Independent Trial HSC 2009 Mathematics Extension 1 Marking Guidelines

Question 1

a. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• rearranges in terms of known trigonometric limit	1
• evaluates limit	1

Answer

$$\lim_{x\to 0}\frac{\sin 3x}{2x}=\tfrac{3}{2}\lim_{x\to 0}\frac{\sin 3x}{3x}=\tfrac{3}{2}\times 1=\tfrac{3}{2}$$

b. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• identifies a and r for the G.P	1
• applies formula for limiting sum	1

Answer

$\left(\frac{e}{e+1}\right)+\left(\frac{e}{e+1}\right)^2+\left(\frac{e}{e+1}\right)^3+\ldots$ is G.P. with $a=\frac{e}{e+1}$, and $r=\frac{e}{e+1}\Rightarrow 0<r<1$

$\therefore$ Limiting sum is $\frac{a}{1-r}=\frac{e}{e+1}\div\frac{1}{e+1}=e$

c. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
• expresses sum of reciprocals of roots in terms of sums of products	1
• evaluates using relationships between roots and coefficients	1

Answer

α, β and γ roots of $x^3+2x^2+3x+6=0$. $\quad\therefore\ \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\beta\gamma+\gamma\alpha+\alpha\beta}{\alpha\beta\gamma}=\frac{3}{-6}=-\frac{1}{2}$

d. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• substitutes values of gradients into formula for tangent of acute angle between the lines	1
• evaluates required angle	1

Answer

Acute angle θ between lines $y=2x$ and $x+y-3=0$ is given by $\tan\theta=\left|\frac{2-(-1)}{1+2\,.\,(-1)}\right|=3$

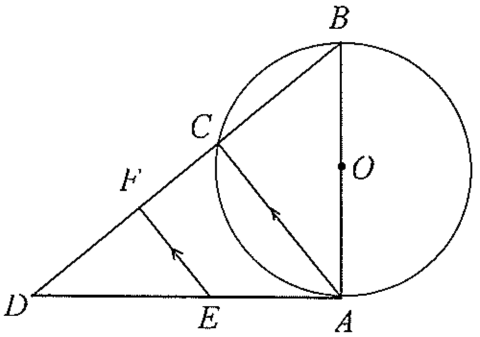
$\therefore\theta\approx 72^\circ$ (to the nearest degree)

e. Outcomes assessed : PE2, PE3

Marking Guidelines

Criteria	Marks
i • quotes alternate segment theorem	1
ii • gives a sequence of deductions resulting in a test for a cyclic quadrilateral	1
• justifies these deductions by quoting geometric properties and tests	1
iii • explains why BE subtends a right angle at A or at F	1

Answer



Let O be the centre of the circle.

i. The angle between the tangent at A and the chord AC is equal to the angle subtended by that chord in the alternate segment, hence $\angle EAC = \angle ABC$.

ii. $\angle EAC = \angle DEF$ (*Corresp. $\angle$'s with parallel lines AC, EF are equal*)

$\therefore \angle DEF = \angle ABC$ (*Both equal to $\angle EAC$*)

$\therefore EABF$ is cyclic (*Exterior $\angle$ equal to interior opp. $\angle$*)

iii. $\angle BAE = 90°$ (*Tangent to circle ABC at A is perpendicular to radius OA drawn to point of contact*)

$\therefore BE$ is a diameter of circle $EABF$. (*subtends right $\angle$ at circumference*)

Question 2

a. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• finds primitive	1
• evaluates in surd form	1

Answer

$$\int_0^{\frac{\pi}{8}} \sec 2x \tan 2x \, dx = \tfrac{1}{2}\left[\sec 2x\right]_0^{\frac{\pi}{8}} = \tfrac{1}{2}\left(\sqrt{2}-1\right)$$

b. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
• counts arrangements for one possible pattern of B's and G's	1
• adds number of arrangements for the second possible pattern of B's and G's	1

Answer

B G B G G B or B G G B G B $\therefore 2 \times 3! \times 3! = 72$ ways

c. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• finds x coordinate of P	1
• finds y coordinate of P	1

