Question 1

a) Solve for x if

(i) $\dfrac{x-3}{|x-2|}=4, \quad x \neq 2$ (3)

(ii) $\dfrac{2}{x}>3$ (3)

(iii) $\dfrac{3}{x-2} \leq 5$ (3)

b) If $f(x)=2x^2-5x+6$, write in simplest form

(i) $f(x+2)$ (2)

(ii) $f(x+h)$ (2)

(iii) $\dfrac{f(x+h)-f(x)}{h}$ (2)

Question 2 (Start a new page)

(a) (i) On the same set of axes sketch the graphs of

$y=2x+1$

and $y=|1-2x|$ (4)

(ii) Hence, or otherwise, solve for x if

$2x+1 \geq |1-2x|$ (1)

(b) (i) Determine the centre and radius of the circle whose equation is

$x^2+y^2-18x+20y+60=0$ (3)

(ii) State the coordinates of the point on the circle which is furthest from the x axis (1)

(c) A circle just touches the positive x and y axes, and also passes through the point $(1,2)$.

(i) Draw a diagram to illustrate this information. (1)

(ii) Determine the centre and radius of all such circles (4)

Please turn over

Question 3 (Start a new page)

(a) Simplify fully

$$\frac{3^{x+1}+3^{x-1}}{3^x}$$ (2)

(b) Solve for a if

$$\left(8^{1-a}\right)\left(2^{a-3}\right)=4$$ (3)

(c) Solve for x and y if

$$\left(125^x\right)\left(5^y\right)=\frac{1}{5}$$

and $$2^x=\frac{4^y}{32}$$ (4)

(d) Sagar believes that for any value of x

$$3^x+3^x+3^x=3^{x+1}$$

(i) Verify that this is true when $x=4$ (1)

(ii) Is this true for any value of x? Justify your answer. (2)

Question 4 (Start a new page)

(a) Consider $y=\frac{3}{x-2}$

(i) State clearly the values of any x or y intercepts. (1)

(ii) State the equations of any vertical asymptotes. (1)

(iii) As the value of x approaches 2 from the positive side, what does the value of y approach? (1)

(iv) Draw a neat sketch of $y=\frac{3}{x-2}$, showing the intercepts and asymptotes. (3)

(b) A function $f(x)$ is given by

$$f(x)=ax^2+bx+c$$

(i) Express $f(-2)$ in terms of a,b and c. (1)

(ii) If the graph of $y=f(x)$ passes through the points $(-1,-6)$, $(-2,-3)$ and $(-3,4)$ find the values of a,b and c. (4)

