

Student name/number: _____



2002
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

Extension 1

Total marks (84)

- Attempt Questions 1 – 7
- All questions are of equal value

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on last page
- All necessary working should be shown in every question

Please note that this is a Trial paper only and cannot in any way guarantee the format or the content of the Higher School Certificate Examination.

QUESTION 1

- a) Use the table of Standard Integrals to find:

$$\int \frac{1}{t^2 + 25} dt \quad (1)$$

- b) The roots of a monic polynomial $P(x)$ of degree 3 are 2, -2 and 1. Find an equation of the polynomial (1)

- c) Differentiate $x^2 \sin^{-1} 4x$ (3)

- d) Use the substitution $u = 4 - x^2$ to find $\int 2x\sqrt{4 - x^2} dx$ (3)

- e) Solve the equation $2\sin^2\theta = \sin 2\theta$ for $0 \leq \theta \leq 2\pi$ (4)

QUESTION 2 (Please start a new page)

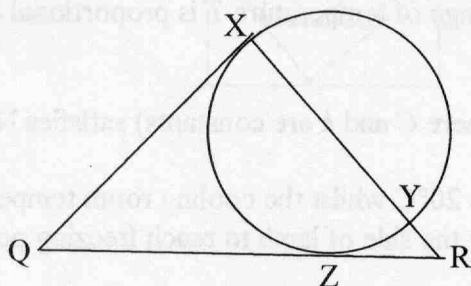
- a) Is $x - 2$ a factor of $x^3 - 5x + 12$? Give a reason for your answer. (1)

- b) Solve $\frac{1}{x} + \frac{x}{4} \leq 1$ (3)

- c) Solve the equation $\sqrt{3} \cos \theta - \sin \theta = \sqrt{3}$, $0 \leq \theta \leq 2\pi$ (4)

- d) XY is a diameter of a circle XYZ . The tangents at X and Z meet at Q . The lines QZ and XY are produced to meet at R .

(Draw this diagram onto your answer sheet)



- (i) Prove $\angle ZXQ = 90^\circ - \angle YZR$ (2)
(ii) Hence, or otherwise, prove that $\angle XQZ = 2\angle YZR$ (2)

QUESTION 3 (Please start a new page)

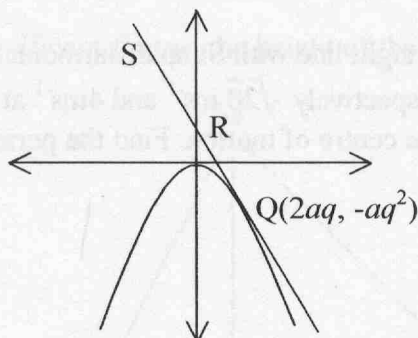
- a) i.) Prove that a root of the equation $x^3 - 6x + 1 = 0$ lies between $x = 2$ and $x = 3$. (1)
- ii) By taking a first approximation $x_1 = 2.5$ use one application of Newton's method to find a better approximation of the root. (3)
- b) Three boys and five girls are at a birthday party.
- i) The children are asked to form a queue to collect some food. In how many ways can the queue be formed? (1)
- ii) After eating the children are asked to sit in a circle for the party games. In how many ways can the children be seated around the circle? (1)
- iii) For the 'Pass the Parcel' game the children remain in a circle, but two of the boys are asked not to sit together. In how many ways may this occur? (2)
- c) For the expansion $(4 + 5x)^{17}$:
- i) Show the ratio of the coefficients of consecutive terms T_{r+1} and T_r is $\frac{5(18-r)}{4r}$. (2)
- ii) Determine which pair of terms has equal coefficients. (2)

QUESTION 4 (Please start a new page)

