NAME:____________________

Examiner: Mrs Williams

# 2011
# ASSESSMENT TASK 1

## Mathematics Extension 1

**General Instructions**

a. Working time – 2 Periods
b. Write using black or blue pen
c. Board – approved calculators may be used
d. All necessary working out should be shown in every question

**Total marks – 70**

a. Attempt all questions 1-6
b. All questions are of equal value

**Office Use Only**

| Topic | Quest 1 | Quest 2 | Quest3 | Quest 4 | Quest 5 | Quest 6 | Total | % |
|---|---|---|---|---|---|---|---|---|
| Mark | /10 | /12 | /12 | /12 | /12 | /12 | /70 | |

"Dear friends, since God so loved us, we also ought to love one another. No one has ever seen God; but if we love one another, God lives in us and His love is made complete in us." - 1 John 4:11-12

vi. A committee of *3* is to be elected from a club of *8* members. How many different committees can be formed?

A $\frac{8!}{3!\,5!}$ B $\frac{8!}{3!}$ C ${}^{8}P_{3}$ D 8

vii. Find the value of $x$ that satisfies the equation $\cos 2x = \sin 80°$

A 5° B 25° C 55° D 10°

viii. We can express $\cos x - \sin x$ in the form $R\cos(x + \alpha)$, where $\alpha$ is in radians. What is the value of $R$?

A 2 B $\sqrt{2}$ C 1 D $\sqrt{3}$

ix. Find the centre and the radius of the circle *C* whose equation is $x^2 + y^2 - 4x + 6y - 12 = 0$.

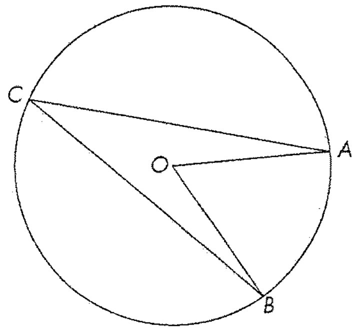
A centre $(2, -3)$ B centre $(-3, 2)$ C centre $(2, 1)$ D centre $(0, 0)$

x. Find the value of $\sum_{k=1}^{4}(-1)^k\, k!$

A 19 B 20 C 19 D 20

**QUESTION 1: Multiple Choice**

i Which of the following statements is incorrect?




**A** $\angle ACB$ is called the angle at the circumference standing on the arc $AB$.

**B** $\angle AOB$ is the angle at the centre standing on the arc $AB$.

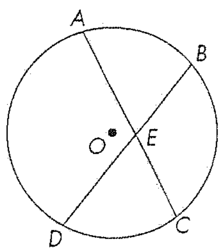
**C** If a chord $AB$ had been drawn we would say that $\angle ACB$ and $\angle AOB$ were standing on the chord $AB$ or they were subtended by the chord $AB$.

**D** $\angle ACB = 2\angle AOB$.

**ii.** A cyclic quadrilateral has one angle measuring 97° and another angle measuring 102°. Another angle in the quadrilateral is:

**A** 80.5° **B** 97° **C** 161° **D** 83°

**iii.** Find the length of $AC$ if $AE = 5$ cm, $BE = 2$ cm, and $DE = 10$ cm.




**A** 12 cm **B** 4 cm **C** 20 cm **D** 9 cm

**iv.** Which of the following are factors of $P(x) = x^3 - 10x^2 + 7x + 18$?

**A** $(x-9)$ **B** $(x-2)$ **C** $(x+1)$ **D** All of the above.

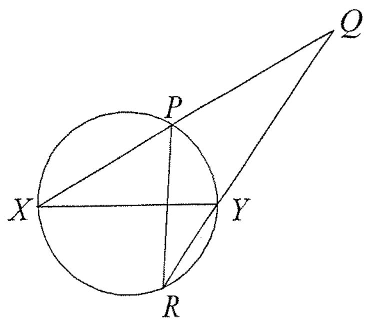
**v.** The serial number on a video cassette recorder (VCR) has 2 letters followed by 5 numbers. How many different VCR's can have this type of serial number?

