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Centre Number

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Student Number



CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2010
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

Morning Session
Monday 9 August 2010

Cath 2010 2U Q&S

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on the back page
- All necessary working should be shown in every question
- Write your Centre Number and Student Number at the top of this page

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

Disclaimer

Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, *Principles for Setting HSC Examinations in a Standards-Referenced Framework* (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and *Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework* (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

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Total marks – 120

Attempt Questions 1-10

All questions are of equal value.

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet.

- (a) Find the value of $\sin 30^\circ - \tan 135^\circ$. 1
- (b) Factorise $x^3 - 1$. 1
- (c) Evaluate $\log_3 63 - \log_3 7$. 2
- (d) Write $\frac{5}{2-\sqrt{3}}$ with a rational denominator. 2
- (e) Solve $|x+3| < 8$. 2
- (f) Solve simultaneously 2
- $$\frac{x}{6} - \frac{y}{4} = 1$$
- $$5x + y = 13.$$
- (g) Given the curve $y = ax^2 - 8x + 9$ has a stationary point at $x = 2$, find the value of a . 2

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Find the equation of the tangent to the curve $f(x) = 2 \sin x$ at the point $(\pi, 0)$. 2

(b) Differentiate

(i) $\frac{1}{(2x+5)^3}$. 2

(ii) $x \ln x$. 2

(c) (i) Find $\int \left(1 + \sec^2 \pi x \right) dx$. 2

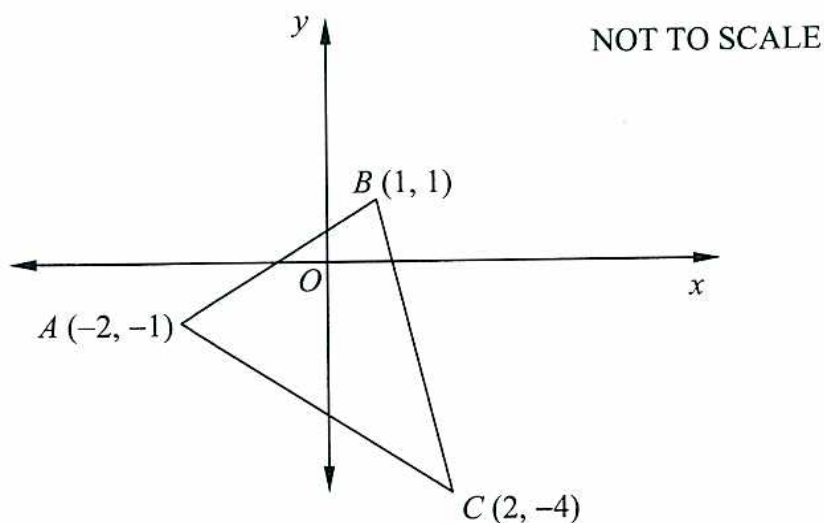
(ii) Evaluate $\int_0^{\ln 2} e^{3x} dx$. 2

(d) Evaluate $\sum_{k=2}^5 (-1)^k \left(\frac{1}{k} \right)$. 2

$\left(\frac{40}{6} \right)$

Question 3 (12 marks) Use a SEPARATE writing booklet.

- (a) A factory is testing a new rechargeable battery.
On the first charging it retains power for 121 hours. It retains power for 88 hours on the second charging and 64 hours on the third.
- (i) The life of the battery continues to decrease in the same ratio. Find the ratio. 1
- (ii) What is the total life of the battery if it continues to be charged in this manner? 2
(Leave your answer in hours and minutes).
- (b) The diagram shows the points $A(-2, -1)$, $B(1, 1)$ and $C(2, -4)$.

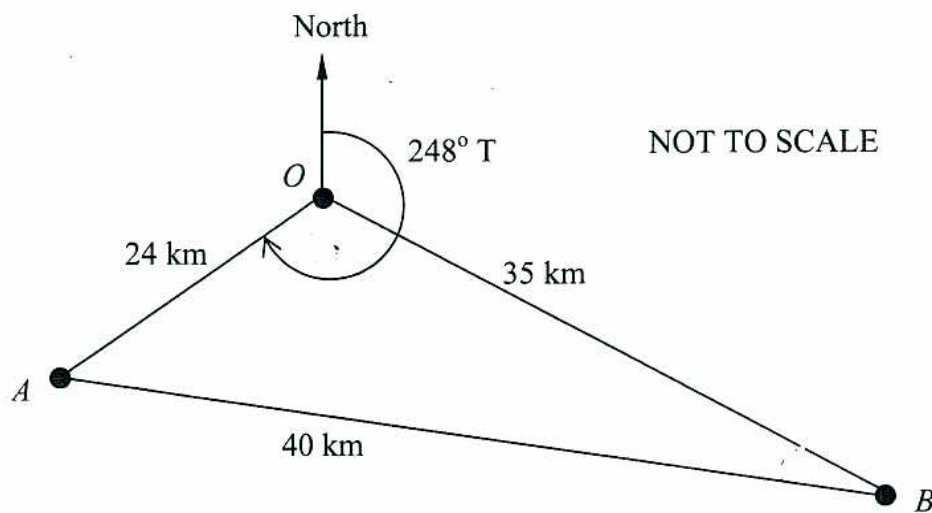


- (i) Calculate the length of the interval AB . 1
- (ii) Find the equation of the line AB . 2
- (iii) Show that the perpendicular distance from C to the line AB is $\frac{17}{\sqrt{13}}$. 1
- (iv) Hence, calculate the area of triangle ABC . 1

Question 3 continues on page 5

Question 3 (continued)

- (c) A section of rainforest is to be designated for a species count. The shape is shown below. The bearing of landmark A from landmark O is 248°T and is 24 km in distance. The distance from landmark A to B is 40 km and from landmark B to O is 35 km.



- | | | |
|-------|--|---|
| (i) | Use the cosine rule to show $\angle AOB$ is 83° . | 1 |
| (ii) | Hence, calculate the area of this section of the rainforest. | 1 |
| (iii) | What is the bearing of landmark O from landmark B ? | 2 |

End of Question 3

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) Solve $\log_3(2x - 1) = 2$. 2

- (b) There are two groups of people at a party and Minh is blind-folded. In the first group there are 4 men, 3 women and 2 children. In the second group there are 7 men and 5 women.

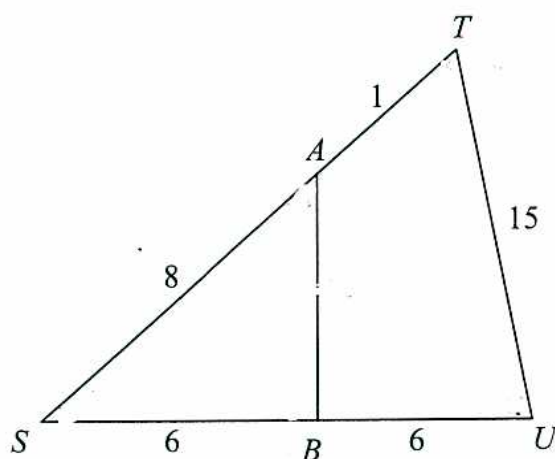
Minh is spun around and asked to select one person at random.

- (i) Find the probability that Minh approaches a person in the first group and then selects a woman. 1
- (ii) Find the probability that a woman from either group is selected. 2

- (c) The first three terms of an arithmetic series are 48, 41 and 34.

- (i) Find an expression for k^{th} term. 1
- (ii) Find the 45th term. 1
- (iii) Find the sum of the first 45 terms. 1

(d)



NOT TO SCALE

- (i) Prove triangle SAB is similar to triangle SUT . 3
- (ii) Hence, find the length of AB . 1

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) Consider the curve with equation $y = x^4 - 8x^2 + 16$.

