School of Mechanical and Manufacturing Engineering

MMAN1300 Engineering Mechanics 1

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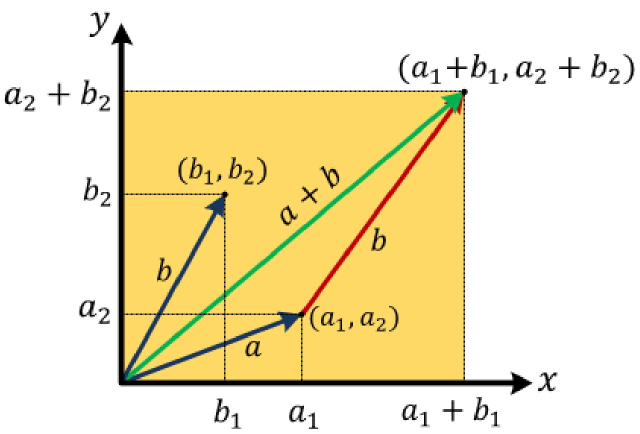


Week 1, L1- Forces and Moments

FUNDAMENTAL CONCEPTS

- Primitive concepts
- Vectors
- Forces
- Dot and cross products
- Newton's Law's

What is Mechanics

Mechanics is the science which describes and predicts the conditions of rest or motion of bodies under the action of forces.

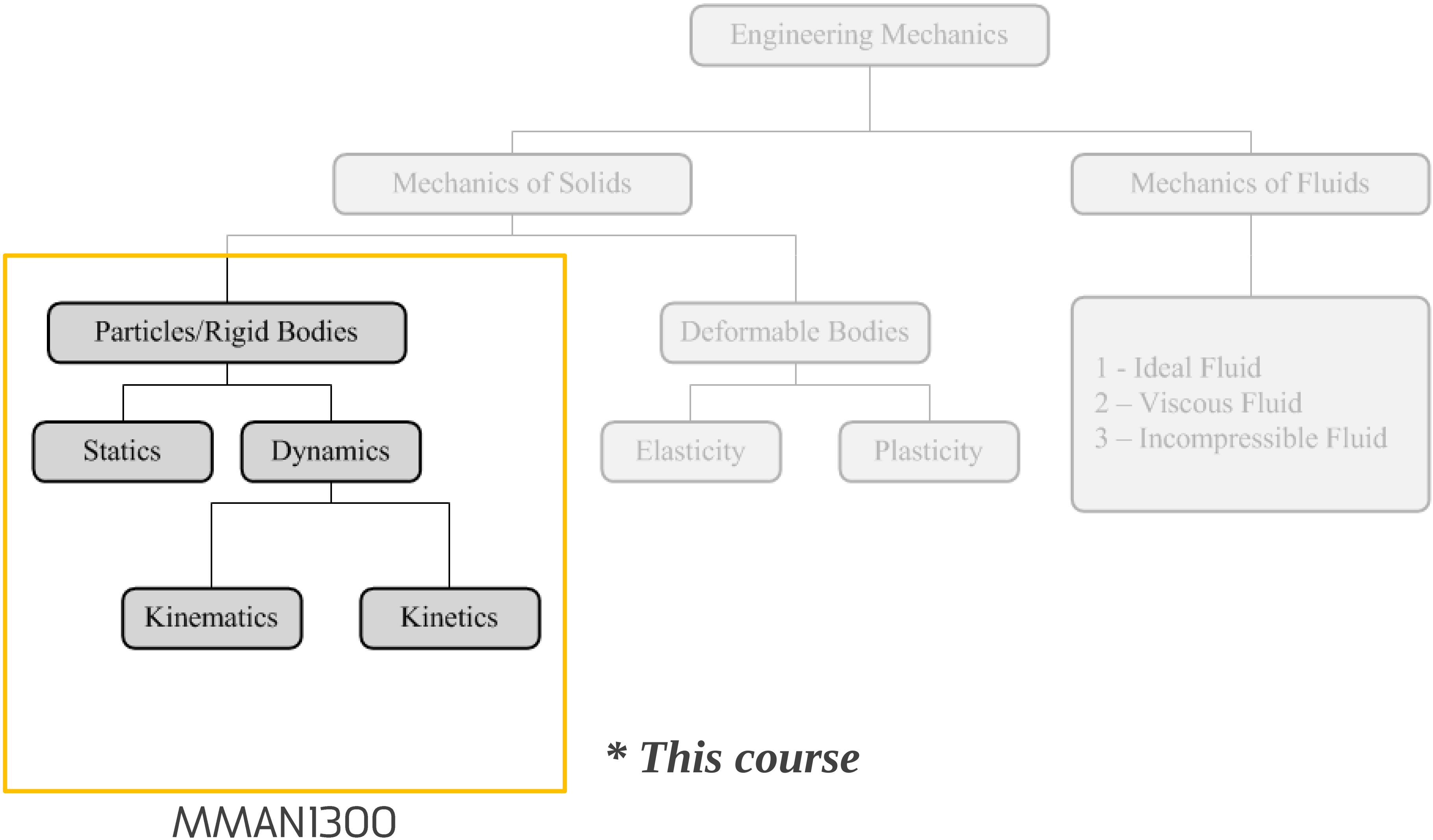
- Categories of Mechanics:
  - Rigid bodies
    - *Statics*
    - *Dynamics*
  - Deformable bodies
  - Fluids

- Mechanics is an applied science - it is not an abstract or pure science but does not have the empiricism found in other engineering sciences.

- Mechanics is the foundation of most engineering sciences and is an indispensable prerequisite to their study.

