Student Name:_____

Saint Ignatius' College
Riverview

2003
YEAR 12
TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Time allowed – 2 hours, + 5 minutes reading time
- Write using blue or black pen
- Board-approved calculators and mathaids may be used
- Show all necessary working
- Answer each question in a separate booklet with your name and teacher's name

Total Marks (84)

- Attempt Questions 1 – 7

Total marks (84)
Attempt Questions 1 – 7
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

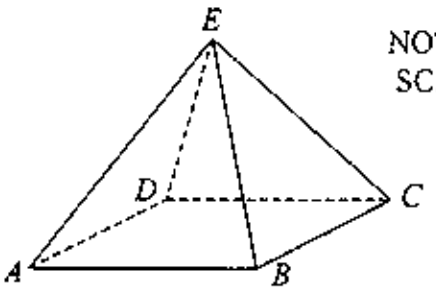
QUESTION 1 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the acute angle between the lines $y = 2x - 5$ and $y = 6 - 3x$. 2

(b) Solve $\frac{x+4}{x} < 3$. 3

(c) Find the general solutions of the equation $\sin 2\theta = \sin^2 \theta$. 4
Give your answer in terms of π.

(d) 3



NOT TO SCALE

A right square pyramid $ABCDE$ has a base of length 6cm and a perpendicular height of 8cm.

Find the angle which the slant edge AE makes with the base $ABCD$.

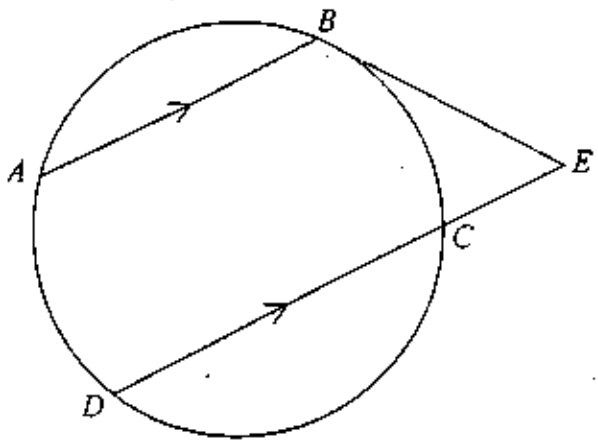
QUESTION 2 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) The point (2, 2) divides the join of (-2, 5) to (a, b) in the ratio 3:2. **2**
Find the values of a and b.

(b) If α, β, γ are the roots of the equation $2x^3 - 6x^2 + 5x - 1 = 0$, **3**
find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.

(c) The polynomial equation $x^3 - 11x^2 + px + q = 0$ has a double root **4**
at $x = \alpha$ and a single root at $x = \alpha + 2$.
Using the formula for the sum of the roots, or otherwise, find the values of α, p and q.

(d) **3**



In the diagram, A, B, C and D lie on a circle.
AB is parallel to DC and the tangent at B meets DC produced at E.

Copy or trace the diagram onto your writing page, and join BC and AC.

Prove that ΔABC is similar to ΔBCE.

QUESTION 3 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the inverse function of the function $f(x) = \dfrac{5-2x}{3}$, expressing your answer in the form $f^{-1}(x) = ...$ **2**

(b) Evaluate $\cos^{-1}\left(\dfrac{1}{2}\tan\dfrac{2\pi}{3}\right)$. **2**

(c) Find the exact value of $\sin\left(2\cos^{-1}\dfrac{2}{3}\right)$. **3**

(d) Prove $\dfrac{\cos A - \sin A}{\cos A + \sin A} = \dfrac{\cos 2A}{1+\sin 2A}$. **2**

(e) Show that there is only one stationary point on the curve $y = x + \cos^{-1} x$, and determine its nature. **3**

QUESTION 4 (12 Marks) Use a SEPARATE writing booklet. Marks

(a) From a standard pack of 52 cards, a hand of 4 cards is dealt.

