1. Earth has a gravitational field that exerts a force on objects both on it and around it.

By David Lin
Mr Ogle

- define weight as the force on an object due to a gravitational field

Sidenote
Law of Universal gravitation is:

$$F = G\frac{m_1 m_2}{r^2}$$

Where G is the universal gravitational constant of 6.67x10^{-11} Nm^{2}kg^{-2}

A **gravitational field** is a **vector field** surrounding a mass (typically a large celestial object), within which another mass will experience a force known as weight.

$$\vec{W} = m\vec{g}$$

Where **W is the weight in newtons (N)** and is a vector quantity, ***m* is the mass in kg** and **g is either**:

a. The *acceleration due to gravity* (=9.8m/s/s at Earth's surface); or
b. The *gravitational field strength* (=9.8N/kg at Earth's surface)

Most commonly (a) in the HSC syllabus.
Since ***g* is different in different places** hence **weight is different in different places** (*i.e. Different altitude, or distance from centre of mass*).

E.g. Our weight on the moon is approximately one sixth of the our weight on Earth.

- explain that a change in gravitational potential energy is related to work done

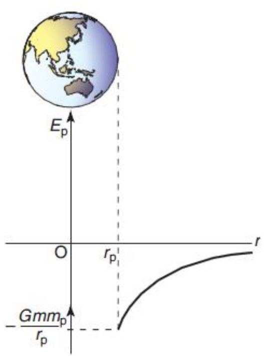
A change in gravitational potential energy means that there would be change in some other form of energy, notably work. In simple terms, **as an object is lifted to a height above the ground, we do work on it** and the **gravitational potential energy** of the object increases. Converse, as an object falls, gravitational potential energy being converted to other forms of energy (i.e. kinetic), and work can be done.
This is in accordance with the law of conservation of energy
I.e. For an object of mass m and height h above the Earth's surface, gravitational potential energy E is given by:

$$E_p = mgh$$

But there is a lot of limitations with this definition and formulae which is **only valid near the surface of the Earth**. A more correct explanation follows...

- define gravitational potential energy as the work done to move an object from a very large distance away to a point in a gravitational field

$$E_p = -G\frac{m_1 m_2}{r}$$



Gravitational potential energy is equivalent to the work done to move the object from a point of reference which is a very large distance away (*i.e. Infinity*) **to a point in the gravitational field.**

At infinity, the object has zero gravitational potential energy. But as the object moves towards the source of the gravitational field from infinity, gravitational potential energy is transferred into kinetic energy, and hence its energy level becomes less than zero (as denoted by the negative sign in the formula).
Conversely, when moving an object against a gravitational field, work is done on the object and energy is transferred as it gains gravitational potential energy.

The general expression for the gravitational potential energy of mass m_1 as it moves from infinity to a distance *r* from a source of a gravitational field (due to mass m_2) is given by...

$$E_p = -G\frac{m_1 m_2}{r}$$

For example, a mass *m* which is distance *r* from the centre of the planet (*M or M_E for Earth) is...*

$$E_p = -G\frac{mM_E}{r}$$

perform an investigation and gather information to determine a value for acceleration due to gravity using pendulum motion or computer-assisted technology and identify reason for possible variations from the value 9.8 ms^{-2}

Preamble

This investigation is based on the fact that when a simple pendulum swings on a small angle it only depends on **two variables**, the **length of the string** and **the acceleration due to gravity**. Thus, it will give very good approximations of the rate of acceleration due to gravity.

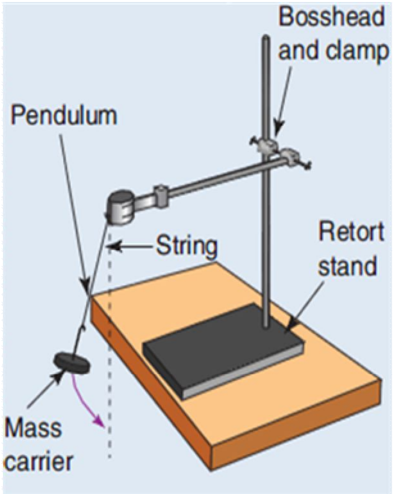
The relationship between the period (T), length (l) and acceleration due to gravity (g)is given by:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Which can be further rearranged to express it in terms of *g*

$$g = \frac{4\pi^2 l}{T^2}$$

Method



Construct the pendulum as shown and *measure the time taken for **10 oscillations***, then **averaging this will give the time taken for one period**.

The lengths of the pendulum can be altered systematically with each trial.

Finally a graph can be plotted with the **time squared (T^2) vs. the length (*m*)**.

Since

$$T^2 = \frac{4\pi^2}{g}l \equiv y = mx + b$$

The **gradient will be equivalent to $4\pi^2/g$** (where b=0). The equation can then be manipulated to find *g.*

Variations

There could be various reasons for variation from the value of 9.8ms^{-2}.

- The Earth is ***not a perfect sphere***, as it is flattened at the poles (where g is approx 9.83ms^{-2}) and different physical features (i.e. mountains, valleys) result in varying distances from the centre of Earth affecting the acceleration due to gravity.
  - *i.e. BHHS 100m above sea level*

At equator: *9.782ms^{-2}*

- The ***variation in lithosphere composition (thickness, structure and density)***
  - *i.e. Density variations due to features such as ore deposits. Or plate boundaries. These alter local g values.*
- The ***Earth's spin*** creates a small centrifugal effect slightly lowering g. *The effect is greatest at the equator and approaches zero as you get closer to the poles.*

However, variations might have simply been from experimental errors, a few of which could be...

- Reaction time in using the stopwatch prevents obtaining the exact value of the oscillations.
- Inability to determine the centre of mass results in inaccurate measure of length.
- Air resistance (which will always be present in school labs), etc

gather secondary information to predict the value of acceleration due to gravity on other planets

A formula can be deriving from the Law of Universal Gravitation, from which we can obtain the acceleration due to gravity *(g)* due to the gravitational field of mass *m*. Where *r* is the distance from the centre of mass.

$$g = \frac{Gm}{r^2}$$

Using this, the theoretical value of acceleration due to gravity on other planets in the solar system can be approximated.

A following table can be constructed from secondary sources to compare the acceleration due to gravity of different planets:

Planet	**Mass (kg)**	**Radius (m)**	**Acceleration Due to Gravity (g) ms^{-2}**	**g / g-Earth**
Mercury	3.18×10^{23}	2.43×10^6	3.59	0.37
Venus	4.88×10^{24}	6.06×10^6	8.87	0.90
Earth	5.98×10^{24}	6.38×10^6	9.81	1.00
Moon	7.36×10^{22}	1.74×10^6	1.62	0.17
Mars	6.42×10^{23}	3.37×10^6	3.77	0.38
Jupiter	1.90×10^{27}	6.99×10^7	25.95	2.65
Saturn	5.68×10^{26}	5.85×10^7	11.08	1.13
Uranus	8.68×10^{25}	2.33×10^7	10.67	1.09
Neptune	1.03×10^{26}	2.21×10^7	14.07	1.43

- *Note: the **value of g for Earth is actually between 9.78-9.82ms^{-2}**. But the exact approximation gives 9.8065...ms^{-2} which is rounded to 9.81ms^{-2} on the table.*
- *Similarly, on other planets, **variations for g will occur** due to planet shape or physical features.*

analyse information using the expression:

$$F = mg$$

to determine the weight force for a body on Earth and for the same body on other planets

$$g = \frac{Gm}{r^2}$$

Subst into F=mg to obtain F.

Newton's second law F=ma can be modified to ***<u>F=mg</u>* <u>or</u> *<u>W=mg</u>*** where W is the weight force acting on a body with a mass *m* under the influence of a gravitational field where the acceleration due to gravity is *g*. **This can be used to estimate the weight force for a mass on Earth and on other planets**.

For example, using the values for acceleration due to gravity of different planets from the above table, we can estimate the weight of a person with the mass of 60kg on the surface of the planet:

Planet	**Mass (kg)**	**Acceleration Due to Gravity - *g* (ms^{-2})**	**Weight (N) 3sf**
Mercury	60	3.59	215
Venus	60	8.87	532
Earth	60	9.81	589
Moon	60	1.62	97.2
Mars	60	3.77	226
Jupiter	60	25.95	1560
Saturn	60	11.08	665
Uranus	60	10.67	640
Neptune	60	14.07	844

2. Many factors have to be taken into account in order to achieve a successful rocket launch, maintain a stable orbit and return to Earth

By David Lin
Mr Ogle

- describe the trajectory of an object undergoing projectile motion within the Earth's gravitational field in terms of horizontal and vertical components

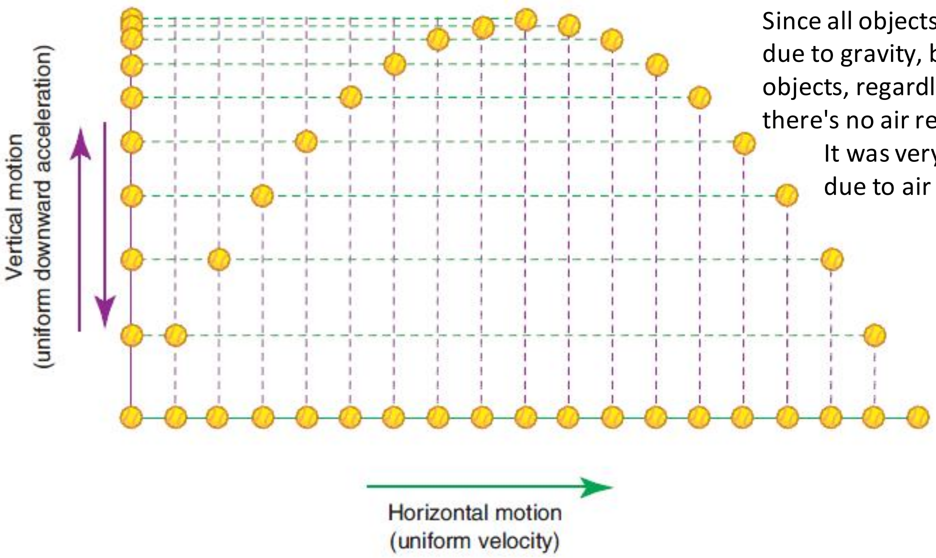
AND

- describe Galileo's analysis of projectile motion

A trajectory is the **path a projectile follows in its flight**. When **air resistance is ignored**, a projectile under the influence of gravity will trace out a **parabolic** trajectory. It doesn't include a rocket which has its own thrust, projectiles complete an unpowered flight.

Galileo observed the parabolic path of a projectile and proposed that the complex motion of any projectile can be regarded as two separate and distinct motions that are superimposed to bring about the overall motion of the object.

One motion is the horizontal motion, which experiences no acceleration. The other motion is the vertical motion subject to acceleration due to gravity. These motions are perpendicular to each other and can be treated as independent to one another. This allows us to analyse them separately.



Since all objects experience the same vertical acceleration due to gravity, by implication, Galileo postulated that all objects, regardless of mass, falls at the same rate; true if there's no air resistance.

It was very difficult for him to prove this conclusively due to air resistance.

However, by rolling balls down a highly polished incline, he minimised the difficulty presented by air resistance and also enabled him to slow the motion down for him to make more accurate measurements using trigonometry to calculate the value of acceleration.

Galileo also proposed the idea of inertial frames of reference, that within each frame, the laws of motion hold.

