SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2004

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK # 2

Mathematics Extension 2

Sample Solutions

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$\boxed{3}$ 1. (a) Method 1:

$$\begin{aligned} \mathrm{I} &= \int_0^3 \frac{xdx}{\sqrt{16+x^2}}, \\ &= \tfrac{1}{2}\int_{16}^{25} u^{-\frac{1}{2}}du, \\ &= \tfrac{1}{2}\left[2u^{\frac{1}{2}}\right]_{16}^{25}, \\ &= 5-4, \\ &= 1. \end{aligned}$$

$$\begin{aligned} \text{put } u &= 16+x^2 \\ du &= 2xdx \\ \text{when } x &= 0, \quad u = 16 \\ x &= 3, \quad u = 25 \end{aligned}$$

Method 2:

$$\begin{aligned} \mathrm{I} &= \int_4^5 \frac{udu}{u}, \\ &= u\Big]_4^5, \\ &= 1. \end{aligned}$$

$$\begin{aligned} \text{put } u^2 &= 16+x^2 \\ 2udu &= 2xdx \\ \text{when } x &= 0, \quad u = 4 \\ x &= 3, \quad u = 5 \end{aligned}$$

Method 3:

$$\begin{aligned} \mathrm{I} &= \int_0^{\tan^{-1}\frac{3}{4}} \frac{4\tan\theta . 4\sec^2\theta d\theta}{4\sec\theta}, \\ &= 4\sec\theta\Big]_0^{\tan^{-1}\frac{3}{4}}, \\ &= 4\left\{\tfrac{5}{4}-1\right\}, \\ &= 1. \end{aligned}$$

$$\begin{aligned} \text{put } x &= 4\tan\theta \\ dx &= 4\sec^2\theta d\theta \\ \text{when } x &= 0, \quad \theta = 0 \\ x &= 3, \quad \theta = \tan^{-1}\tfrac{3}{4} \end{aligned}$$

Method 4:

$$\begin{aligned} \mathrm{I} &= \tfrac{1}{2}\int_0^9 \frac{du}{\sqrt{16+u}}, \\ &= \left[\tfrac{1}{2}\times 2\times\sqrt{16+u}\,\right]_0^9, \\ &= 5-4, \\ &= 1. \end{aligned}$$

$$\begin{aligned} \text{put } u &= x^2 \\ du &= 2xdx \\ \text{when } x &= 0, \quad u = 0 \\ x &= 3, \quad u = 9 \end{aligned}$$

Method 5:

$$\begin{aligned} \mathrm{I} &= \tfrac{1}{2}\int_0^3 \frac{d(x^2)}{\sqrt{16+x^2}}, \\ &= \left[\tfrac{1}{2}\times 2\times\sqrt{16+x^2}\,\right]_0^3, \\ &= 5-4, \\ &= 1. \end{aligned}$$

$\boxed{2}$ (b)
$$\begin{aligned}
\mathrm{I} &= \int \frac{dx}{(x^2+6x+9)+13-9}, \\
&= \int \frac{dx}{(x+3)^2+4}, \\
&= \tfrac{1}{2}\tan^{-1}\left(\frac{x+3}{2}\right) + \mathrm{C}.
\end{aligned}$$

$\boxed{2}$ (c)
$$\begin{aligned}
\mathrm{I} &= \int x e^{-x} dx, \\
&= -xe^{-x} + \int e^{-x} dx, \\
&= -xe^{-x} - e^{-x} + \mathrm{C}.
\end{aligned}$$
$$\begin{aligned} u &= x, & v' &= e^{-x} \\ u' &= 1, & v &= -e^{-x} \end{aligned}$$

$\boxed{3}$ (d) Method 1:
$$\begin{aligned}
\mathrm{I} &= \int \cos^2\theta . \cos\theta d\theta, \\
&= \int (1-\sin^2\theta). \cos\theta d\theta, \\
&= \int (1-u^2) du, \\
&= u - \tfrac{1}{3}u^3 + \mathrm{C}, \\
&= \sin\theta - \tfrac{1}{3}\sin^3\theta + \mathrm{C}.
\end{aligned}$$
$$\begin{aligned} \text{put } \sin\theta &= u \\ \cos\theta d\theta &= du \end{aligned}$$

