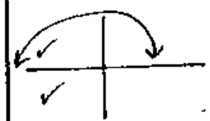
Suggested Solution (s)	Comments
$\lim_{x\to0}\frac{\sin 2x}{4x}$ $=\frac{1}{2}\lim_{x\to0}\frac{\sin 2x}{2x}$ $=\frac{1}{2}$	✓ ✓ or $\underline{aw2}$
$\int_2^3\left(\frac{x^2}{x^3-7}\right)dx$ $=\frac{1}{3}\int_2^3\left(\frac{3x^2}{x^3-7}\right)dx$ $=\left[\frac{1}{3}\ln(x^3-7)\right]_2^3$ $=\frac{1}{3}\left(\ln(27-7)-\ln(8-7)\right)$ $=\frac{1}{3}(\ln 20-\ln 1)$ $=\frac{1}{3}\ln 20$	✓ ✓ ✓
$\frac{2x}{x-1}\le 1$ $(x-1)^2\cdot\frac{2x}{x-1}\le(x-1)^2$ $2x(x-1)\le(x-1)^2$ $2x(x-1)-(x-1)^2\le 0$ $(x-1)(2x-x+1)\le 0$ $(x-1)(x+1)\le 0$ and $x\ne1$. $\therefore -1\le x<1$	✓ ✓ ✓

Suggested Solution (s)	Comments
d) $\frac{d}{dx}\left(\tan^{-1}\frac{x}{3}\right)$ $=\frac{3}{9+x^2}$	✓
e) $x=\frac{mx_2+nx_1}{m+n}$ $19=\frac{-3(x)+2(-2)}{-3+2}$ $-19=-3x-4$ $-3x=-15$ $\underline{x=5}$ and $y=\frac{my_2+ny_1}{m+n}$ $-15=\frac{-3y+2(3)}{-3+2}$ $15=-3y+6$ $3y=-9$ $\underline{y=-3}$ $\therefore B(5,-3)$	✓ ✓ ✓

Suggested Solution (s)	Comments
2a) Let $\angle ACQ=\alpha$ $\therefore\angle QCB=\alpha$ (QC bisects $\angle ACB$). Let $\angle BCP=\beta$. $\therefore\angle CAB=\beta$ ($\angle$ between a tangent and a chord is equal to the $\angle$ in the alt. segment). So $\angle BQC=\alpha+\beta$ (ext. $\angle$ of ΔACQ). also $\angle QCP=\alpha+\beta$. $\therefore\angle BQC=\angle QCP$ (both $=\alpha+\beta$) $\therefore PC=PQ$ (base $\angle$'s of Isos Δ are equal).	✓ ✓ ✓
b) $\int\frac{dx}{e^x+4e^{-x}}$ $=\int\frac{dx}{\frac{e^x}{1}+\frac{4}{e^x}}$ $u=e^x$, $du=e^x dx$ $=\int\frac{e^x\,dx}{e^{2x}+4}$ $=\int\frac{1\,du}{u^2+4}$ $=\frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)+C$ $=\frac{1}{2}\tan^{-1}\left(\frac{e^x}{2}\right)+C.$	✓ ✓ ✓

Suggested Solution (s)	Comments
c) $\int_0^{\frac{\pi}{2}}\cos^2 2x\,dx$ aside: $\cos^2 x=\frac{1}{2}(\cos 2x+1)$ $\therefore\cos^2(2x)=\frac{1}{2}(\cos 4x+1)$ $=\frac{1}{2}\int_0^{\pi/2}(\cos 4x+1)dx$ $=\frac{1}{2}\left[\frac{\sin 4x}{4}+x\right]_0^{\pi/2}$ $=\frac{1}{2}\left[\left(\frac{\sin 2\pi}{4}+\frac{\pi}{2}\right)-(0)\right]$ $=\frac{1}{2}\left(\frac{\pi}{2}\right)$ $=\frac{\pi}{4}$	✓ ✓ ✓
d) let $\alpha=\cos^{-1}\left(\sin\frac{4\pi}{3}\right)$ $\therefore\alpha=\cos^{-1}\left(-\frac{\sqrt3}{2}\right)$ $\cos\alpha=\frac{-\sqrt3}{2}$ $0\le\alpha\le\pi$.  $\therefore\alpha$ is in the 2nd quad. Related $\angle=\pi/6$. $\therefore\alpha=\pi-\frac{\pi}{6}$ $\alpha=\frac{5\pi}{6}$	✓ ✓ ✓

