ST. MARK'S COPTIC ORTHODOX COLLEGE

END OF SEMESTER ONE EXAMINATIONS
PRELIMINARY
2008

Mathematics Extension 1

EXAMINER : MR. WAGDY MICHEAL

General Instructions

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks **(84)**

- Attempt Questions 1-7
- All questions are of equal value

Total Marks – 84
Attempt Questions 1-7
All Questions are of equal value

Begin each question on a NEW SHEET of paper, writing your name and question number at the top of the page. Extra paper is available.

QUESTION 1 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) Divide the interval A (-2, 7) B (12, 0) internally in the ratio 4:3 **2**

b) Simplify $\frac{3ab^2}{5xy} \div \frac{12ab-6a}{x^2y+2xy^2}$ **2**

c) Factorise $27x^6 + \frac{1}{8}$ **2**

d) Solve by completing the square $x^2 + 2x - 7 = 0$, leaving your answer in simplest exact form. **2**

e) Find the horizontal asymptote of the function $y = \frac{2x^2 - 4x + 3}{x^2 - 5}$ **2**

f) For what values of x is $|6x - 3| \le 5$ **2**

QUESTION 2 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

(a) Solve $\frac{3}{x-2} \le 1$. **3**

(b) (i) Write down, in surd form, the values of

$\sin 45^\circ, \cos 45^\circ, \sin 30^\circ, \cos 30^\circ$ **2**

(ii) Hence, show that $\cos 75 = \frac{1}{4}\left(\sqrt{6} - \sqrt{2}\right)$ **(FULL WORKING OUT MUST BE SHOWN, USE OF CALCULATOR WILL RESULT IN A ZERO MARK)** **2**

(c) Micheal who is W metres south of a tower sees the top of it with an angle of elevation of 20°. Gerges is T metres east of the tower. From his position the angle of elevation is 24° to the top of the tower.

The two men are 1400m apart.

(i) Show that $W = h \cot 20^\circ$ **1**

(ii) Find the height of the tower to the nearest metre. **4**

QUESTION 3 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

(a) Show that $\dfrac{\sec^2 x}{\tan x} = \dfrac{1}{\sin x \cos x}$ **2**

(b) (i) Two lines with gradients m_1 and m_2 intersect on the Cartesian Plane. **1**

If the acute angle between the lines is θ, write the formula for $\tan\theta$.

(ii) If $m_1 = 3$, express the exact value(s) of m_2 in the form $a \pm \dfrac{b\sqrt{c}}{d}$

if $\theta = 30^0$. **2**

(c) (i) Express $\cos\theta - 2\sin\theta$ in the form

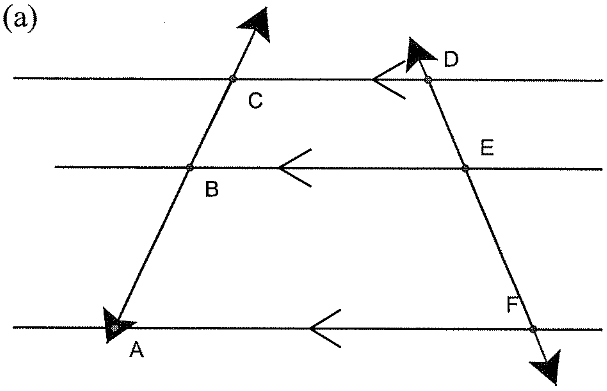
$A\cos(\theta+\alpha)$, $A > 0$, $0 < \alpha < 90^\circ$ **2**

(ii) Hence, solve the equation $\cos\theta - 2\sin\theta = 1$, $0 \le \theta \le 360^\circ$ **2**

(d) Given that $a\sqrt{b} - c = \sqrt{24 - 16\sqrt{2}}$, find the integers a, b and c. **3**

QUESTION 4 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

(a)



