Name: ____________________

Teacher: ____________________

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PYMBLE LADIES' COLLEGE

YEAR 12

MATHEMATICS

TRIAL EXAMINATION

August 2003

Time Allowed: **3 hours plus 5 minutes reading time**

Marking guidelines: **The marks for each part are indicated beside the question.**

Instructions:

- All questions should be attempted.
- All necessary working must be shown.
- Start each question on a new page.
- Write your name and your teacher's name on each page.
- Marks might be deducted for careless or untidy work.
- Only approved calculators may be used.
- All questions are of equal value.
- Diagrams are not drawn to scale.
- A standard integral sheet is attached.
- DO NOT staple different questions together.
- All rough working paper must be attached to the end of the last question.
- Staple a coloured sheet of paper to the back of each question.
- Hand in this question paper with your answers.
- There are ten (10) questions in this paper and twelve (12) pages.

QUESTION 1

a) Evaluate $\dfrac{\sqrt{4\cdot 8+3\cdot 7}}{0\cdot 3\times 8\cdot 9}$ correct to 2 significant figures **2**

b) Solve $\log(x+3)=\log x+\log 3$ **2**

c) Find $\dfrac{d}{dx}\left(\dfrac{1}{e^x-1}\right)$ **2**

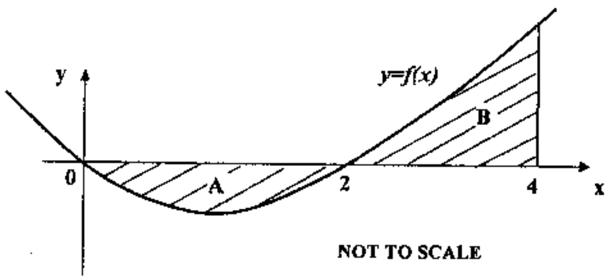
d) Solve $4-\dfrac{2x}{3}\le 6$ **2**

e) Solve $\dfrac{x^2+3x-4}{x^2+x-2}=0$ **2**

f) Given $5^m=4$, find the value of 5^{1-2m} **2**

QUESTION 2 Start a new page

a)



NOT TO SCALE

The diagram shows the curve $y = f(x)$ and the areas bound by the curve and the x axis. Area A = 1.3 units^2 and Area B = 1.8 units^2.

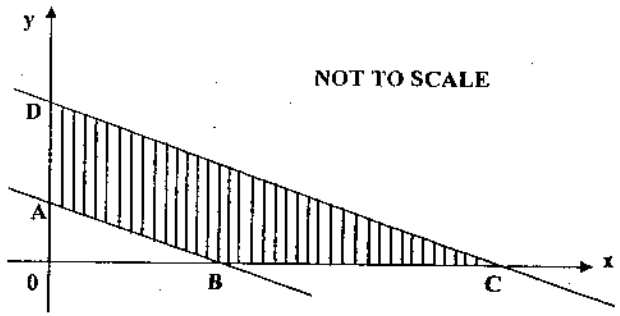
Write down the value of (i) $\int_0^2 f(x)\,dx$ **1**

(ii) $\int_0^4 f(x)\,dx$ **1**

b) (i) Sketch the parabola $(y-1)^2 = x$, showing the intercepts. **2**

(ii) Write down the coordinates of the focus of the parabola. **1**

c)



The diagram shows the straight line AB with equation $x + 2y = 2$.

(i) Show that the equation of the straight line DC which is parallel to AB and passes through the point (2,2) is $x + 2y = 6$. **1**

(ii) Calculate the distance between AB and DC. **2**

(iii) Calculate the area of the trapezium ABCD. **2**

(iv) Write down a set of inequations that uniquely defines the region ABCD. **2**

QUESTION 3. Start a new page

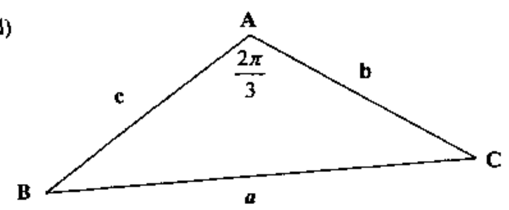
a) Differentiate (i) $\tan^3\left(\frac{x}{3}\right)$ **2**

(ii) $\frac{\ln\sqrt{x}}{x}$ **2**

b) Solve $x - 4 = 3\sqrt{x}$ **3**

c) Evaluate $\int_0^3 \frac{e^x}{e^x + 1}\,dx$ **2**

d) **3**



For the triangle ABC as shown, use the cosine rule to prove

that $\cos C = \frac{2b + c}{2a}$

QUESTION 4. Start a new page

a) In many situations involving environmental pollution much of the pollutant can be removed from the air or water for a reasonable cost, but the last part of the pollutant is very expensive to remove.

One cost function for this is given by $y = \frac{6x}{102 - x}$, where y is the cost in thousands of dollars of removing x percent of a given pollutant.

(i) Using this function find the cost of removing 90% of the pollutant. **1**

(ii) What percentage of pollutant can be removed for $10 000? **2**

(iii) Sketch the cost function, clearly indicating its domain and range. (You may assume that it is a monotonic increasing function) **2**

b) The rate at which a reservoir is being filled is given by

$$V'(t) = 150\,t^{-\frac{1}{4}} + 10 \quad \text{litres/sec.}$$

(i) Find $V(t)$, the volume of water in the reservoir at time t secs, given that the reservoir holds 3 000 litres after 16 seconds. **2**

(ii) How much more will be added to the reservoir, to the nearest 100 litres, by the end of 15 minutes? **2**

c) For a particular curve it is given that $\frac{d^2y}{dx^2} = 12x - 2$. **3**

The tangent at (1, -2) on the curve makes an angle of 45° with the positive direction of the x axis.

Find the equation of the curve.

QUESTION 5. Start a new page

a) Consider the curve $y = 2x^3 + ax^2 + bx + 3$.

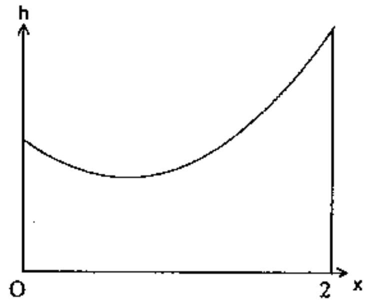
(i) If this curve has stationary points when $x = 1$ and $x = -2$, show that $a = 3$ and $b = -12$. **2**

(ii) Determine the nature of these stationary points and find, if any, the coordinates of any inflection points. **3**

(iii) Sketch the curve showing all important features. **2**

b) A non uniform metal chain hangs between 2 walls that are 2 metres apart. The height h of the chain above the ground is a function of x, where x is the distance along the ground from the left wall.

The function is $h(x) = e^{-2x} + e^{x}$, $0 \le x \le 2$



NOT TO SCALE

(i) Find the exact value of x for which the chain is closest to the ground. **3**

(ii) Find the minimum value of h expressed as an exact value in terms of a power of 2 **2**

QUESTION 6. Start a new page

a) The voltage of an electronic circuit is given by $v = \sin^2 t - \sin t + 2$.

(i) Rearrange $\sin^2 t - \sin t + 2$ in completed square form. **1**

(ii) Hence or otherwise write down the minimum voltage. **1**

(iii) What is the smallest positive t value for which this minimum voltage occurs? **1**

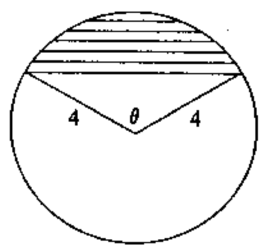
b) A particle travels in a straight line with position x m after t secs given by $x = 12t - 3t^2$.

(i) Where is the particle 3 seconds after it starts? **1**

(ii) How far does the particle travel during the first 5 seconds? **3**

(iii) Find the greatest speed during the first 5 seconds. **1**

c)



(i) Write down an expression in terms of θ for the size of the shaded segment shown. (θ is the angle measure in radians at the centre of the circle) **1**

(ii) Use your expression to discuss the limiting value of $\frac{\sin\theta}{\theta}$ as θ gets closer and closer to zero. **1**

(iii) For the situation where $\theta = 120°$, calculate the exact perimeter of the shaded area. **2**

QUESTION 7. Start a new page

a) A property speculator assumes that for her property the rate of increase in value is directly proportional to the value V.

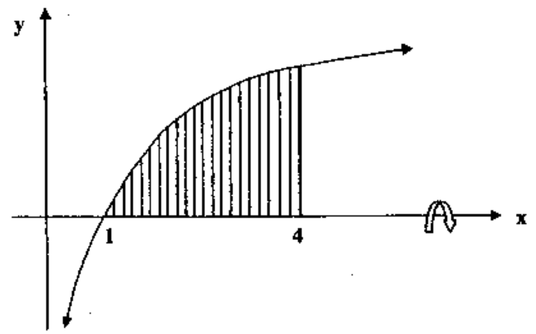
(i) Show that the equation $V = V_0 e^{kt}$ satisfies this assumption, where V_0 and k are constants and t is the time in years. **1**

(ii) She bought an apartment 10 years ago for \$150 000, and calculated that it would be worth \$600 000 after 20 years. **2**

Show that $k = \frac{1}{10} \ln 2$

(iii) If the apartment is currently valued at \$350 000, calculate whether this is more or less than she had anticipated, and by how much. **2**

b) A toy manufacturer produces a spinning top which has the shape generated when the shaded area below is rotated around the x axis.



