Name:____________________ Teacher:______________

ST MARK'S COPTIC ORTHODOX COLLEGE

Mathematics Department

2009

Year 11 Extension 1

Semester One Examination

GENERAL INSTRUCTION

- Reading time 5 minutes
- Working Time – 2 hours
- Write in black or blue pen only
- Approved calculators may be used
- All necessary working must be shown
- Begin each question on a different booklet
- Attempt all questions
- All question are of equal value

Section	1	2	3	4	5	6	7	Total
Mark								/84

"He is gracious and full of compassion and righteous" Ps 112:4

Question 1 (12 marks) *Start work on a new page* **Mark**

a) Factorise, then simplify $\frac{9-x}{81-x^2}$ **2**

b) Simplify $\frac{\sqrt{5}-1}{\sqrt{5}+1} + \frac{\sqrt{5}+1}{\sqrt{5}-1}$ **3**

c) Solve for x: $|2x-1| < 3$ **2**

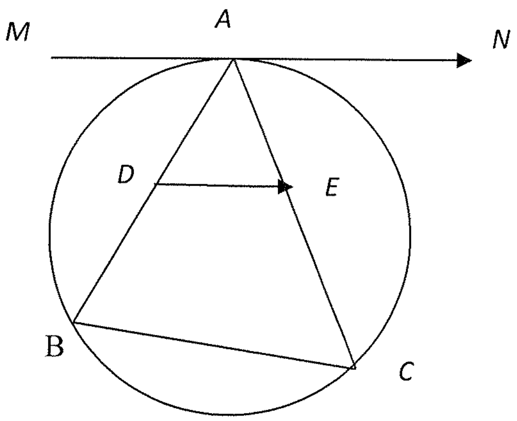
d) Solve for x: $\frac{5}{2x-1} < 1$ **3**

e) Write $\frac{1+\sqrt{7}}{3-\sqrt{7}}$ in the form $a + b\sqrt{7}$, where a and b are rational. **2**

Question 2 (12 marks) *Start work on a new page*

a) A (-2, 5) and B (1, 2) are two fixed points. Find the coordinates of the point P which divides AB externally in the ratio 3:2. **2**

b) ABC is a triangle inscribed in a circle. MAN is the tangent to the circle at A. Points D, E lie on AB, AC respectively, so that DE is parallel to MAN.



i) Explain why angles MAB and ACB are equal **1**

ii) Hence show that $BCED$ is a cyclic quadrilateral **3**

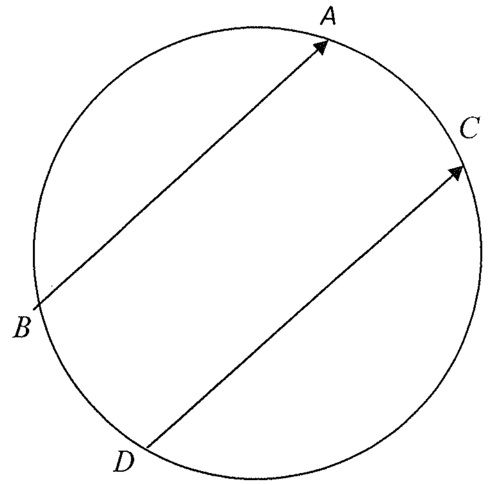
c) Sketch the graph $y = |x-3|$ **2**

d) Solve the equation $2x - 9 = \frac{-9}{x}$ **2**

e) On the same set of axes, sketch the graph of $y = 2x - 9$ and $y = \frac{-9}{x}$ **2**

Question 3 (12 marks) *Start work on a new page* **Marks**

a)



A, B, C and *D* are four points on a circle such that *AB* is parallel to *CD.* Prove that AD = BC **4**

Hint: Let E be the point of intersection of AD and BC.

b) Find the acute angle between the lines $3x - y = 4$ and $2x + 3y = 4$. Write your answer to the nearest minute. **2**