The diagram shows 3 parallel lines; CD || BE || AF and 2 transversals AC and DF

(i) Copy the diagram onto your answers and include the parallel to AC through E. **1**

(ii) Prove $\frac{BC}{BA} = \frac{DE}{EF}$ **2**

(You may assume that the opposite sides of a parallelogram are equal)

(b) A (1, -1) B (-3, 1) C (-3, 4) and D (3, 1) are points on the Cartesian Plane. AB||CD

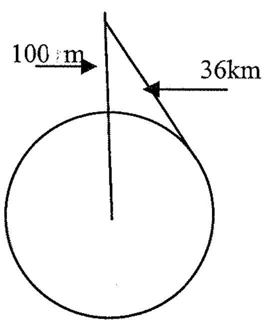
(i) Find the distances AB and DC **2**

(ii) Show that the equation of CD is $x + 2y - 5 = 0$ **2**

(iii) Find the perpendicular distance of A from CD **1**

(iv) Hence or otherwise obtain the area of the trapezium ABCD **1**

(c) From a cliff 100 metres high, the straight line distance to the horizon is 36 kilometres. Calculate the radius of the earth. **3**



QUESTION 5 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

(a) (i) On the same diagram sketch the graphs of $y = \sin x$ and $y = 2\sin x + 1$, where, $0 \le x \le 360^\circ$ **3**

(ii) Hence, or otherwise, solve $2\sin x + 1 \ge \sin x$, where $0 \le x \le 360^\circ$ **2**

(b) (i) Prove $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$ **2**

(ii) Hence, find the exact value of $\tan 15^\circ$ **2**

(c) Boat A sails 15km from port P on a bearing of 055^0. Boat B sails from P for 25 km on a bearing of 135^0.

(i) Show the angle APB = 80^0 **1**

(ii) Calculate their distance apart to 1 decimal place. **2**

QUESTION 6 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

(a) Show that $\dfrac{1-\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} = 2\tan\dfrac{\theta}{2}$ **2**

(b) An Isosceles triangle has base angles of θ° and a base of 12cm. If $\tan\theta = \dfrac{2\sqrt{3}}{3}$, find the exact area of the triangle. **3**

(c) (i) If $t = \tan\dfrac{x}{2}$, show that $3\cos x + 4\sin x + 5 = \dfrac{2t^2 + 8t + 8}{1+t^2}$. **2**

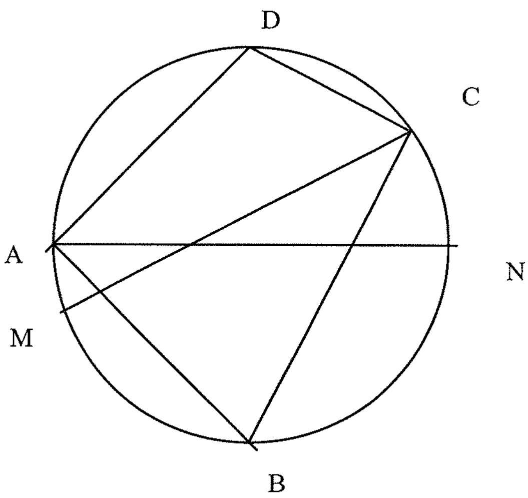
(ii) Hence, solve the equation $3\cos x + 4\sin x + 5 = 0$ for $0 \le x \le 360^\circ$ **2**

(d) Solve $|x-2| + |x+2| > 6$ and graph your solution on a number line. **3**

Question 7 (12 marks) Begin a NEW sheet of writing paper. **Marks**

(a) Find all real solutions for the equation: $\sin 2x = 2\cos^2 x, 0 < x < 360^\circ$. **4**

(b)



In the diagram above , $ABCD$ is a cyclic quadrilateral. M and N are points on the circle through A, B, C and D, such that CM bisects $\angle BCD$ and AN bisects $\angle DAB$.