(i) Prove that the function is even.

1

(ii) Show that $\frac{dy}{dx} = 4x(x - 2)(x + 2)$.

2

(iii) Find the stationary points and determine their nature.

3

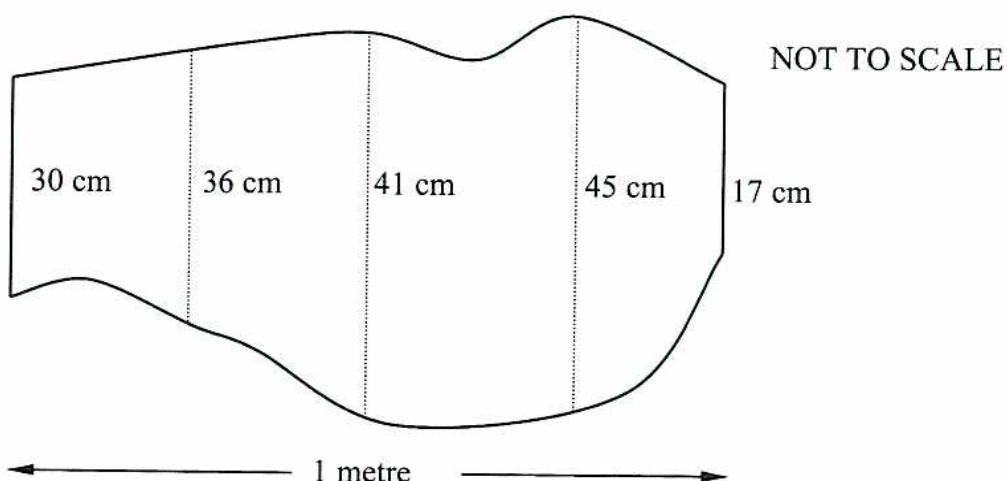
(iv) Sketch the curve showing all important features.

2

(v) Find the values of x for which the curve is decreasing.

1

(b) Suraj needs to estimate the area of the following hole in the wall.



(i) Copy and complete the table below.

1

Distance from left edge (cm)	0				100
Height of hole (cm)	30	36			

(ii) Use Simpson's Rule and all the values from the table to find an approximation for the area of the hole.

2

Question 6 (12 marks) Use a SEPARATE writing booklet.

- (a) The amount of garbage waste W per annum of a small town is known to be increasing exponentially according to the formula $W = W_0 e^{kt}$, where W_0 and k are constants and $t \geq 0$.

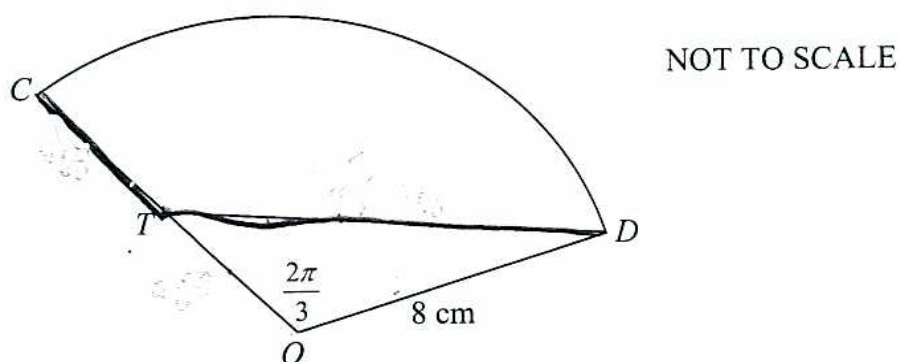
In 2003, the garbage waste weighed 180 000 tonnes while in 2010, it weighed 325 000 tonnes.

- (i) Find the value of W_0 and k . 2
- (ii) Estimate the amount of garbage waste in 2050. 2
- (iii) At what rate will the garbage waste be increasing during 2050? 2

- (b) Consider the curve $y = 3 \cos 2x$ in the domain $-\pi \leq x \leq \pi$.

- (i) State the amplitude of the curve. 1
- (ii) Sketch the curve in the given domain. 2

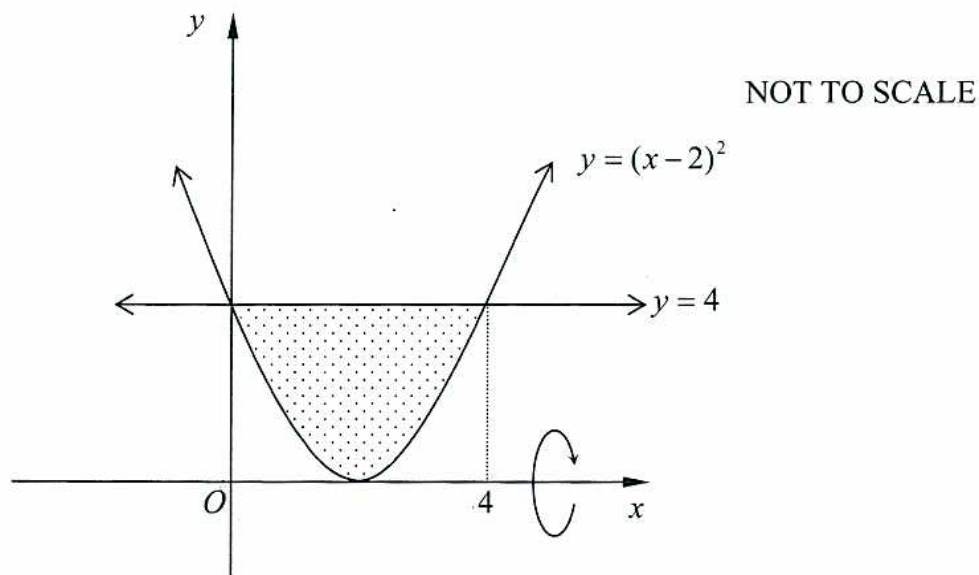
- (c) In the diagram, CD is an arc of a circle with radius 8 cm and centre O . 3
 T is the midpoint of OC . Angle COD is $\frac{2\pi}{3}$.



Find the perimeter of CTD in exact form.

Question 7 (12 marks) Use a SEPARATE writing booklet.

- (a) The shaded region bounded by the graph $y = (x-2)^2$ and the line $y = 4$ is rotated about the x -axis to form a solid of revolution as shown in the diagram.



Find the volume of the solid formed. Leave your answer in exact form.

4

- (b) The velocity, \dot{x} , in m/s of a particle moving in a straight line is given by $\dot{x} = 1 - 2\sin t$ for $0 \leq t \leq 2\pi$, where t is the time in seconds. The particle is initially at $x = 2$.

- | | | |
|-------|---|---|
| (i) | Find the initial velocity. | 1 |
| (ii) | At what time(s) is the acceleration zero? | 2 |
| (iii) | Find the <u>maximum velocity</u> of the particle during this period. | 1 |
| (iv) | Find the first time the particle changes direction during this period. | 2 |
| (v) | Hence, or otherwise, find the distance travelled by the particle between $t = 0$ and the first time the particle changes direction. | 2 |

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) The quadratic equation $x^2 - 4Ax + 6B = 0$ has two different real roots α and β .

(i) Show that $2A^2 > 3B$.

2

(ii) Find the range of values of B if the sum of the roots is equal to the product of the roots.

