# Part A

Hand in questions 1 to 4 as a single bundle.
Ensure your name is on the bundle.

Question 1: (14 marks)

a) Differentiate:

(i) $\sin(3x + 1)$

(ii) $xe^{2x}$

(iii) $\cos^2 x$.

(iv) $\dfrac{\log_e x}{x}$ (in simplest form)

b) Evaluate

(i) $\displaystyle\int_0^3 (e^{3x} + 1)dx$ (leave in exact form)

(ii) $\displaystyle\int_0^{\frac{\pi}{2}} \sin 2x dx$

c) Find a primitive of $\dfrac{2x}{x^2+1}$

Question 2: (11 marks) (*Start a new page*)

a) Given that $\log_a b = 2.75$ and $\log_a c = 0.25$, find the value of:

(i) $\log_a \left(\dfrac{b}{c}\right)$

(ii) $\log_a (bc)^2$.

b) Solve the equation $3^{2x} + 2.3^x - 15 = 0$.

c) Let $\alpha$ and $\beta$ be the roots of the equation $x^2 - 5x + 2 = 0$. Find the values of

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $(\alpha + 1)(\beta + 1)$.

Question 3: (8 marks) (*Start a new page*)

a) Find the acute angle between the lines $2x + y = 4$ and $x - y = 2$, to the nearest degree.

b) Let A(-1, 5) and B(3, 2) be points in the plane. Find the coordinates of the point C which divides the interval AB externally in the ratio 3 : 1.

c) Solve the inequality $\dfrac{x^2-9}{x} > 0$.

<u>Question 4</u>: (17 marks) (*Start a new page*)

a) Consider the function f(x) = x - 3 ln x. There is one turning point for f(x). Find its co-ordinates and determine whether it is a maximum or minimum turning point.

b) A Geiger counter is taken into a region after a nuclear accident and gives a reading of 10 000. One year later, the same Geiger counter gives a reading of 9000. It is known that the reading T is given by the formula $T = T_0e^{-kt}$, where $T_0$ and k are constants and t is the time measured in years from the date of the accident.

(i) Evaluate the constants $T_0$ and k.

(ii) What is the expected Geiger counter reading 10 years after the accident.

c) The displacement (x metres) from a fixed point, O, at time t seconds is given by $x = \frac{2t^3}{3} - \frac{7t^2}{2} - 4t + 1$ . Find

(i) the initial acceleration

(ii) when is the particle at rest

(iii) the distance travelled during the 3rd second.

## <u>PART B</u>

Hand in questions 5 to 7 as a single bundle.
Ensure your name is on the bundle.

<u>Question 5</u>: (16 marks) (*Start a new page*)

a) Write down the exact value of 135° in radians.

b) Solve the equation $2\sin^2\theta = 2\sin\theta\cos\theta$ for $0 \le \theta \le 2\pi$.

c)

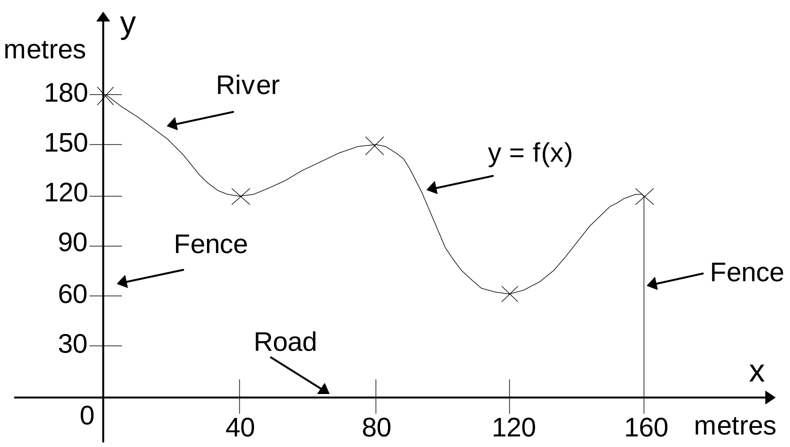
(i) If $\sqrt{3}\sin\theta - \cos\theta = R\sin(\theta - \alpha),\ R > 0, \alpha$ acute, find *R* and $\alpha$ .

