

**2014**  
**TRIAL HIGHER SCHOOL CERTIFICATE**  
**EXAMINATION**

# Mathematics Extension 1

Morning Session  
Friday 8 August 2014

## EXAMINERS

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### General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen  
Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided on a SEPARATE sheet
- In Questions 11–14, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and the Student Number at the top of this page

### Total marks – 70

#### Section I Pages 2–5

#### 10 marks

- Attempt Questions 1–10
- Allow 15 minutes for this section

#### Section II Pages 6–10

#### 60 marks

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

### Disclaimer

Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, *Principles for Setting HSC Examinations in a Standards-Referenced Framework* (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and *Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework* (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

## Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

- 1 The point  $A$  has coordinates  $(-6, 4)$  and the point  $B$  has coordinates  $(5, 1)$ .

Find the coordinates of the point which divides  $AB$  internally in the ratio  $3:4$ .

(A)  $(-39, 13)$

(B)  $\left(-\frac{12}{7}, \frac{23}{7}\right)$

(C)  $\left(-\frac{9}{7}, \frac{19}{7}\right)$

(D)  $\left(\frac{2}{7}, \frac{16}{7}\right)$

- 2 What is the remainder when the polynomial  $P(x) = 5x^3 - 17x^2 - x + 11$  is divided by  $x - 2$ ?

(A)  $-147$

(B)  $-95$

(C)  $-19$

(D)  $11$

- 3 A function is represented by the parametric equations

$$x = 2t + 1$$

$$y = t - 2.$$

Find the Cartesian equation of the function.

(A)  $x - 2y + 3 = 0$

(B)  $x - 2y - 3 = 0$

(C)  $x + 2y + 5 = 0$

(D)  $x - 2y - 5 = 0$

- 4 What is the solution to the inequality  $\frac{2}{3-x} < 1$ ?

(A)  $x < 1$

(B)  $x > 1$

(C)  $1 < x < 3$

(D)  $x < 1$  or  $x > 3$

- 5 Find  $\int \frac{dx}{1+9x^2}$ .

(A)  $\tan^{-1} 3x + C$

(B)  $\frac{1}{3} \tan^{-1} 3x + C$

(C)  $\frac{1}{9} \tan^{-1} 3x + C$

(D)  $\frac{1}{27} \tan^{-1} 3x + C$

- 6 A particle is moving in Simple Harmonic Motion and its displacement,  $x$  units, at time  $t$  seconds is given by the equation

$$x = A \cos(nt) + 2.$$

The period of the motion is  $4\pi$  seconds and the particle is initially at rest, 12 units to the right of the origin.

Find the values of  $A$  and  $n$ .

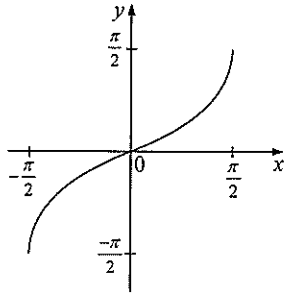
(A)  $A = 10, n = \frac{1}{2}$

(B)  $A = 10, n = 2$

(C)  $A = 12, n = \frac{1}{2}$

(D)  $A = 12, n = 2$

7



Which of the following could be the equation of the graph shown above?

- (A)  $y = \sin^{-1} \frac{\pi x}{2}$
- (B)  $y = \sin^{-1} \frac{2x}{\pi}$
- (C)  $y = \frac{\pi}{2} \sin^{-1} x$
- (D)  $y = \frac{2}{\pi} \sin^{-1} x$

- 8 A Mathematics department consists of 5 female and 5 male teachers.

How many committees of 3 teachers can be chosen which contain at least one female and at least one male?

- (A) 100
- (B) 120
- (C) 200
- (D) 2500

9

A particle is acted on by short bursts of radiation. After each short burst the particle moves forward one unit, backward one unit or does not move.

The probability that the particle moves forward one unit after a single short burst is  $p$  where  $0 < p < \frac{1}{2}$ . The probability that the particle moves backward one unit after a single short burst is also  $p$ .

What is the probability that after two short bursts the particle is at its starting point?

- (A)  $p^2$
- (B)  $2p^2$
- (C)  $5p^2 - 4p + 1$
- (D)  $6p^2 - 4p + 1$

- 10 What is the term independent of  $x$  in the expansion of  $(1+2x)^2 \left(2x + \frac{1}{x}\right)^{12}$ ?

- (A)  $\binom{12}{6} \times 2^6$
- (B)  $\binom{12}{6} \times 2^6 + \binom{12}{7} \times 2^6$
- (C)  $\binom{12}{6} \times 2^6 + \binom{12}{7} \times 2^7$
- (D)  $\binom{12}{6} \times 2^6 + \binom{12}{7} \times 2^9$

## Section II

60 marks

Attempt Questions 11–14

Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

**Question 11** (15 marks) Use a SEPARATE writing booklet.

(a) Use the substitution  $u = 1 + \ln x$  to evaluate  $\int_1^e \frac{1}{x(1+\ln x)^5} dx$ . 3

(b) Find  $\int 4 \sin^2 x dx$ . 2

- (c) Newton's law of cooling states that the rate at which an object loses heat is proportional to the difference in temperatures between the object and its surroundings. That is,

$$\frac{dT}{dt} = k(T - S)$$

where  $T$  is the temperature of the object in degrees Celsius after  $t$  minutes,  $k$  is a constant and  $S$  is the temperature of the surroundings in degrees Celsius.

