



CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW

2012 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

Morning Session Monday, 6 August 2012

General Instructions

- Reading time 5 minutes
- Working time 3 hours
- Write using blue or black pen Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided on a separate sheet
- Show all necessary working for Questions 11-16
- Write your Centre Number and Student Number at the top of this page and page 7

Total marks - 100

Section I

Pages 2-6

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II

Pages 7-17

90 marks

- Attempt Questions 11–16
- Allow about 2 hour and 45 minutes for this section

Disclaimer

Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, Principles for Setting HSC Examinations in a Standards-Referenced Framework (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

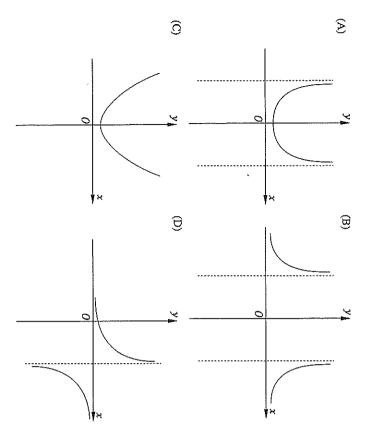
Section I 10 marks

Attempt Questions 1 - 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

Which of the following best represents the graph of $y = \frac{1}{\sqrt{4-x^2}}$?



12 Let z = a + ib, where $a \neq 0$ and $b \neq 0$.

Which of the following statements is false?

(A)
$$z - \overline{z} = 2bi$$

(B)
$$|z|^2 = |z||\overline{z}|$$

(C)
$$|z| + |\overline{z}| = |z + \overline{z}|$$

(D)
$$\arg(z) + \arg(\overline{z}) = 0$$

z satisfies the equation $|z - \sqrt{2} - i\sqrt{2}| = 1$. Find the minimum value of $\arg(z)$.

Find the coordinates of the foci of the hyperbola with equation $\frac{x^2}{9} - \frac{y^2}{4} = 1$.

(A)
$$(\pm\sqrt{5},0)$$

(B)
$$\left(\pm\sqrt{13},0\right)$$

(C)
$$\left(\pm \frac{3\sqrt{5}}{2}, 0\right)$$

 $(D) \quad \left(\pm \frac{3\sqrt{13}}{2}, 0\right)$

2

The substitution $t = \tan \frac{\theta}{2}$ is used to find $\int \frac{d\theta}{\cos \theta}$

Ú

Which of the following gives the correct expression for the required integral?

- $\int \frac{1}{2(1-t^2)} dt$
- ₿ $\int \frac{2}{1-t^2} dt$
- 0 $\int \frac{2t}{1-t^2} dt$
- $(D) \quad \int \frac{4t}{(1+t^2)^2} \, dt$

The equation |z-3|+|z+3|=10 defines an ellipse.

Find the length of the semi-minor axis.

- \mathfrak{S}
- (B) 5
- 3 00
- Θ 10

- 9 In how many ways can 5 letters be chosen from the letters of the world ARRANGE?
- (A) 9
- (B) 12
- 0 21
- Θ 30

Let x be the displacement in metres of the particle above θ at time t seconds after the particle is projected and let g be the acceleration due to gravity.

Which of the following expressions gives the maximum height reached by the particle?

(A)
$$\int_{u}^{0} \frac{dv}{-g + \frac{k}{2}v^2}$$

(B)
$$\int_{u}^{0} \frac{dv}{-g - \frac{k}{2}v^2}$$

(C)
$$\int_{u}^{0} \frac{vdv}{-g + \frac{k}{2}v^{2}}$$

(D)
$$\int_{u}^{0} \frac{vdv}{-g - \frac{k}{2}v^{2}}$$

Find the value of $\alpha^3 + \beta^3 + \gamma^3$. Let α , β and γ be the zeros of the polynomial $x^3 + 5x - 3$.

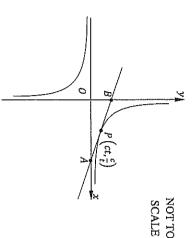
00

- (A) -125
- (B) 0
- 0
- Э

4

is given by $x + t^2y = 2ct$. The tangent cuts the x and y axes at A and B respectively. The equation of the tangent to the rectangular hyperbola $xy=c^2$ at $P\left(ct,\frac{c}{t}\right)$

10



Which of the following statements is false?

- (A) P is the centre of the circle that passes through O, A and B.
- 8 The area of $\triangle AOB$ is $2c^2$ square units.
- 0 The distance AB is $\sqrt{4c^2t^2 + \frac{4c^2}{t^2}}$.
- (D) AP > BP



CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW 2012 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

Section II 90 marks

Attempt Questions 11-16

Student Number

Centre Number

All questions are of equal value.
Allow about 2 hours and 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 11 (15 Marks) Use a SEPARATE writing booklet.

(a) Find
$$\int \frac{(\ln x)^2}{x} dx$$
.

- Evaluate
- Ŧ
- Θ Find real numbers A and B such that

Ŋ

Ç

<u></u>

$$\frac{7x+1}{x^2-x-2} = \frac{A}{x+1} + \frac{B}{x-2}.$$

- (ii) Hence, find
- Using a trigonometric substitution, or otherwise, find

<u>a</u>

@ This region is rotated about the vertical line x = -1. Consider the region in the first quadrant bounded by the curves y = x and $y = x^2$.

Use the method of cylindrical shells to find the volume of the solid formed

Question 12 (15 Marks) Use a SEPARATE writing booklet.

- (a) Let z = -1 + i.
- (i) Express $\frac{1}{z}$ in the form a + ib.
- (ii) Express z in modulus-argument form.
- (iii) Hence, find z^{10} in the form a + ib.

Draw a sketch of the locus specified by |z-i|=|z+3-2i| on an Argand diagram.

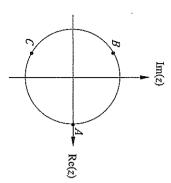
9

(c) The polynomial $P(x) = x^4 - 2x^3 + 8x^2 - 8x + 16$ has $1 + i\sqrt{3}$ as a zero.

Express P(x) as the product of two quadratic factors.

- (d) The roots of $z^3 1 = 0$ are 1, ω and ω^2 where ω is one of the complex roots.
- (i) Find the value of $1 + \omega + \omega^2$.
- (ii) Explain why $z^2 + z + 1 = (z \omega)(z \omega^2)$.

The Argand diagram below shows the points A, B and C which correspond to the roots $1, \omega$ and ω^2 of the equation $z^3 - 1 = 0$.



- (iii) Copy this diagram into your writing booklet and show the vector $1-\omega$ on your diagram, carefully indicating the direction of this vector.
- (iv) Hence, or otherwise, find the product of the lengths of the two chords AB and AC.

Question 13 (15 Marks) Use a SEPARATE writing booklet.

Consider the curve with equation $3x^2 + 3y^2 + 2xy = 24$

(a)

(i) Show that
$$\frac{dy}{dx} = -\left(\frac{y+3x}{3y+x}\right)$$
.

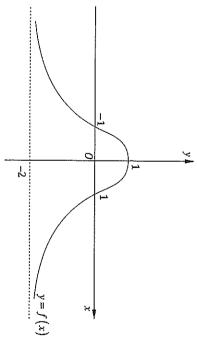
ii) Find the x-coordinates of the stationary points of the curve.

N

The line y = -2 is a horizontal asymptote and f(x) is an even function. y = f(x) has x-intercepts at $(\pm 1, 0)$ and y-intercept at (0, 1).

9

The diagram shows the graph of y = f(x)



Draw separate one-third page diagrams of the graphs of the following, showing any horizontal or vertical asymptotes.

(i)
$$y = |f(x)|$$

(ii)
$$y = [f(x)]^2$$

(iii)
$$y = \ln f(x)$$

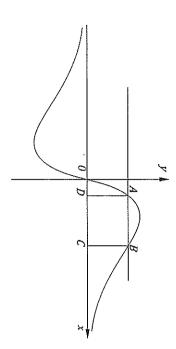
Question 13 continues on page 10

Question 13 (continued)

(c) $P(x) = x^4 + 3x^3 - 6x^2 - 28x + c$ has a zero of multiplicity 3.

Find the value of c.

- (d) Consider the curve $f(x) = \frac{2x}{x^2 + 1}$.
- (i) Show that $f\left(\frac{1}{x}\right) = f(x)$, where $x \neq 0$.
- (ii) The diagram below shows the graph y = f(x).



