

# Space

## 1.1 Gravity and Gravitational Fields

### 1.1.1 Define weight as the force on an object due to a gravitational field

- **Weight** is the force of gravitation exerted on an object by a large celestial object (e.g. the Earth).  

$$W = mg$$
  - $W$  is the weight (N),  $m$  is mass (kg) and  $g$  is the acceleration due to gravity ( $= 9.8 \text{ ms}^{-2}$  at the Earth's surface).

### 1.1.2 Explain that a change in gravitational potential energy is related to work done

- **Gravitational potential energy** ( $E_p$ ) is the energy of a mass due to its position within a gravitational field. This energy can be released and converted into kinetic energy ( $E_k$ ) when the mass is allowed to fall.
- On Earth, the  $E_p$  of an object at a point  $x$  above the ground is equal to the work done in moving the object from the ground up to  $x$ . The object moves against the field, so it gains  $E_p$ .
- Conversely, if an object moves closer to the centre of the Earth, some of its  $E_p$  is converted into  $E_k$  so work is done *by* the object instead of *on* the object.
- For an object of mass  $m$  at height  $h$  above the Earth's surface:  

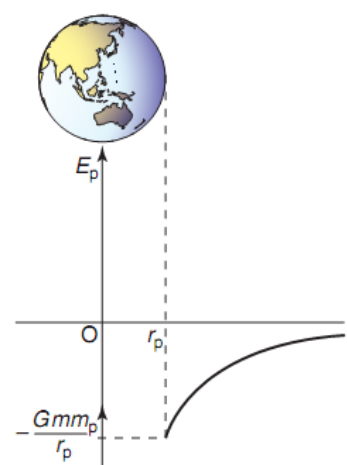
$$E_p = mgh$$
  - This equation is only valid when the object is near the Earth's surface.
- The ground is where  $E_p = 0$ , so when work is done on the object to lift it, it acquires energy and  $E_p > 0$ .

### 1.1.3 Define gravitational potential energy as the work done to move an object from a very large distance away to a point in a gravitational field

- On a large scale, gravitational potential energy is the work done in moving an object from infinity to a point  $x$  within a gravitational field.

$$E_p = -G \frac{m_1 m_2}{r}$$

- $m_1$  and  $m_2$  are two masses separated by distance  $r$  (measured from the centre of the planet)
- Due to the law of universal gravitational attraction ( $F = G \frac{m_1 m_2}{d^2}$ ), the force of attraction between a planet and an object will drop to zero only at an infinite distance from the planet.
- Thus, infinity is defined as the level of zero potential energy ( $E_p = 0$ ). When an object moves from infinity to a point  $x$ , it gains  $E_k$  and must lose  $E_p$ . Therefore,  $E_p$  at point  $x < 0$ .
- In summary:
  - If a **planet's surface** is chosen as the zero level,  $E_p$  at  $x$  is **positive**.
  - If **infinity** is chosen as the zero level,  $E_p$  at  $x$  is **negative**.
- The negative value of  $E_p$  increases with distance up to a maximum value of zero (shown in graph).



- Although the  $E_p$  is negative, the *change* in  $E_p$  can be positive.

$$\begin{aligned}\Delta E_p &= -G \frac{mM}{r_1} - \left( -G \frac{mM}{r_2} \right) \\ &= G \frac{mM}{r_2} - G \frac{mM}{r_1} \\ &= GmM \left( \frac{1}{r_2} - \frac{1}{r_1} \right)\end{aligned}$$

- Since  $r_1 > r_2$ ,  $\frac{1}{r_1} < \frac{1}{r_2}$  and so  $\frac{1}{r_2} - \frac{1}{r_1} > 0$ . Therefore,  $\Delta E_p > 0$ .

### 1.1.4 Gather secondary information to predict the value of acceleration due to gravity on other planets

### 1.1.5 Analyse information using the expression $F = mg$ to determine the weight force for a body on Earth and for the same body on other planets

- For only two masses separated in space, there is a force of attraction between them due to the interaction of their gravitational fields. This is the law of universal gravitational attraction:

$$F = G \frac{m_1 m_2}{d^2}$$

- An object (mass  $m$ ) on the surface of the Earth (mass  $M$ ) will have a weight force of  $F_w = mg$  towards the centre of the Earth. This weight force is created by the universal gravitational attraction force between the object and the Earth.

$$\begin{aligned}F_g &= F_w \\ G \frac{mM}{r^2} &= mg \\ \therefore g &= G \frac{M}{r^2}\end{aligned}$$

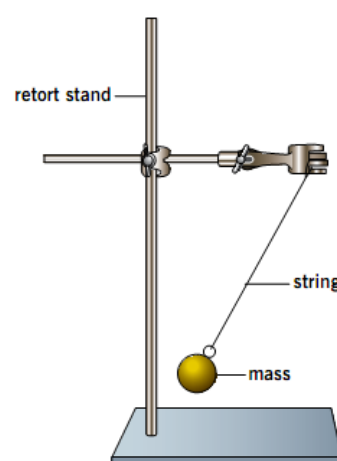
### 1.1.6 Perform an investigation and gather information to determine a value for acceleration due to gravity using pendulum motion or computer-assisted technology and identify reasons for possible variations from the value $9.8 \text{ ms}^{-2}$

- Aim: To determine a value of acceleration due to the Earth's gravity using pendulum motion.
- The period of a pendulum ( $T$ ) is related to the length of the string of the pendulum ( $l$ ):

$$\begin{aligned}T &= 2\pi \sqrt{\frac{l}{g}} \\ g &= \frac{4\pi^2 l}{T^2}\end{aligned}$$

- Method:

- The apparatus was set up as shown.
- The length of the pendulum ( $\approx 1 \text{ m}$ ) was measured, from the top of the string to the centre of the mass.
- The mass was pulled back to the side a set distance, ensuring that the angle between the string and the vertical was  $< 10^\circ$ .
- The mass was released and 10 complete swings were timed using a stopwatch (to increase accuracy).



