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Solution. Hills Grammar

3u 1999 Trial HSC.

Question 1

a) $\dfrac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \dfrac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$= \dfrac{\sqrt{5}-\cancel{\sqrt{3}}+\cancel{\sqrt{10}}-\sqrt{6}+\sqrt{5}+\cancel{\sqrt{3}}-\cancel{\sqrt{10}}-\sqrt{6}}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$

$= \dfrac{2\sqrt{5}-2\sqrt{6}}{2}$

$= \sqrt{5}-\sqrt{6} \qquad a = 1 \qquad b = -1.$ (2)

b) $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n!}{(n-r)!\,r!} \div \dfrac{n!}{(n-r+1)!\,(r-1)!}$

$= \dfrac{\cancel{n!}}{(n-r)!\,r!} \times \dfrac{(n-r+1)!\,(r-1)!}{\cancel{n!}}$

$= \dfrac{n-r+1}{r}$ as reqd. (2)

c) $y = 4x - 2 \qquad m_1 = 4$

$2x + 3y = 9 \qquad m_2 = \dfrac{-2}{3}$

$\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\dfrac{4 + \frac{2}{3}}{1 - \frac{8}{3}}\right|$

$= \dfrac{14}{5}$

$\theta = 70°21'$ (2)

3

2

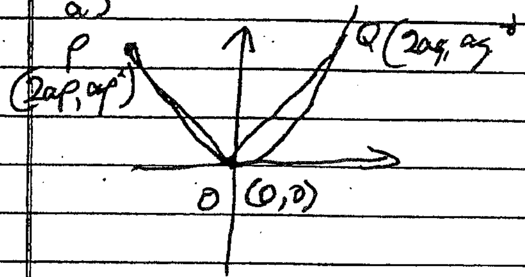
d) $A(-4, 2) \qquad B(6, 5) \qquad P(16, 8)$ (2)

e) $5\sin\theta + 12\cos\theta = 13\left(\frac{5}{13}\sin\theta + \frac{12}{13}\cos\theta\right)$ $\alpha = 67°23'$

$= 13\sin(\theta + \alpha)$ where

Question 3

a)



$PO \perp QO$

$\therefore m_1 \times m_2 = -1$

$\frac{p}{2} \times \frac{q}{2} = -1$

$pq = -4$ (2)

M is $a(p+q), \frac{a(p^2+q^2)}{2}$

$x = a(p+q) \qquad 2y = a(p^2+q^2)$

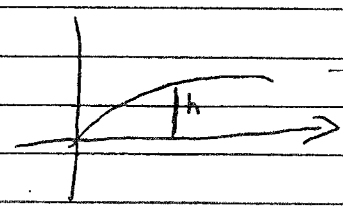
$\frac{x^2}{a^2} = p^2+q^2 - 8$

$\frac{x^2}{a^2} + 8 = \frac{2y}{a}$

$x^2 = 2ay - 8a^2$

$x^2 = 2a(y - 4a)$ (3)

b)



$x = Vt\cos\theta$

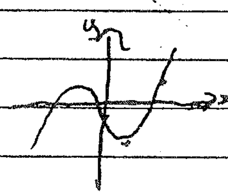
$25 = 20t \times \cos 60°$

$t = 2.5$

$h = -5t^2 + 20 \times 2.5 \sin 60°$

$h = 12.05\text{m}$ (3)

c) (i) $f(x) = x^3 - x^2 - x - 1$



$f(1) = -2$ negative

$f(2) = 1$ positive

$\therefore$ there is a root between 1 and 2 (1)

(ii) $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$f(x) = x^3 - x^2 - x - 1$

$f(1.5) = -1.375$

$f'(x) = 3x^2 - 2x - 1$

$f'(1.5) = 2.75$ (3)

$= 1.5 + \frac{1.375}{2.75}$

$= 2$

Question 5.

a) $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$

Let $x = -1$ (2)

$0 = 1 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots + (-1)^n \, {}^nC_n$ as reqd.

b) (i) If A and B are included, I need only choose 2 from the other eight

$^8C_2 = 28$ ways (3)

(ii) If A and B are excluded, I have to choose 4 from the remaining 8

$^8C_4 = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1}$

$= 70$ ways (3)

c) p = win $\frac{1}{10}$

q = lose $\frac{9}{10}$

$(p+q)^{12}$ select the p^2 term.

$^{12}C_{10} \, p^2 q^{10}$ (4)

$= \frac{12 \times 11}{2 \times 1} \times \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^{10}$

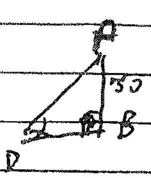
$= \frac{66 \times 9^{10}}{10^{12}}$

Question 6

(i) $X\hat{P}D = P\hat{D}B$ (alternate $XP \parallel PB$) (1)

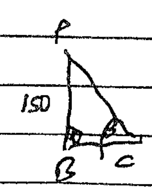
$Y\hat{P}C = P\hat{C}B$ (" $PY \parallel BC$) (1)

$T\hat{O}B = 220° - 120° = 100°$ (given bearings) (1)

(ii) 

$\cot \alpha = \frac{DB}{150}$

$DB = 150 \cot \alpha$



$\cot \beta = \frac{BC}{150}$ (2)

$BC = 150 \cot \beta$

(iii) By cosine rule

$CD^2 = DB^2 + BC^2 - 2\,DB\,BC \cos \theta$

Question 7

a) $\frac{dT}{dt} = k(T-25)$

(i) $T = 25 + Ae^{kt}$ $\qquad Ae^{kt} = T-25$

$\frac{dT}{dt} = kAe^{kt}$

$= k(T-25)$ as reqd. ①

(ii) When $t=0 \quad T=95$

$95 = 25 + A \qquad A = 70$ .. ①

when $t=20 \quad T=65$

$65 = 25 + 70e^{20k}$

$\frac{40}{70} = e^{20k}$

$k = \frac{\ln\frac{4}{7}}{20}$ $\qquad -0.02798$

(iii) when $t=30$

$T = 25 + 70e^{30k}$

$= 55°$ ②

b) $f(x) = x\sin^{-1}x$

(i) $f(x) = x\sin^{-1}x$

$f(-x) = -x \cdot \sin^{-1}(-x)$

$= -x \times -\sin^{-1}(x)$

$= x\sin^{-1}x$

$\therefore$ function is even. ②

(ii) $f'(x) = \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x$

$f'(x) = 0$ when $x=0$

$\therefore$ T.P. @ $(0,0)$

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
y	-1.1	0	1.1
slope	\	—	/

Min T.P. ②

(iii) domain $-1 \le x \le 1$

range $0 \le y \le \frac{\pi}{2}$

(iv)


①