Year 11 (Ext 1) - 2007 - Semester 1 $\frac{}{10}$

Question 1 (10 Marks)

a) $(2x+3)^2 = 25$

$2x + 3 = \pm 5$

$2x = -3 \pm 5$

$x = \frac{-8}{2}$ or $\frac{2}{2}$

$x = -4$ or 1.

1M each.

$\frac{}{2}$

b) $128x - 16x^4$

$= 16x(8 - x^3)$ ½

$= 16x(2-x)(4+2x+x^2)$ 1

½

2

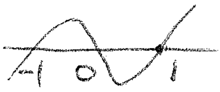
c) $x - \frac{1}{x} \leq 0$

$x \neq 0$ ½

$x^3 - x \leq 0$ ½

$x(x^2-1) \leq 0$

$x(x-1)(x+1) \leq 0$ 1M only



$x \leq -1,\ 0 < x \leq 1$.

½ ½

2

d) (i) $y = \sqrt{x-5}$

$x - 5 \geq 0$

D : $x \geq 5$ 1

R : $y \geq 0$ 1

2

(ii) $y = \frac{1}{2x-4}$

$2x - 4 \neq 0$

D : $x \neq 2$, all real x 1

R : $y \neq 0$, all real y 1

2

Question 2 (10 Marks)

a) $f(x) = x^2 \quad x < -2$

$= 3x \quad -2 \leq x \leq 2$

$= \frac{1}{x} \quad x > 2$.

(i) $f(-1) = 3x - 1$ ½ in correct $f(x)$

$f(-1) = -3$ ½

1

$f(2½) = 1/2½$ ½ $f(x$

$f(2½) = \frac{2}{5}$ ½

1

(ii) at $x = 2$ $f(2) = 3 \times 2$
$= 6$
$f(2) = \frac{1}{2}$ ½ reason

$\therefore$ Discontinuous ½

$f(2^-) \neq f(2^+)$

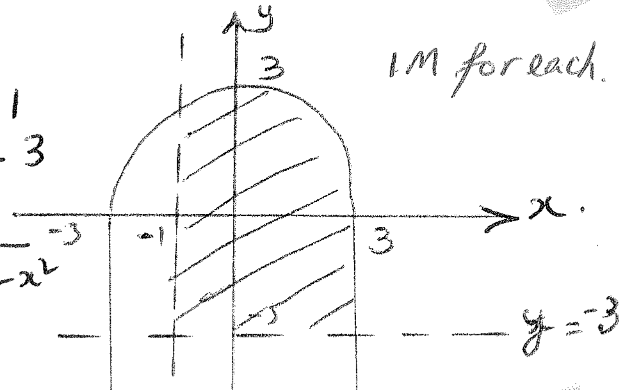
1

b) 1M for each.

$x > -1$

$y \geq -3$

$y \leq \sqrt{9-x^2}$



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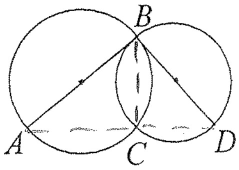
c) (i)



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(ii) Construct the lines joining B to C, A to C and C to D.

$-\frac{1}{2}$ for no construction!!

$\because$ AB is a diameter

$\therefore \angle ACB = 90°$ ($\angle$ in semicircle) 1 (ii)

$\because$ BD is a diameter

$\therefore \angle BCD = 90°$ ($\angle$ in semicircle) $\frac{1}{2}$.

$\therefore \angle ACB + \angle BCD = 180°$.
(by addition). 1

$\therefore$ A, C and D are collinear
$\angle$'s on a straight line. $\frac{1}{2}$.

3

Question 3 (10 Marks)

a) $A(-2,1)$ $B(x,y)$ $P(13,-9)$
external ratio $-5:3$.

$$13 = \frac{3x - 2 + {}^{-}5x}{-2}$$

$$-26 = -6 - 5x$$

$$x = 4$$

$$-9 = \frac{3 \times 1 - 5y}{-2}$$

$$18 = 3 - 5y$$

$$y = -3$$

$$\therefore B(4,-3)$$

3

b)(i) $x^2 + y^2 - 4x + 6y + 4 = 0$.

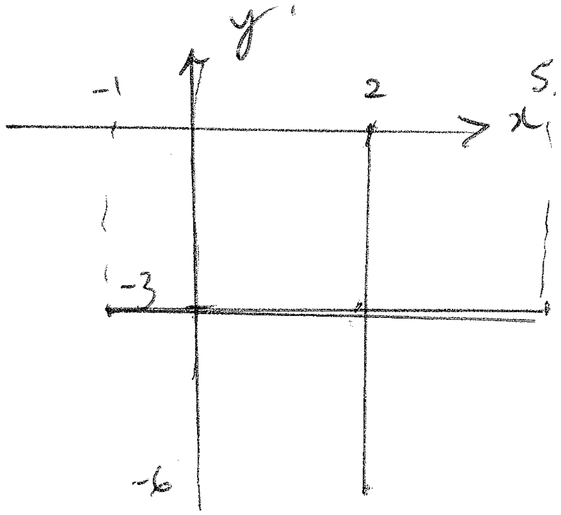
$x^2 - 4x + (-2)^2 + y^2 + 6y + 3^2 = -4 + 4 + 9$

$(x-2)^2 + (y+3)^2 = 9$

Centre $(2,-3)$

radius $= 3u$.

