STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

TRIAL EXAMINATION

2000

MATHEMATICS

2/3 UNIT (COMMON)

Time Allowed - Three hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied.
- Board-approved calculators may be used.
- Each question attempted is to be handed in separately clearly marked Question 1, Question 2 ,... etc..
- ***The question paper must be handed to the supervisor at the end of the examination.***

Write your Student Number/Name on every page.

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Question 1

(a) Evaluate : $\dfrac{28.3+\sqrt{0.512}}{(18.9-2.75)^2}$ correct to 3 significant figures. **2**

(b) Sketch the region defined by : $x^2+y^2 \geq 9$ and $x+y \geq 3$ **2**

(c) Find the exact value of : $\log_3\left(\sqrt{27\sqrt{3}}\right)$ **2**

(d) Given that $x=\sqrt{2}-1$, express $x-\dfrac{1}{x}$ as a surd with rational denominator. **2**

(e) Solve : $-3 \leq 2x+1 \leq 9$. **2**

(f) Solve the pair of simultaneous equations : **2**

$$2x-y=6$$
$$x+3y=10$$

Question 2 *(Start a new page)*

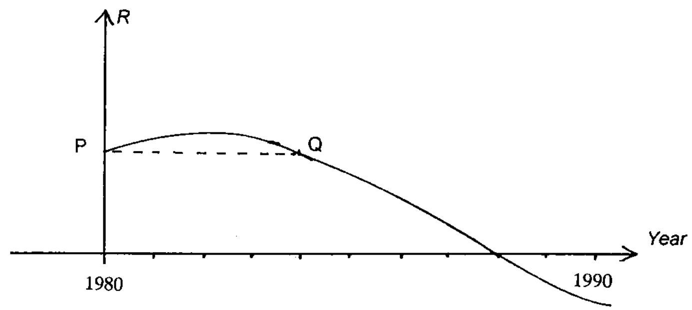
(a) Differentiate each of the following functions : **6**

(i) $x\tan x$.

(ii) $\dfrac{\log_e x}{x}$.

(iii) $\left(e^{-x}-e^{x}\right)^2$

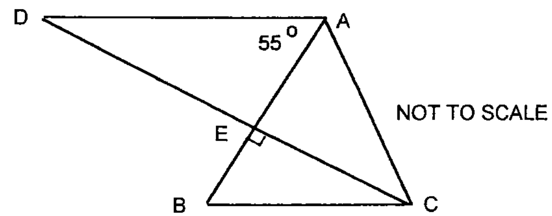
(b) The number of sheep in Australia has gone up and down several times during the 20th Century. The rate of change, R, of the number of sheep during the 1980's is shown in the graph below. **3**



(i) In which year of the 1980's, did the number of sheep begin to decrease?

(ii) Using points P and Q, compare the rate of increase in 1980 with the rate of increase in 1984.

(c) **3**



In the diagram, ΔABC is isosceles with AB = AC.
DA is parallel to BC and AB is perpendicular to DC.
∠DAB = 55°.

Copy the diagram onto your worksheet showing this information.

(i) Show that ∠ACB = 55°.

(ii) Find the size of ∠ACE.

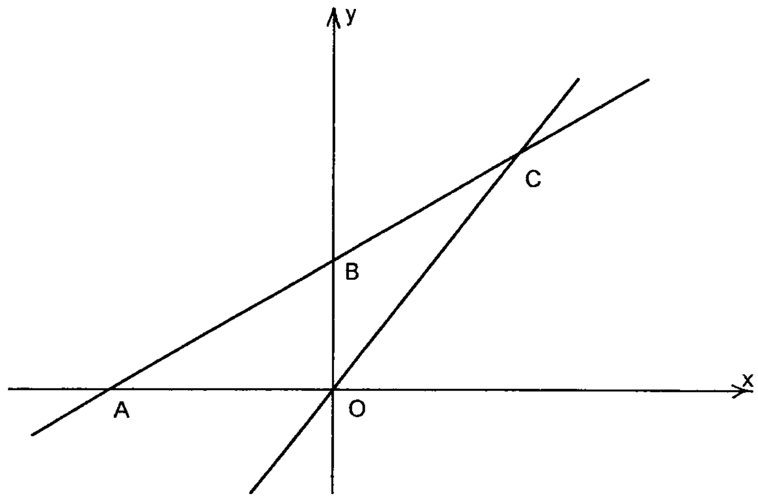
Question 3 *(Start a new page)*

(a) Find **4**

(i) $\int \frac{t^2 - 2t}{t^3}\, dt$.

