Total marks (84)
Attempt questions 1 – 7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) **Marks**

(a) Find: $\frac{d}{dx}\tan^{-1}(2x)$ **1**

(b) A and B are the points $(-5, 12)$ and $(4, 9)$ respectively. P is the point which divides AB internally in the ratio 3 : 2. Find the coordinates of P. **2**

(c) Evaluate: $\lim_{x \to 0}\left(\frac{\tan 3x}{2x}\right)$ **1**

(d) Find the acute angle, in degrees correct to one decimal place, between the two curves $y = x^2$ and $y = x$ at the point of intersection $(1, 1)$. **2**

(e) Find: $\int \sin^2 3x \ dx$ **2**

(f) Using the substitution $u = x - 1$, evaluate $\int_2^5 \frac{x}{\sqrt{x-1}}\, dx$. **4**

Question 2 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Let α, β and γ be the roots of the equation $x^3 - 5x^2 - 2x - 8 = 0$.

Without finding the actual roots, evaluate:

(i) $\alpha + \beta + \gamma$ **1**

(ii) $\alpha\beta + \alpha\gamma + \beta\gamma$ **1**

(iii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

(b) Let $f(x) = \ln x - \sin x$. It is known that the real root of $f(x) = 0$ lies between $x = 2$ and $x = 2.5$. **3**

Using one application of the *'halving the interval'* method, determine whether the root of $f(x) = 0$ is closer to $x = 2$ or $x = 2.5$.

(c) Find the volume of the solid generated when the area under the curve $y = \dfrac{1}{\left(1 - 9x^2\right)^{\frac{1}{4}}}$, above the x-axis and between $x = 0$ and $x = \dfrac{1}{3\sqrt{2}}$, is rotated about the x-axis. **3**

(d) (i) Write down the value of the constant k in the equation $5^x = e^{kx}$, $x \neq 0$. **1**

(ii) Hence or otherwise, find $\dfrac{d}{dx}\left(5^x\right)$. **1**

Question 3 (12 marks) Use a SEPARATE writing booklet **Marks**

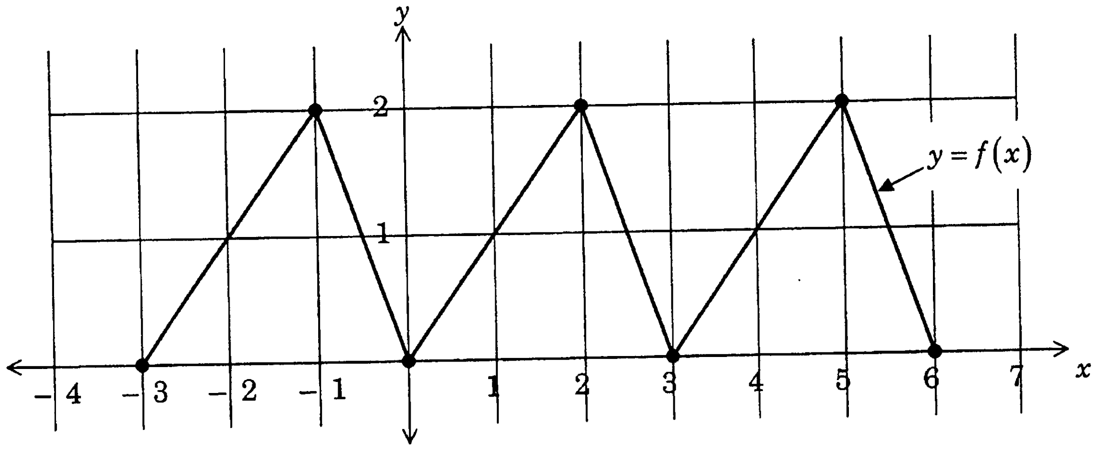
(a) Solve $2\sin x + \cos x = -1$, for $0 \le x \le 2\pi$, by first using the substitution, $t = \tan\frac{x}{2}$. **3**

(b) Prove by mathematical induction that if n is a positive integer, then: **3**

$$\frac{1}{1\times5}+\frac{1}{5\times9}+\frac{1}{9\times13}\cdots\cdots+\frac{1}{(4n-3)(4n+1)}=\frac{n}{4n+1}$$

(c) Without using a calculator, find the exact value of $\sin\left(\cos^{-1}\frac{2}{3}+\sin^{-1}\frac{1}{4}\right)$. **3**

(d)



The diagram above shows the graph of a periodic function $y = f(x)$ over the interval $-3 \le x \le 6$.

