Q.1 TRIAL HSC 2003 SOLNS

$2x^2\sin x\cos x + 2x\sin^2 x$ [PRODUCT RULE]

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{x}{2}\right)^2 + (2y)^2 = 1$

$\frac{x^2}{4} + 4y^2 = 1$

$\sin x = -\frac{1}{2}$

$\sin x = \sin\frac{7\pi}{6}$

$x = n\pi + (-1)^n\left(\frac{-\pi}{6}\right)$

$1 = \frac{3x+4\times2}{7}$ $\qquad$ $1 = \frac{3y+4\times4}{7}$

$x = -5$ $\qquad$ $y = -3$

i.e. $B(-5,-3)$

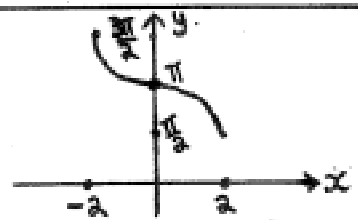
$P(-2) = 7$ [REMAINDER THEOREM]

$\therefore K = 41$

$\frac{1}{4}\left[\sec 4x\right]_0^{\frac{\pi}{6}} = \frac{1}{4}\left[\sec\frac{2\pi}{3} - \sec 0\right]$

$= \frac{1}{4}\left[-2 - 1\right]$

$= -\frac{3}{4}$



(b) $\frac{x-2}{x}\times\frac{x^2}{1} \leqslant 1\times x^2$

$x(x^2-2) - x^2 \leqslant 0$

$x\left[(x^2-2) - x\right] \leqslant 0$

$x(x-2)(x+1) \leqslant 0$

$x \leqslant -1$ OR $0 < x \leqslant 2$

(c) $m_1 = -1$, $m_2 = 2$

$\tan\theta = \left|\frac{-1-2}{1+-1\times2}\right|$

$= 3$

$\theta = 72^\circ$ [NEAREST DEGREE]

(d) $u = x-2 \Rightarrow x = 1 : u = -1$

$\frac{du}{dx} = 1$ $\qquad$ $x = 3 : u = 1$

$du = dx$

$\int_{-1}^{1}(u+2)u^5\,du = \int_{-1}^{1} u^6 + 2u^5\,du$

$= \left[\frac{1}{7}u^7 + \frac{1}{3}u^6\right]_{-1}^{1}$

$= \frac{2}{7}$

3. (a)(i) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

(ii) $\tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$

$= \frac{\sqrt{3}+1}{1-\sqrt{3}\times1}$

$= -2-\sqrt{3}$

(b) $P(x) = x - \tan^{-1}2x \therefore P(1) = 1 - \tan^{-1}2$

$P'(x) = 1 - \frac{2}{1+4x^2} \therefore P'(1) = \frac{3}{5}$

$z_2 = z_1 - \frac{P(1)}{P'(1)}$

$= 1 - \frac{1-\tan^{-1}(2)}{3/5}$

$\doteqdot 1{\cdot}2$

(c)(i) $y' = \frac{1}{2a}y$

At $(2ap, ap^2)$: $m_{TAN} = p$

$\therefore m_{NORM} = -\frac{1}{p}$

EQUN. NORMAL: $y - ap^2 = -\frac{1}{p}(x - 2ap)$

$x + py = 2ap + ap^3$

(ii) $m = p$

EQUN: $y - a = p(x - 0)$

$y = px + a$

(iii) Solve $x + py = 2ap + ap^3$

$y = px + a$

(iii) cont. $N(ap, a(p^2+1))$

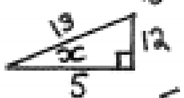
(iv) sub $p = \frac{x}{a}$: $y = a\left[\left(\frac{x}{a}\right)^2 + 1\right]$

$= \frac{1}{a}x^2 + a$

4. (a) $\cos\left(-\sin^{-1}\left(\frac{12}{13}\right)\right) = \cos\left(\sin^{-1}\left(\frac{12}{13}\right)\right)$

let $x = \sin^{-1}\left(\frac{12}{13}\right)$

$\sin x = \frac{12}{13}$

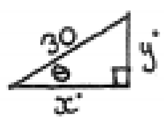


$\therefore \cos x = 5/13$

$= \cos x$

$= 5/13$

(b) (i)



INITIALLY

$\dot{y} = 30\sin\theta$

$\dot{x} = 30\cos\theta$

$\ddot{y} = -10$

$\dot{y} = -10t + C$

sub $t=0, \dot{y} = 30\sin\theta \Rightarrow C = 30\sin\theta$

$\dot{y} = 30\sin\theta - 10t$

$y = 30t\sin\theta - 5t^2 + K$

sub $t=0, y=0 \Rightarrow K=0$

$y = 30t\sin\theta - 5t^2$

(ii) $\ddot{x} = 0$

$\dot{x} = C = 30\cos\theta$

$x = 30t\cos\theta + C$

sub $t=0, x=0 \Rightarrow C=0$

$x = 30t\cos\theta$

(iii) We want $y > 20$ when $x = 60$

$x = 60 \Rightarrow 2 = t\cos\theta \Rightarrow t = \frac{2}{\cos\theta}$

$30t\sin\theta - 5t^2 > 0$

$30 \cdot \frac{2}{\cos\theta} \cdot \sin\theta - 5 \cdot \left(\frac{2}{\cos\theta}\right)^2 > 20$

