Statistics Mid-semester Maple Quiz

<table>
<tr><th>Questions</th><th>Answer</th></tr>
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Question 1: *Score 4/4*

Suppose X and Y are independent random variables with $E(X) = -5$, $Var(X) = 8$, $E(Y) = 1$, $Var(Y) = 9$ and that $W = 2X+2Y$.

(a)

What is $E(W)$? Correct

Your Answer: -8

(b)

What is $Var(W)$? Correct

Your Answer: 68

Comments:

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Question 1: *Score 4/4*

Suppose X and Y are independent random variables with $E(X) = 6$, $Var(X) = 10$, $E(Y) = -4$, $Var(Y) = 8$ and that $W = -2X-4Y$.

(a)

What is the expected value of W? Correct

Your Answer: 4

(b)

What is the standard deviation of W? (Please write to 2 decimal places) Correct

Your Answer: 12.96

Comments:

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Question 2: *Score 4/4*

The amount of time that a surveillance camera will run without having to be reset is a random variable having an Exponential distribution with mean 54 days.

a) Find (to three decimal places) the probability that such a camera will have to be reset in less than 25 days.

<table><tr><th>Your response</th><th>Correct response</th></tr><tr><td>0.371</td><td></td></tr></table>

Grade: 1/1.0

b) A new model of surveillance camera has probability 0.259 that it needs to be reset in less than 25 days. Suppose we have 14 of these new cameras, all put in usage on the same day and working independently of each other.

Use Matlab to find (to three decimal places) the probability that at least 6 of them will have to be reset in less than 25 days.

<table><tr><th>Your response</th><th>Correct response</th></tr><tr><td>0.128</td><td></td></tr></table>

Grade: 1/1.0

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```
%a)

expcdf(days in (a),mean days)
%b)
1-binocdf('at least x' so input (x-1),number of new cameras, probability)
```

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Question 2: *Score 2/4*

A power grid has 80 solar collectors as part of its energy production. On a given day, a solar collector has a 7% chance of failing. If a collector fails it is removed from the grid for the rest of the day.

a) Use Matlab to find (to three decimal places) the chance that there are at least 74 solar collectors working by the end of a given day.

<table><tr><th>Your response</th><th>Correct response</th></tr><tr><td>0.6727</td><td>0.6727±0.002</td></tr></table>

Grade: 1/1.0

b) The amount of daily power that the solar collectors generate, Y (in kWh), depends on the number of working solar collectors each day, X, in the following way:

Y = 50*X - 0.4*X^2

Find the expected power output that the solar collectors will generate on a given day to the nearest kWh.

<table><tr><th>Your response</th><th>Correct response</th></tr><tr><td>1476</td><td>1,504±1</td></tr></table>

Grade: 0/1.0

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```
%a)
 binocdf(6,80,0.07)

 %b)
E(Y)  = E(50*X - 0.4X^2)
      = 50*E(X)-0.4E(X^2)
      = 50*E(X)-0.4(E(X)^2+Var(X))
Where
   -  p = chance of it working
   -  E(X) = np = 80*0.93
   -  Var(X) = np(1-p) = 80*0.93*0.07

E(Y) = power collected
```

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Question 2: *Score 2/4*

The number of machine failures per month in a certain plant has a Poisson distribution with mean equal to 3.5. Present facilities at the plant can repair 4 machines per month. If any additional machines fail then they are repaired by an outside contractor.

a) Use Matlab to find the probability, on a given month, that the contractor is required to work in the plant. Give your answer to three decimal places.

Your response	Correct response
0.275	**0.2746±0.002**

Grade: 1/1.0

b) The cost (in thousands of dollars) to the plant of machine failures can be approximated as:

X = 18+10*Y+Y^2

Find (to one decimal place) the expected cost of machine failures per month (in thousands of dollars).

