

HORNSBY GIRLS HIGH SCHOOL

MATHEMATICS

YEAR 12 TRIAL EXAMINATION

2001

*Time Allowed – 3 hours
(Plus 5 minutes reading time)*

DIRECTIONS TO CANDIDATES

- a) Attempt all 10 questions.
- b) Start each question on a new page.
- c) All questions are of equal value.
- d) All necessary working should be shown. Marks may be deducted for careless or badly arranged work.
- e) Board-approved calculators may be used.

Question 1

- a) Evaluate $\left[\frac{\sqrt{3.12 + 6.9}}{5.03 - 2.9} \right]^3$ correct to two decimal places 1
- b) Evaluate $2|-2|^2 - 2^0$ 1
- c) Rationalise the denominator of $\frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} + \sqrt{2}}$ 2
- d) Express $3x^2 + 27x + 60$ as the product of three factors 2
- e) Solve $6 - (x - 4) = x$ 2
- f) The value of a new car decreased by 12% or \$1500 in one year. What was the original value of the car? 2
- g) If $v^2 = u^2 + 2as$ find all possible values of u when $v = 35$, $a = 9.8$ and $s = 25$ (correct one decimal place). 2

Question 2 (Start a new page)

(a) Differentiate

5

i) $8x^5 - 7x^{-5}$

ii) $\sin 5x$

iii) $\frac{2x}{\log_e 2x}$

(b) Evaluate

5

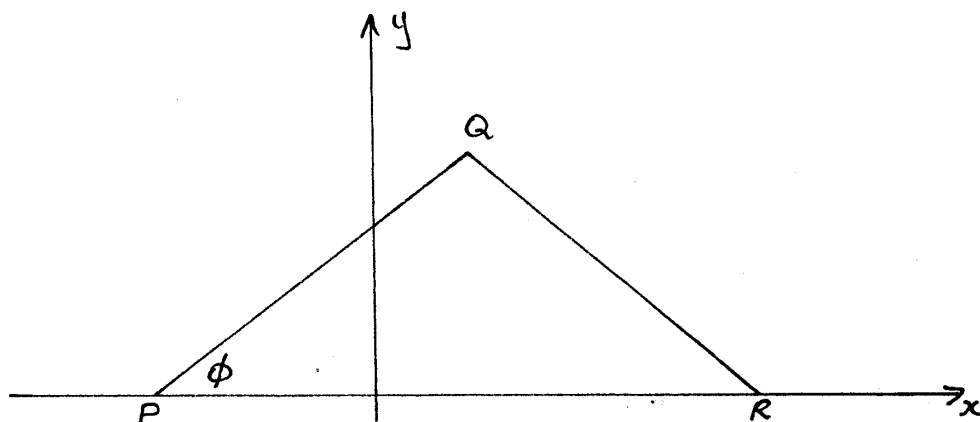
i) $\int_0^5 4e^{2x} dx$

ii) $\int_{-1}^1 (2x+7)^4 dx$

(c) Find $\int \frac{5y}{y^2+8} dy$

2

Question 3 (Start a new page)



In the diagram P , Q and R have coordinates $(-5,0)$, $(1,7)$ and $(7,0)$ and $\angle QPR = \phi$.

a) Find the equation of the line PQ in general form

2

b) Find the mid-point, M , of the interval QR

1

c) Find the distance PQ

2

d) Find the perpendicular distance from the point M to the line passing through the points P and Q

