EXAMINERS

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CATHOLIC TRIAL 2001

Total marks (120)

Attempt Questions 1 – 10

All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Factorise completely $ab - a - bx + x$. **2**

(b) Simplify $|2| + |-5|$. **1**

(c) Find integers a and b such that $\frac{1}{\sqrt{3}+2} = a\sqrt{3} + b$. **2**

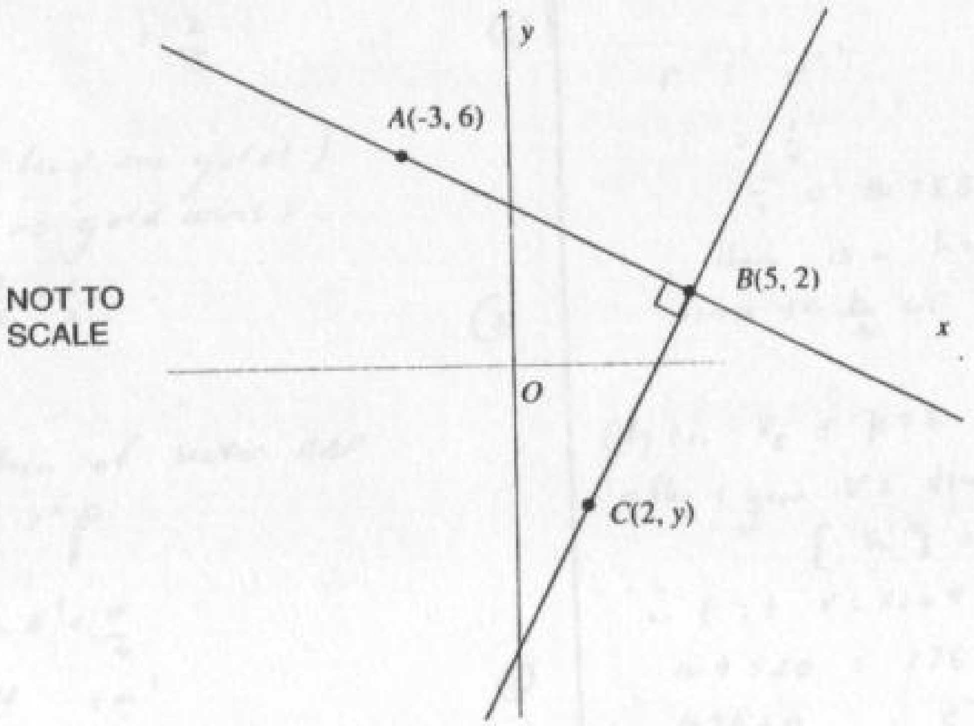
(d) Find the value of $\cos\frac{\pi}{8}$, correct to 3 decimal places. **2**

(e) Solve $\tan\theta = -\frac{1}{\sqrt{3}}$ for $0^\circ \le \theta \le 360^\circ$. **2**

(f) (i) Write down the discriminant of $2x^2 - 3x + k$. **1**

(ii) For what values of k does $2x^2 - 3x + k = 0$ have unequal real roots? **2**

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**



The diagram shows the origin O and the points $A(-3, 6)$, $B(5, 2)$ and $C(2, y)$.
The lines AB and BC are perpendicular.

Copy or trace this diagram onto your writing sheet.

(a) Show that A and B lie on the line $x + 2y = 9$. **2**

(b) Show that the length of AB is $4\sqrt{5}$ units. **1**

(c) Find the perpendicular distance from O to AB. **1**

(d) Find the area of triangle AOB. **1**

(e) Show that C has coordinates $(2, -4)$. **2**

(f) Does the line AC pass through the origin? Explain. **2**

(g) The point D is not shown on the diagram. The point D lies on the x axis and $ABCD$ is a rectangle. Find the coordinates of D. **2**

(h) On your diagram, shade the region satisfying the inequality $x + 2y \geq 9$. **1**