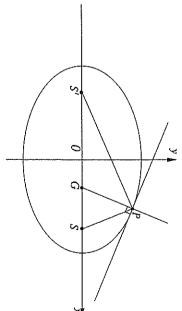
A horizontal line intersects the curve in the first quadrant at A and B. C and D lie on the x-axis directly beneath B and A respectively.

Find the coordinates of C if ABCD is a square.

End of Question 13

Question 14 (15 Marks) Use a SEPARATE writing booklet.

(a) The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ has foci S(ae,0) and S'(-ae,0), and directrices $x = \pm \frac{a}{e}$. $P(a\cos\theta, b\sin\theta)$ is a point on the ellipse with the normal at P meeting the x-axis at G.



(i) Using the focus/directrix definition of an ellipse show that

2

$$\frac{PS'}{PS'} = \frac{1 - e\cos\theta}{1 + e\cos\theta}.$$

(ii) The equation of the normal at P is given by

$$\frac{ax}{\cos \theta} - \frac{by}{\sin \theta} = a^2 - b^2.$$
 (Do NOT prove this.)

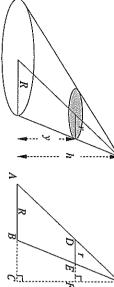
w

Show that $\frac{GS}{GS'} = \frac{PS}{PS'}$.

Question 14 continues on page 12

Question 14 (continued)

(b) The diagram shows an oblique cone of base radius R and perpendicular height h.



A horizontal cross-section of the cone taken at height y, is a circle of radius r shown shaded in the diagram.

(i) By considering the ratio of sides in two pairs of similar triangles, show that

$$= \left(\frac{n-y}{h}\right)R.$$

(ii) Show that the volume of the oblique cone is given by $\frac{1}{3}\pi R^2 h$.

w

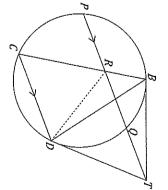
 $\frac{\int_{E} \frac{1}{|r|} \frac{1}{|r|} \frac{1}{r} \frac{1}{r}$

Question 14 continues on page 13



<u>ල</u>

The chords PQ and CD of a circle are parallel. The tangent at D meets PQ produced at T. B is the point of contact of the other tangent from T to the circle. BC meets PQ at R.



Copy or trace the diagram into your writing booklet.

- (i) Prove $\angle BDT = \angle BRT$. Hence, state why B, T, D and R are concyclic points.
- (ii) Prove $\angle BRT = \angle DRT$.
- (iii) Show that ΔRCD is isosceles.

End of Question 14

Question 15 (15 Marks) Use a SEPARATE writing booklet.

(a) A truck of mass m kg moves at speed v ms⁻¹ around a curve banked at angle θ and with radius r. The lateral force down the slope is F, the normal force is N and g is the acceleration due to gravity.

By resolving forces in two directions show that $F = \frac{mv^2}{r}\cos\theta - mg\sin\theta$.

(b) (i) By considering the expansion of $(\cos \theta + i \sin \theta)^5$ and using de Moivre's theorem, show that

w

 $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$

Hence, show that the roots of $16x^5 - 20x^3 + 5x = 0$ are

N

 Ξ

 $x = 0, \pm \cos \frac{\pi}{10}, \pm \cos \frac{3\pi}{10}.$

(iii) Hence, show that

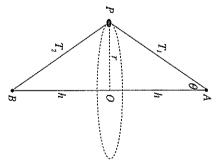
$$\cos^2 \frac{\pi}{10} + \cos^2 \frac{3\pi}{10} = \frac{5}{4}.$$

Question 15 continues on page 15

Question 15 (continued)

(c) Two points A and B are fixed with A vertically above B. Two equal lengths of light inextensible string join A and B to a particle P of mass m. The particle P moves with angular velocity \(\omega\) in a horizontal circle, centre O and radius r, such that AO = BO = h.

This information is shown in the diagram below.



Let T_1 and T_2 be the tensions in AP and BP respectively. Let $\angle PAO = \theta$ and let g be the acceleration due to gravity.

(i) By resolving forces vertically at P, show that

$$(T_1 - T_2)\cos\theta = mg$$

(ii) If $\omega = 2\sqrt{\frac{g}{h}}$, show that $T_1: T_2 = 5:3$.

End of Question 15

Question 16 (15 Marks) Use a SEPARATE writing booklet.

(a) Θ The inequality $\frac{x+y}{2} \ge \sqrt{xy}$ is true for all $x \ge 0$ and $y \ge 0$, with equality when

Use the inequality to show that the minimum value of the function $f(x) = Ae^x + Be^{-x}$ is $2\sqrt{AB}$, where A>0 and B>0 are constants.

Ξ The function $f(x) = Ae^x + Be^{-x}$ has line symmetry about the vertical for some values of k and c. line x = c. That is, we can write f(x) in the form $f(x) = k \left[e^{x-c} + e^{-(x-c)} \right]$

Ŋ

Using part (i), or otherwise, find k and c in terms of A and B.

- ਭ (i) Show that $\frac{x^{2n-1}}{\sqrt{1-x^2}} - \frac{x^{2n+1}}{\sqrt{1-x^2}} = x^{2n-1}\sqrt{1-x^2}$.
- Ξ For every integer $n \ge 1$ let $L_{n-1} = \int_0^1 \frac{x^{2n-1}}{\sqrt{1-x^2}} dx$.

Using integration by parts and the result from part (i), show that for $n \ge 1$

$$I_{2n+1} = \left(\frac{2n}{2n+1}\right) I_{2n-1} \, .$$

Ŋ

(iii) Using part (ii), or otherwise, show that,
$$I_{2n+1} = \frac{2^n \times n!}{1 \times 3 \times 5 \times ... \times (2n+1)}.$$

Question 16 continues on page 17

Question 16 (continued)

(iv) Using part (iii), or otherwise, show that

ы

$$\int_{0}^{1} \frac{x}{\sqrt{1 - x^2}} dx + \int_{0}^{1} \left[\sum_{n=1}^{\infty} \left(C_n \frac{x^{2n+1}}{\sqrt{1 - x^2}} \right) \right] dx = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$$

where
$$C_n = \frac{1 \times 3 \times 5 \times ... \times (2n-1)}{(2n+1)2^n n!}$$

You may assume each infinite series has a limiting sum

(v) The inverse sine function $\sin^{-1} x \operatorname{can}$ be defined by the following series:

N

$$\sin^{-1} x = x + \sum_{n=1}^{\infty} C_n x^{2n+1}$$

where
$$C_n = \frac{1 \times 3 \times 5 \times ... \times (2n-1)}{(2n+1)2^n n!}$$
.

Using this definition of $\sin^{-1} x$ and the result from part (iv), show that

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}.$$

(vi) Hence, find the limiting value of S if

2

$$S = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \dots$$

End of paper



CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW

2012 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS EXTENSION 2

Section I 10 marks

Questions 1-10 (1 mark each)

Question 1 (1 mark)
Outcomes Assessed: E6

Targeted Performance Bands: E2

Solution Solution	Answer	Mark
Asymptotes exist at $x = \pm 2$.		
The graph exists for x values $-2 < x < 2$.	A	
The y-intercept is $\frac{1}{2}$.		

Question 2 (1 mark)

Outcomes Assessed: E3

Targeted Performance Bands: E3

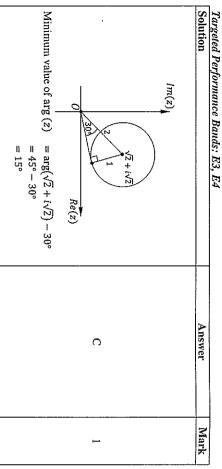
Solution Solution	Answer	Mark
Let $z = a + ib$, then $\bar{z} = a - ib$		
$ z + \bar{z} = 2\sqrt{a^2 + b^2}$	C	1
$ z + \bar{z} = 2a $		
$\neq z + \bar{z} $		

DISCLAIMER

The information contained in this document is intended for the professional assistance of teaching staff. It does not constitute advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible Trial HSC answers. Rather the purpose is to provide teachers with information so that they can better explore, understand and apply HSC marking requirements, as established by the NSW Board of Studies.

