SYDNEY GIRLS HIGH SCHOOL
TRIAL HIGHER SCHOOL CERTIFICATE

1999

MATHEMATICS

3 UNIT (Additional) and 3/4 UNIT (Common)

Time allowed - 2 hours
(Plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES

NAME ____________________

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Each question attempted should be started on a new sheet. Write on one side of the paper only.

QUESTION ONE

a) If $3\cot x = 4$, find the value of

$$\frac{6\sin x - 4\cos x}{\operatorname{cosec} x + \sec x} \quad (x \text{ is acute})$$ [2]

b) Evaluate $\int_0^2 xe^{x^2}\,dx$ [2]

c) Differentiate $x^3 \sin^{-1} 4x$ [2]

d) Given $\log_a b = 0.3$ and $\log_a c = 0.4$, find $\log_a\left(\frac{b}{c}\right) + \log_a ac$ [2]

e) Find the exact value of $\cos 2x$ if $\sin x = \sqrt{3} - 1$ [2]

f) A cosine curve has an amplitude of 5 and a period of 3π. It has a minimum turning point at (0, 5). Find it's equation. [2]

QUESTION TWO

a) Write down the domain of the function

$$y = \frac{1}{x^2 + 5x + 6}$$ [1]

b) The roots of $x^3 + 5x^2 + 8x + 2 = 0$ are $\alpha, \beta,$ and γ [4]

i) Find $(\alpha + 1) + (\beta + 1) + (\gamma + 1)$

ii) Find $(\alpha + 1)(\beta + 1)(\gamma + 1)$

c) The half life of a radioactive substance is 24 hrs. How long will it take for only 15% of the substance to remain. (Assume $M = M_o e^{-kt}$ and give your answer to the nearest hour) [2]

d) Find the equation of the tangent to the curve $y = e^{\tan^{-1} x}$ at the point where it cuts the y-axis. [2]

e) The area of the region below the curve $y = e^{-x}$ and above the x-axis, between x= 0.5 and x=1.5 is rotated about the x-axis. Find the volume of the solid generated. (Answer correct to 2 decimal places) [3]

QUESTION FIVE

a) [9]

i) Find the remainder when $P(x) = x^3 - (k+1)x^2 + kx + 12$ is divided by $A(x) = x - 3$

ii) Find k if $P(x)$ is divisible by $A(x)$

iii) Find the zeros of $P(x)$, for this value of k

iv) Solve $P(x) > 0$

b) It is known that $\log_e x + \sin x = 0$ has one root close to $x = 0.5$. Use one application of Newton's method to obtain a better approximation of the root correct to 3 decimal places. [3]

QUESTION SIX

a) Show that $7^n + 2$ is divisible by 3, for all positive integral n. [3]

b) Find the general solution of $\cos 2x = \sin x$ [3]

c) Find the area bounded by the curve $y = \frac{1}{\sqrt{25 - x^2}}$, the x axis and the ordinates at $x = -2$ and $x = 2$.
(Answer correct to 2 decimal places) [2]

d) Differentiate $\log_e(\sec x + \tan x)$ and hence find $\int_0^{\frac{\pi}{4}} \sec x \, dx$, in simplest exact form. [4]

QUESTION SEVEN

[5]

a) A Particle moving on a horizontal line has a velocity of v m / s given by $v^2 = 64 - 4x^2 + 24x$

i) Prove that the motion is simple harmonic

ii) Find the centre of the motion

iii) Write down the period and amplitude of the motion

iv) Initially the particle is at the centre of the motion and moving to the left. Write down an expression for the displacement as a funtion of time.

[4]

b) i)Write the expression for $\sqrt{2}\cos\theta + \sin\theta$ in terms of t. (where $t = \tan\frac{\theta}{2}$)

ii) Hence or otherwise solve $\sqrt{2}\cos\theta + \sin\theta = 1$ for $0^\circ < \theta < 360^\circ$

c) Find $\int \frac{x\,dx}{(25 + x^2)^{\frac{3}{2}}}$ using the substitution $x = 5\tan\theta$ [3]