MATHEMATICS - TRIAL REVISION

BOOKLET 2

1) NORTH SYDNEY BOYS HIGH 2006
2) CARRINGBAH HIGH 2007
3) SYDNEY TECH. 2008
4) KNOX GRAMMAR 2008
5) CRANBROOK HIGH 2008
6) FORT ST HIGH 2009

NSBHS - 2006

Question 1 (12 marks) **Marks**

(a) Evaluate $\dfrac{12.9}{\sqrt{6.7\times 3.4}}$ correct to 3 significant figures. **2**

(b) Factorise $1-8y^3$. **2**

(c) Find the value of $\dfrac{\log_3 8}{\log_3 2}$. **2**

(d) Find a primitive of $5+\sin 2x$ **2**

(e) Find the values of x for which $x^2-6x+5>0$. **2**

(f) Solve the simultaneous equations : **2**

$$2x+y=3$$
$$x-2y=4$$

Question 2 (12 marks) Start question on a new page. **Marks**

(a) Differentiate with respect to x:

(i) $(x+1)^7$. **1**

(ii) $x\tan x$ **2**

(iii) $\log_e\left(\dfrac{x}{x-1}\right)$ **2**

(b) Find:

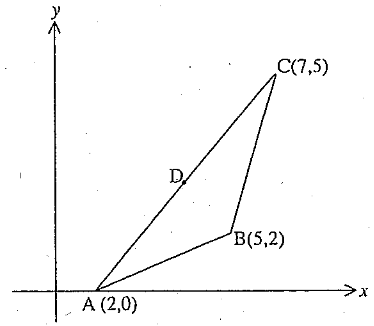
(i) $\displaystyle\int \frac{x}{x^2+6}\,dx$ **2**

(ii) $\displaystyle\int \frac{3}{e^{2x}}\,dx$ **2**

(c) Evaluate $\displaystyle\int_1^e \left(\frac{2}{x}+\frac{x}{2}\right)dx$ leaving your answer in exact form. **3**

Question 3 (12 marks) Start question on a new page. **Marks**

(a) The points A(2,0), B(5,2) and C(7,5) are joined to form a triangle as shown below. D is the midpoint of AC.



(i) Find the length of AC — 1

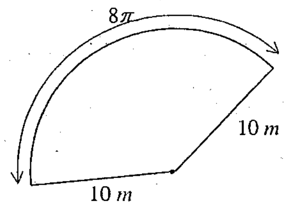
(ii) Find the co-ordinates of D — 1

(iii) Find the slope of DB, and prove that it is perpendicular to AC — 2

(iv) BD is extended to E, so that BD = DE. Find the co-ordinates of the point E. — 1

(v) Find the area of the quadrilateral ABCE — 2

(b)



The diagram shows a garden bed in the shape of a sector. The arc length is 8π *metres* and the radius is 10 *metres*

(i) Show that the angle of the sector is $\frac{4\pi}{5}$ radians — 1

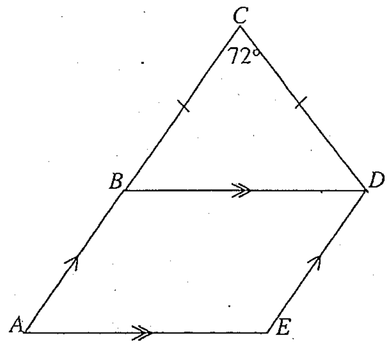
(ii) Calculate the area of this garden bed. — 2

(iii) The garden bed is to be planted with red and yellow tulips. If the tulips can be planted at 15 per square metre, how many tulips can be planted? — 1

(iv) Assuming all tulips flower, what is the expected number of red tulips if the probability of producing a red flower is 0.6? — 1

Question 4 (12 marks) Start question on a new page. **Marks**

(a)



A, B and C are collinear points.
$BD \parallel AE$, $AB \parallel ED$, $BC = \cancel{BD}$ CD
and $\angle BCD = 72°$

Copy this diagram on your answer sheet.

Find the size of $\angle DEA$, giving reasons. — 3

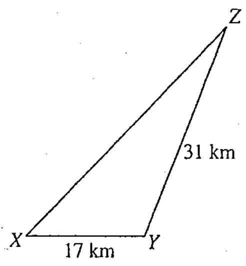
(b) Use Simpson's rule with three function values (i.e. one application) to estimate $\int_1^5 \log_e x \, dx$. — 3

(c) Solve $4^x - 18(2^x) + 32 = 0$ — 3

(d) Solve $2\cos 2x + \sqrt{3} = 0$ for $0 \le x \le 2\pi$ — 3

Question 5 (12 marks) Start question on a new page. **Marks**

(a) In the diagram X, Y and Z represent the locations of three towns. The town Y is due east of X and the bearing of Z from Y is 046°.



(i) Find the size of $\angle XYZ$. **1**

(ii) Find the distance XZ to 1 decimal place. **2**

(iii) What is the bearing of Y from Z? **1**

(b) The root of the equation $x+\frac{1}{x}=7$ are α and β

Find the value of:

(i) $\alpha+\frac{1}{\alpha}$ **1**

(ii) $\alpha+\beta$ **1**

(c) (i) Show that $\frac{3x+4}{x+1}=\frac{1}{x+1}+3$ **1**

Hence:

(ii) Sketch the graph of $y=\frac{3x+4}{x+1}$ showing all the important features. (Do not find stationary points). **2**

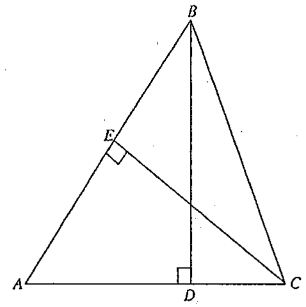
(iii) Find the exact area of the region bounded by the curve $y=\frac{3x+4}{x+1}$, the x and y axes, and the line $x=2$. **3**

Question 6 (12 marks) Start question on a new page. **Marks**

(a) Given that $\log_a 3=0.68$ and $\log_a 2=0.42$, find $\log_a 18$ **2**

(b) Find the limiting sum of the series $\frac{9}{8}+\frac{3}{4}+\frac{1}{2}$ **2**

(c)



The diagram shows BD⊥AC and CE⊥AB

(i) Copy this diagram into your answer booklet and prove $\Delta ECA \parallel\!\mid \Delta DBA$ **3**

(ii) If AB = 10 cm, BD = 7cm and AC = 16 cm find the length of CE. **2**

(d) The rate of water flowing, R litres per hour, into a pond is given by

$$R=65+4t^{\frac{1}{3}}$$

(i) Calculate the initial flow rate. **1**

(ii) If initially there was 15 litres in the pond, find the volume of the water in the pond when 8 hours have elapsed. **2**

Question 7 (12 marks) Start question on a new page. **Marks**

(a) Evaluate $\sum_{k=3}^{5} 2^{4-k}$ 1

(b) A particle moves in a straight so that its distance x, in metres, from a fixed point O at time t, in seconds, is given by

$$x = 5t + \log_e(1-2t), \quad 0 \le t \le \frac{1}{2}.$$

(i) Find the initial velocity and acceleration of the particle. 4

(ii) When does the particle come to rest? 2

(c) A parabola has the equation $x^2 = -12y$

(i) Find the co-ordinates of the vertex of the parabola 1

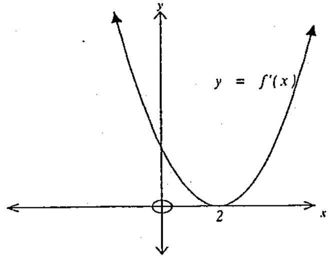
(ii) Write down the focus of the parabola 1

(iii) Find the equation of the tangent of the parabola at the point where $x = 6$. 2

(iv) Find the co-ordinates of Y, the point where the tangent cuts the y-axis 1

Question 8 (12 marks) Start question on a new page. **Marks**

(a) The figure shows the graph of $y = f'(x)$



The curve $y = f(x)$ has a stationary point at (2,0). What is the nature of this stationary point? 1

(b) Consider the curve $y = \frac{1}{x}e^{-x}$:

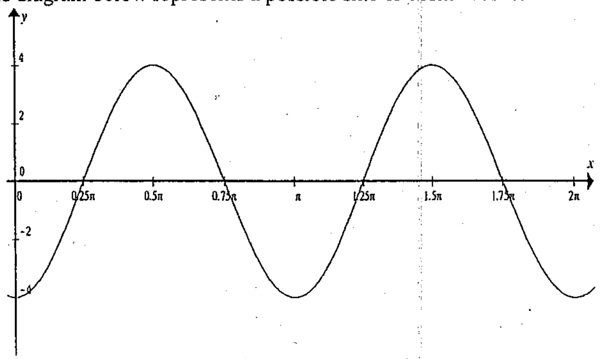
(i) For what values of x is the function defined? 1

(ii) Describe the behaviour of the function as x: 2
(α) approaches zero
(β) increases indefinitely

(iii) Find any stationary points and determine their nature. 3

(iv) Sketch the curve of this function 2

(c) The diagram below represents a possible sine or cosine curve.



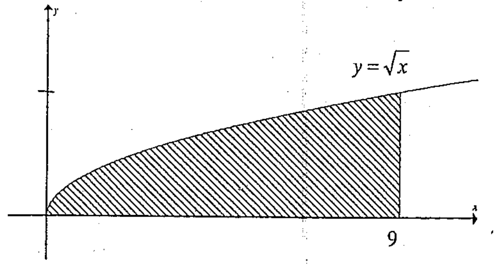
(i) Give the amplitude 1

(ii) Give the period 1

(iii) Write down the possible equation of the curve 1

Question 9 (12 marks) Start question on a new page. **Marks**

(a) Find the volume of the solid formed when the shaded area under the curve $y=\sqrt{x}$, shown below, is rotated around the **y-axis.** 4



(b) (i) Sketch the curve $y=3\sin\frac{\pi x}{2}$ for $-2\le x\le 4$. 1

(ii) Draw on your diagram a line, clearly labelled, which can be used to solve the following equation: 2

$$\sin\frac{\pi x}{2}-\frac{x}{3}=0$$

(iii) Determine the number of solutions to the equation $\sin\frac{\pi x}{2}-\frac{x}{3}=0$ over the domain $-2\le x\le 4$. 1

(c) In a game of chess between two players X and Y, both of approximately equal ability, the player with the *White* pieces, having the first move, has a probability of 0.5 of winning, and the probability that the player with the black pieces for that game winning is 0.3.

(i) What is the probability that the game ends in a draw? 1

(ii) The two players X and Y play each other in chess competition, each player having the *White* pieces once.
In the competition the player who wins the game scores 3 points, the player who loses the game scores 1 point and in a draw each player receives 2 points.
By drawing a tree diagram, or otherwise, find the probability that, as a result of these two games,

(α) X scores 6 points. 1

(β) X scores less than 4 points. 2

Question 10 (12 marks) Start question on a new page. **Marks**

(a) The number N of a certain species is falling according to $N=N_0e^{-0\cdot03t}$ where t is in days and N_0 is the initial number of species present.

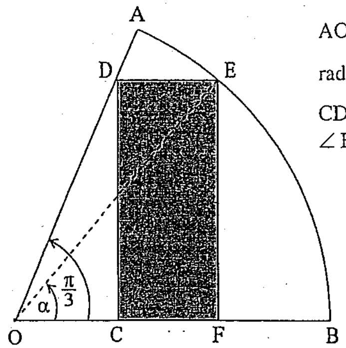
(i) Show that $N=N_0e^{-0\cdot03t}$ is a solution to the differential equation $\frac{dN}{dt}=-0\cdot03N$. 1

(ii) How long, to the nearest day, will it take for the number of species to halve? 1

(iii) Find, in terms of N_0, the rate of change at the time when the number of species has halved. 1

(iv) Find the number of days, to the nearest whole number, for the number of species to fall to just below 5% of the initial number. 2

(b)



NOT TO SCALE

AOB is a sector of a circle with centre at O and radius r such that $\angle \cancel{OAB}$ AOB $=\frac{\pi}{3}$.
CDEF is a rectangle drawn in the sector and $\angle EOF=\alpha$ as shown in the diagram.

(i) Show that $CF=r\cos\alpha-\frac{r\sin\alpha}{\sqrt{3}}$ 2

(ii) Given that $\frac{1}{2}\sin 2\alpha=\sin\alpha\cos\alpha$, show that the area of rectangle CDEF can be expressed as $A=r^2\left(\frac{1}{2}\sin 2\alpha-\frac{\sqrt{3}}{3}\sin^2\alpha\right)$ 2

(iii) Find the value for α which will produce the rectangle of maximum area. 3

Solution of 2006 2 Unit Trial NSBHS

Question 1

a) $2.70279 \doteqdot 2.70$ (3 sign. figures) ✓✓

b) $1-8y^3 = (1-2y)(1+2y+4y^2)$ ✓✓

c) $\dfrac{\log_3 8}{\log_3 2} = \dfrac{3\log_3 2}{\log_3 2}$ ✓

$= 3$ ✓

d) $\int (\sin 2x + 5)\,dx = -\frac{1}{2}\cos 2x + 5x + C$ ✓✓

e) $x^2 - 6x + 5 > 0$

$(x-5)(x-1) > 0$ ✓

$x < 1$ or $x > 5$ ✓

f) $2x + y = 3$

$x - 2y = 4$

$x = 4 + 2y$

$\therefore 2(4+2y) + y = 3$

$8 + 4y + y = 3$

$\therefore y = -1$ ✓

$x = 2$ ✓

Question 2

a) i) $y = (x+1)^7$

$\frac{dy}{dx} = 7(x+1)^6$ ✓

ii) $y = x\tan x$

$\frac{dy}{dx} = 1.\tan x + x\sec^2 x$ ✓✓

iii) $y = \log_e\left(\frac{x}{x-1}\right)$

$= \log_e x - \log_e(x-1)$ ✓

$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x-1}$ ✓

b) i) $\int \frac{x}{x^2+6}\,dx = \frac{1}{2}\ln(x^2+6) + C$ ✓✓

ii) $\int 3e^{-2x}\,dx = -\frac{3}{2}e^{-2x} + C$ ✓✓

c) $\int_1^e \frac{2}{x} + \frac{x}{2}\,dx = \left[2\ln x + \frac{x^2}{4}\right]_1^e$ ✓

$= 2\ln e + \frac{e^2}{4} - \left(0 + \frac{1}{4}\right)$ ✓

$= \frac{7}{4} + \frac{e^2}{4}$ ✓

Question 3

i) $A(2,0)$ $C(7,5)$

$AC = \sqrt{(7-2)^2 + (5-0)^2}$

$= \sqrt{50} = 5\sqrt{2}$ ✓

ii) $D\left(\frac{9}{2}, \frac{5}{2}\right)$ ✓

iii) $m_{DB} = \dfrac{\frac{5}{2} - 2}{\frac{9}{2} - 5}$

$= -1$ ✓

$m_{AC} = \frac{5-0}{7-2}$

$= 1$

$m_{DB} \times m_{AC} = -1$ ✓

$\therefore DB \perp AC$

iv) $E(4,3)$ ✓

v) ABCE is a kite (diag. bisect at 90°)

$\therefore Area = \frac{1}{2} \times AC \times EB$.

$EB = \sqrt{(4-5)^2 + (3-2)^2}$ ✓

$= \sqrt{2}$

$\therefore Area = \frac{1}{2}\, 5\sqrt{2} \times \sqrt{2}$ ✓

$= 5$ sq units

b) i) $l = r\theta$

$8\pi = 10\,\theta$

$\therefore \theta = \frac{4\pi}{5}$ ✓

ii) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}10^2 \times \frac{4\pi}{5}$ ✓

$= 40\pi$ ✓

iii) No of tulips $= 40\pi \times 15 = 188$

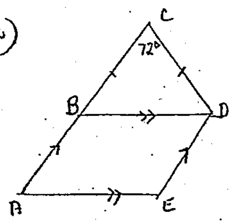
$\doteqdot 1884$ ✓

iv) No of red $\doteqdot 0.6 \times 1884$

$\doteqdot 1130$ ✓

Question 4

a)



$\angle CBD = \frac{180° - 72°}{2}$ (isos. Δ, base ∠s are =) ✓

$= 54°$

$\angle ABD = 180° - 54°$ (st. line) ✓

$= 126°$

$\therefore \angle AED = 126°$ (opp. ∠s of parallelogram) ✓

(b)

x	1	3	5
y	0	1.099	1.609

$y = \log_e x$ ✓

$\int_1^5 \log_e x\,dx = \frac{2}{3}\left[0 + 4 \times 1.099 + 1.609\right]$ ✓

$\doteqdot 4$ ✓

(c) Let $u = 2^x$

$\therefore u^2 - 18u + 32 = 0$ ✓

$(u-16)(u-2) = 0$

$\therefore u = 16$ or $u = 2$

i.e. $2^x = 16$ or $2^x = 2$

$\therefore x = 4$ or $x = 1$ ✓✓

d) $2\cos 2x + \sqrt{3} = 0$

$\cos 2x = -\frac{\sqrt{3}}{2}$

$2x = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}, 3\pi - \frac{\pi}{6}, 3\pi + \frac{\pi}{6}$

$x = \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}$ ✓✓✓

QUESTION 5

i) $\angle XYZ = 46^\circ + 90^\circ$
$= 136^\circ$ ✓

ii) $XZ^2 = 17^2 + 31^2 - 2 \times 17 \times 31 \times \cos 136^\circ$ ✓
$XZ \doteqdot 44.8$ ✓

iii) $180^\circ + 46^\circ = 226^\circ$ ✓

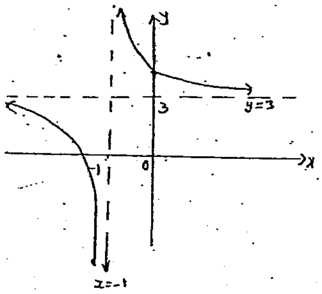
$x + \frac{1}{x} = 7$

i) $\alpha + \frac{1}{\alpha} = 7$ ✓

ii) $x^2 + 1 = 7x$
$x^2 - 7x + 1 = 0$
$\alpha + \beta = \frac{-b}{a}$ ✓
$= 7$

i) $\frac{1}{x+1} + 3 = \frac{1 + 3(x+1)}{x+1}$ ✓
$= \frac{3x+4}{x+1}$

ii)



✓ for asymptotes.
✓ graph.

iii) $A = \int_0^2 \frac{1}{x+1} + 3 \, dx$ ✓
$= \left[\ln(x+1) + 3x\right]_0^2$ ✓
$= \ln 3 + 6 - (0+0)$
$= \ln 3 + 6$ sq units ✓

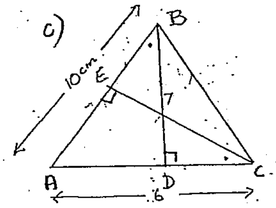
QUESTION 6

a) $\log_a 18 = \log_a(2 \times 3^2)$
$= \log_a 2 + 2\log_a 3$ ✓
$= 0.42 + 2 \times 0.68$ ✓
$= 1.78$

b) $a = \frac{9}{8}$, $r = \frac{3}{4} \div \frac{9}{8}$ ✓
$= \frac{2}{3}$

$S = \frac{a}{1-r}$
$= \frac{\frac{9}{8}}{1-\frac{2}{3}}$ ✓
$= \frac{27}{8}$

c)



✓

(i) $\angle A$ is common
$\angle AEC = \angle BDA = 90^\circ$ (given) ✓
$\therefore \triangle ECA \parallel\!| \triangle DBA$ equiangular ✓

ii) $\frac{CE}{BD} = \frac{AC}{AB}$ (corres sides of similar Δs)
$\frac{CE}{7} = \frac{6}{10}$ ✓
$CE = 4.2$ cm ✓

d) $R = 65 + 4t^{1/3}$

i) $t = 0$, $R = 65$ L/hr ✓

ii) $V = \int 65 + 4t^{1/3} \, dt$
$= 65t + 4 \times \frac{3}{4} t^{4/3} + C$
$= 65t + 3t^{4/3} + C$ ✓
$t = 0$, $V = 15$
$\therefore C = 15$

$\therefore V = 65t + 3t^{4/3} + 15$
When $t = 8$,
$V = 65 \times 8 + 3 \times 8^{4/3} + 15$
$\doteqdot 583$ litres ✓

QUESTION 7

) $\sum_{k=3}^{5} 2^{4-k} = 2 + 2^0 + 2^{-1}$ ✓
$= 3\frac{1}{2}$

b) i) $x = 5t + \log_e(1-2t)$
$\frac{dx}{dt} = 5 + \frac{-2}{1-2t}$ ✓
$= 5 - 2(1-2t)^{-1}$ —(1) ✓
$\frac{d^2x}{dt^2} = 2(1-2t)^{-2} \cdot -2$ ✓
$= \frac{-4}{(1-2t)^2}$ —(2) ✓

$t = 0$, $v = 3$ m/s
$a = -4$ m/s^2

ii) $v = 0$
$\therefore 5 - \frac{2}{1-2t} = 0$ ✓
$5(1-2t) - 2 = 0$
$3 - 10t = 0$
$t = \frac{3}{10}$ sec ✓

c) i) $V(0,0)$ ✓

ii) $-12 = 4a$
$a = -3$
$\therefore F(0,-3)$ ✓

iii) $y = -\frac{x^2}{12}$
$\frac{dy}{dx} = -\frac{x}{6}$ ✓
When $x = 6$, $m_T = -1$
$y = -3$
$y - y_1 = m(x - x_1)$

iv) $x = 0$
$y = 3$
$\therefore A(0,3)$ ✓

QUESTION 8

a) Horizontal point of inflexion ✓

b) i) all real x, $x \neq 0$ ✓

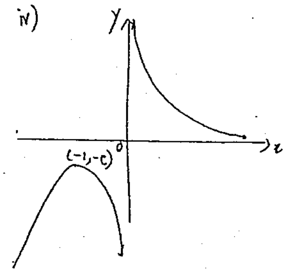
ii) α) $x \to 0$, $y \to \infty$ ✓
β) $x \to \infty$, $y \to 0$ ✓

iii) $y = \frac{e^{-x}}{x}$
$\frac{dy}{dx} = \frac{-e^{-x}x - e^{-x}}{x^2}$ ✓
$\frac{dy}{dx} = 0$, $-e^{-x}x - e^{-x} = 0$
$-e^{-x}(x+1) = 0$
$\therefore x = -1$ ✓
$y = \frac{e}{-1} = -e$

x	-1^-	-1	-1^+
y'	+	0	−

$\therefore$ MAX$(-1,-e)$ ✓

iv)



✓✓

c) i) 4 ✓
ii) π ✓
iii) $y = -4\cos 2x$ ✓

Question 7

a) $V = \pi r^2 h - \int_0^3 \pi x^2 \, dy$ ✓

$y = \sqrt{x}$

$y^4 = x^2$

$\therefore V = \pi \times 9^2 \times 3 - \int_0^3 \pi y^4 \, dy$

$= 243\pi - \pi \left[\frac{y^5}{5}\right]_0^3$ ✓✓

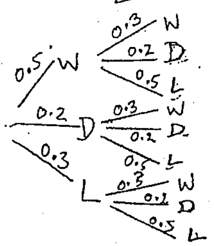
$= 243\pi - \frac{243\pi}{5}$

$= \frac{972\pi}{5}$ ✓

c) i Suppose X has white first

Let W = X wins

L = X loses.



i) $P(D) = 1 - 0.5 - 0.3$

$= 0.2$ ✓

ii) $P(6\cdot) = P(WW)$

$= 0.5 \times 0.3$

$= 0.15$ ✓

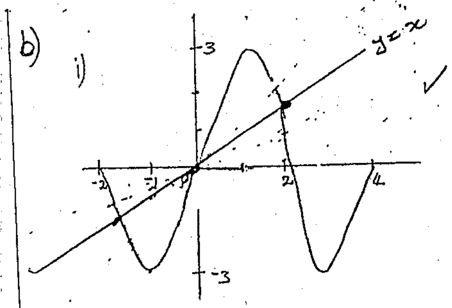
iii) $P(<4) = P(DL) + P(LD) + P(LL)$ ✓

$= 0.2 \times 0.5 + 0.3 \times 0.2 + 0.3 \times 0.5$

$= 0.10 + 0.06 + 0.15$

$= 0.31$ ✓

b) i)



✓

ii) $3\sin\frac{\pi x}{2} = x$ ✓✓

iii) 3 solutions ✓

10) a) i) $N = N_0 e^{-0.03t}$

$\frac{dN}{dt} = -0.03 N_0 e^{-0.03t}$

$= -0.03 N$ ✓

ii) $N = \frac{1}{2} N_0$

$\therefore \frac{1}{2} N_0 = N_0 e^{-0.03t}$

$\ln\left(\frac{1}{2}\right) = -0.03t$

$t = -\frac{\ln\left(\frac{1}{2}\right)}{0.03} = \frac{\ln 2}{0.03}$ ✓

$\doteqdot 23$ days

iii) $\frac{dN}{dt} = -0.03 \times \frac{1}{2} N_0$

$= -0.015 N_0$ ✓

iv) $N < 0.05 N_0$

$\therefore 0.05 N_0 > N_0 e^{-0.03t}$ ✓

$t > \frac{\ln 0.05}{-0.03} \doteqdot 99.9$

i.e. $t = 100$ days ✓

b) i) $CF = OF - OC$

$\frac{OF}{OE} = \cos\alpha$

$\therefore OF = r\cos\alpha$ ✓

$\frac{OC}{DC} = \cot\frac{\pi}{3}$

$OC = \frac{1}{\sqrt{3}} DC$

$DC = EF = r\sin\alpha$ ✓

$\therefore OC = \frac{r\sin\alpha}{\sqrt{3}}$

$CF = r\cos\alpha - \frac{r\sin\alpha}{\sqrt{3}}$ (given)

b) ii) $A = CF \times EF$

$= \left(r\cos\alpha - \frac{r\sin\alpha}{\sqrt{3}}\right) \times r\sin\alpha$ ✓

$= r^2\left(\cos\alpha\sin\alpha - \frac{\sin^2\alpha}{\sqrt{3}}\right)$

$= r^2\left(\frac{1}{2}\sin 2\alpha - \frac{\sqrt{3}\sin^2\alpha}{3}\right)$ ✓

iii) $\frac{dA}{d\alpha} = r^2\left(\cos 2\alpha - \frac{2\sqrt{3}}{3}\sin\alpha\cos\alpha\right)$

$\frac{dA}{d\alpha} = 0$

$\cos 2\alpha - \frac{2\sqrt{3}}{3}\frac{\sin 2\alpha}{2} = 0$

$\therefore \cos 2\alpha = \frac{\sqrt{3}}{3}\sin 2\alpha$ ✓

$\therefore \tan 2\alpha = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{3} = \sqrt{3}$

$2\alpha = \frac{\pi}{3}$

$\therefore \alpha = \frac{\pi}{6}$ ✓

Test for max.

