CATHOLIC SECONDARY SCHOOLS ASSOCIATION

2009 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS – SUGGESTED SOLUTIONS

These marking guidelines show the criteria to be applied to responses along with the marks to be awarded in line with the quality of responses. These guidelines are suggested and not prescriptive. This is not intended to be an exhaustive list but rather an indication of the considerations that students could include in their responses.

Question 1 (12 marks)

(a) (2 marks)

Outcomes Assessed: P3, H3

Targeted Performance Bands: 2-3

Criteria	Marks
• Gives the correct answer.	1
• Correctly rounds to THREE significant figures.	1

Sample answer:

$$\frac{2+\sqrt{2}}{7(e^2-4)} = 0.1439175925$$

$= 0.144$ (3 significant figures)

(b) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 2-3

Criteria	Marks
• Progress towards the correct answer.	1
• Gives the correct answer.	1

Sample answer:

$(y-2)^2 = 9 \quad \therefore y-2 = \pm 3$

$y = 5 \quad , \quad y = -1$

(c) (2 marks)

Outcomes Assessed: H8

Targeted Performance Bands: 3-4

Criteria	Marks
• Correct primitive of ONE of the terms.	1
• Correct primitive of the other term.	1

Sample answer:

Primitive $\frac{x^2}{6}+\frac{x^{-1}}{-1}=\frac{x^2}{6}-\frac{1}{x}+C$

(d) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 2-4

Criteria	Marks
• Writes TWO correct inequations.	1
• Gives the correct answer.	1

Sample answer:

$5a+3\le 13$

$5a\le 10$

$a\le 2$

or

$-(5a+3)\le 13$

$-5a-3\le 13$

$-5a\le 16$

$a\ge -3.2$

(e) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly finds the value of r.	1
• Gives the correct answer.	1

Sample answer:

$$20+4+\frac{4}{5}+\ldots=\frac{20}{1-\frac{1}{5}}=25$$

(f) (2 marks)

Outcomes Assessed: P7, H5

Targeted Performance Bands: 2-3

Criteria	Marks
• Correct derivative.	1
• Gives the correct answer.	1

Sample answer:

$$g(x) = 7x^3 - 3x + 1$$

$$g'(x) = 21x^2 - 3$$

$$g'(2) = 21(2^2) - 3 = 81$$

Question 2 (12 marks)

(a) (i) (1 mark)

Outcomes Assessed: P3, P4, H5

Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct answer.	1

Sample answer:

$$x = \frac{-18+4}{2} = -7 \qquad y = \frac{0+-6}{2} = -3$$

(a) (ii) (1 mark)

Outcomes Assessed: P3, P4 , H5

Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct answer.	1

Sample answer:

$$DC = \sqrt{(-7-0)^2 + (-3-6)^2} = \sqrt{130}$$

(a) (iii) (2 marks)

Outcomes Assessed: P3, P4, H5

Targeted Performance Bands: 2-3

Criteria	Marks
• Finds the correct gradient AC.	1
• Gives the correct equation of line.	1

Sample answer:

$$m = \frac{6 - -6}{0 - 4} = -3$$

$$\therefore y - 6 = -3(x - 0)$$

$$y - 6 = -3x$$

$$3x + y - 6 = 0$$

(a) (iv) (2 marks)

Outcomes Assessed: P3, P4, H5

Targeted Performance Bands: 2-3

Criteria	Marks
• Finds the correct gradient BC.	1
• Correctly shows $m_1 \times m_2 = -1$.	1

Sample answer:

$m_{BC} = \frac{6 - 0}{0 - -18} = \frac{1}{3}$, $\therefore$ h is perpendicular to k since $\frac{1}{3} \times -3 = -1$.

(a) (v) (2 marks)

Outcomes Assessed: P3, P4, H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Finds correct distance DA or DB.	1
• Gives the correct answer.	1

Sample answer:

$DA = \sqrt{(-7-4)^2 + (-3--6)^2} = \sqrt{130}$

Using $(x-h)^2 + (y-k)^2 = r^2$, the centre of the circle is $(-7,-3)$ with radius $\sqrt{130}$.

Then by substitution $(x+7)^2 + (y+3)^2 = 130$

(a) (vi) (1 mark)

Outcomes Assessed: P3, P4, H5

Targeted Performance Bands: 2 - 3

Criteria	Mark
• Gives the correct answer.	1

Sample answer:

$A = \pi r^2 = \pi \times (\sqrt{130})^2 = 130\pi$ units^2

(b) (3 marks)

Outcomes Assessed: P3, H2

Targeted Performance Bands: 3 - 4

Criteria	Marks
• Establishes that $\Delta < 0$.	1
• Solves the quadratic inequation correctly.	1
• Gives the correct answer.	1

Sample answer:

$3qx^2 - 5x + 3q = 0$. For negative definite, $3q < 0$ and $(-5)^2 - 4(3q)(3q) < 0$

