ANSWERS TO SELECTED PROBLEMS

Chapter 6

3. a) $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}.$

7. Axioms 1, 4, 5, 6, 10 are satisfied, others are not. It is not a vector space.

8. a) $2\mathbf{v} = (1+1)\mathbf{v} = 1\mathbf{v} + 1\mathbf{v} = \mathbf{v} + \mathbf{v}$. Identify the axioms that have been used.
   b) Use induction.

9. For part 5: If $\lambda\mathbf{v} = \mu\mathbf{v}$ then $\lambda\mathbf{v} - \mu\mathbf{v} = \mathbf{0}$, so $(\lambda - \mu)\mathbf{v} = \mathbf{0}$. But $\mathbf{v} \neq \mathbf{0}$ so, by part 4 of the proposition, $\lambda - \mu = 0$. So $\lambda = \mu$.

11. The position vectors of the points on a plane which does not pass through the origin do not form a vector subspace.

12. a) For example, $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ are in S
    c) The position vectors of the points on a plane which passes through the origin form a vector subspace.

16. Column 1 belongs to S as $A\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$. Similar arguments apply for columns 2 and 3.

17. a) No. b) Yes. c) Yes.

20. $W = \left\{ \begin{pmatrix} 0 \\ 0 \\ \alpha \\ \beta \\ \gamma \end{pmatrix} : \alpha, \beta, \gamma \in \mathbb{R} \right\}$, a copy of $\mathbb{R}^3$.

23. No, S is not a subspace because the zero polynomial is not in it.

24. b) $x^3 + 3x^2 + 3x + 1$.

25. Hint: Suppose that $S_1 = \left\{ \begin{pmatrix} x \\ 0 \end{pmatrix} : x \in \mathbb{R} \right\}$ and $S_2 = \left\{ \begin{pmatrix} 0 \\ y \end{pmatrix} : y \in \mathbb{R} \right\}$. Show that S_1 and S_2 are subspaces of $\mathbb{R}^2$ but $S_1 \cup S_2$ is not.

28. a) Yes, $\mathbf{a} = 2\mathbf{v}_1 - 3\mathbf{v}_2 + \mathbf{v}_3$. b) Yes.

29. a) No. b) No, $3b_2 - 2b_3 = 0$. $\text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ is a plane in $\mathbb{R}^3$.

30. a) Yes, $\mathbf{a} = \mathbf{v}_1 - \mathbf{v}_2 - 2\mathbf{v}_3$
    b) No, $\mathbf{b} \in \text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ if and only if $b_1 - 2b_2 + b_4 = 0$.
    The span is a 3-dimensional subspace in $\mathbb{R}^4$.

31. Yes, $\mathbf{b} = -10\mathbf{v}_1 + 12\mathbf{v}_2 + 16\mathbf{v}_3$.

32. No, $\text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ is plane in $\mathbb{R}^3$ given by $5b_1 - 4b_2 + b_3 = 0$.

33. Yes. $\mathbf{v} = 3\mathbf{v}_1 - \mathbf{v}_2 - 2\mathbf{v}_3$, where $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are the columns of A.

34. No.

35. Yes.

38. Linearly dependent, coplanar.

39. Linearly independent, not coplanar.

40. Set containing $\mathbf{0}$ is linearly dependent as coefficient of zero vector can be varied to make linear combination non-unique.

41. b) $\mathbf{v}_3 = \mathbf{v}_1 + \mathbf{v}_2$.
    e) $\text{span}(S)$ is plane through origin parallel to $\mathbf{v}_1$ and $\mathbf{v}_2$.

42. b) $\mathbf{v}_4 = -2\mathbf{v}_1 + 2\mathbf{v}_2 + \mathbf{v}_3$ e) $\text{span}\,(S) = \mathbb{R}^3$.

43. No. $-2 - x + 5x^2 = (1 - x + 2x^2) + 3(-1 + x^2)$.

44. No, it is impossible to return to origin.

45. Yes, it is possible to return to origin.

47. Yes, it is a basis.

48. A basis for W is $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \right\}$. $\text{Dim}(W) = 2$.

50. a) True. b) False. c) False. d) True. e) False, True, False, False.
f) False. g) True. h) False. i) True. j) True.

51. a) $n \leqslant \ell$ b) No relation. c) $n \geqslant \ell$. d) $n = \ell$.

53. b) $\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$.

54. A basis for $\text{col}(A)$ is $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 4 \\ 2 \\ 0 \end{pmatrix} \right\}$. $\text{Dim}(\text{col}(A)) = 3$.

55. A basis for $\text{col}(A)$ is $\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 4 \\ 4 \end{pmatrix} \right\}$. $\text{Dim}(\text{col}(A)) = 3$.

56. A basis is $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 5 \\ 5 \\ 5 \end{pmatrix}, \begin{pmatrix} -7 \\ -8 \\ -3 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$.

57. a) $B = \{\mathbf{v}_1, \mathbf{v}_3\}$. b) $\mathbf{x} = -4\mathbf{v}_1 + 5\mathbf{v}_3$, c) $\dim(\text{col}(A)) = 2$.

59. Bases are $\{p_1, p_2, p_4\}$ or $\{p_1, p_3, p_4\}$ or $\{p_2, p_3, p_4\}$. $\{p_1, p_2, p_3\}$ is not a basis; why not?

