SOLUTIONS TO TRIAL H.S.C. EXTENSION II 2004.

QUESTION 1

(a)(i) $z^2 = (-1+\sqrt{3}\,i)^2$
$= 1 - 2\sqrt{3}\,i + 3i^2$
$= -2 - 2\sqrt{3}\,i$
$= 2(-1-\sqrt{3}\,i)$
$\therefore z^2 = 2\bar{z}$

(ii) $|z| = \sqrt{(-1)^2+(\sqrt{3})^2}$
$\therefore |z| = 2$
$\arg z = \tan^{-1}(-\sqrt{3})$
$\therefore \arg z = \dfrac{2\pi}{3}$

(iii) $z^3 = z \cdot z^2$
$= z \cdot 2\bar{z}$ from (i)
$= 2|z|^2$
$\therefore z^3 = 8$ since $|z| = 2$
$\therefore z^3 - 8 = 0$

(b)(i) $\int x \sec^2(x^2)\,dx$
$= \frac{1}{2}\int 2x \sec^2(x^2)\,dx$
$= \frac{1}{2}\int \sec^2(x^2)\,d(x^2)$
$= \frac{1}{2}\int d\tan(x^2)$
$= \frac{1}{2}\tan(x^2) + C$

(ii) $\int \dfrac{x^4}{x^2+1}\,dx$
$= \int\left(\dfrac{x^4-1+1}{x^2+1}\right)dx$
$= \int\dfrac{(x^2-1)(x^2+1)+1}{(x^2+1)}\,dx$
$= \int\left[(x^2-1) + \dfrac{1}{x^2+1}\right]dx$
$= \dfrac{x^3}{3} - x + \tan^{-1}x + C$

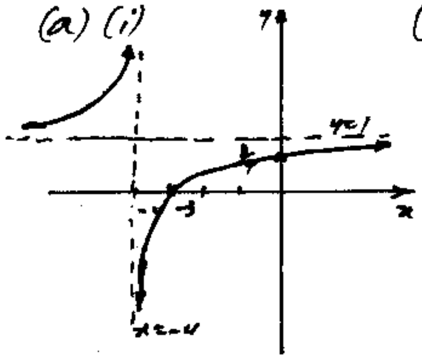
(iii) $\int_0^{\frac{\pi}{3}} \dfrac{\sin^3 x}{\cos^2 x}\,dx$
$= \int_0^{\frac{\pi}{3}}\left(\dfrac{1-\cos^2 x}{\cos^2 x}\right)\sin x\,dx$
$= \int_0^{\frac{\pi}{3}}\left(\dfrac{1-\cos^2 x}{\cos^2 x}\right)(-d\cos x)$

Let $U = \cos x$
when $x = 0$ $U = 1$
$x = \frac{\pi}{3}$ $U = \frac{1}{2}$

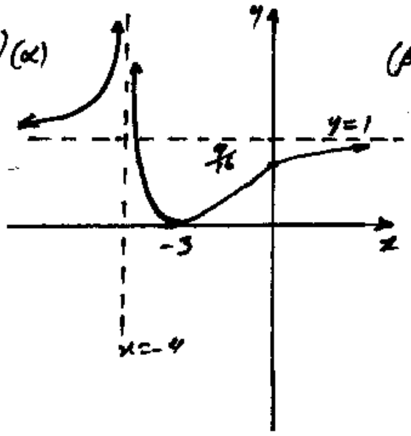
$= \int_1^{\frac{1}{2}}\left(\dfrac{U^2-1}{U^2}\right)dU$
$= \int_1^{\frac{1}{2}}\left(1 - \dfrac{1}{U^2}\right)dU$
$= \left[U + \dfrac{1}{U}\right]_1^{\frac{1}{2}}$
$= 2\frac{1}{2} - 2$
$= \dfrac{1}{2}$

QUESTION 2

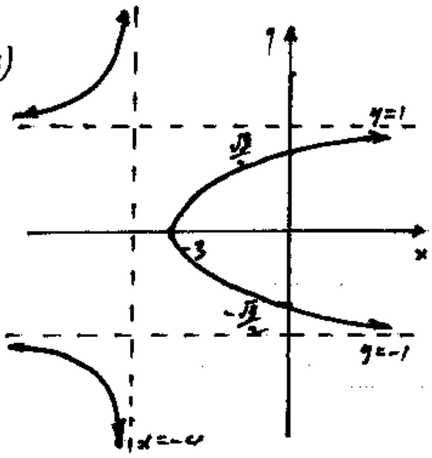
(a) (i)