(i)Copy the diagram. **1**

(ii) Show that MN is a diameter of the circle. **3**

(iii) **Using the fact that** MN **is a diameter**, prove $\angle NMB = 90^\circ - \angle BCM$. **4**

- End of Paper -

Solutions: EXT. One
Y 11 – Half Yearly Exam 2008

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Question One

a) $x = \dfrac{x_2 m + x_1 n}{m+n}$, $y = \dfrac{y_2 m + y_1 n}{m+n}$

$x = \dfrac{12 \cdot 4 + -2 \cdot 3}{4+3}$, $y = \dfrac{0 \cdot 4 + 7 \cdot 3}{4+3}$

$= 6$ $\qquad = 3$

$\therefore$ (6, 3) divides AB in the ratio 4 : 3

b) $\dfrac{3ab^2}{5xy} \div \dfrac{12ab - 6a}{x^2 y + 2xy^2}$

$= \dfrac{\cancel{3}ab^2}{5\cancel{xy}} \times \dfrac{\cancel{xy}(x+2y)}{2\,\cancel{6}a(2b-1)}$

$= \dfrac{b^2(x+2y)}{10(2b-1)}$

c) $27x^6 + \dfrac{1}{8} = (3x^2)^3 + \left(\dfrac{1}{2}\right)^3$

$= \left[3x^2 + \dfrac{1}{2}\right]\left[9x^4 - \dfrac{3}{2}x^2 + \dfrac{1}{4}\right]$

d) $x^2 + 2x - 7 = 0$

$x^2 + 2x = 7$

$x^2 + 2x + 1 = 7 + 1$

$(x+1)^2 = 8$

$x + 1 = \pm 2\sqrt{2}$

$x = -1 \pm 2\sqrt{2}$

e) $\lim\limits_{x \to \infty} \dfrac{\frac{2x^2}{x^2} - \frac{4x}{x^2} + \frac{3}{x^2}}{\frac{x^2}{x^2} - \frac{5}{x^2}} = \dfrac{2 - 0 + 0}{1 - 0}$

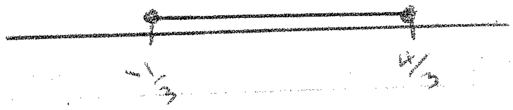
$\therefore$ Horizontal asymptote $y = 2$

f) $|6x - 3| \le 5$

$6x - 3 \le 5$ or $-6x + 3 \le 5$

$6x \le 8$ $\qquad -6x \le 2$

$x \le \frac{4}{3}$ $\qquad x \ge -\frac{1}{3}$



$\therefore -\frac{1}{3} \le x \le \frac{4}{3}$

Question Two

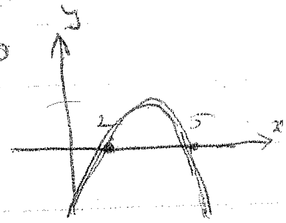
a) $\dfrac{3}{x-2} \times (x-2)^2 \le 1 \times (x-2)^2$

$3(x-2) - (x-2)^2 \le 0$

$(x-2)[3 - (x-2)] \le 0$

$(x-2)(5-x) \le 0$

$x \le 2$, $x \ge 5$



$\therefore x < 2$ and $x \ge 5$ are the solution to $\dfrac{3}{x-2} \le 1$

b) (i) $\sin 45 = \dfrac{1}{\sqrt{2}}$, $\cos 45 = \dfrac{1}{\sqrt{2}}$

$\sin 30 = \dfrac{1}{2}$ $\qquad \cos 30 = \dfrac{\sqrt{3}}{2}$

(ii) $\cos 75^\circ = \cos(30^\circ + 45^\circ)$

$= \cos 30^\circ \cos 45 - \sin 30 \sin 45$

$= \dfrac{\sqrt{3}}{2} \cdot \dfrac{1}{\sqrt{2}} - \dfrac{1}{2} \cdot \dfrac{1}{\sqrt{2}}$

