HORNSBY GIRLS' HIGH SCHOOL

2007
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (84)

- Attempt Questions 1-7
- All questions are of equal value

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All Questions are of equal value

Begin each question on a NEW SHEET of paper, writing your name and question number at the top of the page. Extra paper is available.

Question 1 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) Let A (3,4) and B (-2,5) be points in the plane. Find the co-ordinates of the Point C which divides the interval externally in the ratio 1:3. **1**

(b) A committee of 4 boys and 4 girls is chosen from a class of 8 boys and 6 girls

(i) Calculate the probability that the committee includes the eldest boy and excludes the eldest girl **2**

(ii) Find the number of ways the committee of 4 boys and 4 girls can be arranged in a circle so that each of the girls are separated. **1**

(c) For the function $f(x) = \frac{1}{2}\cos^{-1}\frac{1}{3}x$

(i) State the range and the domain of $f(x)$ **2**

(ii) Sketch the graph of $y = f(x)$ **1**

(d) Using the expansion of $\sin(A-B)$ or otherwise prove the exact value of $\sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$ **2**

(e) The curves $y = \log_e x$ and $y = -x^2 + 1$ intersect at the point P (1,0). Find the acute angle between the tangents to the curves at the point P. Give your answer to the nearest degree. **3**

Question 2 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) Solve $|x-1| \leq |x+1|$ **2**

(b) Express $\cos\left(2 \sin^{-1} \frac{a}{b}\right)$ in terms of a and b. **2**

(c) The graph of $y = \sin x$ for $\pi/_{12} \leq x \leq \pi$ is rotated about the x axis. Calculate the volume of the solid generated. **3**

(d) Write down the general solution of the equation
$\sqrt{3}\ \cos 2x - \sin 2x = 2$ **3**

(e) In how many ways can the letters of the word GEOLOGIST be arranged so that the letters G will be together? **2**

Question 3 (12 marks) Use a SEPARATE sheet of paper. **Marks**

a) Calculate $\int_0^{\sqrt{\frac{27}{2}}} \frac{1}{9+2x^2}\, dx$ **3**

(b) Calculate the area between the curve $y = sin^{-1}x$, *the* x axis and the line $x=1$ **2**

(c) Prove $\dfrac{\sin 2\theta + \sin\theta}{1+\cos 2\theta + \cos\theta} = \tan\,\theta$ **2**

(d) An eight sided die has 5 green faces and 3 blue faces. If the die is tossed 100 times find the most likely number of green faces and the probability of this occurring (*correct to 3 decimal places)* **3**

(e) Find the value of n, if the coefficients of x^5 and x^6 in the expansion of $(3 + 2x)^n$ have the same value **2**

Question 4 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) A particle travelling in a straight line is governed by the equation $v^2 = 15 + 2x - x^2$ where v is the velocity in m/s and x is the distance travelled in time t seconds.

(i) Prove that the particle undergoes simple harmonic motion **1**

(ii) (1) Find the centre of the motion **1**
(2) Find the amplitude of the motion **1**
(3) Find the period of the motion **1**

(iii) Write down the maximum speed and the maximum acceleration **2**

(iv) Given that the particle was originally at its equilibrium position write down an equation for the position $x = f(t)$ and hence or otherwise find the velocity when $t = \pi/4$ s **3**

(b) Prove by Mathematical Induction that:
$2(1!) + 5(2!) + 10(3!) + \ldots\ldots\ldots + (n^2 + 1)n! = n(n+1)!$
for all positive integers $n \geq 1$ **3**

Question 5 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) The Polynomial $2x^3 + ax^2 + bx + 6$ has $(x - 1)$ as a factor and leaves a remainder of -12 when divided by $(x - 2)$. Find the values of a and b. **2**

(b) Solve the equation $x^3 + 2x^2 - 5x - 6 = 0$ given that one of its roots is equal to the sum of the other two roots **2**

(c) Two straight roads intersect at right angles. At a given instant a car is 30 km from the intersection and is travelling towards it at 50 km/h while the truck is 40 km from the intersection and is travelling away from it at 40 km/h.. At what rate is the direct distance between them changing at this instant? **2**

