ASCHAM SCHOOL
2002 TRIAL HSC EXAMINATION

MATHEMATICS EXTENSION 1
FORM VI

General Instructions:

- Reading Time: 5 minutes
- Working Time: 2 hours
- Write using blue or black pen
- Approved calculators and templates may be used.
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question.

Collection:

- Start each question in a new answer book
- Write your name and teacher's name on each book.
- If you use a second book, place it inside the first.

Total Marks : **84**

- Attempt Questions 1 – 7
- All questions are of equal value.

Question 1 **Start a new answer book**

a) Express $\frac{5\pi}{12}$ radians as degrees [1]

b) Find a primitive of e^{-2x} [1]

c) Use the table of standard integrals to find the exact value of

$$\int_0^4 \frac{dx}{\sqrt{x^2+4}}$$ [2]

d) If α, β and γ are roots of the equation $6x^3+7x^2-x-2=0$, find the value of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$ [3]

e) Find the domain and range of $f(x)=4\sin^{-1}\frac{x}{3}$, and sketch the graph of *f(x)* .[3]

f) Find $\frac{d}{dx}e^{\cos x}$ [2]

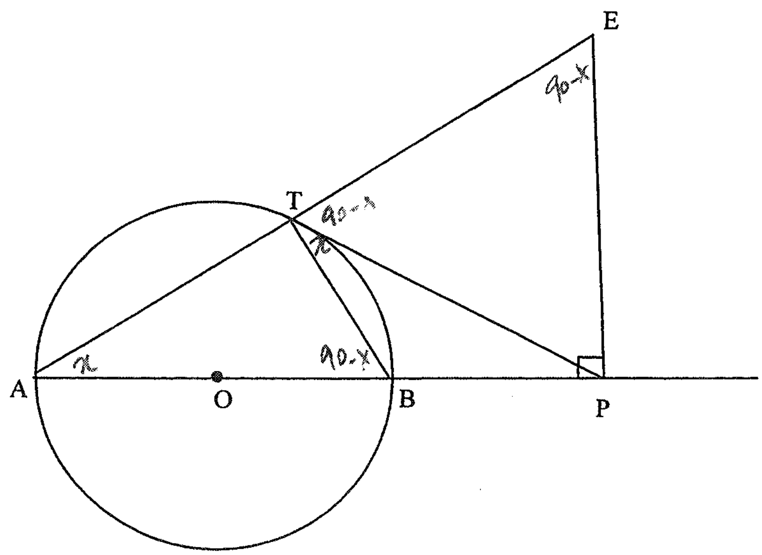
Question 2 **Start an new answer book**

a) Find (i) $\int \frac{x}{4+x^2}\,dx$ [1]

(ii) $\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \tan^2 2x\,dx$ [3]

b) If $\sum_{k=4}^{\infty} 2r^{k-3}=10$, find r if r exists. [3]

c)



Make a large neat copy of the diagram in your answer book.

AB is the diameter of the circle with centre O. TP is a tangent to the circle at T. $EP \perp AP$. Prove:

(i) TBPE is a cyclic quadrilateral [2]
(ii) PT = PE. [3]

Question 3 ***Start a new answer book***

a) Solve : $\dfrac{2x}{5-x} \geq 1$ [3]

b) Prove by mathematical induction that $6^n - 1$ is divisible by 5 for all positive integers.

[5]

c) By substituting $t = \tan\frac{x}{2}$, find the solutions to the equation:

$3\sin x + 4\cos x = 5$ for $0^\circ \leq x \leq 360^\circ$,

giving your answers correct to the nearest degree.

[4]

Question 4 **Start a new answer book**

a) Using the substitution $u = x^3 + 1$, evaluate $\int_{-1}^{1} x^2(x^3+1)\,dx$ [2]

b) (i) Factorise : $x^3 - 3x + 2$

(ii) Hence draw a neat sketch of the polynomial $y = x^3 - 3x + 2$ without the use of calculus, showing all intercepts with the co-ordinate axes.

(iii) Hence solve the inequality $x^3 - 3x + 2 > 0$ [4]

c) Find the value of $\sin\left(2\sin^{-1}\frac{2}{3}\right)$ in exact form
[3]

d) i) Show that the equation $f(x) = x^3 - 8x + 8$ has a zero between -3 and -4 .

ii) Taking $x = -3\cdot5$ as a first approximation of the solution of the equation $f(x) = 0$, use Newton's method once to find a closer approximation, giving your answer to 2 decimal places
[3]

Question 5 **Start a new answer book**

a) A bug is oscillating in simple harmonic motion such that its displacement x metres from a fixed point O at time t seconds is given by the equation $\ddot{x} = -4x$. When $t = 0$, $v = 2$ m/s and $x = 5$.

(i) Show that $x = a\cos(2t + \beta)$ is a solution for this equation, where a and β are constants.

(ii) Find the period of the motion.

(iii) Show that the amplitude of the oscillation is $\sqrt{26}$.