62. Coordinate vector is $\begin{pmatrix} -2 \\ 4 \\ 12 \\ 4 \end{pmatrix}$.

63. $\mathbf{v} = \begin{pmatrix} 18 \\ 7 \\ 11 \\ 19 \end{pmatrix}$.

64. $\mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 10 \end{pmatrix}$.

65. a) $\begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$. b) $\begin{pmatrix} -a_1 - a_2 + 2a_3 \\ a_2 \\ -a_1 + a_3 \end{pmatrix}$.

66. a) $\begin{pmatrix} 12 \\ -8 \\ 21 \end{pmatrix}$; b) $\begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix}$.

67. c) Coordinates are $\lambda_1 = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \cdot \mathbf{v}_1 = -2\sqrt{2}$, $\lambda_2 = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \cdot \mathbf{v}_2 = 2\sqrt{3}$,
$\lambda_3 = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \cdot \mathbf{v}_3 = \sqrt{6}$. Coordinate vector is $\begin{pmatrix} -2\sqrt{2} \\ 2\sqrt{3} \\ \sqrt{6} \end{pmatrix}$.

69. a) For example, $\begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$, $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $\begin{pmatrix} -1 & 0 \\ 0 & 6 \end{pmatrix}$ are in S.
b) S is not a subspace because the 2×2 zero matrix is not in it.

70. a) For example, $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ are in S.
b) Use the Subspace Theorem to prove that S is a subspace.

74. a) $\begin{pmatrix} a_{11} \\ a_{12} \\ a_{21} \\ a_{22} \end{pmatrix}$. b) $\begin{pmatrix} \frac{1}{2}(a_{11} + a_{22}) \\ \frac{1}{2}(a_{12} + a_{21}) \\ \frac{1}{2i}(-a_{12} + a_{21}) \\ \frac{1}{2}(a_{11} - a_{22}) \end{pmatrix}$.

75. a) $\begin{pmatrix} -4 & 2 \\ -1 & -3 \end{pmatrix} = -4\begin{pmatrix} 1 & 0 \\ -2 & 0 \end{pmatrix} + 2\begin{pmatrix} 0 & 1 \\ 3 & 0 \end{pmatrix} - 3\begin{pmatrix} 0 & 0 \\ 5 & 1 \end{pmatrix}$. b) No.

79. Not a subspace.

85. Not a subspace.

87. No.

88. Let $p \in \mathbb{P}_2$ be $p(z) = a_0 + a_1 z + a_2 z^2$. Then the condition is $-\frac{3}{2}a_1 + a_2 = 0$. (An equivalent condition is $p'(\frac{-1}{3}) = 0$.)

89. No for question 87. No for question 88.

90. No. $p_3 = -p_1 + 3p_2$.

91. A basis is $\{p_1, p_2, p_3, p_4, 1, z\}$.

92. $\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$.

93. $\begin{pmatrix} 9 \\ 1 \\ 17 \end{pmatrix}$.

94. $\begin{pmatrix} a_0 - a_1 + a_2 \\ a_0 \\ a_0 + a_1 + a_2 \end{pmatrix}$.

Chapter 7

1. S is not a linear map as the domain $[-1, 1]$ is not a vector space.

2. a) Linear. b) Linear. c) Not Linear. d) Linear. e) Not Linear.

3. a) Domain $\mathbb{C}$, codomain $\mathbb{R}$, linear. b) Domain $\mathbb{C}$, codomain $\mathbb{R}$, linear.
   c) Domain $\mathbb{C}$, codomain $\mathbb{R}_+ = \{x \in \mathbb{R} : x \geqslant 0\}$, not linear.
   d) Domain $\mathbb{C} - \{0\}$, codomain $(-\pi, \pi]$, not linear. e) Domain $\mathbb{C}$, codomain $\mathbb{C}$, linear.

6. No.

7. $T\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 21 \\ 4 \\ -15 \\ 28 \end{pmatrix}$ and $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 - 3x_2 + 4x_3 \\ 2x_1 \\ 3x_1 + x_2 - 5x_3 \\ 4x_1 + 4x_2 + 6x_3 \end{pmatrix}$.

9. $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, and so $T\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

11. a) $\begin{pmatrix} 3 & -1 \\ 2 & 4 \\ -3 & -3 \\ 0 & 1 \end{pmatrix}$. b) $\begin{pmatrix} -2 & 0 & 5 & 0 \\ 6 & -8 & 0 & 2 \\ -2 & 4 & -3 & 0 \end{pmatrix}$. d) $\begin{pmatrix} 1 & 0 & -2 & -4 \\ 3 & -4 & -3 & 1 \\ -2 & 2 & 4 & 0 \end{pmatrix}$.

12. Same as for 11.

13. a) $A\mathbf{e}_1 = 2\mathbf{e}_1$, $A\mathbf{e}_2 = 0.7\mathbf{e}_2$, $A\mathbf{b} = 4\mathbf{e}_1 + 2.1\mathbf{e}_2$. ($\mathbf{e}_1$ is stretched to twice its length, $\mathbf{e}_2$ is compressed to 0.7 of its length and $\mathbf{b}$ is stretched and rotated.)
    d) Notice that $A\mathbf{b} = 3\mathbf{b}$. This means $\mathbf{b}$ is stretched to three times its length.

e) Notice that $A\mathbf{b} = -2\mathbf{b}$. This means the direction of $\mathbf{b}$ is reversed and it is stretched to twice its length.

15. $\frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$.

