

FAMILY NAME: .....  
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THE UNIVERSITY OF NEW SOUTH WALES  
SCHOOL OF MATHEMATICS AND STATISTICS

Example Class Test 4

**MATH2089**  
**Numerical Methods Example Class Test 4**

- (1) TIME ALLOWED – 50 minutes
- (2) TOTAL NUMBER OF QUESTIONS – 4
- (3) ANSWER ALL QUESTIONS
- (4) THE QUESTIONS ARE OF EQUAL VALUE
- (5) THIS PAPER MAY **NOT** BE RETAINED BY THE CANDIDATE
- (6) **ONLY** CALCULATORS WITH AN AFFIXED “UNSW APPROVED” STICKER  
MAY BE USED
- (7) Write your answers on this test paper in the space provided.  
Ask your tutor if you need more paper.

All answers must be written in ink. Except where they are expressly required pencils may only be used for drawing, sketching or graphical work.

1. a) [4 marks] Give the results of the following MATLAB commands when executed on a computer and explain why those results are obtained:

i) `h = 1e-14;`  
`z = 2001 + h > 2001`

*Answer:*

ii) `a = [-2 2];`  
`b = (a-2)./(4 - a.^2)`

*Answer:*

- b) [3 marks] A technician claims the amount of energy  $E$  used in a chemical reaction (in appropriate units) was measured to 2 decimal places. MATLAB gives

```
>> format long
>> E
E =
    2.007358357045850e+02
```

- i) Give an estimate of the absolute error in  $E$ .

*Answer:*

- ii) Give an estimate of the relative error in  $E$ .

*Answer:*

- iii) Give the correctly rounded value for  $E$ .

*Answer:*

- c) [**3 marks**] Estimate the size  $n$  of the largest  $n$  by  $n$  matrix that can be stored in 12Mb cache memory using IEEE double precision floating point arithmetic.

*Answer:*

2. a) A function  $f$  with  $n+1$  continuous derivatives has Taylor series expansion

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \cdots + \frac{h^n}{n!}f^{(n)}(x) + \frac{h^{n+1}}{(n+1)!}f^{(n+1)}(\zeta).$$

where  $\zeta \in [x, x+h]$ . If  $f \in C^3(\mathbb{R})$  then a central difference approximation of  $O(h^2)$  to the first derivative is

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2). \quad (2.1)$$

- i) [4 marks] Use Taylor series expansions of  $O(h^3)$  for both  $f(x+h)$  and  $f(x-h)$  to derive equation (2.1).

*Answer:*

- ii) [2 marks] You are given that the optimal stepsize  $h^* = O(\epsilon^{1/3})$ , where  $\epsilon$  is the relative machine precision. When using double precision floating point arithmetic, give an estimate for the optimal stepsize  $h^*$ .

*Answer:*

- b) [4 marks] Give MATLAB commands for **EITHER** an anonymous function `nrmc` **OR** a function M-file `nrmc.m` to calculate

$$f(x) = \sin(x) e^{-x^2}.$$

Your function should work for an array of inputs `x`, producing an array of output values of the same size.

*Answer:*

3. Consider the problem of finding the sixth root  $a^{1/6}$  of a real number  $a > 1$ .

- a) [1 mark] Convert this into a problem of finding the zero  $x^*$  of a **polynomial**  $p(x)$ .

*Answer:*

- b) [2 marks] Prove that  $p$  has at least one zero in the interval  $(1, a)$

*Answer:*

- c) [2 marks] Prove that  $p$  has at most one zero in the interval  $(1, a)$

*Answer:*

- d) [**4 marks**] Show that Newton's method for finding a zero of  $p(x)$  can be written as

$$x_{k+1} = \frac{1}{6} \left( 5x_k + \frac{a}{x_k^5} \right).$$

*Answer:*

- e) [**1 mark**] Let  $e_k = |x_k - x^*|$  for  $k = 0, 1, \dots$  be the errors produced by Newton's method. If  $e_8 = 2 \times 10^{-9}$ , estimate  $e_9$ .

