NORTH SYDNEY
GIRLS HIGH SCHOOL

2009

HIGHER SCHOOL CERTIFICATE

ASSESSMENT TASK #3

Mathematics Extension 1

Student Name:_____ **Teacher:** _____

General Instructions

- Reading time – 2 minutes.
- Working time – 65 minutes.
- Write using black or blue pen.
- Board approved calculators may be used.
- All *necessary* working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for untidy or badly arranged work.
- Start each **NEW** question on a separate answer sheet.

Total Marks: 57 Marks

- Attempt Questions 1 - 6
- All questions are NOT of equal value.

	Q 1		Q 2		Q 3		Q4			Q5	Q6			Total
	acd	b	abcd	e	a	b	a	b	c		a	b	c	
H 6														/3
H 8														/10
H 9														/31
HE 2														/3
HE 6														/10
	/9		/10		/10		/9			/10	/9			/57

Total marks – 57
Attempt Questions 1 - 6
All questions are NOT of equal value

Start each question on a SEPARATE answer sheet.

Question 1 (9 marks) **Marks**

(a) Evaluate $\int_0^1 \frac{dx}{2x+1}$, leaving your answer in the exact form. **2**

(b) Using the substitution $u = 4 - x^2$, evaluate $\int \frac{x}{\sqrt{4-x^2}}\,dx$ **3**

(c) Let $f(x) = \frac{1}{2}\left(e^x + e^{-x}\right)$ and $F(x) = \frac{1}{2}\left(e^x - e^{-x}\right)$ **2**
Prove that $\left[f(x) + F(x) \right]^n = f(nx) + F(nx)$

(d) Evaluate $\int_0^1 \frac{e^x}{e^x + 1}\,dx$ **2**

Question 2 (10 Marks) Start a NEW answer sheet. **Marks**

(a) Solve $e^x = 5$, leaving your answer correct to 3 decimal places **1**

(b) Find a primitive of $\frac{3x}{1+x^2}$ **2**

(c) Find $\frac{d}{dx}\left(3x \log_e x\right)$ **2**

(d) Evaluate $\int_0^3 3^x \, dx$ **2**

(e) Using the substitution $u = \log_e x$, evaluate $\int_1^e \frac{\left(1+\log_e x\right)^2}{x} \, dx$ **3**

Question 3 (10 Marks) Start a NEW answer sheet. **Marks**

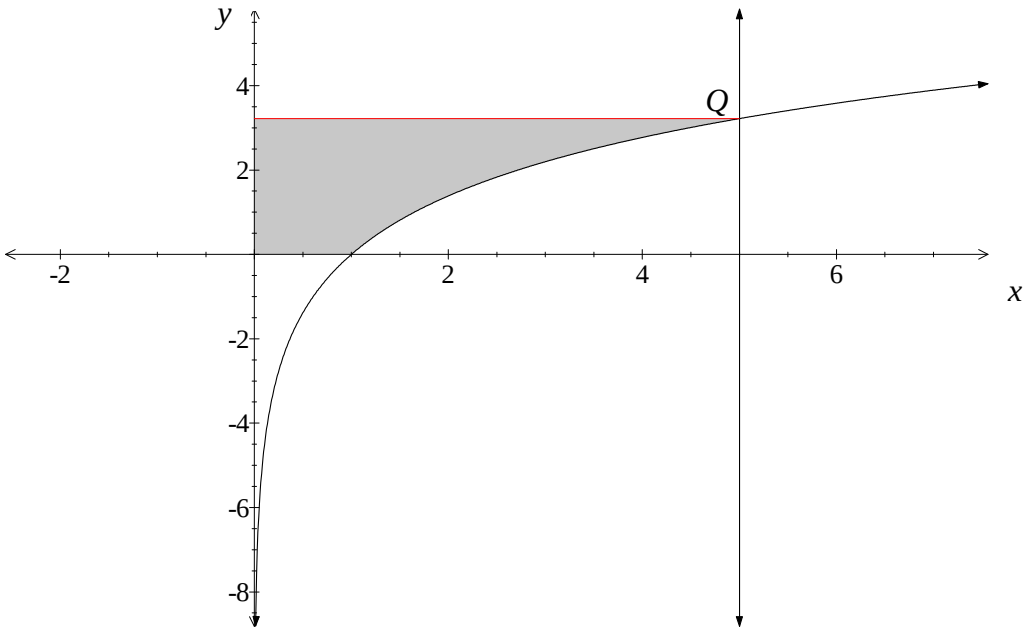
(a) (i) Show that $\dfrac{5}{\sqrt{5x+3}-\sqrt{5x-2}} = \sqrt{5x+3}+\sqrt{5x-2}$ **2**

(ii) Hence find $\displaystyle\int \frac{dx}{\sqrt{5x+3}-\sqrt{5x-2}}$ **2**

(b) (i) Show that $\dfrac{d}{dx}\left(x\ln x - x\right) = \ln x$. **1**

(ii) Hence, or otherwise, find $\displaystyle\int \ln x^2 dx$. **2**

(iii) The graph below shows the curve $y = \ln x^2$ $(x > 0)$ which meets the line $x = 5$ at Q. **3**
Using your answers above, or otherwise, find the area of the shaded region.