(i) State the period of $y = f(x)$. **1**

(ii) Assuming that the period of the function $y = f(x)$ continues to have the same form over the interval $-30 \le x \le 60$, calculate $f(52)$. **1**

(iii) Find $f'(x)$, when $x = 26\frac{1}{2}$. **1**

Question 4 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Consider the function defined by $f(x) = x\left(\sqrt[3]{x^2 - 4}\right)$, where x is any real number, $f'(x) = \dfrac{5x^2 - 12}{3\left(x^2 - 4\right)^{\frac{2}{3}}}$ and $\left(2\sqrt{\dfrac{3}{5}}, -\dfrac{4\sqrt{3}}{5^{\frac{5}{6}}}\right)$ is one of the two stationary points on $y = f(x)$. ***You do not need to verify these facts.***

(i) Show that $f(x)$ is an odd function. **1**

(ii) Write down the coordinates of the second stationary point. **1**

(iii) Explain why there is a vertical tangent at $x = 2$. **1**

(iv) Sketch the graph of $y = f(x)$ and label the axes appropriately. **2**

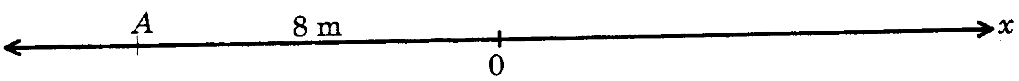
(b) The function f is given by $f(x) = \cos^{-1}\left(\dfrac{x}{3}\right)$.

(i) Find $f^{-1}(x)$. **1**

(ii) Write down the domain and range of $f^{-1}(x)$. **2**

(iii) Sketch the graph of $y = f^{-1}(x)$ and label the axes appropriately. **1**

(c)



A particle is moving in simple harmonic motion about the point O. The point A, as shown in the diagram, is 8 metres from O. When the particle passes through the point A its speed is $3\ \text{ms}^{-1}$. The amplitude of the motion is 10 m.

(i) Calculate the period of the motion. **2**

(ii) If x is the displacement of the particle from O, find the values of x for which the speed is zero. **1**

Question 5 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) The acceleration of a particle is given by $\ddot{x} = 4(1+x)$, where x is the particle's displacement from the origin. The particle is initially at the origin with a velocity of $2\,\text{m/s}$. Let $v = \dfrac{dx}{dt}$.

(i) Prove that $\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = \dfrac{d^2x}{dt^2}$. **2**

(ii) Find an expression for v in terms of x. **2**

(i) Show that $x = e^{2t} - 1$. Note that when $t \geq 0$, $v > 0$. **2**

(b) One hundred grams of cane sugar in water are being converted into dextrose at a rate which is proportional to the amount unconverted at any time t, that is, if M grams are converted in t minutes, then,

$$\frac{dM}{dt} = k(100 - M), \text{ where } k \text{ is a constant}$$

(i) Verify that $M = 100 + Ae^{-kt}$, where A is a constant, satisfies the given differential equation. **2**

(ii) If 40 grams are converted in the first 10 minutes, find A and k. **2**

(iii) How many grams are converted in the first 45 minutes, correct to the nearest whole gram? **2**

Question 6 commences on the next page

Question 6 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Solve: $\dfrac{x}{x-1} \geq 5$ **3**

(b)

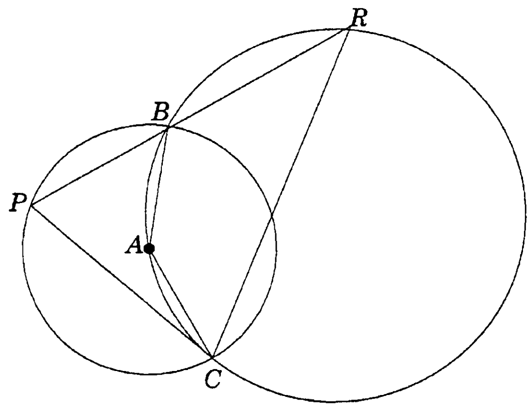


Diagram is not to scale

A is the centre of the circle BCP. The point A lies on another circle BAC. The two circles intersect in B and C as shown in the diagram. PBR is a straight line.

Copy or trace this diagram into your writing booklet.

Prove, with reasons, that $RP = RC$. **3**

(c) Two tangents from the external point $T(x_0, y_0)$ touch the parabola $x^2 = 4ay$ at $P(x_1, y_1)$ and $Q(x_2, y_2)$ respectively.

