MATH2019 PROBLEM CLASS

EXAMPLES 5

ORDINARY DIFFERENTIAL EQUATIONS

$\boxed{1998}$ 1. Use the substitution $y = z^{\frac{1}{3}}$ where y and z are both functions of x to transform the differential equation

$$3y' = e^x y^{-2} + y \qquad (1)$$

into

$$z' = e^x + z$$

and hence find the general solution of (1).

$\boxed{1994}$ 2. A forced vibrating system is represented by

$$y'' + 5y' + 4y = 6\sin(2t)$$

where $6\sin(2t)$ is the driving force and y is the displacement from the equilibrium position. Find the motion of the system corresponding to the following initial displacement and velocity

$$y(0) = 1\,, \qquad y'(0) = 0\,.$$

Then find the steady state oscillations (i.e., the response of the system after a sufficiently long time).

$\boxed{1997}$ 3. Consider the differential equation

$$\frac{1}{2}u'' + cu' + \frac{1}{2}u = 0$$

where c is a non-negative damping constant.

a) What damping constants c produce overdamping, critical damping, underdamping and no damping?

b) Sketch an example of the solution $u(t)$ for the case of overdamping and for the case of underdamping.

$\boxed{1999}$ 4. Consider the vibrating system

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = \sin(\omega t)\,.$$

Will the system in exhibit resonance for any choice of the forcing angular frequency ω? Give reasons for your answer.

$\boxed{2014, S2}$ 5. Use the method of undetermined coefficients to solve the second order differential equation

$$y'' + 2y' + 5y = -25x^2.$$

$\boxed{2015, S1}$ 6. Use the method of undetermined coefficients to solve the second order differential equation

$$y'' - 4y' + 4y = 5\sin t.$$

2015, S2 7. Use the method of variation of parameters to find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 35e^x x^{3/2}.$$

2016, S1 8. Use the substitution $v = x + y$ to solve the ordinary differential equation

$$(x+y)\,\frac{dy}{dx} = \frac{1}{x^2} - x - y, \qquad y(1) = 0.$$

2016, S1 9. Use the method of undetermined coefficients to solve the second order differential equation

$$y'' + 3y' + 2y = \mathrm{e}^{-2t} + 4t^2 + 2.$$

Also describe the long term steady state solution.

2016, S2 10. Use the substitution $v = y + x$ to find the general solution of

$$\frac{dy}{dx} = (y+x)^2.$$

2016, S2 11. Use the method of undetermined coefficients to solve the second order differential equation

$$y'' - 4y = \mathrm{e}^{2t}.$$

2017, S1 12. Use the substitution $v = \dfrac{y}{x}$ to solve the ordinary differential equation

$$x^2\frac{dy}{dx} = 2x^2 + xy + 2y^2.$$

2017, S1 13. Use the method of undetermined coefficients to solve the second order differential equation

$$y'' + 9y = 6\cos 3t + 5\mathrm{e}^t.$$

2017, S2 14. Use the method of undetermined coefficients to solve the second order ordinary differential equation

$$y'' - 2y' - 8y = 8 + 5e^t\cos t.$$

2018, S1 15. An inhomogeneous Euler–Cauchy ordinary differential equation (ODE) is given by

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 2x^2\,, \quad x > 0\,.$$

You are **given** that $y_1 = x$ and $y_2 = x^3$ are solutions to the corresponding **homogeneous** Euler–Cauchy ODE. You **do not** have to check this.

i) Calculate the Wronskian of y_1 and y_2.

ii) Use the method of Variation of Parameters to determine a particular solution y_P for the inhomogeneous Euler–Cauchy ODE.

$\boxed{\text{2018, S2}}$ 16. Consider the following differential equation describing a vibrating system:

$$\frac{d^2y}{dt^2} + 4y = 8\cos(2\pi f t).$$

i) Find the solution y_H to the homogeneous equation.

ii) For which value(s) of f will the system exhibit resonance? Give reasons for your answer.
(Note that you are not being asked to find the particular solution y_P)

$\boxed{\text{2018, S2}}$ 17. Use the substitution $v = \dfrac{y}{x}$ to solve

$$xy' = y + 2x^3\cos^2\left(\frac{y}{x}\right).$$