# YEAR 12 TRIAL HSC EXAMINATION

## 2003

# MATHEMATICS

## EXTENSION 1

*Time Allowed: 2 hours*
*(plus 5 minutes reading time)*

**INSTRUCTIONS TO CANDIDATES:**

- ALL questions should be attempted.

- Board-approved calculators may be used.

- Marks may be deducted for poorly presented work.

- Standard Table of Integrals may be removed, for your convenience, from the back of the examination paper.

- Each question attempted is to be returned in a separate writing booklet.

- Diagrams not to scale.

- All questions are of equal value.

- Write using a blue or black pen.

*Marks*

## QUESTION 1. (START A NEW BOOKLET)

(a)   Find $\lim_{\theta \to 0} \dfrac{\tan\frac{\theta}{3}}{\theta}$                                                                       1

(b)   (i)   In how many ways can the letters of the word  REARRANGE  be arranged?          1

     (ii)   What is the probability of both the E's appearing together?  (Give your answer to
            two decimal places.)                                                                           1

(c)   Show that $(x + 4)$ is a factor of

$$P(x) = x^3 + 2x^2 - 23x - 60$$                                                      2

and hence factorise $P(x)$.

(d)   Solve $\dfrac{x+1}{x-2} < 2$                                                                          3

(e)   Find $\dfrac{d}{dx}\left(x\sin^2 x\right)$                                                                   2

(f)   The positive square root of 48 is approximately 7.  Using one step of
Newton's Method, find a better approximation correct to two decimal places.           2

## QUESTION 2. (START A NEW BOOKLET)

(a)  Find the constant term of $\left(3x+\dfrac{2}{x^2}\right)^9$

**3**

(b)  Show $\cos^{-1}\left(\dfrac{3}{11}\right)-\sin^{-1}\left(\dfrac{3}{4}\right)=\sin^{-1}\left(\dfrac{19}{44}\right)$

**3**
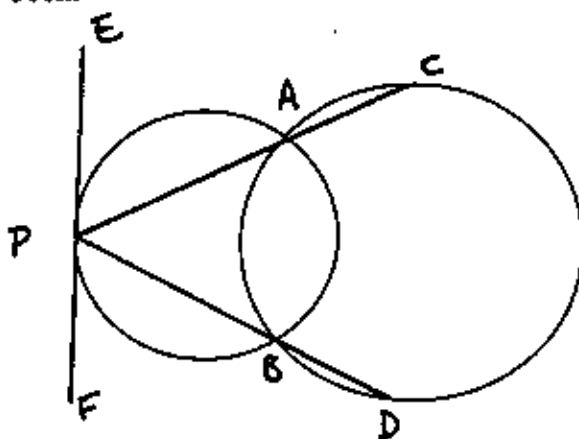
(c)  (i)  Prove $\ddot{x}=v\dfrac{dv}{dx}$

**1**

   (ii)  Suppose that, for a particle, $v=6-2x$ where $v$ is the velocity of the particle in $ms^{-1}$ and $x$ is the displacement from the origin, in metres. Find the acceleration of the particle at the origin.

**1**

   (iii)  Show that $t=-\dfrac{1}{2}\log(1-\dfrac{1}{3}x)$ and find $x$ as a function of $t$.

**4**

## QUESTION 3. (START A NEW BOOKLET)

(a)   Copy this diagram into your answer book.

PAC and PBD
are straight lines



Prove that CD is parallel to the tangent at P.                                          3

(b)   Prove by mathematical induction that:

$$1 \times 2^0 + 2 \times 2^1 + 3 \times 2^2 + ..... + n \times 2^{n-1} = 1 + (n-1)2^n$$

for all $n \geq 1$.                                                                      6

(c)   In how many ways can a committee of 3 women and 4 men be chosen from 8 women
and 7 men?  What is the number of ways when women A refuses to serve if woman B
is a member?                                                                            3

## QUESTION 4. (START A NEW BOOKLET)

(a) It is given that the rate of decrease of temperature of a body hotter than the surrounding air is proportional to the temperature difference. If $A$ is the air temperature, and $T$ the temperature of the body after $t$ minutes, then

$$\frac{dT}{dt} = -k(T-A)$$

(i) Show that, if the initial temperature is $I$, then the following function satisfies this condition

$$T = A + (I-A)e^{-kt} \qquad\qquad 1$$

(ii) An ingot of iron, initially at a temperature of 1500°C, is allowed to cool in the open air, where the temperature is 20°C. If it cools to 1200°C in five minutes, find the temperature of the ingot after one hour, correct to 4 significant figures.

