Shore School

4 unit mathematics

Trial HSC Examination 1986

1. (i) (a) Prove that if a polynomial $P(x)$ has a zero of multiplicity m, then $P'(x)$ has a zero of multiplicity $(m-1)$.
(b) The equation $8x^4+12x^3-30x^2+17x-3=0$ has a root of multiplicity 3. Find all of its roots.
(ii) If in the polynomial $x^3-a^2x^2-2ax-5$, a is an integer and $x-5$ is a factor, find the value of a. For this value of 'a' factorise $x^3-a^2x^2-2ax-5$ over the complex field.
(iii) If α, β, γ are the zero of $2x^3-4x^2-3x-1$ find the value of $\alpha^2\beta^2+\alpha^2\gamma^2+\beta^2\gamma^2$.

2. (i) Find the equation of the circle with AB as a diameter where A, B have coordinates (x_1, y_1) and (x_2, y_2) respectively.
(ii) At a N.S.W state election 30% of the voters favoured party A. If an interviewer selected 5 voters at random, what is approximately the probability that a majority of those selected favoured party A? Give your answer expressed as a percentage correct to two significant figures.
(ii) Two equal light rods AC, CB are hinged (freely joined) at C, the mass of the hinge being m kilograms. The ends A and B are hinged to two points, A being vertically above B and $AB=a$ metres. If the system revolves with angular velocity ω rad/s about AB, prove that both rods are in tension of $\omega > \sqrt{\frac{2g}{a}}$. If the tensions in the rods are in the ratio $5:2$, find the period of the motion.

3. (i) If $P(x,y)$ is a general point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and S and S' are the two foci, prove that $PS+PS'=2a$.
(ii) Show that the equation of the tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point $P(a\sec\theta, b\tan\theta)$ is $\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta=1$. If this tangent at P meets the asymptotes of the hyperbola at the two points A and B, show that P is the midpoint of AB.

4 (i) If z_1 and z_2 are the roots of the equation $z^2-5(1+i)z+17i=0$, show that
(a) the inclination of the line z_1z_2 to the positive direction of the real axis is $\frac{3\pi}{4}$
(b) the area of the triangle Oz_1z_2 is $\frac{15}{2}$ square units.
(ii) If $w=\frac{z-6i}{z+8}$ show that the locus of z is a circle when w is purely imaginary but a straight line when w is real. Find the equations of these loci.

5. (i) Show that $\int_3^6 \frac{dx}{\sqrt{27+6x-x^2}}=\frac{\pi}{6}$
(ii) Evaluate -

(a) $\int_7^{14} \frac{dx}{(x-2)\sqrt{x+2}}$
(b) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos 4x . \cos 2x \; dx$
(iii) Find $\int \sqrt{1+x^2} \; dx$

6. **(i)** Sketch, using the Calculus, $y = \frac{3x}{(x-1)(x-4)}$ showing all asymptotes and all turning points. (There is no need to find the point of inflexion). Distinguish clearly the stationary points.
(ii) A particle of mass 1 kg is projected from a point O with velocity u m/s along a smooth horizontal table in a medium whose resistance is RV^2 Newtons when the particle has velocity V m/s, R being constant. Find its velocity as a function of t. An equal particle is projected from O simultaneously with the first particle but vertically upwards under gravity with velocity u in the same medium. Show that the velocity V of the first particle when the second is momentarily at rest is given by $\frac{1}{V} = \frac{1}{u} + \frac{1}{a}\tan^{-1}(\frac{u}{a})$ where $Ra^2 = g$.

7. **(i)** **(a)** Show that $\int \frac{dx}{5-4\cos x} = \frac{2}{3}\tan^{-1}(3\tan\frac{x}{2}) + C$
(b) Given that $\int_0^{\pi} \frac{dx}{5-4\cos x} = \frac{\pi}{3}$ show that $\int_0^{\pi} \frac{\cos x \; dx}{5-4\cos x} = \frac{\pi}{6}$
(ii) If $u_n = \int_0^{\pi} \frac{\cos nx}{5-4\cos x} \; dx$ show that $u_{n+1} + u_{n-1} - \frac{5}{2}u_n = 0$. Using part **(i)** find the values of u_2 and u_3.
(iii) If $p+q = 1$ and $p^2+q^2 = 2$ determine the values of p^3+q^3 and p^4+q^4 without finding the value of p and q.

8. **(i)** The flat base of a solid is the region bounded by two branches of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and the lines $y = \pm a$. Cross-sections of the solid, perpendicular to the y-axis are equilateral triangles (the ends of their bases are on the hyperbola). Show that the volume of the solid is $\frac{2\sqrt{3}a^3(3b^2+a^2)}{3b^2}$ cubic units.
(ii) Given that $u_n = \frac{1.3.5.....(2n-1)}{2.4.6.....2n}$ for integers $n \geq 1$, prove by Mathematical Induction that $u_n \geq \frac{1}{2\sqrt{n}}$.
(iii) Prove that the expression $\frac{x-a}{x^2-2x+a}$ (for a, x real) can assume all values if $0 < a < 1$.