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Trust in the Lord your God with all your heart, mind and soul.

Preliminary Mathematics

Task Three – Ext. One

General Instructions

- Reading time: 0 minutes
- Working Time: 2 hours
- Write using black or blue pen
- You may use a pencil to draw or complete diagrams
- Attempt ALL questions
- Calculators may be used
- Please write your name on the test paper.

Total Marks - 70

Section 1
Multiple choice questions
10 Marks

Sections 2, 3, 4, 5
Short answer & extended response questions
15 Marks each section

Assessment Weight – 20%

Task Breakdown

Section 1: Mark: _____ / 10

Sections 2, 3, 4 & 5: Mark: _____ / 60

TOTAL MARK: _____ / 70

Task Mark as a Percentage

%

____
Parent Signature

____
Teacher's Name

EXAMINER: MR. WAGDY MICHEAL

Section One (1 mark each)

1)! The statement $|x + 1| + 2|x - 2| < 6$ is equivalent to

(A) $-1 < x < 2$ (B) $0 < x < 1$ (C) $-1 < x < 3$
(D) $x < 2$ (E) $x < -1$ or $x > 2$

2)! $(1 - q^2)(1 + q^2 + q^4)$ equals

(A) $1 - q^6$ (B) $1 - q^2 - q^4 - q^6$ (C) $1 + q^4 + q^6$
(D) $1 + 2q^4 + 2q^6$ (E) $1 - q^2 + q^4 - q^6$

3)! The expression $\frac{k}{3}(k + 1)(k + 2) + (k + 1)(k + 2)$ is equal to

(A) $\frac{(k+1)(k+3)(k+4)}{6}$ (B) $\frac{k(k+1)(k+2)}{3}$ (C) $\frac{(k+1)(k+2)(k+3)}{3}$
(D) $\frac{2k}{3}(k+1)(k+2)$ (E) $\frac{(k+1)(2k+1)(3k+2)}{4}$

4)! If x an y are integers such that $(x - y)^2 + 2y^2 = 27$, then the only numbers x can be are

(A) $3, 5$ (B) $-6, 4$ (C) $0, 4, 6$ (D) $0, -4, 4, -6, 6$
(E) $0, -2, 2, -4, 4, -6, 6$

5)! If $\frac{3}{2 - \frac{x}{2}} = 2$ then x equals

(A) 3 (B) 1 (C) -1 (D) -2 (E) $\frac{1}{2}$

6)! The value of x in the equation $\frac{2}{15} = \frac{1}{8} + \frac{1}{x}$ is

(A) $\frac{15}{8}$ (B) $\frac{1}{7}$ (C) 7 (D) $\frac{120}{31}$ (E) 120

7)! The solution to $3x^2 \leq 5x$ is

(A) $0 \leq x \leq \frac{3}{5}$ (B) $x \geq \frac{5}{3}$ (C) $x \geq 0$ (D) $x \leq \frac{5}{3}$ (E) $0 \leq x \leq \frac{5}{3}$

8)! If $(x - 3)(2x + 1) = 0$ then the possible values of $2x + 1$ are

(A) 0 only (B) 0 and 3 (C) $-\frac{1}{2}$ and 3 (D) 0 and 7 (E) $-\frac{1}{2}$ and $-\frac{7}{2}$

9)! If $(4, 1)$ is the midpoint of the interval from $(x, -2)$ to $(5, y)$, what is the value of xy?

(A) 0 (B) 6 (C) -3 (D) -10 (E) 12

10)! Given $6^{x+y} = 36$ and $6^{x+5y} = 216$ then x is equal to

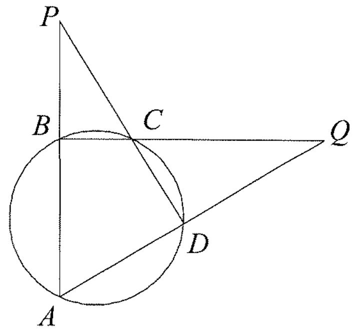
(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{5}{4}$ (D) $\frac{3}{2}$ (E) $\frac{7}{4}$

Section Two

11)! Factorise fully: $a^2 - b^2 + 3a + 3b$. 2

12)! Solve $\dfrac{3x + 2}{x - 1} > 2$. 2

13)!



