Name:______________________ Maths Class:__________________

SYDNEY TECHNICAL HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE

2007

MATHEMATICS

Time Allowed: 3 hours plus 5 mins reading time

Instructions:

- Write your name and class at the top of this page, and at the top of each answer sheet
- At the end of the examination this examination paper must be attached to the front of your answers
- All questions are of equal vale and may be attempted
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.

(For Markers Use Only)

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Total

Question 1 (12 Marks) **Marks**

a) Find the value of $\dfrac{16.2^2}{14.7-8.1}$ correct to 3 significant figures — 2

b) Simplify $4\sqrt{32}-2\sqrt{8}$ — 2

c) Write down the <u>exact</u> value of $\sin\dfrac{5\pi}{4}$ — 2

d) Simplify $4(2x+1)-(x^2+2x-3)$ — 2

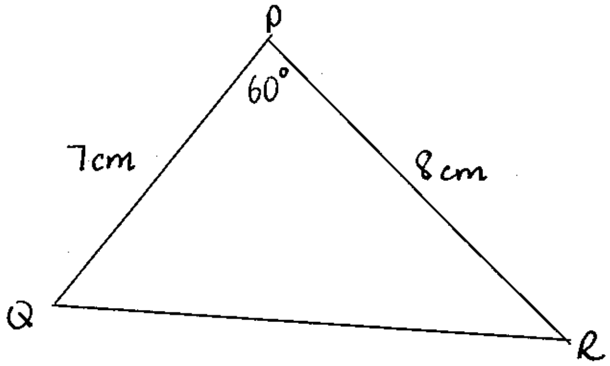
e) Fully factorise $2x^3-2y^3$ — 2

f) Find the primitive of $x^2-2x+\dfrac{1}{x}$ — 2

Question 2 **(12 marks)** **Start a new page** **Marks**

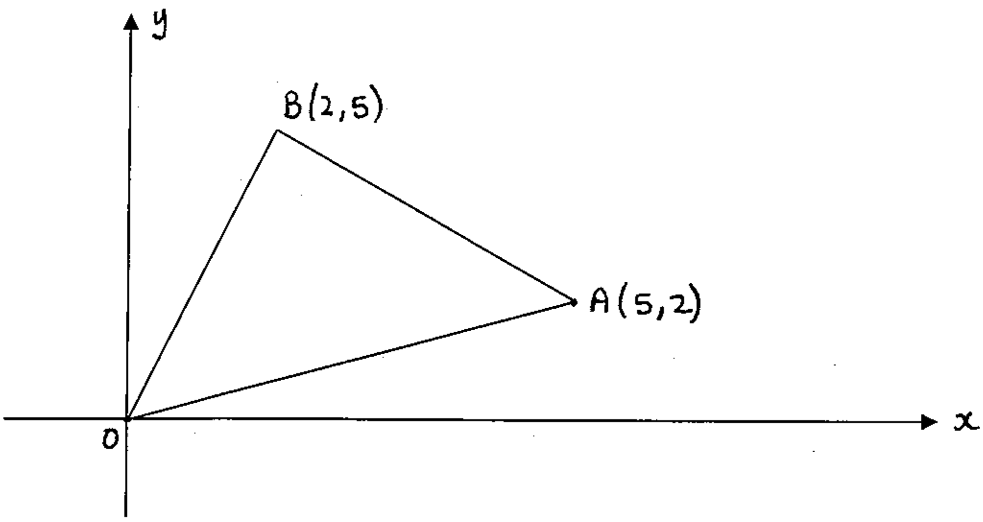
a) Solve $|1-2x| > 7$ 2

b) Find the exact area of ΔPQR 2



Not to scale

c)



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The points **0 (0,0)** ***A (5, 2)*** and ***B (2, 5)*** are the vertices of a triangle ABO.

