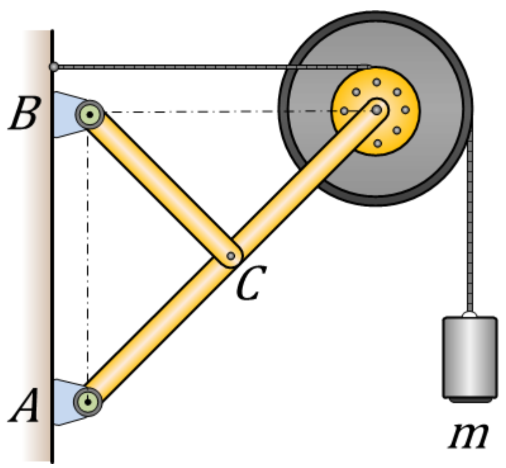
School of Mechanical and Manufacturing Engineering

MMAN1300 Engineering Mechanics 1

Dr. David C. Kellermann and Dr. Muhammad Danish Haneef

Week 4, L1-2: Frames, Machines Friction and Springs



FRAMES

- Rigid vs collapsible structures
- Force analysis and FBDs

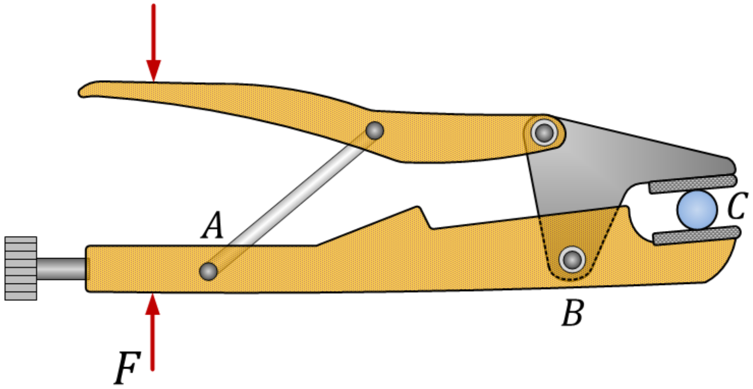
MACHINES AND PULLEY SYSTEMS

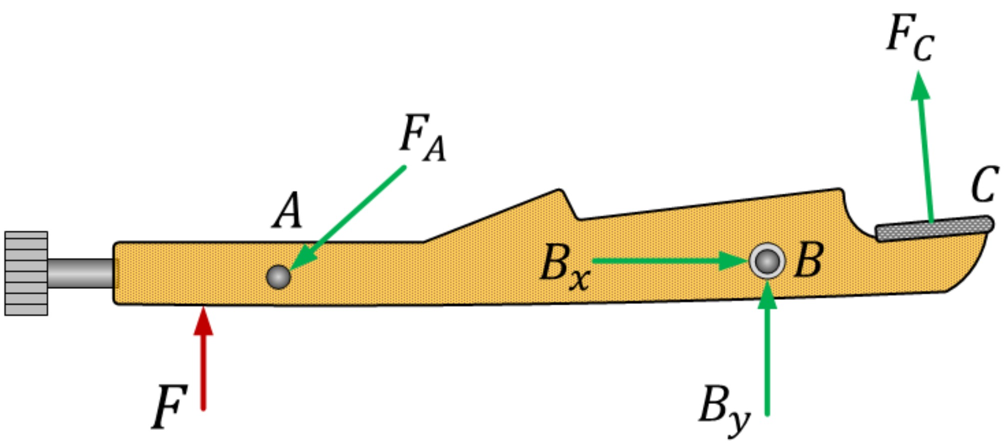
- Definitions: mechanical components
- Pulley systems

FRICTION AND SPRINGS

Frames and Machines are similar to trusses

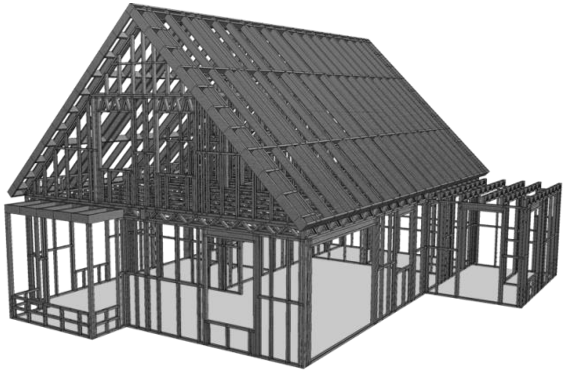
- The difference is that frames and machines contain at least one multi-force member
- A multi-force member is any member with more than two forces





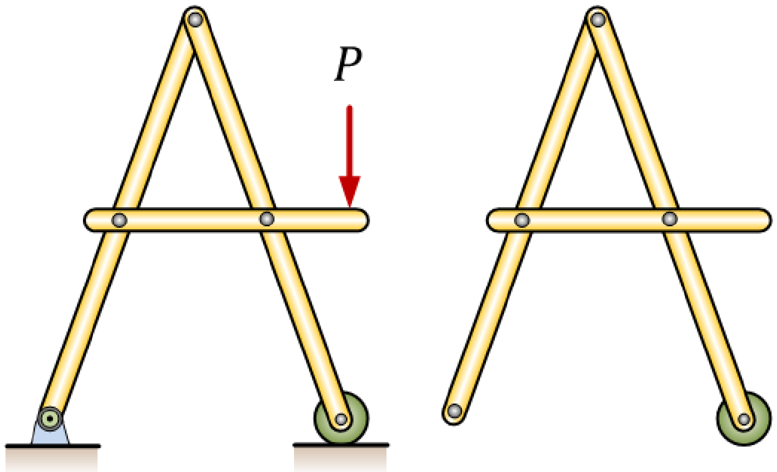
Frames vs. Machines

- Frames are structures which are designed to support loads and are usually fixed in position

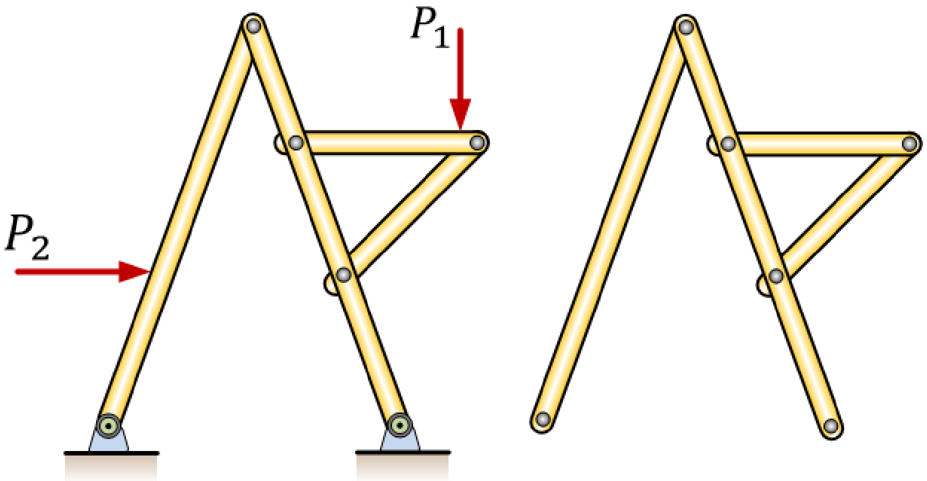


- Machines are structures which contain moving parts and are designed to transmit forces and/or moments

Rigid vs. Collapsible structures



(a) Rigid noncollapsible

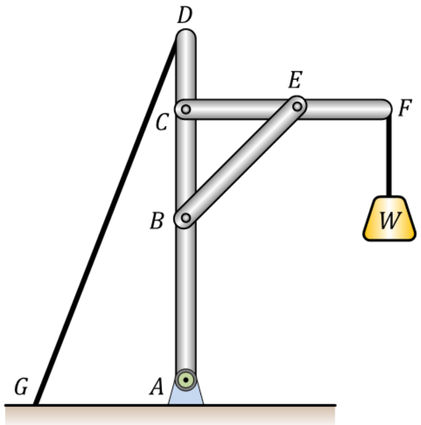


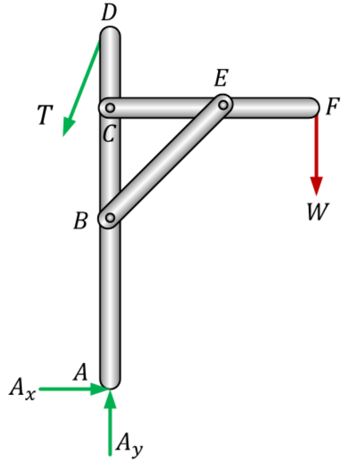
(b) Nonrigid collapsible

- If we disconnect a rigid frame from the rest of the world, its members will remain in the same configuration
- The members of a collapsible frame can move relative to one another if it is disconnected from the rest of the world

