

QUESTION 1 (Use a new sheet)

- (a) Evaluate correct to one decimal place

$$\frac{\sqrt{3.2 + 4.66}}{2 \cdot 33^3}$$

- (b) Factorise $x^2 - 11x + 10$

- (c) A parabola has equation $(x - 3)^2 = 8(y + 1)$. What are the co-ordinates of its focus?

- (d) Write down the gradient of the line $3y = 2x + 6$.

- (e) Solve $(x + 2)^2 = 9$.

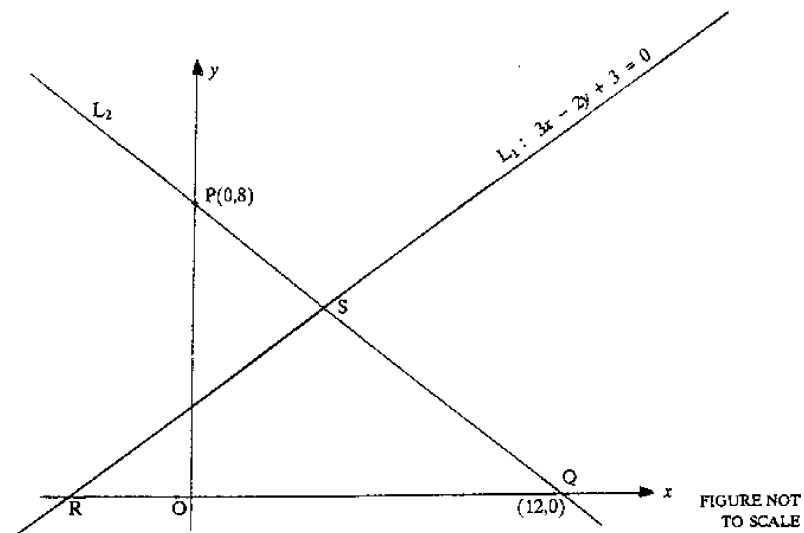
- (f) Express $\frac{3}{\sqrt{5} + 2}$ in the form $a\sqrt{5} + b$

- (g) The value (\$V) of a car after n years is given by the formula :

$$V = V_0(1 - R)^n$$

where V_0 is the initial value of the car and R is the annual percentage rate of depreciation. A car bought 3 years ago for \$15 000 was sold for \$9 000. Calculate the annual rate of depreciation of this car. (Give your answer to the nearest whole number.)

QUESTION 2 (Use a new sheet)



In the diagram the line L_1 has equation $3x - 2y + 3 = 0$ and the line L_2 passes through the points $P(0, 8)$ and $Q(12, 0)$.

- L_1 cuts the x axis at R . Find the co-ordinates of R .
- Find the gradient of the line L_2 .
- Show the line L_1 is perpendicular to the line L_2 .
- Show the equation of the line L_2 is :
 $2x + 3y - 24 = 0$
- Find the coordinates of S , the point of intersection of the lines L_1 and L_2 .
- Hence calculate the area of ΔQRS .
- Shade in the region where the following inequalities hold simultaneously :
 $2x + 3y - 24 \leq 0$; $3x - 2y + 3 \geq 0$ and $y \geq 0$

QUESTION 3

(Use a new sheet)

(a) Differentiate :

(i) $4x^3 + 7$

(ii) xe^{2x}

(iii) $\frac{\sin x}{x}$

(b) (i)

Sketch $f(x) = \begin{cases} -5 & \text{for } x \leq -3 \\ 2x & \text{for } -3 < x < 0 \\ x^2 & \text{for } x \geq 0 \end{cases}$

(ii) Find the value of $f(-3) + f(3)$

(c)

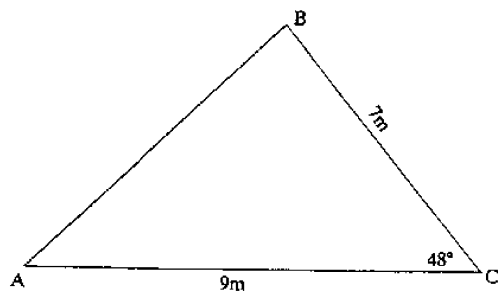


FIGURE NOT TO SCALE

Use the cosine rule to find the length of AB to the nearest metre.

QUESTION 4

(Use a new sheet)

(a)

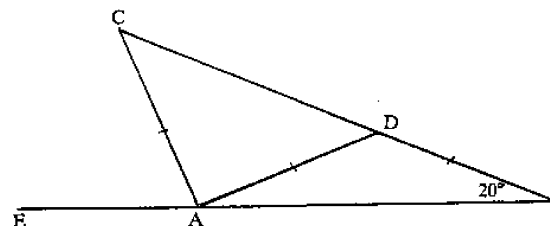


FIGURE NOT TO SCALE

In the diagram $CA = AD = DB$ and $\angle EBD = 20^\circ$.

