Name:________________

Teacher:________________

Saint Mark's Coptic Orthodox College

Mathematics Department

Assessment Task I

Year 11-Extension I

February 2004

Time Allowed: 2 Periods

DIRECTIONS TO CANDIDATE:

- Attempt all questions.
- Give answers in the space provided.
- Show all necessary working. Marks may be deducted for careless or badly arranged work.

Office Use Only			
Section	1	2	Total
Mark	/24	/38	/62

Examiner: Mrs. S. Gerges

Section One: Basic Arithmetic & Algebra (24 Marks)

1) Solve for x: a) $x^2 \geq 2x$ **2 Marks**

b) $22 \leq 5x - 3 \leq 32$.† **2 Marks**

2) Solve for x:

a) $\dfrac{x+4}{x-2} \geq 3$.† **3 Marks**

b) $\dfrac{1}{|x-3|} \geq \dfrac{1}{2}$.† **2 Marks**

3) Solve for x: $|x^2 - 5| = 5x + 9$.† **5 Marks**

4) Solve the equation $x^2 + 2x - 4 + \dfrac{3}{x^2 + 2x} = 0$.† **5 Marks**

5) Solve algebraically for x:

$x + y = 5$

$3x^2 + xy - y^2 = 29$ **5 Marks**

Section Two: Real Functions (38 Marks)

6) If $f(x) = 2ax + b$, find the values of a and b given that $f(-1) = 5$ and $f(2) = -1$.† **3 Marks**

7) The function $f(x)$ is defined as: $f(x) = \begin{cases} 2x & \text{for } -4 \leq x < 0 \\ 9 - x^2 & \text{for } 0 \leq x \leq 3 \end{cases}$

i. Find the value of $f(0)$.
ii. Sketch $y = f(x)$.
iii. State the range of $y = f(x)$.† **4 Marks**

8) Sketch on separate diagrams the following, showing all the essential features and stating their domain and range of:

a. $y = |x| + 2$; **3 Marks**

b. $xy = 4$; **3 Marks**

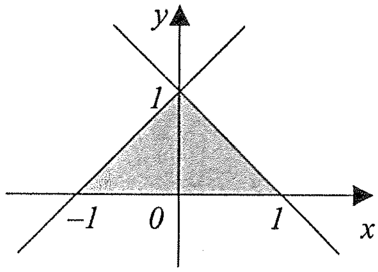
c. $(x - 2)^2 + y^2 = 9$. **4 Marks**

d. $y = \dfrac{1}{x-3}$† **3 Marks**

9) Consider the function $y = \sqrt{9 - x^2}$.

i. State its domain and range.
ii. On the number plane shade in the region where the following inequalities hold simultaneously: $y < \sqrt{9 - x^2}$ and $x \geq 0$.
iii. Does the point $(2, -4)$ lie in this region? Use algebra to justify your answer.† **5 Marks**

10) Write a set of inequalities for which the shaded region is the simultaneous solution.



3Marks

11) Consider the function $f(x)=\dfrac{x}{4-x^2}$.

i. Find the domain of the function.

ii. Show that the function is an odd function.† **3 Marks**

12) Find the centre and the radius of the circle C whose equation is $x^2+y^2-4x+6y-12=0$.† **4 Marks**

13) Describe in geometrical terms the region whose points satisfies the inequalities $x^2+y^2<9$, $x\geq 1$ and $y>-1$. Sketch the region. **4 Marks**

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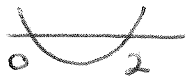
Section One : Arith. + Algebra. (24 Marks)

1/ a/ $x^2 \geq 2x$

$x^2 - 2x \geq 0$

$x(x-2) \geq 0$



$x \leq 0 \quad x \geq 2$

/2.

b/ $22 \leq 5x - 3 \leq 32$

$25 \leq 5x \leq 35$

$5 \leq x \leq 7$

/2.

2/ a) $\dfrac{x+4}{x-2} \geq 3 \qquad x \neq 2.$

$(x-2)(x+4) \geq 3(x-2)^2$

$3(x-2)^2 - (x-2)(x+4) \leq 0$

$(x-2)\left[3x - 6 - x - 4\right] \leq 0$

$(x-2)(2x-10) \leq 0$

$2(x-2)(x-5) \leq 0$

$2 < x \leq 5$

/3

b/ $\dfrac{1}{|x-3|} \geq \dfrac{1}{2} \qquad x \neq 3$

$|x-3| \leq 2.$

$x - 3 \leq 2$ or $x - 3 \geq -2$

$x \leq 5$ or $x \geq 1$

or $1 \leq x \leq 5$ and $x \neq 3$ ½

/2

3/ $|x^2 - 5| = 5x + 9$

$x^2 - 5 = 5x + 9$ or $x^2 - 5 = -5x + 9.$

$x^2 - 5x - 14 = 0$ or $x^2 + 5x + 4 = 0.$

$(x-7)(x+2) = 0 \qquad (x+4)(x+1) = 0$

ie $x = 7$ or -2 or -4 or -1

Test $x = 7 \quad |49 - 5| = 35 + 9$ ✓

$x = -2 \quad |4 - 5| = -10 + 9.$

$1 \neq -1$

$\therefore x \neq -2.$

$x = -4 \quad |16 - 5| = -20 + 9.$

$11 \neq -11$

$\therefore x \neq -4$

$x = -1 \quad |1 - 5| = -5 + 9.$

$4 = 4$ ✓

$\therefore$ Solutions are.

