$$\frac{d}{dx}(\sec 2x) = 2\sec 2x \tan 2x$$ ✓ $\boxed{1}$

$$\log_m \frac{\sqrt{a}}{b^2c^3} = \frac{1}{2}\log_m a - 2\log_m b - 3\log_m c$$ ✓

$$= \frac{1}{2}\times 0.7 - 2\times 0.3 - 3\times 0.2$$ ✓ $\boxed{2}$

$$= -0.85$$

$$\int_e^{e^2} \frac{dx}{x\ln x} = \Big[\ln(\ln x)\Big]_e^{e^2}$$ ✓

$$= \ln(\ln e^2) - \ln(\ln e)$$ ✓ $\boxed{3}$

$$= \ln 2 - \ln 1$$

$$= \ln 2$$ ✓

Alt. Method:

$$\int_e^{e^2} \frac{dx}{x\ln x} = \int_1^2 \frac{du}{u}$$

$$= \Big[\ln u\Big]_1^2$$

$$= \ln 2 - \ln 1$$

$$= \ln 2$$

If $u = \ln x$,

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{dx}{x}$$

When $x = e$, $u = 1$
When $x = e^2$, $u = 2$

$$\lim_{x\to 0} \frac{\sin 2x}{5x} = \frac{2}{5}\lim_{2x\to 0}\frac{\sin 2x}{2x}$$ $\boxed{1}$

$$= \frac{2}{5}\times 1$$

$$= \frac{2}{5}$$

$$= \frac{1}{2}\int \sin 2x \, dx$$ ✓ $\boxed{2}$

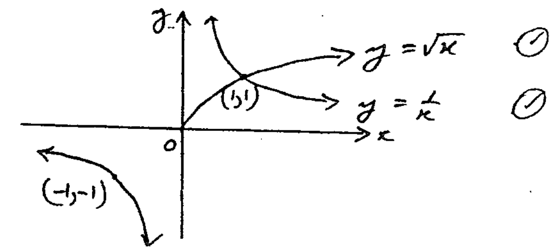
$$= -\frac{1}{4}\cos 2x + c$$ ✓

Alt. methods:

$$\int \sin x \cos x \, dx$$ or $$\int \sin x \cos x \, dx$$

$$= \frac{1}{2}\sin^2 x + c_1$$ $$= -\frac{1}{2}\cos^2 x + c_2$$

(f) (i)



$\boxed{3}$

(ii) $\frac{1}{x} \geq \sqrt{x}$ when $0 < x \leq 1$ ✓

$$= \int_0^{\sqrt{3}} \frac{2\,du}{u^2+1}$$ ✓

$$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\therefore 2\,du = \frac{dx}{\sqrt{x}}$$

When $x=0$, $u=0$
When $x=3$, $u=\sqrt{3}$ ✓

$$= 2\left[\tan^{-1}u\right]_0^{\sqrt{3}}$$ ✓

$$= 2\left[\tan^{-1}\sqrt{3} - \tan^{-1}0\right]$$

$$= \frac{2\pi}{3}$$ ✓

$\boxed{4}$

k) $$\int_0^{\frac{1}{6}} \frac{3\,dx}{\sqrt{1-9x^2}}$$

$$= \int_0^{\frac{1}{6}} \frac{3\,dx}{\sqrt{9\left(\frac{1}{9}-x^2\right)}}$$

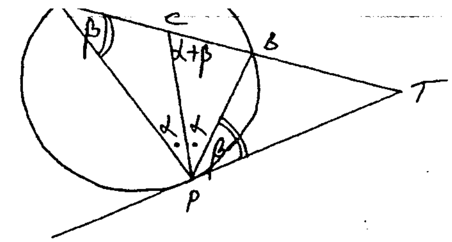
$$= \int_0^{\frac{1}{6}} \frac{dx}{\sqrt{\frac{1}{9}-x^2}}$$ ✓

$$= \left[\sin^{-1}3x\right]_0^{1/6}$$ ✓

$$= \sin^{-1}\tfrac{1}{2} - \sin^{-1}0$$

$$= \frac{\pi}{6}$$ ✓

$\boxed{3}$



(ii) Let $\angle APC = \angle CPB = \alpha$ and let $\angle BPT = \beta$

Now, $\angle BAP = \angle BPT = \beta$ (angle between tangent & chord equals angle in alternate segment)