3

(b) Helen borrows \$25 000 from her local bank. The loan plus interest and charges are to be repaid at the end of each month in equal monthly instalments of \$ F over five years. Interest is charged at 6% p.a. and is calculated on the balance owing at the beginning of each month. Furthermore, at the end of each month a bank charge of \$15 is added to the account balance.

Let A_n be the amount owing after n months.

(i) Write down an expression for A_1 .

1

(ii) Deduce that the amount owing after three months is given by

2

$$A_3 = 25\,000 \times 1.005^3 - (F - 15)(1 + 1.005 + 1.005^2).$$

(iii) Hence write an expression for A_n .

1

(iv) Find Helen's monthly instalment \$ F , correct to the nearest cent.

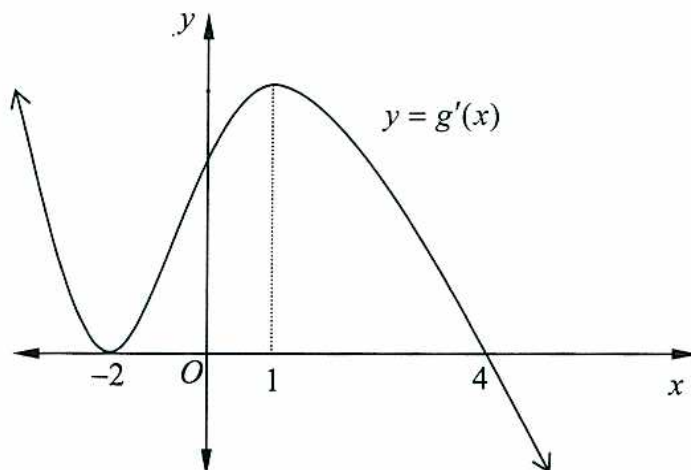
3

Question 9 (12 marks) Use a SEPARATE writing booklet

(a) Use the table of standard integrals provided to find $\frac{d}{dx}(\sec x)$.

1

(b) The graph of $y = g'(x)$ is shown below.



Copy the graph onto your writing booklet.

(i) There are two stationary points on $y = g(x)$.

3

Determine their x -coordinates and nature.

(ii) On the same diagram, draw a possible sketch of $y = g(x)$ showing all important features.

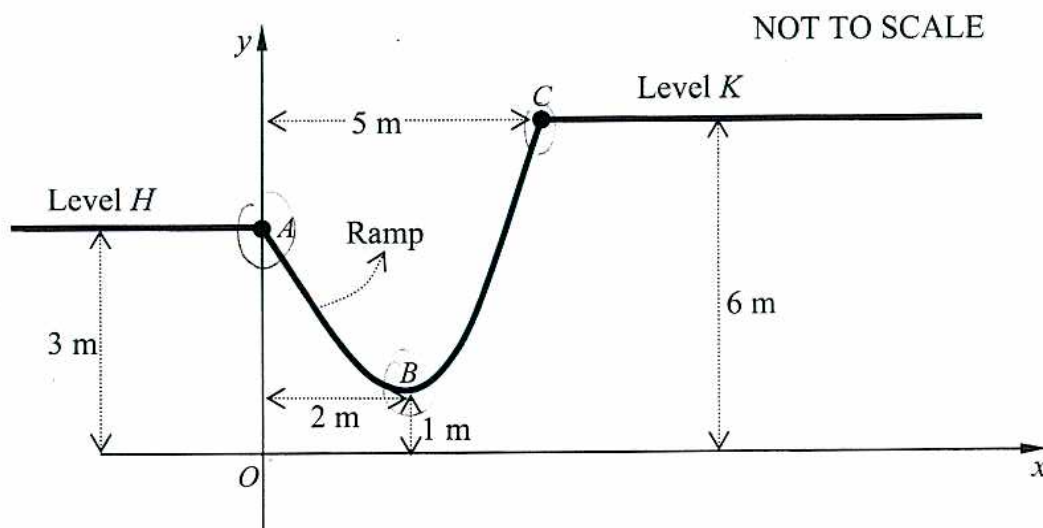
2

Question 9 continues on page 12

Question 9 (continued)

- (c) The city council of Gausstown has decided to build a skateboard ramp for its teenagers. The structure will consist of two levels, H and K and the ramp itself as shown in the diagram. The engineers believe that if the ramp has a gradient of 3 or greater at any point, it will be too dangerous to use.

Below is a cross-section of the proposed ramp.

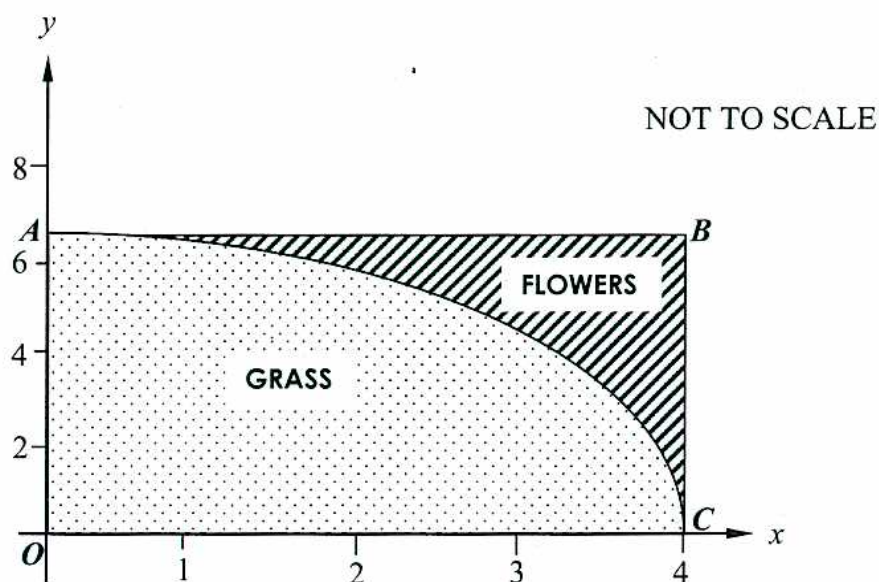


The ramp ABC is given by the equation $y = \frac{8x^2}{15} - \frac{31x}{15} + 3$ for $0 \leq x \leq 5$.

- | | | |
|-------|---|---|
| (i) | Use the information given in the cross-section to write down the coordinates of the points B and C . The point A is $(0,3)$. | 2 |
| (ii) | Determine the maximum gradient possible of the ramp over this domain. | 2 |
| (iii) | What is the greatest height that Level K may be constructed so that the ramp is deemed safe? Give your answer correct to 1 decimal place. | 2 |

End of Question 9

Question 10 (12 marks) Use a SEPARATE writing booklet.



The diagram above represents a rectangular garden $OABC$.

The garden's width AB is 4 metres and its length OA is $4 \log_e 5$ metres.

The garden is divided into two sections by a curve having the form $y = 4 \log_e (ax + b)$.

The lower section contains grass while the upper section contains flowers.

