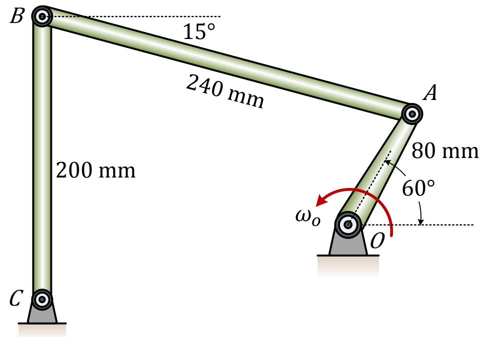
Question 10.6.

A four-bar linkage is shown in the figure (the ground "link" OC is considered the fourth bar). If the drive link OA has a counter-clockwise angular velocity $\omega_o = 10\ \text{rad/s}$, determine the angular velocities of links AB and BC.



Solution

LINK OA

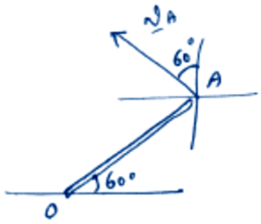
$$v_A = \cancel{v_O}^{\,0} + v_{A/O}$$

$$v_A = v_{A/O} = \omega_O \overline{AO}$$

$$= (10)(0.08)$$

$$= 0.8\ \text{m/s}.$$

Direction is perpendicular to OA



LINK BC

$$v_B = \cancel{v_C}^{\,0} + v_{B/C}$$

$$v_B = v_{B/C} = \omega_{BC} \overline{BC}$$

$$= \omega_{BC}(0.2) \quad \text{— (1)}$$

Direction is perpendicular to BC

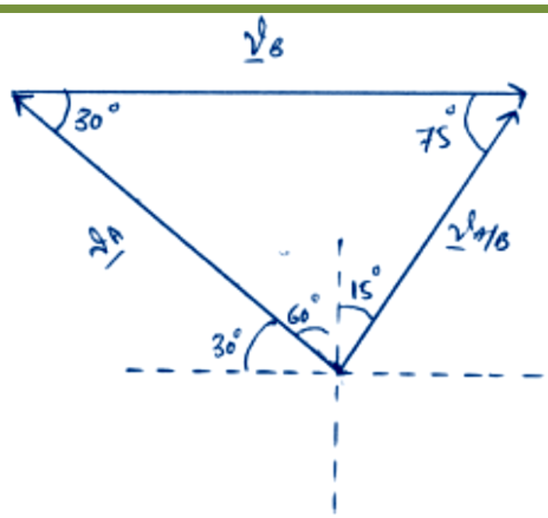
$\longrightarrow$

v_B

LINK AB

$$v_A = v_B + v_{A/B}$$

Construct the velocity triangle for above velocities.



Apply law of sine :

$$\frac{v_A}{\cancel{\sin 75°}} = \frac{v_B}{\cancel{\sin 75°}}$$

$$v_A = v_B = 0.8 \ m/s.$$

from (1)

$$\omega_{BC} = \frac{0.8}{0.2} = 4 \ rad/s \ (ccw)$$

ANSWER

Also,

$$\frac{v_A}{\sin 75°} = \frac{v_{A/B}}{\sin 30°}$$

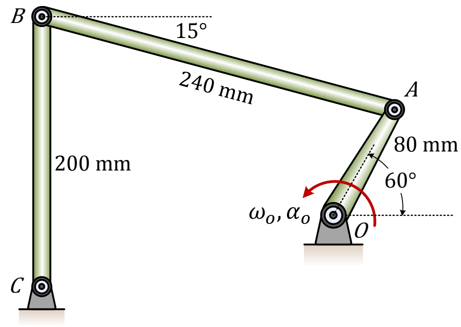
$$v_{A/B} = 0.414 \ m/s$$

$$\because \quad \omega_{AB} = \frac{v_{A/B}}{AB}$$

$$\omega_{AB} = \frac{0.414}{0.24} = 1.725 \ rad/s \ (ccw)$$

ANSWER

Question 10.7.



