Student Number: ....

2003

HIGHER SCHOOL CERTIFICATE
Sample Examination Paper

MATHEMATICS
Extension 1

General Instructions
Reading time - 5 minutes
Working time - 2 hours

- Attempt ALL questions
- Show all necessary working, marks may be deducted for careless or untidy work
- Standard integrals are printed on the last page
- Board-approved calculators may be used
- Additional Answer Booklets are available

Marks

Question 1.

(a) Solve for x: $3^{x+1} = 2$ expressing the answer correct to two decimal places. **2**

(b) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1 + \sin^2 x)dx$ **3**

(c) State the domain and range of the function $g(x) = \frac{1}{2}\cos^{-1}\frac{x}{2}$ **2**

(d) Use the remainder theorem to fully factorise $6x^3 + 17x^2 - 4x - 3$ **3**

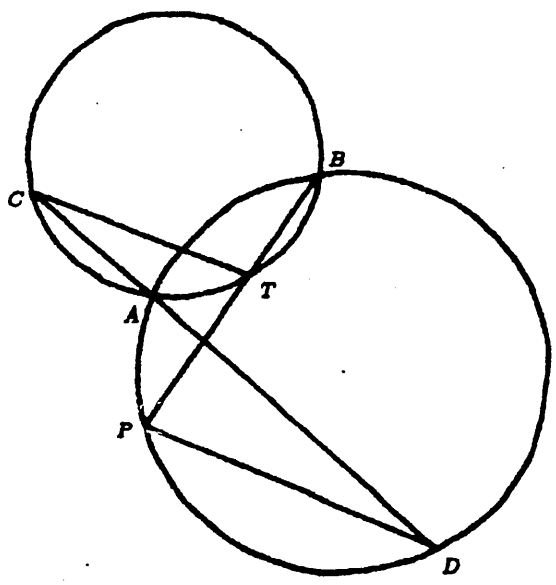
(e) (i) Find the general solution of the equation $\tan\alpha = -\frac{1}{\sqrt{3}}$ expressing your answer in terms of π. **1**

(ii) Hence generate a value of α such that $-\frac{3\pi}{2} < \alpha < -\pi$ **1**

Question 2

(a) Use the substitution $u = 2 - x^2$ to find $\int \frac{x}{(2-x^2)^3}dx$ **2**

(b) **3**



Two circles meet in points A and B. CAD is a double chord and BTP is a chord of the larger circle. Prove that $CT||PD$.

(c) Solve the inequality $\frac{2x-5}{x-4} \geq x$ **3**

(d) Find $\frac{d}{dx}\left(3^{\sqrt{x+1}}\right)$ **2**

(e) A particle moves along the x-axis starting at $x = 1$ at time $t = 0$. The velocity of the particle is $v = \frac{1}{x+3}$. Find the value of t when $x = 5$. **2**

Question 3

(a) Use the method of mathematical induction to prove that $2(n-3)+(n-4)+\cdots+3+2+1=\binom{n}{2}-n$ for $n \geq 4$. **4**

(b) Find the roots of the following equation $4x^3-4x^2-29x+15=0$ given that one root is the difference between the other two roots. **3**

(c) The tangent to the point $P(2ap, ap^2)$ on the parabola $x^2=4ay$ cuts the x-axis in A and the y-axis in B.

(i) Find the coordinates of M, the midpoint of A and B in terms of P. **2**

(ii) Show that the locus of M is a parabola. **2**

(iii) Find the coordinates of the focus of this parabola and the equation of its directrix. **1**

Question 4

(a) (i) How many 11 letter 'words' can be formed from the letters of the word 'PROBABILITY'? **1**

(ii) In how many of these does the word BABY appear? **1**

(b) A surveyor observes two towers, one due north of height 80m, and the other on a bearing of $\theta°$ $(< 90°)$ of height 120m. The angles of elevation of the two towers are 40° and 36° respectively. If the towers are 150m apart on a horizontal plane, calculate the value of θ to the nearest minute. **6**

(c) By considering the expansion of $x(1+x)^n$ or otherwise, show that **4**

$$\sum_{r=0}^{n}(r+1)^nC_r = 2^n\left(\tfrac{n}{2}+1\right)$$

Question 5

(a) A golf ball is to be struck so as to clear a tree 20m away and 6m high on level ground. If the selected club produces an angle of elevation of 40°, (take $g = 10\text{m/s}^2$)

(i) Write down an expression for y, the vertical distance travelled. **1**

(ii) Write down an expression for x, the horizontal distance travelled. **1**

(iii) Hence derive the cartesian equation of the flight path. **1**

(iv) Calculate the speed at which the ball must leave the ground in order to just clear the obstacle. **2**

(b) Given $3x^2 - 5x = -\frac{k}{4}$ calculate value(s) of k if

(i) the real roots are real **1**

(ii) the roots are rational and k is a positive integer. **2**

(c) To promote the sale of Studebaker cars, a dealer offers a special deal in which no interest is charged for the first 3 months and then interest rates are left at 1% per month. Lam Lai buys a 6-cylinder car for \$30 000, pays \$10 000 in cash and agrees to pay the loan plus interest monthly over 3 years. After 20 months, he wins \$10 150 as part of a lotto syndicate. Show that this win is just sufficient to pay off the loan at that time. **4**

Question 6

(a) Tap water at 24°C is placed in a fridge-freezer maintained at a temperature of -11°C. After t minutes the rate of change of temperature T of the water is given by $\frac{dT}{dt} = -k(T+11)$

(i) Show that $T = Ae^{-kt} - 11$ is a solution of the above equation, where A is a constant. **1**

(ii) Find the value of A **1**

(iii) After 15 minutes the temperature of the water falls to 10°C. Find to the nearest minute the time taken for the water to start freezing. (Freezing point of water = 0°C) **2**

(b) Evaluate $\int_{\frac{1}{3}}^{\frac{1}{\sqrt{3}}} \frac{dx}{1+9x^2}$ **3**

(c) In the game of craps 2 dice are thrown and the sum of the dice is noted. The most likely outcome is a total of 7. If two dice are rolled 20 times,

(i) What is the most probable number of sevens thrown? **3**

(ii) Calculate the probability that this number of sevens does indeed occur. **2**

Question 7

(a) A particle moves in a straight line and its position at any time is given by $x = 4.8\cos 2t + 5.5\sin 2t$. Show that the motion is simple harmonic and calculate its greatest speed. **3**

(b) Evaluate $\sin\left[\cos^{-1}\frac{2}{3} + \tan^{-1}\left(-\frac{3}{4}\right)\right]$ giving its exact value. **3**

(c) Consider the function $y = x\sec x$.

(i) Find $\frac{dy}{dx}$ **1**

(ii) By drawing two graphs, show that the function has one stationary point in the domain $\frac{\pi}{2} < x < \frac{3\pi}{2}$. **2**

(iii) Prove that the stationary point lies between $x = 2.5$ and $x = 3.0$. **1**

(iv) Use halving the interval method twice to find a closer approximation of the stationary point. **2**

Solutions

Question 1

(a)

$$3^{x+1} = 2$$
$$\log 3^{x+1} = \log 2$$
$$(x+1)\log 3 = \log 2$$
$$x+1 = \frac{\log 2}{\log 3}$$
$$x = \frac{\log 2}{\log 3} - 1$$
$$= -0.37 \text{ (2 d.p.)}$$

(b)

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1+\sin^2 x)dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1+\tfrac{1}{2}\{1-\cos 2x\})dx$$
$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\tfrac{3}{2} - \tfrac{\cos 2x}{2})dx$$
$$= [\tfrac{3}{2}x - \tfrac{\sin 2x}{4}]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$
$$= \tfrac{\pi}{2} - \tfrac{\sqrt{3}}{8} - (\tfrac{3\pi}{8} - \tfrac{1}{4})$$
$$= \tfrac{\pi}{2} - \tfrac{3\pi}{8} - \tfrac{\sqrt{3}}{8} + \tfrac{1}{4}$$
$$= \tfrac{\pi}{8} - \tfrac{\sqrt{3}-2}{8}$$

(c) Domain $-1 \le \frac{x}{2} \le 1$, $-2 \le x \le 2$. Range $0 \le y \le \frac{\pi}{2}$.

