FAMILY NAME: .....
OTHER NAME(S): .....
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THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF MATHEMATICS AND STATISTICS

Example Class Test 2

MATH2089
Numerical Methods Example Class Test 2

(1) TIME ALLOWED – 50 minutes

(2) TOTAL NUMBER OF QUESTIONS – 4

(3) ANSWER ALL QUESTIONS

(4) THE QUESTIONS ARE OF EQUAL VALUE

(5) THIS PAPER MAY **NOT** BE RETAINED BY THE CANDIDATE

(6) **ONLY** CALCULATORS WITH AN AFFIXED "UNSW APPROVED" STICKER MAY BE USED

(7) Write your answers on this test paper in the space provided.
Ask your tutor if you need more paper.

All answers must be written in ink. Except where they are expressly required pencils may only be used for drawing, sketching or graphical work.

1. a) **[3 marks]** Give the results of the following MATLAB commands when executed on a computer:

i)
```
h = 1e-12;
z = 2 + h > 2
```
Answer:

ii)
```
u = [0  1];
v = u./(u.^2-u)
```
Answer:

b) **[3 marks]** A technician claims the amount of energy used in a chemical reaction (in appropriate units) is

$$E = 1201.469380194205,$$

and that measurements were made to 5 significant figures.

i) Give an estimate of the relative error in E.
Answer:

ii) Give an estimate of the absolute error in E.
Answer:

iii) Give the correctly rounded value for E.
Answer:

Please see over ...

c) **[4 marks]** Estimate the size n of the largest n by n matrix that can be stored in 2Gb RAM using double precision floating point arithmetic.
Answer:

2. a) **[6 marks]** If $f \in C^3(\mathbb{R})$ then

$$f'(x) = \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h} - \frac{h^2}{3} f'''(\zeta), \quad \zeta \in [x, x+2h]$$

You are **not** required to derive this.

i) Give an expression for the truncation error as a function of the stepsize h using "Big-O" $O()$ notation.
Answer:

ii) The rounding error in calculating the finite difference approximation is $O(\frac{\epsilon}{h})$. Estimate the optimal stepsize h^* in terms of the relative machine precision ϵ.
Answer:

iii) When using MATLAB and double precision floating point arithmetic, give an estimate for the optimal stepsize h^*.
Answer:

Please see over ...

b) **[4 marks]** Give MATLAB commands for **EITHER** an anonymous function `myf` **OR** a function M-file `myf.m` to calculate

$$f(x) = xe^{x^2}.$$

Your function should work for an array of inputs `x`, producing an array of output values of the same size.

Answer:

3. Consider the problem of finding the fourth root $a^{1/4}$ of a real number $a > 1$.

a) **[1 mark]** Convert this into a problem of finding the zero x^* of a **polynomial** $p(x)$.
Answer:

b) **[2 marks]** Prove that p has at least one zero in the interval $(1, a)$
Answer:

c) **[2 marks]** Prove that p has at most one zero in the interval $(1, a)$
Answer:

d) **[4 marks]** Show that Newton's method for finding a zero of $p(x)$ can be written as

$$x_{k+1} = \frac{1}{4}\left(3x_k + \frac{a}{x_k^3}\right).$$

Answer:

e) **[1 mark]** Let $e_k = |x_k - x^*|$ for $k = 0, 1, \ldots$ be the errors produced by Newton's method. If $e_4 = 3 \times 10^{-6}$, estimate e_5.

Answer:

4. Consider the data values y_j measured at the times t_j for $j = 1, 2, 3, 4, 5$, given in Table 4.1. The data, which is in column vectors `tdat` and `ydat` produces

j	1	2	3	4	5
t_j	0	0.5	1	1.5	2
y_j	8.1	7.2	4.9	3.1	0.2

Table 4.1: Data

the approximation obtained with the following MATLAB commands

```
A = [ones(size(tdat)) tdat tdat.^2];
[m, n] = size(A);
x = A \ ydat
x =
    8.1800
   -1.9800
   -1.0000
```

The data and the approximation are plotted in Figure 4.1.

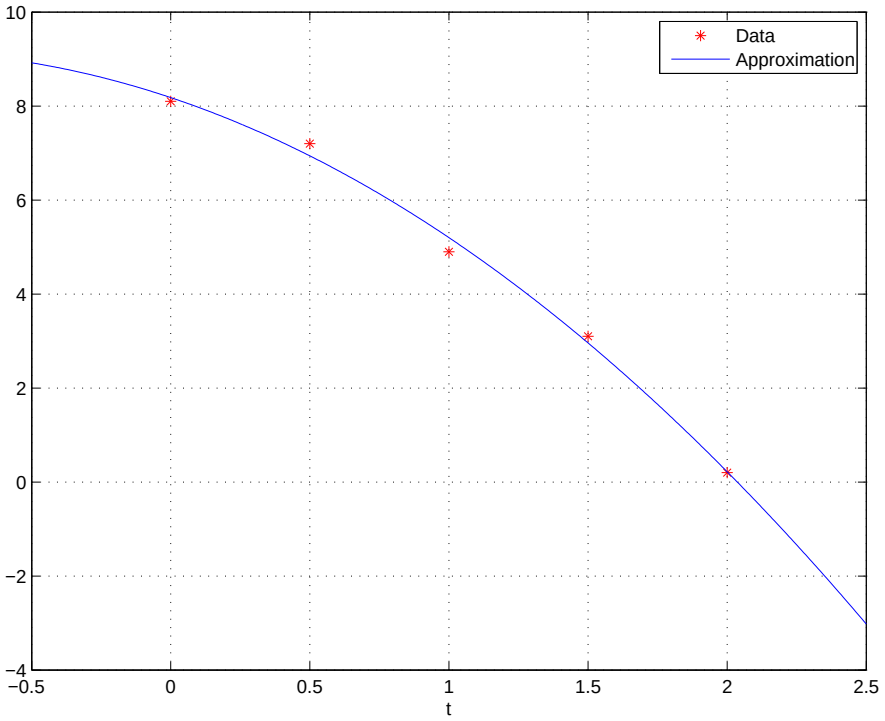

Figure 4.1: Data and approximation

Please see over . . .

a) **[2 marks]** What are the values of `m` and `n` for this example?
Answer:

b) **[1 mark]** It is claimed that the solution $\mathbf{x}$ to the linear system $A\mathbf{x} = \mathbf{y}$ is given by $\mathbf{x} = A^{-1}\mathbf{y}$. Why is this not correct for this example?
Answer:

c) **[1 mark]** What do the MATLAB commands above calculate?
Answer:

d) **[2 marks]** Write down the approximation obtained.
Answer:

e) **[2 marks]** The results of the following MATLAB commands are

```
[m, n] = size(A);
[Q, R] = qr(A);
norm(Q'*Q-eye(m), 1)
ans =
   6.5226e-16
```

It is claimed that this implies that the matrix Q is not orthogonal. Justify or refute this claim.

Answer:

f) **[2 marks]** Show that a square orthogonal matrix Q has condition number $\kappa_2(Q) = 1$.

Answer: