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Centre Number

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Student Number



CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2006
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

Morning Session
Monday 7 August 2006

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- All necessary working should be shown in every question
- Write your Centre Number and Student Number at the top of this page

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value

Disclaimer

Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, *Principles for Setting HSC Examinations in a Standards-Referenced Framework* (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and *Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework* (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

Total marks – 120

Attempt Questions 1-10

All questions are of equal value.

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) Evaluate $\sqrt[3]{\frac{-38.67 \times 7.2}{(11.7)^2 - (1.83)^2}}$ correct to 3 decimal places. 2
- (b) By rationalising the denominator, simplify $\frac{1-\sqrt{2}}{2-\sqrt{8}}$. 2
- (c) Solve $\cos \theta = -\frac{1}{2}$, for $0 \leq \theta \leq 2\pi$. 2
- (d) Completely factorise $4xy + xb + 8ay + 2ab$. 2
- (e) For what values of k does the quadratic equation $4x^2 + kx + 9 = 0$ have equal roots? 2
- (f) Solve the equation $|x - 2| = 3$. 2

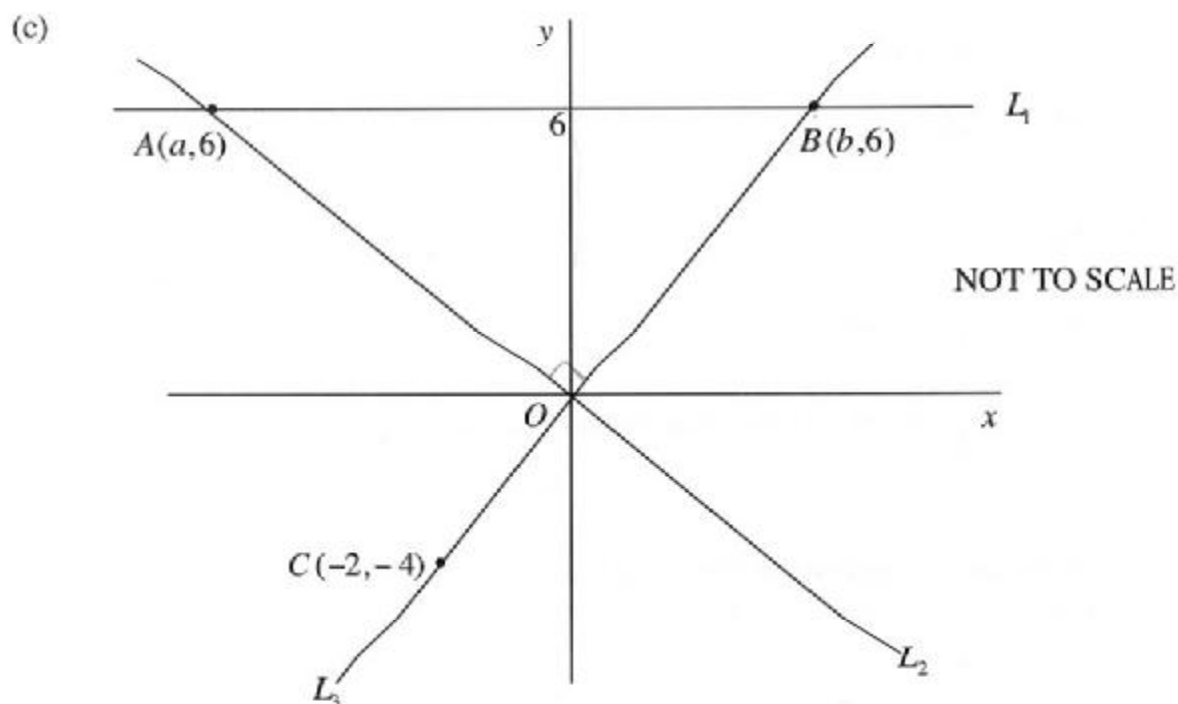
Question 2 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) Sketch the region represented by $x^2 + y^2 < 4$. 2

- (b) The function $f(x)$ is given by $f(x) = \begin{cases} x+1 & \text{for } x \leq 3 \\ x^2-9 & \text{for } x > 3 \end{cases}$ 2

Find $f(3) - f(6)$.



On the diagram above line L_1 is parallel with the x -axis and crosses the y -axis at 6. Lines L_2 and L_3 each pass through the origin, O , and intersect with the line L_1 at the points $A(a, 6)$ and $B(b, 6)$ respectively. The point $C(-2, -4)$ lies on the line L_3 .

- (i) Show that the equation of the line L_3 is $2x - y = 0$. 2
- (ii) Show that the x -ordinate of point B is 3. 1
- (iii) Find the coordinates of point A such that $\angle AOB$ is a right angle. 3
- (iv) Write down the distance between points A and B . Hence, or otherwise calculate the area of $\triangle AOB$. 2

Question 3 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) For the parabola $(x - 2)^2 = 16y$, find the coordinates of the:
- (i) Vertex. 1
 - (ii) Focus. 1
- (b) Differentiate with respect to x the following expressions:
- (i) $3x \log_e x$. 2
 - (ii) $\sin^2 x$. 2
- (c) Find:
- (i) $\int \cos 2006x \, dx$. 2
 - (ii) $\int_0^1 e^{2x} \, dx$. (Leave your answer in exact form). 2
- (d) Show that the equation of the normal to the curve $y = x^3 - 5x$ at the point $(1, -4)$ is given by $x - 2y - 9 = 0$. 2

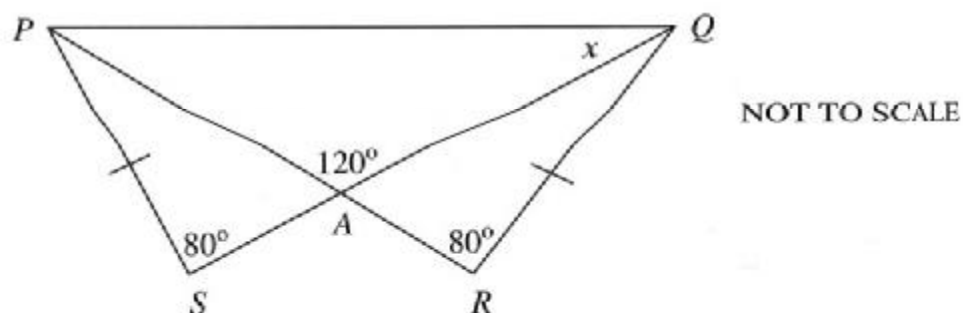
Question 4 (12 marks) Use a SEPARATE writing booklet.

Marks

(a) Consider the curve $f(x) = -\frac{1}{3}x^3 - x^2 + 3x + 1$.

- | | | |
|-------|---|---|
| (i) | Find the coordinates of any stationary points and determine their nature. | 3 |
| (ii) | Find any point(s) of inflexion. | 2 |
| (iii) | Sketch the curve in the domain, $-6 \leq x \leq 3$. | 2 |
| (iv) | What is the maximum value of $f(x)$ in the given domain? | 1 |

(b)



PR and QS are straight lines intersecting at point A . Also $PS = QR$, $\angle PSA = \angle QRA = 80^\circ$, $\angle PAQ = 120^\circ$ and $\angle PQA = x$.

- | | | |
|-------|--|---|
| (i) | Copy the diagram into your writing booklet. | |
| (ii) | Prove that $\triangle PSA$ is congruent to $\triangle QRA$. | 2 |
| (iii) | Hence, show that $x = 30^\circ$. | 2 |

Question 5 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) (i) Write $0.7\dot{5}$ as the sum of an infinite geometric series.

1

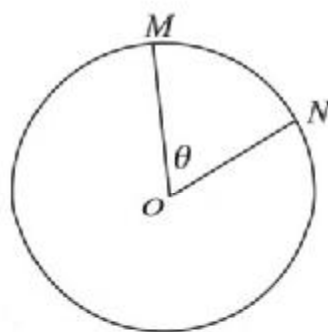
- (ii) Hence, express $0.7\dot{5}$ as a fraction.

2

- (b) Simplify $\frac{1 - \sin^2 x}{\cot x}$.

2

- (c) The circle below has centre O , radius $\frac{6}{\pi}$ cm and arc length $MN = 1$ cm.



NOT TO SCALE

- (i) Find the size of θ in radians.

1

- (ii) Hence, or otherwise, find the exact area of the sector MON .

2

- (d) During qualification for the 2006 World Cup, the Socceroos goalkeeper, Mark, defended many penalty shots at goal. In fact, the probability that he can stop a penalty shot at goal is $\frac{3}{5}$. During a particular match, the opposing team had three penalty shots at goal.

