WORKED EXAMPLES OF MATHEMATICAL PROBLEMS

From 3rd column dot points

Module	Syllabus statement #	3rd column dot point #	Equation(s)	Example numbers
9.1 Space	1	(2nd column)	$E_p = -G\frac{m_1 m_2}{r}$	1, 2
	2	1	Any projectile motion examples: $v_x^2 = u_x^2$; $v = u + at$; $v_y^2 = u_y^2 + 2a_y\Delta y$; $\Delta x = u_x t$; $\Delta y = u_y t + \frac{1}{2}a_y t^2$	3, 4, 5
		4	$F = \frac{mv^2}{r}$	6
		5	$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$	7, 8
	3	2	$F = G\frac{m_1 m_2}{d^2}$	9
	4	5	$E = mc^2$	10
			$l_v = l_0\sqrt{1 - \frac{v^2}{c^2}}$	11
			$t_v = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$	12
			$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$	13
9.3 Motors and generators	1	1	$\frac{F}{l} = k\frac{I_1 I_2}{d}$	14, 15
		3 and 4	$F = BIl \sin\theta$	16
9.4 From ideas to implementation	1	3	$F = qvB\sin\theta$; $F = qE$; $E = \frac{V}{d}$ in context of Thomson's $\frac{q}{m}$ experiments	17, 18
	2	4	$E = hf$ & $c = f\lambda$	19
9.6 Medical physics	1	5	$\frac{I_r}{I_o} = \frac{[Z_2 - Z_1]^2}{[Z_2 + Z_1]^2}$	20

Module	Syllabus statement #	3rd column dot point #	Equation(s)	Example numbers
9.7 Astrophysics	4	1	$M = m - 5\log\left(\frac{d}{10}\right)$	21
			$\frac{I_A}{I_B} = 100^{(m_B - m_A)/5}$	22
	5	2	$m_1 + m_2 = \frac{4\pi^2 r^3}{GT^2}$	23, 24
9.8 Quanta to quarks	1	3	$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$	25
	2	1	$\lambda = \frac{h}{mv}$	26

Points to note:

1. It is important to show the equation being used **before** you substitute values.
2. Keep all your work neat (as you would in maths). If the answer is wrong but the mistake can be traced back to a minor mathematical error, some or most marks would still be awarded in most cases.
3. Equations link many things. If a question asks you to 'describe how A affects B', think of an equation with A and B in it.
4. These worked examples show the answers with the correct number of significant figures. This is the least number of significant figures in given or known values in the example. Round off to this number of significant figures *only* in the final answer.
5. Ensure that the correct units are given with the answers.
6. Units are not included within the calculation stages, but, again, ensure that the right unit is given at the end.
7. The final answer to a physics question usually corresponds to something real. If your answer looks impossible or seems 'silly', check it through carefully. For example, if you find that the speed of an object is 5.0×10^9 m s^{-1}, it is likely that you got it wrong! (Why?)

Examples

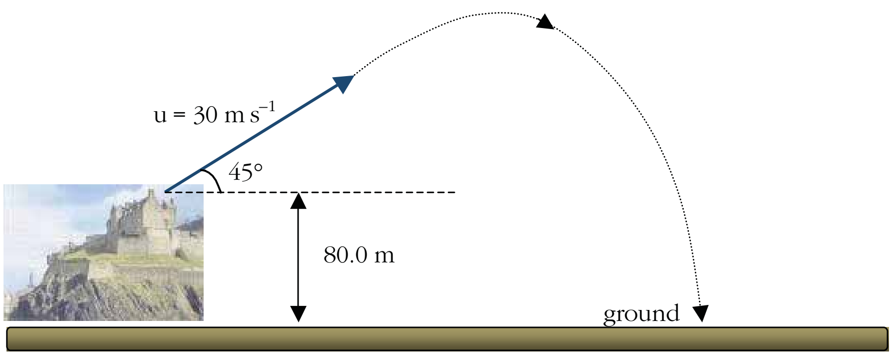
1. A spaceship with a mass of 2.5×10^4 kg moves from an altitude of 300 km to 3000 km. How much work must have been done by the spaceship's engines to do this? (Assume that the spaceship starts at rest and finishes at rest with respect to the Earth, and that Earth's radius is 6370 km. Earth's mass can be taken as 6.0×10^{24} kg, and G, the universal gravitational constant, is 6.67×10^{-11} SI units.)

check solution

2. The escape velocity of the Moon is considerably less than that of the Earth. Building launch facilities on the Moon would mean spacecraft could escape using less fuel. Find the escape velocity for the Moon, using: mass of the Moon = 7.35×10^{22} kg and the Moon's radius = 1737 km.

check solution

3. A clifftop castle is defended by rocks launched by slingshots. The rocks can be launched at an initial speed of 30.0 m s^{-1} at an angle of 45° to the horizontal. They are launched from a height of 80.0 m above the ground. This is shown below:



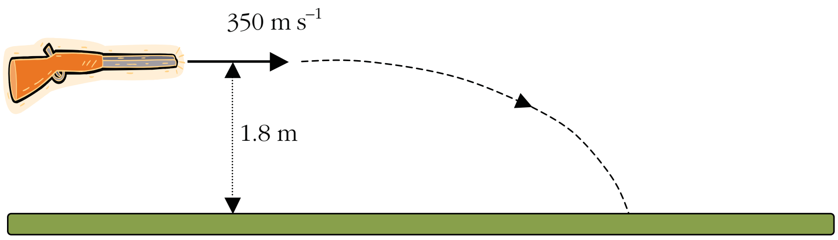
(a) For how long would the rocks remain in the air after being launched before they hit the ground?
(b) What is the speed of the rocks when they hit the ground?

check solution

4. A parachutist lands with a vertical speed of 7.0 m s^{-1} without being hurt. A faster landing speed will cause an injury. What is the highest cliff on the Moon (g_{moon} = 1.6 m s^{-2}) that a person could jump off without being injured?

check solution

5. A bullet is fired from a rifle horizontally over level ground from a height of 1.8 m and has a speed of 350 m s^{-1}. There is no air friction (as is the case in all our projectile motion situations).



(a) Compare the time it would take for a rock to fall from a height of 1.80 m with the time taken for the flight of the bullet. Find the time of flight.
(b) What is the range of the bullet (i.e. how far does it travel before hitting the ground)?

check solution

6. A satellite with a mass of 3.50×10^3 kg has an orbital speed of 4.50 km s^{-1}. It is in a circular orbit 9.00×10^6 m from the centre of the planet it is orbiting. What is the force on the satellite due to the planet's gravity?

check solution

7. The presence of a planet in orbit about a star is suspected, as the planet blocks some of the star's light every 180 days exactly. The mass of the star is 4.00×10^{31} kg. Find the distance between the centre of the planet and the centre of the star.

check solution

8. An asteroid is discovered orbiting the Sun between the Earth and Mars. The asteroid is 1.3 times further than Earth from the Sun. How long, in Earth years, does it take for the asteroid to complete one orbit?

check solution

9. Show that when two cruise ships, each with a mass of 5.00×10^4 tonnes, pass within 100 m of each other, the force of gravity between them will have no effect on their motion.

check solution

10. An electron and a positron, both with a mass of 9.1×10^{-31} kg, annihilate each other when they collide, converting all their mass into energy in the form of two gamma rays. How much energy is possessed by the gamma rays?

check solution

Examples 11–13: Use $\sqrt{1-\frac{v^2}{c^2}}$:

Often the speed of an object is given as a percentage or decimal fraction of the speed of light, c. The mathematics in these examples is much simpler if the following is considered, using the example: $v = 0.90c$:

$$\begin{aligned}\sqrt{1-\frac{v^2}{c^2}} &= \sqrt{1-\left(\frac{v}{c}\right)^2}\\ &= \sqrt{1-\left(\frac{0.90c}{c}\right)^2}\\ &= \sqrt{1-0.90^2}\\ &= \sqrt{1-0.81}\\ &= \sqrt{0.19}\\ &= 0.44\end{aligned}$$

11. A 60.0 m long near-light-speed train passes through a train station that has a 60.0 m long platform. The train is travelling at a constant speed of $0.80c$. To an observer on board the train, how much shorter than the train does the platform appear to be?

check solution

12. The half-life of a muon is 1.56 microseconds. Muons observed travelling towards the ground from the upper atmosphere had half-lives of 10.6 microseconds. Use this information to find the speed of the muons.

check solution

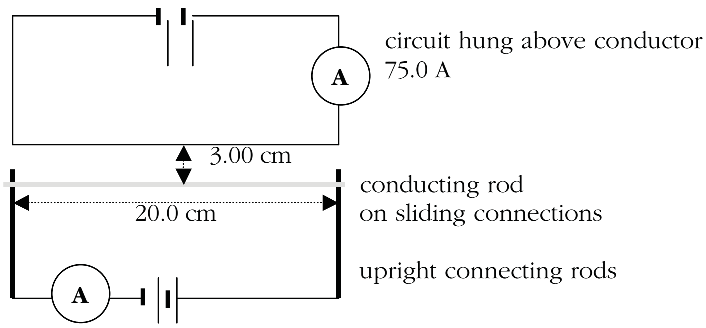
13. The Super Hadron Collider accelerated a proton to a speed of $0.999999c$. What is the kinetic energy of this proton? Hint: Use $E_K = \frac{1}{2}mv^2$. The mass of the proton when it is at rest is 1.673×10^{-27} kg.

check solution

14. How far apart must two parallel wires be placed for the force of attraction between 50.0 cm lengths of the wires to be 3.00×10^{-3} N when they are both carrying a current of 15.0 A?