$x = 7$ or $-1.$

-1 Marks for no testing

/5.

4/ $x^2 + 2x - 4 + \dfrac{3}{x^2 + 2x} = 0$

Let $x^2 + 2x = y.$

$y - 4 + \dfrac{3}{y} = 0$

$y^2 - 4y + 3 = 0$

$(y-3)(y-1) = 0$

$y = 3$ or $y = 1$

$x^2 + 2x = 3$ or $x^2 + 2x = 1$

$x^2 + 2x - 3 = 0 \qquad x^2 + 2x - 1 = 0$

$(x+3)(x-1) = 0 \qquad x = \dfrac{-2 \pm \sqrt{4+4}}{2}$

$x = -3, 1 \qquad = \dfrac{-2 \pm 2\sqrt{2}}{2}$

$= -1 \pm \sqrt{2}$

5/ $x + y = 5$ --- ①
$3x^2 + xy - y^2 = 29$ --- ②

$y = 5 - x$ ①
subst in ②
$3x^2 + x(5-x) - (5-x)^2 = 29$
$3x^2 + 5x - x^2 - (25 - 10x + x^2) = 29$
$3x^2 + 5x - x^2 - 25 + 10x - x^2 = 29$
$x^2 + 15x - 54 = 0$
$(x-3)(x+18) = 0$ 3
$x = 3$ ½ $y = 2$ ½
$x = -18$ ½ $y = 23$ ½
∴ Solutions
$(3, 2)$ $(-18, 23)$

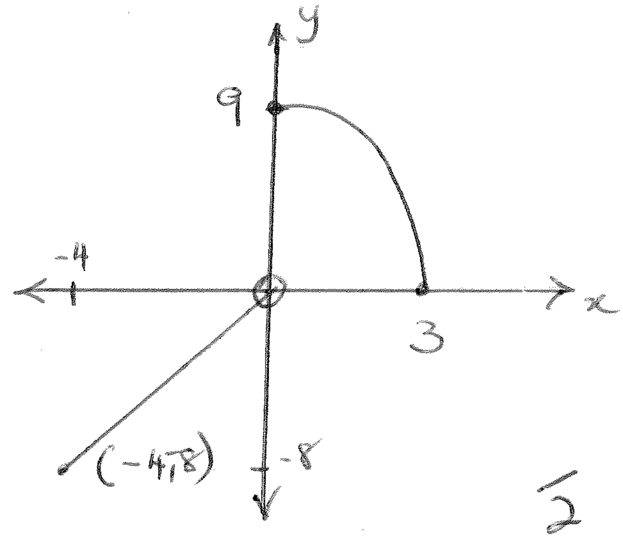
/5

Section 2 : (38 Marks).

6/ $f(x) = 2ax + b.$
$f(-1)$ $-2a + b = 5$ --- ①
$f(2)$ $4a + b = -1$ - ②
② - ① $-6a = 6$
$a = -1$
subst in ①
$b = 3.$
∴ $a = -1$ & $b = 3.$ /3.

7/ $f(x) = 2x$ $-4 \leq x < 0$
$= 9 - x^2$ $0 \leq x \leq 3$
(i) $f(0) = 9 - 0$
$= 9$ /1
(iii)
$R: -8 \leq y \leq 9.$ /1

(ii)



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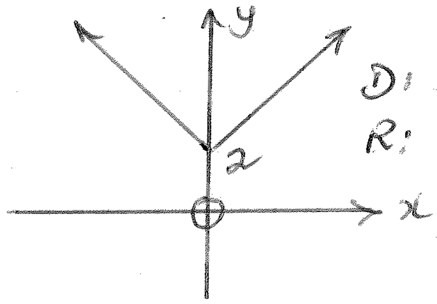
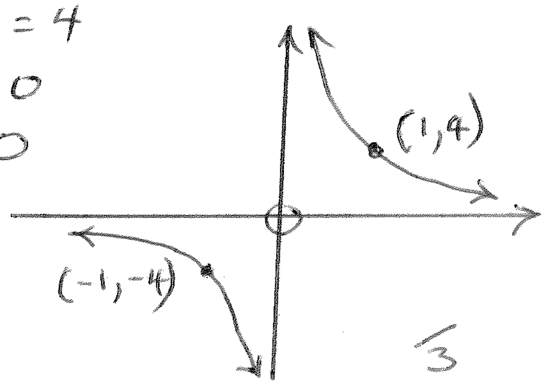
8/ a) $y = |x| + 2$



