National Educational Advancement Programs

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

1999

MATHEMATICS

3 UNIT (ADDITIONAL)
AND
3/4 UNIT (COMMON)

Two

Time Allowed: ~~Three~~ hours
(Plus 5 minutes' reading time)

This paper must be kept under strict security and may only be used on or after the afternoon of Friday 13 August, 1999, as specified in the NEAP Examination Timetable.

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on page 9.
- Approved calculators may be used.
- Each question is to be returned in a separate Writing Booklet clearly labelled, showing your Student Name or Number.
- You may ask for extra Writing Booklets if you need them.

Students are reminded that this is a trial examination only and cannot in any way guarantee the content or the format of the 1999 Mathematics 3 Unit (Additional) and 3/4 Unit (Common) Higher School Certificate Examination.

QUESTION 1. Use a *separate* Writing Booklet.

Marks

(a) Solve $\dfrac{x+1}{x} \geq 2$. **3**

(b) Find the acute angle between the lines $x + 3y = 4$ and $2x - 5y = 0$. **3**
Give the answer to the nearest degree.

(c) Use the "t" results to solve the equation $2\cos\theta - \sin\theta = -1$ for $0 \leq \theta \leq 2\pi$. **4**
Give any answers correct to three significant figures.

(d) Point A is (2, –7) and B is (–6, 9). Find the coordinates of point P which divides AB internally in the ratio 3 : 5. **2**

QUESTION 2. Use a *separate* Writing Booklet. **Marks**

(a) Use the substitution $u = \ln x$ to find $\displaystyle\int \frac{dx}{x\sqrt{1-(\ln x)^2}}$. **2**

(b) Consider the geometric series $1 - \tan^2 x + \tan^4 x - \ldots$, where $0 < x < \frac{\pi}{2}$. **3**

(i) For what values of x does this series have a limiting sum?

(ii) Find the limiting sum in terms of $\cos x$.

(c) It is given that $x^2 + x - 2$ is a factor of $x^3 + rx^2 - 4x + s$, where r and s are constants. **3**

(i) Show that $r + s = 3$.

(ii) Evaluate r and s.

(d) On a certain railway line, there are eleven stations at which a train can stop. The rail authority wishes to print tickets for travel between every possible pair of stations on the line. **1**

How many different one-way tickets must be printed if the ticket specifies which direction the passenger is travelling?

(e) (i) Five men and five women are arranged in a straight line. How many arrangements are there in which men and women alternate? **3**

(ii) Five men and five women are arranged in a circle. How many arrangements are there in which men and women alternate?

(iii) Find the probability that men and women alternate if five men and five women are arranged at random in a circle.

QUESTION 3. Use a *separate* Writing Booklet. **Marks**

(a) (i) Sketch $y = 3\sin x$ and $y = x$, for $0 \le x \le 2\pi$, on the same set of axes. **4**

(ii) By substitution, show that a solution for $3\sin x - x = 0$ lies between $x = 2.2$ and $x = 2.4$.

(iii) Taking $x = 2.3$ as an approximation to a solution of $3\sin x - x = 0$, apply Newton's method once to find a better approximation correct to three decimal places.

(b) (i) Find $\dfrac{d}{dx}(x\tan^{-1}x)$. **4**

(ii) Hence find the exact value of $\displaystyle\int_0^1 \tan^{-1}x\, dx$.

(c) Use mathematical induction to show **4**

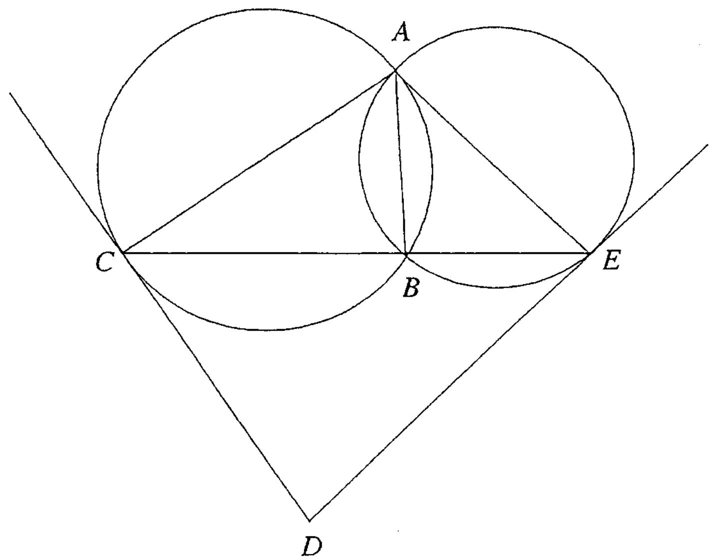
$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1} \quad \text{for } n \ge 1.$$

QUESTION 4. Use a *separate* Writing Booklet. **Marks**

(a) Assume that Antarctica can be approximately represented as a circle, and that its area is decreasing at the rate of 3000 square kilometres per year. **2**

Find the rate at which the radius of the circle is decreasing when the length of the radius is 2100 kilometres.