5. The length of the string was changed and the experiment repeated (to increase reliability).
  6. A graph of  $l$  vs.  $T^2$  was made, and the value of  $g$  was deduced from the gradient of the graph.
- Discussion:
    - If the angle of the swing exceeded  $10^\circ$ , the simple pendulum motion would turn into a conical pendulum, which would not allow for an accurate calculation of  $g$ .
    - The time for 10 oscillations was measured because if the number of swings was too small, there would be a significant amount of error in timing. If the number of swings was too many, the pendulum would not be able to maintain its constant swing due to air resistance, which would also make the experiment less accurate.
    - The main sources of experimental errors were reaction time in measurements and timing, and air friction acting on the mass while it was swinging.
    - To improve accuracy:
      - Use data logging apparatus with position or velocity sensors to more accurately find the time taken for the period of the oscillations of the pendulum.
      - Reduce air currents by closing windows, shielding the pendulum from the surrounding air and using a more streamlined mass bob.
  - The value of  $g$  varies due to:
    - Different **altitudes** at which  $g$  is measured. The further away from the centre of the Earth, the smaller  $g$  becomes. This change only becomes significant at very high altitudes.
    - The **thickness** of the Earth's crust or lithosphere varies due to tectonic plate boundaries and dense mineral deposits.
    - The **Earth is not a perfect sphere**, but is flattened at the poles, so  $g$  will be greater at the poles (since they are closer to the centre of the Earth).
    - The **spinning Earth** creates centrifugal effects that slightly reduce the value of  $g$ . The effect is the greatest at the Equator, and there is no effect at the poles.

## 1.2 Rocket Launches and Orbital Motion

### 1.2.1 Solve problems and analyse information to calculate the actual velocity of a projectile from its horizontal and vertical components using:

Horizontal ( $a = 0$ )	Vertical ( $a = 9.8 \text{ ms}^{-2}$ )	
$u_x = u \cos \theta$	$u_y = u \sin \theta$	
$v_x = u_x = \text{constant } (a_x = 0)$	$v_y = u_y + a_y t$	
$v_x^2 = u_x^2$	$v_y^2 = u_y^2 + 2a_y s_y$	
$s_x = \Delta x = u_x t$	$s_y = u_y t + \frac{1}{2} a_y t^2$	$s = \left(\frac{u+v}{2}\right)t$

### 1.2.2 Describe the trajectory of an object undergoing projectile motion within the Earth's gravitational field in terms of horizontal and vertical components

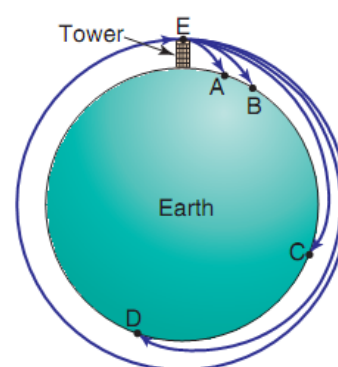
- A **projectile** is any object in free-fall where the *only force* acting on it is *gravity*.
- The horizontal motion of a projectile is independent of its vertical motion, since gravity is the only force acting on it and this always acts towards the centre of the Earth.
- The *horizontal* motion is **constant velocity** (inertia causes it to continue moving in the direction of its motion). The *vertical* motion is **constant acceleration** ( $g$ ).
- The **trajectory** of a projectile is **parabolic** (assuming air resistance is negligible and  $g$  is uniform).

### 1.2.3 Describe Galileo's analysis of projectile motion

- Galileo postulated that all objects, regardless of their mass, fall at the same rate (the acceleration due to gravity,  $g$ , is the same for all objects).
- He deduced that the trajectory of a projectile was a parabola.
- By using an inclined plane in his experiments (so that air resistance was negligible), he concluded that the horizontal and vertical components of a projectile are independent of each other.
  - The horizontal motion of an object experiences no acceleration.
  - The vertical motion of an object is affected by the downward acceleration of gravity.
- Most importantly, he realised the importance of mathematics in analysing the motion of the projectiles.

### 1.2.4 Outline Newton's concept of escape velocity

- Isaac Newton proposed the idea of artificial satellites of the Earth. He considered how a projectile could be launched horizontally from the top of a mountain so that it would not fall to Earth.
- The faster a projectile was fired, the further it would go before it hit the ground. Thus, there must be a specific velocity that would be sufficient to cause the object to orbit the Earth back to its starting point.
- At this speed the **curvature** of the projectile exactly matches the curvature of the Earth. If this specific velocity is exceeded slightly, the object will follow an elliptical orbit around the Earth. If it is exceeded further still, the object will follow a parabolic or hyperbolic path away from the Earth (this is how space probes depart from the Earth and into space).



### 1.2.5 Explain the concept of escape velocity in terms of the gravitational constant and the mass and radius of the planet

- **Escape velocity** is the initial velocity required by a projectile to rise vertically and just escape the gravitational field of a planet so that it does not return.
- For this to happen, the object's kinetic energy due to its velocity must exceed or at least equal its gravitational potential energy.

$$\begin{aligned}
 E_k &\geq E_p \\
 \frac{1}{2}mv^2 &\geq G \frac{mM}{r} \\
 v &\geq \sqrt{\frac{2GM}{r}} \\
 v_e &= \sqrt{\frac{2Gm_{\text{planet}}}{r_{\text{planet}}}}
 \end{aligned}$$

- The escape velocity of a planet does *not* depend on the mass of the escaping object. As  $G$  is a constant, the escape velocity depends on only on the radius  $r$  and the mass  $M$  of the planet. This means that escape velocity is the same for all objects on Earth, and different planets have different escape velocities.

### 1.2.6 Analyse the changing acceleration of a rocket during launch in terms of the Law of Conservation of Momentum and forces experienced by astronauts

- The law of conservation of momentum states that during any interaction in a closed system, the total momentum of the system remains unchanged:

$$\text{Total change in momentum} = 0$$

$$\therefore -\Delta p_{\text{gases}} = \Delta p_{\text{rocket}}$$

$$-\Delta(mv)_{\text{gases}} = \Delta(mv)_{\text{rocket}}$$

- The backward momentum of the gases ( $-\Delta p$ ) is equal to the forward momentum of the rocket ( $+\Delta p$ ). As fuel is burnt and the gases expelled, the mass of the system decreases. The term  $-\Delta(mv)_{\text{gases}}$  is remains relatively constant during a burn.
- Thus, the mass of the rocket decreases as fuel is burnt and the velocity of the rocket increases to compensate, causing the rocket to accelerate.

