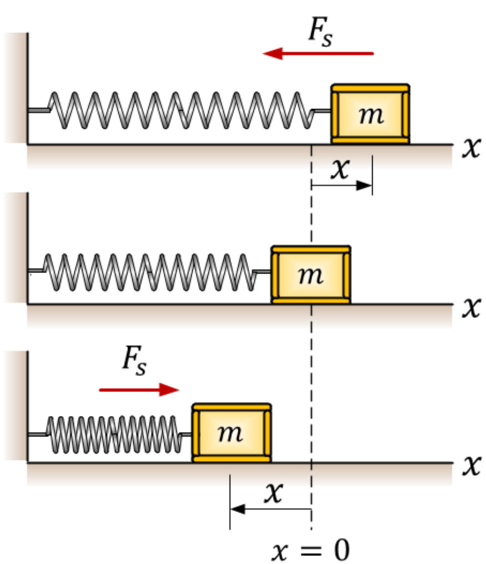
School of Mechanical and Manufacturing Engineering

MMAN1300 Engineering Mechanics 1

Dr. David C. Kellermann



Week 9 L1-2: Particle Energy & Momentum

WORK AND ENERGY

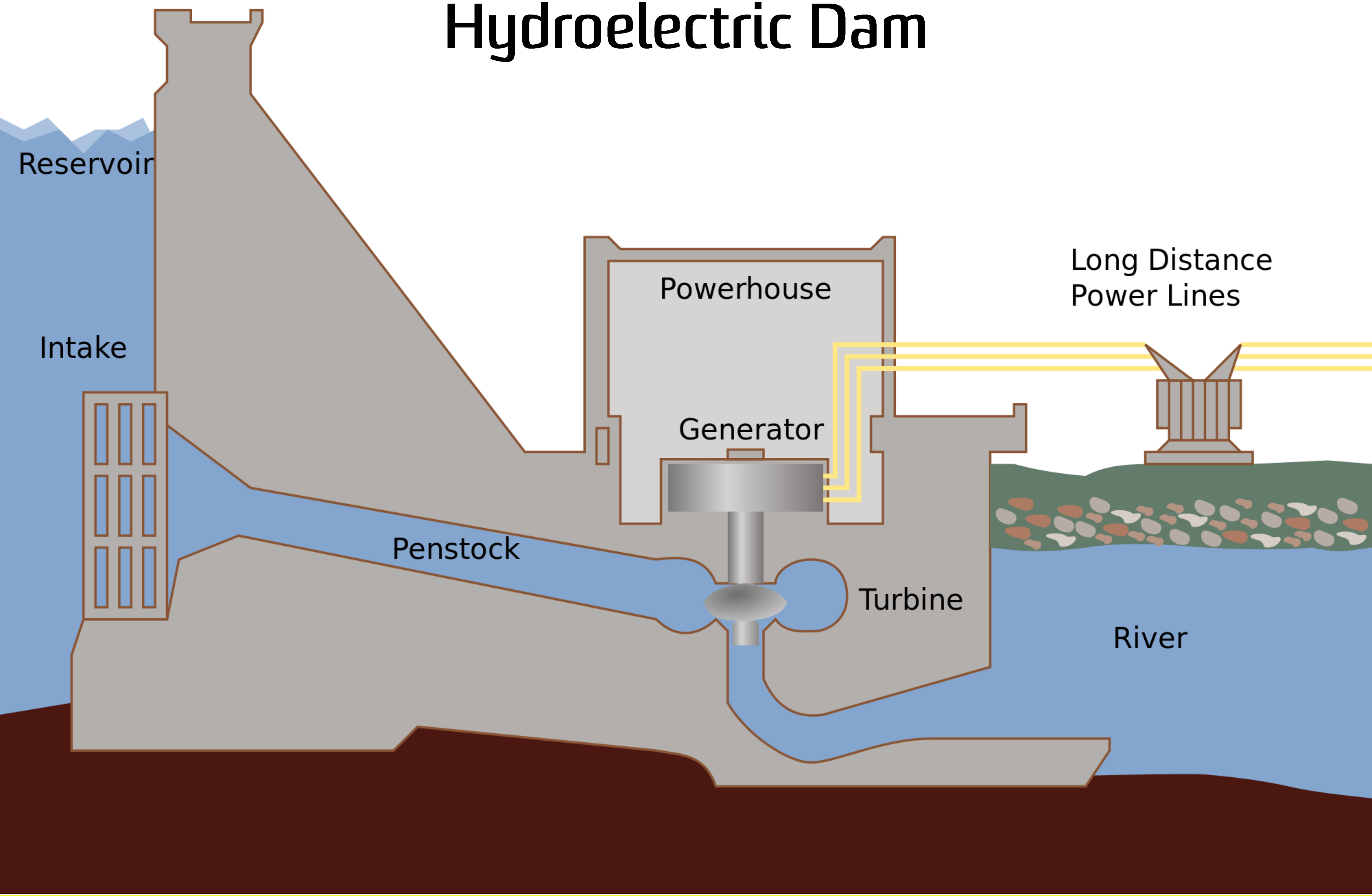
- Work-Energy Equation
- Conservation of energy

PRINCIPLE OF IMPULSE AND MOMENTUM

- Conservation of Linear Momentum
- Impulse equation
- Coefficient of Restitution and impact

Applications: Hydroelectric Dam

Hydroelectric Dam



Topics for Particle Energy and Momentum

Work-energy Method For Particles

- Work-Energy Equation
- Work, Potential Energy and Kinetic Energy
- Conservation of Energy
- Conservative and Non-conservative Systems
- Mechanical Power

Impulse And Momentum Method For Particles

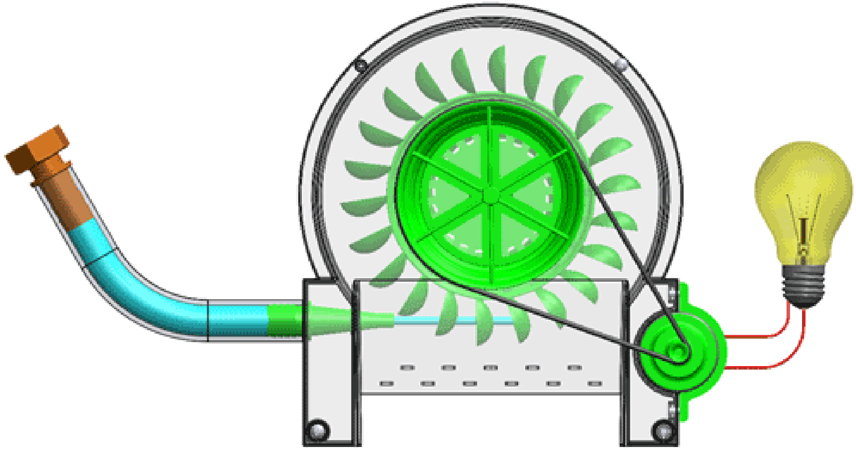
- Linear Impulse
- Conservation of Linear Momentum
- Impact

Conservation of Energy

- A change in the total energy of a system is brought about by either doing work on the system or by adding heat to the system.

- The change in energy of a system between 2 states (from one position to another) is:

$$\boxed{U_{1-2} = U_2 - U_1}$$



Work-Energy

The simplest energy conservation equation including work, heat and internal energy is:

$$W_{1-2} + Q_{1-2} = U_{1-2}$$

- W_{1-2} is the work done on the system between states 1 and 2,
- Q_{1-2} is the heat added to the system between states 1 and 2,
- U_{1-2} is the change in the total internal energy of the system between states 1 and 2.

Work-Energy

Since in Dynamics, we do not consider heat:

$$\boxed{W_{1-2} = U_{1-2}}$$

The work done on a system is equal to its change in internal energy of the system.

Work Done by Conservative Forces

The work done on a system is a function of any external forces and/or moments only.

$$W_{1-2} = F(s_2 - s_1) + M(\theta_2 - \theta_1)$$

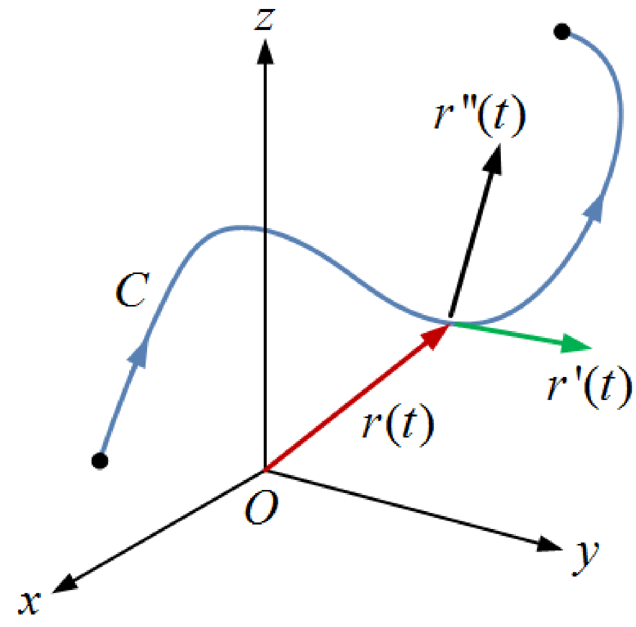
Where:

- W is the work on the system (Nm)
- F is an external force (N)
- s is the displacement (m)
- M is an external moment or torque (Nm)
- θ is the angular displacement (rad).

Conservative Work on Particle Systems

Since we are analysing particle systems, there are no moments and therefore no angular work. The work equation becomes

$$W_{1-2} = F(s_2 - s_1)$$



Where:

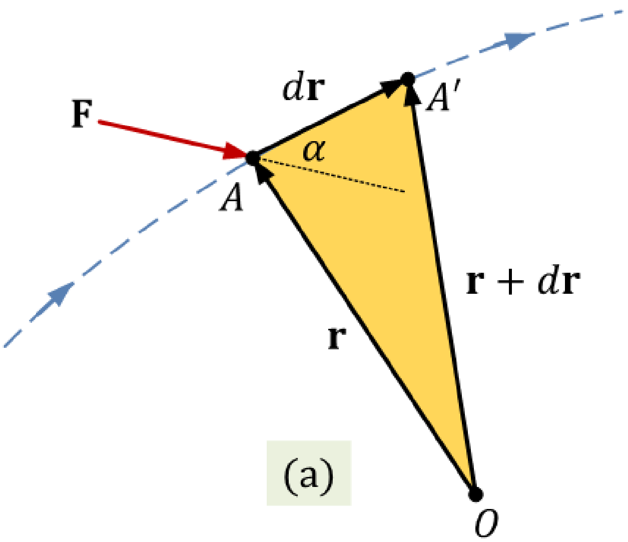
- W is the work on the system (Nm)
- F is an external force (N)
- s is the displacement (m)

The Work of a Force

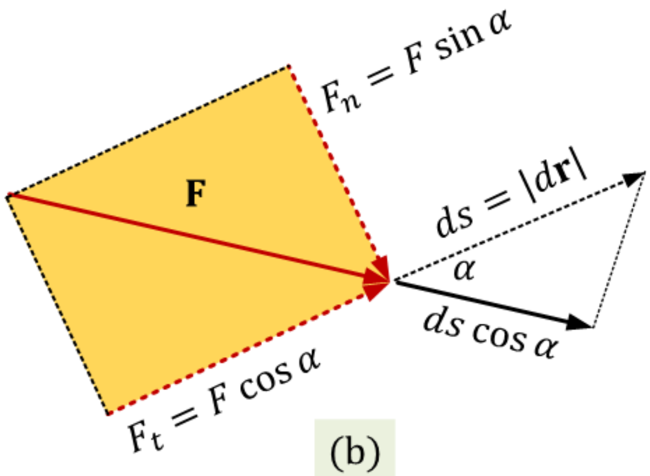
- A force **F** does work on a particle only when the particle undergoes a *displacement in the direction of the force.*

