# THE UNIVERSITY OF NEW SOUTH WALES SCHOOL OF MATHEMATICS AND STATISTICS

### Semester 2 2017

## MATH1241 HIGHER MATHEMATICS 1B

- (1) TIME ALLOWED TWO (2) HOURS
- (2) TOTAL NUMBER OF QUESTIONS 4
- (3) ANSWER ALL QUESTIONS
- (4) THE QUESTIONS ARE OF EQUAL VALUE
- (5) ANSWER EACH QUESTION IN A SEPARATE BOOK
- (6) THIS PAPER MAY BE RETAINED BY THE CANDIDATE
- (7) **ONLY** CALCULATORS WITH AN AFFIXED "UNSW APPROVED" STICKER MAY BE USED
- (8) A SHORT TABLE OF INTEGRALS and A STANDARD NORMAL TABLE ARE APPENDED ON THE LAST PAGES

All answers must be written in ink. Except where they are expressly required pencils may only be used for drawing, sketching or graphical work.

1. i) Evaluate each of the following integrals.

a) 
$$I_1 = \int \frac{1}{\sqrt{x^2 + 4x + 5}} dx$$

b) 
$$I_2 = \int \cos^3 x \sin^2 x \, dx$$

c) 
$$I_3 = \int \frac{x+7}{(x+1)^2(x-2)} dx$$

ii) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 2x.$$

iii) Let 
$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$
,  $\mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ , and  $\mathbf{v}_3 = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$ .

- a) Write  $\mathbf{v}_3$  as a linear combination of  $\mathbf{v}_1$  and  $\mathbf{v}_2$ .
- b) Does there exist a linear map  $T: \mathbb{R}^3 \to \mathbb{R}^3$  such that

$$T(\mathbf{v}_1) = \begin{pmatrix} 1 \\ 6 \\ 3 \end{pmatrix}, \quad T(\mathbf{v}_2) = \begin{pmatrix} -2 \\ 16 \\ 2 \end{pmatrix}, \quad \text{and} \quad T(\mathbf{v}_3) = \begin{pmatrix} -6 \\ -3 \\ -8 \end{pmatrix}?$$

c) What is the relationship between  $\operatorname{span}(\mathbf{v}_1, \mathbf{v}_2)$  and  $\operatorname{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ ?

iv) Let 
$$S = \{ \mathbf{x} \in \mathbb{R}^3 : x_1^2 - x_2^2 = x_3^2 \}.$$

- a) Prove that S is closed under scalar multiplication.
- b) Show that S is not a subspace of  $\mathbb{R}^3$ .

2. i) The volume V of a tumour can be modelled by the differential equation

$$\frac{dV}{dt} = \alpha \left( 1 - \frac{V}{K} \right) V, \tag{*}$$

where t is time, V is the volume of the tumour at time t and  $\alpha$  and K are positive constants. If the initial value  $V(0) = V_0$  is imposed, solving (\*) as a separable equation gives the non constant solution

$$V(t) = \frac{K}{1 + \left(\frac{K}{V_0} - 1\right)e^{-\alpha t}}.$$

- a) Find all constant solutions to equation (\*).
- b) Find the behaviour of V(t) as  $t \to \infty$ .
- c) Give an interpretation of the constants  $\alpha$  and K.
- d) Another model for tumour growth is given by the differential equation

$$\frac{dV}{dt} = -\alpha \ln\left(\frac{V}{K}\right) V. \tag{**}$$

Suppose the same constants  $\alpha$  and K are used in the two models. Without solving (\*\*), explain which model predicts faster tumour growth for tumours when V much smaller than K?

ii) The specific gravity z of a solid heavier than water is given by

$$z = \frac{x}{x - y},$$

where x and y are its weight in air and water respectively. The weights x and y are observed to be 21.3g and 10.2g and each observation is made with an uncertainty whose absolute value is at most 0.1g.

- a) Find  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$ .
- b) Use the total differential approximation for z to estimate the maximum uncertainty in the calculated value of z (to 3 decimal places).

You may find the following Maple session useful.

$$> zy := diff(z,y):$$

-0.08278548821

$$> subs(x=21.3, y=10.2, zy);$$

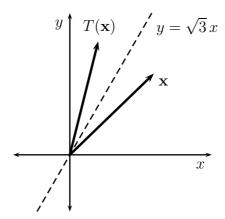
0.1728755783

iii) The probability density function f of a continuous random variable X is given by

$$f(x) = \begin{cases} kx^2 & \text{for } 0 \le x \le 3\\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

- a) Find the value of k.
- b) Evaluate E(X) and Var(X).
- iv) Let  $T: \mathbb{R}^2 \to \mathbb{R}^2$  be the linear map which reflects a vector in the line  $y = \sqrt{3} x$  as show in the diagram.



a) Show that

$$T(\mathbf{e}_1) = \begin{pmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}, \quad T(\mathbf{e}_2) = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}, \quad \text{where } \mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

b) Find the matrix A such that

$$A\mathbf{x} = T(\mathbf{x}), \text{ for all } \mathbf{x} \in \mathbb{R}^2.$$

c) Find  $T\left(\begin{pmatrix} 4\\5 \end{pmatrix}\right)$ .

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- v) Read the following Maple output and use it to answer the questions below.
  - > with(LinearAlgebra):
  - > A := <<1,2,7,4,3>|<-1,6,2,8,1>|<2,-4,5,-4,2>|
    <2,3,-1,5,7>|<-1,14,11,20,5>>;

$$A := \begin{bmatrix} 1 & -1 & 2 & 2 & -1 \\ 2 & 6 & -4 & 3 & 14 \\ 7 & 2 & 5 & -1 & 11 \\ 4 & 8 & -4 & 5 & 20 \\ 3 & 1 & 2 & 7 & 5 \end{bmatrix}$$

> GaussianElimination(A);

> b := <-2,35,16,49,15>:

$$b := \begin{bmatrix} -2 \\ 35 \\ 16 \\ 49 \\ 15 \end{bmatrix}$$

> LinearSolve(A,b);

$$\begin{bmatrix} 1 - _{1}t_{3} - _{1}t_{5} \\ 5 + _{2}t_{3} - 2 _{2}t_{5} \\ _{2}t_{3} \\ 1 \\ _{2}t_{5} \end{bmatrix}$$

Let A be the matrix A defined in the Maple code above.

- a) Give a basis for the kernel of the matrix A.
- b) Find one vector in  $\mathbf{x} \in \mathbb{R}^5$  such that

$$A\mathbf{x} = \begin{pmatrix} -2\\35\\16\\49\\15 \end{pmatrix}.$$

- 3. i) Let  $\mathbb{P}_2$  be the vector space of all real polynomials of degree at most 2.
  - a) Find three polynomials  $f_1, f_2, f_3$  in  $\mathbb{P}_2$  such that  $f_i(0) = 1$  for i = 1, 2, 3 and  $\{f_1, f_2, f_3\}$  is linearly independent.
  - b) Suppose that  $P = \{p_1, p_2, p_3\}$  is a subset of  $\mathbb{P}_2$  such that  $p_i(1) = 0$  for i = 1, 2, 3. Show that P is a linearly dependent set.
  - ii) Let  $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$ .
    - a) Given that the eigenvalues of A are 1, 2, 3, explain why A is diagonalisable.
    - b) Find an eigenvector of A for the eigenvalue  $\lambda = 3$ .
    - c) Let  $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$  and

$$f(x) = (x-1)(x-2)(x-3) = x^3 - 6x^2 + 11x - 6.$$

Show that  $f(D) = D^3 - 6D^2 + 11D - 6I$  is the zero matrix **0**.

