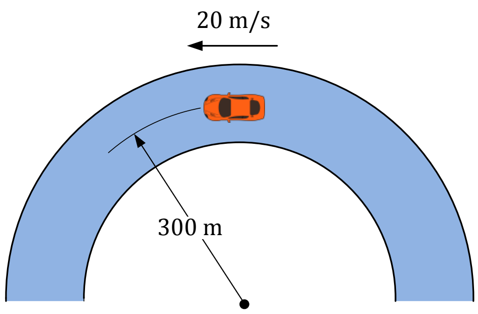
## Question 7.12.

The car travels along the curve having a radius of 300 m. If its speed is uniformly increased from 15 m/s to 27 m/s in 3 s, determine the magnitude of its acceleration at the instant its speed is 20 m/s.




## Solution

$v_1 = 15 \text{ m/s}$

$t = 3 \text{ s}$

$R = 300 \text{ m}$

$v_2 = 27 \text{ m/s}$

$v_3 = 20 \text{ m/s}$

$$a_t = \frac{v_2 - v_1}{t}$$

$$a_t = \frac{27 - 15}{3} = 4 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho}$$

$$a_n = \frac{20^2}{300} = 1.333 \text{ m/s}^2$$

$$|a| = \sqrt{{a_t}^2 + {a_n}^2} = \sqrt{4^2 + 1.333^2} = 4.22 \text{ m/s}^2$$ (Answer)

## Question 7.13.

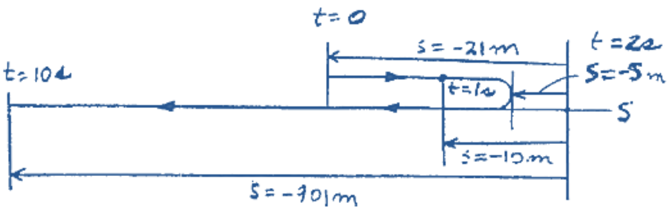
A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where $t$ is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4$ s, the displacement from $t = 0$ to $t = 10$ s, and the distance the particle travels during this time period.

## Solution

$v = (12 - 3t^2)$ ------ (1) (Answer)

$$a = \frac{dv}{dt} = -6t|_{t=4} = -24 \text{ m/s}^2$$

$$\int_{-10}^{s} ds = \int_{1}^{t} v\, dt$$




$$\int_{-10}^{s} ds = \int_{1}^{t} (12 - 3t^2)\, dt$$

$$s + 10 = 12t - t^3 - 11$$

$$s = 12t - t^3 - 21$$

$$s|_{t=0} = -21 \text{ m}$$

$$s|_{t=10} = -901 \text{ m}$$

$\Delta s = -901 - (-21) = -880$ m (Answer)

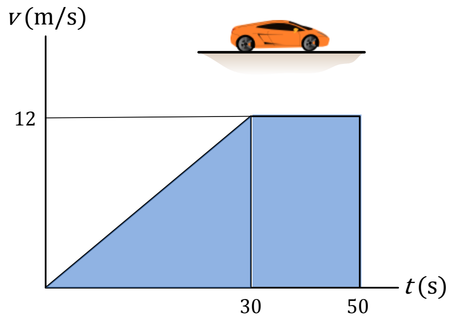
*From Eq. (1):*

$v = 0$ *when* $t = 2$ s

$$s|_{t=2} = 12(2) - (2)^3 - 21 = -5 \text{ m}$$

$s_T = (21 - 5) + (901 - 5) = 912$ m (Answer)

## Question 7.14.




The car moves along a straight course according to the $v-t$ graph. Construct the $s-t$ and $a-t$ graphs for the same 50 s time interval. When $t=0, s=0$.

## Solution

***s − t* Graph:** *The position function in terms of time t can be obtained by applying*

$$v = \frac{ds}{dt}$$

*For time interval* $0 \le t \le 30\ \mathrm{s}$

$$v = \frac{12}{30}t = \left(\frac{2}{5}t\right)\ \mathrm{m/s}$$

$$ds = vdt$$

$$\int_0^s ds = \int_1^t \left(\frac{2}{5}t\right)\,dt$$

$$s = \left(\frac{t^2}{5}\right)\ \mathrm{m}$$

*at* $t = 30\ \mathrm{s}$

$$s = \frac{30^2}{5} = 180\ \mathrm{m}$$

*For time interval* $30\ \mathrm{s} \le t \le 50\ \mathrm{s}$

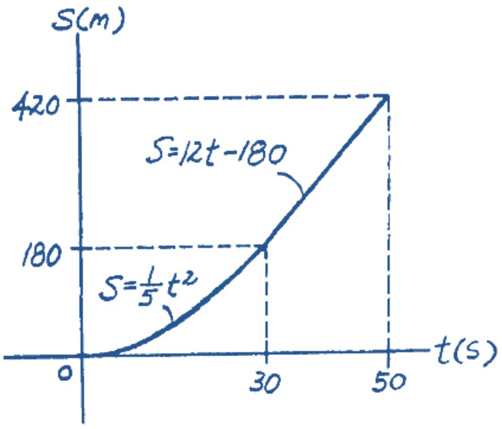
$$ds = vdt$$

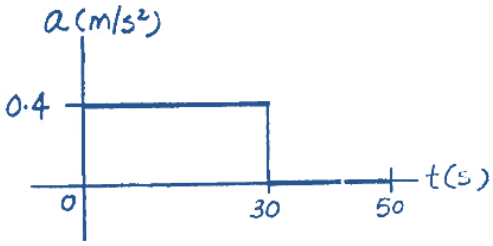
$$\int_{180}^s ds = \int_{30}^t (12)\,dt$$

