JRAHS 2006 TRIAL HSC - EXT I

Question 1. **Marks**

(a) Solve for x: $\dfrac{1}{x-2} \geq 2.$ **3**

(b) Find: $\lim\limits_{h \to 0}\left(\dfrac{\cos 2h - 1}{h}\right).$ **2**

(c) The point P divides $A(-1, 5)$ and $B(3, -2)$ in the ratio $r : 1$.

(i) Find the coordinates of P in terms of r. **2**

(ii) Find the value of r when the line $2x - 3y + 4 = 0$ intersects the interval AB. **2**

(d) Evaluate $\displaystyle\int_0^1 (x^2 + 1)^3 \, dx.$ **3**

Question 2. **[START A NEW PAGE]**

(a) A plate is initially heated to 55^0 *C,* and it then cools to 41^0 *C* in 10 minutes.
If the surrounding temperature, S^0 *C*, is 22^0 *C* and assuming Newton's Law of Cooling:

$$\frac{dT}{dt} = -k(T - S).$$

(i) Find the temperature of the plate 25 minutes from the start of cooling (to 1 decimal place). **3**

(ii) Find the time for the plate to cool to 25^0 *C* (to 1 decimal place). **2**

(iii) Sketch the graph of the rate of temperature, $\dfrac{dT}{dt}$, versus the temperature T. **1**

(b) The displacement x metres of a particle after t seconds, is given by:

$$x = 5\sin 3t - 7\cos 3t.$$

(i) Show that the motion of the particle is SHM. **2**

(ii) Find the maximum displacement. **1**

(iii) Find the time when the particle first passes through the centre of motion (correct to 1 decimal place). **2**

(iv) Sketch the graph of the acceleration $\ddot{x}$ versus displacement x. **1**

Question 3. **[START A NEW PAGE]** **Marks**

(a) Differentiate $\cos^{-1}\left(-\frac{1}{x}\right)$ with respect to x. Answer in simplified form. **3**

(b) (i) On the same set of axes, sketch the graphs of $y = \sin^{-1} x$ and $y = \tan^{-1} x$. **2**

(ii) Given that: $\int_0^1 \sin^{-1} x dx = \frac{\pi}{2} - 1$, find the area of the region bounded by $y = \sin^{-1} x$, $y = \tan^{-1} x$ and $x = 1$. **3**

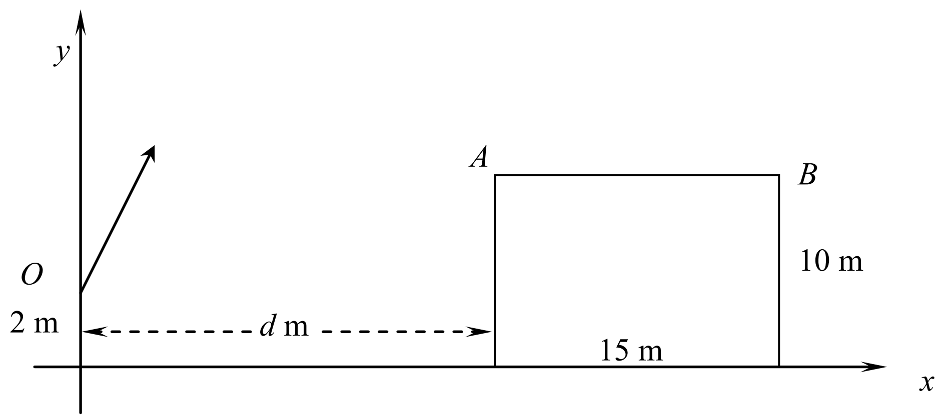
(c) (i) Show that $y = e^{-x}\sin 2x$ is a solution to the differential equation: **3**

$$y'' + 2y' + 5y = 0.$$

(ii) Hence, or otherwise, find $\int e^{-x}\sin 2x\, dx$. **1**

Question 4. **[START A NEW PAGE]**

(a) A fire truck arrives at a burning building 10 metres high and 15 metres wide. The water nozzle hose on the fire truck is 2 metres above the ground and d metres from the building, as shown in the diagram.



The angle of elevation of the hose, α, can be adjusted to range from 10^0 to 45^0. The parametric equations for the water particles from the nozzle are given by:

$x = 30t\cos\alpha$ and $y = 30t\sin\alpha - 5t^2$, where t is the time in seconds when $g = 10$.

(i) Show that the trajectory path of the water is given by the equation: **1**

$$y = x\tan\alpha - \frac{x^2}{180}(1 + \tan^2\alpha).$$

(ii) The hose nozzle is adjusted to an angle of elevation of 45^0. **2**
Find the distance, d, from the building if the water is to reach the furthest point B on top of the building as shown (answer to the nearest centimetre).

