School of Mechanical and Manufacturing Engineering

MMAN1300 Engineering Mechanics 1

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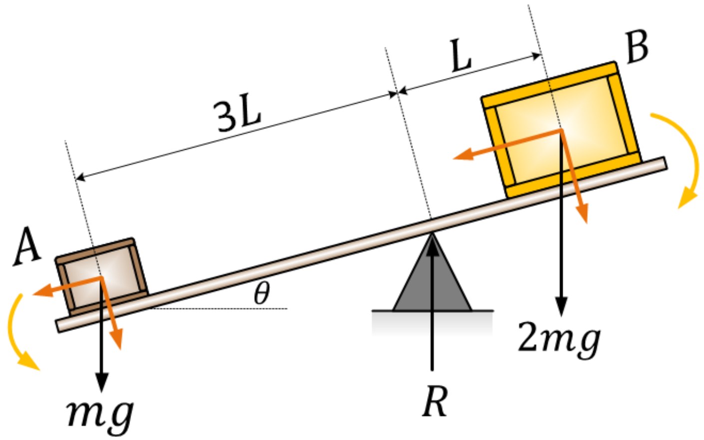


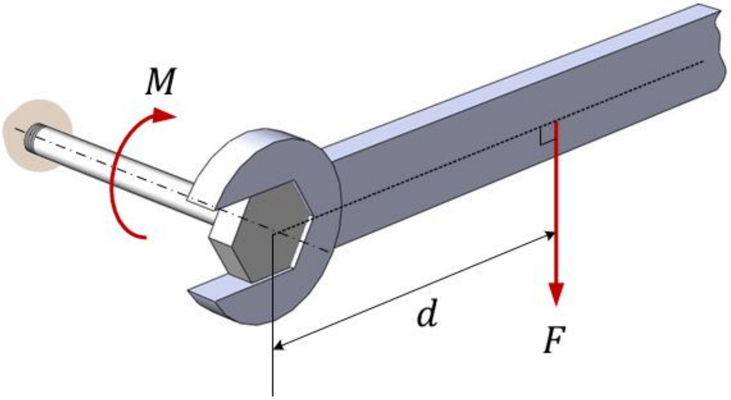
Week 2: FBDs and Equilibrium

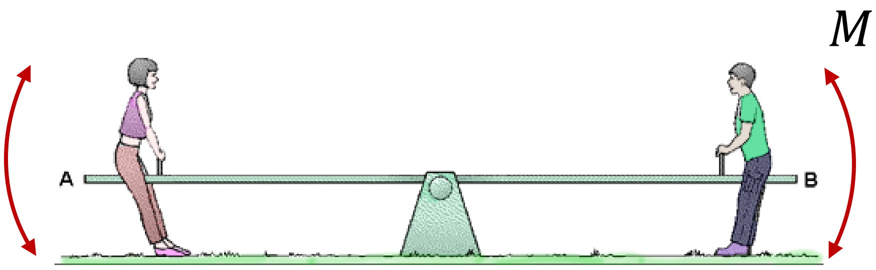
EQUILIBRIUM SYSTEMS

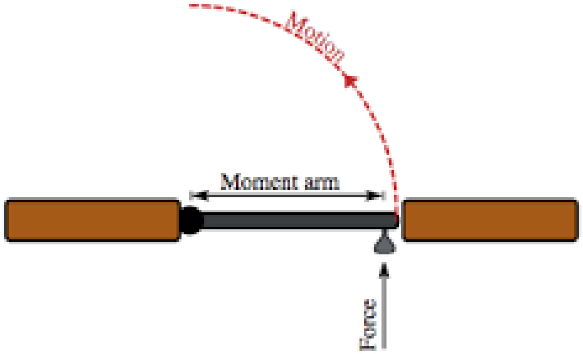
- Moments
- Free Body Diagrams
- Equilibrium
- Equivalent Loads
- Spring Forces

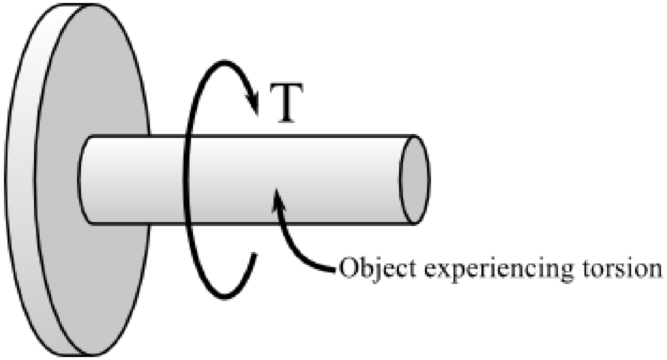
Moments

- In Engineering Mechanics, a moment is our concept of a force that tends to cause turning of an object
- We also sometimes refer to the same concept as torque









Let's review for a moment

- The magnitude of a vector is given by

$$M = rF \sin\theta$$

- The direction is given by the right-hand rule
- This is all very familiar
- We can calculate moments as **cross products**

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

- Recall that the cross products are **NOT** commutative

Free Body Diagrams

Free Body Diagrams may be our **most important tool** in Engineering Mechanics

- In order to draw a good FBD, wee need to

  1. Identify the system
  2. Isolate the system
  3. Determine how the system interacts with its surroundings
  4. Identify all the unknown forces (and moments) acting on the system

- If we do these things correctly, we are well on our way to a solution

1. System Selection

Decide clearly on the choice of the Free Body (system) to be used

- This depends strongly on the physical situation and the problem to be solved

- Once the decision is made, we detach the body from its surroundings
  - ✓ It is important to note that the body in the Free-body diagram may constitute any number (including fractions) of physical bodies

- We then draw the body in isolation

2. External forces (and moments)

Identify all the external forces (and moments) on the isolated body

- Forces are interactions between objects (and pieces of objects)
- If we have detached the Free body from surrounding objects, the action of those objects on the free body are represented by forces on the FBD
- Forces should be drawn on the FBD at the point where the interaction takes place
  - ✓ Usually this is on the boundary, but not for things like weight

3. Known external forces (and moments)

Mark the magnitudes and directions of the **known** external forces and moments

- Remember that we draw the forces which are exerted on the free body, not forces exerted by the free body

- Known forces are typically the weight of the free body and applied forces

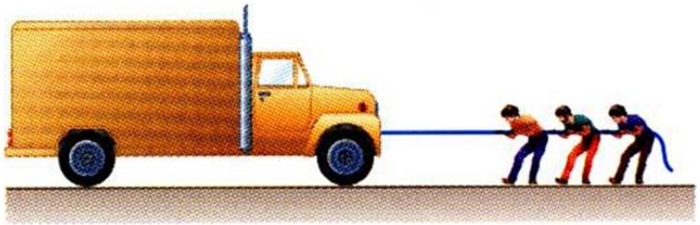
4. Unknown forces and moments

Draw the **unknown** forces and moments on the FBD

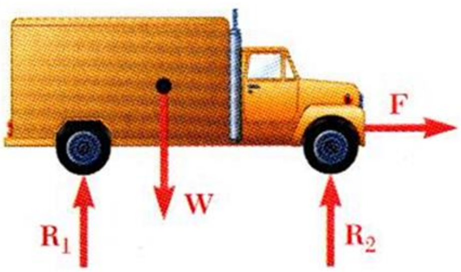
- The unknown forces are typically called reactions or constraint forces
- Usually these arise from the system interacting with its supports or the ground in such a way that its motion is restricted
- We will note how to handle many common supports a bit later
- If the direction or sense of the reaction is not apparent, do not try to determine it at this stage
  - ✓ Instead, arbitrarily assume the direction
  - ✓ The assumption will be verified by solution to the problem

External and Internal Forces

- Forces acting on rigid bodies are divided into two groups
  - ✓ External forces
  - ✓ Internal forces



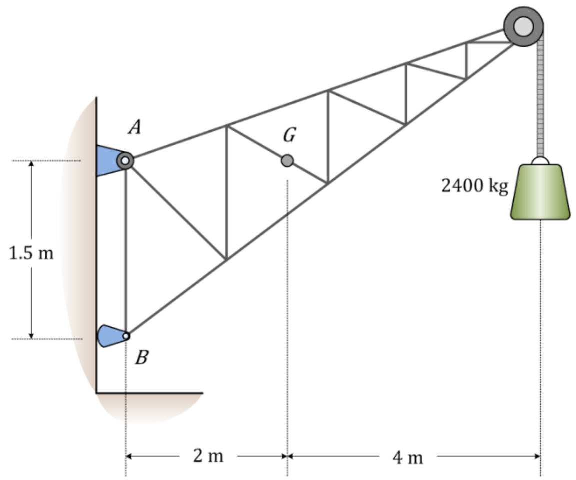
- External forces are shown in a free-body diagram

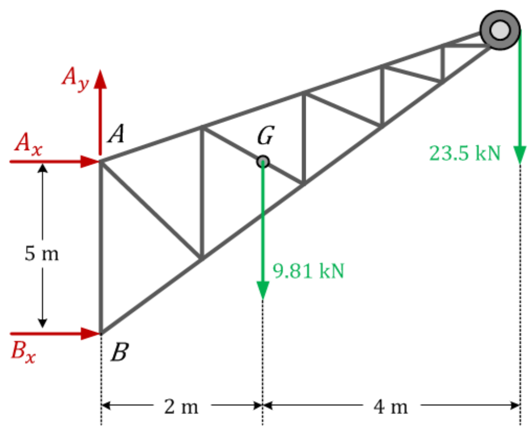


- If unopposed, each external force can impart a motion of translation or rotation, or both

