MMAN1300 Engineering Mechanics 1

2018 Personalised Study Pack

	Surname	First Name
Student Name	Nguyen	Dan
Student ID	z5206032	

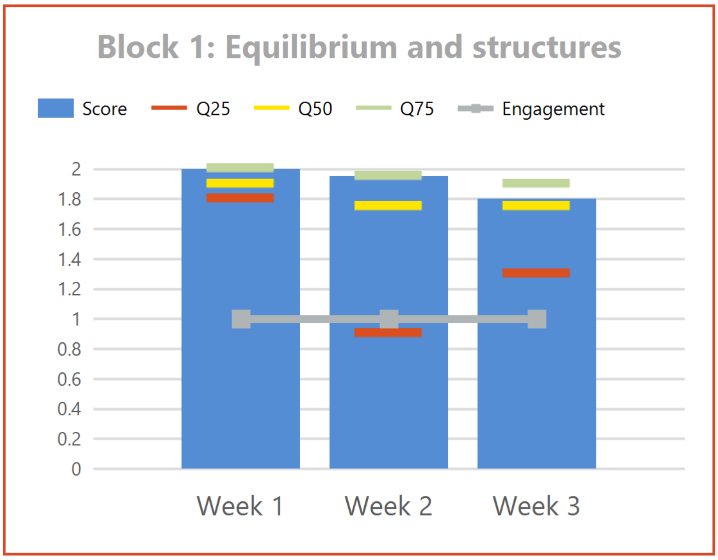
Table of Contents

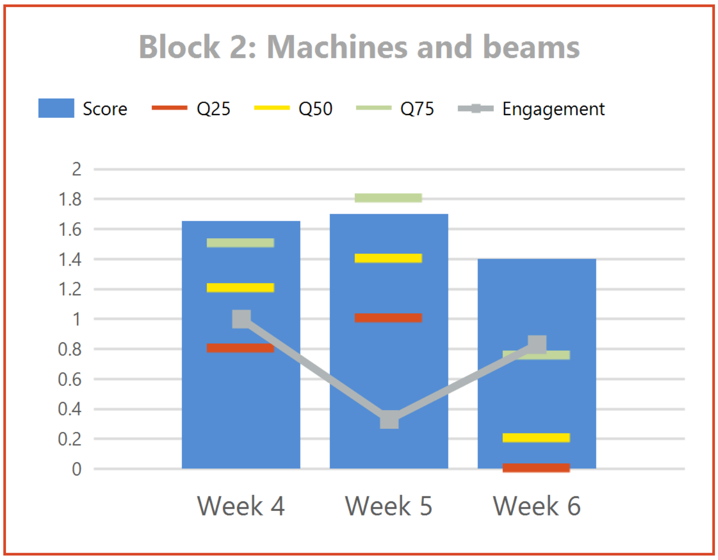
Student Dashboard........ 3
Week 1: Introduction, Vectors and Forces........ 5
Week 2: Free Body Diagrams, Equilibrium and Equivalent Loads........ 8
Week 3: Trusses........ 11
Week 4: Frames, Machines, Friction and Springs........ 14
Week 5: Shear Force and Bending Moment........ 17
Week 6: Centroids, Centre of mass and Moment of Inertia........ 20
Week 7: The Kinematics of Motion........ 24
Week 8: Curvilinear Kinetics of Particles and Relative Motion........ 28
Week 9: Work-Energy and Impulse Momentum Methods........ 31
Week 10: Kinematics of Rigid Bodies........ 34
Week 11: Rigid Body Kinetics........ 37
Week 12: Rigid Body Momentum and Work Energy Method........ 40
Equations........ 44

