Number: ....

Roseville College

Year 12

Trial Higher School Certificate Examination

2001

EXTENSION 1 MATHEMATICS

Time Allowed: **2hours, plus 5 minutes reading time.**

Instructions

- All questions are of equal value.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- **Start each question on a new page.** Write your number on each page.
- Staple each question separately

QUESTION 1. Start a new page (12 marks)

(a) Use the substitution $u = x^2 + 2$ to evaluate

$$\int_0^1 \frac{x}{x^2+2}\,dx$$ (3)

b) Solve for x if $\frac{4}{x-2} > 3$ (3)

(c) Find the exact value of $\tan\left(2\tan^{-1}\frac{3}{4}\right)$ (2)

(d) A box contains 12 jellybeans of which 5 are red, 4 are blue and 3 are white. If 3 jellybeans are picked up at once what is the probability that all three are different colours? (2)

(e) Sketch a continuous smooth curve which satisfies the following conditions (2)

$f(0) = 1$

$f'(x) < 0$ and $f''(x) > 0$ for $0 < x < 2$

$f'(2) = 0$

$f(2) = -2$

$f'(x) < 0$ and $f''(x) < 0$ for $x > 2$

QUESTION 2. Start a new page (12 marks)

(a) State the domain and range

$f(x) = 4\sin^{-1}(\frac{x}{3})$ (3)

(b) (i) Show that the equation $x^3 + x - 3 = 0$ has 1 root between 1.2 and 1.3

(ii) Taking 1.2 as the first approximation to the root, use Newton's method once to find a second approximation. (3)

(c) A polynomial $P(x)$ of degree three, has zeros at $x = -2$, $x = -1$ and $x = 1$ and a remainder of 36 when divided by $(x-2)$. Find $P(x)$, expressing it in the form

$$p_0x^3 + p_1x^2 + p_2x + p_3$$ (3)

(d) The tangent at $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$ meets the directrix at K

(i) Show that the coordinates of K are $(\frac{ap^2 - a}{p}, -a)$ (1)

(ii) Prove that angle PSK is a right angle, where S is the focus (2)

QUESTION 3. Start a new page (12 marks)

(a) The acceleration of a particle is given by $4(1+x)$, where x is the displacement from the origin. If initially, the particle is at the origin with a velocity of $2ms^{-1}$,

(i) show that $v = 2(x+1)$ (2)

(ii) show that $x = e^{2t} - 1$ (2)

(iii) find its acceleration after 1 second (2)

(b) Express the solution to the equation
$\sin 2\theta = \sin\theta$ in general form, θ in radians (2)

(c) Find

(i) $\int \frac{dx}{\sqrt{9-4x^2}}$ (2)

(ii) $\int \sin^2 x\,dx$ (2)

QUESTION 4. Start a new page (12 marks)

(a) Show that

$$\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}$$ (2)

(b) Kool has decided to invest in a superannuation fund. She calculates that she will need \$1 000 000 if she is to retire in 20 years time and maintain her present lifestyle. The superannuation fund pays 12% per annum interest on her investments.

(i) Kool invests \$P at the beginning of each year. Show that at the end of the first year her investment is worth \$P(1.12) (1)

(ii) Show that at the end of the third year the value of her investment is given by the expression \$P(1.12)(1.12^2 + 1.12 + 1) (2)

(iii) Find a similar expression for the value of her investment after 20 years and hence calculate the value of P needed to realise the total of \$1 000 000 required for his retirement. (3)

(c) The daily growth of the population of a colony of insects is 10% of the excess of the population over 1.2×10^6. At $t=0$ the population is 2.7×10^3 (Given $P = N + Ae^{0.1t}$)

(i) Determine the population after $3\frac{1}{2}$ days. (2)

(ii) If a scientist checks the population each day, which is the first day on which she should notice the original population has tripled? (3)

QUESTION 5. Start a new page (12 marks)

(a) A sphere is being heated so that its surface area is increasing at a constant rate of 15mm^2 per second. Find the rate of increase of the volume when the radius is 5mm. (3)

(b) Find the value of the constant m if e^{mx} satisfies the differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$$ (3)

(c) A javelin is thrown across level ground from a height of 2m at a speed of 20m/s at an angle of 60° to the horizontal. Taking acceleration due to gravity as 10m/s^2, find

(i) the height reached (2)
(ii) the time the javelin is in the air (2)
(iii) the length of the throw (2)

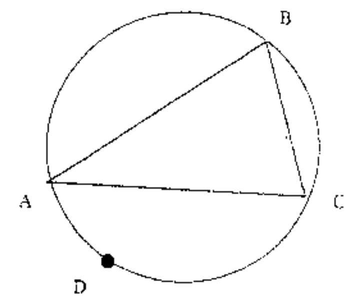
QUESTION 6. Start a new page (12 marks)

(a) A particle moves along a straight line with a velocity given by $\frac{1}{2}v^2 = 18 - 2x^2$, where x is the distance from a fixed point O on the line.

(i) show that the motion is simple harmonic (1)

(ii) find the period and amplitude of the motion of the motion (2)

(b)



ABCD are four points on a circle centre O and radius R units, such that BD is a diameter. A, B, C are joined to form a triangle in which AB=c units, BC=a units and AC=b units. Show, giving reasons, that

(i) $\sin \angle BAC = \frac{a}{2R}$ (3)

(ii) Area $\Delta ABC = \frac{abc}{4R}$ (3)

(c) (i) Express $\sin x + \sqrt{3}\cos x$ in the form $A\sin(x+\alpha)$ (2)

(ii) Use this to solve $\sin x + \sqrt{3}\cos x = \sqrt{3}$ for $0 \le x \le 2\pi$ (2)

QUESTION 7. Start a new page (12 marks)

(a) Prove that for all positive integers n, $9^{n+2} - 4^n$ is divisible by 5. (4)

(b) Evaluate

$$\int_0^{\frac{1}{2}} \frac{dx}{1+4x^2}$$ (3)

(c) The line $y = 2x + 2$ cuts the line segment AB at some point C. If A is the point (-2,3) and B is the point (4,3) find the ratio of AC:CB. (2)

(d) If $y = \frac{1}{2}(e^x - e^{-x})$, show that $x = \log_e(y + \sqrt{y^2 + 1})$ (3)

END OF PAPER