SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2009

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 180 minutes.
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Express your answers in simplest exact form unless otherwise stated.
- Marks may **NOT** be awarded for messy or badly arranged work.
- Start each **NEW** question in a separate answer booklet.

Total Marks - 120 Marks

- Attempt questions 1 - 10
- All questions are of equal value.

Examiner: *E. Choy*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

Total marks – 120
Attempt Questions 1 - 10
All questions are of equal value

Answer each question/section in a SEPARATE writing booklet. Extra writing booklets are available.

SECTION A

Question 1 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Solve $\dfrac{2t}{5}+14=8$. **2**

(b) If $m_1=34$, $m_2=7$, $M=53$ and $g=9\cdot 8$, find correct to 4 significant figures the value of **1**

$$\left(\frac{m_1-m_2}{M+m_1+m_2}\right)g.$$

(c) The line $kx-2y=23$ passes through the point $(3,-1)$. **2**
Find the value of k.

(d) Simplify $\dfrac{x}{4}+\dfrac{3x-1}{3}$. **2**

(e) Factorise $3x^2+5x-12$. **2**

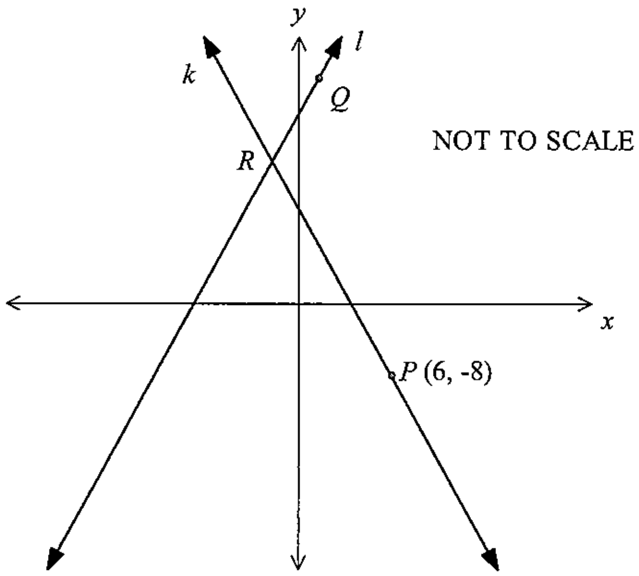
(f) Solve $7-4x>12$. **2**

(g) Write down the exact value of $\operatorname{cosec}\dfrac{\pi}{4}$. **1**

Question 2 (12 marks) **Marks**

(a) Solve $\tan x° = 1$ for $0° \le x° \le 360°$. **2**

(b) The diagram below shows the line $l: 2x - y + 8 = 0$ and the point $Q(2, 12)$ on it.
The line k has gradient -2 and passes through the point $P(6, -8)$.
The lines l and k intersect at R.



(i) Show that the equation of the line k is given by $2x + y - 4 = 0$. **1**

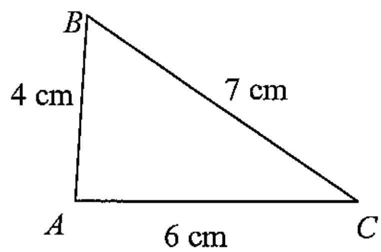
(ii) Show that the coordinates of R are $(-1, 6)$. **1**

(iii) Show that the distance QR is $3\sqrt{5}$. **1**

(iv) Find the perpendicular distance from P to the line l. **2**
Leave your answer in simplified surd form.

(v) Find the area of ΔPQR. **1**

(c) In the diagram below, ABC is a triangle in which $AB = 4$ cm, $BC = 7$ cm, and $CA = 6$cm.



(i) Use the Cosine Rule to show that $\cos C = \dfrac{23}{28}$. **1**

(ii) Write down the size of $\angle C$ correct to the nearest degree. **1**

(iii) Calculate the area of ΔABC. **2**
Leave your answer correct to the nearest square centimetre.