(d) $P(x) = 6x^3 + 17x^2 - 4x - 3$

$P(-3) = -162 + 153 + 12 - 3 = 0$

$\therefore x+3$ is a factor.

$$\begin{array}{r|l} & 6x^2 - x - 1 \\ \hline x+3 & 6x^3 + 17x^2 - 4x - 3 \\ & \underline{6x^3 + 18x^2} \\ & \quad -x^2 - 4x \\ & \quad \underline{-x^2 - 3x} \\ & \qquad -x - 3 \\ & \qquad \underline{-x - 3} \end{array}$$

Factors are $(x+3)(6x^2 - x - 1) = (x+3)(2x-1)(3x+1)$

(e) (i) $\tan\alpha = -\frac{1}{\sqrt{3}}$

$\tan\alpha = \tan(-\frac{\pi}{6})$ or $\tan\alpha = \tan(\frac{5\pi}{6})$

$\therefore \alpha = -\frac{\pi}{6} + n\pi$ or $\alpha = \frac{5\pi}{6} + n\pi$

(ii) $\alpha = -\frac{\pi}{6} - \pi$ or $\alpha = \frac{5\pi}{6} - 2\pi$

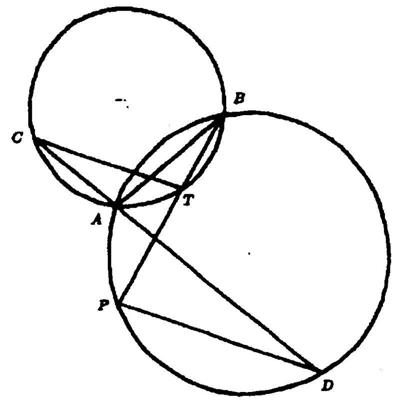
$= -\frac{7\pi}{6}$ $\qquad = -\frac{7\pi}{6}$

Question 2

(a) $\int \frac{x}{(2-x^2)^3}dx$ $\qquad u = 2 - x^2$, $du = -2x\,dx$, $\therefore x\,dx = -\frac{du}{2}$

$$\therefore \int = -\tfrac{1}{2}\int \tfrac{1}{u^3}du$$
$$= -\tfrac{1}{2}\tfrac{u^{-2}}{-2} + C$$
$$= \tfrac{1}{4(2-x^2)^2} + C$$

(b)



Join AB:

$\angle ACT = \angle ABT$ (angles standing on the same arc in smaller circle)

But $\angle ABT = \angle ADP$ (angles standing on the same arc in larger circle)

$\therefore \angle ACT = \angle ADP$

$\therefore CT \| PD$ (alternate angles are equal)

(c)

$$\frac{2x-5}{x-4} \ge x$$
$$(2x-5)(x-4) \ge x(x-4)^2$$
$$(2x-5)(x-4) - x(x-4)^2 \ge 0$$
$$(x-4)[2x-5-x(x-4)] \ge 0$$
$$(x-4)(2x-5-x^2+4x) \ge 0$$
$$(x-4)(-x^2+6x-5) \ge 0$$
$$(x-4)(-5+x)(1-x) \ge 0$$

Test $x = 0$ valid
$\therefore x \le 1$ or $4 < x \le 5$

(d) Let $y = 3^{\sqrt{x+1}}$
Put $u = \sqrt{x+1}$.
$y = 3^u$
$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \ln 3(3^u) \times \frac{1}{2}(x+1)^{-\frac{1}{2}} = \frac{\ln 3(3^{\sqrt{x+1}})}{2\sqrt{x+1}}$

(e) $v = \frac{1}{x+3}$ i.e., $\frac{dx}{dt} = \frac{1}{x+3}$
$\frac{dt}{dx} = x + 3$
$t = \int (x+3)\, dx = \frac{x^2}{2} + 3x + c$
When $t = 0, x = 1$; $0 = \frac{1}{2} + 3 + c \therefore c = -\frac{7}{2}$
$\therefore t = \frac{x^2}{2} + 3x - \frac{7}{2}$
When $x = 5$, $t = \frac{25}{2} + 15 - \frac{7}{2} = 24$

