Question 1

a) Differentiate $\sin^3 2x$. (2)

b) Find i) $\int x e^{x^2} dx$ (2)

ii) $\int \cos^2 x \, dx$ (2)

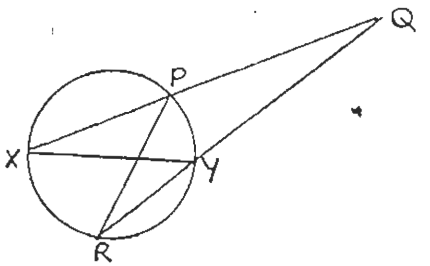
c) Find the co-ordinates of the point which divides the interval AB externally in the ratio 3:2 given that A and B are the points $(-5,2)$ and $(4,5)$ respectively. (3)

d) Solve for x: (3)

$$\frac{x+5}{x-1} > 7$$

Question 2

(a)



XY is the diameter of the circle XP YR. XPQ and RYQ are straight lines. Given that $\angle PXY = 35^0$ and $\angle PQY = 25^0$ find the size of $\angle YPR$, giving reasons. (3)

(b) Express the solution to the equation $\sin 2\theta = \sin\theta$ in general form, θ in radians. (4)

(c) Use mathematical induction to prove that $7^n - 1$ is always divisible by 6 for all positive integral values of n. (5)

Question 3

a) Sketch the graph of $y = \cos^{-1} 2x$ and clearly state the domain and range. (3)

b) Without a calculator, find the value of: $\sin\{2\tan^{-1}(\frac{4}{3})\}$ (4)

c) Find $\frac{d}{dx}\{\frac{2x}{4+x^2} + \tan^{-1}(\frac{x}{2})\}$ (5)

and hence evaluate $\int_0^2 \frac{dx}{(4+x^2)^2}$

Question 4

a) Air is being pumped into a spherical balloon at the rate of 20cm^3/second. Find the rate of increase of the surface area of the balloon when the radius is 5cm.

(Note: $V = \frac{4}{3}\pi r^3$ and $S.A. = 4\pi r^2$) (4)

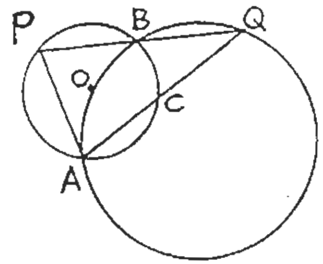
b) Show that the acute angle θ between the tangents at the point $(1,1)$ to the curves $y = x^2$ and $y = x^{\frac{1}{2}}$ is given by $\theta = \tan^{-1}\frac{3}{4}$. (4)

c) It is known that $\log_e x + \sin x = 0$ has a root close to $x = 0.5$. Use one application of Newton's method to find a better approximation. (4)

Question 5

a) $\int^{9} \frac{dx}{x+\sqrt{x}}$ using the substitution $x = u^2$ (4)

b)



In the diagram, O is the centre of the circle APB. PB is produced to Q, on the circle through AOB. PA is not necessarily the tangent to this circle. AQ cuts the first circle, centre O, at C. Prove that PQ = QA. (4)

c) A company assumes that the proportion P of viewers who will buy a new product after it is advertised n times on television satisfies a relation

$$P = 1 - e^{kn} \quad \text{where } k \text{ is a constant.}$$

If 50% of viewers buy the product after 10 advertisements appear, how many times should the company advertise the product if it wants at least 90% of its viewers to buy it? (4)

Question 6

a) A particle moves in a straight line so that its displacement from point O is x metres. Its speed v m/s is given by: $v^2 = 8 - 10x - 3x^2$

i) Find the acceleration as a function of x and hence show that the motion is simple harmonic. (2)

ii) State the period, centre of motion, and amplitude. (3)

b) $P(x) = x^3 - 6x^2 + ax - 4,\ a > 0.$

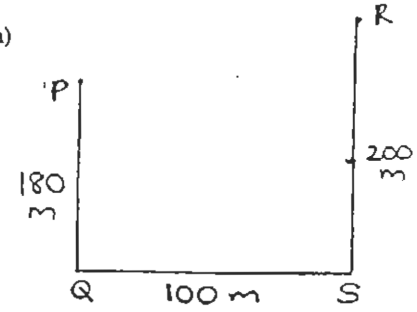
(i) Given that all roots of $P(x) = 0$ are real and positive, and that one of the roots is the product of the other two roots, show that $a = 10$. (3)

(ii) Show that $x - 2$ is a factor of $P(x) = x^3 - 6x^2 + 10x - 4$ (2)

(iii) Find all roots of $x^3 - 6x^2 + 10x - 4$. (2)

Question 7

a)



PQ and RS are 2 buildings situated 100m apart on level ground. Their heights are 180m and 200m respectively. An object is projected from point P at an angle of 45^0 to the horizontal, and the object strikes point R. Take acceleration due to gravity, g, as 10m/s^2. Show that the time taken for an object to get from P to R is 4 seconds and find the value of the initial velocity of projection. (6)

b) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$. The tangent at P and the line through Q parallel to the axis of the parabola meet at point R.

The tangent at Q and the line through P parallel to the axis of the parabola meet at point S.

i) Show that the equations of the tangents at P and Q are $y = px - ap^2$ and $y = qx - aq^2$ respectively. (1)

ii) The co-ordinates of S and R are $(2ap, 2apq - aq^2)$ and $(2aq, 2apq - ap^2)$ respectively. (1)

iii) Show that PQRS is a parallelogram. (2)

iv) Show that the area of this parallelogram is $2a^2|p-q|^3$. (2)

Answers

1(a) $6\sin^2 2x\cos 2x$

(b)(i) $\frac{1}{2}e^{x^2}+c$

(ii) $\frac{1}{4}\sin 2x+\frac{1}{2}x+c$

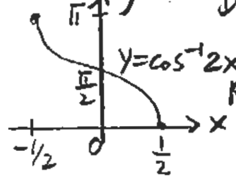
(c) (22, 11)

(d) $1<x<2$

2(a) 30°

(b) $n\pi$ or $2n\pi \pm \frac{\pi}{3}$, n is an integer.

3(a)



$D: -\frac{1}{2} \le x \le \frac{1}{2}$

$R: 0 \le y \le \pi$

(b) $\frac{24}{25}$

(c) $\frac{16}{(4+x^2)^2}$, $\frac{1}{16}\{\frac{1}{2}+\frac{\pi}{4}\}$

4(a) $8\text{cm}^2/\text{s}$

(c) 0.57 (2dp)

5(a) $2\ln 2$

(c) 34 times

6(a)(i) $\ddot{x} = -5-3x$
$= -3(x+5/3)$

(ii) $2\pi\sqrt{3}/3$ s, $x = -5/3$,
$2\frac{1}{3}$ m

6(b)(iii) $2,\ 2\pm\sqrt{2}$

7(a) $25\sqrt{2}$ m/s