-tions Ext. 1 Mathematics 2001 trial. | Comments

) $y = x\ln x - x$, $y' = x \cdot 1/x + \ln x \cdot 1 - 1$ ✓✓
$= 1 + \ln x - 1 = \ln x$

) $\int_2^e \ln x\,dx = x\ln x - x\Big]_2^e = (e\ln e - e) - (2\ln 2 - 2)$
$= 2(1 - \ln 2)$ ✓✓

$\frac{x}{x-2} \le 3$ $\left[\times (x-2)^2\right]$ $x(x-2) \le 3(x-2)^2$ ✓
$x^2 - 2x \le 3(x^2 - 4x + 4)$, $x^2 - 2x \le 3x^2 - 12x + 12$ ✓
$0 \le 2x^2 - 10x + 12$, $0 \le 2(x-3)(x-2)$ ✓
$\therefore$ $x < 2$ or $x \ge 3$ $(x \ne 2)$

$u = x^3 + 1$ $\therefore du/dx = 3x^2$ $\therefore dx = du/3x^2$ ✓
$\int x^2\sqrt{x^3+1}\,dx = \int x^2\sqrt{u}\,du/3x^2 = \frac{1}{3}\int u^{1/2}du$ ✓
$= \frac{1}{3}\left[2u^{3/2}/3 + c\right] = \frac{2\sqrt{(x^3+1)^3}}{9} + c$ ✓

when $x = -2$ $(-2)^3 + 2(-2)^2 + a(-2) + b = 0$
$-8 + 8 - 2a + b = 0$ or $-2a + b = 0$ (1)
when $x = 2$ $(2)^3 + 2(2)^2 + a(2) + b = 12$ ✓
$8 + 8 + 2a + b = 12$ or $2a + b = -4$ (2)
solving (1) and (2) $2b = -4$ $\therefore b = -2$
$a = -1$ ✓

Comments

Q1
(a)(i) Many students need to practice product rule as they made careless errors.

b) Memorize this method. It is the easiest to use

c) ½ mark off for not replacing u with x^3+1 at end of working out.

i) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ ✓

i) $7\pi/12 = \pi/4 + \pi/3 \quad \tan \pi/4 = 1, \ \tan \pi/3 = \sqrt{3}$ ✓

$\tan(\pi/4 + \pi/3) = \dfrac{1 + \sqrt{3}}{1 - (1)(\sqrt{3})} = \dfrac{1+\sqrt{3}}{1-\sqrt{3}} \times \dfrac{1+\sqrt{3}}{1+\sqrt{3}}$ ✓

$= \dfrac{1 + 2\sqrt{3} + 3}{1 - 3} = \dfrac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$ ✓

$8\cos^2 x - 8\sin^2 x = 5, \quad \cos^2 x - \sin^2 x = 5/8$ ✓

$\cos 2x = 5/8 \ \therefore 2x = 51°19',$ ✓✓

$x = 25°40', 154°20', 205°40', 334°20'$

or $n=1 \quad 6^1 - 5 = 5 \ \therefore$ true for $n = 1$

ssume true for $n=k$ ie $6^k - 1 = 5m$ where m ✓

a positive integer.

e true for $n=k+1$

$6^{k+1} - 1 = 6 \times 6^k - 6 + 6 - 1 = 6(6^k - 1) + 5$ ✓

$= 6(5m) + 5 = 5(6m+1) \ \therefore$ divisible by 5 ✓

ie true for $n=1$ and for the next value of n ✓

$n=2$ and so on it is true for all values of $n \geq 1$

$\lim\limits_{x\to 0} \dfrac{\sin 4x}{9x} = \lim\limits_{x\to 0} \dfrac{\sin 4x}{9/4(4x)} = \lim\limits_{x\to 0} \dfrac{4}{9} \times \dfrac{\sin 4x}{4x} = 4/9$

✓

i) $x = 10\sin t/2$, $\dot{x} = 10\left(\tfrac{1}{2}\right)\cos t/2$

$\ddot{x} = -\left(\tfrac{1}{2}\right)^2\left(10\sin t/2\right) = -\tfrac{1}{4}x = -x/4$ ✓✓

ii) amplitude = 10 metres

Period $= 2\pi/\tfrac{1}{2} = 4\pi$ ✓

ii) $\dot{x} = 5\cos t/2$ max speed when $\cos t/2 = \pm 1$

$\therefore = 5$ metres/sec. ✓✓

(i) $y = x^2/4a$ $\quad y' = 2x/4a = \dfrac{x}{2a}$, when $x = 2at$

$y' = M_{tangent} = 2at/2a = t \;\therefore M_{Normal} = -1/t$ ✓✓

using $y - y_1 = m(x - x_1)$ $\quad y - at^2 = -\tfrac{1}{t}(x - 2at)$

$\therefore yt - at^3 = -x + 2at \;\therefore x + ty = 2at + at^3$

ii) If Normal goes through $(0,a)$

$0 + t(a) = 2at + at^3$, $0 = at + at^3$, $0 = at(1+t^2)$

this has only one solution for t, $t=0$ (since $1+t^2=0$ has no solution) ✓✓

$\int_0^{\pi/2} \cos x\,[\sin x]^3\,dx$, if $u = \sin x$ $\quad du/dx = \cos x$ ✓