2



$D: -1 \leq x \leq 5$

$R: -6 \leq y \leq 0$.

2

c) $4\cos^2\theta = 1$ $\quad 0 \leq \theta \leq 360°$

$\cos\theta = \pm\frac{1}{2}$

$\theta = 60°, 120°, 240°, 300°$.

3

Question 4 (10 Marks)

(a) $\dfrac{\cos^2 30°}{\sec 150° \sin 45°}$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 \div \left(\frac{-1}{\cos 30°} \times \sin 45°\right)$$

$$= \frac{3}{4} \div \frac{-2}{\sqrt{3}} \times \frac{1}{\sqrt{2}}$$

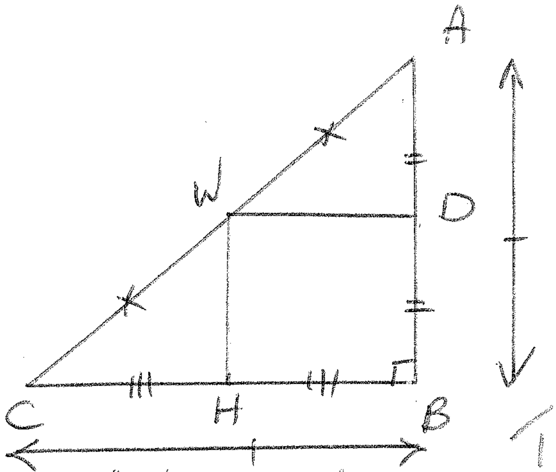
$$= \frac{3}{4} \times \frac{-\sqrt{6}}{2}$$

$$= -\frac{3\sqrt{6}}{8}$$

4

Q4

b)



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One possible proof:

$\frac{AD}{DB} = \frac{AW}{WC} = \frac{BH}{CH}$

(W, D, H are midpoints)

$\therefore WD \parallel CB$ & $WH \parallel AB$

(transversals cut parallel lines in intercepts of equal ratios).

$\therefore \angle ADW = \angle ABC$

$= 90°$

(Corresponding $\angle$'s, $WD \parallel CB$)

$\angle WHC = \angle ABC$

$= 90°$

(Corresp. $\angle$'s, $WH \parallel AB$).

$\because AB = CB$ (given)

$\therefore \triangle ABC$ is an isos. $\triangle$.

$\therefore \angle A = \angle C = 45°$

($\angle$ sum of $\triangle$).

In $\triangle AWD$ & $\triangle CWH$.

$\angle WHC = \angle ADW$

$= 90°$ (proven above)

$\angle C = \angle A = 45°$ (" ")

$CW = AW$ (W bisector of AC)

$\therefore \triangle AWD \equiv \triangle CWH$ (AAS)

$\therefore WH = WD$ (Corresp. sides)

$\therefore WDBH$ is a square

$\because$ 2 pairs of opp sides are parallel and one pair of adjacent $\angle$'s are equal.

/5.

2 Marks for parallel proof
2 Marks " adjacent sides proof
1 Mark for test used.

Question 5 (11 Marks)

a) $f(x) = \frac{x}{9-x^2}$

(i) $9 - x^2 \neq 0$

D.: all real x; $x \neq \pm 3$.

1

(ii) for $f(x)$ to be odd

$f(-x) = -f(x)$

$f(-x) = \frac{-x}{9-(-x)^2}$

$= \frac{-x}{9-x^2} = -f(x)$.

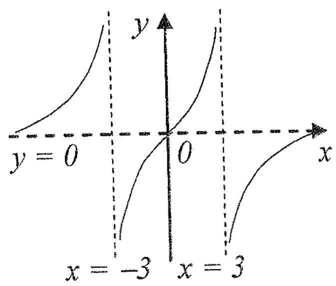
$\therefore$ odd

1

(iii) $x = 0 \quad f(x) = 0. \quad (0,0)$.

$\lim_{x \to +\infty} \frac{x/x^2}{9/x^2 - 1} = 0^+$.

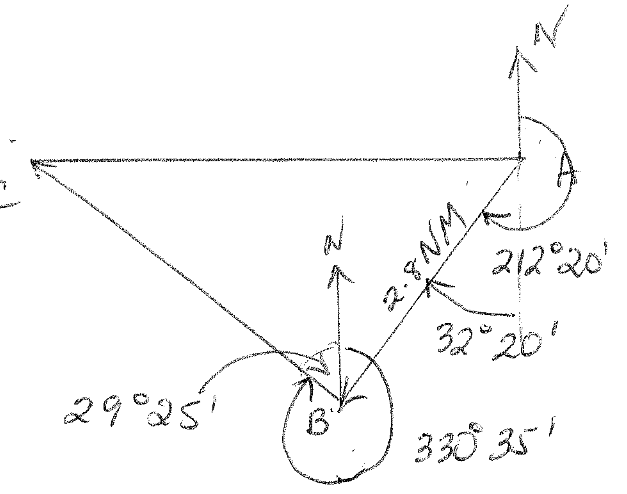
$\lim_{x \to -\infty} f(x) = 0^-$



/3.

