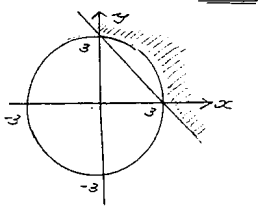
NSW INDEPENDENT TRIAL EXAMS
SUGGESTED SOLUTIONS
2000 MATHEMATICS 2UNIT SOLUTIONS.

Q1

(a) $0.111246314 \doteq \underline{0.111}$

(b)



(c) $\log_3\left((3^{3\frac{1}{4}})^{\frac{1}{2}}\right) = \underline{1\frac{3}{4}}$

(d) $\sqrt{2}-1 - \frac{1}{\sqrt{2}-1}$

$= \sqrt{2}-1 - \frac{(\sqrt{2}+1)}{2-1}$

$= \underline{-2}$

(e) $-4 \le 2x \le 8$

$\underline{-2 \le x \le 4}$

(f) $6x - 3y = 18 \quad \cdots (3)$

$x + 3y = 10 \quad \cdots (2)$

(3)+(2) $7x = 28$

$x = 4$

Sub in (1) $8 - y = 6$

$y = 2$

$\underline{(4,2)}$ solution

Q2

(a) (i) $\underline{x\sec^2 x + \tan x}$

(ii) $\frac{x\cdot\frac{1}{x} - \log_e x \cdot 1}{x^2}$

$= \underline{\frac{1 - \log_e x}{x^2}}$

(iii) $2(e^{-x} - e^{x})(-e^{-x} - e^{x})$

$= 2(-e^{-2x} - 1 + 1 + e^{2x})$

$= \underline{2(e^{2x} - e^{-2x})}$

(b) (i) 1988

(ii) Rate same but increasing 1980 and decreasing 1984

(c) (i) $D\hat{A}B = A\hat{B}C$ (alt $\angle$s $DA \parallel BC$)

$A\hat{B}C = A\hat{C}B$ (base $\angle$s of isos $\triangle ABC$)

$\therefore \underline{A\hat{C}B = 55^\circ}$

(ii) $C\hat{E}B = 90^\circ$ ($AB \perp DC$)

$A\hat{B}C + C\hat{E}B + B\hat{C}E = 180^\circ$ ($\angle$ sum of $\triangle BEC$)

$\therefore 55^\circ + 90^\circ + B\hat{C}E = 180^\circ$

$\therefore B\hat{C}E = 35^\circ$

$\therefore A\hat{C}E + 35^\circ = 55^\circ$

$\therefore \underline{A\hat{C}E = 20^\circ}$

Q3

(a) (i) $\int\left(\frac{1}{t} - 2t^{-2}\right)dt$

$= \ln t + 2t^{-1} + c$

$= \underline{\ln t + \frac{2}{t} + c}$

(ii) $\underline{-\frac{1}{2}\tan(1-2x) + c}$

(b) (i) $A: y = 0 \Rightarrow \sqrt{3}x + \sqrt{3} = 0$

$\underline{A(-1,0)}$

$B: x = 0 \Rightarrow -y + \sqrt{3} = 0$

$\underline{B(0,\sqrt{3})}$

(ii) $m = \underline{\sqrt{3}}$

(iii) $\tan B\hat{A}D = \sqrt{3}$

$\therefore \underline{B\hat{A}D = 60^\circ}$

(iv) $A = \frac{1}{2}ab\sin C$

$3\sqrt{6} = \frac{1}{2}\cdot AC \cdot 1 \cdot \frac{\sqrt{3}}{2}$

$\therefore AC = 12\sqrt{2}$

$AB = \sqrt{1^2 + (\sqrt{3})^2}$

$= 2$

$\therefore \underline{BC = 12\sqrt{2} - 2}$

(c) $\frac{2^n \times 2^{2n+2}}{2^{3n}} = \underline{2^2 = 4}$

Q4

(a) $x = 1 \quad y = e^2$

$y' = 2e^{2x}$

$m_1 = 2e^2$

$m_2 = -\frac{1}{2e^2}$

$\therefore$ Equation of normal is

$y - e^2 = -\frac{1}{2e^2}(x-1)$

$2e^2 y - 2e^4 = -x + 1$

$\underline{x - 2e^2 y - 2e^4 - 1 = 0}$

(b)

x	1	2	3
y	2	$2\sqrt{3}$	1

$\int_1^3 \operatorname{cosec}\frac{\pi x}{6}\,dx$

$\doteq \frac{3-1}{6}\left\{2 + 4\times\frac{2}{\sqrt{3}} + 1\right\}$

$\doteq \underline{1 + \frac{8\sqrt{3}}{9}} \doteq 2.54$

(c) (i) In $\triangle$s ABE, DCB

$AB = DC$ (sides of rhombus)

$BE = CB$ (data)

$A\hat{B}E = D\hat{C}B$ (alt $\angle$s $AB \parallel DC$)

