ANSWERS TO SELECTED PROBLEMS

Chapter 1

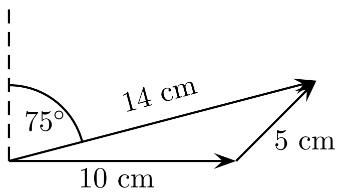
1. a) $\mathbf{a}+\mathbf{h}$, b) $\mathbf{a}-\mathbf{h}$, c) $\mathbf{a}+\frac{1}{2}\mathbf{h}$, d) $\frac{3}{4}\,\mathbf{a}$, e) $\frac{3}{4}\,\mathbf{a}-\frac{1}{2}\,\mathbf{h}$.

2. a) $\mathbf{0}$, b) $2\overrightarrow{CA}$.

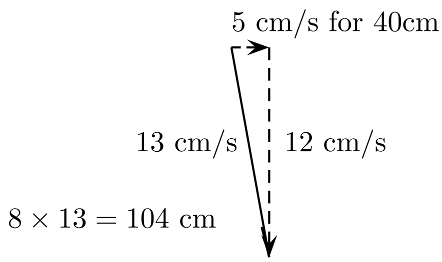
3. a) $-4\mathbf{a}+5\mathbf{b}$, b) $(2p+3r)\mathbf{a}+(2q-3s)\mathbf{b}$.

4. a) $\frac{1}{2}(\mathbf{b}+\mathbf{a})$, $\frac{1}{2}(\mathbf{b}+\mathbf{c})$

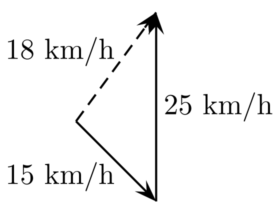
6. a) ≈ 14 cm N 75° E.



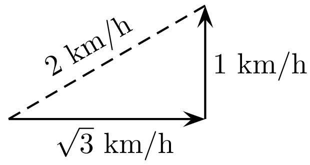
b) ≈ 104 cm S 23° E.



c) ≈ 18 km/h N 36° E.



d) The rower must row 30° upstream.



7. Approximately 28.0 km N $51^\circ\,9'$ E from A.

8. a) $\frac{1}{2}\mathbf{b}+\frac{1}{2}\mathbf{c}$, $\frac{1}{4}\mathbf{a}+\frac{3}{4}\mathbf{c}$, $\frac{1}{4}\mathbf{a}+\frac{3}{4}\mathbf{b}$. b) $\frac{1}{7}\mathbf{a}+\frac{3}{7}\mathbf{b}+\frac{3}{7}\mathbf{c}$.

9. a) $\begin{pmatrix}3\\4\end{pmatrix}$, b) $\begin{pmatrix}16\\15\\-5\end{pmatrix}$, c) $\begin{pmatrix}-7\\2\\-6\\-1\end{pmatrix}$, d) Not possible, e) $7\mathbf{i}-4\mathbf{j}+3\mathbf{k}$.

10. 7.43, N 28° E.

16. a) not parallel, b) parallel, c) parallel.
Only in b) is $ABCD$ a parallelogram.

21. $(4,5,0)$, $(-6,-1,2)$, $(4,7,6)$

22. $\mathbf{d}+\mathbf{e}-\mathbf{f}$, $\mathbf{d}+\mathbf{f}-\mathbf{e}$, $\mathbf{e}+\mathbf{f}-\mathbf{d}$.

23. The midpoint is $(3,-1,3)$. The point Q is $(10,-29,31)$.

24. $\mathbf{t}=\frac{1}{3}\mathbf{a}+\frac{2}{3}\mathbf{b}$

25. $\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$.

26. $6, \frac{1}{6}\begin{pmatrix}4\\-4\\2\end{pmatrix}$; $\sqrt{14}, \frac{1}{\sqrt{14}}\begin{pmatrix}2\\1\\0\\3\end{pmatrix}$; $\sqrt{21}, \frac{1}{\sqrt{21}}\begin{pmatrix}4\\0\\1\\-2\\0\end{pmatrix}$.

27. a) 15, b) 12, c) $\sqrt{62}$.

28. $\sqrt{35}, \sqrt{6}, \sqrt{41}$.

29. A 4–cube has 16 vertices, say, $V=\{(a,b,c,d)\,|\,a,b,c,d=0,1\}$.

30. $(5,0^9)^T$, $(0,5,0^8)^T\ldots,(0^9,5)^T$. Yes, $(\alpha,\alpha,\ldots,\alpha)^T$ where $(5-\alpha)^2+9\alpha^2=50$.

31. a) $\mathbf{x}=\begin{pmatrix}1\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\5\end{pmatrix}$, $\lambda\in\mathbb{R}$; b) $\mathbf{x}=\begin{pmatrix}1\\2\\-1\end{pmatrix}+\lambda\begin{pmatrix}-2\\-3\\6\end{pmatrix}$, $\lambda\in\mathbb{R}$;

c) $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$, $\lambda \in \mathbb{R}$; d) $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \\ -2 \end{pmatrix}$, $\lambda \in \mathbb{R}$.

32. Yes, it corresponds to $\lambda = 1$.

33. a) $\mathbf{x} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\lambda \in \mathbb{R}$; b) $\mathbf{x} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, $\lambda \in \mathbb{R}$;
c) $\mathbf{x} = \lambda \begin{pmatrix} 1 \\ -7 \end{pmatrix}$, $\lambda \in \mathbb{R}$; d) $\mathbf{x} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\lambda \in \mathbb{R}$;
e) $\mathbf{x} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $\lambda \in \mathbb{R}$.

34. a) $\mathbf{x} = \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 1 \\ 0 \end{pmatrix}$ or $\dfrac{x_1 + 4}{6} = x_2 - 1, \quad x_3 = 3.$
b) $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -5 \\ 6 \end{pmatrix}$ or $\dfrac{x_1 - 1}{4} = \dfrac{x_2 - 2}{-5} = \dfrac{x_3 + 3}{6}.$
c) $\mathbf{x} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -1 \\ 2 \end{pmatrix}$ or $\dfrac{x_1 - 1}{5} = \dfrac{x_2 + 1}{-1} = \dfrac{x_3 - 1}{2}.$
d) $\mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix}$ or $x_1 = 1,\ x_2 = x_3.$

35. $\begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$. a) $\mathbf{x} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$, $\lambda \in \mathbb{R}$.

36. a) true, b) false, c) true, d) true.

37. a) $\mathbf{x} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}), \quad 0 \leqslant \lambda \leqslant 1$; b) $\mathbf{x} = \mathbf{b} + \lambda(\mathbf{a} - \mathbf{b}), \quad \lambda \geqslant 0$;
c) $\mathbf{x} = \mathbf{b} + \lambda(\mathbf{a} - \mathbf{b}), \quad \lambda \geqslant 1$; d) $\mathbf{x} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}), \quad \lambda \geqslant \frac{1}{2}$.

38. a) Line segment joining $(1, 3, 6)$ and $(-2, 4, 13)$.
b) Line segment joining $(3, -3, -5, -3, -13)$ and $(-9, 27, 49, 15, 23)$.
c) Line segment joining $(0, 4, 8, 3, -5, 4)$ and $(6, -2, 7, 2, -1, 5)$.
d) Ray from point $(1, 4, -6, 2)$ parallel to $\big(3, 0, -1, 5\big)$.
e) Line through $(3, 1, -4)$ parallel to $\begin{pmatrix} 6 \\ -2 \\ 7 \end{pmatrix}$ with segment from $(-9, 5, -18)$ to $(15, -3, 10)$ removed.

39. a) $\mathbf{x} = \lambda \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 4 \\ -6 \end{pmatrix}; \quad \lambda, \mu \in \mathbb{R}.$

b) $\mathbf{x} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ -14 \\ 5 \end{pmatrix}; \quad \lambda, \mu \in \mathbb{R}.$

40. a) Plane through the origin parallel to $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix}$.

b) Line through $(3, 1, 2, 4)$ parallel to $\begin{pmatrix} -2 \\ 1 \\ 3 \\ 2 \end{pmatrix}$.

c) Line through origin parallel to $\begin{pmatrix} 3 \\ 2 \\ 1 \\ 2 \end{pmatrix}$.

d) Plane through $(1, 2, 3)$ parallel to $\begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ 2 \\ 4 \end{pmatrix}$.

41. a) $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda_1 \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \lambda_2 \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$ for $\lambda_1, \lambda_2 \in \mathbb{R}$;

b) $\mathbf{x} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + \lambda_1 \begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ 6 \\ -6 \end{pmatrix}$ for $\lambda_1, \lambda_2 \in \mathbb{R}$;

c) $\mathbf{x} = \begin{pmatrix} -2 \\ 4 \\ 1 \\ 6 \end{pmatrix} + \lambda_1 \begin{pmatrix} 5 \\ -2 \\ 5 \\ -7 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ 0 \\ -1 \\ -6 \end{pmatrix}$ for $\lambda_1, \lambda_2 \in \mathbb{R}$;

d) $\mathbf{x} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ for $\lambda_1, \lambda_2 \in \mathbb{R}$;

e) $\mathbf{x} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ \frac{6}{5} \\ 1 \end{pmatrix}$ for $\lambda_1, \lambda_2 \in \mathbb{R}$;

f) $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} + \lambda_1 \begin{pmatrix} 4 \\ 0 \\ -4 \\ 5 \end{pmatrix} + \lambda_2 \begin{pmatrix} 7 \\ 2 \\ -3 \\ -5 \end{pmatrix}$ for $\lambda_1, \lambda_2 \in \mathbb{R}$.

