PAST CLASS TESTS

The 2018 Information booklet for MATH1231/1241 lists the material available for examination in the current schedule of class tests.

The following past tests are samples only and are not a replacement for doing tutorial problems. They are intended to give you some idea regarding the style and length of the class tests.

UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1231 MATHEMATICS 1B ALGEBRA S2 2007
TEST 1 VERSION 1B

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*4 marks*)
Consider the set
$$S = \{ \mathbf{x} \in \mathbb{R}^4 \mid x_1 - 5x_3 = 2x_4 \} .$$
   (i) Show that S is a subspace of $\mathbb{R}^4$.
   (ii) Find one non–zero element in S.

2. (*3 marks*)
Let $\mathbb{P}_2$ be the vector space of all polynomials with degree at most 2.
Do the polynomials $p_1(t) = 1 - t + 2t^2$, $p_2(t) = 2 - t + 3t^2$, $p_3(t) = -1 + 5t - 5t^2$ and $p_4(t) = t - 2t^2$ form spanning set for $\mathbb{P}_2$? Give reasons for your answer.

3. (*3 marks*)
Are the vectors $\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 5 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$ linearly independent? Give reasons for your answer.

UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1231 MATHEMATICS 1B ALGEBRA S2 2009
TEST 1 VERSION 2B

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Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*1 mark*)
   Show that
   $$S = \{ \mathbf{x} \in \mathbb{R}^2 \mid 3x_1 - 2x_2 = 1 \}$$
   is **not** a subspace of $\mathbb{R}^2$.

2. (*3 marks*)
   For the polynomials $p_1(x) = 1 - 3x + 2x^2$, $p_2(x) = 3 - 8x + 5x^2$, $p_3(x) = 1 + x - 2x^2$ and $p_4(x) = -x + 8x^2$, **either** write one of the given polynomials as a linear combination of the others, **or** explain why this is not possible.

3. (*3 marks*)
   Do the vectors
   $$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 4 \\ 3 \\ -7 \end{pmatrix}$$
   form a basis for $\mathbb{R}^3$? Show all your working and give full reasons for your answer.

4. (*3 marks*)
   Are the following statements true or false? Give reasons.
   (i) A set of 5 vectors in $\mathbb{R}^5$ **must** be a basis for $\mathbb{R}^5$.
   (ii) A set of 6 vectors in $\mathbb{R}^5$ **cannot** be a basis for $\mathbb{R}^5$.
   (iii) A set of 7 vectors in $\mathbb{R}^5$ **must** be a spanning set for $\mathbb{R}^5$.

UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1231 MATHEMATICS 1B ALGEBRA S2 2011
TEST 1 VERSION 1A

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Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*3 marks*)
   Show that
   $$S = \{ \mathbf{x} \in \mathbb{R}^3 \mid 7x_1 + 3x_2 = 0 \text{ and } x_2 - 4x_3 = 0 \}$$
   is a subspace of $\mathbb{R}^3$.

2. (*3 marks*)
   Let
   $$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \\ -1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} -2 \\ 1 \\ 1 \\ 6 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}.$$
   Find conditions, if any, on b_1, b_2, b_3, b_4 such that $\mathbf{b}$ is an element of span($\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$).

3. (*2 marks*)
   Let P_n be the vector space of polynomials having degree n or less. Are the following statements true or false? Give reasons.
   (i) A set of 5 polynomials in P_4 **must** be a basis for P_4.
   (ii) A set of 5 polynomials in P_4 **may** be a basis for P_4.

4. (*2 marks*)
   Show that the function $T : \mathbb{R} \to \mathbb{R}$ defined by
   $$T(x) = x \cos x$$
   is not a linear transformation.

Please begin your answers below this line, and continue on the other side of the sheet.
Use extra paper if necessary.

UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1231 MATHEMATICS 1B ALGEBRA S2 2011
TEST 1 VERSION 1B

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*2 marks*)
   Let
   $$S = \{\, \mathbf{x} \in \mathbb{R}^2 \mid x_1 - 3x_2 = 5 \,\} \ .$$
   (i) Write down one non-zero element of S.
   (ii) Show that S is not closed under scalar multiplication.

2. (*4 marks*)
   Let $p_1(x) = 1 + x - 2x^2$, $p_2(x) = -4 - 3x + 3x^2$, $p_3(x) = 1 + 2x - 7x^2$ and $q(x) = 2 + x + x^2$.
   (i) Is the polynomial q an element of $\text{span}(p_1, p_2, p_3)$? Give reasons.
   (ii) Does the set $\{\, p_1, p_2, p_3 \,\}$ span P_2, the vector space of polynomials having degree 2 or less? Explain.