(i) Write down the Cartesian equation of the chord of contact in terms of x_0 and y_0. **1**

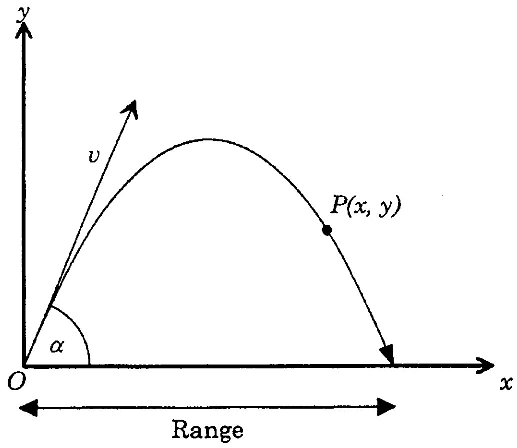
(ii) Show that the x values of the coordinates of P and Q are given by the roots of the equation $x^2 - 2x_0x + 4ay_0 = 0$. **2**

(iii) Show that the midpoint M of QP is given by $\left(x_0, \dfrac{x_0^2}{2a} - y_0\right)$. **2**

(iv) Find the Cartesian equation of the locus of M. **1**

Question 7 (12 marks) Use a SEPARATE writing booklet **Marks**

(a)



A projectile is fired from level ground with an initial velocity, v metres per second, at an angle α to the horizontal. The origin, O, is taken to be at the point of projection on level ground.

(i) Starting with $\ddot{x} = 0$, $\ddot{y} = -g$ and integrating, derive the parametric equations for the position of the projectile $P(x, y)$, after t seconds. Ignore air resistance and assume the acceleration due to gravity is g m/s^2. **3**

(ii) Prove that the horizontal range of the projectile from the point of projection, in metres, is given by $x = \dfrac{v^2 \sin 2\alpha}{g}$. **2**

(iii) A golf ball is driven with a velocity of 50 m/s at an angle α to the horizontal towards the hole on the green 250 metres away on the same horizontal plane as the point of projection. **2**

At what angle should the golf ball be projected in order to achieve a 'hole-in-one', that is without bouncing *or* rolling first? Take $g = 9.8$ m/s^2 and ignore air resistance.

Question 7 continues on the next page

Question 7 continued:

(b)

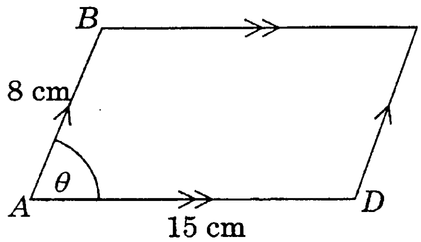


Diagram is not to scale

A parallelogram $ABCD$ has initially sides of length 8 cm and 15 cm. **3**

The angle θ at one of the vertices is decreasing at the rate of $\frac{\pi}{60}$ radians per minute.

Calculate the rate at which the area of the parallelogram is changing when $\theta = \frac{\pi}{6}$. Assume that as θ decreases, $ABCD$ remains a parallelogram.

(c) Gambler buys three tickets in a lottery for which sixty tickets are sold in all. There will be five prizes awarded. Tickets drawn will not be replaced. **2**

Find the probability that Gambler wins at least one prize.

End of Paper

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Q5 ctd.

(ii) when $t=0$, $M=0$

$\therefore M = 100 + Ae^{-kt}$

$\Rightarrow 0 = 100 + Ae^{0}$

$\therefore A = -100$. ✓

$\therefore M = 100 - 100e^{-kt}$

when $t=10$, $M=40$

$\therefore 40 = 100 - 100e^{-10k}$

$\therefore \dfrac{-60}{-100} = e^{-10k}$

ie $e^{10k} = \dfrac{5}{3}$

$\therefore 10k = \ln(5/3)$

ie $k = \frac{1}{10}\ln(5/3)$. ✓

(iii) $M = ?$; $t = 45$

$M = 100 - 100e^{-\frac{1}{10}\ln(5/3)(45)}$

$= 100 - 100e^{\ln(5/3 \times 4.5)}$ ✓

$= 100 - 100 \times (5/3)^{-4.5}$

$= 100 - 100 \times (3/5)^{4.5}$

$\doteqdot 90$ grams. ✓

QUESTION 6: (12 MARKS)

