# TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2010

# MATHEMATICS

*Time Allowed – 3 Hours*
*(Plus 5 minutes Reading Time)*

- *All* questions may be attempted
- *All* questions are of equal value
- Department of Education approved calculators are permitted
- In every question, show all necessary working
- Marks may not be awarded for careless or badly arranged work
- No grid paper is to be used unless provided with the examination paper

**The answers to all questions are to be returned in separate *stapled* bundles clearly labeled Question 1, Question 2, etc. Each question must show your Candidate Number.**



**Question 1.** **Marks**

(a) Evaluate to 2 significant figures : $\frac{3.72\times1.96+\sqrt{4.3+2.7^2}}{3.6\times1.8+3.1^3}$ 1

(b) Rationalise the denominator and write in the form $a+b\sqrt{2}$ : $\frac{3\sqrt{2}+4}{2\sqrt{2}-3}$ 2
where a, b are ~~real~~. rational.

(c) Find the acute angle ( to the nearest minute ) that the line $4x-11y+9=0$ makes with the $x$ axis. 2

(d) Graph $y = 2sin3x$ in the domain $-\pi \le x \le \pi$ . 2

(e) Find $\lim_{h\to0}\left(\frac{4^h-1}{2^h-1}\right)$ 2

(f) Solve : $|x-3| = 4x+2$ 3

**Question 2.**

Three points $A$,$B$ and $C$ lie on the $x$-$y$ plane.
The lines $l$ and $k$ represent the lines $AB$ and $AC$ respectively.
The equations of lines $l$ and $k$ are respectively:

$3x-4y-100=0$ and $16x-63y+175=0$ respectively.

(a) Show that $B(8,-19)$ lies on the line $l$. 1

(b) Find the co-ordinates $A$ of the intersection of lines $l$ and $k$. 3

(c) Find in general form the equation of the line $m$ perpendicular to line $l$ passing through $B$. 2

(d) Show that line $m$ intersects line $k$ at the point $C(-7, 1)$. 2

(e) Find the exact perpendicular distance of $B$ from $AC$. 2

(f) Find the area of triangle $ABC$. 2

**Question 3.** **Marks**

(a) Differentiate : (i) $\frac{3}{\sqrt{1-2x}}$ 2

(ii) $\frac{sinx}{x}$ 2

(iii) $e^{tanx}$ 2

(b) Find (i) $\int \sqrt{e^{2x}}\,dx$ 1

(ii) $\int (\cot x - cosec^2 x)\,dx$ 2

(c) Find in simplest terms : $\frac{d}{dx}\{x^2(2\ lnx - 1)\}$, hence evaluate $\int_1^e x\ lnx\ dx$. 3

**Question 4.**

(a) Given the equation $x^2 = 16\,(y + 4)$

- (i) State the co-ordinates of the vertex. 1
- (ii) Find the focal length 1
- (iii) State the co-ordinates of the focus 1
- (iv) Find in general form the equation of the tangent at ( -12, 5 ) 2
- (v) Find the co-ordinates of the point where the tangent meets the directrix. 1

(b) A jar has 15 red discs and 9 black discs, while another jar has 20 red discs, 15 black discs and 10 white discs. 2
A disc is drawn from each jar.
Find the probability of drawing discs of the same colour ?

(c) A car tyre of diameter 60cm is in contact with the road at the point $P$. 4
After the car has travelled 1000km how high ( to the nearest millimetre ) is the point $P$ from the ground.

**Question 5.**

(a) Given $N = x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \ldots\ldots\ldots\ldots xy^{n-2} + y^{n-1}$

(i) Simplify $N$ in terms of $x$ and $y$. 2

(ii) Hence prove $11^{21} - 5^{21}$ is divisible by 3. 2

(b) Use Simpson's Rule with 3 function values to evaluate to 2 decimal places : 3

$$\int_0^2 \frac{4\,dx}{2sinx + 1}$$

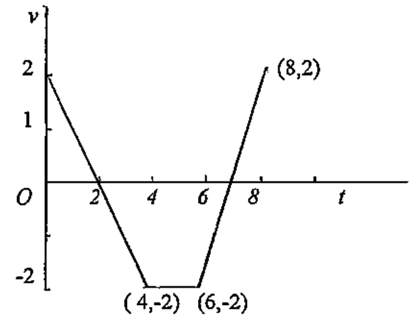
(c) Solve to 2 decimal places : $3^{2x+1} - 3^x = 10$ 3

(d) If the quadratic equation : $(k^2 + l^2)x^2 + 2l\,(k + m)x + l^2 + m^2 = 0$ has equal roots then show $l^2 = km$. 2

**Question 6.** **Marks**

(a) The region bounded by the curve $y = x\,(6 - x)$ and $y = 8$ is rotated around the $x$ axis. 4
Find the exact value of the Volume of revolution.

(b)




A particle of mass 2 kg moves in a straight line with velocity $v$ $m/s$ and displacement $x$ $m$ at time $t$ seconds.

