Sydney Girls High School

LABOR OMNIA VINCIT

2007
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics
Extension 1

This is a trial paper ONLY. It does not necessarily reflect the format or the contents of the 2007 HSC Examination Paper in this subject.

Candidate Number

General Instructions

- Reading Time - 5 mins
- Working time - 2 hours
- Attempt ALL questions
- ALL questions are of equal value
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied
- Board-approved calculators may be used.
- Diagrams are not to scale
- Each question attempted should be started on a new sheet. Write on one side of the paper only.

Marks

Question 1 (12 marks)

(a) Find $\int \cos^2(2x)dx$ 2

(b) Using the substitution $u = e^x$ find $\int \frac{e^x}{1+e^{2x}}dx$ 3

c) Evaluate $\lim\limits_{x \to 0} \frac{\sin\frac{x}{2}}{4x}$ 2

(d) The point $M(-3,8)$ divides the interval AB externally in the ratio $k{:}1$ 3
If $A = (6, -4)$ and $B = (0, 4)$, Find the value of k.

(e) Prove the identity 2

$$\frac{2\tan A}{1+\tan^2 A} = \sin 2A$$

Marks

Question 2 (12 marks)

(a) Consider the function $f(x) = 3\sin^{-1}(\frac{x}{2})$ 4

(i) Evaluate $f(2)$

(ii) Draw the graph of $y = f(x)$

(iii) State the Domain and Range of $y = f(x)$

(b) One root of the polynomial equation $x^3 + 6x^2 - x - 30 = 0$ is equal to the sum of the other two roots. Find all three roots. 3

(c) Use Newton's Method to find a second approximation to the positive root of the equation $x = 2\sin x$ taking $x = 1.7$ as the first approximation. Give answer in radians correct to 1 decimal place. 3

(d) Solve the inequality $\frac{2}{x-1} < 1$ 2

Marks

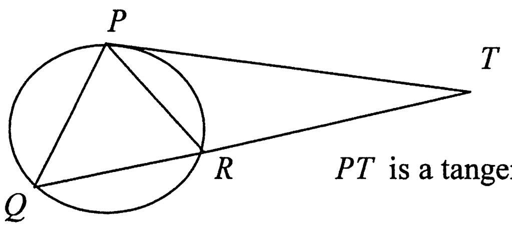
Question 3 (12 marks)

(a) 3

(i) Find the point of intersection of the line $y = x$ with the curve $y = x^3$ in the first quadrant.

(ii) Then find the size of the acute angle between the line and the curve at this point to the nearest degree.

(b)



PT is a tangent and QRT is a secant 5

(i) Copy this diagram onto your answer page.

(ii) Prove that ΔPRT and ΔQPT are similar.

(iii) Hence prove that $PT^2 = QT \times RT$

(c) Let T be the temperature inside a room at time t hours and let A be the constant outside air temperature.
Newton's Law of Cooling states that the rate of change of the temperature T is proportional to (T – A).

(i) Show that $T = A + Ce^{kt}$ where C and k are constants satisfies Newton's Law of Cooling. 1

$$\frac{dT}{dt} = k(T - A)$$

(ii) The outside air temperature is 5^0C when a system failure causes the inside room temperature to drop from 20^0C to 17^0C in half an hour. After how many hours is the inside room temperature equal to 10^0C? Give answer correct to 1 decimal place. 3

Marks

Question 4 (12 marks)

(a) The acceleration of a particle moving in a straight line is given by $\ddot{x} = 2x - 3$ where x is the displacement, in metres, from the origin 0 and t is the time in seconds. Initially the particle is at rest at $x = 4$. 4

(i) If the velocity of the particle is V ms^{-1} show that $V^2 = 2(x^2 - 3x - 4)$

(ii) Show that the particle does **not** pass through the origin.

(iii) Find the position of the particle when $V = 10$ ms^{-1}

(b)

(i) Find the inverse function $f^{-1}(x)$ in terms of x for $f(x) = 2x - x^2$ over the restricted domain $x \geq 1$. Write the Domain and Range of the inverse function. 4

(ii) Find the point common to both $f(x)$ and $f^{-1}(x)$ in this domain.