16. $\mathbf{x}' = T(\mathbf{x}) = \begin{pmatrix} -x_1 \\ x_2 \end{pmatrix}, \qquad A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

17. $\mathbf{x}' = T(\mathbf{x}) = \begin{pmatrix} x_1 \\ x_2 \\ -x_3 \end{pmatrix}, \qquad A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$.

18. $\mathbf{q} = A\mathbf{p}$, where diagonal entries of A are $a_{ii} = -1 + 2d_i^2/|\mathbf{d}|^2$ and off-diagonal entries of A are $a_{ij} = 2d_id_j/|\mathbf{d}|^2$.

19. T is linear. For $T(\mathbf{x}) = A\mathbf{x}$, the matrix is

$$A = \begin{pmatrix} 0 & -b_3 & b_2 \\ b_3 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{pmatrix}.$$

20. $T(\mathbf{a}) = A\mathbf{a}$, where $\quad A = \frac{1}{5}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 4 \end{pmatrix}$.

21. S is not linear.

23. $\begin{pmatrix} a_1' \\ a_2' \\ a_3' \end{pmatrix} = R_\alpha \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} a_1\cos\alpha + a_3\sin\alpha \\ a_2 \\ -a_1\sin\alpha + a_3\cos\alpha \end{pmatrix}$.

25. a) $\ker(A) = \{\mathbf{0}\}$, nullity$(A) = 0$, no basis.

b) Kernel: $\left\{ \lambda \begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} : \lambda \in \mathbb{R} \right\}$, nullity$(B) = 1$.

c) Kernel: $\left\{ \lambda \begin{pmatrix} 2 \\ -2 \\ -1 \\ 1 \end{pmatrix} : \lambda \in \mathbb{R} \right\}$, nullity$(C) = 1$.

26. a) $\left\{ \begin{pmatrix} 1 \\ -3 \\ 1 \\ 0 \end{pmatrix} \right\}$, nullity$(D) = 1$. b) $\{\mathbf{0}\}$, nullity$(E) = 0$.

27. For example $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}$.

28. a) $\ker(A) = \{\mathbf{0}\}$, $\text{nullity}(A) = 0$.

    b) $\ker(A) = \left\{ \lambda \begin{pmatrix} 5 \\ 4 \\ 2 \\ 1 \end{pmatrix} : \lambda \in \mathbb{R} \right\}$, $\text{nullity}(A) = 1$.

    d) $\ker(A) = \left\{ \lambda \begin{pmatrix} 6 \\ 4 \\ 1 \\ 1 \end{pmatrix} : \lambda \in \mathbb{R} \right\}$, $\text{nullity}(A) = 1$.

29. For questions 16, 17 and 18, $\ker(T) = \{\mathbf{0}\}$ and $\text{nullity}(T) = 0$.

    For question 19, $\ker(T) = \{\mathbf{x} \in \mathbb{R}^3 : \mathbf{x} = \lambda\mathbf{b} \text{ for } \lambda \in \mathbb{R}\}$. $\text{Nullity}(T) = 1$. Kernel is set of all vectors parallel to **b**.

    For question 20, $\ker(T) = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{b} \cdot \mathbf{x} = 0\}$. $\text{Nullity}(T) = n - 1$ (Why?). Kernel is set of all vectors orthogonal to **b**.

30. b) 1

31. a) $\mathbf{b} \in \text{im}(A)$ as $A \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 11 \\ 10 \\ 4 \end{pmatrix}$.

    b) **b** is not in $\text{im}(A)$ as $A\mathbf{x} = \begin{pmatrix} 9 \\ -2 \\ -4 \end{pmatrix}$ has no solution.

    c) $\mathbf{b} \in \text{im}(A)$, since, for example, $\mathbf{x} = \begin{pmatrix} -10 \\ 12 \\ 16 \\ 0 \end{pmatrix}$ is a solution of $A\mathbf{x} = \mathbf{b}$.

32. a) No conditions, $\text{im}(A) = \mathbb{R}^3$. b) $3b_2 - 2b_3 = 0$.

    c) No conditions, $\text{im}(A) = \mathbb{R}^3$.

33. $\text{rank}(A) = 3$. Columns 1,2,3 of A form a basis for $\text{im}(A)$.
$\text{rank}(B) = 2$. Columns 1,2 of B form a basis for $\text{im}(B)$.
$\text{rank}(C) = 3$. Columns 1,2,3 of C form a basis for $\text{im}(C)$.
$\text{rank}(D) = 3$. Columns 1,2,4 of D form a basis for $\text{im}(D)$.
$\text{rank}(E) = 3$. Columns 1,2,3 of E form a basis for $\text{im}(E)$.

35. $\text{rank}(A) = 3$. Columns 1,3,4 of A form a basis for $\text{im}(A)$.
$\text{rank}(B) = 3$. Columns 1,3,4 of B form a basis for $\text{im}(B)$.

36. One possible answer is $\left\{ \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$.

37. One possible answer is $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 4 \\ 8 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$.

38. a) $\begin{pmatrix} 3 & 4 & -1 & 0 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. b) $\left\{ \begin{pmatrix} 3 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -3 \\ 0 \\ 1 \end{pmatrix} \right\}$, 3. c) 1. d) No.

46. T is linear.

50. T is a linear function.

53. S is not linear as $\mathbb{Z}$ is not a vector space.
T is not linear as, for example, $T(1.5) + T(1.5) = 2 + 2 = 4$ while $T(1.5 + 1.5) = T(3) = 3$.