- solve problems and analyse information to calculate the actual velocity of a projectile from its horizontal and vertical components using :

$$v_x^2 = u_x^2$$
$$v = u + at$$
$$v_y^2 = u_y^2 + 2a_y\Delta y$$
$$\Delta x = u_x t$$
$$\Delta y = u_y t + \frac{1}{2}a_y t^2$$

Below is a sample problem which involves the standard projectile motion formulas.

1. Find time to the top

[Sol] Vert Half Flight

$u_y = 50\cos70° ms^{-1}$

$v_y = 0ms^{-1}$

$a_y = -9.8ms^{-2}$

$t = ?$

$$v_y = u_y + a_y t$$
$$t = \frac{v_y - u_y}{a_y}$$
$$t = \frac{-50\cos 70^0}{-9.8}$$
$$t = 1.745\ldots s$$
$$= 1.7s\ (2sf)$$

$$v_y^2 = u_y^2 + 2a_y\Delta y$$
$$\Delta x = u_x t$$
$$\Delta y = u_y t + \frac{1}{2}a_y t^2$$

[Sol] Vert Half Flight
u_y=50cos70°ms^{-1}
v_y=0ms^{-1}
a_y=-9.8ms^{-2}
t=?

$v_y = u_y + a_y t$
$$t = \frac{v_y - u_y}{a_y}$$
$$t = \frac{-50\cos 70^0}{-9.8}$$
t = 1.745...s
= 1.7s (2sf)

2. Find max height

[Sol] Vert Half Flight
u_y=50cos70°ms^{-1}
v_y=0ms^{-1}
a_y=-9.8ms^{-2}
t=1.745...s

$$(v_y)^2 = (u_y)^2 + 2a_y\Delta y$$
$$\Delta y = \frac{(v_y)^2 - (u_y)^z}{2ay}$$
$$\Delta y = \frac{(0)^2 - (50\cos 70^0)}{2(-9.8)}$$
$$\Delta y = 14.920\ ...$$
$$\Delta y \approx 15m(2sf)$$

3. Find time of flight

[Sol] Vert. Full flight
$\Delta y = 0$
$u_y = 50\cos 70^0 ms^{-1}$
$a_y = -9.8ms^{-2}$

$$\Delta y = u_y t + \frac{1}{2}a_y t^2$$
$$t = \frac{-u_y \pm \sqrt{u_y{}^2 + 4\Delta y(\frac{a_y}{2})}}{a_y}$$
$$t = \frac{-50\cos 70^0 \pm \sqrt{(50\cos 70^0)^2 + 4(0)(-4.9)}}{-9.8}$$
But $t > 0$
$t = 3.4900\ ...s$
$t \approx 3.5s\ (2sf)$

This is used to demonstrate the above formula only.
An easier way would be to set v_y to be -50cos70° and $v_y = u_y + a_y t$ can be used.

4. Find range

[Sol] Hor. Full flight
$\Delta x = ?$
$u_x = 50\cos 20^0 ms^{-1}$
$t = 3.4900\ ...s$

$$\Delta x = u_x t$$
$$\Delta x = (50\cos 20°)(3.4900\ ...)$$
$$\Delta x = 163.976\ ...m$$
$$\Delta x \approx 100m(3sf)$$

perform a first-hand investigation, gather information and analyse data to calculate initial and final velocity, maximum height reached, range and time of flight of a projectile for a range of situations by using simulations, data loggers and computer analysis

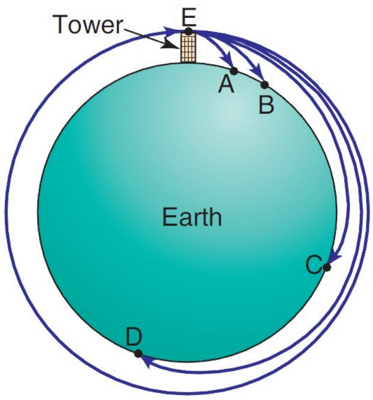
The first two rows below are obtained from the java applet at:
http://zebu.uoregon.edu/nsf/cannon.html
The program's aim was to hit a target a fixed distance out. Hence, when the angel is changed, the initial velocity is also changed to allow for the projectile to hit the target.

Angle (°)	30	45	60
Initial Velocity u (ms^{-1})	64	61	66
Initial Horizontal Velocity u_x	64cos30 =55.425... =55.4 ms^{-1} (3sf)	61cos45 =43.133... =43.1 ms^{-1} (3sf)	66cos60 =33.0 ms^{-1} (3sf)
Initial Vertical Velocity u_y	64sin30 =32.0 ms^{-1} (3sf)	61sin45 =43.133... =43.1 ms^{-1} (3sf)	66sin60 =57.1576... =57.2 ms^{-1} (3sf)
Time of Flight t $\Delta y = u_y t + (1/2)a_y t^2$ where $\Delta y = 0$	32 = 4.9t t=6.530612... t=6.53 s (3sf)	(43.13...)=4.9t t=8.8027... t=8.80 s (3sf)	(57.2...)=4.9t t = 11.664... t = 11.7 s (3sf)
Max Height reached Δy $v_y^2 = u_y^2 - 2a_y\,\Delta y$ where $v_y=0$ $\Delta y = (-u_y^2)/(-2a_y)$	Δy=1024/19.6 Δy = 52.244... Δy = 52.2m (3sf)	Δy=1860.5/19.6 Δy = 94.923... Δy = 94.9m (3sf)	Δy=3267/19.6 Δy = 166.68... Δy = 167m (3sf)
Range Δx	Δx=(55.4..)(6.5...)	Δx=(43.1..)(8.8..)	Δx=(33.0)(11.6...)

$\Delta x = u_x t$	$\Delta x = 361.9632...$ $\Delta x = 362m$ (3sf)	$\Delta x = 379.693...$ $\Delta x = 380m$ (3sf)	$\Delta x = 384.939...$ $\Delta x = 385m$ (3sf)

Although the height of the trajectory differed in each case, it is the range that is about the same.
Due to the program's programming, it must have accepted a range of values over which the target would explode, hence the differing results obtained which were still very close.

- outline Newton's concept of escape velocity



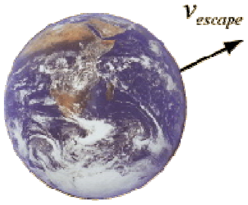
A projectile launched from a height with only horizontal velocity has a range directly proportional to its initial velocity, tracing a parabolic path.

Hence, Newton thought that an object can be launched fast enough to achieve an orbit around the Earth. He though if an object is fired horizontally from a very tall tower, it will cover a considerable range, tracing a parabolic path before striking the ground. Another projectile with more initial velocity would have more range. (As seen from A to B to C then to D on right).

But an object with sufficient initial velocity will never hit the ground, because as the object falls, the Earth's surface will curve away from it, so that it will maintain a circular orbit at a fixed height above the ground.. i.e. Approach E.

Faster velocities will yield an elliptical orbit, and once approaching escape velocity, it follows parabolic and hyperbolic paths.
After initial calculations, Newton found this initial velocity to be too great to be achieved by any means in his time.

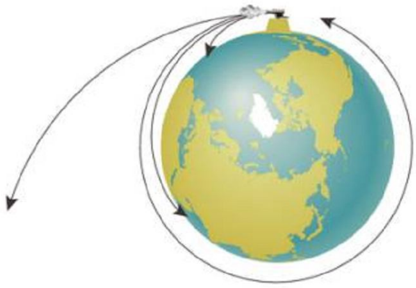
- explain the concept of escape velocity in terms of the:
  - gravitational constant
  - mass and radius of the planet



$$v_{escape} = 11.2\ km/s$$

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

$$v_{escape} = \sqrt{\frac{2GM}{r}}$$



The escape velocity of a celestial body is the initial velocity which an object requires to just escape the planet's gravitational field and go into space, reaching an infinite distance unless acted upon by an another force.

An object thrown up at escape velocity slows down, never stops. The force downwards decreases as it gains displacement. Objects can escape via parabolic, then hyperbolic paths once greater than escape velocity.

To escape, we need to do work equal to or greater than the gravitational potential energy of the object. The kinetic energy has to be "greater-than" the potential energy.

$$E_k \geq Ep$$

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

$$v^2 = \frac{2GM}{r}$$

$$v_E = \sqrt{\frac{2GM}{r}}$$

Where m is the mass of the projectile, M is the mass of the planet, and G is the gravitational constant.

Location	Escape velocity with respect to the own planet's gravity
Earth	11.2 kms^{-1}
Moon	2.4 kms^{-1}
Mars	5.0 kms^{-1}
Jupiter	59.5 kms^{-1}
Sun	617.5 kms^{-1}

Alternatively, when an object comes to rest at an infinite distance, the mechanical energy is zero (kinetic energy is zero [since v=0] and gravitational potential energy is zero [by definition].) Hence...

$$\frac{1}{2}mv^2 = G\frac{Mm}{r}$$

$$v_E = \sqrt{\frac{2GM}{r}}$$

As seen in the above formula, since the gravitational constant stays the same, v_E only directly varies with the mass of the planet and indirectly varies with the mass and radius of the planet. This means that the larger the planet and the closer we are to the planet, the greater the escape velocity.

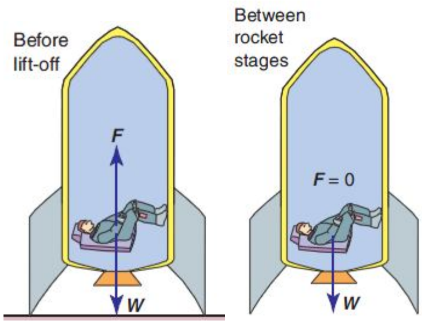
If fired from that velocity, any object will escape, and does not depend on the intrinsic property of the projectile or the direction of the velocity (provided it doesn't intersect with the planet.

- identify why the term 'g forces' is used to explain the forces acting on an astronaut during launch

G-forces is the apparent weight experienced by a person at a given time expressed as multiples of his true weight.

$$g \text{ force } = \frac{\text{apparent weight}}{\text{normal true weight}}$$

Apparent weight is experienced when an external force acts on a person resulting in the sensation of weight. The apparent weight includes the normal reaction force as well as any upwards or downwards acceleration, given by: mg + ma. While the normal true weight is the normal reaction force of the ground acting on the person at the surface of the Earth.



Since apparent weight is given by mg + ma, then…

$$g \text{ force} = \frac{\text{apparent weight}}{\text{normal true weight}}$$

$$= \frac{mg + ma}{9.8\,m}$$

Where g is the acceleration due to gravity (ms^{-2}) and m is the mass of the person.

So that…

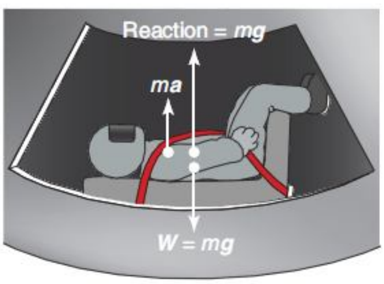
$$g \text{ force} = \frac{g + a}{9.8}$$

Hence we see that g-force is tied into the acceleration that we experience. When no acceleration is experienced, we see that the g force is simply g/9.8 which is 1.