Force Analysis in Frames and Machines

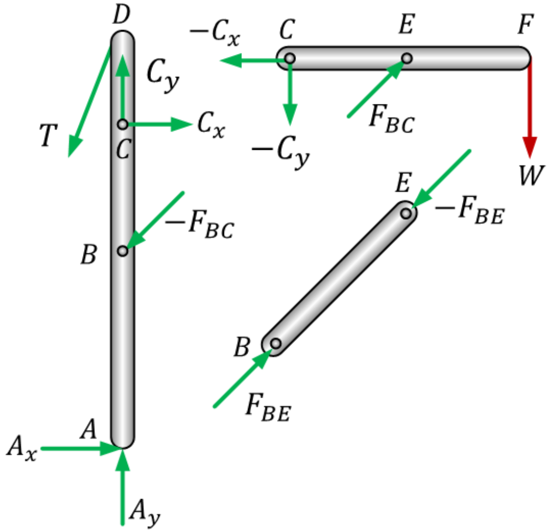
- Frames and machines are structures with at least one multi-force member. Frames are designed to support loads and are usually stationary. Machines contain moving parts and are designed to transmit and modify forces.





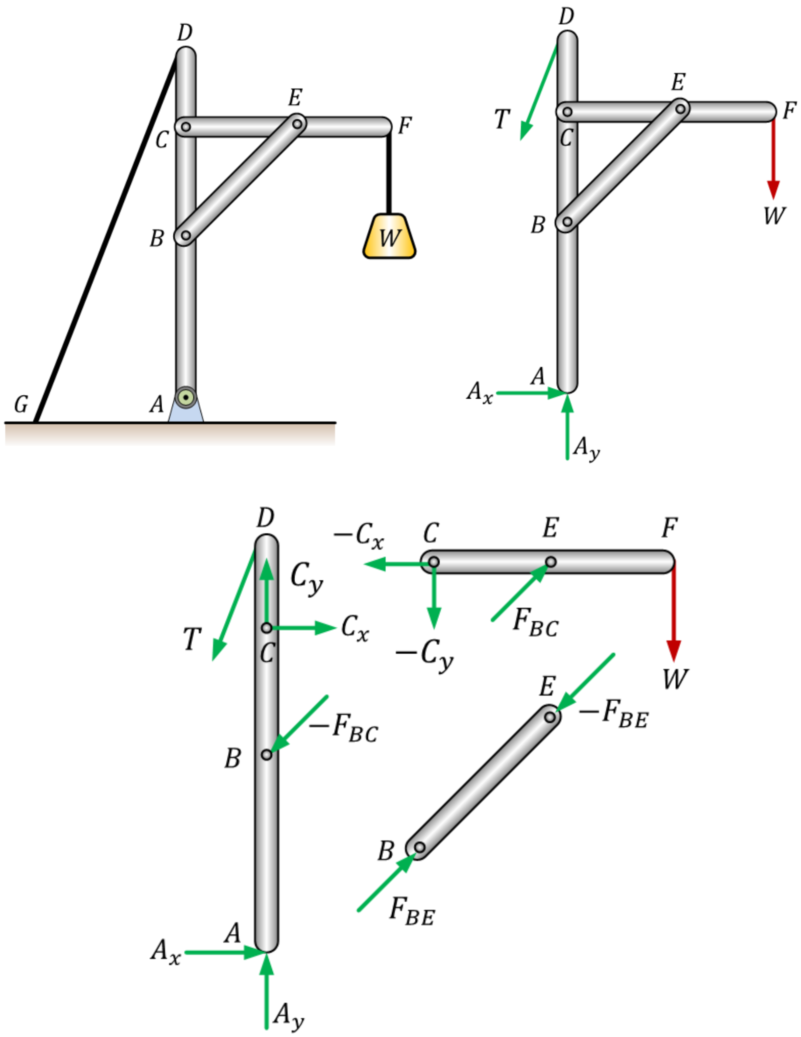
- A free body diagram of the complete frame is used to determine the external forces acting on the frame (similar to the truss analysis)

- Internal forces are determined by dismembering the frame and creating free-body diagram for each component



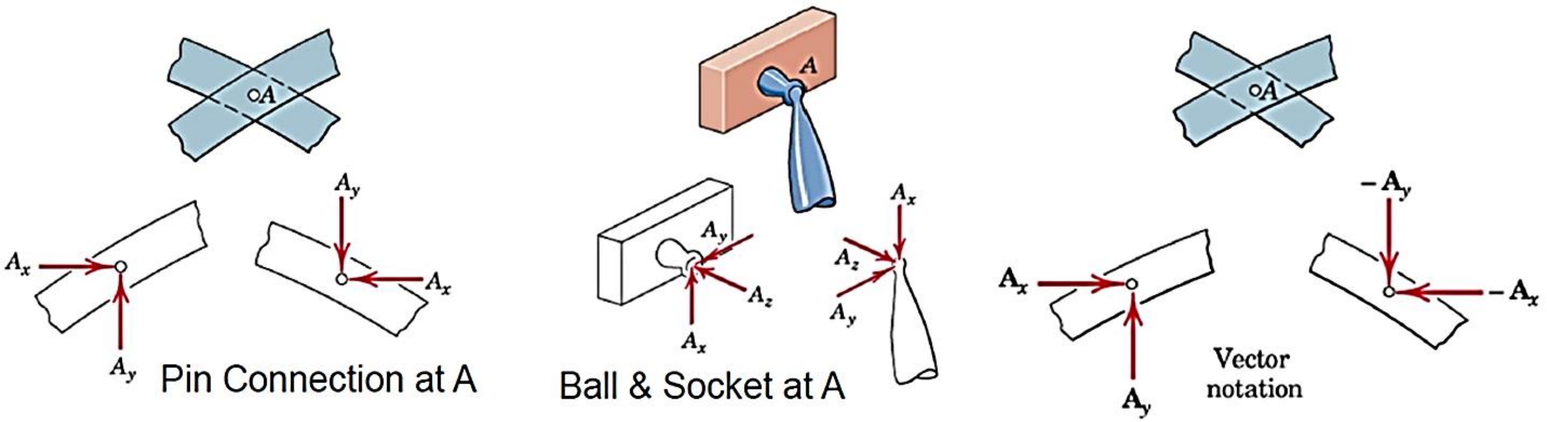
Force Analysis in Frames and Machines

- Forces on two-force members have known lines of action but unknown magnitude and sense
- Forces on multi-force members have unknown magnitude and line of action. They must be represented with two unknown components
- Forces between connected components are equal, have the same line of action, and opposite sense.



Free-Body Diagrams: Forces of Interactions

- Force components must be consistently represented in opposite directions on the separate FBDs (e.g. Pin a A)
- Apply action-and-reaction principle (e.g. Ball and socket at A)
- Vector notation: use plus sign for an action and a minus sign for the corresponding reaction



