CHAPTER 2

Space exploration

Many factors have to be taken into account to achieve a successful rocket launch, maintain a stable orbit and return to Earth

Introduction

In Chapter 2, projectile motion and circular motion are studied. These concepts will provide the basis for understanding the launching, orbiting and safe return of satellites, space probes and spacecraft.

2.1 Projectile motion

- ***Describe the trajectory of an object undergoing projectile motion within the Earth's gravitational field in terms of horizontal and vertical components***

You will often see **projectile motion** in everyday life; for example the motion described by a thrown tennis ball, or by a football kicked over a playing field.

Definition

A **projectile motion** is a motion that is under the influence of only one force—the weight force.

Time-lapse photography shows that the **trajectory** of (the path described by) a projectile is a concave downward-facing **parabola**, as shown in Figure 2.1.

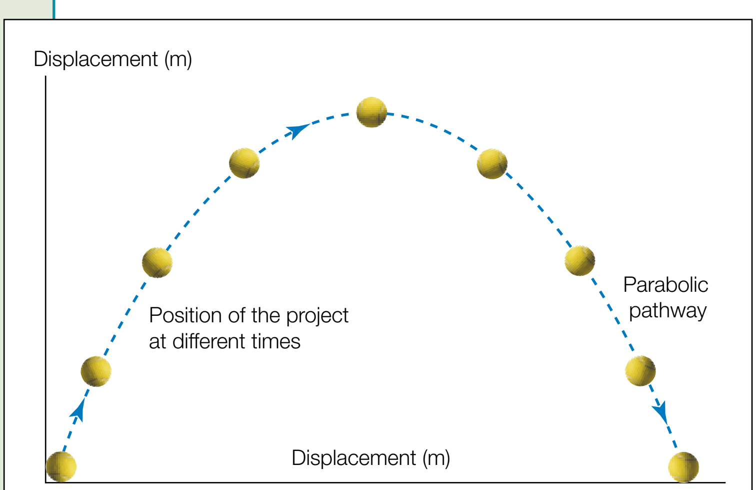


Figure 2.1 The trajectory of a projectile

Although projectile motion may seem complicated at first glance, there are only *two* simple rules that govern it:

1. The horizontal motion and vertical motion are independent. They can be analysed and calculated separately.
2. The horizontal velocity is always constant (neglecting the air friction). The vertical motion has a constant acceleration (downward at 9.8 m s^{-2} at the surface of the Earth) and gravity is the only force acting on the object.

However, before you can perform calculations for projectile motion, you need to understand the concept that a projectile can be considered to be composed of a horizontal and a vertical motion.

Resolving a vector

Consider an object that is fired at ground level at an angle of θ° to the ground with velocity $\boldsymbol{u}$.

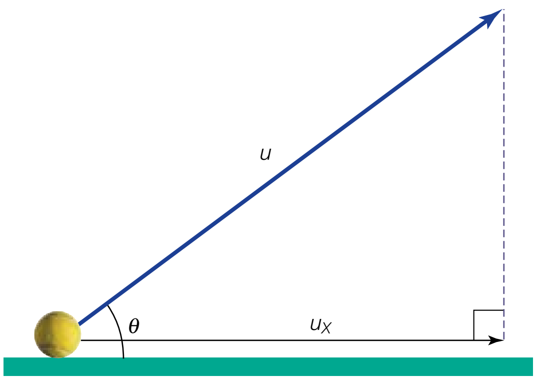


Figure 2.2 Vector resolving horizontal

This velocity vector can be resolved into its horizontal and vertical components:

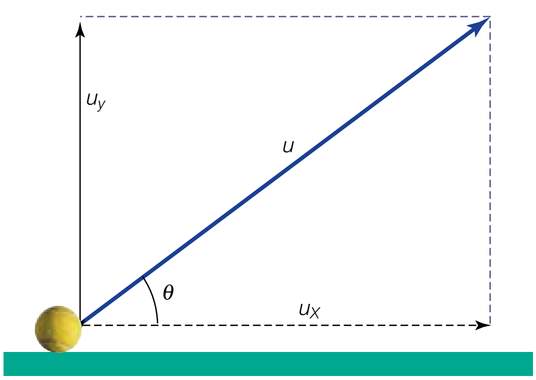


Figure 2.3 Vector resolving vertical

Using trigonometry
The horizontal velocity of the projection: $\boldsymbol{u}_x = u\cos\theta$
The vertical velocity of the projection: $\boldsymbol{u}_y = u\sin\theta$

H14.1G, H
H14.2A, B, D
H14.3B, D

SR

Worked examples
3, 4, 5

Problem solving for projectile motions

■ ***Solve problems and analyse information to calculate the actual velocity of a projectile from its horizontal and vertical components using:***

$$v_x^2 = u_x^2$$
$$v = u + at$$
$$v_y^2 = u_y^2 + 2a_y \Delta y$$
$$\Delta x = u_x t$$
$$\Delta y = u_y t + \frac{1}{2} a_y t^2$$

Five equations are used to solve projectile motion problems. These formulae along with a brief comment on each are summarised in Table 2.1.

Table 2.1

Formula	Relevant motion	Variables (units)	Comment
1. $v_x^2 = u_x^2$ 2. $\Delta x = u_x t$	Horizontal component of the projectile motion	Δx: displacement (m) u_x: initial velocity (m s^{-1}); t: time (s) v_x: final velocity (m s^{-1})	Since the horizontal component of velocity is constant, the final horizontal speed v_x is equal to u_x, the initial horizontal component of velocity.
3. $v = u + at$	Vertical component of the projectile motion	v: final velocity (m s^{-1}) u: initial velocity (m s^{-1}) a: acceleration (m s^{-2}) t: time (s) Δy: displacement (m)	For the vertical component of the projectile motion, the acceleration is constant.
4. $v_y^2 = u_y^2 + 2a_y \Delta y$			It is important to assign the correct sign to the acceleration. For example, if the upwards direction is given a positive sign for then a_y is negative.
5. $\Delta y = u_y t + \frac{1}{2} a_y t^2$			This equation can be derived by making t the subject in equation 3, and then substituting for t in equation 4.

It is standard practice to select positive horizontal and vertical directions as being to the right and upwards respectively. However, in examples where all vertical quantities are downwards, choosing downwards as the positive direction avoids negative signs in the calculation. This should be clearly shown in the working.

Maximum height, time of flight and range for a standard projectile

After finding the horizontal and vertical components of the initial velocity, the known quantities must be substituted into the correctly selected equation, in order to find information such as maximum height, time of flight and range of the projectile.

Whenever a projectile motion question is presented, it is important to consider *what is* known and *what is* required. Given a projectile that is fired at ground level at an angle of θ° to the horizontal with initial speed $\boldsymbol{u}$ as shown in Figure 2.2, the following analyses can be undertaken.

Maximum height

Example

A golf ball is struck at 50 m s^{-1} and leaves the ground at an angle of 30°. What is the maximum height it will reach? It is clear that maximum height relates to the vertical motion of the projectile.

Solution

$$\begin{aligned} \boldsymbol{u_y} &= u\sin\theta \\ &= 50 \times 0.5 \\ &= 25 \text{ m s}^{-1} \text{ (up)} \\ \boldsymbol{a_y} &= -9.8 \text{ m s}^{-2} \text{ (down)} \\ \boldsymbol{v_y} &= 0 \text{ (at the maximum height, the projectile is neither moving up nor down for an instant)} \\ \boldsymbol{\Delta_y} &= \text{? (maximum height is the vertical displacement for the projectile)} \end{aligned}$$

At this point, the suitable equation must be selected:

$$\begin{aligned} \boldsymbol{v_y}^2 &= \boldsymbol{u_y}^2 + 2\boldsymbol{a_y}\boldsymbol{\Delta y} \\ 0 &= 25^2 + 2 \times -9.8\boldsymbol{\Delta y} \\ 19.6\boldsymbol{\Delta y} &= 625 \\ \boldsymbol{\Delta y} &= 32 \text{ m} \end{aligned}$$

Time of flight

Over level ground, the **time of flight** will be twice the time required to reach the maximum height. This is because if air resistance is neglected, the time for a projectile to reach its maximum height and for its return to earth will be the same. The motion is **symmetrical**: the upwards motion mirrors the downwards motion, and a vertical line that passes through the position where the projectile reaches the maximum height is the axis of the parabolic motion.

Using the above example again:

Time of flight = 2 × time of reaching the maximum height.

To find the time taken to reach the maximum height:

$$\begin{aligned} \boldsymbol{u_y} &= 25 \text{ m s}^{-1} \\ \boldsymbol{a_y} &= -9.8 \text{ m s}^{-2} \\ \boldsymbol{v_y} &= 0 \\ t &= ? \end{aligned}$$

The most appropriate equation is the only equation which includes the four quantities:

$$\begin{aligned} \boldsymbol{v_y} &= \boldsymbol{u_y} + \boldsymbol{a_y}t \\ 0 &= 25 - 9.8t \\ 9.8t &= 25 \\ t &= 2.55 \text{ s} \end{aligned}$$

$$\begin{aligned} \text{Therefore the time of flight} &= 2 \times 2.55 \text{ s} \\ &= 5.1 \text{ s} \end{aligned}$$

Range (the horizontal displacement)

Range is related to the horizontal motion of the projectile.

range = initial horizontal velocity × time of the flight

Using the equation:

$$\begin{aligned}\Delta \boldsymbol{x} &= \boldsymbol{u}_x t \\ &= u\cos\theta \times t \\ &= 50\cos 30° \times 5.1 \\ &= 43.3 \times 5.1 \\ &= 221 \text{ m}\end{aligned}$$

Further examples

Example 1

A cricketer strikes a cricket ball and the ball flies off with a velocity of 14 m s^{-1} at 63° to the ground, to the north of the player. Where should the fielder be in order to catch the ball on its return to the ground?

Solution

This question asks for the range of the projectile.

R = horizontal velocity × time of the flight (t_f)

Initial horizontal velocity = 14 × cos63° m s^{-1} = 6.4 m s^{-1}

Initial vertical velocity = 14 × sin63° m s^{-1} = 12.5 m s^{-1}

t_f = 2 × time to reach maximum height

Time to reach maximum height

$$t = \frac{\boldsymbol{v}_y - \boldsymbol{u}_y}{\boldsymbol{a}_y} \quad (\text{since } \boldsymbol{v}_y = \boldsymbol{u}_y + \boldsymbol{a}_y t)$$

$$t = \frac{0 - 12.5}{-9.8}$$

$$t = 1.28 \text{ s}$$

$$\therefore t_f = 1.27 \times 2 = 2.55 \text{ s}$$

$$\begin{aligned}\therefore R &= 6.4 \times 2.55 \\ &= 16.3 \text{ m}\end{aligned}$$

∴ the fielder has to be 16.3 m north of the batter in order to catch the ball.

