CRANBROOK SCHOOL

YEAR 12 MATHEMATICS – 2 UNIT

Term 3 2003   **Time : 3 h / GC, CGH, HRK and SKB**

All questions are of equal value.
All necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.
Approved silent calculators may be used.
Submit your work in ten 4 Page Booklets.

1.   (12marks) (Begin a 4 page booklet.)   **GC**

(a)   Calculate $\dfrac{432 \cdot 5}{18 \cdot 9 \times 4 \cdot 6}$ correct to two decimal places.   **2**

(b)   Factorise $3x^2 - x - 10$ **2**

(c)   Solve for x: $3 - \dfrac{2x}{3} < 4$ **2**

(d)   If $(2\sqrt{3} - 1)(\sqrt{3} + 2) = a + \sqrt{b}$ find the values of a and b.   **2**

(e)   Solve and graph on a number line the values of x for which
        $|x - 2| \le 8$.   **2**

(f)   Express $\dfrac{3^{-1} a^2}{2a^{-3}}$ with positive indices in its simplest form.   **2**

2. (12marks) (Begin a 4 page booklet.)   GC

The points A (-3,2) and B (5,8) lie on a number plane.

  (a)   Find the equation of AB in general form.   **2**

  (b)   Find the midpoint of AB.   **1**

  (c)   Find the length of AB.   **1**

  (d)   Show that the equation of the circle with diameter AB is

$$x^2 + y^2 - 2x - 10y + 1 = 0$$ **2**

  (e)   Prove that the point C (1,10) lies on this circle.   **1**

  (f)   Prove that $AC \perp CB$ **1**

  (g)   Prove that the lines $12x - 5y + 3 = 0$ and $24x - 10y - 7 = 0$ are parallel.   **1**

  (h)   The point A(1, k) lies on the line $12x - 5y + 3 = 0$. Find the value of k.   **1**

  (i)   Hence find the perpendicular distance between the lines $12x - 5y + 3 = 0$ and $24x - 10y - 7 = 0$.   **2**

3.   (12marks) (Begin a 4 page booklet.)   GC

  (a)   Differentiate the following with respect to x.
Leave your answer in its simplest form.

    (i) $(7x^2 - 2)^3$ **2**

    (ii) $\dfrac{3x}{2x + 5}$ **2**

(b)   The tangent to the curve $y = 3x^3 - 8x^2$ at the point of contact,
      P (2,-8), cuts the x-axis at A, and the normal to the curve at the
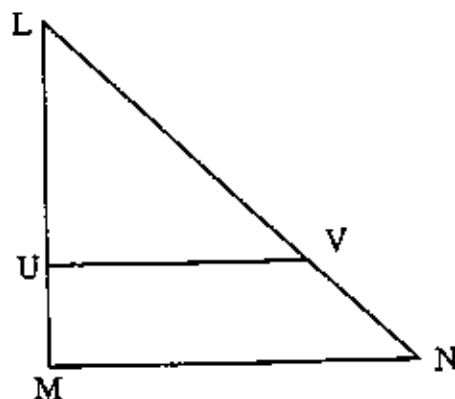   same point of contact cuts the y-axis at B.

    (i)   Find the equation of the tangent at P.   2

    (ii)   Find the equation of the normal at P.   2

    (iii)   Find the coordinates of A and B.   2

(c)   Find $\lim\limits_{x \to 1} \dfrac{3x^2 + x - 4}{x - 1}$ 2

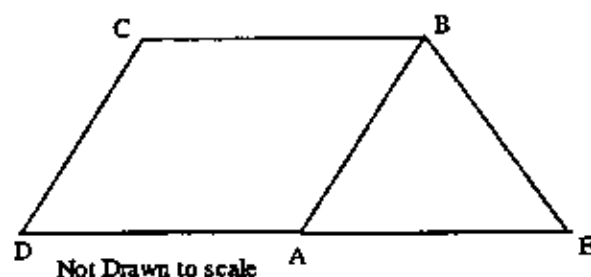4.   (12marks) (Begin a 4 page booklet.)   CGH

    (a)

    (i)   Draw this diagram in your book.   1

    (ii)   Consider triangles LMN and LUV where UV is parallel to MN
    and UM=3m, MN=7m and UV=5m. LM ⊥ MN.

        (A)   Find the length of LU   2

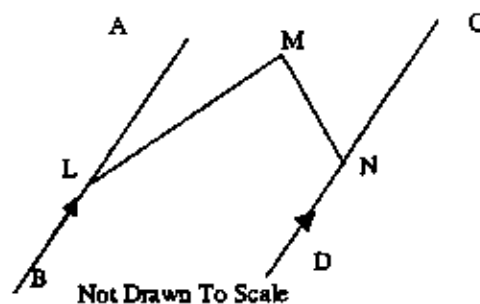   (B)   Find the length of VN   1

(b)

Not Drawn to scale

ABCD is a rhombus with $\angle BCD = 120^0$ while ABE is an equilateral triangle.

    (i)   In your examination booklet draw a neat sketch showing this information.

    (ii)   Find the size of $\angle EBC$ giving reasons for your answer.

    (iii)   Find the size of $\angle ECB$ giving reasons for your answer.   4

(c)

Not Drawn To Scale

    (i)   In your examination booklet draw a neat sketch showing this information.

