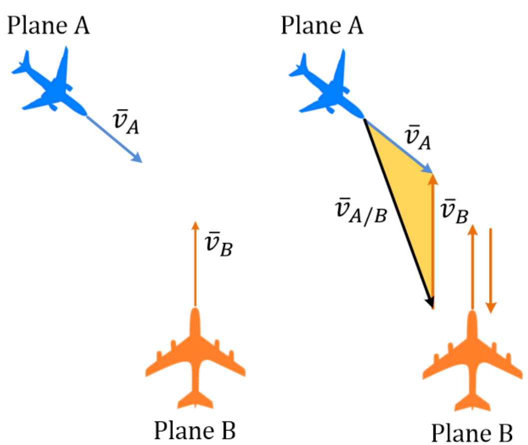
School of Mechanical and Manufacturing Engineering

MMAN1300 Engineering Mechanics 1

Dr. David C. Kellermann



Week 8, L1-2: Particle Kinetics

RELATIVE MOTION (Particle Kinematics)

- Relative Motion
- Relative Velocity

KINETICS OF PARTICLES

- Newton's second law
- Rectilinear motion
- Curvilinear motion

Topics

Contents

- Relative motion
- Relative motion of two particles along a straight line (1-D)
- Relative motion in 2-D
- Relative motion of rigid links

Introduction

We have been describing particle motion using coordinates referred to fixed reference axes.

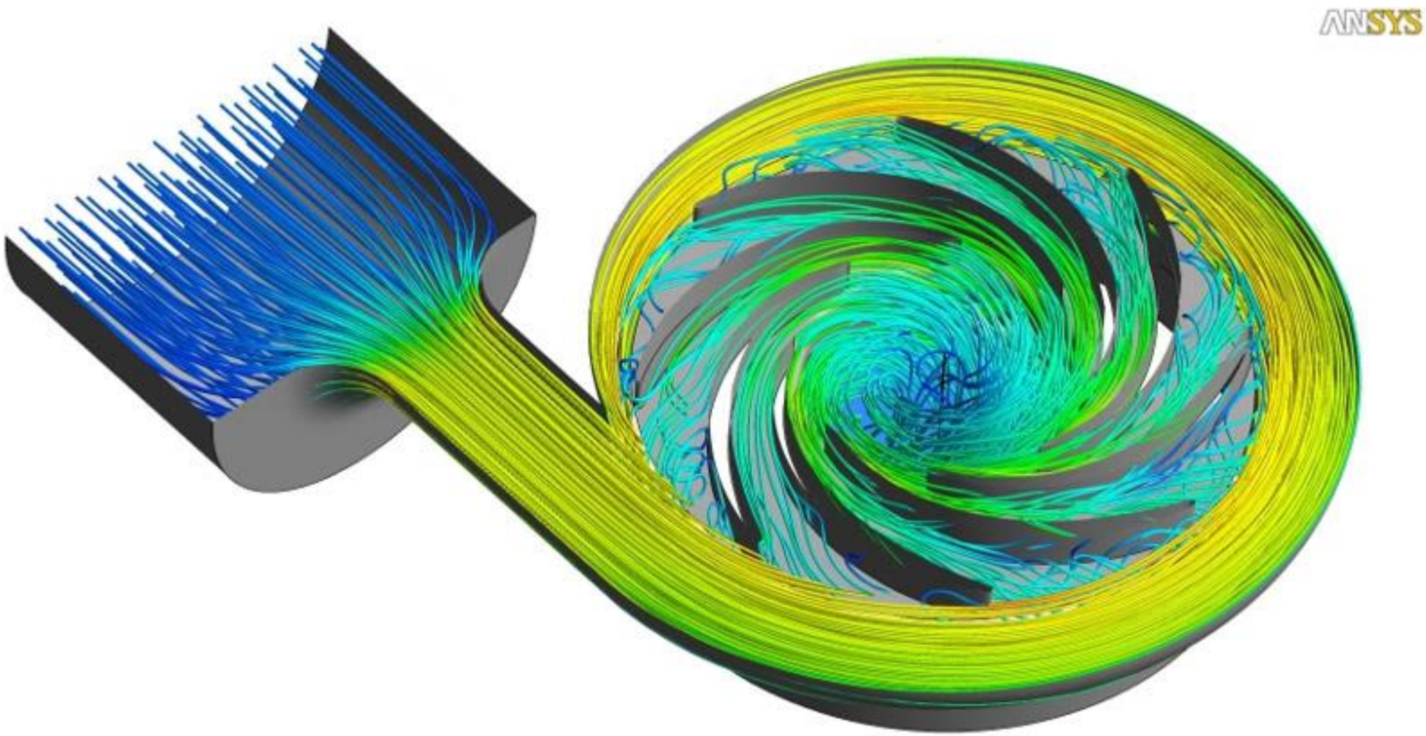
The particle displacements, velocities and accelerations so far determined have been absolute.

Applications

Examples of relative motion: Two cars

Applications

- fluid particles and a point on a turbine blade – the analysis of the relative velocities of fluids and mechanical systems are useful for the design of turbines, pumps, etc.



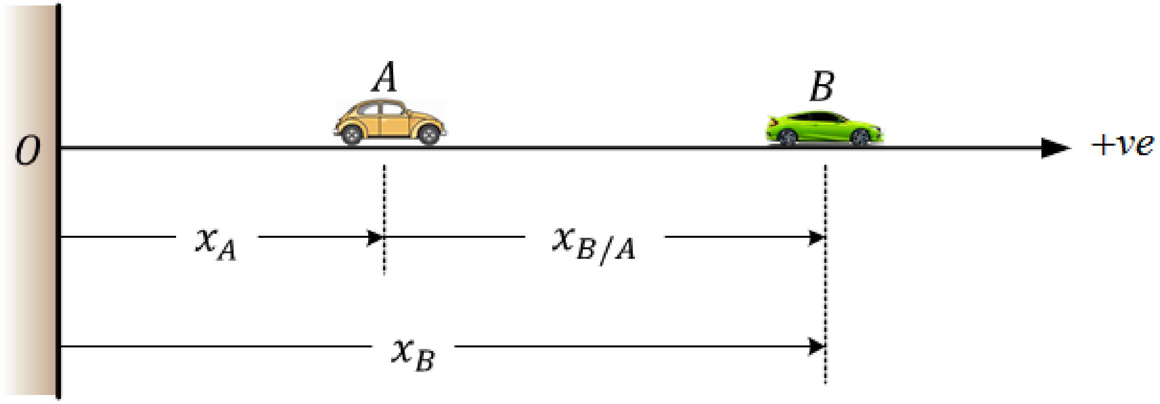
Applications

- two parts of a mechanical linkage, eg. piston and crankshaft in a car engine – the analysis of the relative velocities and accelerations are useful for linkage design (see Kinematics of rigid bodies).

