CHAPTER 3

Gravity, orbits and space travel

The solar system is held together by gravity

3.1

SR

Worked example 9

Gravity: a revision

- ***Describe a gravitational field in the region surrounding a massive object in terms of its effects on other masses in it***
- ***Define Newton's Law of Universal Gravitation:*** $F = G\dfrac{m_1 m_1}{d^2}$
- ***Discuss the importance of Newton's Law of Universal Gravitation in understanding and calculating the motion of satellites***

PFA

H4

'Assesses the impacts of applications of physics on society and the environment'

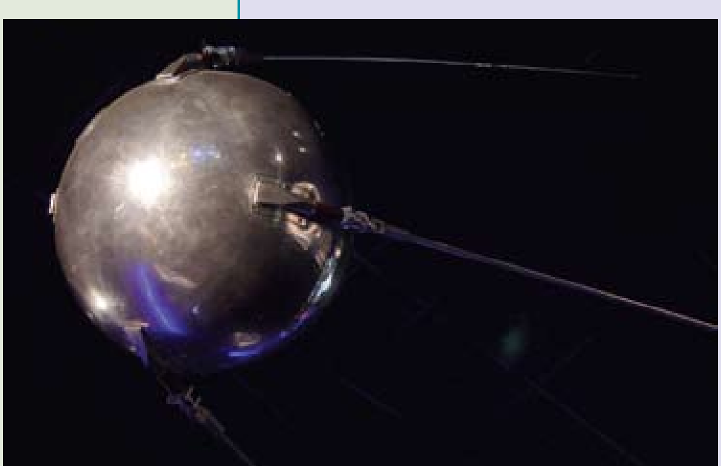
TR

Mapping the PFAs
PFA scaffolds H4

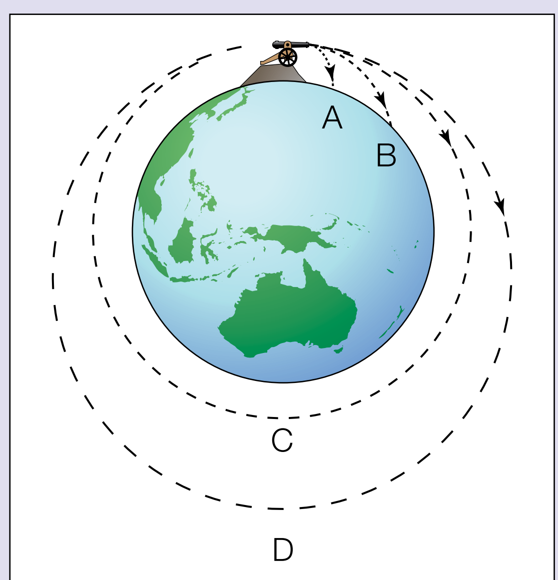
- ***Discuss the importance of Newton's Law of Universal Gravitation in understanding and calculating the motion of satellites***

Sputnik 1, the first artificial satellite to be launched by the human race, sent a wave of fear through the western world, particularly in the US. Its launch by the Soviet Union in October 1957 ignited the Space War between the Soviet Union and the United States, and was a contributing factor to the ensuing Cold War.

The possibility of satellite motion was first conceived by Sir Isaac Newton in the 1680s. Using the previous works of Galileo and Kepler on projectile motion and the motion of the planets respectively, Newton extended the idea of a gravitational force to the motion of the Moon around the Earth. That

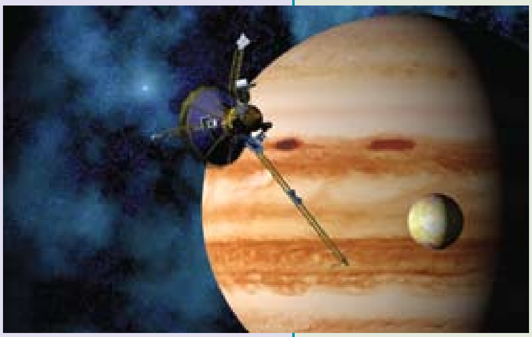


Sputnik 1



Newton's concept of how a satellite might be placed in Earth orbit uses the analogy of cannonballs shot from 'Newton's mountain' gravitational force obeys an inverse square law (i.e. the force is inversely proportional to the square of the distance between two objects, $\boldsymbol{F} \propto \frac{1}{\boldsymbol{d}^2}$).

The Russian scientists responsible for the planning of *Sputnik 1* were able to use Newton's law of universal gravitation to calculate how fast the spacecraft needed to be propelled by the rockets so that it would maintain a stable orbit.


