SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

AUGUST 2005

Trial Higher School Certificate Examination

YEAR 12

Mathematics

Sample Solutions

Section	Marker
A	AF
B	DH
C	PB
D	CK
E	PP

Section A

1.a. $\dfrac{(3.517)^2+(1.763)^2}{(3.517)(1.763)}$

$= 2.50$

b. $(5a-1)(25a^2+5a+1)$

c. $\dfrac{5}{\sqrt{7}-2} \times \dfrac{\sqrt{7}+2}{\sqrt{7}+2}$

$= \dfrac{5\sqrt{7}+10}{7-4}$

$= \dfrac{5\sqrt{7}+10}{3}$

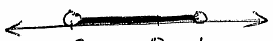
d. $|2x+1| < 3$

$2x+1 < 3$ or $-(2x+1) < 3$

$2x < 2$ $\quad -2x-1 < 3$

$x < 1$ $\quad -2x < 4$

$x > -2$



$-2 \quad 0 \quad 1$

e. $\angle EFI = 42^\circ$ (co-interior angles are supplementary $EG // HI$)

$\angle GFK = 57^\circ$ (co-interior angles are supplementary $EG // JL$)

$\angle IFK = 81^\circ$ (angles on a line are supplementary)

f. $\int (8x - x^{-2})\, dx$

$= \dfrac{8x^2}{2} - \dfrac{x^{-1}}{-1} + c$

$= 4x^2 + \dfrac{1}{x}$

2.a.i. $m = \dfrac{y_2-y_1}{x_2-x_1}$

$m_{AB} = \dfrac{3-0}{-9-(-7)}$

$= -\dfrac{3}{2}$

$m_{BC} = \dfrac{9-3}{0-(-9)}$

$= \dfrac{2}{3}$

$m_{AB} \times m_{BC} = -\dfrac{3}{2} \times \dfrac{2}{3}$

$= -1$

$\therefore AB \perp BC$

ii. $y - y_1 = m(x - x_1)$

$y - 0 = -\dfrac{3}{2}(x+7)$

$2y = -3x - 21$

$3x + 2y + 21 = 0$

iii. $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$AB = \sqrt{(-9+7)^2 + (3-0)^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$ units

iv. $BC = \sqrt{(0-(-9))^2 + (9-3)^2}$

$= \sqrt{81+36}$

$= \sqrt{117}$

Area $= \dfrac{1}{2} \times \sqrt{13} \times \sqrt{117}$

$= \dfrac{39}{2}$ units^2

v. equation of BC

$y = \dfrac{2}{3}x + 9$

$3y = 2x + 27$

$2x - 3y + 27 = 0$

$Pd = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \dfrac{|2(0) + (-3)(0) + 27|}{\sqrt{2^2 + (-3)^2}}$

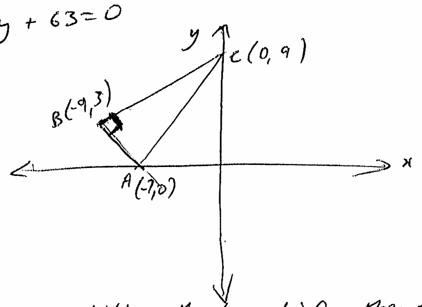
$= \dfrac{27}{\sqrt{13}}$ units.

vi. equation of AB is $3x + 2y + 21 = 0$ Test $(0,0)$ LHS = 21 $\therefore 3x + 2y + 21 > 0$

equation of BC is $2x - 3y + 27 = 0$ Test $(0,0)$ LHS = 27 $2x - 3y + 27 > 0$

equation of AC is $y = \frac{9}{7}x + 9$ Test $(0,0)$ LHS = 63 $9x - 7y + 63 < 0$

