SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2003
HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes.
- Working time – 2 hours.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- Hand in your answer booklets in 3 sections. Section A (Questions 1,2 and 3), Section B (Questions 4 and 5) and Section C (Questions 6 and 7).
- Start each **NEW** section in a separate answer booklet.

Total Marks - 84 Marks

- Attempt Sections A - C
- All questions are of equal value.

Examiner: *B. Opferkuch*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n<0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x>0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a1>0, \quad -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}} \, dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}} \, dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x>0$

Total marks-84.
Attempt Questions 1-7.
All questions are of equal value.

Answer each Section in a SEPARATE writing booklet. Extra writing booklets are available

Section A Use a SEPARATE writing booklet

Question 1 (12 marks) **Marks**

(a) Differentiate

(i) $x\sin 3x$ **1**

(ii) e^{1-x^2} **1**

(b) Find the acute angle between the lines $3y=2x+8$ and $5x-y-9=0$. **2**

(c) Evaluate

(i) $\displaystyle\int_0^2 \frac{dx}{4+x^2}$ **2**

(ii) $\displaystyle\int_0^1 \frac{x^2}{2+x^3}\,dx$ **2**

(d) The letters of the word INTEGRAL are arranged in a row. **2**

If one of these arrangements is selected at random, what is the probability that the vowels are in the same position?

(e) Solve the inequality $\dfrac{\theta-4}{\theta}>0$. **2**

Section A continued.

Question 2. (12 marks) **Marks**

(a) If α, β and γ are the roots of the equation $2x^3 - 5x^2 - 3x + 1 = 0$, evaluate

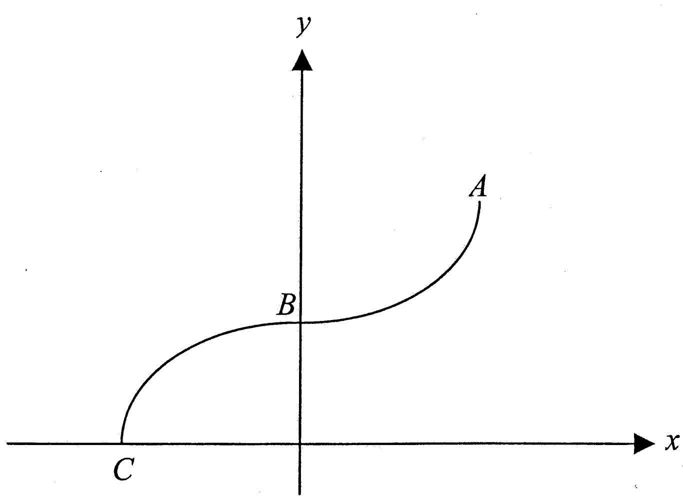
(i) $\alpha + \beta + \gamma$ and $\alpha\beta\gamma$. **1**

(ii) $\alpha^2 + \beta^2 + \gamma^2$. **2**

(b) Use the substitution $u = x^2 + 4$ to find the exact value of $\displaystyle\int_0^{2\sqrt{3}} \frac{x}{\sqrt{x^2+4}}\,dx$. **3**

(c) Determine the exact value of $\cos\left(\tan^{-1}\left(\frac{8}{15}\right)\right)$. **2**

(d)



The diagram shows the graph of $y = \pi + 2\sin^{-1} 3x$.

(i) Find the coordinates of A and C. **2**

(ii) Find the gradient of the tangent at B. **2**

Section A continued.

Question 3. (12 marks) **Marks**

(a) A function is defined as $f(x)=1+e^{2x}$. **2**

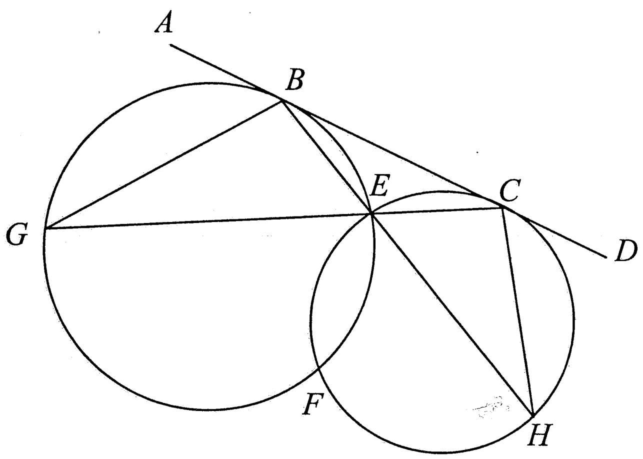
Find the inverse function $f^{-1}(x)$ and state the domain and range.