Adopted from J.W. Oler, NCSU, USA

Engineering Mechanics in a Broader Scope



Fundamental Concepts

Primitive concepts

We have 6 *primitive concepts* in Mechanics

- In Engineering Mechanics, we really make no attempt to define these things at any deeper level

- Everything else in Engineering Mechanics is defined in terms of the following 6 concepts:

1. *Space*
2. *Time*
3. *Body*
4. *Mass*
5. *Force*
6. *Torque*

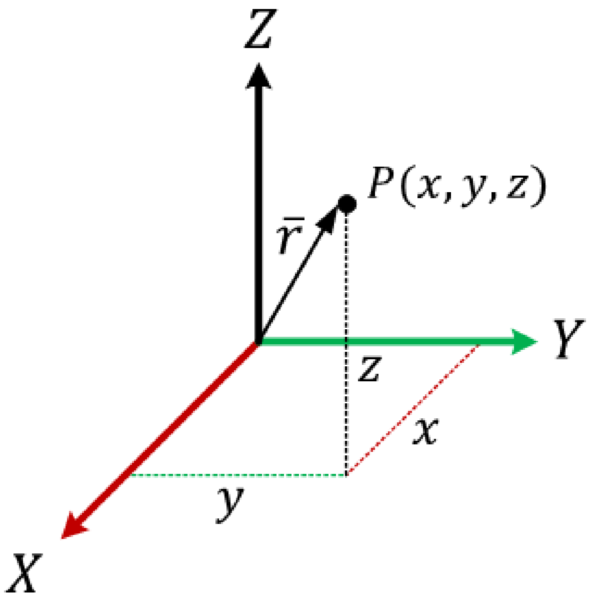
→

Location/Position
Speed/Acceleration
Internal forces
External forces

Space

Space is the environment in which objects exist and move

- It can be viewed as a collection of locations
- The space we use in Engineering Mechanics is the familiar (as we observe around us Euclidean 3-space)
- Any point in this space can be identified with three coordinates (x , y , z)
- Position in space is a vector quantity
- $\bar{r}$ is the position vector



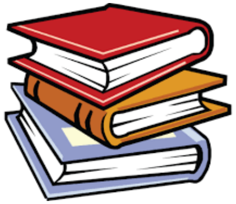
Time

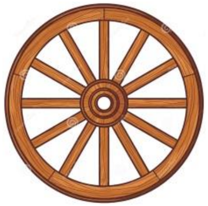
Time is a continuous scalar variable we can use to order a sequence of events

- Time is a Euclidean 1 - space
- In Engineering Mechanics, we regard time as being absolute
- Einstein tells us that space and time are not absolute, and are one (Non-Euclidean) object called space time
- We will ignore Einstein (Both space and time are absolute)

Body

In Mechanics, a **body** is a piece of mater

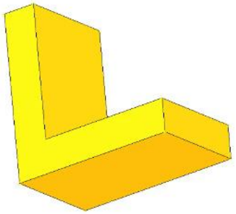




- A body may be a particle, a rigid body or a non-rigid continuum (i.e. a deformable solid or a fluid)

- We will only deal with particles and rigid bodies in this course

- Think of anything around you?



** More on particles and rigid bodies later*

Mass

Mass is a measure of how much matter is in an body

- Mass is closely related to inertia, which is the resistance of a body to changing its state of motion

- Galileo observed that gravitational mass and the inertial mass are the same thing

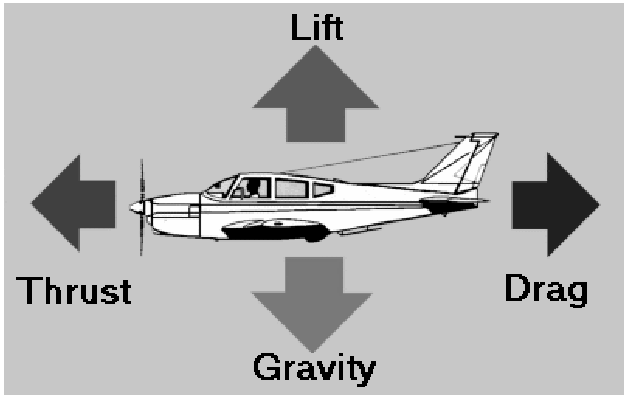
- Why you need to wear a seat belt? (Especially if you are a giraffe)

Force

Force is any interaction between a body and the rest of the world that tends to affect the state of motion of the body

- Forces represents the action of one body on another. A force is characterized by its point of application, magnitude, and direction.

- Forces are either pushes or pulls and result either from direct mechanical contact or from gravitational or electro-magnetic interactions with another body



Torque

Torque is another kind of interaction between bodies that affects their rotational motion

- Torque is a kind of turning force
- In Engineering Mechanics, we tend to refer to torque as moment
- Some people may formulate Engineering Mechanics where moment is derived from force

The concept of vectors

Many important quantities in Mechanics are **vectors**

- Force, moment, torque, velocity, acceleration, displacement, etc.

- In many ways, Engineering Mechanics at the introductory university level is applied vector mechanics

- A firm understanding of vectors, vector representation, vector addition and vector products makes life much easier

Two Kinds of Quantities

1. Scalars:

- Scalars just have a magnitude
- We will regard them as just being real numbers

Mass	Density	Length	Area	Volume
Speed	Power	Energy	Time	Temperature

2. Vectors:

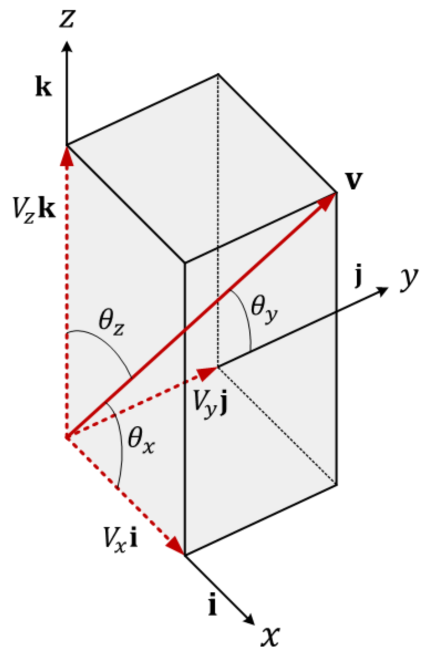
- Vectors have both magnitude and direction
- In print a vector is written in bold: **V**

