2002

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

# Mathematics

**General Instructions**

- Reading Time- 5 minutes
- Working Time – 3 hours
- Write using a blue or black pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (120)

- Attempt Questions 1-10
- All questions are of equal value

**QUESTION 1**

**a)** Find the exact value of $log_2 128$ (1)

**b)** Expand and simplify $5a^2 + 7ab - a(2a+b)$ (1)

**c)** Factorise $49x^2 - y^2$ (1)

**d)** Evaluate $e^4$ correct to 3 significant figures. (1)

**e)** Find the exact value of $\tan\frac{5\pi}{6}$ (2)

**f)** **i)** Express $4\sqrt{27} - \sqrt{243}$ in simplest surd form. (2)

**ii)** Determine the value of $x$ given $4\sqrt{27} - \sqrt{243} = \sqrt{x}$ (1)

**g)** Solve $|3x-2| \geq 7$ and graph the solution on a number line. (3)

**QUESTION 2** *Start a new page*

**a)** Differentiate:

**i)** $x^5 - 2x^3$ (1)

**ii)** $(x+5)^4$ (1)

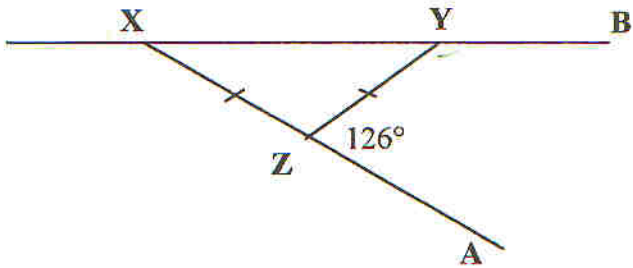
**iii)** $3\sin\frac{2x}{5}$ (2)

**iv)** $xe^{-x}$ (2)

**b)** For the parabola $(x-5)^2 = -8(y-2)$ determine:

**i)** the focal length (1)
**ii)** the co-ordinates of the vertex (1)
**iii)** the co-ordinates of the focus (1)
**iv)** the equation of the directrix. (1)

**c)** In the figure below triangle XYZ is isosceles and $\angle AZY$ is 126°. Determine the size of $\angle ZYB$ giving reasons. (2)



**QUESTION 3** *Start a new page*

A (2,5), B (−2, −1) and C (1, −5) are the vertices of triangle ABC.

**a)** Show this information clearly on a diagram (1)

**b)** Determine the exact length of AB in simplest form. (2)

**c)** Show the gradient of AB is $\frac{3}{2}$. (1)

**d)** Find the equation of AB in general form. (2)

**e)** Calculate the exact distance from C to side AB. (2)

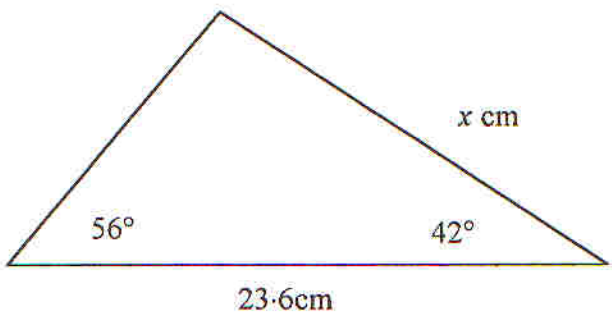
**f)** Find the exact area of triangle ABC. (2)

**g)** Determine the co-ordinates of D such that ABCD is a parallelogram. (1)

**h)** Calculate the angle (nearest minute) that side AB is inclined to the positive direction of the $x$-axis. (1)

**QUESTION 4** *Start a new page*

**a)** Find the value of $x$ correct to 3 significant figures. (3)

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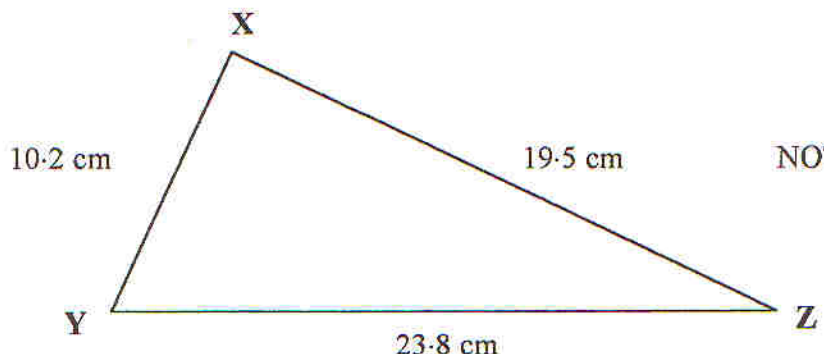
**b)** Without solving, state the largest angle in triangle XYZ, justifying your answer. (1)

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**c)** Consider the arithmetic series $9+13+17+.....$