$= \dfrac{\sqrt{3}}{2\sqrt{2}} - \dfrac{1}{2\sqrt{2}}$

$= \dfrac{\sqrt{3} - 1}{2\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{\sqrt{6} - \sqrt{2}}{4}$

$\cos 75^\circ = \dfrac{1}{4}(\sqrt{6} - \sqrt{2})$

$=$ RHS

) In $\triangle$ ABD,

) $\tan 20 = \frac{h}{w}$

$w = \frac{h}{\tan 20}$

$w = h \cot 20$

ii) In $\triangle$ ADC,

$\tan 24 = \frac{h}{T}$

$T = h \cot 24$

In $\triangle$ BCD,

$\angle BDC = 90^\circ$

$\therefore BC^2 = w^2 + T^2$

$1400^2 = h^2 \cot^2 20 + h^2 \cot^2 24$

$1400^2 = h^2(\cot^2 20 + \cot^2 24)$

$h^2 = \frac{1400^2}{\cot^2 20 + \cot^2 24}$

$= \frac{1400^2}{7.54863 + 5.04468}$

$h \doteq 394.51 \doteq 395\text{m}$

Question 3

(a) $\frac{\sec^2 x}{\tan x} = \frac{1}{\sin x \cos x}$

$L.H.S = \frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}}$

$= \frac{1}{\cos^2 x} \cdot \frac{\cos x}{\sin x}$

$= \frac{1}{\sin x \cos x}$

$= R.H.S$

b) (i) $\tan \theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

(ii) $\tan 30 = \left|\frac{m_2 - 3}{1 + 3m_2}\right|$

$\frac{1}{\sqrt{3}} = \left|\frac{m_2 - 3}{1 + 3m_2}\right|$

$\frac{m_2 - 3}{1 + 3m_2} = \pm \frac{1}{\sqrt{3}}$

$\pm\sqrt{3}(m_2 - 3) = 1 + 3m_2$

$\sqrt{3}m_2 - 3\sqrt{3} = 1 + 3m_2$

$m_2(\sqrt{3} - 3) = 1 + 3\sqrt{3}$

$m_2 = \frac{1 + 3\sqrt{3}}{\sqrt{3} - 3} \times \frac{\sqrt{3} + 3}{\sqrt{3} + 3}$

$= \frac{\sqrt{3} + 3 + 9 + 9\sqrt{3}}{3 - 9}$

$= \frac{10\sqrt{3} + 12}{-6}$

$m_2 = -2 \mp \frac{5\sqrt{3}}{3}$

c) $\cos\theta - 2\sin\theta = A\cos(\theta + \alpha)$

$= A\cos\theta\cos\alpha - A\sin\theta\sin\alpha$

$\therefore \cos\theta = A\cos\theta\cos\alpha$

$1 = A\cos\alpha$ also $2 = A\sin\alpha$

$\frac{A\sin\alpha}{A\cos\alpha} = 2 \quad \therefore \tan\alpha = 2$

$\alpha = 63^\circ 26'$

$A^2\sin^2\alpha + A^2\cos^2\alpha = 5$

$A^2 = 5 \quad (\text{Since } \sin^2\alpha + \cos^2\alpha = 1)$

$A = \sqrt{5}$

$\therefore \cos\theta - 2\sin\theta = \sqrt{5}\cos(\theta + 63^\circ 26')$

ii) $\cos\theta - 2\sin\theta = 1$

$\sqrt{5}\cos(\theta + 63^\circ 26') = 1$

$\therefore \cos(\theta + 63^\circ 26') = \frac{1}{\sqrt{5}}$

$\theta + 63^\circ 26' = 63^\circ 26', 296^\circ 34'$

$\theta = 0, 233^\circ 8'$

3 – d)

$a\sqrt{b} - c = \sqrt{24 - 16\sqrt{2}}$

$a^2b - 2ac\sqrt{b} + c^2 = 24 - 16\sqrt{2}$.