Position	Velocity	Acceleration	Momentum
Impulse	Force	Moment	Torque

Magnitude of a vector

At right is a vector **V** with a known magnitude which is given as:

$$V = \|\mathbf{V}\| = \sqrt{{V_x}^2 + {V_y}^2 + {V_z}^2}$$



- **Note:**

$$V = Magnitude\ i.e.scaler\ (not\ bold)$$

$$\|\quad\| = This\ symbol\ represnts\ magnitude\ of\ a\ vector$$

$$V_x, V_y, V_z = respective\ x, y\ and\ z - components$$

Direction of a vector

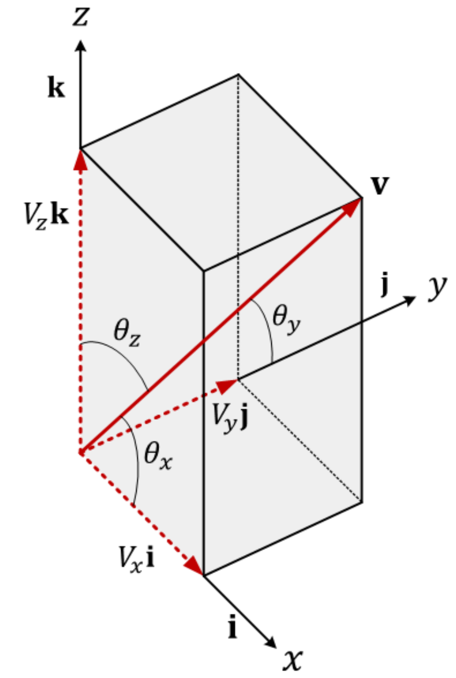
We can specify the direction using space angles

- Space angles are the angles the vectors forms with the coordinate axes

$$\theta_x = \cos^{-1}\left(\frac{V_x}{V}\right)$$

$$\theta_y = \cos^{-1}\left(\frac{V_y}{V}\right)$$

$$\theta_z = \cos^{-1}\left(\frac{V_z}{V}\right)$$



- **Note:**

$V = Magnitude\ of\ \mathbf{V}$

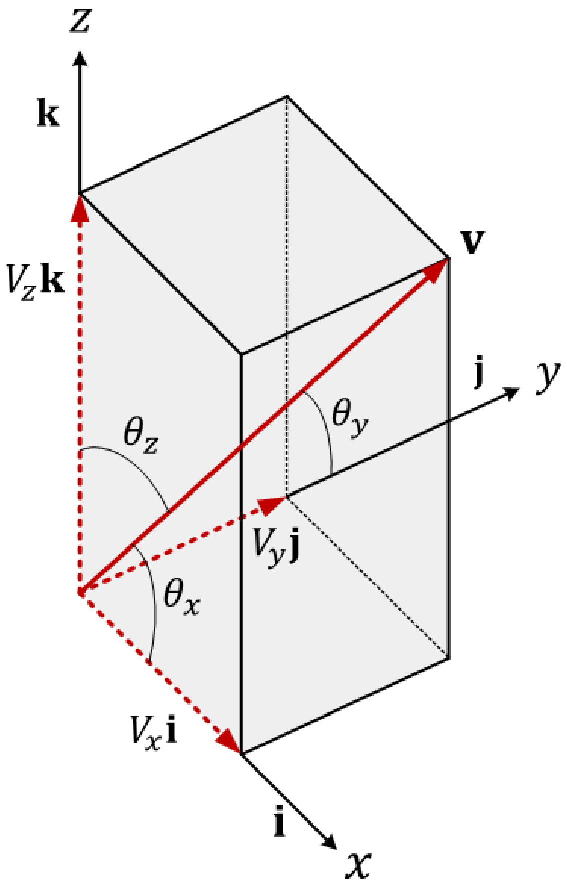
More on space angles

The space angles are not independent

- They must obey

$$\boxed{\sqrt{(\cos\theta_x)^2 + (\cos\theta_y)^2 + (\cos\theta_z)^2} = 1}$$

- Thus, if we know 2, we know the other modulo 180 degrees

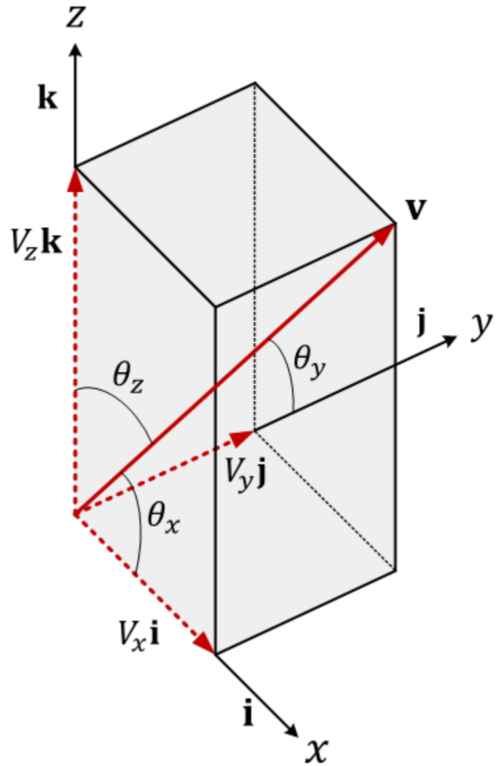


Rectangular Component Representation

It is often convenient to use a rectangular component representation

- Again we start with a right-handed coordinate system
- Now we decompose **V** into its 3 rectangular component vectors i.e. $\mathbf{V_x}, \mathbf{V_y}, \mathbf{V_z}$ such that

$$\mathbf{V} = \mathbf{V_x} + \mathbf{V_y} + \mathbf{V_z}$$



Rectangular Components

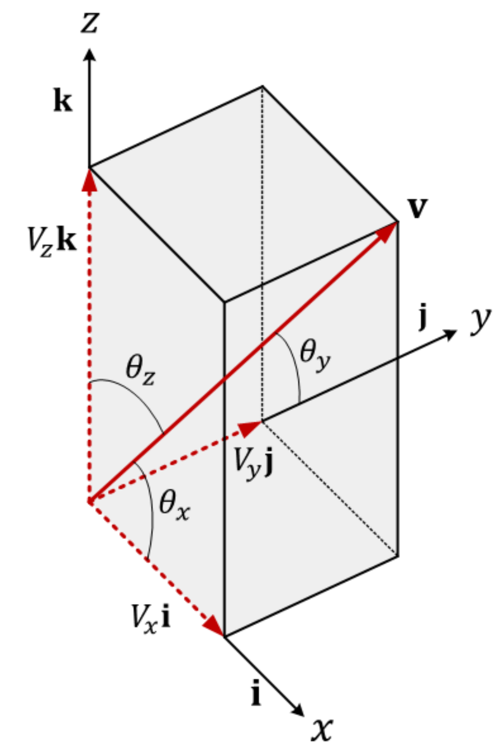
The rectangular components are all directed along one of the coordinate axes

