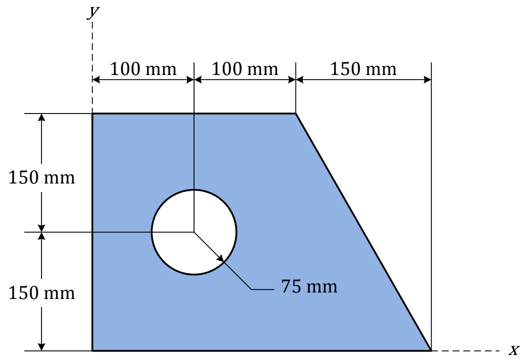
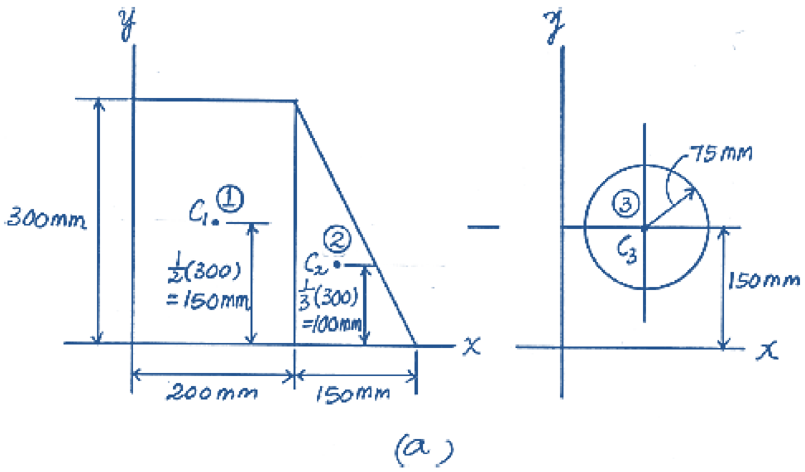
Question 6.11.

Determine the moment of inertia I_x of the shaded area about the x - axis.



Solution

(a) Spilt the object into three regular shapes



(b) Fill in the table below

Segment	A_i (mm^2)	$(d_y)_i$ (mm)	$(\bar{I}_{x'})_i$ (mm^4)	$(Ad_y{}^2)_i$ (mm^4)	$(\bar{I}_x)_i = \bar{I}_{x'} + (Ad_y{}^2)_i$
1	(200)(300)	150	$(1/12)(200)(300)^3$	$1.35(10^9)$	$1.80(10^9)$
2	(1/2)(150)(300)	100	$(1/36)(150)(300)^3$	$0.225(10^9)$	$0.3375(10^9)$
3	$-\pi\ 75^2$	150	$(-\pi\ 75^4/4)$	$-0.3976(10^9)$	$-0.4225(10^9)$

(c) Calculate the Moment of inertia of the

$$\bar{I}_x = \sum(\bar{I}_x)_i$$

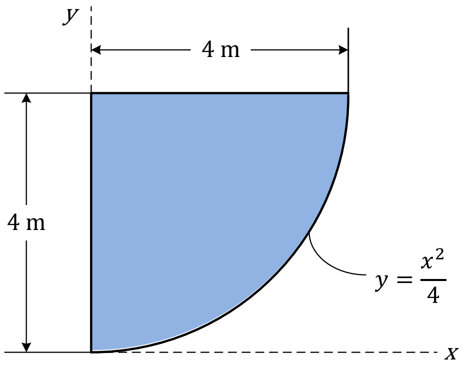
$$\bar{I}_x = (\bar{I}_x)_1 + (\bar{I}_x)_1 + (\bar{I}_x)_1$$

$$\bar{I}_x = 1.8(10^9) \ + \ 0.3375(10^9) - 0.4225(10^9)$$

$\bar{I}_x = 1.72(10^9)\ \mathrm{mm}^4$ (Answer)

Question 6.12.