Example 2

In an indoor soccer match, whenever the ball touches the ceiling of the court following a goal kick from the goalkeeper it is considered to be a foul. If the ceiling of the court is 10 m high, and the keeper always kicks the ball at a speed of 20 m s^{-1}:

(a) What should be the maximum angle the keeper kicks the ball so that a foul is not committed?

(b) Find the velocity of the ball 2.0 s after being kicked by the goalkeeper, assuming it is kicked at 20 m s^{-1} at the angle found in part (a) above.

Solution

(a) To avoid committing a foul, the maximum height achieved by the ball should be less than the height of the ceiling.

$\boldsymbol{v_y}^2 = \boldsymbol{u_y}^2 + 2\boldsymbol{a_y}\boldsymbol{\Delta y}$

$\boldsymbol{v_y} = 0$

$\boldsymbol{u_y}$ = initial vertical velocity

$\boldsymbol{a_y} = -9.8$

$\boldsymbol{s}$ = (more strictly, $s < 10$) 10

$\boldsymbol{u_y}^2 = -2 \times (-9.8) \times 10$

$\boldsymbol{u_y} = y\sqrt{2 \times 9.8 \times 10}$

$\boldsymbol{u_y} = 14 \text{ m s}^{-1}$

Since: $\boldsymbol{u_y} = \boldsymbol{u}\sin\theta$

and $\boldsymbol{v} = 20$

$\therefore \sin\theta\frac{\boldsymbol{u_y}}{\boldsymbol{u}}$

$\therefore \theta < \sin^{-1}\frac{\boldsymbol{u_y}}{\boldsymbol{u}}$

$\theta < \sin^{-1}\frac{14}{20}$

$\theta < 44°$

$\therefore$ The ball must be kicked at an angle of *less* than 44°

(b)

After two seconds

Horizontal velocity is constant: $20 \times \cos 44 \approx 14.38 \text{ m s}^{-1}$

Vertical velocity:

$\boldsymbol{v_y} = \boldsymbol{u_y} + \boldsymbol{a_y}t$

$\boldsymbol{u_y} = 14 \text{ m s}^{-1}$

$\boldsymbol{a_y} = -9.8 \text{ m s}^{-2}$

$t = 2.0 \text{ s}$

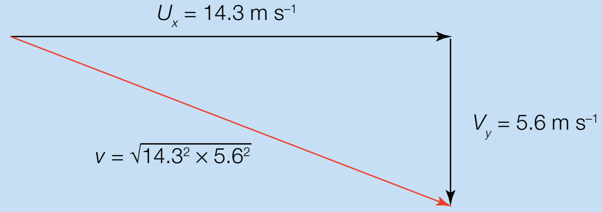
$\boldsymbol{v_y} = 14 + (-9.8) \times 2$

$= -5.6 \text{ m s}^{-1}$

Or

5.6 m s^{-1} down

The overall velocity will be the sum of these two, therefore:



$$V = \sqrt{14.28^2 + 5.6^2}$$

$$V = 15.15 \text{ m s}^{-1}, \textit{ at}$$

$$\theta = \tan^{-1}\left(\frac{5.6}{14.38}\right)$$

$$= 21°$$

∴ After 25, the velocity is 15.15 m s^{-1} at a depression angle of 21°

NOTE: The technique of vector addition has been used here.

Example 3

An F-18 jet plane is given an order to bomb a warehouse. The plane is flying at a speed of 350 m s^{-1}, 3.50 km above the ground. Bombs will be released from the plane. If the air resistance is negligible, how far before the warehouse must the pilot release the bombs in order to hit the target?

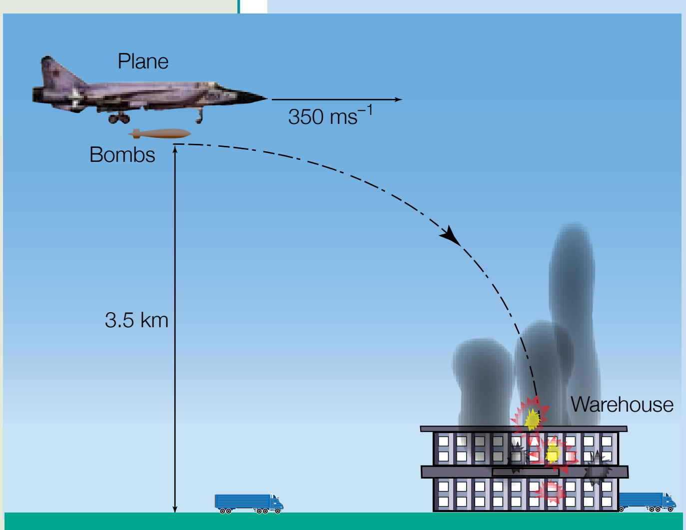


Figure 2.4 Another example of projectile motion

Solution

Obviously, the bombs have to be dropped prior to reaching the warehouse, in order to hit it. The time taken for the bombs to land:

$$\Delta y = u_y t + \frac{1}{2} a_y t^2$$

$$\Delta y = 3.50 \text{ km} = 3500 \text{ m down}$$

$$u_y = 0$$

$$a_y = 9.8 \text{ m s}^{-2} \text{ down}$$

NOTE: *a* is positive in this case because downward direction is positive.

$$\therefore 3500 = \frac{1}{2} \times 9.8t^2$$

$$t = 26.7 \text{ s}$$

Horizontal distance travelled by the bombs within this time:

$$\Delta x = u_x t$$

$$\Delta x = 350 \text{ m s}^{-1} \times 26.7 \text{ s}$$

$$= 9354 \text{ m}$$

∴ The bombs have to be dropped 9354 m ahead of the warehouse in order to hit it.

Galileo's analysis of projectile motion

2.2

PFA
H1

'Evaluates how major advances in scientific understanding and technology have changed the direction or nature of scientific thinking'

■ ***Describe Galileo's analysis of projectile motion***

Why was this discovery a major advance in scientific understanding?

Galileo (1564–1642) was a truly remarkable scientist. Much of the accepted 'knowledge' of his time had not changed significantly since the days of Aristotle, some 2000 years previously. As well as studying astronomy, for which he is most remembered, Galileo furthered the study of the motion of falling objects, including projectiles, by taking meticulous measurements of the time taken for objects to roll down inclined planes. Galileo had found that the speed of objects falling directly was simply too fast for accurate measurements to be made. By using an inclined plane, this motion could be slowed down sufficiently for it to be timed using an ingenious device which relied on weighing the water that could flow into a container. (Accurate clocks or watches were still to be invented—in the early 1600s!)

Galileo repeated his measurements hundreds of times to ensure that any errors were minimised. He came to the conclusion that balls rolling down inclined planes were accelerating. He was even able to put this mathematically in the equation $s = \frac{1}{2}\boldsymbol{a}t^2$, that is, the distance travelled by the ball down the slope was proportional to the square of the time spent rolling.

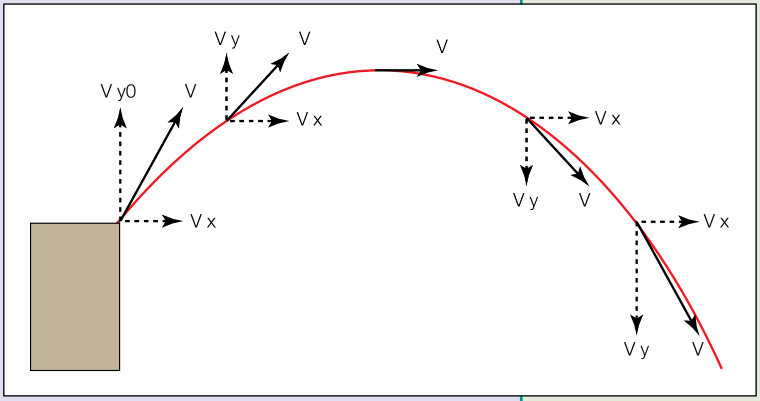
Galileo

How did it change the direction or nature of scientific thinking?

The Aristotelian view of projectile motion was that an object, once set in motion, retained its motion due to its *impetus*, a force from within the moving body. It was believed that an arrow, once fired, travelled in straight lines, and did so due to the continuing force on it from within. Galileo came to the conclusion that objects would retain their motion *unless* a force acted on them, due to their *inertia*.

By analysing the motion of rolling balls down inclined planes after the balls were set in motion with a horizontal component, Galileo concluded that the motion of any projectile (arrows, balls, etc.) was the result of two separate types of motion: a uniform horizontal component that had no acceleration, and at the same time a vertical component that was being accelerated. The resulting path traced by the projectile is a curve, not a straight line. The type of curve had been named by earlier Greek mathematicians—the parabola (Galileo described the path as a 'semi-parabola').

A sketch representing the Aristotelian view of how a cannon ball would be expected to move once set in motion due to its *impetus*



Evaluation of Galileo's analysis of projectile motion

Much of Galileo's work on motion and forces was formalised later by Newton (born in the year Galileo died), who proclaimed that he had seen further only because he had 'stood upon the shoulders of giants' in a reference to Galileo. Newton's work was to form the basis upon which classical physics developed for the next 300 years.

WWW→

USEFUL WEBSITES

Some details of Galileo's study of projectile motion on inclined planes:
http://www.galileo.rice.edu/lib/student_work/experiment95/paraintr.html

Some history of projectile motion:
http://library.thinkquest.org/2779/History.html

To duplicate Galileo's experiment on an inclined plane:
http://www.anselm.edu/homepage/dbanach/h-galileo-projectdata.htm

TR

Mapping PFAs
PFA scaffolds

■ *Describe Galileo's analysis of projectile motion*

Careful study of projectile motion did not begin recently; in fact it began as early as the 17th century. Galileo was the first person to deduce the trajectory of projectiles to be parabolic, through his experiments, which involved projecting an inked ball from tabletops at various heights using an inclined plane placed at the edge of the table. He realised that the horizontal motion of the projectile was totally independent of its vertical motion. More importantly, he realised the importance of mathematics in analysing the motion of projectiles. In his experiments, Galileo did his pioneer work in analysing the projectile motion mathematically, similar to our approach in the previous section.