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

1). $^{12}C_3(x)^9\left(\frac{-2}{x^3}\right)^3$

$= {}^{12}C_3\, x^{9} \cdot \frac{(-2)^3}{(x^3)^3}$

$= {}^{12}C_3 \times (-2)^3$

$= -1760.$

) $y = x^2 - x$

$\frac{dy}{dx} = 2x - 1$

@ $x = 2$ $\frac{dy}{dx} = 3.$

$\therefore y - y_1 = m(x - x_1)$

$y - 2 = 3(x - 2)$

$y = 3x - 6 + 2$

$y = 3x - 4$

) $\frac{x}{3} + \frac{y}{2} = 1$

$2x + 3y = 6$

$3y = 6 - 2x$

$y = 2 - \frac{2}{3}x$

$\therefore m_1 = -2/3.$

$m_2 = 3.$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\tan\theta = \left|\frac{3 - {-2/3}}{1 + 3(-2/3)}\right|$

$\tan\theta = 3\,2/3$

$\therefore \theta = 74°45'$

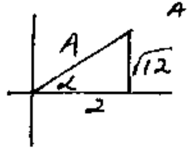
$\therefore$ obtuse $\measuredangle = 105$

c)(i) $\sqrt{12}\sin x + 2\cos x \equiv A\cos(x - \alpha)$

$\equiv A\cos x \cdot \cos\alpha + A\sin x \sin\alpha$

$\therefore A\cos\alpha = 2$ $\quad A\sin\alpha = \sqrt{12}$

$\cos\alpha = \frac{2}{A}$ $\quad \sin\alpha = \frac{\sqrt{12}}{A}.$



$A = 4.$

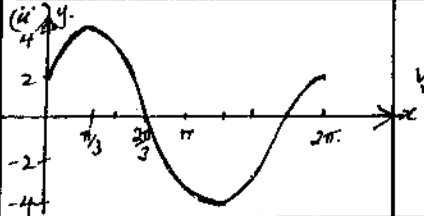
$\tan\alpha = \frac{\sqrt{12}}{2}$

$\tan\alpha = \sqrt{3}$

$\therefore \alpha = \frac{\pi}{3}.$

$\sqrt{12}\sin x + 2\cos x = 4\cos\left(x - \frac{\pi}{3}\right)$

(ii)



(iii) 2

(iv) $4\cos(x - \pi/3) = 1$

$\cos(x - \pi/3) = 1/4.$

$\therefore x = \frac{\pi}{3} + 2m\pi \pm \cos^{-1}\left(\frac{1}{4}\right)$

4/ a) $e^{-x} - \log_e x = 0$

$f(x) = e^{-x} - \log_e(x)$

$f'(x) = -e^{-x} - \frac{1}{x}$

$f(1.4) = e^{-1.4} - \log_e(1.4)$

$f'(1.4) = -e^{-1.4} - \frac{1}{1.4}$

hence $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 1.4 - \left(\frac{e^{-1.4} - \ln(1.4)}{-e^{-1.4} - \frac{1}{1.4}}\right)$

$\doteqdot 1.306$ (3dp)

b) Test that the statement is true for $n = 1$; where n is a positive Integer.

ie $7^1 + 5 = 12 = 6 \times 2$

$\therefore$ divisible by 6.

Assume that the statement is true for $n = k$, ie

$7^k + 5 = 6P$; where P is a positive Integer. —*

Prove that the statement is true for $n = k+1$.

ie $7^{k+1} + 5 = 6Q$; where Q is a positive Integer.

