EXT 2 TERM 2 2005

QUESTION 1 (15 Marks) **Marks**

(a) The base of a solid is the region in the first quadrant bounded by the curve $y = \sin x$, the $x-$ axis and the line $x = \frac{\pi}{2}$. Cross-sections of this solid perpendicular to the $x-$ axis are equilateral triangles with one side lying in the base of the solid. Find the exact volume of the solid. 6

(b) A particle of mass m is projected upwards in a resistive medium where the force against the motion is inversely proportional to v, where v is the velocity of the particle, ie; $m\ddot{x} = -mg - \frac{mK}{v}$, where K is a constant and g the acceleration due to gravity. 9

(i) If the initial velocity of projection is U m/s, show that the time taken by the particle as a function of its velocity is given by an equation of the form $t = A + B \ \ln C$, and find expressions for A, B and C.

(ii) Derive an expression for the time taken to reach its maximum height in this medium.

QUESTION 2 (15 Marks)

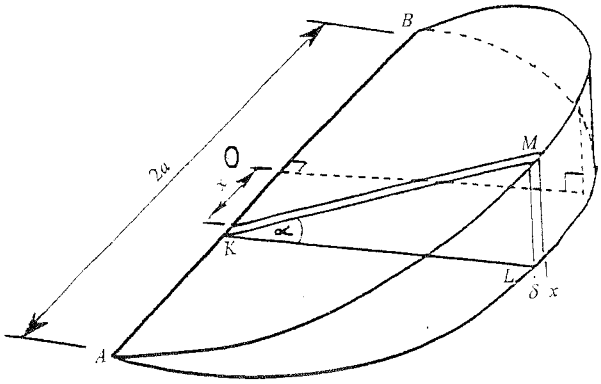
(a) A hole of diameter a centimetres is bored through the centre of a solid sphere of diameter $2a$ centimetres. Use the "method of cylindrical shells" to find the exact volume of the remaining solid. 6

(b) The solid wedge as shown was made by slicing a right cylinder of radius a at an angle α through diameter AB of its base. A triangular slice of thickness δx perpendicular to the base and line AB is positioned at distance x from the centre o as in the diagram. 9

(i) Show that ML, the height of the triangular slice is $\sqrt{a^2 - x^2} \ \tan \alpha$.

(ii) Deduce a formula for the volume of the wedge.

(iii) Given that $\alpha = \frac{2\pi}{n}$ and tan α decreases whilst n increases, find the volume of n identical wedges with a common diameter AB.



QUESTION 3 (15 Marks)

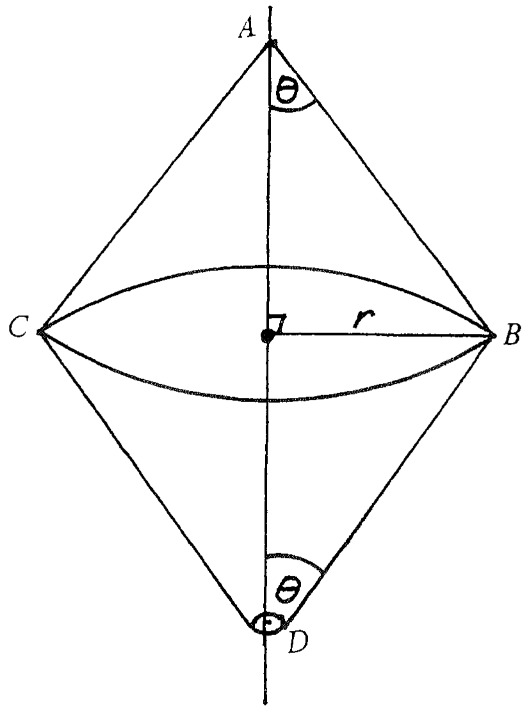
(a) The acceleration of a body of unit mass moving towards earth under gravitational attraction varies inversely as the square of its distance from the centre of the earth, 5

ie; $\ddot{x} = \frac{-k}{x^2}$ (k a constant).If the body starts from rest at a distance a from the centre of the earth, show that its speed at a distance x from the centre of the earth is $v = \sqrt{\frac{2k(a-x)}{ax}}$.

(b) Two equal masses are connected to the ends of two rods AB and AC (in the same plane) of equal length which are hinged together at the point A to a vertical shaft. Two supporting rods DB and DC are also hinged together to a ring D which can slide up and down the shaft. The rods $AB = AC = DB = DC = L$, the masses at B and C are M kg and the ring has mass m kg. 10

(i) Copy the diagram onto your answer sheet showing all the acting forces.