(ii) $\int \sec^2(1-2x)\, dx$.

(b) **6**



In the diagram above, A and B are points on the x and y axes respectively. The line AB has equation $\sqrt{3}x - y + \sqrt{3} = 0$. Point C lies on AB such that the area of ΔAOC is $3\sqrt{6}$ square units.

Copy the diagram onto your worksheet.

(i) Find the coordinates of A and B.

(ii) Find the gradient of AB.

(iii) Find the size of $\angle$BAO.

(iv) Hence or otherwise find the length of BC, leaving your answer in surd form.

(c) Simplify : $\frac{2^n \times 4^{n+1}}{8^n}$ **2**

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Question 4 *(Start a new page)*

(a) Find the equation of the normal to the curve $y = e^{2x}$ at the point where $x = 1$, leaving your answer in exact form. **4**

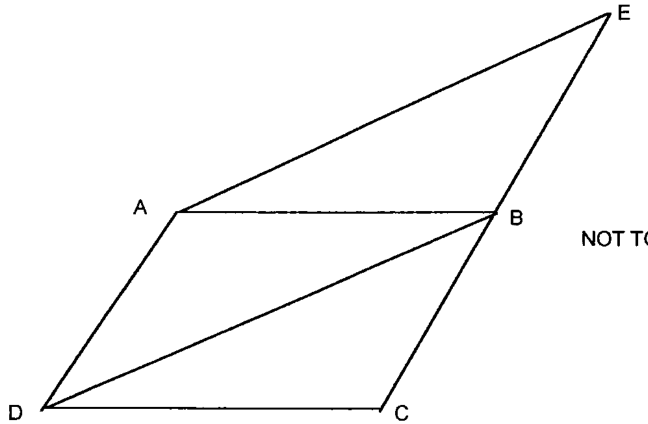
(b) Copy and complete the table below for the function $y = \operatorname{cosec}\frac{\pi x}{6}$ **3**

x	1	2	3
y			

Using one application of Simpson's Rule find an approximate value for

$$\int_1^3 \operatorname{cosec}\frac{\pi x}{6}\, dx$$

(c) **5**



NOT TO SCALE

ABCD is a rhombus. CB is produced to E such that CB = BE.

Copy the diagram onto your worksheet.

(i) Prove that $\Delta ABE \equiv \Delta DCB$.

(ii) Hence show that AE is parallel to DB.

(iii) State, giving reasons,what type of quadrilateral AEBD is.

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Question 5 *(Start a new page)*

(a) For the parabola $y^2 - 6y - 9 = 4x$ **4**

(i) Find the coordinates of the vertex.

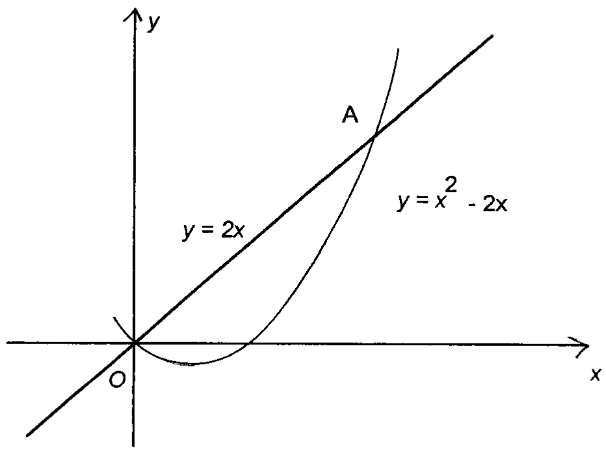
(ii) Find the coordinates of the focus.