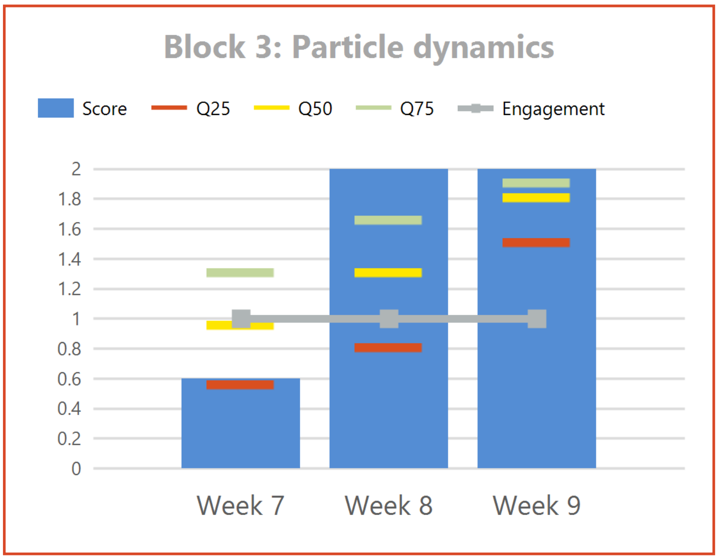
Student Dashboard

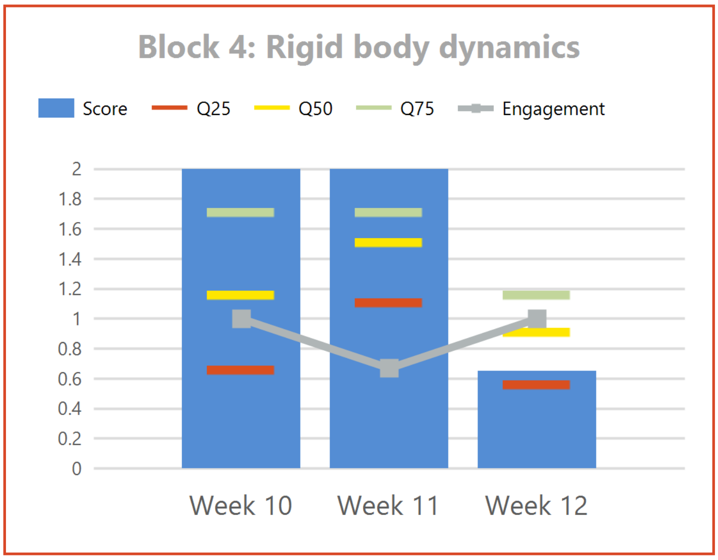
UNSW SYDNEY

MMAN1300
Engineering Mechanics 1









LAB	Score	Class average
LAB2	4.90	5
LAB1	5.64	4.97

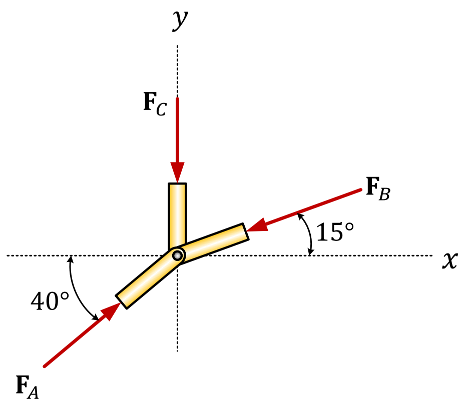
Type	Score
Block Tests (out of 24)	19.75
Labs (out of 12)	10.54
Moodle Quizzes (out of 12)	10.75

Topic	MQ	PSS	ATTD
W1: Vectors & forces	1.00	1.0	0
W2: FBDs & equilibrium	1.00	1.0	0
W3: Structures & trusses	1.00	1.0	0
W4: Frames & machines	1.00	1.0	0
W5: Beams & SF-BM diagrams	0.00	1.0	0
W6: Centroids & MOIs	0.75	1.0	0
W7: Kinematics of motion	1.00	1.0	0
W8: Particle kinetics	1.00	1.0	0
W9: Particle energy & momentum	1.00	1.0	0
W10: Rigid body kinematics	1.00	1.0	0
W11: Rigid body kinetics	1.00	0.0	0
W12: Rigid body momentum & energy	1.00	1.0	0

Week 1: Introduction, Vectors and Forces

Question 1: Vectors and Forces

The figure shows three forces acting on a joint of a structure. The magnitude of $F_c = 60$ kN, and $F_A + F_B + F_C = 0$. What are the magnitudes of F_A and F_B?

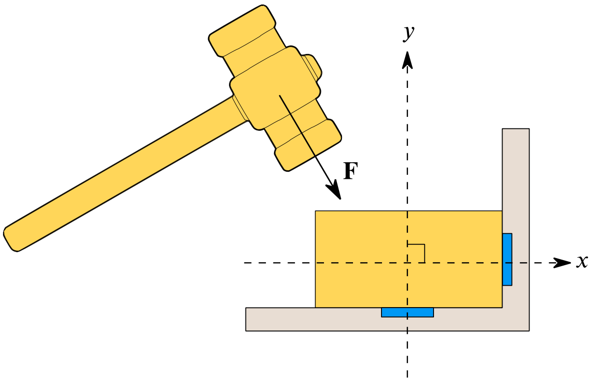


Solution:

Answer: $F_A = 137$ kN and $F_B = 109$ kN

Question 1: Vectors and Forces

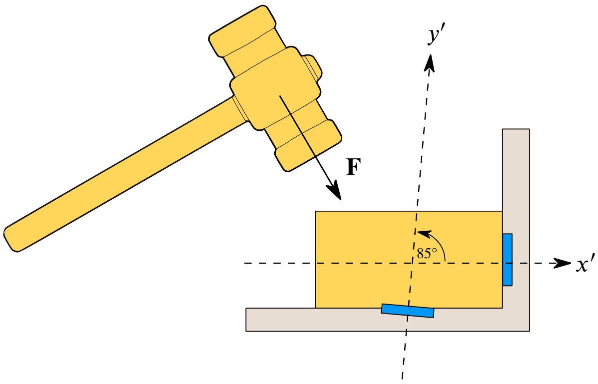
You and your friend decide to have a competition. An apparatus was set up by your friend as shown and you and your friend hit the rigid block with a hammer applying a force which is measured using load cells placed under the block.



The winner of this competition is the one who hits the block the hardest. The following measurements were taken:

$$\mathbf{F}_1 = 625\mathbf{i} - 1150\mathbf{j}$$
$$\mathbf{F}_2 = 150\mathbf{i} - 1275\mathbf{j}$$

With your friend producing $\mathbf{F}_1$ and you producing $\mathbf{F}_2$. Your friend proudly boasts about their victory (check this).

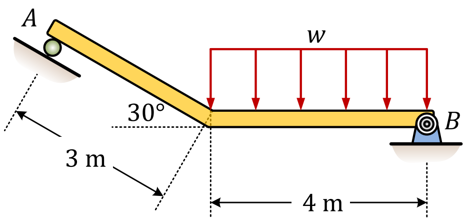


However, upon rechecking the apparatus, you notice that the bottom load cell was not perpendicular to right load cell and was tilted at an angle of 5°. Who really won the competition?