Method 2:
$$\begin{aligned}
\mathrm{I} &= \int \cos^2\theta . \cos\theta d\theta, \\
&= \int (1-\sin^2\theta). d\sin\theta, \\
&= \sin\theta - \tfrac{1}{3}\sin^3\theta + \mathrm{C}.
\end{aligned}$$

Method 3:
$$\begin{aligned}
\mathrm{I} &= \int \cos^2\theta . \cos\theta d\theta, \\
&= \cos^2\theta\sin\theta + 2\int \sin^2\theta\cos\theta d\theta, \\
&= \cos^2\theta\sin\theta + 2\int \cos\theta(1-\cos^2\theta) d\theta, \\
&= \cos^2\theta\sin\theta + 2\sin\theta - 2\int \cos^3\theta d\theta, \\
3\mathrm{I} &= \cos^2\theta\sin\theta + 2\sin\theta + c, \\
\mathrm{I} &= \tfrac{1}{3}\{\cos^2\theta\sin\theta + 2\sin\theta\} + \mathrm{C}.
\end{aligned}$$
$$\begin{aligned} u &= \cos^2\theta, & v' &= \cos\theta \\ u' &= -2\sin\theta\cos\theta, & v &= \sin\theta \end{aligned}$$

$\boxed{3}$ (e) (i) Method 1:

$$\begin{aligned}
x^2-4x-1 &= A(1+x^2)+(Bx+C)(1+2x). \\
\text{Put } x &= -\tfrac{1}{2}, \\
\tfrac{1}{4}+2-1 &= A(1\tfrac{1}{4}), \\
A &= 1. \\
\text{Also, } x^2-4x-1 &= x^2(A+2B)+x(B+2C)+(A+C), \\
\text{so } A+2B &= 1, \\
1+2B &= 1, \\
2B &= 0, \\
B &= 0. \\
\text{And } B+2C &= -4, \\
2C &= -4, \\
C &= -2.
\end{aligned}$$

Method 2:

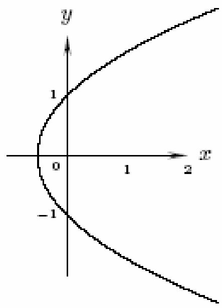
$$\begin{aligned}
\frac{x^2-4x-1}{(1+2x)(1+x^2)} &= \frac{A}{1+2x}+\frac{Bx+C}{x^2+1}. \\
\lim_{x\to-\frac{1}{2}}\left\{\frac{x^2-4x-1}{1+x^2}\right\} &= \lim_{x\to-\frac{1}{2}}\left\{A+\left(\frac{Bx+C}{1+x^2}\right)\times(1+2x)\right\}, \\
\frac{\frac{1}{4}+2-1}{1+\frac{1}{4}} &= A, \\
A &= 1. \\
\lim_{x\to i}\left\{\frac{x^2-4x-1}{1+2x}\right\} &= \lim_{x\to i}\left\{\left(\frac{A}{1+2x}\right)\times(1+x^2)+Bx+C\right\}, \\
\frac{-1-4i-1}{1+2i} &= Bi+C, \\
-2-4i &= Bi+C-2B+2iC, \\
\therefore\ C-2B &= -2, \\
B+2C &= -4, \\
2C-4B &= -4, \\
5B &= 0, \\
B &= 0, \\
C &= -2.
\end{aligned}$$

$\boxed{2}$ (ii)

$$\begin{aligned}
\mathrm{I} &= \tfrac{1}{2}\int\frac{2x\,dx}{1+2x}-2\int\frac{dx}{1+x^2}, \\
&= \tfrac{1}{2}\ln(1+2x)-2\tan^{-1}x+\mathrm{C}.
\end{aligned}$$