- Consider the force acting on the particle;

- If the particle moves along the path from position $\mathbf{r}$ to new position $\mathbf{r}'$ ($= \mathbf{r} + d\mathbf{r}$), displacement $d\mathbf{r} = \mathbf{r}' - \mathbf{r}$



(a)



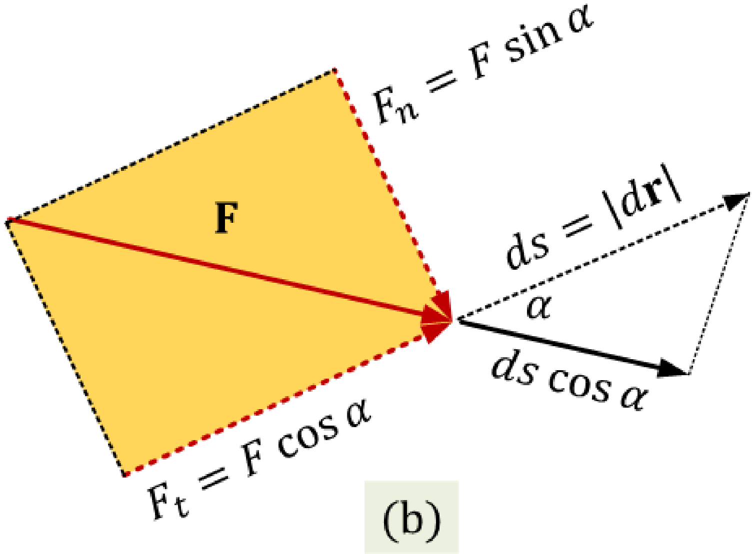
(b)

The Work of a Force

- $d\mathbf{s}$ is the magnitude of $d\mathbf{r}$
- α is the angle between $d\mathbf{r}$ and $\mathbf{F}$
- Work dU done by $\mathbf{F}$ is a scalar

$$dU = F\,ds\cos\theta$$

$$dU = \mathbf{F}\cdot d\mathbf{r}$$



- Resultant interpreted in two ways
  1. Product of F and the component of displacement in the direction of the force $ds \cos\alpha$
  2. Product of ds and component of force in the direction of the displacement $F \cos\alpha$

The Work of a Force

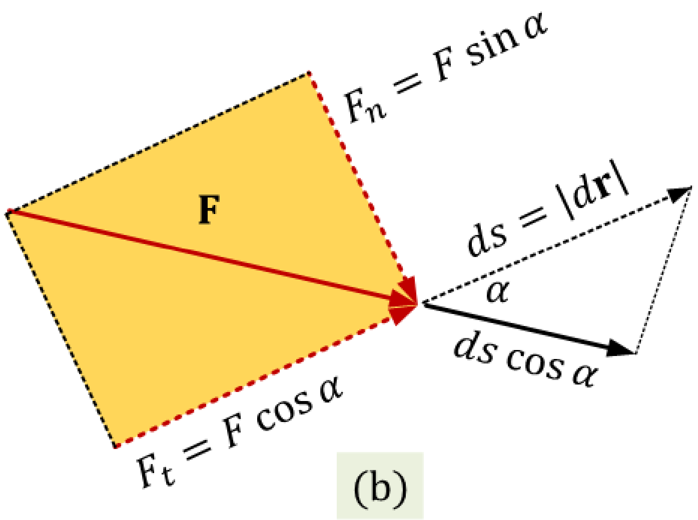
- If $0° < \alpha < 90°$, the work is *positive*

- If $90° < \alpha < 180°$, the work is *negative*

- $dU = 0$ if $\alpha = 90°$, or displacement = 0

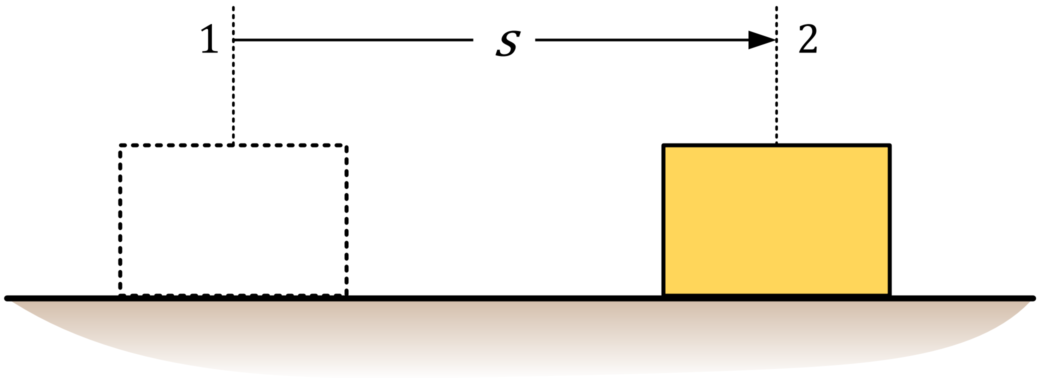
- Basic unit for work in SI units is joule (J)

- 1 *joule of work* is done when a force of 1 Newton moves 1 meter along its line of action, 1J = 1N.m



Example 1

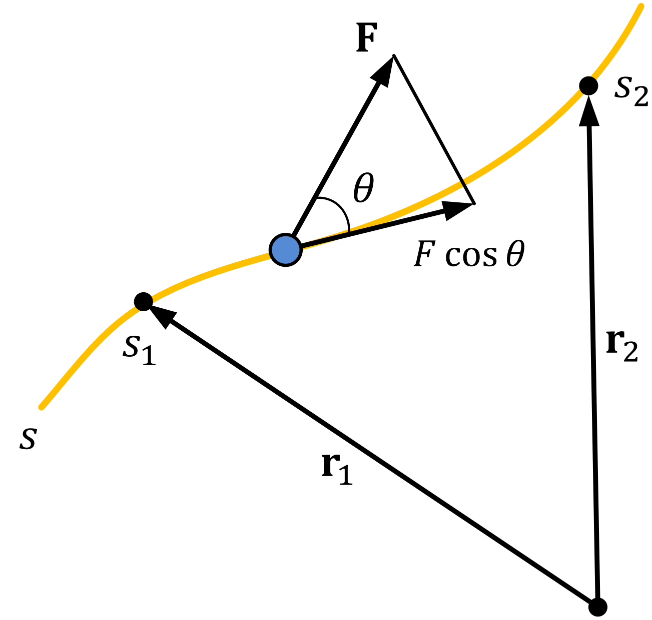
A block slides along a surface from 1 to 2. Find the work done by the block. The coefficient of kinetic friction between the block and the surface is μ_k.



W9 Example 1 (Web view)

The Work of a Variable Force

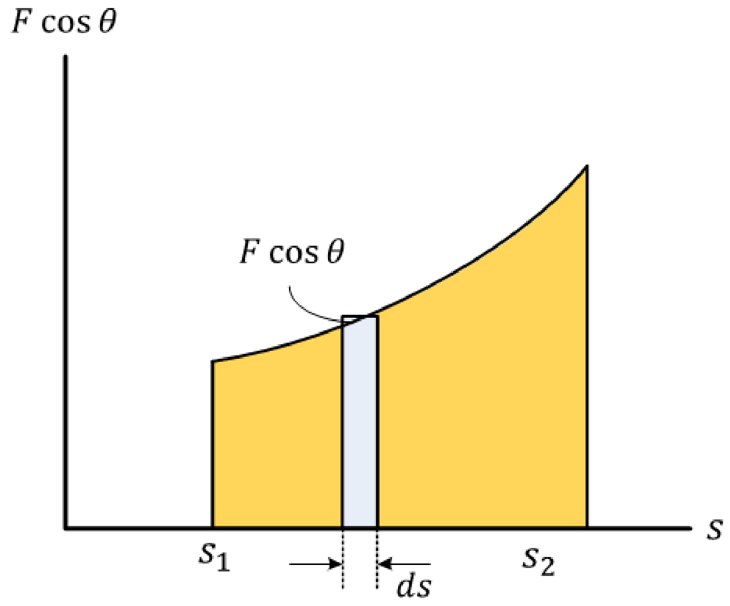
- If the particle undergoes a finite displacement along its path from $\mathbf{r}_1$ to $\mathbf{r}_2$ or s_1 to s_2, the work is determined by integration.
- If $\mathbf{F}$ is expressed as a function of position, $F = F(s)$,



$$U_{1-2} = \int_{r_1}^{r_2} \mathbf{F}.d\mathbf{r} = \int_{s_1}^{s_2} F \cos\theta \, ds$$

The Work of a Variable Force

$$U_{1-2} = \int_{r_1}^{r_2} \mathbf{F}.d\mathbf{r} = \int_{s_1}^{s_2} F\cos\theta\, ds$$



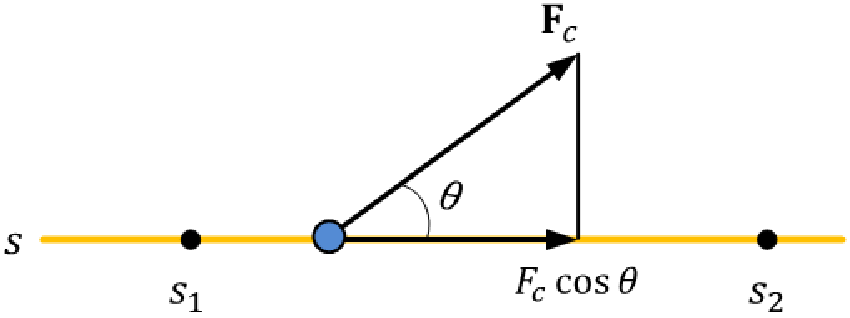
- The work can be interpreted as the *area under the curve* from position s_1 to position s_2

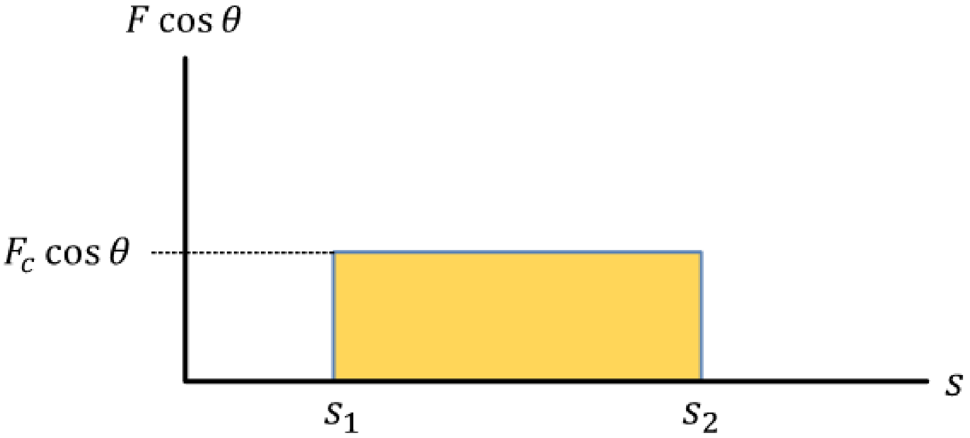
The Work of a Constant Force

- Work of a **Constant** Force Moving Along a **Straight** Line.