2

e) Find the exact area of the triangle PQM

2

f) Find the value of $\tan \phi$

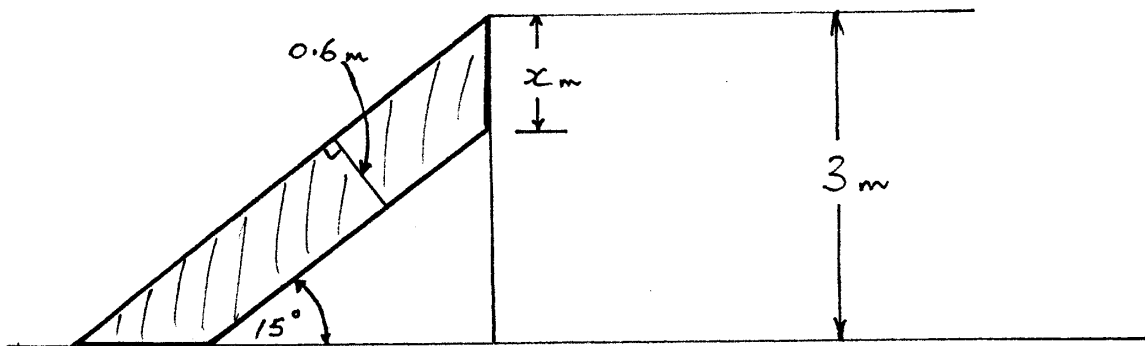
2

g) Prove that the point $(-2,4)$ does not lie on the interval PQ

1

Question 4 (Start a new page)

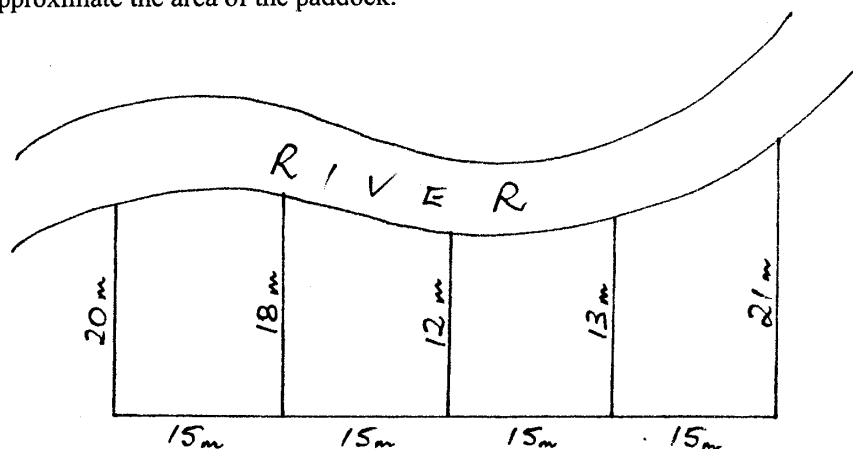
- A) The diagram shows a ramp, inclined at 15 degrees to the horizontal, that a builder is to build to allow cars to drive up a level, 3 metres high, in a car park. The cross-section of the ramp is in the shape of a trapezium and has been shaded on the diagram. The thickness of the ramp is 0.6 metres.



- a) calculate: 4
- i) The value of, x , on the diagram (correct to 3 decimal places)
 - ii) Show that the area of the cross-section of the ramp is 6.2 m^2 (correct to 1 decimal place)
- b) The ramp is 5 metres wide and is to be made of concrete. What volume of concrete will be used to make this ramp ? 1
- B) The population of a small country town grows slowly at a rate proportional to its current population. The population exactly two years ago was 10 000 and is now 10 200.
- Find
- a) The growth constant of the population 2
 - b) The number of years, from now, that it will take the population to exceed 11 500 2
 - c) The population of the town in 5 years from today 1
- C) Solve $8^x = 4^{x-1}$ 2

Question 5 (Start a new page)

- a) The diagram below (not drawn to scale) shows a paddock with one side bounded by a river. Use Simpson's Rule with the five function values shown on the diagram to approximate the area of the paddock. 3



- b) When Jill was born her mother deposited \$180 into a Trust Account earning 12% p.a. interest compounded annually. She continued to deposit \$180 into this account each time Jill had a birthday. The last payment was made on Jill's sixteenth birthday. Calculate the total amount in the account on Jill's twenty-fifth birthday. 4
- c) Consider this arithmetic series $3 + 8 + 13 + 18 + \dots + 488$
- How many terms are in this series 1
 - Find the sum of all the terms in this series 2
- d) For a particular series the sum to n terms is given by $S_n = 2^n + n^2$. What would be the tenth term of this series? 2

Question 6 (Start a new page)