(i) Find the distance ***OA*** and the distance ***OB*** 2

(ii) Show that the equation AB is $x+y-7=0$ 2

(iii) Calculate the perpendicular distance from O to AB 2

(iv) Find the midpoint, M, of AB 1

(v) Without any more calculations what is the distance of OM, give a reason for answer. 1

Question 3 (12 marks) Start a new page **Marks**

a) Differentiate with respect to x:

i) $y = x^2 - 4x + 1$ 1

ii) $y = (e^{2x} + 1)^2$ 2

iii) $y = x^2 \cos 2x$ 2

b) i) Find $\int \frac{4}{4x+1} dx$ 1

ii) Evaluate $\int_0^{\frac{\pi}{4}} 2\sec^2 x \quad dx$ 2

c) The roots of the equation $x^2 + 5x = 7$ are α *and* β

Find the value of

i) $\alpha + \beta$ 1

ii) $\alpha\beta$ 1

iii) $\alpha^2 + \beta^2$ 2

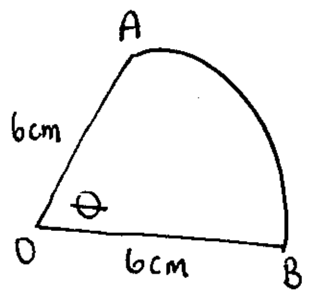
Question 4 **(12 marks)** **Start a new page** **Marks**

a) A ship sails from Port A 70 nautical miles due west to Port B. It then proceeds 40 nautical miles on a bearing of 120°T to Port C.

i) Find the distance of Port C from Port A (correct to 2 decimal places) 2

ii) Find the bearing of Port C from Port A (correct to the nearest degree). 2

b)

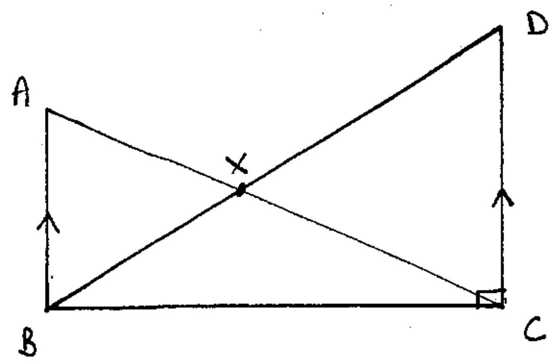


The perimeter of sector AOB is 13.5cm

i) Find the size of $\angle AOB$, correct to the nearest minute 2

ii) Find the area of sector AOB 2

c)



In the diagram AB is parallel to CD and $CD \perp BC$ 2

i) Show that triangle AXB is similar to triangle CXD 2

ii) Given $AB:DC=2:3$ Show that $9(BX)^2 = 4(XD)^2$ 2

Question 5 **(12 marks)** **Start a new page** **Marks**

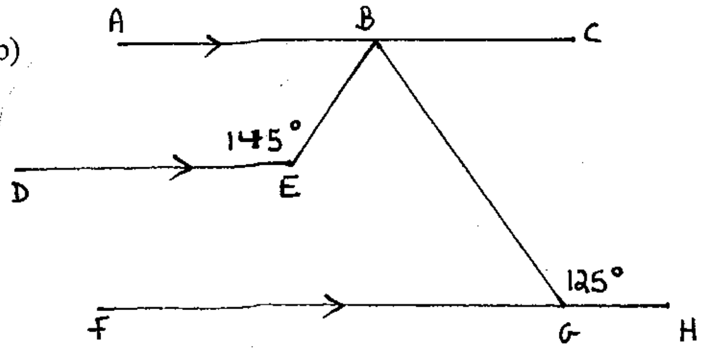
a) For the sequence 95, 91, 87 …………, find,

i) An expression for the *nth* term, ***Tn***, in its simplest form 2

ii) Which term is the first term less than zero 2

iii) What is the sum of all the terms greater than zero 2

b)



In the diagram given

AC // DE and AC // FH

$\angle DEB = 145°$ and $\angle BGH = 125°$

Find the size of $\angle EBG$, giving reasons 2

c) i) For what values of x will a limiting sum exist for the geometric series,

$3 - 12x + 48x^2 -, \ldots\ldots\ldots\ldots\ldots?$ 2

ii) Find the value of x for which the limiting sum is 9. 2

Question 6 **(12 marks)** **Start a new page** **Marks**

a) Find the equation of the *normal* to the curve $y = \ln(2x+3)$ at the point where $x = -1$. 3

b) The function $f(x)$ is given by $f(x) = 2x(x-3)^2$

i) Find the coordinates of the points where the curve $y = f(x)$ cuts the x-axis 2

ii) Find the coordinates of any turning points on the curve $y = f(x)$, and determine their nature 4

iii) Sketch the curve $y = f(x)$ in the domain $-1 \le x \le 4$ 2

iv) Hence solve $2x^3 - 12x^2 + 18x - 8 = 0$ 1

Question 7 (12 marks) Start a new page **Marks**

a) What is the value of $\log_2 \sqrt{8}$ 1

b) Given $3x^2 + 4x + 5 \equiv A(x+1)^2 + B(x+1) + C$

Find the value of the constants A, B and C 3

c) Consider the function $f(x) = x\sin^2 x$

i) Copy and complete the table below in your writing booklet. Values of $f(x)$ are given to 3 decimal places where appropriate.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$f(x)$	0	0.393	1.571		0

