Name : ____________________

Class : 12 MT___

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

1999 AP3

YEAR 12 HALF YEARLY HSC

MATHEMATICS
3/4 UNIT (COMMON)

Time allowed - 2 HOURS
(plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES:

* Attempt ALL questions.
* All questions are of equal value
* All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
* Standard Integrals are provided. Approved calculators may be used.
* Each question attempted is to be returned on a new page clearly marked Question 1, Question 2, etc on the top of the page.

***Each page must show your class and your name.**

QUESTION 1 — **Marks**

(a) Expand $(2x-y)^5$ — **2**

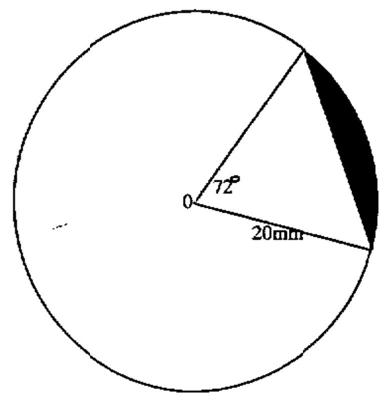
(b) (i) Write down the expansion of $\cos(\alpha-\beta)$. — **3**

(ii) Find the exact values of $\cos 45^0$ and $\cos 30^0$.

(iii) Hence find the exact value of $\cos 15^0$.

(c) (i) Convert 72^0 to radians, giving your answer in terms of π. — **3**

(ii) Hence or otherwise, find the shaded area below correct to 3 significant figures.



(d) Solve $\sin 2x = \sqrt{3}\cos 2x, 0 \le x \le 2\pi$. — **2**

(e) Differentiate with respect to x — **2**

(i) $\sqrt[3]{4x-1}$

(ii) $\dfrac{x}{\cot x}$.

QUESTION 2 — BEGIN A NEW PAGE

Marks

(a) Given $\int_0^3 f(t)\,dt = 6$, evaluate **4**

(i) $\int_0^1 f(t)\,dt + \int_1^3 (f(t) + 1)\,dt$

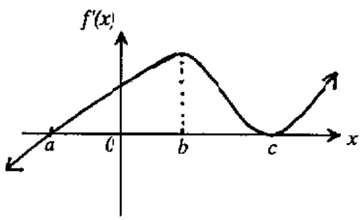
(ii) $\int_3^0 (f(t) + t)\,dt$

(b) Find **2**

(i) $\int \frac{x^4 + 2x^3 + 3}{x^2}\,dx$

(ii) $\int \frac{dt}{(3-t)^2}$.

(c) The *gradient function* of $y = f(x)$ is graphed below. **6**



(i) Copy this diagram onto your answer sheet.

(ii) On the same diagram, sketch and label a possible graph of $y = f''(x)$.

(iii) State the domain where $y = f(x)$ is concave down.

(iv) Find the x values of any points of inflection.

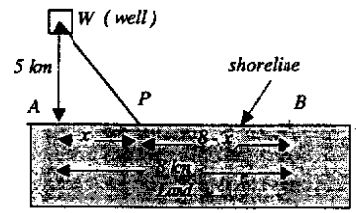
(v) Find any stationary points and determine their nature.

QUESTION 3 — BEGIN A NEW PAGE

Marks

(a) Find the equation of any asymptotes of the curve $y = \frac{x^2 + x + 1}{x}$. **2**

(b)



An offshore oil well is located at a point W, which is 5 *km* from the closest shorepoint A on a straight shoreline.
The oil is to be piped to a shorepoint B that is 8 *km* from A by piping it on a straight line under water from W to some shorepoint P between A and B and then on to B via a pipe along the shoreline.

If the cost of laying the pipe is \$125 000 per *km* under water and \$75 000 per *km* over land.
Let x *km* be the distance between A and P and C (in thousands of dollars) be the cost for the entire pipeline.

(i) Show that the cost is given by $C = 125\sqrt{x^2 + 25} + 75(8 - x)$ **3**

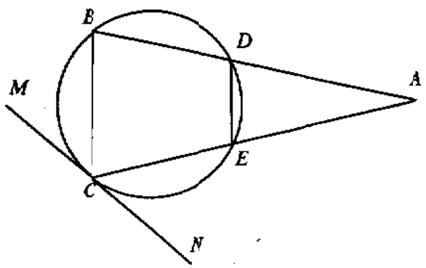
(ii) Find the domain for x **1**

(iii) Find where the point P should be located to minimise the cost of laying the pipe ? **6**

QUESTION 4 — BEGIN A NEW PAGE

Marks

(a) **6**



ABC is a triangle in which $AB = AC$. A circle through B and C cuts AB at D and AC at E. MCN is the tangent at C to the circle through B, C, E, D.