One-Dimensional Relative Motion

Relative motion of two particles along a straight line (1-D)

Consider two particles A and B: for example 2 cars on a road



x_A is the position of particle/car A.

x_B is the position of particle/car B.

$x_{B/A}$ is the position of particle/car B relative to particle/car A.

One-Dimensional Relative Motion

Note: x_A and x_B are with respect to fixed (or absolute) axes. $x_{B/A}$ is with respect to a moving axis.

$$x_B = x_A + x_{B/A}$$

$$x_{B/A} = x_B - x_A$$

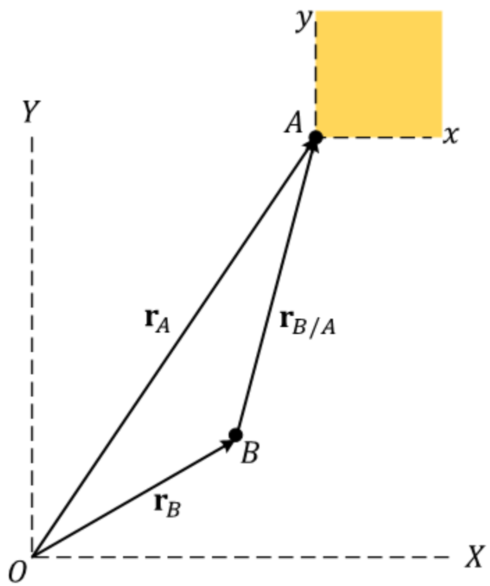
Differentiating with respect to time

$$v_B = v_A + v_{B/A}$$

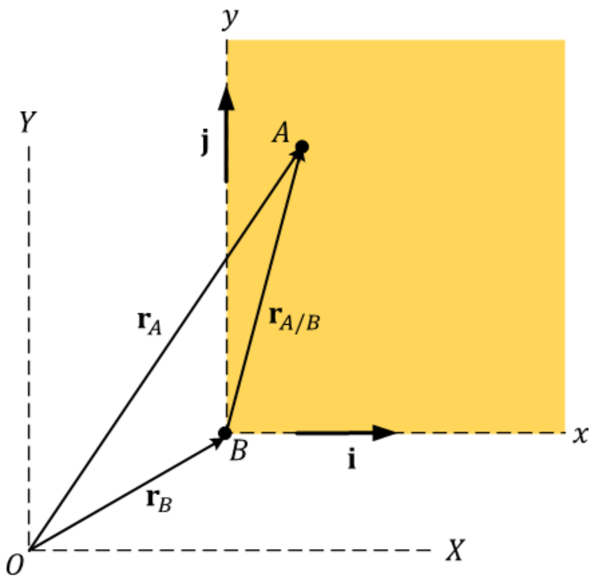
$$a_B = a_A + a_{B/A}$$

In 2-D and 3-D motion, the positions, velocities and accelerations of the particles need to be described in terms of vectors.

Two-Dimensional Relative Motion



$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$



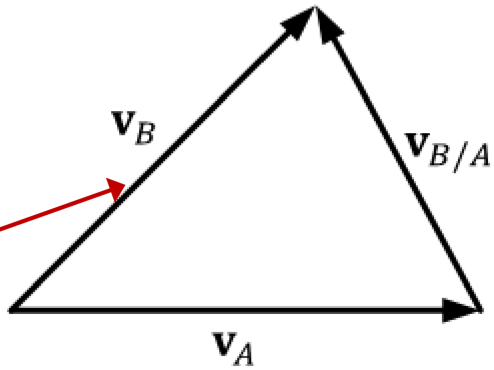
$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$$

$\mathbf{r}_A$ is the position of particle A
$\mathbf{r}_B$ is the position of particle B
$\mathbf{r}_{B/A}$ is the position of particle B relative to particle A
$\mathbf{r}_{A/B}$ is the position of particle A relative to particle B

Two-Dimensional Relative Motion

If $\quad \mathbf{r}_B - \mathbf{r}_A = \mathbf{r}_{B/A} \quad$ and $\quad \mathbf{r}_A - \mathbf{r}_B = \mathbf{r}_{A/B}$

then $\quad \mathbf{r}_{A/B} = -\mathbf{r}_{B/A}$



Differentiating with respect to time:

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \quad \rightarrow \quad \mathbf{v}_{A/B} = -\mathbf{v}_{B/A}$$

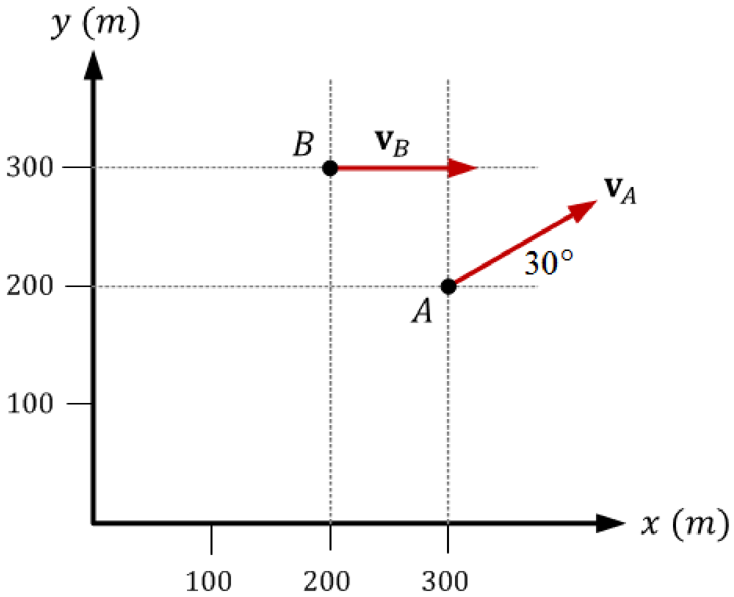
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} \quad \rightarrow \quad \mathbf{a}_{A/B} = -\mathbf{a}_{B/A}$$

In relative motion, it is often convenient to draw a velocity vector diagram.