1

ii) Using Simpson's Rule with five function values, evaluate

$\int_0^{\pi} x\sin^2 x\,dx$, correct to 2 decimal places. 3

d) i) Sketch the curve $y = 1 - \cos 2x$, $0 \le x \le 2\pi$ 2

ii) Find the area bounded by the curve, $y = 1 - \cos 2x$, the x- axis and the lines $x = 0$ and $x = \pi$ 2

Question 8 (12 marks) Start a new page **Marks**

a) Given $\log_a x = 0.417$ and $\log_a y = 0.609$ find the value of

i) $\log_a (ax)$ 2

ii) $\log_a \dfrac{x^2}{y}$ 2

b) The region beneath the curve $y = 3e^{-2x} + 1$ which is above the x – axis and between the lines $x = 0$ and $x = 1$ is rotated about the x – axis

i) Sketch the region 2

ii) Find the volume of the solid revolution 4

c) The price of one gram of gold, \$P, was studied over the period of t days.

i) Throughout the period of study $\frac{dP}{dt} > 0$

What does this say about the price of gold? 1

ii) If it was noted over this time that the rate of change in the price of gold increased. What does this statement imply about $\dfrac{d^2P}{dt^2}$? 1

Question 9 (12 marks) Start a new page **Marks**

a) For what values of k does the equation $x^2 - (k+2)x + 1 = 0$ have;

i) Equal roots **2**

ii) No real roots **1**

b) The population of a town at the end of t years is given by $P = Ae^{kt}$, where A and k are constants.

After 1 year the population is 1060

i) Find the value of A if the population was initially 1020 **1**

ii) Find the value of k **2**

iii) Calculate the population after 12 years **2**

iv) What is the **rate** of increase in the population after 12 years **2**

v) How many years will it take the population to double? **2**

Question 10 (12 marks) Start a new page **Marks**

a) Shrek borrows \$1 000 000 from the Muffin man, at 7.8% p.a. monthly reducible interest to buy a new swamp in Far-Far away land.

He repays the loan in equal monthly repayments of \$8000.

i) Write an expression for the amount Shrek owes immediately **before** the 1^{st} repayment 1

ii) Show that Shrek owes the Muffin man after n months: 3

$$An = 1000\,000(1.0065)^n - 8000\left[\frac{1.0065^n - 1}{0.0065}\right]$$

iii) How many months does Shrek take to repay half the loan to the Muffin man? 2

b) A new grain silo with a capacity of $4000m^3$ is to be constructed on a farm. The silo is a fully enclosed cylinder and is to be constructed from concrete.

To Save costs, the farmer wants to minimise the surface area of the silo.

i) Write an expression for the volume of the silo in terms of radius (r) and height (h) 1

ii) Write an expression for the surface area (A) of the concrete silo in terms of r 2

iii) Show that $\dfrac{dA}{dr} = \dfrac{4\pi r^3 - 8000}{r^2}$ 1

iv) Hence, find the dimensions of the silo to minimise the surface are of the silo. Express your dimensions to 1 decimal place. 2

END OF EXAM

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

2007 - Maths Trial

Answers (2u)

Question 1

a) $39.7636\ldots$ (1)

$= 39.8$ (1)

b) $4\sqrt{32} - 2\sqrt{8} = 16\sqrt{2} - 4\sqrt{2}$ (1)

$= 12\sqrt{2}$ (1)

c) $\sin\frac{5\pi}{4} = -\sin\pi/4$ (1)

$= -1/\sqrt{2}$ (1)

d) $4(2x+1) - (x^2+2x-3)$

$= 8x + 4 - x^2 - 2x + 3$ (1)