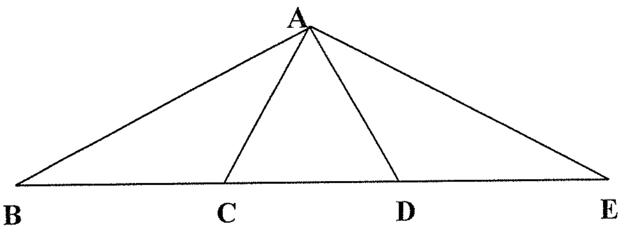
c) Find $\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$ **2**

d) Prove $\frac{\tan x - \tan y}{\tan x + \tan y} = \frac{\sin(x-y)}{\sin(x+y)}$ **4**

Marks

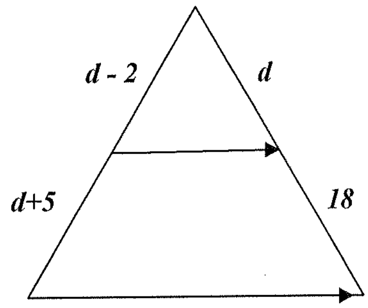
Question 4 (12 marks) *Start work on a new page*

a)

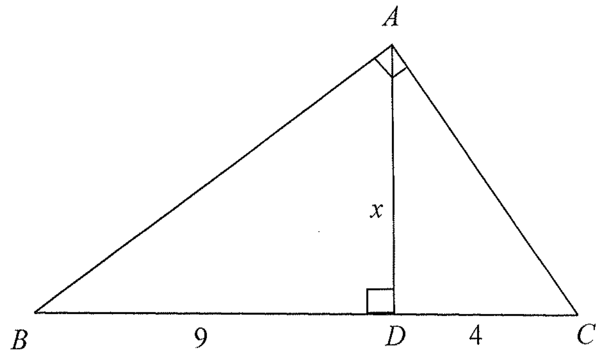


$AC = AD = BC = DE$. Prove $AB = AE$ **4**

b) Find the value of the pronumeral giving reasons **4**



c) Find the value of the pronumeral, giving reasons. *Hint show* ΔABD *is Similar to* ΔADC. **4**



Marks

Question 5 (12 marks) *Start work on a new page*

a) A student lies down on the ground and views the top of a church tower at an angle of elevation of 40^0. If the student is 50m from the foot of the tower, which is on the same level with the student, how high is the tower to 2 decimal places? Draw a neat diagram **3**

b) From a sailboat the window of a light house is seen at an angle of elevation of 40^0. After moving towards the lighthouse a distance of 50m, the angle of elevation is found to be 43^0. How far off is the sailboat from the lighthouse to the nearest metre? **4**

c) From a ship that is running due north the lighthouse is seen at the bearing of $N30^0E$, and after 2km of sailing the lighthouse is seen at $N48^0E$.

(i) Draw a neat diagram, illustrating the above information **2**

(ii) Calculate the distance from the ship to the lighthouse to 2 decimal places. **3**

Marks

Question 6 (12 marks) *Start work on a new page*

a) Sketch $f(x) = \frac{1}{x^2-1}$ Show all essential features. **3**

b) If θ is an acute angle and $tan\theta = \alpha$, express $\cos\theta$, $\sin\theta$ in terms of α **2**

c) Solve $4sin^2x = 1, 0° \leq x \leq 360°$ **2**

d) Solve $3cos^2x = 8sinx, 0 \leq x \leq 360$, giving your answer to the nearest minute **3**

e) If $sin\alpha = \frac{1}{2}$, find the exact value of $\cos 2\alpha$ **2**

Marks

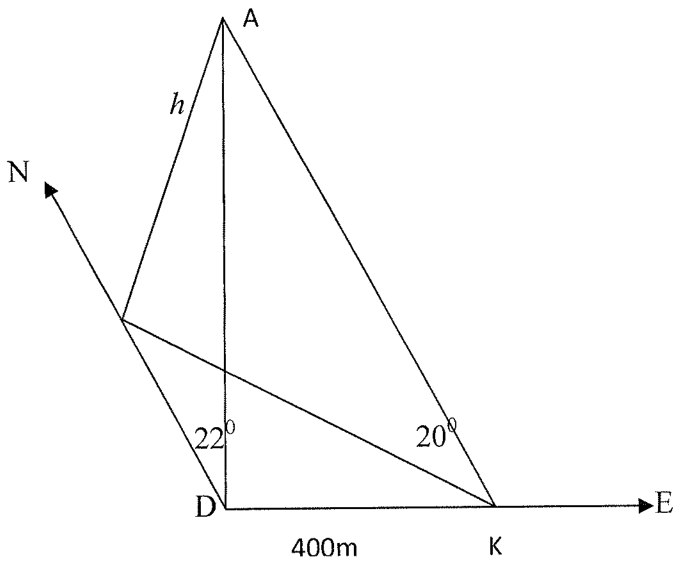
Question 7 (12 marks) *Start work on a new page*

a) (i) Write the expansion for cos $(\propto + \theta)$ **1**

(ii) Hence or otherwise prove that cos $3\theta = 4cos^3\theta - 3cos\theta$ **3**