No guarantee or warranty is made or implied with respect to the application or use of CSSA Marking Guidelines in relation to any specific trial exam question or answer. The CSSA assumes no liability or responsibility for the accuracy, completeness or usefulness of any Marking Guidelines provided for the Trial HSC papers.

Question 3 (1 mark)
Outcomes Assessed: E3



Question 4 (1 mark)

Outcomes Assessed: E3

Targeted Performance Bands: E2

Solution	Answer	Mark
a = 3, b = 2		
$b^2 = a^2(e^2 - 1)$		
$e = \frac{\sqrt{13}}{2}$	В	<u> </u>
The coordinates of the foci are $(\pm\sqrt{13},0)$.		
ĺ		

hund	В	$=\int \frac{2}{1-t^2}dt$
		$\int \frac{d\theta}{\cos \theta} = \int \frac{1+t^2}{1-t^2} \cdot \frac{2}{1+t^2} dt$
Mark	Answer	Solution
		Question 5 (1 mark) Outcomes Assessed: HE6, E8 Targeted Performance Bands: E2

DISCLAMER

The information contained in this document is intended for the professional assistance of teaching stalf. It does not constitute advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible into HSC arcevers. Ruther the purpose is an provide specific marking outcomes for all possible into HSC arcevers. Ruther the purpose is an provide standard with information on Intal they can better explore, understand and apply HSC marking qualitatives, as established by the NSM Board of Studes.

10 generatives or warming is made or implied with inseport to the application or use of CSSA Marking Guidelines in relation to any specific trial exam question or asswer. The CSSA assumes no liability or responsibility for the accuracy, completeness or usefulness of any Marking Guidelines provided for the Trial HSC pupers.

Outcomes Assessed: E3
Targeted Performance Bands: E3 Question 6 (1 mark)

Solution	Answer	Mark
5 4 5 4 (-3,0) 0 (3,0)	A)
Length of the semi-minor axis is 4 units.		

Question 7 (1 mark)

Outcomes Assessed: E5
Targeted Performance 1 7 Z,

Largetea Ferjormance Danas; E3 - E4		
Solution	Answer	Mark
Particle is projected vertically upwards (x>0) and both the		
gravitational force $(1 \times g)$ and resistance $(\frac{kv^2}{2})$ act		
downwards.		
$\therefore 1 \times \ddot{x} = -1 \times g - \frac{kv^2}{2}$		
$\therefore v \frac{dv}{dx} = -g - \frac{kv^2}{2}$		
$\frac{dv}{dx} = \frac{-g - \frac{kv^2}{2}}{v}$	D	1
$\frac{dx}{dv} = \frac{v}{-g - \frac{kv^2}{2}}$		
As the initial velocity is u and velocity at the maximum height is 0 .		
$x = \int_0^0 \frac{v dv}{-g - \frac{k}{2}v^2}$		
- vanishing in the state of the		

OSCLAHER

OSCLAMER

The information contained in this document is intended for the professional assistance of leaching stelf. It does not constitute advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible that HSC areases. Name: be purpose to provide specific marking outcomes for all possible that HSC areases.

HSC areasing requirements, as established by the NSW Board of Studies.

No guaranties or warranty is made or implied with respect to the application or use of CSSA. Marking Guidelines in relation to any specific trial evern question or answer. The CSSA assumes no leability or responsibility for the accuracy, completeness or userfainess of any Marking Guidelines provided for the Tail HSC papers.

Question 8 (1 mark)
Outcomes Assessed: E4

Targeted Performance Bands: E3

Solution	Answer	MARTAI	
$\alpha^3 = 3 - 5\alpha$			
$\beta^3 = 3 - 5\beta$			
$\gamma^3 = 3 - 5\gamma$			
Therefore, $\alpha^3 + \beta^3 + \gamma^3 = 9 - 5(\alpha + \beta + \gamma)$	C	_	
=9-5×0			
= 9.			
•			

Question 9 (1 mark)

Outcomes Assessed: E2, PE3

Targeted Performance Bands: E3

9 **** 1 *** 3 ** * * * * * * * * * * * *		
Solution	Answer	Mark
5 different letters can be chosen in $\binom{5}{5}$ ways.		
2 like letters (RR or AA) and 3 different letters can be chosen in $2 \times {4 \choose 3}$ ways.		
2 pairs of like letters (RR and AA) and 1 different letter can be chosen in $1 \times {3 \choose 1}$ ways.	В) —1
Therefore, 5 letters can be chosen in 12 ways.		

Question 10 (I mark)
Outcomes Assessed: E4

Targeted Performance Bands: E3		
Solution	Answer	Mark
Coordinates are $A(2ct, 0)$, $B(0, \frac{2c}{t})$ and $P(ct, \frac{c}{t})$.		
Therefore, P is the midpoint of AB i.e. $AP = BP$. Hence $AB > BB$ is false	מ	سر
Hence, $AP > BP$ is false.		

ORSCLAMER

ORSCLAMER

The information contained in this document is intended for the professional assistance of teaching staff. If does not constitute atrice to students. Further it is not the intended of CSSA to provide profile making outcomes for all procession. Trial HSC services. Rether the purpose is to provide teachers with information so that they can better emplore, understand and apply HSC marking requirements, consistently by the NSW feared of Studes.

HSC marking requirement, consistently is made or implied with respect to the application or use of CSSA. Marking Guidelines in relation to any specific trial exam question or answer. The CSSA to provide or warranty is made or implied with respect to the application or use of CSSA. Marking Guidelines in relation to any specific trial exam question or answer. The CSSA assumes no liability or responsibility for the accuracy, combinances or usefulness of any Marking Guidelines provided for the Trial HSC papers.

Section II 90 marks

Question 11 (15 marks) (a) (1 mark)

Outcomes assessed: HE6, E8
Targeted Performance Bands: E2

Sample answer:

$$\int \frac{(\ln x)^2}{x} dx = \frac{(\ln x)^3}{3} + \frac{1}{3}$$

Outcomes assessed: E8 (b) (3 marks)

Targeted Performance Bands: E2 - E3

	Criteria
•	Correct simplified solution
•	Correct second integral and substitution of limits
•	Correct use of Integration by Parts

Sample answer:

$$\int_0^{\ln 2} x e^x dx = [x e^x]_0^{\ln 2} - \int_0^{\ln 2} e^x dx$$
$$= 2 \ln 2 - [e^x]_0^{\ln 2}$$
$$= 2 \ln 2 - 1$$

(c) (i) (2 marks)
Outcomes assessed: E8

Targeted Performance Bands: E2

Criteria	Marks
Correctly finds A and B	2
Makes some progress in partial fraction decomposition	

Sample answer:

$$\frac{7x+1}{x^2-x-2} = \frac{A}{x+1} + \frac{B}{x-2}$$
$$= \frac{A(x-2) + B(x+1)}{(x+1)(x+2)}$$

Equating:

$$A+B=7$$
$$B-2A=1$$

Therefore, A = 2 and B = 5.

DISCLAUSER

In be information contained in this document is intended for the professional assistance of teaching staff. It does not constitute advice to students, Further it is not the intention of CSSA to provide specific marking outcomes for all possible trial HSC arewers. Rather the purpose is to provide treatings with information so that they can better explore, understand and apply ISC marking requirements, as established by the NSV Board of States.

No guarantee or warmy is made or implied with insected to the application or use of CSSA Marking Guidelines in relation to any specific trial exam question or asswer. The CSSA assumes no liability or responsibility for the accuracy, completeness or usofutness of any Marking Guidelines provided for the That HSC papers.