$9x - 7y + 63 = 0$

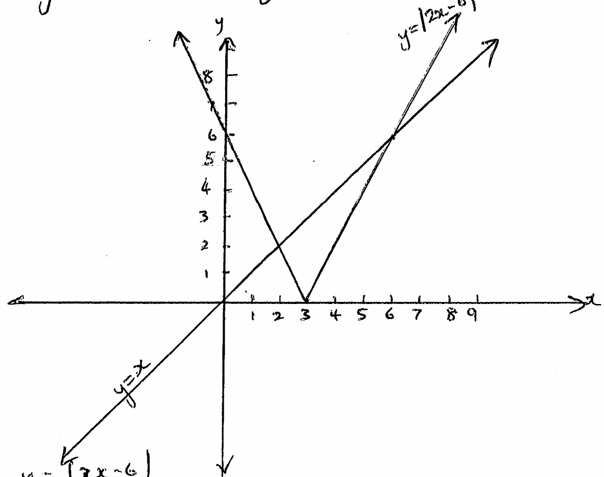


$\therefore$ the 3 inequalities that satisfy the region inside $\triangle ABC$ are $3x + 2y + 21 > 0 \cap 2x - 3y + 27 > 0 \cap 9x - 7y + 63 < 0$.

b. i. $y = |2x - 6|$

when $y = 0$, $2x - 6 = 0$
$2x = 6$
$x = 3$

when $x = 0$, $y = 6$



ii. you can see that $y = x$ and $y = |2x - 6|$ intersect at two points. when $x = 2$ and when $x = 6$. Therefore $|2x - 6| = x$ has two solutions.

$x = 2x - 6$ or $x = -(2x - 6)$

$\underline{x = 6}$ $x = -2x + 6$

$3x = 6$

$\underline{x = 2}$

iii. where is the graph of $y = |2x - 6|$ less than the graph of $y = x$?

$\underline{\underline{2 < x < 6}}$

Section B

3. (a) Differentiate:

   i. $\left(7-3x^2\right)^4$

   Solution: $4\times(-6x)(7-3x^2)^3=-24x(7-3x^2)^3$

   ii. $6\ln x$

   Solution: $\dfrac{6}{x}$

   iii. x^2e^{-x}

   Solution: $2xe^{-x}-x^2e^{-x}$

   (b) Find

   i. $\int e^{3x}\,dx$

   Solution: $\dfrac{e^{3x}}{3}+c$

   ii. $\displaystyle\int 5\cos\left(\frac{x}{2}\right)\,dx$

   Solution: $2\times 5\sin\dfrac{x}{2}+c=1-\sin\dfrac{x}{2}+c$

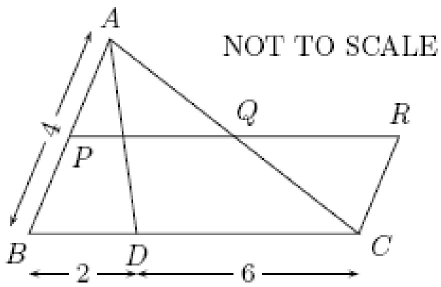
   (c) Evaluate $\displaystyle\int_1^e\frac{dx}{2x}$

   Solution: $\begin{aligned}\tfrac{1}{2}\left[\ln x\right]_1^e &= \tfrac{1}{2}(1-0),\\ &= \tfrac{1}{2}.\end{aligned}$

   (d) If $\dfrac{dy}{dx}=6x-9$ and $y=0$ when $x=1$, express y in terms of x.

   Solution: $y=3x^2-9x+c$.
   $y=0$ when $x=1$, $\therefore\ 0=3-9+c$, so $c=6$.
   Hence $y=3x^2-9x+6$.

(e)



i. In the diagram above prove that $\angle BDA = \angle BAC$.

Solution: $\angle ABD = \angle CBA$ (common)

$\dfrac{AB}{BC} = \dfrac{4}{8} = \dfrac{2}{4} = \dfrac{BD}{AB}$ (data)

$\therefore \ \triangle BAD \,|||\, \triangle BCA$ (2 sides same ratio, included angle equal)

$\therefore \ \angle BDA = \angle BAC$ (corresponding $\angle$s of similar $\triangle$s)

ii. P, Q are the midpoints of sides AB and AC respectively of the triangle ABC.
PQ is produced to R so that $PQ = QR$.