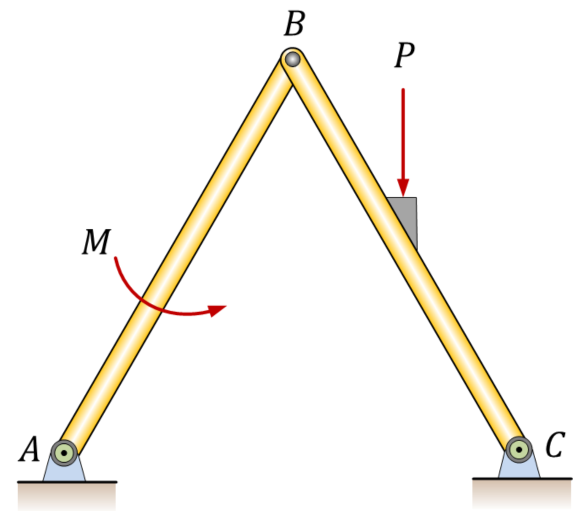
Example: Free-Body Diagram

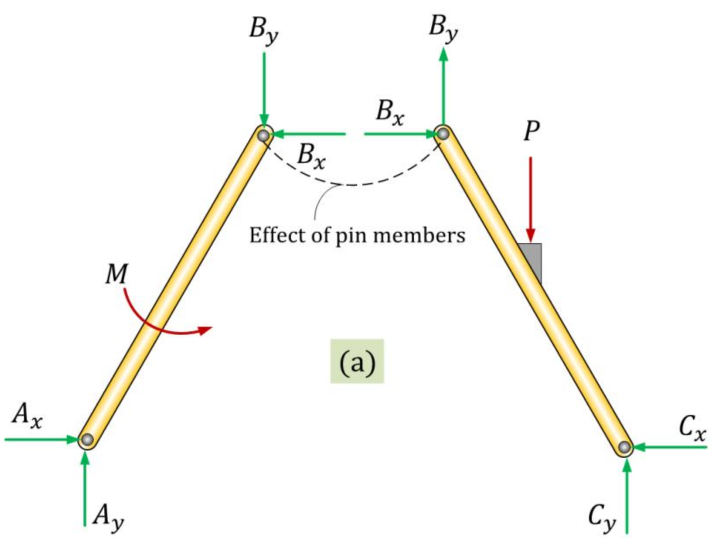
Draw FBD of

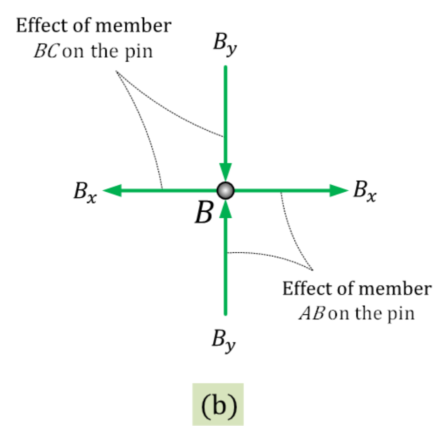
(a) Each member

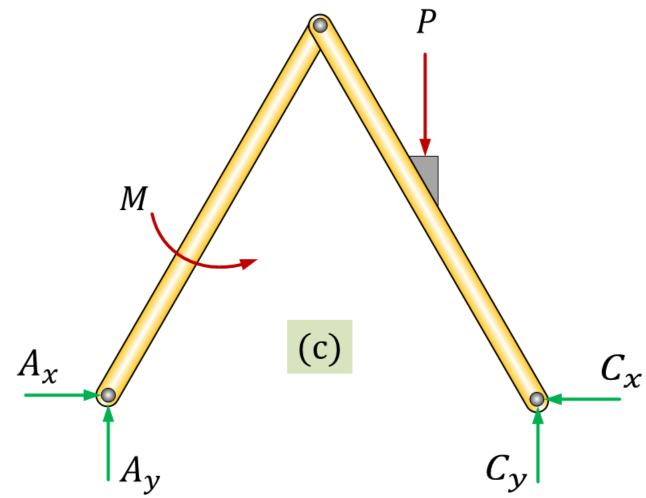
(b) pin at B, and

(c) whole system



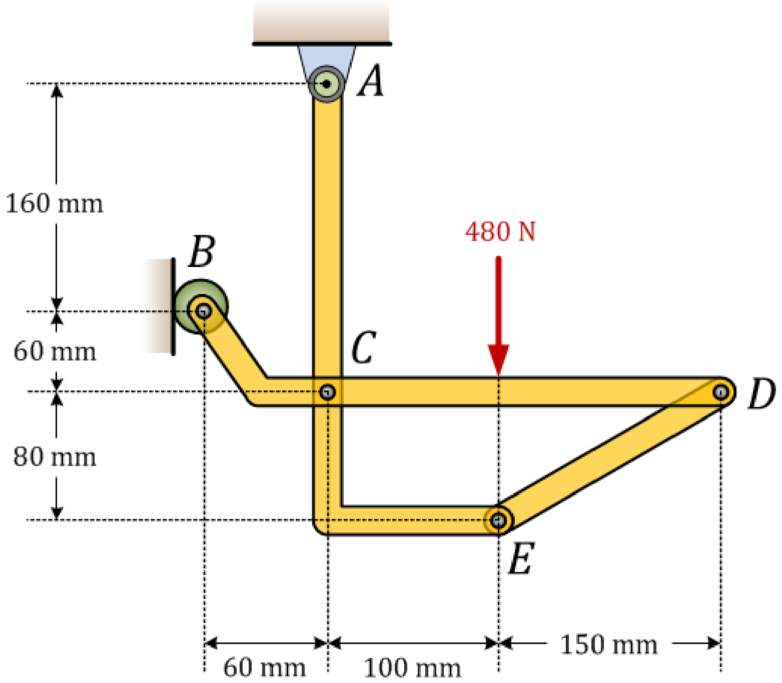






Example 1

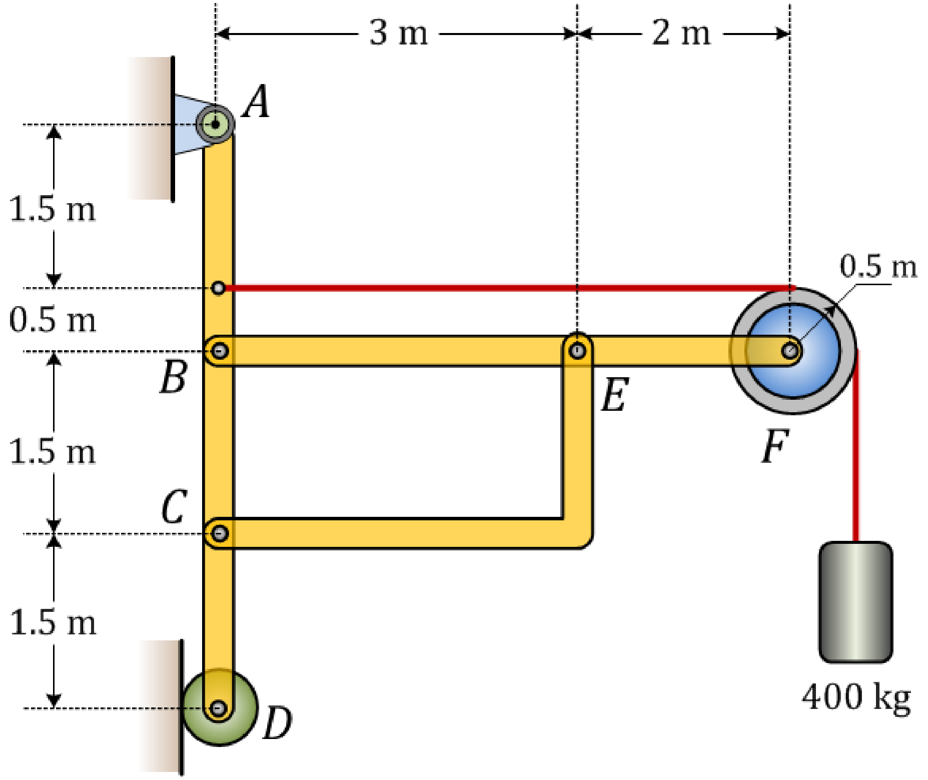
Member ACE and BCD are connected by a pin at C and by the link DE. For the loading shown, determine the force in link DE and the components of the force exerted at C on member BCD.



W4 Example 1 (Web view)

Example 2

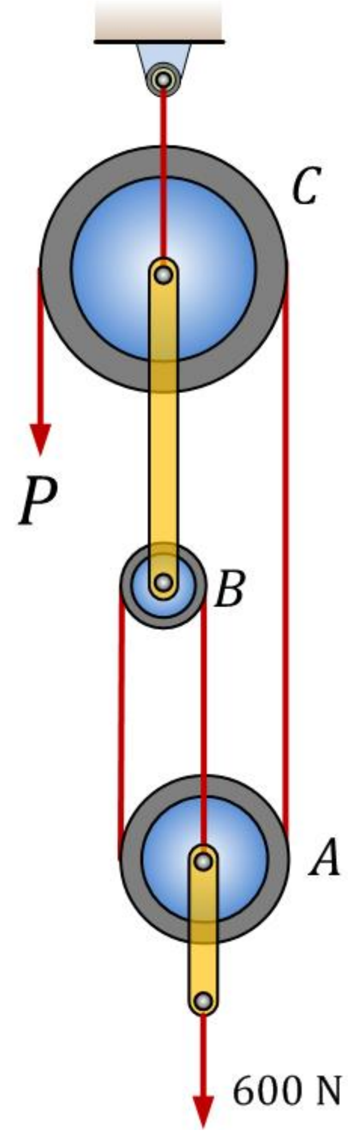
Compute the horizontal and vertical components of all forces acting on each of the members (neglect self weight)



W4 Example 2 (Web view)

Example 3

Find the tension in the rigid link connecting pulleys B and C and the force P required to support the 600 N force using frictionless pulley system (neglect self weight)



