Solutions	Marks	Comments
Question 1		
(a) $\tan x = \frac{1}{\sqrt{3}}$		
$\therefore x = \frac{\pi}{6}, \pi + \frac{\pi}{6}$ for $2\pi \geq x \geq 0$		
$\therefore$ General solution		
$x = \frac{\pi}{6} \pm 2n\pi, \pi + \frac{\pi}{6} \pm 2n\pi$	1	
$= \frac{\pi}{6} + n\pi$ for all integral n	1	Accept $\frac{\pi}{6} \pm n\pi$
(b) $3x^3 + 5x^2 - 7x + 4 = 0$		
$\alpha\beta + \alpha\gamma + \beta\gamma = -\frac{7}{3}$	1	
$\alpha + \beta + \gamma = -\frac{5}{3}$	1	
(c) $\sec^2 x + \tan x - 7 = 0$		
$1 + \tan^2 x + \tan x - 7 = 0$	1	
$\tan^2 x + \tan x - 6 = 0$		
$(\tan x + 3)(\tan x - 2) = 0$		
$\therefore \tan x = -3, 2$	1	
$\therefore x = 108°26', 288°26', 63°26', 243°26'$	1	

Solutions	Marks	Comments
Question 1 (continued)		
(d) $x = \cos\theta + 1$		
$\therefore x-1 = \cos\theta$ — ①		
$y = \sin\theta - 2$		
$y+2 = \sin\theta$ — ②	1	
$①^2 + ②^2$		
$(x-1)^2 + (y+2)^2 = \sin^2\theta + \cos^2\theta$		
$\therefore (x-1)^2 + (y+2)^2 = 1$ is the required equation	1	
(e) $\dfrac{2x}{x-3} \le 1 \qquad x \ne 3$		Some may use another method. if solution is correct give full marks
If $x-3>0$		
$x>3$		
$2x \le x-3$		
$x \le -3$		
No solution	1	
If $x-3<0$		
$x<3$		
$\therefore 2x \ge x-3$		
$x \ge -3$	1	
$\therefore$ Solution is $3 > x \ge -3$	1	

Solutions	Marks	Comments
Question 2		
(a) (i) $x^2+9 \overline{) x^3+3x^2+9x-1}$ quotient $x+3$; $x^3 \quad +9x$; $3x^2 \quad -1$; $3x^2 \quad +27$; -28		
$\therefore Q(x)=x+3 \quad R(x)=-28$	2	
(ii) $\int \frac{x^3+3x^2+9x-1}{x^2+9}\,dx$		
$=\int\left(x+3-\frac{28}{x^2+9}\right)dx$		
$=\frac{1}{2}x^2+3x-\frac{28}{3}\tan^{-1}\frac{x}{3}+C$	2	Do not worry about C
(b) Let $f(n)=7^n-5^n$		
$\therefore f(1)=7-5$ $=2$		
$\therefore$ True for $n=1$		
Assume true for $n=k$	1	
$\therefore f(k)=7^k-5^k=2A$ where A is an integer		
when $n=k+1$		
$f(k+1)=7^{k+1}-5^{k+1}$		Some may test
$=7\times7^k-5\times5^k$		$f(k+1)-f(k)$ even
$=7(7^k-5^k)+2\times5^k$		
$=7\times2A+2\times5^k$	1	
$=2(7A+5^k)$		
$\therefore$ True as $7A+5^k$ is an integer		
$\therefore$ if true for $n=1$ then true $n=2$		
" " " $n=2$ " " $n=3$ etc.	1	
$\therefore$ True for all $n\geq0$ integers		

Solutions	Marks	Comments
Question 2 (continued)		
(c)(i) Number of arrangements		
$= (8-1)!$		
$= 5040$	1	
(ii) Number of couple arrangements $= 2^4 \times (4-1)!$		
$= 96$	1	
$\therefore$ Probability of pairs together		
$= \frac{96}{5040}$		
$= \frac{2}{105}$	1	
(d). Quadrilateral is cyclic		
$\therefore$ Opposite angles are supplementary		
$\therefore \angle C = 180° - \angle A$		
$\angle D = 180° - \angle B$	1	
$\therefore \tan A + \tan B + \tan C + \tan D$		
$= \tan A + \tan B + \tan(180 - A) + \tan(180 - B)$		
$= \tan A + \tan B - \tan A - \tan B$		
$= 0$	1	

