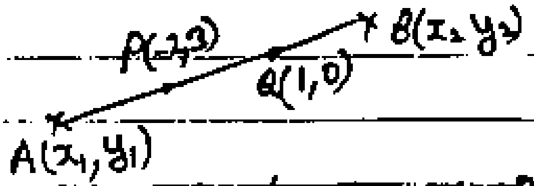
TKS THSC 2004 MATHS EXT 1

1 (a) $\sec^2 x \cdot e^{\tan x}$

(b)




A: $-2 = \frac{x_1+1}{2}$ ; $3 = \frac{y_1+0}{2}$

$x_1 = -5$ ; $y_1 = 6$

$\underline{A(-5,6)}$

B: $1 = \frac{-2+x_2}{2}$ $\quad 0 = \frac{3+y_2}{2}$

$x_2 = 4 \qquad y_2 = -3$

$\underline{B(4,-3)}$

(c) $l_1 : m_1 = 1$

$l_2 : m_2 = -2$

$\tan\theta = \left| \frac{1+2}{1+1\times -2} \right|$

$= 3$

$\theta = 72^\circ$

(d) $u = 1 - x \qquad x = -1, u = 2$

$\frac{du}{dx} = -1 \qquad x = 0, u = 1$

$I = 3\int_2^1 \frac{1-u}{\sqrt{u}} \cdot -du$

$= 3\int_2^1 \frac{u-1}{u^{\frac{1}{2}}}\, du$

$= 3\int_2^1 u^{\frac{1}{2}} - u^{-\frac{1}{2}}\, du$

$= 3\left[\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right]_2^1$

$= 3\left[\frac{2}{3} - 2 - \left(\frac{2}{3}\times\sqrt{8} - 2\sqrt{2}\right)\right]$

$= 3\left[-\frac{4}{3} - \left(\frac{4}{3}\sqrt{2} - 2\sqrt{2}\right)\right]$

$= -4 - 4\sqrt{2} + 6\sqrt{2}$

$= \underline{2\sqrt{2} - 4}$

(e) $T_{n+1} - T_n = 7$

$T_1 = 3$

$T_2 - T_1 = 7$

$\therefore T_2 = 10$

$T_3 - T_2 = 7$

$T_3 = 17$

$T_4 - T_3 = 7$

$T_4 = 24$

$S_n = 3 + 10 + 17 + \cdots + T_n$

AS $a = 3$, $d = 7$

$S_{100} = \frac{100}{2}[2\times 3 + 99$

$= 50(6 + 693)$

$= 699 \times 50$

$= 34\,950$

Q2(a) $\frac{x^2-2}{x} < 1$

c. Values: $x = 0$

$x^2 - 2 = x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $-1$

$-1 \quad 0 \quad 2$

test $x = -\frac{1}{2}$: $\frac{\frac{1}{4}-2}{-\frac{1}{2}} = \frac{-1\frac{3}{4}}{-\frac{1}{2}} = 3\frac{1}{2}$ false

$x = 1$ : $\frac{1-2}{1} < 1$ true

$\therefore \quad x < -1$ or $0 < x < 2$

(b) (i) $\int \frac{e^{2x}}{1+e^{2x}}\,dx$

$= \frac{1}{2}\int \frac{2e^{2x}}{1+e^{2x}}\,dx$

$= \frac{1}{2}\ln(1+e^{2x}) + C$

(ii) $\int \frac{3}{5+x^2}\,dx$

$= 3\int \frac{1}{5+x^2}\,dx$

$= \frac{3}{\sqrt{5}}\tan^{-1}\frac{x}{\sqrt{5}} + C$

(c) $2\ln(3x+1) - \ln(x+1) = \ln(7x+4)$

$\therefore \frac{(3x+1)^2}{x+1} = 7x+4$

$9x^2 + 6x + 1 = 7x^2 + 11x + 4$

$2x^2 - 5x - 3 = 0$

$(2x+1)(x-3) = 0$

$\therefore x = -\frac{1}{2}$ or $3$

but $3x+1 > 0$ & $x+1 > 0$ & $7x+4 > 0$

$x > -\frac{1}{3}$ & $x > -1$ & $x > -\frac{4}{7}$

$\therefore \quad x > -\frac{1}{3}$

$\therefore$ Solution $x = 3$

(d) $2\tan^3\theta - 3\tan^2\theta - 2\tan\theta + 3 = 0$

$\tan^2\theta(2\tan\theta - 3) - 1(2\tan\theta - 3) = 0$

$\therefore (2\tan\theta - 3)(\tan^2\theta - 1) = 0$

$\therefore \tan\theta = \frac{3}{2}$ or $\pm 1$

$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ,$
$56^\circ 19', 236^\circ 19'.$