**i)** Determine the first term and the common difference. (1)

**ii)** Find the sum of the first 20 terms. (1)

**iii)** Which term is the first term greater than 1000 and what is its value? (2)

**d)** 8 black and 5 yellow discs are placed in a bag and 12 black and 10 yellow discs in a second bag. A disc is drawn from each bag. What is the probability of:

**i)** a black disc being drawn from the first bag? (1)

**ii)** a yellow disc being drawn from the second bag? (1)

**iii)** a black and a yellow disc being drawn? (2)

**QUESTION 5** *Start a new page*

**a)** Find a primitive of:

**i)** $3x^4 - e^{-2x}$ (2)

**ii)** $\dfrac{6x}{x^2-1}$ (2)

**iii)** $\sqrt{5x+2}$ (2)

**b)** Find the equation of the normal to the curve $y = e^{2x}$ at the point where $x = 0$ in general form. (3)

**c)** Solve in the domain, $0 \le \alpha \le 2\pi$,

$$2\cos^2\alpha + 3\cos\alpha + 1 = 0$$ (3)

**QUESTION 6** *Start a new page*

**a)** A function is defined by the rule $f(x) = x^3 - 3x^2 - 9x + 6$.

**i)** Find the co-ordinates of any stationary points and determine their nature. (3)

**ii)** Find any point(s) of inflexion. (2)

**iii)** Sketch the curve in the domain $-3 \le x \le 5$ showing clearly important features. (2)

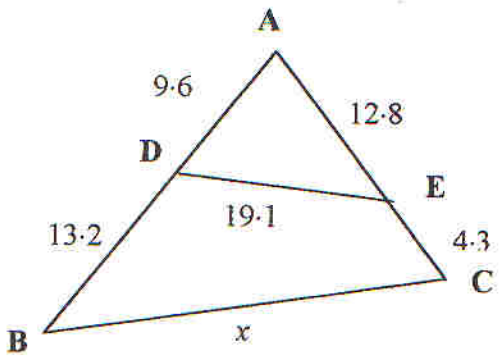
**b)** **i)** Sketch $y = 3\cos 2x$ in the domain $0 \le x \le \pi$. (1)

**ii)** Without solving explain why $\int_0^{\pi} 3\cos 2x \, dx = 0$. (1)

**iii)** Determine the area bounded by the curve $y = 3\cos 2x$, the $x$-axis and values from $x = 0$ to $x = \pi$. (3)

**QUESTION 7** *Start a new page*

**a)** Prove triangle ADE and triangle ACB are similar and determine the value of $x$ (correct to 1 decimal place). Units are mm. (2)

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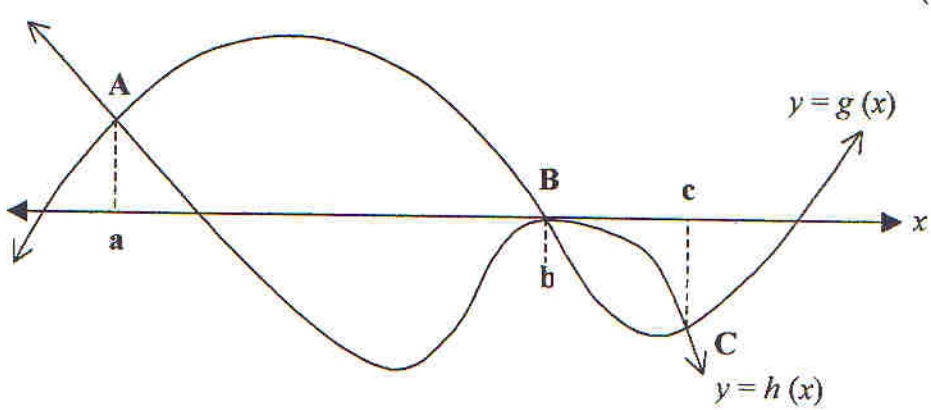
**b)** The volume of water in a tank is given by $V = 200 - 2t + \frac{t^2}{200}$ litres, where t is in minutes.

**i)** How much water is in the tank after 15 minutes? (1)
**ii)** At what rate is water flowing out of the tank after 25 minutes? (2)
**iii)** How long will it take to empty the tank? (1)

**c)** A ball drops from a height of 20 metres and bounces with each successive bounce only $\frac{4}{5}$ the height of the previous bounce.

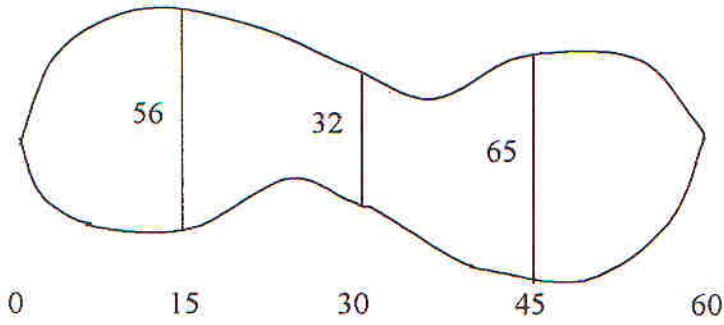
**i)** How far will the ball rebound after it has hit the ground for the 3rd time? (1)
**ii)** Through what distance will the ball eventually travel? (3)