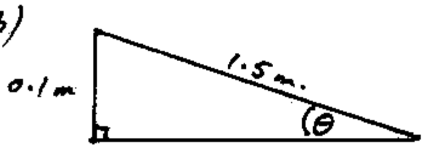
(ii) (α)



(β)



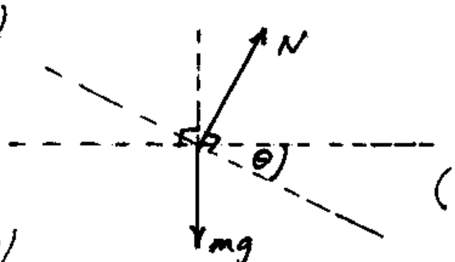
(b)



Motion is in a horizontal circle radius r and constant speed v m/s

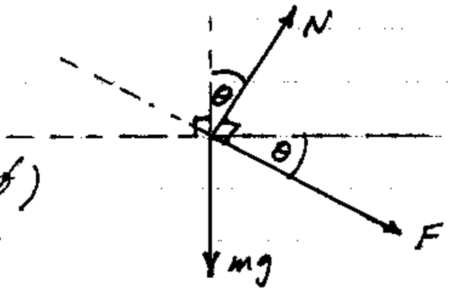
$r = 500$ m.

(i)



(Let weight of train be mg)

(iii)



(ii) (No lateral force on wheels)

$N\cos\theta = mg$ (since vertical component zero)

$N\sin\theta = \dfrac{mv_0^2}{r}$ (since horizontal component $\dfrac{mv^2}{r}$)

$\therefore \tan\theta = \dfrac{mv_0^2}{r} \div mg$

$\therefore \tan\theta = \dfrac{v_0^2}{gr}$

$\therefore v_0^2 = gr\tan\theta$

$\therefore v_0^2 = 500g\tan\theta$

(iv)

$N\cos\theta - F\sin\theta = mg$ —(1)

$N\sin\theta + F\cos\theta = \dfrac{mv^2}{r}$ —(2)

(1) $\times \sin\theta$:

$N\sin\theta\cos\theta - F\sin^2\theta = mg\sin\theta$ —(3)

(2) $\times \cos\theta$:

$N\sin\theta\cos\theta + F\cos^2\theta = \dfrac{mv^2}{r}\cos\theta$ —(4)

(4) − (3):

$F(\cos^2\theta + \sin^2\theta) = \dfrac{mv^2}{r}\cos\theta - mg\sin\theta$

$\therefore F = \dfrac{mv^2}{500}\cos\theta - mg\sin\theta$

(v) Now $F = \dfrac{m\cos\theta}{500} \times 4 \times 500g\tan\theta - mg\sin\theta$ since $v^2 = 4v_0^2$

$\therefore F = 3mg\sin\theta$

$= 3mg \times \frac{1}{15}$

$\therefore F = \frac{1}{5}mg$

QUESTION 3

(a) (i) Let $P(x) = (x-\alpha)^2 Q(x)$

$P'(x) = (x-\alpha)^2 Q'(x) + Q(x) \times 2(x-\alpha)$

$= (x-\alpha)[(x-\alpha)Q'(x) + 2Q(x)]$

$= 0$ when $x = \alpha$ $\therefore \alpha$ is a zero of $P'(x)$.

(ii) Let $P(x) = x^5 + 2x^2 + mx + n$

$P'(x) = 5x^4 + 4x + m$

Since $(x+1)^2$ is a factor of $P(x)$

$\therefore P(-1) = (-1)^5 + 2(-1)^2 + m(-1) + n = 0$ and $P'(-1) = 5(-1)^4 + 4(-1) + m = 0$

