1)

a) $3.28 (21829)$

b) 7

c) $\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} \times \dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \dfrac{5-2\sqrt{10}+2}{3}$

$= \dfrac{7-2\sqrt{10}}{3}$

d) $3x^2 + 27x + 60$

$= 3(x^2 + 9x + 20)$

$= 3(x+5)(x+4)$

e) $6 - (x-4) = x$

$6 - x + 4 = x$

$10 = 2x$

$x = 5$

f) $12\% = \$1500$

$1\% = \$125$

$100\% = \$12500$

g) $v^2 = u^2 + 2as$

$35^2 = u^2 + 2 \times 9.8 \times 25$

$u^2 = 735$

$u = \pm 27.1 (10883)$

2)

a) i) $40x^4 + 35x^{-6}$

ii) $5\cos 5x$

iii) $\dfrac{\ln 2x \cdot 2 - 2x \cdot \frac{1}{x}}{(\ln 2x)^2}$

$= \dfrac{2\ln 2x - 1}{(\ln 2x)^2}$

$OR = \dfrac{\ln 4x^2 - 1}{(\ln 2x)^2}$

b) i) $\left[2e^{2x}\right]_0^5$

$= 2e^{10} - 2e^0$

$= 2e^{10} - 2 \quad OR \quad 2(e^{10} - 1)$

$OR \quad 44050.932$

ii) $\left[\dfrac{1}{10}(2x+7)^5\right]'_{-1} = \dfrac{t}{10}(9^5 - 5^5)$

$= 5592.4$

c) $\int \dfrac{5y}{y^2 + 8} \, dy = \dfrac{3}{2}\int \dfrac{2y}{y^2 + 8} \, dy$

$= \dfrac{3}{2}\ln(y^2 + 8) + C$

3)

a) $\dfrac{y-7}{x-1} = \dfrac{0-7}{-5-1}$

$-6y + 42 = -7x + 7$

$7x + 6y + 35 = 0$

b) $X = \dfrac{1+7}{2} \quad Y = \dfrac{7+0}{2}$

$= 4 \qquad\qquad = 3\frac{1}{2}$

$\therefore (4, 3\frac{1}{2})$

c) $d^2 = 6^2 + 7^2$

$d^2 = 36 + 42$

$d^2 = 78$

$d = \sqrt{78}$

$= 8.8 (317609)$

d) $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= \dfrac{|7\times 4 + 6 \times 3\frac{1}{2} + 35|}{\sqrt{7^2 + 6^2}}$

$= \dfrac{28 + 21 + 35}{\sqrt{78}}$

$= \dfrac{42}{\sqrt{78}}$

$= \dfrac{7\sqrt{78}}{13} = 4.7 (555635)$

e) $A = \dfrac{1}{2} \times \sqrt{78} \times \dfrac{42}{\sqrt{78}}$

$= 21$

f) $\tan\theta = \dfrac{7}{6}$

g) $7x + 6y + 35 = 0$

$7 \times -2 + 6 \times 4 + 35 = 0$

$-14 + 24 + 35 = 0$

$-31 \neq 0$

$\therefore$ not on line.

4)

a) i) $\cos 15° = \dfrac{0.6}{x}$

$x = \dfrac{0.6}{\cos 15}$

$= 0.62 (11657)$

ii) $\sin 15° = \dfrac{3}{H}$

$H = \dfrac{3}{\sin 15} = 11.5911$

$\sin 15° = \dfrac{2.38}{h}$

$h = \dfrac{2.38}{\sin 15} = 9.1956$

$\therefore A = \dfrac{1}{2}(11.5911 + 9.1956) \times 0.6$

$= 10.39335 \times 0.6$

$= 6.236$

$= 6.2$

b) $V = 6.2 \times 5$

$= 31 \text{ m}^3 \quad (31.0 \rightarrow 31.1)$

B)

a) $P = P_0 e^{kt}$

$10200 = 10000 e^{2k}$

$e^{2k} = 1.02$

$2k = \ln 1.02$

$k = 0.0099$

b) $11500 = 10200 e^{0.0099t}$

$0.0099t = \ln\left(\dfrac{11500}{10200}\right)$

$t = 12.1 \text{ years}$

$\therefore 13 \text{ years}$

c) $P = 10200 e^{5 \times 0.0099}$

$= 10717.605$

$\therefore 10717 \text{ or } 10718$

c) $8^x = 4^{x-1}$

$2^{3x} = 2^{2x-2}$

$3x = 2x - 2$

$x = -2$

5)

a) $A = \dfrac{30}{6}\left[20 + 4 \times 18 + 12\right] + \dfrac{30}{6}\left[12 + 4 \times 13 + 21\right]$