Prove that $CR = \frac{1}{2}AB$.

Solution: Method 1:

$$\begin{aligned} AQ &= QC \text{ (data)} \\ PQ &= QR \text{ (construction)} \\ \angle AQP &= \angle CQR \text{ (vertically opposite angles)} \\ \therefore \ \triangle APQ &\equiv \triangle CQR \text{ (SAS)} \\ AP &= CR \text{ (corresponding sides of congruent triangles)} \\ \text{but } AP &= \tfrac{1}{2}AB \text{ (P bisects AB)} \\ \textit{i.e.}, \ CR &= \tfrac{1}{2}AB. \end{aligned}$$

Solution: Method 2:

$\left.\begin{aligned} PQ &\,//\, BC \\ 2PQ &= BC \end{aligned}\right\}$ (midpoint theorem for $\triangle$s)

$2PQ = PR$ (construction)

$\therefore \ PBCR$ is a parm. (opposite sides equal and parallel)

Hence $RC = PB$ (opposite sides of parm.)

$\therefore \ CR = \frac{1}{2}AB$.

4. (a) Let α and β be the roots of the equation $2x^2 - 5x + 1$.
Find the values of

i. $\dfrac{5}{\alpha} + \dfrac{5}{\beta}$

Solution:
$$\begin{aligned} \alpha + \beta &= \frac{5}{2}, \\ \alpha\beta &= \frac{1}{2}. \\ \frac{5}{\alpha} + \frac{5}{\beta} &= 5\left(\frac{\alpha+\beta}{\alpha\beta}\right), \\ &= 5\left(\frac{\frac{5}{2}}{\frac{1}{2}}\right), \\ &= 25. \end{aligned}$$

ii. $(\alpha - \beta)^2$

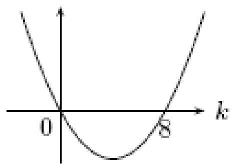
Solution:
$$\begin{aligned} (\alpha-\beta)^2 &= \alpha^2 - 2\alpha\beta + \beta^2, \\ &= \alpha^2 + 2\alpha\beta + \beta^2 - 4\alpha\beta, \\ &= (\alpha+\beta)^2 - 4\alpha\beta, \\ &= \frac{25}{4} - \frac{4}{2}, \\ &= \frac{17}{4}. \end{aligned}$$

(b) Find the values of k for which the equation

$$x^2 - (k-2)x + (k+1) = 0$$

has real roots.

Solution:
$$\begin{aligned} \text{For real roots, } \Delta &\geq 0, \\ (k-2)^2 - 4(k+1) &\geq 0, \\ k^2 - 4k + 4 - 4k - 4 &\geq 0, \\ k^2 - 8k &\geq 0. \\ \therefore\ k \leq 0 \text{ or } k &\geq 8. \end{aligned}$$



(c) Find the equation of the normal to $y = 2x^2 - 3x + 1$ at the point $(-1, 6)$.

Solution:
$$\begin{aligned} \frac{dx}{dy} &= 4x - 3, \\ &= -7 \text{ when } x = -1. \\ \therefore\ y - 6 &= \tfrac{1}{7}(x+1), \\ 7y - 42 &= x + 1, \\ x - 7y + 43 &= 0. \end{aligned}$$

(d) i. By considering a suitable infinite geometric series, express $0{\cdot}\dot{4}$ as a fraction in simplest form.