(b) Consider the quadratic expression $Q(x)=(5k-4)x^2-6x+(6k+3)$, **3**

where k is a constant.

Find the values of k for which $Q(x)=0$ has rational roots.

(c)



ABCD is a common tangent to the two circles.

(i) Prove that $\angle ABG = \angle DCH$. **2**

(ii) Prove that $\Delta\, BCG \;|||\; \Delta\, BCH$. **2**

(d) Consider the series $2^N+2^{N-1}+2^{N-2}+.....+2^{1-N}+2^{-N}$,

where N is a positive integer.

(i) Find an expression in terms of N for the number of terms in the series. **2**

(ii) Find an expression in terms of N for the sum of the series. **1**

Section B Use a SEPARATE writing booklet.

Question 4. (12 marks) **Marks**

(a) Consider the function $f(\theta)=\dfrac{\sin\theta+\sin\dfrac{\theta}{2}}{1+\cos\theta+\cos\dfrac{\theta}{2}}$

(i) Show that $f(\theta)=t$ where $t=\tan\dfrac{\theta}{2}$. **3**

(ii) Write down the general solution of $f(\theta)=1$. **1**

(b) A certain particle moves along the straight line in accordance with the law: $t=2x^2-5x+3$, where x is measured in centimetres and t in seconds.

Initially, the particle is $1\cdot5$ centimetres to the right of the origin O, and moving away from O.

(i) Show that the velocity, v cms^{-1}, is given by **1**

$$v=\frac{1}{4x-5}$$

(ii) Find an expression for the acceleration, a cms^{-2}, of the particle, in terms of x. **2**

(iii) Find the velocity and acceleration of the particle when: **3**

(α) $x=2$ cm

(β) $t=6$ seconds

(iv) Describe carefully in words the motion of the particle. **2**

Section B continued.

Question 5. (12 marks) **Marks**

a) (i) Prove the identity $\dfrac{\cos y - \cos(y+2\alpha)}{2\sin\alpha} = \sin(y+\alpha)$ **2**

(ii) Hence prove by mathematical induction that for positive integers n, **4**

$\sin\alpha + \sin 3\alpha + \sin 5\alpha + \ldots + \sin(2n-1)\alpha = \dfrac{1-\cos 2n\alpha}{2\sin\alpha}$.

(b) (i) Show that the curve $y = \dfrac{x^3+4}{x^2}$ has one stationary point and no points of inflexion. **2**

(ii) Write down the equation(s) of any asymptotes. **1**

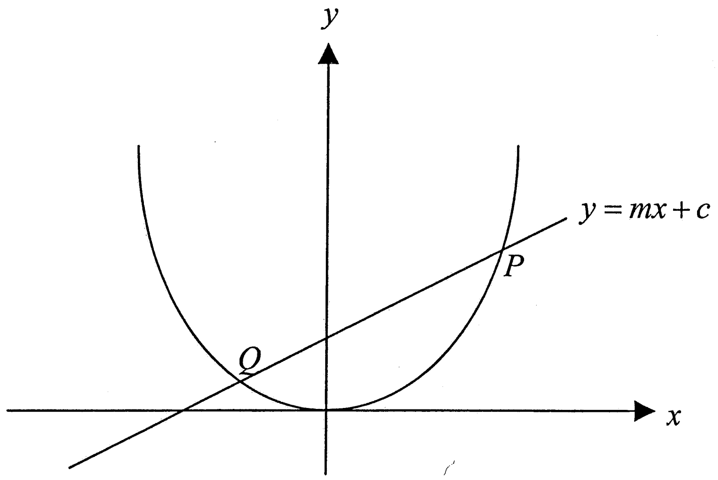
(iii) Sketch the curve. **1**

(iv) Hence, use the graph to find the values of k for which the equation $x^3 - kx^2 + 4 = 0$ has 3 real roots. **2**

Section C Use a SEPARATE writing booklet.

Question 6. (12 marks) **Marks**

The straight line $y = mx + c$ meets the parabola $x = 2t,\ y = t^2$ in real distinct points P and Q which correspond respectively to the values $t = p$ and $t = q$.