α	$\pi/12$	$\pi/6$	$\pi/3$
$\frac{dA}{d\alpha}$	+	0	−

2007

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

MATHEMATICS

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using blue or black pen
- Board approved calculators may be used
- Write your name on each page
- Each question is to be started on a new page.
- This examination paper must NOT be removed from the examination room

- There is a total of ten questions.
- Each question is worth 12 marks.
- Marks may be deducted for careless or badly arranged work.

Question One

Marks

a) Evaluate $e^{1\cdot4}$ correct to 3 significant figures. 2

b) Factorise $2x^2+7x-4$ 2

c) Simplify $\tan 30° \cos 60°$ leaving your answer in surd form. 2

d) Differentiate $x^2 \ln x$. 2

e) Evaluate $\lim_{x \to 3} \frac{x^2-9}{x-3}$ 2

f) Solve $(x+2)^2=9$. 2

Question Two (Start a new page)

Marks

a) Solve $3x^2 - x - 5 = 0$ leaving your answer in surd form. 2

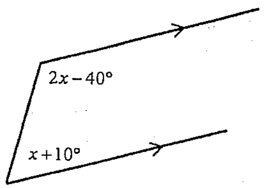
b) Differentiate $\left(4x^3 - 5\right)^6$ 2

c) Find a primitive of $x^2 + \cos 3x$ 2

d) Evaluate $\int_0^1 e^{2x}\,dx$ 2

e) Given that $\frac{12}{\sqrt{6}} = \sqrt{a}$, find the value of '$a$'. 2

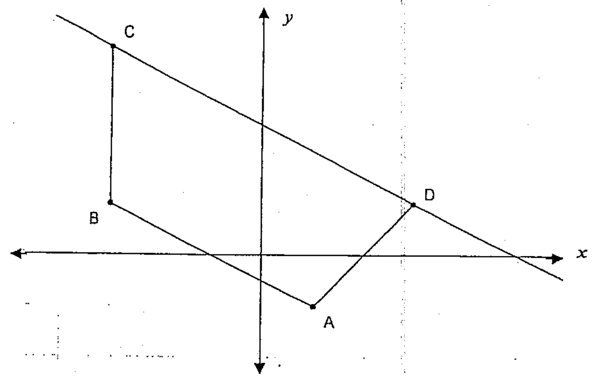
f) Find the value of 'x' giving a reason for your answer. 2



Question Three (Start a new page)

Marks

a) A (1, -1), B (-3, 1), C (-3, 4) and D (3, 1) are points on the Cartesian Plane.



i) Find the distance CD. 1

ii) Show that the equation of the line CD is $x + 2y - 5 = 0$. 2

iii) Find the perpendicular distance of A from CD. 2

iv) Hence or otherwise find the area of the triangle ACD. 1

v) What type of quadrilateral is ABCD? Explain carefully. 2

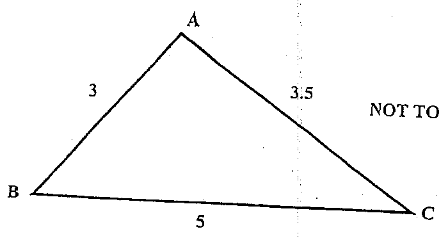
b) Simplify fully $\frac{\sin(180 - \theta) \times \cot\theta}{\sec\theta}$. 2

c) Factorise and simplify $\frac{8x^3 - 27}{4x - 6}$. 2

Question Four (Start a new page)

Marks

a)



In the diagram above AB = 3cm, BC = 5cm and AC = 3.5cm. Find the size of the smallest angle, correct to the nearest degree. 2

b) Find the equation of the tangent to the curve $y = e^x + x$ at the point where $x = 0$. 3

c) State the centre and radius of the circle with equation $(x+3)^2 + y^2 = 16$ 2

d) i) Sketch on the same diagram.

$y = |x-2|$ and $y = 2x$, showing the 'x' and 'y' intercepts. 2

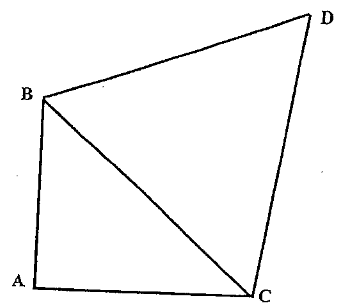
ii) Hence or otherwise solve $|x-2| = 2x$ 2

iii) Using (i) or otherwise, find $\int_0^4 |x-2| \, dx$ 1

Question Five (Start a new page)

Marks

a)



In the above diagram, ABC is an isosceles triangle in which $\angle BAC = 90°$. BCD is an equilateral triangle.

Copy the diagram onto your answer sheet and mark in the given information.

i) Find the size of $\angle ACD$ giving reasons. 2

ii) If BC = 3cm, find the perimeter of ABDC in exact form. 2

b) The quadratic equation $2x^2 - 3x + 6 = 0$ has roots α and β. Find the value of :

i) $\alpha + \beta$ 1

ii) $\alpha\beta$ 1

iii) $a^2 + \beta^2$ 2

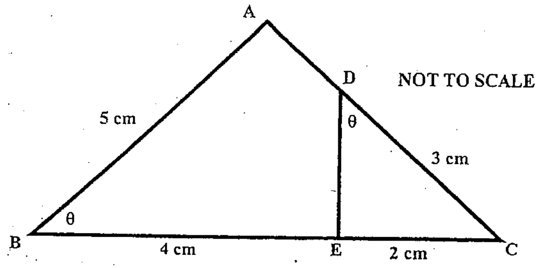
c) Use Simpson's rule with 3 function values to find an approximation for the value of $\int_0^1 10^x \, dx$. Give your answer to 3 decimal places. 2

d) Evaluate $\int_0^{\frac{\pi}{3}} \sec^2 x \, dx$. Give your answer in exact form. 2

Question Six(Start a new page)

Marks

a)



In the above diagram, *ABC* and *DEC* are two triangles in which $\angle ABC = \angle EDC$.
Also AB = 5cm, BE = 4cm, EC = 2cm and CD = 3cm.

Copy the diagram onto your answer sheet.

i) Prove that the two triangles are similar. 2

ii) Hence, giving reasons find the length of *DE*. 2

b) Farmer Brown has hired a driller to drill a borehole to gain access to the underground water on his property. The cost is \$260 for the first 3 metres drilled, \$280 for the next 2 metres, \$300 for the next 2 metres and so on. The price increases by the same amount for each successive 2 metres drilled.

i) Show that the cost of drilling the portion from a depth of 25 metres to to 27 metres is \$500. 2

ii) Calculate the total cost of drilling to a depth of 27 metres. 1

iii) The cost of drilling the borehole to reach water was \$12500. Find the total depth drilled to give access to the water. 3
[To gain full marks all working needs to be shown.]

c) Solve for x: $\log_a 3 = 2\log_a 6 - \log_a x$ 2

Question Seven(Start a new page)

Marks

a) A function $f(x)$ is defined by $f(x) = x^3 - 3x^2 - 9x$.

i) Find $f'(x)$ and $f''(x)$. 1

ii) Find the turning points for the curve and determine their nature. 3

iii) Show that there is one point of inflexion and find its coordinates. 2

iv) Sketch the graph of $y = f(x)$ showing the turning points and the point of inflexion. 2

v) Find the values of 'x' for which the function $f(x)$ is decreasing. 1

b) For what values of 'k' does the quadratic equation $x^2 - (k+3)x + 4k = 0$

i) have one root equal to 2? 1

ii) have no real roots? 2

Question Eight(Start a new page)

Marks

a) A particle is moving in a straight line and its velocity v metres/second at time t seconds is given by:

$$v = \frac{dx}{dt} = 1 - 2\sin 2t, \quad t \geq 0$$

Initially the particle is at the origin.

i) Express the displacement x, as a function of t. 2

ii) Find the position of the particle when $t = \frac{\pi}{6}$. 1

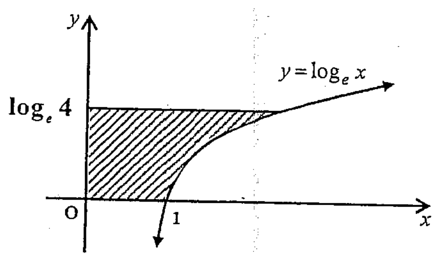
iii) Find an expression for the acceleration in terms of t. 1

iv) Sketch the graph of the acceleration for $0 \leq t \leq \pi$. 2

v) What is the maximum acceleration of the particle? 1

b) In the diagram below, the shaded region bounded by the curve $y = \log_e x$, the x and y axes and the line $y = \log_e 4$ is rotated about the y-axis. 3

Find the exact volume of the solid of revolution formed.



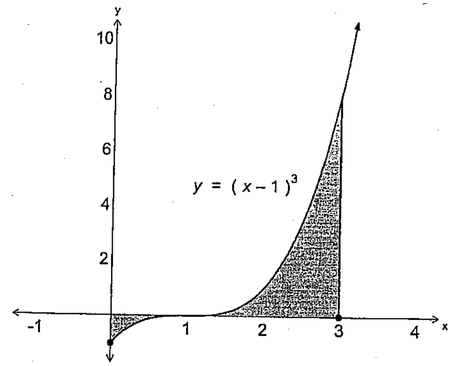
c) The geometric series $1 - x + x^2 - x^3 + \dots$ has a limiting sum of 4. 2

Find the value of x.

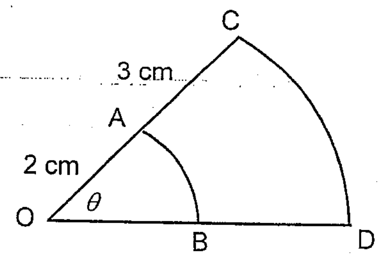
Question Nine(Start a new page)

Marks

a) The shaded area in the diagram below is the region bounded by the curve $y = (x-1)^3$, the x and y axes and the line $x = 3$. Find the shaded area. 4



b)



The arcs AB and CD are parts of concentric circles with centre O. OA = 2 centimetres and AC = 3 centimetres.

i) Find an expression for the area of the sector AOB. 1

ii) Find the ratio of the area of sector AOB : the area of ABDC 2

Question 9 continued over/

Question 9 continued

Marks

c) An industrial plant produces vacuum cleaners. The annual production, P cleaners, at time t years, is given by:

$$P = P_0 e^{kt} \text{ where } P_0 \text{ and } k \text{ are constants}$$

Initially the production of the plant was 2500 cleaners per annum. Five years later it had increased to 4000 cleaners per annum.

i) Find the values of P_0 and k. 3

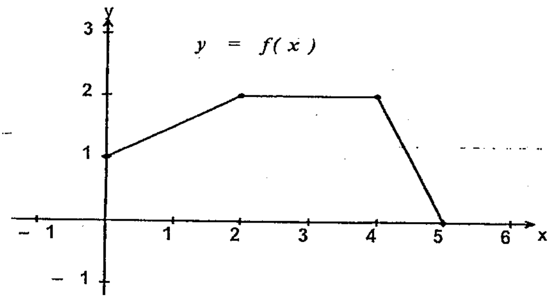
ii) What is the predicted production after 10 years? 1

iii) Find the rate of increase in production after 5 years. 1

Question Ten(Start a new page)

a) The graph of $y = f(x)$ is drawn below.

On a number plane sketch the graph of the derivative function $y = f'(x)$. 2



b) Let A be the point (-2, 0) and B be the point (6, 0).
At the point P(x, y), PA meets PB at right angles.

i) Show that the gradient of PA is $m_1 = \frac{y}{x+2}$. 1

ii) Hence find an equation for the locus of P. 2

Question 10 continued over/

Question 10 continued

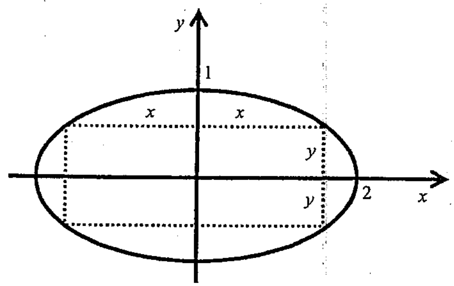
Marks

c) i) Given that $\frac{x^2}{4} + y^2 = 1$,

Show that $y = \frac{\sqrt{4-x^2}}{2}$ for $y \geq 0$ 1

ii) Show that $\frac{dy}{dx} = \frac{-x}{2\sqrt{4-x^2}}$ 1

d) i) The ellipse with equation $\frac{x^2}{4} + y^2 = 1$ is drawn below.



A rectangle of length $2x$ and width $2y$ is to be constructed inside the ellipse with its vertices on the ellipse as shown.

Using part (c) or otherwise show that an expression for the area of the rectangle is given by $A = 2x\sqrt{4-x^2}$. 1

ii) Hence find the value of x so that the area of the rectangle is a maximum. 4

END OF EXAM

CHS 2007 Mathematics (2U) YR 12	HSC SOLUTIONS

1.a) 4.05519967 = 4.06 {3 Sig Fig} ☑☑

b) $(2x-1)(x+4)$ ☑☑

c) $\frac{1}{\sqrt{3}}\times\frac{1}{2}=\frac{1}{2\sqrt{3}}$ ☑☑

d) $x^2\times\frac{1}{x}+2x\times\ln x = x+2x\ln x$ ☑☑

e) $\lim\limits_{x\to 3}\frac{(x-3)(x+3)}{x-3}=\lim\limits_{x\to 3}(x+3)=6$ ☑☑

f) $x+2=\pm 3 \quad \rightarrow \quad x=1$ and $x=-5$ ☑☑

2.a) $x=\frac{1\pm\sqrt{1^2-4\times 3\times -5}}{2\times 3}=\frac{1\pm\sqrt{61}}{6}$ ☑☑

b) $6(4x^3-5)^5\times 12x^2$ ☑

$=72x^2(4x^3-5)^5$ ☑

c) $\frac{x^3}{3}+\frac{1}{3}\sin 3x$ ☑☑

d) $\int_0^1 e^{2x}\,dx = \frac{1}{2}e^{2x}\Big]_0^1$ ☑

$=\frac{1}{2}(e^2-e^0)=\frac{1}{2}(e^2-1)$ ☑

e) $=\frac{12}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}} = 2\sqrt{6}$ ☑

$=\sqrt{24} \Rightarrow a=24$ ☑

f) $(2x-40)+(x+10)=180$ { Co-interior $\angle$'s and || lines } ☑

$3x=210 \Rightarrow x=70°$ ☑

3.a)i) $CD=\sqrt{6^2+(-3)^2}=\sqrt{45}=3\sqrt{5}$ ☑

ii) $m_{CD}=\frac{-3}{6}=-\frac{1}{2}$ ☑

$\therefore eq^n\ CD: y-1=-\frac{1}{2}(x-3)$ ☑

$2y-2=-x+3 \Rightarrow x+2y-5=0$

iii) $a=1, b=2, c=-5, x_1=1, y_1=-1$ ☑

$\therefore d=\frac{|1\times 1+2\times -1+(-5)|}{\sqrt{1^2+2^2}}=\frac{6}{\sqrt{5}}$ ☑

iv) $A=\frac{1}{2}\times\frac{6}{\sqrt{5}}\times 3\sqrt{5}=9u^2$ ☑

v) $m_{AB}=\frac{1-(-1)}{-3-1}=-\frac{1}{2}$

Hence using (ii) AB || CD ☑

So ABCD is a Trapezium ☑

b) $\sin\theta\times\frac{\cos\theta}{\sin\theta}\times\cos\theta = \cos^2\theta$ ☑☑

c) $\frac{(2x-3)(4x^2+6x+9)}{2(2x-3)}$ ☑

$=\frac{4x^2+6x+9}{2}$ ☑

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4. a) $\cos\theta = \frac{3.5^2+5^2-3^2}{2\times 3.5\times 5} \Rightarrow \theta\approx 36°$ ☑☑

b) $y'=e^x+1$ ☑

when $x=0$, $m=y'=2$, $y=1$ ☑

$\therefore y-1=2(x-0) \Rightarrow y=2x+1$ ☑

c) Centre is (-3, 0) and Radius is 4 ☑☑

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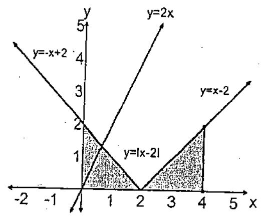
4.d) i)



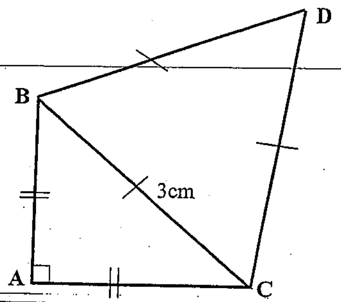
☑☑

ii) $2x=-x+2$ ☑

$3x=2 \Rightarrow x=\frac{2}{3}$ ☑

iii) Shaded area $=\frac{1}{2}(2\times 2)+\frac{1}{2}(2\times 2)=4$ ☑

5.a)



i) $\angle ACB=\frac{180-90}{2}=45°$ { $\angle$ sum isos Δ} ☑

$\angle BCD=60°$ { equilateral Δ } ☑

$\therefore \angle ACD = 45°+60°=105°$

ii) Let AB = AC = x; $\therefore$ $x^2+x^2=3^2$

$\therefore 2x^2=9 \Rightarrow x=\frac{3}{\sqrt{2}}$ ☑

$\therefore P=2\times\frac{3}{\sqrt{2}}+2\times 3=\frac{6}{\sqrt{2}}+6$ cm ☑

b) i) $\alpha+\beta=\frac{3}{2}$ ☑

ii) $\alpha\beta=3$ ☑

iii) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ ☑

$=\left(\frac{3}{2}\right)^2-2\times 3=-3\frac{3}{4}$ ☑

c) $\int_0^1 10^x\,dx=\frac{1-0}{6}\left[f(0)+4f\left(\frac{1}{2}\right)+f(1)\right]$ ☑

$=\frac{1}{6}[1+4\times 3.16228+10]\approx 3.942$ ☑

d) $\int_0^{\frac{\pi}{3}}\sec^2 x\,dx=\left[\tan x\right]_0^{\frac{\pi}{3}}$ ☑

$=\tan\frac{\pi}{3}-\tan 0=\sqrt{3}$ ☑

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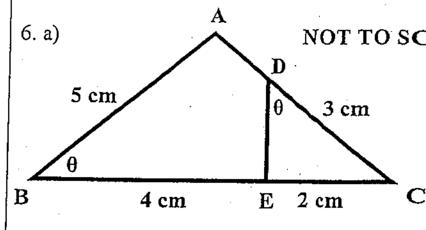
6. a)



i) In the Δ's ABC and EDC

$\angle ABC=\angle EDC$ { given }

$\angle ACB=\angle ECD$ { common } ☑

$\therefore \Delta ABC ||| \Delta EDC$ { equiangular } ☑

ii) $\frac{DE}{AB}=\frac{DC}{BC}$ {corres. sides in similar Δ's} ☑

$\therefore \frac{DE}{5}=\frac{3}{6} \Rightarrow DE=2.5$ cm ☑

b) Set up 2 arithmetic sequences:

Depth: 3, 5, 7, ..., 25, 27, ... ①

Cost: 260, 280, 300, ... ②

i) Using ①: a = 3, d = 2: $T_n=a+(n-1)d$

$\therefore 27=3+(n-1)\times 2 \Rightarrow n=13$ ☑

Using ②: a = 260, d = 20 and n = 13

$T_{13}=260+12\times 20=\$500$ ☑

ii) $S_{13}=\frac{13}{2}[260+500]=\$4940$ ☑

iii) $12500=\frac{n}{2}[2\times 260+(n-1)\times 20]$ ☑

$n^2+25n-1250=0$ {after simplifying} ☑

$(n-25)(n+50)=0 \Rightarrow n=25$

$\therefore T_{25}=3+24\times 2=51$ metres drilled ☑

6.c) $\log_a 3 = \log_a 6^2 - \log_a x$

$\log_a 3 = \log_a \frac{36}{x} \Rightarrow x = 12$ ☑☑

7. i) $f(x) = x^3 - 3x^2 - 9x$

$f'(x) = 3x^2 - 6x - 9$; $f''(x) = 6x - 6$ ☑

ii) For turning points $f'(x) = 0$

$\therefore 3(x^2 - 6x - 9) = 0$

$3(x-3)(x+1) = 0$

$\therefore x = -1$ and $x = 3$ ☑

When $x = 3$: $y = -7, y'' = 12 > 0$

$\therefore (3, -27)$ is a minimum turning point. ☑

When $x = -1$: $y = 5, y'' = -12 < 0$

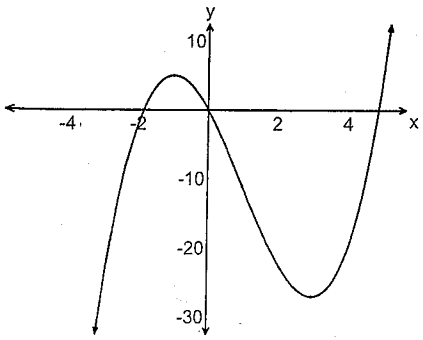
$\therefore (-1, 5)$ is a maximum turning point. ☑

iii) For inflexion points $f''(x) = 0$

$\therefore 6x - 6 = 0 \Rightarrow x = 1$ (Only one point) ☑

It has coordinates $(1, -11)$. ☑

iv) ☑☑



v) For a decreasing function $f'(x) < 0$

Hence from the graph $-1 < x < 3$ ☑

b) i) Substitute $x = 2$ to obtain:

$4 - 2k - 6 + 4k = 0 \Rightarrow k = 1$ ☑

ii) For no real roots $\Delta < 0$, where $\Delta = b^2 - 4ac$

$\therefore (k+3)^2 - 4 \times 1 \times 4k < 0$ ☑

$k^2 - 10k + 9 < 0$

$(k-9)(k-1) < 0 \Rightarrow 1 < k < 9$ ☑

8. a) i) $v = 1 - 2\sin 2t$

$\therefore x = t + \cos 2t + c$ ☑

When $t = 0, x = 0$

$\therefore 0 = 0 + \cos 0 + c$

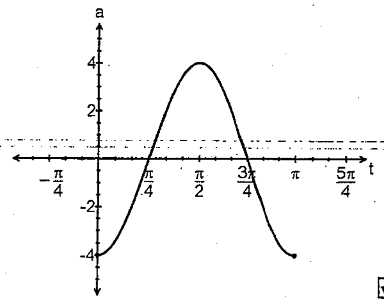
$0 = 1 + c \Rightarrow c = -1$ ☑

$\therefore x = t + \cos 2t - 1$

ii) $x = \frac{\pi}{6} + \cos\frac{\pi}{3} - 1 = \frac{\pi}{6} - \frac{1}{2}$ ☑

iii) $a = \ddot{x} - 4\cos 2t$ ☑

iv)



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v) Maximum acceleration is $4\,m/s^2$ ☑

b) $V = \pi \int_0^{\ln 4} x^2\, dy \quad [y = \ln x \Rightarrow x = e^y]$

$V = \pi \int_0^{\ln 4} (e^y)^2\, dy = \pi \int_0^{\ln 4} e^{2y}\, dy$ ☑

$= \frac{\pi}{2}\left[e^{2y}\right]_0^{\ln 4} = \frac{\pi}{2}\left[e^{2\ln 4} - e^0\right]$ ☑

$\frac{\pi}{2}\left[e^{\ln 16} - 1\right] = \frac{15\pi}{2}\, u^3$ ☑

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8.c) $S_\infty = \frac{a}{1-r}$, with $a = 1$, $r = -x$ ☑

$\therefore 4 = \frac{1}{1+x}$

$4 + 4x = 1 \Rightarrow x = -\frac{3}{4}$ ☑

9. a)

$A = \left|\int_0^1 (x-1)^3\, dx\right| + \int_1^3 (x-1)^3\, dx$ ☑☑

$= \left|\frac{1}{4}\left[(x-1)^4\right]_0^1\right| + \frac{1}{4}\left[(x-1)^4\right]_1^3$ ☑

$= \frac{1}{4} + 4 = 4\frac{1}{4}$ units^2 ☑

b) i) $A = \frac{1}{2}r^2\theta = \frac{1}{2} \times 2^2 \times \theta = 2\theta$ ☑

ii) Area sector $OCD = \frac{1}{2} \times 5^2 \times \theta = 12.5\theta$

$\therefore$ Area $ABCD = 12.5\theta - 2\theta = 10.5\theta$ ☑

Hence ratio $AOB : ABCD = 2\theta : 10.5\theta$

$= 4:21$ ☑

c) i) $P = P_0 e^{kt}$; when $t = 0, P = 2500$

$\therefore 2500 = P_0 e^0 \Rightarrow P_0 = 2500$ ☑

Also when $t = 5, P = 4000$

$\therefore 4000 = 2500e^{5k} \Rightarrow 1.6 = e^{5k}$ ☑

$\ln(1.6) = 5k \Rightarrow k \approx 0.094$ ☑

ii) $P = 2500e^{0.094 \cdot 10} \approx 6400$ ☑

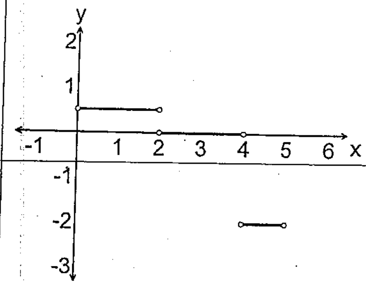
iii) Rate $= \frac{dP}{dt}$: $P = 2500e^{0.094t}$

$\frac{dP}{dt} = 0.094 \times 2500\, e^{0.094t}$

Hence after 5 years $\frac{dP}{dt} = 0.094 \times 2500\, e^{0.094 \times 5}$

≈ 376 *vacuum cleaners / year* ☑

10. a)



☑☑

b) i) $m_{PA} = \frac{y-0}{x-(-2)} \Rightarrow m_1 = \frac{y}{x+2}$ ☑

ii) $m_{PB} = \frac{y-0}{x-6} \Rightarrow m_2 = \frac{y}{x-6}$

Since PA $\perp$ PB then $m_1 \times m_2 = -1$ ☑

$\therefore \frac{y}{x+2} \times \frac{y}{x-6} = -1$

$y^2 = -(x+2)(x-6)$ ☑

$\therefore x^2 + y^2 - 4x - 12 = 0$

10.c) i) $y^2 = 1 - \frac{x^2}{4} = \frac{4 - x^2}{4}$ ☑

$\therefore y = \sqrt{\frac{4-x^2}{4}} = \frac{\sqrt{4-x^2}}{2}$

ii) $y = \frac{1}{2}(4-x^2)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{2} \times (4-x^2)^{-\frac{1}{2}} \times -2x$ ☑

$= \frac{-x}{2\sqrt{4-x^2}}$

d) i) $A = 2x \times 2y$

$= 2x \times 2\frac{\sqrt{4-x^2}}{2} = 2x\sqrt{4-x^2}$ ☑

ii) $A = 2x\sqrt{4-x^2}$

$\frac{dA}{dx} = \sqrt{4-x^2} \times 2 + 2x \times \frac{1}{2}(4-x^2)^{-\frac{1}{2}} \times -2x$ ☑

$= 2\sqrt{4-x^2} - \frac{2x^2}{\sqrt{(4-x^2)}}$

$= \frac{2(4-x^2)}{\sqrt{(4-x^2)}} - \frac{2x^2}{\sqrt{(4-x^2)}}$

$= \frac{8-4x^2}{\sqrt{(4-x^2)}}$ ☑

Stationary points when $\frac{dA}{dx} = 0$

$\therefore 8 - 4x^2 = 0$

$x^2 = 2 \Rightarrow x = \pm\sqrt{2}$ ☑

Since $x > 0$, only need to test $x = +\sqrt{2}$.

x	1.3	$\sqrt{2}$	1.5
A′	0.816	0	-0.756
	/	—	\

☑

$\therefore$ A maximum area occurs when $x = \sqrt{2}$.

SYDNEY TECH HIGH 2008 2U TRIAL

SYDNEY TECHNICAL HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE

2008

MATHEMATICS

Time Allowed: 3 hours plus 5 minutes reading time

Instructions:

- Write your name and class at the top of this page, and at the top of each answer sheet
- At the end of the examination this examination paper must be attached to the front of your answers
- All questions are of equal value and may be attempted
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.

(for Markers Use Only)

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Total

Question 1 (12 marks)

a) Find $e^{-0.6}$ correct to 3 decimal places. 1

b) Expand and simplify $(\sqrt{2}-3)^2$ 2

c) Given $\frac{1}{P} = \frac{1}{Q} + \frac{1}{R}$ make Q the subject of the formula. 2

d) (i) Find $\int_1^2 \frac{dx}{x}$ 1

(ii) Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos\left(\frac{x}{2}\right) dx$. Leave your answer as an exact value. 2

e) Solve the inequality $|2x - 3| \leq 7$ 2

f) Solve the following equations simultaneously

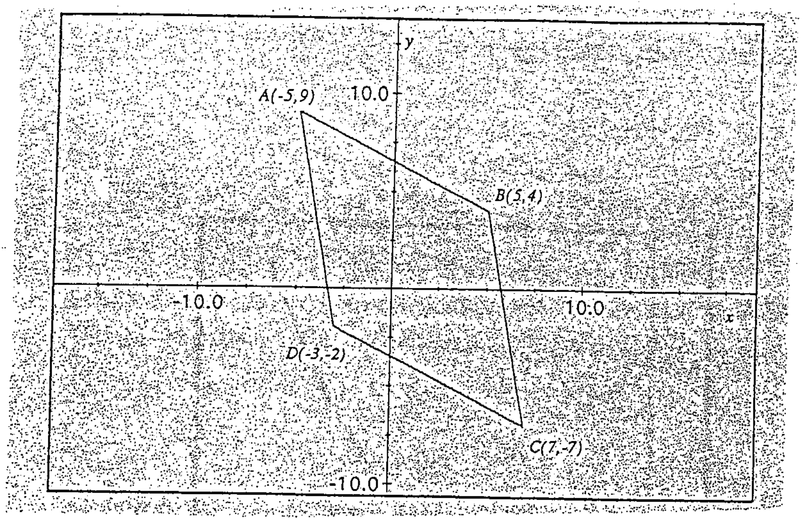
$2x + y = 4$

$5x + 2y = 9$ 2

Question 2 **(Use a separate sheet of paper)** **(12 marks)**

a) A rhombus is a parallelogram with four sides of equal length.

The figure shown below, with vertices $A(-5,9)$, $B(5,4)$, $C(7,-7)$ and $D(-3,-2)$ is a rhombus.



(i) Find the side length of $ABCD$. Give your answer in simplified surd form. 1

(ii) Find the gradient of the longer diagonal. 1

(iii) Show that the diagonals of $ABCD$ are perpendicular. 2

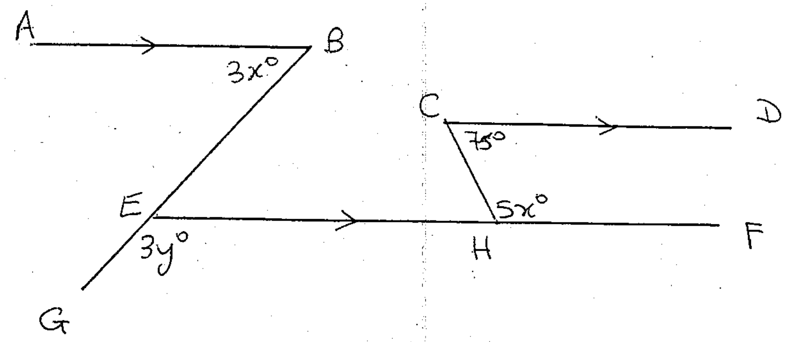
(iv) Find the coordinates of the midpoint of each diagonal. 1

(v) What does this result to part (d) say about the diagonals of this rhombus? 1

(vi) Find the equation of the line passing through AC. 2

(b) In the diagram below the lines AB, CD and EF are parallel. 4

Find the value of x and y. Give reasons for each answer.



Question 3 **(12 marks)** **(Use a separate sheet of paper)**

a) Differentiate

(i) x^2e^x 2

(ii) $\ln\left(\frac{x-5}{x+3}\right)$ 2

b) (i) Find $\int \frac{dx}{3x-1}$ 1

(ii) Evaluate $\int_0^1 e^{4x}\,dx$, leaving your answer in exact form 2

c) For what values of m does the equation $4x^2 + (1+m)x + 1 = 0$ have equal roots. 2

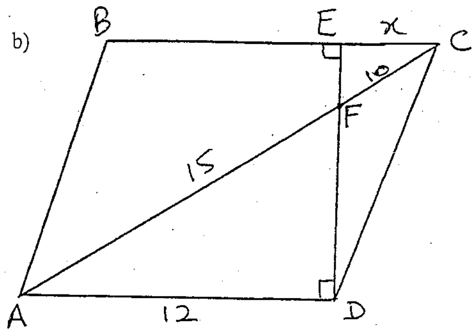
d) For acute angles A and B it is given that $sinA = \frac{12}{13}$ and $cosB = \frac{15}{17}$

Find the exact value of $\sec A + \tan B$. 3

Question 4 **(12 marks)** **(Use a separate sheet of paper)**

a) The sum of the first 4 terms of a geometric progression is 30, and the limiting sum is 32. If the common ratio is negative find the first three terms. 3

b)



$ABCD$ is a parallelogram.

(i) Prove that ΔEFC and ΔDFA are similar.

(ii) Find the value of x. 4

Not to Scale

c) Solve $\sin\left(x + \frac{\pi}{3}\right) = 0$ for $0 \le x \le \pi$ 2

d) α and β are the roots of $2x^2 - 5x + 5 = 0$. Write down the value of

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ 3

Question 5 **(12 marks)** **(Use a separate sheet of paper)**

a) A function is defined by $f(x) = 3x^2 - 2x^3$

(i) Find the coordinates of any turning points and determine their nature 3

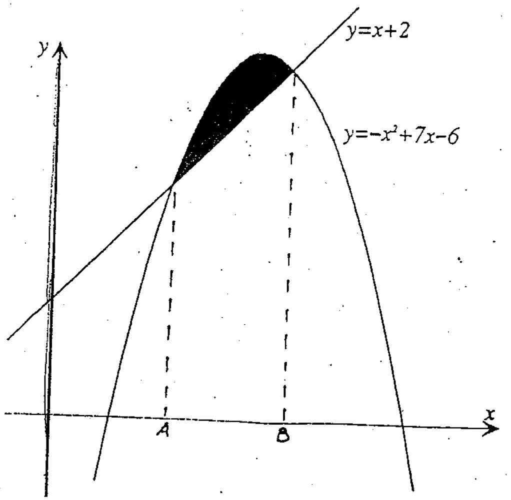
(ii) Sketch the curve, indicating all intercepts and turning points. 2

(iii) State the domain over which both $f(x) > 0$ and $f'(x) > 0$ 1

(iv) On the same set of axes sketch the line $f(x) = \frac{1}{2}$ 1

(v) Hence find the <u>number</u> of solutions to the equation $6x^2 - 4x^3 = 1$ 1 b)

4



The diagram shows the graphs of the functions $y = -x^2 + 7x - 6$ and $y = x + 2$.

(i) Show that the value of A and B is 2 and 4 respectively

(ii) Calculate the area of the shaded region.

Question 6 (12 marks) (Use a separate sheet of paper)

a) Evaluate $\sum_{r=1}^{4} 3^{1-r}$ 1

b) For the arithmetic progression 32, 25, 18, ...

find the (i) the 15[th] term 1

(ii) S_{15} 1

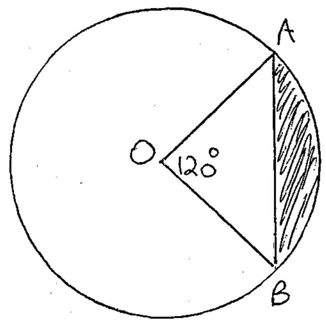
(iii) the sum of the <u>next</u> 20 terms 2

c) The area under the curve $y = 4^x$ between $x = 0$ and $x = 2$ is rotated about the x – axis. Copy and complete the table.

x	0	0.5	1	1.5	2
4^{2x}					

Use your results with Simpson's rule to find an approximate value for the volume of revolution. Use 5 function values and answer correct to 1 decimal place. 3

d)



The circle has a radius of 2cm

(i) Find arc length AB

(ii) Find the shaded area 4

(correct to 1 decimal place)

Question 7 (12 marks) (Use a separate sheet of paper)

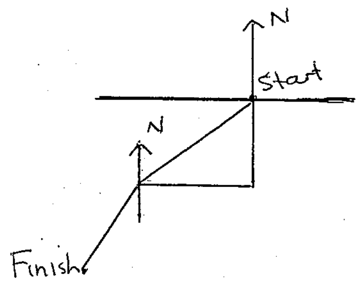
a) $f'(x) = 3x^2 - 4$.

Find $y = f(x)$ if the function passes through (3, 8). 2

b) A boat travels 5km on a bearing of 207° T, then travels 8km on a bearing of 200°T.

Find the straight line distance between the start and finish to 3 significant figures. 4

Copy and complete the given diagram to assist your working.



c) \$30 000 is borrowed to buy a car. Interest is charged at 12% pa, compounding monthly.

The loan is repaid in equal monthly repayments over 4 years. Let A_n be the amount owing after n months.

(i) If M is the monthly payment write an expression for the amount owing

after α) 1 month

β) 3 months

(ii) Find M

(iii) Find the total amount paid over the 4 years. 6

Question 8 (12 marks) (Use a separate sheet of paper)

a) Evaluate $\lim\limits_{x \to 0} \frac{\sin 2x}{x}$ 2

b) Evaluate $\log_5 100 - \log_5 4$ 2

c) A particle moves in such a way that its distance, x metres, from the origin after t seconds is given by

$$x = 2 + 3t - t^3 \text{ for } t > 0$$

(i) Find an equation for its velocity after t seconds. 1

(ii) At what time does the particle stop? 1

(iii) Where is the particle initially? 1

(iv) Find the velocity after 2 seconds. 1

(v) How far has the particle travelled in the first 2 seconds. 2

d) Find the volume of the solid formed when the curve $y = \sqrt{x}$ is rotated about the x axis between $x = 1$ and $x = 5$. (leave the answer in terms of π). 2

Question 9 (12 marks) (Use a separate sheet of paper)

a) If $F(x) = \begin{cases} x^2 - 2 & x \leq -1 \\ 2^x & -1 < x < 2 \\ \log_{10} x & x \geq 2 \end{cases}$

evaluate $f(-1) + f(1) + f(10)$. 2

b) Draw a neat sketch of $y = 3\sin 2x$ within the domain $0 \leq x \leq 2\pi$.

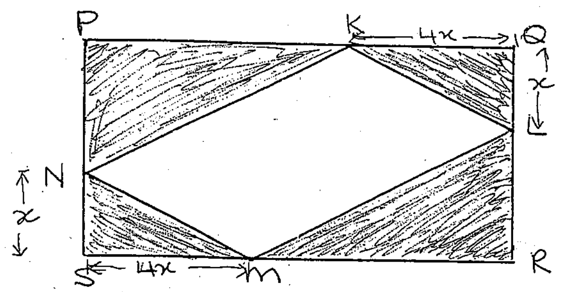
State the (i) period

(ii) amplitude. 4

c) In the diagram, PQRS is a rectangle with PQ=40cm, SP=10cm.

The shaded portions are cut away, leaving the parallelogram KLMN.

QL=SN=x cm and QK=SM = $4x$ cm.



(i) Show that the area of the parallelogram KLMN is given by

$A = 80x - 8x^2$. 3

(ii) Find the allowable values of x 1

(iii) Find the value of x for which A is a maximum 2

Question 10 (12 marks) (Use a separate sheet of paper)

a) For all values of x in the domain of $0 \leq x \leq 6$, a function $f(x)$ satisfies

$f'(x) > 0$ and $f''(x) > 0$.

Sketch a possible graph of $y = f(x)$ in this domain. 2

b) (i) Find the points of intersection of the curve $y = 4 - \sqrt{2x}$ with the x and y axes. 2

(ii) The area enclosed by the curve $y = 4 - \sqrt{2x}$, the x axis and the y axis is rotated about the y axis. Find the volume of the solid of revolution so formed (leave your answer in terms of π) 4

c) The line $x =$ m, cuts the curves $y = log_e x$ and $y = log_e 5x$ at R and S respectively.

Show that the tangents to the curves at R and S are parallel. Also show that the distance RS remains constant for all values of M (ie the distance is independent of m). 4

END OF PAPER

Teacher's Name: Student's Name/N°:

Mathematics 2008
HSC Trial Exam

Question 1

a) $e^{-0.6} = 0.549$ (3dp)

b) $(\sqrt{2}-3)^2 = 2 - 6\sqrt{2} + 9$

$= 11 - 6\sqrt{2}$

c) $\frac{1}{P} = \frac{1}{Q} + \frac{1}{R}$

$\therefore \frac{1}{Q} = \frac{1}{P} - \frac{1}{R}$

$= \frac{R-P}{PR}$

$\therefore Q = \frac{PR}{R-P}$

d) (i) $\int_1^2 \frac{dx}{x} = [\ln x]_1^2$

$= \ln 2 - \ln 1$

$= \ln 2$

(ii) $\int_{\pi/3}^{\pi/2} \cos\left(\frac{x}{2}\right) dx = 2\left[\sin\left(\frac{x}{2}\right)\right]_{\pi/3}^{\pi/2}$

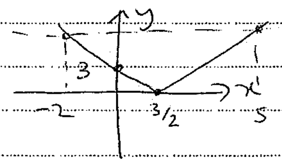
$= 2\left[\sin\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right)\right]$

$= 2\left[\frac{1}{\sqrt{2}} - \frac{1}{2}\right]$

$= 2\left[\frac{\sqrt{2}}{2} - \frac{1}{2}\right]$

$= \sqrt{2} - 1$

e) $|2x-3| \le 7$



$2x - 3 = 7$ $2x - 3 = -7$

$2x = 10$ $2x = -4$

$x = 5$ $x = -2$

$\therefore -2 \le x \le 5$

f) $2x + y = 4$ ①

$5x + 2y = 9$ ②

Teacher's Name: Student's Name/N°:

① × 5 $10x + 5y = 20$ ③

② × 2 $10x + 4y = 18$ ④

③ − ④

$y = 2$

In ① $2x + 2 = 4$

$2x = 2$

$x = 1$.

Question 2

a) (i) Using A and B

side length $= \sqrt{(-5-5)^2 + (9-4)^2}$

$= \sqrt{(10)^2 + (5)^2}$

$= \sqrt{125}$

$= 5\sqrt{5}$ units

(ii) longer diagonal is AC.

gradient AC $= \frac{9 - -7}{-5 - 7}$

$= \frac{16}{-12}$

$= -4/3 = m_1$

(iii) shorter diagonal is DB

gradient DB $= \frac{-2-4}{-3-5}$

$= \frac{-6}{-8}$

$= 3/4 = m_2$

Now $m_1 m_2 = -\frac{4}{3} \times \frac{3}{4}$

$= -1$

$\therefore$ Satisfies condition for perpendicular!

$\therefore$ diagonals perpendicular

(iv)

$$M_{AB} = \left(\frac{-5+7}{2}, \frac{9+-7}{2}\right) \qquad M_{BD} = \left(\frac{-3+5}{2}, \frac{-2+\ldots}{2}\right)$$

$$= (1, 1) \qquad\qquad = (1, 1)$$

(v) Result confirms diagonals bisect, at $(1,1)$

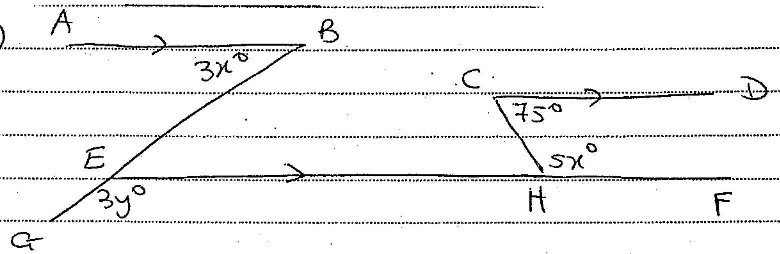
(vi) Gradient $AC = -\frac{4}{3}$

$\therefore$ Eqn AC : $y - 9 = -\frac{4}{3}(x + 5)$

$3y - 27 = -4x - 20$

$4x + 3y - 7 = 0$

b)



Since $CD \parallel HF$ $\quad 75^\circ + 5x^\circ = 180^\circ$

ie cointerior angles supplementary

$\therefore \quad 5x^\circ = 105^\circ$

$x = 21$

Since $AB \parallel EH$, $\angle BEH = 3x^\circ$ (alternate angles equal)

Then $3x + 3y = 180$ (straight angle is 180°)

But $x = 21$

$\therefore \quad 3y = 180 - 63$

$= 117$

$y = 39$

Teacher's Name: Student's Name/N°:

Question 3

a) (i) $y = x^2 e^x$ $\qquad u = x^2 \quad v = e^x$

$y' = x^2 e^x + 2x(e^x)$ $\qquad u' = 2x \quad v' = e^x$

$= x e^x (x + 2)$

(ii) $y = \ln\left(\frac{x-5}{x+3}\right)$

$= \ln(x-5) - \ln(x+3)$

$y' = \frac{1}{x-5} - \frac{1}{x+3}$

$= \frac{\cancel{x} + 3 - (\cancel{x} - 5)}{(x-5)(x+3)}$

$= \frac{8}{(x-5)(x+3)}$

b) (i) $\int \frac{dx}{3x-1} = \frac{1}{3}\ln(3x-1) + c$

(ii) $\int_0^1 e^{4x}\,dx = \left[\frac{1}{4}e^{4x}\right]_0^1$

$= \frac{1}{4}e^4 - \frac{1}{4}e^0$

$= \frac{1}{4}e^4 - \frac{1}{4} = \frac{1}{4}(e^4 - 1)$

c) $4x^2 + (1+m)x + 1 = 0$

Equal roots when $\Delta = 0$.

$\Delta = b^2 - 4ac$

$= (1+m)^2 - 4(4)(1)$

$= 1 + 2m + m^2 - 16$

$= m^2 + 2m - 15$

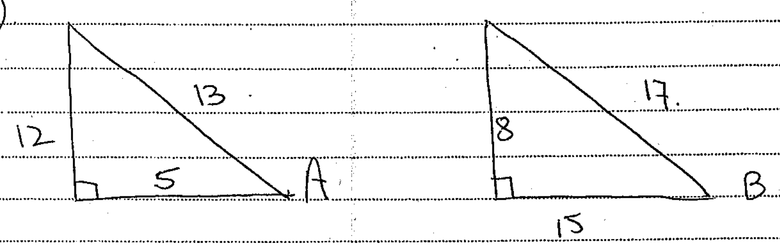
Solve $m^2 + 2m - 15 = 0$

$(m + 5)(m - 3) = 0$

$m = -5$ or $m = 3$

Teacher's Name: **Student's Name/N°:**

d)



Complete each triangle.

$$\sec A + \tan B = \frac{13}{5} + \frac{8}{15}$$

$$= \frac{39+8}{15}$$

$$= \frac{47}{15}$$

Question 4

a) $S_n = \dfrac{a(1-r^n)}{1-r}$

$\therefore S_4 = \dfrac{a(1-r^4)}{1-r} = 30.$

$S_\infty = \dfrac{a}{1-r} = 32$

$\therefore$ In S_4 $\quad 32(1-r^4) = 30$

$$1-r^4 = \frac{30}{32}$$

$$r^4 = \frac{2}{32}$$

$$= \frac{1}{16}$$

$r = \pm \frac{1}{2}$

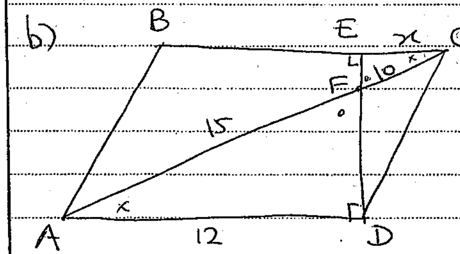
But $r < 0$ $\quad \therefore r = -\frac{1}{2}$ and $a = 48$

$\therefore T_1 = 48$

$T_2 = -24$

$T_3 = 12.$

b)



(i) $\angle FEC = \angle FDA = 90°$ (give

$\angle EFC = \angle AFD$ (verticall

opposite angles equal

$\therefore \Delta EFC$ and ΔDFA are

equiangular.

