2009

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading Time - 5 minutes.
- Working Time - 3 hours.
- Write using a blue or black pen.
- Approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (**120**)

- Attempt Questions 1-10.
- All questions are of equal value.

Question 1 (12 Marks) Use a Separate Sheet of paper **Marks**

(a) Express $3\cdot\dot{5}\dot{3}1$ as a fraction in simplest form. **2**

(b) If $\tan\theta = \frac{7}{8}$ and $\cos\theta < 0$, find the exact value of $\operatorname{cosec}\theta$ **1**

(c) Evaluate $\frac{3.24^2 - 2.1^2}{\sqrt{36 + 2.1}}$ correct to 3 significant figures. **1**

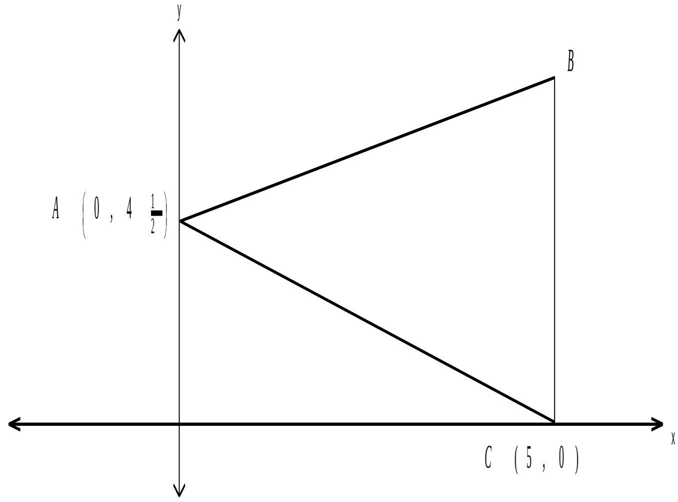
(d) Solve $|15 - 4x| \leq 3$ **2**

(e) If $k = \frac{1}{3}m\left(v^2 - u^2\right)$ find the value of m when $k = 724$, $v = 14\cdot2$ and $u = 7\cdot4$. **2**

(f) Find the period and amplitude for the graph of $3y = \sin\left(2x - \frac{\pi}{4}\right)$. **2**

(g) Paint at the local hardware store is sold at a profit of 30% on the cost price. If a drum of paint is sold for \$67·50, find the cost price. **2**

Question 2 **(12 Marks)** Use a Separate Sheet of paper **Marks**



The lines *AB* and *CB* have equations $x-2y+9=0$ and $4x-y-20=0$ respectively.

(a) Find the coordinates of the point *B*. **2**

(b) Show that the equation of the line *AC* is $9x+10y-45=0$. **2**

(c) Calculate the distance *AC* in exact form. **2**

(d) Find the equation of the line perpendicular to *BC* which passes passes through *A*. **2**

(e) Calculate the shortest distance between the point *B* and the line *AC*. Hence find the area of the triangle *ABC*. **2**

(f) State the inequalities that together define the area bounded by the triangle *ABC*. **2**

Question 3 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) Differentiate with respect to *x*.

i. $3x\sqrt[3]{x}$ **2**

ii. $\dfrac{\sin 2x}{e^{2x}}$ **2**

(b) Find:

i. $\displaystyle\int \frac{dx}{e^{3x}}$ **2**

ii. $\displaystyle\int_0^{\pi} \sec^2 \frac{x}{4}\,dx$. **2**

(c) If α and β are the roots of the equation $3x^2 - 4x - 7 = 0$
Find:

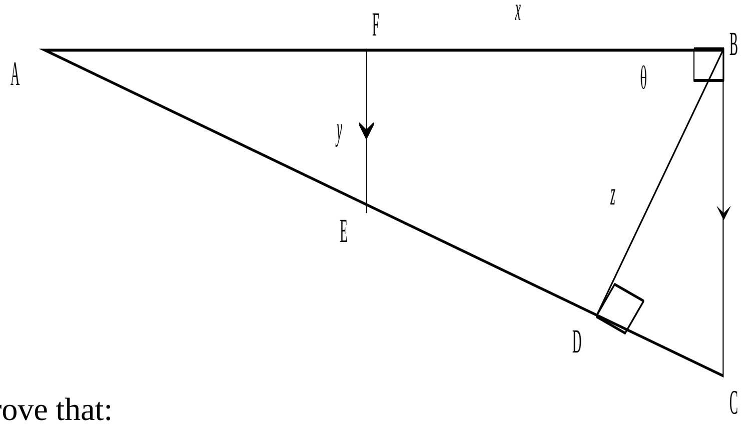
i. $\alpha + \beta$. **1**

ii. $2\alpha^2 + 2\beta^2$. **1**

iii. the equation with roots $2\alpha^2$ and $2\beta^2$. **2**

Question 4 (12 Marks) Use a Separate Sheet of paper **Marks**

(a) The right triangle ABC is shown below. BC $\parallel$ FE, BD $\perp$ AC, $\angle FBD = \theta$, BF = x, EF = y and BD = z.



Prove that:

i. $\angle FEA = \theta$ **2**

ii. AF = $y \tan \theta$ **1**

iii. $z = (x + y \tan\theta) \cos \theta$ **1**

iv. $z = x \cos \theta + y \sin \theta$ **1**

(b) The federal government distributes \$500 million in order to stimulate the economy. Each recipient spends 80% of the money that he or she receives. In turn, the secondary recipient spends 80% of the money that they receive, and so on. What was the total spending that results from the original \$500 million into the economy? **2**

(c) A ship sails from port A, 60 nautical miles due west, to a port B. It then proceeds a distance of 50 nautical miles on a bearing of 210° to a port C.

i. Draw a diagram to illustrate the information given. **1**

ii. Find the distance (nearest nautical mile) and bearing of C from A. **4**

Question 5 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) In a raffle in which 1000 tickets are sold, there is a first prize of \$1000, a second prize of \$500 and a third prize of \$200. The prize winning tickets are drawn consecutively without replacement, with the first ticket winning first prize.

Find the probability that:

i. a person buying one ticket in the raffle wins:

α. first prize. **1**

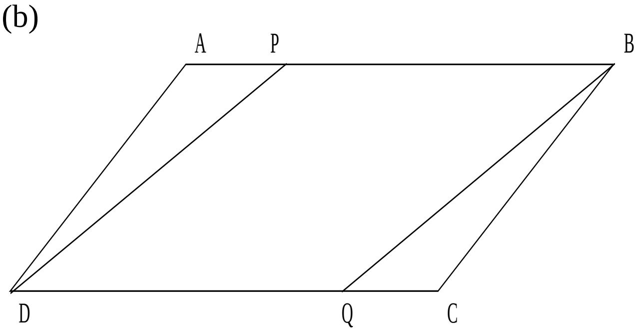
β. at least \$500 **1**

γ. no prizes. **1**

ii. a person buying two tickets in the raffle wins:

α. at least \$500 **1**

(b) **3**



ABCD is a parallelogram, BP = DQ.

Prove DP = BQ

(c) i. Is the series $\log 3 + \log 9 + \log 27 + ...$ arithmetic or geometric? Give reasons for your answer. **2**

iii. Find the sum of the first 10 terms of the series. **1**

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(d) Find the radius and centre of the circle with equation **2**

$$4x^2 - 4x + 4y^2 + 24y + 21 = 0$$

Question 6 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) A curve has a gradient function with equation $\frac{dy}{dx} = 6(x-1)(x-2)$.

i. If the curve passes through the point (1, 2), what is the equation of the curve? **2**

ii. Find the coordinates of the stationary points and determine their nature. **2**

iii. Find any points of inflexion. **2**

iv. Graph the function showing all the main features. **2**

(b) Show that $\frac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta} = \tan\theta$ **3**

(c) Evaluate $\lim_{\theta \to 0} \frac{\sin 2\theta}{3\theta}$ **1**

Question 7 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) The parabola $y = x^2$ and the line $y = x + 2$ intersect at points A and B respectively. Find the coordinates of the points A and B. Hence find the area bounded by the parabola and the line. **4**

(b) The minute hand on a clock face is 12 centimetres long.
In 40 minutes

i. Through what angle does the hand move (in radians)? **1**

ii. How far does the tip of the hand move? **1**

iii. What area does the hand sweep through in this time? **1**

(c) Use Simpson's rule to evaluate $\int_1^{2.5} f(x)\,dx$, to 1 decimal place using the 7 function values in the table below. **2**