$(-5)^2 - 4(3q)(3q) < 0$

$25 - 36q^2 < 0 \quad \therefore (5 - 6q)(5 + 6q) < 0$

$q < \frac{-5}{6}$ & $q > \frac{5}{6}$

But $3q < 0$

$\therefore q < \frac{-5}{6}$

Question 3 (12 marks)

(a) (i) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 2-4

Criteria	Marks
• Finds a correct expression for *PR*.	1
• Gives the correct proof.	1

Sample answer:

$PR^2 = (2w)^2 + w^2 = 5w^2$

$QR^2 = (2w)^2 + 5w^2 = 9w^2$

$QR = \pm 3w \quad \therefore QR = 3w$

(a) (ii) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 3-4

Criteria	Mark
• Finds the correct area of triangle *PSR*.	1
• Finds the correct area of triangle *PQR*.	1

Sample answer:

$$A = \frac{1}{2}(w)(2w) + \frac{1}{2}(2w)(\sqrt{5}w)$$

$$A = w^2 + w^2\sqrt{5} = w^2(1+\sqrt{5}) \quad u^2$$

(b) (i) (2 marks)

Outcomes Assessed: P7, H5

Targeted Performance Bands: 3-4

Criteria	Mar
• Differentiates using the quotient rule with ONE mistake.	1
• Gives the correct answer.	1

Sample answer:

Let $y = \dfrac{\ln x}{x}$ $\qquad u = \ln x \therefore u' = \dfrac{1}{x}$ & $v = x \therefore v' = 1$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2} = \frac{x.\frac{1}{x} - \ln x.1}{x^2} = \frac{1 - \ln x}{x^2}$$

(b) (ii) (2 marks)

Outcomes Assessed: P7, H5

Targeted Performance Bands: 3-4

Criteria	M
• Correctly uses the product rule but has ONE mistake.	
• Gives the correct answer.	

Sample answer:

Let $y = (x-5)^2\, e^x$ $\qquad u = (x-5)^2 \therefore u' = 2(x-5)$ & $v = e^x \therefore v' = e^x$

$$\frac{dy}{dx} = uv' + vu' = (x-5)^2.e^x + e^x.2(x-5) = e^x(x-5)(x-3)$$

(c) (i) (2 marks)

Outcomes Assessed: H8

Targeted Performance Bands: 3-4

Criteria	Marks
• Finds the correct logarithmic primitive but has ONE mistake	1
• Gives the correct answer.	1

Sample answer:

$$\int \frac{3x}{x^2-9}\,dx = \frac{3}{2}\int\left(\frac{2x}{x^2-9}\right)dx$$

$$= \frac{3}{2}\log_e\left(x^2-9\right)+C$$

(c) (ii) (2 marks)

Outcomes Assessed: H8

Targeted Performance Bands: 3-4

Criteria	Marks
• Finds the primitive of $\sqrt{x}$ but has an error in calculating the integral.	1
• Correctly applies Newton–Leibnitz formula to obtain the answer.	1

Sample answer:

$$\int_0^3 \sqrt{x}\,dx$$

$$= \left[\frac{2x^{\frac{3}{2}}}{3}\right]_0^3 = \left(2\left(\frac{3^{1.5}}{3}\right)-2\left(\frac{0^{1.5}}{3}\right)\right) = \frac{2\sqrt{27}}{3} = 3.464$$

Question 4 (12 marks)

(a) (2 marks)

Outcomes Assessed: H4

Targeted Performance Band: 3-4

Criteria	Marks
• Gives correct a, d and n	1
• Gives the correct answer.	1

Sample answer:

$$\sum_{n=2}^{16} 13 - 5n = (13-10) + (13-15) + (13-20) + \ldots + (13-80)$$

$$= 3 - 2 - 7 - \ldots - 67$$

This is an arithmetic series with $a = 3$, $d = -5$ and $n = 15$.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$= \frac{15}{2}(2 \times 3 + 14 \times (-5))$$

$$= -480$$

or

$$S_n = \frac{n}{2}(a + l)$$

$$S_n = \frac{15}{2}(3 - 67)$$

$$= -480$$

(b) (3 marks)

Outcomes Assessed: P3

Targeted Performance Band: 3-4

Criteria	Marks
• Finds correct denominator and numerator.	1
• Changes $(1 - \cos^2\theta)$ to $\sin^2\theta$.	1
• Gives the correct proof.	1

Sample Answer:

$$\text{LHS} = \frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{(1+\cos\theta)\sin\theta + \sin\theta(1-\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{\sin\theta + \sin\theta\cos\theta + \sin\theta - \sin\theta\cos\theta}{1-\cos^2\theta}$$

$$= \frac{2\sin\theta}{\sin^2\theta}$$

$$= \frac{2}{\sin\theta}$$

$$= 2\operatorname{cosec}\theta$$

(c) (2 marks)

