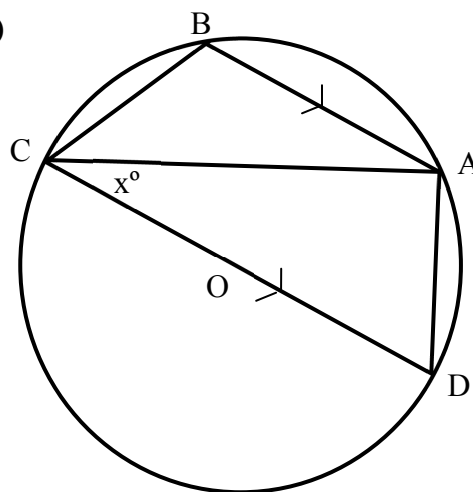


QUESTION 1. (Start on a new sheet of paper)**MARKS**

- a) If $y = x \tan^{-1} x$, find $\frac{dy}{dx}$. 2
- b) If $f(x) = \sin^{-1}(1 - 2x)$, show that $f'(x) = \frac{-1}{\sqrt{x - x^2}}$. 3
- c) $P(x)$ is an odd polynomial of degree 3. It has $(x + 4)$ as a factor and, when it is divided by $(x - 3)$, the remainder is 21. Find $P(x)$. 3
- d) By making the substitution $u = x - 2$, evaluate $\int_4^5 \frac{x(x-4)}{(x-2)} dx$ 4

QUESTION 2. (Start on a new sheet of paper)

- a) The points A, B, C and D lie on the circumference of a circle centred at O . CD is a diameter of the circle and AB is parallel to CD . $\angle ACD = x^\circ$. Find an expression for $\angle ACB$ in terms of x . 3

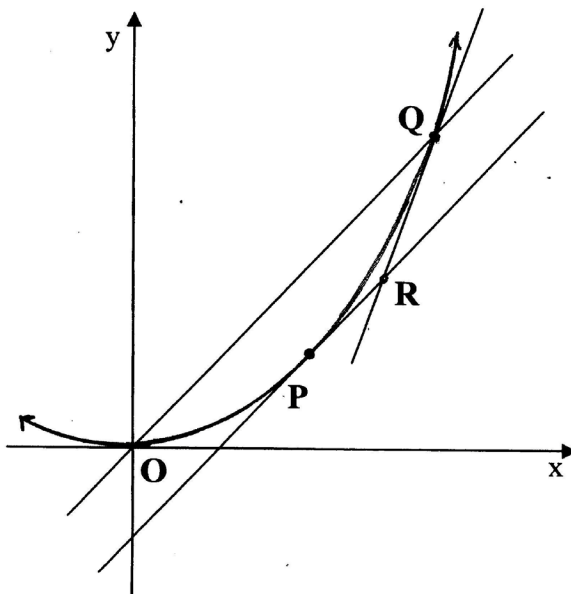


- b) Use the method of mathematical induction to show that the expression $9^n - 8n - 1$ is divisible by 64 for all integers $n \geq 2$. 5
- c) i) Given that ${}^nC_r = \frac{n!}{r!(n-r)!}$, show that $\frac{r \times {}^nC_r}{{}^nC_{r-1}} = n - r + 1$ 1
- ii) Hence show that : 3
- $$\frac{{}^nC_1}{{}^nC_0} + \frac{2 \times {}^nC_2}{{}^nC_1} + \frac{3 \times {}^nC_3}{{}^nC_2} + \dots + \frac{n \times {}^nC_n}{{}^nC_{n-1}} = \frac{n}{2}(n+1).$$

QUESTION 3. (Start on a new sheet of paper)**MARKS**

a) Evaluate the definite integral $\int_0^{\frac{1}{\sqrt{2}}} \frac{x}{\sqrt{1-x^4}} dx$ by using the substitution $u = x^2$. **4**

- b) The point $P(2ap, ap^2)$ is on the parabola $x^2 = 4ay$ and a straight line OQ is drawn through the vertex parallel to the tangent at P . This line meets the parabola again at Q and the tangent to the parabola at Q meets the tangent at P in R , as shown in the diagram.