54. $y_L(s) = \dfrac{s^2 + 9s + 19}{(s+3)^2(s+1)}$.

56. a) $7 - 2x$ c) $\begin{pmatrix} 2 \\ 1 \\ 1 \\ -1 \end{pmatrix}$.

57. $T \begin{pmatrix} i \\ 2 \\ -1 \end{pmatrix} = (2+i) + (7-4i)z + 2z^2 - 3iz^3,$

$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = (x_1 - 2x_3) + [(2+i)x_1 + (4-3i)x_2]z + x_2z^2 - 3x_1z^3.$

58. b) $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}$. c) $\{x, x^2, x^3, x^4\}$. d) The empty set.

59. Let input vector be $\mathbf{b} = \begin{pmatrix} b_1 & b_2 & b_3 & b_4 & b_5 \end{pmatrix}^T$, where b_1, b_2, b_3, b_4 and b_5 are the amounts of steel, plastics, rubber, glass and labour used respectively. Let the output vector be $\mathbf{x} = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \end{pmatrix}^T$, where x_1, x_2, x_3, x_4 are the numbers of station wagons, 4-wheel drives, hatchbacks and sedans made. Then the factory is represented by the linear map

$T_A : \mathbb{R}^4 \to \mathbb{R}^5$, where $T_A(\mathbf{x}) = \mathbf{b} = A\mathbf{x}$ with

$$A = \begin{pmatrix} 1 & 1.5 & 0.8 & 0.9 \\ 0.5 & 0.6 & 0.7 & 0.6 \\ 0.1 & 0.2 & 0.2 & 0.25 \\ 0.2 & 0.15 & 0.3 & 0.3 \\ 1 & 1.5 & 1.1 & 0.9 \end{pmatrix}.$$

60. The matrix is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

61. The matrix is $\begin{pmatrix} -1 & -3 \\ 2 & 4 \end{pmatrix}$.

62. $-6 + 4x$

63. For 48, $\begin{pmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 3 & -1 & 2 & 4 \end{pmatrix}$. For 49, $\begin{pmatrix} 3 & 4 & 0 & 0 \\ 0 & 3 & 8 & 0 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 3 \end{pmatrix}$.

    For 50, $\begin{pmatrix} -8 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$; For 51, $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}$.

64. 48: $\operatorname{im}(T) = \{p \in \mathbb{P}_4(\mathbb{R}) : p(z) = \lambda_0 + \lambda_1 z + \lambda_2 z^3 + \lambda_3 z^4 \text{ for } \lambda_0, \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}\}$, (note z^2 is not in $\operatorname{im}(T)$) rank $= 4$, $\ker(T) = \{0\}$, nullity$(T) = 0$.

    49: $\operatorname{im}(T) = \mathbb{P}_3(\mathbb{R})$, $\operatorname{rank}(T) = 4$, $\ker(T) = \{0\}$, nullity$(T) = 0$.

    50: $\operatorname{im}(T) = \{p \in \mathbb{P}_3(\mathbb{R}) : p(x) = \lambda_0 + \lambda_1 x + \lambda_2 x^3 \text{ for } \lambda_0, \lambda_1, \lambda_2 \in \mathbb{R}\}$, $\operatorname{rank}(T) = 3$, $\ker(T) = \{p \in \mathbb{P}_3(\mathbb{R}) : p(x) = \lambda x^2 \text{ for } \lambda \in \mathbb{R}\}$, nullity$(T)$ =1.

    51: $\operatorname{im}(T) = \{p \in \mathbb{P}_4(\mathbb{R}) : p(x) = \lambda_1 x + \lambda_2 x^2 + \lambda_3 x^3 + \lambda_4 x^4 \text{ for } \lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \mathbb{R}\}$, $\operatorname{rank}(T) = 4$, $\ker(T) = \{0\}$, nullity$(T) = 0$.

65. The matrix A is diagonal with diagonal elements $a_{kk} = (k-1)(k-2) - 3(k-1) + 3$ for $1 \leqslant k \leqslant n+1$. The kernel is $\alpha_1 x + \alpha_2 x^3$ and the nullity is 2. Note that the kernel is the solution of the homogeneous differential equation.

66. The matrix is $\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & -1 & 0 \end{pmatrix}$.

69. b) i) $\begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$. ii) $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$.

70. b) $\lambda \begin{pmatrix} 1 & -3 \\ -3 & 0 \end{pmatrix}$, $\lambda \in \mathbb{F}$; 1. c) 3. d) No. e) $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 3 & 1 & 0 & 0 \end{pmatrix}$.

71. a) $\left\{ \begin{pmatrix} 2 \\ 4 \\ -2 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 2 \\ -2 \end{pmatrix} \right\}$. b) 3. c) $a_4 - a_1 - a_3 - a_2 = 0$.

d) $\left\{ \begin{pmatrix} \frac{3}{2} \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} \\ 0 \\ -3 \\ -3 \\ 1 \end{pmatrix} \right\}$. e) 3, 2.

Chapter 8

1. a) In each case, the eigenvalues are the diagonal entries and the respective eigenvectors are $t\mathbf{e}_1$ and $t\mathbf{e}_2$ $(t \neq 0)$.

   For interpretations of (b), (c) and (d), see part (e) of question.

