JAMES RUSE AGRICULTURAL HIGH SCHOOL
YEAR 12 MATHEMATICS EXTENSION I
TRIAL EXAM 2004

Marks

QUESTION 1

(a) Find $\frac{d}{dx}\left(\ln\left(5+e^{x}\right)\right)$ 2

(b) Find $\int \frac{19\,dx}{4+8x^2}$ 2

(c) Evaluate $\int_{6}^{22} x\sqrt{x+3}\,dx$ using the substitution $u^2 = x+3$ 4

(d) Solve for x : $\frac{x+1}{x-3} \geq 2$ 2

(e) Six identical yellow discs and four identical blue discs are placed in a straight line.

(i) How many arrangements are possible ? 1

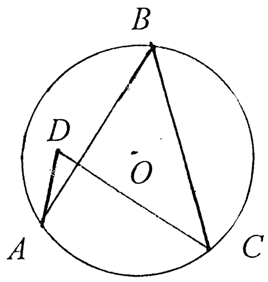
(ii) Find the probability that all the blue discs are together. 1

QUESTION 2 (START A NEW PAGE)

(a) Find the acute angle (to nearest degree) between the lines : 2

$$y = \frac{3x}{8} - \frac{7}{8} \quad \text{and} \quad 2x + y - 5 = 0$$

(b) Points A,B and C lie on the circumference of a circle with centre O, and point D lies inside the circle with $\angle ABC = 17^0$ and $\angle ADC = 34^0$. 3



Prove $ADOC$ is a cyclic quadrilateral.

(c) Find $\int \frac{4x-1}{\sqrt{9-x^2}}\,dx$ 2

(d) Evaluate $\int_{0}^{1} \left(1+x^2\right)^4 dx$ 3

(e) Find $\frac{d}{dx}\left(\cos^{-1}\left(2\cos^2 x - 1\right)\right)$ in simplest terms for $\{ 0 \leq x \leq \frac{\pi}{2} \}$. 2

QUESTION 3 (START A NEW PAGE)

(a)(i) On the same x-y axes graph the functions $y = f(x)$ and $y = f^{-1}(x)$ if $f(x) = e^{x} + e^{2x}$. Show all the y intercepts and asymptotes. 3

(ii) Find the equation of the inverse function $f^{-1}(x)$ if $f(x) = e^{x} + e^{2x}$ stating the domain and range of $f^{-1}(x)$. 4

(b) If α is a multiple root of $P(x)=0$ then $P'(\alpha)=0$. 5

Factorise $P(x) = 12x^3 - 16x^2 + 7x - 1$ if $P(x)$ has multiple zeros.

QUESTION 4 (START A NEW PAGE)

(a) A particle moves in a straight line.

The displacement function x metres in terms of time t seconds is given by :
$x(t) = 6\sin 2t - 6\cos 2t$

(i) Show that the displacement function can be written in the form : 2

$x(t) = R\sin(2t - \alpha)$ where $R>0$ and $\{ 0 < \alpha < 2\pi \}$.

State the exact values of R and α .

(ii) Graph the displacement function $x(t)$ for $\{ 0 < t < 2\pi \}$. 2

(iii) Show that the motion is Simple Harmonic Motion. 2

(iv) Find the expression v^2 in terms of displacement x if v is the velocity of the particle. 2

(v) Find the first time the particle is 2 metres from the centre of motion. 2

(b) Find the constant term in the expression $x^3\left(x^2 + \frac{2}{x}\right)^6$ 2

QUESTION 5

(a) A man has a loan of \$ 15800 with monthly reducible interest of 8% p.a. 5

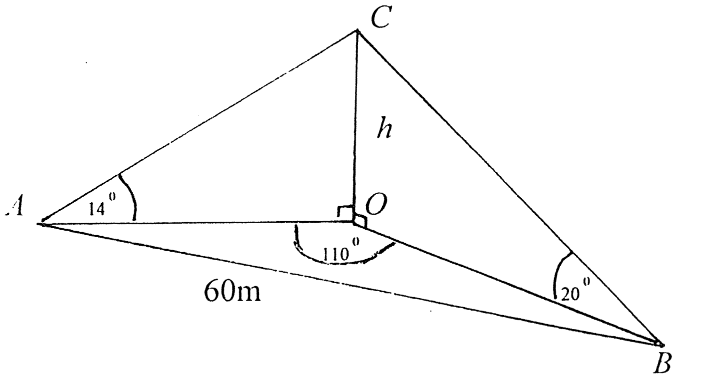
If the repayments are \$1250 per month , find the number of payments to

repay all the loan.

(b) Prove by induction for all positive integers n: 4

$$\frac{5}{6} + \frac{1}{4} + \ldots\ldots\ldots\ldots\ldots\ldots + \frac{n+4}{n(n+1)(n+2)} = \frac{3}{2} - \frac{n+3}{(n+1)(n+2)}$$

(c) 3



A vertical tower shown above has angles of elevation from A and B of 14^0 and 20^0 respectively.

If the distance AB is 60 metres and $\angle AOB = 110^0$, find the height h of the tower to the nearest metre.

QUESTION 6

(a) A bowman fires an arrow with an initial velocity of 50 m/s from 1.5 metres above ground to a target 80 metres away.
The bullseye of the target is 0.3 metres in diameter, and the centre of the bullseye is 1 metre above ground.

(i) Show that the trajectory equation for the flight of the arrow is given by : **3**

$y = x\tan\alpha - \frac{x^2}{500}\left(1+\tan^2\alpha\right)+1.5$ where α is the initial angle of elevation of the arrow, the acceleration due to gravity g is $10 m/s^2$ and the Origin is at ground level .

(ii) Find the range of values of α (to the nearest second) for the arrow to hit the bullseye. **5**

(b) The bowman has a probability of $\frac{3}{5}$ of hitting the bullseye.

(i) Find the probability of hitting the bullseye exactly 7 times from 13 trials. **1**

(ii) By comparing the terms of $\left(\frac{3}{5}+\frac{2}{5}\right)^{13}$ find the most likely outcome of hitting the bullseye from 13 trials. **3**

QUESTION 7

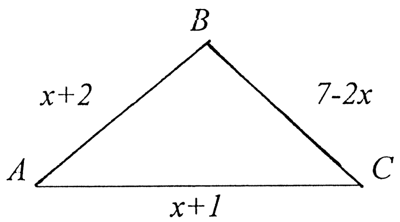
(a) The rate of growth of a population N over t years is given by : $\frac{dN}{dt} = -k\left(N-700\right)$.

(i) Show $N = 700 + Ae^{-kt}$ satisfies $\frac{dN}{dt} = -k\left(N-700\right)$ where A and k are constants. **1**

(ii) The population has decreased from an initial population of 8300 to 5100 in 5 years. **3**

Find the population at the end of the next 5 years.

(b) Triangle ABC is shown .



(i) Show that the domain of x for the triangle to exist is given by { $1<x<3$ }. **2**

(ii) The area A of a triangle with sides a, b and c is given by : **2**

$$A = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where } s = \frac{1}{2}(a+b+c)$$

Show that the expression for the area A of the triangle ABC in terms of x is given by :

$$A = \sqrt{10\left(x^3 - 8x^2 + 19x - 12\right)}$$

(iii) Find the value of x that gives the maximum area of ΔABC . **4**

END OF EXAM