Question 1 **Marks**

(a) Solve the following inequality $\frac{x}{1-x^2} \geq 0$ **3**

(b) (i) Factorise completely $x^4 + x^3 - 2x^2 - 2x$ **1**

(ii) Find the roots of the following equation $4x^3 - 4x^2 - 29x + 15 = 0$, given that one root is the difference between the other two roots. **3**

(b) Solve, showing the solution on the number line, $|x-1| \leq x+2$ **3**

Question 2 (start a new page)

(a) Prove by the Principle of Mathematical Induction that $5^n + 3^n$ is always even for all positive integers n. **4**

(b) i) Show that a solution of $x^3 + x - 1 = 0$ lies between $x = 0$ and $x = 1$. **1**

ii) By using Halving of the Interval Method once, obtain a better approximation. **2**

iii) By using $x = 0.5$ as a first approximation, apply Newton's method once to obtain a better approximation. (give your answer to three decimal places). **2**

iv) Decide which method is more powerful and why is that ? **1**

Question 3 (start a new page)

(a) Use mathematical induction to prove the following result for positive integral values of n: $\sum_{r=1}^{n} 3^{-r} = \frac{3^n - 1}{2(3)^n}$ **6**

(b) Two cars are travelling along roads that intersect at right angles to one another. One starts 200 km away and travels towards the intersection at 80 kmh^{-1}, while the other starts at 120 km away and travels towards the intersection at 60 kmh^{-1}.

i) Show that their distance d apart after t hours is given by:
$d^2 = 10000t^2 - 46400t + 54400$
(note: the distance formula is $d = speed \times time$). **2**

ii) Hence find how long it takes them to reach their minimum distance apart. **3**

iii) Find their minimum distance apart. **1**

Question 4 (start a new page)

(a) Prove by mathematical induction that $4^n \geq 3n+7$, for all integers $n \geq 2$ **4**

(b) Sketch, showing all the important features, the real function $f(x)=\dfrac{x^2}{2x+3}$. **7**

Hence, determine the values of x for which the function is decreasing.

End of Test

Solutions & Marking Scheme

Question 1

a) $\frac{x}{1-x^2} \geq 0 \quad \Big| \; x(1-x^2)^2 \quad (x \neq \pm 1)$

$\therefore x(1-x^2) \geq 0$

$\therefore x(1+x)(1-x) \geq 0$ (1m)

$\boxed{x < -1}$ OR $\boxed{0 \leq x < 1}$
(1m) (1m)

b) i) $x^4 + x^3 - 2x^2 - 2x$
$= x^3(x+1) - 2x(x+1)$
$= (x+1)x(x^2-2)$ (1m)
$= x(x+1)(x+\sqrt{2})(x-\sqrt{2})$ (1m)

ii) let $\alpha, \beta, \alpha-\beta$ be the roots

$\therefore \alpha + \beta + (\alpha - \beta) = -\frac{-4}{4}$

$\alpha\beta + \alpha(\alpha-\beta) + \beta(\alpha-\beta) = \frac{-29}{4}$

$\alpha\beta(\alpha-\beta) = -\frac{15}{4}$

From the first relationship
$\therefore 2\alpha = 1 \therefore \boxed{\alpha = \frac{1}{2}}$ (1m)

From the last relationship
$\therefore \frac{1}{2}\beta\left(\frac{1}{2}-\beta\right) = -\frac{15}{4}$

$\therefore \frac{1}{4}\beta - \frac{1}{2}\beta^2 + \frac{15}{4} = 0$

$\therefore 2\beta^2 - \beta - 15 = 0$

$\therefore \beta = -\frac{5}{2}$ or 3 (1m)

$\therefore$ Solutions are $\boxed{\frac{1}{2}, -\frac{5}{2}, 3}$ (1m)

(by checking for $\alpha - \beta$)

c) $|x-1| \leq x+2$

$\therefore x-1 \leq x+2$ OR $-(x-1) \leq x+2$

$\therefore -1 \leq 2$ (true) OR $x-1 \geq -x-2$ (1m)