(a) $\dfrac{x}{x-1} \geq 5$

$\left(\dfrac{x}{x-1}\right)(x-1)^2 \geq 5(x-1)^2$

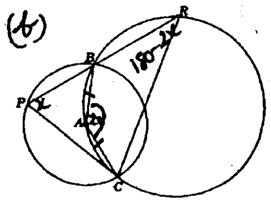
$x(x-1) - 5(x-1)^2 \geq 0$

$(x-1)[x - 5(x-1)] \geq 0$

$(x-1)(5-4x) \geq 0$

$\therefore \{x : 1 < x \leq \frac{5}{4}\}$

(b)



$AB = AC$ (equal radii)

$\angle BAC = 2x^\circ$

$\therefore \angle BPC = x^\circ$

(angle at the centre of a circle ~~on the~~ is twice the angle angle st. on the same arc). ✓

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Q6 ctd.

$\angle BRC = 180 - 2x$

opp. $\angle$s of a cyclic quad. ABRC are supplementary.

$\therefore \angle PCR = 180 - (180 - 2x + x)$

$= 180 - (180 - x)$

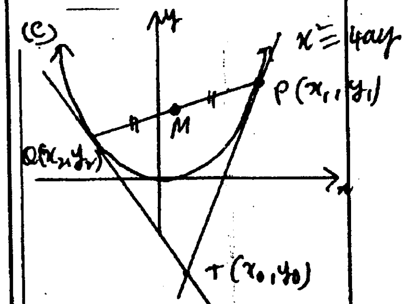
$= x$ ✓

$\therefore \triangle PRC$ is isosceles

$\therefore RP = RC$

(sides opp equal $\angle$'s in a triangle are equal) ✓ reason

(c)



(i) Chord PQ:

$xx_0 = 2a(y + y_0)$. ✓

(ii) Solving $x^2 = 4ay$ ①

$xx_0 = 2a(y + y_0)$ ②

✓ attempt to solve and lead to result.

in ① $y = \frac{x^2}{4a}$

$\therefore$ in ② $xx_0 = 2a\left(\frac{x^2}{4a} + y_0\right)$

ie $xx_0 = \frac{x^2}{2} + 2ay_0$

ie $2xx_0 = x^2 + 4ay_0$

ie $x^2 - 2xx_0 + 4ay_0 = 0$ ③

(iii) Solving ③

$x = \dfrac{2x_0 \pm \sqrt{(-2x_0)^2 - 4(1)(4ay_0)}}{2}$

$= x_0 \pm \sqrt{x_0^2 - 4ay_0}$ ✓

Let $x_1 = x_0 + \sqrt{x_0^2 - 4ay_0}$

$x_2 = x_0 - \sqrt{x_0^2 - 4ay_0}$

$\therefore \dfrac{x_1 + x_2}{2} = \dfrac{2x_0}{2}$

$= x_0$ ④ ✓ or

Sub ④ into $xx_0 = 2a(y + y_0)$

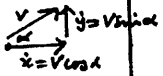
$\therefore \dfrac{x_0^2 - 2ay_0}{2a} = y$ ✓