Drawing an FBD

First step in the static equilibrium analysis of a rigid body is identification of all forces acting on the body with a *free-body* diagram

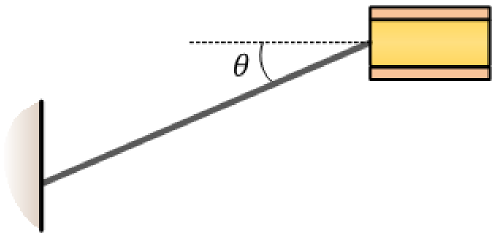




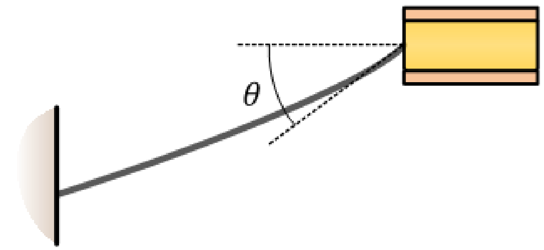
- Select the extent of the free-body and detach it from the ground and all other bodies
- Indicate point of application, magnitude, and direction of external forces, including rigid body’s weight
- Indicate point of application and assumed direction of unknown applied forces. These usually consist of reactions through which the ground and other bodies oppose the possible motion of rigid body
- Include the dimensions necessary to compute the moments of forces

Common types of planar supports

1. Flexible cable, belt, chain, or rope



Weight of cable negligible



Weight of cable not negligible

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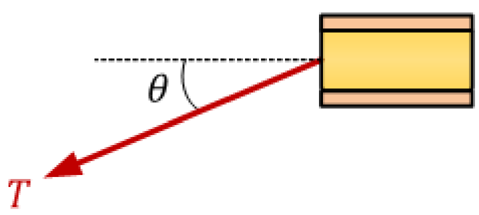
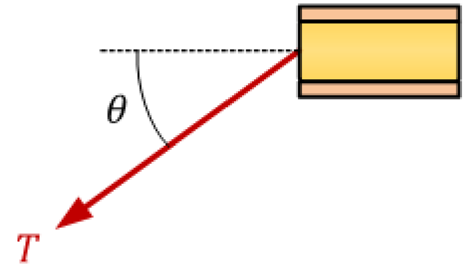
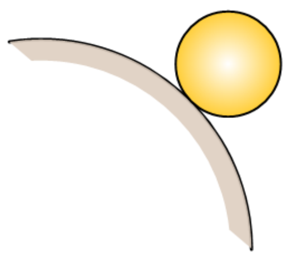




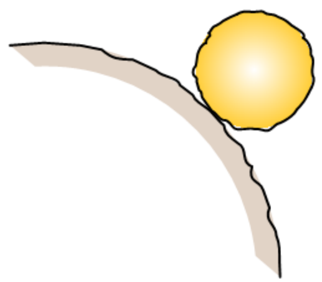
- Force exerted by a flexible cable is always a tension away from the body in the direction of the cable

Common types of planar supports

2. Smooth surfaces



3. Rough surfaces



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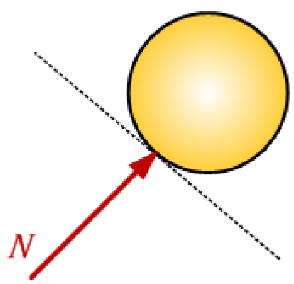

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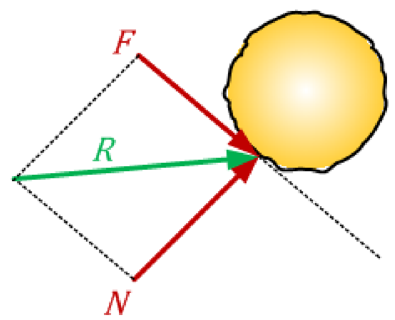
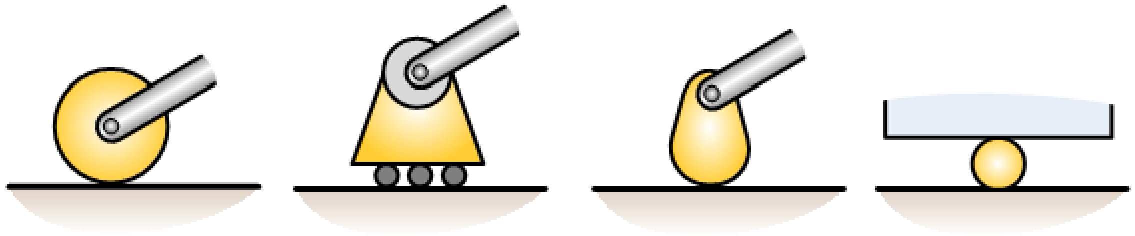


Contact force is compressive and its normal to the surface (plane of contact)

Rough surfaces are capable of supporting a tangential component F as well as a normal component N of the resultant

Common types of planar supports

4. Roller support



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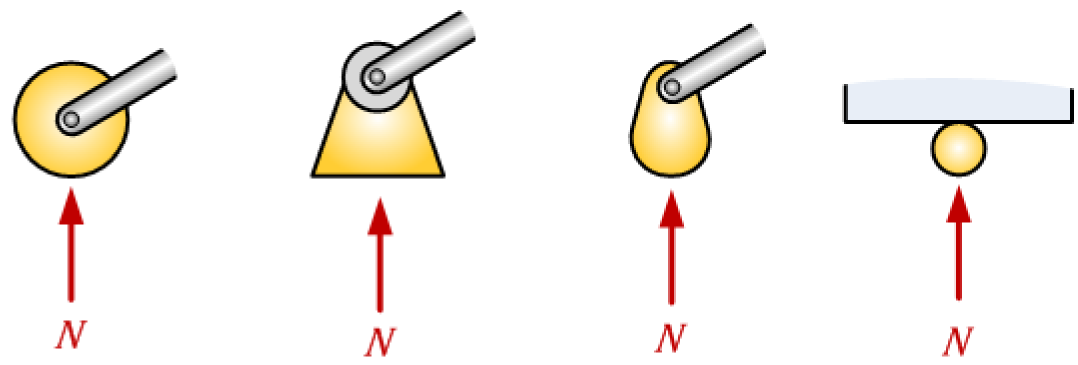
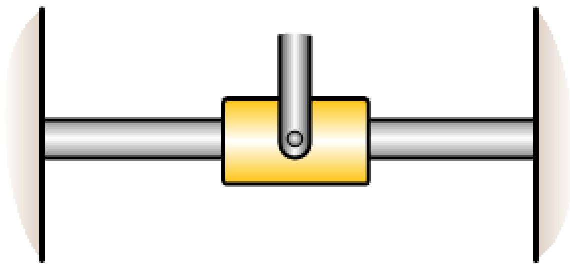
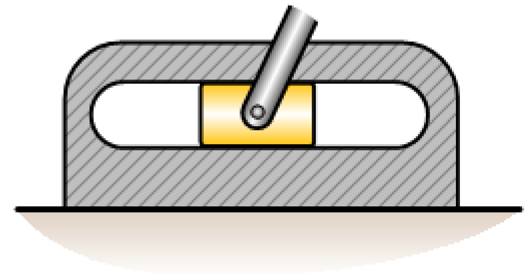


- Roller, rocker, or ball support transmits a compressive force normal to the supporting surface

Common types of planar supports

5. Freely sliding guide





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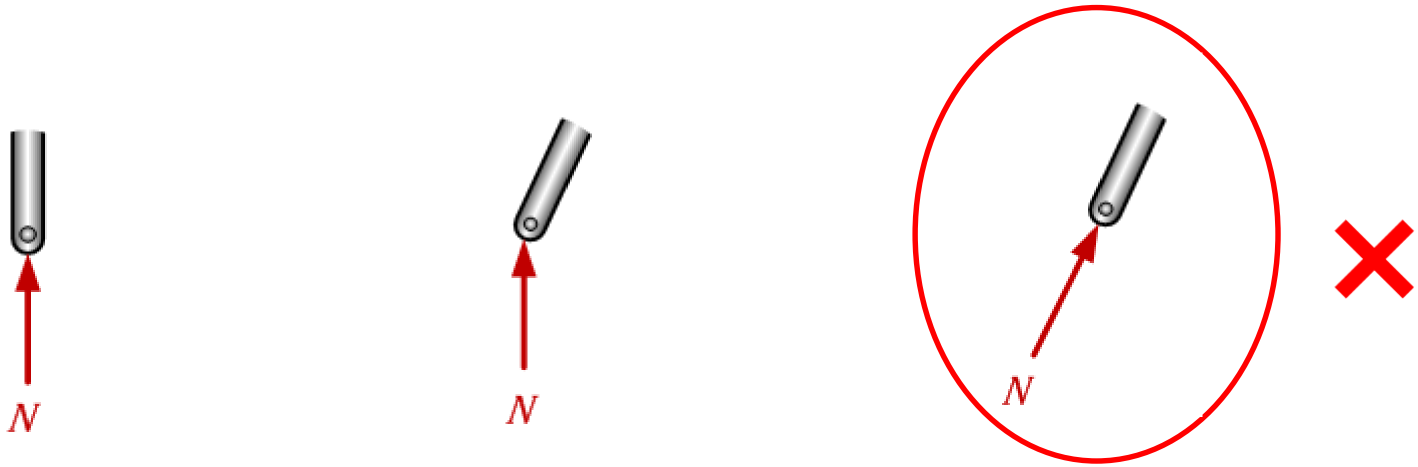
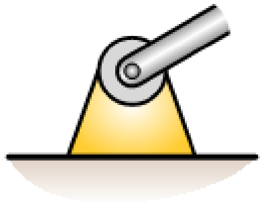


