2006 2U TRIAL.

QUESTION 1

a) $x^2+6x+14$

$= x+6x+9+5.$

$= (x+3)^2+5.$

$\equiv (x+a)^2+b.$

$a=3$ & $b=5$.

b) $e^{2.5} \doteqdot 12.18$ (2 dp)

c) $\cos\frac{7\pi}{6} = -\cos\frac{\pi}{6}$

$= -\frac{\sqrt{3}}{2}.$

d) $|4-x|=7$

$4-x=7$ or $4-x=-7$

$x=-3$ or $x=11$

e) $\frac{4}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \frac{4(\sqrt{5}+\sqrt{3})}{5-3}$

$= 2(\sqrt{5}+\sqrt{3})$

f) $a^2=12a.$

$a(a-12)=0$

$a=12$ or $a=0$.

QUESTION 2.

a) i) $4x+3y-12=0$

when $x=0$ $y=4$

when $y=0$ $x=3$

A(3,0) & B(0,4)

ii) $\tan(180-\theta) = \frac{4}{3}$

$180-\theta = \tan^{-1}(\frac{4}{3})$

$180-\theta = 53°8'$

$\theta = 127°$ nearest degree.

iii) $d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

$C=(x_1,y_1)$
$=(4,2)$

$= \left|\frac{16+6-12}{5}\right|$

$= 2$ units

iv) $A=\frac{1}{2}b\times h.$

$=\frac{1}{2}\times\sqrt{4^2+3^2}\times 2.$

$=5$ units^2.

B) $3x-y=16$ —— ①

$x+4y=1$ —— ② $\Rightarrow x=1-4y$ ③

subs 3 into ①.

$3(1-4y)-y=16$

$-13y=13$ $y=-1$.

subs $y=-1$ into ②.

$x-4=1$ $\therefore x=5$

c) $y=x^2-4x+8.$

$y=(x-2)^2+4.$

$(x-2)^2=4(\frac{1}{4})(y-4)$

vertex $=(2,4).$

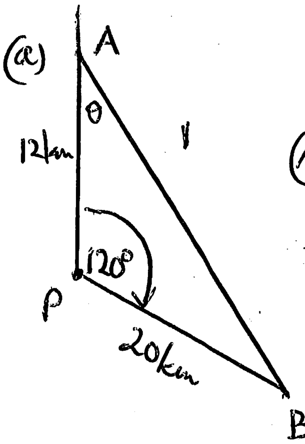
focal length $=\frac{1}{4}.$

$\therefore$ focus $=(2,4\frac{1}{4})$

SECTION B

Question 3

(a)



(i) $(AB)^2 = 12^2 + 20^2 - 2.12.20\cos 120^\circ$ 1

$AB = \cancel{\ldots} = 28\text{ km}$

(ii) $\dfrac{\sin\theta}{20} = \dfrac{AB\sin 120^\circ}{AB}$

$\theta = 38^\circ 13'$ 1

$\therefore$ Bearing is

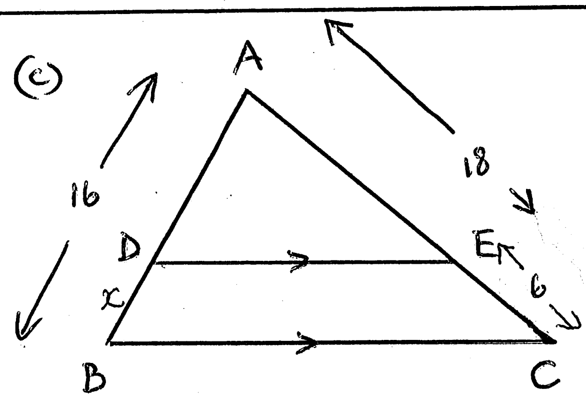
$(180 - \theta)$ ie $141^\circ 47'$ 1

(b) (i) $y = 2x^{-4} \Rightarrow y' = -8x^{-5}$ 1

(ii) $y = \sin(x^3) \Rightarrow y' = 3x^2 \cos x^3$ 1

(iii) $y = x\tan x \Rightarrow y' = x\sec^2 x + 1.\tan x$ 2

(c)



