
Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2012
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Morning Session
Friday, 10 August 2012

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided on a separate sheet
- Show all necessary working for Questions 11-14
- Write your Centre Number and Student Number at the top of this page and page 6

Total marks – 70

Section I Pages 2–5

10 marks
- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II Pages 6–12

60 marks
- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks

Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Let $P(x) = 2x^3 - x^2 + 2$. Find the remainder when $P(x)$ is divided by $x + 1$.

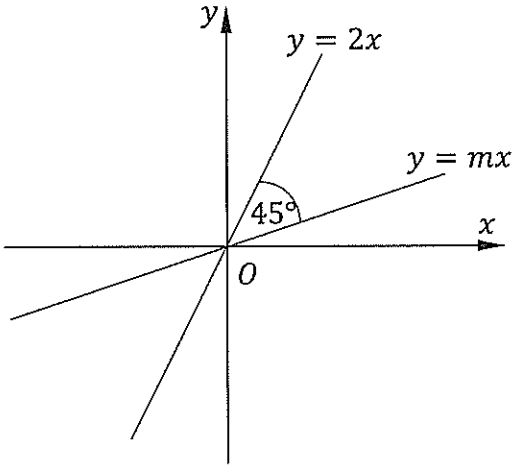
(A) -1

(B) 1

(C) 3

(D) 5

2 The angle between the lines $y = mx$ and $y = 2x$ is 45°, where $m > 0$, as shown in the diagram below.



NOT TO SCALE

Find the value of m.

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) 1

(D) 3

3 Let $t = \tan\frac{\theta}{2}$ where $0 < \theta < \pi$. Which of the following gives the correct expression for $\sin\theta + \cos\theta$?

(A) $\dfrac{3t-1}{1+t}$

(B) $\dfrac{2t-1+t^2}{1+t^2}$

(C) $\dfrac{1+2t-t^2}{1+t^2}$

(D) $\dfrac{t^2-1-2t}{1+t^2}$

4 Let A be the point $(-2,3)$ and B be the point $(3,-4)$. Find the coordinates of the point which divides AB externally in the ratio 3: 2.

(A) $\left(0,\frac{1}{5}\right)$

(B) $\left(1,-\frac{6}{5}\right)$

(C) $(-12,17)$

(D) $(13,-18)$

5 From six girls and four boys, a committee of 3 girls and 2 boys is to be chosen. How many different committees can be formed?

(A) 26

(B) 120

(C) 252

(D) 1440

6 Consider the function $f(x) = \frac{2x}{x+1}$ and its inverse function $f^{-1}(x)$.
Evaluate $f^{-1}(3)$.

(A) -3

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) 3

7 The equation of the normal to the parabola $x^2 = 4ay$ at the variable point $P(2ap, ap^2)$ is given by $x + py = 2ap + ap^3$.

How many different values of p are there such that the normal passes through the focus of the parabola?

(A) 0

(B) 1

(C) 2

(D) 3

8 An advertisement claims that '8 out of 10 people prefer Winky Chocolate Bars'. If the advertisement's claim is accurate and a sample of six people is interviewed, what is the probability that at least five people prefer Winky Chocolate Bars?

(A) $1 - (0.8)^6$

(B) $2(0.8)^5$

(C) $5(0.2)^5$

(D) $(0.8)^5(0.2) + (0.8)^6$

9 What is the coefficient of x^2 in the expansion of $\left(x^2 + \frac{2}{x}\right)^7$?

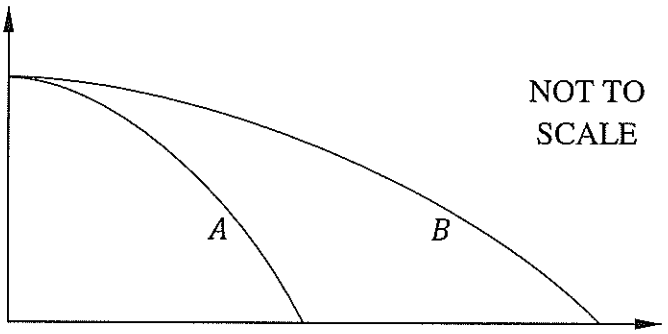
(A) 1

(B) 16

(C) 35

(D) 560

10 Two balls, A and B, are rolled horizontally off a 10 metre cliff at $10\ \text{ms}^{-1}$ and $20\ \text{ms}^{-1}$ respectively.