$\therefore -1 + 2 - m + n = 0$ and $5 - 4 + m = 0$

$\therefore m - n = 1$ $\therefore n = -2$ $\therefore m = -1$

(b) Let $z = x + iy$

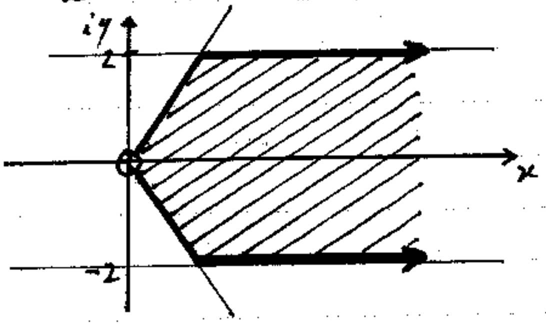
$|z - \bar{z}| = |(x+iy) - (x-iy)|$

$= |2iy|$

$= 2|y|$

$\therefore z$ satisfies $|z - \bar{z}| \leq 4$

iff $|y| \leq 2$. also $-\frac{\pi}{3} \leq \arg z \leq \frac{\pi}{3}$



(c) $x = 2v - \tan^{-1} v$

$\therefore \frac{dx}{dv} = 2 - \frac{1}{1+v^2}$

$\therefore \frac{dx}{dv} = \frac{1+2v^2}{1+v^2}$

$\therefore \frac{dv}{dx} = \frac{1+v^2}{1+2v^2}$

$\therefore \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{(1+v^2)v}{1+2v^2}$

$\therefore \frac{dv}{dt} = \frac{(1+v^2)v}{1+2v^2}$

$\therefore dt = \frac{1+2v^2}{(1+v^2)v}$

$\therefore t = \int \frac{1+2v^2}{(1+v^2)v}\, dv$

$= \int \left(\frac{1}{v} + \frac{v}{1+v^2}\right) dv$

$\therefore t = \ln v + \frac{1}{2}\ln(1+v^2) + C$

when $t = 0$, $v = 1$ $\therefore C = -\frac{1}{2}\ln 2$

$\therefore t = \ln v + \frac{1}{2}\ln(1+v^2) - \frac{1}{2}\ln 2$

$= \ln\left[v\sqrt{\frac{1+v^2}{2}}\right]$

when $v = 7$

$t = \ln\left[7\sqrt{\frac{1+7^2}{2}}\right]$

$\therefore t = \ln 35$

QUESTION 4

(a) (i) Number of ways of choosing 2 from 4 is ${}^4C_2 = 6$.
If players are A, B, C and D then $(A,B)\to(C,D)$ is the same as $(C,D)\to(A,B)$

$\therefore \frac{1}{2}\times {}^4C_2 = \frac{6}{2}$

$= 3$

(b) (i) Since $x = t - y$

$\therefore \frac{dx}{dy} = -1$

$\therefore \int_0^t f(x)\,dx = \int_t^0 f(t-y)\frac{dx}{dy}\cdot dy$

since $x = t - y$

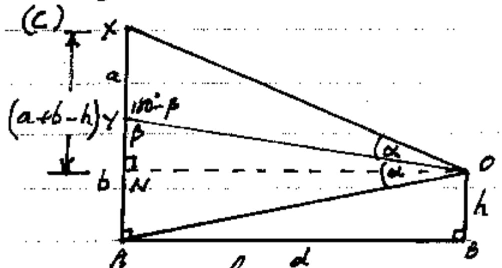
$\therefore \int_0^t f(x)\,dx = -\int_0^t f(t-y)(-1)\,dy$

$= \int_0^t f(t-y)\,dy$

$\equiv \int_0^t f(t-x)\,dx$

$\therefore \int_0^t f(x)\,dx = \int_0^t f(t-x)\,dx$

(c)



(i) From the diagram, in ΔOXY

$\frac{\sin\alpha}{a} = \frac{\sin(180°-\beta)}{OX} = \frac{\sin\beta}{OX}$

$\therefore \frac{OX\sin\alpha}{a} = \sin\beta$ — ①

In ΔOAY

$\frac{\sin\alpha}{b} = \frac{\sin\beta}{OA}$ $\therefore \frac{OA\sin\alpha}{b} = \sin\beta$ — (2)

From (1) and (2)

$\frac{OX\sin\alpha}{a} = \frac{OA\sin\alpha}{b}$

$\therefore \frac{OX}{OA} = \frac{a}{b}$

(ii) There are 4 groups of 2 to be selected, $\therefore$ number of combinations $= \frac{{}^8C_2\times{}^6C_2\times{}^4C_2\times{}^2C_2}{4!}$

$= 105$

Let (A,B), (C,D), (E,F), (G,H) be one set of four combinations. Now (A,B) can play any 3 of the others, leaving the other two pairs to play each other.