42. a) $\mathbf{x} = \lambda_1 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$; $\lambda_1, \lambda_2 \in \mathbb{R}$.

b) $\mathbf{x} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix}$; $\lambda_1, \lambda_2 \in \mathbb{R}$.

c) $\mathbf{x} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ -6 \\ 1 \end{pmatrix}$; $\lambda_1, \lambda_2 \in \mathbb{R}$.

d) $\mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$; $\lambda_1, \lambda_2 \in \mathbb{R}$.

44. a) $(3, 2, 4)$, b) $(3, -4, 11)$.

45. a) $6x - 3y + 2z = -12$, b) $6x - 12y + 13z = 100$.

46. a) $\mathbf{x} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ for $\lambda \in \mathbb{R}$. b) $(-13, 22, 9)$.

47. a) $\mathbf{x} = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 2 \\ -2 \end{pmatrix}$ for $\lambda \in \mathbb{R}$. b) $(1, 2, 3)$.

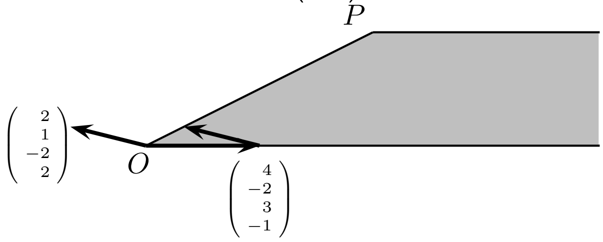
48. a) Parallelogram with vertices $(0, 1)$, $(1, 3)$, $(2, 4)$, $(3, 6)$.

b) Triangle with vertices $(0, 1)$, $(1, 3)$, $(3, 6)$.

c) Parallelogram with vertices $(0, 0, 0)$, $(12, 6, -12)$, $(32, -16, 24)$, $(44, -10, 12)$.

d) Triangle with vertices $(0, 0, 0)$, $(12, 6, -12)$, $(36, -6, 6)$.

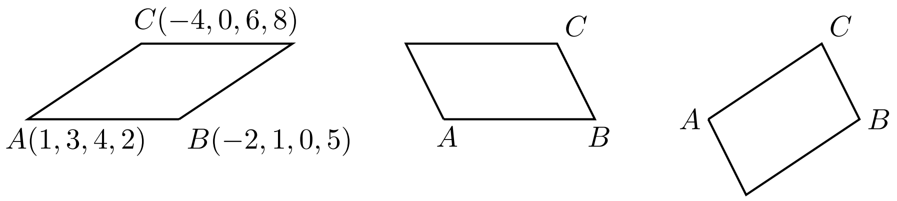
e) An unbounded region with vertices O and P and two of the three sides parallel to $\begin{pmatrix} 4 \\ -2 \\ 3 \\ -1 \end{pmatrix}$. At P, $\lambda_1 = \lambda_2 = 6$ and so $\overrightarrow{OP} = \begin{pmatrix} 36 \\ -6 \\ 6 \\ 6 \end{pmatrix}$.



49. a) See c).

b) $\mathbf{x} = \begin{pmatrix} 1 \\ 3 \\ 4 \\ 2 \end{pmatrix} + \lambda_1 \begin{pmatrix} -3 \\ -2 \\ -4 \\ 3 \end{pmatrix} + \lambda_2 \begin{pmatrix} -2 \\ -1 \\ 6 \\ 3 \end{pmatrix}$ for $0 \leqslant \lambda_1 \leqslant 1,\ 0 \leqslant \lambda_2 \leqslant \lambda_1$.

c) The three parallelograms are:



The algebraic definitions are:

$$\mathbf{x} = \begin{pmatrix} 1 \\ 3 \\ 4 \\ 2 \end{pmatrix} + \lambda_1 \begin{pmatrix} -3 \\ -2 \\ -4 \\ 3 \end{pmatrix} + \lambda_2 \begin{pmatrix} -5 \\ -3 \\ 2 \\ 6 \end{pmatrix} \text{ for } 0 \leqslant \lambda_1 \leqslant 1,\ 0 \leqslant \lambda_2 \leqslant 1.$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ 3 \\ 4 \\ 2 \end{pmatrix} + \lambda_1 \begin{pmatrix} -3 \\ -2 \\ -4 \\ 3 \end{pmatrix} + \lambda_2 \begin{pmatrix} -2 \\ -1 \\ 6 \\ 3 \end{pmatrix} \text{ for } 0 \leqslant \lambda_1 \leqslant 1,\ 0 \leqslant \lambda_2 \leqslant 1.$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ 3 \\ 4 \\ 2 \end{pmatrix} + \lambda_1 \begin{pmatrix} 2 \\ 1 \\ -6 \\ -3 \end{pmatrix} + \lambda_2 \begin{pmatrix} -5 \\ -3 \\ 2 \\ 6 \end{pmatrix} \text{ for } 0 \leqslant \lambda_1 \leqslant 1,\ 0 \leqslant \lambda_2 \leqslant 1.$$

Chapter 2

1. a) $\dfrac{\pi}{4}$ b) $\cos^{-1}\left(\dfrac{1}{10\sqrt{3}}\right) \approx 86^\circ 41'$, c) $\dfrac{\pi}{2}$, d) $\cos^{-1}\left(\dfrac{7}{10\sqrt{13}}\right) \approx 78^\circ 48'$.

2. a) $0, \dfrac{2}{\sqrt{6}}, \dfrac{1}{\sqrt{3}}$; b) $\dfrac{4}{3\sqrt{2}}, -\dfrac{5}{\sqrt{33}}, \dfrac{8}{\sqrt{66}}$; c) $\dfrac{7}{3\sqrt{10}}, -\dfrac{1}{\sqrt{42}}, \dfrac{8}{\sqrt{105}}$.

3. $\cos^{-1}\left(\dfrac{1}{3}\right) \approx 70^\circ 32'$.

7. $\lambda_1 = \mathbf{a} \cdot \mathbf{u}_1 = \dfrac{1}{\sqrt{2}}$, $\lambda_2 = \mathbf{a} \cdot \mathbf{u}_2 = -3$, $\lambda_3 = \mathbf{a} \cdot \mathbf{u}_3 = \dfrac{3}{\sqrt{2}}$.

8. a) $\begin{pmatrix} 5 \\ \frac{5}{2} \\ 1 \end{pmatrix}$. b) $\dfrac{\pi}{2}$. c) $\dfrac{\sqrt{66}}{2}$. d) $\dfrac{1}{17}\begin{pmatrix} 80 \\ 50 \\ 22 \end{pmatrix}$.

9. a) $\dfrac{1}{3}\begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix}$, b) $\dfrac{3}{14}\begin{pmatrix} -1 \\ 3 \\ 0 \\ 2 \end{pmatrix}$, c) $\begin{pmatrix} -3 \\ 3 \\ 6 \end{pmatrix}$.

10. a) 7, b) 3, c) $\sqrt{6}$.

11. $\mathbf{q} = -\mathbf{p} + 2\mathbf{a} + 2\,\mathrm{proj}_{\mathbf{d}}(\mathbf{p} - \mathbf{a})$.

12. b) $q(\lambda_0) = \mathbf{a}\cdot\mathbf{a} - \dfrac{(\mathbf{a}\cdot\mathbf{b})^2}{\mathbf{b}\cdot\mathbf{b}}$

14. a) $\begin{pmatrix} 16 \\ -4 \\ -2 \end{pmatrix}$, b) $\begin{pmatrix} -23 \\ -11 \\ 20 \end{pmatrix}$, c) $\begin{pmatrix} -45 \\ 9 \\ -18 \end{pmatrix}$.

15. $\begin{pmatrix} 12 \\ -8 \\ 6 \end{pmatrix}$.

17. a) $2\sqrt{21}$, $\begin{pmatrix} 8 \\ -4 \\ 2 \end{pmatrix}$; b) $2\sqrt{2}$, $\begin{pmatrix} 0 \\ -2 \\ -2 \end{pmatrix}$.

18. a) $\sqrt{2}$; b) $\dfrac{15}{2}$.

19. a) $-\dfrac{4}{3\sqrt{2}}$. b) $\dfrac{1}{\sqrt{2}}$.

20. a) 2, b) $\dfrac{1}{\sqrt{2}}$, c) 7.

23. a) Line through A and B is $\mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda_1 \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$, $\lambda_1 \in \mathbb{R}$.

Line through C and D is $\mathbf{x} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$, $\lambda_2 \in \mathbb{R}$.

b) Shortest distance is $\dfrac{3}{\sqrt{17}}$.

c) Point P is $\left(-\dfrac{21}{17}, \dfrac{38}{17}, \dfrac{53}{17}\right)$ and Q is $\left(-\dfrac{30}{17}, \dfrac{32}{17}, \dfrac{47}{17}\right)$.

25. a) 14, b) 53.

