SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2000

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Examiner: B. Dowdell

(a) State the domain and range of $4\sin^{-1} 3x$ 2

(b) Solve for x: $(x-2)^2 \leq 4$ 2

(c) Differentiate: 4

(i) $x\cos^{-1} 2x$

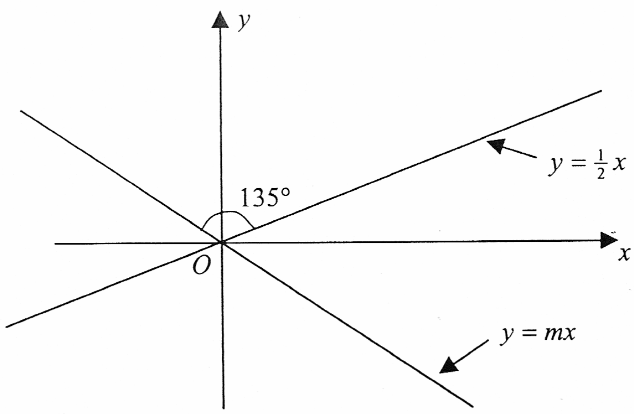
(ii) $\dfrac{1}{4+x^2}$

(d) Find x correct to 3 decimal places if $x^{\frac{3}{4}} = 10$ 2

(e) The point P (11, 7) divides AB externally in the ratio 3:1. If B is (6, 5), find the coordinates of A. 2

Question 2: START A NEW BOOKLET Marks

(a) 2



The angle between the lines $y = mx$ and $y = \frac{1}{2}x$ is 135°. Find the exact value of m.

(b) Using $u = \sqrt{x}$ evaluate $\int_1^4 \frac{dx}{x + \sqrt{x}}$ 2

(c) Write down the exact value of $\cos^{-1}(\cos \frac{4\pi}{3})$ 2

(d) Find a primitive of 4

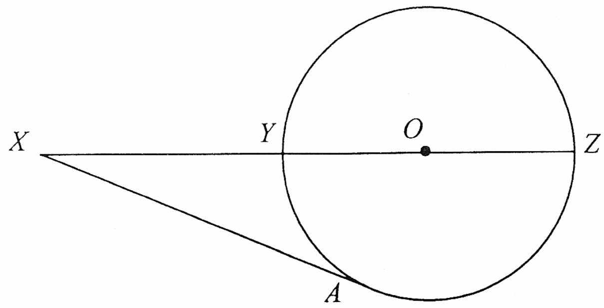
(i) $\dfrac{2}{\sqrt{1-4x^2}}$

(ii) $\dfrac{x}{4+x^2}$

(e) Find the values of a for which $f(x) = e^{-ax}(x-a)$ is stationary at $x = \frac{5}{2}$. 2

Question 3: START A NEW BOOKLET Marks

(a) 3



O is the centre of the circle, XA is a tangent.

$XY = 3$ and $XA = 5$

Calculate the size of $\angle AXY$ correct to the nearest minute.

(b) (i) Sketch the graphs of $y = e^x$ and $y = \cos x$ on the same diagram for $0 \le x \le \frac{\pi}{2}$, clearly showing any points of intersection. 4

Shade the area enclosed by the two curves and the line $x = \frac{\pi}{2}$.

(ii) Calculate the volume of the solid formed when this area is rotated about the x axis.

(c) (i) Prove that $\dfrac{d^2x}{dt^2} = \dfrac{d}{dx}\left(\frac{1}{2}v^2\right)$. 5

(ii) A particle moves in a straight line with velocity given by $v^2 = 36 - 4x^2$ where x is measured in metres and is the displacement from a fixed point O and t is the time measured in seconds.

(α) Show that the motion is simple harmonic

(β) Find the period and amplitude of the motion.

Question 4: START A NEW BOOKLET

(a) When $P(x) = ax^3 + bx + c$ is divided by $x - 1$ the remainder is -4. 3

When $P(x)$ is divided by $x^2 - 4$, the remainder is $-4x + 3$.

Find a, b and c.

(b) Prove by induction that 4

$$1 + (1+2) + (1+2+3) + \dots\dots\dots + (1+2+3+\dots+n) = \tfrac{n}{6}(n+1)(n+2)$$

for all positive integers n.

(c) (i) Show that the point $A\,(6p, 3p^2)$ lies on the parabola $x^2 = 12y$. 5

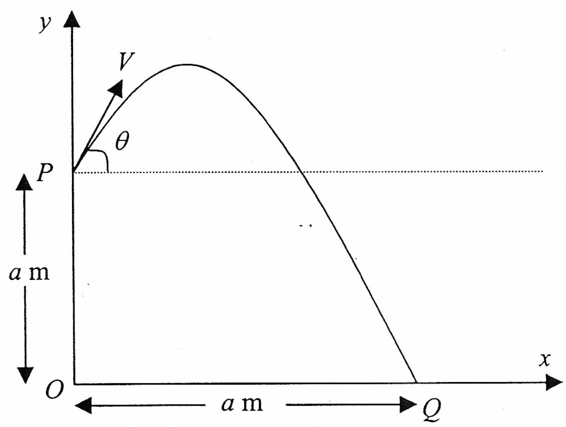
(ii) The chord joining $A\,(6p, 3p^2)$ and $B\,(6q, 3q^2)$, when produced, passes through $C\,(8, 0)$. Show that $4(p+q) = 3pq$ and hence find the locus of M, the midpoint of AB.

