/12	/12	/12	/12	/12	/12	/12	/12	/12	/12	/120	/100

Question One (12 marks)

Marks

a) Write in simplest form the expression — 2

$$2x-(x-2)$$

b) Solve each of the following:-

i) $-3\leq 1-x<4$ — 2

ii) $|x+1|=4$ — 2

c) Rationalise the denominator and express in simple surd form:- — 2

$$\frac{2\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

d) A function is defined by the following rule:-

$$f(x)=\begin{cases} 0 & \text{if } x\leq -2 \\ -1 & \text{if } -2<x<0 \\ x & \text{if } x\geq 0 \end{cases}$$

i) f(-2)+f(-1)+f(0) — 2

ii) $f(a^2)$ — 2

Question Two (12 marks)

a) Find the value of sin 1.7 to 3 decimal places. (N.B. radians) — 1

b) Simplify and evaluate to 3 decimal places.

i) $\cot 65^o \times \sin 65^o$ — 2

ii) $\sin^2 20^o + \sin^2 70^o$ — 2

c) Draw on separate sketches (showing the main features - NOT on graph paper) of:-

i) $x^2+y^2=36$ — 2

ii) $y=x^2+4$ — 2

iii) $y=2^x$ — 2

iv) $xy=1$ — 1

Question Three (12 marks)

Marks

a)

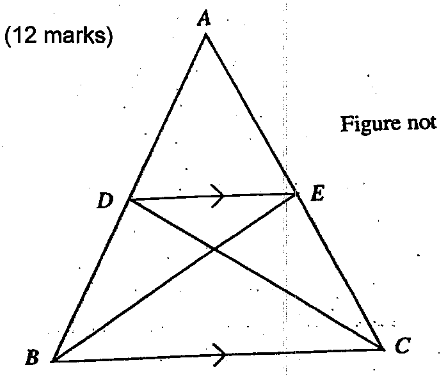


Figure not to scale.

In the diagram ABC is an isosceles triangle where AB = AC and DE is parallel to BC.

i) Show that ADE is an isosceles triangle. — 3

ii) Show that DB = EC — 1

iii) Show that the triangles DBC and ECB are congruent. — 4

b) Find all values of θ such that

i) cos θ = ½ and $0^o \leq \theta \leq 360^o$ — 2

ii) Also find the exact value of sin θ for each of the values of θ. — 2

Question Four (12 marks)

a) Differentiate with to respect to x :-

i) $\dfrac{2x-1}{3x+2}$ — 2

ii) $x\sqrt{1+x}$ — 2

b) The first term of an arithmetic series is 4, and the fifth term Is four times the third term. Find the common difference. — 4

c) A geometric series has second term 6 and the ratio of the seventh term to the sixth term is 3.

i) Find the common ratio. — 1

ii) What is the first term? — 1

iii) Calculate the sum of the first 12 terms. — 2

Question Five (12 marks) **Marks**

a) Two identical perfect cubes (similar to dice) each having faces numbered 0, 1, 2, 3, 4, 5 are rolled. A score for the roll is determined as the product of two numbers on the uppermost faces.

i) The cubes are rolled once. What is the probability that the score is

(α) 0? 2

(β) at least 16? 1

ii) If the cubes are rolled twice and the scores for each roll are added, what is the probability of a combined score of at least 41? 2

b) One hundred tickets are sold in a raffle. Two different tickets are to be drawn out for first and second prizes respectively. A man buys ten tickets. Find the probabilities that:

i) he wins the first prize; 1

ii) he wins both prizes; 1

iii) he wins neither prize; 1

iv) he wins at least one prize. 1

c) Solve the equation $x+\frac{1}{x}=3$. (Leave answers in surd form) 3

Question Six (12 marks) **Marks**

a) Draw on separate sketches (-NOT on graph paper) of:-

i) $y=2\cos x$ for $-2\pi \le x \le 2\pi$ 1

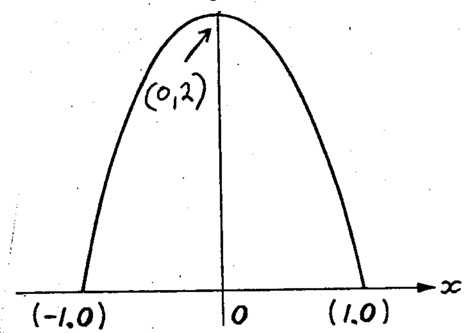
ii) $y=2+2\cos x$ for $-2\pi \le x \le 2\pi$ 1

