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YEAR 12 EXTENSION 1 MATHEMATICS

6th April 2001.

Time Allowed: 2 hours, plus 5 minutes reading time

Name:

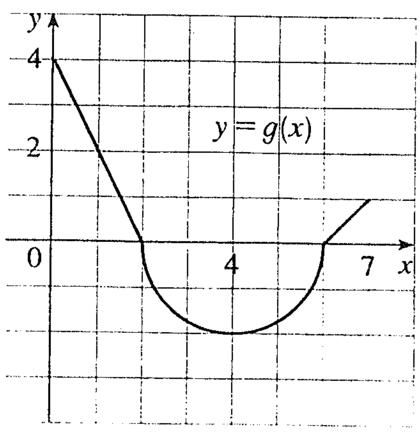
Instructions to students

- Attempt all questions
- Show all necessary working
- Calculators may be used

'UESTION 1. Start a new page. **Marks**

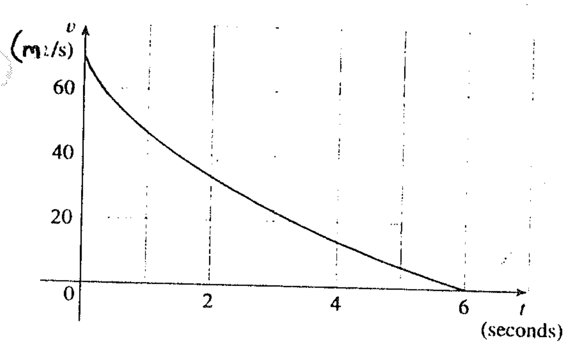
) Find $\frac{d}{dx}\sqrt{2x^2+3}$ hence find $\int\frac{xdx}{\sqrt{2x^2+3}}$. 3

) The graph of $y=f(x)$ is sketched below. It consists of two straight lines and a semi-circle. Use it to evaluate $\int_0^7 f(x)dx$. 3



By changing $\log_2 3x$ to base e differentiate $y=\log_2 3x$. 3

The velocity of a braking car is shown. What meaning is attached to $\int_0^6 vdt$? 1



'or what values of p is $\int_{-1}^{p} x\,dx=0$? Give a geometric interpretation for your ıswer. 2

)UESTION 2. Start a new page. **Marks**

) i) Given that $\cos(\theta+\psi)=\cos\theta\cos\psi - \sin\theta\sin\psi$, show that $\cos 2t = 2\cos^2 t - 1$ **2**

ii) Given that $x=\cos t, y=\cos 2t$ give a geometric description of the locus of $P(x, y)$. **3**

) $P(2p, p^2)$ and $Q(2q, q^2)$ are two points on the parabola $x^2=4y$. PQ is the focal chord of the parabola, with equation $y=\frac{1}{2}x\,(p+q)-pq$.

)

i) Show that $pq=-1$. **2**

ii) Show that the midpoint M of PQ is $(p+q, \frac{1}{2}(p^2+q^2))$. **1**

iii) Given that the point of intersection of the tangents to the parabola at P and Q is $T(p+q, pq)$ find and describe the locus of the point of intersection T and show that MT is always parallel to the y–axis. **4**

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UESTION 3. Start a new page. **Marks**

Consider the function $y = x^{\frac{3}{2}}$ in the domain $0 \le x \le 4$.

i) Write down the range of this function. **1**

ii) Draw a neat sketch of the function. Give reasons as to why the inverse is also a function. **2**

iii) By interchanging x and y and making y the subject write down the equation of the inverse function. **2**

iv) On the same number plane sketch the graph of the inverse. **1**

v) Describe how these two graphs are related geometrically to the line $y = x$. **1**