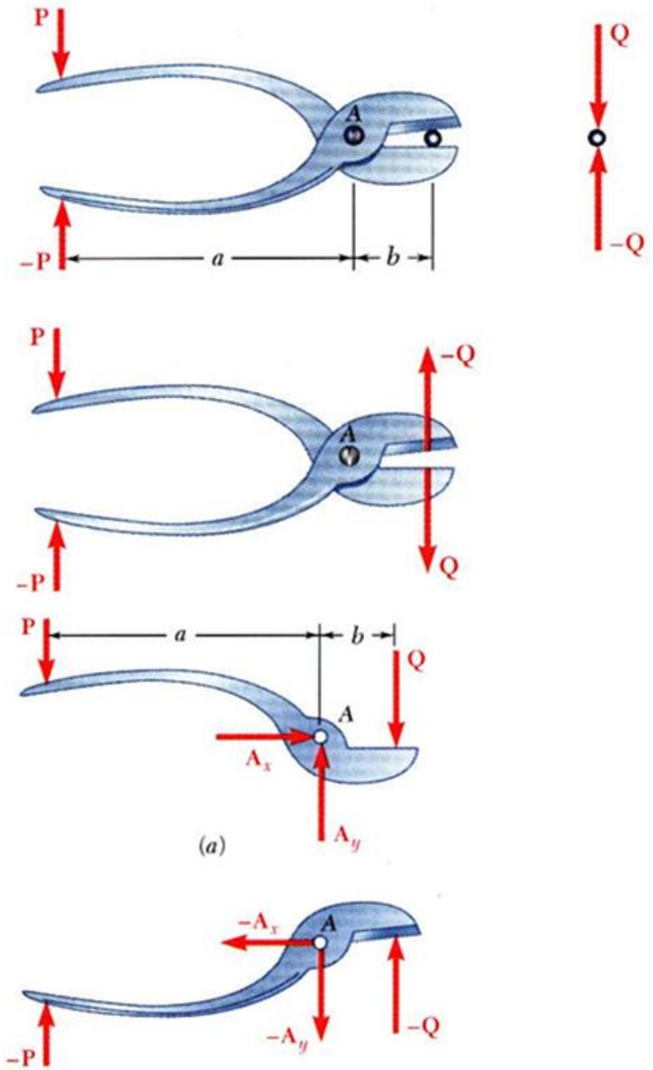
W4 Example 3 (Web view)

Machines

- Machines are structures designed to transmit and modify forces. Their main purpose is to transform input forces into output forces
- Given the magnitude of P, determine the magnitude of Q
- Create a free-body diagram of the complete machine, including the reaction that the wire exerts
- The machine is a nonrigid structure. Use one of the components as a free-body
- Taking moments about A

$$\sum M_A = 0 = aP - bQ$$

$$\boxed{Q = \frac{a}{b} P}$$



Machines

Definitions

- Effort: Force required to overcome the resistance to get the work done by the machine

- Mechanical Advantage: Ratio of load lifted (W) to the effort applied (P)

$$\text{Mechanical Advantage} = \frac{W}{P}$$

- Velocity Ratio: Ratio of the distance moved by the effort (D) to the distance moved by the load (d) in the same interval of time

$$\text{Velocity Ratio} = \frac{D}{d}$$

Machines

Definitions

- Input: Work done by the effort

$$\text{Input} = PD$$

- Output: Useful work got out of the machine i.e. the work done by the load

$$\text{Output} = WD$$

- Efficiency: Ratio of output to the input

Efficiency of an ideal machine is 1, in that case:

$$Wd = PD \longrightarrow \frac{W}{P} = \frac{D}{d}$$

For an ideal machine, mechanical advantage is equal to velocity ratio

Machines

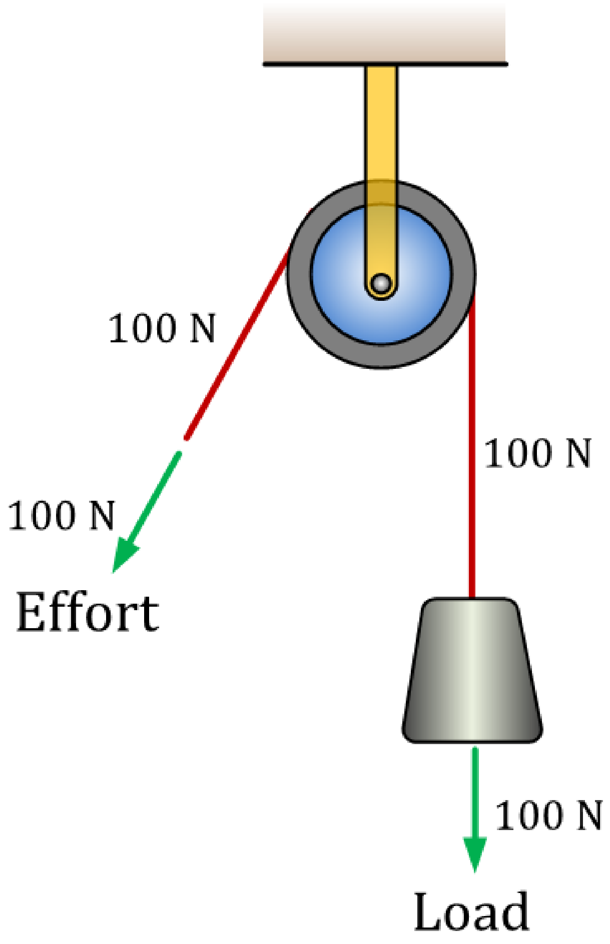
Fixed Pulley

Effort = Load

→ Mechanical Advantage = 1

Distance moved by effort is equal to the distance moved by the load

→ Velocity Ratio = 1



Machines

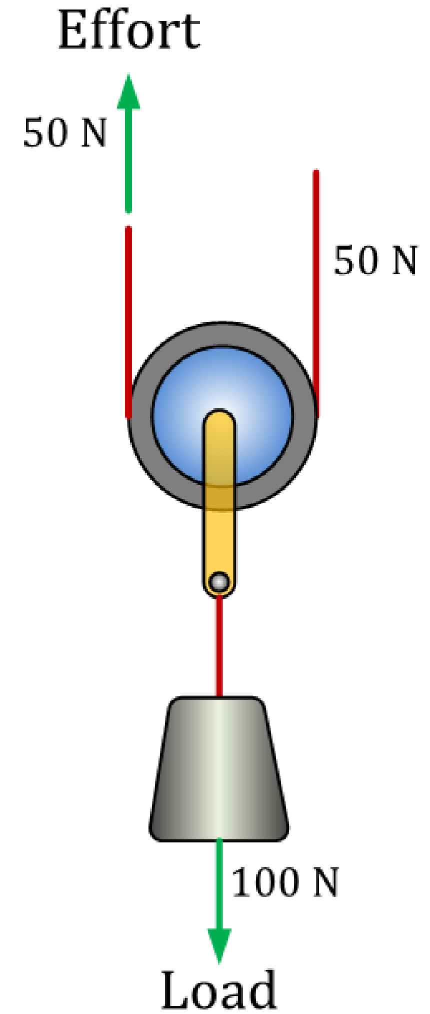
Movable Pulley

Effort = Load/2

→ Mechanical Advantage =2

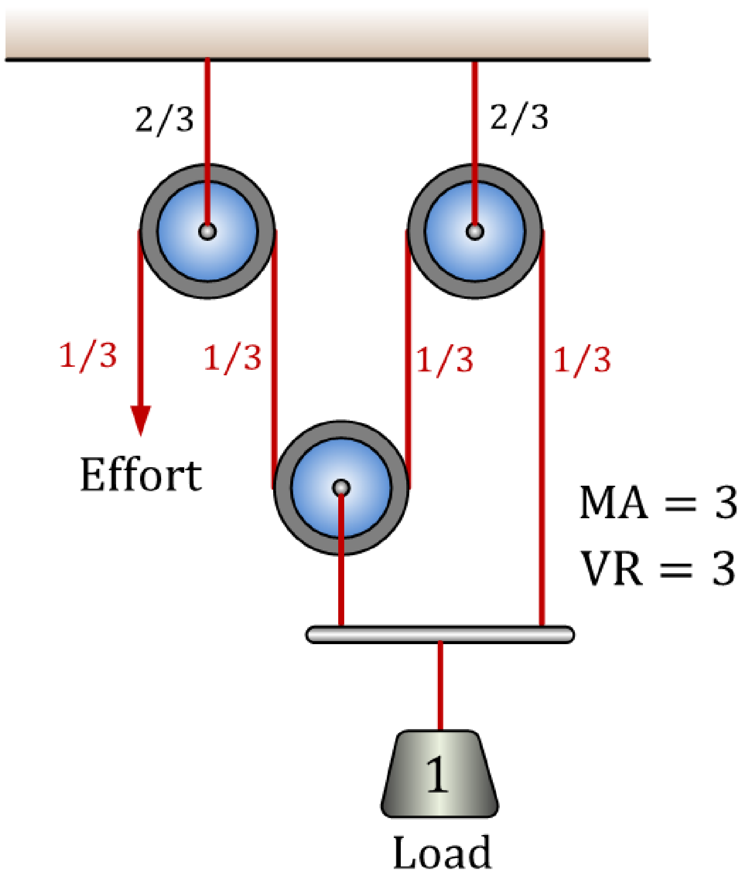
Distance moved by the effort is twice the distance moved by the load (both rope should also accommodate the same displacement by which the load is moved)

