Question 1

a) $y = \log_e(e^{3x}+2)$

$\frac{dy}{dx} = \frac{3e^{3x}}{e^{3x}+2}$ (1) (1)

(ii) $y = x^3\cos 3x$

$y' = \cos 3x \times (3x^2) + x^3 \times (-3\sin 3x)$ (1)

$= 3x^2(\cos 3x - x\sin 3x)$ (1)

b) (i) $\int \frac{dx}{(7x+4)^5}$

$= \int (7x+4)^{-5}\,dx$

$= \frac{(7x+4)^{-4}}{-4\times 7} + c$

$= \frac{-1}{28(7x+4)^4} + c$

(ii) $\int \sin 6x\,dx$

$= -\frac{\cos 6x}{6} + c$ (1)

(iii) $\int 4x e^{x^2}\,dx$

$= 2\times\int 2x e^{x^2}\,dx$ (1)

$= 2e^{x^2} + c$ (1)

c) $\log_a 8 + \log_a 16 = x\log_a 2$

$3\log_a 2 + 4\log_a 2 = x\log_a 2$ (1)

$7\log_a 2 = x\log_a 2$

$\therefore x = 7$ (1)

Question 1 (cont)

d) $\cos 105^\circ$

$= \cos(60^\circ + 45^\circ)$

$= \cos 60^\circ\cos 45^\circ - \sin 60^\circ\sin 45^\circ$ (1)

$= \frac{1}{2}\times\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$ (1)

$\left[= \frac{\sqrt{2}(1-\sqrt{3})}{4}\right]$

Question 2

a) $\frac{\sin x}{\cos x - \sin x} + \frac{\sin x}{\cos x + \sin x}$

$= \frac{\sin x(\cos x + \sin x + \cos x - \sin x)}{(\cos x - \sin x)(\cos x + \sin x)}$ (1)

$= \frac{2\sin x\cos x}{\cos^2 x - \sin^2 x}$

$= \frac{\sin 2x}{\cos 2x}$ (1)

$= \tan 2x$ (1)

b) $\sec x + \tan x$

$= \frac{1+t^2}{1-t^2} + \frac{2t}{1-t^2}$ (1)

$= \frac{1+2t+t^2}{1-t^2}$

$= \frac{(1+t)^2}{1-t^2}$ (1)

$= \frac{(1+t)^2}{(1+t)(1-t)} = \frac{1+t}{1-t}$ (1)

Question 2 (cont)

c) $u = x^2 - 1$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$ (1)

And $x^2 = u+1$

Now $\int x^3(x^2-1)\,dx$

$= \int x^2(x^2-1)x\,dx$

$= \int (u+1)\cdot u\cdot\frac{du}{2}$

$= \frac{1}{2}\int (u^2+u)\,du$

$= \frac{1}{2}\left(\frac{u^3}{3} + \frac{u^2}{2}\right) + c$ (1)

$= \frac{u^2}{2}\left(\frac{u}{3} + \frac{1}{2}\right) + c$

$= \frac{u^2}{2}\left(\frac{2u+1}{6}\right) + c$

$= \frac{(x^2-1)^2(2(x^2-1)+3)}{12} + c$

$= \frac{(x^2-1)^2(2x^2+1)}{12} + c$ (1)

Question 2 (cont)

d) $y = \sin x$

$\frac{dy}{dx} = \cos x$ (1)

$\cos x = \frac{1}{2}$ (1)

when $x = \frac{\pi}{3},\ 2\pi - \frac{\pi}{3}$

$= \frac{\pi}{3},\ \frac{5\pi}{3}$ (1)

Question 3

(a) $P(x) = x^4 + ax^2 + b$

$P(-1) = 0$

$\therefore 1 + a + b = 0$ ① (1)

$b = -a - 1$

and $P(2) = 0$

$16 + 4a + b = 0$ ② (1)

Sub. in ①

$\therefore 16 + 4a - a - 1 = 0$

$15 + 3a = 0$

$\left.\begin{array}{l} a = -5 \\ \therefore b = 4 \end{array}\right\}$ (1)

b) $P(x) = (x-c)^2\cdot Q(x)$ (1)

$\therefore P'(x) = Q(x)\cdot 2(x-c) + (x-c)^2\cdot Q'(x)$ (1

$= (x-c)[2\cdot Q(x) + (x-c)Q'(x$

$\therefore x = c$ is a root of $P'(x)$ (

Question 3 (cont)

c)
$x^3+2x^2+3x+5=0$
$a=1,\ b=2,\ c=3,\ d=5$

i) $p+q+r=-\frac{b}{a}$
$=-2$ (1)

ii) $p^{-1}+q^{-1}+r^{-1}$
$=\frac{1}{p}+\frac{1}{q}+\frac{1}{r}$
$=\frac{qr+pr+pq}{pqr}$ (1)
$=\frac{c/a}{-d/a}$
$=-\frac{c}{d}$ (1)
$=-\frac{3}{5}$ (1)

d) $f(x)=e^x-4x-8$
$f'(x)=e^x-4$

$a_2=a_1-\frac{f(a_1)}{f'(a_1)}$ (1)
$=3-\frac{f(3)}{f'(3)}$
$=3-\frac{e^3-20}{e^3-4}$ (1)
$=2.99468\ldots$
$=2.995$ (3 dp)

Question 4

a) i) $R=\sqrt{2^2+1^2}$
$=\sqrt{5}$ (1)

$\therefore \frac{2}{\sqrt5}\cos\theta-\frac{1}{\sqrt5}\sin\theta$
$=\cos\theta\cos\alpha-\sin\theta\sin\alpha$

$\therefore \cos\alpha=\frac{2}{\sqrt5}$
$\alpha=26°34'$ (1)