(i) $\angle A$ is common

$\angle ADE = \angle ABC$ (corresp. angles on // lines) 1

$\therefore$ Triangles ABC and ADE are equiangular $\Rightarrow$ Similar. 1

(ii) let $BD = x \Rightarrow AD = 16 - x$

$\dfrac{16-x}{16} = \dfrac{18}{24} \Rightarrow \boxed{x = 4}$ 2

$4x = 16$

Question 4

(a) $\left[\log_e(1+x)\right]_0^1 = \log_e 2 - \log_e 1$ 1

$= \log_e 2$ 1

(b) $\sqrt{3}\tan x = 1$

$\Rightarrow \tan x = \dfrac{1}{\sqrt{3}}$ 2

$x = \dfrac{\pi}{6}$ and $\pi + \dfrac{\pi}{6}$

ie $x = \dfrac{\pi}{6}, \dfrac{7\pi}{6}$

(c) $\sqrt{\dfrac{1-\cos^2 A}{1+\tan^2 A}} = \sqrt{\dfrac{\sin^2 A}{\sec^2 A}}$ 2

$= \sqrt{\sin^2 A \cos^2 A}$

$= \sin A \cos A$

(d) $y = \cos\left(x + \frac{\pi}{3}\right)$

$y' = -\sin\left(x + \frac{\pi}{3}\right)$ 2

At $x = 0$ grad. tang. is $-\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

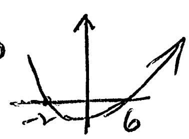
(e) (i) $\int \cos 2x\,dx = \frac{1}{2}\sin 2x + c$ 1

(ii) $\int \dfrac{4}{e^{3x}}\,dx = 4\int e^{-3x}\,dx$ 1

$= -\frac{4}{3}e^{-3x} + c$

(f) $x^2 + (c-2)x + 4 = 0$

for real roots $\Delta \geq 0$



$\Delta = (c-2)^2 - 4(1)4$ 2

$= (c-6)(c+2)$

≥ 0 when $\boxed{c \leq -2 \text{ or } c \geq 6}$

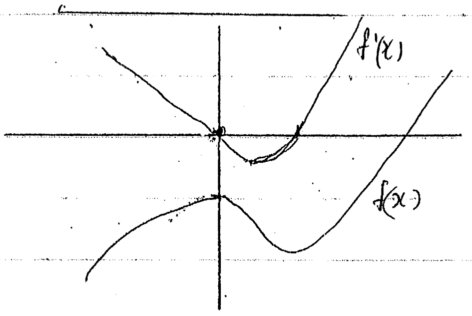
QUESTION 5

(a) $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$x^2 - x(1-\sqrt{3}+1+\sqrt{3}) + (1+\sqrt{3})(1-\sqrt{3})$

$x^2 - 2x - 2 = 0$

(b)



(c)(i) $7 + 7(n-1) < 1200$

$7 + 7n - 1 < 1200$

$n < 171.4$

$n = 171$

multiple $= 1197$

(ii) $\frac{171}{2}(7 + 1197)$

Sum $= 102942$

(d) $V = \pi\int_0^9 (4\sqrt{x})^2\,dx$

$= \pi\int_0^9 16x\,dx$

$= \pi\left[8x^2\right]_0^9$

$= 648\pi\ u^3$

(e) $f'(x) = 3x^2 + 2$

$f(x) = x^3 + 2x + c$

$(2,5)\quad 5 = 8 + 4 + c,\ c = -7$

$f(x) = x^3 + 2x - 7$

QUESTION 6

$y = x^3 - 3x^2 - 9x + 2$

(i) $\frac{dy}{dx} = 3x^2 - 6x - 9$

$\frac{d^2y}{dx^2} = 6x - 6$

stat pts $3x^2 - 6x - 9 = 0$

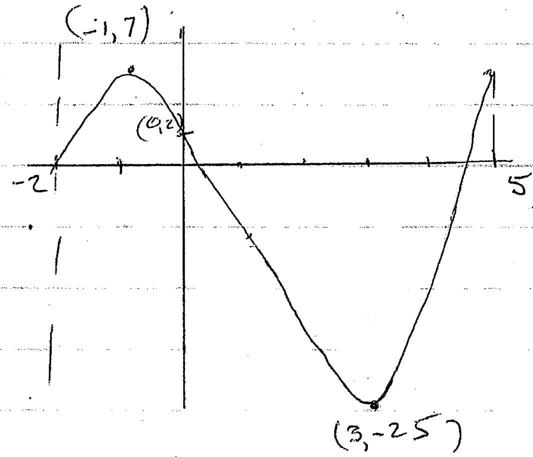
$(x-3)(x+1) = 0$

(ii) pts $(3,-25)$ + $(-1,7)$

$x = 3\quad \frac{d^2y}{dx^2} > 0\ \therefore$ min

$x = -1\quad \frac{d^2y}{dx^2} < 0\ \therefore$ max

(iii)