Your response	Correct response
21.1	**68.75±0.1**

Grade: 0/1.0

```
%2a)
Y = 1-poisscdf(4,3.5)

%2b)
Y = 3.5 (mean)
X = 18+11*Y+Y^2
```

Question 2: *Score 4/4*

The amount of time that a mobile phone will work without having to be recharged is a random variable having the Exponential distribution with mean 3.1 days.

a) Find (to three decimal places) the probability that such a mobile phone will have to be recharged in less than 1 days.

Your response	Correct response
0.276	

Grade: 1/1.0

b) Suppose a new model of phone has probability 0.193 of needing to be recharged in less than 1 days. We have 20 of these new phones, all put in usage on the same day and working independently of each other.

Use Matlab to find (to three decimal places) the probability that at least 5 of them will have to be recharged in less than 1 days.

Your response	Correct response
0.340	

Grade: 1/1.0

```
%a)
expcdf(1,3.1)

%b)
1 - binocdf(4,20,0.193)

4 → 'at least 5…' so input (n-1)
```

Question 2: *Score 4/4*

In the daily production of a certain kind of rope, the number of defects per 10 metres Y is assumed to have a Poisson distribution with mean 5.2.

a) Assume that you randomly select a 10 metre length of rope. Use Matlab to find (to three decimal places) the probability that this rope contains at least 4 defects.

Your response	Correct response
0.762	

Grade: 1/1.0

b) The profit per 10 metres when the rope is sold is given by X (in $) and depends on the number of defects according to the following expression :

X = 50 – 5*Y – 0.01*Y^2

Find the expected profit per 10 metres (to two decimal places).

Your response	Correct response
23.68	

Grade: 1/1.0

```
%a)
1 - poisscdf(3,5.2)

"at least 4 defects…" input (x-1)
%b)
Y = 5.2 (mean)
(Y)^2 = 5.2^2 + 5.2
X = 50 - 5*Y - 0.01*(Y)^2
```

Question 3: *Score 0/2*

Given $Z \sim N(0,1)$ use Matlab to calculate a value c such that $P(Z > c) = 0.011$. Give your answer to three decimal places.

Your response	Correct response
0.014	2.29±0.001

Grade: 0/1.0

Total grade: 0.0×1/1 = 0%

Comment:

```
abs(norminv(c))
```

Question 3: *Score 2/2*

Given $Z \sim N(0,1)$ use Matlab to calculate a value c such that $P(-c < Z < c) = 0.606$. Give your answer to three decimal places.

Your response	Correct response
0.852	

Grade: 1/1.0

```
%3
 -norminv((1-0.606)/2)
```

Question 4: *Score 4/4*

Diameters of the trees in a forest are normally distributed with mean μ = 36.0 cm and standard deviation σ = 9.0 cm. The that can be used as timber must have specific size. The diameters must lie in the interval [25, 47] cm in order to be used timber.

a) In the forest, what fraction of trees cannot be used as timber? Give your answer to three decimal places.

Your response	Correct response
0.222	

Grade: 1/1.0

b) To what value must the standard deviation σ be reduced if it is required that 98.7% of the trees can be used as timbe your answer to two decimal places.

Your response	Correct response
4.43	

Grade: 1/1.0

%a)

1-(normcdf(2nd number of interval, mean, standard deviation)-normcdf(1st number of interval, mean, standard deviation))

%b)

fzero(@(x)normcdf(2nd number of interval, mean, x)-normcdf(1st number of interval, mean, x)-required %, standard deviation)

Question 4: *Score 4/4*

A manufacturer produces bolts that are specified to be between 1.5 and 1.52 cm in diameter. Its production process results in a bolt's diameter being normally distributed with mean 1.51 cm.

a) If the standard deviation of a bolt's diameter is $\sigma = 0.01$ cm, what proportion of bolts will not meet specifications? Give your answer to three decimal places.

