



Year 12 (2001/2002) Extension 1 Mathematics
H.S.C. Assessment Task 1
December 2001

Time Allowed : 1 period
Show All Necessary Working

BUNDLING INSTRUCTIONS:

Hand in your solutions in TWO BUNDLES clearly labelled PART A (questions 1 to 3) and PART B (questions 4 to 6).

Ensure your name is on each bundle.

PART A

Question 1: (9 marks)

- a) Solve $(x^2 - 3x)^2 - 3(x^2 - 3x) - 4 = 0$, for all real values of x (leave answers in simplest surd form, where appropriate)
- b) If α and β are the roots of $2x^2 - 6x + 7 = 0$, write down the value of
- (i) $\alpha + \beta$ (ii) $\alpha\beta$
- (iii) By factorising or otherwise, evaluate $\alpha^3 + \beta^3$

Question 2: (10 marks)

- a) Find all x such that $|x + 1| + x = |x - 1|$
- b) By Mathematical Induction prove
 $1 \times 2^0 + 2 \times 2^1 + 3 \times 2^2 + \dots + n \times 2^{n-1} = 1 + (n-1)2^n$ for all positive integer values of n .
- c) Using Simpson's Rule with 3 function values (i.e. once), find an approximation to the area bounded by the curve $y = \frac{10x}{x^2 + 1}$, the x axis and the lines $x = 1$ and $x = 3$.

Question 3: (14 marks)

- a) (i) If $y = x^2 \sqrt{x^2 + 5}$, show that $\frac{dy}{dx} = \frac{3x^3 + 10x}{\sqrt{x^2 + 5}}$
- (ii) The graph of $y = x^2 \sqrt{x^2 + 5}$ has 1 stationary point at $x = 0$. Determine the NATURE of this stationary point (i.e. maximum, minimum or inflexion).
- b) Find the coordinates of the point P which divides the interval AB internally in the ratio of 2:3 where A and B have coordinates (1,-3) and (6,7) respectively.
- c) Solve the inequality $\frac{x^2 - 4}{x} > 0$
- d) Show the area of the shaded segment subtended by an angle of 30° , in a circle of radius 6 cm equals $(3\pi - 9\sqrt{3}) \text{ cm}^2$

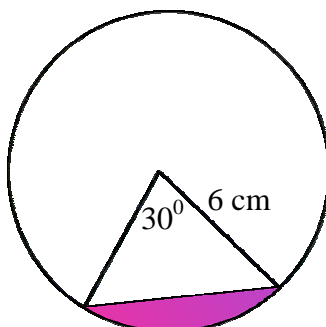


Figure NOT to scale

PART B

Question 4: (5 marks)

Find the exact values of

a) $\sin\left(\frac{\pi}{3}\right)$

b) $\cos\left(\frac{3\pi}{4}\right)$

c) $\cot\left(\frac{11\pi}{6}\right)$

Question 5: (12 marks)

a) Write down primitives (indefinite integrals of)

(i) $8x^3 + 6x^2 - 9$

(ii) $\sqrt[3]{x}$

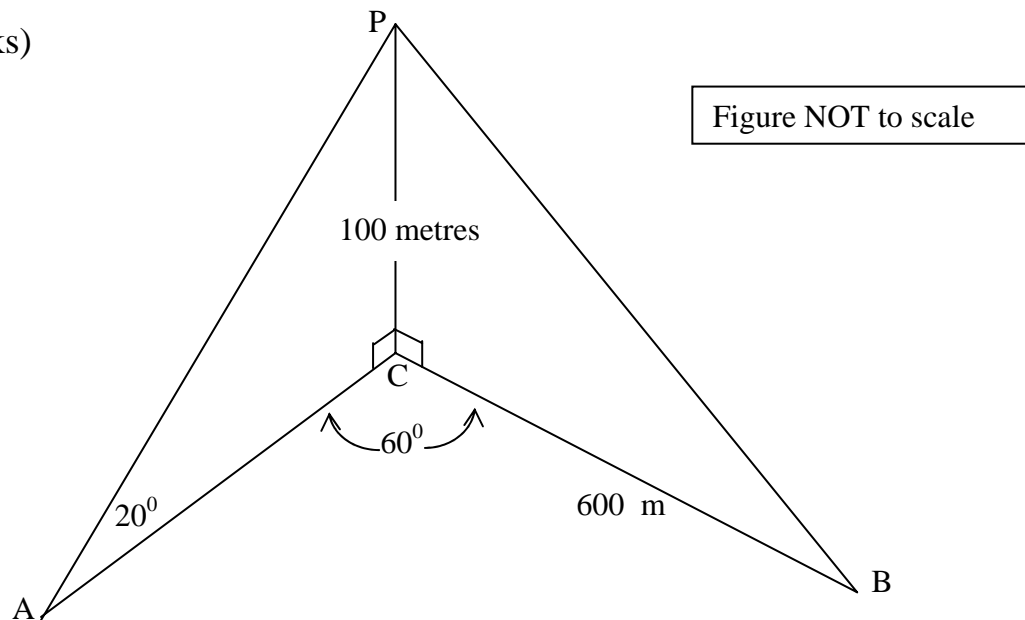
(iii) $\frac{4}{x^2}$

b) Sketch the curve $y = x - x^2$, clearly showing x intercepts. Hence, find the exact area enclosed by the curve, the x axis, $x = 0$ and $x = 2$.

c) The area enclosed by $y = x - x^2$, the x axis, $x = 0$ to $x = 2$ is rotated around the x axis. Find the volume of the solid formed.

Question 6: (14 marks)

a)



Two yachts A and B subtend an angle of 60° at the base of a cliff. From yacht A, the angle of elevation of the point P, 100 metres vertically above C, is 20° . yacht B is 600 metres from C.

- Calculate the length AC
- Calculate the distance between the two yacht.

b) The points $P(2p, p^2)$ and $Q(2q, q^2)$ are two points on the parabola $x^2 = 4y$.

- Show the equation of the tangent at P is given by $y = px - p^2$.
- Find the point of intersection, T, of the tangents at P and Q.
- Given that $p - q = 1$, show the Cartesian equation of the locus of T is $x^2 = 4y + 1$