Galileo's work also provided him with strong evidence to support his heliocentric (Sun-centred) model of the Universe. He was able to offer a satisfactory explanation for why an object dropped from a height, for example from a building on the moving Earth, did not get left behind. Galileo pointed out the reason the ball did not fall away from the building was because both the building and the object shared the same horizontal velocity. The object when falling would have an extra vertical velocity that was independent of its horizontal velocity. Consequently, the object would fall down and at the same time travel with the tower, so relative to the tower, or to anyone on the Earth who was moving with the Earth, the ball fell straight down to the base of the tower. In fact, this demonstrated the fundamental concept of relativity: *motion is seen differently from different frames of reference* (the background to which the motions are compared).

2.3 Circular motion

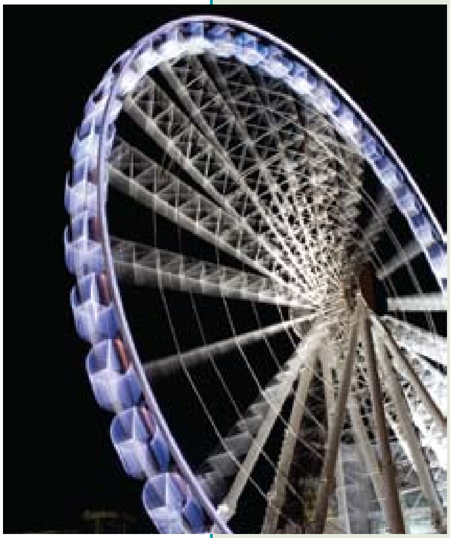
■ *Analyse the forces involved in uniform circular motion for a range of objects, including satellites orbiting the Earth*

Circular motion can be found in systems as small as atoms, where electrons are orbiting nuclei. It can also be found in systems as big as the solar system, where planets are orbiting the Sun.

NOTE: In fact the orbits of the planets are not perfect circles, but ellipses.

For the purposes of this course, only **uniform circular motion** is covered: that is, circular motion with **constant linear orbital speed**, as explained in Figure 2.5. Although the orbital speed of an object undergoing uniform circular motion is constant, its direction is always changing. Consequently its orbital velocity is always changing since velocity includes both the speed and direction. Hence:

Uniform circular motion has constant orbital speed but a changing orbital velocity.



A ferris wheel in circular motion

Centripetal acceleration

Any change in velocity involves acceleration. Acceleration in circular motion is referred to as **centripetal acceleration** (a_c). It is found that this acceleration is related to the orbital speed of the (uniform) circular motion and its direction is *always towards the centre of the circle*, as shown in Figure 2.5. The magnitude of the acceleration can be expressed as:

$$\boldsymbol{a}_c = \frac{\boldsymbol{v}^2}{r}$$

Where:

$\boldsymbol{a}_c$ = the centripetal acceleration, measured in m s^{-2}

$\boldsymbol{v}$ = the linear orbital velocity or speed of the object in motion, measured in m s^{-1}

r = the radius of the circle, measured in m

Centripetal force

Any acceleration is a result of a net force; the force and the acceleration are always in the same direction. Thus the force that provides centripetal acceleration and sustains circular motion is referred to as the **centripetal force ($\boldsymbol{F}_c$)**, and its direction is also always *towards the centre of the circle* as shown in Figure 2.5. For its **magnitude**:

Since $\boldsymbol{F} = m\boldsymbol{a}$

$$\boldsymbol{F}_c = m\boldsymbol{a}_c$$
$$\boldsymbol{F}_c = \frac{m\boldsymbol{v}^2}{r}$$

$$\boldsymbol{F}_c = \frac{m\boldsymbol{v}^2}{r}$$

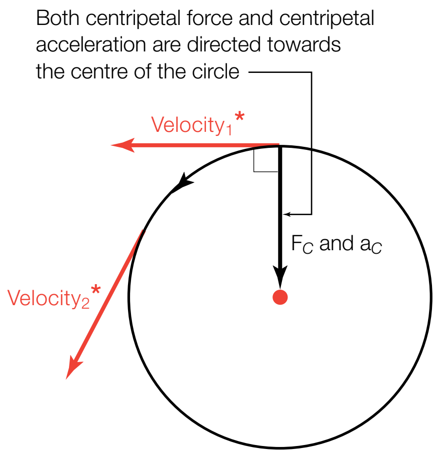
Where:

$\boldsymbol{F}_c$ = the centripetal force, measured in N

m = the mass of the object in motion, measured in kg

$\boldsymbol{v}$ = the linear orbital velocity or speed of the object in motion, measured in m s^{-1}

r = the radius of the circle, measured in m



*These velocities are referred to as **linear orbital velocities**, and are tangential to the circle at any instant. They have different directions but the same magnitude, thus the **linear orbital speed** is **constant**. We can define the **linear orbital speed** as how fast an object moves along the arc of a given size circle.

Figure 2.5 An object undergoing uniform circular motion

Centripetal force is essential to maintain circular motion. If it is insufficient (or has been removed) then circular motion cannot be sustained. For instance, when a car is making a turn, it is describing an arc of a circle. What provides the centripetal force is the friction between the tyres and the road. If the road is covered by ice, the friction between the tyres and the road is greatly reduced. Consequently, the centripetal force may not be sufficient to sustain the circular motion, and the car then skids off the road by maintaining its linear velocity.

H14.1A, G
H14.2A
H14.3D

SR

Worked example 6

Problem solving in circular motion

- ***Solve problems and analyse information to calculate the centripetal force acting on a satellite undergoing uniform circular motion about the Earth using:***

$$F = \frac{mv^2}{r}$$

Example 1

What is the centripetal force required to keep a particle with a mass of 2.0×10^{-8} kg moving at 2.0×10^5 m s^{-1}, in a circular path with a radius of 50 cm?

Solution

$$F_c = \frac{mv^2}{r}$$

$m = 2.0 \times 10^{-8}$ kg

$v = 2.0 \times 10^5$ m s^{-1}

$r = 50$ cm $= 0.5$ m

$$F_c = \frac{2.0 \times 10^{-8} \times (2.0 \times 10^5)^2}{0.50}$$

$= 1.6 \times 10^3$ N towards the centre of the circle

Example 2

A car (mass m) which is turning a corner describes an arc of a circle with a radius of r metres. The friction of the road is just enough for the car to turn at v km h^{-1}. Suppose suddenly the car encounters an oil spill on the road that reduces the surface friction to $\frac{1}{2}$. What is the maximum velocity the car can turn without skidding?

Solution

$$\boldsymbol{F_c} = \frac{m\boldsymbol{v}^2}{r}$$

The orbital speed needs to be in m s^{-1}, and v km h^{-1} = $\frac{\boldsymbol{v}}{3.6}$ m s^{-1}

$$\therefore \boldsymbol{F_c} = \frac{m\left(\frac{\boldsymbol{v}}{3.6}\right)^2}{r}$$

$\boldsymbol{F_c}$ is reduced by $\frac{1}{2}$, and let the new velocity by $\boldsymbol{v}_2$

$$\therefore \frac{\cancel{m}\left(\frac{\boldsymbol{v}_2}{\cancel{3.6}}\right)^2}{\cancel{r}} = \frac{1}{2}F_c = \frac{1}{2}\,\frac{\cancel{m}\left(\frac{\boldsymbol{v}}{\cancel{3.6}}\right)^2}{\cancel{r}}$$

$$\frac{\cancel{m}\left(\frac{\boldsymbol{v}_2}{\cancel{3.6}}\right)^2}{\cancel{r}} = \frac{1}{2}\cdot\frac{\cancel{m}\left(\frac{\boldsymbol{v}}{\cancel{3.6}}\right)^2}{\cancel{r}}$$

$$\boldsymbol{v}_2^{\,2} = \frac{1}{2}\cdot\boldsymbol{v}^2$$

Square root both sides:

$$\boldsymbol{v}_2 = \frac{1}{\sqrt{2}}\boldsymbol{v}$$

$\therefore$ The new maximum velocity is $\frac{1}{\sqrt{2}}\boldsymbol{v}$ km h^{-1}

SR

Simulation: centripetal force

Rotation of the Earth and the variation of *g*

An object that is undergoing circular motion also has an **angular speed** (ω). The angular speed of this object refers to how fast the angle of a line that joins the object to the centre of the circle is changing. The angular speed is related to the linear orbital speed $\boldsymbol{v}$ by $\boldsymbol{v} = \omega r$.

We can now explain how the rotational motion of the Earth affects the size of the Earth's g as mentioned in Chapter 1.

The Earth rotates on its own axis and all latitudes on the Earth will have the same angular speed, that is, the *rate* of rotation is the same (one day is 24 hours long regardless of latitude). However, due to the nearly spherical (but slightly elliptical) shape of the Earth, the equator will be further away from the rotational axis of the Earth than anywhere else on the Earth, the poles being on the axis. Consequently, places on the equator will have a **larger linear speed** than other places on Earth.

Having a larger linear speed also means that any object at this position will experience a greater tendency to be 'flung' off the Earth's surface. This effect counteracts the effect of gravity, and reduces the measured size of g. This apparent reduction of the size of g is greatest at the equator and becomes less, closer to the poles.

ANALOGY: If you are in a car that is making a right turn, the faster it turns the corner, the more you feel you are being thrown to the left.

2.4 Circular motion and satellites

■ *Analyse the forces involved in uniform circular motion for a range of objects, including satellites orbiting the Earth*

An orbiting satellite

Satellites orbiting the Earth, including the Moon, are examples of objects undergoing circular motion. What is special in this circular motion is that the *centripetal force is provided by the gravitational attraction force*. The centripetal acceleration is correlated with the gravitational acceleration.

Satellites orbiting the Earth are in a state of free fall. As a satellite orbits the Earth, it is pulled downwards by the Earth's gravitational field. If the satellite was stationary, it would fall vertically down just as a ripe apple falls straight to the ground. What keeps the satellite from falling is its linear orbital velocity. That is, at the same time the satellite is falling down, it is also moving away from the Earth. This results in its path being circular, as shown in Figure 2.6. Since the horizontal and vertical motions are independent, while the satellite is describing a circle, it can be considered to be in a state of free fall, just like an object undergoing projectile motion.

Astronauts 'floating' inside a spacecraft as a result of their free fall

Indeed, if the orbital speed is not fast enough, the satellite will describe a parabolic path—a projectile motion—and fall back to the Earth. But as the orbital speed increases, the length of the parabola increases. Eventually, when the orbital speed is sufficiently high so that the rate of falling can be matched by the rate of 'moving away', the satellite will describe a circular path (see Fig. 2.7).

The circular motion described by satellites can be said to have two components: one that is constant tangential speed, the other being free fall with constant acceleration of gravity.