We all experience one g force when we are stationary on the surface of the Earth. <i.e. before lift off>

However, when thrust is applied, there is an upward acceleration. Therefore, the force upwards is greater than the true weight. Therefore, There will be increased g forces. <i.e. lift-off>

e.g. an astronaut experiences an acceleration of 39.2 ms^{-2},



$$g\ force = \frac{9.8 + 39.2}{9.8}$$

$$g\ force = \frac{42}{9.8}$$

$$g\ force = 5$$

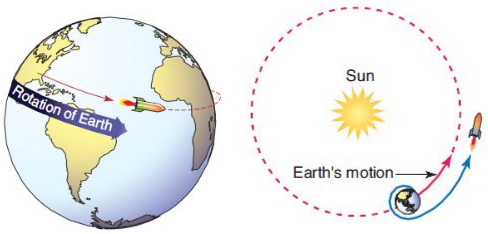
But when the reverse is true and the spaceship is in free fall for a few seconds between rocket stages, astronauts feel a feeling of weightless where there is only downward acceleration and no normal reaction force. Hence during this time, g force is at zero.

If acceleration is in the direction of the astronaut's head, the blood rushes to his feet and he may experience blackouts.

Conversely, if acceleration towards his feet might result in a red-out as the blood rushes to his head.

But the body tolerates more g-forces lying down, during blast astronauts are lying down as in the diagram on the left. Astronauts can handle g forces pretty well in this position. Although excessive g forces may cause bursting of blood vessels in the back of the head.

discuss the effect of the Earth's orbital motion and its rotational motion on the launch of a rocket



Since the Earth has rotational motion around its own axis at around 460ms^{-1} (1700km/h) relative to the sun, engineers can use this to their advantage. By launching the rocket in the direction of the Earth's rotation, i.e. East, the rotational velocity of the launch site will add to the orbital velocity of the rocket relative to the Earth, resulting in a higher orbital velocity relative to the Sun.

Secondly, the Earth's orbital motion around the sun is around 30000ms^{-1} (107000km/h) relative to the sun, and hence, this can be exploited for interplanetary travel. This is only possible when the direction of the Earth's orbital velocity corresponds to the desired direction. In favourable periods known as launch windows, the rocket is first launched into a low earth velocity orbit until the direction of its orbital velocity corresponds with the Earth's, then it accelerates away. Hence the orbital velocity of the Earth adds to the rocket's velocity relative to the sun, saving fuel in achieving the required velocity.

analyse the changing acceleration of a rocket during launch in terms of the:

- Law of Conservation of Momentum
- forces experienced by astronauts

Combustion of rocket fuel propels the rocket forwards. The law of conservation of momentum is fundamental to explaining the operation of a rocket.

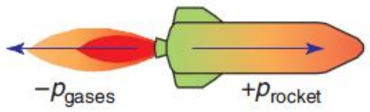
$$\Delta p_{gases} + \Delta p_{rocket} = 0$$

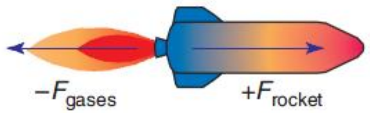
$$\text{Total change in momentum} = 0$$

$$\therefore -\Delta p_{gases} = \Delta p_{rocket}$$

$$-\Delta(mv)_{gases)} = \Delta(mv)_{rocket}$$

Where p is the change in momentum (kg ms^{-1}), m is mass (kg), and v is velocity (ms^{-1}).





Propellant exits the rear of the rocket at very fast velocity. Since, Δp is zero, the rocket, being the other component of the system must move in the opposite direction. As the mass of the rocket is very large compared to the expelled fuel, the velocity of the gases are much larger than that of the rocket.

And since the change in momentum is also the impulse, i.e. Ft, (where F is force(N), and t is time(s)), then…

$$-(Ft)_{gases} = (Ft)_{rocket}$$

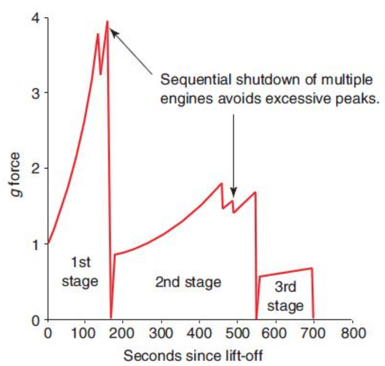
$$-F_{gases} = F_{rocket}$$

This follows Newton's third law where every force has an equal and opposite force. <seen on the left.>

As the rocket expels fuel, significantly more mass is being lost, but the thrust, i.e. the force applied by thrusters, is the same. A typical rocket's mass is 90% fuel, therefore when m is decreasing, since F=ma, and mass is inversely proportional to acceleration when F is constant, then acceleration gets larger.

Therefore, there is an increase in acceleration as the rocket uses up its fuel and the velocity of the rocket will also increase significantly.

Furthermore, g also decreases slightly with increasing altitude, leading to further acceleration of the rocket during its course from the Earth's surface.



When drawing, show multiple peaks, label as above.

Forces experienced by Astronauts

Must relate why to above.

For astronauts, as stated above, they are subject to g forces during a launch. Before launch, they experience one g. And after the ignition of the rocket, this will not change until the thrust has built up to exceed the weight of the rocket, then the rocket will take off and the net acceleration upwards will determine the g forces via the formula given previously.

Since acceleration increases during the flight as stated above <explain>, and the g force steadily increases and reaches maximum when the fuel is exhausted and the g force decreases. During a multistage rocket, sequential shutdown of engine occurs and this process of acceleration and increase in g force followed by the decrease in g force between stages happens several times.

identify data sources, gather, analyse and present information on the contribution of one of the following to

Robert Goddard is considered the father of modern rocket propulsion, and played a significant role in the development of controlled, liquid fuelled rocketry

- identify data sources, gather, analyse and present information on the contribution of one of the following to the development of space exploration: Tsiolkovsky, Oberth, Goddard, Esnault-Pelterie, O'Neill or von Braun

Robert Goddard is considered the father of modern rocket propulsion, and played a significant role in the development of controlled, liquid fuelled rocketry.

Initially in 1914, Goddard submitted patent applications for multistage rockets and liquid fuels which became a milestone in the history of rocketry. Many scientists in his days didn't believe in rocket performance in a vacuum, and in 1915, Goddard experimented in airtight chambers to prove that rockets are propelled and work well in vacuum (Newton's 3rd law) and rocket performance actually decreased under atmospheric pressure.

Furthermore, by using the de Laval steam turbine nozzle, he was able to effectively convert energy of hot gases into forward motion, increasing the engine efficiency from 2% to 64%, attaining exhaust speeds of Mach 7.

In 1926, he launched the world's first liquid fuel rocket which used gasoline and liquid nitrous oxide; hence demonstrating the potential for liquid fuels. He calculated energy-to-weight ratios for various fuels and identified liquid nitrogen and liquid oxygen as ideal propellant combinations, hence publishing "liquid propellant rocket development". This technology is later used to launch rockets into space.

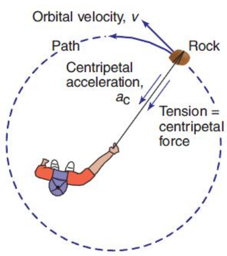
In later designs, he employed gyroscopic controlled steering devices for navigation and added fins to stabilise steering. Some of his rockets reached supersonic velocities and the highest altitude attained was 2.7km. In these designs, to avoid engine burn-in, he employed a cooling method where the engine was cooled by circulated liquid oxygen on the outside of the combustion chamber.

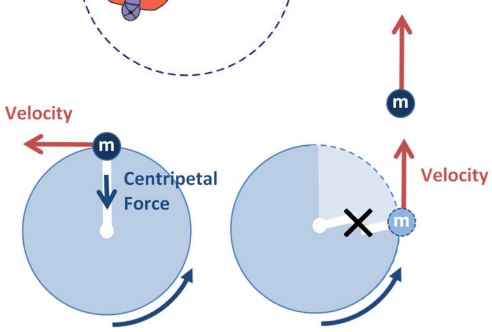
His influential work, *A Method of Reaching Extreme Altitudes*, describes the mathematical theories of rocket flight, solid fuel rockets and possibilities of exploring the world beyond Earth's atmosphere, thought to have inspired scientists such as Oberth and Von Braun.

However, his ideas were apparently stolen by the Germans to make V-series rockets. Throughout most of his lifetime he was under-funded compared to German counterparts. However, he was awarded 214 patents on various technologies.

The world's first liquid fuel rocket, note that the combustion chamber and nozzle are at the top, while the fuel tank is directly beneath, protected by a heatproof cone. He did this for potentially increased stability, but found later that this was no more stable than placing the engine at the base.

- analyse the forces involved in uniform circular motion for a range of objects, including satellites orbiting the Earth





Uniform circular motion refers to the circular motion of an object with uniform orbital speed. A force perpendicular to the velocity of the object, towards the centre, i.e. centripetal force, is required to maintain this circular motion.

This is given by...

$$\text{Centripetal force, } F_C = \frac{mv^2}{r}$$

Where, F_c is the centripetal force (N), m is the mass of object (kg), v is the instantaneous or orbital velocity of the mass (ms^{-1}), and r is the radius of circular motion (m)

And from Newton's second law, we know that any net force acting on an object is associated with its acceleration in the form of F=ma. So for F_c...

$$F_C = \text{mass} \times \text{centripetal acceleration, } a_C$$

Hence, by comparison of the two above statements, we know that.

$$a_C = \frac{v^2}{r}.$$

This is consistent with the fact that any object in uniform circular motion is accelerating towards the centre even though its velocity is tangential to the circle.

The picture of a person with a string attached to a rock is one instance of uniform circular motion in which a_c is provided by tension in the string. However

To find orbital velocity of a satellite, equate:

$$F_g = G\frac{Mm}{r^2} \qquad F_c = \frac{mv^2}{r}$$

$$v_{satellite} = \sqrt{\frac{GM}{r}}$$

it can occur in several ways, the general diagram for these is represented in the second picture on the left.

Another easy to imagine one is a car going around a corner where friction between the tyres and the road provide the centripetal acceleration. Similarly a satellite around a larger body will have the same motion with gravity being a_c. This can be seen with artificial satellites around Earth, or the moon orbiting Earth.

The last example is in a magnetic field, where an electric charge is fired perpendicular to the magnetic field line directions and results in uniform circular motion. Hence, we see that $F_c = (mv^2)/r = qvB$.

- solve problems and analyse information to calculate the centripetal force acting on a satellite undergoing uniform circular motion about the Earth using:

$$F = \frac{mv^2}{r}$$

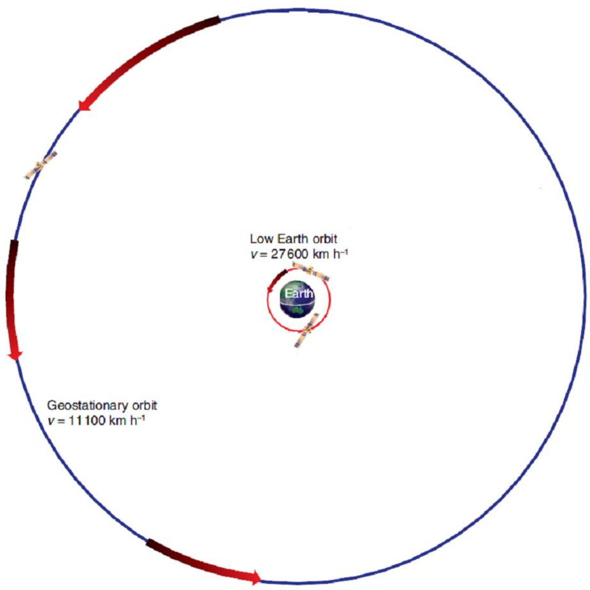
These problems are fairly simple and straightforward. For example...

