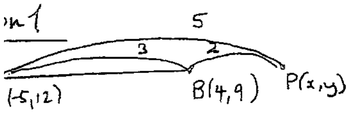
it Trial July 2000

n 1



$\frac{3x+2(-5)}{5}$, $9 = \frac{3y+2(12)}{5}$

$3x - 10$ $\quad 45 = 3y + 24$

$3x$ $\quad 21 = 3y$

10 $\quad y = 7$

$P \equiv (10,7)$

$= 2, \; m_2 = 4$

$\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{2-4}{1+8}\right|$

$\alpha = \frac{2}{9}$

$\alpha = 12^\circ 32'$

$(x) = x^3 + 3x^2 - 10x - 24$

$(3) = 27 + 27 - 30 - 24 = 0$

$3)$ is a factor

$$\begin{array}{r} x^2+6x+8 \\ x-3\,\big)\,x^3+3x^2-10x-24 \\ x^3-3x^2 \\ 6x^2-10x \\ 6x^2-18x \\ 8x-24 \\ 8x-24 \end{array}$$

$\therefore f(x) = (x-3)(x^2+6x+8)$

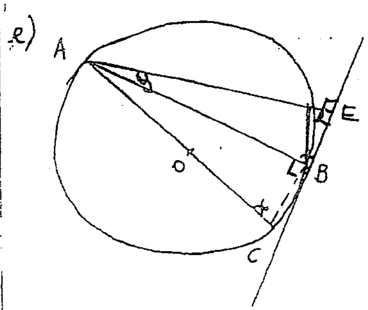
$f(x) = (x-3)(x+2)(x+4)$

d) $\int_0^3 \frac{dx}{\sqrt{9-x^2}} = \left[\sin^{-1}\frac{x}{3}\right]_0^3$

$= \sin^{-1}1 - \sin^{-1}0$

$= \frac{\pi}{2}$

e)



Construction: join BC

$\angle ABC = 90^\circ$ (angle in semicircle)

$\angle AEB = 90^\circ$ (adj suppl $\angle$)

let $\angle EAB = y$, $\angle ABE = x$

then $x + y = 90^\circ$ (angle sum of Δ)

$\angle ACB = x$ (angle in alt segment)

$\therefore \angle CAB = 180 - 90 - x$

$= 90 - x$ (angle sum of Δ)

$= y$

$\therefore \angle CAB = \angle BAE = y$

AB bisects $\angle CAE$

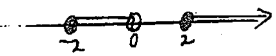
Question 2

a) $x \geq \frac{4}{x}$

$x \neq 0$;

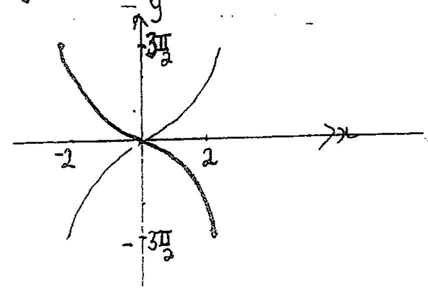
solve $x^2 = 4$

$x = \pm 2$ $\quad$ Try solutions



$\therefore -2 \leq x < 0$ or $x \geq 2$

b) $y = -3\sin^{-1}\frac{x}{2}$



Domain: $-2 \leq x \leq 2$

Range: $-\frac{3\pi}{2} \leq y \leq \frac{3\pi}{2}$

c) $u = 9 - x^2$ $\quad$ limits

$\frac{du}{dx} = -2x$ $\quad$ when $x = 0$, $u = 9$

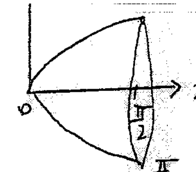
$\frac{du}{-2} = x\,dx$ $\quad$ when $x = 3$, $u = 0$

$\int_0^3 x\sqrt{9-x^2}\,.dx = -\frac{1}{2}\int_9^0 \sqrt{u}\;du$

$= \frac{1}{2}\int_0^9 u^{\frac{1}{2}}\,.du$

$= \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_0^9$

d)