**A** $26^2 \times 10^5$ **B** $26 \times 25 \times 10 \times 9 \times 8 \times 7 \times 6$ **C** ${}^{26}P_2 + {}^{10}P_5$ **D** ${}^{26}P_7$

**Marks**

## Question 2 (12 marks) Use a separate Writing Booklet.

a. Solve $\dfrac{2x+3}{x-2} < 1$ **3**




b. $XY$ is the diameter of the circle $XPYR$. $XPQ$ and $RYQ$ are straight lines. $PR$, $XY$ and $PY$ are joined Given that $\angle PXY = 35°$ and $\angle PQY = 25°$, find the size of $\angle YPR$, giving reasons. **3**

c. Find the point $\boldsymbol{P}$ which divides the interval joining $\boldsymbol{A}\,(2, -4)$ and $\boldsymbol{B}\ (3, -3)$ externally in the ratio $2 : 3$. **2**

d. Find the acute angle between the lines $y = 2x - 7$ and $3x - 5y - 6 = 0$. **2**

e. When $P(x) = x^3 + x^2 - a$ is divided by $x - 2$, the remainder is 4. Find the remainder when $P(x)$ is divided by $x$. **2**

## Question 3 (12 marks) Use a separate Writing Booklet.

a. If $\alpha$, $\beta$ and $\gamma$ are roots of the cubic equation $4x^3 + 2x^2 - x - 2 = 0$, find the value of

i. $\alpha + \beta + \gamma$ **1**

ii. $\alpha\beta + \beta\gamma + \alpha\gamma$ **1**

iii. $\alpha^2 + \beta^2 + \gamma^2$ **2**

b. Show that $\dfrac{\sin(x+y)}{\cos(x-y)} = \dfrac{\tan x + \tan y}{1 + \tan x \tan y}$ **4**

c. Use the $t$ results to show that $\dfrac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = t$ **4**

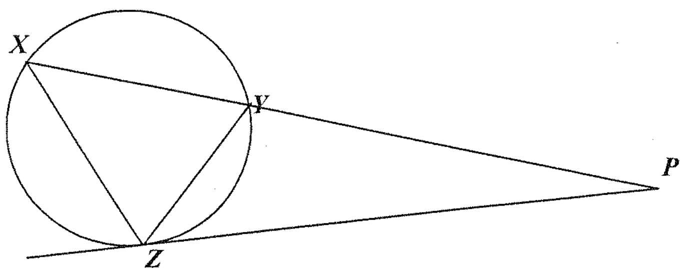
Marks

**Question 4 (12 marks) Use a separate Writing Booklet.**

a. i. If $\sin x - \cos x = A\sin(x-\alpha)$, where $\alpha$ is acute, find $A$ and $\alpha$. 2

ii. Hence or otherwise, solve $\sin x - \cos x = \sqrt{2}$ for $0° \le x \le 360°$. 2

b The chord $PQ$ of the parabola $x^2 = 12y$ passes through the fixed point (4, -3). Show that, if the tangents at $P$ and $Q$ intersect at the point $T$, then the locus of $T$ is the line $2x - 3y + 9 = 0$ 3




c. In the diagram $XP$ is a secant of the circle, cutting the circle at $Y$ and $ZP$ is a tangent to the circle.

i. Prove that $\Delta PYZ$ and $\Delta PZX$ are similar 2

ii. Hence show that $PZ^2 = PY \times PX$ 1

d. Use the expansion of $\tan(A + B)$ to find the value of $\tan 75°$ in simplest form. 2

**Question 5 (12 marks) Use a separate Writing Booklet.**

a. Evaluate $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$ 2

b. $P(2ap, ap^2)$ and $Q(2aq, aq^2)$, where $a > 0$, are two points on the parabola $x^2 = 4ay$. $F$ is the focus of the parabola.

i. Show that the chord $PQ$ has equation $(p+q)x - 2y = 2apq$. 3

ii. If $PQ$ passes through $F$ show that $pq = -1$ and hence find the product of the gradient of $OP$ and $OQ$. 2

c. Find the number of ways in which a committee of 3 people can be chosen from 2 parents, 3 teacher and 5 students

i. Without restrictions 1

ii. So that it contains exactly one student 2

iii. So that it contains an equal number of parents and teachers. 2

Marks

**Question 6 (12 marks) Use a separate Writing Booklet.**

a. Express $\tan 45°$ in terms of $\tan 22\frac{1}{2}^{\circ}$ and hence find the value of $\tan 22\frac{1}{2}^{\circ}$ in simplest form. **4**

b. Use mathematical induction to prove that **4**

$$\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\ldots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$$ for all positive integers n.

