N.S.W. DEPARTMENT OF EDUCATION

HIGHER SCHOOL CERTIFICATE EXAMINATION 1979

MATHEMATICS 3 UNIT (AND 4 UNIT - FIRST PAPER)

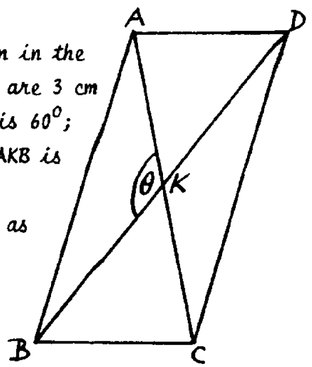
Instructions: Time allowed 3 hours. All questions may be attempted. All questions are of equal value. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.
Mathematical tables will be supplied. Approved slide rules or calculators may be used.

QUESTION 1

(i) Find the exact value of $\frac{2}{\sqrt{5}} - \frac{275}{1000}$ as a fraction in its lowest terms.

(ii) Express $\frac{x+1}{x^2 - x} - \frac{x-1}{x^2 + x}$ as a fraction in its lowest terms.

(iii) ABCD is a parallelogram (as shown in the diagram) in which the lengths of AB and AD are 3 cm and 1 cm respectively, and the angle ABC is 60°; also the diagonals cut at K and the angle AKB is denoted by θ.
Calculate the exact values (giving answers as rational numbers or surds) of:
(a) the length of the diagonal AC
(b) the length of the diagonal BD
(c) $\cos\theta$



QUESTION 2

(i) (a) Find the equation of the line ℓ through $(1, 0)$ which passes through the point of intersection of the line $y = 2x + 1$ and the y-axis.

(b) Find the exact value of the tangent of the acute angle between the line ℓ and the line $y = 2x + 1$.

(ii) (a) Differentiate $x \log x + \sin^2 x$

(b) Find a primitive (indefinite integral) for $\sqrt{x} - e^{-x}$

(iii) Find all real numbers x for which $|x + 1| > |x - 1|$

QUESTION 3

(i) Find (a) $\int_0^{\pi} (2 \sin x - \sin 2x)\, dx$ (b) $\int_0^2 \frac{dx}{x^2 + 4}$

(ii) A plane region is bounded by the curves $y = \sqrt{x}$ and $y = \frac{1}{\sqrt{x}} + \sqrt{x}$, and by the lines $x = 1$ and $x = 9$.
(a) Find the area of the region
(b) Find the volume of the solid obtained by rotating this region about the x- axis.

QUESTION 4

(i) If $0 \le \theta \le \pi/2$, prove that $\tan\theta = \sqrt{\frac{1 - \cos 2\theta}{1 + \cos 2\theta}}$
Hence show that the exact value of $\tan \pi/8$ is $\sqrt{2} - 1$.

(ii) Find the largest positive value of x for which

$(x + \frac{1}{x})^2 - 6(x + \frac{1}{x}) + 8 = 0$, expressing your answer first as a rational number or surd, and then giving its value correct to two decimal places.

QUESTION 5

(i) Find the minimum value of $x + \frac{900}{x}$ for $x > 0$, giving reasons for your answer.

(ii) A cargo service operates by running a ship between port A and port B at a constant speed of v kilometres per hour.
For a given v, the cost per hour of running the ship is $9000 + 10v^2$ dollars.
Find the value of v which minimizes the cost of the trip.

QUESTION 6

(i) A parabola in the Cartesian (x, y) plane has its vertex at $(-1, -2)$ and its focus at $(-1, -3)$. Derive an inequality in x and y which is satisfied by the coordinates of a point P (x, y) if and only if P is closer to the focus of the parabola than it is to the directrix of the parabola.

(ii) The velocity $v(t)$ of a particle moving along the x- axis is given in terms of the time t by: $v(t) = \cos t - \sqrt{3} \sin t$.
If $x(t)$ denotes the position of the particle at time t, show that for every possible choice of times t_1 and t_2, $|x(t_1) - x(t_2)| \le 4$.

OMIT

QUESTION 7

A sphere S has equation $x^2 + y^2 + (z - 1)^2 = 9$. Find its centre and radius.

OMIT

Find the equations of the line through the centre of S and perpendicular to the plane $x + 2y + 2z = c$. Hence, or otherwise, find the positive value of c for which the plane touches S, and the coordinates of the point of contact P.

Also find the value of c for which the corresponding plane intersects S in a circle of radius 3.

QUESTION 8

In a raffle, there is one first prize of \$ 100, one second prize of \$ 20, and one third prize of \$ 10. There are 100 tickets in the raffle, and the prize winning tickets are drawn consecutively without replacement, with the first ticket drawn winning first prize.

Find the probabilities (giving answers as rational numbers) that:

(i) a person buying one ticket in the raffle wins

(a) first prize (b) at least \$ 20 (c) a prize

(ii) a person buying two tickets in the raffle wins

(a) first prize (b) at least \$ 20

QUESTION 9

(i) Sketch (not on graph paper) the curve $y = \cos x$ for $-2\pi \leq x \leq 2\pi$.

(ii) Define the function $\sin^{-1} x$, specifying its domain and range.

Sketch (not on graph paper) the curve $y = \sin^{-1} x$.

(iii) Find the range of the function $\cos(\sin x)$.

(iv) Sketch (not on graph paper) the curve $y = \sin^{-1}(\cos x)$ for $-\pi/2 \leq x \leq \pi/2$

QUESTION 10

(i) The first three terms of an arithmetic series are 50, 43, 36.

(a) Write down a formula for the n-th term

(b) If the last term of the series is -27, how many terms are there in the series?

(c) Find the sum of the series.

(ii) A loan of \$ 1000 is to be repaid by equal annual instalments, repayments commencing at the end of the first year of the loan. Interest, at the rate of 10 per cent, is calculated each year on the balance owing at the beginning of that year, and added to that balance.

1979 HSC 3U (AND 4U - 1ST PAPER) Q 10 – Remainder

If the annual instalment is P dollars, prove that:

(a) the amount owing at the beginning of the second year of the loan is $(1100 - P)$ dollars

(b) the amount owing at the beginning of the third year of the loan is $(1210 - 2.1\,P)$ dollars

(c) if the loan (including interest charges) is exactly repaid at the end of n years, then $P = 100/\left(1 - \frac{1}{(1.1)^n}\right)$