Answer

$$A(-2\,,3) \qquad B(6\,,-1)$$

$$3 \quad : \quad 2$$

$$P\left(\frac{3\times 6+2\times(-2)}{3+2}\,,\,\frac{3\times(-1)+2\times 3}{3+2}\right)$$

$\therefore$ P has coordinates $P\left(\frac{14}{5}\,,\frac{3}{5}\right)$

d. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• simplifies $1-\cos x$ in terms of t	1
• completes simplification of given expression in terms of t to establish required result	1

Answer

$t=\tan\frac{x}{2}$

$$\begin{aligned}1-\cos x &= 1-\frac{1-t^2}{1+t^2}\\ &= \frac{2t^2}{1+t^2}\\ \sin x &= \frac{2t}{1+t^2}\end{aligned}$$

$$\begin{aligned}\therefore \frac{\sin x}{1-\cos x} &= \frac{2t}{1+t^2}\times\frac{1+t^2}{2t^2}\\ &= \frac{1}{t}\\ &= \cot\frac{x}{2}\end{aligned}$$

e. Outcomes assessed : PE3, PE4

Marking Guidelines

Criteria	Marks
i • finds $\frac{dy}{dx}$ as a function of t	1
• finds equation of normal in required form	1
ii • finds coordinates of M	1
• finds equation of locus of M	1

Answer

i.

$$y=at^2 \Rightarrow \frac{dy}{dt}=2at$$

$$x=2at \Rightarrow \frac{dx}{dt}=2a$$

$$\therefore \frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}=t$$

$\therefore$ Normal at P has gradient $-\frac{1}{t}$ and equation

$$\begin{aligned}y-at^2 &= -\tfrac{1}{t}(x-2at)\\ ty-at^3 &= -x+2at\\ x\ +\ ty &= 2at+at^3\end{aligned}$$

ii. $N\left(0\,,2a+at^2\right)$ $\quad\therefore M\left(at\,,a+at^2\right)$

$P\left(2at\,,at^2\right)$

Locus of M has equation $y=a+a\left(\frac{x}{a}\right)^2$

$$x^2=a(y-a)$$

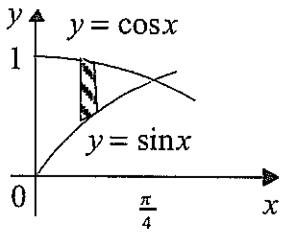
Question 3

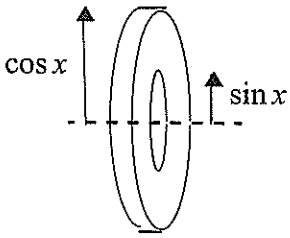
a. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• writes definite integral for the volume in terms of $\cos x$ and $\sin x$	1
• evaluates the integral.	1

Answer





$$V = \pi \int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x)\, dx$$
$$= \pi \int_0^{\frac{\pi}{4}} \cos 2x \, dx$$
$$= \tfrac{1}{2}\pi \left[\sin 2x \right]_0^{\frac{\pi}{4}}$$
$$= \tfrac{1}{2}\pi\,(1-0)$$

Volume is $\frac{\pi}{2}$ cubic units.

b. Outcomes assessed : HE2

Marking Guidelines

Criteria	Marks
• defines an appropriate sequence of statements $S(n)$ and shows the first member is true	1
• writes the LHS of $S(k+1)$ in terms of RHS of $S(k)$, conditional on truth of $S(k)$	1
• rearranges conditional expression for LHS of $S(k+1)$ to obtain RHS	1
• completes proof by Mathematical Induction	1

Answer

Let $S(n)$, $n = 2, 3, 4, \ldots$, be the sequence of statements defined by

$$S(n): \quad 2\times 1 + 3\times 2 + 4\times 3 + \ldots + n(n-1) = \frac{n(n^2-1)}{3}$$

Consider $S(2)$: $\quad LHS = 2\times 1 = 2$; $\quad RHS = \dfrac{2(2^2-1)}{3} = 2$. $\quad$ Hence $S(2)$ is true.

