CSSA

CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW
2015 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION
MATHEMATICS EXTENSION 1-MARKING GUIDELINES

Section I
10 marks

Questions 1-10 (1 mark each)

Question 1 (1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E2

Solution	Mark
Given $p(x) = x^3 + 2x^2 - 5x - 6$ $p(2) = 2^3 + 2\times 2^2 - 5\times 2 - 6$ $= 0.$ Correct Answer is C.	1

Question 2 (1 mark)
Outcomes Assessed: PE2
Targeted Performance Bands: E2

Solution	Mark
$y = \log_e x$. $\dfrac{dy}{dx} = \dfrac{1}{x}$. At $x = 2$, $\dfrac{dy}{dx} = \dfrac{1}{2}$ and at $x = 3$, $\dfrac{dy}{dx} = \dfrac{1}{3}$. Using $\tan\theta = \left\|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right\|$ $= \left\|\dfrac{\frac{1}{2} - \frac{1}{3}}{1 + \frac{1}{2}\times\frac{1}{3}}\right\|$ $= \dfrac{\frac{1}{6}}{\frac{7}{6}}$. $\theta = 8^o$ (to the nearest degree). Correct answer is A.	1

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Question 3 (1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E2

Solution	Mark
$(x_1, y_1) = (1,3)$, $(x_2, y_2) = (4,1)$, $m:n = -1:3$ (or 1 : -3 can be used). Using $x = \dfrac{mx_2 + nx_1}{m+n}$ $y = \dfrac{my_2 + ny_1}{m+n}$ $x = \dfrac{-1\times4+3\times1}{-1+3}$ $y = \dfrac{-1\times1+3\times3}{-1+3}$ $= -\dfrac{1}{2}$. $= 4$. $\therefore P$ is $(-\dfrac{1}{2}, 4)$. Correct answer is C.	1

Question 4 (1 mark)
Outcomes Assessed: HE4,HE5
Targeted Performance Bands: E3

Solution	Mark
$\dfrac{d}{dx}\sin^{-1}\dfrac{2x}{3} = \dfrac{1}{\sqrt{1-\left(\dfrac{2x}{3}\right)^2}} \times \dfrac{2}{3}$ $= \dfrac{1}{\sqrt{1-\dfrac{4x^2}{9}}} \times \dfrac{2}{3}$ $= \dfrac{1}{\dfrac{\sqrt{9-4x^2}}{3}} \times \dfrac{2}{3}$ $= \dfrac{2}{\sqrt{9-4x^2}}$. Correct answer is B.	1

Question 5 (1 mark)
Outcomes Assessed: HE4
Targeted Performance Bands: E2-E3

Solution	Mark
$\int \frac{4}{25+16x^2}\,dx = \int \frac{4}{16\left(\frac{25}{16}+x^2\right)}\,dx$ $= \frac{1}{4}\int \frac{1}{\frac{25}{16}+x^2}\,dx$ $= \frac{1}{5}\tan^{-1}\left(\frac{4x}{5}\right)+C.$ Correct answer is B.	1

Question 6 (1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E2-E3

Solution	Mark
• Number of arrangements if manager and coach sit together $= 2\times 6!$. • Total number of arrangements $= 7!$. • Number of arrangements where manager and coach are not together $=$ $7!-2\times 6! = 3600$. Correct answer is A.	1

Question 7 (1 mark)
Outcomes Assessed: PE2
Targeted Performance Bands: E3

<table>
<tr><th>Solution</th><th>Mark</th></tr>
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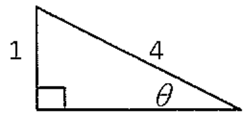
$\sin\theta = \frac{1}{4}$.



Missing side is $\sqrt{16-1} = \sqrt{15}$.

$\cos\theta < 0$ and $\sin\theta = \frac{1}{4} \Rightarrow \theta$ is in the second quadrant.

$$\begin{aligned}
\tan 2\theta &= \frac{2\tan\theta}{1-\tan^2\theta} \\
&= \frac{2\times\frac{-1}{\sqrt{15}}}{1-\left(-\frac{1}{\sqrt{15}}\right)^2} \\
&= \frac{-\frac{2}{\sqrt{15}}}{1-\frac{1}{15}} \\
&= -\frac{2}{\sqrt{15}}\times\frac{15}{14} \\
&= -\frac{15}{7\sqrt{15}}.
\end{aligned}$$

Correct answer is A.

</td><td>1</td></tr>
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Question 8 (1 mark)
Outcomes Assessed: HE3
Targeted Performance Bands: E3-E4

Solution	Mark
Given $\dfrac{dV}{dt} = 100 \text{ cm}^3$ per second. It is required to find $\dfrac{dA}{dt}$. $V = x^3 \Rightarrow \dfrac{dV}{dx} = 3x^2, A = 6x^2 \Rightarrow \dfrac{dA}{dx} = 12x$. [$x$ is the side length of the cube] $\dfrac{dA}{dt} = \dfrac{dV}{dt} \times \dfrac{dx}{dV} \times \dfrac{dA}{dx}$ $= 100 \times \dfrac{1}{3x^2} \times 12x$ $= \dfrac{400}{x}$. When $x = 10, \dfrac{dA}{dt} = \dfrac{400}{10}$ $= 40 \text{ cm}^2$ per second. Correct answer is C.	1

Question 9 (1 mark)
Outcomes Assessed: HE7
Targeted Performance Bands: E3-E4

Solution	Mark
$\dfrac{\sin\theta\cos\theta}{2\cos^2\theta - 1} = -\dfrac{\sqrt{3}}{2}$. $\Rightarrow \dfrac{\sin 2\theta}{\cos 2\theta} = -\sqrt{3}$. $\Rightarrow \tan 2\theta = -\sqrt{3}$. $0 \le \theta \le 2\pi \Rightarrow 0 \le 2\theta \le 4\pi$. $2\theta = \dfrac{2\pi}{3}, \dfrac{5\pi}{3}, \dfrac{8\pi}{3}, \dfrac{11\pi}{3}$ $\theta = \dfrac{\pi}{3}, \dfrac{5\pi}{6}, \dfrac{4\pi}{3}, \dfrac{11\pi}{6}$. $\therefore$ 4 solutions. Correct answer is C.	1

Question 10 (1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E3-E4

Solution	Mark
Boys and girls can each be selected in ${}^{n}C_2$ ways. Number of different committees $= {}^{n}C_2 \times {}^{n}C_2$ $= \dfrac{n!}{2!(n-2)!} \times \dfrac{n!}{2!(n-2)!}$ $= \dfrac{n(n-1)}{2} \times \dfrac{n(n-1)}{2}$ $= \dfrac{n^2(n-1)^2}{4}$ $= \dfrac{n^2(n^2-2n+1)}{4}$. Correct answer is D.	1

Section II
60 marks

Question 11 (15 marks)

(a) (3 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E2

Criteria	Mark
• Correct solution.	3
• Significant progress towards the solution.	2
• Demonstrates a correct method towards finding a solution.	1

Sample Answer:

$\dfrac{2}{1+3x} \leq 1$ where $x \neq -\dfrac{1}{3}$.

Multiply both sides by $(1+3x)^2$.

$\therefore\ 2(1+3x) \leq (1+3x)^2$ 1

$2+6x \leq 1+6x+9x^2$

$0 \leq 9x^2 - 1$

$9x^2 - 1 \geq 0$

$(3x+1)(3x-1) \geq 0$

$x < -\dfrac{1}{3}$ or $x \geq \dfrac{1}{3}$. ← must have $x < -\frac{1}{3}$

(b) (i) (1 marks)
Outcomes Assessed: HE4
Targeted Performance Bands: E2

Criteria	Mark
• Correct solution	1

Sample Answer:

Put $y = \tan^{-1} 2x$.

$\therefore\ \dfrac{dy}{dx} = \dfrac{2}{1+4x^2}$. ←

Question 11(continued)

(b) (ii) (3 marks)

Outcomes Assessed:HE4

Targeted Performance Bands: E3

Criteria	Mark
• Correct solution.	3
• Significant progress towards the solution.	2
• Showing the gradient function equates to 1.	1

Sample Answer:

The gradient of the line $3x+3y-1=0$ is -1.