3. (*4 marks*)
   A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ has the values
   $$T\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 8 \end{pmatrix} \quad \text{and} \quad T\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} \ .$$
   (i) Evaluate $T\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.
   (ii) Find a matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^2$.

Please begin your answers below this line, and continue on the other side of the sheet.
Use extra paper if necessary.

UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1231 MATHEMATICS 1B ALGEBRA S2 2011
TEST 1 VERSION 2A

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*3 marks*)
Let A be a fixed 3×2 matrix. Show that
$$S = \{\, \mathbf{x} \in \mathbb{R}^2 \mid A\mathbf{x} = \mathbf{0} \,\}$$
is a subspace of $\mathbb{R}^2$.

2. (*3 marks*)
Let
$$\mathbf{b} = \begin{pmatrix} -1 \\ 5 \\ 5 \\ -6 \end{pmatrix} \quad \text{and} \quad A = \begin{pmatrix} 1 & 4 \\ -1 & -3 \\ 0 & 2 \\ 2 & 7 \end{pmatrix}.$$
Is the vector $\mathbf{b}$ in the column space of A? Give reasons.

3. (*2 marks*)
Let V be a vector space with dimension n, and let S be a set of m vectors from V. Giving reasons, state the relationship (if any) between the numbers m and n if
   (i) S is a basis for V;
   (ii) S is linearly dependent.

4. (*2 marks*)
A function $T : \mathbb{R}^2 \to \mathbb{R}^3$ has values
$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \quad T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} \quad \text{and} \quad T\begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}.$$
Show that T is not a linear transformation.

Please begin your answers below this line, and continue on the other side of the sheet.
Use extra paper if necessary.

UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1231 MATHEMATICS 1B ALGEBRA S2 2012
TEST 1 VERSION 1A

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*3 marks*)
   Show that
   $$S = \{\, \mathbf{x} \in \mathbb{R}^3 \mid x_1 - 2x_2 = 0 \text{ and } 5x_2 + x_3 = 0 \,\}$$
   is a subspace of $\mathbb{R}^3$.

2. (*3 marks*)
   Let
   $$\mathbf{v}_1 = \begin{pmatrix} 1 \\ -2 \\ 0 \\ -3 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 3 \\ -5 \\ 7 \\ -8 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} -2 \\ 5 \\ 1 \\ 1 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}.$$
   Find conditions, if any, on b_1, b_2, b_3, b_4 such that $\mathbf{b}$ is an element of span$(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

3. (*2 marks*)
   Let P_n be the vector space of real polynomials having degree n or less. Are the following statements true or false? Give reasons for your answer.
   (i) A set of 6 polynomials in P_5 **must** be a basis for P_5.
   (ii) A set of 6 polynomials in P_5 **may** be a basis for P_5.

4. (*2 marks*)
   Show that the function $T : \mathbb{R} \to \mathbb{R}$ defined by
   $$T(x) = x^2 \sin x$$
   is not a linear transformation.

Please begin your answers below this line, and continue on the other side of the sheet.
Use extra paper if necessary.

UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1231 MATHEMATICS 1B ALGEBRA S2 2012
TEST 1 VERSION 1B

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*2 marks*)
   Let
   $$S = \{\, \mathbf{x} \in \mathbb{R}^2 \mid x_1 - 5x_2 < 2 \,\} .$$
   (i) Write down one non-zero element of S.

   (ii) Show that S is not closed under scalar multiplication.

2. (*4 marks*)
   Let $p_1(x) = 1 - x + 2x^2$, $p_2(x) = 3 - x$, $p_3(x) = -1 + x^2$ and $q(x) = 1 + 2x - 7x^2$.

   (i) Is the polynomial q an element of $\text{span}(p_1, p_2, p_3)$? Give reasons for your answer.

   (ii) Does the set $\{\, p_1, p_2, p_3 \,\}$ span P_2, the vector space of real polynomials having degree 2 or less? Explain.

3. (*4 marks*)
   A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ has the values
   $$T\begin{pmatrix}5\\3\end{pmatrix} = \begin{pmatrix}8\\-1\end{pmatrix} \quad \text{and} \quad T\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}3\\-1\end{pmatrix} .$$
   (i) Evaluate $T\begin{pmatrix}1\\0\end{pmatrix}$.

   (ii) Find a matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^2$.

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UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1231 MATHEMATICS 1B ALGEBRA S2 2009
TEST 2 VERSION 3B

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Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*3 marks*)
   Let $\mathbb{P}_n$ be the vector space of polynomials with degree at most n. Prove that the function $T : \mathbb{P}_3 \to \mathbb{R}^2$ defined by
   $$T(p(x)) = \begin{pmatrix} p(1) \\ p(3) \end{pmatrix}$$
   is a linear transformation.