- d) Hence, prove that f(A) = 0.
- e) Compute  $A^{-1}$  as a linear combination of  $A^2$ , A, I.
- iii) Let  $\mathbb{F} = \{0, 1\}$  be the field of 2 elements, where 1 + 1 = 0, and let  $V = \mathbb{F}^3$  be the set of all 3-tuples with elements in  $\mathbb{F}$ . For example,

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

You are **given** that V forms a vector space over the field  $\mathbb{F}$ . Two (not necessarily distinct) vectors  $\mathbf{v}_1$ ,  $\mathbf{v}_2$  are chosen at random from V. Let S denote the sample space. Define E to be the event that  $\mathbf{v}_1$ ,  $\mathbf{v}_2$  are linearly independent.

- a) Find |S|.
- b) Show that  $P(E) = \frac{21}{32}$ .
- c) Consider the discrete random variable  $X = \dim(\operatorname{span}(\mathbf{v}_1, \mathbf{v}_2))$ . Copy and complete the following table for the probability distribution  $p_k = P(X = k)$ .

k	0	1	2		
$p_k$					

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iv) Mo's mobile phone company produces phones with an average lifetime of 4.2 years. Suppose that the lifetime is an approximately normally distributed random variable with standard deviation 1.3 years. Mo wishes to offer a warranty on his phones, but figures that it is only profitable to do so if he replaces fewer than 2% of the stock he sells, during the warranty period. What warranty length should he offer?

4. i) Consider the differential equation

$$x^2y' = xy - \frac{1}{2}e^xy^3.$$

- a) Show that the substitution y(x) = xu(x) leads to a separable equation for u(x).
- b) Solve the inital value problem  $y(1) = 1/\sqrt{e}$ .
- ii) Consider the power series expansion

$$\frac{1}{\sqrt{1 - 2cx + x^2}} = \sum_{n=0}^{\infty} a_n x^n,$$

where c is a constant.

- a) Find the first two coefficients  $a_0$  and  $a_1$ .
- b) Show that

$$(c-x)\sum_{n=0}^{\infty} a_n x^n = (1 - 2cx + x^2)\sum_{n=1}^{\infty} na_n x^{n-1}$$

and, hence, or otherwise, determine  $a_2$ .

iii) Consider the sequence  $\{a_n\}$  defined recursively by

$$a_{n+1} = \frac{a_n^2 + \pi^2}{a_n + \pi}, \quad a_0 = 1.$$

The following Maple session may be useful.

> factor(Pi -  $(a[n]^2 + Pi^2)/(a[n] + Pi));$ 

$$\frac{a_n(\pi - a_n)}{a_n + \pi}$$

- a) Show by induction that  $a_n < \pi$ .
- b) By considering  $a_{n+1} a_n$ , show that the sequence is monotonically increasing.
- c) State clearly why the sequence converges and determine its limit.

iv) Consider the sequence  $\{a_n\}$  given by

$$a_n = \frac{\cos n + n}{n^3 - e^{-n}}.$$

a) Does the series

$$\sum_{n=1}^{\infty} a_n$$

converge? Give reasons for your answer.

b) Determine the interval of convergence of the power series

$$\sum_{n=1}^{\infty} a_n (x+2)^n.$$

You may use the fact that the sequence  $\{a_n\}$  is monotonically decreasing.