The four-bar linkage of Question 5 is repeated here. If the angular velocity and angular acceleration of drive link OA are $\omega_o = 10$ rad/s and $\alpha_o = 5$ rad/s^2 respectively, both counter-clockwise, determine the angular accelerations of bars AB and BC for the instant represented.

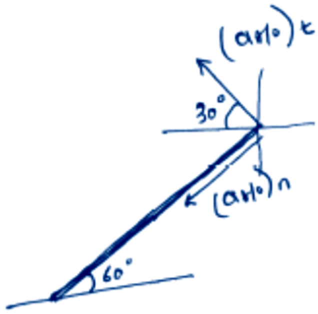
Solution

LINK OA :-

$$\underline{a}_A = \cancel{\underline{a}_o}^{\,0} + \underline{a}_{A/o}$$

$$\underline{a}_A = \underline{a}_{A/o} = (\underline{a}_{A/o})_n + (a_{A/o})_t$$

$$(a_{A/o})_n = \omega_{AO}^2 \, AO$$
$$= (10)^2 (0.08)$$
$$= 8 \text{ m/s}^2$$



$$(\underline{a}_{A/o})_n = -8 \cos 60^\circ \, i - 8 \sin 60^\circ \, j$$
$$= -4 i - 6.93 j$$

$$(a_{A/o})_t = \alpha_{AO} \, \overline{AO} = 0.08 \, \alpha_{AO}$$

$$(\underline{a}_{A/o})_t = -0.08 \, \alpha_{AO} \cos 30^\circ \, \underline{i} + 0.08 \, \alpha_{AO} \sin 30^\circ \, \underline{j}$$
$$= -0.07 \, \alpha_{AO} \, \underline{i} + 0.04 \, \alpha_{AO} \, \underline{j}$$
$$= -0.07 (5) \, \underline{i} + (0.04)(5) \, \underline{j}$$
$$= -0.35 \, \underline{i} + 0.2 \, \underline{j}$$

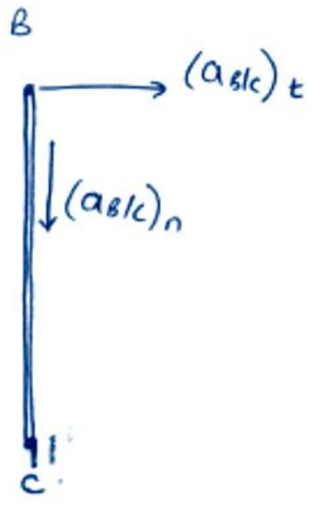
therefore,

$$\underline{a}_A = (-4 \underline{i} - 6.93 \underline{j}) + (-0.35 \underline{i} + 0.2 \underline{j})$$

$$\underline{a}_A = -4.35 \, \underline{i} - 6.73 \, \underline{j}$$

LINK BC :-

$$\underline{a}_B = \cancel{\underline{a}_C}^{\,0} + \underline{a}_{B/C}$$



$$\underline{a}_B = \underline{a}_{B/C} = (a_{B/C})_n + (a_{B/C})_t$$

$$(a_{B/C})_n = \omega_{BC}^2 \, \overline{BC}$$
$$= (4)^2 (0.2)$$
$$= 3.2 \ m/s^2 .$$

$$(\underline{a}_{B/C})_n = -3.2 \, j \quad (m/s^2).$$

$$(a_{B/C})_t = \alpha_{BC} \, \overline{BC}$$
$$= 0.2 \, \alpha_{BC} . \quad (m/s^2)$$

$$(\underline{a}_{B/C})_t = 0.2 \, \alpha_{BC} \, \underline{i} \quad (m/s^2).$$

therefore,

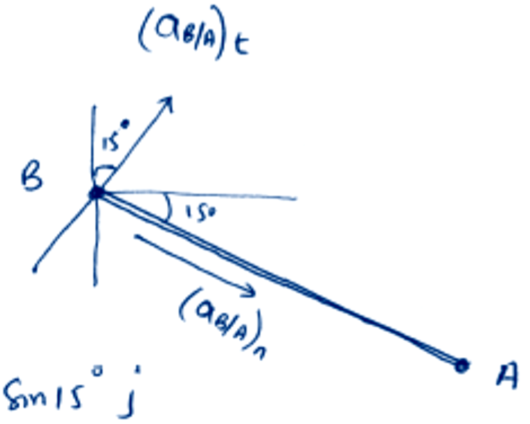
$$\underline{a}_B = -3.2 j + 0.2 \, \alpha_{BC} \, \underline{i}$$