Week 2: Free Body Diagrams, Equilibrium and Equivalent Loads

Question 2: FBDs and Equilibrium

If the roller at A and the pin at B can support a load up to 4 kN and 8 kN, respectively, determine the maximum intensity of the distributed load w, measured in kN/m, so that failure of the supports does not occur.

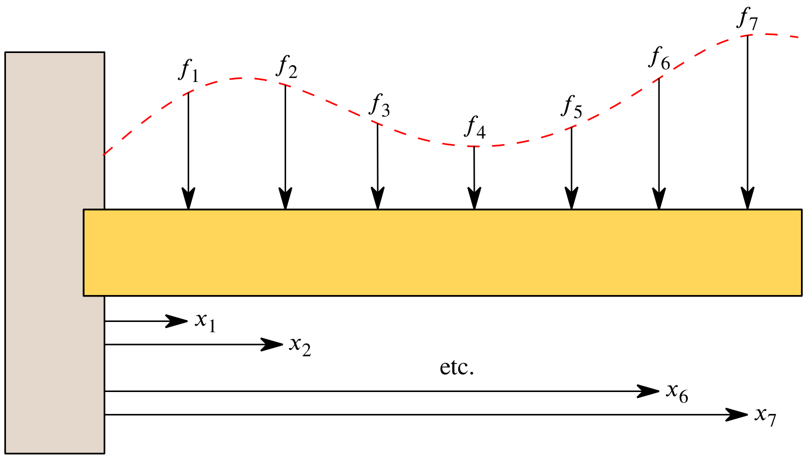


Solution:

Answer: $w = 2.67$ kN/m

Question 2: FBDs and Equilibrium

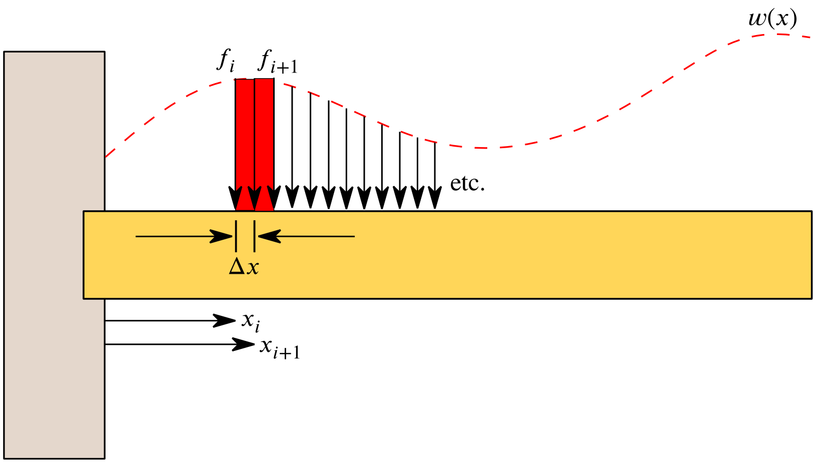
Consider the beam loaded as shown in the figure. Find the equivalent point load magnitude and position along the beam.



Consider the case that there were a large number of forces f_i placed at positions x_i along the beam, with:

$$f_i = w(x_i)\,\Delta x$$
$$\Delta x = x_{i+1} - x_i$$

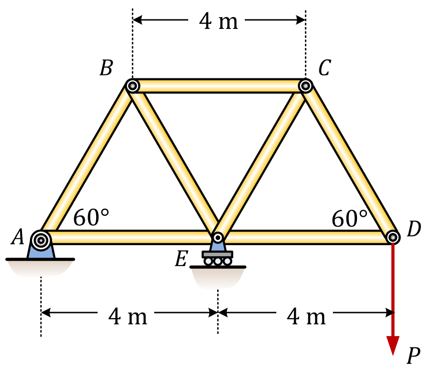
What is the equivalent load magnitude and position as $\Delta x \to 0$?



Week 3: Trusses

Question 3: Structures and Trusses

If the maximum force that any member can support is 8 kN in tension and 6 kN in compression, determine the maximum force P that can be supported at joint D.

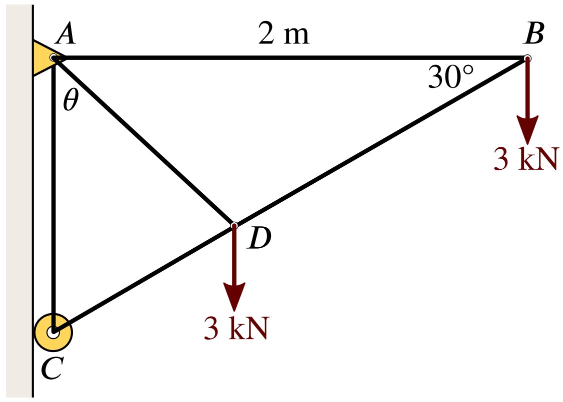


Solution:

Answer: $P = 5.2$ kN (controls)

Question 3: Structures and Trusses

The truss design below can be optimised by choosing an angle θ that minimises the forces in the members.

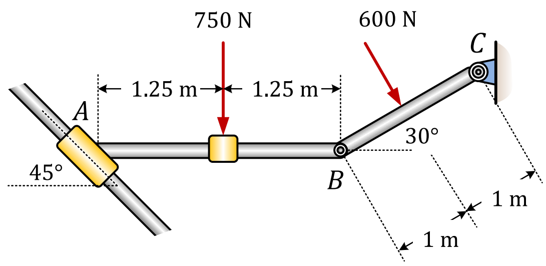


If we want to optimise member AD to have the minimum force magnitude, what angle θ should be chosen? What if we want to minimise the forces in the other members? Do these solutions make sense physically?

