Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2002
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Afternoon Session
Friday 9 August 2002

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- All necessary working should be shown in every question

Total marks **(84)**

- Attempt Questions 1 – 7
- All questions are of equal value

Marks

Question 1 **Begin a new page**

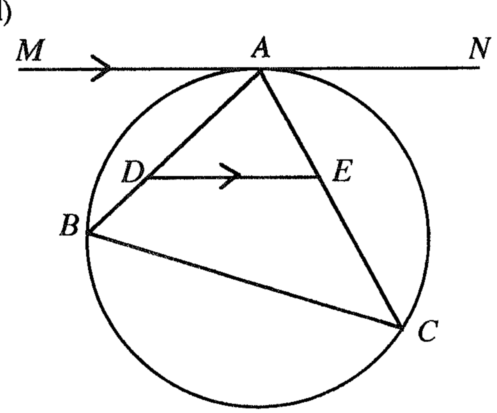
(a) Find $\frac{d^2}{dx^2}\ln\left(e^x+1\right)$ 2

(b) Evaluate $\sum_{k=1}^{4}\frac{(-1)^k}{k!}$ 2

(c) (i) Show that $\frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x$ 2

(ii) Hence find the value of $\tan 22\frac{1}{2}°$ in simplest exact form. 2

(d)



ABC is a triangle inscribed in a circle. MAN is the tangent to the circle at A. D is a point on AB and E is a point on AC such that $DE \parallel MAN$.

(i) Copy the diagram.

(ii) Explain why $M\hat{A}B = A\hat{C}B$. 1

(iii) Hence show that $BCED$ is a cyclic quadrilateral. 3

Question 2 **Begin a new page** **Marks**

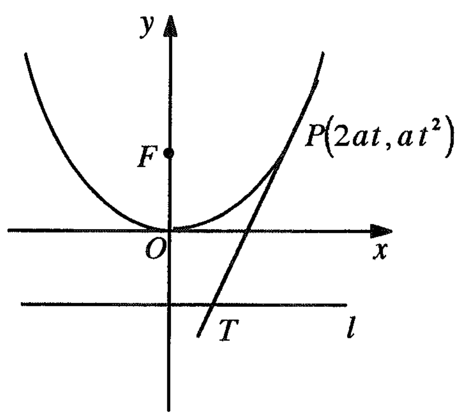
(a) $A(-2,3)$ and $B(4,-5)$ are two fixed points. Find the coordinates of the point $P(x, y)$ which divides the interval AB internally in the ratio $4:1$. **2**

(b) A test consists of 7 multiple choice questions all of which are to be attempted. Each question contains 4 alternative answers of which one and only one is correct. Find the number of ways in which the 7 questions can be attempted so that exactly 2 questions are answered correctly. **2**

(c) (i) Given that $x = 1$ is a zero of the polynomial $P(x) = x^3 - 3x + 2$, express $P(x)$ as a product of three linear factors. **2**

(ii) Hence solve the inequality $x^3 - 3x + 2 \leq 0$ **2**

(d)



The tangent $tx - y - at^2 = 0$ at the point $P(2at, at^2)$ on the parabola $x^2 = 4ay$ cuts the directrix l at T. F is the focus of the parabola.

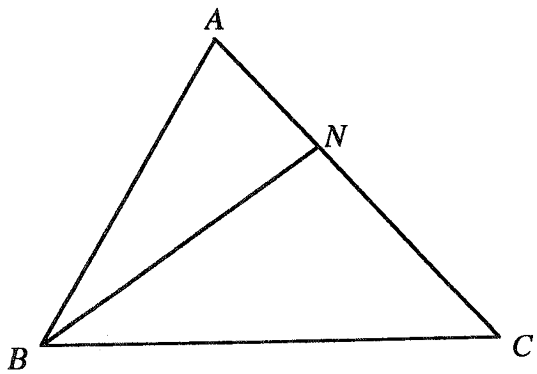
(i) Find the coordinates of T. **1**

(ii) Show that TF is perpendicular to PF. **3**

Marks

Question 3 **Begin a new page**

(a)



ABC is a triangle and N is a point on AC.
$A\hat{B}N = C\hat{B}N = B\hat{C}N$.
$BC = 2a, \quad CA = b, \quad AB = c$.
$BN = CN = d$.