$\therefore$ Gradient of tangents perpendicular to this line is 1.

$$\therefore \frac{2}{1+4x^2}=1.$$

1 mark

$$1+4x^2=2.$$

$$x^2=\frac{1}{4}.$$

$$x=\pm\frac{1}{2}.$$

When $x=\frac{1}{2}$, $y=\tan^{-1}(1)$

$$=\frac{\pi}{4}.$$

When $x=-\frac{1}{2}$, $y=\tan^{-1}(-1)$

$$=-\frac{\pi}{4}.$$

2 marks

Points are $\left(\frac{1}{2},\frac{\pi}{4}\right)$ and $\left(-\frac{1}{2},-\frac{\pi}{4}\right)$.

Question 11(continued)

(c) (i) (2 marks)
Outcomes assessed: HE7
Targeted Performance Bands: E2-E3

Criteria	Mark
• Correctly gives value of A and α .	2
• Only one is correct but not both.	1

Sample Answer:
Let $\cos\theta - \sqrt{3}\sin\theta = A\cos(\theta+\alpha)$.
$\therefore \cos\theta - \sqrt{3}\sin\alpha = A\cos\theta\cos\alpha - A\sin\theta\sin\alpha$.
Equating coefficients of $\cos\theta$ and $\sin\theta$, $A\cos\alpha = 1$ and $A\sin\alpha = \sqrt{3}$.
Squaring and adding, $A^2(\cos^2\alpha + \sin^2\alpha) = 1+3$.
Hence A=2 .

$A = \sqrt{1^2 + (\sqrt{3})^2} = 2$

$$\frac{A\sin\alpha}{A\cos\alpha} = \frac{\sqrt{3}}{1}.$$
$\tan\alpha = \sqrt{3}$.
$$\alpha = \frac{\pi}{3}.$$

$\tan x = \frac{\sqrt{3}}{1}$

$\therefore x = \frac{\pi}{3}$.

(c) (ii) (3 marks)
Outcomes assessed: HE7
Targeted Performance Bands: E3

Criteria	Mark
• Correctly gives all 3 solutions.	3
• Correctly gives 2 solutions only.	2
• Significant progress and is able to find 1 solution.	1

Sample Answer:
Using results from part (i), the equation becomes
$$2\cos(\theta + \frac{\pi}{3}) = 1.$$
Since $0 \le \theta \le 2\pi$, $\frac{\pi}{3} \le \theta + \frac{\pi}{3} \le \frac{7\pi}{3}$
$$\cos(\theta + \frac{\pi}{3}) = \frac{1}{2}.$$
$$\theta + \frac{\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}.$$
$\therefore$ Solutions are $\theta = 0$, $\theta = \frac{4\pi}{3}$, $\theta = 2\pi$.

Question 11(continued)

(d) (3 marks)

Outcomes assessed: PE3
Targeted Performance Bands: E2-E3

Criteria	Mark
• Correct new approximation as required is given.	3
• Correct derivative and use of Newton's Method.	2
• Correct function and derivative.	1

Sample Answer:
The curves intersect when $\log_e x = x^2 - 5$.

$\therefore \log_e x - x^2 + 5x = 0$.

Let $f(x) = \log_e x - x^2 + 5x$.

$$f'(x) = \frac{1}{x} - 2x + 5.$$ ← 1 mark

At $x = 5$, $f(x) = \log_e 5 - 25 + 25$

$$= \log_e 5.$$

$$f'(x) = \frac{1}{5} - 10 + 5$$

$$= -4.8\ .$$ ← 2 marks

From Newton's Method,

$$x_1 = 5 - \frac{f(5)}{f'(5)}$$

$$= 5 - \frac{\log_e 5}{-4.8}$$

$$= 5.3352....\ .$$

$\therefore x_1 \approx 5.3$.