$\therefore \angle BCP = \angle CPA + \angle CAP$ (exterior angle of ΔCAP) ✓

$= \alpha + \beta$

Also, $\angle CPT = \angle CPB + \angle BPT$

$= \alpha + \beta$

$\therefore \angle BCP = \angle CPT$ (both $(\alpha+\beta)$) ✓

$\therefore \Delta TCP$ is isosceles (base angles equal)

$\therefore TP = TC$ (equal sides of isos. Δ)

$\boxed{5}$

(iii) $PT^2 = AT.TB$

$= 9.4$

$= 36$

$\therefore PT = 6$ ✓

$\therefore TC = 6$ $(TP = TC$, from part (ii))

Now, $AC = AT - TC$

$= 9 - 6$

$\therefore AC = 3$ ✓

$$= \int_0^{\frac{\pi}{6}} \frac{1}{2}(1 - \cos 4x)\,dx$$ (1)

$$= \frac{1}{2}\left[x - \frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{6}}$$ (1)

$$= \frac{1}{2}\left[\frac{\pi}{6} - \frac{1}{4}\sin\frac{2\pi}{3}\right]$$ (1) $\boxed{4}$

$$= \frac{1}{2}\left[\frac{\pi}{6} - \frac{1}{4}\cdot\frac{\sqrt{3}}{2}\right]$$

$$= \frac{\pi}{12} - \frac{\sqrt{3}}{16}$$ (1)

(b) $A = \pi r^2$, $C = 2\pi r$, $\frac{dA}{dt} = 4$

$$\therefore \frac{dA}{dr} = 2\pi r$$

Now, $\frac{dA}{dt} = \frac{dA}{dr}\cdot\frac{dr}{dt}$

$$\therefore \frac{dr}{dt} = \frac{\frac{dA}{dt}}{\frac{dA}{dr}}$$

$$= \frac{4}{2\pi r}$$ (1)(1)

Also, $C = 2\pi r$ $\boxed{4}$

$$\therefore \frac{dC}{dr} = 2\pi$$

Now, $\frac{dC}{dt} = \frac{dC}{dr}\cdot\frac{dr}{dt}$

$$= 2\pi \cdot \frac{4}{2\pi \cdot 2}$$ (1) when $r = 2$

$$= 2$$ (1)

$\therefore$ Circumf. is increasing at 2 u/s.

$$= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$$

$$\therefore \left.\begin{aligned} R\cos\alpha &= \sqrt{3} \\ R\sin\alpha &= 1 \end{aligned}\right\}$$

$$R^2\sin^2\alpha + R^2\cos^2\alpha = 1^2 + \sqrt{3}^2$$
$$R^2(\sin^2\alpha + \cos^2\alpha) = 1 + 3$$
$$R^2 = 4$$
$$\therefore R = 2$$

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{1}{\sqrt{3}}$$
$$\tan\alpha = \frac{1}{\sqrt{3}}$$
$$\therefore \alpha = \pi/6$$

$$\therefore \sqrt{3}\cos\theta - \sin\theta = 2\cos(\theta + \pi/6)$$ (1)(1)

(ii) $\sqrt{3}\cos\theta - \sin\theta = 1$ $\boxed{4}$

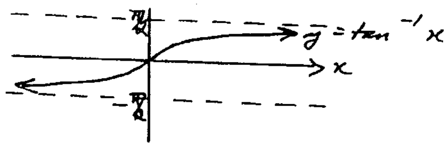
$$\therefore 2\cos(\theta + \pi/6) = 1$$

$$\cos(\theta + \pi/6) = \frac{1}{2}$$

$$\theta + \pi/6 = 2n\pi \pm \pi/3$$ (1)

$$\left.\begin{aligned} \therefore \theta &= 2n\pi + \pi/6 \\ \text{or}\quad \theta &= 2n\pi - \pi/2 \end{aligned}\right\}$$ (1)