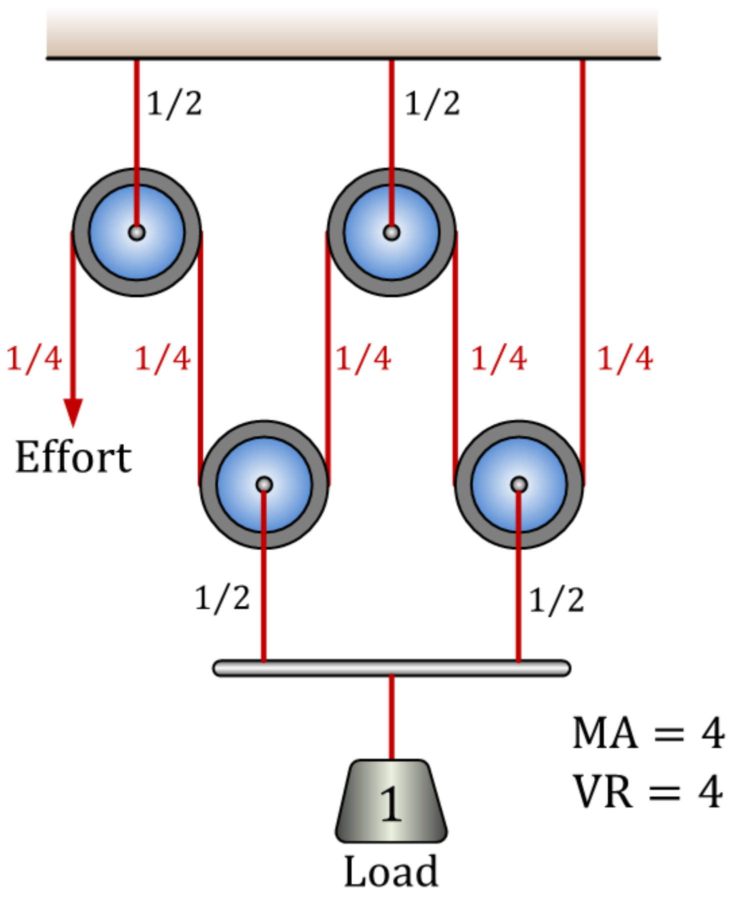
→ Velocity Ratio = 2



Machines

Compound Pulley





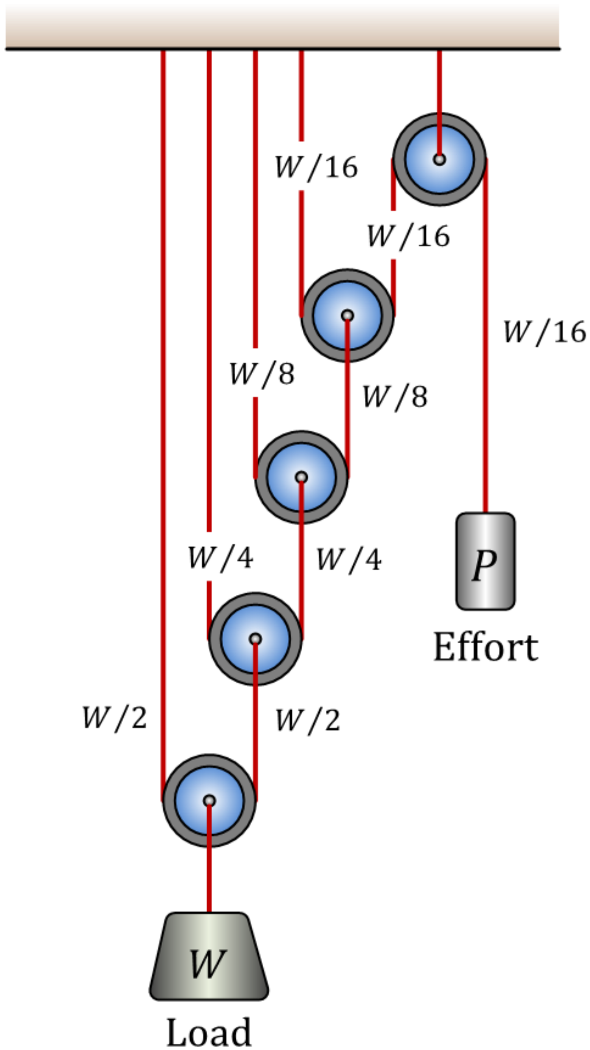
Machines

Compound Pulley

Effort required is 1/16th of the load

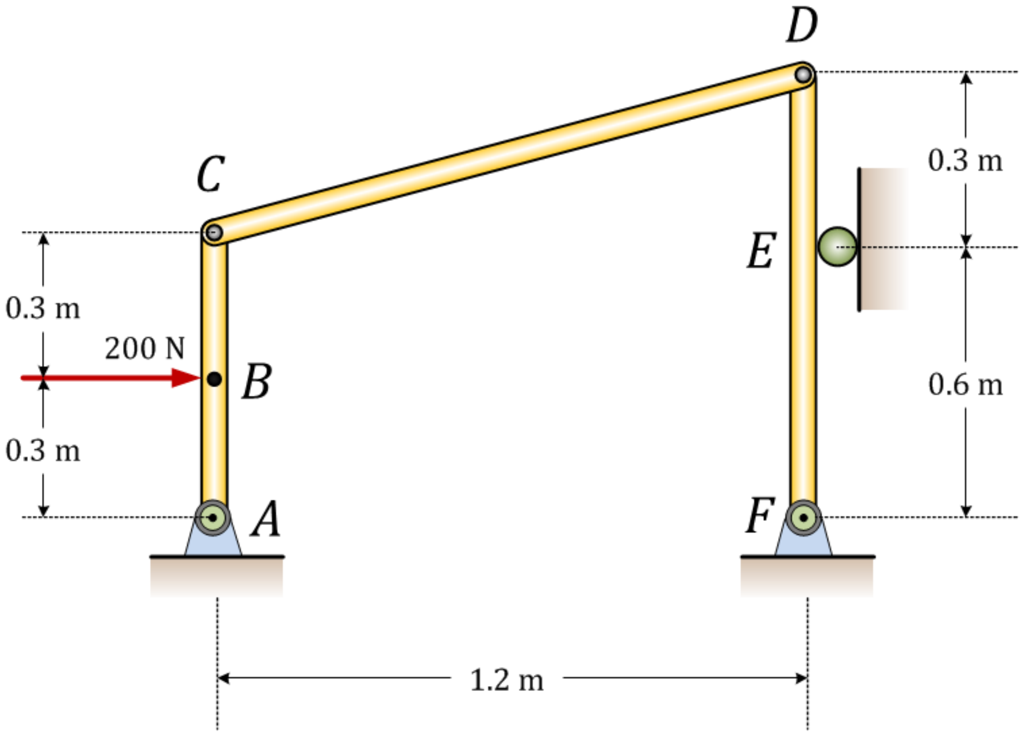
Mechanical Advantage = 16
(Neglecting frictional forces)

Velocity ratio is 16, which means in order to raise a load to 1 unit height, effort has to be moved by a distance of 16 units



Example 4

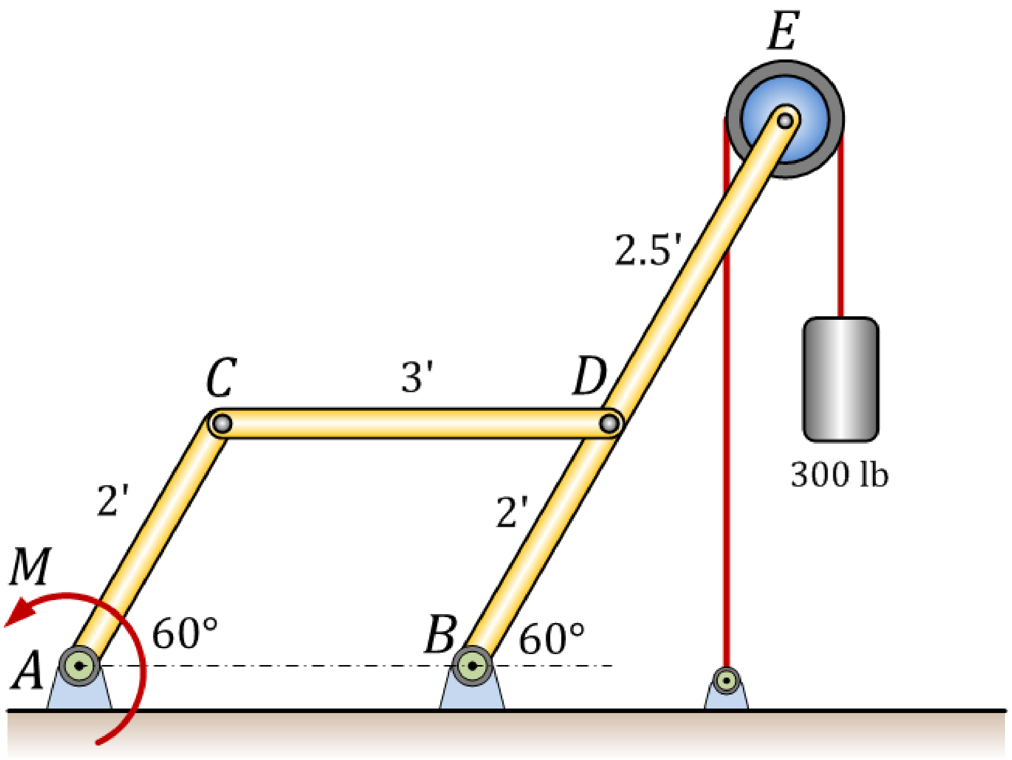
Calculate the force at the point E



W4 Example 4 (Web view)

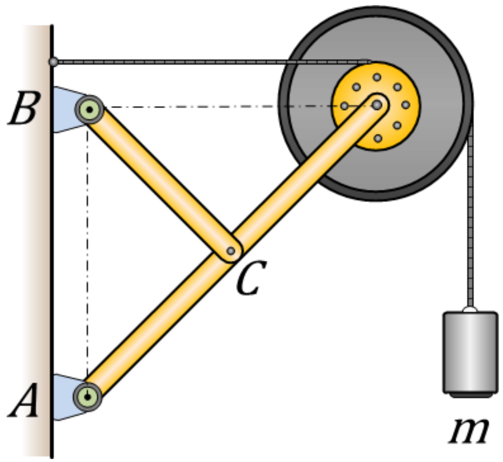
Example 5

Calculate the force and moment at point A

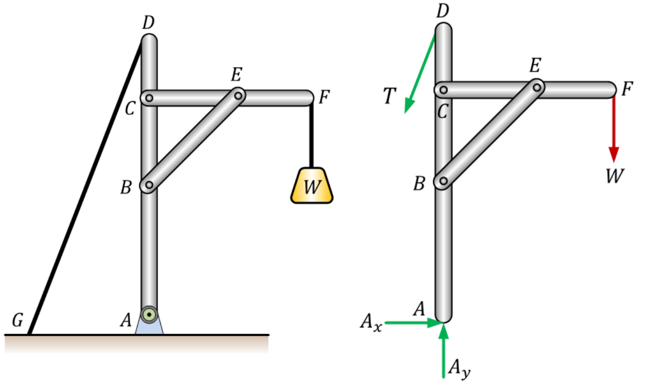


W4 Example 5 (Web view)

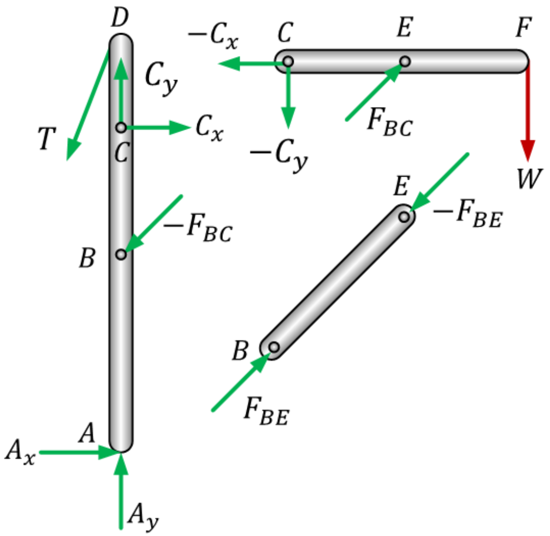
Summary



- A free body diagram of the complete frame is used to determine the external forces acting on the frame (similar to the truss analysis)



- Internal forces are determined by dismembering the frame and creating free-body diagram for each component



Next Topic:

Friction and Springs

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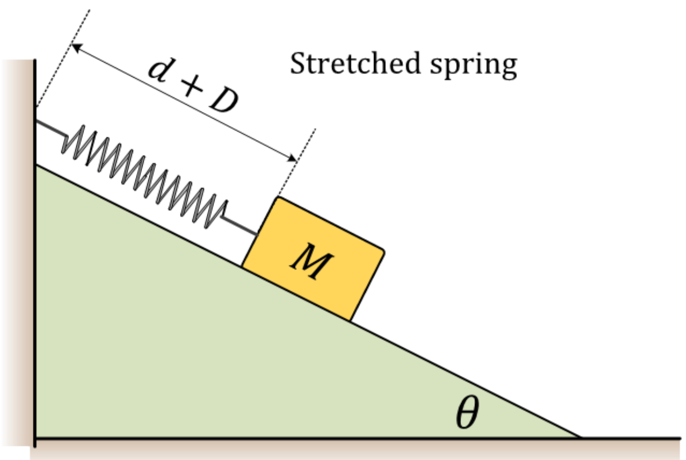
MMAN1300 Engineering Mechanics 1

Dr. David C. Kellermann



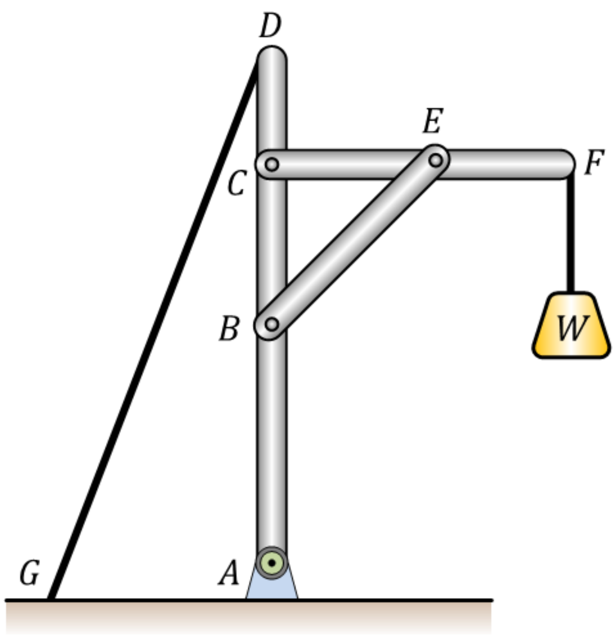
Week 4, L2 – Friction and Springs

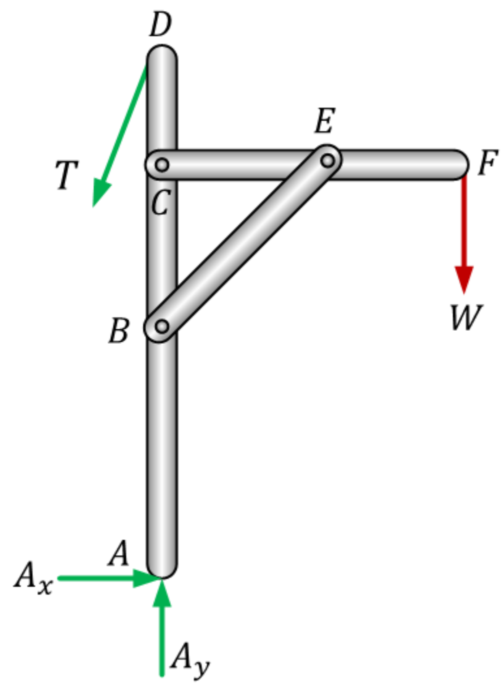
FRICTION
- Static and dynamic coefficient of friction
- Motion threshold
- Actual vs maximum friction force

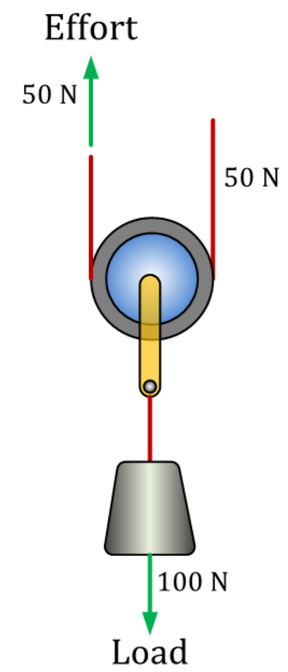
SPRINGS
- Spring equation
- Spring force and stretched lengths

Frames and Machines Review

The mechanical advantage of a machine can be incorporated into a frame







Example 5 – A Frame/Machine

At Central station, from Platform 22, you can see this tensioning device. It is both a Frame and a Machine.