(i) Graph acceleration $\ddot{x}\ m/s^2$ versus time $t$ seconds. 2

(ii) Find the total distance travelled during the motion. 1

(c) Find in general form the equation of the inflexional tangent on the curve : 5

$$y = 15 + 12x + 6x^2 - 2x^3$$

**Question 7.**

(a) (i) On the same axes graph : 4
($\alpha$) the line $y = 1 - 2x$ showing $x$ and $y$ intercepts.
($\beta$) the curve $y = 5 - 2x - x^2$,
showing the co-ordinates of the vertex and $y$ intercept only.

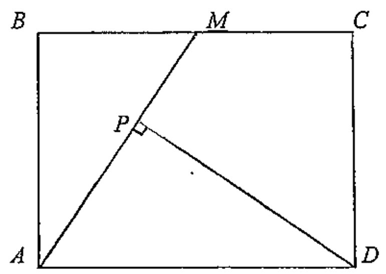
(ii) Find the $x$ values of the points $A$ and $B$ of the intersection of the line $y = 1 - 2x$ and the curve $y = 5 - 2x - x^2$. 2

(iii) Evaluate the enclosed area between the line $y = 1 - 2x$ and the curve $y = 5 - 2x - x^2$. 3

(b) The rate of decay $\frac{dM}{dt}$ of a radioactive substance is proportional to the mass $M$ present. 3
If it takes 51 minutes to decay to $\frac{1}{10}$ of it's original mass find the half-life of the substance ( nearest minute ).

**Question 8.** **Marks**

(a)




*ABCD* is a rectangle in which *AB*=40cm and *AD*=60 cm.
*M* is the midpoint of *BC* and *DP* is perpendicular to *AM*.

Draw a neat sketch of the above diagram.

(i) Prove that triangles *ABM* and *APD* are similar. 2

(ii) Calculate the length of *PD*. 2

(iii) Show that the length of *AP* is 36 cm. Give reasons. 2

(iv) Find the area of the quadrilateral *PMCD*. 3

(b) A plane flies from town *O* to town *A*, 275 km on a bearing of $032^0T$ , then to town *B* 572 km on a bearing of $S\ 26^0E$. 3

(i) Draw a diagram to show the above information.

(ii) Find the final distance ( nearest km ), and bearing (nearest degree) from *O*.

**Question 9.**

(a) A particle of mass *m* kg moves in a straight line with velocity *v m/s* and displacement *x* metres at time *t* seconds.

The velocity of the particle is given by : $v = 3\sqrt{1+9t}$ .

Find (i) the acceleration $\ddot{x}$ in terms of time *t*. 1

(ii) the displacement of the particle as a function of time *t* if the particle is initially 1 metre to the **left** of the origin. 2

(b) A man buys a house and land for $500 000. **Marks**
He pays 20% deposit, and takes a loan for the remainder.

(i) Find the value of the deposit. 1

(ii) If the loan is for 20 years, and the interest rate is 8% p.a. monthly reducible show that the amount owing after the first monthly repayment *R* is : 1

$$\$\,(400\,000(\tfrac{151}{150}) - R\,)$$

(iii) Find the amount owing after *n* months. 2

(iv) Find the monthly repayment. 2

(v) Find the amount owing after the 144th payment. 1

(vi) The value of the land was originally valued at $270 000 . If the value of the land was compounded yearly at 6% p.a. find the value of the land after the 144th payment. 1

(vii) After the 144th payment an earthquake destroys the house. The insurance policy does not cover earthquakes. Could the man sell the land to pay the remainder of the loan? Give reasons. 1

**Question 10.**

A series *S* is given by :

$$S = x + \frac{2x^2}{x+1} + \frac{4x^3}{(x+1)^2} + \frac{8x^4}{(x+1)^3} + \dots\dots\dots\dots\dots\dots$$

(a) Sketch the curve $y = \frac{2x}{x+1}$ , showing all asymptotes and intercepts with the axes. 2

(b) Find the values of $x$ for the sum to infinity to exist. 2

(c) Show that the sum to infinity is given by : 2

$$S_\infty = \frac{x^2+x}{1-x}$$

(d) Show that $\frac{dS_\infty}{dx} = \frac{-x^2+2x+1}{(1-x)^2}$ 2

(e) Find the minimum value of the sum to infinity. Justify your answer. 4

**End of Exam**

Find monthly repayment

$A = 0$ and $n = 20 \times 12 = 240$

$= 400000\left(\frac{151}{150}\right)^{240} - R\left[1 + \frac{151}{150} + \left(\frac{151}{150}\right)^2 + \cdots \left(\frac{151}{150}\right)^{239}\right]$ ✓

$R = \dfrac{400000\left(\frac{151}{150}\right)^{240}}{\dfrac{\left(\frac{151}{150}\right)^{240} - 1}{\frac{151}{150} - 1}}$

$= \dfrac{400000\left(\frac{151}{150}\right)^{240} \cdot \frac{1}{150}}{\left(\frac{151}{150}\right)^{240} - 1}$

$= \$3345.76$ ✓

2 marks

Round off to 2 decimal place

$A_{144} = 400\,000\left(\frac{151}{150}\right)^{144} - 3345.76\left[\dfrac{\left(\frac{151}{150}\right)^{144} - 1}{\left(\frac{151}{150} - 1\right)}\right]$

