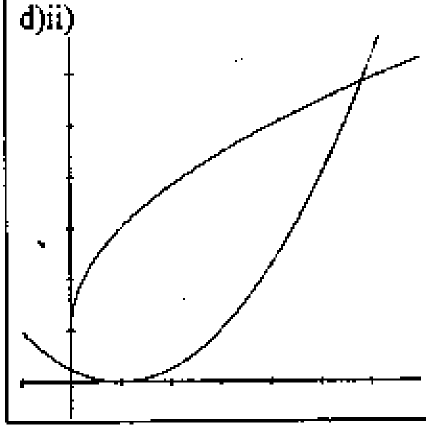
Solutions Question 1 2003	Marks/Comments
1 a) $\lim_{x\to\pm\infty}\frac{3x^2+4x+5}{x^2}$ $=\lim_{x\to\infty}(3+\frac{4}{x}+\frac{5}{x^2})$ $=3+0+0$ $=3$	1 for realising x tends to infinity represents horizontal asymptote 1 for answer
1 b) $\frac{x}{2-x}\le 4, x\ne 2$ $x=8-4x$ $5x=8$ $x=1.6$ $x=2$, 1.6 are critical points test points x=0 ✓, x=5✓, x=1.75 × $x\le 1.6$ *or* $x>2$	1 for CPs by either method 1 for test 1 for statement. not 3rd mark if $x\ge 2$
1 c) $2+5+8+\ldots\ldots+56$ has 19 terms with common difference = 3 $\frac{n}{2}(a+l)=9.5\times 58=551$	1 for clear expression 1 for correct answer
1 d) $\int_2^6\frac{x}{\sqrt{x^2-3}}dx$ $\frac{du}{dx}=2x,\ x=2, u=1,\ x=6, u=33$ $I=\frac{1}{2}\int_1^{33}\frac{du}{\sqrt{u}}$ $=\frac{1}{2}\left[2u^{\frac{1}{2}}\right]_1^{33}$ $=\sqrt{33}-1$	1 not all reqd 1 for clear statement of integral 1 for completion
1 e) i) (15,150)	1
1 e) ii) $9t^2=x^2=1.5y$	1 /12

Solutions Question 2 2003	Marks/Comments
a) i) 1:2	1
a) ii) 3:2	1
b) i) $\log_3 4 = \log_3 \frac{12}{3}$ $= \log_3 12 - \log_3 3$ $= 2.26186 - 1$ $= 1.26186$	1
b) ii) 1.09	1
c) $x^2-1\overline{)x^3+4x^2-2x+3}$ $Q(x) = x+4, \quad R(x) = 7-x$	1 for setting up the division 1
d) i) $\frac{h}{OY} = \tan 15^0$ $OY = \frac{h}{\tan 15^0}$ or $h\cot 15^0$	1
d) ii) Likewise $OX = h\cot 10^0$ Now right angle at O in Δ OXY so $400^2 = h^2(\cot^2 15^0 + \cot^2 10^0)$ $h = \frac{400}{\sqrt{\cot^2 15^0 + \cot^2 10^0}}$ $h = \frac{400}{\sqrt{46.09164071}}$ $= 59m$	1 1 1
d) iii) $\tan \angle OXY$ $= \frac{h\cot 15^0}{h\cot 10^0}$ $= \frac{\tan 10^0}{\tan 15^0}$ $= .658...$ $\angle OXY = 33^0 21'$	1 1 /12

Solutions Question 3 2003	Marks/Comments
a) Now $\cos 2x = 2\cos^2 x - 1$ $\cos^2 x = \frac{\cos 2x + 1}{2}$ $\cos^2 4x = \frac{\cos 8x + 1}{2}$ $I = \frac{1}{2}\int_0^{\pi/16} \cos 8x + 1\,dx$ $= \frac{1}{2}\left[\frac{1}{8}\sin 8x + x\right]_0^{\pi/16}$ $= \frac{1}{2}\left(\frac{1}{8}\sin\frac{\pi}{2} + \frac{\pi}{16} - 0\right)$ $= \frac{2+\pi}{32}$	1 1 1
b) i) 9 letters, E appears 3 times and S appears twice $\frac{9!}{3!2!} = 30240$	1 simplification not reqd
b) ii) The requirement is C V C V C V C V C Vowels can be ordered in $\frac{4!}{3!} = 4$ ways Consonants can be ordered in $\frac{5!}{2!} = 60$ ways Probability $= \frac{240}{30240} = \frac{1}{126}$	1 1 simplification not reqd
c) $4\cos\theta + 3\sin\theta = 2$ then $\frac{4}{5}\cos\theta + \frac{3}{5}\sin\theta = \frac{2}{5}$ noting $\sin(\alpha+\theta) = \sin\alpha\cos\theta + \cos\alpha\sin\theta$ we have $\sin\alpha = \frac{4}{5}$ *and* $\cos\alpha = \frac{3}{5}$ $\alpha = 0.9273^c$ $\therefore 0.9273 + \theta = 0.41151^c$ *or* 1.982313^c and so $\theta = 1.06^c, 5.77^c$	1 1 1 1 ignore failure to conform to accuracy. (Answer in degrees acceptable)
d) i) $x \le 1$ *or* $x \ge 1$	1