(c) (ii) (2 marks)

Outcomes assessed: E8

Targeted Performance Bands: E2

_		Criteria	Marks
	•	Correct integration of both partial fractions found in (i). No penalty for no	ş
		absolute value or no +c or no simplification of answer	t
_	•	Some progress towards the answer	1

Sample answer:

$$\int \frac{7x+1}{x^2-x-2} dx = \int \left(\frac{2}{x+1} + \frac{5}{x-2}\right) dx$$
$$= 2\ln|x+1| + 5\ln|x-2| + c$$
$$= \ln(x+1)^2|x-2|^5 + c$$

(d) (3 marks)

Outcomes assessed: HE6, E8

Targeted Performance Bands: E2 - E3

1111	ingerea i eijoimane pana, 122 - 123	
	Criteria	Marks
•	Correct answer in the variable x , (no penalty for no $+c$)	w
•	Substantial progress towards simplifying the integral	2
•	Correct substitution e.g. $x = \tan \theta$,

Sample answer:

Let
$$x = \tan \theta$$

$$\int \frac{1}{(1+x^2)^{\frac{3}{2}}} dx = \int \frac{\sec^2 \theta \, d\theta}{(1+\tan^2 \theta)^{\frac{3}{2}}}$$

$$= \int \frac{\sec^2 \theta \, d\theta}{\sec^3 \theta}$$

$$= \int \cos \theta \, d\theta$$

$$= \sin \theta + c$$

$$= \frac{x}{\sqrt{1+x^2}} + c$$

φ,

DISCLAMER

The information contained in this document is intended for the professional assistance of teaching staff if does not constant a shoet to students. Further it is not the intention of CSSA to provide prodict making outcomes for all prostable. If Id HSC accesses, Rather the purpose is no provide teachers with information so that they can better explore, understand and apply HSC marking requirement, as established by the NSW legard of Studies.

HSC marking requirement, as established by the NSW legard of Studies, the provided in the proprietable or use of CSSA, Marking Guidelines in relation to any specific trial exam question or answer. The CSSA is to granted or warranty is made or implied with respect to the application or use of CSSA, Marking Guidelines for relating to the Trial HSC papers.

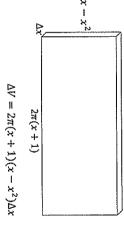
(e) (4 marks)

Outcomes assessed: E7

Targeted Performance Bands: E3 - E4

	The property of the state of th	
	Criteria	Marks
•	Correct Solution	4
•	Expression for the integral with correct limits	3
•	Progress towards finding an expression for the volume of a shell	2
•	Some progress towards finding an expression for the radius or height of a	_
	cylindrical shell	

Sample answer:



$$\Delta V = 2\pi(x+1)(x-x^2)\Delta x$$

$$V = \lim_{\Delta x \to 0} \sum_{x=0}^{1} 2\pi(x+1)(x-x^2) \Delta x$$

$$= 2\pi \int_{0}^{1} (x-x^3) dx$$

$$= 2\pi \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_{0}^{1}$$

$$= \frac{\pi}{2}$$

Therefore, the volume is $\frac{\pi}{2}$ units³.

InSCLAMER

The information contained in this decument is intended for the professional assistance of teaching staff, it does not constitute advice to students. Further it is not the intention of CSSA to The information and making outcomes for all possible Trial HSC analysis. Realize the purpose is to provide specific making outcomes for all possible Trial HSC analysis are information, and the provide specific making requirements, as established by the NSW accord O'Studies.

HSC marking requirements, as established by the NSW accord O'Studies. The CSSA Marking Golddines in relation to any specific trial exam question or answer. The CSSA to gazanties or warranty is made or implied with respect to the application or use of CSSA Marking Golddines provided for the Trial HSC papers.

Question 12 (15 marks)
(a) (i) (1 mark)

Outcomes assessed: E3

Targeted Performance Bands: E2

<u></u>	Correct answer
Mark	Criteria

Sample answer:

$$\frac{1}{z} = \frac{1}{-1+i} \times \frac{-1-i}{-1-i}$$

$$= -\frac{1}{2} - \frac{1}{2}i$$

(a) (ii) (1 mark)

Outcomes assessed: E3

Targeted Performance Bands: E2

Sample Answer:

$$|z| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

$$\arg(z) = \frac{3\pi}{4}$$

$$\therefore z = \sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

(a) (iii) (2 marks)

Outcomes assessed: E3

Targeted Performance Bands: E2

Criteria

Correct simplified answer

$-10 = 10 3\pi 3\pi \setminus 10$	Use of De Moivre's theorem
710	

Sample answer

$$z^{10} = \sqrt{2^{10} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)}$$

$$= 32 \left(\cos \frac{15\pi}{2} + i \sin \frac{15\pi}{2}\right)$$

$$= -32i$$

 ∞

DISCLAMER

DISCLAMER

The information contained in this document is intended for the professional assistance of teaching staff. It does not constitute advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible Trial HSC arowers. Rubber the purpose is to provide seachines with information so had they can better explore, understand and apply 1850 marking equinements, as established by the 1894 Board of Studies.

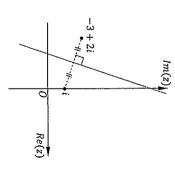
No guaranties or warming is made or implied with respect to the application or use of CSSA. Marking Guidelines in relation to any specific trial exam question or answer. The CSSA assumes no liability or responsibility for the accuracy, completeness or usefulness of warding Guidelines provided for the Trial HSC papers.

(b) (2 marks)

Outcomes assessed: E3
Targeted Performance Bands: E2

•	• [_	
Some manager of the local is a line	Entire correct diagram	Criteria	
_	2	Marks	

Sample answer:



(c) (3 marks)

Outcomes assessed: E4

Targeted Performance Bands: E3

	Criteria	Marks
•	Correct answer	ω
•	Progression towards finding the factor $x^2 - 2x + 4$	2
•	Recognition that $1 - i\sqrt{3}$ is also a root	
I		

Sample answer:

Mark

Since $1 + i\sqrt{3}$ is a root of P(x), $1 - i\sqrt{3}$ is also a root of P(x). (Roots occur in conjugate pairs)

The quadratic factor of P(x) with roots $1 \pm i\sqrt{3}$ is $x^2 - 2x + 4$.

By division:
$$(x^4 - 2x^3 + 8x^2 - 8x + 16) \div (x^2 - 2x + 4) = (x^2 + 4)$$

Therefore,
$$(x^4 - 2x^3 + 8x^2 - 8x + 16) = (x^2 - 2x + 4)(x^2 + 4)$$

(d) (i) (1 mark)

Outcomes assessed: E4

Targeted Performance Bands: E2

•	
Correct answer	Criteria
	Mark
	•

Sample answer:

$$1 + \omega + \omega^2 = \frac{-b}{a}$$
$$= 0$$

9

INSCAMER

The information contained in this document is intended for the professional assistance of teaching staff, it does not constitute advice to students. Further it is not the intendion of CSSA to provide specific marking outcomes for all possible Trial HSC arewers. Rather the purpose is to provide leachers with information so that they can better explore, understand and apply HSC marking squirements, as established by the NSW Board of Studies.

HSC marking squirements, as established by the NSW Board of Studies.

No guarantee or warrantly is made or implied with negacite to the application or use of CSSA Marking Guidelines in relation to any specific brid exam question or answer. The CSSA is summer no liability or responsibility for the accuracy, completaness or usefulness of any Marking Guidelines provided for the Trial HSC papers.

(d) (ii) (2 marks)

Outcomes assessed: E2, E4

Targeted Performance Bands: E3

Criteria	Marks
Correct explanation	2
• Correct factorisation of $z^3 - 1$	_

Sample answer:

$$z^3 - 1 = (z - 1)(z^2 + z + 1)$$

Since 1, ω and ω^2 are roots of $z^3 - 1 = 0$, $z^3 - 1 = (z - 1)(z - \omega)(z - \omega^2)$
 $\therefore (z - 1)(z^2 + z + 1) = (z - 1)(z - \omega)(z - \omega^2)$
Hence, $z^2 + z + 1 = (z - \omega)(z - \omega^2)$

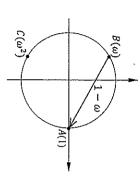
(d) (iii) (1 mark)

Outcomes assessed: E3

Targeted Performance Bands: E3

Criteria	Marks
Correct vector	1

Sample answer:



(d) (iv) (2 marks)

Outcomes assessed: E2, E3

Targeted Performance Bands: E3 - E4

	Criteria	Marks	
•	Correct argument and answer	2	
•	 Recognition that the length of a vector is the modulus of a complex number 	1	

Sample answer:

Product of lengths of chords =
$$|1 - \omega| \times |1 - \omega^2|$$

= $|(1 - \omega)(1 - \omega^2)|$
= $|1^2 + 1 + 1|$ (using part ii)
= 3

10

DISCLAMER
The takematic contained in this document is intended for the professional assistance of teaching staff, it does not constitute advice to students. Further it is not the intendent of CSSA to provide specific making outcomes for all possibly trial HSC areands. Rather the purpose is to provide peachers with information so that they can better explore, understand and apply HSC marking requirements, as established by the NSW Board of Studes.

No guarantee or warranty is made or impided with respect to the application or use of CSSA Marking Guidelines in relation to any specific trial even queetion or answer. The CSSA has been adviced from the property of the accuracy, completeness or usefulness of any Marking Guidelines provided for the Trial HSC papers.

Question 13 (15 marks) (a) (i) (2 marks)

Outcomes assessed: E6

Targeted Performance Bands: E3 Criteria

Criteria	Marks
Correct solution	2
 Demonstrates some knowledge of implicit differentiation 	<u> </u>

Sample answer:

$$3x^{2} + 3y^{2} + 2xy = 24$$

$$6x + 6y \cdot \frac{dy}{dx} + 2x \cdot \frac{dy}{dx} + 2y = 0$$

$$\frac{dy}{dx} (6y + 2x) = -6x - 2y$$

$$\frac{dy}{dx} = -\left(\frac{y+3x}{x+3y}\right)$$

(a) (ii) (2 marks)

Outcomes assessed: E6

Targeted Performance Bands: E2

******	Criteria	Marks
•	Correct solution	2
•	Some progress towards x values or y values	

Sample answer:

Stationary points occur when
$$\frac{dy}{dx} = 0$$
 i.e. $y = -3x$

Substitute into the equation.

$$3x^{2} + 3(9x^{2}) + 2x(-3x) = 24$$

$$x^{2} = 1$$

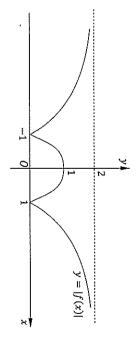
$$x = \pm 1$$

(b) (i) (1 mark)

Outcomes assessed: E6

Targeted Performance Bands: E3

Sample answer:



DISCLAUER

IDISCLAUER

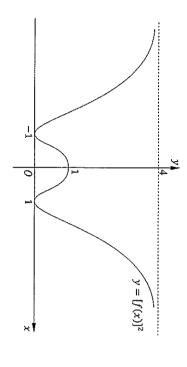
IDISCLA

(b) (ii) (2 marks)
Outcomes assessed: E6

Targeted Performance Bands: E3

Criteria	Marks
	2
Significant progress towards correct graph	-

Sample answer:



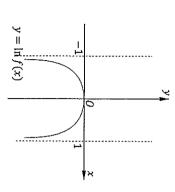
(b) (iii) (2 marks)

Outcomes assessed: E6

Targeted Performance Bands: E3 - E4

aph t progress towards correct graph		Criteria	Marks
aph t progress towards correct graph	annual designation of the last		
t progress towards correct graph	•	aph	2
	•	t progress towards correct graph	-

Sample answer:



DISCUAIMER
The information contained in this document is intended for the professional assistance of teaching staff, it does not constitute advice to students. Furber it is not the intenden of CSSA to provide specific marking outcomes for all possible Trial HSC arowers. Rather the purpose is to provide teachers with information so that they can better explore, understand and apply HSC marking requirements, as established by the NSW Board of Studies.

HSC marking requirements, as established by the NSW Board of Studies.

NS grantee or warranty is made or implied with respect to the application or use of CSSA Marking Guidelines in relation to any specific trial exam question or answer. The CSSA has successful to the Trial HSC papers.

12

(c) (3 marks)
Outcomes assessed: E4

Targeted Performance Bands: E3

	The state of the s	
	Criteria	Marks
•	Correct solution	IJ
•	Justifies that $x = -2$ is the zero of multiplicity 3	2
•	Displays knowledge that if α is a zero of multiplicity 3 then $P''(\alpha) = 0$	<u> </u>

Sample answer:

$$P'(x) = 4x^{3} + 9x^{2} - 12x - 28$$

$$P''(x) = 12x^{2} + 18x - 12$$

$$= 6(x + 2)(2x - 1)$$

P''(x) = 0 when $x = -2, \frac{1}{2}$

Now $P'(\frac{1}{2}) \neq 0$ and P'(-2) = 0, therefore, x = -2 is the zero of multiplicity 3.

P(-2) = 0. Hence c = -24.

(d) (i) (1 mark)

Outcomes assessed: E2
Targeted Performance Bands: E2

•			
Correct solution	- The state of the	Criteria	2
	-	Mark	

Sample answer:

$$f\left(\frac{1}{x}\right) = \frac{2 \times \frac{1}{x}}{\left(\frac{1}{x}\right)^2 + 1}$$

$$=\frac{\left(\frac{1}{x}\right)}{\left(\frac{1+x^2}{x^2}\right)}$$

$$= \frac{2x}{1+x^2}$$
$$= f(x)$$

DISCLAMENT
DISCLAMENT
The information contained in this document is intended for the professional assistance of leaching staff. II does not constitute article to students, Further it is not the intention of CSSA to browde repeate marking outcomes for all postation Trial HSC assistance of the provide specific marking outcomes for all postation and HSC marking equipments, as established by the NSY Board of Studies.

No guaranties or warminy is made or minded with negotic bion or use of CSSA Marking Guidelines in relation to any specific tital exam question or answer. The CSSA is to guaranties or warminy is made or implied with negotic to the application or use of CSSA Marking Guidelines in relation to any specific tital exam question or answer. The CSSA assumes no liability or responsibility for the accuracy, completeness or usefulness of any Marking Guidelines provided for the Trial HSC papers.

(d) (ii) (2 marks)

Outcomes assessed: E2, E9

Targeted Performance Bands: E3-E4

Criteria	Marks
	2
Significant progress to correct answer	ш

Sample answer:

Let the coordinates of C be (x, 0).

DA = CB = f(x) (since ABCD is a square).

Therefore, using part (i), the coordinates of D are $(\frac{1}{x}, 0)$

Given ABCD is a square, CD = CB = f(x)

$$x - \frac{1}{x} = \frac{2x}{x^2 + 1}$$

 $x^4 - 2x^2 - 1 = 0$

$$x^2 = 1 + \sqrt{2} \text{ (since } x^2 > 0\text{)}$$

$$x = \sqrt{1 + \sqrt{2}} \text{ (since } x > 0)$$

 \therefore the coordinates of C are $(\sqrt{1+\sqrt{2}},0)$.

14

DISCLAIMER
The information onlined in this document is intended for the professional assistance of teaching staff, It does not constitute advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible Trial HSC answers. Rather the purpose is to provide teachers with information so that they can better explore, understand and apply HSC marking requirements, as established by the NSW Beact of Studies.

1830 marking requirements, as established by the NSW Beact of Studies.

1840 marking requirements, as established by the NSW Beact of Studies.

1850 marking requirements, as established by the NSW Beact of Studies.

1850 marking requirements, as established by the NSW Beact of Studies.

1850 marking requirements are studied by the NSW Beact of Studies.

1850 marking requirements are studied by the NSW Beact of Studies.

1850 marking requirements are studied by the NSW Beact of Studies.

1850 marking requirements are studied by the NSW Beact of Studies.

Question14 (15 marks)

(a) (i) (2 marks)

Outcomes assessed: E3

Targeted Performance Bands: E3

,	н
	ion of ar
i	SHOWS THE COLLECT LOCAL
s	Shows the correct result
	W
Marks	Criteria
;	

Sample answer

Let M and M' be the feet of the perpendiculars from P to the directrices $x=\frac{a}{e}$ and $x=-\frac{a}{e}$

$$PS = e PM$$

$$= e \left(\frac{a}{e} - a \cos \theta\right)$$

$$= a \left(1 - e \cos \theta\right)$$

$$PS' = e PM'$$

$$= e \left(a \cos \theta + \frac{a}{e}\right)$$

$$= a \left(1 + e \cos \theta\right)$$
Therefore,
$$\frac{PS}{PS'} = \frac{1 - e \cos \theta}{1 + e \cos \theta}$$

(a) (ii) (3 marks)

Outcomes assessed: E4

Targeted Performance Bands: E3

_	Critorio	Morte
Γ	Critera	Marks
٠	Shows the correct result	3
•	Significant progress to result	2
•	Finds the coordinates of G	1

Sample answer: The normal at P meets the x-axis at G.

When
$$y = 0$$
, $x = \frac{(a^2 - b^2)\cos\theta}{a}$

$$= \frac{a^2 e^2 \cos\theta}{a}$$

$$= a e^2 \cos\theta$$
as $b^2 = a^2 (1 - e^2)$

Therefore G has coordinates $(ae^2\cos\theta,0)$.