If $S(k)$ is true: $\quad 2\times 1 + 3\times 2 + 4\times 3 + \ldots + k(k-1) = \dfrac{k(k^2-1)}{3} \quad *$

Consider $S(k+1)$: $\quad LHS = \left\{ 2\times 1 + 3\times 2 + 4\times 3 + \ldots + k(k-1) \right\} + (k+1)k$

$$= \frac{k(k^2-1)}{3} + (k+1)k \qquad \text{if } S(k) \text{ is true, using } *.$$
$$= \frac{k(k+1)\left\{(k-1)+3\right\}}{3}$$
$$= \frac{(k+1)\left\{k^2+2k\right\}}{3}$$
$$= \frac{(k+1)\left\{(k+1)^2-1\right\}}{3}$$
$$= RHS$$

Hence if $S(k)$ is true then $S(k+1)$ is true. But $S(2)$ is true, and hence $S(3)$ is true and so on. Hence by Mathematical Induction, $S(n)$ is true for all positive integers $n \ge 2$.

c. Outcomes assessed : HE4

Marking Guidelines

	Criteria	Marks
i	• rearranges and interchanges x and y to obtain equation of inverse function	1
ii	• sketches graph of $y = f(x)$ showing endpoints and intercepts	1
	• sketches inverse function by reflection in $y = x$	1
	• shows endpoints and intercepts for inverse function	1
iii	• writes equation for x	1
	• solves for x in simplest exact form	1

Answer

i. $f(x) = (x+2)^2 - 9, \quad -2 \le x \le 2.$

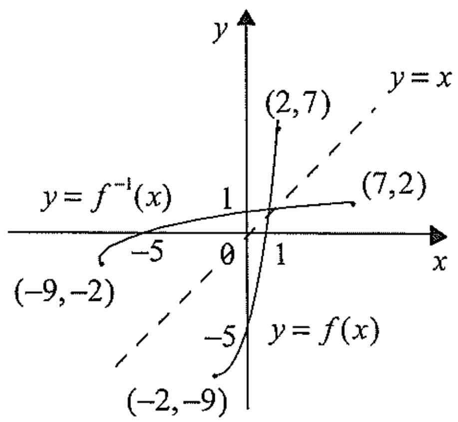
$(x+2)^2 = y+9 \qquad \text{and} \qquad 0 \le x+2 \le 4$

$x + 2 = +\sqrt{y+9}$

$\therefore \quad x = -2 + \sqrt{y+9}\,, \quad -9 \le y \le 7$

$\therefore x \leftrightarrow y \Rightarrow f^{-1}(x) = -2 + \sqrt{x+9}\,, \ -9 \le x \le 7$

ii. Graphs of inverse functions are reflections of each other in $y = x$



iii. Graphs intersect on the line $y = x$.

Hence $(x+2)^2 - 9 = x$

$x^2 + 3x - 5 = 0$

$\therefore x > 0 \Rightarrow x = \dfrac{-3+\sqrt{29}}{2}$

Question 4

a. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
• writes expression for probability in terms of binomial coefficients	1
• evaluates required probability	1

Answer

$P(none\ in\ common) = \dfrac{^{34}C_6}{^{40}C_6} \approx 0\cdot35$ (to 2 decimal places)

b. Outcomes assessed : HE6

Marking Guidelines

Criteria	Marks
• writes du in terms of dx and converts limits for x into limits for u	1
• finds equivalent definite integral in terms of u	1
• finds primitive and substitutes limits	1
• simplifies exact answer	1

Answer

$$\begin{aligned} u &= \sin^2 x \\ du &= 2\sin x\cos x\ dx \\ du &= \sin 2x\ dx \\ x = \tfrac{\pi}{4} &\Rightarrow u = \tfrac{1}{2} \\ x = \tfrac{\pi}{3} &\Rightarrow u = \tfrac{3}{4} \end{aligned}$$