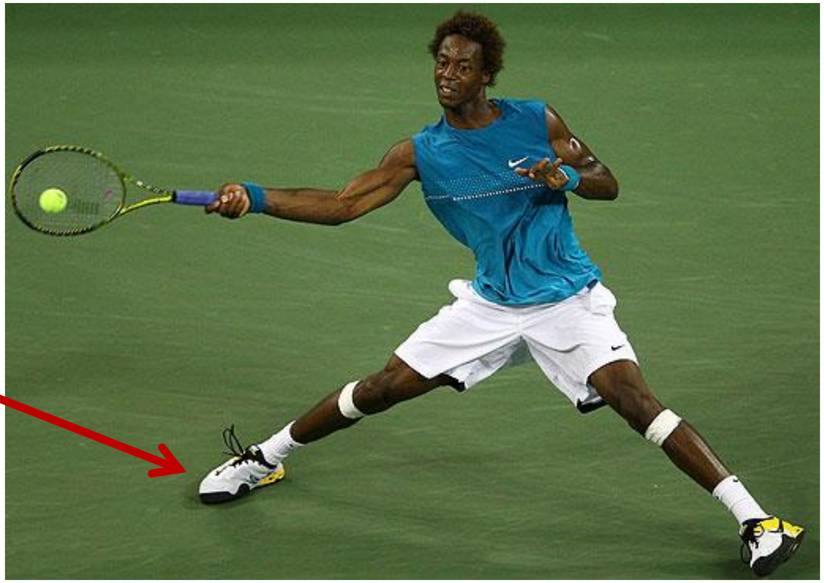
Calculate the declination angle theta of the bracket.

Assume the deadweight is 10 kg, the pulley radii are 100mm and the centre spacings are 300mm.

[W4 Example 6](#) ([Web view](#))

If you try to slide one solid object across another, you will notice a resistance to the sliding

Friction force is acting here



- In mechanics, we call the resistance the **friction force** (it should be obvious that the resistance constitutes a force

Friction force has a magnitude and a direction

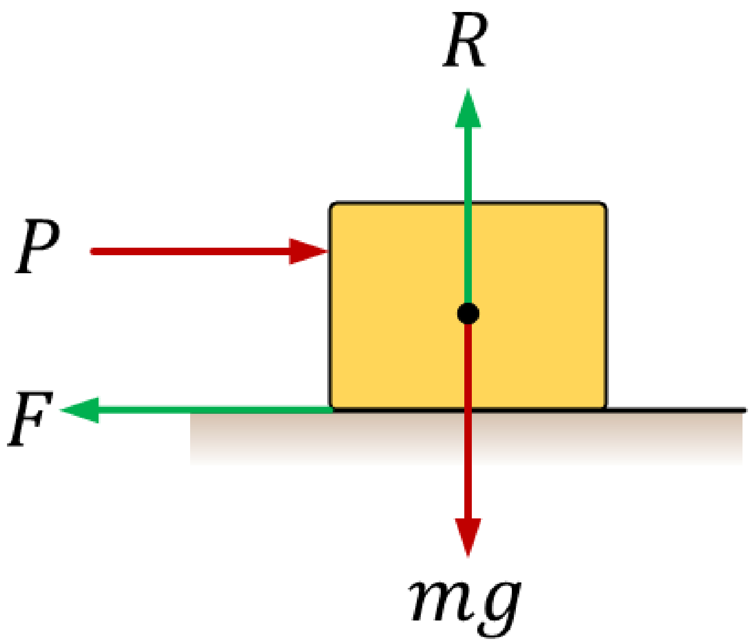
- The direction is parallel to the two contacting surfaces
- The direction is also such that it opposes the sliding
- If the two objects are not sliding with respect to each other, the direction of the force is such that it opposes the impending slip

But how do we find the magnitude?

- In this course, we will use the Coulomb model of friction forces
- This model has limitations, but is widely applicable in engineering
- As you will see, we have to consider 2 different cases: static and dynamic friction

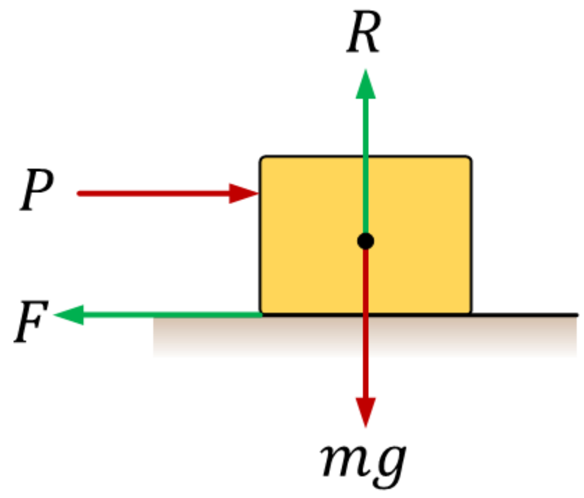
Let's do a little experiment

- Consider a block of mass m that is initially at rest on a horizontal plane
- Initially, the applied force $P = 0$
- Upon increasing P we can make some observations



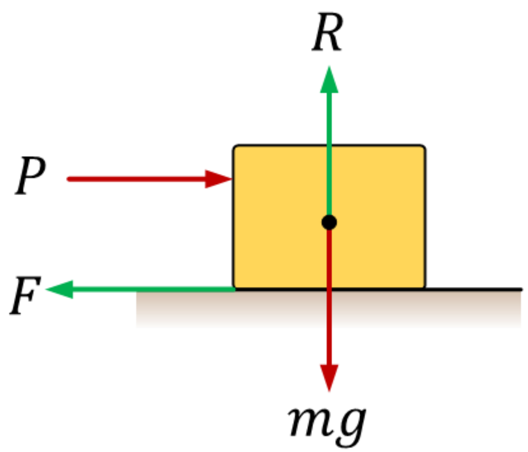
FBD of the block

What relevant observations do we make?



1. For small P, the block remains at rest
2. Beyond a critical value $P = P^*$, the block starts to move
3. Once in motion, a constant value of $P = P^{**}$, is required to move the block at constant speed
4. Both P^* and P^{**} are proportional to the magnitude of the reaction force R and independent of the contact area

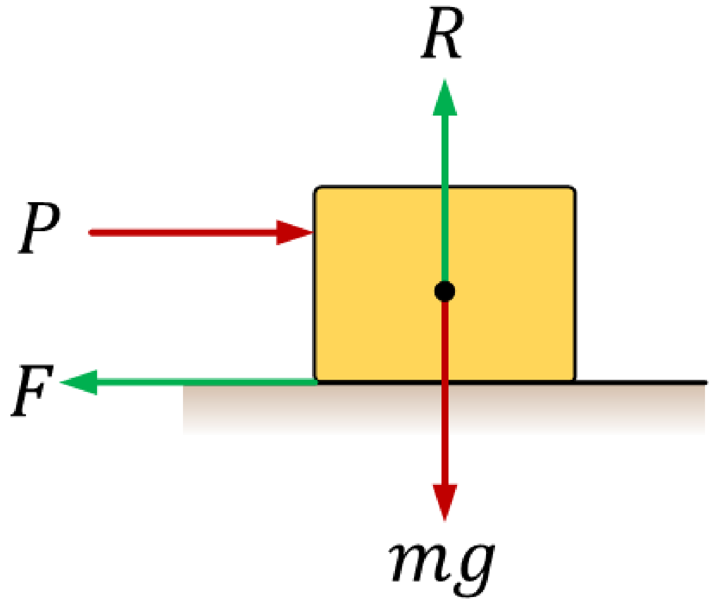
Let's take a closer look at the static case (up until the block starts to move)



- Take $\mathbf{F} = m\mathbf{a}$ in the horizontal (x) direction
- For the static $(\mathbf{a} = 0)$ case, $F = P$
- If P is increased beyond a critical value (P^*), the block starts to move