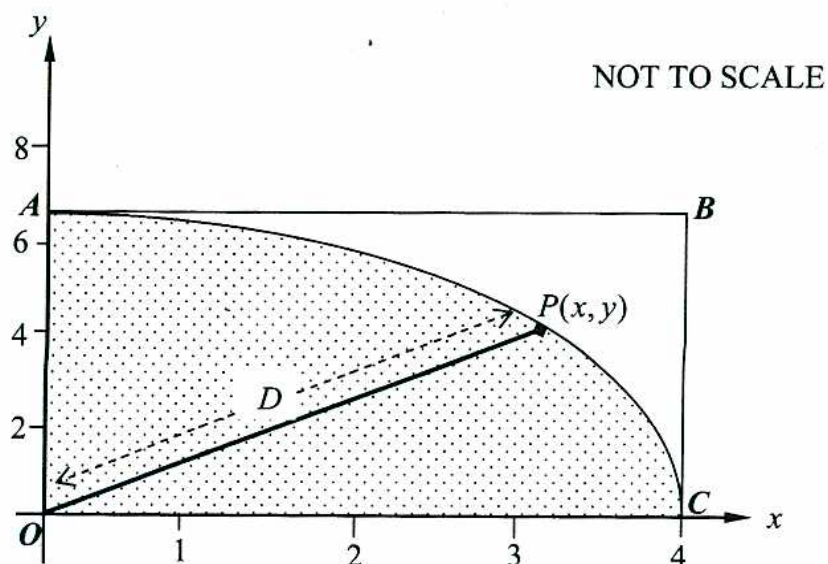
- (i) Find the area of the garden $OABC$ in exact form. 1
- (ii) Show that $a = -1$ and $b = 5$. 2
- (iii) Show that the area of the grassed section is given by 3

$$\int_0^{4 \log_e 5} \left(5 - e^{\frac{y}{4}} \right) dy \text{ and find this area in exact form.}$$

- (iv) Using part (i) and part (iii) to show that the area of the flower section is 1
 $(16 - 4 \log_e 5) \text{ m}^2$.

Question 10 continues on page 14

Question 10 (continued)



A sprinkler system is to be installed by using a straight line hose from the origin O to a point $P(x, y)$ on the curve.

- (v) Show that the distance, D , from the origin O to P is given by 1

$$D^2 = x^2 + 16[\log_e(5-x)]^2.$$

- (vi) Prove that $\frac{dD^2}{dx} = \frac{2x^2 - 10x + 32 \log_e(5-x)}{x-5}$. 2

- (vii) Hence, establish that the minimum length of straight line hose occurs when x is approximately equal to 3.63 metres. 2

End of Question 10

End of Paper

EXAMINERS

Kimon Kousparis (convenor)	Casimir Catholic College, Marrickville
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CATHOLIC SECONDARY SCHOOLS ASSOCIATION
2010 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION
MATHEMATICS – SUGGESTED SOLUTIONS

These marking guidelines show the criteria to be applied to responses along with the marks to be awarded in line with the quality of responses. These guidelines are suggested and not prescriptive. This is not intended to be an exhaustive list but rather an indication of the considerations that students could include in their responses.

Question 1 (12 marks)

(a) (1 mark)

Outcomes Assessed: P3

Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct answer.	1

Sample answer

$$\sin 30^\circ - \tan 135^\circ$$

$$= \frac{1}{2} - (-1)$$

$$= \frac{3}{2}$$

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(b) (1 mark)

Outcomes Assessed: P3

Targeted Performance Bands: 2-3

Criteria	Mark
• Correctly factorises the expression.	1

Sample answer

$$x^3 - 1$$

$$= (x-1)(x^2 + x + 1)$$

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(f) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 3-4

Criteria	Marks
• Eliminates ONE variable by correctly.	1
• Gives the correct answers for x and y.	1

Sample answer

$$\frac{x}{6} - \frac{y}{4} = 1 \quad \textcircled{1}$$

$$5x + y = 13 \quad \textcircled{2}$$

Multiply equation ① by 12, $2x - 3y = 12$ ③

Multiply equation ② by 3, $15x + 3y = 39$ ④

Adding equations ③ and ④, $17x = 51$

Therefore, $x = 3$

Substitute back into equation ②, $y = -2$

(g) (2 marks)
Outcomes Assessed: P7
Targeted Performance Bands: 3-4

Criteria	Marks
• Correct derivative.	1
• Gives the correct answer.	1

Sample answer

$$y = ax^2 - 8x + 9$$

$$\frac{dy}{dx} = 2ax - 8$$

at $x = 2$, $\frac{dy}{dx} = 0$

$$4a - 8 = 0$$

$$4a = 8$$

$$a = 2$$

(c) (2 marks)
Outcomes Assessed: H3
Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly applies logarithms to simplify the expression.	1
• Gives the correct answer.	1

Sample answer

$$\log_5 63 - \log_5 7 = \log_5 \left(\frac{63}{7} \right)$$

$$= \log_5 9$$

$$= \log_5 3^2$$

$$= 2 \log_5 3$$

$$= 2$$

(d) (2 marks)
Outcomes Assessed: P3
Targeted Performance Bands: 3-4

Criteria	Marks
• Multiplies by the conjugate.	1
• Gives the correct answer, with a rational denominator.	1

Sample answer

$$\frac{5}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$= \frac{10 + 5\sqrt{3}}{4 - 3}$$

$$= 10 + 5\sqrt{3}$$

(e) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Writes TWO correct inequations.	1
• Gives the correct answer.	1

Sample answer

$$x + 3 < 8 \quad \text{or} \quad -(x + 3) < 8$$

$$x < 5 \quad -x - 3 < 8$$

$$\quad \quad \quad x > -11 \quad \therefore -11 < x < 5$$

Question 2 (12 marks)

(a)(2 marks)

Outcomes Assessed: P7, P8

Targeted Performance Bands: 3-4

	Criteria	Marks
• Differentiates correctly and determines the gradient at $(\pi, 0)$.		1
• Uses the equation of a line to determine the tangent.		1

Sample answer

$$f(x) = 2 \sin x$$

$$f'(x) = 2 \cos x$$

$$f'(\pi) = 2 \cos \pi$$

$$= -2$$

Substituting into the equation of a line,

$$y - 0 = -2(x - \pi)$$

$$y = -2x + 2\pi$$

(b)(2 marks)

Outcomes Assessed: P7, P8

Targeted Performance Bands: 2-4

	Criteria	Marks
• Differentiates using the chain rule but has ONE mistake.		1
• Gives the correct answer.		1

Sample answer

$$\frac{d}{dx} \left(\frac{1}{(2x+5)^3} \right) = \frac{d}{dx} [(2x+5)^{-3}] = -3(2x+5)^{-4} \times 2$$

$$= \frac{-6}{(2x+5)^4}$$

(b)(2 marks)

Outcomes Assessed: P7, P8, H3

Targeted Performance Bands: 3-4

	Criteria	Marks
• Differentiates using the product rule but has ONE mistake.		1
• Gives the correct result.		1

Sample answer

$$\frac{d}{dx} (x \ln x) = x \left(\frac{1}{x} \right) + \ln x$$

$$= 1 + \ln x$$

(c)(2 marks)

Outcomes Assessed: H5, H8

Targeted Performance Bands: 3-4

	Criteria	Marks
• Finds the correct trigonometric primitive but has ONE mistake.		1
• Gives the correct answer.		1

Sample answer

$$\int (1 + \sec^2 \pi x) dx = x + \frac{1}{\pi} \tan \pi x + c$$

(c)(2 marks)

Outcomes Assessed: H3, H8

Targeted Performance Bands: 3-4

	Criteria	Marks
• Performs the integration correctly.		1
• Substitutes to give the correctly answer.		1

Sample answer

$$\int_0^{\ln 2} e^{3x} dx = \left[\frac{e^{3x}}{3} \right]_0^{\ln 2}$$

$$= \left(\frac{e^{3 \ln 2}}{3} \right) - \left(\frac{e^0}{3} \right)$$

$$= \frac{8}{3} - \frac{1}{3}$$

$$= \frac{7}{3}$$

(d)(2 marks)

Outcomes Assessed: P3, H5, H9

Targeted Performance Bands: 2-4

	Criteria	Marks
• Determines the terms of the sum correctly.		1
• Gives the correct answer.		1