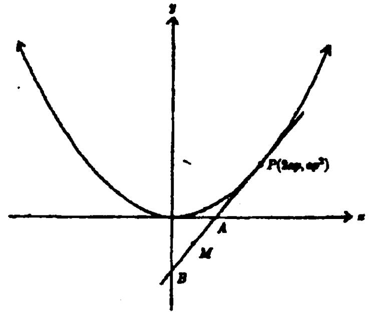
Question 3

(a) Test $n = 4$: LHS $= 2(4-3) = 2$
RHS $= \binom{4}{2} - 4 = 6 - 4 = 2$
$\therefore$ true for $n = 4$.
Suppose that an integer k exists for which the result is true
i.e., $2(k-3) + (k-4) + \cdots + 3 + 2 + 1 = \binom{k}{2} - k$
Consider when $n = k + 1$.
$\therefore$ RHS $= 2(k-2) + (k-3) + (k-4) + \cdots + 3 + 2 + 1$
$= 2(k-2) + 2(k-3) + (k-4) + \cdots + 3 + 2 + 1 - (k-3)$
$= \binom{k}{2} - k + 2(k-2) - (k-3)$
$= \binom{k}{2} - k + k - 1$
$= \binom{k}{2} - 1$
But $\binom{k}{r} = \binom{k+1}{r} - \binom{k}{r-1}$
$\therefore \binom{k}{2} = \binom{k+1}{2} - \binom{k}{1}$
$\therefore$ LHS $= \binom{k+1}{2} - \binom{k}{1} - 1$
$= \binom{k+1}{2} - k - 1$
$= \binom{k+1}{2} - (k+1)$
So the result is true for $n = k + 1$.

But it is true for $n = 4$.
$\therefore$ it is true for $n = 4 + 1 = 5$ and by mathematical induction it is true for all positive integers $n \ge 4$.

(b) Let the roots of the equation be α, β and $\alpha - \beta$.
Sum of roots $= \alpha + \beta + \alpha - \beta = 2\alpha = -\frac{b}{a} = \frac{4}{4} = 1 \therefore \alpha = \frac{1}{2}$
Product of all 3 roots $= \alpha\beta(\alpha - \beta) = -\frac{d}{a}$
$\frac{1}{2}\beta(\frac{1}{2} - \beta) = -\frac{15}{4}$
$\frac{1}{4}\beta - \frac{1}{2}\beta^2 + \frac{15}{4} = 0$
$2\beta^2 - \beta - 15 = 0$
$(2\beta + 5)(\beta - 3) = 0$
$2\beta + 5 = 0$ or $\beta = 3$
$\beta = -\frac{5}{2}, \beta = 3$
$\therefore$ roots are $\frac{1}{2}, 3, -\frac{5}{2}$

(c)



(i) Equation of tangent at P: $y = px - ap^2$
It cuts the x-axis when $y = 0$, i.e., $x = ap \therefore A = (ap, 0)$
It cuts the y-axis when $x = 0$, i.e., $y = -ap^2 \therefore B = (0, -ap^2)$
$\therefore M = (\frac{ap}{2}, -\frac{ap^2}{2})$

(ii) From $x = \frac{ap}{2}, p = \frac{2x}{a}$
Sub in $y = -\frac{ap^2}{2}$
$y = -\frac{a}{2}(\frac{2x}{a})^2$
$y = -\frac{2x^2}{a}$
i.e., $x^2 = -\frac{1}{2}ay$ which is the equation of a parabola.

(iii) $x^2 = -\frac{1}{2}ay$
$= 4(-\frac{1}{8}a)y$
Its focal length is $\frac{a}{8}$

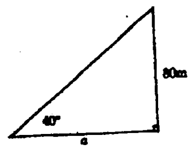
$\therefore$ coordinates of focus are $(0, -\frac{1}{8}a)$
Equation of directrix: $y = \frac{1}{8}a$

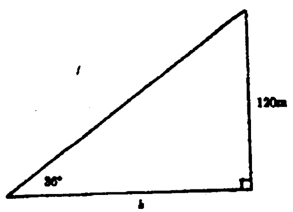
Question 4

(a) **(i)** $N = \frac{11!}{2!2!} = 9\ 979\ 200$

(ii) Consider the letters BABY to be <u>one</u> item, so $N = \frac{8!}{2!} = 20\ 160$

(b) In a vertical plane:

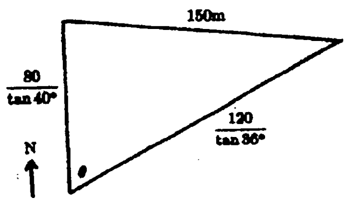




$\tan 40° = \frac{80}{a}$
$a = \frac{80}{\tan 40°}$

$\tan 36° = \frac{120}{b}$
$b = \frac{120}{\tan 36°}$

On a horizontal plane:



$$\cos\theta = \frac{\frac{(80)^2}{\tan^2 40°} + \frac{(120)^2}{\tan^2 36°} - (150)^2}{\frac{2.(80).(120)}{\tan 40° \tan 36°}}$$