- Collar or slider free to move along smooth guides; can support force normal to guide only

Common types of planar supports

6. Pin connection



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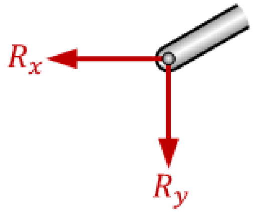
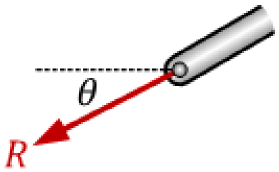
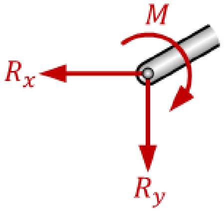




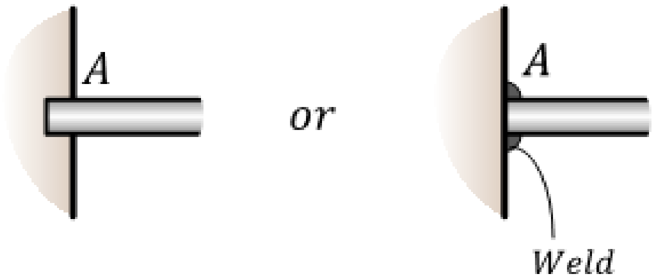


A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and a direction θ

A pin not free to turn also supports a couple M

Common types of planar supports

7. Built-in or fixed support



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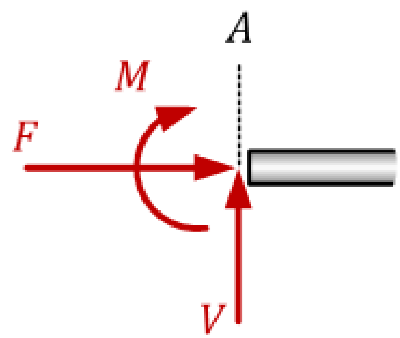
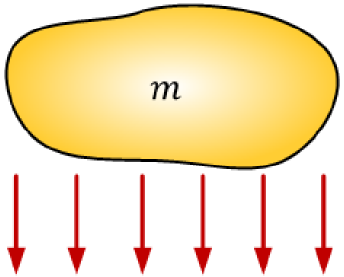


- A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force) and a couple M (bending moment) to prevent rotation

Common types of planar supports

8. Gravitational attraction



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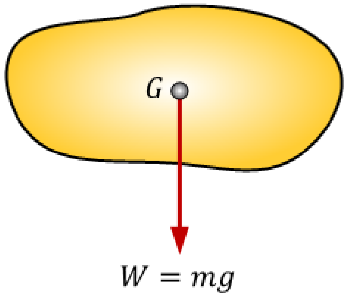
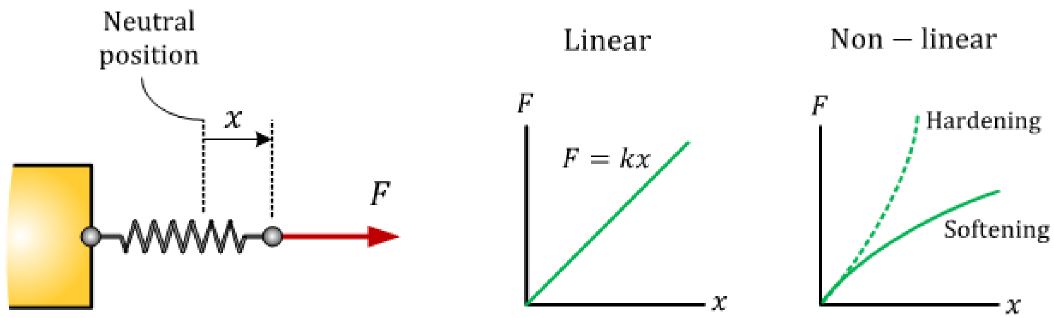


- The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts towards the centre of the earth through the mass centre G

Common types of planar supports

9. Spring action



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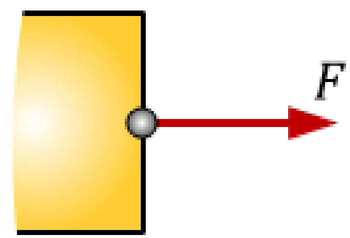


- Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance

Equilibrium

If a system is in equilibrium, it is in a state of balance

- A system in mechanical equilibrium has unchanging linear and angular momentum
  - ✓ Usually in statics, both are equal to zero

- This requires that the loads on the system are balanced in particular ways

Equilibrium

Engineers often care about equilibrium

- We want to make sure that the loads needed to create equilibrium do not exceed the capabilities of the members

- Or we may wish to choose appropriate dimensions for a piece of the structure so that it may carry the necessary load (without spending too much money to make it too big)

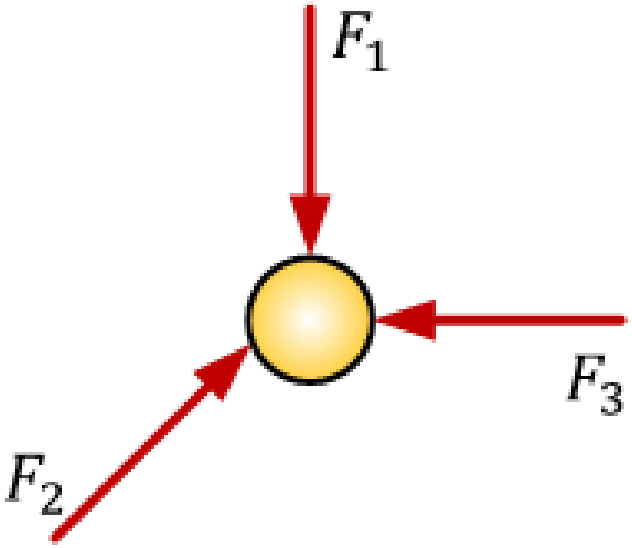
We know how to find equilibrium for a particle

- Particles (by definition) do not have any angular momentum

- So we just have to make sure the linear momentum is unchanging

- This is Newton's 1st Law

$$F_{net} = 0$$



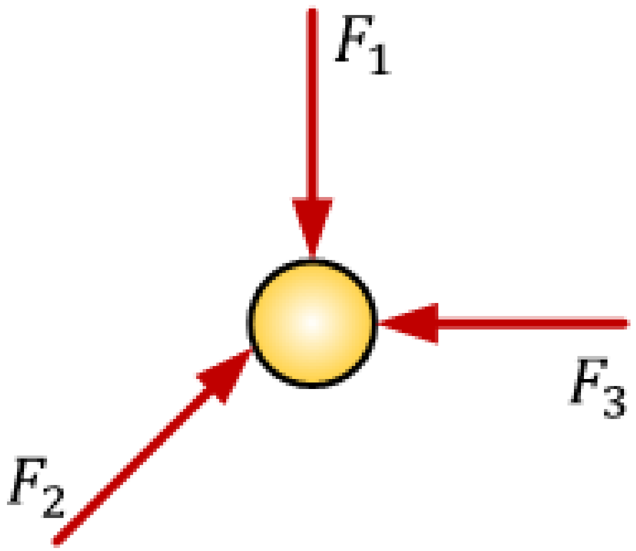
The net force, of course, is the sum of all of the forces acting on the system

- In the simple example at right, we have simply

$$\sum_{\substack{all\\forces}} F = F_1 + F_2 + F_3 = 0$$

- We call this relation the **force equilibrium condition**

- In words, there must be *no net force* or *the vector sum of forces on the system is zero*



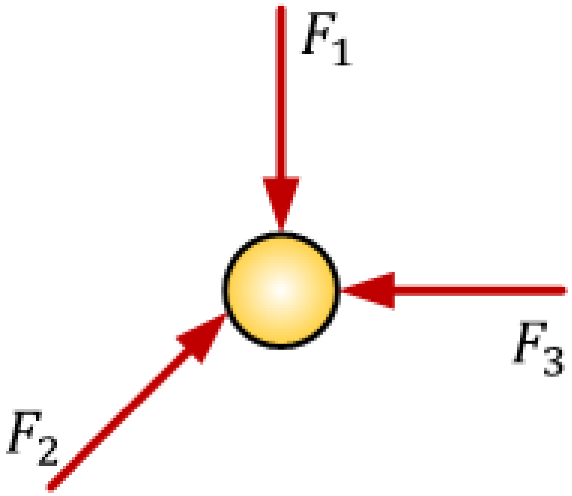
Usually, we deal with the force equilibrium condition in vector-component form