$$U_{1-2} = F_c \cos\theta \int_{s_1}^{s_2} ds$$

$$= F_c \cos\theta (s_2 - s_1)$$





The work of $\mathbf{F}_c$ represents the *area of the rectangle*

The Work of a Weight

Force: $m.\mathbf{g} = \mathbf{W} = -W.\mathbf{j}$

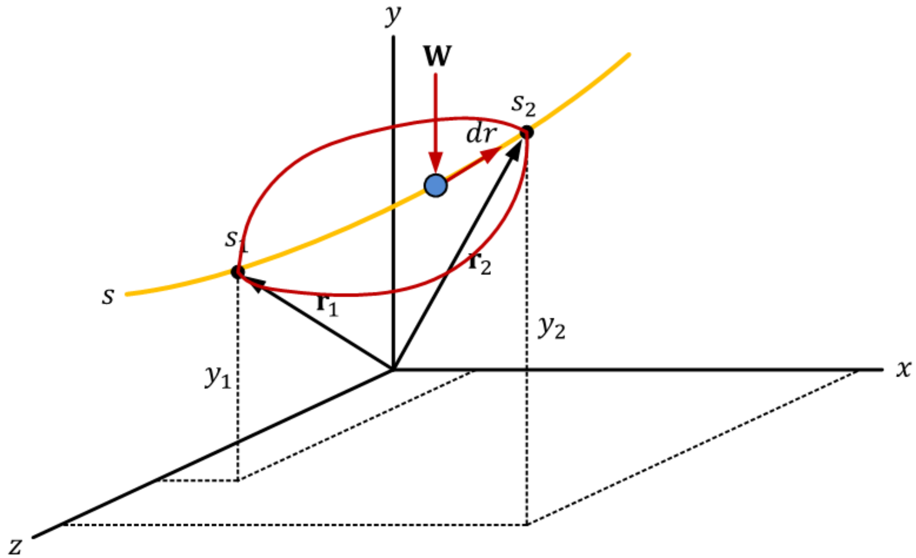
Displacement: $d\mathbf{r} = dx.\mathbf{i} + dy.\mathbf{j} + dz.\mathbf{k}$

$$U_{1-2} = \int \mathbf{F} \cdot d\mathbf{r}$$

$$= \int_{r_1}^{r_2} -W\mathbf{j} \cdot (dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k})$$

$$= \int_{y_1}^{y_2} -W dy = -W(y_2 - y_1)$$

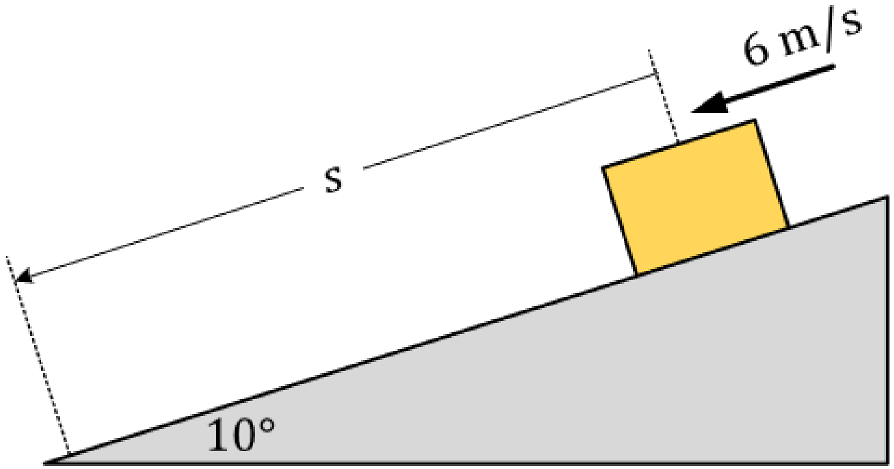
$$U_{1-2} = -W \Delta y$$



$\therefore$ *Independent of the path*

Example 2

The 1750 kg automobile shown in the figure below is travelling down the 10° inclined road at a speed of 6 m/s. If the driver jams on the brakes, causing his wheels to lock, determine how far s the tyres skid on the road. The coefficient of kinetic friction between the wheels and the road is $\mu_k = 0.5$.



[W9 Example 2](#) ([Web view](#))

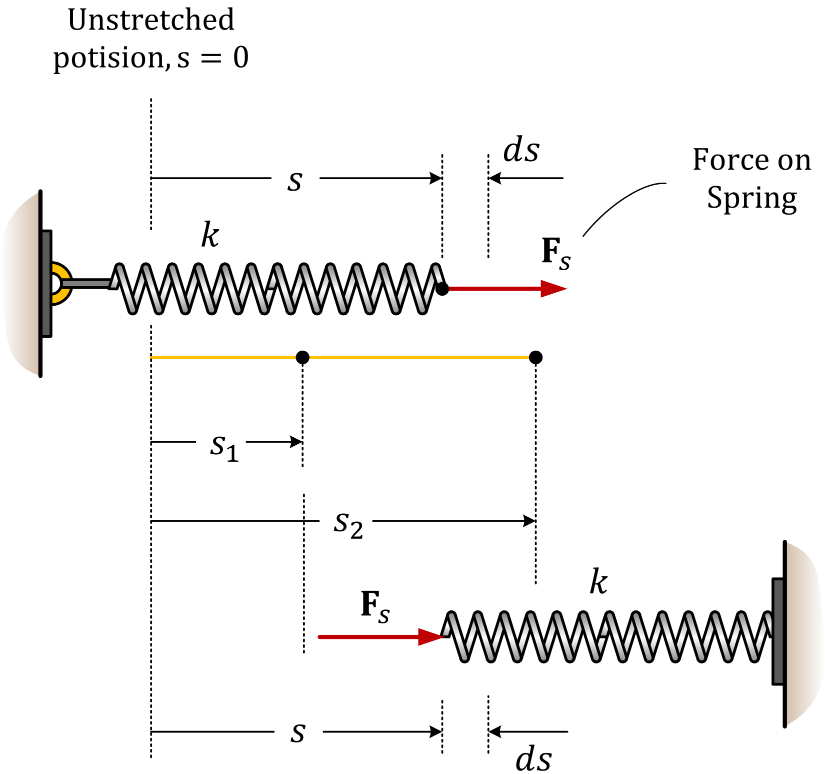
The Work of a Spring Force

- Force on a linear elastic spring

$$F_s = ks$$

k: the spring stiffness

s: distance from its **unstretched** position



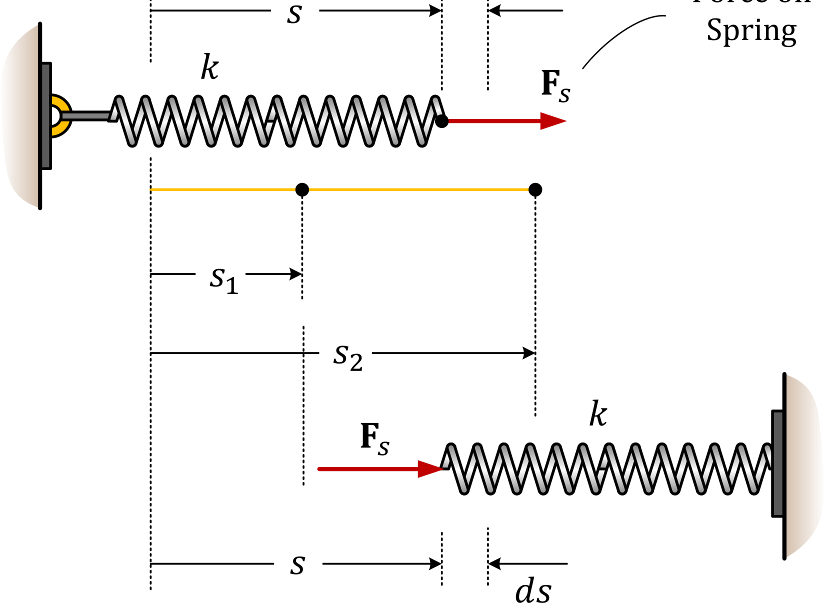
- If the spring is elongated or compressed from a position s_1 to s_2, the work done *on spring* by $\mathbf{F}_s$ is always *positive*, since force and displacement are in the *same direction*.

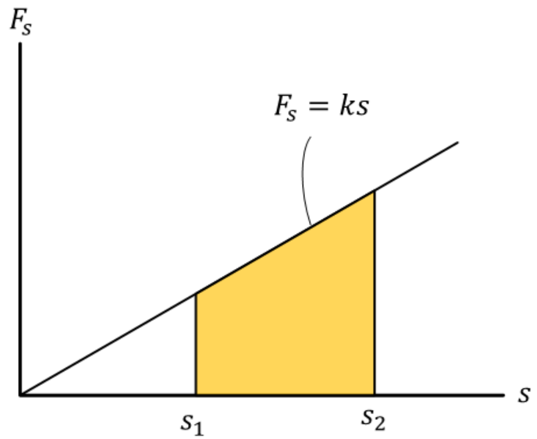
The Work of a Spring Force

$$U_{1-2} = \int_{s_1}^{s_2} F_s \, ds = \int_{s_1}^{s_2} ks \, ds$$

$$= \frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2$$

$$= \frac{1}{2}k(s_2^2 - s_1^2)$$



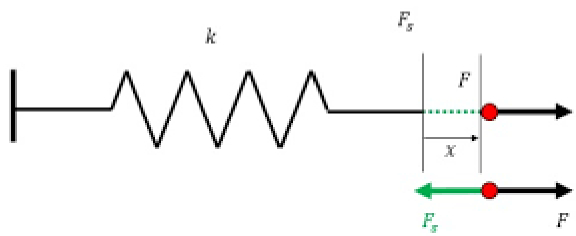


This equation represents the trapezoidal area under the line $F_s = ks$

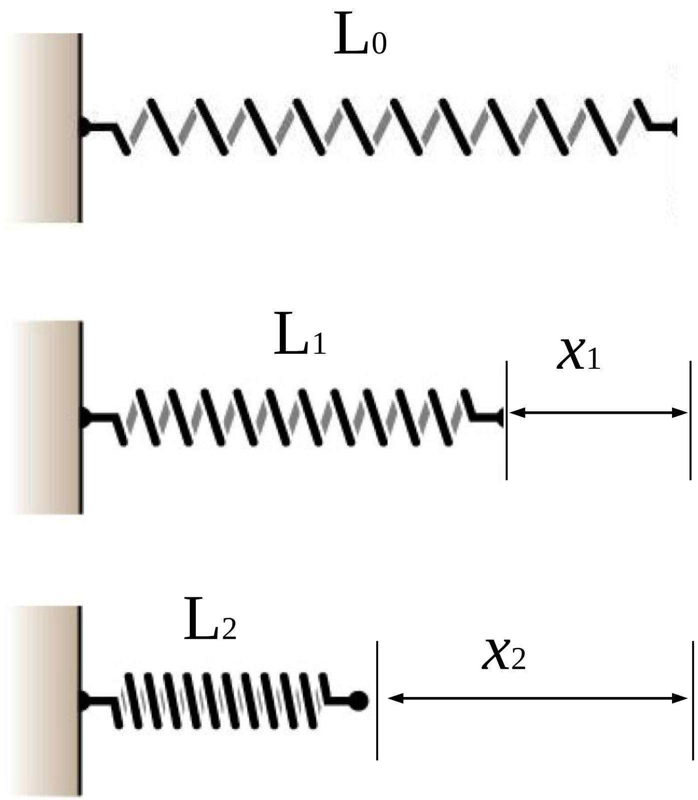
Spring Energy

Change in elastic energy:

$$\Delta V_e = \frac{1}{2}k({x_2}^2 - {x_1}^2)$$



Remember to always give x from the "unstretched length"