Solution:
$$\begin{aligned} 0{\cdot}\dot{4} &= 0{\cdot}4 + 0{\cdot}4 \times 0{\cdot}1 + 0{\cdot}4 \times 0{\cdot}01 + 0{\cdot}4 \times 0{\cdot}001 + \ldots, \\ &= 0{\cdot}4(1 + 0{\cdot}1 + 0{\cdot}1^2 + 0{\cdot}1^3 + \ldots), \\ &= \frac{0{\cdot}4}{1 - 0{\cdot}1}, \\ &= \frac{4}{9}. \end{aligned}$$

ii. Express $\sqrt{0{\cdot}\dot{4}}$ in simplest precise decimal form.

Solution:
$$\begin{aligned} \sqrt{0{\cdot}\dot{4}} &= \sqrt{\tfrac{4}{9}}, \\ &= \tfrac{2}{3}, \\ &= 0{\cdot}\dot{6}. \end{aligned}$$

(e) Find the coordinates of the centre and the radius of the circle with equation

$$x^2 + y^2 - 8x + y + \frac{1}{4} = 0$$

Solution:
$$\begin{aligned} x^2 - 8x \qquad + y^2 + y \qquad &= -\tfrac{1}{4}, \\ x^2 - 8x + 16 + y^2 + y + \tfrac{1}{4} &= -\tfrac{1}{4} + 16 + \tfrac{1}{4}, \\ (x-4)^2 + (y + \tfrac{1}{2})^2 &= 4^2. \end{aligned}$$
$\therefore$ Centre $(4, -\frac{1}{2})$, radius 4.

Section C

QUESTION 5 (a) $y = x^3 + x^2 - x + 1.$

$$\frac{dy}{dx} = 3x^2 + 2x - 1$$

$$\frac{d^2y}{dx^2} = 6x + 2.$$

(i) For turning points: $\frac{dy}{dx} = 0$

$$3x^2 + 2x - 1 = 0.$$

$$(3x-1)(x+1) = 0.$$

$$x = -1, \tfrac{1}{3}.$$

$$\& \; y = 2, \tfrac{22}{27}$$

at $(-1, 2)$ $y'' = -4$ ∴ MAX. TURNING PT.

at $\left(\frac{1}{3}, \frac{22}{27}\right)$ $y'' = 4$ ∴ MIN. TURNING PT.

(ii) For points of inflexion, consider $\frac{d^2y}{dx^2} = 0$

$$6x + 2 = 0$$

$$6x = -2$$

$$x = -\tfrac{1}{3}$$

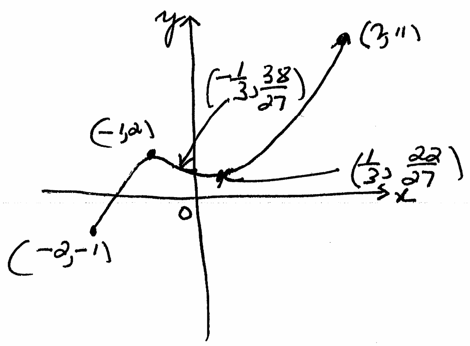
Testing

x	-1	$-\frac{1}{3}$	0
y''	-4	0	2

change in concavity

∴ $\boxed{\left(-\frac{1}{3}, \frac{38}{27}\right)}$ is a point of inflexion.

(iii)



(iv) For concave up

$$\frac{d^2y}{dx^2} > 0.$$

$$6x + 2 > 0$$

$$6x > -2$$

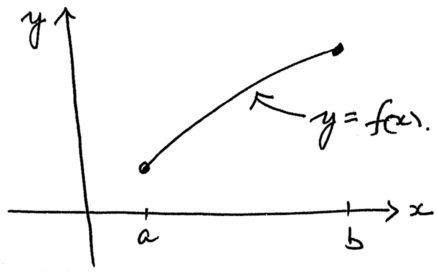
$$\boxed{x > -\tfrac{1}{3}}$$

(b) $4\sin^2 x - 3 = 0.$

$\sin^2 x = \frac{3}{4}$

$\sin x = \pm\frac{\sqrt{3}}{2}.$

$\boxed{x = 60°, 120°, 240°, 300°.}$

(c) 

positive, increasing and concave down.