So $7^{k+1} + 5$

$= 7(7^k) + 5$

$= 7(6P - 5) + 5$; from the assumption —*

$= 42P - 35 + 5$

$= 42P - 30$

$= 6(7P - 5)$

$= 6Q$; where $Q = 7P - 5$ which is divisible by 6.

If the statement is true for $n = k$, then the statement is true for $n = k+1$.

Since the statement is true for $n = 1$, then it is true for $n = 1+1 = 2$, $2+1 = 3$, etc for all positive Integers n.

note: Students must have attempted steps 1, 2, 3 to be awarded marks for step 4.

c) $\frac{d}{dx}\left(x\sin^{-1}\frac{x}{4} + \sqrt{16 - x^2}\right)$

$= x \cdot \frac{1}{\sqrt{16 - x^2}} + \sin^{-1}\left(\frac{x}{4}\right) \cdot 1 + \frac{1}{2}(16 - x^2)^{-1/2} \times -2x$

$= \frac{x}{\sqrt{16 - x^2}} + \sin^{-1}\left(\frac{x}{4}\right) + \frac{-x}{\sqrt{16 - x^2}}$

$= \sin^{-1}\left(\frac{x}{4}\right).$

(ii) $\int_0^4 \sin^{-1}\left(\frac{x}{4}\right) = \left[x\sin^{-1}\left(\frac{x}{4}\right) + \sqrt{16 - x^2}\right]_0^4$ — from (i) —*

$= \left(4\sin^{-1}(1) + \sqrt{16 - 16}\right) - \left(0 + \sqrt{16}\right)$

$= 4\sin^{-1}(1) - 4$

$= 2\pi - 4.$

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

when $t=0$, $T=24°C$,
$C = -40°C$,
$\therefore 24 = -40 + Ae^0$
$\therefore A = 64$ ✓

) $t=5$, $T=19°C$.
$19 = -40 + 64e^{5k}$ ✓
$e^{5k} = \frac{59}{64}$
$\ln(e^{5k}) = \ln(59/64)$
$\therefore 5k = \ln(59/64)$.
$k = \frac{1}{5}\ln(59/64)$ ✓

) when $T = 0°C$;
$0 = -40 + 64e^{kt}$
$\frac{40}{64} = e^{kt}$ ✓
$\ln\left(\frac{40}{64}\right) = \ln(e^{kt})$.
$\therefore kt = \ln\left(\frac{40}{64}\right)$
$t = \dfrac{\ln\left(\frac{40}{64}\right)}{\frac{1}{5}\ln\left(\frac{59}{64}\right)}$ ✓
$t = 28.889\ldots$
$\doteqdot 29$ seconds

$LHS = \dfrac{\tan\frac{\pi}{4} + \tan\theta}{1 - \tan\frac{\pi}{4}\cdot\tan\theta} - \left(\dfrac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\cdot\tan\theta}\right)$ ✓

$= \dfrac{1+\tan\theta}{1-\tan\theta} - \left(\dfrac{1-\tan\theta}{1+\tan\theta}\right)$

$= \dfrac{(1+\tan\theta)^2 - (1-\tan\theta)^2}{(1-\tan\theta)(1+\tan\theta)}$

$= \dfrac{1+2\tan\theta+\tan^2\theta - (1-2\tan\theta+\tan^2\theta)}{(1-\tan\theta)(1+\tan\theta)}$ ✓

$= \dfrac{\cancel{1}+2\tan\theta+\cancel{\tan^2\theta}\cancel{-1}+2\tan\theta-\cancel{\tan^2\theta}}{(1-\tan\theta)(1+\tan\theta)}$

$= \dfrac{4\tan\theta}{1-\tan^2\theta}$

$= \dfrac{2(2\tan\theta)}{1-\tan^2\theta}$ ✓

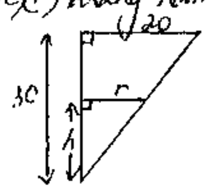
$= \dfrac{2(\tan\theta+\tan\theta)}{1-(\tan\theta)(\tan\theta)}$

$= 2(\tan 2\theta)$

$= RHS.$

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

5c(i) using similar triangles:



$\therefore \frac{r}{h} = \frac{20}{30}$
$\frac{r}{h} = \frac{2}{3}$
$\therefore r = \frac{2h}{3}$ ✓

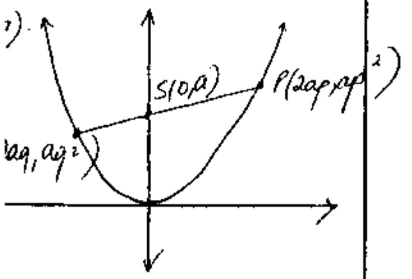
(ii) $V = \frac{1}{3}\pi r^2 h$; $r = \frac{2}{3}h$
$= \frac{1}{3}\pi\left(\frac{2h}{3}\right)^2 h$
$= \frac{1}{3}\pi\left(\frac{4h^2}{9}\right)h$ ✓
$= \frac{4}{27}\pi h^3.$

(iii) $\frac{dV}{dh} = \frac{4}{9}\pi h^2$
$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$
$= \frac{9}{4\pi h^2} \times 24$
$\frac{dh}{dt} = \frac{54}{\pi h^2}$

$A = \pi r^2$
$A = \pi\left(\frac{2h}{3}\right)^2$
$= \frac{4\pi}{9}h^2$ ✓
$\frac{dA}{dh} = \frac{8}{9}\pi h$
$\therefore \frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}$
$= \frac{8\cancel{\pi}\cancel{h}}{9} \times \frac{54}{\cancel{\pi} h^{\cancel{2}}}$
when $h = 16$ cm
$\frac{dA}{dt} = \frac{8\cancel{\pi}}{9} \times \frac{54}{\cancel{\pi}16}$
$= 3\ \text{cm}^2/\text{s}.$ ✓

Suggested Solution (s)	Comments

i)



$m_{PQ} = \dfrac{ap^2-aq^2}{2ap-2aq}$

$= \dfrac{a(p-q)(p+q)}{2a(p-q)}$

$= \dfrac{p+q}{2}$

$y-ap^2 = \dfrac{p+q}{2}(x-2ap)$

$2y-2ap^2 = (p+q)(x-2ap)$

$2y-2ap^2 = px-2ap^2+qx-2apq$

$2y = (p+q)x - 2apq$

$\therefore y = \frac{1}{2}(p+q)x - apq$

ii) Since PQ is a focal chord it passes through $S(0,a)$.

$\therefore a = \frac{1}{2}(p+q)(0) - apq$

$a = -apq$

$\therefore pq = -1.$

iii) $PS = \sqrt{(2ap-0)^2+(ap^2-a)^2}$

$= \sqrt{(2ap)^2 + a^2(p^2-1)^2}$

$= \sqrt{4a^2p^2 + a^2(p^4-2p^2+1)}$

$= \sqrt{4a^2p^2 + a^2p^4 - 2a^2p^2 + a^2}$

$= \sqrt{a^2(p^4+2a^2p^2+1)}$

$= \sqrt{a^2(p^2+1)^2}$

$= a(p^2+1)$

Similarly $QS = a(q^2+1)$

$PQ = PS + SQ$

$= a(p^2+1) + a(q^2+1)$

$= a(p^2+q^2+2)$

Since $pq = -1$

$q = \dfrac{-1}{p}$

$\therefore PQ = a\left(p^2+\dfrac{1}{p^2}+2\right)$

$= a\left(p+\dfrac{1}{p}\right)^2.$

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6b)(i) Using the chain Rule;

$\dfrac{d}{dx}\left(\frac{1}{2}v^2\right) = \dfrac{d}{dv}\left(\frac{1}{2}v^2\right)\times\dfrac{dv}{dx}$

$= v \times \dfrac{dv}{dx}$

$= \dfrac{dx}{dt} \times \dfrac{dv}{dx}$

$= \dfrac{dv}{dt}$

$= \dfrac{d^2x}{dt^2}$

ii) $\dfrac{d^2x}{dt^2} = -4\left(x + \dfrac{16}{x^3}\right)$

$\therefore -4\left(x+\dfrac{16}{x^3}\right) = \dfrac{d}{dx}\left(\frac{1}{2}v^2\right).$

