HB.
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KW

Name: ______________

Class: 12MT3__________

Teacher: ______________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2000 AP3

YEAR 12 HALF YEARLY HSC

MATHEMATICS
3/4 UNIT (COMMON)

Time allowed – 1.5 HOURS
(plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES:

* Attempt ALL questions.
* The value for each question is indicated
* All necessary working should be shown in every question.
  Full marks may not be awarded for careless or badly arranged work.
* Standard Integrals are provided. Approved calculators may be used.
* Each question attempted is to be returned on a new page clearly marked Question 1, Question 2, etc on the top of the page.

***Each page must show your class and your name.**

Marks

Question 1: (12 Marks)

(a) Differentiate the following: **4**

(i) $\log_e(e^{3x}+2)$

(ii) $x^3\cos 3x$.

(b) Find the following indefinite integrals: **4**

(i) $\int \frac{dx}{(7x+4)^5}$.

(ii) $\int \sin 6x\,dx$

(iii) $\int 4xe^{x^2}\,dx$.

(c) Solve for x: **2**

$\log_e 8+\log_a 16 = x\log_a 2$.

(d) Find the exact value of $\cos 105^0$. **2**

Question 2: (Start a New Page) (12 Marks)

(a) Simplify $\frac{\sin x}{\cos x-\sin x}+\frac{\sin x}{\cos x+\sin x}$ **3**

(b) Simplify $\sec x+\tan x$, in terms of t, where $t=\tan\frac{x}{2}$. **3**

(c) Use the substitution $u=x^2-1$ to find **3**

$\int x^3(x^2-1)dx$

(d) Consider the curve $y=\sin x$, for $0\le x\le 2\pi$. **3**

For what values of x is the gradient equal to $\frac{1}{2}$?

Question 3: (Start a New Page) (12 Marks)

Marks

(a) The quartic expression x^4+ax^2+b has factors $(x+1)$ and $(x-2)$. Find the values of a and b. **3**

(b) If $x=c$ is a double root of $P(x)$, show that $x=c$ is a root of $P'(x)$. **3**

(c) p, q and r are the roots of the cubic equation $x^3+2x^2+3x+5=0$. Evaluate: **4**

(i) $p+q+r$.

(ii) $p^{-1}+q^{-1}+r^{-1}$.

(d) The equation $e^x-4x-8=0$ has a root close to $x=3$. Using 3 as a first approximation and one application of Newton's Method to find a better approximation for this root. Give your answer correct to three decimal places. **2**

Question 4: (Start a New Page) (12 Marks)

(a)

(i) Find R and α such that $2\cos\theta-\sin\theta=R\cos(\theta+\alpha)$. (Note: $R>0$ and $0^0<\alpha<90^0$.) **4**

(ii) Hence, solve $2\cos\theta=\sin\theta+1$, for $0^0\le\theta\le360^0$

(b) The curve $y=\cos x$, from $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$, is rotated about the x-axis. **4**

Find the volume of the solid formed. Leave your answer in exact form.

(c) **4**

(i) Find $\frac{d}{dx}(x\log_e x)$.

(ii) Prove that $\int_e^{e^2}\frac{1+\log_e x}{x\log_e x}\,dx=1+\log_e 2$.

Question 5: (Start a New Page) (9 Marks)

Marks

(a)

(i) Sketch $y=\sin 2x$, for $0\le x\le 2\pi$. **4**

(ii) By drawing a suitable straight line, state the number of values of x, in this domain, such that $\sin 2x=\frac{x}{2\pi}$.

(iii) Can there be further solutions beyond $x=2\pi$? Briefly justify your answer.

(b) $A(t,e^t)$ and $B(-t,e^{-t})$ are points on the curve $y=e^x$ and $t>0$. **5**

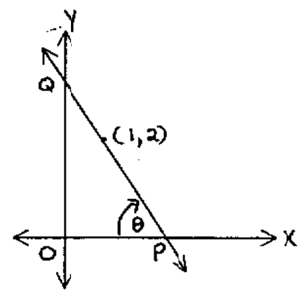
The tangents at A and B form an angle of 45^0.

(i) Prove that $e^t-\frac{1}{e^t}=2$.

(ii) Solve this equation to show that $e^t=1+\sqrt{2}$.

Question 6: (Start a New Page) (10 Marks)

10



A straight line passes through the point (1,2) and meets the x and y axes at P and Q respectively, as shown. The angle OPQ is θ.

(a) Show that the equation of the line PQ is given by $y=\tan\theta+2-x\tan\theta$.

(b) Show that the area (A) of ΔOPQ is given by $A=\frac{\tan\theta}{2}+2+\frac{2}{\tan\theta}$.

(c) Prove that the area is a minimum when $\tan\theta=2$.

(d) Hence, find the minimum area.

End of Exam