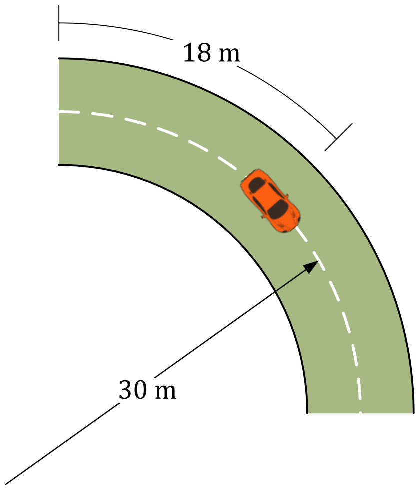
Question 7.7.



The car travels along the circular path such that its speed is increased by $a_t = (0.5e^t)\ \mathrm{m/s^2}$, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18$ m starting from rest. Neglect the size of the car.

Solution

$$\int_0^v dv = \int_0^t 0.5e^t\, dt$$

$$v = 0.5(e^t - 1)$$

$$\int_0^{18} ds = 0.5\int_0^t (e^t - 1)\, dt$$

$$18 = 0.5(e^t - t - 1)$$

Solving,

$$t = 3.7064\ \mathrm{s}$$

Thus,

$$v = 0.5(e^{3.7064} - 1)$$

$v = 19.9\ \mathrm{m/s}$ (Answer)

$$a_t = \dot{v} = |0.5e^t|_{t=3.7064\ \mathrm{s}}$$

$$a_t = 20.35\ \mathrm{m/s^2}$$

$$a_n = \frac{v^2}{\rho} = \frac{19.9^2}{30} = 13.14\ \mathrm{m/s^2}$$

$$a = \sqrt{{a_n}^2 + {a_t}^2} = \sqrt{13.14^2 + 20.35^2}$$

$a = 24.2\ \mathrm{m/s^2}$ (Answer)

Question 7.8.

The acceleration of a particle as it moves along a straight line is given by $= (4t^3 - 1)\ \mathrm{m/s^2}$, where t is in seconds. If $s = 2$ m and $v = 5\ \mathrm{m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 5$ s. Also, determine the total distance the particle travels during this time period.

Solution

$$\int_5^v dv = \int_0^t (4t^3 - 1)\, dt$$

$$v = t^4 - t + 5$$

$$\int_2^s ds = 0.5 \int_0^t (t^4 - t + 5)\, dt$$

$$s = \frac{t^5}{t} - \frac{t^2}{2} + 5t + 2$$

When $t = 5$ s,

$v = 625\ \mathrm{m/s}$ (Answer)

$s = 639.5$ m (Answer)

Since $v \neq 0$, *then*

$d = 639.5 - 2 = 637.5$ m (Answer)

Question 7.9.

A particle starts from $s = 0$ and travels along a straight line with a velocity $v = (t^2 - 4t + 3)$ m/s, where t is in seconds. Construct the $v - t$ and $a - t$ graphs for the time interval $0 \le t \le 4$ s.

Solution

***a* − *t* Graph**

$$a = \frac{dv}{dt} = \frac{d}{dt}(t^2 - 4t + 3)$$

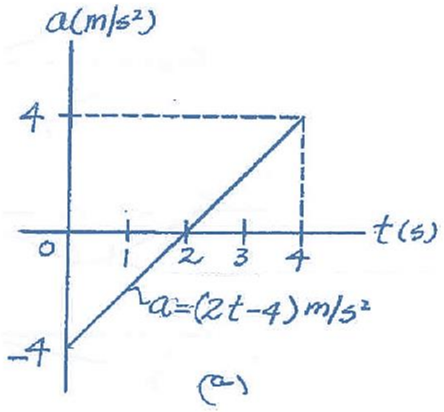
$$a = (2t - 4)\ \mathrm{m/s^2}$$

Thus,

$$a|_{t=0} = 2(0) - 4 = -4\ \mathrm{m/s^2}$$

$$a|_{t=2s} = 2(2) - 4 = 0$$

$$a|_{t=4s} = 2(4) - 4 = 4\ \mathrm{m/s^2}$$



(Answer)

***v* − *t* Graph:** *The slope of the* $v - t$ *graph is zero when*

$$a = \frac{dv}{dt} = 0$$

Thus,

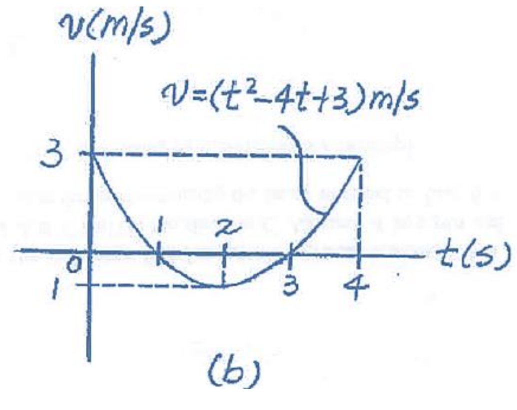
$$a = 2t - 4 = 0 \quad at \quad t = 2\ \mathrm{s}$$