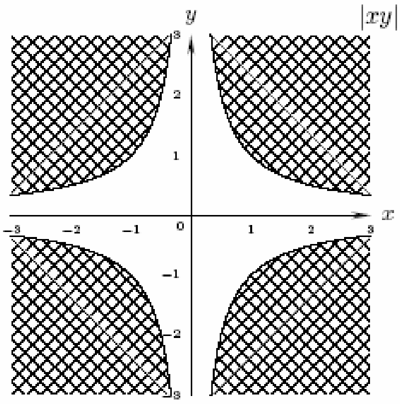
$\boxed{2}$ 2. (a) (i)

$$\begin{aligned} 2\sqrt{x^2+y^2} &= x+iy+x-iy+2, \\ x^2+y^2 &= x^2+2x+1, \\ y^2 &= 4\left(\tfrac{1}{2}\right)\left(x+\tfrac{1}{2}\right). \end{aligned}$$

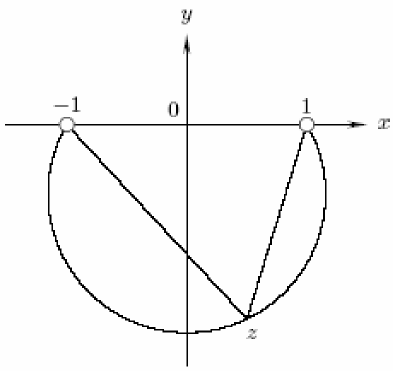


$\boxed{2}$ (ii)

$$\begin{aligned} |(x+iy)^2-(x-iy)^2| &\geq 4, \\ |x^2+2ixy-y^2-(x^2-2ixy-y^2)| &\geq 4, \\ |ixy| &\geq 1, \\ |i||xy| &\geq 1, \\ |xy| &\geq 1. \end{aligned}$$



$\boxed{2}$ (iii)



$\boxed{2}$ (b) (i) $$\begin{aligned}\overrightarrow{BA} &= (10-6)+i(2-8),\\ &= 4-6i.\end{aligned}$$

Note that the question was in error: what was meant was $\overrightarrow{AB}$. Both answers were accepted, $4-6i$ or $-4+6i$.

$\boxed{2}$ (ii) Method 1: $$\begin{aligned}\overrightarrow{AC} &= \overrightarrow{AB}\times 2\times \operatorname{cis}\tfrac{\pi}{4},\\ &= -(4-6i)\times 2\times\left(\tfrac{1}{\sqrt{2}}+\tfrac{i}{\sqrt{2}}\right),\\ &= -\left(4\sqrt{2}+4\sqrt{2}i-6\sqrt{2}i+6\sqrt{2}\right),\\ &= -10\sqrt{2}+2\sqrt{2}i.\\ \therefore\ C &= (10+2i)+\left(-10\sqrt{2}+2\sqrt{2}i\right),\\ &= 10\left(1-\sqrt{2}\right)+2\left(1+\sqrt{2}\right)i.\end{aligned}$$

Method2: $$\begin{aligned}\overrightarrow{OC} &= \overrightarrow{OA}+\overrightarrow{AB}\times 2\times\operatorname{cis}\tfrac{\pi}{4},\\ &= (10+2i)-(4-6i)\times 2\times\left(\tfrac{1}{\sqrt{2}}+\tfrac{i}{\sqrt{2}}\right),\\ &= (10+2i)-\left(4\sqrt{2}+4\sqrt{2}i-6\sqrt{2}i+6\sqrt{2}\right),\\ &= 10+2i-10\sqrt{2}+2\sqrt{2}i.\\ &= 10\left(1-\sqrt{2}\right)+2\left(1+\sqrt{2}\right)i.\end{aligned}$$

$\boxed{1}$ 3. (a) (i) Roots $\alpha,\ \beta$ positive implies $\alpha\beta>0$,
i.e. $\frac{c}{a}>0$ or $K>0$.
Also, for distinct real roots, $$\begin{aligned}\Delta &= 1-4K>0,\\ 1 &> 4K,\\ K &< \tfrac{1}{4}.\end{aligned}$$
So, $0<K<\frac{1}{4}$.

$\boxed{2}$ (ii) $$\begin{aligned}\alpha+\beta &= 1,\\ \alpha\beta &= K,\\ \alpha^2+\beta^2 &= (\alpha+\beta)^2-2\alpha\beta,\\ &= 1-2K.\end{aligned}$$

Method 1:
$K<\frac{1}{4}$,
$\therefore\ \alpha^2+\beta^2>1-2\left(\frac{1}{4}\right)$ ("greater than" as we are subtracting "less than"),
i.e. $\alpha^2+\beta^2>\frac{1}{2}$.