E.g. A string has a breaking strain of 210 Newtons. What is the maximum speed at which a mass of 7kg can be whirled about in a horizontal circle on the end of a piece of that string 0.3m long?

[Sol]
$F_c = 210$N
(assuming max tension)
m = 7kg
r = 0.3m

$$F_c = m\left(\frac{v^2}{r}\right)$$
$$v^2 = \frac{F_c \cdot r}{m}$$
$$v^2 = \frac{210 \cdot 0.3}{7}$$
$$v^2 = 9$$
$$v = 3ms^{-1}, \quad since\ v > 0$$

- compare qualitatively low Earth and geo-stationary orbits



Generally, satellites or spacecrafts placed into orbit, other than GPS satellites, will generally be of low Earth or geo-stationary orbits.

Low Earth Orbit

This kind of orbit is generally between 250km altitude and 1000km altitude. Above 250km altitude as this avoids atmospheric drag. And below 1000km altitude as this is below the Van Allen's belt which traps radiation and is harmful to humans and electronic equipment.

Generally, space shuttles operate in a low Earth orbit of 250-400km depending on mission objectives. At these altitudes, a spacecraft's orbital velocity is approximately 27900kmh^{-1} orbital period is approx 90 min. These type of orbits can provide quick coverage of the whole Earth useful for weather satellites, remote sensing satellites e.g. measuring plant cover, surface water composition or chemical composition. These satellites are cheap to launch, and have stronger signals, but they are significantly affected by drag and orbital decay due to their low altitude.

Geo-stationary orbits

These orbits stays over a fixed point on the Earth's surface regardless of the time of day. It's interesting to note that geostationary orbits are a special kind of geosynchronous orbits (period of revolution equal to 3min56sec less than 24 hours, i.e. 86164s). But geostationary orbits lie on the equatorial plane. Both orbits are around 35800km above the Earth's surface above the upper limits of the Van Allen's belt.

Australia has the OPTUS and AUSSAT satellites in geostationary orbit.

less than 24 hours, i.e. 86164s). But geostationary orbits lie on the equatorial plane. Both orbits are around 35800km above the Earth's surface above the upper limits of the Van Allen's belt.
Geostationary satellites are useful for communications as a receiving dish only needs to be pointed in a fixed point in the sky to remain in satellite contact.

Geosynchronous satellites not on the equatorial plane will trace out a figure eight path over 24 hours rather than being geostationary.

- define the term orbital velocity and the quantitative and qualitative relationship between orbital velocity, the gravitational constant, mass of the central body, mass of the satellite and the radius of the orbit using Kepler's Law of Periods

Orbital velocity is the term given to the instantaneous direction and speed of an object in circular motion along its path. During the uniform circular motion discussed above, we see that the magnitude is constant and it is inversely proportional to the period of orbit. Hence it is given by the formula:

$$\text{orbital velocity } v = \frac{\text{circumference of the circle}}{\text{period } T}$$

$$v = \frac{2\pi r}{T}$$

Where v is the velocity, r is the radius of the orbit of any given satellite and T is the period of satellite orbit.

There is a specific relationship between bodies that orbit the same central mass given by Kepler's third law.

$$\left(\frac{r^3}{T^2}\right) \text{ for planet 1} = \left(\frac{r^3}{T^2}\right) \text{ for planet 2}$$

NOT on formula sheet.

The derivation of Kepler's third law is as follows...

$$F_c = F_G$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v^2 = \frac{GM}{r}$$

Where G is the gravitational constant and M is the mass of the central body.
<--Orbital velocity

$$subst \quad v = \frac{2\pi r}{T} \text{ into LHS}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

Now that we can see that G is constant, $4pi^2$ is constant and so if the central mass is the same, then it will also be a constant and the ratio of r^3/T^2 will be the same.

Qualitatively: Square of orbital period is directly proportional to the cube of orbital radius, mass of central body inversely proportional to cube of orbital radius, constant of proportionality is the gravitational constant times mass of central body divided by $4pi^2$

- solve problems and analyse information using:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

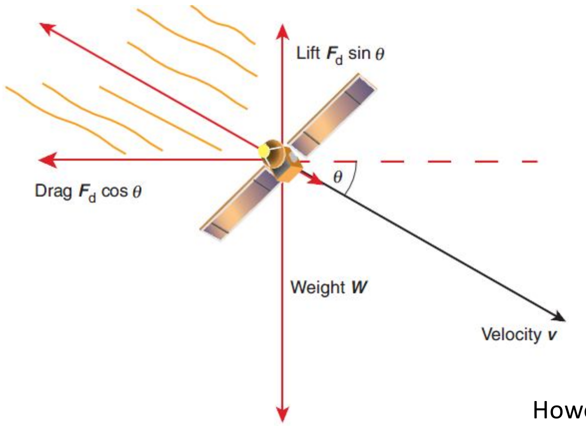
Calculate the period of a satellite orbiting Earth at the altitude of 400km
Given that radius of Earth is 6.38×10^6m and Earth's mass is 5.97×10^{24}kg

$$\frac{r^3}{T^2} = \frac{GM_E}{4\pi^2}$$

$$\frac{(6.38 \times 10^6 + 400 \times 10^3)^3}{T^2} = \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{4\pi^2}$$

$$\therefore \quad T = 5560 \text{ seconds} = 92.7 \text{ minutes}$$

- account for the orbital decay of satellites in low Earth orbit



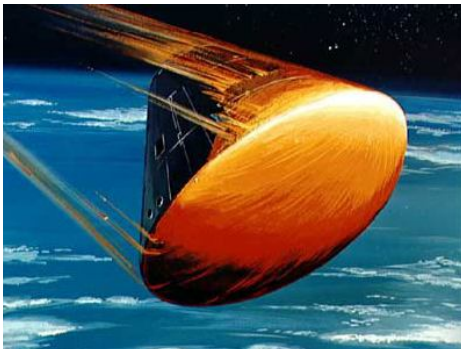
Satellites in low Earth orbit are in the upper atmosphere so they are subject to small some degree of atmospheric drag resulting from friction with air particles. Hence, this gradual friction causes a gradual loss of energy (to heat) which results in orbital decay and the satellite's life span will be limited, potentially spiralling towards the Earth.

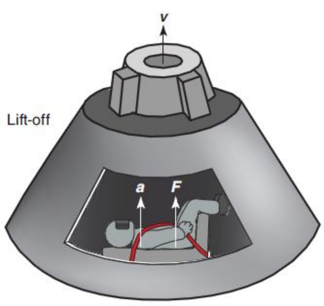
As the satellite drops to a lower altitude, gravitational potential energy is transformed into kinetic energy and this increases the velocity of the satellite. But this, in addition to the denser atmosphere increases the drag, causing further loss of energy (to heat), and orbital decay continues in this cyclic process. This process eventually speeds up, and below 200km altitude, a satellites has a few hours before collision with the Earth. Eventually a violent braking effect takes place, in which the heat produced is sufficient to destroy all but the largest of satellites. Extreme g forces are also generated and definitively destroy any equipment.

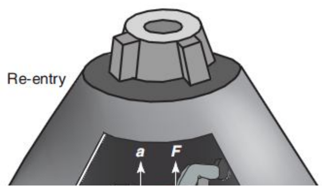
At times, satellites in low Earth orbit can unpredictably go down as factors affecting air density cannot be anticipated: i.e. increased solar activity causing atmosphere to expand and rise, resulting in increased atmospheric density and drag.

However, to counter the drag, designers attach rocket boosters to a satellite with the intention of lifting it back into its intended orbit altitude. These minute adjustments allow satellites to sustain their low Earth orbits for extended periods of time. I.e. the International space station which loses 90m of altitude per day.

- discuss issues associated with safe re-entry into the Earth's atmosphere and landing on the Earth's surface







For a spacecraft to initiate re-entry, a short retroburn of spacecraft thrusters quickly reduce the velocity and kinetic energy resulting in the craft's orbit being altered into a transfer ellipse that intersects the atmosphere at the desired angle. An entry angle that is too steep will lead to extreme heating and g forces while an angle too shallow means that the craft may skip off the atmosphere instead of penetrating it; an angle of 5.2^{o} to 7.2^{o} is the optimum.

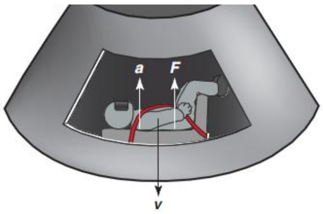
During re-entry, the spacecraft has significant kinetic energy as it moving through the atmosphere at more than 10000km/h, and while it descends considerable gravitational energy it possesses is converted into more kinetic energy. This fast velocity means that friction with atmospheric molecules produces heat as kinetic energy is converted into heat energy. As the kinetic energy is converted to heat, extreme temperatures are reached.

To withstand the heat, a blunt shape produces a shockwave of compressed air in front of the module in which most of the heat is generated, rather than the craft. Heat is carried away by using protective heat shields of ablating tiles (used on Apollo capsules). This idea is also used by the space shuttle, its blunt underside covered with ceramic tiles (e.g. fibreglass), ablated during re-entry, vaporising and carrying away heat. Fibreglass is 90% air with excellent insulation properties, as well as conserving mass. Particularly vulnerable areas such as the nose cone and leading edges are reinforced with carbon composites.

Heating can also be minimised by taking longer to re-enter, lengthening the time over which energy is converted to heat: i.e. space shuttle taking s-banking turns. The shuttle also presents its underside at 40^{o} to the atmosphere, reducing heat.

The friction not only heats up but also **acts against the spacecraft's motion and causes it to decelerate**. Hence the survival of living occupants is a great concern. Research found that a transverse, "eyeballs in" application of g force is the easiest to cope with, and so astronauts should liftoff forwards (facing up) and re-enter backwards (facing down). Therefore, a body suit and supporting fibreglass couches contoured to fit the astronaut's body suit allows the astronaut during launch to lie down as depicted above, and withstand up to as much as 20g.

A further problem was ionisation blackout which resulted from the ionisation of atoms in the air as heat builds up around the spacecraft during re-entry. Radio signals are unable to penetrate this ionised layer of ionised particles and hence radio communication between the ground and the spacecraft is cut off. (Apollo capsules

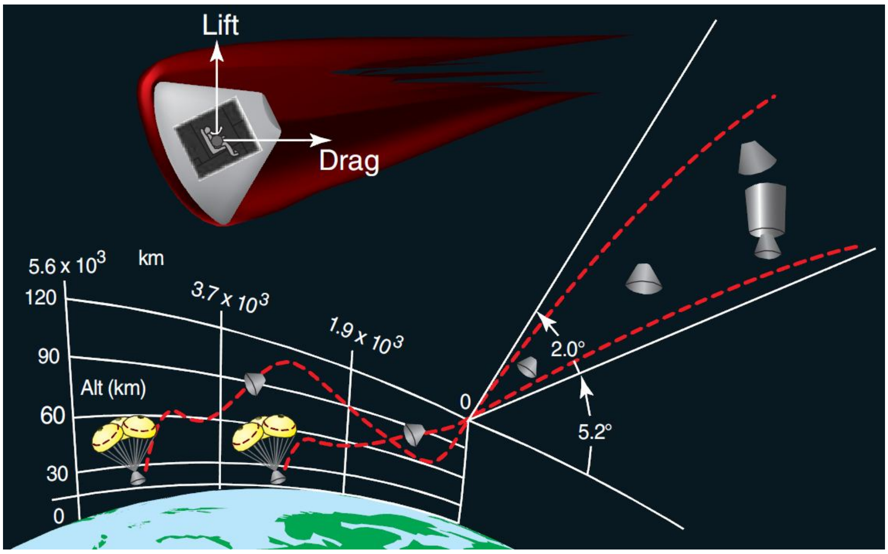


atoms in the air as heat builds up around the spacecraft during re-entry. Radio signals are unable to penetrate this ionised layer of ionised particles and hence radio communication between the ground and the spacecraft is cut off. (Apollo capsules experience ionisation blackouts of 3-4 min while space shuttles experience 16min). But preparation can be made so that astronauts are self-sufficient.

