Blakehurst High School
Mathematics 2 Unit
Half Yearly 2002

PART A:

Question 1: (12 Marks) ***start a new page***

a) Solve the simultaneous equations

$2x + 3y = 4$
$5x + 2y = -1$ (3)

b) Express $0.\dot{2}\dot{7}$ as a simple fraction (2)

c) By rationalising the denominators express

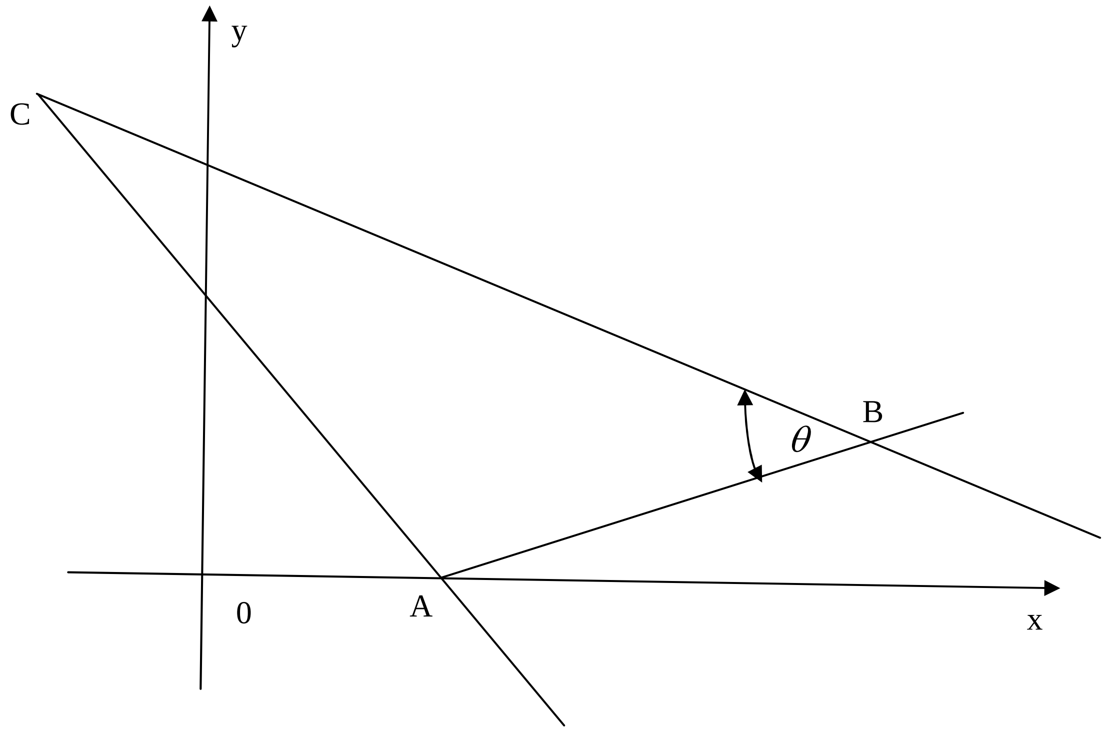
$$\frac{1}{3-\sqrt{2}} + \frac{1}{3+\sqrt{2}}$$ in simplest form. (3)

d) $$f(x) = \begin{cases} 3x + 1 & \text{for } x \geq 1 \\ x^2 + 3 & \text{for } x < 1 \end{cases}$$

(i) find $f(4) - f(0)$ (2)

(ii) sketch the graph of the function. (2)

<u>Question 2:</u> ***start a new page*** **(10 marks)**



The diagram shows points A(1,0) , B(4,1) and C(-1,6) in the Cartesian plane. Angle ABC is θ. Copy this diagram onto your paper.

a) Show that A and C lie on the line $3x + y = 3$ (2)

b) Show that the gradient of AB is $\frac{1}{3}$. (1)

c) Show that the length of AB is $\sqrt{10}$ units. (2)

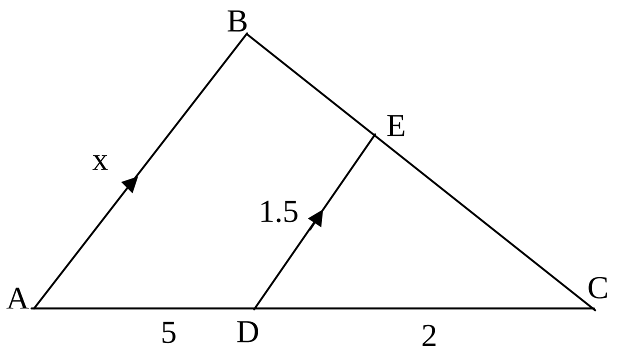
d) (i) Show that AB and AC are perpendicular. (2)

(ii) Hence or otherwise find $\tan\theta$. (2)

e) Find the coordinates of the midpoint BC. (1)

<u>QUESTION 3</u>: *Start a new page* (12 marks)

a) In this diagram, AB is parallel to DE, AD is 5cm, DC is 2cm and DE is 1.5cm. Find the length of AB.



(2)

b) Draw a neat sketch of the function $y = 9 - x^2$. State the domain and range of this function. (3)

c) The third term of an arithmetic sequence is 19 and the 7^{th} term is 63.