- Thus we can write

$$\mathbf{V_x} = V_x\,\mathbf{i} \qquad\qquad V_x = \|\mathbf{V}\|\cos\theta_x$$

$$\mathbf{V_y} = V_y\,\mathbf{j} \qquad \text{where} \qquad V_y = \|\mathbf{V}\|\cos\theta_y$$

$$\mathbf{V_z} = V_z\,\mathbf{k} \qquad\qquad V_z = \|\mathbf{V}\|\cos\theta_z$$



- Of course, the components depend on the chosen coordinate system

- Note that while V is always positive V_x, V_y and V_z can be positive or negative

$$\boxed{\mathbf{V} = V_x\,\mathbf{i} + V_y\,\mathbf{j} + V_z\,\mathbf{k}}$$

There is another representation possible

There is another representation possible

- If we recall the definitions of the rectangular components of **V**

$$V_x = \|\mathbf{V}\| \cos\theta_x \qquad V_y = \|\mathbf{V}\| \cos\theta_y \qquad V_z = \|\mathbf{V}\| \cos\theta_z$$

- We can write

$$\mathbf{V} = V_x\,\mathbf{i} + V_y\,\mathbf{j} + V_z\,\mathbf{k}$$

$$\mathbf{V} = \|\mathbf{V}\| \cos\theta_x\,\mathbf{i} + \|\mathbf{V}\| \cos\theta_y\,\mathbf{j} + \|\mathbf{V}\| \cos\theta_z\,\mathbf{k}$$

$$\boxed{\mathbf{V} = \|\mathbf{V}\| \left(\cos\theta_x\,\mathbf{i} + \cos\theta_y\,\mathbf{j} + \cos\theta_z\,\mathbf{k}\right)}$$

Unit Vectors

The rectangular components are all directed along one of the coordinate axes

- Recall that the space angles must obey

$$\sqrt{(\cos\theta_x)^2 + \left(\cos\theta_y\right)^2 + (\cos\theta_z)^2} = 1$$

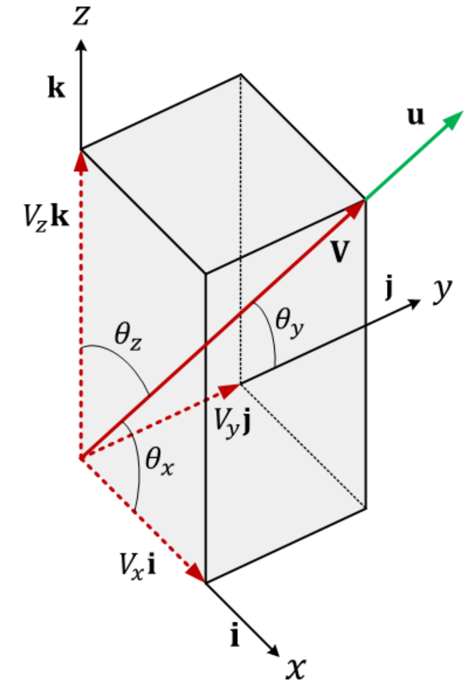
- Thus, if we define a vector **u,** such as

$$\mathbf{u} = \cos\theta_x\,\mathbf{i} + \cos\theta_y\,\mathbf{j} + \cos\theta_z\,\mathbf{k}$$

- Then **u** has a magnitude of 1

  i.e.

  - It is a unit vector
  - It is in the direction of **V**



$$\boxed{\mathbf{V} = \|\mathbf{V}\|\left(\cos\theta_x\,\mathbf{i} + \cos\theta_y\,\mathbf{j} + \cos\theta_z\,\mathbf{k}\right)}$$

Vector Addition

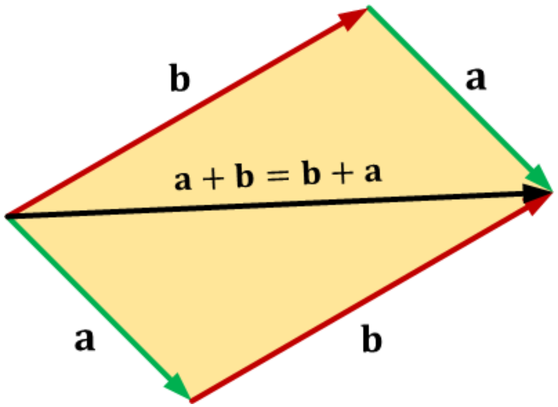
We often need to add vectors

- Vectors obey the parallelogram law of vector addition

- We can see that this implies that vector addition is commutative

  i.e.

$$\mathbf{V_1} + \mathbf{V_2} = \mathbf{V_2} + \mathbf{V_1}$$



- However, it is also clear that vector sums do not work like scalar sums

  i.e.

$$\mathbf{V_1} + \mathbf{V_2} \neq V_1 + V_2$$

Dot and Cross Product

The two most commonly used products of vectors are the dot product and the cross product

Dot product:

- The dot product is also sometimes known as the inner product or the scalar product

Cross product:

- The cross product is sometimes known as the vector product or the wedge product