Outcomes Assessed: P4

Targeted Performance Band: 3-4

Criteria	Marks
• Gives the angle in the regular pentagon.	1
• Gives the correct angle giving reasons.	1

Sample Answer:

Angle in pentagon $= (n-2)\times 180^o$

$= 3\times 180^o$

$= 540^o$

Size of each angle $= 540 \div 5 = 108^o$

$\angle DEA = 108^o$ and $\angle EDC = 108^o$

$\therefore \angle QED = 72^o$ (straight angle) also $\angle EDQ = 72^o$ (similarly)

$\angle PQD = 72^o + 72^o$ (exterior angle of a triangle is equal to the sum of the two interior opposite angles)

$\therefore \alpha = 144^o$

(d) (i) (1 mark)

Outcomes Assessed: H5

Targeted Performance Band: 3-4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer:

Sample space is {1, 2, 3, 3, 4, 5, 6}

$P(E) = \frac{4}{7}$

(d) (ii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Band: 3-4

Criteria	Marks
• Uses the correct outcomes.	1
• Gives the correct answer with required working	1

Sample Answer:

$P(E) = P(1,5) + P(2,4) + P(3,3) + P(4,2) + P(1,5)$

$= 4 \times \frac{1}{7} \times \frac{1}{7} + \frac{2}{7} \times \frac{2}{7}$

$= \frac{8}{49}$

(d) (iii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Band: 4-5

Criteria	Marks
• Uses the correct outcomes.	1
• Gives the correct answer with required working	1

Sample Answer:

P(at least 2 odd numbers) $= P$(3 odd numbers) $+ P$(2 odd and 1 even number)

P(odd number on a fair die) $= \frac{1}{2}$ $\quad$ P(odd number on biased die) $= \frac{4}{7}$

P(even number on a fair die) $= \frac{1}{2}$ $\quad$ P(even number on biased die) $= \frac{3}{7}$

$$P(E) = \frac{1}{2}\times\frac{1}{2}\times\frac{4}{7} + P((\text{odd, odd, even}) \text{ or } (\text{odd, even, odd}) \text{ or } (\text{even, odd, odd}))$$

$$= \frac{1}{7} + \frac{4}{7}\times\frac{1}{2}\times\frac{1}{2} + \frac{4}{7}\times\frac{1}{2}\times\frac{1}{2} + \frac{3}{7}\times\frac{1}{2}\times\frac{1}{2}$$

$$= \frac{15}{28}$$

Question 5 (12 marks)

(a) (2 marks)

Outcomes Assessed: H3

Targeted Performance Band: 4-5

Criteria	Marks
• Correct solution for $\log_{10} x$.	1
• Gives the correct answer.	1

Sample Answer:

$$\log_{10} x^6 - 8 = 3\log_{10} x$$

$$6\log_{10} x - 8 = 3\log_{10} x$$

$$3\log_{10} x = 8$$

$$\log_{10} x = \frac{8}{3}$$

$$10^{\frac{8}{3}} = x$$

$$x = 464.2$$

(b) (i) (3 marks)

Outcomes Assessed: P7, H6

Targeted Performance Band: 3-5

Criteria	Marks
• Finds the stationary points	1
• Finds the nature of ONE stationary point	1
• Finds the nature of the other stationary point	1

Sample Answer :

$y=2x^3-9x^2+12x \quad \therefore \frac{dy}{dx}=6x^2-18x+12$

For stationary points $\frac{dy}{dx}=0 \quad \therefore\ 6(x-2)(x-1)=0 \quad \therefore x=1 \quad \text{or} \quad x=2$

$\therefore$ the stationary points are $(1,\ 5)$ & $(2\ ,\ 4)$

Also for the nature of the stationary points, $\frac{d^2y}{dx^2}=12x-18$

At $x=1$, $\frac{d^2y}{dx^2}=-6<0 \quad \therefore (1,\ 5)$ is a MAXIMUM stationary point

At $x=2$, $\frac{d^2y}{dx^2}=6>0 \quad \therefore (2,\ 4)$ is a MINIMUM stationary point

(b) (ii) (1 mark)

Outcomes Assessed: P6, H6, H7, H9

Targeted Performance Band: 3-5

Criteria	Mark
• $\frac{d^2y}{dx^2}=0$ and shows a change in concavity at $x=\frac{3}{2}$	1

Sample Answer :

For points of inflexion, $\frac{d^2y}{dx^2}=12x-18=0 \quad \therefore x=\frac{3}{2}$

x	1	$\frac{3}{2}$	2
$\frac{d^2y}{dx^2}$	-6	0	6
	concave down		concave up

(b) (iii) (2 marks)

