PAST CLASS TESTS

In the years up to 2007 there were 3 algebra class tests per session. From semester 1 2008 there will be only 2 algebra class tests per semester so the pre-2008 tests included here do not have the same coverage of material as the class tests for 2008 and onwards. The Information booklet for MATH1131/1141 lists the material available for examination in the current schedule of class tests, as does page (240) of these notes. Also there have been some changes to the syllabus for 2008 and onwards and some parts of the questions in the following pre-2008 class tests are no longer examinable. Thus the following pre-2008 tests should only be taken as a guide to the level of difficulty to be expected in class test questions for 2008 and onwards.

Sample class tests from 2008 and onwards are included after all the pre-2008 class tests and these tests correspond to the current syllabus and class test schedule. However, the content of the class tests is specified in the Information booklet for MATH1131/1141.

The following selection of past class tests can be used as a guide to the degree of difficulty of algebra class tests. Due to variations in the timing of the mid-semester breaks the material examined in each class test can vary from semester to semester and from year to year. Thus students must consult the Information booklet for MATH1131/1141, or page (240) of these notes, to ascertain the precise topics that may be examined in each algebra class test.

UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 1 VERSION 1a

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working

All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*2 marks*)
   For the points $A\,(4,2,3)$, $B\,(5,-7,-2)$ and $C\,(7,-25,-10)$.
   (i) Find a parametric vector equation of the straight line AB.
   (ii) Determine, with reasons, whether or not the point C is on the straight line AB.

2. (*2 marks*)
   Find a parametric vector equation of the plane in $\mathbb{R}^3$ with Cartesian equation
   $$2x_1 - 5x_2 + x_3 = 7\ .$$
   Hence give two non–parallel non–zero vectors which are parallel to the plane.

3. (*3 marks*)
   For the points $A\,(1,2,3)$, $B\,(3,4,1)$, $C\,(3,3,4)$ calculate
   (i) $\overrightarrow{AB} \times \overrightarrow{AC}$.
   (ii) Area of $\triangle\, ABC$.

4. (*3 marks*)
   Let ℓ be the straight line in $\mathbb{R}^3$ through the point $P\,(1,2,3)$ and parallel to the vector $\mathbf{v} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$. Let Q be the point with co-ordinates $(1,4,4)$.
   (i) Find $\text{proj}_{\mathbf{v}} \left(\overrightarrow{PQ} \right)$.
   (ii) Find the shortest distance d between the line ℓ and Q.

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MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 1 VERSION 1b

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Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

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Show all your working

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QUESTIONS (*Time allowed: 25 minutes*)

1. (*2 marks*)

   Determine, with reasons, whether or not the 3 points $A\,(3,5,7)$, $B\,(5,-4,3)$ and $C\,(-5,41,22)$ are collinear (i.e. all in a straight line).

2. (*2 marks*)

   Find a parametric vector equation for the plane through the points $A\,(1,2,1), B\,(3,4,2), C\,(5,2,1)$.

3. (*3 marks*)

   For the points $A\,(1,2,3)$, $B\,(5,6,4)$ and $C\,(2,1,3)$ calculate;
   (i) the distance $d(A,B)$ between A and B.
   (ii) the projection $\text{proj}_{\overrightarrow{AC}}\left(\overrightarrow{AB}\right)$.

4. (*3 marks*)

   A triangle has vertices at the origin O, at $A(4,-4,8)$ and at $B\,(0,-3,-6)$.

   Let X be a point on the side OA such that $OX = \frac{3}{4}OA$, and Y a point on the side OB such that $OY = \frac{2}{3}OB$.

   Find parametric vector equations for the lines AY and BX and show that they intersect at the point $P\,(2,-3,2)$.

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SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 1 VERSION 2b

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working

All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*3 marks*)
   Consider the line ℓ and plane Π in $\mathbb{R}^3$ with Cartesian equations:

$$\ell: \quad \frac{x-2}{3} = \frac{y+1}{4} = \frac{z+3}{1}$$

$$\Pi: \quad 3x - 2y - 4z = 11 \ .$$

   (i) Find a parametric equation of the line ℓ.

   (ii) Find the co-ordinates of the point P where ℓ meets Π.