$\frac{1}{2}v^2 = \int(-4x - 64x^{-3})\,dx$

$\frac{1}{2}v^2 = \dfrac{-4x^2}{2} - \dfrac{64x^{-2}}{-2} + C$

$t=0,\ v=0,\ x=2,\ C=0.$

$\therefore v^2 = -4x^2 + 64x^{-2}$

$v^2 = \dfrac{64}{x^2} - 4x^2$

$v^2 = \dfrac{64-4x^4}{x^2}$

$v^2 = \dfrac{4(16-x^4)}{x^2}$

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
) $(x-h)^2 = -4a(y-k)$ vertex $(0, \alpha)$. $(x-0)^2 = -4a(y-\alpha)$ $x^2 = -4a(y-\alpha)$ function passes through $(\pi, 0)$ $\therefore \pi^2 = -4a(0-\alpha)$ $\pi^2 = -4a(-\alpha)$ $\pi^2 = 4a\alpha$ $\therefore 4a = \frac{\pi^2}{\alpha}$	✓ ✓	iii) $\int_{-\pi}^{\pi} \alpha\left(1-\frac{x^2}{\pi^2}\right)dx = 4$ $2\int_0^{\pi} \alpha\left(1-\frac{x^2}{\pi^2}\right)dx = 4$ $\int_0^{\pi}\left(\alpha - \frac{\alpha}{\pi^2}x^2\right)dx = 2$ $\left[\alpha x - \frac{\alpha}{\pi^2}\frac{x^3}{3}\right]_0^{\pi} = 2$ $\left(\alpha\pi - \frac{\alpha}{\pi^2}\frac{\pi^3}{3}\right) - (0-0) = 2$ $\alpha\pi - \frac{\alpha\pi}{3} = 2$. $\frac{2}{3}\alpha\pi = 2$. $\alpha\pi = 3$ $\alpha = \frac{3}{\pi}$.	✓ ✓ ✓
) $x^2 = -4a(y-\alpha)$. since $4a = \frac{\pi^2}{\alpha}$ $\therefore x^2 = -\frac{\pi^2}{\alpha}(y-\alpha)$ $x^2 = -\frac{\pi^2 y}{\alpha} + \pi^2$ $\alpha x^2 = -\pi^2 y + \alpha\pi^2$ $\pi^2 y = \alpha\pi^2 - \alpha x^2$ $y = \frac{\alpha\pi^2 - \alpha x^2}{\pi^2}$ $y = \alpha\left(1 - \frac{x^2}{\pi^2}\right)$	✓ ✓		

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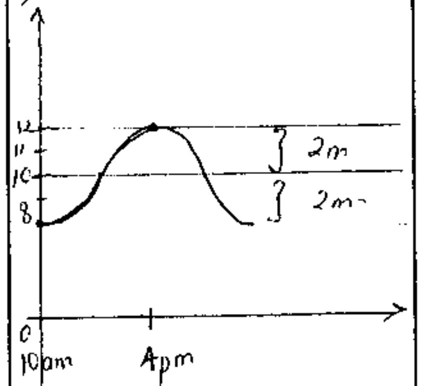
Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
7 b) 6 hours. $\therefore$ wavelength = 12h. period = $\frac{2\pi}{n} = 12$ $\therefore n = \frac{\pi}{6}$ Amplitude = 2m. $x = -2\cos\left(\frac{\pi}{6}t\right) + 10$ when $x = 11$ m; $11 = -2\cos\left(\frac{\pi}{6}t\right) + 10$ $-\frac{1}{2} = \cos\left(\frac{\pi}{6}t\right)$. $\therefore \frac{\pi}{6}t = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$ $t = \frac{2\pi}{3} \times \frac{6}{\pi}, \frac{4\pi}{3} \times \frac{6}{\pi}$ $= 2$ hours. $t = 4$ hours, 8 hours from 10am.	✓ ✓ ✓ ✓ ✓	ie/ the first time period the ship can safely pass through would be between 2pm and 6pm.	