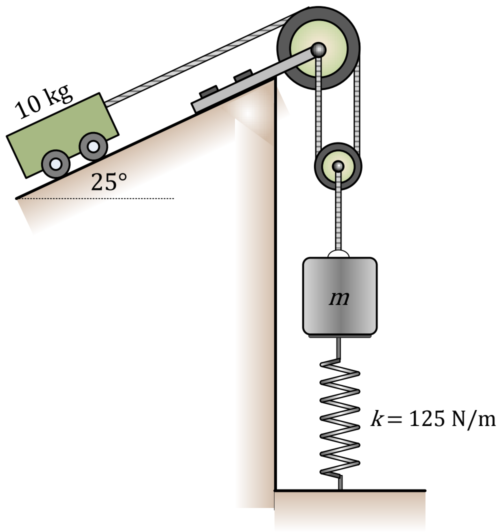
Question 9.8.



The system is released from rest with no slack in the cable and with the spring stretched 200 mm. Determine the distance s travelled by the 10 kg cart before it comes to rest (a) if m approaches zero, and (b) if $m = 2$ kg. Assume no mechanical interferences.

Solution

(a) Let $'s'$ be the slant distance down the incline travelled by the 10 kg cart

$$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

$$W_{1-2} = 0$$

$$\Delta T = \frac{1}{2} m({v_2}^2 - {v_1}^2)$$

$v_1 = 0$ and $v_2 = 0$

therefore

$$\Delta T = 0$$

$$\Delta V_g = mg(h_2 - h_1)$$

$\Delta V_g = -10(9.81)(s)\sin 25°$ *i.e. the change in vertical height for 10 kg cart is* $(-s)\sin 25°$

$$\Delta V_e = \frac{1}{2} k({x_2}^2 - {x_1}^2) = \frac{1}{2} k\left[\left(0.2 + \frac{s}{2}\right)^2 - 0.2^2\right]$$

According to the Work-energy equation

$$0 = -10(9.81)(s)\sin 25° + \frac{1}{2} k\left[\left(0.2 + \frac{s}{2}\right)^2 - 0.2^2\right]$$

$$10(9.81)(s)\sin 25° = \frac{125}{2}\left[\left(0.2 + \frac{s}{2}\right)^2 - 0.2^2\right]$$

Solving which yields,

$s = 1.853$ m (Answer)

(b) Let $'s'$ be the slant distance down the incline travelled by the 10 kg cart

$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$

$W_{1-2} = 0$

$\Delta T = \dfrac{1}{2}m({v_2}^2 - {v_1}^2)$

$v_1 = 0$ and $v_2 = 0$

therefore

$\Delta T = 0$

$\Delta V_g = mg(h_2 - h_1)$

$\Delta V_g = -10(9.81)(s)\sin 25° + 2(9.81)\left(\dfrac{s}{2}\right)$

$\Delta V_e = \dfrac{1}{2}k({x_2}^2 - {x_1}^2) = \dfrac{1}{2}k\ \left[\left(0.2 + \dfrac{s}{2}\right)^2 - 0.2^2\right]$

According to the Work-energy equation

$$0 = -10(9.81)(s)\sin 25° + 2(9.81)\left(\frac{s}{2}\right)\ + \frac{1}{2}k\ \left[\left(0.2 + \frac{s}{2}\right)^2 - 0.2^2\right]$$

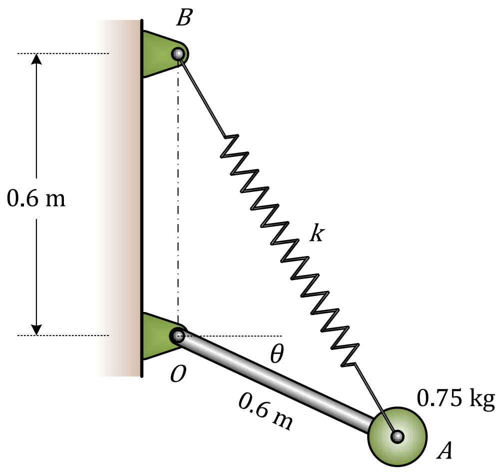
$$10(9.81)(s)\sin 25° - 2(9.81)\left(\frac{s}{2}\right)\ = \frac{125}{2}\ \left[\left(0.2 + \frac{s}{2}\right)^2 - 0.2^2\right]$$

Solving which yields,

$s = 1.226$ m (Answer)

Question 9.9.

The 0.75 kg particle is attached to the light slender rod OA which pivots freely about a horizontal axis through point O. The system is released from rest while in the position $\theta = 0°$ where the spring is unstretched. If the particle is observed to stop momentarily in the position $\theta = 50°$, determine the spring constant k. For your computed value of k, what is the particle speed v at the position $\theta = 25°$?



Solution

Initial length of the spring is given by

$$L_o = \sqrt{0.6^2 + 0.6^2} = 0.8485 \text{ m}$$

At initial position the spring is unstretched i.e. $x_1 = 0$

At $\theta = 50°$

$$L_2{}^2 = 0.6^2 + +0.6^2 - 2(0.6)(0.6)\cos(90° + 50°)$$

$$L_2 = 1.1276 \text{ m}$$

$$x_2 = L_2 - L_o = 1.1275 - 0.8485 = 0.2791 \text{ m}$$

$$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

$W_{1-2} = 0$ (i.e. no external force)

$$\Delta T = \frac{1}{2}m(v_2{}^2 - v_1{}^2)$$

$v_1 = 0$ and $v_2 = 0$

therefore

$$\Delta T = 0$$

$$\Delta V_g = mg(h_2 - h_1)$$

$$\Delta V_g = -0.75(9.81)(0.6\sin 50°)$$

$$\Delta V_e = \frac{1}{2}k(x_2{}^2 - x_1{}^2) = \frac{1}{2}k\,(0.2791^2 - 0^2)$$

