SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

AUGUST 2006
TRIAL HIGHER SCHOOL CERTIFICATE
YEAR 12

Mathematics

General Instructions:

- Reading time—5 minutes.
- Working time—3 hours.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may NOT be awarded for messy or badly arranged work.
- Start each NEW section in a separate answer booklet.
- Hand in your answer booklets in 5 sections:
  Section A(Questions 1 and 2),
  Section B(Questions 3 and 4),
  Section C(Questions 5 and 6),
  Section D(Questions 7 and 8),
  Section E(Questions 9 and 10).

Total marks—120 Marks

- Attempt questions 1–10.
- All questions are of equal value.

Examiner: Mr P.Bigelow

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

Section A — Start a new booklet

Marks

Question 1 (12 marks)

(a) Find integers a and b such that $x^2 + 6x + 14 \equiv (x+a)^2 + b$. $\boxed{2}$

(b) Find $e^{2\cdot 5}$ correct to 2 decimal places. $\boxed{2}$

(c) What is the exact value of $\cos \dfrac{7\pi}{6}$? $\boxed{2}$

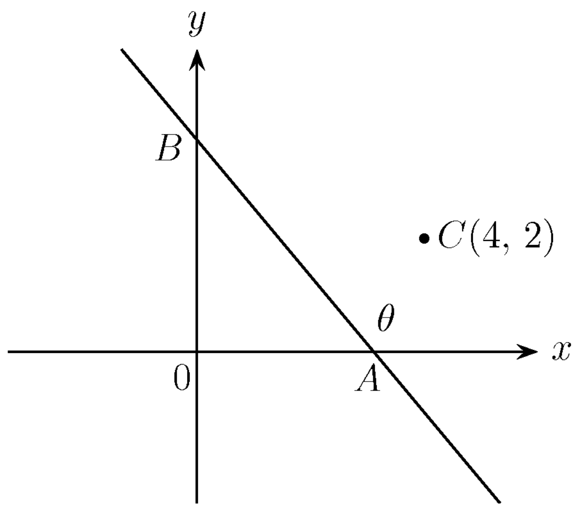
(d) Solve $|4 - x| = 7$. $\boxed{2}$

(e) By rationalising the denominator, express $\dfrac{4}{\sqrt{5} - \sqrt{3}}$ in simplest form. $\boxed{2}$

(f) Solve $a^2 = 12a$. $\boxed{2}$

Question 2 (12 marks)

(a)



The line $4x + 3y - 12 = 0$ has x and y intercepts A and B respectively and makes an angle θ with the positive direction of the x-axis.
C is the point $(4,\ 2)$.

(i) Write down the coördinates of points A and B. $\boxed{2}$

(ii) Find the value of θ to the nearest degree. $\boxed{2}$

(iii) Find the perpendicular distance of C from the line $4x+3y-12=0$. $\boxed{2}$

(iv) Find the area of the triangle ABC. $\boxed{2}$

(b) Solve the pair of simultaneous equations $\boxed{2}$

$$\begin{aligned} 3x - y &= 16, \\ x + 4y &= 1. \end{aligned}$$

(c) Consider the parabola $\boxed{2}$

$$y = x^2 - 4x + 8.$$

Find the coördinates of the focus.

Section B — Start a new booklet

Marks

Question 3 (12 marks)

(a) A vessel sails 12 km due north from a port P to A. A second boat sails 20 km from P to B on a bearing of $120°$.

(i) What is the distance AB? **2**

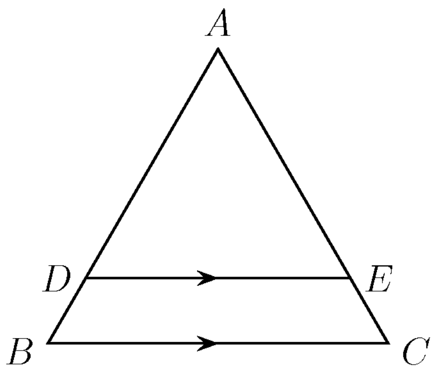
(ii) What is the bearing of B from A, correct to the nearest minute? **2**

(b) Differentiate

(i) $\dfrac{2}{x^4}$ **1**

(ii) $\sin\left(x^3\right)$ **1**

(iii) $x \tan x$ **2**

(c) 

In the diagram $DE // BC$. $AB = 16$ cm, $AE = 18$ cm and $EC = 6$ cm.