$= \$236\,672.36$ ✓ 1 mark

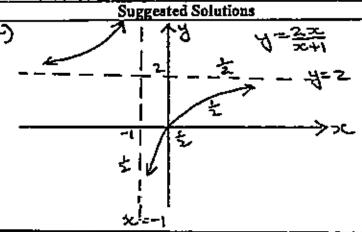
Value of land

value $= 270\,000\,(1.06)^{12}$

$= \$543\,293.05$ ✓ 1 mark

He can sell land as $\$543\,293.05 > \$236\,672.36$. ✓ 1 mark

2010 TRIAL 2U MATHEMATICS: Question 10

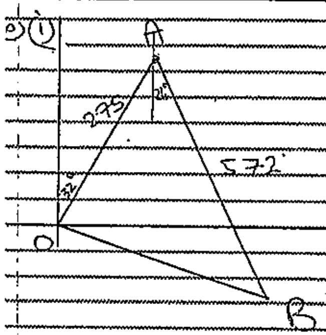
| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (a)   | 2 | Shape ½ mark<br>$y$ intercept labelled gained ½ mark except when there were more than 2 mistakes on the graph. |
| (b) $\left\lvert \frac{2x}{1+x} \right\rvert < 1$<br>$\frac{2x}{1+x} = 1$ or $\frac{2x}{1+x} = -1$<br>$x = 1$ (½) $\quad 3x = -1$<br>$x = -\frac{1}{3}$ (½)<br>∴ For sum to infinity<br>(½) $-\frac{1}{3} < x < 1$ [N.B. $x = 0$, $S = 0$] | 2 | |
| (c) $S_\infty = \frac{a}{1-r}$ where $a = x$, $r = \frac{2x}{1+x}$<br>$= \dfrac{x}{1 - \frac{2x}{1+x}}$<br>$= \dfrac{x(1+x)}{1+x-2x}$ | 1 | |
| $\therefore S_\infty = \dfrac{x^2+x}{1-x}$ | 1 | |
| (d) $S = \dfrac{x^2+x}{1-x}$<br>$\frac{dS}{dx} = \dfrac{(1-x)(2x+1) - (x^2+x)\times(-1)}{(1-x)^2}$<br>$= \dfrac{2x+1-2x^2-x+x^2+x}{(1-x)^2}$ | 1 | |
| $\therefore \frac{dS}{dx} = \dfrac{-x^2+2x+1}{(1-x)^2}$ | 1 | |



2010 TRIAL 2U MATHEMATICS: Question 10

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (e) $\frac{dS}{dx} = \dfrac{-x^2+2x+1}{(1-x)^2}$<br>For minimum $\frac{dS}{dx} = 0$<br>$\therefore x^2 - 2x - 1 = 0$<br>$x = \dfrac{2 \pm \sqrt{4+4}}{2}$<br>$= 1 \pm \sqrt{2}$ | | If the question was made easier by using any other domain except $-\frac{1}{3}<x<1$, max of 2 marks, BUT needed to test endpts of their domain to gain the 2 marks. |
| But $-\frac{1}{3} < x < 1$ from (b) | 1 | |
| But $1-\sqrt{2} < -\frac{1}{3}$ and $1+\sqrt{2} > 1$ then | 1 | |
| Since $S_\infty$ is continuous for $-\frac{1}{3}<x<1$ then the minimum occurs at the endpoints.<br>Test end pts.<br>Minimum $= \lim\limits_{x \to -\frac{1}{3}} S = \dfrac{\left(-\frac{1}{3}\right)^2 - \frac{1}{3}}{1 + \frac{1}{3}}$ and $\lim\limits_{x \to 1} S \to \infty$<br>$= \dfrac{1-3}{9+3}$<br>$= -\frac{1}{6}$ | 2 | When $x = 1-\sqrt{2}$ an answer of $2\sqrt{2}-3$ (−0.1715---) with working received 1½ as question had been greatly simplified<br>i.e. when $x = 1-\sqrt{2}$<br>$S_\infty = \dfrac{(1-\sqrt{2})^2 + 1 - \sqrt{2}}{1-(1-\sqrt{2})}$<br>$= \dfrac{1-2\sqrt{2}+2+1-\sqrt{2}}{\sqrt{2}}$<br>$= \dfrac{4-3\sqrt{2}}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$<br>$= \dfrac{4\sqrt{2}-6}{2}$<br>$= 2\sqrt{2}-3$ |

## 2U MATHEMATICS: Question 8

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| a)(i)  | $\frac{1}{2}$ | If all of the information is correct! |
| ii) $OB^2 = 275^2 + 572^2 - 2(275)(572)\cos 58^\circ$ | $\frac{1}{2}$ | |
| $= 402809 - 314600\cos 58^\circ$ | | |
| $= 402809 - 166712.6005$ | | |
| $= 236\,096.3995$ | | |
| $\therefore OB = 485.8975195$ | $\frac{1}{2}$ | |
| $OB = 486\text{ km}$ (nearest km) | | |
| $572^2 = 275^2 + 485.89^2 - 2(275)(485.89)\cos A\hat{O}B$ | 1 | * If they used the sine rule and didn't find the obtuse angle, they lose $\frac{1}{2}$ mk + lose $\frac{1}{2}$ mk from bearing i.e. +1 off the total. |
| $\therefore \cos A\hat{O}B = \dfrac{275^2 + 485.89^2 - 572^2}{2(275)(485.89)}$ | | |
| $A\hat{O}B = 93^\circ 19'$ | | |
| $\therefore$ Bearing is $(93^\circ + 32^\circ)T$ | $\frac{1}{2}$ | |
| $\approx 125^\circ T$ (nearest degree) | | |