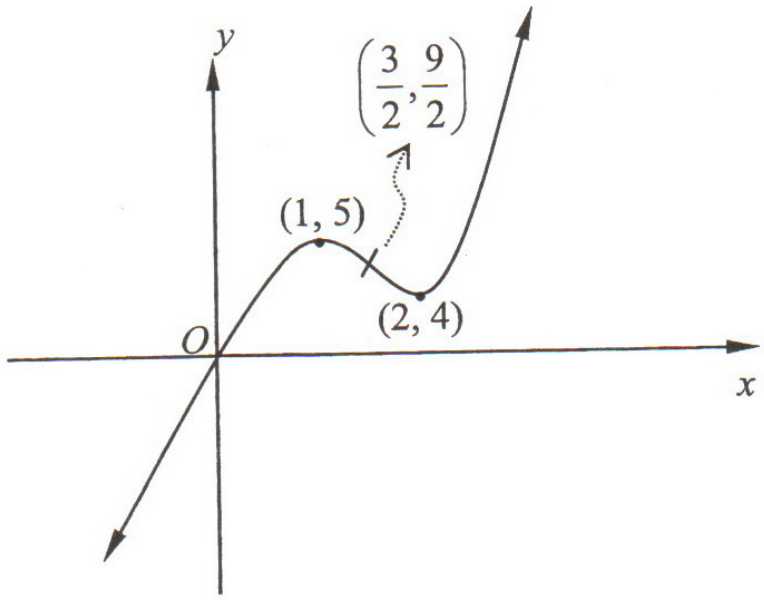
Outcomes Assessed: P6, H6, H7, H9

Targeted Performance Band: 3-5

Criteria	Marks
• Draws the correct cubic curve	1
• Plots all important points	1

Sample Answer :

$y = 2x^3 - 9x^2 + 12x$



(b) (iv) (1 mark)

Outcomes Assessed: P6, H6, H7

Targeted Performance Band: 2-4

Criteria	Mark
• Correctly solves the equation $\frac{d^2y}{dx^2} > 0$ or gives correct answer from graph.	1

Sample Answer :

$\frac{d^2y}{dx^2} = 12x - 18$

For the curve to be concave up, $\frac{d^2y}{dx^2} > 0 \quad \therefore 12x - 18 > 0$

$\therefore x > \frac{3}{2}$

(c) (3 marks)

Outcomes Assessed: P4

Targeted Performance Band: 3-4

Criteria	Marks
• Correctly solves the equation $2A^2 - 19A - 10 = 0$.	1
• Correctly substitutes $(x^2 + 1)$ and then solves for x^2.	1
• Gives the correct solution.	1

Sample Answer :

Let $A = x^2 + 1$

$2A^2 - 19A - 10 = 0$

$(2A + 1)(A - 10) = 0$

$A = -\frac{1}{2}$ or 10

$\therefore x^2 + 1 = -\frac{1}{2}$ or $x^2 + 1 = 10$

$x^2 = -\frac{3}{2}$ No solution $x^2 = 9$

$x = \pm 3$

Question 6 (12 marks) (a) (i) (2 marks)

Outcomes Assessed: P4, H5

Targeted Performance Band: 2-4

Criteria	Marks
• Gives ONE correct answer in radians OR THREE correct answers in degrees.	1
• Gives THREE correct answers in radians.	1

Sample Answer:

$\cos 3x = 0$

Basic angle is $\frac{\pi}{2}$ (First Quadrant).

$\therefore 3x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ $\quad 0 \le 3x \le 3\pi$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ $\quad 0 \le x \le \pi$

(a) (ii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Band: 2-4

Criteria	Marks
• Gives correct amplitude.	1
• Gives correct period.	1

Sample Answer:

Amplitude $= 1$

Period $= \dfrac{2\pi}{n}$

$= \dfrac{2\pi}{3}$

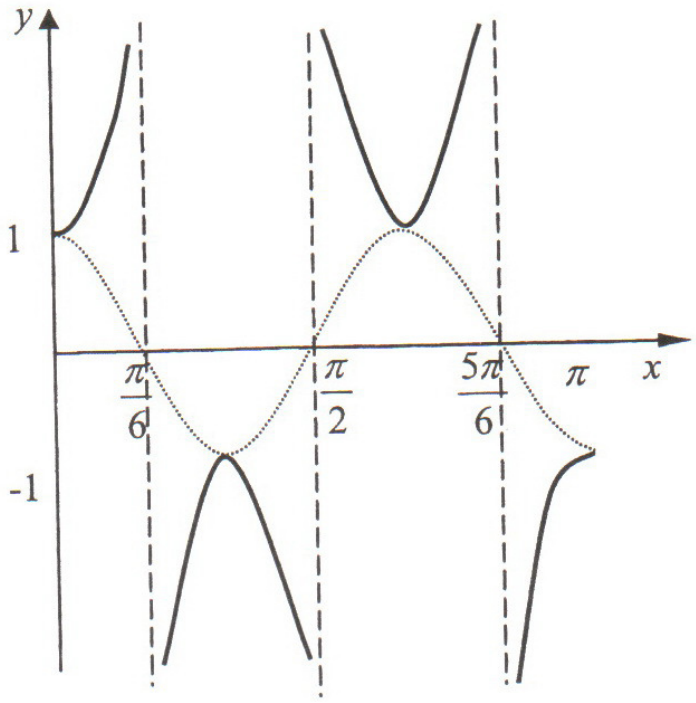
(a) (iii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Band: 2-4

