U11 - Ext I - Semester One 2009

Question 1

Q1 a) $\dfrac{9-x}{81-x^2}$

$= \dfrac{9-x}{(9-x)(9+x)}$

$= \dfrac{1}{9+x}$ 2M

) $\dfrac{\sqrt5-1}{\sqrt5+1} + \dfrac{\sqrt5+1}{\sqrt5-1}$

$\dfrac{(\sqrt5-1)^2 + (\sqrt5+1)^2}{(\sqrt5+1)(\sqrt5-1)}$

$\dfrac{5-2\sqrt5+1 + 5+2\sqrt5+1}{5-1}$

$\dfrac{10+2}{4}$

$= \dfrac{12}{4}$

$\therefore 3$ 3M

$|2x-1| < 3$

$2x-1 < 3$ $\quad$ $2x-1 > -3$

$+1 \quad +1$ $\quad$ $+1 \quad +1$

$\dfrac{2x}{2} < \dfrac{4}{2}$ $\quad$ $\dfrac{2x}{2} > \dfrac{-2}{2}$

$x < 2$ $\quad$ $x > -1$

$\therefore -1 < x < 2$. 2M

d) $\dfrac{5}{2x-1} < 1 \quad \times(2x-1)^2 \quad (2x-1)^2$

$5(2x-1)^2 < (2x-1)^2$

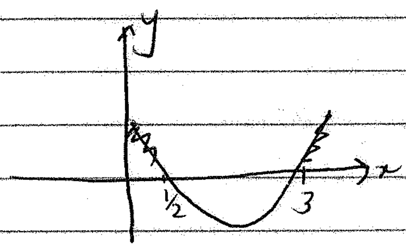
$5(2x-1) < (2x-1)^2$

$(2x-1)^2 - 5(2x-1) > 0$

$(2x-1)(2x-1-5) > 0$

$(2x-1)(2x-6) > 0$.

$2(2x-1)(x-3) > 0$



$x < \tfrac12 \quad x > 3$. 3M

e) $\dfrac{1+\sqrt7}{3-\sqrt7} \times \dfrac{3+\sqrt7}{3+\sqrt7}$

$= \dfrac{(1+\sqrt7)(3+\sqrt7)}{9-7}$

$= \dfrac{3+\sqrt7+3\sqrt7+7}{2}$

$= \dfrac{10+4\sqrt7}{2}$

$= 5+2\sqrt7$ 2M

$\therefore a=5 \quad b=2$.

Question 2

a) $\left(\dfrac{kx_2 + lx_1}{k+l} , \dfrac{ky_2 + ly_1}{k+l} \right)$

$\therefore$ A$(-2,5)$ B$(1,2)$

$3 : -2$

$\left(\dfrac{3 \times 1 + -2 \times -2}{3-2} , \dfrac{3 \times 2 + -2 \times 5}{3-2} \right)$

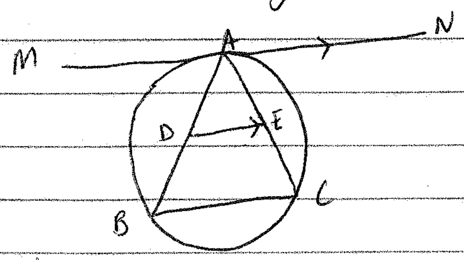
$\left(\dfrac{3+4}{1} , \dfrac{6-10}{1} \right)$

$(7 , -4)$

2M

b) The angle between a tangent and chord drawn to the point of contact is equal to the angle subtended by the chord in the alternate segment.

1M



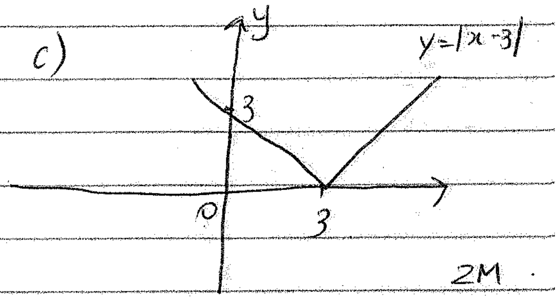
ii) $\angle ADE = \angle MAB$ (alternate $\angle$'s)

$\angle ADE = \angle ECB$ (both equal to $\angle MAB$).

