CHAPTER 4

Special relativity

Answers to revision questions

1. (a) Aether was important because:
   - it was proposed to be a medium through which light could propagate
   - it was the absolute frame of reference to which all motion could be compared.

   (b) Three properties of aether are:
   - permeable to all matters
   - thin and not detectable by touch, smell or sight
   - highly elastic, behaves like a solid to a rapidly changing force such as EMRs and behaves like a medium that can be distorted indefinitely by an uniform and continuous force such as the motion of the Earth.

2. (a) For the detailed procedure of the Michelson–Morley experiment, please refer to the diagram and explanation in Chapter 4. The entire apparatus needed to be rotated by 90 degrees to ensure that the interference pattern was not caused by the distance difference between two pathways; rather it was caused by the difference in velocity of light as it traverses through aether in directions that are perpendicular to each other.

   (b) No change of interference pattern was detected, therefore the presence of aether wind could not be detected. Therefore the aether model still lacked evidence. However, it is important to emphasise that the experiment did not disprove the aether model.

   (c) It led Einstein to disregard the aether model completely and replace it with his theory of special relativity which accounted for the null result in the Michelson–Morley experiment and offered an entirely new perspective of the physics world.

3. The answer will be exactly the same as that obtained in the school laboratory, because both the train and school laboratory are considered to be inertial frames of references.

4. (a) See Chapter 4.

   (b) A turning car; a plane during its lift-off phase.

   (c) Place a ball on the floor within this frame of reference and observe its behaviour. If the ball is in a non-inertial frame of reference, it will undertake motions which cannot be explained by Newton's laws of mechanics, and a fictitious force is required to explain the motions of the ball.

5. (a) The velocity of the ball will be equal to the sum of the two velocities:
   $10 + 5 = 15$ m s^{-1}

   (b) The ball will be 10 m s^{-1} relative to Johnny.

   (c) The velocity of the beam will be c, where c is equal to 3×10^8 m s^{-1}, since the speed of light remains constant irrespective of the motion of the emitter and the motion of the receiver.

   (d) The velocity of the light will be c (same reason as above).

6. No, because they are in a non-inertial frame of reference. They will not be able to use special relativity since it only applies to inertial frames of reference.

7. See Chapter 4.

8. Using the time dilation equation:

$$t_v = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

Known quantities:

$t_o = 2\ \mu s$
$= 2 \times 10^{-6}$ s
t_v = the time measured in an Earth laboratory
$v = 0.99c$

$$t_v = \frac{2 \times 10^{-6}}{\sqrt{1-\frac{0.99^2 \cancel{c^2}}{\cancel{c^2}}}}$$

$= 1.4 \times 10^{-5}$ s

9. (a) Time taken $= \frac{\text{displacement}}{\text{velocity}}$

$$t = \frac{3400000}{0.3 \times 3 \times 10^8}$$

$= 0.038$ s

(b) Due to time dilation, the time taken will be shorter.

$$t_v = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

Known quantities:

$t_v = 0.038$ s
$v = 0.3c$

$$0.038 = \frac{t_0}{\sqrt{1-\frac{(0.3c)^2}{c^2}}}$$

$t_0 = 0.038 \times 0.9539$
$= 0.036$ s

(c) Due to length contraction, the distance will appear shorter to the pilot.

$$l_v = l_o\sqrt{1-\frac{v^2}{c^2}}$$

Known quanitities:

$l_0 = 3400$ km
$v = 0.3c$

$$l_v = 3400 \times \sqrt{1-\frac{(0.3c)^2}{c^2}}$$

$= 3243$ km

(d) They are both correct. Special relativity states that all frames are relative to each other, and none is more correct that the others.

10. (a) The height remains the same, which is 100 m. The width will change according to the length contraction formulae:

$$l_v = l_o\sqrt{1-\frac{v^2}{c^2}}$$

Known quantities:

$l_o = 50\times 2\ \text{m}$

$v = 27000000\ \text{m s}^{-1}$

$$l_v = 50\times 2\times\sqrt{1-\frac{27000000^2}{c^2}}$$

$= 99.6\ \text{m}$

(b) Time runs slower on this UFO since it is travelling at a very high velocity. Therefore, the person will be able to live longer on the UFO according to his relatives on the Earth.

Time dilation:

$$t_v = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

Known quantities:

$t_o = 100$ yrs

$v = 27000000\ \text{m s}^{-1}$

$$t_v = \frac{100}{\sqrt{1-\frac{27000000^2}{c^2}}}$$

$= 100.41$ yrs

11. According to mass dilation, the mass of the hydrogen ion will increase.

$$m_v = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Known quantities:

$m_o = 1.673\times 10^{-27}\ \text{kg}$

$v = 40000000\ \text{m s}^{-1}$

$$m_r = \frac{1.673\times 10^{-27}}{\sqrt{1-\frac{40000000^2}{c^2}}}$$

$= 1.688\times 10^{-27}\ \text{kg}$

12. The mass changes due to mass dilation.

$$m_v = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

First, you need to work out the fuel used; let the maximum speed be V, where $V = xc$.

$$10\times\frac{xc}{0.001c} = 10000x\ \text{kg}$$

$= 10x$ tonnes

Known quantities:

$m_o = (30 - 10x)\,\text{tonnes}$

(m_o is the rest mass when it reaches maximum speed, therefore the mass of the fuel to be consumed during the trip needs to be subtracted from the original mass)

$$m_v = 31 \text{ tonnes}$$

$$\therefore 31 = \frac{30 - 10x}{\sqrt{1 - \left(\frac{x\cancel{c}}{\cancel{c}}\right)^2}}$$

$$\frac{30 - 10x}{31} = \sqrt{1 - x^2}$$

$$(30 - 10x)^2 = 31^2 \cdot (1 - x^2) \left[\textit{square both sides}\right]$$

$$900 + 100x^2 - 600x = 961 - 961x^2$$

$$1061x^2 - 600x - 61 = 0$$

Using the quadratic formulae:

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (where $ax^2 + bx + c = 0$)

$x = 0.65$ (x can't be negative)

The maximum speed is $0.65c$.

13. See Chapter 4.

14. (a) The mass gained in mass dilation is derived from the kinetic energy of the object due to its motion.

(b) The fact that time runs slower for moving objects means that the measured length will be shorter.

15. See Chapter 4.

16. See Chapter 4.