x	1·00	1·25	1·50	1·75	2·00	2·25	2·50
f(x)	3·43	2·17	0·38	1·87	2·65	2·31	1·97

(d) A function is defined by the following features: **3**

$$\frac{d^2y}{dx^2} > 0 \text{ for } x < -1 \text{ and } 1 < x < 3.$$

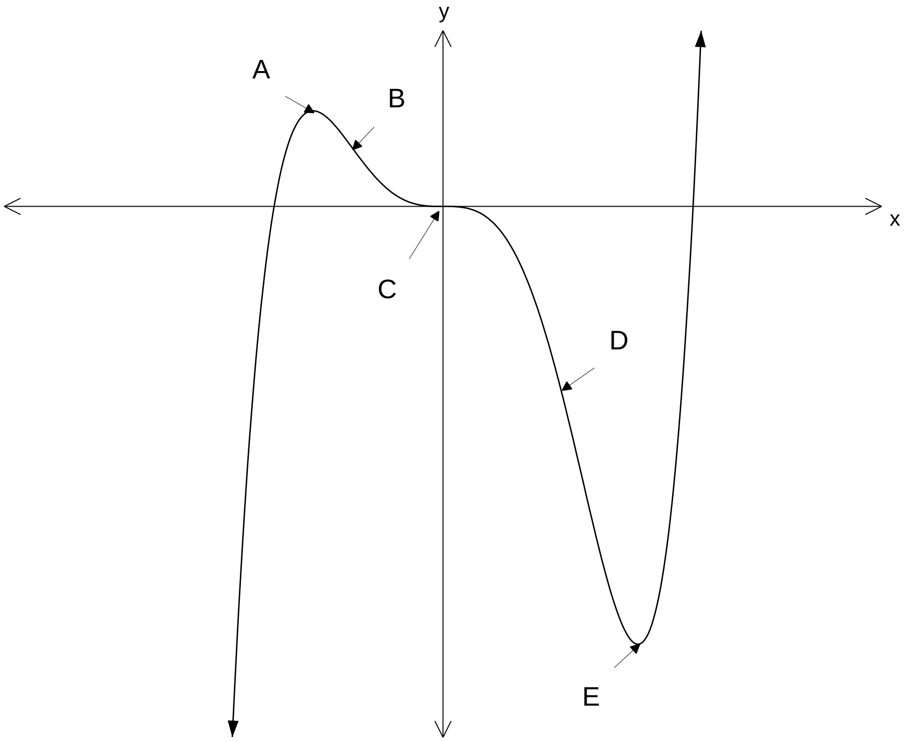
$$\frac{dy}{dx} = 0 \text{ when } x = -3, 1 \text{ and } 5.$$

$$y = 0 \text{ when } x = 1.$$

Sketch a possible graph of the function.

Question 8 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) The graph of the curve $y = f(x)$ is drawn below.



i. Name the points of inflexion. **1**

ii. When is the graph decreasing? **1**

iii. Sketch the gradient function. **1**

(b) Steve borrows \$15 000 for a new car. He decides to repay the loan plus interest at 6% pa compounded monthly. He repays the loan in monthly installments of \$P.

i. Show that after three months the amount that Steve owes is $[15226·13 − P(3·015025)]. **2**

ii. After two years of repaying his loan, Steve still owes \$10 000 on the loan. What was the monthly repayment? **3**

(c) Sketch the graph of the parabola $2x = y^2 - 8y + 4$, showing the vertex, focus and the directrix. **4**

Question 9 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) A particle moves in a straight line so that its displacement (in m) from a fixed point O at time t seconds is given by $x = 2\sin 2t,\ 0 \le t \le 2\pi$.

Find:

i. The initial velocity **1**

ii. The acceleration after $\dfrac{\pi}{12}$ seconds. **1**

iii. When the particle is at rest. **2**

iv. The displacement of the particle when it is at rest. **2**

(b) The area bounded by the curve $y = \sqrt{\dfrac{2x}{3x^2 - 1}}$ between the lines $x = 1$ and $x = 3$ is rotated about the x-axis. Find the volume of the solid of revolution formed. **3**

(c) The rate at which Carbon Dioxide will be produced when conducting an experiment is given by $\dfrac{dV}{dt} = \dfrac{1}{100}\left(30t - t^2\right)$ where V cm^3 is the volume of gas produced after t minutes.

i. At what rate is the gas being produced 15 minutes after the experiment begins. **1**

ii. How much Carbon Dioxide has been produced during this time? **2**

Question 10 (12 Marks) Use a Separate Sheet of paper **Marks**

(a) An open cylindrical can is made from a sheet of metal with an area of 300cm^2.

i. Show that the volume of the can is given by $V = 150r - \frac{1}{2}\pi r^3$. **2**

ii. Find the radius of the cylinder that gives the maximum volume and calculate this volume. **4**

(b) The population of a certain town grows at a rate proportional to the population. If the population grows from 20 000 to 25 000 in two years, find:

i. The population of the town, to the nearest hundred, after a further 8 years. **3**

ii. Calculate the rate of change at this time. **1**

(c) If $\log_a 2 + 2\log_a x - \log_a 6 = \log_a 3$ find the value of *x*. **2**

End of Examination.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2} \right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$