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

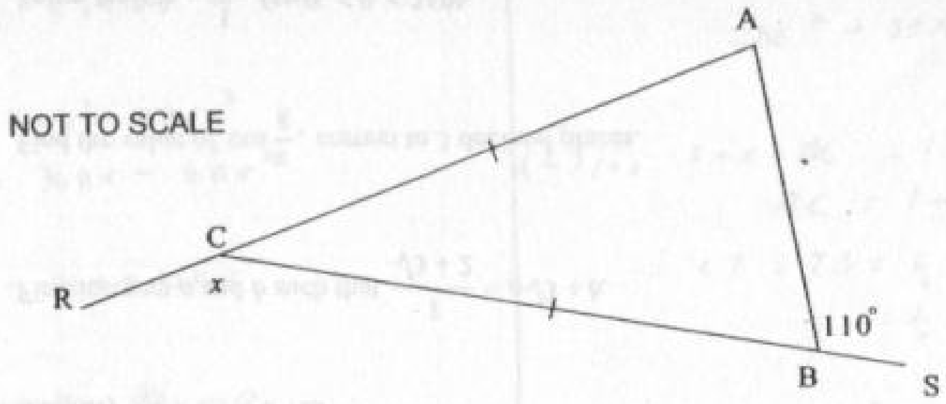
(a) Find:

(i) $\int \sec^2 4x\,dx$ **1**

(ii) $\int \left(\frac{1}{x^2} + \frac{1}{e^{2x}}\right) dx.$ **2**

(b) Evaluate $\int_0^3 \frac{1}{x+1}\,dx.$ **2**

(c)



In the diagram, AC = BC, RCA and CBS are straight lines, $\angle ABS = 110^\circ$ and $\angle BCR = x$.
Copy the diagram onto your writing sheet.
Find the value of x giving reasons. **3**

(d) Differentiate the following

(i) $x^3 \sin x$ **2**

(ii) $\sqrt{1-x^2}$. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The nth term of an arithmetic series is given by $T_n = 3n + 4$.

(i) What is the 12th term of this series? **1**

(ii) What is the sum of the first 20 terms of this series? **2**

(b)

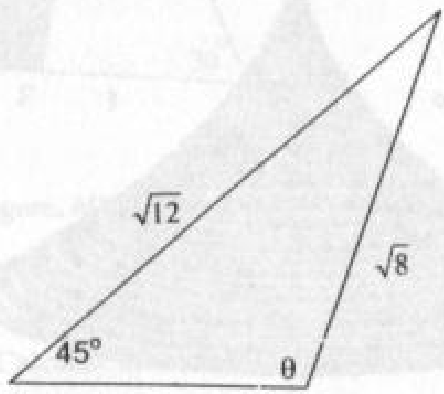
$\boxed{1}$ $\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{3}$ $\boxed{3}$ $\boxed{3}$

Two cards are chosen at random and without replacement from the seven cards above. What is the probability that

(i) both cards show a 1 **1**

(ii) the sum of the two numbers on the cards chosen is greater than 4? **2**

(c) **3**



NOT TO SCALE

Use the sine rule to find the value of θ where θ is obtuse.

(d) The geometric series $a + ar + ar^2 + \ldots$ has a second term of $\frac{1}{4}$ and has a limiting sum of 1.

(i) Show that $a = 1 - r$. **1**

(ii) Solve a pair of simultaneous equations to find r. **2**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the curve given by $y = 2x^3 - 3x^2 - 12x$.

