Solutions Yr 11 Maths Assess.

Qu. 1

(a) $y = -x$ $\qquad \sqrt{3}y = x$

$\therefore m = -1$ $\qquad y = \frac{x}{\sqrt{3}}$

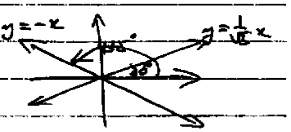
$\qquad \therefore m = \frac{1}{\sqrt{3}}$

$\therefore \alpha = 135^\circ$ $\qquad \therefore \beta = 30^\circ$

$\therefore$ Acute angle $= 180^\circ - (135^\circ - 30^\circ)$

$= 180^\circ - 105^\circ$

$= 75^\circ$.



(b) $\int \frac{dx}{\sqrt{1-9x^2}}$

$= \frac{1}{3}\sin^{-1}(3x) + C$

(c) $2x^3 + x^2 - x - 2 = 0$

(i) $\alpha + \beta + \lambda = -\frac{1}{2}$

(ii) $\alpha\beta\lambda = \frac{2}{2} = 1$

(iii) $\alpha\beta + \alpha\lambda + \beta\lambda = -\frac{1}{2}$

(iv) $(\alpha-1)(\beta-1)(\lambda-1)$

$= (\alpha\beta - \alpha - \beta + 1)(\lambda - 1)$

$= \alpha\beta\lambda - \alpha\lambda - \beta\lambda + \lambda - \alpha\beta + \alpha + \beta - 1$

$= \alpha\beta\lambda - (\alpha\lambda + \beta\lambda + \alpha\beta) + (\alpha + \beta + \lambda) - 1$

$= 1 + \frac{1}{2} + \left(-\frac{1}{2}\right) - 1$

$= 0$

Qu. 1

(d) (i) $x = 12t \implies t = \frac{x}{12}$

$y = 6t^2$

i.e. $y = 6 \cdot \frac{x^2}{144}$

$\therefore x^2 = 24y$

(ii) $x^2 = 24y$

i.e. $x^2 = 4(6)y$

$\therefore$ Focus is $(0, 6)$

(iii) Eqn. of directrix is $y = -6$.

Qu. 2

a) $\sin\theta - \sqrt{3}\cos\theta = 1 \quad , \quad 0 \le \theta \le 2\pi$

$2\sin\left(\theta - \frac{\pi}{3}\right) = 1$

$\sin\left(\theta - \frac{\pi}{3}\right) = \frac{1}{2}$

$\theta - \frac{\pi}{3} = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$\therefore \theta = \frac{\pi}{2}$ or $\frac{7\pi}{6}$

$r = \sqrt{1^2 + (\sqrt{3})^2}$

$r = 2$

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$

$0 \le \theta \le 2\pi$

$-\frac{\pi}{3} \le \theta - \frac{\pi}{3} \le 2\pi - \frac{\pi}{3}$

(b) (i) $\sin 2x = \sin x \quad , \quad 0 \le x \le \pi$

$\sin 2x - \sin x = 0$

$2\sin x\cos x - \sin x = 0$

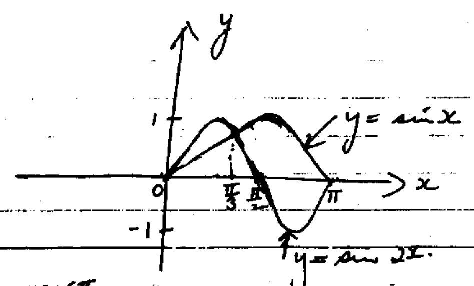
$\sin x\,(2\cos x - 1) = 0$

$\therefore \sin x = 0$ or $\cos x = \frac{1}{2}$

$\therefore x = 0$ or π or $x = \frac{\pi}{3}$

$\therefore x = 0, \frac{\pi}{3}$ or π.

