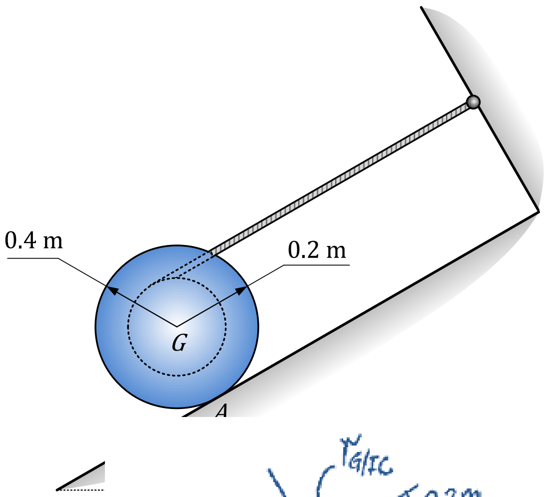
Question 12.10.

The 100 kg spool is resting on the inclined surface for which the coefficient of kinetic friction is $\mu_k = 0.1$. Determine the angular velocity of the spool when $t = 4$ s after it is released from rest. The radius of gyration about the mass center is $k_G = 0.25$ m.



Solution

The IC of the spool is located in the figrue thus,

$v_G = \omega r_{G/IC} = 0.2\omega$

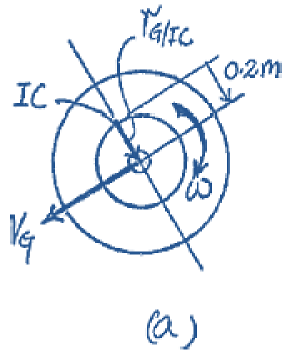
The mass moment of inertia of the spool about its mass centre is,

$I_O = mk_O^2 = (100)(0.25)^2 = 6.25\ \text{kg.m}^2$

Since the spool is required to slip at the point of contact:

$F_f = \mu_k N = 0.1N$

Referring to FBD in Fig. (b)

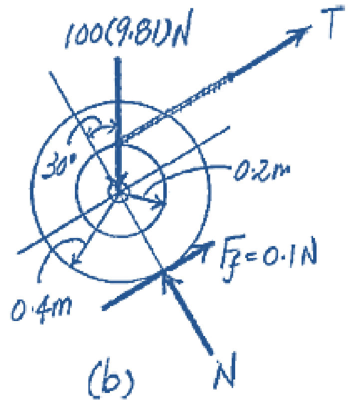


$(+\ \nwarrow)$

$$m[(v_G)_y]_1 + \sum\int_{t_1}^{t_2} F_y dt = m[(v_G)_y]_2$$

$0 + N(4) - 100(9.81)\cos 30°(4) = 0$

$N = 849.57$ N



$(+\ \nearrow)$

$$m[(v_G)_x]_1 + \sum\int_{t_1}^{t_2} F_x dt = m[(v_G)_x]_2$$

$0 + T(4) + (0.1)(849.57)(4) - 100(9.81)\sin 30°(4) = 100[-\omega(0.2)]$

$T + 5\omega = 405.54$ --------------------------------(1)

Subsequently,

$(\circlearrowleft\ +)$

$$I_G\omega_1 + \sum\int M_G dt = I_G\omega_2$$

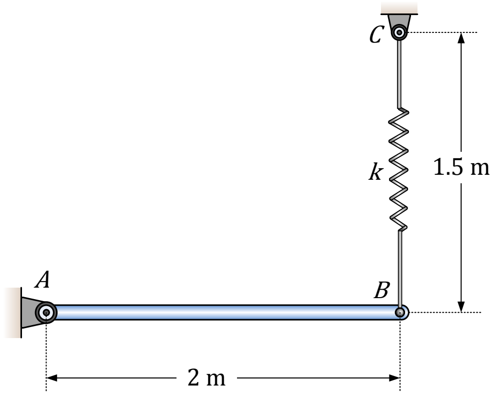
$0 + 0.1(849.57)(0.4)(4) - T(0.2)(4) = -6.25\omega_2$ --------------------------------(2)

Solving (1) and (2)

$T = 313.59$ N

$\omega = 18.4$ rad/s (Answer)

Question 12.11.



