Q.1 (2 Marks) Find $\int \frac{e^{x}}{1+e^{2x}}dx$.

Q.2 (2 Marks) Find $\int \tan^3 x \sec^2 x dx$

Q.3 (3 marks) Show $\int_3^5 \frac{dx}{\sqrt{x^2-9}} = \ln 3$

Q.4 (3 Marks) Evaluate $\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \tan 3x \sec 3x dx$

Q.5 (3 Marks) Evaluate $\int_0^3 x\sqrt{x+1}dx$

Q.6 (3 Marks) Find $\int \sec^3 x \tan^3 x dx$

Q.7 (3 Marks) Evaluate $\int_{-1}^{1} \frac{dt}{5-2t+t^2}$

Q.8 (5 Marks) (a) Find a and b such that $\frac{3-3x}{2+x-x^2} = \frac{a}{1+x} - \frac{b}{2-x}$

(b) Hence find $\int \frac{3-3x}{2+x-x^2}dx$

Q.9 (4 Marks) Evaluate $\int_0^2 \frac{2x-3}{x^2-2x+2}dx$

Q.10 (3 Marks) Evaluate $\int_4^9 \frac{dx}{(x-1)\sqrt{x}}$

Q.11 (5 Marks) Show that $\sin(p-q)+\sin(p+q) = 2\sin p \cos q$

Hence evaluate $\int_0^{\frac{\pi}{10}} 2\sin 5x \cos 2x dx$

Q.12 (5 marks) Use integration by parts or otherwise to find $\int x^2 e^x dx$

Q.13 (8 Marks)

Given $I_n = \int \tan^n x dx$

(a) Show $I_n = \frac{1}{n-1}\tan^{n-1} x - I_{n-2}$ ✓

(b) Hence evaluate $\int_0^{\frac{\pi}{4}} \tan^6 x dx$

Q.14 (7 Marks)

(a) Show that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

(b) Hence show $\int_0^{\frac{\pi}{2}} \frac{\cos^3 x dx}{\cos^3 x + \sin^3 x} = \int_0^{\frac{\pi}{2}} \frac{\sin^3 x dx}{\cos^3 x + \sin^3 x}$

(c) Hence evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos^3 x dx}{\cos^3 x + \sin^3 x}$

Q.15 (10 Marks)

Let $I_n = \int_0^1 x^n \sqrt{1-x} dx$

(a) Show that $I_n = \frac{2n}{2n+3} I_{n-1}$

(b) Evaluate $\int_0^1 x^4 \sqrt{1-x} dx$

(c) Show that $I_n = \frac{n!(n+1)!}{(2n+3)!} 4^{n+1}$