Finally, to land, spacecrafts must decelerate sufficiently by launching parachutes and landing in the ocean (Apollo module) or on land with the aid of retrorockets (Soyuz and Shenzhou). Or landing on an air-strip (space shuttle - with parachutes out the back to slow down the shuttle)

- identify that there is an optimum angle for safe re-entry for a manned spacecraft into the Earth's atmosphere and the consequences of failing to achieve this angle

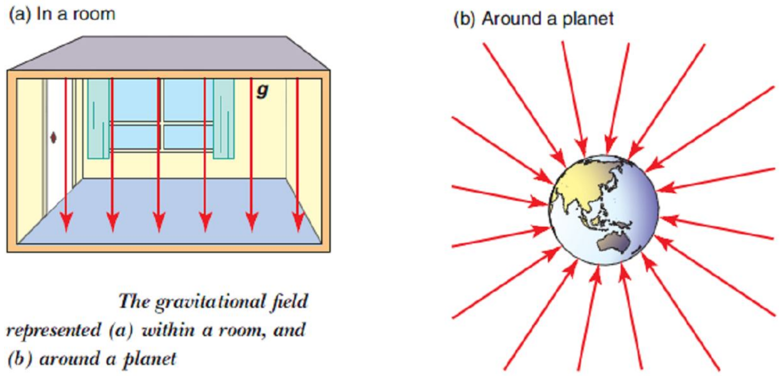
As stated above, the optimum angle for re-entry is 5.2° to 7.2°. If the re-entry angle is too steep, extreme temperatures and high g-forces produced may be too great to ensure survival of the spacecraft and occupants. However, if re-entry angle is too shallow, then the spacecraft may skip off the atmosphere instead of penetrating it due to compression of atmosphere below it. Hence, the optimum angle allows penetration into the atmosphere, yet minimising the heat and g-forces involved in re-entry.



3. The Solar System is held together by gravity.

- describe a gravitational field in the region surrounding a massive object in terms of its effects on other masses in it

A **gravitational field** is the field surrounding a massive object in which another mass experiences an attractive gravitational force. *The field lines representing the gravitational field can be seen in a radial pattern (b) around Earth, or on a small scale approximation, simply parallel vertical lines (a). The further apart the lines, the weaker the field.*

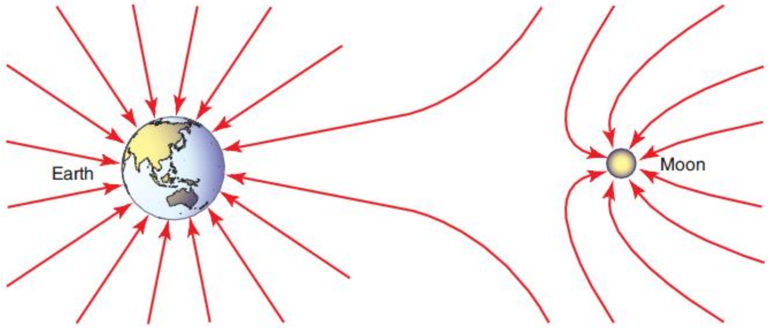


The gravitational field represented (a) within a room, and (b) around a planet

When two large objects combine to form a complex gravitational field as illustrated on the right, there is a point between the two objects where the field strength is exactly zero, being equal and in exactly the opposite direction.
This point is closer to the object with the smaller mass.

A **gravitational field** is a **vector field** and <u>any mass placed within the field will experience a gravitational force towards the large object's centre of mass</u> (*i.e. in the direction of field line arrows*). The closer to the centre of mass, the stronger the gravitational force on masses in the field (*represented by the field lines being closer together*). *(relate this to law of universal gravity - proportional to…)*

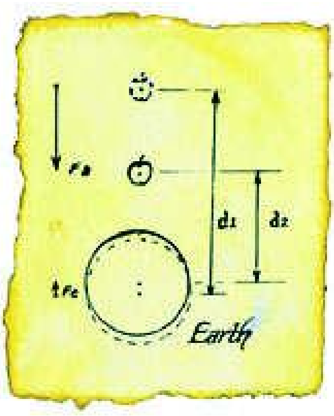
It should be noted, for large objects in the gravitational field, it will have a gravitational field of its own. *I.e. the moon around the Earth*.



As we'll see in the next dot point, the strength of the gravitational field is related to the value of the mass of the object and distance of from centre of the object.

- define Newton's Law of Universal Gravitation:

$$F = G\frac{m_1 m_2}{d^2}$$



Newton noted that a mass attracts every other mass in the universe, and **this attractive force is exerted equally on both masses**. This force (F) is described in what is known as the **Law of Universal Gravitation** given by:

$$F = G\frac{m_1 m_2}{r^2}$$

Where G is the universal gravitational constant of 6.67x10-11 Nm^{2}kg-^{2}, where m_1 and m_2 are the two masses, and r being the distance of separation between their centers of mass.

Hence the **force is inversely proportional to the square of the distance** while being **directly proportional to the mass of the objects**.

- present information and use available evidence to discuss the factors affecting the strength of the gravitational force

So far we can see that the strength of the gravitational force on the object is dependent on a few important variables.

Firstly, the **mass of the central object** (*i.e. a planet*) determines the strength of the gravitational field. The larger the central mass, the stronger the gravitational field. *(e.g. Gravity on Earth is around six times stronger than that on the Moon).*

Secondly, the **mass of the object in the gravitational field**. A larger object will be subject to greater gravitational attraction at a position in the field than a smaller object. *(i.e. At the Earth's surface, a 1 kg mass will experience a weight force of 9.8N, but 1 tonne mass will experience a weight force of 9800N.)*

And lastly, the distance of separation has a great effect on the strength of gravitational force as it has an inverse square relationship. The greater the distance, the less the gravitational force.

In theory, these three are the only variables that affect the gravitational force, as explicated in the formulae. However, it is also important to note uneven mass distributions can also alter the gravitational force in some cases. *(e.g. A person standing over iron ore deposits will experience greater gravitational attraction.)*

Look at prac in first section to have detailed summary on specific factors on variations of gravitational attraction on earth.

- solve problems and analyse information using:

$$F = G\frac{m_1 m_2}{d^2}$$

Sample questions

1) Find the gravitational force between Sam and Ashley standing one metre apart given that:
   a. The mass of the Sam is 7.35×10^1kg
   b. The mass of the Ashley is 5.97×10^1kg
   c. Their distance of separation is 1.00m

   [Sol] Using the formula:

   $$F = G\frac{m_1 m_2}{r^2}$$

   Let m_1 be Sam's mass, let m_2 be Ashley's mass, and r=1. Therefore:

   $$F = (6.67 \times 10^{-11})\frac{(7.35 \times 10^1)(5.97 \times 10^1)}{1.00^2}$$

   Thus F = 2.93×10^{-7}N (3sf)

2) What is the new gravitational force between two objects if the mass of each object is doubled and the distance doubled?

   [Sol] The new force will be as follows.

   $$F = G\frac{2m_1 \times 2m_2}{(2d)^2}$$

   $$F = G\frac{4(m_1 \times m_2)}{4(d^2)}$$

   $$F = G\frac{m_1 m_2}{d^2}$$

   This result is the same as before, thus the gravitational force remains the same.

discuss the importance of Newton's Law of Universal Gravitation in understanding and calculating the motion of satellites

What does this all mean?
When Newton's Law of Universal Gravitation is combined with the expression for centripetal force, we see that orbital velocity required for a particular orbit depends on the mass of the central mass, the radius of the mass and the altitude of the orbit. NOT the mass of the satellite. ****<see comment below>*
Given that the mass and radius of the Earth is fixed, the only variable that determines the orbital velocity is the altitude. And since this is an inverse relationship, the greater the radius of the orbit, the lower the orbital velocity required.

And with derived Kepler's law, we see that as G, M and $4\pi^2$ remains constant, any satellite orbiting the same primary mass will have the same value for:

$$\frac{r^3}{T^2}$$

****With these derivations, we have assumed that M is the mass of the central body, and thus this derivation only works for a small mass performing a circular motion around a larger mass.*

However, when two similarly sized bodies (e.g. two stars) orbit around the common centre of mass of the two body system, the distance of separation of each mass from the centre of its orbit is NOT equal to the separation distances of the masses.

In truth, when these variable distances are allowed to be equal, we find that M is the mass of the system and not just the central mass.

It's just that in the case of a planet and a satellite, the difference is insignificance (i.e. Central mass ≈ System mass).

The law of Universal Gravitation, is used in understanding and calculating the motion of satellites.

Gravity holds together the solar system by **providing the centripetal force that produces the circular motion of a satellite's orbit** (*otherwise, the satellite would fly away tangent to the circle*). This force ties the planets to the Sun and the satellites to the planets, essentially holding the solar system together. These masses trace out an orbital path, and with the Law of Universal Gravitation, we can calculate the orbital velocity of satellites as follows:

i. The gravitational attraction of a satellite around a planet (F_g) would be:

$$F_g = G\frac{Mm_s}{r^2}$$

Where G is the universal gravitational constant $6.22 \times 10^{-11} Nm^2kg^{-2}$, M is the large, central mass, m_s is the mass of the satellite and r is the distance of separation between the centers of mass.

ii. But as stated above, gravitational force also serves as the centripetal force maintaining the circular orbital motion, and we know that the centripetal force (F_c) is given by:

$$F_c = \frac{m_s v^2}{r}$$

Where m_s is the mass of the satellite, v is the velocity of the satellite and r is the distance of separation from the centre.

iii. And since $\boldsymbol{F_g=F_c}$,

$$G\frac{Mm_s}{r^2} = \frac{m_s v^2}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

Where v = orbital velocity (ms^{-1}), r is the distance of separation (sum of the radius of the central mass and the altitude of orbit).

Thus we can calculate the orbital velocity through this derivation.

Sometimes questions will require us to calculate the orbital velocity from the given radius and period of the orbit.

i. Since we know the circumference of the circle to be 2πr and the period (T), and...

$$v = \frac{\Delta r}{\Delta t}$$

ii. Then substituting in, the circumference and the period,

$$v = \frac{2\pi r}{T}$$

And finally, with the above two formulae for orbital velocity, we can derive Kepler's law of periods:

i. Firstly equating the two formulae:

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

ii. Thus, after simplification,

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

Although Kepler couldn't find the value of the constant k, now we know it to be the Right Hand Side of the above formulae.

