2010 Extension 1 - Semester One

Question one

a) $2m^3 - 128 = 2(m^3 - 64)$

$= 2(m-4)(m^2+4m+16)$

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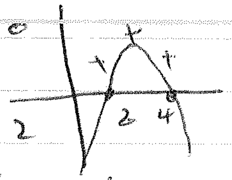
b) $\frac{x}{x-2} \times (x-2)^2 \geq 2 \times (x-2)^2$

$x(x-2) - 2(x-2)^2 \geq 0$

$(x-2)[x - 2(x-2)] \geq 0$

$\quad x - 2x + 4$

$(x-2)(4-x) \geq 0$



$2 \leq x \leq 4, \ x \neq 2$

$\therefore$ solution $2 < x \leq 4$

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c) $x^2 + y^2 - 2y - 4 = 0, \ x - y + 2 = 0$

$\boxed{x = y - 2}$

$\therefore (y-2)^2 + y^2 - 2y - 4 = 0$

$y^2 - 4y + \cancel{4} + y^2 - 2y - \cancel{4} = 0$

$2y^2 - 6y = 0$

$2y(y-3) = 0$

$y = 0$ or $y = 3$

$x = -2 \qquad x = 1$

$(-2, 0), \ (1, 3)$

/2

d) $\cos 2x = \cos x$

$2\cos^2 x - 1 = \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\therefore \cos x = -\frac{1}{2} \qquad \cos x = 1$

$x = 120, 240^\circ \qquad x = 0, 360^\circ$

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e) $\frac{\cos^2 15 - \sin^2 15}{\sin 15 \cos 15}$

$= \frac{\cos 2(15)}{\frac{1}{2}\sin 2(15)}$

$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{4}}$

$= \frac{4\sqrt{3}}{2} = 2\sqrt{3}$

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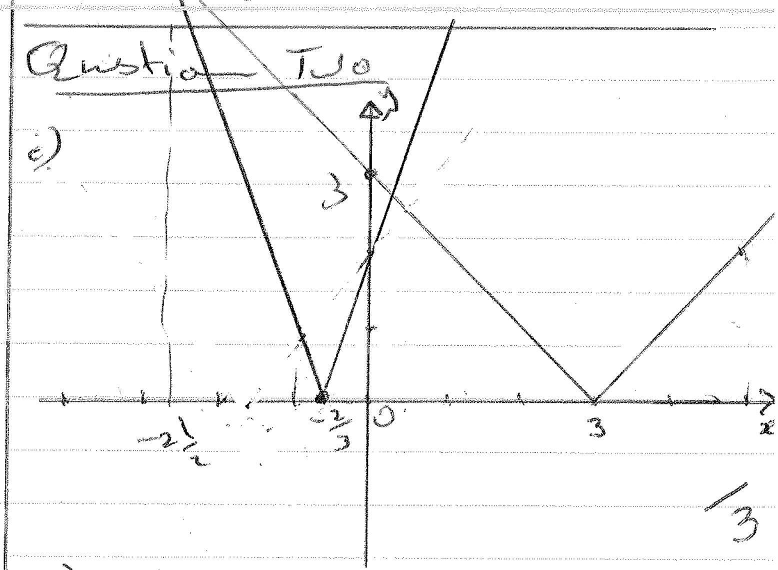
Question Two

i)



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ii) Two solutions.

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iii) $|x-3| = |3x+2|$

$(x-3)^2 = (3x+2)^2$

$x^2 - 6x + 9 = 9x^2 + 12x + 4$

$8x^2 + 18x - 5 = 0$

$(4x-1)(2x+5) = 0$

$x = \frac{1}{4}$ or $x = -2\frac{1}{2}$

/2

iv) Solve $|x-3| < |3x+2|$

$x > \frac{1}{4}, \quad x < -2\frac{1}{2}$

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Q2 Cont.

