School of Mechanical and Manufacturing Engineering

# MMAN1300 Engineering Mechanics 1

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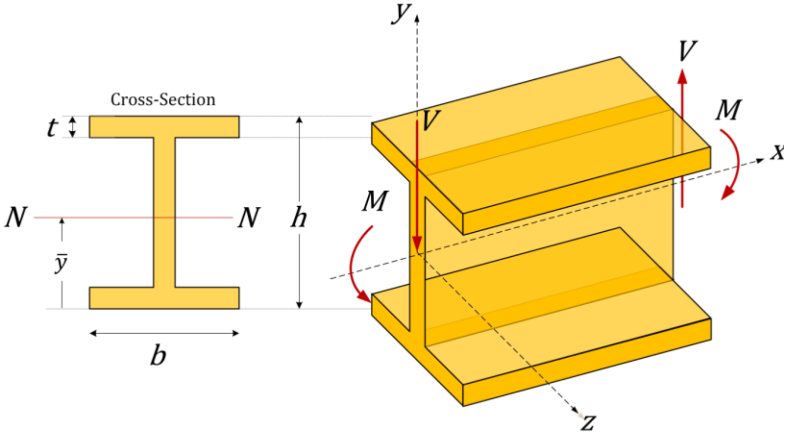
# Week 6 L1-2 – Geometric Properties




CENTROIDS

- Centre of area
- Centre of mass

SECOND MOMENT OF AREA

- Derivation
- Parallel Axis Theorem
- Principal Axes

# Lecture Outline

- Centroids of Areas
- Centroids of Composite Areas
- Second Moment of Area
- Parallel Axis Theorem
- Transformation of Axes
- Principal Axes
- Principal Second Moment of Area
- Mohr's Circle

# Centroids

# Centroids

- Suppose that we want to determine the average position of a group of students sitting in a room:
- Introduce a coordinate system to specify the position of each student
- e.g. align the axes with the walls of the room

# Centroids

- Number the students from 1 to $N$ & denote the position of student 1 by $(x_1, y_1)$, the position of student 2 by $(x_2, y_2)$ & so on

- The average $x$ coordinate, which is denoted by $\bar{x}$, is the sum of their $x$ coordinates divided by $N$:

$$\bar{x} = \frac{x_1 + x_2 + \ \dots\dots\dots + x_N}{N} = \frac{\sum_i x_i}{N}$$

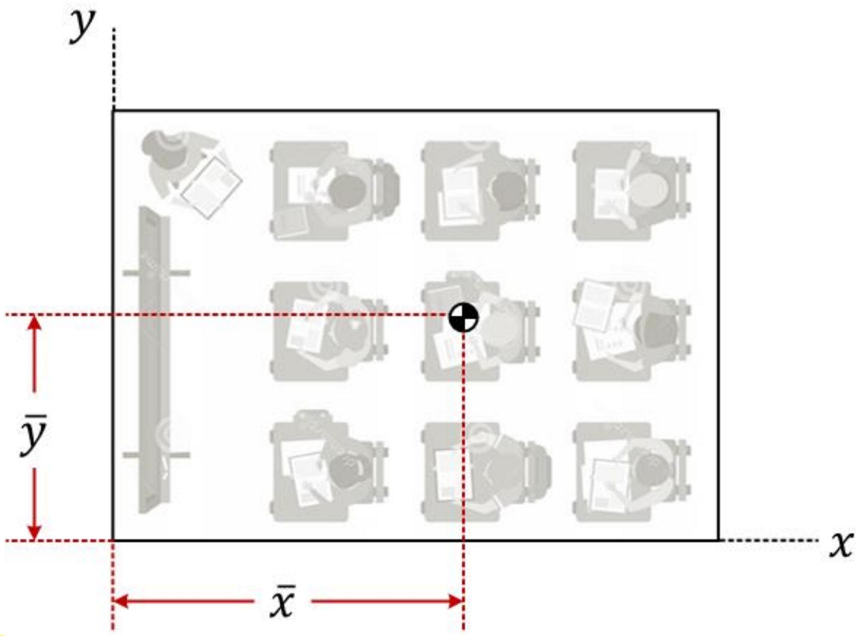
- where $\sum_i$ means " sum over the range of $i$ "

# Centroids

- The average $y$ - coordinate is:

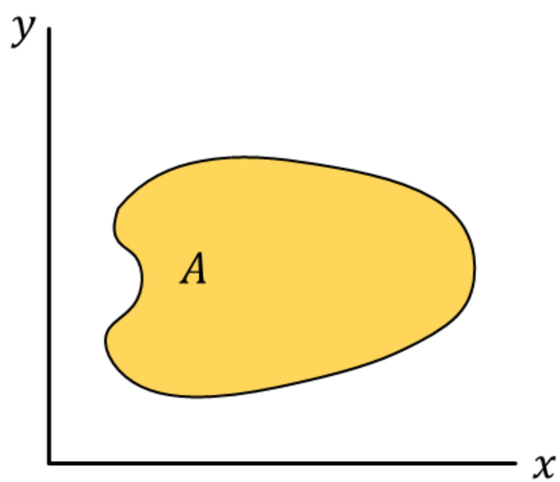
$$\bar{y} = \frac{\sum_i y_i}{N}$$

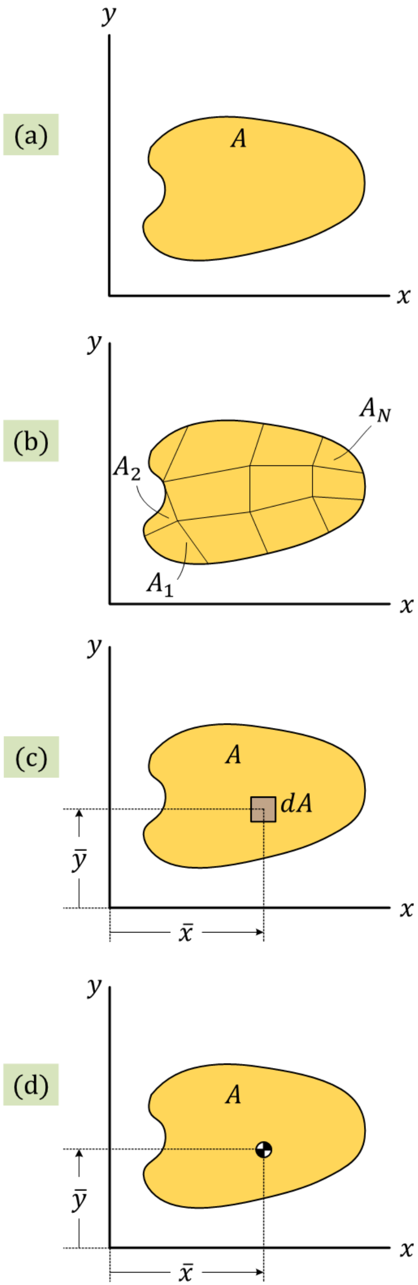
- We indicate the average position by the symbol shown:



# Centroids of Areas

- Consider an arbitrary area $A$ in the $x$-$y$ plane
- Divide the area into parts $A_1$ , $A_2$ , … … … , $A_N$ and denote the positions of the parts by $(x_1, y_1)$, $(x_2, y_2)$, ……… ,$(x_N, y_N)$



# Centroids of Areas

- The centroid or average position of the area:

$$\bar{x} = \frac{\sum_i x_i A_i}{\sum_i A_i} \qquad \bar{y} = \frac{\sum_i y_i A_i}{\sum_i A_i}$$

- To reduce the uncertainty in the positions of areas $A_1$ , $A_2$ , … … … , $A_N$, divide $A$ into smaller parts

- But we would still obtain only approximate values of $x$ and $y$

# Centroids of Areas

- To determine the exact location of the centroid, we must take the limit as the sizes of the parts approach zero

- We obtain this limit by using the integrals:

$$\bar{x} = \frac{\int_A x \, dA}{\int_A dA}$$

$$\bar{y} = \frac{\int_A y \, dA}{\int_A dA}$$

# Centroids of Areas

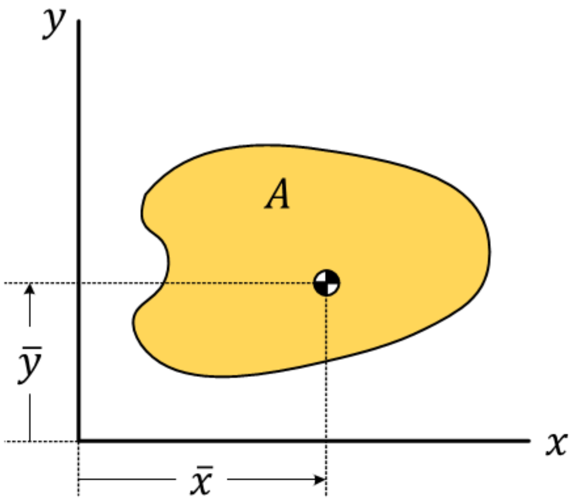
- Where $x$ & $y$ are the coordinates of the differential element of area $dA$

$$\bar{x} = \frac{\int_A x \, dA}{\int_A dA}$$

- The subscript $A$ on the integral sign means the integration is carried out over the entire area

$$\bar{y} = \frac{\int_A y \, dA}{\int_A dA}$$

- The centroid of the area is:

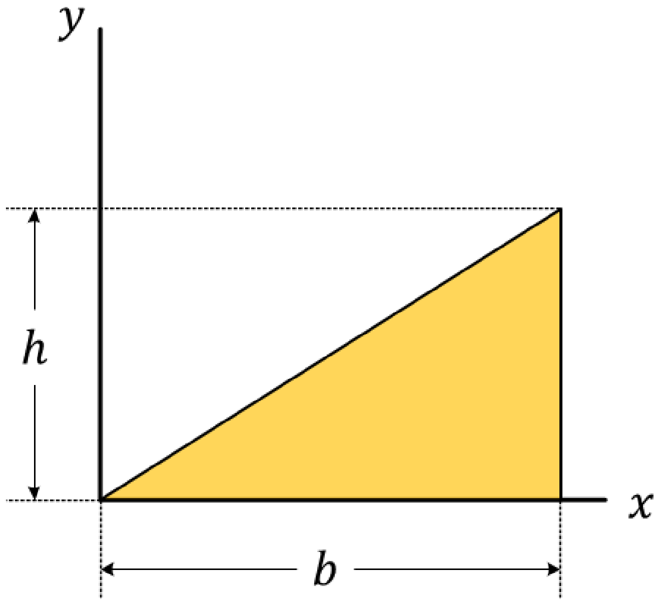


# Centroid of an Area by Integration

- Determine the centroid of the triangular area shown in the figure.

**Strategy**

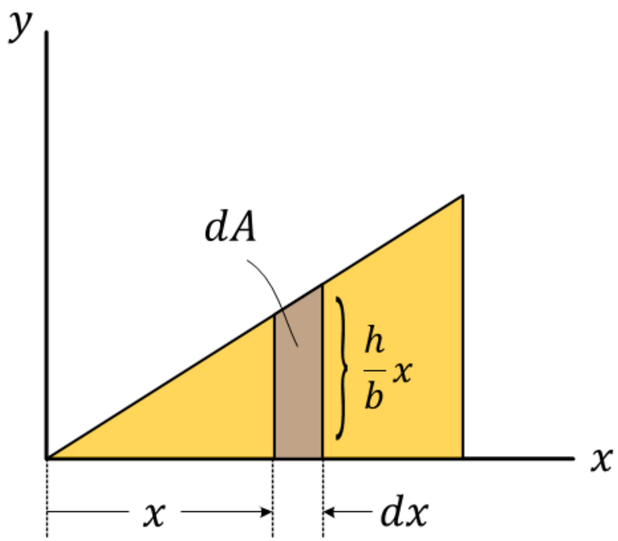
Determine the coordinates of the centroid by using an element of area $dA$ in the form of a “strip” of width $dx$.