$\therefore 2x \geq -1$

$-\frac{1}{2}$ (1m) $\boxed{x \geq -\frac{1}{2}}$ (1m)

Question 2

a) Step 1

For $n=1 \therefore 5^1 + 3^1 = 8$ which is even (1m)

Step 2

Assume that $S_K: 5^K + 3^K = 2m$ (where m is an integer) is true

Hence, prove that S_{K+1} is also true

$S_{K+1}: 5^{K+1} + 3^{K+1} = 5 \times 5^K + 3 \cdot 3^K$

$= 5(2m - 3^K) + 3 \times 3^K$ (Since S_K is true $: 5^K = 2m - 3^K$) (1m)

$= 10m - 2 \times 3^K$

$= 2\underbrace{(5m - 3^K)}_{\text{integer-M}} = 2M$, M integer

$\therefore S_{K+1}$ is also true (1m)

Step 3: Since S_1 is true $\therefore S_2$ is also true
Since S_2 is true $\therefore S_3$ is also true

$\vdots$

S_n is true for all values of n, positive integer

$\therefore 5^n + 3^n$ is even (divisible by 2) for all positive integers n. (1m)

b) i) let $f(x) = x^3 + x - 1$

$\therefore f(0) = -1, \quad f(1) = 1$

$\therefore f(0) \times f(1) < 0 \therefore$ there is (at least) one solution between 0 and 1 (1m)

ii) $x_m = \frac{0+1}{2} = 0.5$

$f(0.5) = -0.375$ (1m)

$\therefore f(0.5) \times f(1) < 0$

$\therefore$ there is (at least) a solution between 0.5 and 1 closer to 0.5 (1m)

iii) $x_{Better} = x - \frac{f(x)}{f'(x)}$

$f'(x) = 3x^2 + 1$

$f(0.5) = -0.375; f'(0.5) = 1.75$

$\therefore x_{better} = 0.5 - \frac{-0.375}{1.75}$ (1m)

$\therefore x_{better} = \boxed{0.714 \text{ (3dp)}}$ (1m)

iv) $f(0.5) = -0.375$

$f(0.714) = 0.07799\ldots$

$\therefore$ Newton's method is more powerful because $f(0.714)$ is closer to 0 (0.714 is closer to the root) (1m)

Question 3

a) $\sum_{R=1}^{n} 3^{-R} = \frac{3^n - 1}{2(3)^n}$

$S_n: 3^{-1} + 3^{-2} + \ldots + 3^{-n} = \frac{3^n - 1}{2(3)^n}$

Step 1

For $R=1 \therefore$ LHS $= 3^{-1} = \frac{1}{3}$

RHS $= \frac{\cancel{2}}{\cancel{2} \times 3} = \frac{1}{3}$

$\therefore$ LHS = RHS $\therefore S_1$ is true (for $n=1$) (1m)

Step 2

Assume $S_K: \sum_{R=1}^{K} 3^{-R} = \frac{3^K - 1}{2(3)^K}$ is true

Prove $S_{K+1}: \sum_{R=1}^{K+1} 3^{-R} = \frac{3^{K+1} - 1}{2(3)^{K+1}}$ is also true

$S_{K+1}: \boxed{3^{-1} + 3^{-2} + \ldots + 3^{-K}} + 3^{-(K+1)}$

$= \frac{3^K - 1}{2(3)^K} + 3^{-K-1}$ (1m)

$= \frac{\overset{3}{\cancel{3^K - 1}}}{2 \times 3^K} + \frac{\overset{2}{\cancel{1}}}{3 \times 3^K}$ (1m)

$= \frac{3 \times 3^K - 3 + 2}{2 \times 3 \times 3^K}$ (1m)

$= \frac{3^{K+1} - 1}{2 \times 3^{K+1}} \therefore S_{K+1}$ is also true (1m)

