UNIVERSITY OF NEW SOUTH WALES
School of Mathematics and Statistics

MATH2089 Numerical Methods and Statistics
Term 2, 2019

Numerical Methods Tutorial – Week 2

1. The speed of light is $2.99792\ldots \times 10^8$ m/sec in a vacuum. Estimate how far an electromagnetic signal in a circuit can move in 1 ns. What are the consequences of this?

2. (a) Calculate the absolute and relative errors when using 1.414 as an approximation to $\sqrt{2}$. Why are the absolute and relative errors similar in this example?

   (b) The volume V of a tank is $1,034.46$ litres measured to two decimal places. Calculate the absolute and relative errors in V.

   (c) An instrument measures the temperature T to 4 significant figures. Calculate the relative and absolute error when $T \approx 200$.

   (d) Let $f(x) = \sin(x) - x$. The **relative error** in x and $\sin(x)$ as stored on a computer is given by the relative machine precision ϵ. When using MATLAB, which uses double precision arithmetic,

      i. Estimate the absolute error in x when $x = 0.1$.

      ii. If the absolute error in $f(x)$ is the same as the absolute error in x, estimate the relative error, and hence the number of significant figures, in $f(x)$.

3. You are working on a 3 GHz dual core computer that can do one flop per core per clock cycle.

   (a) What is the size n of the largest n by n matrices that can be multiplied in 1 hour? Multiplying two n by n matrices requires $2n^3$ flops.

   (b) Assuming each element of a matrix is stored in double precision (8 bytes), how much memory will each of these matrices require?

4. The number of flops required to solve an n by n linear system $A\mathbf{x} = \mathbf{b}$ by Gaussian elimination is $\frac{2n^3}{3}$. On a 3 GHz PC which can do one flop per clock cycle:

   (a) Estimate how long will it take to solve a linear system of size $n = 1000$.

   (b) Estimate the largest linear system than can be solved in

      i. 1 minute,

      ii. 1 hour,

      iii. 1 day.

5. The Fast Fourier Transform (FFT), one of the great algorithms of the 20th century, is used to analyse signals (your mobile phone for example). The FFT takes $n \log_2(n)$ flops to process n data values, rather than n^2 flops for the straightforward Discrete Fourier Transform (DFT). Suppose you have a 2.5GHz quad core computer that can do 4 floating point operations per core per clock cycle.

   (a) Estimate how long will it take to process $n = 2^{30}$ data values using the DFT and the FFT.

   (b) Estimate the largest number n of data values that can be processed in one second by the DFT and the FFT.