(i) Verify that  $T = S + Ae^{kt}$  satisfies the equation  $\frac{dT}{dt} = k(T - S)$ . 1

(ii) A liquid is cooling in a room which has a constant temperature of  $20^\circ\text{C}$ . The initial temperature of the liquid is  $80^\circ\text{C}$  and it cools to  $50^\circ\text{C}$  after 15 minutes. 3

State the value of  $S$  and find the values of  $A$  and  $k$ .

(iii) Draw a clearly labelled graph showing the temperature of the liquid,  $T$ , against time,  $t$ . 2

(d) (i) Find  $\frac{d}{dx} \cos^{-1} \left( \frac{x-10}{10} \right)$ . 2

(ii) Hence, or otherwise, evaluate  $\int_5^{10} \frac{1}{\sqrt{20x-x^2}} dx$ . 2

**Question 12** (15 marks) Use a SEPARATE writing booklet.

(a) Let  $\alpha$ ,  $\beta$  and  $\gamma$  be the roots of the equation  $x^3 - 3x^2 - 6x - 1 = 0$ .

(i) Find  $2\alpha + 2\beta + 2\gamma$ . 1

(ii) Find  $\alpha^2 + \beta^2 + \gamma^2$ . 2

(b) The function  $f(x) = e^{-x} - x$  has a zero near  $x = 0.5$ . 3

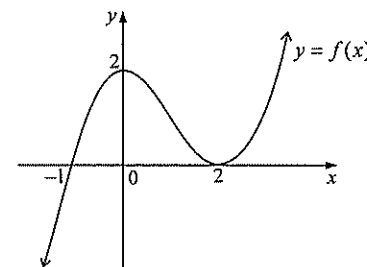
Use one application of Newton's method to obtain another approximation to this zero. Give your answer correct to two decimal places.

(c) A standard die with faces numbered 1 to 6 is tossed 7 times. Find, correct to 3 decimal places,

(i) the probability that exactly 5 sixes are tossed, 2

(ii) the probability that exactly three sixes are tossed, with the third six occurring on the seventh toss. 2

(d) The diagram below shows a sketch of the cubic polynomial function  $y = f(x)$ .



The graph intersects the  $y$ -axis at  $y = 2$  and the  $x$ -axis at  $x = -1$  and  $x = 2$ . The point  $(2, 0)$  is a stationary point.

(i) Find an expression for the cubic polynomial  $f(x)$ . 2

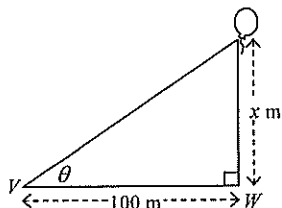
(ii) Let  $g(x) = f(x)$  for  $x \leq 0$ . Sketch a clearly labelled graph of the inverse function  $g^{-1}(x)$ . 1

(iii) Find the gradient of the inverse function  $y = g^{-1}(x)$  at the point where the curve intersects the  $y$ -axis. 2

**Question 13** (15 marks) Use a SEPARATE writing booklet.

- (a) Victor is standing at a point  $V$  watching a weather balloon being released from a point  $W$  which is 100 metres away on the horizontal ground. The weather balloon rises vertically at a constant velocity of  $5 \text{ ms}^{-1}$ .

Let  $\theta$  radians be the angle of elevation of the weather balloon at time  $t$  seconds and let  $x$  metres be the distance the weather balloon has travelled in that time.



Find the rate of change of the angle of elevation of the weather balloon, when  $\theta = \frac{\pi}{4}$ .

- (b) Given that  $\cos \theta = \frac{1}{5}$  and  $0 < \theta < \frac{\pi}{2}$ , find the exact value of  $\tan \frac{\theta}{2}$ .

- (c) (i) Use the binomial theorem to obtain an expression for  $\frac{(1+x)^n}{x}$ .

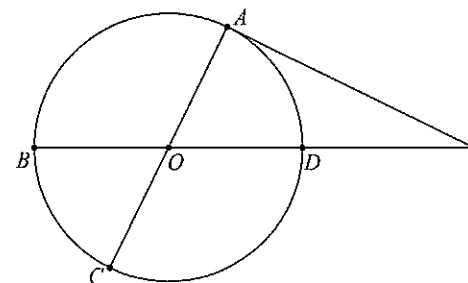
- (ii) By differentiating both sides of the identity obtained in part (i), show that for  $n \geq 2$ ,

$$\binom{n}{2} - 2\binom{n}{3} + 3\binom{n}{4} - 4\binom{n}{5} + \dots + (-1)^n (n-1)\binom{n}{n} = 1.$$

Question 13 continues on page 9

**Question 13** (continued)

- (d) The diagram below shows the diameters  $AC$  and  $BD$  of a circle with centre  $O$ . The tangent to the circle at  $A$  meets  $BD$  produced at  $E$ .



Prove that  $\angle OBC = \angle AED + \angle BAO$ .