$\cos 2x = 1 - 2\sin$

$V = \pi\int_0^{\frac{\pi}{2}} \sin^2 x\,.dx$

$= \frac{\pi}{2}\int_0^{\frac{\pi}{2}} 1 - \cos 2x\,.dx$

$= \frac{\pi}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$

$= \frac{\pi}{2}\left(\left(\frac{\pi}{2} - 0\right) - (0 - 0)\right)$

$= \frac{\pi^2}{4}$ units^3

...on 3

$\ldots s(x-\alpha) = 3\cos x + 4\sin x$

$\ldots s\alpha + A\sin x \sin\alpha = 3\cos x + 4\sin x$

$\ldots \cos\alpha = 3 \quad \ldots (1)$

$\ldots \sin\alpha = 4 \quad \ldots (2)$

$\tan\alpha = 4/3$

$\alpha = 53°$

$A^2(\cos^2\alpha + \sin^2\alpha) = 3^2 + 4^2$

$A^2 = 25$

$A = 5$

$\ldots sx + 4\sin x = 5\cos(x - 53°)$

$5\cos(x-53°) = -3$

$\cos(x - 53°) = -3/5$

$x - 53° = 127°, 233°$

$x = 180°, 286°$

...rls, 7 boys.

x x x x
x x x x
x x x x
x x x x

$\ldots \times {}^5P_4$

$\ldots 6 \times {}^3P_3 \times {}^2P_2$

c) (i) 2 roots

(ii) $f(x) = e^{x-2} - x$

$f'(x) = e^{x-2} - 1$

$a_2 = 3\cdot3 - \frac{f(3\cdot3)}{f'(3\cdot3)}$

$= 3\cdot3 - \frac{0\cdot369}{2\cdot669}$

$= 3\cdot16$

Question 4

a) $\frac{4^x}{16} = 8^{x+y}$ and $2^{2x+y} = 128$

change all to powers of 2

$2^{2x-4} = 2^{3x+3y}$

$2x - 4 = 3x + 3y$ (equating indices)

$-x - 3y = 4 \quad \ldots (1)$

$2^{2x+y} = 2^7$

$2x + y = 7 \quad \ldots (2)$ (equating indices)

solving $(1)\times2$ & (2) ⇒

$-2x - 6y = 8$

$+ \quad 2x + y = 7$

$-5y = 15$

$y = -3 \quad \therefore x = 5$

b) $x = 2 - \cos t \qquad y = 2t + 2\sin t$

$\frac{dx}{dt} = +\sin t \qquad \frac{dy}{dt} = 2 + 2\cos t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{2 + 2\cos t}{\sin t}$

where $\cos t = \frac{1 - t^2}{1+t^2}$

$\sin t = \frac{2t}{1+t^2}$

$\frac{dy}{dx} = \frac{2 + 2\left(\frac{1-t^2}{1+t^2}\right)}{\frac{2t}{1+t^2}}$

$v^2 = 9(26 - 0)$

$v = \pm 3\sqrt{26} \quad \therefore$ max speed is $3\sqrt{\ldots}$ m/...

$\frac{dy}{dx} = \frac{2(1+t^2) + 2(1 - \ldots)}{2t}$

$= \frac{\cancel{2t}\,{}^2 + 2 - 2t^2}{t}$

$= \frac{4}{t}$

c) $x = a\cos(3t + \alpha)$

$\dot{x} = -3a\sin(3t + \alpha)$

$\ddot{x} = -9a\cos(3t + \alpha)$

$\ddot{x} = -9(a\cos(3t+\alpha))$

$\ddot{x} = -9x$

$\therefore$ satisfies the ...

when $t = 0$ and $x = 5$

using (1)

$5 = a\cos\alpha$

using (2)