c. Show that ${}^{n}C_{r}+{}^{n}C_{r-1}={}^{n+1}C_{r}$ where n and r are positive integers and $n > r$. **4**

END OF PAPER

12 Ext 1 Task 1 2011

Q1

(i) D

ii) D

iii) $AE \cdot EC = DE \times BE$

$5 \cdot EC = 2 \times 10$

$EC = 4cm$

D

v) $P(9) = 0 \quad P(2) = 0 \quad P(-1) = 0$

D

i) $26^2 \times 10^5$

A

i) ${}^8C_3$

A

ii) A

ii) $R = \sqrt{a^2+b^2}$

$= \sqrt{2}$

B

x) $x^2 - 4x + y^2 + 6y = 12$ $\quad +4+9$

$(x-2)^2 + (y+3)^2 = 25$

A

i) $-1 + (-1)^2 \times 2 \times 1 + (-1)^3 \cdot 3 \cdot 2 \cdot 1 + (-1)^4 \cdot 4!$

$= 19$

A

Question 2

a) $\frac{2x+3}{x-2} (x-2)^2 < 1 \; (x-2)^2$ where $x \neq 2$

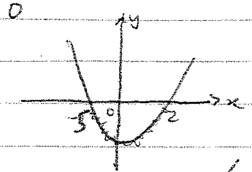
$(2x+3)(x-2) < (x-2)^2$

$(x-2)(2x+3 - x+2) < 0$

$(x-2)(x+5) < 0$

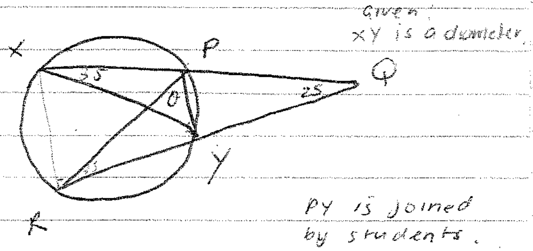
$-(x-2)(x+5) < 0$

$\therefore -5 < x < 2$




/3M

b)




$\angle XPY = 90°$ (angle in a semi-circle)

$\angle PYQ = 90 - 25$

$= 65°$ (ext $\angle$ of $\triangle PQY$)

$\angle PRY = 35°$ (angle in the same segment are $=$)

$\angle YPR = 65 - 35$ (ext. $\angle$ of $\triangle$)

$= 30°$

/3M

c. $A(2,-4) \quad B(3,-3)$

$(2 : -3)$

$x = \frac{2(3) + 2(-3)}{2-3} \qquad y = \frac{2(-3) + (-3)(-4)}{2-3}$

$= 0 \qquad = -6$

$P(0,-6)$

/2M

d. $m_1 = 2 \quad m = \frac{3}{5}$

$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

$= \frac{2 - \frac{3}{5}}{1 + 2 \times \frac{3}{5}}$

$= \frac{20}{33}$

$\theta = 32°28'$

/2M

e) $P(2) = 4$

$2^3 + 2^2 - a = 4$

$12 - a = 4$

$a = 8$

$P(x) = x^3 + x^2 - 8$

also $P(0) = -8$

$\therefore$ remainder is $-8$.

/2M

Question 3

a.

(i) $\alpha + \beta + \gamma = -\frac{2}{4}$

$= -\frac{1}{2}$

/1M

(ii) $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$

$= -\frac{1}{4}$

/1M.

(iii) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma) - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$

$= (-\frac{1}{2})^2 - 2 \times -\frac{1}{4}$

$= \frac{3}{4}$

/2M

b. Show $\frac{\sin(x+y)}{\cos(x-y)} = \frac{\tan x + \tan y}{1 + \tan x \tan y}$

LHS $= \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y + \sin x \sin y}$ $\div \cos x$

$= \frac{\frac{\sin x \cos y}{\cos x} + \sin y \cdot \frac{\cos x}{\cos x}}{\frac{\cos x \cos y}{\cos x} + \frac{\sin x}{\cos x} \sin y}$

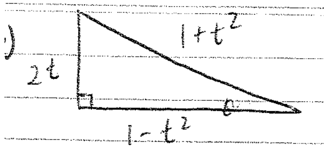
$= \frac{\tan x \cos y + \sin y}{\cos y + \tan x \sin y}$ $\div \cos y$