$\therefore 105\times 3 = 315$ different selections.

(ii) $\int_0^1 x(1-x)^{2004}\,dx$

$= \int_0^1 (1-x)\,x^{2004}\,dx$ from (i)

$= \int_0^1 (x^{2004} - x^{2005})\,dx$

$= \left[\frac{x^{2005}}{2005} - \frac{x^{2006}}{2006}\right]_0^1$

$= \frac{1}{2005} - \frac{1}{2006}$

$= \frac{1}{4{,}022{,}030}$

(ii) Construct $ON \perp AX$ $\therefore XN = (a+b-h)$

Now $OA^2 = h^2 + d^2$ (by Pythagoras')

and $OX^2 = (a+b-h)^2 + d^2$

$\therefore \frac{(OX)^2}{(OA)^2} = \frac{(a+b-h)^2 + d^2}{h^2+d^2} = \frac{a^2}{b^2}$ from (i)

$\therefore \frac{(a+b)^2 - 2h(a+b) + h^2 + d^2}{h^2+d^2} = \frac{a^2}{b^2}$

$\therefore b^2(a+b)^2 - 2hb^2(a+b) + h^2b^2 + b^2d^2 = a^2h^2 + a^2d^2$

$\therefore b^2(a+b)^2 - 2hb^2(a+b) + b^2h^2 - a^2h^2 = a^2d^2 - b^2d^2$

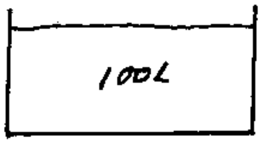
$\therefore b^2(a+b)^2 - 2hb^2(a+b) + h^2(b^2-a^2) = d^2(a^2-b^2)$

Divide both sides by $(a+b)$

$\therefore b^2(a+b) - 2hb^2 - h^2(a-b) = (a-b)d^2$

QUESTION 5

(a) (i)



Now 3L of brine concentration 2 gms/L ie; 6 gms. of salt / minute. Also each litre contains $\frac{Q}{100}$ gms. of salt. Since 3 litres of mixture flows out each minute, $\therefore$ the rate of outflow of salt is $\frac{Q}{100}$ gms/L $\times$ 3L/minute $\therefore \frac{3Q}{100}$ gms/minute.

Since $\frac{dQ}{dt}$ = rate of inflow − rate of outflow

$\therefore \frac{dQ}{dt} = \left(6 - \frac{3Q}{100}\right)$ gms/minute.

(ii) Now $\frac{dQ}{dt} = -\frac{3}{100}(Q-200)$

$\therefore \int \frac{dQ}{Q-200} = \int (-.03)\, dt$

$\therefore \ln(Q-200) = -.03t + C$

when $t=0$, $Q = 300$ (100L with conc. of 3 gms/L)

$\therefore C = \ln 100$

$\therefore \ln\left(\frac{Q-200}{100}\right) = -.03t$

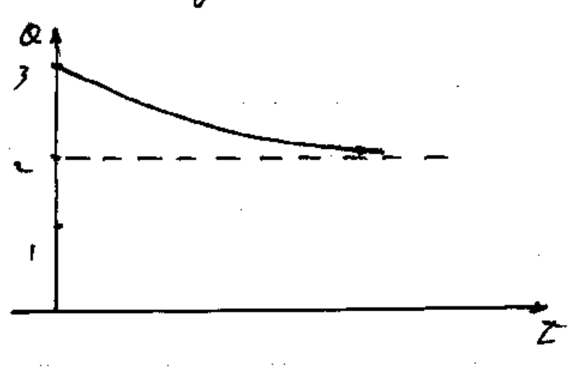
$\therefore \frac{Q-200}{100} = e^{-.03t}$

$\therefore Q = 200 + 100e^{-.03t}$

when $t=0$ $Q = 300$

$t \to \infty$ $Q \to 200$

hence quantity of salt always between 200 gms. and 300 gms.



