CHAPTER 2

Space exploration

Answers to revision questions

1. Luke's brother will see the pen moving in a projectile motion.
   ⇒ Since the train is moving horizontally with respect to where Luke's brother is, the pen is displacing vertically as well as horizontally.
   ⇒ The train is moving at a constant velocity, thus there is no acceleration in the horizontal component.
   ⇒ Vertically there is an acceleration due to gravity.
   Hence Luke's brother sees the pen moving in a projectile motion.

2. (a) The rock is initially moving with a circular motion as it is released, and it then falls with the trajectory of a projectile. The initial velocity of this projectile motion equals the instantaneous velocity when it is released.

$$\boldsymbol{F}_c = \frac{mv^2}{r}$$

Known quantities:

$\boldsymbol{F}_c = 10$ N
$m = 0.01$ kg
$r = 0.25$ m

$$10 = \frac{0.01 \times \boldsymbol{v}^2}{0.25}$$

$$v^2 = 250$$

$$v = 16 \text{ m s}^{-1}$$

(b) The initial velocity is directed horizontally, thus it is also equal to the initial horizontal velocity. To calculate the time of the flight, initially the vertical velocity is zero.

$$s = ut + \frac{1}{2}at^2$$

Known quantities:

$\boldsymbol{s} = -1$ m (taking the top of the young boy as 0)
$\boldsymbol{u} = 0$ m s^{-1}
$\boldsymbol{a} = -9.8$ m s^{-2}

$$\therefore -1 = 0 \cdot t - \frac{1}{2} \cdot 9.8 \cdot t^2$$

$$t^2 = 0.2040816$$

$$t = 0.45 \text{ s}$$

Therefore range $= v \times t$

Known quantities:

$\boldsymbol{v} = 15.81$ m s^{-1} (from part (a))
$t = 0.45$ s (from above)

$$\boldsymbol{s} = 15.81 \times 0.45$$
$$= 7.1 \text{ m}$$

(c) The range of the projectile motion undergone by the rock is 7.1 m, which is bigger than the distance of the tree from the young boy. Thus the rock will hit the tree.

(d) First, you need to determine the time when the rock hits the tree.
Using $\boldsymbol{s} = \boldsymbol{v}t$ in the horizontal component
Where:

$\boldsymbol{s} = 0.6$ m

$\boldsymbol{v} = 15.81$ m s^{-1}

$0.6 = 15.81 \times t$

$t = 0.038$ s

The vertical distance travelled $(s) = ut + \frac{1}{2}at^2$

Known quantities:

$u = 0$

$\boldsymbol{a} = -9.8$ m s^{-2}

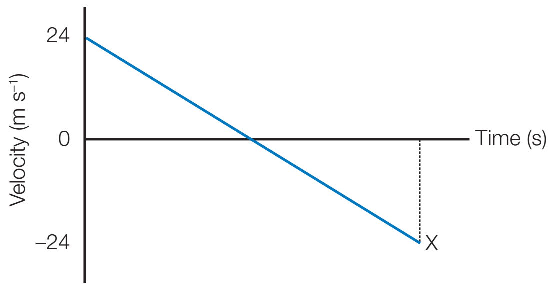
$t = 0.038$ s

$$\boldsymbol{s} = 0 - 9.8 \times \frac{1}{2} \times 0.038^2$$
$$= -7.0572 \times 10^{-3}\,\text{m}$$

$\therefore$ The height of the rock: $1 + \left(-7.0572 \times 10^{-3}\right) = 0.99$ m

$\therefore$ It will hit the tree at a height of 0.99 m.

3.



4. (a) The centripetal force is provided by the force exerted by the athlete, that is:

$$\boldsymbol{F_c} = 20 \times 10^2\,\text{N}$$

$$\boldsymbol{F_c} = \frac{mv^2}{r}$$

Known quantities:

$m = 1.0$ kg

$r = 0.5$ m

$\boldsymbol{F_c} = 20 \times 10^2$ N

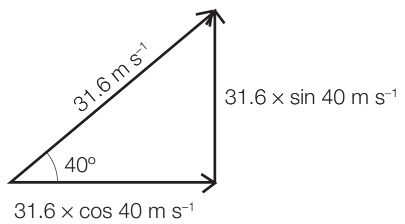
$$20 \times 10^2 = \frac{1 \times v^2}{0.5}$$

$$v^2 = 1000$$

$\boldsymbol{v} = 31.6\,\text{m s}^{-1}$ (32 m s^{-1} to two significant figures)

(b) When the hammer reaches its maximum height the vertical velocity will equal zero.

