



CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NSW

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Centre Number

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Student Number

2013
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics Extension 1

Morning Session
Friday, 9 August 2013

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided on a SEPARATE sheet
- In Questions 11–14, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and the Student Number at the top of this page

Total marks – 70

Section I Pages 2-5

10 marks

- Attempt Questions 1–10
- Allow 15 minutes for this section

Section II Pages 6-11

60 marks

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Disclaimer

Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, *Principles for Setting HSC Examinations in a Standards-Referenced Framework* (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and *Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework* (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

6300-1

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

- 1 Which of the following is an expression for $\int 2\sin^2 x \, dx$?
- (A) $x + \frac{1}{2}\sin 2x + C$
- (B) $x - \frac{1}{2}\sin 2x + C$
- (C) $x + \sin 2x + C$
- (D) $x - \sin 2x + C$
- 2 Let A be the point $(-1, 4)$ and B the point $(9, -6)$. Find the coordinates of the point which divides AB internally in the ratio 3:2.
- (A) $(1, 3)$
- (B) $(3, 1)$
- (C) $(-2, 5)$
- (D) $(5, -2)$
- 3 The polynomial $P(x) = 8x^3 + ax^2 - 4x + 1$ has a factor of $2x + 1$. What is the value of a ?
- (A) -8
- (B) 0
- (C) 3
- (D) 8

- 4 The letters of the word TWITTER are arranged randomly. What is the probability that the three Ts are grouped together?

- (A) $\frac{1}{42}$
- (B) $\frac{1}{35}$
- (C) $\frac{1}{7}$
- (D) $\frac{6}{7}$

- 5 The polynomial equation $x^3 - 5x^2 + x - 4 = 0$ has roots α , β and γ . What is the value of $\alpha^2 + \beta^2 + \gamma^2$?

- (A) 17
- (B) 23
- (C) 25
- (D) 27

- 6 What is the coefficient of x^3 in the expansion of $(1 - 3x)^7$?

- (A) -945
- (B) -35
- (C) 35
- (D) 945

- 7 The expression $\sin x - \sqrt{3} \cos x$ can be written in the form $2\sin(x + \alpha)$.

Find the value of α .

- (A) $\alpha = \frac{\pi}{3}$
- (B) $\alpha = -\frac{\pi}{3}$
- (C) $\alpha = \frac{\pi}{6}$
- (D) $\alpha = -\frac{\pi}{6}$

- 8 Differentiate $\sin^{-1}\left(\frac{3x}{4}\right)$ with respect to x .

- (A) $\sqrt{16 - 9x^2}$
- (B) $\frac{\sqrt{16 - 9x^2}}{3}$
- (C) $\frac{1}{\sqrt{16 - 9x^2}}$
- (D) $\frac{3}{\sqrt{16 - 9x^2}}$

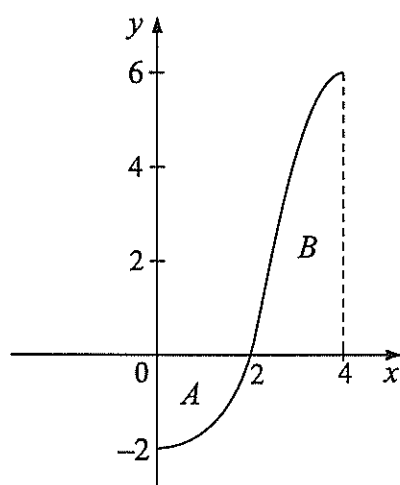
- 9 Three stones, A , B and C , are released from the top of a vertical cliff to reach the horizontal ground below.

Stone A is projected horizontally at 15 ms^{-1} , stone B is projected horizontally at 20 ms^{-1} and stone C is dropped from rest.

Which statement below is correct?

- (A) Stone A reaches the ground first.
- (B) Stone B reaches the ground first.
- (C) Stone C reaches the ground first.
- (D) The three stones all reach the ground at the same time.

- 10 The graph of the function $y = f(x)$ is drawn below.



Area A is equal to 3 square units and Area B is equal to 7 square units.

What is the value of $\int_{-2}^6 f^{-1}(x) dx$?