$$\text{Also, } -(Ft)_{\text{gases}} = (Ft)_{\text{rocket}}$$

$$-F_{\text{gases}} = F_{\text{rocket}}$$

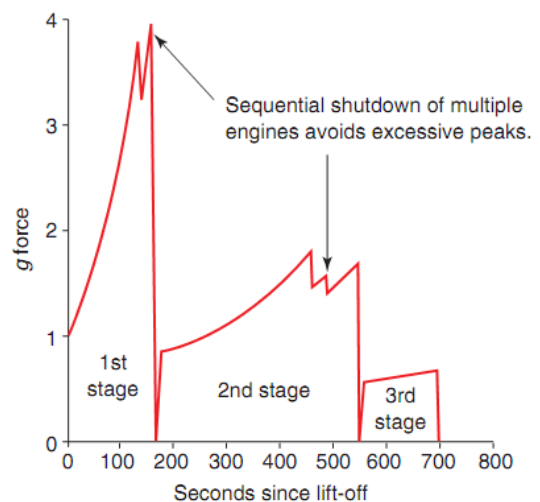
- Rockets generate thrust by burning fuel and expelling the resulting gases. The rocket forces large volumes of gases backwards with very high velocity and the gases, in turn, exert an equal but opposite force on the rocket forward. Although the two forces are equal and opposite, the rocket experiences only the forward push, the thrust. Since the force pair is between the rocket and the expelled gases, rockets are able to accelerate even in a vacuum.
- The forces experienced by astronauts during launch include the **upward thrust** ( $T$ ) and **downward weight** ( $W = mg$ ):

$$a = \frac{\sum F}{m} = \frac{(T - mg)}{m}$$

- A rocket's acceleration will not be constant because fuel constitutes the majority of the mass of a typical rocket. As fuel is burnt, the mass of the rocket decreases so acceleration increases (since  $a \propto \frac{1}{m}$  and  $T$  remains constant). Hence, the astronauts experience higher  $g$  forces.

- Variations in  $g$  forces:

- Before lift-off, a rocket has zero acceleration as the weight force is balanced by the reaction force plus thrust. The astronaut experiences a one  $g$  load.
- When the thrust exceeds the weight, there is a net force upwards on the rocket, which begins to accelerate upwards. The mass of the rocket begins to decrease as fuel is consumed and hence the rate of acceleration increases steadily, reaching maximum values before the rocket has exhausted its fuel.



- A **multi-stage rocket** drops the spent stage away, momentarily experiencing zero  $g$  conditions as it coasts. The second-stage rocket fires and develops the necessary thrust to exceed the effective weight at its altitude, and starts to accelerate again, with the  $g$  forces experienced gradually building to a maximum.
- This process is repeated for each stage of the rocket.
- The peaks on the graph are due to the **sequential shutdown** of the multiple rocket engines of each stage, a technique designed to avoid extreme  $g$  forces.

### 1.2.7 Identify why the term 'g forces' is used to explain the forces acting on an astronaut during launch

- The term **g force** is used to express apparent weight as a proportion of true weight.
$$g \text{ force} = \frac{\text{apparent weight}}{\text{true weight}} = \frac{ma + mg}{mg} = \frac{a + g}{9.8} = \frac{\text{rocket acceleration} + 9.8}{9.8}$$
  - It is used because it is simpler than an absolute force scale and it communicates the same relative forces acting on astronauts of different masses.
- During launch, the astronaut's downward weight force causes the floor to exert an equal upward reaction force, and in addition the floor exerts an upward accelerating force.
- For a rocket experience upward force  $F$  and downward weight  $W$ :
  - Before lift-off,  $F = W$  and  $a = 0$  so  $g \text{ force} = 1$
  - Lift-off:  $F > W$  and  $a > 0$  so  $g \text{ force} > 1$
  - Between rocket stages:  $F = 0$  and  $a = -g$  so  $g \text{ force} = 0$

### 1.2.8 Discuss the effect of the Earth's orbital motion and its rotational motion on the launch of a rocket

- The rotation of the Earth on its axis and the orbital motion of the Earth around the sun can be used to provide a launched rocket with a **velocity boost**. Rockets are launched to the east, the direction of the Earth's rotation. This type of launch is usually located near the equator, where the linear orbital speed of the Earth's spin is the greatest.
- The Earth's orbital velocity (relative to the Sun) adds to the rocket's orbital velocity (relative to the Earth) to produce a higher velocity achieved by the rocket (relative to the Sun). This can either launch a rocket into orbit or further into space.
- This reduces the amount of fuel required to achieve the target orbital velocity, thus making the launching process more economical and efficient.
- Rockets are launched during *launch windows*, which are favourable periods of the year when the direction of the Earth's orbit around the Sun corresponds with the desired direction.

### 1.2.9 Analyse the forces involved in uniform circular motion for a range of objects, including satellites orbiting the Earth

- Uniform circular motion** is circular motion with constant linear orbital speed but changing orbital velocity.
- Centripetal force** is the force that acts to maintain circular motion by causing the moving object to continually change direction so that it follows a circular path. It is perpendicular to the velocity of the object and is directed towards the centre of the circle.

$$F_c = \frac{mv^2}{r}$$

- An object in circular motion is accelerating, since the direction of its velocity is constantly changing, even though its magnitude (speed) remains unchanged.
- Thus, the velocity vector of the object is constantly changing and so **centripetal acceleration** is always present in uniform circular motion and is directed towards the centre of the circle.

$$a_c = \frac{v^2}{r}$$

Motion	Source of centripetal force
Whirling rock on a string	Tension in the string
Electron orbiting atomic nucleus	Electron-nucleus electrical attraction
Car travelling around a corner	Friction between tyres and road
Rider at bottom of curve on roller coaster	Reaction force of seat upwards on rider
Satellite orbiting the Earth	Gravitational attraction between satellite and Earth
Earth orbiting the Sun	Gravitational attraction between Earth and Sun

### 1.2.10 Define the term orbital velocity and the quantitative and qualitative relationship between orbital velocity, the gravitational constant, mass of the central body, mass of the satellite and the radius of the orbit using Kepler's Law of Periods

- **Orbital velocity** is the instantaneous tangential speed and direction of a satellite in circular motion along its path. It can also be defined as the minimum velocity required to maintain a satellite in a given orbit about another body.
- For uniform circular motion:

$$\text{orbital velocity } v = \frac{2\pi r}{T} \dots (1)$$

$$\text{Kepler's Law of Periods: } \frac{r^3}{T^2} = \frac{GM}{4\pi^2} \dots (2) \quad \text{OR} \quad \left(\frac{r^3}{T^2}\right)_{\text{planet 1}} = \left(\frac{r^3}{T^2}\right)_{\text{planet 2}}$$

- Combining (1) and (2),

$$v = \sqrt{\frac{GM}{r}}$$

- $v$  = orbital velocity,  $M$  = mass of central body,  $r$  = distance between centres of masses

- For a satellite orbiting the Earth,

$$v = \sqrt{\frac{GM_E}{r_E + \text{altitude}}}$$

- $M_E$  = mass of Earth,  $r_E$  = radius of Earth

- Since  $v \propto \frac{1}{\sqrt{r}}$ , the smaller the radius, the faster the satellite must travel to stay in orbit.