27. As usual, the answers for equations of planes are not unique.

a) $\mathbf{x} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$; $\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \cdot \left(\mathbf{x} - \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \right) = 0$;
$x_1 - x_2 - 2x_3 = 3$.

b) $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + \lambda_1 \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$; $\begin{pmatrix} -5 \\ 5 \\ -5 \end{pmatrix} \cdot \left(\mathbf{x} - \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \right) = 0$;
$x_1 - x_2 + x_3 = -3$.

c) $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + \lambda_1 \begin{pmatrix} -2 \\ -1 \\ 4 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$, $\begin{pmatrix} -7 \\ 10 \\ -1 \end{pmatrix} \cdot \left(\mathbf{x} - \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \right) = 0$;
$7x_1 - 10x_2 + x_3 = -15$.

d) $\mathbf{x} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1/2 \\ 1 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} -1/4 \\ 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} -4 \\ 2 \\ -1 \end{pmatrix} \cdot \left(\mathbf{x} - \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \right) = 0$;
$4x_1 - 2x_2 + x_3 = -4$.

e) $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + \lambda_1 \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} + \lambda_2 \begin{pmatrix} 14 \\ 2 \\ -6 \end{pmatrix}$, $\begin{pmatrix} 4 \\ 17 \\ 15 \end{pmatrix} \cdot \left(\mathbf{x} - \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \right) = 0$;
$4x_1 + 17x_2 + 15x_3 = 8$.

28. a) $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$, for λ, $\mu \in \mathbb{R}$.

b) $\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$. c) $x_1 + x_2 + x_3 = 7$.

29. a) $\begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}$, b) $\frac{1}{2} \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix}$.

30. a) 3, b) $\sqrt{6}$, c) $\frac{13}{7}$, d) $\frac{25}{7}$.

31. a) $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$, λ, $\mu \in \mathbb{R}$.

b) $\begin{pmatrix}1\\1\\1\end{pmatrix}$. c) $\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot\left(\mathbf{x}-\begin{pmatrix}1\\2\\0\end{pmatrix}\right)=0.$ d) $\dfrac{8}{\sqrt{3}}$.

32. a) $\mathbf{c}=\text{proj}_{\mathbf{a}}\mathbf{v},\ \mathbf{d}=\mathbf{v}-\mathbf{c}$. b) $\mathbf{c}=\begin{pmatrix}\frac{3}{11}\\-\frac{1}{11}\\-\frac{1}{11}\end{pmatrix}$, $\mathbf{d}=\begin{pmatrix}\frac{8}{11}\\\frac{12}{11}\\\frac{12}{11}\end{pmatrix}$.

Chapter 3

1.

	$x\in\mathbb{N}$	$x\in\mathbb{Z}$	$x\in\mathbb{Q}$	$x\in\mathbb{R}$
a)	-	-25	-25	-25
	3	3	3	3
	-	-3	-3	-3
	-	-	$-\frac{10}{3}$	$-\frac{10}{3}$
b)	1	$1,-5$	$1,-5$	$1,-5$
	5	5	$5,\frac{3}{2}$	$5,\frac{3}{2}$
	-	-	-	$\frac{1\pm\sqrt{5}}{2}$
	-	-	-	-
c)	$3j, j\in\mathbb{N}$	$3j, j\in\mathbb{Z}$	$3j, j\in\mathbb{Z}$	$3j, j\in\mathbb{Z}$
	0	0	0	$3k\pi, k\in\mathbb{Z}$

2. No.

3. Yes. The set $\{0\}$ and the empty set $\emptyset=\{\ \}$.

4. Yes.

5. $3z=6+9i,\quad z^2=-5+12i,\quad z+2w=7i,\quad z(w+3)=-2+10i,\quad \dfrac{z}{w}=\dfrac{1}{5}(4-7i),$ $\dfrac{w}{z}=\dfrac{1}{13}(4+7i)$.

6. a) $\dfrac{1}{5}(3-i)$, b) $-\dfrac{1}{2}(1-i)$.

7. a) a^2-b^2+2abi, b) $\dfrac{a}{a^2+b^2}-i\dfrac{b}{a^2+b^2}$, c) $\dfrac{1}{(a-1)^2+b^2}\big((a^2-1+b^2)-2ib\big)$.

8. a) $\frac{1}{2}(-1\pm\sqrt{3}\,i)$, b) $-1\pm\sqrt{2}\,i$, c) $3\pm i$, d) $\frac{1}{2}(3\pm\sqrt{3})i$, e) $\pm i,\ \pm 2i$.

10. 16

11. $\dfrac{8abi(a^2-b^2)}{(a^2+b^2)^2}$

12.

z	$\text{Re}(z)$	$\text{Im}(z)$	$\overline{z}$
$-1+i$	-1	1	$-1-i$
$2+3i$	2	3	$2-3i$
$2-3i$	2	-3	$2+3i$
$\frac{2-i}{1+i}$	$\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1+3i}{2}$
$\frac{1}{(1+i)^2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$

13. $-3+4i, \quad \dfrac{11}{25}-\dfrac{2}{25}i.$

14. $z=2+3i, \quad w=-1+2i.$

17. b) $z^2-6z+13$

18.

z	$\lvert z\rvert$	$\text{Arg}(z)$	Polar Form
$6+6i$	$6\sqrt{2}$	$\frac{\pi}{4}$	$6\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$
-4	4	π	$4(\cos\pi+i\sin\pi)$
$\sqrt{3}-i$	2	$\frac{-\pi}{6}$	$2\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)$
$\frac{-1}{\sqrt{2}}-\frac{i}{\sqrt{2}}$	1	$\frac{-3\pi}{4}$	$\cos\frac{3\pi}{4}-i\sin\frac{3\pi}{4}$
$-7+3i$	$\sqrt{58}$	α	$\sqrt{58}(\cos\alpha+i\sin\alpha)$

Here $\alpha=\pi-\tan^{-1}\frac{3}{7}$.

19. $\sqrt{234}, \quad -1.$

20. $n=4$

21. a) $\dfrac{3}{2}\left(1+\sqrt{3}i\right)$, b) $\dfrac{3}{2}\left(-\sqrt{3}+i\right)$, c) $-\dfrac{3}{2}\left(1+\sqrt{3}i\right)$, d) $\dfrac{3}{2}\left(\sqrt{3}-i\right)$,

e) $\dfrac{3}{2}\left(\sqrt{2+\sqrt{2}}+i\sqrt{2-\sqrt{2}}\right)$ (Double angle formula used).

27. $64, \quad -(1+\sqrt{3})i, \quad \dfrac{1+\sqrt{3}}{2}+\dfrac{\sqrt{3}-1}{2}i.$

28. $\dfrac{7}{2}.$

29. $\pi.$

30. $\operatorname{Arg}(-1+i) = \dfrac{3\pi}{4}$; $\operatorname{Arg}\left(-\sqrt{3}+i\right) = \dfrac{5\pi}{6}$;
$\operatorname{Arg}\left((-1+i)\left(-\sqrt{3}+i\right)\right) = -\dfrac{5\pi}{12}$; $\operatorname{Arg}\left(\dfrac{-1+i}{-\sqrt{3}+i}\right) = -\dfrac{\pi}{12}$.

31. $\sin\dfrac{7\pi}{12} = \dfrac{1+\sqrt{3}}{2\sqrt{2}}$.

32. $zw = 2\sqrt{2}e^{i\pi/12} = 2\sqrt{2}\left[\cos\left(\dfrac{\pi}{12}\right) + i\sin\left(\dfrac{\pi}{12}\right)\right]$; $z^9 = -512$; $\left(\dfrac{z}{\overline{w}}\right)^{12} = 64e^{i\pi} = -64$.

33. a) $16\left(-\sqrt{3}+i\right)$, b) $-i$, c) $-\dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}$.

34. a) $\pm(5-2i)$, b) $\pm(3+5i)$, c) $\pm(7+5i)$.

35. b) $\sqrt{2}e^{\frac{5\pi i}{12}} = \frac{1}{2}\left(\left(\sqrt{3}-1\right) + i\left(\sqrt{3}+1\right)\right)$, c) $\frac{1}{\sqrt{2}}(1+13i)$.

37. a) $2+i,\ 1-i$; b) $4+i,\ 3-2i$; c) $1-2i,\ -5+3i$.

38. $e^{i\pi/7},\ e^{3i\pi/7},\ e^{5i\pi/7},\ e^{i\pi},\ e^{-i\pi/7},\ e^{-3i\pi/7},\ e^{-5i\pi/7}$.

39. $e^{in\pi/12}$ for $n = -11, -7, -3, 1, 5, 9$.

40. $2e^{in\pi/15}$ for $n = -13, -7, -1, 5, 11$.

41. $\dfrac{15}{2} + i\left(\dfrac{3\sqrt{3}}{2} - 1\right)$, $3-i$, $\dfrac{15}{2} - i\left(\dfrac{3\sqrt{3}}{2} + 1\right)$.

48. a) Real part $= \cos(2\theta)$. Imaginary part $= \sin(2\theta)$.

51. a) $\cos 6\theta = \cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta$
$\sin 6\theta = 6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta + 6\cos\theta\sin^5\theta$

b) $\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$

52. $\sin 7\theta = 7\cos^6\theta\sin\theta - 35\cos^4\theta\sin^3\theta + 21\cos^2\theta\sin^5\theta - \sin^7\theta$
$\cos 7\theta = \cos^7\theta - 21\cos^5\theta\sin^2\theta + 35\cos^3\theta\sin^4\theta - 7\cos\theta\sin^6\theta$.

53. a) $\cos\theta = \dfrac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$.

b) $\cos^6\theta = \dfrac{1}{32}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10)$.