- Then we have 3 scalar equations (or 2 in a planar case)

i: $F_{net} = \sum F_x = 0$

j: $F_{net} = \sum F_y = 0$

k: $F_{net} = \sum F_z = 0$

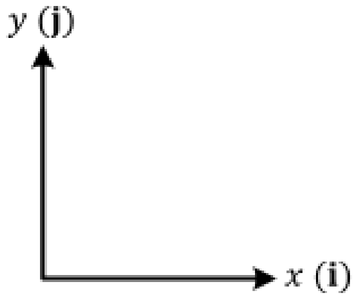


- In our simple example

$$F_{1x} + F_{2x} + F_{3x} = 0$$

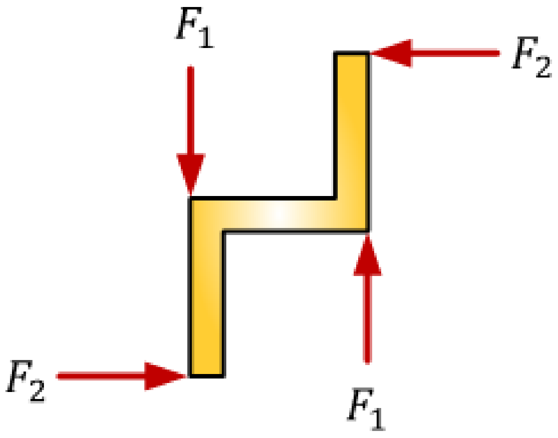
$$F_{1y} + F_{2y} + F_{3y} = 0$$

$$F_{1z} + F_{2z} + F_{3z} = 0$$



But what about things that aren’t particles?

- The vector sum of forces on the object at right is zero
- However, our intuition tells us that it is **not** equilibrium
- It will tend to rotate
  - ✓ That is, its angular momentum will tend to change
- Clearly, the force balance is not enough to guarantee equilibrium

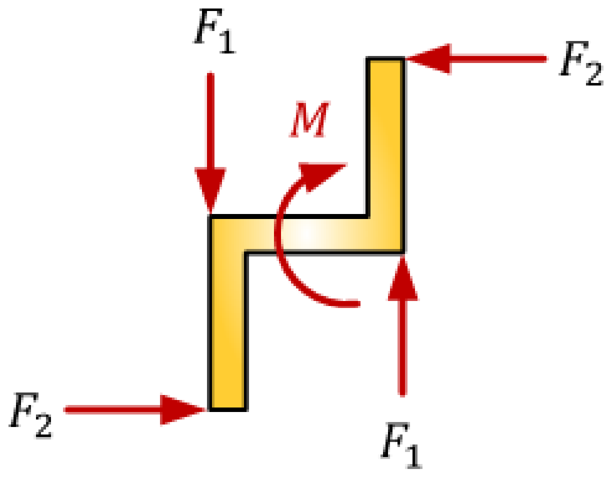


The net moment must be zero

- The net moment is the sum of all the applied moments plus the moments generated by all the applied forces

- Mathematically we write

$$M_{net} = 0$$



Again, it is almost always more convenient to deal with vector components

- We can generate another 3 scalar equations from our moment balance

i: $$M_{net} = \sum M_x = 0$$

j: $$M_{net} = \sum M_y = 0$$

k: $$M_{net} = \sum M_z = 0$$

Altogether, we can generate 6 independent, scalar equations

- 3 from the force balance

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$

- 3 from the moment balance

$$\sum M_x = 0$$

$$\sum M_y = 0$$

$$\sum M_z = 0$$

Why not choose lots of points to take moments?

- If the moments must balance about every point, can't we use lots of points and generate lots of equations and solve for lots of unknowns??

- It is certainly true that we can generate as many moment balance equations as we like

- Unfortunately, a rigid body only has 6 degrees of freedom (or 3 in the planar case)

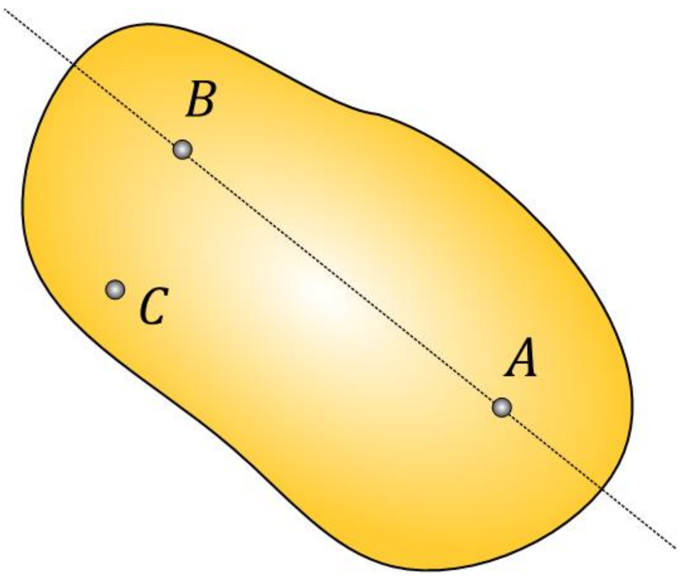
- So at most, 6 (or 3, for planar) of our scalar equations will be independent
  - ✓ This total includes the number we get from our force balance

Force or Moment balance equations

It is sometimes more convenient to replace some force balance equations with extra moment balance equations

- You can eliminate an unknown force from one balance equation by taking moments about a point on its line of action

- Just make sure that if you take more than 2 moment balances, you do not choose more that 2 points that lie on the same line
  - ✓ You equations will not account for any forces acting on that line



For a planar case, we have then 3 options to get our 3 scalar equations:

1

Sum of forces in 2 directions and sum of moments about 1 point (A)

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_A = 0$$

2

Sum of forces in 1 direction and sum of moments about 2 points (A, B)

$$\sum F_x = 0$$

$$\sum M_A = 0$$

$$\sum M_B = 0$$

3

Sum of moments about 3 non-collinear points (A, B, C)

$$\sum M_A = 0$$

$$\sum M_B = 0$$

$$\sum M_C = 0$$

Determinacy

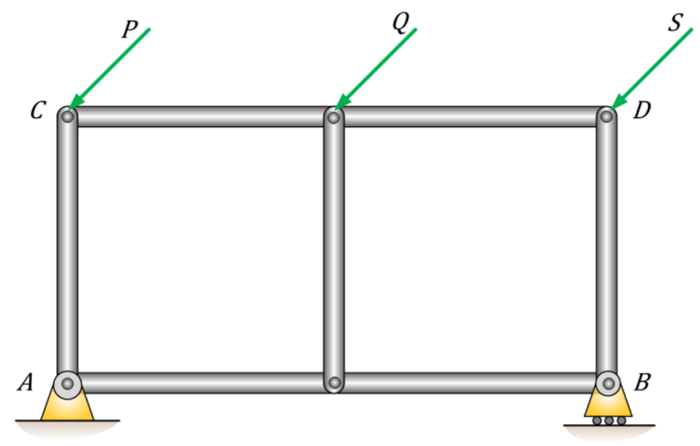
- For all forces and moments acting on a two-dimensional structure

$$F_z = 0 \qquad M_x = M_y = 0 \qquad M_z = M_o$$

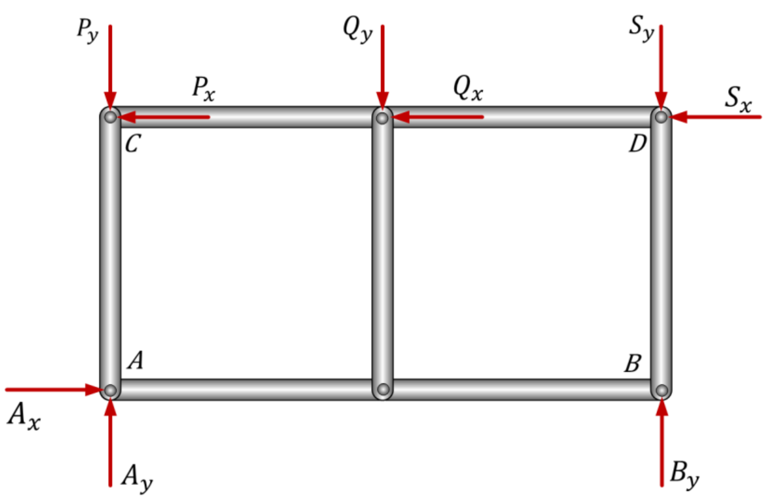
- Equations of equilibrium become

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_A = 0$$

Where A is any point in the plane of structure



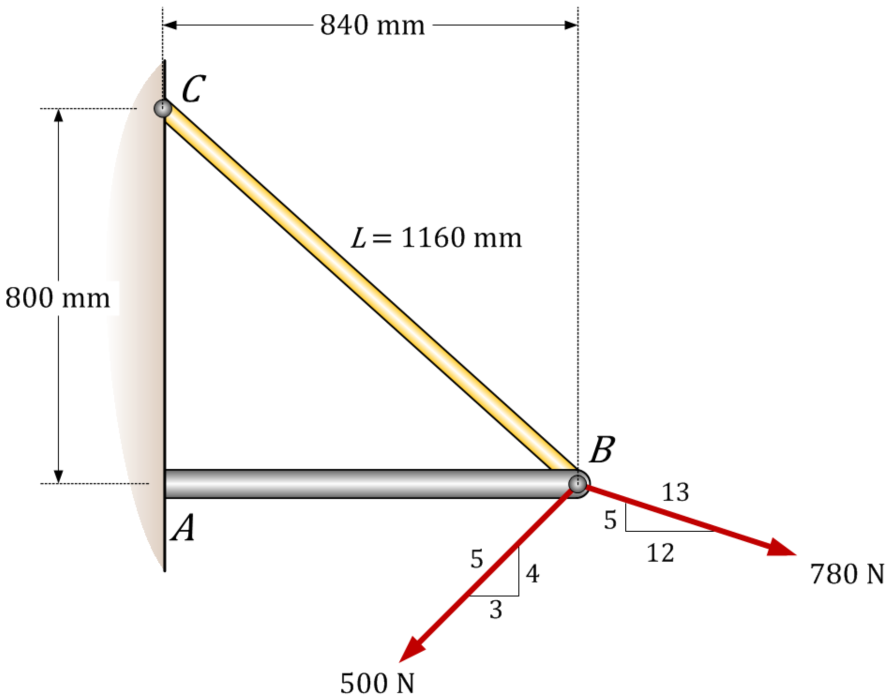
- The 3 equations can be solved for no more than 3 unknowns
- The 3 equations can not be augmented with additional equations, but they can be replaced



