Neap

HSC Trial Examination 2009

Mathematics Extension 1

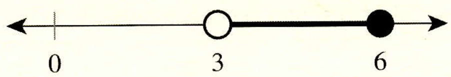
Solutions and marking guidelines

Question 1

Syllabus outcomes and marking guide

(a) $(x-3)^2 \times \dfrac{x}{x-3} \geq 2(x-3)^2,\ x \neq 3$

$(x-3)(-x+6) \geq 0$



$3 < x \leq 6$

PE3
- Gives the correct answer 2

- Gives $x \neq 3$.

OR
- $3 \leq x \leq 6$ 1

(b) $\left(\dfrac{6 \times -1 + 2 \times 4}{2-1}, \dfrac{2 \times -1 + 9 \times 2}{2-1}\right)$

$= (2, 16)$

PE2
- Gives the correct answer 2

- Internal division, i.e. $\left(\dfrac{14}{3}, \dfrac{20}{3}\right)$.

OR
- Minor numerical error in otherwise correct answer 1

(c) $\sin 2x = \dfrac{1}{2}$

$2x = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$ or $\dfrac{13\pi}{6}$ or $\dfrac{17\pi}{6}$

$x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{13\pi}{12}, \dfrac{17\pi}{12}$

General solution:

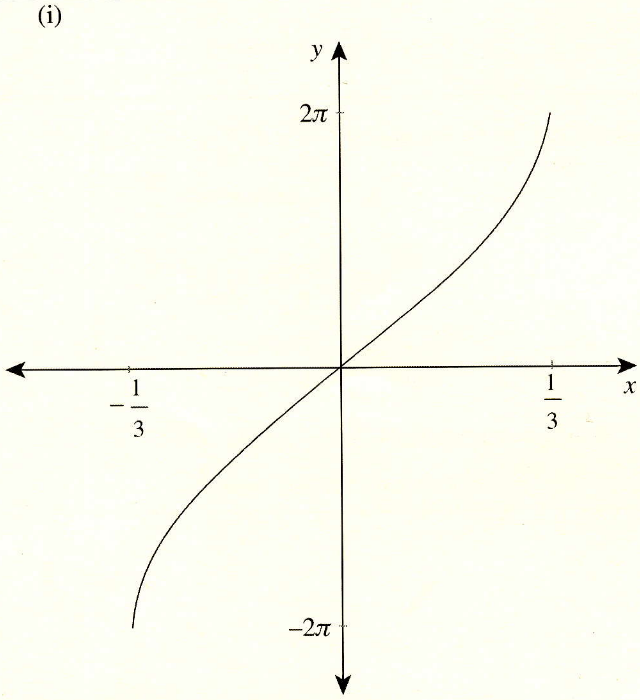
$\dfrac{\pi}{12} + \pi k, \dfrac{5\pi}{12} + \pi k$, k integer

HE3
- Gives correct answer 2

(Note: accept answers in degrees and in the form $30° + 360k$, $150° + 360k$ etc., k integer *)*

- Gives the correct general solution from incorrect values in $0 \leq x \leq 2\pi$ 1

(d) (i)



Graph of function is $y = 4\sin^{-1} 3x$.

P4
- Draws the correct graph. 1

Question 1

	Syllabus outcomes and marking guide
(ii) Domain: $\frac{1}{3} \le x \le \frac{1}{3}$ Range: $-2\pi \le x \le 2\pi$	PE6, HE4 • Gives the correct domain and range. . . . 2 • Gives either the correct domain or the correct range 1
(e) (i) $P(x) = 3x^3 + kx^2 - 5x + 10$ $P(-2) = -24 + 4k + 10 + 10 = 0$ $4k = 4$ $k = 1$	PE3 • Gives the correct answer 1
(ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\lambda}$ $= \frac{-\frac{5}{3}}{-\frac{10}{3}}$ $= \frac{1}{2}$	PE3 • Gives the correct answer . . . . 2 • Uses $\frac{\beta\lambda + \alpha\lambda + \alpha\beta}{\alpha\beta\lambda}$ or equivalent merit. 1