Copy this diagram onto your answer sheet.

(i) Show $\angle ADC = 40^\circ$, giving reasons.

(ii) Hence find the size of $\angle CAE$, giving reasons.

(b) Find :

(i) $\int \frac{dx}{x+5}$

(ii) $\int \sec^2 3x \, dx$

(c) The logo for the company "Top Hats" was designed using parts of the curves $y = 2x^2$ and $y = 12 - x^2$.

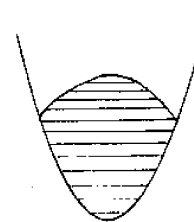


DIAGRAM 1

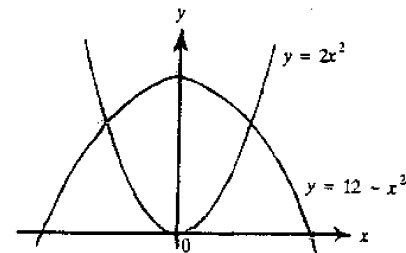


DIAGRAM 2

Diagram 1 shows a drawing of the logo and Diagram 2 shows a sketch of the logo related to the coordinate axes.

(i) Show the curves intersect at $(-2, 8)$ and $(2, 8)$.

(ii) Hence find the area of the shaded part of the logo.

QUESTION 5

(Use a new sheet)

- (a) For what values of x is the graph of the parabola $y = 12 + 4x - x^2$ below the x axis?
- (b) Lisa had 3 similar keys in her pocket. To open her front door she tried the keys at random. She stopped trying when she opened the door. She did not try the same key twice. Find the probability :
- the door opened when she tried the first key.
 - she tried all three keys before the door opened.

(c)

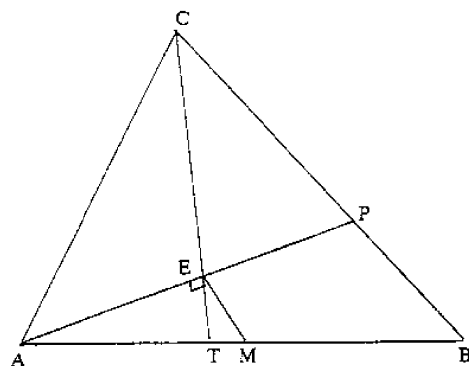


FIGURE NOT TO SCALE

In the diagram CT bisects $\angle ACB$. AE is perpendicular to CT and M is the midpoint of AB. AE produced meets BC at the point P.

- Copy this diagram onto your answer sheet and mark in all the given information.
- Prove that $\triangle ACE$ is congruent to $\triangle PCE$.
- Explain why $AE = EP$.
- Hence prove that EM is parallel to PB.

QUESTION 6

(Use a new sheet)

(a)

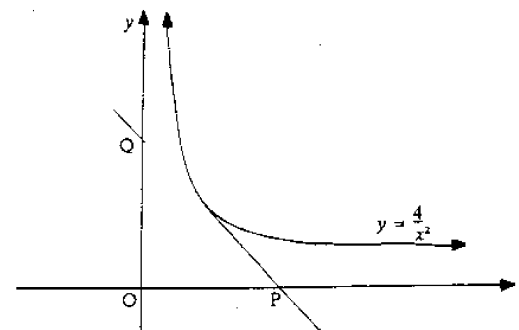


DIAGRAM NOT TO SCALE

A tangent to the curve $y = \frac{4}{x^2}$ intersects the x axis at P and the y axis at Q and $\angle OPQ = \angle OQP$.

- Explain why the gradient of PQ is -1 .
 - Show PQ is a tangent to the curve at $(2, 1)$.
 - Find the equation of this tangent.
- (b) The area bounded by the curve $y = \sqrt{\cos 2x}$, $x = 0$, $x = \frac{\pi}{4}$ and the x axis is rotated about the x axis. Find the exact volume of the solid formed.
- (c) The diagram shows a block of land 60 metres long. At intervals of 10 metres, the width of the block was measured.

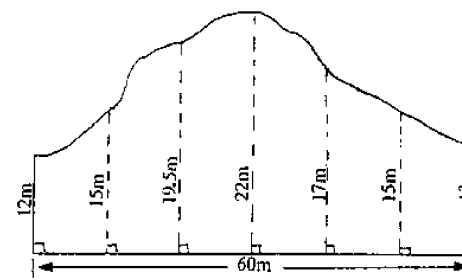


DIAGRAM NOT TO SCALE

Approximate the area of this block of land using the Trapezoidal Rule with the seven heights shown.