Method 2:
$$\begin{aligned}2K &= 1-(\alpha^2+\beta^2),\\ K &= \frac{1-(\alpha^2+\beta^2)}{2}<\frac{1}{4},\\ -(\alpha^2+\beta^2) &< -\frac{1}{2},\\ \therefore\ \alpha^2+\beta^2 &> \tfrac{1}{2}.\end{aligned}$$

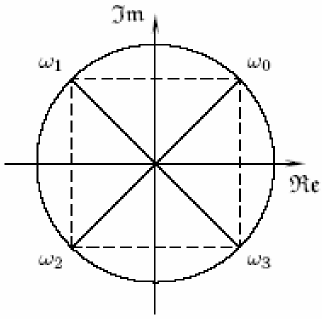
$\boxed{2}$ (iii) $$\frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2}, \\ = \frac{1-2K}{K^2}, \text{ (from above).}$$

Now, also from above,

$$\alpha^2+\beta^2 > \tfrac{1}{2}, \\ \alpha^2\beta^2 < \left(\tfrac{1}{4}\right)^2, \\ \frac{1}{\alpha^2\beta^2} > 16.$$

$$\text{So } \frac{1}{\alpha^2}+\frac{1}{\beta^2} > 16 \times \tfrac{1}{2}, \\ \therefore\ \frac{1}{\alpha^2}+\frac{1}{\beta^2} > 8.$$

$\boxed{3}$ (b) (i) $$\omega = 2\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right), \\ = 2\text{cis}\,\tfrac{\pi}{4}. \\ \omega^4 = \left(2\text{cis}\,\tfrac{\pi}{4}\right)^4, \\ = 16\text{cis}\,\pi, \\ = -16+16\times 0, \\ = -16. \\ \therefore\ \omega_0 = \sqrt{2}+\sqrt{2}i, \\ \omega_1 = -\sqrt{2}+\sqrt{2}i, \\ \omega_2 = -\sqrt{2}-\sqrt{2}i, \\ \omega_3 = \sqrt{2}-\sqrt{2}i.$$



$\boxed{2}$ (ii) $$z^4+16 = \left(z-\sqrt{2}-\sqrt{2}i\right)\left(z-\sqrt{2}+\sqrt{2}i\right)\left(z+\sqrt{2}+\sqrt{2}i\right)\left(z+\sqrt{2}-\sqrt{2}i\right), \\ = \left(z^2-2\sqrt{2}z+4\right)\left(z^2+2\sqrt{2}z+4\right).$$

$\boxed{2}$ (iii) Method 1:

$$\omega = 2\text{cis}\,\tfrac{\pi}{4}, \\ \omega^3 = 2\text{cis}\,\tfrac{3\pi}{4}, \\ \omega^5 = 32\text{cis}\,\tfrac{5\pi}{4}, \\ \omega^7 = 128\text{cis}\,\tfrac{7\pi}{4}.$$

$$\omega+\frac{\omega^3}{4}+\frac{\omega^5}{16}+\frac{\omega^7}{64} = 2\text{cis}\,\tfrac{\pi}{4}+2\text{cis}\,\tfrac{3\pi}{4}+2\text{cis}\,\tfrac{5\pi}{4}+2\text{cis}\,\tfrac{7\pi}{4}, \\ = \omega_0+\omega_1+\omega_2+\omega_3, \text{ the sum of the roots,} \\ = 0.$$

Method 2:

$$\frac{64\omega+16\omega^3+4\omega^5+\omega^7}{64} = \left(16\omega\left(4+\omega^2\right)+\omega^5\left(4+\omega^2\right)\right)\times\tfrac{1}{64}, \\ = \omega\left(16+\omega^4\right)\left(4+\omega^2\right)\times\tfrac{1}{64}.$$

But $16+\omega^4=0$,

$$\therefore\ \omega+\frac{\omega^3}{4}+\frac{\omega^5}{16}+\frac{\omega^7}{64}=0.$$

Method 3:

$\omega^4 = -16$,

$$\begin{aligned}
\omega + \frac{\omega^3}{4} + \frac{\omega^5}{16} + \frac{\omega^7}{64} &= \omega + \frac{\omega^3}{4} + \frac{-16\omega}{16} + \frac{-16\omega^3}{64}, \\
&= \omega - \omega + \frac{\omega^3}{4} - \frac{\omega^3}{4}, \\
&= 0.
\end{aligned}$$

Method 4:

In the geometric series given, $a = \omega$ and $r = \dfrac{\omega^2}{4}$.

$$\begin{aligned}
S_4 &= \frac{\omega\left(1 - \left(\frac{\omega^2}{4}\right)^4\right)}{1 - \frac{\omega^2}{4}}, \text{ note that } \frac{\omega^8}{256} = \frac{(-16)^2}{256} = 1, \\
&= \frac{\omega(1-1)}{1 - \frac{\omega^2}{4}}, \\
&= 0.
\end{aligned}$$

(4)(a)(i) $\int_0^a x\sqrt{a-x}\,dx$

$$u = a - x \Rightarrow x = a - u; dx = -du$$
$$x = 0 \Rightarrow u = a$$
$$x = a \Rightarrow u = 0$$

$$= \int_a^0 (a-u)\sqrt{u}\,(-du)$$
$$= \int_0^a (a-u)u^{1/2}\ du$$
$$= \int_0^a (au^{1/2} - u^{3/2})du$$
$$= \left[\frac{2a}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right]_0^a$$
$$= \left(\frac{2a^2}{3} - \frac{2a^2}{5}\right)\sqrt{a}$$
$$= \frac{4a^2}{15}\sqrt{a} = \frac{4a^{5/2}}{15}$$

(ii) $\int_0^1 \frac{\sin^{-1} x}{\sqrt{1+x}}\,dx$

$$= \int_0^1 \left((1+x)^{-1/2} \times \sin^{-1} x\right)dx$$
$$= 2\sqrt{1+x}\sin^{-1} x\Big|_0^1 - \int_0^1 \frac{2\sqrt{1+x}}{\sqrt{1-x^2}}\,dx$$
$$= \sqrt{2}\pi - 2\int_0^1 \frac{1}{\sqrt{1-x}}\,dx$$
$$= \sqrt{2}\pi + 2\int_0^1 -(1-x)^{-1/2}\,dx$$
$$= \sqrt{2}\pi + 2\times 2\sqrt{1-x}\Big|_0^1$$
$$= \sqrt{2}\pi - 4$$

(b)(i) $\int_0^1 \frac{\left(2dt/1+t^2\right)}{1+\left(1-t^2/1+t^2\right)+\left(2t/1+t^2\right)}$

$$t = \tan\frac{x}{2}$$
$$\cos x = \frac{1-t^2}{1+t^2}, \sin x = \frac{2t}{1+t^2}$$
$$dx = \frac{2dt}{1+t^2}$$

$$= \int_0^1 \frac{2dt}{1+t^2+1-t^2+2t}$$
$$= \int_0^1 \frac{2dt}{2+2t}$$
$$= \int_0^1 \frac{dt}{1+t}$$
$$= \left[\ln|1+t|\right]_0^1$$
$$= \ln 2$$

4 (b) (ii)

$$I=\int_0^{\frac{\pi}{2}}\frac{xdx}{1+\sin x+\cos x}$$

$$=\int_{\frac{\pi}{2}}^{0}\frac{-\left(\frac{\pi}{2}-u\right)du}{1+\sin\left(\frac{\pi}{2}-u\right)+\cos\left(\frac{\pi}{2}-u\right)}$$

$$\boxed{\begin{array}{l} u=\frac{\pi}{2}-x\Rightarrow x=\frac{\pi}{2}-u;\ dx=-du \\ x=0\Rightarrow u=\frac{\pi}{2};x=\frac{\pi}{2}\Rightarrow u=0 \end{array}}$$

$$=\int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}-u\right)du}{1+\cos u+\sin u}$$