A GPS satellite

All other satellites which have been placed in orbit since *Sputnik 1* (of which there have been thousands) have used the same calculations—based on Newton's gravity. In our modern society, satellites are relied upon for many uses, including pay TV and other communications, remote sensing for mineral exploration, detailed weather observations and measurements of atmospheric ozone and other pollutants, mapping and monitoring of land use and modern GPS navigation systems used by aircraft, ships, cars and even missiles.

USEFUL WEBSITES

WWW

Newton's law of universal gravitation:
http://www.glenbrook.k12.il.us/GBSSCI/PHYS/CLASS/circles/u6l3c.html

Projectile motion and satellites:
http://www.glenbrook.k12.il.us/GBSSCI/PHYS/mmedia/vectors/sat.html

More information about Newton, his life and his work:
http://www.galileoandeinstein.physics.virginia.edu/lectures/newton.html

The concepts dealt with in Chapter 3 are closely related to those in Chapter 1. Most of the Part 3 (refer to the syllabus) materials are covered in Chapters 1 and 2 and are not be repeated in detail in Chapter 3.

Recall that any mass, regardless of its size, will have a gravitational field around it. However, gravity, as one of the four forces in nature (the other three being the strong nuclear force, the weak nuclear force and the electromagnetic force), is extremely weak. Unlike the nuclear forces, it can act over very large distances, and is responsible for keeping the atmosphere and oceans (and us) on Earth, and for holding the solar system and indeed our entire galaxy together. Without gravity, stars would not be held together, and would not have the necessary pressure within their cores to enable nuclear fusion. A very different Universe, dark and lifeless, would be the result.

Isaac Newton had defined quantitatively the size of the gravitational force in his law of universal gravitation, which states the size of the attraction force between two objects is proportional to the product of their masses and inversely proportional to the square of their distance of separation (see Chapter 1); and mathematically.

$$\boldsymbol{F} = G\frac{m_1 m_2}{\boldsymbol{d}^2}$$

Where:

m_1 and m_2 = mass of the two objects, measured in kg

$\boldsymbol{d}$ = distance between the two objects measured *from the centre of each object* (m)

G = universal gravitational constant, which is equal to 6.67×10^{-11} N m^2 kg^{-2}

Lastly, recall that the motion of a satellite in orbit around the Earth or other planets is governed by the force of gravity. When space probes are placed in specific orbits, the utilisation of gravity is of paramount importance in determining how the orbit is calculated. These concepts have already been discussed in detail in Chapter 2. A further example is included in this section.

Example

Compare the orbital speed of satellites which have stable orbits with an altitude of 300 km:

(a) Above the Earth.

(b) Above the Moon's surface.

Solution

As the centripetal force required to keep the satellite in orbit $\left(\boldsymbol{F}_c = \frac{mv^2}{r}\right)$ is provided by the force of gravity $\left(\boldsymbol{F}_G = G\frac{m_1 m_2}{\boldsymbol{d}^2}\right)$, the two equations are equated, the mass m_1 (the mass of the satellite) is cancelled and the value for v, the orbital speed, is calculated.

(a) For Earth ($m_{\text{Earth}} = 6.0 \times 10^{24}$ kg):

$$\boldsymbol{F}_c = \boldsymbol{F}_G$$

$$\frac{mv^2}{r} = G\frac{m_1 m_2}{\boldsymbol{d}^2}$$

$$\frac{v^2}{r} = G\frac{m_2}{\boldsymbol{d}^2}$$

$$v^2 = G\frac{m_{\text{Earth}}}{\boldsymbol{d}} \quad (\text{as } r = \boldsymbol{d})$$

$$v = \sqrt{6.67 \times 10^{-11} \times \frac{6.0 \times 10^{24}}{(6370 + 300) \times 10^3}}$$

$$= 7.7 \times 10^3 \text{ m s}^{-1}$$

(b) For the Moon ($m_{\text{Moon}} = 7.4 \times 10^{22}$ kg; radius = 1700 km)

$$v^2 = G\frac{m_{\text{Moon}}}{\boldsymbol{d}}$$

$$v = \sqrt{6.67 \times 10^{-11} \times \frac{7.4 \times 10^{22}}{(1700 + 300) \times 10^3}}$$

$$= 1.6 \times 10^3 \text{ m s}^{-1}$$

The calculations show that a spacecraft orbiting the Moon will have a much slower orbital speed than one orbiting the Earth at the same altitude.

NOTE: In Chapter 1, there are three formulae which students refer to as being similar:

$$F = G\frac{m_1 m_2}{d^2} \qquad g = G\frac{M}{d^2} \qquad E_p = -G\frac{mM}{r}$$

To remember these formulae efficiently and correctly, you should have a sound understanding of the relationship among them and have ideas about the derivations of these formulae. This will also help you to be clear about which formula to use for a specific exam question.

