Western Region
Trial Higher School Certificate Examination
1996

MATHEMATICS
3/4 UNIT COMMON

Solutions and Marking Scheme

Please Note:

* These are suggested solutions. They are not intended to specify the amount of working required or the method to be applied.

  Teachers should accept any valid method of solution providing adequate working is shown

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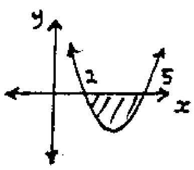
SOLUTIONS	COMMENTS
QUESTION 1. – (12 marks)	
a.) $\displaystyle\int_0^4 \frac{dx}{x^2+16} = \int_0^4 \frac{dx}{x^2+4^2}$	
$= \left[\frac{1}{4}\tan^{-1}\frac{x}{4}\right]_0^4$	1 for integral
$= \left(\frac{1}{4}\tan^{-1}1\right) - \left(\frac{1}{4}\tan^{-1}0\right)$	1 for substitution
$= \frac{1}{4}\times\frac{\pi}{4} - 0$	
$= \boxed{\frac{\pi}{16}}$	1 answer
b.) $\int (1-\cos x)^2 dx = \int (1 - 2\cos x + \cos^2 x)\,dx$	1 for expansion
$= \int 1 - 2\cos x + \frac{1}{2}(1+\cos 2x)$	1 for removal of $\cos^2 x$
$= x - 2\sin x + \frac{1}{2}x + \frac{1}{4}\sin 2x$	1 for integral ACCEPT
$= \boxed{\frac{3x}{2} - 2\sin x + \frac{1}{4}\sin 2x + C}$	~~1 answer~~
c.) $\frac{3}{x-2} \geqslant 1$	
$\frac{(x-2)^{\cancel{2}}.\ 3}{\cancel{x-2}} \geqslant 1.\ (x-2)^2$	1 for method
$3x - 6 \geqslant x^2 - 4x + 4$	
$x^2 - 7x + 10 \leqslant 0$	
$(x-5)(x-2) \leqslant 0$ 	1 for quadratic
$2 \leqslant x \leqslant 5$	
but since $x \neq 2$.	
$\therefore\ \boxed{2 < x \leqslant 5}$	1 answer

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SOLUTIONS	COMMENTS
d.) $y = \log_e \left(\frac{1}{\sqrt{\cos x}} \right)$	
$= \log_e 1 - \log_e \cos x^{\frac{1}{2}}$	1 for splitting. $\log(\frac{a}{b})$
$= \log_e 1 - \frac{1}{2} \log_e \cos x$	
$\frac{dy}{dx} = 0 - \frac{1}{2} \cdot \frac{-\sin x}{\cos x}$	1 for $\frac{dy}{dx}$
$= \boxed{\frac{1}{2} \tan x}$	1 answer.

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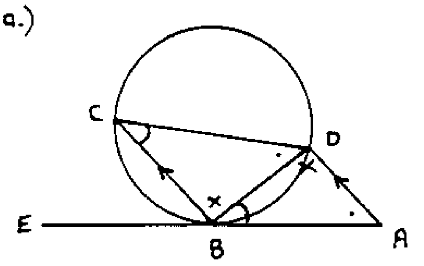
SOLUTIONS	COMMENTS
QUESTION 2 - (12 marks)	
a.) 	
$< DBA = < BCD$ (angle between tangent and chord = to $<$ in alternate segment)	1 (reason 1)
$< CBD = < BDA$ (alternate $<$'s in $\parallel$ BC $\parallel$ AD)	1 (reason 2)
$< CDE = < DAB$ (third $<$ in Δ 180°)	1 (reason 3)
$\therefore$ $\boxed{\Delta BCD \;\vert\vert\vert\; \Delta DBA \text{ (equiangular triangles)}}$	1 answer
b.) $\int \frac{2x}{(x-1)^2}\,dx$ $u = x-1$, $du = dx$	1 for sub and $x = u+1$
$\int \frac{2u+2}{u^2}\,du$ $x = u+1$	1 for $\int \frac{2u+2}{u^2}\,du$
$\int \frac{2}{u} + 2u^{-2}\,du$	
$= 2\ln u - 2u^{-1} + c$	1 for integration
$= 2\ln u - \frac{2}{u} + c$	
$= \boxed{2\ln(x-1) - \frac{2}{x-1} + c}$	1 for answer
c.) STEP 1 : Prove for $n=1$	
LHS $= 5^{n-1} = 5^0 = 1$ RHS $= \frac{5^n - 1}{4} = \frac{4}{4} = 1$	
$\therefore$ LHS = RHS	
$\therefore$ True for $n=1$	1 for step 1

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SOLUTIONS	COMMENTS
STEP 2 : Assume true for $n=k$ ie. $1+5+5^2+\ldots\ldots+5^{k-1}=\dfrac{5^k-1}{4}$	1 statements
and prove true for $n=k+1$ ie. $1+5+5^2+\ldots+5^{k-1}+5^k=\dfrac{5^{k+1}-1}{4}$	
now $1+5+5^2+\ldots+5^{k-1}+5^k$ $=\dfrac{5^k-1}{4}+5^k$ $=\dfrac{5^k-1}{4}+\dfrac{4.5^k}{4}$ $=\dfrac{5.5^k-1}{4}$ $=\dfrac{5^{k+1}-1}{4}$	1 for working
STEP 3 : $\therefore$ If true for $n=k$ then true for $n=k+1$, but it is true for $n=1$. $\therefore$ true for $n=1+1=2$ $n=2+1=3$ etc. $\therefore$ $\boxed{\text{By induction } 1+5+\ldots 5^{n-1}=\dfrac{5^n-1}{4}}$	1 for step 3

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SOLUTIONS	COMMENTS
QUESTION 3 - (12 marks) a.) $^{12}P_r = 120.\ ^{12}C_r$ $^{12}P_r = 120.\ \dfrac{^{12}P_r}{r!}$ $r! = 120.\ \dfrac{^{12}P_r}{^{12}P_r}$ $r! = 120$ $\boxed{r = 5}$	1 for $^{12}C_r = \dfrac{^{12}P_r}{r!}$ 1 solving. 1 answer
b.) i.) $V^2 = 8x - 2x^2$ $\frac{1}{2}V^2 = 4x - x^2$ $a = \dfrac{d}{dx}\left(\frac{1}{2}v^2\right) = 4 - 2x$ $\boxed{= -2(x-2)}$ $\therefore$ the particle is in S.H.M. as the acceleration is proportional to the distance from the centre of motion.	1 for method 1 for statement of S.H.M
b.) ii.) when $a = 0$ $\therefore\ 0 = 2(2 - x)$ $\boxed{x = 2}$ $\therefore$ the centre of motion is 2m to the right of the origin.	1 for centre.
b.) iii.) when $v = 0$ $\therefore\ 0 = 8x - 2x^2$ $0 = 2x(4 - x)$ $\boxed{x = 0,\ 4}$ $\therefore$ the two endpoints are the origin and 4m to the right of the origin.	1 for endpoints

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SOLUTIONS	COMMENTS
b.) iv.) Max speed at centre of motion $\therefore\ x = 2$ $v^2 = 8x - 2x^2$ $= 16 - 8$ $= 8$ $v = \pm\sqrt{8} = \pm 2\sqrt{2}$ $\therefore$ $\boxed{\text{the maximum velocity is } 2\sqrt{2}\text{ m/s}}$	1 for velocity
c.) $P = 3200 + 400e^{kt}$ when $t = 0$ $P = 3200 + 400e^{0} = 3600$ $\therefore$ initial population is 3600. when $t = 20$ $P = 7200$ $\therefore\ 7200 = 3200 + 400e^{20k}$ $4000 = 400e^{20k}$ $10 = e^{20k}$ $\ln 10 = 20k$ $k = \frac{\ln 10}{20} \doteqdot 0.115$ $\therefore\ P = 3200 + 400e^{0.115t}$ when $P = 10800$ $10800 = 3200 + 400e^{0.115t}$ $7600 = 400e^{0.115t}$ $19 = e^{0.115t}$ $\ln 19 = 0.115t$ $t = \frac{\ln 19}{0.115} \doteqdot 25.575$ $\therefore$ $\boxed{\text{to triple } t \doteqdot 25\text{hr } 35\text{mins}}$	1 for initial pop. 1 for k. 1 for setting up equation 1 for answer.