Standard normal probabilities  $P(Z \le z)$ 

z. 9.         .00         .01         .02         .03         .00         .05         .06         .07         .08         .09           -2.9         0.0016         0.0015         0.0015         0.0015         0.0014         0.0014         0.0016         0.015         0.0014         0.0019         0.0021         0.0021         0.0021         0.0021         0.0027         0.0026         0.0019         0.0033         0.0033         0.0033         0.0039         0.0038         0.0037         0.0036         0.0029         0.0006         0.0069         0.0069         0.0064         0.0063         0.00091         0.0064         0.0063         0.0069         0.0069         0.0069         0.0069         0.0064         0.0069         0.0066         0.0064         0.0019         0.0097         0.0074         0.0019         0.0019         0.0019         0.0019         0.0019         0.0068         0.0066         0.0062         0.0024         0.0021         0.0024         0.0024         0.0021         0.0024         0.0021         0.0024         0.0024         0.0034         0.0011         0.0116         0.0134         0.0034         0.0014         0.0014         0.0014         0.0014         0.0014         0.0024         0.0024         <		00	01	00	02	0.1	05	0.0	07	00	00
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−2.2         0.0139         0.0136         0.0132         0.0129         0.0125         0.0122         0.0121         0.0113         0.0113         0.0113           −2.0         0.0228         0.0222         0.0217         0.0212         0.0207         0.0202         0.0150         0.0183         0.0183           1.9         0.0287         0.0281         0.0244         0.0236         0.0329         0.0326         0.0290         0.0210         0.0244         0.0239         0.0231           1.7         0.0446         0.0436         0.0427         0.0418         0.0409         0.0401         0.0392         0.0334         0.0307         0.0307           1.1         0.0468         0.0655         0.0651         0.0505         0.0405         0.0475         0.0455         0.0477         0.0475         0.0485         0.0477         0.0375         0.0371         0.0478         0.0475         0.0451         0.0493											
−2-1         0.0179         0.0174         0.0170         0.0166         0.0162         0.0158         0.0157         0.0150         0.0146         0.0138           −2-0         0.0288         0.0281         0.0274         0.0268         0.0262         0.0256         0.0250         0.0244         0.0239         0.0231           1-1.8         0.0359         0.0351         0.0344         0.0336         0.0329         0.0322         0.0344         0.0307         0.0367           1-1.6         0.0446         0.0436         0.0418         0.0409         0.0401         0.0392         0.0384         0.0367           1-1.5         0.0688         0.0653         0.0618         0.0606         0.0655         0.0495         0.0485         0.0475         0.0465           1-1.4         0.0808         0.0773         0.0778         0.0764         0.0781         0.0694         0.0853           1-1.3         0.0808         0.00931         0.0934         0.0918         0.0901         0.0735         0.0721         0.0708         0.0894         0.0851           1-1.3         0.0808         0.0053         0.0075         0.0852         0.0899         0.0834         0.0552           1-1.4 </td <td></td>											
-2.0 0.0228 0.0221 0.0217 0.0212 0.0207 0.0202 0.0197 0.0192 0.0188 0.0183 -1.9 0.0287 0.0281 0.0274 0.0268 0.0262 0.0256 0.0250 0.0244 0.0239 0.0234 -1.7 0.0446 0.0359 0.0351 0.0344 0.0336 0.0329 0.0322 0.0314 0.0307 0.0301 0.0294 -1.7 0.0446 0.0436 0.0437 0.0526 0.0418 0.0409 0.0401 0.0392 0.0384 0.0375 0.0367 -1.6 0.0548 0.0537 0.0526 0.0516 0.0505 0.0495 0.0485 0.0485 0.0475 0.0465 0.0455 -1.5 0.0668 0.0655 0.0643 0.0630 0.0618 0.0606 0.0594 0.0582 0.0571 0.0556 -1.5 0.0688 0.0695 0.0934 0.0630 0.0618 0.0606 0.0594 0.0582 0.0571 0.0566 -1.3 0.0968 0.0951 0.0934 0.0918 0.0901 0.0885 0.0869 0.0853 0.0864 0.0681 -1.1 0.137 0.1131 0.1112 0.1093 0.1075 0.1056 0.1038 0.1020 0.1003 0.0985 -1.1 0.1537 0.1353 0.1314 0.1292 0.1271 0.1251 0.1230 0.1210 0.1190 0.1170 -1.0 0.1587 0.1562 0.1539 0.1515 0.1492 0.1469 0.1446 0.1423 0.1401 0.1379 -0.9 0.1841 0.1814 0.1788 0.1762 0.1736 0.1717 0.1685 0.1660 0.1635 0.1611 -0.8 0.2119 0.2090 0.2061 0.2033 0.2005 0.1977 0.1949 0.1922 0.1894 0.1864 -0.6 0.2743 0.2798 0.2358 0.2358 0.2327 0.296 0.2266 0.2236 0.2266 0.2216 0.2206 0.2217 0.2148 -0.6 0.2743 0.2379 0.2375 0.3296 0.2266 0.2236 0.2266 0.2217 0.2148 -0.5 0.3085 0.3050 0.3015 0.2981 0.2946 0.2912 0.2877 0.2843 0.2810 0.2716 -0.4 0.3446 0.3499 0.3372 0.3336 0.3300 0.3364 0.3228 0.3192 0.3156 0.3161 -0.5 0.3085 0.3050 0.3015 0.2981 0.2946 0.2912 0.2877 0.2843 0.2810 0.2716 -0.1 0.4602 0.4562 0.4582 0.4483 0.4443 0.4404 0.4364 0.4325 0.4286 0.4241 -0.0 0.5000 0.5040 0.5080 0.5120 0.5160 0.5199 0.5239 0.5279 0.5319 0.5359 -0.1 0.4602 0.4562 0.4582 0.4483 0.4443 0.4404 0.4364 0.4364 0.4395 0.4586 0.4641 -0.0 0.5000 0.5040 0.5080 0.5120 0.5160 0.5199 0.5239 0.5279 0.5319 0.5359 -0.1 0.4602 0.4562 0.4580 0.4580 0.4580 0.4644 0.4364 0.4365 0.4591 0.4561 0											
−1-9         0.0287         0.0281         0.0274         0.0368         0.0262         0.0250         0.0240         0.0239         0.0231           −1.8         0.0359         0.0351         0.0344         0.0336         0.0329         0.0314         0.0307         0.0301         0.0367           −1.6         0.0548         0.0537         0.0526         0.0516         0.0505         0.0495         0.0485         0.0475         0.0465         0.0557           −1.5         0.0688         0.0655         0.0630         0.0618         0.0606         0.0584         0.0527         0.0762         0.0537         0.0680         0.0680         0.0680         0.0681         0.0606         0.0681         0.0606         0.0681         0.0608         0.0831         0.0918         0.0901         0.0885         0.0869         0.0853         0.0834         0.0918         0.0018         0.0608         0.0853         0.0838         0.0681         0.0831         0.1103         0.1103         0.1103         0.1103         0.1103         0.1103         0.1103         0.1103         0.1103         0.1103         0.1103         0.0814         0.0814         0.0814         0.0814         0.0814         0.0814         0.0814         0.081											
-1.8         0.0359         0.0351         0.0344         0.0328         0.0329         0.0322         0.0314         0.0370         0.0361         0.0264           -1.6         0.0548         0.0537         0.0526         0.0516         0.0505         0.0495         0.0485         0.0475         0.0465         0.0455           -1.5         0.0668         0.0657         0.0624         0.0530         0.0618         0.0606         0.0594         0.0582         0.0571         0.0558           -1.4         0.0808         0.0793         0.0764         0.0749         0.0735         0.0721         0.0784         0.0749         0.0735         0.0721         0.0764         0.0749         0.0735         0.0721         0.0681           -1.2         0.1515         0.1314         0.1219         0.1033         0.083         0.0833         0.0833         0.0833         0.0833         0.0833         0.0835         0.1103         0.0904         0.0681         0.0814         0.0144         0.1103         0.0904         0.061         0.1271         0.1251         0.1203         0.10103         0.0904         0.061         0.1271         0.1404         0.1473         0.1410         0.1372         0.1462         0.1462	-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.7         0.0446         0.0436         0.0427         0.0516         0.0505         0.0409         0.0495         0.0485         0.0375         0.0455           -1.5         0.0668         0.0655         0.0630         0.0618         0.0606         0.0545         0.0457         0.0455         0.0455           -1.4         0.0808         0.0763         0.0764         0.0749         0.0735         0.0721         0.0708         0.0694         0.0681           -1.3         0.0968         0.0951         0.0918         0.0901         0.0855         0.0853         0.0838         0.0823           -1.1         0.1517         0.1131         0.1112         0.1093         0.1075         0.1056         0.1038         0.1020         0.1003         0.0985           -1.1         0.1587         0.1552         0.1539         0.1515         0.1492         0.1250         0.1210         0.1103         0.1171         0.1665         0.1633         0.1610         0.1317         0.1466         0.1423         0.1401         0.1313         0.1292         0.1314         0.1292         0.1210         0.1103         0.1617         0.1466         0.1423         0.1401         0.1315         0.1429         0.1426	-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.6         0.0548         0.0537         0.0526         0.0530         0.0618         0.0495         0.0485         0.0455         0.0650         0.0539           −1.4         0.0808         0.0793         0.0778         0.0764         0.0735         0.0721         0.0708         0.0694         0.0581           −1.3         0.0968         0.0951         0.0334         0.0918         0.0901         0.0885         0.0893         0.0833           −1.1         0.15151         0.1131         0.1112         0.1075         0.1066         0.1038         0.1030         0.0785           −1.1         0.1557         0.1539         0.1515         0.1129         0.1240         0.1100         0.1170           −1.0         0.1584         0.1539         0.1515         0.1492         0.1446         0.1420         0.1100         0.1170           −0.9         0.1841         0.11840         0.1788         0.1762         0.1736         0.1711         0.1680         0.1632         0.1660         0.1635           −0.7         0.2420         0.2389         0.2358         0.2376         0.2641         0.2578         0.2546         0.2511         0.2412           −0.5         0.3035	-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.5         0.0668         0.0655         0.0643         0.0630         0.0618         0.0606         0.0594         0.05282         0.0571         0.0559           −1.4         0.0908         0.0951         0.0978         0.0704         0.0735         