THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF MATHEMATICS AND STATISTICS

Example Class Test 1

MATH2089
Numerical Methods Example Class Test 1

(1) TIME ALLOWED – 50 minutes

(2) TOTAL NUMBER OF QUESTIONS – 4

(3) ANSWER ALL QUESTIONS

(4) THE QUESTIONS ARE OF EQUAL VALUE

(5) THIS PAPER MAY **NOT** BE RETAINED BY THE CANDIDATE

(6) **ONLY** CALCULATORS WITH AN AFFIXED "UNSW APPROVED" STICKER MAY BE USED

(7) Write your answers on this test paper in the space provided.

Ask your tutor if you need more paper.

1. a) [**3 marks**] Give the results of the following MATLAB commands when executed on a computer:

```
a = [-1:1]
b = a./(a.^3-a)
```

Answer:

b) [**3 marks**]

i) **Define** the **relative error** in a computed approximation $\bar{x}$ to $x \neq 0$.
Answer:

ii) Estimate the **absolute error** in storing $y = (8.01 \times 10^{12})^{\frac{1}{3}}$ on a computer using double precision floating point arithmetic.
Answer:

c) [**4 marks**] You are asked to calculate the expression

$$D = b + \sqrt{b^2 + \alpha}$$

when $b < 0$ and α is much smaller in magnitude than b

  i) Explain why this expression is/is not good for implementation on a computer.
Answer:

  ii) Find a mathematically equivalent, but numerically preferable, expression for D.
Answer:

2. The computational complexity of some common operations with n by n matrices are given in the Table below.

Operation	Flops
Matrix multiplication	$2n^3$
LU factorization	$\frac{2n^3}{3} + O(n^2)$
Cholesky factorization	$\frac{n^3}{3} + O(n^2)$
Back/forward substitution	$n^2 + O(n)$
Tridiagonal solve	$8n + O(1)$

a) [**4 marks**] You have a 3GHz quad core computer where each core can do two floating point operations per clock cycle. Estimate how long it will take to solve the n by n linear system $A\mathbf{x} = \mathbf{b}$ where A has no special structure and $n = 10^4$.

Answer:

b) [**3 marks**] Estimate the size n of the largest n by n tridiagonal matrix that can be stored in 1Gb RAM using double precision floating point arithmetic.

Answer:

c) [**3 marks**] A programmer claims that as solving a linear system $A\mathbf{x} = \mathbf{b}$ takes around 10 seconds, solving ten linear systems $A\mathbf{x}_j = \mathbf{b}_j$ for $j = 1, \ldots, 10$ will take 100 seconds. Justify or refute this statement.

Answer:

3. a) [**4 marks**] Give MATLAB commands for **EITHER** an anonymous function **osc OR** a function M-file **osc.m** to calculate

$$f(x) = x \sin\left(\frac{1}{x}\right).$$

Your function should work for an array of inputs **x**, producing an array of output values of the same size.

Answer:

b) Consider the function $f(x) = x^3 - \cos(x)$.

i) [**2 marks**] Prove that f has at least one zero in the interval $[0, \pi]$

Answer:

ii) [**2 marks**] Prove that f has at most one zero in the interval $[0, \pi]$

Answer:

c) [**2 marks**] Let `err` be a vector containing the values $e^{(k)} = |x^{(k)} - x^*|$ for $k = 0, 1, \ldots, 10$ produced by an iterative method. The MATLAB commands

```
cv1 = err(2:end) ./ err(1:end-1)
cv2 = err(2:end) ./ err(1:end-1).^2
```

produce

```
cv1 =
  0.8000  0.6400  0.4096  0.1678  0.0281  0.0008  0.0000  0.0000  0.0000
cv2 =
  1.0890  1.1479  1.2500  1.4348  1.7936  2.5735  4.6156 11.8781 54.8251
```

Giving reasons, estimate the rate of convergence.

Answer:

4. You are given the results of the following MATLAB commands and the spy plots in Figure 4.1.

```
size(A)
ans =
     121    121
symchk = norm(A-A',1)
symchk =
     4.5056e-016
ev = eig(A);
evmin = min(ev)
evmin =
     0.9916
evmax = max(ev)
evmax =
         5.5605
p = amd(A);
```
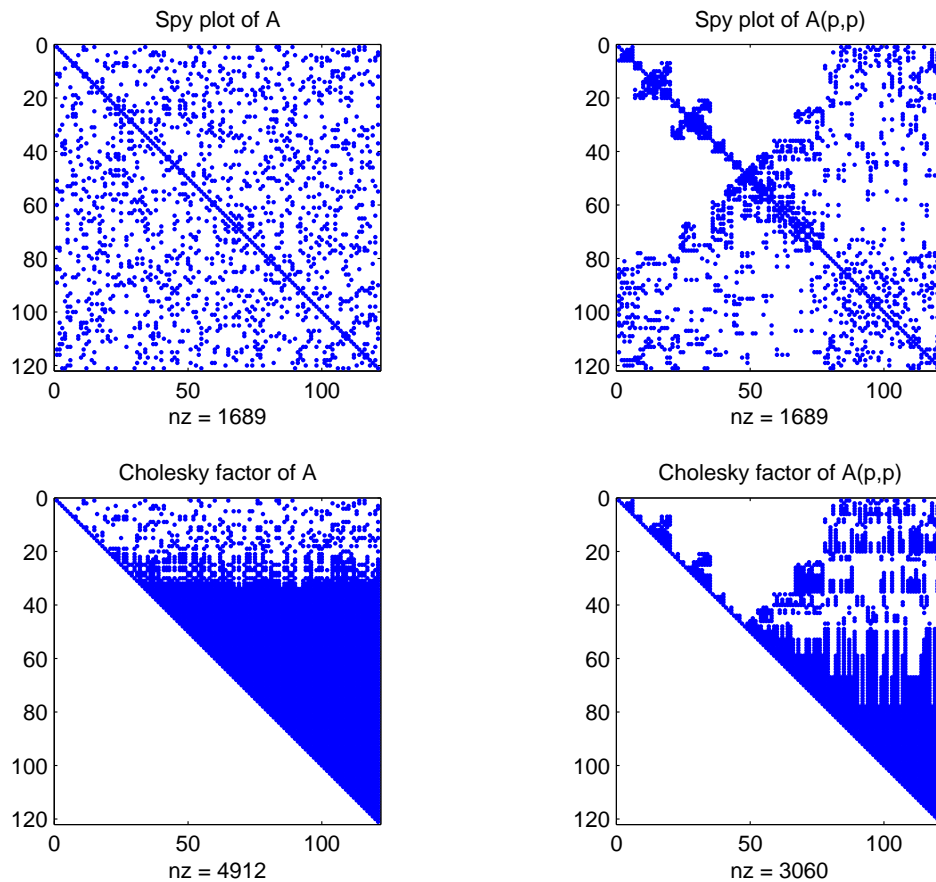

Figure 4.1: Spy plot of A and $A(p, p)$

a) [**2 marks**] A student claims the matrix is not symmetric. Justify or refute this claim.

Answer:

b) [**2 marks**] Calculate the sparsity of A as a percentage.

Answer:

c) [**2 marks**] Calculate the condition number $\kappa(A)$ of A.

Answer:

d) [**4 marks**] The elements of the coefficient matrix A are known exactly and the right-hand-side vector $\mathbf{b}$ is known to 6 significant figures.

    i) Estimate the relative error in the computed solution to $A\mathbf{x} = \mathbf{b}$.
      Answer:

    ii) What confidence do you have in the computed solution?
      Answer: