$8\cos\theta$

$5\sin\theta$

$\frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$5\cos\theta \times \frac{1}{-8\sin\theta}$ (1)

$\frac{5\cos\theta}{-8\sin\theta}$

$\frac{\pi}{4}$

$\frac{5\cos\frac{\pi}{4}}{8\sin\frac{\pi}{4}}$

$(\frac{x}{2})^6$

1 1

3 3 1

4 6 4 1

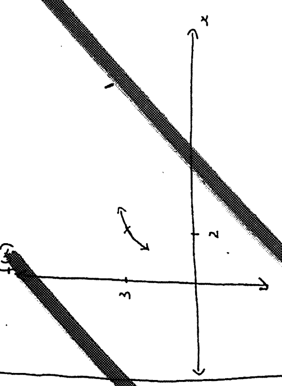
10 10 5 1 (1)

15 20 15 6 1

$1 + \frac{6x}{2} + \frac{15x^2}{4} + \frac{20x^3}{8} + \frac{15x^4}{16} + \frac{6x^5}{32} + \frac{x^6}{64}$ (1)

$1 + 3x + \frac{15x^2}{4} + \frac{5x^3}{2} + \frac{15x^4}{16} + \frac{3x^5}{16} + \frac{x^6}{64}$

c) $f(2) = 3$, $f'(2) = 1$, $f''(2) = -2$



* Goes through the point (2,3)
* Has a positive gradient ∴ leans to the right (1)
* Has a negative second derivative ∴ is concave down near this point. (1)

d) $2x + 3y - 13 = 0$ (1)

$2x - 5y - 5 = 0$ (2)

Rearrange (1): $3y = -2x + 13$, $y = -\frac{2}{3}x + \frac{13}{3}$

Rearrange (2): $5y = 2x - 5$, $y = \frac{2}{5}x - 1$ (1)

gradient (1) $= -\frac{2}{3}$ gradient (2) $= \frac{2}{5}$

∴ $\tan\theta = -\frac{2}{3}$ $\tan\beta = \frac{2}{5}$

$\theta = 146°19'$ $\beta = 21°48'$ (1)

∴ Angle between these two lines $= 55°29'$ (1)

Q1 cont'd.

e) $\int_0^2 \frac{-1}{\sqrt{16-x^2}}\,dx = \left[\cos^{-1}\frac{x}{4}\right]_0^2$ (1)

$= \cos^{-1}\frac{1}{2} - \cos^{-1}0$

$= \frac{\pi}{3} - \frac{\pi}{2}$

$= -\frac{\pi}{6}$ (1)

TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

TRIAL HIGHER SCHOOL CERTIFICATE 1996

MATHEMATICS

3/4 UNIT (COMMON)

Time Allowed - Two hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- Write your Student Number and Class on each answer that you hand in.
- All necessary working should be provided in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied.
- Board Approved calculators may be used.
- *The question paper must be handed to the supervisor at the end of the examination.*

11/

stion 1 (Start a new page)

Marks

(a) If A and B have coordinates (3,-1) and (-2,4) respectively, find the coordinates of the point that divides the interval AB externally in the ratio 2:1 — 2

(b) Find the volume of the solid of revolution formed by rotating the curve $y=\frac{4}{\sqrt{1+x^2}}$ about the x axis from x = 0 to x = 1. — 3

(c) Differentiate and simplify — 4

(i) $[x\cos^{-1}x-\sqrt{1-x^2}]$

(ii) $\log_e(\sin^{-1}x)$

(d) Prove the identity $\frac{2\cos A}{\operatorname{cosec}A-2\sin A}=\tan 2A$ — 3

estion 2 (Start a new page)

(a) Find the term independent of x in the expansion $\left[2x+\frac{1}{x^2}\right]^6$ — 2

(b) If $x^4+2x^3+ax^2+bx-2$ is divisible by x^2+x+2, find the value of a and b. — 3

(c) Find the acute angle between the lines — 2

$$y = 3x + 2$$
$$3x + 2y = 5$$

(d) Evaluate the definite integrals — 5

(i) $\int_0^1 \frac{dx}{\sqrt{9-4x^2}}$

(ii) $\int_1^e \frac{dx}{x(1+\log x)^2}$ by using the substitution u = 1 + log x

Question 3 (Start a new page)

Marks

(a) Tangents are drawn from the point $\left(\frac{1}{2},-\frac{1}{2}\right)$ to the points P and Q on the parabola $x^2=4y$. Find the equation of the chord of contact PQ and the coordinates of P and Q. — 5

(b) P $(4p,2p^2)$ is a point on the parabola $x^2=8y$ and S is the focus. The tangent of the parabola at P meets the Yaxis at M. The perpendicular from the focus S to the tangent PM meets the tangent at N. Find: — 7

(i) The coordinates of the points M and N.