Question 5: START A NEW BOOKLET Marks

(a) Prove that $2\tan^{-1}\theta = \tan^{-1}\left(\dfrac{2\theta}{1-\theta^2}\right)$ provided that $|\theta| < 1$. 2

(b) A balloon is being filled with helium at a constant rate of 30 cm^3/s. Find the rate at which the surface area is increasing when its diameter is 40 cm. 4

(c) 6



A projectile is fired from a point P, a metres above O with an initial velocity V ms^{-1} at an angle of elevation of θ. It is subject to a constant downward acceleration of g ms^{-2}.

(i) Find expressions for the horizontal (x) and vertical (y) displacements from P after t seconds.

(ii) Show that the time taken to reach Q, a metres from O in a horizontal direction is given by $\dfrac{2V(\sin\theta + \cos\theta)}{g}$ seconds.

(iii) Show that $a = \dfrac{V^2(\sin 2\theta + \cos 2\theta + 1)}{g}$ metres.

Question 6: START A NEW BOOKLET

) Eight people attend a meeting. They are provided with two circular tables, one seating 3 people, the other 5 people. 4

(i) How many seating arrangements are possible?

(ii) If the seating is done randomly, what is the probability that a particular couple are on different tables?

) If $f(x) = u(x) - \ln(u(x)+1)$ 4

(i) Show that $f'(x) = \dfrac{u(x).u'(x)}{1+u(x)}$.

(ii) Hence or otherwise evaluate

$$\int_0^{\frac{\pi}{2}} \frac{\cos x.\sin x}{1+\sin x}\,dx$$

) A function $L(x)$ is defined by 4

$L(x) = Pe^{\frac{x}{3}} + Qe^{-\frac{2x}{3}}$ where P and Q are constants.

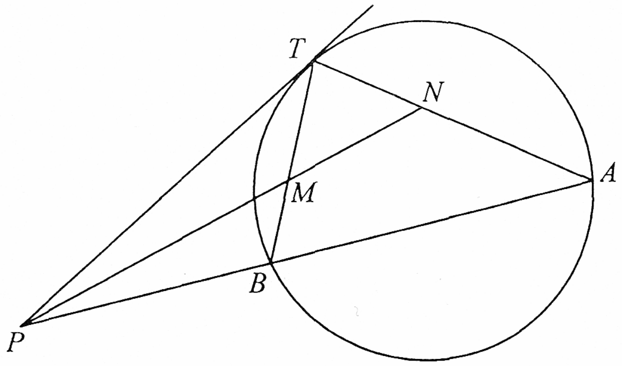
It is given that $L(0) = 30$ and $L'(0) = -14$.

(i) Find the values of P and Q.

(ii) Find $L'(3)$ and explain why $L(x)$ must have a minimum for some value of x between 0 and 3.

Question 7: START A NEW BOOKLET Mark

(a) 3



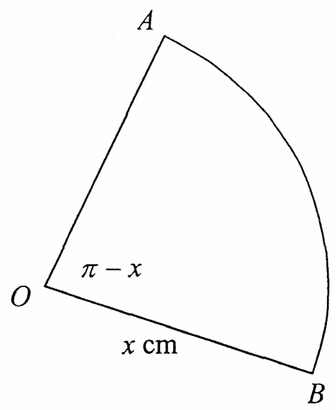
AB is any chord of a circle. AB is produced to P, and PT is a tangent. The bisector of $\angle APT$ meets TB at M and TA at N.

(i) Copy the diagram into your answer booklet.

(ii) Prove that ΔTMN is isosceles.

(b) 9

AOB is a sector of a circle, such that, when the radius is x cm, $\angle AOB = (\pi - x)$ radians and x varies from 0 to π.



(i) Find the maximum value of the perimeter of sector AOB. Comment on the minimum value of the perimeter of the sector.

(ii) If the area of **triangle** AOB is given by $t(x)$

(α) Show that $t(x) = \dfrac{x^2 \sin x}{2}$.

(β) Show that when $t(x)$ is a maximum, $2\tan x = -x$.

(γ) By sketching $y = \tan x$ and a suitable line, show that a solution to the equation in (β) is close to $x = \frac{3\pi}{4}$.

(δ) Taking $\frac{3\pi}{4}$ as a first approximation, use Newton's method once to obtain a better approximation (leave your answer in terms of π).

END OF PAPER