b) Differentiate $\sin\frac{x}{2}$ with respect to x. 1

c) $\int_0^{\frac{\pi}{3}} \sin 2x\,dx$ 2

d) Find the arc length, correct to 2 decimal places, given radius is 5.9 cm and angle subtended is $23°12'$. 2

e)



An ornamental arch window 2 metres wide and 2 metres high is to be made in the shape of an arc of a cosine curve, as illustrated an axes above. 2

If the arch is made in the shape of the curve

$$y=2\cos\frac{\pi}{2}x$$

find the area of the window (your answer may be left in terms of π)

f) Find the volume of the solid formed if the curve $y=\sec x$ is rotated about the x-axis from x = 0 to x = $\frac{\pi}{4}$. 3

Question Seven (12 marks) — Marks

a) Find primitives (i.e. indefinite integrals) of:

i) $x^{\frac{1}{3}}$ — 1

ii) $\dfrac{1}{\sqrt[3]{x}}$ — 1

iii) $\dfrac{1}{\sqrt{7x-1}}$ — 2

b) Sketch the curve $y = x^3$. Find the area enclosed between the curve $y = x^3$, the x-axis and the lines $x = -1$ and $x = 3$. — 4

c) The area under the curve $y = 2x - x^2$ between $x = 0$ and $x = 2$ is rotated about the x-axis through one complete revolution. Find the volume of the solid so formed. — 4

Question Eight (12 marks)

a) Simplify $\log_6 4 + \log_6 63 - \log_6 7$ — 2

b) Solve $2^x = 7$ (to 2 decimal places) — 2

c) Find the equation of the tangent to $y = e^{3x}$ at the point (0,1) — 2

d) Find $\int_0^1 (e^{6x} - 1)dx$ — 2

e) Consider the function y = ln(x-1) for x > 1.

i) Sketch the function, showing its essential features. — 2

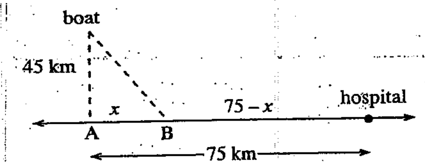
ii) Use Simpson's rule with three function values to find an approximation to (to 2 decimal places) — 2

$$\int_2^4 \ln(x-1)dx.$$

Question Nine (12 marks) — Marks

a) Sketch the curve $y = 4 + 3x - x^3$, showing any turning points or points of inflexion. — 8

b) Imagine that you are the captain of a ship and one of your passengers has been injured and is bleeding internally. Your ship is 45 kilometres from the closest point on the coast. A hospital is a further 75 km down the coast along a straight road from this point. You can contact the hospital to send an ambulance to meet you at any point along the road, The boat travels at 40 km/h and the ambulance averages 70 km/h. Initial conditions are represented by the diagram below.



i) Show that the total time taken is represented by:- — 1

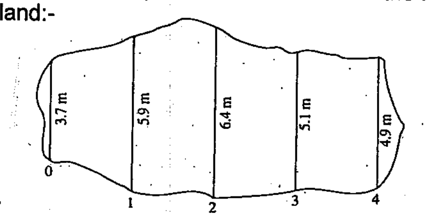
$$T = \frac{\sqrt{45^2 + x^2}}{40} + \frac{75 - x}{70}.$$

ii) You want to get the patient to the hospital as quickly as you can. Determine the point along the road, to 2 decimal places, that the ambulance should meet the boat to minimise travelling time to the hospital. — 3