Step 3

Since S_1 is true $\therefore S_2$ is also true

S_2 is true $\therefore S_3$ is also true

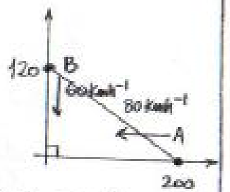
$\vdots$

$S_n : \sum_{R=1}^{n} 3^{-R} = \frac{3^n - 1}{2(3)^n}$ is true

for any positive integer n (1m)

i) $x_A = 200 - 80t$

$x_B = 120 - 60t$



$d^2 = x_A^2 + x_B^2$ (Pythagoras)

$\therefore d^2 = (200 - 80t)^2 + (120 - 60t)^2$ (1m)

$= 40000 - 32000t + 6400t^2 +$
$14400 - 14400t + 3600t^2$

$\boxed{10{,}000t^2 - 46{,}400t + 54{,}400}$ (1m)

ii) $d = \sqrt{10{,}000t^2 - 46{,}400t + 54{,}400}$

$\frac{d(d)}{dt} = \frac{20{,}000t - 46{,}400}{2\sqrt{10{,}000t^2 - 46{,}400t + 54{,}400}}$ (1m)

$\therefore \frac{d(d)}{dt} = 0 \therefore 20{,}000t - 46{,}400 = 0$ (1m)

$\boxed{t = 2.32 \text{ hours}}$

2h 19.2 min

2 h 19 min 12 s (1m)

Test:

x		2.32	
$\frac{d(d)}{dt}$	$-$	0	$+$
d	$\searrow$		$\nearrow$

$\therefore$ distance is a minimum.

iii) For $t = 2.32$ hours

$\therefore \boxed{d = 24 \text{ km}}$ (1m)

Question 4

a) $S_n : 4^n \geq 3n + 7$, $n \geq 2$, integer

Step 1 For $n = 2$ $\therefore$ LHS $= 4^2 = 16$

RHS $= 3 \times 2 + 7$
$= 13$

$\therefore$ LHS $\geq$ RHS

$\therefore S_2$ is true (for $n = 2$) (1m)

Step 2

Assume $S_K : 4^K \geq 3K + 7$ is true

for $n = K$, integer ≥ 2

Prove that $S_{K+1} : 4^{K+1} \geq 3(K+1) + 7$

$4^{K+1} \geq 3K + 10$, for

$n = K+1$, integer ≥ 2

Since S_K is true by multiplying both sides of ineq. by 4

$\therefore 4 \times 4^K \geq 4(3K + 7)$

$\therefore 4^{K+1} \geq 12K + 28 \geq 3K + 10$

$(12K > 3K$ and $28 > 10)$ (1m)

$\therefore 4^{K+1} \geq 3K + 10$, for K integer ≥ 2

$\therefore S_{K+1}$ is also true (1m)

Step 3 Since S_2 is true $\therefore S_3$ is also true

Since S_3 is true $\therefore S_4$ is also true

$\vdots$

S_n is true for all $n \geq 2$, integer

$\therefore 4^n \geq 3n + 7$ for all $n \geq 2$, integer (1m)

b) $f(x) = \frac{x^2}{2x + 3}$

Domain : all real x except $x = -\frac{3}{2}$

x-int : $f(x) = 0 \therefore x = 0$

y-int $x = 0 \therefore y = 0$

1st Derivative $f'(x) = \frac{2x(2x+3) - x^2 \cdot 2}{(2x+3)^2}$

$4x^2 + 6x$

$= \frac{2x^2 + 6x}{(2x+3)^2}$

$= \frac{2x(x+3)}{(2x+3)^2}$ (1m)

$f'(x) = 0 \therefore \frac{2x(x+3)}{(2x+3)^2} = 0$

$\therefore x = -3$ or 0

x		-3		$-3/2$		0	
y'	$+$	0	$-$	$\vert$	$-$		$+$
y	$\nearrow$	-3	$\searrow$		$\searrow$	0	$\nearrow$

Max t.p. $(-3, -3)$; min. t.p. $(0, 0)$ (1m)

V.A. $x \to -\frac{3}{2}^- \therefore f(x) \to -\infty$

$x \to -\frac{3}{2}^+ \therefore f(x) \to +\infty$

$\therefore$ V.A. $\boxed{x = -\frac{3}{2}}$

$x \to +\infty \therefore f(x) \to +\infty$

$x \to -\infty \therefore f(x) \to -\infty$

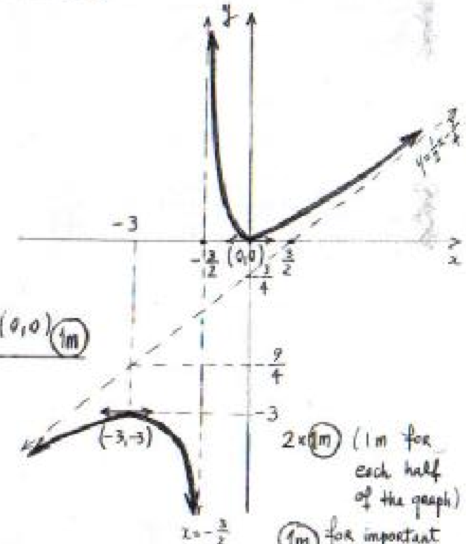
$\therefore$ no H.A.

$$\begin{array}{r|l} & \frac{1}{2}x - \frac{3}{4} \\ \hline 2x+3 & x^2 \\ & -x^2 - \frac{3}{2}x \\ \hline & \quad - \frac{3}{2}x \\ & \quad + \frac{3}{2}x + \frac{9}{4} \end{array}$$

$\therefore f(x) = \frac{1}{2}x - \frac{3}{4} + \frac{9}{4(2x+3)}$

$x \to \pm\infty \therefore f(x) \doteqdot \frac{1}{2}x - \frac{3}{4}$

$\therefore$ Oblique Asymptote is $\boxed{y = \frac{1}{2}x - \frac{3}{4}}$ (1m)



2×(1m) (1m for each half of the graph)

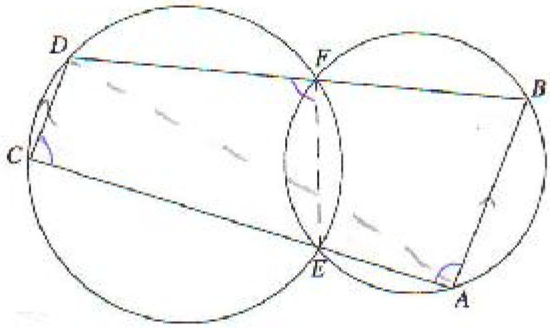
(1m) for important points

Function is increasing for $-3 \leq x \leq 0$ (1m)

Question 1

Marks

(a) Two circles intersect at E and F. AEC and BFD are straight lines. Copy the diagram and prove that AB is parallel to CD. **3**



(b) If $\sin\alpha = \frac{3}{5}$, and $\tan\beta = \frac{12}{5}$, find the exact value of $\tan(\alpha - \beta)$ **2**

(c) (i) Simplify $\frac{\sin 2x}{1+\cos 2x}$ **2**

(ii) Hence, find the exact value of $\tan 15^\circ$. **1**

(d) A surveyor observes two towers, one due north, of height 80m, and the other on a bearing of θ ($\theta < 90^\circ$) of height 120m. The angles of elevation of the two towers are 40° and 36° respectively. If the towers are 150m apart on a horizontal plane, calculate the value of θ to the nearest minute. **4**

Question 2 *(start a new page)*

(a) (i) Find $\int \sin^2 3x\,dx$ **3**

(ii) Use the substitution $u = x-1$ to find $\int 5x\sqrt{x-1}\,dx$ **3**

(b) Find: (i) $\int x^4(2x^5-1)\,dx$ (let $u = 2x^5 - 1$) **3**

(ii) $\int_0^3 \frac{t}{\sqrt{t+1}}\,dt$ **3**

(c) (i) Differentiate $x\cos x$ 1

(ii) Hence find $\int x\sin x\,dx$ 2

Question 3 (start a new page)