(iii) Sketch the curve clearly labelling the vertex and focus

(b) If $y = \ln\left[\frac{1-x}{1+x}\right]$, show that $\frac{dy}{dx} = \frac{-2}{1-x^2}$. **3**

Hence or otherwise evaluate $\int_0^{\frac{1}{2}} \frac{dx}{1-x^2}$.

(c) **3**



The graphs of $y = 2x$ and $y = x^2 - 2x$ are shown in the diagram. They intersect at points O and A.

(i) Find the coordinates of point A.

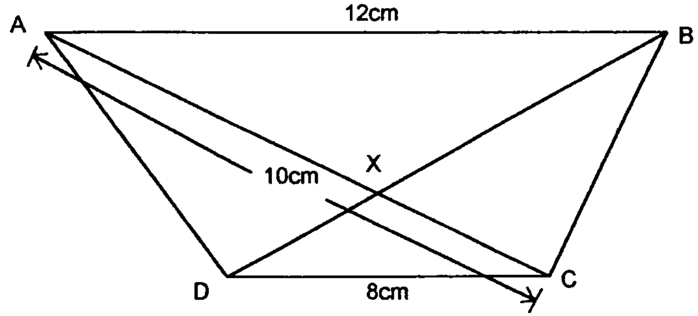
(ii) Find the area completely enclosed by $y = 2x$ and $y = x^2 - 2x$.

(d) Show that $x^2 + kx + k - 1 = 0$ has real roots for all values of k. **2**

Question 6 *(Start a new page)*

(a) For a function $f(x)$, the second derivative is given by $f''(x) = -2x$. **3**
The graph of the function has a minimum turning point at (3,7).
Find the equation of the function.

(b) **5**



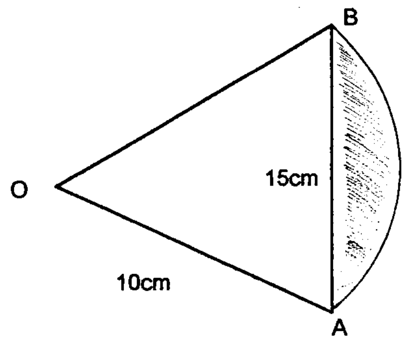
ABCD is a trapezium in which AB is parallel to DC. The diagonals intersect at X.
AB = 12 cm, DC = 8 cm and AC = 10 cm.

Copy the diagram onto your worksheet.

(i) Prove that ΔAXB is similar to ΔCXD.

(ii) Hence find the length of AX.

(c) **4**



The diagram shows a sector of a circle with centre O and radius 10 centimetres.
Chord AB has length 15 centimetres.

(i) Find the size of $\angle$AOB.

(ii) Calculate the shaded area bounded by arc AB and chord AB.

Question 7 *(Start a new page)*

(a) To prepare the Beach Volleyball Courts on Bondi Beach for the Olympics, a truck is tipping loads of sand in the area. For the first load of sand, the truck travels a total distance of 800 metres from the loading point to the courts and back. For the second load the distance is 820 metres and each succeeding load is 20 metres further than the previous one. **5**

(i) How far does the truck travel on the 8^{th} load?

(ii) On which trip would the truck travel 1.4 kilometres?

(iii) At lunchtime the driver found that the truck had travelled a total distance of 27.3 kilometres. How many loads had he tipped?

(b) The number of termites, N, in a colony after t days is given by the formula : **5**

$$N = 10000\, e^{kt} \text{ where } k \text{ is a constant.}$$

The colony doubles in size over a period of 1 week.

(i) Find the number of termites initially in the colony.

(ii) Find the value of k.

(iii) Show that the rate of increase in the number of termites is given by

$$\frac{dN}{dt} = kN$$

(iv) After how many days would the number of termites reach 1 000 000 ?

(v) What is the rate of increase when the number is 1 000 000 ?

(c) If $\cos\beta = \frac{3}{7}$ and $\sin\beta < 0$, find the exact value of $\tan\beta$. **2**

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Question 8 *(Start a new page)*

(a) Consider the curve given by $y = 3x^2 - x^3 + 9x - 2$ **6**

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(ii) Find the coordinates of the stationary points and determine their nature.