$\therefore$ BCED is a cyclic quadrilateral because exterior angle of cyclic quadrilateral is equal to the opposite interior angle)

3M

c)



2M

d) solve $2x - 9 = \dfrac{-9}{x}$

$2x^2 - 9x = -9$

$2x^2 - 9x + 9 = 0$

$2x \quad -3$

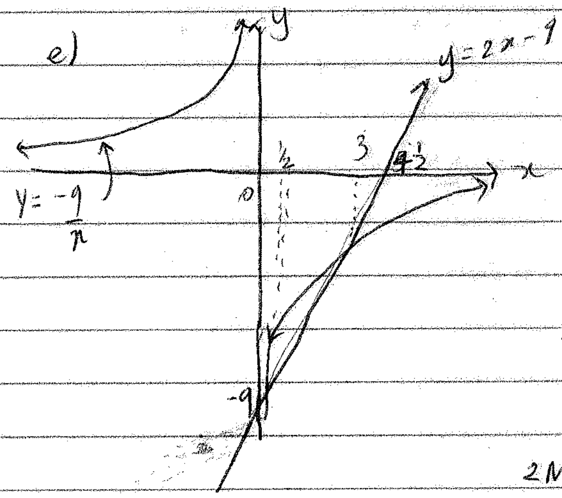
$x \quad -3$

$(2x-3)(x-3) = 0$

$x = \dfrac{3}{2} \qquad x = 3$

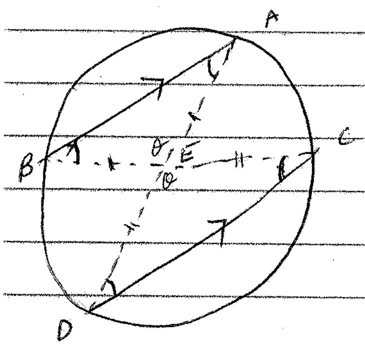
2M

e)



2M

Question 3



Prove $AD = BC$

$\angle BAD = \angle EDC$ (alternate $\angle$'s $AB \parallel CD$)

$\angle AEB = \angle DEC$ (vertically opposite)

$\angle BAD = \angle BCD$ (angles subtended at the circumference by the same arc are equal)

likewise

$\angle ABC = \angle ADC$ (" " " ")

$\therefore$ $\angle BAE = \angle ABE$ (both equal to $\angle BCD$)

$\therefore$ ΔABE is isosceles

then $AE = BE$ (equal sides of isosceles Δ)

likewise

$ED = EC$ (since $\angle EDC = \angle ECD$ & ΔEDC is isosceles)

so $AE + ED = BE + EC$ (sum of equal sides are equal)

4M.

d)

$$\frac{\tan x - \tan y}{\tan x + \tan y} = \frac{\sin(x-y)}{\sin(x+y)}$$

$$LHS = \frac{\frac{\sin x}{\cos x} - \frac{\sin y}{\cos y}}{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}$$

$$= \frac{\frac{\sin x \cos y - \sin y \cos x}{\cos x \cos y}}{\frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y}}$$

$$= \frac{\sin x \cos y - \sin y \cos x}{\sin x \cos y + \sin y \cos x}$$

$$= \frac{\sin(x-y)}{\sin(x+y)}$$

$= RHS$

4M

b)

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

where $m_1 = 3$ since $3x - y = 4$

then $3x - 4 = y$.