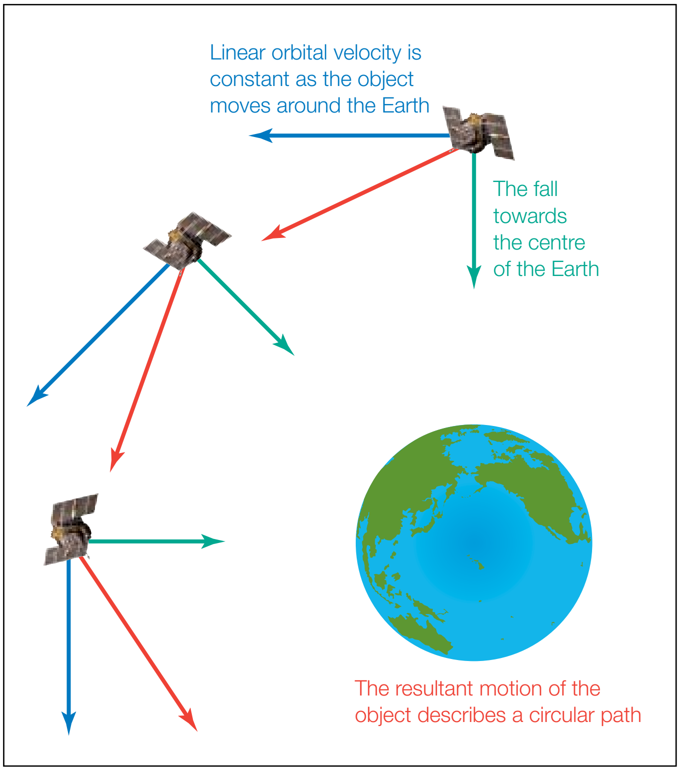


Figure 2.6 A satellite describing a circle

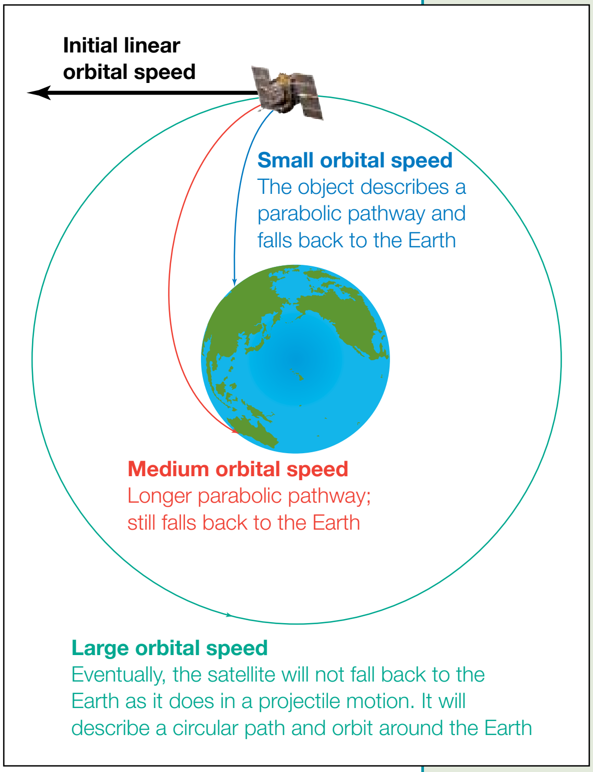


Figure 2.7 An object with different orbital speeds

Example 1

Suppose a manned satellite is orbiting the Earth. An astronaut inside the satellite is fixing a communication device and accidentally lets a screwdriver go.

(a) Describe the subsequent motion of the screwdriver relative to the astronaut.
(b) Explain why this would be so.

Solution

(a) The screwdriver will continue to move with the astronaut, hovering beside the astronaut.
(b) When the satellite is orbiting the Earth, it is in a state of free fall. The astronaut inside the satellite will share the motion of the satellite. The released screwdriver will also enter a state of free fall. However, now the satellite, the astronaut and the screwdriver are all falling at the same rate, since the gravitational acceleration is independent of the mass of the falling object. (A feather and a rock dropped from a tower will land at the same time, if there is no air resistance.) Consequently, relative to the astronaut or the satellite, the screwdriver appears to be stationary and 'float' at the same position.

2.5 A quantitative description of Kepler's third law

■ ***Define the term orbital velocity and the quantitative and qualitative relationship between orbital velocity, the gravitational constant, mass of the central body, mass of the satellite and the radius of the orbit using Kepler's law of periods***

As previously explained, satellites orbiting the Earth and planets orbiting the Sun are all examples of circular motion where the centripetal force is provided by the gravitational attraction force. Hence the centripetal force $\boldsymbol{F_c}$ can be equated with the gravitational attraction force F_g, then:

$$\boldsymbol{F_c} = F_g$$

$$\boldsymbol{F_c} = \frac{m\boldsymbol{v}^2}{r}$$

where m and $\boldsymbol{v}$ are the mass and orbital velocity of the object that is undergoing the circular motion

$$F_g = \frac{GmM}{r^2},\ M \text{ is the central mass, } r \text{ replaces } d$$

$$\frac{\cancel{m}\boldsymbol{v}^2}{\cancel{r}} = \frac{G\cancel{m}M}{r^{\cancel{2}}}$$

$$\boldsymbol{v}^2 = \frac{GM}{r} \Rightarrow \boldsymbol{v} = \sqrt{\frac{GM}{r}}$$ An equation for calculating orbital velocities

Johannes Kepler

Let T = period of the orbit, that is, the time to complete one revolution.

$$\boldsymbol{v} = \frac{\textit{Total circumference}}{\textit{period}} = \frac{2\pi r}{T}$$

Substitute $\boldsymbol{v} = \frac{2\pi r}{T}$ into $\boldsymbol{v}^2 = \frac{GM}{r}$

$$\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}\left(\frac{GM}{4\pi^2} = k\right)$$ **Kepler's third law**

NOTE: k is a constant only for one particular orbital system. It is dependent on the size of the central mass M.

$$v = \sqrt{\frac{GM}{r}}$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

Where:

v = the linear orbital speed, measured in m s^{-1}

r = the radius of the orbit, measured in m

NOTE: r has to be measured from the centre of the mass.

T = the period of the orbit, measured in s

G = the gravitational constant, $= 6.67 \times 10^{-11}$ N m^2 kg^{-2}

M = the central mass, measured in kg

■ *Problem solving using Kepler's law of periods* $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$

■ *Solve problems and analyse information using:* $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$

H14.1A, B, C, D, F, G, H
H14.2A, D
H14.3A, B, D

SR

Worked examples 7, 8

Example 1

If a low Earth orbit satellite is to revolve about the Earth 16 times a day, what must be the altitude of its orbit? Given the mass the Earth is 6.0×10^{24} kg, and its radius is 6378 km:

Solution

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$$r^3 = \left(\frac{GM}{4\pi^2}\right) \cdot T^2$$

$$T = \frac{\textit{total time}}{\textit{total number of revolutions}}$$

$$T = \frac{24 \times 60 \times 60}{16}$$

$$= 5400 \text{ s}$$

$$M = 6.0 \times 10^{24}$$

$$G = 6.67 \times 10^{-11}$$

$$\therefore\ r = \sqrt[3]{\frac{6.0 \times 10^{24} \times 6.67 \times 10^{-11}}{4\pi^2} \times (5400)^2}$$

$$\approx 6.66 \times 10^6 \text{ m}$$

$\therefore$ Its height above the Earth

$$= 6.66 \times 10^6 \text{ m} - 6.378 \times 10^6 \text{ m}$$

$$= 2.83 \times 10^5 \text{ m}$$

Example 2

Satellites X and Y are orbiting planet 'Physics', their period and orbital radii from the centre of the planet are shown in Table 2.2.

Table 2.2

Satellite	Period (units)	Radius (units)
X	2	3
Y	4	?

Find the value for '?'

Solution

Since both satellites are orbiting the same centre mass, $\frac{GM}{4\pi^2}$ can be considered as a constant and is shared between the two satellites.

$$\therefore \frac{r_x^3}{T_x^2} = \frac{GM}{4\pi^2} = K = \frac{r_y^3}{T_y^2}$$

$$\frac{r_x^3}{T_x^2} = \frac{r_y^3}{T_y^2}$$

$$\frac{3^3}{2^2} = \frac{r_y^3}{4^2}$$

$$r_y = \sqrt[3]{108} \approx 4.76 \text{ units}$$

$$\therefore ? = 4.76$$

Animation: Kepler's laws of planetary motion

Example 3

Kepler's third law (law of periods) can also be used to determine the mass of a planet by measuring the orbital radius and period of a moon of the planet.

Io is a moon of Jupiter. It has an orbital period of 1.77 Earth days, and an orbital radius of 4.22×10^8 m. Based on this astronomical data, calculate the mass of Jupiter.

Solution

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$$M = \frac{r^3 \cdot 4\pi^2}{T^2 \cdot G}$$

$$r = 4.22 \times 10^8 \text{ m}$$

$$T = 1.77 \times 24 \times 60 \times 60$$

$$= 152928 \text{ s}$$

$$G = 6.67 \times 10^{-11}$$

$$\therefore M = \frac{(4.22 \times 10^8) \times 4\pi^2}{(152928)^2 \times 6.67 \times 10^{-11}}$$

$$\approx 1.90 \times 10^{27} \text{ kg}$$

Geostationary satellites and low Earth orbit satellites

2.6

■ *Compare qualitatively low Earth and geo-stationary orbits*

Satellites orbits can be classified into different types. Two types of orbit that have specific characteristics and properties are **geostationary** and **low Earth orbits**. Satellites placed in these orbits are designed to perform specific functions made possible by the nature of their orbits. There are thousands of satellites in Earth orbit—the majority of them no longer functioning. The properties of these two very different types of orbits have their own advantages and disadvantages.

Geostationary satellites appear to be stationary in the sky when viewed from the surface of the Earth, hence the name: geo (earth) stationary. However, like other satellites, they are also revolving around the Earth. The reason they appear to stay at one position above the Earth is because:

1. They are situated above the equator.
2. They are orbiting the Earth at the same rate as the Earth's rotation. Thus, the satellites and the Earth have the same **period**, that is, they both complete one revolution or one rotation every 24 hours.

NOTE: The time for the Earth to complete one rotation about its own axis is the length of one day: 24 hours.

Definition

Geostationary satellites are satellites that are situated above the Earth's equator and orbit the Earth with a period of 24 hours, remaining directly above a fixed point on the equator.

Example

What altitude must a geostationary satellite have so that it remains directly over a point on the equator?

