2007
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension I

General Instructions
- Reading time – 5 minutes
- Working time –2 hours
- Write using black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided
- All necessary working should be shown in every question
- Answer each question in a SEPARATE Writing Booklet

Total marks – 84
- Attempt Questions 1 – 7
- All questions are of equal value

. Care has been taken to ensure that this paper is free of errors and that it mirrors the format and style of past HSC papers. The questions have been adapted from various sources, in an attempt to provide students with exposure to a broad range of questions.
However, there is no guarantee whatsoever that the 2007 HSC examination will have similar content, style or format. This paper is intended only as a trial for the HSC examination or as revision leading to the examination.

Question 1 (12 marks) **Marks**

(a) Solve: $x(x^2-1)>0$ **2**

(b) Find the exact value of

(i) $\sin\frac{5\pi}{4}$ **1**

(ii) $\sin\left[2\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$ **2**

(c) Find the Cartesian equation of the curve whose parametric equations are $x=\sin\theta+\cos\theta$ and $y=\sin\theta-\cos\theta$. **1**

(d) Evaluate: $\lim\limits_{x\to 0}\frac{3x}{\sin 2x}$ **2**

(e) Sketch the curve: $y=4\cos^{-1}2x$ **2**

(f) Evaluate: $\int_0^3\sqrt{(9-x^2)}\,dx$ **2**

Question 2 (12 marks) **Use a separate page/booklet** **Marks**

(a) **(i)** Starting from the expansions for sin(A+B) and cos(A+B), prove that $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ **1**

(ii) Solve: $\tan^{-1}x+\tan^{-1}2x=\frac{\pi}{4}$

Give the value of x (x >0) correct to 2 decimal places. **2**

(b) Find the coordinates of the points on the line $y=2x$ which are $\sqrt{2}$ units from the line $y = 4-x$. **2**

(c) Find the coefficient of a^5b^7 in $(2a-b)^{12}$ **2**

(d) Find the coordinates of point P which divides the join of A(3,5) to B(-4,2) externally in the ratio 2:3 **2**

(e) Use Simpson's Rule with 5 function values to find

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos x\,dx$$ **3**

Question 3 (12 marks) **Use a separate page/booklet** **Marks**

(a) How many 11 letter words can be made using the letters of the word MATHEMATICS **2**

(b) A particle moves in a straight line and its position at a time t is given by $x = a\cos(4t + \alpha)$. The particle is initially at the origin moving with a velocity of $6m/s$ in the negative direction.

(i) Show that the particle is undergoing simple harmonic motion. **1**

(ii) Find the constants α and a. **2**

(iii) Find the position of the particle after 4 seconds **1**

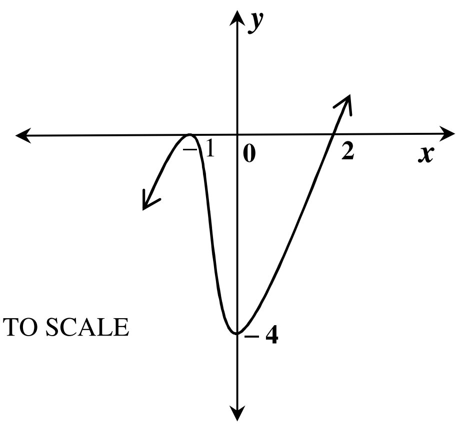
(c) Four blue and four white marbles are arranged in a circle

(i) How many different arrangements are possible? **1**

(ii) How many arrangements are possible if at least two of the blue marbles are together. **1**

(iii) How many arrangements are possible if all the blue marbles are together? **2**

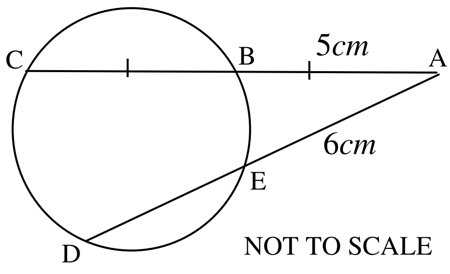
(d) A cubic curve touches the x-axis at -1 and intersects it at 2. It intersects the y-axis at -4 as shown below. Write the equation of the curve. **2**