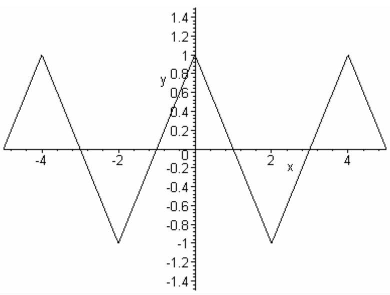
$$\therefore 2I=\int_0^{\frac{\pi}{2}}\frac{\frac{\pi}{2}du}{1+\cos u+\sin u}=\frac{\pi}{2}\int_0^{\frac{\pi}{2}}\frac{du}{1+\cos u+\sin u}=\frac{\pi}{2}\times\ln 2$$

$$\therefore I=\frac{\pi\ln 2}{4}$$

5 (a)

(i) $y=h(x+1)$

Move the curve 1 unit to the left



(ii) $y=\dfrac{1}{h(x)}$

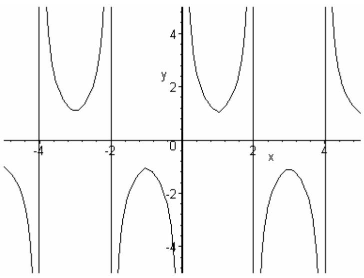
Where $h(x)=0$ there are vertical asymptotes.

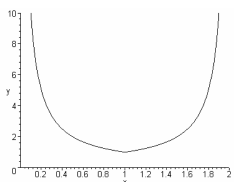
Where $h\to 0^+, y\to\infty$

Where $h\to 0^-, y\to -\infty$

Where h = 1, the reciprocal is *pointed* ie not smooth.

(See the bottom diagram on the right.)

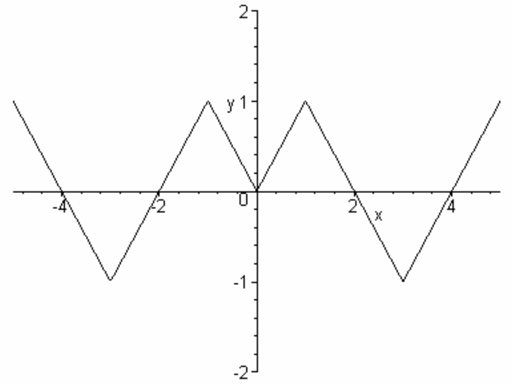




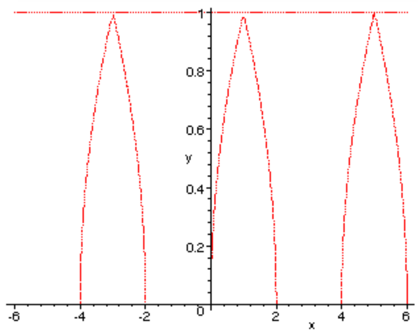
5 (a)

(iii) $y = h(|x|)$.

Erase the LHS of h and then reflect the RHS, so that the result is an even function.



(iv) $y = \sqrt{h(x)}$



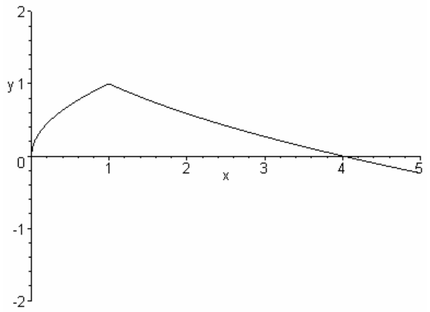
First erase the graph where $h < 0$.

Where $0 < h < 1 \Rightarrow \sqrt{h} > h$

Where $y = 1$, the graph is *pointed*, ie not smooth.

Where $y = 0$, vertical tangents.

(v) $y = h(\sqrt{x})$



Domain: $x \geq 0$

Note that $0 \leq x \leq 4 \Rightarrow 0 \leq \sqrt{x} \leq 2$

So $h\left(\sqrt{4}\right) = h(2) = 0$

The graph for $0 \leq x \leq 4$ will be the same y values for h over $0 \leq x \leq 2$.