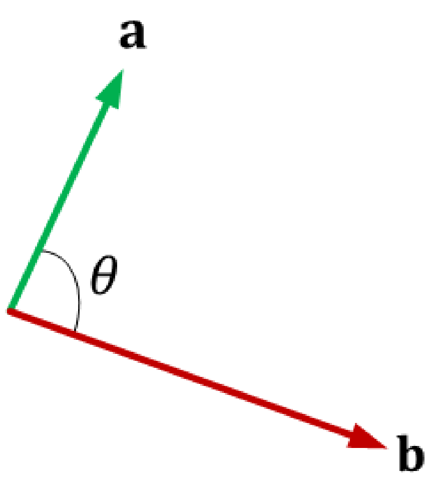
* *We will talk about these in detail in coming slides*

Let's examine the dot product

Let's examine the dot product

- The dot product of two vectors **a** and **b** is defined as

$$\mathbf{a} \,.\, \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\| \cos\theta = ab\cos\theta$$



Where θ is the angle between the two vectors

- The result is the component of **a** in the direction of **b** multiplied by the magnitude of **b**

The dot product has some nice properties

- We can see that the dot product is commutative

$$\mathbf{a}\,.\,\mathbf{b} = ab\cos\theta = ba\cos\theta = \mathbf{b}\,.\,\mathbf{a}$$

- You can also verify that it is distributive

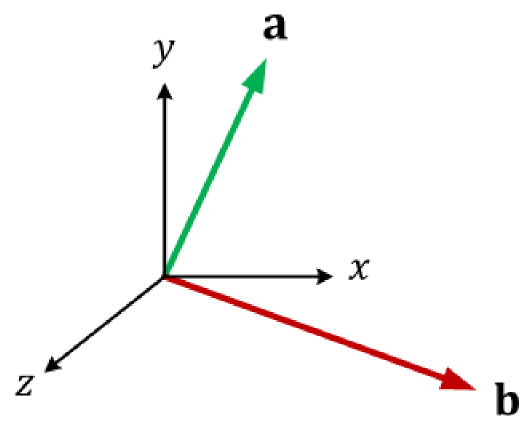
$$\mathbf{a}.\,(\mathbf{b} + \mathbf{c}) = (\mathbf{a}.\,\mathbf{b}) + (\mathbf{a}.\,\mathbf{c})$$

The distributive property is very useful

- Let's write **a** and **b** in rectangular component form

$$\mathbf{a} = a_x\,\mathbf{i} + a_y\,\mathbf{j} + a_z\,\mathbf{k}$$

and $$\mathbf{b} = b_x\,\mathbf{i} + b_y\,\mathbf{j} + b_z\,\mathbf{k}$$



- Now the dot product becomes:

$$\mathbf{a}\,.\,\mathbf{b} = \left(a_x\,\mathbf{i} + a_y\,\mathbf{j} + a_z\,\mathbf{k}\right).\left(b_x\,\mathbf{i} + b_y\,\mathbf{j} + b_z\,\mathbf{k}\right)$$

$$= a_x\,b_x(\mathbf{i}.\,\mathbf{i}) + a_x\,b_y(\mathbf{i}.\,\mathbf{j}) + a_x\,b_z(\mathbf{i}\,.\,\mathbf{k}) +$$

$$a_y\,b_x(\mathbf{j}.\,\mathbf{i}) + a_y\,b_y(\mathbf{j}.\,\mathbf{j}) + a_y\,b_z(\mathbf{j}\,.\,\mathbf{k}) +$$

$$a_z\,b_x(\mathbf{k}.\,\mathbf{i}) + a_z\,b_y(\mathbf{k}.\,\mathbf{j}) + a_z\,b_z(\mathbf{k}\,.\,\mathbf{k})$$

Let's look at the dot products of our mutually orthogonal unit vectors

$$\mathbf{i.\,i} = \mathbf{j.\,j} = \mathbf{k.\,k} = \mathbf{1} \times \mathbf{1}\cos \mathbf{0}° = \mathbf{1}$$

$$\mathbf{i.\,j} = \mathbf{j.\,i} = \mathbf{i.\,k} = \mathbf{k.\,i} = \mathbf{j.\,k} = \mathbf{k.\,j} = \mathbf{1} \times \mathbf{1}\cos \mathbf{90}° = \mathbf{0}$$

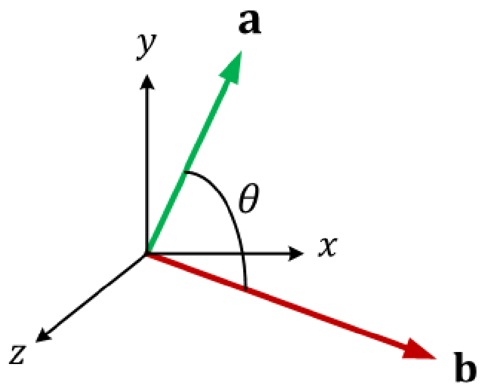
- Now we can greatly simplify our expression for dot product

$$\mathbf{a}\,.\,\mathbf{b} = a_x b_x + a_y b_y + a_z b_z$$

Now we have a way to find the angle between vectors

- If we are given the rectangular components, we can make use of the 2 equivalent definitions of the dot product

$$\mathbf{a}\,.\,\mathbf{b} = ab\,cos\,\theta = a_x b_x + a_y b_y + a_z b_z$$



- Rearranging gives

$$\theta = \cos^{-1}\left(\frac{a_x b_x + a_y b_y + a_z b_z}{ab}\right)$$

We will frequently use the dot product to convert a vector equation into scalar equations

- Say we have a vector equation

$$\mathbf{a} = \mathbf{b} + \mathbf{c}$$

- In component form

$$a_x\,\mathbf{i} + a_y\,\mathbf{j} + a_z\,\mathbf{k} = \left(b_x\,\mathbf{i} + b_y\,\mathbf{j} + b_z\,\mathbf{k}\right) + \left(c_x\,\mathbf{i} + c_y\,\mathbf{j} + c_z\,\mathbf{k}\right)$$

We can take the dot product of both sides with one of our unit vectors

- From the last equation on the previous slide

$$\mathbf{i}.\left(a_x\,\mathbf{i}+a_y\,\mathbf{j}+a_z\,\mathbf{k}\right)=\mathbf{i}.\left[\left(b_x\,\mathbf{i}+b_y\,\mathbf{j}+b_z\,\mathbf{k}\right)+\left(c_x\,\mathbf{i}+c_y\,\mathbf{j}+c_z\,\mathbf{k}\right)\right]$$