Question Ten (12 marks) Marks

a) A tree 3 metres tall grows an 1 metre in the first year and then $\frac{1}{3}$ of the previous year's additional height each year thereafter. 2
What will its ultimate height be?

b) When surveyors need to find the area of an irregular piece of land, they measure regular strips and use an approximation method such as the trapezoidal rule. Consider the following piece of land:- 3



The table below gives the measurements:-

X m	0	1	2	3	4
Y m	3.7	5.9	6.4	5.1	4.9

Use the trapezoidal rule to find its area, correct to 2 decimal places.

c) Some banks offer a "honeymoon" period on their loans. This usually takes the form of a lower interest rate for the first year. Suppose that a couple borrowed $170 000 for their first house, to be paid back monthly over 15 years. They work out that they can afford to pay $1650 per month to the bank. Loan payments are made monthly. The standard rate of interest is 8.4% pa, but the bank also offers a special rate of 6% pa for one year to people buying their first home.

i) Let M be the amount of the monthly payment needed to pay off the loan. Show that the A_{12} the amount owing after twelve months is:-

$$A_{12} = 170000(1.005)^{12} - M(1 + 1.005 + ... + 1.005^{11})$$ 3

ii) Use this value, A_{12}, as the principal of the loan at the standard rate for the next 14 years. Calculate the value of the monthly payment that is needed to pay the loan off. Can the couple afford to agree to the contract? 4

SOLUTIONS MATHS TRIAL 2003

Question 1

a) $2x - (x-2) = 2x - x + 2$ ✓
$= x + 2$ ✓

b) i) $-3 \leq 1 - x < 4$
$-4 \leq -x < 3$ ✓
$4 \geq x > -3$
$-3 < x \leq 4$ ✓

ii) $|x+1| = 4$
$x + 1 = 4$ or $x + 1 = -4$
$x = 3$ ✓ $x = -5$ ✓

c) $\dfrac{2\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \dfrac{2\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ ✓
$= \dfrac{6+3\sqrt{6}+2}{3-2}$
$= 8 + 3\sqrt{6}$ ✓

d) i) $f(-2) + f(-1) + f(0)$
$= 0 + (-1) + 0$ ✓
$= -1$ ✓

ii) $a^2 \geq 0$ ✓
$f(x) = x$ for $x \geq 0$
$f(a^2) = a^2$ ✓

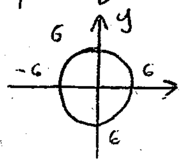
Question 2

a) $\sin 1.7 = 0.9916648 = 0.992$ (3 dp) ✓

b) i) $\cot 65° \times \sin 65°$
$= \dfrac{\cos 65°}{\cancel{\sin 65°}} \times \cancel{\sin 65°}$ ✓
$= \cos 65° = 0.423$ (3 dp) ✓

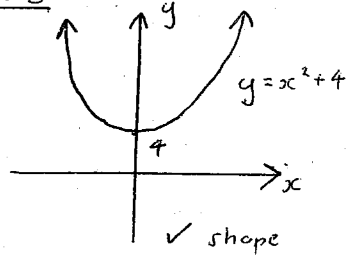
ii) $\sin^2 20° + \sin^2 70°$
$= \cos^2 70° + \sin^2 70°$ ✓
$= 1$ ✓

c) i)



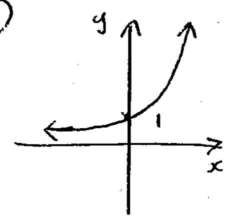
$x^2 + y^2 = 36$
✓ shape
✓ co-ords

ii)



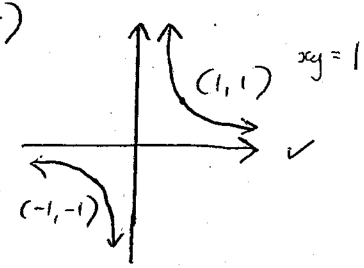
✓ shape
✓ vertex

iii)



✓ shape
✓ y-int

iv)



