School of Mechanical and Manufacturing Engineering

MMAN1300 – ENGINEERING MECHANICS 1

2017 S2 Block Test 3

Instructions:

- Time allowed: 45 minutes
- Total number of questions: 3
- Answer all the questions in the test
- Answer all questions in the spaces provided
- The 6 marks allocations shown are worth 6% of the course overall
- Candidates may bring drawing instruments, rulers and UNSW approved calculators to the test
- Print your name, student ID and all other requested details above
- Record your answers (with appropriate units) in the ***ANSWER BOXES*** provided

Notes:

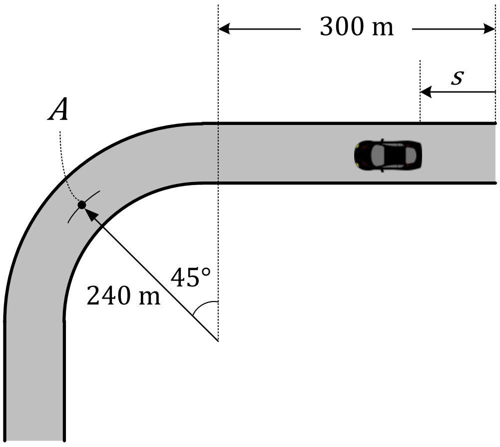
Your work must be complete, clear and logical

Do not skip steps, sign conventions, units and relevant diagrams and clearly state the final answers

No part of this paper is to be retained by candidates until handed back after marking

Question 1: (2 Marks)

The car is originally at rest at $s = 0$. If it then starts to increase its speed at $a = (0.03\ s)\ \text{m/s}^2$, where s is in meters, determine the following at point A.



Solution:

(a) The velocity (v_A) of the car at A

Total distance travelled to arrive at A

$$s_A = 240\left(\frac{\pi}{4}\right) + 300 = 488.5\ \text{m}$$ 0.25

$$vdv = ads$$

$$vdv = (0.03s)ds$$

Integrating both sides

$$\int_0^{v_A} v\,dv = \int_0^{s_A} (0.03s)\,ds$$ 0.25

$$\int_0^{v_A} v\,dv = \int_0^{488.5} (0.03s)\,ds$$

$$\frac{{v_A}^2}{2} = \left(\frac{0.03}{2}\right)(488.5^2 - 0^2)$$

$$v_A = \sqrt{(488.5)^2(0.03)}$$

0.5

$$v_A = 84.61\ \text{m/s}$$

Continue your working for part (a) here:

(c) The magnitude (a_A) and direction (θ) of car's acceleration at A

$$(a_A)_n = \frac{{v_A}^2}{R} = \frac{84.61^2}{240} = 29.83\ \text{m/s}^2$$

0.25

$$(a_A)_t = (0.03)(488.5) = 14.655\ \text{m/s}^2$$

0.25

$$a_A = \sqrt{(29.83)^2 + (14.655)^2} = 33.23\ \text{m/s}^2$$

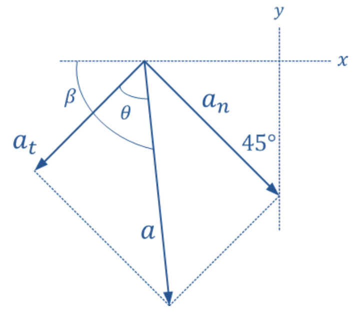
0.25

$$\theta = \tan^{-1}\left(\frac{29.83}{14.655}\right) = 63.83°$$

$$\beta = 45° + 63.83° = 108.84°$$

0.25

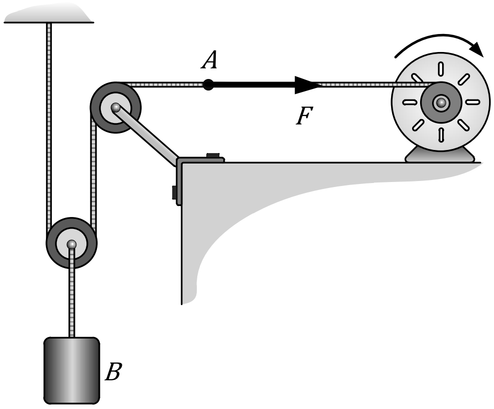
Or from the positive x- axis, angle = 288.84°