(i) Given that ΔABN ||| ΔACB, show that $c^2 = b^2 - 2ac$. **3**

(ii) Hence show that $(a+c)^2 = a^2 + b^2$. **1**

(b) $P(x) = x^3 + 3x^2 + 6x - 5$

(i) Show that the equation $P(x) = 0$ has a root α such that $0 < \alpha < 1$. **2**

(ii) Use one application of Newton's method with a starting value of $x = 0 \cdot 5$ to find an approximation for α, giving the answer to 2 decimal places. **2**

(c) Use the substitution $x = u^2 \quad (u > 0)$ to find the exact value of $\displaystyle\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{\sqrt{x}\sqrt{1-x}}\, dx$. **4**

Question 4 **Begin a new page**

(a) $I = \displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x \; dx$

(i) Find the exact value of I. **2**

(ii) Use Simpson's rule with 3 function values to approximate the value of I. **2**

(b) The numbers $1, 2, 3, \ldots, 9$ are written one on each of 9 cards. 3 of the cards are chosen at random.

(i) Find the probability that the sum of the 3 numbers chosen is equal to 9. **2**

(ii) If it is known that the first number chosen is 2, find the probability now that the sum of the 3 numbers chosen is equal to 9. **2**

(c) A sperical map of the earth is being inflated at a constant rate of $25 \text{ cm}^3 s^{-1}$. Find the rate at which the length of the equator is changing when the radius is 10 cm. **4**

Marks

Question 5 **Begin a new page**

(a) $f(x) = \dfrac{2}{x+1}$, $x > -1$

(i) Find the equation of the inverse function $f^{-1}(x)$. **1**

(ii) On the same diagram sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ showing clearly the coordinates of any points of intersection, the intercepts on the coordinate axes, and the equations of any asymptotes. **3**

(b) A particle is moving in a straight line. O is a fixed point on the line. At time t seconds, the particle has displacement x metres fom O, its velocity v ms^{-1} is given by $v = -x^2$ and its acceleration is a ms^{-2}. Initially the particle is 1 m to the right of O.

(i) Find an expression for a in terms of x. **1**

(ii) Find an expression for x in terms of t. **3**

(c) In the binomial expansion of $\left(x^2 + \dfrac{a}{x}\right)^6$ the term independent of x is 240. Find the value of a. **4**

Question 6 **Begin a new page**

(a) (i) If $\theta = \tan^{-1} A + \tan^{-1} B$ show that $\tan\theta = \dfrac{A+B}{1-AB}$. **1**

(ii) Hence solve the equation $\tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}$. **3**

(b) At time t minutes, the temperature T ° Celcius of a piece of metal is given by $T = 20 + Ae^{-kt}$ for some constants A and k. Initially the temperature of the piece of metal is 100°C, and after 4 minutes its temperature has dropped to 80°C.

(i) Find the exact values of A and k. **2**

(ii) Find how much longer it will take for the temperature to drop to 60°C, giving the answer correct to the nearest second. **2**

(c) A particle moving in a horizontal straight line is performing Simple Harmonic Motion. At time t seconds its displacement x metres from a fixed point O on the line is given by $x = 3\cos 2t + \sin 2t$, where displacements to the right of O are positive.

Marks

Question 7 **Begin a new page**

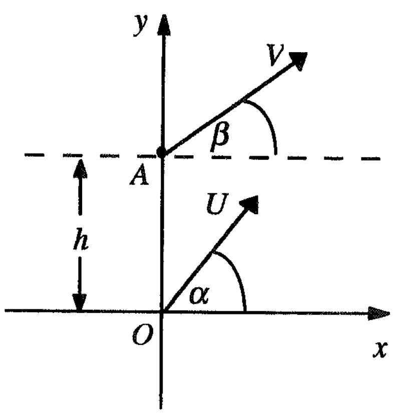
(a) A function $f(x)$ is such that $f(x) > 0$ for all real numbers x and $f(a+b) = f(a).\,f(b)$ for any real numbers a and b.

(i) Show that $f(0) = 1$ and deduce that $f(-x) = \dfrac{1}{f(x)}$. **2**

(ii) Use the method of mathematical induction to show that $f(nx) = [f(x)]^n$ for all positive integers n . **3**

(iii) *Without using mathematical induction again*, deduce that $f(-nx) = [f(x)]^{-n}$ for all positive integers n . **1**

(b)



A particle is projected from a point O with speed $U\,\text{ms}^{-1}$ at an angle of elevation α . At the same instant, another particle is projected from a point A (h metres directly above O) with speed $V\,\text{ms}^{-1}$ at an angle of elevation β , where $\beta < \alpha$. Both particles move freely under gravity in the same plane of motion and collide T seconds after projection.

(i) Write down expressions for the horizontal and vertical displacements of each particle at time t seconds referred to axes Ox and Oy. **2**

(ii) Show that $T = \dfrac{h\cos\beta}{U\sin(\alpha - \beta)}$. **4**