$$\sum F_x = 0 \qquad \sum M_A = 0 \qquad \sum M_B = 0$$

Example 1

Tension in cable *BC* is 725 N, determine the resultant of the three forces exerted at point *B* of beam *AB*

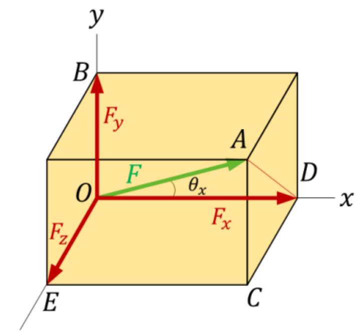


Approach:

- Resolve each force into rectangular components
- Determine the components of the resultant by adding the corresponding force components
- Calculate the magnitude and direction of the resultant

W2 Example 1 (Web view)

Rectangular Components in Space



$F_x = F\cos\theta_x$ $F_y = F\cos\theta_y$ $F_z = F\cos\theta_z$

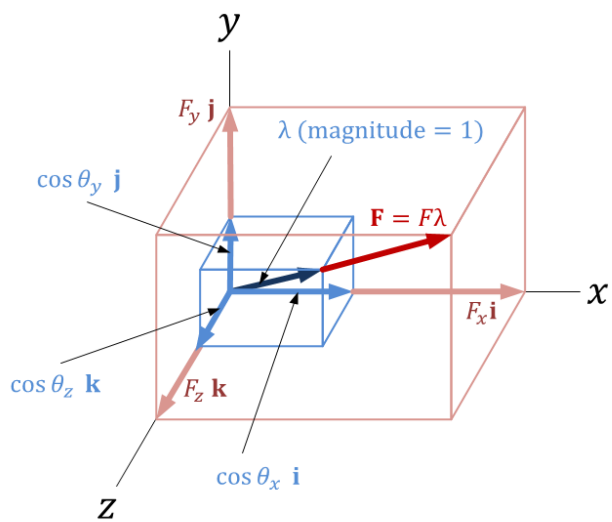
$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}$$

$$\mathbf{F} = F\cos\theta_x\,\mathbf{i} + F\cos\theta_y\,\mathbf{j} + F\cos\theta_z\,\mathbf{k}$$

$$\mathbf{F} = F\left(\cos\theta_x\,\mathbf{i} + \cos\theta_y\,\mathbf{j} + \cos\theta_z\,\mathbf{k}\right)$$

$$\mathbf{F} = F\,\lambda$$

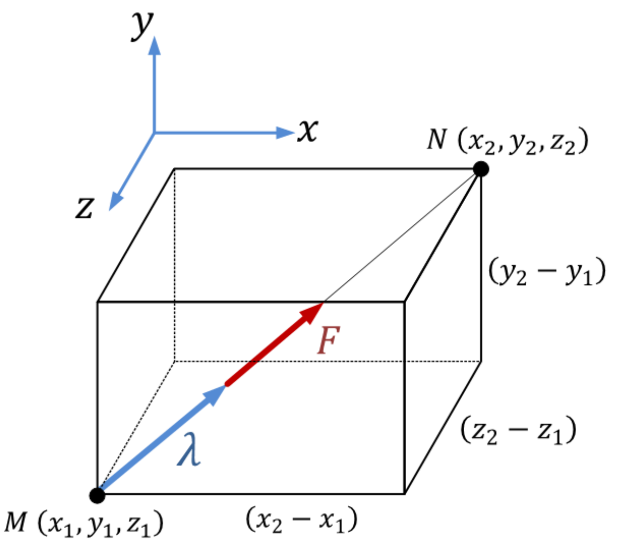
$$\lambda = \cos\theta_x\,\mathbf{i} + \cos\theta_y\,\mathbf{j} + \cos\theta_z\,\mathbf{k}$$



λ is a unit vector along the line of action of $\boldsymbol{F}$ and $\cos\theta_x$, $\cos\theta_y$ and $\cos\theta_z$ are the direction cosine of $\boldsymbol{F}$

Direction of the force is defined by the location of two points:

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$\boldsymbol{d}$ is the vector joining M and N

$$\mathbf{d} = d_x\,\mathbf{i} + d_y\,\mathbf{j} + d_z\,\mathbf{k}$$

$$d_x = (x_2 - x_1) \qquad d_y = (y_2 - y_1) \qquad d_z = (z_2 - z_1)$$

$$\mathbf{F} = F\lambda = F\left(\frac{d_x\,\mathbf{i} + d_y\,\mathbf{j} + d_z\,\mathbf{k}}{d}\right)$$

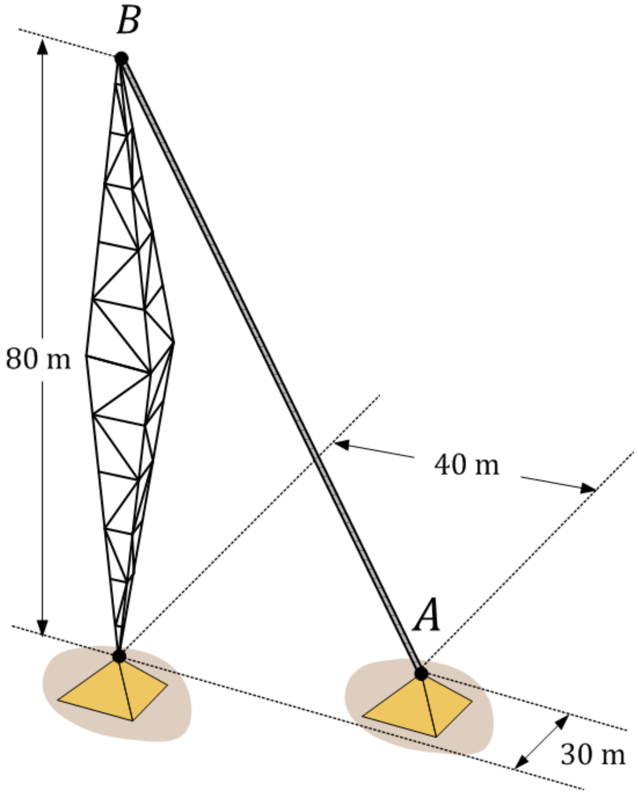
$$F_x = F\frac{d_x}{d} \qquad F_y = F\frac{d_y}{d} \qquad F_z = F\frac{d_z}{d}$$

Example 2

The tension in the guy wire is 2500 N. Determine:

(a) Components F_x, F_y, F_z of the force acting on the bolt at A

(b) The angles $\theta_x, \theta_y, \theta_z$ defining the direction of the force



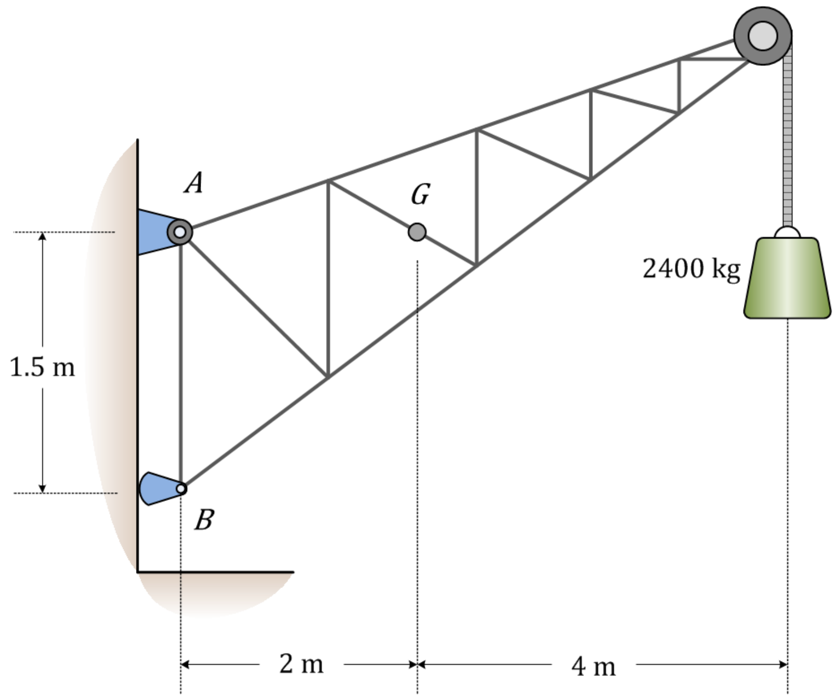
Approach:

- Based on the relative locations of the point *A* and *B*, determine the unit vector pointing from *A* towards *B*
- Apply the unit vector to determine the components of the force acting on *A*
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

W2 Example 2 (Web view)

Example 3

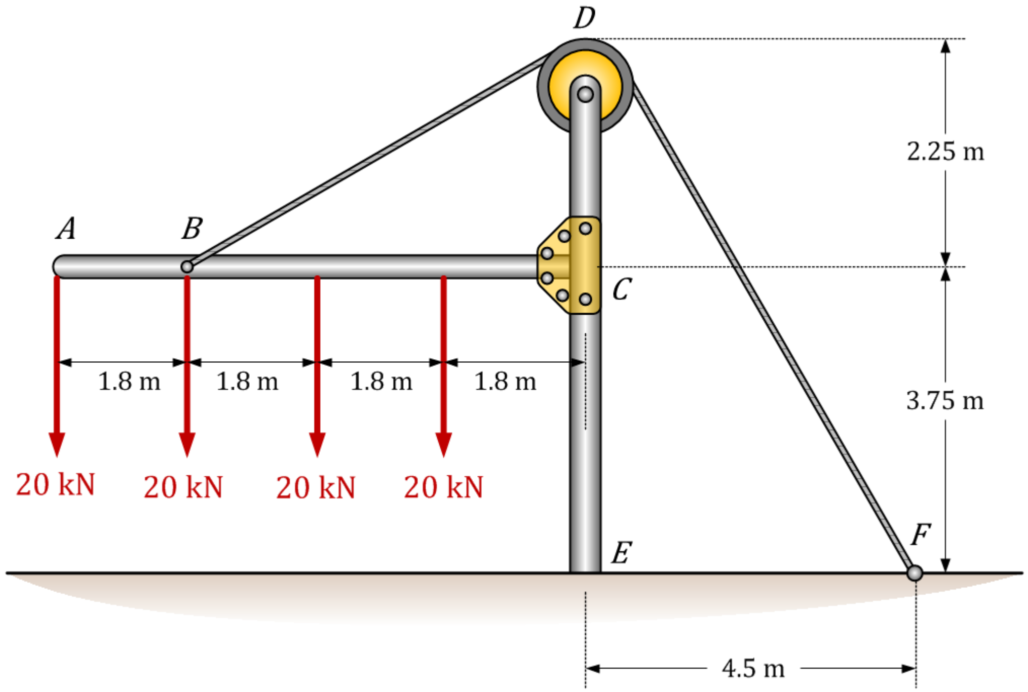
A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at *A* and a rocker at *B*. The centre of gravity of the crane is located at *G*



[W2 Example 3](#) ([Web view](#))

Example 4

The frame supports part of the roof of a small building. The tension in the cable is 150 kN. Determine the reaction at the fixed end *E*.



W2 Example 4 (Web view)

There is one other kind of nice thing:

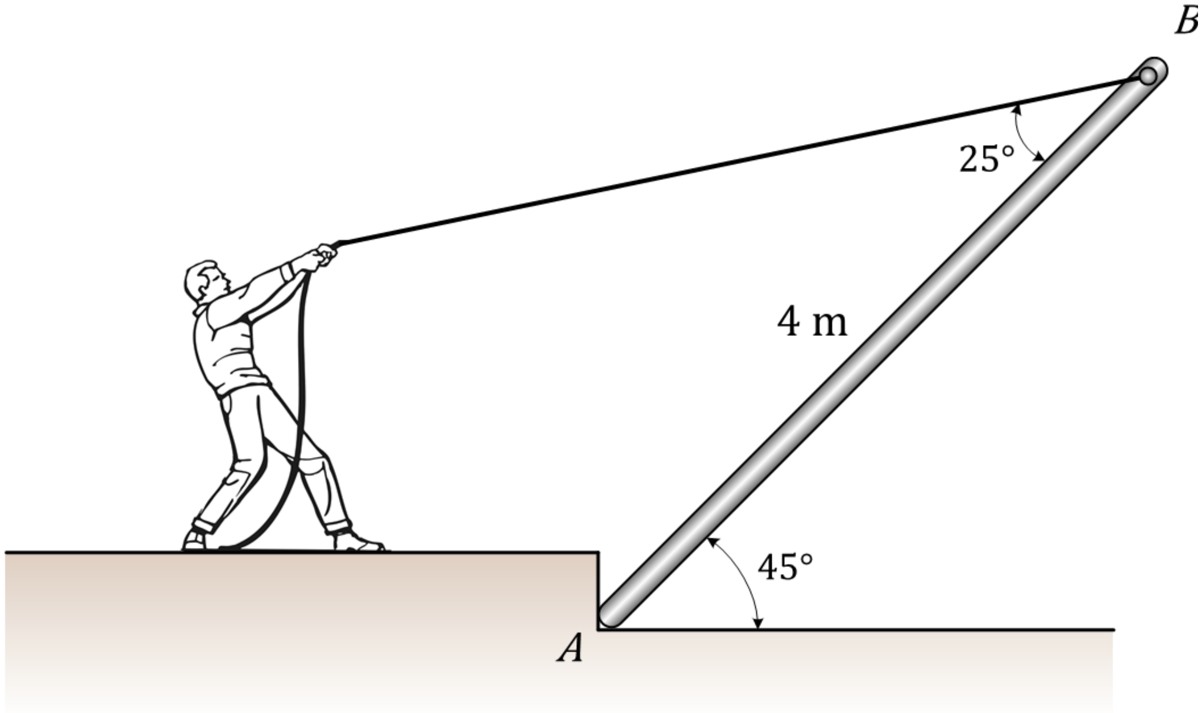
- If a system is in equilibrium, every portion of the system must also be in equilibrium

- That means we can sub-divide a system into any sub-systems that we choose and apply our equilibrium conditions to each sub-system

- Why would we ever want to do that?
  - ✓ To find loads on members of the system
  - ✓ To find internal loads in members
  - ✓ Sometimes we may not have enough equations to find all the unknowns in the full system

Example 5

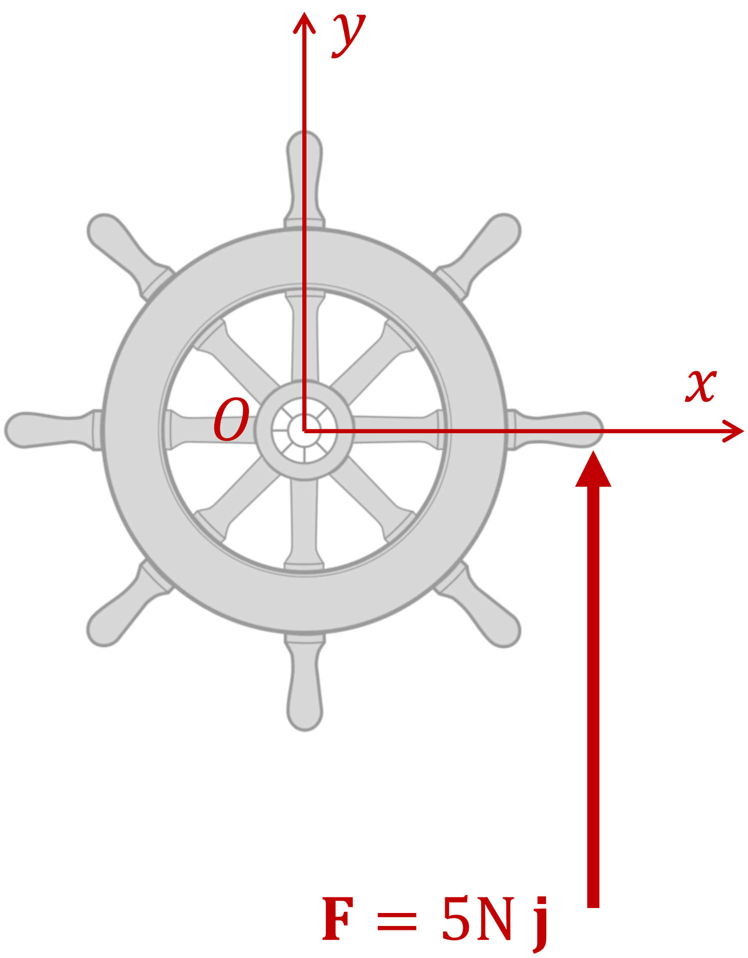
A man raises a 10 kg joist, of length 4 m, by pulling a rope. Find the tension in the rope and reaction at *A*