- Using the distributive property, and knowing that

$$\mathbf{i}.\mathbf{i}=\mathbf{1} \qquad \text{and} \qquad \mathbf{i}.\mathbf{j}=\mathbf{i}.\mathbf{k}=0$$

- We arrive at

$$a_x=b_x+c_x$$

Using similar procedures with j and k gives similar results

- Namely, we arrive at the following three scalar equations from our original vector equation

$$a_x = b_x + c_x \qquad a_y = b_y + c_y \qquad a_z = b_z + c_z$$

Lesson Learnt:

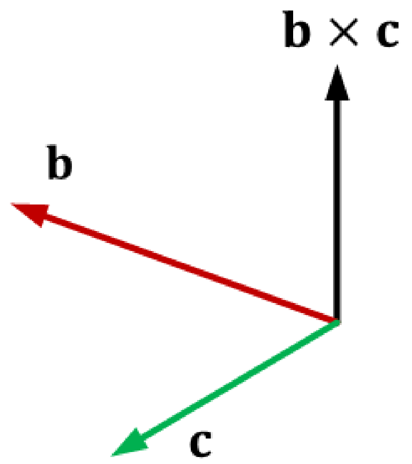
- As long as we have a set of orthogonal coordinates, we can use this procedure to arrive at 3 independent scalar equations for each vector equation

* *We will use this property extensively in problem solving*

The other common vector product we will encounter is the cross product

- The cross product of 2 vectors **b** and **c** is defined as

$$\mathbf{b} \times \mathbf{c} = \|\mathbf{b}\|\|\mathbf{c}\| \sin \boldsymbol{\theta}\, \mathbf{n}$$



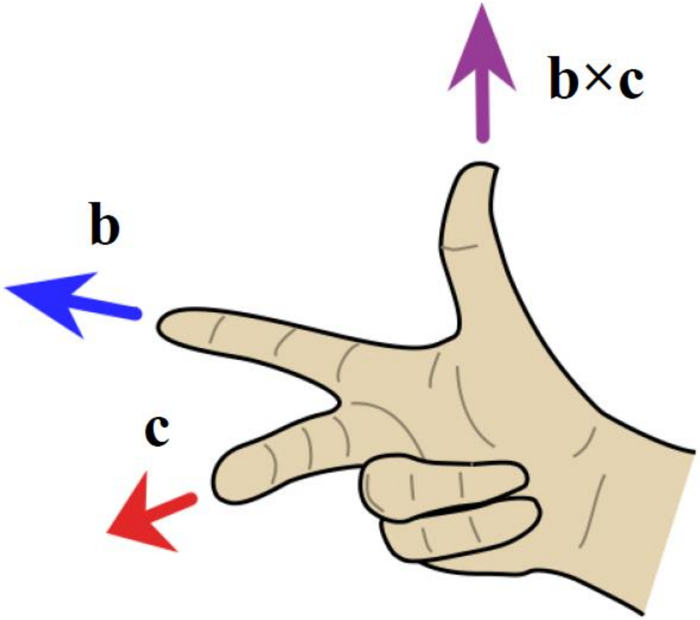
Where **n** is a unit vector that is perpendicular to both **b** and **c**

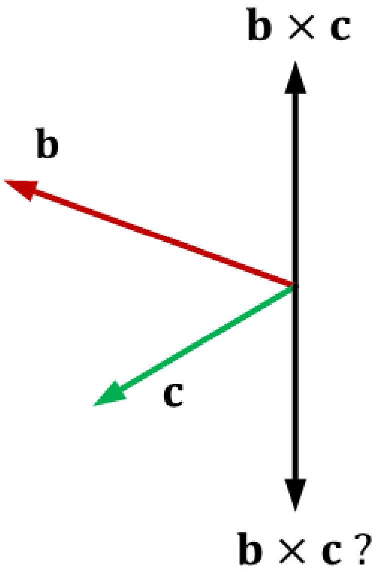
- Notice that the result is a vector
- The magnitude is the area of the parallelogram defined by **b** and **c**

This rule is incomplete

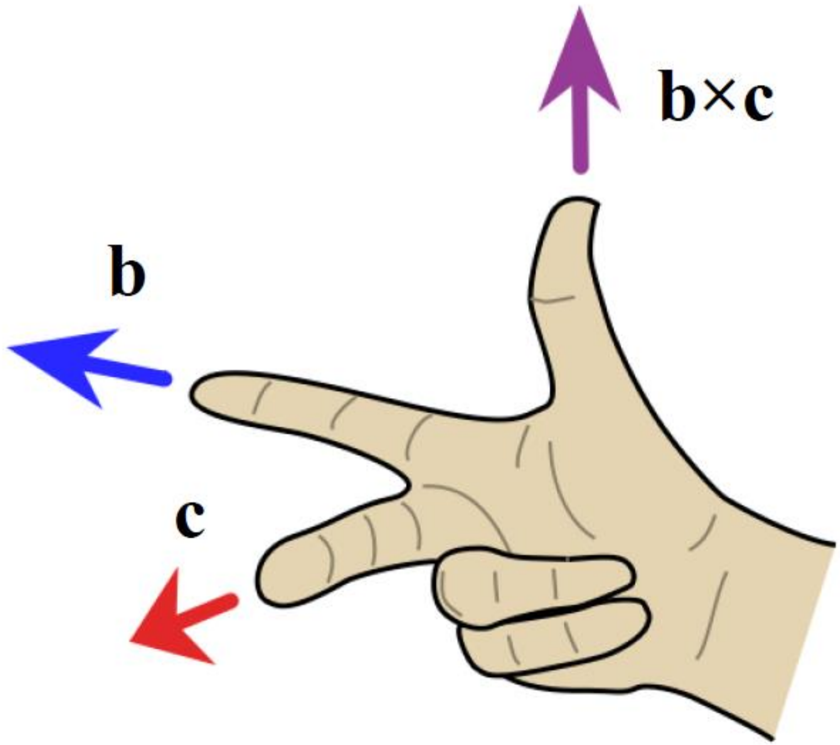
- There are *two* directions that are perpendicular to **b** and **c**

