Neap.
advancing education

HSC Trial Examination 2001

Mathematics Extension 1

This paper must be kept under strict security and may only be used on or after the afternoon of Thursday 9 August, 2001, as specified in the NEAP Examination Timetable.

General Instructions

Reading time 5 minutes

Working time 2 hours

Write using blue or black pen.

Board-approved calculators may be used.

A table of standard integrals is provided on page 10.

All necessary working should be shown in every question.

Examination structure

Total marks 84

Attempt all questions.

All questions are of equal value.

QUESTION 1. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the exact value of $\int_1^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}}$. **3**

(b) Solve $\frac{3}{x^2-1} \geq 0$. **3**

(c) Use the substitution $u^2 = 4 + x$ to evaluate $\int_0^5 \frac{x}{\sqrt{4+x}} dx$. **4**

(d) Simplify $\frac{{}^nP_{r+1}}{{}^nP_r}$. **2**

QUESTION 2. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Differentiate $x\tan^{-1}x$. **1**

(ii) Hence evaluate $\int_0^1 \tan^{-1}x\,dx$. **3**

(b) (i) Sketch the graph of $y=\cos x, -\pi \le x \le \pi$ and use this graph to show that $\cos x + x = 0$ has only one solution. **2**

(ii) Use Newton's method with a first approximation of $x=-1$ to find a second approximation to the root of $\cos x + x = 0$. **2**

(c) There are 12 videotapes arranged in a row on a shelf in a video shop. There are 3 identical copies of *Gone with the Wind*, 4 of *Tootsie* and 5 of *Gladiator*.

(i) How many different arrangements of the videotapes are there? **1**

(ii) How many different arrangements are there if videos with the same title are grouped together? **1**

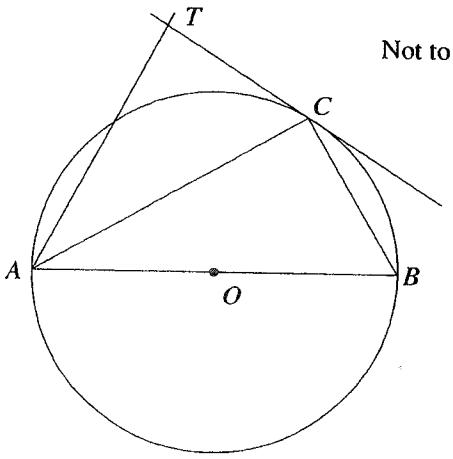
(iii) The 12 videotapes are arranged at random in a row on the shelf. Find the probability that the arrangement has a copy of *Gone with the Wind*, at one end of the row, and a copy of *Gladiator* at the other end. **2**

QUESTION 3. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Write down an expression for $\tan(\alpha+\beta)$. **1**

(ii) Hence evaluate $\alpha+\beta$, for $0 \le \alpha+\beta \le 2\pi$, if $\tan\alpha = \frac{1}{2}$ and $\tan\beta = \frac{1}{3}$. **2**

(b) **3**



In the diagram AOB is the diameter of a circle centre O, and C is the point of contact of the tangent TC such that AC bisects $\angle TAB$.

Prove that AT is perpendicular to TC.

(c) The acceleration of a particle moving in a straight line is given by $\frac{d^2x}{dt^2} = -e^{-2x}$, where x is the displacement, in metres, from the origin O and t is time elapsed in seconds. Initially the particle moves from the origin with velocity +2 metres per second. **3**

Find the velocity of the particle at $x = 1$, correct to 2 decimal places.

(d) The coefficients of x^3 and x^4 in the expansion of $(1+bx)^{12}$, $b \neq 0$, are equal. **3**

Find the value of b.

QUESTION 4. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the expression $P(x) = x^3 + ax^2 + bx + 6$, where a and b are constants.

When $P(x)$ is divided by $(x-2)(x+1)$, the remainder is $4-4x$, and the quotient is $x+k$.

(i) By using the division transformation, or otherwise, find the values of a and b. **4**

(ii) Find the value k for these values of a and b. **1**

(b) (i) Write down an expression for $\cos 2A$ in terms of $\sin A$. **1**

(ii) Hence show that $\sin^2\frac{x}{2} = \frac{1}{2}(1-\cos x)$. **1**

(iii) Sketch $y = \sin^2\frac{x}{2}$ for $0 \le x \le 2\pi$. **2**

(iv) State the amplitude and period of $y = \sin^2\frac{x}{2}$. **1**

(v) Find the exact area of the region bounded by the curve $y = \sin^2\frac{x}{2}$ and the x-axis between $x = 0$ and $x = \frac{\pi}{3}$. **2**