- (e) Prove by mathematical induction that, for all positive integers  $n$ ,

$$\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{2n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2n-1} - \frac{1}{2n}.$$

**End of Question 13**

**Question 14** (15 marks) Use a SEPARATE writing booklet.

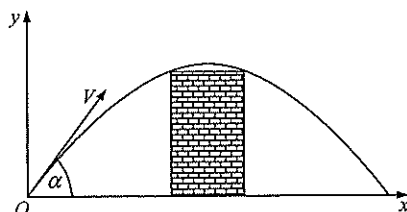
(a) Find the general solution to the equation  $\cos 2\theta + \cos \theta + 1 = 0$ . 3

(b) A particle is moving along the  $x$ -axis, starting from a position 1 metre to the right of the origin (that is,  $x = 1$  when  $t = 0$ ) with an initial velocity of  $2 \text{ ms}^{-1}$  and an acceleration given by  $\ddot{x} = x$ .

(i) Prove that  $v = \sqrt{x^2 + 3}$ , where  $v$  is the velocity of the particle. 2

(ii) Using part (i) and the standard integrals given, find an expression for  $x$  in terms of  $t$ . 3

(c) A projectile is fired from the origin towards the wall of a fort with initial velocity  $V \text{ ms}^{-1}$  at an angle  $\alpha$  to the horizontal.



On its ascent, the projectile just clears one edge of the wall and on its descent it just clears the other edge of the wall, as shown in the diagram.

The equations of motion of the projectile are

$$x = Vt \cos \alpha \text{ and } y = Vt \sin \alpha - \frac{g}{2}t^2. \text{ (Do NOT prove this.)}$$

(i) Show that the horizontal range  $R$  of the projectile is  $\frac{V^2 \sin 2\alpha}{g}$ . 2

(ii) Hence show that the equation of the path of the projectile is  $y = x \left( 1 - \frac{x}{R} \right) \tan \alpha$ . 2

(iii) The projectile is fired at  $45^\circ$  and the wall of the fort is 10 metres high. Show that the  $x$  coordinates of the edges of the wall are the roots of the equation 1

$$x^2 - Rx + 10R = 0.$$

(iv) If the wall of the fort is 4.5 metres thick, find the value of  $R$ . 2

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**End of Paper**









CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW  
2014 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION  
MATHEMATICS EXTENSION 1 – MARKING GUIDELINES

Section I  
10 marks

Questions 1-10 (1 mark each)

Question 1 (1 mark)  
Outcomes Assessed: PE2  
Targeted Performance Bands: E2

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Solution	Answer	Mark
The point dividing $A(-6, 4)$ to $B(5, 1)$ in the ratio 3:4 is $\left( \frac{4 \times -6 + 3 \times 5}{3 + 4}, \frac{4 \times 4 + 3 \times 1}{3 + 4} \right)$ $= \left( -\frac{9}{7}, \frac{19}{7} \right)$	C	1

Question 2 (1 mark)  
Outcomes Assessed: PE3  
Targeted Performance Bands: E2

Solution	Answer	Mark
The remainder when $P(x)$ is divided by $(x-2)$ is $P(2) = 5 \times 2^3 - 17 \times 2^2 - 2 + 11$ $= -19$	C	1

Question 3 (1 mark)  
Outcomes Assessed: PE3  
Targeted Performance Bands: E2

Solution	Answer	Mark
$t = y + 2$ $\therefore x = 2(y + 2) + 1$ $x - 2y - 5 = 0$	D	1

DISCLAIMER

The information contained in this document is intended for the professional assistance of teaching staff. It does not constitute advice to students. Further it is not the intention of CSSA to provide specific marking outcomes for all possible Trial HSC answers. Rather the purpose is to provide teachers with information so that they can better explore, understand and apply HSC marking requirements, as established by the NSW Board of Studies. No guarantee or warranty is made or implied with respect to the application or use of CSSA Marking Guidelines in relation to any specific trial exam question or answer. The CSSA assumes no liability or responsibility for the accuracy, completeness or usefulness of any Marking Guidelines provided for the Trial HSC papers.

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**Question 4 (1 mark)**

**Outcomes Assessed: PE3**

**Targeted Performance Bands: E2-E3**

Solution	Answer	Mark
$\frac{2}{3-x} < 1$ $2(3-x) < (3-x)^2$ $6-2x < 9-6x+x^2$ $x^2-4x+3 > 0$ $(x-3)(x-1) > 0$ $x < 1, x > 3$ $\therefore x < 1 \text{ or } x > 3$	D	1

**Question 5 (1 mark)**

**Outcomes Assessed: HE4**

**Targeted Performance Bands: E2-E3**

Solution	Answer	Mark
$\int \frac{dx}{1+9x^2} = \int \frac{dx}{1+(3x)^2}$ $= \frac{1}{3} \tan^{-1} 3x + C$	B	1

**Question 6 (1 mark)**

**Outcomes Assessed: HE3**

**Targeted Performance Bands: E3**

Solution	Answer	Mark
<p>The centre of the motion is <math>x = 2</math> and an extreme of the motion is <math>x = 12</math>. This gives the amplitude, <math>A = 10</math>.</p> <p>Given the period of the motion is <math>4\pi</math>, <math>\frac{2\pi}{n} = 4\pi</math>, <math>\therefore n = \frac{1}{2}</math>.</p>	A	1