0.0721         0.0708         0.0681           −1.3         0.0968         0.0951         0.0934         0.0901         0.0885         0.0869         0.0838         0.0823           −1.1         0.1135         0.1134         0.1112         0.1033         0.1075         0.1036         0.1030         0.1010         0.1010         0.0910           −0.9         0.1841         0.1816         0.1835         0.1515         0.1492         0.1469         0.1446         0.1423         0.1401         0.1379           −0.8         0.2119         0.2909         0.2061         0.2033         0.0205         0.1977         0.1449         0.1660         0.1635         0.1611           −0.8         0.2119         0.2909         0.2266         0.2266         0.2266         0.2266         0.2266         0.2266         0.2266         0.2266         0.2266         0.2266         0.2266         0.2266         0.2266         0.2266         0.2266         0	-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.4	-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.4	-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.2 0.1951 0.0968 0.0951 0.0934 0.0918 0.0901 0.0885 0.0869 0.0853 0.0828 0.0925 0.12 0.1151 0.1131 0.1112 0.1093 0.1075 0.1056 0.1038 0.1020 0.1003 0.0985 0.11 0.1357 0.1355 0.13314 0.1292 0.1271 0.1251 0.1230 0.1210 0.1190 0.1170 0.1057 0.10 0.1587 0.1562 0.1539 0.1515 0.1492 0.1469 0.1466 0.1423 0.1401 0.1379 0.99 0.1514 0.1814 0.1788 0.1762 0.1736 0.1711 0.1685 0.1660 0.1635 0.1611 0.08 0.2119 0.2090 0.2061 0.2033 0.2005 0.1977 0.1949 0.1922 0.1894 0.1867 0.7 0.2420 0.2389 0.2358 0.2327 0.2296 0.2266 0.2236 0.2206 0.2177 0.2148 0.6 0.06 0.2743 0.2709 0.2676 0.2643 0.2611 0.2578 0.2546 0.2514 0.2483 0.2451 0.0578 0.3085 0.3050 0.3015 0.2981 0.2946 0.2912 0.2877 0.2843 0.2451 0.276 0.3 0.3821 0.3783 0.3745 0.3306 0.3300 0.3264 0.3228 0.3192 0.3156 0.3121 0.3783 0.3821 0.3783 0.3745 0.3069 0.3632 0.3594 0.3557 0.3520 0.3483 0.02 0.00 0.4062 0.4662 0.4522 0.4483 0.4443 0.4404 0.4364 0.4325 0.4266 0.4247 0.0 0.5000 0.4960 0.4920 0.4880 0.4840 0.4801 0.4761 0.4721 0.4681 0.4641 0.0 0.5000 0.5040 0.5080 0.5130 0.5160 0.5190 0.5239 0.5279 0.5319 0.3559 0.1 0.5388 0.5438 0.5478 0.5517 0.5556 0.5566 0.5636 0.5675 0.5714 0.5753 0.2 0.5793 0.5832 0.5871 0.5910 0.5948 0.5987 0.6026 0.6044 0.6103 0.6141 0.3 0.6179 0.6217 0.6255 0.6293 0.6331 0.6386 0.6646 0.6702 0.6808 0.6844 0.6879 0.5948 0.7921 0.7324 0.7357 0.7389 0.7422 0.7454 0.7486 0.6414 0.8750 0.8950 0.5895 0.7019 0.7054 0.7088 0.6844 0.7896 0.6844 0.6879 0.5948 0.7991 0.7324 0.7357 0.7389 0.7422 0.7454 0.7486 0.7517 0.7524 0.66 0.7257 0.7291 0.7324 0.7357 0.7389 0.7422 0.7454 0.7486 0.7517 0.7524 0.6808 0.8916 0.8916 0.8918 0.9918 0.9918 0.9919 0.9911 0.9912 0.9913 0.9914 0.9914 0.9914 0.99											
-1.2 0.1151 0.1183 0.11912 0.1093 0.1075 0.1056 0.1038 0.1020 0.1003 0.0985 -1.1 0.1587 0.1335 0.1314 0.1292 0.1271 0.1251 0.1230 0.1210 0.1190 0.1170 -1.0 0.1587 0.1562 0.1539 0.1515 0.1492 0.1469 0.1446 0.1423 0.1401 0.1379 -0.9 0.1841 0.1814 0.1788 0.1762 0.1736 0.1711 0.1685 0.1660 0.1635 0.1611 0.08 0.2119 0.2090 0.2061 0.2033 0.2005 0.1977 0.1949 0.1922 0.1894 0.1867 -0.7 0.2420 0.2389 0.2358 0.2327 0.2296 0.2266 0.2236 0.2206 0.2177 0.2148 -0.6 0.2743 0.2709 0.2676 0.2643 0.2911 0.2578 0.2546 0.2514 0.2483 0.2451 -0.5 0.3085 0.3050 0.3015 0.2981 0.2946 0.2942 0.2927 0.2287 0.2280 0.2206 0.2177 0.2810 0.2776 -0.4 0.3446 0.3409 0.3372 0.3336 0.3300 0.3264 0.3228 0.3192 0.3156 0.3121 -0.3 0.3821 0.3783 0.3745 0.3707 0.3669 0.3632 0.3594 0.3557 0.3520 0.3483 0.2 0.02 0.0407 0.4168 0.4129 0.4090 0.4052 0.4013 0.3974 0.3396 0.3897 0.3859 -0.1 0.4602 0.4562 0.4522 0.4483 0.4443 0.4404 0.4364 0.4325 0.4286 0.4247 -0.0 0.5000 0.4960 0.4920 0.4880 0.4840 0.4801 0.4761 0.4721 0.4681 0.4641 0.0 0.5000 0.4960 0.4920 0.4880 0.4840 0.4801 0.4761 0.4721 0.4681 0.4641 0.0 0.5000 0.5040 0.5080 0.5120 0.5160 0.5199 0.5239 0.5279 0.5319 0.5359 0.1 0.5398 0.5832 0.5873 0.5832 0.5871 0.5910 0.5948 0.5987 0.6026 0.6064 0.6103 0.6141 0.30 0.6174 0.6628 0.6628 0.6664 0.6700 0.6736 0.6628 0.6614 0.6700 0.6736 0.6628 0.6614 0.6700 0.6736 0.6757 0.7291 0.7324 0.7357 0.7389 0.7422 0.7454 0.7460 0.6517 0.6255 0.6293 0.6331 0.6388 0.6406 0.6443 0.6480 0.6517 0.4810 0.6628 0.6614 0.6700 0.6736 0.6772 0.6808 0.6814 0.6879 0.7995 0.8033 0.8071 0.8790 0.7991 0.7549 0.7580 0.7611 0.7642 0.7673 0.7704 0.7734 0.7764 0.7794 0.7780 0.7549 0.7881 0.7910 0.7939 0.7967 0.7995 0.8033 0.8051 0.8078 0.8610 0.8389 0.8064 0.8440 0.8480 0.8880 0.8810 0.8881 0.9910 0.7996 0.9909 0.9115 0.9131 0.9914 0.9915 0.9915 0.9955 0.9968 0.9968 0.9968 0.9961 0.9965 0.9966 0.9966 0.9967 0.9999 0.9915 0.9915 0.9913 0.9915 0.9913 0.9914 0.9915 0.9915 0.9955 0.9966 0.9967 0.9997 0.9999 0.9901 0.9901 0.9901 0.9901 0.9901 0.9901 0.9901 0.9901 0.9901 0.9901 0.9901 0.9901											
-1.1 0.1357 0.1335 0.1314 0.1292 0.1271 0.1251 0.1230 0.1210 0.1190 0.1170 0.150 0.1587 0.1562 0.1539 0.1515 0.1492 0.1496 0.1446 0.1423 0.1401 0.1379 0.99 0.1841 0.1814 0.1788 0.1762 0.1736 0.1711 0.1685 0.1660 0.1635 0.1611 0.08 0.2119 0.2090 0.2061 0.2033 0.2005 0.1977 0.1949 0.1922 0.1894 0.1867 0.07 0.2420 0.2389 0.2358 0.2327 0.2296 0.2266 0.2236 0.2206 0.2177 0.2148 0.260 0.2743 0.2709 0.2676 0.2643 0.2611 0.2578 0.2546 0.2514 0.2483 0.2451 0.05 0.3085 0.3050 0.3015 0.2981 0.2946 0.2912 0.2877 0.2843 0.2810 0.2776 0.3 0.3821 0.3783 0.3372 0.3336 0.3300 0.3264 0.3228 0.3192 0.3156 0.3121 0.0 0.3821 0.3783 0.3745 0.3707 0.3669 0.3632 0.3594 0.3557 0.3520 0.3483 0.20 0.4602 0.4602 0.4562 0.4523 0.4483 0.4443 0.4404 0.4343 0.3974 0.3936 0.3897 0.3859 0.10 0.2776 0.10 0.5000 0.4960 0.4920 0.4880 0.4840 0.4801 0.4761 0.4721 0.4681 0.4641 0.0 0.5000 0.5040 0.5080 0.4880 0.4840 0.4801 0.4761 0.4721 0.4681 0.4641 0.50539 0.5398 0.5438 0.5438 0.5478 0.5517 0.5557 0.5557 0.5596 0.5636 0.5675 0.5714 0.5753 0.2 0.5793 0.5832 0.5871 0.5910 0.5948 0.5987 0.6026 0.6064 0.6103 0.6141 0.3 0.6179 0.6217 0.6255 0.6293 0.6331 0.6368 0.6406 0.6443 0.6440 0.6517 0.404 0.4654 0.6554 0.6591 0.6628 0.6664 0.6700 0.6736 0.6772 0.6808 0.6844 0.6879 0.7580 0.7580 0.7611 0.7324 0.7357 0.7398 0.7422 0.7454 0.7486 0.7517 0.7590 0.7780 0.7611 0.7638 0.8665 0.8666 0.8708 0.8729 0.8739 0.8810 0.8913 0.9914 0.9006 0.8818 0.8916 0.8915 0.8980 0.8898 0.8896 0.8898 0.8896 0.8898 0.8896 0.8987 0.8925 0.8931 0.8908 0.8916 0.8913 0.9914 0.9906 0.9912 0.9907 0.9922 0.9236 0.9915 0.9950 0.9909 0.9915 0.9919 0.9911 0.9915											
-1.0         0.1587         0.1562         0.1539         0.1515         0.1492         0.1469         0.1446         0.1423         0.1401         0.1379           -0.9         0.1841         0.1814         0.1788         0.1762         0.1762         0.1761         0.1711         0.1660         0.1635         0.1611           -0.7         0.2420         0.2389         0.2358         0.2327         0.2266         0.2266         0.2266         0.2266         0.2266         0.2266         0.2267         0.2420         0.2541         0.2483         0.2317           -0.5         0.3085         0.3050         0.3015         0.2981         0.29246         0.2912         0.2877         0.2543         0.2811         0.2577           -0.4         0.3446         0.3409         0.3372         0.3336         0.3300         0.3264         0.3228         0.3192         0.3156         0.3121           -0.3         0.3821         0.3733         0.3737         0.3669         0.3632         0.3837         0.3356         0.3520         0.3483           -0.2         0.4207         0.4168         0.4129         0.4090         0.4052         0.4013         0.3974         0.3536         0.3520         0.											
-0.9         0.1841         0.1814         0.1788         0.1762         0.1736         0.1711         0.1685         0.1660         0.1635         0.1611           -0.8         0.2119         0.2089         0.2358         0.2327         0.2296         0.2236         0.2236         0.2236         0.2236         0.2236         0.2236         0.2206         0.2177         0.2148           -0.6         0.2743         0.2709         0.2676         0.2643         0.2611         0.2578         0.2546         0.2514         0.2483         0.2451           -0.5         0.3085         0.3505         0.3015         0.2981         0.2946         0.2912         0.2877         0.2843         0.2810         0.2776           -0.4         0.3446         0.3309         0.3372         0.3669         0.3622         0.3594         0.3557         0.3520         0.3156         0.3211           -0.3         0.3821         0.3783         0.3775         0.3669         0.3632         0.3594         0.3557         0.3520         0.3152         0.3156         0.3244         0.3356         0.3859         0.3859         0.3859         0.3239         0.3239         0.3239         0.3239         0.3239         0.3239											
-0.8         0.2119         0.2900         0.2061         0.2033         0.2055         0.1977         0.1949         0.1922         0.1844         0.1867           -0.6         0.2743         0.2709         0.2676         0.2643         0.2611         0.2576         0.2543         0.2611         0.2454         0.2514         0.2483         0.2451           -0.5         0.3085         0.3050         0.3015         0.2981         0.2946         0.2912         0.2877         0.2843         0.2810         0.2776           -0.4         0.3446         0.3403         0.3372         0.3336         0.3300         0.3264         0.3228         0.3192         0.3156         0.3121           -0.3         0.3821         0.3783         0.3745         0.3707         0.3669         0.3632         0.3559         0.3557         0.3520         0.3483           -0.2         0.4207         0.4168         0.4129         0.4090         0.4052         0.4013         0.3974         0.3936         0.3387         0.3859           -0.1         0.4602         0.4562         0.4820         0.4883         0.4443         0.4404         0.4364         0.4325         0.4286         0.4281           0.0	-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.7         0.2420         0.2389         0.2358         0.2327         0.2296         0.2266         0.2206         0.2177         0.2483         0.2451           -0.6         0.2743         0.2709         0.2676         0.2643         0.2611         0.2578         0.2546         0.2514         0.2483         0.2451           -0.5         0.3085         0.3505         0.3015         0.2981         0.2946         