Student name/number: ____________________

2002

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Total marks (84)

- Attempt Questions 1 – 7
- All questions are of equal value

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on last page
- All necessary working should be shown in every question

Total marks 84
Attempt Questions 1–7.
All questions are of equal value.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

QUESTION 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate $x^2\sin^{-1}x$. **2**

(b) Find the value of k if $x+3$ is a factor of $P(x) = 2x^3 - 5kx + 9$. **2**

(c) The interval AB has end points A $(-3, 5)$ and B $(3, 2)$. Find the coordinates of the point P which divides the interval AB externally in the ratio 2:5. **2**

(d) Find the acute angle, to the nearest degree, between the lines $x + y = 5$ and $2y = 3x + 5$. **2**

(e) How many arrangements of the letters in the word PROBABILITY are possible? **1**

(f) Use the table of standard integrals to find the exact value of $\displaystyle\int_0^4 \frac{dx}{\sqrt{x^2+9}}$. **3**

QUESTION 2. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the quotient, $Q(x)$, and the remainder, $R(x)$, when the polynomial $P(x) = 2x^4 - 3x^3 - x^2 + 2x + 1$ is divided by $x^2 + 2x - 1$. **3**

(b) Prove the identity $\dfrac{1 - \tan^2 A}{1 + \tan^2 A} = \cos 2A$. **2**

(c) Find the value of x if $\dfrac{d}{dx}\left(\dfrac{x+2}{\sqrt{x-1}}\right) = 0$. **3**

(d) Use the substitution $u = x - 1$ to evaluate $\displaystyle\int_2^5 \frac{x+1}{\sqrt{x-1}}\,dx$. **4**

QUESTION 3. (12 marks) Use a SEPARATE writing booklet.

Marks

(a) Solve $\frac{x}{x^2-4} \leq 0$. **3**

(b) The function $f(x)$ is given by $f(x) = \cos^{-1}2x + \sin^{-1}2x$.

(i) Show that the derivative of $f(x)$ is zero. **1**

(ii) Sketch the graph of $y = f(x)$. **2**

(c) Consider the function $f(x) = x\log_e x$.

(i) Show that $y = f(x)$ has a minimum turning point at $\left(\frac{1}{e}, -\frac{1}{e}\right)$. **2**

(ii) Hence sketch the curve of $y = f(x)$ for $x \geq \frac{1}{e}$. **1**

(iii) Draw the graph of the inverse function of $y = f(x)$, $x \geq \frac{1}{e}$. **1**

(d) The equation $2x^3 + 12x^2 - 13x - 20 = 0$ has roots $\alpha - d$, α and $\alpha + d$.

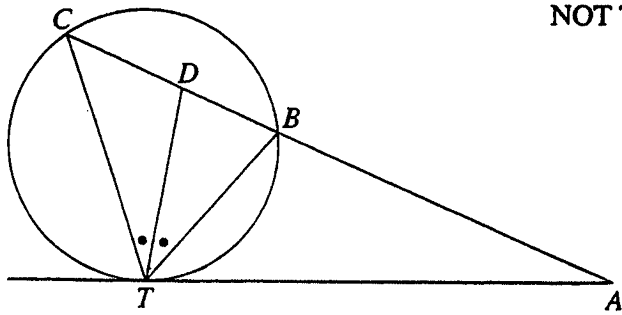
(i) Find the value of α. **1**

(ii) Find a value of d. **1**

QUESTION 4. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the coefficient of x^9 in the binomial expansion of $\left(5x^2 - \frac{1}{2x}\right)^{12}$. **3**

(b) NOT TO SCALE



TA is a tangent to a circle. Line $ABDC$ intersects the circle at B and C. Line TD bisects angle BTC. **3**

Prove $AT = AD$.

(c) Equipment being delivered by a parachute drop is falling at a speed of v m s^{-1}. When the parachute opens, the equipment is falling at 50 m s^{-1}, and thereafter its acceleration is given by $\frac{dv}{dt} = k(2 - v)$, where k is a constant.