2. (*3 marks*)
   For the points $A\,(1,2,1)$, $B\,(3,1,-1)$ and $C\,(2,4,1)$;

   (i) Calculate $\overrightarrow{AB} \times \overrightarrow{AC}$;

   (ii) Find the area of parallelogram with two adjacent sides AB and AC.

3. (*4 marks*)
   Let ℓ be the straight line in $\mathbb{R}^3$ through the point $P\,(1,2,3)$ and parallel to the vector $\mathbf{v} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$. Let Q be the point with co-ordinates $(2,4,4)$.

   (i) Find $\text{proj}_{\mathbf{v}}\left(\overrightarrow{PQ}\right)$;

   (ii) Find the shortest distance d between the line ℓ and Q;

   (iii) Find the co-ordinates $\mathbf{m}$ of the point M on ℓ which is closest to Q.

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SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 1 VERSION 3a

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Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working

All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*2 marks*)
   For the points $A\,(3,2,1)$ and $B\,(6,3,-2)$
   (i) Find a parametric vector equation for the line AB.
   (ii) Find Cartesian equations for the line AB.

2. (*2 marks*)
   Find a parametric vector equation for the plane in $\mathbb{R}^3$ with cartesian equation
   $$7x_1 + 2x_2 - x_3 = 1\ .$$
   Hence give two non-parallel, non-zero vectors which are parallel to the plane.

3. (*2 marks*)
   For the points $A\,(1,4,1)$, $B\,(3,5,-2)$ and $C\,(5,1,2)$,
   (i) Find $\cos(\angle BAC)$.
   (ii) Find $\text{proj}_{\overrightarrow{AC}}\left(\overrightarrow{AB}\right)$.

4. (*4 marks*)
   In the plane with a cartesian co-ordinate system, let $OACB$ be a parallelogram, with O the origin and $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, where $\mathbf{a} \nparallel \mathbf{b}$.
   (i) Write down (and label as such), parametric vector equations of the lines OC and AB in terms of $\mathbf{a}$ and $\mathbf{b}$.
   (ii) Find the co-ordinates of the point P of intersection of lines OC and AB in terms of $\mathbf{a}$ and $\mathbf{b}$.
   (iii) Show that $|\overrightarrow{OP}| = |\overrightarrow{PC}|$ and $|\overrightarrow{PA}| = |\overrightarrow{PB}|$.

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SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 1 VERSION 4a

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Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working

All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*2 marks*)
   For the points $A\,(1,2,3)$, $B\,(5,7,-2)$ and $C\,(8,-3,2)$ in $\mathbb{R}^3$;
   (i) Find the co-ordinates $\mathbf{t}$ of the point T on AB such that $\overrightarrow{AT} = 2\overrightarrow{TB}$.
   (ii) Find the co-ordinates $\mathbf{d}$ of the point D such that the quadrilateral $ABCD$ (named in cyclic order) is a parallelogram.

2. (*2 marks*)
   Find a parametric vector equation for the plane in $\mathbb{R}^3$ with cartesian equation
   $$3x_1 - x_2 + 2x_3 = 8\ .$$
   Hence give two non–parallel non–zero vectors which are parallel to the plane.

3. (*2 marks*)
   For $\mathbf{a} = \begin{pmatrix}1\\2\\3\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}3\\-1\\1\end{pmatrix}$, calculate $\mathbf{a} \times \mathbf{b}$.

4. (*4 marks*)
   Let ℓ be the straight line in $\mathbb{R}^3$ through the point $P\,(1,2,3)$ and parallel to the vector $\mathbf{v} = \begin{pmatrix}2\\3\\1\end{pmatrix}$. Let Q be the point with co-ordinates $(1,4,4)$.
   (i) Find $\text{proj}_{\mathbf{v}}\left(\overrightarrow{PQ}\right)$.
   (ii) Find the shortest distance d between the line ℓ and Q.
   (iii) Find the co-ordinates $\mathbf{m}$ of the point M on ℓ which is closest to Q.

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MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 2 VERSION 1a

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Tutorial Code	Tutor's Name	Mark

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Show all your working

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QUESTIONS (*Time allowed: 25 minutes*)

1. (*2 marks*)
   For the complex numbers $z = 1 + 5i$, $w = 3 - 2i$ calculate

   $$\mathrm{Im}(z + 3iw)\ , \quad z/\overline{w}\ , \quad \mathrm{Arg}(1 - 4i - w)$$

   in simplified cartesian form.

