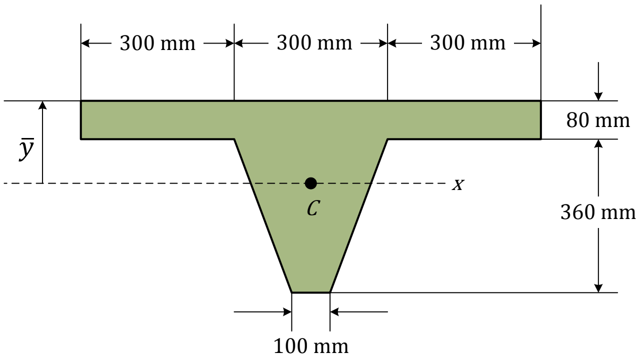
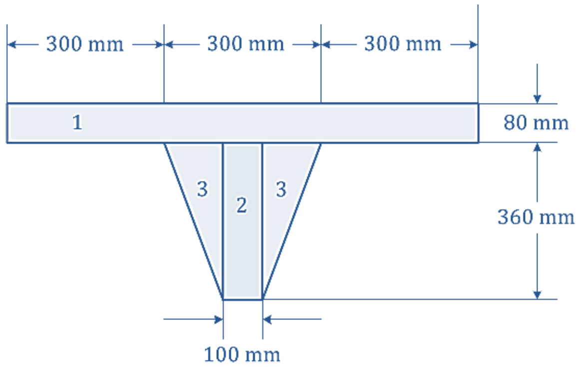
Question 6.7.

Locate the centroid $\bar{y}$ of the concrete beam having the tapered cross section shown.



Solution



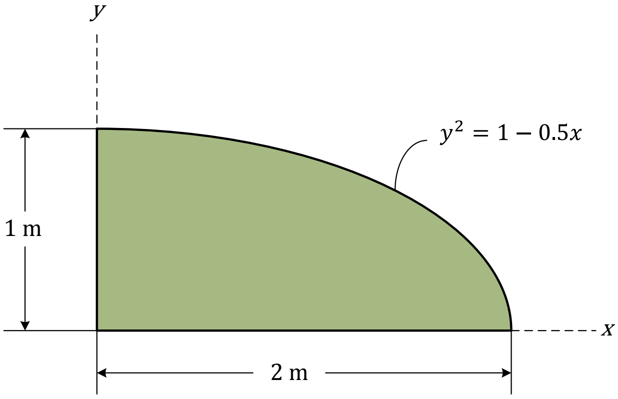
Segment	A (mm^2)	$\tilde{y}$ (mm)	$\tilde{y}A$ (mm^3)
1	(900)(80)	40	2880000
2	(100)(360)	260	9360000
3	(2)(1/2)(360)(100)	200	7200000
$\sum$	144000		19440000

$$\bar{y} = \frac{\sum \tilde{y} A}{\sum A} = \frac{1940000}{144000} = 135 \text{ mm}$$

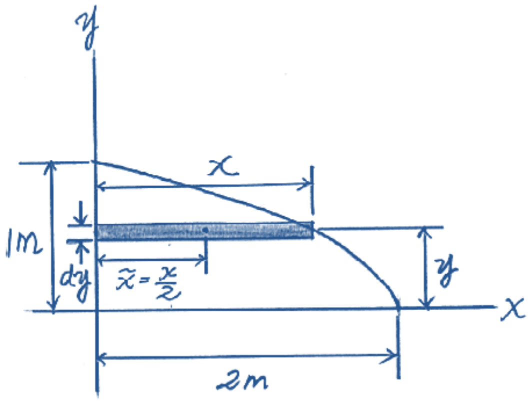
(Answer)

Question 6.8.

Determine the moment of inertia for the shaded area about the y - axis.