$= 6x - x^2 + 7$ (1)

e) $2x^3 - 2y^3$

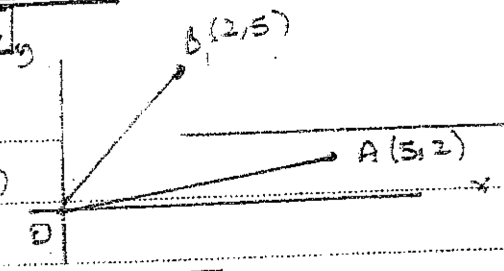
$= 2(x^3 - y^3)$ (1)

$= 2(x-y)(x^2+xy+y^2)$ (1)

f) $\int x^2 - 2x + 1/x \, dx$

$= \frac{x^3}{3} - x^2 + \ln x + c$ (2)

c)



(i) $d_{OA} = \sqrt{29}$ (1)

$d_{OB} = \sqrt{29}$ (1)

(ii) $m_{AB} = \frac{2-5}{5-2}$

$= -1$

$\therefore y - 5 = -1(x-2)$

$y - 5 = -x + 2$

$x + y - 7 = 0$

(iii) pt $(0,0)$ line $x + y - 7 = 0$

$d_\perp = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$ (1)

$= \frac{|0+0-7|}{\sqrt{1^2+1^2}}$

$= \frac{7}{\sqrt{2}}$ (1)

(iv) Midpt $m(3.5, 3.5)$ (1)

v) dist $om = \frac{7}{\sqrt{2}}$ as ΔAOB is isosceles $\therefore$ om is $\perp$ bisector of AB.

(1) $\rightarrow$ must have a suitable reason ...

Question 2

a) $|1-2x| > 7$

$1 - 2x > 7$ $\quad$ $-1 + 2x > 7$

$-2x > 6$ $\quad$ $2x > 8$

$x < -3$ (1) $\quad$ $x > 4$ (1)

b) $A = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 7 \times 8 \times \sin 60°$

$= \frac{1}{2} \times 7 \times 8 \times \sqrt{3}/2$ (1)

$= 14\sqrt{3}$ cm^2 (1)

Question 3

a) i. $\frac{dy}{dx} = 2x - 4$ (1)

ii. $\frac{dy}{dx} = 2(e^{2x}+1)^1 \cdot 2e^{2x}$

$= 4e^{2x}(e^{2x}+1)$ (2)

iii. $\frac{dy}{dx} = \cos 2x\,(2x) + x^2(-2\sin 2x)$

$= 2x\cos 2x - 2x^2 \sin 2x$ (2)

b) i. $\int \frac{4}{4x+1}\,dx = \ln(4x+1) + c$ (1)

ii. $\int_0^{\pi/4} 2\sec^2 x\,dx = \left[2\tan x\right]_0^{\pi/4}$ (1)

$= 2\left[\tan\frac{\pi}{4} - \tan 0\right]$

$= 2\left[1 - 0\right]$

$= 2$ (1)

c) $x^2 + 5x - 7 = 0$ $a=1, b=5, c=-7$

(I) $\alpha + \beta = -b/a$ (II) $\alpha\beta = c/a$

$= -5$ (1) $= -7$ (1)

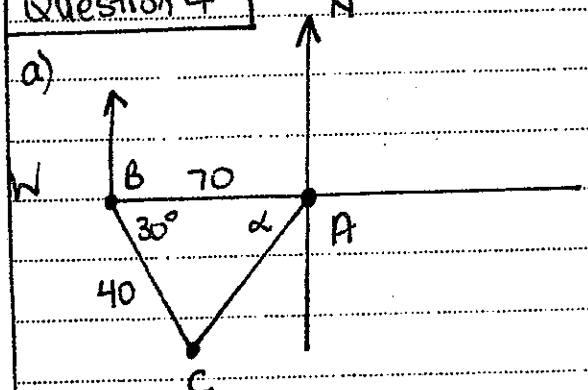
(III) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ (1)

$= (-5)^2 - 2(-7)$

$= 39$ (1)

Question 4

a)



$AC^2 = 70^2 + 40^2 - 2 \times 70 \times 40 \times \cos 30°$

$AC = 40.62336...$

ii. $\frac{\sin\alpha}{40} = \frac{\sin 30°}{40.62}$

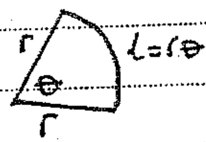
$\sin\alpha = 0.49236...$

$\alpha = 29°30'$

Bearing $= 270° - 29°30'$

$= 240°30'$

b)



$13.5 = 2r + r\theta$

$13.5 = 12 + 6\theta$ (1)