For maximum height:



$$v^2 = u^2 + 2as$$

Where:

$\boldsymbol{u} = 31.6 \times \sin 40^\circ$

$= 20.32\ \text{m s}^{-1}$

$\boldsymbol{v} = 0\ \text{m s}^{-1}$

$\boldsymbol{a} = -9.8\ \text{m s}^{-2}$

$0 = 20.32^2 + 2 \times -9.8 \times s$

$\boldsymbol{s} = 21.05\ \text{m}$

$\therefore$ Maximum height = 21.05 + 1.8 = 22.85 m = 23 m (to two significant figures)

For range:

Use $s = ut + \frac{1}{2}at^2$ to calculate the time of the flight.

Known quantities:

$\boldsymbol{s} = -1.8$ m (taking the top of the athlete as zero)

$\boldsymbol{u} = 20.32\ \text{m s}^{-1}$

$a = -9.8\ \text{m s}^{-2}$

$$-1.8 = 20.32 \cdot t + \frac{1}{2} \cdot (-9.8) \cdot t^2$$

$$4.9 \cdot t^2 - 20.32t - 1.8 = 0$$

Using the quadratic formulae:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ (where } at^2 + bt + c = 0)$$

$$t = 4.23\ \text{s}$$

$\therefore$ Range $(s) = vt$

$\boldsymbol{v} = 31.6 \times \cos 40^\circ$

$= 24.22\ \text{m s}^{-1}$

Range $\boldsymbol{s} = 24.22 \times 4.23$

$= 102.5$ m $= 1.0 \times 10^2$ m (to two significant figures)

(c) The athlete has to be bulky so he will exert a larger centripetal force, hence a higher initial velocity of the projectile motion. Taller athletes have the advantage of a higher point of release, thus increasing the time of flight. According to $\boldsymbol{s} = \boldsymbol{v}t$ this will increase the range of trajectory.

5. (a) The stone is moving in a projectile motion $\therefore$ height

$$\boldsymbol{s} = ut + \frac{1}{2}at^2$$

Known quantities:

$\boldsymbol{u} = 8.60\ \text{m s}^{-1}$ ($\sin 35^\circ \times 15$)

$t = 5.2$ s

$\boldsymbol{a} = -9.8\ \text{m s}^{-2}$

$$\boldsymbol{s} = 8.6 \times 5.2 + \frac{1}{2} \times -9.8 \times 5.2^2$$

$$= -88\ \text{m}$$

$\therefore$ The cliff has a height of 88 m.

(b) Impact velocity is the vector sum of the final vertical velocity and the final horizontal velocity.

Final vertical velocity $v = u + at$

Known quantities:

$\boldsymbol{u} = 8.60\ \mathrm{m\,s^{-1}}$

$\boldsymbol{a} = -9.8\ \mathrm{m\,s^{-2}}$

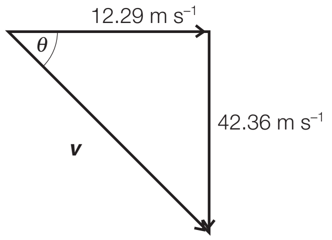
$t = 5.2\ \mathrm{s}$

$\boldsymbol{v} = 8.6 - 9.8 \times 5.2$

$= -42.36\ \mathrm{m\,s^{-1}}$

Horizontally v remains constant

$\boldsymbol{v} = 12.29\ \mathrm{m\,s^{-1}} \left(\cos 35^\circ \times 15\right)$



Impact velocity:

$v^2 = 12.29^2 + 42.36^2$

$\boldsymbol{v} = 44\ \mathrm{m\,s^{-1}}$

$\theta = \tan^{-1}\left(\dfrac{42.36}{12.29}\right) = 74^\circ$

$\therefore$ Impact velocity is 44 m s^{-1} at 74° below the horizontal.

6. See Chapter 2.

7. His contribution to relative motions.

8. Centripetal force is given by:

$$F = \frac{mv^2}{r}$$

$\boldsymbol{v} = 30\ \mathrm{km\,h^{-1}}$

$= 30 \div 3600 \times 1000$

$= 8.3\ \mathrm{m\,s^{-1}}$

The friction between the tyres and the road provides the centripetal force of circular motion. When the friction halves, the maximum centripetal force will also halve:

$$F' = \frac{1}{2}F$$

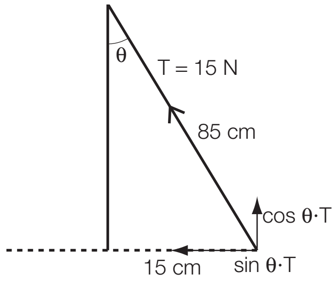
$$\frac{mv'^2}{r} = \frac{1}{2}\left(\frac{mv^2}{r}\right)$$ where v' is the new velocity

$$v'^2 = \frac{1}{2}v^2$$

$$v' = \frac{1}{\sqrt{2}}\boldsymbol{v}$$

$$v' = \frac{8.33}{\sqrt{2}} = 5.89\ \mathrm{m\,s^{-1}}$$

9. To calculate the period, we need to first find the angle between the string and the vertical axis, and then the linear velocity of the circular motion.



$$\theta = \sin^{-1}\left(\frac{0.15}{0.85}\right)$$
$$= 10.16^\circ$$

*The horizontal component of the tension of the string provides the centripetal force of the mass bob.

