

NSW INDEPENDENT TRIAL EXAMS – 2009
MATHEMATICS HSC TRIAL EXAMINATION
MARKING GUIDELINES

Question 1

a) 2 marks

Outcomes assessed: - P3, Targeted Performance Band:- 2

Criteria	Marks
• Correct evaluation of expression.	1
• Correct answer correct to 2 significant figures	1

Sample Answer:-

$$\sqrt{\frac{627}{42+29}} = 2.97169$$

$$= 3.0$$

b) 2 marks

Outcomes assessed: - P3, Targeted Performance Band:- 2

Criteria	Marks
• Correct expansion of expression.	1
• Correct values of a and b .	1

Sample Answer:-

$$(2\sqrt{3}-1)^2 = 12 - 4\sqrt{3} + 1$$

$$= 13 - 4\sqrt{3}$$

$$a = -4$$

$$b = 13$$

c) 2 marks

Outcomes assessed: - P3, Targeted Performance Band:- 2

Criteria	Marks
• Correct multiplication of both sides of the equation by 15.	1
• Correct solution.	1

Sample Answer:-

$$\frac{2x-1}{3} - \frac{1-3x}{5} = 2$$

$$10x - 5 - 3 + 9x = 30$$

$$19x = 38$$

$$x = 2$$

d) 2 marks

Outcomes assessed: - H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct integration of $\sin 2x$	1
• Correct answer including c .	1

Sample Answer:-

$$\int (3 + \sin 2x) dx = 3x - \frac{1}{2} \cos 2x + c$$

e) 2 marks

Outcomes assessed: - P3, Targeted Performance Band:- 3

Criteria	Marks
• Attempting to solve two inequalities and getting one correct	1
• Correct solution.	1

Sample Answer:-

$$|2 - 3x| > 11$$

$$-11 > 2 - 3x$$

$$2 - 3x > 11$$

$$3x > 13$$

$$3x < -9$$

$$x > \frac{13}{3}$$

$$x < -3$$

f) 2 marks

Outcomes assessed: - P3, Targeted Performance Band:- 2

Criteria	Marks
• Correct factorization into two factors.	1
• Correct factorization into three factors.	1

Sample Answer:-

$$x^4 - 16 = (x^2 - 4)(x^2 + 4)$$

$$= (x - 2)(x + 2)(x^2 + 4)$$

Question 2 (12 marks)

a) (i) 2 marks

Outcomes assessed: - P7, Targeted Performance Band:- 2

Criteria	Marks
• Correct application of the product rule with correct differentiation of $\tan x$.	1
• Correct solution.	1

Sample Answer:-

$$\frac{d}{dx}(e^{2x} \tan x) = 2e^{2x} \tan x + e^{2x} \sec^2 x$$

$$= e^{2x}(2 \tan x + \sec^2 x)$$

a) (ii) 2 marks

Outcomes assessed: - P7, Targeted Performance Band:- 2

Criteria	Marks
• Correct application of the quotient rule.	1
• Correct solution.	1

Sample Answer:-

$$\frac{d}{dx}\left(\frac{\sin x}{4 - x}\right) = \frac{(4 - x) \cos x - \sin x(-1)}{(4 - x)^2}$$

$$= \frac{(4 - x) \cos x + \sin x}{(4 - x)^2}$$

b) 2 marks

Outcomes assessed: - H3, Targeted Performance Band:- 3

Criteria	Marks
• Correct differentiation and gradient of tangent.	1
• Correct gradient of normal and equation of normal.	1

Sample Answer:-

$$y = \log_e x - 1$$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\text{at } x = e, m_t = \frac{1}{e}$$

$$m_n = -e \text{ (Gradient of normal)}$$

equation of normal at point $(e, 0)$

$$y - 0 = -e(x - e)$$

$$y = e^2 - ex$$

c) (i) 2 marks

Outcomes assessed: - P8, Targeted Performance Band:- 3

Criteria	Marks
• Write integral as a power.	1
• Correctly evaluates integral (constant c not necessary)	1

