GOD IS LOVE

QUESTION ONE

1) A is the point $(-2, -1)$. B is the point $(1, 5)$. Find the co-ordinates of the point Q, which divides AB externally in the ratio $5 : 3$. 2marks

2) If $(a-3)x^2 - (b-1)x + (c-2) = x^2 + 4x + 5$ for all real x, find a, b and c. 3marks

3) Solve the equation $\cos 2A = \cos A$ where $0 \le A \le 360°$. 3marks

4) i. Express $\cos\theta - \sqrt{3}\sin\theta$ *in the form* $R\cos(\theta+\alpha)$. 2marks

ii. Hence solve the equation $\cos\theta - \sqrt{3}\sin\theta = 1$ for θ in the interval $0 \le \theta \le 360$. 2marks

QUESTION TWO

5) Determine if the roots of the quadratic equation $15x^2 - 41x + 14 = 0$ are real or unreal, rational or irrational, equal or unequal. 2marks

6) Let α and β be the roots of the equation $x^2 + 7x + 3 = 0$. Without solving, find the value of:

a. $\alpha + \beta$; b. $\alpha\beta$; c. $(\alpha + 2)(\beta + 2)$. 2marks

7) Find all angles θ for which $\sin 2\theta = \cos\theta$. 4marks

8) Show that $\dfrac{\cos x - \cos(x + 2\theta)}{2\sin\theta} = \sin(x+\theta)$. 4marks

QUESTION THREE

9) Solve the inequality $\dfrac{x}{x^2 - 1} > 0$. 2marks

10) A is the point $(-4, 1)$ and B is the point $(2, 4)$. Q is the point which divides AB internally in the ratio $2 : 1$ and R is the point which divides AB externally in the ratio $2 : 1$. $P(x, y)$ is a variable point which moves so that $PA = 2PB$.

i. Find the co-ordinates of Q and R. 2marks

ii. Show that the locus of P is a circle on QR as diameter. 2marks

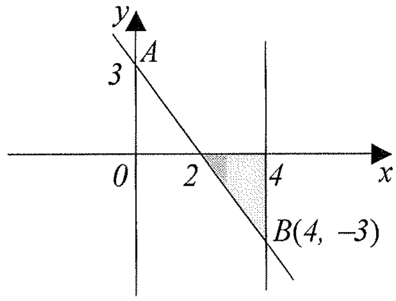
11) Using the "t" results, find all the angles θ with $0 \le \theta \le 360$ for which $\sin\theta + \cos\theta = -1$. 3marks

12) For the equation $4x^2 + 4(r-3)x + (19 - 3r) = 0$:
Find the values of r for which the equation has real roots. 3marks

QUESTION FOUR

14) Solve $3^{2x+1} - 28(3^x) + 9 = 0$ 3marks

15)



A and B are the points $(0, 3)$ and $(4, -3)$ respectively.

a. Find the distance between A and B. 1mark

b. If C is the point $(-5, 0)$, find the co-ordinates of the midpoint of the interval joining B and C. 1mark

c. Show that the equation of the line AB is $3x + 2y - 6 = 0$. 2marks

d. Hence find the equation of the line perpendicular to AB and passing through C. 2marks

e. Find the point of intersection of the line AB with the line $x - 4y + 5 = 0$. 1mark

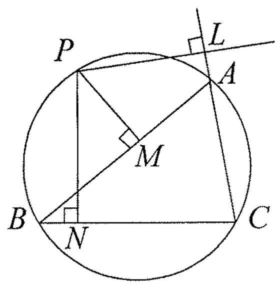
f. Write down three inequalities to describe the shaded region given above. 2marks

QUESTION FIVE

13) One root of the equation $x^2 - (r + 3)x + (5r - 3) = 0$ is twice the other root. Find the two possible values of r. 3marks

17) Prove that $8\cos^4 x \equiv 3 + 4\cos 2x + \cos 4x$. *4marks*

19) ABC is a triangle inscribed in the circle. P is a point on the minor arc AB. The points L, M and N are the feet of the perpendiculars from P to CA produced, AB, and BC respectively. Show that L, M and N are collinear. 5marks



[[End Of Qns]]

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[Answers]

1) $1 \le x < 5$

2) $Q(5\frac{1}{2}, 14)$

3) $A = 0°, 120°, 240°$ or $360°$

4) i) Proof ii) $\theta = 0, \frac{4\pi}{3}$ or 2π

5) Real, rational, unequal

6) a) -7 b) 3 c) -7

7) $\theta = \frac{\pi}{2} \pm n\pi$ or $\theta = n\pi + (-1)^n \sin^{-1}\frac{1}{4}$

8) Proof

9) $-1 < x < 0$ or $x > 1$

10) i) $Q(0, 3)$, $R(8, 7)$ ii) Proof

11) $\theta = 0, \frac{\pi}{2}, 2\pi$

12) a) $r \le -2, r \ge 5$

13) $\frac{3}{2}$ or 15

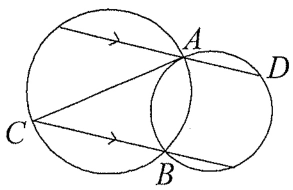
14) a) $2\sqrt{13}$ units b) $(-\frac{1}{2}, -\frac{3}{2})$ c) Proof

d) $2x - 3y + 10 = 0$ e) $(1, \frac{3}{2})$ f) $y \le 0$,

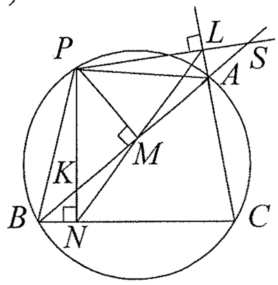
$x \le 4,\ 3x + 2y - 6 \ge 0$

15) $4, -3, 7$

16) i)



ii) iii) Proof



17) In order to prove that L, M and N are collinear, it is sufficient to show that $\angle LMA = \angle NMB$. For this purpose we show, that $\angle NMB = \angle BPN = \angle SPA = \angle LMA$. The first step: $\angle NMB = \angle BPN$. The triangles PKM and BKN are rectangular and $\angle PKM = \angle BKN \Rightarrow \Delta PKM$ are similar $\Delta BKN \Rightarrow \frac{BK}{PK} = \frac{NK}{MK}$. But $\angle PKB = \angle MKN \Rightarrow \Delta PKB$ are similar $\Delta MKN \Rightarrow \angle NMB = \angle BPN$. The second step: $\angle BPN = \angle SPA$. The point P lies on the circle $\Rightarrow PACB$ is a cyclic quadrilateral $\Rightarrow \angle PAC + \angle PBC = 180°$. But $\angle PAC + \angle PAL = 180°$. Hence $\angle PBC = \angle PAL$. From here, as the triangles PNB and PLA are rectangular, we have ΔPNB are similar $\Delta PLA \Rightarrow \angle BPN = \angle APL$.

The third step: $\angle SPA = \angle LMA$. It is obvious that ΔALS is similar ΔPMS, as these rectangular triangles have the common angle $\angle PSM$. Hence $\frac{PS}{AS} = \frac{MS}{LS} \Rightarrow \Delta MLS$ is similar $\Delta PAS \Rightarrow \angle SPA = \angle LMA$.

18) $4, -3, 7$