The centripetal force is given by:

$$\boldsymbol{F_c} = \frac{mv^2}{r}$$

Where $\boldsymbol{F_c} = 15 \times \sin 10.16^\circ = 2.65$ N

*The vertical component of the tension balances the weight of the mass bob.

$$mg = \cos 10.16^\circ \times 15$$
$$\therefore m = 1.51\,\text{kg}$$

*Hence $2.65 = \dfrac{mv^2}{r}$

$$= \frac{1.51 \times v^2}{0.15}$$
$$v^2 = 0.2638$$
$$\boldsymbol{v} = 0.513\ \text{m s}^{-1}$$

Period is given by $\dfrac{2\pi r}{v}$

i.e., $T = \dfrac{2\pi(0.15)}{0.513}$

$$T \approx 1.8\ \text{s}$$

10. If there is no gravity acting on a spacecraft, then there will be no force to hold it in a circular orbit around the Earth. The phenomenon of the astronaut floating can be explained by the fact that when the astronaut and spacecraft are going around the Earth, they are both in a state of freefall, hence relative to each other, so the falling motion of the astronaut will be seen as floating.

11. Newton mathematically derived Kepler's third law. See page 32 of the text.

12. In Kepler's third law $K = \dfrac{GM}{4\pi^2}$ the only non-constant is *M*. Objects in the same orbits share the same centre mass '*M*', and hence *K* is constant.

13. (a) Using Kepler's third law to calculate the distance between the Earth and the Moon:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

Known quantities:

$$T = 27.3 \text{ days}$$
$$= 27.3 \times 24 \times 3600 = 2358720 \text{ s}$$
$$G = 6.67 \times 10^{-11}$$
$$m = 6.0 \times 10^{24} \text{ kg}$$
$$\frac{r^3}{2358720^2} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{4 \times \pi^2}$$
$$r^3 = 5.640 \times 10^{25}$$
$$r = 3.83 \times 10^8 \text{ m}$$

$\therefore$ Distance between the Earth and the Moon is:

$$= 3.83 \times 10^8 - 1738000 - 6378000$$
$$= 375376353.5 \text{ m} \doteqdot 3.75 \times 10^8 \text{ m}$$

$\therefore$ Time of travel $(t) = \frac{s}{v} = \frac{3.75 \times 10^8}{10^6} = 375 \text{ s}$

(b) The gravitational force is providing the centripetal force for the Moon.

That is:

$$F_c = F_G$$
$$\frac{\cancel{m_1} v^2}{\cancel{r}} = \frac{G\cancel{m_1} M_E}{\cancel{r}^2}$$
$$v^2 = \frac{GM_E}{r}$$
$$\text{orbital } v = \sqrt{\frac{GM_E}{r}}$$

Where M_E = the mass of the Earth

For the Moon:

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{3.83 \times 10^8}}$$
$$= 1.02 \times 10^3 \text{ m s}^{-1}$$

(c) The universal gravitational attraction equation is:

$$F = \frac{Gm_1 m_2}{d^2}$$

Known quantities:

$$G = 6.67 \times 10^{-11}$$
$$m_1 = 6.0 \times 10^{24} \text{ kg}$$
$$m_2 = 7.35 \times 10^{22} \text{ kg}$$
$$d = 3.83 \times 10^8 \text{ m}$$
$$F = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 7.35 \times 10^{22}}{(3.83 \times 10^8)^2}$$
$$= 2.00 \times 10^{20} \text{ N}$$

14. Since the satellite is rotating around the same planet, despite a change in radius it should obey Kepler's third law, that is:

$$\frac{r_1^3}{T_1^2} = \frac{r_2^3}{T_2^2}$$

Known quantities:

$$r_1 = 2300\text{ m}$$
$$T_1 = 4.2 \times 3600 = 15120\text{ s}$$
$$r_2 = 1800\text{ m}$$
$$\frac{2300^3}{15120^2} = \frac{1800^3}{T_2^{\ 2}}$$
$$T_2 = 10468\text{ s}$$
$$= 2.9\text{ hrs}$$

15. See Chapter 2.

16. (a) The satellite rotates at 12 revolutions per 24 hours.

$$T = \frac{24}{12} = 2\text{ hrs}$$
$$= 2 \times 3600 \times 7200\text{ s}$$

Using Kepler's third law:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

Known quantities:

$$T = 7200\text{ s}$$
$$G = 6.67 \times 10^{-11}$$
$$M = 6.0 \times 10^{24}\text{ kg}$$

$$\frac{r^3}{7200^2} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{4 \times \pi^2}$$
$$r = 8.07 \times 10^6\text{ m}$$
$$8.07 \times 10^6 - 6.378 \times 10^6 = 1.69 \times 10^6\text{ m}$$
$$= 1692\text{ km}$$

$\therefore$ It is at a height of 1692 km above the ground.