$\therefore \underline{\triangle ABE \equiv \triangle DCB}$ (SAS Test)

(ii) $\therefore A\hat{E}B = D\hat{B}C$ (corr $\angle$s in cong $\triangle$s)

$\therefore AE \parallel DB$ (pair of corr $\angle$s equal)

(iii) Also $AE = DB$ (corr sides in cong $\triangle$s)

$\therefore \underline{AEBO}$ is a parm (pr of opp sides = and $\parallel$)

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Q5

(a) $y^2 - 6y + 9 = 4x + 18$

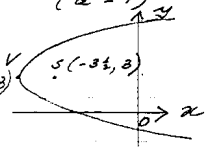
$(y-3)^2 = 4(x + 4\frac{1}{2})$

(i) $V\ \underline{(-4\frac{1}{2}, 3)}$

(ii) Focus $(S)\ \underline{(-3\frac{1}{2}, 3)}$

$(a = 1)$

(iii)



(b) $y = \ln(1-x) - \ln(1+x)$

$= \frac{-1}{1-x} - \frac{1}{1+x}$

$= \frac{-(1+x) - (1-x)}{1-x^2}$

$= \frac{-2}{1-x^2}$

$-\frac{1}{2}\int_0^{\frac{1}{2}} \frac{-2}{1-x^2}\,dx$

$= -\frac{1}{2}\left[\ln\left(\frac{1-x}{1+x}\right)\right]_0^{\frac{1}{2}}$

$= -\frac{1}{2}\ln\left(\frac{\frac{1}{2}}{1\frac{1}{2}}\right)$

$= -\frac{1}{2}\ln\left(\frac{1}{3}\right)$

$= \underline{\frac{1}{2}\ln 3}$

(c) (i) $x^2 - 2x = 2x$

$x(x-4) = 0$

$x = 0, 4 \qquad y = 0, 8$

A is $\underline{(4,8)}$

(ii) $A = \int_0^4 (2x - (x^2 - 2x))\,dx$

$= \int_0^4 (4x - x^2)\,dx$

$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$

$= 32 - \frac{64}{3}$

$= \underline{\frac{32}{3}\ \text{unit}^2}$

(c) $\Delta = k^2 - 4\times1\times(k-1)$

$= k^2 - 4k + 4$

$= (k-2)^2 \ge 0$

$\therefore$ Real Roots.

Q6

(a) $f''(x) = -2x$

$x = 3, f(x) = 7, f'(x) = 0$

$f'(x) = -x^2 + c_1$

$0 = -9 + c_1 \Rightarrow c_1 = 9$

$f(x) = -\frac{x^3}{3} + 9x + c_2$

$7 = -9 + 27 + c_2$

$c_2 = -11$

$\therefore \underline{f'(x) = -\frac{x^3}{3} + 9x - 11}$

(b) (i) In $\triangle$s AXB, CXD

$B\hat{A}X = D\hat{C}X$ (alt $\angle$s $AB \parallel DC$)

Also $A\hat{B}X = C\hat{D}X$

$\therefore \triangle AXB \,///\, \triangle CXD$ (equiangular)

(ii) $\frac{AB}{CD} = \frac{AX}{XC}$ ($AX = x$)

(corr sides in sim $\triangle$s)

$\frac{12}{8} = \frac{x}{10-x}$

$120 - 12x = 8x$

$\underline{Ax = x = 6\ \text{cm}}$

(c) (i) $\cos A\hat{O}B = \frac{10^2 + 10^2 - 15^2}{2\times10\times10}$

$= \underline{-\frac{1}{8}}$

$\therefore \underline{A\hat{O}B \doteq 97^\circ 11'}$

(ii) $A = \frac{1}{2}\cdot r^2\cdot(\theta - \sin\theta)$

$\doteq 50(1.696 - 0.992)$

$\doteq \underline{35.2\ \text{cm}^2}$

Q7

(a) AP $a = 800 \quad d = 20$

(i) $T_8 = 800 + 7\times20$

$= \underline{940\ \text{m}}$

(ii) $1400 = 800 + (n-1)\cdot20$

$20n = 620$

(iii) $S_n = \frac{n}{2}(2a + (n-1)d)$

$27300 = \frac{n}{2}(1600 + (n-1)20)$

$= n(790 + 10n)$

$10n^2 + 790n - 27300 = 0$

$n^2 + 79n - 2730 = 0$

$(n-26)(n+105) = 0$

$\therefore n = 26 \quad (n \ne -105)$

$\underline{26\ \text{trips}}$

(b) (i) $t = 0 \quad \underline{N = 10\,000}$

(ii) $t = 7 \quad N = 20\,000$

$20\,000 = 10000\,e^{7k}$

$e^{7k} = 2$

$k = \ln 2/7$

$\doteq 0.099$

(iii) $\frac{dN}{dt} = k\cdot10000\,e^{kt}$

$= \underline{kN}$

(iv) $1\,000\,000 = 10000\,e^{kt}$

$100 = e^{0.099t}$

$t = \ln 100/0.099$

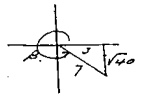
$\doteq 46.5168\ldots$

After $\underline{46.5\ \text{days}}$

(v) $\frac{dN}{dt} = 0.099\times1000000$

$= \underline{99000/\text{day}}$

(c) cos + sin − Quad 4



$\tan\beta = -\frac{\sqrt{40}}{3}$

OR $\underline{-\frac{2\sqrt{10}}{3}}$

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2000 MATHEMATICS
2 unit Solutions

Q8

(a)(i) $y' = 6x - 3x^2 + 9$

$y' = 6 - 6x$

(ii) St Pts $y' = 0$.

