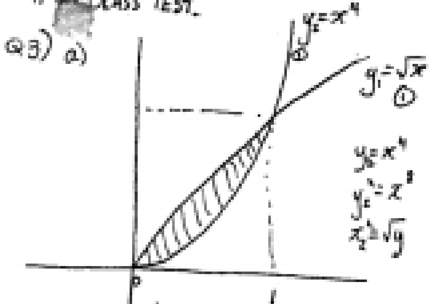
Yr 12 Class Test

Q3) a)



(1)

$y_2 = x^4$ $\quad y_1 = \sqrt{x}$

$y_2^{\frac{1}{4}} = x$ $\quad y_1^2 = x$

$x_2 = \sqrt[4]{y}$ $\quad x^2 = y^4$

b) $A = \int_0^1 \sqrt{x}\, dx - \int_0^1 x^4\, dx$ (1)

$= \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^1 - \left[\frac{1}{5}x^5\right]_0^1$ (1)

$= \left(\frac{2}{3} - 0\right) - \left(\frac{1}{5} - 0\right)$

$= \frac{7}{15}$ sq. units. (1)

c) i) $V = \pi \int_0^1 y_1^2\, dx - \pi \int_0^1 y_2^2\, dx$ (1)

$= \pi \left[\frac{1}{2}x^2\right]_0^1 - \pi\left[\frac{1}{9}x^9\right]_0^1$ (1)

$= \pi\left[\frac{1}{2} - \frac{1}{9}\right]$

$= \frac{7\pi}{18}$ cu. units. (1)

ii) $V = \pi \int_0^1 x_2^2\, dy - \pi \int_0^1 x_1^2\, dy$ (1)

$= \pi \int_0^1 \sqrt{y}\, dy - \pi \int_0^1 y^4\, dy$

$= \pi\left[\frac{2}{3}y^{\frac{3}{2}}\Big|_0^1 - \frac{1}{5}y^5\Big|_0^1\right]$ (1)

$= \pi\left[\left(\frac{2}{3} - 0\right) - \left(\frac{1}{5} - 0\right)\right]$

$= \frac{7\pi}{15}$ cu. units (1)

$\boxed{11}$

Integration and Indices. (2)

Q4. a) $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$

$\underline{n=1}$: LHS $= 1^3$ (1)

$= 1$

RHS $= \frac{1}{4} \cdot 1^2(1+1)^2$

$= \frac{1}{4} \cdot 4$

$= 1 = $ LHS

assume $n = k$ is true:

$\therefore \sum_{r=1}^{k} r^3 = \frac{1}{4}k^2(k+1)^2$ (1)

ie $1^3 + 2^3 + 3^3 + \ldots + k^3 = \frac{1}{4}k^2(k+1)^2$

$\therefore$ Show true for $n = k+1$

ie show: $\sum_{r=1}^{k+1} r^3 = \frac{1}{4}(k+1)^2(k+2)^2$ (1)

ie $1^3 + 2^3 + \ldots + k^3 + (k+1)^3 = \frac{1}{4}(k+1)^2(k+2)^2$

LHS: $1^3 + 2^3 + \ldots + k^3 + (k+1)^3$

$= \frac{1}{4}k^2(k+1)^2 + (k+1)^3$ by assumption

$= (k+1)^2\left[\frac{1}{4}k^2 + (k+1)\right]$ (1)

$= \frac{1}{4}(k+1)^2\left[k^2 + 4(k+1)\right]$

$= \frac{1}{4}(k+1)^2(k^2 + 4k + 4)$

$= \frac{1}{4}(k+1)^2(k+2)^2$ (1)

$=$ RHS

$\therefore$ true for $n = k+1$

$\therefore$ as true for $n = 1$, then true for $n = 2$, and so on for all integer n.

b) $3^n + 7^n$ is divisible by 10, n is odd.

$n=1$: $3 + 7$

$= 10$ $\quad \therefore$ true for $n = 1$ (1)

assume true for $n = k$

ie $3^k + 7^k = 10m$, m an integer

also: $7^k = 10m - 3^k$ (1)

Show true for $n = k+2$ (k must be odd)

$\therefore$ show $3^{k+2} + 7^{k+2} = 10p$, p an integer (1)

Yr 12 Class test Integration and Induction.

LHS $= 3^{k+2} + 7^{k+2}$

$= 3^2 3^k + 7^2 7^k$

$= 9 \cdot 3^k + 49(10m - 3^k)$ from assumption

$= 9 \cdot 3^k + 490m - 49 \cdot 3^k$ (1)

$= 490m - 40 \cdot 3^k$

$= 10(49m - 4 \cdot 3^k)$

$= 10p$ (1)

where $p = 49m - 4 \cdot 3^k$

$\therefore$ true for $n = k+2$

$\therefore$ as true for $n=1$, therefore true for $n = 3$ and so on for all odd integer n.

c) Show $9^n > 4^n + 5^n$ $\quad n \geq 2$

$n=2$: LHS $= 9^2$

$= 81$

RHS $= 4^2 + 5^2$

$= 41$

$\therefore$ LHS > RHS (1)