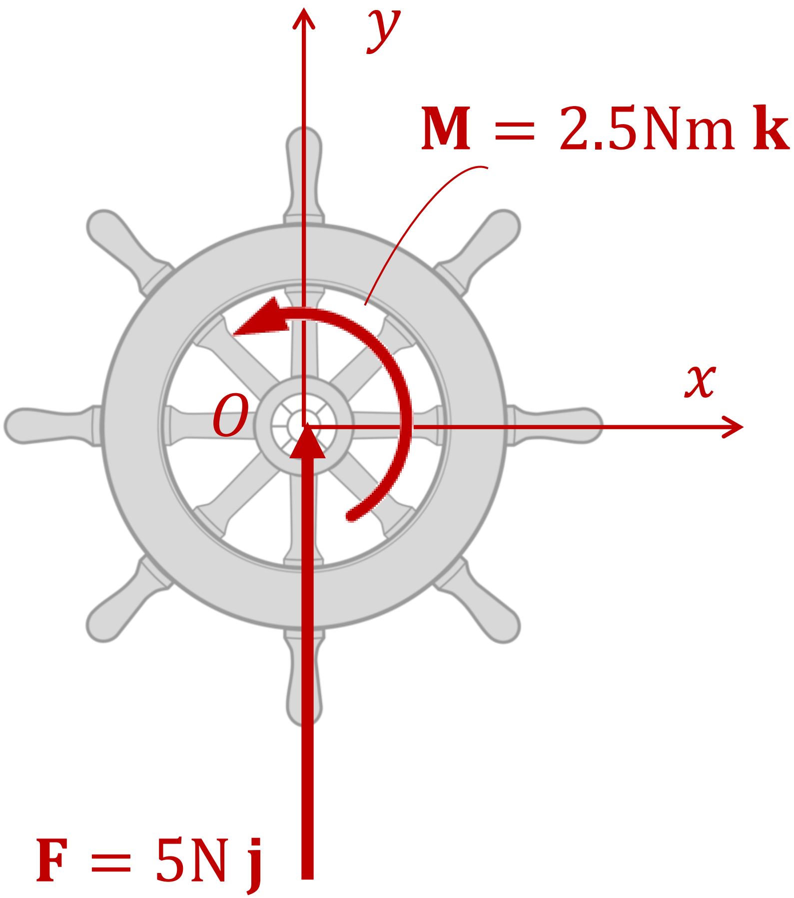
W2 Example 5 (Web view)

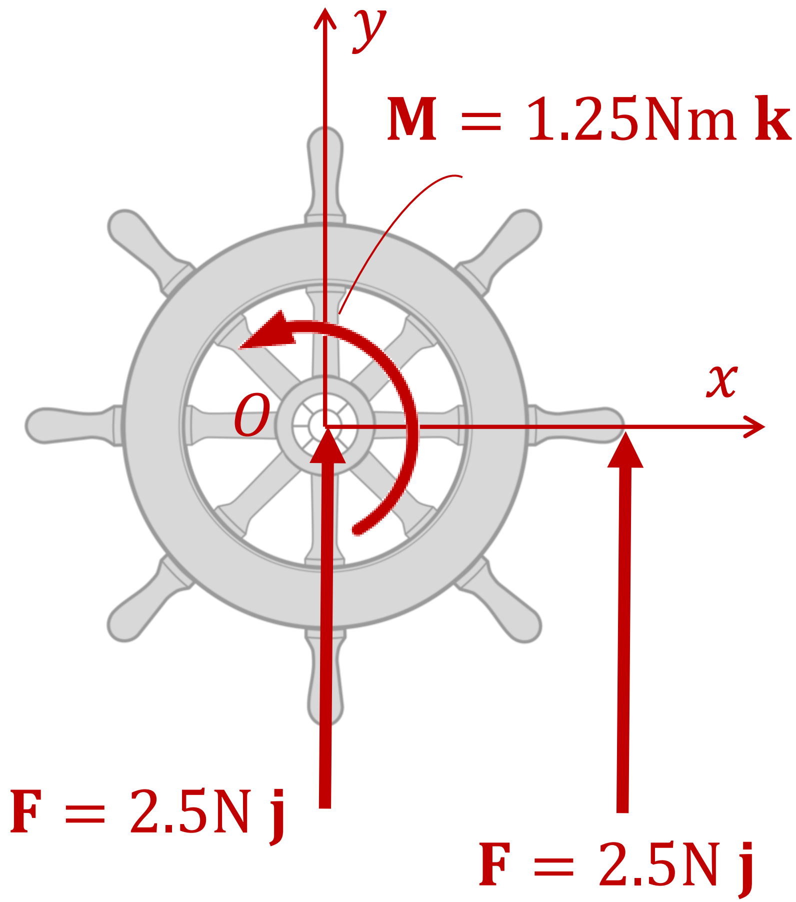
Equivalent Loads

- Let the radius of the wheel be 500 mm
- The if we apply a force of 5 N in the y - direction as shown, we cause a moment of 2.5 Nm about O in the z - direction
- So the wheel experiences a push of 5 N in the y – direction and a twist about O of 2.5 Nm in the z - direction



But there are other ways for the wheel to experience the same effects





We can dream up infinitely many loading conditions that have the same net effect

- Any two sets of loads that have the same net force and moment about a chosen moment centre are said to be **equivalent loads**.

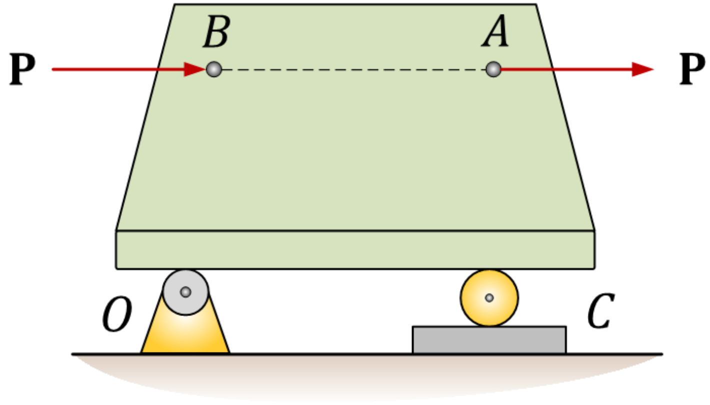
- The **equivalent loading principle** says that we can replace any system of forces and moments with an equivalent load consisting of a single force and a single moment acting at a single point

Line of Action

External effects of a force applied to a rigid body do not change if the point of application is moved along the line of action

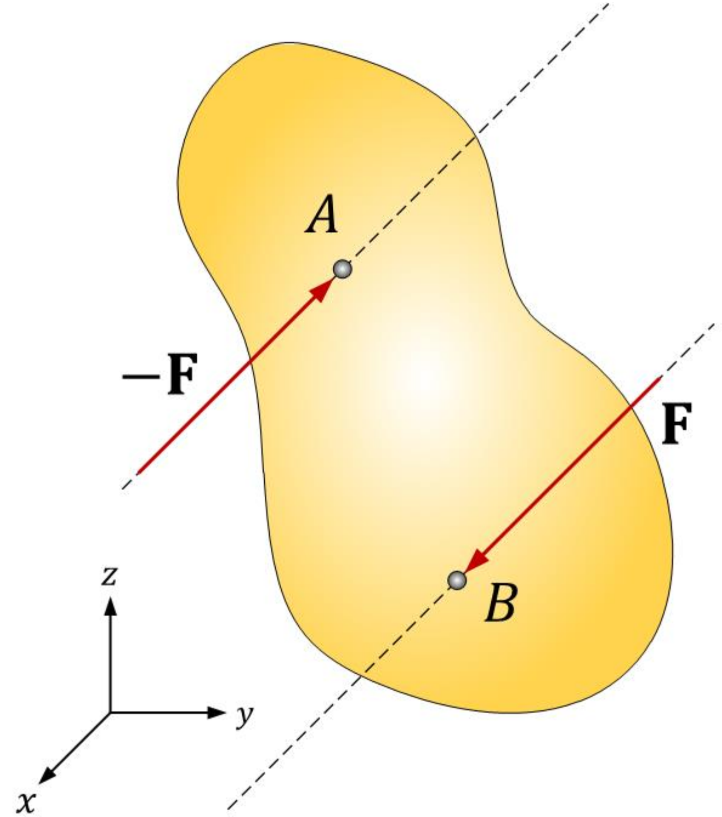
- This is called the **principle of transmissibility**

- The reactions at *O* and *C* are the same whether the force **P** is applied at *A* or *B*



We can see a simple example of equivalence when we introduce the moment of a couple

- A couple is a system of two forces of equal magnitude and opposite direction
  - ✓ That is, $\mathbf{F}$ and $-\mathbf{F}$



- We can see that the moment of the pair about A is

$$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{F}$$

That result does not change if we take *A* to be any point on the line of action of -F

