Answers to selected problems

Chapter 1

1. a) The set of integers between $-\pi$ and π.
   c) The empty set.

3. Answer for both: the interior and boundary of the triangle with vertices at $(0,0)$, $(2,0)$ and $(2,4)$.

4. a) $x < 0$ or $x > 1$ b) $1 < x < 2$ c) $x < -2$ or $x > 0$
   d) $-1 < x < 1$ e) $-2 \leq x < 1$ or $x \geq 3$

5. a) $-4 < x < 2$ b) $x < -5$ or $x > 1$
   c) $-1 < x < -1/3$ d) $x > 0$

6. c) From (a) we have $x^2 + \dfrac{1}{x^2} \geq 2$ with equality if and only if $x = \pm 1$.

7. a) F b) F c) T d) T e) F

8. Hint: $(x^2 + y^2)^2 \geq 4x^2y^2$.

10. a) $-\sqrt{5} \leq x \leq \sqrt{5}$; $0 \leq y \leq \sqrt{5}$
    b) $x \leq -\sqrt{5}$ or $x \geq \sqrt{5}$; $y \geq 0$
    c) $x \neq 8$; $y \neq 0$
    d) $[1, \infty)$; $[0, \infty)$
    e) $(1, \infty)$; $(0, \infty)$
    f) $\{x \in \mathbb{R} : 2n\pi \leq x \leq (2n+1)\pi;\ n \in \mathbb{Z}\}$; $[0, 1]$
    g) The union of the intervals $[-\frac{7\pi}{6} + 2k\pi, \frac{\pi}{6} + 2k\pi]$ where $k \in \mathbb{Z}$; $0 \leq y \leq \sqrt{3}$
    h) $\{x \in \mathbb{R} : x \neq (2n+1)\pi/2,\ n$ an integer$\}$; $[1, \infty)$
    i) $\mathbb{R}$; $[-1, \infty)$

11. a) 22 b) $x^2 + 10x + 22$ c) 6 d) $x^2 + 2$

12. a) $x - 1 + 1/\sqrt{x-1}$ b) $\sqrt{x-1}$ c) $(x-1)^{3/2}$ d) $(1/\sqrt{x-1}) - 1$

16. $[4, 13]$

17. $x = 1, 7$

18. a) If $p(x) = a_0 + a_1 x + \cdots + a_n x^n$ then $p(q(x)) = a_0 + a_1 q(x) + a_2 (q(x))^2 + \cdots + a_n (q(x))^n$. Products and sums of polynomials are again polynomials.

    b) Yes.

Chapter 2

1. a) 1 b) 2 c) 0
   d) Doesn't exist ($\to \infty$). e) 5 f) Doesn't exist.

2. a) 0 b) 0

4. b) 0

5. a) 1 b) $M = 10$ (best possible) c) $M = 1/\sqrt{\epsilon}$ will do.

6. a) 4 b) 0 c) 0 d) 0 e) 0

7. a) Not necessarily, as the information given indicates only that the inequality holds for a subset of (ϵ^{-1}, ∞).

   b) Yes. In fact one can prove that $\lim_{x\to\infty} g(x) = 5$ from the definition of the limit by taking M to be $\frac{1}{\epsilon}$.

8. a) 50 metres per second b) $5 \ln 50 \approx 19.56$ seconds after leaving the plane.

9. a) Yes. If limit of $f(x)$ as $x \to \infty$ does not exist and $f(x) \neq 0$, then $\lim_{x\to\infty} (f(x) - f(x)) = 0$ and $\lim_{x\to\infty} (f(x)/f(x)) = 1$.

   b) Yes, since $g(x) = (f(x) + g(x)) - f(x)$.

   c) No, as in (b).

   d) No. For example if $f(x) = 0$ for all x and $\lim_{x\to\infty} g(x)$ does not exist, we have $\lim_{x\to\infty} (f(x)g(x)) = 0$.

10. a) 10 b) 4 c) 3 d) $-1/9$

11. a) -1 b) 1 c) No

12. a) Doesn't exist. b) Doesn't exist. c) Doesn't exist. d) Doesn't exist.

13. a) 0 b) 0

14. a) $|CB| = \theta$, $|CA| = \sin\theta$, $|DB| = \tan\theta$.