Q 4 *continues over the page*

Q 4 part (a) *continued* **Marks**

(iii) Find the angle of elevation α of the nozzle, for the water to reach position A, when the hose nozzle is 20 metres from the burning building (answer to nearest minute). **2**

(b) Find $\int \frac{4x-7}{2x^2+1}\,dx.$ **3**

(c) (i) For $t > 0$, find the limiting sum of: $e^{-t} + e^{-2t} + e^{-3t} + \ldots$ **1**

(ii) Hence, find an expression for the series; $e^{-t} + 2e^{-2t} + 3e^{-3t} + \ldots$ **1**

(d) A semi-circle of radius r has the equation: $y = \sqrt{r^2 - x^2}$.

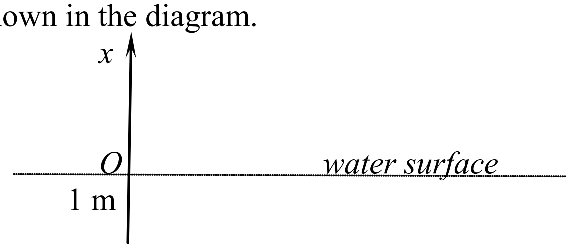
(i) Find $\frac{dy}{dx}$ at the point $P(x, y)$. **1**

(ii) Prove that the tangent, at any point P on the semi-circle, is perpendicular to the radius. **1**

Question 5. **[START A NEW PAGE]**

(a) Find the greatest coefficient in the expansion of $(4x+5)^{11}$. **3**
(Leave the answer in index form).

(b) A ping pong ball is initially placed 1 metre beneath the surface of the water, as shown in the diagram.



The ping pong ball is released in the water with an acceleration of $\ddot{x}$ m/s^2, where $\ddot{x} = -625x$, and where x metres is the displacement of the motion measured from the water surface.

(i) Is the motion of the ping pong ball only SHM? Give reasons. **1**

(ii) Prove that: $\frac{d}{dx}\left(\frac{v^2}{2}\right) = \ddot{x}.$ **2**

(iii) Find the expression for the ping pong ball's velocity v m/s when it is in the water. **2**

(iv) Find the velocity of the ball at the water's surface. **1**

(v) Assuming there is no air resistance and the acceleration due to gravity is 10 m/s^2, derive an expression for the displacement in air in terms of v **2**

(vi) Find the maximum height that the ping pong ball reaches above the surface of the water. **1**

Question 6. **[START A NEW PAGE]** **Marks**

(a) How many groups of 2 men and 2 women can be hosen from 6 men and 8 women? **2**

(b) Six letter words are formed from the letters of the word ***CYCLIC***.

(i) How many different 6-letter words can be formed? **2**

(ii) How many 6 letter words can be formed, if no '***C***'s are together? **2**

(iii) What is the probability of all the '***C***'s together, if it is known a vowel is at the end? **2**

(c) Prove, by the method of mathematical induction that: **4**

$$\sin q + \sin 3q + \sin 5q + \ldots + \sin(2n-1)q = \frac{1-\cos 2nq}{2\sin q}, \text{ for } n = 1, 2, 3, \ldots.$$

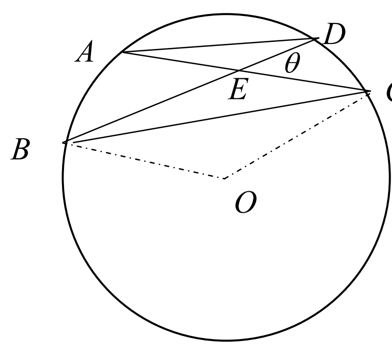
Question 7. **[START A NEW PAGE]**

(a) At the end of each month, for 15 years, a man invests \$400 at an interest rate Which is paid monthly at 6% *pa*.

(i) Show that the value of his first payment, at the end of 15 years, is \$976.75 **2**

(ii) Find the value of the man's total investment at the end of the 15 years. **2**

(b) A circle, centre O with a constant radius r, is such that the chords AC and BD intersect at point E, $\angle CED = \theta$ radians and $\angle BOC = \dfrac{2\pi}{3}$ radians, as shown the diagram.



Not to scale

(i) Show that the sum of the arcs AB and CD equal $2r\theta$, give reasons. **3**

(ii) Show that the perimeter P of the shape $ABCD$, where BC, AD are chords and CD, AB are arc lengths, is given by: **2**

$$P = r\left(2\theta + \sqrt{3} + 2\sin\left(\frac{\pi}{3} - \theta\right)\right).$$

(iii) Find the value of θ, in the domain $0 \le \theta \le \dfrac{\pi}{2}$ for the perimeter of $ABCD$ to have a maximum value. Justify your answer. **3**