5 (b)

First draw $9y = x(x-3)^2$

Clearly x intercepts are at x = 0 and x = 3 **with** x = 3 is a double root.

$$y = x(x-3)^2/9 \Rightarrow z' = \tfrac{1}{9}\left(2x(x-3)+(x-3)^2\right) = \frac{(x-3)}{9}\left(2x+x-3\right)$$

$$\therefore y' = \tfrac{1}{9}(x-3)(3x-3) = \tfrac{1}{3}(x-1)(x-3) = \tfrac{1}{3}(x^2-4x+3)$$

$$\therefore y'' = \tfrac{1}{3}(2x-4)$$

Stationary points when $y' = 0 \Rightarrow x = 1, 3$ ie $\left(1, \tfrac{4}{9}\right) \& (3, 0)$

At x = 1, $y'' < 0 \Rightarrow$ $\left(1, \tfrac{4}{9}\right)$ is a maximum.

The graph in Fig I is the graph of z. The horizontal line is the line y = 1.

So with $y = \tfrac{1}{3}\sqrt{x(x-3)^2}$, the maximum turning point remains the same except it is now $\left(1, \tfrac{2}{3}\right)$.

Any part of the graph in Fig I below the x – axis is not defined for the square root. Where $0 < y < 1$ we get $\sqrt{y} > y$ and where $y > 1$ we get $\sqrt{y} < y$.

The x = 0 intercept will have a vertical tangent, the x = 3 intercept is not smooth. This is shown in Fig 2.

We need to draw $y = \pm\tfrac{1}{3}\sqrt{x(x-3)^2}$: the $\pm$ means that the top part of the graph will be reflected.

The final graph is Fig 3. With turning points $\left(1, \pm\tfrac{2}{3}\right)$

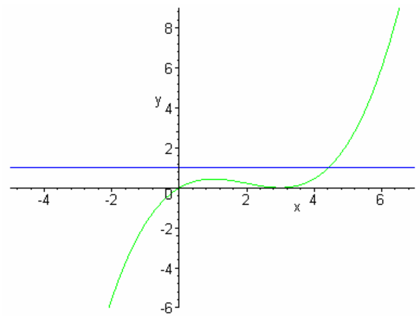


Fig I

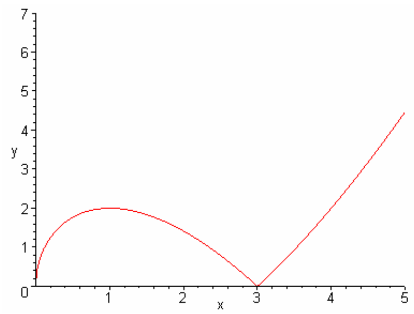


Fig II

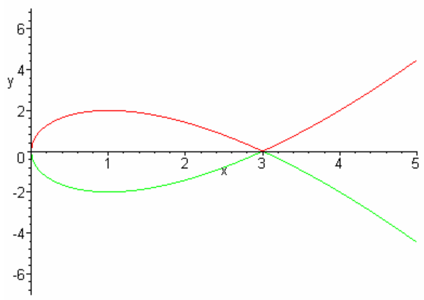
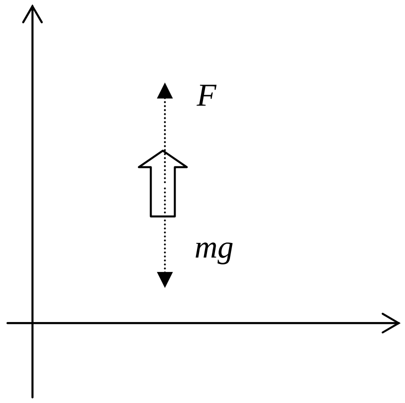


Fig III

Question 6

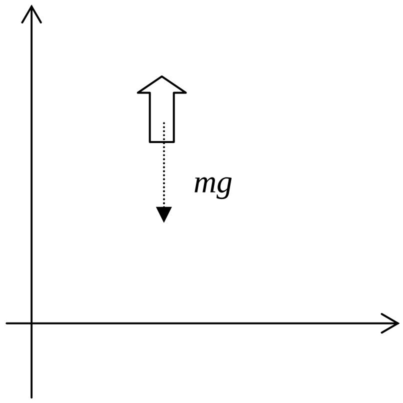
(i)