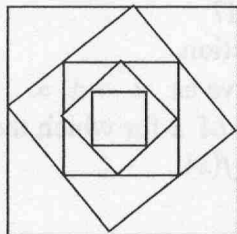
- a) A side of lamb at room temperature T is placed in a cooling room at a time t where A is the constant temperature of the cooling room. Newton's Law of Cooling states that the rate of change of temperature T is proportional to $(T - A)$
- i) Show that $T = A + Ce^{kt}$ (where C and k are constants) satisfies Newton's Law of Cooling. (1)
- ii) The temperature of the room is 20°C whilst the cooling room temperature is -5°C . How long will it take for the side of lamb to reach freezing point (0°) if it drops to 10° in 3 hours? (2)

QUESTION 4 CONTINUED

- b) The point $Q(2aq, -aq^2)$ is a variable point on the parabola $x^2 = -4ay$. The tangent at Q meets the y axis at R . The point S lies on the tangent and divides QR externally in the ratio 3:1.



- i) Show that the equation of the tangent at Q is $qx + y = aq^2$ (2)
 - ii) Find the coordinates of the points R . (1)
 - iii) Show the coordinates of S are $(-aq, 2aq^2)$ (2)
 - iv) Show that the locus of S is a parabola. (1)
- c) The midpoints of a square side $4a$ are joined to give a second square. The midpoints of the sides of this square are joined to give a third square and this process is repeated indefinitely. Calculate the limit of the sum of the areas of all the squares. (3)



QUESTION 5 (Please start a new page)

- a) Prove by induction that for a positive integer

$$\frac{1}{1 \times 5} + \frac{1}{5 \times 9} + \dots + \frac{1}{(4n-3)(4n+1)} = \frac{n}{4n+1} \quad (4)$$

- b) A particle is moving in a straight line with Simple Harmonic Motion. The velocity of the particle is respectively $\sqrt{20} \text{ ms}^{-1}$ and 4 ms^{-1} at distances of 1 metre and 2 metres from the centre of motion. Find the period and amplitude of the motion. (4)

- c) Evaluate $\int_0^{\frac{3}{4}} \frac{dx}{\sqrt{3-4x^2}}$ (4)

QUESTION 6 (Please start a new page)

- a) A spherical hot air balloon is heated so that its radius is expanding at the rate of 0.4 metres per second. At what rate will the volume be increasing when the radius is 3.4 metres? (3)

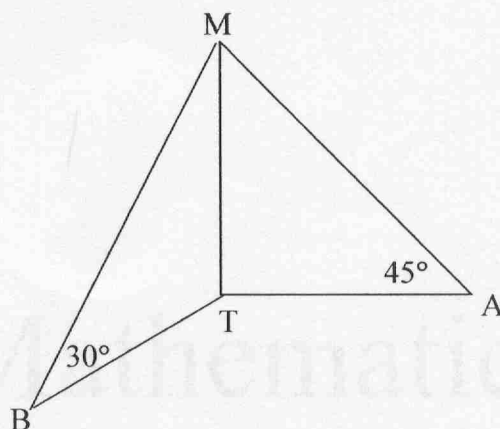
- b) Let $f(x) = \frac{x^2 - 4}{x - 2}$
- For what values is $f(x)$ undefined? (1)
 - Show that $y = f(x)$ is an odd function. (1)
 - Discuss the behaviour of the curve as $x \rightarrow \pm \infty$ (2)
 - Show that $f'(x) > 0$ at all values of x for which the function is defined. (2)
 - Hence, or otherwise, sketch $y = f(x)$. (3)

QUESTION 7 (Please start a new page)

- a) The angle of elevation of a mobile phone tower MT of height h metres at a point A due east is 45° . From another point B , the bearing of the mobile phone tower is $061^\circ T$ and the angle of elevation is 30° . The points A and B are 20 metres apart.

Calculate to 2 significant figures the height of the tower.

(5)



- b) An arrow fired from ground level at velocity 40 m/sec, strikes the ground 80 metres away.

Derive the equations of motion and hence find the angle at which the arrow was be fired. (Assume $g = 10\text{m/sec}^2$)

(7)