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Course : 3/4 Unit Mathematics

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Question 1

Solutions	Marks	Comments
a) $\frac{1+a^{-1}}{1+a^{-3}} = \frac{1+\frac{1}{a}}{1+\frac{1}{a^3}}$	②	
$= \frac{\frac{a+1}{a}}{\frac{a^3+1}{a^3}} = \frac{a+1}{\cancel{a}} \times \frac{a^{\cancel{3}\,2}}{a^3+1}$	1	One mark for correctly forming algebraic fraction
$= \frac{\cancel{(a+1)}(a^2)}{\cancel{(a+1)}(a^2-a+1)}$		
$= \frac{a^2}{a^2-a+1}$	1	One mark for correctly factorisation and simplifying
b) $y = \sec x$ $= (\cos x)^{-1}$	④	
i ∴ $\frac{dy}{dx} = -1.(\cos x)^{-2}.-\sin x$	1	
$= \frac{\sin x}{\cos^2 x}$	1	
$= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$		
$= \sec x . \tan x$	1	
ii $\frac{d^2y}{dx^2} = \tan x . \sec x . \tan x + \sec x . \sec^2 x$		
$= \sec x(\tan^2 x + \sec^2 x)$	1	
c) $\lim_{x \to 0} \frac{1-\cos 2x}{x^2} = \lim_{x \to 0} \frac{2\sin^2 x}{x^2}$	②	
$= 2 \lim_{x \to 0} \left(\frac{\sin x}{x}\right)^2$	1	
$= 2 \times 1^2$		
$= 2$	1	

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Solutions	Marks	Comments
d) $\int_0^1 x^2(1+x^3)^3\,dx$	②	
Let $u = 1+x^3$ When $x=0,\ u=1$		
$du = 3x^2.dx$ $x=1,\ u=2$		
$\therefore I = \frac{1}{3}\int_0^1 3x^2(1+x^3)^3\,dx$		
$= \frac{1}{3}\int_1^2 u^3.du$	1	
$= \frac{1}{3}\left[\frac{1}{4}u^4\right]_1^2$		
$= \frac{1}{3}\left(4-\frac{1}{4}\right)$		
$= 1\frac{1}{4}$	1	
e) $\int_{\pi/3}^{\pi/2} \frac{\sin x}{1-\cos x}\,dx = \left[\log(1-\cos x)\right]_{\frac{\pi}{3}}^{\pi/2}$	② 1	
$= \log 1 - \log\frac{1}{2}$		
$= 0 - \log 1 + \log 2$		
$= \log 2$	1	

Solutions	Marks	Comments
ESTION 2 (a) $\frac{dy}{dx} = 1+y$	②	
$\frac{dx}{dy} = \frac{1}{1+y} = (1+y)^{-1}$		
$\therefore x = \int (1+y)^{-1}.dy$		
$= \log(1+y) + C$	1	One mark for Correctly integrating wrt y
Sub $x=0, y=2$		
$\therefore 0 = \log 3 + c$		
$c = -\log 3$		
$\therefore x = \log\left(\frac{1+y}{3}\right)$		
$\therefore e^x = \frac{1+y}{3}$	1	
$y = 3e^x - 1$		
(b) $\left(t^3 + \frac{1}{t}\right)^7$ Typical term is $T_{K+1} = {}^nC_K\, a^K b^{n-k}$	②	
$\therefore T_{K+1} = {}^7C_K\, t^{3k} . t^{-(7-k)}$		
$= {}^7C_K\, t^{4k-7}$	1	One mark for finding expression of typical term
For a constant $4k-7$ must equal 0		
$4k = 7$		
$\therefore k = \frac{7}{4}$		
since k is NOT an integer there is no constant term	1	One mark for stating k is not an integer.
		Yes or No without justification receives 0 marks