(iv) max $= 7$ at $x = -1, 5$

(b)

x	1	2	3
y	2	$\frac{2}{\sqrt{3}}$	1

$y = \operatorname{Cosec}\left(\frac{\pi x}{6}\right)$

$A \approx \frac{1}{3}\left(1 + 2 + 4\times\frac{2}{\sqrt{3}}\right)$

≈ 2.54

(c)(i) $15000 = 30000e^{-0.08t}$

$\frac{1}{2} = e^{-0.08t}$

$\ln(\tfrac{1}{2}) = -0.08t$

$t = -\frac{\ln\frac{1}{2}}{0.08} = 8.66 \rightarrow 9$ years

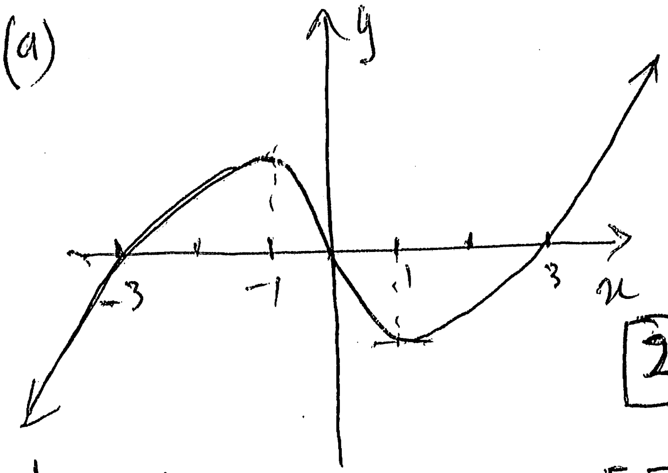
(ii) $30000\left(e^{-0.08\times 8} - e^{-0.08\times 9}\right)$

≈ 1216

decline 1216 people

Question 7

(a)



[2]

(b) $\frac{1}{4}$ [1]

(c) $\int_0^4 f(x)\,dx = \frac{1}{2}\times 3\times 4 - \frac{1}{2}\times 1\times 2$
$= 6-1$
$= 5$ [2]

(d) Let $P = \$20000$
$R = 1.01$
A_n = amount owing after n months
Q = annual instalment.

(i) $A_1 = PR$
$= \$20000 \times 1.01$
$= \$20200$ [1]

(ii) $A_{12} = PR^{12}$
$= \$20000 \times 1.01^{12}$
$= \$22536.50$ [1]

(iii) $A_{13} = (PR^{12} - Q)R$
$A_{36} = PR^{36} - QR^{24} - QR^{12}$
$A_{48} = PR^{48} - QR^{36} - QR^{24} - QR^{12}$
$= PR^{48} - \frac{Q(R^{48}-1)}{R^{12}-1}$

But $A_{48} = Q$

$\therefore Q = \frac{PR^{48}(R^{12}-1)}{R^{48}-1}$

$= \frac{\$20000(1.01)^{48}(1.01^{12}-1)}{1.01^{48}-1}$

$= \$6679.59$ [2]

(iv) Total Interest
$= 4Q - \$20000$
$= \$6718.36$ [1]

(e) GS: $4 - 2\sqrt{2} + 2 - \ldots$

$a = 4$ $r = \frac{-2\sqrt{2}}{4}$
$= -\frac{1}{\sqrt{2}}$

$S_\infty = \frac{a}{1-r}$

$= \frac{4}{1+\frac{1}{\sqrt{2}}}$

$= \frac{4\sqrt{2}}{\sqrt{2}+1}$ [2]

$= 8 - 4\sqrt{2}$

$(\doteqdot 2.343)$

$\left(= \frac{8}{2+\sqrt{2}}\right)$

MATHS (2U) THSC 2006

Question 8

(a) $f(x) = 2 - x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2 - (x+h)^2 - (2 - x^2)}{h}$$

$$= \lim_{h \to 0} \frac{2 - x^2 - 2xh - h^2 - 2 + x^2}{h}$$

$$= \lim_{h \to 0} \frac{h(-2x - h)}{h}$$

$$= -2x$$

When $x = 1$

$f'(1) = -2$ [2]

(b) $$\text{Area} = \int_0^{\pi/2} ((1+\cos x) - \sin x)\,dx + \int_{\pi/2}^{\pi} (\sin x - (1 + \cos x))\,dx$$

$$= \left[x + \sin x + \cos x\right]_0^{\pi/2} + \left[-\cos x - (x + \sin x)\right]_{\pi/2}^{\pi}$$

$$= \left[\left(\tfrac{\pi}{2} + 1 + 0\right) - (0 + 0 + 1)\right] + \left[(-(-1) - (\pi + 0)) - \left(-0 - \left(\tfrac{\pi}{2} + 1\right)\right)\right]$$

$$= \frac{\pi}{2} + \left(1 - \pi - \left(-\tfrac{\pi}{2} - 1\right)\right)$$

$$= 2 \text{ unit}^2$$ [3]

