# HIGHER SCHOOL CERTIFICATE EXAMINATION 1969

## MATHEMATICS - PAPER B (2F) - EQUIVALENT TO 3U AND 4U - 1st PAPER

*Instructions*: Time allowed 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work. Mathematical tables will be supplied.

---

### QUESTION 1 (12 Marks)

(i) Find a primitive function of $x/(1 + x^2)$.

(ii) Find the second derivative of $\tan^{-1}(x/3)$.

(iii) The parabola $y = 4x^2$ is reflected in the horizontal line $y = -2$ (as if this line contained a mirror). Find the equation of the reflected curve.

(iv) Find the area under the curve $y = \sin(2x - 6)$ between $x = 3$ and $x = 4$.

---

### QUESTION 2 (9 Marks)

(i) It is known that at least one of two children was born on a Tuesday. Find the probability that both of them were born on a Tuesday.

(ii) Find the value of the derivative of $\sin^{-1}(\tan x)$ at $x = 0$.

(iii) Use one step of Newton's method to find an improved value of that root of $x^4 - x - 13 = 0$ which lies close to $x = 2$ (express your answer as a fraction).

---

### QUESTION 3 (9 Marks)

(i) State the largest domain and the corresponding range of the function $f$ where $f(x) = \tan^{-1}(\sqrt{1 - x^2})$.

(ii) Let $f(x) = 2x^4 - 4x^3 + 3x^2 - x$. Find $df/dx$ and $d^2f/dx^2$ at $x = \frac{1}{2}$. State what conclusions you can draw from these values of $f'(\frac{1}{2})$ and $f''(\frac{1}{2})$ concerning the nature of the stationary point at $x = \frac{1}{2}$.

(iii) For what values of $x$ does the geometric series

$$1 + \left(\frac{2x - 1}{2 + x}\right) + \left(\frac{2x - 1}{2 + x}\right)^2 + \left(\frac{2x - 1}{2 + x}\right)^3 + \dots$$

converge (that is, a "sum to infinity" exists)?

---

### QUESTION 4 (10 Marks)

$J$ is the integral $\int_0^1 \frac{4}{x^2 + 1}\,dx$.

(i) Show that $J = \pi$.



(ii) Evaluate $J$ approximately, by using Simpson's rule with three function values.

(iii) State (to one decimal place) the percentage error in the result (ii).

---

(OMIT)

### QUESTION 5 (10 Marks)

Describe in geometrical terms the following three sets of points in three-dimensional space, defined respectively by:

(i) the inequality $(x - 2)^2 + y^2 + (z - 1)^2 > 1$;

(ii) the equation $z^2 = 1$;

(iii) the condition that the points of the third set belong to set (ii) but do not belong to set (i).

---

### QUESTION 6 (10 Marks)

A deck consists of four groups of ten cards each, each group having a distinctive colour. The cards within any one group labelled with the integers "1" to "10".

(i) Determine the total number of distinct five-card subsets that can be chosen from this deck.

(ii) Five cards are selected at random. Find the probability that four of them are labelled with the same integer (express your answer as a fraction).

(iii) Find the probability that five cards selected at random all have the same colour (express your answer as a fraction).

---

### QUESTION 7 (10 Marks)

The polynomial $P(x)$ is to have a double root $x = 2$ and a single root $x = 3$.

(i) Write down one possible polynomial which satisfies these conditions.

(ii) State the extent to which $P(x)$ is determined by the above plus the additional condition that $P(x)$ be a monic polynomial of degree 3.

(iii) If now $P(x)$ is to be a monic polynomial of degree 4, discuss carefully the extent to which $P(x)$ is determined.

---

### QUESTION 8 (10 Marks)

Let $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ be points on the parabola $y = x^2/(4a)$. Find the equation of PQ and deduce the condition for PQ to pass through the focus of the parabola. Further, find the locus of the points of intersection of the normals at P and Q, where PQ is a focal chord.

## QUESTION 9 (10 Marks)

(i) By starting from the fact that the derivative of $e^x$ is equal to $e^x$, deduce that

$$\lim_{h \to 0} \left(\frac{e^h - 1}{h}\right) = 1$$

(ii) From the definition of the definite integral $\int_0^2 e^x\,dx$ as the limit of a sum, and the known value of the sum of a geometric progression, evaluate this integral.

---

## QUESTION 10 (10 Marks)

A particle moves along a straight line under the action of a restoring force which tends to force the particle back to the origin, this force being proportional to the cube of the distance from the origin.

(i) Show that the equation of motion has the form $d^2x/dt^2 = -Ax^3$ where A is a positive constant.

(ii) Deduce the fact that the quantity $(dx/dt)^2 + \frac{1}{2}Ax^4$ is a constant of the motion.

(iii) Find this constant, given that the particle starts from rest at time $t = 0$ at a position $x = a$, where $a > 0$. Hence discuss in qualitative terms the nature of the motion (but do not attempt any further integration or calculation to find expressions for x or t).