## Question 9.

a(i)
$$\ddot{x} = \frac{dv}{dt}$$
$$= \frac{d}{dt}\, 3\left[1+9t\right]^{\frac{1}{2}}$$
$$= \frac{3}{2}\cdot 9\,(1+9t)^{-\frac{1}{2}}$$
$$\ddot{x} = \frac{27}{2\sqrt{1+9t}} \checkmark$$

[1 mark]

Do not use decimals $\frac{13.5}{\sqrt{1+9t}}$;

ii)
$$x = \int 3\,(1+9t)^{\frac{1}{2}}\,dt$$
$$x = \frac{2}{9}(1+9t)^{\frac{3}{2}} + C. \checkmark$$

when $t=0,\ x=-1$

$$-1 = \frac{2}{9} + C$$
$$C = -\frac{11}{9}$$
$$\therefore \text{Displacement } x = \frac{2}{9}(1+9t)^{\frac{3}{2}} - \frac{11}{9} \checkmark$$

[2 marks]

b(i) Deposit $= 20\%$ of $\$500000$

$= \$100000$ ✓ [1 mark]

ii) Loan for $\$400000$

Monthly interest $= \frac{8}{12}\% = \frac{8}{1200} = \frac{1}{150}$

Had to show $\frac{8}{12}\%$ to gain a mark.

Amount owing after 1st month.

$$\$400\,000 \times \left(1 + \frac{8}{1200}\right) - R$$
$$= 400\,000\left(\frac{1+150}{150}\right) - R$$
$$= 400000\left(\frac{151}{150}\right) - R \checkmark$$

[1 mark]

iii)
$$A_2 = 400\,000\left(\frac{151}{150} - R\right)\frac{151}{150} - R$$
$$= 400000\left(\frac{151}{150}\right)^2 - R\left(1 + \frac{151}{150}\right)$$
$$A_3 = \left[400\,000\left(\frac{151}{150}\right)^2 - R\left[1 + \frac{151}{150}\right]\right]\frac{151}{150} - R$$
$$= 400000\left(\frac{151}{150}\right)^3 - R\left[1 + \frac{151}{150} + \left(\frac{151}{150}\right)^2\right] - R$$

To show the answer you need to get a pattern for at least 3 months.

$\therefore$ Amount after $n^{th}$ month

$$A_n = 400000\left(\frac{151}{150}\right)^n - R\left[1 + \frac{151}{150} + \left(\frac{151}{150}\right)^2 + \cdots \left(\frac{151}{150}\right)^{n-1}\right.$$

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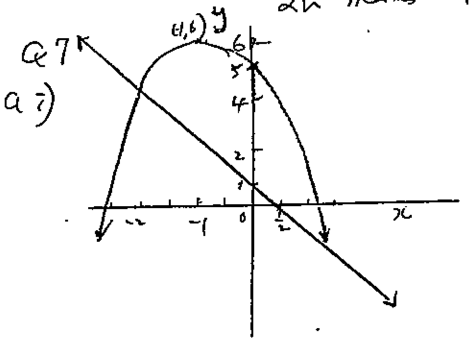
Q7

a i)




intercepts $\frac{1}{2}$m each

$(0,1)$, $(\frac{1}{2},0)$, $(0,5)$

vertex $(-1,6)$ 1m

parabola 1m

straight line $\frac{1}{2}$m.

ii) intersect at $x = 2, -2$ 2m

iii) $\text{Area} = \int_{-2}^{2} (5-2x-x^2)-(1-2x)\,dx$ 1m

$= \left[4x - \frac{x^3}{3}\right]_{-2}^{2}$ 1m

$= 10\frac{2}{3}$ units$^2$ 1m

b) $M = Ae^{-kt}$

$k = \frac{\ln 10}{57}$ 1m

$\frac{A}{2} = A e^{-t \cdot \frac{\ln 10}{57}}$ 1m

$t = 15.35$ $\frac{1}{2}$m

$= 15$ minutes (nearest min) $\frac{1}{2}$m

comments

generally well done.