End of SECTION A

SECTION B

Question 3 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Differentiate with respect to x

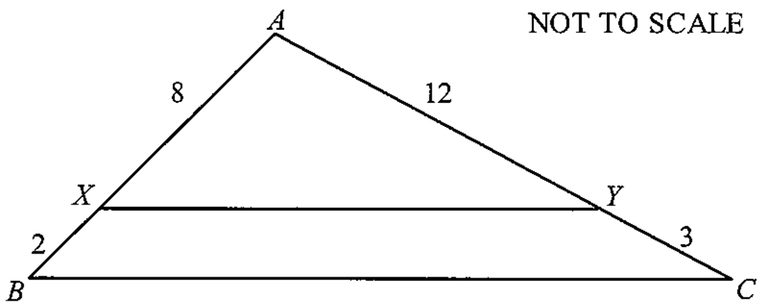
(i) $\left(3-x^2\right)^3$, **2**

(ii) $\log_e\left(x^2+3\right)$, **2**

(iii) $x\cos x$. **2**

(b) The graph of $y=f\left(x\right)$ passes through the point $\left(3,5\right)$ and $f'\left(x\right)=3-2x$. **2**
Find an expression for $f\left(x\right)$.

(c) In the diagram below, AB and AC are straight lines.
$AX = 8$, $BX = 2$, $AY = 12$ and $CY = 3$.



(i) Prove that $\Delta ABC\ |||\ \Delta AXY$. **2**

(ii) Prove that XY is parallel to BC. **1**

(d) Find $\int \sqrt{x-6}\,dx$. **1**

Question 4 (12 marks) **Marks**

(a) Find the value of k if the quadratic equation $(x-3)(x+k)=k(x+2)$ has two equal roots. **2**

(b) After retiring from teaching Mathematics, Eric borrows \$130 000 to start a Shanghai Chinese restaurant. He is charged interest on the balance owing at the rate of $9 \cdot 75\%$ p.a. compounded monthly.
He agrees to repay the loan including the interest by making equal monthly instalments of \$$M$.

(i) How much does Eric owe at the end of the first month just before he pays his first instalment? **1**

(ii) Write an expression involving M for the total amount owed by Eric just after the first instalment is paid. **1**

(iii) Calculate the value of M (to the nearest cent) that which will repay the loan after 13 years. **3**

(iv) In how many months (to the nearest whole month) will the loan be repaid if Eric made instalments of \$1700 per month? **2**

(c) Sketch the parabola which

(i) has a focus of $(2,1)$ and directrix $x=4$. **1**

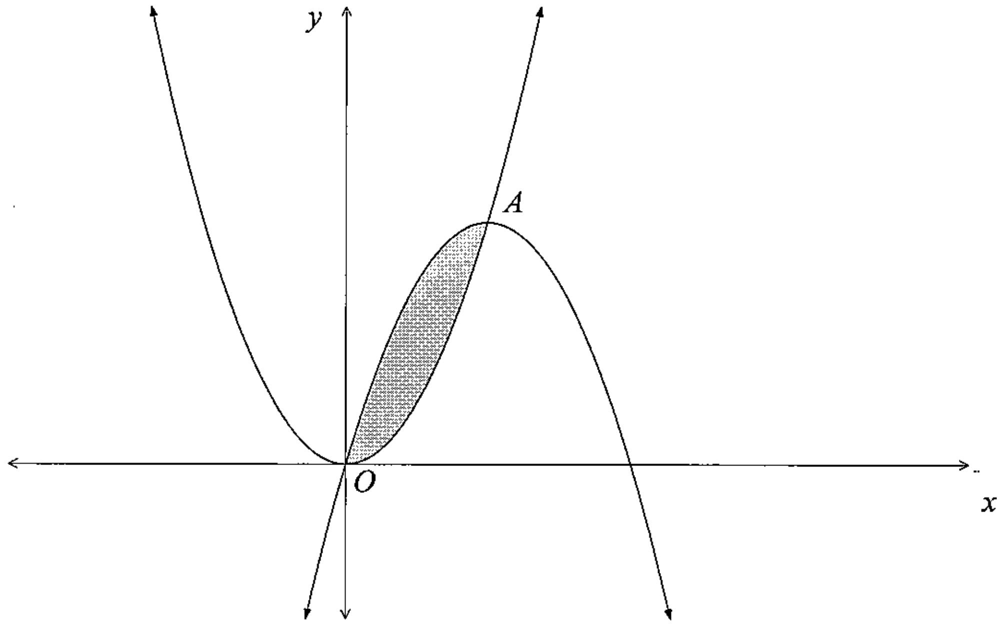
(ii) Find the equation of the parabola. **2**

