CHAPTER 1

Gravity

Answers to revision questions

1. Mass is the quantity of matter: it is an absolute measurement of how much substance is in a body or an object. Weight is the gravitational force acting on a mass in a gravitational field.

2. The equation used to calculate the universal gravitational force of attraction:

$$\boldsymbol{F} = \frac{Gm_1m_2}{\boldsymbol{d}^2}$$

Known quantities:

$m_1 = 1.675 \times 10^{-27}$ kg
$m_2 = 1.675 \times 10^{-27}$ kg
$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
$\boldsymbol{d} = 1 \times 10^{-13}$ m

Unknown: $\boldsymbol{F}$

Substituting into the equation:

$$\boldsymbol{F} = \frac{6.67 \times 10^{-11} \times 1.675 \times 10^{-27} \times 1.675 \times 10^{-27}}{\left(1 \times 10^{-13}\right)^2}$$

$$= 1.87 \times 10^{-38}\,\text{N}$$

3. The equation used to calculate the gravitational force acting on the satellite by the Earth:

$$F = \frac{Gm_1m_2}{d^2}$$

Known quantities:

$G = 6.67 \times 10^{-11}$
$m_1 = 6.0 \times 10^{24}$ kg
$m_2 = 150.0$ kg
$\boldsymbol{d} = \left(6.378 \times 10^6 + 2.3 \times 10^6\right)$ m

Unknown: $\boldsymbol{F}$

Substituting into the equation:

$$\boldsymbol{F} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 150.0}{\left(6.378 \times 10^6 + 2.3 \times 10^6\right)^2\ \text{m}}$$

$$= 797\ \text{N}$$

4. The equation for the universal gravitational attraction:

$$F = \frac{Gm_1m_2}{d^2}$$

Initially $F_I = \dfrac{Gm_1m_2}{d^2}$

Where:

G = the universal gravitational constant
m_1 = initial mass of object A
m_2 = initial mass of object B
$\boldsymbol{d}$ = initial distance between the objects

After the changes are made, the new force is now expressed as:

$$F_{new} = G\frac{M_1M_2}{D^2}$$

G = the universal gravitational constant
M_1 = the new mass of object A
M_2 = the new mass of object B
D = the new distance between the objects

Given that:

$$M_1 = \frac{m_1}{4}$$

$$M_2 = \frac{m_2}{2}$$

$$D = 2d$$

$$F_{new} = G\frac{M_1M_2}{D^2} = G\frac{\left(\frac{m_1}{4}\right)\left(\frac{m_2}{2}\right)}{(2d)^2}$$

$$= G\frac{\frac{m_1m_2}{8}}{4d^2}$$

$$= G\frac{m_1m_2}{32d^2}$$

$$F_{new} = \frac{1}{32}\cdot F_i$$

5. The universal gravitational attraction equation:

$$F = G\frac{m_1m_2}{d^2}$$

On the Earth: (where m = mass of any person)
$\boldsymbol{g} = 9.8\ \text{m s}^{-2}$ $\boldsymbol{F} = ma = 9.8\ m$ N

On the Moon:
$m_1 = 7.35 \times 10^{22}$ kg
$\boldsymbol{d} = 1.738 \times 10^6$ m

$$\boldsymbol{F} = \frac{6.67\times10^{-11}\times7.35\times10^{22}\times m}{\left(1.738\times10^{6}\right)^2}$$

$= 1.6\ m$ N

As seen above, the pull exerted by the moon is much smaller than the Earth, therefore when a person applies the same force on the moon, they will jump instead.

6. The universal gravitational acceleration equation is:

$$\boldsymbol{g} = G\frac{M}{d^2}$$

On the Earth:

Let $\boldsymbol{g}_E = G\frac{M_E}{{d_E}^2}$

The unknown planet has:
mass = 10 M_E
radius = 4d_E
Therefore:

$$\boldsymbol{g} = \frac{10GM_E}{(4d_E)^2} = \frac{5}{8} \cdot G\frac{M_E}{d_E^2}$$

Therefore the unknown planet has a gravitational acceleration $\frac{5}{8}$ times that of the Earth.

7. According to Newton's second law:

$\boldsymbol{F} = ma$

The mass of an object remains constant regardless of its location.

For the Earth: $\boldsymbol{F}_E = ma_E$

For the planet: $\boldsymbol{F}_P = ma_P$

Since m is the same:

$$\frac{\boldsymbol{F}_P}{\boldsymbol{F}_E} = \frac{a_P}{a_E} = \frac{3}{1}$$

Therefore the ratio of the weight of this object on the planet Xero to its weight on the Earth is also 3:1.