Given the mass the Earth is 6.0×10^{24} kg, and its radius is 6378 km:

Solution

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$$r^3 = \left(\frac{GM}{4\pi^2}\right) \cdot T^2$$

$G = 6.67 \times 10^{-11}$

$M = 6.0 \times 10^{24}$ kg

$T = 24 \times 60 \times 60 = 86400$ s

$$r = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{4\pi^2} \times (86400)^2}$$

$r = 42297524$ m

∴ The height above the ground:

$= 4.22 \times 10^7 - 6.378 \times 10^6$ m

$= 3.58 \times 10^7$ m

Or 35800 km

Low Earth orbit satellites are satellites with orbital radii altitude between 200–2000 km (within the van Allen radiation belt, their orbital periods are less than those that of geostationary satellites). They may orbit the Earth many times per day. They do not need to orbit above the equator. Low Earth orbits are often polar so that the satellite obtains a view of the entire surface of the Earth after several orbits.

Definition

Low Earth orbit satellites are satellites with smaller orbital radii than those of geostationary satellites; their orbital periods are greater than those of geostationary satellites. They orbit the Earth many times per day.

Example

If a satellite's orbit is 430 km above the Earth's surface, how many revolutions around the Earth will it make in one day? Given the mass of the Earth is 6.0×10^{24} kg, and its radius is 6378 km:

Solution

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$$T^2 = \frac{4\pi^2 \cdot r^3}{GM}$$

$$G = 6.67 \times 10^{-11}$$

$$M = 6.0 \times 10^{24}$$

$$r = 430000 + 6378000$$

$$= 6808000 \text{ m}$$

$$T = \sqrt{\frac{4\pi^2 \cdot (6808000)^3}{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}}$$

$$= 5579s\ (\div 60 \div 60)$$

$$\approx 1.55 \text{ hr}$$

$\therefore$ The number of revolutions per day:

$$= \frac{24}{1.55}$$

$$\approx 15.5$$

That is, 15 and a half times per day.

Qualitative comparison between geostationary satellites and low Earth orbit satellites

Quantitative comparisons of geostationary and low Earth orbits involve the application of $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$ to solve specific problems. Qualitative comparisons, such as key differences, advantages, disadvantages and the main uses of each type of satellites are summarised in Table 2.3.

Table 2.3

	Geostationary satellites	**Low Earth orbit satellites**
Key differences	Stay at one position directly above a fixed point on the equator. Orbit with the Earth's rotation, so their periods are the same as that of the Earth. Situated at a very high altitude, approximately 35,900 km above the Earth's surface. Situated above the equator. Have a limited view of the Earth's surface (approximately one-third).	Move above the Earth so do not have a fixed position. Periods are much smaller than that of the Earth. May orbit the Earth many times a day. Much lower orbital altitude. Can be made to pass above any point on Earth. Able to view the Earth's entire surface over several orbits.
Advantages	Easy to track since each satellite stays at one position at all times. Do not experience **orbital decay**.	Able to provide scans of different areas of the Earth many times a day. Geographical mappings are made possible. Low altitudes enable a closer view of the surface of the Earth. Low altitudes allow rapid information transmission with little delay. Low altitudes mean the launchings of these satellites are easier and cheaper, as less fuel for the same satellite mass is required.
Disadvantages	Delay in information transmission must be considered. Each satellite has a limited view of the Earth as it only stays at one point above the Earth. (Each satellite can 'see' about one-third of the Earth's surface.) Therefore many geostationary satellites are required to provide coverage of the entire surface of the Earth. Even then, polar regions may still not be properly covered. Their high altitude makes launching processes more difficult and expensive as more fuel is needed. They suffer more damage from incoming energetic cosmic rays due to their high altitude.	Much effort is required to track these satellites, as they move rapidly above the Earth. Atmospheric drag is quite significant and orbital decay is inevitable. The orbital paths of the satellites have to be controlled carefully to avoid interference between one satellite and another. They are more severely affected by the fluctuations in the Earth's van Allen radiation belts.
Main uses	Information relay: information is sent up to one satellite and is bounced off to another place on the Earth. Communication satellites, e.g. Foxtel. Weather monitoring.	Geotopographic studies: including patterns of the growth of crops and spreading of deserts. Remote sensing. Geoscanning and geomapping. Studying weather patterns.

A satellite picture of Sydney CBD taken by a low Earth orbiting satellite (LEOS)

Orbital decay

■ *Account for the orbital decay of satellites in low Earth orbit*

The atmosphere is a gaseous layer that surrounds the Earth. It allows us to breathe and prevents harmful radiation from reaching the Earth's surface. It extends to more than 300 km above the Earth's surface and its density decreases exponentially with distance from the surface. At the summit of Mount Everest (less than 9 km above sea level), the density of the atmosphere is reduced to approximately one-third that at sea level.

A low Earth orbit satellite is usually placed within the upper limits of the Earth's atmosphere. Although the density of the atmosphere is extremely low at such altitudes, friction will still be generated, acting as a resistive force on the moving satellite. This resistive force will slow down the satellite's orbital velocity (see the circular motion section), and causes the satellite to drop to a lower orbit. (Since $\boldsymbol{F}_c = \frac{m\boldsymbol{v}^2}{r}$, if $\boldsymbol{v}$ is decreased, r will be decreased.

The lower altitude means that the satellite is now in a yet denser part of the atmosphere. This leads to an even greater resistive force acting on the satellite. The satellite will then slow down further at a faster rate, and move into an even lower orbit. As this process continues, eventually its orbital velocity will be too small to sustain its circular orbital motion, and the satellite will spiral back to the Earth. In the process, it is usually burnt up in the denser atmosphere, due to the heat caused by air friction, although pieces of residue can pass through the atmosphere and land on Earth.

An unprotected satellite moving at around 7 km s^{-1} will mostly vaporise, however, spacecraft such as the space shuttle use this same air friction to slow down in a controlled manner. The kinetic energy of the craft is transformed into heat energy, which is absorbed by the protective insulating tiles. Damage to these tiles may result in the shuttle burning up upon re-entry into the Earth's atmosphere. This topic is discussed further in Section 2.9.

2.7 Escape velocity

- ***Outline Newton's concept of escape velocity***
- ***Explain the concept of escape velocity in terms of the:***
  - ***gravitational constant***
  - ***mass and radius of the planet***

It has been seen that when the initial horizontal velocity of a projectile increases sufficiently, the object will not fall back to the Earth following a parabolic pathway, but will describe a circular path around the Earth (see Fig. 2.7). If the velocity is increased further, an elliptical path will follow. Eventually, with even greater horizontal speed, it will escape the Earth's gravitational pull and never come back (see Fig. 2.8). This velocity is referred to as the **escape velocity** of the Earth. Sir Isaac Newton is considered to have been the first to think about such a situation, launching a projectile horizontally from a very high tower.

Definition

Escape velocity is the velocity at which an object is able to escape from the gravitational field of a planet.

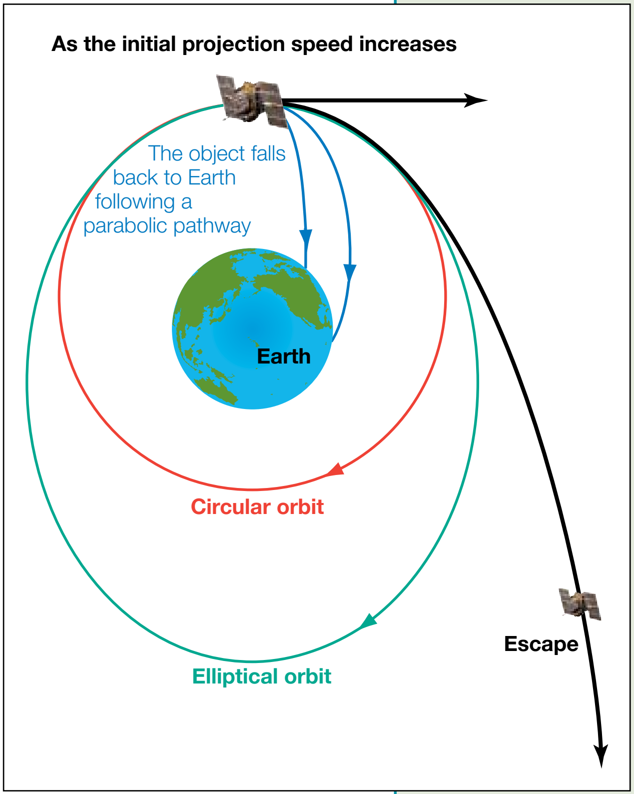


Figure 2.8 An object with different projection speeds

What factors affect the value of the escape velocity?

If an object is to escape the gravitational field of a planet, its **kinetic energy** due to its velocity (v) must exceed or at least equal its gravitational potential energy. Therefore:

$$E_k \geqslant E_p \qquad \text{Kinetic energy } E_k = \frac{1}{2}m\boldsymbol{v}^2$$

$$\frac{1}{2}m\boldsymbol{v}^2 \geqslant G\frac{mM}{r}$$

where r is the radius of the planet, as the object of concern is launched from either at or very close to the surface of the planet.

$$\boldsymbol{v} \geqslant \sqrt{\frac{2GM}{r}}$$

Note that as 'm' is cancelled from both sides of the equation, the escape velocity is not dependent on the mass of the escaping object. As 'G' is also a constant, the only two variables are 'r', the radius of the planet (i.e. how far from the planet's centre is its surface) and 'M', the mass of the planet itself.

This has *two* consequences:

1. Different planets have different escape velocities.
2. A massive body, such as a rocket, will have the same escape velocity as a small object like an atom.

Example 1

Using the data provided in Table 1.1 in Chapter 1, calculate the escape velocity for Mercury. Hence explain why, unlike Earth, Mercury does not have an atmosphere.

Solution

i. $$\frac{1}{2}m\boldsymbol{v}^2 \geq G\frac{mM}{d}$$

$$\boldsymbol{v} \geq \sqrt{\frac{2GM}{r}}$$

$$G = 6.67 \times 10^{-11}$$

$$M = 3.3 \times 10^{23} \text{ kg}$$

$$r = 2439000 \text{ m}$$

$$\therefore \boldsymbol{v} \geq \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 3.3 \times 10^{23}}{2439000}}$$

$$\boldsymbol{v} \geq 4248 \text{ m s}^{-1}$$

$$\text{or } \boldsymbol{v} \geq 4.2 \text{ km s}^{-1}$$

ii. The escape velocity for Mercury as calculated above is quite small (compared to 11.2 km s^{-1} on Earth). Since the escape velocity is independent of an object's mass, under normal circumstances the velocity of gas molecules can easily exceed the escape velocity of Mercury due to their own thermal energies, especially as Mercury is close to the Sun and therefore very hot. Consequently, a small planet like Mercury will not be able to retain its atmosphere, as the majority of its atmospheric gas molecules will escape into space. For the same reason, the Moon cannot retain its own atmosphere as any gas molecules will be able to move faster than the Moon's escape velocity.