(c) $F = t(t-12)^2$

$F' = t \cdot 2(t-12) + (t-12)^2 \cdot 1$

$= (3t - 12)(t - 12)$

$F' = 0$ for $t = 4$ or 12

(i) $F = 0$ when $t = 0, 12, 12$

$\therefore$ Flows for 12 hours [2]

(ii) Stationary points at $t = 4$ or 12

$F'' = 6t - 48$

$F''(4) = -24$ $\quad$ $F''(12) = 24$

$\therefore$ Rel Max $\quad$ $\therefore$ Rel Min

$\therefore$ Max flow when $t = 4$

$F(4) = 256$ ML/hr [2]

(iii) Total flow

$$= \int_0^{12} (t^3 - 24t^2 + 144t)\,dt$$

$$= \left[\frac{t^4}{4} - \frac{24t^3}{3} + \frac{144t^2}{2}\right]_0^{12}$$

$$= \left[\frac{t^4}{4} - 8t^3 + 72t^2\right]_0^{12}$$

$$= 1728 \text{ ML}$$ [3]

Q9 (a) $x = \frac{1}{3}t^3 - 6t^2 + 27t - 18$

(i) $\dot{x} = t^2 - 12t + 27$

$\ddot{x} = 2t - 12$ $\boxed{2}$

(ii) $\ddot{x} = 0$ when $t = 6$ s $\boxed{2}$

(iii) When $t=6$, $x = 0$ m

$\dot{x} = -9$ m/s $\boxed{2}$

(b) (i) $P(WWG) = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{2}$

$= \frac{1}{18}$ $\boxed{2}$

(ii) $P(G) + P(WG) + P(WWG)$

$= \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{18}$

$= \frac{13}{18}$ $\boxed{2}$

(iii) $P(\text{win}) = P(G) + P(WG) + P(WWG)$

$+ P(WWWG) + \ldots$

$= \frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \ldots$

$= \dfrac{\left(\frac{1}{2}\right)}{\left(1 - \frac{1}{3}\right)}$

$= \frac{1}{2} \times \frac{3}{2}$

$= \frac{3}{4}$ $\boxed{2}$

Q10 (a) $3^{x-2} = 50$

$\ln(3^{x-2}) = \ln 50$

$(x-2)\ln 3 = \ln 50$

$x - 2 = \frac{\ln 50}{\ln 3}$

$x = 2 + \frac{\ln 50}{\ln 3}$

$= 5.560876\ldots$

$\simeq 5.56$ $\boxed{2}$

(b) (i) (α) $A_{\text{sector}} = \frac{1}{2}x^2\theta$ m^2 $\boxed{1}$

(β) $A_\Delta = \frac{1}{2}x^2 . \sin\frac{\pi}{3}$

$= \frac{1}{2}x^2 . \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}x^2}{4}$ m^2 $\boxed{1}$

(γ) $l_{\text{arc}} = x\theta$ $\boxed{1}$

(ii) (α) $A = \frac{1}{2}x^2\theta + \frac{\sqrt{3}x^2}{4}$

$= \frac{x^2}{4}(2\theta + \sqrt{3})$ $\boxed{1}$

(β) $P = 3x + x\theta$

$= x(3+\theta)$ $\boxed{1}$

(iii) $12 - 2\sqrt{3} = x(3+\theta)$

$\therefore 3 + \theta = \frac{12 - 2\sqrt{3}}{x}$

$\theta = \frac{12 - 2\sqrt{3}}{x} - 3$

$A = \frac{x^2}{4}(2\theta + \sqrt{3})$

$= \frac{x^2}{4}\left(2\times\left(\frac{12-2\sqrt{3}}{x} - 3\right) + \sqrt{3}\right)$

$= \frac{x^2}{4}\left(\frac{24 - 4\sqrt{3}}{x} - 6 + \sqrt{3}\right)$

$= (6-\sqrt{3})x - (6-\sqrt{3})\frac{x^2}{4}$

$= (6-\sqrt{3})\left(x - \frac{x^2}{4}\right)$

$A' = (6-\sqrt{3})\left(1 - \frac{x}{2}\right)$

$A'' = (6-\sqrt{3}) \times -\frac{1}{2} < 0$ $\boxed{3}$

$\therefore$ Max area when $x = 2$

Max area $= (6-\sqrt{3})(2-1)$

$= 6 - \sqrt{3}$ m^2 $\boxed{2}$