0.2912         0.2377         0.2843         0.2810         0.2776           -0.4         0.3446         0.3409         0.3372         0.3306         0.3300         0.3624         0.3228         0.3192         0.3156         0.3121           -0.2         0.4207         0.4468         0.4129         0.4090         0.4052         0.4013         0.3374         0.3366         0.3897         0.3589           -0.1         0.4602         0.4562         0.4483         0.4444         0.4404         0.4325         0.4286         0.4447           -0.0         0.5000         0.4960         0.4880         0.4840         0.4801         0.4761         0.4721         0.4681         0.4641           -0.0         0.5000         0.5040         0.5080         0.5120         0.5160 <td>-0.9</td> <td>0.1841</td> <td>0.1814</td> <td>0.1788</td> <td>0.1762</td> <td>0.1736</td> <td>0.1711</td> <td>0.1685</td> <td>0.1660</td> <td>0.1635</td> <td>0.1611</td>	-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.6         0.2743         0.2769         0.2643         0.2611         0.2878         0.2546         0.2514         0.2483         0.2476           -0.5         0.3085         0.3015         0.2981         0.2946         0.2912         0.2877         0.2843         0.2810         0.2776           -0.4         0.3446         0.3409         0.3372         0.3336         0.3300         0.3644         0.3228         0.3192         0.3156         0.3121           -0.3         0.3821         0.3783         0.3757         0.3669         0.3632         0.3594         0.3557         0.3520         0.3483           -0.2         0.4207         0.4168         0.4129         0.4090         0.4052         0.4430         0.4334         0.4404         0.4334         0.44325         0.4286         0.4247           -0.0         0.5000         0.4960         0.4820         0.4880         0.4840         0.4801         0.4761         0.4721         0.4681         0.4641           -0.0         0.5000         0.5040         0.5080         0.5120         0.5160         0.5199         0.5239         0.5319         0.5339         0.5319         0.5363         0.5464         0.5714         0.5753	-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.5         0.3085         0.3050         0.3015         0.2981         0.2946         0.2912         0.2877         0.2843         0.2810         0.2776           -0.4         0.3446         0.3409         0.3372         0.3336         0.3300         0.3632         0.3594         0.3557         0.3520         0.3483           -0.2         0.4207         0.4168         0.4129         0.4090         0.4052         0.4013         0.3974         0.3396         0.3897         0.3897           -0.1         0.4602         0.4562         0.4522         0.4483         0.4443         0.4404         0.4364         0.4325         0.4286         0.4247           -0.0         0.5000         0.4960         0.4880         0.4840         0.4801         0.4761         0.4721         0.4681         0.4641           0.0         0.5000         0.5040         0.5080         0.5120         0.5160         0.5199         0.5239         0.5279         0.5319         0.5339           0.1         0.5398         0.5438         0.5478         0.5517         0.5557         0.5596         0.5636         0.5675         0.5714         0.5753           0.2         0.5793         0.5832         0.5871	-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.4         0.3446         0.3409         0.3372         0.3336         0.3300         0.3264         0.3228         0.3192         0.3156         0.3121           -0.3         0.3821         0.3783         0.3707         0.3669         0.3632         0.3594         0.3557         0.3520         0.3483           -0.2         0.4207         0.4168         0.4129         0.4090         0.4052         0.4013         0.3974         0.3936         0.3897         0.3859           -0.1         0.4602         0.4562         0.4522         0.4483         0.4444         0.4401         0.4364         0.4226         0.4247           -0.0         0.5000         0.4960         0.4920         0.4880         0.4840         0.4801         0.4761         0.4721         0.4681         0.4641           0.0         0.5000         0.5040         0.5080         0.5109         0.5199         0.5239         0.5279         0.5319         0.5359           0.1         0.5398         0.5438         0.5478         0.5517         0.5557         0.5596         0.6664         0.6736         0.5665         0.5675         0.5114         0.5736           0.2         0.5793         0.5822         0.5871	-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.4         0.3446         0.3409         0.3372         0.3336         0.3300         0.3264         0.3228         0.3192         0.3156         0.3121           -0.3         0.3821         0.3783         0.3707         0.3669         0.3632         0.3594         0.3557         0.3520         0.3483           -0.1         0.4602         0.4562         0.4522         0.4483         0.4443         0.4404         0.4364         0.4225         0.4286         0.4247           -0.0         0.5000         0.4960         0.4920         0.4880         0.4840         0.4801         0.4761         0.4721         0.4681         0.4641           0.0         0.5000         0.5040         0.5080         0.5120         0.5160         0.5199         0.5239         0.5279         0.5319         0.5359           0.1         0.5398         0.5438         0.5478         0.5517         0.5557         0.5560         0.5665         0.5675         0.5714         0.5759           0.1         0.5398         0.5438         0.5478         0.5517         0.5557         0.5567         0.5666         0.6604         0.6103         0.6141           0.3         0.6179         0.6217         0.6255	-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.3         0.3821         0.3783         0.3745         0.3669         0.3632         0.3594         0.3557         0.3520         0.3483           -0.2         0.4207         0.4168         0.4129         0.4090         0.4052         0.4013         0.3974         0.3936         0.3857         0.3859           -0.1         0.4602         0.4562         0.4483         0.4443         0.4404         0.4361         0.4721         0.4681         0.4641           -0.0         0.5000         0.5040         0.5080         0.5120         0.5160         0.5199         0.5239         0.5279         0.5319         0.5359           0.1         0.5398         0.5438         0.5478         0.5517         0.5557         0.5556         0.5636         0.5675         0.5714         0.5753           0.2         0.5793         0.5832         0.5817         0.5910         0.5948         0.5987         0.6064         0.6130         0.6141           0.3         0.6179         0.6217         0.6255         0.6293         0.6331         0.6368         0.6406         0.6443         0.6480         0.6517           0.4         0.6554         0.6551         0.6628         0.6604         0.6700	0.4	0.3446	0.3400	0.3379	0 3336	0.3300	0.3264	0.3558	0.3102	0.3156	
-0.2         0.4207         0.4168         0.4129         0.4090         0.4052         0.4013         0.3974         0.3936         0.3897         0.3859           -0.1         0.4602         0.4562         0.4483         0.4443         0.4404         0.4364         0.4325         0.4286         0.4247           -0.0         0.5000         0.4560         0.4920         0.4880         0.4840         0.4801         0.4761         0.4721         0.4681         0.4461           0.0         0.5000         0.5040         0.5080         0.5120         0.5160         0.5199         0.5239         0.5279         0.5319         0.5359           0.1         0.5398         0.5438         0.5478         0.5517         0.5557         0.5556         0.5636         0.5675         0.5714         0.5753           0.2         0.5793         0.5821         0.5810         0.5948         0.5987         0.6026         0.6044         0.6103         0.6117           0.4         0.6554         0.6517         0.6628         0.6664         0.6700         0.6736         0.6722         0.7808         0.7123         0.7157         0.7190         0.7224           0.5         0.6915         0.6950											
$\begin{array}{cccccccccccccccccccccccccccccccccccc$											
-0.0         0.5000         0.4960         0.4920         0.4880         0.4840         0.4801         0.4761         0.4721         0.4681         0.4641           0.0         0.5000         0.5040         0.5080         0.5120         0.5160         0.5199         0.5239         0.5279         0.5319         0.5359           0.1         0.5398         0.5438         0.5478         0.5517         0.5557         0.5987         0.6026         0.6064         0.6103         0.6141           0.3         0.6179         0.6217         0.6255         0.6293         0.6331         0.6368         0.6406         0.6443         0.6480         0.6517           0.4         0.6554         0.6591         0.6628         0.6664         0.6700         0.6736         0.6772         0.6808         0.6844         0.6879           0.5         0.6915         0.6950         0.6985         0.7019         0.7054         0.7036         0.7672         0.6808         0.6844         0.6879           0.7         0.7580         0.7611         0.7642         0.7673         0.7734         0.7734         0.7754         0.7754         0.7754         0.7754         0.7754         0.7754         0.7754         0.7754 <td></td>											
0.0         0.5000         0.5040         0.5080         0.5120         0.5160         0.5199         0.5239         0.5279         0.5319         0.5359           0.1         0.5398         0.5438         0.5478         0.5517         0.5557         0.5596         0.5636         0.5675         0.5714         0.5753           0.2         0.5793         0.5832         0.5871         0.5510         0.5948         0.5987         0.6026         0.6064         0.6103         0.6141           0.3         0.6179         0.6217         0.6255         0.6293         0.6331         0.6368         0.6406         0.6443         0.6480         0.6517           0.4         0.6554         0.6591         0.6628         0.6664         0.6700         0.6736         0.6772         0.8808         0.6844         0.6877           0.5         0.6915         0.6950         0.6985         0.7019         0.7054         0.7088         0.7123         0.7157         0.7190         0.7524         0.7881         0.7442         0.7454         0.7446         0.7754         0.7517         0.7549         0.7823         0.7852         0.823         0.8051         0.8078         0.8106         0.8121         0.8238         0.8											
