N.S.W. DEPARTMENT OF EDUCATION

HIGHER SCHOOL CERTIFICATE EXAMINATION 1974

MATHEMATICS - PAPER B (2F) - (EQUIVALENT OF 3U AND 4U - 1ST PAPER)

Instructions: Time allowed 3 hours. All questions may be attempted. (Questions are not of equal value.) In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

Mathematical tables will be supplied. Approved slide rules or calculators may be used.

QUESTION 1 (12 Marks)

(i) Find a primitive of e^{2x}.

(ii) Find the second derivative of $\log_e (\cos x)$.

(iii) Evaluate $\int_1^4 t\sqrt{t}\, dt$.

(iv) Use Simpson's Rule with three points to approximately evaluate $\int_2^4 \log_{10} x\, dx$. (Use mathematical tables)

QUESTION 2 (9 Marks)

(i) Find the equation of the tangent to the curve $y = x^4 + 1$ at the point where $x = 1$.

(ii) Find the acute angle between the lines $2y - x - 1 = 0$ and $y - 3x + 2 = 0$.

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(iii) The equations of two given planes p and q are $ax - 2y + z = 1$ and $3x + by + z = 0$.

(a) If p is parallel to q, find a and b.

(b) Find a pair of values of a and b which would instead make p perpendicular to q.

QUESTION 3 (9 Marks)

(i) Find the area under the curve $y = \dfrac{x}{x^2 + 1}$ between $x = 0$ and $x = 1$.

(ii) Find the x- coordinates of all the stationary points of $y = \cos (x^2)$.

(iii) Find the set of values of x for which the expression $12 + x - x^2$ is positive.

QUESTION 4 (10 Marks)

(i) Find the equation of the curve $y = \dfrac{-1}{x^2 + 4x + 5}$ in the new (X, Y) coordinate system in which the origin is at $x = -2$, $y = -1$, and the directions of the axes are unchanged.

(ii) Find the focus and vertex of the parabola $2y = x^2 + 6x + 11$.

(iii) Prove by mathematical induction that

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{n(n+1)} = 1 - \frac{1}{n+1}$$

QUESTION 5 (10 Marks)

(i) Given the formula $\sin (x + y) = \sin x \cos y + \cos x \sin y$

(a) deduce that $\sin (x + \pi/2) = \cos x$

(b) express $\sqrt{3} \sin x + \cos x$ in the form $C \sin (x + \alpha)$ where C is positive (giving numerical values for C and α).

(ii) Find all solutions in the interval $-\pi \leq x \leq \pi$ of the equation $\cos x + \sin^2 x = 5/4$.

QUESTION 6 (10 Marks)

(i) Sketch (not on graph paper) the graph of $y = \sin^{-1} x$. State the domain and range of the function.

(ii) If $f(x) = \sin^{-1} x$, show that $f'(x) = (1 - x^2)^{-\frac{1}{2}}$, and sketch the graph of $f'(x)$. Describe the behaviour of this graph near $x = +1$ and $x = -1$.

(iii) State the remainder theorem for polynomials. Hence, or otherwise, factorise $x^3 + 3x^2 - 9x + 5$ into linear factors.

QUESTION 7 (10 Marks)

(i) Two balls are drawn in succession (without replacement) from a bag containing 18 red, 14 blue, and 4 white balls. What is the probability that

(a) the first ball is red?

(b) both balls are red?

Express your answers as fractions.

(ii) A biased coin has a probability $p = 1/3$ of giving the result 'heads'.

(a) If the coin is thrown n times, state without proof the expected number of heads.

(b) If the coin is thrown 3 times, find the probability of obtaining each of the results:

(1) 3 heads (2) 2 heads (3) 1 head

Hence determine the expected number of heads for 3 throws. Verify that this result for 3 agrees with your answer to part (a).

QUESTION 8 (10 Marks)

A particle moves on a line so that its distance from the origin at time t is x and its velocity is v.

(i) Prove that $\frac{d^2x}{dt^2} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

(ii) If the acceleration satisfies $\frac{d^2x}{dt^2} = n^2(3 - x)$ where n is a constant, and if the particle is released from rest at $x = 0$, show that $\frac{1}{2}v^2 - n^2(3x - \frac{1}{2}x^2) = 0$.

Hence show that the particle never moves outside a certain interval.

(iii) Show that the expression for the acceleration given in (ii) can be simplified by an appropriate change of origin. Hence state without proof the period of the motion.

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QUESTION 9 (10 Marks)

(i) Find the point where the line $\frac{x - 3}{5} = \frac{2y - 1}{7} = \frac{z - 1}{4}$ meets the plane $2x - 5y + 3z = 20$.

(ii) (a) A sphere of radius 5 is centred at the origin, and a second sphere of the same radius is centred at $(8, 0, 0)$. Write down the equations of these two surfaces.

(b) Find the equation of the plane containing the circle of intersection of the two spheres in part (a). Also find the radius of the circle.

QUESTION 10 (10 Marks)

A canned fruit producer wishes to minimize the area of sheet metal used in manufacturing cans of a given volume. Find the ratio of radius to height for the desired can. (Treat the can as a circular cylinder with closed ends.)