FILE

Centre Number

Student Number

SCEGGS Darlinghurst

2004
Higher School Certificate
Trial Examination

Mathematics Extension 1

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1–7
- All questions are of equal value

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Attempt Questions 1–7
All questions are of equal value

Answer each question on a NEW page

Question 1 (12 marks) **Marks**

(a) Solve for x: **3**

$$\frac{3}{x-2} \le 1$$

(b) Find, to the nearest minute, the acute angle between the lines $y = 4x + 5$ and $3x + 2y - 1 = 0$. **2**

(c) Find $\lim\limits_{x \to 0} \dfrac{\sin 4x}{8x}$ **1**

(d) Evaluate $\displaystyle\int_0^{\frac{\pi}{3}} \sin^2 3x \; dx$ **3**

(e) Evaluate $\displaystyle\int_0^1 x(1-x)^4 \; dx$ using the substitution $u = 1 - x$. **3**

Question 2 (12 marks) **START A NEW PAGE** **Marks**

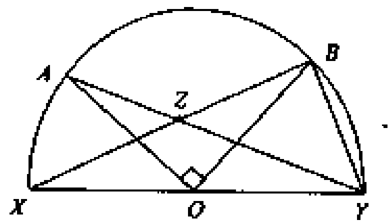
(a) Differentiate $x^2 \sin^{-1} 3x$ with respect to x. **2**

(b) How many different arrangements of the letters of the word PARABOLA are possible? **2**

(c) Find all real values of a for which $P(x) = ax^3 - 8x^2 - 9$ is divisible by $x - a$. **2**

(d) The two curves $y = \cos^{-1} x$ and $y = 2\tan^{-1}(1 - x)$ both cut the y-axis at the point $\left(0, \frac{\pi}{2}\right)$. Both curves also share a common tangent at $\left(0, \frac{\pi}{2}\right)$. Find the equation of this tangent. **2**

(e)



Not to scale

O is the centre of a semicircle, diameter XY.
OA and OB are perpendicular, AY and XB intersect at Z.

Copy the diagram onto your answer sheet.

(i) Explain why $\angle AYB = 45°$. **1**

(ii) Prove that $BY = BZ$. **3**

Question 3 (12 marks) START A NEW PAGE — Marks

(a) (i) Express $\sqrt{3}\cos x - \sin x$ in the form $R\cos(x+\alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. — **2**

(ii) Hence, sketch the graph of the equation $y = \sqrt{3}\cos x - \sin x$ for

$$\frac{-\pi}{6} < x < 2\pi .$$

— **1**

(iii) Solve the equation $\sqrt{3}\cos x - \sin x = \sqrt{2}$ for $0 \le x \le 2\pi$. — **2**

(b) On a particularly windy day, a sock pegged on a clothes line is oscillating in simple harmonic motion such that its displacement, x centimetres, from the origin, O, is given by the equation:

$$\ddot{x} = -16x \quad \text{where } t \text{ is the time in seconds.}$$

(i) Show that $x = a\cos(4t+\alpha)$, where a and α are constants, is a solution of motion for the sock. — **1**

(ii) Initially, the sock is 5cm to the right of the origin with a velocity of -4cms^{-1}. Show that the amplitude of the oscillation is $\sqrt{26}$ cm. — **2**

(iii) Find the maximum speed of the sock. — **1**

(c) Prove that $5^n + 11$ is divisible by 4 for all integers $n \ge 0$, by mathematical induction. — **3**

Question 4 (12 marks) START A NEW PAGE — Marks

(a) Consider the function $f(x) = \pi + 2\sin^{-1}\left(\frac{x}{3}\right)$

(i) State the domain and range of $y = f(x)$. — **2**

(ii) Sketch the graph of $y = f(x)$, marking clearly any endpoints. — **2**

(b) Two roots of the equation $x^3 + px^2 + q = 0$ (p, q real) are reciprocals of each other.

(i) Show that the third root is equal to $-q$. — **1**

(ii) Show that $p = q - \frac{1}{q}$. — **2**

(c) A forklift is driving down a warehouse aisle. The acceleration of the forklift is given by the equation:

$$\ddot{x} = -\frac{1}{2}\mu^2 e^{-x}$$

where x is the displacement from the origin and μ is the initial velocity at the origin.

