BH Ext 1 2004.

QUESTION 1

(a) Find the co-ords of the point P that divides the interval A(–3, 4) and B(2, – 3) externally in the ratio 1 : 2.

(b) Solve $\frac{4}{x-3} < 1$.

(c) Evaluate $\lim_{x \to 0} \frac{\sin 2x \cos 2x}{3x}$.

(d) A curve has parametric equations $x = 2t - 2$, $y = t^2 + 1$. Find the cartesian equation for this curve.

(e) Use the substitution $u = 2 + x$ to evaluate $\int_{-2}^{2} x\sqrt{2+x}\,dx$.

QUESTION 2

(a) Find (i) $\int \tan x\,dx$.

(ii) $\int_{-3/2}^{3/4} \frac{1\,dx}{\sqrt{9-4x^2}}$.

(b) Find the term independent of x in the binomial expansion $\left(x^2 + \frac{1}{x}\right)^9$.

(c) (i) Express $\sin 4t + \sqrt{3}\cos 4t$ in the form $R\sin(4t + \alpha)$, where α is in radians.

(ii) Hence solve $\sin 4t + \sqrt{3}\cos 4t = 0$ for $0 \le t \le \pi$.

QUESTION 3

(a) Prove by induction $9^{n+2} - 4^n$ is divisible by 5 for $n \ge 1$.

(b) Consider the function $f(x) = 2\tan^{-1}x$.

(i) State the range of the function $y = f(x)$.

(ii) Sketch the graph of $y = f(x)$.

(iii) Find the gradient of the tangent to the curve $y = f(x)$ at $x = \frac{1}{\sqrt{3}}$.

(c) (i) By equating the coefficients of sin x and cos x, or otherwise, find constants A and B satisfying the identity.

$$A(2\sin x + \cos x) + B(2\cos x - \sin x) \equiv \sin x + 8\cos x.$$

(ii) Hence evaluate $\int \frac{\sin x + 8\cos x}{2\sin x + \cos x}\,dx$.

QUESTION 4

(a) If $x^3 - 8x^2 + kx - 12 = 0$ has one root equal to the sum of the other two; find k.

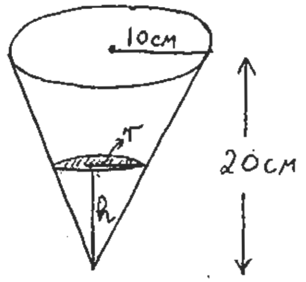
(b) Taking $x = 0.5$ as the first approximation, use Newtons method to find a second approximation to the root of:

$$x - 3 + e^{2x} = 0.$$

Write your answer to 2 significant figures.

QUESTION 4 (Continued)

(c)



Water is poured into a conical vessel at a rate of $30cm^3/s$.

(i) What is the rate of increase of the radius of water when $r = 5$.

(ii) Hence find the rate of increase of the area of the surface of the liquid when $r = 5$.

(d) Using $\sin 3\theta = \sin(2\theta + \theta)$. Prove $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

QUESTION 5

(a) A particle moves in a straight line such that its position x from a fixed point 0 at time 't' is given by $x = 5 + 8\sin 2t + 6\cos 2t$.

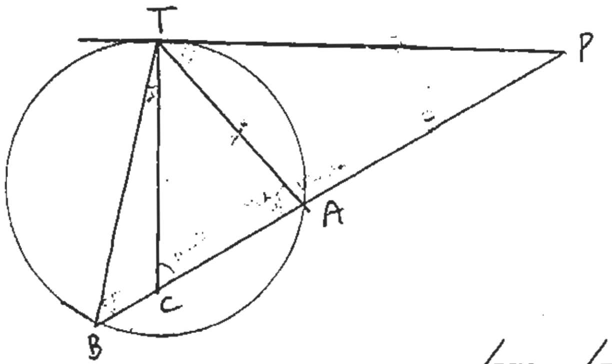
(i) Prove the motion is simple harmonic motion.

(ii) Find the period and amplitude of the motion.

(iii) Find the greatest speed of the particle.

(b) State the largest positive domain for which $y = x^2 - 4x + 7$ has an inverse function.

(c)



PT is a tangent and PAB is a secant. TC = TA. Prove $\angle BTC = \angle TPA$

(d) By using the expansion $(1+x)^n$. Prove $\sum_{k=0}^{n} 2^{3k}\binom{n}{k} = 3^{2n}$.

