2004
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Board approved calculators may be used
- Write using black or blue pen
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in every question
- Write your student number and/or name at the top of every page

Total marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: ……………………..

STUDENT NAME / NUMBER ....

Marks

Question 1 (12 marks)

(a) Evaluate correct to 3 significant figures : $\log_e\left(\frac{1}{2.75}\right)$ **2**

(b) Simplify : $\frac{x}{x^2-4}-\frac{2}{x-2}$ **2**

(c) Differentiate $3x+e^{3x}$ with respect to x. **2**

(d) Kate gets a $7\frac{1}{2}\%$ discount from the clothing store where she works. The discount on a pair of jeans was \$6.00. What did she pay for the jeans? **2**

(e) Solve the pair of simultaneous equations **2**

$$3x-y=5$$
$$x+2y=-3$$

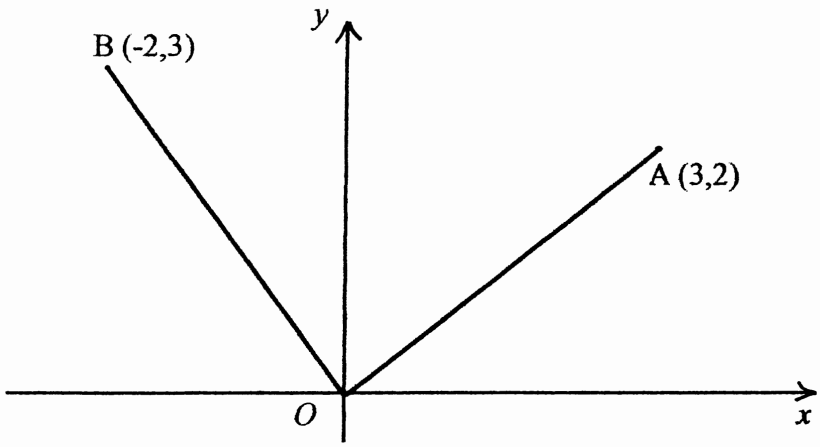
(f) Find a primitive for: $\frac{3}{x}-\sin 3x$ **2**

STUDENT NAME / NUMBER ....

Marks

Question 2 (12 marks) ***Start a new page***

(a)



NOT TO SCALE

The diagram above shows two points A(3,2) and B(-2,3) on the number plane.

Copy the diagram onto your worksheet.

(i) Find the gradient of the line BO. **1**

(ii) Show that AO is perpendicular to BO. **1**

(iii) OACB is a trapezium in which OB is parallel to AC. Show that the equation of AC is $3x+2y-13=0$. **2**

(iv) The point C lies on the line $x=-1$. Find the coordinates of C and mark the position of C on your diagram on your worksheet. **1**

(v) Show that the length of the line AC is $2\sqrt{13}$ units. **1**

(vi) Find the area of trapezium OACB **2**

(b) Find the equation of the tangent to the curve $y=\cos x$ at the point $\left(\frac{\pi}{2},0\right)$ **2**

(c) Connor bought a new car costing \$19 600. The value of the car depreciates by 15% of the new price in its first year and at 12.5% of the previous year's value in each succeeding year. What is the value of the car (to the nearest \$10) after 8 years? **2**

STUDENT NAME / NUMBER ....

Marks

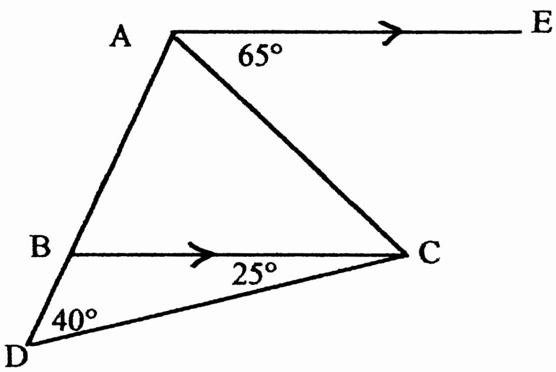
Question 3 (12 marks) ***Start a new page***

(a) Differentiate:

(i) $x\, e^{\sin x}$ **2**

(ii) $\dfrac{1}{(5-3x)^5}$ **2**

(b) **3**



In the diagram above, AE is parallel to BC.
$\angle BDC = 40°$, $\angle BCD = 25°$ and $\angle EAC = 65°$.
Copy this diagram onto your worksheet.