NOT TO SCALE

Question 4 (12 marks) **Use a separate page/booklet** **Marks**

(a) In the circle below, the chords CB and DE are extended to intersect at A. CB=BA=5*cm* and EA=6*cm*. Find DE. **1**

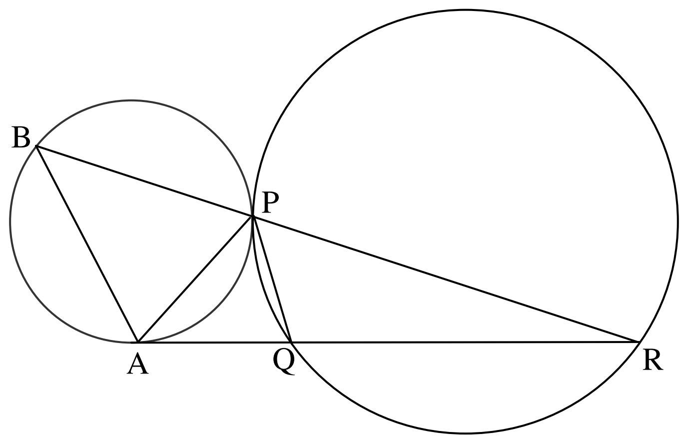


(b) **(i)** Show that one root of the equation $x+\sin x=\frac{\pi}{3}$ lies between $\frac{\pi}{6}$ and $\frac{\pi}{4}$ **1**

(ii) By letting $x=\frac{\pi}{6}$ find, using one application of Newton's method, a better approximation. **2**

(c) Two roots of $2x^3-x^2+k\,x+4=0$ are reciprocals of each other. Find the value of *k*. **2**

(d) Two circles touch externally at P. The tangent drawn at a point A on one of these circles meets the other circle in Q and R. RP meets the other circle in B. Prove ∠BPA = ∠QPA. **4**



(e) Solve for: $\frac{2x-3}{x}\le 1, \qquad x\neq 0$ **2**

Question 5 (12 marks) **Use a separate page/booklet** **Marks**

(a) Find : $\int \sin^2 3x \, dx$ **2**

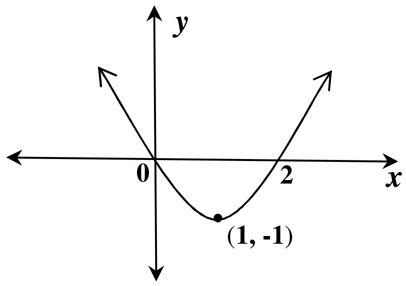
(b) The figure below shows a sketch of $f(x) = (x-1)^2 - 1$

(i) Explain why $f(x)$ does not have an inverse function. **1**

(ii) Sketch the graph of the inverse function, $g^{-1}(x)$, of $g(x)$, where $g(x) = (x-1)^2 - 1, \quad x \geq 1$. **1**

(iii) State the domain of $g^{-1}(x)$. **1**

(iv) Find the expression for $y = g^{-1}(x)$ in terms of x. **1**



(c) Some bacteria are introduced into a solution. After 't' hours there are y grams of bacteria present, where $y = \dfrac{Pe^{qt}}{1 + Pe^{qt}} \quad (t \geq 0)$ and P and q are constants.