(a) (i) Find the exact value for $2\cos^2 22.5^\circ - 1$ 2

(ii) Show that $\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} = \tan\dfrac{\theta}{2}$ 2

(b) Express $\sqrt{3}\sin x + \cos x$ in the form $r\sin(x+\theta)$ and hence solve the trigonometric equation $\sqrt{3}\sin x + \cos x = 1$, for $0 \le x \le 360^\circ$ (Give answers to the nearest degree). 5

Question 4 (start a new page)

(a) (i) Sketch $f(x) = 2\sqrt{x} + 5$. State the Domain and Range for this function. 2

(ii) State if it is a one-one function in its domain, and if it is, find its inverse function $y = f^{-1}(x)$ 2

(iii) Sketch $y = f^{-1}(x)$. State the Domain and Range for this function. 2

(b) A curve has $\dfrac{dy}{dx} = 6\sin 2x$, and passes through the point $\left(\dfrac{\pi}{4}, 3\right)$. Find the equation of this curve. 3

(c) Find:

(i) $\displaystyle\int_0^2 \frac{x}{\sqrt{1+x^2}}\,dx$ 3

(ii) $\displaystyle\int_0^1 \sqrt{4-x^2}\,dx$ (use the substitution $x = 2\cos\theta$) 4

End of Test

Solutions & Marking Scheme

Question 1

) $\angle BAE + \angle BFE = 180^\circ$ (opp ∠s in the cyclic quad ABFE)

$\angle DFE + \angle BFE = 180^\circ$ (supplem ∠s)

$\therefore \angle BAE = \angle DFE$ (1m)

$\angle DFE + \angle DCE = 180^\circ$ (opp ∠ in the cyclic quad DFEC) (1m)

$\therefore \angle BAE + \angle DCE = 180^\circ$

$\therefore AB \parallel DC$ (Since $\angle BAE$ and $\angle DCE$ are cointerior ∠s) (1m)

b) if $\sin\alpha = \frac{3}{5}$ $\therefore \cos^2\alpha = 1 - \frac{9}{25}$

$\therefore \cos\alpha = \frac{4}{5}$

$\therefore \tan\alpha = \frac{3}{4}$ (1m)

$\therefore \tan(\alpha - \beta) = \dfrac{\frac{3}{4} - \frac{12}{5}}{1 + \frac{3}{4} \times \frac{12}{5}}$

$\therefore \boxed{\tan(\alpha - \beta) = -\frac{33}{56}}$ (1m)

i) $\dfrac{\sin 2x}{1+\cos 2x} = \dfrac{2\sin x \cos x}{\cancel{\sin^2 x} + \cos^2 x + \cos^2 x - \cancel{\sin^2 x}}$ (1m)

$= \dfrac{\cancel{2}\sin x \cos x}{\cancel{2}\cos^2 x}$

$= \dfrac{\sin x}{\cos x} = \boxed{\tan x}$ (1m)

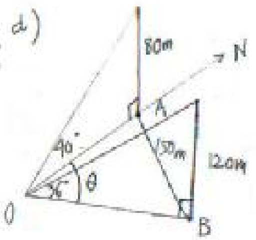
ii) $\tan 15^\circ = \dfrac{\sin 2\times 15^\circ}{1+\cos 2\times 15^\circ}$

$= \dfrac{\sin 30^\circ}{1+\cos 30^\circ}$

$= \dfrac{\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}}$

$= \dfrac{1}{2+\sqrt{3}} = \boxed{2-\sqrt{3}}$ (1m)

d)



$\tan 40^\circ = \dfrac{80}{OA}$ $\therefore OA = \dfrac{80}{\tan 40^\circ}$ (1m)

$\tan 36^\circ = \dfrac{120}{OB}$ $\therefore OB = \dfrac{120}{\tan 36^\circ}$ (1m)

in $\triangle OAB$ apply cosine Rule :

