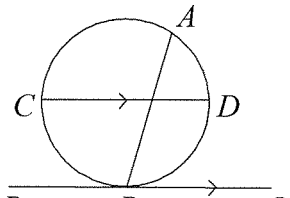
Circle Geometry HSC

1)! Yr11-3U\circleg.hsc Qn7) 3U89-5a

AB and CD are two intersecting chords of a circle and CD is parallel to the tangent to the circle at B.

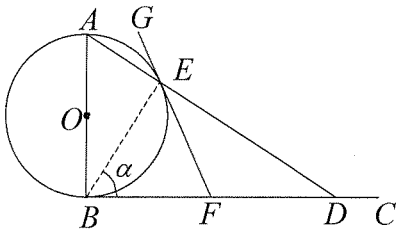
i. Draw a neat sketch of the above information in your writing booklet.

ii. Prove that AB bisects $\angle CAD$.¤



«1)→ i) ii) Proof »

2)! Yr11-3U\circleg.hsc Qn11) 3U93-4a



In the diagram, AB is a diameter of the circle, centre O, and BC is tangential to the circle at B. The line AED intersects the circle at E and BC at D. The tangent to the circle at E intersects BC at F. Let $\angle EBF = \alpha$.

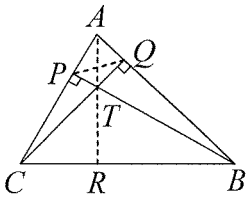
i. Copy the diagram into your Writing Booklet.

ii. Prove that $\angle FED = \frac{\pi}{2} - \alpha$.

iii. Prove that $BF = FD$.¤

«2)→ Proof »

3)! Yr11-3U\circleg.hsc Qn21) 3U03-4d



In the diagram, CQ and BP are altitudes of the triangle ABC. The lines CQ and BP intersect at T, and AT is produced to meet CB at R. Copy or trace the diagram.

i. Explain why $CPQB$ is a cyclic quadrilateral.

ii. Explain why $PAQT$ is a cyclic quadrilateral.

iii. Prove that $\angle TAQ = \angle QCB$.

iv. Prove that $AR \perp CB$.¤

«3)→ Proof »

4)! Yr11-3U\circleg.hsc Qn3) 3U86-2ii

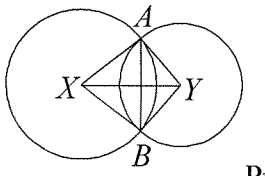
Two circles with centres X and Y intersect at two points A and B.

a. Draw a neat sketch joining XA, XB, YA, YB, XY, AB. Let P be the point where XY meets AB.

b. Prove that the triangles AXY and BXY are congruent.

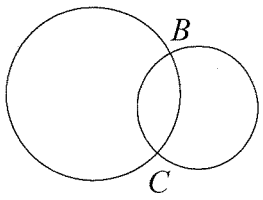
c. Prove that $AP = BP$.

d. Given that XA is also a tangent to the circle with centre Y, prove that $XAYB$ is a cyclic quadrilateral.¤



«4)→ Proof »

5)! Yr11-3U\circleg.hsc Qn4) 3U87-2i

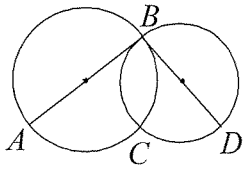


NOT TO SCALE

Two circles cut at points B and C as shown in the diagram. A diameter of one circle is AB while BD is a diameter of the other.

a. Draw a neat sketch in your answer book showing the given information.

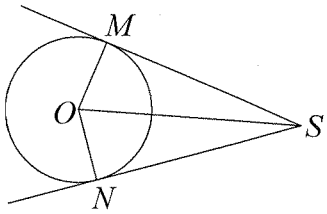
b. Prove that A, C and D are collinear, giving reasons.¤



«5)→ a) b) Proof »

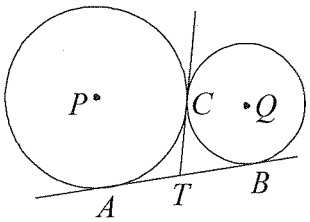
6)! Yr11-3U\circleg.hsc Qn5) 3U88-4a

i.



SM and SN are tangents drawn from an external point S to a circle with centre O. The points of contact of these tangents with the circle are M and N. Copy this diagram into your writing booklet. By proving triangle OMS and ONS are congruent show that $SM = SN$.

ii.



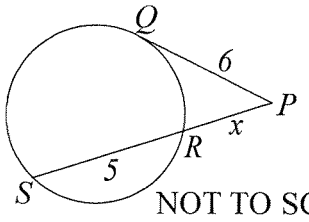
Two circles touch externally at C. The circles, which have centres P and Q, are touched by a common tangent at A and B respectively. The common tangent at C meets AB in T.

α. Copy this diagram in your writing booklet. Using the result from (i) prove that $AT = TB$.