(4)



$y = \tan^{-1} x$ $\boxed{3}$

$\frac{dy}{dx} = \frac{1}{1+x^2}$

Now, $1+x^2 \geq 1$ for all real values of x

$\therefore \frac{1}{1+x^2} \leq 1$

$\therefore$ Max. value of gradient $= 1$

(b) (i) In $\triangle BCF$, $\sin 30° = \frac{BF}{BC}$

$\therefore BF = BC \sin 30°$

$= 0.8 \times \frac{1}{2}$ (as $BC = AD$)

$\therefore$ length of $BF = 0.4$ m

(ii) In $\triangle ABC$, $AC^2 = (1.2)^2 + (0.8)^2$

$\therefore AC = \sqrt{2.08} = 1.4422\ldots\ldots$

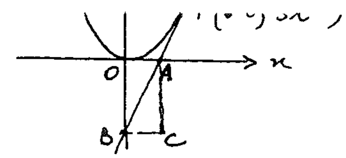
$\therefore$ length of $AC = 1.44$ m (correct to 2 d.p.)

(iii) Required angle is $\angle ACE$ $\boxed{3}$

$\sin \angle ACE = \frac{AE}{AC}$

$= \frac{0.4}{1.44}$ (as $AE = BF$)

$\therefore \angle ACE = 16°$ (to nearest degree)



(i) $y = \frac{1}{12}x^2$

$\frac{dy}{dx} = \frac{1}{6}x$

$\therefore$ Grad. of tangent at $P = \frac{1}{6}.6t = t$

$\therefore$ Eqn. of tangent at P is

$y - 3t^2 = t(x - 6t)$

$y - 3t^2 = tx - 6t^2$

$\therefore tx - y - 3t^2 = 0$

(ii) Cuts x axis when $y = 0$ $\boxed{6}$

$\therefore A$ is $(3t, 0)$

Cuts y axis when $x = 0$

$\therefore B$ is $(0, -3t^2)$

As OACB is a rectangle,

C is $(3t, -3t^2)$

(iii) $x = 3t$, $y = -3t^2$

$\therefore t = \frac{x}{3}$

$y = -3\left(\frac{x}{3}\right)^2$

$y = -\frac{x^2}{3}$

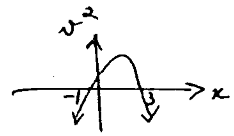
$\therefore x^2 = -3y$ is locus of C.

(i) For motion to exist, $v^2 \geq 0$

$6 + 4x - 2x^2 \geq 0$

$2(3-x)(1+x) \geq 0$

$\therefore -1 \leq x \leq 3$



i.e. Particle is oscillating between $x = -1$ and $x = 3$. 2

(ii) Amplitude of motion = 2 metres 1

(iii) Acceleration $= \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dx}\left(3 + 2x - x^2\right)$ 1

$= 2 - 2x$ 1

$= -2(x-1)$

(iv) Period $= \frac{2\pi}{n}$ $\boxed{8}$

$= \frac{2\pi}{\sqrt{2}}$

$= \sqrt{2}\,\pi$ seconds 1

(v) Max. speed occurs as particle passes through centre of motion $x = 1$ 1

$v^2 = 6 + 4 - 2 = 8$

$v = \pm\sqrt{8}$

$\therefore$ Max. speed $= 2\sqrt{2}$ ms^{-1} 1

Alt. method :

$v^2 = 6 + 4x - 2x^2$

$= -2\left[x^2 - 2x - 3\right]$

$= -2\left[(x^2 - 2x + 1) - 3 - 1\right]$

$= -2\left[(x-1)^2 - 4\right]$

$= -2(x-1)^2 + 8$

$\therefore v_{max}^2 = 8$

$\therefore$ Max. speed $= 2\sqrt{2}$ ms^{-1}.