Solution



Here $x = 2(1 - y^2)$

The moment of inertia of the differential element parallel to the x - axis shown shaded in the Fig about the y - axis is

$$dI_y = d\bar{I}_{y'} + dA\tilde{x}^2$$

$$dI_y = \frac{1}{12}(dy)(x^3) + xdy\left(\frac{x}{2}\right)^2$$

$$dI_y = \frac{x^3}{3}dy$$

$$dI_y = \frac{[2(1 - y^2)]^3}{3}dy$$

$$dI_y = \frac{8}{3}\ (-y^6 + 3y^4 - 3y^2 + 1)\ dy$$

Perform the integration

$$I_y = \int dI_y = \frac{8}{3}\int_0^{1\text{ m}} (-y^6 + 3y^4 - 3y^2 + 1)\ dy$$

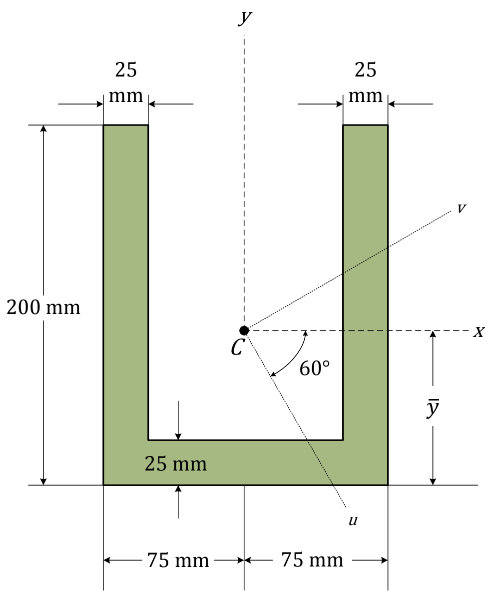
$$I_y = \frac{8}{3}\left| -\frac{y^7}{7} + \frac{3y^5}{5} - y^3 + y \right|_0^{1\text{ m}}$$

$$I_y = \frac{128}{105} = 1.22\ \text{m}^4$$

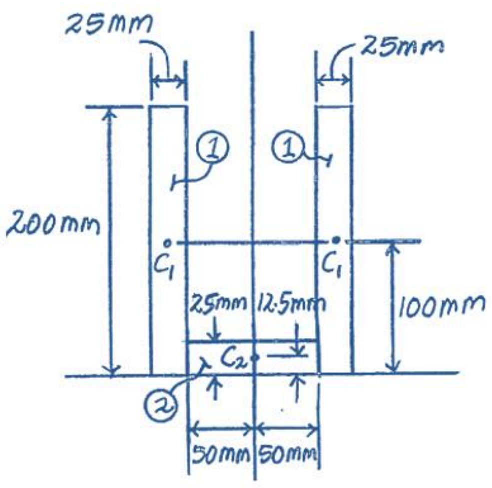
(Answer)

Question 6.9.

Locate the centroid $\bar{y}$ of the beam's cross-sectional area and then determine the moments of inertia of this area and the product of inertia with respect to u and v axes. The axes have their origin at the centroid C. (Use Mohr's Circle)



Solution



(a)

$$\bar{y} = \frac{\sum \tilde{y} A}{\sum A} = \frac{2[(100)(200)(25)] + 12.5(25)(100)}{2(200)(25) + (25)(100)} = 82.5 \text{ mm}$$

(Answer)

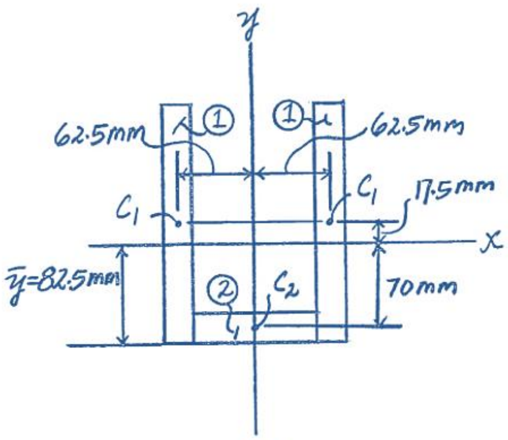
The perpendicular distances measured from the centroid of each segment to the x and y axes are indicated in Fig. b. Using the parallel - axis theorem

$$I_x = 2\left[\frac{1}{12}(25)(200^3) + (25)(200)(17.5^2)\right] + \left[\frac{1}{12}(100)(25^3) + (100)(25)(70^2)\right]$$

$$I_x = 48.78\ (10^6)\ \text{mm}^4$$

$$I_y = 2\left[\frac{1}{12}(200)(25^3) + (200)(25)(62.5^2)\right] + \left[\frac{1}{12}(25)(100^3)\right]$$

$$I_y = 41.67\ (10^6)\ \text{mm}^4$$



(b)