(ii) Hence or otherwise, find all angles $\theta$, where $0 \le \theta \le 2\pi$, for which $\sqrt{3}\sin\theta - \cos\theta = 1$

d) Prove the following identity: $\frac{\sin A}{\cos A + \sin A} + \frac{\sin A}{\cos A - \sin A} = \tan 2A$

Question 6: (12 marks) (*Start a new page*)

a)



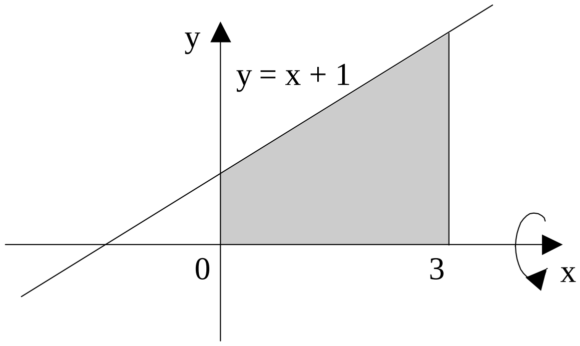

The diagram is a scale drawing of a paddock bounded by a river, a road, and two fences perpendicular to the road. A farmer wishes to calculate the area of this paddock and has measured the perpendicular distances of the river from the road at intervals of 40 metres. These distances can be read off the diagram.

(i) Take the road as the x axis, the fences as the y axis and the line $x = 160$, and the river as $y = f(x)$. Copy and complete the following table of values in your examination booklet:

| x | 0 | 40 | 80 | 120 | 160 |
|---|---|---|---|---|---|
| y = f(x) | | | | | |

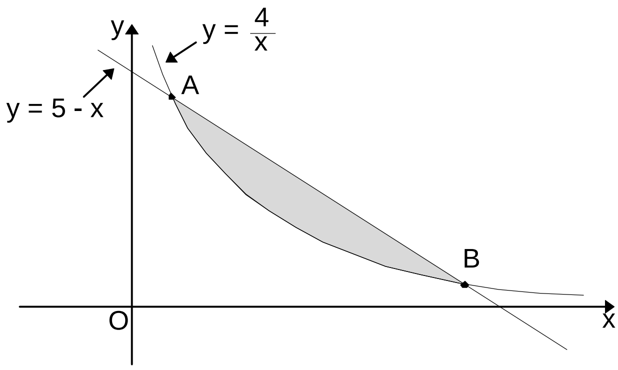
(ii) Estimate the area of the paddock using Simpson's Rule with five function values.

b)




The region which lies between the x axis and the line $y = x + 1$ from $x = 0$ to $x = 3$ is rotated about the x axis to form a solid. Find the volume of the solid.

c)




NOT TO SCALE

The diagram shows the graphs of $y = \frac{4}{x}$ and $y = 5 - x$. The graphs intersect at the points A and B as shown.

(i) Find the x coordinates of the points A and B.

(ii) Find the area of the shaded region between $y = \frac{4}{x}$ and $y = 5 - x$ (leave in exact form)

<u>Question 7</u>: (14 marks) (*Start a new page*)

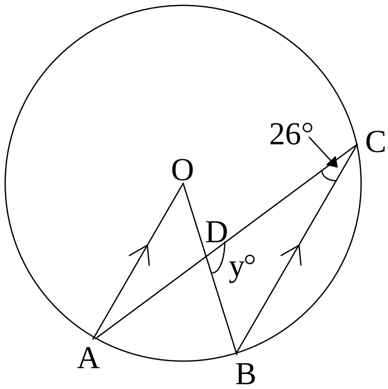
a) The tenth term of an arithmetic sequence is 29 and the fifteenth term is 44.

(i) Find the value of the common difference and the value of the first term.

(ii) Find the sum of the first 75 terms

b) Find the number which when added to each of 2, 6 and 13 will give a set of three numbers in geometric progression.

c)




The points A, B and C lie on a circle with centre O. The lines AO and BC are parallel, and OB and AC intersect at D. Also, $\angle ACB = 26°$ and $\angle BDC = y°$, as shown in the diagram. Copy or trace the diagram into your Writing Booklet.

(i) State why $\angle AOB = 52°$.

(ii) Find y. Justify your answer.

d) Let $S_n = 1 \times 2 + 2 \times 3 + \ldots + (n - 1) \times n$. Use mathematical induction to prove that, for all integers n with $n \geq 2$, $S_n = \frac{1}{3}(n-1)\,n(n+1)$

# STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \ x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \ a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \ a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \ a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \ a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \ a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln(x + \sqrt{x^2 + a^2}), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE: $\ln x = \log_e x, \ x > 0$