Neap.

HSC Trial Examination 2000

Mathematics

3 Unit (Additional)
and
3/4 Unit (Common)

Time Allowed – Two hours
(Plus 5 minutes reading time)

This paper must be kept under strict security and may only be used on or after the afternoon of Thursday 3 August, 2000, as specified in the NEAP Examination Timetable.

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on page 9.
- Board-approved calculators may be used.
- Answer each question in a SEPARATE Writing Booklet.
- You may ask for extra Writing Booklets if you need them.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2000 Mathematics 3 Unit (Additional) and 3/4 Unit (Common) Higher School Certificate Examination.

QUESTION 1. Use a SEPARATE writing booklet.

Marks

(a) Let $A\ (-3, 6)$ and $B\ (1, 10)$ be points on the number plane. Find the coordinates of the point C, which divides the interval AB externally in the ratio $5 : 3$. **2**

(b) Find the obtuse angle between the lines $3y = 2x + 1$ and $y = -3x + 5$, correct to the nearest degree. **3**

(c) Use the substitution $u = 2x - 1$ to evaluate $\int_0^1 x(2x-1)^4 dx$. **4**

(d) Solve the inequality $\frac{x}{x-3} < 4$. **3**

QUESTION 2. Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\int_{-3}^{3} \frac{1}{9+x^2}\,dx$. **3**

(b) Consider the function $y = \cos^{-1}(2x) - \frac{\pi}{2}$. **3**

(i) State the domain of this function.

(ii) State the range of this function.

(iii) Sketch the graph of this function.

(c) Find $\lim_{\theta \to 0} \frac{\theta + \sin 2\theta}{3\theta}$. **2**

(d) Use the table of standard integrals to find $\int \frac{dx}{\sqrt{x^2-4}}$. **1**

(e) Consider the polynomial $P(x) = x^3 - 5x + c$. **3**

(i) Find the value of c if $x+2$ is a factor of $P(x)$.

(ii) For this value of c, find $Q(x)$ such that $P(x) = (x+2)Q(x)$.

QUESTION 3. Use a SEPARATE writing booklet. Marks

(a) If α, β, γ are the roots of the equation $x^3 + 2x^2 - x - 5 = 0$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. **2**

(b) Find the coefficient of x^2 in the expansion of $(3 - 2x)(2 + x)^4$. **3**

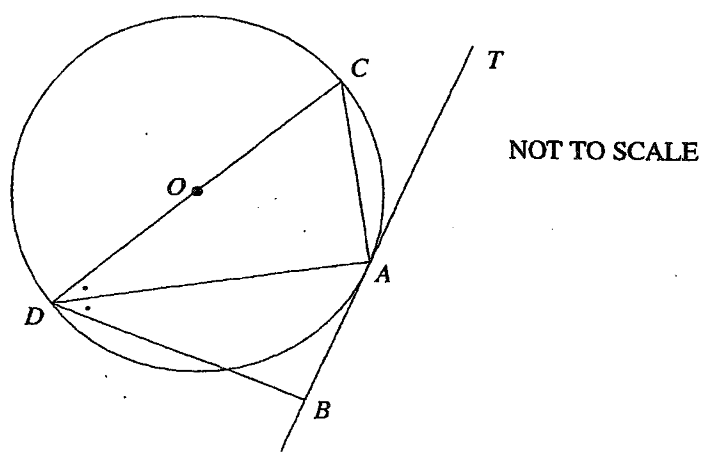
(c) A tennis club consists of 20 members, 12 men and 8 women. A committee of four people is to be chosen randomly. How many committees are possible if **4**

(i) there is to be equal numbers of men and women?

(ii) there is to be a majority of women on the committee?

(iii) the youngest member of the club must be on the committee?

(d) **3**



O is the centre of a circle. TAB is a tangent to the circle at A. AD bisects the angle CDB.
Copy or trace the diagram into your Writing Booklet.
Prove that the angle ABD is a right angle.

QUESTION 4. Use a SEPARATE writing booklet. **Marks**

(a) Due to the general ageing of the community, the numbers in the local high school were declining at a rate proportional to the amount by which the numbers in the school exceeded 600. This is expressed by the equation **4**

$$\frac{dN}{dt} = k(N - 600),$$

where N is the number of students enrolled t years after 1990.

