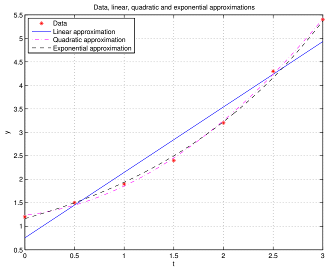
UNIVERSITY OF NEW SOUTH WALES
School of Mathematics and Statistics

MATH2089 Numerical Methods and Statistics
Term 2, 2019

Numerical Methods Laboratory – Week 7

1. Consider approximating the $m = 7$ data values

i	1	2	3	4	5	6	7
t_i	0	0.5	1	1.5	2	2.5	3
y_i	1.2	1.5	1.9	2.4	3.2	4.3	5.4



(a) Consider approximating the data by a linear function

$$y(t) = \alpha + \beta t.$$

In this case there are $n = 2$ parameters $\mathbf{x} = [\alpha \quad \beta]^T \in \mathbb{R}^2$ and the residuals are

$$r_i = y(t_i) - y_i = \alpha + \beta t_i - y_i \quad i = 1, \ldots, m.$$

Write a MATLAB script to do the following

i. Define the column vectors `tdat` and `ydat` containing the data.

ii. Define the coefficient matrix A

$$\begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ \vdots & \vdots \\ 1 & t_m \end{bmatrix}$$

iii. Find the least squares solution to $A\mathbf{x} = \mathbf{y}$ using each of the following methods:

A. The MATLAB backslash \ operator

B. The normal equations

$$(A^T A)\mathbf{x} = A^T \mathbf{y}$$

C. The QR factorization

$$R\mathbf{x} = Y^T \mathbf{y}$$

where

$$A = Q \begin{bmatrix} R \\ 0 \end{bmatrix}, \qquad Y = Q(:, 1:n)$$

iv. In each case calculate the

A. condition number of the linear system being solved.

B. sum of squares ($\mathbf{r}^T\mathbf{r} = \|\mathbf{r}\|_2^2$) of the residuals $\mathbf{r} = A\mathbf{x} - \mathbf{y}$

v. Plot the data and your approximation. Include a grid, legend, title and axis labels.

(b) Approximate the data by the quadratic function

$$y(t) = a_0 + a_1 t + a_2 t^2,$$

find the residual sun of squares and add the quadratic approximation to the plot.

(c) Approximate the data by the exponential function

$$z(t) = \lambda e^{\mu t}$$

i. Convert this into a linear problem by taking logs.
ii. Find the least squares approximation.
iii. Find the sum of squares (2-norm squared) of the residuals.
iv. Add the exponential approximation to the plot.

2. Consider the 156 by 156 matrix A from the chemical plant model illustrated in Figure 1. See `http://math.nist.gov/MatrixMarket/data/Harwell-Boeing/chemwest/west0156.html`

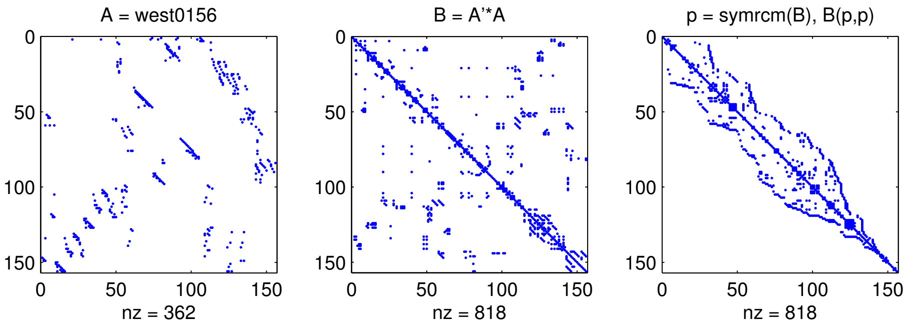


Figure 1: Spy plots of `west0156` matrices

(a) Download the data file `west0156.dat` from the course web page. This file contains the data for the coefficient matrix A in sparse format (i, j, A_{ij}) for the non-zero entries.
(b) Clear all variables from your MATLAB workspace using the `clear` command.
(c) Load the data using `load west0156.dat`. This should produce a 371 by 3 array – check using the `whos` command. (There are in fact 9 rows with value 0 included!)
(d) Store the first column (row indices) in the variable `I`, the second column (column indices) in the variable `J` and the third column (values) in the column vector `V`.
(e) Create the sparse matrix A using the command `A = sparse(I, J, V);`
(f) Check that A is a 156 by 156 sparse matrix, using either the `whos` command or the `size` and `issparse` commands.
(g) Find the values of $A_{i,j}$ for $i = 146, \ldots, 156$ and $j = 1, \ldots, 5$.
(h) Check if A is symmetric by calculating $\|A - A^T\|_\infty$.
(i) Create a spy plot of the non-zero elements in A in figure 1.
(j) Calculate the number of non-zero elements in A and the sparsity of A (as a %).
(k) Form the matrix $B = A^T A$
i. Check that B is symmetric.
ii. Create a spy plot of the non-zero elements of B in figure 2.
iii. Calculate the number of non-zero elements in B and the sparsity of B (as a %).
iv. Calculate `p = symrcm(B)` and create a spy plot of $B(p, p)$ in figure 3.
v. What does $B(p, p)$ give and why is it useful?
vi. Try to calculate the Cholesky factorization of B using `[R, k] = chol(B);`. What is the value of k and what does this mean?
vii. Calculate the smallest eigenvalue of B. Does this agree with the theory that $B = A^T A$ is symmetric positive semi-definite?