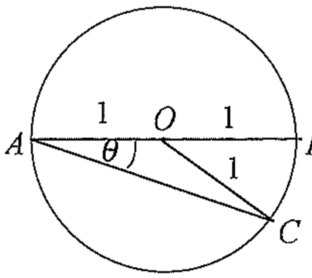
$$\begin{aligned} \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin 2x}{1+\sin^2 x}\,dx &= \int_{\frac{1}{2}}^{\frac{3}{4}} \frac{1}{1+u}\,du \\ &= \Big[\ln(1+u)\Big]_{\frac{1}{2}}^{\frac{3}{4}} \\ &= \ln\tfrac{7}{4} - \ln\tfrac{3}{2} \\ &= \ln\tfrac{7}{6} \end{aligned}$$

c. Outcomes assessed : H5, PE3

Marking Guidelines

Criteria	Marks
i • finds area of ΔAOC in terms of $\sin 2\theta$	1
• uses area information to complete equation for θ	1
ii • shows that $f(0\cdot 4)$, $f(0\cdot 5)$ have opposite signs	1
• notes that f is continuous, and deduces equation has one root θ, $0\cdot 4<\theta<0\cdot 5$	1
iii • applies Newton's rule to write numerical expression for next approximation	1
• evaluates this approximation	1

Answer



i. $\angle OCA = \theta$ $\quad$ *($\angle$'s opp. equal sides are equal in ΔAOC)*

$\angle AOC = \pi - 2\theta$ $\quad$ *($\angle$ sum of Δ is π)*

$\angle BOC = 2\theta$ $\quad$ *(adj. supp. $\angle$'s add to π)*

Area sector BOC + Area $\Delta AOC = \frac{1}{4}$ Area circle

$$\therefore \tfrac{1}{2}\times 1^2\times 2\theta + \tfrac{1}{2}\times 1^2\times \sin(\pi-2\theta) = \tfrac{1}{4}\times\pi\times 1^2$$

$$\theta + \tfrac{1}{2}\sin 2\theta - \tfrac{\pi}{4} = 0$$

ii. Let $f(\theta) = \theta + \frac{1}{2}\sin 2\theta - \frac{\pi}{4}$

$f(0\cdot 4) \approx -0\cdot 03 < 0$

$f(0\cdot 5) \approx \ 0\cdot 14 > 0$ $\quad$ and f is continuous

Also $f'(\theta) = 1+\cos 2\theta > 0 \Rightarrow f$ monotonic increasing

$\therefore f(\theta)=0$ for exactly one value of θ, $0\cdot 4<\theta<0\cdot 5$

iii. Since $f'(\theta) = 1+\cos 2\theta$,

$$\begin{aligned} \theta &\approx 0\cdot 4 - \frac{f(0\cdot 4)}{f'(0\cdot 4)} \\ &\approx 0\cdot 4 - \frac{-0\cdot 0267}{1\cdot 6967} \\ &\approx 0\cdot 42 \quad \text{(to 2 dec. pl.)} \end{aligned}$$

Question 5

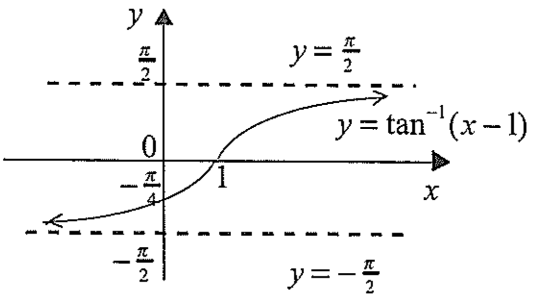
a. Outcomes assessed : HE4

Marking Guidelines

Criteria	Marks
i • shows correct shape and asymptotes	1
• shows intercepts on coordinate axes	1
ii • finds $\frac{dy}{dx}$ and evaluates for $x=1$	1
• finds equation of tangent	1

Answer

i.



ii. $y=\tan^{-1}(x-1)$

$\frac{dy}{dx}=\frac{1}{1+(x-1)^2}$

$\therefore \frac{dy}{dx}=1$ when $x=1$

$\therefore$ Tangent at $(1,0)$ has gradient 1 and equation $y=x-1$

b. Outcomes assessed : HE5

Marking Guidelines

Criteria	Marks
i • shows by differentiation that a is constant	1
ii • integrates to find a primitive function for t in terms of x	1
• evaluates constant of integration using initial conditions then writes x as a function of t	1
iii • evaluates x at $t=2$ and $t=3$ to find distance travelled in third second.	1