$\therefore$ The required x-coordinate is $x = 5.3$ (to one decimal place).

must give exact answer to
obtain full solution

Question 12 (15 marks)

(a) (3 marks)
Outcomes Assessed: HE6
Targeted Performance Bands: E2-E3

Criteria	Mark
• Gives the correct solution .	3
• Able to integrate the function with an incorrect solution.	2
• Correctly changes $\int_0^{\frac{\pi}{2}} \cos^2 2x + \sin^2 \frac{x}{2}\, dx$ to $\frac{1}{2}\int_0^{\frac{\pi}{2}} 2 + \cos 4x - \cos x\, dx$..	1

Sample Answer:

$\cos^2 2x = \dfrac{\cos 4x + 1}{2}$ and $\sin^2 \dfrac{x}{2} = \dfrac{1 - \cos x}{2}$. ✓ /

$$\therefore \int_0^{\frac{\pi}{2}} \cos^2 2x + \sin^2 \frac{x}{2}\, dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} 2 + \cos 4x - \cos x\, dx$$

$$= \frac{1}{2}\left[2x + \frac{1}{4}\sin 4x - \sin x\right]_0^{\frac{\pi}{2}}$$ 2

$$= \frac{1}{2}\left((2 \times \frac{\pi}{2} + \frac{1}{4}\sin 2\pi - \sin\frac{\pi}{2}) - 0\right)$$

$$= \frac{1}{2}(\pi - 1).$$

?
3.

(b) (3 marks)
Outcomes Assessed: HE6
Targeted Performance Bands: E3

Criteria	Mark
• Correct solution.	3
• Correct integration with incorrect substitution.	2
• Correct use of substitution including limits.	1

Sample Answer:

$u = 2t + 1 \qquad du = 2\, dt$.

When $t = 4$, $u = 9$ and when $t = 0$, $u = 1$. /

$$\therefore \int_0^4 \frac{2t}{\sqrt{2t+1}} dt = \int_1^9 \frac{u-1}{\sqrt{u}} \frac{du}{2}$$

$$= \frac{1}{2}\int_1^9 u^{\frac{1}{2}} - u^{\frac{-1}{2}}\, du$$

$$= \frac{1}{2}[(\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}})]_1^9$$ 2.

$$= 6\frac{2}{3}. = \frac{20}{3}$$ 3

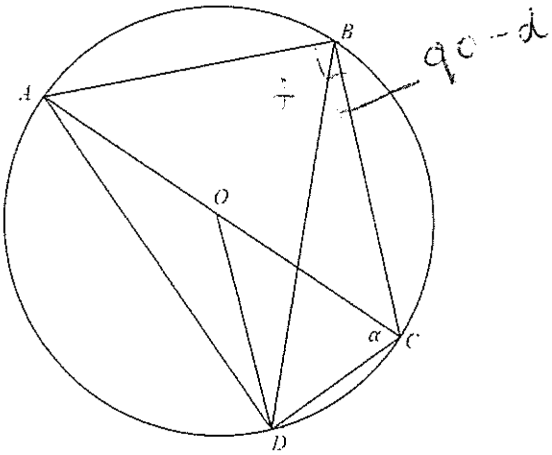
?

Question 12(continued)

(c) (2 marks)
Outcomes Assessed: PE3,PE6
Targeted Performance Bands: E3

Criteria	Mark
•Correct proof.	2
• Significant progress towards a correct proof.	1

Sample Answer:



Join BC.
Let $\angle DCA = \alpha$.

$\angle ABC = 90^\circ$ (Angle at the circumference in a semicircle).

$\angle ABD = \angle DCA = \alpha$ (Angles at the circumference subtended by the same arc are equal).

$\therefore \angle DBC = 90^\circ - \alpha$.

$\therefore 90^\circ - \angle DBC = 90^\circ - (90^\circ - \alpha) = \alpha = \angle DCA$.

$\therefore$ Proven as required that $\angle DCA = 90^\circ - \angle DBC$.

Question 12(continued)

(d) (i) (1 mark)

Outcomes Assessed: PE4,PE6
Targeted Performance Bands: E2

Criteria	Mark
• Correctly shows that $pq = -2$.	1

Equation of the chord is $y - ap^2 = \dfrac{(p+q)}{2}(x - 2ap)$.

The chord passes through $(0, 2a)$.

$$\begin{aligned}\therefore 2a - ap^2 &= \frac{(p+q)}{2}(-2ap)\\ &= (p+q)(-ap)\\ &= -ap^2 - apq.\end{aligned}$$

Must show proper substitution.

$apq = -2a$.

$\therefore pq = -2$.