(ii) The coordinates of the midpoint of the interval MN

(iii) The equation of the locus of the midpoint of MN as P varies.

Question 4 (Start a new page)

(a) Show that $3\sin\theta+4\cos\theta$ may be expressed in the form $R\sin(\theta+\alpha)$ where $R>0$ and $0\le\alpha\le 90^0$ — 6

(i) Find the values of R and α (to the nearest minute)

(ii) Hence or otherwise solve $3\sin\theta+4\cos\theta=1$ for $0\le\theta\le 360^0$

(b) If α,β and γ are the roots of the equation $x^3+2x^2+3x+4=0$ find the value of :

(i) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

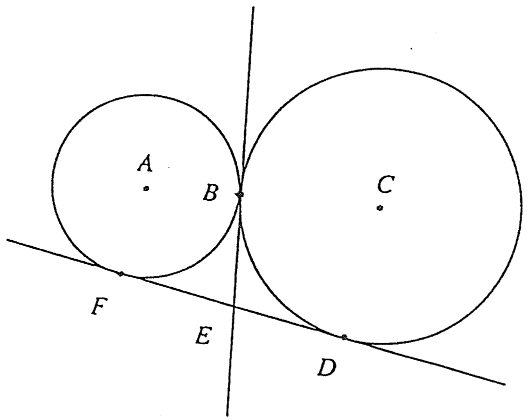
(ii) $\frac{1}{\alpha^2\beta^2}+\frac{1}{\alpha^2\gamma^2}+\frac{1}{\beta^2\gamma^2}$

(iii) $(\alpha-1)(\beta-1)(\gamma-1)$

uestion 5 (Start a new page)

Marks

(a) **4**



Tangents BE and FD are common to the circles with centres A and C. Prove that angle FBD = 90^0

(b) For the function $y=2\cos^{-1}\left(\frac{x}{4}\right)$ **3**

(i) State the domain
(ii) State the range
(iii) Sketch the function

(c) Write in terms of "n" where n is an integer, the general solution of the equation: **2**

$$\sin 2x=\frac{1}{2}$$

(d) Use one step of Newton's method to find an approximation for a root of **3**

$$f(x)=\ln x-\cos x, \text{ near } x=1$$

Question 6 (Start a new page)

Mark

(a) A steady wind is blowing with a speed of 36 kilometres per hour. From clouds moving horizontally with the wind, heavy raindrops fall to the ground 200 metres below. **8**

(i) Find the time taken for a drop to reach the ground.

(ii) Find the speed and angle at which a drop hits the ground (assumed horizontal)

(iii) At what angle does a drop hit the ground when the wind speed is doubled?
(Air resistance may be neglected and the approximate value $g = 10ms^{-1}$ may be assumed.)

(b) Prove by mathematical induction that : **4**

$$5^n + 2(11^n)$$

is a multiple of 3 for all positive integers n

uestion 7 (Start a new page)

Marks

(a) A particle under Simple Harmonic Motion has its displacement, x metres, after t seconds given by **7**

$$x=5\cos\left[\frac{\pi}{2}\left(t+\frac{1}{3}\right)\right]$$

(i) Show that $\ddot{x}=-\frac{\pi^2}{4}x$

(ii) State the period and amplitude of the motion

(iii) Find the speed and acceleration when $x=-\frac{5}{2}$

(b) It is given that the decrease of temperature of a body hotter than surrounding air is proportional to the temperature difference. If A is the air temperature, and T the temperature of the body after t minutes then, **5**

$$\frac{dT}{dt}=-k(T-A)$$

(i) Show that if I is the initial temperature of the body, then the following function satisfies this condition:

$$T=A+(I-A)e^{-kt}$$

(ii) An ingot of pig iron initially at a temperature of $1500^\circ C$ is allowed to cool in the open, where the temperature is $20^\circ C$. If it cools to $1200^\circ C$ in five minutes, find the temperature of the ingot after one hour. (Correct to four significant figures)

END OF EXAM

1996 3 UNIT TRIAL (TRINITY)

Q1 a) $x = \frac{-4-3}{1}$, $y = \frac{8+1}{1}$

$(-7, 9)$

b) $V = \pi \int y^2\,dx$

$= \pi \int_0^1 \frac{16}{1+x^2}\,dx$

$= 16\pi \left[\tan^{-1} x\right]_0^1$

$V = 4\pi^2$ units^3

c) i) $y = x\cos^{-1}x - \sqrt{1-x^2}$

$\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}} + \cos^{-1}x + \frac{x}{\sqrt{1-x^2}}$

$\frac{dy}{dx} = \cos^{-1}x$

ii) $y = \log_e(\sin^{-1}x)$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}\cdot\sin^{-1}x}$

d) Prove $\frac{2\cos A}{\operatorname{cosec}A - 2\sin A} = \tan 2A$

LHS $= \frac{2\cos A}{\frac{1}{\sin A} - 2\sin A}$