Example 1: Relative Motion of Particles

Two ships pass in the night

$v_A = 2\ \text{m/s}$
$v_B = 3\ \text{m/s}$



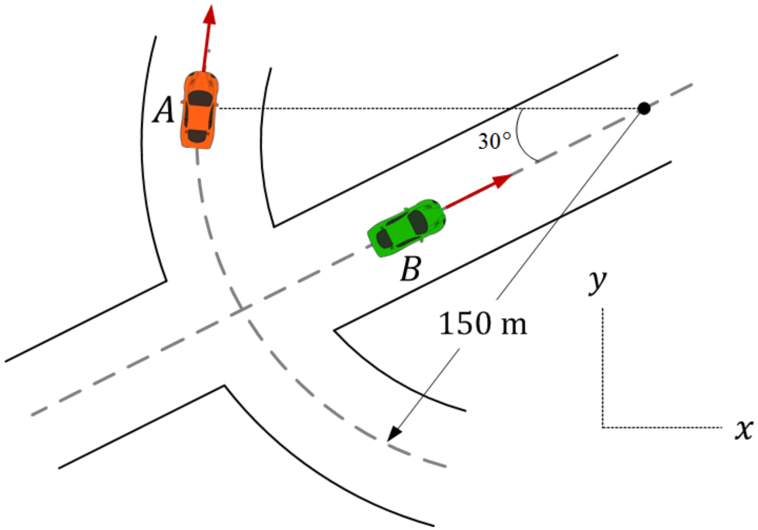
$a_A = 0.5\ \text{m/s}^2$ in the same direction of v_A
$a_B = -0.5\ \text{m/s}^2$ in the direction of v_B

Find the relative accelerations of ship B relative to ship A.

W8 Example 1 (Web view)

Example 2: Relative Circular Motion

For the instant shown below, car A is rounding the circular curve at a constant speed of 50 km/h, while car B with an instantaneous speed of 60 km/h is slowing down at the rate of 8 km/h per second (i.e., 2.22 m/s^2).

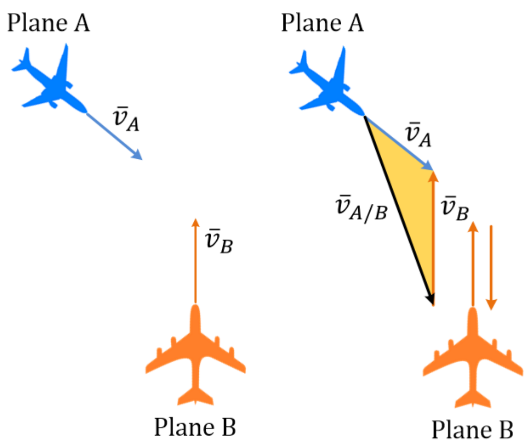


Determine:

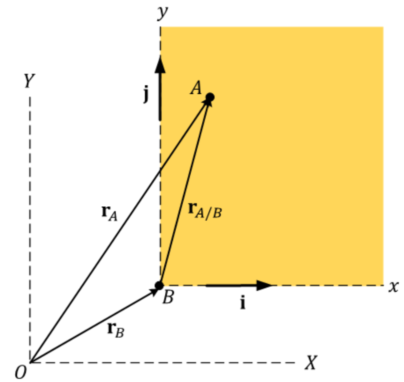
a) the velocity of car A relative to car B,
b) the acceleration that car A appears to have to an observer in car B.

W8 Example 2 (Web view)

Summary



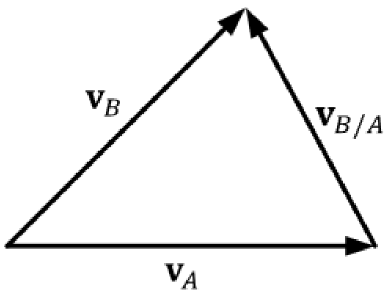
$\mathbf{r}_A$ and $\mathbf{r}_B$ are the positions of particle A and B
$\mathbf{r}_{B/A}$ is the position of particle B relative to A
$\mathbf{r}_{A/B}$ is the position of particle A relative to B



$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$$

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

Differentiating with respect to time:



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Next Topic:

Particle Kinetics

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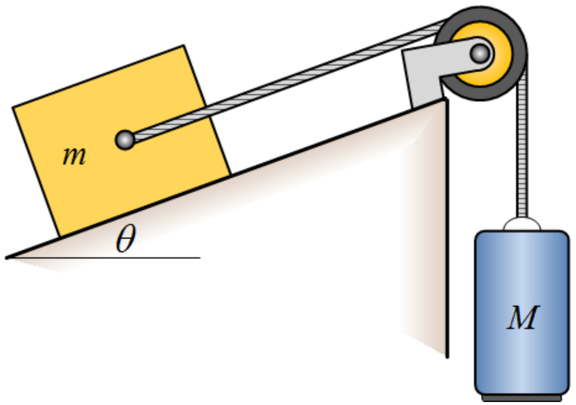


Week 8, L2: Particle Kinetics

RELATIVE MOTION (Particle Kinematics)

- Relative Motion
- Relative Velocity

KINETICS OF PARTICLES

- Newton's second law
- Rectilinear motion
- Curvilinear motion

Kinetics- Introduction

Kinetics is concerned with the relationship between the forces acting on an object and the motion of the object

- At this stage, we are only interested in particles
- Therefore, we are only concerned with the translational motion of the centre of mass of the particle (*i.e.*, rectilinear translation and curvilinear translation)
- We are NOT yet interested in the rotation of the object about its centre of mass

Kinetics- Introduction

To work in kinetics, we need both forces and kinematics (motion)

- Kinetics looks at the forces needed to maintain a particular motion or the motion caused by particular forces

- We can look at Statics as a special case of Dynamics where the acceleration is zero