Prove that triangle ABC is isosceles.

(c) Find :

(i) $\int e^{3x}\,dx$ **1**

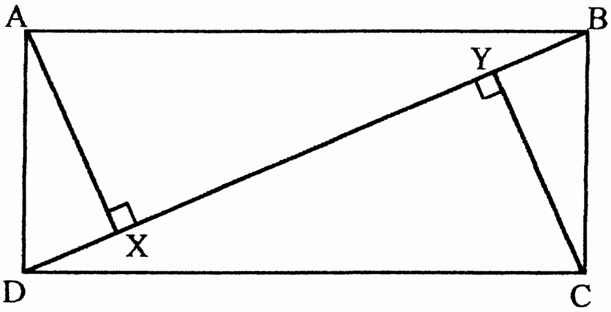
(ii) $\int_0^{\frac{\pi}{4}} \left(\sec^2 x - x\right) dx$ **2**

(d) Solve: $5 - \dfrac{2-3x}{3} = -1$ **2**

Marks

Question 4 (12 marks) ***Start a new page***

(a)



In the diagram above, ABCD is a rectangle. X and Y are points on diagonal DB such that AX and CY are perpendicular to DB.
Copy the diagram onto your worksheet.

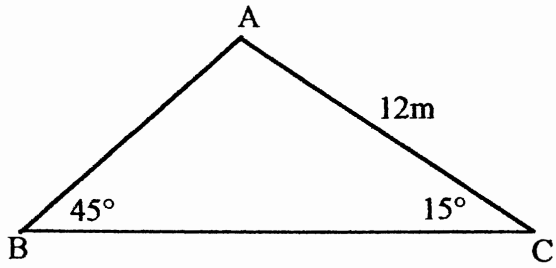
(i) Explain why $\angle ADX = \angle CBY$. **1**

(ii) Prove that $\Delta ADX \equiv \Delta CBY$ **3**

(iii) Show that $AX = CY$ **1**

(iv) What type of quadrilateral is AXCY? (Give reasons) **1**

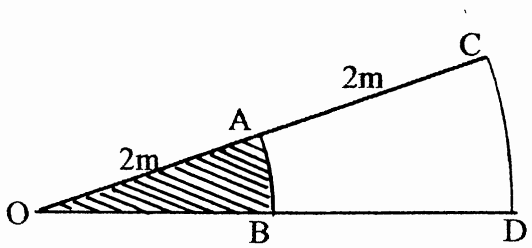
(b) **3**



NOT TO SCALE

In ΔABC, $\angle B = 45°, \angle C = 15°$ and $AC = 12$ metres.
Find the length of BC, giving your answer in exact form.

(c) **3**



NOT TO SCALE

AB and CD are arcs of concentric circles with centre O. OA = AC = 2 metres. The shaded section has area π centimetres^2. Calculate the size of $\angle AOB$ (in degrees).

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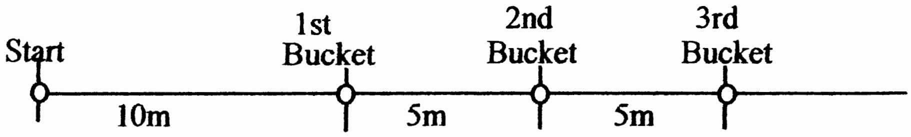
Marks

Question 5 (12 marks) ***Start a new page***

(a) In a novelty egg-and-spoon race, a series of n buckets are placed in a straight line as in the diagram below. The first bucket is placed 10 metres from the start while each successive bucket is 5 metres from the previous one.