### 1.2.11 Compare qualitatively low Earth and geo-stationary orbits

- Low Earth orbit:
  - A **low Earth orbit** is an orbit that lies above the Earth's atmosphere but below the van Allen radiation belts. They move above the Earth and so do not have a fixed position. Since their periods are smaller than that of the Earth, they may orbit the Earth many times a day. This allows them to pass above any point on Earth and view the Earth's entire surface over several orbits.
  - LEO's are used in studying weather patterns, mapping ecological threats and military and civilian surveillance.
  - Advantages: Low altitudes present several advantages, including a closer and more *detailed view of the Earth's surface*, *rapid information transmission* with little delay and more *efficient* and *cheaper* satellite launches (as less fuel for the same satellite mass is required).
  - Disadvantages: Atmospheric drag is quite significant and orbital decay is inevitable. The orbital paths of the satellites must be controlled carefully to avoid interference between one satellite and another.

- Geostationary orbit:
  - A **geostationary orbit** is at an altitude at which the period of the orbit matches that of the Earth. If over the Equator, a geostationary satellite appears to be stationary in the sky as it is always located in the same direction regardless of the time of day. This is because it is orbiting the Earth at the same rate as the Earth's rotation, so the satellite and the Earth have the same period. A *geosynchronous* orbit is not positioned over the Equator, so it will not remain fixed at one point in the sky (from the Earth it will appear to trace out a 'figure of eight' path).
  - Geostationary orbits are used in communications satellites and information relay because receiving dishes only need to point to a fixed spot in the sky. Information is sent up to one satellite and is bounced off to another place on the Earth.
  - Advantages: Geostationary satellites are *easy to track* as each satellite stays at one position at all times. They do not experience orbital decay.
  - Disadvantages: Each satellite has a limited view of the Earth's surface as it only stays at one point above the Earth. Thus, many geostationary satellites are required to cover the Earth's entire surface. Their high altitude makes launching processes more difficult and expensive as more fuel is needed.

### 1.2.12 Account for the orbital decay of satellites in low Earth orbit

- **Orbital decay** refers to the loss of altitude of a spacecraft due to the friction between the spacecraft and the atmosphere at the orbit altitude.
- All satellites in low Earth orbit are subject to some degree of atmospheric drag (friction) that will eventually **decay** their orbit and limit their lifetimes. This friction causes a gradual loss of orbital energy which leads to a loss of altitude. The satellite will be moving faster than before (lower orbits require faster orbital velocities) as the extra  $E_K$  is derived from the lost  $E_p$ .
- As the satellite descends, it encounters slightly denser atmosphere, which leads to further friction and loss of energy as it moves into an even lower orbit. Eventually the satellite's orbital velocity will be insufficient to sustain its circular orbital motion, and it will spiral back to the Earth.
- The amount of atmospheric drag depends on the size of the satellite and the density of the air along the orbit.
- Satellites in low orbits can counteract decay by firing rocket boosters so that the satellite can be lifted periodically back up to its intended orbital altitude.

### 1.2.13 Identify that there is an optimum angle for safe re-entry for a manned spacecraft into the Earth's atmosphere and the consequences of failing to achieve this angle

- During re-entry, a spacecraft enters the atmosphere at the **optimum re-entry angle** (which was between  $5.2^\circ$  and  $7.2^\circ$  for Apollo capsules).
- If the angle of re-entry is too shallow (i.e.  $< 5.2^\circ$ ):
  - The spacecraft will skip off the atmosphere instead of entering it, and return back into space.
- If the angle of re-entry is too steep (i.e.  $> 7.2^\circ$ ):
  - The upward resistive force experienced by the spacecraft will be too large. The spacecraft will decelerate too quickly, producing excessive heat which would cause the spacecraft to burn up.
  - The  $g$  forces experienced by the crew will also be too large to tolerate and will be fatal.
- The re-entry angle varies depending on the shape of the craft and its re-entry speed.



### 1.2.14 Discuss issues associated with safe re-entry into the Earth's atmosphere and landing on the Earth's surface

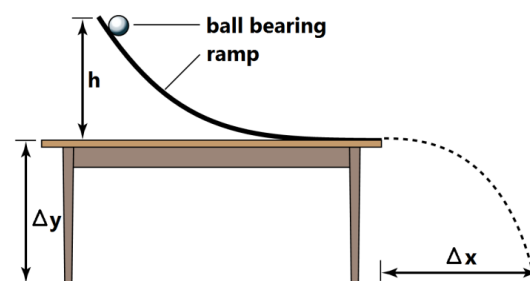
- *Orbital decay* is an unintended, gradual process resulting in a spiral path downwards.
- *De-orbiting* is a deliberate orbital manoeuvre resulting in an elliptical path down to the atmosphere.
- Issues associated with safe re-entry include:
  - Heat:
    - As the spacecraft re-enters, atmospheric friction causes it to experience intense heat, which can vaporise the spacecraft (as kinetic energy is converted to heat).
    - The best shape for re-entry is a **blunt** one, because it produces a shockwave of air in front of itself and this absorbs most of the frictional heat. The space shuttle keeps its nose up during re-entry and presents its flat underbelly to the atmosphere to create the shockwave.
    - However, the blunt shape would still need to cope with high temperatures.
      - » The heat can be *minimised* by taking longer to re-enter (lengthening the time over which the energy is converted to heat).
      - » The heat can be *tolerated* by using **heat shields** that use ablating surfaces or insulating surfaces. The technique of **ablation** involves covering the nose cone with a ceramic material, which is vaporised or 'ablated' during re-entry. The vaporising of the surface dissipates the heat and carries it away.
      - » The space shuttle uses **insulating tiles** made of porous silica fibre, giving them excellent thermal insulation properties.
  - *g* forces:
    - The deceleration of a re-entering spacecraft also produces *g*-forces, typically greater than those experienced during launch.
    - High *g* forces can be better *tolerated* by **reclining** astronauts so that they are oriented perpendicular to, and facing ('eyeballs in'), the direction of acceleration. This ensures that blood is not forced away from the brain, which would cause unconsciousness and death if it is prolonged.
    - The *g*-forces can be *minimised* by **extending the re-entry**, slowing the rate of descent.
  - Ionisation blackout:
    - **Ionisation blackout** is a period of no communication with a spacecraft due to a surrounding layer of ionised particles forming in the heat of re-entry. This can be a safety issue if contact is needed between the spacecraft and Earth.
    - This problem has been solved for the space shuttle by communicating via a satellite above it, since only the bottom of the shuttle has significant ionisation.
  - Landing:
    - Even after surviving re-entry, the spacecraft must land safely. If it does not sufficiently decelerate, its final velocity is very high and impact with land would create high forces due to the short collision time.
    - **Parachutes** can be used to slow the spacecraft down to speeds where it can descend onto land or splash into the ocean. The space shuttle can land on an **air strip**, as its flight control structures allow adjustment of the landing path.

