CSSA - NSW

4 unit mathematics

Trial HSC Examination 1982

1. (i) (a) Show that $(x-3)$ is a factor of the polynomial $4x^3 - 15x^2 + 8x + 3$
(b) Given that the equation $x^4 - 5x^3 + 4x^2 + 3x + 9 = 0$ has a root of multiplicity 2, solve the equation completely.
(ii) The equation $x^4 + px^3 + qx^2 + rx + t = 0$ has roots a, b, c and d. Obtain the monic, quartic (degree 4) equation which has roots $2a, 2b, 2c, 2d$, in terms of x, p, q, r, t.

2. (a) (i) Sketch $y = |x| - 2$ and $y = 4 + 3x - x^2$ on the same number plane.
(ii) Hence or otherwise, solve $\frac{|x|-2}{4+3x-x^2} > 0$.
(b) Sketch showing the main features, the graphs of
(1) $x^2y + y = -4$
(2) $y = x^2 \ln(\frac{1}{x^3})$

3. (a) Show that $\int_0^{\frac{\pi}{2}} \frac{\sin^2\theta}{1+\cos\theta}\, d\theta = \frac{\pi}{2} - 1$.
(b) Decompose $\frac{1}{x^3-1}$ into partial fractions. Hence determine $\int \frac{dx}{x^3-1}$
(c) (i) Show that $\frac{d}{d\theta}[\ln(\sec\theta + \tan\theta)] = \sec\theta$
(ii) By making a suitable substitution show that $\int \frac{dx}{\sqrt{x^2+1}} = \ln(x + \sqrt{x^2+1}) + C$
(iii) Find $\int \sqrt{1+x^2}\, dx$, using the method of integration by parts.

4. (a) The complex number Z is given by $Z = 1 + \frac{1+i}{1-i}$.
Find: **(i)** $\Re(Z)$
(ii) $\Im(Z)$
(iii) $|Z|$
(iv) $\arg Z$
(b) Draw neat labelled sketches to indicate each of the subsets of the Argand diagram described below:
(1) $\{Z : |Z - 2 - i| = 4\}$
(2) $\{Z : \Re(Z + iZ) \geq 2\}$
(c) Determine the locus of the complex number Z given $\arg(Z-2) = \frac{\pi}{4} + \arg(Z+2)$. Sketch this locus on an Argand diagram.
(d) Solve completely $Z^2 + 16 = 30i$

5. (a) Obtain the following results, using the "addition" formulæ or otherwise.
(1) $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$
(2) $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

(b) Given $\sin x + \sin y = a$

$\cos x + \cos y = b$

obtain expressions (in terms of (a, b)) for

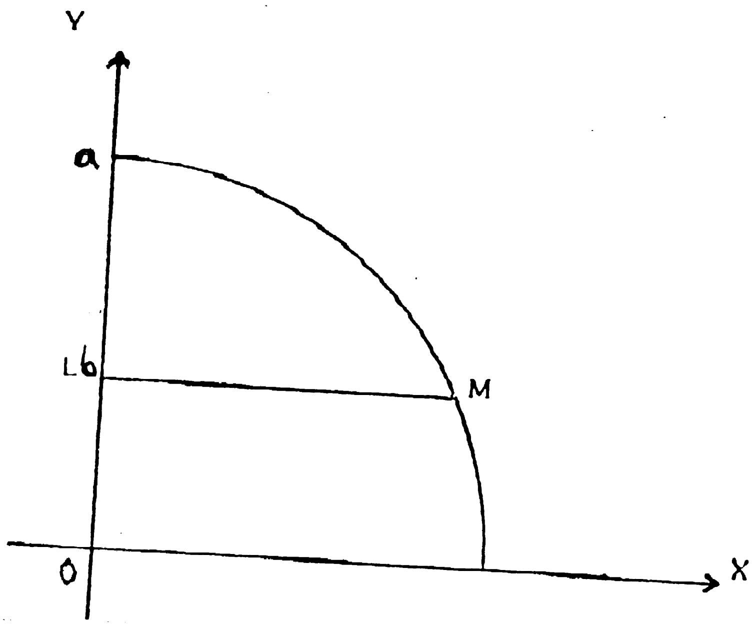
(i) $\tan(\frac{x+y}{2})$

(ii) $\sin(x + y)$

(iii) $\cos \frac{x-y}{2}$

(c) Show that $\frac{1}{\cos(x+h)} - \frac{1}{\cos x} = \frac{2\sin(x+\frac{h}{2})\sin\frac{h}{2}}{\cos x\cos(x+h)}$ where $0 < h < \frac{\pi}{2}$. Hence deduce $\lim\limits_{h\to 0} \frac{1}{h}[\frac{1}{\cos(x+h)} - \frac{1}{\cos x}]$. Interpret the result. Prove that in any triangle ABC, $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

6. The diagram below shows that part of the circle $x^2 + y^2 = a^2$ in the first quadrant. If the horizontal line LM through $(0, b)$, where $0 < b < a$, divides the area, between the curve and the coordinate axes, into two equal parts show that $\sin^{-1}\frac{b}{a} + \frac{b\sqrt{a^2-b^2}}{a^2} = \frac{\pi}{4}$. If the radius of the circle is 1 unit, show that b can be found by solving the equation $\sin 2\theta = \frac{\pi}{2} - 2\theta$ where $\theta = \sin^{-1} b$.



7. The area below the curve $y = bx - ax^2$ (where $a > 0$ and $b > 0$) and above the x-axis is rotated about the y-axis through a complete revolution. Show, using a "slice" technique or otherwise, that the volume of the solid so formed is $\frac{\pi b^4}{6a^3}$ cubic units.

8. (a) Obtain the Cartesian equation for the curve represented parametrically by $x = 5\cos\theta$, $y = \sin\theta$ for $0 \le \theta \le 2\pi$. Identify the curve and sketch it, showing its main features.

(b) A point P is moving in an anti-clockwise direction in a circular path with radius r, as shown below. Given $\frac{d\theta}{dt} = k$ (where k is a positive constant)

(i) Show that the velocity of P at any instant has magnitude rk.

(ii) If S is the projection of P on the x-axis and T is the projection of P on the y-axis prove that both S and T execute simple harmonic motion as P moves around the circle.
(iii) Determine the maximum speed of S and T and where these maximum speeds occur.
(iv) Show that the distance ST is constant and determine its value.
(v) If M is the midpoint of ST determine the locus of M as P moves around the circle.