Answer

i. $v=\sqrt{x} \Rightarrow \frac{1}{2}v^2=\frac{1}{2}x$

$\therefore a=\frac{d}{dx}(\frac{1}{2}v^2)=\frac{1}{2}$ for all x

Hence a is independent of x.

ii. $\frac{dx}{dt}=x^{\frac{1}{2}}$

$\frac{dt}{dx}=x^{-\frac{1}{2}}$

$t=2x^{\frac{1}{2}}+c$

$\left.\begin{array}{l} t=0 \\ x=1 \end{array}\right\} \Rightarrow c=-2$

$\therefore t=2\sqrt{x}-2$

$x=\frac{1}{4}(t+2)^2$

iii. Between $t=2$ and $t=3$, particle moves right from $x=4$ to $x=\frac{25}{4}$

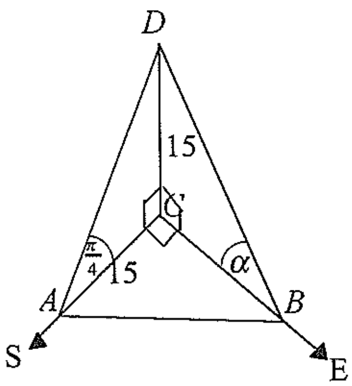
Distance travelled in third second is $2\cdot 25$ m.

c. Outcomes assessed : H5, HE5, HE7

Marking Guidelines

Criteria	Marks
i • finds AC and finds BC in terms of $\cot\alpha$	1
• uses Pythagoras' theorem and an appropriate trig. identity to find AB in terms of $\operatorname{cosec}\alpha$	1
ii • differentiates AB with respect to t using chain rule or implicit differentiation	1
• substitutes given values and interprets result	1

Answer



i. In ΔACD,

$\angle DAC=\angle ADC=\frac{\pi}{4}$

$\therefore AC=15$.

In ΔBCD, $BC=15\cot\alpha$.

$\therefore$ In ΔABC,

$AB^2=15^2+15^2\cot^2\alpha$

$=15^2(1+\cot^2\alpha)$

$=15^2\operatorname{cosec}^2\alpha$

$\therefore AB=15\operatorname{cosec}\alpha$

ii. When $\alpha=\frac{\pi}{3}$,

$\frac{dAB}{dt}=-15\operatorname{cosec}\alpha\cot\alpha\frac{d\alpha}{dt}$

$=-15\times\frac{2}{\sqrt{3}}\times\frac{1}{\sqrt{3}}\times 0\cdot 01$

$=-0\cdot 1$

$\therefore AB$ is decreasing at a rate of $0\cdot 1$ ms^{-1}

Question 6

a. Outcomes assessed : HE3

Marking Guidelines

	Criteria	Marks
i	• integrates v with respect to t to find expression for x	1
	• uses initial conditions to evaluate the constant of integration, giving x as a function of t	1
	• differentiates v with respect to t to get $\ddot{x}$ then expresses $\ddot{x}$ in terms of x	1
ii	• states period	1
	• states extremities	1
iii	• solves trig. equation to find time to first return	1

Answer

i. $v = -12\sin(2t + \frac{\pi}{3})$

$x = 6\cos(2t + \frac{\pi}{3}) + c$

$t = 0,\ x = 5 \Rightarrow c = 2$

$\therefore x = 2 + 6\cos(2t + \frac{\pi}{3})$

$\ddot{x} = -24\cos(2t + \frac{\pi}{3})$

$\therefore \ddot{x} = -4(x - 2)$

ii. Period is π seconds. $-4 \le x \le 8$

iii. $x = 5 \Rightarrow \cos(2t + \frac{\pi}{3}) = \frac{1}{2}$

$2t + \frac{\pi}{3} = \frac{\pi}{3},\ 2\pi - \frac{\pi}{3}, \ldots$

$t = 0,\ \frac{2\pi}{3},\ \ldots$

First return after $\frac{2\pi}{3}$ seconds.

