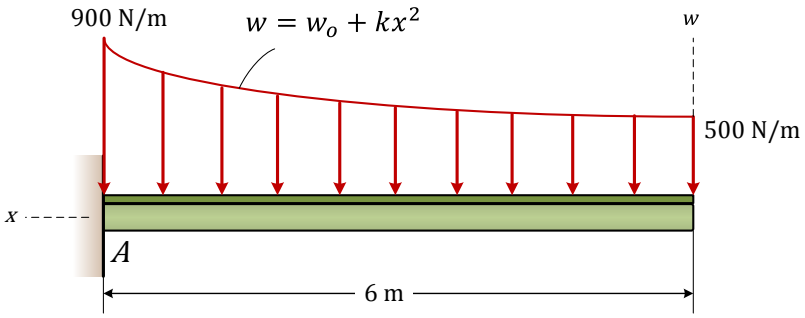
Question 5.7.

Draw the shear force and bending moment diagrams for the loaded cantilever beam shown. What is the shear force V and bending moment M at the midlength of the beam?



Solution

At $A = 0$

$w = w_o = 500\ \text{N/m}$

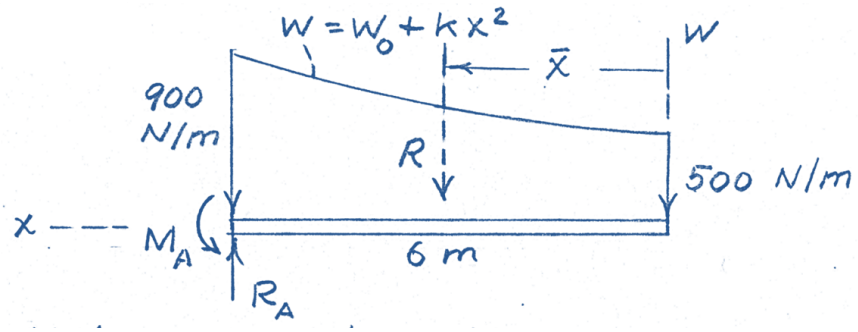
At $x = 6\ \text{m}$

$w = 900 = w_o + kx^2$

$900 = 500 + k(6)^2$

$k = \left(\frac{100}{9}\right)\ \text{N/m}^3$

Therefore,



$w = 500 + \frac{100}{9}\,x^2 \quad (\text{N/m})$

$R = \int w\,dx$

$R = \int_0^6 \left(500 + \frac{100}{9}\,x^2\right) dx$

$R = \left|500x + \frac{100}{27}\,x^3\right|_0^6 = 3800\ \text{N}$

$\bar{x} = \frac{\int x\,w\,dx}{R}$

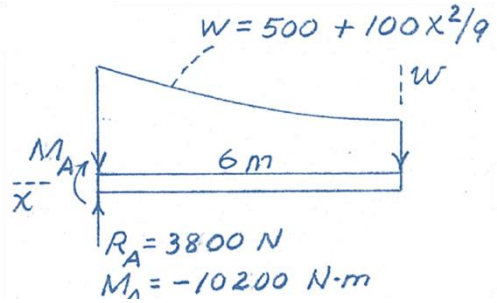
$\bar{x} = \int_0^6 \left(500x + \frac{100}{9}\,x^3\right) dx\ /3800$

$\bar{x} = \frac{\left|250x^2 + \frac{25}{9}\,x^4\right|_0^6}{3800} = 3.32\ \text{m}$

$+\uparrow \sum F_y = 0$: $\quad R_A - 3800 = 0$ $\quad R_A = 3800\ \text{N} = 3.8\ \text{kN}$

$+\circlearrowleft \sum M_A = 0$: $\quad M_A - 3800(6 - 3.32) = 0$ $\quad M_A = 10200\ \text{N} = 10.2\ \text{kN.m}$

For an element of length x,

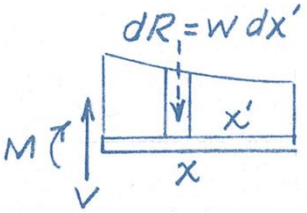


$+\uparrow \sum F_y = 0$:

$$V - \int w\, dx' = 0$$

$$V = \int_0^x \left(500 + \frac{100}{9}\, x'^2\right) dx'$$

$$V = \left(500x + \tfrac{100}{27}\, x^3\right) \qquad \text{(N)}$$

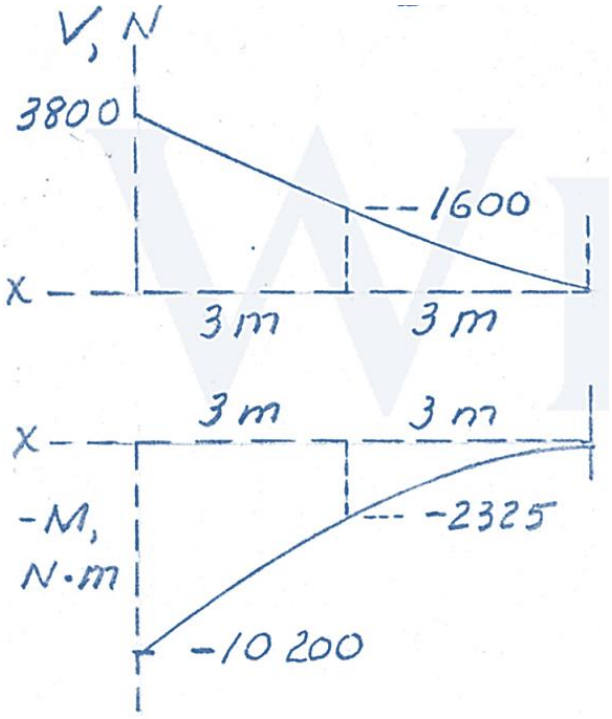


$+\circlearrowleft \sum M = 0$:

$$M + \int_0^x w\, dx'(x - x') = 0$$

$$M = -\int_0^x \left(500 + \frac{100}{9}\, x'^2\right)(x - x')\, dx' = 0$$

$$M = -250x^2 - \frac{25}{27}\, x^4 \qquad \text{(N. m)}$$



at $x = 3$ m

$V = 1600$ N

$V = 1.6$ kN (Answer)

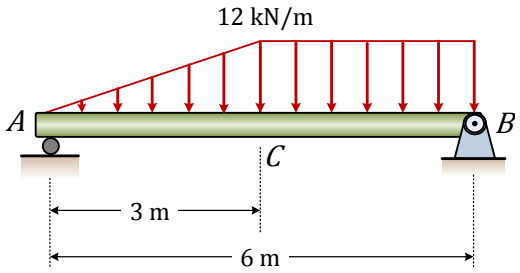
and

$M = -2325$ N. m

$M = -2.33$ kN. m (Answer)

Question 5.8.

For the loaded beam shown, draw the shear force and bending moment diagrams.



Solution

Referring to FBD of entire beam in Fig. (a),

$+\circlearrowleft \sum M_B = 0$:

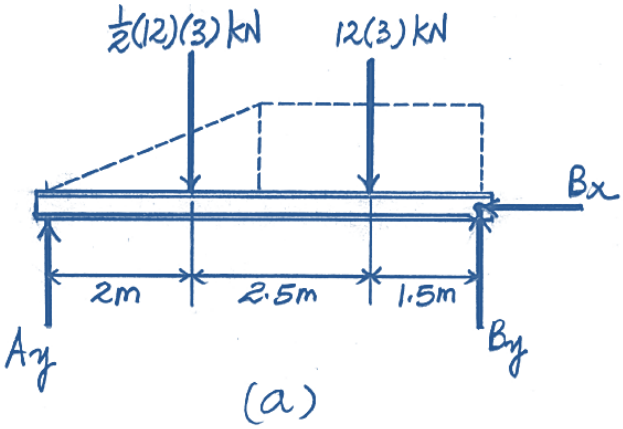
$$12(3)(1.5) + \frac{1}{2}(12)(3)(4) - A_y(6) = 0$$

$$A_y = 21 \text{ kN}$$

$+\circlearrowleft \sum M_A = 0$:

$$B_y(6) - \frac{1}{2}(12)(3)(2) - 12(3)(4.5) = 0$$

$$B_y = 33 \text{ kN}$$



Taking section cut AC $(0 \le x < 3 \text{ m})$

$+\uparrow \sum F_y = 0$:

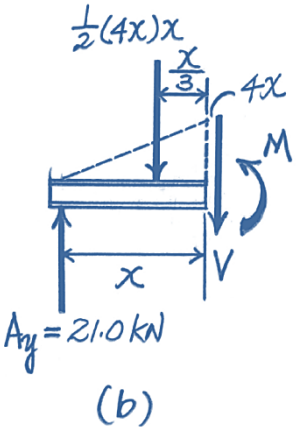
$$-V + 21 - 2x^2 = 0$$

$$V = 21 - 2x^2 \quad \text{(kN)}$$

$+\circlearrowleft \sum M = 0$:

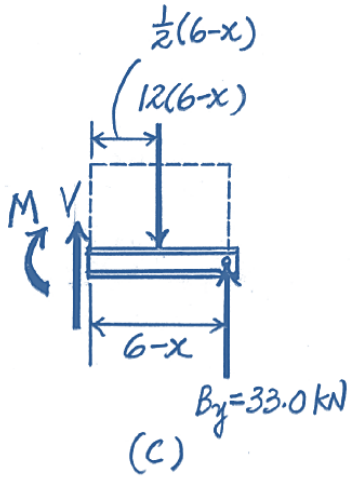
$$M + \left[\frac{1}{2}(4x)(x)\right]\left(\frac{x}{3}\right) - 21(x) = 0$$

$$M = 21x - \frac{2}{3}x^3 \quad \text{(kN. m)}$$



Taking section cut CB $(3\ \text{m} \leq x < 6\ \text{m})$

$+\uparrow \sum F_y = 0$:



$V + 33 - 12(6 - x) = 0$

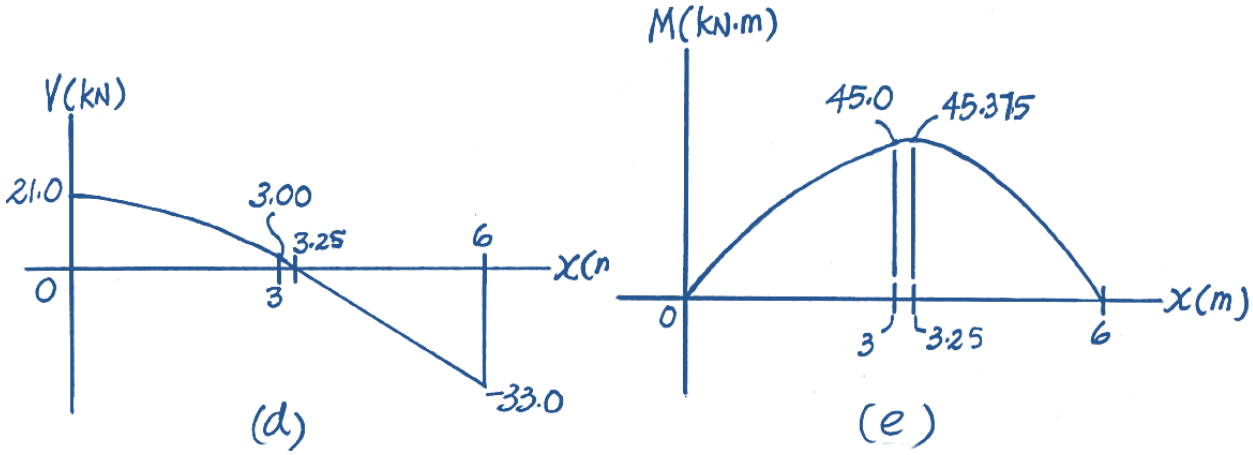
$V = 39 - 12x \quad (\text{kN})$

$+\circlearrowleft \sum M = 0$:

$33(6 - x) - M - [12(6 - x)]\left[\frac{1}{2}(6 - x)\right] = 0$

$M = -6x^2 + 39x - 18 \quad (\text{kN.m})$

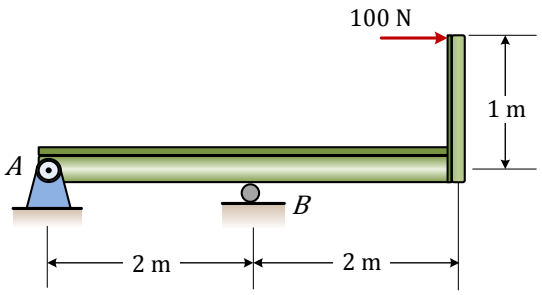
Now plot the shear and moment functions obtained i.e. shown in Fig (d) and Fig. (e)



(Answer)

Question 5.9.

Draw the shear and moment diagrams for the beam loaded by a force of 100 N applied to the strut welded to the beam as shown. Specify the bending moment at point B.

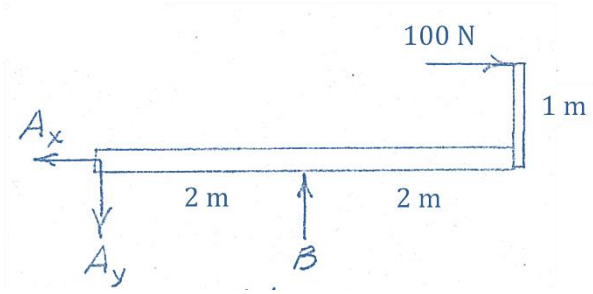


Solution

$+\circlearrowleft \sum M_A = 0$:

$B(2) - 100(1) = 0$

$B = 50$ N

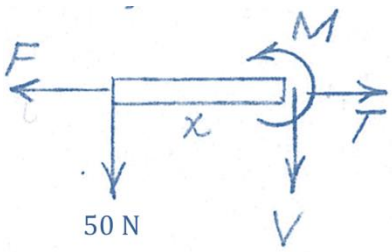


$+\uparrow \sum F_y = 0$:

$B - A_y = 0$

$A_y = B = 50$ N

Taking section cut



$+\uparrow \sum F_y = 0$:

$V = -50$ N

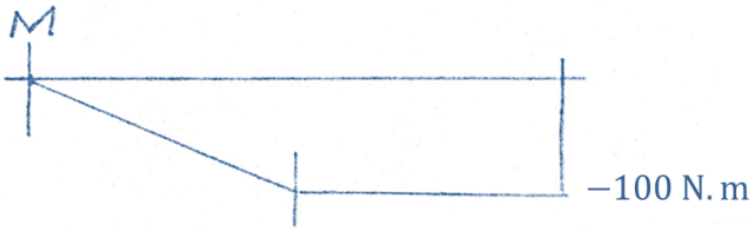
$+\circlearrowleft \sum M = 0$:

$M + 50(x) = 0$

$M = -50x$ N.m

at B

$M = -100(1)$ N.m

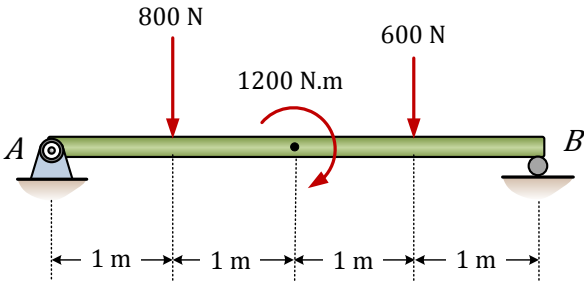


$M = -100$ N.m (Answer)

(Answer)

Question 5.10.

For the loaded beam shown, draw the shear force and bending moment diagrams.



Solution

$+\circlearrowleft \sum M_A = 0$:

$-800(1) - 600(3) - 1200 + B_y(4) = 0$

$B_y = 950$ N

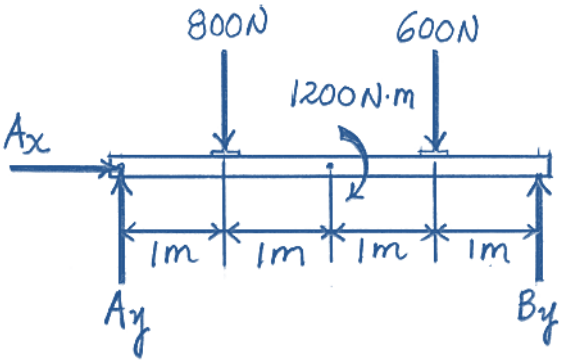
$+\circlearrowleft \sum M_B = 0$:

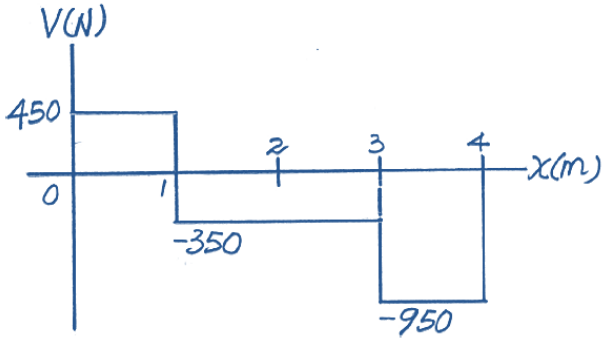
$800(3) + 600(1) - 1200 - A_y(4) = 0$

$A_y = 450$ N

$+\rightarrow \sum F_x = 0$:

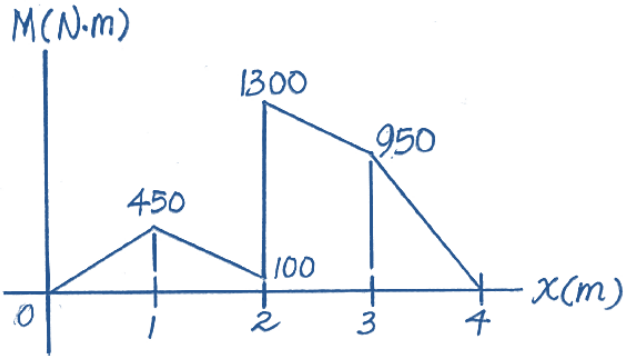
$A_x = 0$ N





Draw the SF and BM diagrams:

NOTE: This is an easy example to learn drawing the SF and BM diagrams by inspection, since there are only point loads and moment involved.



Attempt the problem also by using taking section cuts (as in previous problems) and validate your answers.

(Answer)