(i) Prove that $pq = -c$. **2**

(ii) Prove that $p^2 + q^2 = 4m + 2c$. **2**

(iii) Show that the equation of the normal to the parabola at P is $x + py = 2p + p^3$. **2**

(iv) The point N is the point of intersection of the normals to the parabola at P and Q. **2**
Show that the coordinates at N are $\left(-pq(p+q),\ \left(2 + p^2 + pq + q^2\right)\right)$

(v) If the chord PQ is free to move while maintaining a fixed gradient.

(α) Show that the locus of N is a straight line. **2**

(β) Hence, or otherwise, show that this straight line is a normal to the parabola. **2**

Section C continued.

Question 7. (12 marks) **Marks**

(a) When the polynomial $P(x)$ is divided by $(x+4)$ the remainder is 5 and when $P(x)$ is divided by $(x-1)$ the remainder is 9. Find the remainder when $P(x)$ is divided by $(x-1)(x+4)$. **3**

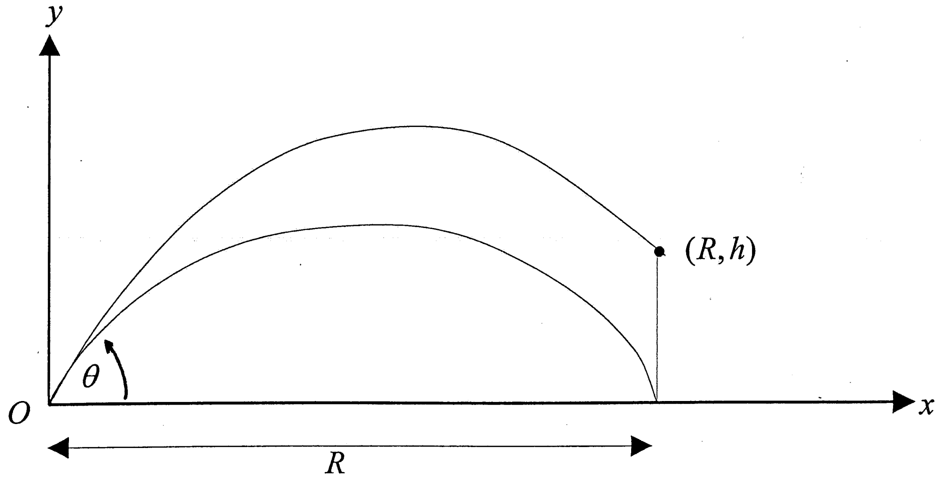
(b) A projectile is fired from a point on horizontal ground with initial speed V ms^{-1} and angle of projection θ.
The cartesian equation of the path is given by

$$y = x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$$

where x and y are the horizontal and vertical displacements of the particle from O, the point of projection.
The acceleration due to gravity is g and air resistance has been neglected.

(i) Use the given equation to show that the maximum range R on the horizontal plane is given by $R = \dfrac{V^2}{g}$. **2**

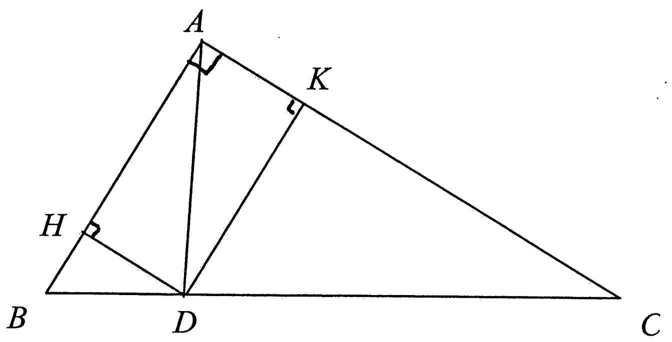
(ii) Show that to hit a target h metres above the ground at the same horizontal distance R using the same angle of projection θ, the speed of projection must be increased to $\dfrac{V^2}{\sqrt{V^2 - gh}}$. **4**



Section C continued.

Question 7. **Marks**

(c)



In the triangle ABC, $< BAC = 90^{\circ}$. AD bisects $< BAC$. $DH \perp AB$ and $DK \perp AC$.

Copy the diagram.

(i) Show that $\dfrac{AD}{DH} = \sqrt{2}$. **1**

(ii) By considering the areas of the triangles or otherwise, show that $\dfrac{\sqrt{2}}{AD} = \dfrac{1}{AB} + \dfrac{1}{AC}$. **2**

THIS IS THE END OF THE PAPER