Teacher's Name: **Student's Name/N°:**

SOLUTIONS – 2003 2 UNIT TRIAL

QUESTION 1:

(a) $-\frac{\sqrt{3}}{2}$ (1)

(b) $(3+2\sqrt{2})+(3-2\sqrt{2})$ (1)

$= 6$ (1)

(c) $\frac{1}{(x+1)(x-1)} - \frac{x-1}{(x+1)(x-1)}$ (1)

$= \frac{2-x}{(x+1)(x-1)}$ (1)

(d) $3x > -6$

$x > -2$ (1)

$-2 \quad 0 \quad 2$ (1) (1)

(e) $0{\cdot}2079$ (1)

(f) $9-\sqrt{80} = 9-4\sqrt{5}$

$\therefore \begin{cases} a = 9 & (1) \\ b = -4 & (1) \end{cases}$

(g) (i) $f(3)+f(-1) = 3+3$

$= 6$ (1)

(ii) $f(a^2) = a^2$ (1)

QUESTION 2:

(a) (i) $\frac{3}{2}x^{\frac{1}{2}}$ OR $\frac{3\sqrt{x}}{2}$ (1)

(ii) $2\sin x(-\sin x) + 2\cos x\cos x$

$= 2\cos^2 x - 2\sin^2 x$ (1) either

(iii) $\frac{d}{dx}\{\log(x+1)-\log(x-1)\}$ (1)

$= \frac{1}{x+1} - \frac{1}{x-1}$ (1)

1 off if no constants are shown in all questions

(b) (i) $\frac{1}{3}\log_e(x^3-2)+k$ (1)

(ii) $\frac{3}{\pi}\sin\frac{\pi x}{3} + k$ (1) (1)

(iii) $-\frac{5}{2}(2x-1)^{-1}+k$ OR $-\frac{5}{2(2x-1)}+k$ (2) 1 off for each bit wrong accept equivalent answers

2 (c) $2\tan\frac{3x}{2}\Big]_0^{\pi/2} = 2\tan\frac{3\pi}{4} - 2\tan 0$ (1) (1)

$= -2$

QUESTION 3:

(a)(i) $AC = \sqrt{(7-2)^2+5^2}$

$= \underbrace{\sqrt{50} \text{ OR } 5\sqrt{2}}_{\text{either } (1)}$

(ii) D is $\left(\frac{9}{2}, \frac{5}{2}\right)$ (1)

(iii) $m_{DB} = \frac{\frac{1}{2}}{-\frac{1}{2}}$

$= -1$ (1)

$m_{AC} = \frac{5}{5} = 1$

$\therefore m_{AC}\cdot m_{DB} = -1$

$\therefore AC \perp DB$ (1)

(iv) $E = (4, 3)$ (1)

(v) ABCE is a kite since AE and BE intersect at 90° bisect.

$\therefore$ Area $= \frac{1}{2}AC \times BE$ (1)

AND $BE = \sqrt{(1)^2+(1)^2}$

$= \sqrt{2}$

$\therefore$ Area $= \frac{1}{2}\times 5\sqrt{2}\times\sqrt{2}$

$= 5\ u^2$ (1)

(b) $y = e^{x^2}$

(i) $\frac{dy}{dx} = 2xe^{x^2}$ (1)

(ii) $\frac{d^2y}{dx^2} = 2e^{x^2} + 2x.2xe^{x^2}$

$= 2e^{x^2}[1+2x^2]$ either (1)

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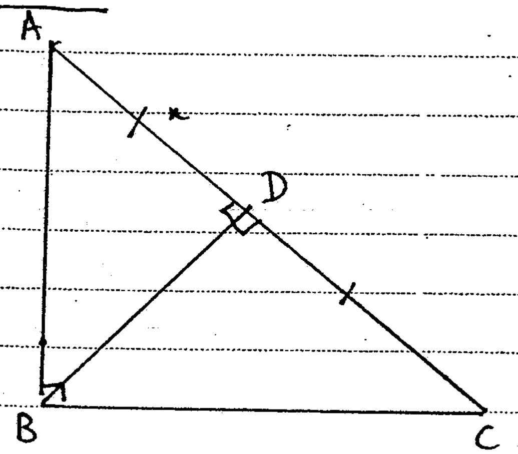
QUESTION 3 (i)

$$12y = x^2 - 4x + 16$$
$$12y = (x-2)^2 + 12$$
$$12(y-1) = (x-2)^2 \leftarrow 1$$
$$(x-2)^2 = 4.3.(y-1) \leftarrow 1$$
$$V: (2,1) \leftarrow 1$$

QUESTION 4:

(a)



