Solutions Yr 11 Maths Assess.

Qu. 1

(a) $y = -x$, $\sqrt{3}y = x$

$\therefore m = -1$, $y = \frac{x}{\sqrt{3}}$

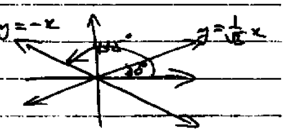
$\therefore m = \frac{1}{\sqrt{3}}$

$\therefore \alpha = 135^\circ$, $\therefore \beta = 30^\circ$

$\therefore$ Acute angle $= 180^\circ - (135^\circ - 30^\circ)$

$= 180^\circ - 105^\circ$

$= 75^\circ$.



(b) $\int \frac{dx}{\sqrt{1-9x^2}}$

$= \frac{1}{3}\sin^{-1}(3x) + C$

(c) $2x^3 + x^2 - x - 2 = 0$

(i) $\alpha + \beta + \lambda = -\frac{1}{2}$

(ii) $\alpha\beta\lambda = \frac{2}{2} = 1$

(iii) $\alpha\beta + \alpha\lambda + \beta\lambda = -\frac{1}{2}$

(iv) $(\alpha-1)(\beta-1)(\lambda-1)$

$= (\alpha\beta - \alpha - \beta + 1)(\lambda - 1)$

$= \alpha\beta\lambda - \alpha\lambda - \beta\lambda + \lambda - \alpha\beta + \alpha + \beta - 1$

$= \alpha\beta\lambda - (\alpha\lambda + \beta\lambda + \alpha\beta) + (\alpha + \beta + \lambda) - 1$

$= 1 + \frac{1}{2} + \left(-\frac{1}{2}\right) - 1$

$= 0$

Qu. 1

(d) (i) $x = 12t \Rightarrow t = \frac{x}{12}$

$y = 6t^2$

i.e. $y = 6 \cdot \frac{x^2}{144}$

$\therefore x^2 = 24y$

(ii) $x^2 = 24y$

i.e. $x^2 = 4(6)y$

$\therefore$ Focus is $(0, 6)$

(iii) Eqn. of directrix is $y = -6$.

Qu. 2

a) $\sin\theta - \sqrt{3}\cos\theta = 1$, $0 \le \theta \le 2\pi$

$2\sin\left(\theta - \frac{\pi}{3}\right) = 1$

$\sin\left(\theta - \frac{\pi}{3}\right) = \frac{1}{2}$

$\theta - \frac{\pi}{3} = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$\therefore \theta = \frac{\pi}{2}$ or $\frac{7\pi}{6}$

$r = \sqrt{1^2 + (\sqrt{3})^2}$

$r = 2$

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$

$0 \le \theta \le 2\pi$

$-\frac{\pi}{3} \le \theta - \frac{\pi}{3} \le 2\pi - \frac{\pi}{3}$.

(b) (i) $\sin 2x = \sin x$, $0 \le x \le \pi$

$\sin 2x - \sin x = 0$

$2\sin x\cos x - \sin x = 0$

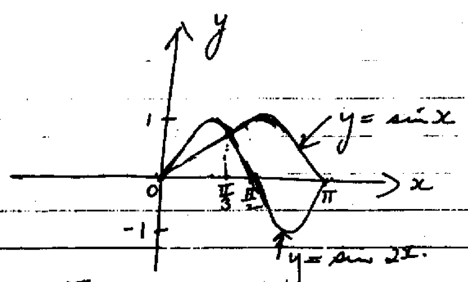
$\sin x(2\cos x - 1) = 0$

$\therefore \sin x = 0$ or $\cos x = \frac{1}{2}$

$\therefore x = 0$ or π or $x = \frac{\pi}{3}$

$\therefore x = 0, \frac{\pi}{3}$ or π.