0.1         0.5398         0.5438         0.5478         0.5517         0.5597         0.5596         0.5636         0.5675         0.5714         0.5753           0.2         0.5793         0.5832         0.5871         0.5910         0.5948         0.5987         0.6026         0.6064         0.6103         0.6141           0.3         0.6179         0.6217         0.6255         0.6293         0.6331         0.6368         0.6406         0.6443         0.6480         0.6517           0.4         0.6554         0.6591         0.6628         0.6664         0.6700         0.6736         0.6772         0.6808         0.6844         0.6879           0.5         0.6915         0.6950         0.6985         0.7019         0.7054         0.7088         0.7123         0.7157         0.7190         0.7224           0.6         0.7257         0.7291         0.7324         0.7357         0.7389         0.7422         0.7454         0.7794         0.7784         0.7764         0.7794         0.7783         0.7850         0.7811         0.7662         0.7995         0.8233         0.8051         0.8078         0.8106         0.8133         0.9861         0.8843         0.8661         0.8843         0.											
0.2         0.5793         0.5832         0.5871         0.5910         0.5948         0.5987         0.6026         0.6064         0.6103         0.6117           0.3         0.6179         0.6217         0.6255         0.6293         0.6331         0.6368         0.6406         0.6443         0.6480         0.6517           0.4         0.6554         0.6591         0.6628         0.6664         0.6700         0.6736         0.6772         0.6808         0.6844         0.6879           0.5         0.6915         0.6950         0.6985         0.77019         0.7054         0.7088         0.7123         0.7157         0.7190         0.7224           0.6         0.7257         0.7291         0.7327         0.7389         0.7422         0.7454         0.7486         0.7517         0.7549           0.7         0.7580         0.7611         0.7642         0.7673         0.7704         0.7734         0.7764         0.7794         0.7794         0.7794         0.7844         0.7794         0.7893         0.8016         0.8133           0.8         0.7881         0.7910         0.7939         0.7967         0.7995         0.8023         0.8315         0.8078         0.8133											
0.3         0.6179         0.6217         0.6255         0.6293         0.6331         0.6368         0.6406         0.6443         0.6480         0.6517           0.4         0.6554         0.6591         0.6628         0.6664         0.6700         0.6736         0.6772         0.6808         0.6844         0.6879           0.5         0.6915         0.6950         0.6985         0.7019         0.7054         0.7088         0.7123         0.7157         0.7739         0.7389         0.7422         0.7454         0.7486         0.7517         0.7549           0.7         0.7580         0.7611         0.7642         0.7633         0.7704         0.7764         0.7794         0.77823         0.7852           0.8         0.7881         0.7910         0.7939         0.7967         0.7995         0.8023         0.8051         0.8078         0.8106         0.8133           0.9         0.8159         0.8186         0.8212         0.8238         0.8264         0.8289         0.8315         0.8340         0.8365         0.8389           1.0         0.8413         0.8438         0.8461         0.8485         0.8508         0.8531         0.8540         0.8577         0.8599         0.8621 <td></td>											
0.4         0.6554         0.6591         0.6628         0.6664         0.6700         0.6736         0.6772         0.6808         0.6844         0.6879           0.5         0.6915         0.6950         0.6985         0.7019         0.7054         0.7088         0.7123         0.7157         0.7190         0.7224           0.6         0.7257         0.7291         0.7324         0.7357         0.7389         0.7422         0.7454         0.7486         0.7517         0.7549           0.7         0.7580         0.7611         0.7642         0.7667         0.7794         0.7734         0.7764         0.7794         0.7794         0.7819         0.8106         0.8133           0.9         0.8186         0.8212         0.8238         0.8264         0.8289         0.8315         0.8340         0.8365         0.8389           1.0         0.8413         0.8438         0.8461         0.8485         0.8508         0.8531         0.8554         0.8577         0.8599         0.8621           1.1         0.8643         0.8665         0.8686         0.8708         0.8729         0.8749         0.8770         0.8790         0.8810         0.8830           1.2         0.88499	0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.5         0.6915         0.6950         0.6985         0.7019         0.7054         0.7088         0.7123         0.7157         0.7190         0.7224           0.6         0.7257         0.7291         0.7324         0.7357         0.7389         0.7422         0.7454         0.7486         0.7517         0.7549           0.7         0.7580         0.7611         0.7642         0.7673         0.7704         0.7734         0.7764         0.7794         0.7823         0.7852           0.8         0.7881         0.7910         0.7939         0.7967         0.7995         0.8023         0.8051         0.8078         0.8106         0.8133           0.9         0.8159         0.8186         0.8212         0.8238         0.8264         0.8289         0.8315         0.8340         0.8365         0.8389           1.0         0.8413         0.8461         0.8485         0.8508         0.8554         0.8577         0.8599         0.8621           1.1         0.8643         0.8665         0.8686         0.8708         0.8749         0.8770         0.8790         0.8810         0.8330           1.2         0.8849         0.8886         0.8907         0.8925         0.8944	0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.6         0.7257         0.7291         0.7324         0.7357         0.7389         0.7422         0.7454         0.7486         0.7517         0.7549           0.7         0.7580         0.7611         0.7642         0.7673         0.7704         0.7734         0.7764         0.7794         0.7823         0.7852           0.8         0.7881         0.7910         0.7939         0.7967         0.7995         0.8023         0.8051         0.8078         0.8166         0.8133           0.9         0.8159         0.8186         0.8212         0.8238         0.8264         0.8289         0.8315         0.8340         0.8365         0.8389           1.0         0.8413         0.8461         0.8485         0.8508         0.8531         0.8557         0.8599         0.8621           1.1         0.8643         0.8665         0.8686         0.8708         0.8729         0.8749         0.8770         0.8790         0.8810         0.8830           1.2         0.8849         0.8869         0.8888         0.8907         0.8925         0.8944         0.8962         0.8980         0.8997         0.9015           1.3         0.9932         0.9949         0.9926         0.99279	0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.6         0.7257         0.7291         0.7324         0.7357         0.7389         0.7422         0.7454         0.7486         0.7517         0.7549           0.7         0.7580         0.7611         0.7642         0.7673         0.7704         0.7734         0.7764         0.7794         0.7823         0.7852           0.8         0.7881         0.7910         0.7939         0.7967         0.7995         0.8023         0.8051         0.8078         0.8166         0.8133           0.9         0.8159         0.8186         0.8212         0.8238         0.8264         0.8289         0.8315         0.8340         0.8365         0.8389           1.0         0.8413         0.8461         0.8485         0.8508         0.8531         0.8557         0.8599         0.8621           1.1         0.8643         0.8665         0.8686         0.8708         0.8729         0.8749         0.8770         0.8790         0.8810         0.8830           1.2         0.8849         0.8869         0.8888         0.8907         0.8925         0.8944         0.8962         0.8980         0.8997         0.9015           1.3         0.9932         0.9949         0.9926         0.99279	0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.7         0.7580         0.7611         0.7642         0.7673         0.7704         0.7734         0.7764         0.7794         0.7823         0.7852           0.8         0.7881         0.7910         0.7939         0.7967         0.7995         0.8023         0.8051         0.8078         0.8106         0.8133           0.9         0.8159         0.8186         0.8212         0.8238         0.8264         0.8289         0.8315         0.8340         0.8365         0.8389           1.0         0.8413         0.8438         0.8461         0.8485         0.8508         0.8531         0.8577         0.8599         0.8621           1.1         0.8643         0.8665         0.8686         0.8708         0.8749         0.8770         0.8790         0.8810         0.8830           1.2         0.8849         0.8886         0.8888         0.8907         0.8925         0.8944         0.8962         0.8980         0.8897         0.9015           1.3         0.9032         0.99049         0.9066         0.9082         0.9999         0.9131         0.9147         0.9162         0.9177           1.4         0.9192         0.9207         0.9222         0.9236         0.9251											