End of SECTION B

SECTION C

Question 5 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) The diagram shows the curves $y = x^2$ and $y = 4x - x^2$, which intersect at the origin and at the point A.



(i) Show that the coordinates of the point A are $(2, 4)$ **2**

(ii) Hence find the area enclosed between the curves. **2**

(b) (i) Copy and complete the table of values for $y = \dfrac{1}{1+x^2}$.
Express your values in exact form. **2**

x	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2
y					

(ii) Use Simpson's Rule with the five function values from part (i) to estimate **3**

$$\int_0^2 \frac{dx}{1+x^2}.$$

Give your answer correct to four decimal places.

(c) The sum of the first and third terms of a geometric series is 13. The sum of the second and fourth terms is $19\frac{1}{2}$. **3**
Find the first term and the common ratio.

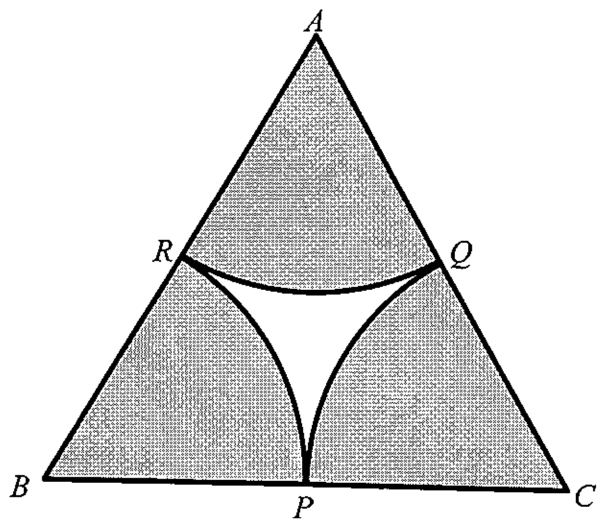
Question 6 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Prove $\dfrac{\sin\theta}{1-\cos\theta}=\dfrac{1+\cos\theta}{\sin\theta}$. **2**

(b) (i) Sketch the curves $y=\sin x$ and $y=\cos x$ for $0\le x\le 2\pi$ on the same set of axes. **2**

(ii) Find the enclosed area bounded by the curves in part (i). **2**

(c) In the diagram below, triangle ABC is equilateral with a side length of 12 cm.
P, Q and R are the midpoints of BC, AC and AB respectively.
RP, PQ, and QR are arcs of circles centred at B, C and A respectively.