$60\tan\theta - 20\sec^2\theta > 20$

$60\tan\theta - 20 - 20\tan^2\theta > 20$

$\tan^2\theta - 3\tan\theta + 1 > -1$

$(\tan\theta - 2)(\tan\theta - 1) > 0$

$1 < \tan\theta < 2$

$\frac{\pi}{4} < \theta < 1.11^c$

or $45° < \theta < 63°26'$

(c) (i) sub $t=0, T=90 \Rightarrow 90 = 25 + Ae^0$

$A = 65$

sub $t=2, T=80 \Rightarrow 80 = 25 + 65e^{-2k}$

$\frac{11}{13} = e^{-2k}$

$k = \ln\left(\frac{11}{13}\right) \div -2$

$= 0.0835$

(ii) sub $t=7$: $T = 25 + 65e^{-0.0835\times7}$

$= 61$ (nearest degree)

5. (a) (i) $P(3) = 0 \Rightarrow k = 7$

(ii) $P(x) = (x-3)(2x+1)(x-2)$

roots of $P(x) = 0$

$x = 3, -\frac{1}{2}, 2$

(b) (i) $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$\frac{1}{2}v^2 = \frac{15}{2} + 2x - 2x^2$

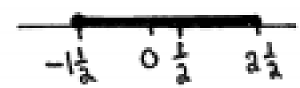
$\ddot{x} = 2 - 4x$

(ii) $V = \sqrt{15 + 4x - 4x^2}$

$15 + 4x - 4x^2 \geq 0$

$4x^2 - 4x - 15 \leq 0$

$(2x+3)(2x-5) \leq 0$



i.e. $-1\frac{1}{2} \leq x \leq 2\frac{1}{2}$

$\therefore$ centre of motion is $\frac{1}{2}$

amplitude $= 2$ m

period $= \frac{2\pi}{n} = \frac{2\pi}{2} = \pi$ s

(c) STEP 1 : Prove true for $n=1$

$n=1$: LHS $= \cos(x + \pi)$

$= -\cos x$

RHS $= -\cos x$

5.(c) cont. $= LHS$

$\therefore$ True for $n=1$

STEP 2: Assume true for $n=k$.

ie. $\cos(x+k\pi) = (-1)^k \cos x$

Hence prove true for $n=k+1$

i.e. $\cos(x+(k+1)\pi) = (-1)^{k+1}\cos x$

Now $\cos(x+(k+1)\pi) = \cos(x+k\pi+\pi)$

$= \cos(x+k\pi)\cos\pi - \sin(x+k\pi).\sin\pi$

$= -\cos(x+k\pi) - 0$

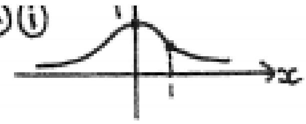
$= -1.(-1)^k \cos x$ by our assumption

$= (-1)^{k+1}\cos x.$

ie. if true for $n=k$ then true for $n=k+1$.

STEP 3: We assumed true for $n=k$ and hence proved true for $n=k+1$. Since true for $n=1$ then true for $n=2$. Since true for $n=2$ then true for $n=3$ and so on for all positive integers $n \geq 1$.

6. (a)(i)



largest domain:
$x \geq 0$

(ii) SWAP x AND y: $x = \dfrac{1}{1+y^2}$

$f^{-1}(x) = \sqrt{\dfrac{1-x}{x}}$

(b) $\angle BAC = 90^\circ$ ($\angle$ in semicircle)

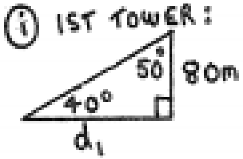
$\angle ACB = \alpha^\circ$ (alternate segment theorem)

$\angle ABC = \alpha^\circ$ (alternate $\angle$s, $AT \parallel BC$)

$\therefore 2\alpha^\circ + 90^\circ = 180^\circ$ ($\angle$ sum of $\triangle ABC$)

$\alpha = 45^\circ$

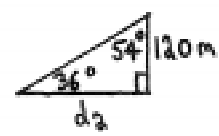
(c)(i) 1ST TOWER:



$\tan 50^\circ = \dfrac{d_1}{80}$

$d_1 = 80\tan 50^\circ$

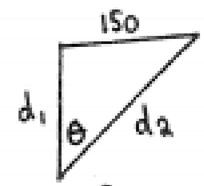
(ii) 2ND TOWER



$\tan 54^\circ = \dfrac{d_2}{120}$

$d_2 = 120\tan 54^\circ$

(iii) On ground:



$\cos\theta = \dfrac{(80\tan 50^\circ)^2 + (120\tan 54^\circ)^2 - 150^2}{2 \cdot 80\tan 50^\circ \cdot 120\tan 54^\circ}$

