Shahbaz Skah.

Homebush Boys' High School

2002
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics
Extension 1

+ Answers

General Instructions

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a blue or black pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.

Instructions for use of booklets

Use the answer booklets as follows

- Booklet 1 Question 1
- Booklet 2 Question 2
- Booklet 3 Question 3
- Booklet 4 Questions 4 & 5
- Booklet 5 Questions 6 & 7

Total marks (84)

- Attempt Questions 1-7
- All questions are of equal value

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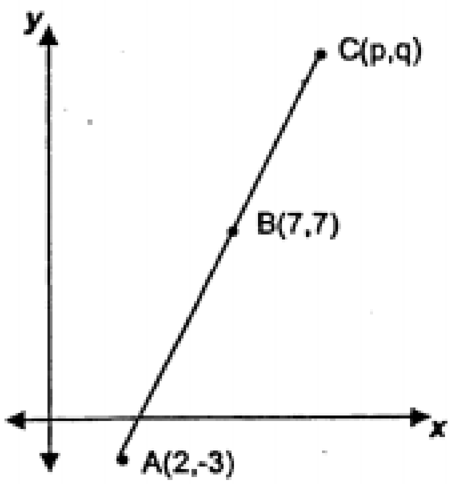
Attempt Questions 1-7
All Questions are of equal value

Begin each question on a NEW SHEET of paper, writing your name and question number at the top of the page. Extra paper is available.

QUESTION 1 (12 MARKS) Answer Question 1 in Booklet 1 **Marks**

(a) Find the co-ordinates of the point C (p, q) below, given that AC : CB = 8:3 **2**



(b) Evaluate $\sum_{n=3}^{16} 3n-1$ **1**

(c) The times when a particle is stationary are given by the solutions to the equation $e^t - 4t^2 = 0$. Given that one solution is approximately t = 4, find a better approximation (correct to one decimal place) using one step of Newtons Method. **2**

(d) Differentiate $(1+x^2)\tan^{-1} x$ **2**

(e) Use the table of Standard Integrals to evaluate: $\int_5^8 \frac{dx}{\sqrt{x^2-16}}$ **2**

(f) Solve $\frac{x+3}{x-1} \le 2$ **3**

$e^t \approx 4t^2$

QUESTION 2 (12 MARKS) **Answer Question 2 in Booklet 2** **Marks**

(a) Joanne has 12 coloured beads, 4 are red and the remainder are 8 other different colours.

(i) Joanne lays the beads out on a table in a straight line. How many distinct arrangements of the beads in a straight line are possible? **1**

(ii) She then threads them onto a string to make a bracelet. How many distinct arrangements of the beads on the bracelet are possible? **1**

(iii) How many arrangements of the bracelet are possible if the four red beads must be together? **1**

(b) (i) By substituting $x = 1$ into the the binomial expansion : **1**

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots\ldots \binom{n}{n}x^n$$

show that :

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots\ldots \binom{n}{n} = 2^n$$

(ii) By substituting $x = -1$ and using the result from (i) show that : **2**

$$\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \ldots\ldots = 2^{n-1}$$

(c) Show that $(x-1)$ is a factor of $P(x) = x^3 + 4x^2 + x - 6$ and hence factorise $P(x) = x^3 + 4x^2 + x - 6$ fully. **3**

(d) Use the substitution $u = 2 + x^4$ to evaluate $\int_0^1 \frac{x^3}{(2+x^4)^2}\,dx$ **3**

QUESTION 3 (12 MARKS) **Answer Question 3 in Booklet 3** **Marks**

a) (i) Use the definition $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ **2**

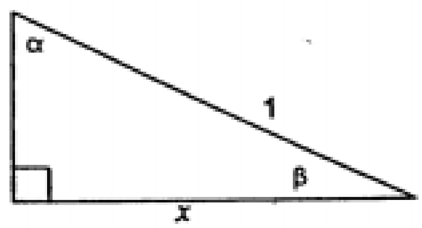
to show that the derivative of $f(x)=2^x$

is $f'(x)=c.2^x$, where c is a constant.

(ii) Estimate the value of the constant c, correct to 2 decimal places **1**

b) (i) Using the right triangle shown below, or otherwise, show that **2**

$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$



(ii) Hence evaluate $\int_2^5 \sin^{-1}x+\cos^{-1}x\ dx$ **1**

c) A particle is moving in simple harmonic motion with a velocity (in m/s) given by $v^2=2-x-x^2$ where x is the displacement (in metres) from a point O.