Question 2

	Solution	Syllabus outcomes and marking guide
(a)	$\tan\theta = \left\lvert \dfrac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$ $m_1 = -1,\ m_2 = 2 \times 3 = 6$ $\tan\theta = \left\lvert \dfrac{6+1}{1 + 6 \times -1} \right\rvert$ $= \dfrac{7}{5}$ $\theta = 54°28'$	PE3 • Gives the correct answer .... 2 *(Note: accept 54° or similar rounding.)* • Use of correct formula with one correct gradient. OR • Gradient of curve = 6. OR • Similar merit 1
(b)	$u = x^{\frac{1}{2}}$ $\dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$ $= \dfrac{1}{2\sqrt{x}}$ $x = 1, u = 1$ $x = 9, u = 3$ $\dfrac{1}{2}\int_1^3 e^u\,du = \dfrac{1}{2}[e^u]_1^3$ $= \dfrac{1}{2}(e^3 - e)$	HE6 • Correct answer, MUST show change of limits .... 3 • Correct expression for integral with change of limits .... 2 • Makes some progress, e.g. changes limits, or determines $du = \dfrac{1}{2\sqrt{x}}dx$ or similar merit........................ 1
(c)	$\angle QRT = 80°$ (alternate angles, $PQ \parallel ST$) $\angle PRS = 55°$ (angle at the circumference is half the angle at the centre, on arc PS) $\angle PRQ + 55° + 80° = 180°$ (angles making a straight line add to 180°) $\therefore \angle PRQ = 45°$	PE2 • Gives correct answer with supporting reasons .... 3 • Uses two relevant, correct geometrical facts with reason........................ 2 • Uses one relevant, correct geometrical fact with reason. OR • $\angle PRQ = 45°$ without giving adequate reasons 1

Question 2 (Continued)

(d) To prove

$$\frac{1}{4}+\frac{1}{28}+\frac{1}{77}+\ldots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$$

Test for $n = 1$

$$LHS = \frac{1}{(3\times 1-2)(3\times 1+1)} = \frac{1}{4}$$

$$RHS = \frac{1}{3\times 1+1} = \frac{1}{4} \quad \therefore \text{ true for } n = 1$$

Assume

$$\frac{1}{4}+\frac{1}{28}+\ldots+\frac{1}{(3k-2)(3k+1)}=\frac{k}{3k+1},$$

where k is a positive integer.

Hence, test for $n = k + 1$.

$$\frac{1}{4}+\frac{1}{28}+\ldots+\frac{1}{(3k-2)(3k+1)}+\frac{1}{[3(k+1)-2][3(k+1)+1]}$$

$$=\frac{k}{3k+1}+\frac{1}{[3k+1][3k+4]}$$

$$=\frac{k(3k+4)+1}{(3k+1)(3k+4)}$$

$$=\frac{(k+1)(3k+1)}{(3k+4)(3k+1)}$$

$$=\frac{k+1}{3(k+1)+1}$$

$\therefore$ if the formula is true for a value of n, then it is true for the following value of n.

It is true for $n = 1$, $\therefore$ it will be true for $n = 2$, $n = 3$, etc.
$\therefore$ true for all n.

Syllabus outcomes and marking guide

HE2

- Gives the correct, complete answer 4
- Gives an essentially correct solution with a minor omission or flaw 3
- Makes significant progress, e.g. tests for $n = 1$.

AND

- Attempts to show true for $n = k + 1$ assuming $n = k$ is true 2
- Makes some progress, e.g. tests for $n = 1$.