Sample Answer:-

$$\int \frac{3}{\sqrt{2x-1}} dx = 3 \int (2x-1)^{-\frac{1}{2}} dx$$

$$= 3 \frac{(2x-1)^{\frac{1}{2}}}{\frac{1}{2} \times 2} + c$$

$$= 3\sqrt{2x-1} + c$$

c) (ii) 2 marks

Outcomes assessed: - H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct integral	1
• Correct evaluation of definite integral	1

Sample Answer:-

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{9}} \sec^2 3x dx = \frac{1}{3} [\tan 3x]_{\frac{\pi}{12}}^{\frac{\pi}{9}}$$

$$= \frac{1}{3} \left(\tan \frac{\pi}{3} - \tan \frac{\pi}{4} \right)$$

$$= \frac{1}{3} (\sqrt{3} - 1)$$

d) 2 marks

Outcomes assessed: - P4, Targeted Performance Band:- 2

Criteria	Marks
• Correct rearrangement of trigonometric equation and one correct solution.	1
• Two correct solutions.	1

Sample Answer:-

$$2\sin\theta + 1 = 0$$

$$2\sin\theta = -1$$

$$\sin\theta = -\frac{1}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

Question 3 (12 marks)

a) (i) 1 mark

Outcomes assessed: - P4, Targeted Performance Band:- 2

Criteria	Marks
• Correct length of interval AB .	1

Sample Answer:-

$$d = \sqrt{(2 - -1)^2 + (4 - 0)^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$= 5$$

a) (ii) 1 mark

Outcomes assessed: - P4, Targeted Performance Band:- 2

Criteria	Marks
• Correct equation of circle.	1

Sample Answer:-

$$(x - 2)^2 + (y - 4)^2 = 25$$

a) (iii) 2 marks

Outcomes assessed: - P4, Targeted Performance Band:- 2

Criteria	Marks
• Correct acute angle.	1
• Correct answer (ignore accuracy)	1

Sample Answer:-

$$m = \frac{4-0}{2-(-1)}$$

$$= \frac{4}{3}$$

$$\tan \theta = \frac{4}{3}$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\theta = 53^\circ$$

$$\angle CAB = 180 - 53$$

$$= 127^\circ$$

a) (iv) 1 mark

Outcomes assessed: - P4, Targeted Performance Band:- 2

Criteria	Marks
• Correct midpoint.	1

Sample Answer:-

$$\text{Midpoint} = \left(\frac{2+(-6)}{2}, \frac{4+0}{2} \right)$$

$$= (-2, 2)$$

a) (v) 1 mark

Outcomes assessed: - P4, Targeted Performance Band:- 2

Criteria	Marks
• Correctly shown	1

Sample Answer:-

$$\frac{y-0}{4-0} = \frac{x+6}{2+6}$$

$$\frac{y}{4} = \frac{x+6}{8}$$

$$2y = x+6$$

$$x-2y+6=0$$

a) (vi) 2 marks

Outcomes assessed: - P4, Targeted Performance Band:- 2

Criteria	Marks
• Correct substitution into correct formula.	1
• Correct answer	1

Sample Answer:-

$$d = \frac{|(-1) - 2(0) + 6|}{\sqrt{1+2^2}}$$

$$= \frac{5}{\sqrt{5}}$$

$$= \sqrt{5}$$

a) (vii) 2 marks

Outcomes assessed: - P4, Targeted Performance Band:- 3

Criteria	Marks
• Correctly identifies rhombus.	1
• Correct reasoning	1

Sample Answer:-

Rhombus.

