1) a) $l = r\theta$

$= 12 \times \frac{2\pi}{3}$

$= 8\pi$

b) i) $y = e^{4x} + 3x$

$y' = 4e^{4x} + 3$

ii) $y = \sin 3x$

$y' = 3\cos 3x$

iii) $y = x^2 \log_e (2-5x)$

$y' = 2x \log_e (2-5x) + x^2 \times \frac{-5}{2-5x}$

$= 2x \log_e (2-5x) - \frac{5x^2}{2-5x}$

iv) $y = \tan^2 5x$

$y' = 2\tan 5x \times 5 \sec^2 5x$

v) $y = \log_e \sqrt{x+1}$ $\quad y = \log_e (x+1)^{\frac{1}{2}}$

$y' = \frac{\frac{1}{2}(x+1)^{-\frac{1}{2}}}{\sqrt{x+1}} = \frac{1}{2\sqrt{x+1} \times \sqrt{x+1}}$

$= \frac{1}{2(x+1)}$

1) d $3^2 = 4x$

$x = 2\frac{1}{4}$

c) i) $\int \frac{4}{1+3x}\,dx$

$= \frac{4}{3} \log_e 1+3x + c$

ii) $\int \tan^2 \frac{x}{3}\,dx$

$\int -1 + \sec^2 \frac{x}{3}\,dx$

$= -x + 3\tan \frac{x}{3} + c$

iii) $\int 5e^{2x+3}\,dx$

$5\int e^{2x+3}\,dx$

$= 5\left(\frac{1}{2} e^{2x+3}\right) + c$

$= \frac{5}{2} e^{2x+3} + c$

2) a) i) $\int_1^e \frac{4\,dx}{x}$

$= 4\left[\log_e x\right]_1^e$

$= 4\left[\log_e e - \log_e 1\right]$

$= 4[1 - 0]$

$= 4$

ii) $\int_0^1 \frac{e^{2x}+1}{e^x}\,dx$

$= \int_0^1 e^x + e^{-x}\,dx$

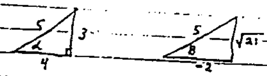
$= \left[e^x - e^{-x}\right]_0^1$

$= \left[e^1 - e^{-1}\right] - \left[e^0 - e^{-0}\right]$

$= e^1 - \frac{1}{e} - (1 - 1)$

$= e - \frac{1}{e}$

2) c



i) $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2 \times \frac{3}{5} \times \frac{4}{5}$

$= \frac{24}{25}$

ii) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{4}{5} \times \frac{-2}{5} - \frac{3}{5} \times \frac{\sqrt{21}}{5}$

$= \frac{-8}{25} - \frac{3\sqrt{21}}{25}$

$= \frac{-8 - 3\sqrt{21}}{25}$

3) a) $\frac{\sin 2x}{1+\cos 2x}$

LHS $= \frac{2\sin x\cos x}{2\cos^2 x}$

$= \frac{\sin x}{\cos x}$

$= \tan x$

$=$ RHS

c) $\cos\theta = \frac{8}{9}$

$\cos\theta = \frac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}$

$\frac{8}{9} = \frac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}$

$8 + 8\tan^2\frac{\theta}{2} = 9 - 9\tan^2\frac{\theta}{2}$

$17\tan^2\frac{\theta}{2} = 1$

$\tan^2\frac{\theta}{2} = \frac{1}{17}$

d) $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$

$= \frac{1}{2}\left[\left(x + \frac{1}{2}\sin 2x\right)\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{2}\left[\frac{\pi}{2} + \frac{1}{2}\sin\pi - 0 - \frac{1}{2}\sin 0\right]$

$= \frac{1}{2}\left[\frac{\pi}{2} + 0\right]$

$= \frac{\pi}{4}$

e) $y' = \frac{2}{x+1}$

$y = 2\log_e(x+1) + c$

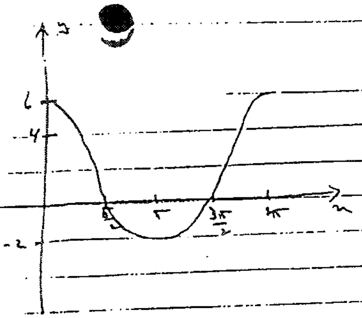
$1 = 2\log_e 1 + c$ $\qquad y = 2\log_e(x+1) + 1$

f) $\left[\ln x\right]_a^e = 5$

$\ln e - \log_e a = 5$ $\qquad \log_e a = -4$

$1 - \log_e a = 5$ $\qquad a = e^{-4}$

Q4 a)



b) $\frac{x+2}{2} = 2x$

$x + 2 = 4x$

$3x = 2$

$x = \frac{2}{3}$

c) $\sqrt{3}\cos x - \sin x = R\cos(x+\alpha)$

$= R(\cos x\cos\alpha - \sin x\sin\alpha)$

$R = 2$

$R\cos\alpha = \sqrt{3}$

$R\sin\alpha = 1$

$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}$

$2\cos\left(x+\frac{\pi}{6}\right) = 1$

$\cos\left(x+\frac{\pi}{6}\right) = \frac{1}{2}$

$x + \frac{\pi}{6} = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{3} - \frac{\pi}{6}, \frac{5\pi}{3} - \frac{\pi}{6}$