Your response	Correct response
0.317	

Grade: 1/1.0

b) What is the maximum allowable value of σ that will permit no more than 1.0% of the bolts to be outside specifications? Give your answer to four decimal places

Your response	Correct response
0.0039	

Grade: 1/1.0

```
%a)
A = normcdf(1.5,1.51,0.01);
B =
normcdf(1.22,1.51,0.01);
C = 1 - B;
A + C

%b)
fzero(@(x)normcdf(1.52,
1.51, x)-normcdf(1.5, 1.51,
x)-(1-0.01), 0.01)
```

Question 4: *Score 4/4*

Extruded plastic rods are automatically cut into nominal lengths of 14.0 cm. Actual lengths are normally distributed about a mean of 14.0 cm and their standard deviation is 0.4 cm.

a) What proportion of the rods have lengths that are outside the tolerance limits of 13.3 to 14.7 cm? Give your answer to three decimal places.

Your response	Correct response
0.080	

Grade: 1/1.0

b) To what value does the standard deviation (in cm) need to be reduced if 98.5% of the rods must be within tolerance? Answer to two decimal places.

Your response	Correct response
0.29	

Grade: 1/1.0

```
%4
 %a)

A = normcdf(13.3,14,0.4);
B = normcdf(14.7,14,0.4);
C = 1-B;
A + C
```

(1^{st} limit → 2^{nd} limit)

```
%b)
fzero(@(x)normcdf(14.7,14,x
)-normcdf(13.3,14,x)-0.985,
0.4)
```

(2^{nd} limit → 1^{st} limit)

Question 4: *Score 4/4*

Consider resistors with a nominal resistance of 2.0 kiloohms, and which are required to be within 4.0% of this value, that is, within 1,920 to 2,080 ohms. Assume the resistance is normally distributed around the nominal value.

a) If the standard deviation of the resistance of these resistors is 60 ohms, what proportion of them are within the specifications? Answer to three decimal places.

Your response	Correct response
0.818	

Grade: 1/1.0

b) To what value must this standard deviation be reduced (in ohms) if it is required that 99.5% of the resistors are within specifications? Answer to one decimal place.

Your response	Correct response
28.5	

Grade: 1/1.0

```
%a)
normcdf(2080,2000,60)-
normcdf(1920,2000,60);


%b)
fzero(@(x)normcdf(2080,2000
,x)-normcdf(1920,2000,x)-
0.995,60);
```

Question 5: *Score 4.5/6*

Paper is being manufactured continuously before being cut and wound into large rolls. The process is monitored for thickness (using a caliper). A sample of 12 measurements on paper, in micrometres, yielded:

11.56, 10.21, 11.54, 11.73, 11.90, 11.36, 12.72, 13.36, 13.04, 11.56, 12.13, 12.20

a. Find the five-number summary for these observations (to two decimal places)

Your response	Correct response
10.21	10.21±0.005

Grade: 40/40.0

Your response	Correct response
11.55	11.55±0.005

Grade: 40/40.0

Your response	Correct response
11.815	11.82±0.005

Grade: 40/40.0

Your response	Correct response
12.46	12.46±0.005

Grade: 40/40.0

Your response	Correct response
13.36	13.36±0.005

Grade: 40/40.0

b.Find the average thickness (to two decimal places)

Your response	Correct response
11.94	11.9425±0.005

Grade: 200/200.0

c.Find the standard deviation of thickness (to two decimal places)

Your response	Correct response
0.84	0.840034±0.005

Grade: 200/200.0

d.Construct a visual display of the data. Which description best matches the distribution of paper thickness:

Your response	Correct response
Bimodal, fairly symmetric, no outliers	Unimodal, fairly symmetric, no outliers

Grade: 0/200.0

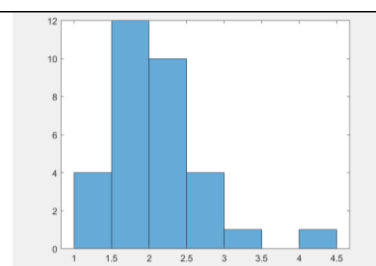
```
%a)
x = [11.56 10.21 11.54
11.73 11.90 11.36 12.72
13.36 13.04 11.56 12.13
12.20]
quantile(x,[0:0.25:1])

%b)
mean(x)
%c)
std(x)
%d)
histogram(x)
```

d. Construct a visual display of the data. Which description best matches the distribution of ignition time

