SBHS MATHEMATICS TRIAL 2002

1

(a) $4x = 15$

$x = \frac{15}{4}\ ,\ 3\frac{3}{4}$ — 2

(b) 2.94 — 2

(c) $c(d-1) - y(d-1)$

$(c-y)(d-1)$ — 2

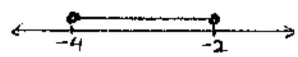
(d) $4k + 1 = 29$

$k = 7$ — 2

(e) $x + 3 \le 1$ or $-x - 3 \le 1$

$x \le -2$ $\quad -x \le 4$

$x \ge -4$



2

(f) $\frac{1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2}$

$= \frac{\sqrt{3}-2}{-1}$

$-\sqrt{3} + 2 = a\sqrt{3} + b$

$a = -1 \quad b = 2$ — 2

2(a) $(4,-1)\ (7,3)$

$m = \frac{4}{3}$

$y + 1 = \frac{4}{3}(x - 4)$

$3y + 3 = 4x - 16$

$4x - 3y - 19 = 0$ as req'd — 2

(b) $BC\ (7,3)(8,1)$ $\quad AD\ (4,-1)(-1,9)$

$m_1 = -\frac{2}{1} = -2 \quad m_2 = \frac{10}{-5} = -2$

$m_1 = m_2 \quad BC \parallel AD$ — 2

(c) $DC\ (1,9)\ \ (7,3)$

$m = -\frac{6}{8} = -3/4$

grad $AC = \frac{4}{3} \rightarrow$ from (a)

$m_1 m_2 = -1 \quad DC \perp AC$

$\therefore \angle ACD = 90°$ — 2

(d) $AC\ (4,-1)\ \ (7,3)$

$d = \sqrt{(7-4)^2 + (3 - -1)^2}$

$= \sqrt{25} = 5u$ — 2

(e) $d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right|$

$4x - 3y - 19 = 0 \quad (8,1)$

$d = \left| \frac{4\times8 - 3\times1 - 19}{\sqrt{16+9}} \right|$

$= \frac{10}{5} = 2u$ — 2

(f) length DC $(-1,9)\ (7,3)$

$d = \sqrt{8^2+6^2} = 10u$

Area $= \triangle DCA + \triangle ACB$

$A = \frac{1}{2}\times10\times5 + \frac{1}{2}\times5\times2$

$= 30u^2$ — 2

Mathematics Trial, Assessment Task #4, 2002

[1] 3. (a) i. $7\times 2(2x-1)^6 = 14(2x-1)^6$.

[2] ii. $\frac{5}{4+5x}$.

[2] iii. $2x\sin x + x^2\cos x$.

[1] (b) $\int \sec^2 3x\,dx = \frac{1}{3}\tan 3x + c$.

[3] (c) $\int_1^{e^2} \frac{3}{x}dx = 3\left[\ln x\right]_1^{e^2}$,
$= 3\{2-0\}$,
$= 6$.

[3] (d) Area $= \int_0^{\frac{3}{2}} e^{2x}dx$,
$= \left[\frac{e^{2x}}{2}\right]_0^{\frac{3}{2}}$,
$= \frac{1}{2}\{e^3 - 1\}$.

[2] 4. (a) i. $x^2 - 2x + 1 = 8y - 25 + 1$,
$\therefore (x-1)^2 = 4\times 2(y-3)$.

[1] ii. (1,3).

[1] iii. (1,5).

[1] iv. $y = 1$.

[4] (b) First method: using λ,
$x + 2y - 6 + \lambda(x-y) = 0$,
$(2-\lambda)y = -(1+\lambda)x + 6$.
The slope of line $3x - 2y + 7 = 0$ is $\frac{3}{2}$.
i.e. $\frac{3}{2} = \frac{\lambda+1}{\lambda-2}$,
$3k - 6 = 2\lambda + 2$,
$\lambda = 8$.
$\therefore$ New line is $x + 2y - 6 + 8x - 8y = 0$.
i.e. $9x - 6y - 6 = 0$,
$3x - 2y - 2 = 0$.

Second method: first find the intersection,
$-y = 2y - 6$,
$y = 2$,
and $x = 2$.
$\therefore$ Intersection is $(2,2)$.
The slope of line $3x - 2y + 7 = 0$ is $\frac{3}{2}$.
$\therefore$ New line is : $y - 2 = \frac{3}{2}(x-2)$,
$2y - 4 = 3x - 6$,
i.e. $3x - 2y - 2 = 0$.