S has coordinates (ae, 0), and S has coordinates (-ae, 0)

$$GS = ae - ae^{2}\cos\theta$$
$$GS' = ae + ae^{2}\cos\theta$$

$$\frac{GS}{GS'} = \frac{ae - ae^2 \cos \theta}{ae + ae^2 \cos \theta}$$

$$= \frac{1 - e \cos \theta}{1 + e \cos \theta}$$

$$= \frac{PS}{PS'}$$
15

DISCLAIMER
The information contained in this document is intended for the professional assistance of teaching staff, it does not constitute palvec to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible Trial HSC acrowest. Bather the purpose is to provide toachers with information so that they can better explore, understand and apply HSC marking requirements, as established by the HSW Board of Studies.

HSC marking requirements, as established by the HSW Board of Studies.

The CSSA Marking Suidelines or relation to any specific trial eram question or answer. The CSSA so guarantee or warranty is made or impaid with respect to the application or use of CSSA Marking Suidelines provided for the Trial HSC papers.

(b) (i) (1 mark)

Outcomes assessed: E7

Targeted Performance Bands: E3

		_
Correctly shows the result	CITEFIA	
L	VIPTAT	101

Sample answer:

 $(\Delta GDE \text{ is similar to } \Delta GAB)$

 $(\Delta GEF \text{ is similar to } \Delta GBC)$

$$r = \left(\frac{h - y}{h}\right) \cdot R$$

(b) (ii) (3 marks)

Outcomes assessed: E7

Targeted Performance Bands: E3

,	, m 6	-
	Criteria	Marks
	Shows the correct result	ယ
	Makes substantial progress towards required result	2
— ₁	Establishes a correct expression for the volume of a slice	

Sample answer: Let the thickness of the slice be Δy

Volume of Slice =
$$\pi \left(\frac{h-y}{h}\right)^2 R^2 \Delta y$$

 \therefore Total Volume = $\lim_{\Delta y \to 0} \sum_{y=0}^{h} \pi \left(\frac{h-y}{h}\right)^2 R^2 \Delta y$
= $\pi \int_{0}^{h} \left(\frac{h-y}{h}\right)^2 R^2 dy$

 $= \frac{-\pi R^2}{3h^2} [(h-y)^3]_0^h$

 $=\frac{1}{3}\pi R^2 h$

Therefore, the volume of the oblique cone is given by $\frac{1}{3}\pi R^2 h$ units³

16

DISCLAMER

To ISCLAMER

The Information contained in this document is intended for the professional assistance of tranching staff, it does not constitute advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible intal HSC anawers. Rather the purpose is to provide seachers with information so that they can better explore, understand and apply HSC marking requirements, are stabilished by the NSW Board of Studies.

No gazarather or warranty is made or implied with respect to the application or use of CSSA, Marking Guidelines in relation to any specific trial exam question or answer. The CSSA was assumed to a suppossibility for the accuracy, completeness or usefulness of any Marking Guidelines provided for the Trial HSC papers.

(c) (i) (3 marks)

Outcomes assessed: E2, E9

Targeted Performance Bands: E3-E4

ľ	O O	
Į.	Criteria	Marks
•	Gives reason for concyclic points	3
•	Proves the required pair of angles are equal	2
•	Establishes 1 correct pair of equal angles leading to result	1

Sample answer:

Let
$$\angle BDT = x^{\circ}$$

 $\angle BCD = x^{\circ}$ (angle between tangent and chord equals angle in alternate segment)

 $\angle BRT = x^{\circ}$ (corresponding angles, PQ parallel to CD)

 $\angle BDT = \angle BRT$.

the interval BT. Therefore, B, T, D and R are concyclic points as $\angle BRT$ and $\angle BDT$ are equal angles subtended by

(c) (ii) (2 marks)

Outcomes assessed: E2, E9

Targeted Performance Bands: E2-E3

Progress towards required result	Correct proof	
		Criteria
_	2	Marks

Sample answer:

$$\angle TBD = \angle BCD = x^{\circ}$$
 (angle between tangent and chord equals angle in alternate segment)

$$\angle TBD = \angle DRT = x^{\circ}$$
 (angle in the same segment of the circle passing through B, T, D and R)

Therefore,
$$\angle BRT = \angle DRT = x^{\circ}$$

(c) (iii) (1 mark)

Outcomes assessed: E2, E9

Targeted Performance Bands: E2 – E3

Correctly shows the result	Criteria	
_	Mark	

Sample answer:

$$\angle CDR = \angle DRT = x^{\circ}$$
 (a

(alternate angles, PQ parallel to CD)

$$\angle DCR = \angle BCD = x^{\circ}$$
 (from part (i))

Therefore, ΔRCD is an isosceles triangle (two angles equal)

17

DISCLALLER

DISCLALLER

The information contained in this document is intended for the professional assistance of teaching staff, it does not constitute advice to students. Further it is not the intention of CSSA to provide specific making outcomes for all possible Tried HSC archivers. Rather the purpose is to provide teaches with information so that they can better explore, understand and apply HSC marking requirements, are established by the NSW Board of Studies.

No guarantee or warrantly is made or implied with insection to the accuracy, completeness or usefulness of any Marking Guidelines provided for the Triad HSC papers.

Question 15 (15 marks)
(a) (3 marks)
Outcomes assessed: E5
Targeted Performance R.

mde: F3-F4

Car	targetea x erjormance banas: 25-24	
l	Criteria	Marks
•	Shows the correct result	သ
•	Resolves forces correctly in TWO directions	2
•	Resolves forces correctly in ONE direction	1

Sample answer:

Resolving horizontally and vertically

$$F\cos\theta + N\sin\theta = \frac{mv^2}{r} \qquad \text{Eqn (i)}$$

$$N\cos\theta - mg - F\sin\theta = 0$$
 Eqn (ii)

From Eqn (i):
$$F\cos\theta = \frac{mv^2}{r} - N\sin\theta$$
 Eqn (ii)
From Eqn (ii): $F\sin\theta = N\cos\theta - mg$ Eqn (iv)

Eqn (iii)
$$\times \cos \theta$$
: $F \cos^2 \theta = \frac{mv^2}{r} \cos \theta - N \sin \theta \cos \theta$

Eqn (iv)
$$\times \sin \theta$$
: $F \sin^2 \theta = N \cos \theta \sin \theta - mg \sin \theta$

$$F(\cos^2\theta + \sin^2\theta) = \frac{mv^2}{r}\cos\theta - mg\sin\theta$$
$$F = \frac{mv^2}{r}\cos\theta - mg\sin\theta$$

(b) (i) (3 marks)

Outcomes assessed: E3, E4

Ta	Targeted Performance Bands: E3-E4	
٦	Criteria	Marks
•	Shows the correct result	3
•	Correctly equates TWO expressions	2
•	Correctly expands binomial product	_
ĺ		

Sample answer:

$$(\cos\theta + i\sin\theta)^{5}$$

$$= \cos^{5}\theta + 5i\cos^{4}\theta \sin\theta - 10\cos^{3}\theta \sin^{2}\theta - 10i\cos^{2}\theta \sin^{3}\theta + 5\cos\theta \sin^{4}\theta$$

$$+ i\sin^{5}\theta$$
By De Moivre:
$$(\cos\theta + i\sin\theta)^{5} = \cos 5\theta + i\sin 5\theta$$

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

 $\cos 5\theta = \cos^5 \theta - 10\cos^3 \theta (1 - \cos^2 \theta) + 5\cos \theta (1 - \cos^2 \theta)^2$

Equating real parts:

DISCUALLER

DISCUALLER

The information contained in this document is intended for the professional assistance of teaching steff. It does not constitute achieve to students, Further it is not the intention of CSSA to provide specific marking outcomes for all possible trial HSC answers. Rather the purpose is to provide trachers with information so that they can better explore, understand and apply HSC marking requirements, as established by the NSW Board of Stories.

No guarantees or warrantly is made or imposed with respect to the application or use of CSSA Marking Guidelines in relation to any specific trial exam question or arower. The CSSA long granutes or warrantly is made or imposed with respect to the application or usefulness or usefulness or usefulness of any Marking Guidelines provided for the Trial HSC papers.