generally well done

A few students wrote

$k = -\frac{\ln \frac{9}{10}}{57}$

and

$t = 336$ min

get 2m

## 2u MATHEMATICS: Question 8

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (i) In $\triangle$s ABM, APD | 1/2 | |
| $\angle ABM = \angle APD$ (both 90°) | 1/2 | |
| $\angle BMA = \angle PAD$ (alternate angles are equal BM ∥ AD) | 1/2 | |
| $\therefore \triangle ABM \,\vert\vert\vert\, \triangle APD$ (equiangular) | 1/2 | |
| (ii) $\frac{PD}{AB} = \frac{AO}{AM}$ (corresponding sides in similar triangles are in the same ratio) | 1/2 | $-\frac{1}{2}$ if ratio is written incorrectly. |
| $\frac{PD}{40} = \frac{60}{50}$ | 1/2 | |
| $PD = \frac{6}{5} \times 40$ ($AB^2 + BM^2 = AM^2$, $30^2 + 40^2 = AM^2$, $AM = 50$ (by Pythagoras)) 1/2 | | |
| $PD = 48$cm | 1/2 | |
| (iii) $AP^2 + PD^2 = AD^2$ (by Pythagoras) | 1/2 | (or could have done it using ratio of corresponding sides in similar triangles) |
| $AP^2 + 48^2 = 60^2$ | 1/2 | |
| $AP^2 = 1296$ | 1/2 | |
| $AP = 36$cm | 1/2 | |
| (iv) Area PMCD $= A_{ABCD} - A_{ABM} - A_{APD}$ | 1 | (1/2 off for each error). |
| $= (60 \times 40) - \left(\frac{40 \times 30}{2}\right) - \left(\frac{48 \times 36}{2}\right)$ | 1 | |
| $= 936$ units$^2$ | 1 | |
| OR PM = AM − PA $= 50 - 36$ $= 14$ | 1 | |
| $\text{Area}_{DCM} = \frac{1}{2} \times 30 \times 40 = 600$ | 1/2 | |
| $\text{Area}_{PMD} = \frac{1}{2} \times 14 \times 48 = 336$ | 1 | |
| $\therefore \text{Area}_{PMCD} = \text{Area}_{DCM} + \text{Area}_{PMD}$ $= 600 + 336$ $= 936$ units$^2$ | 1/2 | |

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## 2U MATHEMATICS: Question 6 ..

**Suggested Solutions** | **Marks** | **Marker's Comments**

(a) Intersection points: $x(6-x)=8$

$x^2-6x+8=0$

$(x-4)(x-2)=0$

$x=4$ or $x=2$ — **1**

If they had the wrong limits, 1 mark off

Volume $=\pi\int_2^4 x^2(6-x)^2\,dx-\pi\int_2^4 8^2\,dx$ — **½**

$=\pi\int_2^4 x^2(36-12x+x^2)\,dx-\pi\int_2^4 64\,dx$

$=\pi\int_2^4(36x^2-12x^3+x^4-64)\,dx$

$=\pi\left[12x^3-3x^4+\frac{x^5}{5}-64x\right]_2^4$ — **1**

$=\pi\left[\frac{4^5}{5}+12(4)^3-3(4)^4-64(4)\right]-\pi\left[12\times8-3(16)+\frac{32}{5}-128\right]$ — **1**

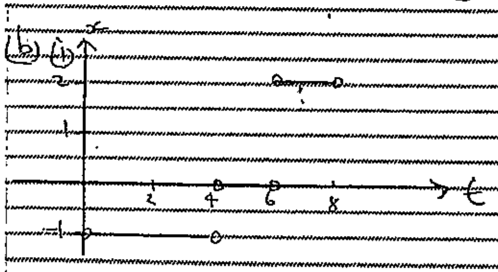
$=\pi(204\tfrac45-768+768-256)-\pi(96-48+6\tfrac25-128)$

$=\pi(198\tfrac45-176)$ — **½**

$=22\tfrac45\pi$ or $\frac{112\pi}{5}$ units$^3$ or $22.4\pi$ units$^3$

* 1½ mk off if they forgot to square the fns.
* ½ mk off if the fns are around the wrong way
* ½ mk off if squared as one big function
* ½ mk off for every calculator error
* 1 mk off if they forgot $\pi\int_2^4 8^2\,dx$.

(b) (i)




**2 mks**

* ½ mk off for no open circles
* ½ mk off if lines joined up
* 0 for curves!

(ii) Total distance $=\frac{2\times2}{2}+\frac{2\times2}{2}+2\times2+\frac{1\times2}{2}+\frac{1\times2}{2}$

$=2+2+4+1+1$

$=10$ metres. — **1**

* ½ mk if they got 8 metres.

(c) $y=15+12x+6x^2-2x^3$

$\frac{dy}{dx}=12+12x-6x^2$ — **½**



## MATHEMATICS: Question 6 .. continued

**Suggested Solutions** | **Marks** | **Marker's Comments**

$\frac{d^2y}{dx^2}=12-12x$ — **½**

possible points of inflexion when $\frac{d^2y}{dx^2}=0$ — **½**

$12-12x=0$

$x=1$ — **½**

when $x=1$, $y=15+12+6-2$

$y=31$ $(1,31)$ — **½**

when $x=1$, $y'=12+12-6$

$=18$

$\therefore m=18$ — **½**

Test for change in concavity

| $x$ | 0.9 | 1 | 1.1 |
|---|---|---|---|
| $\frac{d^2y}{dx^2}$ | 1.2 | 0 | −1.2 |
| concavity | up | — | down |

$\therefore$ change in concavity at $x=1$

$\therefore$ $(1,31)$ is a point of inflexion — **1**

* ½ mk off if they didn't use numbers or if they didn't state there's a change in concavity

eqn. of inflexional tangent is:

$y-y_1=m(x-x_1)$ — **½**

$y-31=18(x-1)$

$18x-y+13=0$ — **½**

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Q5

a) $N = \dfrac{x^{n-1}\left[\left(\frac{y}{x}\right)^n - 1\right]}{\frac{y}{x} - 1}$ 1m