Week 4: Frames, Machines, Friction and Springs

Question 4: Frames and Machines

Determine the reactions on the collar at A and the pin at C. The collar fits over a smooth rod, and rod AB is fixed connected to the collar.

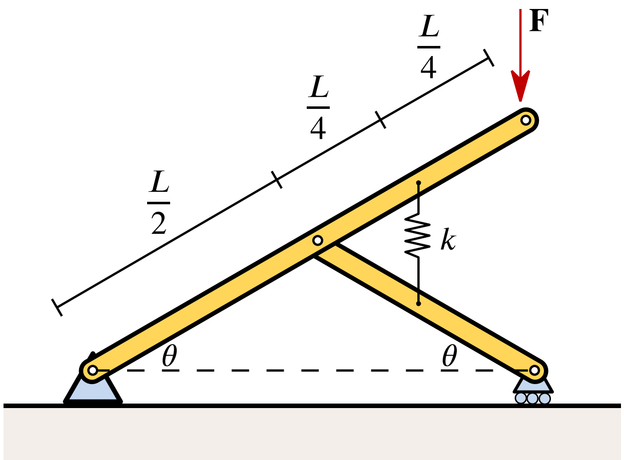


Solution:

Answer: $M_A = 5.55\ \text{kN.m}, C_x = 2.89\ \text{kN}$ and $C_y = 1.32\ \text{kN}$

Question 4: Frames and Machines

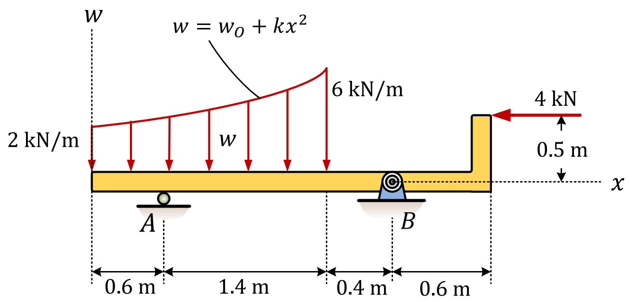
For the system below, determine the equilibrium condition for an arbitrary angle θ. What is the force $\mathbf{F}$ required to make the distance between the pin support and roller support a maximum? Take the unstretched length of the spring as l.



Week 5: Shear Force and Bending Moment

Question 5: Distributed Loads, Shear Force and Bending Moments

Determine the reactions at A and B for the beam subjected to the distributed and concentrated loads as shown.



Solution:

Answer: $A_y = 5.56$ kN. m, $B_x = 4$ kN and $B_y = 1.11$ kN

Question 5: Distributed Loads, Shear Force and Bending Moments

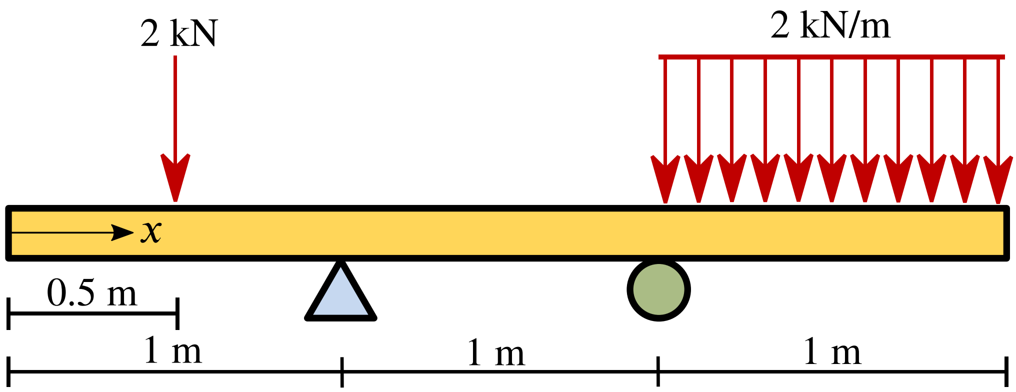
We know the following equations relating bending moment M, shear force V and the load distribution w:

$$V = \frac{dM}{dx}$$
$$q = \frac{dV}{dx}$$

We can directly integrate the load distribution q to obtain the shear force and bending moment functions. However, discontinuities in q give us problems when integrating. A work around is possible by modelling point loads using the delta function $\delta(x)$ and distributed loads using the step function $H(x)$. The functions have the following properties:

$$\int_a^b \delta(x-k) = 1 \text{ if } k \in (a,b)$$
$$\int_{-\infty}^{x} \delta(x-k)\,dx = H(x-k) = \begin{cases} 1 \text{ if } x \geq k \\ 0 \text{ if } x < k \end{cases}$$

Consider the load distribution on a beam shown below.



The expression for $q(x)$ on the beam is (check this):

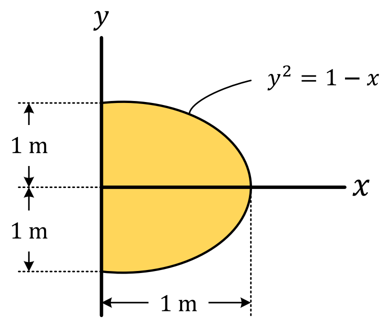
$$q(x) = -2\delta(x-0.5) + 2\delta(x-1) + 2\delta(x-2) + 2H(x-2) - 2H(x-3)$$

Find the shear force $V(x)$ and bending moment $M(x)$ by directly integrating q.