Sample answer

$$\sum_{k=2}^5 (-1)^k \left(\frac{1}{k} \right) = (-1)^2 \left(\frac{1}{2} \right) + (-1)^3 \left(\frac{1}{3} \right) + (-1)^4 \left(\frac{1}{4} \right) + (-1)^5 \left(\frac{1}{5} \right)$$

$$= \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5}$$

$$= \frac{13}{60}$$

Question 3 (12 marks)

(a) (i) (1 mark)

Outcomes Assessed: P3, H5

Targeted Performance Bands: 2-3

Criteria	Mark
• Determines the ratio correctly.	1

Sample answer

$$r = \frac{t_2}{t_1} = \frac{88}{121} = \frac{8}{11}$$

(a) (ii) (2 marks)

Outcomes Assessed: P3, H5

Targeted Performance Bands: 2-3

Criteria	Mark
• Substitutes correctly into the limiting sum formula.	1
• Determines the answer correctly, in hours and minutes.	1

Sample answer

$$S_{\infty} = \frac{121}{1 - \frac{8}{11}} = \frac{121}{\frac{3}{11}} = \frac{1331}{3} = 443 \text{ hours and 40 minutes}$$

(b) (i) (1 mark)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 3-4

Criteria	Mark
• Uses the distance formula to find the correct answer.	1

Sample answer

$$AB = \sqrt{(1+1)^2 + (1+2)^2} = \sqrt{2^2 + 3^2} = \sqrt{13}$$

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(b) (ii) (2 marks)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 3-4

Criteria	Mark
• Uses the gradient formula to give the correct answer.	1
• Substitutes correctly into the equation of a line to give the correct answer.	1

Sample answer

$$m = \frac{1+1}{1+2} = \frac{2}{3}$$

$$y-1 = \frac{2}{3}(x-1)$$

$$3y-3 = 2x-2$$

$$\therefore 2x-3y+1 = 0$$

(b) (iii) (1 mark)

Outcomes Assessed: P3, P4

Targeted Performance Bands: 3-4

Criteria	Mark
• Substitutes correctly into the perpendicular distance to give the correct answer.	1

Sample answer

$$\text{Perpendicular distance} = \frac{|2(2) - 3(-4) + 1|}{\sqrt{2^2 + 3^2}}$$

$$= \frac{|4+12+1|}{\sqrt{13}}$$

$$= \frac{17}{\sqrt{13}}$$

(b) (iv) (1 mark)

Outcomes Assessed: P3, P4

Targeted Performance Band: 3-4

Criteria	Mark
• Finds the area correctly.	1

Sample answer

$$\text{Area} = \frac{1}{2} \times \sqrt{13} \times \frac{17}{\sqrt{13}}$$

$$= \frac{17}{2}$$

$$= 8\frac{1}{2} \text{ units}^2$$

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(c) (i) (1 mark)

Outcomes Assessed: P3, P4

Targeted Performance Band: 3-4

Criteria	Mark
• Applies the Cosine rule correctly to give the answer.	1

Sample Answer

$$\cos(\angle AOB) = \frac{24^2 + 35^2 - 40^2}{2 \times 24 \times 35}$$

$$\angle AOB = \cos^{-1}\left(\frac{24^2 + 35^2 - 40^2}{2 \times 24 \times 35}\right)$$

$$= 83^\circ$$

(c) (ii) (1 mark)

Outcomes Assessed: P3, P4

Targeted Performance Band: 3-4

Criteria	Mark
• Applies the Area formula correctly to give the answer.	1

Sample Answer

$$A = \frac{1}{2} \times 24 \times 35 \times \sin 83^\circ$$

$$= 416.8693\dots$$

$$= 417 \text{ km}^2$$

(c) (iii) (2 marks)

Outcomes Assessed: P3, P4

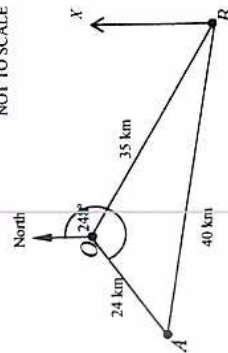
Targeted Performance Band: 3-4

Criteria	Mark
• Correctly determines the size of the angle between the North line and the interval AB.	1
• Determines the correct bearing.	1

Sample Answer

The angle between the North line and the interval AB is $248 - 83 = 165^\circ$.

NOT TO SCALE



$\angle XBO = 15^\circ$ (co-interior angles).
Therefore the bearing of O from B
is $360 - 15 = 345^\circ$.

Question 4 (12 marks)

(a) (2 marks)

Outcomes Assessed: H3

Targeted Performance Band: 3-5

Criteria	Mark
• Writes as an exponential equation.	1
• Gives the correct answer.	1

Sample Answer

$$\log_3(2x - 1) = 2$$

$$3^2 = 2x - 1$$

$$9 = 2x - 1$$

$$x = 5$$

(b) (i) (1 mark)

Outcomes Assessed: H5, H9

Targeted Performance Band: 3-4

Criteria	Mark
• Gives correct answer.	1

Sample Answer

Group 1: 4 men, 3 women, 2 children
Group 2: 7 men, 5 women

$$P(\text{Group 1, woman}) = \frac{1}{2} \times \frac{3}{9}$$

$$= \frac{1}{6}$$

(b) (ii) (2 marks)

Outcomes Assessed: H5, H9

Targeted Performance Band: 3-4

Criteria	Mark
• Gives the TWO probabilities.	1
• Gives the correct answer.	1

Sample Answer

$P(\text{choosing a woman}) = \text{probability of choosing a woman from group 1 OR group 2}$

$$= \frac{1}{6} + \frac{1}{2} \times \frac{5}{12}$$

$$= \frac{3}{8}$$

(c) (i) (1 mark)

Outcomes Assessed: H5

Targeted Performance Band: 3-4

Criteria	Mark
• Gives the correct answer in simplest form.	1

Sample Answer

$$48 + 41 + 34 + \dots \quad d = -7, \quad a = 48$$

$$T_k = a + (k-1)d$$

$$T_k = 48 + (k-1)(-7)$$

$$T_k = 48 - 7k + 7$$

$$T_k = 55 - 7k$$

(c) (ii) (1 mark)

Outcomes Assessed: H5

Targeted Performance Band: 3

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$$T_{45} = 55 - 7(45)$$

$$T_{45} = -260$$

(c) (iii) (1 mark)

Outcomes Assessed: H5

Targeted Performance Band: 3-4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$$S_n = \frac{n}{2}(a + l)$$

$$S_{45} = \frac{45}{2}(48 + (-260))$$

$$S_{45} = -4770$$

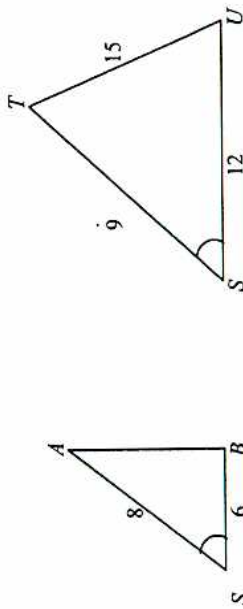
(d) (i) (3 marks)

Outcomes Assessed: P2, H2

Targeted Performance Band: 4-5

Criteria	Mark
• Gives the common angle.	1
• Gives the proportion statement.	1
• Gives the correct conclusion.	1

Sample Answer



$\angle ASB = \angle TSU$ (common angle)

$$\frac{SB}{ST} = \frac{6}{9} = \frac{2}{3} \quad \frac{SA}{SU} = \frac{8}{12} = \frac{2}{3} \quad (\text{Sides are in the same proportion})$$

\therefore Triangles ASB and STU are similar as two sides are in same proportion and the included angles are equal.