# Centroid of an Area by Integration

**Solution strategy:**

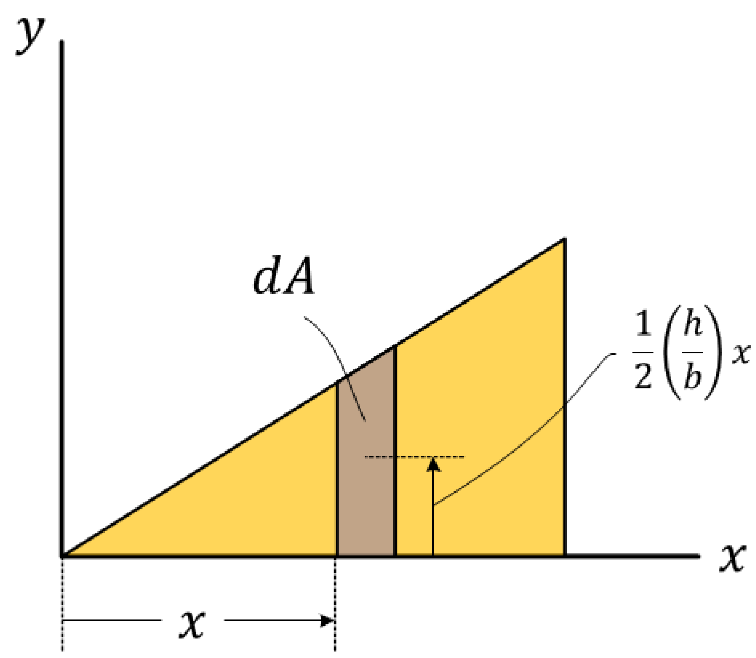
Let $dA$ be the area of the vertical strip. The height of the strip is $(h/b)x$, so $dA = (h/b)x\,dx$. To integrate over the entire area, we must integrate with respect to $x$ from $x = 0$ to $x = b$. The $x$ coordinate of the centroid is:



# Centroid of an Area by Integration

**Solution strategy:**

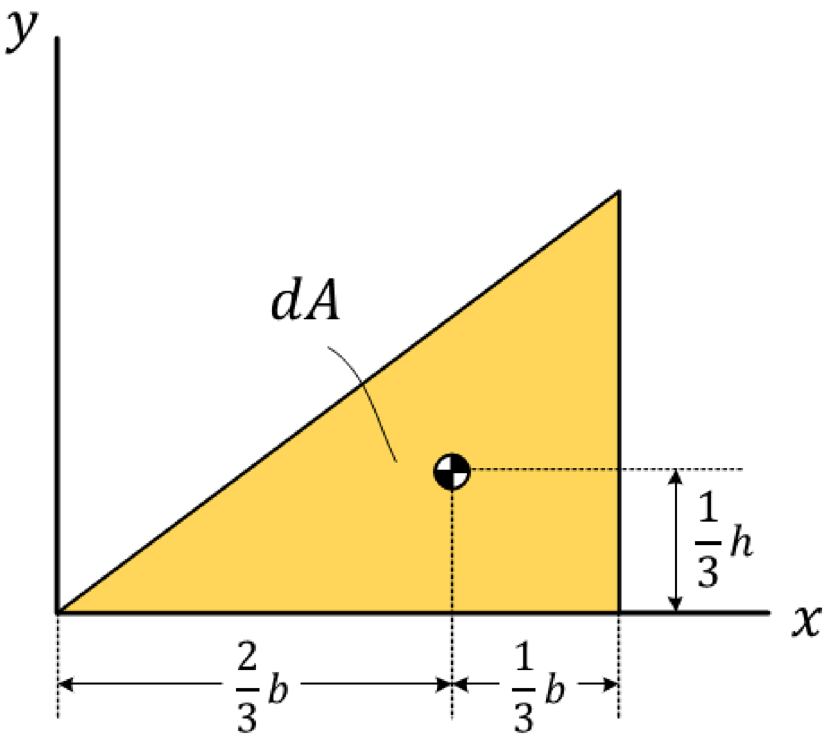
- To determine $\bar{y}$, we let $y$ be the $y$-coordinate of the midpoint of the strip.
- This is used to formulate an integral equation.
- We can solve this to find the centroidal distance in y.



# Centroid of an Area by Integration

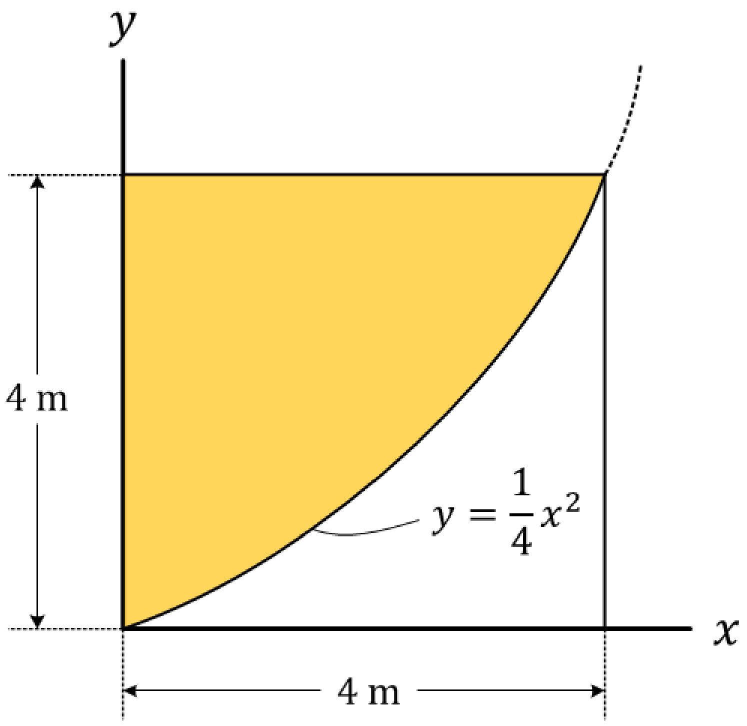
**Solution strategy:**

- The centroid is shown:



# Example 1

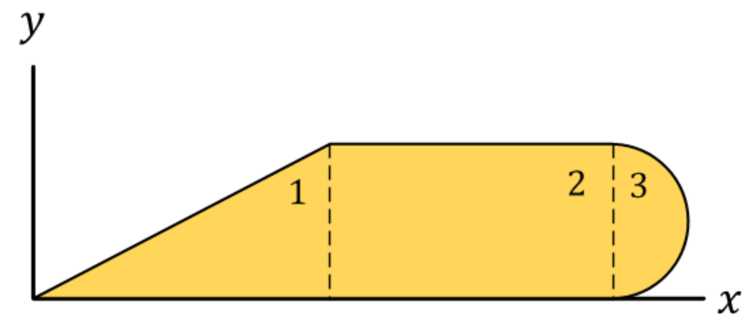
Locate the x centroid of the shaded area.




W6 Example 1 (Web view)

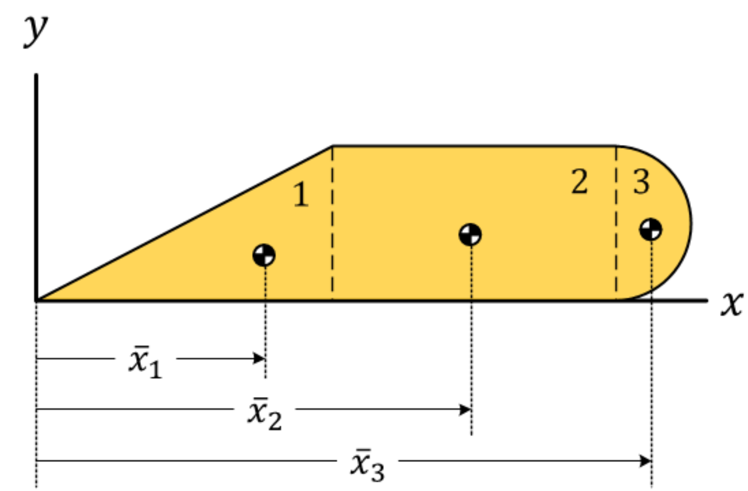
# Centroids of Composite Areas

## Composite Area:




- An area consisting of a combination of simple areas

- The centroid of a composite area can be determined without integration if the centroids of its parts are known




- The area in the figure consists of a triangle, a rectangle & a semicircle, which we call parts 1, 2 & 3

# Centroids of Composite Areas

- The $x$ coordinate of the centroid of the composite area is:

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\int_{A1} x\,dA + \int_{A2} x\,dA + \int_{A3} x\,dA}{\int_{A1} dA + \int_{A2} dA + \int_{A3} dA}$$

- From the equation for the x coordinate of the centroid of part 1:

$$\bar{x}_1 = \frac{\int_{A1} x\,dA}{\int_{A1} dA}$$

- We obtain:

$$\int_{A1} x\,dA = \bar{x}_1 A_1$$

# Centroids of Composite Areas

- Using this equation and equivalent equations for parts 2 and 3, we can write

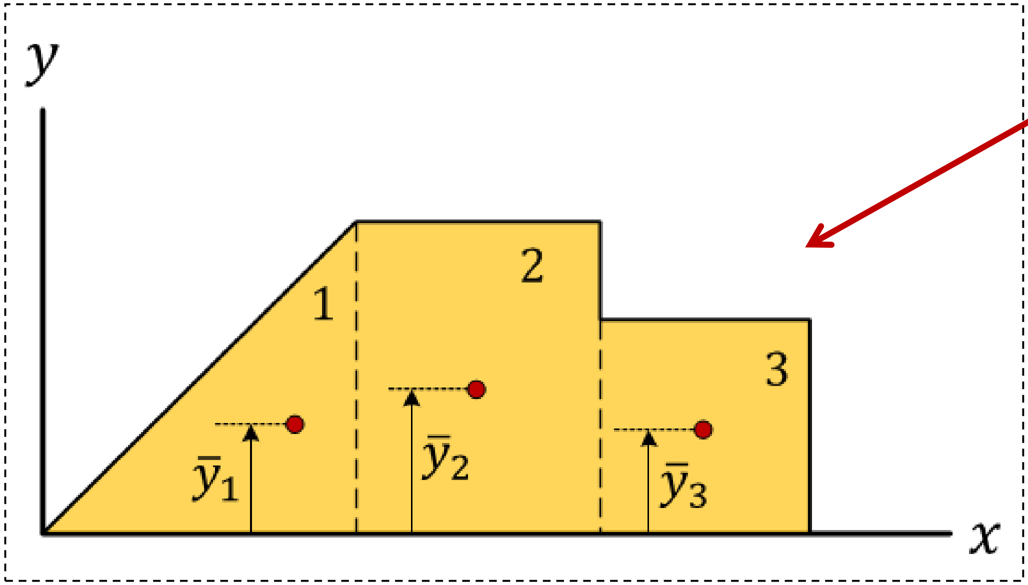
$$\bar{x} = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2 + \bar{x}_1 A_3}{A_1 + A_2 + A_3}$$

- The coordinates of the centroid of a composite area with an arbitrary number of parts are:

$$\bar{x} = \frac{\sum_i \bar{x}_i A_i}{\sum_i A_i} \qquad \bar{y} = \frac{\sum_i \bar{y}_i A_i}{\sum_i A_i}$$

# Centroid – Composite Sections

- For composite cross-sections made of elements:




Composite cross-section made of elements

$i$ is the $i^{th}$ element

- Centroid of composite shapes

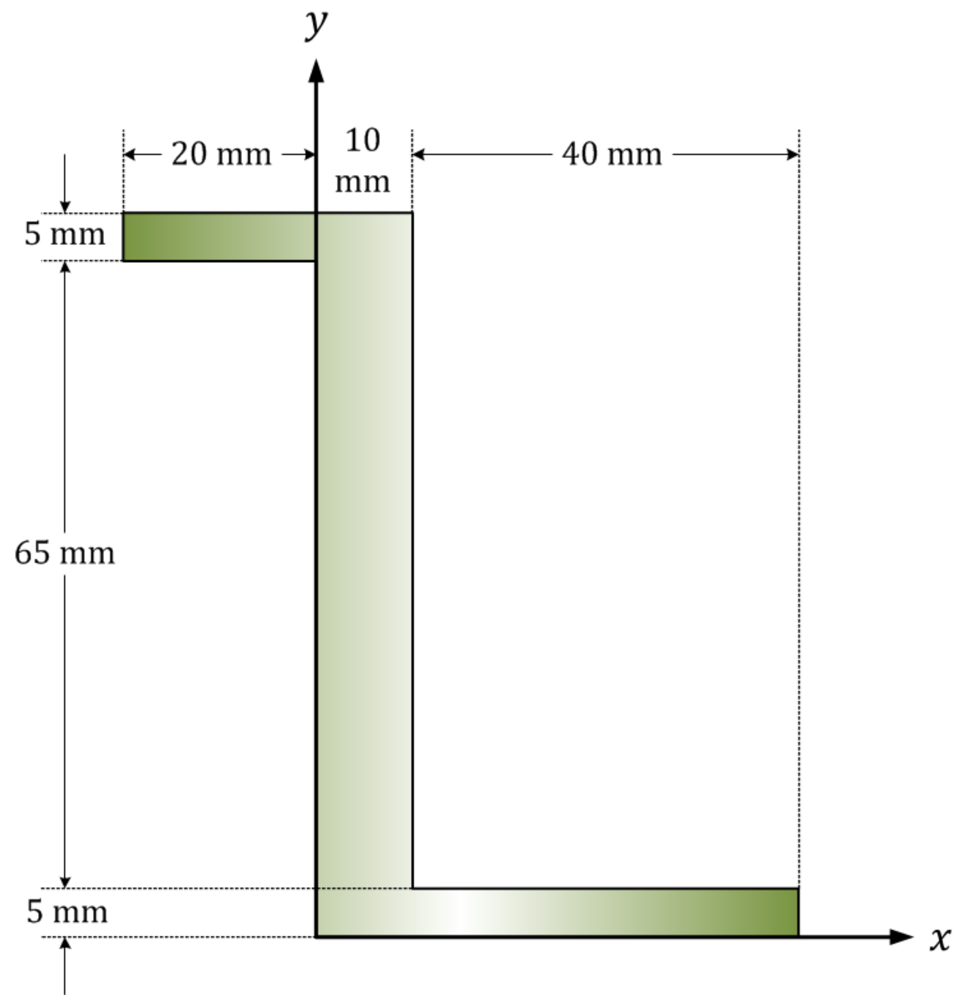
$$\bar{x} = \frac{\sum x_i A_i}{\sum A_i}$$

$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i}$$

$x_i, y_i$ - is the distance of the centroid of the $i^{th}$ element to the reference axis ($x$)

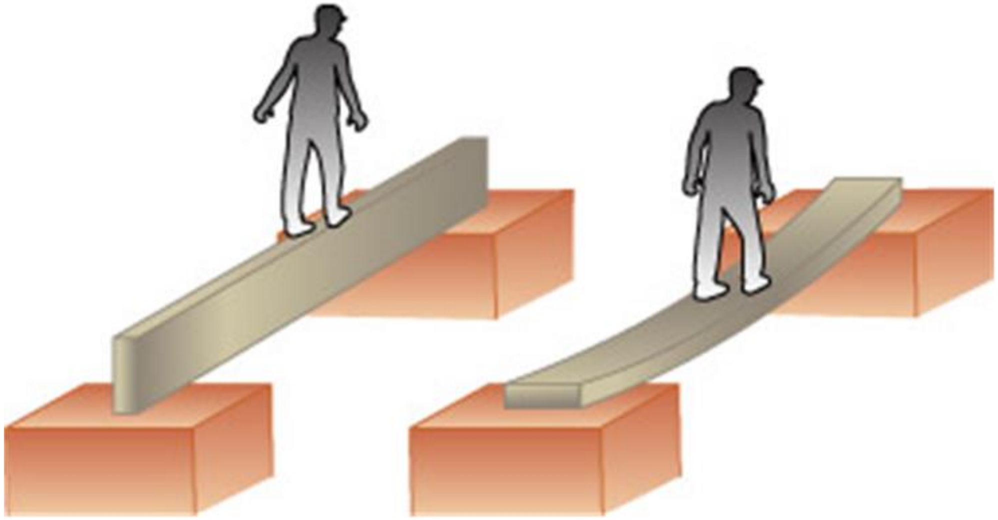
# Example 2

Find the centroid of the section shown:



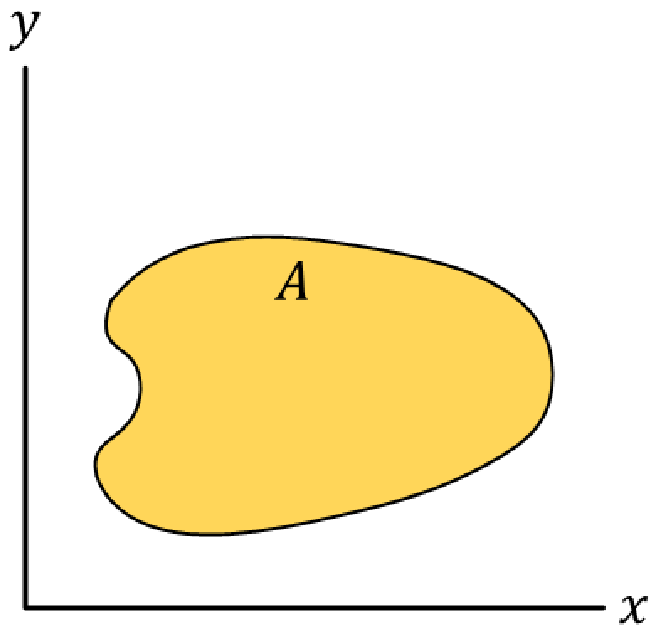

W6 Example 2 (Web view)

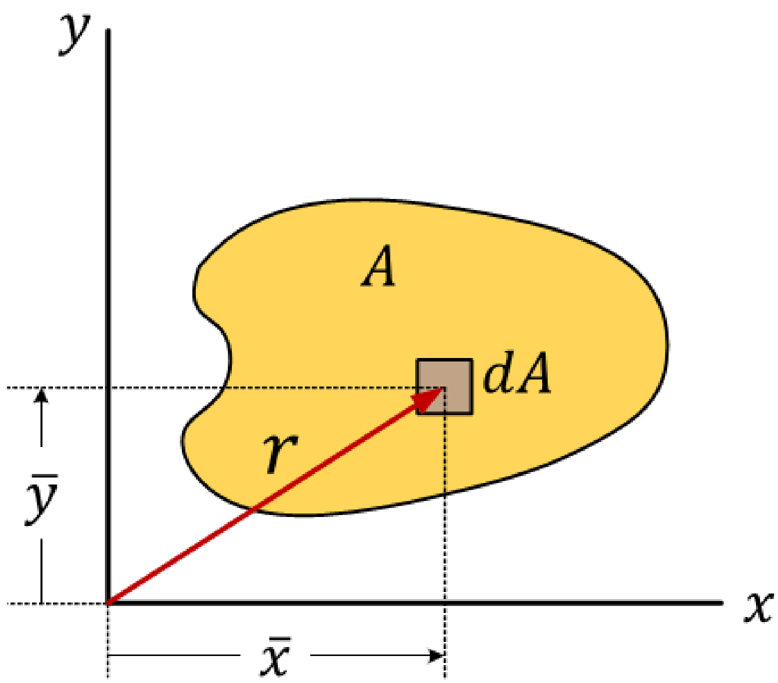
# Moment of Inertia
# *AKA*
# Second Moment of Area



# Moment of Inertia (Second Moment of Area)

- Consider an area $A$ in the $x$-$y$ plane



# Moment of Inertia (Second Moment of Area)

Moments of inertia of $A$ are defined:

1. Moment of inertia about the $x$-axis

$$I_x = \int_A y^2 dA$$

Where $y$ is the $y$-coordinate of the differential element of area $dA$

# Moment of Inertia (Second Moment of Area)

Moments of inertia of $A$ are defined:

2. Moment of inertia about the $y$-axis

$$I_y = \int_A x^2 dA$$

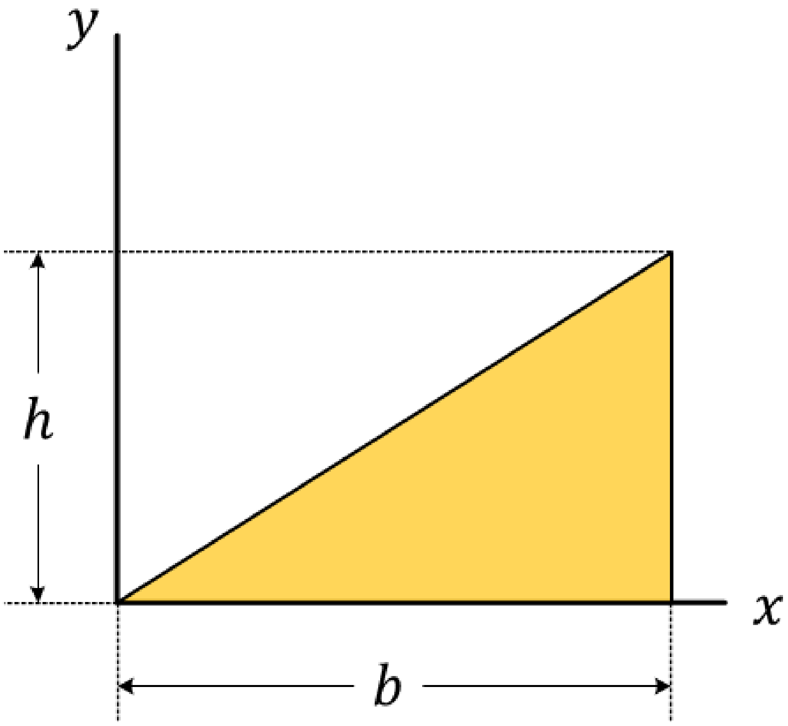
Where $x$ is the $x$-coordinate of the differential element of area $dA$

3. Product of Inertia

$$I_{xy} = \int_A xy dA$$

# Moments of Inertia of a Triangular Area

Determine $I_x, I_y$ and $I_{xy}$ for the triangular area shown below:



# Example - Moments of Inertia of a Triangular Area

**Solution strategy:**

The moment of inertia about the $y$ - axis is very similar to the equation for the $x$ - coordinate of the centroid of an area & it can be evaluated for this triangular area in exactly in the same way: by using a differential element of area $dA$ in the form of a vertical strip of width $dx$. Then show that $I_{xx}$ & $I_{xy}$ can be evaluated by using the same element of area.