The shaded area lies between the curve $y = 4 - \frac{4}{x}$, the x axis and the line $x = 4$.

(i) Express this volume as a definite integral. **1**

(ii) Use the trapezoidal rule with 4 function values to estimate this volume (to 1 decimal place). **3**

(iii) Show that the exact volume is $4\pi(15 - 16\ln 2)$. **3**

QUESTION 8 Start a new page

a) A curve has equation $y = 1 + 3\cos\frac{x}{2}$.

(i) State the amplitude and period of this curve. **1**

(ii) Sketch this curve for $-2\pi \le x \le 2\pi$. **2**

(iii) Hence determine the number of solutions to the equation **1**

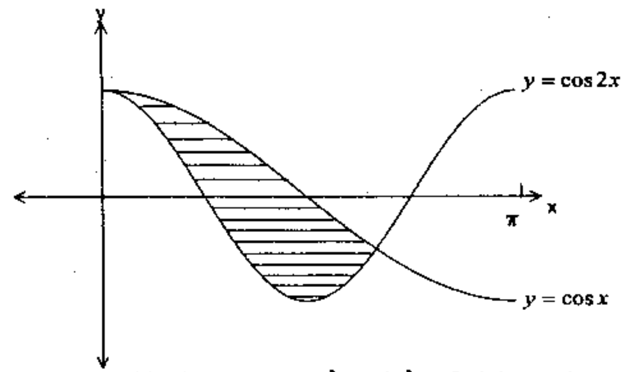
$$3\cos x = \frac{3}{2}$$

b) (i) Write down an expression for the difference between the limiting sum and the sum of n terms of the infinite series **2**

$$1 + 0\cdot 4 + 0\cdot 16 + \ldots$$

(ii) Find the least number of terms that must be taken for this difference to be less than 10^{-6}. **1**

c) The sketch shows the curves $y = \cos 2x$ and $y = \cos x$ over the domain $0 \le x \le \pi$.



(i) Using the identity $\cos 2x = \cos^2 x - \sin^2 x$, find the x values at the points of intersection. **2**

(ii) Find the exact size of the shaded area. **3**

QUESTION 9 Start a new page

a) An orchestra wished to build a concert hall to accommodate an audience of 750 people. One of the alternatives they considered for raising funds was to sell each seat for \$3 000.

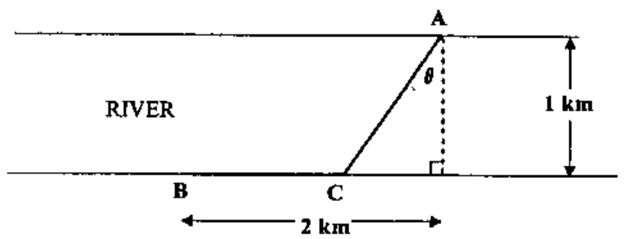
(i) How much money would they raise by this method? **1**

(ii) A second alternative was to sell each seat for a different amount, so that the amounts formed the terms of an arithmetic sequence. If the cheapest seat were to be sold for \$1 000, how much would the most expensive seat be if the same total amount would be raised? **2**

b) (i) For the domain $0 \le \theta < \frac{\pi}{2}$ sketch on the same axes the graphs of $y = 5\tan\theta$ and $y = \sec\theta$. **2**

(ii) Calculate to 1 decimal place the value of θ where the curves intersect. **2**

c) Two towns A and B lie on opposite sides of a straight river which is 1 km wide, A being 2 km further upstream than B, as shown below.



A road ACB is to be built as shown passing through some point C, where θ is the angle between the road AC and the line at right angles to the river. It costs five times as much to construct the road over the river as it does to construct the same length of road on land.

(i) Let \$ k be the cost of constructing 1 km of road on land. Write down an expression in terms of θ for the total cost \$C of constructing the road. **2**

(ii) Find the value of θ that would minimise the cost of construction. (You might wish to refer to your answer for (b) above.) **3**

QUESTION 10. Start a new page

a) α and β are the roots of the equation $x^2 + 4x + 2 = 0$.

(i) Find the value of $\alpha^2 + \beta^2$. **2**

(ii) Hence or otherwise write down the equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. **4**

b) For some $a > 0$ two curves $f(x) = a^x$ and $g(x) = \log_a x$ are drawn on the same axes so that they touch on $y = x$.

(i) Write down expressions for $f'(x)$ and $g'(x)$. **2**

(ii) Write down an equation involving natural logarithms whose solution is the x value at their point of contact. **1**

(iii) Find the coordinates of the point of contact. **2**

(iv) What is the value of a? **1**

END OF PAPER