$x = \frac{\pi}{6}, \frac{3\pi}{2}$

d) $\frac{1}{\tan 2x} = \tan 2x$

$\tan^2 2x = 1$

$\tan 2x = \pm 1$

$2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

$\frac{7\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}$

$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$

4) e) LHS $= \frac{\tan\frac{\pi}{4} + \tan\frac{\theta}{2}}{1 - \tan\frac{\pi}{4}\tan\frac{\theta}{2}}$

$= \frac{1+t}{1-t}$ $(t = \tan\frac{\theta}{2})$

RHS $= \frac{1+t^2}{1-t^2} + \frac{2t}{1-t^2}$

$= \frac{(1+t)^2}{1-t^2} = \frac{1+t}{1-t}$

$=$ LHS

Q5 a) i) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= 16$

ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= 0$

iii) $\alpha\beta\gamma + 2(\alpha\beta + \alpha\gamma + \beta\gamma) + 4(\alpha+\beta+\gamma) + 8$

$= 33$

b) $P(0) = 11$

$P(2) = 5$

c) $y_1 = x^4 + 2x + 2$

$y' = 4x^3 + 2$

$m_1 = 4 + 2$

$= 6$

$y_2 = \frac{1}{x}$

$y' = -x^{-2}$

$= \frac{-1}{x^2}$

$m_2 = -1$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{7}{-5}\right|$

$\theta = 54^\circ 28'$

$= 54.5^\circ$

d) $\frac{5\left(\frac{3}{\sqrt{13}}\right) - 7\left(\frac{2}{\sqrt{13}}\right)}{\frac{\sqrt{13}}{3} \times \frac{\sqrt{13}}{2}}$

$= \frac{6}{65}$

Q6. a) $\frac{d}{dx}(x \ln x) = x \cdot \frac{1}{x} + \ln x$

$= 1 + \ln x$

$\therefore \int (1 + \ln x) = x \ln x + c$

$\therefore \int \ln x \, dx = x \ln x - x + c$

$\therefore \int_{\sqrt{e}}^{e} \ln x \, dx = \left[x \ln x - x \right]_{\sqrt{e}}^{e}$

$= e \ln e - e - \sqrt{e} \ln \sqrt{e} + \sqrt{e}$

$= e - \frac{\sqrt{e}}{2} - e + \sqrt{e}$

$= \sqrt{e}/2$

b) $y = 2\cos x$

$y = \frac{1}{2}\sec x$

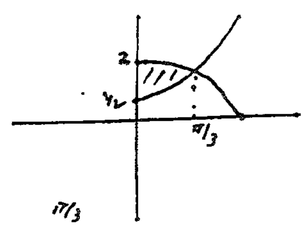
$\therefore 2\cos x = \frac{1}{2\cos x}$

$4\cos^2 x = 1$

$\cos^2 x = 1/4$

$\cos x = \pm 1/2$

$\therefore x = \pi/3$



$V_1 = \pi \int_0^{\pi/3} 4\cos^2 x \, dx$

$= 2\pi \int_0^{\pi/3} (1 + \cos 2x) \, dx$

$= 2\pi \left[\frac{\pi}{3} + \frac{1}{2}\sin \frac{2\pi}{3} \right]$

$V_2 = \frac{\pi}{4} \int_0^{\pi/3} \sec^2 x \, dx$

$= \frac{\pi}{4} \cdot \left[\tan x \right]_0^{\pi/3}$

$= \frac{\pi\sqrt{3}}{4}$

$\therefore \text{Volume} = 2\pi \left[\frac{\pi}{3} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \right] - \frac{\pi\sqrt{3}}{4}$

$= \left(\frac{2\pi^2}{3} + \frac{\pi\sqrt{3}}{4} \right) u^2.$

c) $4x^3 - 12x^2 + 11x - 3 = 0$

Roots are $\alpha - \beta, \alpha, \alpha + \beta$

$3\alpha = \cancel{\beta}\,3 \quad \alpha = \cancel{4}\,1$

$\therefore x - \cancel{4}\,1$ is a factor.

$\therefore P(x) = (x-1)(4x^2 - 8x + 3)$

$= (x-1)(2x-1)(2x-3)$

$\therefore$ Roots are $1/2, 1, 3/2$

d) $\cos x = \cos\left(\frac{x}{2} + \frac{x}{2}\right)$

$= \cos^2 \frac{x}{2} - \sin^2 x/2$

$= 1 - 2\sin^2 x/2$

$\therefore 1 - \cos x = 2\sin^2 x/2$

$\therefore \lim_{x \to 0} \frac{1 - \cos x}{3x^2} = \lim_{x \to 0} \frac{2\sin^2 x/2}{3x^2}$

$= \lim_{x \to 0} \frac{2}{3} \cdot \frac{\sin^2 x/2}{x^2}$

$= \lim_{x \to 0} \frac{2}{3} \cdot \frac{\sin x/2}{x/2} \cdot \frac{\sin x/2}{x/2}$

$= \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{6}$