### 1.2.15 Identify data sources, gather, analyse and present information on the contribution of Goddard to the development of space exploration

- **Robert Goddard** (1882-1945) was an American professor and physicist, acknowledged as the 'father' of modern rocketry, who invented and tested many practical aspects of rockets. He was inspired by Jules Verne and decided to dedicate his life to rocketry.
- He was the first to recognise the scientific importance of space travel and also worked experimentally in order to achieve this.
- He measured the fuel values for various rocket fuels, such as liquid hydrogen and oxygen.
- Main contribution: In 1926, Goddard successfully launched the world's first liquid-propellant rocket (all previous rockets had used solid fuels). This dramatically increased the efficiency of fuel conversion from 2% to 60%.
- Goddard confirmed experimentally that rockets would work in a vacuum and did not need air to 'push against' for propulsion.
- He launched the first scientific payload that parachuted back to Earth, and was the first to steer rockets using vanes to direct exhaust gas and a pivoted nozzle under the automatic control of a gyroscope.
- Mostly unrecognised in his lifetime, Goddard strongly influenced other German scientists in the development of their wartime rockets.

### 1.2.16 Perform a first-hand investigation, gather information and analyse data to calculate initial and final velocity, maximum height reached, range and time of flight of a projectile for a range of situations by using simulations, data loggers and computer analysis

- Aim: To analyse an example of projectile motion and compare the calculated value for its horizontal displacement with the one measured experimentally.
- Method:
  - The apparatus was set up as shown.
  - A ball was projected from a fixed height on a ramp. The range was marked using carbon paper and measured with a ruler to obtain an experimental value of  $\Delta x$ .
  - The ball bearing's horizontal launch speed ( $u_x$ ) was calculated by using the conservation of energy ( $E_p$  is converted to  $E_k$ , so  $mgh = \frac{1}{2}mu_x^2$  and  $u_x = \sqrt{2gh}$ ).
  - The expected time of flight ( $t$ ) of the ball bearing was calculated using  $\Delta y = \frac{1}{2}a_y t^2$  (as  $u_y = 0$ ), where  $\Delta y$  is the height of the table.
  - The expected value for  $\Delta x$  was calculated using  $\Delta x = u_x t$ , and compared with the experimental value of  $\Delta x$ .
- Discussion:
  - If the angle of the slope to the horizontal is increased, the time taken for the ball to reach the projection point decreases.
  - It was assumed that there was no friction (drag) and that the value of  $g$  was  $9.8 \text{ ms}^{-2}$ .
  - The accuracy of the experimental data can be improved by using a data logger to more accurately record measurements. A smooth surface would also reduce frictional effects.



## 1.3 Gravitational Force and Planetary Motion

### 1.3.1 Describe a gravitational field in the region surrounding a massive object in terms of its effects on other masses in it

- A **gravitational field** is a field in which any mass will experience a gravitational force. It takes on a radial pattern with field lines pointing towards the *centre* of the object.
- Any massive object is surrounded by a gravitational field in which other objects experience an attractive force.
- There is a point between two large objects at which the strength of the field is zero. The gravitational attraction of the objects are equal but opposite in direction.

### 1.3.2 Define Newton's Law of Universal Gravitation

- Newton's *Law of Universal Gravitation* states that the force of gravitational attraction is proportional to the product of the masses of two bodies and inversely proportional to the square of their distance of separation.

$$F = G \frac{m_1 m_2}{d^2}$$

- Every object in the universe attracts every other object with a gravitational force. This gravitational force is exerted equally on both masses.
- For the gravitational attraction between a satellite and the Earth:

$$F_G = G \frac{m_E m_S}{r^2}$$

- $m_E$  = mass of Earth,  $m_S$  = mass of satellite,  $r$  = radius of orbit

### 1.3.3 Discuss the importance of Newton's Law of Universal Gravitation in understanding and calculating the motion of satellites

- A centripetal force maintains the orbital motion of a satellite around the Earth, which is assumed to be uniform circular motion. This is provided by the force of gravitational attraction between the Earth and the satellite.

$$\begin{aligned} F_G &= F_C \\ G \frac{m_E m_S}{r^2} &= \frac{m_S v^2}{r} \\ v &= \sqrt{\frac{G m_E}{r}} \end{aligned}$$

- This shows that the greater the radius of the orbit, the lower the orbital velocity required.
- The constant in Kepler's Law of Periods can be derived by substituting  $v = \frac{2\pi r}{T}$  into  $v = \sqrt{\frac{GM}{r}}$

$$\begin{aligned} \frac{2\pi r}{T} &= \sqrt{\frac{GM}{r}} \\ \frac{r^3}{T^2} &= \frac{GM}{4\pi^2} \end{aligned}$$

- The importance of Newton's Law of Universal Gravitation:
  - The gravitational force is responsible for 'weightlessness'.
  - Gravitational attraction initiates the slingshot effect.

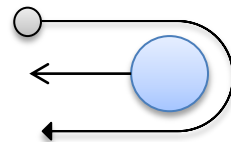
- The stability of the gravitational force enables accurate calculations of launch windows.
- Gravitational force is the reason rocket launches are so expensive and require such massive engines to overcome it.

### 1.3.4 Present information and use available evidence to discuss the factors affecting the strength of the gravitational force

- From  $F_G = G \frac{m_1 m_2}{d^2}$ , the strength of gravitational force depends on the mass of the object and the distance from the centres of mass.
- Factors affecting gravitational force include:
  - **Altitude:** The higher the altitude, the less the gravitational force ( $F_G \propto \frac{1}{d^2}$ )
  - **Position on the Earth's surface:** The gravitational force is smaller on top of a mountain. Since the Earth is not a perfect sphere,  $d$  will be a maximum at the Equator and decrease towards the poles.
  - **Variations in mass distribution** such as the presence of ore bodies or oil and gas fields.