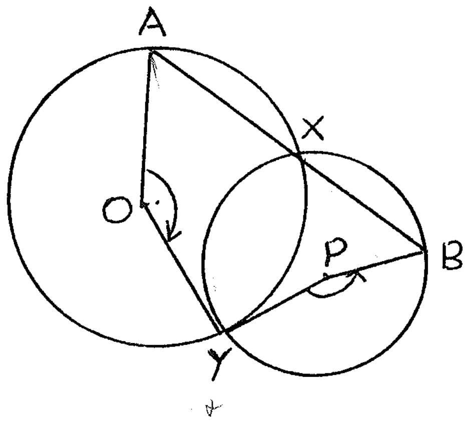
vi) Write an integral to find the area bounded by the x-axis and the curve $y = x^{\frac{2}{3}}$ between $x = 0$ and $x = 8$. Also write another integral to find the area bounded by the y-axis and the curve $y = x^{\frac{3}{2}}$ between $y = 0$ and $y = 8$.
Give reasons as to why these two integrals are equal. State the value of the integrals. **3**

vii) A student stated that the domain of the function $y = x^{\frac{3}{2}}$ is $x \ge 0$ and that the domain of the function $y = x^{\frac{2}{3}}$ is all real numbers. Is the student correct? **2**

Give reasons.

QUESTION 4. Start a new page. **Marks**

a) 4



O and P are the centres of the circles. AXB is straight line.
Prove that $\angle AOY = \angle BPY$.

b) i) Show that $\ln e = \int_1^3 \frac{1}{x}\,dx - \int_e^3 \frac{1}{x}\,dx$. 2

ii) Show that by one application of the Trapezoidal Rule $\int_e^3 \frac{1}{x}\,dx = \frac{1}{6e}(9- e^2)$. 2

iii) You are given that $\ln 3 = \frac{11}{10}$. Using this result and combining 2

parts (i), (ii) and (iii) show that e may be approximated with the equation

$5e^2 + 3e - 45 = 0$.

iv) Solve this equation to find an approximation for e correct to 3 decimal places 2

QUESTION 5. Start a new page. **Marks**

a) Let $y = x \ln x$.

i) State the natural domain of this function. **1**

ii) find $\lim_{x \to 0} x \ln x$. **1**

iii) Show that a stationary point exists at $x = \frac{1}{e}$ **2**

iv) Determine the nature of this stationary point using the second derivative. **2**

v) Show that no points of inflexion exist. **1**

vi) Draw a sketch of $y = x \ln x$ showing all critical points. **2**

b) i) Using one of the logarithmic laws, expand $\ln \frac{x+1}{\sqrt{x-1}}$. **2**

ii) Hence differentiate $y = \ln \frac{x+1}{\sqrt{x-1}}$. **1**

QUESTION 6. Start a new page.

a) i) Find the value of the integral $\int_1^T \frac{1}{x^2}\,dx$, in terms of T. **2**

ii) By taking the limit as T tends to infinity, evaluate the improper integral **2**

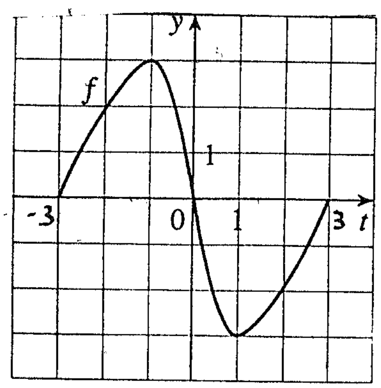
$\int_1^\infty \frac{1}{x^2}\,dx$.

iii) Explain the geometric meaning of your answer. **2**

b) Prove by Mathematical Induction that $3^{2n} + 2^{n+2}$ is divisible by 5 for $n \geq 1$ **6**

QUESTION 7. Start a new page. **Marks**

a) Let $g(x) = \int_{-3}^{x} f(t)\,dt$, where f is the function whose graph is shown.



i) Evaluate $g(3)$ and $g(-3)$. **2**

ii) Estimate $g(0)$. **1**

iii) On what interval is g increasing? **1**

iv) Where does g have a maximum value ? **1**

v) Draw a sketch of $g(x)$. **2**

b) i) Sketch the graph of $y = \dfrac{1}{1+x^2}$. **2**

ii) The region enclosed by the curve , the y axis and the line $y = \dfrac{1}{2}$ in the first quadrant is rotated about the y axis. Find the exact volume of the solid so formed. **3**