(ii)



(iii) Area $= \int_0^{\frac{\pi}{3}} (\sin 2x - \sin x)\,dx$

$= \left[-\frac{\cos 2x}{2} + \cos x\right]_0^{\frac{\pi}{3}}$

$= \left(-\frac{\left(-\frac{1}{2}\right)}{2} + \frac{1}{2}\right) - \left(-\frac{1}{2} + 1\right) = \frac{1}{4}$ unit^2

Q3 a) $6^2 = x(5+x)$ (square of tangent = product of intercepts)

$x^2 + 5x - 36 = 0$

$x = 4$ or -9

but $x > 0$ (or x is a length)

$\therefore x = 4$

b) $\hat{KNL} = \hat{JML}$ (ext. $\angle$ = int. opp in cyclic quad)

$\hat{NKL} = \hat{JML}$ (opp $\angle$'s of parallelogram)

$\therefore \triangle NKL$ is isosceles ($\hat{NKL} = \hat{KNL}$ base angles equal)

$\therefore NL = KL$ (sides opp equal angles of $\triangle$)

c) $z = \hat{CBE}$ ($\angle$ between chord and tangent $= \angle$ in alternate segment)

$= 50$

$\hat{ABE} = 90°$ (Tangent $\perp$ to radius)

$\therefore y + 50° = 90°$

$y = 40°$

$x = 2 \times \hat{FCB}$ ($\angle$ at centre = twice $\angle$ at circumference)

$= 2 \times 35$

$= 70°$

Common errors : incomplete or missing reasons

: false statements, especially in part (a).

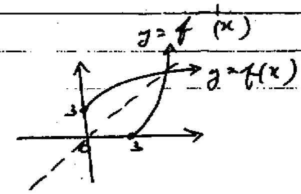
Qu.4

(a) $\tan\theta = \frac{1}{\sqrt{2}}$

$\therefore\ \theta = n\pi + \tan^{-1}\frac{1}{\sqrt{2}}$

1

(b) $f(x) = \sqrt{x} + 3$



(i) $y = \sqrt{x} + 3$

$y - 3 = \sqrt{x}$

$x = (y-3)^2$

$\therefore\ f^{-1}(x) = (x-3)^2, \ x \geq 3, \ y \geq 0$

(ii) Domain: $x \geq 3$

(c) (i) $y = \tan^{-1}\frac{1}{x}, \ x \neq 0$

$y' = \dfrac{-x^{-2}}{1+\left(\frac{1}{x}\right)^2}$

$= \dfrac{-\frac{1}{x^2}}{\frac{x^2+1}{x^2}}$

2

$= \dfrac{-1}{x^2+1}$

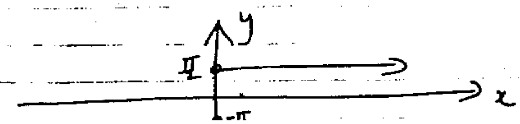
(ii) $\frac{d}{dx}\left[\tan^{-1}x + \tan^{-1}\frac{1}{x}\right]$

$= \frac{1}{1+x^2} + \left(-\frac{1}{1+x^2}\right)$

1

$= 0.$

(iii)



Qu-5.

(a) (i) $f(x)$ has a zero between 1 & 2.

$f'(x) = 3x^2 - 2x - 1$

then $x = 2 - \dfrac{1}{3(4) - 2(2) - 1}$

$= 2 - \frac{1}{7}$

$= 1\frac{6}{7}$

(ii) take $x = 1$ as 1^{st} approx^n

then $x = 1 - \dfrac{-2}{3(1)^2 - 2(1) - 1}$

$= 1 - \frac{2}{3-2-1}$

$= 1 - \frac{2}{0}$

Since $f'(1) = 0$, $f(x)$ has a stat. pt. at $x = 1$. Hence it is unsuitable as a first approx^n.

* and therefore tangent at $x = 1$ is parallel to x axis

Qu-5.