$\therefore y = \dfrac{x_0^2}{2a} - y_0$

$\therefore M = \left(x_0, \dfrac{x_0^2}{2a} - y_0\right)$ ⑤

(iv) $x = x_0$ & $y = \frac{x_0^2}{2a} - y_0$ from ⑤ ✓

$\therefore y = \frac{x^2}{2a} - y_0$ ie $\boxed{2a(y + y_0) = x^2}$

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
QUESTION 7: (12 MARKS)		$\therefore gt^2 - 2vt\sin\alpha = 0$	
(a)(i) $\ddot{x} = 0$ 		$\therefore t[gt - 2v\sin\alpha] = 0$	
$\therefore \dot{x} = c_1$		$\therefore$ either $t = 0$ or	
when $t=0$, $\dot{x} = V\cos\alpha = c_1$		$t = \frac{2v\sin\alpha}{g}$ — ③	✓
$\therefore \dot{x} = V\cos\alpha$		sub ③ into ①	
$\therefore x = Vt\cos\alpha + c_2$		$x = v\left[\frac{2v\sin\alpha}{g}\right](\cos\alpha)$	
when $t=0$, $x=0$		$= \frac{2v^2\sin\alpha\cos\alpha}{g}$	✓
$\therefore c_2 = 0$			
$\therefore x = Vt\cos\alpha$ ①	✓	$x = \frac{v^2\sin 2\alpha}{g}$ as required. ④	
$\ddot{y} = -g$			
$\therefore \dot{y} = -gt + c_3$		(iii) From ④, $g=9.8$, $x=250$, $v=50$.	
when $t=0$, $\dot{y} = V\sin\alpha$		$250 = \frac{50^2}{9.8}\sin 2\alpha$	
$\therefore \dot{y} = -gt + V\sin\alpha$	✓	$\therefore \sin 2\alpha = 0.98$	✓
$y = -\frac{gt^2}{2} + Vt\sin\alpha + c_4$		$\therefore 2\alpha \doteqdot 78.52°\ldots$	
when $t=0$, $y=0$ $\therefore c_4 = 0$		$\alpha \doteqdot 39.26°\ldots$	
$\therefore y = -\frac{gt^2}{2} + Vt\sin\alpha$ ②	✓	or $2\alpha = \sin^{-1} 0.98$	
$\therefore P(x, y)$		$\therefore \alpha = \frac{1}{2}\sin^{-1}(0.98)$	✓ or equivalent
$= \left[Vt\cos\alpha, Vt\sin\alpha - \frac{gt^2}{2}\right]$		(in exact form)	
(ii) when $y = 0$, the particle has reached the ground or is on the ground to start with. Solving for $y=0$ in ②			
$\frac{gt^2}{2} - Vt\sin\alpha = 0$			

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Q7 ctd:		(c) P(win at least one prize)	
(b) Area of a parallelogram.		$= 1 - P(\text{win no prize})$	
$A = ab\sin\theta$		$= 1 - \left[\frac{57}{60} \times \frac{56}{59} \times \frac{55}{58} \times \frac{54}{57} \times \frac{53}{56}\right]$	✓
where a, b are the adjacent sides including θ.		$= \frac{1597}{6844}$ $(\doteqdot 0.2\dot{3})$	✓
$\therefore A = 8 \times 15 \sin\theta$	✓		
$A = 120\sin\theta$			
Given $\frac{d\theta}{dt} = -\pi/60$ rads/min		or $1 - \left(\frac{55}{60} \times \frac{54}{59} \times \frac{53}{58}\right)$	
We want: $\frac{dA}{dt}$		or $= 1 - \frac{^{55}C_3}{^{60}C_3}$	
$\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt}$	✓	or $= 1 - \frac{^{57}C_3}{^{60}C_5}$	
$= 120\cos\theta \times \left(-\frac{\pi}{60}\right)$		etc....	
$\frac{d}{dt}(A)$ when $\theta = \pi/6$			
$= 120 \times \cos\frac{\pi}{6} \times \left(-\frac{\pi}{60}\right)$			
$= 120 \times \frac{\sqrt{3}}{2} \times -\frac{\pi}{60}$	✓		
$= -\pi\sqrt{3}$ cm²/min			
$\therefore$ The area is decreasing at the rate of $\pi\sqrt{3}$ cm² per minute.			

MARKERS: Q1: DS Q2: EH Q3: IM Q4: RD Q5: AJ Q6: DS Q7: AJ

Suggested Solution (s) | **Comments**

QUESTION 1: (12 MARKS)

(a) $\frac{d}{dx}\tan^{-1}2x = \frac{2}{1+4x^2}$

(b) $k:l = 3:2$

$A(-5,12)$ $B(4,9)$, with $A = (x_1, y_1)$, $B = (x_2, y_2)$

$\therefore P(x,y) = \left[\frac{kx_2+lx_1}{k+l}, \frac{ky_2+ly_1}{k+l}\right]$

$= \left[\frac{3\times4+2\times-5}{3+2}, \frac{3\times9+2\times12}{3+2}\right]$

$= \left[\frac{2}{5}, \frac{51}{5}\right]$

(c) $\lim_{x\to0}\left(\frac{\tan 3x}{2x}\right)$

$= \lim_{x\to0}\left(\frac{\tan 3x}{3x}\right)\times\left(\frac{3x}{2x}\right)$

$= \frac{3}{2}\times 1$

$= 3/2$

(d) $y_1 = x^2 \therefore y_1' = 2x$

$y_2 = x \therefore y_2' = 1$

when $x = 1$, $\begin{cases} y_1' = 2 \\ y_2' = 1 \end{cases}$

Let θ = acute angle

$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\tan\theta = \left|\frac{2-1}{1+2}\right|$

$\tan\theta = \frac{1}{3}$

$\therefore \theta = 18.4°$

(e) $\int \sin^2 3x\, dx$.