OR

$\frac{dQ}{dt} = -.03(Q-200)$

$\therefore Q = 200 + Ae^{-.03t}$

[Solution to $\frac{dQ}{dt} = K(Q-Q_0)$ is $Q = Q_0 + Ae^{Kt}$]

when $t=0$, $Q = 300 (\ldots)$

$\therefore 300 = 200 + A$

$\therefore A = 100$

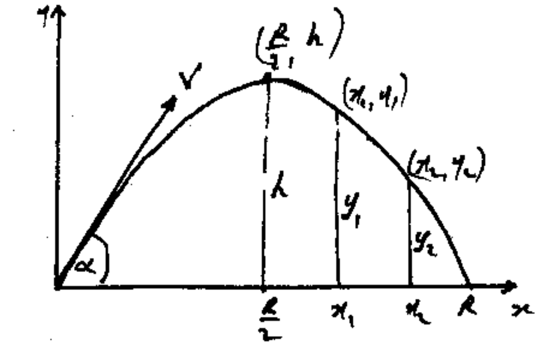
$\therefore Q = 200 + 100e^{-.03t}$

As $t \to 0$ $Q \to 300$

As $t \to \infty$ $Q \to 200$

$\therefore Q$ is always between 200 gms and 300 gms.

5(b)



Let $x = Vt\cos\alpha$ — (i)

$y = Vt\sin\alpha - \frac{gt^2}{2}$ — (ii)

Solving (i) and (ii)

$y = x\tan\alpha - \frac{g x^2}{2V^2}\sec^2\alpha$

Let $b = \tan\alpha$ and $c = \frac{g\sec^2\alpha}{2V}$

$\therefore y = bx - cx^2$ — (1)

Substituting $(\frac{R}{2}, h)$, (x_1, y_1), (x_2, y_2) and $(R, 0)$ into (1):

$h = \frac{bR}{2} - \frac{cR^2}{4}$ — (2)

$y_1 = x_1 b - c x_1^2$ — (3)

$y_2 = x_2 b - c x_2^2$ — (4)

$0 = Rb - R^2 c$ — (5)

For $R > 0$, $b = Rc$ from (5)

Subst. into (2)

$h = \frac{R}{2} \times Rc - \frac{R^2}{4} \times c$

$\therefore h = \frac{R^2 c}{4}$

Let the required distance be $x_2 - x_1$.

From (3): $x_1^2 c - x_1 b + y_1 = 0$

$\therefore x_1 = \frac{b \pm \sqrt{b^2 - 4cy_1}}{2c}$

From (4): $x_2^2 c - x_2 b + y_2 = 0$

$\therefore x_2 = \frac{b \pm \sqrt{b^2 - 4cy_2}}{2c}$

Now $x_2 - x_1 = \left(\frac{b \pm \sqrt{b^2 - 4cy_2}}{2c}\right) - \left(\frac{b \pm \sqrt{b^2 - 4cy_1}}{2c}\right)$

(Note that only positive root required, since $x_1 > \frac{R}{2}$ and $x_2 > \frac{R}{2}$)

$\therefore x_2 - x_1 = \left(\frac{b + \sqrt{b^2 - 4cy_2}}{2c}\right) - \left(\frac{b + \sqrt{b^2 - 4cy_1}}{2c}\right)$

$= \sqrt{\frac{b^2 - 4cy_2}{4c^2}} - \sqrt{\frac{b^2 - 4cy_1}{4c^2}}$

$= \sqrt{\frac{b^2}{4c^2} - \frac{y_2}{c}} - \sqrt{\frac{b^2}{4c^2} - \frac{y_1}{c}}$

$= \sqrt{\frac{R^2}{4} - \frac{y_2 \cdot R^2}{4h}} - \sqrt{\frac{R^2}{4} - \frac{y_1 \cdot R^2}{4h}}$ (since $b = Rc$ and $h = \frac{R^2 c}{4}$)

$\therefore x_2 - x_1 = \frac{R}{2}\left[\sqrt{1 - \frac{y_2}{h}} - \sqrt{1 - \frac{y_1}{h}}\right]$