Question 12(continued)

(d) (ii) (3 marks)
Outcomes Assessed: PE3,PE4,PE6
Targeted Performance Bands: E3

Criteria	Mark
• Correct equation.	3
• Correct relation of x and y in terms of p and q.	2
• Correct midpoint.	1

Sample Answer:
At M

$$x=\frac{2ap+2aq}{2} \quad \ldots\ (1)$$
$$=a(p+q).$$
$$y=\frac{a(p^2+q^2)}{2}. \quad \ldots\ (2)$$

1

Re-arranging (1) and (2).

$$\frac{x}{a}=p+q. \quad \ldots\ (3)$$
$$\frac{2y}{a}=p^2+q^2. \quad \ldots\ (4)$$

Square (3)

$$\frac{x^2}{a^2}=p^2+q^2+2pq.$$

2

$$\frac{x^2}{a^2}=p^2+q^2-4, \text{ using } pq=-2.$$

$$\frac{x^2}{a^2}+4=p^2+q^2. \quad \ldots\ (5)$$

Equating (4) and (5)

$$\frac{x^2}{a^2}+4=\frac{2y}{a}.$$
$$x^2+4a^2=2ay.$$
$$x^2=2ay-4a^2.$$
$$x^2=2a(y-2a).$$

3

Question 12(continued)

(e) (3 marks)
Outcomes Assessed: HE3
Targeted Performance Bands: E3

Criteria	Mark
• Correct solution.	3
• Finds correct value for k.	2
• States correctly the general term T_{k+1} for the expansion or equivalent.	1

Sample Answer:

Let T_{k+1}, where $k = 0,1,2,\ldots 12$ represent terms in the expansion of $\left(\frac{x}{3}-\frac{3}{x}\right)^{12}$.

$$\begin{aligned} T_{k+1} &= {}^{12}C_k\left(\frac{x}{3}\right)^{12-k}\left(-\frac{3}{x}\right)^k \\ &= {}^{12}C_k\left(\frac{1}{3}\right)^{12-k}(-3)^k(x)^{12-k}(x)^{-k} \\ &= {}^{12}C_k\left(\frac{1}{3}\right)^{12-k}(-3)^k(x)^{12-2k}. \end{aligned}$$

For term containing x^8, $12-2k=8$.

$\therefore k=2$. — 2

$$\begin{aligned} \text{Coefficient of } x^8 &= {}^{12}C_2\left(\frac{1}{3}\right)^{10}(-3)^2 \\ &= {}^{12}C_2\left(\frac{1}{3}\right)^8 \\ &= \frac{22}{2187}. \end{aligned}$$

3

Question 13 (15 marks)

(a) (3 marks)

Outcomes Assessed: HE2

Targeted Performance Bands: E3

Criteria	Mark
• Complete proof.	3
• Correct proof that P_1 is true and correct assumption that P_k is true with some progress of showing that P_{k+1} is true.	2
• Proof that P_1 is true.	1

Sample Answer:

Let P_n be the statement $1^2+3^2+...+(2n-1)^2=\dfrac{n(2n-1)(2n+1)}{3}$.

Step 1: Show P_1 is true.

For $n=1$, L.H.S $=1^2=1$.

$$\text{R.H.S}=\frac{1\times(2-1)\times(2+1)}{3}=\frac{1\times1\times3}{3}=1.$$

L.H.S = R.H.S $\quad\therefore P_1$ is true.

Step 2: Assume that P_k is true for some positive integer k.

i.e. Assume that $1^2+3^2+...+(2k-1)^2=\dfrac{k(2k-1)(2k+1)}{3}$.

We need to prove that P_{k+1} is true.

$\therefore$ We prove that $1^2+3^2+...+(2(k+1)-1)^2=\dfrac{(k+1)(2(k+1)-1)(2(k+1)+1)}{3}$.

i.e. $1^2+3^2+...+(2k+1)^2=\dfrac{(k+1)(2k+1)(2k+3)}{3}$.

$$\begin{aligned}
\text{L.H.S. of } P_{k+1} &= 1^2+3^2+...+(2k-1)^2+(2k+1)^2\\
&=\frac{k(2k-1)(2k+1)}{3}+(2k+1)^2 \quad \text{by the assumption}\\
&=\frac{(2k+1)}{3}(k(2k-1)+3(2k+1))\\
&=\frac{(2k+1)}{3}(2k^2-k+6k+3)\\
&=\frac{(2k+1)}{3}(2k^2+5k+3)\\
&=\frac{(2k+1)}{3}(2k+3)(k+1)\\
&=\frac{(k+1)(2k+1)(2k+3)}{3}\\
&=\text{R.H.S of } P_{k+1}.
\end{aligned}$$

$\therefore$ If P_k is true then P_{k+1} is true.