2. (*3 marks*)
   Let
   $$A = \begin{pmatrix} -1 & 2 & 1 & 0 & 1 \\ 1 & -1 & 0 & -1 & 0 \\ 3 & -2 & 1 & -4 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix} .$$
   (i) Find, giving reasons, rank(A) and nullity(A).
   (ii) Is $\mathbf{b}$ in the image of A? Explain.

3. (*1 mark*)
   The 2×2 matrix $A = \begin{pmatrix} 11 & -3 \\ 8 & 1 \end{pmatrix}$ has eigenvalues and eigenvectors
   $$\lambda_1 = 5 , \quad \mathbf{v}_1 = \alpha \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \lambda_2 = 7 , \quad \mathbf{v}_2 = \alpha \begin{pmatrix} 3 \\ 4 \end{pmatrix} .$$
   Write down the general solution of the system of differential equations
   $$\frac{dy_1}{dt} = 11y_1 - 3y_2 , \qquad \frac{dy_2}{dt} = 8y_1 + y_2 .$$

4. (*3 marks*)
   A discrete random variable X has probability distribution given by
   $$p_k = ck^2 \quad \text{for } 1 \le k \le 5 .$$
   (i) Find the value of the constant c.
   (ii) Calculate $P(X = 3)$.
   (iii) Calculate $P(X \le 4)$.

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TEST 2 VERSION 7A

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Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*4 marks*)
   For the matrix
   $$A = \begin{pmatrix} 1 & -3 & -1 & 1 \\ 2 & -5 & 0 & 1 \\ -3 & 5 & -6 & 3 \end{pmatrix}$$
   (i) find, giving reasons, rank(A) and nullity(A);
   (ii) find a basis for the kernel of A.

2. (*3 marks*)
   Find all the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} -7 & 5 \\ 2 & -4 \end{pmatrix}$.

3. (*3 marks*)
   A new diagnostic test is being developed for a disease which affects 6% of the population. A person who has the disease tests positive in 90% of cases, and a person who does not have the disease tests positive in 5% of cases. If the test is administered to a randomly chosen person, find the probability that
   (i) the person has the disease and tests positive;
   (ii) the person tests positive;
   (iii) the person has the disease, given that the test is positive (leave your answer as a fraction).

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MATH1231 MATHEMATICS 1B ALGEBRA S2 2011
TEST 2 VERSION 1A

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Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*3 marks*)
   Let P_n be the vector space of polynomials with degree at most n. Prove that the function $T : P_2 \to \mathbb{R}^2$ defined by
   $$T(p(x)) = \begin{pmatrix} p(1) \\ p'(2) \end{pmatrix}$$
   is a linear transformation.

2. (*4 marks*)
   For the matrix
   $$A = \begin{pmatrix} 1 & 2 & 1 & -1 & 3 \\ -2 & -5 & -1 & 3 & -8 \\ 1 & 5 & -2 & -3 & 5 \end{pmatrix}$$
   (i) find, giving reasons, rank(A) and nullity(A);
   (ii) find a basis for the kernel of A.

3. (*3 marks*)
   A bag contains four red dice and three blue dice: each of the red dice has six faces marked $1, 1, 1, 1, 2, 3$ and each of the blue dice has six faces marked $1, 2, 2, 3, 3, 3$. If one of the dice in the bag is selected at random and thrown and the number on the top face is noted, find the probability that
   (i) one of the red dice was selected and a 1 was thrown;
   (ii) a 1 was thrown;
   (iii) one of the red dice was selected, given that a 1 was thrown.

Please begin your answers below this line, and continue on the other side of the sheet.
Use extra paper if necessary.

UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1231 MATHEMATICS 1B ALGEBRA S2 2011
TEST 2 VERSION 1B

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*3 marks*)
   Let
   $$A = \begin{pmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 3 & 1 \\ 2 & -1 & 7 & 5 \\ -1 & 1 & -2 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 6 \\ -7 \end{pmatrix}.$$
   (i) Find, giving reasons, rank(A) and nullity(A).
   (ii) Is $\mathbf{b}$ in the image of A? Explain.

2. (*4 marks*)
   (i) Find all the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 7 & -1 \\ 6 & 2 \end{pmatrix}$.
   (ii) Is A diagonalisable? Give reasons.

3. (*3 marks*)
   Show that the sequence defined by
   $$p_k = \frac{20}{5^k} \qquad \text{for } k = 2, 3, 4, \ldots$$
   is a probability distribution.