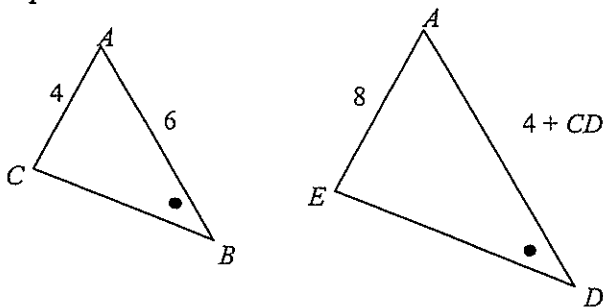
Diagonals bisect (common midpoint D) and two adjacent sides equal ($AB = AC = 5$).

b) 2 marks

Outcomes assessed: - P4, H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct algebraic/numerical expression involving corresponding sides.	1
• Correct answer	1

Sample Answer:-



$$\frac{4}{8} = \frac{6}{4+CD}$$

$$\frac{1}{2} = \frac{6}{4+CD}$$

$$4+CD = 12$$

$$CD = 8$$

Question 4 (12 marks)

a) 3 marks

Outcomes assessed: - H5, Targeted Performance Band:- 3

Criteria	Marks
• Correctly generating the series	1
• Recognising an AP and correct number of terms	1
• Correct solution	1

Sample Answer:-

$$\sum_{x=2}^{20} (3x-5) = 1+4+7+\dots\dots\dots +55$$

$$n = 19$$

$$S_{19} = \frac{19}{2}(1+55)$$

$$= 532$$

b) (i) 2 marks

Outcomes assessed: - P4, Targeted Performance Band:- 3

Criteria	Marks
• Correctly completing the square	1
• Correctly shown	1

Sample Answer:-

$$4y = x^2 - 2x + 5$$

$$x^2 - 2x = 4y - 5$$

$$x^2 - 2x + 1 = 4y - 4$$

$$(x-1)^2 = 4(y-1)$$

$$\therefore \text{Vertex} = (1, 1)$$

b) (ii) 2 marks

Outcomes assessed: - P4

Targeted Performance Band:- 3

Criteria	Marks
• Correct value of a	1
• Correct Focus	1

Sample Answer:-

$$a = 1$$

$$\text{Focus} = (1, 2)$$

c) (i) 1 mark

Outcomes assessed: - H5, Targeted Performance Band:- 2

Criteria	Marks
• Correct answer	1

Sample Answer:-

$$l = \theta r$$

$$10 = 15\theta$$

$$\theta = \frac{2}{3}$$

c) (ii) 1 mark

Outcomes assessed: - H5, Targeted Performance Band:- 2

Criteria	Marks
• Correct answer for area.	1

Sample Answer:-

$$A = \frac{1}{2} \theta r^2$$

$$= \frac{1}{2} \times \frac{2}{3} \times 15^2$$

$$= 75\text{cm}^2$$

d) 3 marks

Outcomes assessed: - H2, H5, Targeted Performance Bands:- 3/4

Criteria	Marks
• Correct conclusion of $\angle ACB = x$	1
• Correct conclusion of two expressions for $\angle BDC$ (must have second expression)	1
• Correct answer	1

Sample Answer:-

$$\angle DBC = x$$

$$\angle ABD = 3x$$

$$\angle ABC = 4x \quad (\angle DBC + \angle ABD)$$

$$\angle ACB = 4x \quad (\text{base angle of isosceles } \triangle ABC)$$

$$\angle BDC = 4x \quad (\text{base angle of isosceles } \triangle BCD)$$

$$\angle BDC = 180 - (x + 4x) \quad (\text{angle sum of } \triangle BCD)$$

$$= 180 - 5x$$

$$4x = 180 - 5x \quad (= \angle BDC)$$

$$9x = 180$$

$$x = 20^\circ$$

Question 5 (12 marks)

a) 2 marks

Outcomes assessed: - H5, Targeted Performance Bands:- 3

Criteria	Marks
• Correct value of a	1
• Correct value of n	1

Sample Answer:-

From graph $a = 2$

$$\text{Period} = \frac{4\pi}{3}$$

$$\text{Period} = \frac{2\pi}{n}$$

$$\frac{2\pi}{n} = \frac{4\pi}{3}$$

$$n = \frac{3}{2}$$

b) (i) 2 marks

Outcomes assessed: - H5, Targeted Performance Bands:- 2

Criteria	Marks
• Correct substitution into a correct formula	1
• Correct answer	1