$\therefore dx = du/\cos x$

$\int \cos x\, u^3\, du/\cos x = \int u^3 dx = u^4/4 = \left[\sin x\right]^4/4\Big]_0^{\pi/2}$ ✓

$= \dfrac{[\sin \pi/2]^4 - [\sin 0]^4}{4} = 1/4$ ✓

i) Generally well done. Important to use calculus rather than qua SHM equations.

ii) Some mechani errors here, esp. finding the peri

iii) A few differen methods employe here. Some studen lost marks for not taking absolute value.

(b) (i) very well done by most students.

ii) relatively poo response here

(c) (i) few problem encountered here if correc substitution was used.

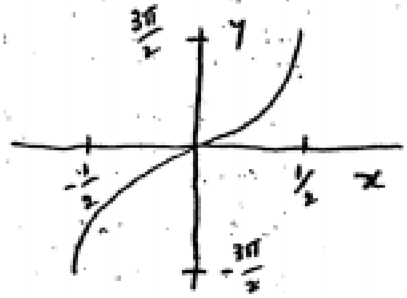
L Beevers.

) $f(\tfrac{1}{4}) = 3\sin^{-1}\tfrac{1}{2} = 3(\tfrac{\pi}{6}) = \tfrac{\pi}{2}$

) $-1 \le 2x \le 1 \quad \therefore$ domain $-\tfrac{1}{2} \le x \le \tfrac{1}{2}$

Range $y = \sin^{-1}\theta$ is $-\tfrac{\pi}{2} \le y \le \tfrac{\pi}{2}$

$\therefore$ range $-\tfrac{3\pi}{2} \le y \le \tfrac{3\pi}{2}$



$f'(x) = 3 \times \dfrac{2}{\sqrt{1-(2x)^2}} = \dfrac{6}{\sqrt{1-4x^2}}$

i) If a G.P. $\beta/\alpha = \delta/\beta \quad \therefore \beta^2 = \alpha\delta$

ii) $\alpha\beta\delta = 54/2 = 27$

) $\beta^2 = \alpha\delta \quad \therefore \beta^2 \times \beta = \beta^3 = 27 \quad \therefore \beta = 3$

$\therefore \alpha\delta = 9$

Also $\alpha + \beta + \delta = -\tfrac{9}{2} \quad \therefore \alpha + 3 + \tfrac{9}{\alpha} = -\tfrac{9}{2}$

$\therefore 2\alpha^2 + 15\alpha + 18 = 0$

$(2\alpha + 3)(\alpha + 6) = 0$

$\therefore \alpha = -6$ or $\alpha = -\tfrac{3}{2}$

$\delta = -\tfrac{3}{2}$ or $\delta = -6$

$\therefore \alpha, \beta, \delta$ are $-6, 3, -\tfrac{3}{2}$

Q4

(a)

(ii) If domain and range incorrect but graph is ok 2/4

(iv) Best to use $y = \sin^{-1} f(x)$ $\quad y' = \dfrac{f'(x)}{\sqrt{1-(f(x))^2}}$

(iv) If the student did not use $\beta^2 = \alpha\delta$ then the question is difficult to '–

(i) $d^2x/dt^2 = -72/x^2$

acc. $= d\left(\tfrac{1}{2}v^2\right)/dx = -72x^{-2}$

$\therefore \tfrac{1}{2}v^2 = \int -72x^{-2}\,dx = -72x^{-1}/-1 + C = 72/x + C$

when $x = 9$, $v = 4$ $\quad \tfrac{1}{2}(4)^2 = 72/9 + C$

$\therefore C = 0 \quad \therefore \tfrac{1}{2}v^2 = 72/x \quad v^2 = 144/x$

$v = \pm 12/\sqrt{x}$. Since when $t = 0$ v is $+$ $\therefore$ use

$v = 12/\sqrt{x}$ ✓✓✓

) $dx/dt = v = 12/\sqrt{x} \quad \therefore dt/dx = x^{1/2}/12$

$\therefore t = \int \frac{x^{1/2}}{12}\,dx = \frac{2x^{3/2}}{3 \times 12} + C = \frac{\sqrt{x^3}}{18} + C$

when $t = 0$, $x = 9$ $\therefore 0 = 27/18 + C \therefore C = -27/18 = -3/2$

$\therefore t = \sqrt{x^3}/18 - 3/2$ ✓✓

) $t = \sqrt{35^3}/18 - 3/2 \doteqdot 10$ seconds ✓

$\frac{\operatorname{cosec}^2 A}{\cot^2 A - 1} \equiv \sec 2A$ ✓✓

$LHS = \frac{1/\sin^2 A}{\cos^2 A/\sin^2 A - 1} = \frac{1/\sin^2 A}{\frac{\cos^2 A - \sin^2 A}{\sin^2 A}} = \frac{1}{\cos^2 A - \sin^2 A} = \frac{1}{\cos 2A} = \sec 2A$