(d) (i) $11 + A = 9$

$\therefore A = -2.$

$\therefore$ Area $= \boxed{2 u^2.}$

(ii)

$$\int_{-1}^{5} f(x)\,dx = \int_{-1}^{3} f(x)\,dx + \int_{3}^{5} f(x)\,dx$$

$$9 = 11 + \int_{3}^{5} f(x)\,dx.$$

$$\therefore \boxed{\int_{3}^{5} f(x)\,dx = -2}$$

QUESTION 6.

(a) (i) $3x^2 - 8 = 2x^2$

$x^2 = 8$

$x = 2\sqrt{2}$ (NB. $x > 0$)

$y = 16$

$\therefore \boxed{P \text{ is } (2\sqrt{2}, 16)}$

(ii) $A = \int_0^{2\sqrt{2}} \left[2x^2 - (3x^2 - 8)\right] dx.$

$= \int_0^{2\sqrt{2}} (8 - x^2)\, dx$

$= \left[8x - \frac{x^3}{3}\right]_0^{2\sqrt{2}}$

$= 16\sqrt{2} - \frac{16\sqrt{2}}{3}$

$= \boxed{\frac{32\sqrt{2}}{3} \text{ u}^2}$

(b) $LHS = \frac{\sin\theta}{1-\cos\theta}$

$= \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$

$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$

$= \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}$

$= \frac{1+\cos\theta}{\sin\theta}$

$= RHS$

(c) (i) $S_n = \dfrac{a(1-r^n)}{1-r}$

$= \dfrac{\sin^2\theta\,(1-(\cos^2\theta)^n)}{1-\cos^2\theta}$

$= \dfrac{\sin^2\theta\,(1-\cos^{2n}\theta)}{\sin^2\theta}$

$= 1-\cos^{2n}\theta$

(ii) $r = \cos^2\theta$ and $0 < \cos\theta < 1$ for $0 < \theta < \frac{\pi}{2}$

$\therefore \boxed{0 < \cos^2\theta < 1}$

(NB. limiting sum exists where $|r| < 1$.)

(iii) $S = \dfrac{\sin^2\theta}{1-\cos^2\theta}$ (ie $\frac{a}{1-r}$)

$= \dfrac{\sin^2\theta}{\sin^2\theta}$

$= 1$

$\therefore S - S_n = 1 - (1-\cos^{2n}\theta)$

$= \boxed{\cos^{2n}\theta.}$

(iv) If $\theta = \frac{\pi}{3}$, $(\cos^2\theta)^n = \left(\left(\frac{1}{2}\right)^2\right)^n$

$= 4^{-n}$

$\therefore$ If $S - S_n < 10^{-6}$ then $4^{-n} < 10^{-6}$

$4^n > 10^6$

$$\log_{10} 4^n > \log_{10} 10^6$$

$$n \log_{10} 4 > 6.$$

$$n > \frac{6}{\log_{10} 4}$$

$$> 9.965$$

$\therefore \boxed{n = 10}$ is the best value.

Section D

Question 7

(a) W_1 = win 1st prize

W_2 = win 2nd prize

(i) $P(W_1 \text{ and } W_2) = \frac{2}{20} \cdot \frac{1}{19} = \frac{1}{190}$ 1

(ii) $P(W_1 \text{ and } \tilde{W_2}) = \frac{2}{20} \cdot \frac{18}{19} = \frac{9}{95}$ 1

(iii) $P(\tilde{W_1} \text{ and } \tilde{W_2}) = \frac{18}{20} \cdot \frac{17}{19} = \frac{153}{190}$ 1

(iv) $1 - P(\text{no. phones}) = 1 - \frac{153}{190}$ 1

$= 37/190$

(b) (i) $2\sin x = \tan x$ 2

$2\sin x = \frac{\sin x}{\cos x}$

$\Rightarrow 2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\therefore \sin x = 0$ or $\cos x = \frac{1}{2}$

