ST. IVES HIGH SCHOOL

TRIAL EXAMINATION

2001

MATHEMATICS

YEAR 12

3 unit/4 unit common paper

Time allowed - TWO hours

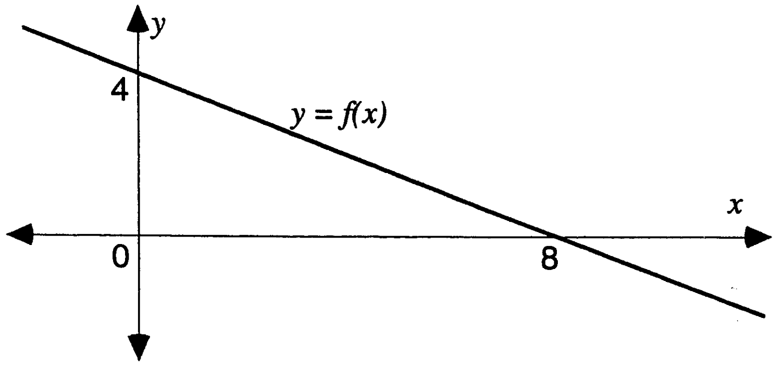
DIRECTIONS:

- Answer all questions.
- Begin each question on a new page.
- All answer pages should be stapled together in the top left hand corner.

THIS IS A TRIAL PAPER ONLY AND DOES NOT NECESSARILY REFLECT THE CONTENT OR FORMAT OF THE FINAL HIGHER SCHOOL CERTIFICATE EXAMINATION IN THIS SUBJECT.,

QUESTION 1. Marks

(a) In the figure below evaluate $\int_4^8 f(x)\,.dx$ **2**



(b) Find the derivative of $\log_e(\cos^2 x)$ **2**

(c) Find $\lim_{x \to 0} \dfrac{\sin 4x}{3x}$ **2**

(d) Use the substitutrion $u = 3\sin x - 1$ to find the indefinite integral **3**

$$\int \frac{\cos x.dx}{(3\sin x - 1)^2}$$

(e) Solve $\dfrac{x^2 - 4}{x} \geq 0$ **3**

QUESTION 2. *(Begin on a new page)* Marks

(a) (i) Show that $\dfrac{3x - 7}{x - 2} = 3 - \dfrac{1}{x - 2}$ **1**

(ii) Find $\displaystyle\int \frac{3x - 7}{x - 2}\,.dx$ **2**

(b) (i) Differentiate $\cos^{-1}\left(\dfrac{1}{x}\right)$ **2**

(ii) Hence, find $\displaystyle\int_{\frac{2}{\sqrt{3}}}^{2} \frac{dx}{x\sqrt{x^2 - 1}}$ **2**

(c) (i) Sketch the curve $y = \cos^{-1} x$ and state its domain and range. **2**

(ii) A region $\Re$ is defined by the curve $y = \cos^{-1} x$, the x-axis and the y-axis in the first quadrant. Find the exact area of this region. **3**

QUESTION 3. *(Begin on a new page)* **Marks**

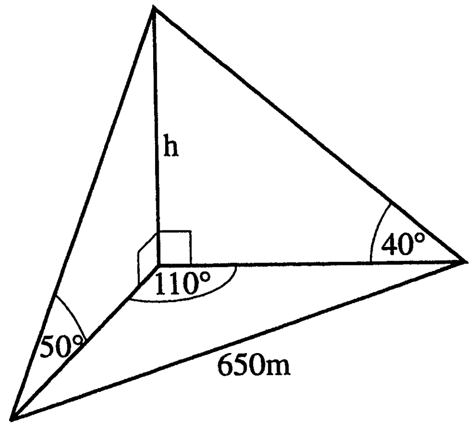
(a) A committee of five is chosen at random from 5 men and 3 women.

(i) Find the probability that it contains 3 men and 2 women. **2**

(ii) The comittee of 3 men and 2 women sit at random on one side of a rectangular table. Find the probability that the women are separated. **2**

(b) Use mathematical induction to prove that $n! > 2^n$, for $n > 3$. **4**

(c) Find the value of h, to the nearest metre: **4**



QUESTION 4. *(Begin on a new page)* **Marks**

(a) The height of a closed metal cylinder is 3 metres and stays constant while the cylinder is expanding under heating. The base radius is increasing at a constant rate of 0·02 metres per minute.
When the radius is 2·5 metres, find the rate at which:

(i) the volume is increasing, (correct to 2 decimal places) **2**

(ii) the total surface area is increasing, (correct to 2 decimal places)

(b) On a particular day, the depth of water above a sandbank at high tide at 3.20 pm is 10 metres. At low tide, $6\frac{1}{4}$ hours later, the depth is 7 metres.
Assuming the rise and fall of the water to be simple harmonic, find the first time during the afternoon after which a boat requiring 9 metres depth of water is prevented from safely passing over the sandbank. **5**

(c) Find the area bounded by the curve $y = 8\cos^2 x$ and the x-axis from $x = 0$ to $x = \pi/4$. **3**

QUESTION 5. *(Begin on a new page)* **Marks**

(a) The acceleration of a particle moving in a straight line is given by

$$\ddot{x} = \frac{-900}{x^3}$$

where x metres is the displacement from the origin after t seconds.
Initially the particle is 10 metres to the right of the origin and moving with velocity $3ms^{-1}$.