Criteria	Marks
• Shows x-intercepts as asymptotes on $y = \cos 3x$.	1
• Gives correct reciprocal curve.	1

Sample Answer:



(a) (iv) (2 marks)

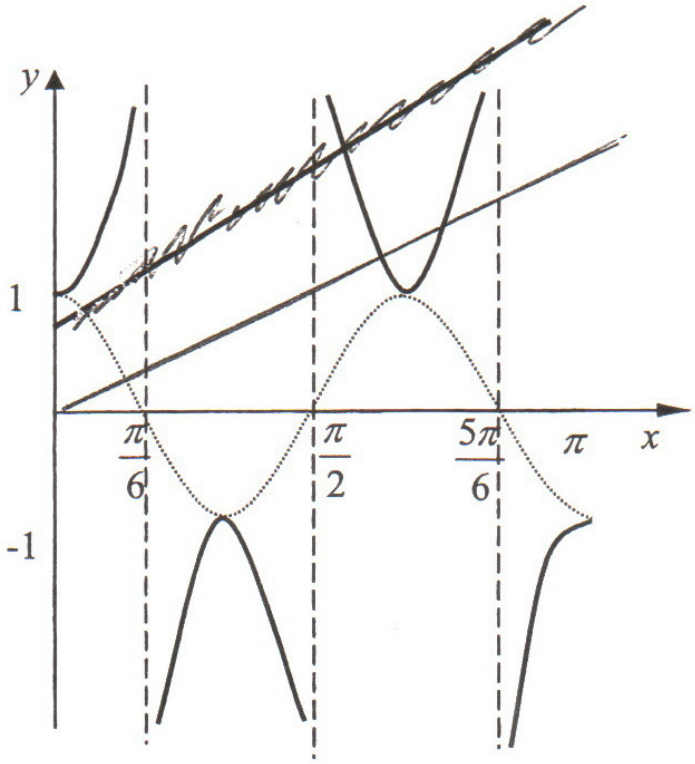
Outcomes Assessed: H5

Targeted Performance Band: 2-4

Criteria	Marks
• Correctly draws $y=x$ on graph.	1
• Shows TWO solutions.	1

Sample Answer:

2 solutions



(b) (i) (2 marks)

Outcomes Assessed: P4

Targeted Performance Band: 2-4

Criteria	Marks
• Correctly substitutes into cosine rule.	1
• Gives correct solution.	1

Sample Answer:

$$\cos \angle ADC = \frac{10.1^2 + 10.1^2 - 4^2}{2 \times 10.1 \times 10.1}$$

$$= 0.92$$

$$\angle ADC = 23^\circ$$

(b) (ii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Band: 2-3

Criteria	Marks
• Gives angle in radians.	1
• Gives correct solution.	1

Sample Answer:

$$23° = \frac{\pi}{180} \times 23 \text{ radians}$$
$$= 0.4 \text{ radians}$$

$$l = r\theta$$
$$= 10.1 \times 0.4$$
$$= 4.04 \text{ metres}$$

Question 7 (12 marks)

(a)(i) (1 mark)

Outcomes Assessed: H4, H5

Targeted Performance Band: 4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer:

Vertex = (0, –2)

(a)(ii) (1 mark)

Outcomes Assessed: H4, H5

Targeted Performance Band: 4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer:

Focus = (0, 0)

(a) (iii) (2 marks)

Outcomes Assessed: H4, H5

Targeted Performance Band: 4-5

Criteria	Marks
• Correctly differentiates to find the gradient.	1
• Gives the correct equation.	1

Sample Answer:

$x^2 = 8(y+2)$

$y = \frac{x^2}{8} - 2 \qquad \therefore \frac{dy}{dx} = \frac{2x}{8} = \frac{x}{4} \qquad \therefore \text{ at } \left(2, -\frac{3}{2}\right) \Rightarrow m = \frac{1}{2}$

$\therefore y + \frac{3}{2} = \frac{1}{2}(x-2) \Rightarrow x - 2y - 5 = 0$

$\therefore$ The tangent to the parabola at $\left(2, -\frac{3}{2}\right)$ is $x - 2y - 5 = 0$.

