CSSA MATHEMATICS 3 UNIT SOLUTIONS 1998

QUESTION 1

(a) (i) $\dfrac{1+\cos 2A}{\sin 2A} = \dfrac{1+(2\cos^2 A-1)}{2\sin A\cos A}$

$= \dfrac{\cancel{2}\cos^{\cancel{2}} A}{\cancel{2}\sin A\cancel{\cos A}} = \cot A$.

(ii) $\cot 15^\circ = \dfrac{1+\cos 30^\circ}{\sin 30^\circ} = \dfrac{1+\sqrt{3}/2}{1/2} = 2+\sqrt{3}$

(b) $x^3 - mx + 2 = 0$. Let the roots be α, α, β.

(i) sum of roots : $\alpha+\alpha+\beta = -(0)/(1)$

$\therefore 2\alpha+\beta = 0 \quad \therefore \beta = -2\alpha$

product of roots : $(\alpha)(\alpha)(\beta) = -(2)/(1)$

$\therefore \alpha^2\beta = -2$

(ii) $\alpha^2\beta = -2 \quad \therefore \alpha^2(-2\alpha) = -2$

$\therefore -2\alpha^3 = -2 \quad \therefore \alpha^3 = 1$

$\therefore \alpha = 1 \quad \therefore \beta = -2$

sum of roots taken two at a time

$(\alpha)(\alpha) + (\alpha)(\beta) + (\alpha)(\beta) = (-m)/(1)$

$\therefore (1)(1) + (1)(-2) + (1)(-2) = -m$

$\therefore 1-2-2 = -m \quad \therefore -3 = -m$

$\therefore m = 3$.

(c) $y = e^{-kx}$. $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 3y = 0$

(i) $\therefore \dfrac{dy}{dx} = -ke^{-kx}$, $\dfrac{d^2y}{dx^2} = -k(-ke^{-kx}) = k^2e^{-kx}$

$\therefore k^2e^{-kx} + 4(-ke^{-kx}) + 3(e^{-kx}) = 0$

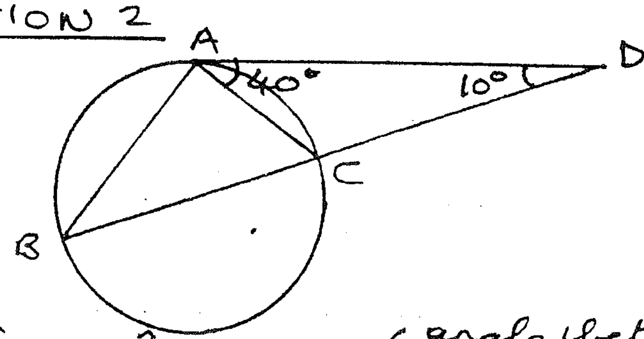
$\therefore k^2e^{-kx} - 4ke^{-kx} + 3e^{-kx} = 0$

$\therefore e^{-kx}(k^2-4k+3) = 0 \quad \therefore k^2-4k+3 = 0$.

(ii) $\therefore (k-1)(k-3) = 0 \quad \therefore k=1$ or $k=3$.

QUESTION 2

(a) (i)



(ii) $A\hat{B}C = D\hat{A}C = 40^\circ$ (angle between tangent and chord is equal to angle in alternate segment)

$A\hat{B}D + B\hat{D}A + D\hat{A}B = 180^\circ$ (angle sum of $\Delta ABD = 180^\circ$)

$\therefore A\hat{B}D + B\hat{D}A + (D\hat{A}C + C\hat{A}B) = 180^\circ$

$\therefore C\hat{A}B = 180^\circ - A\hat{B}D - B\hat{D}A - D\hat{A}C$

$= 180^\circ - 40^\circ - 10^\circ - 40^\circ = 90^\circ$

$\therefore$ BC is a diameter (a right angle stands in a semicircle)

(b) $f(x) = x/4-x^2$

(i) $4 - x^2 \neq 0 \quad \therefore x^2 \neq 4 \quad \therefore x \neq -2, x \neq 2$

$\therefore$ domain is all real numbers x except -2 and 2.

(ii) $f(-x) = \dfrac{(-x)}{4-(-x)^2} = -\dfrac{x}{4-x^2} = -f(x)$

for all values of x in the domain $\therefore$ odd function

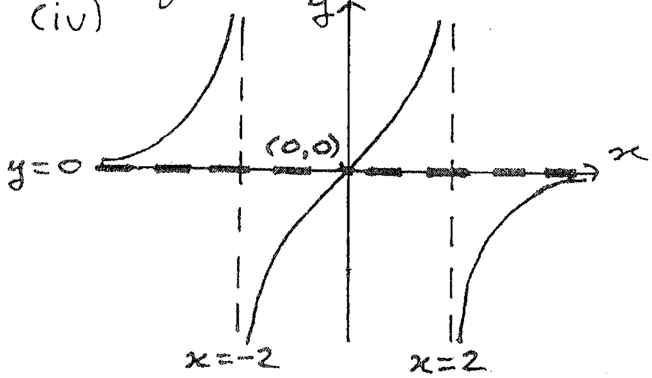
(iii) $f'(x) = \dfrac{(4-x^2)(1) - (x)(-2x)}{(4-x^2)^2}$

$= \dfrac{4-x^2+2x^2}{(4-x^2)^2} = \dfrac{4+x^2}{(4-x^2)^2}$

$\therefore f'(x) > 0$ for all values of x in the domain.

$\therefore$ function is increasing throughout its domain

(iv)



(v) The inverse function does not exist. The function is not a one to one function. Horizontal lines can be drawn to cut the graph in more than one point.