(iii) Solve $8cos^3\theta - 6cos\theta - \sqrt{3} = 0$ for $0° \leq \theta \leq 360°$ **3**

b) Donna is standing at D and observes the angle of elevation of the tip of a flagpole A, on top of a building to be 22^0. Her friend Kate, who is standing at K, 400 metres due east of Donna, finds the angle of elevation of the tip of the flagpole to be 20^0. The building is due north of Donna and B is the base of the building. The points B, D and K are all on level ground.



(i) Show that the height h, of the flagpole above the ground is given by **3**

$$h = \frac{400}{\sqrt{(cot^2 20 - cot^2 22)}}$$

(ii) Find the value of h, correct to 3 significant figures. **2**

"He is gracious and full of compassion and righteous" Ps 112:4

END OF EXAM

equiangular ∴ similar & not congruent.

Question 1

Q1 a) $\frac{9-x}{81-x^2}$

$= \frac{9-x}{(9-x)(9+x)}$

$= \frac{1}{9+x}$ 2M

$\frac{\sqrt5-1}{\sqrt5+1} + \frac{\sqrt5+1}{\sqrt5-1}$

$\frac{(\sqrt5-1)^2 + (\sqrt5+1)^2}{(\sqrt5+1)(\sqrt5-1)}$

$\frac{5-2\sqrt5+1+5+2\sqrt5+1}{5-1}$

$\frac{10+2}{4}$

$\frac{12}{4}$

3 3M

$|2x-1| < 3$

$2x-1<3 \quad 2x-1>-3$

$+1 \quad +1 \quad\quad +1 \quad +1$

$\frac{2x}{2} < \frac{4}{2} \quad\quad \frac{2x}{2} > \frac{-2}{2}$

$x<2 \quad\quad x>-1$

$\therefore -1<x<2.$ 2M

½ for solving w/o inequality.

d) $\frac{5}{2x-1} < 1 \quad \times(2x-1)^2$

$5(2x-1)^2 < (2x-1)^2$

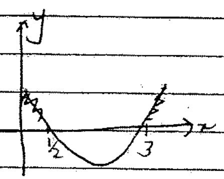
$5(2x-1) < (2x-1)^2$

$(2x-1)^2 - 5(2x-1) > 0$

$(2x-1)(2x-1-5) > 0$

$(2x-1)(2x-6) > 0$

$2(2x-1)(x-3) > 0$



$x<½ \quad x>3.$ 3M

e) $\frac{1+\sqrt7}{3-\sqrt7} \times \frac{3+\sqrt7}{3+\sqrt7}$ ½

$= \frac{(1+\sqrt7)(3+\sqrt7)}{9-7}$ ½

$= \frac{3+\sqrt7+3\sqrt7+7}{2}$ ½

$= \frac{10+4\sqrt7}{2}$

$= 5+2\sqrt7$ ½

$\therefore a=5 \quad b=2.$ 2M

Question 2

a) $\left(\frac{kx_2+lx_1}{k+l}, \frac{ky_2+ly_1}{k+l}\right)$

$\therefore A(-2,5) \quad B(1,2)$

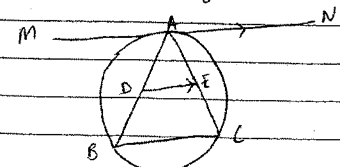
$3 \quad : \quad -2$

$\left(\frac{3\times1+-2\times-2}{3-2}, \frac{3\times2+-2\times5}{3-2}\right)$

$\left(\frac{3+4}{1}, \frac{6-10}{1}\right)$

$(7, -4)$ 2M

b) The angle between a tangent and chord drawn to the point of contact is equal to the angle subtended by the chord in the alternate segment. 1M



