THE KING'S SCHOOL

2004
Higher School Certificate
Trial Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown for every question

Total marks – 84

- Attempt Questions 1-7
- All questions are of equal value
- Start a new booklet for each question
- Put your Student Number and the question number on the front of each booklet

Total marks – 84
Attempt Questions 1-7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\frac{d}{dx}\left(e^{\tan x}\right)$. 2

(b) The interval joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is trisected by the points $(-2,3)$ and $Q(1,0)$. Write down the coordinates of A and B. 3

(c) Find the acute angle, to the nearest degree, between the lines $x - y = 2$ and $2x + y = 1$. 2

(d) Use the substitution $u = 1 - x$ to evaluate $3\int_{-3}^{0} \frac{x}{\sqrt{1-x}}\,dx$. 3

(e) For a given series $T_{n+1} - T_n = 7$ and $T_1 = 3$. Find the value of S_{100}, where $Sn = T_1 + T_2 + \ldots + T_n$. 2

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $\dfrac{x^2-2}{x}<1$. 3

(b) Find

(i) $\displaystyle\int \frac{e^{2x}}{1+e^{2x}}\,dx$. 1

(ii) $\displaystyle\int \frac{3}{5+x^2}\,dx$. 2

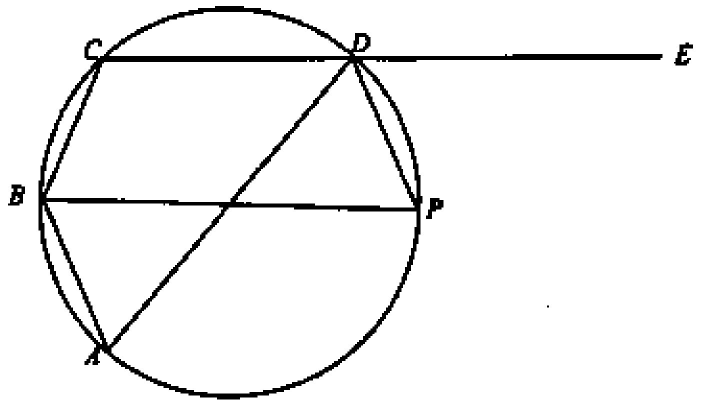
(c) Solve the equation $2\ln(3x+1)-\ln(x+1)=\ln(7x+4)$. 3

(d) Solve $2\tan^3\theta-3\tan^2\theta-2\tan\theta+3=0$ for $0\le\theta\le 360°$, giving your answers to the nearest minute, where necessary. 3

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Use one application of Newton's Method to approximate the root of the equation $e^x + x = 2$ which is near 0.5, correct to two decimal places. **3**

(b)



In the diagram above $ABCD$ is a cyclic quadrilateral. CD is produced to E. P is a point on the circle through A, B, C, D such that $\angle ABP = \angle PBC$.

(i) Copy the diagram showing the above information.

(ii) Explain why $\angle ABP = \angle ADP$. **1**

(iii) Show that PD bisects $\angle ADE$. **2**

(iv) If $\angle BAP = 90°$ and $\angle APD = 90°$, explain where the centre of the circle is located. **2**

(c) (i) Write $\cos x - \sqrt{3}\sin x$ in the form $A\cos(x+\alpha)$ where $A > 0$, $0 < \alpha < \pi$. **2**

(ii) Hence or otherwise, solve $\cos x - \sqrt{3}\sin x = 1$ for all values of x. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Find the polynomial $P(x)$, if $P(x)$ has

(α) degree 4;

(β) factors of $(x+3)^2$ and $(x-3)^2$; and

(γ) a remainder of -50 when divided by $x+2$. **2**

(ii) Sketch the curve. **1**

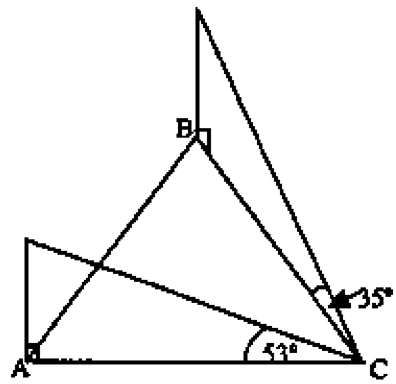
(b) The speed v cm/sec of a particle moving with simple harmonic motion in a straight line is given by $v^2 = 6 + 4x - 2x^2$, where x cm is the magnitude of the displacement from a fixed point O.