**Question 7 (1 mark)**

**Outcomes Assessed: HE4**

**Targeted Performance Bands: E3**

Solution	Answer	Mark
$y = \sin^{-1}\left(\frac{2x}{\pi}\right)$ gives an inverse sine graph with a range $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ and a domain $-1 \leq \frac{2x}{\pi} \leq 1$ , i.e., $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ .	B	1

**Question 8 (1 mark)**

**Outcomes Assessed: PE3**

**Targeted Performance Bands: E3**

Solution	Answer	Mark
<p>If the committee of 3 is to contain at least one female and at least one male, it can either have 2 females and 1 male, or 1 female and 2 males.</p> <p>The number of ways of doing this is <math>\binom{5}{2}\binom{5}{1} + \binom{5}{1}\binom{5}{2} = 100</math>.</p>	A	1

**Question 9 (1 mark)**

**Outcomes Assessed: HE3, HE7**

**Targeted Performance Bands: E4**

Solution	Answer	Mark
<p>To be at its starting point after two short bursts, the particle could have:</p> <ul style="list-style-type: none"> <li>• moved forward one unit then backward one unit; this would have happened with probability <math>p \times p = p^2</math>.</li> <li>• moved backward one unit then forward one unit; this would have also happened with probability <math>p \times p = p^2</math>.</li> <li>• not moved for either of the short bursts; this would have happened with probability <math>(1-2p) \times (1-2p) = 1-4p+4p^2</math>.</li> </ul> <p>Therefore, the probability that after two short bursts the particle is at its starting point is <math>(p^2) + (p^2) + (1-4p+4p^2) = 6p^2 - 4p + 1</math>.</p>	D	1

**Question 10** (1 mark)

**Outcomes Assessed:** HE3

**Targeted Performance Bands:** E4

Solution	Answer	Mark
$(1+2x)^2 \left(2x + \frac{1}{x}\right)^{12}$ $= (1+4x+4x^2) \left( (2x)^{12} + \binom{12}{1}(2x)^{11}\left(\frac{1}{x}\right) + \dots + \binom{12}{6}(2x)^6\left(\frac{1}{x}\right)^6 + \right.$ $\left. \binom{12}{7}(2x)^5\left(\frac{1}{x}\right)^7 + \dots + \left(\frac{1}{x}\right)^{12} \right)$ $= (1+4x+4x^2) \left( 2^{12}x^{12} + \binom{12}{1}2^{11}x^{10} + \dots + \binom{12}{6}2^6 + \binom{12}{7}2^5\frac{1}{x^2} + \dots + \frac{1}{x^{12}} \right)$ <p>The term independent of <math>x</math> is <math>\binom{12}{6}2^6 + 4 \times \binom{12}{7}2^5 = \binom{12}{6}2^6 + \binom{12}{7}2^7</math></p>	C	1

**Section II**

**60 marks**

**Question 11** (15 marks)

(a) (3 Marks)

**Outcomes assessed:** HE6

**Targeted Performance Bands:** E2-E3

Criteria	Mark
• Correct solution	3
• Correct primitive, or equivalent merit	2
• Correct integrand in terms of $u$ , or equivalent merit	1

**Sample answer:**

$$u = 1 + \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{dx}{x}$$

$$x = 1 \Rightarrow u = 1$$

$$x = e \Rightarrow u = 2$$

$$\int_1^e \frac{dx}{x(1+\ln x)^5} = \int_1^2 \frac{du}{u^5}$$

$$= \left[ -\frac{1}{4u^4} \right]_1^2$$

$$= \left( -\frac{1}{64} \right) - \left( -\frac{1}{4} \right)$$

$$= \frac{15}{64}$$

(b) (2 Marks)

**Outcomes assessed:** HE6

**Targeted Performance Bands:** E3

Criteria	Mark
• Correct solution	2
• Correctly uses the identity $\cos 2x = 1 - 2\sin^2 x$ to simplify the integrand	1

**Sample answer:**

$$\cos 2x = 1 - 2\sin^2 x \Rightarrow 4\sin^2 x = 2 - 2\cos 2x$$

$$\int 4\sin^2 x \, dx = \int 2 - 2\cos 2x \, dx$$

$$= 2x - \sin 2x + C$$

Question 11 (continued)

(c) (i) (1 Mark)

Outcomes assessed: HE3

Targeted Performance Bands: E2-E3

Criteria	Mark
• Correct proof	1

Sample answer:

$$T = S + Ae^{kt}$$

$$\frac{dT}{dt} = kAe^{kt}$$

$$\frac{dT}{dt} = k(T - S), \text{ since } Ae^{kt} = T - S$$

(c) (ii) (3 Marks)

Outcomes assessed: HE3

Targeted Performance Bands: E2-E3

Criteria	Mark
• All three correct values for $S$ , $A$ and $k$	3
• Two correct values for $S$ , $A$ or $k$ , or equivalent merit	2
• One correct value for $S$ , $A$ or $k$ , or equivalent merit	1

Sample answer:

$$\text{Since } S \text{ is the surrounding temperature, } S = 20. \quad \therefore T = 20 + Ae^{kt}$$

$$\text{When } t = 0, T = 80$$

$$\therefore 80 = 20 + Ae^{k \times 0} \Rightarrow A = 60.$$

$$\therefore T = 20 + 60e^{kt}$$

$$\text{When } t = 15, T = 50$$

$$\therefore 50 = 20 + 60e^{k \times 15} \Rightarrow k = \frac{1}{15} \ln\left(\frac{30}{60}\right) = \frac{1}{15} \ln\left(\frac{1}{2}\right)$$

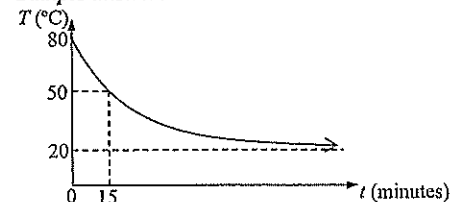
(c) (iii) (2 Marks)

Outcomes assessed: HE3, HE7

Targeted Performance Bands: E2-E3

Criteria	Mark
• Correct graph	2
• Correct general shape of graph showing the vertical intercept, or equivalent merit	1

Sample answer:



Question 11 (continued)

(d) (i) (2 Marks)

Outcomes assessed: HE4

Targeted Performance Bands: E3

Criteria	Mark
• Correct solution	2
• Makes some progress differentiating $\cos^{-1}\left(\frac{x-10}{10}\right)$	1

Sample answer:

$$\begin{aligned} \frac{d}{dx} \cos^{-1}\left(\frac{x-10}{10}\right) &= \frac{-1}{\sqrt{1 - \left(\frac{x-10}{10}\right)^2}} \times \frac{1}{10} \\ &= \frac{-1}{\sqrt{100 - (x-10)^2}} \\ &= \frac{-1}{\sqrt{20x - x^2}} \end{aligned}$$

(d) (ii) (2 Marks)

Outcomes assessed: HE4

Targeted Performance Bands: E3

Criteria	Mark
• Correct solution	2
• Correct primitive and some progress substituting limits	1

Sample answer:

$$\begin{aligned} \int_5^{10} \frac{dx}{\sqrt{20x - x^2}} &= \left[ -\cos^{-1}\left(\frac{x-10}{10}\right) \right]_5^{10} \\ &= -\cos^{-1}(0) - -\cos^{-1}\left(-\frac{1}{2}\right) \\ &= -\frac{\pi}{2} + \frac{2\pi}{3} \\ &= \frac{\pi}{6} \end{aligned}$$

**Question 12 (15 marks)**

(a) (i) (1 Mark)

**Outcomes assessed: PE3**

**Targeted Performance Bands: E2**

Criteria	Mark
• Correct answer	1

**Sample answer:**

$$2\alpha + 2\beta + 2\gamma = 2(\alpha + \beta + \gamma)$$

$$= 2\left(-\frac{b}{a}\right)$$

$$= 2\left(-\frac{-3}{1}\right)$$

$$= 6$$

(a) (ii) (2 Marks)

**Outcomes assessed: PE3**

**Targeted Performance Bands: E2-E3**

Criteria	Mark
• Correct answer	2
• Correct value of $\alpha\beta + \alpha\gamma + \beta\gamma$ and makes some progress	1

**Sample answer:**

$$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$$

$$= (3)^2 - 2(-6)$$

$$= 21$$

(b) (3 Marks)

**Outcomes assessed: HE7**

**Targeted Performance Bands: E3**

Criteria	Mark
• Correct solution	3
• Correctly applies Newton's method	2
• Correctly finds $f'(x)$ and attempts to apply Newton's method	1

**Sample answer:**

$$f(x) = e^{-x} - x, \quad f'(x) = -e^{-x} - 1$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.5 - \frac{e^{-0.5} - 0.5}{-e^{-0.5} - 1}$$

$$= 0.56631... \approx 0.57 \text{ (2 d.p.)}$$

**Question 12 (continued)**

(c) (i) (2 Marks)

**Outcomes assessed: HE3**

**Targeted Performance Bands: E2-E3**

Criteria	Mark
• Correct solution	2
• Makes some progress	1

**Sample answer:**

$$P(\text{tossing a six}) = \frac{1}{6}, \quad P(\text{not tossing a six}) = \frac{5}{6}$$

$$P(5 \text{ sixes in 7 tosses}) = \binom{7}{5} \left(\frac{1}{6}\right)^5 \left(\frac{5}{6}\right)^2$$

$$= 0.001875... \approx 0.002 \text{ (3 d.p.)}$$

(c) (ii) (2 Marks)

**Outcomes assessed: HE3**

**Targeted Performance Bands: E3-E4**

Criteria	Mark
• Correct solution	2
• Identifies $P(7^{\text{th}} \text{ toss gives the 3}^{\text{rd}} \text{ six}) = P(2 \text{ sixes in 6 tosses}) \times P(\text{tossing a six})$ , or equivalent merit	1