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SOLUTIONS	COMMENTS
QUESTION 4 - (12 marks)	
a.) $y = \cos^{-1} x$ $\quad m = \frac{-2}{\sqrt{3}}$	
$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$	1 for derivative
$\frac{-2}{\sqrt{3}} = \frac{-1}{\sqrt{1-x^2}}$	
$-2\sqrt{1-x^2} = -\sqrt{3}$	
$4(1-x^2) = 3$	
$4 - 4x^2 = 3$	
$4x^2 = 1$	
$x^2 = \frac{1}{4}$	
$x = \pm\frac{1}{2}$	1 for solving.
when $x = -\frac{1}{2}$ $\quad y = \cos^{-1}(-\frac{1}{2}) = \frac{2\pi}{3}$	
when $x = \frac{1}{2}$ $\quad y = \cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$	
$\therefore$ points are $\boxed{\left(\frac{1}{2}, \frac{\pi}{3}\right) \text{ and } \left(-\frac{1}{2}, \frac{2\pi}{3}\right)}$	1 for points
b.) $\left(x^3 - \frac{1}{3x}\right)^8$	
Note :- middle term is the fifth term.	1 for 5th term
$T_{k+1} = {}^nC_k (a)^{n-k} . b^k$	1 for Theorem
$T_{4+1} = {}^8C_4 (x^3)^4 . \left(\frac{-1}{3x}\right)^4$	
$T_5 = 70x^{12} . \frac{1}{81x^4}$	1 for substitution
$= \frac{70x^8}{81}$	
$\therefore$ $\boxed{\text{the middle term is } \frac{70}{81}x^8}$	1 for answer

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SOLUTIONS	COMMENTS
c.) i.) $V = 16$ $\therefore \quad \frac{4}{3}\pi r^3 + 4\pi r^2 = 16$ $\frac{1}{3}\pi r^3 + \pi r^2 = 4$ $\pi r^3 + 3\pi r^2 = 12$ $r^3 + 3r^2 = \frac{12}{\pi}$ $\therefore \quad \boxed{r^3 + 3r^2 = \frac{12}{\pi}}$	1 for showing.
c.) ii.) $r^3 + 3r^2 = \frac{12}{\pi}$ $f(r) = r^3 + 3r^2 - 12/\pi$ $f(0) = -12/\pi$ $f(1) = 0.18028$ change in sign. $\therefore$ one root lies between 0 and 1.	1 for sub 0 and 1 1 for change of sign and reason
c.) iii.) $f(r) = r^3 + 3r^2 - 12/\pi$ $f'(r) = 3r^2 + 6r$ if $a = 0.9$ then a closer approx of a is given by $a_1 = a - \frac{f(a)}{f'(a)}$ $a_1 = 0.9 - \frac{f(0.9)}{f'(0.9)}$ $a_1 = 0.9 - \left(\frac{-0.6607186}{7.83}\right)$ $a_1 = 0.9 + 0.0843$ $a_1 = 0.9843$ $\therefore$ 0.9843 is a better approx	1 for formula and substitution 1 for calculation

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SOLUTIONS	COMMENTS
QUESTION 5 - (12 marks)	
a.) Let the roots be α, β and γ	
Product of roots	
$\therefore \alpha\beta\gamma = \frac{-12}{3} = -4$	1 for product
but $\alpha\beta = 4$ or $\beta = \frac{4}{\alpha}$	
$\therefore 4\gamma = -4$	
$\gamma = -1$	
Sum of roots	
$\alpha + \beta + \gamma = \frac{17}{3}$	1 for sum
$\alpha + \beta - 1 = \frac{17}{3}$	
$\alpha + \frac{4}{\alpha} = \frac{20}{3}$	
$3\alpha^2 - 20\alpha + 12 = 0$	
$(3\alpha - 2)(\alpha - 6) = 0$	
$\alpha = 6$ or $\frac{2}{3}$	1 for 3 solutions
$\therefore$ the roots are $6, \frac{2}{3}$ or -1	
$\boxed{\text{ie. } x = 6, \frac{2}{3}, -1}$	
b.) i.) P(fail at least once)	
$= 1 -$ P(doesn't fail)	1 for $P(E) = 1 - P(\bar{E})$
$= 1 - \left(\frac{29}{30}\right)^m$	1 for expression
b.) ii.) P (fail at least once) $> \frac{9}{10}$	
$\therefore 1 - \left(\frac{29}{30}\right)^m > \frac{9}{10}$	1 for setting up equation
$\left(\frac{29}{30}\right)^m < \frac{1}{10}$	
$\left(\frac{30}{29}\right)^m > 10$	
$\log_{10}\left(\frac{30}{29}\right)^m > \log_{10}10$	1 for working
$m(\log_{10}30 - \log_{10}29) > 1 \quad \therefore m > \frac{1}{\log_{10}30 - \log_{10}29}$	