OR

- Attempts to show true for $n = k + 1$ assuming $n = k$ is true, or similar merit.......................... 1

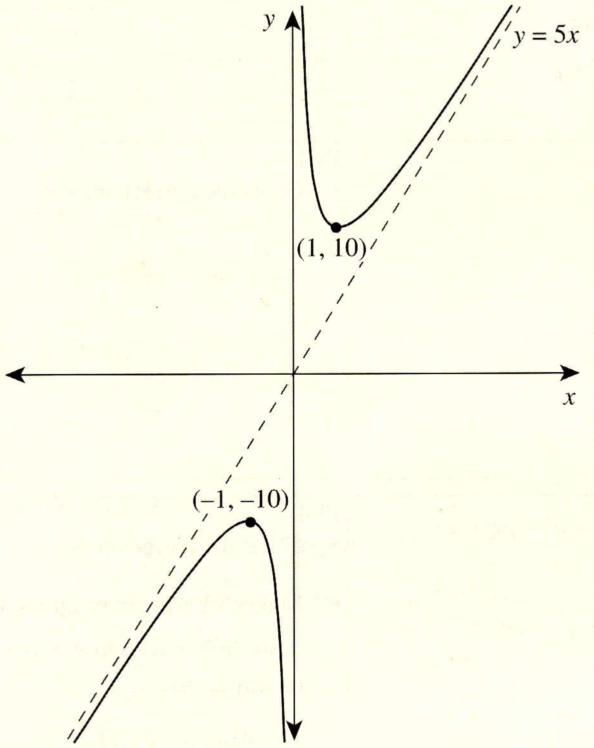
Question 3

	Sample answer	Syllabus outcomes and marking guide
(a)	$y = x\tan^{-1}x$ let $u = x$ $\quad v = \tan^{-1}x$ $\therefore u' = 1 \quad v' = \dfrac{1}{1+x^2}$ $\therefore \dfrac{d}{dx}(x\tan^{-1}x)$ $= uv' + vu'$ $= \dfrac{x}{1+x^2} + \tan^{-1}x$ $= \dfrac{x + (1+x^2)\tan^{-1}x}{1+x^2}$	HE4 • Obtains the correct answer in any form . . 2 • Shows $\dfrac{d}{dx}\tan^{-1}x = \dfrac{1}{1+x^2}$ 1
(b)	$\displaystyle\int_0^1 \frac{dx}{\sqrt{4-x^2}} = \int_0^1 \frac{dx}{\sqrt{2^2-x^2}}$ $= \left[\sin^{-1}\dfrac{x}{2}\right]_0^1$ $= \left[\sin^{-1}\left(\dfrac{1}{2}\right) - \sin^{-1}0\right]$ $= \left[\dfrac{\pi}{6} - 0\right]$ $= \dfrac{\pi}{6}$	HE4 • Gives the correct answer in any form . . . 2 • Shows $\displaystyle\int \frac{dx}{\sqrt{4-x^2}} = \sin^{-1}\frac{x}{2}$. 1
(c) (i)	$\sin 3\theta = \sin(2\theta + \theta)$ $= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$ $= 2\sin\theta\cos\theta\cos\theta + (\cos^2\theta - \sin^2\theta)\sin\theta$ $= 3\sin\theta\cos^2\theta - \sin^3\theta$ $= 3\sin\theta(1 - \sin^2\theta) - \sin^3\theta$ $= 3\sin\theta - 4\sin^3\theta$	HE3, HE6 • Correct proof . . . . 2 • Demonstrates the correct use of at least one multiple angle formula e.g. $\sin(A+B)$ or $\sin 2\theta$ or $\cos 2\theta$ 1
(ii)	$\displaystyle\int 2\sin^3\theta\, d\theta$ $4\sin^3\theta = 3\sin\theta - \sin 3\theta$ $2\sin^3\theta = \dfrac{1}{2}(3\sin\theta - \sin 3\theta)$ $\displaystyle\int 2\sin^3\theta\, d\theta = \frac{1}{2}\int(3\sin\theta - \sin 3\theta)\,d\theta$ $= \dfrac{1}{2}\left(-3\cos\theta + \dfrac{1}{3}\cos 3\theta\right) + C$ $= \dfrac{1}{6}\cos 3\theta - \dfrac{3}{2}\cos\theta + C$	HE6 • Gives the correct answer, ignore $+ C$. . . . 1