(d) (ii) (1 mark)

Outcomes Assessed: P2, H2

Targeted Performance Band: 4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$$\frac{AB}{UT} = \frac{SB}{ST}$$

(matching sides of similar triangles are in the same proportion)

$$\frac{AB}{15} = \frac{2}{3} \quad AB = 10 \text{ units}$$

Question 5 (12 marks)

(a) (i) (1 mark)

Outcomes Assessed: P2

Targeted Performance Band: 3-4

Criteria	Mark
• Shows that $f(-x) = f(x)$.	1

Sample Answer

For an even function, $f(-x) = f(x)$.

$$f(x) = x^4 - 8x^2 + 16$$

$$f(-x) = (-x)^4 - 8(-x)^2 + 16$$

$$= x^4 - 8x^2 + 16$$

$= f(x) \therefore$ function is even.

(a) (ii) (2 marks)

Outcomes Assessed: P7, H6

Targeted Performance Band: 3-5

Criteria	Marks
• Differentiates correctly.	1
• Finds common factor.	1

Sample Answer

$$y = x^4 - 8x^2 + 16 \therefore \frac{dy}{dx} = 4x^3 - 16x$$

$$= 4x(x^2 - 4)$$

$$= 4x(x - 2)(x + 2)$$

(a) (iii) (3 marks)

Outcomes Assessed: H6

Targeted Performance Band: 3-5

Criteria	Marks
• Finds TWO stationary points.	1
• Determines the nature of TWO stationary points.	1
• Determines the nature of the THIRD stationary point.	1

Sample Answer

For stationary points $\frac{dy}{dx} = 0 \therefore 4x(x - 2)(x + 2) = 0 \therefore x = 0, x = -2 \text{ or } x = 2$

\therefore the stationary points are $(0, 16), (-2, 0)$ & $(2, 0)$

Also for the nature of the stationary points, $\frac{d^2y}{dx^2} = 12x^2 - 16$

At $x = -2, \frac{d^2y}{dx^2} = 32 > 0 \therefore (-2, 0)$ is a MINIMUM stationary point

At $x = 0, \frac{d^2y}{dx^2} = -16 < 0 \therefore (0, 16)$ is a MAXIMUM stationary point

Since the curve is even, the y-axis is an axis of symmetry, $\therefore (2, 0)$ is also a MINIMUM stationary point.

(a) (iv) (2 marks)

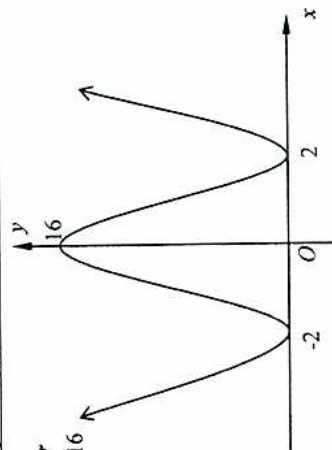
Outcomes Assessed: P6, H6, H9

Targeted Performance Band: 3-5

Criteria	Marks
• Draws the correct quartic curve.	1
• Plots all important points.	1

Sample Answer

$$y = x^4 - 8x^2 + 16$$



(a) (v) (1 mark)

Outcomes Assessed: P6, H6, H7

Targeted Performance Band: 3-4

Criteria	Mark
Correctly solves the equation $\frac{dy}{dx} < 0$ or gives correct answer from graph.	1

Sample Answer

$$\frac{dy}{dx} = 4x(x-2)(x+2)$$

For the curve to be decreasing, $\frac{dy}{dx} < 0 \therefore 4x(x-2)(x+2) < 0$

$$\therefore x < -2 \text{ and } 0 < x < 2$$

(b) (i) (1 mark)

Outcomes Assessed: H4

Targeted Performance Band: 2-3

Criteria	Mark
Completes the table correctly.	1

Sample Answer

Distance from left edge (cm)	0	25	50	75	100
Height of hole (cm)	30	36	41	45	17

(b) (ii) (2 marks)

Outcomes Assessed: H8, H9

Targeted Performance Band: 3-5

Criteria	Mark
Substitutes correctly into Simpson's Rule.	1
Gives the correct answer.	1

Sample Answer

$$\text{Area} = \frac{h}{3}(y_0 + y_4 + 4(y_1 + y_3) + 2y_2), \quad h = 25$$

$$= \frac{25}{3}[30 + 17 + 4(36 + 45) + 2(41)]$$

$$= 3775 \text{ m}^2$$

Question 6 (12 marks)

(a) (i) (2 marks)

Outcomes Assessed: H3, H4

Targeted Performance Band: 4-5

Criteria	Mark
Correctly shows $W_0 = 180000$.	1
Correctly shows $k = 0.0844$.	1

Sample Answer

$$180000 = W_0 e^{k(0)} \therefore W_0 = 180000$$

$$325000 = 180000 e^{k(7)} \therefore e^{7k} = 1.806$$

$$7k = \log_e 1.806$$

$$k = 0.0844 \text{ (to three significant figures)}$$

(2004 is year 1 \therefore 2010 is year 7)

(a) (ii) (2 marks)

Outcomes Assessed: H3, H4

Targeted Performance Band: 4-5

Criteria	Mark
Correctly substitutes 47 with progress towards the answer.	1
Correctly determines the amount of garbage waste in 2050.	1

Sample Answer

$$W = 180000 e^{0.0844 \times 47}$$

$$\text{(The year 2050 corresponds to } t = 47 \text{)}$$

$$W = 9506745.24 \text{ tonnes}$$

(a) (iii) (2 marks)

Outcomes Assessed: H3, H4

Targeted Performance Band: 4-5

Criteria	Mark
Correctly differentiates to get the rate $\frac{dW}{dt}$.	1
Correctly substitutes $t = 47$ into $\frac{dW}{dt}$ and has correct answer.	1

Sample Answer

$$W = 180000 e^{0.0844t}$$

$$\frac{dW}{dt} = 0.0844 \times 180000 e^{0.0844t}$$

$$\text{Let } t = 47, \frac{dW}{dt} = 0.0844 \times 180000 e^{0.0844 \times 47}$$

$$\frac{dW}{dt} = 802369.3$$

During 2050, the garbage rate will be increasing at a rate of 802 369 tonnes per year.

(b) (i) (1 mark)

Outcomes Assessed: H5

Targeted Performance Band: 2-4

Criteria	Mark
• Gives correct amplitude.	1

Sample Answer

$$\text{Amplitude} = 3$$

(b) (ii) (2 marks)

Outcomes Assessed: H5

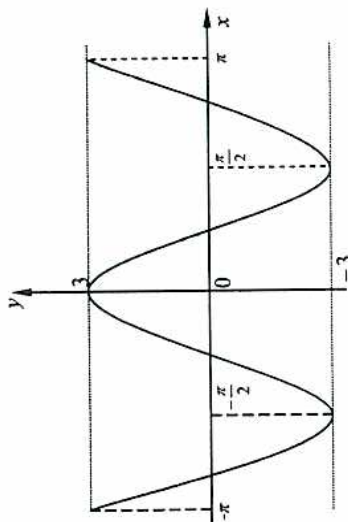
Targeted Performance Band: 2-4

Criteria	Marks
• Shows a sketch with the correct shape.	1
• Shows the correct period and range.	1

Sample Answer

$$\text{Amplitude} = 3$$

$$\text{Period} = \frac{2\pi}{n} = \frac{2\pi}{2} = \pi$$



(c) (3 marks)