(i) Show that the area of triangle ABC is $36\sqrt{3}$ cm^2. **2**

(ii) Find the exact area of sector ARQ. **2**

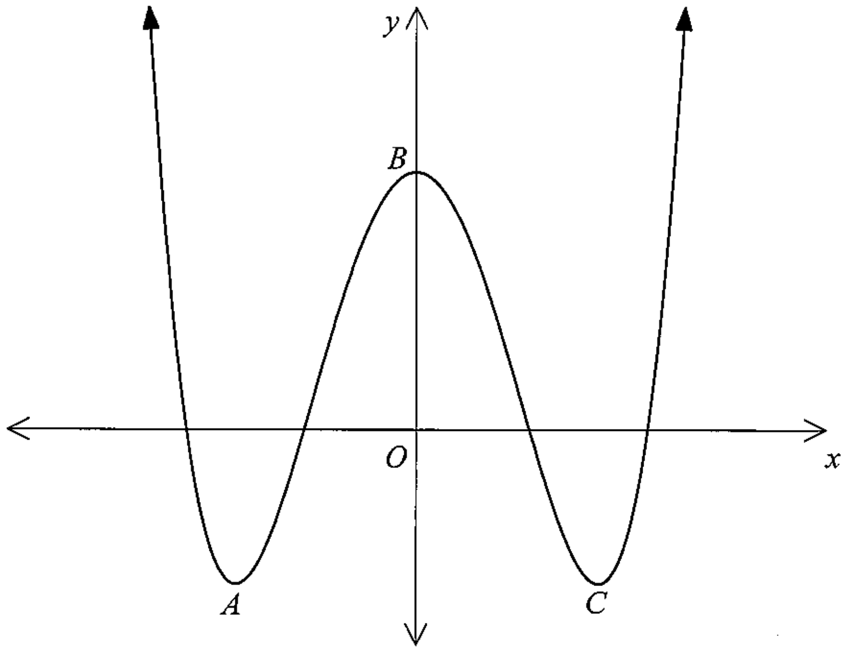
(iii) Hence find the area of the **unshaded part**, correct to three significant figures. **2**

End of SECTION C

SECTION D

Question 7 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) The graph below is of the function $y = f(x)$ where $f(x) = x^4 - 8x^2 + 10$.
The points A and C are minimum turning points and B is the maximum turning point where the graph cuts the y-axis.



(i) Find the coordinates of B. **1**

(ii) Find $f'(x)$. **1**

(iii) Show that $f'(0) = f'(2) = f'(-2) = 0$. **2**

(iv) Hence find the coordinates of A and C. **2**

(b) Two bags contain respectively 5 red and 2 white balls, and 4 red and 1 white ball. One ball is drawn at random from each bag.

(i) Draw a probability tree diagram to show all the possibilities. **2**

(ii) Find the probability that the two balls drawn out are of different colours. **2**

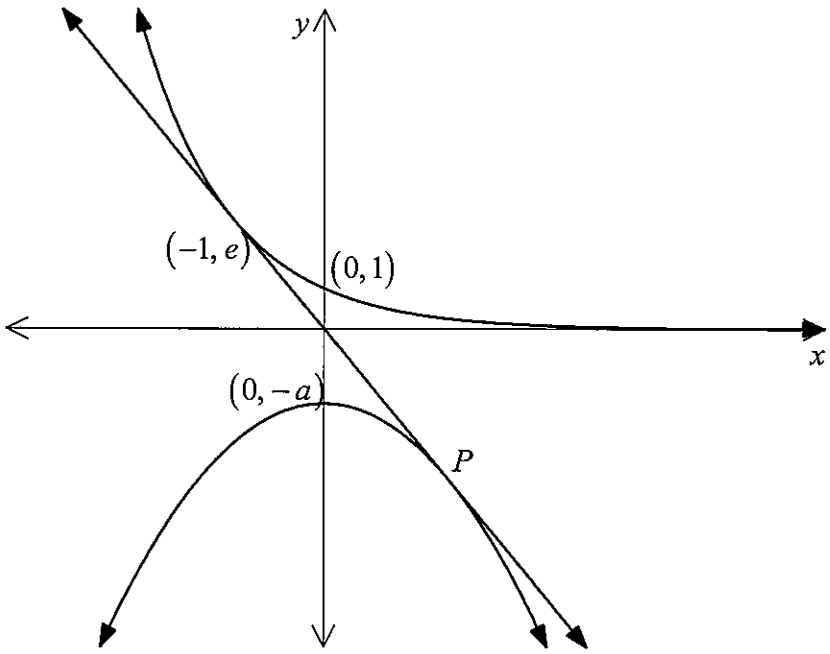
(c) A continuous curve $y = f(x)$ has the following properties for the closed interval $a \le x \le b$:

$$f(x) > 0,\ f'(x) > 0,\ f''(x) < 0.$$

Sketch a curve satisfying these conditions. **2**

Question 8 (12 marks) **Marks**

(a) The diagram below shows the graph of $y = e^{-x}$ and the parabola $y = -x^2 - a$.
The tangent to $y = e^{-x}$ through the point $(-1, e)$ is also the tangent to the parabola at P.



