Name ....

KILLARA HIGH SCHOOL

1999
TRIAL EXAMINATION

MATHEMATICS
3 Unit (Additional)
and
3/4 Unit (Common)

Time Allowed - Two (2) hours
(plus 5 minutes reading time)

Directions to Candidates

- Attempt ALL questions
- Show all necessary working, marks may be deducted for careless or untidy work
- Standard integrals are printed on the last page
- Board-approved calculators may be used
- Additional Answer Booklets are available

Question One

(a) For the function $f(x) = e^{x+1}$ find the inverse function $f^{-1}(x)$ and hence show that $f[f^{-1}(x)] = f^{-1}[f(x)] = x$ *(3 marks)*

(b) Solve the inequality $\dfrac{1}{x+2} \geq \dfrac{2}{x-3}$ and represent the solution on a number line. *(3 marks)*

(c) If $\sum_{r=1}^{n} \frac{3}{2}(2)^{r-1} = 766\frac{1}{2}$, find n *(3 marks)*

(d) The word **EQUATION** contains all five vowels. How many 3-letter 'words' consisting of at least 1 vowel and 1 consonant can be made from the letters of **EQUATION**? *(3 marks)*

Question Two

(a) Prove that $\dfrac{2\cos A}{\operatorname{cosec} A - 2\sin A} = \tan 2A$ *(3 marks)*

(b) Show that $\tan^{-1} x = \sin^{-1} \dfrac{x}{\sqrt{1+x^2}}$ *(3 marks)*

(c) Write down the expansion for $\sin(\alpha - \beta)$ and hence prove that $\sin(-\theta) = -\sin\theta$ *(3 marks)*

(d) Find ${}^{d}\!/\!_{dx}\left[\dfrac{\ln x}{x}\right]$ and hence find the primitive function of $\dfrac{2 - \ln x}{x^2}$ *(4 marks)*

Question Three

(a) The sides of a square sheet of cardboard are each 12m long. At each corner a square of $x^2\text{m}^2$ is cut away. The sides of the sheet are then turned up to form a box. Calculate:

(i) the values of x so that the box has a volume of 108m^3

(ii) the value of x so that the box has a maximum volume *(5 marks)*

(c) Use mathematical induction to show that for all positive integers n, $\sum_{r=1}^{n} a^{-r} = \dfrac{a^n - 1}{(a-1)a^n}$ *(4 marks)*

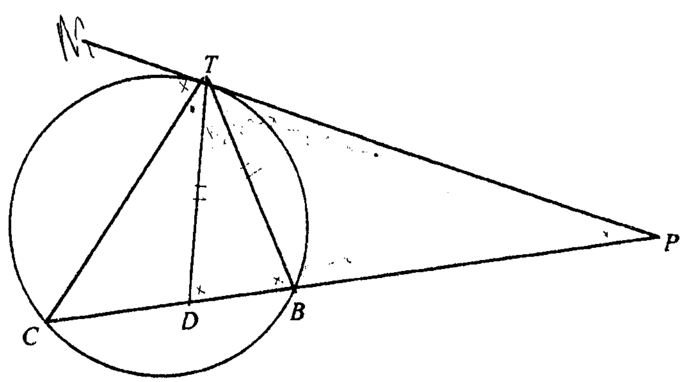
(d) A polynomial $P(x) = ax^3 + bx^2 + cx + d$ has zeroes at -2, 2 and $\frac{3}{2}$.
It leaves a remainder of 12 when divided by x - 1.
Find the values of a, b, c and d. *(3 marks)*

Question Four

(a) The tangent at the point $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$ cuts the y-axis at T. The line through the focus S parallel to this tangent cuts the directrix at V. M is the mid-point of TV. Find the locus of M as P moves on the parabola. *(5 marks)*

(b) If $3^x = 2^y = 6^z$, prove that $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$ *(3 marks)*

(c)

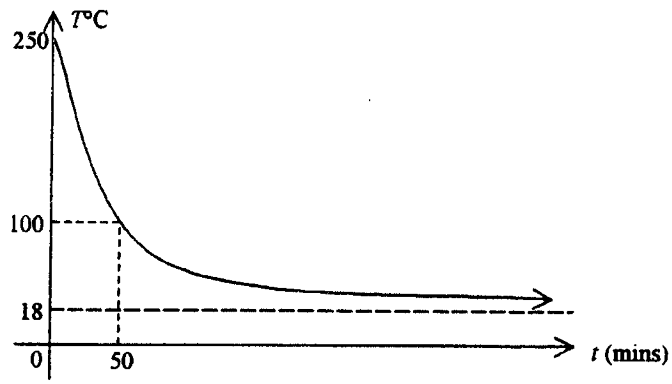


PT is a tangent to the circle and PBC is a secant. D is a point on PBC such that $TD = TB$.
Prove that $\angle CTD = \angle P$. *(4 marks)*

Ling Ling it would be much better if this were on your answer sheet.

Question Five

(a) The graph shown below shows the cooling curve for a container of paraffin oil which as been heated to a temperature of 250° then allowed to cool in air whose temperature is 18°C.



It is known that the rate at which the temperature T of the oil is changing is given by $\frac{dT}{dt} = k(T - M)$ where M is the temperature of the surrounding air and t is the time elapsed after cooling begins, in minutes.

(i) Show that $T = M + Ae^{kt}$ is a solution to the given equation.

(ii) Use the graph to write down the values of M and A.

(iii) Find the value of k to one significant figure if the temperature of the oil drops to 103.3°C in 50 minutes of cooling. *(6 marks)*

(b) Write the equation $2\tan\theta - 3\sec\theta = -2\sqrt{3}$ in the form $a\sin\theta + b\cos\theta = c$, and then by expressing the left hand side as sine of a compound angle, solve the equation for $0 \le \theta \le 360°$ *(6 marks)*

Question Six

(a) A particle moving in simple harmonic motion, passes through the centre of oscillation O with a velocity of 5cm/s. If it has a period of π seconds, find

(i) the value of n

(ii) the amplitude of the motion

(iii) the time taken for the particle to first reach $x = 1.5$cm *(5 marks)*

(b) Express $\sec(\sin^{-1} x)$ in terms of x and hence write down $\int \sec(\sin^{-1} x)dx \quad (-1<x<1)$ *(2 marks)*

(c) The acceleration of a body moving in a straight line is given by

$$\frac{d^2x}{dt^2} = -\frac{24}{x^2}$$

where x is the displacement from the origin after t seconds. When $t = 0$ the body is 3 metres to the right of the origin with a velocity of 4m/s.

(i) Show that the velocity v of the body in terms of x, is

$$v = \frac{4\sqrt{3}}{\sqrt{x}}$$

(ii) Find an expression for t in terms of x

(iii) How long does it take for the body to reach a point 10m to the right of the origin? *(5 marks)*

Question Seven

(a) (i) Show that the range of flight of a projectile fired at an angle of α to the horizontal and at a velocity v is

$$\frac{v^2 \sin 2\alpha}{g}$$

where g is the acceleration due to gravity

(ii) A cannon fires a shell at an angle of 45° to the horizontal and strikes a point 50m beyond its target. When fired with the same velocity at an angle of 30° it hits a point 20m in front of the target. Calculate

(I) the distance of the target from the cannon

(II) the correct angles required to hit the target *(7 marks)*

(b) By considering the value of $(1+x)^{2n}$ when $x = 1$, prove that

$$\sum_{k=0}^{n} \binom{2n}{k} = 2^{2n-1} + \frac{(2n)!}{2(n!)^2}$$

(5 marks)