STUDENT NUMBER: ______________________________

TEACHER'S NAME: ______________________________

BAULKHAM HILLS HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

2007

MATHEMATICS

EXTENSION 1

Time allowed – Two hours
(Plus five minutes reading time)

GENERAL INSTRUCTIONS:

- Attempt **ALL** questions.
- Start each of the 7 questions on a new page.
- All necessary working should be shown.
- Write your student number at the top of each page of answer sheets.
- At the end of the exam, staple your answers in order behind the cover sheet.

QUESTION 1 (START ON A NEW PAGE)

Marks

(a) Find $\int \frac{dx}{\sqrt{9-25x^2}}$. **2**

(b) Find the acute angle between the lines $y=3x+4$ *and* $2x+3y=6$. **2**

(c) When the polynomial $P(x)=2x^3-ax+1$ is divided by $x+1$, the remainder is 2. Find a. **2**

(d) Show that $\cot\theta-\cot 2\theta = \operatorname{cosec} 2\theta$ **3**

Hence find the exact value of $\cot 15^o$

(e) Use the substitution $u = 2x + 1$ to find $\int x\,(2x+1)^{10}\ dx$ **3**

QUESTION 2 (START ON A NEW PAGE)

(a) In the expansion of $\left(3x-\frac{1}{x^3}\right)^{12}$, find the term independent of x. **3**

(b) (i) Sketch $y = 2\sin^{-1}(x-3)$, stating clearly the domain and range. **3**

(ii) Find the exact gradient of the function at the point where $x = 3.5$ **2**

(c) A curve has parametric equations:

$x = 3\tan 2\theta.$

$y = \tan\theta$ **2**

Find its Cartesian equation.

(d) Find $\int \cos^2 2x\,dx.$ **2**

QUESTION 3 (START ON A NEW PAGE) **Marks**

(a) Simplify $\dfrac{6^x+4^x}{3^x+2^x}$ **2**

(b) Solve the equation

$2x^3-17x^2+40x-16 = 0$ given that it has a double root which is an integer. **4**

(c) Prove by mathematical induction that for all integers n ≥ 1

$2\times2^0+3\times2^1+4\times2^2+\ldots\ldots\ldots\ldots\ldots\ldots+(n+1)\times2^{n-1} = n\times2^n$ **3**

(d) Find the range of values of x if the series

$\dfrac{2}{1+3x}+\dfrac{6}{(1+3x)^2}+\dfrac{18}{(1+3x)^3}+\ldots\ldots$ has a limiting sum. **3**

QUESTION 4 (START ON A NEW PAGE)

(a) (i) Write $2\sqrt{3}\cos2t-2\sin2t$ in the form of $R\cos(2t+\alpha)$ **2**

(ii) A particle moves so that its displacement x metres is given by:

$x=2\sqrt{3}\cos2t-2\sin2t$

Show that the motion is simple harmonic and state its period and amplitude **3**

(b) The equation $x+2\tan x=0$ has a root near $\dfrac{3\pi}{4}$.

With $x=\dfrac{3\pi}{4}$ as a first approximation, find using Newton's method once, a second approximation to the root in terms of π **3**

(c) Show that $\dfrac{d}{d\theta}\left(\dfrac{1}{3}\tan^3\theta\right)=\sec^4\theta-\sec^2\theta$ **2**

Hence or otherwise find the value of $\displaystyle\int_0^{\frac{\pi}{4}}\sec^4\theta\ d\theta$ **2**

QUESTION 5 (START ON A NEW PAGE)

Marks

(a) Find $\frac{d}{dx}\log_{10}\left(x^2+1\right)$ **2**

(b) The acceleration $\ddot{x}\ m/s^2$ of a particle moving in a straight line is given by

