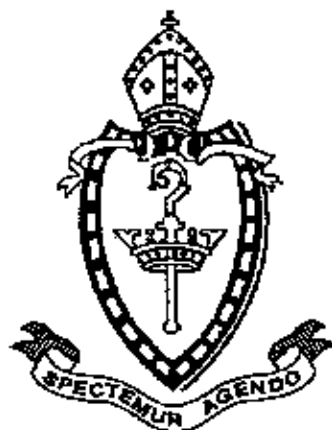


NEWCASTLE GRAMMAR SCHOOL



YEAR 12 2003 MATHEMATICS TRIAL EXAMINATION

*Time allowed – Three hours
(Plus 5 minutes reading time)*

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question.
Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on page 10.
- Board-approved calculators may be used.
- Answer each question in a SEPARATE Writing Booklet.
- You may ask for extra Writing Booklets if you need them.

QUESTION 1 Use a SEPARATE Writing Booklet.

Marks

- a) Calculate the value of $\frac{\sqrt{4\pi}}{3 \cdot 6^2 - 9 \cdot 8}$ correct to four significant figures 2
- b) Express $\frac{6}{\sqrt{3}-1}$ with a rational denominator 2
- c) Differentiate $6 - x^3$ 2
- d) Solve $\frac{x}{2} + \frac{x}{3} = 1$ 2
- e) Integrate $\frac{4}{x}$ 2
- f) Factorise completely $9 - 16t^2$ 2

QUESTION 2 Use a SEPARATE Writing Booklet.

- a) Differentiate:
- i) $y = e^{3x} + \frac{x^4}{2}$ 3
- ii) $y = \frac{\log_e x}{x}$ 3
- b) Sketch the graph with the equation $y = x - x^2$ showing all intercepts: 2
- c) Solve $|x + 4| = 1$ 2
- d) Give the exact value for $\sec 210^\circ$ 2

QUESTION 3 Use a SEPARATE Writing Booklet.**Marks**

- a) The first term of an arithmetic sequence is 6 and the common difference is 9. 3
- i) Write down the expression for the n^{th} term
- ii) Which term of this sequence is 4623 ?
- b) Consider the points O (0, 0), A (-1, 3) and B (11, -6)
- i) Find the gradient of line AB. 1
- ii) Show that the equation of AB is $3x + 4y - 9 = 0$ 2
- iii) Find the equation of line L, which passes through O and is parallel to line AB 2
- iv) The point P, (4, k), lies on line L. Find the value of k 2
- v) Calculate the perpendicular distance from P to AB 2

QUESTION 4 Use a SEPARATE Writing Booklet.

Marks

a) Find

i) $\int \cos 2x \, dx$ 2

ii) $\int \frac{dx}{2x+3}$ 2

iii) $\int e^{3x} \, dx$ 2

- b) Bank X pays compound interest, compounded annually. Bank Y pays simple interest. \$5000 is invested in Bank X and also in Bank Y at 9% p.a. for 6 years - at both banks. Find the difference between the compound interest and simple interest earned at each bank. 3

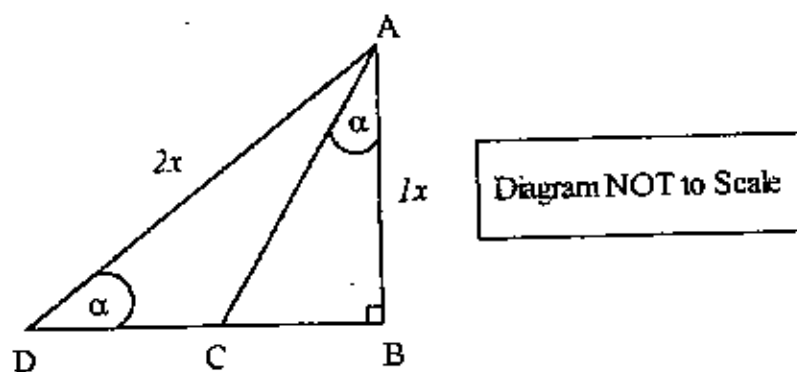
- c) For what values of k is $(3-k)x^2 + (3-k)x + 1$ positive definite? 3

QUESTION 5 Use a SEPARATE Writing Booklet.

Marks

- a) In the diagram below, $AD = 2 \times AB$ and $\angle ADC = \angle BAC$

5



- By writing an expression for $\sin \alpha$, show that $\alpha = 30^\circ$
- Hence find the size of $\angle DAC$
- If $DC = 2$ cm find the length of AB

- b) Solve $9^x + 6 \times 3^x - 27 = 0$

3

- c) There are five nominees for President and Vice President of a club. Three are women and two are men. The first name, selected at random, will be the President and the second name will be the Vice President.

4

- Draw a tree diagram to represent all possible outcomes
- Determine the probability that the two positions will be filled by a woman and a man, in either order.

