GOD IS LOVE

Student Name:___________________ **Teacher Name:**_________________

Saint Mark's Coptic Orthodox College

Mathematics Department
Year 11 Mathematics Extension 1

PRELIMINARY TASK THREE
8^{TH} JUNE, 2007

Time Allowed: TWO PERIODS

EXAMINER: Mr. Wagdy Micheal

DIRECTIONS TO CANDIDATE:

- Attempt all questions.
- Show all necessary working. Marks may be deducted for careless or badly arranged work.
- Only approved calculators may be used.
- This paper contains 5 questions in 2 pages.

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QUESTION ONE

1) A is the point $(-2, -1)$. B is the point $(1, 5)$. Find the co-ordinates of the point Q, which divides AB externally in the ratio $5 : 3$. 2marks

2) If $(a-3)x^2 - (b-1)x + (c-2) = x^2 + 4x + 5$ for all real x, find a, b and c. 3marks

3) Solve the equation $\cos 2A = \cos A$ where $0 \le A \le 360°$. 3marks

4) i. Express $\cos\theta - \sqrt{3}\sin\theta$ *in the form* $R\cos(\theta+\alpha)$. 2marks

ii. Hence solve the equation $\cos\theta - \sqrt{3}\sin\theta = 1$ for θ in the interval $0 \le \theta \le 360$. 2marks

QUESTION TWO

5) Determine if the roots of the quadratic equation $15x^2 - 41x + 14 = 0$ are real or unreal, rational or irrational, equal or unequal. 2marks

6) Let α and β be the roots of the equation $x^2 + 7x + 3 = 0$. Without solving, find the value of:

a. $\alpha + \beta$; b. $\alpha\beta$; c. $(\alpha+2)(\beta+2)$. 2marks

7) Find all angles θ for which $\sin 2\theta = \cos\theta$. 4marks

8) Show that $\dfrac{\cos x - \cos(x+2\theta)}{2\sin\theta} = \sin(x+\theta)$. 4marks

QUESTION THREE

9) Solve the inequality $\dfrac{x}{x^2-1} > 0$. 2marks

10) Using the "t" results, find all the angles θ with $0 \le \theta \le 360$ for which $\sin\theta + \cos\theta = -1$. 3marks

11) For the equation $4x^2 + 4(r-3)x + (19-3r) = 0$:
Find the values of r for which the equation has real roots. 3marks

12) Prove that $8\cos^4 x \equiv 3 + 4\cos 2x + \cos 4x$. *4marks*

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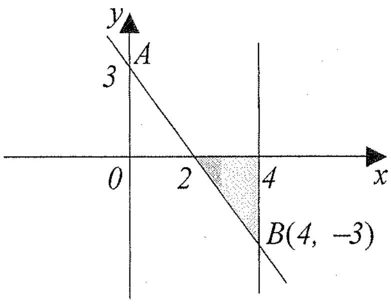
QUESTION FOUR

13) Solve $3^{2x+1} - 28(3^x) + 9 = 0$ 3marks

14)



A and *B* are the points $(0, 3)$ and $(4, -3)$ respectively.

a. Find the distance between *A* and *B*. 1mark

b. If *C* is the point $(-5, 0)$, find the co-ordinates of the midpoint of the interval joining *B* and *C*. 1mark

c. Show that the equation of the line *AB* is $3x + 2y - 6 = 0$. 2marks

d. Hence find the equation of the line perpendicular to *AB* and passing through *C*. 2marks

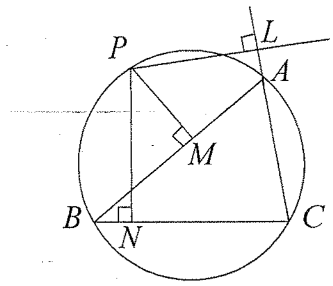
e. Find the point of intersection of the line *AB* with the line $x - 4y + 5 = 0$. 1mark

f. Write down three inequalities to describe the shaded region given above. 2marks