2. Eigenvalue is 2.

3. $\lambda = (\det A)^{1/3}$.

4. b) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

5. a) $\lambda = 2$ with eigenvectors $\left\{ t \begin{pmatrix} 1 \\ 2 \end{pmatrix} : t \neq 0 \right\}$ and $\lambda = 3$ with eigenvectors $\left\{ t \begin{pmatrix} 2 \\ 3 \end{pmatrix} : t \neq 0 \right\}$,

   b) $\lambda = -3$ with eigenvectors $\left\{ t \begin{pmatrix} 1 \\ 1 \end{pmatrix} : t \neq 0 \right\}$ and $\lambda = 1$ with eigenvectors $\left\{ t \begin{pmatrix} 1 \\ 3 \end{pmatrix} : t \neq 0 \right\}$.

7. a) $\lambda = 3$ with eigenvectors $\left\{ t \begin{pmatrix} 1 \\ 1 \end{pmatrix} : t \neq 0 \right\}$ and

   $\lambda = -1$ with eigenvectors $\left\{ t \begin{pmatrix} -1 \\ 1 \end{pmatrix} : t \neq 0 \right\}$.

   b) Only one eigenvalue $\lambda = 2$ with multiplicity 2 and eigenvectors $\left\{ t \begin{pmatrix} 1 \\ 0 \end{pmatrix} : t \neq 0 \right\}$.

c) $\lambda = 3$ with eigenvectors $\left\{ t\begin{pmatrix}1\\0\end{pmatrix} : t \neq 0 \right\}$ and
$\lambda = -6$ with eigenvectors $\left\{ t\begin{pmatrix}-5\\9\end{pmatrix} : t \neq 0 \right\}$.

d) $\lambda = 1 \pm i$ with eigenvectors $\left\{ t\begin{pmatrix}-1 \pm i\\1\end{pmatrix} : t \neq 0 \right\}$.

e) $\lambda = 5 \pm i\sqrt{3}$ with eigenvectors $\left\{ t\begin{pmatrix}\pm\frac{\sqrt{3}}{2} + \frac{1}{2}i\\1\end{pmatrix} : t \neq 0 \right\}$.

f) $\lambda = 5 \pm \sqrt{5}$ with eigenvectors $\left\{ t\begin{pmatrix}\frac{1}{2}(1 \mp \sqrt{5})i\\1\end{pmatrix} : t \neq 0 \right\}$.

9. The eigenvalues are the diagonal entries, 2, -2, 3, 5. Corresponding eigenvectors are

$$\mathbf{v}_1 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \qquad \mathbf{v}_2 = \begin{pmatrix}1\\1\\0\\0\end{pmatrix}, \qquad \mathbf{v}_3 = \begin{pmatrix}1\\1\\5\\0\end{pmatrix}, \qquad \mathbf{v}_4 = \begin{pmatrix}25\\-3\\21\\14\end{pmatrix}.$$

10. a) $-1, 4, 6$; $\begin{pmatrix}-3\\2\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$. b) $2, -3, 3$; $\begin{pmatrix}0\\-1\\6\end{pmatrix}, \begin{pmatrix}0\\-1\\1\end{pmatrix}, \begin{pmatrix}1\\0\\0\end{pmatrix}$.

11. In each of the following answers, the diagonal entries in D and the columns in M may be rearranged in the same way and the answer is still correct. Also, any column in M may be multiplied by a scalar and the new M is still correct.

For Question 7:

a) $D = \begin{pmatrix}3 & 0\\0 & -1\end{pmatrix}, \qquad M = \begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}$.

b) The matrix is not diagonalisable.

c) $D = \begin{pmatrix}3 & 0\\0 & -6\end{pmatrix}, \qquad M = \begin{pmatrix}1 & -5\\0 & 9\end{pmatrix}$.

d) $D = \begin{pmatrix}1+i & 0\\0 & 1-i\end{pmatrix}, \qquad M = \begin{pmatrix}1+i & 1-i\\1 & 1\end{pmatrix}$.

e) $D = \begin{pmatrix}5+i\sqrt{3} & 0\\0 & 5-i\sqrt{3}\end{pmatrix}, \qquad M = \begin{pmatrix}\frac{\sqrt{3}}{2} + \frac{1}{2}i & -\frac{\sqrt{3}}{2} + \frac{1}{2}i\\1 & 1\end{pmatrix}$.

f) $D = \begin{pmatrix}5+\sqrt{5} & 0\\0 & 5-\sqrt{5}\end{pmatrix}, \qquad M = \begin{pmatrix}\frac{1}{2}(1-\sqrt{5})i & \frac{1}{2}(1+\sqrt{5})i\\1 & 1\end{pmatrix}$.

For Question 9:

$$D = \begin{pmatrix}2 & 0 & 0 & 0\\0 & -2 & 0 & 0\\0 & 0 & 3 & 0\\0 & 0 & 0 & 5\end{pmatrix}, \qquad M = \begin{pmatrix}1 & 1 & 1 & 25\\0 & 1 & 1 & -3\\0 & 0 & 5 & 21\\0 & 0 & 0 & 14\end{pmatrix}.$$

For Question 10:

a) $D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{pmatrix}, \qquad M = \begin{pmatrix} -3 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$

b) $D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 3 \end{pmatrix}, \qquad M = \begin{pmatrix} 0 & 0 & 1 \\ -1 & -1 & 0 \\ 6 & 1 & 0 \end{pmatrix}.$

15. If $\mathbf{v}$ is an eigenvector of T then the coordinate vector $[\mathbf{v}]_B$ of $\mathbf{v}$ with respect to the basis B is the corresponding eigenvector for the matrix A.

