BARKER

ANS (3u) Trial 2000.

Q1.(a) (i) method 1: $\times (x-2)^2$: $(x+4)(x-2) > 5(x-2)^2$

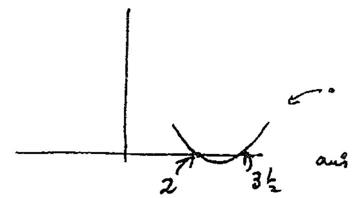
$x^2+2x-8 > 5(x^2-4x+4)$

$0 > 4x^2-22x+28$

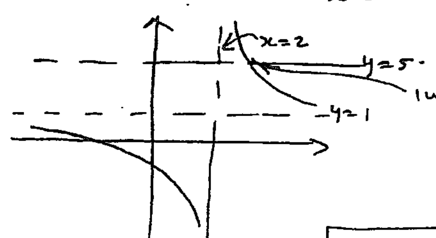
i.e. $2x^2-11x+14 < 0$

$(2x-7)(x-2) < 0$

$\therefore \boxed{2 < x < 3\tfrac{1}{2}}$



method 2: sketch $y = \dfrac{x-2+6}{x-2} = 1 + \dfrac{6}{x-2}$.



inters: $x+4 = 5(x-2)$.

$= 5x-10$

$4x-14=0$

$x = \frac{14}{4} = \frac{7}{2} = 3\tfrac{1}{2}$.

$\therefore \boxed{2 < x < 3\tfrac{1}{2}}$

method 3: cases:

for $x > 2$: $x+4 > 5(x-2) \Rightarrow x+4 > 5x-10$.

i.e. $4x-14 < 0$ $\therefore x < 3\tfrac{1}{2}$

$\therefore 2 < x < 3\tfrac{1}{2}$ is part sol.

for $x < 2$: $x+4 < 5(x-2) \;\ldots\ldots \Rightarrow x > 3\tfrac{1}{2}$

no part sol. here

$\therefore \boxed{2 < x < 3\tfrac{1}{2}}$

(ii) $y^2-5y+6=0 \Rightarrow (y-2)(y-3)=0$

$\therefore x + \frac{1}{x} = 2, 3$

$x^2-2x+1=0$ or $x^2-3x+1=0$

$(x-1)^2=0$ $\quad x = \dfrac{3 \pm \sqrt{9-4}}{2}$

$\therefore x = 1, \dfrac{3\pm\sqrt{5}}{2}$

Q1.(b) (i) $y = \cos^3 2x$

$y' = 3\cos^2 2x \cdot -\sin 2x \cdot 2$ ← 2 marks, 1 off [illegible] mistake

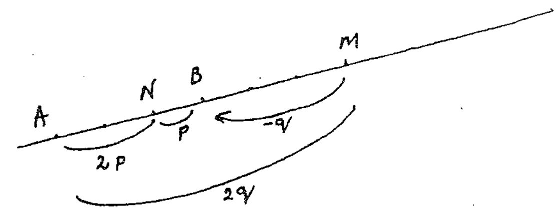
$= -6\sin 2x \cdot \cos^2 2x$

(ii) $y = e^{x\ln x}$

$y' = (1.\ln x + x.\frac{1}{x})e^{x\ln x}$ ← 2 marks, 1 off [illegible]

$= (1+\ln x)e^{x\ln x}$.

(c)

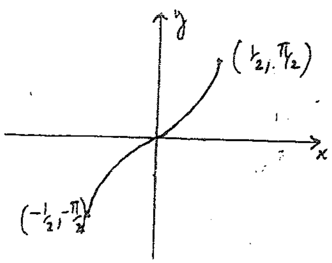


B divides MN in ratio 3:1

~~Q2. (a)~~ $\cancel{L = \lim_{x \to 0} \frac{\sin 5x}{2x} = \lim_{5x \to 0} \frac{\sin 5x}{5x} \cdot \frac{5}{2}}$