Solutions	Marks	Comments
Question 3		
(a) $\int_0^1 \frac{2x\,dx}{(2x+1)^2}$ $u = 2x+1$, $\frac{du}{dx} = 2$, $dx = \frac{du}{2}$, $x=1$ $u=3$, $x=0$ $u=1$		
$\therefore \int_1^3 \frac{u-1}{u^2}\frac{du}{2}$	1	
$= \frac{1}{2}\int_1^3 \left(\frac{1}{u} - \frac{1}{u^2}\right) du$		
$= \frac{1}{2}\left[\ln u + \frac{1}{u}\right]_1^3$	1	
$= \frac{1}{2}\left(\ln 3 + \frac{1}{3}\right) - \frac{1}{2}(\ln 1 + 1)$		
$= \frac{1}{2}(\ln 3 - 2/3)$	1	
(b) $\cos(A+B) = \cos A\cos B - \sin A\sin B$		
$\cos 75^\circ = \cos(45^\circ + 30^\circ)$	1	
$\therefore \cos 75^\circ = \cos 45^\circ\cos 30^\circ - \sin 45^\circ \sin 30^\circ$		
$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\cdot\frac{1}{2}$		
$= \frac{\sqrt{3}-1}{2\sqrt{2}}$	1	
(c) $\sqrt{3}\cos x - \sin x$		
$= R\cos x\cos\alpha - R\sin x\sin\alpha$		
$\therefore R\cos\alpha = \sqrt{3}$, $R\sin\alpha = 1$	1	
$\frac{R\sin\alpha}{R\cos\alpha} = \frac{1}{\sqrt{3}}$ $R^2\sin^2\alpha + R^2\cos^2\alpha = 1+3$		
$\tan\alpha = \frac{1}{\sqrt{3}}$ $R^2 = 4$		
$\alpha = 30^\circ$ $\therefore R = 2$	2	1 each

Solutions	Marks	Comments
Question 3 (continued)		
(d) General Term $= {}^{20}C_r (2x)^{20-r}\left(\frac{-1}{x^3}\right)^r$		
Ignoring coefficient to find r		
$x^{20-r} \cdot (x^{-3})^r = x^0$		
$x^{20-4r} = x^0$	1	
$\therefore 4r = 20$		
$r = 5$		
$\therefore$ Term is ${}^{20}C_5 \cdot 2^{15} \cdot (-1)^5$		
$= -{}^{20}C_5 \cdot 2^{15}$	1	
-508035072		
(e) $f(x) = \sin^{-1}x + \cos^{-1}x$		
$= \frac{\pi}{2}$		
(i) $f'(x) = 0$	1	
(ii) $\int_0^1 f(x)\,dx = \int_0^1 \frac{\pi}{2}\,dx$		
$= \left[\frac{\pi}{2}x\right]_0^1$		
$= \frac{\pi}{2} - 0$		
$= \frac{\pi}{2}$	1	

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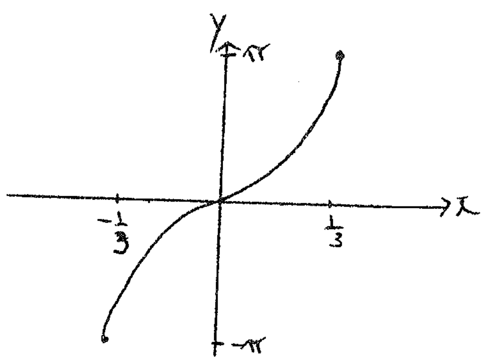
Solutions	Marks	Comments
Question 4		
(a)(i) Let $P(S)=0.4$ and $P(F)=0.6$		
$\therefore P(S=6) = {}^{10}C_6(0.6)^4(0.4)^6$		
$=0.111$ (3sig figs)	2	
(ii) Let there be n grafts		
$\therefore P(F=n)=(0.6)^n$		
$\therefore P(S\geq 1) = 1-(0.6)^n \geq 0.999$	1	
$\therefore 0.001 \geq (0.6)^n$		
$\frac{\ln(0.001)}{\ln(0.6)} \leq n$	1	
$13.523 \leq n$		
$\therefore$ 14 or more grafts are required.	1	
(b)(i) Given $V=\frac{4}{3}\pi r^3$ $\frac{dV}{dt}=1000$		
$\frac{dV}{dr}=4\pi r^2$	1	
$\therefore \frac{dr}{dt} = \frac{dV}{dt} \div \frac{dV}{dr}$		
$= \frac{1000}{4\pi r^2}$		
$= \frac{250}{\pi r^2}$	1	
Radius is changing at $\frac{250}{\pi r^2}$ cm s^{-1}		