β. Show that ACB is a right angle.¤

«6)→ Proof »

7)! Yr11-3U\circleg.hsc Qn6) 3U89-2a



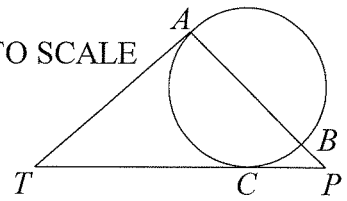
NOT TO SCALE

PQ is a tangent to a circle QRS, while PRS is a secant intersecting the circle in R and S, as in the diagram. Given that $PQ = 6$, $RS = 5$, $PR = x$, find x.¤

«7)→ $x = 4$ »

8)! Yr11-3U\circleg.hsc Qn8) 3U90-3a

NOT TO SCALE



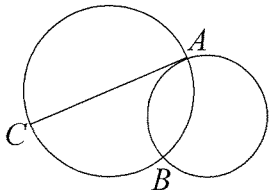
AB is a diameter of a circle ABC. The tangents at A and C meet at T. The lines TC and AB are produced to meet at P. Copy the diagram into your examination booklet. Join AC and CB.

i. Prove that $\angle CAT = 90° - \angle BCP$.

ii. Hence, or otherwise, prove that $\angle ATC = 2\angle BCP$.¤

«8)→ Proof »

9)! Yr11-3U\circleg.hsc Qn9) 3U91-2c

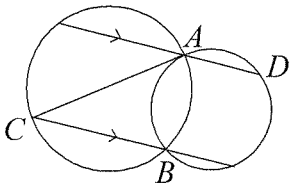


The diagram shows two circles intersecting at A and B. The diameter of one circle is AC. Copy this diagram into your examination booklet.

i. On your diagram draw a straight line through A, parallel to CB, to meet the second circle in D.

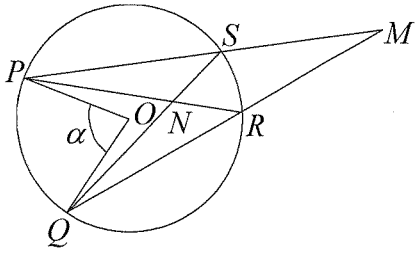
ii. Prove that BD is a diameter of the second circle.

iii. Suppose that BD is parallel to CA. Prove that the circles have equal radii.¤



«9)→ i) ii) iii) Proof »

10)! Yr11-3U\circleg.hsc Qn10) 3U92-3c



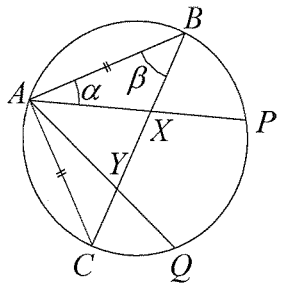
In the diagram P, Q, R, and S are points on a circle centre O, and $\angle$ POQ = α. The lines PS and QR intersect at M and the lines QS and PR intersect at N.

i. Explain why $\angle PRM = \pi - \frac{1}{2}\alpha$.

ii. Show that $\angle PNQ + \angle PMQ = \alpha$.¤

«10)→ Proof »

11)! Yr11-3U\circleg.hsc Qn12) 3U94-2b



Let $ABPQC$ be a circle such that $AB = AC$, AP meets BC at X, and AQ meets BC at Y, as in the diagram. Let $\angle BAP = \alpha$ and $\angle ABC = \beta$.

i. Copy the diagram into your Writing Booklet and state why $\angle AXC = \alpha + \beta$.

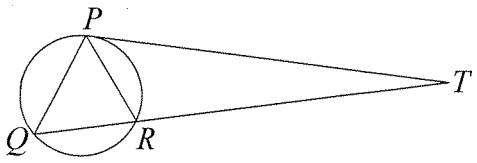
ii. Prove that $\angle BQP = \alpha$.

iii. Prove that $\angle BQA = \beta$.

iv. Prove that $PQYX$ is a cyclic quadrilateral.¤

«11)→ i) $\angle AXC$ is the exterior angle of ΔAXB and as such is equal to the sum of the two interior opposite angles. ii) iii) iv) Proof »

12)! Yr11-3U\circleg.hsc Qn13) 3U95-6a



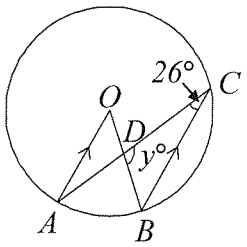
PT is a tangent to the circle PRQ, and QR is a secant intersecting the circle in Q and R. The line QR intersects PT at T. Copy or trace the diagram into your Writing Booklet.

i. Prove that the triangles PRT and QPT are similar.

ii. Hence prove that $PT^2 = QT \times RT$.¤

«12)→ Proof »