$\therefore \cos\theta = \dfrac{\left(\frac{80}{\tan 40^\circ}\right)^2 + \left(\frac{120}{\tan 36^\circ}\right)^2 - 150^2}{2 \times \frac{80}{\tan 40^\circ} \times \frac{120}{\tan 36^\circ}}$

$= 0.44\ldots$ (1m)

$\therefore \boxed{\theta = 63^\circ 52'}$ (nearest min.) (1m)

Question 2

a) i) $\int \sin^2 3x\,dx = \int \dfrac{1-\cos 6x}{2}\,dx$ (1m)

$= \frac{1}{2}x - \frac{1}{2}\int \cos 6x\,dx$ (1m)

$= \boxed{\frac{1}{2}x - \frac{1}{12}\sin 6x + C}$ (1m)

ii) let $u = x-1$ $\therefore du = dx$

$\therefore x = u+1$

$\therefore \int 5x\sqrt{x-1}\,dx$

$= 5\int (u+1)\,u^{\frac{1}{2}}\,du$ (1m)

($u^{\frac{3}{2}} + u^{\frac{1}{2}}$)

$= 5\dfrac{u^{\frac{5}{2}}}{\frac{5}{2}} + 5\dfrac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$ (1m)

$= 2u^{\frac{5}{2}} + \frac{10}{3}u^{\frac{3}{2}} + C$

$= \boxed{2(x-1)^{5/2} + \frac{10}{3}(x-1)^{3/2} + C}$

$= 2u^2\sqrt{u} + \frac{10}{3}u\sqrt{u} + C$

$= \boxed{2(x-1)^2\sqrt{x-1} + \frac{10}{3}(x-1)\sqrt{x-1} + C}$ (1m)

b) i) let $u = 2x^5 - 1$

$\therefore du = 10x^4\,dx$ $\therefore dx = \dfrac{du}{10x^4}$

$\therefore \int x^4(2x^5-1)\,dx$

$= \frac{1}{10}\int u\,du$ (1m)

$= \frac{1}{20}u^2 + C$ (1m)

$= \boxed{\frac{1}{20}(2x^5-1)^2 + C}$ (1m)

ii) $\int_0^3 \dfrac{t}{\sqrt{t+1}}\,dt$ let $\sqrt{t+1} = u$

$\therefore t+1 = u^2$

$= \int \dfrac{u^2-1}{\cancel{u}} \cdot 2\cancel{u}\,du$ $\therefore dt = 2u\,du$ (1m)

$= 2\dfrac{u^3}{3} - 2u$ (1m)

$= 2\left[\dfrac{(\sqrt{t+1})^3}{3} - \sqrt{t+1}\right]_0^3$

$= \boxed{\frac{8}{3}}$ (1m)

c) i) $(x\cos x)' = \boxed{\cos x - x\sin x}$ (1m)

ii) Since $(x\cos x)' = \cos x - x\sin x$

integrating both sides

$\therefore x\cos x = \int \cos x\,dx - \int x\sin x\,dx$ (1m)

$\therefore \boxed{\int x\sin x\,dx = \sin x - x\cos x}$ (1m)

Question 3

a) i) $2\cos^2 22.5^\circ - 1 = \cos 2\times 22.5^\circ$

$= \cos 45^\circ$ (1m)

$= \frac{1}{\sqrt{2}}\left(= \frac{\sqrt{2}}{2}\right)$ (1m)

n) let $t = \tan\frac{\theta}{2}$

$\therefore LHS = \dfrac{1 + \frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}}{1 + \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}$

$= \dfrac{\cancel{1}+t^2 + 2t - \cancel{1}+t^2}{1+t^2 + 2t + 1-t^2}$ (1m)

$= \dfrac{2t(1+t)}{2(1+t)} = t = \tan\frac{\theta}{2} = RHS$ (1m)