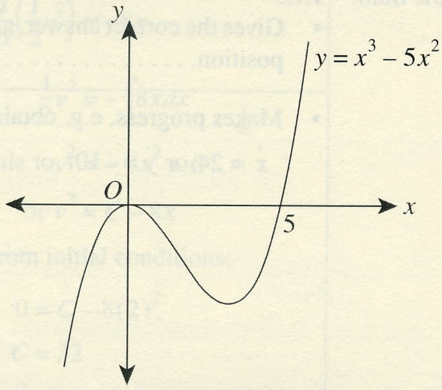
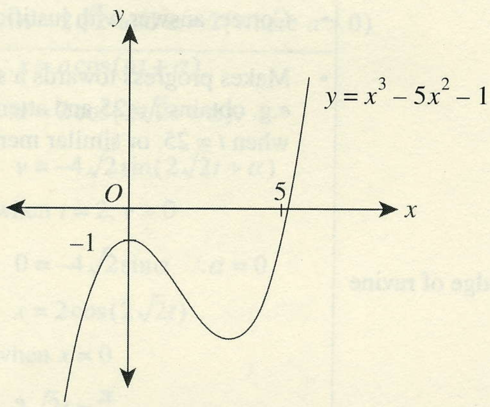
Question 3 (Continued)

	Sample answer	Syllabus outcomes and marking guide
(d) (i)	$m_{PQ} = \dfrac{ap^2 - aq^2}{2ap - 2aq}$ $= \dfrac{a(p^2 - q^2)}{2a(p - q)}$ $= \dfrac{a(p - q)(p + q)}{2a(p - q)}$ $= \dfrac{p + q}{2}$ Use point P (or Q) $y - ap^2 = \dfrac{p + q}{2}(x - 2ap)$ $2y - 2ap^2 = (p + q)x - 2ap(p + q)$ $2y - 2ap^2 = (p + q)x - 2ap^2 - 2apq$ $2y = (p + q)x - 2apq$	PE4 • Gives the correct answer . . . . 2 • Shows $m_{pq} = \dfrac{p + q}{2}$ or equivalent merit. . 1
(ii)	Substitute point $(0, -2a)$ into: $2y = (p + q)x - 2apq$. When $x = 0$ and $y = -2a$ $2(-2a) = (p + q) \times 0 - 2apq$ $-4a = -2apq$ $\dfrac{-4a}{-2a} = pq$ $pq = 2$	PE4 • Gives the correct answer 1
(iii)	$x = -apq(p + q)$ and $y = a(p^2 + q^2 + pq + 2)$ $pq = 2$ $\therefore x = -2a(p + q)$ and $y = a(p^2 + q^2 + 2pq)$ $\dfrac{x}{-2a} = p + q, y = a(p + q)^2$ $y = a\left(\dfrac{x}{-2a}\right)^2$ $= a \times \dfrac{x^2}{4a^2}$ $4ay = x^2$ therefore, K is on the original parabola $x^2 = 4ay$.	PE4 • Gives the correct answer . . . . 2 • Makes significant progress, e.g. obtains $x = -2a(p + q)$ and $y = a(p^2 + q^2 + 2pq)$ or similar merit. 1

Question 4

	Sample answer	Syllabus outcomes and marking guide
(a)	(i) $y = 5x + 5x^{-1}, x \neq 0$ $y' = 5 - 5x^{-2}$ $= 0$ $x^{-2} = 1$ $x = \pm 1, (1, 10)(-1, -10)$ $y'' = 10x^{-3}$ $f''(1) = 10 > 0, \therefore \text{min } (1, 10)$ $f''(-1) = -10 < 0 \therefore \text{max } (-1, -10)$	H6 • Gives the correct answer .... 2 • Gives the correct stationary points. OR • Gives one stationary point with correct nature 1
	(ii) As $x \to \infty, y \to 5x$ and $x \neq 0$  	H6 • Correct graph showing $(1, 10), (-1, -10)$ and asymptotes at $x = 0$ and $y = 5x$ ..... 1 *Note: accept equation of asymptotes shown anywhere in Question 4(a)(i), (ii) or (iii).*
	(iii) Range: $y \geq 10$ and $y \leq -10$	H5 • Gives the correct answer 1
(b)	(i) ${}^{20}C_{14} = 38\ 760$	PE3 • Gives the correct answer (accept correct numerical expression) 1
	(ii) ${}^{8}C_4 \times {}^{12}C_{10} = 4620$	PE3 • Gives the correct answer (accept correct numerical expression) 1
	(iii) ${}^{8}C_5 \times {}^{12}C_9 \times 14! = 1.074 \times 10^{15}$	PE3 • Gives the correct answer in any form (accept correct numerical expression) .... 2 • Multiplying combinations by 14! 1