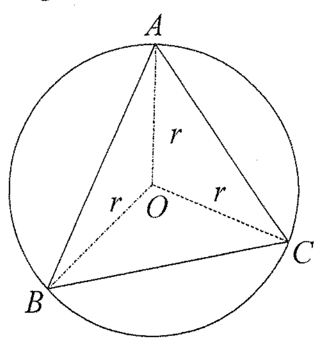
QUESTION FIVE

15) One root of the equation $x^2 - (r + 3)x + (5r - 3) = 0$ is twice the other root. Find the two possible values of r. 2marks

16) ABC is a triangle inscribed in the circle. P is a point on the minor arc AB. The points L, M and N are the feet of the perpendiculars from P to CA produced, AB, and BC respectively. Show that L, M and N are collinear. 5 marks



17) The circle through the vertices of triangle *ABC* has centre *O* and radius *r*.



i. Show that $BC = 2r \sin A$. 2 marks

ii. Show that $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$. 3 marks

[[End Of Qns]]

EXT1 : Yr 11 – Task 3, 8/6/2007 Solutions

1) $A(\overset{x_1}{-2},\overset{y_1}{-1})$, $B(\overset{x_2}{1},\overset{y_2}{5})$, $\overset{m}{5} : \overset{n}{-3}$

$x = \frac{x_2 m + x_1 n}{m+n}$ $\quad$ $y = \frac{y_2 m + y_1 n}{m+n}$

$= \frac{1 \cdot 5 + -2 \cdot -3}{5 + -3}$ $\quad$ $= \frac{5 \cdot 5 + -1 \cdot -3}{5 + -3}$

$= \frac{11}{2}$ $\quad$ $= \frac{28}{2}$

$= 5\frac{1}{2}$ $\quad$ $= 14$

$Q(5\frac{1}{2}, 14)$

2) $(a-3)x^2 - (b-1)x + (c-2) = x^2 + 4x + 5$

$a - 3 = 1$, $-b + 1 = 4$, $c - 2 = 5$

$\underline{a = 4}$ $\quad$ $-b = 3$ $\quad$ $\underline{c = 7}$

$\underline{b = -3}$

3) $\cos 2A = \cos A$

$2\cos^2 A - 1 = \cos A$

$2\cos^2 A - \cos A - 1 = 0$

$(2\cos A + 1)(\cos A - 1) = 0$

$\cos A = -\frac{1}{2}$ or $\cos A = 1$

$A = 120^\circ, 240^\circ$ $\quad$ $A = 0^\circ, 360^\circ$

4) i) $\cos\theta - \sqrt{3}\sin\theta \Rightarrow R\cos(\theta + \alpha)$

<u>method 1</u>

$R = \sqrt{1^2 + (\sqrt{3})^2}$

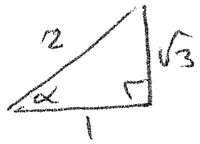
$= 2$

$\cos\theta - \sqrt{3}\sin\theta = 2\left[\cos\theta \cdot \frac{1}{2} - \sin\theta \cdot \frac{\sqrt{3}}{2}\right]$

$= 2\left[\cos\theta \cdot \cos\alpha - \sin\theta \cdot \sin\alpha\right]$

where $\cos\alpha = \frac{1}{2}$, $\sin\alpha = \frac{\sqrt{3}}{2}$

$\therefore \alpha = 60^\circ$



$\therefore \cos\theta - \sqrt{3}\sin\theta = 2\cos(\theta + \alpha)$

$= 2\cos(\theta + 60^\circ)$

<u>method 2</u>

$\cos\theta - \sqrt{3}\sin\theta = R\cos(\theta + \alpha)$

$= R[\cos\theta\cos\alpha - \sin\theta\sin\alpha]$

$= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

$\therefore R\cos\theta\cos\alpha = \cos\theta$, $R\sin\theta\sin\alpha = \sqrt{3}\sin\theta$