$\boxed{x = 0}$ or $\boxed{\frac{\pi}{3}, -\frac{\pi}{3}}$

(ii) $\int_0^{\pi/3} \frac{\sin x}{\cos x}\,dx = \left[-\ln(\cos x)\right]_0^{\pi/3}$ 2

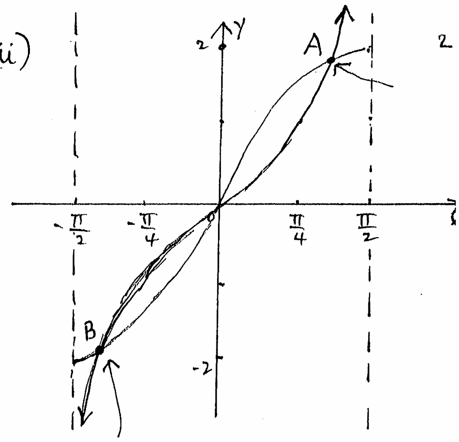
$= \left[\left(-\ln\cos\frac{\pi}{3}\right) - (-\ln\cos 0)\right]$

$= -\ln\frac{1}{2} + \ln 1$

$= -\ln 2^{-1} + 0$

$= \ln 2$

(iii) 2



$A\left(\frac{\pi}{3}, \sqrt{3}\right)$ $B\left(-\frac{\pi}{3}, -\sqrt{3}\right)$

(iv) Area required 2

$= 2 \times \int_0^{\pi/3} (2\sin x - \tan x)\,dx$

$= 2 \times \left[\int_0^{\pi/3} 2\sin x\,dx - \int_0^{\pi/3} \tan x\,dx\right]$

$= 2 \times \left[-2\cos x\right]_0^{\pi/3} - 2 \times \ln 2$

$= 2 - 2\ln 2$ from (ii)

$= \cancel{4}\ 2 - 2\ln 2$

Question 8

(a) Vol. about Y axis

$$V = \pi\int_c^d [g(y)]^2\,dy$$

$\cancel{x=}$ $\quad y = \dfrac{4}{1+x^2}$

$y + yx^2 = 4$

$x^2 = \dfrac{4-y}{y}$

$x^2 = \dfrac{4}{y} - 1$

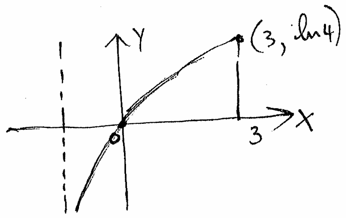
When $x=0$, $y=4$

$\Rightarrow \quad V = \pi\int_1^4 \left(\dfrac{4}{y} - 1\right)dy$

$= \pi\Big[4\log_e y - y\Big]_1^4$

$= \pi\Big[8\log_e 2 - 3\Big]$

(b) (i)



$f(x) = [\ln(x+1)]^2$

(ii)

$$\pi\int_0^2 [\ln(x+1)]^2\,dx \doteqdot \pi\left\{\frac{1}{6}\left[f(0)+4f(\tfrac{1}{2})+f(1)\right] + \frac{1}{6}\left[f(1)+4f(1\tfrac{1}{2})+f(2)\right]\right\}$$

ie $\doteqdot \dfrac{\pi}{6}\left[0 + 4(\ln\tfrac{3}{2})^2 + (\ln 2)^2 + (\ln 2)^2 + 4(\ln 2.5)^2 + (\ln 3)^2\right]$