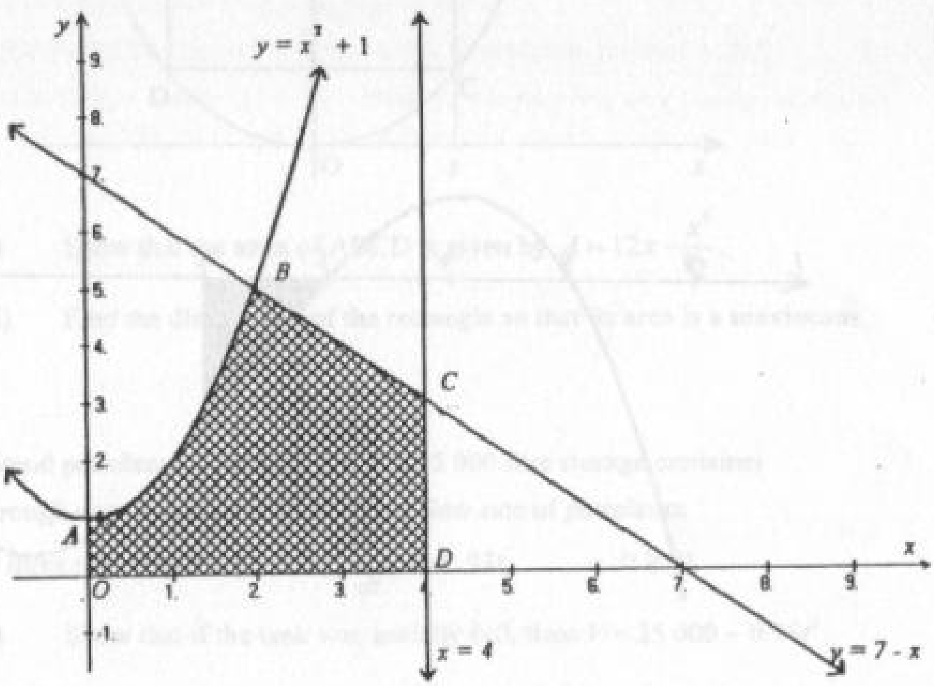
(i) Find $\dfrac{dy}{dx}$. **1**

(ii) Find the coordinates of the two stationary points. **3**

(iii) Determine the nature of the stationary points. **2**

(iv) Sketch the curve for $-2 \le x \le 3$. **2**

(b)



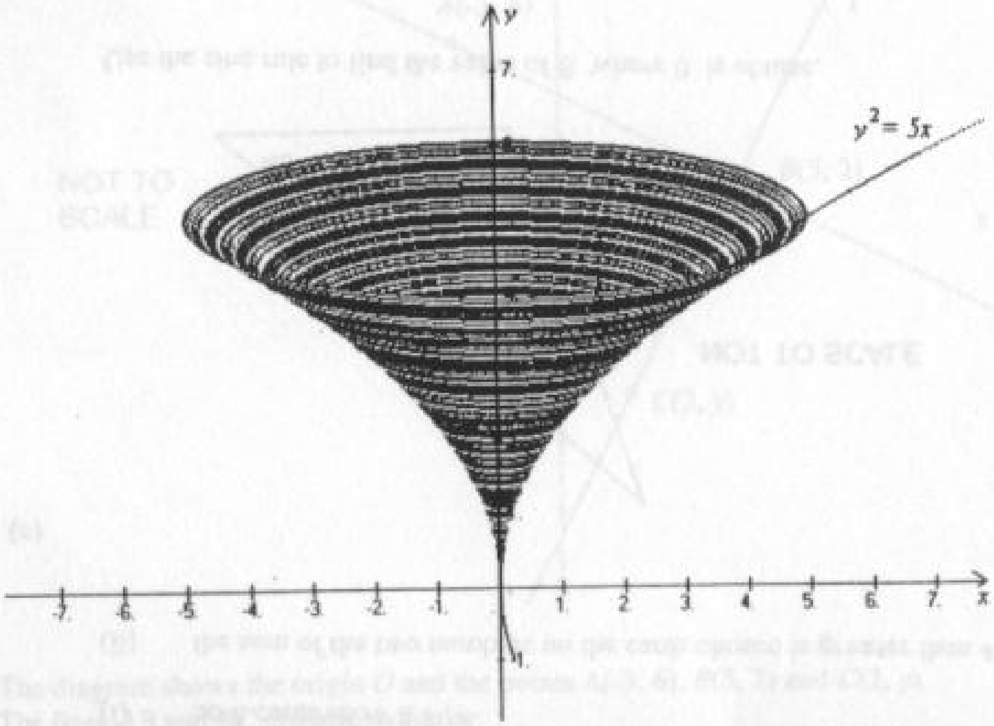
In the diagram, the shaded region $OABCD$ is bounded by $y = x^2 + 1$ the lines $y = 7 - x$, $x = 4$ and the x and y axes.