- We use the **right hand rule** to determine which the correct direction





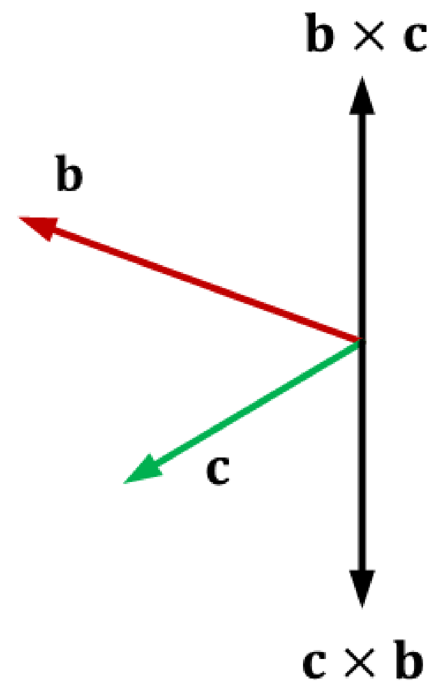
An illustration of the right-hand rule



This means the cross product is *NOT* commutative

- In fact, we can see that reversing the order of the product gives us the negative of the original result, i.e.

$$\mathbf{b} \times \mathbf{c} = -\mathbf{c} \times \mathbf{b}$$



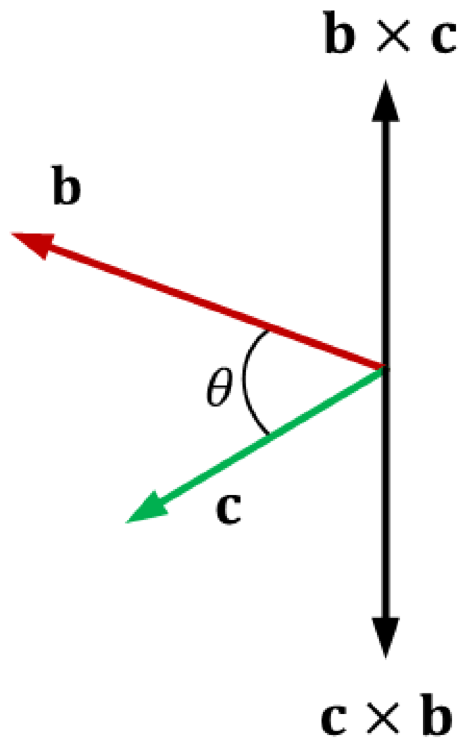
- Sometimes the above definition is not very handy for computing the product

$$\mathbf{b} \times \mathbf{c} = bc \sin\theta \, \mathbf{n}$$

Sometimes the above definition is not very handy for computing the product

- If we know the magnitudes of **b** and **c** and the angle between them, we can quickly find the magnitude of the product

$$||\mathbf{b} \times \mathbf{c}|| = bc \sin \theta$$



- However, it is not trivial to find the direction of **n** in general

Furthermore, we often are dealing with vectors in component form

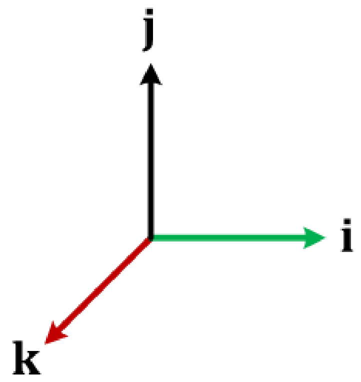
- Luckily, while the cross product is not commutative, it is distributive

- So we can compute $\mathbf{b} \times \mathbf{c}$ as

$$\mathbf{b} \times \mathbf{c} = \left(b_x\,\mathbf{i} + b_y\,\mathbf{j} + b_z\,\mathbf{k}\right) \times \left(c_x\,\mathbf{i} + c_y\,\mathbf{j} + c_z\,\mathbf{k}\right)$$

And compute the product component-wise

It is helpful to examine the cross products of the Cartesian unit vectors



- Using the definition of the cross product and remembering that $\sin 0° = 0$ and $\sin 90° = 1$, we can find the following

$$\mathbf{i} \times \mathbf{i} = 0 \qquad \mathbf{j} \times \mathbf{i} = -\mathbf{k} \qquad \mathbf{k} \times \mathbf{i} = \mathbf{j}$$

$$\mathbf{i} \times \mathbf{j} = \mathbf{k} \qquad \mathbf{j} \times \mathbf{j} = 0 \qquad \mathbf{k} \times \mathbf{j} = -\mathbf{i}$$

$$\mathbf{i} \times \mathbf{k} = -\mathbf{j} \qquad \mathbf{j} \times \mathbf{k} = \mathbf{i} \qquad \mathbf{k} \times \mathbf{k} = 0$$

Now we can arrive at an expression for the cross product in component form

$$\mathbf{b} \times \mathbf{c} = \left(b_y c_z - b_z c_y\right)\mathbf{i} + \left(b_z c_x - b_x c_z\right)\mathbf{j} + \left(b_x c_y - b_y c_x\right)\mathbf{k}$$

- Hold on, that doesn't seem very convenient either
- How are we supposed to remember all that?

If you are confident with doing determinants of 3x3 matrices, it is simpler to compute some cross products in this way:

$$\mathbf{b} \times \mathbf{c} = \det \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{bmatrix}$$

Let's try re-writing above equation

Cross Products

- If you are confident with doing determinants of 3x3 matrices, it is simpler to compute some cross products in this way:

$$\mathbf{b} \times \mathbf{c} = \det \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{bmatrix}$$

Fortunately, many vectors lie in a coordinate plane or along a coordinate axis

- If **a** lies in the *x*-*y* plane, it is $\mathbf{a} = a_x\,\mathbf{i} + a_y\,\mathbf{j}$

- If **a** lies in the *y*-*z* plane, it is $\mathbf{a} = a_y\,\mathbf{j} + a_z\,\mathbf{k}$

- If **a** lies along the *y* axis, it is $\mathbf{a} = a_y\,\mathbf{j}$ Etc.