(a) (iv) (1 mark)

Outcomes Assessed: H4, H5

Targeted Performance Band: 4-5

Criteria	Mark
• Shows that the tangent meets the directrix at $(-3,-4)$.	1

Sample Answer:

Directrix has equation $y = -4$

Substituting into the tangent, $x - 2(-4) - 5 = 0$, $x = -3$

Therefore the tangent cuts the directrix at (–3, –4).

(b) (i) (1 mark)

Outcomes Assessed: H5

Targeted Performance Band: 4

Criteria	Mark
• Determines the initial velocity correctly.	1

Sample Answer:

When $t = 0$, $V = 18 - 2e^0$

$\therefore V = 18 - 2 = 16 m/s.$

(b) (ii) (2 marks)

Outcomes Assessed: H4

Targeted Performance Band: 4-5

Criteria	Marks
• Substitutes $V=0$ to determine t with some progress towards the answer.	1
• Correctly solves for t.	1

Sample Answer:

At rest when $V=0$

$\therefore 0=18-2e^t$

$e^t=9$

$\therefore t=\log_e 9$

$=2\log_e 3$

(b) (iii) (1 mark)

Outcomes Assessed: H4, H5

Targeted Performance Band: 4

Criteria	Mark
• Correctly determines the coordinates of A and B.	1

Sample Answer:

$A=(0,16)$ and $B=(2\log_e 3,\ 0)$

(b) (iv) (3 marks)

Outcomes Assessed: H4, H5

Targeted Performance Band: 4-5

Criteria	Marks
• Splits the integral to allow for the absolute value or equivalent working.	1
• Integrates the expression for velocity correctly.	1
• Correctly substitutes to find the distance travelled.	1

Sample Answer:

$$\text{Distance} = \int_0^{2\log_e 3}(18-2e^t)\,dt + \left|\int_{2\log_e 3}^{3\log_e 3}(18-2e^t)\,dt\right|$$

$$=\left[18t-2e^t\right]_0^{2\log_e 3} + \left|\left[18t-2e^t\right]_{2\log_e 3}^{3\log_e 3}\right|$$

$$=\left[36\log_e 3-2e^{2\log_e 3}\right]-[0-2]+\left|\left[54\log_e 3-2e^{3\log_{3e} 3}\right]-\left[36\log_e 3-2e^{2\log_e 3}\right]\right|$$

$$=36\log_e 3-18+2\ +\left|54\log_e 3-54-36\log_e 3+18\right|$$

$$=36\log_e 3-16+\left|18\log_e 3-36\right|$$

$$=36\log_e 3-16+36-18\log_e 3$$

$$=18\log_e 3+20$$

Question 8 (12 marks)

(a) (i) (2 marks)

Outcomes Assessed: H1, H5

Targeted Performance Band: 4-5

Criteria	Marks
• Progress towards A_2	1
• Shows that $A_2 = 5\times10^5(1.08)^2 - 5.6\times10^4(1.08+1)$.	1

Sample Answer:

$A_1 = 500000(1.08) - 56000$

$= 5\times10^5(1.08) - 5.6\times10^4$

$A_2 = A_1(1.08) - 5.6\times10^4$

$= \left[5\times10^5(1.08) - 5.6\times10^4\right](1.08) - 5.6\times10^4$

$= 5\times10^5(1.08)^2 - 5.6\times10^4(1.08+1)$ as required.

(a)(ii) (2 marks)

Outcomes Assessed: H4, H5

Targeted Performance Band: 4-6

Criteria	Marks
• Gives the correct expression for A_n.	1
• Correctly simplifies A_n to give the expression as required.	1

Sample Answer:

$A_n = 5\times10^5(1.08)^n - 5.6\times10^4\left(1+1.08+...+1.08^{n-1}\right)$

$= 5\times10^5(1.08)^n - 5.6\times10^4\left[\dfrac{1.08^n-1}{0.08}\right]$

$= 5\times10^5(1.08)^n - 7\times10^5(1.08^n-1)$

$= 5\times10^5(1.08)^n - 7\times10^5(1.08)^n + 7\times10^5$

$= 7\times10^5 - 2\times10^5(1.08)^n$

$= 10^5\left[7-2(1.08)^n\right]$ as required.

(a) (iii) (3 marks)

Outcomes Assessed: H3, H4, H5

Targeted Performance Band: 4-5

Criteria	Marks
• Equates $A_n = 0$	1
• Progress towards finding n.	1
• Finds the correct value for n and hence the year 2025.	1

Sample Answer:

When $A_n = 0$, the fund will have reached zero.

$0 = 10^5\left[7 - 2(1.08)^n\right]$

$0 = \left[7 - 2(1.08)^n\right]$

$2(1.08)^n = 7$

$$n = \frac{\log_e 3.5}{\log_e (1.08)}$$

$= 16.27788...$

During the 17th year the fund will reach zero.

Therefore during 2025 the fund will reach zero.