check solution

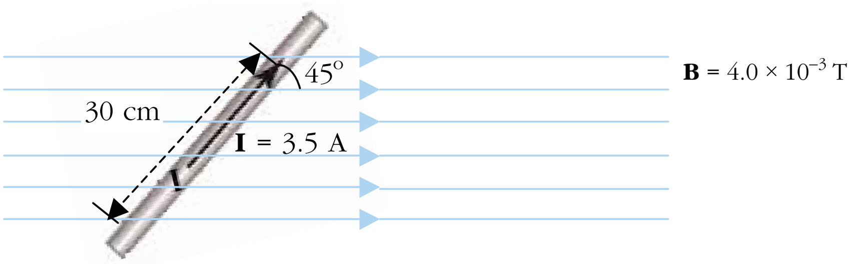
15. A 20.0 cm length of a conductor is suspended against its weight, as shown in the diagram below:



The mass of the conducting rod is 4.50 g. What is the reading on the lower ammeter?

check solution

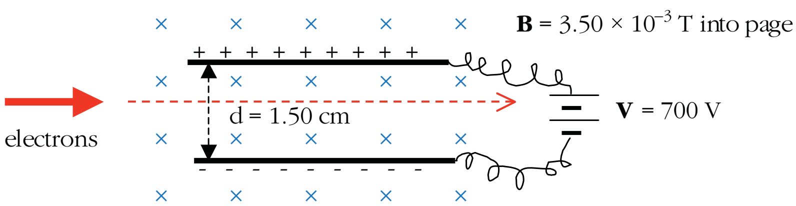
16. A wire carrying a current of 3.5 A passes through a magnetic field with an intensity of 4.0×10^{-3} T at an angle of 45°, as shown below. What is the force on this wire due to the magnetic field?



check solution

17. A beam of electrons is fired from an electron gun through an electric field and a magnetic field that are perpendicular to each other, as shown in the diagram below. The required information is provided in the diagram.

Some of the electrons pass through the perpendicular fields undeflected. Find the speed of these undeflected electrons. (This arrangement is known as a velocity selector.)



check solution

18. When Thomson performed a similar investigation using cathode ray particles, the charge of the electron was not known. Thomson found the charge to mass ratio of the particles by passing them through a velocity selector and then directing them into a second, known, magnetic field.

If Thomson had used the velocity selector shown in example 17, what would be the radius of curvature of the electrons' path when they entered a second magnetic field perpendicular to their velocity with an intensity of 9.00×10^{-2} T?

check solution

19. Find the frequency, f, and the wavelength, λ, of a photon with energy of 4.15×10^{-19} J.

check solution

20. Given that the acoustic impedance of water = 1.43×10^{6} kg m^{-2} s^{-1}, and the acoustic impedance of compact bone = 5.30×10^{6} kg m^{-2} s^{-1}, calculate the ratio of the reflected intensity to the original intensity of an ultrasound signal at an interface between water and compact bone.

check solution

21. From observations of a star's spectrum, its absolute magnitude is believed to be –6.5. When measured from Earth, its apparent magnitude is +14.7. How distant is this star?

check solution

22. What is the ratio of the brightness of two stars with apparent magnitudes of +4.5 and +7.3?

check solution

23. Two distant stars making a binary system are observed to orbit each other with a period of 7.35 Earth years. Their distance apart is calculated as 3.45×10^{11} m. What is the total mass of these two stars?

check solution

24. Given that the mass of Earth is 5.97×10^{24} kg, the average distance to the Moon is 384,400 km, and the time taken for the Moon to revolve around Earth is 27.3 days, calculate the mass of the Moon.

check solution

25. What is the wavelength of a photon of light emitted when an electron jumps from an orbital with $n = 6$ to an orbital with $n = 3$ in a hydrogen atom?

check solution

26. For a particle to have a de Broglie wavelength of 1.00×10^{-8} m when moving at 2.50×10^{7} m s^{-1}, what must be its mass?

check solution

Solutions

Example 1

A spaceship with a mass of 2.5×10^4 kg moves from an altitude of 300 km to 3000 km. How much work must have been done by the spaceship's engines to do this? (Assume that the spaceship starts at rest and finishes at rest with respect to the Earth, and that Earth's radius is 6370 km. Earth's mass can be taken as 6.0×10^{24} kg, and G, the universal gravitational constant, is 6.67×10^{-11} SI units.)

Solution

The gravitational potential energy of the spaceship changes by the same amount as the work done, since work done, W, causes a change in energy of an object. To find the work done, the change in the gravitational energy, E_p, of the spaceship must be found.

$$\begin{aligned}
\text{Work done } &= \Delta E_P \\
&= E_{P_{final}} - E_{P_{initial}} \\
&= -G\frac{m_1 m_2}{r_1} - -G\frac{m_1 m_2}{r_2} \\
&= -6.67\times10^{-11}\frac{6.0\times10^{24}\times2.5\times10^{4}}{(6370+3000)\times10^{3}} + 6.67\times10^{-11}\frac{6.0\times10^{24}\times2.5\times10^{4}}{(6370+300)\times10^{3}} \\
&= -1.07\times10^{12} + 1.50\times10^{12} \\
&= 4.3\times10^{11}\text{ J}
\end{aligned}$$

Example 2

The escape velocity of the Moon is considerably less than that of the Earth. Building launch facilities on the Moon would mean spacecraft could escape using less fuel. Find the escape velocity for the Moon, using: mass of the Moon = 7.35×10^{22} kg and the Moon's radius = 1737 km.