Sample Answer:-

$$A = \frac{1}{3}(6 + 2 + 4(10 + 8) + 2 \times 11)$$

$$= \frac{1}{3} \times 102$$

$$= 34$$

b) (ii) 1 marks

Outcomes assessed: - H5, Targeted Performance Band:- 4

Criteria	Marks
• Correct answer	1

Sample Answer:-

Distance travelled by the particle in the first 4 seconds of motion.

c) 4 marks

Outcomes assessed: - H5, Targeted Performance Band:- 2/3

Criteria	Marks
• Correct angle at O	1
• Correct use of cosine rule for distance traveled PQ	1
• Correct answer ms^{-1} (Or changing units before use of cosine rule)	1
• Correct answer km/h	1

Sample Answer:-

$$\angle O = 73 - 7$$

$$= 66^\circ$$

$$d^2 = 750^2 + 3000^2 - 2 \times 750 \times 3000 \cos 66$$

$$= 7732185.106$$

$$d = 2780.680691$$

$$\text{Speed} = \frac{2780.680691}{8}$$

$$= 347.5850864 \text{ m/s}$$

$$= 347.5850864 \times 60 \times 60 \div 1000$$

$$= 1251.3063$$

$$= 1251 \text{ km/h}$$

$$d^2 = 0.75^2 + 3^2 - 2 \times 0.75 \times 3 \cos 66$$

$$= 7.732185106$$

$$= 2.780680691$$

$$\text{Speed} = \frac{2.780680691}{(8 / (60 \times 60))}$$

$$= 1251.306311$$

$$= 1251 \text{ km/h}$$

d) 3 marks

Outcomes assessed: - H3, H5, Targeted Performance Band:- 3/4

Criteria	Marks
• Correct integration of function	1
• Correct substitution of point to find c	1
• Correct evaluation of c and equation of curve.	1

Sample Answer:-

$$\frac{dy}{dx} = \frac{2x}{x^2 + e}$$

$$y = \ln(x^2 + e) + c$$

Sub the point (0, 2)

$$2 = \ln(e) + c$$

$$c = 1$$

$$y = \ln(x^2 + e) + 1$$

Question 6 (12 marks)

a) (i) 3 marks

Outcomes assessed: - H6, Targeted Performance Band:- 2

Criteria	Marks
• Correct differentiation and evaluation of x values of turning points.	1
• Correct turning points.	1
• Correct determination of nature of turning points using a recognized method.	1

Sample Answer:-

$$f(x) = x^3 - 3x^2 - 9x + 22$$

$$f'(x) = 3x^2 - 6x - 9$$

Stationary points occur when $f'(x) = 0$

$$0 = 3x^2 - 6x - 9$$

$$0 = x^2 - 2x - 3$$

$$0 = (x-3)(x+1)$$

$$x = -1, 3$$

$$f(-1) = -1 - 3 + 9 + 22$$

$$= 27$$

$$f(3) = 27 - 27 - 27 + 22$$

$$= -5$$

Turning points $(-1, 27), (3, -5)$

Test nature of points.

$$f''(x) = 6x - 6$$

Test $(-1, 27)$

$$f''(-1) = -12 < 0 \therefore \text{concave down. } (-1, 27) \text{ is a maximum}$$

Test $(3, -5)$

$$f''(3) = 12 > 0 \therefore \text{concave up. } (3, -5) \text{ is a minimum.}$$

a) (ii) 2 marks

Outcomes assessed: - H6, Targeted Performance Band:- 2

Criteria	Marks
• Correct identification of a possible point of inflexion	1
• Correct test and confirmation of point of inflexion	1

Sample Answer:-

$$f''(x) = 6x - 6$$

Possible points of inflexion occur when $f''(x) = 0$

$$6x - 6 = 0$$

$$6x = 6$$

$$x = 1$$

$$f(1) = 1 - 3 - 9 + 22$$

$$= 11$$

Possible point of inflexion (1, 11)

Test point of inflexion.