(i) Show that $v^2 = 4e^{-x}$ if $\mu = 2\text{ms}^{-1}$. — **1**

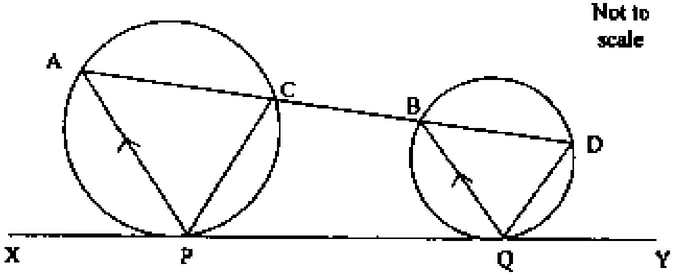
(ii) Explain why $v > 0$. — **1**

(iii) Find an equation for x in terms of t. — **2**

(iv) Describe the motion of the particle as $t \to \infty$. — **1**

Question 5 (12 marks) START A NEW PAGE **Marks**

(a)



In the diagram, XY is a common tangent to two non-intersecting circles.
This tangent touches one circle at P and the other circle at Q.
AP is a chord in one circle and BQ, a chord in the other circle, is parallel to AP.
AD is a straight line, cutting one circle at A and C and the other circle at B and D.

Copy the diagram onto your answer sheet.

Prove that:

(i) $PC \parallel QD$. **3**

(ii) PQBC is a cyclic quadrilateral. **2**

(b) The equation of the tangent to the parabola $y = x^2$ at the point $P(t, t^2)$ is $y = 2tx - t^2$.

(i) Show that the line passing through the focus of the parabola, perpendicular to this tangent, has equation $y = \frac{t - 2x}{4t}$ **2**

(ii) Show that the foot of the perpendicular from the focus to the tangent is the point $F\left(\frac{t}{2}, 0\right)$. **2**

(iii) Find the locus of M, the midpoint of PF. **3**

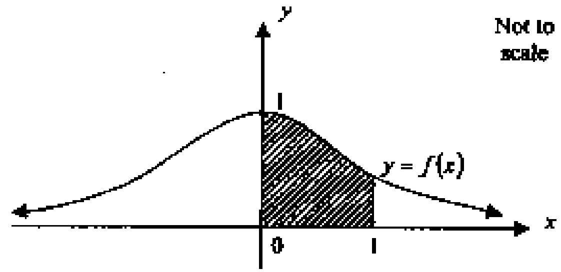
Question 6 (12 marks) START A NEW PAGE **Marks**

(a) A crew of four rowers is to be chosen from five boys and six girls. How many different crews are possible if:

(i) there are no restrictions? **1**

(ii) the shortest girl and the tallest boy must be included? **1**

(b) Consider the graph of the function $f(x) = \frac{1}{1 + x^2}$.



(i) Find the area bounded by this curve, the x axis and the two ordinates $x = 0$ and $x = 1$ using Simpson' Rule with three function values. Answer correct to 4 decimal places. **2**

(ii) Find the exact value of the area bounded by $y = f(x)$, the x-axis and the two ordinates $x = 0$ and $x = 1$. **2**

(iii) Hence find an approximation for π correct to 2 decimal places. **1**

(c) Surveyors have marked out two points, A and B, in St Peter's St. The points are 52m apart and B is due east of A. **5**

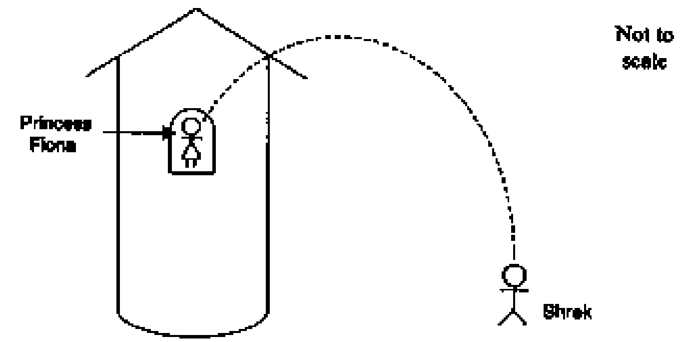
The bearings of A and B from the tallest point of the Great Hall are 230°T and 110°T respectively. The angles of elevation of the tallest point of the Great Hall from A and B are 30° and 60° respectively.

Show that the tallest point of the Great Hall is $4\sqrt{39}$ m high.

Question 7 (12 marks) START A NEW PAGE **Marks**

(a) Find all the values of θ for which $\cos^2\theta + \frac{\sqrt{3}}{2}\sin 2\theta = 0$. **4**

(b)



Princess Fiona is locked up in a tower, 80m above the ground. To gain the attention of Shrek, Princess Fiona throws a lentil at an angle of elevation of θ and an initial velocity of 50ms^{-1}.

(i) Derive the equations for the horizontal and vertical displacements of the lentil t seconds after it is thrown. (Use $g = 10\text{ms}^{-2}$.) **4**

(ii) Shrek is 300m from the base of the tower when he is hit by the lentil. Find the values of the initial angle of projection, θ, correct to the nearest degree, if Shrek is 2m tall. **4**

End of Paper

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