Question 4 **(Continued)**

Sample answer	Syllabus outcomes and marking guide
(c) (i) 	PE3 HE3 • Sketches correct graph 1
(ii) $y = x^3 - 5x^2 - 1$ has only one root as it is the graph of $y = x^3 - 5x^2$ shifted down one unit. Since $y = x^3 - 5x^2$ has a maximum turning point as a repeated root at $x = 0$, a downwards shift of the graph results in this root disappearing, leaving only one root near $x = 5$. 	PE3 HE3 • Correct translation of graph in part (i). OR • Correct explanation without graph 1
(iii) Let $f(x) = x^3 - 5x^2 - 1$ $f'(x) = 3x^2 - 10x$ Using Newton's method: $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ $= 5 - \frac{(5)^3 - 5(5)^2 - 1}{3(5)^2 - 10(5)}$ $= 5 - \frac{-1}{25}$ $= 5 + \frac{1}{25}$ $= 5.04$	HE7, HE3 • Gives answer correct to two decimal places . . . . 2 • Correctly finds $f(5)$ and $f'(x)$ or similar merit 1

Question 5

Syllabus outcomes and marking guide

(a) (i) Position the origin where the weight falls off the train.

$\ddot{x} = 0$

$\dot{x} = v$

$\dot{x} = 24 \text{ m s}^{-1}$

$x = 24t + C_1$

$C_1 = 0$

$x = 24t$

$\ddot{y} = -g = -10$

$\dot{y} = -10t + C_2$

$C_2 = 0$ (starts from vertical rest)

$\dot{y} = -10t$

$y = -5t^2 + C_3$

$C_3 = 0$ (starts at origin)

$\therefore y = -5t^2$

HE3
- Gives the correct answer, allow origin in any position. 2
- Makes progress, e.g. obtains $\dot{x} = 24$ or $\dot{y} = -10t$ or similar merit . . . 1

(ii) The value of y when $x = 600$ is required.

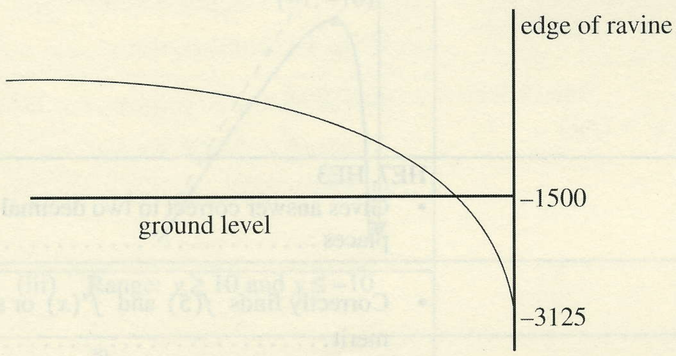
$24t = 600$

$t = 25$

$y = -\frac{1}{2}g \times 25^2$

$y = -5 \times 25^2$

$= -3125$ m



The weight will hit the ground, not the vertical side of the ravine.

HE3
- Correct answer with justification 2
- Makes progress towards a solution, e.g. obtains $t = 25$ and attempts to find y when $t = 25$, or similar merit 1