& $m_2 = -2/3$ since $2x + 3y = 4$

$3y = 4 - 2x$

$y = \frac{4}{3} - \frac{2x}{3}$

$$\tan\theta = \left|\frac{3 + 2/3}{1 + 3 \times -2/3}\right|$$

$$= \frac{11}{3}$$

$\theta = 74^\circ 45'$

c)

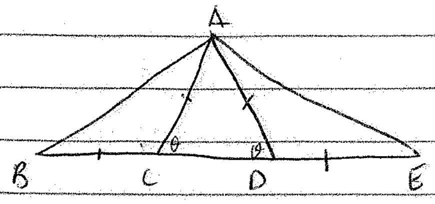
$$\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$$

$$\lim_{x \to 3} \frac{\cancel{(x-3)}(x^2 + 3x + 9)}{\cancel{x-3}}$$

$$= \lim_{x \to 3} x^2 + 3x + 9$$

$= 27$

Question 4



$AC = AD = BC = DE$ prove $AB = AE$

In $\triangle ACD$ $\angle ACD = \angle ADC$ (base $\angle$'s of isosceles $\triangle$ are equal)

$\angle ADE = \angle ACB = 180 - \theta$ (adjacent supplementary $\angle$'s)

$AC = AD$ (given)

$BC = DE$ (given)

$\therefore \triangle ACB \equiv \triangle ADE$ (SAS)

$\therefore AB = AE$ (corresponding sides of congruent $\triangle$'s)

4M

b) $\frac{d-2}{d} = \frac{d+5}{18}$ (A line parallel to one side of a triangle divides the other sides proportionally)

$18d - 36 = d^2 + 5d$
$-18d \qquad\qquad -18d$

$d^2 - 13d + 36 = 0$

$(d-9)(d-4) = 0$

$d = 9$ & $d = 4$

c) In $\triangle ABD$ and $\triangle ADC$

$\angle ADC = 90°$ (adjacent suplementary $\angle$'s)

$\angle ADC = \angle ADB$ (both equal to $90°$)

$\angle ACD = 180 - 90 - \angle ABD$ ($\angle$ sum of $\triangle ABC$)

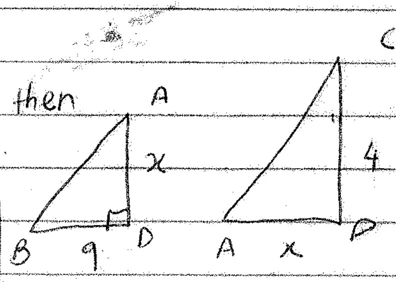
$= 90 - \angle ABD$

$\angle BAD = 180 - 90 - \angle ABD$ ($\angle$ sum of $\triangle ABD$)

$= 90 - \angle ABD$

$\therefore \triangle ABD \parallel\!\mid \triangle ADC$ (equal angular)

then



$\frac{AD}{BD} = \frac{CD}{AD}$

$\frac{x}{9} = \frac{4}{x}$

$x^2 = 36$

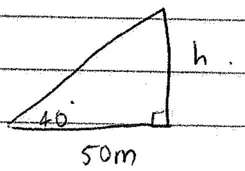
$x = \pm 6$

$\therefore x = 6$ only.

4M

Question 5

a)

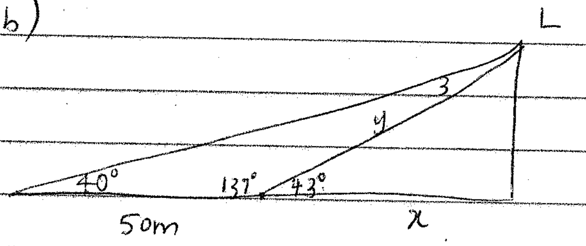


$\frac{h}{50} = \tan 40^\circ$

$h = 50 \tan 40^\circ$

$= 41 \cdot 95m$ [3M]

b)



$\frac{y}{\sin 40} = \frac{50}{\sin 3^\circ}$

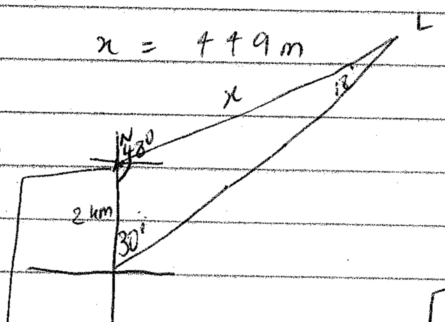
$y = \frac{50 \times \sin 40}{\sin 3^\circ}$