$\theta = 63°52'$

(c) $x(1+x)^n = x({}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \cdots + {}^nC_n x^n)$
Differentiating both sides of the equation gives

$$\begin{aligned}(1+x)^n + nx(1+x)^{n-1} &= {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \cdots + {}^nC_n x^n \\ &\quad + x({}^nC_1 + 2{}^nC_2 x + \cdots + n{}^nC_n x^{n-1}) \\ &= {}^nC_0 + 2x{}^nC_1 + 3{}^nC_2 x^2 + \cdots + (n+1){}^nC_n x^n\end{aligned}$$

Put $x = 1$

$\therefore 2^n + n(2^{n-1}) = {}^nC_0 + 2{}^nC_1 + 3{}^nC_2 + \cdots + (n+1){}^nC_n$

$$2^n + \frac{n(2^n)}{2} = \sum_{r=0}^{n}(r+1)\cdot {}^nC_r$$

$$\text{i.e., } 2^n(\tfrac{n}{2} + 1) = \sum_{r=0}^{n}(r+1)\cdot {}^nC_r$$

Question 5

(c) **(i)** $y = vt\sin 40° - 5t^2$

(ii) $x = vt\cos 40°$

(iii) From (ii) $t = \frac{x}{v\cos 40°}$

$$\begin{aligned}y &= \frac{v\sin 40°.x}{v\cos 40°} - 5(\frac{x}{v\cos 40°})^2 \\ &= x\tan 40° - \frac{5x^2}{v^2\cos^2 40°}\end{aligned}$$

$\therefore y = x\tan 40° - \frac{5x^2}{v^2}(1 + \tan^2 40°)$

(iv) $x = 20\text{m}$, $y = 6\text{m}$

$\therefore 6 = 20\tan 40° - \frac{5(20)^2}{v^2}(1 + \tan^2 40°)$

$\therefore v^2 = \frac{5(20)^2(1+\tan^2 40°)}{20\tan 40° - 6}$

$\therefore v = 17.8\,\text{m/s}$

(b) $3x^2 - 5x = -\frac{k}{4}$
$12x^2 - 20x + k = 0$

(i) For real roots $\Delta \geq 0$

$$\begin{aligned}b^2 - 4ac &\geq 0 \\ 400 - 4(12)k &\geq 0 \\ 25 - 3k &\geq 0 \\ k &\leq \frac{25}{3}\end{aligned}$$

(ii) For rational roots Δ is a perfect square i.e., $25 - 3k$ is a perfect square. Since k is a positive integer, less than $\frac{25}{3}$, the only possible values of $25 - 3k$ are 0, 1, 4, 9, 16, 25, yielding solutions $k = 3$, 7 and 8.

(c) Let P = monthly repayment.
A_n = amount owing after n months.

$A_1 = 20\,000 - P$

$A_2 = 20000 - 2P$

$A_3 = 20\,000 - 3P$

$A_4 = A_3(1.01) - P$

$= (20\,000 - 3P)(1.01) - P$

$A_5 = [(20\,000 - 3P)(1.01) - P](1.01) - P$

$= (20\,000 - 3P)(1.01)^2 - P(1 + 1.01)$

$A_6 = (20\,000 - 3P)(1.01)^3 - P(1 + 1.01 + 1.01^2)$

$A_{36} = (20\,000 - 3P)(1.01)^{33} - P(1 + 1.01 + 1.01 + \cdots + 1.01^{32})$

$0 = (20\,000 - 3P)(1.01)^{33} - P\frac{[1.01^{33}-1]}{1.01-1}$

$(1.01^{33} - 1)P = 0.01(20\,000 - 3P)(1.01)^{33}$

$(1.01^{33} - 1)P = 0.01(1.01)^{33}20\,000 - 3P(0.01)(1.01)^{33}$

$P[1.01^{33} - 1 + 3(0.01)(1.01)^{33}] = 0.01(1.01)^{33} \times 20\,000$

$P = \frac{0.01(1.01)^{33}\times 20\,000}{1.01^{33}-1+3(0.01)(1.01)^{33}}$

$= \frac{0.01(1.01)^{33}\times 20\,000}{1.01^{33}[1+0.03]-1}$

$\approx \$645.38$

$A_{20} = (20\,000 - 3P)(1.01)^{20} - \frac{P(1.01^{17}-1)}{1.01-1}$

Sub $P = 645.38$

$A_{20} = (20\,000 - 3(645.38))(1.01)^{20} - \frac{645.38(1.01^{17}-1)}{0.01}$

$= 10\,146.70$

$\therefore$ \$10 150 is just sufficient to pay off the loan at 20 months.

