PETRUS KY COLLEGE
NEW SOUTH WALES

in partnership with

VIETNAMESE COMMUNITY IN AUSTRALIA
NSW CHAPTER

JULY 2006

MATHEMATICS EXTENSION 1

PRE-TRIAL TEST

HIGHER SCHOOL CERTIFICATE (HSC)

Student Number: | | | | |

Student Name: _____

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided on Page 2.
- All necessary working should be shown in every question
- Write your Student Number and your Name on all working booklets.

Total marks – 72

- Attempt Questions 1–7
- Questions are not of equal value

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Marks

Question 1 **10**

(A) By using the substitution method, or otherwise, find the integration of

(i) $\int x.\sqrt{4-x}\ dx$ 2

(ii) $\int \frac{1-\tan x}{1+\tan x}\ dx$ 2

(iii) $\int \frac{3e^x dx}{4+2e^{2x}}$ 2

(B) (i) If $\alpha = \sin^{-1} x$ and $\beta = \tan^{-1} x$ and $\alpha + \beta = \frac{\pi}{9}$, Show that 2

$$\cos(\alpha+\beta) = \frac{\sqrt{1-x^4}-x^2}{\sqrt{1+x^2}}$$

(ii) Solve the following equation 2

$\tan^{-1} 3x - \tan^{-1} x = \tan^{-1} \frac{1}{2}$

Question 2 **10**

(A) Two points P($2ap, ap^2$) and Q($2aq, aq^2$) are on the parabola P : $x^2 = 4ay$

(i) Find the equations of the two tangents to the parabola at P and at Q. Hence find their intersection point T. 2

(ii) Find the equation of the two normal at P and Q and their intersection point N. 2

(iii) If the two tangents are perpendicular, find the locus of point M, which is the midpoint of T and N . 2

(B) Show that $\frac{1}{n-1} - \frac{1}{n+1} = \frac{2}{n^2-1}$ 4

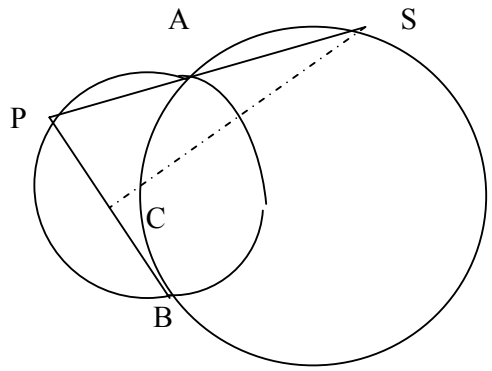
Hence find, as a fraction in lowest terms, the sum of the first 100 term of the series

$$\frac{1}{3} + \frac{1}{8} + \frac{1}{15} + \frac{1}{24} + \ldots$$

Obtain an expression for $\sum_{r=2}^{n} \frac{1}{r^2-1}$ and hence find the limiting sum of the series.

Question 3 **12**

(A) Two different radii circles come across at 2 points A & B. The centre C of smaller circle stays on the circumference of the bigger one. P is a point on the alternate segment (of the smaller circle) outside the common region. From P draw a line through A, that line cuts the bigger circle at S. Show that CS perpendicular to PB. 6



(B) A research party is held by electing 7 scientists from a department of C.S.I.R.O. There are 7 men and 5 women in that department, and the party will contain 4 men and 3 women.

(i) How many ways the party can be formed ? 2

(ii) Find the probability of gaining of party if the oldest man can not be selected together with the youngest woman. 2

(iii) Find the probability of gaining a party if both the oldest man and youngest woman present in the party with the condition that no refraction of number of men and women in that 7 people but there are must be at least 3 women present. 2

Question 4 **12**

(A) A ball is thrown with initial velocity 20 m/s at the angle of elevation of $\tan^{-1}\frac{4}{3}$

(i) Show that the parabolic path of the ball has the parametric equation 2

$$\begin{cases} x = 12t \\ y = 16t - 5t^2 \end{cases}$$

Find the range of the ball and its greatest height. 2

(ii) Show that in order to reach ¾ of the greatest hight (on the way up), the ball just spends ¼ of the total time. 2

(iii) Suppose that the ball is thrown up a road inclined at angle $\alpha = \tan^{-1}\frac{1}{3}$ to the horizontal. Find the time, distance along the road and the angle when the ball hit the road. 3

(B) Using the mathematic induction method of proving to show that $7^n + 11^n$ is divisible by 9 for odd $n \geq 1$. 3

Question 5 **10**

(A) A cylinder is inscribed in a cone whose base diameter is 10 cm and whose hight is 12 cm.

If the highest of the cylinder is h cm and the radius of its base is r cm ,

Show that:

(i) $5h + 12r = 60$ 2

(ii) Show that the volume of the cylinder is: $V = \frac{\pi r^2(60-12r)}{5}$ 2

Hence find the dimension r and h of which the volume of that cylinder is maximum. Find the maximum volume. 2

(B) What is the domain and range of the function 2

$$y = \frac{\pi}{6} - 2\sin^{-1} x^2$$

Sketch that curve. 2

Question 6 **8**

(A) If the equation $6x^4 - 13x^3 - 90x^2 + 208x - 96 = 0$ have 4 distinct roots of α, $-\alpha$, β and $\frac{1}{\beta}$, then solve the equation. 3

(B) By sketching the 2 separate functions, show that the equation $x + \ln x = 0$ has only one root from [0, 1]

(i) By using the half-interval method three times, find the approximate value of the root. 2

(ii) By using the approximation Newton's method 2 times, find the closest root of this equation. 2

(iii) By comparison the two answers of i) and ii) above, which method is more appropriate? 1

Question 7 **10**

(A) Using the binomial expansion or else show that 2

$$(3+\sqrt{5})^6+(3-\sqrt{5})^6=20608$$

(B) The position x cm of a particle relative to a fixed point 0 at any time t is:

$x = 5 - 2\cos^2 t$

(i) Show, by finding its acceleration in term of x that the motion is simple harmonic. 2

(ii) Find the centre of the motion, the period and the amplitude. 2

(iii) Find the initial velocity and acceleration. 2

(iv) Find the velocity when the particle passing the centre of motion. 2

END OF TEST