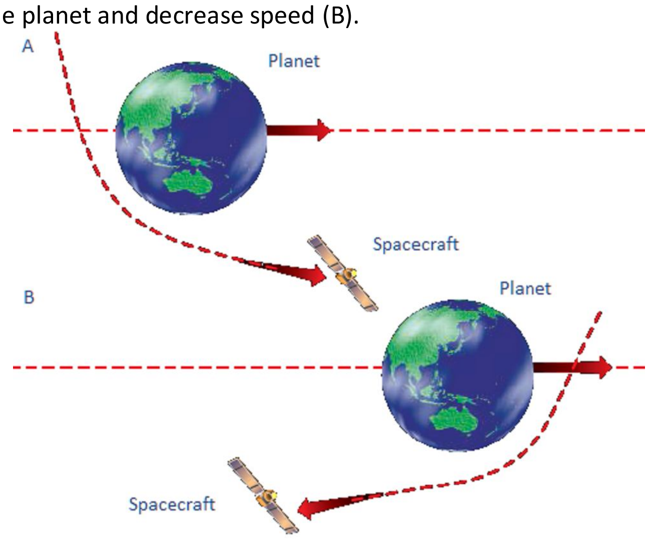
Hence, with the law of universal gravitation, many mathematical derivations can be made to understand and calculate the motion of satellites around a large central mass.

- identify that a slingshot effect can be provided by planets for space probes

The **slingshot effect** is used by space probes to **achieve a change in velocity with little fuel expenditure** as it angles in close to a mass and flings itself around the planet. *This is also known as a planetary swing-by or a gravity-assist manoeuvre.*

During the manoeuvre, the spacecraft deliberately passes close to a planet such that the planet's gravitational field catches the spacecraft, pulling it into an arc of circular motion. Such a manoeuvre will result in a **change in the speed relative to the Sun**. The speed acquired is sufficient to throw the spacecraft away from the planet. By controlling the approach, **the manoeuvre can be manipulated to achieve specific changes in speed and direction**.

For instance, when a spacecraft approaches the planet at an angle to its orbital path, it can swing behind the planet and increase speed (A), or swing in front of the planet and decrease speed (B).



We can look at this swing as a perfectly elastic collision.

a) **Applying the law of conservation of momentum**, we find that:

Initial momentum = final momentum

p_i of planet + p_i of spacecraft = p_f of planet + p_f of spacecraft

$$KmV_i + m(-v_i) = KmV_f + mv_f$$

$$KV_i + v_i = KV_f + v_f$$

Where Km is the mass of the planet, V is the velocity of the planet, m is the mass of the spacecraft and v is the velocity of the spacecraft.

So after this interaction, the ***spacecraft would increase in speed by acquiring kinetic energy*** *while the planet would have lost an equal amount of kinetic energy. But since:*

$$E_k = \frac{1}{2}mv^2$$

The slight loss in kinetic energy will result in an imperceptible decrease in the planet's velocity because the mass of the planet is so large.

b) And **applying the conservation of kinetic energy**.

Initial kinetic energy = final kinetic energy

E_{ki} of planet + E_{ki} of spacecraft = E_{kf} of planet + E_{kf} of spacecraft

$$\frac{1}{2}KmV_i^2 + \frac{1}{2}m(-v_i)^2 = \frac{1}{2}KmV_f^2 + \frac{1}{2}mv_f^2$$

$$KV_i + (-v_i) = KV_f + v_f$$

Finally, by **solving the two above equations in (a) and (b) simultaneously**,

$$v_f - v_i + 2V_i$$

We see that energy and momentum has been conserved. This is the maximum velocity that can be achieved by the slingshot effect and is achieved by a head-on course

4. Current and emerging understanding about time and space has been dependent on earlier models of the transmission of light

By David Lin
Mr Ogle

- outline the features of the aether model for the transmission of light

After light was concluded to be a waveform, 19th century physicists theorised that light waves required a medium in which to travel. This was based on observations of other waveforms such as sound and water waves. However, despite this proposition, the existence of the medium couldn't be proved.

But it was believed that the medium, called "luminiferous aether", was perfectly transparent, had low density, filling all space and stayed stationary in space. It supposedly permeated all matter whilst having great elasticity to support the propagation of light waves.

- describe and evaluate the Michelson-Morley attempt to measure the relative velocity of the Earth through the aether

It was theorised that the Earth in its orbit also moves through the supposed stationary aether. This flow of aether past Earth is known as the 'aether wind'. But aether is tenuous and has very low density, making any aether winds very hard to detect. Most experiments failed to detect aether, and it was assumed that instruments weren't sensitive enough.

However, the Michelson-Morley experiment designed an experiment which would attempt to measure the relative velocity of the Earth through the aether.

As wind rushing past an observer would change the speed of sound, the speed of light which was constant relative to the aether wind, would vary as seen by the observer. This difference should be detectable by using an interferometer to compare the speed of light rays heading into and across the supposed aether wind.

A light ray from a source is split by a half-silvered mirror. One ray travels into the supposed aether wind (slower) and back (faster). While the second ray travels across the wind as it is perpendicular to ray one. These rays are overlapped and their interference pattern observed. If aether wind does actually exist, when the apparatus is rotated 90^{o} a shift in the observed interference pattern (of light and dark bands) should be apparent since the speed of each ray will be different. This change in interference pattern can be used to calculate the relative velocity of Earth through the aether. *Expected fringe shift was 0.04, but a shift of less than 0.01 was detected, within experimental error, thus no significant difference between speed of light rays were detected.* With the expected shift not observed; the null result of the experiment disproved the existence of aether winds as no motion of the Earth relative to the aether was detectable.

The Michelson-Morley experiment was sufficiently sensitive, and very reliable, producing identical results no matter where, when it was done, or who it was conducted by (even with more sensitive equipment, i.e. Illingworth - detected fringe shift was 0.0002, even less noticeable). Therefore, no one was able to demonstrate the existence of the aether using this experiment.

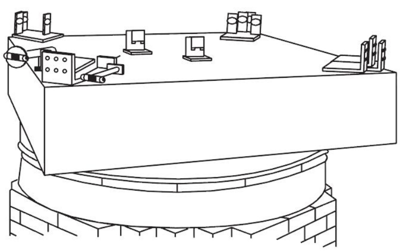
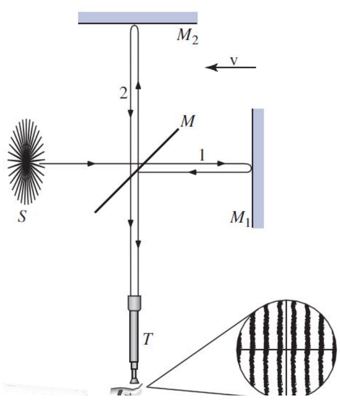
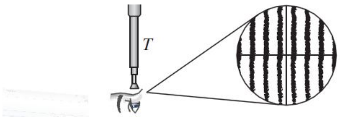


Figure 5.4 *The apparatus of the Michelson–Morley experiment set up on a large stone block, to keep it rigid, and floating on mercury so that it can*



The experiment is set up such that light sent from a source (S) is split into two perpendicular beams by the half silvered mirror in the centre. One ray travels with and against the supposed aether wind (ray 1), while the other travels across the aether (ray 2). Rays are reflected off respective mirrors (M_1, M_2), and then recombined in a telescope (T) for observation.

Michelson–Morley experiment set up on a large stone block, to keep it rigid, and floating on mercury so that it can be easily rotated 90 degrees



(M_1, M_2), and then recombined in a telescope (T) for observation.

■ gather and process information to interpret the results of the Michelson-Morley experiment

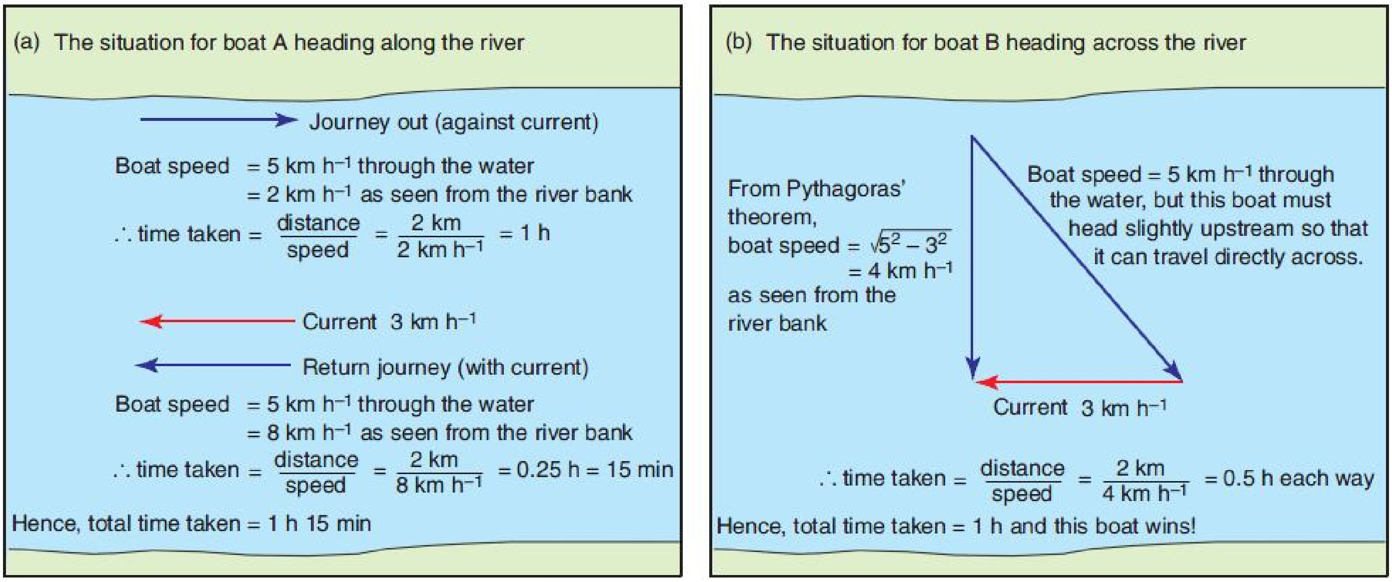
The Michelson-Morley experiment can be interpreted by using a model of boats through a river current rather than light waves through an aether.

Consider two boats (both capable of 5kmh^{-1}) heading on different 2km courses in a river. Boat A heads upstream against currents of 3kmh^{-1} for 2km at 2kmh^{-1}. But returns with the current at 8kmh^{-1}. Meanwhile, boat B heads directly across the shore, but having to head upstream slightly and still cross the river, ultimately taking 15min less than boat A.

Similarly, if the aether did exists, it must have an effect on the time taken for the two light rays raced around the Michelson-Morley experiment such that one light wave arrives earlier than the other. And the relative velocity of Earth through the aether can be calculated.

And by interposing the boats, repeating the experiment will eliminate any difference between the boats as a cause for time difference and boat A should win by the same margin. Similarly light rays are also interposed by rotating the experiment by 90º and aether current direction is changed, creating a different disturbance pattern. This difference between rays as they finished the course can be used to calculate the value for aether wind.

However, a change in disturbance pattern was not observed meaning light did not vary in velocity despite rotation as predicted. And hence, the only explanation, is that there is no aether affecting the light waves. Therefore, no motion of the Earth relative to the aether was detectable.



