SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2008

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – 180 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Start each **NEW** question in a separate answer booklet.
- Marks may **NOT** be awarded for messy or badly arranged work.
- All necessary working should be shown in every question.

Total Marks – 120

- Attempt questions 1-10.
- All questions are of equal value.

Examiner: *D.McQuillan*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax},\ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a} \sin ax,\ \ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a} \cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a} \tan ax,$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a} \sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a},\ \ a > 0,\ \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

Total marks – 120
Attempt Questions 1–10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) How many degrees, to the nearest minute, are in 1 radian? **2**

(b) Rationalise the denominator of $\dfrac{2\sqrt{2}}{\sqrt{7}-\sqrt{3}}$. **2**

(c) Sketch a graph of $y = |2x-3|$. **2**

(d) Solve the inequality $2x^2 + 7x - 15 \geq 0$. **2**

(e) Evaluate $\sum_{k=0}^{19}(3k-1)$. **2**

(f) If $\log_e 5x - \log_e 2 = 2\log_e x$ find all real values of x. **2**

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\frac{dy}{dx}$ for the following

(i) $y = \tan(x^2)$ **2**

(ii) $y = 2x\sin(2x)$ **2**

(b)

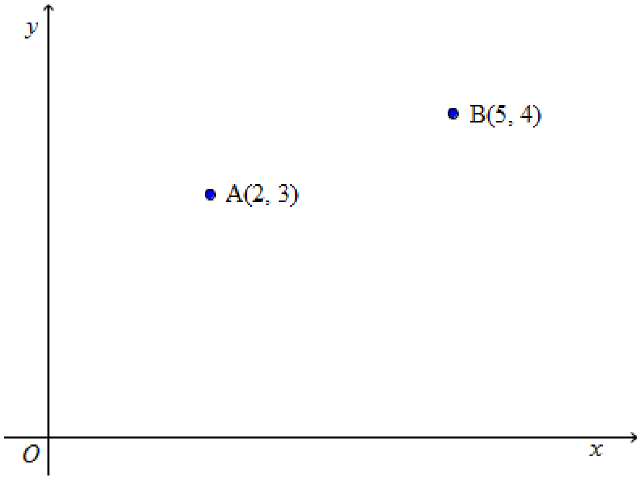
(i) Find $\int \frac{x^2}{x^3 - 1}\,dx$. **2**

(ii) Evaluate $\int_{\frac{\pi}{2}}^{\pi} \cos\left(\frac{1}{2}x\right)dx$ in exact form. **3**

(c) Find the equation of the tangent to $y = \sin\left(x + \frac{\pi}{3}\right)$ at the point where $x = \pi$. **3**

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The diagram shows the points A(2, 3) and B(5, 4)



(i) Show that the equation of AB is $x - 3y + 7 = 0$. **2**

(ii) Find the coordinates of M, the midpoint of AB. **1**

(iii) Show that the equation of the perpendicular bisector of AB is $3x + y - 14 = 0$. **2**

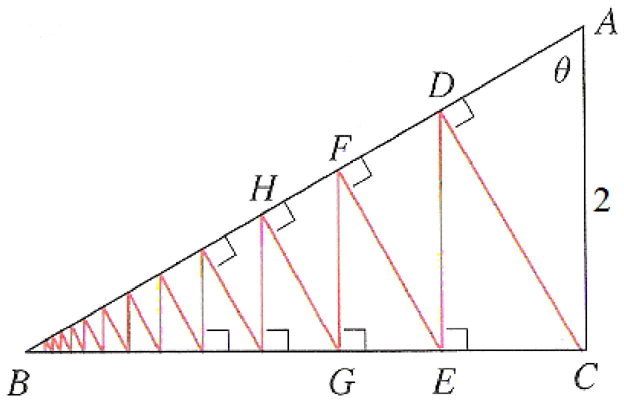
(iv) The perpendicular bisector of AB cuts the x-axis at C. Find the coordinates of C. **1**

(v) Find the area of triangle BCO. **2**

Question 3 continues on page 4

Question 3 (continued)

(b)



A right triangle ABC is given with $\angle A = \theta$ and $|AC| = 2$. CD is drawn perpendicular to AB, DE is drawn perpendicular to BC, $EF \perp AB$, and this process is continued indefinitely as in the figure. Find the total length of all the perpendiculars

$|CD| + |DE| + |EF| + |FG| + \cdots$

in terms of θ. **4**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) In Lower Warkworth the local doctor, based on years of data research, estimates that the probability of an adult catching influenza was 0.1 while the probability of a child catching the dreaded influenza was 0.3. The Blott family consists of Dad, Mum and two young Blotts. Calculate the probability that:

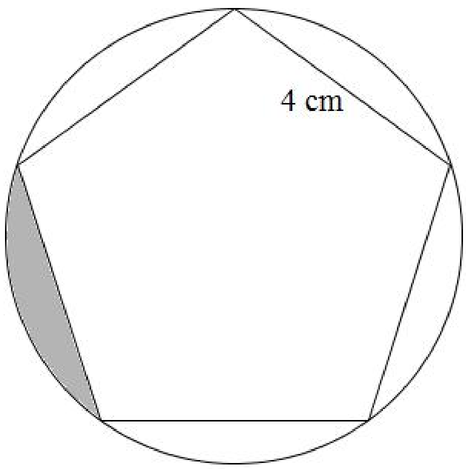
(i) both adults catch influenza **1**

(ii) only one child catches influenza **1**

(iii) exactly one adult and one child catches influenza **2**

(iv) at least one family member catches influenza. **2**

(b)



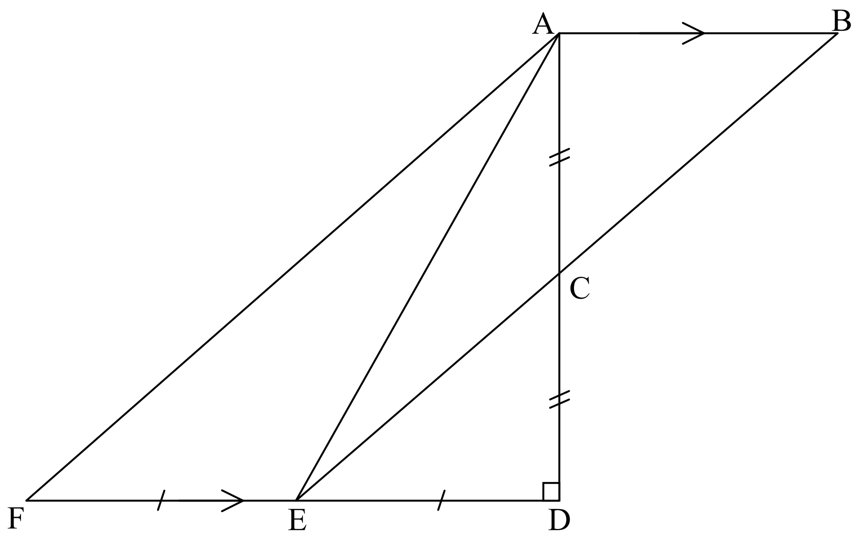
(i) Find an expression for the area of the regular pentagon with side length 4 cm. **3**

(ii) Find the radius of the circle to two decimal places. **2**

(iii) Hence or otherwise find the area of the shaded segment to two decimal places. **1**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



In the diagram AB $\parallel$ FD, ADF is a right-angled triangle, C is the midpoint of AD and E is the midpoint of FD.