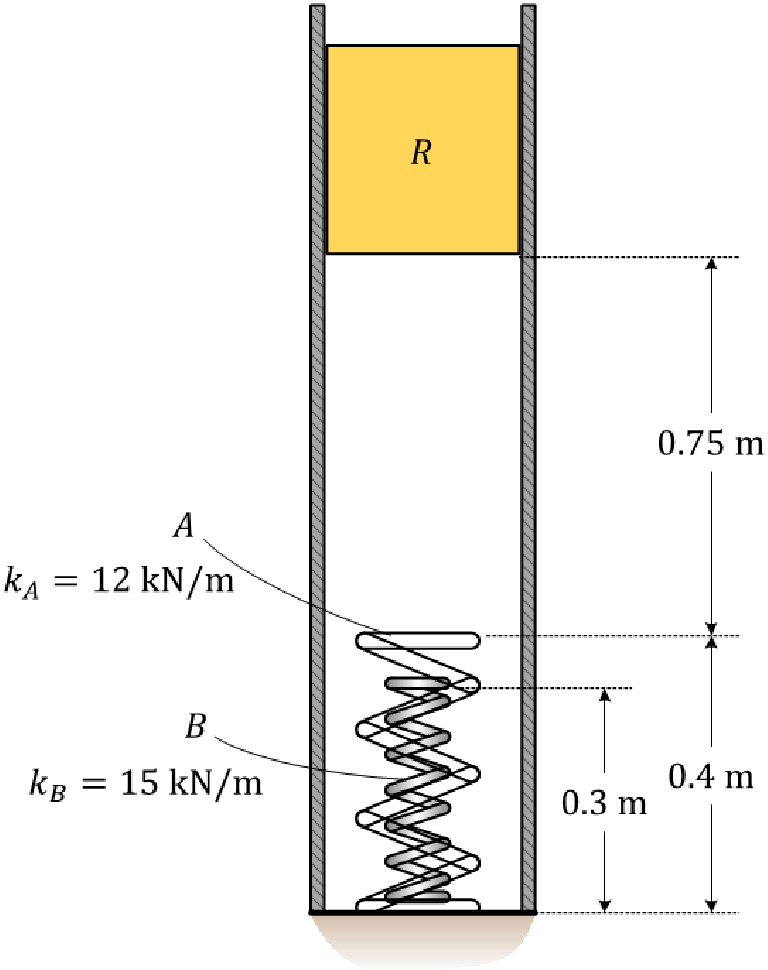


$$x_1 = |L_o - L_1|$$

$$x_2 = |L_o - L_2|$$

Example 3

The ram R has a mass of 100 kg and is released from rest 0.75 m from the top of a spring, A, that has a stiffness $k_A = 12$ kN/m. If a second spring B, having a stiffness $k_B = 15$ kN/m is "nested" in A, determine the max displacement of A needed to stop the downward motion of the ram.



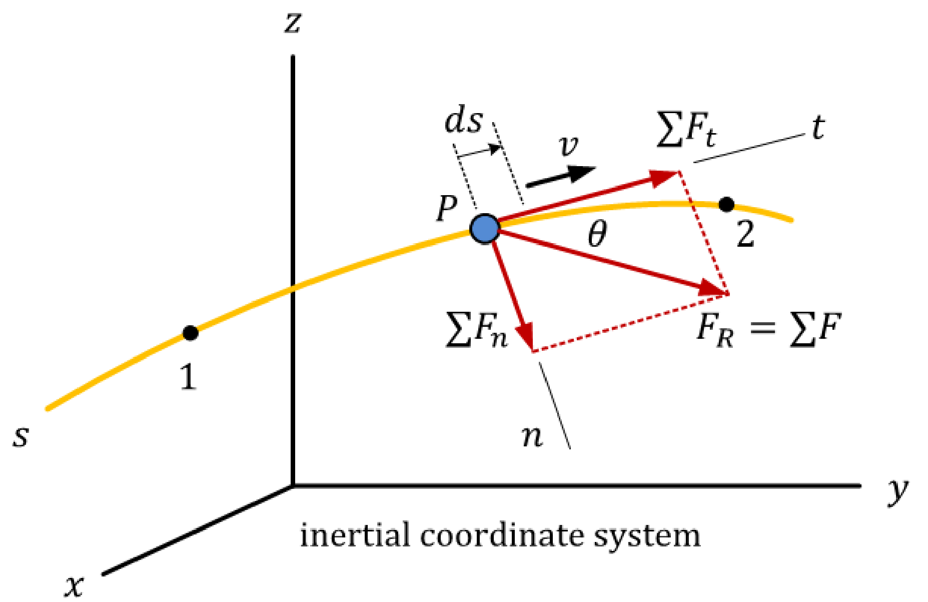
W9 Example 3 (Web view)

The Principle of Work and Energy

- For the particle P moving in the tangential direction,

$$\sum F_t = ma_t$$

- Using $a.ds = v.dv$ and integrating over ds



$$\sum \int_{s_1}^{s_2} F_t ds = \int_{v_1}^{v_2} mvdv$$

$$= \frac{1}{2} m v_2^2$$

- **Kinetic Energy** of a particle $T = \frac{1}{2} m v^2$

The Principle of Work and Energy

- Principle of work and energy for the particle,

$$\sum U_{1-2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = T_2 - T_1$$

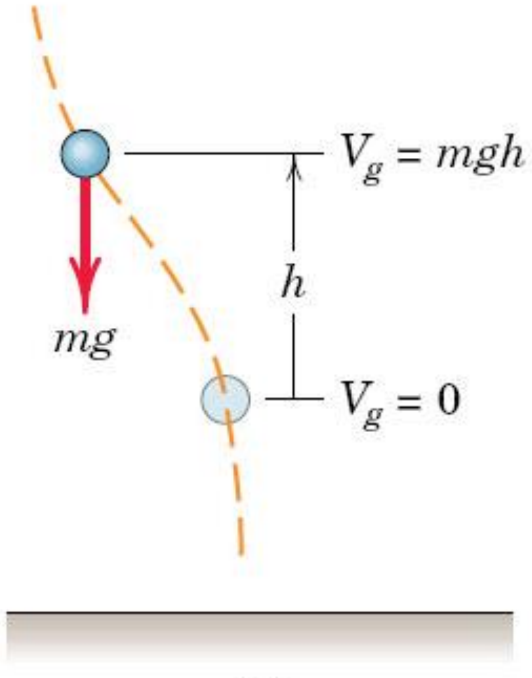
or $$T_1 + \sum U_{1-2} = T_2$$

- $\Sigma U_{1\text{-}2}$: the sum of work done by all the forces acting on the particle as the particle moves from point 1 to point 2
- T_1: the particle's initial kinetic energy.
- T_2: the particle's final kinetic energy.

Conservative Forces and Potential Energy

Conservative Force.

- It is defined by the work done in moving a particle from one point to another that is *independent of the path* followed by the particle.



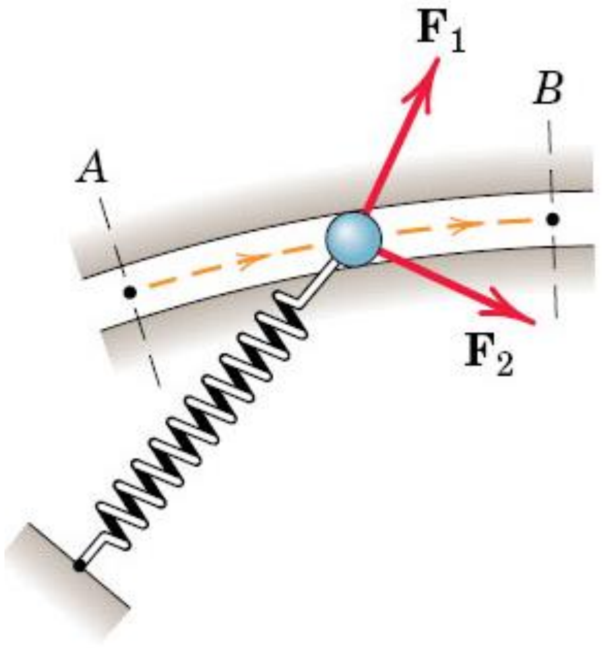
Potential Energy. (capacity of an object to do work)

- It is the measure of the amount of work a conservative force will do when it moves from a given position to the datum.

Conservative Forces and Potential Energy

Potential Function.

- If a particle is subjected to both gravitational and elastic forces, the particle's potential energy can be expressed as a potential function



$$V = V_g + V_e$$
$$= Wy + \frac{1}{2}ks^2$$

Conservative vs Non-Conservative Forces

- Work done by *conservative forces* written in terms of the difference in their potential energies

$$\left(\sum U_{1-2}\right)_{cons.} = V_1 - V_2$$

- Work done by non-*conservative forces*

$$\left(\sum U_{1-2}\right)_{noncons.}$$

- The principle of work and energy can be written as

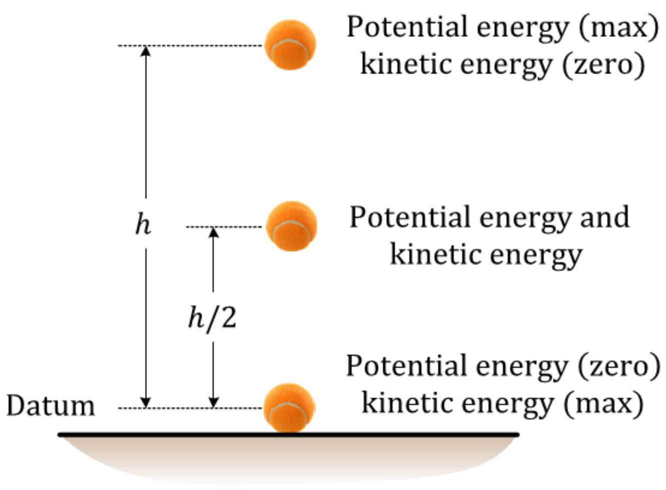
$$T_1 + V_1 + \left(\sum U_{1-2}\right)_{noncons.} = T_2 + V_2$$

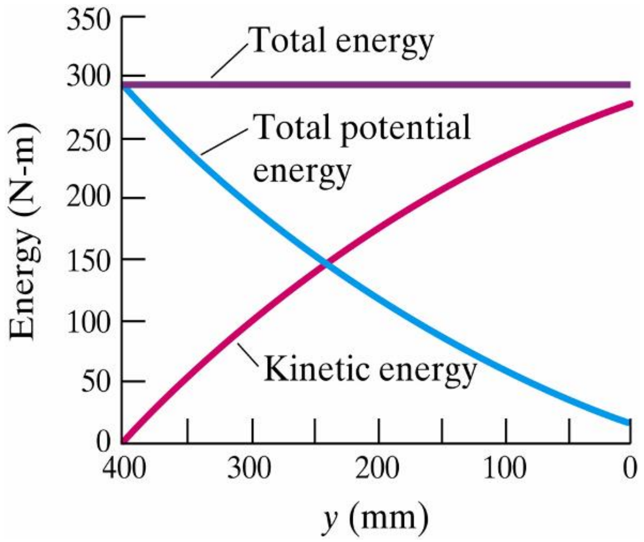
Conservation of Energy - Conservative

- If only conservative forces are applied, we have the law of the conservation of mechanical energy

$$T_1 + V_1 = T_2 + V_2$$

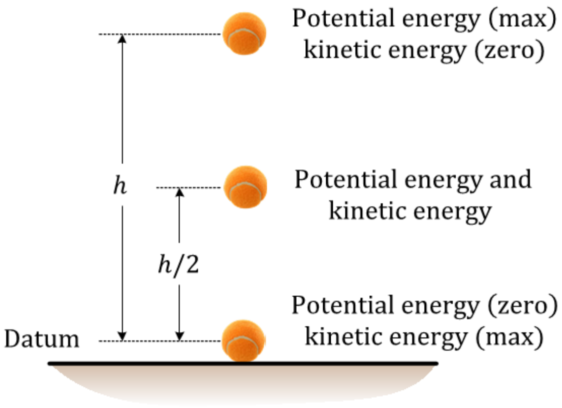
- It is used to solve problem involving *velocity, displacement* and *conservative force systems*.