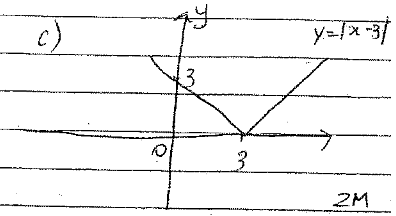
ii) $\angle ADE = \angle MAB$ (alternate ∠'s)

$\angle ADE = \angle ECB$ (both equal to $\angle MAB$)

∴ BCED is a cyclic quadrilateral

because exterior angle of cyclic quadrilateral is equal to the opposite interior angle) 3M

c)



2M

d) solve $2x-9 = \frac{-9}{x}$

$2x^2-9x=-9$

$2x^2-9x+9=0$

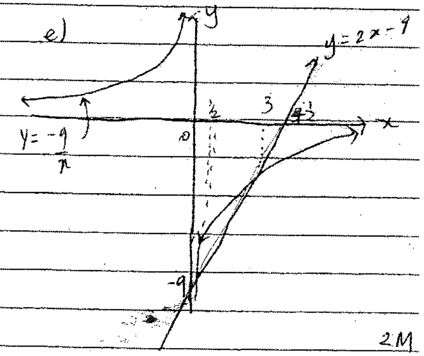
$2x \quad -3$

$x \quad -3$

$(2x-3)(x-3)=0$

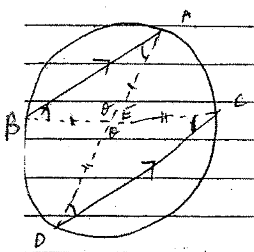
$x=\frac32 \quad x=3$ 2M

e)



2M

uestion 3



Prove $AD = BC$

$\angle BAD = \angle EDC$ (alternate $\angle$'s $AB \parallel CD$)

$\angle AEB = \angle DEC$ (vertically opposite)

$\angle BAD = \angle BCD$ (angles subtended at the circumference by the same arc are equal)

ewise

$\angle ABC = \angle ADC$ (" " " ")

$\therefore \angle BAE = \angle ABE$ (both equal to $\angle BCD$)

$\therefore \triangle ABE$ is isosceles

then $AE = BE$ (equal sides of isosceles $\triangle$)

wise

$ED = EC$ (since $\angle EDC = \angle ECD$ & $\triangle EDC$ is isosceles)

so $AE + ED = BE + EC$ (sum of equal sides are equal)

4M.

d)

$$\frac{\tan x - \tan y}{\tan x + \tan y} = \frac{\sin(x-y)}{\sin(x+y)}$$

$$LHS = \frac{\frac{\sin x}{\cos x} - \frac{\sin y}{\cos y}}{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}$$

$$= \frac{\frac{\sin x \cos y - \sin y \cos x}{\cos x \cos y}}{\frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y}}$$

$$= \frac{\sin x \cos y - \sin y \cos x}{\sin x \cos y + \sin y \cos x}$$

$$= \frac{\sin(x-y)}{\sin(x+y)}$$

$= RHS$

4M

1 mark if expanded

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$ where $m_1 = 3$ since $3x - y = 4$

then $3x - 4 = y$.

& $m_2 = -\frac{2}{3}$, since $2x + 3y = 4$

$3y = 4 - 2x$

$y = \frac{4}{3} - \frac{2x}{3}$

$$\tan\theta = \left|\frac{3 + \frac{2}{3}}{1 + 3 \times -\frac{2}{3}}\right|$$

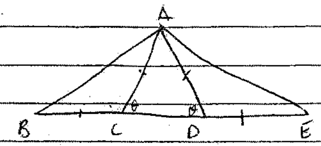
$$= \frac{11}{3}$$

$\theta = 74^\circ 45'$

$$\lim_{x \to 3} \frac{x^3 - 27}{x - 3} \quad \lim_{x \to 3} \frac{(\cancel{x-3})(x^2 + 3x + 9)}{\cancel{x-3}}$$

$= \lim_{x \to 3} x^2 + 3x + 9$ ← cancelled.

