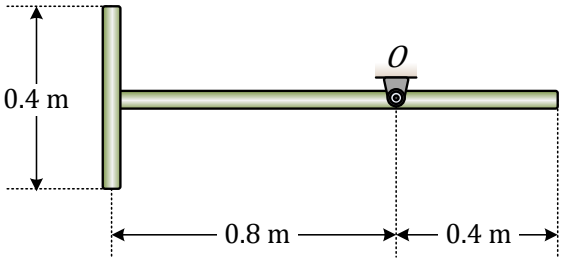
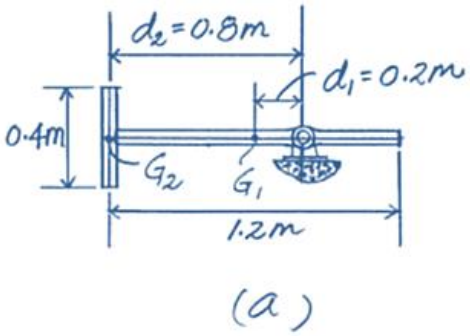
Question 11.7.

The assembly is made of the slender rods that have a mass per unit length of 3 kg/m. Determine the mass moment of inertia of the assembly about an axis perpendicular to the page and passing through point O.



Solution



(a)

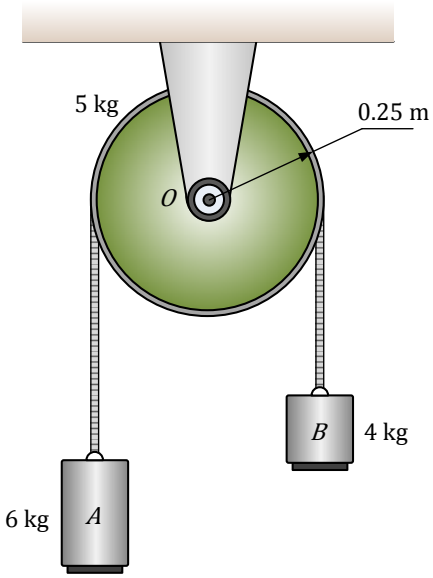
Using the parallel axis theorem referring to Fig. (a)

$$I_O = \sum(I_G + md^2)$$

$$I_O = \left\{\frac{1}{12}[3(1.2)](1.2)^2 + [3(1.2)](0.2)^2\right\} + \left\{\frac{1}{12}[3(0.4)](0.4)^2 + [3(0.4)](0.8)^2\right\}$$

$$I_O = 1.36 \text{ kg.m}^2 \qquad \text{(Answer)}$$

Question 11.8.



Determine the angular acceleration of the uniform disk if (a) the rotational inertia of the disk is ignored and (b) the inertia of the disk is considered. The system is released from rest, the cord does not slip on the disk, and bearing friction at O may be neglected.

Solution

(a)

$I_O = 0$

$a_A = a_B = a$

From $\sum M_O = 0$: $T_A = T_B$

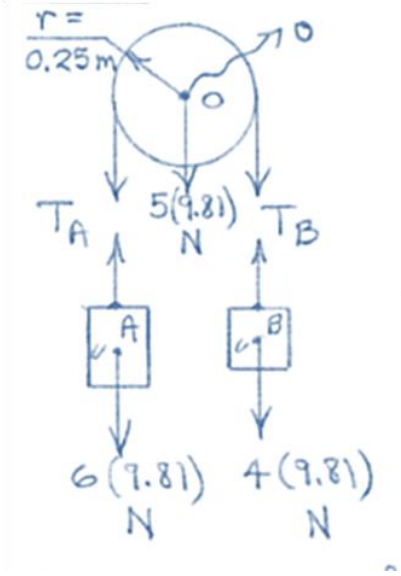
$+\downarrow \sum F_y = ma_A$

$6(9.81) - T = 6a$ ---------- (1)

$+\uparrow \sum F_y = ma_B$

$T - 4(9.81) = 4a$ ---------- (2)

Solving Eqs. (1) and (2)



$T = 47.1$ N

$a = 1.962$ m/s^2

$\alpha = \dfrac{1.962}{0.25} = 7.85$ rad/s^2 (CCW)

(Answer)

(b)

$I_O = \frac{mr^2}{2} = \frac{5(0.25)^2}{2} = 0.15625 \text{ kg. m}^2$

$a_A = a_B = a$

$T_A \neq T_B$

$+\downarrow \sum F_y = ma_A$

$6(9.81) - T_A = 6a$ ---------- (1)

$+\uparrow \sum F_y = ma_B$

$T_B - 4(9.81) = 4a$ ---------- (2)

$+\circlearrowleft \sum M_O = I_O\alpha$

$0.25(T_A - T_B) = 0.15625\alpha$ ---------- (3)

Solving Eqs. (1) - (3)

$T_A = 49.4 \text{ N}$

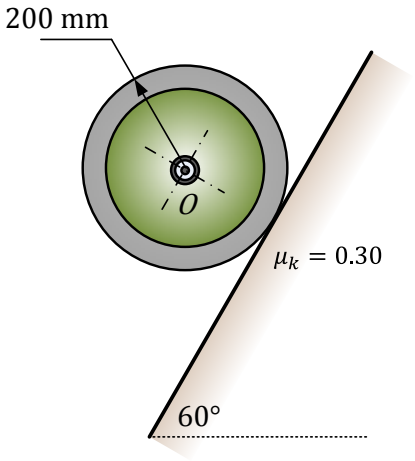
$T_B = 45.5 \text{ N}$

$a = 1.570 \text{ m/s}^2$

$\alpha = \frac{1.570}{0.25} = 6.28 \text{ rad/s}^2$ (CCW)

(Answer)

Question 11.9.



