Sample Statistics Class Test

Question 1.

i) Suppose that Z has a standard normal distribution (i.e. $N(0,1)$). Use the tables to determine the value of a such that $P(-a < Z < a) = 0.4$. (Include in your answer a sketch of a normal curve and shade the area corresponding to the given probability.)

ii) The yield strength of A36 steel (in ksi) is normally distributed with $\mu = 43$ and $\sigma = 4.5$.

    a) What proportion of specimens have a yield strength between 40 and 50 ksi?

    b) How strong must the yield strength of a specimen be if it is to be among the strongest 1% of all specimens?

Question 2. The amount of aluminium contamination (ppm) in plastic of a certain type was determined for a sample of 24 plastic specimens, resulting in the following data.

$$89, 97, 107, 119, 124, 128, 132, 132, 134, 135, 137, 137,$$
$$139, 139, 141, 147, 150, 159, 163, 167, 168, 171, 180, 186.$$

i) Compute the 5 number summary for this data set.

ii) Use the $1.5 \times$IQR rule to determine which, if any, of the data values are possible outliers.

iii) Construct a (horizontal) boxplot from the 5 number summary determined in part i) and, if applicable, be sure to properly label any potential outliers found in part ii).

iv) Construct a stemplot for the data set.

v) Comment on the shape of the distribution of the data set by inspecting your boxplot and stemplot. Be sure to use terms such as unimodal, bimodal, symmetric and skewed while specifying a specific direction.

Question 3.

i) Determine the value of c such that the function

$$f(x) = \begin{cases} c\, x^2(1-x)^2, & \text{if } 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

is a probability density function.

ii) Assume the random variable X has continuous density function

$$f(x) = \begin{cases} \frac{3}{4}(1-x^2), & \text{if } -1 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Calculate $P(|X| < 0.5)$. (Hint: You might find a sketch of the density curve to be useful.)

Question 4. A dashboard warning light is supposed to flash red if a car's oil pressure is too low. On a certain model, the probability of the light flashing when it should is 0.99. When the oil pressure is normal, the warning light incorrectly flashes 2% of the time. There is a 10% chance that the oil pressure really is low.

i) What is the probability that the warning light goes on?

ii) When the warning light goes on, what is the probability that the oil pressure is low?

Question 5.

i) The proportion of faulty items produced by a production line is 10%. For quality control purposes, a sample of 20 items is taken each week and if the number of faulty items in the sample is two or more, the line is shut down for repairs. Compute the probability that the line is shut down in a certain week.

ii) What is the probability that the line is shut down at least 5 times in 10 weeks.

End of paper.