2010

TRIAL
HIGHER SCHOOL CERTIFICATE

GIRRAWEEN HIGH SCHOOL

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board – approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 120
Attempt Questions 1 – 10
All questions are of equal value

Total marks – 120
Attempt Questions 1 – 10
All questions are of equal value

Marks

Question 1 (12 marks).

(a) Simplify $1-\dfrac{p-q}{p+q}$. **2**

(b) Solve $\dfrac{4x-5}{x}=2$. **2**

(c) Solve $|x-1|=5$. **2**

(d) Find the gradient of the tangent to the curve $y=x^3-4x$ at the point $(1,-3)$. **2**

(e) Find the exact value of θ such that $2\sin\theta=1$, where $0\le\theta\le\dfrac{\pi}{2}$. **2**

(f) Solve the equation $\ln x=3$. Give your answer correct to three decimal places. **2**

Marks

Question 2 (12 marks). Start on a SEPARATE page.

(a) Differentiate with respect to x:

(i) $x \tan x$. **2**

(ii) $\left(e^{x}+1\right)^{3}$. **2**

(b) (i) Find $\int 4dx$. **1**

(ii) Find $\int \frac{2}{(x-5)^{2}}dx$. **2**

(iii) Evaluate $\int_{0}^{3} \sqrt{5x+1}\,dx$. **3**

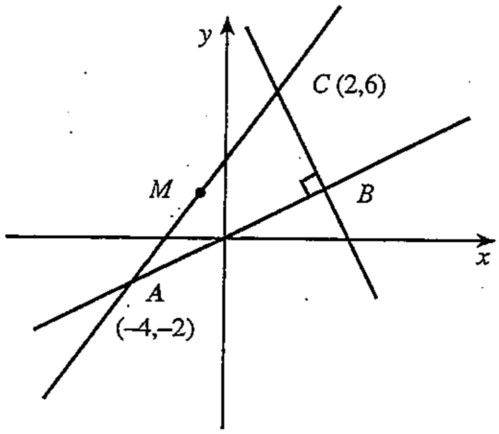
(c) Evaluate $\sum_{k=2}^{5} \frac{(-1)^{k}}{k+1}$. **2**

Marks

Question 3 (12 marks). Start on a SEPARATE page.

(a) An arithmetic series has 20 terms. The first term is 1 and the common difference is 7. Find the sum of the series. **2**

(b)



NOT TO SCALE

(i) Find the equation of the line AB, given that it passes through the origin. **2**

(ii) The line BC is perpendicular to AB. **2**

Show that its equation is $y=-2x+10$.

(iii) By solving the equations in (i) and (ii) above, find the coordinates of B. **2**

(iv) Find the length of AC . **1**

(v) Find the coordinates of M, the midpoint of AC . **1**

(vi) Explain why a circle, centre M, can be drawn to pass through A, B and C . **1**

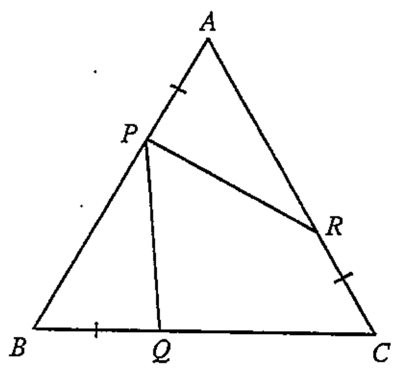
(vii) Write down the equation of this circle . **1**

Marks

Question 4 (12 marks). Start on a SEPARATE page.

(a) A man undertook to pay \$200 to a charity one year, \$150 the next year, three-quarters of \$150 the third year and so on until he died. What is the greatest sum of money the charity may expect from these donations ? **2**

(b) NOT TO SCALE



Δ ABC is equilateral. AP = BQ = CR.

Copy or trace the diagram onto your answer page.

(i) Prove that triangles APR and BQP are congruent. **4**

(ii) Prove that $\angle QPR = 60°$. **3**

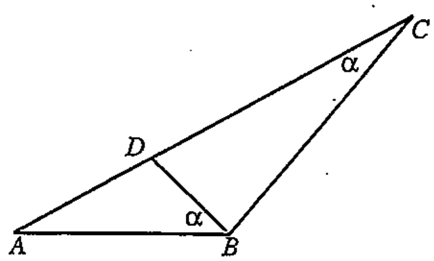
(iii) Prove that triangle PQR is equilateral. **3**

Marks

Question 5 (12 marks). Start on a SEPARATE page.

(a) Find the values of k for which the quadratic equation $x^2 - k(x-1) + 3 = 0$ has equal roots . **3**

(b) NOT TO SCALE



Copy or trace the diagram onto your answer page.

(i) Prove that triangles ABC and ADB are similar . **2**

(ii) If AD = 4 cm and DC = 12 cm, find the length of AB . **2**

(c) A certain soccer team has a probability of 0.5 of winning a match and a probability of 0.2 of drawing the match. If the team plays two matches, find the probability that it will :

(i) draw both matches . **1**

(ii) win at least one match . **2**

(iii) not win either match . **2**

Marks

Question 6 (12 marks). Start on a SEPARATE page.

(a) An arc AB of a sector of a circle is of length $\frac{\pi}{4}$ metres and subtends an angle of $30°$ at the centre, O, of the circle.