(i) Show that B has coordinates $(2, 5)$. **1**

(ii) Use Simpson's rule with 5 function values to estimate the **area** of the shaded region. **3**

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) 3



The diagram shows the shape of a vessel obtained by rotating about the y axis, the part of the parabola $y^2 = 5x$ between $y = 0$ and $y = 5$.

Show that the **volume** of the vessel is 25π *units*^{3}.

(b) The number N of bacteria in a colony is growing at a rate that is proportional to the current number. The number at time t hours is given by

$N = N_0 e^{kt}$ where N_0 and k are positive constants.

(i) If the size of the colony doubles every half hour, find the value of k. 2

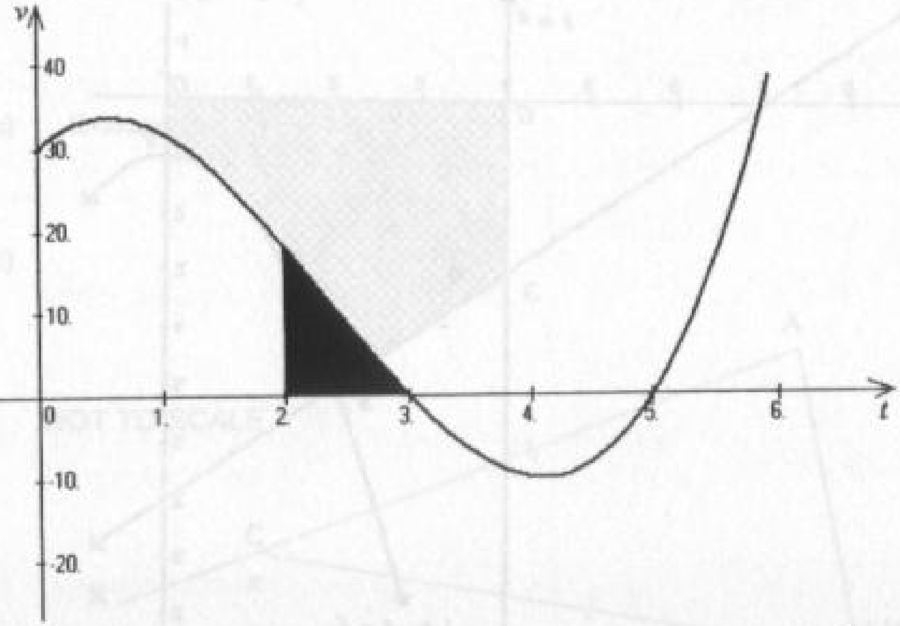
(ii) If the colony now contains 600 million bacteria, how long ago did the colony contain 3 million bacteria? 2

(iii) Show that the numbers of bacteria present at consecutive integer hours form a geometric sequence. 2

Question 6 continues on page 9

Question 6 (continued) **Marks**

(c)



A particle moves along a straight line for 6 seconds. The particle's velocity v at time t seconds is shown on the graph above.

(i) When is the particle at rest? **1**

(ii) What does the shaded region represent? **1**

(iii) Is this particle further from its initial position at $t = 3$ or at $t = 5$? Explain briefly. **1**

End of Question 6

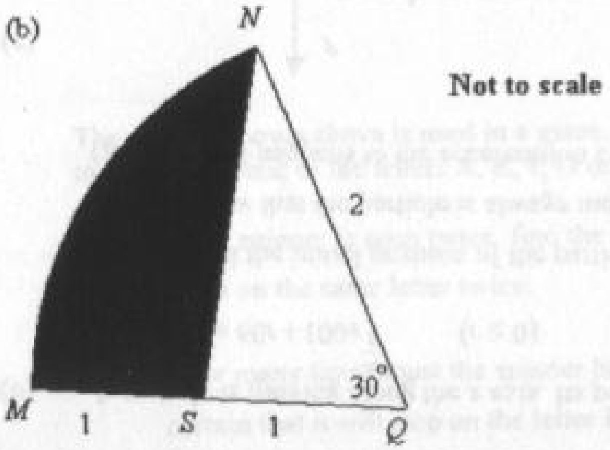
Question 7 (12 marks) Use a SEPARATE writing sheet. **Marks**

