Pittwater House Schools

4 unit mathematics

Trial HSC Examination 1992

1. **(a)** Find the indefinite integrals of **(i)** $e^x \sin 2x$ **(ii)** **(iii)** $\frac{e^x + e^{2x}}{1+e^{2x}}$
(b) Evaluate $\int_0^1 \frac{x^2\ dx}{\sqrt{4-x^2}}$.

2. **(a)** Express $\sqrt{5-12i}$ in the form $a+ib$ where a and b are real and rational
(b) Solve $2x^2 - 6ix - 3 = 0$
(c) Simplify $\frac{(\cos\theta + i\sin\theta)^5(\cos 2\theta + i\sin 2\theta)^{-2}}{(\sin 3\theta - i\cos 3\theta)^4}$
(d) If q is real and $z = \frac{3+iq}{3-iq}$ show that as q varies, the point in the complex plane which represents z lies on a circle. Find the centre and radius of this circle.

3. **(a)** Find the equation of the ellipse with its centre at the origin passing through the point $(\frac{9}{4}, 4)$ and one focus at the point $(0, 4)$.
(b) Given the ellipse $\frac{x^2}{225} + \frac{y^2}{144} = 1$, prove that the section of the tangent between the point of contact and its point of intersection with the directrix subtends a right angle at the corresponding focus.

4. **(a)** Sketch the graphs of **(i)** $y = |\tan x|$ **(ii)** $y = \frac{1}{1-e^{-x}}$ **(iii)** $y = x + \sin x$
(b) Sketch $y = \frac{4(2x-7)}{(x-3)(x+1)}$ showing clearly the points of intersection with the x and y axis, the coordinates of any maximum or minimum points and the equation of any asymptotes.

5. **(a)** The circle $x^2 + y^2 = a^2$ is rotated about the x axis to form a sphere. A hole of diameter a is bored through the centre of the sphere. Find the remaining volume using cylindrical shells.
(b) A vase is such that any cross section parallel to the base is an ellipse of eccentricity $\frac{4}{5}$. If the semi minor axis of height y is equal to the distance of the curve $y^2 = 50(x-4)$ from the y axis and the height of the vase is 20 centimetres, find the volume.

6. **(a)** Find the factor of $P(x) = (x^2 - 2x)^2 - 4$ over the rational, real and complex fields.
(b) If α, β and γ are the roots of the equation $z^3 - z - 4 = 0$, form the equation with roots $\frac{\alpha+1}{\alpha}$, $\frac{\beta+1}{\beta}$, $\frac{\gamma+1}{\gamma}$.

7. **(a)** A car is travelling round a section of a race track which is banked at $15°$. The radius of the track is 100 metres. What is the speed at which the car can travel

without tending to slip?
(b) The effect of putting a golf ball is to give the ball an initial velocity of V m/s at the origin. The effect of the green is to give the ball a retardation of $\frac{1}{2}Ve^{-\frac{t}{2}}$. To sink a putt, the golfer must judge V so that the ball reaches the cup with a speed v where $0 < v < \frac{1}{2}$. Find the initial speed for a golfer to sink a 10 metre putt.
(c) A projectile is fired from the origin with initial velocity having x component v_1 and y component v_2. Prove that the time of flight is independent of v_1 and derive a formula for the time of flight T. (The x axis is at ground level and the y axis points vertically upwards.)

8. (a) If $x^m y^n = k$ where k is a constant, show $\frac{dy}{dx} = \frac{my}{-nx}$
(b) (i) Factorise $1 + x + x^2 + x^3$.
(ii) Prove that the equation $\frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + c = 0$ has no real roots if $c > \frac{5}{12}$. How many real roots are there if $c \leq \frac{7}{12}$?
(c) If x, y and z are real numbers, prove that $x^2y^2 + y^2z^2 + z^2x^2 \geq xyz(x + y + z)$.