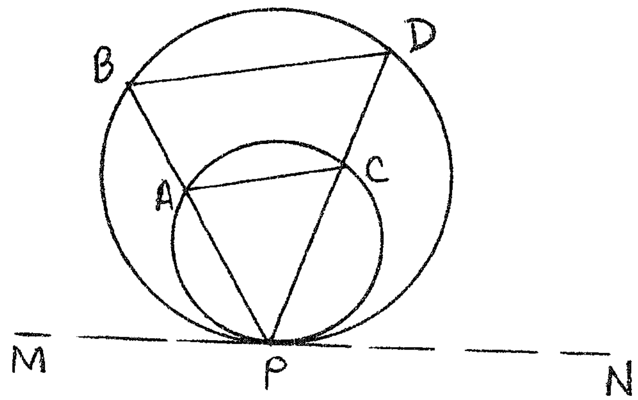
Solutions	Marks	Comments
Question 4 (continued)		
(b) (ii) Given $A = 4\pi r^2$		
$\frac{dA}{dr} = 8\pi r$		
$\therefore \frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$	1	
$= 8\pi r \times \frac{250}{\pi r^2}$		
$= \frac{2000}{r}$	1	
when $r = 10$ $\frac{dA}{dt} = \frac{2000}{10} = 200$		
Area is changing at $200 \text{ cm}^2 \text{s}^{-1}$	1	
(c) $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} (1 + \cos 2x)\, dx$		
$= \frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}$	1	
$= \frac{1}{2}\left[\left(\frac{\pi}{2} + 0\right) - 0\right]$		
$= \frac{\pi}{4}$	1	

Solutions	Marks	Comments
Question 5		
(a) $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$		
$f(x) = \frac{1}{2}x - \sin x$	1	
$f'(x) = \frac{1}{2} - \cos x$		
$x_1 = 2 \quad \therefore x_2 = 2 - \frac{\frac{1}{2}\cdot 2 - \sin 2}{\frac{1}{2} - \cos 2}$	1	
$= 1.90$		
(b)(i) Gradient of tangent $= p$		
$\angle PMX = 90 - \alpha$		
$\therefore$ Gradient of $MN = \tan(90-\alpha)$	1	
$\therefore \quad p = \tan(90-\alpha)$		
$= \cot\alpha$		
$\frac{1}{p} = \tan\alpha$	1	
(ii) $\angle PLM + \angle LPM = \angle PMX$ (Exterior $\angle$ $\triangle PLM$)	1	
$\therefore \angle PLM + \alpha = 90 - \alpha$		
$\angle PLM = 90 - 2\alpha$ ✓	1	
$\therefore$ Gradient $LP = \tan(90 - 2x)$		
$= \cot 2x$	1	
$= \frac{1 - \tan^2 x}{2\tan x}$		
$= \frac{1 - \frac{1}{p^2}}{2 \cdot \frac{1}{p}}$		
$= \frac{p^2 - 1}{2p}$ ✓	1	