x	-1	1	3
$f''(x)$	-12	0	12

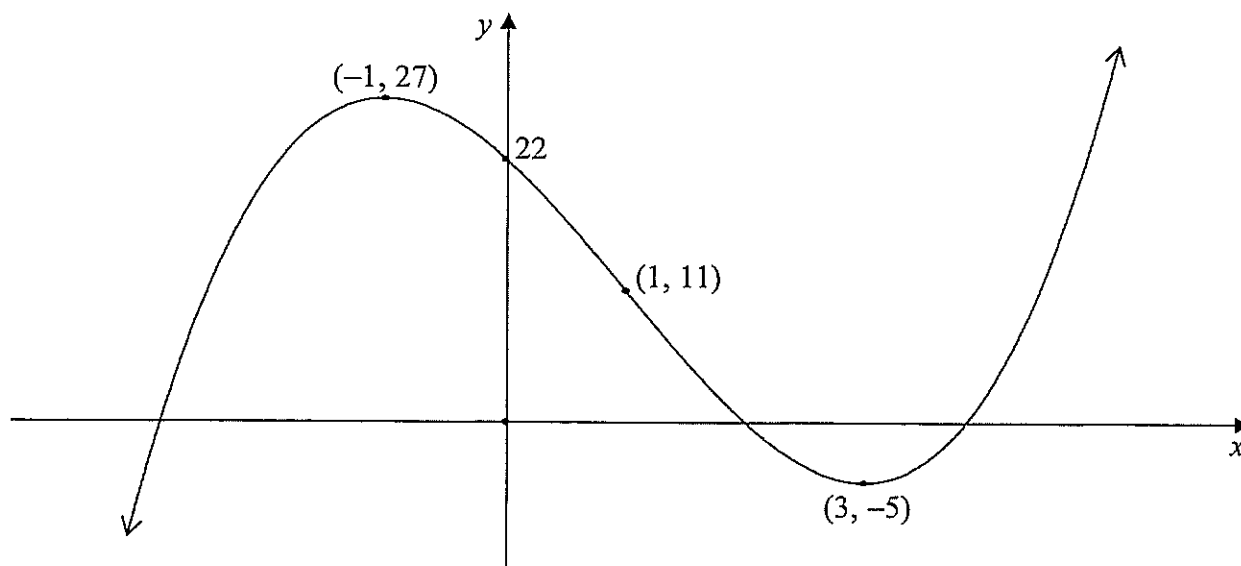
Change in concavity, $\therefore (1, 11)$ is a point of inflexion.

a) (iii) 2 marks

Outcomes assessed: - H6, Targeted Performance Band:- 3

Criteria	Marks
• Correct shape graph	1
• Correct graph with all 4 points.	1

Sample Answer:-



a) (iv) 1 marks

Outcomes assessed: - H6, Targeted Performance Band:- 2

Criteria	Marks
• Correct answer	1

Sample Answer:-

$$x > 1$$

b) 2 marks

Outcomes assessed: - P6, Targeted Performance Band:- 2

Criteria	Marks
• Correct discriminant and assumption $\Delta \geq 0$	1
• Correct solution of inequality.	1

Sample Answer:-

For real roots $\Delta \geq 0$.

$$\Delta = k^2 - 4 \times 2 \times 5$$

$$k^2 - 40 \geq 0$$

$$k^2 \geq 40$$

$$k \leq -2\sqrt{10} \text{ or } k \geq 2\sqrt{10}$$

c) 2 marks

Outcomes assessed: - H4, Targeted Performance Band:- 4/5

Criteria	Marks
• Correct probability for not surviving.	1
• Correct solution.	1

Sample Answer:-

$$P(\text{Die}) = 1 - 0.35$$

$$= 0.65$$

$$P(\text{at least one survives}) = 1 - P(\text{all die})$$

$$= 1 - (0.65)^5$$

$$= 0.8839709375$$

Question 7 (12 marks)

a) (i) 2 marks

Outcomes assessed: - H3, H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct differentiation of $\log_e f(x)$	1
• Correct differentiation of $\cos x$ and simplification to $-\tan x$	1