$\therefore$ Similar

(ii) Corresponding sides are in the same ratio

$\therefore \dfrac{x}{12} = \dfrac{10}{15}\,\dfrac{2}{3}$

$x = \dfrac{24}{3}$

$= 8$

c) $\sin(x + \pi/3) = 0 \qquad 0 \le x \le \pi$

$x + \pi/3 = 0, \pi, 2\pi, 3\pi, \ldots$

$x = -\pi/3, 2\pi/3, 5\pi/3, 8\pi/3, \ldots$

For given domain:

$x = 2\pi/3.$

d) $2x^2 - 5x + 5 = 0$

(i) $\alpha + \beta = 5/2$

(ii) $\alpha\beta = 5/2$

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\beta+\alpha}{\alpha\beta}$

$$= \frac{5/2}{5/2}$$

$$= 1$$

Teacher's Name: Student's Name/N°:

Question 5

a) $f(x) = 3x^2 - 2x^3$

(i) $f'(x) = 6x - 6x^2$

for turning points (stationary) $f'(x) = 0$

$\therefore$ Solve $6x(1-x) = 0$

$x = 0, \quad x = 1$

$f''(x) = 6 - 12x$

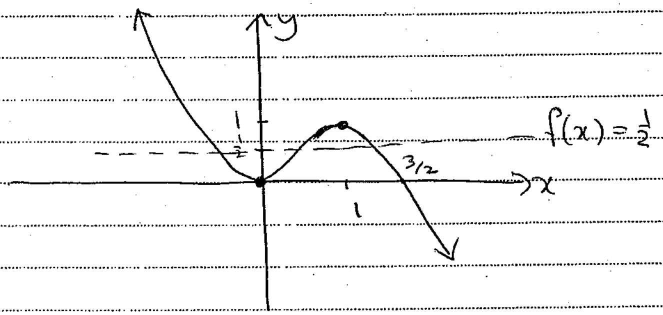
$f''(0) = 6 > 0 \Rightarrow \min$

$f''(1) = 6 - 12 < 0 \Rightarrow \max$

$\therefore$ min at $(0, 0)$

max at $(1, 1)$

(ii)



$f(x) = 0$ when $x^2(3-2x) = 0$.

ie $x = 0$ or $x = 3/2$

(iii) $f(x) > 0$ above y axis
$f'(x) > 0$ increasing
$\}$ Both hold for $0 < x < 1$

(iv) $f(x) = \frac{1}{2}$ (above)

(v) $6x^2 - 4x^3 = 1 \Rightarrow 3x^2 - 2x^3 = \frac{1}{2}$

Since $f(x) = 3x^2 - 2x^3$ and $f(x) = \frac{1}{2}$ intersect 3 times, there will be 3 solutions.

Teacher's Name: Student's Name/N°:

b) $y = -x^2 + 7x - 6$, $y = x + 2$

(i) Intersect when

$-x^2 + 7x - 6 = x + 2$

ie $x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$.

$x = 2$ or $x = 4$

From graph $A = 2$

$B = 4$

(ii) Area $= \int_2^4 (-x^2 + 7x - 6) - (x+2)\, dx$

$= \int_2^4 (-x^2 + 6x - 8)\, dx$

$= \left[-\frac{1}{3}x^3 + 3x^2 - 8x\right]_2^4$

$= -\frac{1}{3}(64) + 3(16) - 32 - \left(-\frac{8}{3} + 12 - 16\right)$

$= -\frac{64}{3} + 48 - 32 + 8/3 - 12 + 16$

$= -\frac{56}{3} + 20$

$= 1\frac{1}{3}\, u^2$

Question 6

a) $\sum_{r=1}^{4} 3^{1-r} = 3^0 + 3^{-1} + 3^{-2} + 3^{-3}$

$= 1 + 1/3 + 1/9 + \frac{1}{27}$

$= 1\, 13/27$

b) $32, 25, 18 \ldots$ $\quad a = 32, \; d = -7$

(i) $T_{15} = a + 14d$

$= 32 + 14 \times (-7)$

$= 32 - 98$

$= -66$

(ii) $S_{15} = \frac{15}{2}[2a + 14d]$

$= \frac{15}{2}[64 + 14(-7)]$

$= 15[32 - 49]$

$= 15 \times -17$

(iii) Sum next 20 terms

$= S_{35} - S_{15}$

$= \frac{35}{2}[64 + 34\times(-7)] - (-255)$

$= 35[32 + 17\times(-7)] + 255$

$= -3045 + 255$

$= -2790$

c)

x	0	0.5	1	1.5	2
4^{2x}	1	4	16	64	256
	y_0	y_1	y_2	y_3	y_4

$Vol = \pi\int_0^2 4^{2x}\,dx$

$\doteqdot \pi\left[\frac{h}{3}(y_0 + y_4 + 4\times(y_1+y_3) + 2(y_2))\right]$

$= \pi\left[\frac{1}{6}(1 + 256 + 4(68) + 2(16))\right]$

$Vol \doteqdot \pi\left[\frac{1}{6}(561)\right]$

$= 293.7\ u^3$ (1 dp)

d) (i) $120^\circ = \frac{2\pi}{3}^c$

$l = r\theta^c$

$= 2\left(\frac{2\pi}{3}\right)$

$= \frac{4\pi}{3}$ cm.

(ii) Area $= \frac{1}{2}r^2(\theta^c - \sin\theta^c)$

$= \frac{1}{2}(4)\left[\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right]$

$= 2\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$ cm^2

Teacher's Name: **Student's Name/N°:**

Question 7

a) $f'(x) = 3x^2 - 4$

$f(x) = x^3 - 4x + c$

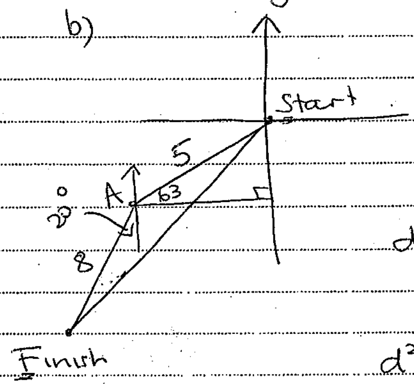
$(3,8)$ satisfies

$\therefore 8 = 3^3 - 4(3) + c$

$8 = 27 - 12 + c \Rightarrow c = -7$

$\therefore y = x^3 - 4x - 7$

b)



Angle at A $= 63 + 90 + 20$

$= 173^\circ$

d = distance S → F

$\therefore$ By cosine rule

$d^2 = 5^2 + 8^2 - 2\times5\times8\cos173^\circ$

$= 25 + 64 - 80\cos173^\circ$

$= 89 - 80\cos173^\circ$

$d^2 = 168.4036921$

$\therefore d = 12.97704482$

$= 13.0$ km (3 sig figs)

c) \$30000 12% pa = 1% per month

48 repayments

(i) α) $A_1 = 30000(1.01) - M$

β) $A_2 = [30000(1.01) - M](1.01) - M$

$= 30000(1.01)^2 - M(1.01 + 1)$

Similarly

$A_3 = 30000(1.01)^3 - M(1.01^2 + 1.01 +$

(ii) $A_{48} = 0$ since fully repaid

$0 = A_{48} = 30000(1.01)^{48} - M(1.01^{47} + 1.01^{46} + \cdots +$

ie $30000(1.01)^{48} = M(1 + 1.01 + \cdots + 1.01^{47})$

GP with $a=1$ $r=1.1$

$n=48$

Teacher's Name: ______ Student's Name/N°: ______

$$\therefore M = \frac{30000(1.01)^{48}}{\left(\frac{1-1.01^{48}}{1-1.01}\right)}$$

$$= \frac{30000(1.01)^{48}(0.01)}{1.01^{48}-1}$$

$= \$790.02$ (nearest cent).

(iii) Total repaid $= M \times 48$

$= \$37920.72$ (nearest cent).

Question 8

a) $\lim_{x \to 0} \frac{\sin 2x}{x} = \lim_{x \to 0} \left(\frac{\sin 2x}{2x}\right) \times 2$

$= 2$

b) $\log_5 100 - \log_5 4 = \log_5\left(\frac{100}{4}\right)$

$= \log_5 25$

$= \log_5 5^2$

$= 2$.

c) $x = 2 + 3t - t^3, \quad t > 0$

(i) $\frac{dx}{dt} = 3 - 3t^2$

vel $= 3 - 3t^2$

(ii) Stops when $v = 0$

ie solve $3 - 3t^2 = 0$

$t = 1 \quad (t > 0)$

Stops after 1 second.

(iii) $t = 0$ in $x = 2 + 3t - t^3$

$= 2$

$\therefore$ Initially 2 m to the right of O.

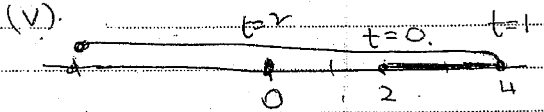
(iv) When $t = 2$

$v = 3 - 3(2)^2$

$= -9$

ie $v = -9$ m/sec (travelling to the left).

(v)



When $t=1$, $x = 2 + 3 - 1$

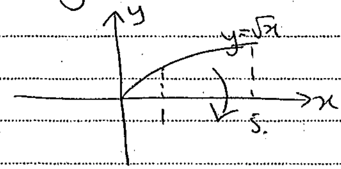
$= 4$

$t = 2$, $x = 2 + 6 - 8$

$= 0$

$\therefore$ Has travelled $2 + 4 = 6$ m.

d) $y = \sqrt{x}$



$\text{Vol} = \pi \int_1^5 x \, dx$

$= \pi \left[\frac{1}{2}x^2\right]_1^5$

$= \frac{\pi}{2}[25 - 1]$

$= 12\pi \ u^3$

Question 9

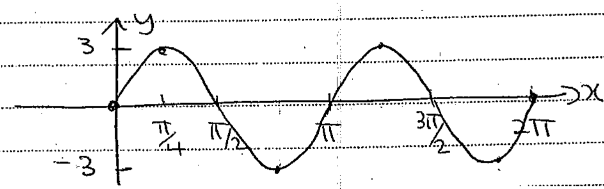
a) $f(-1) = (-1)^2 - 2 = -1$

$f(1) = 2^1 = 2$

$f(10) = \log_{10} 10 = 1$

$\therefore f(-1) + f(1) + f(10) = 2$.

b) $y = 3 \sin 2x \qquad 0 \le x \le 2\pi$.

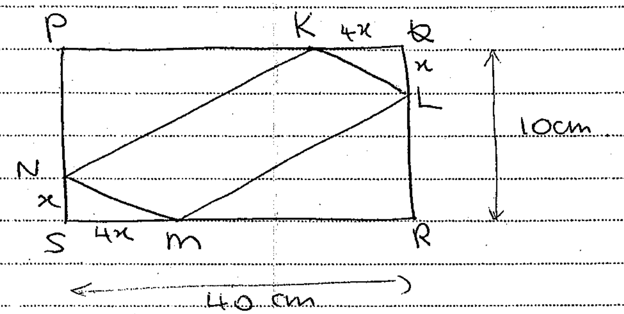


(i) Period $= \frac{2\pi}{2}$

$= \pi$

(ii) Amplitude $= 3$

c)



(i) Area parallelogram KLMN

$= 40 \times 10 - \cancel{2} \times \frac{1}{\cancel{2}}(4x)(x)$

$- \cancel{2} \times \frac{1}{\cancel{2}}(40-4x)(10-x)$

$= \cancel{400} - 4x^2 - (\cancel{400} - 40x - 40x + 4x^2)$

$= 80x - 8x^2$

(ii) $0 < x < 10$

(iii) $\frac{dA}{dx} = 80 - 16x$

$\frac{dA}{dx} = 0$ when $16x = 80$

$x = 5$

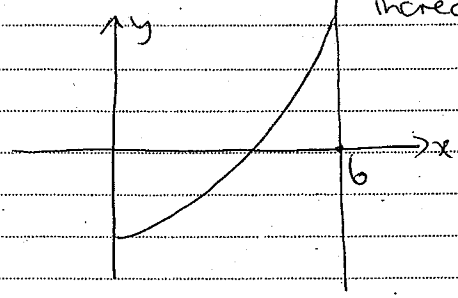
$\frac{d^2A}{dx^2} = -16 < 0 \Rightarrow$ max

$\therefore$ Area max when $x = 5$.

Teacher's Name: **Student's Name/Nº:**

Question 10

a) $0 \le x \le 6$ $f'(x) > 0$ increasing $f''(x) > 0$ concave up.



b) (i) $y = 4 - \sqrt{2x}$.

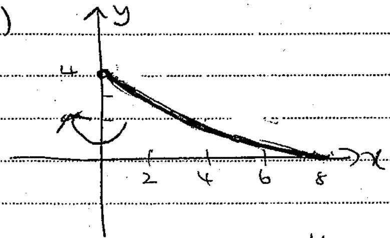
x axis: $y = 0$ ie $\sqrt{2x} = 4$

$2x = 16$

$x = 8$.

y axis: $x = 0$. ie $y = 4$

(ii)



$y = 4 - \sqrt{2x}$

$\sqrt{2x} = 4 - y$

$2x = (4-y)^2$

$x = \frac{(4-y)^2}{2}$

$\text{Vol} = \pi \int_0^4 x^2 \, dy$

$= \pi \int_0^4 \frac{(4-y)^4}{4} \, dy$

$= \frac{\pi}{4} \left[\frac{(4-y)^5}{-5} \right]_0^4$

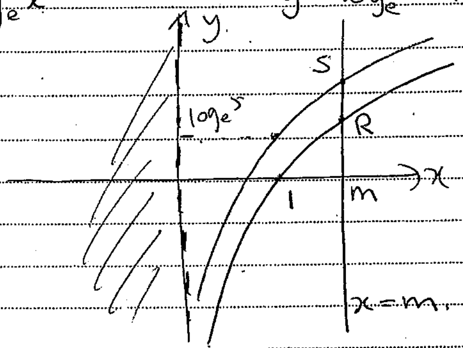
$= -\frac{\pi}{20} \left[\cancel{(4-4)^5} - (4-0)^5 \right]$

$= -\frac{\pi}{20} (-4)^5$

$= \frac{\pi}{\cancel{20}_5} \times 4^{\cancel{5}\,4} = \frac{256\pi}{5} u^3$

Teacher's Name: **Student's Name/N°:**

c) $y = \log_e x$ $\qquad$ $y = \log_e 5 + \log_e x$



$y = \log_e x$

$y' = \frac{1}{x}$

At R, $x = m$

$\therefore$ grad $= \frac{1}{m}$

$y = \log_e 5 + \log_e x$

$y' = 0 + \frac{1}{x}$

$= \frac{1}{x}$

At S, $x = m$

$\therefore$ grad $= \frac{1}{m}$

$\therefore$ They have the same gradient.

Tangents are parallel.

$R = (m, \log_e m)$ $\qquad$ $S = (m, \log_e 5 + \log_e m)$

$RS = \sqrt{(\cancel{m-m})^2 + (\cancel{\log_e m} - (\log_e 5 + \cancel{\log_e m}))^2}$

$= \sqrt{(\log_e 5)^2}$

$= \log_e 5$ $\qquad$ $\therefore$ RS remains constant

END

Student Number

Knox Grammar School

2008

Trial Higher School Certificate Examination

Mathematics

General Instructions
- Reading time – 5 minutes
- Working time - 3 hours
- Write using blue or black pen only
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120
- Attempt Questions 1 – 10
- Answer each question in a separate writing booklet
- All questions are of equal value

Subject Teachers
Mr A. Johansen
Mr J. Harnwell
Mr I. Mulray
Miss L. Schultz
Miss F. Yamamer

This paper MUST NOT be removed from the examination room

Number of Students in Course: 76

Number of Writing Booklets Per Student (Four Page) 10

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Total marks – 120
Attempt Questions 1–10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\dfrac{0.1}{\sqrt{e+1}}$ correct to two significant figures. **2**

(b) Factorise $2x^2-4x+2$ completely. **2**

(c) Write down the primitive function of $\dfrac{3}{x}+5$. **2**

(d) Solve $\dfrac{x}{4}=3-\dfrac{x-2}{3}$, leaving your answer as an improper fraction. **2**

(e) If $a+\sqrt{b}=4(7+\sqrt{5})$ find a and b if they are both integers. **2**

(f) Sketch the graph of $y=|4-x|$ **2**

Question 2 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Draw a neat sketch of a number plane and plot the points $A(-4, 0)$, $B(4, 0)$ and $C(0, 8)$ on it. **1**

(b) Find the gradient of AC and show that the equation of AC is $2x-y+8=0$. **2**

(c) Find the perpendicular distance of AC from $Z(0, 3)$. **2**

(d) If X is the midpoint of AC, and Y is the midpoint of BC, find the coordinates of X and Y. **1**

(e) Show that XZ is perpendicular to AC. **2**

(f) Show that the lengths $AZ = BZ = CZ = 5$ units. **2**

(g) Find the equation of the circle passing through A, B and C. **2**

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate the following with respect to x.

(i) $y = \dfrac{\sin x}{x}$ **1**

(ii) $y = \left(x^2 + 3\right)^6$ **1**

(iii) $y = x \ln x$ **1**

(b) Find an expression for each of the following integrals.

(i) $\displaystyle\int \frac{8}{x^2}\, dx$ **1**

(ii) $\displaystyle\int \sec^2 \pi x \, dx$ **1**

(c) Evaluate $\displaystyle\int_0^1 e^{2x} - e^{-x} \, dx$ **3**

(d) The gradient function of a curve is given by $\dfrac{dy}{dx} = 6x^2 - 4$. The curve passes through the point (1, 8). Determine the equation of the curve. **2**

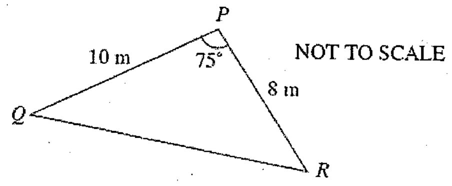
(e) The exterior angle of a regular polygon is $\dfrac{\pi}{10}$ radians.

(i) What is the size of each interior angle in radians? **1**

(ii) How many sides does this regular polygon have? **1**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



(i) Determine the length of QR, correct to 2 decimal places. **2**

(ii) What is the area of triangle PQR? Answer correct to 2 decimal places. **1**

(b) A function $f(x)$ is defined as $f(x) = x^4 - 8x^2$.

(i) Locate all stationary points and any points of inflexion. Distinguish between them. **4**

(ii) Determine the coordinates of the points where $y = f(x)$ crosses the x-axis. **2**

(iii) On a half-page diagram, sketch the function $y = f(x)$. Clearly label the stationary points, points of inflexion and intercepts with the x-axis. **2**

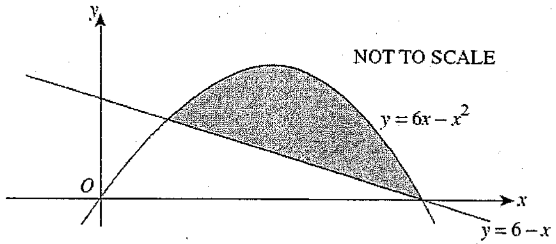
(iv) What is the maximum value of $f(x)$ in the interval $-2 \le x \le 3$? **1**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Simplify $\log_b a^m \div \log_m a$ as a single expression in a logarithm of base b. 2

(b) The roots of the quadratic equation $2x^2 + kx + D = 0$ are α and β. $\alpha\beta = -5$ and $\alpha + \beta = 3$. Determine the values of k and D. 2

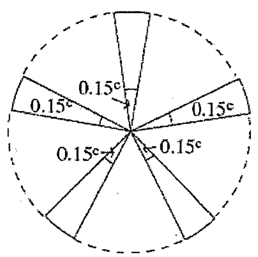
(c) The diagram shows the graph of $y = 6x - x^2$ and $y = 6 - x$.



(i) Use simultaneous equations to show that $y = 6x - x^2$ and $y = 6 - x$ intersect at (1, 5) and (6, 0). 1

(ii) Use calculus to determine the size of the shaded area. 3

(d)



The five blades on a windmill are identical sectors of the same circle. The angle of each blade at the centre of the circle is 0.15^c and the radius is 1.2 metres.

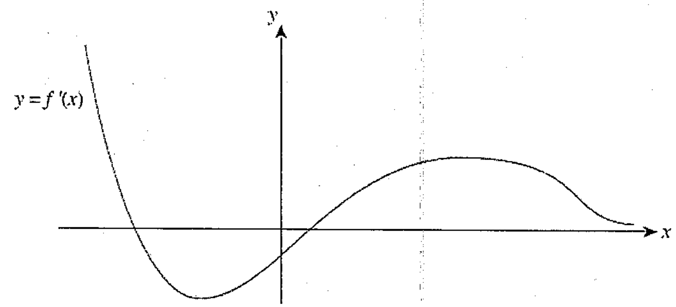
(i) All the edges on each of the blades are to be covered by a protective metal strip. Calculate the total length of metal strip required to protect the edges of all five blades. 2

(ii) The front and back surface of each blade is to be painted with a metal protector. A 100 mL container of the metal protector covers 400 cm^2. Calculate the quantity of metal protector required. 2

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $\sin\theta = \frac{-\sqrt{3}}{2}$ for $0 \le \theta \le 2\pi$. 2

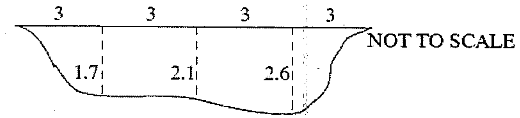
(b)



The diagram shows the gradient function $y = f'(x)$. Copy or trace the diagram into your answer booklet.

The curve $y = f(x)$ passes through the origin. Sketch the function $y = f(x)$ on the same set of axes. Clearly indicate any turning points, points of inflexion, and the behaviour of the graph for very large positive and negative values of x. 3

(c)

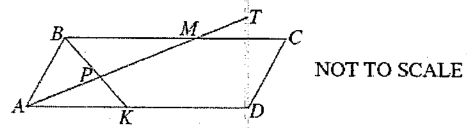


The diagram shows the cross-section of a 12-metre-wide pond. The depths are taken every 3 metres.

(i) Use Simpson's rule with five function values to find an approximate value for the area of the cross-section. 2

(ii) The pond is 25 metres long. Calculate the approximate quantity of water in the pond. Express the volume in cubic metres. 1

(d)



$ABCD$ is a parallelogram. Line AT bisects $\angle BAD$ and cuts BC at M. Line BK bisects $\angle ABC$. AT and BK intersect at P.

Copy the diagram onto your answer page and prove that

(i) $\angle BPA = 90°$. 2

(ii) $AB = BM$. 2

Question 7 (12 marks) Use a SEPARATE writing booklet. Mark

(a) Solve the equation $e^{2x} - 28e^{x} + 27 = 0$. Leave your answer in exact form. 3

(b) An ambulance is delivering a patient to the hospital who is unconscious from a drug overdose. The medical staff don't know how much of the drug the unconscious patient has taken.

The rate of change of the concentration of the drug (C) in the blood is proportional to the concentration, i.e. $\frac{dC}{dt} = kC$.

(i) Prove that $C = C_0 e^{kt}$ is a solution to $\frac{dC}{dt} = kC$. 1

(ii) Three hours after the patient took the overdose, the blood concentration of the drug was 2.45 mg/L. Half an hour later the concentration was 1.84 mg/L. Determine the initial concentration of the drug in the patient's blood. Give your answer correct to two decimal places. 3

(iii) If the medical staff don't give the patient any further medication, when will the drug concentration fall below the critical value of 0.5 mg/L? 1

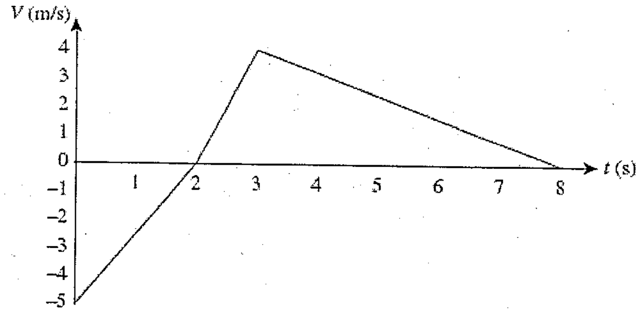
(c) Two particles moving in a straight line are initially at the origin. The velocity of one particle is $\frac{2}{\pi}$ m/s and the velocity of the other particle at time t seconds is given by $v = -2\cos t$ m/s.

(i) Determine equations that give the displacements, x_1 and x_2 metres, of the particles from the origin at time t seconds. 2

(ii) Hence, or otherwise, show that the particles will never meet again. 2

Question 8 (12 marks) Use a SEPARATE writing booklet. Marks

(a)



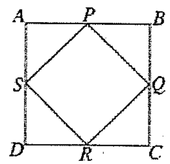
The graph shows the velocity of a particle moving in a straight line for 8 seconds.

(i) When does the particle change direction? 1

(ii) Determine the total distance covered by the particle during the 8 seconds. 2

(iii) What is the particle's position relative to its starting position when $t = 8$ seconds? 1

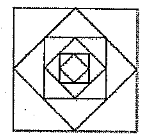
(b)



$ABCD$ is a square with sides 16 cm long. P, Q, R and S are the midpoints of the sides of the square $ABCD$. P, Q, R and S are joined to make another square.

(i) Show that $PS = 8\sqrt{2}$ and that the area of $PQRS$ is 128 cm^2. 2

A 'squares within squares' pattern is produced by joining midpoints of the sides of successive squares.