$= \frac{\tan x + \frac{\sin y}{\cos y}}{1 + \tan x \tan y}$

$= \frac{\tan x + \tan y}{1 + \tan x \tan y}$ = RHS

/4M

)




show $\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} = t$

LHS $= \dfrac{1+\frac{2t}{1+t^2}-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}$

$= \dfrac{\frac{1+t^2+2t-1+t^2}{\cancel{1+t^2}}}{\frac{1+t^2+2t+1-t^2}{\cancel{1+t^2}}}$

$= \dfrac{2t^2+2t}{2+2t}$

$= \dfrac{\cancel{2}t(\cancel{t+1})}{\cancel{2}(\cancel{1+t})}$

$= t$

= RHS

/4M

Question 4

a. $A\sin(x-\alpha) = \sin x - \cos x$

$A\sin(x-\alpha) = A\sin x\cos\alpha - A(\cos x\sin\alpha)$

so

$A\cos\alpha = 1 \qquad A\sin\alpha = 1$

$\dfrac{A\sin\alpha}{A\cos\alpha} = 1$

$\tan\alpha = 1$

$\alpha = 45$

also

$A^2(\sin^2\alpha+\cos^2\alpha) = 1^2+1^2$

$A = \sqrt{2}$

so

$\sin x - \cos x = \sqrt{2}\sin(x-45)$

/2M

ii) $\sin x - \cos x = \sqrt{2}$

$\sqrt{2}\sin(x-45) = \sqrt{2}$

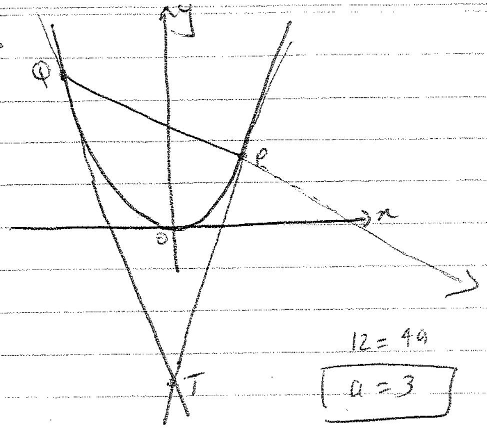
$\sin(x-45) = 1$

$(x-45) = 90^\circ$

$x = 135^\circ$

/2M.

b.




$12 = 4a$

$\boxed{a=3}$

Let $T(x_0, y_0)$, then the equation of the chord of contact from T is:

$xx_0 = 2a(y+y_0)$

$xx_0 = 6(y+y_0)$

But this passes through $(4,-3)$

so

$4x_0 = 6(-3+y_0)$

now replace $x_0 + y_0$ with $x, y$

$4x = 6(-3+y)$

$4x = -18+6y$

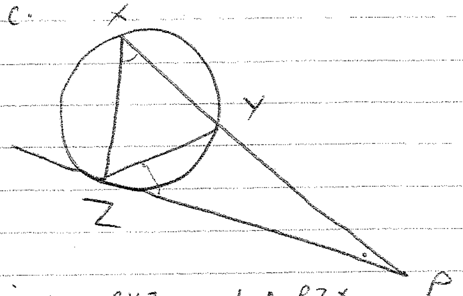
$4x-6y+18=0 \qquad (\div 2)$

$2x-3y+9=0$

/3M

Q4 continued.

c.




i. In $\Delta PYZ$ and $\Delta PZX$

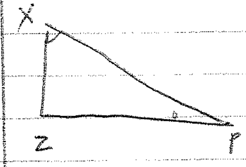
$\angle YPZ$ is common

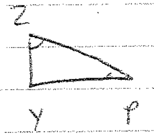
$\angle PZY = \angle ZXY$ ($\angle$ between a tangent and chord = to $\angle$ in the alternate segment).

$\therefore \Delta PYZ \parallel\!\mid \Delta PZX$

(corresp $\angle$'s are equal)

/2M




$\dfrac{PZ}{PX} = \dfrac{PY}{PZ}$ (corresp sides of similar $\Delta$'s are in the same ratio)

$PZ^2 = PY \cdot PX$

/1M

1.