(i) In $\triangle ABD$ and $\triangle ACB$,

$\angle A$ is common 1

$\angle ADB = \angle ABC = 90^\circ$ 1

$\therefore \triangle ABD \,|||\, \triangle ACB$ (equiangular)

(ii) Because of similarity,

$$\left.\begin{array}{l} \dfrac{AD}{AB} = \dfrac{AB}{AC} \\ \therefore AD.AC = AB^2 \end{array}\right\} 1$$
$$\therefore x.2x = AB^2$$
$$\therefore AB = x\sqrt{2} \quad 1$$

(iii) Using Pythagoras in $\triangle ABD$,

$$BD^2 = AB^2 - AD^2$$
$$= 2x^2 - x^2$$
$$\therefore BD = x$$
$\therefore \triangle ABD$ is isosceles,

$\therefore \angle BAD = 45^\circ$ (angle sum) 2

OR

$$\cos\angle BAD = \frac{AD}{AB}$$
$$= \frac{x}{x\sqrt{2}}$$
$$= \frac{1}{\sqrt{2}}$$
$$\therefore \angle BAD = 45^\circ$$
2

(iv) If $\angle BAD = 45^\circ$, then $\triangle ABC$ is isosceles 1

$\therefore AB/BC = 1$ 1 or 2

4 (b)

$$x^2 + y^2 = 25$$
$$y = \sqrt{25 - x^2} \quad 1$$
$$\therefore \frac{dy}{dx} = \tfrac{1}{2}(-2x)(25-x^2)^{-\frac{1}{2}} \quad 1$$
At (3,4) $m_T = \dfrac{-3}{4}$ 1

Equation is:

$$y - 4 = -\tfrac{3}{4}(x-3)$$
$$4y - 16 = -3x + 9$$
$$3x + 4y - 25 = 0 \quad 1$$

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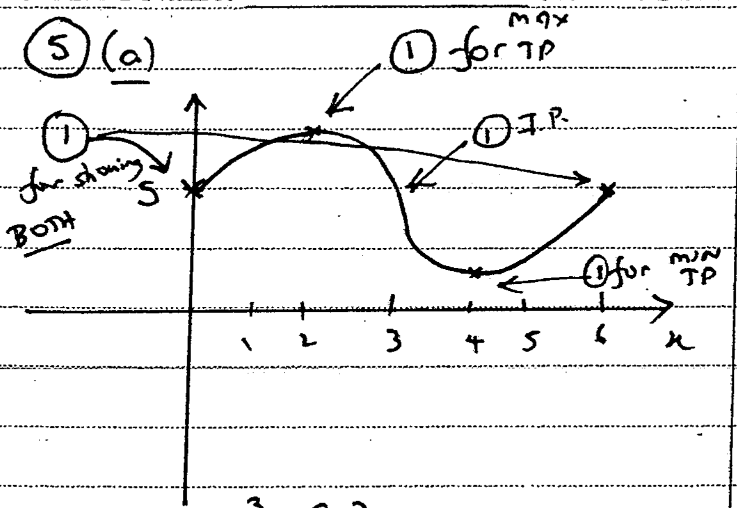
5 (a)



(b) $y = x^3 - 9x^2 + 24x$,

(i) $\frac{dy}{dx} = 3x^2 - 18x + 24$, $\frac{d^2y}{dx^2} = 6x - 18$ (1)

(ii) At S.P.'s $\frac{dy}{dx} = 0$ (1)

$\therefore x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$ (1)

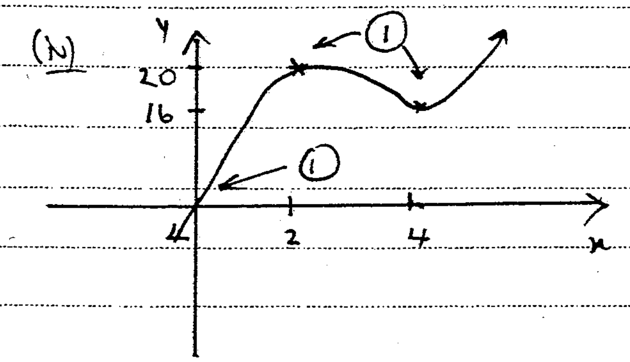
$\begin{cases} x = 4 \\ y = 16 \\ y'' > 0 \end{cases}$ or $\begin{cases} x = 2 \\ y = 20 \\ y'' < 0 \end{cases}$

$\therefore$ MIN T.P. at $(4,16)$ (1) MAX T.P. at $(2,20)$ (1)

(iii) At I.P. $\frac{d^2y}{dx^2} = 0$

$\therefore \begin{cases} x = 3 \\ y = 18 \end{cases}$ (1) accept just $x = 3$

(iv)



6

(a) $A_1 = \int_{-1}^{0} g(x)\,dx$

$= 1 \times 3 = 3u^2$ (1)

$A_2 = \int_0^3 g(x)\,dx$

$= \frac{1}{4} \times \pi \times 9$ (1)

$= 9\pi/4$

$A_3 = \int_3^4 g(x)\,dx$

$= \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$ (1)

$\therefore$ Area $= 3\frac{1}{2} + 9\pi/4$.