54.
$$\sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$$
$$\int \sin^5\theta\, d\theta = \frac{1}{16}\left(-\frac{1}{5}\cos 5\theta + \frac{5}{3}\cos 3\theta - 10\cos\theta\right) + C,$$

$$\cos^4\theta = \frac{1}{8}\left[3 + 4\cos(2\theta) + \cos(4\theta)\right]$$
$$\int \cos^4\theta\, d\theta = \frac{1}{8}\left[3\theta + 2\sin(2\theta) + \frac{1}{4}\sin(4\theta)\right] + C.$$

55. a) $\cos 5\theta = 16x^5 - 20x^3 + 5x$

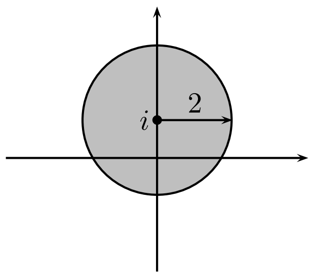
d) $-1,\ \cos\frac{\pi}{5},\ \cos\frac{3\pi}{5},\ \cos\frac{7\pi}{5},\ \cos\frac{9\pi}{5}.$

56. The sum is n when k is an integer multiple of n and 0 otherwise.

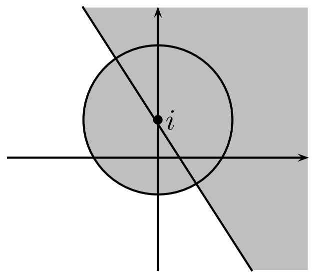
58. $\dfrac{\sin\left(\frac{1}{2}(n+1)\theta\right)\sin\left(\frac{1}{2}n\theta\right)}{\sin\frac{1}{2}\theta}.$

59. a) $\dfrac{9e^{i\theta}}{9 + e^{i2\theta}}.$

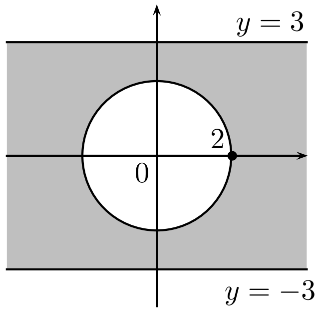
60. a) $|z - i| \leqslant 2$



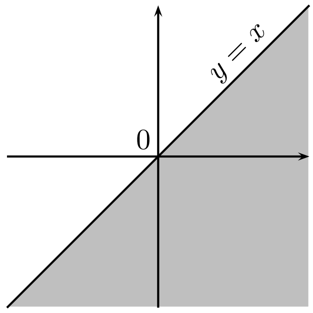
b) $|z - i| \leqslant 2$ or $-\dfrac{\pi}{3} \leqslant \operatorname{Arg}(z - i) \leqslant \dfrac{2\pi}{3}$



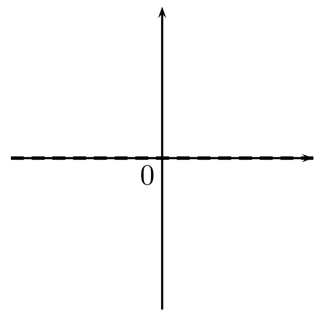
c) $|z| \geqslant 2$ and $|\operatorname{Im}(z)| \leqslant 3$



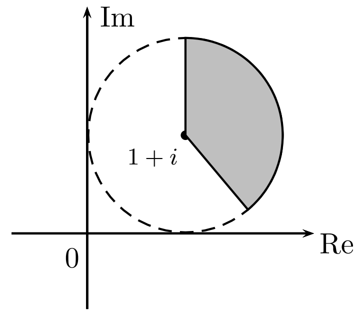
d) $y \leqslant x$



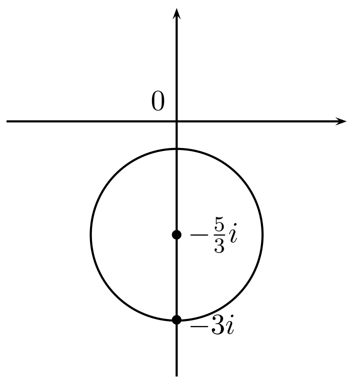
e) The real axis



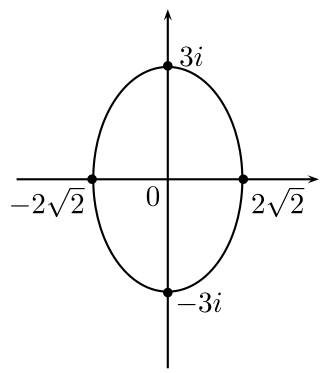
f) $|z-1-i|<1$ & $-\dfrac{\pi}{4}<\operatorname{Arg}(z-1-i)\leqslant\dfrac{\pi}{2}$



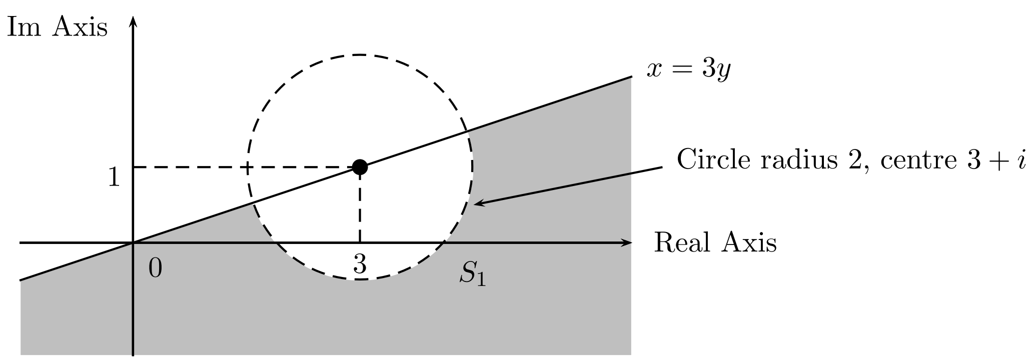
g) Circle: $x^2+\left(y+\dfrac{5}{3}\right)^2=\left(\dfrac{4}{3}\right)^2$



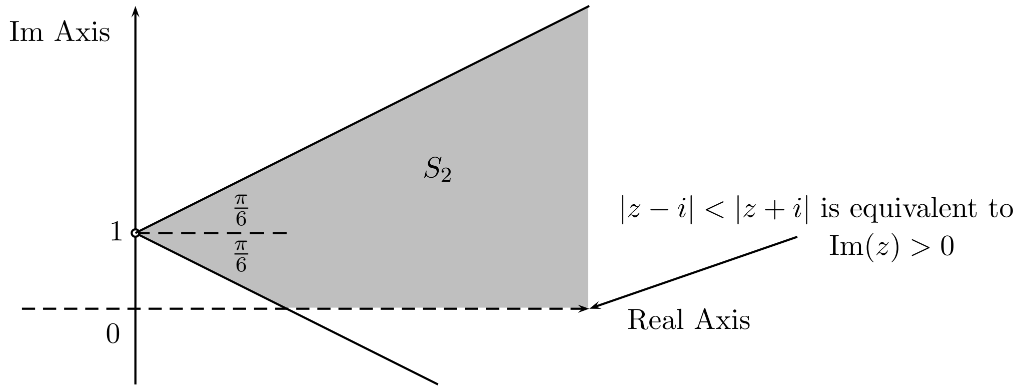
h) Ellipse: $\left(\dfrac{x}{2\sqrt{2}}\right)^2+\left(\dfrac{y}{3}\right)^2=1$



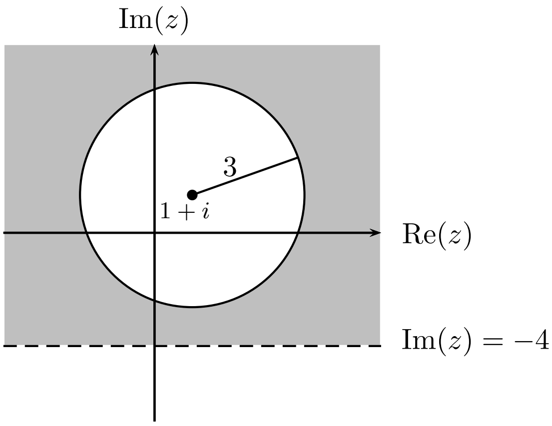
61. a) $\operatorname{Re}(z)\geqslant 3\operatorname{Im}(z)$ and $|z-(3+i)|>2$



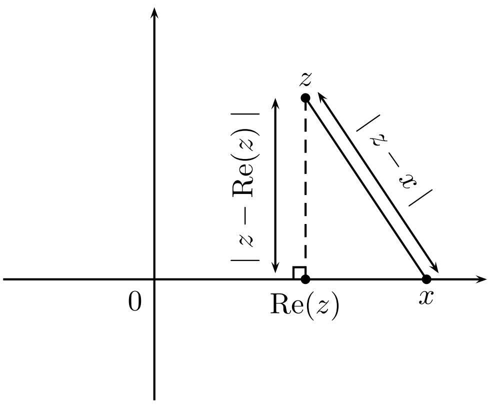
b) $|z-i| < |z+i|$ and $-\dfrac{\pi}{6} \leqslant \operatorname{Arg}(z-i) \leqslant \dfrac{\pi}{6}$



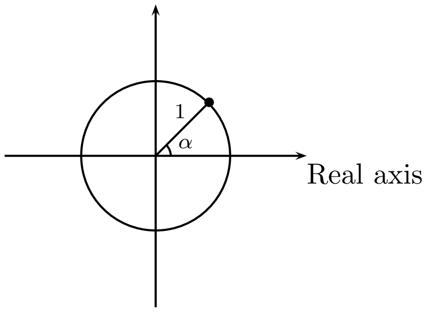
62. a) $\operatorname{Im}(z) > -4$ and $|z-1-i| \geqslant 3$ b) Yes.