In the diagram above ABP, DCP, BCQ, and ADQ are all straight lines and $\angle APD = \angle BQA$.

a. Show that $\angle ABC = \angle ADC$. 2

b. Prove that AC is a diameter of the circle. 2

14)! $A(-2, -5)$ and $B(1, 4)$ are two points. Find the acute angle θ between the line AB and the line $x + 2y + 1 = 0$, giving the answer correct to the nearest minute. 3

15)! i. By expanding $cos(2A + A)$, show that $cos\ 3A = 4cos^3A - 3cos\ A$. 2

ii. Hence show that if $2\cos A = x + \dfrac{1}{x}$ then $2\cos 3A = x^3 + \dfrac{1}{x^3}$. 2

Section Three

16)! Find all values of θ, Using the "t" results with $0 \le \theta \le 180$, such that $2\sin\theta + \cos\theta = 1$. 3

17)! A and B are the points $(-5, 12)$ and $(4, 9)$ respectively. P is the point which divides AB externally in the ratio $5 : 2$. Find the co-ordinates of P and show that if Q is the point $(0, 2)$, then triangle APQ is both right-angled and isosceles. 3

18)! Find, for $0 \le x \le 360$, all solutions of the equation $\sin 2x = \cos x$. 3

19)! If α, β are the roots of $x^2 - 10x + 21 = 0$, find the value of $\alpha^2 + \beta^2$. 2

20)! A vertical tower of height h metres stands on horizontal ground. From a point P on the ground due east of the tower the angle of elevation of the top of the tower is $45°$. From a point Q on the ground due south of the tower the angle of elevation of the top of the tower is $30°$.
If the distance PQ is 40 metres, find the exact height of the tower. 4

Section Four

21)! Show that $tan^2A - tan^2B = \dfrac{cos^2 B - cos^2 A}{cos^2 A cos^2 B}$. 2

22)! Solve the equation $x^2 + 2x - 4 + \dfrac{3}{x^2 + 2x} = 0$. 2

23)! Find the coordinates of the point(s) on the curve $y = \dfrac{x^2 + x}{x - 1}$ where the tangent is perpendicular to the line $y = -2x + 3$. 3

24)! The tangent to the curve $y = x^2$ at the point $A(1, 1)$ meets the y-axis at B. The normal at A meets the y-axis at C. Find the area of ΔABC. 3

25)! Solve simultaneously $3x + 2 \leq 7 - 2x \leq 8 - x$. 2

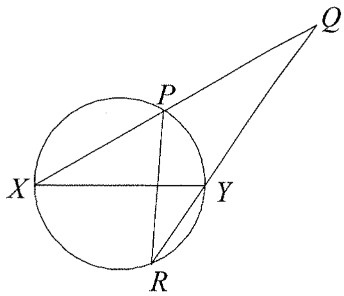
26)! i. Write $8cosx + 6sinx$ in the form $Acos(x - \alpha)$, where $A > 0$ and $0 \leq \alpha \leq 90°$. 1

ii. Hence, or otherwise, solve the equation $8cosx + 6sinx = 5$ for $0 \leq x \leq 360$. Give your answers correct to three decimal places. 2

Section Five

27)! Simplify $\frac{2}{x^2-9} - \frac{1}{x-3} - \frac{4}{x+3}$. 2

28)!



XY is the diameter of the circle $XPYR$. XPQ and RYQ are straight lines. PR, XY and PY are joined. Given that $\angle PXY = 35°$ and $\angle PQY = 25°$, find the size of $\angle YPR$, giving reasons. 3

29)! By using the substitution $t = tan\frac{\theta}{2}$, or otherwise, show that $\frac{1-cos\,\theta}{sin\,\theta} = tan\frac{\theta}{2}$. 2

30)! It can be shown that $sin3\theta = 3sin\theta - 4sin^3\theta$ for all values of θ. (Do NOT prove this.)
Use this result to solve $sin3\theta + sin2\theta = sin\theta$ for $0 \le \theta \le 360$ 3

31)! Consider the function $f(x) = \frac{x}{4-x^2}$.

i. Find the domain of the function. 1
ii. Show that the function is an odd function. 1
iii. Show that the function is increasing throughout its domain. 1
iv. Sketch the graph of the function showing clearly the coordinates of any points of intersection with the x-axis or the y-axis and the equations of any asymptotes. 2

[[End Of Qns]]