15. Neither the left-hand nor right-hand limits exist due to wild oscillatory behaviour.

Chapter 3

1. b) Yes

2. a) Continuous everywhere. b) Continuous everywhere except at $\pi/2$.

3. $k = 8$

5. Use the intermediate value theorem.

9. a) Yes b) Yes c) No d) Yes

Chapter 4

2. a) $5(4x^3 + 21x^6)$ b) $(4x^3-2)(4x^2+2x+4)+(x^4-2x)(8x+2)$
   c) $(16y - y^4)/(y^3 + 8)^2$ d) $(2x^2 - 4)/(x^2 - 4)^{1/2}$
   e) $-4/(t^2 - 4)^{3/2}$ f) $3\cos 3y + 12\cos 2y \sin 2y$
   g) $(4x^3 - x^4)e^{-x}$ h) $x\ln(x^3 + 1) + 3x^2(x^2 + 1)/2(x^3 + 1)$
   i) $\sec^2 x$ j) $-\tan x$

3. a) 0 b) 0 c) $f'(0) = 0$

4. a) i) $x \neq 0$ ii) all x b) i) all x ii) all x
   c) i) $x \neq -2$ ii) $x \neq -2$

7. $2pf'(a)$

8. a) $x + 17\pi + \cos 2x$ b) $1 - 2\sin 2x$ c) $2-x^2+\cos 2(2-x^2)$
   d) $1 - 2\sin 2(2 - x^2)$ e) $-2x(1 - 2\sin 2(2 - x^2))$

9. a) $\dfrac{dy}{dx} = \dfrac{3x^2 - y}{x - 3y^2}$ b) $\dfrac{dy}{dx} = (y - 4x\sqrt{xy})/(4y\sqrt{xy} - x)$

11. $y = 2$

12. a) (i) $b = 0$ (ii) $a = 1$, $b = 0$
    b) (i) $b = 1$ (ii) $a = 2$, $b = 1$.

13. $a = 1$, $b = 0$

14. a) $f(8.01) \approx f(8) = 2$
    b) i) $y = (x - 8)/12 + 2$
       ii) $f(8.01) \approx (8.01 - 8)/12 + 2 = 2 + \frac{1}{1200}$
    c) The approximation in (b) is much better.

15. $\sqrt{3}\,ar/2$

16. 7/8

17. a) $\frac{1}{8\pi}$
    b) $\frac{32000\pi}{81}\text{cm}^3$

18. a) $\frac{dh}{dt} = \frac{2}{125\pi}$ when $h = 50$.

Chapter 5

1. $\sqrt{\frac{7}{3}}$ b) $\frac{1}{2}$

5. b) 0

7. a) By the Mean Value Theorem, for some c with $16 < c < 17$, $\sqrt{17} - \sqrt{16} = \frac{1}{2\sqrt{c}} < \frac{1}{2\sqrt{16}} = 0.125$.
   b) 0.008
   c) 2×10^{-6}.

8. -1, 1 and 4 are stationary points; 4 is a local minimum point;-1 is a local maximum point.

9. No

10. a) $11, -61$ b) $3, -253$ c) $27/256, -750$
    d) $250, -54$ e) $2, 0$

11. $(12/13, 18/13)$

12. $p_n'(x) = p_{n-1}(x)$, and if $p_{n-1}(x) = 0$ then $p_n(x) = x^n/n!$. These hints are all you need!

13. a) $(400)/(4+\pi)$, $100\pi/(4+\pi)$ b) $0, 100$

14. The greatest distance is $a + 2$; the least distance is $\begin{cases} \sqrt{1 - a^2/3} & \text{if } 0 \le a \le 3/2 \\ |a - 2| & \text{if } a > 3/2. \end{cases}$

15. $a = \frac{\pi}{2}, x = \frac{3\pi}{4}, \frac{7\pi}{4}$; $a = \frac{3\pi}{2}, x = \frac{\pi}{4}, \frac{5\pi}{4}$. The Maple commands
```
with(plots):
animate(plot,[cos(a) + 2*cos(2*x) + cos(4*x-a), x=0..2*Pi],a=0..2*Pi);
```
should confirm your answers.