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(45+30) = \dfrac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30}$

$= \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}}$

$= \dfrac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$

$= \dfrac{\sqrt{3}+1}{\sqrt{3}-1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \dfrac{3 + 2\sqrt{3} + 1}{2}$

$= \dfrac{4 + 2\sqrt{3}}{2}$

$= 2+\sqrt{3}$

/2M

Question 5

a) $\lim_{x \to 2} \dfrac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}}$

$= 2^2 + 2 \cdot 2 + 4$

$= 12$

b. i. $m_{pq} = \dfrac{ap^2 - aq^2}{2ap - 2aq}$

$= \dfrac{\cancel{a}\cancel{(p-q)}(p+q)}{2\cancel{a}\cancel{(p-q)}}$

$= \frac{1}{2}(p+q)$

$y - ap^2 = \frac{1}{2}(p+q)(x - 2ap)$

$2y - 2ap^2 = (p+q)(x - 2ap)$

$2y - 2ap^2 = (p+q)x - 2ap(p+q)$

$2y - \cancel{2ap^2} = (p+q)x - \cancel{2ap^2} - 2apq$

$(p+q)x - 2y = 2apq$ /2M

(ii) $F(0,a)$ sub into above equ

$(p+q) \times 0 - 2a = 2apq$

$-2\cancel{a} = 2\cancel{a}pq$

$\boxed{-1 = pq}$

$m_{op} = \dfrac{ap^2 - 0}{2ap}$

$= \dfrac{p}{2}$

$\therefore m_{oq} = \dfrac{q}{2}$

$\therefore \dfrac{p}{2} \cdot \dfrac{q}{2} = \dfrac{pq}{4} = \dfrac{-1}{4}$ /2M

Q5

c. (i) ${}^{10}C_3 = 120$ /1M

(ii) ${}^5C_1 \cdot {}^5C_2$

1 student . 2 others

$= 50$ /1M

(iii) ${}^5C_1 \cdot {}^2C_1 \cdot {}^3C_1$

1S 1P 1T

or 3 Students $= {}^5C_3$

${}^5C_3 + {}^5C_1 \times {}^2C_1 \times {}^3C_1$

$= 40$ /2M

Question 6

a.

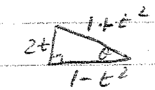
$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$= \dfrac{2\tan 22\frac{1}{2}}{1 - \tan^2 22\frac{1}{2}}$

/1M

let $\tan 22\frac{1}{2} = t$

$\tan 45 = \dfrac{2t}{1-t^2}$




$1 = \dfrac{2t}{1-t^2}$

$1 - t^2 = 2t$

$t^2 + 2t - 1 = 0$

$t = \dfrac{-2 \pm \sqrt{2^2 - 4 \times 1 \times -1}}{2}$

$= \dfrac{-2 \pm \sqrt{8}}{2}$

$= -1 \pm \sqrt{2}$

$= -1 + \sqrt{2}$ since $t > 0$

Hence $\tan 22\frac{1}{2} = \sqrt{2} - 1$

b) Show true for $n=1$

$$LHS = \frac{1}{(2(1)-1)(2(1)+1)}$$
$$= \frac{1}{1\times 3}$$
$$= \frac{1}{3}$$

$$RHS = \frac{1}{2\times 1+1}$$
$$= \frac{1}{3}$$

$\therefore$ true for $n=1$

Assume true for $n=k$

$$\frac{1}{1\times 3} + \frac{1}{3\times 5} + \dots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1} \quad \text{—(A)}$$

prove true for $n=k+1$

$$\frac{1}{1\times 3} + \frac{1}{3\times 5} + \dots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2k+1)(2k+3)} = \frac{(k+1)}{2k+3}$$

From (A)

$$LHS = \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$$
$$= \frac{k(2k+3)+1}{(2k+1)(2k+3)}$$
$$= \frac{2k^2+3k+1}{(2k+1)(2k+3)}$$
$$= \frac{(2k+1)(k+1)}{(2k+1)(2k+3)}$$
$$= \frac{k+1}{2k+3}$$
$$= RHS$$

Step 3: statement!

/4M

c) Show ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$$LHS = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$
$$= \frac{n!}{r(r-1)!(n-r)!} + \frac{n!}{(r-1)!(n-r)!(n-r+1)}$$
$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$
$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right]$$
$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{n+1}{r(n-r+1)}\right]$$
$$= \frac{n!(n+1)}{(r-1)!(n-r)!\,r(n-r+1)}$$
$$= \frac{(n+1)!}{r!(n-r+1)!}$$
$$= {}^{n+1}C_r$$
$$= RHS$$

/4m.