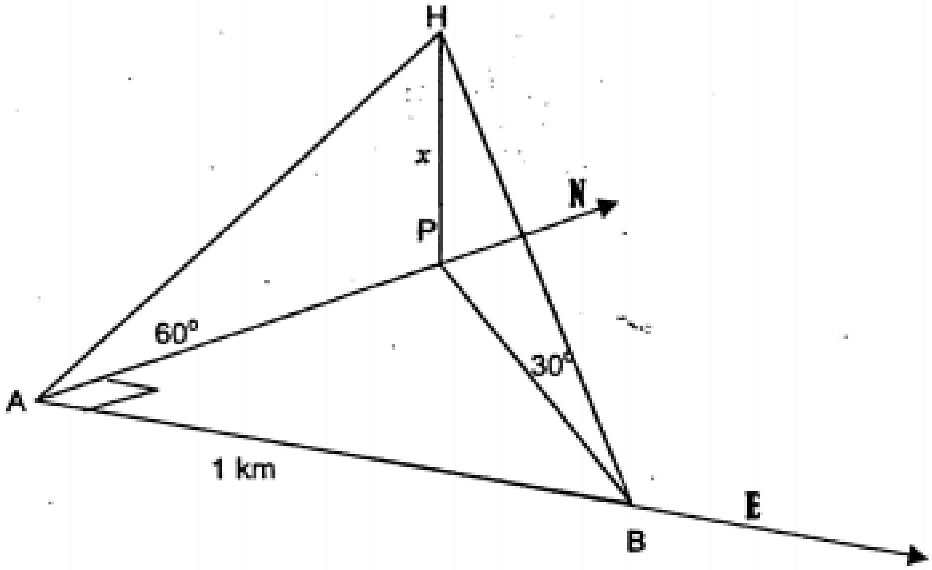
(i) What are the end points of the particles oscillation? **1**

(ii) Find the maximum velocity of the particle. **1**

Question 3 continues on the next page.

Marks

d) Anna (A) is standing due south of Phillip (P) who is assisting an injured bushwalker. A rescue helicopter (H) is hovering directly over P and lowering a stretcher. Anna measures the angle of elevation of the helicopter to be 60° from her position. Belinda (B) is 1 kilometre due east of A and measures the angle of elevation of the helicopter to be 30°. The height of the helicopter above P is x metres.



(i) Write expressions for both AP and BP in terms of x. **1**

(ii) Hence or otherwise find the height of the helicopter (x) correct to the nearest 10 m. **3**

QUESTION 4 (12 marks) **Answer Question 4 & 5 in Booklet 4** Marks

a) Prove by Induction that

$$\sum_{k=1}^{n} k^2 = \tfrac{1}{6}n(n+1)(2n+1)$$ **3**

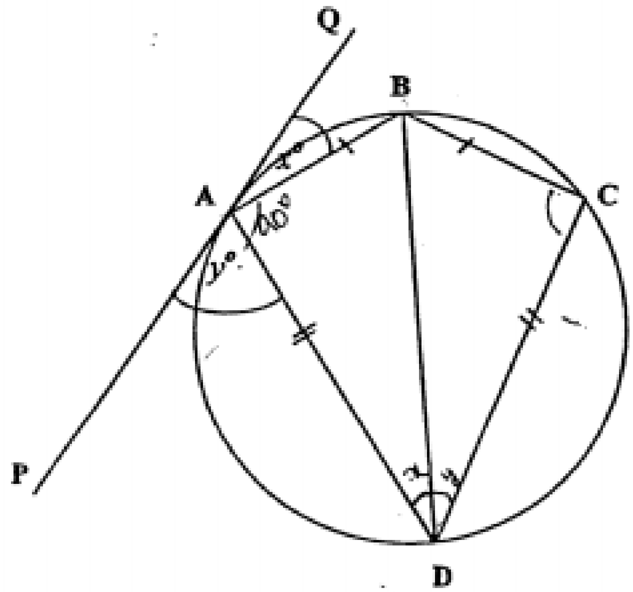
b) Solve $\sqrt{3}\sin x + \cos x = 1$ for $0 \le x \le 2\pi$ **3**

c) Find the coefficient of x^2 in the expansion of $(1+x)^2(1+x)^8$ **3**

d) Evaluate $\int_0^{\pi} \sin^2 3x - x \; dx$ **3**

QUESTION 5 (12 marks) **Answer Question 5 in Booklet 4** **Marks**

a) The line **PQ** is a tangent to the circle **ABCD** at the point **A**.
AB=BC and **AD=DC**. $\angle QAB = x°$ and $\angle PAD = y°$



(i) Explain why $\angle ADB = \angle QAB = x°$ **1**

(ii) Deduce that $x° + y° = 90°$ **3**

(iii) Explain why BD is a diameter of the circle. **1**

b) The probability of Michael hitting the bulls eye on a target with a single shot in a rifle competition is 0.9. In the competition each entrant has 10 shots at the target.