$R\cos\alpha = 1$ $\quad$ $R\sin\alpha = \sqrt{3}$

$R^2\cos^2\alpha + R^2\sin^2\alpha = 1 + 3$

$R^2(\cos^2\alpha + \sin^2\alpha) = 4$

$R^2 = 4$

$\boxed{R = 2}$

<u>Also</u> $\frac{R\sin\alpha}{R\cos\alpha} = \frac{\sqrt{3}}{1}$

$\tan\alpha = \frac{\sqrt{3}}{1}$

$\therefore \boxed{\alpha = 60^\circ}$

$\therefore \underline{\cos\theta - \sqrt{3}\sin\theta = 2\cos(\theta + 60^\circ)}$

ii) $\cos\theta - \sqrt{3}\sin\theta = 1$

$2\cos(\theta + 60) = 1$

$\cos(\theta + 60) = \frac{1}{2}$

$\theta + 60 = \cancel{\cos} 60^\circ, 300^\circ$

$\theta = 0^\circ, 240^\circ$

Q2

5. $15x^2 - 41x + 14 = 0$

$\Delta = b^2 - 4ac$

$= (-41)^2 - 4 \times 15 \times 14$

$= 841$

$= (29)^2 > 0$ and perfect square.

$\therefore$ Real, Rational, Unequal.

6. $x^2 + 7x + 3 = 0$

a. $\alpha + \beta = \frac{-b}{a}$
$= -7$

b. $\alpha\beta = \frac{c}{a}$
$= 3$

c. $(\alpha + 2)(\beta + 2) = \alpha\beta + 2\alpha + 2\beta + 4$

$= \alpha\beta + 2(\alpha + \beta) + 4$

$= 3 + 2(-7) + 4$

$= -7$

7. $\sin 2\theta = \cos\theta$

$2\sin\theta\cos\theta - \cos\theta = 0$

$\cos\theta(2\sin\theta - 1) = 0$

$\cos\theta = 0$ oR $2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\cos\theta = \cos 90^\circ$

$\sin\theta = \sin 30$

$\theta = 90^\circ \pm n \times 90^\circ$

$\theta = n \times 180 + (-1)^n \times 30^\circ$

8) $\frac{\cos x - \cos(x + 2\theta)}{2\sin\theta} = \sin(x + \theta)$

$LHS = \frac{\cos x - [\cos x \cos 2\theta - \sin x \sin 2\theta]}{2\sin\theta}$

$= \frac{\cos x - \cos x(1 - 2\sin^2\theta) + \sin x \, 2\sin\theta\cos\theta}{2\sin\theta}$

$= \frac{\cos x - \cos x + 2\sin^2\theta\cos x + 2\sin x\sin\theta\cos\theta}{2\sin\theta}$