$= 1 \times \frac{5}{2} = 2\frac{1}{2}$

(b) (i) $-1 \le 2x \le 1 \Rightarrow -\frac{1}{2} \le x \le \frac{1}{2}$



$y = \sin^{-1} 2x$

(ii) Domain $-\frac{1}{2} \le x \le \frac{1}{2}$

Range $-\frac{\pi}{2} \le y \le \pi/2$

(c) $I = \int_0^2 \frac{4\,dx}{\sqrt{4-x^2}} = 4\left[\sin^{-1}\frac{x}{2}\right]_0^2$

$= 4\{\sin^{-1} 1 - \sin^{-1} 0\}$

$= 4\{\pi/2 - 0\}$

$= 2\pi$

(d) $m_1 = 3/4 \quad , \quad m_2 = -\frac{1}{2}$

$\tan\theta = \frac{|3/4 - -\frac{1}{2}|}{1 + \frac{3}{4}(-\frac{1}{2})} = \frac{5/4}{5/8} = 2$

acute.

$\therefore \theta = 180^\circ - 63^\circ 26' = 116^\circ 34'$

Q3.

(a) $LHS = \frac{\sin\theta + 2\sin\theta\cos\theta}{1+\cos\theta + 2\cos^2\theta - 1}$

$= \frac{\sin\theta(1+2\cos\theta)}{\cos\theta}$

$= \tan\theta$

$= RHS.$

(b) $u = \cos x \quad , \quad x = \pi/3 \Rightarrow u = \frac{1}{2}$

$du = -\sin x\,dx \qquad x = 0 \Rightarrow u = 1$

$I = -\int_0^{\frac{\pi}{3}} \frac{-\sin x\,dx}{\cos x}$

$= -\int_1^{\frac{1}{2}} \frac{du}{u}$

$= \int_{\frac{1}{2}}^1 \frac{du}{u}$

$= \left[\ln u\right]_{\frac{1}{2}}^1$

$= \ln 1 - \ln\frac{1}{2}$

$= 0 - -\ln 2$

$= \ln 2$

(c) $LHS = \frac{9!}{5!4!} + \frac{9!}{4!5!} = \frac{2\times 9!}{5!4!} \times \frac{5}{5} = \frac{10!}{5!5!} = {}^{10}C_5$

$\therefore m = 5$ [note bald answer OK]

(d) (i) $\frac{d}{dx}\left(\ln(\sec 3x)\right) = \frac{3 \times \sec 3x \tan 3x}{\sec 3x}$

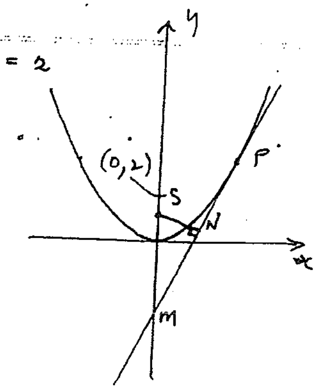
$= 3\tan 3x$

(ii) $\frac{d}{dx}\left(\tan^{-1}(2\tan x)\right) = \frac{2\sec^2 x}{1+4\tan^2 x}$.

Q4.

(a)(i) PM is $x \cdot 4p = 4(y+2p^2)$ $\quad a = 2$

i.e. $px = y + 2p^2$



cuts y axis: $x = 0 \therefore y = -2p^2$

i.e. M is $\underline{(0, -2p^2)}$

(ii) gr. PM = p

$\therefore$ SN is $y - 2 = -\frac{1}{p}(x - 0)$

i.e. $\underline{y = 2 - \frac{x}{p}}$.