D: all real x
R: $y \geq 2.$

/3

b) $xy = 4$
D: $x \neq 0$
R: $y \neq 0$



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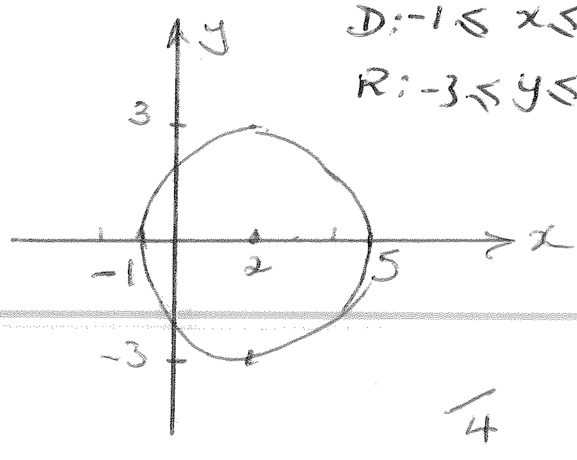
c) $(x-2)^2 + y^2 = 9$

D: $-1 \leq x \leq 5$
R: $-3 \leq y \leq 3$



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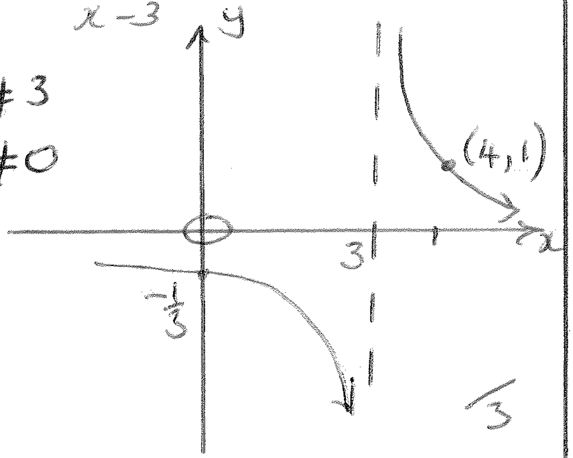
d) $y = \frac{1}{x-3}$

D : $x \neq 3$
R : $y \neq 0$



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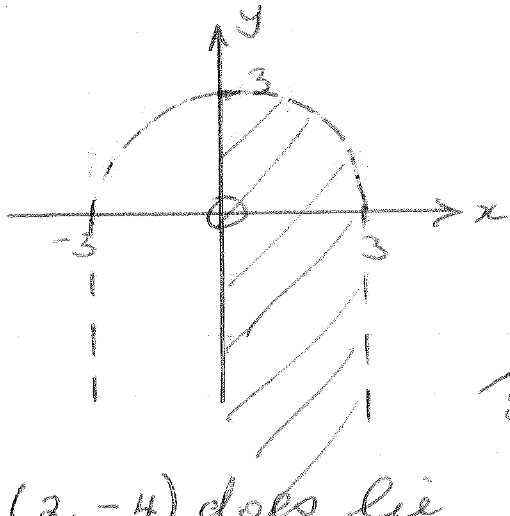
9/ $y = \sqrt{9-x^2}$

(i) D : $-3 < x < 3$
R : $y < 3$.

/2

(ii) $y < \sqrt{9-x^2}$ $x \geq 0$



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(iii) (2, −4) does lie

$-4 < \sqrt{9-(2)^2}$

$-4 < \sqrt{5}$ True.

∴ (2, −4) lies in Region.

/1

Region inside the circle of
tre (0,0) and radius 3, to the
to r $x = 1$ and a above the
$y = -1$

/2

10/ $y \leq x + 1$

and $y \leq 1 - x$

and $y \geq 0$.

/3

11/ $f(x) = \frac{x}{4-x^2}$

(i) Domain

$4 - x^2 \neq 0$

$(2-x)(2+x) \neq 0$

D : $x \neq \pm 2$

1½

(ii) $f(-x) = \frac{-x}{4-(-x)^2}$

$= \frac{-x}{4-x^2}$

$= -f(x)$

∴ odd.

1½

12/ $x^2 + y^2 - 4x + 6y - 12 = 0$

$x^2 - 4x + 4 + y^2 + 6y + 9 = 12 + 4 + 9$

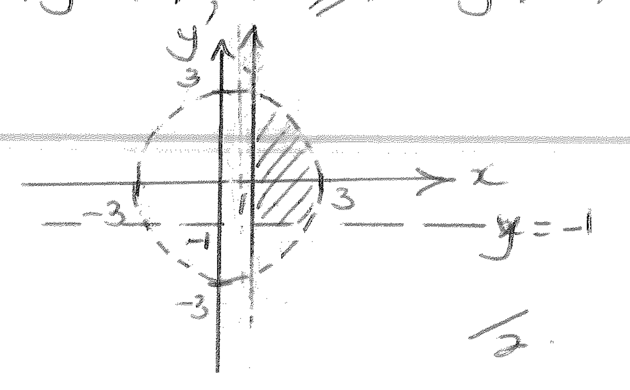
$(x-2)^2 + (y+3)^2 = 25$.

Centre (2, −3)

radius = 5u.

/4

13/ $x^2 + y^2 < 9$, $x \geq 1$ & $y > -1$



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