(i) Show the common difference is 11. (3)

(ii) Find the first term of the sequence. (1)

(iii) Find the sum of the first 20 terms of the sequence. (2)

(iv) Find an expression for the nth term of the sequence. (1)

<u>Question 4</u>: ***start a new page*** **(10 marks)**

a) Differentiate the following expressions with respect to x:

(i) $3xe^{x}$ (2)

(ii) $\sqrt{x} + 5x^3 + 1$ (2)

(iii) $3 \ln x + \dfrac{1}{x}$ (2)

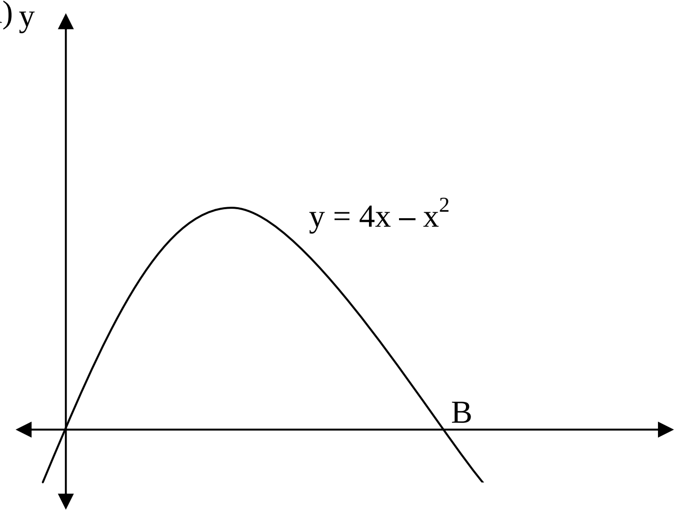
b) Find

(i) $\displaystyle\int \frac{1}{\sqrt{x}}\, dx$ (2)

(ii) $\displaystyle\int \frac{t^3 - 3t + t}{2t}\, dx$ (2)

<u>Question 5</u> :*Start a new page*   (12 marks)

a)



The diagram shows the graph function $y = 4x - x^2$.

(i) Find the x coordinate of the point B where the curve crosses the positive x- axis. (1)

(ii) Find the area of the shaded region contained by the curve $y = 4x - x^2$ and the x – axis. (3)

b) Find the volume of the solid formed when the curve $y = \sqrt{x+3}$ is rotated about the x- axis from x = 1 to x = 6. (3)

c) Use the trapezoidal rule with 2 subintervals to find an approximation for

$$\int_3^5 \frac{dx}{x}$$ (2)

d) A car manufacturer randomly tests new cars for defects. The probability of any car having a defect is 3%. If 3 cars are tested at random, find the probability as a percentage to 1 decimal place.

(i) exactly two cars will have defects. (1)

(ii) at least one car will have a defect. (2)

PART B:

Question 6: ***Start a new page*** **(14 marks)**

a) If the roots of the quadratic equation $2x^2 - 5x + 4 = 0$ are α and β find the value of:

(i) $\alpha + \beta$ (1)

(ii) $\alpha\beta$ (1)

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ (2)

(iv) $\alpha^2 + \beta^2$ (2)

b) Solve $3^{2x} + 4.\ 3^x - 21 = 0$ (3)

c) (i) Write down the discriminant of $3x^2 + 2x + k = 0$. (1)

(ii) For what values of k does $3x^2 + 2x + k = 0$ have real roots. (2)

d) If $\dfrac{dy}{dx} = 18x^2 - 6x + 12$, find the equation of the curve if it passes through (1,4). (2)

<u>Question 7</u>:*Start a new page* (9 marks)

a) A closed cylinder is to have a volume of 800 cm^3. If h is the height and r is the radius

(i) Show that $h = \dfrac{800}{\pi r^2}$ (1)

(ii) Show that the surface area of the cylinder is given by

$$S = 2\pi r^2 + \frac{1600}{r}.$$ (2)

(iii) Find the value of r that gives the minimum surface area. (3)

b) Kim invests \$500 at the beginning of each year in a superannuation fund. The money earns 12% interest per annum. If she starts the fund at the beginning of 1996 what will the fund be worth at the end of 2025? (3)

<u>Question 8</u> :*Start a new page* (7 marks)

A function is given by $y = x^3 - 3x^2 - 9x + 2$.

(i) Find the first and second derivative. (2)

(ii) Find any stationary points and their nature. (4)

(iii) Draw a **<u>neat</u>** sketch of the curve showing these essential features. (1)

<u>Question 9</u>: *Start a new page* (12 marks)

a) A plane flies from Sydney for 1500km on a bearing of 125^0.

(i) What is the bearing of Sydney from the plane? (1)

(ii) How far east is the plane from Sydney to 1 decimal place? (2)

b) Find the value of e^3 correct to 3 significant figures. (1)

c) Find $\int_0^1 e^{2x} - e^{-x}\,dx$ in terms of 'e'. (3)

d) (i) Differentiate $(2x^2 - 3)^4$. (2)

(ii) Find $\int_1^2 x(2x^2 - 3)^3\,dx$. (3)

STANDARD INTEGRALS

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln(x + \sqrt{x^2 - a^2}),\quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE: $\ln x = \log_e x,\ x > 0$