Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NEW SOUTH WALES

2002
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics
Extension 1

Afternoon Session
Friday 9 August 2002

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- All necessary working should be shown in every question

Total marks (**84**)
- Attempt Questions 1 – 7
- All questions are of equal value

Disclaimer

Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, *Principles for Setting HSC Examinations in a Standards-Referenced Framework* (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and *Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework* (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

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Marks

Question 1   **Begin a new page**

(a) Find $\dfrac{d^2}{dx^2}\ln\left(e^x+1\right)$ 2

(b) Evaluate $\displaystyle\sum_{k=1}^{4}\frac{(-1)^k}{k!}$ 2
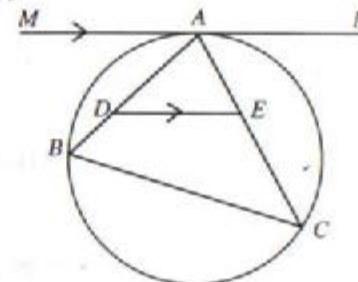
(c) (i) Show that $\dfrac{1-\cos 2x}{1+\cos 2x}=\tan^2 x$ 2

    (ii) Hence find the value of $\tan 22\tfrac{1}{2}°$ in simplest exact form.   2

(d)

ABC is a triangle inscribed in a circle.
MAN is the tangent to the circle at *A*.
D is a point on *AB* and *E* is a point on
AC such that *DE* ‖ *MAN*.

(i) Copy the diagram.

(ii) Explain why $M\hat{A}B = A\hat{C}B$.   1

(iii) Hence show that *BCED* is a cyclic quadrilateral.   3

Marks

Question 2   **Begin a new page**

(a) $A(-2,3)$ and $B(4,-5)$ are two fixed points. Find the coordinates of the point $P(x, y)$ which divides the interval AB internally in the ratio $4:1$.   **2**

(b) A test consists of 7 multiple choice questions all of which are to be attempted. Each question contains 4 alternative answers of which one and only one is correct. Find the number of ways in which the 7 questions can be attempted so that exactly 2 questions are answered correctly.   **2**

(c) (i) Given that $x = 1$ is a zero of the polynomial $P(x) = x^3 - 3x + 2$, express $P(x)$ as a product of three linear factors.   **2**

     (ii) Hence solve the inequality $x^3 - 3x + 2 \le 0$ **2**

(d)
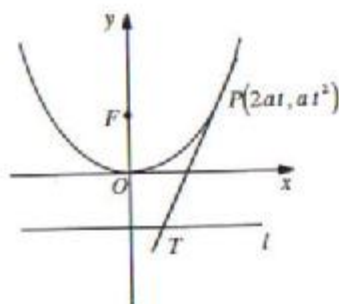
The tangent $tx - y - at^2 = 0$ at the point $P(2at, at^2)$ on the parabola $x^2 = 4ay$ cuts the directrix l at T.
F is the focus of the parabola.

(i) Find the coordinates of T.   **1**

(ii) Show that TF is perpendicular to PF.   **3**
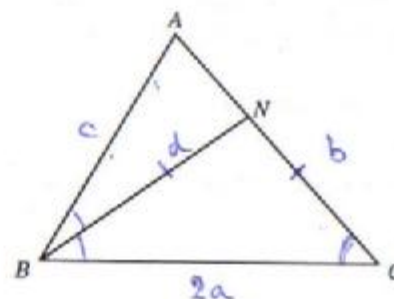

Marks

Question 3   **Begin a new page**

(a)

ABC is a triangle and N is a point on AC.
$\hat{ABN} = \hat{CBN} = \hat{BCN}$.
$BC = 2a$, $CA = b$, $AB = c$.
$BN = CN = d$.

(i) Given that $\triangle ABN \; ||| \; \triangle ACB$, show that $c^2 = b^2 - 2ac$.   **3**

(ii) Hence show that $(a+c)^2 = a^2 + b^2$.   **1**

(b) $P(x) = x^3 + 3x^2 + 6x - 5$

(i) Show that the equation $P(x) = 0$ has a root α such that $0 < \alpha < 1$.   **2**

(ii) Use one application of Newton's method with a starting value of $x = 0.5$ to find an approximation for α, giving the answer to 2 decimal places.   **2**

(c) Use the substitution $x = u^2 \; (u > 0)$ to find the exact value of $\displaystyle\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{\sqrt{x}\sqrt{1-x}} \, dx$.   **4**

Question 4   **Begin a new page**

(a) $I = \displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$

(i) Find the exact value of I.   **2**

(ii) Use Simpson's rule with 3 function values to approximate the value of I.   **2**

(b) The numbers $1, 2, 3, \dots, 9$ are written one on each of 9 cards. 3 of the cards are chosen at random.