$\therefore LHS = RHS$

(i) Domain $-1 \le 2x \le 1 \quad \therefore -\tfrac{1}{2} \le x \le \tfrac{1}{2}$

Range of $y = \cos^{-1}\theta$ is $0 \le \theta \le \pi \quad \therefore \frac{\pi}{2} - 0 = \frac{\pi}{2},\ \frac{\pi}{2} - \pi = -\frac{\pi}{2}$

$\therefore$ Range is $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ ✓✓

ii) $y = \frac{\pi}{2} - \cos^{-1}\tfrac{1}{2} = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$ ✓

ii)

✓

$y' = e^{-x}\cos x + e^{-x}(-1)\sin x$ for St. points

$0 = e^{-x}\cos x - e^{-x}\sin x = e^{-x}(\cos x - \sin x)$

$e^{-x} \neq 0$ ∴ only solutions $\cos x - \sin x = 0$ ✓

- y' solns

$\cos x = \sin x$ ∴ $\tan x = 1$ $\quad x = \pi/4,\ 5\pi/4$ ✓

· testing it is a max.

testing for turning points ✓

$x = \pi/4$

x	0.7	$\pi/4$	0.8
y'	+	0	−

∴ when $x = \pi/4$ this is a maximum turning point

$x = \frac{5\pi}{4}$

x	3.5	$\frac{5\pi}{4}$	4
y'	−	0	+

∴ when $x = 5\pi/4$ this is a minimum turning point

∴ max value $y = e^{-\pi/4}\sin\pi/4 \doteqdot 0.32$

. most students thought that the max would occur when sin x was a max.

$\dot{x} = 12\cos\alpha$, $\quad \dot{y} = 12\sin\alpha$ at $t = 0$

$x = 0 \qquad y = -1$

$\ddot{x} = 0 \qquad \ddot{y} = -10$

$x = (12\cos\alpha)t \qquad acc = \ddot{y} = dv/dt = -10$ ✓✓

- eqtns

- derivation

∴ $v = \dot{y} = \int -10\,dt = -10t + c$

$c = 12\sin\alpha$ ∴ $v = \frac{dy}{dt} = 12\sin\alpha - 10t$

$y = \int 12\sin\alpha - 10t\,dt = (12\sin\alpha)t - 5t^2 + c$

when $t = 0$ $y = -1$ ∴ $c = -1$

∴ $y = -5t^2 + (12\sin\alpha)t - 1$

. a number of students quoted the eqtns rather than deriving them.

Ball lands when $y = 0$, $\sin 30 = 0.5$

$0 = -5t^2 + 6t - 1 \qquad (5t-1)(t-1) = 0$, $t = \frac{1}{5}$ and 1

∴ $x = (12\cos 30°)(1) \doteqdot 10.39$

∴ distance from A $\doteqdot 10.39 - 4 = 6.39$ m

$6\sqrt{3} - 4$

- x

- distance from A ✓

. students are not solving quadratic eqtn correctly.

. students are not reading the question to see what is required but just quoting formulae which requires some reference to question eg in this part having range.

(cont)

Max height when $\dot{y} = 0$ $\therefore 0 = -10t + 12\sin 30°$
or t $\qquad 0 = -10t + 6$
or height $\qquad \therefore t = 0.6$ sec

$y = -5t^2 + (12\sin\alpha)t - 1$ $\therefore y = -5(0.6)^2 + 6(0.6) - 1$
$= 0.8$m ✓✓

Point A is $(4, 0)$
then $x = 4$ $\quad 4 = (12\cos\alpha)t$ $\therefore t = \frac{1}{3\cos\alpha}$ ✓ ①
then $y = 0$ $\quad 0 = -5t^2 + (12\sin\alpha)t - 1$ ②
sub ① into ②

$$-5\left(\frac{1}{3\cos\alpha}\right)^2 + 12\sin\alpha \times \frac{1}{3\cos\alpha} - 1 = 0$$

$$5\sec^2\alpha - 36\tan\alpha + 9 = 0 \quad (\sec^2\alpha = \tan^2\alpha + 1)$$

$$\therefore 5\tan^2\alpha - 36\tan\alpha + 14 = 0$$

$$\tan\alpha = 36 \pm \sqrt{36^2 - 4(5)(14)}/10 \doteqdot 0.4125 \text{ or } 6.7875$$

$$\therefore \alpha = 22.4 \text{ or } 81.6$$

since when rounding to nearest degree $\alpha \neq 22°$ or $82°$

$$\therefore 23° \leq \alpha \leq 81°$$ ✓✓✓

1 for $t = \frac{1}{3\cos\alpha}$

1 for a quad.

1 for correct angles.

• most students did not know what to do with this question

• those that formed the quadratic could not solve it correctly.