$r = 6$ $0.25 = \theta$ (rad)

$\theta = 0.25 \times \frac{180°}{\pi} = 14°19'$ (1)

ii) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 6^2 \times 0.25$

$= 4.5\text{ cm}^2$

c) In ΔAXB and ΔCXD

(1) $\angle BAX = \angle DCX$ (alternate angles $AB \parallel CD$)

$\angle AXB = \angle CXD$ (vertically opposite)

$\therefore \Delta AXB \,|||\, \Delta CXD$ (equiangular) (1)

ii) $\frac{AB}{CD} = \frac{XB}{XD}$ corresponding sides of $|||$ Δ's in propositio (1)

$\frac{2}{3} = \frac{XB}{XD}$

$2XD = 3BX$

$4(XD)^2 = 9(BX)^2$

both = (1)

Question 5

a) 95, 91, 87, ... $a = 95$, $d = -4$ AP

i) $T_n = a + (n-1)d$ (1)

$= 95 + (n-1)(-4)$

$= 95 - 4n + 4$ (1)

$= 99 - 4n$

ii) $T_n < 0$

$99 - 4n < 0$

$4n > 99$

$n > 24.75$ (1)

$\therefore$ 25th term is 1st negative. (1)

iii) ie 24 terms > 0 $n = 24$ $a = 95$ $d = -4$

$S_n = \frac{n}{2}(2a + (n-1)d)$ (1)

$= \frac{24}{2}(2(95) + 23(-4))$ (1)

$= 1176$

b) $\angle CBG + 125° = 180°$ (co-interior angles AC ∥ FH)

$\angle CBG = 55°$ (1) both

$\angle ABE + 145° = 180°$ (co-interior angles AC ∥ DE)

$\angle ABE = 35°$

$35 + 55 + \angle EBG = 180°$ (straight) (1)

$\angle EBG = 90°$

c) $a = 3$ $r = -4x$

i) $\therefore$ $-1 < r < 1$ (1)

$-1 < -4x < 1$

$1/4 > x > -1/4$

$\therefore$ $-1/4 < x < 1/4$ (1)

ii) $S_\infty = \frac{a}{1-r}$ $9 = \frac{3}{1+4x}$ (1)

$9(1+4x) = 3$

$x = -1/6$ (1)

Question 6

a) $\frac{dy}{dx} = \frac{2}{2x+3}$ (1) at $x = -1$ $y = 0$

$m_T = 2$

$m_N = -1/2$ (1)

eq: $y - 0 = -1/2(x+1)$

$2y = -x - 1$ (1)

$x + 2y + 1 = 0$

b) $f(x) = 2x(x-3)^2 = 2x^3 - 12x^2 + 18x$

(i) x-int $y = 0$

(0, 0) (1) and (3, 0) (1)

(ii) Stat pts $f'(x) = 0$

$f'(x) = 6x^2 - 24x + 18 = 0$

$6(x-3)(x-1) = 0$

$x = 3$ $x = 1$

$y = 0$ $y = 8$

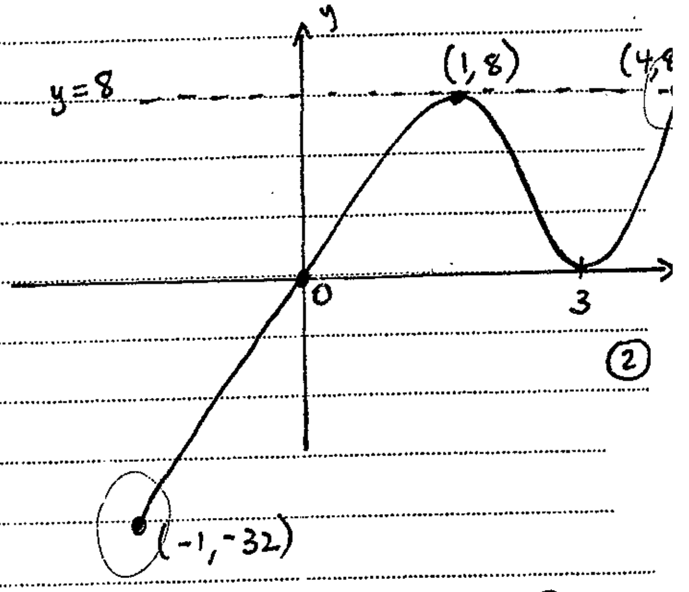
test

x	0	1	2
y'	14	0	-12

x	2	3	4
y'	-12	0	/

MAX (1, 8) 1 MIN (3, 0) 1

(iii) End pts (−1, −32) & (4, 8)