2

(b) Sketch $y = \dfrac{x-1}{x^2}$ showing any stationary points, inflexions, asymptotes or any other features in the range, $-3 \le x \le 4$

6

(c) Find the integral $I = \displaystyle\int_1^9 \frac{dx}{x+\sqrt{x}}$ using the substitution $\sqrt{x} = u$

3

## QUESTION 5. (START A NEW BOOKLET)

(a)   The points $P(2ap,\ ap^2)$ and $Q(2aq,\ aq^2)$ lie in the parabola $x^2 = 4ay$.

  (i)   Find the equation of the chord in general form.                    **2**

  (ii)   If $PQ$ is a focal chord, prove that the normals to the parabola at $P$ and $Q$
       are perpendicular to each other.                                  **3**
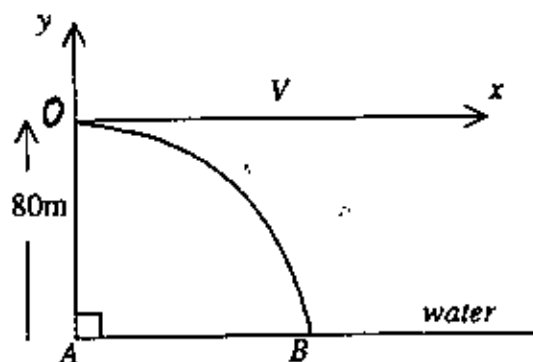
(b)   Find the general solution of:

$$2\sin 2x \cos x = \sqrt{3}\sin 2x$$                                     **3**

(c)



A stone is thrown horizontally with velocity 30 m/s from a point 0 on the top of a vertical
cliff so as to hit boat $B$. If O is 80m above the water level, find:

  (i)   the horizontal distance of the boat $B$ from the foot of the cliff.    **3**

  (ii)   The speed of the stone when it hits the boat.                     **1**

[Use acceleration due to gravity, $g = 10ms^{-2}$]

## QUESTION 6. (START A NEW BOOKLET)

(a) A sphere is expanding so that its surface area is increasing at a rate of 24 cm$^2$/s. Find the rate of increase of its:

**4**

    (i)   radius,

    (ii)  volume,

where the radius is 12cm. (Give your answer in terms of $\pi$.)

(b) Find $\displaystyle\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{dx}{\sqrt{4-9x^2}}$

**3**

(c) Differentiate $y = \cos^{-1}(\sin x)$ with respect to $x$

**2**

(d) $PQ$ meets the line $2x + 3y = 4$ at $C$, where $P$ is the point $(2, 2)$ and $Q$ is the point $(-1, -2)$. Find the ratio of $PC : CQ$.

**3**

## QUESTION 7. (START A NEW BOOKLET)

(a) The speed $v$ m/s of a point moving along the $x$ axis is given by:

$$v^2 = 36 - 6x - 2x^2$$

where $x$ is in metres.

   (i) Prove that the motion is simple harmonic and find the centre of motion.     **3**

   (ii) Find the period and amplitude of the motion.     **2**

   (iii) Find the maximum speed. (Give your answer to 1 decimal place.)     **1**

(b) Find $n$, if the coefficients of $x^5$ and $x^6$ in the expansion of $(3+2x)^n$ have the same value.     **3**

(c) (i) Write down the inverse function of:

$$y = e^x + 3$$     **1**

   (ii) Sketch the graph of $y = 3\cos^{-1}(x+1)$ clearly showing the domain and range of the graph.     **2**