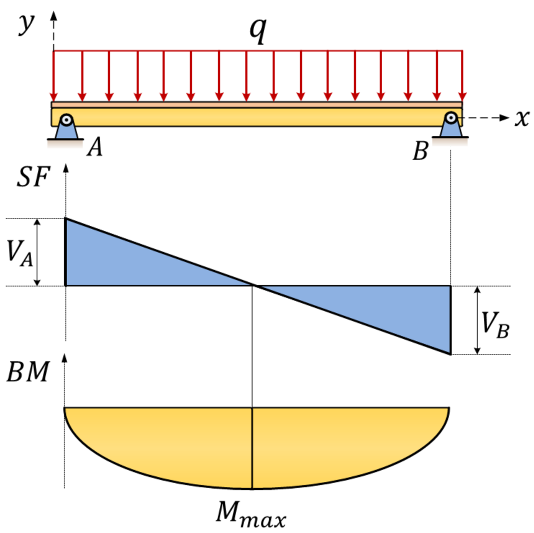
School of Mechanical and Manufacturing Engineering

MMAN1300 Engineering Mechanics 1

Dr. David C. Kellermann



Week 5, L1-2: Distributed Loads, Shear Force and Bending

DISTRIBUTED LOADING

- Uniform, linearly varying and other load cases

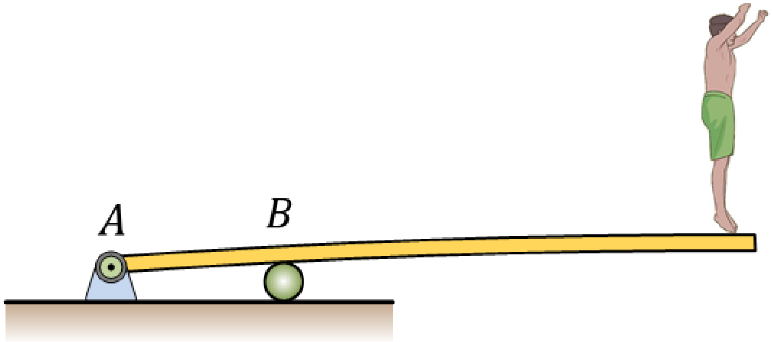
BEAM BENDING AND DIAGRAM SKETCHING

- Axial/shear force, bending moment and slope
- Shear Force diagrams
- Bending Moment diagram

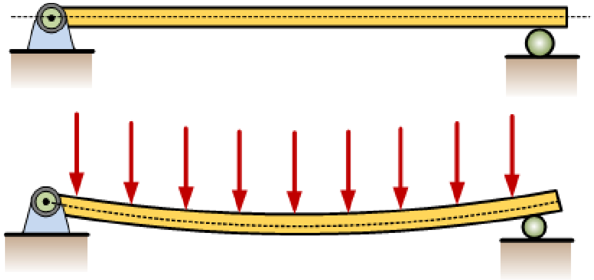
Applications: Beams

Beams are structural members that offer resistance to bending due to applied load



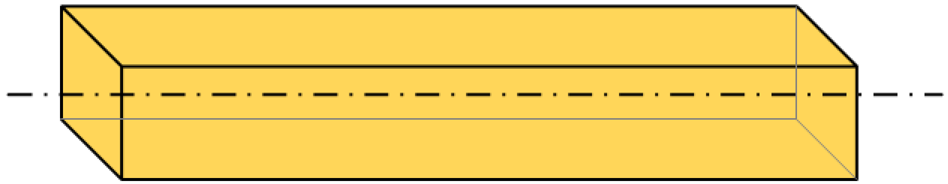




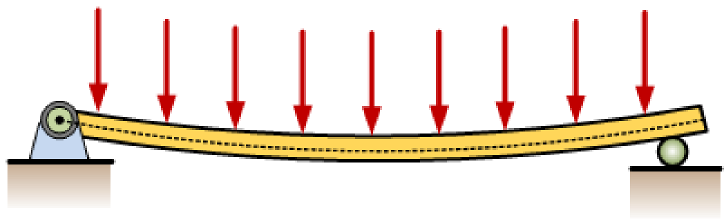


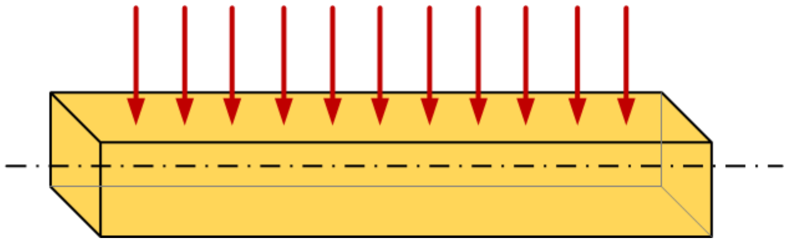
Beams: Prismatic Members

- Long prismatic members
- Non-prismatic members also possible
- Each cross-section dimension << Length of member



- Loading is perpendicular to the member axis (Neutral Axis)

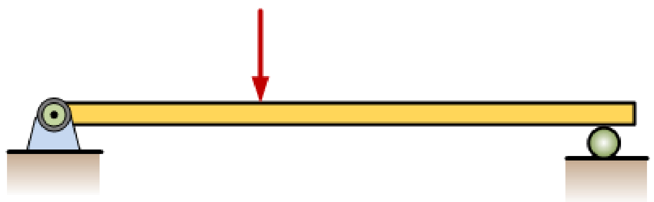


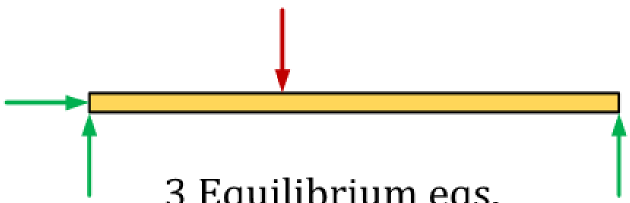


Static Determinacy in Beams

How do we define determinacy?

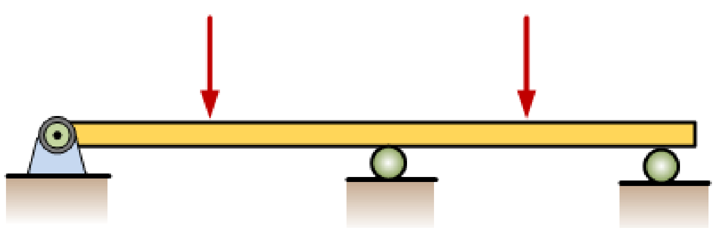
- **Statically determinate beam** i.e. only equilibrium equations required to obtain support reactions

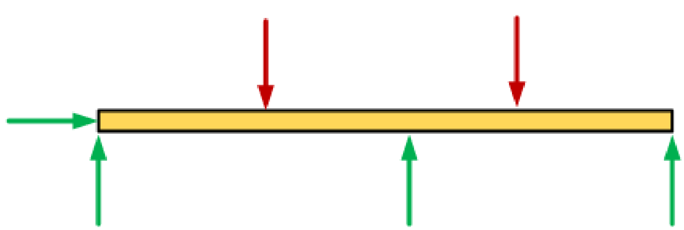




3 Equilibrium eqs.
(One redundant)

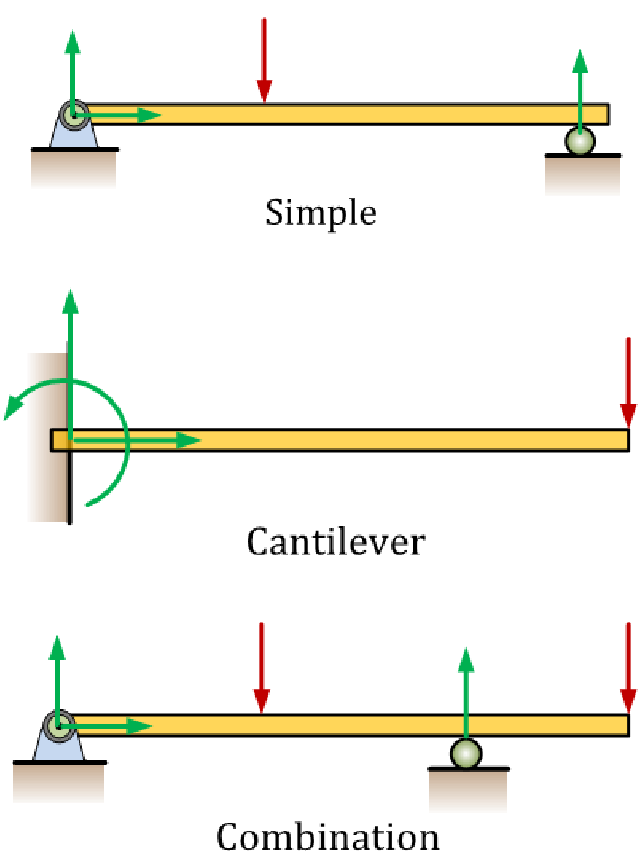
- **Statically indeterminate beam** i.e. deformability required to obtain support reactions