(i) How many different hands can be selected? 1

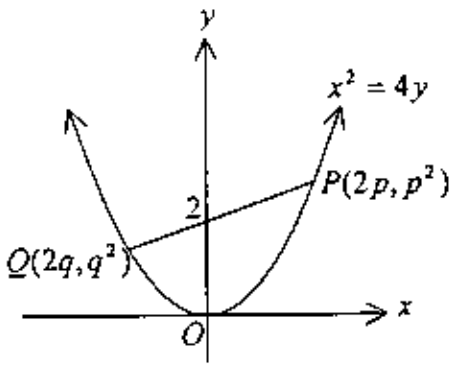
(ii) What is the probability I will be dealt exactly two aces? 2

(b) The letters of the word C A L C U L U S are arranged in a row.

(i) How many different arrangements are possible? 2

(ii) In how many of the arrangements will the letters U be at each end? 1

(c)



Points $P(2p, p^2)$ and $Q(2q, q^2)$ lie on the parabola $x^2 = 4y$.

(i) Show that the equation of the chord PQ is $(p+q)x - 2y - 2pq = 0$. 2

(ii) Find the coordinates of M, the midpoint of PQ. 1

(iii) Hence find the equation of the locus of M if the chord PQ crosses the y axis at (0, 2). 3

QUESTION 5 (12 Marks) Use a SEPARATE writing booklet. Marks

(a) Find the following indefinite integrals:

(i) $\int \frac{1}{4+x^2}\,dx$ 1

(ii) $\int \frac{x}{4+x^2}\,dx$ 1

(b) Find $\int_0^{\frac{\pi}{6}} \sin^2 x\,dx$. 3

(c) Find $\int \frac{e^{2x}}{e^x-2}\,dx$ using the substitution $u = e^x - 2$. 3

(d) The acceleration of a particle moving in a straight line at position x is given by $\ddot{x} = -\frac{6}{(x+1)^2}$. Initially it has velocity 4 units when it is at the origin. 4

Show that the velocity v at position x is given by $v = \pm 2\sqrt{\frac{x+4}{x+1}}$.

QUESTION 6 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Use the table of standard integrals to find $\int \frac{1}{\sqrt{x^2+16}}\,dx$. **1**

(b) Prove by mathematical induction, for positive integers n, that **4**

$$\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\ldots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}.$$

(c) Use one application of Newton's method to find an approximation to the root of $2x-4\sin 3x=0$ near $x=1$. **3**
Write your answer to two decimal places.

(d) (i) On the same set of axes, sketch the graphs of the equations $y=|2x|$ and $y=x^2-3$. **2**

(ii) Hence or otherwise solve the inequality **2**

$$|2x|>x^2-3.$$

QUESTION 7 (12 Marks) Use a SEPARATE writing booklet. Marks

(a) An object is projected from level ground at an angle θ to the horizontal, with a velocity of V m/s. The object returns to the ground after 4 seconds and 100 metres from its point of projection.
Assume acceleration due to gravity is 10 m/s², and neglect air resistance.

(i) From the equations for acceleration in the x and y directions, find expressions for x and y in terms of time t ($t \leq 4$). 2

(ii) Hence find the values of V and θ. 2

(iii) What is the maximum height reached by the object? 2

(b) Newton's Law of Cooling states that the rate at which a body cools is proportional to the difference between the temperature of the body and that of the surrounding medium.

$$\text{i.e. } \frac{dT}{dt} = -k(T - T_0)$$

where T is the temperature of the body at time t and T_0 is the temperature of the surrounding medium, assumed constant.

(i) Show that $T = T_0 + Ae^{-kt}$ is a solution to this equation. 1

(ii) A body whose temperature is 150°C is cooled by placing it in a liquid at 25°C. In one minute, the temperature of the body had cooled to 100°C. 5
How long will it take for the body to cool to 50°C?

End of paper