The slender 6 kg bar AB is horizontal and at rest and the spring is unstretched. Determine the stiffness k of the spring so that the motion of the bar is momentarily stopped when it has rotated clockwise 90° after being released.

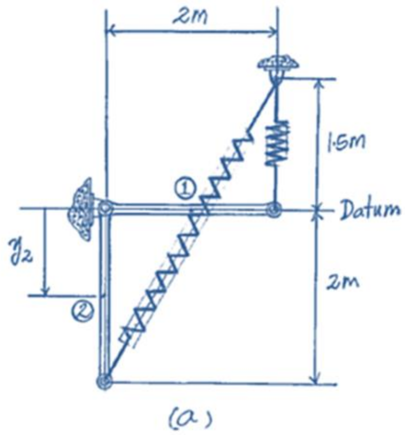
Solution

$$I_A = \frac{ml^2}{12} + md^2 = \frac{(6)(2)^2}{12} + 6(1)^2 = 8\ \mathrm{kg.m^2}$$

$$\Delta T = 0$$

$$W_{1-2} = 0 \quad \text{(i.e. no external force)}$$

$$\Delta V_g = 6(9.81)(-1) = -58.86\ \mathrm{J}$$



$$x_1 = 0$$

$$L_2 = \sqrt{3.5^2 + 2^2} = 4.0311\ \mathrm{m}$$

therefore

$$x_2 = 4.0311 - 1.5 = 2.5311\ \mathrm{m}$$

Thus:

$$\Delta V_e = \frac{1}{2}k\,(x_2^2 - x_1^2) = \frac{1}{2}(k)(2.5311^2 - 0^2) = 3.2k\ \mathrm{J}$$

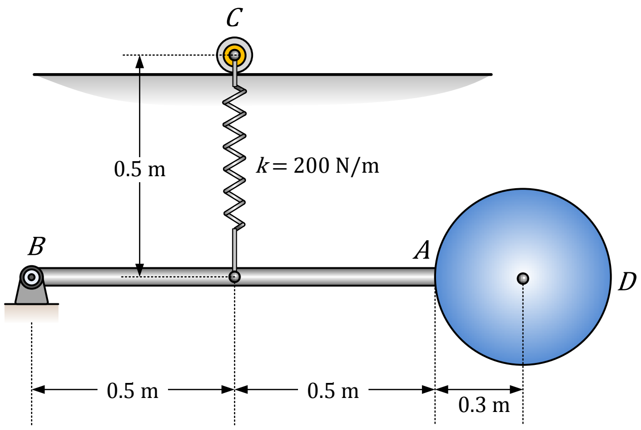
$$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

$$0 = 0 - 58.86 + 3.2k$$

$k = 18.4\ \mathrm{N/m}$ (Answer)

Question 12.12.

The pendulum consists of a 6 kg slender rod fixed to a 15 kg disk. If the spring has an unstretched length of 0.2 m, determine the angular velocity of the pendulum when it is released from rest and rotates clockwise 90° from the position shown. The roller at C allows the spring to always remain vertical.



Solution

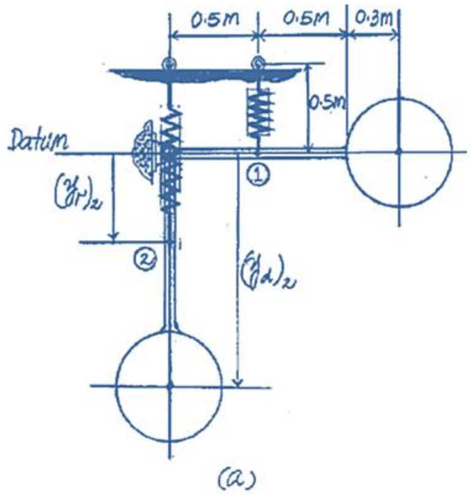
$W_{1-2} = 0$

The mass moment of inertia of the spool about B is,

$$I_B = \left[\frac{1}{12}(6)(1)^2 + (6)(0.5)^2\right] + \left[\frac{1}{2}(15)(0.3)^2 + (15)(1.3)^2\right] = 28.025 \text{ kg.m}^2$$

Therefore,

$$\Delta T = \frac{1}{2} I_B \omega^2 = \frac{1}{2}(28.025)\omega^2 = 14.0125\omega^2$$