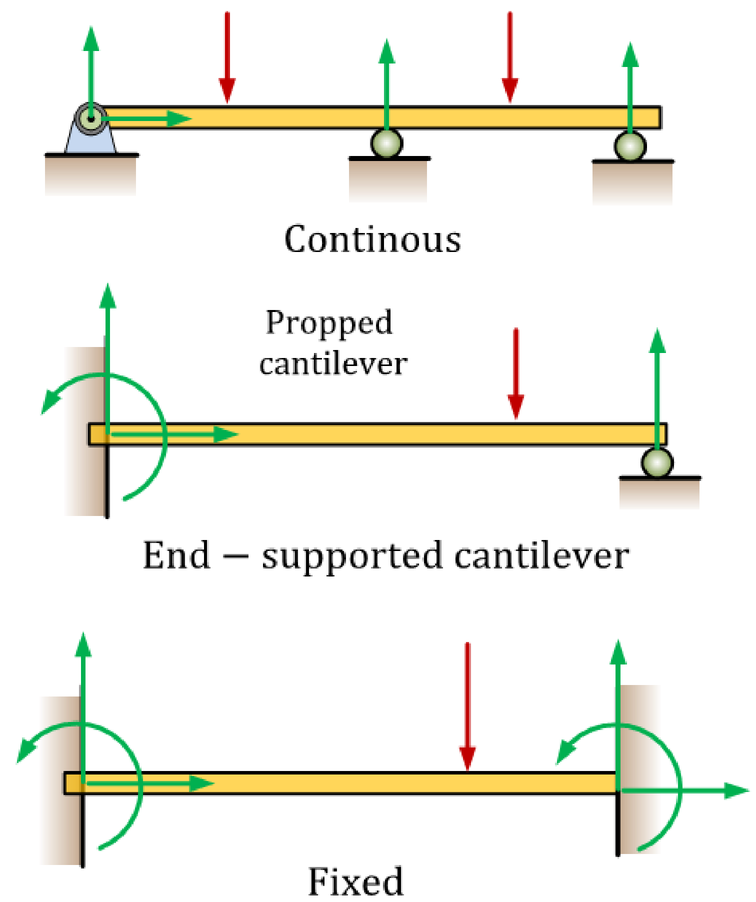


3 Equilibrium eqs.
(insufficient)

Beam Types



Statically determinate beams

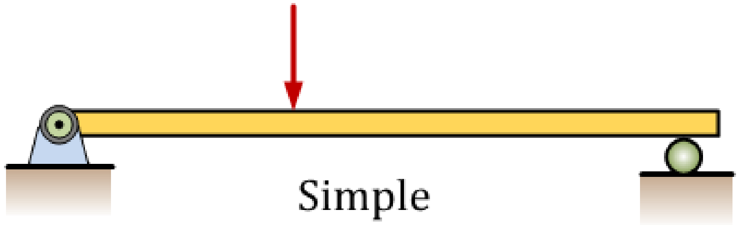


Statically indeterminate beams

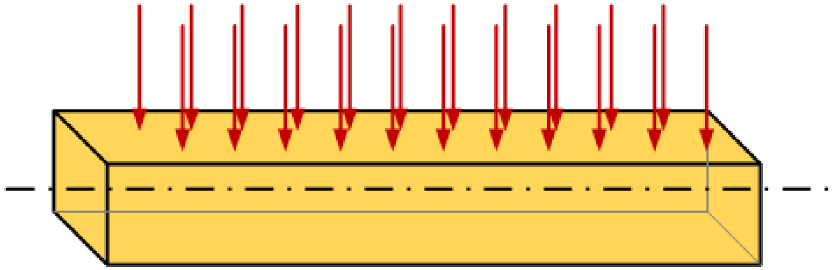
Applied Loads

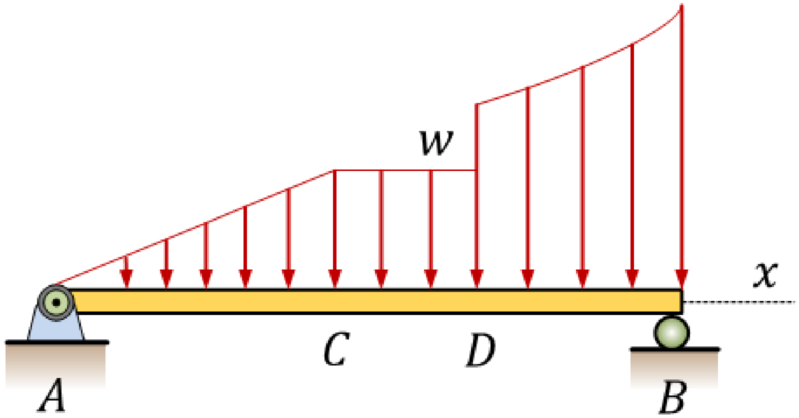
Based on pattern of external loading

- **Concentrated load** OR point load



- **Distributed load**
  - ✓ Intensity (w) expressed force per unit length of the beam





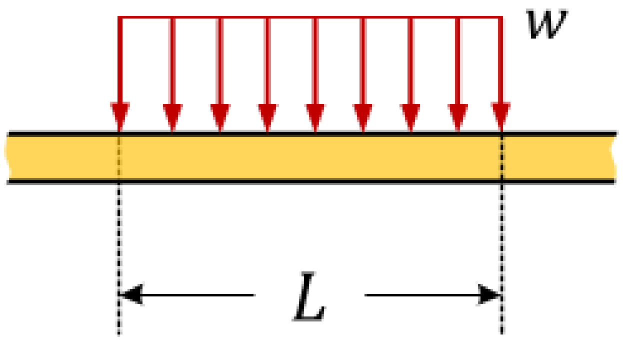
Equivalent (Resultant) Forces

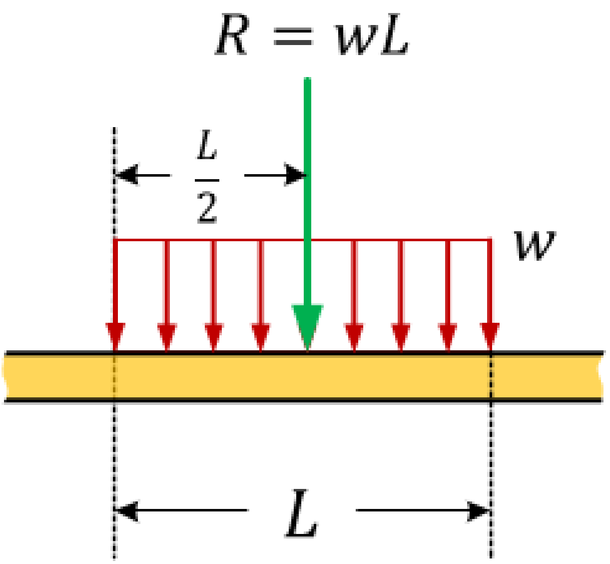
Resultant force (R) on beams

R = the area formed by w along L

L = the length of the beam over which the load is distributed

Note: R passes through the centroid of this area





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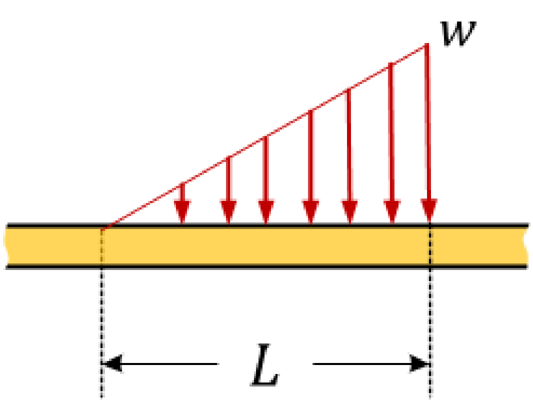