2.8 Leaving Earth

Launching a rocket and the conservation of momentum

- ***Analyse the changing acceleration of a rocket during launch in terms of the:***
  - ***law of conservation of momentum***
  - ***forces experienced by astronauts***

Newton proposed three laws of mechanics. The third law states: when one object exerts a force on another, it will itself receive an equal force but opposite in direction. Newton's third law is also known as the law of conservation of momentum.

Rockets usually combust hydrogen in oxygen to produce thrust. These gases are liquified for storage to reduce the storage volume.

In the engine, which is situated at the end of a rocket, the two gases mix and then burn. The combustion of these gases produces enormous amounts of energy and pushes the gases at the end of the rocket backwards or downwards with a very high velocity. As these gases are pushed backwards, the rocket also receives an equal but opposite force, in accordance with Newton's third law. Hence the rocket receives a forward or upward force that lifts it off the ground.

ANALOGY: When you are swimming, in order to move forward, you push the water backwards with your hands and feet.

Because the force pair is between the rocket and the expelled gases, rockets are able to accelerate even in a vacuum. Unlike aeroplane jet engines, the oxygen required for the combustion of the fuel (hydrogen) must be carried onboard the rocket itself so combustion can continue in space.

Physics background: The vast majority of the mass of a spacecraft (such as the space shuttle) at lift-off is the fuel (hydrogen) and the oxygen. Storing these gases as liquids requires very low temperatures (as well as high pressure), which in itself causes problems. The fuel tanks cool down so much that ice forms on the exterior of the tanks, which can fall off during launch and damage the shuttle. The insulating foam designed to keep the tank cold (and to help prevent ice from building up) often falls off during launch. The shuttle *Columbia* had its heat-shielding tiles damaged so badly that the heat upon re-entry caused the spacecraft to burn up, killing all on board.

Liquefied gases occupy less than 0.1% of the volume occupied at normal temperatures and pressure. The oxygen required weighs 16 times more than the fuel itself!

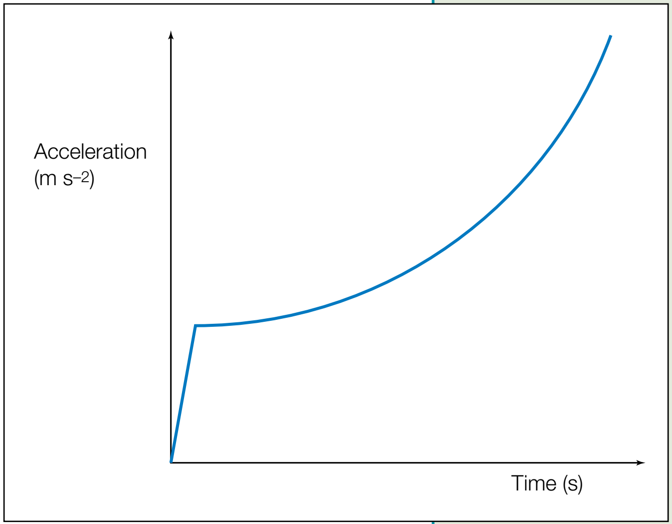
Increase in acceleration during lift-off

The upward acceleration of a rocket as it ascends does not remain constant but rather it *increases* gradually for three reasons.

The first reason is that as the rocket ascends, fuel is consumed, which results in a decrease in mass of the rocket. The upward thrust force of the rocket remains constant (because the engine power does not change). Therefore, according to Newton's second law: $\boldsymbol{F} = m\boldsymbol{a}$, if the mass m decreases, and the thrust $\boldsymbol{F}$ is unchanged, then $\boldsymbol{a}$, the acceleration, gradually increases.

If we plot the acceleration versus time for a single-stage rocket as it ascends (most modern rockets are **multistage rockets**), the graph would look similar to Figure 2.9.

Figure 2.9 Acceleration of a single-stage rocket versus time



The straight-line portion of the graph indicates that shortly after the engine is turned on, there will be acceleration. The curved portion shows the acceleration increases gradually over time. Note for multistage rockets, the graph will be similar, except the acceleration drops to '$-g$' at the end of each stage and rises in a similar way as before when a new staged is turned on.

The second reason is that the direction of the velocity changes from being vertical to horizontal as the rocket goes into orbit—thus its acceleration is no longer reduced by g.

Third, as the rocket ascends, it moves further and further away from the planet. This results in

An ascending multistage rocket

a decrease in the force of gravity. Consequently, the net upward force acting on the rocket increases, hence its acceleration increases. The magnitude of gravity at typical low Earth orbit altitudes is still around 90%–95% of the magnitude on the Earth's surface. However, at the altitude of geostationary satellites, the magnitude of gravity is about 1% of the value on the surface of the Earth.

A word about multistage rockets

A schematic drawing of a multistage rocket is shown in Figure 2.10. When the fuel in the first stage is used up, this stage is discarded and the engine of the second stage is turned on. A similar process takes place when the fuel of the second stage is depleted.

The purpose of the multiple stages is to allow rockets to avoid carrying empty fuel tanks, which act as unnecessary mass and decrease the efficiency of the launching process.

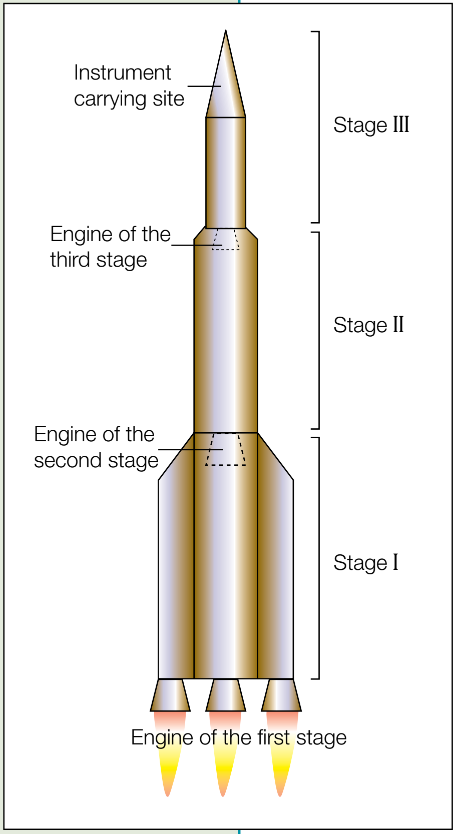


Figure 2.10 A schematic drawing of a multistage rocket

Acceleration and *g* forces

- ***Identify why the term 'g forces' is used to explain the forces acting on an astronaut during launch***

Acceleration during lift-off can also be assessed using '*g* forces'.

Definition

***g* force** is a measure of acceleration force using the Earth's gravitaitonal acceleration as the unit.

As we stand, sit or walk we experience 1 *g*, due to the normal force that pushes upwards, and we feel our 'normal' weight. However, there are certain situations where the *g* forces we experience will deviate from 1, and the apparent weight we experience will be different. This is summarised in Figure 2.11 (a) to (e).

NOTE: The situation shown in Figure 2.11 (b) is commonly experienced during lift-offs. However, the size of the *g* force has to be carefully controlled, because large *g* forces can be lethal, as discussed below.

A **positive *g* force** is one that is directed from the feet to the head (upwards), whereas a **negative *g* force** is in the other direction (downwards). If a person experiences the sensation of feeling *more* weight than normal, the *g* force is positive. Feeling *less* weight (or even negative weight) is due to negative *g* forces.

Effects of large *g* forces

Although the human body can tolerate moving at any speed, it cannot withstand very high accelerations or *g* forces. The maximum *g* force on shuttle astronauts is limited to around 4 *g*, while astronauts in their training may be subjected to 10 *g* forces. An enormous g force acting along the longitudinal axis of the astronaut is fatal:

- *Enormous positive g force*: The extreme case of the situation shown in Figure 2.11 (b), will tend to drain the blood away from the head and brain, causing unconciousness or 'blackout' and death if it is prolonged.
- *Enormous negative g force*: The extreme case of the situation shown in Figure 2.11 (e), causes the blood to rush from the feet to the head and brain, which leads to excessive bleeding through orifices and brain damage. This is known as a 'red-out'. It too can be fatal.

Astronaut inside a spacecraft

To help astronauts withstand extremely large g forces during lift-offs, the astronauts lie down so that the forces act in the direction along their back-to-front axis and special cushions are used. Pilots of fighter planes, routinely subjected to large g forces during tight turns and manoeuvres, wear 'g suits' which apply pressure to lower parts of the body, preventing blood from being pulled away from the brain. As astronauts move around inside the spacecraft once in orbit, and the g forces only act for several minutes from launch until orbit is attained, g suits are impractical.

NOTE: During the re-entry of a rocket, the astronauts again experience positive g forces. This is because decelerating downwards is equivalent to accelerating upwards.