Section one

1) C 2) A 3) C 4) D 5) B

6) E 7) E 8) D 9) E 10) E

Section Two

11) $a^2 - b^2 + 3a + 3b$

$= (a-b)(a+b) + 3(a+b)$ 1

$= (a+b)[a-b+3]$ 1

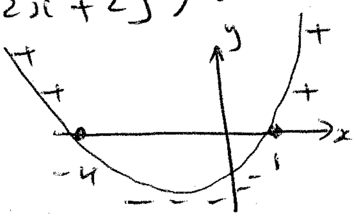
12) $\frac{3x+2}{x-1} > 2$

$(3x+2)(x-1) > 2(x-1)^2$

$(x-1)[(3x+2)-2(x-1)] > 0$

$(x-1)[3x+2-2x+2] > 0$

$(x-1)(x+4) > 0$ 1



$x < -4, \quad x > 1$ 1

13)

14) $A(-2,-5)$ $B(1,4)$, $x + 2y + 1 = 0$

$2y = -x - 1$

$y = -\frac{1}{2}x - \ldots$

$m_{AB} = \frac{4-(-5)}{1-(-2)} = \frac{9}{3} = 3$ 1

$\tan\theta = \left|\frac{3-(-\frac{1}{2})}{1+3\times-\frac{1}{2}}\right|$ 1

$= \left|\frac{3\frac{1}{2}}{-\frac{1}{2}}\right|$

$= 7$

$\theta = \cancel{1^\circ 26'}$ 1 $81^\circ 52'$

15) i) $\cos(2A+A) = \cos 2A\cos A - \sin 2A\sin A$

1 $= (2\cos^2 A - 1)\cos A - 2\sin A\cos A\cdot\sin A$

$= 2\cos^3 A - \cos A - 2\sin^2 A\cos A$

$= 2\cos^3 A - \cos A - 2(1-\cos^2 A)\cos A$

1 $= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

ii) $2\cos 3A = 2[4\cos^3 A - 3\cos A]$

1 $= 2\cdot\left[4\left(\frac{1}{2}\left(x+\frac{1}{x}\right)\right)^3 - 3\left(\frac{x}{2}+\frac{1}{2x}\right)\right]$

$= \left[\cancel{8}x\frac{1}{\cancel{8}}\left(x^2+2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right) - \frac{3x}{4} - \frac{3}{4x}\right]$

$= \left[x^3 + 2x + \frac{1}{x} + x + \frac{2}{x} + \frac{1}{x^3} - 3x - \frac{3}{x}\right]$

1 $= x^3 + \frac{1}{x^3} = RHS$

Section Three

16) $2\sin\theta + \cos\theta = 1$

$2\left(\frac{2t}{1+t^2}\right) + \left(\frac{1-t^2}{1+t^2}\right) = 1$

$\frac{4t}{1+t^2} + \frac{1-t^2}{1+t^2} = 1$

$4t + 1 - t^2 = 1 + t^2$

$2t^2 - 4t = 0$

$2t(t-2) = 0$

$t = 0$ oR $t = 2$

$\tan\frac{\theta}{2} = 0$

$\frac{\theta}{2} = 0, 180, 360$

$\theta = 0, 360$

$\tan\frac{\theta}{2} = 2$

$\frac{\theta}{2} = 1^\circ\ 6', 181^\circ\ 6'$

$\theta = 2^\circ\ 12'$

17) $A(\overset{x_1}{-5}, \overset{y_1}{12})$ $B(\overset{x_2}{4}, \overset{y_2}{9})$, $P(x, y)$

$5 : -2$

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

$= \frac{5(4) + -2(-5)}{5 + -2}$

$= \frac{30}{3}$

$x = 10$

$= \frac{5(9) + -2(12)}{5 + -2}$

$= \frac{21}{3}$

$= 7$

$P(10, 7)$, $Q(0, 2)$, $A(-5, 12)$

$AP = \sqrt{(-5-10)^2 + (12-7)^2} = 5\sqrt{10}$

$PQ = \sqrt{(0-10)^2 + (2-7)^2} = 5\sqrt{5}$

$AQ = \sqrt{(-5-0)^2 + (12-2)^2} = 5\sqrt{5}$

$AQ = PQ$ $\therefore \triangle APQ$ is an Isos.

$m_{AQ} = \frac{12-2}{-5-0} = -2$, $m_{PQ} = \frac{7-2}{10-0} = \frac{1}{2}$

$m_{AQ} \cdot m_{PQ} = -1$

$\therefore \angle AQP = 90^\circ$. $\therefore \triangle APQ$ is a right angled Isos. Triangle.