✓

Question 3

a) i) $AB = AC$ (given)
$\angle ABC = \angle ACB$ ✓
(angles opp equal sides)
$\angle ABC = \angle ADE$
(corres $\angle$s $BC \parallel DE$)
$\angle ACB = \angle AED$
(corres $\angle$s $BC \parallel DE$)
$\therefore \angle ADE = \angle AED$
$AD = AE$ ✓
(sides opp equal angles)
$\therefore \triangle ADE$ is isosceles triangle

ii) $AB = AC$ (given)
$AD = AE$ (proved above)
$AB - AD = AC - AE$
ie $DB = EC$ ✓

iii) In $\triangle$s DBC and ECB
BC is common ✓
$\angle DBC = \angle ECB$ (isosceles $\triangle$) ✓
$DB = EC$ (proved above) ✓
$\therefore \triangle DBC \equiv \triangle ECB$ (SAS) ✓

b) $\cos\theta = \frac{1}{2}$ $0° \leq \theta \leq 360°$
$\theta = 60°, 300°$ ✓✓
$\sin 60° = \frac{\sqrt{3}}{2}$ ✓ $\sin 300° = -\sin 60°$
$= -\frac{\sqrt{3}}{2}$ ✓

Question 4

a) i) $\dfrac{d}{dx}\left(\dfrac{2x-1}{3x+2}\right) = \dfrac{2(3x+2) - 3(2x-1)}{(3x+2)^2}$ ✓
$= \dfrac{7}{(3x+2)^2}$ ✓

ii) $\dfrac{d}{dx}\left(x\sqrt{1+x}\right) = x\,\frac{1}{2}(1+x)^{-\frac{1}{2}} + (1+x)^{\frac{1}{2}}$ ✓
$= \dfrac{x}{2\sqrt{1+x}} + \sqrt{1+x}$ ✓
$= \dfrac{x}{2\sqrt{1+x}} + \dfrac{2(1+x)}{2\sqrt{1+x}}$
$= \dfrac{3x+2}{2\sqrt{1+x}}$

b) $t_n = a + (n-1)d$
$t_1 = a = 4$
$t_3 = 4 + 2d$ ✓
$t_5 = 4 + 4d$ ✓

now $t_5 = 4t_3$
$4 + 4d = 16 + 8d$ ✓
$-4d = 12$
$d = -3$ ✓

c) i) $t_n = ar^{n-1}$
$\dfrac{t_7}{t_6} = 3$
$\dfrac{ar^6}{ar^5} = 3$
$r = 3$ ✓

ii) $t_2 = ar = 6$
$a = 2$ ✓

iii) $S_n = \dfrac{a(r^n - 1)}{r-1}$
$= \dfrac{2(3^{12}-1)}{3-1}$ ✓
$= 3^{12} - 1$
$= 531\,440$ ✓

Question 5

a)

		Cube 2					
		0	1	2	3	4	5
Cube 1	0	0	0	0	0	0	0
	1	0	1	2	3	4	5
	2	0	2	4	6	8	10
	3	0	3	6	9	12	15
	4	0	4	8	12	16	20
	5	0	5	10	15	20	25

✓

i) α) $P(0) = \frac{11}{36}$
β) $P(\geq 16)$
$= \frac{4}{36} = \frac{1}{9}$ ✓

ii) The following combinations give scores of at least 41:
(16, 25) (25, 16)
(20, 25) (25, 20)
(20, 25) (25, 20)
(25, 25) ✓
$P(>41) = \dfrac{7}{36 \times 36}$
$= \dfrac{7}{1296}$ ✓