# Moment of Inertia of a Triangular Area

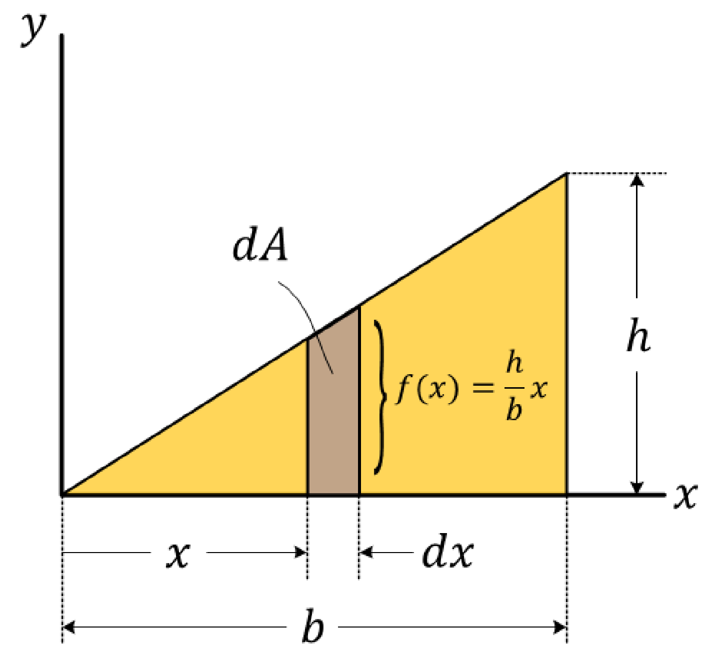
## Approach

Let $dA$ be the vertical strip. The equation describing the triangular area's upper boundary is

$$f(x) = (h/b)x,$$

so

$$dA = f(x)dx = \left(\frac{h}{b}\right)x\,dx$$



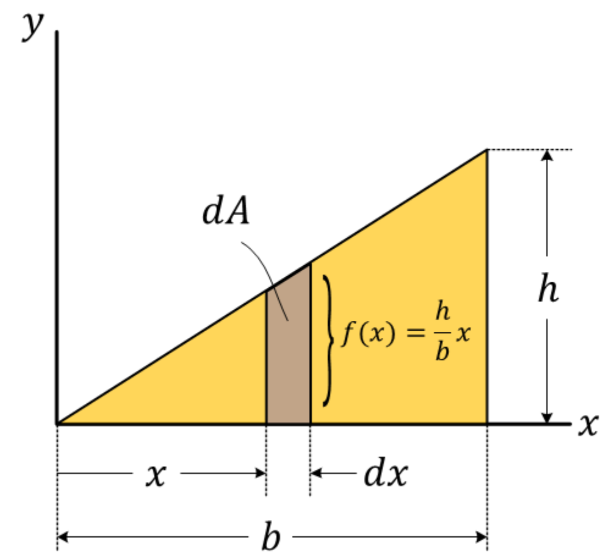

To integrate over the entire area, we must integrate with respect to $x$ from $x = 0$ to $x = b$.

# Moment of Inertia of a Triangular Area:

## Solution strategy:

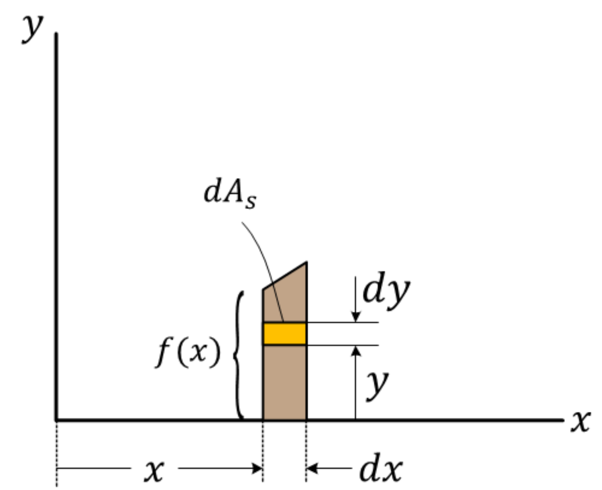
**Moment of inertia about the $y$ – axis:**

- Find $I_{yy}$ in terms of the element area ($dA_s = dx\ dy$)
- Integrate this expression with respect to $x$ from $x = 0$ to $x = b$, we obtain the value of $I_{xx}$ for the entire area:




**Product of Inertia:**

- $1^{st}$ evaluate the product of inertia of the strip $dA$, holding $x$ and $dx$ fixed:
- Integrate this expression with respect to $x$ from $x = 0$ to $x = b$ to obtain the value of $I_{xy}$ for the entire area:



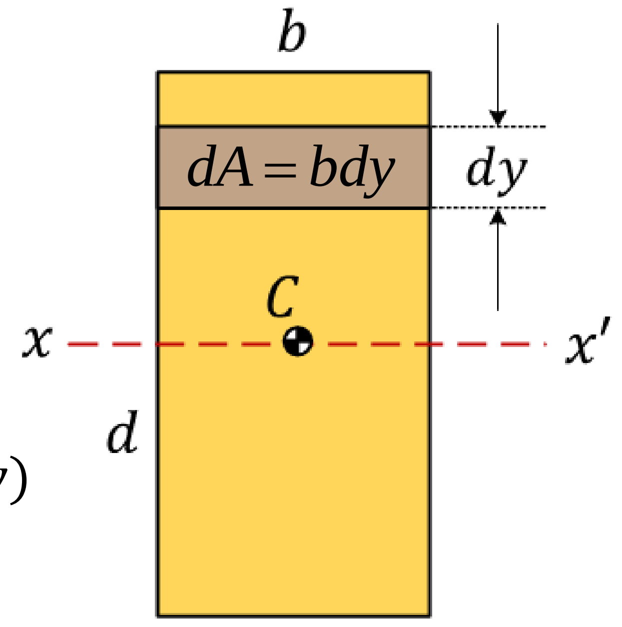
# Moment of Inertia of a Rectangular Section

## Solution strategy:

**Moment of inertia about the $y$ – axis:**

- Start with the definition of Moment of Inertia:

$$\int_A y^2 dA = I_{xx}$$




- Find $I_{xx}$ in terms of the element area ($dA = b\,dy$)
- Integrate this expression with respect to y from $y = -d/2$ to $y = \frac{d}{2}$

$$\int_{-d/2}^{d/2} by^2 dy = \left[by^3/3\right]_{-d/2}^{d/2}$$

- we obtain the value of $I_{xx}$ for the entire area:

$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

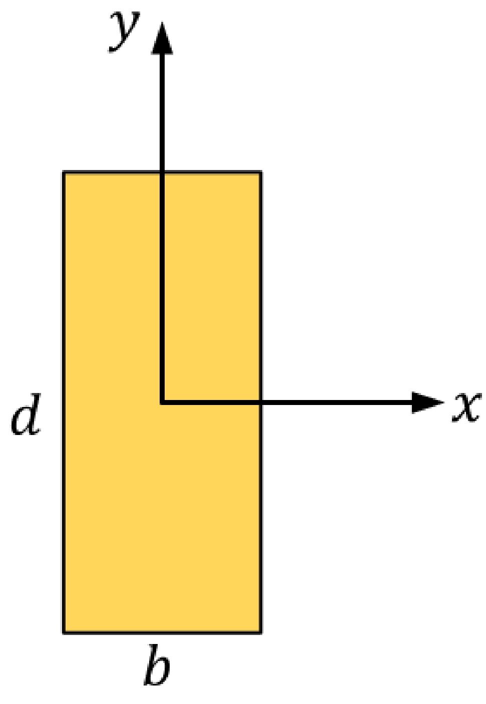
# Moment of Inertia of a Rectangular section

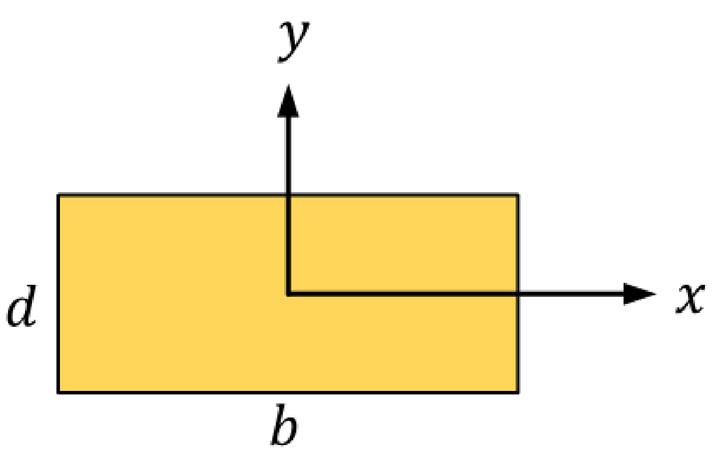
$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

where:

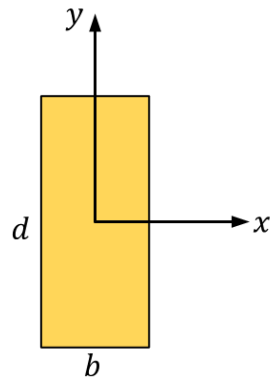
$b$ – $width$

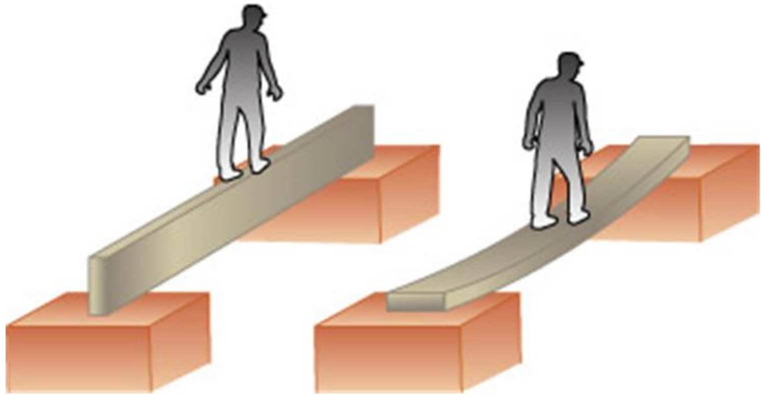
$d$ – $depth$

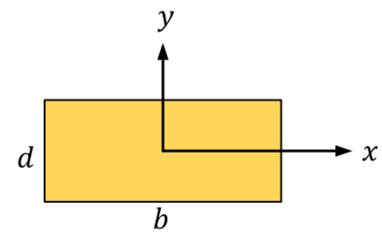


# Moment of Inertia of a Rectangular section





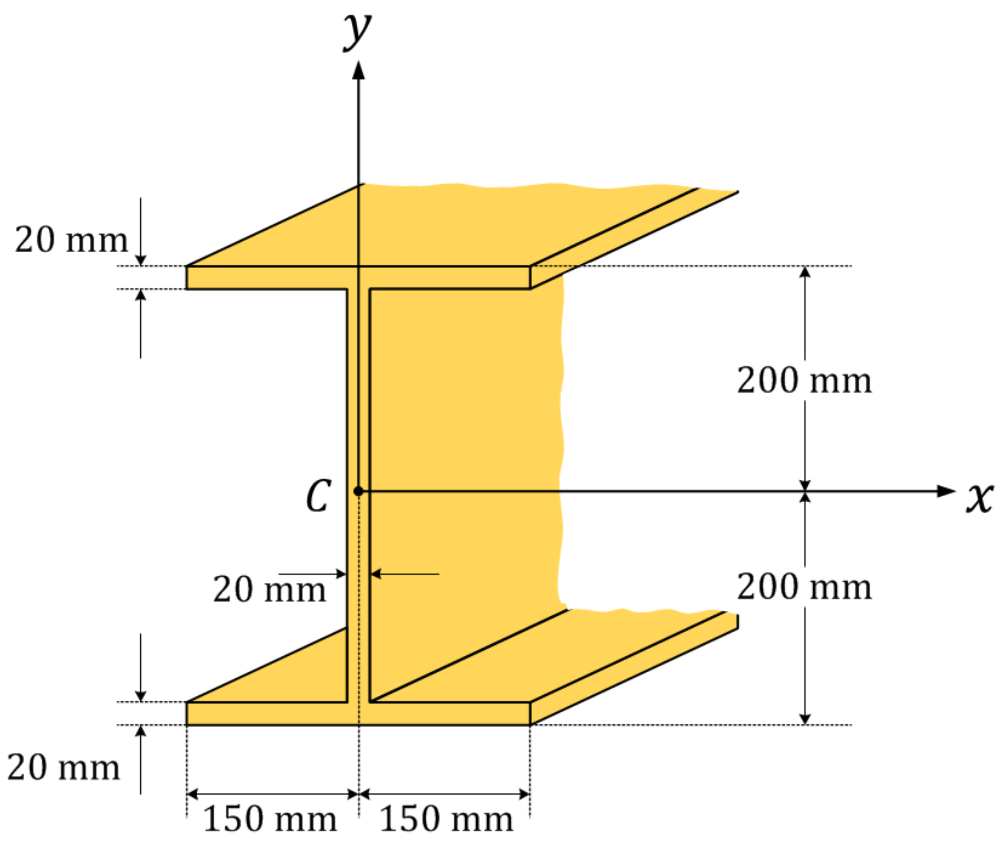

$$I_{xx} = \frac{bd^3}{12}$$

Compare rigidity $EI$…

$$I_{xx} = \frac{bd^3}{12}$$

# Example 3

Determine the moment of inertia about the $x$ - axis:




W6 Example 3 (Web view)

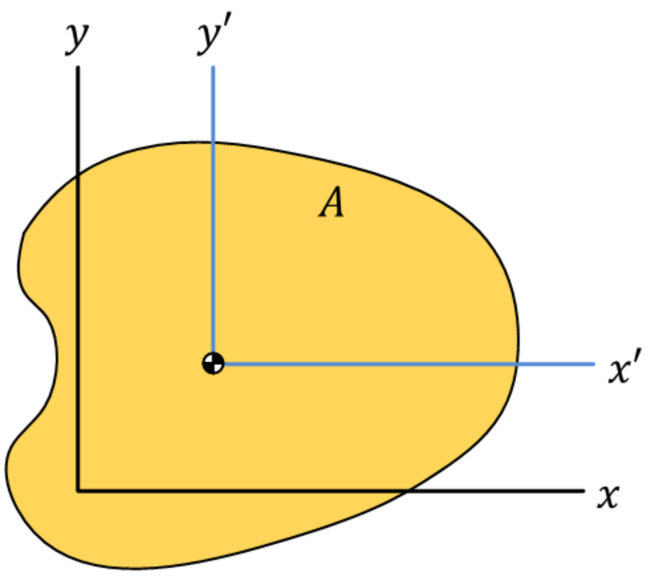
# Parallel Axes Theorem

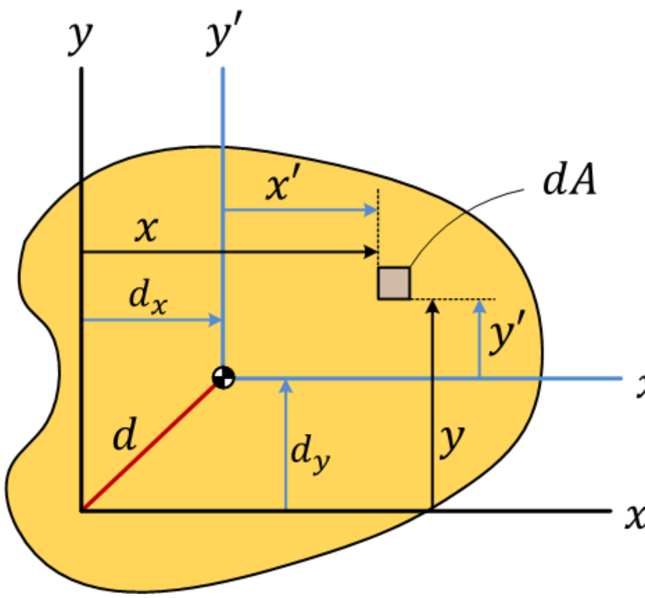
# Parallel-Axis Theorems

- The values of the moments of inertia of an area depend on the position of the coordinate system relative to the area
- In some situations the moments of inertia of an area are known in terms of a particular coordinate system but we need their values in terms of a different coordinate system
- When the coordinate systems are parallel, the desired moments of inertia can be obtained using the parallel-axis theorems:
- Possible to determine the moments of inertia of a composite area when the moments of inertia of its parts are known

# Parallel-Axis Theorems

Suppose that we know the moments of inertia of an area $A$ in terms of a coordinate system $x'y'$ with its origin at the centroid of the area & we wish to determine the moments of inertia in terms of a parallel coordinate system $xy$:



# Parallel-Axis Theorems

Denote the coordinates of the centroid $A$ in the $xy$ coordinate system by:

$$(d_x, d_y) \text{ and } d = \sqrt{{d_x}^2 + {d_y}^2}$$

$d$ is the distance from the origin of the $xy$ coordinate system to the centroid. In terms of $x'y'$ coordinate system, the coordinates of the centroid of $A$ are:

$$\bar{x}' = \frac{\int_A x' dA}{\int_A dA} \qquad \bar{y}' = \frac{\int_A y' dA}{\int_A dA}$$

# Parallel-Axis Theorem

But the origin of $x'y'$ coordinate system is located at the centroid of A, so $\bar{x}' = 0$ and $\bar{y}' = 0$ , Therefore,

$$\bar{x}' = \frac{\int_A x' dA}{\int_A dA} \qquad \bar{y}' = \frac{\int_A y' dA}{\int_A dA}$$

**Moment of Inertia about the *x*-axis:**

In terms of the $xy$ coordinate system, the moment of inertia of $A$ about the $x$ - axis is:

$$I_{xx} = \int_A y^2 \, dA$$

where $y$ is the $y$ coordinate of the element $dA$ relative to the $xy$ coordinate system

# Parallel-Axis Theorem

From the figure, $y = y' + d_y$ where $y'$ is the coordinate of $dA$ relative to the $x'y'$ coordinate system

Substituting this expression into equation $\rightarrow$ $\boxed{I_{xx} = \int_A y^2 \, dA}$

We obtain:

$$I_{xx} = \int_A (y' + d_y)^2 \, dA$$

$$= \int_A (y')^2 \, dA + 2d_y \int_A y' dA + {d_y}^2 \int_A dA$$

The 1st integral on the right is the moment of inertia of $A$ about the $x'$ axis

From Eq.

$$\bar{x}' = \frac{\int_A x' dA}{\int_A dA} \qquad \bar{y}' = \frac{\int_A y' dA}{\int_A dA}$$

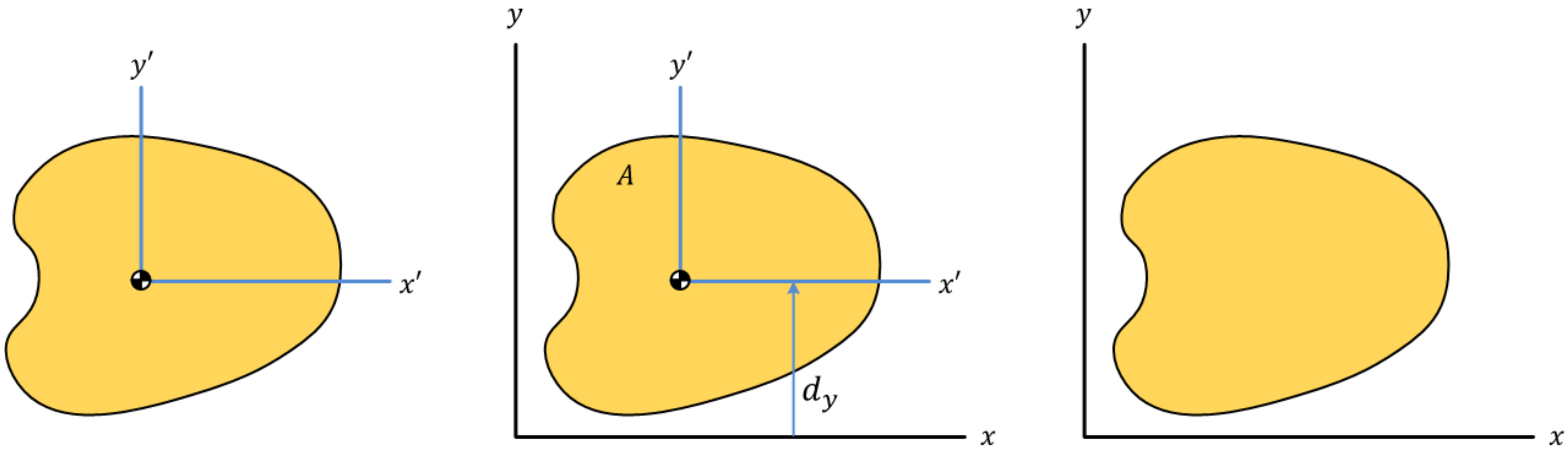
the 2nd integral on the right equals zero

# Parallel-Axis Theorem

Therefore we obtain:

$$\boxed{I_{xx} = I_{x'x'} + {d_y}^2 A}$$

This is a **parallel-axis theorem,** it relates the moment of inertia of $A$ about the $x'$ axis through the centroid to the moment of inertia about the parallel axis $x$




$$I_{x'} \quad + \quad {d_y}^2 \quad = \quad I_x$$

# Parallel-Axis Theorem

Moment of inertia about the $y$ – axis:

In terms of the $xy$ coordinate system, the moment of inertia of A about the $y$ - axis is:

$$I_{yy} = \int_A x^2 \, dA = \int_A (x' + d_x)^2 \, dA$$

$$= \int_A (x')^2 \, dA + 2d_x \int_A x' dA + {d_x}^2 \int_A dA$$

the 2nd integral on the right equals zero

# Parallel-Axis Theorem

Therefore, the **parallel-axis theorem** that relates the moment of inertia of $A$ about the $y'$ axis through the centroid to the moment of inertia about the parallel axis $y$ is:

$$\boxed{I_{yy} = I_{y'y'} + {d_x}^2 A}$$

# Parallel-Axis Theorems

## Product of Inertia:

In terms of the *xy* coordinate system, the product of inertia is

$$I_{xy} = \int_A xy\, dA = \int_A (x' + d_x)(y' + d_y)\, dA$$

$$= \int_A x'y' dA + d_y \int_A x' dA + d_x \int_A y' dA + d_x d_y \int_A dA$$

the 2nd and 3rd integrals equal zero

Thus, the parallel-axis theorem for product of inertia is:

$$\boxed{I_{xy} = I_{x'y'} + d_x d_y A}$$

# Parallel-Axis Theorems

**Determining a moment of inertia of a composite area in terms of a given coordinate system involves 3 steps:**

1. Choose the parts — try to divide the composite area into parts whose moments of inertia you know or can easily determine.

2. Determine the moments of inertia of the parts — determine the moment of inertia of each part in terms of a parallel coordinate system with its origin at the centroid of the part & then use the parallel-axis theorem to determine the moment of inertia in terms of the given coordinate system.

3. Sum the results — sum the moments of inertia of the parts (or subtract in the case of a cutout) to obtain the moment of inertia of the composite area.

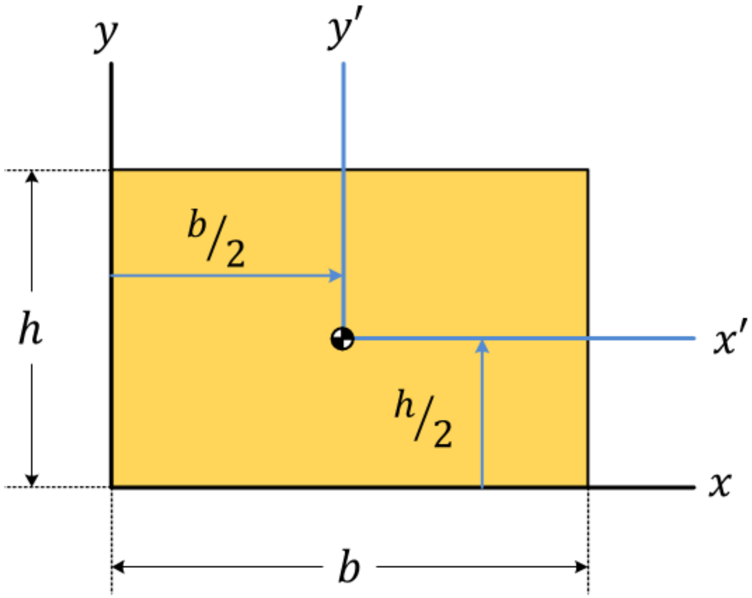
# Example - Parallel-Axis Theorems

The moments of inertia of the rectangular area
In the figure in terms of the $x'y'$ coordinate
system are

$$\boxed{I_{x'} = \frac{1}{12} b h^3}$$

$$\boxed{I_{y'} = \frac{1}{12} h b^3}$$

$$\boxed{I_{x'y'} = 0}$$



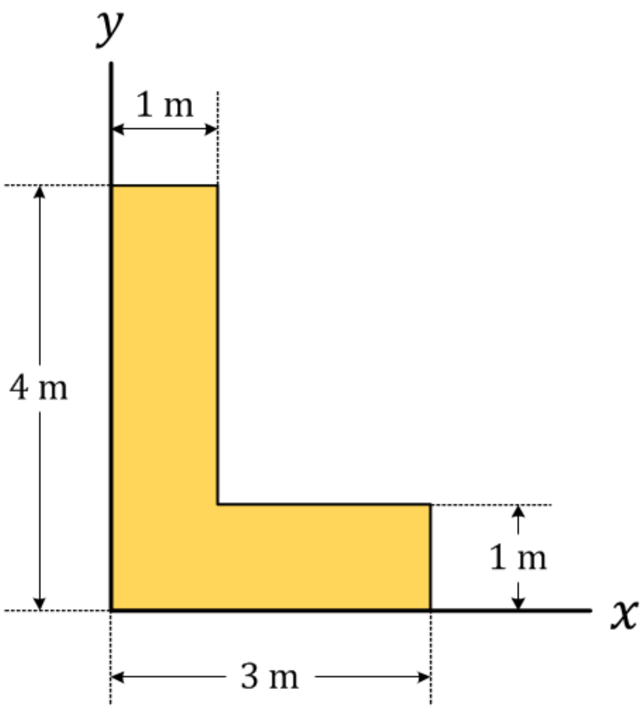

Determine its moment of inertia in terms of the $xy$ coordinate system.