N.B. $\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\therefore \sin^2 3x = \frac{1}{2}(1-\cos 6x)$

$\therefore \int \sin^2 3x\, dx$

$= \frac{1}{2}\int 1 - \cos 6x\, dx$

$= \frac{1}{2}\left[x - \frac{1}{6}\sin 6x\right] + c$

(f) Let $u = x-1$ ① NB: $\frac{dx}{du} = 1$

when $x = 2$, $u = 1$

$x = 5$, $u = 4$.

Also $x = u+1$ from ①

$\therefore \int_2^5 \frac{x}{\sqrt{x-1}}\, dx$

$= \int_2^5 \frac{x}{\sqrt{x-1}}\frac{dx}{du}\, du$

$= \int_1^4 \frac{u+1}{\sqrt{u}}\, du$

$= \int_1^4 u^{1/2} + u^{-1/2}\, du$

$= \left[\frac{2u\sqrt{u}}{3} + 2\sqrt{u}\right]_1^4$

$= \left[\frac{16}{3} + 4 - \frac{2}{3} - 2\right]$

$= \frac{20}{3}$

Suggested Solution (s) | **Comments**

QUESTION 2: (12 MARKS)

(a)(i) $\alpha+\beta+\gamma = 5$

(ii) $\alpha\beta+\alpha\gamma+\beta\gamma = -2$

(iii) $\alpha^2+\beta^2+\gamma^2$

$= (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\gamma\beta)$

$= 5^2 - 2\times(-2)$

$= 25 + 4$

$= 29$

Comment: ✓ only if includes $\Sigma\alpha\beta$.

(b) Let $f(x) = \ln x - \sin x$

$f(2) = \ln 2 - \sin 2$
$= -0.216\ldots$
< 0

$f(2.5) = \ln 2.5 - \sin 2.5$
$= 0.3178\ldots$
> 0

Since $|-0.216\ldots| < |0.3178\ldots|$ then we conclude the root lies closer to $x = 2$.

or Consider $f\left(\frac{2+2.5}{2}\right) = f(2.25)$
$= 0.0328$
> 0

$\therefore$ The desired interval is $\{x: 2 < x < 2.25\}$ hence, the root lies closer to $x = 2$.

Comments: In radians. ✓ $f(2.25)$ & conclusion

(c) $V = \pi\int_0^{\frac{1}{3\sqrt{2}}}\left[\frac{1}{\sqrt[4]{1-9x^2}}\right]^2 dx$

$V = \pi\int_0^{\frac{1}{3\sqrt{2}}}\frac{1}{\sqrt{1-9x^2}}\, dx$

Comment: ✓ only if correct formula applied.

$\therefore V = \pi\int_0^{\frac{1}{3\sqrt{2}}}\frac{1}{\sqrt{9(\frac{1}{9}-x^2)}}\, dx$

$= \frac{\pi}{3}\int_0^{\frac{1}{3\sqrt{2}}}\frac{1}{\sqrt{(\frac{1}{3})^2-x^2}}\, dx$

$= \frac{\pi}{3}\left[\sin^{-1}3x\right]_0^{\frac{1}{3\sqrt{2}}}$

$= \frac{\pi}{3}\left[\sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1}0\right]$

$= \frac{\pi}{3}\times\frac{\pi}{4}$

$V = \frac{\pi^2}{12}$ cubic units.

(d)(i) If $5^x = e^{kx}$ then $k = \log_e 5$.

(ii) $\frac{d}{dx}(5^x)$

$= \frac{d}{dx}e^{(\ln 5)x}$

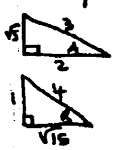
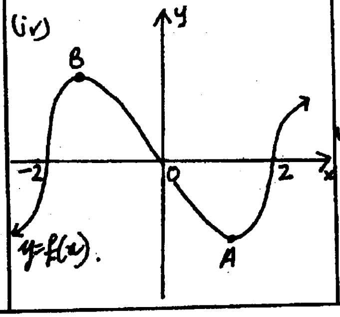
$= \ln 5\, e^{(\ln 5x)}$

$= \ln 5.(5^x)$.

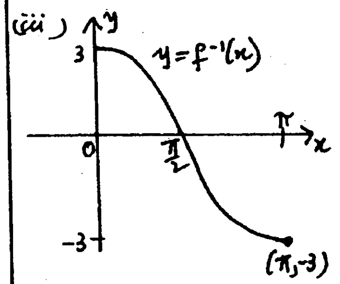
(or $\ln 5 \cdot e^{x\ln 5}$ or $\ln 5 \cdot e^{(\ln 5)x}$)

Comment: will not accept $\frac{\ln 5^x}{x}$ but $\ln 5$ is the OK.

