CRANBROOK
SCHOOL

Year 12 Extension 1 Mathematics

Mini Examination

Wednesday April 8, 2009

Instructions

- There are four (4) questions, each worth 15 marks
- Attempt all questions
- Answer each question in a new booklet
- Show all necessary working
- Calculators are allowed in all sections
- 5 minutes reading time

Time Allowed: 90 minutes Total Marks: 60

Question 1 (15 Marks) **START A NEW BOOKLET** **Marked by SKB**

(a) Consider the function $P(x) = x - \ln 10x$.

(i) Show that a root exists between $x = 3$ and $x = 4$. **1**

(ii) By choosing $x = 3.6$ as a first approximation and applying Newton's Method once determine a second approximation to this root. **2**

(iii) Comment on the accuracy of your second approximation. **1**

(iv) Why would Newton's Method have failed if $x = 1$ had been chosen as the first approximation? **1**

(b) If α, β and γ are the roots of $x^3 + 4x^2 + 8x + 16 = 0$, find the value of

(i) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ **2**

(ii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

(c) The polynomial $P(x) = x^5 + 3x^4 - 10x^3 + 2x^2 + 9x - 5$ has a triple root at $x = 1$ and two other single roots. Determine the values of these other roots and express $P(x)$ as a product of its factors. **3**

(d) A polynomial $Q(x) = x^4 + px^3 + qx^2 - 5x + 1$ has a zero at $x = 1$.

When $Q(x)$ is divided by $x^2 + 2$ it has a remainder of $1 - 7x$. Find p and q. **3**

EXAMINATION CONTINUES OVER THE PAGE

Question 2 (15 Marks) **START A NEW BOOKLET** **Marked by SKB**

(a) **(i)** Use the substitution $u = 1 - x^6$ to find $\int \frac{x^5}{\sqrt{1-x^6}}\,dx$ **3**

(ii) Use the substitution $u = 1 + \log_e x$ to evaluate $\int_1^e \frac{dx}{x(1+\log_e x)^2}$ **3**

(b) $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ lie on the parabola $x^2 = 4ay$. Normals to this parabola at P and Q meet at the point R.

(i) Prove that R has coordinates $[-apq(p+q), a(p^2+pq+q^2+2)]$ **4**

(ii) If the normals intersect at right angles prove that the locus of R is the parabola $x^2 = a(y-3a)$. **4**

(iii) Hence find the coordinates of the focus of the locus of R. **1**

EXAMINATION CONTINUES OVER THE PAGE

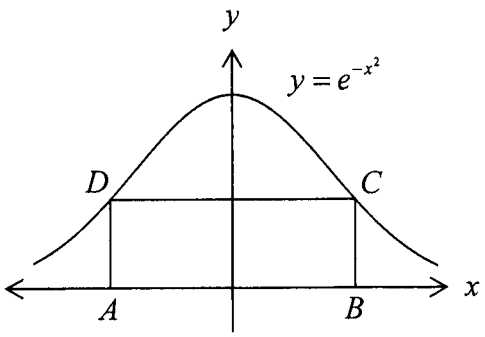
Question 3 (15 Marks) **START A NEW BOOKLET** **Marked by HRK**

(a) Differentiate $y = \ln\left(x^3\sqrt{x^2+1}\right)$ **2**

(b) Evaluate $\int_1^3 \left(2x + \frac{3}{x^2}\right)^2 dx$ **3**

(c) Find the exact value of the area enclosed by the curve $y = \frac{e^x}{1+e^x}$, the x-axis, and the lines $x = 0$ and $x = 1$. **3**

(d) $ABCD$ is a rectangle drawn between the curve $y = e^{-x^2}$ and the x-axis. **4**



(i) Show that $ABCD$ has area $2xe^{-x^2}$ $units^2$

(ii) Hence find the maximum area of such a rectangle.

(e) Write down the derivative of $(x-1)e^x$ and use your result to evaluate $\int_{-1}^{1} xe^x\,dx$ **3**

EXAMINATION CONTINUES OVER THE PAGE

Question 4 (15 Marks) **START A NEW BOOKLET** **Marked by HRK**

(a) Prove by mathematical induction where n is a positive integer, $3^{3n} + 2^{n+2}$ is divisible by 5. **6**

(b) For the curve $y = xe^{-x}$, **9**

(i) Determine the stationary point and the point of inflexion.

(ii) Sketch the curve.

(iii) From your sketch, show that the equation $xe^{-x} = k$ has

(α) Two roots if $0 < k < \frac{1}{e}$

(β) One real root if $k \leq 0$

(γ) No real roots if $k > \frac{1}{e}$

END OF EXAMINATION