$N = \dfrac{x^n - y^n}{x - y}$ 1m

$x^n - y^n = (x-y)N$

$11^{21} - 5^{21} = (11-5)\left[11^{20} + 11^{19}\cdot 5 + \cdots + 5^{20}\right]$ 1m

$= 6 \times (11^{20} + 11^{19}\cdot 5 + \cdots + 5^{20})$

$= 3 \times 2 \times \underbrace{(11^{20} + 11^{19}\cdot 5 + \cdots + 5^{20})}_{\text{integer}}$

$\therefore 11^{21} - 5^{21}$ is divisible by 3 # 1m

b)

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $\frac{4}{2\sin x+1}$ | 4 | 1.4909 | 1.4191 |

1m

$\int_0^2 \frac{4\,dx}{2\sin x+1} = \frac{2-0}{6}\left[4 + 4\left[\frac{4}{2\sin 1+1}\right] + \frac{4}{2\sin 2+1}\right]$ 1m

$= 3.79425$

$= \underline{\underline{3.79\ (2dp)}}$ 1m

c) $3\cdot 3^{2x} - 3^x - 10 = 0$

Put $u = 3^x$

$3u^2 - u - 10 = 0$

$(3u+5)(u-2) = 0$ 1m

$u = 2$ or $u = -5/3$

$\therefore 3^x = 2$ or $3^x = -5/3$ 1m

but $3^x > 0$ $\therefore 3^x = 2$ only

$x = \frac{\ln 2}{\ln 3} = 0.6309\ldots$

$x = \underline{\underline{0.63\ (2dp)}}$ 1m

Comments

many students did not simplify completely to lowest term.

many students did not justify $11^{20} + 11^{19}\cdot 5 + \cdots + 5^{20}$ is an integer and simply say $N$ is an integer get 1m only.

2u Maths Trial 2010

5 d) $\Delta = 0$ for equal roots

$[2l(k+m)]^2 - 4[k^2+l^2][l^2+m^2] = 0$ 1m

$4l^2[\cancel{k^2} + 2km + \cancel{m^2}] - 4[\cancel{k^2l^2} + k^2m^2 + l^4 + \cancel{l^2m^2}] = 0$

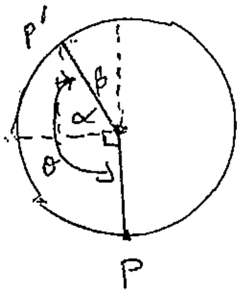
$\cancel{4}l^2\cdot 2km - \cancel{4}k^2m^2 - \cancel{4}l^4 = 0$

$l^4 + k^2m^2 - 2kml^2 = 0$

$(l^2 - km)^2 = 0$ 1m

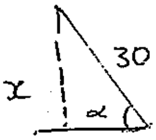
$l^2 = km$ #

many students made mistakes & cannot complete squa max 1m



$\alpha = \theta - \frac{\pi}{2}$

$= 1.426283065$



$\sin\alpha = \frac{x}{30}$

$x = 30 \sin\alpha$

$\cong 29.8$

∴ Total height $= 30 + x$

$= 30 + 29.68658014$

$= 59.68658014$

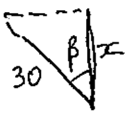
$= 597$ mm

$\boxed{\text{OR}}$

$\beta = \pi - \theta$

$= \pi - 171.72^\circ$

$= 8.28^\circ$



$\cos 8.28 = \frac{x}{30}$

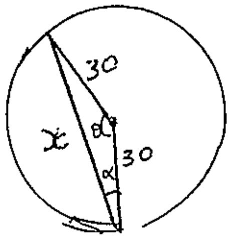
$x = 30 \cos 8.28^\circ$

$= 29.68728356$

∴ height $29.68728356 + 30$

$= 59.68728356$

height $= 597$ mm



$\alpha = \frac{180 - 171.72}{2}$

$= 4.14^\circ$

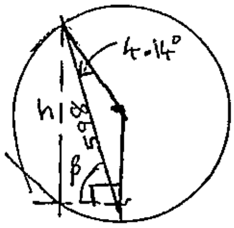
Sine Rule

$\frac{x}{\sin 171.72^\circ} = \frac{30}{\sin 4.14^\circ}$

$x = \frac{30 \sin 171.72^\circ}{\sin 4.14^\circ}$

$= 59.843$ cm

$= 598$ mm



$\beta = 90 - 4.14^\circ$

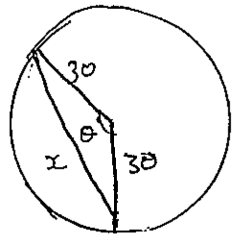
$= 85.86$

$\sin\beta = \frac{h}{598}$

$h = 598 \sin 85.86$

$h = 596.4395939$

$h = 596$ mm.



