NAME OF CANDIDATE: ....
STUDENT NUMBER: ....

THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF MATHEMATICS AND STATISTICS

November 2010

MATH2089
Numerical Methods and Statistics

(1) TIME ALLOWED – 3 Hours

(2) TOTAL NUMBER OF QUESTIONS – 6

(3) ANSWER ALL QUESTIONS

(4) THE QUESTIONS ARE OF EQUAL VALUE

(5) THIS PAPER MAY **NOT** BE RETAINED BY THE CANDIDATE

(6) **ONLY** CALCULATORS WITH AN AFFIXED "UNSW APPROVED" STICKER MAY BE USED

(7) STATISTICAL FORMULAE ARE AT THE START OF PART B
STATISTICAL TABLES ARE ATTACHED AT END OF PAPER

Part A – consists of questions 1 – 3

Part B – **Statistics** consists of questions 4 – 6

Both parts must be answered

All answers must be written in ink. Except where they are expressly required pencils may only be used for drawing, sketching or graphical work.

Part A – Numerical Methods Component

1. Answer in a separate book marked Question 1

a) Use linear regression to fit the following data points.

x_i	1	2	3	4	5
y_i	0.7	2.2	2.8	4.4	4.9

b) Use forward and centered difference approximations to estimate the first derivative of $y = \exp(x)$ at $x = 1$ for $\triangle x = 0.1$. Then estimate the percentage relative errors E_t by comparing with the exact solution and explain your results.

c) Integrate the following function both analytically and using Simpson's rule, with $n = 4$

$$\int_{-3}^{5} (4x+5)^3 \, dx$$

Discuss the results.

2. **Answer in a separate book marked Question 2**

The fourth-order Runge-Kutter method can be written as:

$$y_{i+1} = y_i + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

where

$$\begin{aligned} k_1 &= f(x_i, y_i) \\ k_2 &= f(x_i + 0.5h, y_i + 0.5hk_1) \\ k_3 &= f(x_i + 0.5h, y_i + 0.5hk_2) \\ k_4 &= f(x_i + h, y_i + hk_3) \end{aligned}$$

a) Find the solution of the initial value problem:

$$y' = 2ty^2$$

with $y(0) = 1$ at $t = 0.2$.

i) Using Euler's method with $h = 0.2$.

ii) Using the fourth-order Runge-Kutta method with $h = 0.2$.

iii) Compare the results with the exact solution $y(t) = \frac{1}{(1-t^2)}$ and find the percentage of errors for the results obtained in i) and ii)

iv) Why do you have improvement in the case of the Runge-Kutta method?

b) Consider the second order differential equation

$$x''(t) + 10x'(t) + 7x(t) = 0.$$

Convert this into a system of first order differential equations.

3. **Answer in a separate book marked Question 3**

Radial heat transfer in a thick hollow cylinder is given by

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0.$$

You are required to formulate the boundary-value problem for an infinitely long hollow cylinder whose inner surface ($r = 50$ mm) is maintained at 50 degrees C, while the outer surface ($r = 150$ mm) is maintained at 25 degress C. Use $h = 20$ mm and perform following steps.

a) Write the central-difference formulas for the derivatives of Equation (1) at the interior nodes.

b) Using the boundary conditions; $T(r_0) = 50$ and $T(r_N) = 25$ write the coefficients A_i,B_i,C_i and D_i to obtain the system of linear equations for all interior points

$$A_i y_{i-1} + B_i y_i + C_i y_{i+1} = D_i, \ i = 1, 2, \ldots, N-1.$$

c) Express the system of equations in matrix form and discuss which method you can use for solving this system of equations. It is not required to solve this system of equations.