LINK AB :-

$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$$

Here $\underline{a}_B$ and $\underline{a}_A$ are Known

$$\underline{a}_{B/A} = (a_{B/A})_n + (a_{B/A})_t$$



$$(a_{B/A})_n = \omega_{AB}^2 \, \overline{AB}$$
$$= (1.725)^2 (0.24)$$
$$= 0.714 \ m/s^2$$

$$(\underline{a}_{B/A})_n = 0.714 \cos 15^\circ \, \underline{i} - 0.714 \sin 15^\circ \, \underline{j}$$

$$(\underline{a}_{B/A})_n = 0.69 \, \underline{i} - 0.185 \, \underline{j}$$

$$(a_{B/A})_t = \alpha_{AB} \, \overline{AB}$$
$$= \alpha_{AB} (0.24).$$

$$(\underline{a}_{B/A})_t = 0.24 \, \alpha_{AB} \sin 15^\circ \, \underline{i} + 0.24 \, \alpha_{AB} \cos 15^\circ \, \underline{j}$$

$$(\alpha_{B/A})_t = (0.062 \, \underline{i} + 0.232 \, \underline{j}) \alpha_{AB}$$

therefore,

$$(\underline{a}_{B/A}) = (0.69 \, \underline{i} - 0.185 \, \underline{j}) + (0.062 \, \alpha_{AB} \, \underline{i} + 0.232 \, \alpha_{AB} \, \underline{j})$$

Now using the equation.

$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$$

Group the i terms together

$$0.2\,\alpha_{BC} = -4.35 + 0.69 + 0.062\,\alpha_{AB}$$

(1)

Group the j terms together.

$$-3.2 = -6.73 - 0.185 + 0.232\,\alpha_{AB}$$

(2)

From (2).

$$\alpha_{AB} = 16.02 \text{ rad/s}^2 \text{ (cw)}.$$

ANSWER

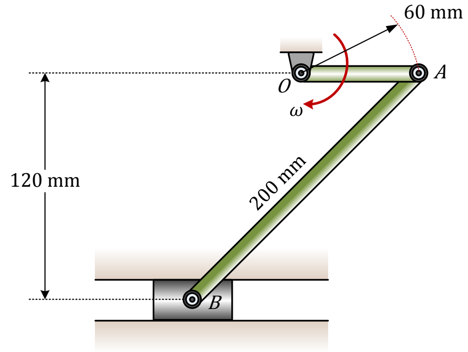
From (1)

$$\alpha_{BC} = 13.31 \text{ rad/s}^2 \text{ (ccw)}$$

ANSWER

Question 10.8.

For a short interval of motion, link OA has a constant angular velocity of $\omega = 4\,\mathrm{rad/s}$. Determine the angular acceleration α_{AB} of the link AB for the instant when OA is parallel to the horizontal axis through B.

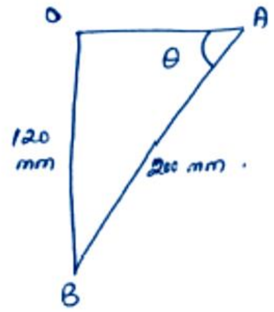


Solution

Determine the angle OAB first

$$\sin\theta = \frac{120}{200}.$$

$$\theta = 36.87^\circ$$



LINK OA :-

$$\underline{v}_A = \cancel{\underline{v}_O}^{\,0} + \underline{v}_{A/O}$$

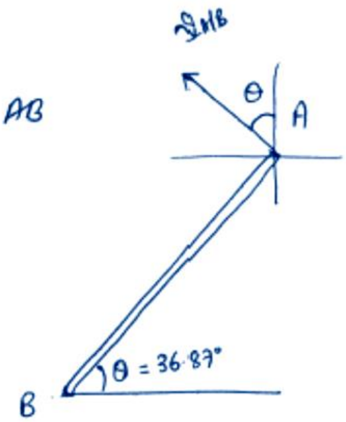
$$\underline{v}_A = \underline{v}_{A/O} = (4)(0.06) = 0.24\ \mathrm{m/s}\ (\downarrow)$$

or $\underline{v}_A = -0.24\,\underline{j}$

also it is known that

$$\underline{v}_B = v_B\,\underline{i}$$

$\underline{v}_{A/B} \longrightarrow$ is perpendicular to AB



therefore,

$$\underline{v}_{A/B} = -v_{A/B}\sin 36.87^\circ\,\underline{i} + v_{A/B}\cos 36.87^\circ\,\underline{j}$$

$$\underline{v}_{A/B} = -0.6\,v_{A/B}\,\underline{i} + 0.8\,v_{A/B}\,\underline{j}$$

LINK AB :-

$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} \quad \text{--- (1)}$$

Group the i terms in (1).

$$0 = \underline{v}_B - 0.6\, v_{A/B} \quad \text{—— (2)}.$$

Group the j terms in (1).

$$-0.24 = 0.8\, v_{A/B} \quad \text{—— (3)}.$$

$$v_{A/B} = -0.3 \text{ m/s}$$

From (2).

$$v_B = -0.18 \text{ m/s} \quad (\longrightarrow)$$

or $$v_B = 0.18 \text{ m/s} \quad (\longleftarrow).$$

$$\omega_{AB} = \frac{v_{A/B}}{AB} = \frac{0.3}{0.2} = 1.5 \text{ rad/s (cw)}$$

Now perform the acceleration analysis :-

LINK OA :-

$$\underline{a}_A = \overset{0.}{\cancel{\underline{a}_O}} + \underline{a}_{A/O}$$

$$\underline{a}_A = \underline{a}_{A/O} = (a_{A/O})_n + (a_{A/O})_t.$$

Since ω_{OA} is constant, therefore.

$$(a_{A/O})_t = 0.$$

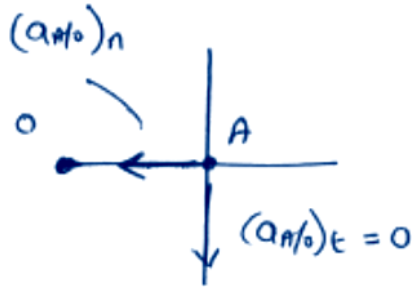
And,

$$(a_{A/O})_n = \omega_{OA}^2\, \overline{OA} = 0.96 \text{ m/s}^2$$

or

$$\underline{a}_A = 0.96 \; m/s^2.$$

$$\underline{a}_A = -0.96\,\underline{i} \quad (m/s^2)$$



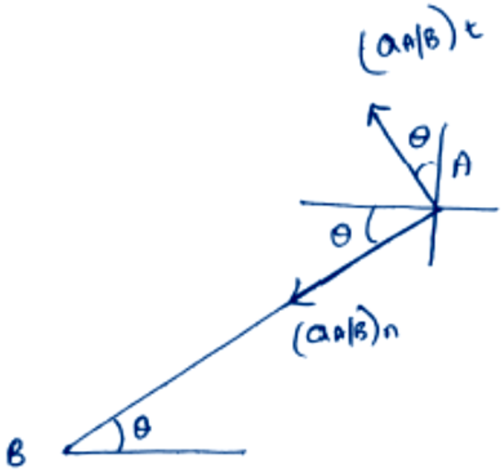
LINK AB :-

$$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B} \qquad (4)$$

where $\underline{a}_A$ is known and lets assume :-

$$a_B = -a_B\,\underline{i} \qquad (\leftarrow).$$

$$(\underline{a}_{A/B}) = (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$$



$$\begin{aligned}(a_{A/B})_n &= \omega_{AB}^2\,\overline{AB}\\ &= (1.5)^2(0.2)\\ &= 0.45 \; m/s^2.\end{aligned}$$

$$(a_{B/A})_n = -0.45\cos 36.87°\,\underline{i} - 0.45\sin 36.87°\,\underline{j}$$

$$(a_{B/A})_n = -0.36\,i - 0.27\,\underline{j}$$

Also,

$$(a_{B/A})_t = \alpha_{AB}\,\overline{AB} = 0.2\,\alpha_{AB}.$$

$$(a_{B/A})_t = -0.2\,\alpha_{A/B}\sin 36.87° + 0.2\,\alpha_{AB}\cos 36.87°\,\underline{j}$$

Group the i terms in (4)

$$-0.96 = -a_B - 0.36 - 0.2\,\alpha_{AB} \sin 36.87^\circ \quad \text{— (5)}$$

Group the j terms in (4)

$$0 = 0 - 0.27 + 0.2\,\alpha_{AB} \cos 36.87^\circ \quad \text{— (6)}.$$

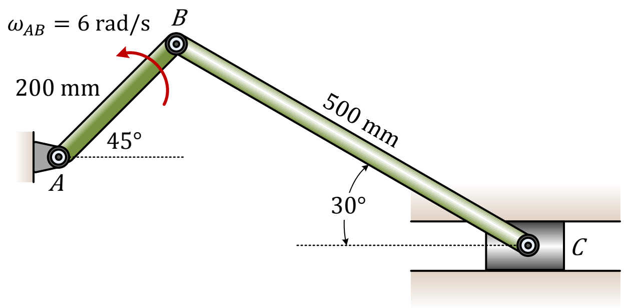
From (6)

$$\alpha_{AB} = 1.687 \text{ rad/s}^2 \quad (\text{ccw})$$

ANSWER

Question 10.9.

If bar AB has an angular velocity $\omega_{AB} = 6\ \text{rad/s}$, determine the velocity of the slider block C at the instant shown.



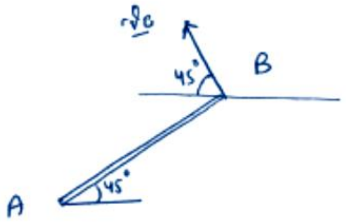
Solution

Link AB :-

$$\underline{v}_B = \cancel{\underline{v}_A}^{\,0} + \underline{v}_{B/A}$$

$$v_B = v_{B/A} = \omega_{AB}\,\overline{AB}$$

$$= (6)(0.2) = 1.2\ \text{m/s} \quad \perp \text{ to } AB$$



Link BC :-

$$\underline{v}_B = \underline{v}_C + \underline{v}_{B/C} \quad \text{—— (1)}$$

where $\underline{v}_B$ is known and $\underline{v}_{B/C}$ is $\perp$ to BC

Construct the velocity triangle (or use i and j components).

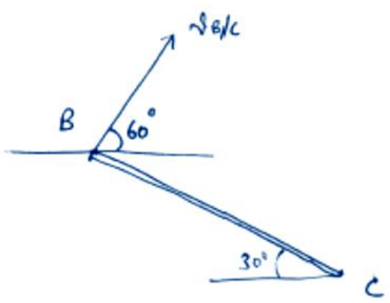
$$\underline{v}_B = -1.2\cos 45^\circ\,\underline{i} + 1.2\sin 45^\circ\,\underline{j}$$

$$\underline{v}_B = -0.85\,\underline{i} + 0.85\,\underline{j}$$

$$\underline{v}_C = -v_C\,\underline{i} \quad \text{(Horizontal motion)}.$$

$$\underline{v}_{B/C} = v_{B/C}\cos 60^\circ\,\underline{i} + v_{B/C}\sin 60^\circ\,\underline{j}$$

$$\underline{v}_{B/C} = 0.5\,v_{B/C}\,\underline{i} + 0.866\,v_{B/C}\,\underline{j}$$



Now from equation (1)

Group the i terms together.

$$-0.85 = -v_c + 0.5\, v_{B/C} \quad \text{— (1)}.$$

Group the j terms together.

$$0.85 = 0.866\, \underline{v}_{B/C} \quad \text{— (2)}.$$

From (2).

$$v_{B/C} = 0.98 \text{ m/s}$$

From (1).

$$v_c = 1.34 \text{ m/s} \quad (\leftarrow)$$

ANSWER

NOTE :-

The positive answer for $\underline{v}_c$ indicates that the direction assumed was correct