Week 6: Centroids, Centre of mass and Moment of Inertia

Question 6: Centroids and Moment of Inertia

Determine the moment of inertia of the shaded area about the y-axis.

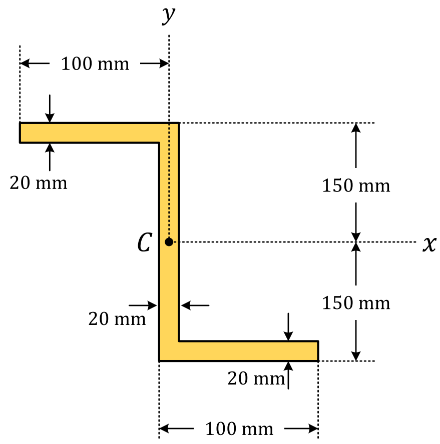


Solution:

Answer: $I_{max} = 113(10^{-6})\ \mathrm{m}^4, I_{min} = 5.03(10^{-6})\ \mathrm{m}^4, \left(\theta_p\right)_1 = 12.3°$ and $\left(\theta_p\right)_2 = -77.7°$

Question 6: Centroids and Moment of Inertia

Determine the orientation of the principal axes, which have their origin at centroid C of the beam's cross-sectional area. Also, find the principal moments of inertia.

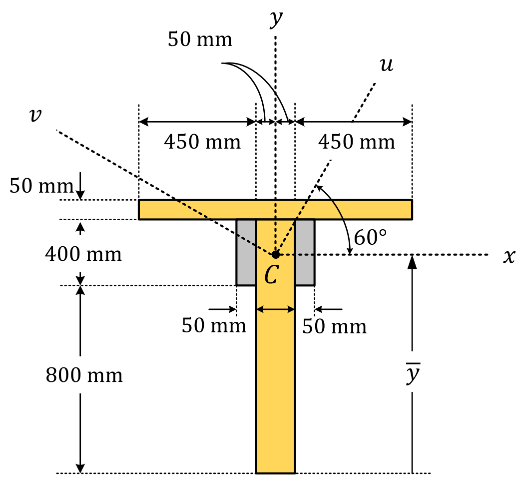


Solution:

Answer: $I_{max} = 113(10^6)\ \text{mm}^4, I_{min} = 5.03(10^6)\ \text{mm}^4, (\theta_p)_1 = 12.3°$ and $(\theta_p)_2 = -77.7°$

Question 6: Centroids and Moment of Inertia

Locate the centroid $\overline{y}$ of the beam's cross-sectional area and then determine the moments of inertia and the product of inertia of this area with respect to the u and v axes. (USE MOHR'S CIRCLE)



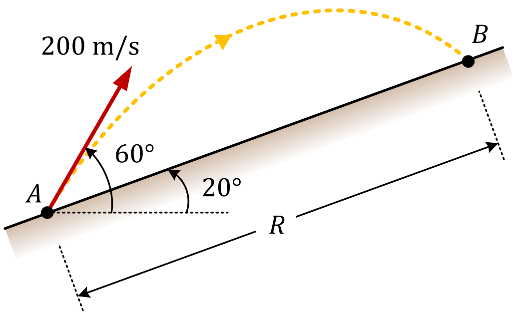
Solution:

Answer: $\overline{y} = 825$ mm, $I_u = 109(10^8)$ mm^4, $I_v = 238(10^8)$ mm^4, $I_{uv} = 111(10^8)$ mm^4

Week 7: The Kinematics of Motion

Question 7: Particle Kinematics

A projectile is launched with an initial speed of 200 m/s at an angle of $\theta = 60°$ with respect to the horizontal. Compute the range R as measured up the incline.



Solution

(a) Calculate the horizontal component of initial velocity

(b) Calculate the vertical component of initial velocity

(c) Write down the displacement equation in x- direction - Eq.(1)

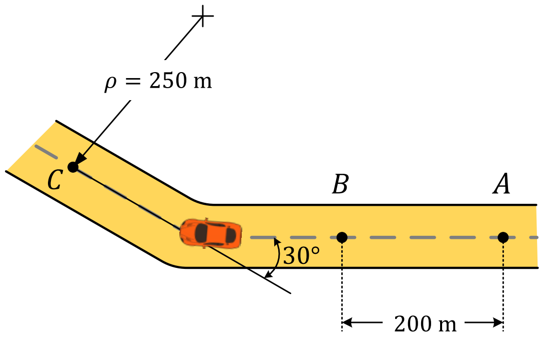
(d) Write down the displacement equation in y- direction - Eq.(2)

(e) Substitute value of t from (1) in (2) to obtain the value of R:

Answer: $R = 2970$ m

Question 7: Particle Kinematics

When the cars reaches point A, it has a speed of 25 m/s. If the brakes are applied, its speed is reduced by $a_t = (0.001s - 1)\ \mathrm{m/s^2}$. Determine the magnitude of acceleration of the car just before it reaches point C.