Outcomes Assessed: H4, H9

Targeted Performance Band: 3-4

Criteria	Marks
• Correctly finds the length DT in exact form.	1
• Finds the arc length CD .	1
• Gives the correct perimeter in exact form.	1

Sample Answer

Length DT

$$OT = 4 \text{ cm (midpoint of } OC)$$

$$DT^2 = 4^2 + 8^2 - 2 \times 4 \times 8 \cos\left(\frac{2\pi}{3}\right)$$

$$DT^2 = 112$$

$$DT = 4\sqrt{7}$$

$$\text{Perimeter} = DT + \text{arc } CD + CT$$

$$= \left(4\sqrt{7} + \frac{16\pi}{3} + 4\right) \text{ cm}$$

Arc length CD

$$l = r\theta$$

$$CD = 8 \times \frac{2\pi}{3}$$

$$= \frac{16\pi}{3}$$

Question 7 (12 marks)

(a) (4 marks)

Outcomes Assessed: H8, H9

Targeted Performance Band: 4–5

	Criteria	Marks
• Finds V_1		1
• Correctly applies Newton–Leibnitz formula/substitution finds V_2		2
• Calculates the correct volume.		1

Sample Answer

$$V_1 = \pi \times 4^2 \times 4 = 64\pi$$

$$V_2 = \pi \int_0^4 (x-2)^4 dx = \pi \left[\frac{(x-2)^5}{5} \right]_0^4 = \pi \times \left[\frac{(4-2)^5}{5} - \frac{(0-2)^5}{5} \right] = \frac{64\pi}{5}$$

$$\therefore \text{Total volume is } V_1 - V_2 = 51\frac{1}{5}\pi \text{ u}^3$$

(b) (i) (1 mark)

Outcomes Assessed: H5

Targeted Performance Band: 4

	Criteria	Mark
• Gives the correct answer.		1

Sample Answer

$$\dot{x} = 1 - 2\sin t = 1 - 2\sin(0) = 1 \text{ m/s}$$

Therefore the initial velocity is 1 m/s.

(b) (ii) (2 marks)

Outcomes Assessed: H4, H5

Targeted Performance Band: 4–5

	Criteria	Marks
• Finds an expression for \ddot{x} .		1
• Gives the correct answer.		1

Sample Answer

$$\ddot{x} = -2\cos t$$

$$0 = -2\cos t \quad \therefore t = \frac{\pi}{2}, \frac{3\pi}{2}$$

Therefore the acceleration is zero at $\frac{\pi}{2} \text{ s}, \frac{3\pi}{2} \text{ s}$.

(b) (iii) (1 mark)

Outcomes Assessed: H4, H5

Targeted Performance Band: 4–5

	Criteria	Mark
• Gives the correct answer.		1

Sample Answer

Max velocity = 3 m/s from amplitude of \dot{x} or sub $t = \frac{3\pi}{2}$ into \dot{x} .

(b) (iv) (2 marks)

Outcomes Assessed: H4

Targeted Performance Band: 4–5

	Criteria	Marks
• Equates $\dot{x} = 0$ and progress towards answer.		1
• Gives the correct answer.		1

Sample Answer

Particle changes direction when $\dot{x} = 0$.

$$\dot{x} = 1 - 2\sin t = 0$$

$$\sin t = \frac{1}{2} \Rightarrow t = \frac{\pi}{6} \text{ sec}$$

(b) (v) (2 marks)

Outcomes Assessed: H4, H5

Targeted Performance Band: 4–5

	Criteria	Marks
• Gives a correct expression for x or the distance travelled		1
• Gives the correct answer		1

Sample Answer

Distance travelled is given by

$$\int_0^{\frac{\pi}{6}} 1 - 2\sin t \, dt = \left[t + 2\cos t \right]_0^{\frac{\pi}{6}} = \left[\frac{\pi}{6} + 2\cos \frac{\pi}{6} \right] - \left[0 + 2\cos 0 \right] = \frac{\pi}{6} + \sqrt{3} - 2 \approx 0.26 \text{ m}$$

Question 8 (12 marks)

(a) (i) (2marks)

Outcomes Assessed: P3, H2

Targeted Performance Band: 3-4

Criteria	Marks
• Establishes that $\Delta > 0$.	1
• Shows the correct answer of $2A^2 > 3B$.	1

Sample Answer

Since the roots are real and different $\therefore \Delta > 0$ i.e. $b^2 - 4ac > 0$

$$16A^2 - 24B > 0 \quad \therefore 2A^2 > 3B \quad \dots \textcircled{1}$$

(a) (ii) (3marks)

Outcomes Assessed: P3, H9

Targeted Performance Band: 3-5

Criteria	Marks
• Gives correct expression $4A = 6B$.	1
• Finds the correct inequality $3B^2 - 2B > 0$.	1
• Gives the correct answer.	1

Sample Answer

Since the sum of the roots is equal to the product of the roots

$$a + \beta = a\beta \quad \therefore 4A = 6B \quad \therefore A = \frac{3}{2}B \quad \therefore \text{sub into } \textcircled{1}$$

$$\therefore 2\left(\frac{3}{2}B\right)^2 > 3B \quad \frac{9}{2}B^2 > 3B \quad \Rightarrow 3B^2 - 2B > 0 \quad \Rightarrow B(3B - 2) > 0$$

\therefore the range of values of B is $B < 0$ and $B > \frac{2}{3}$.

(b) (i) (1mark)

Outcomes Assessed: H5

Targeted Performance Band: 3-5

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$$A_1 = 25\,000(1.005) - F + 15$$

$$= 25\,000(1.005) - (F - 15)$$

(b) (ii) (2marks)

Outcomes Assessed: H4, H5

Targeted Performance Band: 3-5

Criteria	Marks
• Progress towards A_2 .	1
• Shows that $A_3 = 25\,000 \times 1.005^3 - (F - 15)(1 + 1.005 + 1.005^2)$.	1

Sample Answer

$$A_2 = [25\,000(1.005) - (F - 15)(1.005)] - (F - 15)$$

$$= 25\,000(1.005)^2 - (F - 15)(1 + 1.005)$$

$$\therefore A_3 = 25\,000(1.005)^3 - (F - 15)[1 + 1.005 + 1.005^2]$$

(b) (iii) (1mark)

Outcomes Assessed: H4

Targeted Performance Band: 4-5

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$$A_n = 25\,000(1.005)^n - (F - 15)[1 + 1.005 + 1.005^2 + \dots + 1.005^{n-1}]$$

(b) (iv) (3marks)

Outcomes Assessed: H4, H5

Targeted Performance Band: 4-6

Criteria	Marks
• Equates $A_{60} = 0$.	1
• Progress towards finding F .	1
• Finds the correct value for F .	1

Sample Answer

Let $n = 60$ and $A_{60} = 0$

$$\therefore A_{60} = 25\,000(1.005)^{60} - (F - 15)[1 + 1.005 + 1.005^2 + \dots + 1.005^{59}]$$

$$\therefore 0 = 25\,000(1.005)^{60} - (F - 15)\left[\frac{1(1.005^{60} - 1)}{1.005 - 1}\right]$$

$$(F - 15)(69.77003) = 33721.25381$$

$$F - 15 = 483.32$$

$$\therefore F = \$498.32$$

Question 9 (12 marks)

(a) (1 mark)

Outcomes Assessed: H5

Targeted Performance Band: 3-5

Criteria	Marks
• Gives the correct answer.	1

Sample Answer

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

(b) (i) (3 marks)