According to the Work-energy equation

$$0 = 0 - 0.75(9.81)(0.6\sin 50°) + \frac{1}{2}k\,(0.2791^2 - 0^2)$$

Solving which yields,

$k = 86.862 \text{ N/m}$ (Answer)

When $\theta = 25°$

Initial length of the spring is given by

$L_o = \sqrt{0.6^2 + 0.6^2} = 0.8485 \text{ m}$

At initial position the spring is unstretched i.e. $x_1 = 0$

At $\theta = 50°$

$L_2{}^2 = 0.6^2 + +0.6^2 - 2(0.6)(0.6)\cos(90° + 25°)$

$L_2 = 1.012 \text{ m}$

$x_2 = L_2 - L_o = 1.012 - 0.8485 = 0.1635 \text{ m}$

$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$

$W_{1-2} = 0$ (i.e. no external force)

$\Delta T = \frac{1}{2}m(v_2{}^2 - v_1{}^2)$

$v_1 = 0$

therefore

$\Delta T = \frac{1}{2}mv^2$

$\Delta V_g = mg(h_2 - h_1)$

$\Delta V_g = -0.75(9.81)(0.6\sin 25°)$

$\Delta V_e = \frac{1}{2}k(x_2{}^2 - x_1{}^2) = \frac{1}{2}(86.862)\ (0.1635^2 - 0^2)$

According to the Work-energy equation

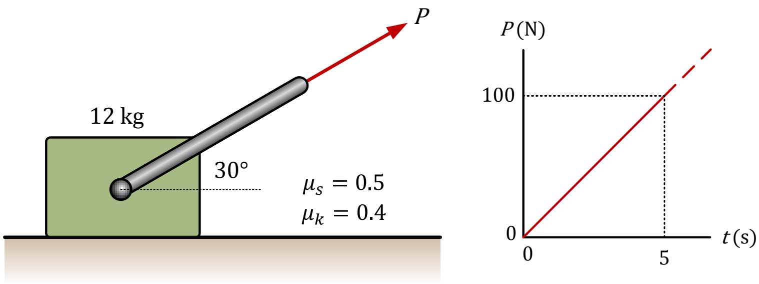
$0 = \frac{1}{2}mv^2 - 0.75(9.81)(0.6\sin 25°) + \frac{1}{2}(86.862)\ (0.1635^2 - 0^2)$

Solving which yields,

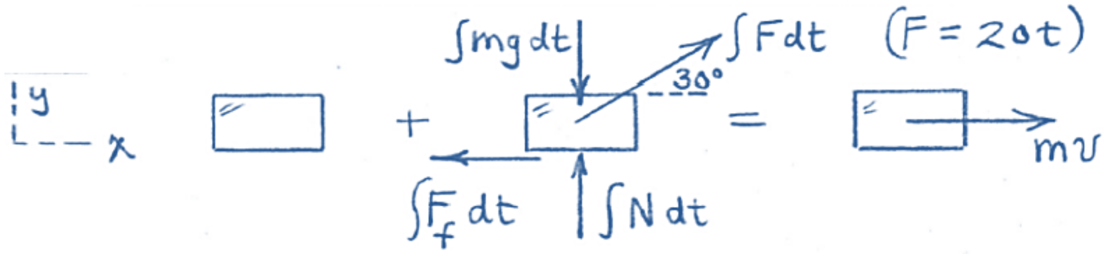
$v = 1.3713 \text{ m/s}$ (Answer)

Question 9.10.

The initially stationary 12 kg block is subjected to the time-varying force whose magnitude P is shown in the plot. The 30° angle remains constant. Determine the block speed at (a) $t = 1$ s and (b) $t = 4$ s.



Solution



The block begins to move when:

$\sum F_x = 0$ i.e. $20t \cos 30° - 0.5N = 0$ ----------- (1)

$\sum F_y = 0$ i.e. $N - 12(9.81) + 20t \sin 30° = 0$ ----------- (2)

$N = 117.7 - 10t$ ----------- (3)

Solving (1) and (3) yields,

$t = 2.64$ s (start time)

(a) Speed at $t = 1$ s is $v_1 = 0$ (Answer)

(b) Speed at $t = 4$ s

$$\int_{t_1}^{t_2} \sum F\, dt = mv_4 - mv_1$$

$$\int_{2.64}^{4} [-N\mu_k + F\cos 30°]\, dt = mv_4$$

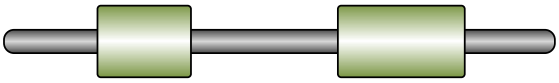
$$\int_{2.64}^{4} [-(117.7 - 10t)(0.4) + 20t\cos 30°]\, dt = 12v_4$$

Solving which yields,

$v_4 = 2.69$ m/s (Answer)

Question 9.11.

Compute the final velocities v_1' *and* v_2' after collision of the two cylinders which slide on the smooth horizontal shaft. The coefficient of restitution is $e = 0.6$.



Solution

Linear momentum of the system:

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$(2)(7) + (3)(-5) = 2v_1' + 3v_2' \qquad \text{-----------} \quad (1)$$

Coefficient of restitution:

$$e = \frac{v_2' - v_1'}{v_2 - v_1}$$

$$0.6 = \frac{v_2' - v_1'}{7 - (-5)}$$

$$v_2' - v_1' = 7.5 \qquad \text{-----------} \quad (2)$$

Solving (1) and (2) yields,

$v_1' = -4.52$ m/s (Answer)

$v_2' = 2.68$ m/s (Answer)