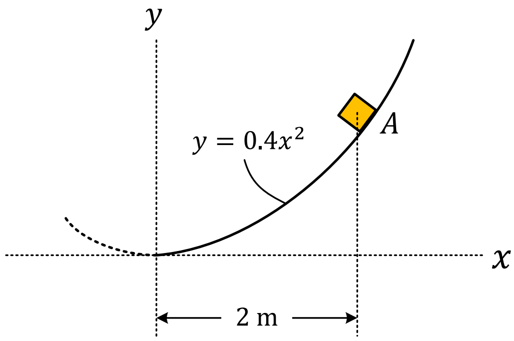


Solution:

Answer: $a = 0.730\ \mathrm{m/s^2}$

Question 7: Particle Kinematics

The box of negligible size is sliding down along a curved path defined by the parabola $y = 0.4x^2$. When it is at A ($x_A = 2$ m and $y_A = 1.6$ m), the speed is $v = 8\ \mathrm{m/s}$ and the increase in speed is $dv/dt = 4\ \mathrm{m/s^2}$. Determine the magnitude of the acceleration of the box at this instant.



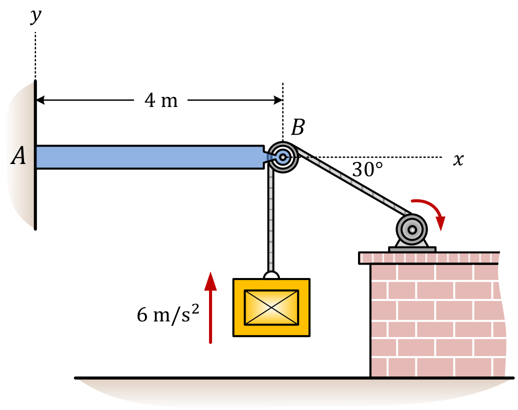
Solution:

Answer: $a = 8.61\ \mathrm{m/s^2}$

Week 8: Curvilinear Kinetics of Particles and Relative Motion

Question 8: Particle Kinetics

The motor lifts the 50 kg crate with an acceleration of $6\ \mathrm{m/s^2}$. Determine the components of force reaction and the couple moment at the fixed support A.



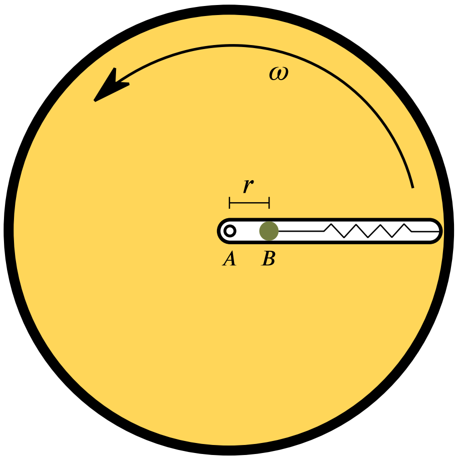
Solution:

Answer: $M_A = 4.74\ \mathrm{kN.m}$, $A_x = 0.685\ \mathrm{kN}$ and $A_y = 1.19\ \mathrm{kN}$

Question 8: Particle Kinetics

The slotted disk below rotates about its centre, point A in the horizontal plane at an angular velocity of $\omega = 10$ rad/s. A ball B with mass 1 kg moves in the slot and is attached to a spring of stiffness k and is undeformed when $r = 0$.

If the radial velocity of the ball is zero when it is released in the position $r = 0.5$ m, where will the ball be at $t = 0.1$ s? Take $k = 100$ N/m and repeat for $k = 200$ N/m. Also find the spring force $\mathbf{F}_s$ and normal force $\mathbf{F}_n$. You may assume the slot and ball are smooth.



Week 9: Work-Energy and Impulse Momentum Methods

Question 9: Work-Energy Methods for Particles

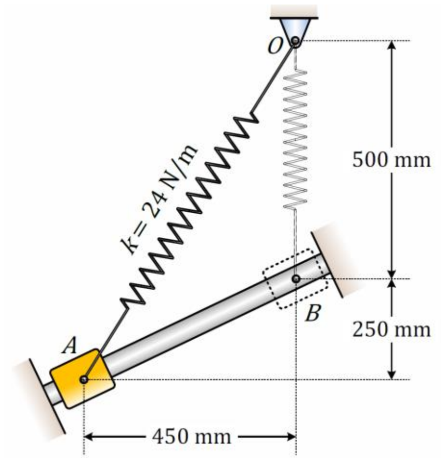
The 20 g bullet is traveling at 400 m/s when it becomes embedded in the 2 kg stationary block. Determine the distance the block will slide before it stops. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$.

Solution:

Answer: $s = 4$ m

Question 9: Work-Energy Methods for Particles

The 0.9kg collar is released from rest at A and slides freely up the inclined rod, striking the stop at B with a speed v. The spring has a stiffness $k = 24$ N/m and has an unstretched length of 375 mm. Previously you have shown that $v = 1.159$ m/s.



After building the above apparatus and testing you find that the collar hits B at a speed greater than 1.159 m/s. You suspect the spring force does not exactly follow Hooke's law which states that the spring force is $F_s = -kx$.

To begin, you analyse the effect of replacing the spring force F_s with another function.

The relationship between a potential energy function and the associated force is given by:

$$F = -\frac{dU}{dx}$$

Obtain the linear spring potential energy function by integrating the force $F_s = -kx$.