Question 6

(a) (i) $T = Ae^{-kt} - 11$ $(Ae^{-kt} = T + 11)$

$\frac{dT}{dt} = -kAe^{-kt}$

$= -k(T + 11)$

(ii) When $t = 0, T = 24°$

$\therefore T = Ae^{-kt} - 11$

$24 = Ae^0 - 11$

$A = 35$

(iii) $T = 35e^{-kt} - 11$

When t =15 min, $T = 10$

$\therefore 10 = 35e^{-15k} - 11$

$e^{-15k} = \frac{21}{35} = 0.6$

$-15k = \ln(0.6)$

$k = \frac{\ln(0.6)}{-15} \approx 0.034$

$\therefore T = 35e^{-0.034t} - 11$

When $T = 0$

$0 = 35e^{-0.034t} - 11$

$e^{-0.034t} = \frac{11}{35}$

$t = \frac{\ln(0.6)}{-0.034}$

≈ 33.98

$= 34$ min to the nearest minute.

(b) $\int_{\frac{1}{3}}^{\frac{1}{\sqrt{3}}} \frac{dx}{1+9x^2} = \int_{\frac{1}{3}}^{\frac{1}{\sqrt{3}}} \frac{dx}{9(\frac{1}{9}+x^2)}$

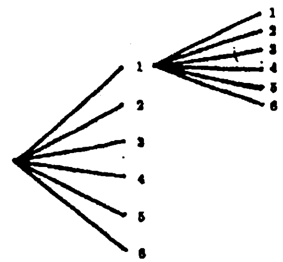
$= \frac{1}{9}.3\int_{\frac{1}{3}}^{\frac{1}{\sqrt{3}}} \frac{\frac{1}{3}}{\frac{1}{9}+x^2}\,dx$

$= \frac{1}{3}[\tan^{-1} 3x]_{\frac{1}{3}}^{\frac{1}{\sqrt{3}}}$

$= \frac{1}{3}(\tan^{-1}\sqrt{3} - \tan^{-1} 1)$

$= \frac{1}{3}(\frac{\pi}{3} - \frac{\pi}{4}) = \frac{\pi}{36}$

(c)



P(a total of 7) $= \frac{6}{36} = \frac{1}{6}$

Let p = success (a total of 7)

$= \frac{1}{6}$

q = failure (any other total)

$= \frac{5}{6}$

(i) Consider $(q + p)^{20}$ i.e., $(\frac{5}{6} + \frac{1}{6})^{20}$

To find greatest coefficient in this expansion,

$\frac{C_{r+1}}{C_r} = \frac{n-r+1}{r} \cdot \frac{b}{a} \geq 1$

$\frac{21-r}{r} \cdot \frac{1}{5} \geq 1$

$21 - r \geq 5r$

$6r \leq 20$

$r = 3$

i.e., most probable number of total of 7 is 3.

(ii) Consider $(q + p)^{20} = \sum_{r=0}^{20} {}^{20}C_r q^{20-r}.p^r$

$r = 3$ produces

$P(E) = {}^{20}C_3(\frac{5}{6})^{17}(\frac{1}{6})^3$

≈ 0.238

Question 7

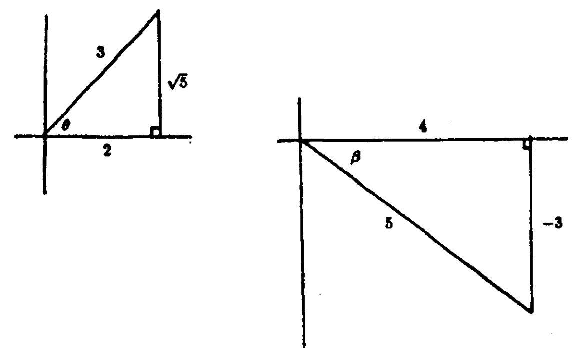
(a) $x = 4.8\cos 2t + 5.5\sin 2t = A\sin(2t+\alpha)$
$A = \sqrt{(4.8)^2 + (5.5)^2} = 7.3$
$\tan\alpha = \frac{4.8}{5.5}\ (0 < \alpha < \frac{\pi}{2})$
$\alpha = 0.72^c$
$\therefore x = 7.3\sin(2t+0.72)$
$\dot{x} = 2(7.3)\cos(2t+0.72)$
$\ddot{x} = -2^2(7.3)\sin(2t+0.72)$
$= -2^2 x$
Since $\ddot{x} = -n^2x$, the motion is simple harmonic.