16. $A^5 = \begin{pmatrix} -78 & 330 \\ 55 & -133 \end{pmatrix}.$

17. a) $6, \quad t\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \ t \neq 0, \ t \in \mathbb{R}; \qquad -4, \quad \begin{pmatrix} -3 \\ 4 \end{pmatrix}, \ t \neq 0, \ t \in \mathbb{R}.$

b) $P = \begin{pmatrix} 1 & -3 \\ 2 & 4 \end{pmatrix}, \quad D = \begin{pmatrix} 6 & 0 \\ 0 & -4 \end{pmatrix}.$ c) $\begin{pmatrix} 6^n & (-3)\,(-4)^n \\ 2 \times 6^n & 4(-4)^n \end{pmatrix}.$

18. When A is diagonalisable, $A^k = MD^kM^{-1}$ and $\mathbf{x}_k = A^k\mathbf{x}_0$. As a check, if you put $k = 0$ in the answers below you should get $A^0 = I$, whereas if you put $k = 1$ you should get $A^1 = A$.

For question 7:

a) $A^k = \dfrac{1}{2}\begin{pmatrix} 3^k + (-1)^k & 3^k + (-1)^{k+1} \\ 3^k + (-1)^{k+1} & 3^k + (-1)^k \end{pmatrix}.$

c) $A^k = \dfrac{1}{9}\begin{pmatrix} 3^{k+2} & 5(3^k - (-6)^k) \\ 0 & 9(-6)^k \end{pmatrix}.$

19. $\alpha_1, \ \alpha_2, \ \alpha_3$ and α_4 are arbitrary real numbers.

For question 5:

a) $\mathbf{y}(t) = \alpha_1 e^{3t}\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \alpha_2 e^{2t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}.$

b) $\mathbf{y}(t) = \alpha_1 e^{t}\begin{pmatrix} 1 \\ 3 \end{pmatrix} + \alpha_2 e^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}.$

For question 9:

$$\mathbf{y}(t) = \alpha_1 e^{2t}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \alpha_2 e^{-2t}\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \alpha_3 e^{3t}\begin{pmatrix} 1 \\ 1 \\ 5 \\ 0 \end{pmatrix} + \alpha_4 e^{5t}\begin{pmatrix} 25 \\ -3 \\ 21 \\ 14 \end{pmatrix}.$$

For question 10:

a) $\mathbf{y}(t) = \alpha_1 e^{-t} \begin{pmatrix} -\frac{3}{2} \\ 1 \\ 0 \end{pmatrix} + \alpha_2 e^{4t} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \alpha_3 e^{6t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

b) $\mathbf{y}(t) = \alpha_1 e^{2t} \begin{pmatrix} 0 \\ -\frac{1}{6} \\ 1 \end{pmatrix} + \alpha_2 e^{-3t} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \alpha_3 e^{3t} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.

20. a) 1, $\lambda \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, $\lambda \neq 0$; 5, $\lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\lambda \neq 0$. b) $\begin{cases} x_1 = 3\alpha e^t + \beta e^{5t}, \\ x_2 = -\alpha e^t + \beta e^{5t}. \end{cases}$

21. a) $x = 300e^t - 200e^{3t}$, $y = 150e^t - 50e^{3t}$. b) $x = -500 + 600e^{-2t}$, $y = -100 + 200e^{-2t}$.

22. The solutions by the two methods are:

a) $\mathbf{y}(t) = \begin{pmatrix} y(t) \\ \dot{y}(t) \end{pmatrix} = \alpha_1 e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \alpha_2 e^{t/5} \begin{pmatrix} 1 \\ \frac{1}{5} \end{pmatrix}$, (matrix method)

$y(t) = \alpha_1 e^t + \alpha_2 e^{t/5}$. (calculus method)

b) $\mathbf{y}(t) = \begin{pmatrix} y(t) \\ \dot{y}(t) \end{pmatrix} = \alpha_1 e^{4t} \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \alpha_2 e^{-4t} \begin{pmatrix} 1 \\ -4 \end{pmatrix}$, (matrix method)

$y(t) = \alpha_1 e^{4t} + \alpha_2 e^{-4t}$. (calculus method)

23. The matrix method given in notes is not applicable as the matrix is not diagonalisable.

24. a) $A = \begin{pmatrix} 0 & 1 \\ -\frac{c}{a} & -\frac{b}{a} \end{pmatrix}$.

25. a) 500 years — 0.9048 : 0.0928 : 0.0024
1000 years — 0.8187 : 0.1722 : 0.0091
1000000 years — 1.4×10^{-87} : 7×10^{-44} : 1

b) The associated matrix is not diagonalisable.

26. In the 12th: $\begin{pmatrix} 0.98 & 0.02 & 0.03 \\ 0.01 & 0.96 & 0.03 \\ 0.01 & 0.02 & 0.94 \end{pmatrix}^{11} \begin{pmatrix} 300 \\ 300 \\ 300 \end{pmatrix} \approx \begin{pmatrix} 378 \\ 293 \\ 229 \end{pmatrix}$,

In the 24th: $\begin{pmatrix} 0.98 & 0.02 & 0.03 \\ 0.01 & 0.96 & 0.03 \\ 0.01 & 0.02 & 0.94 \end{pmatrix}^{23} \begin{pmatrix} 300 \\ 300 \\ 300 \end{pmatrix} \approx \begin{pmatrix} 426 \\ 280 \\ 194 \end{pmatrix}$.

