NSW INDEPENDENT TRIAL EXAMS –2001
MATHEMATICS (2 unit) SUGGESTED ANSWERS

Q1. (a) $0.156410\ldots$

$\doteqdot 0.156$

(b) $7-3x < 5$

$-3x < -2$

$x > 2/3$

$2/3$

(c) $\dfrac{(\sqrt{3}+2)-(\sqrt{3}-2)}{(\sqrt{3})^2 - 2^2}$

$= \dfrac{4}{-1} = -4$

(d) $3\pi = r \cdot 2\pi/3$

$r = 9\pi/2\pi$

$= 4.5\,cm$

(e) $(a-3)^2 + (-2+7)^2 = (5\sqrt{2})^2$

$a^2 - 6a + 34 = 50$

$a^2 - 6a - 16 = 0$

$(a-8)(a+2) = 0$

$a = 8, -2$

(f) $6x - 3y = 21$

$x + 3y = 0$

$7x = 21$

$x = 3$

$y = -1$

Q2. (a)

(i) $7(2x^3-5)^6 \cdot 6x^2$

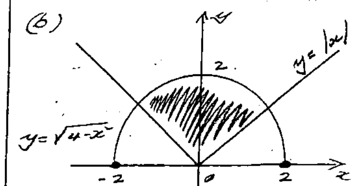
$= 42x^2 \cdot (2x^3-5)^6$

(ii) $e^{3x} \cdot 2x + x^2 \cdot 3e^{3x}$

$xe^{3x}(2+3x)$

(iii) $\dfrac{x\cos x - \sin x}{x^2}$

(b)



(c) Sub $(1,7)$

$7 = a + b$ — ①

$y' = 3ax^2 + b$

$m = 2$ (// $y = 2x - 6$)

$\therefore\ 2 = 3a + b$ — ②

② − ① $\quad -5 = 2a$

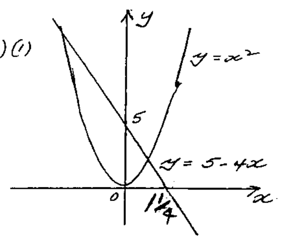
$a = -2\frac{1}{2}$

$b = 9\frac{1}{2}$

Q3. (a) (i) $\frac{1}{7}\tan 7x + c$

(ii) $\frac{1}{3}\ln(x^2+3) + C$

(b) (i)



(ii) $x^2 = 5 - 4x$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = -5, 1$

(iii) $A = \int_{-5}^{1} (5 - 4x - x^2)\,dx$

$= \left[5x - 2x^2 - \frac{x^3}{3}\right]_{-5}^{1}$

$= 2\frac{2}{3} - (-33\frac{1}{3})$

$= 36$ units^2

①

(c) (i) $\left[\dfrac{x^{-1}}{-1}\right]_{-2}^{-1} = 1 - \frac{1}{2}$

$= \frac{1}{2}$

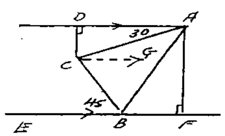
(ii) $\left[\frac{1}{2}\sin(2x+\pi)\right]_0^{\pi/3}$

$= \frac{1}{2}(\sin\frac{5\pi}{3} - \sin\pi)$

$= \frac{1}{2}(-\frac{\sqrt{3}}{2} - 0)$

$= \dfrac{-\sqrt{3}}{4}$

Q4. (a)



(i) Draw $CG \parallel DA$ & EF

$A\hat{C}G = 30°$ (alt ∠'s $DA \parallel CG$)

sim $B\hat{C}G = 45°$

$\therefore\ A\hat{C}B = 75°$

$A\hat{B}C = 75°$

(ii) (Base ∠'s isos △ ACB)

$\therefore\ C\hat{A}B + 2 \times 75° = 180°$

(∠ sum △ ACB)

$C\hat{A}B = 30°$

(iii) $D\hat{A}F = 90°$ ($AF \perp EF$

$\therefore AF \perp DA \parallel EF$)

$\therefore\ B\hat{A}F = 30°$

In △'s ACD, ABF

$B\hat{A}F = C\hat{A}D = 30°$

$AC = AB$ (data)

$A\hat{D}C = 90° = A\hat{F}B$

($CD \perp DA$ & $AF \perp EF$)

$\therefore$ △ ACD ≡ △ ABF (AAS)