[3] (c) Area $\Delta XYZ = \frac{1}{2}\times 4\times 4$,
$= 8\,\text{unit}^2$.
Area of Sector $XYW = \frac{1}{2}\times 4^2 \times \frac{\pi}{4}$,
$= 2\pi\,\text{unit}^2$.
$\therefore$ Shaded area $= 8 - 2\pi\,\text{unit}^2$.

TRIAL 2U HSC 2002
SOLUTIONS

QUESTION 5

a) $y = x^2 - 3x + 5$

(2marks) $y' = 2x - 3$

at $(3,5)$

$y' = 2\times 3 - 3$

$= 3$

Now $m_2 = \frac{-1}{m_1}$

$= \frac{-1}{3}$

using

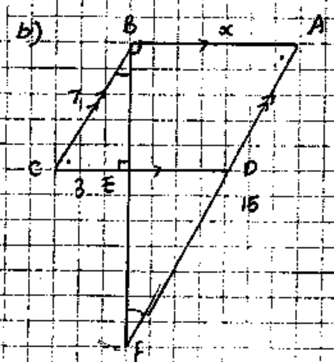
$y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{3}(x-3)$

$3y - 15 = -x + 3$

$\therefore x + 3y - 18 = 0$

b)



i) (3marks) In ΔCBE and ΔAFB

$\angle CEB = \angle ABF$ (alt. $\angle$s, AB||

$\angle BCE = \angle BAF$ (opp. $\angle$s, || gr

$\angle CBE = \angle AFB$ ($\angle$sum of Δ)

$\therefore \Delta CBE\ |||\ \Delta AFB$ (equiangular

ii) (2marks) In ΔCBE and ΔAFB

$\frac{CB}{CE} = \frac{AF}{AB}$ (corr. sides of $|||$ Δs)

$\frac{7}{3} = \frac{15}{x}$

$7x = 45$

$x = 6\frac{3}{7}$

$\therefore AB = 6\frac{3}{7}$ cm

c) $y = x^2 - 1$

(6 marks) $y^2 = (x^2-1)^2$

$= x^4 - 2x^2 + 1$

$V = \pi \int y^2 \, dx$

$= 2\pi \int_0^1 (x^4 - 2x^2 + 1) \, dx$

$= 2\pi \left[\frac{x^5}{5} - \frac{2x^3}{3} + x \right]_0^1$

$= 2\pi \left[\left(\frac{(1)^5}{5} - \frac{2(1)^3}{3} + 1 \right) - \left(\frac{(0)^5}{5} - \frac{2(0^3)}{3} - 0 \right) \right]$

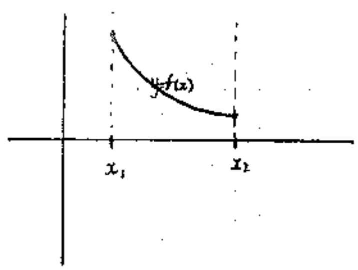
$= 2\pi \left(\frac{1}{5} - \frac{2}{3} + 1 \right)$

$= 2\pi \left(\frac{8}{15} \right)$

$V = \frac{16\pi}{15}$ units^3

d) (2 marks) $y = f(x)$ $\quad x_1 \le x \le x_2$

$f(x) > 0, \quad f'(x) < 0, \quad f''(x) > 0$



$f(x)$ above x-axis
$f'(x)$ negative gradient
$f''(x)$ concave up

QUESTION 6

a) (3 marks) $y = x^3 + mx + n$ $\qquad P(1,5)$ when $y' = 0$

$y' = 3x^2 + m$

at $P(1,5)$

$0 = 3(1)^2 + m$

$\therefore m = -3$

$y = x^3 - 3x + n$

at $P(1,5)$

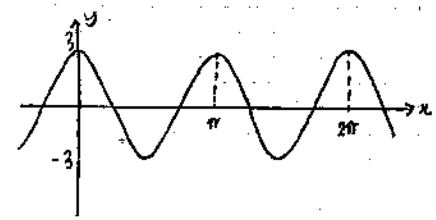
$5 = (1)^3 - 3(1) + n$

$5 = 1 - 3 + n$

$7 = n$

$\therefore n = 7$

b) (2marks)



