

ST IGNATIUS COLLEGE RIVERVIEW



TASK 4

YEAR 12

2004

EXTENSION 2

TRIAL HSC EXAMINATION

Time allowed: 3 hours + 5 minutes reading time.

Instructions to Candidates

- Attempt **all** questions
- Show all necessary working.
- Marks may be deducted for missing or poorly arranged work.
- Board approved calculators may be used.
- Each question attempted must be returned in a *separate* writing booklet clearly marked Question 1, Question 2 etc, on the cover
- **Each booklet must have your name and the name of your mathematics teacher written on the cover.**

Question 1	{15 marks} Use a SEPARATE writing booklet.	Marks
a	<p>If $Z_1 = 1 + 2i$, $Z_2 = 2 - i$ and $Z_3 = 1 - \sqrt{3}i$, Express in the form $(a + bi)$ where a and b are real.</p> <p>(i) $Z_1 + Z_2$</p> <p>(ii) $\frac{1}{Z_2}$</p> <p>(iii) $(Z_1)^3$</p>	<p>1</p> <p>1</p> <p>2</p>
b	Express $\frac{4+3i}{3+i}$ in the form $(a + bi)$ where a and b are real numbers.	2
c	<p>(i) Express $Z = \sqrt{3} + i$ in modulus- argument form.</p> <p>(ii) Hence, show that $Z^7 + 64Z = 0$.</p>	<p>1</p> <p>3</p>
d	<p>(i) Find the square root(s) of $(-8 + 6i)$.</p> <p>(ii) Hence, solve the equation</p> <p>$2Z^2 - (3 + i)Z + 2 = 0$, expressing Z in the form $(a + bi)$ where a and b are real.</p>	<p>3</p> <p>2</p>

Question 2 (15 marks) Use a SEPARATE writing booklet.

Marks

a Evaluate

(i) $\int_0^{\frac{\pi}{4}} x \sin 2x \, dx$. 3

(ii) $\int_0^1 \frac{dx}{\sqrt{4-x^2}}$. 2

(iii) $\int_0^{\frac{\pi}{3}} \frac{\tan x}{1+\cos x} \, dx$. (using $t = \tan \frac{x}{2}$). 4

b Show that , if $I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$. 6

Then $I_n + I_{n-2} = \frac{1}{n-1}$, where n is an integer and $n \geq 3$

Hence evaluate I_7 .

Question 3 (15 marks) Use a SEPARATE writing booklet.

Marks

a The point $A(a \cos \alpha, b \sin \alpha)$ lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

B is the foot of the perpendicular from A to the x -axis. The normal at A cuts the x -axis at C .

(i) Represent this information with a suitable diagram.

1

(ii) Derive the equation of the normal AC .

3

(iii) Show that the length of CB is $\left| \frac{b^2 \cos \alpha}{a} \right|$.

3

b Consider the hyperbola H with equation $4x^2 - 9y^2 = 36$. The point $R(x_1, y_1)$ is an arbitrary point on H .

(i) Prove that the equation of the tangent l at R is $4x_1x - 9y_1y = 36$.

3

(ii) Find the co-ordinates of the point K at which l cuts the x -axis.

1

(iii) Hence, prove that $\frac{SR}{PR} = \frac{SK}{PK}$ where S and P are the foci of H .

4

Question 4 (15 marks) Use a SEPARATE writing booklet.

Marks

- a The equation $x^3 - 3x + 3 = 0$ has roots which are α, β and γ . Find the equation in x where the roots are α^2, β^2 and γ^2 . 4
- b The base of a solid is a circle of radius 2 units. A diameter runs through the centre of the base. Any cross section of the solid formed by a plane perpendicular to the given diameter is an equilateral triangle. 5
Show that the volume of the solid is $\frac{32\sqrt{3}}{3}$ units³.
- c The region bounded by the curve $y = \log_e x$, the straight lines $y = 1$ and $x = 3$ is rotated about the y -axis. Find the volume of the resulting solid using the method of cylindrical shells. 6

Question 5	(15 marks) Use a SEPARATE writing booklet.	Marks
a	Find the four fourth roots of -16 in the form $(a + bi)$.	4
b	A function is defined by $f(x) = \frac{\log_e x}{x}$ for $x > 0$.	
	(i) Find the x intercept.	1
	(ii) Find the turning point.	2
	(iii) Find the point of inflection.	2
	(iv) Sketch the graph of $y = f(x)$.	2
c	Consider the function in part (b) sketch	
i	$y = f(x) $.	2
ii	$y = \frac{1}{f(x)}$.	2

- a Consider the polynomial $Q(x) = ax^4 + bx^3 + cx^2 + dx + e$ where a, b, c, d and e are integers. Suppose α is an integer such that $Q(\alpha) = 0$.

(i) Prove that α is a factor of e .