# Demonstration of the Parallel-Axis Theorem

Determine $I_x$ and $I_y$ for the composite area shown in the figure:

**Strategy**

This area can be divided into 2 rectangles. Use the parallel-axis theorems to determine $I_x$ & $I_{xy}$ for each rectangle in terms of the $xy$ coordinate system & sum the results for the rectangles to determine $I_x$ & $I_{xy}$ for the composite area.

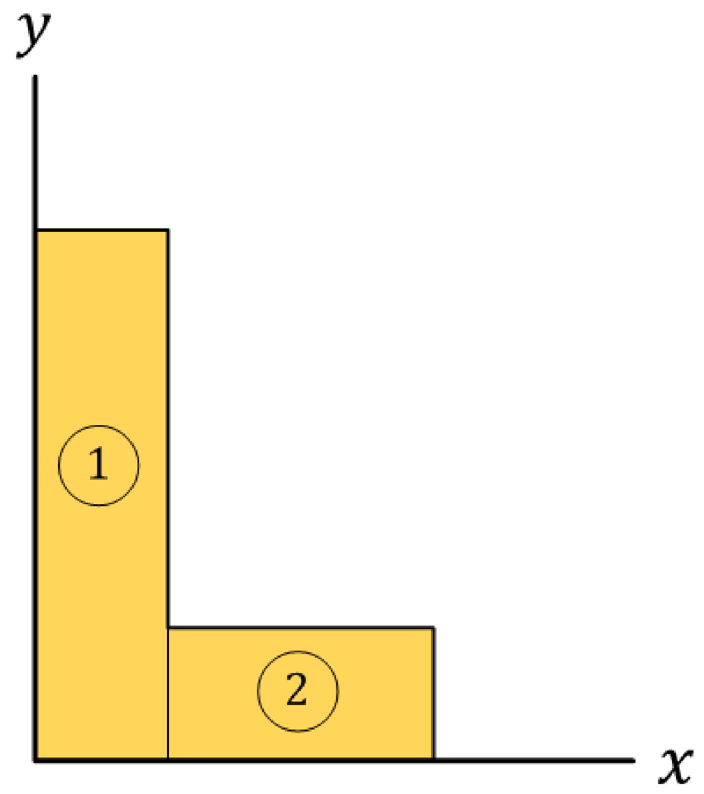


# Moment of Inertia of a Composite Area

## Solution strategy

**Choose the parts:**

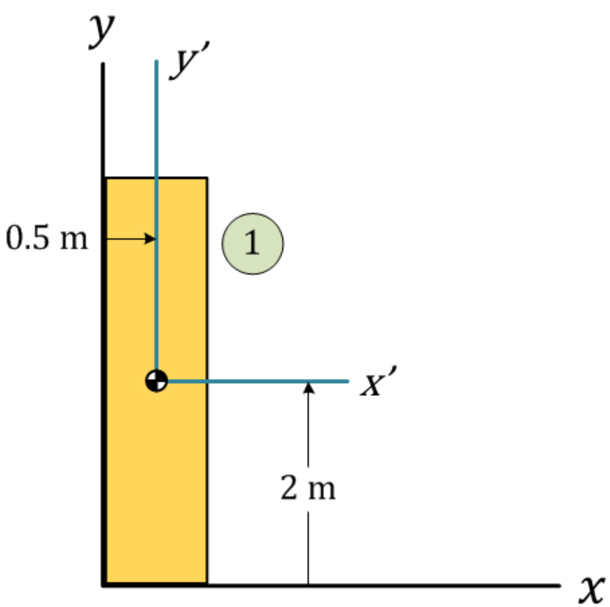
Determine the moments of inertia by dividing the area in 2 rectangular parts 1 and 2

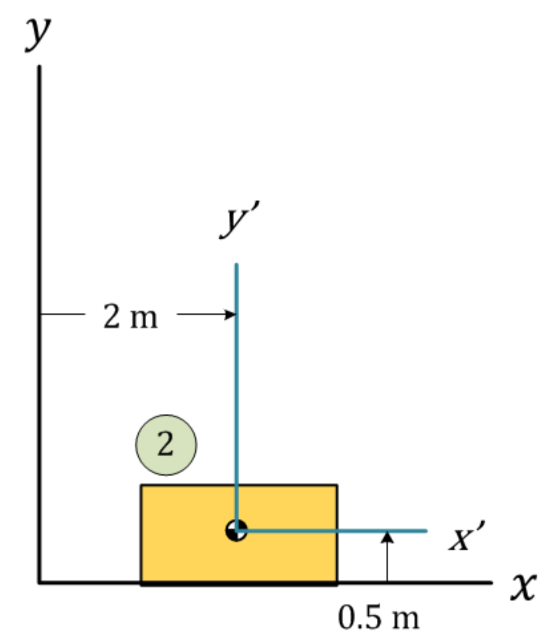


# Moment of Inertia of a Composite Area

## Solution

- Determine the Moments of Inertia of the Parts
- For each part, introduce a coordinate system *x'y'* with its origin at the centroid of the part:



# Moments of Inertia of a Composite Area

## Solution strategy

- Use the parallel-axis theorem to determine the moment of inertia of each part about the *x*-axis:

| | $d_y$(m) | $A$ ($\mathrm{m}^2$) | $I_{x'}$($\mathrm{m}^4$) | $I_x = I_{x'} + {d_y}^2 A$ |
|---|---|---|---|---|
| **Part 1** | 2 | $(1)(4) = 4$ | $\frac{1}{12}(1)(4)^3 = 5.33$ | 21.33 |
| **Part 2** | 0.5 | $(2)(1) = 2$ | $\frac{1}{12}(2)(1)^3 = 0.16$ | 0.67 |

# Moment of Inertia of a Composite Area

## Solution strategy

- Sum the results
- Thus, the moment of inertia of the composite area about the $x$-axis is:

$$I_x = (I_x)_1 + (I_x)_2 = 21.33\ \mathrm{m}^4 + 0.67\ \mathrm{m}^4 \boxed{= \quad 22.00\ \mathrm{m}^4}$$

# Moment of Inertia of a Composite Area

## Solution strategy

- Repeating this procedure, determine $I_{xy}$ for each part in table

| | $d_y$(m) | $d_x$(m) | $A$ (m$^2$) | $I_{x'y'}$(m$^4$) | $I_{xy} = I_{x'y'} + d_x d_y A$ (m$^4$) |
|---|---|---|---|---|---|
| **Part 1** | 2 | 0.5 | $(1)(4) = 4$ | 0 | 4 |
| **Part 2** | 0.5 | 2 | $(2)(1) = 2$ | 0 | 2 |

- The product of inertia of the composite area is:

$$I_{xy} = \left(I_{xy}\right)_1 + \left(I_{xy}\right)_2 = 4\ \mathrm{m}^4 + 2\ \mathrm{m}^4 \boxed{= \ 6\ \mathrm{m}^4}$$

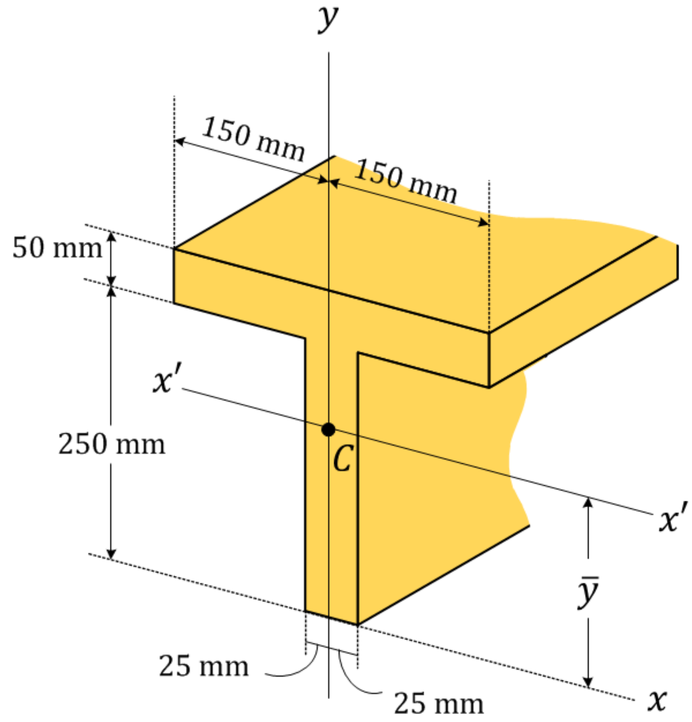
# Moment of Inertia of a Composite Area

**Note:**

**The moments of inertia** you obtain do not depend on how you divide a composite area into parts & you will often have a choice of convenient ways to divide a given area.

# Example 4

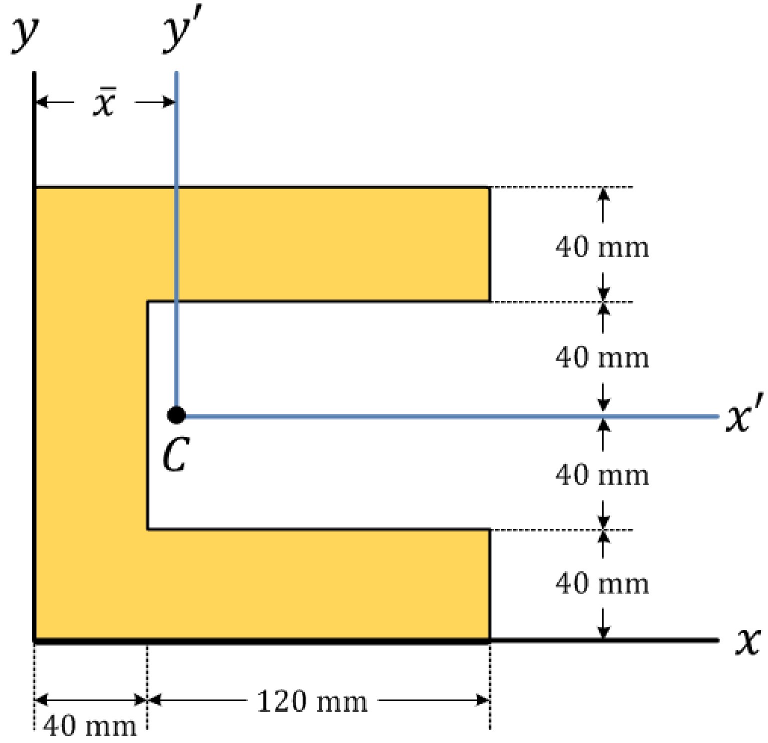
Determine $\bar{y}$, which locates the centroidal axis $x'$ for the cross-sectional area of the T-beam, and then find the moment of inertia about the $x'$ axis




W6 Example 4 (Web view)

