2010

TRIAL
HIGHER SCHOOL CERTIFICATE

GIRRAWEEN HIGH SCHOOL

Mathematics Extension 1

General Instructions:

- Reading Time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen.
- Board - approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown in every question.

Total marks – 84

- Attempt Questions 1 – 7
- All questions are of equal value

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Attempt Questions 1 –7

All questions are of equal value.

Question 1 (12 marks). Start on a SEPARATE page.

Marks

(a) The line $y = mx$ makes an angle of $45°$ with the line $y = 2x - 3$. Find the possible values of m. 2

(b) Find the coordinates of the point $P(x, y)$ which divides the interval joining $A(-4,-6)$ and $B(6,-1)$ externally in the ratio 3:2. 2

(c) Solve for x: $\dfrac{2x+1}{x-1} \geq 3$ 2

(d) Differentiate $y = x\tan^{-1}\dfrac{x}{2}$ 3

(e) Use the substitution $u = \sqrt{x}$ to evaluate $\displaystyle\int_1^4 \frac{dx}{x+\sqrt{x}}$ 3

Question 2 (12 marks). Start on a SEPARATE page.

(a) Find the coefficient of x^9 in the expansion of $\left(x^2+\frac{2}{x}\right)^{12}$ 2

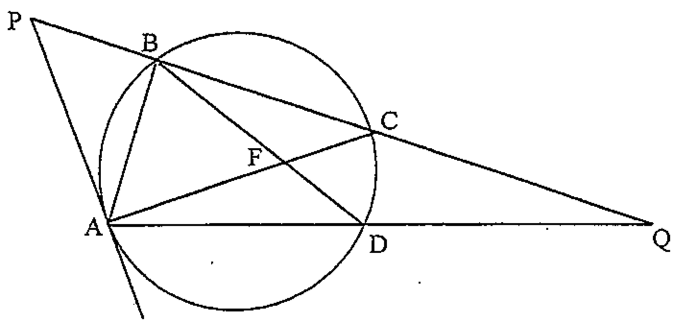
(b) Evaluate: $\lim\limits_{x \to 0} \frac{\sin 6x}{7x}$ 2

(c) If $f(x)=4\cos^{-1}\frac{x}{3}$, find

(i) the domain and range of $f(x)$. 2

(ii) Sketch the curve. 2

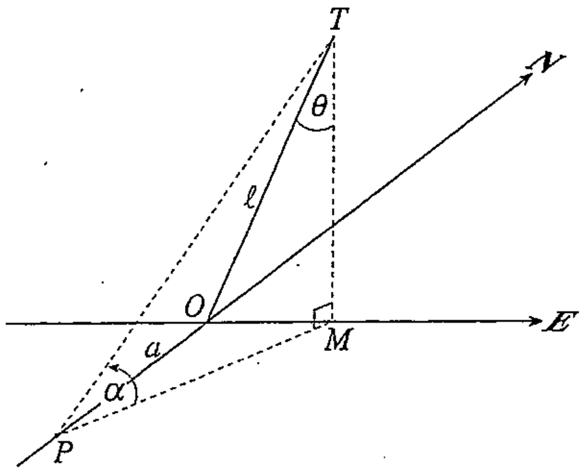
(d)



In the above figure, AP is a tangent to the circle at A. $PBCQ$ and ADQ are straight lines. Prove that $\angle PAB=\frac{1}{2}(\angle CFD+\angle CQD)$ 4

Question 3 (12 marks) Start on a SEPARATE page.

(a) A pole, OT, of length l metres stands on horizontal ground. The pole leans towards the east, making an angle of θ with the vertical. From P, a metres south of O, the elevation of T is α.



(i) Copy the diagram above onto your booklet.Find expressions, in terms of l and θ for OM and MT. 2

(ii) Prove that $PM = l\cos\theta\cot\alpha$. 1

(iii) Prove that $l^2 = \dfrac{a^2}{\cos^2\theta\cot^2\alpha - \sin^2\theta}$ 3

(iv) Find the length of the pole, to the nearest metre, if $a = 25$, $\theta = 20°$ and $\alpha = 24°$. 1

(b) A tea party is arranged for 16 people along two sides of a long table with 8 chairs on each side. Four persons wish to sit on one particular side and two on the other side. In how many ways can they be seated? 2

(c) In an election 40% of the voters favoured Party A. If an interviewer selected 5 voters at random, what is the probability that

(i) exactly three of them favoured Party A.

(ii) A majority of those selected favoured Party A

(iii) At most two favoured Party A. 3

Question 4 (12 marks). Start on a SEPARATE page.

(a) Prove the following by the Principle of mathematical induction.

$\log 2 + \log\left(\frac{3}{2}\right) + \log\left(\frac{4}{3}\right) + \ldots \quad + \log\left(\frac{n}{n-1}\right) = \log n$ for all integers $n \geq 2$. 3

(b) $P(2ap, ap^2)$ is a point on the parabola $x^2 = 16y$. The equation of the normal at P is given by $x + py = 4p^3 + 8p$.

(i) Find the point of intersection R of the normals at P and Q, the end points of focal chord PQ. 2

(ii) Find the locus of R. 2

(c) For the function $y = \dfrac{2x^2 - 2}{x^2 - 9}$

(i) Write down the equations of horizontal and vertical asymptotes. 2

(ii) Sketch the curve showing intercepts with axes and asymptotes. 3

Question 5 (12 marks)

(a) By expanding both sides of the identity $(1+x)^5(1+x)^5 = (1+x)^{10}$, show

that $\sum_{k=0}^{5} ({}^5C_k)^2 = {}^{10}C_5$ 3

(b) (i) Write the expansion of $(1+x)^n$. 1

(ii) By integrating, show that

$${}^nC_0 + \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 + \ldots\ldots \quad + \frac{1}{n+1}\,{}^nC_n = \frac{2^{n+1}-1}{n+1}$$ 3

(c) The rate at which an object warms in air is proportional to the difference between its temperature T and the constant temperature A of the surrounding air. This rate can be expressed by the differential equation

$$\frac{dT}{dt} = k(T - A),$$

Where t is the time in minutes, T and A are measured in degrees centigrade, and k is aconstant.

(i) Show that $T = A + Ce^{kt}$, where C is a constant is a solution of the differential equation. 2

(ii) An object warms from $10°C$ to $15°C$ in 20 minutes. The air temperature surrounding the object is $25°C$. Determine the temperature of the object after a further 30 minutes have passed. Give your answer to the nearest degree. 2

(iii) Using the equation for T, given in part (i), explain the behaviour of T as t increases to large values. 1

Question 6 (12 marks). Start on a SEPARATE page.

(a) (i) Given $f(x) = x\sin^{-1}x + \sqrt{1-x^2}$. Find $f'(x)$. 3

(ii) Hence evaluate $\int_0^{\frac{1}{2}} \sin^{-1}x \; dx$ 2

(b) (i) show that there exists a root of the equation $\tan x - x = 0$ between $x = 4$ and $x = 4.5$. 1

(ii) By halving the interval twice find an approximate value of the root Correct to 1 decimal place. 2

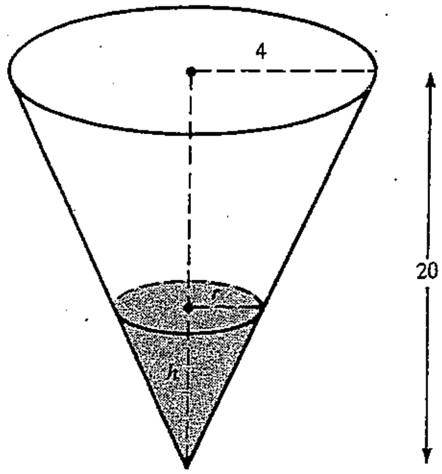
(c) Assume tides at a harbour rise and fall in SHM. At low tide the harbour is 12 m deep, and at high tide 17 m deep. Low tide is at 9-00 am and high tide at 3.00 pm.Assuming a ship needs 14 m to go safely,

(i) at what time can the ship go into the harbour. 3

(ii) if the ship take 30 minutes to go out, before what time must it depart the harbour. 1

Question 7 (12 marks). Start on a SEPARATE page.

(a)



A small funnel in the shape of a cone is being emptied of fluid at the rate of $12\,cm^3/s$. The height of the funnel is $20\ cm$ and the radius of the top is $4\ cm$. How fast is the fluid level dropping when the level stands $5\ cm$ above the vertex of the cone? 3

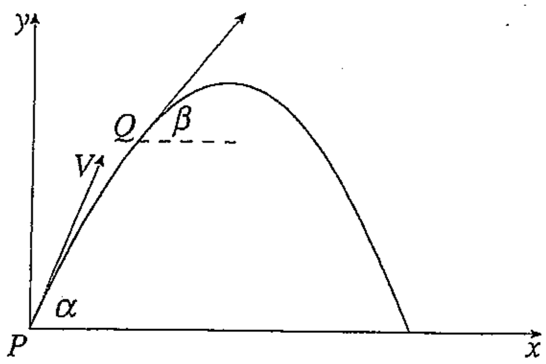
(b) Given that $x^3 + x^2 - 10 = 0$ has a root between 1 and 2. By taking 2 as the initial value find an approximation to the root using Newton's method, correct to one decimal place. 2

(c) A particle is projected from a point P on horizontal ground, with initial speed $V\ m/s$ at an angle of elevation α to the horizontal. It's equations of motion are $x = 0$ and $y = -g$. The horizontal and vertical component of velocity and displacement of the particle at any time t are given by

$$\frac{dx}{dt} = V\cos\alpha \quad \text{and} \quad \frac{dy}{dt} = V\sin\alpha - gt$$

$$x = Vt\cos\alpha \quad \text{and} \quad y = Vt\sin\alpha - \frac{1}{2}gt^2 \text{ (do not prove these)}$$

(i) Determine the time of flight of the particle. 2



(ii) The particle reaches a point Q, as shown, where the direction of the flight makes an angle β with the horizontal. Find an expression for $\tan\beta$. 1

(iii) Hence show that the time taken to travel from P to Q is

$\dfrac{V\sin(\alpha-\beta)}{g\cos\beta}$ seconds. 2

(iv) Consider the case where $\beta = \dfrac{\alpha}{2}$. If the time taken to travel from P to Q is one third of the total time of flight, find the value of α. 2

End of paper