MMAN1300 Engineering Mechanics 1

Lab: 2 - Motion of a Rolling Disk

Location: Undergraduate Teaching Laboratory, Room 116, Willis Annex

Submission Individual: Due on Friday, October 19th, 2018 (Week 12)

Report writing: Use the template provided for writing your report

Method: Online submission on TEAMS (submission box will be provided)

Introduction

The circular disk is a solid form that is widely used in engineering. It constitutes a fundamental component in many mechanical systems. It is convenient to consider the motion of any rigid body as being made up of two components:

- rigid body translation of the centre of mass of the body, and
- rotation of the rigid body about its centre of mass.

In any particular case, the relationship between the translation and rotation of the body will be determined by the constraints that are imposed by the mechanism in which the component is located. In this case, the body is a simple disk with concentric axles and it rolls without slipping down an incline.

Getting into the Laboratory

You can obtain access to the undergraduate teaching laboratory, room 116 of the Willis Annex building (J18), during your booked time slot in Week 09.

Before you enter the laboratory, you must gain competency in a Safe Work Procedure and Risk Management Form for the rigs prior to using the experimental equipment. The documents may be accessible only after you have been invited. Enclosed footwear is required to enter the laboratory.

The report will be given a mark out of 40, which will be scaled to a mark out of 6 for the course total.

Experimental Apparatus

The equipment involves a steel disk having two concentric axles and a ramp with an adjustable slope, as shown in the figure below. The apparatus is in the undergraduate teaching laboratory, room 116 of the Willis Annex building.

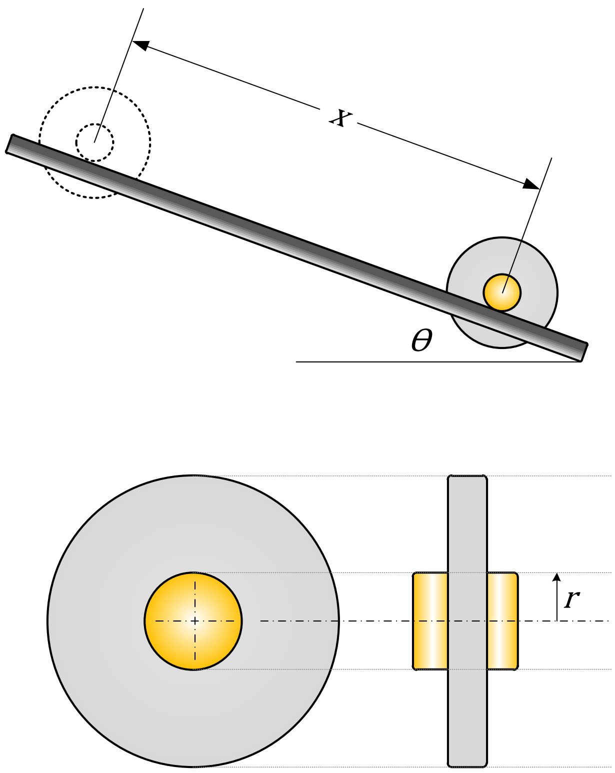


Figure 1: Schematic diagram of a disk rolling down an incline

Experimental Part

1. Measure the angle of the slope θ by measuring the length and height
2. For your measured angle, measure the time required for the disk to travel the following distances down the slope: $x = 0.2\ \mathrm{m},\ 0.4\ \mathrm{m}, 0.6\ \mathrm{m},\ 0.8\ \mathrm{m}$ and $1\ \mathrm{m}$. Repeat your measurements at least 3 times and record the average.

Disk parameters

- Large disk diameter 300 mm
- Axle diameter 20 mm (effective)
- Disk thickness 20 mm
- Total mass of disk and axles 11.6 kg
- Material: stainless steel. The density of stainless steel is $7700\ \mathrm{kg/m^3}$

Analytical Part

It is possible to derive an equation for the disk to travel a distance x down the ramp as a function of time t, using both Newton's 2nd Law and Work-Energy methods. In both methods, the friction force is considered negligible. The expression for the distance travelled x will be a function of time t, the dimensions of the disk, the moment of inertia of the disk and the ramp slope angle.

<u>Moment of inertia</u>

I (units: $\mathrm{kg.m^2}$)

For a solid disk rotating about its centre of mass:

$$I_G = \frac{1}{2}mr^2$$

For a composite disk:

$$I_{G,total} = \frac{1}{2}m_1r_1^2 + \frac{1}{2}m_2r_2^2 + \frac{1}{2}m_3r_3^2$$

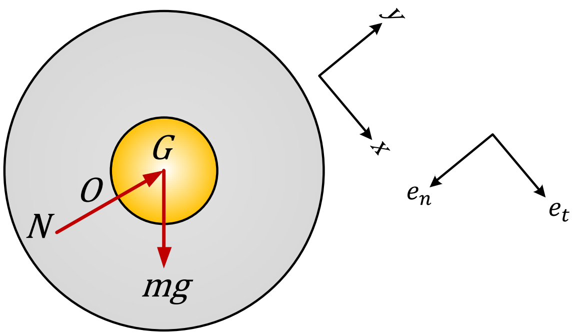
For a disk which is not rotating about its centre of mass, use the Parallel Axis Theorem:

$$I_O = I_{G,total} + m_{total}d^2$$

where d is the distance from the centre of mass (*G*) to the point of rotation (*O*).