There were 1100 students enrolled at the beginning of 1990 and 900 students enrolled at the beginning of year 2000.

(i) Prove that $N = 600 + Ae^{kt}$ satisfies this equation.

(ii) Find the value of A.

(iii) Find the value of k correct to 4 significant figures.

(iv) How many students would you expect to be enrolled at the beginning of the year 2010 if the decline continued under the same conditions?

(b) Prove, using mathematical induction, that $7^n - 4^n$ is divisible by 3, where n is a positive integer. **4**

(c) (i) Using the identities for the expansions of $\sin(A + B)$, $\sin 2A$ and $\cos 2A$, prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. **4**

(ii) Hence solve the equation $3\sin\theta - 4\sin^3\theta = -1$ for $0 \le \theta \le 2\pi$.

QUESTION 5. Use a SEPARATE writing booklet.

Marks

(a) A particle P moves in a straight line in simple harmonic motion. The acceleration in metres per second per second is given by **6**

$$\ddot{x} = 2 - 3x$$

where x metres is the displacement of the particle from the origin.

Initially the particle is at $x = 1$ moving with a velocity of $\sqrt{5}$ m s^{-1}.

(i) Using integration show that the velocity v m s^{-1} of the particle is given by

$$v^2 = 4 + 4x - 3x^2.$$

(ii) Find the amplitude of motion.

(iii) Find the centre of motion.

(iv) Find the maximum speed of the particle.

(v) Find the period of the motion.

(b) (i) Prove that $e^{2x} - e^x = 56$ has a root between 2 and 3. **6**

(ii) Taking $x = 2$ as an approximation, use one application of Newton's method to find a better approximation correct to three significant figures.

(iii) By considering $e^{2x} - e^x = 56$ as a quadratic equation in e^x, solve the equation, giving your answer correct to three significant figures.

QUESTION 6. Use a SEPARATE writing booklet.

Marks

(a) Twelve students, six boys and six girls, sat for the HSC French examination. After the exam, they sat randomly in a circle to discuss the exam. Find the probability that: **3**

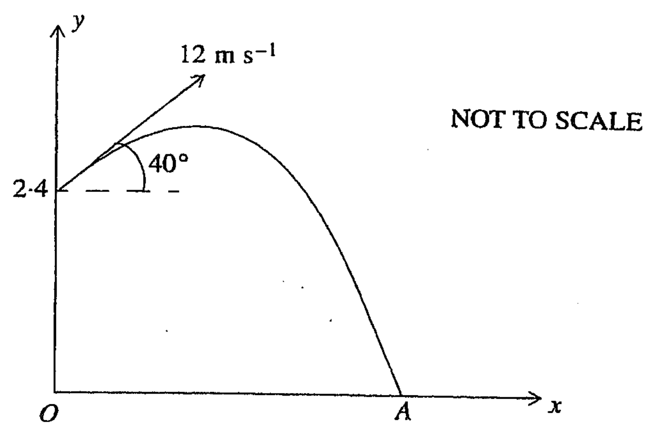
(i) no two boys are sitting next to each other.

(ii) the two top students (based on their school assessment) are sitting next to each other.

(b) If r is a positive integer and $1 \le r \le 10$, find the largest value of r which satisfies **3**

$$\binom{10}{r} 3^{10-r} \times 2^r > \binom{10}{r-1} 3^{11-r} \times 2^{r-1}.$$

(c) **6**



In an Olympic trial, a shot putter releases the shot from a height of 2·4 metres above ground level at an angle of 40° to the horizontal, and with a speed of 12 metres per second.

Take the origin O at a point on the ground directly under the point of release of the shot.

The equations of motion of the shot are

$$\ddot{x} = 0, \quad \ddot{y} = -g.$$

(i) Using calculus, show that the position of the shot at time t is given by

$$x = 12\cos 40° t, \quad y = 2{\cdot}4 + 12\sin 40° t - \frac{1}{2}gt^2.$$

(ii) The shot lands at a point A on the ground. Find the length of OA to the nearest centimetre. (Take $g = 9{\cdot}8$).