$-3x^2 + 6x + 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$.

$x = 3$, $y = 25$

$y'' = -12 < 0$

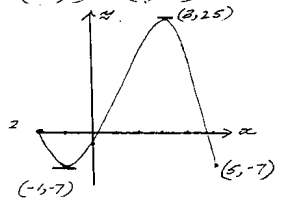
$(3, 25)$ Max T.P.

$x = -1$ $y = -7$

$y'' = 12 > 0$

$(-1, -7)$ Min T.P.

(iii) $(-3, 0)$ $(5, -7)$



(iv) Min Value is -7

(b)(i) $270\pi = 2\pi r^2 + 2\pi r h$

$135 = r^2 + rh$

$h = \frac{135 - r^2}{r}$

(ii) $V = \pi r^2 \left(\frac{135 - r^2}{r}\right)$

$V = 135\pi r - \pi r^3$

(iii) $V' = 135\pi - 3\pi r^2 = 0$

$r^2 = 45$

$r = 3\sqrt{5}$ $(r > 0)$

$V'' = -6\pi r < 0$ Max

When $r = 3\sqrt{5}$ cm

(iv) $h = \frac{135 - (3\sqrt{5})^2}{3\sqrt{5}}$

$= 30/\sqrt{5}$

$\therefore$ Max $V = \pi \cdot (3\sqrt{5})^2 \cdot 30/\sqrt{5}$

$= \frac{1350\pi}{\sqrt{5}}$ cm^3

Q9

(a)(i) $y = \log_e x$

$x = e^y$

$x = 1$ $y = 0$.

$x = 5$ $y = \ln 5$.

$V = \pi \int_a^b x^2 dy$

$= \pi \int_0^{\ln 5} e^{2y} dy$

(ii) $V = \frac{\pi}{2}\left[e^{2y}\right]_0^{\ln 5}$

$= \frac{\pi}{2}\left(e^{2\ln 5} - e^0\right)$

$= 12\pi$ units^3

(iii) GP $a = 4000$ $r = 3/4$

(i) $S_\infty = \frac{a}{1-r}$

$= \frac{4000}{1 - 3/4}$

$= \$16\,000$

(ii) 20 donations

$S_{20} = \frac{4000\left(1 - \left(\frac{3}{4}\right)^{20}\right)}{1/4}$

$\doteqdot \$15949$

$\$51$ short

(c) $y = xe^{-2x}$.

$y' = e^{-2x} \cdot 1 + x \cdot -2e^{-2x}$

$= e^{-2x}(1 - 2x)$

$y'' = -2e^{-2x} - 2e^{-2x}(1-2x)$

$= 4e^{-2x}(x - 1)$

$\therefore y'' + 4y' + 4y$

$= e^{-2x}(4x - 4 + 4 - 8x + 4x)$

$= e^{-2x}(0)$

$= 0$

Q10

(a)(i) $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10000}$

(ii) $1 - P(0 \text{ correct})$

$1 - \left(\frac{9}{10}\right)^4$

$= 1 - \frac{6561}{10000}$

$= \frac{3439}{10000}$

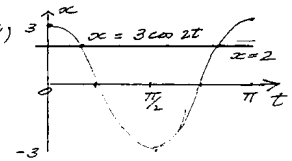
(iii) $\frac{1}{10} \cdot \frac{1}{2} \cdot \frac{1}{10} \cdot \frac{1}{10} + \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{2} \cdot \frac{1}{10}$

$= \frac{1}{1000}$

(iv) $\frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot 1$

$= \frac{1}{24}$

(b)(i)



(ii) $v = \dot{x}$

$v = -6 \sin 2t$

(iii) $t = 2$ secs

$v = -6 \sin 4$

$\doteqdot -6 \times -0.757$

$\therefore$ direction positive

(iv) At rest $v = 0$

$-6 \sin 2t = 0$

$2t = 0, \pi, 2\pi, \ldots$

second time $t = \pi/2$ secs

(v) $\ddot{x} = dv/dt = a$

$a = -12 \cos 2t$

$= -4(3 \cos 2t)$

$= -4x$

(vi) Max x $\cos 2t = 1$

$\therefore$ Max $x = 3$ cm

(vii) Twice