Solution

To escape a planet or the Moon, the spacecraft must be given an initial speed such that its kinetic energy, E_K, is exactly equal to (or greater than) the absolute value of its E_P at the launch site (i.e. on the surface).

$E_K = E_P$ at the surface

$\frac{1}{2} m_1 v^2 = -G \frac{m_1 m_2}{r}$, and since m_1 is the mass of the spaceship, we get:

$$\frac{1}{2} v^2 = G \frac{m_{moon}}{r_{moon}}$$

$$v^2 = 2 \times 6.67 \times 10^{-11} \frac{7.35 \times 10^{22}}{1737 \times 10^3}$$

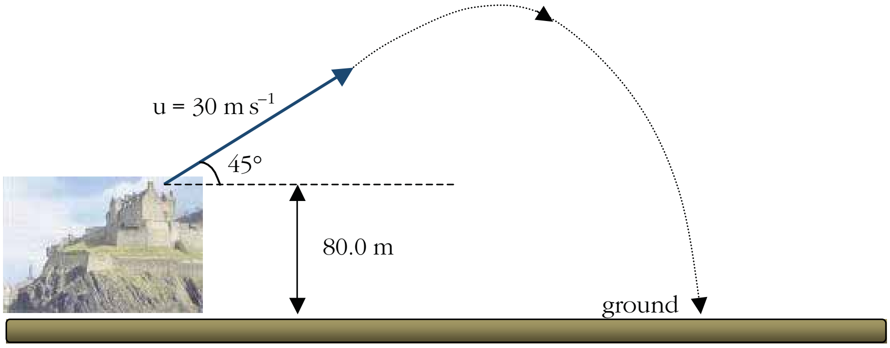
$$v = \sqrt{5.645 \times 10^6}$$

$$= 2376 \text{ m s}^{-1}$$

$$= 2.38 \text{ km s}^{-1}$$

Example 3

A clifftop castle is defended by rocks launched by slingshots. The rocks can be launched at an initial speed of 30.0 m s^{-1} at an angle of 45° to the horizontal. They are launched from a height of 80.0 m above the ground. This is shown below:



(a) For how long would the rocks remain in the air after being launched before they hit the ground?
(b) What is the speed of the rocks when they hit the ground?

Solution

(a) To begin solving this, the initial vertical speed, u_y is required. The horizontal speed is not relevant to this part of the example.

$$\begin{aligned} u_y &= u\sin\theta \\ &= 30\sin 45° \\ &= 21.21 \text{ m s}^{-1} \end{aligned}$$

Next, write down all pieces of information known, and what is needed:

$$\begin{aligned} u_y &= +21.21 \text{ m s}^{-1} \uparrow \\ a_y &= -9.8 \text{ m s}^{-2} \downarrow \\ \Delta y &= -80 \text{ m} \downarrow \\ t &= ? \end{aligned}$$

The appropriate equation is then selected, i.e.

$$\Delta y = u_y t + \frac{1}{2}a_y t^2$$

and then substitute into this equation:

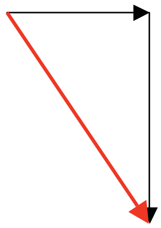
$$-80 = 21.21t + \frac{1}{2}\times -9.8\times t^2$$

$4.9t^2 - 21.21t - 80 = 0$ (a quadratic equation to solve – unlikely in major examinations)

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{21.21 \pm \sqrt{(21.21)^2 - 4\times 4.9\times -80}}{9.8} \\ &= 6.75 \text{ s (the only positive solution)} \end{aligned}$$

(b) The solution to this example requires the vector addition of the final vertical speed, v_y, and the final horizontal speed, v_x. A right-angled triangle results.

$v_x = u_x = u \cos \theta = 21.21 \text{ m s}^{-1}$



$$v = \sqrt{21.21^2 + 44.94^2}$$
$$= 49.7 \text{ m s}^{-1}$$

$$v_y = u_y + a_y t$$
$$= 21.21 + -9.8 \times 6.75$$ (using results from part (a))
$$= -44.94 \text{ m s}^{-1}$$

The example asked for the *speed* of the rocks, not the velocity, so an angle is not required.

Example 4

A parachutist lands with a vertical speed of 7.0 m s^{-1} without being hurt. A faster landing speed will cause an injury. What is the highest cliff on the Moon (g_{moon} = 1.6 m s^{-2}) that a person could jump off without being injured?