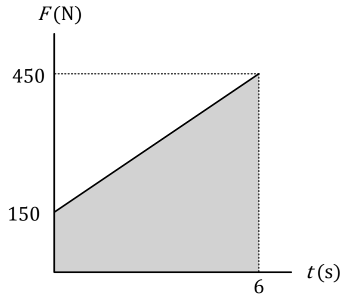


Answers:	$v_A = 84.61\ \text{m/s}$	$a_A = 33.23\ \text{m/s}^2$	$\theta = 108.84°\ or\ 288.84°$

Question 2: (2 Marks)

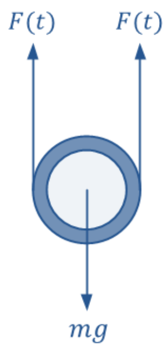
A spring winder exerts a force F on the cable as shown in the graph. When $t = 0$, the 40 kg crate moves downward at 10 m/s, thus overcoming winder (spinning it CCW). At time t_r, the direction reverses and the winder begins to spin CW





Solution:

(a) Draw the free body diagram of the 40 kg mass at B (including the pulley)



0.25

(b) Calculate how long it takes for the mass B, to reach a velocity of 5.68 m/s (downwards)

From the graph

0.25

$$F(t) = 150 + 50t$$

From the FBD,

$$+\uparrow \sum F_y = ma_y$$

$$2F - mg = ma_y$$

$$2(150 + 50t) - (40)(9.81) = (40)a_y$$

$$a_y(t) = 2.5t - 2.31 \quad \mathrm{m/s^2}$$

0.5

Since

$$a_y(t) = \frac{dv_y(t)}{dt}$$

Continue your working for part (b) here:

$$\int_{-10}^{v_y} dv_y(t) = \int_0^t (2.5t - 2.31)\, dt$$

$$v_y + 10 = \frac{2.5}{2}t^2 - 2.31t$$

$$v_y = 1.25t^2 - 2.31t - 10$$

0.5

Time it takes to reach a velocity of $v_y = -5.68\ \mathrm{m/s}$

$$-5.68 = 1.25t^2 - 2.31t - 10$$

0.25

$$t = 3\ \mathrm{s}$$

(c) At what instant of time (t_r) *the velocity will reverse its direction?*

Instant where the velocity will reverse its direction will be at $v_y = 0$

$$1.25{t_r}^2 - 2.31t_r - 10 = 0$$

Solving the quadratic equation

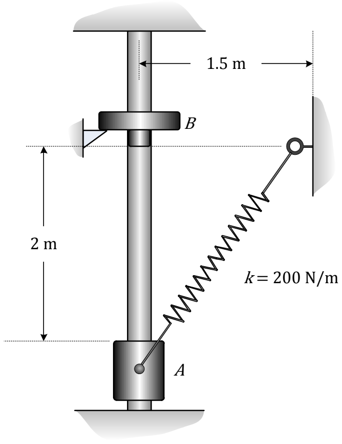
0.25

$$t_r = 3.9\ \mathrm{s}$$

Answers:	$t = 3\ \mathrm{s}$	$t_r = 3.9\ \mathrm{s}$

Question 3: (2 Marks)

The spring has a stiffness $k = 200$ N/m and an unstretched length of 0.5 m. It is attached to the 3 kg smooth collar A and the collar is released from rest. The 2 kg mass B is also at rest initially, held by the stopper. If the two masses A and B coalesce upon impact, determine the combined velocity (v_{AB}) of the masses. Also, calculate the average force (F_{av}) imparted upon the contact for a duration of 50 ms. Neglect the size of the collar.