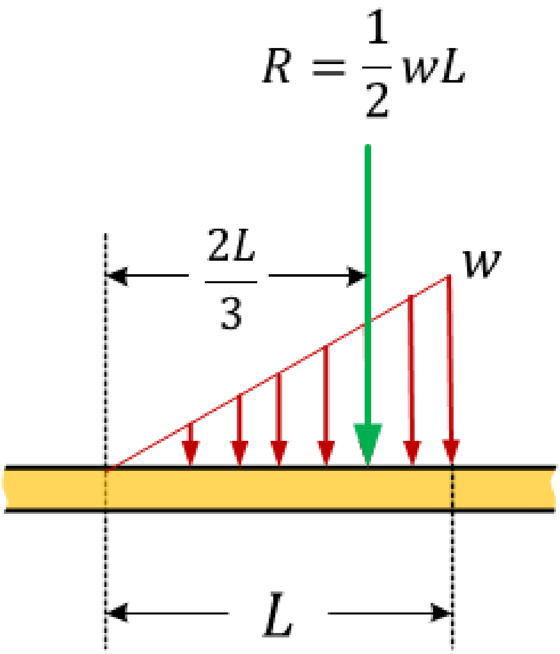

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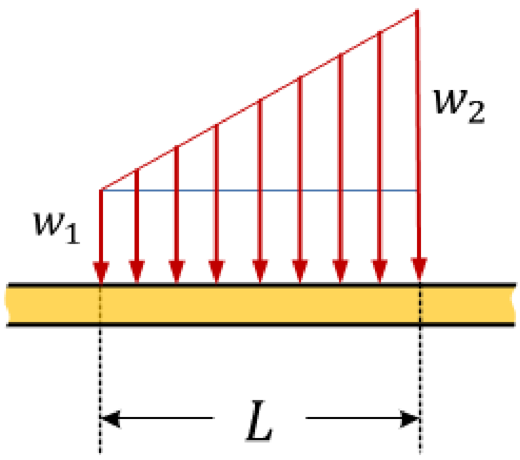
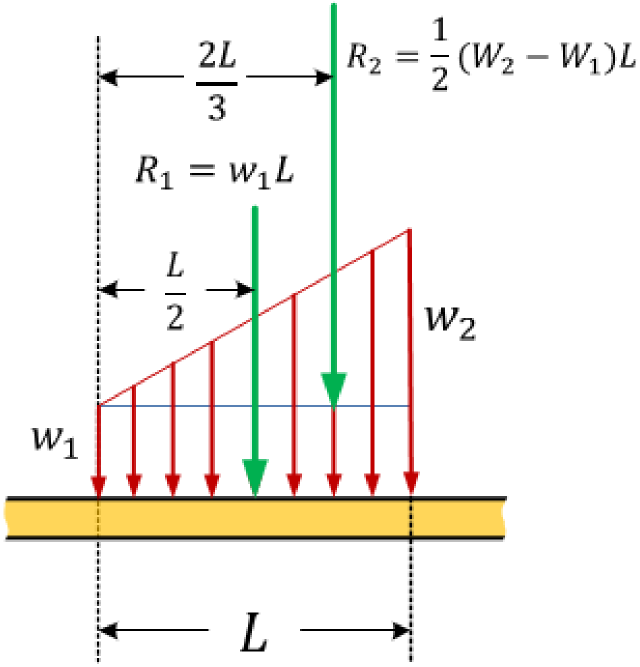
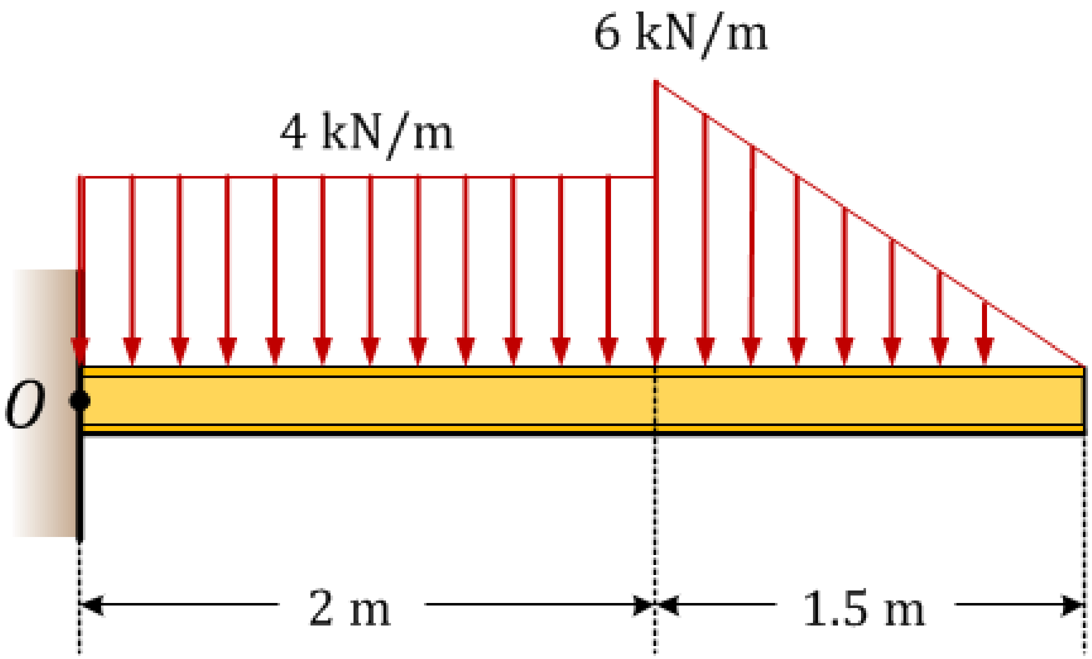




Example 1

Replace this loading by an equivalent resultant force and specify its location, measured from point *O*.



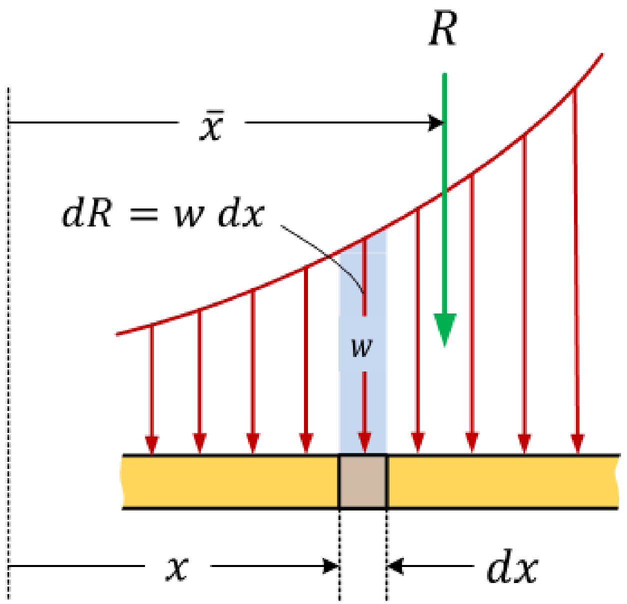
W5 Example 1 (Web view)

Distributed Loading

General load distribution

Differential increment of force is

$$dR = w\,dx$$



The total load R is the sum of all the differential forces

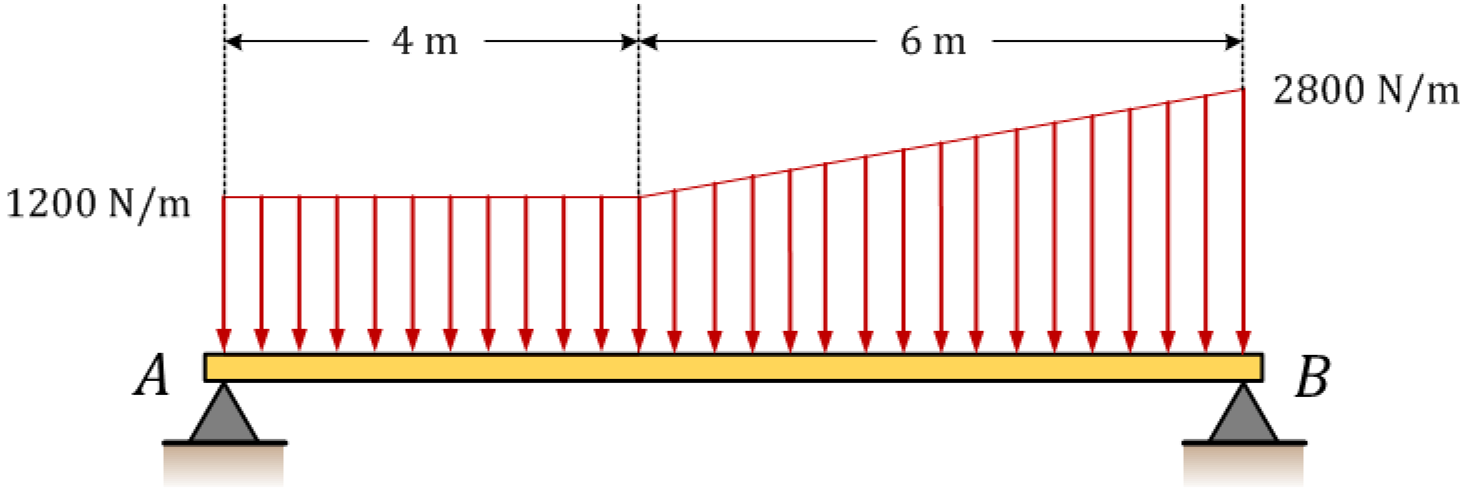
$$R = \int w\,dx$$ (acting at the centroid of the area under consideration)