18) $\sin 2x = \cos x$

$\sin 2x - \cos x = 0$

$2\sin x \cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ or $\sin x = \frac{1}{2}$

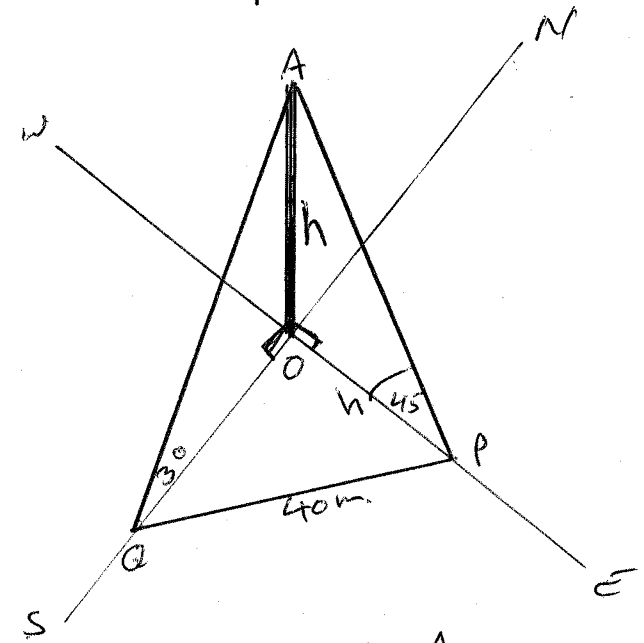
$x = 90^\circ, 270^\circ$ $\quad x = 30^\circ, 150^\circ$

19) $x^2 - 10x + 21 = 0$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{+10}{1}\right)^2 - 2 \times \frac{21}{1}$

$= 58$.



In $\triangle AOP$,

$\angle AOP = 90^\circ$

$\angle APO = 45^\circ$

$\therefore \angle OAP = 45^\circ$

$\therefore \triangle AOP$ is an Isos. $\triangle$

$\therefore OP = h$

In $\triangle QOA$

$\angle AQO = 30^\circ$

$\therefore \tan 30^\circ = \frac{h}{OQ}$

$OQ = \frac{h}{\tan 30} = \frac{h}{1/\sqrt{3}}$

$OQ = \sqrt{3}h$

In $\triangle QOP$, $\angle QOP = 90^\circ$

$\therefore QP^2 = QO^2 + PO^2$

$40^2 = (\sqrt{3}h)^2 + h^2$

$1600 = 3h^2 + h^2$

$4h^2 = 1600$

$h^2 = 400 \Rightarrow \boxed{h = 20m}$

21) $\tan^2 A - \tan^2 B = \dfrac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B}$

$L.H.S = \dfrac{\sin^2 A}{\cos^2 A} - \dfrac{\sin^2 B}{\cos^2 B}$

$= \dfrac{\sin^2 A\cos^2 B - \sin^2 B\cos^2 A}{\cos^2 A\cos^2 B}$ |

$= \dfrac{(1-\cos^2 A)\cos^2 B - (1-\cos^2 B)\cos^2 A}{\cos^2 A \cos^2 B}$

$= \dfrac{\cos^2 B - \cancel{\cos^2 A\cos^2 B} - \cos^2 A + \cancel{\cos^2 A\cos^2 B}}{\cos^2 A \cos^2 B}$

$= \dfrac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B} = R.H.S$ |

22) $x^2 + 2x - 4 + \dfrac{3}{x^2+2x} = 0$

$y - 4 + \dfrac{3}{y} = 0$

$y^2 - 4y + 3 = 0$

$(y-3)(y-1) = 0$ |

$y - 3 = 0$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$\boxed{x = -3,\ x = 1}$