$\doteqdot 3.238$ units^3

(c) $f(1) = 1$ and

$f'(1) = 0$

$\Rightarrow \quad f''(x) = 2x - 3$

$f'(x) = x^2 - 3x + c$

$\therefore \quad f'(1) = -2 + c = 0$ — ①

$f(x) = \dfrac{x^3}{3} - \dfrac{3x^2}{2} + cx + c_1$

and $f(1) = \dfrac{1}{3} - \dfrac{3}{2} + 2 + c_1 = 1$ — ②

$\therefore \quad c_1 = \dfrac{1}{6}$

$\Rightarrow f(x) = \dfrac{x^3}{3} - \dfrac{3x^2}{2} + 2x + \dfrac{1}{6}$

(d) (i) $y = e^{kx}$

$y' = ke^{kx}$,

$y'' = k^2e^{kx}$

(ii) $e^{kx} = 2ke^{kx} - k^2e^{kx}$

$1 = 2k - k^2$

$\Rightarrow k^2 - 2k + 1 = 0$

$(k-1)^2 = 0 \quad \therefore \boxed{k=1}$

Section E

(9)(a) (i) $f(x) = x^2 - \ln(2x-1) \Rightarrow 2x - 1 > 0 \quad [\because \ln u \text{ is defined for } u > 0]$

$\therefore x > \dfrac{1}{2}$

(ii) $f'(x) = 2x - \dfrac{2}{2x-1}$

$= 2x - 2(2x-1)^{-1}$

$f''(x) = 2 + 2(2x-1)^{-2} \times 2$

$= 2 + \dfrac{4}{(2x-1)^2}$

(iii) Stationary points are when $f'(x) = 0$

$f'(x) = 2x - \dfrac{2}{2x-1} = 0$

$\therefore 2x(2x-1) - 2 = 0$

$\therefore x(2x-1) - 1 = 0$

$\therefore 2x^2 - x - 1 = 0$

(iv) $2x^2 - x - 1 = 0$

$\therefore (2x+1)(x-1) = 0$

$\therefore x = 1 \Rightarrow y = 1$

$f''(x) > 0$ for $x > \dfrac{1}{2}$, so $y = f(x)$ is **always** concave up.

So $(1,1)$ is the minimum point on the function.

So the minimum value is 1.

(b) (i) Even though it is interest free, the repayments are required each month.

$A_1 = 50\,000 - M$

$A_2 = A_1 - M = 50\,000 - 2M$

and so on for 6 months so that

$A_6 = 50\,000 - 6M$.

(ii) $A_7 = A_6(1\cdot005) - M$

$= (50\,000 - 6M)(1\cdot005) - M$

$A_8 = A_7(1\cdot005) - M$

$= [(50\,000 - 6M)(1\cdot005) - M](1\cdot005) - M$

$= (50\,000 - 6M)(1\cdot005)^2 - M(1 + 1\cdot005)$

(iii) For $n > 6$

$$A_n = (50\,000 - 6M)(1\cdot005)^{n-6} - M\left(1 + 1\cdot005 + \cdots + 1\cdot005^{(n-6)-1}\right)$$

$$= (50\,000 - 6M)(1\cdot005)^{n-6} - M\left(1 + 1\cdot005 + \cdots + 1\cdot005^{n-7}\right)$$

$$A_{120} = (50\,000 - 6M)(1\cdot005)^{114} - M\left(\underbrace{1 + 1\cdot005 + \cdots + 1\cdot005^{113}}_{114 \text{ terms}}\right)$$

$$= (50\,000 - 6M)(1\cdot005)^{114} - M \times \left(\frac{1\cdot005^{114} - 1}{1\cdot005 - 1}\right)$$

$$= (50\,000 - 6M)(1\cdot005)^{114} - M \times \left(\frac{1\cdot005^{114} - 1}{0\cdot005}\right)$$

$$= (50\,000 - 6M)(1\cdot005)^{114} - 200M\left(1\cdot005^{114} - 1\right)$$

(iv) $A_{120} = 0$

$$\therefore (50\,000 - 6M)(1\cdot005)^{114} - 200M\left(1\cdot005^{114} - 1\right) = 0$$

$$\therefore 50\,000(1\cdot005)^{114} - 206M(1\cdot005)^{114} + 200M = 0$$

$$\therefore M\left[206(1\cdot005)^{114} + 200\right] = 50\,000(1\cdot005)^{114}$$

$$\therefore M = \frac{50\,000(1\cdot005)^{114}}{206(1\cdot005)^{114} + 200} \approx \$539\cdot18$$

(10) (i) Let X be the intersection of the diagonals.