Sample Answer:-

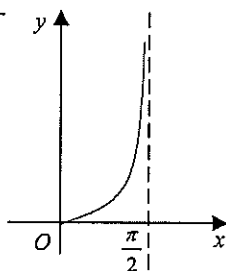
$$\begin{aligned}\frac{d}{dx} \log_e (\cos x) &= \frac{-\sin x}{\cos x} \\ &= -\tan x\end{aligned}$$

a) (ii) 1 marks

Outcomes assessed: - H5, Targeted Performance Band:- 2

Criteria	Marks
• Correct graph	1

Sample Answer:-

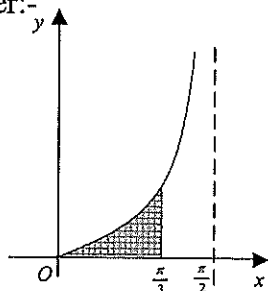


a) (iii) 3 marks

Outcomes assessed: - H3, H8, Targeted Performance Band:- 4

Criteria	Marks
• Correct statement for area and correct integration	1
• Correct solution to give $-\log_e\left(\frac{1}{2}\right)$	1
• Correct solution in correct form. $\log_e(2)$	1

Sample Answer:-



$$A = \int_0^{\pi/3} \tan x \, dx$$

$$= -\left[\ln(\cos x)\right]_0^{\pi/3}$$

$$= -\left(\ln\left(\cos\frac{\pi}{3}\right) - \ln(\cos 0)\right)$$

$$= -\left(\ln\left(\frac{1}{2}\right) - \ln 1\right)$$

$$= -\ln\left(\frac{1}{2}\right)$$

$$= \ln 2$$

b) 3 marks

Outcomes assessed: - P5, Targeted Performance Band:- 3

Criteria	Marks
• Correct domain	1
• $0 \leq y \leq 3$ (i.e. ignore 2)	1
• Correct range	1

Sample Answer:-

Domain: -

$$9 - x^2 \geq 0$$

$$x^2 \leq 9$$

$$-3 \leq x \leq 3$$

Range:-

$$0 \leq y \leq 6$$

c) 3 marks

Outcomes assessed: - H3, H8, Targeted Performance Band:- 4/5

Criteria	Marks
• Correctly changes subject of formula to $x = e^y$	1
• Correct expression for volume	1
• Correct solution	1

Sample Answer:-

$$x = e^y$$

$$V = \pi \int_0^2 e^{2y} dy$$

$$= \frac{\pi}{2} [e^{2y}]_0^2$$

$$= \frac{\pi}{2} [e^4 - e^0]$$

$$= \frac{\pi}{2} (e^4 - 1) \text{ units}^3$$

Question 8 (12 marks)

a) (i) 2 marks

Outcomes assessed: - H5

Targeted Performance Band:- 3

Criteria	Marks
• Correct value for r .	1
• Correct solution	1

Sample Answer:-

$$3 - 6y + 12y^2 - 24y^3 \dots\dots\dots$$

$$r = -2y$$

$$|-2y| < 1$$

$$-1 < 2y < 1$$

$$-\frac{1}{2} < y < \frac{1}{2}$$

a) (ii) 2 marks

Outcomes assessed: - H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct expression for $S_{\infty} = 2\frac{1}{4}$	1
• Correct solution	1

Sample Answer:-

$$S_{\infty} = \frac{3}{1+2y}$$

$$\frac{9}{4} = \frac{3}{1+2y}$$

$$9+18y=12$$

$$18y=3$$

$$y = \frac{1}{6}$$

b) (i) 1 marks

Outcomes assessed: - H3, H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct solution	1

Sample Answer:-

$$A = 20$$

b) (ii) 2 marks

Outcomes assessed: - H3, H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct substitution into the formula	1
• Correct solution (Ignore decimal places)	1