### 1.3.5 Identify that a slingshot effect can be provided by planets for space probes

- The **slingshot effect** is the change in velocity of a spacecraft as it enters the gravitational field of a planet and flies past it. This allows a long distance space probe to increase its velocity every time it passes a planet **without spending any fuel**.
- During a slingshot, a spacecraft deliberately passes close to a planet so that the planet's gravity pulls the spacecraft in towards it. This causes the spacecraft to accelerate and it heads around the planet and departs with a **different trajectory**.
- The approach speed of the spacecraft is the same as the departure speed, except its pathway has been changed. As the spacecraft moves around the planet, the orbital speed of the planet adds to the original speed of the spacecraft. Hence, the speed of the spacecraft relative to the planet does not change, but its **speed increases relative to the Sun**.
- The **maximum speed** the probe can gain is twice the orbital speed of the planet, and occurs when the spacecraft and planet approach each other **head-on** and the spacecraft passes behind the planet.
- The increase in speed of the spacecraft can be interpreted in terms of the law of conservation of energy and momentum:
  - The slingshot effect involves a non-contact elastic collision between the spacecraft and the planet. By the **law of conservation of energy**, the rotational kinetic energy of the planet is transferred to the spacecraft as translational kinetic energy, giving the spacecraft its increase in velocity.
  - The extra velocity gained by the spacecraft is also due to the **law of conservation of momentum**. When the slingshot effect occurs, the total initial momentum of the probe and planet must equal the total final momentum. Since the probe has now sped up and gained momentum, the planet must lose an equal amount of momentum and slow down. Because  $p = mv$ , and the mass of the planet is much larger than that of the probe, the speed lost by the planet is insignificant compared to the speed gained by the probe.



## 1.4 Relativity and the Speed of Light

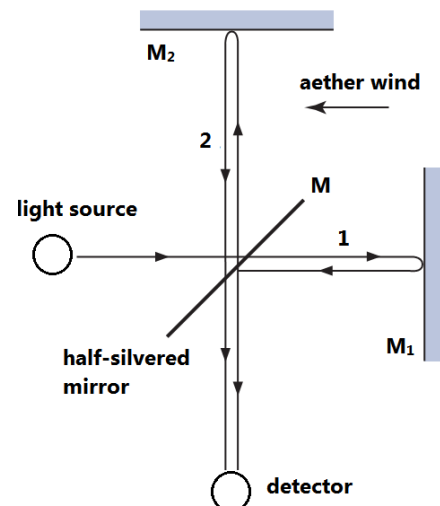
### 1.4.1 Outline the features of the aether model for the transmission of light

- The aether was proposed to be the **medium** through which light propagates. Light had been shown to be a wave in the 19<sup>th</sup> century. Like all other waves, physicists assumed that it required a medium to travel in.
- The aether was also thought to be the **absolute frame of reference** to which all motion was compared.
- The aether **filled all of space**, had **low density** and was perfectly **transparent**. It **permeated all matter** and yet was completely permeable to material objects. The aether had great **elasticity** to support and propagate the light waves.

### 1.4.2 Describe and evaluate the Michelson-Morley attempt to measure the relative velocity of the Earth through the aether

### 1.4.3 Gather and process information to interpret the results of the Michelson-Morley experiment

- Aim: The aim of the **Michelson-Morley experiment** was to measure the velocity of the Earth relative to the aether. This was done by comparing the light rays using the effect of **interference of light waves**.
- The apparatus used was an *interferometer* mounted on a large stone black floating on mercury so that it could be rotated to repeat the experiment in different directions.
- Hypothesis: When looking into a telescope, interference patterns would indicate differences in speed of light and give evidence for the existence of the aether.
- Procedure:
  - An interferometer is set up in which light from a light source is split into two perpendicular beams by passing it through a half-silvered mirror.
  - Beam 1 travels against the aether wind to reach mirror  $M_1$ , then reflects back to travel with the aether wind.
  - Beam 2 travels across the aether wind to reach mirror  $M_2$ , and then reflects back also across the aether wind.
  - The two light beams reunite at the half-silvered mirror and are reflected to the detector, where an interference pattern can be observed.
  - The entire apparatus is then rotated 90° to observe for a *shift in the interference pattern*, which would show that the phase difference is due to the effect of aether on the velocity of light.
- Results:
  - When the apparatus was rotated, no shift in the interference pattern was observed.
  - The experiment was repeated many times at different places and times, with the interferometer set in various orientations, and by different groups with more sensitive equipment, but still a **null result** was recorded.



- This meant that the motion of the Earth through the aether could not be detected. Consequently, the existence of the aether could not be proven, and the aether model was still lacking in experimental evidence.
- Impact of the experiment:
  - The inability to provide evidence for the existence of the aether meant that the aether model was an invalid physics theory. However, because it formed the basic of many physics theories and laws, scientists at the time found it hard to discard the model.

#### **1.4.4 Discuss the role of the Michelson-Morley experiments in making determinations about competing theories**

- Many proposals were put forward to attempt to explain the null result of the experiment. It was suggested that the Earth carried the aether with it, and thus there was not relative motion (and hence aether wind) detected. Another suggestion was that objects contract in the direction of the aether wind. However, none of the modifications survived close scrutiny as no physical theory supported them.
- Einstein later proposed his theory of relativity, in which the aether model was not needed. The theory not only successfully accounted for the null result of the Michelson-Morley experiment, but it could be tested to see if its predictions proved true.
- As technology has improved, the predictions have been tested and found to be correct. Thus, scientists had a choice of continuing to follow a theory for which no predictions proved true (aether) or to follow an alternative theory for which predictions proved true (relativity).
- The result of the Michelson-Morley experiment has been able to help scientists of the 20<sup>th</sup> century to reject the aether model and accept Einstein's relativity. Importantly, it contributed to a shift in scientific thinking from classical theory to relativity.

#### **1.4.5 Perform an investigation to help distinguish between non-inertial and inertial frames of reference**

- Aim: To distinguish between an inertial and non-inertial frame of reference using an accelerometer.
- Hypothesis: With a non-inertial frame of reference, the mass will swing backwards when accelerating forwards. With an inertial frame of reference, the mass does not move or moves with constant motion (i.e. it hangs vertically).
- Theory:
  - When the accelerometer undergoes positive acceleration, the mass swings backwards from the direction of the velocity.
  - The mass will swing forwards towards the direction of the velocity when there is negative acceleration.
  - It is only possible to measure velocity relative to another frame, not within your own frame of reference.
- Method:
  - The string was attached with a mass to the centre of a protractor.
  - The accelerometer was subject to both inertial and non-inertial frames of reference, and the motion of the mass was observed.

- Discussion:
  - When the accelerometer is held in a non-inertial frame of reference, there is an additional force, a fictitious force, which causes the mass to move backwards. This is proportional to the acceleration of the object.
  - In a non-inertial frame, depending on the direction of the object, the mass will either move forwards or backwards.
  - In an inertial frame, when there is no movement of the mass, the motion is constant or there is no motion.

