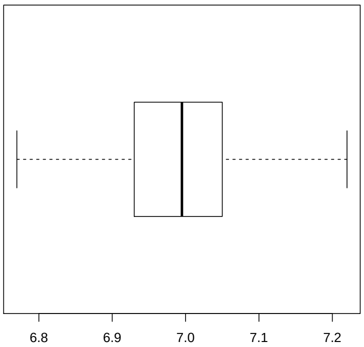
4. [**20 marks**]

(a) i. [**2 marks, 1/2 mark for min/max, q1, q2, q3**] five number summary {6.77,6.93,6.995,7.05,7.22}

ii. q_1-1.5×iqr=6.75, q_3+1.5×iqr=7.23 [**1/2 mark**], hence no outliers [**1/2 mark**] (please be aware of carry-over mistakes)

iii. [**2 marks, 1 mark for boxplot, 1 mark for comment**] Fairly sym-



metric, no outliers.

iv. [**2 marks, 1 mark for using t-quantile, 1 mark for correct answer**] 99% CI for μ:
$(\bar{x} \pm t_{9,0.995} \times s/\sqrt{n}) = (6.987 \pm 3.25 \times 0.1259/\sqrt{10}) = (6.858, 7.116)$

v. [**2 marks, 1 mark each**]
- random independent sample, no real way of checking this assumption
- normal population, check by normal quantile plot of observations which is not provided here. Can get an idea of symmetry from the boxplot.

(b) i. [**1 mark**] $S_p^2 = \frac{11 \times 5100^2 + 11 \times 5900^2}{12+12-2} = 30410,000$

ii. [**5 marks, 1 mark for hypotheses, 1 mark for conclusion, 3 marks for EITHER rejection region approach (2nd bullet point) OR p-value approach (3rd bullet point)**]

- State hypotheses: H_0: $\mu_1 = \mu_2$ vs H_a: $\mu_1 \neq \mu_2$
- Rejection region: reject H_0 if $\bar{x}_1 - \bar{x}_2 \notin (\pm 2.074 \times \sqrt{30410,000} \times \sqrt{1/12 + 1/12}) = (-4669.19, 4669.19)$. $\bar{x}_1 - \bar{x}_2 = -1900$, hence do not reject H_0 (3 marks: 1 mark for $\bar{x}_1 - \bar{x}_2$, 1 mark for $t_{22,0.975} = 2.074$, 1 mark for correct interval)
- $t_0 = -0.84$, p-value is $2 \times P(t_{22} \geq |-0.84|) \in (0.4, 0.5)$ (3 marks: 1 mark for test statistic, 1 mark for t_{22}, 1 mark for p-value)
- Do not reject H_0, no significant difference in the average wear of these two brands, at 5% level.

iii. independent samples from normal populations [**1 mark**] assuming equal variances [**1 mark**]

(c) i. [**2 marks, 1 mark for stating normal distribution, 1 mark for correct mean and sd**] $X + Y \sim N(0, \sqrt{5})$

ii. [**1 mark**] $P(X+Y < 1) = 0.6736$ (please be aware of carry-over mistakes)

5. [**20 marks**]

(a) [3 marks, 1 mark for each point]

- The observed costs appear higher for car 2 than cars 1 and 3. There is considerable overlap between cars 1 and 3.
- The observed variability for car 1 appears smaller than the others.
- The distribution of observed operating costs for each hybrid car is fairly symmetrical.

(b) [3 marks, 1/2 mark for each assumption and 1/2 mark for an appropriate comment]

- the observations for the cost of operating each hybrid car were drawn from normal distributions; there is no way of checking this here (a quantile/qq-plot would be needed, a symmetrical boxplot does not tell anything about normality)
- the observations are independent; there is no way of checking this here
- the variances of the cost of operating each hybrid car are the same; using the rule-of-thumb (the ratio of the largest sample standard deviation to the smallest one is smaller than 2) this assumption is acceptable here

(c) [3 marks, 1/2 mark for each missing value in the table]

$$\begin{aligned}
(1) &= k - 1 = 2 \\
(2) &= 52.0/2 = 26 \\
(3) &= n - k = 15 \\
(4) &= 74.7 - 52.0 = 22.7 \\
(5) &= 22.7/15 = 1.513 \\
(6) &= n - 1 = 17
\end{aligned}$$

This yields the full table:

Source	df	SS	MS	F
Treatment	2	52.0	26.0	17.18
Error	15	22.7	1.513	
Total	17	74.7		

Note: please pay attention to carry-over mistake. For example, if (1) is wrong but (2) is calculated as 52.0/(1), the 1/2 mark for (2) should be granted.