(i) What is the probability (to 2 significant figures) that Michael will hit the bulls eye with all 10 shots? **1**

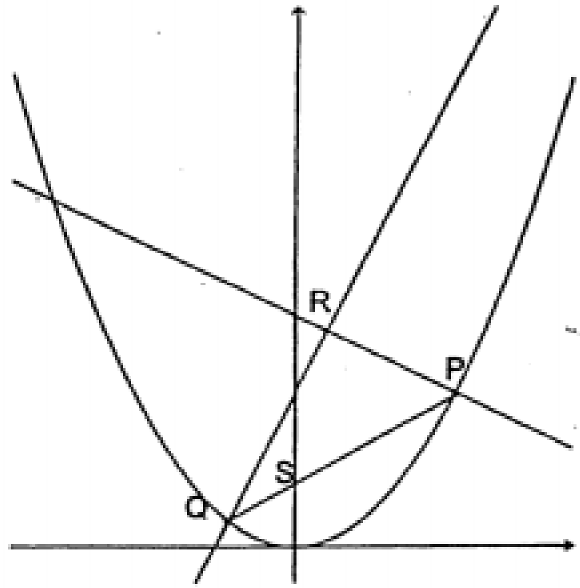
(ii) What is the probability (to 2 significant figures) that Michael will hit the bulls eye with at least 8 shots? **2**

c) (i) Explain why the function $f(x) = x^3 + 1$ has an inverse function $f^{-1}(x)$ **1**

(ii) Find the inverse function $f^{-1}(x)$ and sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes. **3**

QUESTION 6 (12 marks) **Answer Question 6 & 7 in Booklet 5** **Marks**

a) Two points P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$ with focus S $(0, a)$



(i) Show that the equation of the chord PQ is **1**

$\cancel{(p + q)x - 2y + apq = 0}$. $(p+q)x - 2y - 2apq = 0$

(ii) Show that if PQ is a focal chord then p.q = -1 **1**

(iii) Show that the equation of the normal at P is **2**

$x + py = ap^3 + 2ap$

(iv) Show that the normals at P and Q intersect at the point R **2**
where R has coordinates:

$[\,-apq(p + q),\ a(p^2 + pq + q^2 + 2)]$

(v) Find the locus of R, if PQ is a focal chord. **3**

b) The polynomial $P(x) = x^4 + x^3 + x^2 + x - 2$ has roots, α, β, γ and δ.

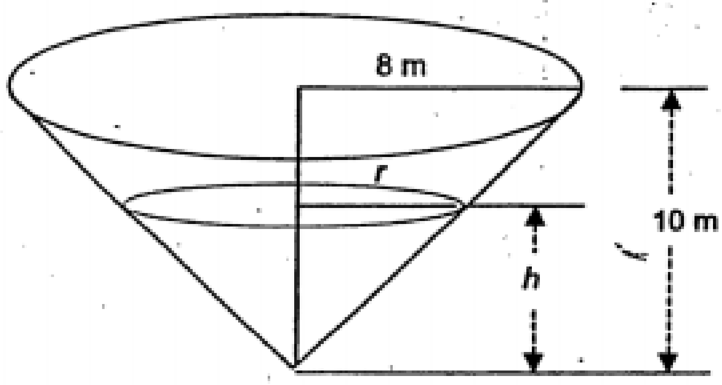
(i) Find the value of $\alpha\beta\gamma\delta$. **1**

(ii) If $\alpha = 1$, find the value of $\dfrac{1}{\beta} + \dfrac{1}{\gamma} + \dfrac{1}{\delta}$. **2**

QUESTION 7 (12 marks) **Answer Question 7 in Booklet 5** **Marks**

a) A bulk container for emptying grain into rail trucks is in the shape of an inverted cone with base radius 8 metres and height 10 metres. The grain is released from the apex of the cone at a constant rate of 35 m^3 / s. The depth of grain in the container at any given time is h metres and the radius of the circle formed by the top of the grain at that same time is r metres. **5**

If I release continuously until the container is empty calculate the ~~[illegible]~~ rate what radius r is decrea when depth page is 6.5m.



b) A radioactive element decomposes such that the amount R changes at a rate $\frac{dR}{dt} = -kR$

(i) Show that $R = Ae^{-kt}$ satisfies this differential equation. **1**

(ii) Given that half of the original amount remains after 10 years. calculate the percentage remaining after 4 years. **3**

c) In Year 12 there are 40 students, of whom 18 are boys and 22 girls.

(i) In how many ways can four students be selected from Year 12 to be on the student council, if there are no restrictions as to who is selected and the order is not important? **1**

(ii) In how many ways can the four students be selected from Year 12 for the student council if there must be 2 boys and 2 girls ? **2**