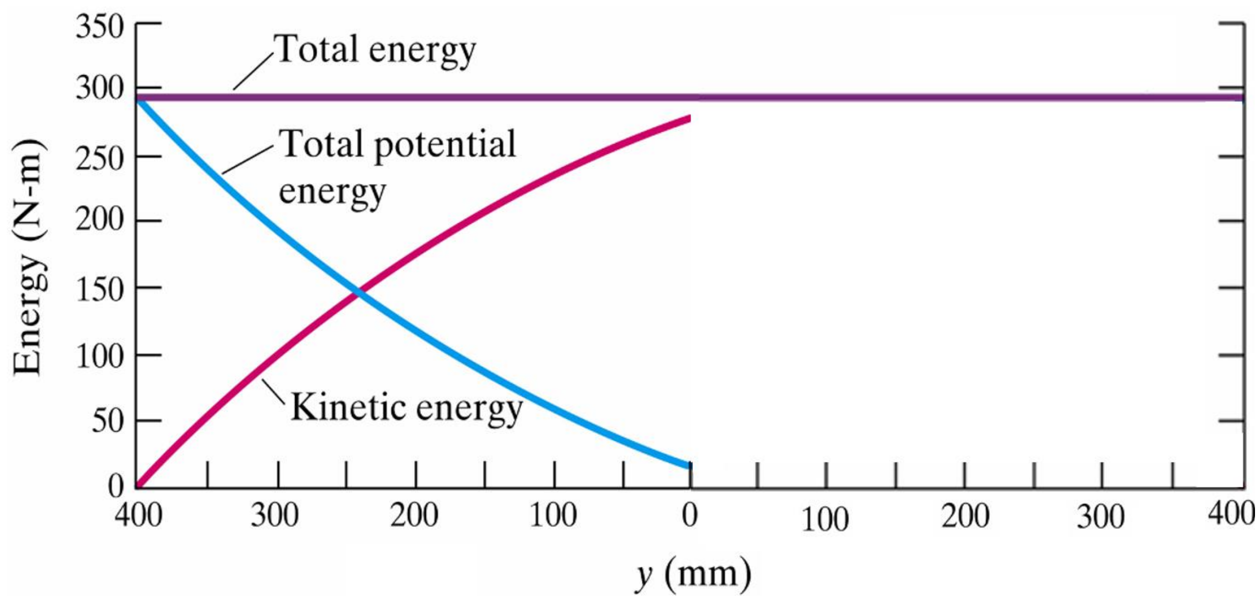


Conservation of Energy - Conservative

- If there is a moment when the ball is stationary as it bounces, then the kinetic energy must be momentarily zero. Where is the energy?



$$T_1 + V_{e1} + V_{g1} = T_2 + V_{e2} + V_{g1}$$



Procedure for Analysis

Potential Energy.

- Draw two diagrams showing the particle located at its initial and final points along the path
- If the particle is subjected to a vertical displacement, establish the fixed horizontal datum.
- Determine the elevation y of the particle from the datum and the extension or compression s of any connecting springs

- Gravitational potential energy $V_g = Wy$

- Elastic potential energy $V_e = \frac{1}{2}ks^2$

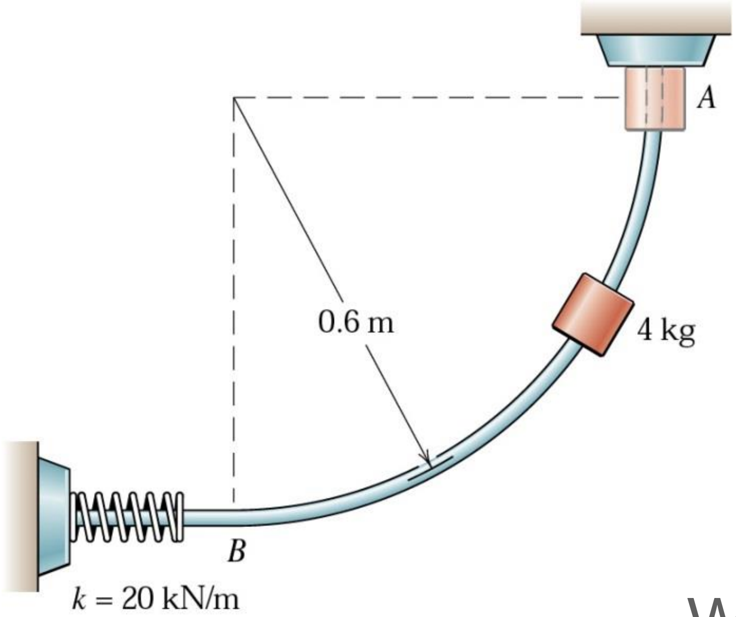
Example 4

A 4 kg slider is released from rest at *A* and slides with negligible friction down the circular rod in the vertical plane. Determine:

a) the maximum velocity of the slider as it reaches *B*;

b) the maximum deformation of the spring.

At point *B*, there is no compression of the spring.



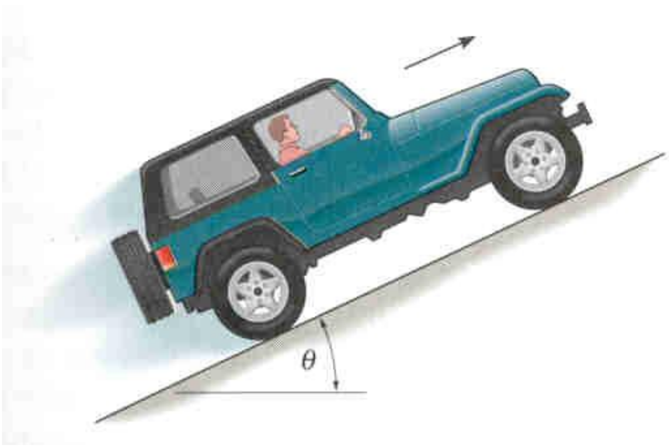
W9 Example 4 (Web view)

Mechanical Power

- Engines and motors are often rated in terms of their power output.
- The power requirements of the motor lifting this elevator depend on the vertical force ***F*** that acts on the elevator, causing it to move upwards.

Given the desired lift velocity for the elevator, how can we determine the power requirement of the motor?

Mechanical Power



Concept Example:

The speed at which a vehicle can climb a hill depends in part on the power output of the engine and the angle of inclination of the hill.

For a given angle, how could we determine the speed of this jeep, knowing the power transmitted by the engine to the wheels?

Mechanical Power

- **Power** is defined as the amount of work performed per unit of time.
- If a machine or engine performs a certain amount of work, dU, within a given time interval, dt, the power generated can be calculated as

$$P = dU/dt$$

- Since the work can be expressed as $dU = \mathbf{F} \bullet d\mathbf{r}$, the power can be written

$$P = dU/dt = (\mathbf{F} \bullet d\mathbf{r})/dt = \mathbf{F} \bullet (d\mathbf{r}/dt) = \mathbf{F} \bullet \mathbf{v}$$

- *Thus, power is a scalar defined as the dot product of the force and velocity components!*

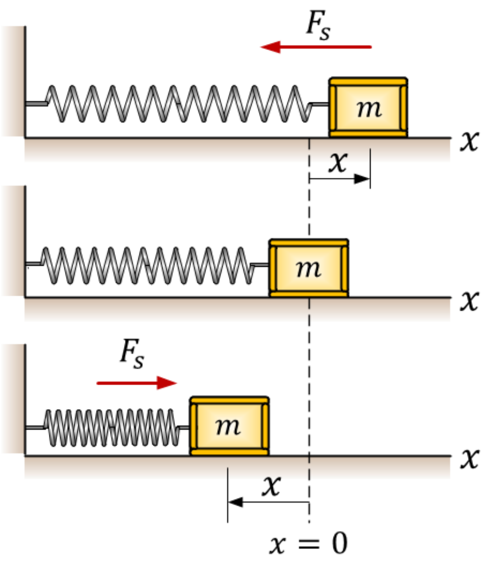
Mechanical Power

- So if the velocity of a body acted on by a force ***F*** is known, the power can be determined

- The unit of power in the SI system is the watt (W) where

$$1\ \mathrm{W} = 1\ \mathrm{J/s} = 1\ (\mathrm{N}\cdot\mathrm{m})/\mathrm{s}$$

Summary of Particle Work–Energy



- The Work–Energy equation gives us

$$W_{1-2} = F(s_2 - s_1) = \Delta T + \Delta V_g + \Delta V_e$$

where F is an *external* force

- The principle of conservation of energy yields the equation:

$$T_1 + V_1 + \left(\sum U_{1-2}\right)_{noncons.} = T_2 + V_2$$

- Power can be written

$$P = dU/dt = (\mathbf{F} \bullet d\mathbf{r})/dt = \mathbf{F} \bullet d\mathbf{r}/dt = \mathbf{F} \bullet \mathbf{v}$$

Next Topic:

Particle Impulse and Momentum

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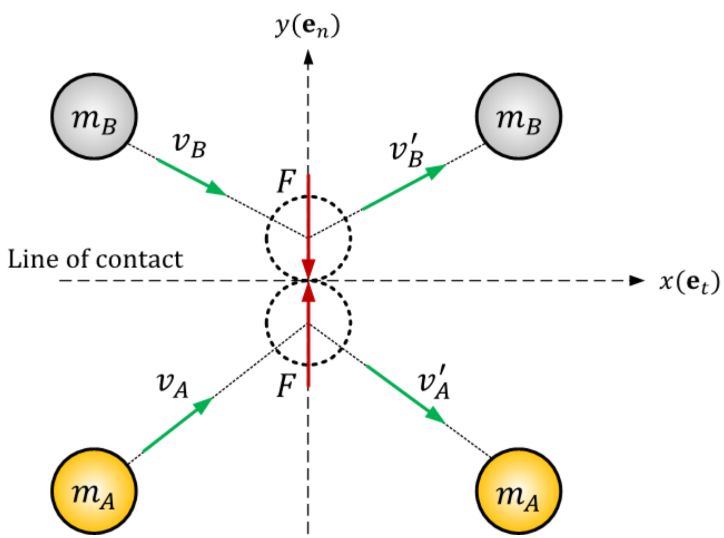
Week 9 L2: Particle Momentum



PRINCIPLE OF IMPULSE AND MOMENTUM

- Conservation of Linear Momentum
- Impulse equation

PARTICLE IMPACT

- Direct Central Impact
- Coefficient of Restitution
- Oblique Central Impact

Topics

Principle of Impulse and Momentum

- Conservation of Linear Momentum
- Impulse equation

Impact

- Direct Central Impact
- Coefficient of Restitution
- Oblique Central Impact

Recall Newton's 2nd Law

Newton's statement (translated from Latin) is

The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed

We say this means $\mathbf{F} = m\mathbf{a}$, but he doesn't mention acceleration anywhere

Newton's 2nd Law

The "change in motion" actually sounds like $\Delta\mathbf{v}$, not $\mathbf{a}$

1. We know how to relate $\Delta\mathbf{v}$ to $\mathbf{a}$
2. It is just the integral of $\mathbf{a}$ with respect to time
3. We can also integrate force with respect to time

Newton's 2nd Law

We might infer that Newton was actually talking about an integrated form of $\mathbf{F} = m\mathbf{a}$

These days, we refer to this form as the Impulse/Momentum formulation of Newton's 2nd law

In some ways, this is a more generally applicable form of the law

This problem is hard to treat with $\mathbf{F} = m\mathbf{a}$

Newton's 2nd Law

Previously, we integrated $\mathbf{F} = m\mathbf{a}$ with respect to displacement, what if we integrated it with respect to time?