OR : $x = 4.8\cos 2t + 5.5\sin 2t$
$\dot{x} = -2(4.8)\sin 2t + 2(5.5)\cos 2t$
$\ddot{x} = -2^2(4.8)\cos 2t - 2^2(5.5)\sin 2t$
$= -2^2(4.8\cos 2t + 5.5\sin 2t)$
$= -2^2 x$

The speed is greatest when $\ddot{x} = 0$
i.e., when $-4(7.3)\sin(2t+0.72) = 0$
$2t + 0.72 = 0, \pi, 2\pi, \ldots$
The smallest positive value of t for which the speed is a maximum is given by $2t + 0.72 = \pi$
$\dot{x} = 2(7.3)\cos\pi$
$= -14.6$
The maximum speed has magnitude 14.6.

(b) $\sin[\cos^{-1}\frac{2}{3} + \tan^{-1}(-\frac{3}{4})]$
Let $\cos^{-1}\frac{2}{3} = \alpha,\ 0 \le \alpha \le \pi$
and $\tan^{-1}(-\frac{3}{4}) = \beta,\ -\frac{\pi}{2} < \beta < \frac{\pi}{2}$
$\therefore \cos\alpha = \frac{2}{3}$ and $\tan\beta = -\frac{3}{4}$

α may be represented as an angle in the first question and β may be represented as an angle in the fourth quadrant.



$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$= \frac{\sqrt{5}}{3}\cdot\frac{4}{5} + \frac{2}{3}(-\frac{3}{5})$
$= \frac{4\sqrt{5}-6}{15}$

(c) (i) $y = x\sec x$
$\frac{dy}{dx} = \sec x + x\sec x\tan x$

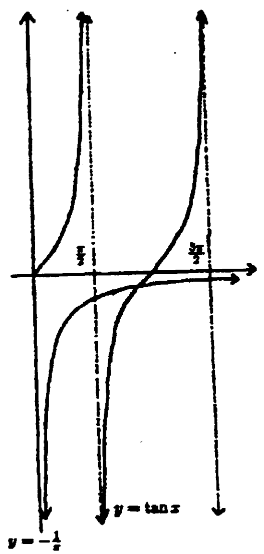
(ii) $\sec x + x\sec x\tan x = 0$ for stationary points

$\sec x(1 + x\tan x) = 0$

$\sec x \neq 0, 1 + x\tan x = 0$

$x\tan x = -1$

$\tan x = -\frac{1}{x}$



(iii) Let $f(x) = 1 + x\tan x$

$f(2.5) = -0.867... < 0$

$f(3.0) = 0.572... \quad > 0$

$\therefore$ change in sign between 2.5 and 3 so that stationary point lies between 2.5 and 3.

(iv) Consider $x = 2.75$

$f(2.75) = -0.1355... < 0$

$\therefore$ stationary point lies between 2.75 and 3.

Consider $x = 2.875$

$f(2.975) = 0.2148... > 0$

$\therefore$ stationary point lies between 2.75 and 2.875

$\therefore$ closer approximation of the stationary point is $\frac{2.75+2.875}{2} = 2.8125$

Marking Guidelines

1 (a)		1	Log form
		1	Correct answer, 2d.p.
1 (b)		1	Conversion to multiple angle
		1	Integration
		1	Exact value
1 (c)		1	Domain
		1	Range
1 (d)		1	First factor
		1	Second factors by long division
		1	Factorisation
1 (e)		1	General solution
		1	Specific value
2 (a)		1	Substitution
		1	Integration
2 (b)		1	Statement with reason
		1	Second statement and reason
		1	Conclusion
2 (c)		1	Various methods
		1	Testing a value or graphical method
		1	Solution
2 (d)		1	Chain rule
		1	Answer
2 (e)		1	t in terms of x
		1	Substitution and answer
3 (a)		1	Test n=4
		1	Integer k
		1	n=k+1 and algebra
		1	Conclusion
3 (b)		1	Sum of roots
		1	Product of roots
		1	Solution
3 (c)	(i)	1	Coordinates of A and B
		1	Coordinates of M
	(ii)	1	Elimination of parameter
		1	Conclusion
	(iii)	1	Focal length
4 (a)	(i)	1	Correct answer
	(ii)	1	Correct answer
(b)		2	Triangles in vertical plane
		1	Triangle on horizontal plane
		1	Expressions for a and b
		1	Cosine rule
		1	Correct angle (nearest minute)
(c)		1	Expansion
		1	Differentiation
		1	x=1
		1	Algebra and answer