Question 4



$AC = AD = BC = DE$ Prove $AB = AE$

In $\triangle ACD$ $\angle ACD = \angle ADC$ (base $\angle$'s of isosceles $\triangle$ are equal)

$\angle ADE = \angle ACB = 180 - \theta$ (adjacent supplementary $\angle$'s)

$AC = AD$ (given)

$BC = DE$ (given)

$\therefore \triangle ACB \equiv \triangle ADE$ (SAS)

$\therefore AB = AE$ (corresponding sides of congruent $\triangle$'s)

4M

b) $\frac{d-2}{d} = \frac{d+5}{18}$ (A line parallel to one side of a triangle divides the other sides proportionally)

$18d - 36 = d^2 + 5d$
$-18d \qquad\qquad -18d$

$d^2 - 13d + 36 = 0$

$(d-9)(d-4) = 0$

$d = 9$ & $d = 4$

c) In $\triangle ABD$ and $\triangle ADC$

$\angle ADC = 90^\circ$ (adjacent suplementary $\angle$'s)

$\angle ADC = \angle ADB$ (both equal to 90°)

$\angle ACD = 180 - 90 - \angle ABD$ ($\angle$ sum of $\triangle ABC$)

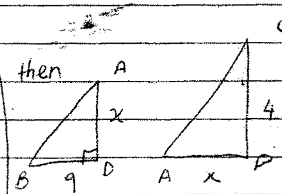
$= 90 - \angle ABD$

$\angle BAD = \cancel{180}\ 90 - \angle ABD$ ($\angle$ sum of $\triangle ABD$)

$= 90 - \angle ABD$

$\therefore \triangle ABD \ |||\ \triangle ADC$ (equal angular)

then



$\frac{AD}{BD} = \frac{CD}{AD}$

$\frac{x}{9} = \frac{4}{x}$

$x^2 = 36$

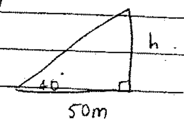
$x = \pm 6$

$\therefore x = 6$ only.

4M

Question 5

a)

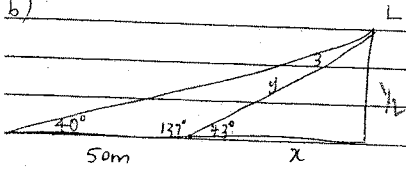


$\frac{h}{50} = \tan 40^\circ$

$h = 50 \tan 40^\circ$

$= 41.95m$ [3M]

b)



½ for diag if an wrong!

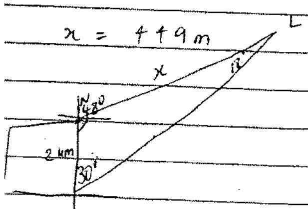
$\frac{y}{\sin 40} = \frac{50}{\sin 3^\circ}$

$y = \frac{50 \times \sin 40}{\sin 3^\circ}$

$= 614.0975114$ 2

$\cos 43 = \frac{x}{614.0975114}$

$x = 449m$ [4M]



[2M]

$180 - 48$

$= 132^\circ$

$= 180 - 132 - 30$

$= 18^\circ$

$\frac{x}{\sin 30} = \frac{2}{\sin 18}$

$x = \frac{2 \sin 30}{\sin 18} = 3.24km$ [3M]

Question 6

a) y-int let $x=0$ $f(x) = -1$

no x-intercept.