$\therefore R=\sqrt5,\ \alpha=26°34'$

ii) $2\cos\theta-\sin\theta=1$
$\sqrt5\cos(\theta+\alpha)=1$
$\cos(\theta+\alpha)=\frac{1}{\sqrt5}$ (1)

$(\theta+\alpha)=63°26',\ 296°34'$
$\theta=63°26'-26°34'$
$296°34'-26°34'$
$\theta=36°52',\ 270°$ (1)

b) $V=\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2x\,dx$ (1)
$=2\pi\int_0^{\frac{\pi}{2}}\cos^2x\,dx$
$=2\pi\int_0^{\frac{\pi}{2}}\frac{\cos2x+1}{2}\,dx$ (1)
$=\pi\int_0^{\frac{\pi}{2}}(\cos2x+1)\,dx$
$=\pi\left[\frac{\sin2x}{2}+x\right]_0^{\frac{\pi}{2}}$ (1)
$=\pi\left(\frac{\sin\pi}{2}+\frac{\pi}{2}\right)=\frac{\pi^2}{2}$ (1)

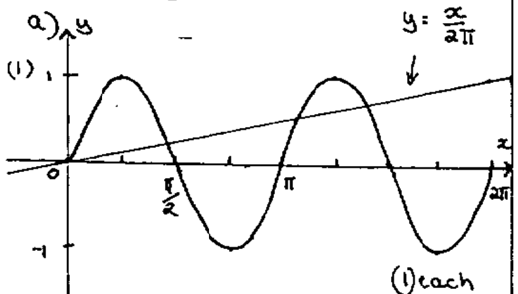
Question 4 (cont)

c) (i)
$\frac{d}{dx}(x\log_ex)$
$=(\log_ex)\times1+x\times\frac{1}{x}$
$=1+\log_ex$ (1)

(ii) $\int_e^{e^2}\frac{1+\log_ex}{x\log_ex}\,dx$
$=\left[\log_e(x\log_ex)\right]_e^{e^2}$ (1)
$=\log_e(e^2.2)-\log_e(e)$
$=\log_ee^2+\log_e2-1$ (1)
$=2+\log_e2-1$
$=1+\log_e2$ (1)

Question 5

a) (1)



(1)each

ii) There are 4 values (1)

iii) No. $\frac{x}{2\pi}>1$ when $x>2\pi$ (1)
$\therefore$ no further solutions because max. value of $\sin 2x$ is 1.

Question 5 (cont)

b)
(i) Let θ = angle between the tangents.

At A, $m_1=e^t$
B, $m_2=e^{-t}$ (1)

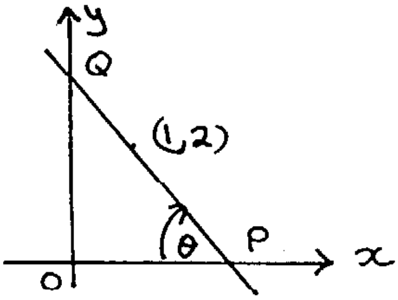
$\therefore \tan\theta=\left|\frac{e^t-(e^{-t})}{1+e^t.e^{-t}}\right|$ (1)
$1=\frac{e^t-e^{-t}}{2}$
i.e. $2=e^t-e^{-t}$
or $e^t-\frac{1}{e^t}=2$ (1)

(ii) $e^t-\frac{1}{e^t}-2=0$
$(e^t)^2-2e^t-1=0.$ (1)
$\therefore e^t=\frac{2\pm\sqrt{4+4}}{2}$ ($\frac12$)
$=1\pm\sqrt2$ ($\frac12$)

(Quest 6 on next page)

Question 6

a)



For PQ $m = -\tan\theta$ (1)

Now, $y - y_1 = m(x - x_1)$

$y - 2 = -\tan\theta\,(x - 1)$

$y - 2 = -x\tan\theta + \tan\theta$

OR $y = 2 + \tan\theta - x\tan\theta$ (1)

b) $A = \frac{1}{2} \times OP \times OQ$

At P $y = 0$

$\therefore \quad 0 = \tan\theta + 2 - x\tan\theta$

$x\tan\theta = \tan\theta + 2$

$x = 1 + \frac{2}{\tan\theta}$ (1)

$\therefore OP = 1 + \frac{2}{\tan\theta}$

At Q, $x = 0$

$\therefore y = 2 + \tan\theta$

$\therefore OQ = 2 + \tan\theta$ (1)

$\Rightarrow A = \frac{1}{2}\left(1 + \frac{2}{\tan\theta}\right)(2 + \tan\theta)$

$= \frac{1}{2}\left(2 + \tan\theta + \frac{4}{\tan\theta} + 2\right)$

$= \frac{\tan\theta}{2} + 2 + \frac{2}{\tan\theta}$ (1)

Question 6 (cont)

c) Let $t = \tan\theta$

$\therefore A = \frac{t}{2} + 2 + \frac{2}{t}$

Now $\frac{dA}{dt} = \frac{1}{2} - \frac{2}{t^2}$ (1)

$\frac{d^2A}{dt^2} = \frac{4}{t^3}$ (1)

$\frac{dA}{dt} = 0$ when $\frac{1}{2} = \frac{2}{t^2}$

$t^2 = 4$

$t = \pm 2$

But $t > 0$, since $A > 0$ (1)

$\therefore$ when $t = 2$, $\frac{d^2A}{dt^2} = \frac{4}{2^3} > 0$ (1)

$\therefore$ min. value when $t = 2$

d) When $t = 2$

$A = \frac{2}{2} + 2 + \frac{2}{2}$

$= 4$ (1)

$\therefore$ Min area is 4 sq. units.