8. i. The equator is further away from the centre of the earth than the poles, as the Earth is elliptical in shape. Therefore according to $\boldsymbol{g} = G\frac{M}{\boldsymbol{d}^2}$, as d increases, the gravitational acceleration decreases.

ii. The rotation of the Earth is the fastest at the equator, that is, it has the greatest rotational speed. A small proportion of '$\boldsymbol{g}$' is required to keep an object in circular motion (preventing it lifting off the surface), so that the weight force measured is slightly smaller.

9. Work done = gravitational potential energy

That is, $W = E_p = mgh$

Known quantities:

m = 10 g = 0.01 kg

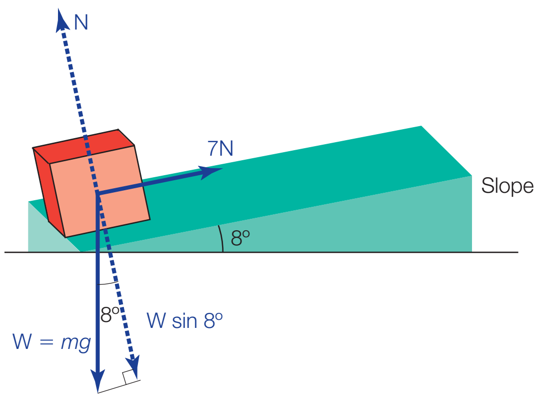
$\boldsymbol{g}$ = 9.8 m s^{-2}

$\boldsymbol{h}$ = 1.2 m

$W = 0.01 \times 9.8 \times 1.2$

$= 0.12$ J (0.1176)

10. (a)



$$\begin{aligned}\text{Work done} &= F \times d \\ &= (\text{net force}) \times (\text{distance}) \\ \text{Net force} &= 7 - W\sin 8^\circ \\ &= 7 - mg\sin 8^\circ \\ &= 7 - (0.39)\times(9.8)\times(\sin 8^\circ) \\ &= 6.468\ \text{N} \\ \text{Therefore, work} &= 6.468 \times 8 = 52\ \text{J}\end{aligned}$$

(b) The change in potential energy is 52 J since the work done on the object is translated to the potential energy gained during this time.

11. Gravitational potential energy:

$$E_p = -\frac{GmM}{r}$$

Known quantities:

$m = 68$ kg
$M = 6.0 \times 10^{24}$ kg
$r = \left(8848 + 6.378 \times 10^6\right)$ m & 6378000 m
$E = 6.67 \times 10^{-11}$

$\Delta E_p = E_p$ at the summit – E_p at the surface

$$\Delta E_p = -\frac{\left(6.67\times10^{-11}\right)(68)\left(6.0\times10^{24}\right)}{(8848+6378000)} - \left[-\frac{\left(6.67\times10^{-11}\right)(68)\left(6.0\times10^{24}\right)}{6378000}\right]$$

$$\approx 5.91 \times 10^6\ \text{J}$$

12. Work done = change in gravitational potential energy

That is, $W = \Delta E_p = -\dfrac{GmM}{d}$

Known quantities:

$G = 6.67 \times 10^{-11}$
$M = 6.0 \times 10^{24}$ kg
$m = 0.198$ kg
$d = 6378000 + 200000$ m
$d_2 = 6378000 + 3500000$ m
$\Delta E_p = E_p$ at 3500 km – E_p at 200 km
$= 4.02 \times 10^6$ J

13. The mechanical energy of an object is equal to the sum of its kinetic and potential energy.

$$E_k = \frac{1}{2}mv^2 \qquad E_p = -\frac{GmM}{r}$$

Known quantities:

$m = 15$ tonnes $= 15000$ kg
$v = 250$ m s^{-1}
$G = 6.67 \times 10^{-11}$
$M = 6.0 \times 10^{24}$ kg
$r = \left(6.378\times10^6 + 1.02\times10^4\right)$ m

$$E_k = \frac{1}{2} \times 15000 \times (250)^2$$

$$= 468750000\ \text{J}$$

$$E_p = -\frac{6.67 \times 10^{-11} \times 15000 \times 6.0 \times 10^{24}}{\left(6.378 \times 10^{6} + 1.02 \times 10^{4}\right)}$$

$$= -9.39701 \times 10^{11}\ \text{J}$$

$$\text{Total } E_m = 468750000 + \left(-9.39701 \times 10^{11}\right)$$

$$= -9.39 \times 10^{11}\ \text{J}$$

14. At a point of an infinite distance from the Earth, the gravitational field is zero and an object will not experience a force. Thus the gravitational potential energy is zero at infinity. Work is to be done (energy input) in order to move an object away from the Earth, until E_p reaches zero at infinity. Hence any point below infinity (e.g. near the Earth) must have a negative energy.

15.