■ discuss the role of the Michelson-Morley experiments in making determinations about competing theories

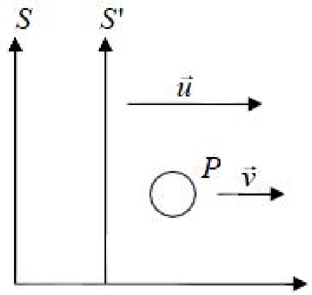
From a hypothesis, predictions are made of what should happen in a particular experiment. If results are in disagreement with predictions, the hypothesis (in this case the aether model) is invalid.

Originally, the set of predictions from the aether model couldn't be tested. But with improved technology, the Michelson-Morley experiment produced a null result despite being sufficiently sensitive, and very reliable, producing identical results no matter where, when it was done, or who it was conducted by (even with more sensitive equipment). Although the aim was to measure relative velocity of Earth through the aether, no shift in disturbance pattern was detected and hence no aether wind was detected.

Physicists initially didn't want to abandon the aether model. Their assumption of aether was based on decades of correct scientific thinking as their prediction is true for any other wave. Furthermore, the concept of absolute frame of reference underpinned by Newtonian dynamics, supported their hypothesis; a successful and unquestioned theory of the time. Many adapted theories such as large objects (i.e. a planet) dragging the aether along with it, or objects contracting in the direction of the aether wind. But these proposals did not survive under scrutiny and finally in 1905, Einstein opened up a revolutionary view of space and time where aether was not necessary at all.

In hindsight, Michelson-Morley's experiment helped physicists reject the incorrect hypothesis of the aether model, and accept Einstein's relativity model.

- outline the nature of inertial frames of reference



Galileo's principle of relativity states that all steady motion is relative and cannot be detected without reference to an outside point. Therefore, we use frames of reference, which are objects or coordinate systems, with respect to which we measure position, displacement, velocity, etc.

For instance, in maths, the frame of reference can be the Cartesian coordinate system with x, y, z axes. Or a school experiment would use the laboratory as the frame of reference.

Despite which frame of reference is used, firstly, when an object, P, travels with velocity, v, with respect to the reference frame S, an alternate reference frame, S' , travelling at the velocity u relative to S' can also be used. Although the velocity of P relative to S is v, relative to S' it is v-u. Therefore, velocity depends on reference frames.

For an inertial frame of reference, it is one that is moving at constant velocity or at rest (i.e. a non-accelerated environment). There is only steady motion or no motion allowed, and it is the frame in which no mechanical experiments can reveal whether one is moving with constant velocity or being stationary.

I.e. interior of car or train at constant velocity, a class room, etc.

On the other hand, non-inertial frames of references is one that is accelerating.

- perform an investigation to help distinguish between non-inertial and inertial frames of reference

To distinguish between inertial and non-inertial frames of reference requires the use of an accelerometer, the simplest of which is a plumb bob. In an inertial (non-accelerated) frame of reference, it will hang vertically down. It was seen on a bus, that when the bus was stationary or in steady motion (i.e. at cruising speed), the plumb bob remained vertical. Hence in those moments, the bus is an inertial frame of reference.

But in a non-inertial frame of reference, plumb bob would deviate from its vertical position. So when the bus accelerated, the plumb bob would move backwards, or when turning, it would move sideways; it no longer hangs vertically. It's not the plumb bob's movement which indicates a frame is non-inertial, but its deviation from its vertical position. In these instances of acceleration, the plumb bob is not vertical but at an angle, hence it is a non-inertial frame of reference.

Hence, the plumb bob acts as an accelerometer in distinguishing between non-inertial and inertial frames of reference.

- discuss the principle of relativity

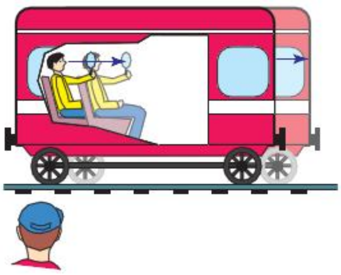
Galileo's proposed principle of relativity stated that all steady motion (stationary or constant velocity) is relative and cannot be detected without reference to an outside points (i.e. another frame of reference). This notion is also found built into Newton's first law of motion.

The principle of relativity only applies to inertial (non-accelerated) frames of reference. And that within all inertial frames of reference, the laws of physics are the same and one cannot perform any mechanical experiment or observation to reveal whether one was standing still or at constant velocity.

Ultimately, there is no absolute motion and it all must be measured relative to another object.

- analyse and interpret some of Einstein's thought experiments involving mirrors and trains and discuss the relationship between thought and reality

Einstein used thought experiments to investigate/illustrate situations which could not be tested experimentally, stemming from limitations in current technology. Thought is a powerful tool when our ability to observe reality is limited. It allows for hypothetical situations which are based on logic and fact, which are usually correct.

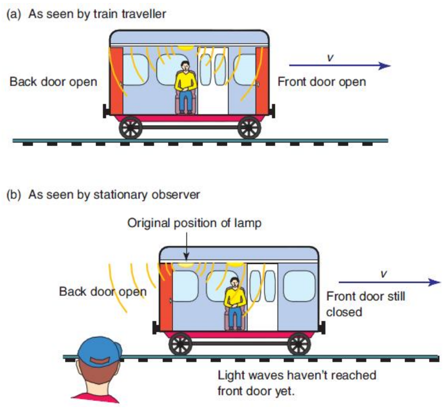


The mirror and the train: An apparent dilemma was reduced by Einstein into a thought experiment. If a train was travelling at light speed with an observer inside holding a mirror, according to the aether model, light has a fixed velocity to the stationary aether. and light rays would never catch up to the mirror. This means that no reflection can be observed, providing a way to determine motion within an inertial frame of reference, violating the principle of relativity. Furthermore, a stationary observer outside the train would view the light at twice its speed.

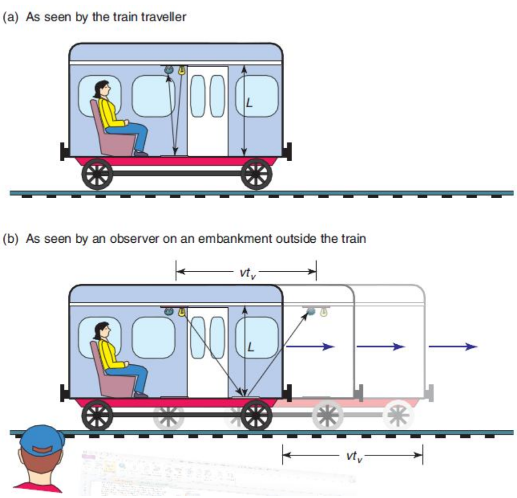
Instead Einstein though that the principle of relativity must hold, and therefore, he postulated that light's observed speed from any frame of reference was constant and the principle of relativity is not violated. Therefore the reflection can be seen by the traveller, and the observer outside the train will also see light at the same speed.

Consequently, since c=distance/speed and the light is constant speed. Then distance and time witnessed by observers are different under relativistic speeds. Hence the aether was not needed to explain light speeds, and deemed superfluous.

This consequently affects the judgement of event simultaneity, and a thought experiment is offered by Einstein. Lightning bolts strike both ends of a moving train carriage simultaneously as seen by an stationary observer outside the train standing perpendicular to the centre of the carriage. Light from each end of the carriage takes the same time to reach this outside observer. However, the traveller inside the train is moving forwards and first sees the light coming from the front because it has less distance to travel. Whereas, the light from the back has to travel more than half the length of the carriage and is seen later. Although events from the perspectives of both observers seem to differ in simultaneity, both observers have judged correctly from their different frames of reference.

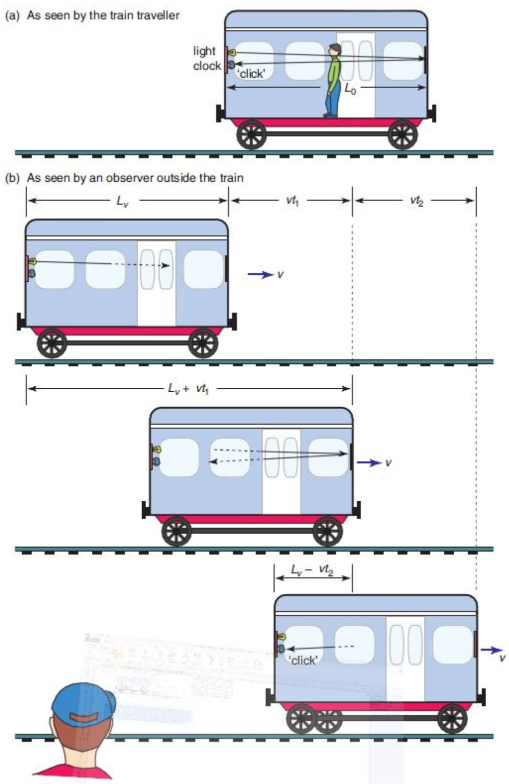


On the other hand, if light activated doors at both ends of a moving train are closed and a light in the centre of the carriage is turned on. As seen by the observer inside the train, doors open simultaneously. But seen by a stationary observer, the back door opens before the front door. This is because after the instance the light was turned on, the train has moved forward and the light waves have less distance to travel to the back door whereas, the forward journey to the front door is now longer. But since, the speed of light is a constant, the backward journey takes less time than the forward journey. This demonstrates simultaneity is dependent on the observer's motion.



Another consequence is time dilation, and it follows in a thought experiment that if a light clock runs the height of a moving train, the distance the light travels seen from inside and outside the train are perceived differently. A stationary observer sees that light has to travel further to complete one second. But since the velocity of light is the same for that observer, therefore, time within the moving environment is dilated. In other words, any measurements of time from any other inertial frame of reference in relative motion to the first is always greater, such that time passes slower in the first inertial frame of reference.

Finally, another thought experiment is proposed to illustrate the consequence of length contraction parallel to the direction of travel. If a light clock runs the length of a moving train, with lamp and sensor placed at the rear, then as the train moves, a stationary responder sees forward leg of the light pulse's journey is lengthened, while the return journey is shortened. This results in the person outside the train seeing the train shorter than the person inside the train.



describe the significance of Einstein's assumption of the constancy of the speed of light

As opposed to the aether model in which light's velocity was constant relative to the aether (an universal reference frame), in 1905, one of Einstein's fundamental postulates for the theory of relativity is that the speed of light was constant and independent of the motion of the source and the observer. This means that all observers see light travelling at the same speed, and explained the negative result of the Michelson-Morley experiment and showed that the concept of aether was superfluous.

Unlike the aether, there is no absolute frame of reference with all inertial frames of reference equivalent (all motion is relative). Hence, laws of physics are the same in all frames of references, so that relativity always holds. Therefore, time, mass and length are no longer absolute quantities, but are relative.

identify that if c is constant then space and time become relative

For instance, if B moves from A at c/3, and A sends a light ray to B, the light ray overtakes B at c and not 2c/3 as expected with Newtonian physics.

Since speed = distance/time, and the speed of light is constant, then constancy of the speed of light inevitably leads to the difference of distance and time witnessed by the observers.

In Newtonian physics, space is relative to an observer. Therefore, distance and velocity are relative terms, and even the speed of light was considered to be relative to an observer. But time was an absolute and fundamental quantity passing identically for all observers.

In Einstein's thought experiment concerning a traveller inside a moving light speed train, if the traveller were to hold a mirror and a reflection is seen; the light ray's forward journey from the face to the mirror seen by an outside stationary observer is double the distance seen by the traveller. Instead of the light ray being seen twice as fast by the outside observer, the speed of light is postulated to be a constant to both observers. Hence the distance and time perceived by both observers must be different (c= distance/time).