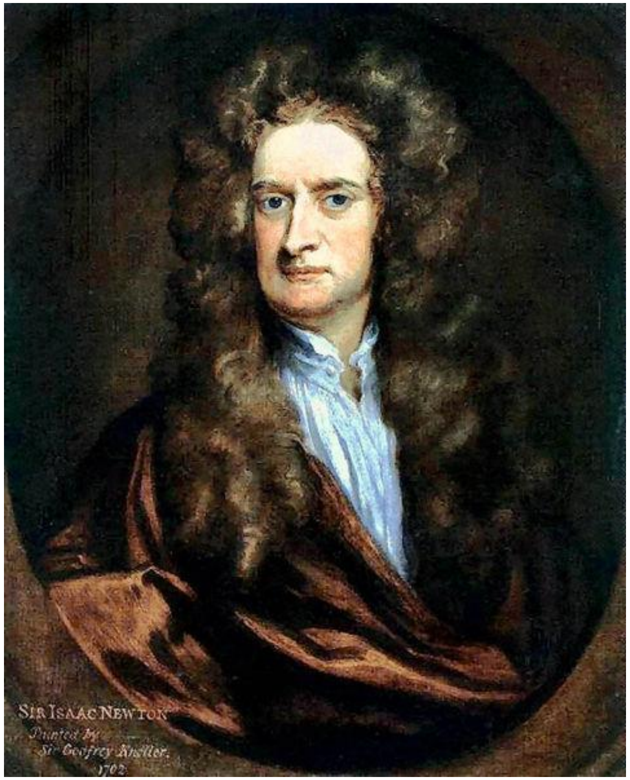
Particle Kinetics- What We Need to Learn

1. How do we apply Newton's 2nd Law along with Free Body Diagrams to particles?
2. How can we model forces from springs, strings, friction, gravity, etc. in a Dynamics setting?
3. How do Work/Energy and Impulse/Momentum principles apply in Dynamics?
4. How do we know when to apply each of the 3 approaches (Newton's laws, Work/Energy & Impulse/Momentum)?

Newton's 2nd Law:

We are probably due a reminder on Newton's 2nd Law:

"*The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*" – Sir Isaac Newton



Isaac Newton

Euler's Equation

Newton's 2nd Law is also known as "The Balance of Linear Momentum":

- Newton's statement of the law is actually in a form that we will call Impulse/Momentum
- Leonhard Euler wrote the law in its differential form and referred to it as the Balance of Linear Momentum
- Where $G = mv$ is the linear momentum

Leohnard Euler

$$\mathbf{F} = \frac{d\mathbf{G}}{dt} \quad \text{or} \quad \mathbf{F} = \dot{\mathbf{G}}$$

Newtonian vs. Eulerian Formulations

For a particle, the mass m is constant ($dm/dt = 0$)

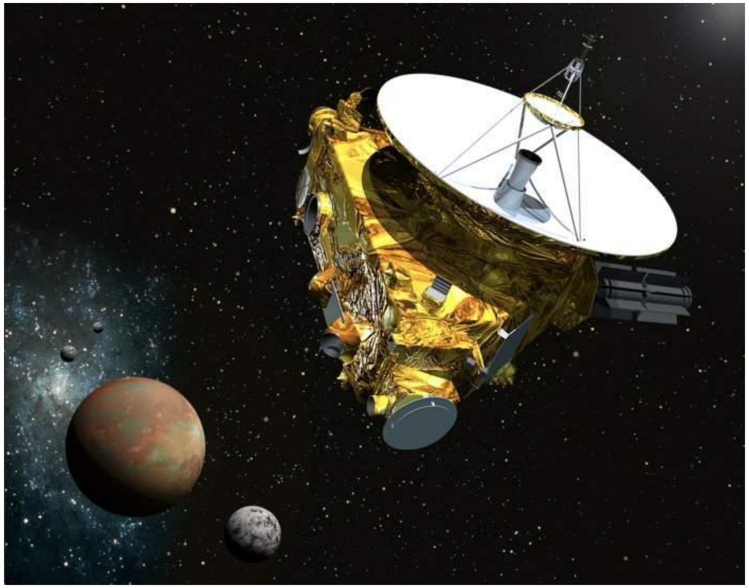
Then the balance of linear momentum reduces to

$$\boxed{\mathbf{F} = m\mathbf{a}}$$

- Note that F and a are vectors
- This is called an "equation of motion"
- Also note that F is the sum of the forces (net force) if a is the total acceleration
- The acceleration and the sum of the forces are in the same direction

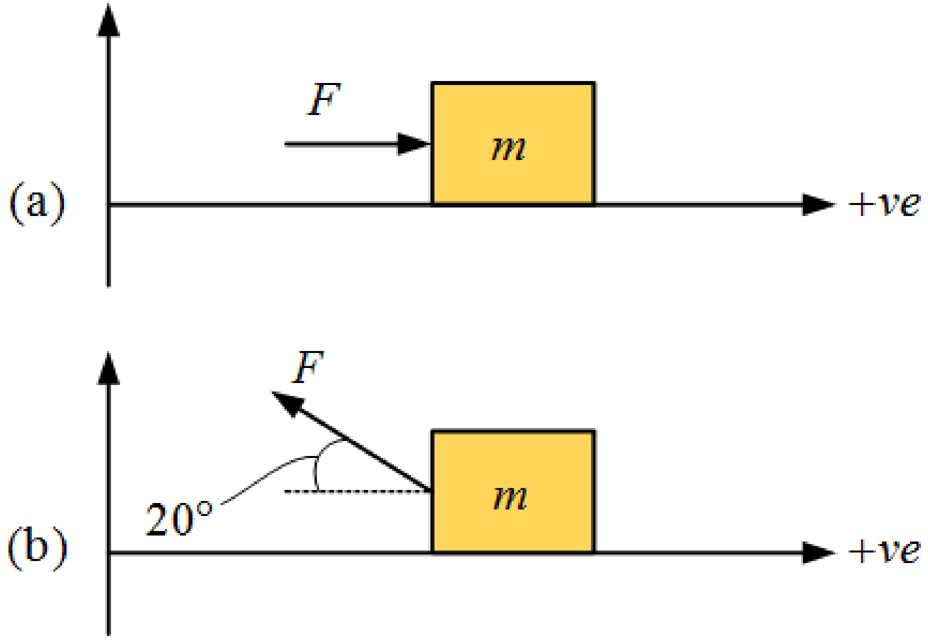
What Does Kinetics Enable?

- So if we know the motion of the particle from kinematics, we can find the net force
- Likewise, if we can find the net force, we can use kinematics to find the path



Example 3: Rectilinear motion

A block of mass $m = 5$ kg is acted on by a force of 20 N. Neglecting friction, find the acceleration of the block in each case.