(b) (i) $0.5 \times 0.5 \times 0.25$

(ii) G.P. $a = 4 \quad r = \frac{1}{8}$

$S_\infty = \dfrac{4}{1 - \frac{1}{8}}$

$= \frac{32}{7}\ m^3$

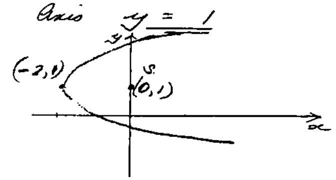
Q4 (c)

$(y-1)^2 = 4 \times 2\,(x+2)$

$V = (-2, 1)$

$S = (0, 1)$

Axis $y = 1$



Q5 (a)

$A = \frac{4}{6}(4 + 4\times 9 + 8)$

$- \frac{4}{6}(4 + 4\times 5 + 8)$

$= \frac{4}{6}(4 \times 4)$

$= \frac{32}{3}$ units^2

(b) (i) $M(2, 1)$

(ii) $m_1 = \dfrac{2-0}{0-4} = -\frac{1}{2}$

(iii) $m_2 = 2$

$y - 1 = 2(x-2)$

$y = 2x - 3$

(iv) $7 = 2 \times 5 - 3$

(v) Isos △ as ⊥ bisector of base passes through the vertex.

(c) Let $X = 2^x$

$X^2 - 15X - 16 = 0$

$(X-16)(X+1) = 0$

$2^x = 16, -1$

$x = 4 \quad (2^x \neq -1)$

Q6.

(a) (i) $y' = 3x^2 + 8x - 3$

$x = 1 \quad m_1 = 3 + 8 - 3 = 8$

(ii) $y = 1 + 4 - 3 = 2$

$m_2 = -\frac{1}{8}$

$\therefore\ y - 2 = -\frac{1}{8}(x-1)$

$8y - 16 = -x + 1$

$x + 8y - 17 = 0$

(b) (i) 0.05

(ii) $(0.05)^2 = 0.0025$

(iii) $1 - 0.0025$

$= 0.9975$

(iv) Arrive late day 1 only

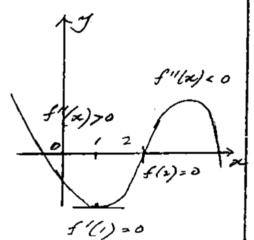
$= 0.05 \times 0.95 \times 0.95$

Sim for days 2, 3

$\therefore\ 3 \times 0.05 \times 0.95 \times 0.95$

$= 0.135375$

(c)



(d) (i) $T_{20} = a + 19 \times (2a - 1)$

$= 39a - 19$

(ii) $S_{20} = \frac{20}{2}(a + 39a - 19)$

$= 10(40a - 19)$

②

Q7 (a) (i) $S = (2n-4) \times 90$

$= 540°$

$\therefore$ Each ∠ $= 108°$

(ii) $E\hat{D}C + C\hat{D}F = 180°$

$\therefore\ C\hat{D}F = 72°$

Sim $D\hat{C}F = 72°$

△ FCD isosceles

(iii) $FC = FD$ (isos △)

$CB = DE$ (reg pent.)

$\therefore\ FB = FE$

$\therefore\ \dfrac{FC}{FB} = \dfrac{FD}{FE}$

$\hat{F}$ is common

$\therefore$ △ FCD ||| △ FBE

(iv) $\dfrac{FC}{FB} = \dfrac{DC}{BE}$

Let $FC = x$ cm

$\therefore\ \dfrac{x}{x+5} = \dfrac{5}{8}$

$8x = 5x + 25$

$CF = x = 8\frac{1}{3}$ cm

(b) (i) $y' = 3x^2 + 6x = 0$

(ii) $3x(x+2) = 0$

$x = 0, -2$

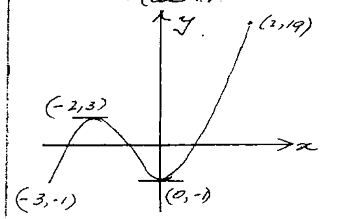
$y'' = 6x + 6$

$(0, -1) \quad y'' = 6$

$\therefore$ Min T.P.

$(-2, 3) \quad y'' = -6$

Max T.P.



Q8

(a) (i) $t = \frac{1}{2}$

$x = 6(\frac{1}{2}) + e^{-4(\frac{1}{2})}$

$= \underline{\underline{3 + e^{-2}}}$

or $\underline{\underline{3 + \frac{1}{e^2}}}$

$(3 \cdot 135335283 \ldots)$

(ii) $v = \underline{\underline{6 - 4e^{-4t}}}$

$t = 0 \quad v = 6 - 4e^{0}$

$= \underline{\underline{2 \text{ m/sec}}}$

(iii) $a = 16e^{-4t}$

$t = 0 \quad a = 16.$

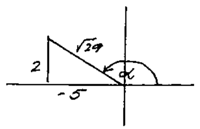
(iv) As $t \to \infty \quad e^{-4t} \to 0$

$\therefore \underline{v \to 6}$

$\therefore$ Never stops

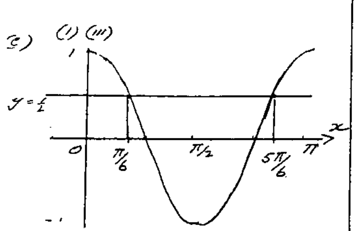
(b) $\cos < 0 \quad \tan < 0$

$\therefore$ Quad 2.



$\text{cosec}\, \alpha = \underline{\underline{+\frac{\sqrt{29}}{2}}}$

c) (i) (iii)



(ii) $\cos 2x = \frac{1}{2}$

$0 \le 2x \le 2\pi$

$2x = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

Q9 (a) (i) $\underline{\underline{R(-2,0)}} \quad \underline{\underline{S(2,0)}}$

(ii) $A = \frac{h}{2}(a + b)$

$= \frac{y}{2}(4 + 2x)$

$= \left(\frac{4 - x^2}{2}\right)(4 + 2x)$

$= \underline{(2 + x)(4 - x^2)}$

(iii) $A = 8 + 4x - 2x^2 - x^3$

$A' = 4 - 4x - 3x^2 = 0$

for Max, Min

$(2 - 3x)(2 + x) = 0$

$x = \frac{2}{3}, -2.$

$A'' = -4 - 6x$

$x = \frac{2}{3} \quad A'' = -4 - 6(\frac{2}{3})$

< 0

$\therefore$ Max value of A

when $\underline{\underline{x = \frac{2}{3}}}$

(b) (i) $t = 0 \quad \underline{\underline{P = 3000}}$

(ii) $4000 = 3000\, e^{k}$

$e^{k} = 4/3$

$k = \ln(4/3)$

$\underline{\underline{k \doteqdot 0 \cdot 29}}$

(iii) $t = 2 \quad P = 3000\, e^{2(0 \cdot 29)}$

$= \underline{\underline{5358}}$

(iv) $12000 = 3000\, e^{0 \cdot 29t}$

$4 = e^{0 \cdot 29t}$

$0 \cdot 29t = \ln 4$

$t = \frac{\ln 4}{0 \cdot 29}$

$\doteqdot 4 \cdot 78$

ie $\underline{\text{after 5 days}}$

Q10 (a)

(i) $x = 8 \quad y = \frac{1}{2}$

$V = \pi r^2 h + \pi \int_{\frac{1}{2}}^{4} x^2 \, dy$

$= \pi (8)^2 \cdot \frac{1}{2} + \pi \int_{\frac{1}{2}}^{4} 16y^{-2} \, dy$

(ii) $= 32\pi + \pi \left[-16y^{-1}\right]_{\frac{1}{2}}^{4}$

$= 32\pi + \pi\left[-4 - (-32)\right]$

$= \underline{\underline{60\pi \text{ units}^3}}$

(b) (i) $\Delta = (-5)^2 - 4(3)(6)$

$= \underline{-47.}$

As $\Delta < 0$

$\therefore$ Unreal roots

(c) (i) Let I_n = value of investment after n yrs.

$I_1 = \$P + 12\% \text{ of } \$P$

$= \underline{\$P(1 \cdot 12)}$

(ii) $I_2 = P(1 \cdot 12) + P$

$+ 12\%(P(1 \cdot 12) + P)$

$= (1 \cdot 12)(P(1 \cdot 12) + P)$

$= P(1 \cdot 12)(1 \cdot 12 + 1)$

Sim. $I_3 = P(1 \cdot 12)(1 \cdot 12 + 1) + P$

$+ 12\%\{P(1 \cdot 12)(1 \cdot 12 + 1) + P\}$

$= \underline{P(1 \cdot 12)\{(1 \cdot 12)^2 + 1 \cdot 12 + 1\}}$

(iii) $I_{20} = 1\,000\,000$

$= P(1 \cdot 12)(1 \cdot 12^{19} + \ldots 1 \cdot 12^2 + 1 \cdot 12 + 1)$

$1\,000\,000 = \frac{P(1 \cdot 12)(1 \cdot 12^{20} - 1)}{(1 \cdot 12 - 1)}$

$P = \frac{1\,000\,000 \times 0 \cdot 12}{1 \cdot 12(1 \cdot 12^{20} - 1)}$

$= \underline{\underline{\$12391 \cdot 77}}$