$$\bar{x} = \frac{\int xw\,dx}{R}$$

Once R is known reactions can be found out from statics

Example 2

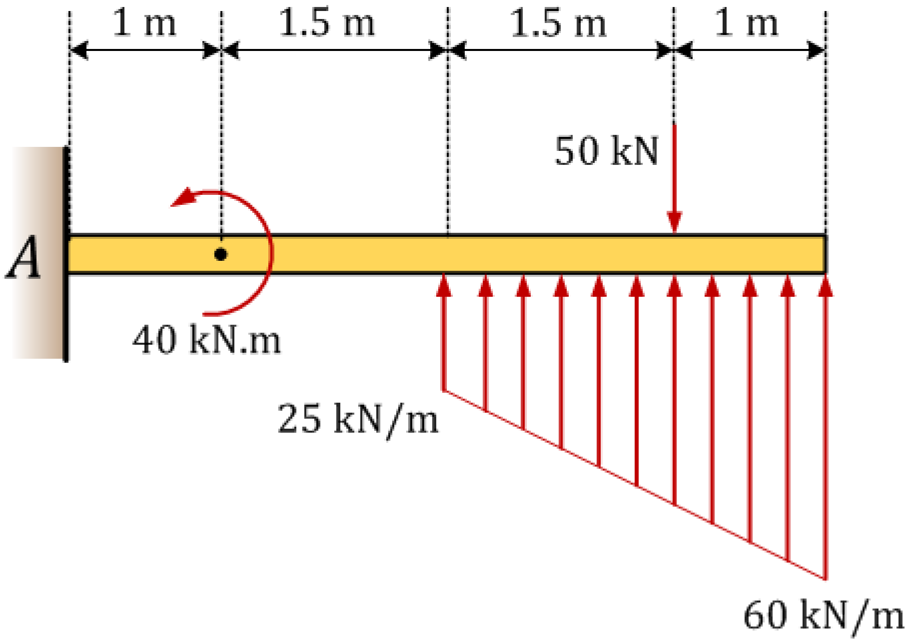
Determine the external reactions for the loaded beam



W5 Example 2 (Web view)

Example 3

Determine the external reactions for the loaded beam

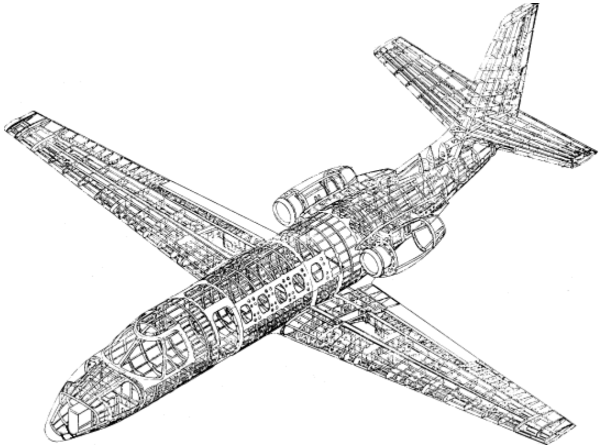


W5 Example 3 (Web view)

Internal Loads

Internal loads are forces and moments inside a member due to applied external loads

- We need to know the internal loads experienced by a member so that in design we can specify
  - ✓ The material
  - ✓ The dimensions
  - ✓ The connections or attachment to other members



- A the internal forces get larger, we must specify stronger materials or larger members

Assumptions

In this course we will be concerned with internal loads in slender members

- Slender members are those whose length is much greater (at least 10 times) that their cross-sectional dimensions (i.e. width and thickness)

- We call slender members that experience transverse loads "beams"

Assumptions

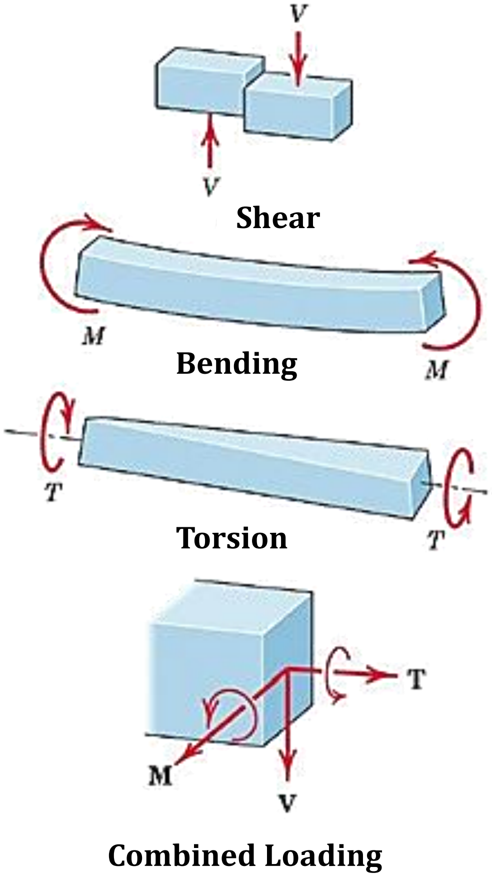
Why slender members?

- Slender members are common in engineering as they can often carry a lot of load without weighing a lot
  - ✓ Meriam and Kraige say "beams are undoubtedly the most important of all structural members"
- Also, finding internal loads in non-slender members is much, much more difficult
  - ✓ We'll leave that for Mechanics of Solids

Shear Force and Bending Moment

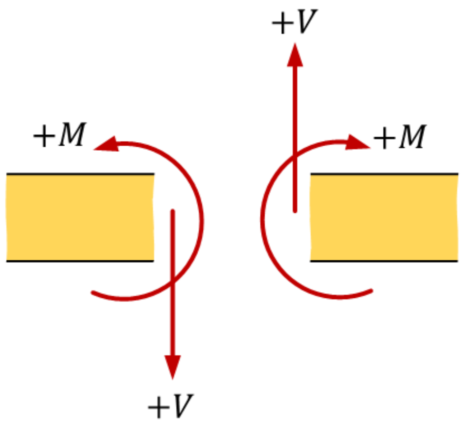
Internal loads include shear force, bending moment, torsion and axial force (compression or tension)

- In this course, we will be mostly concerned with shear force and bending moments in beams
- We have already looked a little at the axial forces
- Torsion will be dealt with in subsequent courses (MMAN2400)



Shear Force and Bending Moment

We need to introduce a sign convention:

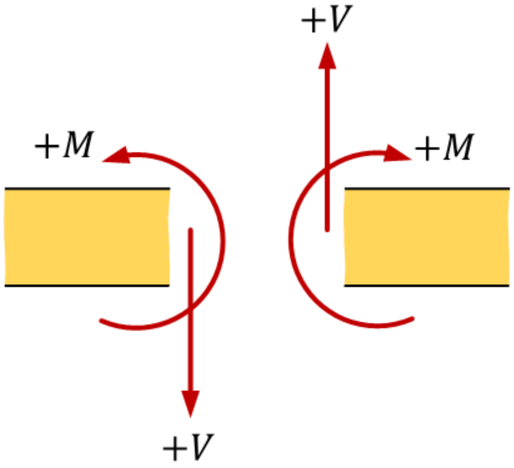


- Newton's 3rd law tells us that the loads act in opposite directions on opposite sides of the section

- We arbitrarily take the loads drawn above to be positive
  - ✓ Shear force is positive when it acts down on the left side of the section
  - ✓ Bending moment is positive when it acts counter-clockwise on the left side of the section

Shear Force and Bending Moment

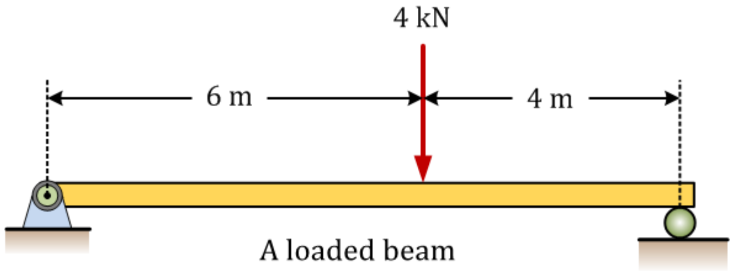
It is often not obvious in which direction the loads will be acting