(ii)



(iii) Area $= \int_0^{\frac{\pi}{3}} (\sin 2x - \sin x)\,dx$

$= \left[-\frac{\cos 2x}{2} + \cos x\right]_0^{\frac{\pi}{3}}$

$= \left(-\frac{\left(-\frac{1}{2}\right)}{2} + \frac{1}{2}\right) - \left(-\frac{1}{2} + 1\right) = \frac{1}{4}$ unit^2

Q3 a) $6^2 = x(5+x)$ (square of tangent = product of intercepts)

$x^2 + 5x - 36 = 0$

$x = 4$ or -9

but $x > 0$ (or x is a length)

$\therefore x = 4$

b) $\hat{KNL} = \hat{JML}$ (ext. < = int. opp in cyclic quad)

$\hat{NKL} = \hat{JML}$ (opp <'s of parallelogram)

$\therefore \triangle NKL$ is isosceles ($\hat{NKL} = \hat{KNL}$ base angles equal)

$\therefore NL = KL$ (sides opp equal angles of $\triangle$)

c) $z = \hat{CBE}$ (< between chord and tangent = < in alternate segment)

$= 50$

$\hat{ABE} = 90°$ (Tangent ⊥ to radius)

$\therefore y + 50° = 90°$

$y = 40°$

$x = 2 \times \hat{FCB}$ (< at centre = twice < at circumference)

$= 2 \times 35$

$= 70°$

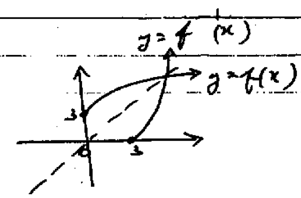
Common errors : incomplete or missing reasons
: false statements, especially in part (a).

Qu. 4

(a) $\tan\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = n\pi + \tan^{-1}\frac{1}{\sqrt{2}}$

(b) $f(x) = \sqrt{x} + 3$



(i) $y = \sqrt{x} + 3$

$y - 3 = \sqrt{x}$

$x = (y-3)^2$

$\therefore f^{-1}(x) = (x-3)^2, \; x \geq 3, \; y \geq 0$

(ii) Domain: $x \geq 3$

(c) (i) $y = \tan^{-1}\frac{1}{x}$, $x \neq 0$

$y' = \dfrac{-x^{-2}}{1+\left(\frac{1}{x}\right)^2}$

$= \dfrac{-\frac{1}{x^2}}{\frac{x^2+1}{x^2}}$

$= \dfrac{-1}{x^2+1}$

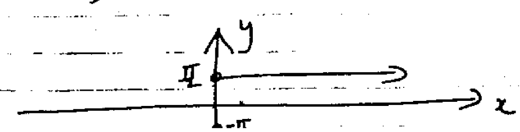
2

(ii) $\frac{d}{dx}\left[\tan^{-1}x + \tan^{-1}\frac{1}{x}\right]$

$= \frac{1}{1+x^2} + \left(-\frac{1}{1+x^2}\right)$

$= 0.$

(iii)



Qu-5.

(a) (i) $f(x)$ has a zero between 1 & 2.

$f'(x) = 3x^2 - 2x - 1$

then $x = 2 - \dfrac{1}{3(4) - 2(2) - 1}$

$= 2 - \frac{1}{7}$

$= 1\frac{6}{7}$

(ii) take $x = 1$ as 1^{st} approx^n

then $x = 1 - \dfrac{-2}{3(1)^2 - 2(1) - 1}$

$= 1 - \frac{2}{3-2-1}$

$= 1 - \frac{2}{0}$

Since $f'(1) = 0$, $f(x)$ has a stat. pt. at $x = 1$. Hence it is unsuitable as a first approx^n.

* and therefore tangent at $x = 1$ is parallel to x axis

Qu-5.