) $\sqrt{3}\sin x + \cos x = R\sin(x+\theta)$

$\therefore R = \sqrt{3+1}$ $\quad \tan\theta = \frac{1}{\sqrt{3}}$

$= 2$ (1m) $\quad \therefore \theta = \frac{\pi}{6}$ (1m) 30°

$\therefore \sqrt{3}\sin x + \cos x = 2\sin(x + \frac{\pi}{6})$

$\therefore 2\sin(x + \frac{\pi}{6}) = 1$

$\therefore \sin(x + \frac{\pi}{6}) = \frac{1}{2}$ (1m)

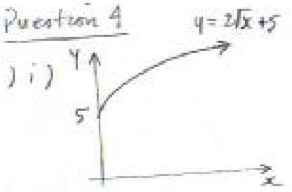
$x + \frac{\pi}{6} = \frac{\pi}{6}, \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}$ $\quad (30^\circ, 180^\circ - 30^\circ, 360^\circ + 30^\circ)$ (1m)

$\therefore x = 0, \frac{2\pi}{3}, 2\pi$

$\therefore \boxed{x = 0, 120^\circ, 360^\circ}$ (1m)

Question 4

) i)



(1m)

Domain $x \geq 0$

Range $y \geq 5$ (1m)

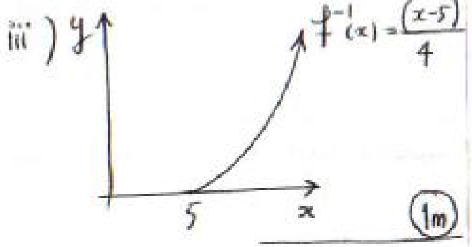
ii) $f(x) = 2\sqrt{x} + 5$ is an one-one fn (HLT) (1m)

$x = 2\sqrt{y} + 5$

$\therefore 2\sqrt{y} = x - 5$

$\therefore \boxed{f^{-1}(x) = \left(\frac{x-5}{4}\right)^2}$ (1m)

iii)



(1m)

Domain : $x \geq 5$

Range : $y \geq 0$ (1m)

b) $y = \int 6\sin 2x\,dx$

$\therefore y = -3\cos 2x + C$ (1m)

For $x = \frac{\pi}{4}$, $y = 3$

$\therefore 3 = -3\cos\frac{\pi}{2} + C \therefore \cancel{C=3}$ (1m)

$\therefore C = 3$

$\therefore \boxed{y = -3\cos 2x + 3}$ (1m)

c) i) let $1+x^2 = u$

$\therefore 2x\,dx = du$

$\therefore \frac{1}{2}\int \frac{du}{\sqrt{u}} = \frac{1}{2}\int u^{-\frac{1}{2}}du$

$= \frac{1}{\cancel{2}} \cdot \dfrac{u^{\frac{1}{2}}}{\frac{1}{\cancel{2}}}$ (1m)

$= \left[\sqrt{1+x^2}\right]_0^2$ (1m)

$= \boxed{\sqrt{5} - 1}$ (1m)

ii) let $x = 2\cos\theta$

$\therefore dx = -2\sin\theta\,d\theta$

when $x = 1 \therefore \theta = \frac{\pi}{3}$

$x = 0 \therefore \theta = \frac{\pi}{2}$

$\therefore \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} \sqrt{4 - 4\cos^2\theta} \times (-2\sin\theta)\,d\theta$ (1m)

$= 4\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{1-\cos^2\theta}\,\sin\theta\,d\theta$ $\quad |\sin\theta| (= \sin\theta, 0 \leq \theta \leq \pi)$

$= 4\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^2\theta\,d\theta$ (1m)

$= 4\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1-\cos 2\theta}{2}\,d\theta$

$= 2\left[\theta - \frac{1}{2}\sin 2\theta\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ (1m)

$= 2\left[\frac{\pi}{2} - \frac{1}{2}\sin\pi - \frac{\pi}{3} + \frac{1}{2}\sin\frac{2\pi}{3}\right]$

$= \boxed{\frac{\pi}{3} + \frac{\sqrt{3}}{2}}$ (1m)