Your response	Correct response
Unimodal, right-skewed, with some apparent outliers	

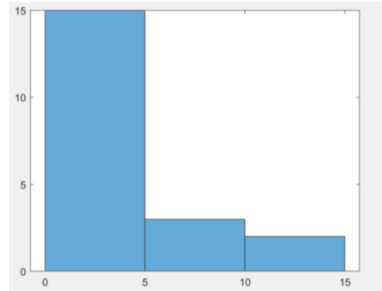
Grade: 200/200.0



d. Construct a visual display of the data. Which description best matches the distribution of crack length:

Your response	Correct response
Unimodal, right-skewed, no outliers	

Grade: 200/200.0



Extra Question	Answers
A check of rooms in a particular university college, called "Old College", revealed that 65% had refrigerators, 35% had TVs, and 32% had both a refrigerator and a TV. • What is the probability that a room at Old College had a refrigerator or a TV (or both)? Give your answer to three decimal places. Number • A general survey of all college rooms across the university has been conducted. The survey showed that 75% of college rooms had a refrigerator or a TV (or both). Old College has 22% of the college rooms in the university. What is the probability that a randomly selected college room which had a refrigerator or a TV (or both) is an Old College room? Give your answer tc three decimal places. Number	Venn diagram setup %a) P(A)=0.65, P(B)=0.35, P(A & B)= 0.32 Therefore: $P(A^c)$= P(A)-P(A & B)=0.33 $P(B^c)$= P(B)-P(A & B)=0.03 P(A OR B) OR P(A&B) = $P(A^c)$+ $P(A^c)$+ (A&B) =0.33+0.03+0.32 =0. 68 %b) (0.22/0.75)*0.68=0.199
We know the following about a colorimetric method used to test lake water for nitrates. If water specimens contain nitrates, a solution dropped into water will cause the specimen to turn red 90% of the time. When used on water specimen without nitrates, the solution causes the water to turn red 15% of the time (because chemicals other than nitrates might also be present and react to the agent). Past experience in a lab indicates that nitrates are contained in 25% of the water specimens that are sent to the lab for testing. a) If a water specimen is randomly selected from among those sent to the lab, what is the probability that it will turn red when tested? Give your answer to three decimal places. b) If a water specimen is randomly chosen and turns red when tested, what is the probability that it actually contains the nitrates? Give your answer to three decimal places.	Tree diagram %a) (0.25*0.9) + (0.75*0.15) = 0.338 %b) P(A)/P(A&B) = (0.25*0.9)/(0.25*0.9+0.75*0.15) =0.666
Suppose the number of cyclones Y near Porpoise Spit has a Poisson distribution with a mean of 1.11 per year. a) For a randomly selected year, use Matlab to find (to three decimal places) the probability that there are at least 2 cyclones. Number b) Property damage (in millions of dollars) in the Porpoise Spit area, per year, is given by X and it depends on the number of cyclones according to the following expression : X = 2 + 2.4*Y − 0.1*Y^2 Find, to two decimal places, the expected damage per year (in millions of dollars).	a) 1 - poisscdf(1,1.11) b) Y = 1.11 E(X)= E(2+2.4*Y-0.1*Y^2) E(X)=2+E(2.4*Y)-E (0.1*Y^2) E(X)=2+2.4*E(Y)-0.1*E (Y^2) AS Var(Y) = E(Y^2)-E(Y)^2 Then E(Y^2)=Var(Y)+E(Y)^2 E(X)=2+2.4*E(Y)-0.1*Var(Y)-0.1*E(Y)^2 E(X) = X = 2 + 2.4*(1.11) - 0.1*(1.11^2 + 1.11) Since you can think about it as: E(X) = 2 + 2.4*E(Y) - 0.1*(E(Y^2)) E(Y) = mean E(Y^2) = mean^2 + mean (See lecture notes L5s20)