Sample Answer:-

$$G = 20e^{-0.00005d}$$

$$= 20e^{-0.00005 \times 2000}$$

$$= 18.10ms^{-2}$$

b) (iii) 2 marks

Outcomes assessed: - H3, H5, Targeted Performance Band:- 4/5

Criteria	Marks
• Correct substitution into the formula	1
• Correct expression $\ln\left(\frac{3}{4}\right) > -0.00005d$	1
• Correct solution	1

Sample Answer:-

$$15 > 20e^{-0.00005d}$$

$$\frac{3}{4} > e^{-0.00005d}$$

$$\ln\left(\frac{3}{4}\right) > -0.00005d$$

$$d > \ln\left(\frac{3}{4}\right) \div (-0.00005)$$

$$d > 5753.64119$$

$$d = 5800 \text{ metres}$$

b) (iv) 2 marks

Outcomes assessed: - H3, H5, Targeted Performance Band:- 3

Criteria	Marks
• Correctly recognizes the use of the derivative	1
• Correct solution (ignore units)	1

Sample Answer:-

$$G = 20e^{-0.00005d}$$

$$\frac{dG}{dd} = -0.001e^{-0.00005d}$$

$$\text{at } d = 2000$$

$$\frac{dG}{dd} = -0.001e^{-0.00005 \times 2000}$$

$$= -0.001e^{-0.1}$$

$$= -0.0009 \text{ ms}^{-2} \text{ per metre.}$$

At 2000 m, the gravitational pull is decreasing at a rate of 0.0009 ms^{-2} per metre.

Question 9 (12 marks)

a) (i) 2 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4/5

Criteria	Marks
• Correct integration of derivative	1
• Correct equation for volume of water	1

Sample Answer:-

$$\frac{dV}{dt} = \frac{1}{8}t - 35$$

$$V = \frac{1}{16}t^2 - 35t + c$$

At $t = 0$, $V = 4900$

$$4900 = \frac{1}{16}(0)^2 - 35(0) + c$$

$$c = 4900$$

$$V = \frac{1}{16}t^2 - 35t + 4900$$

a) (ii) 1 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 3

Criteria	Marks
• Correct solution	1

Sample Answer:-

At $t = 30$

$$V = \frac{1}{16}(30)^2 - 35(30) + 4900$$

$$= 3906.25 \text{ litres.}$$

a) (iii) 1 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4

Criteria	Marks
• Correct solution	1

Sample Answer:-

At $t = 16$

$$\frac{dV}{dt} = \frac{1}{8}(16) - 35$$

$$= -33$$

Water is flowing out of the tank at the rate of 33 litres per minute.

a) (iv) 2 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4/5

Criteria	Marks
• Correct interpretation using either $\frac{dV}{dt} = 0$ or $V = 0$	1
• Correct solution for either method.	1

Sample Answer:-

$$\begin{aligned} \frac{dV}{dt} &= 0 & V &= 0 \\ \frac{1}{8}t - 35 &= 0 & \frac{1}{16}t^2 - 35t + 4900 &= 0 \\ \frac{1}{8}t &= 35 & t &= \frac{35 \pm \sqrt{1225 - 1225}}{2\left(\frac{1}{16}\right)} \\ t &= 280 \text{ minutes} & t &= 280 \text{ minutes} \end{aligned}$$

b) (i) 1 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 5

Criteria	Marks
• Correct solution.	1

Sample Answer:-

$$\begin{aligned} \text{Jonah deposited} &= 31 \times 400 \\ &= \$12400 \end{aligned}$$

b) (ii) 3 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4/5

Criteria	Marks
• Correctly identifying G.P and 31 terms	1
• Correct substitution into correct formula	1
• Correct solution.	1

Sample Answer:-

$$\begin{aligned} S &= 400(1.004) + 400(1.004)^2 + 400(1.004)^3 + \dots + 400(1.004)^{31} \\ \text{G.P, } a &= 400(1.004), r = 1.004, n = 31 \\ S &= \frac{400(1.004)(1.004^{31} - 1)}{1.004 - 1} \\ &= 13226.28557 \\ &= \$13226 \end{aligned}$$

b) (iii) 2 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 2/3

Criteria	Marks
• Correct formula and interest rate.	1
• Correct solution.	1

Sample Answer:-

$$A = P \left(1 + \frac{r}{100} \right)^n, P = 10000, r = 5/4 = 1\frac{1}{4}\%, n = 8$$

$$\begin{aligned} A &= 10000(1.0125)^8 \\ &= 11044.86101 \\ &= \$11045 \end{aligned}$$