(3)

b) i) P (wins first prize) $= \frac{10}{100} = \frac{1}{10}$ ✓

ii) P (wins both) $= \frac{10}{100} \times \frac{9}{99} = \frac{1}{110}$ ✓

iii) P (wins neither) $= \frac{90}{100} \times \frac{89}{99} = \frac{89}{110}$ ✓

iv) P (at least one prize) $= 1 - P(LL)$

$= 1 - \frac{89}{110}$

$= \frac{21}{110}$ ✓

c) $x + \frac{1}{x} = 3$

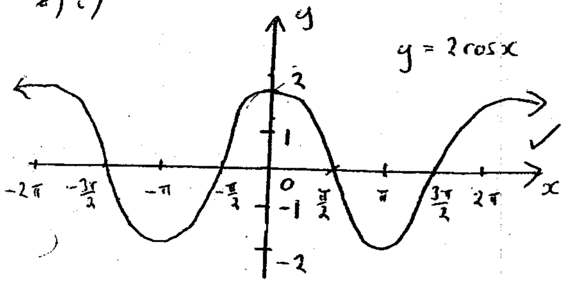
$x^2 - 3x + 1 = 0$ ✓

$x = \frac{3 \pm \sqrt{(-3)^2 - 4}}{2}$ ✓

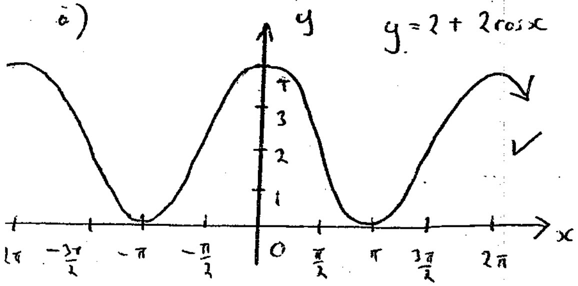
$= \frac{3 \pm \sqrt{5}}{2}$ ✓

Question 6

a) i)



ii)



b) $\frac{d}{dx}\left(\sin \frac{x}{2}\right) = \frac{1}{2} \cos \frac{x}{2}$ ✓

c) $\int_0^{\frac{\pi}{3}} \sin 2x \, dx$

$= -\frac{1}{2}\left[\cos 2x\right]$

$= -\frac{1}{2}\left(\cos \frac{2\pi}{3} - \cos 0\right)$

$= -\frac{1}{2}\left(-\frac{1}{2} - 1\right)$

$= \frac{3}{4}$ ✓

d) $l = 5.9 \times 23°12' \times \frac{\pi}{180}$

$= 2.39$ cm (to 2 dp)

e) $A = 2\int_0^1 2\cos \frac{\pi}{2} x \, dx$

$= \frac{8}{\pi}\left[\sin \frac{\pi}{2}x\right]_0^1$ ✓

$= \frac{8}{\pi}\left(\sin \frac{\pi}{2} - \sin 0\right)$

$= \frac{8}{\pi}$ metres^2 ✓

f) $V = \pi \int_0^{\frac{\pi}{4}} \sec^2 x \, dx$ ✓

$= \pi \left[\tan x\right]_0^{\frac{\pi}{4}}$ ✓

$= \pi (1 - 0)$

$= \pi$ units^3 ✓

(5)