Outcomes Assessed: H6, H7

Targeted Performance Band: 4-6

Criteria	Marks
• Correctly determines the MAX stationary point at $x = 4$	1
• Correctly determines the stationary point at $x = -2$	1
• Correctly determines the HPOI at $x = -2$	1

Sample Answer

At $x = 4$ $g'(x) = 0$ \therefore stationary point. Moving from left to right $g'(x) = + 0 - \therefore$ MAX TP

At $x = -2$ $g'(x) = 0$ \therefore stationary point. Moving from left to right $g'(x) = + 0 + \therefore$ HPOI

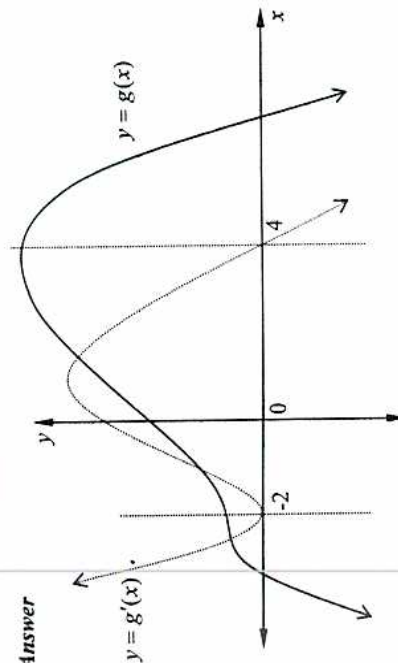
(b) (ii) (3 marks)

Outcomes Assessed: H6, H7

Targeted Performance Band: 4-6

Criteria	Marks
• Correctly draws the MAX stationary point at $x = 4$.	1
• Gives the correct graph.	1

Sample Answer



(c) (i) (2 marks)

Outcomes Assessed: P5,

Targeted Performance Band: 3-5

Criteria	Marks
• Correctly determine the coordinates of B.	1
• Correctly determine the coordinates of C.	1

Sample Answer

B (2, 1) and C (5, 6).

(c) (ii) (2 marks)

Outcomes Assessed: P7, H6, H9

Targeted Performance Band: 3-5

Criteria	Marks
• Differentiates correctly.	1
• Gives the correct answer.	1

Sample Answer

$$y = \frac{8x^2}{15} - \frac{31x}{15} + 3 \dots \textcircled{1}$$

$$\therefore \frac{dy}{dx} = \frac{16}{15}x - \frac{31}{15} \dots \textcircled{2} \quad \text{Since } 0 \leq x \leq 5 \quad \therefore \text{the maximum gradient occurs when } x = 5$$

$$\therefore \frac{dy}{dx} = \frac{16}{15} \times 5 - \frac{31}{15} = 3.27 \text{ (correct to two decimal places)}$$

(c) (iii) (2 marks)

Outcomes Assessed: H2, H5, H6, H9

Targeted Performance Band: 3-5

Criteria	Marks
• Establishes that $\frac{dy}{dx} = 3$.	1
• Gives the correct answer.	1

Sample Answer

$$\text{When } \frac{dy}{dx} = 3 \quad \therefore \text{sub into } \textcircled{2} \quad \therefore 3 = \frac{16}{15}x - \frac{31}{15}$$

$$\therefore 45 = 16x - 31 \quad \therefore x = 4.75 \quad \therefore \text{sub into } \textcircled{1}$$

$$\therefore y = \frac{8 \times 4.75^2}{15} - \frac{31 \times 4.75}{15} + 3 = 5.2$$

\therefore The height is 5.2 metres.

Question 10 (12 marks)

(i) (1 mark)

Outcomes Assessed: P4

Targeted Performance Band: 2-4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

Area of the garden $\Rightarrow 4 \times 4 \log_e 5 = 16 \log_e 5$

(ii) (2 marks)

Outcomes Assessed: H3, H9

Targeted Performance Band: 3-5

Criteria	Mark
• Progress towards the correct answer	1
• Correctly determines the values of a and b .	1

Sample Answer

Since $y = 4 \log_e(ax + b)$ \therefore when $x = 4, y = 0 \Rightarrow 0 = 4 \log_e(4a + b)$
 $\therefore e^0 = 4a + b \quad \therefore 1 = 4a + b \dots \textcircled{1}$

When $x = 0, y = 4 \log_e 5 \quad 4 \log_e 5 = 4 \log_e b \quad \therefore b = 5$
 Now sub into $\textcircled{1} \quad 1 = 4a + 5 \quad \therefore a = -1$

(iii) (3 marks)

Outcomes Assessed: H3, H8

Targeted Performance Band: 4-6

Criteria	Mark
• Establishes correct integral $A = \int_0^{4 \ln 5} (5 - e^{\frac{y}{4}}) dy$	1
• Integrates correctly	1
• Evaluates correctly to find the area in exact form.	1

Sample Answer

Since $y = 4 \log_e(ax + b) \quad \therefore \frac{y}{4} = \log_e(5 - x) \quad \therefore e^{\frac{y}{4}} = 5 - x \quad \therefore x = 5 - e^{\frac{y}{4}}$

\therefore the area of the grass section is: $A = \int_0^{4 \ln 5} (5 - e^{\frac{y}{4}}) dy$

$$A = \left[5y - \frac{e^{\frac{y}{4}}}{\frac{1}{4}} \right]_0^{4 \ln 5} = \left[5y - 4e^{\frac{y}{4}} \right]_0^{4 \ln 5} = 5 \times 4 \log_e 5 - 4e^{\frac{4 \log_e 5}{4}} - [0 - 4 \times 1]$$

$$= [20 \log_e 5 - 20] + 4$$

$$= (20 \log_e 5 - 16) \text{ m}^2$$

(iv) (1 mark)

Outcomes Assessed: H3

Targeted Performance Band: 3-5

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

Area of the flowers section $\Rightarrow 16 \log_e 5 - (20 \log_e 5 - 16) = (16 - 4 \log_e 5) \text{ m}^2$

(v) (1 mark)

Outcomes Assessed: H1, H4

Targeted Performance Band: 3-5

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$D^2 = x^2 + y^2$, and since $y = 4 \log_e(ax + b)$ with $a = -1$ and $b = 5$

$\therefore y = 4 \log_e(5 - x)$

$\therefore D^2 = x^2 + 16[\log_e(5 - x)]^2$

(vi) (2 marks)

Outcomes Assessed: H3, H5

Targeted Performance Band: 3-5

	Criteria	Marks
• Correctly uses the chain rule but has ONE mistake.		1
• Gives correct answer.		1

Sample Answer

$$D^2 = x^2 + 16[\log_e(5-x)]^2$$

$$\therefore \frac{dD^2}{dx} = 2x + 16 \times 2[\log_e(5-x)] \times \frac{-1}{5-x}$$

$$= 2x - \frac{32 \log_e(5-x)}{5-x}$$

$$= 2x + \frac{32 \log_e(5-x)}{x-5}$$

$$= \frac{2x^2 - 10x + 32 \log_e(5-x)}{x-5}$$

(vii) (2 marks)

Outcomes Assessed: P8, H6

Targeted Performance Band: 3-5

	Criteria	Marks
• Shows $\frac{dD^2}{dx} \approx 0$ for $x = 3.63$.		1
• Proves a minimum length.		1

Sample Answer

$$\frac{dD^2}{dx} = \frac{2(3.63)^2 - 10(3.63) + 32 \log_e(5-3.63)}{3.63-5} = -0.09 \text{ which is approximately zero.}$$

x	3	3.63	4
$\frac{dD^2}{dx}$	-5.09	0	8
	<	0	>

\therefore minimum length of hose occurs at $x = 3.63$.