(b) (i) (1 mark)

Outcomes Assessed: H3, H4

Targeted Performance Band: 4-5

Criteria	Mark
• Correctly determines $\frac{dN}{dt}$ as a rate proportional to the number of kangaroos alive.	1

Sample Answer:

$N = N_0 e^{-kt}$

$$\therefore \frac{dN}{dt} = N_0 \times -ke^{-kt}$$

$$= -k \times N_0 e^{-kt}$$

$$= -kN$$

$\therefore$ The number of kangaroos decreases at a rate proportional to the number of kangaroos alive.

(b) (ii) (2 marks)

Outcomes Assessed: H3, H4

Targeted Performance Band: 4-5

Criteria	Marks
• Correctly shows $N_0 = 2000$.	1
• Correctly shows $k = 0.0351$.	1

Sample Answer:

$2000 = N_0 e^{-k(0)} \qquad \therefore N_0 = 2000$

$1800 = 2000e^{-k(3)} \qquad \therefore e^{-3k} = 0.9$

$-3k = \log_e 0.9$

$k = 0.0351$

(b) (iii) (2 marks)

Outcomes Assessed: H3, H4

Targeted Performance Band: 4-5

Criteria	Marks
• Correctly substitutes 1000 with progress towards the answer.	1
• Correctly determines the time when the population halves.	1

Sample Answer:

$1000 = 2000e^{-0.0351t} \qquad \therefore e^{-0.0351t} = 0.5$

$-0.0351t = \log_e 0.5$

$t = 19.7 \text{ years}$

Question 9 (12 marks)

(a) (i) (2 marks)

Outcomes Assessed: H4, H9

Targeted Performance Band: 4-5

Criteria	Marks
• Solves for t correctly.	1
• Correctly determines the first time after 8.55 pm	1

Sample Answer:

When $\frac{dV}{dt} = 0$

$$20\sin\frac{\pi}{35}t = 0$$

$$\sin\frac{\pi}{35}t = 0$$

$$\frac{\pi}{35}t = \pi$$

$\therefore t = 35$ minutes

Therefore the first time the flow rate is zero after 8.55 pm is $8.55 + 35$ minutes $= 9.30$ pm

(a) (ii) (3 marks)

Outcomes Assessed: H8, H9

Targeted Performance Band: 4-5

Criteria	Marks
• Correctly integrates the given rate.	1

(a) (iii) (2 marks)

Outcomes Assessed: H4, H5

Targeted Performance Band: 4-5

Criteria	Marks
• Correctly substitutes 35 minutes into the expression for volume.	1
• Correctly determines the maximum volume.	1

Sample Answer:

From part (i), the filtering cycle is 35 minutes.

$$\therefore V = \frac{700}{\pi} - \frac{700}{\pi}\cos\frac{\pi}{35}\times 35 \quad = \frac{700}{\pi} + \frac{700}{\pi} \quad = \frac{1400}{\pi} \text{ litres}$$

(b) (i) (2 marks)

Outcomes Assessed: H8, H9

Targeted Performance Band: 4-6

Criteria	Marks
• Correctly applying the volume formula.	1
• Gives the expression.	1

Sample Answer:

Use the formula $V = \pi \int y^2 dx$

$$V = \pi \int_1^2 \left[\frac{4}{\sqrt{4x-1}}\right]^2 dx = \pi \int_1^2 \frac{16}{4x-1} dx = 4\pi \int_1^2 \frac{4}{4x-1} dx$$

(b) (ii) (3 marks)

Outcomes Assessed: H8, H9

Targeted Performance Band: 4-6

Criteria	Marks
• Correctly integrates the volume.	1
• Correctly applies Newton–Leibnitz formula/substitution.	1
• Finds the volume of the solid, in exact form.	1

Sample Answer:

$$V = 4\pi \int_1^2 \frac{4}{4x-1} dx$$

$$= 4\pi \left[\log_e(4x-1)\right]_1^2 = 4\pi\left[(\log_e 7) - (\log_e 3)\right]$$

$$= 4\pi \log_e\left(\frac{7}{3}\right) u^3$$

Question 10 (12 marks)

(a) (2 marks)

Outcomes Assessed: H8, H9

Targeted Performance Band: 5-6

Criteria	Marks
• Performs the integration correctly	1
• Correctly substitutes into the integral to find the area.	1

Sample Answer :

$$\int_{-1}^{4}[f(x)+4]dx = \int_{-1}^{4} f(x)dx + \int_{-1}^{4} 4\,dx$$

$$= 7\frac{1}{2} + [4x]_{-1}^{4}$$

$$= 7\frac{1}{2} + [4\times 4 - 4\times -1]$$

$$= 7\frac{1}{2} + [16+4]$$

$$= 27\frac{1}{2}$$

(b) (3 marks)

Outcomes Assessed: H3, H9

Targeted Performance Band: 5-6

Criteria	Marks
• Correctly integrates the expression.	1
• Correctly substitutes into the integral.	1
• Correctly determines the value of the integral.	1