2

(ii) Prove that the polynomial equation $P(x) = 0$,

2

where $P(x) = 4x^4 - x^3 + 3x^2 + 2x - 3$ does not have an integer root.

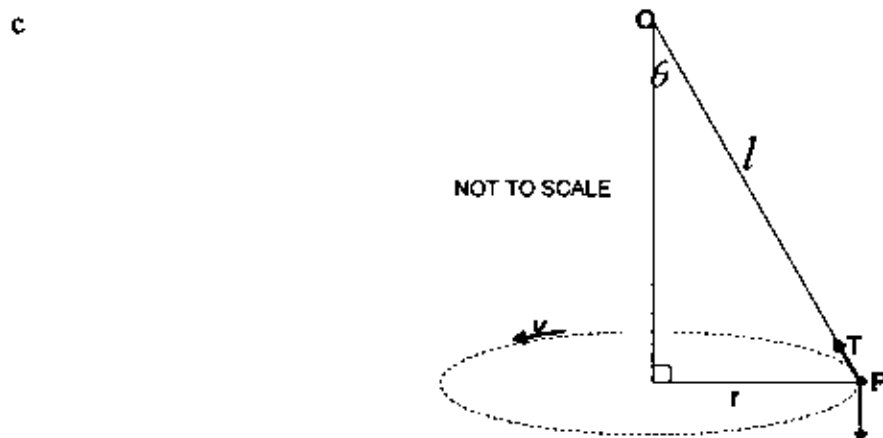
- b It is estimated that the probability that a torpedo will hit its target is $\frac{1}{3}$.

(i) If 5 torpedoes are fired, what is the probability of 3 successes.

2

(ii) How many torpedoes must be fired so that the probability of at least one success should be greater than 0.9?

2



The above diagram shows a light string of length l , fixed at O , and making an angle θ with the vertical as shown in the above diagram. A particle is attached at P . The particle moves with uniform speed v metres / second in a horizontal circle of radius r . The centre of the circle is directly below O .

If the particle is to maintain its motion in a horizontal circle, show by resolving forces vertically and horizontally, that the particle's velocity is given by

4

$v = \sqrt{rg \tan \theta}$. (Note: g is the acceleration due to gravity)

- d When a polynomial $P(x)$ is divided by $(x - 3)$ the remainder is 5 and when it is divided by $(x - 4)$ the remainder is 9. Find the remainder when $P(x)$ is divided by $(x - 4)(x - 3)$.

3

Question 7 (15 marks) Use a SEPARATE writing booklet. Marks

a If $\sin^{-1} x$, $\cos^{-1} x$ and $\sin^{-1}(1-x)$ are acute, show that 6

$$\sin(\sin^{-1} x - \cos^{-1} x) = 2x^2 - 1.$$

Hence, solve the equation

$$\sin^{-1} x - \cos^{-1} x = \sin^{-1}(1-x).$$

b Find the general solution of the equation $3 \tan^2 x = 2 \sin x$. 5

c Each of the following statements is either true or false. Write 'True' or 'False' for each statement giving a brief reason for your answers. (You are not required to evaluate the integrals).

(i) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^3 \cos x \, dx = 0.$ 2

(ii) $\int_{-1}^1 e^{-x^2} \cos^{-1} x \, dx = 0.$ 2

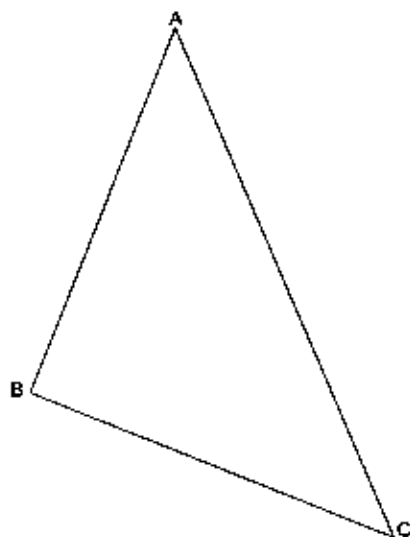
Question 8 (15 marks) Use a SEPARATE writing booklet.

Marks

- a In the Argand diagram, the points A , B and C represent the complex numbers Z_1 , Z_2 and Z_3 respectively.

What can you say about triangle ABC if $i(Z_3 - Z_2) = (Z_1 - Z_2)$.

2



- b Solve for x if $|3x + 3| + |x - 1| \leq 4x + 3$.

5

- c A particle, projected vertically upward with initial speed u is subjected to forces which create a constant vertical downward acceleration of magnitude g and an acceleration, directed against the motion, of magnitude kv when the speed is v .

8

(i) Show that the acceleration function is given by $\ddot{x} = -g - kv$.

(ii) Prove that the maximum height reached by the particle after a time T is given

$$\text{by } T = \frac{1}{k} \log_e \left(\frac{g + ku}{g} \right).$$

(iii) Prove that the maximum height reached is $\frac{1}{k}(u - gT)$.