N: $px = \left(2 - \frac{x}{p}\right) + 2p^2$

i.e. $p^2x = 2p - x + 2p^3$

$x(p^2+1) = 2p(p^2+1)$

$\therefore x = 2p$, since $p^2+1 > 0$

& $y = 2 - \frac{2p}{p} = 0$

$\therefore$ $\underline{N \text{ is } (2p, 0)}$

(iii) midpt of MN: $\left(\frac{0+2p}{2}, \frac{-2p^2+0}{2}\right)$

i.e. $\underline{(p, -p^2)}$

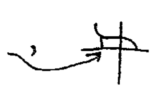
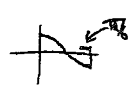
(iv) locus: $y = -p^2 = -x^2$

i.e. $\underline{y = -x^2}$

(b) $(1+2x)^8 = \binom{8}{0} + \binom{8}{1}(2x) + \dots + \binom{8}{5}(2x)^5 + \dots + (2x)^8$

$\therefore$ coeff of x^5 is $\binom{8}{5} \times 2^5 = \frac{8 \times 7 \times 6}{1 \times 2 \times 3} \times 32 = 7 \times 256$

$= 1792$.

(c) $\sin \frac{4\pi}{3} = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$,  $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ 

$\therefore$ Ans $= \pi - \frac{\pi}{\dots}$ ...

Q5.(a) $n = 1 \Rightarrow 3^{2 \times 1} - 1 = 9 - 1 = 8 \therefore$ div by 8 when $n = 1$

Say $(n = k)$, $3^{2k} - 1 = 8P$ for some pos. int. k, P

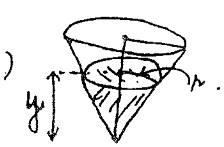
then $3^{2(k+1)} - 1 = 3^{2k+2} - 1$

$= 9 \times (3^{2k} - 1) + 8$

$= 9 \times 8P + 8$

$= 8(9P + 1)$ & $(9P+1)$ is an int.

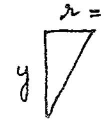
$\therefore$ If div by 8 for some value of n then div by 8 for next value of n, and shown true for $n = 1$ $\therefore$ true for all pos. int. n.

(b) 

$V = \frac{1}{3}\pi r^2 y$

$\frac{dV}{dt} = 5 \text{ cm}^3/\text{h}$

Want $\frac{dy}{dt}$ when $y = 3.5$



$\therefore V = \frac{\pi}{3} \cdot \frac{y^3}{3} = \frac{\pi y^3}{9}$

$\frac{dV}{dy} = \frac{\pi y^2}{3}$

And $\frac{dV}{dt} = \frac{dV}{dy} \cdot \frac{dy}{dt}$

$5 = \frac{\pi y^2}{3} \cdot \frac{dy}{dt}$

$\therefore \frac{dy}{dt} = 5 \times \frac{3}{\pi \times (3.5)^2}$ at $y = 3.5$

$\underline{\doteqdot 1.2 \text{ cm/h}}$.

(c) $2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\frac{1}{2}, 1$

$\theta = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}, 0, 2\pi$

Q6 (a) $\frac{1}{x}\cdot\binom{7}{k}(3x)^{7-k}\left(-\frac{1}{2x}\right)^{k}$

we want $x^{-1}\times x^{7-k}\times x^{-k} = 1 = x^{0}$

i.e. $6-2k=0$

$\therefore \underline{k=3}$

$\therefore \binom{7}{3}3^{7-3}\left(-\frac{1}{2}\right)^{3}$ is the req. term

i.e. $-\frac{7\times 6\times 5}{1\times 2\times 3}\times\frac{3^4}{2^3} = -\frac{35\times 81}{8}$

$\therefore \underline{-354\frac{3}{8}}$

(b) $x = 2\cos 3t - 5\sin 3t$

$\dot{x} = -6\sin 3t - 15\cos 3t$

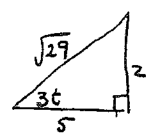
$\ddot{x} = -18\cos 3t + 45\sin 3t$

(i) $\therefore \underline{\ddot{x} = -9x}$ which is SHM

(ii) max speed is when $\ddot{x}=0$

i.e. $x=0$ $\therefore$ $2\cos 3t = 5\sin 3t$

$\therefore \frac{2}{5} = \tan 3t$



& max speed $= \left|-6\times\frac{2}{\sqrt{29}} - 15\times\frac{5}{\sqrt{29}}\right|$