W8 Example 3 (Web view)

Free Body Diagram for Kinetics

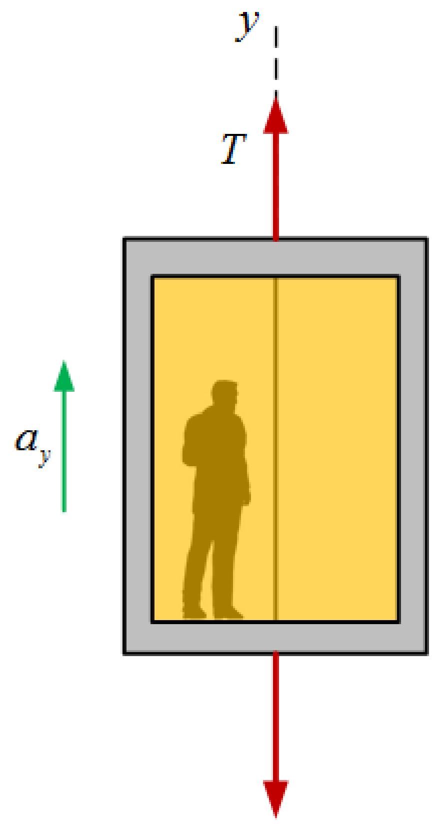
When we draw a FBD, we draw the body free of any attachments. At the same time, we draw ALL the forces that are acting on the free body. There forces will include:

- force due to gravity
- forces from any attachments or contacts
- externally applied forces
- Indicate velocity or acceleration

Example 4: Kinetics and Gravity

Consider an elevator without a passenger. The elevator is accelerating upwards.

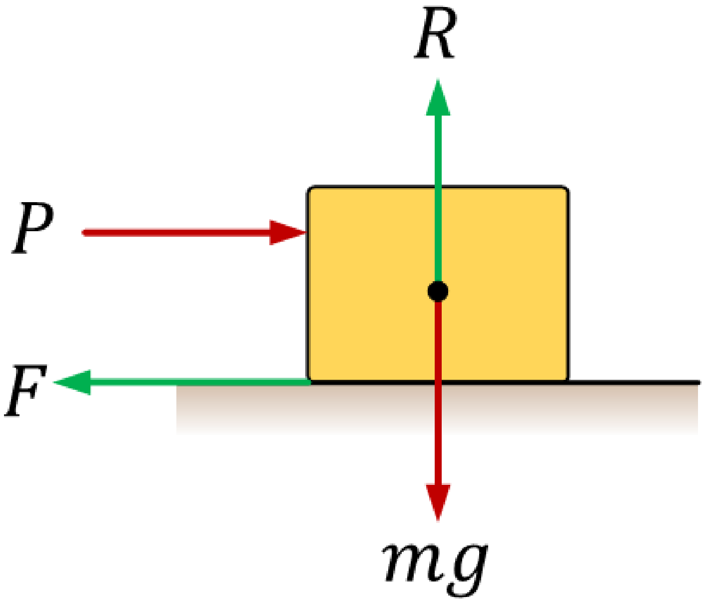
1. Draw the FBD of the elevator.
2. Find the tension in the elevator cable when the acceleration of the elevator $a = 0.6\ \text{m/s}^2$, upwards. The mass of the elevator $m = 400$ kg.
3. If a passenger with a mass of 80 kg stands on a set of scales on the floor, what mass will the scales read the elevator moves up with $a = 0.6\frac{\text{m}}{\text{s}^2}(\uparrow)$?



W8 Example 4 (Web view)

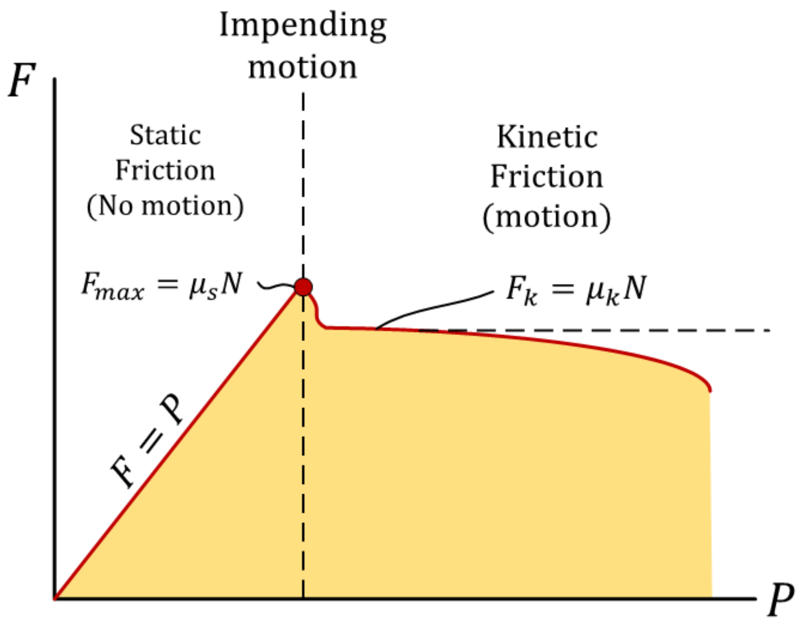
Review: Static and Dynamic Friction

- We denote the constant of proportionality by μ_s
- Hence, $|F| \leq \mu_S |R|$
- When the block starts moving, the friction opposes motion
- The direction is opposite
- As noted above, its magnitude is proportional to the magnitude of R, the reaction (normal) force
- Let's call the constant of proportionality μ_k
- Now Newton's 2nd Law gives us $F = \mu_k$