You are given that the tangent at P has equation $y = px - ap^2$.

- | | | |
|------|---|----------|
| i) | Write down the equation of the line OQ . | 1 |
| ii) | Find the coordinates of Q in terms of a and p . | 2 |
| iii) | Show that the equation of the tangent at Q is $y = 2px - 4ap^2$. | 1 |
| iv) | Find the coordinates of R . | 2 |
| v) | Show that, as P varies on the parabola, R moves on another parabola whose equation is $x^2 = \frac{9}{2}ay$. | 2 |

QUESTION 4. (Start on a new sheet of paper)**MARKS**

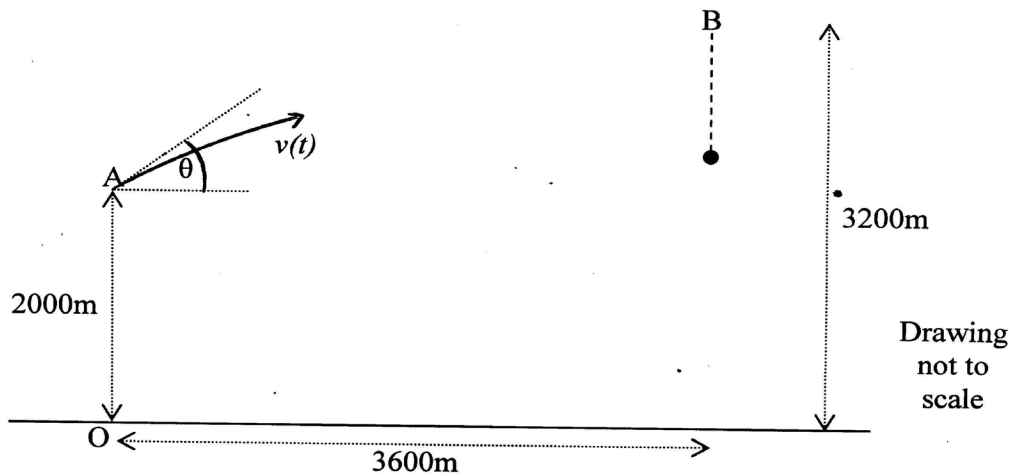
- a) Consider the function $f(x) = \frac{e^x}{(1 + e^x)}$.
- i) Find $f'(x)$ and deduce that $f(x)$ is increasing for all x . **2**
 - ii) State the range of $f(x)$. **1**
 - iii) Find the inverse function $f^{-1}(x)$ **2**
 - iv) Draw $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram. **2**
- b) A particle moves in a straight line on the x axis. At time t its velocity is v and its acceleration is a .
- i) If $a = 4x - 4$ and initially $x = 6$ and $v^2 = 64$, show that $v^2 = 4x^2 - 8x - 32$. **2**
 - ii) Use this expression for v^2 to find the possible values of x . **1**
 - iii) Describe the motion of the particle if $v = -8$ initially. **2**

QUESTION 5. (Start on a new sheet of paper)

- a) A particle moves in a straight line with displacement in centimetres from the point $x = 0$ at time t seconds given by $x = \sin 3t + 2 \cos 3t$ for $t \geq 0$.
- i) Express x in the form $R \sin(3t + \alpha)$ where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$. **2**
 - ii) Show that the motion is simple harmonic. **1**
 - iii) Write down the period of motion. **1**
 - iv) Find at what time, to the nearest tenth of a second, the particle first reaches $x = 1$. **2**
- b) Fred deposited \$20,000 at the beginning of January into an account which paid interest at the rate of 0.5% per month compounded monthly. He withdrew \$50 each month from the account each month, immediately after the interest was paid.
- i) How much money was in the account immediately after the first withdrawal? **1**
 - ii) Show that, after making the n^{th} withdrawal, his account balance is given by the expression $\$(10,000 \times 1.005^n + 10,000)$ **3**
 - iii) Find the number of months it will take for his account balance to be \$50,000 **2**