(b)

$f(x) = ax^3 + bx^2 + cx + d$

$f(1) = a + b + c + d = 0 \quad \text{--- (1)}$

$f(-1) = -a + b - c + d = -4 \quad \text{--- (2)}$

$f'(x) = 3ax^2 + 2bx + c$

$f'(-1) = 3a - 2b + c = 0 \quad \text{--- (3)}$

$f'(1) = 3a + 2b + c = 0 \quad \text{--- (4)}$

$(3) - (4) \Rightarrow -4b = 0$

$\therefore b = 0$

Subst for b in (1), (2) & (3)

$(1) \Rightarrow a + c + d = 0 \quad \text{--- (5)}$

$(2) \Rightarrow -a - c + d = -4 \quad \text{--- (6)}$

$(3) \Rightarrow 3a + c = 0 \quad \text{--- (7)}$

$(5) + (6) \Rightarrow 2d = -4$

$d = -2$

Subst. for d in (5)

$a + c - 2 = 0$

$a = 2 - c \quad \text{--- (8)}$

Subst (8) into (7)

$3(2 - c) + c = 0$

$6 - 3c + c = 0$

$6 - 2c = 0$

$2c = 6$

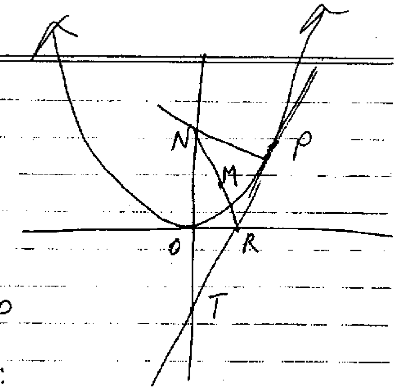
$c = 3$

$a + c - 2 = 0$

$a + 3 - 2 = 0$

$\therefore a = -1,\ b = 0,\ c = 3$ and $d = -2$

Qu-6



$x^2 = 4ay$

(a) $y = \frac{1}{4a} x^2$

$y' = \frac{1}{2a} x$

When $x = 2ap$, $y' = p$

Eqn of tangent at P is:

$p = \frac{y - ap^2}{x - 2ap}$

$px - 2ap^2 = y - ap^2$

$\therefore y = px - ap^2$

(ii) When $x = 0$

$y = 0 - ap^2 = -ap^2$

$\therefore T = (0, -ap^2)$

(iii) $m(\text{normal}) = -\frac{1}{p}$

Eqn of normal at P is:

$-\frac{1}{p} = \frac{y - ap^2}{x - 2ap}$

$-x + 2ap = py - ap^3$

$x + py - 2ap - ap^3 = 0$

(iv) When $x = 0$

$py = 2ap + ap^3$

$y = 2a + ap^2$

$\therefore N = (0, 2a + ap^2)$.

Qu 6 (contd)

(v) $\angle NPT = 90°$ (angle between tangent + normal is $90°$)

& an angle in a semicircle is $90°$

$\therefore$ NT is a diameter of the circle passing thro PTN

Centre of this circle $= \left(0, \frac{2a + ap^2 - ap^2}{2}\right)$

$= (0, a)$

Radius $= \frac{2a + ap^2 + ap^2}{2} = a + ap^2$

$\therefore$ Eqn of circle is:

$$x^2 + (y-a)^2 = (a + ap^2)^2$$

(vi) Eqn. of tangent at P is: $y = px - ap^2$

when $y = 0$, $px - ap^2 = 0$

$px = ap^2$

$x = ap$

$\therefore R = (ap, 0)$

$\therefore M = \left(\frac{0 + ap}{2}, \frac{2a + ap^2 + 0}{2}\right)$

$= \left(\frac{ap}{2}, \frac{2a + ap^2}{2}\right)$

(vii) $\begin{cases} x = \frac{1}{2}ap \\ y = a + \frac{1}{2}ap \end{cases}$

Subst. $p = \frac{2x}{a}$ into y

$y = a + \frac{1}{2}a\left(\frac{2x}{a}\right)^2$

$= a + \frac{1}{2}a\left(\frac{4x^2}{a^2}\right)$

$= a + \frac{2x^2}{a}$

$2x^2 = a(y-a)$

$x^2 = \frac{a}{2}(y-a)$

$\therefore$ locus of M is a parabola with vertex at $(0, a)$ & a focal length at $\frac{1}{8}a$