$= 614 \cdot 0975114$

$\cos 43 = \frac{x}{614 \cdot 0975114}$

$x = 449m$ [4M]

c)



[2M]

i) $180 - 48$

$= 132^\circ$

$\angle L = 180 - 132 - 30$

$= 18^\circ$

$\frac{x}{\sin 30} = \frac{2}{\sin 18}$

$x = \frac{2 \sin 30}{\sin 18} = 3 \cdot 24 km$ [3M]

Question 6

a) y-int let $x = 0$ $f(x) = -1$

no x-intercept.

vertical asymptotes at

$x - 1 = 0$ $x + 1 = 0$

at $x = \pm 1$

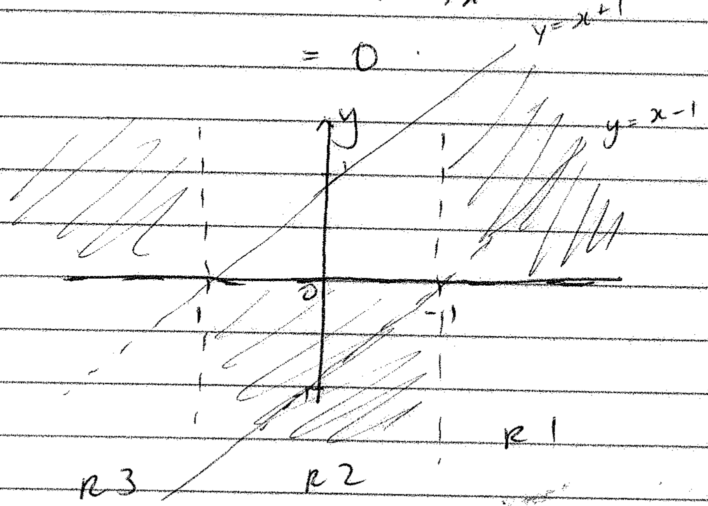
horizontal asymptotes

at $\lim\limits_{x \to \infty} \frac{1}{x^2 - 1}$

$\lim\limits_{x \to \infty} \frac{1/x^2}{x^2/x - 1/x^2}$

$= \lim\limits_{x \to \infty} \frac{1/x^2}{x - 1/x^2}$

$= 0$.



$R1 = \frac{1}{(x-1)(x+1)}$

$= \frac{1}{+ \times +}$

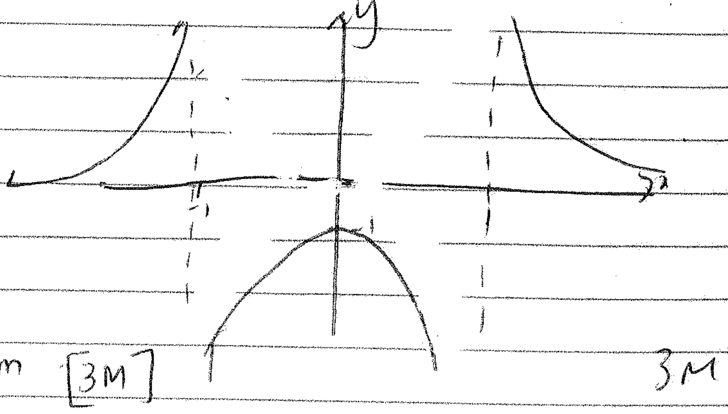
$= +$

$R_2 : = \frac{1}{+ \times -}$

$= -$

$R3 = \frac{1}{- \times -}$

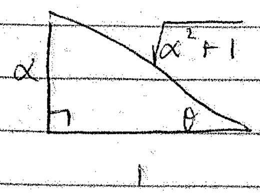
$= +$



3M

Question 6 cont

b)



$$\cos\theta = \frac{1}{\sqrt{\alpha^2+1}}$$

$$\sin\theta = \frac{\alpha}{\sqrt{\alpha^2+1}}$$

[2M]

c) $4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

$x = 30, 150, 210, 330^\circ$

(2M)

d) $3\cos^2 x = 8\sin x$

$3\cos^2 x - 8\sin x = 0$

~~$3\cos^2 x - 8(1-\sin$~~

$3(1-\sin^2 x) - 8\sin x = 0$

$3 - 3\sin^2 x - 8\sin x = 0$

$3\sin^2 x + 8\sin x - 3 = 0$

$3x \quad -1$

$x \quad +3$

$(3\sin x - 1)(\sin x + 3) = 0$

$\sin x = \frac{1}{3}$ $\quad \sin x = -3$

↑ no solution.

