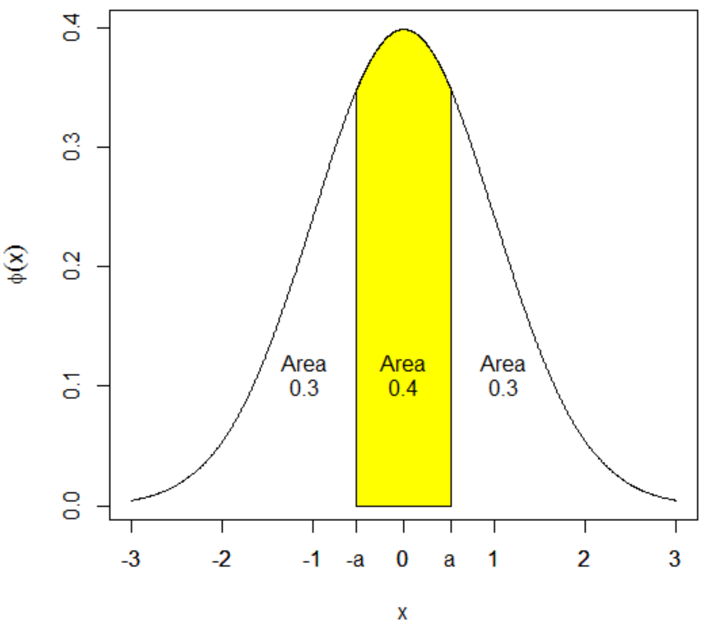
Sample Statistics Class Test

Question 1.

i)

$$
\begin{aligned}
P(-a < Z < a) = 0.4 \quad &\Rightarrow \quad P(Z < -a) = 0.3 \\
&\Rightarrow \quad -a = -0.52 \\
&\Rightarrow \quad a = 0.52
\end{aligned}
$$



ii) Let X be the yield strength of a random specimen of A36 steel. Then $X \sim N(43, 4.5)$.

a)

$$
\begin{aligned}
P(40 < X < 50) &= P\left(\frac{40-43}{4.5} < \frac{X-43}{4.5} < \frac{50-43}{4.5}\right) \\
&= P(-0.67 < Z < 1.56) \\
&= 0.9406 - 0.2514 \\
&= 0.6892
\end{aligned}
$$

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b)

$$
\begin{aligned}
P(X > c) &= 0.01 \\
P\left(\frac{X-43}{4.5} > \frac{c-43}{4.5}\right) &= 0.01 \\
P\left(Z > \frac{c-43}{4.5}\right) &= 0.01 \\
\frac{c-43}{4.5} &= 2.33 \\
c &= 53.485 \text{ ksi}
\end{aligned}
$$

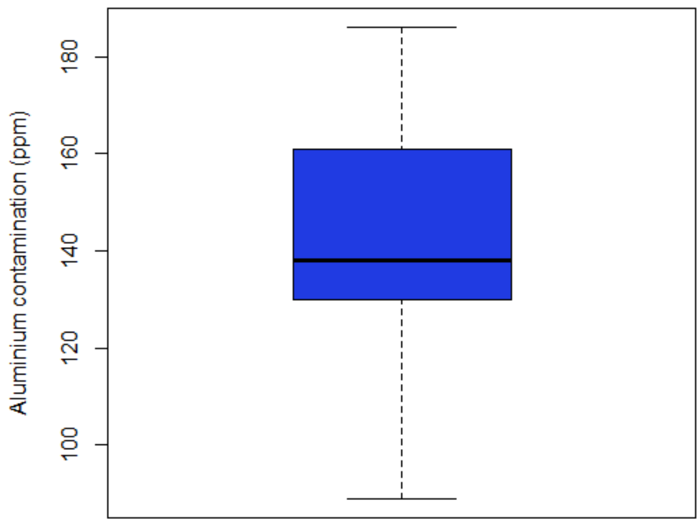
Question 2.

i) $\{89, 130, 138, 161, 186\}$

ii)

$$
\begin{aligned}
IQR &= 161 - 130 \\
IQR &= 31 \\
q_1 - 1.5IQR &= 83.5 \\
q_3 + 1.5IQR &= 207.5
\end{aligned}
$$

No observations are outside of this range. Hence, there are no suspected outliers.



iii)

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iv)

8	9
9	7
10	7
11	9
12	4 8
13	2 2 4 5 7 7 9 9
14	1 7
15	0 9
16	3 7 8
17	1
18	0 6

v) The distribution is unimodal and roughly symmetric.

Question 3.

i)

$$\begin{aligned}
c\int_0^1 x^2(1-x)^2dx &= 1\\
c\int_0^1 x^2(1-2x+x^2)dx &= 1\\
c\int_0^1 (x^2-2x^3+x^4)dx &= 1\\
c\left[\frac{x^3}{3}-\frac{x^4}{2}+\frac{x^5}{5}\right]_0^1 &= 1\\
c\left(\frac{1}{3}-\frac{1}{2}+\frac{1}{5}-0\right) &= 1\\
c\left(\frac{1}{30}\right) &= 1\\
c &= 30
\end{aligned}$$

ii)

$$\begin{aligned}
P(|X| < 0.5) &= P(-0.5 < X < 0.5) \\
&= \int_{-0.5}^{0.5} \frac{3}{4}(1-x^2)dx \\
&= \frac{3}{4}\left[x - \frac{x^3}{3}\right]_{-0.5}^{0.5} \\
&= \frac{3}{4}\left[\frac{1}{2} - \frac{1}{24} - \left(-\frac{1}{2} + \frac{1}{24}\right)\right] \\
&= \frac{11}{16}
\end{aligned}$$

Question 4. Let L be the event that the oil pressure is low. Let F be the event that the warning light flashes. Then $P(F|L) = 0.99$, $P(F|L^C) = 0.02$, and $P(L) = 0.1$.

i) L and L^C form a partition of the sample space, so we can use the Law of Total Probability to find $P(F)$:

$$\begin{aligned}
P(F) &= P(F|L)P(L) + P(F|L^C)P(L^C) \\
&= (0.99)(0.1) + (0.02)(0.9) \\
&= 0.117
\end{aligned}$$

ii) We can use Bayes' Rule:

$$\begin{aligned}
P(L|F) &= \frac{P(F|L)P(L)}{P(F)} \\
&= \frac{(0.99)(0.1)}{0.117} \\
&\approx 0.846
\end{aligned}$$

Question 5.

i) Let X be the number of faulty items in the sample. Then $X \sim Bin(20, 0.1)$. We seek $P(X \geq 2)$.

$$\begin{aligned}
P(X \geq 2) &= 1 - P(X \leq 1) \\
&= 1 - P(X = 0) - P(X = 1) \\
&= 1 - \binom{20}{0}(0.1)^0(1-0.1)^{20-0} - \binom{20}{1}(0.1)^1(1-0.1)^{20-1} \\
&= 1 - 0.1216 - 0.2702 \\
&= 0.6082
\end{aligned}$$

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ii) Let Y be the number of times the line is shut down over 10 weeks. Then $Y \sim Bin(10, 0.6082)$.

$$\begin{aligned}
P(Y \geq 5) &= 1 - P(Y \leq 4) \\
&= 1 - P(X = 0) - \cdots - P(X = 4) \\
&= 1 - \binom{10}{0}(0.6082)^0(1 - 0.6082)^{10-0} - \cdots - \binom{10}{4}(0.6082)^4(1 - 0.6082)^{10-4} \\
&= 1 - 0.0001 - 0.0013 - 0.0092 - 0.0383 - 0.1039 \\
&= 0.8472
\end{aligned}$$