GIRRAWEEN HIGH SCHOOL

EXAMINATION

2007

MATHEMATICS

EXTENSION 1

Time allowed – Two hours
(Plus 5 minutes' reading time)

Question 1 *(12 Marks)* Use a separate piece of paper — Marks

a) Evaluate $\displaystyle\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}}$ — 2

b) Let A be the point (-8 , -3) and B the point (4 , 7). Find the coordinates of the point that divides AB externally in the ratio 1 : 2. — 2

c) Use the substitution $u = \tan x$ to evaluate $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^3 x \sec^2 x\,dx$ — 3

d) State the domain and range of the function $f(x) = 2\sin^{-1}\dfrac{x}{3}$ — 2

e) Solve for x $\quad \dfrac{3x}{x-1} \le 2$ — 3

Question 2 *(12Marks)* Use a separate piece of paper

a) Find $\dfrac{d}{dx} x\tan^{-1} x^2$ — 3

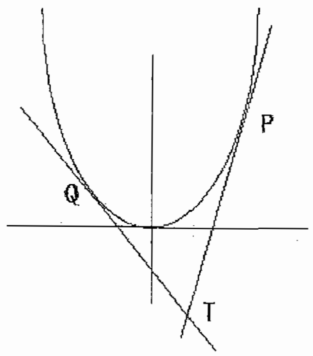
b) (i) Write $5\sin x + 3\cos x$ in the form $R\sin(x+\alpha)$ where $0 \le \alpha \le 90^0$ and $R \ge 0$ — 2

(ii) Hence or otherwise solve the equation $5\sin x + 3\cos x = 4$ for x to the nearest degree for $0 \le x \le 360^0$ — 2

c) Find the term independent of x in the expansion of $\left(2x - \dfrac{1}{x^2}\right)^9$ — 3

d) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sin^2 x\,dx$ — 2

Question 3 *(12 Marks)* Use a separate piece of paper — Marks



a) The diagram above shows the parabola $x^2 = 4ay$ the points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are points on the parabola.

(i) Find the equation of the tangent at P. — 3

(ii) State the equation of the tangent at Q. — 1

(iii) Find the coordinates of T the point of intersection of the two tangents — 2

b) When $P(x)$ is divided by ($x + 1$) the remainder is 3, when $P(x)$ is divided by ($x - 2$) the remainder is -5. What is the remainder when $P(x)$ is divided by ($x + 1$) ($x - 2$). — 3

c) In the figure, M, N, E and D are the points on the circle. MN is a diameter. NE is produced to C and NM is produced to A such that the CA $\perp$ AN. AE meets the circle at D. ND is produced to meet CA at B.

(i) Prove ΔMEN is similar to ΔCAN — 1

(ii) Hence or otherwise show that B, C, E and D are concyclic — 2

Question 4 *(12 Marks)* Use a separate piece of paper — Marks

a) Assume that the rate at which a body cools in air is proportional to the difference between its temperature T and the constant temperature of the surrounding air A . This can be expressed as

$\dfrac{dT}{dt} = -k(T - A)$ where t is time in minutes and k a constant

(i) Show that $T = A + Ce^{-kt}$ where C is a constant is a solution to the differential equation. — 1

(ii) If molten steel cools from an initial temperature of 500^0 to 200^0 in 15 minutes with an air temperature of 20^0. Find the values of A, C and k. — 3

(iii) How long does it take, to the nearest minute, for the steel to cool to 100^0. — 1

b) Prove by Mathematical induction that

$$1^3+2^3+3^3+......+n^3=\frac{1}{4}n^2(n+1)^2 \qquad 3$$

c) The polynomial $P(x)=x^3-x^2-2x+5$ has a root between -1 and -2. Using x = -1 as the first value use Newton's Method once to find a better approximation. (Give your answer to two decimal places) 2

d) Use the definition of the derivative $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

to find $f'(x)$ when $f(x)=\sqrt{x}$ 2

Question 5 ***(12 Marks)*** Use a separate piece of paper

a) How many arrangements of the letters of the word WALLABY are possible. 1

b) For positive integers n and r with $r<n$ show that

$${}^nC_r+{}^nC_{r+1}={}^{n+1}C_{r+1}$$

$$where\ {}^nC_r=\frac{n!}{r!(n-r)!} \qquad 3$$

c) A gambling game consists of three fair dice being rolled and betting on a particular number appearing on one of the uppermost faces. If such a game is played and the chosen number is 6

