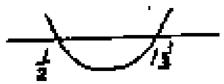
(2004 Trials)

MATHEMATICS EXTENSION 1 - QUESTION 1

(a)
$$\frac{5}{2x-1} < 3$$
$$5(2x-1) < 3(2x-1)^2$$
$$3(2x-1)^2 - 5(2x-1) > 0$$
$$(2x-1)\left[3(2x-1)-5\right] > 0$$
$$(2x-1)(6x-8) > 0$$
$$x < \tfrac{1}{2},\ x > 1\tfrac{1}{3}$$



③

(b)
$$2x - y + 1 = 0 \qquad m_1 = 2$$
$$x + 3y - 4 = 0 \qquad m_2 = -\tfrac{1}{3}$$
$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$
$$= \left|\frac{2 - (-\frac{1}{3})}{1 + 2(-\frac{1}{3})}\right|$$
$$= \frac{2\frac{1}{3}}{\frac{1}{3}}$$
$$= 7$$
$$\theta = 82^\circ$$

③

(c)
$$(-2, 5),\ (8, -9) \qquad 2:3$$
$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right) = \left(\frac{2\times 8 + 3\times(-2)}{2+3}, \frac{2\times(-9) + 3\times 5}{2+3}\right)$$
$$= \left(2, -\tfrac{3}{5}\right)$$

②

(d)
$$x^3 - 5x^2 - 3x + 2 = 0$$
$$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = \frac{\gamma + \alpha + \beta}{\alpha\beta\gamma} = \frac{5}{-2} = -2.5$$

②

(e)
$$\cos\theta = -\tfrac{1}{2}$$
$$\theta = 2n\pi \pm \tfrac{2\pi}{3}$$
$$\text{OR} \quad \theta = (2n+1)\pi \pm \tfrac{\pi}{3}$$
or equivalent.

②

Mathematics Extension One: Question Number 1 Marker: CJF

Marks Awarded	Marker's Comments
(a). 1 mark 1 mark 1 mark	$5(2x-1) < 3(2x-1)^2$ ... or critical points. $2(2x-1)(3x-4) > 0$ $x < \frac{1}{2}$, $x > 4/3$. or correctly solving the inequality obtained (unless trivial).
(b). 1 mark 1 mark 1 mark.	$m_1 = 2$ / $m_2 = -\frac{1}{3}$. $\tan\theta = \left\lvert\frac{m_1 - m_2}{1 + m_1m_2}\right\rvert$ ... understanding this is formula to use even if not stated in this form. number crunching → $82°$.
(c). 1 mark. 1 mark	$\frac{(2\times 8) + 3(-2)}{2+3}$ $\quad \frac{2\times(-9) + (3\times 5)}{2+3}$. $(2, -3/5)$
(d) 1 mark 1 mark	$\frac{\gamma + \beta + \alpha}{\alpha\beta\gamma}$ $\alpha + \beta + \gamma = 5$ and $\alpha\beta\gamma = -2$.
(e) 1 mark. 1 mark	$2n\pi$ or equivalent $\pm\frac{2\pi}{3}$ or equivalent (must be $\pm$)

Question 2

a) $\int_1^5 (x+1)\sqrt{x-1}\,dx$

$x = u^2+1$

$\frac{dx}{du} = 2u$

$dx = 2u\,du$. $\boxed{1}$

$x = 1 \quad u = 0$

$x = 5 \quad u = 2$

$= \int_0^2 (u^2+2)\sqrt{u^2}\;2u\,du$

$= \int_0^2 (2u^4 + 4u^2)\,du \quad \boxed{2}$

$= \left[\frac{2u^5}{5} + \frac{4u^3}{3}\right]_0^2$

$= \left[\left(\frac{64}{5} + \frac{32}{3}\right) - (0)\right]$

$= \frac{352}{15} = 23\frac{7}{15} \quad \boxed{1}$

b) $\int_0^{\frac{\pi}{4}} \sin^2\left(\frac{1}{2}x\right)dx = \int_0^{\frac{\pi}{4}} \frac{1}{2}\left(1 - \cos x\right)dx \quad \boxed{1}$

$= \frac{1}{2}\left[x - \sin x\right]_0^{\frac{\pi}{4}} \quad \boxed{1}$

$= \frac{1}{2}\left[\frac{\pi}{4} - \frac{1}{\sqrt{2}}\right] \quad \boxed{1}$

c) Prove $9^{n+2} - 4^n$ is divisible by 5.