(iv) $f(x) = ax^3 + bx^2 + cx + d$

$f(1) = a + b + c + d = 0$ -- (1)

$f(-1) = -a + b - c + d = -4$ -- (2)

$f'(x) = 3ax^2 + 2bx + c$

$f'(-1) = 3a - 2b + c = 0$ -- (3)

$f'(1) = 3a + 2b + c = 0$ -- (4)

(3)-(4) $\Rightarrow -4b = 0$

$\therefore b = 0$

Subst for b in (1), (2) & (3)

(1) $\Rightarrow a + c + d = 0$ -- (5)

(2) $\Rightarrow -a - c + d = -4$ -- (6)

(3) $\Rightarrow 3a + c = 0$ -- (7)

(5)+(6) $\Rightarrow 2d = -4$

$d = -2$

Subst. for d in (5)

$a + c - 2 = 0$

$a = 2 - c$ -- (8)

Subst (8) into (7)

$3(2-c) + c = 0$

$6 - 3c + c = 0$

$6 - 2c = 0$

$2c = 6$

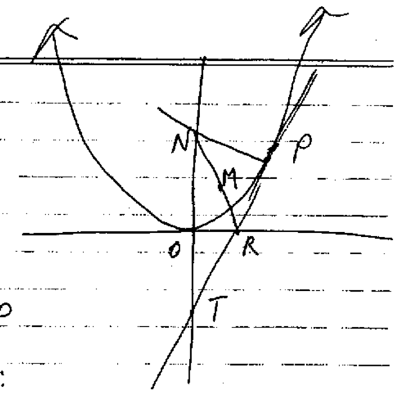
$c = 3$

$a + c - 2 = 0$

$a + 3 - 2 = 0$

$\therefore a = -1,\ b = 0,\ c = 3$ and $d = -2$

Qu-6



$x^2 = 4ay$.

(i) $y = \frac{1}{4a}x^2$

$y' = \frac{1}{2a}x$

When $x = 2ap$, $y' = p$

Eqn of tangent at P is:

$p = \frac{y - ap^2}{x - 2ap}$

$px - 2ap^2 = y - ap^2$

$\therefore y = px - ap^2$

(ii) When $x = 0$

$y = 0 - ap^2 = -ap^2$

$\therefore T = (0, -ap^2)$

(iii) $m(\text{normal}) = -\frac{1}{p}$

Eqn of normal at P is:

$-\frac{1}{p} = \frac{y - ap^2}{x - 2ap}$

$-x + 2ap = py - ap^3$

$x + py - 2ap - ap^3 = 0$

(iv) When $x = 0$

$py = 2ap + ap^3$

$y = 2a + ap^2$

$\therefore N = (0, 2a + ap^2)$.

[redacted]

(Contd)

$\angle NPT = 90°$ (~~angle between~~ tangent + normal is $90°$)

angle in a semicircle is $90°$

$\therefore$ NT is a diameter of the circle passing thro PTN

Centre of this circle $= \left(0, \dfrac{2a + ap^2 - ap^2}{2}\right)$

$= (0, a)$

Radius $= \dfrac{2a + ap^2 + ap^2}{2} = a + ap^2$

Eqn of circle is:

$x^2 + (y-a)^2 = (a + ap^2)^2$

Eqn. of tangent at P is: $y = px - ap^2$

when $y = 0$, $px - ap^2 = 0$

$px = ap^2$

$x = ap$

$R = (ap, 0)$

$M = \left(\dfrac{0 + ap}{2}, \dfrac{2a + ap^2 + 0}{2}\right)$

$= \left(\dfrac{ap}{2}, \dfrac{2a + ap^2}{2}\right)$

$\begin{cases} x = \frac{1}{2}ap \\ y = a + \frac{1}{2}ap \end{cases}$

Sub. $p = \dfrac{2x}{a}$ into y

$= a + \frac{1}{2}a\left(\dfrac{2x}{a}\right)^2$

$= a + \frac{1}{2}a\left(\dfrac{4x^2}{a^2}\right)$

$= a + \dfrac{2x^2}{a}$

$2x^2 = a(y-a)$

$x^2 = \frac{a}{2}(y-a)$

$\therefore$ locus of M is a parabola with vertex at $(0, a)$ + a focal length at $\frac{1}{8}a$