#### 1.4.6 Outline the nature of inertial frames of reference

- A frame of reference is a rigid framework or coordinate system relative to which motion is measured and described.
- An **inertial frame of reference** is a non-accelerated environment, where only steady motion or no motion is allowed. Motion cannot be detected. In an inertial frame of reference, the laws of motion are always valid.
  - For example, the interior of a plane flying smoothly at a steady speed is an inertial frame of reference. If a ball is dropped, it falls vertically under gravity and no other force. Without observing out the window, it is not possible to tell whether the plane is in flight or stationary on the ground.
- A **non-inertial frame of reference** experiences acceleration, so motion is detectable. For a non-inertial frame of reference, the laws of motion do not hold true. Objects can appear to change velocity without a true net external force. This is due to *fictitious forces* or *pseudo-forces*.
  - For example, in a car turning a corner, a passenger experiences the sensation of being 'thrown outwards' by a fictitious centrifugal force.

#### 1.4.7 Discuss the principle of relativity

- The **principle of relativity** states that all steady motion is relative and cannot be detected without reference to an outside point (or another frame of reference).
- It only applies to **inertial frames of reference** (non-accelerated steady motion), i.e. standing at rest or moving with uniform velocity.
  - For example, when you are inside a moving vehicle, you cannot tell if you are moving at a steady velocity or standing still without looking out the window.
- Within an inertial frame of reference, it is not possible to perform any mechanical experiment or observation to detect the motion of the frame of reference. The only way to detect the motion of an inertial frame of reference is by referring to another frame of reference.
  - For example, a string with a mass on the end acts as a simple accelerometer, but it is unable to distinguish between being motionless and steady motion.
- The laws of motion are thus the same in all inertial frames of reference. All velocities are relative. There are no absolute velocities and there is no special absolutely stationary inertial frame. In general, there is no absolute motion and it all must be measured relative to another object.
- While the principle of relativity was accepted for most events, the aether theory meant that it did not hold for light. If the aether did exist, measurements of the speed of light from an object moving with constant velocity would give different values depending on which way the object was

moving relative to the aether. This would enable the observer to determine that they were in an inertial frame of reference, which would violate the principle of relativity.

#### 1.4.8 Describe the significance of Einstein's assumption of the constancy of the speed of light

- The speed of light is assumed to be constant relative to all observers so that the principle of relativity cannot be violated by using optical experiments.
- Einstein concluded that if two observers at relative motion to each other observe the speed of light to be constant, since  $speed = \frac{distance}{time}$ , then the distance and time witnessed by both observers must be different – i.e. time dilation and length contraction result.
- These ideas were published by Einstein in his **theory of special relativity** which presented:
  - **The first postulate:** The laws of physics are the same in all frames of reference (i.e. the principle of relativity always holds).
  - **The second postulate:** The speed of light is constant regardless of the observer's frame of reference.
  - **The statement:** The aether is not needed to explain the behaviour of light, and in fact does not exist.
- This successfully accounted for the null result of the Michelson-Morley experiment, as the constancy of the speed of light led to no change in the interference pattern when the apparatus was rotated through 90°.

#### 1.4.9 Identify that if $c$ is constant then space and time become relative

- In classical physics, space (position, displacement and velocity) can be relative to an observer but time is an absolute quantity.
- Einstein's special relativity proposes that stationary and moving observers perceive space and time differently. As  $c$  is now constant, space and time become relative quantities that depend upon the motion of the observer.
- This means that on a train moving at relativistic speeds relative to a platform, observers would note that their perceptions of mass, time, and length are different.

#### 1.4.10 Discuss the concept that length standards are defined in terms of time in contrast to the original metre standard

- The metre as a unit of length was initially defined by the French government as a fraction of the length of the Earth's quadrant passing through Paris. The Systeme Internationale later defined the metre to be the distance between two lines scribed on a single bar of platinum-iridium alloy.
- There is always a need for the accuracy of a unit of measure to keep pace with improvements in technology and science.
- The **current definition of the metre** is the length of the path travelled by light in a vacuum during the time interval of  $\frac{1}{299792458}$  of a second. This modern definition takes advantage of the constancy of the speed of light, as well as the capability technology has given us to measure time and the speed of light with great precision
- The 'light-year' is a similar distance unit, being the length of the path travelled by light in a time interval of one year.



**1.4.11 Explain qualitatively and quantitatively the consequence of special relativity in relation to the relativity of simultaneity, the equivalence between mass and energy, length contraction, time dilation and mass dilation**

- In all equations, if speeds are expressed as fractions of light speed, the term  $\sqrt{1 - \frac{v^2}{c^2}}$  reduces to  $\sqrt{1 - v^2}$ .
- All effects of special relativity (including time dilation and mass dilation) only become apparent at relativistic velocities, i.e. when the speed is a significant proportion of the speed of light.
- Relativity of simultaneity:
  - **At relativistic speeds, the relativity of simultaneity states that simultaneous events in one frame of reference are not necessarily observed to be simultaneous in a different frame of reference.**
  - If an observer sees two events to be simultaneous, then any other observer, in relative motion to the first, generally will not judge them to be simultaneous.
  - *Einstein's thought experiment:*
    - An operator of a lamp rides in the middle of a train that is fitted with light operated doors. When the train passes alongside an observer outside, the operator switches on the lamp, which opens the doors.
    - The operator of the lamp will see the two doors opening simultaneously. The distance of each door from the lamp is the same and light will travel at the same speed both forward and backward.
    - However, an observer standing outside will see the back door opening before the front. Before the light has reached both doors, the train has moved so that the front door is further away and the back door is closer. The light travels forwards and backwards at the same speed, but the forward journey is now longer than the backward journey, so the back door is seen to open before the front door does.
    - Both observers judged the situation correctly from their different frames of reference. This is a direct consequence of the constancy of the speed of light.
      - » Note: The relativity of simultaneity demonstrates that even absolutes are now relative and there is no 'better' inertial frame of reference in special relativity.
- Time dilation:
  - **Time dilation** is the slowing down of events as observed from a reference frame in relative motion. It can be summarised as '*moving clocks appear to run slow*'.
  - The time taken for an event to occur within its rest frame is called the *proper time* ( $t_0$ ).
  - Observers in any other inertial reference frame in relative motion will always judge the time taken ( $t_v$ ) to be *longer*.

$$t_v = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- The term  $\sqrt{1 - \frac{v^2}{c^2}}$  is always less than one, so  $t_v$  is always greater than  $t_0$ .
- *Example:* If a light pulse is released on a train, time passes more slowly on the train as observed by a person outside the train. The observer sees the light travel further but with the same speed, hence time is slowed down on the train. Since there is no absolute frame of reference, each observer sees time dilated in the other frame of reference.