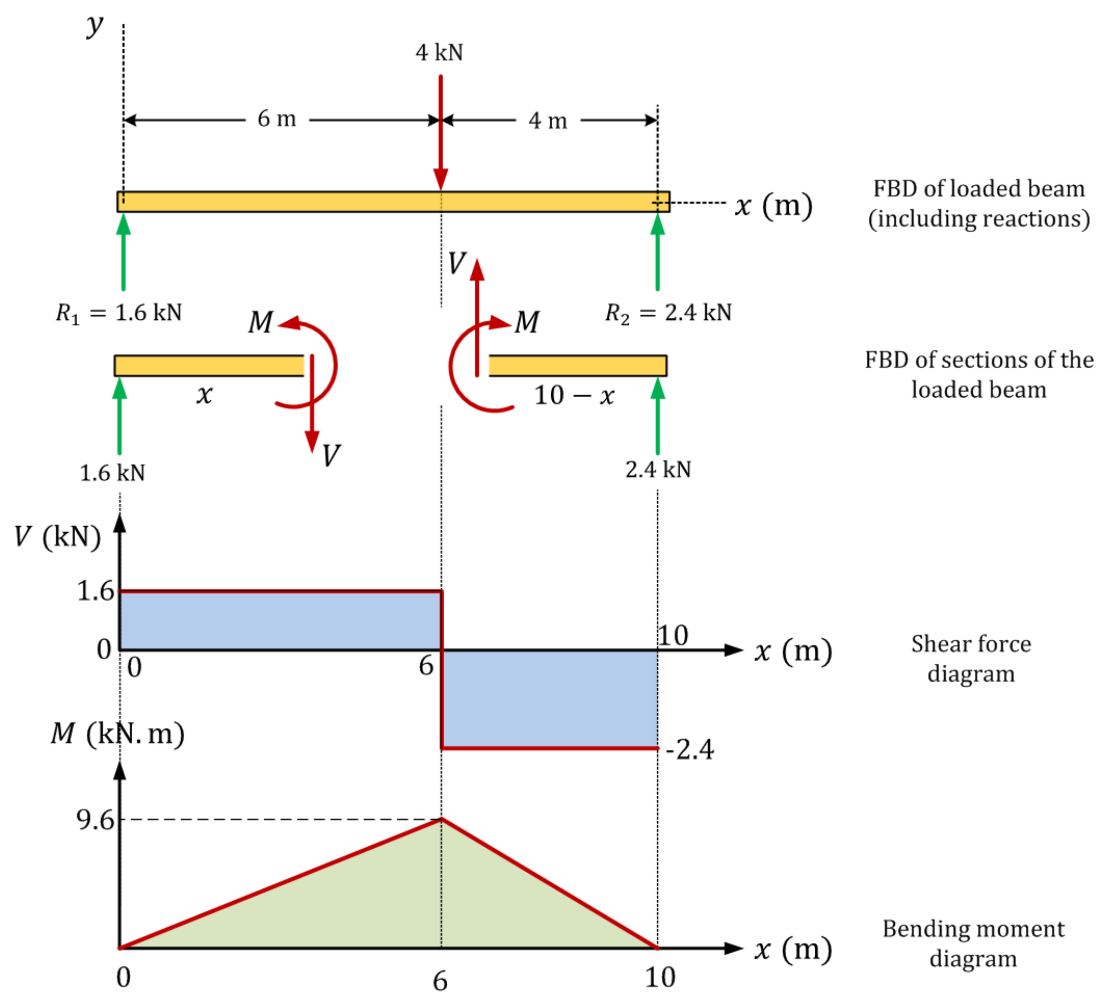
- Therefore it is almost always better to draw the loads as positive in the FBD

- We will let their algebraic sign in the solution determine the correct direction

Shear Force and Bending Moment Diagrams

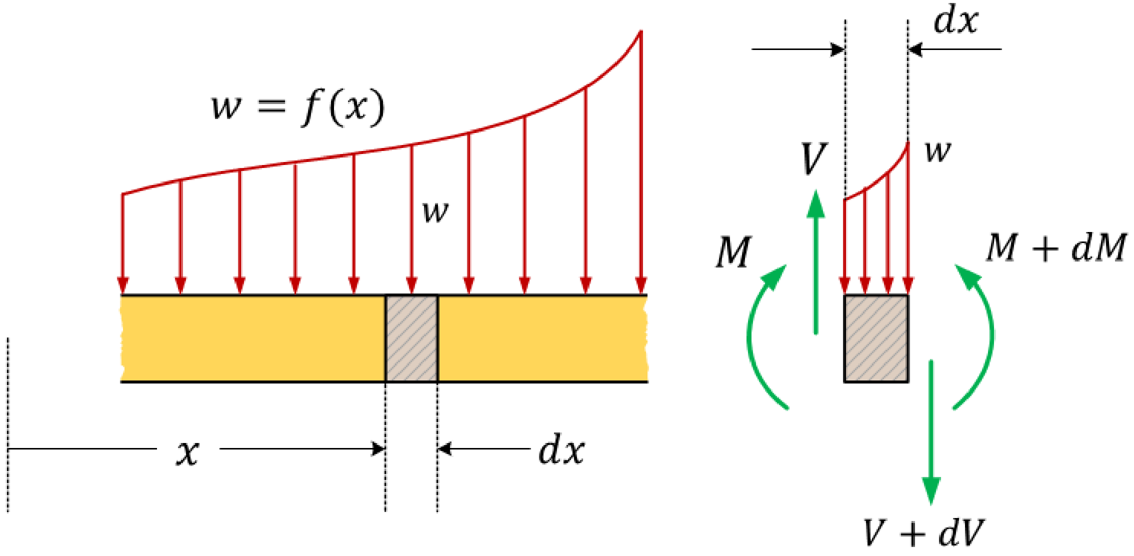


- It is much easier to draw these diagrams if we first establish some mathematical relationships



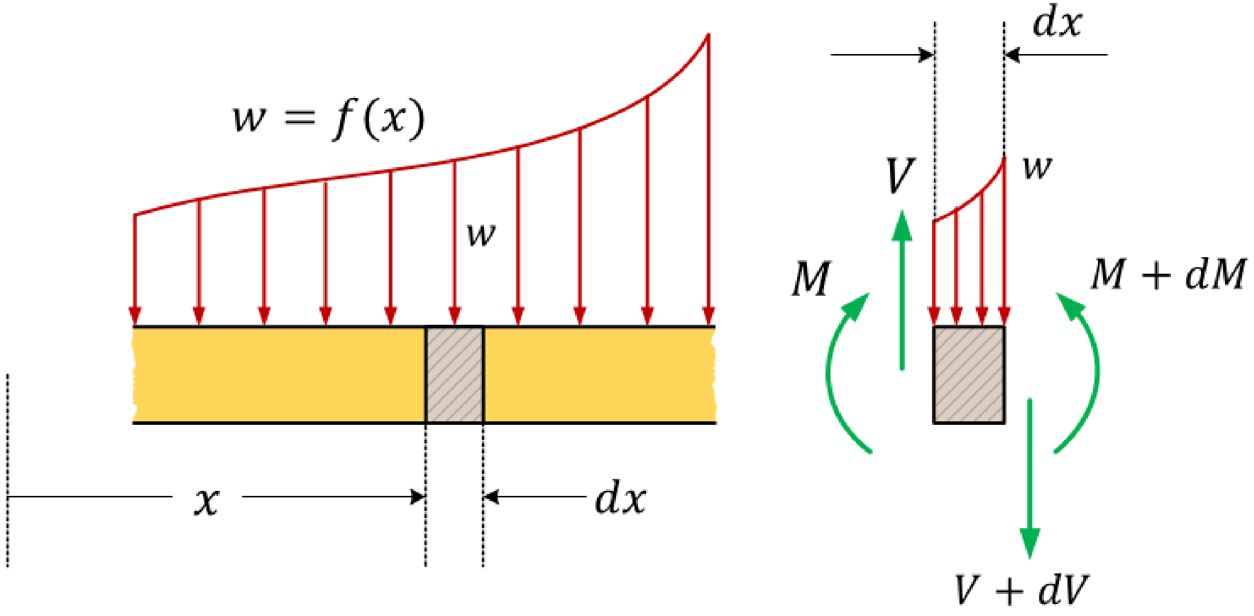
Elemental Equilibrium

Let's look at the equilibrium of an infinitesimal element of a beam



- A piece of the beam including the element is shown at the left
- The infinitesimal element (of length dx) is shown at right

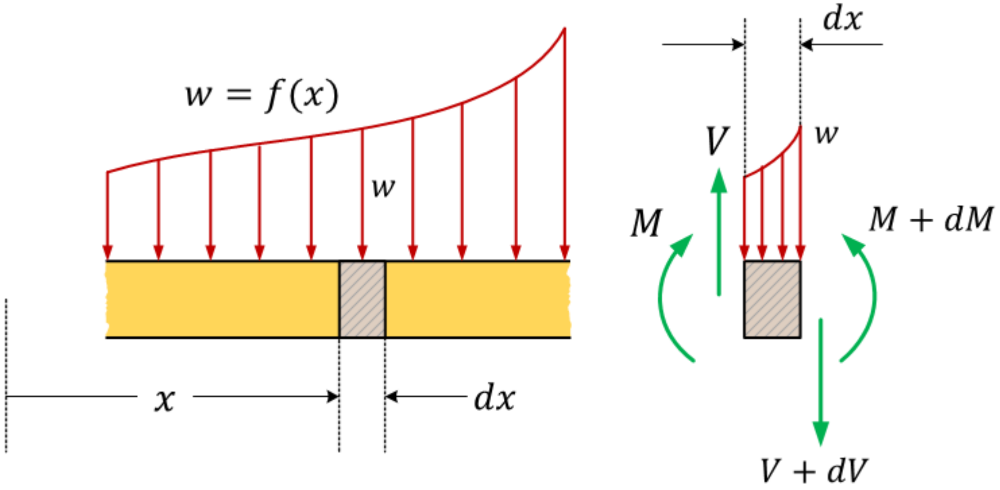
Elemental Equilibrium



- Notice that the internal loads are drawn in the positive direction according to our sign convention
- Also notice that the loads on the right end differ from those on the left end by differential amounts dV and dM