$\therefore a^2b + c^2 = 24$ ①, $2ac\sqrt{b} = 16\sqrt{2}$

$ac\sqrt{b} = 8\sqrt{2}$

$a^2c^2b = 128$

$a^2 = \frac{128}{c^2b}$ ②

Sub. ② into ①

$\frac{128}{c^2\cancel{b}} \cdot \cancel{b} + c^2 = 24$

$128 + c^4 = 24c^2$

$c^4 - 24c^2 + 128 = 0$

$(c^2 - 16)(c^2 - 8) = 0$

$\therefore c = \pm 4$, $c = \pm 2\sqrt{2}$

$\therefore \boxed{c = \pm 4}$ since a, b, and c are integers.

① → $a^2b + 16 = 24$ ② → $a^2 = \frac{128}{16b}$

$a^2b = 8$

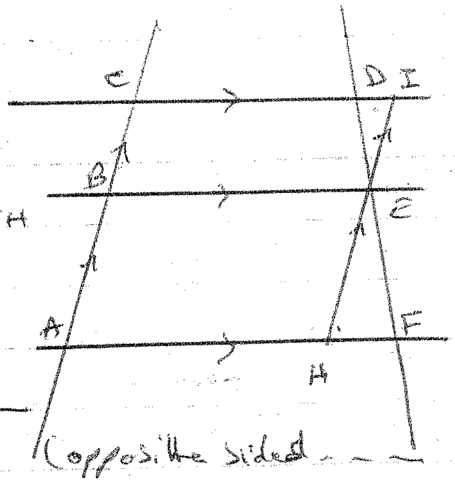
$ac = 8$

$a \times \pm 4 = 8$

$\boxed{a = \pm 2}$

$\therefore b = \frac{8}{(\pm 2)^2} = 2$

Question 4



(i)

(i) Since ABC ∥ IEH

and CD ∥ BE

∴ CBIE is a parallelogram

∴ BC = EI (opposite sides ...)

Similarly BA = EH

Now in Δ's DEI, HEF

∠DEI = ∠HEF (vertically opposite angles are equal).

∠IDE = ∠EFH (alternate angles are equal, DI ∥ HF)

∠DIE = ∠EHF (alternate angles are equal, DI ∥ HF)

∴ ΔDEI ||| ΔHEF

∴ Corresponding sides are in the same ratio.

$\therefore \frac{DE}{EF} = \frac{IE}{EH}$

But IE = BC and EH = BA from above.

$\therefore \frac{DE}{EF} = \frac{BC}{BA}$

(b)(i) A(1, −1), B(−3, 1)

$AB = \sqrt{(1+3)^2 + (-1-1)^2}$

$= \sqrt{16 + 4} = 2\sqrt{5}$

C(−3, 4), D(3, 1)

$CD = \sqrt{(3+3)^2 + (1-4)^2}$

$= \sqrt{36 + 9} = 3\sqrt{5}$

Q4 (b) Cont.

b (iii) equ of CD, $C(-3,4)$, $D(3,1)$

$$CD \Rightarrow \frac{y_2-y_1}{x_2-x_1} = \frac{y-y_1}{x-x_1}$$

$$\frac{1-4}{3+3} = \frac{y-4}{x+3}$$

$$\frac{-3}{6} = \frac{y-4}{x+3}$$

$$-\frac{1}{2} = \frac{y-4}{x+3}$$

$$\therefore \quad x+3 = -2y+8$$

$$x+2y-5=0$$

(iii) $A(1,-1)$, $CD: x+2y-5=0$

$$\text{Perp dis} = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

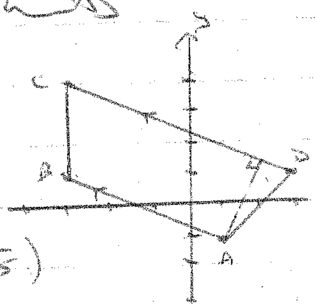
$$= \left|\frac{1\times 1+2\times -1-5}{\sqrt{1+4}}\right|$$