(i) Prove that $\triangle ADE /// \triangle ABC$. **2**

(ii) Find the length of DB. **2**

Marks

Question 4 (12 marks)

(a) Evaluate $\displaystyle\int_0^1 \frac{dx}{1+x}$ (leave your answer in exact form). **2**

(b) Solve $\sqrt{3}\tan x = 1$ for $0 \le x \le 2\pi$. **2**

(c) Simplify $\displaystyle\sqrt{\frac{1-\cos^2 A}{1+\tan^2 A}}$. **2**

(d) Find the slope of the tangent to the curve $y = \cos\left(x + \frac{\pi}{3}\right)$ at the point $\left(0,\, \frac{1}{2}\right)$. **2**

(e) Find

(i) $\int \cos 2x\, dx$ **1**

(ii) $\displaystyle\int \frac{4}{e^{3x}}\, dx$ **1**

(f) Find the values of c for which the equation $x^2 + (c-2)x + 4 = 0$ has real roots. **2**

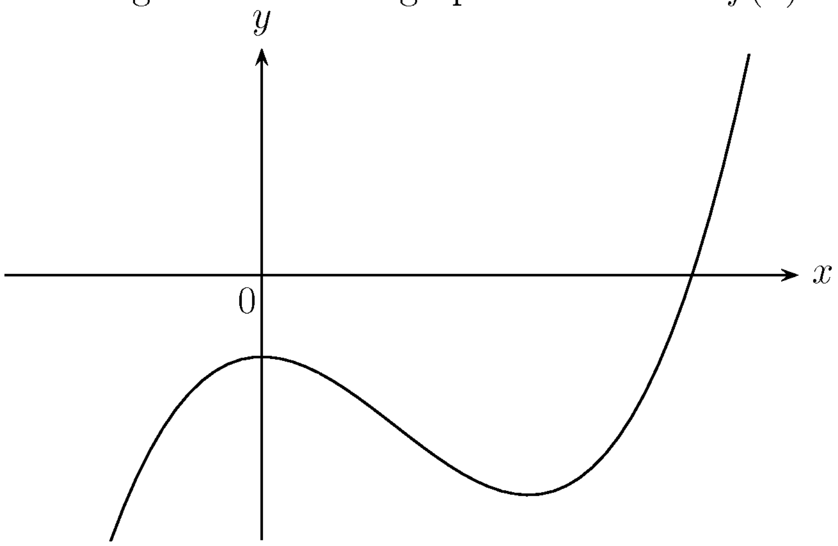
Section C — Start a new booklet

Marks

Question 5 (12 marks)

(a) Write down a quadratic equation with roots $1+\sqrt{3}$ and $1-\sqrt{3}$. **2**

(b) The diagram shows the graph of a function $f(x)$. **2**



(i) Copy this graph.

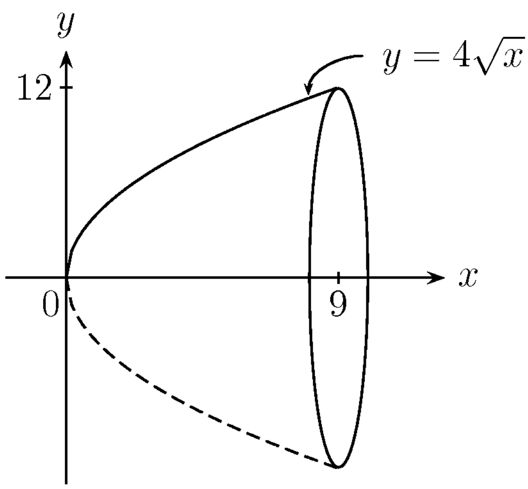
(ii) On the *same* set of axes, draw a sketch of the derivative $f'(x)$ of the function.

(c) The positive multiples of 7 are 7, 14, 21, ...