Elemental Equilibrium

Since the element is in equilibrium, we know that forces will sum to zero



- A simple force balance (in the y - direction) gives:

$$V - wdx - (V + dV) = 0$$

- Note that the length of the element is only dx so the distributed force is effectively constant over the element

Elemental Equilibrium – shear force

V and $-V$ cancel out and we are left with a useful result

$$w = -\frac{dV}{dx}$$

- The slope of the shear diagram must be equal to the negative value of the applied loading

- We can rearrange this to find shear force in terms of applied loading

$$\int_{V_o}^{V} dV = -\int_{x_o}^{x} w\, dx \longrightarrow \boxed{V = V_o - \int_{x_o}^{x} w\, dx}$$

Elemental Equilibrium – shear force

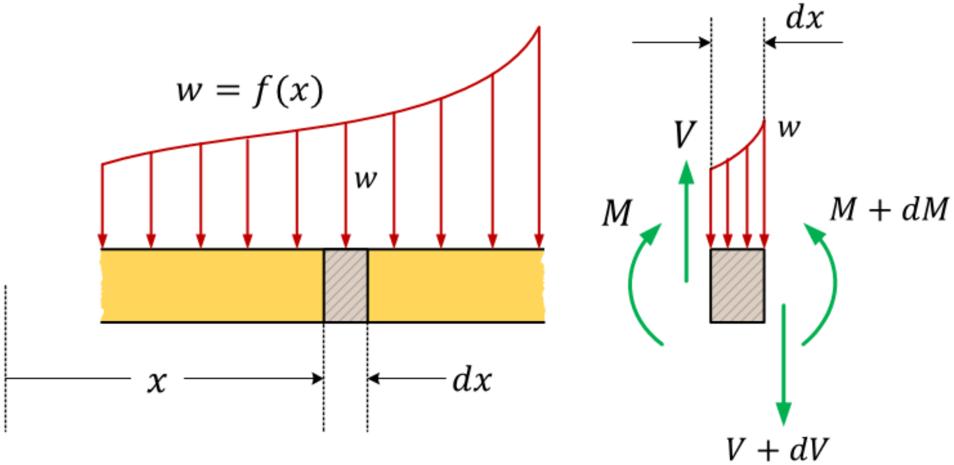
In other words,

$$\boxed{V = V_o - \int_{x_o}^{x} w\, dx}$$

- The shear force V at some point x is equal to the shear force V_o at x_o minus the area under the loading curve from x_o to x

Elemental Equilibrium – internal moment

Now let's look at the moments

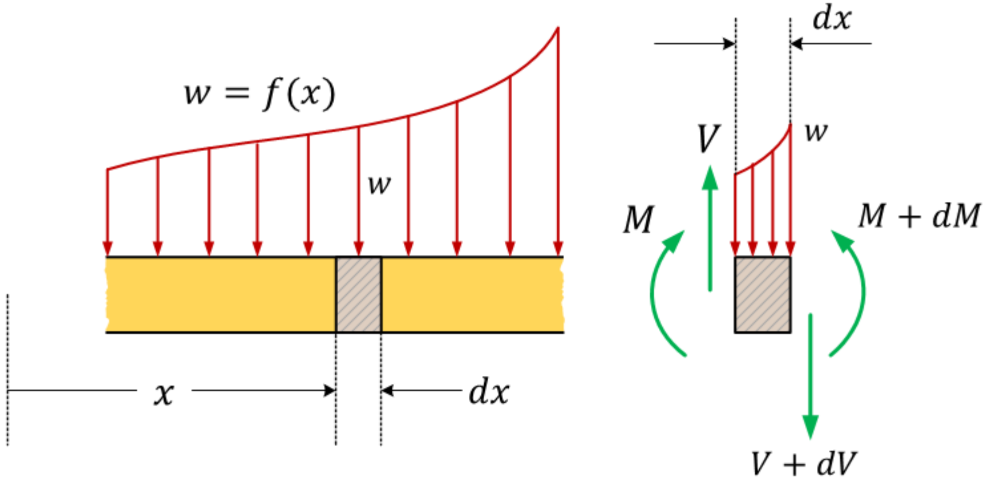


- The element is still in equilibrium, so we can sum moments to equal zero
- We will sum the moments about left end of the element
- The moment balance gives

$$M + wdx\frac{dx}{2} + (V + dV)dx - (M + dM) = 0$$

Elemental Equilibrium – internal moment

M and $-M$ cancels out



- Also, the terms that include dx^2 and $dV dx$ are negligibly small
- We are left with another useful result

$$V = \frac{dM}{dx}$$

Interrelation – SF and BM

The shear force is the derivative of the moment

$$V = \frac{dM}{dx}$$

- Again, we can rearrange in order to find M from V

$$\int_{M_o}^{M} dM = -\int_{x_o}^{x} V\, dx \longrightarrow \boxed{M = M_o + \int_{x_o}^{x} V\, dx}$$

- The bending moment M at some point x is equal to the bending moment M_o at x_o plus the area under the shear curve from x_o to x

Bending Moment

Now we can find the bending moment from a given loading

- Usually we choose x_o to be a point where the shear and bending moment are known or can be easily determined

i.e.

✓ At an end of the beam

✓ At a support with a known reaction

Sketching SFDs and BMDs

- In regions of the beam where there is no loading, shear force is constant and bending moment is linear

Sketching SFDs and BMDs

- In regions where distributed force is constant, shear force is linear and bending moment is quadratic

Sketching SFDs and BMDs

- In regions where the distributed force is linear, shear force is quadratic and bending moment is cubic

Concentrated Loads

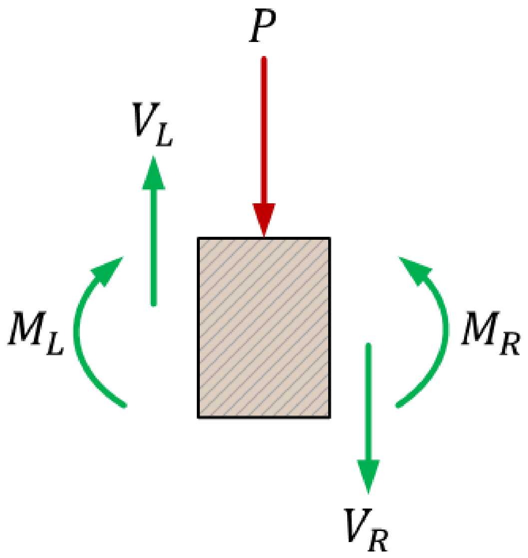
But what about concentrated loads?

- All these things we have determined are for distributed loads

- There will often be concentrated loads – potentially both as support reactions and as applied loads

Concentrated Loads

Let's look at a concentrated force P on a beam element of length dx



- The shear force and bending moment on the left end of the element are V_L and M_L, respectively, and are assumed in the positive direction

- The shear force and bending moment on the right end of the element are V_R and M_R, respectively, and are assumed in the positive direction

- The difference between the left end and right end is no longer just dV and dM as there is a finite (not just wdx) amount of load applied to the element

Concentrated Loads

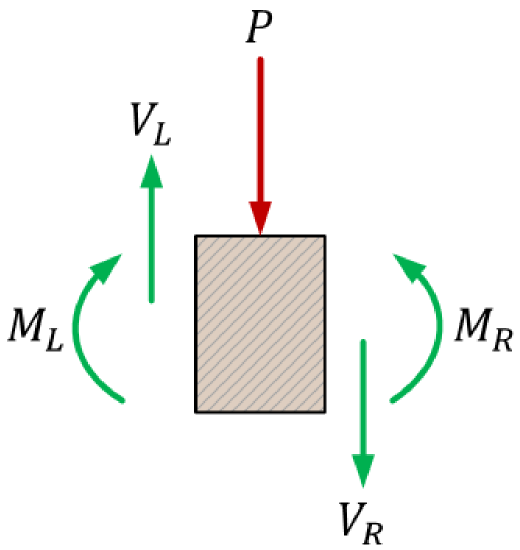
Now let's balance forces in the vertical direction