- a) A particle starts from O and moves along a straight line so that after t seconds its distance from O is x cm, where

$$x = 6t - \frac{t^3}{2}.$$

- After how many seconds does it return to O and what is its velocity at that time? 2
 - What is its distance from O when its velocity is zero and what is its acceleration here? 2
 - What is its average velocity during the first two seconds of the motion? 2
- b) The continuous curve corresponding to the function $y = f(x)$ has the following properties in the closed interval $a \leq x \leq b$

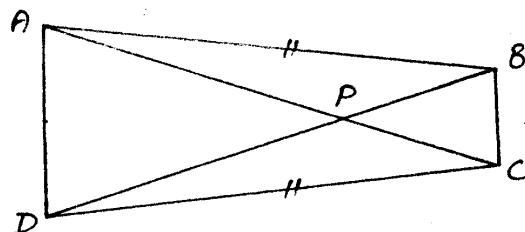
$$f(x) > 0, \quad f'(x) < 0, \quad f''(x) > 0$$

- Sketch neatly a curve satisfying these conditions. 2
 - State the least value of $f(x)$ in this interval 1
- c) Find the stationary points and any points of inflexion of $y = x^3 - x^2 - x - 1$. 3

Question 7 (Start a new page)

- a) ABCD is a quadrilateral in which $AB = DC$ and $\angle BAC = \angle BDC$.
P is the point of intersection of the diagonals.

5



Prove that:

- i) $\triangle APB \equiv \triangle DPC$
 - ii) $\triangle PBC$ is isosceles
- b) A parabola has equation $y^2 - 6y + 25 = 8x$. Express this in the form $(y - p)^2 = 4a(x - q)$ and hence find:
- i) The coordinates of its vertex
 - ii) The equation of its axis of symmetry
 - iii) Its focal length
 - iv) The coordinates of its focus
 - v) The equation of its directrix

7

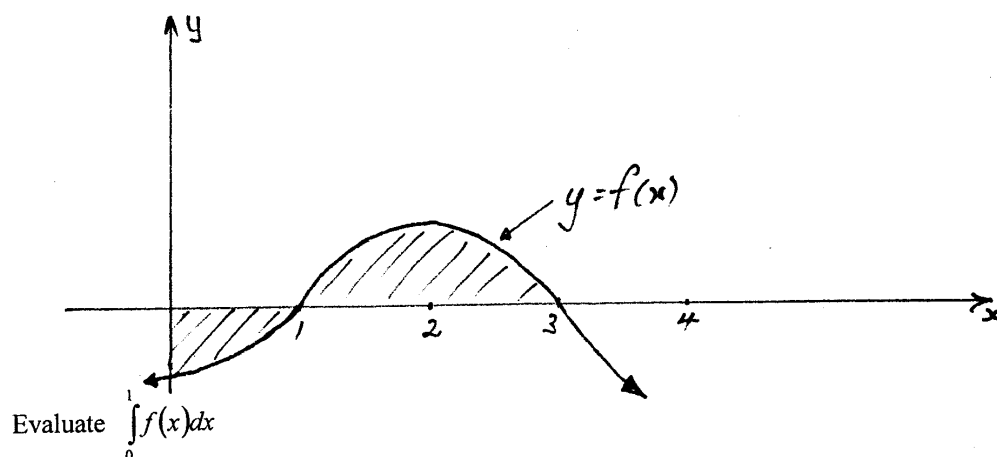
Question 8 (Start a new page)

- a) A athlete knows that she has a 20% chance of winning the 100 metre sprint event and a 30% chance of winning the 200 metres sprint. If she competes in both events at an athletics carnival what is the probability that she will:
- i) Win both events
 - ii) Not win both events
 - iii) Win one event only
 - iv) Win at least one event

4

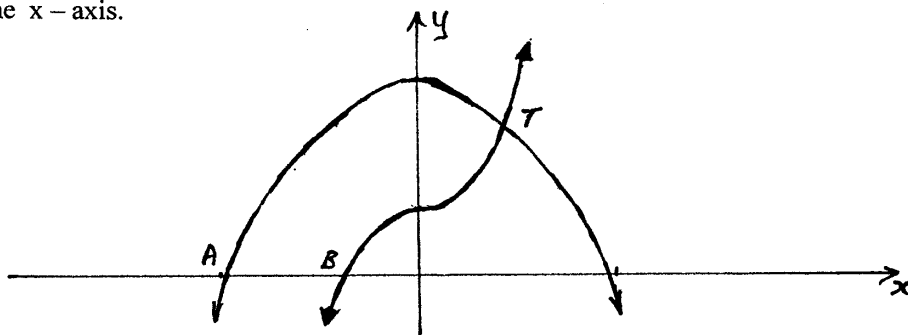
- b) In the diagram below $\int_1^3 f(x) dx = 5$ and the area of the shaded region is 7 units².