i) Let $n=1$ $\quad 9^3 - 4 = 725$ $\therefore$ True for $n=1$ $\quad \boxed{1}$

ii) Assume true for $n=k$ ie $9^{k+2} - 4^k = 5m$ (m is pos integer) $\boxed{1}$

iii) When $n = k+1$ $\quad 9^{k+3} - 4^{k+1} = 9(9^{k+2}) - 4(4^k)$

$= 9(5m + 4^k) - 4(4^k)$

$= 45m + 9.4^k - 4.4^k$

$= 45m + 5.4^k \quad \boxed{2}$

$= 5[9m + 4^k]$

This is divisible by 5.

$\therefore$ if true for $n=k$, then true for $n=k+1$

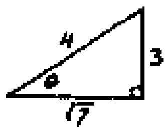
Mathematics Extension One: Question Number 2 Marker: NM.

Marks Awarded	Marker's Comments
1	Many mucked up the conversion.
2	Those with correct step one went on to get the correct integral
1	and find the correct numeric value.
1	Many did not know $\sin^2 x = \frac{1}{2}(1-\cos 2x)$
1	most integrated correctly.
1	most had correct evaluation.
1	Almost all proved true for $n=1$.
1	$9^{k+2} - 4^k = 5m$ well stated. most DID NOT state that m was a positive integer.
2	Many could not set out the correct steps for this section. Many used "m" again. Is it the same "M" used earlier?
1	Many were lazy in their final statement. $\therefore$ most did not get this mark.

MATHEMATICS EXTENSION I - QUESTION 3

(a) $\tan\left(2\sin^{-1}\frac{3}{4}\right)$

Let $\theta = \sin^{-1}\frac{3}{4}$ $\therefore \sin\theta = \frac{3}{4}$



$\tan\left(2\sin^{-1}\frac{3}{4}\right) = \tan 2\theta$

$= \dfrac{2\tan\theta}{1-\tan^2\theta}$

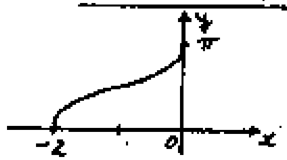
$= \dfrac{2\times\frac{3}{\sqrt7}}{1-\frac{9}{7}}$

$= -3\sqrt7$ (3)

(b) $f(x) = \sin^{-1}(x+1) + \frac{\pi}{2}$

(i) Domain: $-1 \le x+1 \le 1$ $\therefore -2 \le x \le 0$ (1)

(ii)



(2)

(c) $f(x) = \log_e(2x+1)$.

(i) Domain: $2x+1>0$ $\therefore x > -\frac{1}{2}$ (1)

(ii) $y = \log_e(2x+1)$

$2x+1 = e^y$

Inverse is: $2y+1 = e^x$

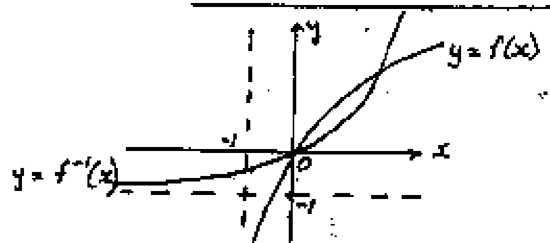
$y = \frac{1}{2}(e^x - 1)$

$f^{-1}(x) = \frac{1}{2}(e^x - 1)$ (2)

(iii) $f'(x) = \frac{2}{2x+1}$; $f'(0) = 2$

$\frac{d}{dx}f^{-1}(x) = \frac{1}{2}e^x$; At $x=0$, $\frac{d}{dx}f^{-1}(x) = \frac{1}{2}$. (1)

(iv)



(2)

Mathematics Extension One: Question Number 3 Marker: Mr M'Car

	Marks Awarded	Marker's Comments
(a)	1	$\tan\theta = \frac{3}{\sqrt7}$
	1	double angle formula substit^{n}
	1	correct final answer
		Note - 1 mark awarded for calculator answer ~ 7.94
(b) (i)	1	Correct domain $-2 \le x \le 0$
(ii)	1	Correct shape
	1	Correct position
		Note - Use a stencil !!
(c) (i)	1	Correct domain $x > -\frac{1}{2}$
(ii)	1	Interchange $x \Leftrightarrow y$
	1	Make y the subject (Generally well done)
(iii)	1	Either answer $f'(0) = 2$ or $\frac{d}{dx}f^{-1}(0) = \frac{1}{2}$
(iv)	1	One correct function
	or	or
	2	- Must pass thru' origin - Show both asymptotes - intersect twice } required for FULL marks