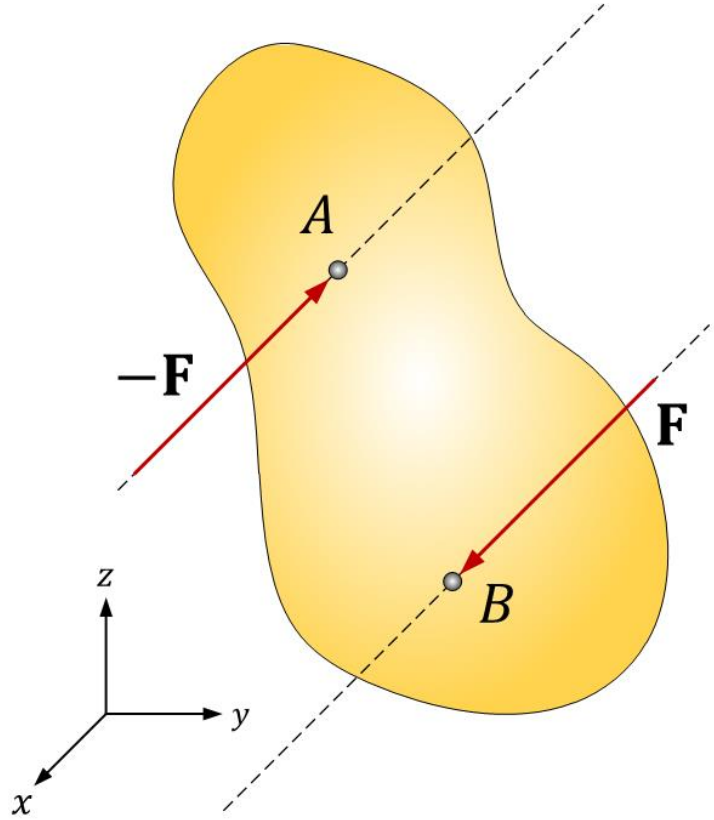
- Additionally, if we take the moment of the pair of forces about any point on the line of action of **F**, we find

$$\mathbf{M}_B = \mathbf{r}_{A/B} \times (-\mathbf{F})$$

$$\mathbf{M}_B = \left(-\mathbf{r}_{B/A}\right) \times (-\mathbf{F})$$

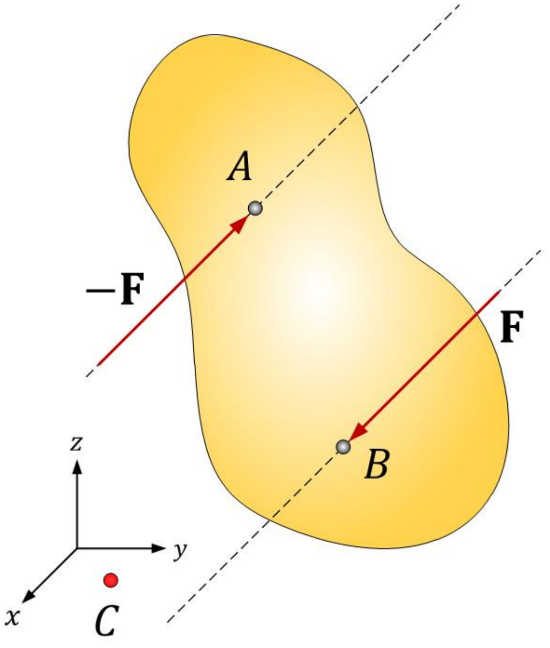
$$\mathbf{M}_B = \mathbf{r}_{B/A} \times \mathbf{F} = \mathbf{M}_A$$

OR $\mathbf{M}_B = \mathbf{M}_A$



Even more interesting is if we try some other, arbitrary point *C*

$$\mathbf{M}_C = \mathbf{r}_{A/C} \times (-\mathbf{F}) + \mathbf{r}_{B/C} \times \mathbf{F}$$



- We can see that

$$\mathbf{r}_{B/C} = \mathbf{r}_{A/C} + \mathbf{r}_{B/A}$$

and then

$$\mathbf{M}_C = \mathbf{r}_{A/C} \times (-\mathbf{F}) + \left(\mathbf{r}_{A/C} + \mathbf{r}_{B/A}\right) \times \mathbf{F}$$

$$\mathbf{M}_C = \mathbf{r}_{A/C} \times (-\mathbf{F}) + \mathbf{r}_{A/C} \times \mathbf{F} + \mathbf{r}_{B/A} \times \mathbf{F}$$

$$\mathbf{M}_C = \mathbf{r}_{B/A} \times \mathbf{F} = \mathbf{M}_A$$ OR $$\mathbf{M}_C = \mathbf{M}_A$$

The moment of a couple is independent of the moment centre

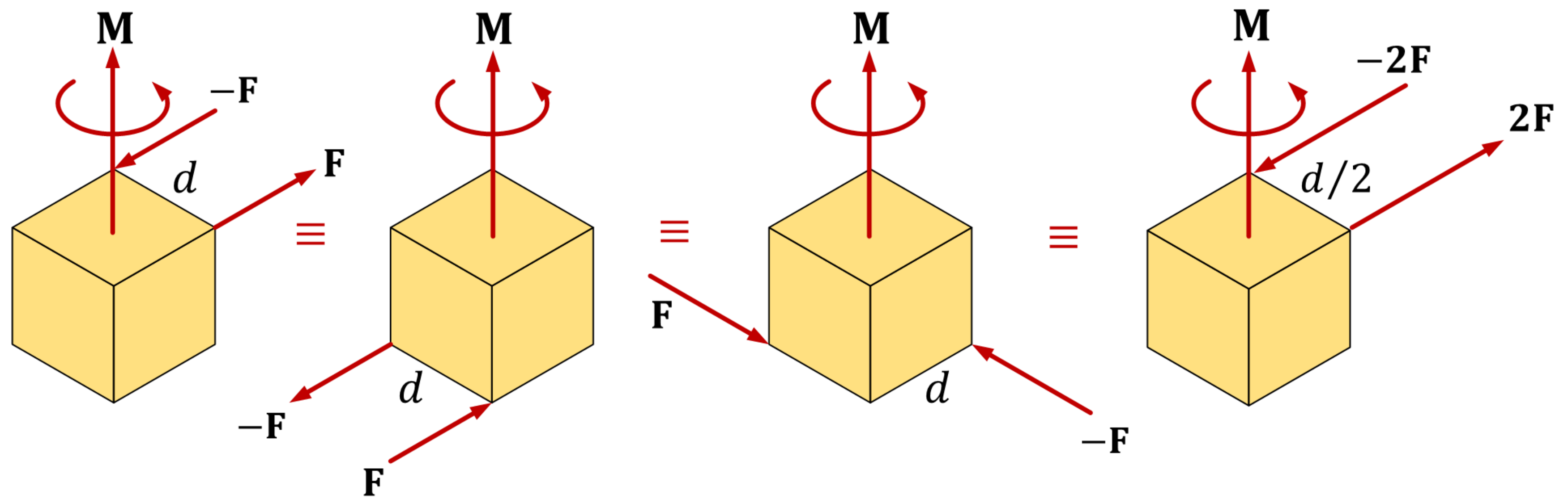
- A couple is equivalent to a moment

- The moment has magnitude $M = Fd$
  - ✓ Where d is the perpendicular distance between the lines of action of the two forces and F is the magnitude of a force in the pair

- The moment is in the direction perpendicular to the plane containing the forces

- For any given moment centre, the couple and moment it produces are equivalent loads.

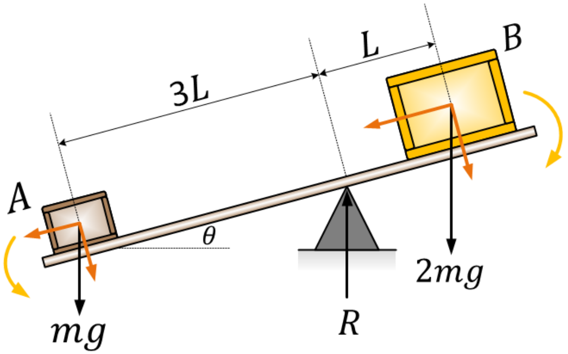
We can also see that there are many different pairs of forces that produce the same couple



- Any couple that exists in the same plane and results in the same product Fd is entirely equivalent in terms of the external effects

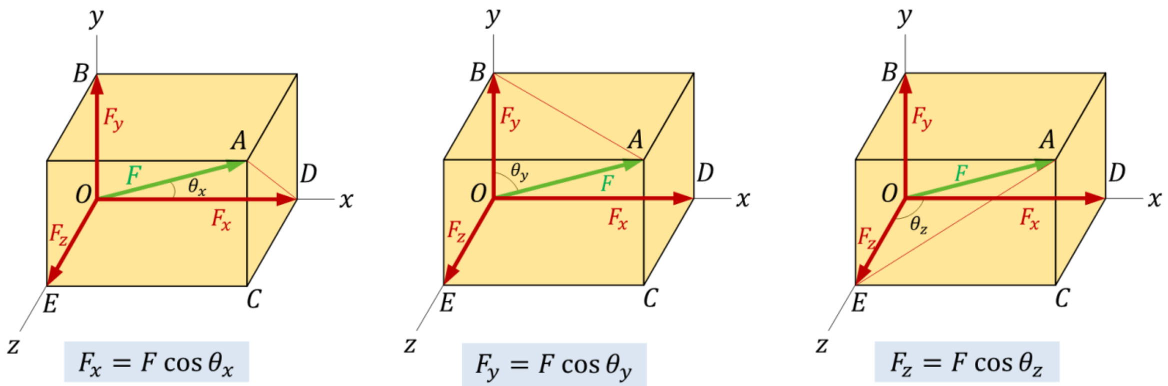
Summary

- Moment is a vector: $\mathbf{M} = \mathbf{r} \times \mathbf{F}$



- There are an infinite number of equivalent Force-couple systems

- Forces can be decomposed into scalar components: $\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}$



Next topic:

Structures and Trusses