Change in height of the rod and disc is,

$h_r = -0.5 \text{ m}$ *and* $h_d = -1.3 \text{ m}$

The total change in gravitational potential energy is thus

$$\Delta V_g = 6(9.81)(-0.5) + 15(9.81)(-1.3) = -220.725 \text{ J}$$

$x_1 = 0.5 - 0.2 = 0.3 \text{ m}$

$x_2 = 1 - 0.2 = 0.8 \text{ m}$

$$\Delta V_e = \frac{1}{2} k\,(x_2^2 - x_1^2) = \frac{1}{2}(200)(0.8^2 - 0.3^2) = 55 \text{ J}$$

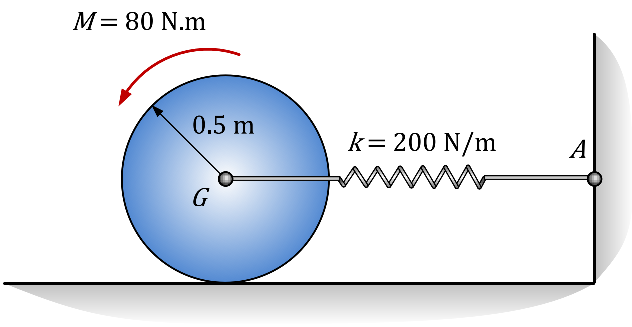
According to work-energy relation

$$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

$$0 = 14.0125\omega^2 - 220.725 + 55$$

$\omega = 3.44 \text{ rad/s}$ (Answer)

Question 12.13.



The 30 kg disk is originally at rest, and the spring is unstretched. A couple moment of $M = 80\ \mathrm{N.m}$ is then applied to the disk as shown. Determine its angular velocity when its mass center G has moved 0.5 m along the plane. The disk rolls without slipping.

Solution

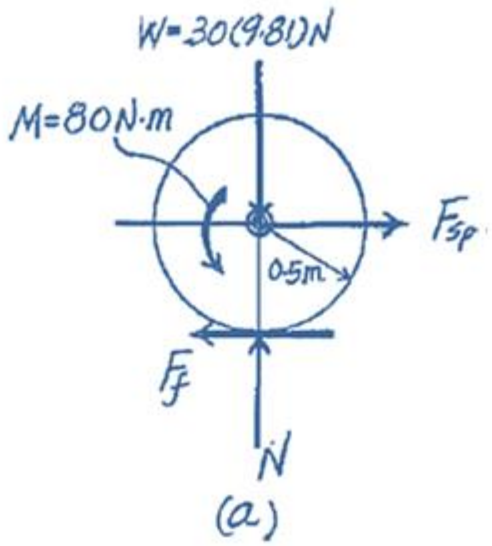
The disc rolls without slipping, thus

$$v_G = \omega r_{G/IC} = 0.5\omega$$

Also $F_f = 0$

The mass moment of inertia of the spool about its mass centre is,

$$I_A = \frac{mr^2}{2} = \frac{(30)(0.5)^2}{2} = 3.75\ \mathrm{kg.m^2}$$



$$\Delta T = \frac{1}{2}m(v_{G2}^2 - v_{G1}^2) + \frac{1}{2}I_G(\omega_2^2 - \omega_1^2)$$

$\omega_1 = 0$ and $v_{G1} = 0$

$$\Delta T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$$

$$\Delta T = \frac{1}{2}(30)(0.5\omega)^2 + \frac{1}{2}(3.75)\omega^2$$

$$\Delta T = 5.625\ \omega^2 \quad \mathrm{J}$$

$$\Delta V_g = 0$$

Angular displacement

$$\theta = \frac{s}{r} = \frac{0.5}{0.5} = 1\ \mathrm{rad}$$

$$W_{1-2} = M\theta = 80(1) = 80\ \mathrm{J}$$

$$\Delta V_e = \frac{1}{2}k\ (x_2^2 - x_1^2) = \frac{1}{2}(200)(0.5^2 - 0^2) = 25\ \mathrm{J}$$

$$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

$$80 = 5.625\omega^2 + 0 + 25$$

$\omega = 3.127\ \mathrm{rad/s}$ (Answer)