Factors that affect the size of gravitational attraction

SECONDARY SOURCE INVESTIGATION

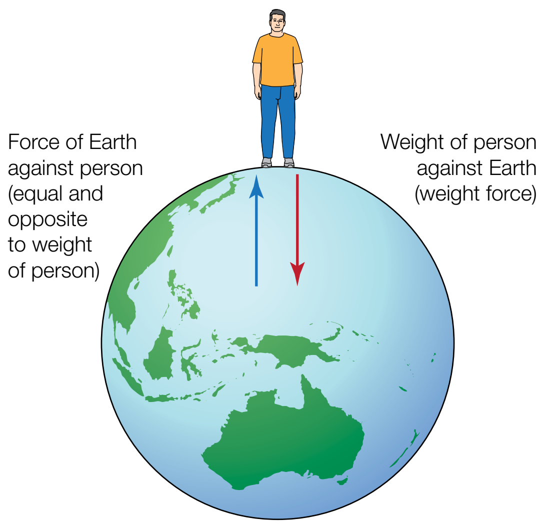
PHYSICS SKILLS

H14.1A, C, D, F, G, H
H14.2A, D
H14.3B

■ *Present information and use available evidence to discuss the factors affecting the strength of the gravitational force*

The law of universal gravitation states that the size of the attraction force is proportional to mass and inversely proportional to the square of distance; therefore the bigger the mass and the closer the distance is from the mass object, the larger the gravitational force.

Due to gravity's weakness (the small value of G), only very massive bodies produce noticeable gravitational fields. For example, a 50 kg person on Earth's surface, being approximately 6370 km from the centre of the Earth (which has a mass of 6.0×10^{24} kg) only produces an attractive force of 490 N. This force may be thought of as either being a force on the person, or an equal (but opposite) force on the Earth. (Both views are correct.)



What will be the size of the attraction force if this person is on Jupiter?

■ *Solve problems and analyse information using:* $F = G\dfrac{m_1 m_2}{d^2}$

Example 1

(a) Two large asteroids, both with a mass of 1.5×10^9 kg, pass within 200 km of each other. What force of attraction would there be between the asteroids? This result is the equivalent to the weight of a small grain of sand!

Solution

$$\begin{aligned} \boldsymbol{F} &= G\frac{m_1 m_2}{\boldsymbol{d}^2} \\ &= 6.67 \times 10^{-11}\,\frac{1.5 \times 10^9 \times 1.5 \times 10^9}{(300 \times 10^3)^2} \\ &= 1.7 \times 10^{-3}\ \text{N} \end{aligned}$$

Example 2

Jupiter has a mass of 1.9×10^{27} kg. This is approximately 200 times the mass of Earth. Some astronomers believe that Jupiter has saved Earth from being bombarded by many more asteroids than it actually has in the past. Why is this?

Solution

The answer stems from the way in which Jupiter's gravity can attract asteroids that stray too close to the planet, pulling them in and thus preventing them from continuing in an orbit that may pass close to Earth or colliding with Earth.

Example 3

Tides on Earth are caused by a combination of the Moon's and the Sun's gravity acting on the oceans. Compare the strength of the Moon's gravity on Earth's surface (approximately 400 000 km from the Moon's centre) to that on the Moon's surface. The radius of the Moon is 1700 km.

Solution

Newton's law of universal gravitation is an example of the inverse square law: that is, the strength of the gravitational field is inversely proportional to the square of the distance, so:

$$\text{Strength of gravity on Moon's surface is} = \left(\frac{400000}{1700}\right)^2$$
$$= 5.5 \times 10^4$$

greater than that exerted at Earth's surface.

3.2 The slingshot effect

■ *Identify that a slingshot effect can be provided by planets for space probes*

The principle of the slingshot effect is to use a planet's gravitational field and orbital speed to help a space probe gain extra speed by flying past the planet.

This allows a long distance space probe to increase its velocity every time it passes a planet without spending any fuel (other than a small amount for maneuvering). This consequently reduces the time of its trip as well as the fuel requirement.

A space probe

How does the slingshot effect work?

When a space probe is passing close to a planet, say Jupiter, the probe accelerates (speeds up) due to the force of the planet's gravitational field. However, when the space probe moves away from Jupiter, it decelerates (slows down) for the same reason. Effectively, the incoming speed of the probe is about the same as its receding speed except the trajectory or pathway of the probe has now been changed (see Fig. 3.1).