(b) **4**



In the diagram, two circles intersect at A and B. Points C and E lie on the circles and C, B and E are collinear. Tangents at C and E meet at D.

Copy the diagram into your booklet and show that quadrilateral $AEDC$ is cyclic.

(c) A particle moves in a straight line and at time t seconds, its velocity, v metres per second, is related to its displacement, x metres, by **6**

$$v^2 = 108 + 36x - 9x^2.$$

(i) By deriving an expression for acceleration, show that the motion is simple harmonic.

(ii) Find the period of the motion.

(iii) Find the amplitude of the motion.

(iv) For what value of x does the particle have maximum speed?

QUESTION 5. Use a *separate* Writing Booklet. **Marks**

(a) Consider the function $f(x) = \dfrac{1}{1+x^2}$ for $x \leq 0$. **4**

(i) Sketch $y = f(x)$. (It is not necessary to show working.)

(ii) Find the inverse function $f^{-1}(x)$.

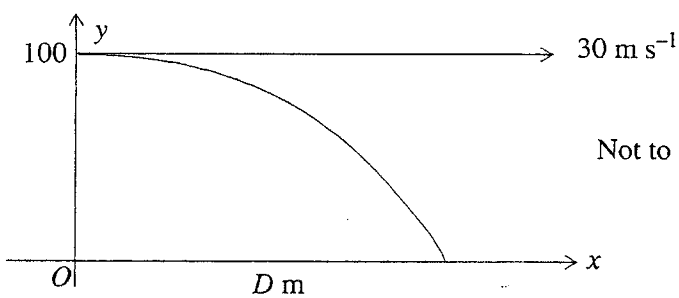
(iii) State the domain of $f^{-1}(x)$.

(b) Fish are taken at random from a large number of fish in a dam. For this particluar kind of fish, one quarter of the population is male. **4**

(i) Find the probability that if ten fish are taken, then exactly four are male.

(ii) How many fish have to be taken from the dam for the probability of there being at least one male to be greater than 99%?

(c) A plane designed to drop water "bombs" flies in a horizontal line towards a spot fire. It maintains a steady speed of 30 metres per second, and an altitude of 100 metres. The plane releases its water when at a horizontal distance D metres from the fire. **4**



Let the origin O be as shown in the diagram, and let x and y metres be the horizontal and vertical displacements of the centre of the water bomb at time t seconds after its release. Then $\dot{x}$ and $\dot{y}$ are the horizontal and vertical components of its velocity in metres per second. Let $g = 10$ metres per second per second, and assume that air resistance has negligible effect.

(i) Use calculus to derive expressions for $\dot{x}$, $\dot{y}$, x and y in terms of t.

(ii) Hence determine the value of D correct to the nearest metre.

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QUESTION 6. Use a *separate* Writing Booklet. **Marks**

(a) Solve $|2x-1| < 3x+2$. **3**

(b) (i) On the same set of axes, sketch $y = \sin^{-1}x$ and $y = \cos^{-1}x$, showing all essential information. **4**

(ii) Let $f(x) = \sin^{-1}x + \cos^{-1}x$. By referring to the graphs in part (i), or otherwise, explain why $f(x)$ is a constant function, and find its value.

(iii) Hence evaluate $\int_{-1}^{1} (\sin^{-1}x + \cos^{-1}x)dx$.

(c) At time t seconds after the parachute has opened, a parachutist falls with speed v metres per second and with acceleration given by **5**

$$\frac{dv}{dt} = k(4-v)$$

where k is constant.

(i) Show that $v = 4 + Ae^{-kt}$, where A is constant, satisfies the above equation.

(ii) At the instant the parachute opens, the parachutist is falling at 25 metres per second. Find the value of A.

(iii) Two seconds after the parachute has opened, the parachutist's speed is 12 metres per second. Evaluate k correct to three significant figures.

(iv) As t increases indefinitely, v approaches a fixed value. Show that after 20 seconds v differs from this fixed value by less than 0.1%.

QUESTION 7. Use a *separate* Writing Booklet. **Marks**

(a) An expression of the form $(1-ax)^n$, where a is a constant and n a positive integer, is expanded. The first three terms of the expansion are **4**

$$1-4x+\frac{20}{3}x^2.$$

Find the values of a and n.

(b) (i) Show that $\cos 3x = 4\cos^3 x - 3\cos x$ by using the expansion of $\cos(A+B)$. **8**

(ii) Show that the solution of $\cos 3x - \sin 2x = 0$ for $0 < x < \frac{\pi}{2}$ is given by $\sin x = \frac{\sqrt{5}-1}{4}$.

(iii) Use a trigonometric identity to explain why $x = \frac{\pi}{10}$ is a solution to $\cos 3x = \sin 2x$.

(iv) Hence, using the results referred to in parts (ii) and (iii), prove $\sin\frac{\pi}{5}\cos\frac{\pi}{10} = \frac{\sqrt{5}}{4}$.

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The following list of standard integrals may be used:

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq 1; \; x \neq 0, \; \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2}).$$

Note: $\ln x = \log_e x, \quad x > 0$

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