2. (*4 marks*)
   Determine what conditions on b_1, b_2, b_3, b_4 are needed to ensure that $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$ belongs to the span of the vectors $\begin{pmatrix} 1 \\ -2 \\ -2 \\ 6 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -5 \\ -4 \\ 3 \end{pmatrix}$, $\begin{pmatrix} -3 \\ 4 \\ 2 \\ 12 \end{pmatrix}$.

3. (*4 marks*)
   Use the identity

   $$\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$$

   to write $\sin^5\theta$ in terms of $\sin\theta,\ \sin 2\theta,\ \sin 3\theta, \ldots$.

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TEST 2 VERSION 1b

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Student's Family Name	Initials	Student Number
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Show all your working

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QUESTIONS (*Time allowed: 25 minutes*)

1. (*3 marks*)
   For the complex numbers $z = -2 - 3i$, $w = 1 - i$ calculate

   $$\mathrm{Re}\big((1+3i)z\big)\ , \quad |z^2|\ , \quad \frac{z+1}{w}$$

   in simplified cartesian form.

2. (*4 marks*)
   Determine what conditions on b_1, b_2, b_3, b_4 are needed to ensure that $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$ belongs to the span of the vectors $\begin{pmatrix} 1 \\ 2 \\ 4 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} -2 \\ 1 \\ -3 \\ -7 \end{pmatrix}$.

3. (*3 marks*)
   Use the identity

   $$\cos\theta = \tfrac{1}{2}(e^{i\theta} + e^{-i\theta})$$

   to write $\cos^5\theta$ in terms of $\cos\theta$, $\cos 2\theta$, $\cos 3\theta$, ...

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TEST 2 VERSION 2a

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Student's Family Name	Initials	Student Number
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Show all your working

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QUESTIONS (*Time allowed: 25 minutes*)

1. (*3 marks*)
   Find the complex square roots of $-24-70i$ by solving $(x+iy)^2 = -24-70i$ for x, y real.

2. (*3 marks*)
   Determine, with reasons, whether or not the lines

$$\ell_1: \quad \mathbf{x} = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

   and

$$\ell_2: \quad \mathbf{x} = \begin{pmatrix} 0 \\ -1 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ -1 \\ 6 \end{pmatrix}$$

   intersect.

3. (*4 marks*)

   (i) Find the complex roots of $z^6 + 64 = 0$.

   (ii) Hence factorise $p(z) = z^6 + 64$ into real linear and real irreducible quadratic factors.

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TEST 2 VERSION 2b

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Student's Family Name	Initials	Student Number
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Show all your working

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QUESTIONS (*Time allowed: 25 minutes*)

1. (*3 marks*)
   Find the complex square roots of $16 - 30i$ by solving $(x + iy)^2 = 16 - 30i$ for x, y real.

2. (*3 marks*)
   Determine, with reasons, whether or not the lines

$$\ell_1 : \quad \mathbf{x} = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} , \quad \lambda \in \mathbb{R}$$

   and

$$\ell_2 : \quad \mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix} , \quad \mu \in \mathbb{R}$$

   intersect.

3. (*4 marks*)

   (i) Find the complex roots of $z^5 - 32 = 0$.

   (ii) Hence factorise $p(z) = z^5 - 32$ into real linear and real irreducible quadratic factors.

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SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2014
TEST 2 VERSION 3a

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working

All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*3 marks*)
   For the complex numbers $z = -1 - i$, $w = -11 + 7i$ find
   $$(-5 - i)\overline{z} + 2w \ , \quad \frac{w}{1 + 3i} \ , \quad \text{Arg}(2z) \ .$$

2. (*3 marks*)
   Let $z = -\sqrt{3} + 3i$. Find a polar form for z and the principal argument and "$a + ib$" form of z^{19}.

   Powers of real numbers may be left unsimplified.

3. (*4 marks*)
   Find the general solution for the following linear system of equations by setting up an augmented matrix, performing Gaussian Elimination and solving by back substitution.
   $$\begin{aligned} x_1 + 3x_2 - 2x_3 + 4x_4 &= 2 \\ -2x_1 - 4x_2 + 5x_3 - 9x_4 &= 0 \\ -x_1 + x_2 + 4x_3 - 6x_4 &= 6 \end{aligned}$$

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