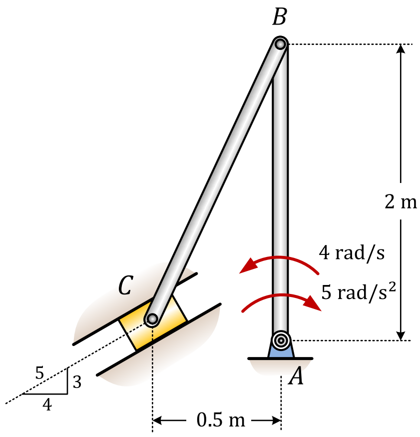
You decide that since the collar hit B with a greater speed the spring must be producing more force than anticipated. You decide to replace the spring force with $F_s = -k(x + x^3)$. What speed will the collar hit the stop at B, using the new spring potential? Use the same value of k as above.

Note that the function chosen is an odd function, would it make sense to choose an even function such as $F_s = -kx^2$?

Week 10: Kinematics of Rigid Bodies

Question 10: Kinematics of Rigid Bodies

Member AB has the angular motions shown. Determine the velocity and acceleration of the slider block C at this instant.

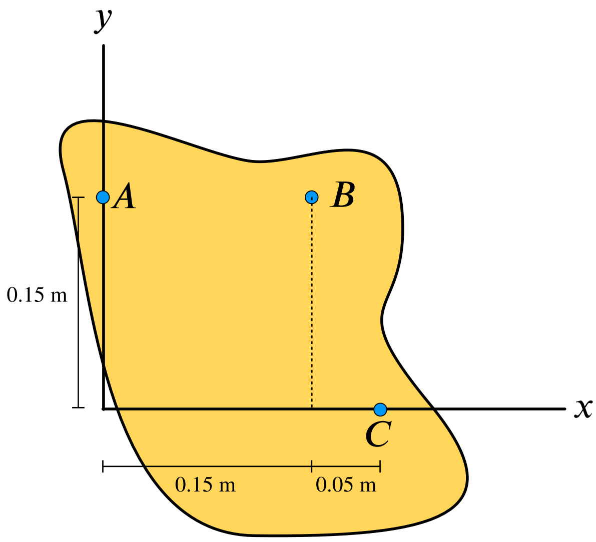


Solution:

Answer: $v_C = 2.5 \text{ m/s}$ and $a_C = 13 \text{ m/s}^2$

Question 10: Kinematics of Rigid Bodies

A measurement system is used to track the motion of a rigid body. The system can measure speed along one direction, the x direction.



The system measures the x component of velocities at A, B and C as:

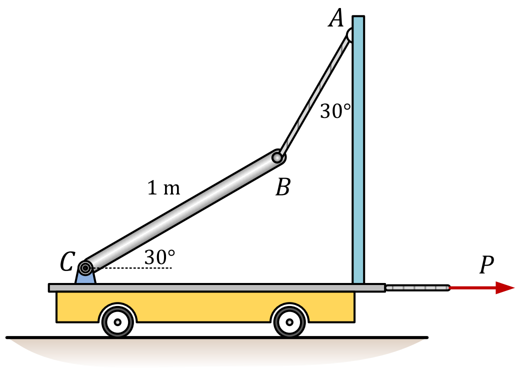
$$v_{A,x} = 0.25 \text{ m/s}$$
$$v_{B,x} = -0.45 \text{ m/s}$$
$$v_{C,x} = -0.5 \text{ m/s}$$

Is it possible to find the angular velocity of the rigid body and the velocities of points A, B and C? If so, find the above quantities.

Week 11: Rigid Body Kinetics

Question 11: Kinetics of Rigid Bodies

If the cart's mass is 30 kg and it is subjected to a horizontal force of $P = 90$ N, determine the tension in cord AB and the horizontal and vertical components of reaction on end C of the uniform 15 kg rod BC.



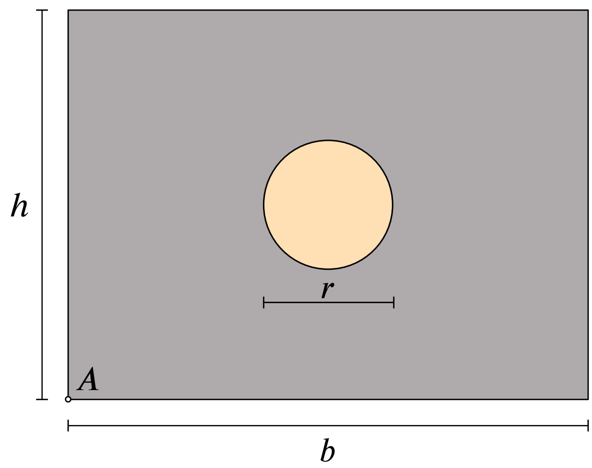
Solution:

Answer: $F_{AB} = 112$ N, $C_x = 26.2$ N and $C_y = 49.8$ N

Question 11: Kinetics of Rigid Bodies

A composite plate is constructed by taking a rectangular sheet of steel (density ρ_s) and cutting a circle out of the centre of the plate. A brass (density ρ_b) circle is then pressed into the centre of the rectangular sheet.

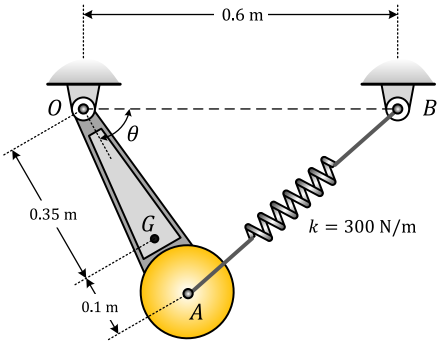
Take the dimensions of the rectangle and circle as shown, you may assume that the steel and brass components are the same thickness and the component is thin. Calculate the moment of inertia of the plate about the point A.