# Review Problem

Describe three strategies you could use to determine the moment of inertia for the beam's cross-sectional area about the axis $x'$




1)

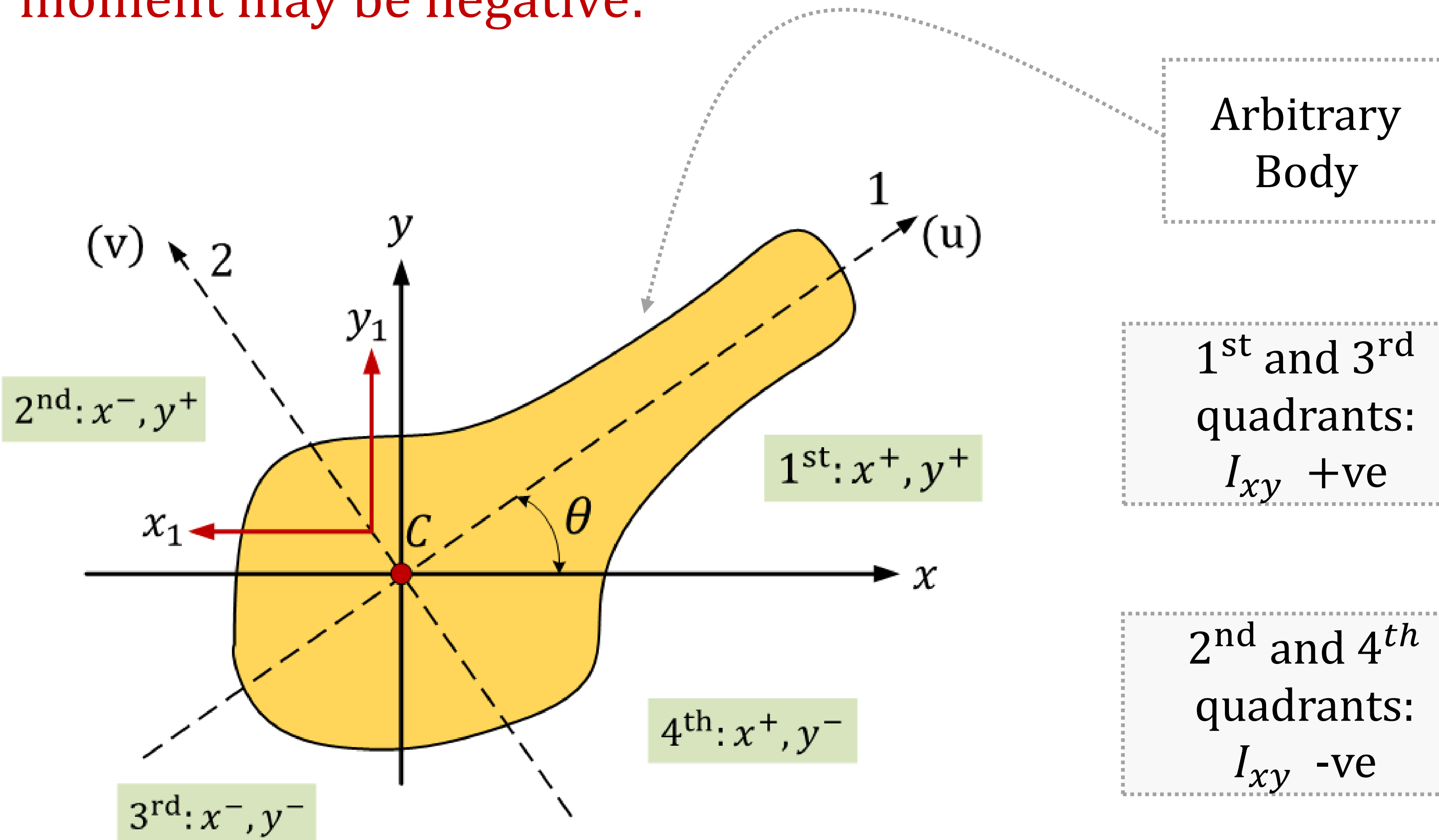
2)

3)

# Principal Axes and Mohr's Circle

# Product Second Moment of Area

Unlike second moments of area, the product second moment may be negative:




$1^{st}$ and $3^{rd}$ quadrants: $I_{xy}$ +ve

$2^{nd}$ and $4^{th}$ quadrants: $I_{xy}$ -ve

# Product Second Moment of Area

- The product second moment of the figure about the $xy$ axis is positive, since the predominant area lies in the first and third quadrants.
- If we consider the axes $x_1y_1$, however, the product of area is negative.

$$I_{x_1y_1} = -I_{xy}$$

- Thus, rotation of the axes by 90o has reversed the sign of $Ixy$. The change of Ixy as the axes rotate is a gradual one, hence, for some intermediate position of the axes, $Ixy = 0$.
- These are called the PRINCIPAL AXES.

# Rotation of Axes

Definition:

- The centroidal principle axes are those axes through the centroid for which the product second moment of area $I_{xy}$ is zero

*Suppose we know*

(i) Centroid and

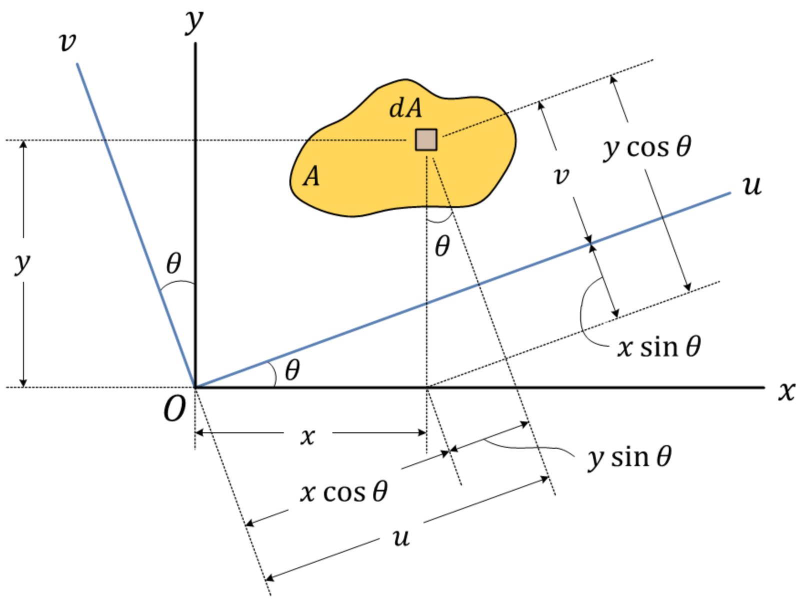
(ii) $I_{xx}, I_{yy}$ and $I_{xy}$

About an axes system through the centroid

We want to determine the angle of rotation of the axes $\theta$ such that

$$I_{uv} = 0$$

# Rotation of Axes




$$u = x\cos\theta + y\sin\theta$$

$$v = y\cos\theta - x\sin\theta$$

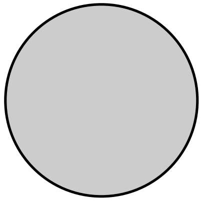
$$Want\ p\ such\ that\ I_{uv} = 0$$

**i.e.**

- Rectangle

$$I_{xy} \neq 0$$

**i.e.**



- Circle

$$I_{xy} = 0$$

# Transformation Of Second Moment Of Area

The moments and product of inertia becomes:

$$\boxed{I_{uu} = I_{yy}\sin^2\theta - 2I_{xy}\sin\theta\cos\theta + I_{xx}\cos^2\theta}$$

$$\boxed{I_{vv} = I_{yy}\cos^2\theta + 2I_{xy}\sin\theta\cos\theta + I_{xx}\sin^2\theta}$$

$$\boxed{I_{uv} = \left(I_{xx} - I_{yy}\right)\sin\theta\cos\theta + I_{xy}\left(\cos^2\theta - \sin^2\theta\right)}$$

# Transformation Of Second Moment Of Area

$$I_{uu} = \frac{I_{xx} + I_{yy}}{2} + \frac{I_{xx} - I_{yy}}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_{vv} = \frac{I_{xx} + I_{yy}}{2} - \frac{I_{xx} - I_{yy}}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

**Note:** $\boxed{I_{uu} + I_{vv} = I_{xx} + I_{yy}}$

"The sum of the second moments of area with respect to all pairs of rectangular axes having a **common origin**, is a constant"

$$I_{uv} = \frac{I_{xx} - I_{yy}}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

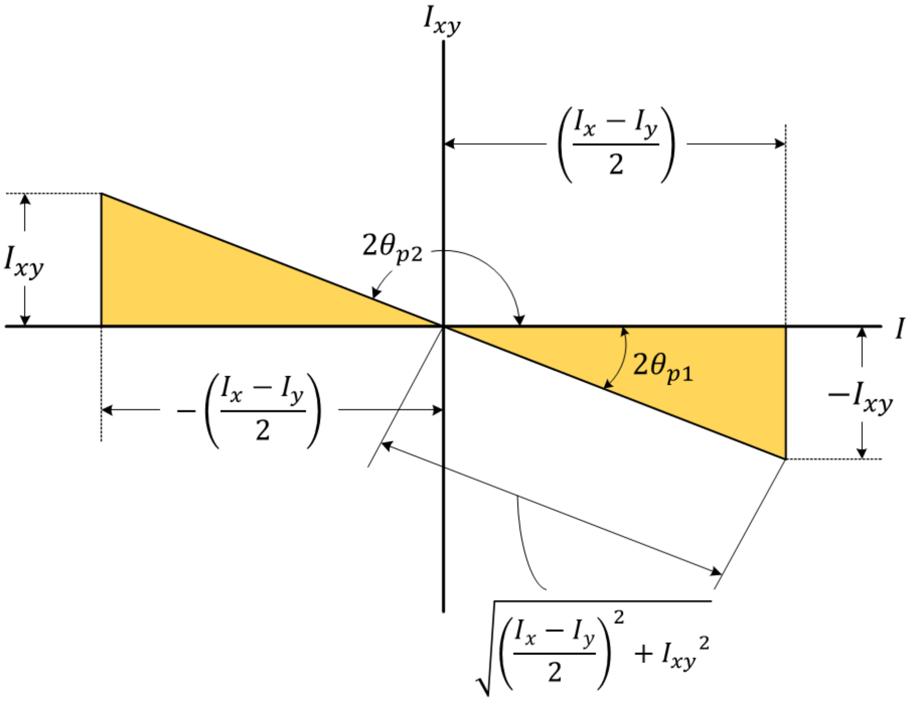
# Principal Moment of Inertia

$$I_{uv} = \frac{I_{xx} - I_{yy}}{2} \sin 2\theta_p + I_{xy} \cos 2\theta_p = 0$$

$$\tan 2\theta_p = \frac{-I_{xy}}{(I_{xx} - I_{yy})/2}$$

$$\sin 2\theta_{p1} = \frac{-I_{xy}}{\sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + I_{xy}{}^2}}$$

$$\sin 2\theta_{p2} = \frac{I_{xy}}{\sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + I_{xy}{}^2}}$$