QUESTION 6

(a) $\displaystyle\int \frac{5}{16+9\cos^2 x}\,dx$

$= \displaystyle\int \frac{5}{16(\sin^2 x+\cos^2 x)+9\cos^2 x}\,dx$

$= \displaystyle\int \frac{5}{16\sin^2 x+25\cos^2 x}\,dx$

$= \displaystyle\int \frac{5/\cos^2 x}{(16\sin^2 x+25\cos^2 x)/\cos^2 x}\,dx$

$= \displaystyle\int \frac{5\sec^2 x}{16\tan^2 x+25}\,dx$

$= \displaystyle\int \frac{5\,d\tan x}{16\tan^2 x+25}$

$= \dfrac{1}{4}\tan^{-1}\left(\dfrac{4}{5}\tan x\right)+c$

Note: Let $v=\tan x$

$\displaystyle\int \frac{5\,dv}{25+16v^2} = \frac{5}{16}\int \frac{dv}{\left(\frac{5}{4}\right)^2 - v^2}$

$= \dfrac{1}{4}\tan^{-1}\left(\dfrac{4v}{5}\right)+c$

$= \dfrac{1}{4}\tan^{-1}\left(\dfrac{4\tan x}{5}\right)+c$

(b)

$|z| = \left|\dfrac{t-i}{t+i}\right|$

$= \dfrac{\sqrt{t^2+1}}{\sqrt{t^2+1}}$

$\therefore |z| = 1$

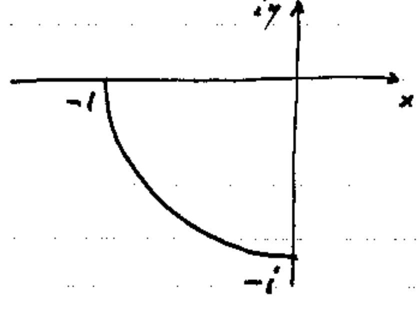
$\therefore$ z lies on the unit circle centre O.

Now $z = \dfrac{t-i}{t+i}\times\dfrac{t-i}{t-i} = \dfrac{t^2-1-2ti}{t^2+1}$

Let $z = X+iY$ where $X=\dfrac{t^2-1}{t^2+1}$ and $Y=\dfrac{-2t}{t^2+1}$

$\therefore$ X and Y are both non-positive for $0\le t\le 1$. Hence the locus is in the 3rd quadrant.

Also $\tan\arg z = \dfrac{Y}{X} = \dfrac{2t}{1-t^2}$. Hence for $0\le t\le 1$ $\tan\arg z$ varies from 0 to ∞, $\therefore$ $\arg z$ varies from π to $\frac{3\pi}{2}$: $\therefore$ the required locus is part of the unit circle in the 3rd quadrant where, for $0\le t\le 1$, z travels from -1 to $-i$.



QUESTION 7

(a) (i) $LHS = \tan\left(A + \frac{\pi}{2}\right)$ $\qquad$ $R.H.S = -\cot A$

$$= \frac{\sin\left(A + \frac{\pi}{2}\right)}{\cos\left(A + \frac{\pi}{2}\right)}$$

$$= \frac{\sin A \cos\frac{\pi}{2} + \cos A \sin\frac{\pi}{2}}{\cos A \cos\frac{\pi}{2} - \sin A \sin\frac{\pi}{2}}$$

$$= -\cot A = RHS.$$

(ii) Test for $n = 1$

$LHS = \tan\frac{3\pi}{4}$

$= -1 = RHS.$

Assume true for $n = k$

$$\tan\left[(2k+1)\frac{\pi}{4}\right] = (-1)^k$$

Prove true for $n = k+1$

$$LHS = \tan\left\{[2(k+1)+1]\frac{\pi}{4}\right\}$$

$$= \tan\left[(2k+3)\frac{\pi}{4}\right]$$

$$= \tan\left[(2k+1)\frac{\pi}{4} + \frac{\pi}{2}\right]$$

$$= -\cot\left[(2k+1)\frac{\pi}{4}\right]$$ from (a) where $A = (2k+1)\frac{\pi}{4}$

$$= -\frac{1}{\tan\left[(2k+1)\frac{\pi}{4}\right]}$$

$$= -\frac{1}{(-1)^k} = (-1).\frac{1}{(-1)^{-k}(-1)^{2k}}$$

$$= (-1)^{k+1} = RHS$$

Since proven true for $n=1$ $\therefore$ true for $n=2$. Since true for $n=k$, proven true for $n=k+1$ $\therefore$ true for all $n = 1, 2, 3 \cdots$ by M.I.