(i) Show $\dfrac{d^2x}{dt^2} = -2(x-1)$. **2**

(ii) Find the period of the motion. **1**

(iii) Find the amplitude of the motion. **2**

(c) A and B are the feet of two towers of equal height. B lies due North of A. From a point C, 40m East of A and in the same horizontal plane, the angle of elevation of the top of the tower A is 53°. From the same point the angle of elevation of tower B is 35°. Find the distance between the towers, AB, correct to the nearest metre. **4**



Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

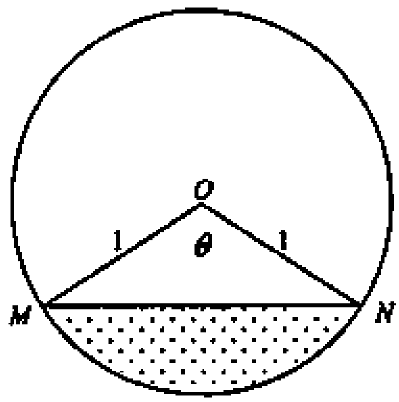
(a) For the function $y = 3\cos^{-1}\frac{x}{2}$

(i) Find the domain and the range. **2**

(ii) Sketch the curve. **1**

(iii) Find the equation of the tangent to the curve at the point on the curve where $x = 0$. **3**

(b) O is the centre of a circle, radius 1m and $\angle MON = \theta$ radians. The shaded segment formed by MN has an area A square metres and perimeter P metres.



(i) Prove $A = \frac{1}{2}(\theta - \sin\theta)$. **1**

and $P = \theta + 2\sin\frac{\theta}{2}$. **1**

(ii) P is increasing at a constant rate of R m/s. Find, in terms of R, the rate of increase of

(α) θ when $\angle MON = \frac{2\pi}{3}$; and **2**

(β) A when $\angle MON = \frac{2\pi}{3}$. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the coordinates of the focus and the equation of the directrix of the parabola $x^2 = 4(x+y)$. **2**

(b) Prove by Induction that $3^{3n} + 2^{n+2}$ is divisible by 5 for all positive integers n. **4**

(c) Consider the variable point $T(-2t, t^2)$ on the parabola $y = \frac{1}{4}x^2$.

(i) Prove that the equation of the tangent at T is $y + tx + t^2 = 0$ **2**

(ii) If A is the x intercept of the tangent at T, find the coordinates of A. **1**

(iii) Find the coordinates of M, the midpoint of the interval TA. **1**

(iv) Find the equation in Cartesian form of the locus of the point M given in part (iii). **2**

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A stone is thrown from the top of a building 15m high with an initial velocity of 26 m/s at an angle of $\tan^{-1}\dfrac{5}{12}$ to the horizontal.

If the acceleration due to gravity is 10m/sec^2, find

(i) the greatest height above the ground reached by the stone **2**

(ii) the time of flight **2**

(iii) the range of the stone **1**

(iv) the velocity after 2 seconds **2**

(b) Two of the roots of the equation $x^3 + ax^2 + b = 0$ are reciprocals of each other where a and b are real numbers.

Show that

(i) the third root is equal to $-b$; **1**

(ii) $a = b - \dfrac{1}{b}$; and **2**

(iii) the two roots, which are reciprocals, will be real if $-\dfrac{1}{2} \le b \le \dfrac{1}{2}$. **2**

End of Examination