Q5 b)

(i)



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(ii) $\angle CAB = 270 - 212°20'$
$= 57°40'$

$\angle CBA = 29°25' + 32°20'$
$= 61°45'$

$\therefore \angle C = 180 - (57°40' + 61°45')$
$= 60°35'$

$$\frac{BC}{\sin 57°40'} = \frac{2.8}{\sin 60°35'}$$

$$BC = \frac{2.8 \sin 57°40'}{\sin 60°35}$$

$= 2.716$

~ 2.7 NM (nearest 1/10)

/5

Question 6 (8 Marks)

a) $\cos x = \frac{-2}{5}$

$\sin x < 0$

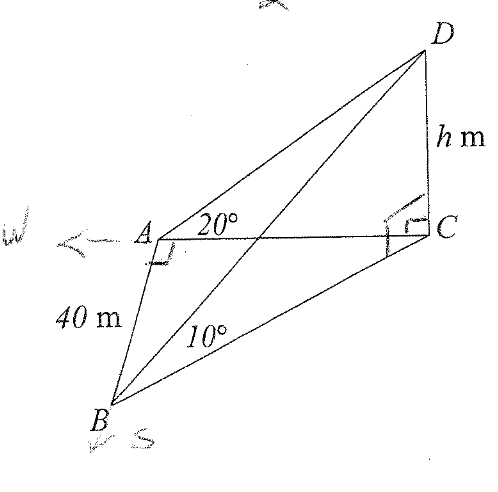
X 1

5, $\sqrt{25-4} = \sqrt{21}$, 2, x 1

$\tan x = \frac{\sqrt{21}}{2}$ 1

/3

b)



$$\tan 20° = \frac{h}{AC}$$

$AC = h \cot 20°$ or $h \tan 70°$ 1

$$\tan 10° = \frac{h}{BC}$$

$BC = h \cot 10°$ or $h \tan 80°$ 1

$\triangle ABC$ is right-angled.

$\therefore$

$BC^2 = 40^2 + AC^2$ 1

$h^2 \cot^2 10° = 40^2 + h^2 \cot^2 20°$

$h^2(\cot^2 10 - \cot^2 20°) = 40^2$

$$h^2 = \frac{1600}{\cot^2 10 - \cot^2 20°}$$ 1

$= 65.00\ldots$

$h = 8.06\ldots$

$h \simeq 8$ m. (nearest metre)

1

/5

Question 7 (18 Marks)

a) Prove.

$\cot^2\theta \sin\theta + \sin\theta = \operatorname{cosec}\theta$

$LHS = \sin\theta(\cot^2\theta + 1)$ 1

$= \sin\theta \times \operatorname{cosec}^2\theta$ 1

$= \sin\theta \times \frac{1}{\sin^2\theta}$ 1/2

$= \operatorname{cosec}\theta$ 1/2

$= RHS.$

/3

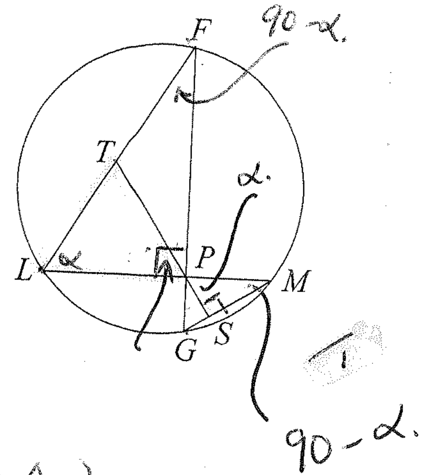
Q7(b)(ii) Show ΔLTP is isosceles. 1)

Let $\angle MLF = \alpha$.

$\therefore \angle LFG = 90 - \alpha$ ($\angle$ sum of Δ)

$\angle LFG = \angle LMG$ ($\angle$ on same chord)

$= 90 - \alpha$.



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In ΔPMS

$\angle MPS = 90 - (90 - \alpha)$

$= \alpha$. ($\angle$ sum of Right Δ)

$\therefore \angle MPS = \angle TPL$ (Vert. Opp $\angle$'s)

$= \alpha$.

$\therefore \Delta LTP$ is isos. (base $\angle$'s are equal).

4

(iii) Show T is mid-pt. of LF

$LT = TP$ (sides of isosc Δ).

~~Fr~~ $\angle TPF = 90 - \alpha$. (Comp. $\angle$'s).

$= \angle LFP$ (proven above).

$\therefore \Delta TFP$ is isosceles (base $\angle$'s are equal)

$\therefore TP = TF$ (sides of isos Δ).

$\therefore LT = TF = TP$.

$\therefore T$ is a mid-pt of LF

3