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UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1231 MATHEMATICS 1B ALGEBRA S2 2011
TEST 2 VERSION 2A

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Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*3 marks*)
Let
$$A = \begin{pmatrix} 1 & 3 & 1 & 2 & 0 \\ 1 & 5 & 0 & 3 & 1 \\ -2 & 4 & -7 & 2 & 4 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

   (i) Find, giving reasons, rank(A) and nullity(A).

   (ii) Find condition(s), if any, on b_1, b_2, b_3 for $\mathbf{b}$ to belong to im(A).

2. (*4 marks*)
For
$$A = \begin{pmatrix} 2 & -5 \\ -3 & 4 \end{pmatrix}$$
find a diagonal matrix D and an invertible matrix M such that $A = MDM^{-1}$.

3. (*3 marks*)
At Angry Jack's takeaway outlets 60% of the customers buy hamburgers. Of the customers who buy hamburgers, 30% also buy drinks; of those who do not buy hamburgers, 20% buy drinks. If a customer is chosen at random, find the probability that

   (i) the customer bought a hamburger and a drink;

   (ii) the customer bought a drink;

   (iii) the customer bought a hamburger, given that he/she bought a drink (leave your answer as a fraction).

Please begin your answers below this line, and continue on the other side of the sheet.
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UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1231 MATHEMATICS 1B ALGEBRA S2 2012
TEST 2 VERSION 1A

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number

Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*3 marks*)
   Let P_n be the vector space of polynomials with degree at most n. Prove that the function $T : P_2 \to \mathbb{R}^3$ defined by
   $$T(p(x)) = \begin{pmatrix} p(0) \\ p(1) \\ p(2) \end{pmatrix}$$
   is a linear transformation.

2. (*4 marks*)
   For the matrix
   $$A = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 3 & 5 & 0 & 4 \\ 2 & 6 & 8 & 1 \end{pmatrix}$$
   (i) find, giving reasons, rank(A) and nullity(A);
   (ii) find a basis for the kernel of A.

3. (*3 marks*)
   In a hypothetical city 40% of drivers drive red cars. Each month 30% of the red cars are caught speeding, while 5% of the non–red cars are caught. If one car in the city is selected at random, find the probability that
   (i) the car was red and it was caught speeding;
   (ii) the car was caught speeding;
   (iii) the car was red, given that it was caught speeding.

Please begin your answers below this line, and continue on the other side of the sheet.
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UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1231 MATHEMATICS 1B ALGEBRA S2 2012
TEST 2 VERSION 1B

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*3 marks*)
   Let
   $$A = \begin{pmatrix} 1 & 2 & -1 \\ -3 & -3 & 2 \\ 1 & 2 & 1 \\ 2 & 7 & -3 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 3 \\ -8 \\ 4 \\ 0 \end{pmatrix}.$$
   (i) Find, giving reasons, rank(A) and nullity(A).
   (ii) Is $\mathbf{b}$ in the image of A? Explain.

2. (*4 marks*)
   (i) Find all the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 7 & -1 \\ 8 & 1 \end{pmatrix}$.
   (ii) Is A diagonalisable? Give reasons.

3. (*3 marks*)
   Show that the sequence defined by
   $$p_k = \frac{6}{3^k} \qquad \text{for } k = 2, 3, 4, \ldots$$
   is a probability distribution.

Please begin your answers below this line, and continue on the other side of the sheet.
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UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1231 MATHEMATICS 1B ALGEBRA S2 2012
TEST 2 VERSION 2A

This sheet must be filled in and stapled to the front of your answers

Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

QUESTIONS (*Time allowed: 20 minutes*)

1. (*3 marks*)
   Let
   $$A = \begin{pmatrix} -1 & 1 & 3 & 2 & -1 \\ 2 & 0 & -5 & -3 & -1 \\ 1 & 1 & -2 & -1 & 3 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} .$$
   (i) Find, giving reasons, rank(A) and nullity(A).
   (ii) Find condition(s), if any, on b_1, b_2, b_3 for $\mathbf{b}$ to belong to im(A).

2. (*4 marks*)
   For
   $$A = \begin{pmatrix} -3 & 4 \\ 2 & 4 \end{pmatrix}$$
   find a diagonal matrix D and an invertible matrix M such that $A = MDM^{-1}$.

3. (*3 marks*)
   At Angry Jack's takeaway outlets 60% of the customers buy hamburgers. Of the customers who buy hamburgers, 30% also buy drinks; of those who do not buy hamburgers, 20% buy drinks. If a customer is chosen at random, find the probability that
   (i) the customer bought a hamburger and a drink;
   (ii) the customer bought a drink;
   (iii) the customer bought a hamburger, given that he/she bought a drink (leave your answer as a fraction).

Please begin your answers below this line, and continue on the other side of the sheet.
Use extra paper if necessary.