(b)(i) $(x-2)^2 = 4x - x^2 - 2$ (1)

$x^2 - 4x + 4 = 4x - x^2 - 2$

$\therefore 2x^2 - 8x + 6 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$\therefore x = 3$ or $x = 1$ (1)

(ii) $A_1 = \int_1^3 4x - x^2 - 2\,dx$

$= \left[2x^2 - \frac{1}{3}x^3 - 2x\right]_1^3$

$= (18 - 9 - 6) - (2 - \frac{1}{3} - 2)$

$= 3\frac{1}{3}u^2$ (1)

$A_2 = \int_1^3 x^2 - 4x + 4\,dx$

$= \left.\frac{1}{3}x^3 - 2x^2 + 4x\right]_1^3$

$= (9 - 18 + 12) - (\frac{1}{3} - 2 + 4)$

$= \frac{2}{3}u^2$ (1)

$\therefore$ Area $= 3\frac{1}{3} - \frac{2}{3}$

$= 2\frac{2}{3}u^2$ (1)

OR $\int_1^3 (2x^2 - 8x + 6)\,dx$ (1)

$= \left[\frac{2}{3}x^3 - 4x^2 + 6x\right]_1^3$ (1)

$= 2\frac{2}{3}$ (1)

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6 (c) $\text{Vol}_{\text{cylinder}} = \pi r^2 h$

$= \pi (9)^2 (3)$ (1)

$= 243\pi \quad (763.41)$

Vol under curve $= \pi \int_0^3 x^2 \, dy$ (1)

$= \pi \int_0^3 y^4 \, dy$

$= \pi \left[\frac{1}{5} y^5\right]_0^3$

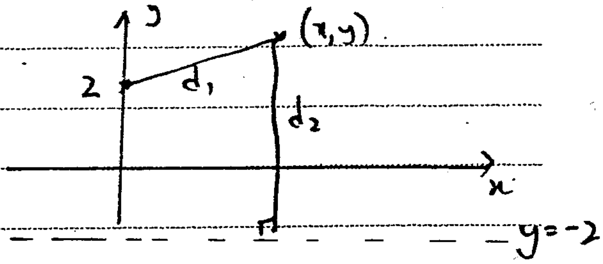
$= \pi \cdot \frac{243}{5}$ (1)

$= \frac{243\pi}{5} \quad (152.68)$

$\therefore$ Vol $= 243\pi - \frac{243\pi}{5}$

$= \begin{cases} \frac{972\pi}{5} \\ 610.73 \ u^3 \end{cases}$ (1)

(c)



$\left.\begin{aligned} d_1 &= \sqrt{x^2 + (y-2)^2} \\ d_2 &= y + 2 \end{aligned}\right\}$ (1)

$d_1 = 3 d_2$ (1)

$d_1^2 = 9 d_2^2$

$x^2 + y^2 - 4y + 4 = 9y^2 + 36y + 36$

$\therefore 8y^2 + 40y - x^2 + 32 = 0$ (1)

(or equivalent)

7 (a)(i) $\Delta = 1 - 4(2)(3)$

< 0

$\therefore$ no real roots (1)

(ii) (I) $\alpha + \beta = \frac{1}{2}$ (1)

(II) $\alpha\beta = \frac{3}{2}$ (1)

(III) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \frac{1}{4} - 3$

$= -2\frac{3}{4}$ (1)

(IV) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

$= \frac{1}{3}$ (1)

(iii) $\left(x - \frac{1}{\alpha}\right)\left(x - \frac{1}{\beta}\right) = 0$

$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha\beta} = 0$ (1)

$\left.\begin{aligned} \therefore x^2 - \tfrac{1}{3}x + \tfrac{2}{3} &= 0 \\ 3x^2 - x + 2 &= 0 \end{aligned}\right\}$ (1) either

(b) $\underbrace{40^c + 80^c + \cdots}_{16 \text{ months}}$ (1)

$= 0.40 \left(1 + 2 + \cdots + 2^{15}\right)$

$= 0.40 \left(\frac{2^{16} - 1}{1}\right) \approx \$26{,}214$ (1)