1



3

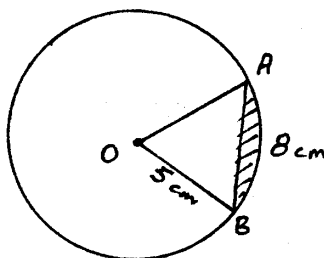
- c) The diagram below shows the shaded region enclosed by the curves $y = 3 - x^2$, $y = x^3 + 1$ and the x -axis.



- i) Find the coordinates of the points A and B. 2
- ii) Show that the point (T) of intersection between the curves is (1,2) 2
- iii) Hence find the area of the shaded region in exact form. 3

Question 9 (Start a new page)

- a) In the circle with center O, arc AB is 8cm and the radius OB is 5cm. Find: 3
 - i) The size of angle AOB, giving your answer in radians.
 - ii) The area of the shaded segment, giving your answer correct to 2 decimal places.



- b) The area under the curve $y = 1 - e^{-x}$, above the x -axis and between $x = 0$ and $x = 1$, is rotated about the x -axis. Prove that the volume of the generated solid is 3

$$\frac{\pi}{2} (4e^{-1} - e^{-2} - 1) \text{ cubic units}$$

- c) Sharon was driving a car in an off-road rally competition. From the start, S, Sharon drove 55 km due east to, A. At A, she proceeded on a bearing of 055° for 100 km to B. At B, she changed course to a bearing of 130° and continued in this direction until she reached the finish at C. (C is due east of A).

- i) Draw a diagram representing all this information on your answer sheet. 1
- ii) Show that angle ACB = 40° . 2
- iii) Find the distance from B to C. Give your answer to the nearest kilometre. 2
- iv) It took Sharon 24 minutes to travel from the start to the finish. What was her average speed in km/h? 1

Question 10 (Start a new page)

- a) A mechanic borrows \$P to buy new equipment for his business. The interest is compounded monthly at the rate of 18% p.a. The mechanic intends to repay the loan by making repayments of \$M per month (at the end of each month).

- i) Write an expression for the amount owed by the mechanic at the end of the first month ? 1
- ii) Write an expression for the amount owed at the end of N months. 2
- iii) If the mechanic had borrowed \$40 000 calculate the amount of the monthly repayment (\$M) if he wishes to repay the loan in 5 years. 3

- b) The mass, m grams, of a raindrop falling for, t, seconds in a humid cloud, is increasing at a rate

$$\frac{dm}{dt} \text{ where } \frac{dm}{dt} = \frac{1}{100} \left[t + \frac{t^2}{10} \right] \text{ gs}^{-1}$$

- i) If the initial mass of the raindrop is zero, what is the mass of the raindrop after 20 seconds ? 2
- ii) If the raindrop started as a smoke particle of mass 0.001 g, how much heavier would it be after 20 seconds than the raindrop in part (i) ? 1

- c) Observe that:

$$\begin{aligned} 1 &= 1 \\ 3x &= x + 2x \\ 5x^2 &= x^2 + 2x^2 + 2x^2 \\ 7x^3 &= x^3 + 2x^3 + 2x^3 + 2x^3 \\ 9x^4 &= x^4 + 2x^4 + 2x^4 + 2x^4 + 2x^4 \end{aligned}$$

By studying the above arrangement, or otherwise, find in simplest algebraic form, an expression for the limiting sum of the series

$$1 + 3x + 5x^2 + 7x^3 + 9x^4 + \dots + (2n-1)x^{n-1} + \dots \quad \text{3}$$

END OF PAPER