



2009

TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 84

Attempt Questions 1–7

All questions are of equal value

At the end of the examination, place your solution booklets in order and put this question paper on top.

Submit one bundle.

The bundle will be separated before marking commences so that anonymity will be maintained.

Student Number: _____

Teacher: _____

Student Name: _____

QUESTION	MARK
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
TOTAL	/84

Total Marks – 84

Attempt Questions 1–7

All questions are of equal value

Begin each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) Solve $\frac{x-2}{x+3} > -2$. 3
- (b) Find the exact value of $\cos 2x$ if $\sin x = \sqrt{3} - 1$. 2
- (c) The graphs of $y = \frac{1}{x}$ and $y = x^3$ intersect at $x = 1$. Find the size of the acute angle between these two curves at $x = 1$. 3
- (d) Use the table of standard integrals to find $\int \sec 3x \tan 3x dx$. 1
- (e) Using the substitution $u = 1+t$, find the exact value of $\int_0^1 \frac{t}{\sqrt{1+t}} dt$. 3

Question 2 (12 marks) Use a SEPARATE writing booklet.

- (a) Given α, β, γ are the roots of the equation $2x^3 + 3x^2 - 2x - 4 = 0$, evaluate
- (i) $\alpha + \beta + \gamma$ 1
 - (ii) $\alpha^2 + \beta^2 + \gamma^2$ 1
- (b) The point $P(2ap, ap^2)$ lies on the parabola $x^2 = 4ay$
- (i) Show that the equation of the normal to the curve of the parabola at the point P is $x + py = 2ap + ap^3$. 2
 - (ii) Find the co-ordinates of the point Q where the normal at P meets the y -axis. 1
 - (iii) The point R divides PQ externally in the ratio 3:1. Show that the co-ordinates of the point R are $(-ap, 3a + ap^2)$. 2
 - (iii) Find the cartesian equation of the locus of R . 1
- (c) (i) Find the co-efficient of x^2 in the expansion of $\left(ax + \frac{b}{x^2}\right)^{11}$. 2
- (ii) If this co-efficient of x^2 is equal to the co-efficient of x^{-1} , show that $a - 2b = 0$. 2

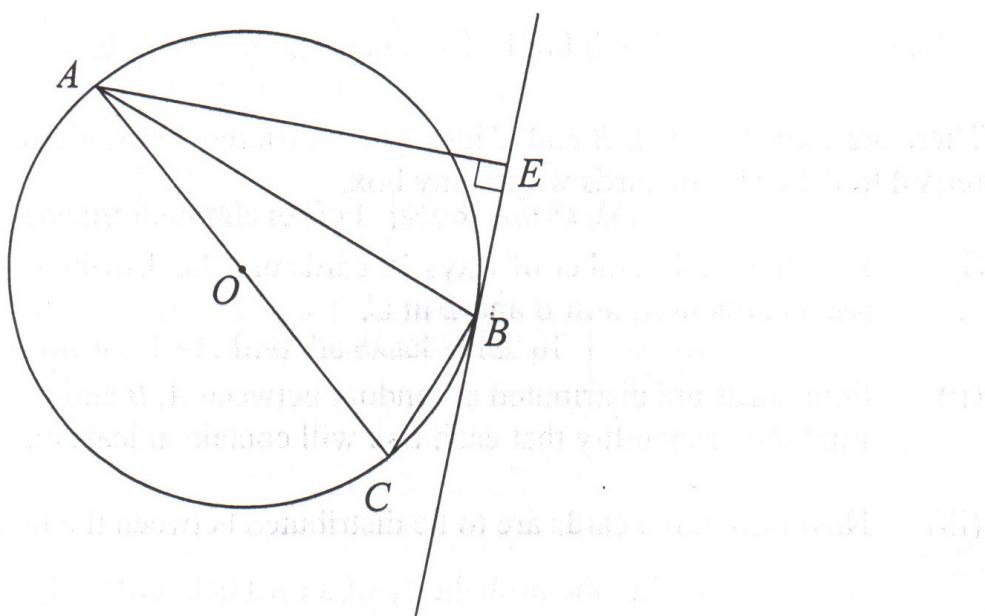
Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

- (a) Find $\frac{d}{dx} \sin(\log_e x)$. 2
- (b) (i) Show that $(k+1)(k+2)$ is even for any positive integer. 1
- (ii) Prove by mathematical induction that $n(n+1)(n+2)$ is divisible by 6 for all positive integers n . 3
- (c) There are three boxes A , B and C into which different cards are placed without regard to the order of cards within any box.
- (i) Find the total number of ways 10 cards may be distributed so that there are 5 cards in A , 3 in B and 2 in C . 1
- (ii) Four cards are distributed at random between A , B and C .
Find the probability that each box will contain at least one card. 3
- (iii) Now only three cards are to be distributed between the boxes A , B and C . Assuming the probability of a card being placed in box A is $\frac{3}{10}$ and in box B is $\frac{2}{5}$, find the probability that each of the three boxes will contain a card. 2

- (a) (i) Find the value of the constant a for which the polynomial $P(x) = x^4 + 2x^3 - x^2 - 8x - a$ is divisible by $Q(x) = x^2 - 4$. 1
- (ii) Hence or otherwise find all real zeros of the polynomial $P(x)$ with that particular value of a . 2

- (b) Two points A and B lie on a circle and AC is the diameter. AE is perpendicular to the tangent at B .