Solution

First, assume that there are cliffs on the Moon. Then state all the known information and what needs to be found:

$$v_y = -7.0 \text{ m s}^{-1}$$

$u_y = 0$ (assume the person jumps straight off the cliff)

$$a_y = -1.6 \text{ m s}^{-2}$$

$$\Delta y = ?$$

Select the appropriate equation:

$$v_y^2 = u_y^2 + 2a_y\Delta y$$

$$-7.0^2 = 0 + 2 \times -1.6 \times \Delta y$$

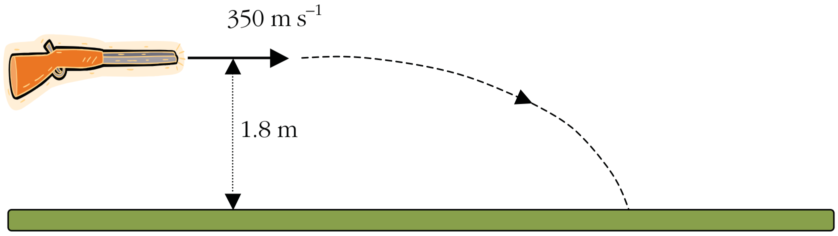
$$3.2\Delta y = -49$$

$\Delta y = -15$ m (the minus sign indicates a downward vertical displacement)

Therefore the highest cliff on the moon that a person could jump off without being injured is 15 m.

Example 5

A bullet is fired from a rifle horizontally over level ground from a height of 1.80 m and has a speed of 350 m s^{-1}. There is no air friction (as is the case in all our projectile motion situations).



(a) Compare the time it would take for a rock to fall from a height of 1.80 m with the time taken for the flight of the bullet. Find the time of flight.

(b) What is the range of the bullet (i.e. how far does it travel before hitting the ground)?

Solution

(a) The horizontal motion of the bullet has no effect on its vertical motion, as the vertical acceleration will always be 9.8 m s^{-2} downwards. The rock and the bullet will fall with the same acceleration and therefore they both hit the ground at the same time. Following the same procedure as used previously:

$$u_y = 0$$
$$a_y = -9.8 \text{ m s}^{-2}$$
$$\Delta y = -1.8 \text{ m}$$
$$t = ?$$

Select the appropriate equation:

$$\Delta y = u_y + \frac{1}{2}a_y t^2$$
$$-1.8 = 0 + \frac{1}{2} \times -9.8t^2$$
$$t^2 = \frac{1.8}{4.9}$$
$$t = 0.606 \text{ s}$$

(b) The range is found by multiplying the horizontal speed of the bullet (a projectile has a constant horizontal speed throughout its flight) by the time of the flight:

$$\Delta x = u_x t$$
$$= 350 \times 0.606$$
$$= 212 \text{ m}$$

Example 6

A satellite with a mass of 3.50×10^3 kg has an orbital speed of 4.50 km s^{-1}. It is in a circular orbit 9.00×10^6 m from the centre of the planet it is orbiting. What is the force on the satellite due to the planet's gravity?

Solution

The planet's gravity is providing the force keeping the satellite in its circular orbit, so we use:

$$\begin{aligned} F &= \frac{mv^2}{r} \\ &= \frac{3.50 \times 10^3 \times \left(4.5 \times 10^3\right)^2}{9.00 \times 10^6} \\ &= 7.88 \times 10^3 \text{ N towards the centre of the planet} \end{aligned}$$

Example 7

The presence of a planet in orbit about a star is suspected, as the planet blocks some of the star's light every 180 days exactly. The mass of the star is 4.00×10^{31} kg. Find the distance between the centre of the planet and the centre of the star.

Solution

The orbital period, T, of the planet must be converted to SI units. The distance is then found using:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}, \text{ where M is the mass of the star}$$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

$$= \frac{6.67 \times 10^{-11} \times 4.00 \times 10^{31} \times (180 \times 24 \times 60 \times 60)^2}{4\pi^2}$$

$$r = 2.54 \times 10^{11} \text{ m}$$

Example 8

An asteroid is discovered orbiting the Sun between the Earth and Mars. The asteroid is 1.3 times further than Earth from the Sun. How long, in Earth years, does it take for the asteroid to complete one orbit?

Solution

The Sun's mass is not provided, nor is the distance of the Earth from the Sun, so the equation used in Example 7 cannot be used. However, as the same central mass, M, is being orbited by both the Earth and the asteroid, the value of $\frac{GM}{4\pi^2}$ is the same in both cases. Converting to SI units is not required, as the answer is to be in Earth years.

$$\frac{r^3_{asteroid}}{T^2_{asteroid}} = \frac{r^3_{earth}}{T^2_{earth}}$$

$$\frac{1.3^3}{T^2_{asteroid}} = \frac{1^3}{1^2}$$

$$T^2_{asteroid} = 1.3^3$$

$$T_{asteroid} = 1.5 \text{ Earth years}$$

Example 9

Show that when two cruise ships, each with a mass of 5.00×10^4 tonnes, pass within 100 m of each other, the force of gravity between them will have no effect on their motion.

Solution

Be careful using tonnes: 1 tonne = 1000 kg.

Use Newton's Law of Universal Gravity equation to find the force due to gravity between the two ships:

$$F = G\frac{m_1 m_2}{d^2}$$
$$= 6.67 \times 10^{-11} \frac{\left(5.00 \times 10^4 \times 10^3\right)^2}{100^2}$$
$$= 16.7 \text{ N}$$

However, using $F = ma$, it can be shown that this force will only produce an acceleration of:

$$a = \frac{F}{m}$$
$$= \frac{16.7 \text{ N}}{5.00 \times 10^7 \text{ kg}}$$
$$= 3.34 \times 10^{-7} \text{ m s}^{-2}$$

The ships' motion is undetectably altered by their acceleration due to their mutual gravitational attraction.