Q3 (a) $y = e^x + x - 2$

$\frac{dy}{dx} = e^x + 1$

let $x_1 = 0.5$

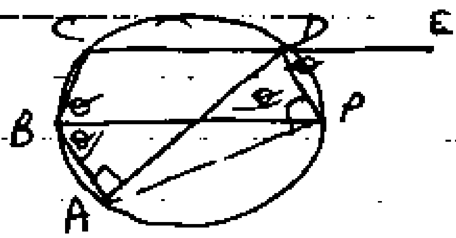
$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.5 - \frac{e^{0.5} + 0.5 - 2}{e^{0.5} + 1}$

$= 0.5 - \frac{0.148\ldots}{2.648\ldots}$

$= 0.44$

(b)




let $\angle ABP = \angle PBC = \theta$

(ii) $\angle ABP = \angle ADP = \theta$ (angles in same segment)

(iii) $\angle EDP = \angle PBC = \theta$ (ext. angle of cyclic quad = int. opp)

but $\angle ADP = \theta$ (from (ii))

$\therefore$ PD bisects $\angle ADE$

(iv) BP and AD are both diameters (angles in semi circle = $90°$)

$\therefore$ centre of circle is the point of intersection of BP & AD.

(c) (i) $\cos x - \sqrt{3}\sin x \equiv A\cos(x \pm \alpha)$

$\equiv A[\cos x.\cos\alpha - \sin x$

$A\cos\alpha = 1, \; A\sin\alpha = \sqrt{3}$

$A = \sqrt{1^2 + (\sqrt{3})^2} = 2$

$\sin\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{3}$

$\therefore \cos x - \sqrt{3}\sin x = 2\cos(x +$

$\therefore 2\cos(x + \frac{\pi}{3}) = 1$

$\cos(x + \frac{\pi}{3}) = \frac{1}{2}$

$x + \frac{\pi}{3} = 2n\pi \pm \cos^{-1}(\frac{1}{2}$

$= 2n\pi \pm \frac{\pi}{3}$

$\therefore x = 2n\pi$ or $2n\pi -$

Q4 (a)(i) $P(x) = a(x+3)^2(x-3)^2$

$P(-2) = -50$

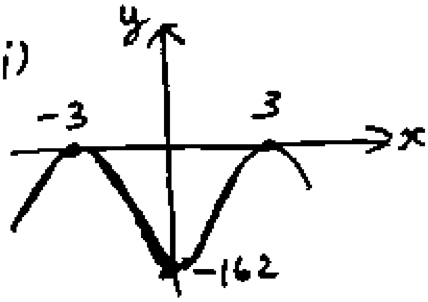
$\therefore -50 = a(1)^2(-5)^2$

$-50 = 25a$

$a = -2$

$P(x) = -2(x+3)^2(x-3)^2$

(ii)




(b)(i) $v^2 = 6+4x-2x^2$

$\frac{d^2x}{dt^2} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dx}(3+2x-x^2)$

$= 2-2x$

$= -2(x-1)$

(ii) $\ddot{x} = -n^2x$

$n = \sqrt{2}$

$\therefore$ Period $= \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$ secs.

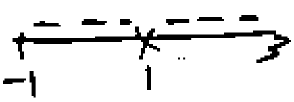
(iii) $v = 0$

$\therefore 2x^2-4x-6=0$

$x^2-2x-3=0$

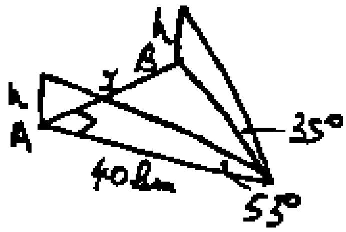
$(x-3)(x+1)=0$

$\therefore x=-1$ or $3$




Amplitude = 2 cm.

(c)




$\tan 53^\circ = \frac{h}{40}$

$h = 40\tan 53^\circ$

$\tan 35^\circ = \frac{h}{BC}$

$BC = \frac{h}{\tan 35^\circ}$

$\therefore BC = \frac{40\tan 53^\circ}{\tan 35^\circ}$

$BC^2 = x^2+40^2$

$= \left(\frac{40\tan 53^\circ}{\tan 35^\circ}\right)^2 + 40$

$= 4146.94..$

$x = 63.396..$

$= 64$ m.