$$= \left|\frac{-6}{\sqrt5}\right|$$

$$= \frac{6}{\sqrt5}\times\frac{\sqrt5}{\sqrt5}$$

$$= \frac{6\sqrt5}{5} \text{ units}$$

(iv) $A = \frac{1}{2}\times h\times(a+b)$

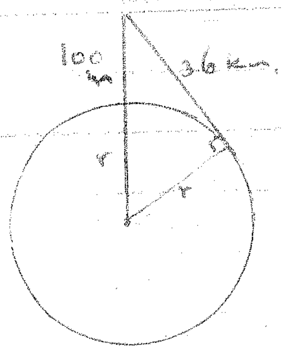


$$= \frac{1}{2}\times\frac{6\sqrt5}{5}(2\sqrt5+3\sqrt5)$$

$$= \frac{3\sqrt5}{5}\times 5\sqrt5$$

$$= 15 \text{ units}^2$$

c)



$$(r+0.1)^2 = r^2+36^2$$

$$r^2+0.2r+(0.1)^2 = r^2+36^2$$

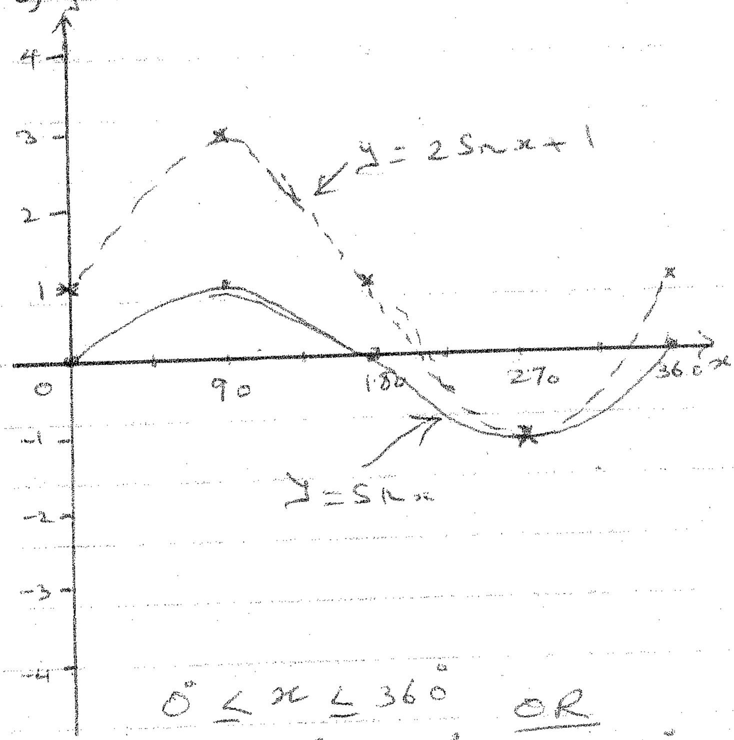
$$200r = 36^2-(0.1)^2$$

$$200r = 1295.99$$

$$r = 6.47995 \text{ km}$$

Question 5

a) i)



$0° \le x \le 360°$ OR

ii) $0° \le x \le 270°$, $270° \le x \le 360°$

b) i) $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$

$$\text{LHS} = \frac{2\sin x\cos x}{1+2\cos^2 x-1}$$

$$= \frac{\sin x\,\cancel{\cos x}}{\cos x\,\cancel{\cos x}}$$

$$= \tan x = \text{RHS}$$

5 - cont.