(i) What is the probability that no 6's will appear 1

(ii) What is the probability that at least one 6 will appear 1

(iii) If five such games are played, using a binomial expansion or otherwise, find the probability that exactly three turns will include at least one 6. (leave answer in index form) 2

(iv) Find the probability that at least one game will include at least one 6 . (leave answer in index form) 2

d) Show that $\cos^{-1}\frac{1}{\sqrt{5}}+\cos^{-1}\frac{1}{\sqrt{10}}=\frac{3\pi}{4}$ 2

Question 6*(12 marks)*Use a separate piece of paper

Marks

a) By putting $\frac{d^2x}{dt^2}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ in the equation of S.H.M. $\frac{d^2x}{dt^2}=-n^2x$

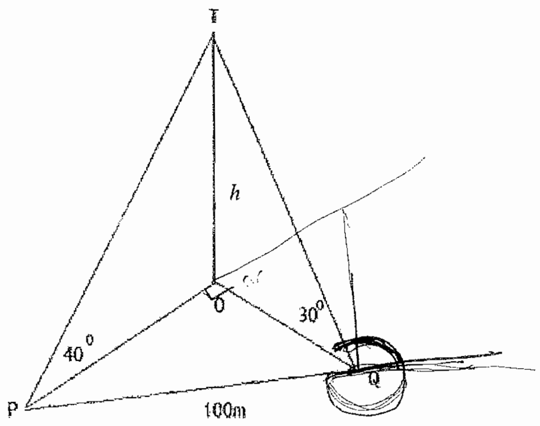
(i) Show that $v^2=n^2\left(a^2-x^2\right)$ where a is the amplitude. 2

(ii) A particle P performing S.H.M. in a straight line about a point O has speeds of 5m/s and 3m/s at two points A and B which are 0.2 m and 0.6 m respectively from O.

Find the amplitude and frequency of the motion. 2

(iii) Find the length PO at the instant the velocity v the particle is 2/3 the maximum velocity of the motion. 2

b)



A surveyor stands at a point P due south of a tower OT of height h , and finds the angle of elevation of the top of the tower to be 40^0 . And then walks 100m to a point Q, so that the angle POQ is 90°, and finds that the angle of elevation from Q is 30°

(i) Find expressions for OP and OQ in terms of h. 2

(ii) Show that $h=\frac{100(\tan 40^\circ \tan 30^\circ)}{\sqrt{\tan^2 40^\circ+\tan^2 30^\circ}}$ 2

(iii) Find the bearing of P from Q. 2

<u>Question 7</u> ***(12 Marks)***Use a separate piece of paper — Marks

a) A coal loader is stacking coal on a flat surface, in the shape of cone.
The cone has a semi vertical angle of 30^0. If the coal is being deposited at the rate of $1\text{m}^3/\text{min}$, find

(i) An expression for the volume of the cone in terms of the radius only — 2

(ii) The rate at which the radius is changing when the radius is 2m. — 2

b) (i) Find the largest positive domain of the function $f(x) = x^2 - 4x + 5$ for which $f(x)$ has an inverse function $f^{-1}(x)$ — 1

(ii) Find $f^{-1}(x)$ and hence sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same set of axes. — 2

c) A particle is projected with a velocity of V m/s at an angle of θ^0. Using

$$x = V\cos\theta t$$

$$y = V\sin\theta t - \frac{1}{2}gt^2$$

$$\dot{y} = V\sin\theta - gt$$

(There is no need to prove these results)

(i) Find an expression for the maximum height reached by the projectile — 1

(ii) Prove that the Cartesian equation of the particle is

$$y = \tan\theta x - \frac{g\sec^2\theta x^2}{2V^2}$$ — 2

(iii) If the particle passes through a point at height ***b***, and horizontal distance ***a*** from the origin, prove that the maximum height reached is given by

$$\frac{1}{4}\left[\frac{a^2\tan^2\theta}{a\tan\theta - b}\right]$$ — 2