UNIVERSITY OF NEW SOUTH WALES
School of Mathematics and Statistics

MATH2089 Numerical Methods and Statistics
Term 2, 2019

Numerical Methods Laboratory – Week 1

You will need to become familiar with MATLAB, downloading example M-files from the course web page and editing and creating M-files. The MATH2089 course web page is available through UNSW Moodle. Here you will find material on the Numerical Methods component, as well as material for the Statistics component and MATLAB.

0. Download the MATLAB M-file `ex00.m` from the the MATH2089 course web page, run it in MATLAB and open it in the MATLAB editor.

   (a) Open the MATH2089 course web page in UNSW Moodle, then find the M-file `ex00.m`. Try left-clicking on the name of the file, and choosing the option to save the file (make sure you save it in an appropriate folder). Alternatively, right-click the file name and choose `Save Target As ...` (or `Save Link As ...` if using Firefox).

   (b) Start MATLAB and run this M-file by typing `ex00` in the MATLAB command window. Note that the file `ex00.m` must be in MATLAB's current working directory.

   (c) Use the MATLAB editor to open this file. You do not need to change any of the commands.

The following exercises can be done interactively, that is the commands typed directly into the MATLAB command window. However it is better to create small M-files containing these commands using the MATLAB editor. If the commands for question 1 are in the file `nm02q1.m` then simply typing `nm02q1` in the MATLAB command window will run all the commands in this file.

1. MATLAB has built-in values for some common mathematical constants, such as `pi` for π.

   (a) What do you get if you type `pi` in the MATLAB command window?

   (b) What do you get if you enter `format long` and then `pi`?

   (c) What is the value of π rounded to 8 significant figures? (Do by hand)

   (d) Define the variable `p = 3.1416`. What is the absolute error and the relative error in using `p` as an approximation to π.

   (e) Calculate $x = 2 \times 10^{30}$, $y = \sin(x)$. (**[H]** The correct value is $y \approx 0.1795$ using MuPAD or Maple and extended precision)

   (f) **[H]** Plot the values of $y_k = \sin(\pi 10^k)$ for $k = 1, \ldots, 100$. Compare with the true value.

2. Floating point number systems are only finite approximations to the real number system, based on a finite binary representation of numbers. Thus powers of 2 are represented exactly, but powers of 10, for example 0.1, may not have a finite binary representation so must be approximated by the closest floating point number.

   A logical true is represented by 1 (or any non-zero number in some systems) and a logical false by 0.

   Try the following operations in MATLAB or C, Fortran, Excel, ...

(a) `0.75*0.2 - 0.15`

(b) `floor(6/1), floor(0.6/0.1)` (`floor(`x`)` is the largest integer $\leq x$.)

(c) `h = 1e-14, 100+h-100`

(d) `h = 1e-15, 100+h-100`

(e) `eps, realmax, realmin`

(f) `eps, 1+eps >1, 1+eps/2>1`

3. IEEE floating point arithmetic has a special representation for `NaN`, (standing for Not a Number) which is used for quantities that are not mathematically defined. Some systems (for example MATLAB) also have special representations `Inf` for $+\infty$ and `-Inf` for $-\infty$. If an operation produces one of these quantities then they can propagate throughout subsequent calculations.

   Try the following operations in MATLAB or C, Fortran, Excel, ...

   (a) `1/0,    -1/0,    log(0),    log(-1)`

   (b) `0/0,    Inf - Inf.    0*Inf,    0*NaN`

4. The MATLAB function `exp` gives the exponential function.

   (a) Find the largest integer value of $t > 0$ such that on your computer system e^t is a finite number.

   (b) Calculate the value of e^{-t} for $t = 700, 710, 720, \ldots, 800$.

5. **[H]** For positive integer values of n, the integral

$$J_n = \int_0^1 x^n e^{x-1} dx \geq 0$$

   can be calculated using the recursion formula (established using integration by parts)

$$J_1 = e^{-1}, \quad J_n = 1 - nJ_{n-1}. \tag{1}$$

   Calculate J_n for $n = 2, \ldots, 10$ using the recursion formula (1) and

   (a) the approximate initial value $J_1 = 0.367879$ (e^{-1} correct to 6 significant figures).

   (b) the approximate initial value $J_1 =$ `exp(-1)` provided by the package you are using (MATLAB, Excel, C, Fortran, etc).

   This is an example of a calculation that is sensitive to error in the initial data. There are other calculations, such as the calculation of e^{-1} from the Taylor series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots,$$

   which are sensitive to roundoff error, although the double precision arithmetic used in many packages (MATLAB, Excel for example) hides much of this sensitivity.

6. Use the MATLAB online help system,

   - `help` command or
   - MATLAB documentation using **Help** menu item or `helpdesk` command,

   to find out information on the commands used in the previous exercises, including

   (a) `eps`, `realmax`, `realmin`

   (b) `floor`, `ceil`

   (c) `log`, `log10`, `log2`

   (d) `ops`