QUESTION 1:

(a) Solve $\dfrac{x}{x-3} > 10$

(b) Solve, for $0^\circ \le x \le 360^\circ$,
$\sin x + \cos x + 1 = 0$

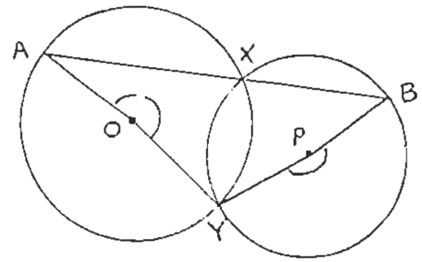
(c) Find the acute angle between the lines $5x + 4y + 3 = 0$ and $3x + 8y - 1 = 0$

(d) If A and B are the points (-3, -4) and (2, -1), find the coordinates of the point P dividing AB externally in the ratio 4:7

(e) Show that $(x-3)$ is a factor of $2x^3 - 11x^2 + 12x + 9$ and hence find the factors of this polynomial.

QUESTION 2:

(a)



O and P are the centres of the circles; AXB is a straight line.
Prove $\angle AOY = \angle BPY$.

(b) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$. PQ subtends a right angle at the vertex O.

(i) Show that $pq = -4$

(ii) Prove that the equation of the normal at P is given by $x + py = 2ap + ap^3$

(iii) Write down the equation of the normal at Q, and hence determine the point of intersection, R, of these normals.

(iv) Find the equation of the locus of R and describe it geometrically.

QUESTION 3:

(a) Find $\int x\sqrt{3+x^2}\,dx$, using the substitution $u = 3 + x^2$

(b) Find $\int_0^{\pi} 2\sin^2 x\,dx$

(c) Assume that the rate at which a body warms in air is proportional to the difference between its temperature T and the constant temperature A of the surrounding air.

This rate can be expressed by the differential equation $\dfrac{dT}{dt} = K(T - A)$

Where t is the time in minutes and K is a constant.

(i) Show that $T = A + Ce^{kt}$, where C is a constant, is a solution of the differential equation.

(ii) A cooled body warms from 10^0 C to 15^0 C in 20 minutes. The air temperature around the body is 28^0 C. Find the temperature of the body after a further 20 minutes have elapsed. Give your answer to the nearest degree.

(iii) By referring to the equation for T, explain the behaviour of T as t becomes large.

QUESTION 4:

(a) The acceleration of a body P is given by $a = 18x\left(x^2+1\right)$ where x cm is the displacement at time t sec. Initially P starts from the origin with velocity 3cm/s

(i) Show that $v = 3\left(x^2+1\right)$

(ii) Find x in terms of t.

(b) A ball is projected from a horizontal plane with initial velocity V m/s and angle of projection α where $\tan\alpha = \frac{3}{4}$. The ball just clears a wall which is 27m high and 96m from the point of projection. Let g, the acceleration due to gravity = 10 m/s^2

(i) Show that the horizontal and vertical displacements are given by $x = \frac{4}{5}Vt$ and $y = \frac{3}{5}Vt - 5t^2$

(ii) Find the time to reach the wall in seconds.

(iii) Show that the speed of projection is 40 m/s.

(iv) Find the greatest height to which the ball will rise above the plane.

QUESTION 5:

(a) A particle moves along the *x-axis* with acceleration, $\ddot{x} = 4\cos 2t$. If the particle is initially at rest at the origin O, find expressions for

(i) the velocity v in terms of t

(ii) the position x in terms of t

(iii) Express $\ddot{x}$ in terms of x, and hence show that the motion is simple harmonic.

(iv) Find the centre and period of the motion.

(v) Sketch the graph of x in terms of t for $0 \le t \le \pi$

(b) (i) Write down a primitive function of $e^{f(x)} \bullet f'(x)$

(ii) Hence, evaluate $\int_0^1 \frac{e^{\cos^{-1} x}}{\sqrt{1-x^2}}$ dx (Leave your answer in exact form)

(c) Find the inverse function f^{-1} of the function f, defined by $f(x) = 2\log_e x + 3$. Express the result in the form y in terms of x.

QUESTION 6:

(a) Prove by induction that $n(n+3)$ is divisible by 2 for all positive integers n.

(b) Find the term independent of x in the expansion of $\left(2x^2 + \frac{1}{x}\right)^{12}$

(c) Find the relationship between p, q, r if the roots of the equation $x^3 + px^2 + qx + r = 0$ are in an arithmetic progression.

QUESTION 7:

(a) Use Newton's Method once and a first approximation of x =2 to solve $x^2 - 2 - \sqrt{x} = 0$ to 2 dec. places.

(b) A right circular cone with vertex downwards and semi-vertical angle 60^0 is being filled with water.

(i) Show that when the height of the water in the cone is h cm, then the volume of water is $\pi h^3 cm^3$

(ii) If the height of the water is increasing at the constant rate of $\frac{1}{2}$ cm/s, find the rate of increase of the volume when the height is 6 cm.

(c) (i) Prove that $\dfrac{2}{(x^2+1)(x^2+3)} = \dfrac{1}{x^2+1} - \dfrac{1}{x^2+3}$

(ii) Hence determine the value of $\int_1^{\sqrt{3}} \dfrac{dx}{(x^2+1)(x^2+3)}$

Answers

1(a) $3<x<3\frac{1}{3}$

(b) 180° or 270°

(c) 30.78°(2dp)

(d) $(-9\frac{2}{3}, -8)$

(e) $(x-3)(x-3),(2x+1)$

2(b)(iii) $x+qy=2aq+aq^3$

$(4a(p+q), a(p^2+q^2-2)))$

(iv) $x^2=16a(y-6a)$

parabola, focal length $=4a$

vertex $=(0,6a)$, focus $=(0,10a)$

directrix : $y=2a$

3(a) $\frac{1}{3}(3+x^2)^{3/2}+c$

(b) π

(c) (ii) 19°

(iii) As $t\to\infty$, $T\to 28°C$

4(a)(ii) $x=\tan 3t$

(b) (ii) 3 s

(iv) 28.8 m

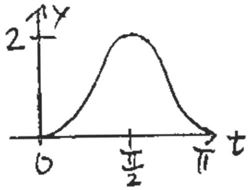
5(a) (i) $v=2\sin 2t$

(ii) $x=1-\cos 2t$

(iii) $\ddot{x}=-4(x-1)$

(iv) $x=1$, π s

(v)



5(b)(i) $e^{f(x)}+c$

(ii) $e^{\pi/2}-1$

(c) $f^{-1}(x)=e^{\frac{x-3}{2}}$

6(b) 7920

(c) $2p^3=9pq-27r$

7(a) 1.84

(b) (ii) 54π cm³/s

(c) (ii) $\frac{\pi}{72}\{21-5\sqrt{3}\}$