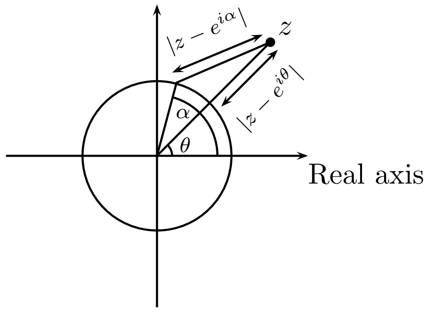
63. $|\, z - x \,| \geqslant |\, z - \operatorname{Re}(z) \,|$



64. a) $w = e^{i\alpha}$, $-\pi < \alpha \leqslant \pi$



c) $\mid z - e^{i\alpha} \mid \geqslant \mid z - e^{i\theta} \mid$, $\theta = \mathrm{Arg}(z)$



65. a) 742, b) 129, c) $1 + 9i$.

66. $p(z) = (z-2)(2z-5)(z+3)$.

67. $p(z) = (z-1)(z+1)(z+2)(z+4)$.

68. a) $\left(z - e^{-\frac{i\pi}{10}}\right)\left(z - e^{\frac{3\pi i}{10}}\right)\left(z - e^{\frac{7\pi i}{10}}\right)\left(z - e^{-\frac{i\pi}{2}}\right)\left(z - e^{-\frac{9\pi i}{10}}\right)$.

b) $\left(z - \sqrt{2}e^{\frac{i\pi}{6}}\right)\left(z - \sqrt{2}e^{\frac{i\pi}{2}}\right)\left(z - \sqrt{2}e^{\frac{5\pi i}{6}}\right)\left(z - \sqrt{2}e^{-\frac{i\pi}{6}}\right)\left(z - \sqrt{2}e^{-\frac{i\pi}{2}}\right)\left(z - \sqrt{2}e^{-\frac{5\pi i}{6}}\right)$.

69. a) $(x-1)(x+1)(x^2+1)(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)$.

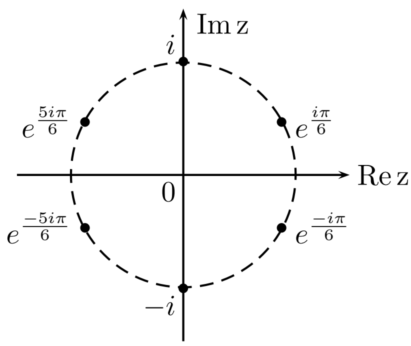
b) $(x^2+2)(x^2+\sqrt{6}x+2)(x^2-\sqrt{6}x+2)$.

70. $(z^2+2z+2)(z^2-2z+2)$

71. $(z - e^{-i\pi/8})(z - e^{i3\pi/8})(z - e^{i7\pi/8})(z - e^{-i5\pi/8})$

72. a) $e^{-\frac{5\pi i}{6}}$, $e^{-\frac{\pi i}{2}}$, $e^{-\frac{\pi i}{6}}$, $e^{\frac{\pi i}{6}}$, $e^{\frac{\pi i}{2}}$, $e^{\frac{5\pi i}{6}}$.

b) Note that the solutions are evenly spaced around the unit circle centred on 0.



c) $\left(z - e^{-\frac{5\pi i}{6}}\right)\left(z - e^{-\frac{\pi i}{2}}\right)\left(z - e^{-\frac{\pi i}{6}}\right)\left(z - e^{\frac{\pi i}{6}}\right)\left(z - e^{\frac{\pi i}{2}}\right)\left(z - e^{\frac{5\pi i}{6}}\right)$.

d) $\left(z^2+1\right)\left(z^2+\sqrt{3}\,z+1\right)\left(z^2-\sqrt{3}\,z+1\right)$.

73. a) $e^{i\pi/4}$, $e^{i\pi/2}$, $e^{i3\pi/4}$, $e^{-i\pi/4}$, $e^{-i\pi/2}$, $e^{-3\pi/4}$.
b) $\left(z-e^{i\pi/4}\right)\left(z-e^{-i\pi/4}\right)\left(z-e^{i3\pi/4}\right)\left(z-e^{-i3\pi/4}\right)\left(z-e^{i\pi/2}\right)\left(z-e^{-i\pi/2}\right)$
c) $\left(z^2-\sqrt{2}z+1\right)\left(z^2+\sqrt{2}z+1\right)\left(z^2+1\right)$.

74. a) $(z-e^{2i\pi/5})(z-e^{-2i\pi/5})(z-e^{4i\pi/5})(z-e^{-4i\pi/5})$
b) $\left(z^2-2z\cos\left(\frac{2\pi}{5}\right)+1\right)\left(z^2-2z\cos\left(\frac{4\pi}{5}\right)+1\right)$

75. a) $(t+1-i)\ (t+1+i)\ (t-2)\ (t+1)\ (t+i)\ (t-i)$,
b) $(t^2+2t+2)\ (t-2)\ (t+1)\ (t^2+1)$.

76. $1+i$, $\quad 1-i$, $\quad \sqrt[3]{5}$, $\quad \dfrac{\sqrt[3]{5}}{2}\left(-1+i\sqrt{3}\right)$, $\quad \dfrac{\sqrt[3]{5}}{2}\left(-1-i\sqrt{3}\right)$.

77. a) $(z^2+z+1)(z^6+z^3+1)$. b) $e^{\pm 2i\pi/9}$, $e^{\pm 4i\pi/9}$, $e^{\pm 8i\pi/9}$.

79. a) One of the roots is $(-2+2i)^{1/3}+(-2-2i)^{1/3}$.

80. d) -2, $2\sqrt{2}\cos\dfrac{\pi}{12}$, $2\sqrt{2}\cos\dfrac{7\pi}{12}$.

81. a) 1; b) -1, $\dfrac{5}{7}$, $\dfrac{1}{4}$; c) 4, $\pm\dfrac{1}{5}$.

90. `evalc((sqrt(2)+7*I)^13);`

Chapter 4

1. a) $\left\{\frac{5}{2}\right\}$, $\left\{\begin{pmatrix}\frac{5}{2}\\ \lambda\end{pmatrix} : \lambda\in\mathbb{R}\right\}$, $\left\{\begin{pmatrix}\frac{5}{2}\\ \lambda\\ \mu\end{pmatrix} : \lambda,\mu\in\mathbb{R}\right\}$

b) $\left\{\begin{pmatrix}4-2\lambda\\ \lambda\end{pmatrix} : \lambda\in\mathbb{R}\right\}$, $\left\{\begin{pmatrix}4-2\lambda\\ \lambda\\ \mu\end{pmatrix} : \lambda,\mu\in\mathbb{R}\right\}$

c) $\left\{\begin{pmatrix}\lambda\\ \mu\\ 2-2\lambda+3\mu\end{pmatrix} : \lambda,\mu\in\mathbb{R}\right\}$

2. a) No solution. b) Unique solution $\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}8\\ -9\end{pmatrix}$.

c) Infinite number of solutions on the line $\mathbf{x}=\begin{pmatrix}5\\ 0\end{pmatrix}+\lambda\begin{pmatrix}5\\ 1\end{pmatrix}$, $\lambda\in\mathbb{R}$.

3. For $a_{11} \neq 0$ the conditions are as follows.

   a) If $a_{11}a_{22} - a_{12}a_{21} \neq 0$, then solution is unique.
   b) If $a_{11}a_{22} - a_{12}a_{21} = 0$ and $a_{11}b_2 - a_{21}b_1 \neq 0$, then there is no solution.
   c) If $a_{11}a_{22} - a_{12}a_{21} = 0$ and $a_{11}b_2 - a_{21}b_1 = 0$, then there are an infinite number of solutions.

4. The general conditions are as follows.

   a) If $a_{11}a_{22} - a_{12}a_{21} \neq 0$, then solution is unique.
   b) There is no solution if $a_{11}a_{22} - a_{12}a_{21} = 0$ and either
      i) $a_{11}b_2 - a_{21}b_1 \neq 0$, or
      ii) $a_{12}b_2 - a_{22}b_1 \neq 0$, or
      iii) $a_{11} = a_{12} = a_{21} = a_{22} = 0$ and b_1, b_2 are not both zero.
   c) There are an infinite number of solutions otherwise.

5. a) Solution set $= \left\{ \begin{pmatrix} 1+\lambda \\ 2-2\lambda \\ \lambda \end{pmatrix} : \lambda \in \mathbb{R} \right\}$.

   Planes intersect in line $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, $\lambda \in \mathbb{R}$.

   b) No solution. Planes are parallel.

   c) Solution set $= \left\{ \begin{pmatrix} 4 - \frac{5}{4}\lambda + \frac{1}{2}\mu \\ \lambda \\ \mu \end{pmatrix} : \lambda,\ \mu \in \mathbb{R} \right\}$. Equations represent the same plane.