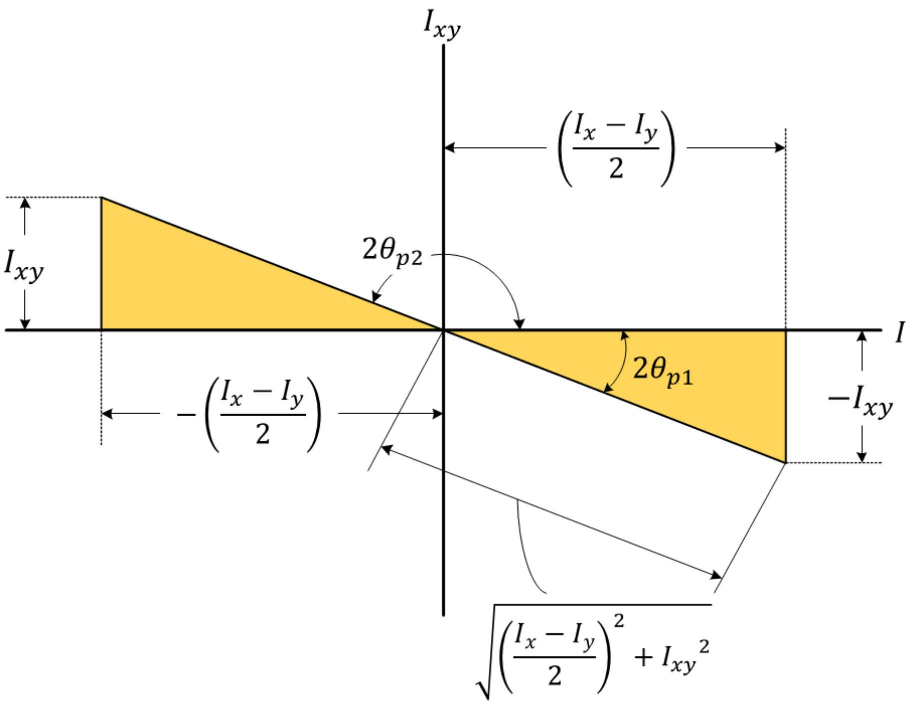
# Principal Moment of Inertia

$$I_{uv} = \frac{I_{xx} - I_{yy}}{2} \sin 2\theta_p + I_{xy} \cos 2\theta_p = 0$$

$$\tan 2\theta_p = \frac{-I_{xy}}{(I_{xx} - I_{yy})/2}$$

$$\cos 2\theta_{p1} = \frac{\left(\frac{I_{xx} - I_{yy}}{2}\right)}{\sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + I_{xy}{}^2}}$$

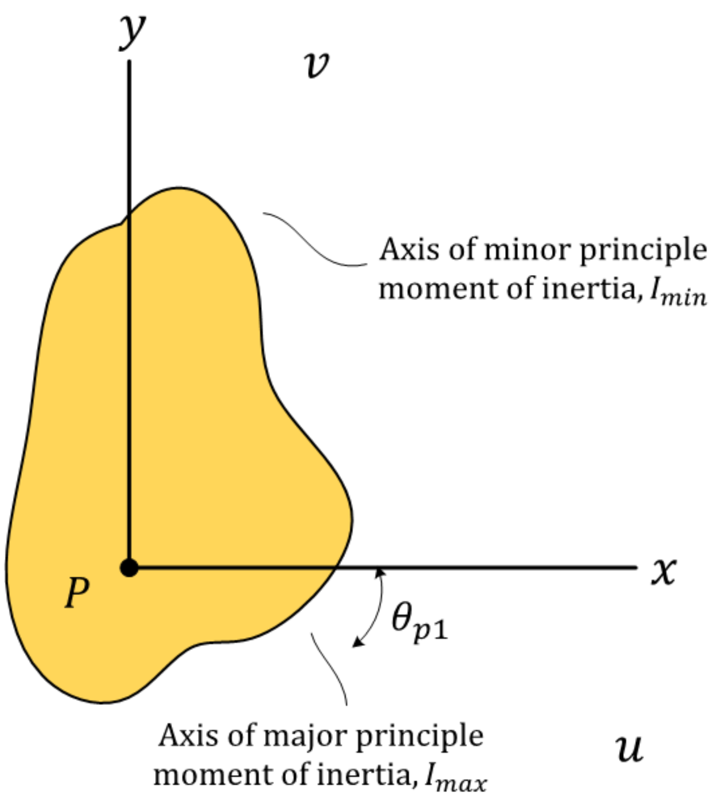
$$\cos 2\theta_{p2} = \frac{-\left(\frac{I_{xx} - I_{yy}}{2}\right)}{\sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + I_{xy}{}^2}}$$



# Mohr's Circle

$$I_{max} = \left(\frac{I_{xx} + I_{yy}}{2}\right) + \sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + {I_{xy}}^2}$$

$$I_{min} = \left(\frac{I_{xx} + I_{yy}}{2}\right) - \sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + {I_{xy}}^2}$$



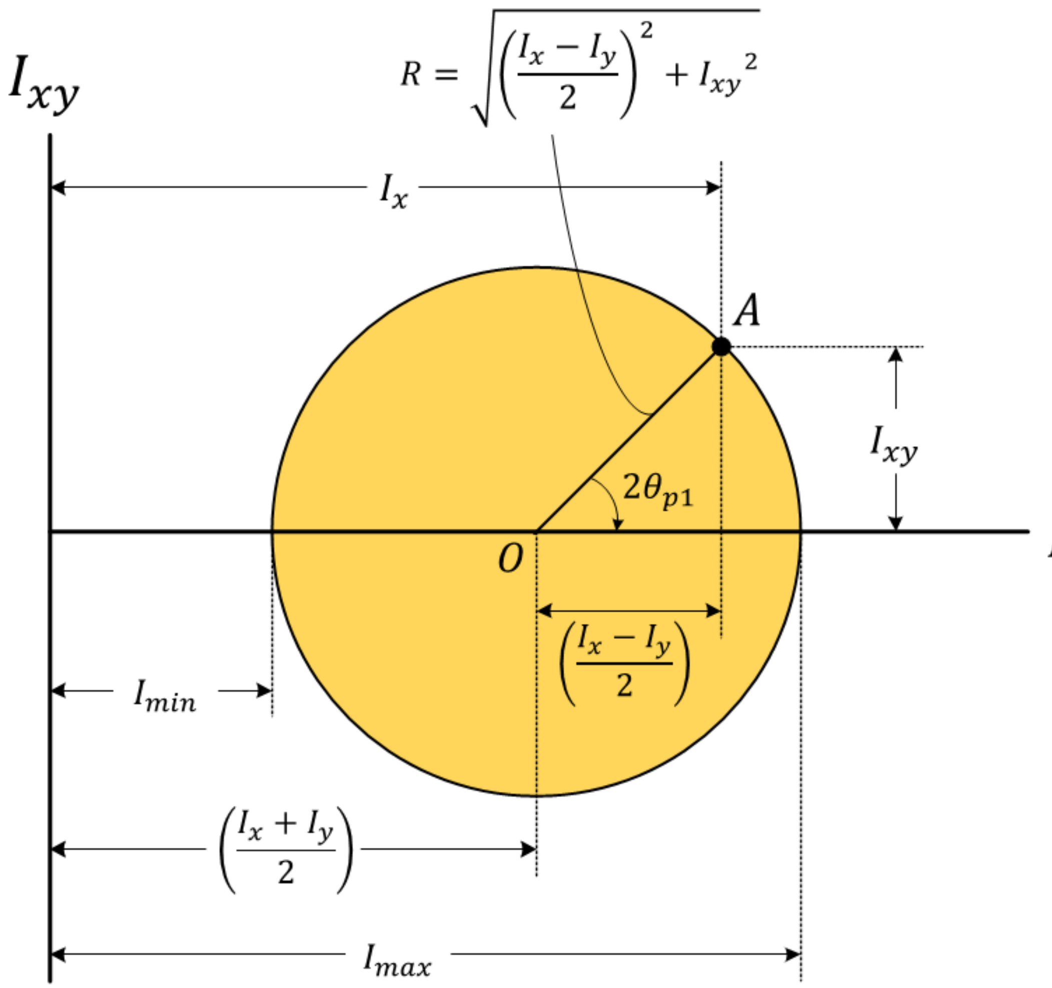

$$\boxed{\left(I_{uu} - \frac{I_{xx} + I_{yy}}{2}\right)^2 + {I_{uv}}^2 = \left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + {I_{xy}}^2}$$

# Mohr's Circle

$$(I_{uu} - c)^2 + I_{uv}{}^2 = R^2$$

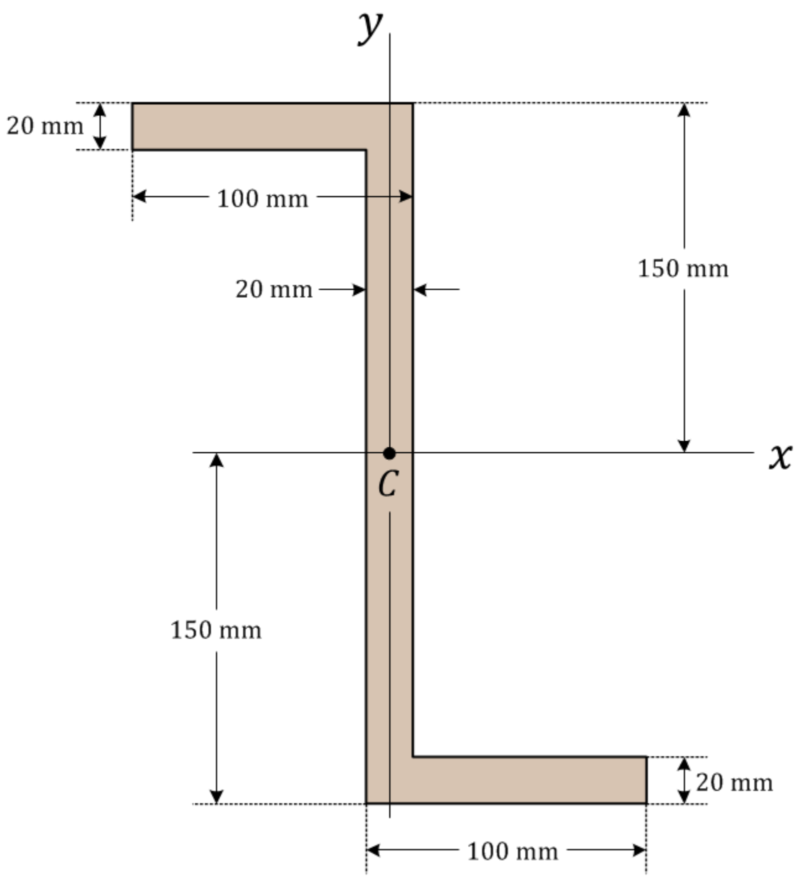
$$c = \left(\frac{I_{xx} + I_{yy}}{2}\right)$$

$$R = \sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + I_{xy}{}^2}$$



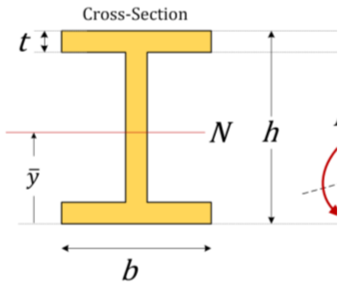
# Example 5

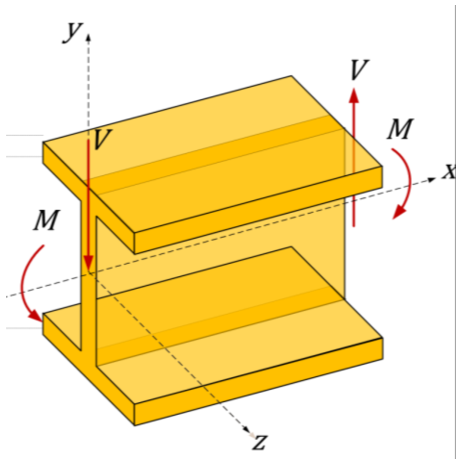
Determine the orientation of the principal axes, which have their origin at centroid $C$ of the beam's cross-sectional area. Also, find the principal moments of inertia.




W6 Example 5 (Web view)

# Summary




- Centroids $\bar{x} = \frac{\sum_i \bar{x}_i A_i}{\sum_i A_i}$ $\bar{y} = \frac{\sum_i \bar{y}_i A_i}{\sum_i A_i}$
- Moments of Inertia

$$I_y = \int_A x^2 dA \qquad I_{xx} = \frac{bd^3}{12}$$

- Parallel axis theorem

$$I_{xx} = I_{x'x'} + {d_y}^2 A \qquad I_{xy} = I_{x'y'} + d_x d_y A$$

- Principal axes of moment of inertia

$$I_{min/max} = \left(\frac{I_{xx} + I_{yy}}{2}\right) +/- \sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + {I_{xy}}^2}$$

# Next Topic:

*Particle Kinematics*