17. Three real zeros

18. a) $f(t) = -\cos t + t^2/2 + 3$ b) No

19. a) 0 b) 8/3

20. a) $\frac{1}{3}$ b) $\frac{m}{n}$ c) -1 d) $-\frac{1}{2}$ e) $\frac{1}{4}$ f) $\frac{1}{3}$

21. a) $\to 0$ b) $\to \infty$ c) $\to 0$
    d) $\to 1$ e) $\to 1$ f) $\to \frac{3}{2}$

22. Combine the two fractions and apply l'Hôpital twice only. You will need to simplify the quotient obtained after the first application of l'Hôpital. Maple can confirm your answer.

23. $(a, b) = (-\sqrt{2}, \sqrt{2})$ or $(\sqrt{2}, -\sqrt{2})$

26. a) $-1/2$
    b) $a = -1/2$, $b = 1$

27. c) $a = b = 0$

Chapter 6

2. a) $f^{-1}(x) = \dfrac{1}{3}(x - 1)$
   b) $g^{-1}(x) = -\sqrt{x-1}$, Dom$(g^{-1}) = [1, \infty)$,
   Range$(g^{-1}) = (-\infty, 0]$, $(g^{-1})'(x) = \frac{-1}{2\sqrt{x-1}}$

4. b) $1/3$

5. b) The restriction of f to $(-\infty, -1]$ has an inverse with domain $(-\infty, 3]$, the restriction of f to $[-1, 1]$ has an inverse with domain $[-1, 3]$, and the restriction of f to $[1, \infty)$ has an inverse with domain $[-1, \infty)$.

6. a) No b) Yes

7. a) The graph is symmetric about $x = -\frac{1}{2}$, which surely gives a local maximum of $f(x)$. There will be four (maximal) intervals where f will have an inverse. Try this exercise on `Maple`. The commands `plot, diff` and `solve` should suffice.
   b) f is one-to-one; $f^{-1}(x) = x^{1/17} - 1$ is not differentiable when $x = 0$.
   c) I can be one of four intervals.

8. a) $\pi/3$ b) $2/5$ c) $-\pi/3$ d) $\pi/3$
   e) $4/5$ f) $3/\sqrt{34}$ g) $\pi/3$ h) $\pi - x$

11. a) $-2/\sqrt{1-4x^2}$ b) $1/(2\sqrt{x-x^2}\,)$ c) $2/(4x^2 - 12x + 10)$

12. Differentiate; $-1 \le x \le 1$; $\pi/2$.

13. b) $f(x) = \pi/2$ when $x > 0$ and $f(x) = -\pi/2$ when $x < 0$.

14. b) The derivative of the inverse is $-1/x\sqrt{x^2-1}$ when $x > 1$.

16. $a = \pi/2$, $b = 0$

17. a) 24π km/min b) $104\pi/3$ km/min

18. $\sqrt{48}$ metres

Chapter 7

1. $[-1, 5]$, $[0, 3]$, upper half of circle.

2. a) period $2\pi/3$, odd b) period 3π, neither
   c) not periodic, even d) period $\pi/3$, odd
   e) period π, even f) 2π

3. odd, even, neither, odd, odd, even.

4. The asymptotes are

   a) $x = 3$, $y = x + 2$ b) $x = -1$, $y = x - 1$ c) $x = -3$, $x = 2$, $y = x - 1$.

6. a) $x \geq 3$, $-\frac{1}{3} < x \leq \frac{1}{3}$ b) $x = -\frac{1}{3}, x = -3, y = 1$ c) $(1, -\frac{1}{4})$, $(-1, -4)$ d) Domain: $x \neq 3, -\frac{1}{3}$, Range: $(-\infty, -4], [-\frac{1}{4}, \infty)$.

7. a) $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$, ellipse b) $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$, hyperbola
   c) $y = x^{2/3}$ d) spiral

8. a) ii) $(2, 0)$ iii) -1
   b) ii) $(5, 0)$, $(-1, 0)$ iii) $4t^3/3$
   c) ii) $(1, 0)$, $(-1, 0)$ iii) $-\cot t$

9. a) $3x - 27y + 52 = 0$ b) $\frac{1}{9}$

10. a) $y = 3x^{\frac{2}{3}}$.

11. b) Hint: the length of one particular arc of the larger circle equals the length of one arc on the smaller circle.
    d) $x^{2/3} + y^{2/3} = 1$

12. a) $\mathbf{p}(t) = \mathbf{a} + t(\mathbf{b} - \mathbf{a}), \mathbf{p}(0) = \mathbf{a}, \mathbf{p}(1) = \mathbf{b}$ b) $y = 4 - x, \mathbf{q}(1/2)$ is the coodinate vector for the midpoint of B and C c) $p_0(t) = (1-t)^2, p_1(t) = 2t(1-t), p_2(t) = t^2$

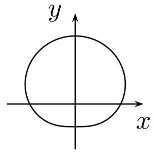
13. a) $(3, 0)$ b) $(-3\sqrt{3}, -3)$ c) $(\sqrt{2}, -\sqrt{2})$

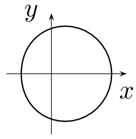
14. a) $(3, \pi)$ b) $(\sqrt{2}, -3\pi/4)$ c) $(4, 2\pi/3)$
    d) $(1, \pi/2)$ e) $(4, 5\pi/6)$ f) $(4, -5\pi/6)$

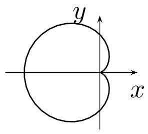
15. a) Circle, centre (0,0), radius 4
    b) A ray in the second quadrant
    c) A spiral of Archimedes

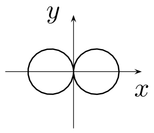
16. a) Circle, centre (0,3), radius 3
    b) Circle, centre (1,0), radius 1

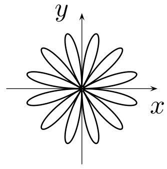
17. The following sketches are a guide to shape only.

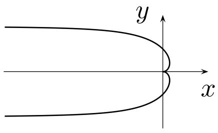
a) 

b) 

c) 

d) 

e) 

f) 

19. $\dfrac{(x-2)^2}{9} + \dfrac{y^2}{5} = 1$

Chapter 8

1. a) i) $\overline{S}_{\mathcal{P}_n}(f) = \underline{S}_{\mathcal{P}_n}(f) = 1$
   ii) $\underline{S}_{\mathcal{P}_n}(f) = \frac{1}{2}\left(1 - \frac{1}{n}\right), \qquad \overline{S}_{\mathcal{P}_n}(f) = \frac{1}{2}\left(1 + \frac{1}{n}\right)$
   iii) $\underline{S}_{\mathcal{P}_n}(f) = \frac{1}{6}\left(1 - \frac{1}{n}\right)\left(2 - \frac{1}{n}\right), \qquad \overline{S}_{\mathcal{P}_n}(f) = \frac{1}{6}\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)$
   v) $\overline{S}_{\mathcal{P}_n}(f) = 1, \qquad \underline{S}_{\mathcal{P}_n}(f) = 0$
   b) i) 1 ii) $\frac{1}{2}$ iii) $\frac{1}{3}$ iv) $\frac{1}{4}$ (v) Not Riemann integrable

2. a) $\sqrt{1365} = 36.95$
   b) $\sqrt{1690.9} = 41.12$ and the lower bound is $\sqrt{1078.9} = 32.85$

4. 4.5

5. a) 82.4 b) 10

6. $f(x) = \dfrac{1}{x^2 + x + 1}$

7. $\frac{1}{x}$ is not differentiable on all of $[-1, 1]$ so the FTC doesn't apply.

8. a) Draw a picture! b) $5\pi/12 - \sqrt{3}/2$

10. F is continuous everywhere, but not differentiable at the integers.

12. a) $\sin x^2$ b) $3x^2 \sin x^6$ c) $-3x^2 \sin x^6$ d) $3x^2 \sin x^6 - \sin x^2$

13. $-(5 - 4x)^5$

14. biii) $\frac{\pi}{6}$.