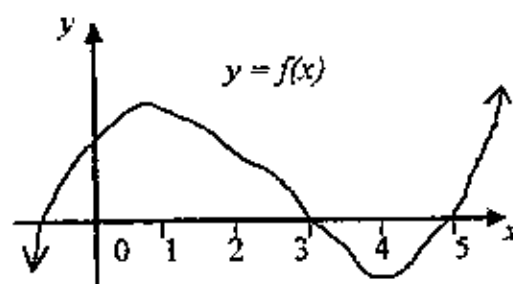
QUESTION 6 Use a SEPARATE Writing Booklet.

Marks

- a) Given the graph of $y = f(x)$, EXPLAIN why

3

$$\int_0^4 f(x) dx \text{ is LESS than } \int_0^3 f(x) dx$$



- b) A sector of a circle, of radius 1 cm, has a perimeter of 4 cm.

4

- i) Show that the angle at the centre of the sector is 2 radians
- ii) Find the area of the sector

- c) Use Simpson's Rule with 5 function values (i.e. 4 strips) to find an approximation for $\int_1^3 (\log_e x) dx$ correct to 3 decimal places

5

QUESTION 7 Use a SEPARATE Writing Booklet.

Marks

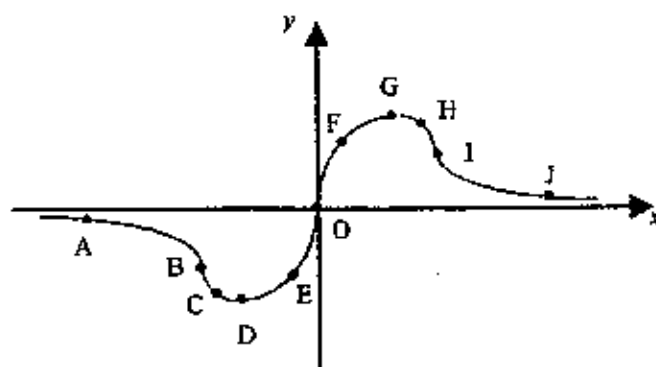
- a) The rate of decay of a radioactive substance is proportional to the mass, M , present at time, t years, i.e. $\frac{dM}{dt} = -kM$

5

- i) Show that $M = M_0 e^{-kt}$ satisfies $\frac{dM}{dt} = -kM$
- ii) If the half-life of the substance is 17 600 years, find k (correct to 6 decimal places)
- iii) How long will it take for $\frac{2}{3}$ (two thirds) of the substance to decay

- b) For the given graph of $y = f(x)$ write down which of the labelled point(s) best demonstrate the properties below:

7



- i) $f(x) = 0$
- ii) $f'(x) = 0$
- iii) $f''(x) = 0$
- iv) $f(x) > 0$
- v) $f'(x) > 0$
- vi) $f''(x) > 0$
- vii) $\lim_{x \rightarrow \infty} f(x) = 0$

QUESTION 8 Use a SEPARATE Writing Booklet.

Marks

- a) i) Differentiate $y = \cos^3 x$ 5
- ii) Hence, evaluate $\int_0^{\frac{\pi}{2}} (\cos^2 x \sin x) dx$
- b) Consider the parabola with the equation $x^2 - 8x = 12y - 28$ 5
- i) Show that the equation can be written as $(x - 4)^2 = 12(y - 1)$
- ii) Find the coordinates of the vertex
- iii) Find the coordinates of the focus
- iv) Find the equation of the directrix
- c) Find k if $\int_1^k \left(\frac{1}{x}\right) dx = 1$ 2

QUESTION 9 Use a SEPARATE Writing Booklet.

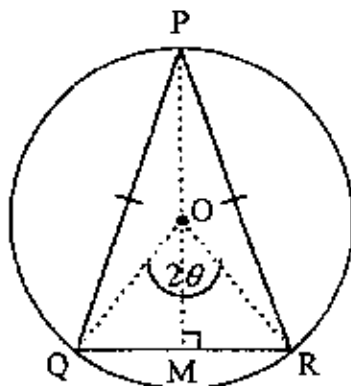
A particle moves along the x -axis so that its displacement, x metres, after t seconds is given by $x = 3 - 2 \cos t$ 12

- i) Find the initial displacement
- ii) Show that the particle starts from rest
- iii) When does the particle next come to rest?
- iv) Find the velocity when the particle passes through $x = 2$ for the second time
- v) Find the particle's greatest velocity
- vi) Find the particle's position when it is NOT being accelerated

a) i) Show that $\frac{1}{x^2-9} = \frac{1}{6} \left(\frac{1}{x-3} - \frac{1}{x+3} \right)$ 5

ii) Hence find the exact volume generated by revolving
 $y = \frac{1}{\sqrt{x^2-9}}$ around the x -axis from $x=5$ to $x=6$

- b) Isosceles triangle PQR is in a circle of radius 1 unit, centre O. 7
 $\angle QOR = 2\theta$ (θ is acute). PO is extended to meet QR at M
 such that $\angle OMR = 90^\circ$.



- i) Prove that $QM = \sin \theta$ and $OM = \cos \theta$
- ii) Show that the area, A , of $\triangle PQR$ is given by $A = \sin \theta (\cos \theta + 1)$
- iii) Hence show that $\triangle PQR$ has a maximum area when it is equilateral