Cosine rule

$x^2 = 30^2 + 30^2 - 2(30)(30)\cos 0.954$

$= 3581.237013$

$x = 59.843$

$= 598$ mm

Similarily as in previous example

## Question 3

| | |
|---|---|
| a) Differentiate<br>i) $\frac{3}{\sqrt{1-2x}} = \frac{d}{dx} 3(1-2x)^{-\frac{1}{2}} = 3 \times -\frac{1}{2} \cdot -2(1-2x)^{-\frac{3}{2}}$<br>$= 3(1-2x)^{-\frac{3}{2}}$<br>OR $\frac{3}{(1-2x)^{\frac{3}{2}}}$<br>or $\frac{3}{(1-2x)(\sqrt{1-2x})}$ | $\frac{-3}{2\sqrt{(1-2x)^3}}$ earned (1) mark<br>$\frac{-3}{\sqrt{1-2x}}$ earned (1) mark<br>2 marks |
| ii) $\frac{d}{dx} \frac{\sin x}{x} = \frac{x\cos x - \sin x}{x^2}$ | ($1\frac{1}{2}$) mark if $\frac{\sin x - x\cos x}{x^2}$<br>2 marks |
| (iii) $\frac{d}{dx} e^{\tan x} = \sec^2 x\, e^{\tan x}$ | 2 marks |
| b (i) $\int \sqrt{e^{2x}}\, dx = \int e^x\, dx$<br>$= e^x + c$ 1 mark | $-\frac{1}{2}$ if no C<br>Note cannot apply chain rule for $\int$. |
| (ii) $\int (\cot x - \operatorname{cosec}^2 x)\, dx = \int \left(\frac{\cos x}{\sin x} - \operatorname{cosec}^2 x\right) dx$<br>$\int \frac{\cos x}{\sin x}\, dx = \ln \sin x + c$<br>we know, the derivative of $\cot x = -\operatorname{cosec}^2 x$<br>proof $\cot x = \frac{\cos x}{\sin x}$ let $u = \cos x \to u' = -\sin x$<br>$v = \sin x \to v' = \cos x$<br>Using Quotient Rule $\frac{\sin x(-\sin x) - \cos^2 x}{\sin^2 x}$<br>$= \frac{-1(\sin^2 x + \cos^2 x)}{\sin^2 x}$<br>$= \frac{-1}{\sin^2 x}$<br>$= -\operatorname{cosec}^2 x$<br>$\therefore \int -\operatorname{cosec}^2 x\, dx = \cot x + c$<br>$\therefore \int \cot x - \operatorname{cosec}^2 x = \ln \sin x + \cot x + C$ | $-\frac{1}{2}$ wrong sign<br>2 marks |
| c) $\frac{d}{dx} x^2(2\ln x - 1) = x^2 \cdot \frac{2}{x} + (2\ln x - 1)2x$<br>$= 4x\ln x$<br>$\int_1^e x\ln x\, dx = \left\{\frac{1}{4} x^2 [\therefore 2\ln x - 1]\right\}_1^e$<br>$= \frac{1}{4}\left[e^2(2-1) - 1\cdot(-1)\right]$<br>$= \frac{e^2+1}{4}$ | Use product rule<br>3 marks. |

## Question 4

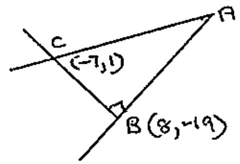
| | |
|---|---|
| a) (i) Vertex $(0,-4)$ | 1 mark |
| ii) $4a = 16$<br>$a = 4$ | 1 mark |
| iii) focus $(0,0)$ | 1 mark |
| iv) $y = \frac{x^2}{16} - 4$<br>$\frac{dy}{dx} = \frac{x}{8}$<br>at $x = -12$ $m = \frac{-12}{8}$<br>$m = \frac{-3}{2}$<br>Equation of tangent $y - y_1 = m(x - x_1)$<br>$y - 5 = -\frac{3}{2}[x + 12]$<br>$2y - 10 = 3x - 36$<br>$3x + 2y + 26 = 0$ | few did not simplify $\frac{-12}{8} = \frac{-3}{2}$<br>2 marks |
| v) Directrix $y = -8$<br>$3x + 2(-8) + 26 = 0$<br>$3x = -10$<br>$x = -3\frac{1}{3}$<br>$\therefore$ point is $(-3\frac{1}{3}, -8)$ | 1 mark |
| b) $P(R,R) + P(B\,B) = \frac{15}{24} \times \frac{20}{45} + \frac{9}{24} \times \frac{15}{45}$<br>$= \frac{5}{8} \times \frac{4}{9} + \frac{3}{8} \times \frac{3}{9}$<br>$= \frac{29}{72}$ | 2 marks |
| c) Circumference of Tyre $= 60\pi$<br>Distance travelled $= 1000\text{ km} = 1000000\text{ m}$<br>$= 100000000\text{ cm}$<br>$= 1\,000\,000\,000\text{ mm}$<br>Number of revolutions $= \frac{\text{Distance travelled}}{\text{circumference tyre}}$<br>$= \frac{100000000\text{ cm}}{60\pi\text{ cm}}$<br>$= 530516.477$ rev.<br>There is 530,516 complete revolutions with 0.477 of a revolution left over<br>Hence 0.477 of circumference ($60\pi$ cm) is left over<br>ie $0.477 \times 60\pi$<br>$= 89.91238175$ cm.<br>Calculate $\theta$<br>$l = r\theta$<br>$89.91238175 = 30\theta$<br>$\theta = 2.997079392^c$ or $0.954\pi$<br>or $171.7°$ | • few $r = 60$ cm instead of 30 cm.<br>• Did not change both distance and radius into same unit<br>4 marks |