15. a) $\frac{1}{2}\, e^{x^2} + C$ b) $-2\cos\sqrt{x} + C$ c) $15/4$
    d) $4\sqrt{2}\, a^{9/2}/9$ e) $1/4$ f) $(2\sqrt{2} - 1)/3$

16. a) $2\sqrt{x} - 2\ln(1 + \sqrt{x}) + C$ b) $\frac{1}{25}\left(\frac{1}{21}(5x-1)^{21} + \frac{1}{20}(5x-1)^{20}\right) + C$
    c) $x/(x+1)^2 + C$ d) $4 - 10\ln(7/5)$

17. $\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{t^2-1}{\sqrt{2}\,t}\right)$ for $t \neq 0$

18. a) $\frac{4e^5+1}{25}$ b) $x^2\sin x + 2x\cos x - 2\sin x + C$
c) $x(\ln(x)-1)+C$ d) $\frac{\pi}{12}+\frac{\sqrt{3}}{2}-1$
e) $\frac{7e^8+1}{64}$ f) $\frac{\pi}{2}$
g) $\frac{e^x}{2}(\cos x + \sin x)$ h) $x\tan^{-1}x - \ln\sqrt{1+x^2}+C$
i) $\frac{\sqrt{2}}{2}+\frac{1}{2}\ln(1+\sqrt{2})$

19. a) $1/5$ b) diverges c) $\pi/4$
d) 0 e) 2 f) diverges

21. a) 0 b) $\ln 2$ c) No

22. a) convergent b) divergent c) divergent

23. a) convergent b) divergent c) convergent

24. a) convergent b) divergent c) convergent

25. $s < 0$

26. $p > 1$

27. The integral converges whenever $2a - b > 1$.

28. a) 4, 8, 2
c) $\mathrm{Li}'(x) = \frac{1}{\ln x} > 0$ so Li is an increasing function; $\mathrm{Li}(2) = 0$.
d) $\mathrm{Li}(10^6) \geq \frac{10^6-2}{6\ln 10}$.
e) $\frac{\pi(10^6)}{x} \gtrapprox 0.07238$.

29. a) $\frac{2}{\sqrt{\pi}}e^{-x^2}$
d) (i) $0.749 < \mathrm{erf}(1) < 0.928$ (iii) $1/e$ (iv) 1.344

Chapter 9

2. a) A partition into 7 equal parts will suffice

3. a) $3x^2/2(x^3+1)$ b) e^x for $x > 0$, $-e^{-x}$ for $x < 0$
c) $\frac{1}{(\ln(\ln\, x))(\ln\, x)x}$ d) $5x^4$ (where $x > -6^{1/5}$)

4. a) $\frac{1}{2}\ln(1+e^{2x})$ b) $-e^{1/x}$
c) $3^x/\ln 3$ d) $\frac{e^{\sqrt{x}}}{4}$
e) $\frac{(\ln x)^2}{2}$ f) $\ln|\sin x|$

7. b) e

8. a) $3^x \ln 3$ b) $\left(\dfrac{x^3-3}{x^2+1}\right)^{1/5}\left(\dfrac{3x^2}{5(x^3-3)}-\dfrac{2x}{5(1+x^2)}\right)$
   c) $(\sin x)^{\sin x}\, \cos x\,(1+\ln(\sin x))$ d) $\cos(x^{\sin x})\, x^{\sin x}\left(\cos x\ln x+\dfrac{\sin x}{x}\right)$

9. a) 0 b) 0 c) 1 d) e^2 e) 1
   f) 1 g) e^a h) 0 i) 0

Chapter 10

2. a) $\operatorname{sech}^2 x$ b) $-\operatorname{sech} x \tanh x$ c) $-\operatorname{cosech}^2 x$

3. a) $6x\cosh(3x^2)$ b) $\frac{-\sinh(1/x)}{x^2}$ c) $\frac{1}{2}+\frac{1}{2x^2}$

4. a) $\sinh 2x = 2\,\cosh x \sinh x\,;$ $\cosh 2x = \cosh^2 x + \sinh^2 x$
   b) $\frac{1}{4}(\frac{1}{3}\,\cosh 3x - 3\,\cosh x)$ or $\frac{1}{3}\,\cosh^3 x - \cosh x$

7. a) $\frac{\sinh 4x}{4}$ b) $\frac{1}{12}$ c) $(2x+\sinh 2x)/4$ d) $2\,\cosh\sqrt{x}$