$\therefore$ By the principle of mathematical induction, P_n is true for all positive integers n.

Question 13 (continued)

(b) (i) (2 marks)
Outcomes Assessed: HE4
Targeted Performance Bands: E3

Criteria	Mark
• Correct explanation is given.	2
• Correct derivative of $f(x)$.	1

Sample Answer:

$$f(x) = \log_e(\frac{2-x}{x})$$
$$= \log_e(2-x) - \log_e x .$$
$$f'(x) = \frac{-1}{2-x} - \frac{1}{x}$$
$$= \frac{-x-2+x}{x(2-x)}$$
$$= \frac{-2}{x(2-x)} .$$

For $0 < x < 2, \quad 2 - x > 0$.

$\therefore$ For $0 < x < 2$, $f'(x) < 0$.

Hence $f(x)$ is a monotonic decreasing and has an inverse function.

(b) (ii) (2 marks)
Outcomes Assessed: HE4
Targeted Performance Bands: E3

Criteria	Mark
• Correct inverse function.	2
• Correct change from logarithmic to exponential form.	1

Sample Answer:

Let $y = \log_e(\frac{2-x}{x})$.

The inverse function can be written as $x = \log_e(\frac{2-y}{y})$.

$$\therefore e^x = \frac{2-y}{y} .$$
$$ye^x = 2 - y .$$
$$ye^x + y = 2 .$$
$$y(e^x + 1) = 2 .$$
$$y = \frac{2}{e^x+1} . \qquad \therefore \text{ inverse function is } f^{-1}(x) = \frac{2}{e^x+1} .$$

Question 13 (continued)

(c) (i) (1 mark)
Outcomes Assessed: HE3
Targeted Performance Bands: E3

Criteria	Mark
• Correct answer.	1

Sample Answer:

Centre of motion occurs when the acceleration is zero.

$5 - x = 0$.

$\therefore$ Centre of motion is at $x = 5$.

(c) (ii) (2marks)
Outcomes Assessed: HE3
Targeted Performance Bands: E3

Criteria	Mark
• Correct answer.	2
• Correct expression for v^2.	1

Sample Answer:

$\frac{d}{dx}(\frac{1}{2}v^2) = 5 - x$.

$v^2 = 10x - x^2 + c$ (where c is a constant).

When $x = 4$ $v = \sqrt{3}$.
$\therefore$ $3 = 40 - 16 + c$
$c = -21$.
$\therefore$ $v^2 = 10x - x^2 - 21$.

At the extremes the velocity is equal to zero.
$\therefore x^2 - 10x + 21 = 0$.
$(x-7)(x-3) = 0$.
$x = 7$, $x = 3$.
With the centre of motion at $x = 5$ the amplitude is 2 m.

Question 13 (continued)

(c) (iii) (2 marks)
Outcomes Assessed: HE3
Targeted Performance Bands: E3

Criteria	Mark
• Correct maximum speed.	2
• Realisation that the maximum speed occurs at $x = 5$.	1

Sample Answer:
Maximum speed occurs at the centre of motion $x = 5$.

When $x = 5$
$v^2 = 10 \times 5 - 5^2 - 21$.

$v = 2$.

$\therefore$ Maximum speed is 2 ms^{-1}.

(d) (3 marks)
Outcomes Assessed: HE3, HE7
Targeted Performance Bands: E3-E4

Criteria	Mark
• Correct solution as a fraction or decimal.	3
• Correct sum of terms in binomial coefficients and fractions (or decimals).	2
• Demonstrates the understanding that a sum that is greater than 20 can only be obtained by either drawing a 6 each time, or drawing a 6 three times and a 4 once.	1

Sample Answer:

$P(\text{drawing a } 6) = \frac{2}{5}$.