- We get

$$V_L - V_R - P = 0$$

- Which leads us to see

$$\boxed{V_R = V_L - P}$$



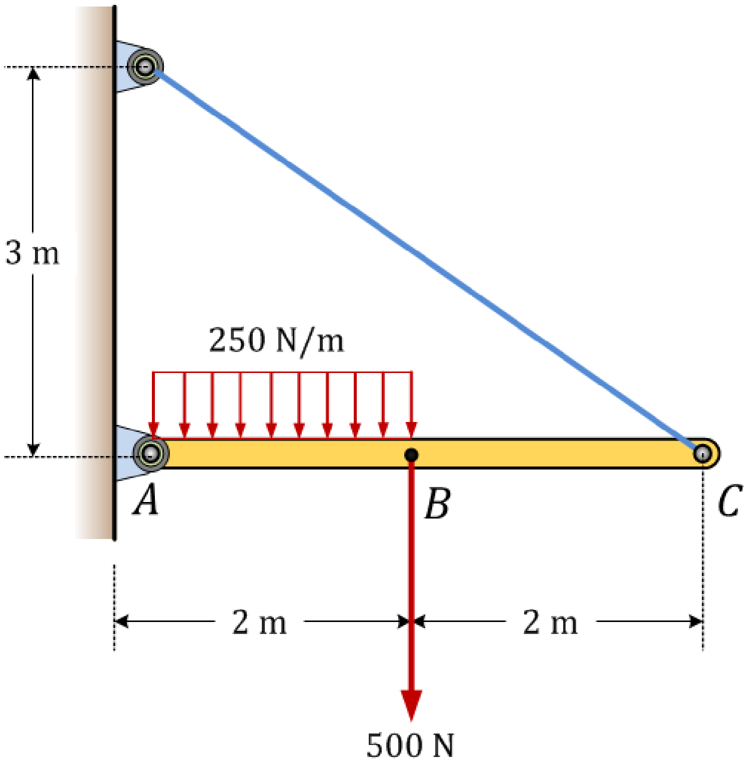
- The shear force on the right side of the element is reduced by the value of the concentrated load (if it acts downward)

- If the concentrated load acts upward

$$\boxed{V_R = V_L + P}$$

Example 4

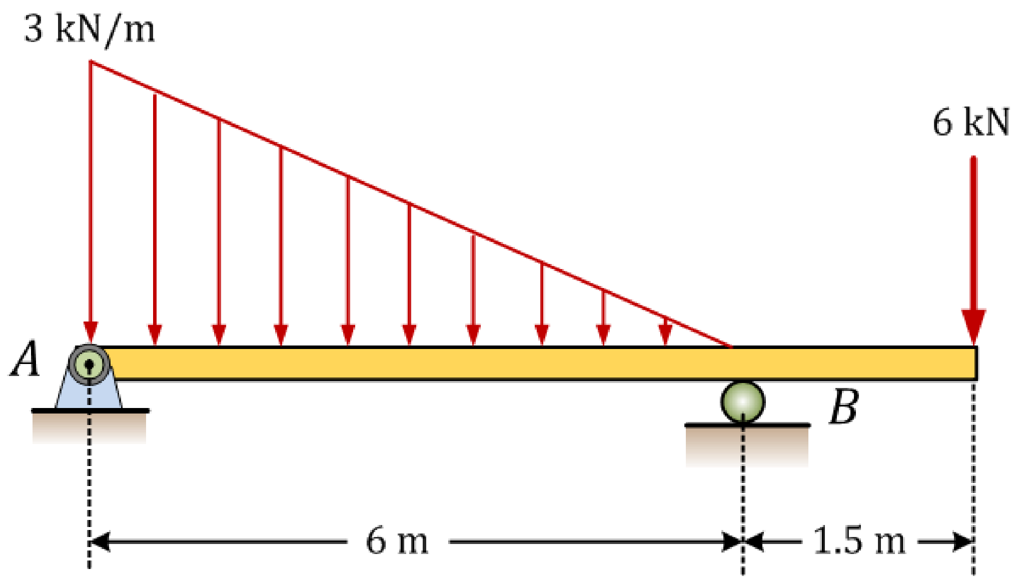
Obtain the shear force and bending moment values along the length of the beam.



W5 Example 4 (Web view)

Example 5

Obtain the shear force and bending moment values along the length of the beam.

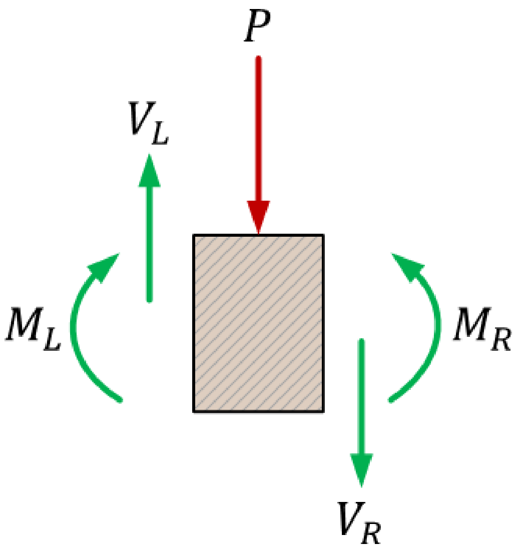


W5 Example 5 (Web view)

Concentrated Moments

What about moments?

- Write a moment balance about the left end



$$M_L + P\frac{dx}{2} + V_R dx - M_R = 0$$

- The terms that include dx are negligibly small, so we are left with

$$M_L = M_R$$

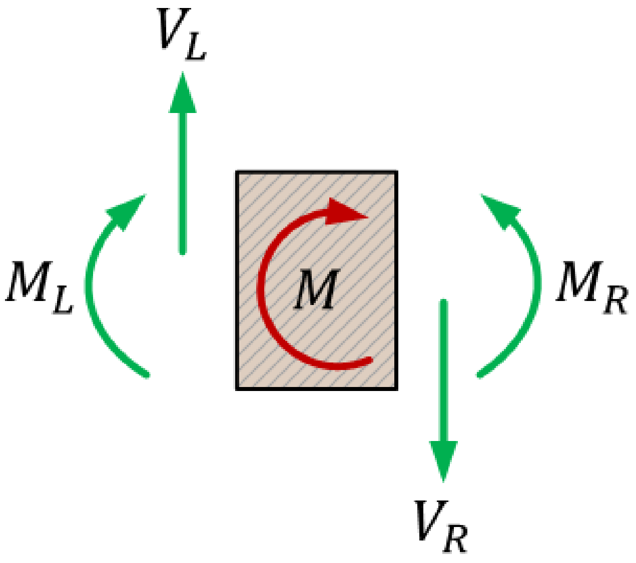
- The bending moment remains constant across a section with a concentrated load

Concentrated Moments

What if there is a concentrated moment applied?

- A force balance quickly shows that $V_L = V_R$
  (there are no other forces)

- This implies that, a concentrated moment does not affect the shear force

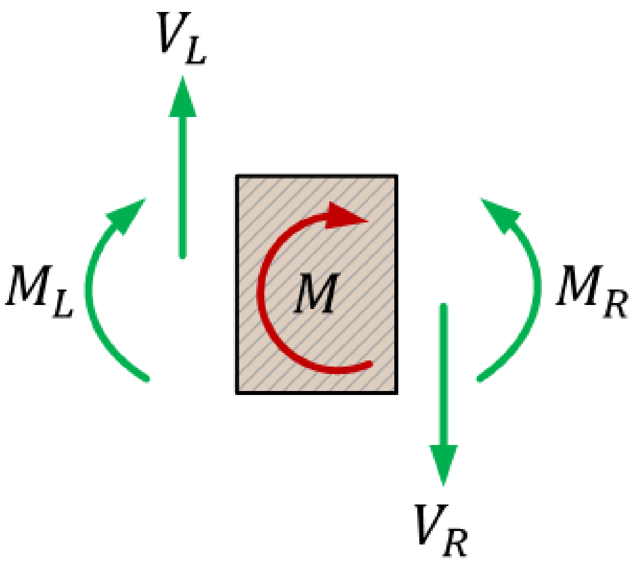


Concentrated Moments

What about a moment balance for this case?

- Write a moment balance about the left end

$$M_L + M + V_R dx - M_R = 0$$



- $V_R dx$ is infinitesimal, while the other terms are finite, so we may ignore it, then:

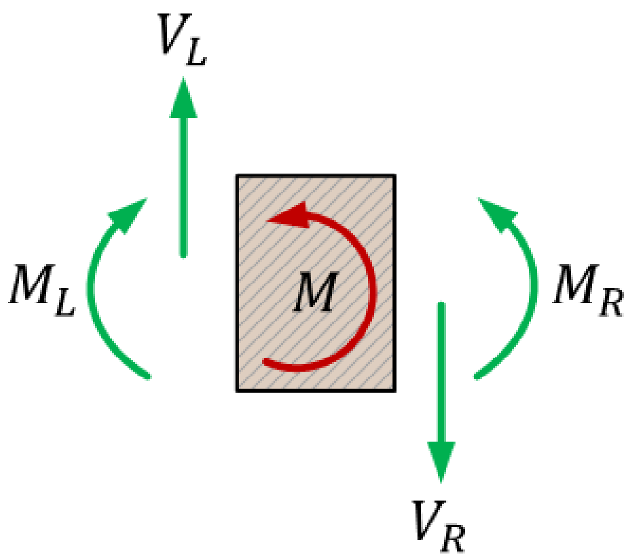
$$\boxed{M_R = M_L + M}$$

Concentrated Moments

If the concentrated moment is in the other direction?