<u>Newton's 2nd Law</u>

$$\sum M_O = I_O\alpha$$



$$\Rightarrow \qquad mg\sin\theta.r = I_O\alpha$$

$$\Rightarrow \qquad \alpha = \frac{mg\sin\theta.r}{I_O}$$

Now the centre of mass (*G*) has both velocity and acceleration. The velocity of *G* is acting parallel to the slope (i.e. the tangential direction is coinciding with the x – direction). The normal direction is acting towards the centre of rotation (point *O*), which is coinciding with the negative y – direction. The acceleration of the centre of mass has both normal and tangential components.

The normal acceleration is:

$$a_n = \omega^2 r$$

The tangential acceleration is:

$$a_t = \alpha r$$

Since the angular acceleration α is a constant, then a_t is also constant. Hence, the acceleration down the slope is constant, and we can use constant acceleration equations to determine x as a function of t.

$$x = x_o + v_o t + \frac{1}{2}at^2$$

where x_o , v_o are zero and $a = a_t$.

Hence:

$$x = \frac{mg\sin\theta.r^2}{2I_O}t^2$$

<u>Work-Energy Methods</u>

$$W_{1-2} = F(s_2 - s_1) + M(\theta_2 - \theta_1) = \Delta T + \Delta V_g + \Delta V_e$$

If the friction force is considered negligible, then there are no external forces acting on the disk. There are no external moments acting on the system. Also, there are no springs in this system. Hence, the Work-Energy equation reduces to:

$$\Delta T + \Delta V_g = 0$$

The disk is a rigid body, and is described as having:

- translation of its centre of mass (straight line motion down the slope)
- rotation of its centre of mass (rotating down the slope).

The change in kinetic energy can be written as:

$$\Delta T = \frac{1}{2}m(v_2^2 - v_1^2) + \frac{1}{2}I_G(\omega_2^2 - \omega_1^2)$$

{translation and rotation of the centre of mass}

Hence the change in kinetic energy can be written as:

$$\Delta T = v^2\left(\frac{m}{2} + \frac{I_G}{2r^2}\right)$$

The change in potential energy can be written as:

$$\Delta V_g = mg(h_2 - h_1) = -mg\sin\theta.x$$

The Work-Energy equation becomes:

$$\Delta T + \Delta V_g = v^2\left(\frac{m}{2} + \frac{I_G}{2r^2}\right) - mg\sin\theta.x = 0$$

This can be rearranged to give:

$$v = c\sqrt{x}$$

where c is simply a constant and is given by

$$c = \sqrt{\frac{2mg\sin\theta.r^2}{I_G + mr^2}}$$

Use:

$$v = \frac{dx}{dt}$$

which gives:

$$dx = vdt = c\sqrt{x}dt$$

Divide both sides by $\sqrt{x}$ and then integrating gives:

$$\int_0^x \frac{dx}{\sqrt{x}} = \int_0^t cdt$$

which results in

$$x = \frac{c^2 t^2}{4}$$

which can be expanded as:

$$x = \frac{mg\sin\theta.r^2}{2(I_G + mr^2)}t^2$$

Noting that $I_O = I_G + mr^2\ (r = d)$, the expression for the distance travelled down the slope obtained using Work-Energy methods is an identical expression to the one obtained using Newton's 2nd law.

Matlab script

Using the given analytical equations, use either Excel or Matlab to obtain a theoretical plot of the distance travelled x as a function of time t. The following Matlab code is provided to help:

```
% define the variables;

%  m = total mass of disk and axles

% r = radius

% Io = moment of inertia about point o

% theta = ramp slope angle

% g=gravity

% a=acceleration


m=…;  (input mass here)

r=…..;  (input radius here)

Io=…;  (input the mass moment of inertia here) – THEN JUST RUN THE CODE

pi=3.141592654;

theta=degrees*2*pi/360;

g=9.81;

a=(m*g*sin(theta)*r^2)/(Io);


% time range from 1 to 20 seconds;

for i = 1:2001

time(i)=(i-1)/100;

x(i)=0.5*a*time(i)^2;

end


plot(time,x)

xlabel('time (s)')

ylabel('x(t)')

title('x(t)=0.5at^2')
```

Superimpose your experimental results on the theoretical plot obtained using Matlab or Excel. Explain any differences between the theoretical and calculated values. Plot the velocity and acceleration as functions of time. Discuss your results.

Notes for writing a laboratory report

When writing any technical report, you must provide information which is specific and precise. As engineers, you must be able to communicate technical details and calculations to your colleagues, supervisors and clients. A very important goal of a lab is to train you to have good written communication skills through giving you practice in technical report writing.

Where appropriate, you should follow the rules below in writing your lab report.

- Type your report (including equations) in the main body of the report
- Use tables where possible
- Label all figures

In general, a lab report should include the following content:

- Title
- Introduction (which includes the aims of the lab)
- Apparatus
- Procedure
- Sample calculation (where appropriate)
- Results and discussion (including discussion on sources of error where appropriate)
- Conclusions
- References (where appropriate)
- Appendix (where appropriate)

Avoid using statements with “I” and “we”. Avoid starting a sentence with “And” and “Because”.