Extension I

Higher School Certificate
TRIAL EXAMINATION
2005

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Use Board approved calculators
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may NOT be awarded for messy or badly arranged work
- Use writing booklets provided
- ALL questions are NOT of equal Value.

Total Marks – 74 Marks

Examiner: Patrick Loi

Disclaimer:	**This does not necessarily reflect the content or format of the Higher School Certificate.**

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Question 1 ***(12 Marks)*** **Marks**

(a) Use the table of standard integrals to evaluate, in simplest form,

$$\int_0^4 \frac{1}{\sqrt{x^2+9}}\,dx$$ **2**

(b) P divides the interval from (-4, 2) to (2, -1) externally in the ratio 5 : 2.

Find the coordinates of P. **2**

(c) P is the point, other than the origin, where $y = ax^2$ meets the line

$y = ax^2$ meets the line $y = x$.

(i) Find the coordinates of P. **1**

(ii) Find, to the nearest minute, the size of the acute angle formed by

the line $y = x$ and the tangent to $y = ax^2$ at P. **2**

(d) Evaluate $\lim_{h \to 0} \dfrac{\sin \frac{h}{2}}{h}$ **1**

(e) Solve $\dfrac{2x+5}{x} \le 1$ **2**

(f) How many different arrangements can be made using the seven letters of

the word **2**

A R R A N G E

Question 2 (12 Marks) **Marks**

(a) Use the substitution $u = \sqrt{x}$ to evaluate

$$\int_0^3 \frac{1}{\sqrt{x}(1+x)}dx$$ **3**

(b) (i) Show that the function $f(x) = x^2 - e^{-x}$ has a root between 0 and 1. **1**

(ii) Determine whether this root lies closer to 0 or 1. **1**

(iii) Take 0.5 as an approximation to this root and use Newton's method to find this root correct to one decimal place. **2**

(c) Prove that

$$\frac{\sin 5x}{\sin x} - \frac{\cos 5x}{\cos x} = 4\cos 2x$$ **3**

(d) A sequence of numbers is defined by $u_1 = \frac{1}{3}$, and, if n is any positive integer,

$$u_{n+1} = \frac{1+3u_n}{3+u_n}$$

(i) Find u_n **1**

(ii) Prove, by induction, that **2**

$$u_n = \frac{2^n - 1}{2^n + 1}$$

Question 3 ***(12 Marks)***

Marks

(a) **(i)** **Show that**

$$\int_0^{\pi/4} \sin^2 x\,dx = \frac{\pi - 2}{8}$$ **2**

(ii) **Show that**

$$\frac{d}{dx}(\sin x - x\cos x) = x\sin x$$ **1**

Hence, evaluate

$$\int_0^{\pi/4} x\sin x\,dx = \frac{\sqrt{2}(4-\pi)}{8}$$ **2**

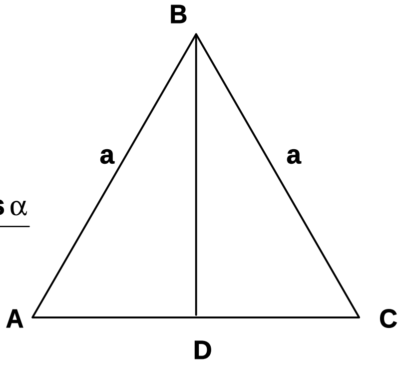
(iii) **Show that, if** $0 < x < \frac{\pi}{4}$

$$\sin^2 x < x\sin x$$ **1**

(iv) **Use the above results to prove that** $\pi < 2(3 - \sqrt{2})$ **1**

(b) **The triangle ABC is isosceles, with** $AB = BC = a$**, and BD is perpendicular to AC.**

Let $\angle ABD = \angle CBD = \alpha$



(i) **Show that the area of** $\triangle ABD$ **is** $\dfrac{a^2 \sin\alpha\cos\alpha}{2}$ **3**

(ii) **By considering the area of** $\triangle ABC$**,**

Prove that,

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$ **2**

Question 4 ***(12 Marks)*** **Marks**

(a) **A container of water, heated to 100^o, is placed in a coolroom where the temperature is maintained at -5^o. After t minutes, the rate of change of the temperature, H^o, of water is given by**

$$\frac{dH}{dt} = -k(H+5) \qquad \text{where } k \text{ is a constant}$$

(i) **Show that the function $H = Ae^{-kt} - 5$, where A is a constant, provides this rate of change.** **1**