(i) Show that $0 < y < 1$ for all values of t. **1**

(ii) Show that $\dfrac{dy}{dt} = q\,y(1-y)$ **2**

(iii) The maximum value of $\dfrac{dy}{dt}$ occurs when $y = 0{\cdot}5$, if $P = 0{\cdot}01$and q= $0{\cdot}7$ find, to the nearest hour, when the bacteria will be increasing at the fastest rate. **3**

Question 6 (12 marks) **Use a separate page/booklet** **Marks**

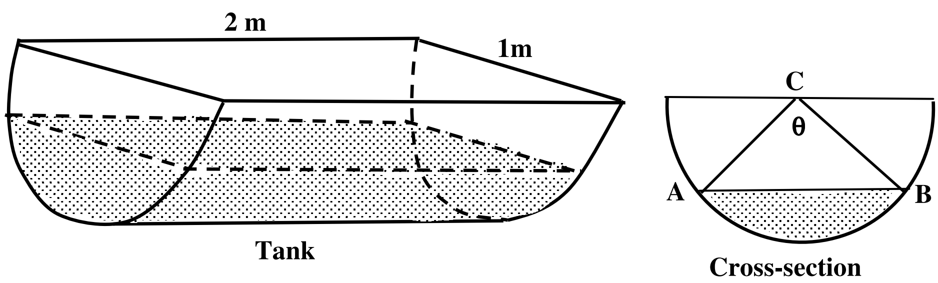
(a) The acceleration of a particle P is given by the equation $\frac{d^2x}{dt^2} = 8x(x^2+1)$ where the displacement in *cm* of P from a fixed point O, after 't' seconds, is given by x

Initially, P is projected from O with velocity 2*cm/s* in the negative direction.

(i) Show that the speed in any position x is $2(x^2+1)\ cm/s$ and hence find an expression for x in terms of t. **3**

(ii) Determine the displacement of P after $\frac{\pi}{8}$ seconds. **1**

(b) An aquarium tank is 2m long and has a semi-circular cross-section of diameter 1m as shown in the diagram.



NOT TO SCALE

C is at the centre of the top edge. AB represents the water level and ∠ACB = θ where θ is measured in radians.

(i) Show that the volume of the water in the tank is given by

$$V = \frac{1}{4}(\theta - \sin\theta)$$ **2**

(ii) Water is poured into the tank at the rate of $0{\cdot}1\ m^3/\text{min}$. Find the rate at which the water level is rising when the depth of water is 0·2m. Give answer correct 3 sig. fig. **4**

(c) Find the values of x for which $f(x) = xe^{-2x}$ decreases. **2**

Question 7 (12 marks) **Use a separate page/booklet** **Marks**

(a) A tennis club consists of $2n$ members, n being female and n male.

(i) How many possible committees are there consisting of 2 females and 1male ? **1**

(ii) How many possible committees are there consisting of 3 females? **1**

(iii) A committee consists of three people. Using the above results, or otherwise, prove that

$$n\left({}^nc_2\right)+{}^nc_3=\frac{1}{2}\left({}^{2n}c_3\right)$$ **1**

(b) Find $\int x\sqrt{x^2+9}dx$ using $u=x^2+9$ **2**

(c) A particle is projected with a velocity **V** ***m/s*** from a point at a height ***3h*** metres above the ground, the direction of projection making an angle α with the horizontal. The greatest height above the point of projection is ***h*** metres.

(i) Show that $V\sin\alpha=\sqrt{2gh}$ **2**

(ii) Show that the horizontal distance travelled by the particle before hitting the ground is $6h\cot\alpha$. **3**

(d) By expanding $[x+(1-x)]^n$, for all real numbers x and all positive integers n, show that

(i) ${}^nc_0x^n+{}^nc_1x^{n-1}\left(1-x\right)+{}^nc_2x^{n-2}\left(1-x\right)^2+\cdots+{}^nc_n\left(1-x\right)^n=1$ **,** when n is even **1**

(ii) Deduce that ${}^nc_0+{}^nc_1+{}^nc_2+\cdots+{}^nc_n=2^n$ **1**

End of paper

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x,\quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax},\quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax,\ a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax,\quad a \neq 0$$

$$\int \sec^2 ax dx, = \frac{1}{a} \tan ax,\quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a},\quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}),\quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE : $\ln x = \log_e x, \qquad x > 0$