$P(\text{drawing a } 4) = \frac{3}{5}$.

A sum that is greater than 20 can only be obtained by either drawing a 6 each time, or drawing a 6 three times and a 4 once.

$$\therefore\ P(\text{sum} > 20) = \left(\frac{2}{5}\right)^4 + {}^4C_1\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^1$$

$$= \frac{112}{625}.$$

Question 14 (15 marks)

(a) (i) (1 mark)
Outcomes Assessed: HE4
Targeted Performance Bands: E3-E4

Criteria	Mark
• Correct value of $f(-2)$ is given.	1

Sample Answer:

$$f(-2) = 2\sin^{-1}(1) + \frac{\pi}{2}$$
$$= \frac{3\pi}{2}.$$

(a) (ii) (1 mark)
Outcomes Assessed: HE4
Targeted Performance Bands: E3-E4

Criteria	Mark
• Correct domain is given.	1

Sample Answer:

$$-1 \le \frac{1-x}{3} \le 1.$$

$\therefore\ -3 \le 1 - x \le 3$

$-4 \le -x \le 2$

$4 \ge x \ge -2$

$-2 \le x \le 4$.

$\therefore$ Domain of $f(x)$ is $\{x : -2 \le x \le 4\}$.

(a) (iii) (1 mark)
Outcomes Assessed: HE4
Targeted Performance Bands: E3-E4

Criteria	Mark
• Correct range is given.	1

Sample Answer:

Range of $\sin^{-1}(\frac{1-x}{3})$ is $\{y: -\frac{\pi}{2} \le y \le \frac{\pi}{2}\}$.

$\therefore$ Range of $2\sin^{-1}(\frac{1-x}{3})$ is $\{y: -\pi \le y \le \pi\}$.

$\therefore$ Range of $2\sin^{-1}(\frac{1-x}{3}) + \frac{\pi}{2}$ is $\{y: -\frac{\pi}{2} \le y \le \frac{3\pi}{2}\}$.

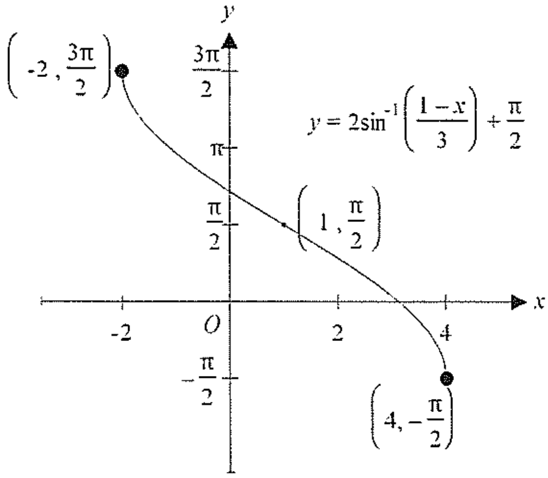
Question 14 (continued)

(a) (iv) (2marks)

Outcomes Assessed: HE7

Targeted Performance Bands: E3-E4

Criteria	Mark
• Correct graph with endpoints and 'centre' of graph.	2
• Significant progress towards a correct sketch.	1



(b) (i) (2 marks)

Outcomes Assessed: HE3

Targeted Performance Bands: E3

Criteria	Mark
• Correct relation between x and y.	2
• Correct expression for t in terms of x.	1

Sample Answer:

$y = -5t^2 + Ut\sin\alpha$. (1)

$x = Ut\cos\alpha$.

$t = \dfrac{x}{U\cos\alpha}$. (2)

Substitute (2) into (1)

$$y = -5\left(\frac{x}{U\cos\alpha}\right)^2 + U\left(\frac{x}{U\cos\alpha}\right)\sin\alpha$$

$$= \frac{-5x^2}{U^2\cos^2\alpha} + x\tan\alpha$$

$$= \frac{-5x^2\sec^2\alpha}{U^2} + x\tan\alpha \text{, as required.}$$

Question 14 (continued)

(b) (ii) (3 marks)
Outcomes Assessed: HE3, HE7
Targeted Performance Bands: E4

Criteria	Mark
• Complete solution.	3
• Correct method of solving simultaneous equations.	2
• Correct simultaneous equations for h in terms of a and b.	1