5 (a) (i) 1 Vertical distance
(ii) 1 Horizontal distance
(iii) 1 Derivation
(iv) 1 Substitution
1 Correct answer
(b) (i) 1 k for real roots
(ii) 1 k for rational roots
1 Integral values
(c) 1 A_1-A_3
1 A_{36}
1 Value of instalment
1 A_{20} and conclusion

6 (a) (i) 1 Differentiation
(ii) 1 Value of A
(iii) 1 Value of k
1 Time taken
(b) 1 Adjustment
1 Integration
1 Evaluation
(c) (i) 1 P(total of 7)
1 Greatest coefficient
1 Answer
(ii) 2 Binomial probability

7 (a) 1 Transformation
1 Differentiation and conclusion
1 Maximum speed
(b) 2 Two triangles
1 Compound angle
(c) (i) 1 Differentiation
(ii) 1 Equation
1 Graphs
(iii) 1 Proof
(iv) 2 Halving the interval twice

Mathematics Extension 1

HSC TRIAL EXAMINATION MAPPING GRID

Question	Marks	Content	Syllabus Outcomes	Targeted Performance Bands
1(a)	2	Logarithmic and Exponential functions	P5, H9	E2-E3
1(b)	3	Integration: Trigonometric functions	H8	E2-E3
1(c)	2	Inverse Trigonometric functions	HE4	E2-E3
1(d)	3	Polynomials	PE3	E2-E3
1(e)(i)	1	Trigonometric functions	P4	E2-E3
(ii)	1	Trigonometric functions	P4	E2-E3
2(a)	2	Integration	HE6	E2-E3
(b)	3	Circle Geometry	PE2, PE3	E2-E3
(c)	3	Inequalities	PE3	E2-E3
(d)	2	Derivative of a function	P7, H5	E2-E3
(e)	2	Applications of Calculus	HE3	E3-E4
3(a)	4	Mathematical Induction	HE2	E3-E4
(b)	3	Polynomials	PE3	E2-E3
(c)(i)	2	Parametric representation	PE4	E2-E3
(c)(ii)	2	Parametric representation	PE4	E2-E3
(c)(iii)	1	Parametric representation	PE4	E2-E3
4(a)(i)	1	Permutations and Combinations	HE3	E2-E3
(ii)	1	Permutations and Combinations	HE3	E2-E3
(b)	6	Trigonometric functions	PE3	E2-E3
(c)	4	Binomial Theorem	PE3, PE6, HE7	E2-E4
5(a)(i)	1	Applications of Calculus to the Physical World	HE3, HE7	E2-E3
(ii)	1	Applications of Calculus to the Physical World	HE3, HE7	E2-E3
(iii)	1	Applications of Calculus to the Physical World	HE3, HE7	E2-E3
(iv)	2	Applications of Calculus to the Physical World	HE3, HE7	E2-E3
5(b)(i)	1	Quadratic theory	PE3	E2-E3
(ii)	2	Quadratic theory	PE3	E2-E4
(c)	4	Series and Applications	H5, H9	E2-E4
6(a)(i)	1	Applications of Calculus to the Physical World	HE3, HE7	E2-E3
(ii)	1	Applications of Calculus to the Physical World	HE3, HE7	E2-E3
(iii)	2	Applications of Calculus to the Physical World	HE3, HE7	E2-E3
(b)	3	Integration: inverse trigonometric functions	H8	E2-E4
(c)(i)	3	Binomial theorem	PE3, PE6	E2-E4
(ii)	2	Binomial probability	HE7	E2-E4
7(a)	3	Applications of Calculus to the Physical World	HE3, HE7	E2-E4
(b)	3	Inverse trigonometric functions	H8	E3-E4
(c)(i)	1	Differentiation of trigonometric functions	P4, H5	E2-E3
(ii)	2	Differentiation of trigonometric functions	PE2	E2-E4
(iii)	1	Differentiation	PE2	E2-E3
(iv)	2	Polynomials	PE3	E2-E4