Impulse-Momentum Equation

- Take Newton's 2nd Law: $\mathbf{F} = m\mathbf{a}$ (for constant mass)
- Now integrate with respect to time:

$$\int \mathbf{F}\,dt = \int m \frac{d\mathbf{v}}{dt}\,dt \qquad \text{or} \qquad \int \mathbf{F}\,dt = \int m\,d\mathbf{v}$$

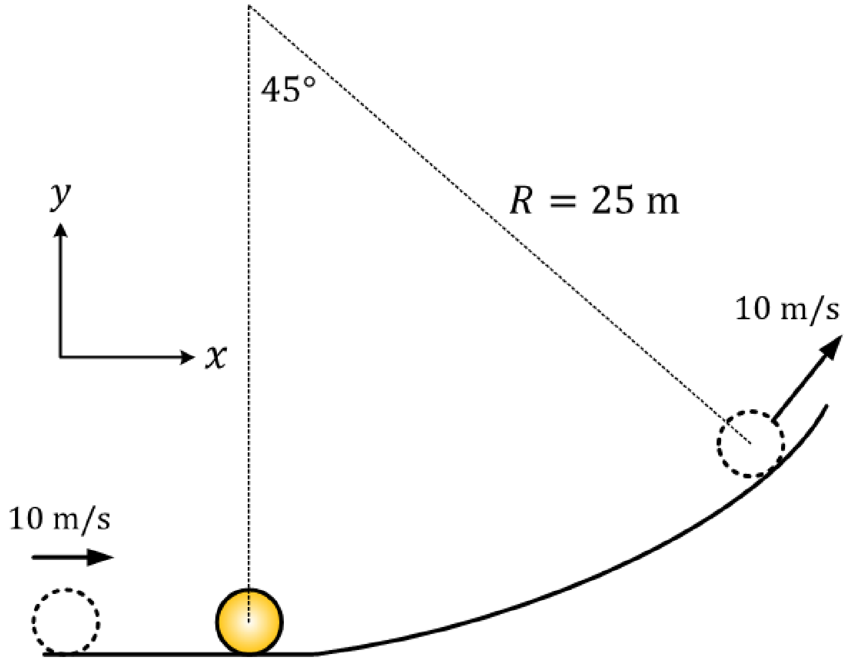
So:

$$\boxed{\int \mathbf{F}\,dt = m\mathbf{v}_2 - m\mathbf{v}_1}$$

- In words: **impulse = change in momentum**
- This is a **vector** equation

Example 5

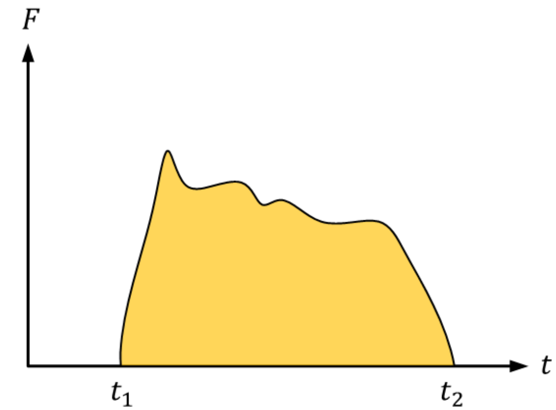
A smooth 3 kg particle travelling at a constant speed of 10 m/s encounters a circular guide as shown below. Determine the average force exerted by the guide on the particle.



W9 Example 5 (Web view)

The time integral of a Force is its Linear Impulse

- Impulse is often useful when a large magnitude force is applied over a very small time interval
  - Golf club hitting a golf ball
  - Space vehicle transfer orbits
  - Explosives
  - Impacts
- Especially if we only care about the velocity before and after the event (and not the acceleration during)

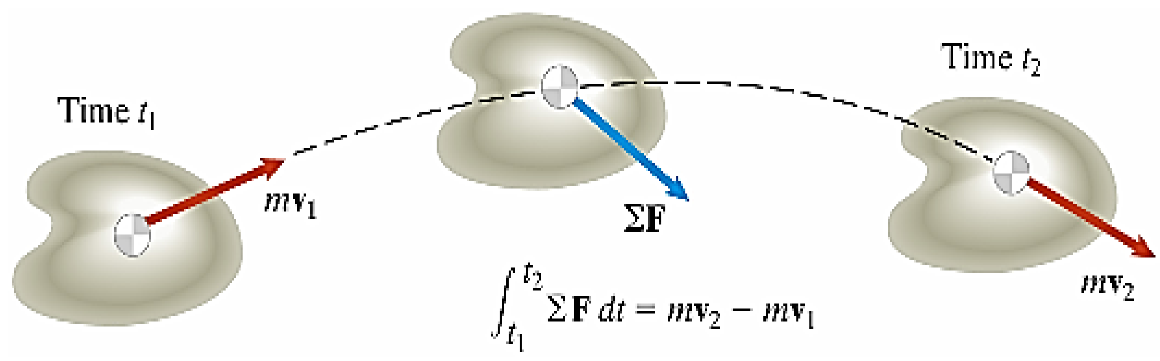


$$\int_{t_1}^{t_2} F\,dt = \text{area under the curve}$$

Linear Impulse-Momentum Equation

Remember that impulse is a vector quantity

- The change in velocity (or linear momentum, $m\mathbf{v}$) is also a vector quantity
- We should still be drawing FBDs when we use Impulse/Momentum



A general guideline for using Impulse/Momentum

- When we know (or want to know) about the motion of an object at some initial and final conditions

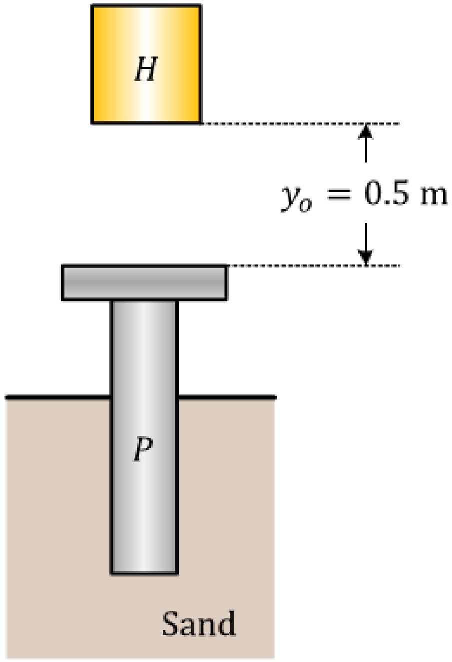
    - *Over some given time interval (effectively infinitesimal)*

    - *We do not care about instantaneous accelerations or forces*

- Impacts, collisions, and short-duration but high-magnitude forces often are treated well using this method

Example 6

An 800 kg rigid pile P shown in the figure in driven into the ground using a 300 kg hammer H. The hammer falls from rest at a height $y_0 = 0.5$ m and strikes the top of the pile. Assume the pile is surrounded entirely by loose sand so that after striking, the hammer does not rebound off the pile.



Determine:

a) the velocity of the hammer immediately before and after it strikes the pile,

b) the impulse which the hammer imparts on the pile.

[W9 Example 6](#) ([Web view](#))

What if impulse is zero?

We can quickly reformulate the impulse momentum equation to consider the case when there is no impulse on a 'system'

$$\int \mathbf{F}dt = m\mathbf{v}_f - m\mathbf{v}_i$$

$$\mathbf{0} = m\mathbf{v}_f - m\mathbf{v}_f$$

$$m\mathbf{v}_f = m\mathbf{v}_i$$

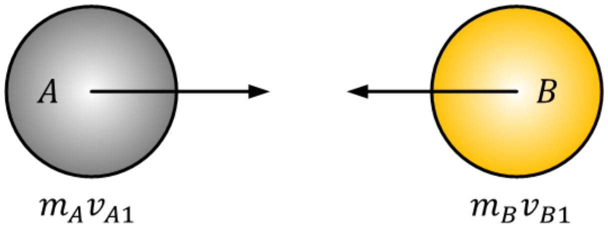
$$\boxed{\therefore \sum m_f\mathbf{v}_f = \sum m_i\mathbf{v}_i}$$

Conservation of Linear Momentum

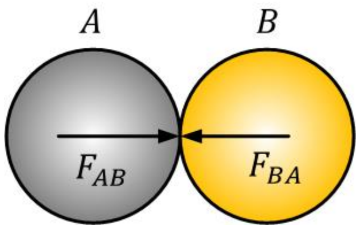
From the principle of impulse and momentum, it follows that if there are no external forces acting on a system, then the impulse term is zero, and the total momentum of the system does not change (momentum is conserved).

Consider the impact of two balls

At time t_1



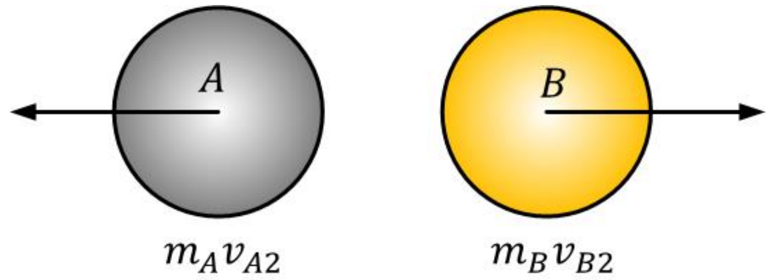
During impact



F_{AB} = the force exerted by ball A on ball B = $-F_{BA}$

Conservation of Linear Momentum

At time t_2



For each ball we can write

Ball B $$\int_{t_1}^{t_2} \boldsymbol{F}_{AB} dt = m_B \boldsymbol{v}_{B_2} - m_B \boldsymbol{v}_{B_1} \qquad (1)$$

Ball A $$\int_{t_1}^{t_2} \boldsymbol{F}_{BA} dt = m_A \boldsymbol{v}_{A_2} - m_A \boldsymbol{v}_{A_1} \qquad (2)$$

$$\int_{t_1}^{t_2} \boldsymbol{F}_{AB} dt = -\int_{t_1}^{t_2} \boldsymbol{F}_{BA} dt$$

$$m_B \boldsymbol{v}_{B_2} - m_B \boldsymbol{v}_{B_1} = -(m_A \boldsymbol{v}_{A_2} - m_A \boldsymbol{v}_{A_1})$$

Conservation of Linear Momentum

$$\underbrace{m_A \mathbf{v}_{A_1} + m_B \mathbf{v}_{B_1}}_{\text{total momentum of system before impact}} = \underbrace{m_A \mathbf{v}_{A_2} + m_B \mathbf{v}_{B_2}}_{\text{total momentum of system after impact}}$$

The total momentum of the system is a constant.

Conservation of Linear Momentum is defined as

$$\boxed{m_A \mathbf{v}_A + m_B \mathbf{v}_B = constant}$$

Impact

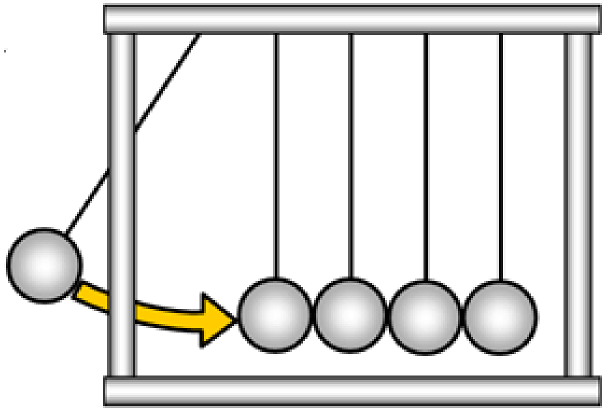
- The principle of impulse and momentum is useful to describe the behaviour of colliding bodies. Impact refers to the collision between two bodies, generating large contact forces that act over a very short interval of time.

Impact

- Impact is a complex event and involves material (plastic) deformation and (elastic) recovery, and the generation of heat and sound. Hence, impact calculations are an indication only.
- If no external forces act on the impacting bodies, then momentum will be conserved, that is, total linear momentum before impact will equal total linear momentum after impact for the system.

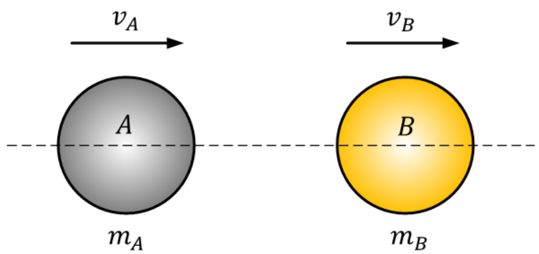
Direct Central Impact

- Centres of mass of two objects travel along the same straight line.
- Contact forces acting on each object during impact are directed along the same straight line (through the centres of mass of each object).