Q5 (a) $y = 3\cos^{-1}\frac{x}{2}$

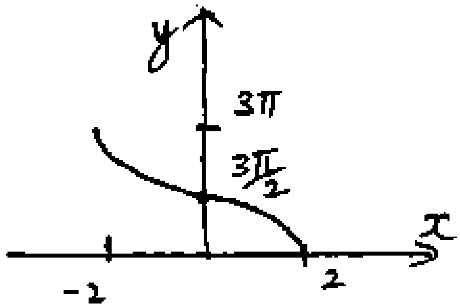
(i) $D: -1 \le \frac{x}{2} \le 1$

$-2 \le x \le 2$

$R: 0\times 3 \le y \le 3\times\pi$

$0 \le y \le 3\pi$

(ii)




(iii) $y = 3\cos^{-1}\frac{x}{2}$

$y' = \frac{-3}{\sqrt{1-\frac{x^2}{4}}} \times \frac{1}{2}$

at $x=0$, $y' = \frac{-3}{2\sqrt{1}} = -\frac{3}{2}$

$y = \frac{3\pi}{2}$

eqn of tangent is

$y - \frac{3\pi}{2} = -\frac{3}{2}(x-0)$

$2y - 3\pi = -3x$

$3x + 2y - 3\pi = 0$

(b) (i) $A = \frac{1}{2}\times1\times1\times\theta - \frac{1}{2}\times1\times1\times\sin\theta$

$= \frac{1}{2}(\theta - \sin\theta)$

$P = 1\times\theta + 2\times\sin\frac{\theta}{2}$

(ii) $R = \frac{dP}{dt}$

(α) $\frac{d\theta}{dt} = ?$

$P = \theta + 2\sin\frac{\theta}{2}$

$\frac{dP}{dt} = 1 + \cos\frac{\theta}{2}$

when $\theta = \frac{2\pi}{3}$, $\frac{dP}{dt} = 1 + \cos$

$=$

$\frac{d\theta}{dt} = \frac{d\theta}{dP} \times \frac{dP}{dt}$

$= \frac{2}{3} \times R$

$= \frac{2R}{3}$ radians/s

(β) $\frac{dA}{dt} = ?$

$A = \frac{1}{2}(\theta - \sin\theta)$

$\frac{dA}{d\theta} = \frac{1}{2}(1 - \cos\theta)$

when $\theta = \frac{2\pi}{3}$ $\frac{dA}{d\theta} = \frac{1}{2}(1-c$

$= \frac{1}{2} \times 1$

$=$

$\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt}$

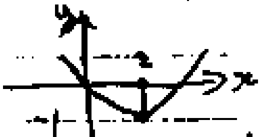
$= \frac{3}{4} \times \frac{2R}{3}$

$= \frac{R}{2}$ m²/se

Q6 (a) $x^2 = 4x + 4y$

$x^2 - 4x + 4 = 4y + 4$

$(x-2)^2 = 4(y+1)$




Focus $(2, 0)$

Directrix $y = -2$

(b) When $n=1$, $3^{3n} + 2^{n+2} = 3^3 + 2^3 = 35$

which is divisible by 5

assume true for $n = k$

i.e. $\frac{3^{3k} + 2^{k+2}}{5} = c$ (an integer)

$\therefore 3^{3k} = 5c - 2^{k+2}$

Prove true for $n = k+1$

i.e. $\frac{3^{3k+3} + 2^{k+3}}{5} = c_1$ (an integer)

$LHS = \frac{3^{3k} . 3^3 + 2^{k+3}}{5}$

$= \frac{(5c - 2^{k+2}).27 + 2^{k+2}.2^1}{5}$

$= \frac{5 \times 27c - 27 \times 2^{k+2} + 2 \times 2^{k+2}}{5}$

$= \frac{5[27c - 5.2^{k+2}]}{5}$

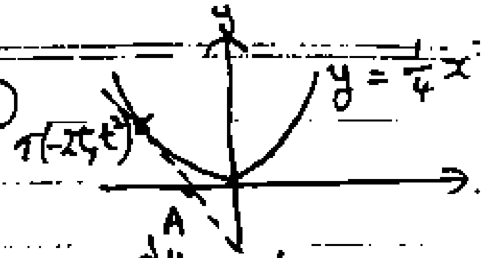
which is an integer if c is an integer

$\therefore$ if it is true for $n = k$, it is true for $n = k+1$

since it is true for $n = 1$, it is true $n = 2$

and so on for all $n$.