When $n = 2$, LHS $= 1 - \frac{1}{2^2}$, RHS $= \frac{2+1}{2.2}$

$= 1 - \frac{1}{4}$ $= \frac{3}{4}$

$= \frac{3}{4}$ 1

$\therefore$ True for $n = 2$

Assume true for $n = k$

i.e. Assume that $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \ldots \left(1 - \frac{1}{k^2}\right) = \frac{k+1}{2k}$

When $n = k+1$,

$\underbrace{\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right) \ldots \left(1 - \frac{1}{k^2}\right)}\left(1 - \frac{1}{(k+1)^2}\right)$

$= \left(\frac{k+1}{2k}\right) \cdot \left(\frac{(k+1)^2 - 1}{(k+1)^2}\right)$ 1 $\boxed{4}$

$= \left(\frac{k+1}{2k}\right) \cdot \left(\frac{k^2 + 2k}{(k+1)^2}\right)$

$= \left(\frac{\cancel{k+1}}{2\cancel{k}}\right) \cdot \left(\frac{\cancel{k}(k+2)}{(k+1)^{\cancel{2}}}\right)$ 1

$= \frac{k+2}{2(k+1)}$

$\therefore$ Statement is true for $n = k+1$ if true for $n = k$. 1

As true for $n = 2$, it is true for $n = 2 + 1 = 3$

As true for $n = 3$, it is true for $n = 3 + 1 = 4$

etc.

$\therefore$ True for all $n \geq 2$.

sequence, let roots be $\alpha - d, \alpha, \alpha + d$.

Sum of roots $= -\frac{b}{a}$

$(\alpha - d) + \alpha + (\alpha + d) = 6$

$3\alpha = 6$

$\therefore \alpha = 2$ $\boxed{5}$

As $\alpha = 2$ is one of the roots, it must satisfy equation

$\therefore 2^3 - 6.2^2 + 3.2 + k = 0$

$8 - 24 + 6 + k = 0$

$\therefore k = 10$

(k) $f(x) = \frac{x-4}{x-2}$ for $x > 2$

(i) $f'(x) = \frac{(x-2).1 - (x-4).1}{(x-2)^2}$

$= \frac{2}{(x-2)^2}$

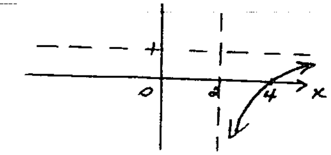
Now, $(x-2)^2 \geq 0$ for all real x

$\therefore f'(x) > 0$ for all x in domain $x > 2$

$\therefore f(x)$ is an increasing function

(ii) As $f(x)$ is an increasing function it is a one-one function

$\therefore$ The inverse function $f^{-1}(x)$ exists.



Domain is $x > 2$

Range is $y < 1$

$\therefore$ For $f^{-1}(x)$, Domain is $x < 1$

Range is $y > 2$

(iv) $f'(x) = \frac{2}{(x-2)^2}$ (from part (i)) $\boxed{7}$

$f'(4) = \frac{2}{(4-2)^2} = \frac{1}{2}$

$\therefore$ Grad. of tangent to $y = f^{-1}(x)$ at the point $(0, 4) = 2$.

Alt. method:

Inverse is $x = \frac{y-4}{y-2}$

$xy - 2x = y - 4$

$xy - y = 2x - 4$

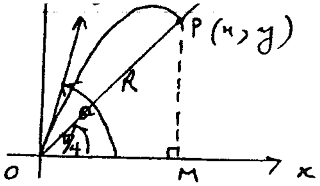
$\therefore y = \frac{2(x-2)}{x-1}$

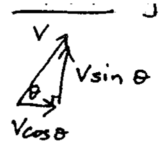
$\frac{dy}{dx} = \frac{(x-1).2 - (2x-4).1}{(x-1)^2}$

$= \frac{2}{(x-1)^2}$

$= 2$ when $x = 0$

$\therefore$ Gra of tangent to inverse fn. $= 2$.