$3 = -3a\sin\alpha$

$-1 = a\sin\alpha$

$(3)^2 + (4)^2 \Rightarrow 25 + 1 = c\ldots$

$a = \sqrt{2\ldots}$

$(4) \div (3) \Rightarrow \tan\alpha = -1/5$

$\alpha = -0\cdot1\ldots$

(iii) max speed at centre

from (1) $0 = \sqrt{26}\cos(3t - 0\cdot\ldots)$

$3t - 0\cdot18 = \frac{\pi}{\ldots}$

$\therefore v = -3\sqrt{26}\sin(3(0\cdot56) - 0\cdot1\ldots)$

on 5

$\alpha+\beta+\gamma = -3/2$

$\alpha\beta\gamma = -(-4/2) = 2$

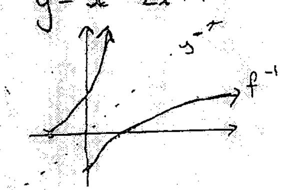
$\beta+\beta\gamma+\alpha\gamma) = 0$

$\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\alpha\gamma)$

$= (-3/2)^2$

$= 9/4$

$y = x^2-2x+1$



gest domain of f: $x \geq -1$

let $y = x^2-2x+1$

ange $x = y^2-2y+1$

ke y the subject

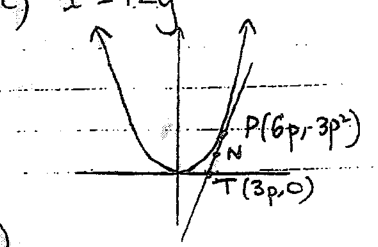
$x = (y-1)^2$

$\sqrt{x} = y-1$

$y = \sqrt{x}+1$

nge: $y \geq -1$

c) $x^2 = 12y$



(i) $\frac{x^2}{12} = y$

$\frac{dy}{dx} = \frac{2x}{12} = \frac{x}{6}$

at $x = 6p$, $\frac{dy}{dx} = \frac{6p}{6} = p$

$\therefore$ eqn of tangent is

$(y - 3p^2) = p(x-6p)$

$y - 3p^2 = px - 6p^2$

$y = px - 3p^2$

(ii) tangent meets x-axis when y=0

$px = 3p^2$

$x = 3p$

$\therefore T(3p, 0)$

(iii) N is mid pt of PT

$N \equiv \left(\frac{6p+3p}{2}, \frac{3p^2}{2}\right)$

$N \equiv \left(\frac{9p}{2}, \frac{3p^2}{2}\right)$

(iv) let $X = \frac{9p}{2}$, $Y = \frac{3p^2}{2}$

$X^2 = \frac{81p^2}{4}$

$X^2 = \frac{27}{2}\left(\frac{3p^2}{2}\right) = \frac{27}{2}Y$

Question 6

a) $\lim_{x\to 0}\left(\frac{\sin 3x}{3x} \times \frac{3}{5}\right)$

$= \frac{3}{5}(1)$

$= \frac{3}{5}$

b) $\frac{dN}{dt} = 0.1(N - 1.2\times10^6)$

Solution is

$N = A \mp Be^{kt}$

where $A = 1.2\times10^6$

$k = 0.1$

when $t=0$, $N = 2.7\times10^6$

$2.7\times10^6 = 1.2\times10^6 + Be^0$

$B = 1.5\times10^6$

$N = 1.2\times10^6 + 1.5\times10^6 e^{0.1t}$

(i) when $t = 3\frac{1}{2}$

$N = 1.2\times10^6 + 1.5\times10^6 e^{0.35}$

$N = 3328\ 601$

(ii) when $N = 2.7\times10^6 \times 3$

$N = 8.1\times10^6$

$8.1\times10^6 = 1.2\times10^6 + 1.5\times10^6 e^{0.1t}$

$\frac{6.9\times10^6}{1.5\times10^6} = e^{0.1t}$

$\ln 4.6 = t$ $\quad t = 15.26$

c)(i) $\ddot{x} = 0$ $\quad \ddot{y} = -10$

$\dot{x} = V\cos\alpha$ $\quad \dot{y} = -10t + V\sin$

$\dot{x} = 30\sqrt{3}\cos 60°$ $\quad \dot{y} = -10t + 30\sqrt{3}\sin$

$\dot{x} = 15\sqrt{3}$ $\quad \dot{y} = -10t + 45$

$x = 15\sqrt{3}t + 0$ $\quad y = -5t^2 + 45$

(ii) For time of flight

when $y=0$, $5t^2 - 45t = 0$

$5t(t-9) = 0$

$t = 0$, $t = 9$

$\therefore$ 9 seconds

For range;