(i) What is the largest multiple of 7 less than 1200? **2**

(ii) What is the sum of the positive multiples of 7 which are less than 1200? **2**

(d)



The region enclosed by the curve $y = 4\sqrt{x}$ and the x-axis between $x = 0$ and $x = 9$ is rotated about the x-axis, as shown in the diagram. Find the volume of revolution. **2**

(e) The graph of $y = f(x)$ passes through $(2, 5)$ and $f'(x) = 3x^2+2$. Find $f(x)$. **2**

Marks

Question 6 (12 marks)

(a) Given the curve with equation

$$y = x^3 - 3x^2 - 9x + 2.$$

(i) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. 2

(ii) Find the coördinates of the stationary points and determine their nature. 2

(iii) Sketch the graph of the function for the domain $-2 \le x \le 5$. 1

(iv) State the maximum value of the function over this domain. 1

(b) (i) Copy and then complete the table for $y = \operatorname{cosec} \dfrac{\pi x}{6}$. 1

x	1	2	3
y			

(ii) Using Simpson's Rule with three function values find an approximate value for 2

$$\int_1^3 \operatorname{cosec} \frac{\pi x}{6}\, dx.$$

(c) The population of Goldtown is given by $P = 30\,000e^{-0{\cdot}08t}$.

(i) Find the time to the nearest year for the population to halve. 1

(ii) Find the decline in the population of Goldtown during the ninth year. 2

Section D — Start a new booklet

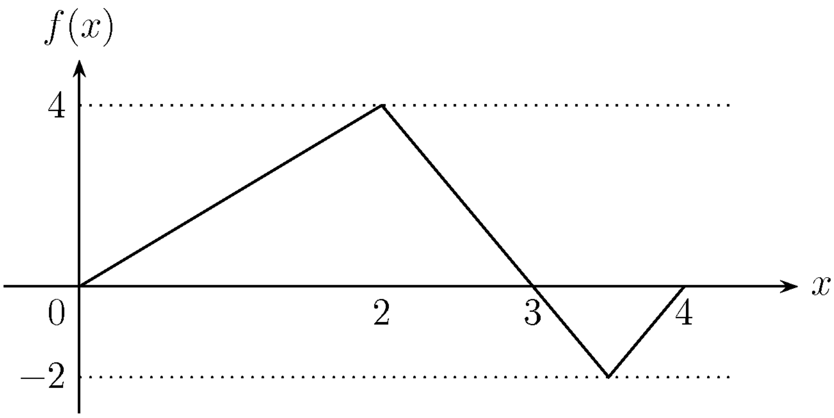
Marks

Question 7 (12 marks)

(a) Make a sketch of a continuous curve $y = f(x)$ that has the following properties: 2
$f(x)$ is odd, $f(3) = 0$, $f'(1) = 0$.
$f'(x) > 0$ for $x > 1$,
$f'(x) < 0$ for $0 \le x < 1$.

(b) A bag contains three times as many red marbles as white marbles. If a marble is chosen at random, what is the probability that it is white? 1

(c) Find $\int_0^4 f(x)\,dx$ for the following function. 2



(d) Simone borrows \$20 000 over 4 years at a rate of 1% compound interest per month. If she pays off the loan in 4 equal yearly instalments find

(i) the amount she will owe after one month. 1

(ii) the amount she will owe after the first year, just before she pays the first instalment. 1

(iii) the amount of each instalment. 2

(iv) the total amount of interest she will pay. 1

(e) Find the limiting sum of the geometric series 2

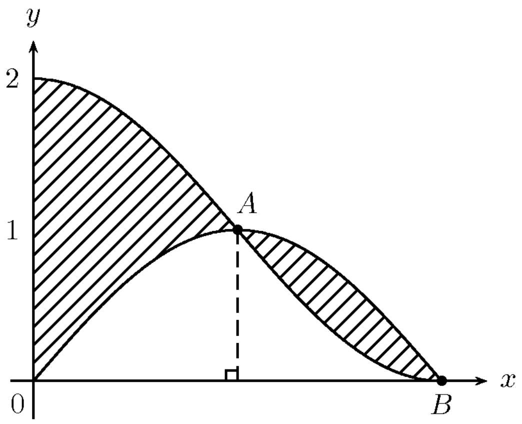
$$4 - 2\sqrt{2} + 2 - \cdots$$

Marks

Question 8 (12 marks)

(a) Evaluate $\displaystyle\int_0^{\ln 4} e^{-2x}\,dx$. 2

(b) 3



The graphs of $y = \sin x$ and $y = 1 + \cos x$ are shown intersecting at $A(\frac{\pi}{2}, 1)$ and $B(\pi, 0)$.