(i) Find an equation for the velocity of the particle at displacement x metres. **3**

(ii) Find the velocity of the particle when it is 100 metres from the origin. **1**

(iii) Find the time taken for the particle to reach this point. **2**

(b) When a body falls, the rate of change of its velocity is given by:

$$\frac{dv}{dt} = -k(v - P)$$

where k and P are constants.

(i) Show that $v = P + Ae^{-kt}$ is a solution. **1**

(ii) If the initial velocity is zero and $P = 500$, and the velocity after 5 seconds is $21ms^{-1}$, prove $A = -500$ and find k, correct to three significant figures. **2**

(iii) Find the velocity after 20 seconds. **1**

(iv) Find the maximum possible velocity of the particle. **2**

QUESTION 6. *(Begin on a new page)*

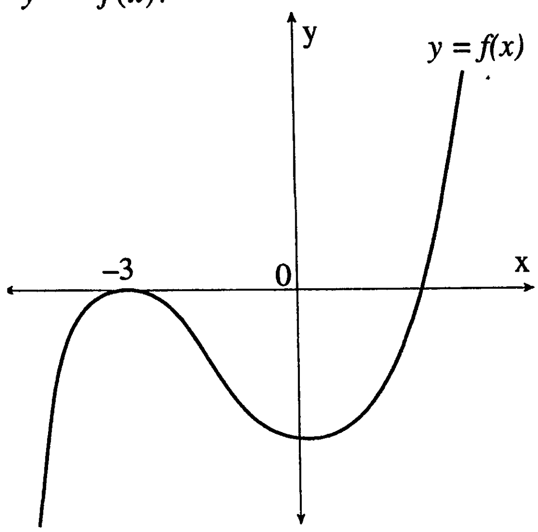
Marks

(a) If $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$, solve for R and α: **2**

$$R\cos\alpha = 3$$
$$R\sin\alpha = 2$$

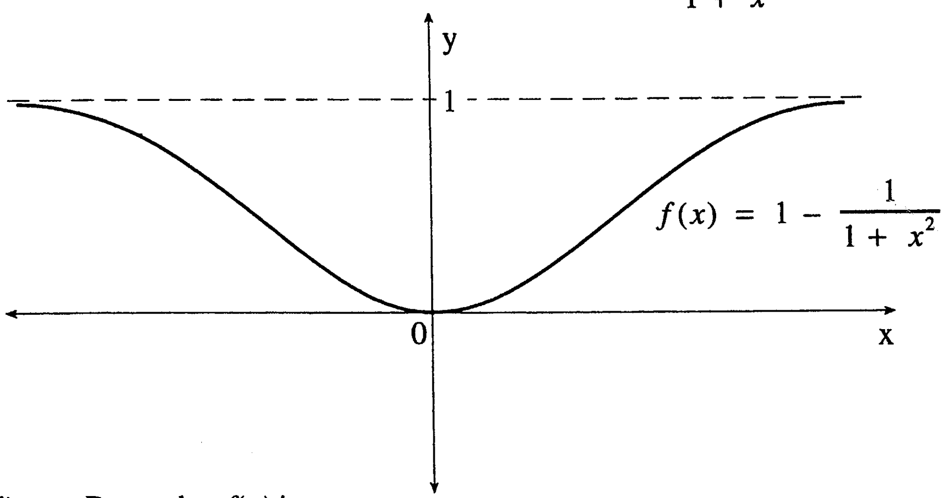
(b) The circle $x^2 + y^2 = r^2$ is rotated about the x-axis. **4**
Use calculus to find the volume of the sphere so generated.

(c) The graph shows $y = f(x)$. **2**



Make a copy of the graph on your answer sheet.
Then sketch the graph $y = f'(x)$ on the same set of axes.

(d) The diagram below shows the function $f(x) = 1 - \dfrac{1}{1 + x^2}$.



(i) Prove that $f(x)$ is even. **1**

(ii) Find the area bounded by the asymptote, $x = 1$, $x = -1$ and the curve. **3**

QUESTION 7. *(Begin on a new page)* **Marks**

(a) (i) Find the centre and radius of the circle S with equation **2**

$$x^2 + y^2 - 2x - 14y + 25 = 0.$$

(ii) A line $y = mx$ is drawn on the same diagram a S. Solve these equations simultaneously to find the values of m for which this line is a tangent to S. **3**

(iii) Illustrate on a clear diagram. **1**

(b) A particle is projected from the origin O with velocity $15ms^{-1}$ at an angle θ.

(i) Taking $g = 10ms^{-1}$, show that the equations for the horizontal (x) and vertical (y) components of displacement of the particle from O after t seconds are given by **3**

$$x = 15t\cos\theta, \quad y = 15t\sin\theta - 5t^2$$

(ii) Hence determine the cartesian equation of the path. **1**

(iii) The particle just clears an object 2 metres high standing 5 metres from O. Find two possible values of θ. **2**