b) $f(x) = \frac{x}{x^2-1}$

i) $x = \pm 1$ /1

ii) $f(-x) = \frac{-x}{(-x)^2 - 1}$

$= \frac{-x}{x^2-1}$

$-f(x) = -\left[\frac{x}{x^2-1}\right]$

$= \frac{-x}{x^2-1}$ /1

$\therefore f(-x) = -f(x)$

$\therefore$ it's an odd function

iii) $f(x) = \lim_{x \to \infty} \frac{x}{x^2-1}$

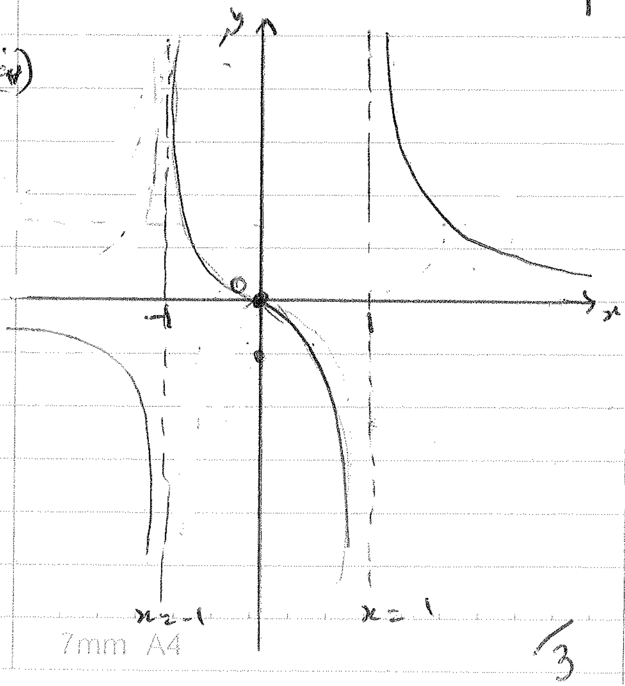
$= \lim_{x\to\infty} \frac{\frac{1}{x}}{1 - \frac{1}{x^2}}$

$= \frac{\frac{1}{\infty}}{1 - \frac{1}{\infty^2}}$

$y = \frac{0}{1-0} = 0$

$\therefore$ horizontal asymptote $y = 0$ /1

(iv)



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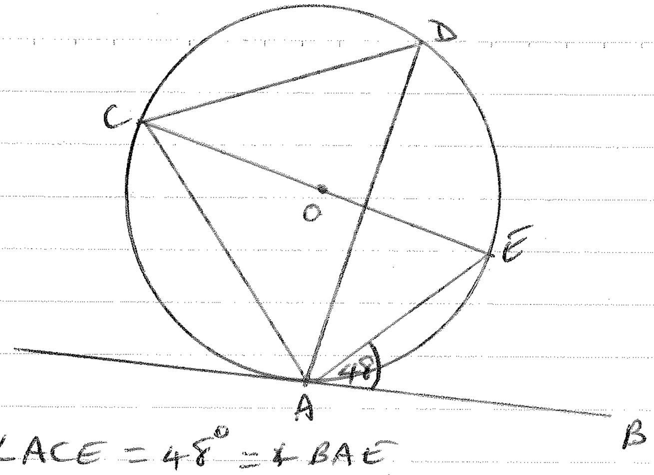
Q3

a)



i) $\angle ACE = 48^\circ = \angle BAE$
~~(angle between the tangent and the~~ chord equal to the angle in the alternate segment) /1

ii) In $\triangle CAE$,
$\angle CAE = 90^\circ$ (angle in a semi circle is a right angle)
and $\angle ACE = 48^\circ$ proven above

$\therefore \angle CEA = 180 - (90 + 48)$
$= 42^\circ$

$\therefore \angle CDA = \angle CEA = 42^\circ$
angles in the same segment are equal.

$\therefore \angle CDA = 42^\circ$ /2

b) $\frac{\tan 85 - \tan 25}{1 + \tan 85 \tan 25}$

$= \tan(85 - 25)$

$= \tan 60$

$= \frac{\sqrt{3}}{1}$ /1

c) i) $\theta = 270^\circ, -90$ 1

ii) $\dfrac{1}{1+\sin\theta} = \sec^2\theta - \sec\theta\tan\theta$

$RHS = \dfrac{1}{\cos^2\theta} - \dfrac{1}{\cos\theta}\cdot\dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1-\sin\theta}{\cos^2\theta}$