**d)** The diagram below shows 2 curves $y = g(x)$ and $y = h(x)$ which intersect at points A, B and C. Write down an expression which could be used to evaluate the area between the 2 curves. (2)



**QUESTION 8** *Start a new page*

**a)** The diagram shows the widths of a garden bed in a park at 15m intervals. Use Simpson's Rule to estimate the surface area of the garden bed. Units are metres. (3)

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**b)** **i)** Find the derivative of $y = ln\,(ln\,x)$. (1)

**ii)** Hence or otherwise evaluate $\int_e^{e^3} \frac{2dx}{x \ln x}$ (2)

**c)** A block of ice is melting at a rate proportional to its mass, M kg, left in the block after time $t$ minutes, i.e. $\frac{dM}{dt} = -kM$

After 25 minutes, 15% of the block has melted.

**i)** Show that $M = M_o e^{-kt}$ is a solution of the equation $\frac{dM}{dt} = -kM$ (1)

**ii)** Determine the value of $k$. (2)

**iii)** How long, to the nearest minute, will it take for 70 % of the block of ice to have melted? (2)

**d)** Evaluate $\lim_{x \to 0} \frac{\sin \frac{3x}{2}}{6x}$ (1)

**QUESTION 9** *Start a new page*

**a)** The Smith's borrow \$150 000 from Friendly Bank at 9% p.a. charged monthly for 25 years. Friendly Bank has a special deal to help customers, **no repayments are required for the first 9 months**.

**i)** Show that the amount owing after 1 month is 150 000 (1.0075) (1)

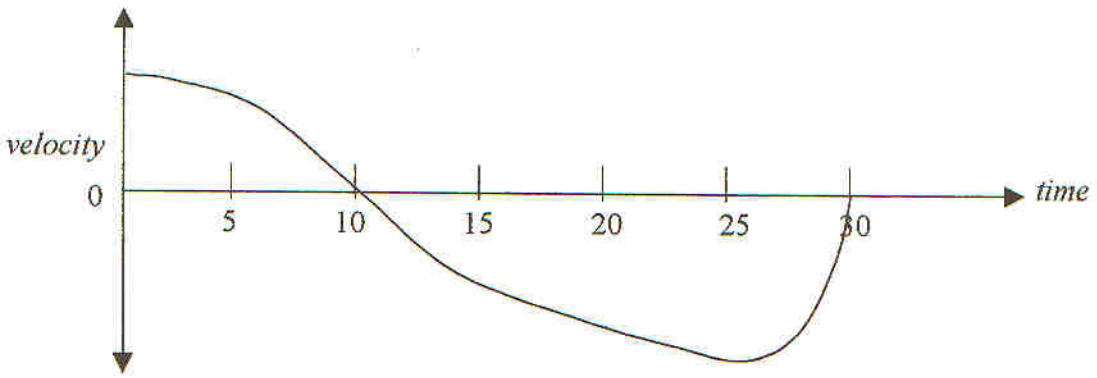
**ii)** Show an expression for the amount owing after 10 months assuming the monthly repayment is \$M. (1)

**iii)** Derive a similar expression for the amount owing after n months. (1)

**iv)** Calculate (to nearest cent) the amount of each monthly payment. (2)

**b)** The diagram shows a velocity-time graph for particle P for its first 30 seconds of motion.

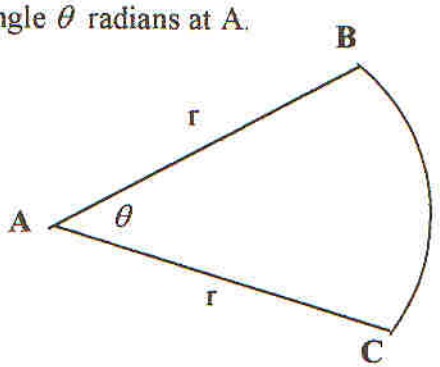
**i)** When is P moving forwards? (1)
**ii)** When is the acceleration positive? (1)
**iii)** When is P furthest from its starting position in a positive direction? (1)
**iv)** When is it furthest in the negative direction? (1)
**v)** About when does it return to its starting position? (1)
**vi)** Assuming the particle starts at the origin, sketch graphs of its displacement and its acceleration. (2)




**QUESTION 10** *Start a new page*

**a)** **i)** Shade the region between the curve $y = \log_e x$ and the $x$-axis from $x = 1$ to $x = 2$. (1)
**ii)** This region is rotated about the $y$-axis. Determine the exact volume. (4)

**b)** In the figure AB and AC are radii of length r metres of a circle centre A. Arc BC subtends an angle $\theta$ radians at A.




**i)** If the perimeter of the sector is 8 metres, show that the area of the sector is given by $A = \dfrac{32\theta}{(\theta+2)^2}$ (3)

**ii)** Hence determine the maximum area of the sector. (4)

**END OF PAPER**