Example 10

An electron and a positron, both with a mass of 9.1×10^{-31} kg, annihilate each other when they collide, converting all their mass into energy in the form of two gamma rays. How much energy is possessed by the gamma rays?

Solution

The equivalence between mass and energy can be calculated using $E = mc^2$.

$$
\begin{aligned}
E &= mc^2 \\
&= 2 \times 9.1 \times 10^{-31} \times \left(3.00 \times 10^{8}\right)^2 \\
&= 1.6 \times 10^{-13} \text{ J}
\end{aligned}
$$

Example 11

A 60.0 m long near-light-speed train passes through a train station that has a 60.0 m long platform. The train is travelling at a constant speed of 0.80*c*. To an observer on board the train, how much shorter than the train does the platform appear to be?

Solution

This is a length contraction equation, with v = 0.80c and l_o = 60.0 m.

Use: $$l_v = l_o\sqrt{1-\frac{v^2}{c^2}}$$

$$= 60.0\sqrt{1-\left(\frac{0.80c}{c}\right)^2}$$

$$= 60.0\sqrt{1-0.64}$$

$$= 36.0 \text{ m}$$

So the platform appears to be: 60.0 – 36.0 = 24.0 m shorter than the train.

Example 12

The half-life of a muon is 1.56 microseconds. Muons observed travelling towards the ground from the upper atmosphere had half-lives of 10.6 microseconds. Use this information to find the speed of the muons.

Solution

Using $t_o = 1.56 \times 10^{-6}$ s and $t_v = 10.6 \times 10^{-6}$ s, find v:

$$t_v = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$10.6 \times 10^{-6} = \frac{1.56 \times 10^{-6}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$\sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{1.56}{10.6}$$

$$1 - \left(\frac{v}{c}\right)^2 = 0.02166$$

$$\left(\frac{v}{c}\right)^2 = 1 - 0.02166$$

$$\frac{v}{c} = \sqrt{0.97834}$$

$$v = 0.989c$$

Example 13

The Super Hadron Collider accelerated a proton to a speed of 0.999999c. What is the kinetic energy of this proton? Hint: Use $E_K = \frac{1}{2}mv^2$. The mass of the proton when it is at rest is 1.673×10^{-27} kg.

Solution

First, the relativistic mass of the proton must be found using the mass dilation equation. Then this mass is used to find the proton's kinetic energy.

$$\begin{aligned} m_v &= \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \\ &= \frac{1.673\times10^{-27}}{\sqrt{1-0.999999^2}} \\ &= 1.183\times10^{-24}\ \text{kg} \end{aligned}$$

Then we find E_K:

$$\begin{aligned} E_K &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}\times8.365\times10^{-22}\times\left(0.999999\times3.00\times10^{8}\right)^2 \\ &= 5.323\times10^{-8}\ \text{J} \end{aligned}$$

Example 14

How far apart must two parallel wires be placed for the force of attraction between 50.0 cm lengths of the wires to be 3.00×10^{-3} N when they are both carrying a current of 15.0 A?

Solution

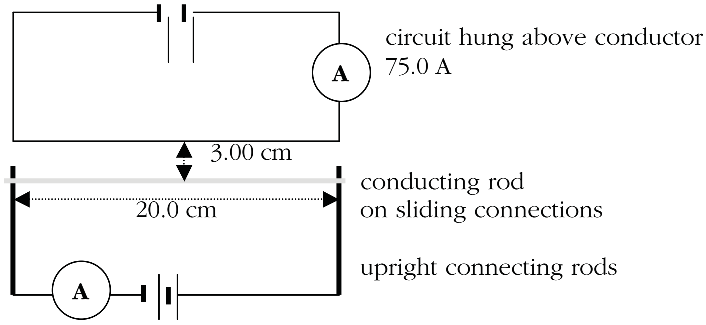
Using $l = 0.50$ m, $F = 3.00 \times 10^{-3}$ N, $I_1 = I_2 = 15.0$ A:

$$\frac{F}{l} = k\frac{I_1 I_2}{d}$$

$$\begin{aligned} d &= \frac{kI_1 I_2 l}{F} \\ &= \frac{2.0\times10^{-7}\times15\times15\times0.50}{3.00\times10^{-3}} \\ &= 7.5\times10^{-3}\ \text{m} \end{aligned}$$

Example 15

A 20.0 cm length of a conductor is suspended against its weight, as shown in the diagram below:



The mass of the conducting rod is 4.50 g. What is the reading on the lower ammeter?