- (A) 4
- (B) 10
- (C) 20
- (D) 22

Section II

60 marks

Attempt Questions 11–14

Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\int_0^1 \frac{dx}{\sqrt{4-x^2}}$, giving your answer in terms of π . 2

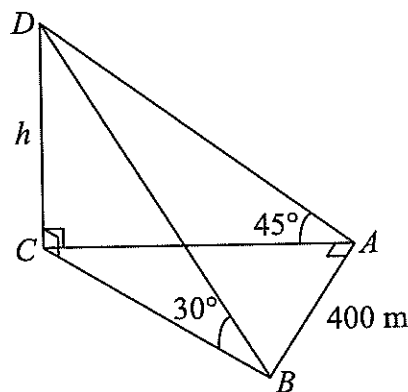
(b) Solve the inequality $\frac{t-2}{t+3} > -2$. 3

(c) Find $\int \frac{e^{3\sqrt{x}}}{\sqrt{x}} dx$ using the substitution $u = \sqrt{x}$. 3

(d) (i) Find $\frac{d}{dx}(x \sin 2x)$. 2

(ii) Hence, or otherwise, find $\int x \cos 2x dx$. 2

(e) 3



The angle of elevation of a tower, CD , from a point A due East of the tower is 45° . From a point B due South of A , the angle of elevation is 30° . The distance from A to B is 400 metres.

Find the height, h , of the tower.

Question 12 (15 marks) Use a SEPARATE writing booklet.

- (a) A bowl of soup is cooling in a room that has a constant temperature of 20°C . At time t , measured in minutes, the temperature, T , of the soup is decreasing according to the differential equation

$$\frac{dT}{dt} = -k(T - 20)$$

where k is a positive constant.

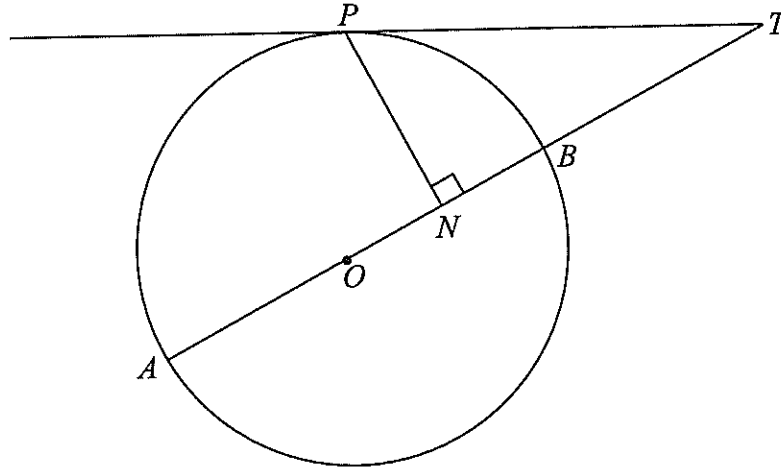
The initial temperature of the soup is 100°C and it cools to 70°C after 5 minutes.

- | | | |
|-------|---|---|
| (i) | Verify that $T = 20 + Ae^{-kt}$ is a solution to the differential equation, where A is a constant. | 1 |
| (ii) | Find the values of A and k . | 2 |
| (iii) | Find the temperature of the soup after 15 minutes.
Give your answer correct to the nearest degree. | 1 |
- (b) Sketch the graph of $y = \sin^{-1} \frac{x}{3}$. 2
- (c) In how many ways can seven students sit in a circle if Jack and Jill refuse to sit next to each other? 2
- (d) (i) Show that the function 2
- $$f(x) = \log_e x - \sin x + 1$$
- has a zero between 0.7 and 0.8.
- (ii) Hence use *halving-the-interval* method to find the value of this zero, correct to one decimal place. 2

Question 12 continues on page 8

Question 12 (continued)

- (e) AB is a diameter of the circle, centre O , shown below and P is a point on the circumference. The tangent to the circle at P meets AB produced at T . Let N be the foot of the perpendicular from P to AB .



Prove that BP bisects $\angle NPT$.

3

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

- (a) The velocity $v \text{ ms}^{-1}$ of a particle moving in simple harmonic motion along the x -axis is given by

$$v^2 = -x^2 - 4x + 12.$$

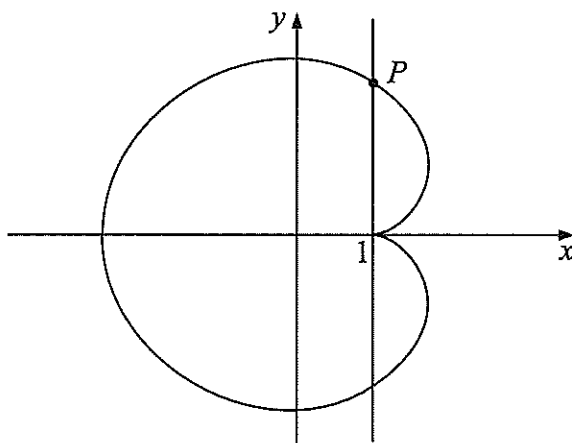
- | | | |
|-------|---|---|
| (i) | Find the acceleration of the particle in terms of x . | 1 |
| (ii) | State the centre and period of the motion. | 2 |
| (iii) | What is the maximum speed of the particle? | 1 |