Which of the following statements is FALSE?

(A) A and B are in the air for the same length of time.

(B) A and B are travelling with the same vertical speed on impact.

(C) B is travelling at twice the speed of A on impact with the ground.

(D) B lands twice as far from the base of the cliff as A.

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2012 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Centre Number

Mathematics Extension 1

Student Number

Section II
60 marks

Attempt Questions 11–14
All questions are of equal value.
Allow about 1 hour and 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\lim_{x\to 0}\dfrac{\sin 2x}{x}$. **1**

(b) Use the table of standard integrals to evaluate $\displaystyle\int_0^4 \frac{dx}{\sqrt{x^2+9}}$. **2**

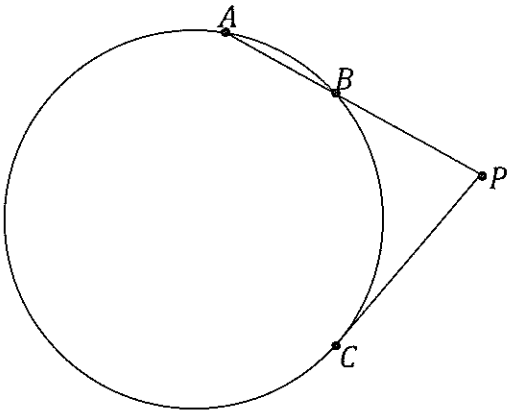
(c) Use the substitution $u = x - 8$ to find $\displaystyle\int_8^{8.5} \frac{dx}{\sqrt{(7-x)(x-9)}}$. **3**

(d) Solve $\dfrac{2t}{1-t} \geq t$. **3**

Question 11 continues on page 7

Question 11 (continued)

(e)



NOT TO SCALE

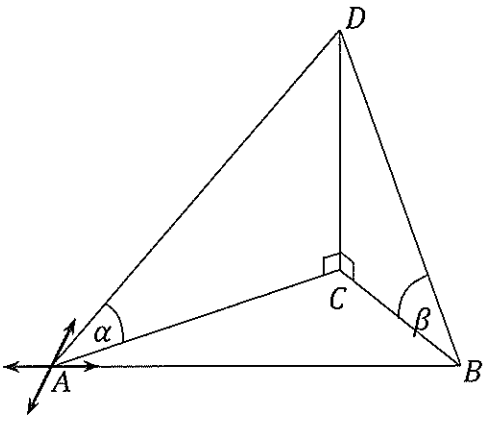
In the diagram the points A, B and C lie on the circle and AB produced meets the tangent from C at the point P.

(i) Given that $PC = 12$, $AB = 7$ and $PB = x$, find x. **2**

(ii) BC is the diameter of the circle passing through P, B and C. **1**

Find the length of BC.

(f) **3**



A vertical pole, CD, is positioned so that the angles of elevation of the top of the pole from the points A and B on the ground are α and β respectively.

The ground is a level horizontal surface and the triangle ABC is right-angled at C. Point B is due east of point A and point C is on a bearing of 060°T from point A.

Show that $\dfrac{\tan\alpha}{\tan\beta} = \dfrac{1}{\sqrt{3}}$.

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Consider the cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$ where a, b, c and d are real numbers and $a \neq 0$. Let α, β and γ be zeros of $f(x)$.

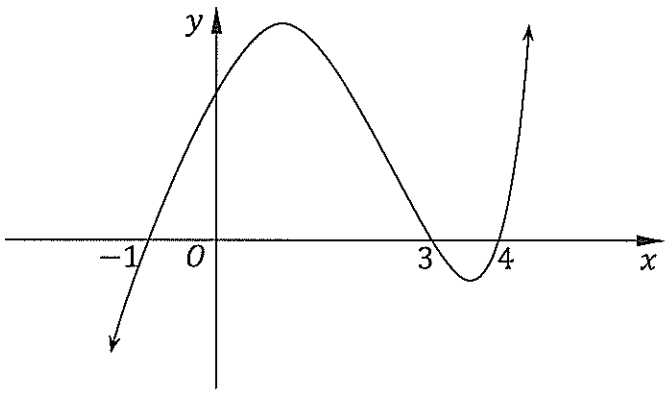
(i) Write down an expression for $\alpha + \beta + \gamma$. **1**

All cubic polynomial functions have a single point of inflexion when the second derivative is equal to zero.

(ii) Using part (i), or otherwise, show that the x-coordinate of the point of inflexion on the curve $y = f(x)$ is given by **2**

$$x = \frac{\alpha + \beta + \gamma}{3}.$$

(iii) The cubic polynomial below has x-intercepts at -1, 3 and 4. Find the x-coordinate of the point of inflexion of the cubic polynomial. **1**



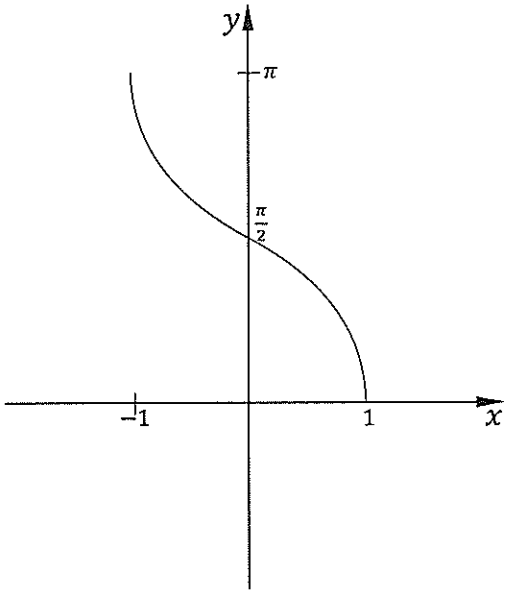
Question 12 continues on page 9

Question 12 (continued)

(b) (i) Given $\sin^{-1}x = \alpha$ and $\cos^{-1}x = \frac{\pi}{2} - \alpha$, solve $\sin^{-1}x = \cos^{-1}x$. **2**

(ii) The curve $y = \cos^{-1}x$ is sketched below. **1**

Copy the diagram into your writing booklet and draw a sketch of the curve $y = \sin^{-1}x$ on the same set of axes. Clearly show the point of intersection of the two curves.



The region bounded by $y = \sin^{-1}x$, $y = \cos^{-1}x$ and the y-axis is rotated about the y-axis to form a solid of revolution.

(iii) Explain why the volume V of the solid formed is given by: **2**

$$V = 2\pi \int_0^{\frac{\pi}{4}} \sin^2 y \, dy.$$

(iv) Hence, find the volume V of the solid formed. **2**

(c) (i) Use mathematical induction to prove that for $n \geq 2$ **3**

$$\left(1 - \frac{1}{2^2}\right) \times \left(1 - \frac{1}{3^2}\right) \times \left(1 - \frac{1}{4^2}\right) \times \ldots \times \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}.$$

(ii) Hence evaluate $\frac{3}{4} \times \frac{8}{9} \times \frac{15}{16} \times \ldots \times \frac{9999}{10000}$. **1**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) When an egg is placed in a pot of boiling water the rate at which the temperature, T (in degrees Celsius), of the egg increases after t minutes is given by $\frac{dT}{dt} = 0.2(100 - T)$.

(i) Show that $T = 100 - Ae^{-0.2t}$ satisfies this equation, for some constant A. **1**

(ii) Find the value of the constant A, for an egg taken from the refrigerator with an initial temperature of 4°C. **1**

(iii) It is known from experience that it will take $4\frac{1}{2}$ minutes for an egg taken from the refrigerator to cook. **1**

Determine the temperature of the egg after $4\frac{1}{2}$ minutes. Give your answer correct to 3 significant figures.

(iv) If an egg is initially at room temperature, the temperature of the egg can be modelled by the equation $T = 100 - 79e^{-0.2t}$. **2**

How much less time will it take for an egg initially at room temperature to reach the temperature of part (iii)?

(b) A particle moves in Simple Harmonic Motion. Initially, the particle is 4 metres to the right of the origin, moving with a velocity of $-8\sqrt{3}$ ms^{-1}. The displacement x is given by $x = A\cos(2t + \alpha)$ for some constants A and α.

(i) Find the values of A and α. **3**

(ii) When does the particle first reach the centre of motion? **2**

(c) The acceleration of a particle P is given by the equation $\frac{d^2x}{dt^2} = 2x^3 + 18x$, where x is its displacement in metres from the origin after t seconds. Initially the particle is at the origin and has velocity 9 ms^{-1}.

(i) Show that the velocity of the particle is given by $v = x^2 + 9$. **3**

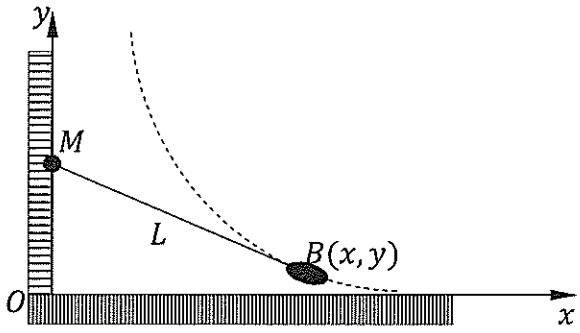
(ii) Hence, find an expression for x as a function of t. **2**

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) The equation $3\sin x = \ln x$ has a number of positive solutions, with the smallest solution being close to $x = 3$. **3**

Use ONE application of Newton's Method to find another approximation to the smallest positive solution. Give this approximation correct to TWO decimal places.

(b) A man M walks along a pier, represented by the positive y-axis, pulling on a boat B by a rope of length L. The man is initially at the origin O and the boat is initially on the x-axis, L metres from O. The man keeps the rope taut and the path followed by the boat is such that the rope is always tangent to the curve tracing its path.



(i) Let the path followed by the boat be the graph of the function $y = f(x)$. **1**
By considering the gradient of the line MB, show that

$$\frac{dy}{dx} = \frac{-\sqrt{L^2 - x^2}}{x}.$$

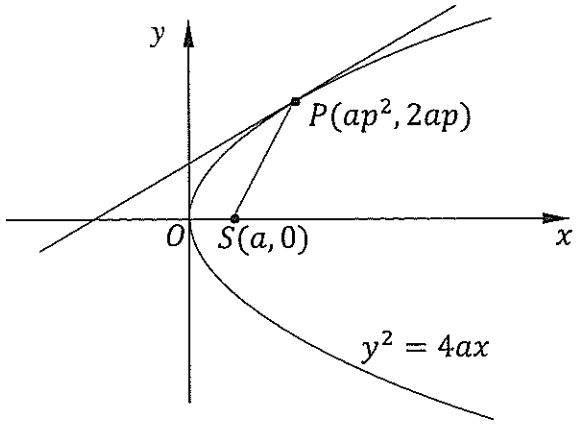
(ii) The man walks along the pier such that the boat moves in the y direction at a constant rate of $3\ \text{ms}^{-1}$. **2**

Find the rate $\frac{dx}{dt}$ at which the boat approaches the pier, when it is a distance $\frac{L}{2}$ metres horizontally from the pier.

Question 14 continues on page 12

Question 14 (continued)

(c) The tangent at the point $P(ap^2, 2ap)$ on the parabola $y^2 = 4ax$, with the focus $S(a, 0)$ is shown in the diagram below. **3**



The gradient function is given by $\frac{dy}{dx} = \frac{2a}{y}$. (Do NOT prove this.)

Prove that the tangent to the parabola at P is equally inclined to the axis of the parabola and the focal chord through P.

(d) (i) Show that $\binom{n}{r} = \binom{n}{n-r}$ for $r = 0, 1, 2, \ldots, n$. **1**

(ii) Let $f(r) = \binom{n}{0}\binom{n}{r} + \binom{n}{1}\binom{n}{r+1} + \cdots + \binom{n}{n-r}\binom{n}{n}$ for $r = 0, 1, 2, \ldots, n$. **3**

By considering the coefficient of x^{n-r} in the expansions of $(1+x)^{2n}$ and $(1+x)^n(1+x)^n$ show that

$$f(r) = \binom{2n}{n-r}$$

(iii) Show that $\binom{n}{0}f(0) + \binom{n}{1}f(1) + \cdots + \binom{n}{n}f(n) = \binom{3n}{n}$. **2**

End of Paper

Examiners

Gerry Sozio (Convenor)	St Mary Star of the Sea College, Wollongong
Jenny Bell	St Joseph's Catholic High School, Albion Park
Frank Reid	University of New South Wales, Australian Catholic University
Thanom Shaw	SCEGGS, Darlinghurst
Greg Wagner	Kesser Torah College, Dover Heights