b) ii) $\tan 15^\circ = \dfrac{\sin 2(15)}{1+\cos 2(15)}$

$= \dfrac{\sin 30}{1+\cos 30}$

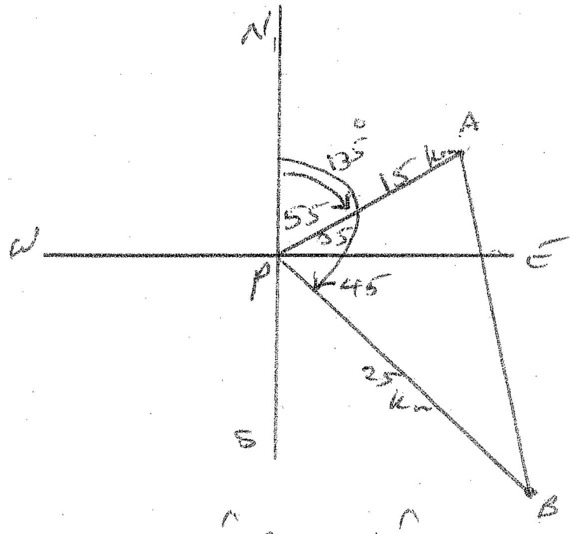
$= \dfrac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}$

$= \dfrac{\frac{1}{\cancel{2}}}{\frac{2+\sqrt{3}}{\cancel{2}}}$

$= \dfrac{1}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \dfrac{2-\sqrt{3}}{1}$

$= 2-\sqrt{3}$ #



) $\angle APB = N\hat{P}B - N\hat{P}A$

$= 135^\circ - 55$

$= 80^\circ$

) $AB^2 = 15^2 + 25^2 - 2 \times 15 \times 25 \times \cos 80$

$AB^2 = 719.7638668$

$AB = 26.82841529$ km

$AB \doteqdot 26.8$ km (1 d.p.)

Question 6

a) $\dfrac{1-\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} = 2\tan\frac{\theta}{2}$

$L.H.S = \dfrac{1-\frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}} + \dfrac{\frac{2t}{1+t^2}}{1+\frac{1-t^2}{1+t^2}}$

$= \dfrac{\frac{\cancel{1}+t^2-\cancel{1}+t^2}{\cancel{1+t^2}}}{\frac{2t}{\cancel{1+t^2}}} + \dfrac{\frac{2t}{\cancel{1+t^2}}}{\frac{1+\cancel{t^2}+1-\cancel{t^2}}{\cancel{1+t^2}}}$

$= \dfrac{\cancel{2}t\cancel{t}}{\cancel{2t}} + \dfrac{\cancel{2}t}{\cancel{2}}$

$= 2t$

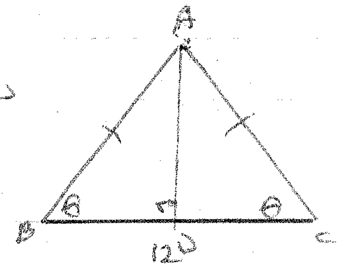
$= 2\tan\frac{\theta}{2} = RHS$

b) construct

$AD \perp BC$

Since $\triangle ABC$ is an Isos. $\triangle$

$\therefore$ AD bisects BC



$\therefore \tan\theta = \dfrac{AD}{DC}$

$\dfrac{2\sqrt{3}}{3} = \dfrac{AD}{6}$

$AD = 4\sqrt{3}$ cm

Area $= \frac{1}{2} \times 12 \times 4\sqrt{3}$

$= 24\sqrt{3}$ cm^2

1.