    (ii)   If $\angle MLB = 130^0$ and $\angle MND = 110^0$ find, giving reasons for your answers:

        (A) $\angle LMN$

        (B) $\angle MNC$ 4

5. (12marks) (Begin a 4 page booklet.)

(a) Sketch the graph of $y = 3\sin 2x$ *for* $|x| \le \pi$ and find the area between the x- axis and the curve $y = 3\sin 2x$ *for* $|x| \le \pi$.   **4**

(b) ABCD is a square of side x cm, with P a point within the square such that PC=6cm, PB=2cm and AP= $2\sqrt{5}$ cm. Let $\angle PBC = \alpha$.
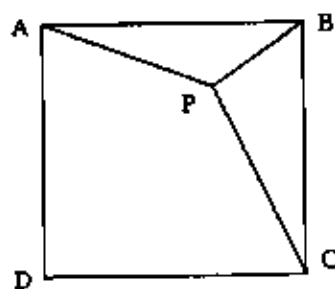
Not to Scale

(i) Using the cosine rule in the triangle PBC show that
$$\cos\alpha = \frac{x^2 - 32}{4x}.$$ **3**

(ii) By considering triangle PBA, show that $\sin\alpha = \dfrac{x^2 - 16}{4x}$.   **2**

(iii) Hence or otherwise show that the value of x is a solution of
$$x^4 - 56x^2 + 640 = 0$$ **1**

(iv) Find x. Give reasons for your answer.   **2**

6. (12marks) (Begin a 4 page booklet.) **CGH**

(a) If $y = (x-3)^3$ find all stationary points and determine their nature. **3**

(b) If $y = x^3 - 9x$:

    (i) Find any x intercepts. **1**

    (ii) Find both $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. **1**

    (iii) Determine whether $y = x^3 - 9x$ is an odd or an even function. **1**

    (iv) Find any maximum or minimum turning points. **3**

    (v) Find any points of inflexion. **2**

    (vi) Graph $y = x^3 - 9x$ **1**

7. (12marks) (Begin a 4 page booklet.) **HRK**

(a) To calculate the area of the region bounded by the curve $y = x^2 - 2x$ and the x axis and between the lines $x = 0$ and $x = 4$, Ernie used $\int_0^4 (x^2 - 2x)\, dx$.

    (i) Explain why Ernie's method of calculating this area is incorrect. **1**

    (ii) Find the area of the required region. **3**

(b) Use Simpson's Rule with 3 function values to approximate the area enclosed between the curve $y = \dfrac{1}{(x+1)^2}$ and the lines $x = 0$ and $x = 4$ correct to 2 significant figures. **4**

(c) Find the volume of the solid formed when the area bounded by the curve $y = 5 - x^2$ for $x \ge 0$, the y axis and the line $y = 1$ is rotated about the y axis. **4**

8. (12marks) (Begin a 4 page booklet.)

(a) For the equation $x^2 + (k+6)x - 2k = 0$ find the:

    (i)   discriminant in terms of k;   2

    (ii) values of k for which this equation has real roots.   2

(b) Given the equation $3x^2 + 4x - 3 = 0$ has roots α and β, evaluate the following without finding α or β:

    (i) $\alpha + \beta$ 1
    (ii) $\alpha\beta$ 1
    (iii) $2\alpha^2 + 2\beta^2$ 2

(c) A parabola has equation $x^2 + 6x - 33 = 12y$.

    (i)   Find the coordinates of its vertex.   1
    (ii) Find the focal length.   1
    (iii) Find the equation of its directrix.   1
    (iv) Show that the line $x - 2y + 7 = 0$ is not a focal chord of this parabola.   1

9. (12marks) (Begin a 4 page booklet.)

(a) Differentiate the following with respect to x (do not simplify your answer):

    (i) $x^3 e^{5x-1}$ 2

    (ii) $\log_e \left[\dfrac{3x^3 - 4}{6x^3 - 5} \right]$ 2

(b) Evaluate the following integral:

$$\int_0^1 \frac{24x^2 - 14}{4x^3 - 7x - 5}\, dx \qquad 4$$

(c) Find the exact area bounded by the curve $y = 2\ln x$, the x-axis and line $x = 2$.   4

10.   (12marks) (Begin a 4 page booklet.)   **SKB**

(a) If x, y and 9 are the first three terms of a geometric series and
     y, x and 2 are the first three terms of an arithmetic sequence, find
     the values of x and y.   **4**

(b) An investor wants to borrow $1 000 000 to purchase a block of units at
     Penrith from financial institution X which offers an interest rate of 6% p.a.
     reducible. The investor is to repay the loan in equal monthly instalments
     M, over 10 years.

   (I)   If A_n is the amount owing after n instalments develop expressions
          for A_1, A_2, A_3 and A_n .Hence show that the monthly instalment, M is

          given by : $M = \dfrac{5000(1.005)^{120}}{1.005^{120} - 1}$ **3**

   (ii)   Calculate the value of the monthly instalment, M to the nearest
          cent.   **1**

   (iii) Determine the amount still owing to institution X after 5 years, to
          the nearest cent.   **2**

   (iv)   If the investor borrows the same amount of money over 10 years
          from F.B.Knightly Investments Ltd. which offers simple interest at
          4.5% p.a., would the investor have been worse off? Explain your
          answer.   **2**