0.8         0.7881         0.7910         0.7939         0.7967         0.7995         0.8023         0.8051         0.8078         0.8106         0.8133           0.9         0.8159         0.8186         0.8212         0.8238         0.8264         0.8289         0.8315         0.8340         0.8365         0.8389           1.0         0.8413         0.8438         0.8461         0.8485         0.8508         0.8531         0.8554         0.8577         0.8599         0.8621           1.1         0.8643         0.8665         0.8686         0.8708         0.8729         0.8749         0.8770         0.8790         0.8810         0.8830           1.2         0.8849         0.8869         0.8888         0.8907         0.8925         0.8944         0.8962         0.8980         0.8997         0.9015           1.3         0.9032         0.9049         0.9066         0.9082         0.9099         0.9115         0.911         0.9147         0.9162         0.9177           1.4         0.9192         0.9207         0.9222         0.9236         0.9291         0.9265         0.9279         0.9292         0.9306         0.9319           1.5         0.9332         0.9345											
0.9         0.8159         0.8186         0.8212         0.8238         0.8264         0.8289         0.8315         0.8340         0.8365         0.8389           1.0         0.8413         0.8438         0.8461         0.8485         0.8508         0.8531         0.8554         0.8577         0.8599         0.8621           1.1         0.8643         0.8665         0.8686         0.8708         0.8729         0.8749         0.8770         0.8790         0.8810         0.8830           1.2         0.8849         0.8869         0.8888         0.8907         0.8925         0.8944         0.8962         0.8980         0.8997         0.9015           1.3         0.9032         0.9049         0.9066         0.9082         0.9099         0.9115         0.9131         0.9147         0.9162         0.9177           1.4         0.9192         0.9207         0.9222         0.9236         0.9251         0.9265         0.9279         0.9292         0.9306         0.9319           1.5         0.9332         0.9345         0.9357         0.9370         0.9382         0.9394         0.9406         0.9418         0.9429         0.9441           1.6         0.9452         0.9463											
1.0         0.8413         0.8438         0.8461         0.8485         0.8508         0.8531         0.8554         0.8577         0.8599         0.8621           1.1         0.8643         0.8665         0.8686         0.8708         0.8729         0.8749         0.8770         0.8790         0.8810         0.8830           1.2         0.8849         0.8869         0.8888         0.8907         0.8925         0.8944         0.8962         0.8980         0.8997         0.9015           1.3         0.9032         0.9049         0.9066         0.9082         0.9099         0.9115         0.9131         0.9147         0.9162         0.9177           1.4         0.9192         0.9207         0.9222         0.9236         0.9251         0.9265         0.9279         0.9292         0.9306         0.9319           1.5         0.9332         0.9345         0.9357         0.9370         0.9382         0.9394         0.9406         0.9418         0.9429         0.9441           1.6         0.9452         0.9463         0.9474         0.9484         0.9495         0.9505         0.9515         0.9525         0.9535         0.9545           1.7         0.9554         0.9564											
1.1       0.8643       0.8665       0.8686       0.8708       0.8729       0.8749       0.8770       0.8790       0.8810       0.8830         1.2       0.8849       0.8869       0.8888       0.8907       0.8925       0.8944       0.8962       0.8980       0.8997       0.9015         1.3       0.9032       0.9049       0.9066       0.9082       0.9099       0.9115       0.9131       0.9147       0.9162       0.9177         1.4       0.9192       0.9207       0.9222       0.9236       0.9251       0.9265       0.9279       0.9292       0.9306       0.9319         1.5       0.9332       0.9345       0.9357       0.9370       0.9382       0.9394       0.9406       0.9418       0.9429       0.9441         1.6       0.9452       0.9463       0.9474       0.9484       0.9495       0.9505       0.9515       0.9525       0.9535       0.9545         1.7       0.9554       0.9564       0.9573       0.9582       0.9591       0.9599       0.9608       0.9616       0.9625       0.9633         1.8       0.9641       0.9649       0.9656       0.9664       0.9671       0.9678       0.9686       0.9693       0.9699											
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$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
$\begin{array}{cccccccccccccccccccccccccccccccccccc$											
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$\begin{array}{cccccccccccccccccccccccccccccccccccc$											
2.0         0.9772         0.9778         0.9783         0.9788         0.9793         0.9798         0.9803         0.9808         0.9812         0.9817           2.1         0.9821         0.9826         0.9830         0.9834         0.9838         0.9842         0.9846         0.9850         0.9854         0.9857           2.2         0.9861         0.9864         0.9868         0.9871         0.9875         0.9878         0.9881         0.9884         0.9887         0.9890           2.3         0.9893         0.9896         0.9898         0.9901         0.9904         0.9906         0.9909         0.9911         0.9913         0.9916           2.4         0.9918         0.9920         0.9922         0.9925         0.9927         0.9929         0.9931         0.9932         0.9934         0.9936           2.5         0.9938         0.9941         0.9943         0.9945         0.9946         0.9948         0.9949         0.9951         0.9952           2.6         0.9953         0.9955         0.9956         0.9957         0.9959         0.9960         0.9961         0.9962         0.9963         0.9964           2.7         0.9965         0.9966         0.9967											
2.1       0.9821       0.9826       0.9830       0.9834       0.9838       0.9842       0.9846       0.9850       0.9854       0.9857         2.2       0.9861       0.9864       0.9868       0.9871       0.9875       0.9878       0.9881       0.9884       0.9887       0.9890         2.3       0.9893       0.9896       0.9898       0.9901       0.9904       0.9906       0.9909       0.9911       0.9913       0.9916         2.4       0.9918       0.9920       0.9922       0.9925       0.9927       0.9929       0.9931       0.9932       0.9934       0.9936         2.5       0.9938       0.9940       0.9941       0.9943       0.9945       0.9946       0.9948       0.9949       0.9951       0.9952         2.6       0.9953       0.9955       0.9956       0.9957       0.9959       0.9960       0.9961       0.9962       0.9963       0.9964         2.7       0.9965       0.9966       0.9967       0.9968       0.9969       0.9970       0.9971       0.9972       0.9973       0.9974         2.8       0.9974       0.9975       0.9976       0.9977       0.9977       0.9978       0.9979       0.9979       0.9980											
2.2     0.9861     0.9864     0.9868     0.9871     0.9875     0.9878     0.9881     0.9884     0.9887     0.9890       2.3     0.9893     0.9896     0.9898     0.9901     0.9904     0.9906     0.9909     0.9911     0.9913     0.9916       2.4     0.9918     0.9920     0.9922     0.9925     0.9927     0.9929     0.9931     0.9932     0.9934     0.9936       2.5     0.9938     0.9940     0.9941     0.9943     0.9945     0.9946     0.9948     0.9949     0.9951     0.9952       2.6     0.9953     0.9955     0.9956     0.9957     0.9959     0.9960     0.9961     0.9962     0.9963     0.9964       2.7     0.9965     0.9966     0.9967     0.9968     0.9969     0.9970     0.9971     0.9972     0.9973     0.9980     0.9981       2.8     0.9974     0.9975     0.9976     0.9977     0.9977     0.9978     0.9979     0.9979     0.9980     0.9981											
2.3     0.9893     0.9896     0.9898     0.9901     0.9904     0.9906     0.9909     0.9911     0.9913     0.9916       2.4     0.9918     0.9920     0.9922     0.9925     0.9927     0.9929     0.9931     0.9932     0.9934     0.9936       2.5     0.9938     0.9940     0.9941     0.9943     0.9945     0.9946     0.9948     0.9949     0.9951     0.9952       2.6     0.9953     0.9955     0.9956     0.9957     0.9959     0.9960     0.9961     0.9962     0.9963     0.9964       2.7     0.9965     0.9966     0.9967     0.9968     0.9969     0.9970     0.9971     0.9972     0.9973     0.9981       2.8     0.9974     0.9975     0.9976     0.9977     0.9977     0.9978     0.9979     0.9979     0.9980     0.9981											
2.4     0.9918     0.9920     0.9922     0.9925     0.9927     0.9929     0.9931     0.9932     0.9934     0.9936       2.5     0.9938     0.9940     0.9941     0.9943     0.9945     0.9946     0.9948     0.9949     0.9951     0.9952       2.6     0.9953     0.9955     0.9956     0.9957     0.9959     0.9960     0.9961     0.9962     0.9963     0.9964       2.7     0.9965     0.9966     0.9967     0.9968     0.9969     0.9970     0.9971     0.9972     0.9973     0.9974       2.8     0.9974     0.9975     0.9976     0.9977     0.9977     0.9978     0.9979     0.9979     0.9980     0.9981											
2.5     0.9938     0.9940     0.9941     0.9943     0.9945     0.9946     0.9948     0.9949     0.9951     0.9952       2.6     0.9953     0.9955     0.9956     0.9957     0.9959     0.9960     0.9961     0.9962     0.9963     0.9964       2.7     0.9965     0.9966     0.9967     0.9968     0.9969     0.9970     0.9971     0.9972     0.9973     0.9974       2.8     0.9974     0.9975     0.9976     0.9977     0.9977     0.9978     0.9979     0.9979     0.9980     0.9981											
2.6     0.9953     0.9955     0.9956     0.9957     0.9959     0.9960     0.9961     0.9962     0.9963     0.9964       2.7     0.9965     0.9966     0.9967     0.9968     0.9969     0.9970     0.9971     0.9972     0.9973     0.9974       2.8     0.9974     0.9975     0.9976     0.9977     0.9977     0.9978     0.9979     0.9979     0.9980     0.9981	2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.6     0.9953     0.9955     0.9956     0.9957     0.9959     0.9960     0.9961     0.9962     0.9963     0.9964       2.7     0.9965     0.9966     0.9967     0.9968     0.9969     0.9970     0.9971     0.9972     0.9973     0.9974       2.8     0.9974     0.9975     0.9976     0.9977     0.9977     0.9978     0.9979     0.9979     0.9980     0.9981	2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			0.9955		0.9957	0.9959				0.9963	
2.8  0.9974  0.9975  0.9976  0.9977  0.9977  0.9978  0.9979  0.9979  0.9980  0.9981					0.9968	0.9969	0.9970				
			0.9975		0.9977			0.9979			