$= \dfrac{1-\sin\theta}{1-\sin^2\theta}$

$= \dfrac{\cancel{(1-\sin\theta)}1}{\cancel{(1-\sin\theta)}(1+\sin\theta)}$

$= \dfrac{1}{1+\sin\theta} = LHS$ /2

d) $\cos\theta = \frac{2}{3}$, $\tan\frac{\theta}{2}$

$\tan\frac{\theta}{2} = t$

$\therefore \dfrac{1-t^2}{1+t^2} = \dfrac{2}{3}$

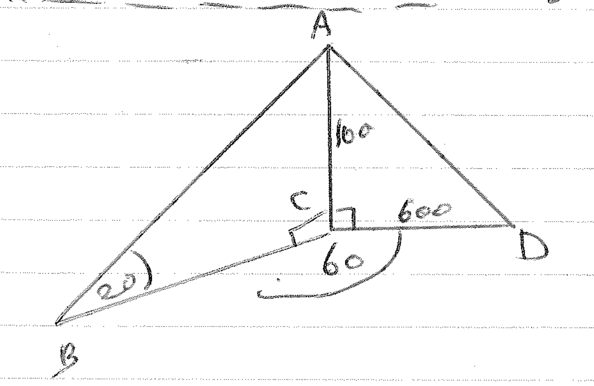
$3 - 3t^2 = 2 + 2t^2$

$1 = 5t^2 \Rightarrow 5t^2 - 1 = 0$

$t = \pm\dfrac{1}{\sqrt{5}}$

$\therefore \tan\frac{\theta}{2} = \pm\dfrac{1}{\sqrt{5}}$ /3

e)



$\tan 20 = \dfrac{100}{BC}$ $\therefore BC = \dfrac{100}{\tan 20}$

$BC = 274.7$ 1

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ii) $BD^2 = 600^2 + \left(\dfrac{100}{\tan 20}\right)^2 - 2\times 600 \times \left(\dfrac{100}{\tan 20}\right) \times \cos 60$

$BD = 520.23$

$\doteqdot 520$ m. /2

Question four

a) i) $\sqrt{3}\cos x - \sin x \equiv R\cos(x+\alpha)$

$\sqrt{3}\cos x - \sin x \equiv R\cos x\cos\alpha - R\sin x\sin\alpha$

$\sqrt{3} = R\cos\alpha$, $1 = R\sin\alpha$

$\sqrt{3}^2 + 1^2 = R^2(\cos^2\alpha + \sin^2\alpha)$

$\boxed{R = 2}$ $\dfrac{R\sin\alpha}{R\cos\alpha} = \dfrac{1}{\sqrt{3}}$

$\tan\alpha = \dfrac{1}{\sqrt{3}}$

$\therefore \alpha = 30^\circ$ /2

$\therefore \sqrt{3}\cos x - \sin x = 2\cos(x+30)$

ii) Solve $2\sqrt{3}\cos x - 2\sin x - 2 = 0$

$\sqrt{3}\cos x - \sin x = 1$

$2\cos(x+30) = 1$

$\cos(x+30) = \frac{1}{2}$

$x + 30 = 60, 300$

$x = 30^\circ, 270^\circ$ /2

b) i) $\cos 2B = 1 - 2\sin^2 B$ (prove)

$\cos(x+y) = \cos x\cos y - \sin x\sin y$

Let $x = y = B$

$\therefore \cos 2B = \cos B\cos B - \sin B\sin B$

$= \cos^2 B - \sin^2 B$

$= 1 - \sin^2 B - \sin^2 B$

$= 1 - 2\sin^2 B$ 1

ii) $\cos 2(15) = 1 - 2\sin^2 15$

$\cos 30 = 1 - 2\sin^2 15$

$\dfrac{\sqrt{3}}{2} = 1 - 2\sin^2 15$

$2\sin^2 15 = 1 - \dfrac{\sqrt{3}}{2}$ /2

Q4 (b) (ii) Cont.

$2\sin^2 15 = \frac{2-\sqrt{3}}{2}$

$\sin^2 15 = \frac{2-\sqrt{3}}{4}$

$\sin 15 = \pm\frac{\sqrt{2-\sqrt{3}}}{2}$

$\therefore \sin 15 = +\frac{\sqrt{2-\sqrt{3}}}{2}$ or $\frac{\sqrt{6}-\sqrt{2}}{4}$

Since $\sin 15 > 0$ $\quad$ /2

c) Prove $\frac{\cos B - \cos(B+2A)}{2\sin A}$

$= \sin(B+A)$

$LHS = \frac{\cos B - [\cos B\cos 2A - \sin B\sin 2A]}{2\sin A}$

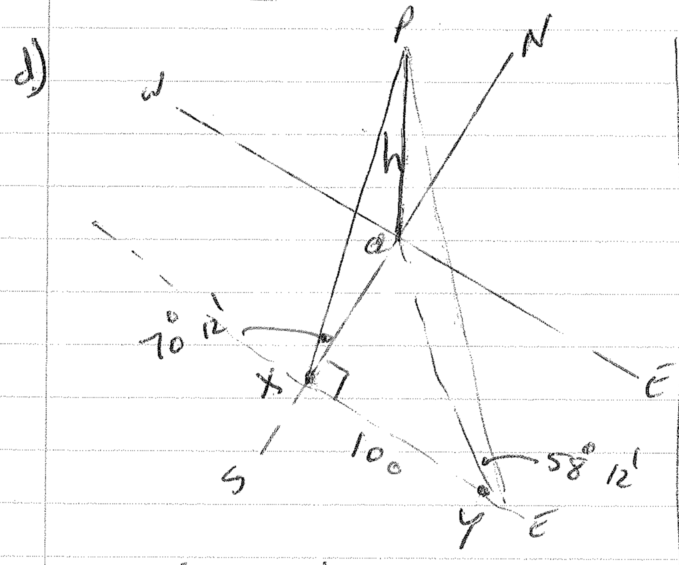
$= \frac{\cos B - \cos B(1-2\sin^2 A) + \sin B \cdot 2\sin A\cos A}{2\sin A}$

$= \frac{\cancel{\cos B} - \cancel{\cos B} + 2\cos B\sin^2 A + 2\sin A\sin B\cos A}{2\sin A}$

$= \frac{\cancel{2\sin A}(\cos B\sin A + \sin B\cos A)}{\cancel{2\sin A}}$

$= \sin(B+A) = RHS$ $\quad$ /2

d)



$\tan 70^\circ 12' = \frac{h}{QX}$

$QX = \frac{h}{\tan 70^\circ 12'}$

$= h\tan 19^\circ 48'$

$\tan 58^\circ 12' = \frac{h}{QY}$

$QY = \frac{h}{\tan 58^\circ 12'} = h\tan 31^\circ 48'$

$100^2 = QY^2 - QX^2$

$= h^2\tan^2 31^\circ 48' - h^2\tan^2 19^\circ 48'$

$= h^2(\tan^2 31^\circ 48' - \tan^2 19^\circ 48')$

$h = 198.10$

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Question 5

a) $\sin x + \cos x = 1$

$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = 1$

$2t + 1 - t^2 = 1 + t^2$

$2t^2 - 2t = 0$

$2t(t-1) = 0$

$t = 0$ or $t = 1$

$\tan\frac{\theta}{2} = 0$ $\quad$ $\tan\frac{\theta}{2} = 1$

$\tan\frac{\theta}{2} = 0$ $\quad$ $\tan\frac{\theta}{2} = 45$

$\frac{\theta}{2} = 180n + 0$ $\quad$ $\frac{\theta}{2} = 180n + 45$

$\theta = 360n$ $\quad$ $\theta = 360n + 90°$

/5

b) $\lim_{x\to\infty} \frac{3x^2 - x + 1}{x^2 - 1}$

$\lim_{x\to\infty} \frac{3 - \frac{1}{x} + \frac{1}{x^2}}{1 - \frac{1}{x^2}}$

$= \frac{3}{1}$

k) $x + 2y - z = -5$ ①

$2x - 3y + 4z = 28$ ②

$4x + 5y - 3z = -10$ ③

① $\xrightarrow{\times 4}$ $4x + 8y - 4z = -20$

② $\longrightarrow$ $2x - 3y + 4z = 28$

$6x + 5y = 8 \rightarrow$ ④

① $\xrightarrow{\times 3}$ $3x + 6y - 3z = -15$

③ $\longrightarrow$ $4x + 5y - 3z = -10$ (−)

