Student Name:.....

St Mark's Coptic Orthodox College

2008

PRELIMINARY COURSE

Semester two Exam

MATHEMATICS
Extension 1

Examiner Mr. Talat GERGES

General Instructions

- Reading time - 5 minutes
- Working time - Two (2) hours
- Attempt all questions.
- Questions are of equal value.
- All necessary working must be shown in every question.
- Marks may be deducted for careless or badly arranged work.
- Board approved calculators may be used.
- Each question is to be started on a new page.
- This examination paper must NOT be removed from the examination room.

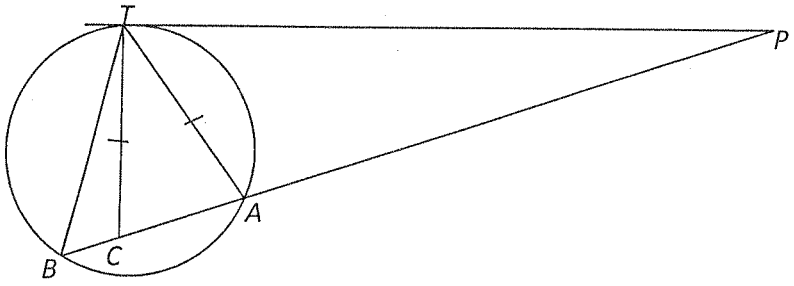
Q 1	Q2	Q3	Q4	Q5	Q6	Q7	TOTAL

QUESTION 1 (12 Marks) *(Start a new booklet)* MARKS

a) Solve the inequation $\frac{1}{x-3} < 3$ 3

b) Find, correct to the nearest degree, the acute angle between the straight lines $y=x$ and $y = -\frac{5}{3}x + 2$ 3

c)



PT is a tangent to a circle and *PAB* is a secant. *C* is a point on *BA* such that $TC = TA$. Prove that $\angle BTC = \angle TPA$. *3*

(d) The interval *PQ* has end points *P*(–3,4) and *Q*(11,12). Find the co–ordinates of k which divides *PQ* externally in the ratio 3:2 *3*

QUESTION 2 **(12 Marks)** *(Start a new booklet)* **MARKS**

a) Solve for x: $3x + 3 = |\, 1 - 2x \,|$ 2

b) If α, β and γ are the roots of the equation $x^3 - 2x^2 + 4x + 1 = 0$, evaluate $(\alpha + 1)(\beta + 1)(\gamma + 1)$ 3

c) Factorise $x^2 - y^2 + x - y$. 2

d) Find in surd form the exact value of $\cos 75°$. 2

e) Prove that
$\sin(A + B) + \cos(A - B) = (\sin A + \cos A)(\sin B + \cos B)$. 3

QUESTION 3 (12 Marks) *(Start a new booklet)* MARKS

a) The polynomial equation f(x) $= 8x^3 + 12x^2 - 18x - 20$, has a root at $x = -2$. 3

Find all roots of f(x) = 0

b) Prove the trigonometric identity: $\frac{\cos 2x}{(\cos x + \sin x)^3} = \frac{\cos x - \sin x}{1 + \sin 2x}$. 3

c) Express 2 sin θ + cos θ in the form R sin($\theta + \alpha$) and hence solve 2 sin θ + cos θ = 1,

$0 \leq \theta \leq 180$. 3

d) The quadratic equation $x^2 + Lx + M = 0$ has one root which is twice the other. 3

Prove that $2L^2 = 9M$.

QUESTION 4 (12 Marks) *(Start a new booklet)* MARKS

a) A vertical flagpole CD of height h metres stand with its base C on horizontal ground. A is a point on the ground due East of C and B is a point on the ground on a bearing 030° from C such that the distance AB is 84 metres. The angles of elevation of D from A and B are α and β respectively where $\tan\alpha = \frac{1}{3}$ and $\tan\beta = \frac{1}{8}$. Find the exact value of h. 4

b) Derive expressions for both $\sin\Theta$ *and* $\cos\Theta$ in terms of t {where $t=\tan\frac{\Theta}{2}$} 3

c) The roots of $x^2 - 6x + k = 0$; differ by 1. Find the value of k. 2

d) A line passing through (7,5) makes a right angle with the line 4x-y=6

At the point P. Find the co-ordinates of P. 3

QUESTION 5 (12 Marks) *(Start a new booklet)* MARKS

(a) Differentiate each of the following expressions with respect to x.

(i) $5x^4 + 3x - 7x^{-1} + 6$. 1

(ii) $7\sqrt{x} + 6$. 1

(iii) $(7 - 4x)^5$. 1

(iv) $\dfrac{4x+1}{x-2}$. 1

(b) The points P and Q have the coordinates $(-1, 0)$ and $(3, 3)$ respectively. 3
If PR is perpendicular to QR, show that the locus of R is
$x^2 + y^2 - 2x - 3y - 3 = 0$.

(c) A parabola has the equation $x^2 - 6x - 8y - 7 = 0$.

(i) Find its vertex. 1

(ii) Find its focus. 1

(iii) What is the equation of the directrix? 1

(d) Solve for x if $4^x - 5.2^x + 4 = 0$. 2

QUESTION 6 (12 Marks) *(Start a new booklet)* MARKS

a) In what ratio does the point (3 , -2) divide the interval AB, where A = (1,-6) and B = (6,4)? 2

b) If $3x^2 + 5x + 4 \equiv (x+1)(ax+b) + c(x^2+1)$, $find\ a, b\ and\ c.$ 3

c) AB is a chord of a circle and PAQ a tangent at A. If R is the mid-point of the major arc cut off by the chord AB and perpendiculars are drawn from R to the tangent and the chord, prove that the perpendicular are equal in length. 4

d) Solve the cubic equation $3x^3 - x^2 - 38x - 24 = 0$ given that one root is double the reciprocal of a second root. 3

QUESTION 7 **(12 Marks)** *(Start a new booklet)* MARKS

a) If $\cos x = \frac{3}{4}$ and $\sin x < 0$ find the exact value of $\sin 2x$. 2

b) The lines $Ax + By = 11$ and $Bx - Ay = 2$ meet at P (2,1). Find the values of A *and* B. 2

c) Sketch the graph of $P(x) = (x+1)(x-3)(x-5)$.

Hence, solve $(x+1)(x-3)(x-5) \geq 0$. 3

d) $2x^3 - 3x^2 - 4x + 6 = 0$ has roots α, β, γ. Without solving the equation, evaluate

i. $(\alpha+2)(\beta+2)(\gamma+2)$

ii. $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

iii. $\alpha^2 + \beta^2 + \gamma^2$ 5

End of paper