(i) Show that the equation of the tangent is $y = -ex$. **2**

(ii) Show that the value of x for which the tangent to $y = -x^2 - a$ has gradient $-e$ is $\frac{1}{2}e$. **2**

(iii) Find the coordinates of the point P, and hence find the value of a in exact form **3**

(b) The electrical charge Q retained by a capacitor t minutes after charging is given by $Q = Ce^{-kt}$, where C and k are constants.

The charge after 20 minutes is one half of the initial charge.

(i) Show that $k = \dfrac{1}{20}\ln 2$ **2**

(ii) How long will it be before one tenth of the original charge is retained? Answer to the nearest minute. **3**

End of SECTION D

SECTION E

Question 9 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) A jet engine uses fuel at the rate of R litres per minute.
The rate of fuel use t minutes after the engine starts operating is given by

$$R = 15 + \frac{10}{1+t}.$$

(i) What is R when $t = 0$? **1**

(ii) What is R when $t = 9$? **1**

(iii) What value does R approach as t becomes very large? **1**

(iv) Draw a sketch of R as a function of t. **2**

(v) Calculate the total amount of fuel burned during the first 9 minutes. **2**
Give your answer correct to the nearest litre.

(b) The position x cm at time t seconds of a particle moving in a straight line is given by

$$x = 3t + e^{-3t}.$$

(i) Find the position of the particle when $t = 1$. **1**
Give your answer correct to 3 significant figures.

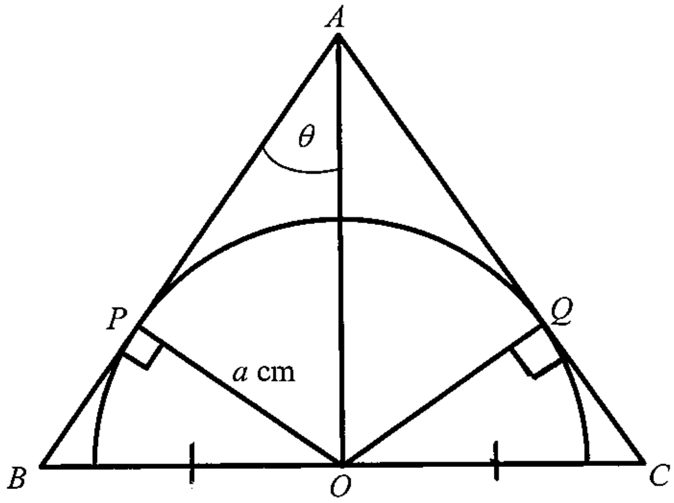
(ii) By finding an expression for the velocity of the particle, show that initially the particle is at rest. **2**

(iii) Find an expression for the acceleration of the particle. **1**

(iv) Find the limiting velocity of the particle as $t \to \infty$. **1**

Question 10 (12 marks) Use a SEPARATE writing booklet **Marks**

ABC is a variable isosceles triangle with $AB = AC$.
The sides AB and AC touch a semicircle of radius a cm at P and Q.
O is the centre of the semicircle and BOC is a straight line.



Let S cm^2 be the area of ΔABC and $\angle BAO = \theta$.

It is given that $\sin 2\theta = 2\sin\theta\cos\theta$.

(a) Show that $S = \dfrac{2a^2}{\sin 2\theta}$, where $0 < \theta < \dfrac{\pi}{2}$. **3**

(b) Determine the range of values of θ for which S is

(i) Increasing, **2**

(ii) Decreasing. **2**

(c) Sketch the curve of S against θ for $0 < \theta < \dfrac{\pi}{2}$. **2**

(d) If $2a < OA < 3a$, find the greatest value of S. **3**

End of paper

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ \ x \neq 0,\ \text{if}\ n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ \ x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax},\ \ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax,\ \ a \neq 0$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$