Locate the y - coordinate $(\bar{y})$ of the centroid of the shaded area



Solution

The area of the differential element dA is shown shaded in Fig. (a) and is given as

$dA = x\,dy$

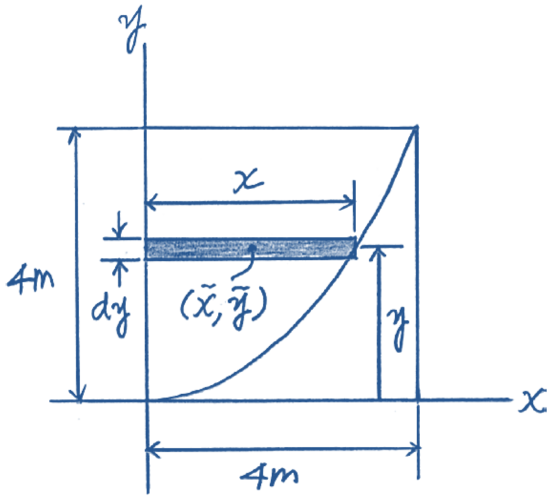
The centroid of the differential element is at

$\tilde{y} = y$

where,

$x = 2y^{1/2}$

Perform the integration



(a)

$$\bar{y} = \frac{\int \tilde{y}\,dA}{\int dA}$$

$$\bar{x} = \frac{\int_0^4 y(2y^{\frac{1}{2}}\,dy)}{\int_0^4 (2y^{\frac{1}{2}}\,dy)}$$

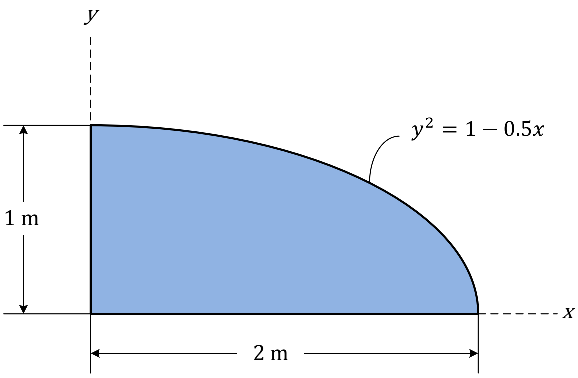
$$\bar{y} = \frac{\left(\frac{4}{5}\,y^{5/2}\right)\Big]_0^4}{\left(\frac{4}{3}\,y^{3/2}\right)\Big]_0^4}$$

$$\bar{y} = \frac{12}{5}\text{ m}$$

(Answer)

Question 6.13.

Determine the moment of inertia for the shaded area about the x - axis.



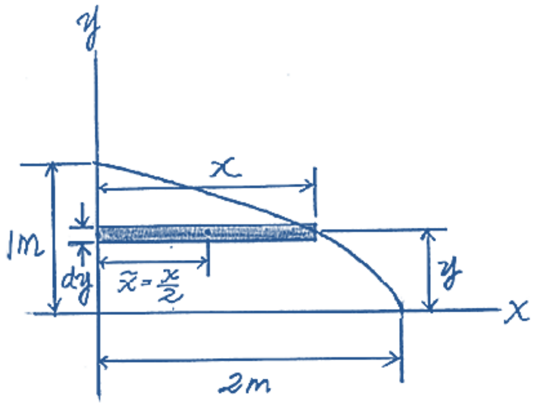
Solution

Here $x = 2(1 - y^2)$

The area of the differential element dA is shown shaded in Fig. (a) and is given as

$dA = x\,dy$

$dA = 2(1 - y^2)\,dy$



$$I_x = \int y^2\,dA = \int_0^{1\,m} y^2\,[2(1 - y^2)\,dy]$$

$$I_x = \int_0^{1\,m} (y^2 - y^4)\,dy$$

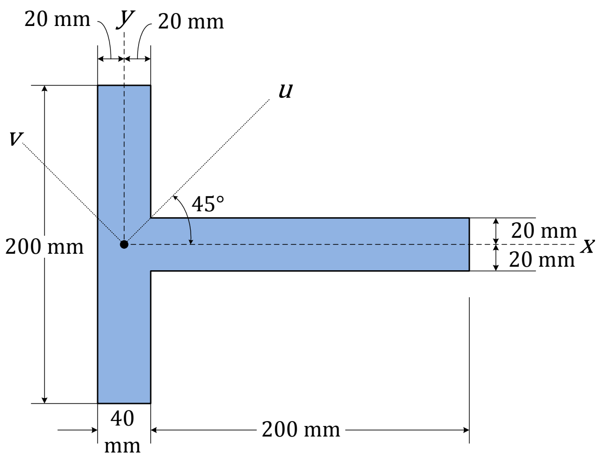
$$I_x = 2\left|\frac{y^3}{3} - \frac{y^5}{5}\right|_0^{1\text{ m}}$$

$$I_x = \frac{4}{15} = 0.267\text{ m}^4$$

(Answer)

Question 6.14.