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SOLUTIONS	COMMENTS
c.) i.) $\frac{dV}{dh} = \frac{\pi}{12}(h^2+12h+36)$ $V = \frac{\pi}{12}\int h^2+12h+36\,dh$ $= \frac{\pi}{12}\left(\frac{h^3}{3}+6h^2+36h\right)+c$ $= \frac{\pi}{36}(h^3+18h^2+108h)+c$ $= \frac{\pi h}{36}(h^2+18h+108)+c$ when $h=0$, $V=0$ and $c=0$ $\therefore \boxed{V = \frac{\pi h}{36}(h^2+18h+108)}$	integral 1 for correct form
c.) ii.) when $h=6$ $V = \frac{6\pi}{36}(252) = \boxed{42\pi \text{ cm}^3}$ 131.9 cm^3	1 for V when h = 6
c.) iii.) $\frac{dV}{dt} = 8$ $\frac{dh}{dt} = \frac{dV}{dt}\times\frac{dh}{dV}$ $= 8 \times \frac{12}{\pi(h+6)^2}$ $\boxed{\frac{dh}{dt} = \frac{96}{\pi(h+6)^2}}$	1 for chainrule and $\frac{dh}{dt}$
c.) iv.) when $h=6$ $\frac{dh}{dt} = \frac{96}{144\pi} = \frac{2}{3\pi}$ $\cancel{\frac{2\pi}{3}}$ $\therefore$ the depth is increasing at a rate of $\boxed{\frac{2\pi}{3}\text{ cm/sec.}}$ (see over page)	1 for rate

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SOLUTIONS	COMMENTS
c.) iv.) con't. $\frac{dh}{dt} = \frac{96}{\pi(h+6)^2}$ $\frac{dt}{dh} = \frac{\pi}{96}(h^2+12h+36)$ $t = \frac{\pi}{96}\int h^2+12h+36\,dh$ $t = \frac{\pi}{96}\left(\frac{h^3}{3}+6h^2+36h\right)+c$ when $h=0$, $t=0$ and $c=0$ $\therefore t = \frac{\pi}{288}(h^3+18h^2+108h)$ when $h=6$ $t = \frac{\pi}{288}((6)^3+18(6)^2+108(6))$ $= \boxed{16\cdot 49 \text{ seconds}}$	1 for working to 16·49 se
OR $\frac{dV}{dt} = 8$ $V = 8t+c$ $t=0, V=0 \therefore c=0$ $V=8t$ $42\pi = 8t$ $t = 16{\cdot}49s.$	

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SOLUTIONS	COMMENTS
QUESTION 6 - (12 marks) a.) i.) number of arrangements $n = \dfrac{10!}{2!2!2!}$ $= \boxed{453600}$	1 for arrangements
a) ii.) P (vowels and consonants alt) $= \dfrac{2 \times \frac{5!}{2!2!} \times \frac{5!}{2!}}{453600}$ $= \dfrac{360\cancel{0}}{45360\cancel{0}}$ CVCVCVCVCV or VCVCVCVCVC $= \dfrac{1}{126}$ $\therefore \boxed{P(\text{V and C alternate}) = \dfrac{1}{126}}$ Note:- $\dfrac{5!}{2!2!} \rightarrow$ Arrange 5 vowels with 2 vowels repeated. $\dfrac{5!}{2!} \rightarrow$ Arrang 5 consonants with 1 repeated.	1 for accounting for 2 combinations 1 for correct prob.
b.) i.) $\boxed{(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n}$	1 for expression
b.) ii.) $(1+x)^n = {}^nC_0 + {}^nC_1 x + \ldots\ldots + {}^nC_n x^n$ integrate both sides w.r.t x. $\dfrac{(1+x)^{n+1}}{n+1} = {}^nC_0 x + {}^nC_1 \dfrac{x^2}{2} + \ldots\ldots + \dfrac{{}^nC_n x^{n+1}}{n+1} + C$ Let $x = 0$ $\dfrac{1}{n+1} = C$ $\therefore (1+x)^{n+1} = {}^nC_0 x + {}^nC_1 \dfrac{x^2}{2} + \ldots + \dfrac{{}^nC_n x^{n+1}}{n+1} + \dfrac{1}{n+1}$	1 for integration