(poorly answered)

Question 4

a) $(2-x)(1+x)^5 = (2-x)(1+5x+10x^2+10x^3\ldots)$ [1]

$= 2(10x^3) + (-x \times 10x^2)$

$= 20x^3 - 10x^3$ [1]

$= 10x^3 \quad \therefore$ Coefficient $= 10$ [1]

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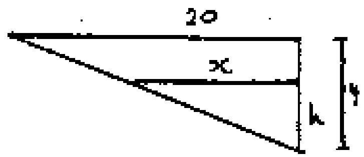
b) i)



$\frac{h}{x} = \frac{4}{20}$

$20h = 4x$

$x = 5h$ [1]

$V = \frac{1}{2}hx \times 8$

$= \frac{1}{2}h.5h \times 8$

$= 20h^2$. [1]

ii) $\frac{dV}{dt} = 2 \quad h = 1$

$\frac{dh}{dt} = ?$

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ [1]

$2 = 40h \times \frac{dh}{dt}$

$\frac{dh}{dt} = 0.05$ [1]

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c) $\frac{dV}{dt} = k(4000 - v)$

i) $V = 4000 - Ae^{-kt}$

$\frac{dv}{dt} = -k(-Ae^{-kt})$ [1]

$= +k(4000 - V)$.

ii) $t = 0 \quad V = 2000 \quad 2000 = 4000 - Ae^0$ [1]

$A = 2000$

$t = 5 \quad V = 3000 \quad 3000 = 4000 - 2000e^{-k5}$.

$e^{-5k} = \frac{1}{2}$ [1]

$k = -\frac{1}{5}\ln 0.5 = 0.1386$

iii) $3800 = 4000 - 2000e^{-kt}$ [1]

$e^{-k.t} = 0.1$ [1]

Mathematics Extension One: Question Number 4. Marker: NM

Marks Awarded	Marker's Comments
1	Correct expansion
1	Correct collection of coefficients
1	10. (Well done)
1	Correct explanation of why ratio was 5 : 1.
1	Correct explanation of why $V = 20h^2$. (Poorly done)
1	$\frac{dv}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ or equivalent.
1	0.05 or $\frac{1}{20}$ or equivalent (Well done)
1	Use of $Ae^{-kt} = 4000 - V$.
1	Evaluate A
1	Evaluate k (correct dp's).
1	Correct equation.
1	Correct value of t.

MATHEMATICS EXTENSION I - QUESTION 5.

(a) $y = x^2 e^{-x}$

(i) $\frac{dy}{dx} = e^{-x}(2x) + x^2 \times (-e^{-x})$

$= 2x e^{-x} - x^2 . e^{-x}$

$= x e^{-x}(2-x)$ (1)

(ii) Let $f(x) = x^2 e^{-x} - 0.4$: $f(1) = e^{-1} - 0.4 = -0.030$

$f(2) = 4e^{-2} - 0.4 = 0.14 > 0$

Since $f(x)$ is continuous and $f(1), f(2)$ have opposite signs, a root lies between 1 and 2. (1)

(iii) $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ $\qquad f'(1) = e^{-1} \times 1 = 0.3679$

$= 1 - \frac{-0.03}{0.3679}$

$= 1.08$ (3)

b) Let $\angle ADF = \theta$

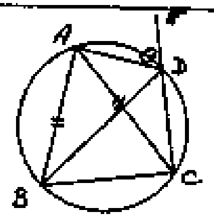
$\therefore \angle ABC = \theta$ (exterior angle cyclic quad)

$\therefore \angle ACB = \theta$ (base angles isosceles triangle)

$\therefore \angle ADB = \theta$ (angles in same segment)

$\therefore \angle ADF = \angle ADB$

$\therefore$ AD bisects $\angle BDF$. (3)



c) $(3+2x)^8$

(i) $\frac{c_r}{c_{r-1}} = \frac{\binom{8}{r} 3^{8-r} 2^r}{\binom{8}{r-1} 3^{9-r} 2^{r-1}}$