(c) From the top of a mountain 200 metres above ground an observer sights two landmarks A and B. Point A has a bearing of 300^0T at an angle of depression of 10^0. Point B has a bearing of 040^0T at an angle of depression of 15^0 Calculate the distance from A to B given that both points are at ground level. (to the nearest metre). 4

Marks

Question 5 (12 marks)

(a) 3

(i) Express $\sqrt{3}\sin\Theta - \cos\Theta$ in the form $A\sin(\Theta - \alpha)$ where α is in radians and $A > 0$

(ii) Hence, or otherwise find all angles Θ, where $0 \le \Theta \le 2\pi$ for which $\sqrt{3}\sin\Theta - \cos\Theta = 1$

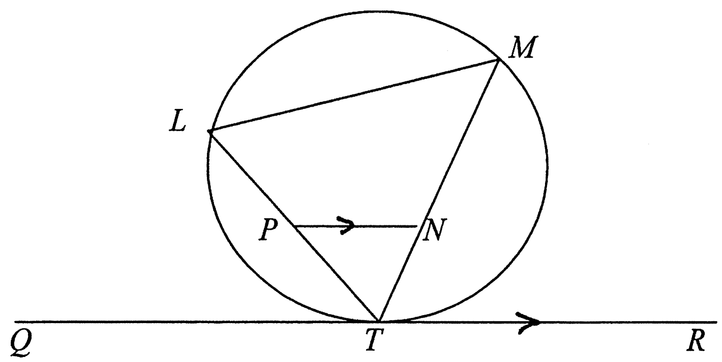
(b) Consider the parabola $x^2 = 4ay$ where $a > 0$. 5
The tangents at $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ intersect at the point T.
Let $S(o,a)$ be the focus of the parabola.

(i) Find the coordinates of T. (You may assume the equation of the tangent at P is $px - y - ap^2 = 0$)

(ii) Show that $SP = ap^2 + a$

(iii) Now P and Q move along the parabola in such a way that $SP + SQ = 4a$
Find the locus of T under this condition.

(c) 4



QR is a tangent touching the circle at T

(i) Copy this diagram onto your answer page.

(ii) Prove that $LMNP$ is a cyclic quadrilateral

Marks

Question 6 (12 marks)

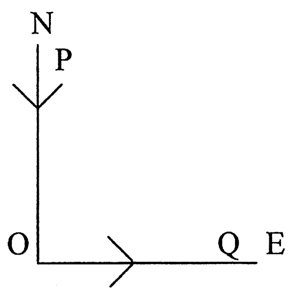
(a) Prove by mathematical induction that $n^3 + 2n$ is divisible by 3 for all positive integers n. 4

(b) 4

(i) Find the exact area bounded by the curve $y = \dfrac{x-1}{\sqrt{x+1}}$, the x axis and the lines $x = 3$ and $x = 8$.
Use the substitution $u^2 = x + 1$

(ii) Now find the volume of the solid of revolution formed by rotating this area about the x axis. Give answer correct to 1 decimal place.

(c) 4



Car P is North of an intersection and travelling towards O
Car Q is moving away from the intersection eastwards at 60 km / hour
The distance between the two cars at any given time is 10 km.
Find the rate in km per hour at which car P is moving when car Q is 8 km away from the intersection.

Marks

Question 7 (12 marks)

(a) A particle's displacement is given by $x = 2\cos(t + \frac{\pi}{4})$ metres at time t seconds 5

(i) Show that acceleration is proportional to the displacement and hence describe its motion.

(ii) Find the initial position

(iii) Find the period of the motion

(iv) Find the maximum displacement

(v) Find the particle's position after $\frac{\pi}{2}$ secs.

(b) A sky rocket is fired vertically into the air. At a height of 28 metres it explodes and is projected at an angle of 60^0 to the horizontal with a velocity of 30 ms^{-1}. Take g = 10 ms^{-2} 7

(i) How long from the time of the explosion will it take to fall back to the ground?

(ii) How far from its launching site will it land?

(iii) At what velocity will it strike the ground? To nearest whole number.

(iv) What acute angle will it make with the ground on impact? To nearest degree.

End of Exam

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$