Jupiter is not stationary but it is orbiting around the Sun. Therefore, as the probe is pulled along and swung around the

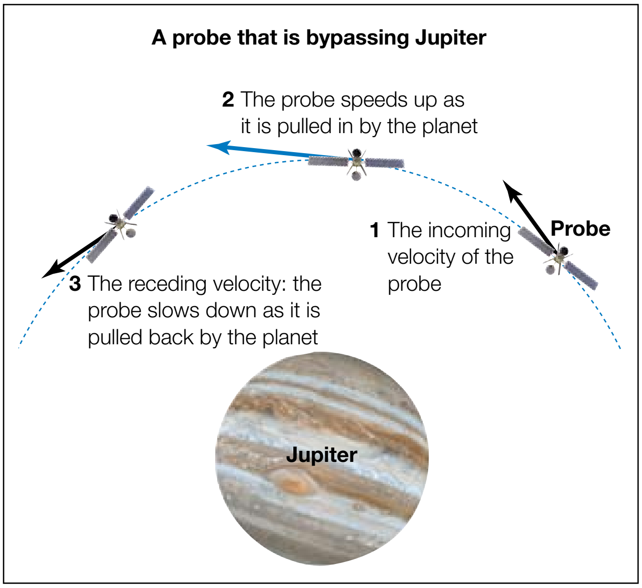


Figure 3.1 A probe passing Jupiter

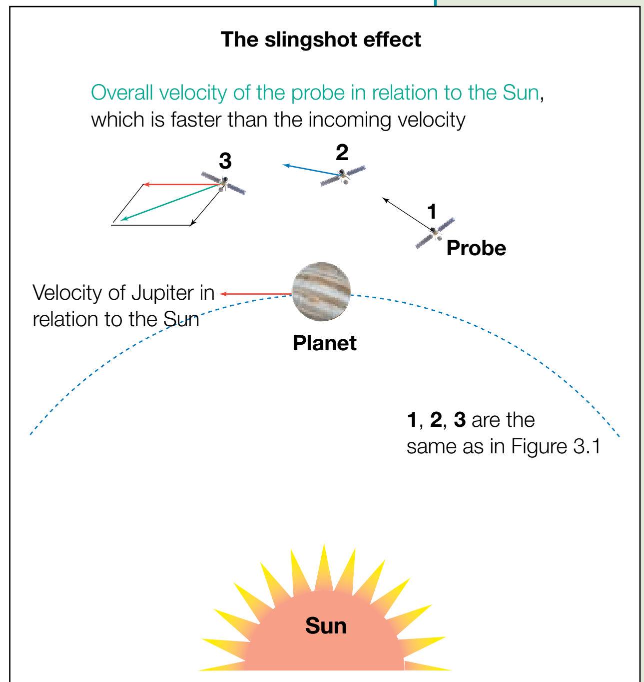


Figure 3.2 The slingshot effect

planet by the gravitational field of Jupiter, it will also have the speed of Jupiter around the Sun added to its original speed; hence, the probe speeds up with respect to the Sun (see Fig. 3.2).

The maximum speed the probe can gain will be twice the speed of the planet around the Sun, however, this is not often achieved. Nevertheless, by allowing the probe to by-pass many planets during its trip in a specific pattern, the probe will be able to travel towards the destination (not necessary in a straight path) at a significantly higher speed. This also means in order to use this type of slingshot effect, all planets have to be at the right place at the right time. This very narrow **launching window** requires careful calculations and planning.

The extra velocity gained by the probe does not come for 'free'. When the slingshot effect takes place, momentum will still have to be conserved, that is, the total initial (angular) momentum of the probe *and* planet must equal the total final momentum. Since the probe has now sped up and gained momentum, so the planet must lose an equal amount of momentum and slow down. Because momentum is the product of mass and velocity, and the mass of the planet is much larger than that of the probe, the speed lost by the planet will be insignificant compared to the speed gained by the probe.

CHAPTER REVISION QUESTIONS

For all the questions in this chapter, take the mass of the Earth to be 6.0×10^{24} kg, and the radius of the Earth to be 6378 km.

1. Define the term 'gravitational field'.
2. (a) With the aid of a diagram, briefly describe the meaning of the 'slingshot effect' and the physics principle behind it.
   (b) With respect to the slingshot effect, explain the meaning of 'conservation of angular momentum'.
3. The Earth revolves around the Sun once every 365 days. The Sun has a mass of 1.99×10^{30} kg and a radius of 696 000 km.
   (a) Determine the distance between the Earth and the Sun.
   (b) Calculate the force of attraction between the Earth and the Sun.

Answers to chapter revision questions