The velocity of the particle at $t = 0$ s, 2 s *and* 4 s *are*

$$v|_{t=0} = 0^2 - 4(0) + 3 = 3\ \mathrm{m/s}$$

$$v|_{t=2s} = 2^2 - 4(2) + 3 = -1\ \mathrm{m/s}$$

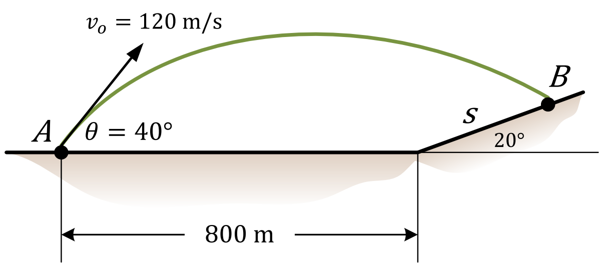
$$v|_{t=4s} = 4^2 - 4(4) + 3 = 3\ \mathrm{m/s}$$



(Answer)

Question 7.10.

A projectile is launched from point A with the initial conditions shown in the figure. Determine the slant distance s which located the point B of impact. Calculate the time of flight.



Solution

Setup x-y origin coordinates with origin at A

$$x = x_o + v_{x_o} t$$

at B

$$800 + s\cos 20° = (120\cos 40°)\, t \quad \text{------------ (1)}$$

$$y = y_o + v_{y_o} t - \frac{1}{2} g t^2$$

at B

$$s\sin 20° = (120\sin 40°)t - \frac{1}{2}(9.81)t^2$$

------------ (2)

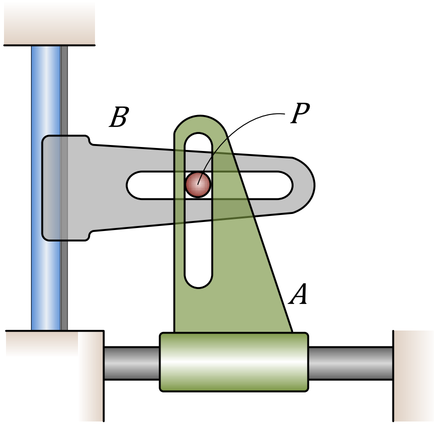
Solve (1) and (2)

$s = 455$ m (Answer)

$t = 13.35$ s (Answer)

Question 7.11.

The pin P is constrained to move in the slotted guides which move at right angles to one another. At the instant represented, A has a velocity to the right of $0.2\ \mathrm{m/s}$ which is decreasing at the rate of $0.75\ \mathrm{m/s}$ each second. At the same time, B is moving down with a velocity of $0.15\ \mathrm{m/s}$ which is decreasing at the rate of $0.5\ \mathrm{m/s}$ each second. For the instant determine the radius of curvature of the path followed by P.



Solution

Given:

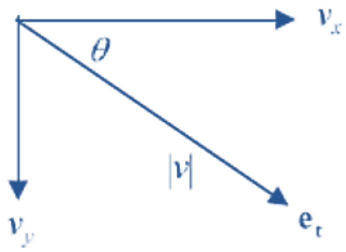
$v_x = 0.2\ \mathrm{m/s}$, $\quad v_y = 0.15\ \mathrm{m/s}$, $\quad a_x = -0.75\ \mathrm{m/s^2}$, $\quad a_y = -0.5\ \mathrm{m/s^2}$

Note that velocity defines the tangential direction. Perpendicular to the tangential direction is the normal direction (towards the centre of curvature of the path).

$$|v| = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{0.2^2 + 0.15^2} = 0.25\ \mathrm{m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{0.15}{0.2}\right) = 36.87°$$

The tangential component of acceleration can be found by resolving a_x and a_y components of the acceleration into the tangential direction $\mathbf{e}_t$.

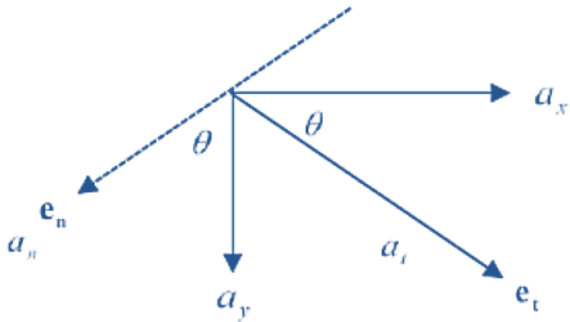


$$a_t = a_x \cos\theta + a_y \sin\theta$$

$$a_t = (-0.75)\cos 36.87° + (-0.5)\sin 36.87°$$

$$a_t = -0.9\ \mathrm{m/s^2}$$

Similarly the normal component of acceleration can be found by resolving a_x and a_y components of the acceleration into the normal direction $\mathbf{e}_n$.



$$a_n = -a_x \sin\theta + a_y \cos\theta$$

$$a_n = -(-0.75)\sin 36.87° + (-0.5)\cos 36.87°$$

$$a_n = 0.05\ \mathrm{m/s^2}$$

$$a_n = \frac{v^2}{\rho}$$

$$\rho = \frac{v^2}{a_n} = \frac{0.25^2}{0.05} = 1.25\ \mathrm{m}$$

(Answer)