Since the cross - sectional area is symmetrical about the y axis

$$I_{xy} = 0$$

The Coordinates of centre O of the circle are

$$\left(\frac{I_x + I_y}{2}, 0\right) = \left(\frac{48.78 + 41.67}{2}, 0\right)(10^6) = (45.22,0)(10^6)$$

And the reference point A is

$$\left(I_x, I_{xy}\right) = (48.78,0)(10^6)$$

Thus the radius of the circle is,

$$R = \overline{OA} = \sqrt{(48.78 - 45.22)^2 + (0)^2} = 3.56\ (10^6)\ \mathrm{mm}^4$$

Using these Results, the circle shown in Fig. a can be constructed. Rotate radial line OA clockwise $2\theta = 120°$ to coincide with radial line OP where coordinate of point P is (I_u, I_{uv}). Then

$$I_u = (45.22 - 3.56\cos 60°)(10^6) = 43.4\ (10^6)\ \mathrm{mm}^4$$

(Answer)

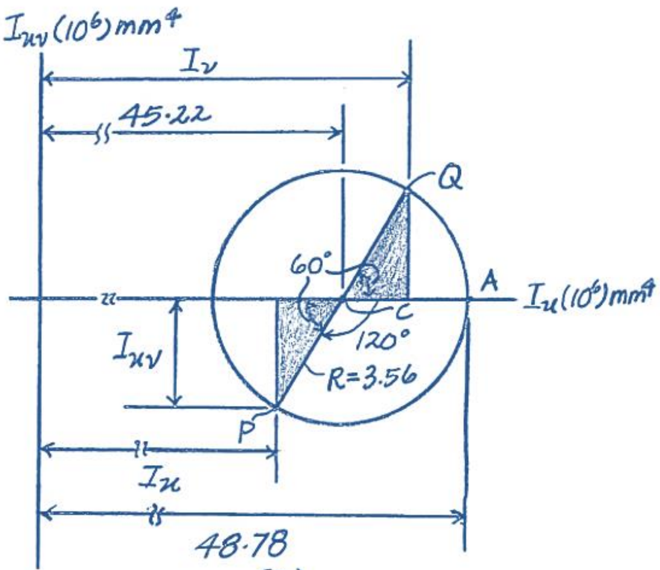
$$I_{uv} = -3.56(10^6)\sin 60° = -3.08\ (10^6)\ \mathrm{mm}^4$$

(Answer)

I_v is represented by the coordinate of point Q. Thus,

$$I_v = (45.22 + 3.56\cos 60°)(10^6) = 47.0\ (10^6)\ \mathrm{mm}^4$$

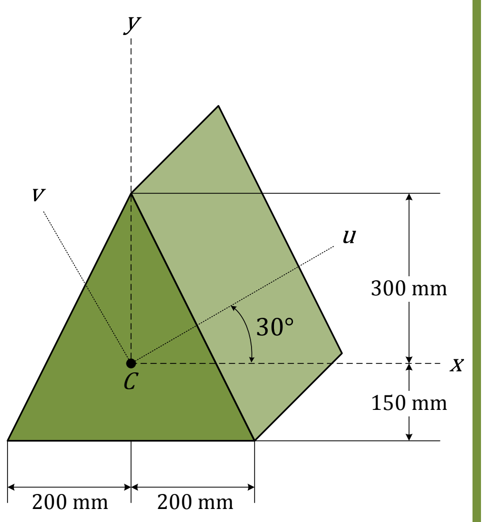
(Answer)



(c)