Using a tree diagram, find the probability that:

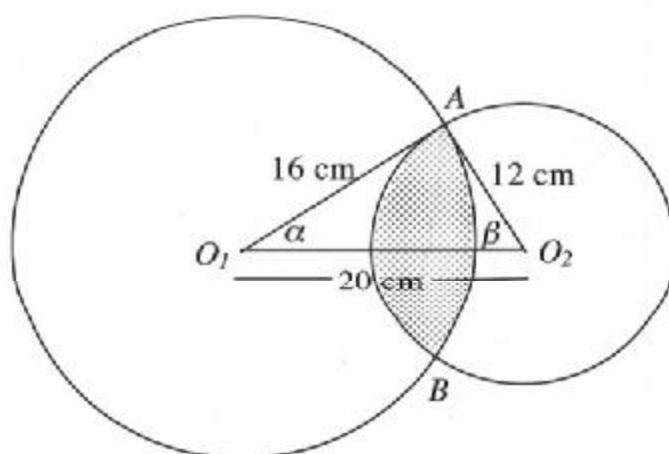
- (i) the goalkeeper will stop all shots at goal.

2

- (ii) the goalkeeper will stop at least 1 shot at goal.

2

(a)



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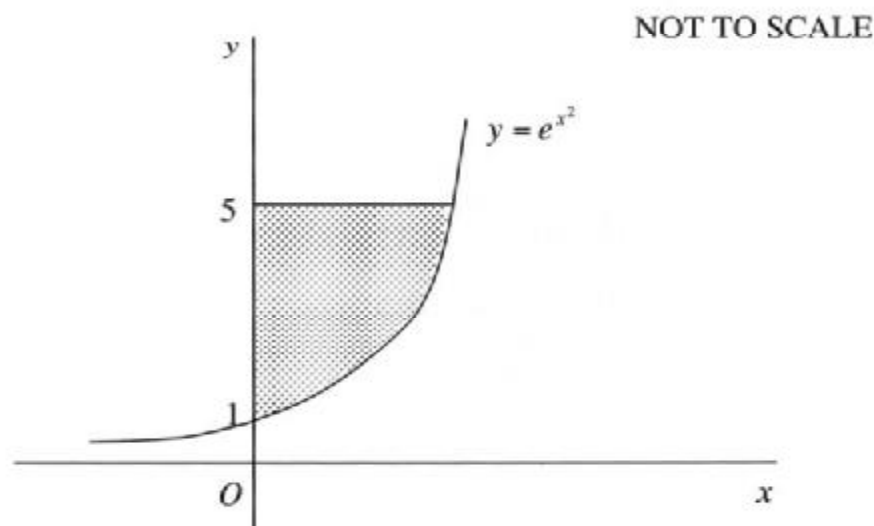
A circle with centre at O_1 and radius 16 cm intersects with another circle with centre at O_2 and radius 12 cm. Their points of intersection are A and B and the distance between their centres, O_1O_2 , is 20 cm.

$\angle AO_1O_2 = \alpha$ and $\angle AO_2O_1 = \beta$.

- | | | |
|-------|---|---|
| (i) | Show that $\triangle O_1AO_2$ is a right angled triangle. | 1 |
| (ii) | Find the size of the angles α and β . | 2 |
| (iii) | Find the shaded area enclosed by these circles. (Give your answer correct to 2 decimal places). | 3 |

Question 6 continues on page 8

(b)



The shaded region bounded by the graph $y = e^{x^2}$, the line $y = 5$ and the y-axis is rotated about the y-axis to form a solid of revolution.

- (i) Show that the volume of the solid is given by $V_y = \pi \int_1^5 \log_e y \, dy$. 2
- (ii) Copy and complete the following table into your writing booklet. Give all answers correct to three decimal places. 1

y	1	2	3	4	5
$\log_e y$	0	0.693	1.099		1.609

- (iii) Use Simpson's Rule with five function values to approximate the volume of the solid of revolution V_y , correct to three decimal places. 3

End of Question 6

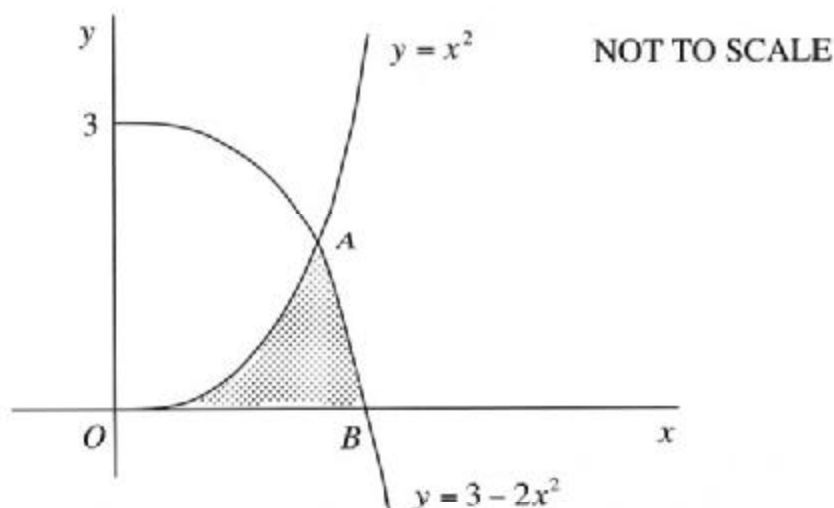
Question 7 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) Solve the following equation: $\log_2 x + \log_2(x+7) = 3$, for $x > 0$.

3

(b)



The shaded region OAB is bounded by the parabolas $y = x^2$, $y = 3 - 2x^2$ and the x -axis. Point A is the intersection of the two parabolas and point B is the x -intercept of the parabola $y = 3 - 2x^2$.

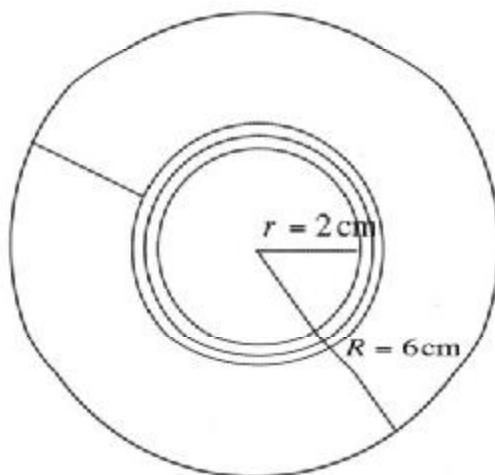
- (i) Find the x -ordinates of points A and B .
- (ii) By considering the sum of two areas, show that the exact area of the shaded region OAB is given by $2\sqrt{\frac{3}{2}} - 2$ square units.

2

3

Question 7 continues on page 10

(c)



NOT TO SCALE

The playing track of a *CD* is made out of a number of concentric circles with the inner circle having a radius of 2 cm and the outer circle having a radius of 6 cm. The *CD* is rotating at 5 revolutions per second and takes 25 minutes to completely play.

- | | | |
|------|---|---|
| (i) | Find the total number of revolutions for this <i>CD</i> . | 1 |
| (ii) | Find the total length of the playing track in km, correct to one decimal place. | 3 |

End of Question 7

Question 8 (12 marks) Use a SEPARATE writing booklet

Marks

- (a) If $A^m = 3$, find the value of $A^{4m} - 5$. 2

- (b) A 100 mg tablet is dissolved in a glass of water. After t minutes the amount of *undissolved* tablet, U in mg, is given by the formula:

$$U = 100 e^{-kt}, \text{ where } k \text{ is a constant.}$$

- (i) Calculate the value of k , correct to 4 decimal places, given that 2 mg of the tablet remain after 10 minutes. 2
- (ii) Find the rate at which the tablet is dissolving in the glass of water after 12 minutes. Give your answer correct to two decimal places. 2

- (c) Mr. Egan borrows $\$P$ from a bank to fund his house extensions. The term of the loan is 20 years with an annual interest rate of 9%. Each month, interest is calculated on the balance at the beginning of the month and added to the balance owing. Mr. Egan repays the loan in equal monthly instalments of $\$1\,050$.

- (i) Write an expression for the amount, A_1 , Mr. Egan owes immediately at the end of the first month. 1
- (ii) Show that at the end of n months the amount owing, A_n , is given by: 3
- $$A_n = P(1.0075)^n - 140\,000(1.0075)^n + 140\,000$$
- (iii) If at the end of 20 years the loan has been repaid, calculate the amount Mr. Egan originally borrowed, correct to the nearest dollar. 2

Question 9 (12 marks) Use a SEPARATE writing booklet.

