SOLUTIONS EXT 1 TRIAL 2002 CRANBROOK

($\checkmark$ = 1 MARK) HMX1 HRK

Q1 a) (i) (2) $T_5 = \frac{1}{(2(5)-1)(10+1)}$

$= \frac{1}{99}$ $\checkmark$

$T_{k+1} = \frac{1}{(2k+1)(2k+3)}$ $\checkmark$

ii) (3)

$S_{k+1} = S_k + T_{k+1}$

$= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$ $\checkmark$

$= \frac{k(2k+3)+1}{(2k+1)(2k+3)}$

$= \frac{2k^2+3k+1}{(2k+1)(2k+3)}$ $\checkmark$

$= \frac{\cancel{(2k+1)}(k+1)}{\cancel{(2k+1)}(2k+3)}$

$= \frac{k+1}{2k+3}$ $\checkmark$

iii) (3) Let $n = 1$

$T_1 = \frac{1}{(2-1)(2+1)}$ $\quad S_1 = \frac{1}{2(1)+1}$

$= \frac{1}{3}$ $\quad = \frac{1}{3}$

$\therefore$ true for $n=1$ $\checkmark$

assume true for $n = k$

ie $S_k = \frac{k}{2k+1}$

Prove true for $n = k+1$

ie $S_k + T_{k+1} = S_{k+1}$

$LHS = \frac{k+1}{2k+3}$ from (ii) $\checkmark$

$RHS = \frac{k}{\ldots} = LHS$

$\therefore$ if true for $n = k$ then true for $n = k+1$

But it is true for $n=1$

$\therefore$ it is true for $n = 1+1 = 2$

& since true for 2 it is true for $n = 3$ & so on

$\therefore S_n = \frac{n}{2n+1}$ is proven true by M.I. $\checkmark$

b) (4) Prove true for $n=1$

$5^n \geq 1+4n$

$5^1 = 5 \quad 1+4(1) = 5$

$\therefore 5^1 \geq 1+4(1)$ $\checkmark$

ie true for $n=1$

Assume true for $n=k$

ie $5^k \geq 1+4k$

Prove true for $n=k+1$ $\checkmark$

ie $5^{k+1} \geq 1+4(k+1)$

$5^{k+1} \geq 5+4k$

using assumption

$5^k \geq 1+4k$

$\times 5$

$5 \times 5^k \geq 5(1+4k)$

$\therefore 5^{k+1} \geq 5+20k$ $\checkmark$

but $5+20k \geq 1+4k$

ie $16k \geq -4$

$\therefore 5^{k+1} \geq 1+4k$

$\therefore$ if true for .... $\checkmark$

See (iii) above