(i) Find the length of the radius . **2**

(ii) Find the area of the sector AOB . Give your answer correct to two decimal places . **1**

(iii) Find the length of the chord AB. Give your answer correct to two decimal places. **2**

(b) Find the perpendicular distance from the point $(2, -1)$ to the line $5x-12y+4=0$. **2**

(c) Solve $2\log x = \log(5x+6)$ **2**

(d) The section of the curve $y = \sec x$, from $x=\frac{\pi}{4}$ to $x=\frac{\pi}{3}$ is rotated about the x – axis. Find the exact value of the volume of the solid of revolution so formed. **3**

Marks

Question 7 (12 marks). Start on a SEPARATE page.

(a) Solve $-x^2 + 13x - 36 = 0$. **2**

(b) Find the equation of the tangent to the parabola $y = -x^2 + 13x - 36$ at the point where $x = 6$. **3**

(c) Draw a diagram showing the parabola and the tangent. Shade the region bounded by the parabola, the tangent and the x – axis . **3**

(d) Find the area shaded in the diagram above. **4**

Question 8 (12 marks). Start on a SEPARATE page.

(a) Let $f(x) = \frac{1}{3}x^3 + x^2 - 3x + 5$.

(i) Find the stationary points and determine their nature. **4**

(ii) Find any points of inflection. **2**

(iii) Sketch the graph of $f(x)$. **1**

(iv) For what values of x is $f(x)$ concave upwards ? **1**

(b) The mass, M , in grams of a radioactive substance is expressed as $M = 175e^{-kt}$ where k is a positive constant and t the time in days. The mass of the substance halved in 6 days.

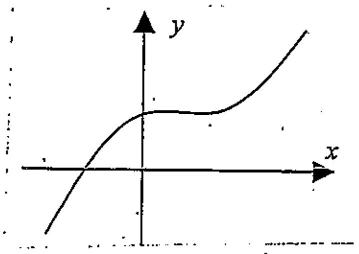
(i) Find the value of k correct to 5 decimal places. **2**

(ii) At what rate is the mass disintegrating after 10 days ? **2**

Marks

Question 9 (12 marks). Start on a SEPARATE page.

(a) The diagram shows the graph of a function $y = f(x)$.



Sketch the graph of $y = f'(x)$. **2**

(b) The gradient function of a curve is given by $6x - \dfrac{2}{2x-1}$. **2**

Find the equation of the curve if it passes through the point $(1,7)$.

(c) An amount of \$10 000 is borrowed and an interest rate of 1% per month is charged monthly. An amount M is repaid every month.

(i) If A_n is the amount owing after n months, show that **4**

$$A_n = \$10000(1.01)^n - M\left(\frac{1.01^n - 1}{0.01}\right).$$

(ii) Find the value of M, to the nearest cent, if the loan is repaid at the end of 5 years. **2**

(iii) How much extra, in total, will be repaid if the loan is taken over 7 years ? **2**

Marks

Question 10 (12 marks). Start on a SEPARATE page.

(a) Use Simpson's rule, with five function values, to approximate **3**

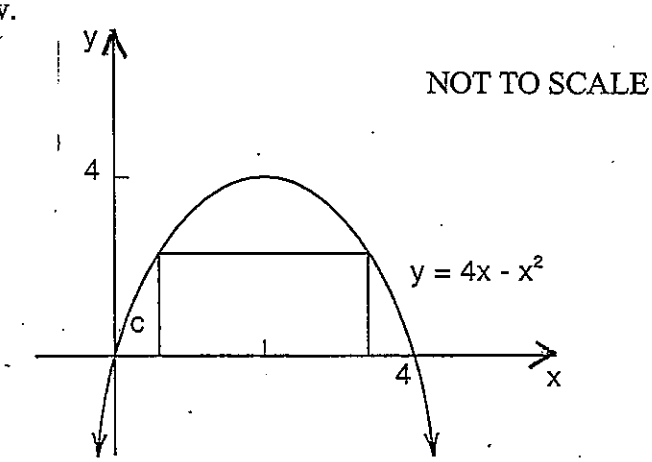
$$\int_0^2 \sqrt{x^2+4}\,dx .$$

(b) A particle, initially at the origin, moves so that after t seconds its velocity, v m/s, is given by $v = \dfrac{6}{\sqrt{2t+1}}$.

(i) Show that the position of the particle is given by $x = 6\sqrt{2t+1} - 6$. **1**

(ii) Find the particle's average velocity in moving from $x = 0$ to $x = 24$. **2**

(c) A rectangle has two of its vertices on the curve $y = 4x - x^2$ and the other two vertices on the x – axis in the interval $0 \le x \le 4$ as shown in the diagram below.



(i) If the height of the rectangle is c cm, show that its area is $2c\sqrt{4-c}$ square centimetres. **3**

(ii) Show that the greatest value of this area is $\dfrac{32\sqrt{3}}{9}$ square centimetres. **3**

End of paper

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ \ n \neq -1;\ \ x \neq 0, \text{if } \ n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ \ x < 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax},\ \ a \neq 0$$

$$\int \cos ax dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax,\ \ a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

NOTE : $\ln x = \log_e x,\ \ x > 0$