(2)

iv) $2x^3 - 12x^2 + 18x = 8$

$\therefore$ $x = 1$, $x = 4$ (1)

Question 7

a) $\log_2 \sqrt{8} = \frac{1}{2}\log_2 8$

$= \frac{1}{2} \times 3\log_2 2$

$= 1.5$ (1)

b) $3x^2 + 4x + 5 \equiv A(x^2 + 2x + 1) + Bx + B + C$

equating

$\underline{3 = A}$

$4 = 2A + B$

$4 = 6 + B$

$\underline{B = -2}$

$5 = A + B + C$

$5 = 3 - 2 + C$

$\underline{C = 4}$

$\therefore A = 3$ (1), $B = -2$ (1), $C = 4$ (1)

c) | $\frac{3\pi}{4}$ |
|---|
| 1.178 |

(1)

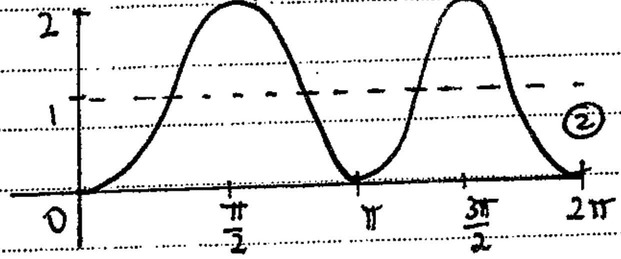
ii. $\frac{h}{3}\left[F + L + 4m\right]$ (1)

$\frac{\pi/4}{3}\left[0 + 1.571 + 4 \times 0.393\right]$ (1)

$+ \frac{\pi}{12}\left[1.571 + 0 + 4 \times 1.178\right]$

$= 2.46772\ldots$ (1)

$= 2.47$ (2dp)

c) 

$y = 1 - \cos 2x$ (2)

ii. $\int_0^{\pi} 1 - \cos 2x \, dx$ (1)

$= \left[x - \frac{1}{2}\sin 2x\right]_0^{\pi}$

$= \pi - \frac{1}{2}\sin 2\pi - \left[0 - 0\right]$

Question 8

a) $\log_a(ax) = \log_a a + \log_a x$ (1)

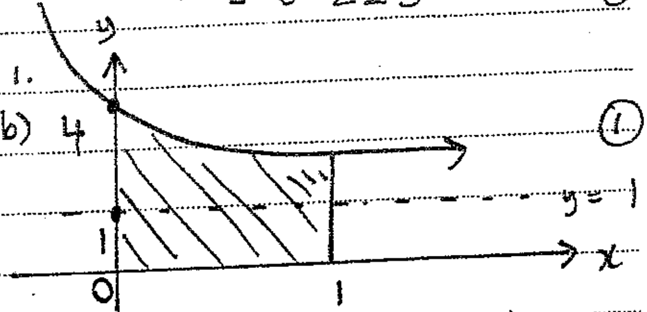
$= 1 + 0.417$

$= 1.417$ (1)

ii) $\log_a \frac{x^2}{y} = 2\log_a x - \log_a y$ (1)

$= 2(0.417) - 0.609$

$= 0.225$ (1)

b) i. 

(1)

ii. $V_x = \pi \int y^2 \, dx$ (1)

$= \pi \int_0^1 (3e^{-2x} + 1)^2 \, dx$ (1)

$= \pi \int_0^1 9e^{-4x} + 6e^{-2x} + 1 \, dx$

$= \pi \left[\frac{9}{-4}e^{-4x} + \frac{6e^{-2x}}{-2} + x\right]_0^1$ (1)

$= \pi \left[\frac{9}{-4}e^{-4} - 3e^{-2} + 1 - \left(\frac{-9}{4} - 3\right)\right]$

$= \pi \left[-\frac{9}{4}e^{-4} - 3e^{-2} + \frac{25}{4}\right]$

c) $\frac{dP}{dt} > 0$ price of gold increasing (1)

ii) $\frac{d^2P}{dt^2} > 0$ (1)

Question 9

a) $x^2 - (k+2)x + 1 = 0$

i) Equal roots $\Delta = 0$

$b^2 - 4ac = 0$

$(k+2)^2 - 4(1)(1) = 0$ (1)