vertical asymptotes at

$x-1=0$ $x+1=0$

at $x = \pm 1$

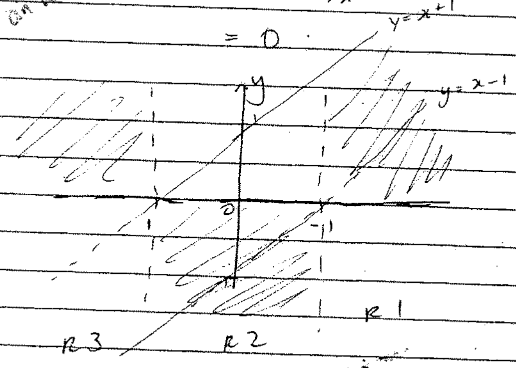
horizontal asymptotes

at $\lim_{x \to \infty} \frac{1}{x^2-1}$

$\lim_{x \to \infty} \frac{1/x^2}{x^2/x - 1/x^2}$

$= \lim_{x \to \infty} \frac{1/x^2}{x - 1/x^2}$

$= 0$



$R1 = \frac{1}{(x-1)(x+1)}$

$= \frac{1}{-x+}$

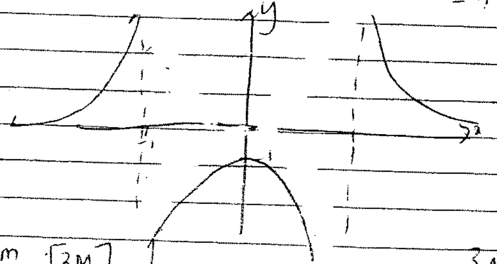
$= +$

$R_2 : = \frac{1}{+x-}$

$= -$

$R3 = \frac{1}{-x-}$

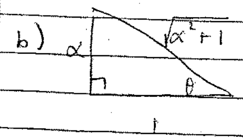
$= +$



3M

Question 6 cont

b)



$\cos\theta = \frac{1}{\sqrt{\alpha^2+1}}$

$\sin\theta = \frac{\alpha}{\sqrt{\alpha^2+1}}$

[2M]

c) $4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

$x = 30, 150, 210, 330^\circ$

[2M]

d) $3\cos^2 x = 8\sin x$

$3\cos^2 x - 8\sin x = 0$

~~$3\cos^2 x - 8(1-\sin$~~

$3(1-\sin^2 x) - 8\sin x = 0$

$3 - 3\sin^2 x - 8\sin x = 0$

$3\sin^2 x + 8\sin x - 3 = 0$

$3x \quad -1$

$x \quad +3$

$(3\sin x - 1)(\sin x + 3) = 0$

$\sin x = \frac{1}{3}$ $\sin x = -3$

↑ no solution.

$x = 19^\circ 28'$, $180 - 19^\circ 28'$

$= 19^\circ 28'$, $160^\circ 32'$

3M

e) $\sin\alpha = \frac{1}{2}$ find the exact value of $\cos 2\alpha$.

$\cos 2\alpha = 1 - 2\sin^2\alpha$.

$= 1 - 2\left(\frac{1}{2}\right)^2$

$= 1 - 2 \times \frac{1}{4}$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$ [2M]

Question 7

$\cos(\alpha+\theta) = \cos\alpha\cos\theta - \sin\alpha\sin\theta$ [1M]

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\cos 3\theta = \cos(2\theta+\theta)$

$= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$ 1

$= (\cos^2\theta - \sin^2\theta)\cos\theta - 2\sin\theta\cos\theta \times \sin\theta$

$= \cos^3\theta - \sin^2\theta\cos\theta - 2\sin^2\theta\cos\theta$

$= \cos^3\theta - 3\sin^2\theta\cos\theta$

$= \cos^3\theta - 3(1-\cos^2\theta)\cos\theta$ 1/2

$= \cos^3\theta - (3 + 3\cos^2\theta)\cos\theta$ 1/2

$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta$ 1/2

$= RHS.$

[3M]

Solve $8\cos^3\theta - 6\cos\theta - \sqrt{3} = 0$

$= 2(4\cos^3\theta - 3\cos\theta) = \sqrt{3}$

$(4\cos^3\theta - 3\cos\theta) = \frac{\sqrt{3}}{2}.$ 1

$\cos 3\theta = \frac{\sqrt{3}}{2}$

$3\theta = 30°, 330, 390, 690, 750, 1050$ 1

$\theta = 10°, 110, 130, 230, 250, 350$ 1

[3M]

b) In $\triangle ABD$

$\tan 68 = \frac{BD}{h}.$

$h\tan 68 = BD$ 1/2

In $\triangle ABK$ $\tan 70 = \frac{BK}{h}$

$h\tan 70 = BK$ 1/2

In $\triangle BDK$.

In $\triangle BDK$

$400^2 = BK^2 - BD^2$

$400^2 = h^2\tan^2 70 - h^2\tan^2 68$

$\frac{400^2}{\tan^2 70 - \tan^2 68} = \frac{h^2(\tan^2 70 - \tan^2 68)}{\tan^2 70 - \tan^2 68}$

h^2

$h =$

$h = \frac{400}{\sqrt{\cot^2 20 - \cot^2 22}}$

$= 335\,m$ accept $23.9\,m.$