$= \frac{87}{\sqrt{29}} \doteq 16.155 \doteq$ <u>16 speed units</u>

(c)(i) $\frac{d}{dx}\left[e^{x}(\sin x+\cos x)\right] = e^{x}(\sin x+\cos x)+e^{x}(\cos x-\sin x)$

$= 2e^{x}\cos x.$ <u>as req.</u>

(ii) $I = \int_{1}^{\pi/2} e^{x}\cos x\,dx$

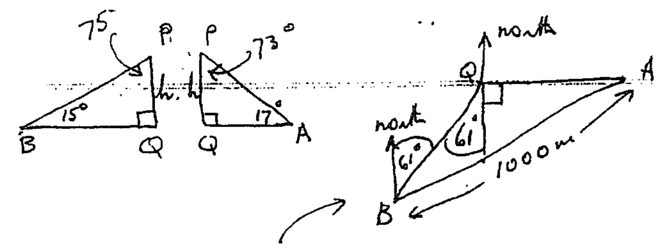
$= \frac{1}{2}\int_{1}^{\pi/2} 2e^{x}\cos x\,dx$

$= \frac{1}{2}\left[e^{x}(\sin x+\cos x)\right]_{1}^{\pi/2}$

$= \frac{1}{2}\left\{e^{\pi/2}(1+0) - e(\sin 1+\cos 1)\right\}$ $\frac{1}{2}\{4.81047\ldots - 3.75605\ldots\}$

$\doteq 0.527$

Q7. (a)



(i) $\angle AQB = 61^\circ + 90^\circ = 151^\circ$

(ii) in $\triangle APQ$: $\tan 73^\circ = \frac{AQ}{h}$ $\therefore AQ = h\tan 73^\circ$

(iii) in $\triangle BPQ$: $\tan 75^\circ = \frac{BQ}{h}$ $\therefore BQ = h\tan 75^\circ$

(iv) in $\triangle ABQ$:

$1000^2 = (h\tan 73^\circ)^2 + (h\tan 75^\circ)^2 - 2(h\tan 73^\circ)(h\tan 75^\circ)\cos 151^\circ$

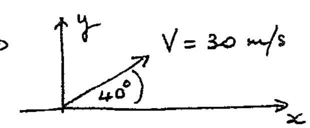
$= h^2\left[\tan^2 73^\circ + \tan^2 75^\circ - 2\tan 73^\circ\tan 75^\circ\cos 151^\circ\right]$

$= h^2\times 45.9796\ldots$

$\therefore h = \frac{1000}{\sqrt{45.97\ldots}} = 147.47\ldots$

$= \underline{147\text{ m}.}$

(b) $\rightarrow$



$\ddot{x} = 0$ | $\ddot{y} = -10$

$\dot{x} = 30\cos 40^\circ$ | $\dot{y} = -10t + 30\sin 40^\circ$

$x = 30t\cos 40^\circ$ | $y = -5t^2 + 30t\sin 40^\circ$

(i) max ht: when $\dot{y}=0$: $t = 3\sin 40^\circ$

then ht $= -5(3\sin 40^\circ)^2 + 90\sin^2 40^\circ$

$= 45\sin^2 40^\circ$

$\doteq \underline{18.6\text{ m}.}$

(ii) Speed at top pt $(\dot{y}=0)$ $= \dot{x} = 30\cos 40^\circ \doteq 23$ m/s

(iii) $x = 40 \Rightarrow t = \frac{4}{3\cos 40^\circ} \Rightarrow y = -5\times\left(\frac{4}{3\cos 40^\circ}\right)^2 + \frac{4}{3\cos 40^\circ}\times 30\sin 40^\circ$

$= -15.14745\ldots + 33.56\ldots$

$\doteq \underline{18.4\text{ m}.}$