Calculate the total area of the two shaded regions.

(c) Water is being released from a dam. The rate of flow, F megalitres per hour is given by $F = t(t-12)^2$, where t is the numbere of hours since the flow began.
The function applies until the flow ceases.

(i) For how long does the water flow? 2

(ii) Find the maximum rate of flow. 2

(iii) What is the total volume of water released? 3

Section E — Start a new booklet

Marks

Question 9 (12 marks)

(a) The displacement of a particle x metres from the origin, at time t seconds, is given by

$$x = \frac{1}{3}t^3 - 6t^2 + 27t - 18.$$

(i) Find expressions for velocity and acceleration. $\boxed{2}$

(ii) When is the acceleration zero? $\boxed{2}$

(iii) Where is the particle at this time and what is its velocity? $\boxed{2}$

(b) A uniform cube has three green faces, two white faces, and one red face. If a player throws a green face they win; if red, they lose; and if white they throw again. Robert will throw until he either wins or loses. What is the probability that

(i) Robert wins with his third throw? $\boxed{2}$

(ii) Robert wins with his first, second, or third throw? $\boxed{2}$

(iii) Robert wins? $\boxed{2}$

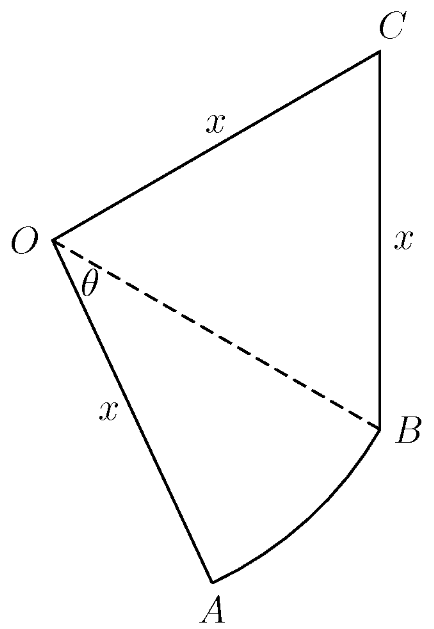
Marks

Question 10 (12 marks)

(a) Solve for x (correct to 3 significant figures) $\boxed{2}$

$$3^{x-2} = 50$$

(b)



The diagram shows a sector OAB of a circle, centre O, and radius x metres. Arc AB subtends an angle θ radians at O. An equilateral triangle BCO adjoins the sector.

(i) Write down expressions for the

(α) area of sector OAB $\boxed{1}$

(β) area of the triangle BCO $\boxed{1}$

(γ) length of the arc AB. $\boxed{1}$

(ii) Hence write down expressions for the

(α) area $\boxed{1}$

(β) perimeter of the figure $OABC$. $\boxed{1}$

(iii) The perimeter of this figure is $(12 - 2\sqrt{3})$ metres.

(α) For what value of x is the area a maximum? $\boxed{3}$

(β) Show that the maximum area is $(6 - \sqrt{3})\,\text{m}^2$. $\boxed{2}$

End of Paper

STANDARD INTEGRALS

$$
\begin{aligned}
\int x^n dx &= \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0 \\
\int \frac{1}{x} dx &= \ln x, \quad x > 0 \\
\int e^{ax} dx &= \frac{1}{a}e^{ax}, \quad a \neq 0 \\
\int \cos ax dx &= \frac{1}{a}\sin ax, \quad a \neq 0 \\
\int \sin ax dx &= -\frac{1}{a}\cos ax, \quad a \neq 0 \\
\int \sec^2 ax dx &= \frac{1}{a}\tan ax, \quad a \neq 0 \\
\int \sec ax \tan ax dx &= \frac{1}{a}\sec ax, \quad a \neq 0 \\
\int \frac{1}{a^2+x^2} dx &= \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0 \\
\int \frac{1}{\sqrt{a^2-x^2}} dx &= \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a \\
\int \frac{1}{\sqrt{x^2-a^2}} dx &= \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0 \\
\int \frac{1}{\sqrt{x^2+a^2}} dx &= \ln\left(x+\sqrt{x^2+a^2}\right)
\end{aligned}
$$

NOTE: $\ln x = \log_e x, \quad x > 0$