Question 5 **(Continued)**

	Syllabus outcomes and marking guide
(b) (i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -8x$ $\frac{1}{2}v^2 = -\int 8x\,dx$ $v^2 = -8x^2 + C$ or $v^2 = C - 8x$ From initial conditions: $0 = C - 8(2)^2$ $C = 32$ $\therefore v^2 = 32 - 8x^2$ $\therefore v^2 = 8(4 - x^2)$	HE3, HE5 • Obtains the correct solution . . . . 2 • Uses $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ 1
(ii) From (i), $v^2 = 8(4 - x^2)$ $= n^2(a^2 - x^2)$ since the particle is exerting simple harmonic motion $\therefore n^2 = 8$ and $a^2 = 4$ $\therefore n = 2\sqrt{2}$ and $a = 2$ (where $a > 0$) $\therefore x = a\cos(nt + \alpha)$ $\therefore x = 2\cos(2\sqrt{2}t + \alpha)$. $v = -4\sqrt{2}\sin(2\sqrt{2}t + \alpha)$ when $t = 2$, $v = 0$ $0 = -4\sqrt{2}\sin\alpha \quad \therefore \alpha = 0$ $x = 2\cos(2\sqrt{2}t)$ when $x = 0$ $2\sqrt{2}t = \frac{\pi}{2}$ $t = \frac{\sqrt{2}\pi}{8}$	HE3, HE5 • Obtains the correct answer. . . . . 2 • Shows $x = 2\cos(2\sqrt{2}t)$ or similar merit. 1

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Question 5 (Continued)	Syllabus outcomes and marking guide
(c) (i) $T = S - Ae^{kt} \Rightarrow Ae^{kt} = S - T$ $\frac{dT}{dt} = -Ae^{kt} \times k$ $= -(S - T) \times k$ That is, the rate is proportional to $S - T$. $\therefore T = S - Ae^{kt}$ is a solution to $\frac{dT}{dt} \propto (S - T)$.	HE3 • Correct demonstration 1
(ii) $S = 160$ °C $t = 0, T = 4$ °C $t = 30, T = 60$ °C $t = ?, T = 150$ °C $T = 160 - Ae^{kt}$ $4 = 160 - Ae^{0}$ $\therefore A = 156$ $t = 30$ and $T = 60$ $60 = 160 - 156e^{30k}$ $e^{30k} = \frac{100}{156}$ $30k = \log_e\left(\frac{100}{156}\right)$ $k = \frac{1}{30}\log_e\left(\frac{100}{156}\right)$ $t = ?$ and $T = 150$ $150 = 160 - 156e^{kt}$ $e^{kt} = \frac{10}{156}$ $t = \frac{\log_e\left(\frac{10}{156}\right)}{k}$ $t = 185.34$ $t \approx 186$ min	HE3 • Gives the correct answer . . . . 3 • Makes significant progress, e.g. determines the value of S, A and k, or makes a minor error . . . . 2 • Makes some progress, e.g. determines two of the values for S, A and k 1

