N.S.W. DEPARTMENT OF EDUCATION

HIGHER SCHOOL CERTIFICATE EXAMINATION 1967

MATHEMATICS - PAPER B (2F) - EQUIVALENT TO 3U AND 4U - 1st PAPER

Instructions: Time allowed 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work. Mathematical tables will be supplied.

QUESTION 1 (30 Marks)

(i) The curve $y = (x - 1)^2$ is rotated around the x-axis. Find the volume of the solid of revolution between the planes $x = 0$ and $x = 2$.

(ii) Find the primitive function of $(x + 1)/(x + 2)$.

(iii) Find the second derivative of $y = \sin(x^3)$. Give a sketch (not on graph paper) of this curve in the neighbourhood of $x = 0$.

(iv) Two points P and Q move along the parabola $y = x^2/4a$ in such a way that the x-coordinates of P and Q differ by the constant amount $2a$. Find the equation of the locus of the midpoint of the chord PQ, and describe in words this locus.

(v) An urn contains 2 red balls and 3 blue balls. 6 drawings are made, the ball being placed back into the urn after each drawing. What is the probability, expressed as a rational number, that exactly 2 red balls are drawn?

(vi) Transform the curve $x^2 + y^2 - 2x - 4y + 1 = 0$ to parallel axes through the point $(1, 2)$. Hence, or otherwise, describe the curve in words.

(vii) Write $5\cos\theta + (75)^{\frac{1}{2}}\sin\theta$ in the form $R\sin(\theta + \varepsilon)$, $R > 0$.

(viii) State carefully the range and the domain of the function $y = 2\sin^{-1}(x/2)$. Draw a rough sketch of this function.

(ix) Evaluate the integral $\int_0^2 (4 - x^2)^{-\frac{1}{2}}dx$.

(x) (a) For arbitrary positive integral n, find the sum
$2^n + 2^{n-1} + 2^{n-2} + \ldots + 2^{-n+1} + 2^{-n}$.

(b) State whether this sum has a limit as n approaches infinity. Give reasons.

QUESTION 2 (10 Marks)

(i) Find the quotient and the remainder obtained by dividing $P(x) = x^3 - bx^2 - bx + 4$ by $A(x) = x - 2$.

(ii) Hence, or otherwise, find a value of the constant b such that $P(x)$ is divisible by $A(x)$.

(iii) Find all the zeros of $P(x)$ for this value of b.

QUESTION 3 (10 Marks)

(i) Factorize the polynomial $2n^2 + 7n + 6$.

(ii) By use of the principle of mathematical induction, prove the relation $6(1^2 + 2^2 + 3^2 + \ldots + n^2) = n(n + 1)(2n + 1)$.

(iii) Hence find the value of the limit $\lim_{n\to\infty} \left[\frac{1^2 + 2^2 + \ldots + n^2}{n^3}\right]$.

QUESTION 4 (10 Marks)

(i) Starting from the definition of the derivative as a limit, prove the product rule $(uv)' = uv' + vu'$.

(ii) Find the derivatives of: (a) $(x^2 + 1)\log_e(x^2 + 1)$;

(b) $(x^2 + 1)^{(x^2+1)}$.

QUESTION 5 (10 Marks)

Let $J = \int_1^5 (9x^2 + 8x)dx$.

(i) Find approximate values of J by:

(a) the midordinate rule, using one interval;

(b) the midordinate rule, using two intervals;

(c) Simpson's rule, using two intervals.

(ii) Compare your results with the exact value of J, and comment on the magnitudes of the errors.

QUESTION 6 (10 Marks)

(i) A plane S_1 contains the three points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$. Find an equation for S_1.

(ii) The perpendicular from the origin O to another plane, S_2 meets it at $(1, 1, 0)$. Find an equation for S_2.

(iii) Find the points at which the line of intersection of S_1 and S_2 meets:

(a) the plane containing the y-axis and the z-axis;

(b) the plane containing the x-axis and the z-axis.

QUESTION 7 (10 Marks)

A particle of mass m is free to move along the x-axis. At time $t = 0$, the particle is at rest at the origin. A force $F(t) = A\cos(nt)$ acts on the particle.

(i) Assuming Newton's law of motion F = ma, find the acceleration a at any time t.

(ii) Find the velocity v at any time t.

(iii) Find the position x at any time t.

(iv) Draw a rough sketch of x as a function of t, and give in words a detailed description of the motion.

QUESTION 8 (10 Marks)

A small deck of cards contains four aces (namely the ace of hearts, the ace of spades, the ace of diamonds, and the ace of clubs) and four kings, eight cards in all.

(i) A man draws two cards and announces: "I hold the ace of hearts." Find the probability that he has:

(a) two aces;

(b) one ace and one king.

(ii) The two cards are replaced, the deck is reshuffled (mixed), the man again draws two cards, and announces: "I hold at least one ace." Find the probability that he holds two aces in his hand.