(b) Similarity: They will both have the same orbital velocity since this velocity does not depend on the mass of the satellite. Difference: The heavier satellite will require more fuel since it needs more force to achieve the same velocity as the lighter satellite.

17. Geostationary satellites are placed at an altitude of approximately 36 000 km above the surface of the Earth. At this height, it is outside the Earth's atmosphere, and therefore will not experience any atmospheric drag as it moves, because it is moving through almost a vacuum.

18. The equation for the escape velocity is:

$$v = \sqrt{\frac{2GM}{r}}$$

The mass of Saturn is huge compared to the Earth, so according to the equation the escape velocity will also be much larger. It is difficult to achieve such a high velocity.

19. See Chapter 2.

20. For the definition of *g* force, see Chapter 2. During the early stage of rocket launching, the rocket has an enormous upward acceleration. This results in a very large positive *g* force experienced by astronauts. This may result in blackouts and may be fatal if it is prolonged.

21. (a) The normal force equals the weight force: $W = mg$

$F = 68 \times 9.8$
$= 666.4\ \text{N}$

(b) The normal force will now equal the sum of the original normal force and the upward acceleration force.

$\text{N} = 666.4 + 20.8 \times 68$
$= 2.08 \times 10^3\ \text{N}$

(c) The weight is the same as the normal force in part (ii), that is, 2.08×10^3 N.

(d) If the g forces are too large the astronaut can blackout.

(e) Lie the astronaut down so that the force will act over the transversal axis rather than the longitudinal one. Train the astronauts so they're used to these high g forces.

22. (a) The net force will be zero since the weight force is balanced by the normal force. The velocity is constant, thus it has no force.

(b) The net force experienced will be the force due to the acceleration.

$a = \dfrac{0-6}{3} = -2\ \text{m s}^{-2}$, $2\ \text{m s}^{-2}$ up

$F = ma = 70 \times 2 = 140\ \text{N}$ up

(c) Apparent weight $= 70 \times 9.8 + 140$
$= 826\ \text{N}$

23. See Chapter 2 for definition.
The formulae used to calculate the escape velocity is:

$$v = \sqrt{\frac{2GM}{r}}$$

Known quantities:

$G = 6.67 \times 10^{-11}$
$M = 1.90 \times 10^{27}\ \text{kg}$
$r = 71492\ \text{km}$

$$v = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 1.90 \times 10^{27}}{71492000}}$$
$$= 5.95 \times 10^4\ \text{m s}^{-1}$$

This velocity is much higher than the one on the Earth due to its huge mass. It is difficult to achieve this large velocity.

24. (a) See Chapter 2.

(b) Even if everything is done correctly during re-entry, the high velocity of the spacecraft means the friction force it experiences as it goes down through the atmosphere is still very large. Consequently, there will still be enormous amounts of heat produced and a large positive g force. The large g force can be prevented by the same means as during ascent. To protect the astronauts from the heat, special tiles are used on the outside of the spacecraft to insulate against the heat, while internal reflective plates are used to reflect the heat back to space. An air conditioner is also used to fine-tune the temperature inside the spacecraft. At the nozzle and front surface of the wings where the heat is extreme, special carbon–carbon reinforcement materials are used.

25. (a) The statement is false. Geostationary satellites not only have to be placed at a certain altitude but also need to be placed in the plane of the equator.

(b) The statement is false. While the satellite is orbiting around the Earth, there is a net force acting on it, that is, its weight force, which provides the centripetal force for the circular orbit.

26. See Chapter 2.

27. Only a brief concept is described here. A ball can be projected from a table edge with a measured height, using a ramp. The ball should be released from a fixed height on the ramp so it can be projected with a fixed horizontal velocity. Note that the height on the ramp where the ball is projected from will determine the projectile velocity. With each projection, range can be marked using carbon papers and measured with a ruler. Repeat the projection, but change the height of the table as this will give a different time of flight – hence the range. Record the range and time of flight (height of the table), and plot range versus the time of flight. A straight line plot should be obtained, the gradient of which is the horizontal projection velocity.