$= \frac{2}{3} \times \frac{8!}{r!(8-r)!} \times \frac{(r-1)!(9-r)!}{8!}$

$= \frac{2}{3} \times \frac{9-r}{r}$

$= \frac{18-2r}{3r}$ (2)

(ii) $\frac{18-2r}{3r} > 1$

$18 > 5r$

$r < 3\frac{3}{5}$

Greatest coefficient = coefficient of x^3

$= \binom{8}{3} 3^5 2^3$

or 108864 (2)

Mathematics Extension One: Question Number 5 Marker: Mr M

	Marks Awarded	Marker's Comments
(a) (i)	1	Correct use of product rule
(ii)	1	Show a change of sign
(iii)	1	$f(1) = -0.03$
	1	$f'(1) = 0.368$
	1	Correct estimate $x_1 = 1.087$
		Note . must use $f(x) = x^2 e^{-x} - 0.4$
		Poorly answered by many students.
(b)	1	Exterior $\angle$ of cyclic quad.
	1	Base $\angle$s of isos. Δ
	1	Angles in same segment.
		Note - very poor structure - drawing a diagram helps.
(c) (i)	1	$\frac{{}^8C_r 3^{8-r} 2^r}{{}^8C_{r-1} 3^{9-r} 2^{r-1}}$ or $\frac{n-r+1}{r} \cdot \frac{b}{a}$
	1	Use of factorial definition to correctly simplify.
(ii)	1	Solving inequality $r < 3\frac{3}{5}$
	1	Finding greatest term 108 864
		Note : part (i) poorly answered.

Question 6

a) i) $x = 3\sin 2t - 4\cos 2t$

$\dot{x} = 6\cos 2t + 8\sin 2t$ [1]

$\ddot{x} = -12\sin 2t + 16\cos 2t$

$= -4[3\sin 2t - 4\cos 2t]$ [1]

$= -4x \quad (\text{ie } -n^2x)$

ii) $t = 0 \quad \dot{x} = 6\cos 0 + 8\sin 0$

$= 6$ [1]

iii) $\dot{x} = 0 \quad 6\cos 2t + 8\sin 2t = 0$ [1]

$8\sin 2t = -6\cos 2t$

$\tan 2t = \frac{-3}{4}$ [1]

$2t = \pi - 0.6435$

$t = 1.249$ [1]

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b) i) $\ddot{x} = -\frac{1}{4x^3}$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -\frac{1}{4}x^{-3}$ [1]

$\frac{1}{2}v^2 = \frac{1}{8}x^{-2} + C$

$v = \frac{1}{2}\ x = 1 \quad \frac{1}{2}\left(\frac{1}{4}\right) = \frac{1}{8} + C$ [1]

$\therefore C = 0$

$\frac{1}{2}v^2 = \frac{1}{8x^2}$ [1]

$v = \frac{1}{2x}$

ii) $\frac{dx}{dt} = \frac{1}{2x} \quad \frac{dt}{dx} = 2x$ [1]

[1] $t = x^2 + C$

Mathematics Extension One: Question Number 6. Marker: NM.

Marks Awarded		Marker's Comments
a) i)	1	Correct expression for $\dot{x}$
	1	Correct manipulation to $\ddot{x} = -n^2x$ (well done)
ii)	1	$\dot{x} = 6$ (Very well done)
iii)	1	Let $\dot{x} = 0$ (Generally OK.
	1	$\tan 2t = -3/4$ Some poor solutions to
	1	$t = 1.249$ $\tan 2t = -3/4$)
b) i)	1	Correct version of $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
	1	Evaluate $C = 0$
	1	Correct "tidy up" to $v = \frac{1}{2x}$ (Many forgot C)
ii)	1	For $\frac{dt}{dx} = 2x$
	1	Evaluate C.
	1	Manipulate to $x = \sqrt{t+1}$.

MATHEMATICS EXTENSION 1 - QUESTION 7

a) Initially, $\dot{x} = V\cos\theta = 40\cos 60^\circ = 20$; $\dot{y} = V\sin\theta = 40\sin 60^\circ = 20\sqrt{3}$.