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
QUESTION 3: (12 MARKS) (a) let $t = \tan\frac{x}{2}$ $\therefore 2\sin x + \cos x = -1$ $2\left[\frac{2t}{1+t^2}\right] + \frac{1-t^2}{1+t^2} = -1$ ✓ ie $4t = -2$ (N.B. t^2 term "cancels") ie $t = -\frac{1}{2}$ $\therefore \tan\frac{x}{2} = -\frac{1}{2}$ $\therefore \frac{x}{2} = n\pi + \tan^{-1}(-\frac{1}{2})$ ie $x = 2n\pi - 2\tan^{-1}(\frac{1}{2})$ general solution & $x = (2n+1)\pi$ where n is an integer. *We only want $x \in [0, 2\pi]$* $\therefore x = \pi$ or let $n = 1$ in $x = 2n\pi - 2\tan^{-1}\frac{1}{2}$ $\therefore x = 2\pi - 2\tan^{-1}\frac{1}{2}$ ($\doteqdot 5.4$ radians to 2d.p) These are the only two required solutions. (b) Let $S(n)$ be the statement: $\sum_{k=1}^{n} \frac{1}{(4k-3)(4k+1)} = \frac{n}{4n+1}$ where n is a positive integer.	substitution ✓✓ ✓ ✓ ✓ = all stages of induction completed correctly.	Proving $S(1)$ is true: $LHS = \frac{1}{1 \times 5}$ $= \frac{1}{5}$ $RHS = \frac{1}{4 \times 1 + 1}$ $= \frac{1}{5}$ $= LHS$ $\therefore S(1)$ is true. Assume $S(k)$ is true where $1 \le k \le n$ & k, n are positive integers. Proving $S(k+1)$ is true: We assume: $S(k) = \frac{k}{4k+1}$ RTP: $S(k) + \sum_{k=k+1}^{k+1} \frac{1}{(4k-3)(4k+1)}$ $= \frac{k+1}{4k+5}$ $LHS = \frac{k}{4k+1} + \frac{1}{(4k+1)(4k+5)}$ $= \frac{4k^2+5k+1}{(4k+1)(4k+5)}$ $= \frac{(4k+1)(k+1)}{(4k+1)(4k+5)}$ $= \frac{k+1}{4k+5}$ ✓ $= RHS$ $\therefore S(n)$ is true for $n = 1$. Whenever, $S(k)$ is true the statement $S(n)$ is also true for all positive integer values of $n \geq 1$.	(from assumption) ✓ or equivalent correct simplification

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Q3 ctd: (c) let $A = \cos^{-1}\frac{2}{3}$ $B = \sin^{-1}\frac{1}{4}$ $\therefore \cos A = \frac{2}{3}$ $\sin B = \frac{1}{4}$  $\therefore \sin(A+B)$ $= \sin A \cos B + \sin B \cos A$ $= \frac{\sqrt{5}}{3} \times \frac{\sqrt{15}}{4} + \frac{1}{4} \times \frac{2}{3}$ $= \frac{5\sqrt{3}}{12} + \frac{2}{12}$ $= \frac{5\sqrt{3}}{12} + \frac{1}{6}$. (d) (i) PERIOD = 3 (ii) N.B: $f(x) = \begin{cases} x & 0 \le x \le 2 \\ 6-2x & 2 \le x \le 3 \end{cases}$ $\therefore f(52) = f(1)$ $= 1$ (iii) $f'(26\frac{1}{2}) = f'(2\frac{1}{2})$ $= -2$.	✓ or implied ✓ ✓ ✓ ✓	**QUESTION 4: (12 MARKS)** (a)(i) $f(x) = x\left[\sqrt[3]{x^2-4}\right]$ $f(-x) = (-x)\left[\sqrt[3]{(-x)^2-4}\right]$ $= -x\left[\sqrt[3]{x^2-4}\right]$ $= -f(x)$ $\therefore f(x)$ is an odd f^n in x about $x = 0$. (ii) Since $f(x)$ is an odd f^n about $(0,0)$ and one of the SP's is at $A\left[2\sqrt{\frac{3}{5}}, -\frac{4\sqrt{3}}{5^{5/6}}\right]$ then the other S.P is at: $B\left[-2\sqrt{\frac{3}{5}}, \frac{4\sqrt{3}}{5^{5/6}}\right]$. (iii) Among the reasons, one could include: • $f'(x)$ is undefined when $x = 2$ • the slope of the curve is "vertical" (iv)  	✓ ✓ must be stated. ✓ ✓ intercepts ✓ vertical feature and correct shape.