$y - 1 = 0$

$x^2 + 2x - 1 = 0$

$x = \dfrac{-2 \pm \sqrt{4+4}}{2}$

$= \dfrac{-2 \pm 2\sqrt{2}}{2}$

$\boxed{x = -1 \pm \sqrt{2}}$ |

23) $y = \dfrac{x^2 + x}{x - 1}$

$y' = \dfrac{(2x+1)(x-1) - 1(x^2+x)}{(x-1)^2}$

$= \dfrac{2x^2 - 2x + \cancel{x} - 1 - x^2 - \cancel{x}}{(x-1)^2}$ |

$m_T = y' = \dfrac{x^2 - 2x - 1}{(x-1)^2}$ |

$\cancel{+}2 \times \dfrac{x^2-2x-1}{(x-1)^2} = \cancel{+}1$ |

$2x^2 - 4x - 2 = x^2 - 2x + 1$

$x^2 - 2x - 3 = 0$

$x = +3$ or $x = -1$ |

$x = \dfrac{\qquad}{2}$

$= \dfrac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$

$x =$

24) $y = x^2$

$\therefore \dfrac{dy}{dx} = 2x$ at $(1,1)$

$m_T = 2 \Rightarrow y - 1 = 2(x-1)$

$y = 2x - 1$

at B, $x = 0$

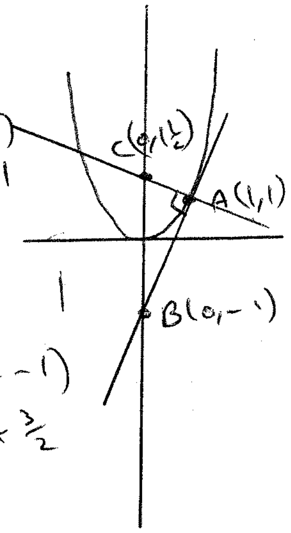
$y = -1 \Rightarrow B(0,-1)$ |

$m_N = -\frac{1}{2} \Rightarrow y - 1 = -\frac{1}{2}(x-1)$

$y = -\frac{1}{2}x + \frac{3}{2}$

at C, $x = 0 \Rightarrow y = 1\frac{1}{2}$

$C(0, 1\frac{1}{2})$ |



Area of $\triangle ABC = \frac{1}{2} \times AC \times AB$

$AB = \sqrt{(1-0)^2 + (1+1)^2} = \sqrt{5}$

$AC = \sqrt{(1-0)^2 + (1-1\frac{1}{2})^2} = \sqrt{5/4}$

Area $= \frac{1}{2} \times \frac{\sqrt{5}}{2} \times \sqrt{5} = \frac{5}{4}$ |

25) $3x + 2 \leq 7 - 2x \leq 8 - x$

$3x + 2 \leq 7 - 2x$

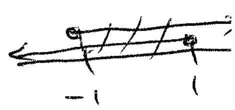
$5x \leq 5$

$x \leq 1$

$7 - 2x \leq 8 - x$

$-x \leq 1$

$x \geq -1$



$\therefore \boxed{-1 \leq x \leq 1}$

26) $8\cos x + 6\sin x = A\cos(x - \alpha)$

$8\cancel{\cos x} = A\cancel{\cos x}\cos\alpha$, $6\cancel{\sin x} = A\cancel{\sin x}\sin\alpha$

$100 = A^2(\cos^2\alpha + \sin^2\alpha)$

$A = 10$

$\dfrac{6}{8} = \dfrac{\cancel{A}\sin\alpha}{\cancel{A}\cos\alpha} \Rightarrow \tan\alpha = \frac{3}{4}$

$\alpha = \cancel{0.644} = 37^0$

$8\cos x + 6\sin x = 10\cos(x - \cancel{0.644}\,37)$ |

$5 = 10\cos(x - \cancel{0.644}\,37)$

$\cos(x - \cancel{0.644}\,37) = \frac{1}{2}$

$x - \cancel{0.644}\,37 = 60, 300$

$x = \cancel{97}, 97^0, 337^0$

2

Section Five

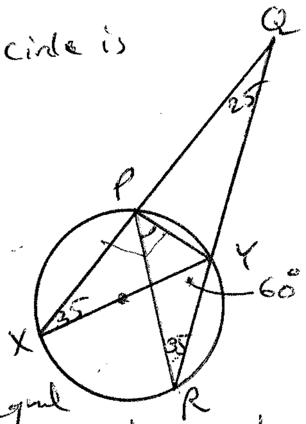
27) $\frac{2}{x^2-9} - \frac{1}{x-3} - \frac{4}{x+3}$

$= \frac{2}{(x-3)(x+3)} - \frac{1}{x-3} - \frac{4}{x+3}$

$= \frac{2 - 1(x+3) - 4(x-3)}{(x-3)(x+3)}$ |

$= \frac{2 - x - 3 - 4x + 12}{(x-3)(x+3)} = \frac{11 - 5x}{x^2-9}$ |

28) $\angle XPY = 90^\circ \rightarrow (1)$ (angle in a semi-circle is a right angle).