**Sample answer:**

$$P(7^{\text{th}} \text{ toss gives the 3}^{\text{rd}} \text{ six}) = P(2 \text{ sixes in 6 tosses}) \times P(\text{tossing a six})$$

$$= \binom{6}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^4 \times \left(\frac{1}{6}\right)$$

$$= 0.033489... \approx 0.033$$

(d) (i) (2 Marks)

**Outcomes assessed: PE3**

**Targeted Performance Bands: E2-E3**

Criteria	Mark
• Correct solution	2
• Recognises $(x+1)$ and $(x-2)^2$ are factors of $f(x)$ .	1

**Sample answer:**

$$f(x) = k(x+1)(x-2)^2$$

$$f(0) = 2 \Rightarrow 2 = k(0+1)(0-2)^2 \therefore k = \frac{1}{2}$$

$$\therefore f(x) = \frac{1}{2}(x+1)(x-2)^2$$

Question 12 (continued)

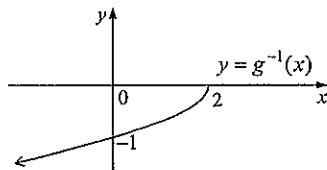
(d) (ii) (1 Mark)

Outcomes assessed: HE4

Targeted Performance Bands: E3

Criteria	Mark
• Correct graph	1

Sample answer:



(d) (iii) (2 Marks)

Outcomes assessed: HE4

Targeted Performance Bands: E3-E4

Criteria	Mark
• Correct solution	2
• Finds an expression for the inverse function and attempts to differentiate	1

Sample answer:

Inverse function  $y = g^{-1}(x)$  is given by

$$x = \frac{1}{2}(y+1)(y-2)^2$$

$$\frac{dx}{dy} = \frac{1}{2}(y+1) \times 2(y-2) + \frac{1}{2}(y-2)^2$$

$$= \frac{3y}{2}(y-2)$$

$$\frac{dy}{dx} = \frac{2}{3y(y-2)}$$

$\therefore$  gradient of the inverse function at the y-intercept,  $y = -1$ , is  $\frac{2}{3(-1)(-1-2)} = \frac{2}{9}$

Question 13 (15 marks)

(a) (3 Marks)

Outcomes assessed: HE5, HE7

Targeted Performance Bands: E3

Criteria	Mark
• Correct solution	3
• Correctly applies the chain rule to determine an expression for $\frac{d\theta}{dt}$	2
• Identifies $\tan \theta = \frac{x}{100}$ and makes progress differentiating	1

Sample answer:

The weather balloon rises at  $5\text{ms}^{-1} \Rightarrow \frac{dx}{dt} = 5$

$$\tan \theta = \frac{x}{100}$$

$$x = 100 \tan \theta$$

$$\frac{dx}{d\theta} = 100 \sec^2 \theta$$

$$\frac{d\theta}{dx} = \frac{\cos^2 \theta}{100}$$

Therefore,

$$\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt}$$

$$= \frac{\cos^2 \theta}{100} \times 5$$

$$= \frac{\cos^2 \frac{\pi}{4}}{100} \times 5$$

$$= \frac{1}{40}$$

$$= 0.025 \text{ radians per second}$$

Question 13 (continued)

(b) (2 Marks)

Outcomes assessed: PE2, HE7

Targeted Performance Bands: E3

Criteria	Mark
• Correct solution	2
• Correctly applies $t$ -formulae, or equivalent merit	1

Sample answer:

$$\text{Let } t = \tan \frac{\theta}{2}, \text{ then } \cos \theta = \frac{1-t^2}{1+t^2}.$$

$$\cos \theta = \frac{1}{5}$$

$$\frac{1-t^2}{1+t^2} = \frac{1}{5}$$

$$5-5t^2 = 1+t^2$$

$$6t^2 = 4$$

$$t^2 = \frac{2}{3}$$

$$t = \sqrt{\frac{2}{3}} \text{ (since } \theta \text{ is acute, } t = \tan \frac{\theta}{2} > 0)$$

$$\therefore \tan \frac{\theta}{2} = \sqrt{\frac{2}{3}}$$

(c) (i) (1 Mark)

Outcomes assessed: HE3

Targeted Performance Bands: E2

Criteria	Mark
• Correct expression	1

Sample answer:

$$\begin{aligned} \frac{(1+x)^n}{x} &= \frac{\binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \dots + \binom{n}{n}x^n}{x} \\ &= \binom{n}{0}\frac{1}{x} + \binom{n}{1} + \binom{n}{2}x + \binom{n}{3}x^2 + \dots + \binom{n}{n}x^{n-1} \end{aligned}$$

Question 13 (continued)

(c) (ii) (2 Marks)

Outcomes assessed: HE3

Targeted Performance Bands: E3-E4

Criteria	Mark
• Correct proof	2
• Substantial progress differentiating both sides of the identity	1

Sample answer:

Differentiating both sides of the identity in part (i), we get

$$\frac{x \times n(1+x)^{n-1} - (1+x)^n}{x^2} = \binom{n}{0}\frac{-1}{x^2} + \binom{n}{2} + 2\binom{n}{3}x + 3\binom{n}{4}x^2 + \dots + (n-1)\binom{n}{n}x^{n-2}$$