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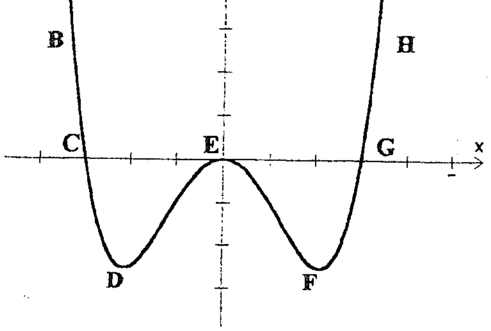
Solutions	Marks	Comments
(c) Proof: Join PQ $\angle TRS = \angle RPQ$ (Angle in alternate segment) $\angle TSR = \angle SPQ$ (" " " ")	③	N.B. There may be alternate proofs
$\therefore \angle RPS = \angle TRS + \angle TSR$	1	One mark for this stage
$\angle RTS = 180 - (\angle TRS + \angle TSR)$ (Angle sum $\triangle RTS$) $= 180 - \angle RPS$ (from above) $\therefore \angle RTS$ and $\angle RPS$ are supplementary	1	One mark for this stage
$\therefore \angle PST$ and $\angle PRT$ are supplementary (angle sum of quad. PSTR) $\therefore$ TSPR is a cyclic quad (opp. angle supplementary)	1	
(d)	⑤	
i Function is odd. Graph is asymetrical about the y-axis ie reflected in line $y = x$ and passes through Origin	1	
ii a) D to E and F to G	1	Both answers required for mark
b) A to B and E to F	1	"
iii 	1 1	One mark for correct graph One mark for correct labelling of critical pts C, D, E, F, G

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Solutions	Marks	Comments
(a) (i) To keep S, S, T together, there are 6 groups.	④	
ie M, O, N, E, R, SST	1	
No. of ways $= 6! \times \frac{3!}{2!}$		
$= 2160$	1	
(ii) Total number of juries that can form $= {}^{10}C_7$		
Number of juries containing majority of females: Must choose 4 females ∴ Need to select 3 males in ${}^{6}C_3$ ways.	1	One mark for recognition of this.
$\therefore P_{\text{Majority females}} = \frac{{}^{6}C_3}{{}^{10}C_7}$ $= \frac{1}{6}$	1	
(b) (i) In $\triangle ABC$; $\frac{AC}{\sin 34} = \frac{100}{\sin 94}$	③ 1	
$AC = \frac{100 \sin 34}{\sin 94}$		
$= 56 \cdot 05584$	1	
∴ A is 56m from foot of pole		
(ii) In $\triangle ADC$; $\frac{DC}{AC} = \tan 58^0$		
$DC = 56 \tan 58^\circ$	1	
$= 89 \cdot 618733$		
∴ Height of pole is 89·6m		

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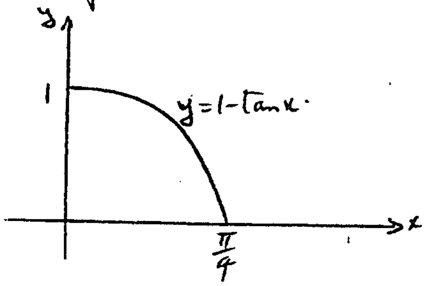
Solutions	Marks	Comments
(C)(i) $f(x) = 1 - \tan x \qquad 0 \le x \le \frac{\pi}{4}$	⑤	
	1	One mark for graph.
(ii) $A = \int_0^{\pi/4} (1 - \tan x)\,dx = \int_0^{\pi/4} \left(1 - \frac{\sin x}{\cos x}\right) dx$		
$= \left[x + \log(\cos x)\right]_0^{\pi/4}$	1	one mark for correct integration
$= \frac{\pi}{4} + \log\left(\frac{1}{\sqrt{2}}\right) - 0$		
$= \frac{\pi}{4} + \log 1 - \log(2)^{\frac{1}{2}}$		
$= \frac{\pi}{4} - \frac{1}{2} \log 2$		
$= \frac{\pi}{4} - \frac{2}{4} \log 2$		
$= \frac{\pi}{4} - \frac{1}{4} \log 2^2$	1	
$= \frac{\pi - \log 4}{4}$ units^2		
(iii) $V = \pi \int_0^{\pi/4} (1 - \tan x)^2\,dx$		
$= \pi \int_0^{\pi/4} (1 - 2\tan x + \tan^2 x)\,dx$		
$= \pi \int_0^{\pi/4} (\sec^2 x - 2\tan x)\,dx$		
$= \pi \left[\tan x + 2\log(\cos x)\right]_0^{\pi/4}$	1	One for this stage
$= \pi\left(1 + 2\log\left(\frac{1}{\sqrt{2}}\right) - 0\right)$		
$= \pi(1 - \log 2)$ units^3	1	