Solution

First, find the weight of the 4.50 g conducting rod:

$$\begin{aligned} W &= mg \\ &= 4.41 \times 10^{-2} \text{ N} \end{aligned}$$

Using $I_1 = 75.0$ A, $l = 0.200$ m, $F = 4.41 \times 10^{-2}$ N, $d = 0.030$ m, apply the equation to find I_2:

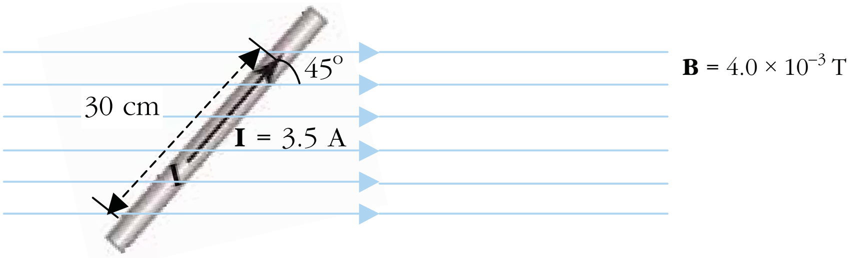
$$\frac{F}{l} = k\frac{I_1 I_2}{d}$$

$$\begin{aligned} I_2 &= \frac{Fd}{klI_1} \\ &= \frac{4.41 \times 10^{-2} \times 0.030}{2.0 \times 10^{-7} \times 0.200 \times 75.0} \\ &= 441 \text{ A} \end{aligned}$$

(This very large current is why the effect of conductors carrying parallel currents is not normally noticed in electrical appliances.)

Example 16

A wire carrying a current of 3.5 A passes through a magnetic field with an intensity of 4.0×10^{-3} T at an angle of 45°, as shown. What is the force on this wire due to the magnetic field?



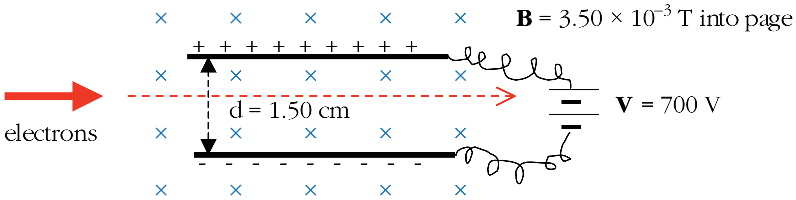
Solution

$$F = BIl \sin\theta$$
$$= 4.0 \times 10^{-3} \times 3.5 \times 0.30 \sin 45°$$
$$= 3.0 \times 10^{-3} \text{ N}$$

Example 17

A beam of electrons is fired from an electron gun through an electric field and a magnetic field that are perpendicular to each other, as shown in the diagram below. The required information is provided in the diagram.

Some of the electrons pass through the perpendicular fields undeflected. Find the speed of these undeflected electrons. (This arrangement is known as a velocity selector.)



Solution

The undeflected electrons experience the same force due to the electric field, i.e. $F = qE$, as the force due to the magnetic field, i.e. $F = qvB$. To find the magnitude of E, we need to use $E = \frac{V}{d}$ first:

$$E = \frac{700}{0.0150}$$
$$= 46.67 \text{ V m}^{-1}$$

Next equate the two forces and solve for v:

$$F_B = F_E$$
$$qvB = qE$$
$$vB = E$$
$$v = \frac{E}{B}$$
$$= \frac{46.67}{3.5 \times 10^{-3}}$$
$$= 1.33 \times 10^4 \text{ m s}^{-1}$$

Example 18

When Thomson performed a similar investigation using cathode ray particles, the charge of the electron was not known. Thomson found the charge to mass ratio of the particles by passing them through a velocity selector and then directing them into a second, known, magnetic field.

If Thomson had used the velocity selector shown in Example 17, what would be the radius of curvature of the electrons' path when they entered a second magnetic field perpendicular to their velocity with an intensity of 9.00×10^{-2} T?

Solution

The force from the magnetic field is providing centripetal force: $F = \frac{mv^2}{r}$.

The magnetic force is equated to the centripetal force to find r:

$$F_B = F_C$$

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv^2}{qvB}$$

$$= \frac{mv}{qB}$$

$$= \frac{9.1\times10^{-31}\times1.33\times10^{4}}{1.6\times10^{-19}\times9.00\times10^{-2}}$$ (using previously found values and known mass and charge of electron)

$$= 8.40\times10^{-7}\text{ m}$$

Example 19

Find the frequency, f, and the wavelength, λ, of a photon with energy of 4.15×10^{-19} J.

Solution

Use $E = hf$

$$f = \frac{E}{h}$$

$$= \frac{4.15 \times 10^{-19}\ \text{J}}{6.626 \times 10^{-34}\ \text{J s}}$$

$$= 6.26 \times 10^{14}\ \text{Hz}$$

Next, find λ:

$$c = f\lambda$$

$$\lambda = \frac{c}{f}$$

$$= \frac{3.00 \times 10^{8}}{6.26 \times 10^{14}}$$

$$= 4.79 \times 10^{-7}\ \text{m}$$

Example 20

Given that the acoustic impedance of water = 1.43×10^6 kg m^{-2} s^{-1}, and the acoustic impedance of compact bone = 5.30×10^6 kg m^{-2} s^{-1}, calculate the ratio of the reflected intensity to the original intensity of an ultrasound signal at an interface between water and compact bone.