8. 5/4, 3, 5/12

11. a) $2/\sqrt{1+4x^2}$ b) $\frac{1}{1-x^2}$ for $|x|>1$ c) $\sec x$

12. a) $\frac{1}{2}\,\sinh^{-1} 2x$ b) $\tanh^{-1}\frac{1}{2}=\frac{1}{2}\,\ln 3$ c) $\sinh^{-1}\left(\frac{x+2}{3}\right)$

PAST CLASS TESTS

In the years up to 2007 there were 3 calculus class tests per session. From semester 1 2008 there will be only 2 calculus class tests per semester so the pre-2008 tests included here do not have the same coverage of material as the class tests for 2008 and onwards. The Information booklet for MATH1131/1141 lists the material available for examination in the current schedule of class tests. Also there have been some changes to the syllabus for 2008 and onwards and some parts of the questions in the following pre-2008 class tests are no longer examinable. Thus the following pre-2008 tests should only be taken as a guide to the level of difficulty to be expected in class test questions for 2008 and onwards.

Sample class tests from 2008 and onwards are included after all the pre-2008 class tests and these tests correspond to the current syllabus and class test schedule. However, the content of the class tests is specified in the Information booklet for MATH1131/1141.

The following selection of past class tests can be used as a guide to the degree of difficulty of calculus class tests. Due to variations in the timing of the mid-semester breaks the material examined in each class test can vary from semester to semester and from year to year.

UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131 Mathematics 1A Calculus S1 2008
TEST 1 VERSION 5a

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*2 marks*)
   Solve $|2-3x| \leq 1$.

2. (*2 marks*)
   Find the (maximal) domain and the range of the function $f(x) = \dfrac{1}{\sqrt{3-x}}$.

3. (*2 marks*)
   Sketch the graph $y = x^2 - 3x - 10$, and hence sketch the graph $y = \dfrac{1}{x^2 - 3x - 10}$.

4. (*2 marks*)
   For $f(x) = \dfrac{|x^2-9|}{x-3}$ and $a = 3$, discuss the limiting behaviour of $f(x)$ as $x \to a^+$, as $x \to a^-$ and as $x \to a$.

5. (*2 marks*)
   Determine the limiting behaviour of $f(x) = \dfrac{2x + 3x^2 + e^{-x}}{2x^2 + \cos x}$ as $x \to \infty$.

UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131 Mathematics 1A Calculus S2 2008
TEST 1 VERSION 2b

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*2 marks*)
   Let $f(x) = x^2 + 4$, and $g(x) = \dfrac{1}{\sqrt{x+1}}$. Give the explicit forms of $(f \circ g)(x)$ and $(g \circ f)(x)$.

2. (*2 marks*)
   Find the limiting value of $f(x) = \dfrac{x^2 - 5x + 6}{2x^2 - 5x + 2}$ as x tends to 2.

3. (*2 marks*)
   For $f(x) = \dfrac{|x^2 - 4x + 3|}{x - 1}$ and $a = 1$, discuss the limiting behaviour of $f(x)$ as $x \to a^+$, as $x \to a^-$ and as $x \to a$.

4. (*2 marks*)
   Solve $|2 - 3x| \leq 1$.

5. (*2 marks*)
   Find the (maximal) domain and the range of the function $f(x) = \ln(x^2 - 5)$.

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SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Calculus S1 2009
TEST 1 VERSION 8a

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*2 marks*)

   Sketch the graph $y = \sqrt{x+2}$, and hence sketch the graph $y = \dfrac{1}{\sqrt{x+2}}$.

2. (*2 marks*)

   Solve $|3x+2| \geq 1$.

3. (*2 marks*)

   Find the (maximal) domain and range of the function $f(x) = \dfrac{1}{\sqrt{9-x^2}}$.

4. (*2 marks*)

   Determine the limiting behaviour of $f(x) = \dfrac{e^{-x} + 3x^2 - 2}{4x^2 + 3x + \sin x}$ as $x \to \infty$.

5. (*2 marks*)

   For $f(x) = \dfrac{x-2}{|x^2-4|}$ and $a = 2$, discuss the limiting behaviour of $f(x)$ as $x \to a^+$, as $x \to a^-$ and as $x \to a$.

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MATH1131 Mathematics 1A Calculus S1 2009
TEST 1 VERSION 6a

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Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*2 marks*)
   Solve $\dfrac{1}{x+1} \le -\dfrac{1}{2}$.

