Student Number: .....

2003

HIGHER SCHOOL CERTIFICATE
Sample Examination Paper

MATHEMATICS
Extension 1

General Instructions
Reading time - 5 minutes
Working time - 2 hours

- Attempt ALL questions
- Show all necessary working, marks may be deducted for careless or untidy work
- Standard integrals are printed on the last page
- Board-approved calculators may be used
- Additional Answer Booklets are available

Directions to School or College

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Question 3

(a) Use the method of mathematical induction to prove that $2(n-3)+(n-4)+\cdots+3+2+1=\binom{n}{2}-n$ for $n \geq 4$. 4

(b) Find the roots of the following equation $4x^3-4x^2-29x+15=0$ given that one root is the difference between the other two roots. 3

(c) The tangent to the point $P(2ap, ap^2)$ on the parabola $x^2=4ay$ cuts the x-axis in A and the y-axis in B.

(i) Find the coordinates of M, the midpoint of A and B in terms of P. 2

(ii) Show that the locus of M is a parabola. 2

(iii) Find the coordinates of the focus of this parabola and the equation of its directrix. 1

Question 4

(a) (i) How many 11 letter 'words' can be formed from the letters of the word 'PROBABILITY'? 1

(ii) In how many of these does the word BABY appear? 1

(b) A surveyor observes two towers, one due north of height 80m, and the other on a bearing of θ° ($< 90^\circ$) of height 120m. The angles of elevation of the two towers are 40° and 36° respectively. If the towers are 150m apart on a horizontal plane, calculate the value of θ to the nearest minute. 6

(c) By considering the expansion of $x(1+x)^n$ or otherwise, show that 4

$$\sum_{r=0}^{n}(r+1)^nC_r = 2^n\left(\tfrac{n}{2}+1\right)$$

Question 5

(a) A golf ball is to be struck so as to clear a tree 20m away and 6m high on level ground. If the selected club produces an angle of elevation of 40°, (take $g = 10\text{m/s}^2$)

(i) Write down an expression for y, the vertical distance travelled. 1

(ii) Write down an expression for x, the horizontal distance travelled. 1

(iii) Hence derive the cartesian equation of the flight path. 1

(iv) Calculate the speed at which the ball must leave the ground in order to just clear the obstacle. 2

(b) Given $3x^2 - 5x = -\frac{k}{4}$ calculate value(s) of k if

(i) the real roots are real 1

(ii) the roots are rational and k is a positive integer. 2

(c) To promote the sale of Studebaker cars, a dealer offers a special deal in which no interest is charged for the first 3 months and then interest rates are left at 1% per month. Lam Lai buys a 6-cylinder car for \$30 000, pays \$10 000 in cash and agrees to pay the loan plus interest monthly over 3 years. After 20 months, he wins \$10 150 as part of a lotto syndicate. Show that this win is just sufficient to pay off the loan at that time. 4

Question 6

(a) Tap water at 24°C is placed in a fridge-freezer maintained at a temperature of −11°C. After t minutes the rate of change of temperature T of the water is given by $\frac{dT}{dt} = -k(T+11)$

(i) Show that $T = Ae^{-kt} - 11$ is a solution of the above equation, where A is a constant. 1

(ii) Find the value of A 1

(iii) After 15 minutes the temperature of the water falls to 10°C. Find to the nearest minute the time taken for the water to start freezing. (Freezing point of water = 0°C) 2

(b) Evaluate $\int_{\frac{1}{3}}^{\frac{1}{\sqrt{3}}} \frac{dx}{1+9x^2}$ 3

(c) In the game of craps 2 dice are thrown and the sum of the dice is noted. The most likely outcome is a total of 7. If two dice are rolled 20 times,

(i) What is the most probable number of sevens thrown? 3

(ii) Calculate the probability that this number of sevens does indeed occur. 2

Question 7

(a) A particle moves in a straight line and its position at any time is given by $x = 4.8\cos 2t + 5.5\sin 2t$. Show that the motion is simple harmonic and calculate its greatest speed. 3

(b) Evaluate $\sin\left[\cos^{-1}\frac{2}{3} + \tan^{-1}\left(-\frac{3}{4}\right)\right]$ giving its exact value. 3

(c) Consider the function $y = x\sec x$.

(i) Find $\frac{dy}{dx}$ 1

(ii) By drawing two graphs, show that the function has one stationary point in the domain $\frac{\pi}{2} < x < \frac{3\pi}{2}$. 2

(iii) Prove that the stationary point lies between $x = 2.5$ and $x = 3.0$. 1

(iv) Use halving the interval method twice to find a closer approximation of the stationary point. 2