$k^2 + 4k + 4 - 4 = 0$

$k^2 + 4k = 0$

$k(k+4) = 0$

$k = 0, \; k = -4$ (1)

ii) $\Delta < 0, \quad -4 < k < 0$ (1)

b) $t = 0 \quad P = 1020$

$\therefore A = 1020$ (1)

ii. $t = 1 \quad P = 1060$

$1060 = 1020e^{k(1)}$

$\frac{1060}{1020} = e^k$ (1)

$\ln\left(\frac{106}{102}\right) = k$

$k = \ln\left(\frac{106}{102}\right)$ (1)

$\doteq 0.038466\ldots$

iii) $t = 12 \quad P = ?$ (1)

$P = 1020e^{k \cdot 12} \quad k = \ln\left(\frac{106}{102}\right)$

$= 1618.335\ldots$

$\doteq 1618$ (1)

iv) rate $= \frac{d}{dt}$

(1)

$\frac{dP}{dt} = k.(1020e^{kt}) \quad k = \ln\left(\frac{106}{102}\right) \quad t = 12$

$= 62.2513\ldots$

$= 62.25$ people/yr. (1)

v) $t = ? \quad P = 2A$

$2A = Ae^{kt} \quad k = \ln\left(\frac{106}{102}\right)$

$2 = e^{kt}$ (1)

$\ln 2 = \ln e^{kt}$

$\ln 2 = k.t$

$t = \ln 2 \div k$

$= 18.0196\ldots$ (1)

$\doteq 18$ years.

Question 10

a) monthly repayment = 8000

Principal = 1 000 000

rate = 7.8% ÷ 12 (monthly)

= 0.0065

(i) $1\,000\,000(1.0065)$ ①

(ii) $A_1 = 1\,000\,000(1.0065) - 8000$

$A_2 = A_1(1.0065) - 8000$

$= 1\,000\,000(1.0065)^2 - 8000(1.0065) - 8000$ ①

$A_n = 1\,000\,000(1.0065)^n - 8000\left[1.0065^{n-1} + 1.0065^{n-2} \dots + 1\right]$ ①

$= 1\,000\,000(1.0065)^n - 8000\left[\frac{a(r^n - 1)}{r-1}\right]$ ①

$a = 1 \quad r = 1.0065 \quad n = n$

$= 1\,000\,000(1.0065)^n - 8000\left[\frac{1.0065^n - 1}{0.0065}\right]$

(iii)

$500\,000 = 100\,000(1.0065)^n - 1230769\left[1.0065^n - 1\right]$

① $500\,000 = 1\,000\,000(1.0065)^n - 1230769(1.0065)^n + 1230769$

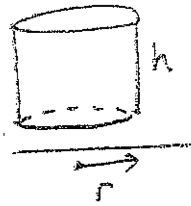
$230769(1.0065)^n = 730769$

$1.0065^n = 3.1666\dots$

$\log 1.0065^n = \log 3.1666\dots$

$n\left[\log 1.0065\right] = \log 3.166\dots$

$n = \log 3.166 \div \log 1.0065$

b) 

$V = 4000\text{ m}^3$

$V = \pi r^2 h$

(i) $4000 = \pi r^2 h$ ①

(ii) $\therefore A = 2\pi r^2 + 2\pi r h$ ① $\quad h = \frac{4000}{\pi r^2}$

$\therefore A = 2\pi r^2 + 2\pi r\left[\frac{4000}{\pi r^2}\right]$

$= 2\pi r^2 + \frac{8000}{r}$ ①

$= 2\pi r^2 + 8000 r^{-1}$

(iii) $\frac{dA}{dr} = 4\pi r - 8000 r^{-2}$ ①

$= \frac{4\pi r^3 - 8000}{r^2}$

(iv) Min Surface Area $dA/dr = 0$

$\frac{4\pi r^3 - 8000}{r^2} = 0$

$4\pi r^3 = 8000$

$r^3 = \frac{8000}{4\pi}$

$r = \sqrt[3]{\frac{2000}{\pi}}$ ①

$\doteqdot 8.6025\dots$

$\doteqdot 8.6$ (1dp)

test

r	8	8.6025...	9
$\frac{dA}{dr}$	\	—	/

min

∴ dimensions are ①

$r \doteqdot 8.6\text{ m} \quad h = 17.2\text{ m}$