*Answer:*

4. Consider the data values  $y_j$  measured at the times  $t_j$  for  $j = 1, 2, 3, 4, 5, 6$ , given in Table 4.1. The data, which is in column vectors `tdat` and `ydat`,

$j$	1	2	3	4	5	6
$t_j$	0	0.2	0.4	0.6	0.8	1.0
$y_j$	0.91	0.88	0.84	0.45	0.32	0.12

Table 4.1: Data

produces the approximation obtained with the following MATLAB commands

```
A = [ones(size(tdat)) tdat];
[m, n] = size(A);
x1 = (A'*A) \ (A'*ydat)
x1 =
    1.0167
   -0.8600
x2 = A \ ydat
x2 =
    1.0167
   -0.8600
```

The data and the approximation are plotted in Figure 4.1.

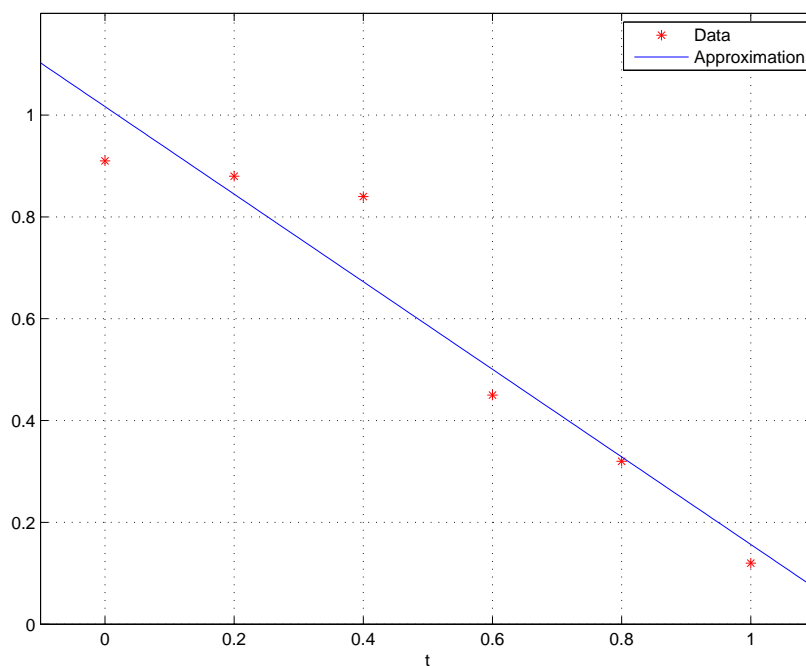


Figure 4.1: Data and approximation



- a) [**2 marks**] What are the values of  $m$  and  $n$  for this example?

*Answer:*

- b) [**1 mark**] It is claimed that the solution  $\mathbf{x}$  to the linear system  $A\mathbf{x} = \mathbf{y}$  is given by  $\mathbf{x} = A^{-1}\mathbf{y}$ . Why is this not correct for this example?

*Answer:*

- c) [**1 mark**] Why are the values of the variables  $x_1$  and  $x_2$  the same?

*Answer:*

- d) [**2 marks**] Write down the approximation obtained.

*Answer:*

- e) [**2 marks**] If  $A$  has condition number  $\kappa_2(A) \approx 3 \times 10^4$ , estimate the condition number of  $A^T A$ .

*Answer:*

- f) [**2 marks**] The QR factorization of an  $m$  by  $n$  matrix  $A$  with  $m > n$  gives

$$A = Q \begin{bmatrix} R \\ 0 \end{bmatrix},$$

where  $Q$  is an  $m$  by  $m$  orthogonal matrix and  $R$  is an  $n$  by  $n$  upper triangular matrix. Let  $Y$  be the  $m$  by  $n$  orthogonal matrix obtained from the first  $n$  columns of  $Q$ , so  $A = YR$ .

How can the QR factorization be used to calculate the least squares solution to  $A\mathbf{x} = \mathbf{b}$ ?

*Answer:*