Week 12: Rigid Body Momentum and Work Energy Method

Question 12: Work-Energy Methods for Rigid Bodies

The 30 kg pendulum has its mass centre at G and a radius of gyration about point G of $\bar{k}_G = 300$ mm. If it is released from rest when $\theta = 0°$, determine its angular velocity at the instant $\theta = 90°$. Spring AB has a stiffness of $k = 300$ N/m and is unstretched when $\theta = 0°$.



Solution

(a) Calculate the stretched length in position 1 ($\theta = 0°$)

(b) Calculate the stretch of the spring in position 1

(c) Calculate the stretched length in position 2 ($\theta = 90°$)

(d) Calculate the stretch of the spring in final position

(e) Calculate the work done

(f) Calculate the change in gravitational potential energy

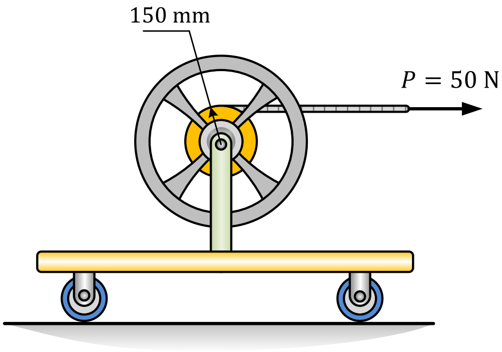
(g) Calculate the change in kinetic energy

(h) Calculate the change in elastic potential energy

(i) Assemble all terms from (a) – (h) and substitute all known values in Eq. (1) to and solve for the angular velocity ω

Answer: $\omega = 3.921$ rad/s

Question 12: Work-Energy Methods for Rigid Bodies

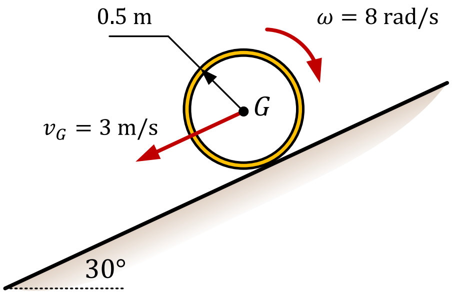


The 30 kg reel is mounted on the 20 kg cart. If the cable wrapped around the inner hub of the reel is subjected to a force of $P = 50$ N, determine the velocity of the cart and the angular velocity of the reel when $t = 4$ s. The radius of gyration of the reel about its centre of mass O is $\overline{k}_O = 250$ mm. Neglect the size of the small wheels.

Solution:

Answer: $\omega = 16$ rad/s and $v = 4$ m/s

Question 12: Work-Energy Methods for Rigid Bodies



The hoop (thin ring) has a mass of 5 kg and is released down the inclined plane such that it has a backspin $\omega = 8$ rad/s and its centre has a velocity $v_G = 3$ m/s as shown. If the coefficient of kinetic friction between the hoop and the plane is $\mu_k = 0.6$ determine how long the hoop rolls before it stops slipping.

Solution:

Answer: $t = 1.32$ s

Equations

Equation Sheet

Linear motion

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt} \qquad vdv = ads$$

Constant linear acceleration equations $(t_o = 0)$

$$v = v_o + at \qquad v^2 = v_o^2 + 2a(s - s_o) \qquad s = s_o + v_o t + \frac{1}{2}at^2$$

Angular motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} \qquad \omega d\omega = \alpha d\theta$$

Displacement, velocity and acceleration components

Rectangular coordinates

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} \qquad \mathbf{v} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} \qquad \mathbf{a} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$

Normal and tangential coordinates

$$\mathbf{v} = v\mathbf{e_t} \qquad \mathbf{a} = a_t\mathbf{e_t} + a_n\mathbf{e_n} \qquad v = \omega r \qquad a_t = \dot{v} = \alpha r \qquad a_n = \frac{v^2}{\rho} = \omega^2 r$$

Relative motion

$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B} \qquad \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} \qquad \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

Equation of motion (Newton's 2nd law)

$$\sum \mathbf{F} = m\mathbf{a}$$

Work-Energy

$$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e \qquad W_{1-2} = F\Delta s \text{ and/or } M\Delta\theta$$

$$\Delta T = \frac{1}{2}m\left(v_2^2 - v_1^2\right) \text{ and/or } \frac{1}{2}I\left(\omega_2^2 - \omega_1^2\right)$$

$$\Delta V_g = mg(h_2 - h_1)$$

$$\Delta V_e = \frac{1}{2}k\left(x_2^2 - x_1^2\right) \quad \text{for a linear spring}$$

For a rigid body in plane motion

$$\sum \mathbf{F} = m\mathbf{a} \qquad \sum M = I\alpha$$

Mass moment of inertia $I = \int r^2 dm$

Centroid of a cross-section:

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\sum_i x_i A_i}{\sum_i A_i} \quad , \quad \bar{y} = \frac{\int y\,dA}{\int dA} = \frac{\sum_i y_i A_i}{\sum_i A_i} \quad \mathrm{I} = \frac{1}{12}\mathrm{m}(\mathrm{w}^2 + \mathrm{h}^2)$$

DATA: Acceleration in free fall due to gravity g = 9.81 m/s^2

Quadratic formula:

For: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$