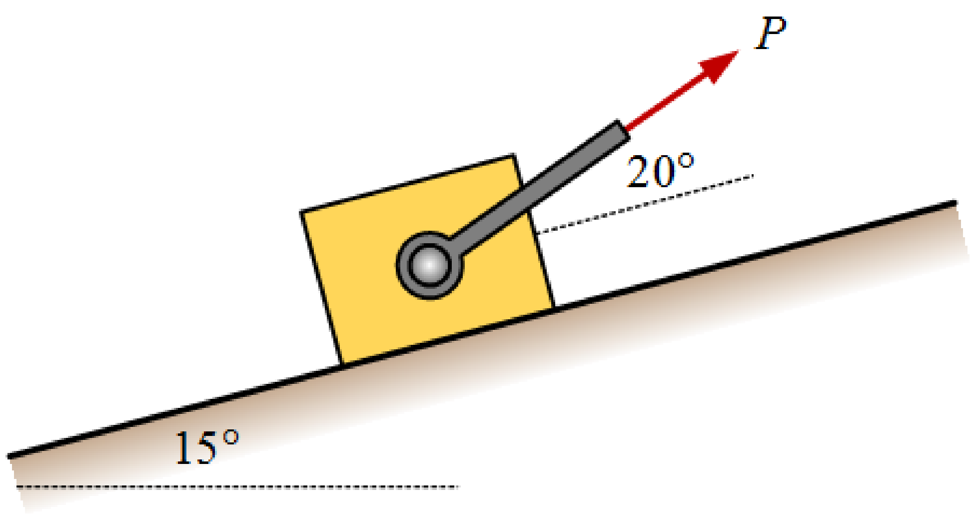
Don't use $F_f = \mu_s N$ to calculate static friction force

- The motion is known (i.e. there is no relative motion between the contacting objects)
- So we can use $\mathbf{F} = m\mathbf{a}$ to find F_f even if $\mathbf{a} = 0$
- The equation $F_f = \mu_s N$ is only valid at impending slip



Example 5: Dynamic Friction

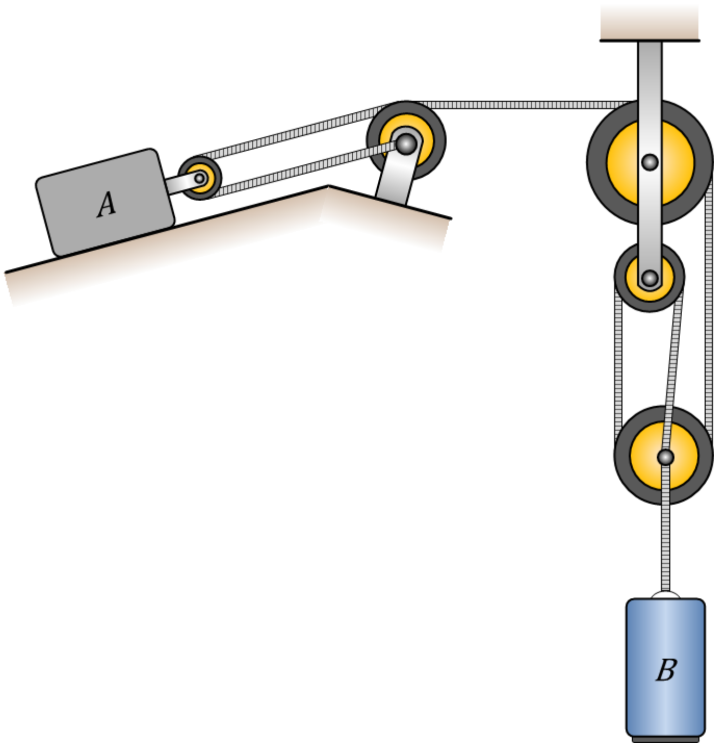
Consider a block of mass m on a fixed inclined surface. The block has an external force P applied to it. Assuming the friction coefficient between the block and the incline is $\mu_d = 0.3$, find the acceleration of the block.

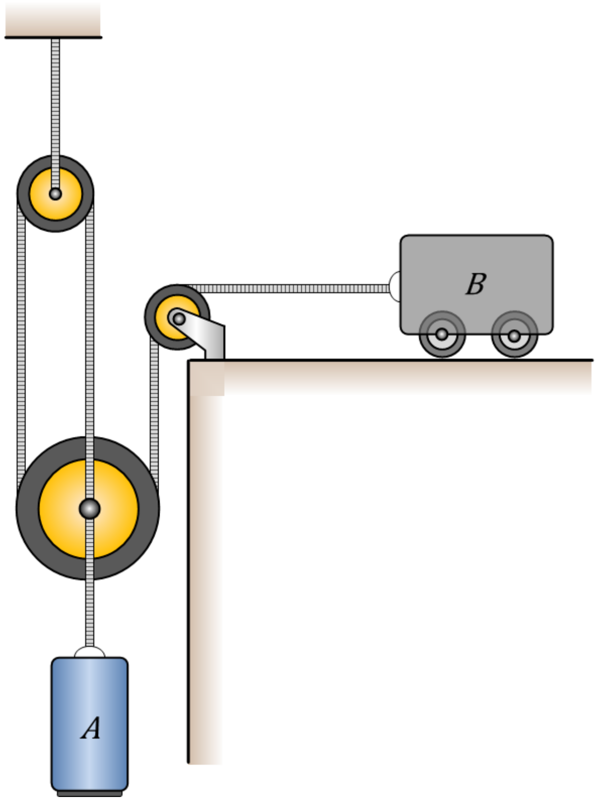


W8 Example 5 (Web view)

Review: Strings, Ropes and Pulleys

Strings, ropes and cables are often used to connect particles, and often the connection is via a pulley.

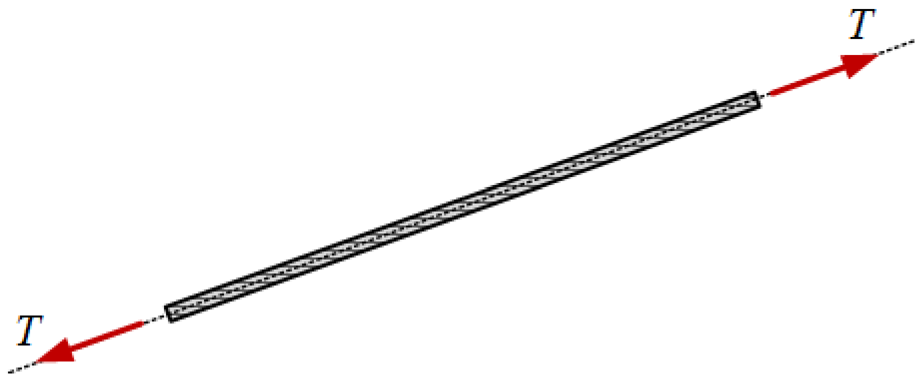




Review: Strings, Ropes and Pulleys

Strings and cables are considered inextensible (ie. not stretching), massless, and can only transit forces while in tension.

