JAMES RUSE AGRICULTURAL HIGH SCHOOL

TERM 2 ASSESMENT 2001 – YEAR 12

MATHEMATICS – EXTENSION II

INSTRUCTIONS:
Time allowed: 85 minutes.
Write your student number on all answer sheets.
Attempt all questions.
All questions are of equal value.
Approved silent calculators may be used.
Resource material may be used.

QUESTION 1: (20 MARKS)

(i) (a) Sketch the region R bounded by $y = x^2$, the positive x-axis and the line x = 2.

(b) Find the volume of the solid formed when the region R is rotated one revolution about the line x = 2.

(ii) From the letters of the word ELECTED five letters are chosen to make a five letter "word".

(a) How many different "words" can be formed?

(b) Find the probability that a five letter "word" formed at random will not have any E's adjacent to each other.

(iii) A particle moves to the right on a horizontal surface under the effect of an acceleration (in kmh^{-2}) given by $\ddot{x} = \frac{36}{v} + v$ where v is the velocity of the particle in kmh^{-1}. Initially the particle is observed at the origin travelling with velocity $2\sqrt{3}\ kmh^{-1}$.

(a) Show that the position of the particle when its velocity is $v\ kmh^{-1}$ is given by

$$x = v - 6\tan^{-1}\left(\frac{v}{6}\right) + \pi - 2\sqrt{3}.$$

(b) Find an expression for the time taken to reach velocity $v\ kmh^{-1}$.

(c) Find the time of travel and the distance travelled by the particle when it attains its minimum acceleration.

QUESTION 2: (20 MARKS) (START A NEW PAGE)

(i) Twelve players turn up for a game of crazyball. In crazyball two teams of 5 play against each other.

(a) If the coach selects two teams at random to play against each other, how many selections can be made?

(b) If the Bovine twins (Angus and Murray) are amongst the 12 players, find the probability that they will play on opposing teams. (Express your answer as a fraction in simplest form)

(ii) (a) Find the three stationary points on the curve $y = x^2\sqrt{32 - x^4}$.

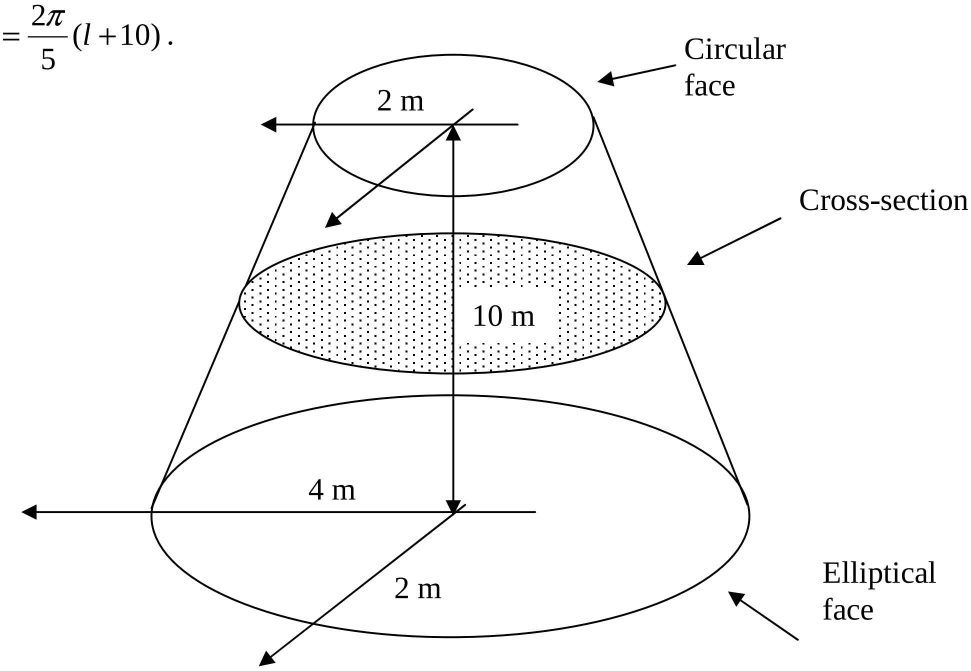
(b) Sketch the curve showing all stationary points and intercepts with the co-ordinate axes.

(c) The area bounded by this curve and the x-axis is rotated one revolution about the y-axis. Using the method of evaluating volumes by cylindrical shells, find the volume of the solid generated.

(iii) (a) Explain with the aid of a diagram why $\int_0^a \sqrt{a^2 - x^2}\, dx = \frac{\pi a^2}{4}$.

(b) Show that the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ equals πab .

(c) A right solid has a circular face with radius 2 *m* at one end and an elliptical face with semi-major axis 4 *m* and semi-minor axis 2 *m* at the other end (see diagram). Cross-sections perpendicular to the axis of the solid are ellipses. If the solid is 10 meters long, show that the area ($A\,m^2$) of a cross-section distance *l* meters from the circular end is given by $A = \frac{2\pi}{5}(l + 10)$.



(d) Calculate the volume of the solid

QUESTION 3: (20 MARKS) START A NEW PAGE

(i) (a) Show that the equation of the tangent to $xy = 16$ at the point $T\left(4t, \frac{4}{t}\right)$ is given by

$x + t^2 y = 8t$.

(b) Find the co-ordinates of the point Q where the tangent from T meets the x-axis.

(c) Find the equation of the line through Q and perpendicular to the tangent from T.

(d) If the line in (c) meets the hyperbola at points R and S, show that the co-ordinates of M, the midpoint of RS, has co-ordinates $M(4t, -4t^3)$.

(e) Find the locus of the point M giving any restrictions that may apply to the locus.

(ii) A small object of mass m kg is released from rest in a medium whose resistance is mkv where v is the velocity (in ms^{-1}) of the object.

(a) If the terminal velocity of the object is V_T, show that the acceleration is given by

$$\ddot{x} = \frac{g}{V_T}(V_T - v).$$

(b) Hence show that the velocity of the object t seconds after it is released is given by

$$v = V_T\left(1 - e^{-\frac{g}{V_T}t}\right).$$

(c) At the instant the first object is released, a second object is projected vertically in the same medium and with initial velocity V_O. Show that when the second object reaches its maximum height the first object is falling with velocity $v = \dfrac{V_O V_T}{V_O + V_T}$.

THIS IS THE END