(i) Explain why $\angle CED = \angle ABC$. **1**

(ii) Show that $\Delta CDE \equiv \Delta CAB$. **2**

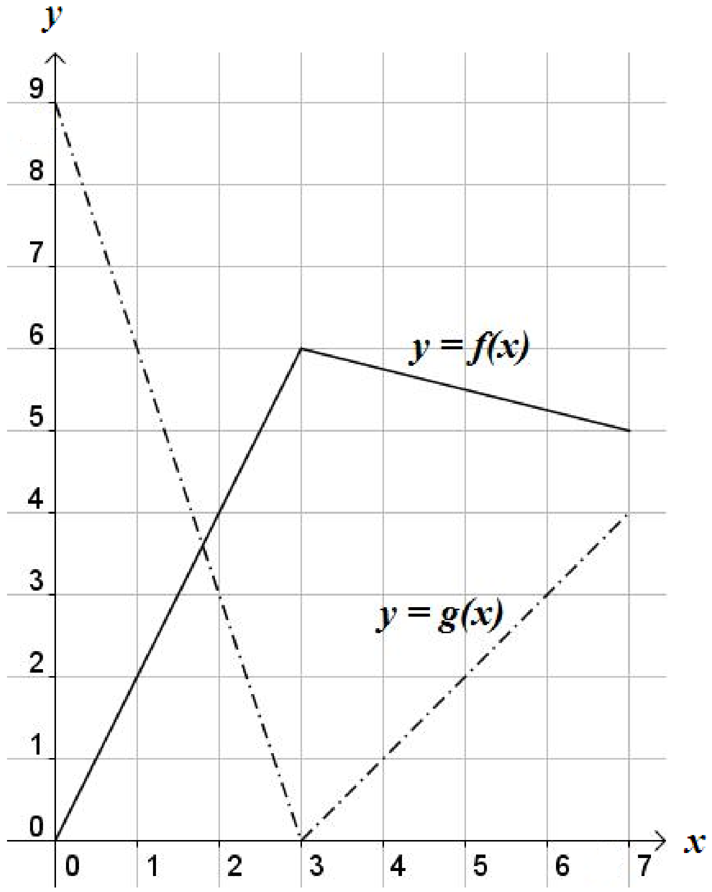
(iii) Show that $AF = 2BC$. **2**

(iv) Show that $\angle ACB = \angle DAF$. **1**

Question 5 continues on page 7

Question 5 (continued)

(b)



If $f(x)$ and $g(x)$ are the functions whose graphs are shown, let $u(x) = f(x)g(x)$ and $v(x) = f(g(x))$ find the value of

(i) $u'(1)$ **2**

(ii) $v'(1)$ **2**

(c) Show that if $|x+3| < \frac{1}{2}$, then $|4x+13| < 3$. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) For the curve $y = \dfrac{x}{x^2+1}$.

(i) Find the turning points and determine their nature. **3**

(ii) Find the points of inflection. **2**

(iii) Since x^2+1 is never zero the curve has no vertical asymptotes. Find the horizontal asymptotes by evaluating $\lim\limits_{x\to\infty}\dfrac{x}{x^2+1}$. **1**

(iv) Sketch the curve. **2**

(b) Tom is 60 years old and about to retire at the beginning of the year 2009. He joined a superannuation scheme at the beginning of 1969. He invested \$750 at the beginning of each year. Compound interest is paid at 9% per annum on the investment, calculate to the nearest dollar:

(i) The amount to which the 1969 investment will have grown by the beginning of 2009. **1**

(ii) The amount to which the total investment will have grown by the beginning of 2009. **3**

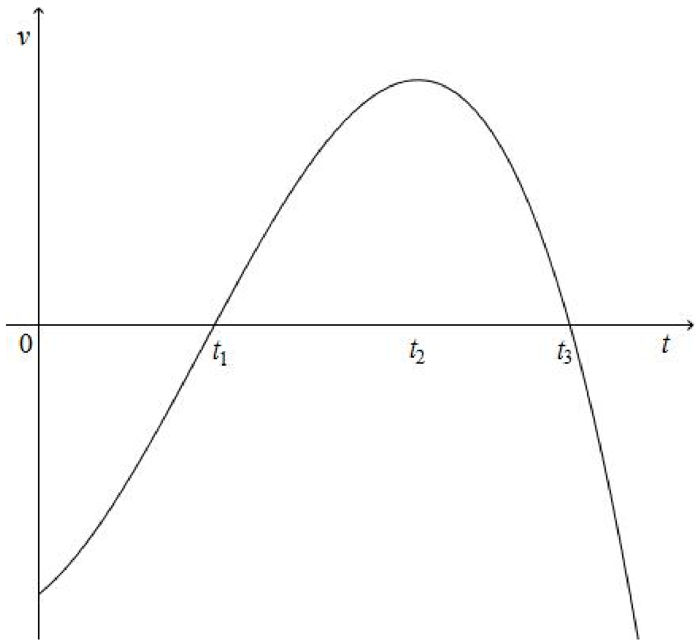
Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) If α and β are the roots of the equation $3x^2 - 12x - 9 = 0$, find the values of:

(i) $\dfrac{1}{\alpha^3\beta^3}$ **1**

(ii) $\dfrac{\beta}{\alpha} + \dfrac{\alpha}{\beta}$ **2**

(b) A particle moves in a straight line and the graph shows the velocity *v* of the particle after time *t*.



(i) What is happening to the particle at t_1? **1**

(ii) What is happening to the particle at t_2? **1**

(iii) Sketch the graph of displacement *x*, as a function of *t*, if the particle is initially at the origin. **3**

(c) The locus of the point P(*x*, *y*) such that the sum of the squares of its distances from the points A(2, 4) and B(6, –8) is 118, is a circle. Find the centre and radius of the circle. **4**

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate $10^x + 10x$. **2**

(b) A particle moves in a straight line. At time *t* seconds its displacement *x* cm from a fixed point O on the straight line is given by:

$$x = t + \frac{1}{t+1}$$

(i) What is the initial displacement of the particle? **1**

(ii) When is the particle at rest? **2**

(iii) What is the acceleration after 5 seconds. **2**

(iv) What happens to the acceleration as *t* increases? What does this tell you about the velocity as *t* becomes large. **2**