(ii) Show that the semi-vertical angle θ when rotating at a speed of ω radians/second is given by $\sec\theta = \frac{ML\omega^2}{(M+m)g}$.



QUESTION 4 (15 Marks)

(a) A square $ABCD$ has sides of length 1 unit. The asymptotes of hyperbola $xy = c^2$ lie on sides AB and AD of the square, and the point $C(1,1)$ is one of the foci. Show that the hyperbola bisects the other two sides. **3**

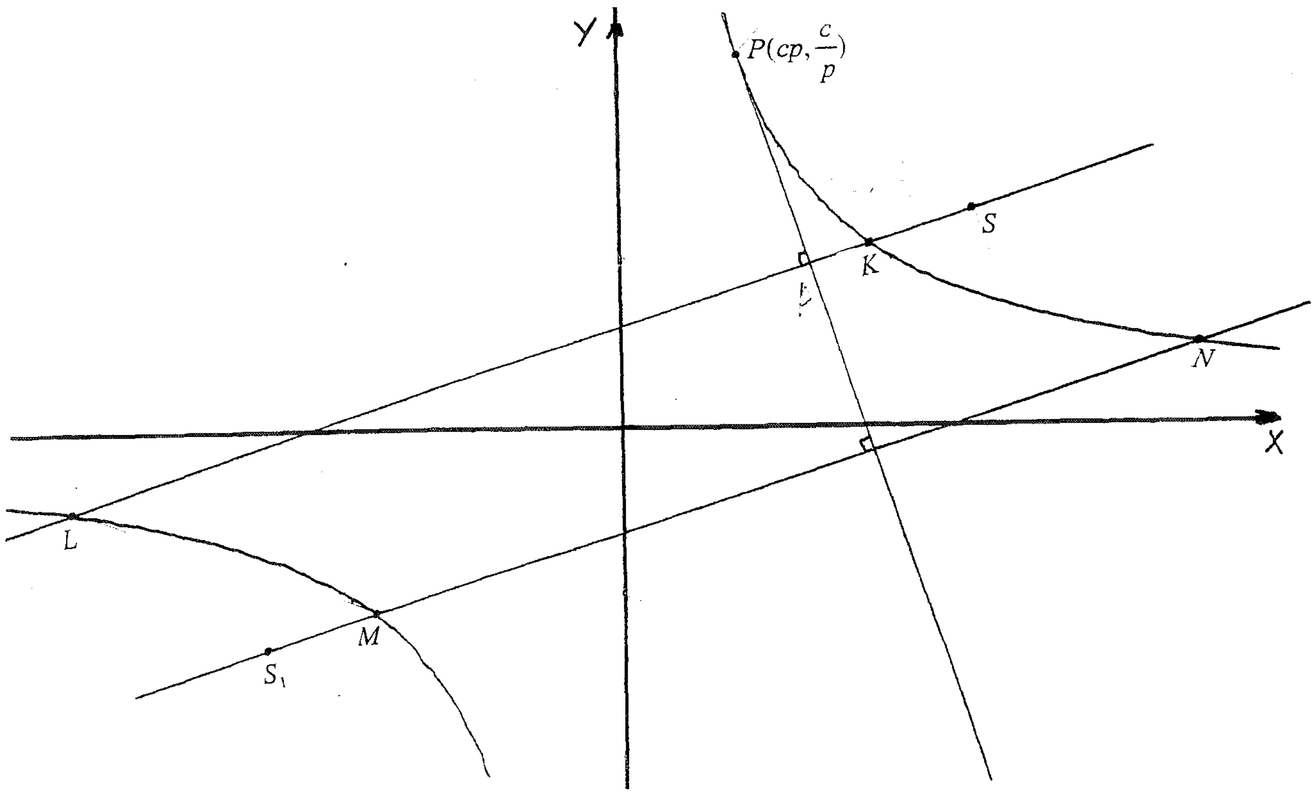
(b) The perpendiculars drawn from the foci S and S_1 of hyperbola $xy = c^2$ to the tangent at a point $P(cp, \frac{c}{p})$ meet the curve at K, L, M, N as shown in the diagram. **12**

(i) Find the equation of the line SKL.

(ii) Find, as a function of p, an expression for the parameter "k" at the point K.

(iii) Hence, or otherwise, prove that $KLMN$ is a parallelogram.

(iv) Show that the sides KN and LM of the parallelogram are perpendicular to the diameter through P.



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