QUESTION 7

(Use a new sheet)

- (a) Simplify

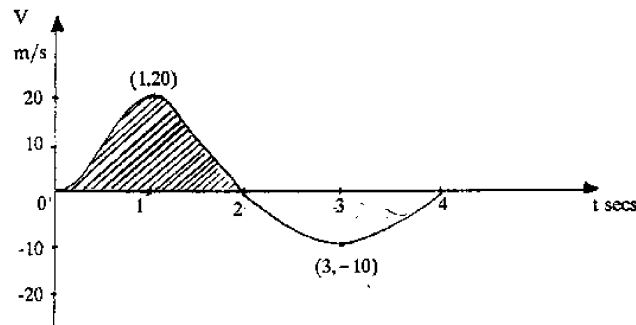
$$\frac{16}{2^{3x} \times 8^{1-x}}$$

- (b) Solve for x :

$$3^{2x} - 10(3^x) + 9 = 0$$

- (c) If $6x^2 - 11 \equiv A(x+2)^2 + Bx + C$, find the values of A , B and C .

- (d) The graph represents the velocity (v m/s) of a particle after t seconds. The particle is moving in a straight line starting from rest.



- What is the velocity of the particle after 1 second?
- What is the acceleration of the particle after 3 seconds?
- When does the particle change directions?
- Explain what is represented by the area of the shaded region in the diagram.

QUESTION 8

(Use a new sheet)

- (a) The length of the arc AB of the sector OAB is 2π cm.

- Show the length of OA is 6 cm.
- Calculate the exact area of the sector OAB .

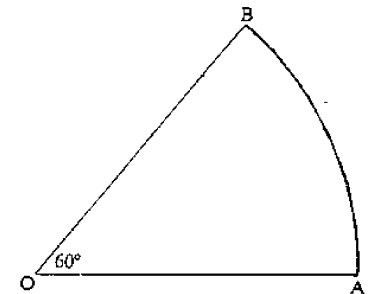
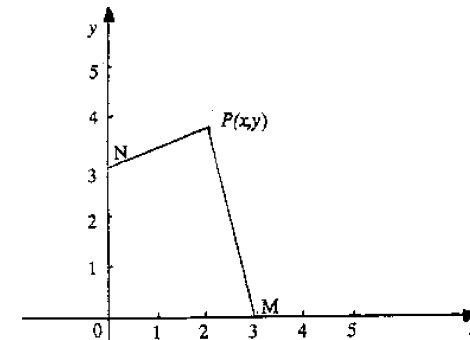


FIGURE NOT TO SCALE

- (b)



$P(x,y)$ moves so that its distance from $M(3,0)$ is always twice its distance from the point $N(0,3)$

- Show that the equation of the locus of all points $P(x,y)$ is :
$$x^2 + 2x + y^2 - 8y + 9 = 0$$
- Give a geometric description of the locus of all points $P(x,y)$.

- (c) In a chemical experiment, the amount of crystals, x grams, that dissolved in a solution after t minutes was given by :

$$x = 20(1 - e^{-kt})$$

- After 3 minutes it was found that 10 grams of the crystals had dissolved. Show the value of k correct to 3 significant figures was 0.231.
- At what rate were the crystals dissolving after 5 minutes? Give your answer to the nearest gm/minute.

QUESTION 9

(Use a new sheet)

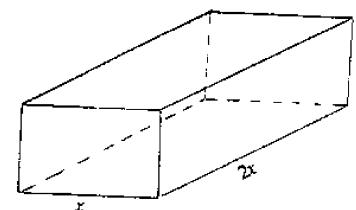
- (a) The curve $f(x) = x^3 + 3x^2 - 9x - 1$ is defined in the domain $-4 \leq x \leq 2$.
- Find the coordinates of the two stationary points and determine their nature.
 - Show a point of inflexion occurs at $x = -1$.
 - Sketch this curve.
- (b) When Kim entered University, her parents borrowed \$30 000 to pay for her education. They plan to repay the loan by making 48 equal monthly instalments. Interest is charged at the rate of 1.1% per month on the balance owing.
- Show that immediately after making two monthly instalments of \$M, the balance owing is given by :

$$$(30\,663.63 - 2 \cdot 011M)$$$
 - Calculate the value of each monthly instalment.

QUESTION 10

(Use a new sheet)

- (a) If $\cos \alpha = -\frac{3}{5}$ and $\sin \alpha < 0$, find the exact value of $\tan \alpha$.
- (b) x , x^2 and $5x$ are three consecutive terms of an arithmetic series.
- Show that $2x^2 - 6x = 0$.
 - What is the common difference of this arithmetic series?
- (c) Boxes in the shape of rectangular prisms are to be constructed from special materials. The width (x metres) of the base is to be half the length of the base and each box is to hold a volume of 4 cubic metres.



Material that is used to build the base and top costs \$15 per m^2 . A cheaper material at \$10 per m^2 is used for the four sides.

- Show that the total cost (\$C) of building each box is given by :

$$C = 60x^2 + \frac{120}{x}$$
- What is the width of the base of the cheapest box that can be constructed?