(c) A petrol tank is designed by the rotation of the curve $y = \frac{1}{5}x(x-40)$ about the *x* axis between the planes $x = 0$, $x = 40$. If the units are in centimetres, how many litres would the tank hold? **3**

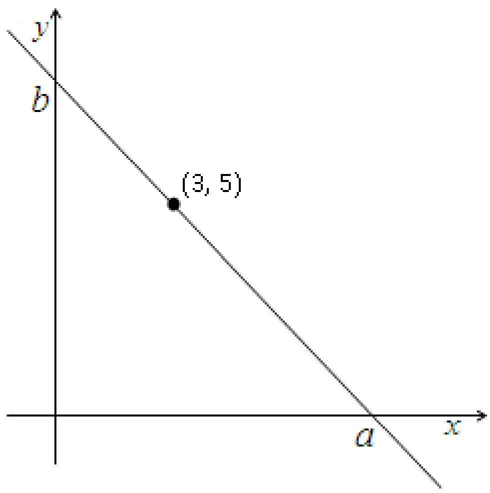
Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The population of a small town grows from 9000 to 11000 in 10 years.

(i) Find the annual growth rate to the nearest per cent, assuming it is proportional to the population. **2**

(ii) Calculate the population of the town 25 years after the initial count. **1**

(b)



(i) For the given figure show that $a = \dfrac{3b}{b-5}$. **2**

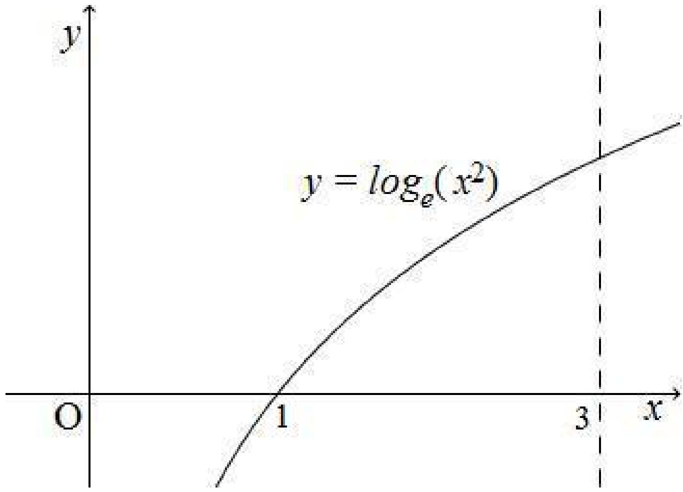
(ii) Find the equation of the line through the point (3, 5) that cuts off the least area from the first quadrant. **4**

(c) A ladder 2 metres long rests against a vertical wall. Let θ be the angle between top of the ladder and the wall and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \dfrac{\pi}{3}$. **3**

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) If $x\sin\pi x = \int_0^{x^2} f(t)dt$ find $f(4)$. **2**

(b) The graph of the function $y = \log_e(x^2)$ is shown below.



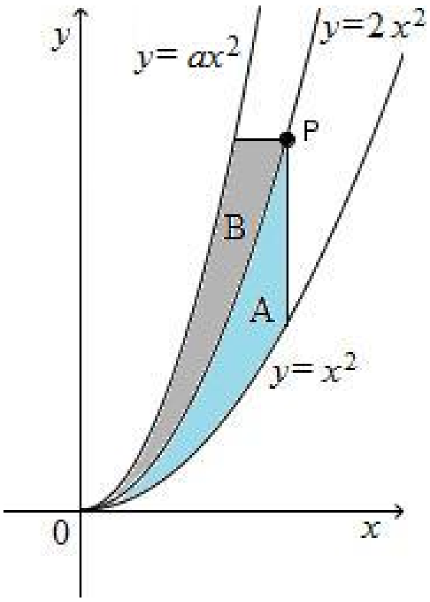
(i) Use the Trapezoidal rule with 5 function values to approximate $\int_1^3 \log_e\left(x^2\right)dx$ and explain why this approximation underestimates the value of the integral. **3**

(ii) Find $\int_0^{\ln 9} e^{\frac{y}{2}}dy$ and hence find the exact value of $\int_1^3 \log_e\left(x^2\right)dx$. **3**

Question 10 continues on page 13.

Question 10 (continued)

(c)



The figure shows a function $y = ax^2$ with the property that, for every point P on the middle function $y = 2x^2$, the area A and B are equal. Find the value of a. **4**

End of Paper