8. a) In vector form,

$$x_1 \begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix} + x_2 \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix} + x_3 \begin{pmatrix} 4 \\ -3 \\ 6 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \\ 8 \end{pmatrix}.$$

   As a matrix equation and augmented matrix,

$$\begin{pmatrix} 3 & -3 & 4 \\ 5 & 2 & -3 \\ -1 & -1 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \\ 8 \end{pmatrix}; \qquad (A|\mathbf{b}) = \left(\begin{array}{rrr|r} 3 & -3 & 4 & 6 \\ 5 & 2 & -3 & 7 \\ -1 & -1 & 6 & 8 \end{array} \right).$$

   b) In vector form,

$$x_1 \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} + x_3 \begin{pmatrix} 7 \\ -5 \\ 6 \end{pmatrix} + x_4 \begin{pmatrix} 8 \\ -1 \\ -6 \end{pmatrix} = \begin{pmatrix} -2 \\ 7 \\ 5 \end{pmatrix}.$$

   As a matrix equation and augmented matrix,

$$\begin{pmatrix} 1 & 3 & 7 & 8 \\ 3 & 2 & -5 & -1 \\ 0 & 3 & 6 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2 \\ 7 \\ 5 \end{pmatrix}; \qquad (A|\mathbf{b}) = \left(\begin{array}{rrrr|r} 1 & 3 & 7 & 8 & -2 \\ 3 & 2 & -5 & -1 & 7 \\ 0 & 3 & 6 & -6 & 5 \end{array} \right).$$

9. The system of equation is

$$\begin{array}{rcrcrcr} x_1 & - & 3x_2 & & & = & 10 \\ & & 6x_2 & + & 6x_3 & = & -2 \\ -6x_1 & - & x_2 & - & 4x_3 & = & 0 \\ 7x_1 & + & 9x_2 & + & 11x_3 & = & 5 \end{array}$$

The augmented matrix form is

$$A = \left(\begin{array}{rrr|r} 1 & -3 & 0 & 10 \\ 0 & 6 & 6 & -2 \\ -6 & -1 & -4 & 0 \\ 7 & 9 & 11 & 5 \end{array}\right).$$

10. a) $R_2 = R_2 - 2R_1,\ R_3 = R_3 - 4R_1;$ b) $R_1 = R_1 - R_2,\ R_2 = \frac{1}{2}R_2.$

11. $\left(\begin{array}{rrr|r} 2 & 4 & 1 & 2 \\ 9 & 14 & 7 & 7 \\ 1 & 3 & 1 & 3 \end{array}\right).$

12. All but c) and h) are in row-echelon form.

13. a) $\mathbf{x} = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix}$. Point of intersection of 3 planes.

b) $\mathbf{x} = \begin{pmatrix} 2 \\ 3 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 0 \\ 2 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R}.$

A line in $\mathbb{R}^4$ through the point $(2, 3, -2, 0)$ and parallel to $\begin{pmatrix} -1 \\ 0 \\ 2 \\ 1 \end{pmatrix}$.

14. a) $\mathbf{x} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$. b) $\mathbf{x} = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}, \quad \lambda \in \mathbb{R}.$ c) $\mathbf{x} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$.

d) No solution. e) $\mathbf{x} = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}, \quad \lambda \in \mathbb{R}.$ f) No solution.

g) $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 2 \end{pmatrix}$. h) $\mathbf{x} = \begin{pmatrix} -3 \\ 6 \\ 5 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ -1 \\ 1 \end{pmatrix}, \quad \lambda \in \mathbb{R}.$

15. a) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right)$.

Solution: $\mathbf{x} = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$, which is the position vector of a point in $\mathbb{R}^3$.

b) $\left(\begin{array}{cccc|c} 1 & 0 & 0 & -75 & -34 \\ 0 & 1 & 0 & 29 & 13 \\ 0 & 0 & 1 & 7 & 3 \end{array}\right)$.

Solution: $\mathbf{x} = \begin{pmatrix} -34 \\ 13 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 75 \\ -29 \\ -7 \\ 1 \end{pmatrix}, \quad \lambda \in \mathbb{R}$, which is a line in $\mathbb{R}^4$.

16. a) Unique solution, b) no solution, c) infinitely many solutions, d) infinitely many solutions, e) unique solution.

17. a) $k \neq 3$, b) no such value of k, c) $k = 3$.

18. a) $\lambda = \pm 2$, b) $\lambda = 1$, c) all other values of λ.

19. a) $a \neq 0$, b) $a = 0$, $b \neq 0$, c) $a = b = 0$, d) $\mathbf{x} = \begin{pmatrix} 5 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ \frac{5}{2} \\ -1 \\ 3 \end{pmatrix}$ $\lambda \in \mathbb{R}$.

20. Perhaps, if the costs are negative or very large then you can be sure that someone is cheating.

21. No.

22. a)
$$\begin{aligned} x_1 &= 7b_1 + 5b_2 + 3b_3 \\ x_2 &= 6b_1 + 4b_2 + 3b_3 \\ x_3 &= 2b_1 + b_2 + b_3 \end{aligned}$$
b)
$$\begin{aligned} x_1 &= \tfrac{3}{2}b_1 - 2b_2 - 2b_3 \\ x_2 &= -\tfrac{7}{2}b_1 + 5b_2 + 4b_3 \\ x_3 &= \tfrac{1}{2}b_1 - b_2 - b_3 \end{aligned}$$

24. a) $b_3 - \frac{1}{2}b_1 + b_2 = 0$. b) $b_1 - b_2 + b_3 = 0$ and $-2b_1 + b_2 + b_4 = 0$.

26. Yes.

27. No.

28. Yes, since $\begin{pmatrix} 1 \\ 1 \\ 4 \\ 12 \end{pmatrix} = 3 \begin{pmatrix} 3 \\ -1 \\ 4 \\ 6 \end{pmatrix} - 2 \begin{pmatrix} 4 \\ -2 \\ 4 \\ 3 \end{pmatrix}$.

29. No.

30. Yes, at $(6, 13, 11)$.

31. Yes, since $\begin{pmatrix} 5 \\ 7 \\ -1 \end{pmatrix} = 3 \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix} - 4 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$.

33. Meet at $(6, 9, 4)$.

34. The planes intersect at the line $\mathbf{x} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 4 \\ 8 \end{pmatrix}$ $\lambda \in \mathbb{R}$.

35. Planes are not parallel as $\lambda_1 \begin{pmatrix} 2 \\ 1 \\ -2 \\ 7 \end{pmatrix} + \lambda_2 \begin{pmatrix} -3 \\ 1 \\ 5 \\ 2 \end{pmatrix} = \mu_1 \begin{pmatrix} 3 \\ -1 \\ 2 \\ 4 \end{pmatrix} + \mu_2 \begin{pmatrix} -1 \\ 4 \\ 2 \\ 6 \end{pmatrix}$
only when $\lambda_1 = \lambda_2 = \mu_1 = \mu_2 = 0$.

37. a) $\mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \\ 1 \end{pmatrix}$, $\lambda \in \mathbb{R}$. b) The planes intersect in a line.

38. $p(x) = 2x^2 - 4x + 7$

39. I am 42, my brother is 46 and my sister is 52.

40. 6 days in Bangkok, 4 each in Singapore and Kuala Lumpur.

41. $3, 1, 2$.

42. a) Letting x_1 be the number of hectares of wheat, x_2 be the number of hectares of oats and x_3 be the number of hectares of barley gives the equations

$$\begin{array}{rcrcrcr} x_1 & + & x_2 & + & x_3 & = & 12 \\ 6x_1 & + & 6x_2 & + & 2x_3 & = & 48 \\ 150x_1 & + & 100x_2 & + & 70x_3 & = & 700 \\ 72x_1 & + & 48x_2 & + & 36x_3 & = & 612 \end{array}$$

b) There is no solution.

c) The inequalities are

$$\begin{array}{rcrcrcr} x_1 & + & x_2 & + & x_3 & \leqslant & 12 \\ 6x_1 & + & 6x_2 & + & 2x_3 & \leqslant & 48 \\ 150x_1 & + & 100x_2 & + & 70x_3 & \leqslant & 700 \\ 72x_1 & + & 48x_2 & + & 36x_3 & \leqslant & 612 \end{array}$$

and with slack variables s_1, s_2, s_3, s_4 the equations are

$$\begin{array}{rcrcrcccccccrcr} x_1 & + & x_2 & + & x_3 & + & s_1 & & & & & & & = & 12 \\ 6x_1 & + & 6x_2 & + & 2x_3 & & & + & s_2 & & & & & = & 48 \\ 150x_1 & + & 100x_2 & + & 70x_3 & & & & & + & s_3 & & & = & 700 \\ 72x_1 & + & 48x_2 & + & 36x_3 & & & & & & & + & s_4 & = & 612 \end{array}$$

d) Some sensible solutions are to either plant $4\frac{2}{3}$ hectares wheat and no oats and barley, or 7 hectares oats and no wheat and barley, or 10 hectares barley and no wheat and oats. There are also an infinite number of other reasonable solutions. In each case it is the fertiliser which is restricting the planting.

44. a) Π_1 is $x + 2y - z = 2$, Π_2 is $3x + 6y - z = 12$, Π_3 is $2x + 4y - z = 7$.

b) $\mathbf{x} = \begin{pmatrix} 5 - 2t_2 \\ t_2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + t_2 \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \quad t_2 \in \mathbb{R}$

The intersection is a line through $(5, 0, 3)$ and parallel to $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$.

c) $x = -2y + 5$ and $z = 3$.

45. a) $\begin{cases} x - 2y + z = a \\ 3x + 6y + 8z = b \\ 4x + 2y + 7z = c \\ 7x - 8y + 6z = d \end{cases}$. b) $d - a + 2b - 3c = 0$. c) $\left(\frac{4}{7}, -\frac{1}{7}, \frac{1}{7}\right)$.