$= 5 \times 104 + 5 \times 85$

$= 945 (\text{m}^2)$

b) $A = 180(1.12)^{35} + \ldots + 180(1.12)^1$

$= 180\left(\dfrac{1.12^9(1.12^{17} - 1)}{1.12 - 1}\right)$

$= \$24400.49$

c) i) $a = 3 \quad d = 5$

$T_n = a + (n-1)d$

$488 = 3 + (n-1)5$

$485 = 5n - 5$

$5n = 490$

$n = 98$

ii) $S_n = \dfrac{n}{2}(2a + (n-1)d)$

$S_{98} = \dfrac{98}{2}(2\times 3 + 97 \times 5)$

$= 24059$

d) $S_{10} = 2^{10} + 10^2 = \ldots \quad 1124$

$S_9 = 2^9 + 9^2 = 593$

$\therefore T_{10} = 1124 - 593$

$= 531$

6)

i) $x = 6t - \dfrac{t^3}{2}$

$\dfrac{dx}{dt} = 6 - \dfrac{3t^2}{2}$

when $x = 0 \quad 6t - \dfrac{t^3}{2} = 0$

$12t - t^3 = 0$

$t(12 - t^2) = 0$

$\therefore t = 0 \text{ or } \sqrt{12}$

when $x = \sqrt{12} \quad V = 6 - \dfrac{3 \times 12}{2}$

$= -12$

$\therefore$ returns when $t = \sqrt{12} \quad V = -12$.

Column 1

ii) $V = 6 - \frac{3t^2}{2}$

when $V = 0$, $\frac{3t^2}{2} = 6$

$3t^2 = 12$

$t^2 = 4$

$t = 2$
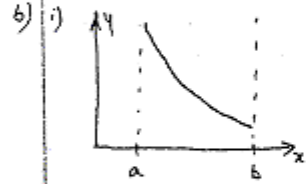
when $t = 2$, $x = 6 \times 2 - \frac{2^3}{2}$

$= 8$

$a = -3t$

when $t = 2$, $a = -6$

iii) $S = \frac{D}{T}$

$S = \frac{8}{2}$

$= 4 \text{ cm/s}$

b) i)

ii) $f(b)$

c) $y = x^3 - x^2 - x - 1$

$\frac{dy}{dx} = 3x^2 - 2x - 1$

$\frac{d^2y}{dx^2} = 6x - 2$

stat pt when $\frac{dy}{dx} = 0$

i.e. $3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$3x = -1 \quad x = 1$

$x = -\frac{1}{3}$

when $x = 1$, $y = 1 - 1 - 1 - 1$

$= -2$

$x = -\frac{1}{3}$, $y = (-\frac{1}{3})^3 - (-\frac{1}{3})^2 + \frac{1}{3} - 1$

$= -\frac{22}{27}$

Column 2

pt inflexion when $\frac{d^2y}{dx^2} = 0$

i.e. $6x - 2 = 0$

$6x = 2$

$x = \frac{1}{3}$

when $x = \frac{1}{3}$, $y = (\frac{1}{3})^3 - (\frac{1}{3})^2 - \frac{1}{3} - 1$

$= -1\frac{5}{27}$

∴ stat pts $(1, -2)$ + $(-\frac{1}{3}, -\frac{22}{27})$

inflexion pt $(\frac{1}{3}, -1\frac{5}{27})$

7) a) i) In Δ's APB + DPC

AB = DC (given)

∠BAC = ∠BDC (given)

∠APB = ∠DPC (vert opp)

∴ ΔAPB ≡ ΔDPC (AAS)

ii) In ΔPBC

PB = PC corresponding sides of congruent Δs.

b) $y^2 - 6y + 25 = 8x$

$y^2 - 6y + 9 = 8x - 16$

$(y-3)^2 = 8(x-2)$

i) $(2,3)$

ii) $y = 3$

iii) 2

iv) $(4,3)$

v) $x = 0$

8) a) i) $.2 \times .3 = 0.06 \qquad 6\%$

ii) $.8 \times .7 = 0.56 \qquad 56\%$

iii) $.2 \times .7 + .8 \times .3 = .14 + .24 = 0.38 \qquad 38\%$

iv) $1 - 0.56 = 0.44 \qquad 44\%$

b) ... -2.