- (i) Copy or trace the diagram onto your paper. 1
- (ii) Prove that AB bisects $\angle CAE$. 3

- (c) A function is defined by the equation $y = \frac{e^x}{x^2 + 1}$.
- (i) Describe the behaviour of the function for large positive and negative values of x . 1
- (ii) Show that $\frac{dy}{dx} = \frac{e^x(x-1)^2}{(x^2+1)^2}$. 2
- (iii) Determine the co-ordinates of the stationary point(s), without considering the second derivative. 1
- (iv) Sketch the curve showing all important features. 2

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

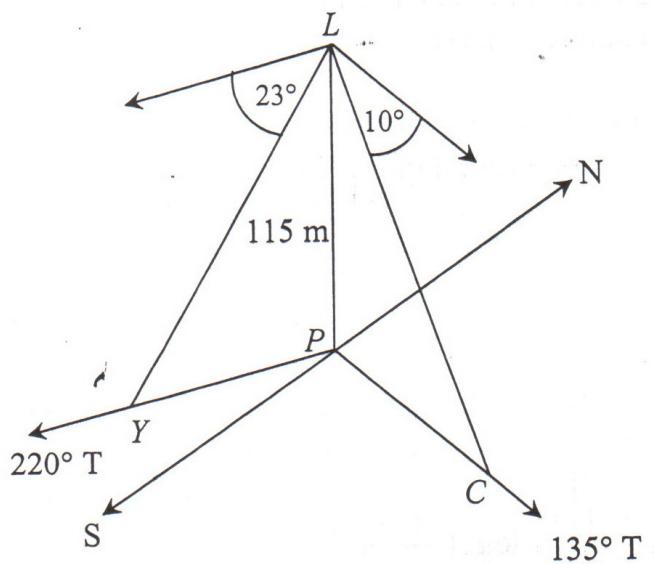
- (a) Fiona is one of 7 executive members of a travel company. Each year one member is selected at random to win a round the world ticket.
- (i) What is the probability that in the first 4 years Fiona will win at least one ticket? 1
- (ii) Show that in the first 25 years Fiona has a greater chance of winning exactly 3 tickets than exactly 2 tickets. 2
- (iii) How many years should Fiona work for the travel company to be 90% certain of winning at least one round the world ticket? 2
- (b) (i) For $x > 0$, show that 2
- $$\log_2\left(\frac{1}{x}\right) + \log_2\left(\frac{1}{x^2}\right) + \log_2\left(\frac{1}{x^3}\right) + \dots$$
- is an arithmetic series, and hence find the common difference.
- (ii) If the sum of the first ten terms is 440, find the value of x . 2
- (c) Use the identity $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$ to solve the equation $\sin 3\theta = 2 \sin \theta$ for $0 \leq \theta \leq 2\pi$. 3

Question 6 (12 marks) Use a SEPARATE writing booklet.

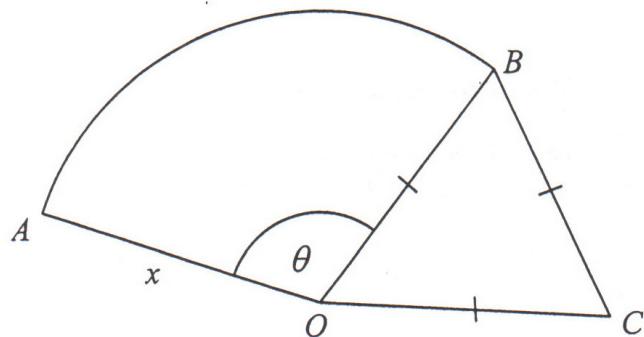
- (a) From the top of a lighthouse, L , 115 metres above sea level, a container ship, C , is seen on a bearing of $135^\circ T$, at an angle of depression of 10° . A yacht, Y is also sighted on a bearing of $220^\circ T$, at an angle of depression of 23° . This is illustrated in the diagram below.

3

Calculate the distance between the two vessels, correct to the nearest metre.



- (b) The diagram shows a sector OAB of a circle, centre O and radius x metres. Arc AB subtends an angle of θ radians at O . An equilateral triangle BCO adjoins the sector.



- (i) Write an expression in terms of θ and x for

1

(α) the perimeter of $OABC$.

2

(β) the area of $OABC$.

2

- (ii) Given that the perimeter has the value $(12 - 2\sqrt{3})$ metres, show that the area A is given by

$$A = \frac{(6 - \sqrt{3})^2 (2\theta + \sqrt{3})}{(\theta + 3)^2}$$

4

- (iii) For which value of θ is the area a maximum? Justify your answer.

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) Given $\frac{dy}{dx} = \cos^2 x$ and $y = 1$ when $x = 0$ find y in terms of x . 2

(b) The surface area A of a solid of revolution generated by rotating the part of a curve between $x = a$ and $x = b$ about the x axis is given by

$$\int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx.$$

(i) The curve $y = 2\sqrt{x}$ is rotated about the x axis. Show that the surface area generated over the interval $0 \leq x \leq 8$ is given by 2

$$4\pi \int_0^8 (x+1)^{\frac{1}{2}} dx.$$

(ii) Hence find the required surface area. 2

(c) (i) Show that, for all positive integers n , 2

$$1 + (1+x) + (1+x)^2 + \dots + (1+x)^n = \frac{(1+x)^{n+1} - 1}{x}.$$

(ii) Hence, or otherwise, show that for all integers $n \geq 2$, 2

$$\binom{2}{2} + \binom{3}{2} + \binom{4}{2} + \dots + \binom{n}{2} = \binom{n+1}{3}.$$

(iii) The polynomial $1 - x + x^2 - x^3 + \dots + x^{16} - x^{17} + x^{18}$ may be written in the form 2

$$b_0 + b_1 w + b_2 w^2 + \dots + b_{17} w^{17} + b_{18} w^{18}$$

where $w = x+1$ and b_0, b_1, \dots, b_{18} are real numbers.

Using the results of parts (i) and (ii) above, or otherwise, find the value of b_2 .