Part A – Numerical Methods Component

1. Answer in a separate book marked Question 1

a) Use linear regression analysis to find an equation relating the heat-transfer coefficient (h) to the velocity of the fluid (v) based on the following data points:

$v_i(m/s)$	2	4	6	8	10
$h_i(W/m^3K)$	4,000	8,000	11,000	13,000	14,000

b) Apply the forward difference approximation and central difference approximation to evaluate the first derivative of $f(x) = e^{4x} + x^2 + x$ at $x = 1.5$ for $\Delta x = 0.2$. Calculate the percentage relative errors for each approximation by comparing with the exact solution. Explain your results.

c) Integrate the following function:

$$\int_{-2}^{6} (2x + 3)^3 dx$$

using (i) Simpson's and (ii) trapezoidal rules, with $n = 4$. Determine the percentage relative errors for the numerical solutions obtained in (i) and (ii) with respect to exact solution, which can be determined analytically. Discuss your results.

2. Answer in a separate book marked Question 2

The fourth-order Runge-Kutter method can be written as:

$$y_{i+1} = y_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$k_1 = f(x_i, y_i)$$
$$k_2 = f(x_i + 0.5h, y_i + 0.5hk_1)$$
$$k_3 = f(x_i + 0.5h, y_i + 0.5hk_2)$$
$$k_4 = f(x_i + h, y_i + hk_3)$$

a) Find the solution of the initial value problem:

$$y' = \frac{dy}{dt} = 4t^2y^2$$

with $y(0) = 1$ at $t = 0.2$.

i) Using Euler's method with $h = 0.2$.

ii) Using the fourth-order Runge-Kutta method with $h = 0.2$.

iii) Compare the results with the exact solution $y(t) = \frac{3}{3-4t^3}$ and find the percentage of errors for the results obtained in i) and ii)

iv) Why do you have improvement in the case of the Runge-Kutta method?

b) Consider the second order differential equation

$$3y''(t) - 7y'(t) - 2y(t) = 30e^{2t}.$$

Reformulate this equation as a system of two first order differential equations.

3. Answer in a separate book marked Question 3

The heat conduction equation which models the temperature with an insulated rod of 1m with ends held at constant temperatures can be written in the form as:

$$\frac{\partial T(x,t)}{\partial t} = \frac{\partial^2 T(x,t)}{\partial x^2}$$

a) Write a finite difference approximation of this equation using the Forward-Time, Central-Space (FTCS) scheme and rearrange it to be solved by an explicit method.

b) Using a step size $\Delta x = 0.5m$ and a time step $\Delta t = 0.1s$, solve the finite difference equation and calculate the temperature $T(x,t)$ at $t = 0.3s$ and $x = 0.5m$ if the initial condition is set at

$$T(x, t = 0s) = 35°C \ \ (0m < x < 1m)$$

And the boundary conditions at the ends of the rod are

$$T(x = 0m, t) = 20°C \text{ and } T(x = 1m, t) = 50°C.$$

c) If the calculations in the previous part where repeated with $\Delta x = 0.05m$ and $\Delta t = 0.05s$ to reduce truncation error, what difficulty would be encountered? Do not repeat the finite difference calculations to determine your answer.