Marks

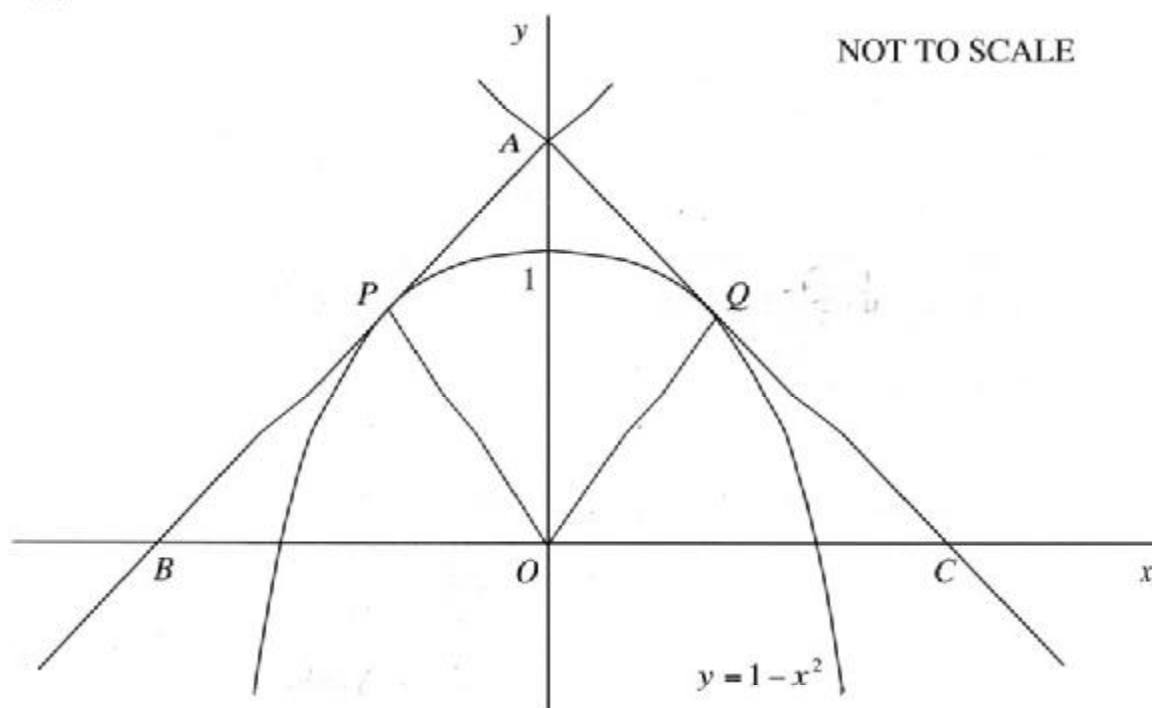
- (a) Two particles, A and B , move along a straight line so that their displacements, x_A and x_B , in metres, from the origin at time t seconds are given by the following equations respectively:

$$x_A = 12t + 5$$

$$x_B = 6t^2 - t^3$$

- | | | |
|-------|--|---|
| (i) | Find two expressions for the velocities of particles A and B . | 2 |
| (ii) | Which of the two particles is travelling faster at $t = 1$ second? | 1 |
| (iii) | At what time does particle B come to rest? | 1 |
| (iv) | Find the maximum positive displacement of particle B . | 2 |

(b)



In the diagram above P and Q are two points on the parabola $y = 1 - x^2$. The tangents to the parabola at P and Q intersect each other on the y -axis at A and the x -axis at B and C respectively. Triangle ABC is an equilateral triangle.