$$s = (12t - 180)\ \mathrm{m}$$

*at* $t = 50\ \mathrm{s}$

$$s = 12(50) - 180 = 420\ \mathrm{m}$$




***a − t* Graph:** *The acceleration function in terms of time t can be obtained by applying*

$$a = \frac{dv}{dt}$$

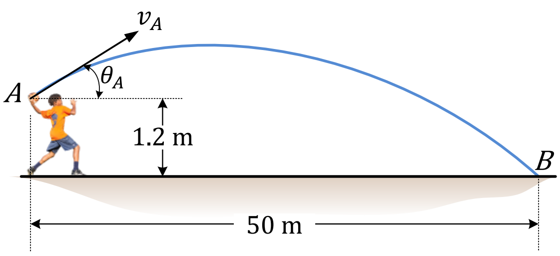
*For time interval* $0 \le t \le 30\ \mathrm{s}$ *and* $30\ \mathrm{s} \le t \le 30\ \mathrm{s}$, *respectively*

$$a = \frac{dv}{dt} = \left(\frac{2}{5}\right) = 0.4\ \mathrm{m/s^2}$$

$$a = \frac{dv}{dt} = 0$$

## Question 7.15.

It is observed that the time for the ball to strike the ground at $B$ is 2.5 s. Determine the speed $v_A$ and angle $\theta_A$ at which the ball was thrown.




## Solution

***Coordinate System:*** *The $x-y$ coordinate system will be set so that its origin coincides with point A.*

***Horizontal Motion:*** *Here, $(v_A)_x = (v_A \cos\theta_A)$, $x_A = 0$ m, $x_B = 50$ m and $t = 2.5$ s. Thus,*

$(+\rightarrow)$

$x_B = x_A + (v_A)_x\, t$

$50 = 0 + (v_A \cos\theta_A)\,(2.5)$

$v_A \cos\theta_A = 20$ ------------- (1)

***Vertical Motion:*** *Here, $(v_A)_y = (v_A \sin\theta_A)$, $y_A = 0$ m, $y_B = -1.2$ m and $a_y = -9.81\ \text{m/s}^2$. Thus,*

$(+\uparrow)$

$y_B = y_A + (v_A)_y\, t + \frac{1}{2}\, a_y\, t^2$

$-1.2 = 0 + (v_A \sin\theta_A)\,(2.5) + \frac{1}{2}\,(-9.81)\,(2.5)^2$

$v_A \sin\theta_A = 11.7825$ ------------- (2)

*Solving (1) and (2) yields,*

$\theta_A = 30.5°$ (Answer)

$v_A = 23.2$ m/s (Answer)

## Question 7.16.

A train enters a curved horizontal section of track at a speed of $100\ \mathrm{km/h}$ and slows down with constant deceleration to $50\ \mathrm{km/h}$ in 12 seconds. An accelerometer mounted inside the train records a horizontal acceleration of $2\ \mathrm{m/s^2}$ when the train is 6 seconds into the curve. Calculate the radius of curvature of the track for this instant.

## Solution

$$v = v_o + a_t\, t$$

$$\frac{50}{3.6} = \frac{100}{3.6} + 12\, a_t$$

$$a_t = -1.157\ \mathrm{m/s^2}$$

$$a = \sqrt{{a_t}^2 + {a_n}^2}$$

$$2 = \sqrt{(-1.157)^2 + {a_n}^2}$$

$$a_n = 1.631 \mathrm{m/s^2}$$

$$v_6 = v_o + a_t\, t$$

$$v_6 = \frac{100}{3.6} - (1.157)(6) = 20.8\ \mathrm{m/s}$$

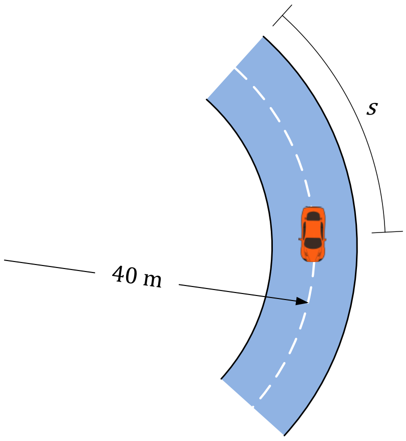
$$a_n = \frac{v^2}{\rho}$$

$$\rho = \frac{v^2}{a_n} = \frac{(20.8)^2}{1.631}$$

$\rho = 266\ \mathrm{m}$ (Answer)

## Question 7.17.

The car starts from rest at s $= 0$ and increases its speed at $a_t = 4 \text{ m/s}^2$. Determine the time when the magnitude of acceleration becomes $20 \text{ m/s}^2$. At what position $s$ does this occur?




## Solution

***Acceleration:*** *The normal component of the acceleration can be determined from:*

$$a_n = \frac{v^2}{\rho} = \frac{v^2}{40}$$

$$a = \sqrt{{a_t}^2 + {a_n}^2}$$

$$20 = \sqrt{(4)^2 + \left(\frac{v^2}{40}\right)^2}$$

$$v = 28 \text{ m/s}$$

***Velocity:*** *Since the car has a constant tangential acceleration of* $a_t = 4 \text{ m/s}^2$

$$v = v_o + a_t\, t$$

$$28 = 0 + (4)\, t$$

$t = 7 \text{ s}$ (Answer)

$$v^2 = v_o^2 + 2\, a_t\, s$$

$$(28)^2 = (0)^2 + 2\,(4)\, s$$

$s = 98 \text{ m}$ (Answer)