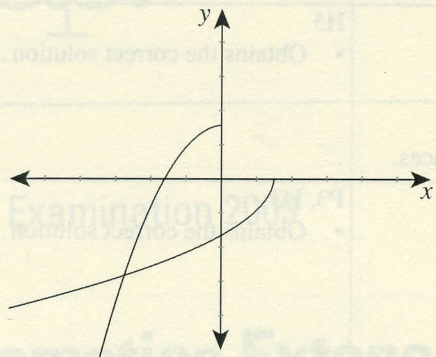
Question 6

Sample answer	Syllabus outcomes and marking guide
(a) $T_5 = {}^{15}C_4(1)^{11}(2x)^4$ $= 1365 \times 2^4 x^4$ $= 21\,840x^4$	HE3 • Gives the correct answer 1
(b) (i) $P(\text{red}) = \frac{3}{8}$ and $P(\text{black}) = \frac{5}{8}$ Terms in $\left(\frac{3}{8}+\frac{5}{8}\right)^{12}$ are required. A quarter black means 3 black, i.e. 9 red. ${}^{12}C_3\left(\frac{3}{8}\right)^9\left(\frac{5}{8}\right)^3 = 0.007877$	HE3 • Gives the correct answer, ignore rounding . . . . 2 • Uses $\left(\frac{3}{8}+\frac{5}{8}\right)^{12}$ 1
(ii) The most likely number $\rightarrow$ the term with the largest probability. Need $C_{k+1} > C_k$ $\dfrac{{}^{12}C_{k+1}\left(\frac{3}{8}\right)^{11-k}\left(\frac{5}{8}\right)^{k+1}}{{}^{12}C_k\left(\frac{3}{8}\right)^{12-k}\left(\frac{5}{8}\right)^{k}} > 1$ $\dfrac{\frac{12!}{(k+1)!(11-k)!}\times\frac{5}{8}}{\frac{12!}{(12-k)!k!}\times\frac{3}{8}} > 1$ $\frac{5}{k+1}\times\frac{12-k}{3} > 1$ $8k < 57$ $k < 7\frac{1}{8}$ Need $k = 7$, that is the term involving ${}^{12}C_8$. $T_9 = {}^{12}C_8\left(\frac{3}{8}\right)^4\left(\frac{5}{8}\right)^8$ The most likely combination is 4 red and 8 black.	HE3 • Gives the correct answer, accept any method . . . . 4 • Obtains $k < 7\frac{1}{8}$, or similar value through a different approach . . . . 3 • Makes some progress in the solution to $C_{k+1} > C_k$ e.g. correctly uses the ${}^{n}C_{r}$ formula, or similar standard with a different approach. . . . . 2 • Attempts to use: term with ${}^{12}C_{k+1}$ > term with ${}^{12}C_k$ or $\frac{T_{k+1}}{T_k} > 1$ 1
(c) (i) Let A_n = value of investment at the end of n years. $A_1 = 1000(1 + 0.056)^1$ $A_2 = 1000(1.056)^2 + 1.2 \times 1000(1.056)^1$ Let $M = 1.056$ $A_3 = 1000M^3 + 1.2 \times 1000M^2 + 1.2^2 \times 1000M$ $A_3 = 1000M(1.2^2 + 1.2M + M^2)$	H5 • Correct demonstration 1

Question 6 **(Continued)**

Sample answer	Syllabus outcomes and marking guide
(ii) $A_n = 1000M(M^{n-1} + 1.2M^{n-2} + 1.2^2M^{n-3} \ldots 1.2^{n-1})$ $= 1000M \times \dfrac{a(r^n - 1)}{r - 1}$ $= 1000M \times 1.2^n \times \dfrac{\left[\left(\dfrac{M}{1.2}\right)^n - 1\right]}{\dfrac{M}{1.2} - 1}$ $= 1000 \times 1.056 \times 1.2^{n-1} \times \dfrac{\left(\dfrac{1.056}{1.2}\right)^n - 1}{\dfrac{1.056}{1.2} - 1}$ $= 1056 \times (1.2)^{n-1} \times \dfrac{(0.88)^n - 1}{-0.12}$ $= 8800(1.2)^{n-1}[1 - (0.88)^n]$	H5 • Correct demonstration . . . . 2 • Makes substantial progress, e.g. attempts to sum the correct sequence or finds a correct sequence for A_n 1
(iii) $A_{30} = \$1\,703\,156.94$ The amount Cassie invested $= 1000 + 1000(1.2) + 1000(1.2)^2 \ldots 1000(1.2)^{29}$ $= 1000 \times \dfrac{[(1.2)^{30} - 1]}{1.2 - 1}$ $= \$1\,181\,881.57$ interest $= \$521\,275.37$ % interest $= \dfrac{521\,275.37}{1\,703\,156.94} \times 100$ $= 30.6\%$	H5 • Gives correct answer . . . . 2 • Determines that Cassie invested \$1 181 881.57 (ignore rounding) or equivalent merit 1

