NAME ________

Teacher ________

St Marys Senior High School
YEAR 12 EXTENSION
Half Yearly EXAM 2005

Time Allowed: 2 HOURS

DIRECTIONS

- Answer all questions on the paper supplied.
- Place your name and your teachers name on every piece of paper handed in

OUTCOMES ASSESSED

HE3 uses a variety of strategies to investigate mathematical models of situations involving binomial probability, projectiles, simple harmonic motion, or exponential growth and decay

HE5 applies the chain rule to problems including those involving velocity and acceleration as functions of displacement

HE6 determines integrals by reduction to a standard form through a given substitution

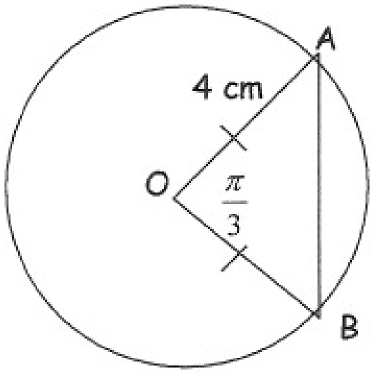
HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form

	HE3	HE5	HE6	HE7	TOTAL
MARKS	19	10	12	38	79
RESULT					

	Question	MARK	OUTCOME
1	Find the constant term in the expansion of $\left(x-\frac{1}{2x^3}\right)^{20}$	3	HE3
2	Use mathematical induction to prove that $1\times2^0+2\times2^1+3\times2^2+\ldots\ldots.+n\times2^{n-1}=1+(n-1)2^n$	4	HE7
3			
	(i) $\int xe^{x^2}\,dx$	2	HE6
	(ii) $\int\left(1+e^{-x}\right)^2\,\mathrm{dx}$	3	
	(iii) $\int_0^1\frac{x}{1+x^2}\,dx$	2	
4	Consider the function $y=f(\theta)$ where $f(\theta)=\cos\theta-\frac{1}{4\sqrt{3}\sin\theta}$		HE5
	(i) Verify that $f'\left(\frac{\pi}{6}\right)=0$	3	
	(ii) Sketch the curve $y=f(\theta)$ for $0<\theta\le\frac{\pi}{2}$ given that $f''(\theta)<0$. On your sketch, write the coordinates of the turning point in exact form and label the asymptote.	4	
5	Find the general solution for $\cos\theta=0.245$	2	HE7

6	Use mathematical induction to prove that, for every positive integer *n*, $13 \times 6^n + 2$ is divisible by 5.	3	HE7
7	Given that $0 < x < \frac{\pi}{4}$ prove that $\tan\left(\frac{\pi}{4} + x\right) = \frac{\cos x + \sin x}{\cos x - \sin x}$	3	HE7
8	Evaluate (i) $\int_0^{\pi} (\sin\theta + 1)d\theta$	2	HE6
	(ii) $\int_0^{\frac{\pi}{2}} \sin^2 3x$ dx	3	
9	For the curve $y = x^2 + \log_e x$ find : (i) any stationary points and points of inflexion, and (ii) hence, sketch the curve.	4 2	HE7
10	Show that the point $A\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is a stationary point of the curve $y = x + \cos x$, $0 \le x \le \pi$. Determine the nature of the stationary point and hence sketch the curve in the given domain.	5	HE7
11	Find the volume of the solid formed if the area bounded by the curve $y = e^{3x}$, the *x*-axis and the lines $x = 1$ and $x = 2$ is rotated about the x-axis.	3	HE7

12	(i) Find the general term in the expansion of $\left(x^2 - \frac{3}{x}\right)^{2n}$, where n is a positive integer. (ii) What value of k (in terms of n) results in the power of x being equal to 6? (iii) What is the coefficient of x^6 in the expansion of $\left(x^2 - \frac{3}{x}\right)^{12}$?	1 2 2	HE3
13	Evaluate $\log_a 50$ if $\log_a 5 = 1.3$ and $\log_a 2 = 0.43$	2	HE7
14	Find the area bounded by the curve $y = x \ln x$, the x-axis and the lines $x = 1$ and $x = 5$ by using Simpson's Rule and 5 function values, correct to 2 decimal places.	3	HE7
15	The probability that a piece of space junk will land in Australia is estimated at 0.01. If 18 pieces of space junk are due to crash, find the probability that 10 of them will crash in Australia. (Leave your answer in index form.)	2	HE3
16	Find the derivative of $(1 + x^3)\ln(1 + x^3)$	3	HE5
17	Sketch the curves $y = 7\sin 3x$ and $2x - y - 1 = 0$ on the same number plane for $\frac{-2\pi}{3} \le \theta \le \frac{2\pi}{3}$ and from your sketch state how many solutions there are to the equation $7\sin 3x = 2x - 1$ for $\frac{-2\pi}{3} \le \theta \le \frac{2\pi}{3}$	4	HE7

18	 O is the centre of the circle with radius 4 cm. Find the area of the major segment cut off by the chord AB.	3	HE7
19	Six cards are drawn at random from a pack of 52 playing cards, each being replaced before the next is drawn. Find, as a fraction with denominator 4^6, the probability at least four are clubs.	3	HE3
20	Assume $(2+5x)^{12} = \sum_{k=0}^{12} t_k x^k$ (i) Use the binomial theorem to write an expression for t_k, $0 \le k \le 12$ (ii) Show that $\frac{t_{k+1}}{t_k} = \frac{5(12-k)}{2(k+1)}$ (iii) Hence or otherwise, find the largest coefficient t_k (you may leave your answer in binomial form)	1 3 2	HE3