When  $x = -1$

$$0 = -\binom{n}{0} + \binom{n}{2} - 2\binom{n}{3} + 3\binom{n}{4} - \dots + (n-1)\binom{n}{n}(-1)^{n-2}$$

$$0 = -\binom{n}{0} + \binom{n}{2} - 2\binom{n}{3} + 3\binom{n}{4} - \dots + (n-1)\binom{n}{n}(-1)^n \quad (\text{since } (-1)^{n-2} = (-1)^n)$$

$$\binom{n}{2} - 2\binom{n}{3} + 3\binom{n}{4} - \dots + (-1)^n(n-1)\binom{n}{n} = 1 \quad (\text{since } \binom{n}{0} = 1)$$

(d) (3 Marks)

Outcomes assessed: PE2, PE3

Targeted Performance Bands: E3

Criteria	Mark
• Correct proof	3
• Substantial progress	2
• Applies a relevant theorem in circle geometry, leading towards a correct proof	1

Sample answer:

Let  $\angle AED = \alpha$  and  $\angle BAO = \beta$

$\angle OAE = 90^\circ$

$\angle ABE = 90^\circ - \alpha - \beta$

$\angle ABC = 90^\circ$

$\angle OBC = \angle ABC - \angle ABE$

$= \alpha + \beta$

$= \angle AED + \angle BAO$

(radii are perpendicular to tangents at their point of contact)

(angle sum of  $\triangle BAE$  is  $180^\circ$ )

(angle in a semicircle is  $90^\circ$ )

Question 13 (continued)

(e) (4 Marks)

Outcomes assessed: HE2

Targeted Performance Bands: E3-E4

Criteria	Mark
• Correct proof	4
• Applies the induction step and progress towards a complete proof	3
• Writes the correct expression for $n = k + 1$ and progress towards using the assumption	2
• Proves case for $n = 1$ , or equivalent merit	1

Sample answer:

Let  $P(n)$  be the given proposition.  $P(1)$  is true since  $RHS = 1 - \frac{1}{2} = \frac{1}{2} = LHS$ .

Assume  $P(k)$  is true for some positive integer  $k$ .

$$\text{i.e. } \frac{1}{k+1} + \frac{1}{k+2} + \frac{1}{k+3} + \dots + \frac{1}{2k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2k-1} - \frac{1}{2k}$$

Prove  $P(k+1)$  is true:

$$\begin{aligned} & \frac{1}{k+2} + \frac{1}{k+3} + \dots + \frac{1}{2k} + \frac{1}{2k+1} + \frac{1}{2k+2} \\ &= \left( \frac{1}{k+1} + \frac{1}{k+2} + \frac{1}{k+3} + \dots + \frac{1}{2k} \right) - \frac{1}{k+1} + \frac{1}{2k+1} + \frac{1}{2k+2} \\ &= \left( 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2k-1} - \frac{1}{2k} \right) + \frac{1}{2k+1} - \left( \frac{1}{k+1} - \frac{1}{2k+2} \right) \text{ (using the assumption)} \\ &= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2k-1} - \frac{1}{2k} + \frac{1}{2k+1} - \left( \frac{2-1}{2k+2} \right) \\ &= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2k-1} - \frac{1}{2k} + \frac{1}{2k+1} - \frac{1}{2k+2} \end{aligned}$$

Therefore, by the Principle of Mathematical Induction,  $P(n)$  is true for all positive integers  $n$ .

Question 14 (15 marks)

(a) (3 Marks)

Outcomes assessed: H5, HE7

Targeted Performance Bands: E3

Criteria	Mark
• Correct general solution for $\theta$	3
• Makes substantial progress finding solutions for $\theta$	2
• Uses the identity $\cos 2\theta = 2\cos^2 \theta - 1$ and finds solutions for $\cos \theta$ , or equivalent merit	1

Sample answer:

$$\cos 2\theta + \cos \theta + 1 = 0$$

$$2\cos^2 \theta - 1 + \cos \theta + 1 = 0$$

$$\cos \theta (2\cos \theta + 1) = 0$$

$$\cos \theta = 0, \cos \theta = -\frac{1}{2}$$

$$\theta = \pm \frac{\pi}{2} + 2\pi n \text{ or } \pm \frac{2\pi}{3} + 2\pi n, \text{ for all integers } n.$$

(b) (i) (2 Marks)

Outcomes assessed: HE3, HE5, HE7

Targeted Performance Bands: E3-E4

Criteria	Mark
• Correct proof	2
• Correct expression for $v^2$	1

Sample answer:

$$\ddot{x} = x$$

$$\frac{d}{dx} \left( \frac{1}{2} v^2 \right) = x$$

$$\frac{1}{2} v^2 = \frac{1}{2} x^2 + C$$

$$x = 1, v = 2 \Rightarrow C = \frac{3}{2}$$

$$\therefore v^2 = x^2 + 3$$

Since the particle has a positive displacement, and hence positive acceleration, initially (as well as a positive velocity) the particle will continue to move to the right. The particle's displacement will always be positive, and so its acceleration will always be positive and it will move with increasing velocity to the right.

$$v > 0 \Rightarrow v = \sqrt{x^2 + 3}$$

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Question 14 (continued)

(b) (ii) (3 Marks)