2. (*2 marks*)
   Find the (maximal) domain and the range of the function $f(x) = \sqrt{2 - e^{-x}}$.

3. (*2 marks*)
   Let $f(x) = 3x + 4$, and $g(x) = \dfrac{1}{\sqrt{x-2}}$. Give the explicit forms of $(f \circ g)(x)$ and $(g \circ f)(x)$.

4. (*2 marks*)
   Find the limiting value of $f(x) = \dfrac{2x^2 - x - 6}{3x^2 - 2x - 8}$ as x tends to 2.

5. (*2 marks*)
   For $f(x) = \dfrac{|x^2 + 3x - 18|}{x - 3}$ and $a = 3$, discuss the limiting behaviour of $f(x)$ as $x \to a^+$, as $x \to a^-$ and as $x \to a$.

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MATH1131 Mathematics 1A Calculus S2 2009
TEST 1 VERSION 1a

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Student's Family Name	Initials	Student Number

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Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*2 marks*)
   Sketch the set of points in the (x, y) plane satisfying $0 < x < 3y$ and $0 < y < 2$.

2. (*2 marks*)
   Solve $\left|\dfrac{3x+1}{2}\right| \leq 2$.

3. (*2 marks*)
   Find the (maximal) domain and the range of the function $f(x) = \dfrac{1}{\sqrt{x-1}}$.

4. (*2 marks*)
   For $f(x) = \dfrac{|x^2 - 4x + 3|}{3 - x}$ and $a = 3$, discuss the limiting behaviour of $f(x)$ as $x \to a^+$, as $x \to a^-$ and as $x \to a$.

5. (*2 marks*)
   Consider the function
   $$f(x) = -\cos x$$
   on the interval $(-\pi/2, \pi/2)$. Determine whether f attains a maximum value on the interval. Give reasons for your answer.

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TEST 1 VERSION 7b

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Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*2 marks*)
   Sketch the graph $y = \sqrt{x-1}$, and hence sketch the graph $y = \dfrac{1}{\sqrt{x-1}}$.

2. (*2 marks*)
   For $f(x) = \dfrac{|x^2+x-2|}{x-1}$ and $a = 1$, discuss the limiting behaviour of $f(x)$ as $x \to a^+$, as $x \to a^-$ and as $x \to a$.

3. (*2 marks*)
   Let $p(x) = x^3 - 3x^2 - 4x + 2$. Use the Intermediate Value Theorem to show that p has a root between -2 and 0.

4. (*2 marks*)
   Solve $\dfrac{1}{x+1} > -\dfrac{1}{2}$.

5. (*2 marks*)
   Find the (maximal) domain and the range of the function $f(x) = \sqrt{3+x}$.

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TEST 2 VERSION 8a

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Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*2 marks*)
   Show that the function f given by $f(x) = x^3 - 2x^2 - 3x + 3$ has a zero in each of the intervals $[0, 1]$ and $[2, 3]$.

2. (*2 marks*)
   Using the definition of the derivative, show that if $f(x) = -2x^2+x$ then $f'(x) = -4x+1$.

3. (*2 marks*)
   The length L of a rectangle is decreasing at the rate of 2 cm per second, and the width W is increasing at the rate of 4 cm per second. Find the rate of change of the area when L=13 cm and W=10 cm.

4. (*2 marks*)
   State the Mean Value Theorem. Find a point which satisfies the conclusions of the Mean Value Theorem for $f(x) = x^3 - 2x^2 + 5$ on the interval $[0, 2]$.

5. (*2 marks*)
   Find $\displaystyle\lim_{x\to 0} \frac{x \sin x}{1 - \cos x}$.

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TEST 2 VERSION 4b

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Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*2 marks*)

   State the Mean Value Theorem and find a point which satisfies the conclusions of the Mean Value Theorem for $f(x) = \sqrt{x-1}$ on the interval $[1, 3]$.

2. (*2 marks*)

   Find $\lim_{x\to 0} \dfrac{1-\cos 3x}{x^2}$.

3. (*2 marks*)

   Show that the function f given by $f(x) = x^3 - 3x^2 - 2x + 5$ has a zero in each of the intervals $[-2, -1]$ and $[1, 2]$.