$\ddot{x} = 0$ $\qquad$ $\ddot{y} = -10$

$\dot{x} = c$ $\qquad$ $\dot{y} = -10t + k$

$\dot{x} = 20$ $\qquad$ when $t=0, \dot{y} = 20\sqrt{3}$ $\therefore k = 20\sqrt{3}$

$x = 20t + c'$ $\qquad$ $\therefore \dot{y} = 20\sqrt{3} - 10t$

When $t=0, x=0 \therefore c'=0$ $\qquad$ $y = 20\sqrt{3}t - 5t^2 + k'$

$\therefore x = 20t$ (A) $\qquad$ When $t=0, y=0 \therefore k'=0$

$\therefore y = 20\sqrt{3}t - 5t^2$ (B)

From (A), $t = \frac{x}{20}$ $\therefore y = 20\sqrt{3}\left(\frac{x}{20}\right) - 5\left(\frac{x}{20}\right)^2$

$y = \sqrt{3}x - \frac{x^2}{80}$ $\qquad$ (4)

b) Equation of slope: $y = \frac{1}{4}x$ $\quad \therefore \sqrt{3}x - \frac{x^2}{80} = \frac{1}{4}x$

$80\sqrt{3}x - x^2 = 20x$

$(80\sqrt{3} - 20)x - x^2 = 0$

$x\left[80\sqrt{3} - 20 - x\right] = 0$

$x = 0$ or $x = 80\sqrt{3} - 20$

$\therefore$ Horizontal distance is $80\sqrt{3} - 20$ $\qquad$ (2)

c) From $y = \frac{1}{4}x$, $y = \frac{1}{4}(80\sqrt{3} - 20) = 29.64$

$x = 80\sqrt{3} - 20 = 118.56$

Distance $OA = \sqrt{118.56^2 + 29.64^2}$

$= 122$ metres (nearest metre) $\qquad$ (2)

d) Height above slope: $H = \sqrt{3}x - \frac{x^2}{80} - \frac{1}{4}x$

$\frac{dH}{dx} = \sqrt{3} - \frac{1}{4} - \frac{x}{40}$

$\frac{dH}{dx} = 0$: $\quad x = 40\left(\sqrt{3} - \frac{1}{4}\right)$

This is for maximum value of H (concave down parabola).

When $x = 40\left(\sqrt{3} - \frac{1}{4}\right)$, $H = \left(\sqrt{3} - \frac{1}{4}\right)40\left(\sqrt{3} - \frac{1}{4}\right) - \frac{1}{80}\cdot 40^2\left(\sqrt{3} - \frac{1}{4}\right)^2$

$= 40\left(\sqrt{3} - \frac{1}{4}\right)^2 - 20\left(\sqrt{3} - \frac{1}{4}\right)^2$

$= 20\left(\sqrt{3} - \frac{1}{4}\right)^2$

$= 20\left(3 - \frac{\sqrt{3}}{2} + \frac{1}{16}\right)$

$= 61.25 - 10\sqrt{3}$

Maximum height is $(61.25 - 10\sqrt{3})$ m. $\qquad$ (4)

Mathematics Extension One: Question Number 7 $\qquad$ Marker: CJF.

Marks Awarded	Marker's Comments
(a)	
1 mark.	$\dot{x} = 20$, $\dot{y} = 20\sqrt{3}$ > in whatever form and wherever stated in answer
1 mark.	$x = 20t$ ..... deriving equation / finding c'
1 mark.	$y = 20\sqrt{3}t - 5t^2$ ... deriving equation and finding k and k' (mark not awarded if k and k' ignored)
1 mark.	$t = \frac{x}{20} \rightarrow y = 20\sqrt{3}\left(\frac{x}{20}\right) - 5\left(\frac{x}{20}\right)^2$.
(b)	
1 mark	$\frac{1}{4}x = \sqrt{3}.x - \frac{x^2}{80}$.
1 mark.	$x = 80\sqrt{3} - 20$ / derived from above.
(c)	
1 mark	$y = \frac{1}{4}(80\sqrt{3} - 20)$ and $x = 80\sqrt{3} - 20$.
1 mark	distance $OA = 122$ m.
(d) 1 mark	$H = \sqrt{3}x - \frac{x^2}{80} - \frac{x}{4}$
1 mark	$\frac{dH}{dt} =$
1 mark	$x = 40\left(\sqrt{3} - \frac{1}{4}\right)$
1 mark	$H = 61.25 - 10\sqrt{3}$
	Note: $t = 2\sqrt{3} - \frac{1}{2}$ Not $t \neq 2\sqrt{3}$. This is because ramp is on a slope and maximum height is not in middle