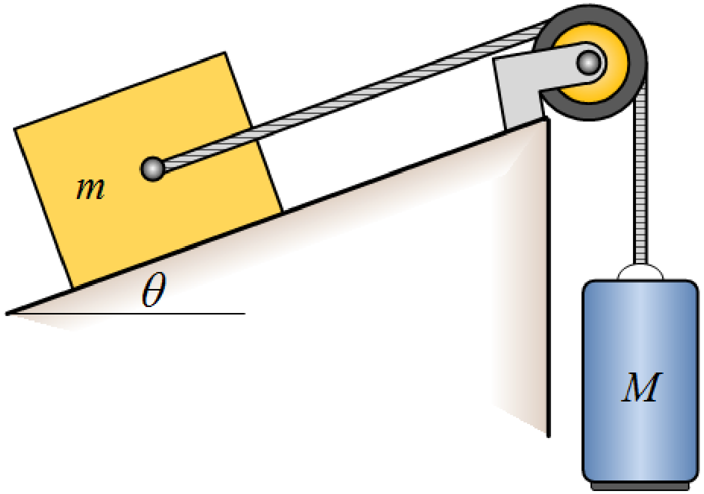
A FBD of a short length of string is:



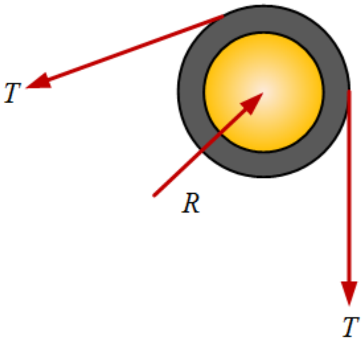
The tension force T must be equal and opposite.

Review: Strings, Ropes and Pulleys

Simplifying assumptions are also made about pulleys. (Unless otherwise stated), we assume pulleys are massless and frictionless. A FBD of a pulley shown in the figure below is shown. Since the pulley is massless and frictionless, there can be no resultant motion. Hence, the magnitude and direction of the reaction force on the pulley from its bearing is such that the sum of the forces on the pulley is zero.



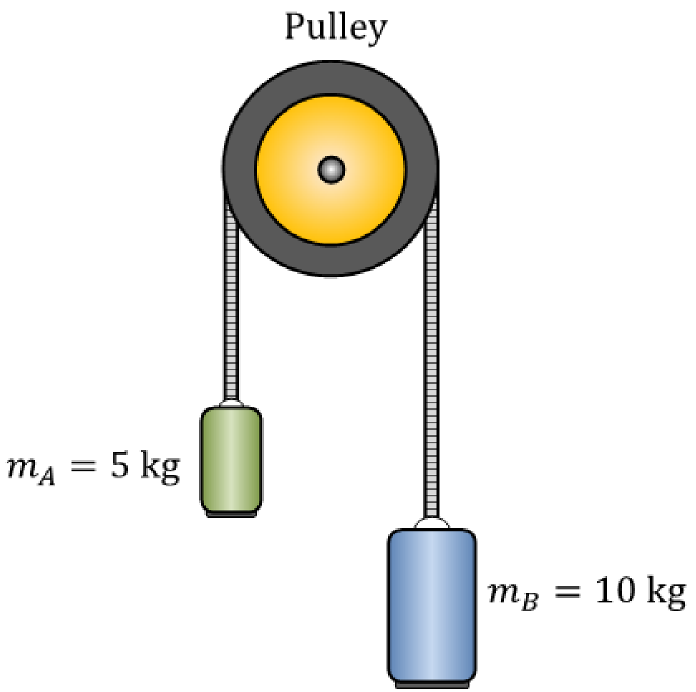
FBD of Pulley



Example 6: Pulleys

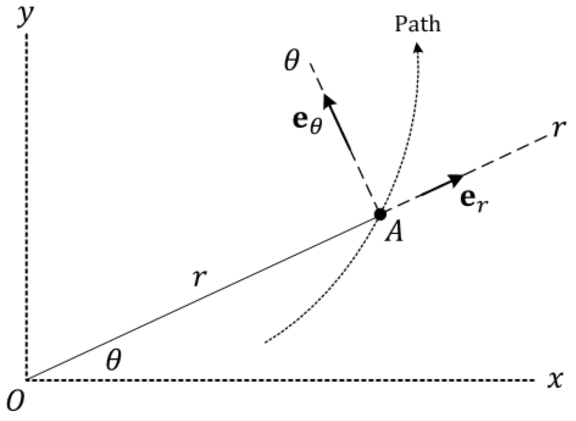
A 10 kg mass and a 5 kg mass are connected by a cable which runs over a massless pulley. The system is released. Determine:

(a) the acceleration of each mass,
(b) the tension in the cable.



W8 Example 6 (Web view)

Review: Curvilinear motion

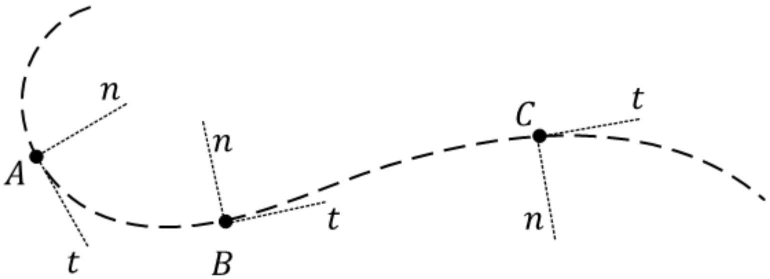


- Acceleration in polar coordinates

$$\mathbf{a} = a_r \mathbf{e}_r + a_\theta \mathbf{e}_\theta$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

