UNIVERSITY OF NEW SOUTH WALES
School of Mathematics and Statistics

MATH2089 Numerical Methods and Statistics
Term 2, 2019

Numerical Methods Laboratory – Week 3

1. The easiest way to define a simple function in MATLAB is to use an anonymous function.
   (a) Write an anonymous function `myf` to evaluate $f(x) = e^{-x^2}$.
   (b) Create a vector `x` of 21 equally spaced points on $[-3, 3]$ (Hint: `linspace`).
   (c) Use your anonymous function to plot f on $[-3, 3]$. Does it look correct?
   (d) Repeat using 101 points.
   (e) Zoom in (Magnifying glass with a + at the top of the figure window) around $x = 0$.

2. Write a MATLAB script to calculate the forward difference approximation

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} \tag{1}$$

   to the first derivative of $f(x) = e^{x^2/2}$ at the point $x = 1$.

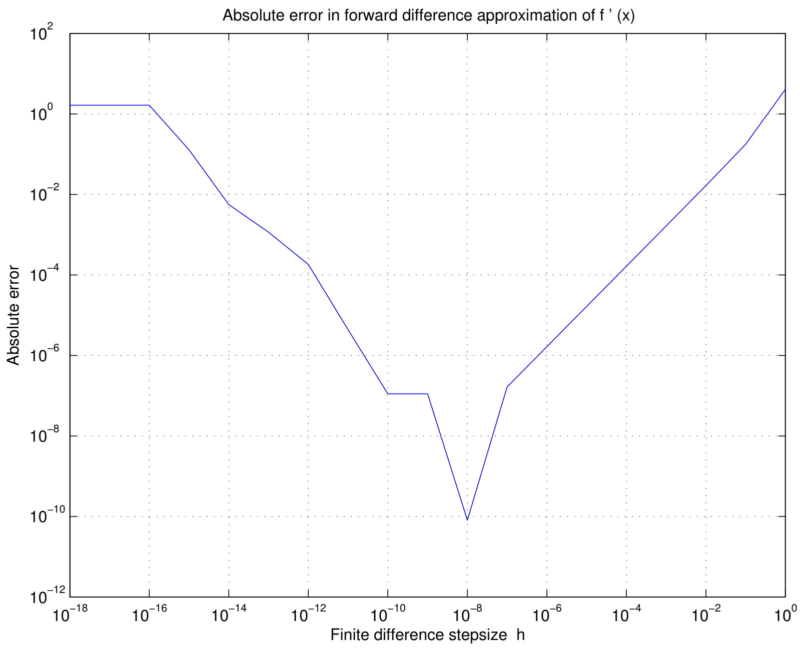

Figure 1: Error in forward difference estimate of the derivative of $f(x) = e^{x^2/2}$ at $x = 1$

   (a) What is the smallest sensible stepsize $h > 0$?
   (b) Write anonymous functions for $f(x)$ and the exact derivative $f'(x)$.
   (c) Calculate the finite difference approximation (1) for $h = 1, 10^{-1}, 10^{-2}, 10^{-3}, \ldots$.
   (d) Calculate the error between the approximation and the exact derivative.
   (e) Plot the absolute error against the stepsize h on a log-log scale, as in Figure 1.

(f) i. Find the value of h that gives the smallest error in this example.

ii. Theoretically the best value of h is approximately $\epsilon^{\frac{1}{2}}$. Does this agree with your answer to the previous part?

(g) The error in Figure 1 is constant for $h = 10^{-16}, 10^{-17}, 10^{-18}$. Why?

(h) Print out your results as a table, similar to the one below. Hint: `fprintf`.

```
   h          f'(x)    forward difference      error
----------------------------------------------------
 1.00e+00    1.648721        5.740335          4.0916e+00
 1.00e-01    1.648721        1.825309          1.7659e-01
 1.00e-02    1.648721        1.665319          1.6598e-02
 1.00e-03    1.648721        1.650371          1.6498e-03
 1.00e-04    1.648721        1.648886          1.6488e-04
 1.00e-05    1.648721        1.648738          1.6487e-05
 1.00e-06    1.648721        1.648723          1.6486e-06
 1.00e-07    1.648721        1.648721          1.6423e-07
 1.00e-08    1.648721        1.648721         -8.0582e-11
 1.00e-09    1.648721        1.648721          1.1094e-07
 1.00e-10    1.648721        1.648721         -1.1110e-07
 1.00e-11    1.648721        1.648726          4.3298e-06
 1.00e-12    1.648721        1.648903          1.8197e-04
 1.00e-13    1.648721        1.647571         -1.1503e-03
 1.00e-14    1.648721        1.643130         -5.5912e-03
 1.00e-15    1.648721        1.776357          1.2764e-01
 1.00e-16    1.648721        0.000000         -1.6487e+00
 1.00e-17    1.648721        0.000000         -1.6487e+00
 1.00e-18    1.648721        0.000000         -1.6487e+00
```

3. The standard expressions for the solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{2}$$

(a) Show that equivalent expressions are

$$r_1 = \frac{2c}{-b - \sqrt{b^2 - 4ac}}, \quad r_2 = \frac{2c}{-b + \sqrt{b^2 - 4ac}} \tag{3}$$

(b) Which expression avoids catastrophic cancellation when

i. $b > 0$

ii. $b < 0$

(c) Write and test a MATLAB function `quadsolve` to solve the quadratic in a numerically stable way.

i. The MATLAB function `quadsolve` must be in the file `quadsolve.m`

ii. The specifications are

```
[r1, r2] = quadsolve(a, b, c)
```

iii. Include comments at the beginning of your function giving the calling sequence and the purpose of the function

iv. Write a MATLAB M-file to test your function `quadsolve` on a range of examples, including

A. $x^2 + 3x + 2 = 0$ (this quadratic factors)

B. $0.01x^2 + 2000x - 0.001 = 0$

C. $x^2 - 1 = 0$

D. $a = 1, b = 200, c = -0.000015$ (example in WIKIPEDIA)

E. Any other cases you can think of?