(i) Use differentiation to show that the equation for $\frac{dv}{dt}$ is satisfied by $v = 2 + Ae^{-kt}$, where A is a constant. **1**

(ii) Find the value of A. **1**

(iii) One second after the parachute opens, the speed of the equipment has decreased to 35 m s^{-1}. Determine the value of k correct to four decimal places. **2**

(iv) After a number of seconds, the equipment continues to fall with a speed that is very nearly constant, and which is called the 'terminal speed'. Find the terminal speed for this particular parachute drop. **1**

(v) How many seconds does it take for the speed of the equipment to be 5% more than the terminal speed? **1**

QUESTION 5. (12 marks) Use a SEPARATE writing booklet.

Marks

(a) (i) Express $\sqrt{3}\cos 2t - \sin 2t$ in the form $R\cos(2t + \alpha)$, where $0 < \alpha < \frac{\pi}{2}$. **2**

(ii) Hence or otherwise find all positive solutions of $\sqrt{3}\cos 2t - \sin 2t = 0$. **2**

(b) A particle moves in a straight line and is x metres from a fixed point O after t seconds, where $x = 5 + \sqrt{3}\cos 2t - \sin 2t$.

(i) Prove that the acceleration of the particle is $-4(x - 5)$. **2**

(ii) Between which two points does the particle oscillate? You may use your answers from part (a). **1**

(iii) At what time does the particle first pass through the point $x = 5$? **1**

(c) Use mathematical induction to prove that $1^2 + 3^2 + \ldots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$ **4**

for all positive integers $n = 1, 2, 3, \ldots$.

QUESTION 6. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) When a particle moving in a straight line has displacement x metres from a fixed point O, its acceleration in metres per second per second is given by $\ddot{x} = \sqrt{3x+4}$.

(i) Show that $v^2 = \frac{4}{9}(3x+4)^{\frac{3}{2}} + c$, where v is the velocity of the particle in metres per second, and c is a constant. **1**

(ii) Given that the particle starts from rest at O, evaluate c. **1**

(iii) Explain why the motion of the particle is always in the positive direction. **1**

(b) In a breed of rabbits, 20% have long hair and 45% have grey eyes. The two characteristics are independent of each other.

(i) Find the probability that one rabbit chosen at random has both long hair and grey eyes. **1**

(ii) In a random sample of 10 rabbits, what is the probability of there being exactly three rabbits that have both long hair and grey eyes? Give the answer correct to three decimal places. **2**

(c) (i) By using graphs or otherwise, show that the curves $y = \ln x$ and $y = 2 - x$ have a point of intersection for which the x coordinate is close to 1.5. **1**

(ii) Use $x = 1.5$ and one application of Newton's method to find a better approximation for the x coordinate of this point of intersection, correct to two decimal places. **2**

(d) A solid is formed by rotating the curve $y = 1 + \sqrt{2}\cos x$, between $x = 0$ and $x = \frac{\pi}{4}$, about the x-axis.

(i) Show the volume of the solid can be expressed as **1**

$$V = \pi \int_0^{\frac{\pi}{4}} (2 + 2\sqrt{2}\cos x + \cos 2x)\,dx\ .$$

(ii) Hence find the exact volume of the solid. **2**

QUESTION 7. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Use the definition $f'(c) = \lim_{x \to c} \dfrac{f(x) - f(c)}{x - c}$ to find the derivative of x^3 at $x = a$. **2**

(b) Ten people arrive to eat at a restaurant. The only seating available for them is at two circular tables, one that seats six persons, and another that seats four.

(i) Using these tables, how many different seating arrangements are there for the ten people? **2**

(ii) Assuming that the seating arrangement is random, what is the probability that a particular couple will be seated at the same table? **2**

(c) (i) Prove that $(1 + x)^{2n}\left(1 - \frac{1}{x}\right)^{2n} \equiv \left(x - \frac{1}{x}\right)^{2n}$. **1**

(ii) By expanding both sides of the identity in part (i), and equating the terms independent of x, show that $\left({}^{2n}C_0\right)^2 - \left({}^{2n}C_1\right)^2 + \left({}^{2n}C_2\right)^2 - \ldots + \left({}^{2n}C_{2n}\right)^2 = (-1)^n\, {}^{2n}C_n$. **3**

(iii) Hence show $\sum_{r=0}^{n} (-1)^r \left({}^{2n}C_r\right)^2 = \frac{(-1)^n}{2}\left({}^{2n}C_n\right)\left(1 + {}^{2n}C_n\right)$. **2**

End of paper