(ii) $ABCD$ is the first square and $PQRS$ is the second square. What is the area of the 10th square? 2

(iii) Which square has a perimeter of $\sqrt{2}$ cm? 2

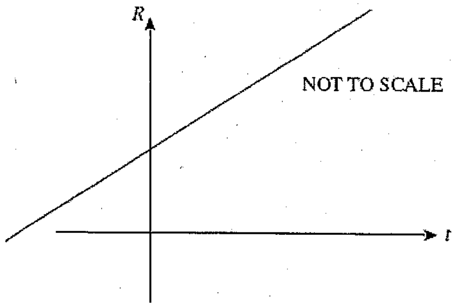
(iv) Imagine the pattern can be repeated infinitely. What is the relationship between the sum of the areas of all the squares and the original square $ABCD$? Use a calculation to justify your answer. 2

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) During a famine in Europe in the 19th century people in a small rural city ate an increasing quantity of potatoes each month as other food became increasingly scarce.

The rate at which potatoes were eaten (R) was given by $R = 15 + 2t$ tonnes per month, where t is the time in months after the beginning of the famine.

(i)



The diagram shows the graph of $R = 15 + 2t$. Copy the graph onto your answer page and show on the graph the region representing the total quantity of potatoes eaten in the city in 12 months. **1**

(ii) Calculate the total amount of potatoes that were eaten in the city during the 12-month famine. **3**

(b) Beth and Cathy are best friends who work in the same office. Each year on January 1, they each receive a cash bonus of \$5000. They received their first bonuses in 1997. Every year Beth invests her \$5000 in superannuation at 9% p.a. compounding interest. Each year Cathy spends her bonus on an overseas trip.

(i) Show that the expression $5000(1.09^{10} + 1.09^{9} + 1.09^{8} + \ldots + 1.09)$ represents the amount in Beth's superannuation account on January 1, 2007, immediately before her 2007 bonus was added to the account. **2**

(ii) Show that Beth had almost \$88 000 in her superannuation account on January 1, 2007, after her 2007 bonus was credited to her account. **2**

Cathy decides that on January 1, 2007, she will start saving for her retirement, which will occur in 20 years' time. She would like to have the same amount that Beth will have in 20 years' time from saving her annual \$5000 bonus. Cathy's account also pays 9% p.a. compounding interest.

(iii) How much will Cathy need to save each year to have the same total amount as Beth will have in 20 years' time (i.e. including the amounts Beth invested in the first 10 years)? **3**

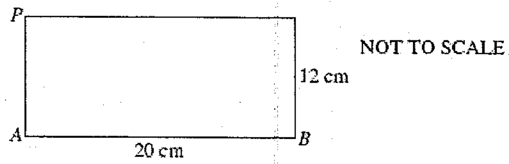
(iv) How much more will Cathy have to invest over the 20 years than Beth will have invested over the 30 years? **1**

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

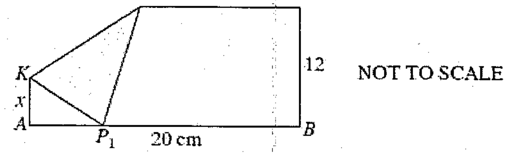
(a) (i) Shade the region bounded by $y \leq 4 - x^2$, $x \geq 0$ and $y \geq 0$. **2**

(ii) Find the volume of the solid of revolution formed when the region defined in part (i) above is rotated about the x-axis. **4**

(b)



I have a rectangular sheet of paper 12 cm high by 20 cm long. I take the vertex labelled P and place it on the side AB. P now lies on top of P_1.



At the bottom left of the rectangle there is a small triangle AKP_1. Let the length of KA be x cm.

(i) Explain why KP_1 is $(12 - x)$ cm long. **1**

(ii) Show that the area of ΔAKP_1 is given by $A = x\sqrt{36 - 6x}$. **2**

(iii) Hence show that when x is one-third the length of PA the area of ΔAKP_1 is a maximum. **3**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Note: $\ln x = \log_e x, \quad x > 0$

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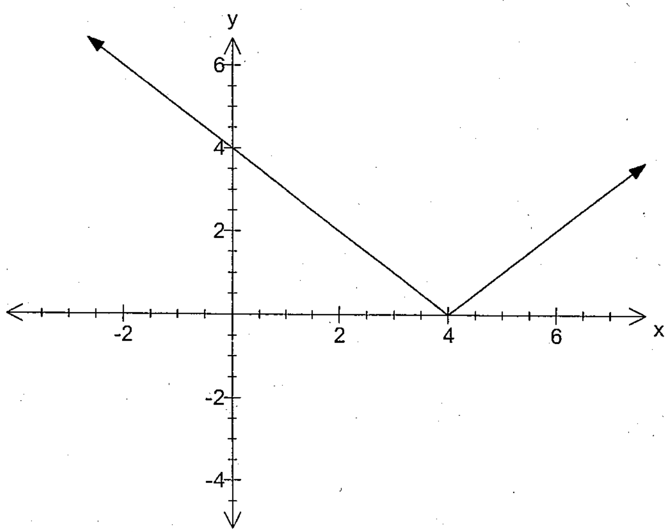
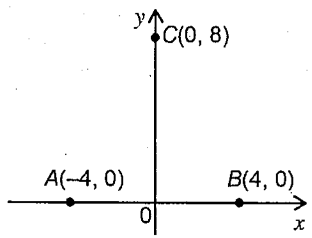
Marking guidelines

Question 1

Criteria	Marks
(a) $\dfrac{0.1}{\sqrt{e+1}} = 0.0518\ldots$ $= 0.052$ (correct to 2 significant figures)	1 value 1 rounding 2 full answer
(b) $2x^2 - 4x + 2 = 2(x^2 - 2x + 1)$ $= 2(x-1)^2$	1 common factor 2 full
(c) $\int\left(\dfrac{3}{x}+5\right)dx = 3\ln x + 5x + c$	1 for each part 2 full
(d) $\dfrac{x}{4} = 3 - \dfrac{x-2}{3}$ $3x = 36 - 4(x-2)$ $3x = 36 - 4x + 8$ $7x = 44$ $x = \dfrac{44}{7}$	1 for 2nd line 2 for +8 3 full
(f) 	1 for y = 4-x 2 for complete graph

Question 2

Criteria	Marks
(a) 	1
(b) $m = \dfrac{8-0}{0-(-4)} = 2$ C is the y-intercept, so $b = 8$ $y = 2x + 8$ $2x - y + 8 = 0$	1 for mostly correct 2 full answer
(c) $d = \dfrac{\lvert 2x - y + 8\rvert}{\sqrt{2^2 + (-1)^2}}$ $= \dfrac{\lvert 2\times 0 - 3 + 8\rvert}{\sqrt{5}}$ $= \dfrac{5}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = \sqrt{5}$	1 for mostly correct 2 full answer
(d) $X\left(\dfrac{-4+0}{2}, \dfrac{0+8}{2}\right)$ and $Y\left(\dfrac{4+0}{2}, \dfrac{0+8}{2}\right)$ $X(-2, 4)$ and $Y(2, 4)$	1
(e) $m_{XZ} = \dfrac{3-4}{0-(-2)} = \dfrac{-1}{2}$ $m_{XZ} \times m_{AC} = 2 \times \dfrac{-1}{2} = -1$, so XZ is perpendicular to AC.	1 for gradient 1 for test 2 full answer
(f) $d_{AZ} = \sqrt{(0--4)^2 + (3-0)^2}$ $\quad d_{BZ} = \sqrt{(0-4)^2 + (3-0)^2}$ $= 5 \quad\quad = 5$ $d_{CZ} = \sqrt{(0-0)^2 + (3-8)^2}$ $= 5$	1 for mostly correct 2 full answer
(g) From part (f), Z is the centre of the circle with radius 5 passing through A, B and C. $x^2 + (y-3)^2 = 25$	1 for circle equation 2 full answer

Question 3

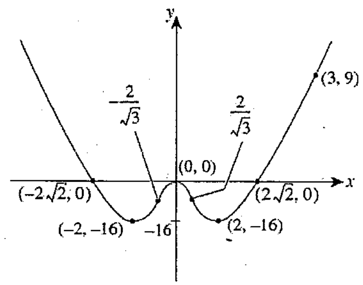
Criteria	Marks
(a) (i) $\dfrac{dy}{dx}=\dfrac{x\cos x-\sin x}{x^2}$	1
(ii) $\dfrac{dy}{dx}=12x(x^2+6)^5$	1
(iii) $\dfrac{dy}{dx}=1+\ln x$	1
(b)	
(i) $\displaystyle\int \frac{8}{x^2}\,dx=-\frac{8}{x}+C$	1
(ii) $\displaystyle\int \sec^2 \pi x\,dx=\frac{1}{\pi}\tan \pi x+C$	1
(c) $\displaystyle\int_0^1 e^{2x}-e^{-x}\,dx=\left[\frac{1}{2}e^{2x}+e^{-x}\right]_0^1$	1 for each primitive
$=\left(\frac{1}{2}e^2+e^{-1}\right)-\left(\frac{1}{2}e^0+e^0\right)$	
$=\frac{1}{2}e^2+e^{-1}-\frac{3}{2}$	3 for correct answer.
(d) $y=2x^3-4x+C$ at $x=1, y=8\ \therefore C=2$ $\therefore y=6x^2-4x+2$	1 for primitive 2 marks correct equation.
(e) (i) $\dfrac{9\pi}{10}$	1
(ii) $2\pi\div\dfrac{\pi}{10}=20$ sides	1

Question 4

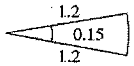
		Sample answer	Syllabus outcomes and marking guide
(a)	(i)	$x^2=10^2+8^2-2\times10\times8\cos75$ $\therefore x=11.07$ to 2 decimal places	P4 • Gives the correct answer .... 2 • Makes the correct substitution into the correct formula.................... 1
	(ii)	area $=\frac{1}{2}\times10\times8\times\sin75=38.64\text{ m}^2$	• Gives the correct answer 1
(b)	(i)	$f(x)=x^4-8x^2$ $f'(x)=4x^3-16x$ stationary points occur when $f'(x)=0$ i.e. $4x^3-16x=0$ $4x(x^2-4)=0$ $\therefore$ stationary points occur at $(0,0)$, $(2,-16)$ and $(-2,-16)$ Testing $f''(x)=12x^2-16$ At $(0,0)$, $f''(x)=-16<0$. $\therefore$ maximum turning point. At $x=\pm2$, $f''(x)=32>0$. $\therefore$ minimum turning points. For inflexions, $f''(x)=0$ and a change in concavity occurs. $\therefore x=\pm\sqrt{\frac{16}{12}}=\pm\frac{2}{\sqrt{3}}$	H6, H9 • Gives the correct solutions........... 4 • Locates stationary points and determines nature or equivalent progress......... 3 • Locates stationary points or equivalent progress .... 2 • Correctly identifies the x-values of a cubic derivative 1

x	-2	$-\frac{2}{\sqrt{3}}$	0	$\frac{2}{\sqrt{3}}$	2
$f''(x)$	>0	0	<0	0	>0

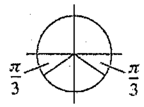
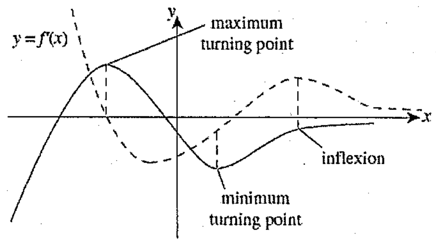
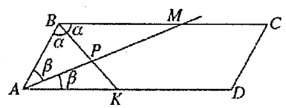
Therefore, points of inflexion are at $x=\pm\frac{2}{\sqrt{3}}$.

		Sample answer	Syllabus outcomes and marking guide
	(ii)	crosses x-axis at $x^4-8x^2=0$ $x^2(x^2-8)=0$ $x=0$ or $\pm2\sqrt{2}$	P4, H9 • Gives the correct answers .... 2 • Gives one correct answer, or attempts to solve $x^4-8x^2=0$ 1
	(iii)		H6, H9 • Gives a correct sketch showing all features (or correct from previous answer) (point (3, 9) not required) .... 2 • Gives any quartic-shaped sketch (or correct from previous answer)...... 1
	(iv)	maximum value in $-2\le x\le3$ is 9	P4, H9 • Gives correct answer (or correct from previous answer) 1

Question 5

Sample answer	Syllabus outcomes and marking guide
(a) $\log_b a^m \div \log_m a$ $= m\log_b a \div \dfrac{\log_b a}{\log_b m}$ $= m\log_b a \times \dfrac{\log_b m}{\log_b a}$ $= m\log_b m$	H3, H9 • Gives the correct answer .... 2 • Uses the change of base law 1
(b) $\alpha + \beta = -\dfrac{b}{a}$ $\quad \alpha\beta = \dfrac{c}{a}$ $3 = -\dfrac{k}{2}$ $\quad -5 = \dfrac{D}{2}$ $k = -6$ $\quad D = -10$	P4 • Gives the correct answers .... 2 • Gives the correct answer for either D or k. 1
(c) (i) $6 - x = 6x - x^2$ $x^2 - 7x + 6 = 0$ $(x-1)(x-6) = 0$ $\therefore x = 1$ or 6 $y = 6 - 1, 6 - 6$ Therefore, the points are (1, 5) and (6, 0).	P4, H9 • Gives the correct answers 1
(ii) $\int_1^6 [6x - x^2 - (6 - x)]dx$ $= \int_1^6 (7x - x^2 - 6)dx$ $= \left[\frac{7}{2}x^2 - \frac{1}{3}x^3 - 6x\right]_1^6$ $= 7 \times 18 - \frac{1}{3} \times 6 \times 36 - 36 - \left(\frac{7}{2} - \frac{1}{3} - 6\right)$ $= 20\frac{5}{6}$ square units	H8 • Gives the correct solution .... 3 • Makes significant progress .... 2 • Gives the correct expression for area or equivalent merit 1
(d) (i)  length for 1 blade = $2 \times 1.2 + 1.2 \times 0.15 = 2.58$ m length for 5 blades = 12.9 m	H4 • Gives the correct answer .... 2 • Gives the correct length for one blade OR • Gives 0.18×5 as the length for the five arcs 1
(ii) area $= 10 \times \frac{1}{2} \times (120)^2 \times 0.15$ $= 10\,800$ cm^2 quantity $= 10\,800 \div 400 \times 100$ mL $= 2.7$ L	H4 • Gives the correct quantity .... 2 • Gives the correct area or equivalent merit 1

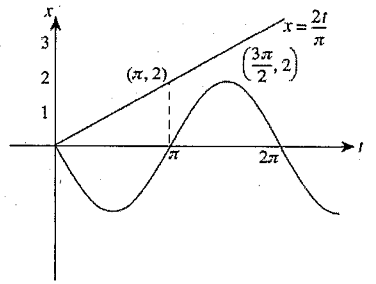
Question 6

Sample answer	Syllabus outcomes and marking guide
(a) $\sin\theta = \dfrac{-\sqrt{3}}{2}$  $\theta = \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3} = \frac{4\pi}{3}, \frac{5\pi}{3}$	H5 • Gives the correct solutions............ 2 • Gives one correct solution 1
(b) 	H7 • Gives a correct sketch which shows all features.......................... 3 • Gives a sketch which shows two correct features.......................... 2 • Gives a sketch showing one correct feature 1
(c) (i) $A = \frac{h}{3}\{(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2)\}$ $= \frac{3}{3}\{0 + 0 + 4(1.7 + 2.6) + 2 \times 2.1\}$ $= 21.4$ m^2	H8, H9 • Gives the correct answer .... 2 • Gives a correct evaluation for h or equivalent merit..................... 1
(ii) $V = Ah = 21.4 \times 25 = 535$ m^3	H8 • Gives the correct answer 1
(d) (i)  Let $\angle BAP = \beta$, $\angle ABP = \alpha$ $\therefore \angle MBP = \alpha$ $\angle PAK = \beta$ (given BK and PA bisect $\angle ABM$ and $\angle BAK$ respectively) Now $2\alpha + 2\beta = 180°$ (co-int angles $BM \parallel AK$ $ABCD$ parm) $\therefore \alpha + \beta = 90°$ $\therefore \angle BPA = 90°$ (angles in a triangle add to 180°)	P2, P4 • Gives the correct proof............... 2 • Makes some progress................ 1
(ii) In ΔBAP and ΔBPM, $\angle BPA = \angle BPM = 90°$ (proved in (i)) BP is common $\angle ABP = \angle MBP$ (given PB bisects ΔABC) $\therefore \Delta ABP \equiv MPB$ (AAS) $\therefore AB = BM$ (corresponding sides in congruent triangles)	P2, P4 • Gives the correct proof............... 2 • Makes some progress (e.g. establishes congruency without reasons).......... 1

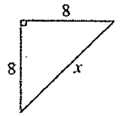
Question 7

Sample answer	Syllabus outcomes and marking guide
(a) Let $k = e^x$ $k^2 - 28k + 27 = 0$ $(k-27)(k-1) = 0$ $\therefore k = 27$ or 1 Hence, $e^x = 27$ or $e^x = 1$ $x = \log_e 27$ or $x = 0$	H3 • Gives the correct solutions . . . . 3 • Reduces equation to quadratic, and correctly factorises and solves for k. . . . . 2 • Reduces equation to a quadratic or equivalent merit 1
(b) (i) $C = C_0 e^{kt}$ $\frac{dC}{dt} = k \times C_0 e^{kt}$ $= kC$ as required	H3 • Gives the correct proof. 1
(ii) $t = 3$ $C = 2.45$ $t = 3.5$ $C = 1.84$ $2.45 = C_0 e^{3k} \Rightarrow C_0 = \frac{2.45}{e^{3k}}$ $1.84 = C_0 e^{3.5k} \Rightarrow C_0 = \frac{1.84}{e^{3.5k}}$ $\frac{2.45}{e^{3k}} = \frac{1.84}{e^{3k} \times e^{0.5k}}$ $e^{0.5k} = \frac{1.84}{2.45}$ $0.5k = \log_e \frac{1.84}{2.45}$ $k = 2\log_e \frac{1.84}{2.45}$ $= -0.5726$ $\therefore C_0 = \frac{2.45}{e^{3 \times -0.2487}}$ $= 13.65$ mg/L	H3, H4 • Gives the correct solution (ignore rounding). . . . . 3 • Makes significant progress . . . . 2 • Establishes two values for C_0 or equivalent merit 1
(iii) $t = ?$ $C = 0.5$ $0.5 = 13.65 \times e^{-0.5726t}$ $0.03663 = e^{-0.5726t}$ $\therefore t = \log_e 0.09677 \div -0.5726$ $= 5.78$ hours $\therefore$ after 5.78 hours	H3, H4 • Gives the correct answer 1

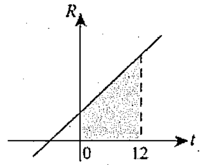
Question 7 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) (i) $t = 0, x = 0$ $v_1 = \frac{2}{\pi}$ $v_2 = -2\cos t$ $\therefore x_1 = \frac{2t}{\pi} + C_1$ $x_2 = -2\sin t + C_2$ when $t = 0, x = 0 \Rightarrow C_1 = 0$ when $t = 0, x = 0 \Rightarrow C_2 = 0$ $\therefore x_1 = \frac{2t}{\pi}$ $\therefore x_2 = -2\sin t$	H4, H5 • Gives the correct answers . . . . 2 • Gives one correct answer. 1
(ii)  The graphs don't intersect again. $x = \frac{2t}{\pi}$ has a value greater than 2 for $x > \pi$, and the maximum value of $x = -2\sin t$ is 2.	H2, H4, H5 • Gives the correct justification and explanation. . . . . 2 • Draws graphs, with no justification or explanation. 1

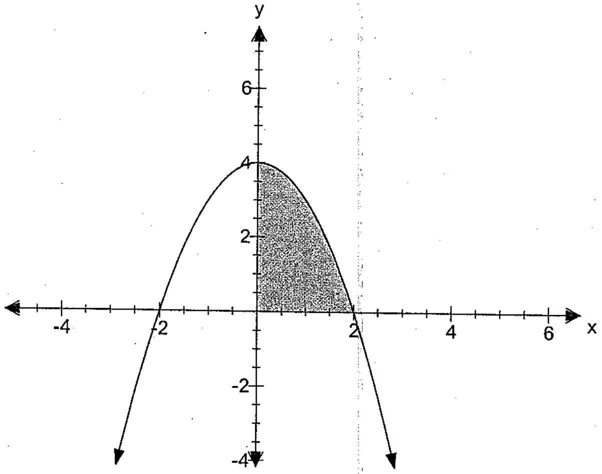
Question 8

		Sample answer	Syllabus outcomes and marking guide
(a)	(i)	At $t = 2$, because $v = 0$.	H4, H5 • Gives the correct answer 1
	(ii)	Total distance covered equals the area under the v–t graph. Therefore, the distance covered $= \frac{1}{2} \times 5 \times 2 + \frac{1}{2} \times 6 \times 4$ $= 5 + 12$ $= 17$ m	H4, H5, H8 • Gives the correct distance .... 2 • Correctly calculates one area or equivalent merit.................... 1
	(iii)	7 metres on the positive side of the starting position.	H4, H5 • Gives the correct answer 1
(b)	(i)	 $x^2 = 64 + 64$ $= 64 \times 2$ $x = 8\sqrt{2}$ area of $PQRS = (8\sqrt{2})^2$ $= 128$ cm^2	H4, H5 • Gives the correct proof and the correct area......................... 2 • Gives one correct proof 1
	(ii)	The areas are 256, 128, ... This is a geometric sequence: $a = 256$, $r = \frac{1}{2}$. $T_{10} = ar^9$ $= 256 \times \left(\frac{1}{2}\right)^9$ $= \frac{1}{2}$ cm^2	H5 • Gives the correct answer .... 2 • Identifies the correct $r = \frac{1}{2}$ 1
	(iii)	The perimeters are 64, $32\sqrt{2}$, 32 ... $T_n = \sqrt{2} = 64 \times \left(\frac{1}{\sqrt{2}}\right)^{n-1}$ $(\sqrt{2})^n = 64$ $2^{\frac{1}{2}n} = 2^6$ $n = 12$	H5 • Gives the correct answer .... 2 • Determines a correct equation, or solves an incorrect, non-trivial exponential equation for n...................... 1
	(iv)	areas $= 256, 128 ..., r = \frac{1}{2}$ $S_\infty = \frac{a}{1 - \frac{1}{2}}$ $= 2a$ The sum of the areas of all the squares is twice the area of the original square.	H5 • Gives the correct answer .... 2 • Uses limiting sum 1

Question 9

		Sample answer	Syllabus outcomes and marking guide
(a)	(i)		P4, H4, H9 • Gives the correct answer (12 must be shown)......................... 1
	(ii)	$\int_0^{12} (15 + 2t)\,dt = [15t + t^2]_0^{12}$ $\frac{15+39}{2} \times 12$ $= 15 \times 12 + 12^2$ $= 324$ tonnes OR area $= \frac{15+39}{2} \times 12$ $= 324$ tonnes	H4, H8, H9 • Gives the correct answer .... 3 • Makes significant progress .... 2 • Makes limited progress 1
(b)	(i)	Let a_n = amount in the account at the end of n years immediately before the next addition. $a_1 = 5000(1.09)$ $a_2 = [5000(1.09) + 5000](1.09)$ $= 5000(1.09)^2 + 5000(1.09)$ $= 5000[(1.09)^2 + 1.09]$ $a_3 = [5000\{(1.09)^2 + 1.09\} + 5000](1.09)$ $= 5000[1.09^3 + 1.09^2 + 1.09]$ $a_n = 5000[1.09^n + 1.09^{n-1} ... 1.09]$ $\therefore a_{10} = 5000[1.09^{10} + 1.09^9 ... 1.09]$	H5, H9 • Gives the correct demonstration .... 2 • Makes some progress................ 1
	(ii)	$A_{10} + 5000$ $= 5000 + 5000 \times \frac{1.09(1.09^{10} - 1)}{1.09 - 1}$ $= \$87\ 801.46$ Beth has almost \$88 000 in her account.	H5, H9 • Gives the answer \$87 801.46.......... 2 • Uses the sum of geometric series....... 1
	(iii)	In 20 more years, Beth will have $5000 \times \frac{1.09(1.09^{30} - 1)}{1.09 - 1} = \$742\ 876.09$ Cathy $742\ 876.09 = A \times \frac{1.09(1.09^{20} - 1)}{1.09 - 1}$ $= A \times 55.7645$ $\therefore A = 13\ 321.66$ Cathy will need to invest \$13 321.66 each year.	H5, H9 • Gives the answer \$13 321.66.......... 3 • Makes significant progress .... 2 • Makes some progress (e.g. determines \$742 876.09) 1
	(iv)	\$13 321.66 × 20 – \$5000 × 30 = \$116 433.19 Cathy will have to invest \$116 433.19 more than Beth.	• Gives the correct answer (accept correct from previous answer)............... 1

Question 10

Sample Answer	Marking Guide
(i) 	2 for correct region 1 for parabola
(ii) $V=\pi\int_0^2 y^2\,dx$ where $y=4-x^2$ so $y^2=16-8x^2+x^4$ $=\pi\int_0^2 16-8x^2+x^4dx$ $=\pi\left[16x-\frac{8}{3}x^3+\frac{1}{5}x^5\right]_0^2$ $=\frac{256\pi}{15}$	4 for correct answer 3 for correct primitive prior to evaluation 2 for correct integral with integrand in terms of x. 1 for formula with correct limits of integration.