$x = 19^\circ 28'$, $180 - 19^\circ 28'$

$= 19^\circ 28'$, $160^\circ 32'$

3M

e) $\sin\alpha = \frac{1}{2}$ find the exact value of $\cos 2\alpha$.

$\cos 2\alpha = 1 - 2\sin^2\alpha$.

$= 1 - 2\left(\frac{1}{2}\right)^2$

$= 1 - 2\times\frac{1}{4}$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

[2M]

Question 7

(i) $\cos(\alpha+\theta) = \cos\alpha\cos\theta - \sin\alpha\sin\theta$

[1M]

i) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

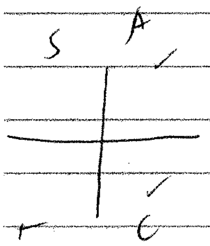
$\cos 3\theta = \cos(2\theta+\theta)$
$= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$
$= (\cos^2\theta - \sin^2\theta)\cos\theta - 2\sin\theta\cos\theta \times \sin\theta$
$= \cos^3\theta - \sin^2\theta\cos\theta - 2\sin^2\theta\cos\theta$
$= \cos^3\theta - 3\sin^2\theta\cos\theta$
$= \cos^3\theta - 3(1-\cos^2\theta)\cos\theta$
$= \cos^3\theta - (3+3\cos^2\theta)\cos\theta$
$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta$
$= 4\cos^3\theta - 3\cos\theta$
$=$ RHS.

[3M]

ii) solve $8\cos^3\theta - 6\cos\theta - \sqrt{3} = 0$

$= 2(4\cos^3\theta - 3\cos\theta) = \sqrt{3}$
$(4\cos^3\theta - 3\cos\theta) = \frac{\sqrt{3}}{2}$.

$\cos 3\theta = \frac{\sqrt{3}}{2}$



$3\theta = 30^\circ, 330, 390, 690, 750, 1050$
$\theta = 10^\circ, 110^\circ, 130, 230, 250, 350$

[3M]

b) In $\triangle ABD$

$$\tan 68 = \frac{BD}{h}$$

$$h \tan 68 = BD$$

In $\triangle ABK$ $\quad \tan 70 = \frac{BK}{h}$

$$h \tan 70 = BK$$

In $\triangle BDK$.

In $\triangle BDK$

$$400^2 = BK^2 - BD^2$$

$$400^2 = h^2 \tan^2 70 - h^2 \tan^2 68$$

$$\frac{400^2}{\tan^2 70 - \tan^2 68} = \frac{h^2(\tan^2 70 - \tan^2 68)}{\tan^2 70 - \tan^2 68}$$

$$h^2 = 400^2 / (\cot^2 20 - \cot^2 22)$$

$$\cancel{h = \sqrt{400^2 (\cot^2 20 - \cot^2 22)}}$$

$$= \frac{400}{\sqrt{\cot^2 20 - \cot^2 22}}$$ 3M

ii) . 2M

$$= 335$$

$$400^2 = h^2 \tan^2 68 + h^2 \tan^2 70$$

$$\frac{400^2}{\tan^2 68 + \tan^2 70} = \frac{h^2(\tan^2 68 + \tan^2 70)}{\tan^2 68 + \tan^2 70}$$

$$h^2 = \frac{400^2}{\tan^2 68 \tan^2 70}$$

$$h = \sqrt{\frac{400^2}{\tan^2 68 \tan^2 70}}$$

$= \cancel{58\ 321}$

$= 2720.098$.

$= 2720$ m. 2M

$\therefore \cancel{h = \sqrt{400^2 (\cot^2 22 - \cot 2}}$