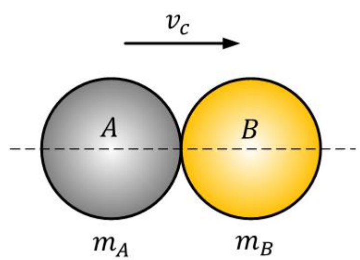


(a) Before impact (t_1)

$v_A > v_B$

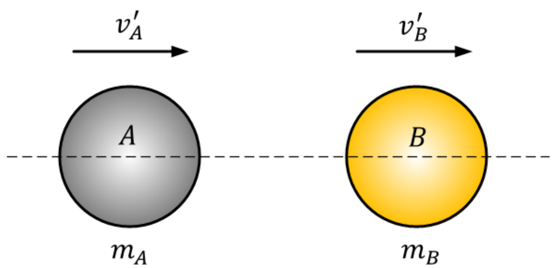


(b) Maximum deformation during impact (t_c)



Deformation period

(c) After impact (t_2)



Restoration period

Direct Central Impact

At time t_c during maximum deformation, the centres of mass of the two objects are closest, and m_A, m_B will have the same velocity v_c.

v_A, v_B: velocity before impact

v'_A, v'_B: velocity after impact

Since the contact forces are equal and opposite during impact, the linear momentum of the system remains unchanged. Since it is colinear, the equations are scalar

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

Coefficient of Restitution (*e*)

While the linear momentum of a system is conserved before and after impact, the kinetic energy of a system is not necessarily conserved.

Kinetic energy may be lost as a result of:

- plastic deformation of the bodies
- sound radiation
- heat generated
- friction, etc.

Coefficient of Restitution (e)

Perfectly elastic impact
Zero kinetic energy loss

Perfectly plastic impact
Maximum kinetic energy loss
Bodies remain together after impact

All impacts lie somewhere between these two extremes

Coefficient of Restitution (e)

The coefficient of restitution e is a measure of the kinetic energy loss during impact. Coefficient of restitution e is defined as:

$$e = \frac{\text{Impulse during restoration}}{\text{Impulse during deformation}} = \frac{\int_{t_c}^{t_2} F dt}{\int_{t_1}^{t_c} F dt}$$

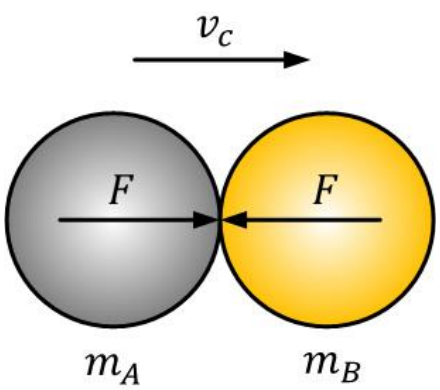
Coefficient of Restitution (e)

We can apply the principle of impulse and momentum to each particle (A and B) for

(i) the deformation period $\{t_1 \text{ to } t_c\}$

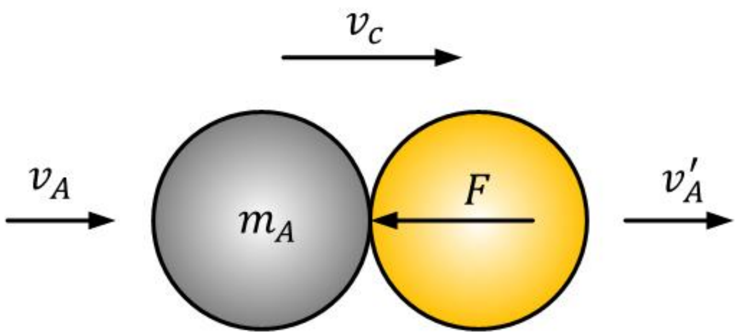
(ii) the restoration period $\{t_c \text{ to } t_2\}$

At time t_c



Note: at time t_c the contact forces are equal and opposite.

Coefficient of Restitution (e)

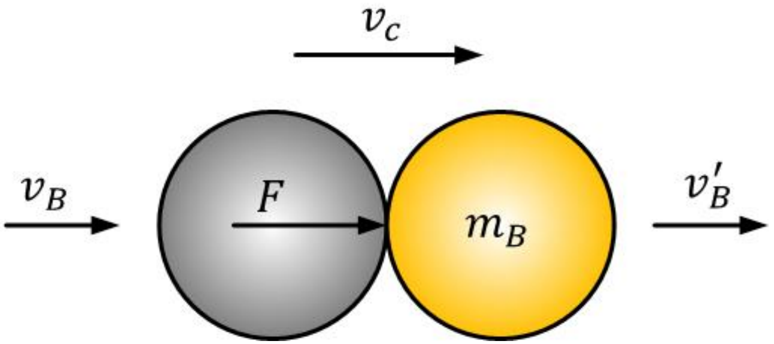


Particle A

$$\int_{t_1}^{t_c} -F dt = m_A v_c - m_A v_A \qquad (1) \qquad \text{Deformation}$$

$$\int_{t_c}^{t_2} -F dt = m_A v'_A - m_A v_c \qquad (2) \qquad \text{Restoration}$$

Coefficient of Restitution (e)

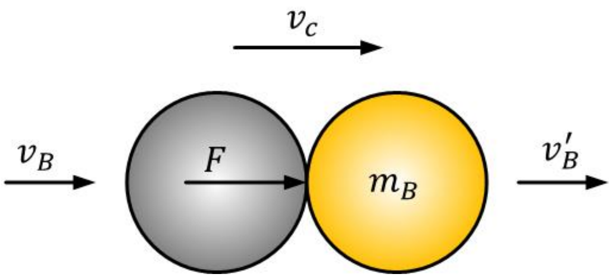


Particle B

$$\int_{t_1}^{t_c} F\,dt = m_B v_c - m_B v_B \qquad (3) \qquad \text{Deformation}$$

$$\int_{t_c}^{t_2} F\,dt = m_B v_B' - m_B v_c \qquad (4) \qquad \text{Restoration}$$

Coefficient of Restitution (e)



$$e = \frac{\int_{t_c}^{t_2} F dt}{\int_{t_1}^{t_c} F dt}$$

For particle A
$$e = \frac{v'_A - v_c}{v_c - v_A} \quad (5)$$

For particle B
$$e = \frac{v'_B - v_c}{v_c - v_B} \quad (6)$$

Coefficient of Restitution (e)

Eliminating v_c from (5) and (6) gives

$$e = \frac{v'_B - v'_A}{v_A - v_B} = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}} = \frac{\text{Difference in velocity after impact}}{\text{Difference in velocity before impact}}$$

Coefficient of restitution equation

$$\boxed{e = \frac{|v'_B - v'_A|}{|v_A - v_B|}}$$

Note:

$e \leq 1$ always

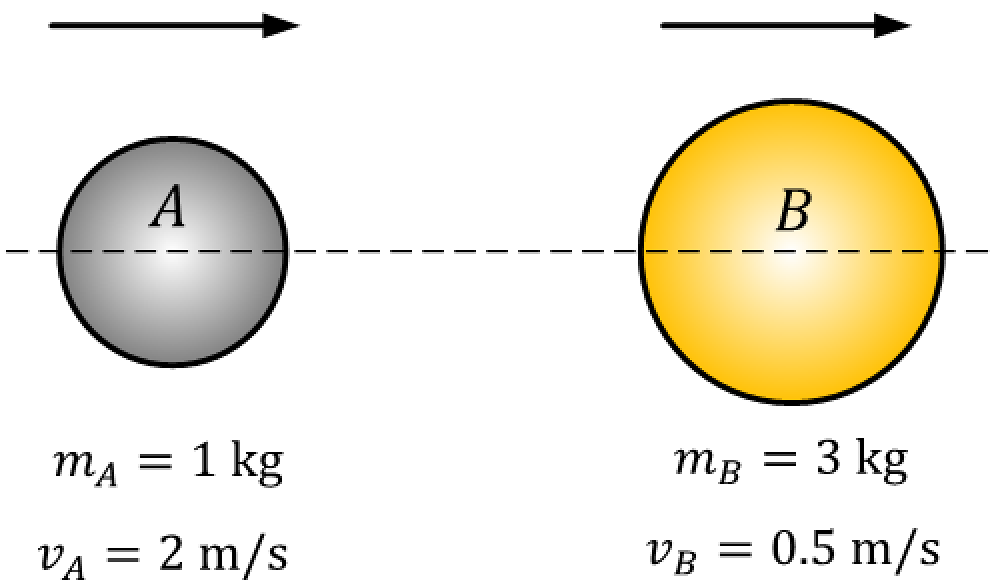
$e = 1 \Rightarrow$ perfectly elastic

$e = 0 \Rightarrow$ perfectly plastic

$0 \leq e \leq 1$

Example 7

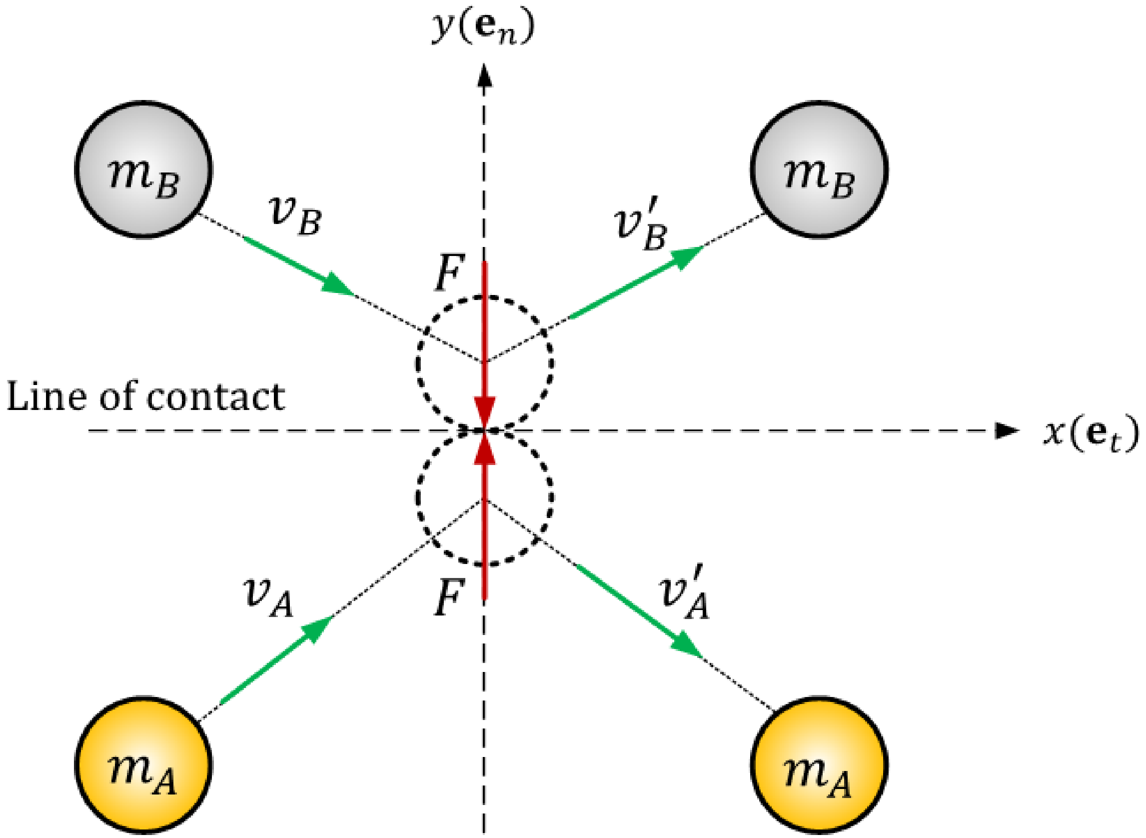
Calculate the velocity of each mass after impact. Find the change in total kinetic energy of the system.
Given: $e = 0.6$



W9 Example 7 (Web view)

Oblique Central Impact

Now consider two bodies approaching at an angle. The contact forces still pass through the centre of mass of each object (central impact).