QUESTION 6

(a) A particle is projected horizontally with velocity Vms^{-1}, from a point h metres above the ground. Take $g\ ms^{-2}$ as the acceleration due to gravity.

(i) Taking the origin as the point on the ground immediately below the projection point, find expressions for x and y, the horizontal and vertical displacements of the particle at time 't' secs.

(ii) Show the equation of the path is given by $y = \dfrac{2hV^2 - gx^2}{2V^2}$.

(iii) Find the range of the particle.

(b) Assume that the rate at which a body warms in air is proportional to the difference between its temperature T and the constant temperature A of the surrounding air. The rate can be expressed as:

$$\frac{dT}{dt} = K(T - A) \text{ where 't' is in minutes and K is constant.}$$

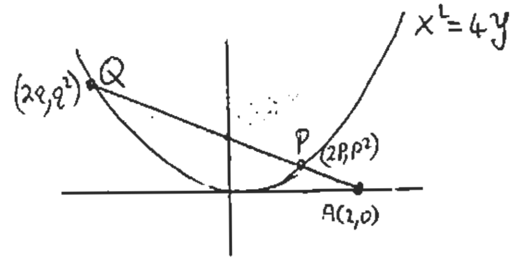
(i) Show $T = A + Ce^{kt}$, where C is constant, is a solution of the differential equation.

(ii) A cooled body warms from 5^0C to 10^0C in 20 minutes. The air temperature is 20^0C. Find the temperature of the body after a further 30 minutes have elapsed.

(iii) Explain the behaviour of T as t becomes large.

(c) Differentiate from 1^{st} Principles $f(x) = x^2 - 2x + 1$.

QUESTION 7



(a) The chord PQ joining the points $P(2p,p^2)$ and $Q(2q,q^2)$ on $x^2 = 4y$ always passes through the point A(2,0) when produced.

(i) Show $(p + q) = pq$.

(ii) Find the co-ordinates of M, the mid-point of PQ.

(iii) Find the equation of the locus of M as P and Q vary on the parabola.

QUESTION 7 (Continued)

(b) Two circles C_1 and C_2 are members of a set of circles defined by the equation:
$x^2 + y^2 - 6x + 2ky + 3k = 0$ where k is real. The centre of C_1 lies on the line $x - 3y = 0$ and C_2 touches the x – axis. Find the equations of C_1 and C_2.

(c) Use Simpson's Rule with 3 function values to approximate the volume when $y = \ln x$ is rotated about the x – axis between $x = 1$ and $x = 3$.

Answers

1(a) $(-8, 11)$

(b) $x < 3$ or $x \geq 7$

(c) $2/3$

(d) $y = \frac{x^2}{4} + x + 2$

(e) $2\frac{2}{15}$

2(a)(i) $-\ln\cos x + c$

(ii) $\pi/3$

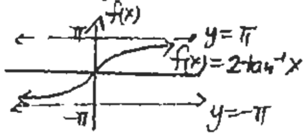
(b) 84

(c)(i) $2\sin(4t + \pi/3)$

(ii) $\pi/6, 5\pi/12, 2\pi/3, 11\pi/12$ s

3(b)(i) $-\pi < f(x) < \pi$

(ii)



(iii) $1\frac{1}{2}$

(c)(i) $A = 2, B = 3$

(ii) $2x + 3\ln(2\sin x + \cos x) + c$

4(a) 19

(b) 0.47

(c)(i) $\frac{3}{5\pi}$ cm/s (ii) 6 cm²/s

5(a)(ii) π s, 10

(iii) 20 ms^{-1}

(b) $x \geq 2$

6(a)(i) $x = Vt$

$y = -\frac{1}{2}gt^2 + h$

(iii) $V\sqrt{\frac{2h}{g}}$ m

(b)(ii) 14.56°C (2dp, by using $e^{20K} = 2/3$)

(iii) As t becomes large, the body warms up to the temp. of the surrounding air, 20°C.

(c) $2x - 2$

7(a)(ii) $(p+q, \frac{1}{2}(p^2+q^2))$

(iii) $x^2 - 2x - 2y = 0$

(b) C_1: $x^2 + y^2 - 6x - 2y - 3 = 0$

C_2: $x^2 + y^2 - 6x + 6y + 9 = 0$

(c) 3.28 units^3 (2dp)