8

(b) (ii) (2 marks)

Outcomes assessed: E3, E4

Targeted Performance Bands: E3-E4

	9	
	Criteria	Marks
•	hows correctly the required roots	2
•	Substantial progress towards the solution	1

Sample answer:

Let $x = \cos \theta$: $16x^5 - 20x^3 + 5x = 0 \implies 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta = 0$

$$\cos 5\theta = 0$$

$$5\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$$

$$\theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{2}, \frac{7\pi}{10}, \frac{9\pi}{10}$$

$$\therefore x = \cos \theta = \cos \frac{\pi}{10}, \cos \frac{3\pi}{10}, \cos \frac{\pi}{2}, \cos \frac{7\pi}{10}, \cos \frac{9\pi}{10}$$

$$= \cos \frac{\pi}{10}, \cos \frac{3\pi}{10}, \cos \frac{3\pi}{10}, -\cos \frac{7\pi}{10}, \cos \frac{9\pi}{10}$$

$$= \cos \frac{\pi}{10}, \cos \frac{3\pi}{10}, \cos \frac{3\pi}{10}, -\cos \frac{\pi}{10}$$
Hence the roots of the equation are $x = 0, \pm \cos \frac{\pi}{10}, \pm \cos \frac{3\pi}{10}$.

(b) (iii) (2 marks)

Outcomes assessed: E3, E4

Targeted Performance Bands: E3-E4 Shows correctly the required roots Criteria Marks 0

Sample answer:

Substantial progress towards the solution

$$\sum \alpha \beta = \frac{-20}{16}$$

$$= \frac{-5}{4}$$

$$\sum \alpha \beta = -\cos^2 \frac{\pi}{10} + \cos \frac{\pi}{10} \cdot \cos \frac{3\pi}{10} - \cos \frac{\pi}{10} \cdot \cos \frac{3\pi}{10} - \cos \frac{\pi}{10} \cos \frac{3\pi}{10} + \cos \frac{\pi}{10} \cos \frac{3\pi}{10} - \cos^2 \frac{3\pi}{10}$$

$$= -\left(\cos^2 \frac{\pi}{10} + \cos^2 \frac{3\pi}{10}\right)$$

Therefore
$$\left(\cos^2\frac{\pi}{10} + \cos^2\frac{3\pi}{10}\right) = \frac{5}{4}$$

61

DISCLAMER

DISCLAMER

This information contained in this document is intended for the professional assistance of teaching staff. It does not consider advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible trial HSC asswers. Rather the purpose is to provide teachers with information so that they can better explore, understand and apply HSC marking equirements, as established by the NSW Board of Studes.

No guarantee or warranty is made or implied with respect to the application or use of CSSA Marking Guidelines in relation to any specific trial exam question or answer. The CSSA assumes no flability or responsibility for the accuracy, completeness or usefulness of any Marking Guidelines provided for the Trial HSC papers.

(c) (i) (1 mark)

Outcomes assessed: E5

Targeted Performance Bands: E3

Shows the correct result	Criteria	
-	Marks	

Sample answer:

Resolving vertically: $T_1 \cos \theta - mg - T_2 \cos \theta = 0$ $T_1 \cos \theta - T_2 \cos \theta = mg$

$$T_1 - T_2)\cos\theta = mg$$

(c) (ii) (4 marks)

Outcomes assessed: E5

Targeted Performance Bands: E4	
Criteria	Marks
Correctly shows the required result	4
Significant progress towards the result	သ
 Attempts to solve simultaneously the correct equations 	2
Descrives forces horizontally	-

Sample answer:

Resolving horizontally: $T_1 \sin \theta + T_2 \sin \theta = mr\omega^2$

$$(T_1 + T_2)\sin\theta = mr\omega^2$$

Using part (i) result:

$$\frac{(T_1+T_2)\sin\theta}{(T_1-T_2)\cos\theta} = \frac{mr\omega^2}{mg}$$

 $r = h \tan \theta$

 $\frac{(T_1 + T_2) \tan \theta}{(T_1 - T_2)} = \frac{mh \tan \theta \omega^2}{m}$

 $(T_1 - T_2)$

mg

$$\therefore \frac{T_1 + T_2}{T_1 - T_2} = \frac{\hbar \omega^2}{g}$$
$$= \frac{h(\frac{4g}{\hbar})}{g} \operatorname{since} \omega = 2\sqrt{\frac{g}{\hbar}}$$

Hence,
$$\frac{T_1+T_2}{T_1-T_2} = 4$$

 $T_1 + T_2 = 4T_1 - 4T_2$
 $5T_2 = 3T_1$
 $\frac{T_1}{T_2} = \frac{5}{3}$
 $\therefore T_1 : T_2 = 5 : 3$

20

DISCLAMER

The Information contained in this document is intended for the professional assistance of teaching staff, it does not constained and the subvients. Further it is not the intention of CSSA to provide specific marking outcomes for all possible Trial HSC arismets. Rather the purpose is to provide teachers with information so that they can before explore, understand and apply HSC marking equirements, as established by the NSW Board of Studies.

14.9C marking equirements, as established by the NSW Board of Studies.

14.9C marking requirements, as established by the NSW Board of Studies.

14.9C marking equirements is made on the PSW Board of Studies.

14.9C marking equirements is made on the profession of the specific trial exam question or answer. The CSSA harking Guidelines in relation to any specific trial exam question or answer. The CSSA is suggested to the profession of the profession of the profession of the profession of the PSW and the PS

Question 16 (15 marks)
(a) (i) (1 mark)

Outcomes assessed: E2, E9

Targeted Performance Bands: E3-E4

The second contract of	Correctly shows the result	Criteria	
	1	Marks	

Sample answer:

$$\frac{Ae^{x} + Be^{-x}}{2} \ge \sqrt{Ae^{x} \cdot Be^{-x}} \qquad \left(\text{using } \frac{x+y}{2} \ge \sqrt{xy}\right)$$

$$Ae^{x} + Be^{-x} \ge 2\sqrt{Ae^{x} \cdot \frac{B}{e^{x}}}$$

$$Ae^x + Be^{-x} \ge 2\sqrt{AB}$$

: The minimum value of $f(x) = Ae^x + Be^{-x}$ is $2\sqrt{AB}$

(a) (ii) (2 marks)

Outcomes assessed: E2, E9

Targeted Performance Bands: E4

Γ	Criteria	Marks
•	Correct answers	2
•	Significant progress towards the solution	-

Sample answer:

value of f(x) which is $2\sqrt{AB}$ from part (i). If f(x) is symmetrical about x = c, then c must be the x-value corresponding to the minimum

:. The minimum value of f(x) occurs when x = c and is $2\sqrt{AB}$

$$f(c) = 2\sqrt{AB}$$

$$k[e^{c-c} + e^{-(c-c)}] = 2\sqrt{AB}$$

$$2k = 2\sqrt{AB}$$

$$\therefore k = \sqrt{AB}$$

Equality exists when $Ae^x = Be^{-x}$

$$\therefore e^{2x} = \frac{B}{A} \implies x = \frac{1}{2} \log_e \left(\frac{B}{A}\right)$$

$$\therefore c = \frac{1}{2} \log_e \left(\frac{B}{A}\right)$$

$$= \log_e \sqrt{\frac{B}{A}}$$

DISCLAMER
The internation of comment is intended for the professional assistance of teaching staff. If does not constitute advise to students. Further it is not the intention of CSSA to the intention contained in this document is intended for the profession assistance of teaching staff. If does not constitute advise to that they can better explore, understand and apply profess specific requirements, as established by the NSW Board of Studes.

No guarantee or warranty is made or impried with respect to the application or use of CSSA Marking Guidelines in relation to sor specific bial exam question or answer. The CSSA locations are provided for the Trial HSC papers.