(a) (i) Show that $x = \frac{2\pi}{3}$ is a solution of $\cos x = \cos 2x$. **1**

(ii) On the same set of axes, sketch the graphs of $y = \cos x$ and $y = \cos 2x$ for $0 \le x \le \pi$, showing the x coordinate of all points of intersection. **2**

(iii) Find the exact area of the region bounded by the curves $y = \cos x$ and $y = \cos 2x$ over the interval $0 \le x \le \frac{2\pi}{3}$. **3**

(b)



In the figure, MNQ is the sector of a circle, $\angle MQN = 30^\circ$, $NQ = 2$ cm and $MS = SQ = 1$ cm.

(i) Calculate the exact length of NS. **2**

(ii) Find the perimeter of the shaded region MSN. **1**

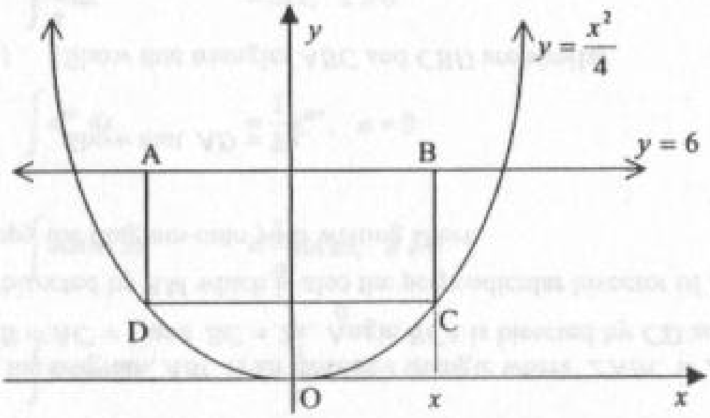
(c) (i) Without using calculus, sketch the graph of $y = e^x - 3$. **2**

(ii) On the same sketch, find graphically the number of solutions of the equation $e^x - 3 = -x^2$. **1**

Question 8 (12 marks) Use a SEPARATE writing sheet. **Marks**

(a) Differentiate $y = \log_2 x$. **2**

(b) The diagram shows a rectangle ABCD inscribed in the region bounded by the parabola $y = \dfrac{x^2}{4}$ and the line $y = 6$.



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(i) Show that the area of ABCD is given by $A = 12x - \dfrac{x^3}{2}$. **2**

(ii) Find the dimensions of the rectangle so that its area is a **maximum**. **4**

(c) Liquid petroleum is pumped out of a 25 000 litre storage container through a valve such that the volume flow rate of petroleum in litres per second is given by $\dfrac{dV}{dt} = -1\cdot 92t$ $\quad(t \geq 0)$.

(i) Show that if the tank was initially full, then $V = 25\,000 - 0.96t^2$. **2**

(ii) How long before the tank is only 40% full? **2**

Question 9 (12 marks) Use a SEPARATE writing sheet. **Marks**

(a) Mia would like to save \$60 000 for a deposit on her first home. She decides to invest her net monthly salary of \$3000 in a bank account that pays interest at a rate of 6% per annum compounded monthly. Mia intends to withdraw $\$E$ at the end of each month from this account for living expenses, immediately after the interest has been paid.