26 Cont.

i) i) $3\cos x + 4\sin x + 5 = \frac{2t^2+8t+8}{1+t^2}$

$LHS = 3\frac{1-t^2}{1+t^2} + 4\frac{2t}{1+t^2} + 5$

$= \frac{3-3t^2+8t+5+5t^2}{1+t^2}$

$= \frac{2t^2+8t+8}{1+t^2} = RHS$

ii) $3\cos x + 4\sin x + 5 = 0$

$\therefore \frac{2t^2+8t+8}{1+t^2} = 0$

$\therefore 2t^2+8t+8=0,\ 1+t^2 \neq 0$

$2(t^2+4t+4)=0$

$2(t+2)^2=0$

$\therefore t=-2$

$\therefore \tan\frac{\theta}{2} = -2$

$\theta/2 = 116^\circ 34',\ 296^\circ 34'$

$\theta = 233^\circ 8',$

$|x-2| + |x+2| > 6$

j) $2x > 6$ | $-2x > 6$

$x > 3$ | $x < -3$

$-3 \qquad 3$

Question 7

(a) $\sin 2x = 2\cos^2 x$

$2\sin x\cos x - 2\cos^2 x = 0$

$2\cos x(\sin x - \cos x) = 0$

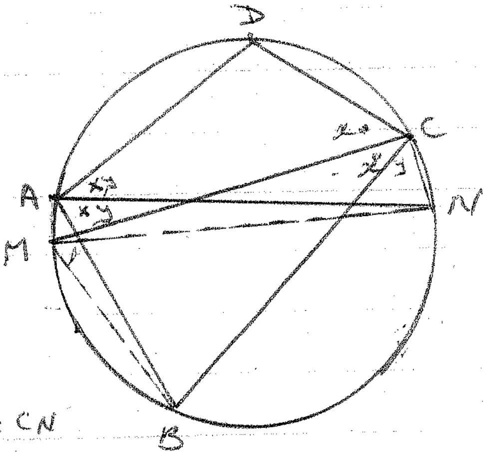
$\therefore 2\cos x = 0$ or $\sin x = \cos x$

$\cos x = 0 \qquad \tan x = 1$

$x = 90^\circ, 270^\circ \qquad x = 45^\circ, 225^\circ$

b)

i)



Join MN & CN

ii) Let $\angle DCM = \angle BCM = x$ (Say)

Since MC bisects $\angle DCB$.

Let $\angle DAN = \angle BAN = y$ (Say)

Since NA bisects $\angle DAB$

Now, $\angle NCB = \angle NAB = y^\circ \longrightarrow (1)$

(angles in the same segment are equal).

$\angle DCB + \angle DAB = 180^\circ$

(opposite angles of a cyclic Quad. are Supplementary).

$\therefore 2x + 2y = 180$

$x + y = 90^\circ$

But $\angle NCB + \angle BCM = y + x$

$\therefore \angle NCM = 90^\circ$

$\therefore$ MN is a diameter of the circle since angle in a semi-circle is a right angle.

OR

ii) Since $\angle DCM = \angle MCB$

(MC bisects $\angle DCB$)

$\therefore$ arc $\overset{\frown}{BM}$ = arc $\overset{\frown}{MA}$ + arc $\overset{\frown}{AD}$ = arc $\overset{\frown}{MD}$

Also $\angle DAN = \angle BAN$
(AN bisects $\angle DAB$)

$\therefore$ arc $\overset{\frown}{BN}$ = arc $\overset{\frown}{NC}$ + arc $\overset{\frown}{DC}$

arc $\overset{\frown}{BN}$ = arc $\overset{\frown}{ND}$ $\longrightarrow$ 2

arcs subtended by equal angles at the circumference are equal.

$\therefore$ from 1 & 2

$\overset{\frown}{BM} + \overset{\frown}{BN} = \overset{\frown}{MD} + \overset{\frown}{ND}$

$\therefore$ arc $\overset{\frown}{MBN}$ = arc $\overset{\frown}{MADCN}$

$\therefore$ MN Divides the circumference of the circle in two equal halfs.

$\therefore$ MN is a diameter.

(ii) Since MN is a diameter

$\therefore \angle NCM = 90^\circ$

i.e $\angle NCB = 90 - \angle BCM$

But $\angle NCB = \angle NMB$

$\therefore \angle NMB = 90^\circ - \angle BCM$.