Question 7

Sample answer	Syllabus outcomes and marking guide
(a) (i) 	HE4 • Draws a graph that is the mirror image of $y = f(x)$ in the line $y = x$ 1
(ii) $x = -\sqrt{3-2y}$ $\quad x \le 1\frac{1}{2}, y \le 0$ $x^2 = 3 - 2y$ $\quad x \le 0$ $y = \frac{1}{2}(3 - x^2)$	HE4 • Gives the correct answer 1
(b) (i) $(1+x)^n(1+x)^{2n} = (1+x)^{3n}$ Terms in x^2 $RHS = {}^{3n}C_2x^2$ On *LHS* $({}^nC_0 + {}^nC_1x + {}^nC_2x^2 \ldots)({}^{2n}C_0 + {}^{2n}C_1x + {}^{2n}C_2x^2 \ldots$ The product of the paired terms give x^2 $\therefore$LHS term in x^2 $= {}^nC_0\,{}^{2n}C_2x^2 + {}^nC_1\,{}^{2n}C_1x^2 + {}^nC_2\,{}^{2n}C_0x^2$ Equation coefficients ${}^nC_0\,{}^{2n}C_2 + {}^nC_1\,{}^{2n}C_1 + {}^nC_2\,{}^{2n}C_0 = {}^{3n}C_2$	HE4 • Correct demonstration . . . . 2 • Determines the expression for x^2 on either side of the equation 1
(ii) Consider terms in x^n on the RHS ${}^{3n}C_nx^n$ $\sum_{k=0}^{n}\binom{n}{k}\binom{2n}{n-k} = {}^nC_0\,{}^{2n}C_n + {}^nC_1\,{}^{2n}C_{n-1}\ldots + {}^nC_n\,{}^{2n}C_0$ Terms in x^n on LHS $= {}^nC_0\,{}^{2n}C_nx^n + {}^nC_1x\,{}^{2n}C_{n-1}x^{n-1} \ldots {}^nC_nx^n\,{}^{2n}C_0$ $= [{}^nC_0\,{}^{2n}C_n + {}^nC_1\,{}^{2n}C_{n-1}\ldots + {}^nC_n\,{}^{2n}C_0]x^n$ Coefficients of x^n on both sides of the equation are equal. $\therefore {}^nC_0\,{}^{2n}C_n + {}^nC_1\,{}^{2n}C_{n-1}\ldots + {}^nC_n\,{}^{2n}C_0 = {}^{3n}C_n$ i.e. $\sum_{k=0}^{n}\binom{n}{k}\binom{2n}{n-k} = \binom{3n}{n}$	HE3 • Correct demonstration . . . . 2 • Makes some progress, e.g. identifies the significance of x^n to the solution 1

Question 7 **(Continued)**

Sample answer	Syllabus outcomes and marking guide
(c) (i) Compounded monthly $A = P\left(1 + \frac{0.06}{12}\right)^{12}$ $= P \times 1.0616778\ldots$ Effective interest rate is 6.168% to 3 decimal places.	H5 • Obtains the correct solution 1
(ii) (α) Gradient of $PQ = \dfrac{\log_e\left(1 + \frac{r}{n}\right)}{\frac{r}{n}}$ $= \frac{n}{r}\log_e\left(1 + \frac{r}{n}\right)$ $= \frac{1}{r}\log_e\left(1 + \frac{r}{n}\right)^n$	P3, H3 • Obtains the correct solution 1
(β) Using $\frac{d}{dx}(\log_e x) = \frac{1}{x}$, the gradient of the tangent P, where $x = 1$, will be 1. Using the definition of a derivative, the gradient of the tangent at P: $\lim\limits_{n \to \infty} \frac{1}{r}\log_e\left(1 + \frac{r}{n}\right)^n$ $\lim\limits_{n \to \infty} \frac{1}{r}\log_e\left(1 + \frac{r}{n}\right)^n = 1$ $\lim\limits_{n \to \infty} \log_e\left(1 + \frac{r}{n}\right)^n = r$ $\lim\limits_{n \to \infty} \left(1 + \frac{r}{n}\right)^n = e^r$	H3, P6, P8 • Obtains the correct solution . . . . 3 • Makes significant progress towards the solution. . . . . 2 • Attempts to use the gradient at $x = 1$ for the derivative and the limiting position of the secant 1
(γ) $A = P\left(1 + \frac{0.06}{n}\right)^n$ when interest is compounded continuously, i.e. $n \to \infty$. $\lim\limits_{n \to \infty} \left(1 + \frac{0.06}{n}\right)^n = e^{0.06}$ $A = P \times e^{0.06}$ $1 + r = e^{0.06}$ The effective rate when the investment is compounded continuously is $e^{0.06} - 1$.	H5, H9 • Obtains the correct solution 1