27. In the 12th: $\begin{pmatrix} 339 \\ 262 \\ 205 \end{pmatrix}$ total = 806; In the 24th: $\begin{pmatrix} 339 \\ 222 \\ 154 \end{pmatrix}$ total = 715.

28. The population settles to the proportions $1.156 : 1.124 : 1.116 : 1.086 : 1$ but eventually dies out.

31. a) 6, $t\begin{pmatrix}1\\-1\\1\end{pmatrix}$, $t \in \mathbb{R}$; 7, $t\begin{pmatrix}2\\0\\1\end{pmatrix}$, $t \in \mathbb{R}$; 8, $t\begin{pmatrix}2\\1\\1\end{pmatrix}$, $t \in \mathbb{R}$.

b) $A = \begin{pmatrix}1 & 2 & 2\\-1 & 0 & 1\\1 & 1 & 1\end{pmatrix}$, $D = \begin{pmatrix}6 & 0 & 0\\0 & 7 & 0\\0 & 0 & 8\end{pmatrix}$.

c) $A^{-1} = \begin{pmatrix}-1 & 0 & 2\\2 & -1 & -3\\-1 & 1 & 2\end{pmatrix}$,

$$M^k = \begin{pmatrix}-6^k + 4\times 7^k - 2\times 8^k & -2\times 7^k + 2\times 8^k & 2\times 6^k - 6\times 7^k + 4\times 8^k\\ 6^k - 8^k & 8^k & -2\times 6^k + 2\times 8^k\\ -6^k + 2\times 7^k - 8^k & -7^k + 8^k & 2\times 6^k - 3\times 7^k + 2\times 8^k\end{pmatrix}$$

Chapter 9

1. a) $\{a, c\}$, b) $\{f\}$, c) S, d) $\emptyset$,
e) $\{a, b, c, d, e\}$, f) $\{f\}$, g) $\{b\}$, h) $\{b\}$.

2. 32.

3. 81%, 95.3%.

4. 26.

5. a) $A \cap B^c \cap C^c$, b) $A \cup B \cup C$, c) $(A \cap B) \cup (A \cap C) \cup (B \cap C)$,
d) $(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C)$,
e) $(A^c \cap B \cap C) \cup (A \cap B^c \cap C) \cup (A \cap B \cap C^c)$.

6. a) $\frac{5}{36}$. b) $\frac{1}{6}$. c) $\frac{3}{4}$.

7. $\frac{2}{3}$.

8. a) $\frac{3}{50}$, b) $\frac{1}{2}$, c) $\frac{47}{50}$.

9. 32%, $\frac{5}{17}$.

10. a) $\frac{19}{45}$, b) $\frac{11}{25}$, c) $\frac{6}{11}$.

11. a) 25.24%, b) 0.0131, c) 0.000545.

12. No.

13. a) p^n; b) $1-(1-p)^n$.

15. $P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_n|A_1 \cap \cdots \cap A_{n-1})P(A_{n-1}|A_1 \cap \cdots \cap A_{n-2}) \cdots P(A_2|A_1)P(A_1)$, 56%, 33.6%, 22.4%.

19.

x	0	1	2
$P(X=x)$	$\frac{1}{15}$	$\frac{8}{15}$	$\frac{2}{5}$

20. a) 0.214, b) 0.713.

21. $\dfrac{13}{51}$.

22. a) $c = \dfrac{1}{e}$. b) $P(X=2) = \dfrac{1}{2e}$. c) $P(X<2) = \dfrac{2}{e}$. d) $P(X \geqslant 4) = 1 - \dfrac{8}{3e}$.

23. b) $\dfrac{(\lfloor \alpha n \rfloor)^2 + 3\lfloor \alpha n \rfloor + 2}{n^2+3n+2}$, c) $n=5$.

24. a) $\dfrac{\theta(1-\theta^{2n})}{(1-\theta^{2n+1})(1+\theta)}$, b) $\dfrac{1-\theta^{n+1}}{1-\theta^{2n+1}}$.

25. a) $c = 0.1$, b) $E(X) = 2.5$, $\mathrm{Var}(X) = 2.05$, c) $E(Y) = -9$, $\mathrm{Var}(Y) = 32.8$.

26. $\mu = \dfrac{n+1}{2}$; $\sigma^2 = \dfrac{n^2-1}{12}$.

28. $\mu = \dfrac{\alpha}{1-\alpha}$; $\sigma^2 = \dfrac{\alpha}{(1-\alpha)^2}$.

29. 0.1123.

30. $70p^4q^4 + 56p^5q^3 + 28p^6q^2 + 8p^7q + p^8$ where $p = \frac{1}{4}$, $q = \frac{3}{4}$. This evaluates to $\dfrac{7459}{65536} \simeq 0.1138$.

31. 0.383.

32. 11.

34. $P(X=k) = \dfrac{2}{3^k}$ for $k = 0, 1, 2, 3, \ldots$.

35. a) $\dbinom{19}{2}\left(\dfrac{1}{6}\right)^3\left(\dfrac{5}{6}\right)^{17}$. b) $\dbinom{n-1}{k-1}\left(\dfrac{1}{6}\right)^k\left(\dfrac{5}{6}\right)^{n-k}$.