(i) Find the probability that the sum of the 3 numbers chosen is equal to 9.   **2**

(ii) If it is known that the first number chosen is 2, find the probability now that the sum of the 3 numbers chosen is equal to 9.   **2**

(c) A spherical map of the earth is being inflated at a constant rate of $25 \text{ cm}^3 s^{-1}$. Find the rate at which the length of the equator is changing when the radius is 10 cm.   **4**

Marks

Question 5   **Begin a new page**

(a) $f(x) = \dfrac{2}{x+1}$, $x > -1$

   (i) Find the equation of the inverse function $f^{-1}(x)$.   **1**

   (ii) On the same diagram sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ showing clearly the coordinates of any points of intersection, the intercepts on the coordinate axes, and the equations of any asymptotes.   **3**

(b) A particle is moving in a straight line. O is a fixed point on the line. At time t seconds, the particle has displacement x metres fom O, its velocity v ms^{-1} is given by $v = -x^2$ and its acceleration is a ms^{-2}. Initially the particle is 1 m to the right of O.

   (i) Find an expression for a in terms of x.   **1**

   (ii) Find an expression for x in terms of t.   **3**

(c) In the binomial expansion of $\left(x^2 + \dfrac{a}{x}\right)^6$ the term independent of x is 240. Find the value of a.   **4**

Question 6   **Begin a new page**

(a) (i) If $\theta = \tan^{-1}A + \tan^{-1}B$ show that $\tan\theta = \dfrac{A+B}{1-AB}$.   **1**

   (ii) Hence solve the equation $\tan^{-1}3x + \tan^{-1}2x = \frac{\pi}{4}$.   **3**

(b) At time t minutes, the temperature T °Celcius of a piece of metal is given by $T = 20 + Ae^{-kt}$ for some constants A and k. Initially the temperature of the piece of metal is 100°C, and after 4 minutes its temperature has dropped to 80°C.

   (i) Find the exact values of A and k.   **2**
   (ii) Find how much longer it will take for the temperature to drop to 60°C, giving the answer correct to the nearest second.   **2**

(c) A particle moving in a horizontal straight line is performing Simple Harmonic Motion. At time t seconds its displacement x metres from a fixed point O on the line is given by $x = 3\cos 2t + \sin 2t$, where displacements to the right of O are positive.

   (i) Explain whether the particle is initially moving to the right or to the left, and whether it is speeding up or slowing down.   **2**

   (ii) Find the time taken by the particle to first reach O, giving the answer in seconds correct to   **2**

Marks

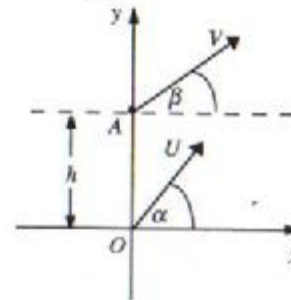**Question 7**   **Begin a new page**

(a)   A function $f(x)$ is such that $f(x) > 0$ for all real numbers x and $f(a+b) = f(a) \cdot f(b)$ for any real numbers a and b.

   (i) Show that $f(0) = 1$ and deduce that $f(-x) = \dfrac{1}{f(x)}$.   **2**

   (ii) Use the method of mathematical induction to show that $f(nx) = [f(x)]^n$ for all positive integers n.   **3**

   (iii) *Without using mathematical induction again*, deduce that $f(-nx) = [f(x)]^{-n}$ for all positive integers n.   **1**

(b)

A particle is projected from a point O with speed U ms^{-1} at an angle of elevation α. At the same instant, another particle is projected from a point A (h metres directly above O) with speed V ms^{-1} at an angle of elevation β, where $\beta < \alpha$. Both particles move freely under gravity in the same plane of motion and collide T seconds after projection.

   (i) Write down expressions for the horizontal and vertical displacements of each particle at time t seconds referred to axes Ox and Oy.   **2**

   (ii) Show that $T = \dfrac{h\cos\beta}{U\sin(\alpha - \beta)}$.   **4**