Question 9

a) $y = 4 + 3x - x^3$ ✓

$\frac{dy}{dx} = 3 - 3x^2$ ✓ $\frac{d^2y}{dx^2} = -6x$

For stationary points $\frac{dy}{dx} = 0$

$3(1 - x^2) = 0$

$x = \pm 1$ ✓ ✓

∴ Stationary pts at $(1, 6)$ $(-1, 2)$

at $(1, 6)$ $\frac{d^2y}{dx^2} < 0$ ∴ $(1, 6)$ max pt ✓

$(-1, 2)$ $\frac{d^2y}{dx^2} > 0$ ∴ $(-1, 2)$ min pt ✓

For pts of inflexion

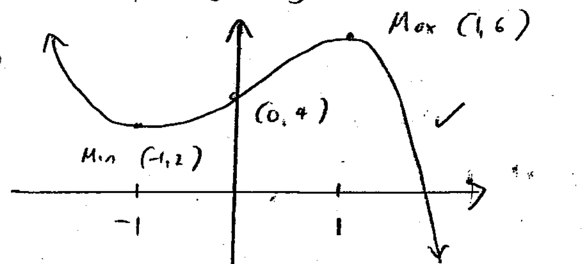
$\frac{d^2y}{dx^2} = 0$ $\quad -6x = 0$

$x = 0 \quad y = 4$ ✓

when x a little less than 0 $\frac{d^2y}{dx^2} > 0$

x a little more than 0 $\frac{d^2y}{dx^2} < 0$ ✓

Thus pt of inflexion



b) Time = $\frac{\text{Distance}}{\text{Speed}}$

Total Time = Time$_{Ship}$ + Time$_{Ambulance}$

$T = \frac{\sqrt{45^2 + x^2}}{40} + \frac{75 - x}{70}$ ✓

$T' = \frac{\frac{1}{2} \times 2x \, (45^2 + x^2)^{-\frac{1}{2}}}{40} -$

$= \frac{x}{40\sqrt{45^2 + x^2}} - \frac{1}{70}$

For $T' = 0$

$\frac{x}{40\sqrt{45^2 + x^2}} = \frac{1}{70}$

$\frac{7}{4} x = \sqrt{45^2 + x^2}$

$\frac{49}{16} x^2 = 45^2 + x^2$

$x^2 = 45^2 \times \frac{16}{33}$

$x = 31.33$ m

when x is a little less than 31.33 $T' < 0$

when x is a little less than 31.33 $T' > 0$ ✓

∴ min at $x = 31.33$

∴ ship should proceed to a point 31.33 km from the closest point on coast

Question 10

a) $S_\infty = \frac{a}{1-r} = \frac{3}{1-\frac{1}{3}} = \frac{3}{\frac{2}{3}}$ ✓

$= \frac{9}{2} = 4\frac{1}{2}m$ ✓

$\therefore$ ultimate height is $4\frac{1}{2}m$

b) $\int_a^b f(x)\,dx \doteq \frac{h}{2}\left[(y_0+y_4)+2(y_1+y_2+y_3)\right]$ ✓

where $h = \frac{b-a}{n} = \frac{4-0}{4} = 1$

$\int_0^4 f(x)\,dx \doteq \frac{1}{2}\left[3.7+4.9+2(5.9+6.4+5.1)\right]$ ✓

$= 21.70\,m^2$ (to 2dp) ✓

c) i) let M be monthly repayment

after 1 mth $A_1 = 170000(1.005) - M$ ✓

" 2 mths $A_2 = (170000(1.005)-M)1.005 - M$

$= 170000(1.005)^2 - M(1+1.005)$ ✓

" 3 mths $A_3 = 170000(1.005)^3 - M(1+1.005+1.005^2)$

12 mths $A_{12} = 170000(1.005)^{12} - M(1+1.005+\ldots+1.005^{11})$

ii) After 15 years

$A_{180} = A_{12}(1.007)^{168} - M(1+1.007+\ldots+1.007^{167}) = 0$ ✓

$170000(1.005^{12}) - M(1+1.005+\ldots+1.005^{11}) \times 1.007^{168}$
$= M(1+1.007+\ldots 1.007^{167})$

$\left[170000(1.005^{12}) - M\frac{(1.005^{12}-1)}{1.005-1}\right] \times 1.007^{168} = \frac{M(1.007^{168}-1)}{1.007-1}$

$170000(1.005^{12}) \times (1.007)^{168} - M\frac{(1.005^{12}-1)}{0.005} \times 1.007^{168}$
$= M\frac{(1.007^{168}-1)}{0.007}$

$M\left[\frac{1.005^{12}-1}{0.005} \times 1.007^{168} + \frac{1.007^{168}-1}{0.007}\right]$
$= 170000(1.005)^{12} \times (1.007)^{168}$

$M = \dfrac{170000(1.005)^{12} \times (1.007)^{168}}{\frac{1.005^{12}-1}{0.005} \times (1.007)^{168} + \frac{1.007^{168}-1}{0.007}}$ ✓

$\doteq \dfrac{170000 \times 1.0617 \times 3.2281}{\left(\frac{0.0617}{0.005} \times 3.2281\right) + \frac{2.2281}{0.007}}$

$= \$1626.86$ ✓

Yes the couple can afford the loan. ✓