$\theta = 63^\circ 52'$

7. (a) $2\sqrt{3}\left(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right) = -3$

$\cos\alpha\cos\theta - \sin\alpha\sin\theta = -\dfrac{\sqrt{3}}{2}$

where $\left.\begin{array}{l}\cos\alpha = \frac{\sqrt{3}}{2} \\ \sin\alpha = \frac{1}{2}\end{array}\right\}$ $\alpha = \dfrac{\pi}{6}$

$\cos\left(\theta + \dfrac{\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$

$\theta + \dfrac{\pi}{6} = \dfrac{5\pi}{6}$ or $\dfrac{7\pi}{6}$

$\theta = \dfrac{2\pi}{3}$ or π

(b)(i) $A_1 = PR - M$

$A_2 = A_1 R - M$

$= (PR-M)R - M$

$= PR^2 - MR - M$

$= PR^2 - M(1+R)$

(ii) $A_n = PR^n - M(1+R+R^2+\dots+R^{n-1})$

$= PR^n - \dfrac{M(R^n-1)}{R-1}$ USING $S_n = \dfrac{a(r^n-1)}{r-1}$

(iii) $\dfrac{PK}{100} = PR^n - \dfrac{M(R^n-1)}{R-1}$

$\dfrac{PK(R-1)}{100} = PR^n(R-1) - M(R^n-1)$

11(b) cont.

$$\frac{PK(R-1)}{100} = PR^{n+1} - PR^n - MR^n + M$$

$$\frac{PK(R-1)}{100} - M = R^n(PR-P-M)$$

$$\frac{PK(R-1)-100M}{100} = R^n(PR-P-M)$$

$$R^n = \frac{PK(R-1)-100M}{100[PR-P-M]}$$

$$= \frac{PK(R-1)-100M}{100[P(R-1)-M]}$$

(iv) $$1{\cdot}0075^n = \frac{40000\times 20(0{\cdot}0075)-80000}{100[40000\times 0{\cdot}0075-800]}$$

$$= 1{\cdot}48$$

$$\ln 1{\cdot}0075^n = \ln 1{\cdot}48$$

$$n = \frac{\ln 1{\cdot}48}{\ln 1{\cdot}0075}$$

$\doteqdot$ 52·5 months

$\doteqdot$ 5 years

Q1 a) – Poorly set out, many differentiated $(\sin x)^2 \neq \sin^2 x$ incorrectly by not using chain rule.

b) – Usually well done, but many could not link $\sin\theta$, $\cos\theta$ using $\sin^2\theta + \cos^2\theta = 1$.

c) Well done, but a number forgot general form.

d) Very straight forward, caused no problem

e) Easy for people who used remainder theorem.

f) This was also done well by most students.

Q2 (a) Some candidates lost marks for not showing y-intercept or "end points" of the curve.

(b) N.B. Here $x \neq 0$!

(c) Some need to learn formula

(d) Generally well done.

Q3 a) Usually done well, some found it easier to convert to degrees. Some careless errors with exact values cost some students a mark.

b) This caused many problems! Very few could correctly differentiate $\tan^2 2x$!! Poor setting out meant that marks could not be awarded for showing what you could do. Far too many did not use rads on calculator!

c) i), ii) Usually well attempted.

iii) poor algebra skills cost marks – check working!

iv) poorly attempted because of algebra probs in part iii). Many did not realise they simply had to eliminate p using $p = \frac{3}{2}$.

Q4 comments not available.

Q5 (a) Many factorised correctly but failed to continue and write down the roots.

(b) $\ddot{x} = -4(x - \frac{1}{2})$ of form $\ddot{x} = -n^2(x - \frac{1}{2})$

so $n = 2$, Period $= \frac{2\pi}{n} = \frac{2\pi}{2} = \pi$

(c) N.B. $\cos[(x + k\pi) + \pi] = \cos(x + k\pi)\cos\pi - \sin(x + k\pi)\sin\pi$

Q6 Inverse trig question not well done from an algebraic point of view (ie making y the subject when swapping x + y).

Circle geometry well done, most students got it out. 3D trig was OK, but not knowing the Cosine Rule let some students down.

Q7. a) Half of the students who used the "t" results neglected to check $\theta = \pi$. Of those that used $R\cos(\theta \pm \alpha)$ or $R\sin(\theta \pm \alpha)$ most found R correctly but a lot of students had difficulty with α.

b) i) Well done.

ii) Show means to insert every line of working — too many students assumed that $S_n = \frac{a(r^n-1)}{r-1}$ and that $a=1$, $r=R$ and $n=n$. A lot of students extrapolated from $A_2 = \ldots$ to $A_n = PR^n - \frac{M(R^n-1)}{R-1}$ without writing $A_n = PR^n - M(1 + R + R^2 + \cdots + R^{n-1})$.

iii) Half of all candidates did not realise that K% of 100n meant $\frac{PK}{100}$. Very few students completed this section correctly.

iv) Very few students gained full marks. A lot could not solve $1{\cdot}0075^n = 1{\cdot}48$.