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(8) (a) $x = 40 + 10t - 5t^2$

$v = 10 - 10t$

$a = -10$

(i) At $t = 0$

$x = 40\,m$ (1)

(ii) At $x = 0$

$5t^2 - 10t - 40 = 0$

$\therefore t^2 - 2t - 8 = 0$

$(t-4)(t+2) = 0$

$\therefore t = 4$ secs (1)

(iii) At rest $v = 0$

$\therefore t = 1$ sec (1)

(iv) acceleration is constant / UNIFORM (1)

(v) (α) At $t = \frac{1}{2}$, $v = 5$ m/s (1)

At $t = 2$, $v = -10$ m/s (1)

(vi) It turns around / it stops and turns around / changes direction — ANY (1)

(b) $4\sin^2\theta = 3$

$\sin\theta = \frac{\pm\sqrt{3}}{2} \leftarrow$ (1)

$\therefore \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \underbrace{\frac{4\pi}{3}, \frac{5\pi}{3}}$

(1) (1) (1)

(c) $\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots$

$= \dfrac{2/3}{1 - 2/3}$

$= 2$ (1)

(9) (a) $SA = \pi r^2 + 2\pi r h = 75\pi$ (1)

(i) $\therefore h = \dfrac{75\pi - \pi r^2}{2\pi r}$

$= \dfrac{75 - r^2}{2r} \leftarrow$ (1)

(ii) $V = \pi r^2 h$

$= \pi r^2 \left(\dfrac{75 - r^2}{2r}\right)$

$= \dfrac{\pi r(75 - r^2)}{2}$ (1)

(iii) $\dfrac{dV}{dr} = \dfrac{75\pi}{2} - \dfrac{3\pi r^2}{2}$ (1)

$\dfrac{d^2V}{dr^2} = -\,6\pi r/2$

At MAX

$\dfrac{dV}{dr} = 0$ (1)

$\therefore 25 = r^2$

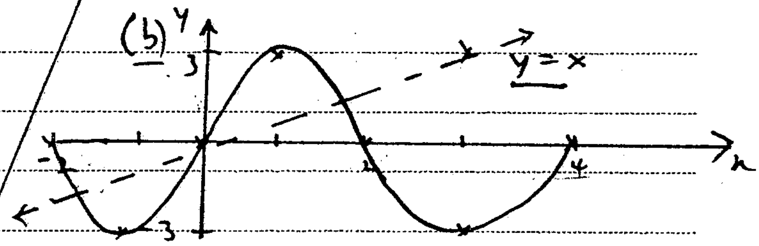
$\therefore r = 5, \; h = 5$ (1)

$V'' < 0 \leftarrow$ (1)

$\Rightarrow$ maximum. $V = \dfrac{5\pi(50)}{2}$

(1) $\rightarrow = 125\pi$ cu

(b)



(1) for $y = 3\sin\frac{\pi x}{2}$

(1) for drawing $y = x$

(1) for getting $y = x$ as the line.

(iii) 3 SOLUTIONS (1)

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QUESTION 10:

(a)(i). $A = \frac{1}{3} \times \frac{1}{2} \times \left[1 + 4\left(\frac{2}{3}\right) + \frac{1}{2}\right]$ (1)

$= \frac{25}{36}$ (1)

(ii) $\int_1^2 \frac{1}{x}\,dx = \ln x \Big]_1^2$ (1)

$= \ln 2 - \ln 1$

$= \ln 2$ (1)

(iii) Areas approximate:

$\therefore \frac{25}{36} \approx \log 2$ (1)

$\therefore 2 = e^{25/36}$.

$\therefore e = 2^{36/25}$

≈ 2.7132 (1)

(b) $y = x^2 \ln x$

$\frac{dy}{dx} = (\ln x)2x + x^2 \cdot \frac{1}{x}$

$= x\left[2\ln x + 1\right]$ (1) or equivalent

$\frac{d^2y}{dx^2} = (2\ln x + 1) + x\left(\frac{2}{x}\right)$

$= 2\ln x + 3$ (1)

For concave up $\frac{d^2y}{dx^2} > 0$

$\therefore 2\ln x + 3 > 0$

$\ln x > -\frac{3}{2}$

$x > e^{-3/2}$ (1)

(c) $\frac{d}{dx} e^{\cos x} = -\sin x \, e^{\cos x}$ (1)

$\therefore \int_0^{\pi/2} \sin x \, e^{\cos x}\,dx = \left[-e^{\cos x}\right]_0^{\pi/2}$ (1)

$= -e^{\cos \pi/2} + e^{\cos 0}$

$= -1 + e^1$

$= e - 1$ (1)