Question 1.

Answer in a separate book marked Question 1.

(a) The heat-transfer coefficient (h) in a forced convection heat transfer in cross-flow past a cylinder at room temperature is found to vary with the velocity of the fluid (v) flowing past the cylinder as follows:

v_i (m/s)	2	4	6	8
h_i (W/m^{2}K)	6,000	10,000	13,000	15,000

Use linear regression analysis and find an equation relating h and v.

(b) Compute forward difference approximation of $O(\Delta x)$ and central difference approximation of $O(\Delta x)^2$ for the first derivative of $f(x) = e^x + x$ at $x = 1$, using a value of $\Delta x = 0.25$.

Calculate the percentage relative errors for each approximation by comparing with exact solution and discuss your results.

The exact solution is: $f'(1) = e^1 + 1 = 3.71828$

(c) Integrate the following function:

$$\int_{-3}^{5} (4x+5)^3 \, dx$$

using (i) Simpson's and (ii) trapezoidal rules, with $n = 4$.

Compute the percentage relative errors for the numerical solutions obtained in (i) and (ii) with respect to exact solution and discuss your results.

The exact solution is: $\int_{-3}^{5} (4x+5)^3 \, dx = 24264$

Question 2.

Answer in a separate book marked Question 2.

The fourth-order Runge-Kutta method can be written as:

$$y_{i+1} = y_i + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

where

$$k_1 = f(x_i, y_i)$$
$$k_2 = f(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}hk_1)$$
$$k_3 = f(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}hk_2)$$
$$k_4 = f(x_i + h, y_i + hk_3)$$

(a) Find the solution of the initial value problem:

$y' = 3y + 3t$ with $y(0) = 1$

at t=0.2

(i) Using Euler's method with $h = 0.2$.

(ii) Using the fourth-order Runge-Kutta method with $h = 0.2$.

(iii) Compare the results with the exact solution $y(t) = \frac{4}{3}e^{3t} - t - \frac{1}{3}$ and find the percentage errors for the results obtained in (i) and (ii).

(iv) Why do you have improvement in the case of the Runge-Kutta method?

(b) Consider the second order differential equation

$$2x''(t) - 5x'(t) - 3x(t) = 45e^{2t}$$

Reformulate this equation as a system of two first order differential equations.

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Question 3.

Answer in a separate book marked Question 3.

The heat conduction equation which models the temperature in an insulated rod with ends held at constant temperatures can be written in the dimensionless form as:

$$\frac{\partial \Theta(x,t)}{\partial t} = \frac{\partial^2 \Theta(x,t)}{\partial x^2}$$

(a) Write a finite difference approximation of this equation using the Forward-Time, Central-Space (FTCS) scheme and rearrange it to be solved by an explicit method.

(b) Solve this equation and calculate the temperature $\Theta(x,t)$ at t = 0.3 and x = 0.5 if the initial condition is

$$\Theta(x,t=0) = 1 \qquad (0 < x \leq 1)$$

and the boundary conditions at the ends of the rod are

$$\Theta(x=0,t) = 0; \ \Theta(x=1,t) = 1.$$

Use value of 0.5 for the step in space, Δx, and value of 0.1 for the time step, Δt.

(c) If the calculations in the previous part were repeated with Δx = 0.1 to reduce truncation error and Δt kept equal to 0.1, what difficulty would be encountered? Do not repeat the finite difference calculations to determine your answer.