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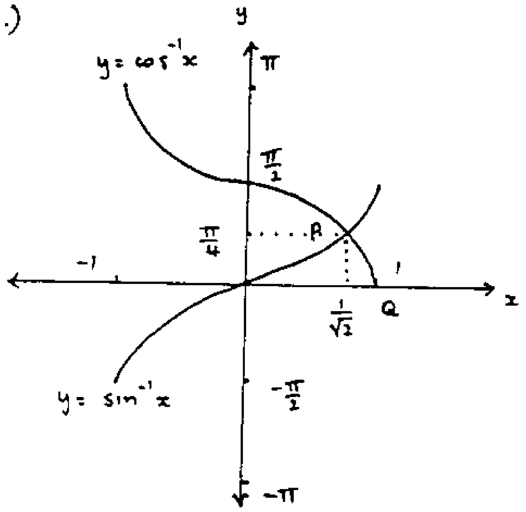
SOLUTIONS	COMMENTS
b.) ii.) Con't Let $x = 1$ $\therefore$ $\frac{2^{n+1}}{n+1} = {}^nC_0 + \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 + \dots + \frac{1}{n+1}\,{}^nC_n + \frac{1}{n+1}$ $\frac{2^{n+1}-1}{n+1} = {}^nC_0 + \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 + \dots + \frac{1}{n+1}\,{}^nC_n.$ $\boxed{\frac{2^{n+1}-1}{n+1} = {}^nC_0 + \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 + \dots + \frac{1}{n+1}\,{}^nC_n}$	1 for working
c.) i.)  	1 for correct graphs
c.) ii.) when $x = \frac{1}{\sqrt{2}}$ $y = \cos^{-1}x = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$ $y = \sin^{-1}x = \sin^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$ $\therefore \boxed{P\left(\frac{1}{\sqrt{2}}, \frac{\pi}{4}\right)}$ Thus $y = \cos^{-1}x$ and $y = \sin^{-1}x$ intersect at $\left(\frac{1}{\sqrt{2}}, \frac{\pi}{4}\right)$	1 for substitution

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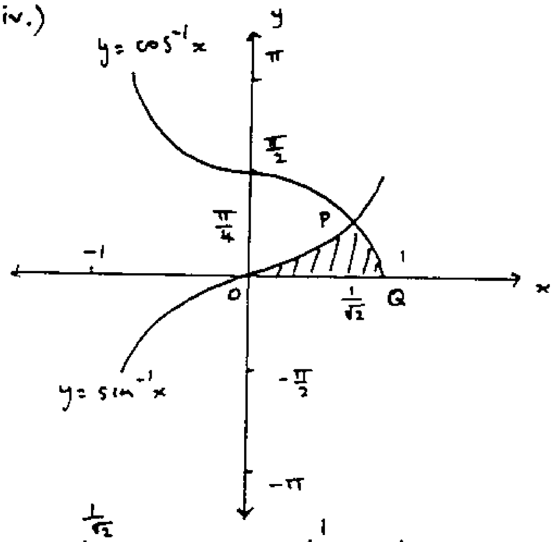
SOLUTIONS	COMMENTS
c.) iii) $\frac{d}{dx}\left(x\sin^{-1}x + \sqrt{1-x^2}\right) = \sin^{-1}x$ LHS $= \sin^{-1}x.(1) + \frac{x \cdot 1}{\sqrt{1-x^2}} + \frac{1}{\cancel{2}}(1-x^2)^{-\frac{1}{2}} \cdot -\cancel{2}x$ $= \sin^{-1}x + \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}$ $= \sin^{-1}x$ $=$ RHS.	1 for derivative 1 for proof LHS = RHS.
c.) iv.)  Area $= \int_0^{\frac{1}{\sqrt{2}}} \sin^{-1}x\,dx + \int_{\frac{1}{\sqrt{2}}}^{1} \cos^{-1}x\,dx$ $= \left[x\sin^{-1}x + \sqrt{1-x^2}\right]_0^{\frac{1}{\sqrt{2}}} + \left[x\cos^{-1}x - \sqrt{1-x^2}\right]_{\frac{1}{\sqrt{2}}}^{1}$ $= \left[\left(\frac{1}{\sqrt{2}}\cdot\frac{\pi}{4} + \frac{1}{\sqrt{2}}\right) - (1)\right] + \left[(0) - \left(\frac{1}{\sqrt{2}}\cdot\frac{\pi}{4} - \frac{1}{\sqrt{2}}\right)\right]$ $= \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 - \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}}$ $= \frac{2}{\sqrt{2}} - 1 = \frac{2-\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}-2}{2}$ $= \boxed{\sqrt{2} - 1 \text{ units}^2 \text{ is area.}}$	1 for integral 1 for sub and area