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#### BASIC INTEGRALS

$$\int \frac{1}{x} dx = \ln|x| + C = \ln|kx|, \qquad C = \ln k$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\int a^x dx = \frac{1}{\ln a} a^x + C, \qquad a \neq 1$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax + C$$

$$\int \cos ax dx = \frac{1}{a} \sin ax + C$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax + C$$

$$\int \cot ax dx = \frac{1}{a} \ln|\sec ax| + C$$

$$\int \cot ax dx = \frac{1}{a} \ln|\sec ax| + C$$

$$\int \sinh ax dx = \frac{1}{a} \ln|\sec ax| + C$$

$$\int \sinh ax dx = \frac{1}{a} \ln|\sec ax| + C$$

$$\int \sinh ax dx = \frac{1}{a} \sinh ax + C$$

$$\int \cosh ax dx = \frac{1}{a} \sinh ax + C$$

$$\int \cosh ax dx = \frac{1}{a} \tanh ax + C$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{a} \tanh^{-1} \frac{x}{a} + C, \quad |x| < a$$

$$= \frac{1}{a} \coth^{-1} \frac{x}{a} + C, \quad |x| > a > 0$$

$$= \frac{1}{2a} \ln \left| \frac{a + x}{a - x} \right| + C, \quad x^2 \neq a^2$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1} \frac{x}{a} + C, \quad x \geqslant a > 0$$