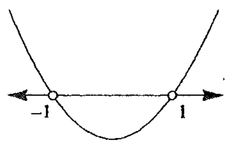
Neap.
advancing education

HSC Trial Examination 2001

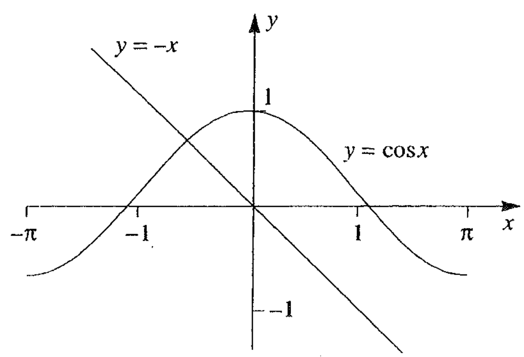
Mathematics Extension 1

Solutions and suggested marking scheme

QUESTION 1

Sample Answer	Outcome listing and mark guide
(a) $\displaystyle\int_1^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} = \left[\sin^{-1}\frac{x}{2}\right]_1^{\sqrt{3}}$ $= \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{2}$ $= \frac{\pi}{3} - \frac{\pi}{6}$ $= \frac{\pi}{6}$	HE4 • Gives correct answer . . . . 3 • Gives correct integral and gives at least one correct $\sin^{-1}$ evaluation . . . . 2 • Gives correct integral 1
(b) $\frac{3}{x^2-1} \geq 0$ $x^2 - 1 \neq 0$ i.e. $x \neq 1$ or $x \neq -1$ $(x^2-1) > 0$ $(x-1)(x+1) > 0$ $x < -1$ or $x > 1$ 	PE3 • Gives correct solution . . . . 3 • Gives solution of $x \leq -1, x \geq 1$ . . . . 2 • Gives correct factorisation 1
(c) $u^2 = 4 + x$ $x = u^2 - 4$ $dx = 2u\,du$ If $x = 5, u^2 = 9$ $\therefore u = 3$ If $x = 0, u^2 = 4$ $\therefore u = 2$ $\displaystyle\int_0^5 \frac{x}{\sqrt{4+x}}dx = \int_2^3 \frac{u^2-4}{\sqrt{u^2}}2u\,du$ $\displaystyle= 2\int_2^3 (u^2-4)du$ $= 2\left[\frac{1}{3}u^3 - 4u\right]_2^3$ $= 2\left\{[9-12] - \left[\frac{8}{3} - 8\right]\right\}$ $= \frac{14}{3}$ or $4\frac{2}{3}$ or 4.67 (2 d.p.)	HE6 • Gives correct solution . . . . 4 • Finds correct integral. . . . . 3 • Gives correct expression to be integrated . 2 • Shows significant progress in finding the correct expression to be integrated 1
(d) $\frac{{}^nP_{r+1}}{{}^nP_r} = \frac{n!}{(n-(r+1))!} \div \frac{n!}{(n-r)!}$ $= \frac{n!}{(n-r-1)!} \times \frac{(n-r)(n-r-1)!}{n!}$ $= n - r$	PE3 • Gives correct answer . . . . 2 • Gives correct expression for ${}^nP_{n+1}$ and ${}^nP_r$ 1

QUESTION 2

Sample Answer	Outcome listing and mark guide
(a) (i) $\dfrac{d}{dx}x\tan^{-1}x = \tan^{-1}x + \dfrac{x}{1+x^2}$	HE4, H5 • Gives correct derivative 1
(ii) $\displaystyle\int_0^1 \tan^{-1}x\,dx = \int_0^1 \frac{d}{dx}x\tan^{-1}x\,dx - \int_0^1 \frac{x\,dx}{1+x^2}$ $= \Big[x\tan^{-1}x\Big]_0^1 - \Big[\frac{1}{2}\ln(1+x^2)\Big]_0^1$ $= [(1)\tan(1) - 0]$ $\quad - \Big[\frac{1}{2}\ln(1+1) - \frac{1}{2}\ln(1+0)\Big]$ $= \Big[\frac{\pi}{4} - 0\Big] - \Big[\frac{1}{2}\ln 2 - 0\Big]$ $= \frac{\pi}{4} - \frac{1}{2}\ln 2$	HE4, H5 • Gives correct answer . . . . 3 • Gives the two correct integrals or • Gives the correct answer to one of the integrals . . . . 2 • Gives correct expression to integrate 1
(b) (i)  To solve: $\cos x + x = 0$ $\cos x = -x$ $\therefore$ draw $y = \cos x$ and $y = -x$, $-\pi \le x \le \pi$ There is only one point of intersection. $\therefore \cos x + x = 0$ has only one solution.	P4 • One mark for drawing two correct graphs. • One mark for giving a satisfactory explanation 2
(ii) Newton's method Let $f(x) = \cos x + x$ $f(-1) = \cos(-1) - 1$ $= -0.459697\ldots$ $f'(x) = -\sin x + 1$ $f'(-1) = -\sin(-1) + 1$ $= 1.84147\ldots$ $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$ $= -1 - \dfrac{-0.459697\ldots}{1.84147\ldots}$ $= -0.75036\ldots$ $= -0.75$ (to 2 decimal places)	PE3, H5 • Gives correct answer . . . . 2 • Finds correct values of $f(-1)$ and $f'(-1)$ and quotes correct formula 1

QUESTION 2 (Continued)

		Sample Answer	Outcome listing and mark guide
(c)	(i)	Number of arrangements $= \dfrac{12!}{3!4!5!}$ $= 27\ 720$	PE3 • Gives correct answer 1
	(ii)	Number of arrangements $= 3!$ $= 6$	PE3 • Gives correct answer 1
	(iii)	Number of arrangements with *Gone with the Wind* at one end and *Gladiator* at the other end $= 2! \times \dfrac{10!}{2! \times 4! \times 4!}$ $= 6300$ $P(\text{above arrangement}) = \dfrac{6300}{27\ 720}$ $= \dfrac{5}{22}$	PE3, H5 • Gives correct answer . . . . 2 • Gives correct answer for number of arrangements 1