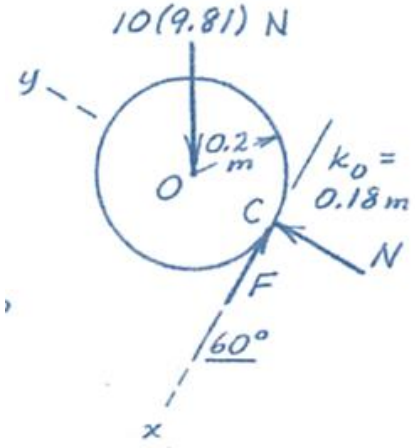
The 10 kg wheel with a radius of gyration of 180 mm about its centre O is released from rest on the 60° incline and slips as it rolls. If the coefficient of kinetic friction is $\mu_k = 0.30$, calculate the acceleration (a_O) of the centre O of the wheel and its angular acceleration (α).

Solution

$+\uparrow \sum F_y = 0$

$N - 98.1 \cos 60° = 0$

$N = 49$ N



$F = \mu_k N = 0.3(49) = 14.72$ N

$+\rightarrow \sum F_x = ma_x$

$10(98.1 \sin 60°) - 14.72 = 10a_O$

$a_O = 7.02$ m/s^2

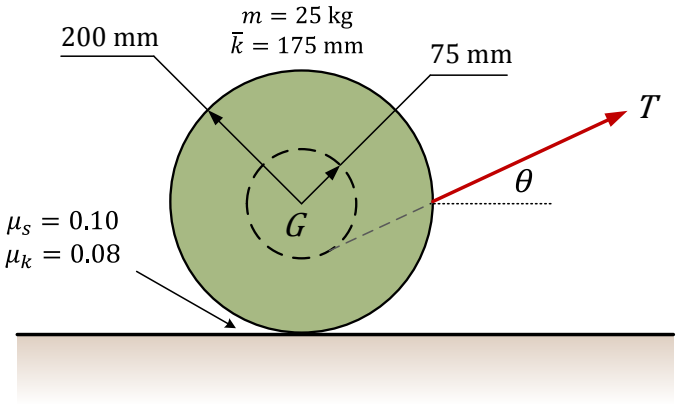
$+\circlearrowleft \sum M_O = I_O \alpha$

$14.72(0.2) = 10(0.18)^2 \alpha$

$\alpha = 9.08$ rad/s^2 (Answer)

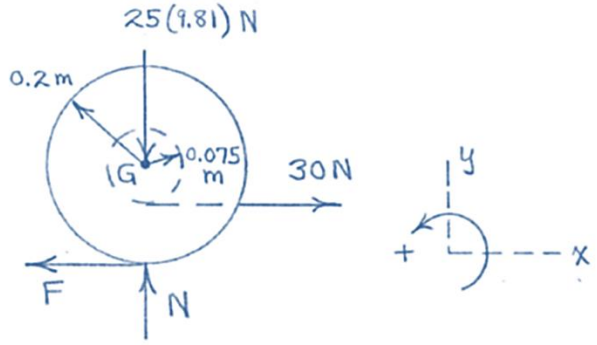
Question 11.10.

The circular disk of 200 mm radius has a mass of 25 kg with centroidal radius of gyration $\bar{k} = 175$ mm and has a concentric circular groove of 75 mm radius cut into it. A steady force T is applied at an angle θ to a cord wrapped around the groove as shown. If $T = 30$ N, $\theta = 0°$, $\mu_s = 0.10$ and $\mu_k = 0.08$, determine the angular acceleration α of the disk, the acceleration a_G of its mass centre G, and the friction force F which the surface exerts on the disk.



Solution

$\bar{k} = 0.175$ m, $\mu_s = 0.10$ and $\mu_k = 0.08$



$+\uparrow \sum F_y = 0$

$N = 25(9.81) = 245$ N

$+\rightarrow \sum F_x = ma_x$

$30 - F = 25\,a$ ---------- (1)

$+\circlearrowleft \sum M_O = I_G\alpha$

$30(0.075) - F(0.2) = 25(0.175)^2\alpha$ ---------- (2)

Assuming rolling with no slip:

$a = -r\alpha$ ---------- (3)

Solving Eqs. (1) - (3)

$F = 19.38$ N (Answer)

$a = 0.425$ m/s^2 (Answer)

$\alpha = \dfrac{1.570}{0.25} = -2.12$ rad/s^2

(Answer)

$F_{max} = \mu_s N = (0.1)(245) = 24.5$ N $> F$ (Thus the assumption was correct)