$y = a\cos mx$ $\qquad$ where amplitude $(a) = 3$

$\therefore y = 3\cos 2x$ $\qquad$ period $T = \frac{2\pi}{m}$

$\pi = \frac{2\pi}{m}$

$\therefore m = 2$

c) (2marks) $\ln 2 + 2\ln 18 - \frac{3}{2}\ln 36$

$= \ln 2 + \ln 324 - \ln 216$

$= \ln \left(\frac{2 \times 324}{216} \right)$

d) (marks)

$$\sqrt{\frac{1-\cos^2\theta}{1+\tan^2\theta}}$$

Using $\cos^2\theta + \sin^2\theta = 1$

$\sin^2\theta = 1-\cos^2\theta$

$1 + \tan^2\theta = \sec^2\theta = \frac{1}{\cos^2\theta}$

$$= \sqrt{\frac{\sin^2\theta}{\frac{1}{\cos^2\theta}}}$$

$$= \sqrt{\sin^2\theta\cos^2\theta}$$

$$= \underline{\sin\theta\cos\theta}$$

e) (3marks)

$$\int_{-1}^{7} f(x)\,dx \doteq \frac{h}{3}\left[(y_0 + y_8) + 4(y_1 + y_3) + 2(y_2)\right]$$

where $h = \frac{b-a}{n}$

$= \frac{7--1}{4}$

$= 2$

$y_0 = 5$
$y_1 = 9$
$y_2 = 2$
$y_3 = -1$
$y_4 = -6$

$$\int_{-1}^{7} f(x)\,dx \doteq \frac{2}{3}\left[(5-6) + 4(9-1) + 2(2)\right]$$

$$\doteq \frac{2}{3}(-1+32+4)$$

$$\doteq \frac{70}{3}$$

$$\doteq \underline{23\tfrac{1}{3}}$$

QUESTION 7

a) i) $V = 50000e^{-0.25t}$

(1mark)

when $t=0$, $V=?$

$V = 50000e^{-0.25\times 0}$

$= 50000$

$\therefore$ value of new car is $\underline{\$50000}$

ii) when $t=3$, $V=?$

(2marks)

$V = 50000e^{-0.25\times 3}$

$= 23618.33$

$\therefore$ value of car after 3 years is $\underline{\$23600}$ (nearest $100)

iii) $V = 16000$ $t=?$

(2marks)

$15000 = 50000e^{-0.25t}$

$\frac{15000}{50000} = e^{-0.25t}$

$0.3 = e^{-0.25t}$

$\ln 0.3 = -0.25t$

$t = \frac{\ln 0.3}{-0.25}$

$= 4.815$

$\therefore$ time to taken for the car to be worth $15000 is $\underline{4.8 \text{ years}}$ (nearest tenth of a year).

b) i)
(2 marks)

$S_n = a + (a+d) + (a+2d)$

where $S_n = 15600$

$a = 1600$

$15600 = 1600 + (1600+d) + (1600+2d)$

$10800 = 3d$

$d = 3600$

$\therefore$ $A = \$1600$ $B = \$5200$ $C = \$8800$

ii) $S_n = \frac{a(r^n - 1)}{r-1}$
(2 marks)

$S_n = a + ar^1 + ar^2$

$15600 = 1600 + 1600r + 1600r^2$

$14000 = 1600r^2 + 1600r$

$0 = 8r^2 + 8r - 70$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-8 \pm \sqrt{64 + 2240}}{16}$

$\therefore r = \pm 2\frac{1}{2}$

$r = 2.5$

$\therefore$ $A = \$1600$ $B = \$4000$ $C = \$10000$

c) i) $P(2,2) = \frac{2}{7} \times \frac{1}{6}$
(1 mark)

$= \frac{1}{21}$

ii) $P(4+6) = \frac{1}{7} \times \frac{4}{6}$
(2 marks)

$= \frac{4}{42}$

$P(6+4) = \frac{4}{7} \times \frac{1}{6}$

$= \frac{4}{42}$

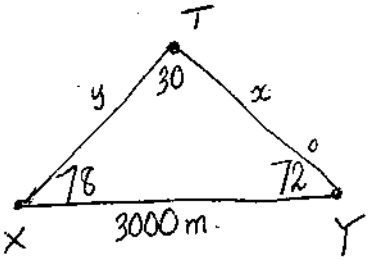
$P(6+6) = \frac{4}{7} \times \frac{3}{6}$