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SOLUTIONS	COMMENTS
QUESTION 7. - (12 marks)	
a.) Vertical Motion	
$\frac{d^2y}{dt^2} = -g$	
$\therefore \frac{dy}{dt} = -gt + c$	
when $t=0$ $V = V \sin\theta = V \sin 45 = \frac{V}{\sqrt{2}}$	1 for integrals
$\therefore c = \frac{V}{\sqrt{2}}$	
$\therefore \frac{dy}{dt} = -gt + \frac{V}{\sqrt{2}}$	
$y = \frac{-gt^2}{2} + \frac{Vt}{\sqrt{2}} + c$	1 for $y = \frac{-gt^2}{2} + \frac{Vt}{\sqrt{2}}$
but $t=0$ $y=0$ $\therefore c=0$	
$\therefore \boxed{y = -\frac{gt^2}{2} + \frac{Vt}{\sqrt{2}}}$	
Horizontal Motion	
$\frac{dx}{dt} = V \cos 45^\circ = \frac{V}{\sqrt{2}}$	
$\therefore x = \frac{Vt}{\sqrt{2}} + c$ when $t=0$, $x=0$ $\therefore c=0$	1 for $x = \frac{Vt}{\sqrt{2}}$
$\therefore \boxed{x = \frac{Vt}{\sqrt{2}}}$	

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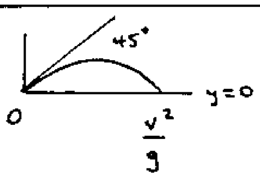
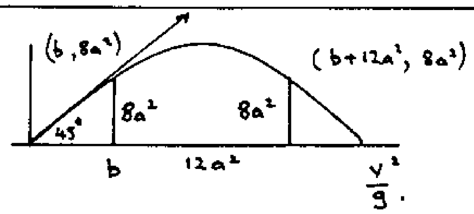
SOLUTIONS	COMMENTS
b.) $t = \frac{\sqrt{2}\,x}{V}$ $y = \frac{-gt^2}{2} + \frac{Vt}{\sqrt{2}}$ $y = \frac{-g\left(\frac{\sqrt{2}\,x}{V}\right)^2}{\cancel{2}} + \frac{\cancel{V}\frac{\cancel{\sqrt{2}}\,x}{\cancel{V}}}{\cancel{\sqrt{2}}}$ $\boxed{y = x - \frac{gx^2}{V^2}}$	1 for substitution 1 for equation
c.) when $y = 0$  $0 = x - \frac{gx^2}{V^2}$ $0 = xV^2 - gx^2$ $0 = x(V^2 - gx)$ $x = 0$ or $V^2 = gx$ $x = \frac{V^2}{g}$ $\therefore$ $\boxed{\text{range of projectile } \frac{V^2}{g}}$	1 for solving equation 1 for correct range
d.) i)  $\therefore$ $b + 12a^2 + b = \frac{V^2}{g}$ $\boxed{\frac{V^2}{g} = 2b + 12a^2}$	(via diagram) ↑ 1 for showing $\frac{V^2}{g} = 2b + 12a^2$

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SOLUTIONS	COMMENTS
d.) ii.) the first post has co-ordinates $(b, 8a^2)$ $\therefore \quad y = x - \frac{gx^2}{V^2}$ sub $x = b$ $y = 8a^2$ $8a^2 = b - \frac{gb^2}{V^2}$ $\therefore \quad \boxed{8a^2 = b - \frac{gb^2}{V^2}}$	1 for sub of (x,y) and getting equation
e.) $\frac{V^2}{g} = 2b + 12a^2$ (i) $8a^2 = b - \frac{gb^2}{V^2}$ (ii) sub (1) into (2) $8a^2 = b - \frac{gb^2}{g(2b+12a^2)}$	1 for substitution
$8a^2 = b - \frac{b^2}{2b+12a^2}$ $b - 8a^2 = \frac{b^2}{2b+12a^2}$ $(b-8a^2)(2b+12a^2) = b^2$ $2b^2 - 4a^2b - 96a^4 = b^2$ $b^2 - 4a^2b - 96a^4 = 0$ $(b-12a^2)(b+8a^2) = 0$ $b = 12a^2$, $b = -8a^2$ (only positive for length). $\therefore b = 12a^2$	1 for solving equation
sub into (i) $\frac{V^2}{g} = 24a^2 + 12a^2 = 36a^2$ $V^2 = 36a^2g$ $\quad \therefore \boxed{V = 6a\sqrt{g}}$	1 for substitution and answer.