(i) Copy the diagram onto your answer sheet.

(ii) Show that $DE \parallel BC$.

(iii) Show that $\angle ACN = \angle BCD$.

(b) $P(2at, at^2)$ is a variable point on the parabola $x^2 = 4ay$, whose focus is S. **6**
$Q(x,y)$ divides the interval from P to S in the ratio $t^2 : 1$.

(i) Find x and y in terms of a and t.

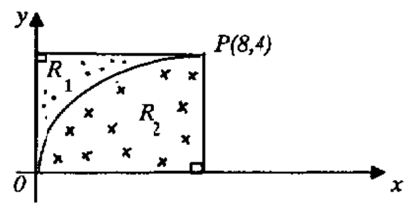
(ii) Verify that $\frac{y}{x} = t$.

(iii) Prove that as P moves on the parabola, Q moves on a circle, and state its centre and radius.

QUESTION 5 — BEGIN A NEW PAGE

Marks

(a) **7**



OP is an arc of the curve $y^3 = x^2$.
Calculate the volume of the solids generated when

(i) Region R_1 revolves around the y- axis

(ii) Region R_2 revolves around the x- axis

(iii) Region R_2 revolves about the y-axis.

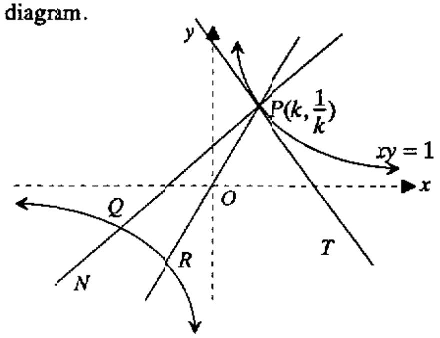
(b) (i) Express $\sin x - \cos x$ in the form $A\sin(x - \alpha)$ with $A > 0$ and $0 < \alpha < \frac{\pi}{2}$. **5**

(ii) Determine $\lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$.

QUESTION 6 — BEGIN A NEW PAGE

Marks

$P(k, \frac{1}{k})$ is a point on the curve $xy = 1$ where k is a real number, $k \neq 0$.
PT is the tangent to the curve at P and PN is the normal at P.
POR is the line passing through P, the Origin O and R as shown on the diagram.



(a) Find the equation of the line passing through O and P. **1**

(b) The line in part (a) intersects the curve again at R. Find the coordinates of R. **2**

(c) Show that the equation of the tangent at P is given by: **2**

$$x + k^2 y = 2k.$$

(d) Find the equation of the normal line at P. **2**

(e) Show that when the normal intersects the curve again at Q, the equation formed to solve is the quadratic equation given by: **3**

$$k^3x^2 - (k^4 - 1)x - k = 0.$$

Hence find the coordinates of point Q

(f) Show that $QR \perp PR$. **2**

QUESTION 7 — BEGIN A NEW PAGE

Marks

(a) Given the polynomial function $P(x) = x^3 - 2x^2 - 6x + 4$, when $P(x) = 0$, $P(x)$ has one rational root and two irrational roots.

(i) Find the rational root of $P(x) = 0$. 1

(ii) Without finding the irrational roots of $P(x)$, show that one of the irrational roots of this equation lies between $x = 3$ and $x = 4$. 1

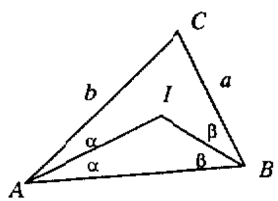
(iii) Using $x = 3\cdot 5$ as a first approximation, apply Newton's Method once to find a better approximation to the root, to 2 decimal places. 3

(iv) Sketch $P(x) = x^3 - 2x^2 - 6x + 4$. 1

(v) Explain why $x = 2$ would not be a good approximation to use when solving $P(x) = 0$ using Newton's Method. 1

(vi) Find the area bounded by the curve, $x = -3$, $x = -2$ and the x-axis. 2

(b) IA and IB bisect angles CAB and CBA as shown in the diagram below. 3



Prove that $\frac{IB}{IA} = \frac{a\cos\beta}{b\cos\alpha}$.