EXTENSION 1 SOLUTIONS

1 (a) (i) $\dfrac{x-3}{|x-2|} = 4$

$x|x-2| \quad x-3 = 4|x-2|$

$x-3 = 4x-8$ or $x-3 = -4x+8$

$-3x = 5$ $\qquad 5x = 11$

$x = -\frac{5}{3}$ $\qquad x = 2\frac{1}{5}$

Test LHS < 0 $\qquad$ LHS < 0

∴ **no solution**

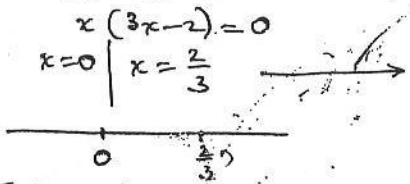
(ii) $\dfrac{2}{x} > 3$

$x x^2 \quad 2x > 3x^2$

$3x^2 - 2x = 0$

$x(3x-2) = 0$

$x = 0 \quad | \quad x = \frac{2}{3}$



Test $x = 1$ ✗

Test $x = \frac{1}{2}$ ✓

Test $x = -1$ ✗

$\therefore \boxed{0 < x < \frac{2}{3}}$

(iii) $\dfrac{3}{x-2} \leq 5$

$x(x-2)^2 \quad 3(x-2) \leq 5(x-2)^2$

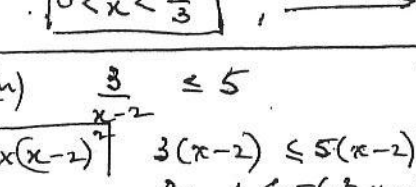
$3x - 6 \leq 5(x^2 - 4x + 4)$

$3x - 6 \leq 5x^2 - 20x + 20$

$5x^2 - 23x + 26 = 0$

$(5x - 13)(x - 2) = 0$

$x = \frac{13}{5}, \; 2$



Test $x = 0$ ✓

Test $x = 2\frac{1}{2}$ ✗

Test $x = 3$ ✓

$\therefore \boxed{x < 2 \text{ or } x \geq 2\frac{3}{5}}$

(b) (i) $f(x+2) = 2(x+2)^2 - 5(x+2) + 6$

$= 2(x^2 + 4x + 4) - 5x - 10 + 6$

$= \boxed{2x^2 + 3x + 4}$

(ii) $f(x+h) = 2(x+h)^2 + 5(x+h) + 6$

$= 2(x^2 + 2xh + h^2) + 5x + 5h + 6$

$= \boxed{2x^2 + 4xh + 2h^2 + 5x + 5h + 6}$

(iii) $\dfrac{f(x+h) - f(x)}{h}$

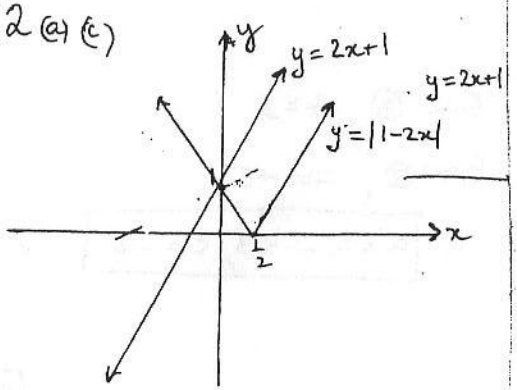
$\dfrac{2x^2 + 4xh + 2h^2 + 5x + 5h + 6 - (2x^2 - 5x + 6)}{h}$ ①

$= \dfrac{4xh + 2h^2 + 5h}{h}$

$= \dfrac{h(4x + 2h + 5)}{h}$

$= \boxed{4x + 2h + 5}$ ①

2 (a) (i)



(ii) $x \geq 0$ by inspection.

(b) $x^2 - 18x + 81 + y^2 + 20y + 100 = -60 + 81 + 100$

$(x-9)^2 + (y+10)^2 = 121$

$\boxed{\text{Centre } (9, -10) \quad \text{Radius} = 11}$

Point furthest from the x axis is $\boxed{(9, -21)}$

3 (a) $\dfrac{3^{x+1} + 3^{x-1}}{3^x}$

$= \dfrac{3^x(3 + 3^{-1})}{3^x}$

$= \boxed{3\frac{1}{3}}$

(b) $(8^{1-a})(2^{a-3}) = 4$

$(2^3)^{1-a} \, 2^{a-3} = 2^2$

$\therefore 2^{3-3a+a-3} = 2^2$

$-2a = 2$

$\boxed{a = -1}$

(c) $(125^x)(5^y) = \frac{1}{5}$ →

$\therefore (5^3)^x 5^y = 5^{-1}$

$\boxed{3x + y = -1}$ ① →

$2^x = \dfrac{4^y}{32}$

$2^x = \dfrac{(2^2)^y}{2^5}$

$\boxed{x = 2y - 5}$ ② → 1

By substitution,

$3(2y - 5) + y = -1$

$6y - 15 + y = -1$

$7y = 14$

$\boxed{y = 2}$

$\boxed{x = -1}$ } → ①

(d) $3^4 + 3^4 + 3^4 = 81 + 81 + 81$

$= 243$

$3^{4+1} = 3^5 = 243$ } ∴ true

(ii) $3^x + 3^x + 3^x = 3(3^x)$ → ①

$= 3^{1+x}$

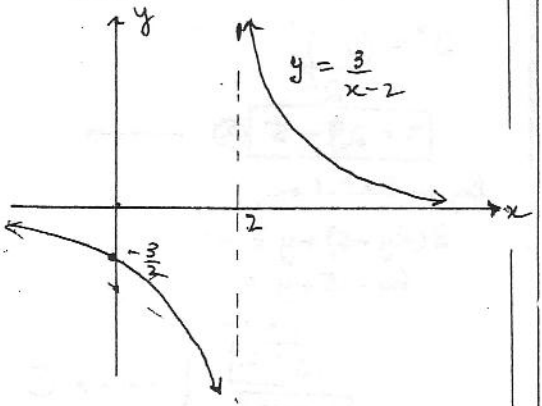
∴ true → ①

4 (a)(i) If $x=0$, $y=-\frac{3}{2}$

(ii) $x=2$

(iii) y approaches ∞

(iv)



(b)(i) $f(-2) = a(-2)^2 + b(-2) + c$

$= 4a - 2b + c$

(ii) $f(-1) = a - b + c = -6$ ①

$f(-2) = 4a - 2b + c = -3$ ②

$f(-3) = 9a - 3b + c = 4$ ③

② − ① $\quad 3a - b = 3$ ④

③ − ② $\quad a - b = \quad$ ⑤

④ − ⑤ $\quad 2a = 4$

$a = 2$

From ④, $b = 3$

From ①, $c = -5$

$\therefore$ $a=2, b=3, c=-5$