$x = 15\sqrt{3}\times 9 = 135\sqrt{3}$ me

(iii) equation of hill, $y = \frac{1}{\sqrt{3}}x$

$-5t^2 + 45t = \frac{1}{\sqrt{3}}(15\sqrt{3})t$

$-5t^2 + 30t = 0$

$-5t(t-6) = 0$

$t=0$ or $t=6$

$\therefore$ time of flight is 6 secon

tion 7

r $n=1$

s $= \frac{1}{2!} = \frac{1}{2}$

$= \frac{(1+1)!-1}{2!} = \frac{2-1}{2} = \frac{1}{2}$

ue for $n=1$

sume true for $n=k$

$\frac{2}{3!} + \cdots \frac{k}{(k+1)!} = \frac{(k+1)!-1}{(k+1)!}$

for $n=k+1$

$\underbrace{\frac{1}{2!} + \frac{2}{3!} + \cdots \frac{k}{(k+1)!}} + \frac{k+1}{(k+2)!}$

$\frac{(k+1)!-1}{(k+1)!} + \frac{k+1}{(k+2)!}$

$= \frac{(k+2)[(k+1)!-1]}{(k+2)!} + \frac{k+1}{(k+2)!}$

$= \frac{(k+2)!-(k+2)+(k+1)}{(k+2)!}$

$= \frac{(k+2)!-1}{(k+2)!}$

$\frac{(k+2)!-1}{(k+2)!}$

. true for $n=k+1$

true for $n=1$

$\therefore$ true for all n, positive integral

b) (i) $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\frac{dy}{dx} = \frac{(e^x+e^{-x})(e^x+e^{-x}) - (e^x-e^{-x})(}{(e^x+e^{-x})^2}$

$= \frac{(e^{2x}+2+e^{-2x}) - (e^{2x}-2+e}{(e^x+e^{-x})^2}$

$\frac{dy}{dx} = \frac{4}{(e^x+e^{-x})^2}$

but $\frac{dy}{dx} \neq 0$ $\therefore$ no stat pts

as $4 \neq 0$ and $e^x + e^{-x} \neq 0$

(ii) $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\lim_{x\to\infty} \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)$

$\lim_{x\to\infty} \left(\frac{\frac{e^x}{e^x} - \frac{e^{-x}}{e^x}}{\frac{e^x}{e^x} + \frac{e^{-x}}{e^x}}\right)$

$\left(\frac{1-0}{1+0}\right) = 1$

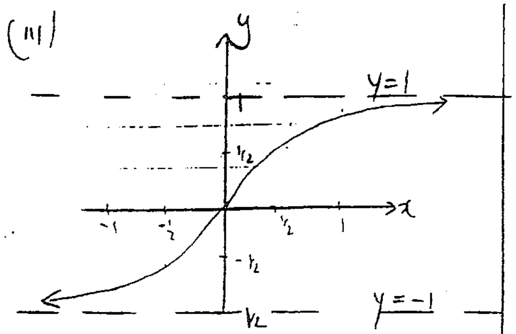
$\therefore$ $y=1$ is an asymptote

Similarly $\lim_{x\to-\infty} \left(\frac{\frac{e^x}{e^{-x}} - \frac{e^{-x}}{e^{-x}}}{\frac{e^x}{e^{-x}} + \frac{e^{-x}}{e^{-x}}}\right)$

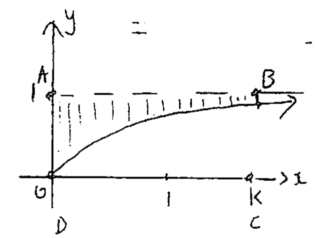
$= \frac{-1}{1} = -1$

$y=-1$ is an asymptote

(iii)



(iv)



Area of rectangle ABCD $= k \times 1$
$= k$

Area under curve $= \int_0^k \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

$= \ln(e^x + e^{-x})\Big]_0^k$

$= \ln(e^k + e^{-k}) - \ln 2$

$\therefore$ ~~Shaded area~~ $= k - (\ln(e^k + e^{-k}) - \ln 2)$

Area $= k - \ln(e^k + e^{-k}) + \ln 2$

(v)

$= \log_e e^k - \ln(e^k + e^{-k}) + \ln 2$

$= \log_e \frac{e^k}{e^k + e^{-k}} + \ln 2$

Now, $\log_e\left[\frac{e^k}{e^k + e^{-k}}\right] < 0$

as $\frac{e^k}{e^k + e^{-k}} < 1$

$\therefore$ max value is $\log_e 2$