4. (*2 marks*)

   Using the definition of the derivative, show that if $f(x) = -x^3$ then $f'(x) = -3x^2$.

5. (*2 marks*)

   Find the equation of the line tangent to $x^2 + y^3 - x^2y = 1$ at $(1, 1)$.

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TEST 2 VERSION 8b

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Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

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QUESTIONS (*Time allowed: 20 minutes*)

1. (*2 marks*)

   Carefully state the Mean Value Theorem. Find a point which satisfies the conclusions of the Mean Value Theorem for $f(x) = x^3 - x^2 + 3$ on the interval $[0, 1]$.

2. (*2 marks*)

   Find $\lim_{x \to 1} \dfrac{3x^3 - 5x^2 + x + 1}{x^2 - 2x + 1}$.

3. (*2 marks*)

   Determine the values of x at which the function

   $$f(x) = \begin{cases} x^3 & \text{for } x < 1 \\ (x-1)^3 + 2 & \text{for } x \geq 1 \end{cases}$$

   is continuous. Give reasons for your answer.

4. (*2 marks*)

   Using the definition of the derivative, show that if $f(x) = -x^3$ then $f'(x) = -3x^2$.

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TEST 2 VERSION 1a

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QUESTIONS (*Time allowed: 20 minutes*)

1. (*2 marks*)
The function
$$f(x) = \frac{x-2}{x^2-3x+2}$$
is not defined for $x = 2$. Find a value to be given to $f(2)$ that will make f continuous at 2.

2. (*3 marks*)
Determine all real values of a and b such that the function
$$f(x) = \begin{cases} ax+b & \text{for } x \leq 1 \\ \tan\dfrac{\pi x}{4} & \text{for } 1 < x < 2 \end{cases}$$
is differentiable at $x = 1$.

3. (*3 marks*)
Let $f(x) = |x^2 - 2x|$.
(i) Giving reasons for your answer, find all critical points of f on the interval $[0, 5]$.
(ii) Find the absolute maximum and absolute minimum values of $f(x)$ on the given interval.

4. (*2 marks*)
Find $\displaystyle\lim_{x\to 1} \frac{2x^4 - 3x^3 + x}{(x-1)^2}$.

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TEST 2 VERSION 3a

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Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*2 marks*)

   Find the equation of the line tangent to $x + \ln x = y + 2\ln y$ at $(1, 1)$.

2. (*2 marks*)

   A ladder of length 3 metres is leaning against a vertical wall. The foot of the ladder is pulled away from the wall at the rate of 0.5 metres per second. How fast is the top of the ladder moving down the wall when the foot is 1 metre away from the wall? (Leave your answer in surds.)

3. (*2 marks*)

   Carefully state the Mean Value Theorem and find a point which satisfies the conclusions of the Mean Value Theorem for $f(x) = \sqrt{x-1}$ on the interval $[1, 3]$.

4. (*3 marks*)

   Let $f(x) = (x-1)^{2/3}$.

   (i) Giving reasons for your answer, find all critical points of f on the interval $[0, 2]$.

   (ii) Find the absolute maximum and absolute minimum values of $f(x)$ on tthe given interval.

5. (*1 mark*)

   Find $\lim\limits_{x\to 0} \dfrac{\tan x}{x}$.

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TEST 1 VERSION 2a

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Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*3 marks*)
   Determine all real values of a and b such that the function
   $$f(x) = \begin{cases} ae^x + b & \text{for } x < 0, \\ \sin x & \text{for } x \geq 0 \end{cases}$$
   is differentiable at $x = 0$.

2. (*1 mark*)
   Find $\displaystyle\lim_{x\to 0} \frac{\tan x}{e^{3x} - 1}$.

3. (*2 marks*)
   Find the equation of the line tangent to $x^3 + y^3 - x - y^2 = 0$ at $(1, 1)$.

4. (*3 marks*)
   Determine how many real numbers satisfy the equation $x^3 - 6x^2 + 1 = 0$. Give reasons for your answer, naming any theorems you use. (Hint: find the stationary points of the polynomial function.)

5. (*1 mark*)
   Differentiate $\tan^{-1}(4x + 1)$.

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