Determine the moments of inertia I_u and I_v of the shaded area.



Solution

Moment and Product of Inertia about x and y axes. Since the rectangular area is symmetrical about the x axis, $I_{xy} = 0$

$$I_x = \left[\frac{1}{12}(200)(40^3) + \frac{1}{12}(40)(200^3)\right] = 27.73\ (10^6)\ \text{mm}^4$$

$$I_y = \left[\frac{1}{12}(40)(200^3) + 40(200)(120^2) + \frac{1}{12}(200)(40^3)\right] = 142.93\ (10^6)\ \text{mm}^4$$

Moment and product of inertia about the inclined u and v axes. With $\theta = 45°$

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

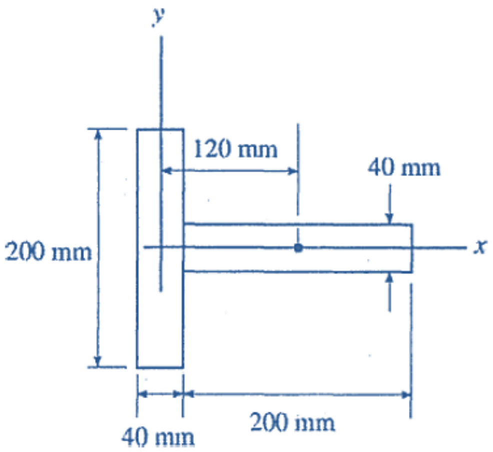
$$I_u = \left[\frac{27.73 + 142.93}{2} + \frac{27.73 - 142.93}{2}\cos 90°\right](10^6) = 85.3\ (10^6)\ \text{mm}^4$$

(Answer)

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

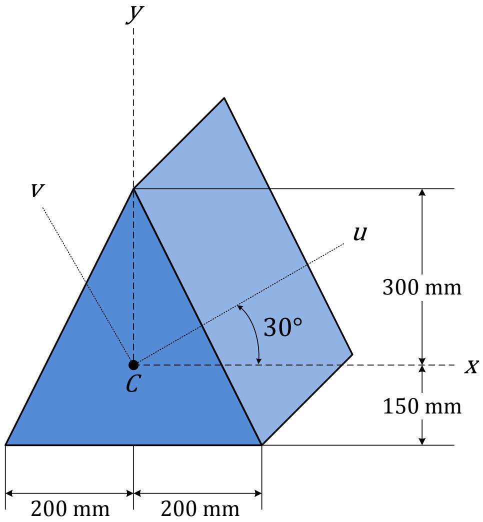
$$I_v = \left[\frac{27.73 + 142.93}{2} - \frac{27.73 - 142.93}{2}\cos 90°\right](10^6) = 85.3\ (10^6)\ \text{mm}^4$$

(Answer)



Question 6.15.

Determine the moments of inertia and the product of inertia of the beam's cross sectional area with respect to the u and v axes. (Use transformation equations)



Solution

Moment and Product of Inertia about x and y axes. Since the rectangular area is symmetrical about the y axes, $I_{xy} = 0$

$$I_x = \frac{1}{36}(400)(450^3) = 1012.5\ (10^6)\ \text{mm}^4$$

$$I_y = 2\left[\frac{1}{36}(450)(200^3) + \frac{1}{2}(450)(200)(66.67^2)\right] = 600\ (10^6)\ \text{mm}^4$$

Moment and product of inertia about the inclined u and v axes. With $\theta = 30°$

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_u = \left[\frac{1012.5 + 600}{2} + \frac{1012.5 - 600}{2}\cos 60°\right](10^6) = 909\ (10^6)\ \text{mm}^4$$

(Answer)

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$I_v = \left[\frac{1012.5 + 600}{2} - \frac{1012.5 - 600}{2}\cos 60°\right](10^6) = 703\ (10^6)\ \text{mm}^4$$

(Answer)

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$I_{uv} = \frac{1012.5 - 600}{2}\sin 60° = 179\ (10^6)\ \text{mm}^4$$

(Answer)