Solution:

Present your solution to Question-3 here:

For mass A

0.2

$$W_{1-2} = 0 \text{ (no external force)}$$

$$\Delta T = \frac{1}{2} m_A ({v_{A2}}^2 - {v_{A1}}^2)$$

$$\Delta T = \frac{1}{2} m_A ({v_{A2}}^2 - 0^2)$$

$$\Delta T = \frac{1}{2}(3)({v_{A2}}^2 - 0^2)$$

$$\Delta T = 1.5\, {v_{A2}}^2$$

0.2

$$\Delta V_g = mg(h_2 - h_1)$$

$$\Delta V_g = (3)(9.81)(2 - 0)$$

$$\Delta V_g = 58.86 \text{ J}$$

0.2

$$\Delta V_e = \frac{1}{2} k ({x_2}^2 - {x_1}^2)$$

where,

$$x_1 = \sqrt{(2)^2 + (1.5)^2} - 0.5 = 2 \text{ m}$$

$$x_2 = 1.5 - 0.5 = 1 \text{ m}$$

Continue your solution to Question-3 here:

Therefore,

$$\Delta V_e = \frac{1}{2}(200)(1^2 - 2^2)$$

$$\Delta V_e = -300 \text{ J}$$

0.2

Using Work-Energy equation

$$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

$$0 = 1.5\, {v_{A2}}^2 + 58.86 - 300$$

0.2

$$v_{A2} = 12.68 \text{ m/s}$$

Using law of conservation of momentum,

Total momentum before impact = total momentum after impact

$$m_A v_{A2} + m_B v_{B2} = (m_A + m_B) v_{AB}$$

$$v_{AB} = \frac{(3)(12.68) + 0}{(3+2)}$$

0.5

$$v_{AB} = 7.608 \text{ m/s}$$

Impulse imparted by A to B is equal to the loss of momentum of A

$$F_{av} \times t = m_A v_{A2} - m_A v_{AB}$$

$$F_{av} = \frac{3(12.68 - 7.608)}{50 \times 10^{-3}}$$

0.5

$$F_{av} = 304.32 \text{ N.s}$$

Answers:	$v_{AB} = 12.68 \text{ m/s}$	$F_{av} = 304.32 \text{ N.s}$

Equation Sheet

Linear motion

$v = \frac{ds}{dt}$ $\qquad a = \frac{dv}{dt}$ $\qquad v\,dv = a\,ds$

Constant linear acceleration equations $(t_o = 0)$

$v = v_o + at$ $\qquad v^2 = v_o^2 + 2a(s - s_o)$ $\qquad s = s_o + v_o t + \frac{1}{2}at^2$

Angular motion

$\omega = \frac{d\theta}{dt}$ $\qquad \alpha = \frac{d\omega}{dt}$ $\qquad \omega\,d\omega = \alpha\,d\theta$

Displacement, velocity and acceleration components

Rectangular coordinates

$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ $\qquad \mathbf{v} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$ $\qquad \mathbf{a} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$

Normal and tangential coordinates

$\mathbf{v} = v\mathbf{e_t}$ $\qquad \mathbf{a} = a_t\mathbf{e_t} + a_n\mathbf{e_n}$ $\qquad v = \omega r$ $\qquad a_t = \dot{v} = \alpha r$ $\qquad a_n = \frac{v^2}{\rho} = \omega^2 r$

Relative motion

$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$ $\qquad \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$ $\qquad \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$

Equation of motion (Newton's 2nd law)

$\sum \mathbf{F} = m\mathbf{a}$

Work-Energy

$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$ $\qquad W_{1-2} = F\Delta s$ and/or $M\Delta\theta$

$\Delta T = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$ and/or $\frac{1}{2}I\left(\omega_2^2 - \omega_1^2\right)$

$\Delta V_g = mg(h_2 - h_1)$

$\Delta V_e = \frac{1}{2}k\left(x_2^2 - x_1^2\right)$ for a linear spring

For a rigid body in plane motion

$\sum \mathbf{F} = m\mathbf{a}$ $\qquad \sum M = I\alpha$

Mass moment of inertia $\quad I = \int r^2\,dm$

Centroid of a cross-section:

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\sum_i x_i A_i}{\sum_i A_i} \quad , \quad \bar{y} = \frac{\int y\,dA}{\int dA} = \frac{\sum_i y_i A_i}{\sum_i A_i}$$

DATA: Acceleration in free fall due to gravity g = 9.81 m/s^2

Quadratic formula:

For: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$