JRAHS Ext 1 T2 2007

QUESTION 1 (9 Marks) **Marks**

(a) (i) How many ways could the letters of the word SOCIETY be arranged if each arrangement begins with C and ends with E. **1**

(ii) If an arrangement is selected at random, find the probability that it contains the word SOY. **1**

(b) The displacement function of a particle moving x metres along a straight line after t seconds is given by $x = \sqrt{2}\cos 5t - \sin 5t$. Show that its acceleration function is of the form $\ddot{x} = -n^2 x$ and find the value of n. **2**

(c) A plane travelling at a constant height of 1500 metres at a speed of 600 *km/hr* releases a bomb. What is the horizontal distance the bomb has travelled when it hits the ground. (Take g=10 m/s^2). **3**

(d) (i) How many different ways could four cards be selected from a regular pack of 52 playing cards. **1**

(ii) How many of these selections will contain exactly two Aces. **1**

QUESTION 2 (9 Marks)

(a) A sky-diver opens his parachute when falling at 30 *m/s*. Thereafter, his acceleration is given by $\frac{dv}{dt} = k(6 - v)$, where k is a constant.

(i) Show that this condition is satisfied when $v = 6 + Ae^{-kt}$, and find the value of A. **2**

(ii) One second after opening his parachute, his velocity has fallen to 10.7 *m/s*. Find k to two decimal places. **2**

(iii) Find, to one decimal place, his velocity two seconds after his parachute has opened. **2**

(iv) If, with the same acceleration, the sky-diver opens his parachute when falling at 6 *m/s*, briefly describe his subsequent motion. **1**

(b) Persons A,B,C,D,E,F and G are to be seated at a round table. How many arrangements are possible if A refuses to sit next to B or C. **2**

QUESTION 3 (9 Marks)

(a) A particle moves in Simple Harmonic Motion. When it is 2 metres and 3 metres respectively from its centre of motion, its velocity is respectively 6 *m/s* and 4 *m/s*. Find the period of its motion and its amplitude. **3**

(b) A function *N(t)* is given by $N(t) = Ae^{\frac{t}{3}} + Be^{\frac{-2t}{3}}$, where A and B are constants.

(i) If *N(0)* = 30 and $N'(0) = -14$, find A and B. **2**

(ii) Find, to 2 decimal places, the value of t for which *N(t)* is a minimum, and find this minimum value. **3**

(iii) Briefly describe the behaviour of *N(t)* as t increases. **1**

QUESTION 4 (9 Marks)

(a) How many arrangements of the letters of the word CONTAINER are possible if:

(i) there are no restrictions. **1**

(ii) the vowels are together. **1**

(b) In a herd of 500 cows, the number N infected with a disease at time t years is given by $N = \dfrac{500}{1 + Ae^{-500t}}$.

(i) Briefly explain why all the cows will eventually be infected. **1**

(ii) Initially, only one cow was infected. After how many days will 200 cows be infected. **3**

(iii) Show that $\dfrac{dN}{dt} = N(500 - N)$. **3**

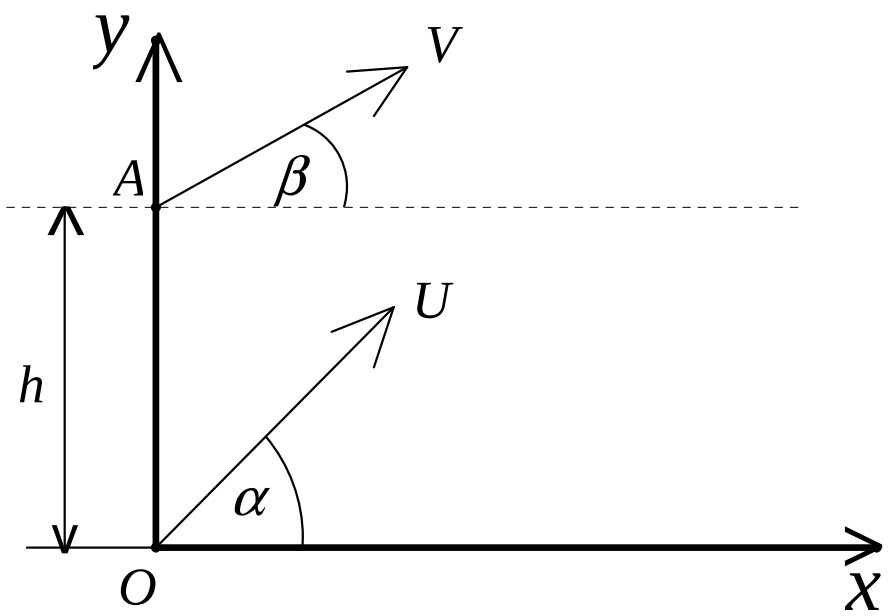
QUESTION 5 (9 Marks)

(a) The equation of motion of a particle moving in Simple Harmonic Motion is given by $x = a\cos(nt + \alpha)$, where x metres is its displacement from origin 0 after t minutes. It is initially 6 metres right of 0 and moving towards it. The period of its motion is 8 minutes and its maximum speed is 3π m/min. Find:

(i) the values of n, a and α. **3**

(ii) the first time when it passes through the origin. **1**

(b) In the diagram below, a particle is projected from the origin 0 with a speed of U m/s at an angle of elevation α .At the same instant, another particle is projected from A, h metres above 0 with a speed of V m/s at an angle of elevation β $(\beta < \alpha)$. The particles move in the same plane of motion and collide T seconds after projection. **5**



The horizontal and vertical components of displacement t seconds after the particle is projected from 0 are given by $x_o = Ut\cos\alpha$ and $y_o = Ut\sin\alpha - \frac{1}{2}gt^2$ respectively, and the horizontal and vertical components of displacement t seconds after the particle is projected from A are given by $x_A = Vt\cos\beta$ and $y_A = h + Vt\sin\beta - \frac{1}{2}gt^2$ respectively.

Show that $T = \dfrac{h\cos\beta}{U\sin(\alpha - \beta)}$.

QUESTION 6 (9 Marks)

(a) The displacement function of a particle moving x metres along a straight line after t seconds is given by $x = 3\cos^2 4t$. Show that its motion is Simple Harmonic and find its centre of motion. **3**

(b) The acceleration of a particle moving along a straight line is given by $\ddot{x} = 3x(x-2)$, where x metres is its displacement from the origin 0 after t seconds. Initially it is at 0 and its velocity is 2 *m/s* .

(i) Show that $v = 2(x^3 - 3x^2 + 2)$, where v is its velocity. **2**

(ii) Find its velocity and acceleration at $x = 1$. **2**

(iii) Briefly describe its motion after it moves from $x = 1$. **2**

QUESTION 7 (9 Marks)

(a) A velocity function is given by $\frac{dx}{dt} = (4-3x)^2$. Find $\frac{d^2x}{dt^2}$. **2**

(b) A team of FIVE is to be selected from a group of FOUR boys and FOUR girls.

(i) How many teams are possible if there is to be a majority of girls. **1**

(ii) What is the probability of a particular girl being included in the team and a particular boy not included, still assuming a majority of girls in the team. **2**

(c) On a certain day, the depth of water in a harbour at high tide is 11metres. **4**
At low tide $6\frac{1}{4}$ hours later, the depth of water is 7 metres. If high tide is due at 2.50 AM, what is the earliest time after midday that a ship requiring a depth of at least 10 metres of water can enter the harbour.

END of PAPER