(b)(i) $P(x) = (x^2 - a^2)\,Q(x) + px + q$

$= (x-a)(x+a)\,Q(x) + px + q$

$\therefore P(a) = pa + q \quad$ — (1)

and $P(-a) = -pa + q \quad$ — (2)

(1) − (2): $\quad P(a) - P(-a) = 2pa$

$$\therefore p = \frac{1}{2a}\left[P(a) - P(-a)\right]$$

(1) + (2): $\quad 2q = P(a) + P(-a)$

$$\therefore q = \frac{1}{2}\left[P(a) + P(-a)\right]$$

(ii) when $P(x) = x^n - a^n$ is divided by $x^2 - a^2$ for n EVEN

$\therefore P(a) = a^n - a^n = 0$, $P(-a) = (-a)^n - a^n = 0$

$\therefore$ remainder is zero, since

$$px + q = \frac{1}{2a}[0 - 0]x + \frac{1}{2}[0 + 0] = 0$$

If n is ODD, then

$P(a) = (+a)^n - a^n = 0$ and

$P(-a) = (-a)^n - a^n = -a^n - a^n = -2a^n$

and remainder is

$$px + q = \frac{1}{2a}\left[0 - -2a^n\right]x + \frac{1}{2}\left[0 - 2a^n\right]$$

$$\therefore px + q = a^{n-1}x - a^n$$

6 (c)

$$xy(x+y)+16=0$$

$$\therefore x^2y+xy^2+16=0$$

Differentiating w.r.t. x:

$$\therefore x^2\frac{dy}{dx}+2xy+2xy\frac{dy}{dx}+y^2=0$$

$$\therefore (x^2+2xy)\frac{dy}{dx}=-(2xy+y^2)$$

When $\frac{dy}{dx}=-1$ $\quad (x^2+2xy)(-1)=-2xy-y^2$

$$\therefore -x^2-\cancel{2xy}=-\cancel{2xy}-y^2$$

$$\therefore x^2=y^2$$

$$\therefore x=\pm y$$

If $x=-y$

$\therefore xy(x+y)+16\neq 0$

$\therefore x\neq -y$

If $x=y$

$\therefore x^2(2x)+16=0$

$\therefore x^3=-8$

$\therefore x=-2$

at $x=-2$ $\quad (-2y)(-2+y)+16=0$

$$\therefore y^2-2y-8=0$$

$$\therefore (y-4)(y+2)=0$$

$$\therefore y=4 \text{ or } -2$$

Since $x=y$ $\therefore y=-2$

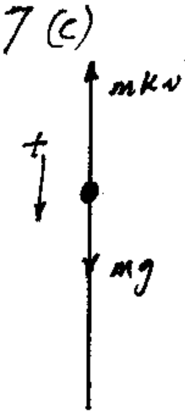
$\therefore$ required point is $(-2,-2)$

Equation of tangent at $(-2,-2)$ is

$$y+2=-1(x+2)$$

$$\therefore x+y+4=0$$

7 (c)



From Newton's 2nd Law:

$$F = m\ddot{x} = mg - mkv^2$$

$$\therefore \ddot{x} = g - kv^2 \quad \text{for unit mass}$$

$$\therefore v\frac{dv}{dx} = g - kv^2$$

$$\therefore \frac{v\,dv}{g-kv^2} = dx$$

$$\therefore \int \frac{v\,dv}{g-kv^2} = \int dx$$

$$\therefore -\frac{1}{2k}\int \frac{-2kv\,dv}{g-kv^2} = \int dx$$

$$\therefore -\frac{1}{2k}\ln(g-kv^2) = x + c$$

When $x=0$, $v=0$ $\therefore c = -\frac{1}{2k}\ln g$

$$\therefore -\frac{1}{2k}\ln(g-kv^2) = x - \frac{1}{2k}\ln g$$

$$\therefore -\frac{1}{2k}\ln\left(\frac{g-kv^2}{g}\right) = x$$

$$\therefore \ln\left(\frac{g-kv^2}{g}\right) = -2kx$$

$$\therefore \frac{g-kv^2}{g} = e^{-2kx}$$

$$\therefore g - kv^2 = ge^{-2kx}$$

$$\therefore v^2 = \frac{g}{k}\left(1-e^{-2kx}\right)$$

When $x = d$ and $v > 0$

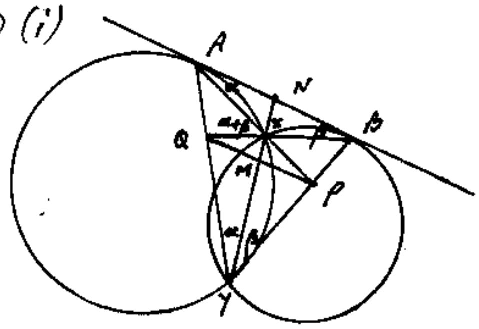
$$v = \sqrt{\frac{g}{k}}\sqrt{1-e^{-2kd}}$$

Since $V = \sqrt{\frac{g}{k}}$

$$\therefore v = V\sqrt{1-e^{-2kd}}$$

QUESTION 8

(a) (i)



(ii) $B\hat{A}X = A\hat{Y}X$ ($B\hat{A}X$ formed by chord AX and tangent equal to $A\hat{Y}X$ in alternate segment)

Similarly $A\hat{B}X = B\hat{Y}X$

$\therefore Q\hat{Y}P = \alpha + \beta$

In ΔABX, $A\hat{X}Q = \alpha + \beta$ (Exterior $A\hat{X}Q$ equal to interior opposite $A\hat{B}X$ and $B\hat{A}X$)

$\therefore PXQY$ is a cyclic quad. (Exterior $A\hat{X}Q$ equal to interior remote $P\hat{Y}Q$)

(iii) $B\hat{A}X = X\hat{Y}Q$ from (ii)

$X\hat{Y}Q = A\hat{P}Q$ (angles at circumference in same segment)

$\therefore B\hat{A}X = A\hat{P}Q$

$\therefore AB \parallel PQ$ (alternate angles equal)

(iv) Let YX intersect PQ at M.

Extend YX to meet AB at N.

Now $AN^2 = YN \cdot NX = BN^2$ (Square of tangent equal to product of intercepts of intersecting secant)

$\therefore AN = BN$ ie; N bisects AB

Since $\Delta ABY ||| \Delta QPY$, $\therefore$ M bisects PQ.

(b) (i) Since $(K-1)(K+1) < K^2 \quad K \geq 3$

$\therefore (K-1)K(K+1) < K^3$

$$\therefore \frac{1}{(K-1)K(K+1)} > \frac{1}{K^3}$$

(ii) Let
$$\frac{1}{(K-1)K(K+1)} = \frac{A}{(K-1)K} + \frac{B}{K(K+1)} = \frac{\left(\frac{1}{2}\right)}{(K-1)K} - \frac{\left(\frac{1}{2}\right)}{K(K+1)} \quad \text{--- (1)}$$

Now $S_n = \frac{1}{3^3} + \frac{1}{4^3} + \frac{1}{5^3} + \dots + \frac{1}{n^3}$

$$\therefore S_n < \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \frac{1}{4 \cdot 5 \cdot 6} + \dots + \frac{1}{(n-1)n(n+1)} = \sum_{K=3}^{n} \frac{1}{(K-1)K(K+1)}$$

ie; $S_n < \frac{1}{2} \sum_{K=3}^{n} \left[\frac{1}{(K-1)K} - \frac{1}{K(K+1)}\right]$ from (1)

$$\therefore 2S_n < \left[\left(\frac{1}{2 \cdot 3} - \frac{1}{3 \cdot 4}\right) + \left(\frac{1}{3 \cdot 4} - \frac{1}{4 \cdot 5}\right) + \dots + \left(\frac{1}{(n-1)n} - \frac{1}{n(n+1)}\right)\right]$$

Now $\left(\frac{1}{2 \cdot 3} - \frac{1}{3 \cdot 4}\right) + \left(\frac{1}{3 \cdot 4} - \frac{1}{4 \cdot 5}\right) + \dots + \left(\frac{1}{(n-1)n} - \frac{1}{n(n+1)}\right) = \frac{1}{2 \cdot 3} - \frac{1}{n(n+1)}$

$\therefore 2S_n < \frac{1}{6} - \frac{1}{n(n+1)}$

$\therefore 2S_n < \frac{1}{6}$ for $n \geq 3$

$\therefore S_n < \frac{1}{12}$