- (b) A cardioid curve is defined by the following pair of parametric equations 3

$$x = 2 \cos \theta - \cos 2\theta$$

$$y = 2 \sin \theta - \sin 2\theta$$

for $0 \leq \theta < 2\pi$. The graph is drawn below.



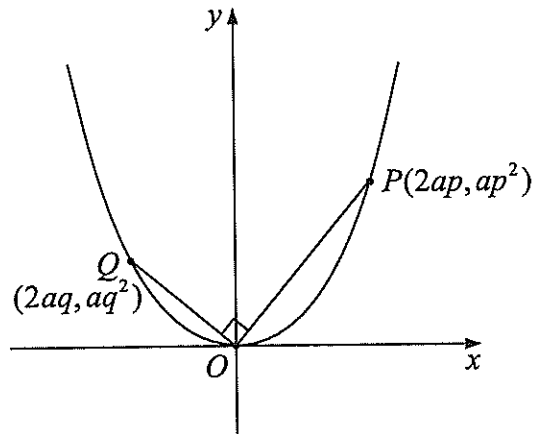
The cardioid curve and the vertical line $x = 1$ intersect in the 1st quadrant at a point P .

Find the coordinates of the point P .

Question 13 continues on page 10

Question 13 (continued)

- (c) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$ such that OP is perpendicular to OQ .



- (i) Prove that $pq = -4$. 2
- (ii) Let R be the point such that $OPRQ$ is a rectangle. 2
 R has coordinates $(2a(p+q), a(p^2+q^2))$. (Do not prove this).
 Show that the locus of R is a parabola.

Suppose the point $P(2ap, ap^2)$, $p > 0$, is moving along the original parabola $x^2 = 4ay$ such that its y coordinate, y_p , is increasing at a rate of 1 unit per second.

- (iii) Using part (i) show that $\frac{dq}{dt} = \frac{-q}{p} \times \frac{dp}{dt}$. 2
- (iv) Hence, or otherwise, show that the y coordinate of Q , y_Q , is decreasing at a 2
 rate of $\frac{q^2}{p^2}$ units per second.

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Prove by induction that $n^3 + 5n$ is divisible by 6, for integers $n \geq 1$. 4

(b) Let n be a positive even integer.

(i) Expand and simplify $(a+b)^n + (a-b)^n$. 2

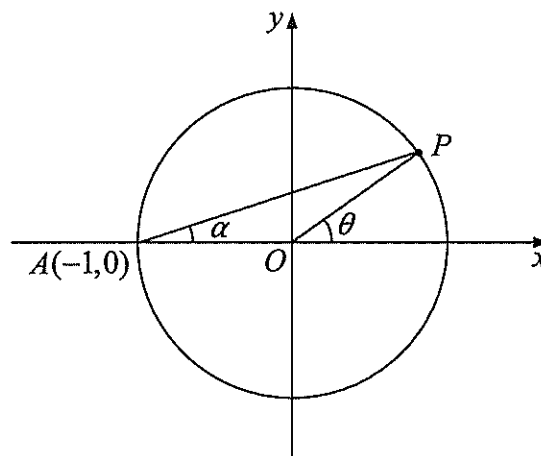
Suppose n fair dice are thrown, each with faces numbered 1 to 6.

(ii) Write down the probability of obtaining exactly two sixes. 1

(iii) Show that the probability of obtaining an even number of sixes is given by 2

$$\frac{1}{2} \left(1 + \left(\frac{2}{3} \right)^n \right).$$

(c)



Let A be the point $(-1, 0)$ on the unit circle and suppose that the line through A with gradient t meets the circle at P in the first quadrant. OP and AP make angles of θ and α respectively with the positive x -axis.

(i) What range of values can t take? 1

(ii) Prove that P has x -coordinate $\frac{1-t^2}{1+t^2}$ and find the y -coordinate of P . 3

(iii) Deduce that $\tan \theta = \frac{2t}{1-t^2}$, for $t \neq 1$. 1

(iv) Find the value of t if $\alpha = 22\frac{1}{2}^\circ$. 1

End of Paper

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Examiners

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