Outcomes assessed: HE3, HE5, HE7

Targeted Performance Bands: E3-E4

Criteria	Mark
• Correct solution	3
• Correctly finds the value of the constant and attempts to make $x$ the subject	2
• Correctly uses the standard integrals to find an expression for $t$ in terms of $x$ (with an unknown constant)	1

Sample answer:

$$\frac{dx}{dt} = \sqrt{x^2 + 3}$$

$$\frac{dt}{dx} = \frac{1}{\sqrt{x^2 + 3}}$$

$$t = \ln(x + \sqrt{x^2 + 3}) + C$$

$$t = 0, x = 1 \Rightarrow 0 = \ln(1 + \sqrt{1^2 + 3}) + C$$

$$\Rightarrow C = -\ln 3$$

$$t = \ln(x + \sqrt{x^2 + 3}) - \ln 3$$

$$t = \ln\left(\frac{x + \sqrt{x^2 + 3}}{3}\right)$$

$$3e^t = x + \sqrt{x^2 + 3}$$

$$3e^t - x = \sqrt{x^2 + 3}$$

$$9e^{2t} - 6xe^t + x^2 = x^2 + 3$$

$$6xe^t = 9e^{2t} - 3$$

$$x = \frac{9e^{2t} - 3}{6e^t}$$

$$x = \frac{3e^{2t} - 1}{2e^t}$$

$$x = \frac{3}{2}e^t - \frac{1}{2}e^{-t}$$

Question 14 (continued)

(c) (i) (2 Marks)

Outcomes assessed: HE3, HE7

Targeted Performance Bands: E3

Criteria	Mark
• Correct proof	2
• Finds an expression for the time of flight, or equivalent merit	1

Sample answer:

For the horizontal range  $R$ , first solve  $y = 0$  to find the time of flight

$$0 = t\left(V \sin \alpha - \frac{g}{2}t\right)$$

$$t = \frac{2V \sin \alpha}{g}, \text{ since } t \neq 0 \text{ for the horizontal range}$$

$$\text{When } t = \frac{2V \sin \alpha}{g},$$

$$x = V\left(\frac{2V \sin \alpha}{g}\right) \cos \alpha$$

$$= \frac{V^2 2 \sin \alpha \cos \alpha}{g}$$

$$= \frac{V^2 \sin 2\alpha}{g}$$

$$\text{Therefore, the horizontal range } R \text{ is } \frac{V^2 \sin 2\alpha}{g}.$$

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Question 14 (continued)

(c) (ii) (2 Marks)

**Outcomes assessed: HE3, HE7**

**Targeted Performance Bands: E3-E4**

Criteria	Mark
• Correct proof	2
• Eliminates the parameter $t$ from the equations of motion	1

**Sample answer:**

$$x = Vt \cos \alpha \Rightarrow t = \frac{x}{V \cos \alpha}$$

Substituting into  $y = Vt \sin \alpha - \frac{g}{2} t^2$  gives the equation of the path of the projectile:

$$y = V \left( \frac{x}{V \cos \alpha} \right) \sin \alpha - \frac{g}{2} \left( \frac{x}{V \cos \alpha} \right)^2$$

$$y = x \frac{\sin \alpha}{\cos \alpha} - x^2 \frac{\sin \alpha}{\cos \alpha} \left( \frac{g}{2V^2 \sin \alpha \cos \alpha} \right)$$

$$y = x \tan \alpha - x^2 \tan \alpha \left( \frac{1}{R} \right)$$

$$y = x \left( 1 - \frac{x}{R} \right) \tan \alpha$$

(c) (iii) (1 Mark)

**Outcomes assessed: HE3, HE7**

**Targeted Performance Bands: E3**

Criteria	Mark
• Correct solution	1

**Sample answer:**

$$\alpha = 45^\circ$$

For the  $x$  coordinates of the edge of the wall, solve  $y = 10$ :

$$10 = x \left( 1 - \frac{x}{R} \right) \times \tan 45^\circ$$

$$10R = x(R - x)$$

$$x^2 - Rx + 10R = 0$$

Question 14 (continued)

(c) (iv) (2 Marks)

**Outcomes assessed: HE3, HE7**

**Targeted Performance Bands: E4**

Criteria	Mark
• Correct solution	2
• Makes some progress using the result in part (iii) and the information given to find $R$	1

**Sample answer:**

Let  $x_1$  and  $x_2$  be the  $x$  coordinates of the edge of the wall, with  $x_1 > x_2$ .

Since  $x_1, x_2$  are the roots of the quadratic  $x^2 - Rx + 10R = 0$ , using the quadratic formula we have

$$x_1 = \frac{R + \sqrt{R^2 - 40R}}{2} \text{ and } x_2 = \frac{R - \sqrt{R^2 - 40R}}{2}.$$

Since the wall of the fort is 4.5 metres thick,  $x_1 - x_2 = 4.5$

$$\therefore \sqrt{R^2 - 40R} = 4.5$$

$$R^2 - 40R - 20 \frac{1}{4} = 0$$

$$4R^2 - 160R - 81 = 0$$

$$(2R + 1)(2R - 81) = 0$$

Since  $R$  is the horizontal range,  $R > 0$ ,  $\therefore R = 40.5$  metres