QUESTION 6. (Start on a new sheet of paper)**MARKS**

- a) Find the term independent of x in the expansion of $\left(\frac{2x^2}{3} - \frac{3}{2x}\right)^9$. 3
- b) An aeroplane, A , flying at a height of 2000m observes a stationary blimp, B , at a height of 3200m drop an object. As the object is dropped, the plane fires a projectile towards it at a speed of 240m/s and at an angle θ to the horizontal. The horizontal distance between the plane and the blimp is 3600m at the time that the projectile is fired.



The origin of coordinates, O , is taken to be the point on the ground below A .

The particle's coordinates at time t (secs) are given by : $x = 240t \cos \theta$,
 $y = 2000 + 240t \sin \theta - \frac{gt^2}{2}$

The coordinates of the dropped object at time t are : $x = 3600$,
 $y = 3200 - \frac{gt^2}{2}$

(You may use $g=10\text{m/s}^2$)

- i) What is the angle θ at which the projectile must be fired to intercept the object, and how long does it take to reach it? 3
- ii) At what height does the projectile intercept the object? 1
- c) A man notices two towers, one due North and one in a direction $N\theta E$ (i.e. at an angle θ east of north). The angle of elevation β of both towers is the same but the height of one tower is twice the height of the other. Show that

$$\cos \theta = \frac{5 \cot^2 \beta - \cot^2 \alpha}{4 \cot^2 \beta} \quad \text{5}$$

where α is the angle of elevation of the top of the taller tower from the top of the shorter.

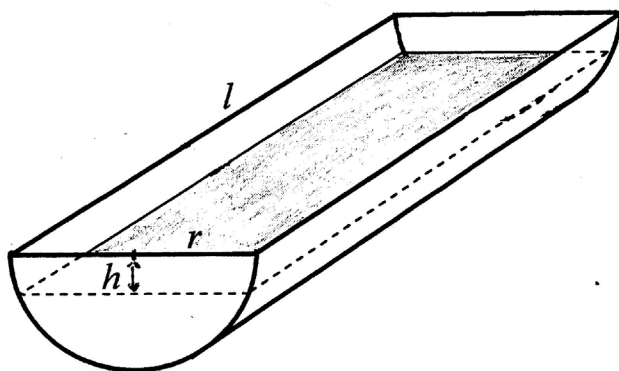
QUESTION 7. (Start on a new sheet of paper)**MARKS**

- a) A group of four contestants for a quiz game are to be selected at random from a class of eight girls and five boys.

i) What is the probability that the team comprises three girls and one boy? **2**

ii) Find the probability that there are more girls chosen than boys. **2**

- b) A water trough takes the shape of a hollow semi-circular prism with length l and radius r . It is placed on horizontal ground and filled with water. The surface of the water is at a distance h below the top of the trough, as shown in the diagram.



- i) Show that the area A of the flat surface area of water is given by

$$A = 2l\sqrt{r^2 - h^2} \quad \mathbf{2}$$

- ii) Show that the volume V of water in the trough is given by

$$V = l \left(r^2 \cos^{-1} \left(\frac{h}{r} \right) - h\sqrt{r^2 - h^2} \right) \quad \mathbf{2}$$

- iii) If the water level is falling, show that $\frac{dV}{dt} = -2l\sqrt{r^2 - h^2} \frac{dh}{dt} = -A \frac{dh}{dt}$. **3**

- iv) On a sunny day, the rate of evaporation at any time (and hence $-\frac{dV}{dt}$) is proportional to A . Show that the water level falls at a constant rate. **1**

END OF THE PAPER