37. a) $B(15, 0.25)$. b) 0.08018. c) No.

38. a) $B(23, 0.5)$. b) 2.86×10^{-6}. c) Yes.

39. a) 6. b) $B(8, 0.5)$ c) 0.1445. d) No.

40. a) 10. b) 12. c) $B(12, 0.5)$. d) 0.01929. e) Yes.

42. a) $\mu = \frac{1}{2}(a+b)$; $\sigma^2 = \frac{1}{12}(a-b)^2$.

b) $\mu = \dfrac{k}{k-1}$ for $k > 1$; $\sigma^2 = \dfrac{k}{(k-1)^2(k-2)}$ for $k > 2$.

c) $\mu = n + 1$, $\sigma^2 = n + 1$.

d) $\mu = 0$, $\sigma^2 = 2$.

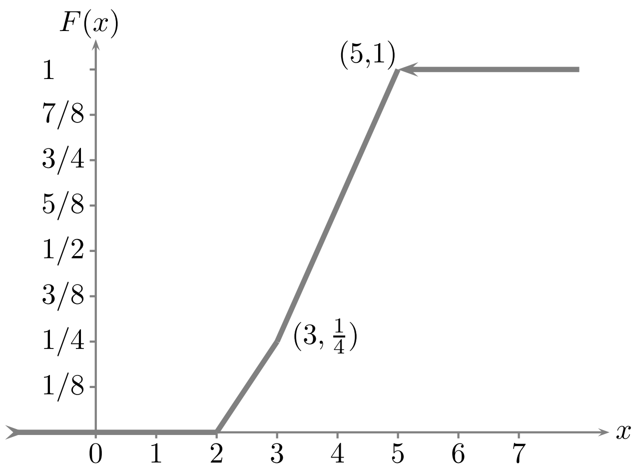
43. a) $\alpha = \dfrac{1}{\pi}$. b) $c = -1$. c) neither exists.

44. a) $\dfrac{9}{16}$, b) $\dfrac{7}{16}$, c) 0.4106, d) $\dfrac{49}{12}$.

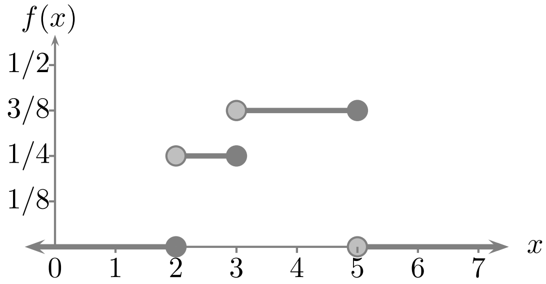
45. a) $\dfrac{1}{\log 10}$. b) $F(y) = \begin{cases} 0 & y < 10 \\ \frac{\log(y/10)}{\log 10} & 10 \leqslant y \leqslant 100 \\ 1 & y > 100 \end{cases}$.

c) $10^{3/2} \approx 31.62$. d) $\dfrac{90}{\log 10} \approx 39.09$.

46. a) The graph of $F(x)$ is ...



b) The function $f(x) = \begin{cases} 0 & x \leqslant 2 \\ \frac{1}{4} & 2 < x \leqslant 3 \\ \frac{3}{8} & 3 < x \leqslant 5 \\ 0 & x > 5 \end{cases}$ will do. Its graph is shown below.



c) $E(X) = 3\frac{5}{8}$.

47. a) $E(Y) = \dfrac{2}{\alpha} + 3$; $\quad \mathrm{Var}(Y) = \dfrac{4}{\alpha^2}$.

48. a) 0.8907. b) 0.0107. c) 0.3594 d) 0.8925. e) 0.1359 f) 0.3830

49. a) 0.8413. b) 0.2514. c) 0.2514 d) 0.6915. e) 0.2789 f) 0.8854

50. a) 0.93. b) -1.65. c) 47 d) 32

51. a) 0.7299. b) 81

52. 0.0401.

53. a) 1.2%, b) 6.7%, c) 75.9%.

54. 2.3% over, 0.4% under.

55. 0.0122

56. a) Binomial(288,0.25). b) $\displaystyle\sum_{k=88}^{288} \binom{288}{k} \left(\frac{1}{4}\right)^k \left(\frac{3}{4}\right)^{288-k}$. c) 0.017. d) Yes.

58. $E(X) = \mathrm{Var}(X) = \frac{1}{2}$.

60. a) $-\frac{1}{\lambda}\log(1-p)$ b) $\frac{1}{\lambda}\log 2$.

62. a) 0.487 b) 0.146 c) 0.264 d) 62.4 min.

63. a) 0.6703 b) 0.1353 c) more than 5.76 hours.

64. 0.4493

65. Let $\lambda = \lambda_1 + \cdots + \lambda_n$

a) $P(T \leqslant t) = 1 - e^{-\lambda t}$, $\quad \begin{cases} 1 - e^{-\lambda t}, & t > 0 \\ 0, & t \leqslant 0. \end{cases}$

b) $\begin{cases} \lambda e^{-\lambda t}, & t > 0 \\ 0, & t \leqslant 0. \end{cases}$

c) The exponential distribution $\mathrm{Exp}(\lambda)$, mean $= \frac{1}{\lambda}$, variance $= \frac{1}{\lambda^2}$.