$= \frac{2\sin\theta(\sin\theta\cos x + \cos\theta\sin x)}{2\sin\theta}$

$= \sin\theta\cos x + \cos\theta\sin x$

$= \sin(x + \theta) = RHS$

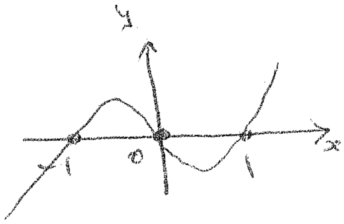
Q3

9) $\dfrac{x}{(x^2-1)} \times (x^2-1)^2 > 0 \times (x^2-1)^2$

$x(x^2-1) > 0$

$x(x-1)(x+1) > 0$

$-1 < x < 0, \; x > 1$



10) $\sin\theta + \cos\theta = -1$

$\dfrac{2t}{1+t^2} + \dfrac{1-t^2}{1+t^2} = -1$

$2t + 1 - \cancel{t^2} = -1 - \cancel{t^2}$

$2t = -2$

$t = -1$

$\tan\frac{\theta}{2} = -1$

$\frac{\theta}{2} = 135^\circ, 315^\circ$

$\theta = 270^\circ, 630^\circ$

$\theta = 270^\circ, 180^\circ$

11) $4x^2 + 4(r-3)x + (19-3r) = 0$

$\Delta \geq 0 \Rightarrow b^2 - 4ac \geq 0$

$16(r-3)^2 - 4 \times 4 \times (19-3r) \geq 0$

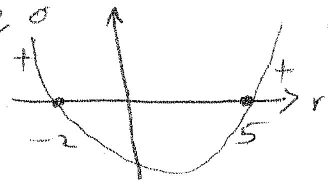
$16(r^2-6r+9) - 304 + 48r \geq 0$

$16r^2 - 96r + 144 - 304 + 48r \geq 0$

$16r^2 - 48r - 160 \geq 0$

$r^2 - 3r - 10 \geq 0$

$(r-5)(r+2) \geq 0$



$r \leq -2, \; r \geq 5$

12) $8\cos^4 x \equiv 3 + 4\cos 2x + \cos 4x$

L.H.S $= 8\cos^4 x$

$= 8(\cos^2 x)^2$

$= 8\left(\frac{1}{2}(\cos 2x + 1)\right)^2$

$= 2\left[\cos^2 2x + 2\cos 2x + 1\right]$

$= 2\left[\frac{1}{2}(\cos 4x + 1) + 2\cos 2x + 1\right]$

$= \cos 4x + 1 + 4\cos 2x + 2$

$= 3 + 4\cos 2x + \cos 4x$

$=$ R.H.S

Question 4

13) $3^{2x+1} - 28(3^x) + 9 = 0$

$3(3^{2x}) - 28(3^x) + 9 = 0$

Let $3^x = y$

$3y^2 - 28y + 9 = 0$

$(3y-1)(y-9) = 0$

$3y - 1 = 0$ OR $y = 9$

$y = \frac{1}{3}$ OR $y = 9$

$3^x = \frac{1}{3}$ $\qquad 3^x = 9$

$x = -1$ $\qquad x = 2$

14) $A(0,3)$, $B(4,-3)$

a) $AB = \sqrt{(4-0)^2+(-3-3)^2}$

$= \sqrt{16+36}$

$= \sqrt{52} = 2\sqrt{13}$

b) $B(4,-3)$, $C(-5,0)$

mid-pt of $BC = \left(\frac{4+-5}{2}, \frac{-3+0}{2}\right)$

$= \left(-\frac{1}{2}, -1\frac{1}{2}\right)$

c) $A(0,3)$, $B(4,-3)$

Eqn. of $AB \Rightarrow \frac{y_2-y_1}{x_2-x_1} = \frac{y-y_1}{x-x_1}$

$\frac{-3-3}{4-0} = \frac{y-3}{x-0}$

$\frac{-6}{4} = \frac{y-3}{x}$

$4y-12 = -6x$

$\therefore 6x+4y-12=0$

AB: $\underline{3x+2y-6=0}$

d) $m_{AB} = \frac{-3}{2}$ $\therefore m_{\text{line} \perp AB} = \frac{2}{3}$

$C(-5,0)$

eqn. $y-0 = \frac{2}{3}(x+5)$

$3y = 2x+10$

e) AB: $3x+2y-6=0$ ①

$x-4y+5=0$ ②

① $\xrightarrow{\times 2}$ $6x+4y-12=0$

② $\longrightarrow$ $x-4y+5=0$

$7x \quad -7=0$

$\boxed{x=1}$

$x-4y+5=0$

$1-4y+5=0$

$6=4y$

$\boxed{y=\frac{3}{2}}$

pt of intersection $(1, \frac{3}{2})$

f) $y \leq 0$, $x \leq 4$

$3x+2y-6 \geq 0$

Question five

15) $x^2-(r+3)x+(5r-3)=0$

$\alpha, \beta \Longrightarrow \alpha, 2\alpha$

$\alpha+2\alpha = +(r+3)$ | $2\alpha^2 = 5r-3$

$3\alpha = r+3$ | $2\alpha^2 = 5r-3$ ②

$\alpha = \frac{r+3}{3}$ ①

Sub. ① into ②

$2\left(\frac{r+3}{3}\right)^2 = 5r-3$

$2\left(\frac{r^2+6r+9}{9}\right) = 5r-3$

$2r^2+12r+18 = 45r-27$

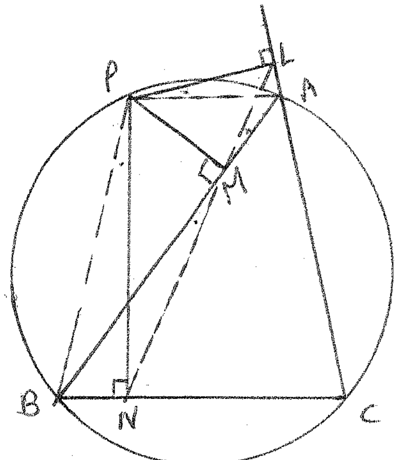
$2r^2-33r+45=0$

$(2r-3)(r-15)=0$

$r=\frac{3}{2}$ or $r=15$

16) Constructions

Join LM, MN, PB & PA



Proof

$\angle PLA = 90°$ and

$\angle PMA = 90°$

(adj. suppl. $\angle$s)

$\therefore PMAL$ is a Cyclic Quad

Since opp. angles are suppl.

$(\angle PLA + P\hat{M}A = 180°)$

Also $\angle PMB = \angle PNB = 90°$ (given)

$\therefore PMNB$ is a cyclic Quad.

(angles in the same segment are equal)

$\angle BPN = \angle B\underline{MN}$ ① (angles in the same segment are equal, PMNB is a cyclic Quad (proven above)

Cont. (16)

$\angle LPA = \angle LMA$ (angles in the same segment are equal, PMAL is a cyclic Quad (proven above). (2)

Now In Δ's PBN, PAL

$\angle PLA = \angle PNB = 90°$ (given)

$\angle PAL = \angle PBN$ (Since PACB is a cyclic Quad, therefore exterior angle of a cyclic Quad. equal to the interior opp. angle).

$\therefore \Delta PBN \,|||\, \Delta PAL$

$\therefore \angle LPA = \angle BPN$ (Corresp. $\angle$'s of Isos Δ are equal, OR angle sum of a Δ) (3)

$\therefore$ from 1, 2, 3

$\therefore \angle LMM = \angle BMN = x°$ (say)

(equals to equals are equal)

and $\angle PMA = 90°$

$\therefore \angle PML = 90 - x°$

$\therefore \angle PML + P\hat{M}N = 90 - x + 90 + x$

$= 180°$

$\therefore \angle LMN = 180°$

$\therefore$ L, M & N are Collinear.

Q17

i) $\angle BOC = 2\angle BAC = 2A$

(angle at the centre is twice the size of the angle at the circumference)

$\therefore BC^2 = r^2 + r^2 - 2r^2 \cos B\hat{O}C$

$= 2r^2 - 2r^2 \cos 2A$

$= 2r^2(1 - \cos 2A)$

$= 2r^2[1 - (1 - 2\sin^2 A)]$

$= 2r^2 \times 2\sin^2 A$

$BC^2 = 4r^2 \sin^2 A$

$\boxed{BC = 2r \sin A}$

ii) Also $BA = 2r \sin C$

$AC = 2r \sin B$

Area of $\Delta ABC = \frac{1}{2} \cdot AB \cdot AC \cdot \sin A$

$= \frac{1}{2} \cdot 2r\sin C \cdot 2r\sin B \cdot \sin A$

$= 2r^2 \sin A \sin B \sin C$

Area of $\Delta OBC = \frac{1}{2} r^2 \sin 2A$

" " $\Delta OCA = \frac{1}{2} r^2 \sin 2B$

" " $\Delta OAB = \frac{1}{2} r^2 \sin 2C$

Area of ΔABC = Areas($\Delta OBC + \Delta OCA + \Delta OAB$)

$\therefore 2r^2 \sin A \sin B \sin C = \frac{1}{2} r^2 (\sin 2A + \sin 2B + \sin$

$4 \sin A \sin B \sin C = \sin 2A + \sin 2B + \sin 2C$