(b) (i) (1 Mark)

Outcomes assessed: E2

Targeted Performance Bands: E2

Correctly shows the result 1	•		1 111 7 6
Mark 1		eria	
	-	Mark	

Sample answer:

LHS =
$$\frac{x^{2n-1}-x^{2n+1}}{\sqrt{1-x^2}}$$

$$=\frac{x^{2n-1}(1-x^2)}{\sqrt{1-x^2}}$$

$$= \chi^{2n-1}\sqrt{1-\chi^2}$$

= RHS

(b) (ii) (3 Marks)

Outcomes assessed: E8

Targeted Performance Bands: E4

1	I W Select I el la minusa manage ma	***************************************
	Criteria	Marks
•	Correctly shows the result	ယ
•	Significant progress towards the result	2
•	Attempts to apply integration by parts or attempts to use the results from part (i)	-

Sample Answer:

$$\begin{split} I_{2n+1} &= \int_0^1 \frac{x^{2n+1}}{\sqrt{1-x^2}} dx \\ &= \int_0^1 x^{2n} \times x (1-x^2)^{-\frac{1}{2}} dx \\ &= \left[-x^{2n} \sqrt{1-x^2} \right]_0^1 + 2n \int_0^1 x^{2n-1} \sqrt{1-x^2} dx \\ &= 2n \int_0^1 \left(\frac{x^{2n-1}}{\sqrt{1-x^2}} - \frac{x^{2n+1}}{\sqrt{1-x^2}} \right) dx \\ &= 2n [I_{2n-1} - I_{2n+1}] \end{split}$$

$$l_{2n+1}(1+2n) = 2n l_{2n-1}$$

$$\vdots l_{-n} = \frac{2n}{n-1} l_{-n}$$

$$\therefore l_{2n+1} = \frac{2n}{2n+1} l_{2n-1}$$

23

INSICIALHER

INSICIALHER

The information continued in this document is intended for the professional assistance of leaching staff, it does not constitute advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible. This HSC analysis provides provide toucheds with information so that they can better explore, understand and apply HSC marking qualifornities, as established by the NSFR Board of Studes.

No guaranties or wastering is made or implied with respect to the application or use of CSSA. Marking Guidelines in relation to any specific trial exam question or answer. The CSSA to guaranties or wastering is made or tampled with respect to the application or usefulness of usefulness of usefulness for usefulness of usefulness for usefulness of usefulness for usefulness of safety. Marking Guidelines provided for the Trial HSC papers.

(b) (iii) (2 Marks)
Outcomes assessed: E2
Targeted Performance Bands: E3-E4

• Correct solution 2	Tal Secon Leaf of manage Samues From France	Critorio	Marke
2		Criteria	
	 Correct solution 		2

Significant progress towards expression for I_{2n+1} .

Sample answer:

$$I_{2n+1} = \left(\frac{2n}{2n+1}\right)I_{2n-1} \text{ from (ii)}$$

$$= \left(\frac{2n}{2n+1}\right)\left(\frac{2n-2}{2n-1}\right)I_{2n-3}$$

$$= \left(\frac{2n}{2n+1}\right)\left(\frac{2n-2}{2n-1}\right) \cdots \frac{2}{3} \cdot I_{1}$$

$$I_{1} = \int_{0}^{1} \frac{x}{\sqrt{1-x^{2}}} dx$$

$$= \left[-\sqrt{1-x^{2}}\right]_{0}^{1}$$

$$= 1$$

$$\therefore I_{2n+1} = \left(\frac{2n}{2n+1}\right)\left(\frac{2n-2}{2n-1}\right) \cdots \frac{2}{3}$$

$$= \frac{2^{n}n!}{1\times 3\times 5\times ... \times (2n-1)\times (2n+1)}$$

23

(b) (iv) (2 Marks)

Outcomes assessed: E2, E9

Targeted Performance Bands: E4

The state of the s	 Significant progress towards result 	Correctly shows the result	Criteria
	1	2	Marks

Sample answer:

$$\int_{0}^{1} \frac{x}{\sqrt{1-x^{2}}} dx + \int_{0}^{1} \left[\sum_{n=1}^{\infty} \left(C_{n} \frac{x^{2n+1}}{\sqrt{1-x^{2}}} \right) \right] dx = 1 + C_{1}l_{3} + C_{2}l_{5} + \dots + C_{n}l_{2n+1} + \dots$$

$$Now \ C_{n}l_{2n+1} = \frac{1 \times 3 \times 5 \times \dots \times (2n-1)}{(2n+1)^{2}n_{n!}} \times \frac{2^{n}n!}{1 \times 3 \times 5 \times \dots \times (2n+1)}$$

$$= \frac{1}{(2n+1)^{2}}$$

$$\therefore LHS = 1 + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^{2}}$$

$$= \sum_{n=0}^{\infty} \frac{1}{(2n+1)^{2}}$$

24

O ISCLAMER

The information contained in this document is benefied for the professional assistance of texching staff, it does not constitute advice to students. Further it is not the intention of CSSA to provide prodict making outcomes for all possion Trial HSC answers, faither the purpose is to provide teachers with information so that they can better explore, understand and apply HSC marking requirement, as established by the NSW popur of Studies. It is a contained to a smaller is made or implied with respect to the application or use of CSSA. Marking Guidelines in relation to any specific trial exam question or asswer. The CSSA to guidelines for relation to any specific trial exam question or asswer. The CSSA is summary to relation to the first trial HSC papers.

(b) (v) (2 Marks)
Outcomes assessed: E2, E9 (2 Marks)

Targeted Performance Bands: E4

•	•	2
Progress towards correct result	Correct answer to the limiting sum	Criteria
ı	2	Marks

Sample answer:

$$\sin^{-1} x = x + \sum_{n=1}^{\infty} C_n x^{2n+1}$$

$$\int_{0}^{1} \frac{\sin^{-1} x}{\sqrt{1 - x^{2}}} dx = \int_{0}^{1} \frac{x + \sum_{n=1}^{\infty} C_{n} x^{2n+1}}{\sqrt{1 - x^{2}}} dx$$

$$= \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$$

$$=1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\dots$$

$$= 1 + \frac{1}{3^2} + \frac{1}{8^2} + \frac{1}{7^2} + \dots$$

$$\text{Now } \int_0^1 \frac{\sin^{-1}x}{\sqrt{1 - x^2}} dx = \left[\frac{1}{2} (\sin^{-1}x)^2 \right]_0^1$$

$$= \frac{\pi^2}{8}$$

$$\therefore 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$$

25

DISCLAMENT
DISCLAMENT
The information contained in this document is intended for the professional assistance of teaching steff. It does not constitute advice to students. Further it is not the intendent of CSSA to The information containing outcomes, for all possible Trial HISC carding equitements, as established by the HISM Board of Studies.

HISC marking equitements, as established by the HISM Board of Studies.

No guaranties or warranty is made or implied with respect to the application or use of CSSA, Musking Guidelines in relation to any specific trial exam question or answer. The CSSA is assumed no liability or responsibility for the accuracy, completeness or usefulness of any Masking Guidelines provided for the Trial HISC papers.

(b) (vi) (2 Marks)
Outcomes assessed: E2, E9 (2 Marks) Targeted Performance Bands: E4 Criteria

Sample answer:

Progress towards correct result

Correct answer

Marks 2

Let
$$S = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$S = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$= 1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \dots$$

$$= 1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots + \frac{1}{2^2} \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots \right)$$

$$\therefore S = \frac{\pi^2}{8} + \frac{1}{4} S$$

$$\frac{3}{4} S = \frac{\pi^2}{8}$$

: S =

6 72

26

In ISCCLALED.

The information contained in this document is intended for the professional assistance of teaching staff, it does not constitute advice to suidents. Further it is not the intention of CSSA to provide specific marking outcomes for all possible Trial HSC answers. Raiher the purpose is to provide teachers with information so that they can better explore, understand and apply 185C marking outcomest, as established by the NSW Board of Studies.

185C marking outcomest, as established by the NSW Board of Studies.

186C marking outcomest, as established by the NSW Board of Studies.

186C marking outcomest, as established by the NSW Board of Studies.

186C marking outcomest is easier intended with respect to the application or use of CSSA. Marking Guidelines in staffon to any specific trial exam question or answer. The CSSA assumes no liability or responsibility for the accuracy, completeness or usefulness of any Marking Guidelines provided for the Trial HSC papers.

BLANK PAGE

DISCLAMER

DISCLAMER

The information contained in this document is manded for the professional assistance of teaching stelf II does not constitute advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible Trial HSC arowers. Rother the purpose is to provide teachers with information so that they can better explore, understand and apply HSC marking equatements, as extablished by the NSM Board of Studies.

No guaranties or warranty is made or implied with inseptic to the application or use of CSSA. Marking Guidelinas in relation to any specific bild exam question or answer. The CSSA assumes no liability or responsibility for the accuracy, completeness or usefulness of any Marking Guidelinas provided for the Trial HSC papers.