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Solutions	Marks	Comments
Question 4 (a) i Area $\triangle ABO = \frac{1}{2}a^2 \sin x$	③	
Area of sector OPQ $= \frac{1}{2}r^2 x$	1	One mark for stating both results
$\therefore \frac{1}{2}a^2 \sin x = 2 \times \frac{1}{2} r^2 x$		
$r^2 = \frac{a^2 \sin x}{2x}$	1	One mark for expressing r^2 correctly
ii $x = \frac{\pi}{2}^c$		
$\therefore r^2 = \frac{a^2 \sin\frac{\pi}{2}}{\pi}$ $= \frac{a^2}{\pi}$ $\therefore r = \frac{a}{\sqrt{\pi}}$	1	
(b) (i) $P(x) = 6x^3 - 7x^2 + ax + b$	⑤	
$P(-1) = 0 \quad \therefore -6 - 7 - a + b = 0$ $\therefore -a + b = 13$	1	One mark for this stage
Let remaining roots be α and $\frac{1}{\alpha}$ $\therefore \alpha \times \frac{1}{\alpha} \times -1 = \frac{-b}{6}$ $b = 6$ $\therefore a = -7$	1	One mark for values of a and b
ii If -1 is a root then $x+1$ is a factor $6x^2 - 13x + 6$ $x+1 \,\overline{)\, 6x^3 - 7x^2 - 7x + 6}$ $6x^3 + 6x$ $-13x^2 - 7x$ $-13x^2 - 13x$ $6x + 6$ $6x + 6$ 0	1	One for correct division
$\therefore P(x) = (x+1)(6x^2 - 13x + 6)$ $= (x+1)(3x-2)(2x-3)$	1	One for correct factors
Zeros of $P(x)$ are $-1, \frac{2}{3}, \frac{3}{2}$	1	One for correct zeros.

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4 c) Solutions	Marks	Comments
To prove: $\sum_{r=1}^{n} r(r+1) = n\frac{(n+1)(n+2)}{3}$		
Step 1 when $n=1$		
$LHS = 1(2) = 2$ $\quad$ $RHS = 1\frac{(2)(3)}{3} = 2$		
∴ true when $n=1$	1	
Step 2 assume true when $n=k$		
ie $1\times2 + 2\times3 + \dots\dots + k(k+1) = k\frac{(k+1)(k+2)}{3}$	1	for correctly phrasing statement in algebra
ie $S_k = k\frac{(k+1)(k+2)}{3}$		
Step 3 now prove for $n=k+1$		
ie $1\times2 + 2\times3 + \dots\dots + k(k+1) + (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3}$		
$LHS = k\frac{(k+1)(k+2)}{3} + (k+1)(k+2)$ from Step 2		
$= \frac{k(k+1)(k+2) + 3(k+1)(k+2)}{3}$		
$= \frac{(k+1)(k+2)(k+3)}{3}$ as required	1	
Thus if true when $n=k$ statement follows when $n=k+1$.		
Step 4 Since statement is true when $n=1$ it follows that the statement holds for $n=2$ from Step 3. Since it is true when $n=2$ it also holds for $n=3$ etc. ∴ the statement holds for all $n \in \mathbb{N}$	1	for invoking the process of induction

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Solutions	Marks	Comments
(a) $y = m_1 x + c_1$ $y = m_2 x + c_2$	③	
Angle between 2 lines is given by $\tan\theta = \left\lvert \frac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$ if $\theta = 45°$ then $\tan\theta = 1$ $\therefore \left\lvert \frac{m_1 - m_2}{1 + m_1 m_2} \right\rvert = 1$	1	One mark for here
$\therefore \frac{m_1 - m_2}{1 + m_1 m_2} = 1$ or $\frac{m_1 - m_2}{1 + m_1 m_2} = -1$		
$m_1 - m_2 = 1 + m_1 m_2$ $\quad$ $m_1 - m_2 = -1 - m_1 m_2$	1	One mark for each correct solution
$m_1 m_2 = m_1 - m_2 - 1$ $\quad$ $\therefore m_1 m_2 = m_2 - m_1 - 1$	1	
(b)	⑤	
(i) $\frac{dx}{dt} = -0.075$	1	One mark for correct expression of $\frac{dx}{dt}$ incl. neg. sign
(ii) $\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$ $\qquad$ Volume $= x^3$ $= 3x^2 \cdot -0.075$	* 1	One mark here
when $x = 4$ $\frac{dv}{dt} = 3 \times 16 \times -0.075$ $= -3.6$ $\therefore$ rate of change in volume is decreasing at $3.6\,\text{cm}^3/\text{min}$	1	One mark here.
(iii) Surface Area $= 6x^2$ if Surface area $= 100\,\text{cm}^2$ then $6x^2 = 100$ $x = \frac{10}{\sqrt{6}}$ cm	1	One mark here