1) <u>Horizontal motion</u>: ✓

$\ddot{x} = 0$

Integrating w.r.t. t,

$\dot{x} = c_1$

When $t = 0$, $\dot{x} = V\cos\theta$

$\therefore c_1 = 5\cos\theta$

$\therefore \dot{x} = 5\cos\theta$

Integrating w.r.t. t,

$x = 5t\cos\theta + c_2$

When $t = 0$, $x = 0$

$\therefore c_2 = 0$

$\therefore x = 5t\cos\theta$

<u>Vertical motion</u>: ✓

$\ddot{y} = -10$

Integrating w.r.t. t,

$\dot{y} = -10t + c_3$

When $t = 0$, $\dot{y} = V\sin\theta$

$\therefore c_3 = 5\sin\theta$

$\therefore \dot{y} = -10t + 5\sin\theta$ $\boxed{2}$

Integrating w.r.t. t,

$y = -5t^2 + 5t\sin\theta + c_4$

When $t = 0$, $y = 0$

$\therefore c_4 = 0$

$\therefore y = -5t^2 + 5t\sin\theta$

In ΔPOM, $\cos\frac{\pi}{4} = \frac{x}{R}$ and $\sin\frac{\pi}{4} = \frac{y}{R}$

$\therefore x = R\cos\frac{\pi}{4}$ and $y = R\sin\frac{\pi}{4}$ ✓

$= R \times \frac{1}{\sqrt{2}}$ $\quad = R \times \frac{1}{\sqrt{2}}$ $\boxed{1}$

$= \frac{R}{\sqrt{2}}$ $\quad = \frac{R}{\sqrt{2}}$

i.e. $x = y = \frac{R}{\sqrt{2}}$

$\therefore t = 0$ (at O) or $t = \sin\theta - \cos\theta$ (at P)

Now, $x = 5t\cos\theta = \frac{R}{\sqrt{2}}$ ✓

$\therefore R = 5\sqrt{2}\,t\cos\theta$ $\boxed{2}$

$\therefore R = 5\sqrt{2}(\sin\theta - \cos\theta)\cos\theta$ ✓

$\therefore R = 5\sqrt{2}(\sin\theta\cos\theta - \cos^2\theta)$

(iv) $\frac{dR}{d\theta} = 5\sqrt{2}\left[\cos\theta.\cos\theta + \sin\theta.-\sin\theta - 2\cos\theta.-\sin\theta\right]$

$= 5\sqrt{2}\left[\cos^2\theta - \sin^2\theta + 2\sin\theta\cos\theta\right]$ ✓

$= 5\sqrt{2}\left[\cos 2\theta + \sin 2\theta\right]$ ✓

When $\frac{dR}{d\theta} = 0$, $\sin 2\theta = -\cos 2\theta$

$\tan 2\theta = -1$ ✓ $\boxed{6}$

$\therefore 2\theta = \frac{3\pi}{4}$ ✓ (as $\frac{\pi}{2} < 2\theta < \pi$)

$\therefore \theta = \frac{3\pi}{8}$ ✓ (as $\frac{\pi}{4} < \theta < \frac{\pi}{2}$)

When $\theta < \frac{3\pi}{8}$, $\frac{dR}{d\theta} > 0$

When $\theta > \frac{3\pi}{8}$, $\frac{dR}{d\theta} < 0$

$\therefore$ MAX. Distance R when $\theta = \frac{3\pi}{8}$ ✓

(v) When $\theta = \frac{3\pi}{8}$, $R = 5\sqrt{2}\left(\sin\frac{3\pi}{8}\cos\frac{3\pi}{8} - \cos^2\frac{3\pi}{8}\right)$

$\therefore R = 1.464\ldots$ ✓ $\boxed{1}$

As $R < 1.8$, cat will need to run up the slope.