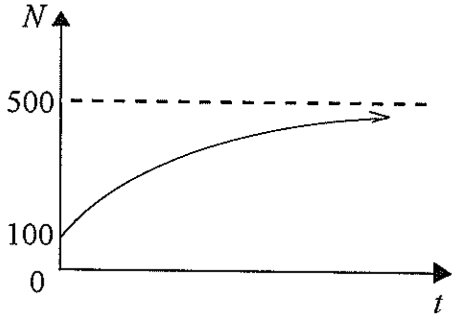
b. Outcomes assessed : HE3

Marking Guidelines

	Criteria	Marks
i	• sketches graph of correct shape with correct vertical intercept	1
	• shows asymptote for limiting population size	1
ii	• differentiates with respect to t	1
iii	• writes and solves equation for N	1

Answer

i.



ii.

$N = 500 - 400e^{-0 \cdot 1t}$

$\frac{dN}{dt} = 0 \cdot 1 \times 400e^{-0 \cdot 1t}$

$= 0 \cdot 1\,(500 - N)$

iii.

Initial rate of growth is

$0 \cdot 1\,(500 - 100) = 0 \cdot 1 \times 400$

$\therefore$ want N such that

$0 \cdot 1\,(500 - N) = 0 \cdot 1 \times 200$

$500 - N = 200$

$N = 300$

c. Outcomes assessed : H5, HE4

Marking Guidelines

Criteria	Marks
• uses inverse trig. identity to simplify equation	1
• uses trig. expansion to evaluate x in terms of k	1

Answer

$\cos^{-1} x - \sin^{-1} x = k, \quad -\frac{\pi}{2} \le k \le \frac{3\pi}{2}$

$\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$

$\therefore 2\cos^{-1} x = k + \frac{\pi}{2}$

$\cos^{-1} x = \frac{k}{2} + \frac{\pi}{4}$

$x = \cos(\frac{k}{2} + \frac{\pi}{4})$

$\therefore x = \cos\frac{k}{2}\cos\frac{\pi}{4} - \sin\frac{k}{2}\sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}(\cos\frac{k}{2} - \sin\frac{k}{2})$

Question 7

a. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
i • uses integration to find expressions for $\dot{x}$ and x	1
• uses integration to find expressions for $\dot{y}$ and y	1
ii • writes simultaneous equations for V and θ	1
• finds the value of V	1
• finds the value of θ	1
iii • finds the values of $\dot{x}$ and $\dot{y}$ just before impact	1
• uses Pythagoras' theorem to find the magnitude of v	1
• uses trigonometry to find the direction of v as an angle relative to the horizontal	1

Answer

i.

$$\begin{aligned}\ddot{x}&=0\\ \dot{x}&=c_1\\ \therefore \dot{x}&=V\cos\theta\end{aligned}$$

$$x=Vt\cos\theta+c_2 \qquad \left.\begin{aligned}t&=0\\x&=0\end{aligned}\right\}\Rightarrow c_2=0 \qquad \therefore x=Vt\cos\theta$$

$$\ddot{y}=-10 \qquad \dot{y}=-10t+c_3 \qquad \left.\begin{aligned}t&=0\\ \dot{y}&=V\sin\theta\end{aligned}\right\}\Rightarrow c_3=V\sin\theta \qquad \therefore \dot{y}=-10t+V\sin\theta$$

$$y=-5t^2+Vt\sin\theta+c_4 \qquad \left.\begin{aligned}t&=0\\y&=0\end{aligned}\right\}\Rightarrow c_4=0 \qquad \therefore y=Vt\sin\theta-5t^2$$

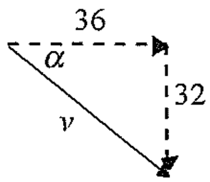
ii. When $t=8$

$$\left.\begin{aligned}x&=288\\y&=64\end{aligned}\right\}\Rightarrow \begin{aligned}8V\cos\theta&=288\\8V\sin\theta&=384\end{aligned} \qquad \therefore V^2(\cos^2\theta+\sin^2\theta)=36^2+48^2 \quad V=60 \qquad \tan\theta=\tfrac{48}{36}=\tfrac{4}{3} \quad \theta=\tan^{-1}\tfrac{4}{3}$$

iii. When $t=8$

$$\dot{x}=60\times\tfrac{3}{5}=36$$

$$\dot{y}=-80+60\times\tfrac{4}{5}=-32$$



$$v^2=36^2+32^2 \qquad v=4\sqrt{145}$$

$$\tan\alpha=\tfrac{8}{9} \qquad \alpha\approx 41\cdot 6^\circ$$

Velocity of rocket just before impact is approximately $48\ \text{ms}^{-1}$ inclined at 42° below the horizontal.

b. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
i • writes a typical term in x^r in the expansion of the RHS of the identity	1
• collects like terms to find coefficient of x^r, then equates to coefficient of x^r on LHS	1
ii • writes single binomial coefficient for sum on LHS	1
• writes single binomial coefficient for sum on RHS then deduces result	1

Answer

i. $(1+x)^{m+n}=(1+x)^m(1+x)^n$

For $0\le r\le m$ and $0\le r\le n$,

terms in x^r in expansion of the RHS have the form ${}^mC_k x^k\times{}^nC_{r-k}x^{r-k}$, $k=0,1,2,\dots,r$.

Collecting such like terms gives the coefficient of x^r as $\sum_{k=0}^{r}{}^mC_k\,{}^nC_{r-k}$.

The coefficient of x^r in the expansion of the LHS is ${}^{m+n}C_r$.

Hence equating coefficients of x^r on both sides of the identity gives ${}^{m+n}C_r=\sum_{k=0}^{r}{}^mC_k\,{}^nC_{r-k}$.

ii. Using i., for $m\ge 2$ and $n\ge 2$,

$${}^{m+1}C_0\,{}^nC_2+{}^{m+1}C_1\,{}^nC_1+{}^{m+1}C_2\,{}^nC_0={}^{(m+1)+n}C_2 \quad\text{and}\quad {}^mC_0\,{}^{n+1}C_2+{}^mC_1\,{}^{n+1}C_1+{}^mC_2\,{}^{n+1}C_0={}^{m+(n+1)}C_2$$

$$\therefore\ {}^{m+1}C_0\,{}^nC_2+{}^{m+1}C_1\,{}^nC_1+{}^{m+1}C_2\,{}^nC_0={}^mC_0\,{}^{n+1}C_2+{}^mC_1\,{}^{n+1}C_1+{}^mC_2\,{}^{n+1}C_0={}^{m+n+1}C_2$$

Independent Trial Examination 2009 Mathematics Extension 1 Mapping Grid

Question	Marks	Content	Syllabus Outcomes	Targeted Performance Bands
1 a	2	Trigonometric functions	H5	E2-E3
b	2	Series and applications	H5	E2-E3
c	2	Polynomials	PE3	E2-E3
d	2	Angle between two lines	H5	E2-E3
e i	1	Circle geometry	PE3	E2-E3
ii	2	Circle geometry	PE2, PE3	E2-E3
iii	1	Circle geometry	PE3	E2-E3
2 a	2	Trigonometric functions	H5	E2-E3
b	2	Permutations and combinations	PE3	E2-E3
c	2	Division of an interval	H5	E2-E3
d	2	Trigonometric functions	H5	E2-E3
e i	2	Parametric representation	PE3, PE4	E2-E3
ii	2	Parametric representation	PE3, PE4	E2-E3
3 a	2	Trigonometric functions	H5	E2-E3
b	4	Induction	HE2	E3-E4
c i	1	Inverse functions	HE4	E2-E3
ii	3	Inverse functions	HE4	E2-E3
iii	2	Inverse functions	HE4	E2-E3
4 a	2	Further probability	HE3	E2-E3
b	4	Methods of integration	HE6	E2-E3
c i	2	Trigonometric functions	H5	E2-E3
ii	2	Polynomials	PE3	E2-E3
iii	2	Polynomials	PE3	E2-E3
5 a i	2	Inverse functions	HE4	E2-E3
ii	2	Inverse functions	HE4	E2-E3
b i	1	Velocity and acceleration as functions of displacement	HE5	E2-E3
ii	2	Velocity and acceleration as functions of displacement	HE5	E2-E3
iii	1	Velocity and acceleration as functions of displacement	HE5	E2-E3