Column 3

c) i) when $y = 0$

$3 - x^2 = 0$

$x = \pm\sqrt{3}$

∴ $A = (-\sqrt{3}, 0)$

$x^3 + 1 = 0$

$x^3 = -1$

$x = -1$

∴ $B = (-1, 0)$

ii) $3 - x^2 = x^3 + 1$

$x^3 + x^2 - 2 = 0$

when $x = 1$

$1 + 1 - 2 = 0$

$0 = 0$

True ∴ $x = 1$ is a soln.

when $x = 1$ $y = 3 - 1^2$

$= 2$

∴ $(1, 2)$ is pt T

iii) $A = \int_{-\sqrt{3}}^{1} (3 - x^2 - x^3 - 1) \, dx$

$= \int_{-\sqrt{3}}^{1} 2 - x^2 - x^3 \, dx$

$= \left[2x - \frac{x^3}{3} - \frac{x^4}{4}\right]_{-\sqrt{3}}^{1}$

$= \left(2 - \frac{1}{3} - \frac{1}{4}\right) - \left(-2\sqrt{3} + \frac{3\sqrt{3}}{8} - \frac{9}{4}\right)$

$= 2 - \frac{1}{3} - \frac{1}{4} + \sqrt{3} + \frac{9}{4}$

$= \frac{11}{3} + \sqrt{3}$ or $3\frac{2}{3} + \sqrt{3}$
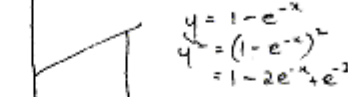
$= (5.3987175)$

9) a) i) $l = r\theta$

$8 = 5\theta$

$\theta = \frac{8}{5}$

Column 4

ii) $A = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$= \frac{1}{2} \times 25 \times \frac{8}{5} - \frac{1}{2} \times 25 \times \sin\frac{8}{5}$

$= 7.51 \quad (7.50533)$

b)

$y = 1 - e^{-x}$

$y^2 = (1 - e^{-x})^2$

$= 1 - 2e^{-x} + e^{-2x}$

$V = \pi \int_0^1 y^2 \, dx$
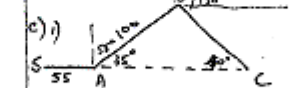
$= \pi \int_0^1 1 - 2e^{-x} + e^{-2x} \, dx$

$= \pi \left[x + 2e^{-x} - \frac{1}{2}e^{-2x}\right]_0^1$

$= \pi\left[1 + 2e^{-1} - \frac{1}{2}e^{-2} - 0 - 2 + \frac{1}{2}\right]$

$= \pi\left(-\frac{1}{2} + \frac{1}{2}e^{-2} + 2e^{-1}\right)$

$= \frac{\pi}{2}\left(4e^{-1} - e^{-2} - 1\right)$

c) i)

ii) $\angle CBX = 130 - 90$

$= 40$

$\angle ACB = \angle CBX = 40$ (alt ∠'s BX ∥ AC)

iii) $\frac{BC}{\sin 35} = \frac{100}{\sin 40}$

$BC = \frac{100 \sin 35°}{\sin 40°}$

$= 89.(2 \; 32653)$

iv) $S = \frac{89 + 100 + 55}{24/60}$

$= 610 \; (km/h)$

10) i) $A_1 = 1.015P - M$

ii) $A_2 = 1.015^2P - 1.015M - M$

$A_3 = 1.015^3P - 1.015^2M - 1.015M - M$

$\vdots$

$A_n = 1.015^nP - M\left(1.015^{n-1} + 1.015^{n-2} + \cdots + 1.015^1 + 1.015^0\right)$

iii) $1.015^{60} \times 40000 - M\left(\dfrac{1\left(1.015^{60} - 1\right)}{1.015 - 1}\right) = 0$

$M = \dfrac{1.015^{60} \times 40\,000 \times 0.015}{1.015^{60} - 1}$

$= \$1015.74 \qquad (\$1015 - \$1016)$

b) i) $\dfrac{dm}{dt} = \dfrac{1}{100}\left[t + \dfrac{t^2}{10}\right]$

$m = \dfrac{1}{100}\left[\dfrac{t^2}{2} + \dfrac{t^3}{30}\right] + C$

when $t = 0$, $m = 0$ $\therefore C = 0$

$\therefore$ when $t = 20$

$m = \dfrac{1}{100}\left[\dfrac{400}{2} + \dfrac{8000}{30}\right]$

$= 4\dfrac{2}{3}\,g$ or $4.7g$ or $4.6g$

ii) when $t = 0$, $m = 0.001$ $\therefore C = 0.001$

$\therefore m = \dfrac{1}{100}\left[\dfrac{t^2}{2} + \dfrac{t^3}{30}\right] + 0.001$

$\therefore 0.001\,g$

c) $S = \dfrac{1}{1-x} + \dfrac{2x}{1-x} + \dfrac{2x^2}{1-x} + \cdots$

$S = \dfrac{1}{1-x}\left(1 + 2x + 2x^2 + \cdots\right)$

$= \dfrac{1}{1-x}\left(1 + \dfrac{2x}{1-x}\right)$

$= \dfrac{1}{1-x} + \dfrac{2x}{(1-x)^2}$

$= \dfrac{1-x+2x}{(1-x)^2} = \dfrac{1+x}{(1-x)^2}$