Question 10 (Continued)

(b) (i)

Sample answer:

$kA+kP_1=\text{length of }AP$

$kA+kP_1=12$

$kP_1=12-kA$

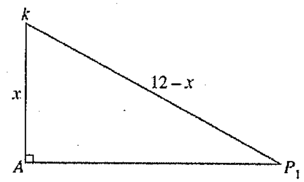
$=12-x$

Syllabus outcomes and marking guide:

H2
- Gives the correct explanation. 1

(ii)

Sample answer:



$(AP_1)^2=(12-x)^2-x^2$

$=4(36-6x)$

$AP_1=2\sqrt{36-6x}$

$\text{area}=\frac{1}{2}\times Ak\times AP_1$

$=\frac{1}{2}x\times2\sqrt{36-6x}$

$=x\sqrt{36-6x}$

Syllabus outcomes and marking guide:

H2, H4
- Gives the correct demonstration 2
- Makes progress (e.g. shows $(AP_1)^2=4(36-6x)$) 1

(iii)

Sample answer:

For a maximum $\frac{dA}{dx}=0$

$\frac{dA}{dx}=\sqrt{36-6x}+\frac{1}{2}\times x(36-6x)^{-\frac{1}{2}}\times-6=0$

$\sqrt{36-6x}-\frac{3x}{\sqrt{36-6x}}=0$

$36-6x-3x=0$

$9x=36$

$x=4$

test in the first derivative

x	3	4	5
y'	$\frac{9}{\sqrt{18}}$ ↗	0 —	$\frac{-9}{\sqrt{6}}$ ↖

Therefore it is a maximum.

When $x=4$, which is $\frac{1}{3}$ of AP.

The area of the triangle is a maximum.

Syllabus outcomes and marking guide:

H4, H5, H9
- Gives the correct proof. 3
- Makes significant progress 2
- Makes some progress, e.g. equates the correct expression for $\frac{dA}{dx}$ to 0 1

CRANBROOK
SCHOOL

Term 3, 2008

Year 12 Mathematics
Trial Examination

Wednesday July 23, 2008

Time Allowed: 3 hours, plus 5 minutes reading time

Total Marks: 120

There are 10 questions, all of equal value
Submit your work in ~~twelve~~ ten 4 Page Booklets.
All necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.
Board of Studies approved calculators may be used.
A list of standard integrals is attached to the back of this paper.

Total marks available – 120
Attempt all questions

Question 1 (12 marks) **Marks**

(a) Evaluate $\dfrac{2}{8+2\times(8-1)}$ correct to 4 significant figures. **2**

(b) Solve $|x-1|\le 2$ and graph the solution on a number line. **2**

(c) Simplify $\dfrac{2}{x(x-3)}-\dfrac{1}{x}$. **2**

(d) Solve $x^2-3=3x+1$. **2**

(e) Integrate $\dfrac{-1}{\sqrt{x}}$. **2**

(f) Sketch the graph of $y=-x+2$ on a set of axes, showing any x and y intercepts. **2**

Question 2 (12 marks) **Marks**

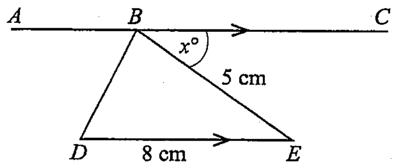
(a) Differentiate:

(i) $\cos\left(1-x^3\right)$ **2**

(ii) $\dfrac{x+1}{e^x}$. **2**

(b) If α and β are the roots of $2x^2-3x+7=0$, find the value of $\alpha^2+\beta^2$ **2**

(c)



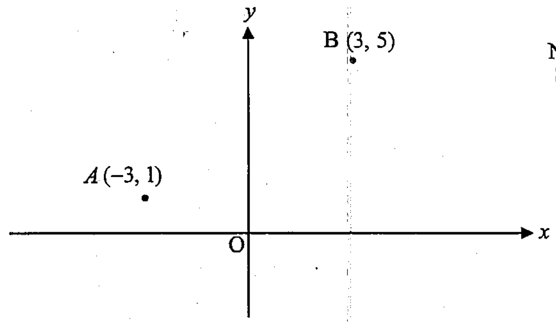
In the diagram, AC is parallel to DE, $BE = 5$ cm, $DE = 8$ cm and $\angle CBE = x°$. The area of triangle BDE is 10 square cm. Find the value of x, giving reasons for your answer. **3**

(d) Find the equation of the tangent to the curve $y = 2\sqrt{x}$ at the point (1, 2). **3**

Question 3 (12 marks) **Marks**

(a) Find the exact value of $\displaystyle\int_2^4 \frac{2}{x-1}\,dx$. **2**

(b)



NOT TO SCALE

The diagram shows the points $A(-3,1)$ and $B(3,5)$ on the Cartesian plane. Copy or trace this diagram onto your writing page.

(i) Show that the equation of AB is $2x-3y+9=0$. **2**

(ii) Show that the point C, which is the midpoint of AB is the y-intercept of AB. **1**

(iii) Calculate the perpendicular distance from the point $D(2,0)$ to the line AB and mark the point D on your diagram. **2**

(iv) The point E, lies on the line $y=-1$ and the line BE is perpendicular to the line AB. Show that E has the coordinates $(7,-1)$ and mark point E on your diagram. **2**

(v) Show that $BCDE$ is a trapezium. **1**

(vi) Find the area of $BCDE$. **2**

Question 4 (12 marks) **Marks**

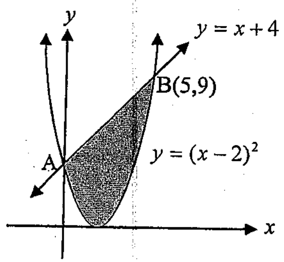
(a) If $\log_x 128 = \frac{7}{3}$, find x. 1

(b) (i) Sketch the graph of $y = 5\cos\frac{x}{2}$ for $-360° \le x \le 360°$. 2

(ii) Mark clearly on your graph the point or points where $5\cos\frac{x}{2} = -1$. 1

(iii) Calculate the value(s) of x which satisfy the equation $5\cos\frac{x}{2} = -1$. Express your answer(s) to the nearest minute. 2

(c)



NOT TO SCALE

The graphs of $y = (x-2)^2$ and $y = x + 4$ intersect at the point A and the point $B(5,9)$.

(i) Show that the point A lies on the y-axis. 2

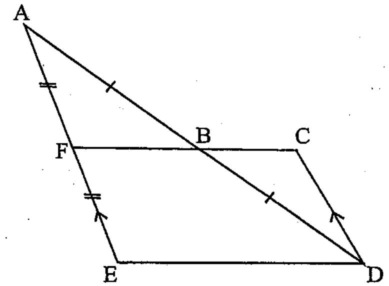
(ii) Write down the two inequalities whose intersection describes the shaded area shown in the diagram above. 1

(iii) Find the area of the shaded region bounded by the graphs of $y = (x-2)^2$ and $y = x + 4$. 3

Question 5 (12 marks) **Marks**

(a) Sketch the graph of the function $y = \frac{1}{x+1}$ and state the domain and the range of the function. 3

(b)



NOT TO SCALE

In the diagram, the line FC bisects AE at F and AD at B.
The line AE is parallel to CD.
Copy the diagram onto your working page.

(i) Explain why $ED = 2BF$. 1

(ii) Prove that $\Delta ABF \equiv \Delta DBC$. 3

(c) Organizers of a music festival issued 750 tickets in the first year of the festival. The number of tickets issued increased by 150 each year after that.

(i) How many tickets were issued in the fifteenth year of the festival? 1

(ii) In the first 20 years that the festival ran, what was the total number of tickets issued? 2

(iii) In which year of the festival did the number of tickets issued for that year first exceed 5000? 2

Question 6 (12 marks) **Marks**

(a) Find the equation of the parabola which has its vertex at (2,0) and its directrix is given by $x = 5$. **2**

(b) The number of subscribers S, to a pay-TV company t years after its launch is given by

$$S = S_0 e^{kt}$$

where S_0 and k are constants. Initially the pay TV company had 50 000 subscribers and after 3 years it had 200 000.

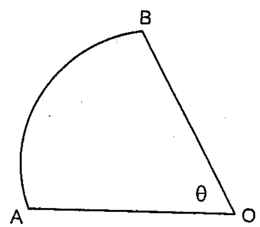
(i) Find the value of S_0. **1**

(ii) Find the value of k. Express your answer correct to 4 decimal places. **2**

(iii) After how many years will the number of subscribers first exceed one million? Express your answer correct to 1 decimal place. **2**

(iv) After 3 years, what is the rate at which the number of subscribers is increasing? Express your answer to the nearest whole number. **1**

(c) The area of a sector AOB of a circle centre O, radius 8cm, is $56\,cm^2$.



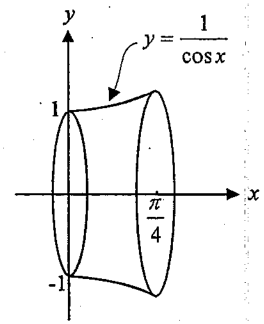
(i) Calculate the length of the minor arc AB **2**

(ii) The straight edges OA and OB are joined to form a cone. Find the exact value of the base radius of the cone. **2**

Question 7 (12 marks) **Marks**

(a) For the function $f(x)$ over the domain $0 \le x \le 5$, it is the case that $f'(x) > 0$ and $f''(x) < 0$.
Sketch a graph which could be that of $y = f(x)$ over this domain. **2**

(b) **4**



The graph of $y = \dfrac{1}{\cos x}$ between $x = 0$ and $x = \dfrac{\pi}{4}$ is rotated around the x-axis.
Find the volume of the solid of revolution.

(c) A particle moves in a straight line so that its displacement x, in metres from a fixed origin at time t seconds is given by

$$x = \log_e(t+1), \qquad t \ge 0$$

(i) Find the initial position of the particle. **1**

(ii) Explain how many times the particle is at the origin. **1**

(iii) Find an expression for the velocity and the acceleration of the particle. **2**

(iv) Explain whether or not the particle is ever at rest. **2**

Question 8 (12 marks) **Marks**

(a) Use Simpson's rule with 3 function values to find an approximate value of **2**

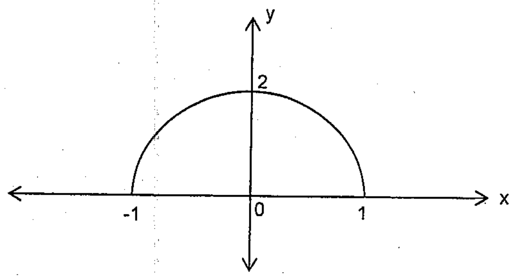
$$\int_0^2 \frac{5}{9-x^2}\,dx.$$

(b) Consider the function $y = x\ln x - x$, for $x > 0$.

(i) Find the x-intercept of the graph of the function. **1**

(ii) Find the coordinates of the turning point of the graph of the function. **3**

(c) An ornamental arch window 2 metres wide and 2 metres high is to be made in the shape of either a cosine curve or a parabola as illustrated on the axes below.



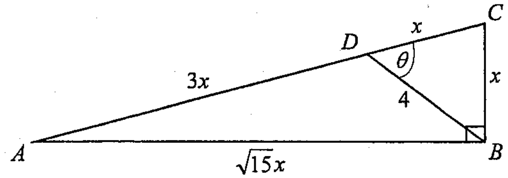
(i) If the arch is made in the shape of the curve $y = 2\cos\dfrac{\pi x}{2}$, find the exact area of the window. **2**

(ii) If the arch is made in the shape of a parabola, find the equation of the parabola. **2**

(iii) Hence find the area of this parabolic window. **2**

Question 9 (12 marks) **Marks**

(a)



NOT TO SCALE

In the diagram, ABC is a right angled triangle where $AB = \sqrt{15}x$ cm and $BC = x$ cm. The point D lies on AC and $CD = BC = x$ cm, $AD = 3x$ cm and $BD = 4$ cm. Let $\angle BDC = \theta$.

(i) Use the cosine rule to show that $\cos\theta = \dfrac{2}{x}$. **1**

(ii) Use the sine rule in triangle BCD to show that $\sin\theta = \dfrac{\sqrt{15}x}{16}$. **2**

(iii) Hence show that $15x^4 - 256x^2 + 1024 = 0$. **1**

(iv) Explain why one of the solutions to the equation in part (iii), namely $x = 2.53$ (to 2 decimal places), could not be the value of x indicated in the diagram above. **1**

(b) Gayle has a superannuation fund, which pays 5% per annum interest compounding annually. Gayle pays \$12 000 into the fund on 1 July each year.

(i) What is the value of Gayle's superannuation fund on 30 June one year after she makes her first payment? **1**

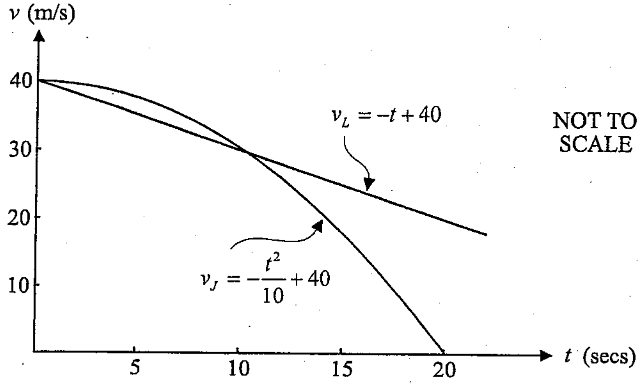
(ii) What is the value of Gayle's superannuation fund on 30 June ten years after she makes her first payment? **3**

(iii) After making her tenth payment, Gayle considers increasing her payment to M dollars per year. **3**
Show that if Gayle does this, then the value of her superannuation fund twenty years after her first payment of \$12 000 was made, would be approximately given by

$$13\cdot2068\left(12\ 000\times1\cdot05^{10}+M\right).$$

Question 10 (12 marks) **Marks**

(a) Larry and Jack are each speeding down a straight stretch of freeway and are side by side, when they spot a police car. They each brake. The velocity of Larry's car during this braking phase is given by $v_L = -t + 40$ and the velocity of Jack's car during this phase is given by $v_J = \dfrac{-t^2}{10} + 40$.



(i) When are the velocities the same during this braking phase? **1**

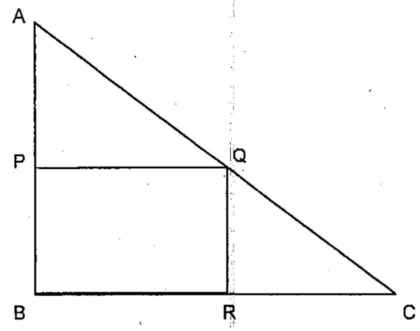
(ii) When are the two cars level with one another during this braking phase? **2**

(iii) State the times when Larry's car is further ahead of Jack's car during this braking phase? Give reasons for your answer. **1**

(b) Find the values of m for which the equation $x^2 + (m-2)x + 4 = 0$ has real roots. **3**

Question 10 continues on the next page.

(c) In ΔABC, AB = $20m$, BC = $15m$ and $\angle ABC = 90^0$. BPQR is a rectangle inscribed in ΔABC. PQ = x metres.
Copy and complete the diagram, showing all information given.



(i) Using similar triangles, or otherwise, find an expression for the length of AP in terms of x. **1**

(ii) Show that the area of the rectangle BPQR is given by **1**

$$A = x\left(20 - \frac{4}{3}x\right) \text{ square metres.}$$

(iii) Hence find the minimum possible area of the rectangle BPQR. **3**

END OF PAPER

12 4U Trial 2008

Question 1

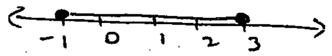
a) 0.09091 ✓✓ (Give 1 mark if not correct to 4 sig figs) (to 4 sig figs)

b) $x-1 \le 2$ $\quad$ $-x+1 \le 2$

$x \le 3$ $\quad$ $-x \le 1$

$x \ge -1$

$\therefore -1 \le x \le 3$ ✓



✓

c) $\dfrac{2}{x(x-3)} - \dfrac{1}{x}$

$= \dfrac{2-(x-3)}{x(x-3)}$ ✓

$= \dfrac{2-x+3}{x(x-3)}$

$= \dfrac{5-x}{x(x-3)}$ ✓

d) $x^2 - 3 = 3x + 1$

$x^2 - 3x - 4 = 0$ ✓

$(x-4)(x+1) = 0$

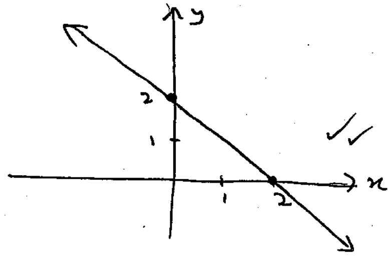
$x = 4,\ x = -1$ ✓

e) $\int \dfrac{-1}{\sqrt{x}}\,dx = \int -x^{-\frac{1}{2}}\,dx$ ✓

$= -2x^{\frac{1}{2}} + c$

$= -2\sqrt{x} + c$ ✓ (+c is not necessary)

f) $y = -x + 2$



✓✓

Question 2

a) (i) $y' = 3x^2 \sin(1-x^3)$ ✓

(ii) $y' = \dfrac{e^x \cdot 1 - (x+1)\cdot e^x}{e^{2x}}$ ✓

$= \dfrac{1-x-1}{e^x}$

$= \dfrac{-x}{e^x}$ ✓

b) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓

$= \left(\frac{3}{2}\right)^2 - 2\left(\frac{7}{2}\right)$

$= -4.75$ ✓

c) $\angle BED = x^\circ$ alternate ∠'s are equal as $AC \parallel DE$ ✓

Area $= \frac{1}{2}ab\sin C$

$10 = \frac{1}{2} \times 8 \times 5 \times \sin x^\circ$ ✓

$\sin x^\circ = \frac{1}{2}$

$x^\circ = 30^\circ$ ✓

d) $y = 2\sqrt{x}$

$y = 2x^{\frac{1}{2}}$

$\dfrac{dy}{dx} = x^{-\frac{1}{2}} = \dfrac{1}{\sqrt{x}}$ ✓

at $x = 1$ $\dfrac{dy}{dx} = 1$ ✓

$\therefore$ eqn. of tangent is:

$y - 2 = 1(x-1)$

$y = x + 1$ ✓

Question 3

a) $\int_2^4 \dfrac{2}{x-1}\,dx$

$= \left[2\ln(x-1)\right]_2^4$ ✓

$= 2\ln 3 - 2\ln 1$

$= 2\ln 3$ ✓

b) (i) $m_{AB} = \dfrac{5-1}{3+3}$

$= \dfrac{4}{6} = \dfrac{2}{3}$ ✓

$y - 1 = \frac{2}{3}(x+3)$ ✓

$3y - 3 = 2x + 6$

$\therefore 2x - 3y + 9 = 0$

(ii) M.P. of $AB = \left(\dfrac{-3+3}{2}, \dfrac{1+5}{2}\right)$

$= (0, 3)$ ✓

y int of $2x - 3y + 9 = 0$

sub $x = 0$ $\quad -3y + 9 = 0$

$3y = 9$

$y = 3$

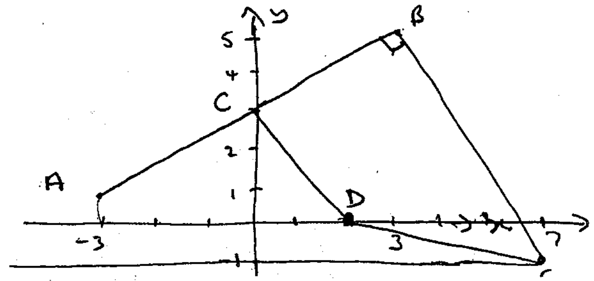
$\therefore C$ is midpt of AB and y int of line

(iii) perp d $= \left|\dfrac{2x-3y+9}{\sqrt{2^2+(-3)^2}}\right|$ ✓

$= \dfrac{|2(2)-3(0)+9|}{\sqrt{13}}$

$= \dfrac{13}{\sqrt{13}}$ ✓

$= \sqrt{13}$ units



(iv) $m_1 m_2 = -1$

Eqn. of BE

$y - 5 = -\frac{3}{2}(x-3)$

$2y - 10 = -3x + 9$

$3x + 2y - 19 = 0$

For E sub $y = -1$

$\therefore 3x + 2(-1) - 19 = 0$ ✓

$3x - 21 = 0$

$3x = 21$

$x = 7$

$\therefore E(7, -1)$ as required.

(v) BCDE is a trapezium if $BE \parallel C$

$m_{BE} = -\frac{3}{2}$

$m_{CD} = \dfrac{0-3}{2-0} = -\dfrac{3}{2}$

$\therefore m_{BC} = m_{CD}$ ✓

$\therefore BC \parallel CD$

$\therefore$ BCDE is a trapezium

(vi) $CD = \sqrt{2^2 + 3^2}$

$= \sqrt{13}$

$BE = \sqrt{(7-3)^2 + (-1-5)^2}$

$= \sqrt{16 + 36}$

$= \sqrt{52}$

$= 2\sqrt{13}$

$CB = \sqrt{3^2 + (5-3)^2}$

$= \sqrt{13}$

✓

Area $= \frac{1}{2}h(a+b)$

$= \frac{1}{2}\sqrt{13}\left(\sqrt{13} + 2\sqrt{13}\right)$

$= 19.5$ units^2 ✓

Question 4

a) $\log_x 128 = \frac{7}{3}$

$128 = x^{\frac{7}{3}}$

$x = \sqrt[7]{128}^{\,3}$

$= 2^3$

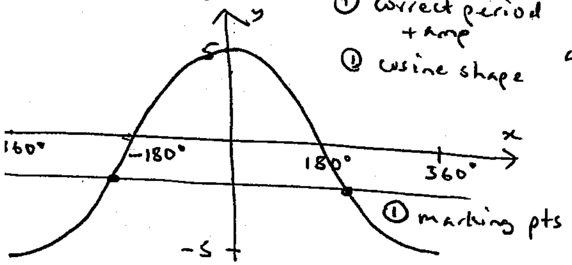
$= 8$ ✓

b)(i) $y = 5\cos\frac{x}{2} \quad -360° \le x \le 360°$

Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

① correct period + amp

① cosine shape



① marking pts

(ii) $5\cos\frac{x}{2} = -1$

$\cos\frac{x}{2} = -\frac{1}{5}$

$\frac{x}{2} = \cos^{-1}\left(-\frac{1}{5}\right)$ ✓

$\frac{x}{2} = 101°32', -101°32'$

$x = 203°4', -203°4'$ ✓

c) (i) $(x-2)^2 = x+4$

$x^2 - 4x + 4 = x + 4$

$x^2 - 5x = 0$ ✓

$x(x-5) = 0$

$x = 0, \quad x = 5$

$y = 4 \quad y = 9$

A is $(0,4)$ which is on y axis ✓
x coord is zero.

(ii) $y \le x+4$ and $y \ge (x-2)^2$

① for both correct

(iii) $A = \int_0^5 x+4-(x-2)^2\,dx$ ✓

$= \int_0^5 x+4-(x^2-4x+4)\,dx$

$= \int_0^5 5x - x^2\,dx$

$= \left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_0^5$ ✓

$= 62\frac{1}{2} - \frac{125}{3}$

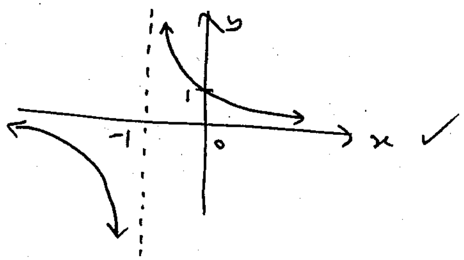
$= 20\frac{5}{6}$ units^2 ✓

Question 5

a) $y = \frac{1}{x+1}$

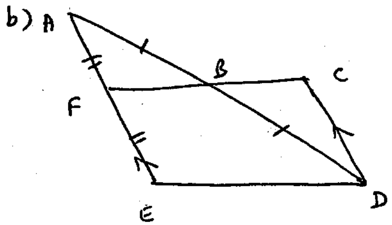
D: all x, $x \neq -1$ ✓

R: all y, $y \neq 0$ ✓



✓

b)



B is midpt of AD
F is midpt of AE
∴ BF joins 2 midpts
∴ BF is parallel to ED and half its length ✓
∴ $BF = \frac{1}{2}ED$
∴ $ED = 2BF$

(ii) In $\triangle ABF$ and $\triangle DBC$

$AB = BD$ given ✓

$\angle FAB = \angle BDC$ alt ∠'s equal AE∥CD

$\angle ABF = \angle CBD$ vert. opp ✓

∴ $\triangle ABF \equiv \triangle DBC$ by AAS ✓

c) 750, 900, 1050 . . .

$a = 750$

$d = 150$

i) $T_{15} = ?$

$T_n = a + (n-1)d$

$T_{15} = 750 + 14 \times 150$

$= 2850$ ✓

∴ 2850 tickets were issued

ii) $S_n = \frac{n}{2}(2a + (n-1)d)$ ✓

$S_{20} = 10(1500 + 19 \times 150)$

$= 43\,500$ ✓

iii) $T_n > 5000$

∴ $750 + (n-1)150 > 5000$

$750 + 150n - 150 > 5000$

$600 + 150n > 5000$

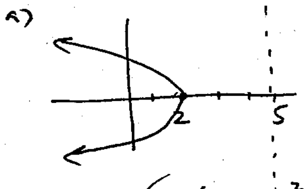
$150n > 4400$

$n > 29.3$ ✓

∴ in the 30th year ✓

Question 6

a)



$a = 3$ ✓ $(y-k)^2 = -4a(x-h)$

$(y-0)^2 = -12(x-2)$ ✓

$y^2 = -12x + 24$

b) $S = S_0 e^{kt}$

(i) $S_0 = 50\,000$ ✓

(ii) $200\,000 = 50\,000 e^{3k}$ ✓

$4 = e^{3k}$

$3k = \ln 4$

$k = \frac{1}{3}\ln 4$

$k = 0.4621$ (to 4 d.p.) ✓

(iii) $1\,000\,000 = 50\,000 e^{0.4621t}$ ✓

$20 = e^{0.4621t}$

$0.4621t = \ln 20$

$t = 6.48289$

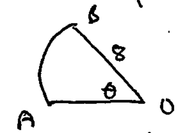
$t = 6.5$ years ✓

(iv) $\frac{dS}{dt} = 50\,000k\,e^{3k}$

$= 92419.6$

$= 92420$ subscribers / year ✓
(also give mark for 92421)

c)



$A = 56$

(i) $A = \frac{1}{2}r^2\theta$

$56 = \frac{1}{2}(8)^2\theta$

$\theta = 1.75$ ✓

$l = r\theta$

$= 8 \times 1.75$

$= 14$ cm ✓

(ii) arc length = circum of base ✓

$14 = 2\pi r$

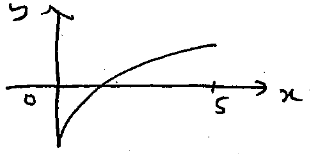
$r = \frac{14}{2\pi}$

$r = \frac{7}{\pi}$ cm ✓

Question 1

a) $f'(x) > 0$ increasing ✓

$f''(x) < 0$ concave down



✓

b) $V = \pi \int y^2 \, dx$

$= \pi \int_0^{\frac{\pi}{4}} \left(\frac{1}{\cos x} \right)^2 dx$ ✓

$= \pi \int_0^{\frac{\pi}{4}} \sec^2 x \, dx$ ✓

$= \pi \left[\tan x \right]_0^{\frac{\pi}{4}}$ ✓

$= \pi \left(\tan \frac{\pi}{4} - \tan 0 \right)$

$= \pi$ units^3 ✓

c) $x = \log_e (t+1)$

i) $t = 0 \quad x = \ln 1$

$x = 0$

∴ initial position is at the origin ✓

ii) $x = 0 \quad \log_e (t+1) = 0$

$t + 1 = e^0$

$t + 1 = 1$

$t = 0$

∴ particle only at origin once ✓

iii) $v = \frac{1}{t+1} = (t+1)^{-1}$ ✓

$a = -(t+1)^{-2}$

$= \frac{-1}{(t+1)^2}$ ✓

iv) at rest when $v = 0$

$\therefore \frac{1}{t+1} = 0$ ✓

$1 = 0$!!

∴ particle never at rest ✓

Question 8

a)

x	0	1	2
y	$\frac{5}{9}$	$\frac{5}{8}$	1

$h = 1$

$\therefore \int_0^2 \frac{5}{9 - x^2} dx \doteqdot \frac{1}{3} \left(\frac{5}{9} + 1 + 4 \left(\frac{5}{8} \right) \right)$ ✓

$= \frac{1}{3} \left(4 \frac{1}{18} \right)$

$= \frac{73}{54}$ ✓

b) $y = x \ln x - x$

(i) x int when $y = 0$

$\therefore x \ln x - x = 0$

$x(\ln x - 1) = 0$

$x = 0, \quad \ln x = 1$

$x = e$

but $x > 0$

∴ x int is $(e, 0)$ ✓

(ii) $\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x \cdot 1 - 1$

$= 1 + \ln x - 1$

$= \ln x$ ✓

turning pt when $\frac{dy}{dx} = 0$

$\therefore \ln x = 0$

$x = e^0$

$x = 1$ ✓

$y = -1$ ✓

∴ turning pt is $(1, -1)$

c) $A = 2 \int_0^1 2 \cos \frac{\pi x}{2} dx$ ✓

$= 4 \left[\frac{2}{\pi} \sin \frac{\pi x}{2} \right]_0^1$

$= 4 \left[\frac{2}{\pi} \sin \frac{\pi}{2} - 0 \right]$

$= \frac{8}{\pi}$ units^2 ✓

$y = ax^2 + 2$ ✓

sub $(1, 0)$

$0 = a + 2$

$a = -2$

$\therefore y = -2x^2 + 2$ ✓

iii) $A = 2 \int_0^1 -2x^2 + 2 \, dx$ ✓

$= 2 \left[-\frac{2x^3}{3} + 2x \right]_0^1$

$= 2 \left(-\frac{2}{3} + 2 \right)$

$= \frac{8}{3}$ units^2 ✓

Question 9

a) (i) $\cos \theta = \frac{x^2 + 4^2 - x^2}{2(x)(4)}$ ✓

$= \frac{16}{8x}$

$= \frac{2}{x}$

(ii) let $\angle BCD = \alpha$

$\therefore \frac{\sin \theta}{x} = \frac{\sin \alpha}{4}$

$\sin \theta = \frac{x \sin \alpha}{4}$ ✓

In $\triangle ABC$

$\sin \alpha = \frac{\sqrt{15} x}{4x} = \frac{\sqrt{15}}{4}$

$\therefore \sin \theta = \frac{x}{4} \cdot \frac{\sqrt{15}}{4}$ ✓

$\sin \theta = \frac{\sqrt{15} x}{16}$ as required

(iii) Now $\sin^2 \theta + \cos^2 \theta = 1$

$\left(\frac{\sqrt{15} x}{16} \right)^2 + \left(\frac{2}{x} \right)^2 = 1$ ✓

$\frac{15x^2}{256} + \frac{4}{x^2} = 1$

$15x^4 + 1024 = 256x^2$

$\therefore 15x^4 - 256x^2 + 1024 = 0$

as required

$\therefore \theta = 37°46'$

Also $\sin \alpha = \frac{\sqrt{15}}{4}$

$\alpha = 75°31'$

In $\triangle DCB$: $\theta + \theta + \alpha$ should be 180°

But $37°46' + 37°46' + 75°31'$

$= 151°3'$

$\neq 180°$ ✓

$\therefore x \neq 2.53$

b) $A_1 = 12000(1.05)$

(i) $= \$12\,600$ ✓

(ii) $A_1 + A_2 + \cdots + A_{10}$

$= 12000(1.05 + 1.05^2 + \cdots + 1.05^{10})$ ✓

$a = 1.05$

$r = 1.05$

$n = 10$ ✓

$S_n = \frac{1.05(1.05^{10} - 1)}{0.05}$

$= 12000 \times S_n$

$= \$158\,481.45$ ✓

(iii) $A_1 + A_2 + \cdots + A_{19} + A_{20}$

$= 12000(1.05^{20} + 1.05^{19} + \cdots + 1.05^{11})$

$+ M(1.05^{10} + 1.05^9 + \cdots + 1.05)$

$= 12000(1.05)^{10}(1.05^{10} + 1.05^9 + \cdots$

$+ M(1.05^{10} + 1.05^9 + \cdots + 1.05)$

$= \left(12000(1.05)^{10} + M \right)(1.05^{10} + \cdots$

$n = 10$

$a = 1.05$

$r = 1.05$

$S_{10} = \frac{1.05(1.05^{10} - \cdots}{0.05}$

$= \left(12000(1.05)^{10} + M \right)(13.2068 \cdots$

Question 10

) (i) $-t+40 = -\frac{t^2}{10} + 40$

$t = \frac{t^2}{10}$

$t^2 = 10t$

$t^2 - 10t = 0$

$t(t-10) = 0$

$t = 0, \quad t = 10$

locities are same initially and ter 10 seconds ✓

) The cars are level when their splacements are equal. Let is time be T secs

$\int_0^T -t+40\,dt = \int_0^T -\frac{t^2}{10} + 40\,dt$ ✓

$\left[-\frac{1}{2}t^2 + 40t\right]_0^T = \left[-\frac{t^3}{30} + 40t\right]_0^T$

$\frac{1}{2}T^2 + 40T = -\frac{T^3}{30} + 40T$

$30T^2 = 2T^3$

$T^3 - 15T^2 = 0$

$T^2(T-15) = 0$

$T = 0, \quad T = 15$

after 15 seconds cars are vel ✓

Cars are at same place fter 15 seconds. By looking at e graph we can see there more area under the curve ler Larry's graph after $t=15$ arry is further ahead after secs. ✓

b) Real roots $\Delta \geq 0$

$(m-2)^2 - 4(1)(4) \geq 0$ ✓

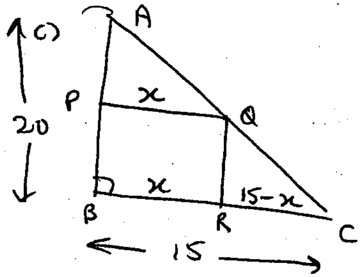
$m^2 - 4m + 4 - 16 \geq 0$

$m^2 - 4m - 12 \geq 0$

$(m-6)(m+2) \geq 0$ ✓

$\therefore m \leq -2, \; m \geq 6$ ✓

c)



(i) $\frac{AP}{20} = \frac{x}{15}$

$AP = \frac{20x}{15} = \frac{4x}{3}$ ✓

(ii) Area of rect $= PQ \cdot PB$ ✓

$= x \cdot \left(20 - \frac{4x}{3}\right)$

(iii) $A = 20x - \frac{4x^2}{3}$

$\frac{dA}{dx} = 20 - \frac{8x}{3}$

maximum when $\frac{dA}{dx} = 0$ and $\frac{d^2A}{dx^2} < 0$

$\therefore 20 - \frac{8x}{3} = 0$

$60 - 8x = 0$

$8x = 60$

$x = 7.5$ ✓

$\frac{d^2A}{dx^2} = -\frac{8}{3} < 0 \quad \therefore$ max as required ✓

$\therefore A = 20(7.5) - \frac{4}{3}(7.5)^2$

$= 75 \text{ m}^2$ ✓

Marker's Notes - BMM. 2U HSC TRIAL 2008.

Question 1.

1) A lot of variations to answers for this question. Students are reminded that 4 sig fig ≠ 4 dp.

) done well.

) Take care in algebraic fractions, particularly when subtracting. Remember to bracket any binomial numerators or denominators as this will make a difference when expanding.

) done well.

) done well.

) done well.

Question 2.

) (i) In $\cos(1-x^3)$ cos and $(1-x^3)$ are not 2 seperate fns! There were too many students that used the product rule here!

(ii) There were too many students who simplified incorrectly!

Can you see the error here?? $\frac{e^x - e^x(x+1)}{e^{2x}} \neq \frac{-e^x(x+1)}{e^x}$

Name: ______________________

Teacher: ______________________

Class: ______________________

FORT STREET HIGH SCHOOL

2009

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 4: TRIAL HSC

Mathematics

TIME ALLOWED: 3 HOURS
(PLUS 5 MINUTES READING TIME)

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1,2,6	
Manipulates algebraic expressions to solve problems from topic areas such as functions, quadratics, trigonometry, probability and logarithms	5,7,	
Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	3,4,10	
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	8,9	

Question	1	2	3	4	5	6	7	8	9	10	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/12	/12	/12	/120	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started in a new booklet

2009 TRIAL

QUESTION 1 (12 marks) Start a NEW booklet. **Marks**

(a) Factorise $16x^2-25$ **1**

(b) Find the value of $17^{-0.5}$ to two decimal places **2**

(c) Convert $\frac{4\pi}{5}$ radians to degrees **1**

(d) Simplify $\frac{x}{2}+\frac{3x-1}{3}$ **2**

(e) Evaluate $\int_1^2 4x+7\,dx$ **2**

(f) Express $0.\dot{2}\dot{3}$ as a fraction. Show working. **2**

(g) Solve $5-3x<9$ **2**

QUESTION 2 (12 marks) Start a NEW booklet. **Marks**

(a) For the points A (3,2) and B (-5,-5),

(i) Find gradient between A and B **1**

(ii) Find midpoint of A and B **1**

(iii) Find distance between A and B. Answer as a surd. **1**

(iv) Show that the equation of the line l through A and B is $7x-8y-5=0$ **2**

(v) Show that the point C (-3,4) does not lie on the line l **1**

(vi) Find the perpendicular distance from the line l to (-3,4) **2**

(b) Find the equation of the line through (2,3) and the point of intersection of $x+2y-3=0$ and $2x+3y-7=0$ **4**

QUESTION 3 (12 marks) Start a NEW booklet.

(a) Differentiate (i) $(x^2-1)^{11}$ **1**

(ii) $\tan(3x)$ **1**

(b) Find the equation of the tangent to the curve $y=xe^x$ at the point (1,e) **4**

(c) Differentiate $y=\frac{\sin x}{1+\cos x}$ **3**

and hence show that $\frac{dy}{dx}=\frac{1}{1+\cos x}$

(d) The curve $y=3x+\frac{a}{x^2}$ has a turning point at $x=3$. **3**

Find the constant a

QUESTION 4 **(12 marks)** Start a NEW booklet. **Marks**

(a) Find the primitives (i.e. indefinite integrals) of:

(i) e^{2x} **1**

(ii) $\sin 6x$ **1**

(b) Evaluate

(i) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x dx$ **2**

(ii) $\int_{9}^{13} \frac{dx}{x-7}$ **2**

(c) The following gives values of $f(x) = x \log x$

x	1	2	3	4	5
f(x)	0	1.39	3.30	5.55	8.05

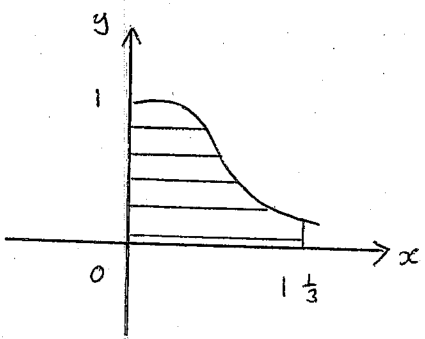
Use Simpson's rule with these five values to find an approximation to two decimals places of **3**

$$\int_{1}^{5} x \log x dx$$

(d) Find the area between the curve $y = \frac{1}{(1+3x)^2}$, **3**

the x-axis and the ordinates $x = 0$ and $x = 1\frac{1}{3}$ as

shown in the sketch below.



QUESTION 5 **(12 marks)** Start a NEW booklet. **Marks**

(a) (i) The co-ordinates of P are (2,1). Show that P lies on both the parabolas $4y = x^2$ and $4y = (x-4)^2$. Show that P is the only point of intersection of the two curves. **3**

(ii) Find the equation of the tangent at P to the parabola $4y = (x-4)^2$. **2**

(iii) Find the co-ordinates of the other point Q at which this tangent intersects the parabola $4y = x^2$ **3**

(b) The roots of $2x^2 - 3x - 7 = 0$ are α and β. Find:-

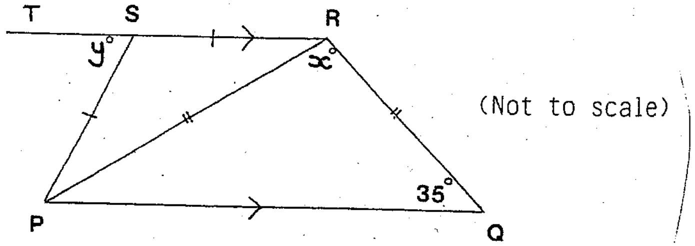
(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2+\beta^2$ **2**

QUESTION 6 (12 marks) Start a NEW booklet. **Marks**

(a)

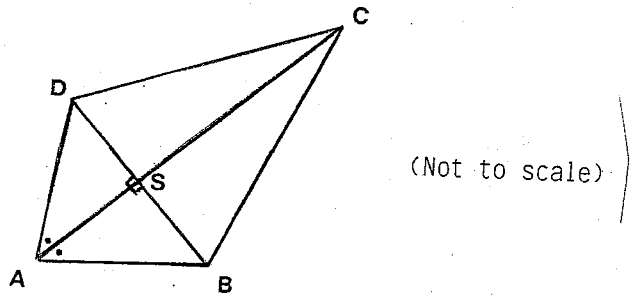


The diagram (not to scale) shows a quadrilateral PQRS, in which $PQ \parallel SR$, $PS = SR$, and $PR = RQ$. Also, T is a point on RS produced. Draw a neat sketch of this diagram in your answer book.

(i) Given that $\angle RQP = 35^0$, and $\angle PRQ = x^0$, find x, giving reasons. **3**

(ii) If also $\angle TSP = y^0$, find y, giving reasons **3**

(b)



In the diagram (not to scale), ABCD is a quadrilateral. The diagonals AC, BD interest at right angles, and $\angle DAS = \angle BAS$. Draw a neat sketch of the above diagram in your answer book.

(i) Explaining the reason for each step, use congruent triangles to prove that DA=AB. **3**

(ii) Hence prove that DC=CB **3**

QUESTION 7 (12 marks) Start a NEW booklet **Marks**

(a) Two ordinary dice, with the numbers 1 to 6 on their faces are thrown. What is the probability that:-

(i) they both show 6? **1**

(ii) they show a 1 and a 6? **1**

(iii) at least one of them shows a 1? **2**

(iv) they show a total of six? **1**

(b) On a destroyer there are two lines of defence against aircraft attack. These are a surface-to-air missile and a 15mm rapid-firing gun. The probability of success in hitting an attacking aircraft with each line of defence is respectively 0.9 and 0.8. Find the probability of hitting an attacking aircraft before it penetrates both defences. **3**

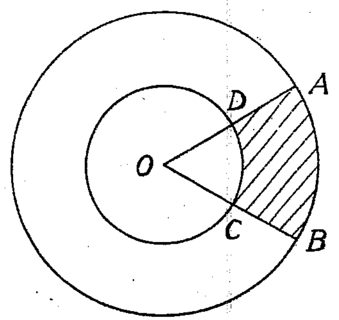
(c) Given $\log_2 3 = 1.58496$, find, correct to two decimal places:-

(i) $\log_2 9$ **2**

(ii) $\log_2 12$ **2**

QUESTION 8 **(12 marks)** Start a NEW booklet **Marks**

(a)



The diagram shows two concentric circles centre 0 and radii 20 cm and 10 cm respectively. ODA and OCB are straight lines and the angle between OA and OB is 60^0.

Find, correct to 3 significant figures:-

(i) the perimeter of the shaded region ABCD **2**

(ii) the area of the shaded region ABCD **2**

(b) From a point O the point P bears 120^0 from North and is 12.3 km away. The point Q is 15.2 km South West of O.

(i) Mark the relative positions of O, P, Q on a sketch. **1**

(ii) What is the size of $\angle POQ$? **1**

(iii) Calculate the distance PQ in kilometres (rounded off correct to one decimal place). **2**

(c) The area under the curve $y = \sqrt{9-x^2}, -3 \le x \le 3,$ is rotated about the x - axis. Find the volume of the solid of revolution thus obtained. Name the solid. **4**

QUESTION 9 **(12 marks)** Start a NEW booklet. **Marks**

(a) The first three terms of an arithmetic series are 50, 43, 36.

(i) Write down a formula for the nth term. **1**

(ii) If the last term of the series is -27, how many terms are there in the series? **1**

(iii) Find the sum of the series. **2**

(b) A loan of $1000 is to be repaid by equal annual instalments, repayments commencing at the end of the first year of the loan. Interest , at the rate of 10 per cent, is calculated each year on the balance before each repayment, and added to that balance.

If the annual instalment is P dollars, prove that:

(i) the amount owing at the beginning of the second year of the loan is (1100 - P) dollars. **2**

(ii) the amount owing at the beginning of the third year of the loan is (1210 - 2.1P) dollars **2**

(iii) if the loan (including interest charges) is exactly repaid at the end of n years, then **4**

$$P = \frac{100(1.1)^n}{(1.1)^n - 1}$$

QUESTION 10 (12 marks) Start a NEW booklet. **Marks**

(a) A function *f(x)* is defined by the rule

$$f(x) = 9x(x-2)^2$$

in the domain $-1 \le x \le 3$.

(i) Draw a sketch of the graph of $y = f(x)$, showing clearly the turning points, the intercepts with x and y axes, and the values at the end-points of the domain. **6**

(ii) What is the range of *f(x)?* **1**

(b) A cylindrical can is to hold a volume of 600cm^3.

(i) Show that the can's surface area can be expressed in terms of radius *r* as:- **1**

$$SA = \frac{1200}{r} + 2\pi r^2$$

(ii) Find the radius *r* and height *h* for the minimum surface area to hold a volume of 600cm^3. (Answer to 2 decimal places.) **4**

(For a cylinder $V = \pi r^2 h$, $SA = 2\pi rh + 2\pi r^2$)

END OF EXAMINATION

FORT STREET HIGH SCHOOL
TRIAL HSC 2009
MATHEMATICS 2U
SOLUTIONS

QUESTION ONE

(a) $16x^2 - 25 = (4x-5)(4x+5)$ ✓

(b) $17^{-0.5} = 0.242535625......$ ✓

$= 0.24$ (to 2dp) ✓

(c) $\frac{4\pi^c}{5} = \frac{4}{5} \times 180^\circ$

$= 144^\circ \quad (\pi^c = 180^\circ)$ ✓

(d) $\frac{x}{2} + \frac{3x-1}{3}$

$= \frac{3x}{6} + \frac{2(3x-1)}{6}$ ✓

$= \frac{3x + 6x - 2}{6}$

$= \frac{9x-2}{6}$ ✓

(e) $\int_1^2 (4x+7)\,dx$

$= \left[2x^2 + 7x\right]_1^2$ ✓

$= (8+14) - (2+7)$

$= 13$ ✓

Although a very basic integration, students had some difficulty.

(f) Let $x = 0.2323...$ ✓ (1)

$100x = 23.2323...$ (2)

$99x = 23 \quad (2) - (1)$

$x = \frac{23}{99}$ ✓

OR $0.\dot{2}\dot{3} = 0.23 + 0.0023 + 0.000023 ...$

∴ Infinite sum of a geometric progression

where $a = 0.23 \quad r = 0.01$

$S = \frac{a}{1-r} = \frac{0.23}{1-0.01}$ ✓

$= \frac{0.23}{0.99} = \frac{23}{99}$

ie $0.\dot{2}\dot{3} = \frac{23}{99}$ ✓

Usually well done, however, some students did not know how to proceed.

(g) $5 - 3x < 9$

$-3x < 9$ ✓

$x > -\frac{4}{3}$ ✓

QUESTION TWO

(a) (i) grad $AB = \frac{2-(-5)}{3-(-5)} = \frac{7}{8}$ ✓

(ii) midpoint $AB = \left(\frac{3+(-5)}{2}, \frac{2+(-5)}{2}\right)$

$= \left(-1, -\frac{3}{2}\right)$ ✓

(iii) distance $AB = \sqrt{(3-(-5))^2 + (2-(-5))^2}$

$= \sqrt{8^2+7^2}$

$= \sqrt{113}$ ✓

(iv) line thru' $(3, 2)$ with gradient $\frac{7}{8}$

$y - 2 = \frac{7}{8}(x-3)$ ✓

$8y - 16 = 7x - 21$ ✓

$7x - 8y - 5 = 0$

(v) Substitute $(-3, 4)$ into

$7x - 8y - 5 = 0$ ✓

$7 \times -3 - 8 \times 4 - 5 = -58 \neq 0$

$\therefore (-3, 4)$ does not lie on line ℓ.

(vi)

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \frac{|7 \times -3 + -8 \times 4 - 5|}{\sqrt{7^2+8^2}}$ ✓

$= \frac{|-58|}{\sqrt{113}}$

$= \frac{58}{13}$ ✓

Some students had difficulty with substitution into the formula.

(b) using "k" method.

required line is

$x + 2y - 3 + k(2x + 3y - 7) = 0$ ✓

substitute $(2, 3)$

$2 + 6 - 3 + k(4 + 9 - 7) = 0$

$5 + 6k = 0$

$k = -\frac{5}{6}$ ✓

substitute $k = -\frac{5}{6}$

$x + 2y - 3 - \frac{5}{6}(2x + 3y - 7) = 0$ ✓

$6x + 12y - 18 - 10x - 15y + 35 = 0$

$-4x - 3y + 17 = 0$

$4x + 3y - 17 = 0$ ✓

OR

$x + 2y - 3 = 0$ (1)

$2x + 3y - 7 = 0$ (2)

$2x + 4y - 6 = 0$ (1) $\times 2 =$ (3)

(3) $-$ (2) $y + 1 = 0$

$y = -1$

Substitute $y = -1$ in (1)

$x - 2 - 3 = 0$

$x = 5$ ✓

gradient from $(2, 3)$ to $(5, -1)$

$= \frac{3-(-1)}{2-5} = -\frac{4}{3}$ ✓

required equation

$y - 3 = -\frac{4}{3}(x - 2)$ ✓

$3y - 9 = -4x + 8$ ✓

$4x + 3y - 17 = 0$

Most students preferred to use the solution involving simultaneous equations.

corrections please.