(i) Show that the amount of money in the account following the second withdrawal of $\$E$ is given by **2**

$$\$3000(1\cdot 005^2+1\cdot 005)-\$E(1\cdot 005+1).$$

(ii) Calculate the value of E if Mia is to reach her goal after 4 years. **3**

(b) A particle is moving along the x axis. Its position x at time t is given by

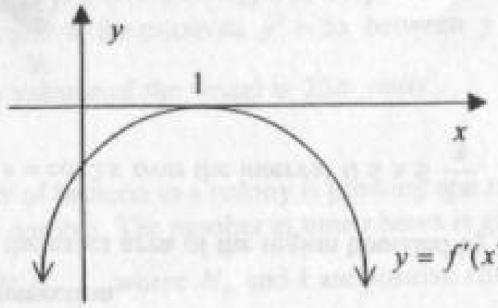
$$x=60t+100e^{\frac{-t}{5}} \qquad (t\geq 0).$$

(i) Find the initial position of the particle. **1**

(ii) Show that the particle is always moving to the right. **2**

(iii) What happens to the acceleration eventually? **2**

(c)



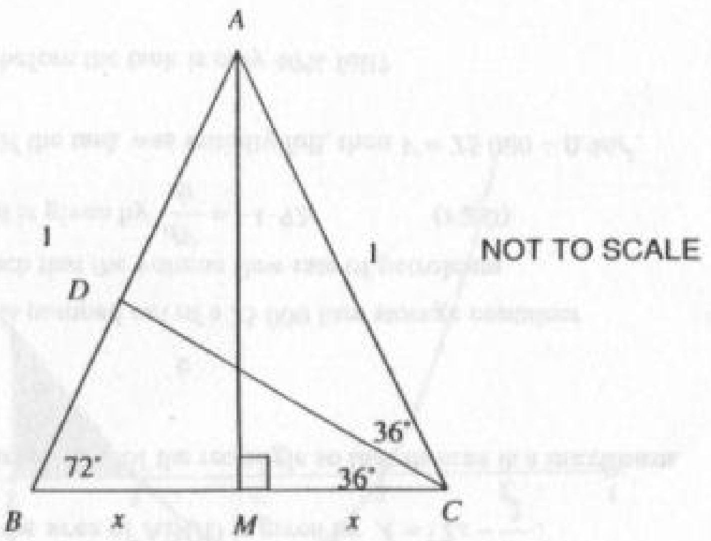
The diagram shows the graph of the **derivative** of the curve $y=f(x)$.

(i) The curve $y=f(x)$ has a stationary point of inflexion at $x=1$. Justify this statement by reference to the graph. **1**

(ii) Draw a possible graph of $y=f(x)$ if $f(1)=-3$. **1**

Question 10 (12 marks) Use a SEPARATE writing sheet. **Marks**

(a)



In the diagram, ABC is an isosceles triangle where $\angle ABC = \angle BCA = 72^\circ$, $AB = AC = 1$ and $BC = 2x$. Angle BCA is bisected by CD and angle BAC is bisected by AM which is also the perpendicular bisector of BC.

Copy the diagram onto your writing sheet.

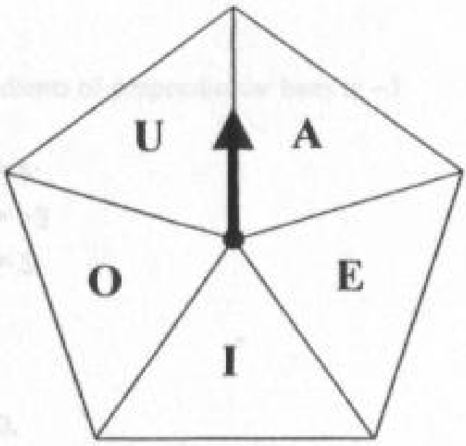
(i) Show that $AD = 2x$. **2**

(ii) Show that triangles ABC and CBD are similar. **2**

(iii) By using (ii), find the **exact** value of x. **3**

(iv) Hence find the **exact** value of $\sin 18^\circ$. **1**

(b)



The spinner shown above is used in a game. Once spun, it is equally likely to stop at any one of the letters **A**, **E**, **I**, **O** or **U**.

(i) If the spinner is spun twice, find the probability that it stops on the same letter twice. **2**

(ii) How many times must the spinner be spun for it to be 99% certain that it will stop on the letter **E** at least once? **2**

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$