Chapter 5

1. a) $3A = \begin{pmatrix} 6 & -9 & 12 \\ 9 & 6 & -6 \\ 3 & -3 & 9 \end{pmatrix}$.

   b) $-2B = \begin{pmatrix} 4 & -2 \\ -6 & -8 \\ 2 & -10 \end{pmatrix}$.

   c) $A + B$ is not defined.

   d) $B + C = \begin{pmatrix} -5 & 3 \\ 4 & 0 \\ 5 & 7 \end{pmatrix}$.

   e) $A + 3I = \begin{pmatrix} 5 & -3 & 4 \\ 3 & 5 & -2 \\ 1 & -1 & 6 \end{pmatrix}$.

   f) $B + 3I$ is not defined.

   g) $AB = \begin{pmatrix} -17 & 10 \\ 2 & 1 \\ -8 & 12 \end{pmatrix}$.

   h) BA is not defined.

   i) BC is not defined.

   j) $CD = \begin{pmatrix} -4 & -13 & -9 \\ -2 & 11 & 13 \\ 14 & 14 & 0 \end{pmatrix}$.

   k) $A^2 = \begin{pmatrix} -1 & -16 & 26 \\ 10 & -3 & 2 \\ 2 & -8 & 15 \end{pmatrix}$.

   l) B^2 is not defined.

   m) $(BD)^2 = \begin{pmatrix} -86 & 81 & 167 \\ -47 & 38 & 85 \\ -187 & 171 & 358 \end{pmatrix}$.

7. $96A + 205I$.

8. $N^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\quad N^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

11. a) $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, b) $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$, c) $\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$, d) $\begin{pmatrix} 1 & -2 & 3 \end{pmatrix}$.

13. $A^T = \begin{pmatrix} 1 & -3 & 4 \\ -2 & 0 & 5 \end{pmatrix}$, $\quad B^T = \begin{pmatrix} 2 & -4 & 5 \\ -5 & 6 & 0 \\ 4 & 5 & 8 \\ 3 & 5 & 6 \end{pmatrix}$, $\quad C^T = \begin{pmatrix} 1 & 4 & 2 \\ 4 & -3 & 6 \\ 2 & 6 & 7 \end{pmatrix} = C$.

14. $\mathbf{a}^T\mathbf{b} = \mathbf{b}^T\mathbf{a} = 8$, $\mathbf{ab}^T = \begin{pmatrix} 0 & 4 & 2 \\ 0 & 12 & 6 \\ 0 & -8 & -4 \end{pmatrix}$, $\mathbf{ba}^T = \begin{pmatrix} 0 & 0 & 0 \\ 4 & 12 & -8 \\ 2 & 6 & -4 \end{pmatrix}$, $\mathbf{ab}$ and $\mathbf{a}^T\mathbf{b}^T$ are not defined.

17. A possible $G = \begin{pmatrix} 3 & 6 \\ -4 & 2 \end{pmatrix}$.

19. a) $\begin{pmatrix} 4 & -7 \\ -1 & 2 \end{pmatrix}$, b) $\begin{pmatrix} 5 & 7 \\ 3 & 4 \end{pmatrix}$, c) no inverse, d) $\frac{1}{5}\begin{pmatrix} 4 & -9 \\ -3 & 8 \end{pmatrix}$, e) $\begin{pmatrix} -7 & 1 \\ 1 & 0 \end{pmatrix}$.

20. $A^{-1} = \begin{pmatrix} 1 & 3 & -4 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$, $B^{-1} = \begin{pmatrix} 8 & -2 & -3 \\ \frac{1}{2} & 0 & 0 \\ -3 & 1 & 1 \end{pmatrix}$, C is not invertible,

$$D^{-1} = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 \\ 5 & -3 & 1 \\ -17 & 11 & -5 \end{pmatrix}.$$

21. a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{6} \end{pmatrix}$ b) $\begin{pmatrix} 0 & 0 & -\frac{1}{2} \\ 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{pmatrix}$

22. $A^{-1} = \begin{pmatrix} 4 & -3 & -2 & 0 \\ -1 & 1 & 1 & 0 \\ 1 & -2 & -2 & 1 \\ 0 & 1 & 2 & -1 \end{pmatrix}$; $B^{-1} = \begin{pmatrix} 6 & -2 & 1 & 0 \\ 9 & -4 & 3 & -1 \\ 25 & -11 & 8 & -2 \\ -14 & 6 & -4 & 1 \end{pmatrix}$;

C^{-1} does not exist.

23. a) $\left(B^{-1}\right)^2$, b) AB^6A^{-1}, c) $(A+A^{-1})^2$, d) $I-(I-A)^{m+1}$.

24. a) $A^{-1}B$. b) $\begin{pmatrix} 2 & 4 & 4 \\ 1 & -2 & 3 \\ 1 & 0 & 3 \end{pmatrix}$.

25. b) i) B^TB, ii) $C^{-1}C^T$.

26. a) $\begin{pmatrix} -2 & 0 & 1 \\ 2 & 1 & -1 \\ 5 & 1 & -2 \end{pmatrix}$. b) $\begin{pmatrix} -2c_1+c_3 \\ 2c_1+c_2-c_3 \\ 5c_1+c_2-2c_3 \end{pmatrix}$.

27. $\mathbf{x} = Q^T\mathbf{b}$.

29. e.g. $\begin{pmatrix} \frac{1}{\sqrt{2}}i & -\frac{1}{\sqrt{2}}i \\ \frac{1}{\sqrt{2}}i & \frac{1}{\sqrt{2}}i \end{pmatrix}$.

33. From Question 29, Q is invertible, and hence $Q\mathbf{x} = \mathbf{b}$ has the solution $\mathbf{x} = Q^{-1}\mathbf{b} = \overline{Q}^T\mathbf{b}$.

34. a) $\frac{1}{ab}\begin{pmatrix} b & 0 \\ -c & a \end{pmatrix}$. b) $\begin{pmatrix} A^{-1} & 0 \\ -B^{-1}CA^{-1} & B^{-1} \end{pmatrix}$.

35. a) 1, b) -1, c) 0, d) 5, e) -2. All are invertible except $\begin{pmatrix} 5 & 2 \\ 10 & 4 \end{pmatrix}$.

36. a) -9, b) 0, c) 56.

37. -126.

38. a) -30, b) 5, c) 5, d) $5 \times 7^3 = 1715$.

39. a) -2, b) $-\frac{1}{2}$, c) -32.

40. -83, -108, 8964.

41. $a \neq 1$.

42. a) $\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 5 & 1 & -3 \end{pmatrix}$. b) $\begin{pmatrix} -2 & -1 & 1 \\ 1 & 2 & -1 \\ -3 & -1 & 1 \end{pmatrix}$. c) 1.

45. a) $(\alpha - 3)\ (\alpha + 1)\ (\alpha + 2)$. b) -1, -2, 3.

46. For example, $A = B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

47. For example, $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\lambda = -1$.

54. $2(x + y + z)^3$.

55. $(z - 1)(z^2 + 2z - 4)$, $x = -1, y = 1 \pm \sqrt{5}, z = -1 \pm \sqrt{5}$.

PAST CLASS TESTS

In the years up to 2007 there were 3 algebra class tests per session. From semester 1 2008 there will be only 2 algebra class tests per semester so the pre-2008 tests included here do not have the same coverage of material as the class tests for 2008 and onwards. The Information booklet for MATH1131/1141 lists the material available for examination in the current schedule of class tests, as does page (240) of these notes. Also there have been some changes to the syllabus for 2008 and onwards and some parts of the questions in the following pre-2008 class tests are no longer examinable. Thus the following pre-2008 tests should only be taken as a guide to the level of difficulty to be expected in class test questions for 2008 and onwards.

Sample class tests from 2008 and onwards are included after all the pre-2008 class tests and these tests correspond to the current syllabus and class test schedule. However, the content of the class tests is specified in the Information booklet for MATH1131/1141.

The following selection of past class tests can be used as a guide to the degree of difficulty of algebra class tests. Due to variations in the timing of the mid-semester breaks the material examined in each class test can vary from semester to semester and from year to year. Thus students must consult the Information booklet for MATH1131/1141, or page (240) of these notes, to ascertain the precise topics that may be examined in each algebra class test.

UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 1 VERSION 1a

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working

All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*2 marks*)
   For the points $A\,(4,2,3)$, $B\,(5,-7,-2)$ and $C\,(7,-25,-10)$.
   (i) Find a parametric vector equation of the straight line AB.
   (ii) Determine, with reasons, whether or not the point C is on the straight line AB.

2. (*2 marks*)
   Find a parametric vector equation of the plane in $\mathbb{R}^3$ with Cartesian equation
   $$2x_1 - 5x_2 + x_3 = 7\ .$$
   Hence give two non–parallel non–zero vectors which are parallel to the plane.

3. (*3 marks*)
   For the points $A\,(1,2,3)$, $B\,(3,4,1)$, $C\,(3,3,4)$ calculate
   (i) $\overrightarrow{AB} \times \overrightarrow{AC}$.
   (ii) Area of $\triangle\,ABC$.

4. (*3 marks*)
   Let ℓ be the straight line in $\mathbb{R}^3$ through the point $P\,(1,2,3)$ and parallel to the vector $\mathbf{v} = \begin{pmatrix}2\\3\\1\end{pmatrix}$. Let Q be the point with co-ordinates $(1,4,4)$.
   (i) Find $\text{proj}_{\mathbf{v}}\left(\overrightarrow{PQ}\right)$.
   (ii) Find the shortest distance d between the line ℓ and Q.

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UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 1 VERSION 1b

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working

All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*2 marks*)

   Determine, with reasons, whether or not the 3 points $A\,(3,5,7)$, $B\,(5,-4,3)$ and $C\,(-5,41,22)$ are collinear (i.e. all in a straight line).