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Solutions	Marks	Comments
$\therefore \frac{dV}{dt} = 3 \times \frac{100}{6} \times -0.075$ $= -3.75$ ∴ volume decreasing at rate of 3.75 cm³/min	1 * ④	One mark here NOTE: If in Part (i) students did not have neg. sign and parts (ii) + (iii) are correctly worked then 4 marks are awarded.
(c)(i) $\frac{dv}{dt} = k(6-v)$ $\therefore \frac{dv}{6-v} = k\,dt$ On integrating both sides $-\log(6-v) = kt + c$ $\log(6-v) = -kt + c$ $6 - v = e^{-kt+c}$ $v = 6 - e^{-kt+c}$ $= 6 - e^{-kt} \cdot e^{c}$ $= 6 + Ae^{-kt}$ (where $A = -e^{c}$) When $t = 0, v = 30$ $\therefore 30 = 6 + Ae^{0}$ $A = 24$ $\therefore v = 6 + 24e^{-kt}$	1 1	One mark here Accept $A = -e^{-c}$ One mark for correct value of A

15 cont.

Solutions	Marks	Comments
(c) (ii) when $t=1$, $v=10.7$ $10.7 = 6+24e^{-k}$ $e^{-k} = \frac{4.7}{24}$ $k = 1.63$ (2 decimal places)	1	One mark here.
(iii) $v = 6+24\, e^{-1.63 \cdot 2}$ $\doteqdot 6.92$ $\therefore$ velocity $\doteqdot 6.9\,ms^{-1}$	1	

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Solutions	Marks	Comments
(a) $\sin x = x^2 - 10$	②	
Let $f(x) = x^2 - \sin x - 10$		
$\therefore f'(x) = 2x - \cos x$	1	One mark for expressing $f(x)$ + $f'(x)$ correctly
$\therefore a_1 = a - \frac{f(a)}{f'(a)}$		
$= \pi - \frac{f(\pi)}{f'(\pi)}$		
$= \pi - \frac{\pi^2 - \sin\pi - 10}{2\pi - \cos\pi}$		
$= \pi - \frac{\pi^2 - 10}{2\pi + 1}$		
$= 3.1595$ (4 d.p)	1	
(b) $x = a\cos nt + b\sin nt$	⑥	
(i) $\dot{x} = -na\sin nt + nb\cos nt$		
$\ddot{x} = -n^2 a\cos nt - n^2 b\sin nt$	1	One mark for getting here
$= -n^2(a\cos nt + b\sin nt)$		
$= -n^2 x$		
∴ Motion is Simple Harmonic	1	One mark here.
(ii) $\dot{x} = v = -na\sin nt + nb\cos nt$		
$\therefore v^2 = n^2a^2\sin^2 nt + n^2b^2\cos^2 nt - 2n^2ab\sin nt\cos nt$		
Also $x^2 = a^2\cos^2 nt + b^2\sin^2 nt + 2ab\cos nt\sin nt$		
$\therefore n^2x^2 = a^2n^2\cos^2 nt + n^2b^2\sin^2 nt + 2n^2ab\sin nt\cos nt$	1	
$\therefore v^2 + n^2x^2 = a^2n^2(\sin^2 nt + \cos^2 nt) + b^2n^2(\sin^2 nt + \cos^2 nt)$		
$= a^2n^2 + b^2n^2$		
$= n^2(a^2 + b^2)$ which is independent of t ∴ is a constant throughout motion	1	

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Solutions	Marks	Comments
(b)(iii) $\dot{x} = -na\sin nt + nb\cos nt$ $= 0$ when $na\sin nt = nb\cos nt$ $\tan nt = \frac{b}{a}$ $nt = \tan^{-1}\left(\frac{b}{a}\right)$ $t = \frac{1}{n}\tan^{-1}\left(\frac{b}{a}\right)$ $\therefore x = a\cos\left(\cancel{n}\cdot\frac{1}{\cancel{n}}\tan^{-1}\left(\frac{b}{a}\right)\right) + b\sin\left(n\cdot\frac{1}{n}\tan^{-1}\left(\frac{b}{a}\right)\right)$	1	
Let $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$ $\therefore \cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$, $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$ $\therefore x = a\cdot\frac{a}{\sqrt{a^2+b^2}} + b\cdot\frac{b}{\sqrt{a^2+b^2}}$		
$= \frac{a^2+b^2}{\sqrt{a^2+b^2}} = \sqrt{a^2+b^2}$	1	
$\therefore$ amplitude of motion is $\sqrt{a^2+b^2}$ cm		