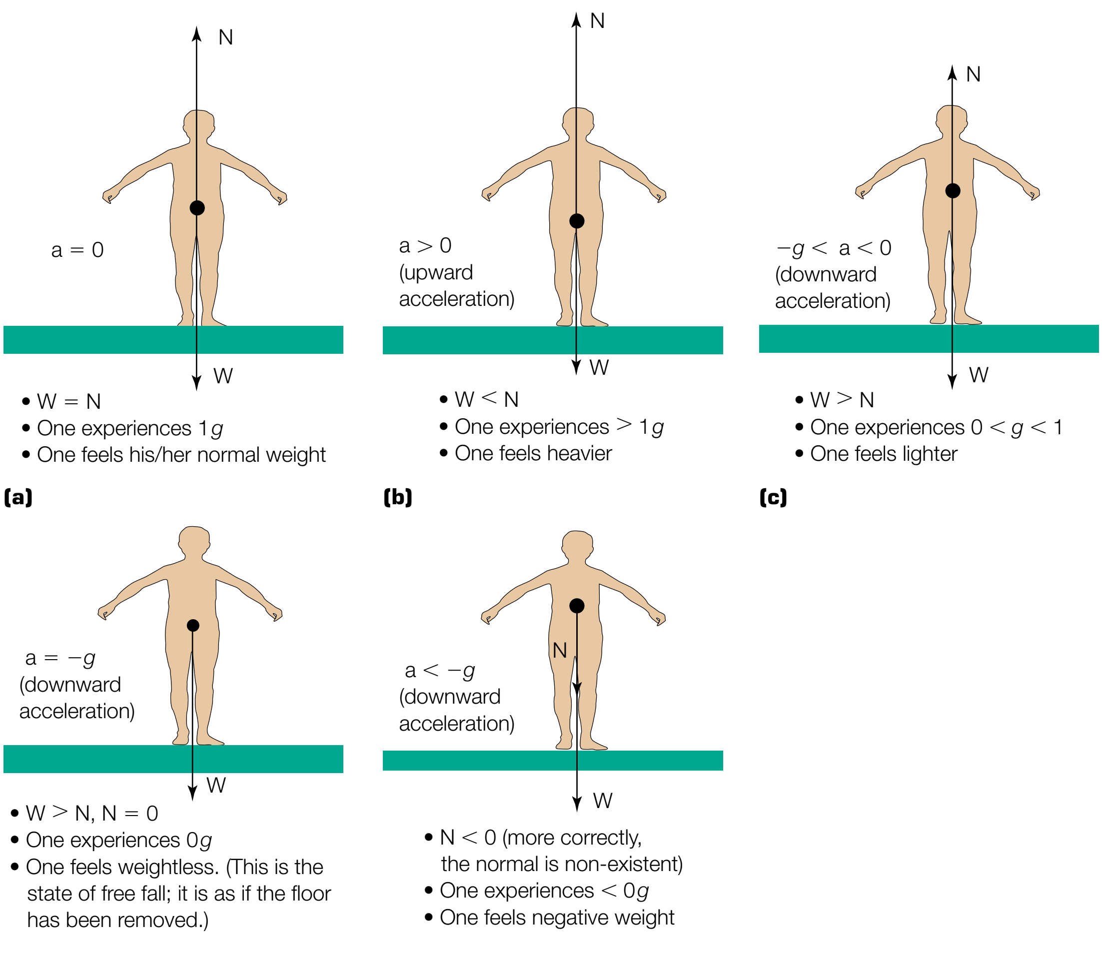


Figure 2.11 Acceleration and g force

Note: W = weight force; N = normal force

Earth's rotational motion and rocket launching

■ ***Discuss the effect of the Earth's orbital motion and its rotational motion on the launch of a rocket***

The Earth rotates on its axis from west to east at the quite fast speed of 465 m s^{-1} at the equator. At the same time it is also orbiting the Sun with a speed of 29.8 km s^{-1}. These high speeds can contribute to rocket launchings.

To launch a satellite that is to orbit the Earth, the rocket is usually launched in an eastward direction, the same direction as the Earth's rotation. This is to take advantage of the rotational speed of the Earth so that the rocket can gain an **additional velocity** during its lift-off without requiring fuel. Also, this type of launch is usually located at places near the equator, where the linear orbital speed of the Earth's spin is the greatest (see page 29 in this chapter).

A launch site at the equator

This reduces the amount of fuel required to achieve the same orbital velocity, thus making the launch more economical and efficient.

Similarly, to launch a satellite that is to probe around the Sun, the rocket is launched in the same direction as the direction the Earth is orbiting the Sun. This again enables the rocket to pick up an additional velocity component during its lift-off, making the launching process more efficient.

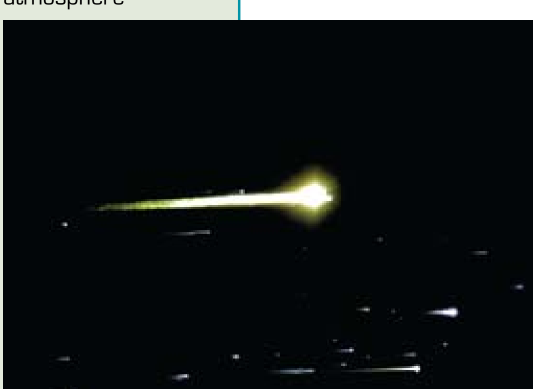
Most countries with satellite launching capabilities have their launch sites as close to the equator as possible. The United States of America launches its satellites and space shuttles from Florida. There have been plans to construct a new Australian launch facility somewhere in Cape York, as close to the equator as possible.

2.9 Coming back to Earth

■ ***Discuss issues associated with safe re-entry into the Earth's atmosphere and landing on the Earth's surface***

■ ***Identify that there is an optimum angle for safe re-entry for a manned spacecraft into the Earth's atmosphere and the consequences of failing to achieve this angle***

A spacecraft re-entering the Earth's atmosphere



Spacecraft or space shuttles on their return to Earth are purely under the influence of gravity, and only the atmosphere provides a frictional medium to slow them down. High technologies and precise planning are essential in order to overcome the difficulties experienced during the re-entry of manned spacecraft.

For a spacecraft to return safely, it is most important to ensure the spacecraft enters the atmosphere at a certain angle called the **optimum re-entry angle**, which lies within the very narrow range of about 5.2° to 7.2° to the atmosphere.

If the re-entry angle is too big, that is if it exceeds 7.2°, then the upward resistive force (the friction between the spacecraft and the atmosphere) experienced by the spacecraft will be too large, so that it will be decelerated too rapidly. The disastrous consequences are that the *g* force experienced by the astronauts will be too large to tolerate and will be fatal. Also, the huge friction produces an enormous amount of heat too rapidly so that it will cause the spacecraft to melt or burn up.

ANALOGY: If you throw a small piece of rock straight down into a lake, it will be slowed down very rapidly. Whereas if you throw it with the same force but at an angle, it will be slowed down at a much slower rate.

If the re-entry angle is too small, that is less than 5.2°, then the spacecraft will *not* enter but will **bounce** off the atmosphere, and return to space. Because a spacecraft that is about to land does not have enough fuel reserve to actively guide its return, a spacecraft that is bounced off the atmosphere may not have sufficient fuel to re-align itself for a second attempt at a controlled re-entry—and burn up.

ANALOGY: When you throw a shell into the sea at a very shallow angle (almost horizontally), the shell does not sink into the sea, but rather it skims off the surface of the water.

Even when the re-entry angle is met, the friction between the spacecraft and atmosphere will still produce an enormous amount of heat. Therefore special materials have to be employed to shield the spacecraft and astronaut from this heat blast. On a modern spacecraft, external porous silicon complex tiles are employed to insulate against the heat, while internal reflective aluminium plates are used to reflect the excessive heat back into space. Final adjustments are made by air conditioners, which keep the interior of the spacecraft at normal room temperature. The Apollo missions used an ablative shield that burned away—taking the heat energy with it.

Another phenomenon that occurs during re-entry into the atmosphere is known as 'ionisation blackout'. This blackout refers to a loss in radio communications for between 30 seconds to several minutes during the high-speed initial phase of re-entry. The intense heat caused by the high speed of the vehicle produces a plasma layer (hot charged particles) that prevents radio waves being received by or transmitted from the spacecraft. This issue is being studied closely by scientists in an endeavour to minimise the effect or to eliminate it entirely by designing new antennas and spacecraft shapes.

The work of rocket scientists

- ***Identify data sources, gather, analyse and present information on the contribution of one of the following to the development of space exploration: Tsiolkovsky, Oberth, Goddard, Esnault-Pelterie, O'Neill or von Braun***

Five rocket scientists who have made many great contributions to the development rocketry as well as space exploration include:

- Konstantin Tsiolkovsky
- Robert H. Goddard
- Hermann Oberth
- Wernher von Braun
- Robert Esnault-Pelterie

An outline of their contributions to the development of space exploration is given here. Further research into *one* chosen contributor should be undertaken using secondary sources such as the internet by typing in appropriate words into a search engine.

SECONDARY SOURCE INVESTIGATION

PFAs

H1

PHYSICS SKILLS

H13.1A, C
H14.1H

Konstantin Tsiolkovsky

Robert H. Goddard

General resources—How to evaluate a website

German rocket experts (L to R, foreground) Dr Ernst Stuhlinger, Professor Hermann Oberth, Wernher von Braun and Robert Lusser

Robert Esnault-Pelterie

Konstantin Tsiolkovsky: a Russian theorist (1857–1935)

- Built wind tunnels to study the aerodynamics of a variety of aircraft.
- Was the first to propose the idea of liquid fuels instead of solid fuels for rockets.
- Proposed the idea of mounting one rocket engine on top of another, with the first discarded after it is used. This was the forerunner of modern multistage rockets.
- Proposed solutions to problems in navigation, heating as a consequence of air friction and maintaining fuel supply for rockets.

Robert H. Goddard: an American experimentalist (1882–1945)

- Measured the fuel values for various rocket fuels, such as liquid hydrogen and oxygen.
- Launched the world's first liquid-fuel-powered instrument rocket.
- Launched the first liquid-fuelled supersonic rocket.
- Developed pumps for liquid fuels, as well as rocket engines that have automatic cooling systems.

Hermann Oberth: an Austrian theorist (1894–1989)

- Simulated the effects of weightlessness.
- Calculated the velocity a rocket must achieve in order to escape the Earth's gravitational pull.
- In his book *Ways to Spaceflight*, for which he won the Robert Esnault-Pelterie–André Hirsch Prize, he proposed the idea of ion and electric repulsion rocket engines.
- Employed von Braun to help him to launch his first rocket near Berlin. Thus he was responsible for influencing future generations in the area of rocketry.

Wernher von Braun: a German engineer (1912–1977)

- When still very young, helped Oberth to launch his first rocket near Berlin.
- Led the development of V2 rockets for Germany, which caused massive destruction in other countries during World War II.
- Helped the US to develop advance missiles for military purposes and later rocketry for high altitude studies and space exploration (after surrendering to US armies at the end of World War II).
- Led the development of Saturn series rocketry, which let the first American walk on the Moon.

Robert Esnault-Pelterie: French aircraft engineer and spaceflight theorist (1881–1957)

- Designed and developed ailerons—moveable surfaces on the trailing edge of an aircraft wing. These became an essential element on most modern aircraft.

- Developed and trialled monoplanes. In 1908 his *Pelterie II* set a record, flying 1200 m at 30 m altitude.
- Inventor of the 'joy stick' flight control used in many planes built during World War I.
- Calculated in 1913 the energy required to reach the Moon and nearby planets.
- Proposed the use of atomic energy and nuclear power for interplanetary flight.

Projectile motion

FIRST-HAND INVESTIGATION

PHYSICS SKILLS

H11.1A, B, E
H11.2A, B, C, D, E
H11.3A, B, C
H12.1A, D
H12.2A, B
H12.4A, B, C, D, E
H13.1D
H14.1A, D, E, G
H14.2A, D

- ***Perform a first-hand investigation, gather information and analyse data to calculate initial and final velocity, maximum height reached, range and time of flight of a projectile for a range of situations by using simulations, data loggers and computer analysis***

A number of commercial packages offer computer simulation programs; data logging products are supplied with software to capably analyse projectile motion experiments.

