CRANBROOK SCHOOL

Form VI/Year 12 MATHEMATICS - 3 Unit (Second Paper), 4 Unit (First Paper)

Term 3 1997

Time: 2 hrs (CGH / GPP)

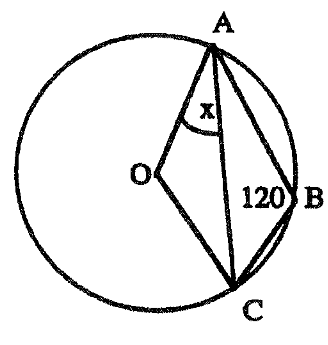
All questions may be attempted.
All questions are of equal value.
All necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.
Standard integrals are provided at the end of the paper.
Approved silent calculators may be used.
Begin each question on a new page. Submit your work in two booklets
(i) qq. 1 - 4
(ii) qq. 5 - 7

1.

(a) Solve for x : $\dfrac{2x-5}{x+3} \geq 1$

(b) Find the point P which divides the interval AB externally in the ratio 3 : 2, given that A is the point (5, 3) and B is the point (1, -3).

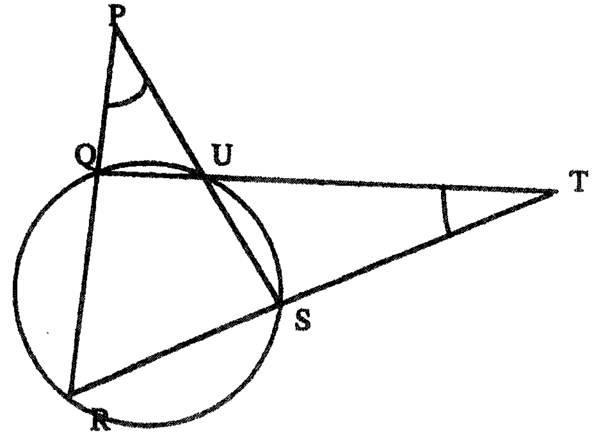
(c) In the diagram opposite find x, giving full reasons at each step.



(d) In the diagram opposite $\angle RPS = \angle QTR$, and PQR, TUQ and TSR are straight lines.

(i) Prove that $\angle UQR = \angle USR$.

(ii) Hence explain why UR is a diameter of the circle.



5. (new page please)

(a) (i) Show that there is a root of the equation $3\sin 2x = x$ between 1.3 and 1.4.
(ii) Use Newton's method to find this root correct to two decimal places.

(b) Given that $f(x) = ax^3 + bx^2 + cx + d$ is a function with a double zero at $x = 1$ and with a minimum value of -4 when $x = -1$, find the value of each coefficient in $f(x)$.

(c) Prove by the principle of mathematical induction that $n^3 + 2n$ is divisible by 3 for all positive integers n.

6. (new page please)

(a) In the expansion of $\left(x^2 + \frac{2}{x}\right)^{10}$ find the coefficient of x^2.

(b) (i) Assuming that, for all real numbers x and for all positive integers n

$$(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r \qquad \left[\text{where } \binom{n}{r} = {}^nC_r\right]$$

show that :

$$\sum_{r=0}^{n} (-1)^r \binom{n}{r} = 0$$

(ii) Similarly, evaluate :

$$\sum_{r=0}^{n} 2^r \binom{n}{r} \qquad \text{and} \qquad \sum_{r=0}^{n} r\binom{n}{r}$$

(c) Using the substitution $u = x^4$, or otherwise, show that $\int_0^1 \frac{x^3}{1+x^8}\,dx = \frac{\pi}{16}$

7. (new page please)

The distinct points P, Q on the parabola $x = 2t$, $y = t^2$ have parameters equal to p and q respectively.

(i) Write down the equation of the tangent to the parabola at P.
(ii) Show that the equation of the chord PQ is $2y - (p+q)x + 2pq = 0$
(iii) Show that M, the point of intersection of the tangents to the parabola at P and at Q has the co-ordinates $(p+q\,,\,pq)$.
(iv) Prove that for any value of p , except $p = 0$, there are exactly two values of q for which M lies on the parabola $x^2 = -4y$ and find these two values in terms of p. Find also the co-ordinates of the corresponding point M.
(v) Show that, for these values of q, the chord PQ is a tangent to the parabola $x^2 = -4y$.

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