Solution

Use $Z_1 = 1.43 \times 10^6$ and $Z_2 = 5.30 \times 10^6$, noting that the common 10^6 can be cancelled as a common factor in the equation:

$$\frac{I_r}{I_o} = \frac{[Z_2 - Z_1]^2}{[Z_2 + Z_1]^2}$$

$$= \frac{[5.30 - 1.43]^2}{[5.30 + 1.43]^2}$$

$$= \frac{14.98}{45.29}$$

$$= 0.331$$

That is, the reflected intensity is 33.1% of that of the original intensity of the ultrasound signal.

Example 21

From observations of a star's spectrum, its absolute magnitude is believed to be –6.5. When measured from Earth, its apparent magnitude is +14.7. How distant is this star?

Solution

Use M = –6.5 and m = +14.7:

$$M = m - 5\log\left(\frac{d}{10}\right)$$

$$-6.5 = 14.7 - 5\log\left(\frac{d}{10}\right)$$

$$5\log\left(\frac{d}{10}\right) = 14.7 + 6.5$$

$$\log\left(\frac{d}{10}\right) = \frac{21.2}{5}$$

$$\frac{d}{10} = 10^{4.24}$$

$$d = 10^{5.24} \text{ (as } 10 \times 10^{4.24} = 10^{5.24}\text{)}$$

$$d = 1.74 \times 10^{5} \text{ parsecs}$$

Example 22

What is the ratio of the brightness of two stars with apparent magnitudes of +4.5 and +7.3?

Solution

Use:

$$\frac{I_A}{I_B} = 100^{(m_B - m_A)/5}$$

$$= 100^{(7.3-4.5)/5}$$

$$= 100^{0.56}$$

$$= 13.2$$

That is, the star with m = 4.5 is 13.2 times brighter from Earth.

Example 23

Two distant stars making a binary system are observed to orbit each other with a period of 7.35 Earth years. Their distance apart is calculated as 3.45×10^{11} m. What is the total mass of these two stars?

Solution

Use the equation but first convert all measurements to SI units:

7.35 years = $7.35 \times 365 \times 24 \times 60 \times 60$ seconds
= 2.318×10^{8} s (ignoring the effect of leap years)

Now:

$$m_1 + m_2 = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4\pi^2 \left(3.45 \times 10^{11}\right)^3}{6.67 \times 10^{-11} \times \left(2.318 \times 10^{8}\right)^2}$$

$$= 4.52 \times 10^{29} \text{ kg}$$

Example 24

Given that the mass of the Earth is 5.97×10^{24} kg, the average distance to the Moon is 384,400 km, and the time taken for the Moon to revolve around the Earth is 27.3 days, calculate the mass of the Moon.

Solution

Use the equation from Example 23, but, again, convert all measurements to SI units first:

$$\begin{aligned} 27.3 \text{ days} &= 27.3 \times 24 \times 60 \times 60 \text{ seconds} \\ &= 2.359 \times 10^{6} \text{ s} \end{aligned}$$

Now:

$$m_1 + m_2 = \frac{4\pi^2 r^3}{GT^2}$$

$$m_{moon} + m_{earth} = \frac{4\pi^2 r^3}{GT^2}, \text{ where } r = \text{distance between centres of Earth and Moon}$$

$$m_{moon} + 5.97 \times 10^{24} = \frac{4\pi^2 \left(384,400 \times 10^3\right)^3}{6.67 \times 10^{-11} \times \left(2.359 \times 10^6\right)^2}$$

$$= 6.041 \times 10^{24} \text{ kg}$$

$$m_{moon} = (6.041 - 5.97) \times 10^{24} \text{ kg}$$

$$= 7.13 \times 10^{22} \text{ kg}$$

(The mass of the Moon is actually 7.35×10^{22} kg, the difference is due to the rounding-offs during the calculations.)

Example 25

What is the wavelength of a photon of light emitted when an electron jumps from an orbital with $n = 6$ to an orbital with $n = 3$ in a hydrogen atom?

Solution

Use the Rydberg equation to find λ, with $n_f = 3$ and $n_i = 6$:

$$\begin{aligned}
\frac{1}{\lambda} &= R\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right) \\
&= 1.097\times10^{7}\left(\frac{1}{3^2} - \frac{1}{6^2}\right) \\
&= 1.097\times10^{7}\left(8.333\times10^{-2}\right) \\
&= 9.142\times10^{5} \\
\lambda &= 1.094\times10^{-6}\ \text{m}
\end{aligned}$$

Example 26

For a particle to have a de Broglie wavelength of 1.00×10^{-8} m when moving at 2.50×10^{7} m s^{-1}, what must be its mass?

Solution

This example illustrates why the de Broglie wavelength of particles is normally not detectable:

$$
\begin{aligned}
\lambda &= \frac{h}{mv} \\
mv\lambda &= h \\
m &= \frac{h}{v\lambda} \\
&= \frac{6.626 \times 10^{-34}}{2.50 \times 10^{7} \times 1.00 \times 10^{-8}} \\
&= 2.65 \times 10^{-33} \text{ kg}
\end{aligned}
$$

(This is about 1/30th the mass of an electron.)