$\angle XAD = \angle XAC = \theta$ [property of rhombi]

$AX = 2\cos\theta \Rightarrow AC = 4\cos\theta$

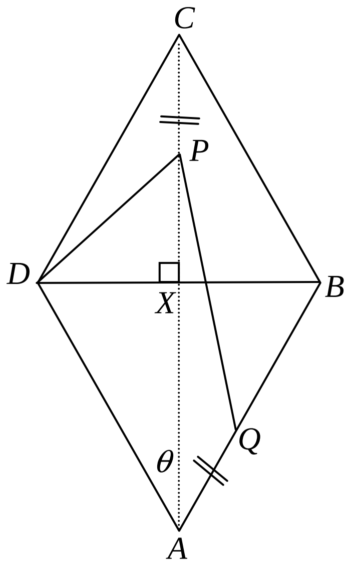
$\therefore AP = 4\cos\theta - x$

The shaded area is the sum of triangles ADP and APQ.

$$S = \frac{1}{2}\times 2\times\left(4\cos\theta - x\right)\sin\theta + \frac{1}{2}\times\left(4\cos\theta - x\right)\times x\sin\theta$$

$$= \frac{\sin\theta}{2}\left(4\cos\theta - x\right)\left(x+2\right)$$

[**NB** S is a concave down parabola in x]



(ii) $$S = \frac{\sin\theta}{2}\left[8\cos\theta + \left(4\cos\theta - 2\right)x - x^2\right]$$

$$\frac{dS}{dx} = \frac{\sin\theta}{2}\left[\left(4\cos\theta - 2\right) - 2x\right] = \sin\theta\left(2\cos\theta - 1 - x\right)$$

$$\therefore \frac{dS}{dx} = 0 \Rightarrow x = 2\cos\theta - 1 \qquad \left[\because \sin\theta \neq 0\right]$$

(iii) $$\frac{dS}{dx} = \sin\theta\left(2\cos\theta - 1 - x\right)$$

$$\therefore \frac{d^2S}{dx^2} = -\sin\theta \qquad \left[< 0 \text{ for } 0 < \theta < \frac{\pi}{2}\right]$$

(iv) $\theta = \dfrac{\pi}{6}$

$$\frac{dS}{dx} = 0 \Rightarrow x = 2\cos\left(\frac{\pi}{6}\right) - 1 = \sqrt{3} - 1$$

$$\frac{d^2S}{dx^2} < 0 \Rightarrow S \text{ is a maximum}$$

$$AC = 4\cos\left(\frac{\pi}{6}\right) = 2\sqrt{3}$$

$$\therefore \frac{PC}{AC} = \frac{\sqrt{3}-1}{2\sqrt{3}}$$

(v) $\theta = \dfrac{\pi}{4}$

$$\frac{dS}{dx} = 0 \Rightarrow x = 2\cos\left(\frac{\pi}{4}\right) - 1 = \sqrt{2} - 1$$

$$\frac{d^2S}{dx^2} < 0 \Rightarrow S \text{ is a maximum}$$

$$AC = 4\cos\left(\frac{\pi}{4}\right) = 2\sqrt{2}$$

$$\therefore \frac{PC}{AC} = \frac{\sqrt{2}-1}{2\sqrt{2}}$$

So the statement is **FALSE.**

(vi) If $\theta = \dfrac{\pi}{3}$ then

$$\frac{dS}{dx} = 0 \Rightarrow x = 2\cos\left(\frac{\pi}{3}\right) - 1 = 0$$

$$\frac{d^2S}{dx^2} < 0 \Rightarrow S \text{ is a maximum}$$

So if $\theta = \dfrac{\pi}{3}$ then S **STARTS** at its maximum value and then decreases to 0.

So the statement is **FALSE**.