(iv) What is the maximum speed of the bug? [5]

b) (i) Prove that $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \ddot{x}$ [2]

(ii) The acceleration of a creature is given by $\ddot{x} = -\frac{1}{2}u^2 e^{-x}$ where x is the displacement from the origin, and u is the initial velocity at the origin. Given that $u = 2$ m/s:

(α) Show that $v^2 = 4e^{-x}$

(β) Explain why $v > 0$, and find x in terms of t.

(γ) Describe the subsequent motion of the creature as $t \to \infty$.
[5]

Question 6 **Start a new answer book**

a) A ladder is slipping down a vertical wall. The ladder is 4 metres long. The top of the ladder is slipping down at a rate of 3 m/s. How fast is the bottom of the ladder moving along the ground when the bottom is 2 metres away from the foot of the wall? [4]

b) The point $P(2ap, ap^2)$ lies on the parabola $x^2 = 4ay$. The focus S is the point $(0, a)$. The tangent at P meets the y-axis at Q.

(i) Find the equation of the tangent at P and the co-ordinates of Q.
(ii) Prove that $SP = SQ$
(iii) Hence show that $\angle PSQ + 2\angle SQP = 180^\circ$ [4]

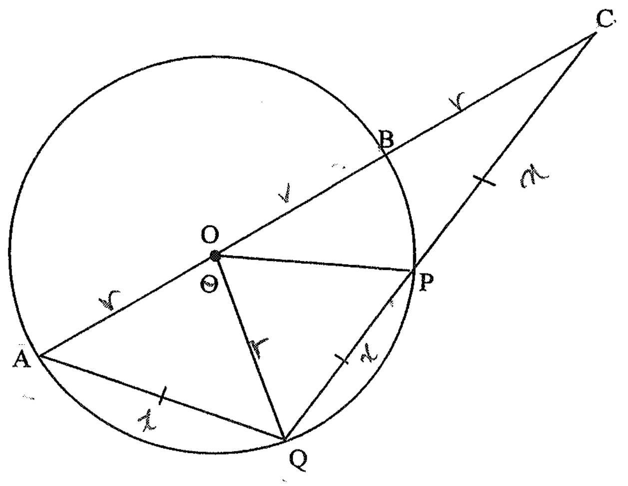
c) In a town in Mathsland, a 'flu epidemic is spreading at a rate proportional to the population that have it, such that is it predicted that the number of people who have the disease will double in 3 weeks, i.e. $\frac{dA}{dt} = kA$, where A is the number of people with 'flu in time t weeks.

(i) Show that $A = A_0 e^{kt}$, where A_0 is the initial number of people with 'flu, satisfies the above differential equation.

(ii) Find k in exact form

(iii) In the neighbouring town with a population of 20,000, three people have the 'flu. How many weeks (to the nearest week) will it take for the whole population to contract the disease?
[4]

Question 7 Start a new book



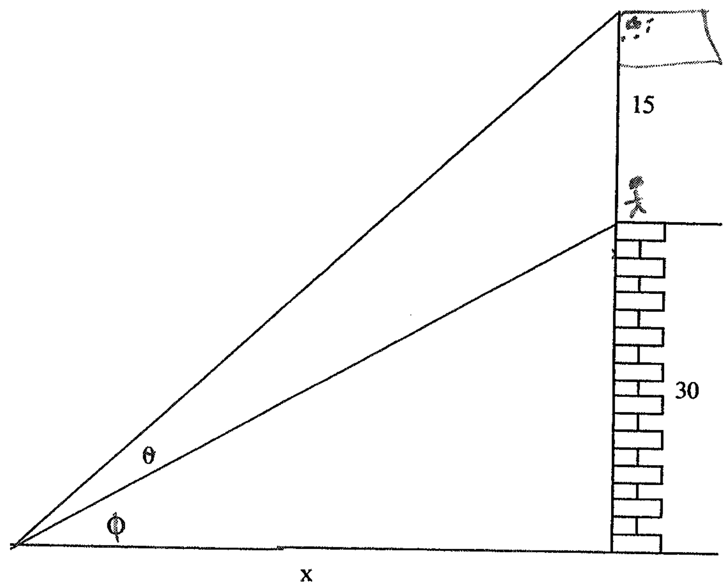
a) *Make a large neat copy of the diagram in your answer book.*

AB is the diameter of the circle with centre O and radius r. BC = r, AQ = QP = PC, and $\angle AOQ = \theta$

(i) Prove that $\cos\theta = \dfrac{1}{4}$ (Hint: use the cosine rule in triangles AQO and QOC) [5]

(ii) Hence prove that QC = $r\sqrt{6}$ [2]

b) A 15 metre high flagpole stands on top of a building which is 30 m high. The flagpole subtends an angle of θ degrees to a point x metres from the foot of the building, and the building subtends an angle of ϕ degrees to the



same point.

i) Show that $\theta = \tan^{-1}\left(\dfrac{15x}{x^2+1350}\right)$

ii) Hence find the value of x which will make θ a maximum. [6]

End of Examination

Standard Integrals

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\;dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\;dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\;dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\;dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$