d)ii) 	1 /12

Solutions Question 4 2003	Marks/Comments
a) BX =DX (tangents drawn from external point) $\therefore \angle$DBX=$\angle$BDX likewise AX = CX and $\angle$CAX = $\angle$ACX (Both these pairs of equal angles are equal since $\angle$X is common in both triangles) $\therefore$ AC ‖ BD (corresponding angles equal)	1 1 1
b) $\cos(\theta+2\theta)$ $=\cos\theta\cos2\theta-\sin\theta\sin2\theta$ $=\cos\theta(2\cos^2\theta-1)-\sin\theta(2\sin\theta\cos\theta)$ $=2\cos^3\theta-\cos\theta-2(1-\cos^2\theta)\cos\theta$ $=4\cos^3\theta-3\cos\theta$	1 1
c) $\sin x=\dfrac{10\pm\sqrt{100-96}}{16}=0.5$ *or* 0.75 Then $x=.524^c$ or $.848^c$ Minimum when first deriv. $=0$ $16\sin x\cos x-10\cos x=0$ $16\sin x=10$ since $\cos x\neq0$ $\left(x=\frac{\pi}{2}\text{ but }\frac{\pi}{2}>1,\ \therefore\text{ not a solution}\right)$ sinx=0.625 , $x=.675\ldots$ $y=8\left(\dfrac{5}{8}\right)^2-10\times\dfrac{5}{8}+3=-0.125$	1 1 1 1 1
d) i) $y=\lvert 3x-2\rvert$	1
d) ii) $0\le x\le2$	1 /12

Solutions Question 5 2003	Marks/Comments
a) i) If $n=1$, $1=1^3$ true when $n=1$ Assume when $n=k$ ie. $1+7+19+.....+(3k^2-3k+1)=k^3$ Reqd to prove $1+7+...+(3k^2-3k+1)+(3(k+1)^2-3(k+1)+1)=(k+1)^3$ LHS $=k^3+3(k+1)^2-3(k+1)+1$ $=k^3+3k^2+6k+3-3k-3+1=k^3+3k^2+3k+1$ $=(k+1)^3$ The proposal holds when $n=1$. If assumed for a number it will hold for the next number, so it holds for n = 2 etc. Hence by induction the proposal holds for all $n \in J$, $n \geq 1$	1 1 1
a) ii) $n^3-(n-1)^3$ $=n^3-n^3+3n^2-3n+1$ $=3n^2-3n+1$	1
b) i) $\frac{dv}{dt}=\frac{dv}{dx}\cdot\frac{dx}{dt}$ $=v\cdot\frac{dv}{dx}$ $=\frac{d}{dv}\left(\frac{1}{2}v^2\right)\frac{dv}{dx}$ $=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$	1
b) ii) $-g=\frac{-k}{R^2}$ $\therefore k=gR^2$	1
b) iii) $a=\frac{-gR^2}{x^2}$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right)=-gR^2x^{-2}$ $\frac{1}{2}v^2=\int -gR^2x^{-2}dx$ $v^2=2gR^2x^{-1}+c$ but when $x=R$, $v=u$, $\therefore c=u^2-2gR$ $\therefore\ v^2=\frac{2R^2g}{x}+u^2-2gR$ as reqd	1 1

b) iv) $v = 0$ for max distance $0 = \frac{2R^2g}{x} + u^2 - 2Rg$ $2Rg - u^2 = \frac{2R^2g}{x}$ $x = \frac{2R^2g}{2Rg - u^2}$	1
b) v) $as \quad x \to \infty, \quad u^2 = 2gR$ $= 2 \times 9.8 \times 6400000$ $u = 11200ms^{-1}$	1
c) $f(x) = ax^3 + bx^2 + cx + d \qquad f^{1}(x) = 3ax^2 + 2bx + c$ $f(-1) = -a + b - c + d$ ...(1) $\quad f^{1}(-1) = 3a - 2b + c$...(2) (double root at x = -1) min value at (1,-4) $\therefore\ f(1) = a + b + c + d = -4$...(3) $\quad f^{1}(1) = 3a + 2b + c = 0$...(4) (min turning point) Solving (2) + (4), (2) – (4), (1) + (3), (3) – (1) and subbing we get $a = 1,\ b = 0,\ c = -3,\ d = -2$	1 1
	/12