Sample Answer:

When $x = a, y = h$, hence $h = \dfrac{-5a^2 \sec^2 \alpha}{U^2} + a \tan \alpha$. (1)

When $x = b, y = h$, hence $h = \dfrac{-5b^2 \sec^2 \alpha}{U^2} + b \tan \alpha$. (2)

$(1) \times b^2 \quad \Rightarrow b^2 h = \dfrac{-5a^2 b^2 \sec^2 \alpha}{U^2} + ab^2 \tan \alpha$. (3)

$(2) \times a^2 \quad \Rightarrow a^2 h = \dfrac{-5a^2 b^2 \sec^2 \alpha}{U^2} + a^2 b \tan \alpha$. (4)

$(4) - (3) \ \Rightarrow h(a^2 - b^2) = ab \tan \alpha (a - b)$.

$$\therefore \tan \alpha = \frac{h(a-b)(a+b)}{ab(a-b)}$$

$$= \frac{h(a+b)}{ab} \text{, as required.}$$

(b) (iii) (1 mark)
Outcomes Assessed: HE3
Targeted Performance Bands: E2

Criteria	Mark
• Correct answer.	1

Sample Answer:

$h = 20, a = 40, b = 80$.

$$\tan \alpha = \frac{h(a+b)}{ab}$$

$$= \frac{20(40+80)}{40 \times 80}$$

$$= \frac{3}{4}.$$

$$\alpha = 36.869^\circ$$

$$\approx 37^\circ.$$

$\therefore$ Required angle of projection is 37°, to the nearest degree.

Question 14 (continued)

(c) (i) (2 marks)
Outcomes Assessed: PE6,HE7
Targeted Performance Bands: E4

Criteria	Mark
• Correct proof.	2
• Correct coefficients in the term involving x^n on one side of the identity.	1

Sample Answer:
Consider the term involving x^n on the left hand side of the identity.
This term is
$${}^nC_nx^n + {}^{n+1}C_nx^n + ... + {}^{n+m}C_nx^n \quad = ({}^nC_n + {}^{n+1}C_n + ... + {}^{n+m}C_n)x^n.$$
$\therefore$ the coefficient is ${}^nC_n + {}^{n+1}C_n + ... + {}^{n+m}C_n$.

The term involving x^n on the right hand side will involve the term with x^{n+1} on the numerator.
This must come from $(1+x)^{n+m+1}$ since it's not able to come from $(1+x)^n$.

Hence, the required term involving x^n is ${}^{n+m+1}C_{n+1}x^n$ with coefficient ${}^{n+m+1}C_{n+1}$.

Hence, ${}^nC_n + {}^{n+1}C_n + ... + {}^{n+m}C_n = {}^{n+m+1}C_{n+1}$, due to the identity.

Question 14 (continued)

(c) (ii) (2 marks)
Outcomes Assessed: PE6,HE7
Targeted Performance Bands: E4

Criteria	Mark
• Correct proof.	2
• Significant progress towards the proof.	1

Sample Answer:
Put $n = 4$ into statement of part (i).

$$\therefore {}^{4}C_4 + {}^{5}C_4 + \ldots.. + {}^{m+4}C_4 = {}^{4+m+1}C_{4+1}.$$

$$\therefore {}^{5}C_4 + {}^{6}C_4 + \ldots + {}^{m+4}C_4 = {}^{m+5}C_5 - {}^{4}C_4.$$

$$\therefore \sum_{r=5}^{m+4} {}^{r}C_4 = {}^{m+5}C_5 - {}^{4}C_4.$$

$$\therefore \sum_{r=5}^{m+4} \frac{r!}{(r-4)!4!} = {}^{m+5}C_5 - 1.$$

$$\therefore \frac{1}{4!}\sum_{r=5}^{m+4} r(r-1)(r-2)(r-3) = {}^{m+5}C_5 - 1.$$

$$\therefore \sum_{r=5}^{m+4} r(r-1)(r-2)(r-3) = 4!({}^{m+5}C_5 - 1)$$

$$= 24({}^{m+5}C_5 - 1)\text{, as required.}$$