Sample Answer:

$$\int_{0}^{\pi} \pi^{x}\,dx$$

$$= \left[\frac{\pi^{x}}{\ln \pi}\right]_{0}^{\pi}$$

$$= \left[\frac{\pi^{\pi}}{\ln \pi}\right] - \left[\frac{\pi^{0}}{\ln \pi}\right]$$

$$= \frac{\pi^{\pi}}{\ln \pi} - \frac{1}{\ln \pi}$$

$$= 30.9786 = 31.0$$

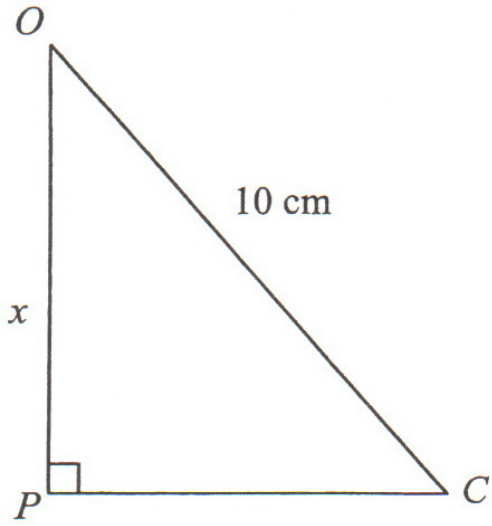
(c) (i) (2 marks)

Outcomes Assessed: H1, H4

Targeted Performance Band: 5-6

Criteria	Marks
• Correct application of Pythagoras' Theorem.	1
• Gives the correct expression for area.	1

Sample Answer:



In ΔOPC, $OC = 10$ (radius of circle)

Therefore, by Pythagoras' Theorem $PC = \sqrt{100 - x^2}$

In ΔABC, height $= 10 + x$ and base $= 2 \times \sqrt{100 - x^2}$

$$\therefore \text{Area } A = \frac{1}{2} \times 2\sqrt{100 - x^2} \times (10 + x)$$

$$= (10 + x)\sqrt{100 - x^2} \quad \text{(as required)}$$

(c) (ii) (2 marks)

Outcomes Assessed: H1, H4

Targeted Performance Band: 5-6

Criteria	Marks
• Correctly applying the product rule.	1
• Correctly simplifying to the required expression.	1

Sample Answer:

$$\frac{dA}{dx} = (100 - x^2)^{\frac{1}{2}} - \frac{x(10 + x)}{(100 - x^2)^{\frac{1}{2}}}$$

$$= \frac{(100 - x^2) - x(10 + x)}{\sqrt{100 - x^2}}$$

$$= \frac{100 - x^2 - 10x - x^2}{\sqrt{100 - x^2}}$$

$$= \frac{100 - 10x - 2x^2}{\sqrt{100 - x^2}} \quad \text{as required}$$

(c) (iii) (3 marks)

Outcomes Assessed: H1, H4, H9

Targeted Performance Band: 5-6

Criteria	Marks
• Determines the maximum correctly.	1
• Shows a side is $10\sqrt{3}$.	1
• Shows that ΔABC is equilateral when $x = 5$.	1

Sample Answer:

For maximum area, $\dfrac{dA}{dx} = 0$

$$\frac{100 - 10x - 2x^2}{\sqrt{100 - x^2}} = 0$$

$$2x^2 + 10x - 100 = 0$$

$$x^2 + 5x - 50 = 0$$

$$(x - 5)(x + 10) = 0$$

$\therefore x = 5$ (Cannot have a negative value for a side length)

Show that it is a maximum area at $x = 5$

x	4	5	6
$\dfrac{dA}{dx}$	>0	$=0$	<0

Therefore a maximum area occurs at $x = 5$.

Dimensions of ΔABC are as follows for maximum area,

$$AC = 2 \times \sqrt{100 - 5^2}$$

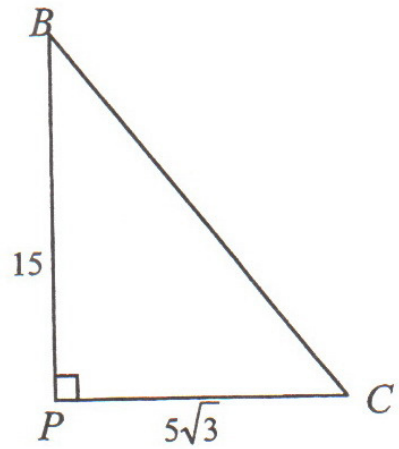
$$= 2\sqrt{75}$$

$$= 10\sqrt{3}$$

By Pythagoras' Theorem

$BC = 10\sqrt{3}$

Similarly $AC = 10\sqrt{3}$



Therefore the triangle with maximum area is equilateral with side length $10\sqrt{3}$ cm.