USEFUL WEBSITES

A simulation with user inputs:
http://galileo.phys.virginia.edu/classes/109N/more_stuff/Applets/ProjectileMotion/enapplet.html

A simulation showing component velocities as the projectile moves:
http://www.walter-fendt.de/ph11e/projectile.htm

A straightforward projectile motion analysis:
http://www.walter-fendt.de/ph11e/projectile.htm

WWW

Aim

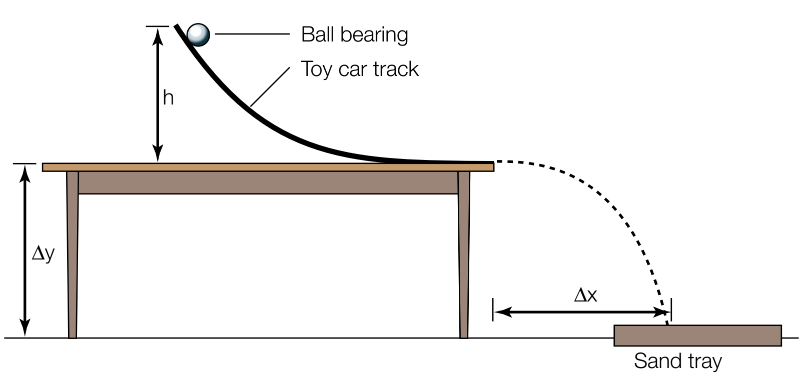
To analyse an example of projectile motion and compare the calculated value for its horizontal displacement to the one measured experimentally.

Equipment/apparatus

Toy car track, ball bearing, sand tray (or similar), metre ruler

Procedure

1. Set up the experiment as shown in the diagram:



Set-up of an experiment to analyse an example of projectile motion

SR

Projectile motion simulation spreadsheet

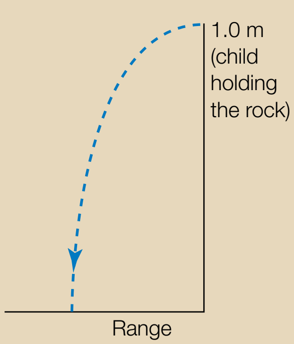
2. Calculate the ball bearing's horizontal launch speed ($\boldsymbol{u_x}$) by using the conservation of energy, that is, gravitational potential energy is converted to kinetic energy:
   - $\boldsymbol{u_x} = \sqrt{2gh}$. (Note: the ball bearing's rotational kinetic energy and friction results in $\boldsymbol{u_x}$ being slightly less than the value given by this equation. A closer value is $\boldsymbol{u_x} = \sqrt{1.43gh}$).
3. Calculate the expected *time of flight* of the ball bearing, using $\boldsymbol{a_y}$ = 9.8 m s^{-2} (down), $\Delta \boldsymbol{y}$ is the height of the table and $\boldsymbol{u_y}$ = 0.
4. Hence calculate the *expected* value for $\boldsymbol{\Delta_x}$, and compare this with the experimental value of $\boldsymbol{\Delta_x}$ obtained by performing the experiment (measuring the horizontal distance from the impression made on the sand tray to a point directly below the edge of the table). Be sure that all other variables are kept constant.
5. Present your results in a suitable format.
6. Assess the *reliability* of the experiment by repeating it several times. (If it is reliable, the results of each trial will not differ significantly. The experiment is *not* made more reliable by simply repeating it!)
7. Write a suitable conclusion.

An excellent task to extend this first-hand investigation involves using a spreadsheet (Excel) to calculate the X,Y position of a projectile at small time increments; use this to produce a 'strobe photograph'.

CHAPTER REVISION QUESTIONS

For all the questions in this chapter, take the mass of the Earth to be 6.0×10^{24} kg, and the radius of the Earth to be 6378 km.

1. Luke is in a train which drives past a platform at a constant velocity v m s^{-1}. He is tossing a pen vertically up and down. Incidentally Luke's brother is on the platform. Describe the motion of this pen seen by Luke's brother and justify your answer.
2. A young child is practising his sling shot. He swings a rock with mass of 10 g with a rope of 25 cm long so that it makes a horizontal circle just above his head, and the rope develops a tension of 10 N. He then releases the rock, and the rock shoots towards a tree which is 1.2 m tall and 0.60 m away.
   (a) At what speed will the rock be projected initially?
   (b) Assuming this young child is 1.0 m tall, calculate the range of the sling shot.



(c) Will the rock hit the tree?
(d) Calculate the position on the tree where the rock will hit the tree.

3. Sketch a velocity-time graph to describe the motion of a marble stone that has been projected vertically upwards from the ground level, which then returns to the original position, if it is projected at a velocity of 24 m s^{-1}. Label your axis.

4. An Australian athlete is competing in the 2008 Olympic hammer-throw. Assume the string is 0.50 m long and the mass of the ball (hammer) is 1.0 kg.
(a) If the athlete is exerting a force of 20×10^2 N while swinging the ball, how fast will the ball be released?
(b) Assuming the athlete is 1.8 m tall, and the ball leaves just above his head at an angle of 40° to the ground, calculate how high the ball will reach and how far the ball will land.
(c) Justify why such athletes are usually quite tall and bulky in build.

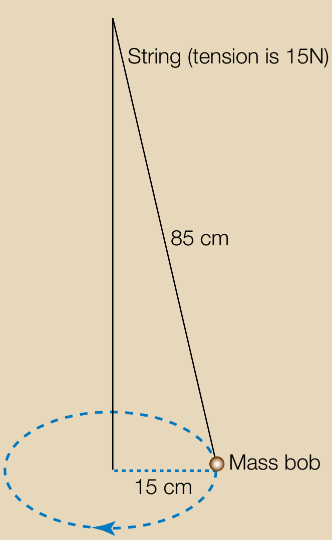
5. A stone is thrown from the top of a cliff at 15 m s^{-1}, 35° to the horizontal. If the stone takes 5.2 seconds to land:
(a) determine the height of the cliff
(b) calculate the impact velocity

6. Outline Galileo's analysis of projectile motions.

7. Describe one other of Galileo's contributions apart from his work on projectile motions.

8. On a normal day, a car is able to turn a corner whose arc has a radius of 10 m at 30 km h^{-1}. On a rainy day, the slipperiness of the road reduces the friction between the tires and road by a half. If this car is still going to turn safely without skidding, how fast should it turn?

9. A mass bob is undergoing a conical pendulum motion as shown in the diagram. If the string is 85 cm long and the tension in the string is 15 N, and the radius of the circle is 15 cm, determine the period of this conical pendulum. Note: Period is defined as the time for one complete revolution or oscillation.



10. A student watched a documentary on TV that seemed to show that astronauts in a spacecraft orbiting around the Earth were able to 'float' inside the spacecraft. The student explained this by saying there is no gravity acting on these astronauts, thus they are not falling. What is the problem with this statement? How would you explain such a phenomenon?

11. Using equations, describe Newton's contributions to Kepler's third law.

12. Kepler's third law can be written as $\frac{r^3}{T^2}$ = k, where k is a constant. Justify the following statement: K is only a constant if one is dealing with objects in the same orbital system.

13. The Moon orbits the Earth every 27.3 days. The Moon has a mass of 7.35×10^{22} kg, and a radius of 1738 km.
    (a) How long would it take a rocket with a constant velocity of 1.0×10^6 m s^{-1} to reach the Moon?
    (b) Determine the orbital velocity of the Moon.
    (c) Determine the size of the gravitational attraction force between the Moon and the Earth.

14. A satellite orbiting the planet Physics with an orbital radius of 2300 km, measured from the centre of the planet. It completes one revolution around the planet in 4.2 hours. If the same satellite is to descend to a radius of 1800 m, how long will it take to orbit the planet now?

15. During your study, you have qualitatively evaluated the key features of low Earth orbit satellites and geostationary satellites. List two advantages of low Earth orbit satellites and geostationary satellites respectively.

16. A low Earth orbit satellite that has a mass of 1.0 tonne is to be placed into an orbit around the Earth such that it will go around the Earth exactly 12 times a day.
    (a) Calculate the height above the ground at which the satellite needs to be.
    (b) Describe one similarity and one difference of launching another satellite which has a mass of 2.0 tonnes into the same orbit.

17. Justify the fact that geostationary satellites do not experience orbital decay.

18. Critically evaluate one technical difficulty of launching a space probe from the surface of Saturn. (Assume Saturn has a solid surface.)

19. The acceleration gradually increases during the launching of a modern rocket. Analyse two factors that could contribute to this increase in acceleration.

20. Define the term '*g* force', and discuss the relevance of *g* force during the early stage of rocket launching.

21. A spacecraft is to be launched vertically, and an astronaut is standing upright in the spacecraft.
    (a) If the astronaut weighs 68 kg, what is the size of the normal force acting on him or her before the spacecraft takes off?
    (b) Assuming a constant upward acceleration of 20.8 m s^{-2}, what will be the normal force acting on the astronaut now?
    (c) What is the weight of this person if it is measured while the spacecraft is accelerating at 20.8 m s^{-2}?
    (d) Name one possible health hazard that may be experienced by this astronaut.
    (e) Name two ways of minimising this health hazard.

22. Suppose you are inside a lift that is going down from level 10 to the ground level at a constant velocity of 6.00 m s^{-1}. Three seconds before reaching the ground level, the lift starts to decelerate to prepare to stop at the ground level. If your mass is 70.0 kg:
(a) What is the net force you experience when the lift is going down at a constant velocity of 6.00 m s?
(b) What is the net force you experience in the last three seconds?
(c) What will your apparent weight be during the last three seconds?

23. Define escape velocity. Calculate escape velocity for the planet Jupiter and comment on its magnitude. The mass of Jupiter is 1.90×10^{27} kg and its radius is 71492 km.

24. (a) Discuss the safety issues involved in re-entries of satellites or spacecrafts and evaluate the significance of the optimum re-entry angle(s).
(b) Even if everything is done correctly during the re-entry, there are still technical difficulties. Name one of these difficulties and comment on how this can be controlled.

25. Identify the following statement as true or false. If false please provide a correction to such a statement:
(a) A geostationary satellite can be placed anywhere above the Earth as long as its altitude will allow it to have a period of exactly 24 hours as according to Kepler's third law.
(b) There is no net force acting on a satellite while it is orbiting the Earth.

26. Choose two scientists from the following list:
(a) Konstantin Tsiolkovsky
(b) Robert H. Goddard
(c) Hermann Oberth
(d) Wernher von Braun
(e) Robert Esnault-Pelterie
Using point form, list three significant contributions these two scientists have made to rocketry and space exploration.

27. Design an experiment that allows you to measure one unknown horizontal projection velocity of a projectile (assume friction is negligible). In this experiment, you may assume the gravitational acceleration is 9.8 m s^{-2}.

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Answers to chapter revision questions