Question 10 (12 marks)

a) (i) 1 mark

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4

Criteria	Marks
• Correct integration of derivative	1

Sample Answer:-

At $t = 0$

$$x = 0 + \frac{16}{1}$$

= 16 metres to the right of O .

a) (ii) 2 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4

Criteria	Marks
• Correct velocity equation	1
• Correct acceleration equation	1

Sample Answer:-

$$x = t + \frac{16}{t+1}$$

$$= t + 16(t+1)^{-1}$$

$$\dot{x} = 1 - 16(t+1)^{-2}$$

$$= 1 - \frac{16}{(t+1)^2}$$

$$\ddot{x} = 32(t+1)^{-3}$$

$$= \frac{32}{(t+1)^3}$$

a) (iii) 2 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4

Criteria	Marks
• Correctly equating velocity to 0 and finding time t	1
• Correctly find where particle is at rest.	1

Sample Answer:-

$$\dot{x} = 1 - \frac{16}{(t+1)^2}$$

$$0 = 1 - \frac{16}{(t+1)^2}$$

$$1 = \frac{16}{(t+1)^2}$$

$$(t+1)^2 = 16$$

$$t+1 = \pm 4$$

$$t = 3 \text{ seconds}$$

$$x = t + \frac{16}{t+1}$$

$$x = 3 + \frac{16}{4}$$

$$x = 3 + 4$$

$x = 7$ metres to the right of the origin.

a) (iv) 1 mark

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4

Criteria	Marks
• Correct answer	1

Sample Answer:-

As $t \rightarrow \infty$

$$\dot{x} = 1 - \frac{16}{(\infty+1)^2}$$

$$\dot{x} = 1 \text{ ms}^{-1}$$

b) (i) 1 mark

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4

Criteria	Marks
• Correct answer	1

Sample Answer:-

$$\text{Base radius} = \sqrt{r^2 - x^2}$$

b) (ii) 2 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 4

Criteria	Marks
• Correct height of $2x$	1
• Correct formula	1

Sample Answer:-

$$V = \pi (\text{Base Radius})^2 \times \text{height}$$

$$= \pi (r^2 - x^2) \times 2x$$

$$= 2\pi x (r^2 - x^2)$$

b) (iii) 3 marks

Outcomes assessed: - H4, H5, Targeted Performance Band:- 5/6

Criteria	Marks
• Equating derivative to 0 and finding x .	1
• Testing maximum	1
• Correct maximum volume.	1

Sample Answer:-

$$V = 2\pi (xr^2 - x^3)$$

$$\frac{dV}{dx} = 2\pi (r^2 - 3x^2)$$

Maximum occurs when $\frac{dV}{dx} = 0$

$$0 = 2\pi (r^2 - 3x^2)$$

$$3x^2 = r^2$$

$$x^2 = \frac{r^2}{3}$$

$$x = \frac{r}{\sqrt{3}}$$

Test for Maximum

$$\frac{d^2V}{dx^2} = -12\pi x$$

$$\text{at } x = \frac{r}{\sqrt{3}}$$

$$\frac{d^2V}{dx^2} = -\frac{12\pi r}{\sqrt{3}} < 0 \quad \therefore \text{ a maximum volume when } x = \frac{r}{\sqrt{3}}$$

Maximum Volume

$$V = 2\pi \cdot \frac{r}{\sqrt{3}} \left(r^2 - \frac{r^2}{3} \right)$$

$$= \frac{2\pi r}{\sqrt{3}} \cdot \frac{2r^2}{3}$$

$$= \frac{4\pi r^3}{3\sqrt{3}}$$

$$= \frac{4\sqrt{3}\pi r^3}{9} \text{ cubic units}$$