13)! Yr11-3U\circleg.hsc Qn15) 3U97-2a



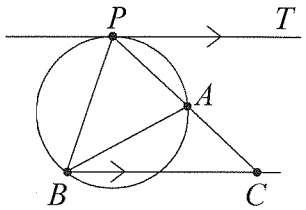
The points A, B and C lie on a circle with centre O. The lines AO and BC are parallel, and OB and AC intersect at D. Also, $\angle ACB = 26°$ and $\angle BDC = y°$, as shown in the diagram. Copy or trace the diagram into your Writing Booklet.

i. State why $\angle AOB = 52°$.

ii. Find y. Justify your answer.¤

«13)→ i) The angle subtended by an arc at the centre of a circle is twice the angle subtended by the arc at the circumference ii) $y = 102°$ »

16)! Yr11-3U\circleg.hsc Qn18) 3U00-5a



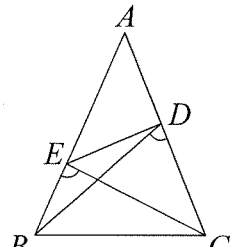
In the diagram, A, P and B are points on the circle. The line PT is tangent to the circle at point P, and PA is produced to C so that BC is parallel to PT. Copy the diagram into your writing booklet.

i. Show that $\angle PBA = \angle PCB$.

ii. Deduce that $PB^2 = PA \times PC$.¤

«16)→ Proof »

14)! Yr11-3U\circleg.hsc Qn16) 3U98-4c



Not to scale

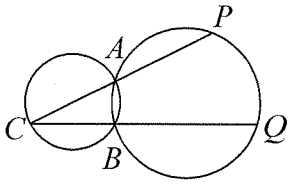
ABC is an acute-angled triangle. D is a point on AC, E is a point on AB, and $\angle BEC = \angle BDC$, as shown in the diagram. Sonya was asked to prove that $\angle AED = \angle ACB$. She provided a two-step proof but did not give reasons.

i. State a reason for her correct statement that $EDCB$ is a cyclic quadrilateral.

ii. State a reason why she could then correctly conclude that $\angle AED = \angle ACB$.¤

«14)→ i) The end points of an interval which subtends two equal angles at two points on the same side of it, are concyclic. ii) The exterior angle of a cyclic quadrilateral equals the interior opposite angle. »

15)! Yr11-3U\circleg.hsc Qn17) 3U99-3c

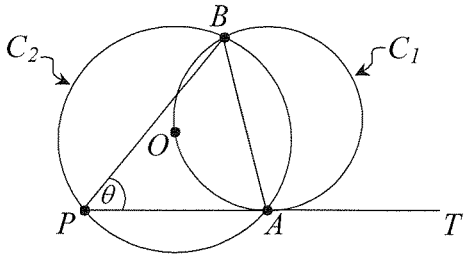


Two circles intersect at two points A and B as shown in the diagram. The diameter of one circle is CA and this line intersects the other circle at A and P. The line CB intersects the second circle at B and Q. Copy or trace the diagram into your Writing Booklet.

Prove $\angle CPQ$ is a right angle.¤

«15)→ Proof »

17)! Yr11-3U\circleg.hsc Qn19) 3U01-3b



Two circles, C_1 and C_2, intersect at points A and B. Circle C_1 passes through the centre O of circle C_2. The point P lies on circle C_2 so that the line PAT is tangent to circle C_1 at point A. Let $\angle APB = \theta$. Copy or trace the diagram into your writing booklet.

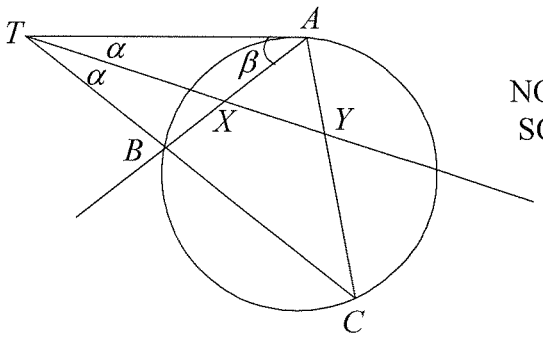
i. Find $\angle AOB$ in terms of θ. Give a reason for your answer.

ii. Explain why $\angle TAB = 2\theta$.

iii. Deduce that $PA = BA$.¤

«17)→ i) $\angle AOB = 2\theta$ (Angle at the centre is twice the angle at the circumference standing on the same arc) ii) iii) Proof »

18)! Yr11-3U\circleg.hsc Qn20) 3U02-2e



NOT TO SCALE

In the diagram the points A, B and C lie on the circle and CB produced meets the tangent from A at the point T. The bisector of the angle ATC intersects AB and AC at X and Y respectively. Let $\angle TAB = \beta$. Copy or trace the diagram into your writing booklet.

i. Explain why $\angle ACB = \beta$.

ii. Hence prove that triangle AXY is isosceles.¤

«18)→ i) $\angle ACB = \angle BAT$ (Angle between a tangent and a chord is equal to the angle in the alternate segment)

ii) Proof »

[[End Of Qns]]