QUESTION THREE

(a) (i) $\frac{d}{dx}(x^2-1)^{11}$

$= 11(x^2-1)^{10} \times 2x$

$= 22x(x^2-1)^{10}$ ✓

Some students forgot to write the power of 10.

(ii) $\frac{d}{dx}\tan(3x)$

$= 3\sec^2(3x)$ ✓

Some wrote x instead of $3x$

(b) (i) $y = xe^x$

$\frac{dy}{dx} = xe^x + e^x$ ✓

Some did not learn Product rule to differentiate $y = uv$

at $x = 1$ $\frac{dy}{dx} = e + e$

$= 2e$ ✓

equation of tangent through $(1, e)$ with gradient $= 2e$ is

ok

$y - e = 2e(x-1)$ ✓

$y - e = 2ex - 2e$

$y = 2ex - e$ ✓

(c) $y = \frac{\sin x}{1+\cos x}$

$\frac{dy}{dx} = \frac{(1+\cos x)(\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$ ✓

$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$

$= \frac{1+\cos x}{(1+\cos x)^2}$ $(\sin^2 x + \cos^2 x = 1)$ ✓

$= \frac{1}{1+\cos x}$ ✓

Some had problems differentiating $y = \frac{u}{v}$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

(6)

(d) $y = 3x + \frac{9}{x^2}$

$= 3x + ax^{-2}$

$\frac{dy}{dx} = 3 - \frac{2a}{x^3}$

At the turning point when $x = 3$

$\frac{dy}{dx} = 0$

$3 - \frac{2a}{3^3} = 0$

$3 = \frac{2a}{27}$

$2a = 3 \times 27$

$a = \frac{81}{2}$

$= 40\frac{1}{2}$

some had proble with the derivat of $y = x^{-2}$, $\frac{dy}{dx} = x^{-3}$ and

NOT x^{-1}

OK if they wer correct in $\frac{dy}{dx}$

QUESTION FOUR

(a) (i) $\int e^{2x}\,dx = \frac{1}{2}e^{2x} + c$ ✓

(1) Answer + c

[Lose 1 mark if "+c" omitted in either (i) or (ii)]

(ii) $\int \sin 6x\,dx = \frac{1}{6}\left(-\cos 6x\right) + c$

$= -\frac{1}{6}\cos 6x + c$ ✓

(1) Answer + c

[some mistakes with $\frac{1}{2}, \frac{1}{6}$ or $\ominus$]

(b) (i) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x\,dx = \left[\sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ ✓

1. Integration

[Some students wrote $\ominus\sin x$.]

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$ ✓

1. Subst. to get answer

(2)

(ii) $\int_{6}^{13} \frac{dx}{x-7} = \left[\log_e (x-7)\right]_{9}^{13}$ ✓

1. Integration

[zero marks if failed to recognise "Log Integral".]

$= \log_e 6 - \log_e 2$

[Some students were unable to simplify $\ln 6 - \ln 2$, making errors in evaluation.]

$= \log_e \frac{6}{2}$

$= \log_e 3$ ✓

$(= 1.0986\ldots)$

1. Simplified answer

(2) [Allow decimal if rounded correctly]

(c) $\int_1^5 x\log x\,dx \doteqdot \frac{3-1}{6}\left[f(1) + 4f(2) + f(3)\right]$

$+ \frac{5-3}{6}\left[f(3) + 4f(4) + f(5)\right]$

[Students using $\frac{h}{3}$ were more successful than those using $\frac{b-a}{6}$.]

$= \frac{1}{3}\left[f(1) + f(5) + 2f(3) + 4\left(f(4) + f(5)\right)\right]$ ✓

1. Using correct h in a correct version of Simpson's Rule.

$= \frac{1}{3}\left(0 + 8.05 + 2(3.30) + 4(1.39 + 5.55)\right)$ ✓

1. Substituting y-values into correct formula.

$= 14.1366\ldots$ ✓

$= 14.14$ (to 2 dec pls)

1. Answer

(3) [allow other rounding]

or use $h = 1$ (by inspection, or $\frac{b-a}{n} = \frac{5-1}{4} = 1$)

in $\frac{h}{3}\left[y_0 + 4(y_1 + y_3) + 2(y_2) + y_4\right]$ etc.

[No carry-on mark for evaluating after incorrect formula.]

[Some students mixed up "odds" and "evens" or failed to get ...]

(d) $A = \int_0^{\frac{4}{3}} \frac{1}{(1+3x)^2}\,dx$ ✓

1. Setting up a correct definite integral expression

[N.B. Many students use poor integral notation e.g. missing out "dx", although no mark was deducted for this (Generous!)]

$= \int_0^{\frac{4}{3}} (1+3x)^{-2}\,dx$

$= \left[\frac{(1+3x)^{-1}}{-1\times 3}\right]_0^{\frac{4}{3}}$ ✓

1. Correct Integration step.

[Very poorly done. Many students attempted a log integral or expanded $\frac{1}{(1+3x)^2} = \frac{1}{1+6x+9x^2}$. Many students missed the factor of 3 in the denominator.]

$= -\frac{1}{3}\left[\frac{1}{1+3x}\right]_0^{\frac{4}{3}}$

$= -\frac{1}{3}\left(\frac{1}{1+4} - \frac{1}{1+0}\right)$

$= -\frac{1}{3}\left(\frac{1}{5} - 1\right)$

$= -\frac{1}{3} \times -\frac{4}{5}$

1. Substitution to get answer.

$= \frac{4}{15}$ units^2 ✓

[Many students thought incorrectly that $F(0) = \frac{1}{1+0} = 0$ This is wrong!]

(3)

any corrections please

QUESTION FIVE

(a) (i) Substituting coordinates of P into

$4y = x^2$

$4 \times 1 = 2^2$

$\therefore$ P lies on $4y = x^2$

Substituting coordinates of P into

$4y = (x-4)^2$

$4 \times 1 = (2-4)^2$

$4 = (-2)^2$

$\therefore$ P lies on $4y = (x-4)^2$ ✓

Solving simultaneously

$4y = x^2$ (1)

$4y = (x-4)^2$ (2)

Substituting (1) into (2)

$x^2 = (x-4)^2$

$x^2 - (x-4)^2 = 0$ ✓

$x^2 - (x^2 - 8x + 16) = 0$

$8x - 16 = 0$

$8x = 16$

$x = 2$

$\therefore y = 1$ ✓

Hence P is the only point of intersection of the two curves

Some students did not show that P satisfied both equations.

Some students did not state that P only satisfied but implied it.

Most students tried to solve 2 equations simultaneously.

Mainly well done.

(ii) $4y = (x-4)^2$

$y = \frac{1}{4}(x-4)^2$

$\frac{dy}{dx} = \frac{1}{2}(x-4)$ ✓

$\therefore m = \frac{1}{2} \times -2 = -1$ at $x = 2$

Some students found the differentiation difficult.

Equation of tangent at $(2, 1)$ with gradient $= -1$ is

$y - 1 = -1(x-2)$

$y - 1 = -x + 2$

$y = 3 - x$ or $x + y - 3 = 0$ ✓

Incorrect general form by some stud

(iii) Solving simultaneously

$4y = x^2$ (1) $\qquad y = 3 - x$ (2)

Substituting (2) into (1)

$4(3-x) = x^2$ ✓

$12 - 4x = x^2$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$\therefore x = -6$ or 2 ✓

at $x = -6$ $\quad y = 3 - (-6)$

$= 9$

$\therefore$ Q has co-ordinates $(-6, 9)$ ✓

Co-ordinates for Q we incorrect because of incorrect equation c tangent.

(b) (i) $\alpha + \beta = \frac{3}{2}$ ✓

Well done

(ii) $\alpha\beta = -\frac{7}{2}$ ✓

Well done.

(iii) $(\alpha + \beta)^2 - 2\alpha\beta$ ✓

$= \left(\frac{3}{2}\right)^2 - 2 \times -\frac{7}{2}$

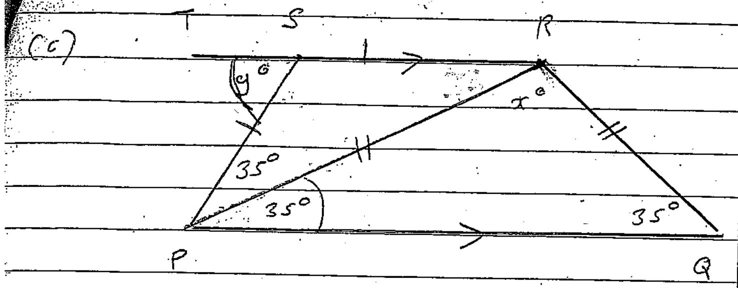
$= \frac{9}{4} + 7$

$= \frac{37}{4} = 9\frac{1}{4}$ ✓

Many incorrectly expanded $(\alpha+\beta)^2$ and said $\alpha^2 + \beta^2 = (\alpha+\beta$

QUESTION SIX

(c)



(i) $\angle RPQ = \angle RQP = 35°$

($\triangle RPQ$ is isosceles) ✓

$\angle PRQ = 180° - 2 \times 35°$

(angle sum $\triangle RPQ$) ✓

$= 110°$

$\therefore x = 110°$ ✓

(ii) $\angle SRP = \angle RPQ = 35°$

(alternate angles $SR \parallel PQ$) ✓

$\angle SPR = \angle SRP = 35°$

($\triangle SPR$ is isosceles) ✓

$\angle TSP = \angle SRP + \angle SPR$

(exterior angle)

$= 35° + 35°$

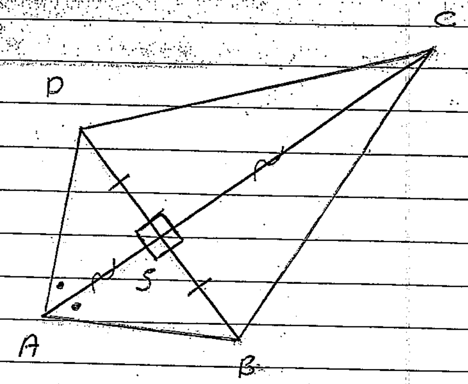
$= 70°$

$\therefore g = 70°$ ✓

* some students ignored the request to make a neat sketch for both parts. (this makes it hard to see exactly what each student is referring to)

a) generally well done although setting out and reasoning need to be refined.

(b)



(i) In $\triangle$'s ASD and ASB

AS is a common side ✓

$\angle DAS = \angle BAS$ (given)

$\angle DSA = \angle BSA = 90°$ (diagonals intersect at rt. $\angle$'s given) ✓

$\therefore \triangle ASD \equiv \triangle ASB$ (AAS)

$\therefore DA = AB$ (corres sides in congt $\triangle$s) ✓

and $DS = BS$

(ii) In $\triangle$'s DCS and BCS

CS is a common side ✓

$DS = BS$ (proved above)

$\angle CSD = \angle CSB$ (diagonals intersect at rt. angles)

$\triangle DCS \equiv \triangle BCS$ (SAS)

$\therefore DC = CB$ (corres sides in congt $\triangle$'s) ✓

* Marks may well be deducted in the H.S.C if they are presented in the same manner as they were submitted in the trial

Some general knowledge of setting out congruence proofs although in many cases it was sloppy!

- Reasoning need to be succinct and correct.
- need to start off with In $\triangle$'s ___
- state tests correctly eg SAS not ASS (no such congruence test of AAA)
- notation for is congruent to is $\equiv$

ii) some students stated that ABCD was a kite $\therefore$ DC = BC.

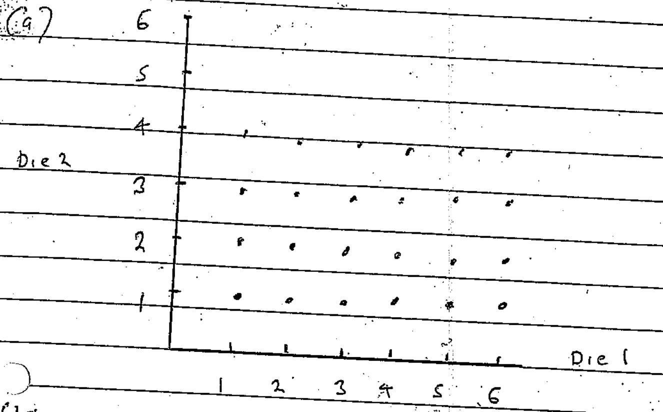
BUT the defn of a kite is two pairs of adj. sides equal and diagonals intersect at rt. $\angle$'s and it is the 2nd pair of adj. sides we are trying to prove equal!

* some students used $\triangle$'s ADC & ABC which works as well

Comments (13)

QUESTION SEVEN

(a)



Using a graphical representation to determine the number of favourable outcomes in each case:

(i) 1 favourable outcome (6, 6)

P(both show 6) = $\frac{1}{36}$ since $n(S) = 36$ ✓

(ii) 2 favourable outcomes (1, 6) and (6, 1)

P(show a 1 and a 6) = $\frac{2}{36} = \frac{1}{18}$ ✓

(iii) 11 favourable outcomes (1, 1) (1, 2) (2, 1) (1, 3) (3, 1) (1, 4) (4, 1) (1, 5) (5, 1) (1, 6) (6, 1) ✓

P(at least one 1) = $\frac{11}{36}$ ✓

(iv) 5 favourable outcomes (1, 5) (2, 4) (3, 3) (4, 2) and (5, 1)

P(total 6) = $\frac{5}{36}$ ✓

* Part (a) was fairly well done.

- Some students just wrote answers withou[t] working out or drawi[ng] a diagram.

(14)

(b) P(hitting attacking aircraft)

= 1 − P(missing aircraft with both defences) ✓

= 1 − (P(missile missing) × P(gun missing))

= 1 − 0.1 × 0.2 ✓

= 1 − 0.02

= 0.98 ✓

* Some students solved this as $0.9 \times 0.8 = 0.7$[2] because they di[d] not make a tree diagram.

(c) (i) $\log_2 9 = \log_2 3^2$

$= 2 \log_2 3$ ✓

$= 2 \times 1.584962$

$= 3.16992$ ✓

$= 3.17$ (to 2 dec pls)

(ii) $\log_2 12 = \log_2 (4 \times 3)$

$= \log_2 4 + \log_2 3$ ✓

$= \log_2 2^2 + 1.584962$

$= 2 \log_2 2 + 1.584962$

$= 2 + 1.584962$

$= 3.584962$

$= 3.58$ (to 2 dec pl) ✓

* mostly well do[ne]. Some students tried to solve th[is] by change of base. this was not appropriate because of the wording in the question.

Also some simp[le] mistakes eg. $\log_2 3^2 = \log_2 \ldots \times \log$

QUESTION EIGHT

(a) (i) Let $r_1 = OD$ and $r_2 = OA$

Perimeter $= AD + BC + r_1\theta + r_2\theta$

$= 10 + 10 + \frac{10\pi}{3} + \frac{20\pi}{3}$ ✓

$= 20 + 10\pi$

$= 51.4$ cm (to 3 sig figs) ✓

(ii) Area $= \frac{1}{2}r_2^2\theta - \frac{1}{2}r_1^2\theta$

$= \frac{\theta}{2}(r_2^2 - r_1^2)$

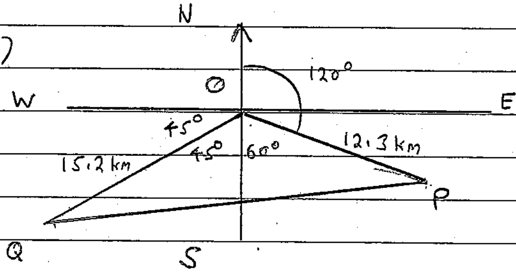
$= \frac{\pi}{6}(400 - 100)$ ✓

$= \frac{300\pi}{6} = 50\pi$ ✓

$= 157\text{ cm}^2$ (to 3 sig figs)

Many students did not provide answers to three significant figures.

(b) (i)



✓

(ii) $\angle POQ = 105°$ (from sketch) ✓

(iii) By the cosine rule in $\triangle QOP$

$PQ^2 = OQ^2 + OP^2 - 2 \times OQ \times OP \times \cos\angle POQ$

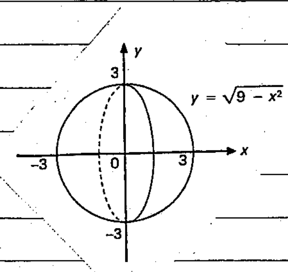
$= (15.2)^2 + (12.3)^2 - 2 \times 15.2 \times 12.3 \times \cos 105°$

$PQ = 21.9$ km to 1 dec pl

* Many students had difficulty doing diagram.

* Some students had a plus sign instead of a minus sign in cosine rule.

(c)



$$V = \pi\int_{-3}^{3} y^2\,dx$$

$$= \pi\int_{-3}^{3}\left(\sqrt{9-x^2}\right)^2 dx \quad ✓$$

$$= 2\pi\int_0^3 (9 - x^2)\,dx$$

$$= 2\pi\left[9x - \frac{x^3}{3}\right]_0^3 \quad ✓$$

$$= 2\pi\left[\left(27 - \frac{27}{3}\right) - (0 - 0)\right]$$

$$= 2\pi \times 18$$

$$= \underline{36\pi \text{ units}^3} \quad ✓$$

The shape of the solid is a sphere ✓

OR Since the shape of the volume is a sphere ✓

$$V = \frac{4}{3}\pi r^3 \quad ✓ \quad r = 3$$

$$= \frac{4}{3}\pi 3^3$$

$$= 4 \times \pi \times 9 \quad ✓$$

$$= \underline{36\pi \text{ units}^3} \quad ✓$$

Many students forgot to write down the shape produced or identified it as a hemisphere or circle.

QUESTION NINE

(a) (i) $t_n = a + (n-1)d$

$a = 50 \quad d = -7$

$t_n = 50 + (n-1)(-7)$ ✓

$= 50 - 7n + 7$

$= 57 - 7n$

① Using formula correctly.

(ii) $57 - 7n = -27$

$-7n = -84$

$n = 12$ ✓

There are 12 terms in the series

① Answer.

(iii) $S_n = \frac{n}{2}(a + L)$

[or can use $S_n = \frac{n}{2}[2a + (n-1)d]$]

$n = 12 \quad a = 50 \quad L = -27$

$S_{12} = \frac{12}{2}(50 - 27)$ ✓

$= 6 \times 23$

$= 138$ ✓

1. Using formula correctly.

1. Answer.

②

Part (a) generally well done.

Part (b) was very poorly done. Many students did not set out their ~~algebra and reasoning~~ clearly enough to "PROVE THAT..."

(b) (i) Amount owing at the beginning of the second year

$A_1 = 1000\left(1 + \frac{10}{100}\right) - P$ ✓

$= 1000(1.1) - P$ ✓

$= 1100 - P$

IMPORTANT. If a Qn says "Prove that..." or "Show that..." you must set out algebra and reasoning clearly.

1. Getting ~~from interest rate~~ $\longrightarrow \times(1.1)$

1. Showing $1000(1.1) - P \longrightarrow$ answer.

②

(ii) Amount owing at the beginning of the third year.

$A_2 = (1000(1.1) - P)1.1 - P$ ✓

or can use $= 1000(1.1)^2 - P1.1 - P$

$A_2 = A_1(1.1) - P \quad = 1000(1.1)^2 - P(1 + 1.1)$

$= 1210 - 2.1P$ ✓

Observation: "Beginning of 2^{nd} Yr" means immediately after repayment at end of 1^{st} Yr.

1. Showing $A_2 = A_1(1.1) - P$.

1. Simplifying $\longrightarrow$ answer.

②

OR $(1100 - P)(1.1) - P$ ✓

$= 1100(1.1) - (1.1)P - P$

$= 1100(1.1) - P(1 + 1.1)$

$= 1210 - 2.1P$ ✓

(iii) Continuing from part (ii)

Amount owing after 3 years

$A_3 = \left[1000(1.1)^2 - P(1 + 1.1)\right](1.1) - P$

$= 1000(1.1)^3 - P(1 + 1.1 + 1.1^2)$ ✓

⋮ continuing the pattern

Amount owing after n years

$= 1000(1.1)^n - P(1 + 1.1 + 1.1^2 + \dots 1.1^{n-1})$ ✓

$1 + 1.1 + (1.1)^2 + \dots + (1.1)^{n-1}$ is a geometric series with $a = 1 \quad r = 1.1$

If after n years the loan is repaid

$1000 \times (1.1)^n - P\left(\frac{(1.1)^n - 1}{1.1 - 1}\right) = 0$ ✓

$1000 \times (1.1)^n - 10P\left[(1.1)^n - 1\right] = 0$

$10P\left[(1.1)^n - 1\right] = 1000 \times (1.1)^n$ ✓

$P = \frac{1000 \times (1.1)^n}{10\left[(1.1)^n - 1\right]}$

$= \frac{100 \times (1.1)^n}{(1.1)^n - 1}$

N.B. Part (b)(iii) was clearly understood by many students but very poorly set out to "Prove that...".

For full marks you needed:

1. Expression for A_3 or equ[ivalent] to show pattern leading to A[n]

Not enough just to write down $A_n = \dots$

1. Correct expression for A_n or equivalent.

Some students made errors such as omitting P or writi[ng] $-P(1 + 1.1 + 1.1^2 + \dots + 1.1^{n-2})$

This is wrong for end of n^{th} ye[ar]

1. Identifying and using a Geometric Series correctly

Note: best to state clearly "Geom. Series with n terms, $a = 1$, $r = 1.1$, so $S_n = \frac{a(r^n - 1)}{r - 1}$".

Few students showed this clearly but mark was given for a part of this information

1. Using $A_n = 0$ and simplifying $\longrightarrow$ answer.

④

QUESTION TEN

(a) (i) $f(x) = 9x(x-2)^2$

$= 9x(x^2 - 4x + 4)$

$= 9x^3 - 36x^2 + 36x$

$f'(x) = 27x^2 - 72x + 36$ ✓ — OK.

$f''(x) = 54x - 72$ ✓

stationary points occur when $f'(x) = 0$

$27x^2 - 72x + 36 = 0$

$9(3x^2 - 8x + 4) = 0$

$9(3x-2)(x-2) = 0$

$x = \frac{2}{3}$ or 2 ✓ — ok

$f\left(\frac{2}{3}\right) = 9\left(\frac{2}{3}\right)\left(\frac{2}{3} - 2\right)^2$

$= 9 \times \frac{2}{3} \times \left(\frac{-4}{3}\right)^2 = \frac{32}{3}$

$f''\left(\frac{2}{3}\right) < 0$ ✓

$\therefore$ max turning pt at $\left(\frac{2}{3}, \frac{32}{3}\right)$

$f(2) = 18(2-2)^2 = 0$

$f''(2) > 0$

$\therefore$ min turning pt at $(2, 0)$ ✓

students lost 1 mark for failing to investiga the max and MIN turning points using the second derivatives.

at $x = 0$ $y = 0$

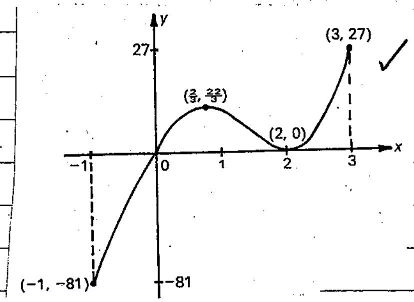
at $y = 0$ $x = 0$ $x = 2$

Values of the end-points of domain

$f(-1) = -9 \times (-3)^2 = -81$

$f(3) = 27 \times 1^2 = 27$

(i) continued



✓

Some students lost 1 mark for not sketching the curve nicely and not showing critical points on the graph.

(ii) Range $-81 \le f(x) \le 27$ ✓ — ok

(b)(i) $V = \pi r^2 h = 600$

$h = \frac{600}{\pi r^2}$

$SA = 2\pi r h + 2\pi r^2$

$= 2 \times \pi \times r \times \frac{600}{\pi r^2} + 2\pi r^2$ ✓ — ok

$= \frac{1200}{r} + 2\pi r^2$

(ii) $\frac{dSA}{dr} = -1200 r^{-2} + 4\pi r$ ✓

$= \frac{-1200}{r^2} + \frac{4\pi r^3}{r^2}$

good as SA was given if students knew the $\frac{dy}{dx}$ of $\frac{1}{x}$ is $\frac{-1}{x^2}$

stationary point occurs when $\frac{dSA}{dr} = 0$

$4\pi r^3 - 1200 = 0$

$r^3 = \frac{1200}{4\pi}$

$r = \sqrt[3]{\frac{300}{\pi}}$

$= 4.57$ cm ✓

Some had problems with re-arranging the formula and finding the cube root of r.

$\frac{d^2SA}{dr^2} = \frac{2400}{r^3} + 4\pi > 0$ at $r = 4.57$

$\therefore$ min SA at $r = 4.57$ cm ✓

$h = \frac{600}{\pi \times 4.57^2} = 9.14$ cm ✓

Students lost 1 mark if the value of r was not stated as min by not investigating $\frac{d^2SA}{dr^2}$