

UNIVERSITY OF NEW SOUTH WALES
School of Mathematics and Statistics

MATH2089 Numerical Methods and Statistics
Term 2, 2019

Numerical Methods Tutorial – Week 6

1. Let

$$A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 1 & -1 \\ 2 & -1 & 3 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- (a) Show that A is symmetric.
- (b) Show that $A = LDL^T$.
- (c) Find the Cholesky factorization $A = R^T R$, where R is upper triangular.
- (d) Show that A is positive definite.
- (e) Using the Cholesky factorization, find the inverse A^{-1} .
- (f) Compute the condition numbers $\kappa_\infty(A)$ and $\kappa_1(A)$.
- (g) Estimate the relative error in the computed solution to $A\mathbf{x} = \mathbf{b}$ if \mathbf{b} is known to 4 significant figures,

2. The MATLAB script `tut06q2.m` produces the output

- (a) What row operations does MATLAB do to produce zeros in the first column of A ?
- (b) Why is `chk` not equal to zero?
- (c) Calculate the sparsity of A
- (d) What is the value of B ?
- (e) Why are `x1T` and `x2T` not the same?

3. Consider the spy plots of the 156 by 156 matrix A from the chemical plant model
<http://math.nist.gov/MatrixMarket/data/Harwell-Boeing/chemwest/west0156.html>

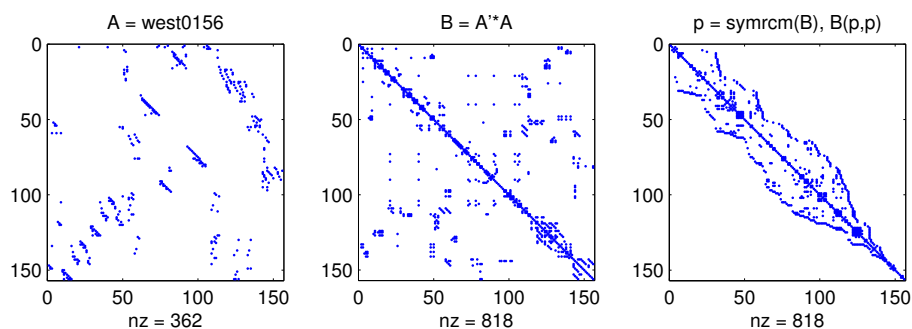


Figure 1: Spy plots of `west0156` matrices

- (a) Are A and $B = A^T A$ symmetric?
- (b) How many non-zero elements do A and B have?
- (c) Calculate the sparsity of A and B .
- (d) What is the sparsity of $B(p, p)$?
- (e) What does $B(p, p)$ give and why is it useful?