QUESTION 1 (9 Marks)

Marks

(a) In a set of 7 letters, some of the letters are T 's and all other letters are different. If the number of different arrangements of these letters is 210, how many letters are T 's. **2**

(b) In a colony of bacteria, the rate of change of the colony is given by:

$$\frac{dP}{dt} = kP - r,$$

where P is the number of bacteria at time t minutes, r is the constant rate per minute at which the bacteria die and k is a constant.

(i) Verify that $P = \frac{r}{k} - \frac{A}{k}e^{kt}$ is the solution to the rate equation $\frac{dP}{dt} = kP - r$, given A is a constant. **2**

(ii) Find the time when the population of the bacteria colony is reduced to zero, given that when $t = 0$, $P = 5000$, $k = 0 \cdot 2$ and $r = 1500$. Give your answer to the nearest second. **3**

(iii) Find P when $t = 2$, (answer to the nearest bacteria). **2**

QUESTION 2 (9 Marks) START A NEW PAGE

Marks

(a) The velocity v cms^{-1} of a particle is given by $v = 2x + 5$. If the initial displacement is 1cm to the right of the origin, find the displacement as a function of time. **3**

(b) (i) A Brine solution contains 1kg of salt per 10 litres. **2**
It runs into a tank, initially filled with 500 litres of fresh water, at a rate of 25 litres per minute. At the same time, the mixture runs out of the tank at the same rate.
If A kg is the amount of salt in the tank at time t minutes,
Explain why: $\frac{dA}{dt} = 2.5 - \frac{A}{20}$.

(ii) Find the amount of salt in the tank at the end of 60 minutes, assuming the mixture is kept homogenous (to the nearest 10 grams). **3**

(iii) Find the maximum concentration of salt in the mixture. **1**

QUESTION 3 (9 Marks)   START A NEW PAGE

Marks

(a) Sixteen of the chickens on the James Ruse School Farm are separated at random into 4 pens of 4 chickens for a feed trial. **3**
What is the probability that 4 particular chickens, *A, B, C* and *D* are in 4 separate pens?

(b) The velocity of a body, v ms^{-1}, moving in a straight line is given as $v = e^t - e^{-t}$, where t is the time in seconds.
The initial position of the body is at the origin.

(i) Find the displacement x as a function of time t. **2**

(ii) Find the acceleration when $t = 2$. **2**
Give your answer correct to 2 decimal places.

(iii) Show that the body does not have a zero acceleration. **2**

QUESTION 4 (9 Marks)   START A NEW PAGE

Marks

The depth of water in y metres on a tidal creek is given by:

$$4\frac{d^2y}{dt^2} = 5 - y$$, where time t is measured in hours.

(i) Prove that the vertical motion of the water level is simple harmonic and hence find the centre of motion. **2**

(ii) Find the period of the motion. **1**

(iii) Given that $y = 2$ at low tide and $y = 8$ at high tide, and that $y = a + b\cos nt$ is the solution of the equation: $4\ddot{y} = 5 - y$, write down the values of a, b and n. **3**

(iv) If the low tide is at 10 am, what is the earliest time after low tide that a fishing boat requiring a depth of 4 metres of water can enter the creek? **3**

QUESTION 5 (9 Marks)   START A NEW PAGE

Marks

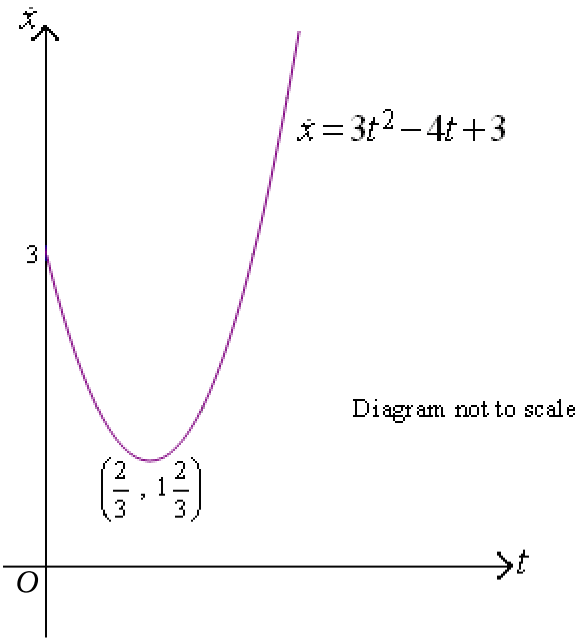
(a) Calculate the number of arrangements of the letters ***DESCARTES***:

(i) If the two ***S***'s are adjacent. **1**

(ii) If no two vowels are together. **2**

(iii) If the conditions from part (i) and (ii) hold simultaneously. **2**

(b) The graph below illustrates the velocity of a particle as a function of time.



(i) Sketch the graph of the particle to illustrate the acceleration as a function of time, given that the particle is initially 1 m to the left of the origin O. **2**

(ii) Hence write a description of the motion. **2**

QUESTION 6 (9 Marks) START A NEW PAGE

Marks

(a) The velocity v ms^{-1} of a particle moving along the x-axis is given by: $v = \sqrt{2 + 2\cos 2x}$. Initially the particle is located at the origin.

(i) Find the initial velocity and acceleration. **3**

(ii) Assuming that the particle reaches the position of $\frac{\pi}{2}$ metres from the origin, determine what would happen to the particle after this time. **2**

(b) In a certain experiment recording the number of bees N pollinating flowers in a given area, it was found that the rate of change of N is given by: $\frac{dN}{dt} = kN\left(1 - \frac{N}{2000}\right)$,
where t is the time in days and k is a constant.
At the beginning of the experiment 1000 bees were introduced to the area.

(i) Verify that $N = \frac{2000}{1 + e^{-kt}}$ is the solution of the equation. **2**

(ii) If N =1500 when t = 10, determine the time in days, when N = 1800. **2**

QUESTION 7 (9 Marks) START A NEW PAGE

Marks

(a) A shell is detonated on level ground throwing fragments with a speed V ms^{-1} in all directions.
After a time T, a fragment hits the ground at a distance M from the shell.
You may assume these parametric equations of motion:

$$x = Vt\cos\alpha \text{ and } y = Vt\sin\alpha - \frac{1}{2}gt^2$$

(i) Show that: $g^2T^4 - 4V^2T^2 + 4M^2 = 0$. **2**

(ii) Hence find, to 2 decimal places, the shortest period of time during which a man, standing 20 metres from the place where the shell bursts, is in danger when V = 25. Take g = 10. **3**

(b) Twelve politicians are seated at a round table. A committee of five is to be chosen. If each politician, for one reason or another, dislikes their immediate neighbours and refuses to serve on a committee with them, in how many ways can a compatible group of five politicians be chosen? **4**

END OF EXAMINATION