The Scots College

2001
TRIAL HSC EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using a blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on page 8
- All necessary working should be shown in every question
- Start each question in a new booklet.

Total Marks: (84)
Weighting: 35% HSC

- Attempt Questions 1 - 7
- All questions are of equal value

Total marks (84)
Attempt Questions 1 – 7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet.

a. Evaluate $\displaystyle\int_0^{2\sqrt{3}} \frac{dx}{4+x^2}$ **2**

b. Differentiate $\cos^3 x$ **2**

c. Find the point which divides the line joining (4, 6) and (13, 5) externally in the ratio 4:1 **2**

d. Write down the equation of the vertical asymptote of $y = \dfrac{2x}{3x-1}$ **1**

e. Solve for x: $\dfrac{3}{x+5} \le 1$ **2**

f. Evaluate $\displaystyle\int_0^{\frac{1}{\sqrt{2}}} \frac{2x^3}{\sqrt{1-x^4}}\,dx$ using the substitution $u = x^4$ **3**

End of Question 1

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

a. Using all the letters, how many different arrangements can be made from the word MATHEMATICS ? **2**

b. Find all values of θ in the range $0 \le \theta \le 2\pi$ for which $\sin\theta + \sqrt{3}\cos\theta = 1$ **4**

c. i. Show that the function $f(x) = 2x^2 + x - 2$ cuts the x axis between $x = 0$ and $x = 1$ **1**

ii. Use the method of halving the interval twice to find an approximation to the root of this equation. **3**

iii. Starting with a value of $x = 0.7$ use Newton's method once to find an approximation to this root correct to 3 decimal places. **2**

End of Question 2

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

a. The region R is bounded by the curve $y = \cos x$, $x = 0, x = \frac{\pi}{2}$ and the x – axis.

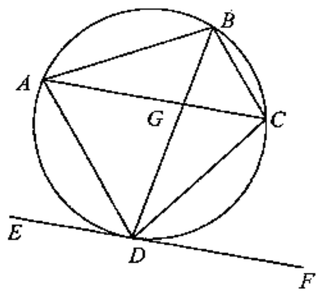
i. Sketch R **1**

ii. Find the exact volume of the solid generated when the region R is rotated about the x – axis. **2**

b. If α, β, γ, are the roots of the cubic polynomial equation $x^3 + 4x^2 - 6x - 8 = 0$

Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **2**

c.



$ABCD$ is a cyclic quadrilateral. EF is a tangent at D. If BD bisects $\angle ABC$, prove that AC is parallel to EF **2**

d. i. By equating coefficients, find the values of A and B in the identity

$A(2\sin x + \cos x) + B(2\cos x - \sin x) \equiv 7\sin x + 11\cos x$ **2**

ii. Hence show that $\int_0^{\frac{\pi}{2}} \frac{7\sin x + 11\cos x}{2\sin x + \cos x}\,dx = \frac{5\pi}{2} + \ln 8$ **3**

End of Question 3

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

a. P is a variable point on the parabola $x^2 = 8y$ with parameter p. The normal at P cuts the y axis at A and R is the midpoint of AP.

i. Show that the normal at P has equation $x + py = 4p + 2p^3$ **2**

ii. Show that R has coordinates $(2p, 2p^2 + 2)$ **2**

iii. Show that the locus of R is a parabola and show that the vertex of this parabola is the focus of the parabola $x^2 = 8y$. **3**

b. i. Evaluate $\int_1^3 \frac{dx}{x}$ **1**

ii. Use Simpson's rule with 3 function values to approximate $\int_1^3 \frac{dx}{x}$ **2**

iii. Use your results to parts i and ii to obtain an approximation for e. Give your answer correct to 3 decimal places. **2**

End of Question 4

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

a. Prove by induction that, for all integers $n \geq 1$,

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$ **3**

b. i. Find the domain over which the function $y = x^2 + 6x$ is monotonic increasing. **2**

ii. Find the inverse function over this restricted domain, and sketch a graph of this inverse function clearly showing its domain and range. **3**

iii Evaluate $\cos\left[\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right]$ **1**

iv. Sketch the graph of $y = 3\sin^{-1}\left(\frac{x}{2}-1\right)$ **3**

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

a. When the temperature T of a certain body is 65°C it is cooling at the rate of 1°C per minute.

Assuming Newton's law of cooling: $\frac{dT}{dt} = -k(T - S)$ where

T is the temperature of the body at time t minutes
S is the temperature of the surrounding medium
k is a constant

i. Verify that $T = S + Ae^{-kt}$ is a solution of the given differential equation, where A is a constant. **2**

ii. Determine the value of k given that S, which is constant, is 15°C. **2**

iii. Find T when $t = 20$ minutes, giving your answer to the nearest degree **2**

iv. How long will it take for the temperature of the body to fall to 35°C? **2**

b. The acceleration of a particle P, moving along a straight line has an acceleration given by

$$\frac{d^2x}{dt^2} = -4\left(x+\frac{16}{x^3}\right)$$

i. Given that P is initially at rest at the point $x = 2$ m, show that the velocity v m/s at any time is given by **3**

$$v^2 = 4\left(\frac{16-x^4}{x^2}\right)$$

ii. Hence, or otherwise, show that when P is halfway to the origin, the speed is given by $2\sqrt{15}$ m/s **1**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

a. An arrow is fired horizontally at 60ms^{-1} from the top of a 20m high wall. Taking $g = 10\text{ ms}^{-2}$

i. Show, using calculus, that the horizontal and vertical components of the arrows motion are given by

$x = 60t$
$y = -5t^2 + 20$ **3**

ii. Find the time taken for the arrow to hit the ground. **2**

iii. Find the distance that the point of impact will be from the base of the wall. **1**

iv. Find the angle with which the arrow will strike the ground. **2**

b. A squad of 8 is chosen at random from 3 baseball teams A, B and C with 10 players in each team.

i. If 5 of the squad are chosen from the A team, 2 from the B team and 1 is chosen from the C team, in how many ways can the squad be formed? **2**

ii. Find the probability that Joe from the B team and Fred from the A team will be chosen. **2**

End of paper

The Scots College Mathematics Department 2001
Written by: D Hamaty
Assessed by: D Scardino

Standard Integrals

$\int x^n dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1; x \neq 0, \text{if } n < 0$

$\int \frac{1}{x} dx = \ln x + C, \quad x > 0$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + C, \quad a \neq 0$

$\int \cos ax\, dx = \frac{1}{a}\sin ax + C, \quad a \neq 0$

$\int \sin ax\, dx = -\frac{1}{a}\cos ax + C, \quad a \neq 0$

$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax + C, \quad a \neq 0$

$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax + C, \quad a \neq 0$

$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C, \quad a \neq 0$

$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a} + C, \quad a > 0, \ -a < x < a$

$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right) + C, \quad x > a > 0$

$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$

NOTE : $\ln x = \log_e x, \quad x > 0$