Solutions	Marks	Comments
Question 5 (continued)		
(b)(iii) Coords. of S are $(0, a)$		
Gradient $PS = \dfrac{ap^2 - a}{2ap}$		Other ways are O.K.
$= \dfrac{p^2 - 1}{2p}$	1	
Gradient PS = Gradient LP		
$\therefore$ LP passes through S		
(c) $x = 36t$ $\quad y = 15t - \frac{1}{2}gt^2$		
$\dfrac{dx}{dt} = 36$ $\quad \dfrac{dy}{dt} = 15 - gt$		
$\therefore \dfrac{dy}{dx} = \dfrac{dy}{dt} \cdot \dfrac{dt}{dx}$		Give marks for correct method
$= \dfrac{15 - gt}{36}$	2	
If θ is the angle of projection		
$\tan\theta = \dfrac{15 - gt}{36}$ when $t = 0$		
$= \dfrac{15}{36}$		
$\therefore \theta = \tan^{-1}\dfrac{15}{36}$	1	
$= 22°37'$		

Solutions	Marks	Comments
Question 6		
(a)(i) $y = 2\sin^{-1}(3x)$.		
$1 \geq 3x \geq -1$		
$\therefore$ Domain $\frac{1}{3} \geq x \geq -\frac{1}{3}$	1	
Range $\pi \geq y \geq -\pi$	1	
(ii) 	1	
(b)(i) $T = S + Be^{kt}$ — ①		
$\frac{dT}{dt} = Bke^{kt}$	1	
Now $Be^{kt} = T - S$ from ①		
$\therefore Bke^{kt} = k(T-S)$		
$\therefore$ It is a solution.	1	

Solutions	Marks	Comments
Question 6 (continued)		
(ii) $T = S + Be^{kt}$		
$S = 30°$, $T = 150°$ when $t = 0$		
$150° = 30° + Be^0$		
$120° = B$	2	
When $t = 3$ $T = 90°$		
$\therefore 90 = 30 + 120e^{3k}$		
$60 = 120e^{3k}$		
$0.5 = e^{3k}$		
$\therefore K = \frac{1}{3}\ln 0.5$		
$= -0.231$ (3 d.p.)	2	
(iii) When $t = 6$		
$T = 30° + 120 \times e^{-0.231 \times 6}$		
$= 60°$		
Temperature is 60°	1	
(c) $x = \frac{kx_2 + lx_1}{k+l}$ $\quad y = \frac{ky_2 + ly_1}{k+l}$		
$= \frac{3 \times 5 + 2 \times -1}{3+2}$ $\quad = \frac{3 \times -3 + 2 \times 2}{3+2}$		
$= 2\frac{3}{5}$ $\quad = -1$		
$\therefore$ Coordinates of P are		
$(2\frac{3}{5}, -1)$		

Solutions	Marks	Comments
Question 7		
(a)(i) 	1	
(ii) Draw tangent MN through P as shown		
$\angle ACP = \angle APM$ (Angle in alternate segment.)	1	
$\angle BDP = \angle BPM$ (" " ")		
$\angle BPM = \angle APM$ (Same angle)	1	
$\therefore \angle ACP = \angle BDP \; (= \angle APM)$	1	
$\therefore BD \parallel CA$ corresponding angles equal	1	
(b) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dv}\left(\frac{1}{2}v^2\right) \cdot \frac{dv}{dx}$		
$= v \cdot \frac{dv}{dx}$	1	
$= \frac{dx}{dt} \times \frac{dv}{dx}$		
$= \frac{dv}{dt}$		
$= a$	1	

Solutions	Marks	Comments
<u>Question 7</u> (Continued)		
(c)(i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -e^{-2x}$		
$\frac{1}{2}v^2 = \frac{1}{2}e^{-2x} + C$		
Given $v = 1$ when $x = 0$	1	
$\frac{1}{2} = \frac{1}{2}e^0 + C$		
$\therefore C = 0$		
$\frac{1}{2}v^2 = \frac{1}{2}e^{-2x}$		
$v^2 = e^{-2x}$		
$v = e^{-x}$	1	strictly speaking $v = \pm e^{-x}$
(ii) $\frac{dx}{dt} = e^{-x}$		
$\therefore \frac{dt}{dx} = e^{x}$	1	
$t = e^{x} + D$		
When $t = 0$ $x = 0$		
$\therefore D = -1$	1	
$\therefore t = e^{x} - 1$		
$t + 1 = e^{x}$		
$\therefore x = \ln(t+1)$	1	
(iii) $v = \frac{dx}{dt} = \frac{1}{t+1}$	1	