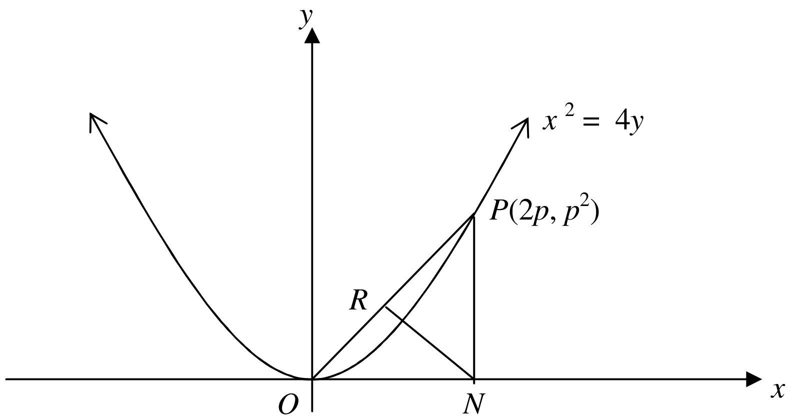
$\ddot{x} = 6\left(1-x^2\right)$. Initially the particle is at $x=-3$ and is moving with velocity

$4\ m/s$. **(i)** By using the result $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ show that

$$v^2 = 12x - 4x^3 - 56$$ **3**

(ii) Does the particle pass through the origin? Justify your answer.

(c)



$P(2p, p^2)$ is a variable point on the parabola $x^2 = 4y$. N is the foot of the perpendicular from P to the x axis. NR is perpendicular to OP

(i) Find the equation of OP **1**

(ii) Find the equation of NR **1**

(iii) Show that the R has coordinates $\left(\frac{8p}{p^2+4}, \frac{4p^2}{p^2+4}\right)$ **2**

(iv) Show that the locus of R is a circle and state its centre and radius **3**

QUESTION 6 (START ON A NEW PAGE)

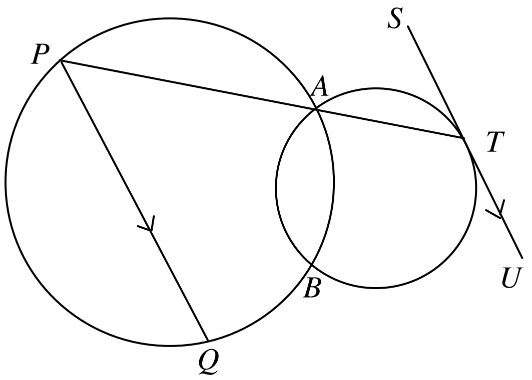
(a) Solve the equation $e^x - e^{-x} = 1$ for x in the exact form. **3**

(b) Given that $(1+ax)^7 + (1+bx)^7 = 2+21x+609x^2+\ldots\ldots\ldots\ldots$ **4**
find the values of a and b

(c) Find the exact value of $\sin(2\tan^{-1}\frac{3}{5})$ **2**

(d) Two circles intersect at A and B. STU is a tangent and is parallel to PQ.

Prove that the points Q, B, T are collinear. **3**



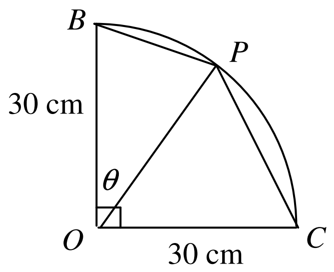
QUESTION 7 (START ON A NEW PAGE)

(a) If $f(x)=\ln(2x+3)$, find an expression for the inverse function $f^{-1}(x)$ **2**

(b) P rotates about O along the arc BC at a constant rate of $\frac{\pi}{60}$ radians/minute.

(i) Show that the area of $OBPC$ is given by $A=450(\sin\theta+\cos\theta)$ **2**

(ii) Find the rate at which the area is changing when $\theta=\frac{\pi}{6}$ **2**



(c) A projectile is fired from a point O on the ground with speed V m/s at an angle α. Given that the equations of motion are

$$x=V\cos\alpha\, t \qquad \text{and} \qquad y=V\sin\alpha\, t-\frac{gt^2}{2}$$

(i) Find the time of flight of the projectile. **1**

(ii) The projectile is climbing at an angle of 45^o after a time T. Show that

$$T=\frac{V\sin\alpha-V\cos\alpha}{g}$$ **2**

(iii) If T is $\frac{1}{3}$ of the time of flight, find α to the nearest degree. **3**

End of Paper