$\angle PRY = \angle PXY = 35^\circ$ (angles in the same segment are equal)

$\angle RYX = 35^\circ + 25^\circ$
$= 60^\circ$

Exterior angle of a $\triangle$ equal the sum of the interior opposite angles

$\angle XPR = \angle RYX = 60$ (angles in the same segment are equal) $\rightarrow (2)$

$\therefore \angle YPR = 90^\circ - 60^\circ$
$= 30^\circ$ (from 1 and 2)

29) $\frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$

L.H.S $= \frac{1 - \frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}} = \frac{1+t^2-1+t^2}{2t} = \frac{2t^2}{2t}$

$= t = \tan\frac{\theta}{2} =$ R.H.S |

30) $\sin 3\theta + \sin 2\theta = \sin\theta$

$3\sin\theta - 4\sin^3\theta + 2\sin\theta\cos\theta - \sin\theta = 0$

$2\sin\theta - 4\sin^3\theta + 2\sin\theta\cos\theta = 0$ |

$2\sin\theta[1 - 2\sin^2\theta + \cos\theta] = 0$

$\sin\theta = 0$ or

$\theta = 0, 180, 360^\circ$ |

$1 - 2(1-\cos^2\theta) + \cos\theta = 0$

$1 - 2 + 2\cos^2\theta + \cos\theta = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$ |

$\cos\theta = \frac{1}{2}$, $\cos\theta = -1$

$\theta = 60, 300^\circ$, $\theta = 180^\circ$

31) $f(x) = \frac{x}{4-x^2}$ |

i) Domain all real x and $x \neq \pm 2$

ii)
$f(-x) = \frac{-x}{4-(-x)^2} = \frac{-x}{4-x^2}$

$-f(x) = -\frac{x}{4-x^2}$

$\therefore f(-x) = -f(x) \therefore$ Odd |

iii) $f(x) = \frac{x}{4-x^2}$

$f'(x) = \frac{1(4-x^2) - (-2x)x}{(4-x^2)^2}$

$= \frac{4 - x^2 + 2x^2}{(4-x^2)^2}$

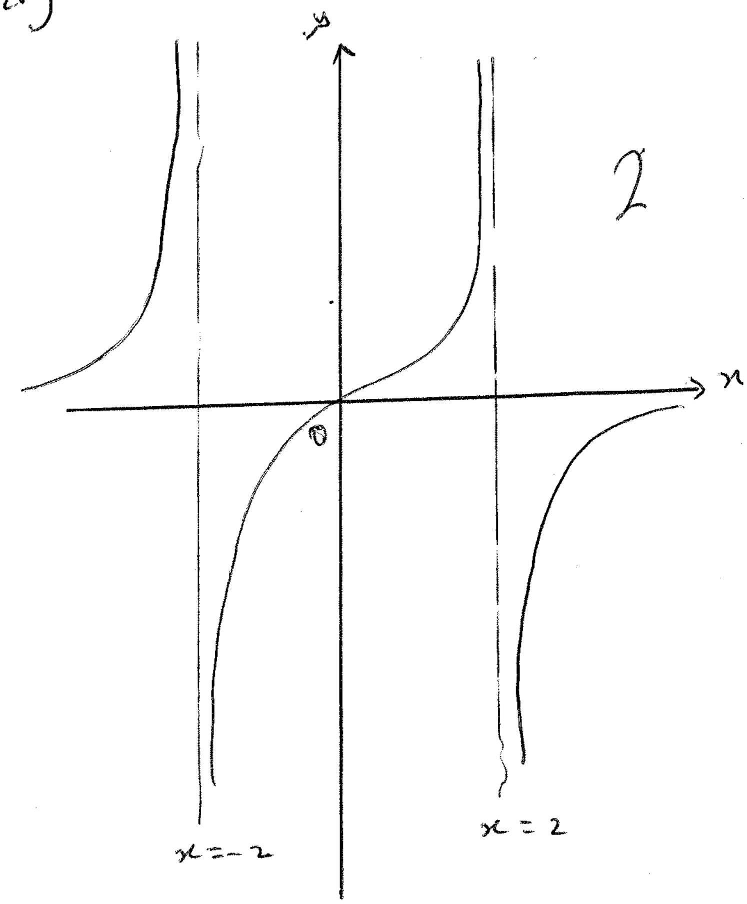
$= \frac{4+x^2}{(4-x^2)^2}$

Since $4 + x^2 > 0$ for all x-values and $(4-x^2)^2 > 0$ for the domain in (i). |

$\therefore f'(x) > 0$ throughout the domain.

$\therefore f(x)$ is an increasing function throughout its domain.

iv)



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