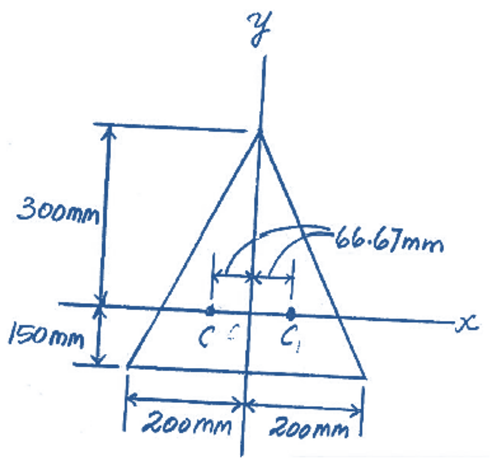
Question 6.10.

Determine the moments of inertia and the product of inertia of the beam's cross sectional area with respect to the u and v axes.



Solution

The perpendicular distances measured from the centroid of the triangular segment to the y axis are indicated in Fig. a.



(a)

$$I_x = \frac{1}{36}(400)(450^3) = 1012.5\ (10^6)\ \text{mm}^4$$

$$I_y = 2\left[\frac{1}{36}(450)(200^3) + \frac{1}{2}(450)(200)(66.67^2)\right] = 600\ (10^6)\ \text{mm}^4$$

Since the rectangular area is symmetrical about the y axis, $I_{xy} = 0$

The Coordinates of centre O of the circle are

$$\left(\frac{I_x + I_y}{2}, 0\right) = \left(\frac{1012.5 + 600}{2}, 0\right)(10^6) = (806.25\ , 0)(10^6)$$

And the reference point A is

$\left(I_x, I_{xy}\right) = (1012.5\,, 0)(10^6)$

Thus the radius of the circle is,

$R = \overline{OA} = \sqrt{(1012.5 - 806.25)^2 + (0)^2} = 206.25\ (10^6)\ \mathrm{mm}^4$

Using these Results, the circle shown in Fig. a can be constructed. Rotate radial line OA counterclockwise $2\theta\ =\ 60°$ *to coincide with radial line* OP *where coordinate of point* P *is* (I_u, I_{uv}). *Then*

$I_u = (806.25 + 206.25\cos 60°)(10^6) = 909\ (10^6)\ \mathrm{mm}^4$

(Answer)

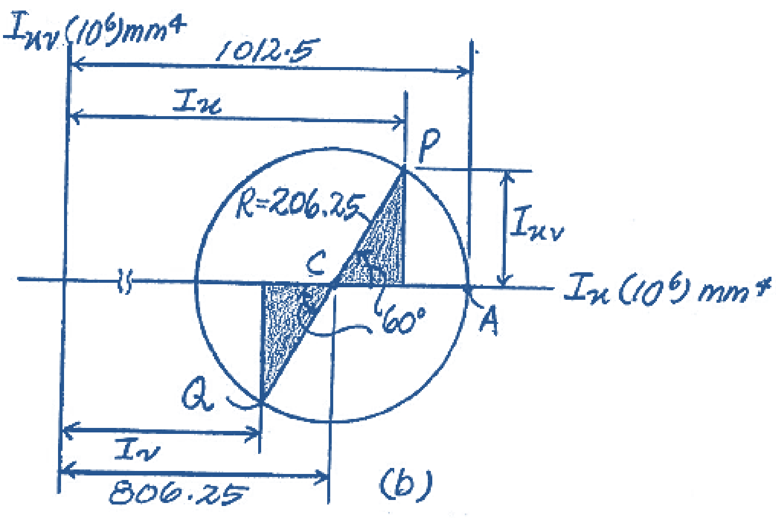
$I_{uv} = 206.25(10^6)\sin 60° = 179\ (10^6)\ \mathrm{mm}^4$

(Answer)

I_v *is represented by the coordinate of point* Q. *Thus,*

$I_v = (806.25 - 206.25\cos 60°)(10^6) = 703\ (10^6)\ \mathrm{mm}^4$

(Answer)



(b)