Contestants run from the starting point carrying an egg on a spoon and place the egg in the first bucket. They run back to the start, pick up a second egg and run to place it in the second bucket. This process continues until they have placed an egg in each bucket and returned to the starting point. Thus a contestant runs 20 metres to place the first egg in the first bucket, 30 metres for the second egg and so on. A contestant who drops an egg is disqualified

(i) How far does a contestant run to place the fifth egg in its bucket and return to the start? **1**

(ii) How far does a contestant run to place the nth egg in its bucket and return? **1**

(iii) Express, in terms of n, the distance a contestant runs to complete the race. **1**

(iv) To complete the race, a contestant runs 900 metres. How many eggs are carried to complete the race? **2**

(b) Solve $2\cos^2\theta - 1 = 0$, where $0° \le \theta \le 360°$. **2**

(c) Solve: $|7 - 2x| \ge 5$ and graph your solution on the number line. **2**

(d) The roots of the equation $2x^2 - 5x + 12 = 0$ are α and β.

Find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **1**

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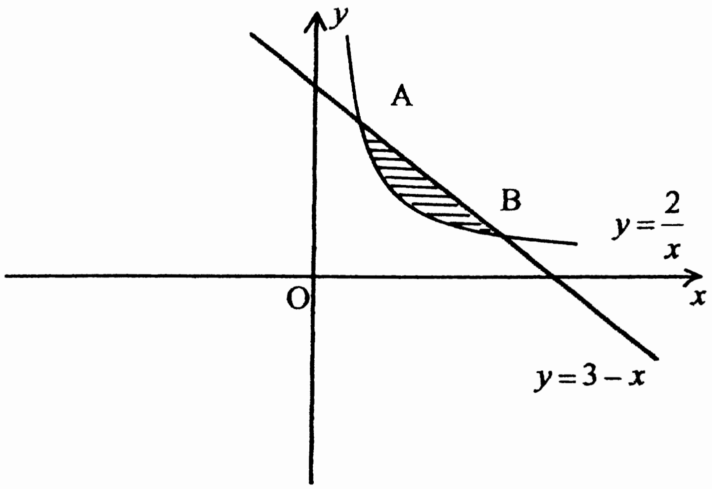
Marks

Question 6 (12 marks) ***Start a new page***

(a) The diagram below shows the graphs $y = \frac{2}{x}$ and $y = 3 - x$, which intersect at A and B.



NOT TO SCALE

(i) Find the coordinates of A and B. **2**

(ii) Calculate the shaded area bounded by $y = 3 - x$ and $y = \frac{2}{x}$. **3**

(b) Consider the parabola $4y = 12 - 4x - x^2$. Find

(i) the coordinates of the vertex. **2**

(ii) the minimum value of the quadratic expression $x^2 + 4x - 12$. **1**

(c) A farmer is constructing an open rectangular tank with a square base of side x metres. The tank is to be made from sheet metal with a capacity of 4 metres^3.

(i) Show that the height, h metres, of the tank is given by $h = \frac{4}{x^2}$. **1**

(ii) Find an expression for the surface area of the tank, in terms of x. **1**

(ii) Calculate the minimum area of sheet metal which can be used to construct the tank **2**

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Marks

Question 7 (12 marks) ***Start a new page***

(a) (i) Copy and complete the table of values below for the function $y = \log_{10}(x+2)$ **1**

$$y = \log_{10}(x+2)$$

x	-1	-0.5	0	+0.5	+1
y					

(ii) State the domain of $\log_{10}(x+2)$ **1**

(iii) Sketch the graph of $y = \log_{10}(x+2)$ **1**

(iv) Using the five function values in the table and Simpson's Rule, find an approximate value for $\int_{-1}^{1} \log_{10}(x+2)\,dx$ (correct to 3 significant figures) **3**

(b) The local model power boat club has eight equally matched boats, 3 green, 4 blue and 1 red

If two races are held, find the probability that:

(i) both winners are blue **1**

(ii) neither winner is green **2**

(iii) the winners are different colours. **1**

(iv) assuming that the winner of the first race is not allowed to compete in the second, both winners are the same colour. **2**

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Marks

Question 8 (12 marks) ***Start a new page***

(a) Consider the function $f(x)=3x^2-x^3$

(i) Find the values of x for which $f'(x)=0$. **2**

(ii) Find the coordinates of the stationary points of the curve $y=(x)$ and determine their nature **2**

(iii) Sketch the graph of the curve $y=f(x)$, showing these stationary points. **1**