Year 12- 2005 Trial HSC Mathematics EXTENSION 1 Assessment Task 4
Suggested Solutions and Marking Scheme

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Q4 ctd:			
(b) $f(x) = \cos^{-1}\left(\frac{x}{3}\right)$			
here: D: $\left\lvert\frac{x}{3}\right\rvert \le 1$			
i.e. $\lvert x\rvert \le 3$			
& R: $0 \le y \le \pi$.			
(i) we seek $f^{-1}(x)$ such that: $f(f^{-1}(x)) = x$			
i.e. $\cos^{-1}\left[\frac{f^{-1}(x)}{3}\right] = x$			
i.e. $\cos x = \frac{f^{-1}(x)}{3}$			
$\therefore f^{-1}(x) = 3\cos x$.	✓		
(ii) D: $0 \le x \le \pi$	✓	(c)(i) $v^2 = n^2(a^2 - x^2)$ let $v=0$, $a=10$, $x=-8$	
R: $\lvert y\rvert \le 3$	✓ or equivalent	$\therefore 9 = n^2(100-64)$	✓
(iii) 	✓ correct shape position, and intercepts	$\frac{9}{36} = n^2$	
		$\therefore n = \frac{3}{6} = \frac{1}{2}$ $(n>0)$	
		$\therefore T = \frac{2\pi}{n} = \frac{2\pi}{\frac{1}{2}}$ $= 4\pi$ seconds.	✓
		(ii) $v=0$, $a=10$, $n=\frac{1}{2}$	
		$\therefore 0 = \frac{1}{4}(100-x^2)$	
		$\therefore \lvert x\rvert = 10$ i.e. $x = \pm 10$ m. (both must be stated).	✓

Year 12- 2005 Trial HSC Mathematics EXTENSION 1 Assessment Task 4
Suggested Solutions and Marking Scheme

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
QUESTION 5: (12 MARKS)		i.e. $\frac{dx}{dt} = 2(x+1)$	
(a) $\ddot{x} = 4(1+x)$ when $t=0$, $x=0$, $\dot{x}=2$		i.e. $\frac{dt}{dx} = \frac{1}{2(x+1)}$	
(i) $LHS = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$		$\therefore t = \frac{1}{2}\ln(x+1) + C_2$	✓
$= \frac{1}{2} \times \frac{d}{dx}v^2$		when $x=0$, $t=0$	
$= \frac{1}{2} \times 2v\frac{dv}{dx}$	✓	$\therefore 0 = \frac{1}{2}\ln 1 + C_2$	
$= v\frac{dv}{dx}$		$\therefore C_2 = 0$	
$= \frac{dx}{dt} \times \frac{dv}{dx}$	✓	$\therefore t = \frac{1}{2}\ln(x+1)$	
$= \frac{dv}{dt}$		i.e. $e^{2t} = x+1$	✓
$= \ddot{x}$		$\therefore x = e^{2t} - 1$ as required.	
$= RHS$.		(b)(i) We know: $\frac{dM}{dt} = k(100-M)$.	
alternatively one could should show RHS=LHS.		Suppose $M = 100 + Ae^{-kt}$ ①	
(ii) Let $\frac{d}{dx}\frac{1}{2}v^2 = 4(1+x)$		then $\frac{dM}{dt} = 0 - Ake^{-kt}$ $= -k(Ae^{-kt})$	✓
$\therefore \int \frac{d}{dx}\frac{1}{2}v^2\,dx = 4\int 1+x\,dx$		from ① $M - 100 = Ae^{-kt}$.	
i.e. $\frac{1}{2}v^2 = \frac{4(1+x)^2}{2} + C_1$	✓	$\therefore \frac{dM}{dt} = -k(M-100)$	✓
when $x=0$, $v=2$		i.e. $\frac{dM}{dt} = k(100-M)$	
$\therefore 2 = \frac{4 \times 1}{2} + C_1$		or We know $\frac{dM}{dt} = k(100-M)$	
$\therefore C_1 = 0$		then $\frac{dM}{dt} = k[100 - (100 + Ae^{-kt})]$	
$\therefore v^2 = 4(x+1)^2$	✓	$= k[-Ae^{-kt}]$	
$\therefore v = \pm 2(x+1)$.		$= k(M-100)$ as expected.	
(iii) we are told $v>0$ $\therefore$ let $v = 2(x+1)$			