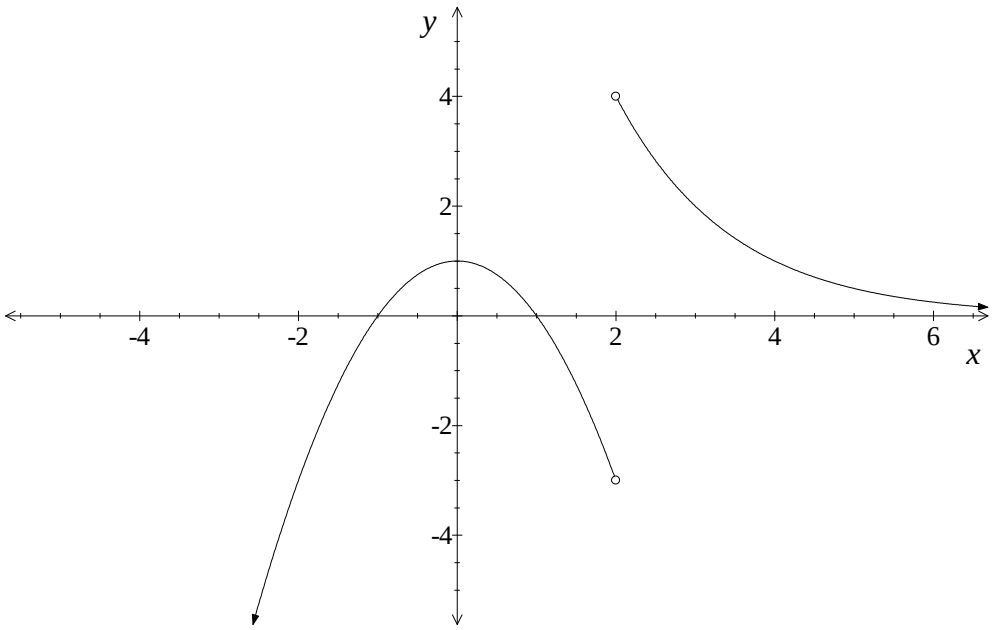
Question 4 (9 Marks) Start a NEW answer sheet. **Marks**

(a) Find $\int \frac{x+1}{x^2}\,dx$ **2**

(b) The following graph shows the gradient function $y = f'(x)$. **3**

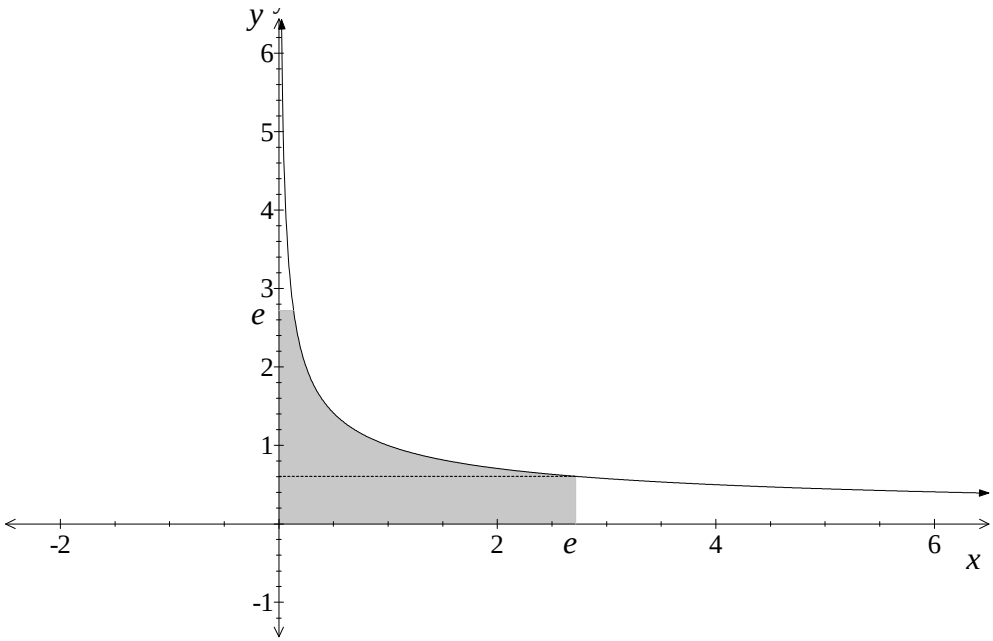
The graph shows that $f'(1) = f'(-1) = 0$.

Sketch the graph of $y = f(x)$, given that $f'(x)$ is continuous everywhere except at $x = 2$ and that $f(0) = 1$ and $f(-1) = -2$



(c) The shaded region below is that bounded by $y = \frac{1}{\sqrt{x}}$, the coordinate axes and the lines $x = e$ and $y = e$. **4**

Find the volume when the shaded region is rotated about the y-axis, correct to 2 significant figures.



Question 5 (10 Marks) Start a NEW answer sheet. **Marks**

Consider the function $y = \dfrac{\ln x}{x}$

(a) What is the domain of this function? **1**

(b) Show that $\dfrac{d}{dx}\left(\dfrac{\ln x}{x}\right) = -\left(\dfrac{\ln x - 1}{x^2}\right)$ **1**

(c) Describe the behaviour of the function as x

(i) approaches zero. **1**

(ii) increases indefinitely **1**

(d) Find any stationary points and determine their nature. **2**

(e) Sketch the curve of this function. **2**

(f) Hence find the value(s) of k for which $e^{kx} = x$ has no solutions. **2**

Question 6 (9 Marks) Start a NEW answer sheet. **Marks**

(a) Use mathematical induction to show that the following statement is true **3**

$$n^3 + 2n \text{ is a multiple of } 12$$

where n is an <u>even</u> positive integer

(b) By use of an appropriate diagram and reasons, evaluate the following sum. **2**
Do NOT evaluate any primitive functions.

$$\int_0^1 e^x dx + \int_1^e \ln x \, dx$$

(c) (i) Show $\frac{1}{u} - \frac{1}{u+1} = \frac{1}{u(u+1)}$ **1**

(ii) Using the substitution $x = \ln u$, find $\int \frac{dx}{1+e^x}$ **3**

End of paper

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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$