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Solutions	Marks	Comments
(c) $(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \dots + {}^{2n}C_n x^n + \dots + {}^{2n}C_{2n} x^{2n}$	④	
$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$		
$\therefore (1+x)^n(1+x)^n = ({}^nC_0 + {}^nC_1 x + \dots + {}^nC_n x^n)({}^nC_0 + {}^nC_1 x + \dots + {}^nC_n x^n)$	1	One mark for getting here
Now, in the expansion of $(1+x)^{2n}$, the coeff. of x^n is ${}^{2n}C_n$ or $\binom{2n}{n}$		
and, in the expansion of $(1+x)^n(1+x^n)$ the terms in x^n is given by ${}^nC_0 \cdot {}^nC_n x^n + {}^nC_1 \cdot {}^nC_{n-1} x^n + {}^nC_2 \cdot {}^nC_{n-2} x^n + \dots + {}^nC_n \cdot {}^nC_0$	1	One mark for here
And since ${}^nC_0 = {}^nC_n$, ${}^nC_1 = {}^nC_{n-1}$ ... etc		
then the coeff of x^n are $({}^nC_0)^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \dots \binom{n}{n}^2$	1	On mark here
$\therefore \binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \dots \binom{n}{n}^2$	1	

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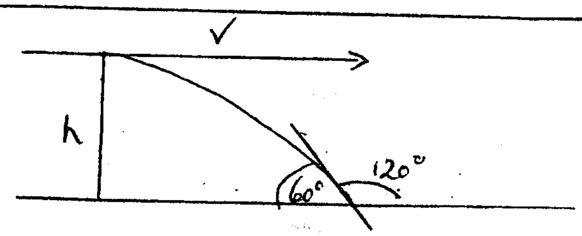
Solutions	Marks	Comments
(a) 	5	
(i) $\ddot{x}=0$ $\dot{x}=c_1$ $=V\cos\alpha$ $=v$ since $\alpha=0$ $\therefore x=Vt+c_2$ when $t=0$, $x=0$ $\therefore c_2=0$ $\therefore x=vt$	1	One mark for getting $x=vt$
$\ddot{y}=-g$ $\dot{y}=-gt+c_3$ when $t=0$, $\dot{y}=V\sin\alpha$ $=0$ since $\alpha=0$ $\therefore \dot{y}=-gt$ $\therefore y=-\frac{1}{2}gt^2+c_4$ $t=0$, $y=h$ $\therefore y=-\frac{1}{2}gt^2+h$	1	One mark for getting $y=-\frac{1}{2}gt^2+h$
Projectile will hit ground when $y=0$ $\therefore -h=-\frac{1}{2}gt^2$ $t^2=\frac{2h}{g}$ $t=\sqrt{\frac{2h}{g}}$ (negative t is ignored)	1	One mark for $t=\sqrt{\frac{2h}{g}}$
(ii) If projectile strikes ground at 60°, the angle to positive x-axis is 120° $\therefore \frac{dy}{dx}=\tan 120°$ $\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}$ $=-gt\cdot\frac{1}{v}$ $=-\frac{gt}{v}$ $=-\frac{g\sqrt{\frac{2h}{g}}}{v}$	1	One mark here

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Solutions	Marks	Comments
$\therefore$ since $\tan 120^\circ = -\sqrt{3}$ $-\sqrt{3} = \frac{-g\sqrt{\frac{2h}{g}}}{v}$ $3v^2 = g^{\cancel{2}}\frac{2h}{\cancel{g}}$ $= 2gh$	1	One mark here
(b) i $y = \log_e(e^x \sin^2 x) = \log_e e^x + \log_e \sin^2 x$ $= x + 2\log_e \sin x$	④	
$\frac{dy}{dx} = 1 + 2\frac{\cos x}{\sin x}$ $= 1 + 2\cot x$	1	One mark here.
(ii) $\frac{dy}{dx} = 1 + 2\cot x$ When $x = \frac{\pi}{2}$, $\frac{dy}{dx} = 1 + 2\cot\frac{\pi}{2}$ $= 1$	1	
Also when $x = \frac{\pi}{2}$, $y = \frac{\pi}{2} + 2\log\sin\frac{\pi}{2}$ $= \frac{\pi}{2}$	1	
Eqtn of normal $\frac{y - \frac{\pi}{2}}{x - \frac{\pi}{2}} = -1$ $y - \frac{\pi}{2} = -x + \frac{\pi}{2}$ $x + y = \pi$	1	

Solutions	Marks	Comments
c)(i) $y = \sin^{-1}x$		
$x = \sin y$		
$\frac{dx}{dy} = \cos y$	1	one Mark for $\frac{dx}{dy} = \cos y$
$= \sqrt{1-\sin^2 y}$		
$= \sqrt{1-x^2}$		
$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$	1	
(ii) $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx = \left[\sin^{-1}x\right]_0^{\frac{1}{2}}$		
$= \frac{\pi}{6} - 0$		
$= \frac{\pi}{6}$	1	