Assume true for $n = k$

ie $9^k > 4^k + 5^k$ (1)

or $9^k - 4^k - 5^k > 0$

Show true for $n = k+1$

ie show $9^{k+1} > 4^{k+1} + 5^{k+1}$ (1)

LHS $= 9^{k+1}$

$= 9 \cdot 9^k$

$> 9(4^k + 5^k)$ (1)

$> 9\left(\frac{1}{4} \cdot 4 \cdot 4^k + \frac{1}{5} \cdot 5 \cdot 5^k\right)$

$> 9\left(\frac{1}{4}4^{k+1} + \frac{1}{5}5^{k+1}\right)$

$> 9\left(\frac{5 \cdot 4^{k+1} + 4 \cdot 5^{k+1}}{20}\right)$

$> \frac{9}{20}\left(5 \cdot 4^{k+1} + 4 \cdot 5^{k+1}\right)$

$> \frac{45}{20} \cdot 4^{k+1} + \frac{36}{20} 5^{k+1}$

$> 1.4^{k+1} + 1.5^{k+1}$ (1)

$\therefore$ true for $n = k+1$

$\therefore$ as true for $n = 2$, t[…]

for $n = 3$, and so on for […]

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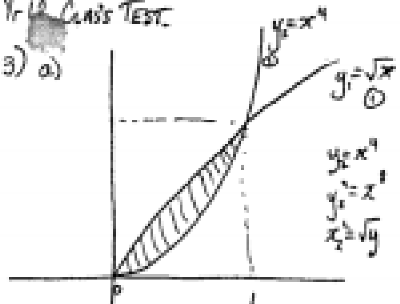

$y=x^4$ $\quad$ $y=\sqrt{x}$

$y^2=x^8$ $\quad$ $y^2=x$

$x=\sqrt{y}$ $\quad$ $x^2=y^4$

b) $A=\int_0^1 \sqrt{x}\,dx - \int_0^1 x^4\,dx$ (1)

$= \left[\frac{2}{3}x^{3/2}\right]_0^1 - \left[\frac{1}{5}x^5\right]_0^1$ (1)

$= \left(\frac{2}{3}-0\right) - \left(\frac{1}{5}-0\right)$

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c) i) $V = \pi\int_0^1 y_1^2\,dx - \pi\int_0^1 y_2^2\,dx$ (1)

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ii) $V = \pi\int_0^1 x_2^2\,dy - \pi\int_0^1 x_1^2\,dy$ (1)

$= \pi\int_0^1 \sqrt{y}\,dy - \pi\int_0^1 y^2\,dy$

$= \pi\left[\frac{2}{3}y^{3/2}\Big|_0^1 - \frac{1}{3}y^3\Big|_0^1\right]$ (1)

$= \pi\left[\left(\frac{2}{3}-0\right)-\left(\frac{1}{3}-0\right)\right]$

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$n=1$: LHS $= 1^3$ (1)

$= 1$

RHS $= \frac{1}{4}.1^2(1+1)^2$

$= \frac{1}{4}.4$

$= 1 \;=$ LHS

assume $n=k$ is true:

$\therefore \sum_{r=1}^{k} r^3 = \frac{1}{4}k^2(k+1)^2$ (1)

ie $1^3+2^3+3^3+\ldots+k^3 = \frac{1}{4}k^2(k+1)^2$

$\therefore$ Show true for $n=k+1$

ie show: $\sum_{r=1}^{k+1} r^3 = \frac{1}{4}(k+1)^2(k+2)^2$ (1)

ie $1^3+2^3+\ldots+k^3+(k+1)^3 = \frac{1}{4}(k+1)^2(k+2)^2$

LHS: $1^3+2^3+\ldots+k^3+(k+1)^3$

$= \frac{1}{4}k^2(k+1)^2 + (k+1)^3$ by assumption

$= (k+1)^2\left[\frac{1}{4}k^2 + (k+1)\right]$ (1)

$= \frac{1}{4}(k+1)^2\left[k^2+4(k+1)\right]$

$= \frac{1}{4}(k+1)^2(k^2+4k+4)$

$= \frac{1}{4}(k+1)^2(k+2)^2$ (1)

$=$ RHS

$\therefore$ true for $n=k+1$

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$n=1$: $\quad 3+7$ (1)

$= 10 \quad \therefore$ true for $n=1$

assume true for $n=k$

ie $3^k+7^k = 10m$, m an integer (1)

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Show true for $n=k+1$

ie show $9^{k+1} > 4^{k+1}+5^{k+1}$ (1)

LHS $= 9^{k+1}$

$= 9.9^k$

$> 9(4^k+5^k)$ (1)

$> 9\left(\frac{1}{4}.4.4^k+\frac{1}{5}.5.5^k\right)$

$> 9\left(\frac{1}{4}4^{k+1}+\frac{1}{5}5^{k+1}\right)$

$> 9\left(\frac{5.4^{k+1}+4.5^{k+1}}{20}\right)$

$> \frac{9}{20}(5.4^{k+1}+4.5^{k+1})$

$> \frac{45}{20}.4^{k+1}+\frac{36}{20}5^{k+1}$

$> 1.4^{k+1}+1.5^{k+1}$ (1)

$\therefore$ true for $n=k+1$

$\therefore$ as true for $n=2$, then for $n=3$, and so on for all n

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