We can notice (observation #4 above) that P^* is proportional to R

- We denote the constant of proportionality by μ_s
- Hence, $|F| \leq \mu_S|R|$



Now let's examine the block once it starts moving

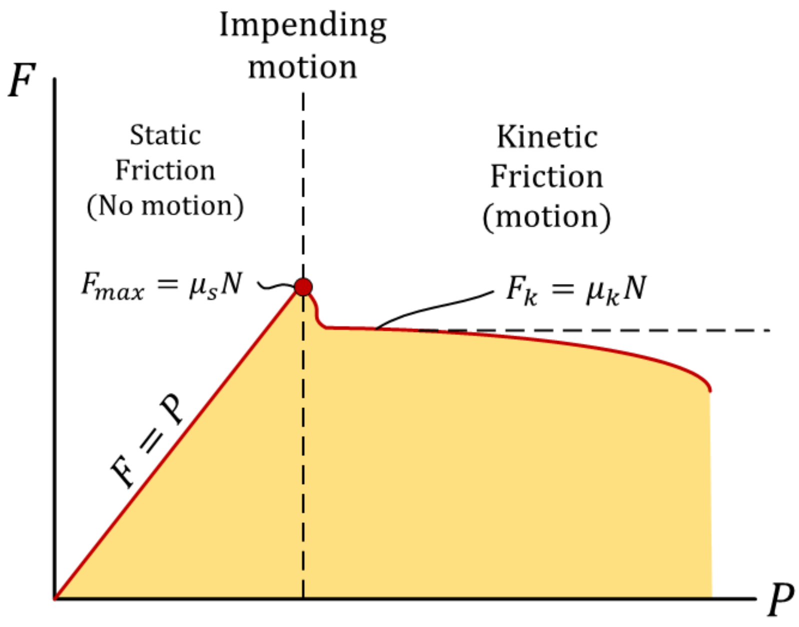
- When the block starts moving, the friction opposes motion
- The direction is opposite **v**
- As noted above, its magnitude is proportional to the magnitude of R, the reaction (normal) force
- Let's call the constant of proportionality μ_k
- Now Newton's 2$^{\text{nd}}$ Law gives us $F = \mu_k R$

Now we need some terminology

- We will call μ_s "The coefficient of **static** friction"
- We will call μ_k "The coefficient of **kinetic** friction"
- Generally $\mu_k < \mu_s$
- Coefficients of friction are particular to pairs of materials (i.e. steel on steel, brass on steel, plastic on wood etc.)

We cannot use $F_f = \mu_s N$ to calculate friction force in the static case

- The motion is known (i.e. there is no relative motion between the contacting objects)
- So we can use $\mathbf{F} = m\mathbf{a}$ to find F_f even if $\mathbf{a} = 0$
- The equation $F_f = \mu_s N$ is only valid at impending slip



Static Friction is a reaction force!

DO NOT use $F_f = \mu_s N$ to find the static friction force

$(Except\ for\ at\ impending\ slip)$

Linear Springs Hooke's Law

- "Ut tensio sic vis"

  –*Robert D. Hooke FRS*

- Translation: The power of any spring is in the same proportion with the tension thereof

- This is known as Hooke's Law

Robert Hooke

Physicist, Academic, Scientist, Scholar

(1635–1703)

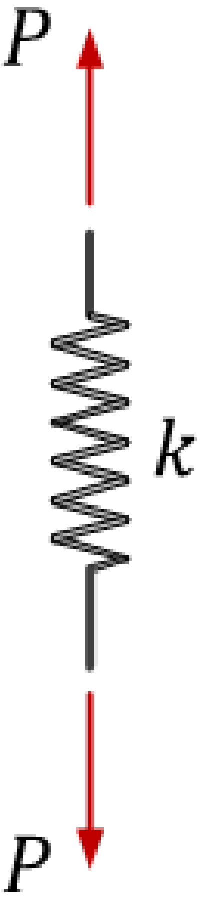
Hooke's Law

- Consider a FBD of a length of spring

- Springs are considered to be massless, nut unlike strings, springs maybe stretched or compressed

- If the spring extends by an amount δ from its unstretched length, then the stiffness of the spring is defined as:

$$k = \frac{P}{\delta}$$



Linear Springs

- With Hooke's Law, the relationship between the force and the displacement is linear

- The magnitude is given as $F_s = k\delta$
  - Where $\delta = L - L_0$
  - L is the current length of the spring
  - L_0 is the unstretched length of the spring

- The direction is opposite the stretch (or compression) of the spring

Some notes on Springs

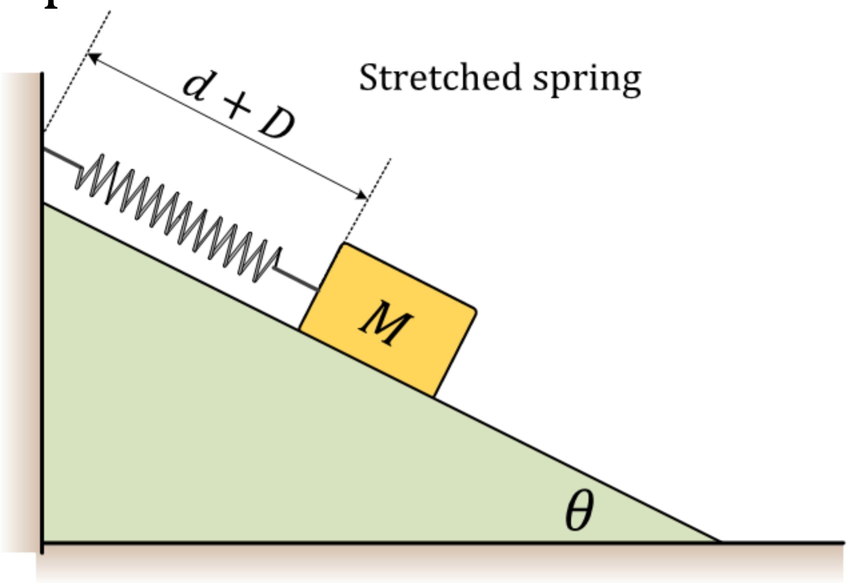
- It is easy to think of situations where Hooke's Law does not hold (but we will not deal with them in this course)

- In this course, we will solve most problems involving springs using Work/Energy Methods

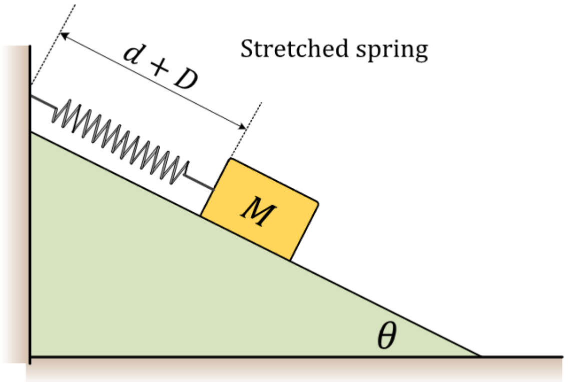
- These will be covered in the coming weeks

Example 6

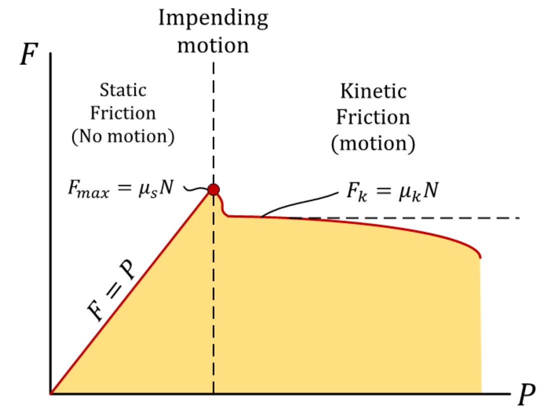
A spring with coefficient k is stretched by and amount d from it's unstretched length D by a mass M on an incline of angle θ. Calculate the minimum friction coefficient required to maintain equilibrium on the assumption that the friction is acting up the slope.



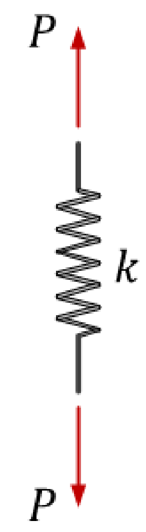
Summary



- Potential friction force- the equation $F_f = \mu_s N$ is only valid at impending slip



- Spring Force is given as $F_s = k\delta$
  - Where $\delta = L - L_0$
  - L is the current length of the spring
  - L_0 is the unstretched length of the spring



Next Topic:

Distributed Loads, Shear Force and Bending Moments