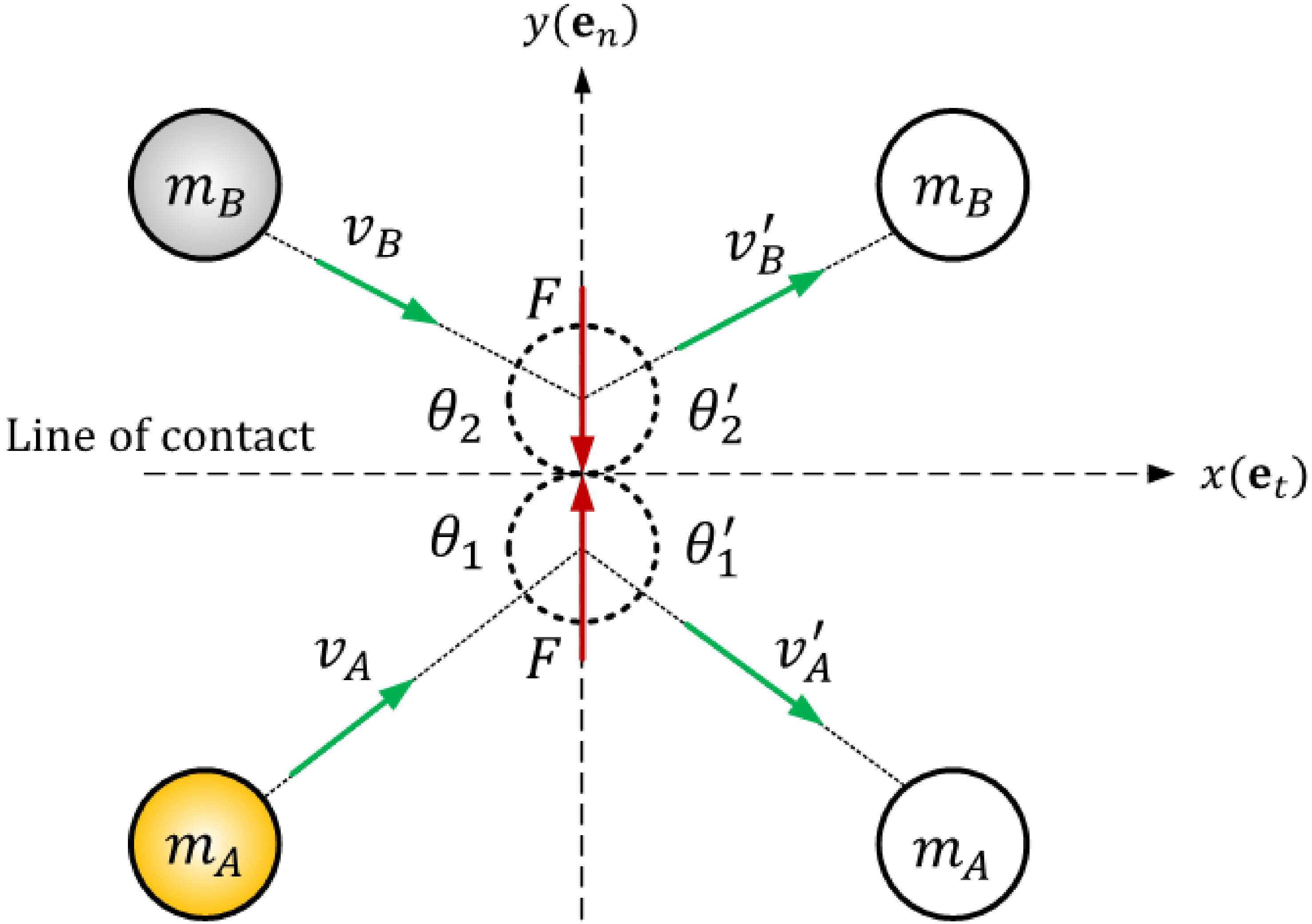
Oblique Central Impact

Conservation of linear momentum

$$m_A \mathbf{v}_A + m_B \mathbf{v}_B = m_A \mathbf{v'}_A + m_B \mathbf{v'}_B$$

Attach a normal ($\mathbf{e}_n$) and tangential ($\mathbf{e}_t$) coordinate system. The contact force during impact acts purely in the normal ($\mathbf{e}_n$) direction, that is, normal to the line of contact. The line of contact is in the tangential ($\mathbf{e}_t$) direction.

Oblique Central Impact



Oblique Central Impact

Hence we can write two scalar conservation of momentum equations (one in the $\boldsymbol{e}_n$ direction and the other in the $\boldsymbol{e}_t$ direction).

$$m_A v_{An} + m_B v_{Bn} = m_A v'_{An} + m_B v'_{Bn}$$

$$m_A v_{At} + m_B v_{Bt} = m_A v'_{At} + m_B v'_{Bt}$$

where

$$v_{At} = v_A \cos\theta_1 \qquad v'_{At} = v'_A \cos\theta'_1$$

$$v_{An} = v_A \sin\theta_1 \qquad v'_{An} = -v'_A \sin\theta'_1$$

$$v_{Bt} = v_B \cos\theta_2 \qquad v'_{Bt} = v'_B \cos\theta'_2$$

$$v_{Bn} = -v_B \sin\theta_2 \qquad v'_{Bn} = v'_B \sin\theta'_2$$

Oblique Central Impact

If all of the contact force is acting in the normal ($\mathbf{e}_n$) direction, for each particle, the impulse in the tangential ($\mathbf{e}_t$) direction is zero.

Since the Impulse-Momentum equation states

$$\int_{t_1}^{t_2} \sum \boldsymbol{F}\, dt = m\boldsymbol{v}_2 - m\boldsymbol{v}_1 = 0$$

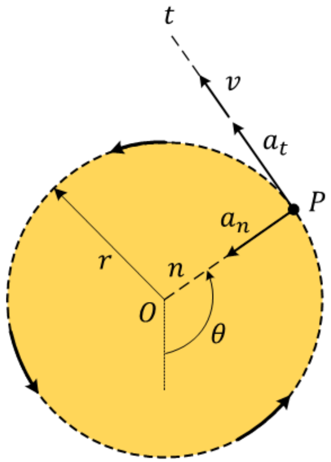
in the $\mathbf{e}_t$ direction

$$\Rightarrow \quad m_A v_{At} = m_A v'_{At} \qquad \Rightarrow \quad v'_{At} = v_{At}$$

$$m_B v_{Bt} = m_B v'_{Bt} \qquad \Rightarrow \quad v'_{Bt} = v_{Bt}$$

Oblique Central Impact

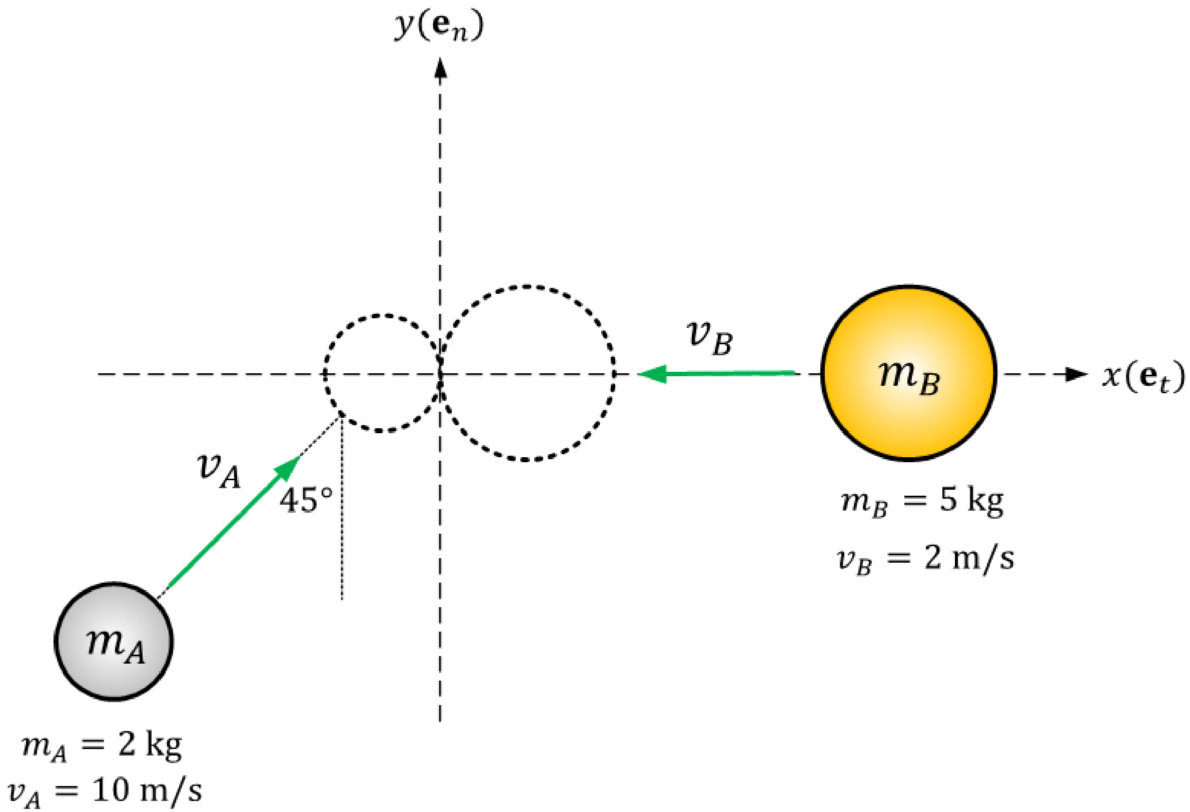
The coefficient of restitution, as before, is the ratio of recovery impulse/deformation impulse, and now only applies to the $\mathbf{e}_n$ direction (direction of contact forces).



$$e = \frac{|v'_{Bn} - v'_{An}|}{|v_{An} - v_{Bn}|}$$

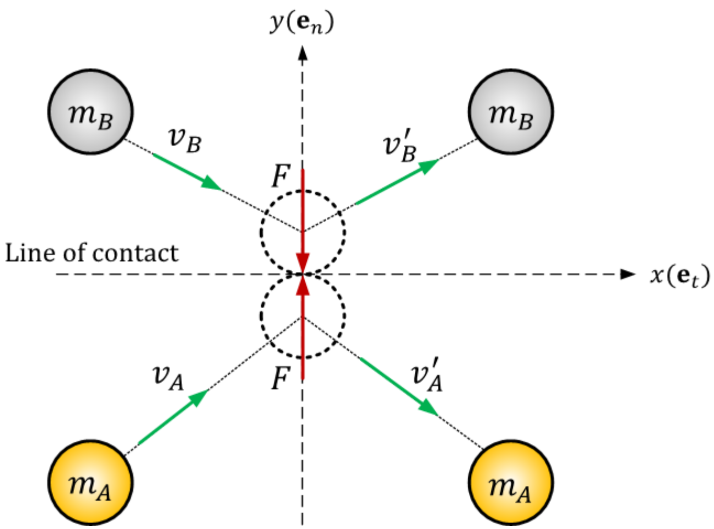
Example 8

Calculate the final velocity of each mass after impact.
Given: $e = 0.8$



W9 Example 8 (Web view)

Summary of Particle Momentum



- By integrating Newton's law, we get the Impulse–Momentum equation

$$\int \mathbf{F} dt = m\mathbf{v}_2 - m\mathbf{v}_1$$

- For closed systems, we deduce the conservation of linear momentum

$$\sum m_f \mathbf{v}_f = \sum m_i \mathbf{v}_i$$

- Imperfect elasticity yields the coefficient of restitution

$$e = \frac{|v'_{Bn} - v'_{An}|}{|v_{An} - v_{Bn}|}$$

Next Topic:

Rigid Body Kinematics