2010 TRIAL 2U MATHEMATICS: Question..2...

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| 2(a) Show $B(8,-19)$ lies on $3x-4y-100=0$<br>$LHS = 3\times 8-4\times -19-100$<br>$=24+76-100$<br>$=0$<br>$\therefore LHS = RHS$, hence $B(8,-19)$ lies on $\ell$. | 1 | $0=0$ without saying $LHS=RHS$ lost $\frac{1}{2}$ mark |
| (b) $\ell: 3x-4y-100=0$ ....(1)<br>$k: 16x-63y+175=0$ ....(2)<br>(1)×16 and (2)×3<br>$48x-64y=1600$ ....(3)<br>$48x-189y=-525$ ....(4)<br>(3)−(4)<br>$125y=2125$<br>$y=17$<br>Subst. into (1)<br>$3x-68=100$<br>$3x=168$<br>$x=56$<br>$\therefore A(56,17)$ | 1<br>1<br>1 | 1 mark deducted for each error |
| (c) $m_\ell = \frac{3}{4}$<br>Gradient of a line perpendicular to $\ell$ is $-\frac{4}{3}$ as $m_1\times m_2=-1$ for perp. lines<br>Eqn is: $y+19=-\frac{4}{3}(x-8)$<br>$3y+57=-4x+32$<br>$\therefore$ Eqn. of the line in general form is:<br>$4x+3y+25=0$ | 1<br>1 | Not in general form, max of 1 mark only |
| (d) For C $4x+3y=-25$ ....(1)<br>$16x-63y=-175$ ....(2)<br>(1)×4<br>$16x+12y=-100$ ....(3)<br>(3)−(2)<br>$75y=75$<br>$y=1$ ....(4)<br>Sub (4) in (1)<br>$4x+3=-25$<br>$4x=-28$<br>$x=-7$ $\therefore C(-7,1)$ | 1<br>1 | |

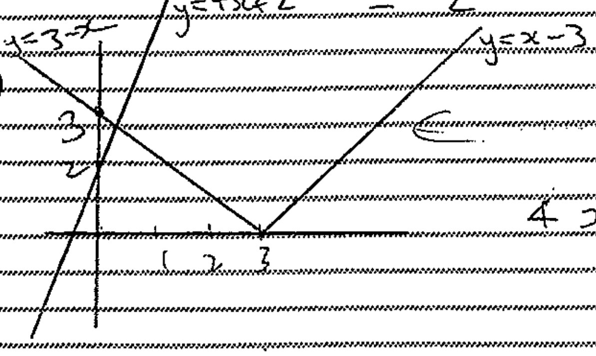


2010 TRIAL 2U MATHEMATICS: Question..2...

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| 2(e) $D=\left\|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right\|$<br>$=\frac{16\times 8-63\times -19+175}{\sqrt{16^2+63^2}}$<br>$=\frac{1500}{65}=\frac{300}{13}$<br>$\therefore D=23\frac{1}{13}$ units | 1<br>1 | No penalty for units missing |
| (f) <br>$AC=\sqrt{63^2+16^2}$<br>$=65$ units<br>$\therefore$ Area $\Delta ABC=\frac{1}{2}\times 65\times 23\frac{1}{13}$<br>$A=750$ units$^2$ | 1<br>1 | |

JRAHS 2U MATHS TRIAL SOLUTIONS

2U **MATHEMATICS: Question** 1

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (a) $\dfrac{3.72 \times 1.96 + \sqrt{4.3} + 2.7^2}{3.6 \times 1.8 + 3.1^3}$ | | If they write out the calculator display and it was wrong but they rounded up correctly they got ½ mk. |
| $= 0.29488$ | ½ | |
| $= 0.29$ (2 sig figures) | ½ | |
| (b) $\dfrac{(3\sqrt{2}+4)}{(2\sqrt{2}-3)} \cdot \dfrac{(2\sqrt{2}+3)}{(2\sqrt{2}+3)} = \dfrac{12+9\sqrt{2}+8\sqrt{2}+12}{8-9}$ | ½ | |
| ½ | ½ | |
| $= -24 - 17\sqrt{2}$ | ½ | |
| (c) $m = 4/11$ | 1 | |
| $\tan\theta = \frac{4}{11}$ | ½ | |
| $\theta = 19°59'$ (nearest minute) | ½ | |
| (d)   | | ½ intercepts<br>½ - shape<br>½ - amplitude<br>½ - period |
| (e) $\lim_{h\to 0} \dfrac{(4^h - 1)}{2^h - 1} = \lim_{h\to 0} \dfrac{(2^h-1)(2^h+1)}{2^h - 1}$ | 1 | an answer of "2" with no working = ½ mk. |
| $= \lim_{h\to 0} (2^h + 1)$ | ½ | |
| $= 1 + 1$<br>$= 2$ | ½ | |
| (f)   | 2mks | * scored 2 mks if you solved algebraically but forgot to check the answers. |
| $4x + 2 = 3 - x$ | | |
| $5x = 1$ | 1mk | |
| $x = 1/5$ | | |