(d) Show that $\frac{d}{dx}\left[\sin^{-1}(\tfrac{1}{2}\sin x)\right] = \frac{\cos x}{\sqrt{4-\sin^2 x}}$

hence evaluate $\int_0^{\pi/2} \frac{\cos x}{\sqrt{4-\sin^2 x}}\,dx$ **3**

(e) Using the substitute $u = 1 - 3x$

evaluate $\int_0^{\frac{1}{3}} 3x(1-3x)^4\,dx$ **3**

Question 6 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) A rocket is fired at a speed of V m/s at an angle θ to the horizontal where $\tan\theta = \frac{4}{3}$. Neglecting air resistance and using acceleration due to gravity as $g = 10\text{m/s}^2$

(i) Show that the horizontal position x and the vertical position y of the rocket at any time t is given by $x = \frac{3Vt}{5}$ and $y = \frac{4Vt}{5} - 5t^2$ **2**

(ii) The rocket hits a target which has the coordinates (324,27) Find the value of V **3**

(b) Using the expansion of $(1+x)^n$

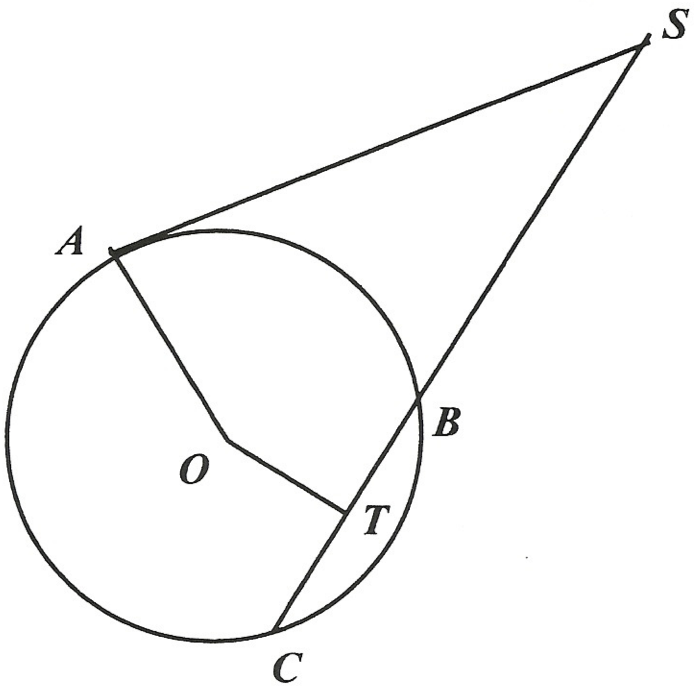
Show $\frac{-1}{n+1} = -{}^nc_0 + \frac{1}{2}({}^nc_1) - \frac{1}{3}({}^nc_2) + \text{-------------} + \frac{(-1)^{n+1}}{n+1}({}^nc_n)$ **3**

(c) A, B, C are three points on the circumference of a circle, centre O. The tangent at A meets CB produced at S. T is the mid point of BC.

Prove that

(i) $TOAS$ is a cyclic quadrilateral **2**

(ii) $\angle OAT = \angle OST$ **2**



Question 7 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) The tangent at $P(2ap, ap^2)$ To the parabola $x^2 = 4ay$ meets the x axis at A.

(i) Find the co-ordinates of A **2**

(ii) If S is the Focus of this parabola. Prove SA is perpendicular to AP **1**

(iii) Show that the equation of the locus of the Centre C of the circle which passes through the three points P, S and A is a parabola and write down the coordinates of its vertex **3**

(b) The rate at which a body warms or cools in air is proportional to the difference between its temperature T and the constant temperature of its surroundings S. The temperature obeys the differential equation $\frac{dT}{dt} = k(T - S)$. You may assume the solution $T = S + Ae^{kt}$

(i) A cup of boiling water at $100°$C and a cup of iced water at $0°$C are placed simultaneously in a room which has a temperature of $25°$C. After 5 minutes the temperature of the boiling water has fallen to $55°$C and the temperature of the iced water has risen to $15°$C. Find the time at which the temperature of the two liquids differs by $10°$C. *(Give your answer correct to two decimal places)* **4**

(ii) Draw a graph of the behaviour of the temperature T of both liquids as t becomes large. **2**

End of Examination