- Length contraction:

- **Length contraction** is the shortening of an object in the direction of its motion as observed from a reference frame in relative motion. It can be summarised as '*moving objects shorten in the direction of their motion*'.
  - The length of an object measured within its rest frame is called its *proper length* ( $L_0$ ).
  - Observers in any other inertial reference frame in relative motion will always measure the length ( $L_v$ ) to be shorter.

$$L_v = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

- The term  $\sqrt{1 - \frac{v^2}{c^2}}$  is always less than one, so  $L_v$  is always less than  $L_0$ .
- *Example:* If a light pulse is released that runs the length of the train, the length of the train shortens as observed by a person outside the train.
- Like time dilation, the effect is symmetrical – the observers can be swapped. Length contraction is only observed in the direction of the speed and only in a frame in relative motion to the rest frame.

- Mass dilation:

- **Mass dilation** is the increase in the mass of an object as observed from a reference frame in relative motion.
  - The mass of an object within its own rest frame is called its *proper mass* ( $m_0$ ).
  - Observers in any other inertial reference frame in relative motion will always measure the mass ( $m_v$ ) to be greater.

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- The relativistic mass of an object increases as its speed relative to the observer increases. As speed approaches  $c$ , the mass approaches  $\infty$ , so the force required to accelerate an object to the speed of light becomes infinite ( $a = \frac{F}{m}$ ).
- Conversely, if the acting net force remains constant, then its acceleration decreases to 0 as mass approaches  $\infty$ . Thus, its velocity can no longer increase, so the speed of light cannot be reached.

- Equivalence between mass and energy:

- **Rest energy** is the energy equivalent of a stationary objects' mass, measured within the object's rest frame.

$$E = mc^2$$

- In special relativity, the Law of Conservation of Energy and the Law of Conservation of Mass are replaced by the *Law of Conservation of Mass-Energy*.

#### 1.4.12 Discuss the implications of mass increase, time dilation and length contraction for space travel

- Mass dilation:
  - As speed increases, energy that would otherwise increase the speed is converted into mass. For this reason, space travel at relativistic speeds is highly impractical.
  - Accelerating to such speeds would require considerable energy and fuel costs, due to the conversion of energy into mass.
- Time dilation:
  - Time dilation is the main factor which allows the possibility of space travel within human lifetimes.
  - If relativistic speeds could be reached, the nearest stars should be able to be reached in several years. The lengthy space travel as observed by people on Earth would be reduced considerably according to the astronauts. When the astronauts return to Earth, they will have aged less time compared to the time that has actually passed on Earth.
  - However, there are issues with communication over light year distances and sending radio signals, as messages become garbled and will take a long time to reach their destination.
- Length contraction:
  - From the point of view of the astronauts, the length of the journey of space travel has contracted to a significantly shorter distance, compared with that being measured by people on Earth.
  - Thus, it will take the astronauts a shorter time to reach their destination. At the same time, time passes slower in the astronauts' frame of reference, as measured by an external stationary observer. Once again, this shows that length is related to time.

#### 1.4.13 Analyse information to discuss the relationship between theory and the evidence supporting it, using Einstein's predictions based on relativity that were made many years before evidence was available to support it

- A proposed theory usually needs experimental evidence before it is taken seriously.
- At the time they were put forward, Einstein's ideas strongly contradicted classical science developed by scientists like Newton.
- When Einstein proposed his special theory of relativity, the experimental and technological capability to verify the predictions did not exist. As technology improved in the 20<sup>th</sup> century, relativity theory predictions became testable.
- Experimental evidence for special relativity:
  - **Time dilation** can be verified by comparing **atomic clocks** that have been flown over long journeys with clocks that remained stationary for the same period. These experiments are possible now because of the extreme accuracy of modern atomic clocks. The clocks that were flown around were slow compared to those that stayed on the ground.
  - There is an abundance of **mesons** (created in the upper atmosphere by incoming cosmic rays) striking the ground. These mesons have an average proper lifetime (as measured in their rest frame) of  $2.2 \mu\text{s}$ , but they have been detected to have lifespans of  $16 \mu\text{s}$  when travelling at  $0.996 c$ . This is explained by **time dilation** at relativistic speeds. The fact that mesons are detected on Earth provides strong evidence for the validity of special relativity.

- Accurate measurements of the masses of nuclides and the energy released in transmutations have verified  $E = mc^2$ .
- **Mass dilation** due to relativity has been verified using particle accelerators.

#### 1.4.14 Analyse and interpret some of Einstein's thought experiments involving mirrors and trains and discuss the relationship between thought and reality

- Einstein's 1st thought experiment: *If I was travelling in a train at the speed of light and I held up a mirror, would I be able to see my own reflection?*
  - If the aether model was right, light could go no faster than the train so it would never catch up with the mirror to return as a reflection. The principle of relativity is thus violated as seeing one's reflection disappear would be a way to detect motion.
  - If the principle of relativity were not to be violated, the reflection must be seen normally (i.e. it is moving away from the mirror holder at the speed of light). However, this would mean that a stationary observer standing next to the train would see light travelling at twice its normal speed, which is impossible.
  - In both cases, some principle of Newtonian physics is violated.
  - This was a considerable dilemma but Einstein concluded that the principle of relativity can never be violated and the reflection in the mirror must always be seen. This in turn meant that the aether did not exist, and the speed of light is constant regardless of the motion of the observer. If both observers were to see the same speed of light, and since  $\text{speed} = \frac{\text{distance}}{\text{time}}$ , then the distance and time witnessed by both observers must be different.
- Einstein's 2nd thought experiment: *A light on the ceiling of a train, moving at the speed of light, shines onto a mirror on the floor. The light reflects from the mirror and travels back to the ceiling to a detector that records the time of its arrival.*
  - The observer inside the train sees the light travel from the floor to the ceiling and back again.
  - A stationary observer outside the train sees the train move forwards during the time it takes the light to travel from the ceiling to the floor and back again.
  - He sees the positions of the mirror and light detector change, and so sees the light take a longer route.
  - The obvious conclusion is that time for the observer inside the train is passing more slowly than time for the observer outside the train.
- The effects of relativity – relativity of simultaneity, time, length and mass – are reversible by looking at a situation from a different reference frame.
- Two observers moving with relative velocity may judge a situation to be different, but both are correct in their own frames of reference.