(c)




(i) $\frac{dy}{dx} = \frac{1}{2}x$

at T $\frac{dy}{dx} = -t$

eq'n of tangent at T

$y - t^2 = -t(x + 2t)$

$y - t^2 = -tx - 2t^2$

$y + tx + t^2 = 0$

(ii) $A(-t, 0)$

(iii) $M\left(-\frac{3t}{2}, \frac{t^2}{2}\right)$

(iv) $x = \frac{-3t}{2}$, $y =$

$t = \frac{-2x}{3}$

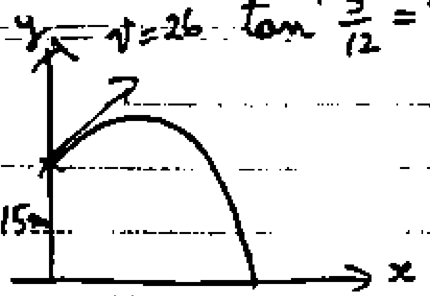
$\therefore y = \frac{\left(\frac{-2x}{3}\right)^2}{2}$

$= \frac{4x^2}{18}$

$y = \frac{2}{9}x^2$

Q7 (a)

$v = 26 \quad \tan^{-1}\frac{5}{12} = \theta$





$t = 0,\ x = 0,\ y = 15,\ \dot{x} = 26 \times \frac{12}{13} = 24;\ \dot{y} = 26 \times \frac{5}{13} = 10$

$\ddot{y} = -10$

$\dot{y} = -10t + c_1$

$t = 0,\ \dot{y} = 10,\ c_1 = 10$

$\dot{y} = -10t + 10$

$y = -5t^2 + 10t + c_2$

$t = 0,\ y = 15,\ c_2 = 15$

$y = -5t^2 + 10t + 15$

$\ddot{x} = 0$

$\dot{x} = c_3 = 24$

$x = 24t + c_4$

$t = 0,\ x = 0\ \ c_4 = 0$

$x = 24t$

(i) greatest height $\dot{y} = 0,\ t = 1$

$y = -5 + 10 + 15$

$= 20m$

(ii) time of flight $y = 0$

$t^2 - 2t - 3 = 0$

$(t+1)(t-3) = 0$

$\therefore\ t = -1$ or $3$

Time of flight = 3 secs

(iii) when $t = 3$ $\quad x = 24 \times 3$

$= 72m$

(iv) $t = 2 \quad \dot{x} = 24,\ \dot{y} = 10 - 20 = -10$

$v = \sqrt{24^2 + (-10)^2}$

$= 26$ m per sec.

Q7 (b) $x^3 + ax^2 + b = 0$

let roots be $\alpha, \frac{1}{\alpha}$ & $\beta$

$\therefore \alpha + \frac{1}{\alpha} + \beta = -a$ — ①

$\alpha \times \frac{1}{\alpha} + \alpha\beta + \frac{1}{\alpha} \times \beta = 0$ — ②

$\alpha \times \frac{1}{\alpha} \times \beta = -b$ — ③

(i) $\therefore \beta = -b$ from ③

(ii) $\alpha + \frac{1}{\alpha} - b = -a$ from ①

$\alpha + \frac{1}{\alpha} = b - a$

$1 - \alpha b - \frac{b}{\alpha} = 0$ from ②

$1 - b(\alpha + \frac{1}{\alpha}) = 0$

$\therefore 1 - b(b-a) = 0$

$1 = b(b-a)$

$\frac{1}{b} = b - a$

$a = b - \frac{1}{b}$.

(iii) $\alpha + \frac{1}{\alpha} = b - a$

$= b - b + \frac{1}{b}$

$\alpha + \frac{1}{\alpha} = \frac{1}{b}$

$\alpha^2 - \frac{1}{b}\alpha + 1 = 0$

real roots $\Delta \geq 0$

$\therefore \frac{1}{b^2} - 4 \geq 0$

$1 - 4b^2 \geq 0$

$b^2 \leq \frac{1}{4}$

$-\frac{1}{2} \leq b \leq \frac{1}{2}$