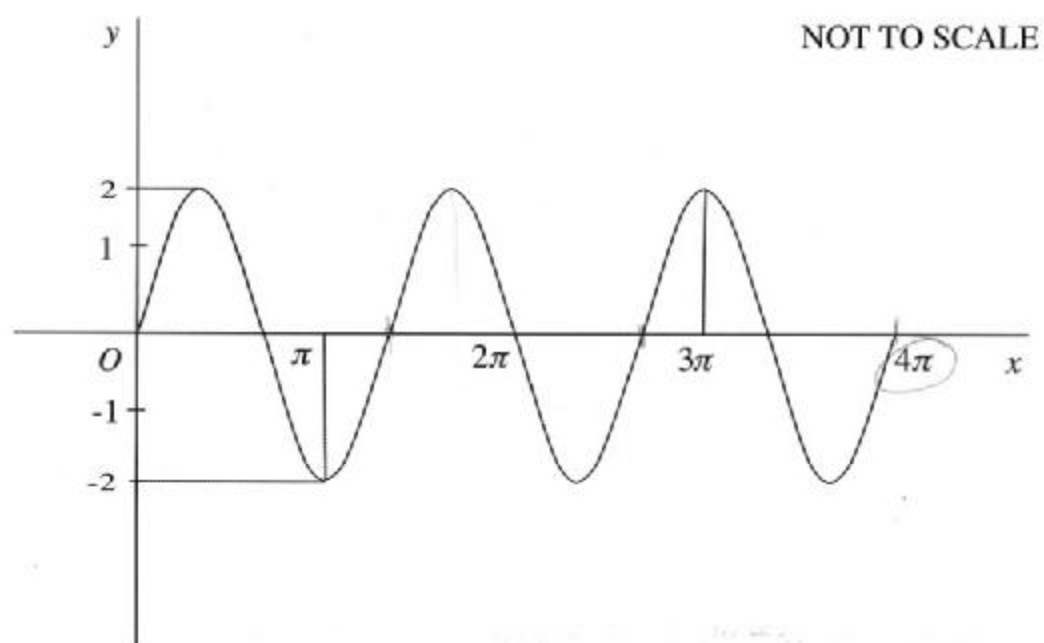
- | | | |
|-------|--|---|
| (i) | Show that the gradient of the tangent at point P is equal to $-\sqrt{3}$. | 1 |
| (ii) | Show that the coordinates of point P are $\left(-\frac{\sqrt{3}}{2}, \frac{1}{4}\right)$. | 2 |
| (iii) | Find the value of $\angle POQ$, in radians, correct to one decimal place. | 3 |

Question 10 (12 marks) Use a SEPARATE writing booklet.

Marks

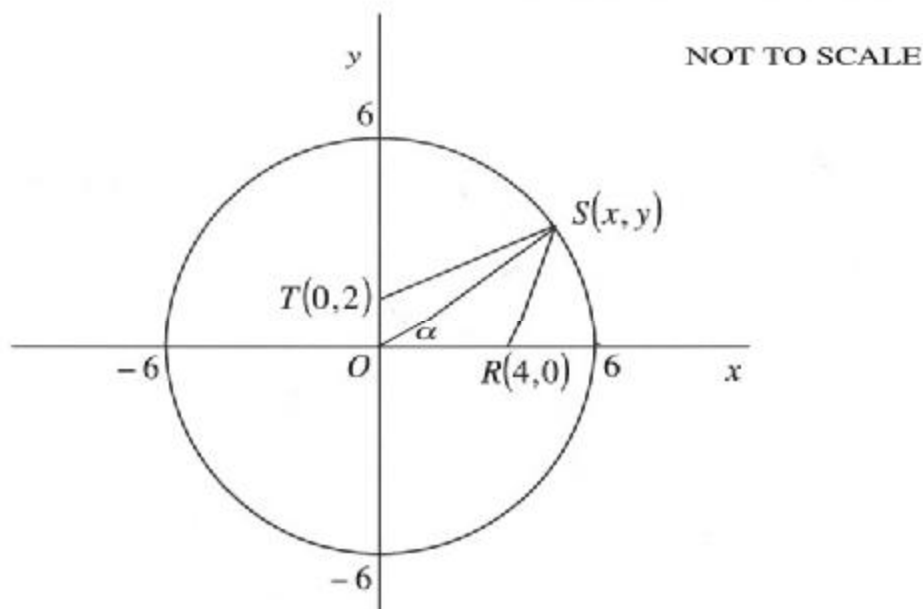
- (a) Find the trigonometric equation for the graph below:

2



Question 10 continues on page 14

(b)



The diagram above shows the circle $x^2 + y^2 = 36$. The point $S(x, y)$ lies on the circle in the first quadrant. O is the origin, $R(4, 0)$ lies on the x -axis and $T(0, 2)$ lies on the y -axis. The size of $\angle ROS$ is α radians, where $0 < \alpha < \frac{\pi}{2}$.

- (i) Show that the area of triangle SOR is $12 \sin \alpha$. 1
- (ii) Hence show that the area, A , of the quadrilateral $ORST$ is given by: 3

$$A = 6 \cos \alpha (2 \tan \alpha + 1)$$

- (iii) Find the value of $\tan \alpha$ for which the area A is a maximum. 3
- (iv) Hence, show that for this maximum area, the coordinates of point S are $\left(\frac{6}{5}\sqrt{5}, \frac{12}{5}\sqrt{5}\right)$ 3

End of Paper