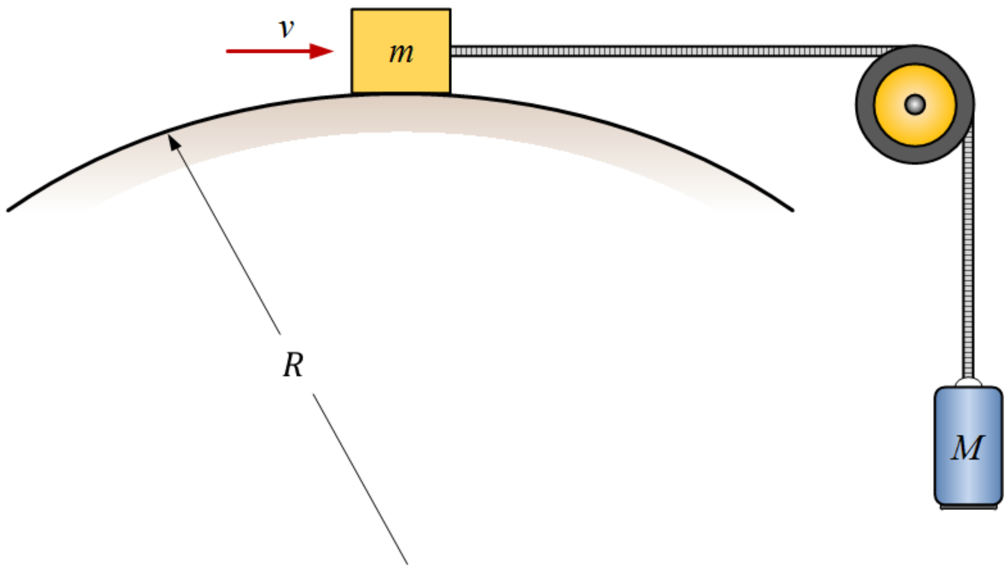


- Acceleration in curvlinear n-t coordinates

$$\boldsymbol{a} = \dot{v}\boldsymbol{e}_t + \frac{v^2}{\rho}\boldsymbol{e}_n = a_t\boldsymbol{e}_t + a_n\boldsymbol{e}_n$$

Example 7: Circular Motion

The mass m (2.3 kg) has a velocity of 5.4 m/s.
The coefficient of kinetic friction for the mass m is 0.5.
The pulley is massless and frictionless.
The curved surface is circular with a radius $R = 0.9$ m.
Find the acceleration of both masses. Mass $M = 5$ kg.



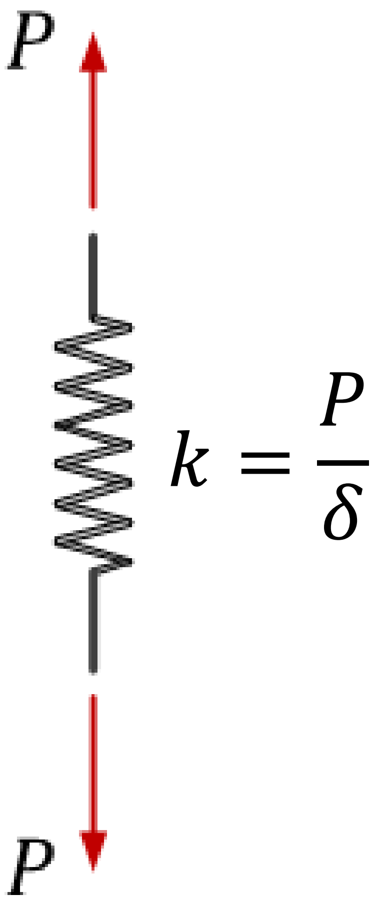
W8 Example 7 (Web view)

Review: Linear Springs

- With Hooke's Law, the relationship between the force and the displacement is linear

- The magnitude is given as $F_s = k\delta$
  - Where $\delta = L - L_0$
  - L is the current length of the spring
  - L_0 is the unstretched length of the spring

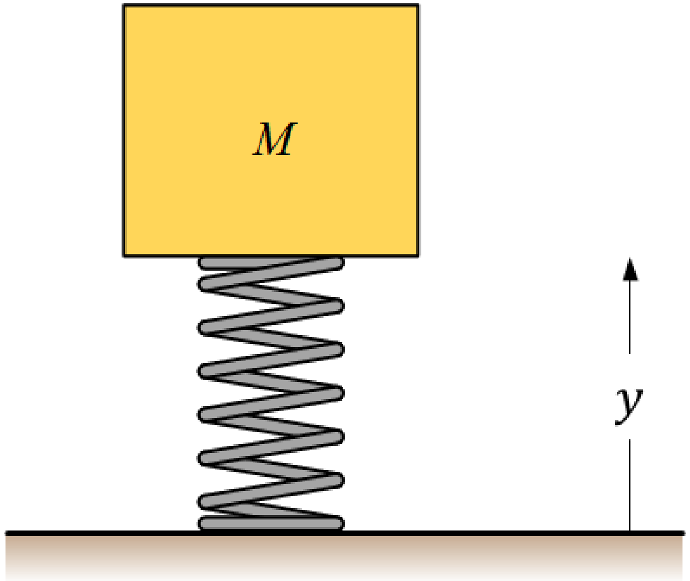
- The direction is opposite the stretch (or compression) of the spring



Example 8: Spring Kinetics

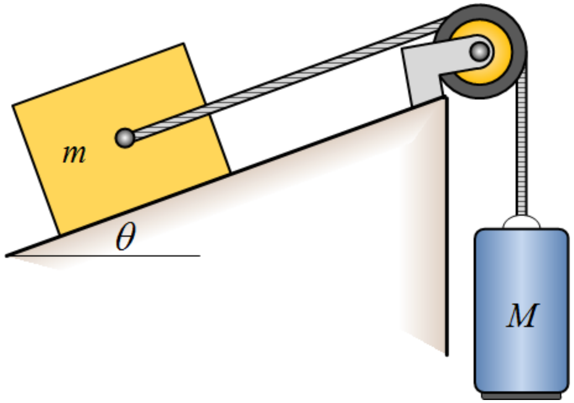
Given: $M = 10\ \text{kg}$ and stiffness of the spring $k = 1000\ \text{N/m}$
Find the deformation of the spring when

(a) $a = 0\ \text{m/s}^2$,
(b) $a = 1\ \text{m/s}^2 \uparrow$.



W8 Example 8 (Web view)

Summary



- Newton's statement of the law is actually in a form that we will call Impulse/Momentum

$$\mathbf{F} = m\mathbf{a}$$

- Leonhard Euler wrote the law in its differential form and referred to it as the Balance of Linear Momentum, where $G = mv$ is the linear momentum

$$\mathbf{F} = \frac{d\mathbf{G}}{dt} \quad \text{or} \quad \mathbf{F} = \dot{\mathbf{G}}$$

- For particle kinetics, we must be competent with cables/pully systems, springs, friction, gravity and curvilinear motion

Next Topic:

Particle Work and Energy