$0 \le t \le 2$



$$F = g(2-t)$$
$$\therefore ma = F - mg = g(2-t) - mg$$
$$\therefore a = \frac{g(2-t)}{0\cdot 2} - g$$
$$\therefore a = 5g(2-t) - g = g(9-5t)$$

$t > 2$



After 2 seconds the only force acting is gravity

$$\therefore a = -g$$

6(ii) (α) For $0 \le t \le 2$

$$\frac{dv}{dt} = g(9-5t)$$

$$v = 9gt - \frac{5}{2}gt^2 + c_1$$

$\left.\begin{matrix} t=0 \\ v=0 \\ c_1=0 \end{matrix}\right\}$ $\therefore$ $v = gt\left[9 - \frac{5}{2}t\right]$ $\quad$ & $x = \frac{9gt^2}{2} - \frac{5}{6}gt^3 + c_3$ (*)

Max speed when $a = g(9-5t) = 0$

ie when $t = 9/5$

$\therefore$ Max speed $v = \frac{81g}{10}$ m/s

(β) When $t=2$, $v = 8g$

For $t \ge 2$, $a = -g$

$$\frac{dv}{dt} = -g$$

$$v = -gt + c_2$$

$c_2 = 10g$ $\qquad$ ie $v = 10g - gt$ — (**)

For max height $v=0 \Rightarrow \underline{t=10}$

From (*) when $t=2$, $x = \frac{34g}{3}$

From (**) $x = 10gt - \frac{gt^2}{2} + c_4$

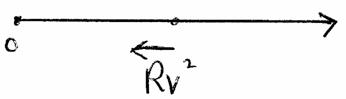
When $t=2$, $x = \frac{34g}{3} \Rightarrow c_4 = -\frac{20g}{3}$

$$\therefore \quad x = -\frac{gt^2}{2} + 10gt - \frac{20g}{3}$$

When $t=10$, $x = \frac{130g}{3}$ m

Question 7

(i)



$m\ddot{x} = -Rv^2$ 1

since $m = 1 \Rightarrow \ddot{x} = -Rv^2$

(ii) $\dfrac{dv}{dt} = -Rv^2$

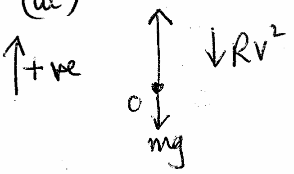
$$\int_u^v \frac{dv}{v^2} = -R\int_0^t dt$$

$$\left[-\frac{1}{v}\right]_u^v = -Rt \qquad 3$$

$$-\frac{1}{v} + \frac{1}{u} = -Rt$$

$$t = \frac{1}{R}\left(\frac{1}{v} - \frac{1}{u}\right)$$

(iii)



1

$m\ddot{y} = -mg - Rv^2$

$m = 1 \Rightarrow \ddot{y} = -(g + Rv^2)$

(iv) $\ddot{y} = -(g + Rv^2)$

Since $g = Ra^2$

$$\Rightarrow \ddot{y} = -(Ra^2 + Rv^2)$$

$$\text{i.e. } \ddot{y} = -R(a^2 + v^2)$$

$$\frac{dv}{dt} = -R(a^2 + v^2)$$

$$\int_u^0 \frac{dv}{a^2 + v^2} = -R\int_0^t dt$$

$$\left[\frac{1}{a}\tan^{-1}\frac{v}{a}\right]_u^0 = -Rt$$

$$0 - \frac{1}{a}\tan^{-1}\frac{u}{a} = -Rt$$

$$\Rightarrow t = \frac{1}{Ra}\tan^{-1}\frac{u}{a} \qquad 3\tfrac{1}{2}$$

At this time 2nd particle is at rest.

Subst. into (ii) with $v = V$ $\quad 1\tfrac{1}{2}$

$$\Rightarrow \frac{1}{R}\left[\frac{1}{V} - \frac{1}{u}\right] = \frac{1}{Ra}\tan^{-1}\frac{u}{a} \qquad 4$$

$$\therefore \frac{1}{V} = \frac{1}{u} + \frac{1}{a}\tan^{-1}\frac{u}{a}$$

where V = vel. of 1st particle when the second is momentarily at rest