Hence, this new notion of the speed of light being constant and absolute, requires a radical change from previous thoughts in Newtonian physics. With the speed of light being the same as seen by any observer from any reference frame, this means space and time become relative quantities based on the motion of the observer.

discuss the concept that length standards are defined in terms of time in contrast to the original metre standard

Initially, the metre was defined as 1×10^{-7} times the length of the Earth's quadrant passing through Paris. After surveying, three platinum standards were made along with several iron copies. When the survey was found to be incorrect, the metre was redefined to be the distance between two marks on a bar.

With the introduction of SI units in 1875, the metre was defined as the distance between two lines scribed on a single bar of platinum-iridium alloy.

In the 1960s it was defined as a certain multiple of the wavelength of the orange light emitted by krypton 86 atoms when subject to electrical discharge.

From 1983, we have used a much more precise definition is the distance travelled by light in a vacuum during the time interval of 1/299792458th of a second. This takes advantage of the constancy of the speed of light and technology which can make very small accurate measurements. This standard can be reproduced in any laboratory.

Interestingly, the light year is a similar distance unit, being the length of the path travelled by light in a time interval of one year, being approximately 9.46728×10^{12} km.

explain qualitatively and quantitatively the consequence of special relativity in relation to:
- the relativity of simultaneity
- the equivalence between mass and energy
- length contraction
- time dilation
- mass dilation

Relativity of Simultaneity

Observers in relative motion will disagree on the simultaneity of events separated by space. And hence although events from the perspectives of both observers seem to differ in simultaneity, both observers have judged correctly from their different frames of reference. This is a direct consequence of the constancy of light. *<Thought Experiment as above>*

Equivalence between Mass and Energy

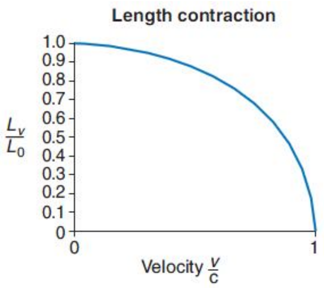
An object's rest mass is equivalent to a certain quantity of energy. Under certain circumstances, mass can be converted to energy and energy to mass. This equivalence is expressed in Einstein's equation $E=mc^2$.

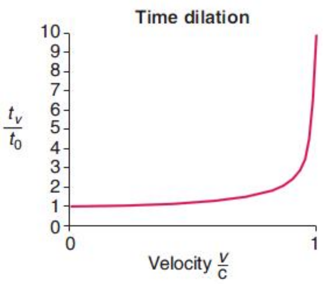
In nuclear fission for instance, mass is transformed to energy. And if force is applied to an object such that it is approaching to light speed, the applied force does not mean the object acquires the kinetic energy we would expect. Instead, it acquires extra mass through mass dilation.

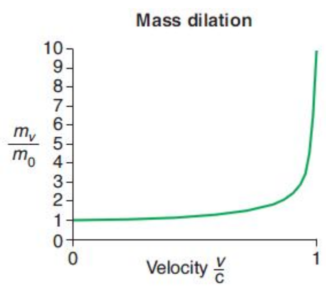
In relativity, the conservation of mass and conservation of energy is replaced by the conservation of mass-energy.

Relativity results in the new definition:

$E = E_k + mc^2$ where *E = total energy*, E_k *= kinetic energy, m = mass, c = speed of light*. But at rest, the object as no kinetic energy and its rest energy is given by $E = mc^2$.







Length Contraction

As a consequence of perceiving time differently, there is an effect called length contraction. If L_o is the length of the object measured within its rest frame, then measurements of this length, L_v made from any other inertial frame of reference in relative motion parallel to that length, are always less.

This is expressed by the equation:

$$L_v = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

I.e. A train is measured to be 100m long at rest. When it travels at 0.8c, a person on the train will measure it to be 100m long, but an observer at the side of the track will see it as just 60m long.

Time Dilation

Time is perceived differently by observers in relative motion to each other (i.e. in different frames of reference). Specifically, if t_0 is the time taken for an event to take place within its own rest frame, any measurements of time, t_v, made from any other inertial frame of reference in relative motion to the first is always greater, such that time dilation occurs within the observed frame of reference.

This is given by:

$$t_v = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

I.e. a traveller on a 0.8c train picks up a newspaper, taking 1.0 seconds. A person standing on the *side of the tracks observe this to take 1.7 seconds.*

Mass Dilation

Since c is the maximum speed in the universe, it follows that a steady force applied to an object cannot accelerate it to exceed light speed.

Applied force to an object gives it kinetic energy, but in an effect known as mass dilation, instead gaining the expected kinetic energy, the object also acquires some extra mass. This is particularly evident when approaching light speed and Einstein made the inference that the extra mass of the object contained the energy.

Overall, the mass of an object within its own rest frame is m_0, *(i.e. rest mass)*. When measurements are made, m_v, from other inertial frames of reference in relative motion to the first, are always greater.

This is given by formula:

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

It is this increase in mass which prevents objects

Time Dilation and Mass Dilation as an inhibition to approaching light speed

It is mass dilation which prevents any object from exceeding the speed of light. Because as they are accelerated to higher velocities, mass increases requiring a greater force for further acceleration. Furthermore, this is complicated by time dilation where speed increases approaching light speed means the applied force has less and less time in which to act.

The combined effect as an object approaches light speed is that mass approaches infinite while time dilates to approach zero. Therefore, an infinite force would be required to achieve any acceleration at all.

Hence, *sufficient force can never be supplied to accelerate an object beyond the speed of light*.

- analyse information to discuss the relationship between theory and the evidence supporting it, using Einstein's predictions based on relativity that were made many years before evidence was available to support it

Proposed theories should explain all existing observations and produce theoretical results generating hypothesises which can be tested experimentally to verify the theory's validity.

Einstein's general and special theories of relativity predicted many previously unknown phenomenon which relied on thought experiments which some deemed unsatisfactory. When the theories were proposed in 1905 and 1915, there lacked technological capability to verify the predictions.

But technology did become available over the 20th century and both relativity theory's predictions became testable. In 1919, the prediction of starlight deflection by the sun was confirmed during a solar eclipse.

Following this, further technology advances allowed atomic clocks to be flown at high speeds to confirm the existence of time dilation. Similarly, high speed mesons penetrating the atmosphere at 0.996c last 16µs rather than the rest time of 2.2µs in laboratories, a further confirmation of time dilation.

By detecting charge/mass ratios in emitted beta radiation of electrons travelling at significant speeds confirmed mass dilation. The accelerated particles decreased in charge/mass ratio, suggesting it was heavier as velocities approached c.

Therefore, for a theory to be verified, its predictions can be tested and hence evidence can be used to support it. Predictions were made by Einstein's theories of special and general relativity, many years before technology was available to test them, but over time, experiments consistently agreed with the predictions, verifying Einstein's theories.

solve problems and analyse information using:

$$E = mc^2$$

$$l_v = l_0\sqrt{1-\frac{v^2}{c^2}}$$

$$t_v = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m_v = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Length

When stationary, the carriages on the state's new VVFT (very, very fast train) are each 20 m long. How long would each carriage appear to a person standing on a station platform as this express train speeds through at half the speed of light?

The proper length, L_0, of a carriage is 20 m, while the length as seen from the platform is L_v.

$$L_v = L_0\sqrt{1-\frac{v^2}{c^2}}$$
$$= 20\sqrt{1-(0.5)^2}$$
$$= 17.32 \text{ m}$$

Time

A train traveller sneezes just as his train passes through a station. The sneeze takes precisely 1.000 s as measured by another person seated next to the sneezer. If the train is travelling at half the speed of light, how long does the sneeze take as seen by a person standing on the platform of the station?

The rest time, t_0, is the time as observed within the sneezer's rest frame, and therefore is 1.000 s. The time dilation equation is needed to determine the time, t_v, as observed from the frame in relative motion; that is, the platform:

$$t_v = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$= \frac{1.000}{\sqrt{1-\left(\frac{0.5c}{c}\right)^2}}$$
$$= 1.155 \text{ s.}$$

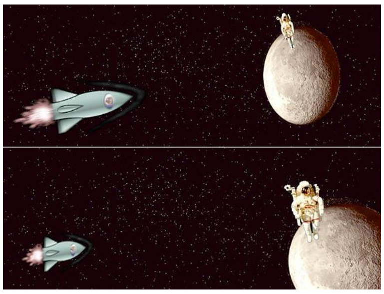
Mass

The rest mass of an electron is 9.109×10^{-31} kg. Calculate its mass if it is travelling at 80 per cent of the speed of light.

$$m_v = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$= \frac{9.109 \times 10^{-31}}{\sqrt{1-\frac{(0.8c)^2}{c^2}}}$$
$$= \frac{9.109 \times 10^{-31}}{0.6}$$
$$= 1.518 \times 10^{-30} \text{ kg}$$

That is, the electron's mass is approximately 1.7 times its rest mass.

■ discuss the implications of mass increase, time dilation and length contraction for space travel



Predicted changes from normal conditions as consequences of relativity, presents new perspectives and further possibilities concerning space travel.

Length contraction means that from the point of view of spacecraft occupants, length of the journey has contracted to a significantly shorter distance, which they cover in less time. Travel to Alpha Centauri at 0.5 c takes 7 years, but would take 8 years without length contraction. Occupants measure distance to the star as less, hence reaching it faster.

Another implication is that time dilation means that occupants experience significantly less time than when observed from Earth's point of view. Therefore, due to a combination of the above factors, as spacecrafts reach relativistic speeds, interstellar space travel becomes possible in human lifetimes.

However, it is very difficult to achieve relativistic speeds, one reason being mass dilation. As speed increases, more of the energy that would otherwise increase the speed instead creates more mass. When accelerating to speeds above 0.1c (relativistic), this effect becomes noticeable and approaching light speed, almost all energy would go into creating mass, hence it's impossible to reach c.

Today, current speeds are still far from relativistic and further technological development is required to reach fast speeds. Furthermore, due to the constancy of the speed of light, two way communication technology based on EM waves will be very difficult as the lag time in response is still many years between stars.

Einstein previously considered the twins paradox, concerning the strange implications of time dilation. As one twin on a speeding spaceship would think time is dilating on Earth, from the perspective of the Earth twin, time would be dilating on the spacecraft. Hence a paradox as no inertial frames of references are considered over the other, and in both cases, it appears that the other twin is accelerating away.

However, Einstein challenges this notion, as the perspective is not reversible, because the spaceship twin's frame of reference has not remained inertial, as it is accelerated. Therefore, the two frames are not equivalent, and the spacecraft twin is younger and survives while the Earth on dies.

SPACECRAFT	SPEED (km h^{-1})	RATIO $\frac{v}{c}$	TIME PASSED ON SPACECRAFT IN ONE EARTH DAY			CONTRACTED LENGTHS AS % OF ORIGINAL
			HOURS	MINUTES	SECONDS	
Space shuttle	28 000	0.000 026	23	59	59.999 972	99.999 999 97
Fast space probe	100 000	0.000 093	23	59	59.999 630	99.999 999 6
Light sail	108 000 000	0.1	23	52	46.92	99.499
Starship *Intastella*	972 000 000	0.9	10	27	40.89	43.59
Starship *Galactica*	1 079 892 000	0.999 9	0	20	21.85	1.4