Solutions Question 6 2003	Marks/Comments
a) i) $-1 \le \sin t \le 1$ $0 \le \sin^2 t \le 1$ $0 \le 2\sin t \le 2$ $\therefore$ extremities are between $x = 0$ and $x = 2$	1
a) ii) $\frac{dx}{dt} = 2 \times 2\sin t \cos t$ $= 4\sin t \cos t$ $\frac{d^2x}{dt^2} = vu' + uv'$ $= 4(\cos^2 t - \sin^2 t)$ $= 4(1 - 2\sin^2 t)$ $= 4(1 - x)$	1 for clear intention to differentiate wrt t 1 for completion
a) iii) Particle has SHM since its acceleration has form $-n^2X$	1
a) iv) Maximum speed when $x = 1$, $t = \sin^{-1}(1/\sqrt{2})$ Then $\frac{dx}{dt} = 2 \times 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 2\ \text{ms}^{-1}$	1
b) $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$ differentiating both sides wrt x $n(1+x)^{n-1} = {}^nC_1 + 2\,{}^nC_2 x + 3\,{}^nC_3 x^2 + \dots + n\,{}^nC_n x^{n-1}$ letting $x = 1$ RHS $= n \times 2^{n-1}$	1 for clear expression of bin. th. and differentiating or letting x equal something 1
c) ${}^7C_5 \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^2 = 0.1640625$	1
d) i) By inspection $m_{max} = 2$ when $x = 0$	1
d) ii) The curve represents $\frac{dy}{dx} = \frac{2}{1+x^2}$ which equals $\frac{1}{3}$ when $x = \pm\sqrt{5}$	1
d)iii) $\int_{-\infty}^{\infty} \frac{2}{1+x^2}\,dx = 2\int_0^{\infty} \frac{2}{1+x^2}\,dx$ $= 4\int_0^{\infty} \frac{1}{1+x^2}\,dx$ $= 4\left[\tan^{-1} x\right]_0^{\infty}$ $= 4 \times \frac{\pi}{2}$ $= 2\pi$	1 1

Solutions Question 7 2003	Marks/Comments
a) i) $\frac{dy}{dt}=-10t+c$ but when t = 0 $y'=V\sin\theta$ so $y'=V\sin\theta-10t$ Also $x'=V\cos\theta$ $x=Vt\cos\theta$ $y=\int V\sin\theta-10t\,dt=Vt\sin\theta-5t^2+c$ and from the initial conditions $c=10$	1 clear intention to integrate both wrt t 1 1 for correct constants
a) ii) By Pythagoras $\sin\theta=5/13$ and $\cos\theta=12/13$ We require x when $y=0$ $y=13t\frac{5}{13}-5t^2+10$ which =0 when $t=\frac{-5\pm\sqrt{25+4\times5\times10}}{-10}$ $=\frac{-20}{-10}$ *or* $\frac{10}{-10}$ When $t=2$ $x=13\times2\times\frac{12}{13}=24$ m	1 1
b) i) PQ has eqn $\frac{aq^2-ap^2}{2aq-2ap}=\frac{y-ap^2}{x-2ap}$ $=\frac{q+p}{2}$ which becomes $2y-2ap^2=(p+q)x-2apq-2ap^2$ when $x=0$, $y=a$ $2a=-2apq$ $\therefore pq=-1$	1 1
b) ii) Tangent at P $y=px-ap^2$ Tangent at Q $y=qx-aq^2$ $q\times$ Tangent at P $qy=pqx-ap^2q$ $p\times$ Tangent at Q $py=pqx-aq^2p$ whence $(q-p)y=apq(q-p)$ $y=-a$ subbing $-a=px-ap^2$ $pqa+ap^2=px$ $x=a(p+q)$	1 1
b)iii) $PQ=\sqrt{(2ap-2aq)^2+(ap^2-aq^2)^2}$	1

$=\sqrt{4a^2(p-q)^2+a^2(p^2-q^2)^2}$ $=a\sqrt{4(p-q)^2+(p-q)(p+q)^2}$ $=a(p-q)\sqrt{4+(p+q)^2}\ ,\ p>q$ $=a(p-q)\sqrt{-4pq+(p+q)^2}\ ,\quad (pq=-1)$ $=a(p-q)\sqrt{(p-q)^2}$ $=a(p-q)^2$ but $q=\frac{-1}{p}$ $=a\left(p+\frac{1}{p}\right)^2$ as reqd	 1 1 or 3 marks for other method