- We can see that M_R is reduced by the amount of M

$$M_L - M + V_R dx - M_R = 0$$

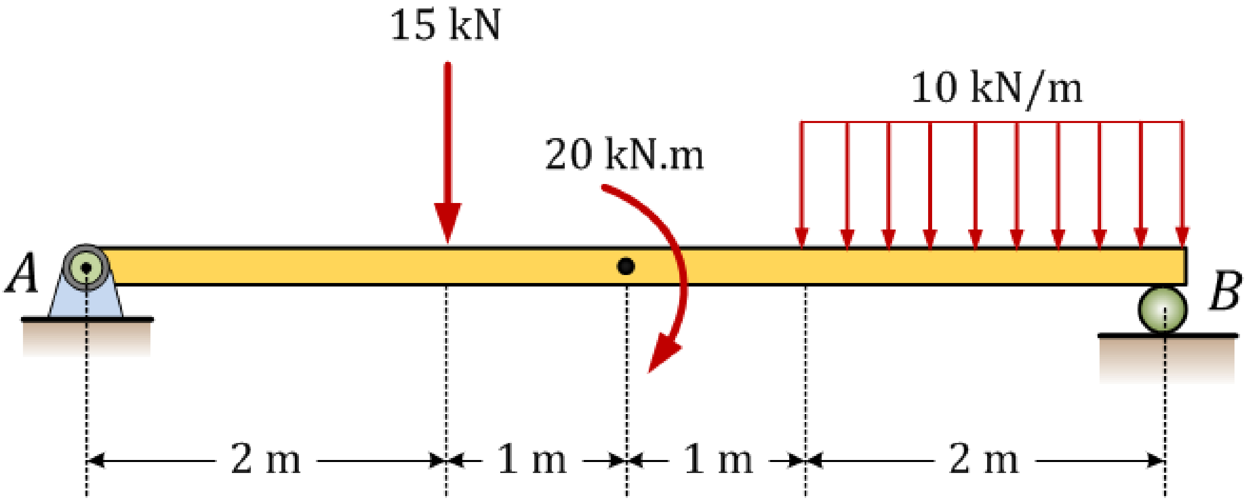


- $V_R dx$ is infinitesimal, while the other terms are finite, so we may ignore it, then:

$$\boxed{M_R = M_L - M}$$

Example 6

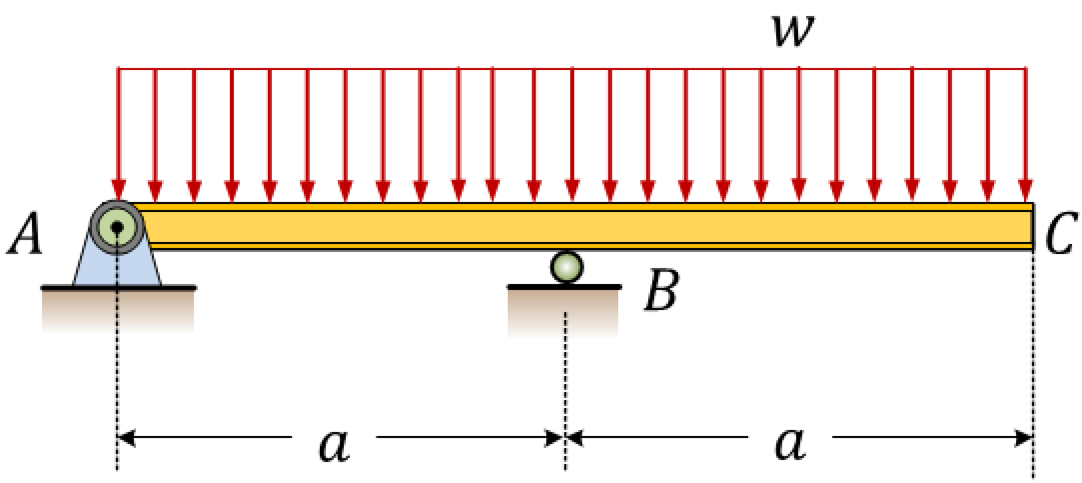
Draw the shear force and bending moment diagrams for the beam AB.



W5 Example 6 (Web view)

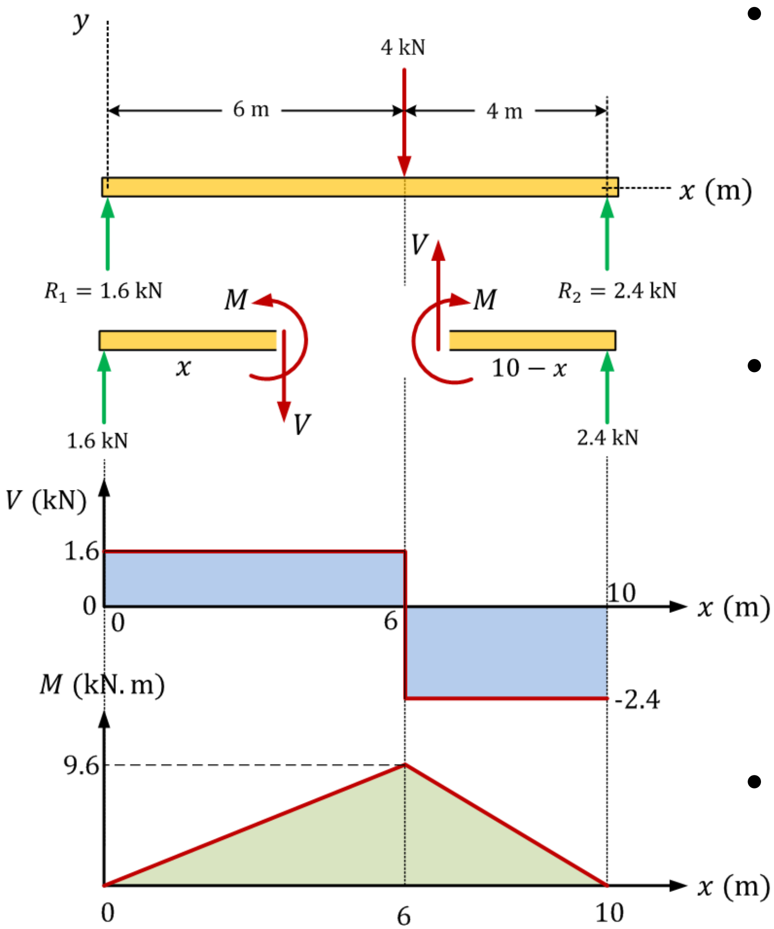
Example 7

Derive the mathematical functions for shear force and bending moment for the beam AB. Sketch to confirm.

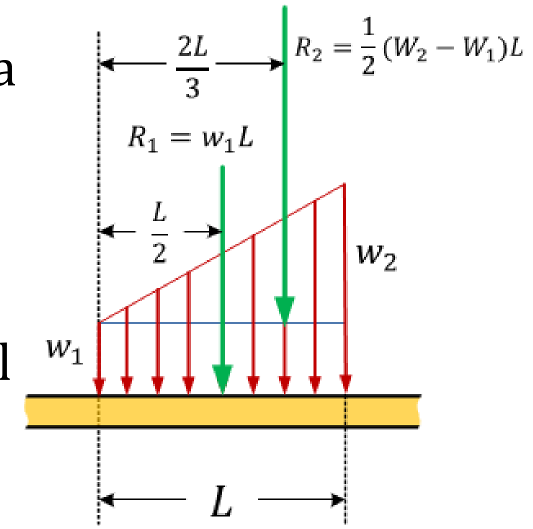


W5 Example 7 (Web view)

Summary



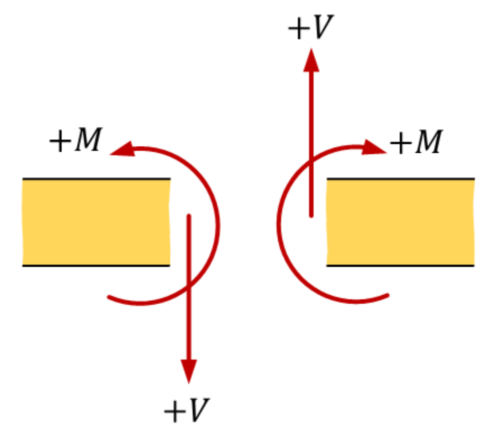
- Distributed loads can be temporarily replaced by a resultant R to calculate reaction forces



- Shear force is the integral of the loading function

$$\boxed{V = V_o - \int_{x_o}^{x} w\, dx}$$

- Bending moment is the integral of the shear force



$$\boxed{M = M_o + \int_{x_o}^{x} V\, dx}$$

Next Topic:

Geometric Properties of Sections