$= \frac{12}{42}$

$P(18) = \frac{8}{42} + \frac{12}{42}$

$= \frac{10}{21}$

(a)



$\dfrac{y}{\sin 72°} = \dfrac{3000}{\sin 30}$

$y = \dfrac{3000 \times \sin 72°}{\sin 30°}$ (3)

$\doteqdot 5706\,m \doteqdot 5.706\,km$

(b) $mx^2 - 20x + m = 0$

let $x = -\frac{1}{2}$ $\quad \frac{m}{4} + 10 + m = 0$

$m + 40 + 4m = 0$

$5m = -40$

$m = -8$

So we have $-8x^2 - 20x - 8 = 0$

and $-\frac{1}{2} + \alpha = \frac{20}{-8}$

$\alpha = -\frac{20}{8} + \frac{1}{2}$

$= -2$

r $(2x+1)(x+2) = 0$
$x = -\frac{1}{2},\ x = -2$

Other root is -2. (3)

(c) $y = x^3 + 3x^2 - 9x$
$= x(x^2 + 3x - 9)$

$y' = 3x^2 + 6x - 9$
$= 3(x^2 + 2x - 3)$

$y'' = 6x + 6 = 6(x+1)$

$3(x^2 + 2x - 3) = 0$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3,\ x = 1$

$y = 27\quad y = -5$

stat points are $(-3, 27)$ and $(1, -5)$ (2)

(ii) Curve is concave upwards when $y'' > 0$

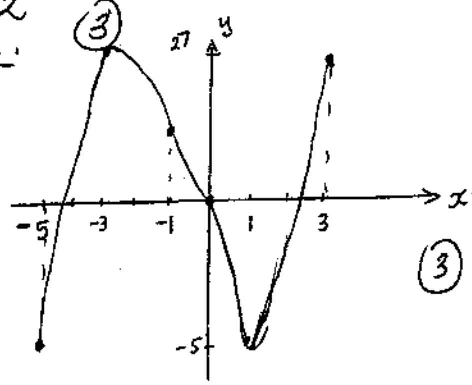
$6(x+1) > 0$

$x + 1 > 0$

$x > -1$ (1)

(iii) $(-3, 27)$ $y'' < 0$ MAX
$(1, -5)$ $y'' > 0$ min
$(-1, 11)$ inflection

At $x = -5,\ y = -5$
$x = 3,\ y = 27$.



(3)

(a) $|x+1| = 2x + 7$

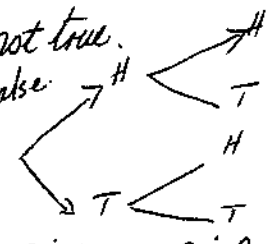
$x + 1 = 2x + 7$
$-6 = x$
LHS $\neq$ RHS

$x + 1 = -2x - 7$
$3x = -8$
$x = -\frac{8}{3}$

LHS $= 1\frac{2}{3}$
RHS $= 1\frac{2}{3}$.

$x = -\frac{8}{3} = -2\frac{2}{3}$ $(-2.\dot{6})$ (3)

(b) not true.
ie: false



4 possibilities not 3. (2)

(c) (i) $a = 2\pi^2 \cos \pi t$

data $t = 0$
$x = 0$
$V = \pi$

$V = \int 2\pi^2 \cos \pi t \, dt$.

$= \frac{2\pi^2}{\pi} \int \pi \cos \pi t \, dt$.

$V = 2\pi \sin \pi t + C_1$

data $\pi = C_1$

$V = 2\pi \sin \pi t + \pi$ $(1\frac{1}{2})$

$x = \int (2\pi \sin \pi t + \pi)\, dt$.

$x = -2\cos \pi t + \pi t + C_2$

C (ii) when $V = 0$

$2\pi \sin \pi t + \pi = 0$

$2\pi \sin \pi t = -\pi$

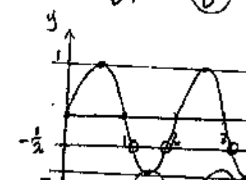
$\sin \pi t = -\frac{\pi}{2\pi}$

$\sin \pi t = -\frac{1}{2}$

$\pi t = \frac{7\pi}{6}, \frac{11\pi}{6}$

$t = \frac{7\pi}{6\pi} = \frac{7}{6}$ sec

$t = \frac{11\pi}{6\pi} = \frac{11}{6}$



also $t = \frac{19}{6}, \frac{23}{6}$

(iii) $t = \frac{19}{6}$ $\quad x = -2\cos \frac{19\pi}{6}$
$= -2 \times -\frac{\sqrt{3}}{2} + \frac{19}{6}$
$= 2 + \sqrt{3} + \frac{19}{6}$

$t = \frac{23}{6}$
$x = -2\cos \frac{23\pi}{6} + \frac{23\pi}{6}$
$= -2 \times \frac{\sqrt{3}}{2} + \frac{23\pi}{6} +$
$= 2 + \frac{23\pi}{6} - \sqrt{3}$

At $t = \frac{19}{6}$ x is larg

So $x = -2\cos \pi t + \pi t$ (1

(a) $C(n) = 1000(0.8)^n + 10{,}000[1 + .8 + .8^2 + \cdots + .8$

as $n \to \infty$ $1000(0.8)^n \to 0$

now $10000[1 + .8^1 + .8^2 + \cdots + .8^{n-1}]$

GP $\left.\begin{array}{l} a = 1 \\ r = .8 \\ n = n \end{array}\right\}$ $S_n = \dfrac{1(.8^n - 1)}{.8 - 1} = \dfrac{.8^n - 1}{-.2} = \dfrac{1 - .8^n}{.2}$

so we have $10{,}000 \times 5(1 - .8^n)$

$= 50{,}000(1 - .8^n)$ (3)

as $n \to \infty$ $50{,}000(1 - .8^n) \to 50{,}000$

(b) (i) $\varphi(t) = A(1+t)e^{-0.5t}$

$\varphi(t) = Ae^{-0.5t} + Ate^{-0.5t}$

$\varphi'(t) = -0.5Ae^{-0.5t} + At \times -0.5e^{-0.5t} + Ae^{-0.5t}$

$= 0.5Ae^{-0.5t} - 0.5Ate^{-0.5t}$

$\varphi''(t) = -0.25Ae^{-0.5t} - (0.5At \times -0.5e^{-0.5t} + 0.5Ae^{-0.5t})$

$= -0.25Ae^{-0.5t} + 0.25Ate^{-0.5t} - 0.5Ae^{-0.5t}$

$= -0.75Ae^{-0.5t} + 0.25Ate^{-0.5t}$

$4(-0.75Ae^{-0.5t} + 0.25Ate^{-0.5t}) + 4(0.5Ae^{-0.5t} - 0.5Ate^{-0.5t}) + Ae^{-0.5t} + Ate^{-0.5t}$

$= 0$ (4)

(ii) $\varphi(0) = 10$

So when $t = 0$ $A(1+t)e^{-0.5t} = 10$

$A(1+0)e^{-0.5 \times 0} = 10$

$A = 10$

So $\varphi(t) = 10(1+t)e^{-0.5t}$

$\varphi'(t) = 0.5 \times 10e^{-0.5t} - 0.5 \times 10te^{-0.5t}$

$= 5e^{-0.5t} - 5te^{-0.5t}$

$= 5e^{-0.5t}(1-t)$

let $\varphi'(t) = 0 \Rightarrow t = 1$

Sub $t=1$, $\varphi''(t) = -0.75 \times 10e^{-0.5} + 0.25 \times 10 \times e^{-0.5}$

$= -7.5e^{-0.5} + 2.5e^{-0.5}$

$= -5e^{-0.5} < 0$

at $t=1$, $\varphi(t)$ is a MAX.

When $t=1$, $\varphi(t) = 10(2)e^{-0.5}$

$= \dfrac{20}{e^{.5}} \doteq 12.13$ (4)

(iii) $\varphi(t) = 10(1+t)e^{-0.5t}$

$= 10e^{-0.5t} + 10te^{-0.5t}$

$= \dfrac{10}{e^{0.5t}} + \dfrac{10t}{e^{0.5t}}$ as $t \to \infty$ $\varphi(t) \to 0$