(iii) Sketch the graph of the function for the domain $-2 \le x \le 5$.

(iv) State the minimum value of the function over this domain.

(b) A cylindrical can of radius r centimetres and height h centimetres is to be made from a sheet of metal with area 300π centimetres^2. There is 10% wastage of the sheet in manufacturing the can. **6**

(i) Show that $h = \frac{135 - r^2}{r}$.

(ii) Find an expression for the volume V as a function of r.

(iii) Find the value of r which gives the maximum volume.

(iv) Calculate the maximum volume.

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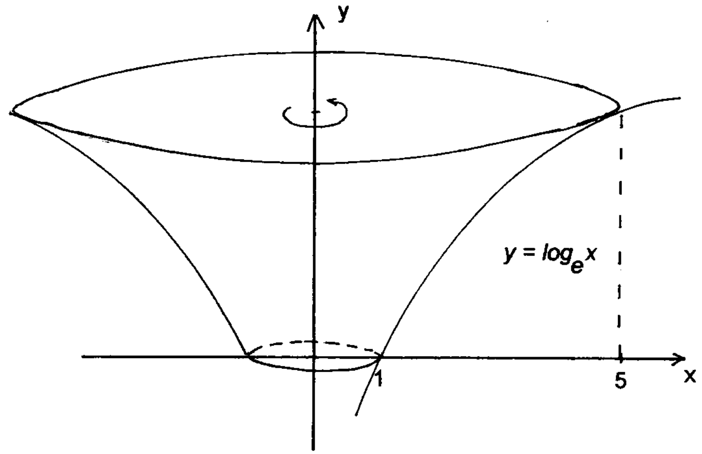
Question 9 *(Start a new page)*

(a) **4**



The interior of the bowl which is to be used to hold the Olympic flame is shaped by rotating the arc of the curve $y = \log_e x$ from $x = 1$ to $x = 5$ about the y-axis.

(i) Show that its volume is given by $V = \pi \int_0^{\ln 5} e^{2y}\,dy$

(ii) Calculate the capacity of the bowl.

(b) Sula undertook to donate \$4 000 to the Salvation Army Red Shield Appeal on her 60th birthday. On her 61st birthday she would donate \$3 000, and on each succeeding birthday, three-quarters of the amount donated on the previous birthday. **4**

(i) What is the greatest sum of money the Salvation Army could expect to receive from Sula?

(ii) If Sula died after her 79th birthday, by how much, correct to the nearest dollar, would the total received fall short of this sum?

(c) Given that $y = x e^{-2x}$, prove that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$. **4**

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Question 10 *(Start a new page)*

(a) A credit card holder requires a 4 digit Personal Identification Number (PIN) to enable her to withdraw money from her bank account. The digits can be chosen from the integers 0, 1, 2, 3, ... 9. **5**

(i) If the credit card holder could not remember her PIN, what is the probability of guessing it correctly?

(ii) What is the probability that she will guess at least one digit correctly?

(iii) If she remembered that either the second ***or*** third digit was either a 7 or a 6, what is the probability of guessing the PIN correctly?

(iv) A member of the card holder's family has borrowed the card. He can remember the four digits but not their order. What is the probability that he will guess the PIN correctly?

(b) The motion of a particle moving in a straight line, is defined by the function $x = 3\cos 2t$, where x is the displacement, measured in metres from a fixed point O, and t is the time elapsed measured in seconds. **7**

(i) Draw a neat sketch of the displacement-time graph from $t = 0$ to $t = \pi$.

(ii) Find an expression for the particle's velocity.

(iii) In which direction is the particle moving after two seconds?

(iv) The particle is intially at rest. Find when it next comes to rest.

(v) Show that the particle acceleration of the particle can be described by the equation $a = -4x$.

(vi) Find the maximum displacement of the particle.

(vii) By drawing a straight line on your graph, show the number of times in the period $t = 0$ to $t = \pi$, the particle would be 2 metres on the positive side of O.

End of Paper

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