(d) [5 marks, 1 mark for each of the following point]

- $H_0 : \mu_1 = \mu_2 = \mu_3$ vs. H_a : not all the means are equal (an alternative hypothesis stated as $H_a : \mu_1 \neq \mu_2 \neq \mu_3$ is obviously not correct)
- Rejection criterion: reject H_0 if $f_0 > f_{2,15;0.95} = 3.68$
- The observed value of the test statistic is $f_0 = 17.18$, which is much larger than $3.68 \rightarrow$ reject H_0
- The p-value is $p = \mathbb{P}(X > 17.18)$ for $X \sim \mathbf{F}_{2,15}$. From the table, it can be concluded that $p < 0.005$
- Conclusion: there is clear evidence that the cost of operating each hybrid car are not all the same

(e) [3 marks; 1 mark for a correct expression of the CI, 1 mark for the correct values and 1 mark for a correct conclusion on $H_0 : \mu_1 = \mu_2$] A 95% confidence interval for $\mu_1 - \mu_2$ is

$$\left[\bar{x}_1 - \bar{x}_2 \pm t_{n-k;1-\alpha/2}\sqrt{\text{ms}_{\text{Er}}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\right] = \left[20.0 - 23.0 \pm 2.131\sqrt{1.513 \times \left(\frac{1}{6}+\frac{1}{6}\right)}\right]$$
$$= [-4.51336, -1.48664]$$

Note: in the ANOVA context, students were asked to always use ms_{Er} as estimate of the common standard deviation σ, that's why $t_{n-k;1-\alpha/2} = t_{15;0.975} = 2.131$ is used as critical value. Consequently, answers using a classical two-sample t-confidence interval (hence based on a $t_{n_1+n_2-2}$ sampling distribution) are not totally correct. Half marks (1/2) may still be granted if all but this is correct.

Conclusion: 0 does not belong to that 95% confidence interval for $\mu_1 - \mu_2$, which indicates that there is a significant difference (at significance level 95%) between μ_1 and μ_2.

(f) [3 marks; 1 mark for mentioning (or effectively using) a Bonferonni adjustment, 1 mark for comparing p to 0.01667 and 1 mark for a correct conclusion] No. This two-sample t-test does not show clear evidence that μ_1 is not equal to μ_3, with a p-value of 0.17. To relate this to an overall test for $H_0 : \mu_1 = \mu_2 = \mu_3$ at significance level $\alpha = 0.05$, we should compare that p-value to $\alpha/\binom{3}{2} = 0.05/3 = 0.01667$ (Bonferonni adjustment)

Here the p-value is $0.17 > 0.01667$, so H_0 cannot be rejected at significance level $\alpha = 0.05$

6. [**20 marks**]

(a) [7 marks: 2 for hypotheses, 1 for conclusion, 4 for EITHER rejection region approach (2nd bullet point) OR p-value approach (3rd bullet point)]

- $H_0 : \beta_1 = 0$ vs. $H_a : \beta_1 \neq 0$ [2 marks: 1 mark for H_0 and 1 mark for H_a]
- Rejection criterion: reject H_0 if $\hat{b}_1 \notin \left[-t_{22,0.975}\frac{s}{\sqrt{s_{XX}}}, t_{22,0.975}\frac{s}{\sqrt{s_{XX}}}\right] = [-(2.074)(3.2551), (2.074)(3.2551)] = [-6.751, 6.751]$. As $\hat{b}_1 = 16.5378 \notin [-6.751, 6.751]$, reject H_0 at the 5% level. [4 marks: 1 mark for $\hat{b}_1$, 1 mark for $t_{22,0.975} = 2.074$, 1 mark for correct interval, 1 mark for rejecting H_0]
- The test statistic is given by $t_0 = \frac{\hat{b}_1}{s/\sqrt{s_{XX}}} = 5.081$. The p-value is $p = 2\mathbb{P}(t_{22} > 5.081) < 0.001$ (0.0000434 from regression output). As $p < 0.05$, reject H_0. [4 marks: 1 mark for $t_0 = 5.081$, 1 mark for t_{22}, 1 mark for p-value, 1 mark for rejecting H_0]
- There is evidence that `permeability` is significant in predicting `absorption`. [1 mark]

(b) [1 mark] $r = \sqrt{r^2} = \sqrt{0.54} = 0.7348$

(c) [2 marks, 1 for margin of error and 1 for correct answer] A 95% C.I. for β_1: $\hat{b}_1 \pm t_{22,0.975}\frac{s}{\sqrt{s_{XX}}} = 16.5378 \pm (2.074)(3.2551) = [9.7867, 23.2889]$

(d) [1 mark] $\hat{y}(x) = 1.1136 + 16.5378(0.6) = 11.0363$

(e) [1 mark] A 95% C.I. for $\mu_{Y|X=0.4}$ is $(6.3176, 9.1398)$

(f) [1 mark] A 95% P.I. for $Y^*(X = 0.4)$ is $(2.5865, 12.871)$

(g) [1 mark] The confidence interval accounts for the variation in estimating the regression line, while the prediction interval accounts also accounts for the variability of points about the regression line.

(h) [6 marks] We assume that the errors are independent [1 mark], Normally distributed [1 mark] and have constant variance [1 mark]. We can check the Normality and constant variance assumptions [1/2 mark for each] with the provided plots. Because the residuals vs. fitted values plot shows no pattern, the assumption of constant variance is satisfied [1 mark]. Because the errors follow a linear pattern in the Normal quantile plot, the Normality assumption is satisfied [1 mark].