(ii) **Find the value of A.** **2**

(iii) **After 20 minutes, the temperature of the water falls to 30^o. Find, to the nearest degree, the temperature of the water after a further 10 minutes.** **3**

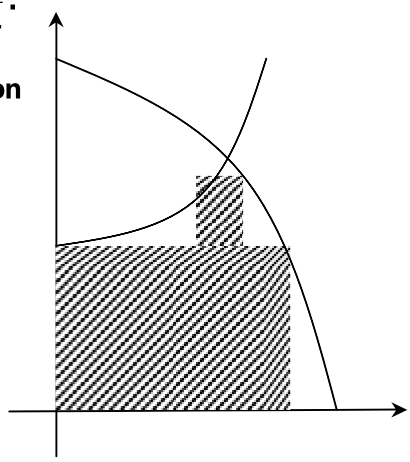
(iv) **Find, to the nearest minute, the time the water will need to be in the coolroom before its temperature reaches 0^O.** **2**

Marks

...Question 4 Continues

(b) **P is the point of intersection between x = 0 and $x = \frac{\pi}{2}$ of the graphs of $y = \sec x$ and $y = 2\cos x$, as shown.**

(i) **Verify that the x coordinate of P is $\frac{\pi}{4}$.** **1**



(ii) **The shaded region makes a revolution about the x-axis. Show that the volume of the resulting solid is $\frac{\pi^2}{2}$ units^3.** **3**

Question 6 (12 Marks)

(a) **Let $f(x) = 3 + \sqrt{x - 1}$**

(i) **State the domain of f (x)**

(ii) **Prove that f (x) is an increasing function and hence state its range.**

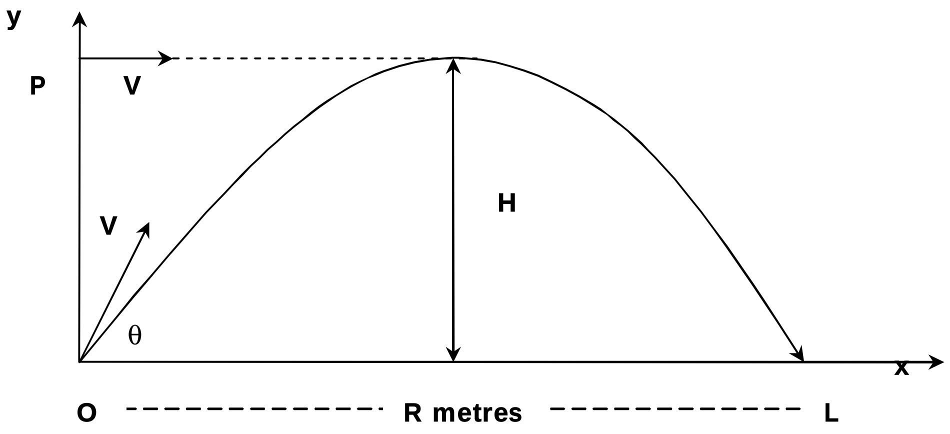
(iii) **Since f (x) is monotonic, an inverse function exists. State the domain and range of $f^{-1}(x)$**

(iii) **Find $f^{-1}(x)$ and sketch, on one number plane, graphs of $y = f^{-1}(x)$.**

(v) **Find where the graphs in (iv) intersect.**

Marks

Question 7 *(12 Marks)*



A particle is projected from a point O on horizontal ground with speed V ms^{-1} at an angle of elevation of θ, landing at L, as shown.

You may assume that the displacements of this particle after t seconds are given by:

$$y = (V\sin\theta)t - \frac{1}{2}gt^2$$
$$x = (V\cos\theta)t$$

(a) **Show that the range, R, and the greatest height reached are given by**

$$R = \frac{V^2\sin 2\theta}{g} \text{ and } R = \frac{V^2\sin^2\theta}{2g}$$ **3**

(b) **A second particle is projected at the same time as the first, with speed V ms^{-1}, horizontally from the point P,**

Prove that its displacements are given by

$$x = Vt \text{ and } y = H - \frac{1}{2}gt^2$$ **3**

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Marks

(c) **Prove that, when the second particle lands, the first is at the top of its flight.** **2**

(d) **Let the range of the second particle be S metres and find the value of θ for which $R = S$.** **2**

(e) **Describe the manner in which $|R - S|$ varies as θ increase from 0 to $\frac{\pi}{2}$.** **2**

[Hint: Using the graphs of $y = \sin\theta$ and $y = \sin 2\theta$ for $0 \le x \le \frac{\pi}{2}$]

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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