(iv) Determine the values of x for which $f'(\mathrm{x})\geq 0$ **1**

(b) A water tank holds 12 500 litres of water when full. The water is allowed to flow out of the tank through a valve. If the volume of water in the tank is V litres at time t minutes, the flow rate is given by the formula:

$$\frac{dV}{dt}=-10(50-t)$$

(i) Find an expression for the volume of water V in the tank at time t **2**

(ii) How long does it take for the tank to empty? **1**

(iii) What is the initial flow rate? **1**

(iv) Find the time taken for 80% of the water in the tank to flow out. Give your answer correct to the nearest minute. **2**

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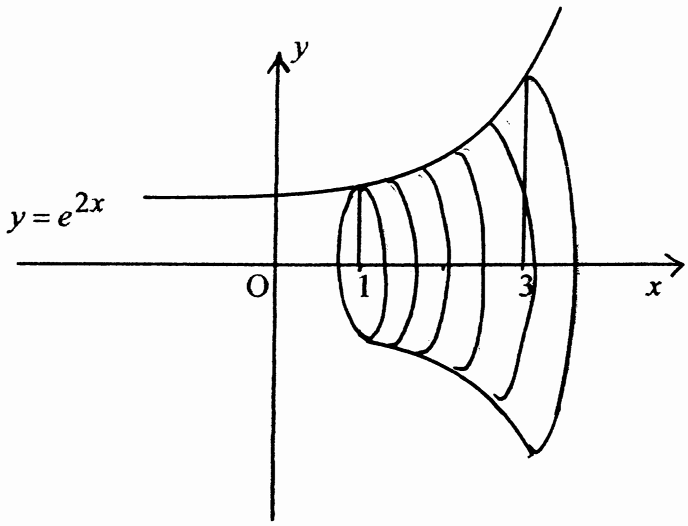
Marks

Question 9 (12 marks) ***Start a new page***

(a) A wood turner is making a wooden bowl. He determines the shape by rotating the curve $y = e^{2x}$ from $x = 1$ to $x = 3$ about the x-axis as shown in the diagram below. **4**



Calculate the volume of the above solid, leave your answer in exact form.

(b) Scientists studying a colony of moths in the Northern Territory, have found that the number of moths, W, after t days, is given by

$$W = W_0 e^{kt}$$

where W_0 and k are constants.

Initially they estimated that the colony contained 800 moths. Three days later, the number had increased by 50%.

(i) Find the values of W_0 and k. **2**

(ii) How many moths (to the nearest 10) were present after a further 3 days? **1**

(iii) How many days will it take for the number of moths to increase to 8 000? Give your answer correct to the nearest day. **2**

(c) Simplify: $\dfrac{1+\cot\theta}{\operatorname{cosec}\theta} - \dfrac{\sec\theta}{\tan\theta + \cot\theta}$ **3**

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Marks

Question 10 (12 marks) ***Start a new page***

(a) Emilie has decided that she needs to set up a superannuation fund. Her financial advisor told her that she needs \$500 000 in the fund when she retires in 25 years time. She decides that she will make equal payments of $\$P$ at the beginning of each year. The fund pays 6%p.a. interest compounded annually.

(i) Show that at the end of the first year (before she makes her payment for the second year) her account balance, \$B, is given by: **1**

$$B = P(1.06)$$

(ii) Show that her account balance after 3 years (before making the fourth payment) is given by: **1**

$$B = P(1.06)\left((1.06)^2 + (1.06) + 1\right)$$

(iii) Find a similar expression for the balance when she retires after 25 years. **1**

(iv) Hence find the amount Emilie will need to pay each year to satisfy her retirement requirements (to the nearest dollar). **2**

(b) A particle is moving in a straight line. Its distance x metres at time t seconds from a fixed point O is given by

$$x = t + 2\sin t \text{ for } 0 \le t \le 2\pi .$$

(i) Find an expression for the velocity, v metres per second, of the particle. **1**

(ii) At what times is the particle at rest? **2**

(iii) Sketch the graph of x as a function of t. **2**

(iv) What important feature on the graph indicates those times when the particle is at rest. Show this feature on the graph. **1**

(v) Describe the motion of the particle for the first π seconds. **1**

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$