$-x + y = -5 \rightarrow$ ⑤

④ $\longrightarrow$ $6x + 5y = 8$

⑤ $\xrightarrow{\times 6}$ $-6x + 6y = -30$

$11y = -22$

$\boxed{y = -2}$

$\therefore -x + y = -5$

$-x - 2 = -5$

$\boxed{x = 3}$

$x + 2y - z = -5$

$3 + 2(-2) - z = -5$

$-z = -4$

$\boxed{z = 4}$

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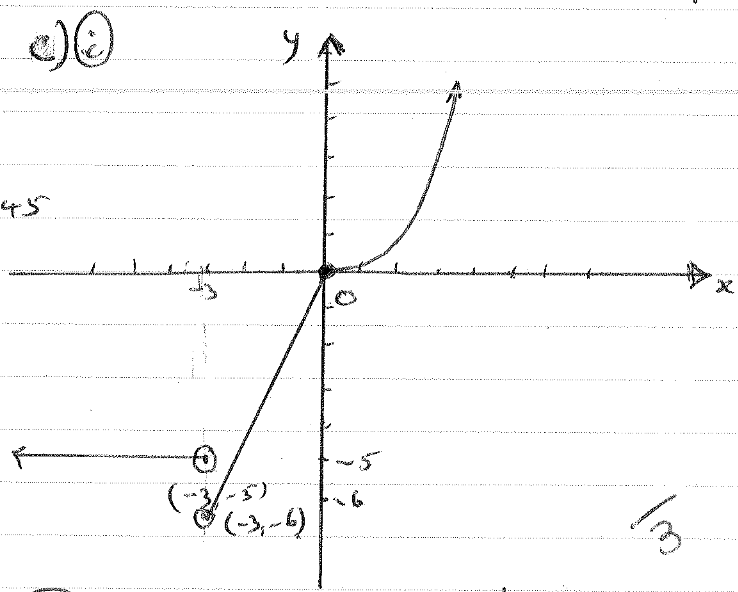
c) (i)



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(ii) Domain all real x, $x \neq -3$

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Question 6

a) $\cos\theta + 3\sin\frac{\theta}{2} - 2 = 0$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$

$\therefore 1 - 2\sin^2\frac{\theta}{2} + 3\sin\frac{\theta}{2} - 2 = 0$

$2\sin^2\frac{\theta}{2} - 3\sin\frac{\theta}{2} + 1 = 0$

$(2\sin\frac{\theta}{2} - 1)(\sin\frac{\theta}{2} - 1) = 0$

$\sin\frac{\theta}{2} = \frac{1}{2}$ or $\sin\frac{\theta}{2} = 1$

$\frac{\theta}{2} = 30°, 150$ $\quad \frac{\theta}{2} = 90°$

$\theta = 60°, 300$ $\quad \theta = 180°$

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b) $\frac{x+1}{x-3} \leq 1$

$(x+1)(x-3) \leq (x-3)^2$

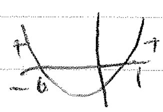
$(x-3)[(x+1)-(x-3)] \leq 0$

$(x-3)[4] \leq 0$

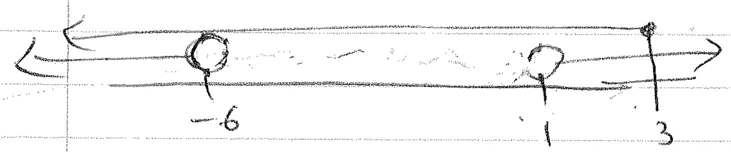
$x \leq 3$

$x^2 + 5x - 6 > 0$

$(x+6)(x-1) > 0$



$x < -6, x > 1$



$x < -6, 1 < x < 3$

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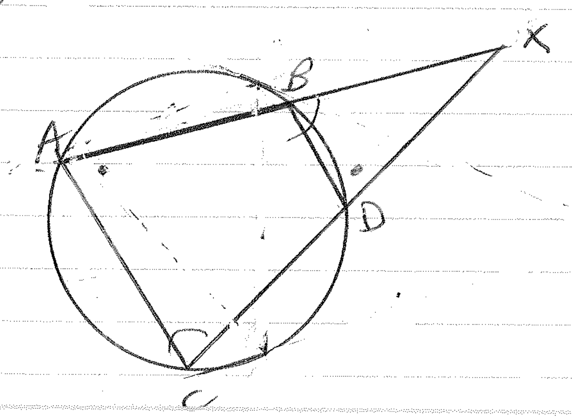
(c)



(i) In Δ's AXC, BXD

$\angle X$ is a common angle

$\angle XBD = \angle XCA$ (Exterior angle of a cyclic Quad. equal to the angle in the interior opposite angle.

Also $\angle XDB = \angle XAC$ (Exterior angle of a cyclic Quad. equal to the angle in the interior opposite angle.

$\therefore \Delta AXC ||| \Delta BXD$ (equiangular)

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(ii) Since $\Delta AXC ||| \Delta BXD$ (proven above)

$\therefore$ corresponding sides are in the same ratio.

$\frac{XB}{XC} = \frac{XD}{XA}$

$\therefore XA \cdot XB = XC \cdot XD$

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