3 UNIT TRIAL CRANBROOK 2000.

1(a) $5x^3-6x^2-29x+6=0$, has roots α, β, γ.

$\therefore \alpha+\beta+\gamma = \frac{--6}{5} = \frac{6}{5}$

$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{-29}{5}$

$\alpha\beta\gamma = \frac{-6}{5}$

Now $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$

$= \left(\frac{6}{5}\right)^2 - 2\left(\frac{-29}{5}\right)$

$= \frac{36}{25} + \frac{58}{5}$

$= \frac{326}{25}$

(b) (i) If $P(x)$ is divisible by $Q(x)$ then $P(2)=P(-2)=0$

as $Q(x) = x^2-4$
$= (x-2)(x+2)$.

Now $P(2) = 16+16-4-16-b$
$= 12-b$

$\therefore$ If $P(2)=0$ then $b=12$

Also $P(-2) = 16-16-4+16-b$
$= 12-b$

$\therefore$ if $P(-2)=0$ then $b=12$ again.

ie there exists only 1 value of the constant b if $P(x)$ is divisible by $Q(x)$.

(ii) $P(x) = x^4+2x^3-x^2-8x-12$

$= (x^2-4)(x^2+2x+3)$

$= (x-2)(x+2)(x^2+2x+3)$

$\therefore$ Roots are $x = 2, -2$.

($x^2+2x+3=0$ has no real roots)

c (i) Let $y = \operatorname{cosec} x \cot x$

$\therefore \frac{dy}{dx} = \operatorname{cosec} x \cdot -\operatorname{cosec}^2 x + \cot x \cdot -\operatorname{cosec} x \cot x$

$= -\operatorname{cosec}^3 x - \operatorname{cosec} x(\cot^2 x)$

$= -\operatorname{cosec}^3 x - \operatorname{cosec} x(\operatorname{cosec}^2 x - 1)$

$= -2\operatorname{cosec}^3 x + \operatorname{cosec} x$

$\therefore \frac{d}{dx}(\operatorname{cosec} x \cot x) = -2\operatorname{cosec}^3 x + \operatorname{cosec} x$

or let $y = \operatorname{cosec} x \cot x$

$= \frac{1}{\sin x \tan x}$

$= \frac{\cos x}{\sin^2 x}$

$\therefore \frac{dy}{dx} = \frac{\sin^2 x \cdot -\sin x - \cos x \cdot 2\sin x \cos x}{\sin^4 x}$

$= \frac{-\sin^3 x - 2\sin x(1-\sin^2 x)}{\sin^4 x}$

$= \frac{-\sin^3 x - 2\sin x + 2\sin^3 x}{\sin^4 x}$

$= \frac{\sin^3 x - 2\sin x}{\sin^4 x}$

$= \frac{1}{\sin x} - \frac{2}{\sin^3 x}$

$= \operatorname{cosec} x - 2\operatorname{cosec}^3 x$.

ie $\frac{d}{dx}(\operatorname{cosec} x \cot x) = \operatorname{cosec} x - 2\operatorname{cosec}^3 x$.

(ii) $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{cosec} x(\cot^2 x + \operatorname{cosec}^2 x)\,dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{cosec} x(\operatorname{cosec}^2 x - 1 + \operatorname{cosec}^2 x)\,dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\operatorname{cosec}^3 x - \operatorname{cosec} x \,dx$

$= -\left[\operatorname{cosec} x \cot x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ (using result of part (i))

$= -\left[\frac{1}{\sin x \tan x}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$