2. (*2 marks*)

   Find a parametric vector equation for the plane through the points $A\,(1,2,1)$, $B\,(3,4,2)$, $C\,(5,2,1)$.

3. (*3 marks*)

   For the points $A\,(1,2,3)$, $B\,(5,6,4)$ and $C\,(2,1,3)$ calculate;
   (i) the distance $d(A,B)$ between A and B.
   (ii) the projection $\text{proj}_{\overrightarrow{AC}}\left(\overrightarrow{AB}\right)$.

4. (*3 marks*)

   A triangle has vertices at the origin O, at $A(4,-4,8)$ and at $B\,(0,-3,-6)$.

   Let X be a point on the side OA such that $OX = \frac{3}{4}OA$, and Y a point on the side OB such that $OY = \frac{2}{3}OB$.

   Find parametric vector equations for the lines AY and BX and show that they intersect at the point $P\,(2,-3,2)$.

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UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 1 VERSION 2b

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working

All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*3 marks*)
   Consider the line ℓ and plane Π in $\mathbb{R}^3$ with Cartesian equations:
   $$\ell: \quad \frac{x-2}{3} = \frac{y+1}{4} = \frac{z+3}{1}$$
   $$\Pi: \quad 3x - 2y - 4z = 11 \ .$$
   (i) Find a parametric equation of the line ℓ.
   (ii) Find the co-ordinates of the point P where ℓ meets Π.

2. (*3 marks*)
   For the points $A\,(1,2,1)$, $B\,(3,1,-1)$ and $C\,(2,4,1)$;
   (i) Calculate $\overrightarrow{AB} \times \overrightarrow{AC}$;
   (ii) Find the area of parallelogram with two adjacent sides AB and AC.

3. (*4 marks*)
   Let ℓ be the straight line in $\mathbb{R}^3$ through the point $P\,(1,2,3)$ and parallel to the vector $\mathbf{v} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$. Let Q be the point with co-ordinates $(2,4,4)$.
   (i) Find $\text{proj}_{\mathbf{v}}\left(\overrightarrow{PQ}\right)$;
   (ii) Find the shortest distance d between the line ℓ and Q;
   (iii) Find the co-ordinates $\mathbf{m}$ of the point M on ℓ which is closest to Q.

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UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 1 VERSION 3a

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working

All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*2 marks*)
   For the points $A\,(3, 2, 1)$ and $B\,(6, 3, -2)$
   (i) Find a parametric vector equation for the line AB.
   (ii) Find Cartesian equations for the line AB.

2. (*2 marks*)
   Find a parametric vector equation for the plane in $\mathbb{R}^3$ with cartesian equation
   $$7x_1 + 2x_2 - x_3 = 1\ .$$
   Hence give two non-parallel, non-zero vectors which are parallel to the plane.

3. (*2 marks*)
   For the points $A\,(1, 4, 1)$, $B\,(3, 5, -2)$ and $C\,(5, 1, 2)$,
   (i) Find $\cos(\angle BAC)$.
   (ii) Find $\text{proj}_{\overrightarrow{AC}}\left(\overrightarrow{AB}\right)$.

4. (*4 marks*)
   In the plane with a cartesian co-ordinate system, let $OACB$ be a parallelogram, with O the origin and $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, where $\mathbf{a} \nparallel \mathbf{b}$.
   (i) Write down (and label as such), parametric vector equations of the lines OC and AB in terms of $\mathbf{a}$ and $\mathbf{b}$.
   (ii) Find the co-ordinates of the point P of intersection of lines OC and AB in terms of $\mathbf{a}$ and $\mathbf{b}$.
   (iii) Show that $|\overrightarrow{OP}| = |\overrightarrow{PC}|$ and $|\overrightarrow{PA}| = |\overrightarrow{PB}|$.

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SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 1 VERSION 4a

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working

All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*2 marks*)
   For the points $A\,(1,2,3)$, $B\,(5,7,-2)$ and $C\,(8,-3,2)$ in $\mathbb{R}^3$;
   (i) Find the co-ordinates $\mathbf{t}$ of the point T on AB such that $\overrightarrow{AT} = 2\overrightarrow{TB}$.
   (ii) Find the co-ordinates $\mathbf{d}$ of the point D such that the quadrilateral $ABCD$ (named in cyclic order) is a parallelogram.

2. (*2 marks*)
   Find a parametric vector equation for the plane in $\mathbb{R}^3$ with cartesian equation
   $$3x_1 - x_2 + 2x_3 = 8\ .$$
   Hence give two non–parallel non–zero vectors which are parallel to the plane.

3. (*2 marks*)
   For $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$, calculate $\mathbf{a} \times \mathbf{b}$.

4. (*4 marks*)
   Let ℓ be the straight line in $\mathbb{R}^3$ through the point $P\,(1,2,3)$ and parallel to the vector $\mathbf{v} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$. Let Q be the point with co-ordinates $(1,4,4)$.
   (i) Find $\text{proj}_{\mathbf{v}}\left(\overrightarrow{PQ}\right)$.
   (ii) Find the shortest distance d between the line ℓ and Q.
   (iii) Find the co-ordinates $\mathbf{m}$ of the point M on ℓ which is closest to Q.

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SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 2 VERSION 1a

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working

All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*2 marks*)
   For the complex numbers $z = 1 + 5i$, $w = 3 - 2i$ calculate
   $$\mathrm{Im}(z + 3iw)\ , \quad z/\overline{w}\ , \quad \mathrm{Arg}(1 - 4i - w)$$
   in simplified cartesian form.

2. (*4 marks*)
   Determine what conditions on b_1, b_2, b_3, b_4 are needed to ensure that $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$ belongs to the span of the vectors $\begin{pmatrix} 1 \\ -2 \\ -2 \\ 6 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -5 \\ -4 \\ 3 \end{pmatrix}$, $\begin{pmatrix} -3 \\ 4 \\ 2 \\ 12 \end{pmatrix}$.

3. (*4 marks*)
   Use the identity
   $$\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$$
   to write $\sin^5\theta$ in terms of $\sin\theta,\ \sin 2\theta,\ \sin 3\theta,\ \ldots$.

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UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 2 VERSION 1b

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working

All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*3 marks*)
   For the complex numbers $z = -2 - 3i$, $w = 1 - i$ calculate
   $$\mathrm{Re}\big((1+3i)z\big)\ , \quad |z^2|\ , \quad \frac{z+1}{w}$$
   in simplified cartesian form.

2. (*4 marks*)
   Determine what conditions on b_1, b_2, b_3, b_4 are needed to ensure that $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$ belongs to the span of the vectors $\begin{pmatrix} 1 \\ 2 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ -3 \\ -7 \end{pmatrix}$.

3. (*3 marks*)
   Use the identity
   $$\cos\theta = \tfrac{1}{2}(e^{i\theta} + e^{-i\theta})$$
   to write $\cos^5\theta$ in terms of $\cos\theta$, $\cos 2\theta$, $\cos 3\theta$, $\ldots$.

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MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 2 VERSION 2a

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working

All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*3 marks*)
   Find the complex square roots of $-24-70i$ by solving $(x+iy)^2 = -24-70i$ for x, y real.

2. (*3 marks*)
   Determine, with reasons, whether or not the lines

$$\ell_1 : \quad \mathbf{x} = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

   and

$$\ell_2 : \quad \mathbf{x} = \begin{pmatrix} 0 \\ -1 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ -1 \\ 6 \end{pmatrix}$$

   intersect.

3. (*4 marks*)

   (i) Find the complex roots of $z^6 + 64 = 0$.

   (ii) Hence factorise $p(z) = z^6 + 64$ into real linear and real irreducible quadratic factors.

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UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 2 VERSION 2b

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working

All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*3 marks*)
   Find the complex square roots of $16 - 30i$ by solving $(x + iy)^2 = 16 - 30i$ for x, y real.

2. (*3 marks*)
   Determine, with reasons, whether or not the lines

$$\ell_1 : \quad \mathbf{x} = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \quad , \quad \lambda \in \mathbb{R}$$

   and

$$\ell_2 : \quad \mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix} \quad , \quad \mu \in \mathbb{R}$$

   intersect.

3. (*4 marks*)

   (i) Find the complex roots of $z^5 - 32 = 0$.

   (ii) Hence factorise $p(z) = z^5 - 32$ into real linear and real irreducible quadratic factors.

Please write your answers on lined A4 paper and staple to this cover sheet.

UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 2 VERSION 3a

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working

All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*3 marks*)

   For the complex numbers $z = -1 - i$, $w = -11 + 7i$ find

   $$(-5-i)\overline{z} + 2w \ , \quad \frac{w}{1+3i} \ , \quad \mathrm{Arg}(2z) \ .$$

2. (*3 marks*)

   Let $z = -\sqrt{3} + 3i$. Find a polar form for z and the principal argument and "$a + ib$" form of z^{19}.

   Powers of real numbers may be left unsimplified.

3. (*4 marks*)

   Find the general solution for the following linear system of equations by setting up an augmented matrix, performing Gaussian Elimination and solving by back substitution.

   $$\begin{array}{rcrcrcrcl} x_1 &+& 3x_2 &-& 2x_3 &+& 4x_4 &=& 2 \\ -2x_1 &-& 4x_2 &+& 5x_3 &-& 9x_4 &=& 0 \\ -x_1 &+& x_2 &+& 4x_3 &-& 6x_4 &=& 6 \end{array}$$

Please write your answers on lined A4 paper and staple to this cover sheet.