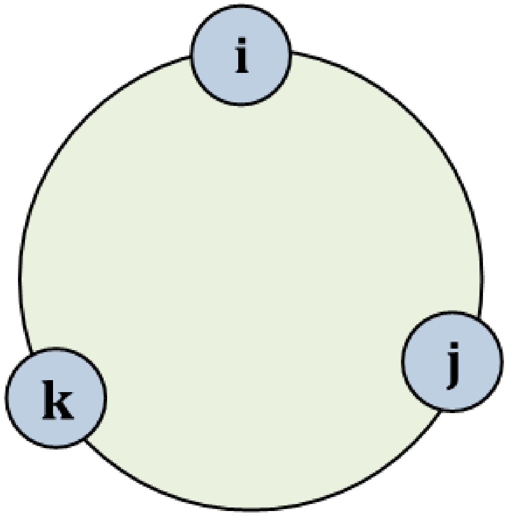
- That is, vectors in coordinate planes or on coordinate axes do not have all three components
- In fact, if we do a good job of choosing coordinates, we can maximize the number of vectors like this

We often take cross products of vectors with fewer that 3 components

- In these cases, it may actually be easier and faster to do the products term by term instead of doing the full determinant
- A little mnemonic diagram is often very useful

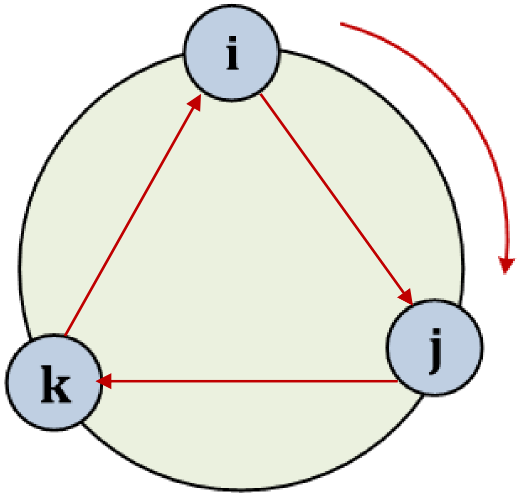
How to use it?

- Draw a circle
- Label **i, j** and **k** in alphabetical order along the circumference in clockwise direction



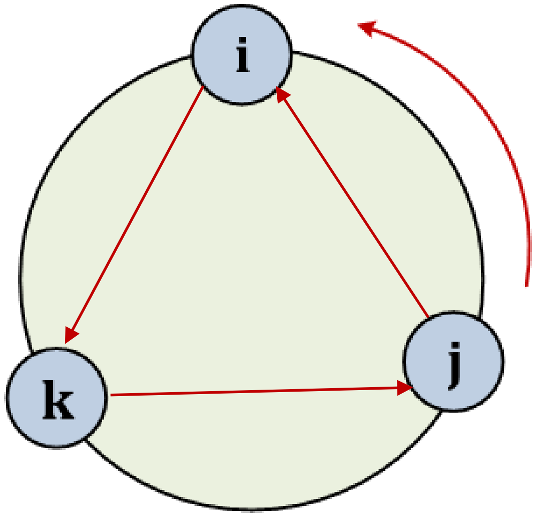
Sign Conventions

- Cross product between vectors in clockwise direction yields positive results



$\mathbf{i} \times \mathbf{j} = \mathbf{k}$ $\quad$ $\mathbf{j} \times \mathbf{k} = \mathbf{i}$ $\quad$ $\mathbf{k} \times \mathbf{i} = \mathbf{j}$

- Cross product between vectors in counterclockwise direction yields negative results



$\mathbf{i} \times \mathbf{k} = -\mathbf{j}$ $\quad$ $\mathbf{k} \times \mathbf{j} = -\mathbf{i}$ $\quad$ $\mathbf{j} \times \mathbf{i} = -\mathbf{k}$

A Comparison

Dot Product	Cross Product
Gives a scalar answer	Gives a vector answer
Is zero when the two vectors are perpendicular	Is zero when the two vectors are parallel
Is maximum when the two vectors are parallel	Is maximum when the two vectors are perpendicular
Is commutative	Is anti-commutative

Newton's Laws

Engineering Mechanics is based on Newton's Laws

- Newton's laws hold when
  - ✓ We are in an absolute (inertial) reference frame
  - ✓ We are talking about things "medium" in size i.e. not on the scale of atoms or galaxies
  - ✓ We are talking about things that aren't moving at a significant fraction of the speed of light
- Most of you should be familiar with the Newton's laws

Newton's 1st Law (law of inertia)

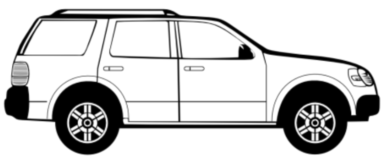
This is regarded as the basis for the field of **Statics**

- A particle remains at rest if there is no unbalanced force acting on it

At rest

Engine force

Motion



- A particle continues to move with *uniform velocity* (in a straight line with a constant speed) if there is no unbalanced force acting on it

Moving car

Braking force

At rest

** Both states tend to continue*

Newton's 1st Law (Examples)

- Notice the jerks during the to and fro motion of the car?

The object tends to resists change in its state

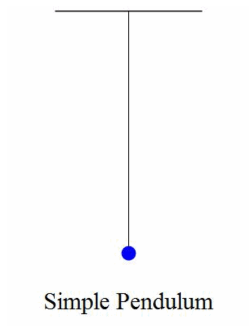
The heavier the object, the higher the inertia

Think of a car and train moving at same speed?

Can you relate it to the giraffe example? YES

Lesson: ALWAYS wear a seat belt on a vehicle

- Will this pendulum continue to swing forever?



YES (in a vacuum only)

There is no air resistance

** Any other examples you can think of*

Newton's 2nd Law

This is regarded as the basis for the field of **Dynamics**

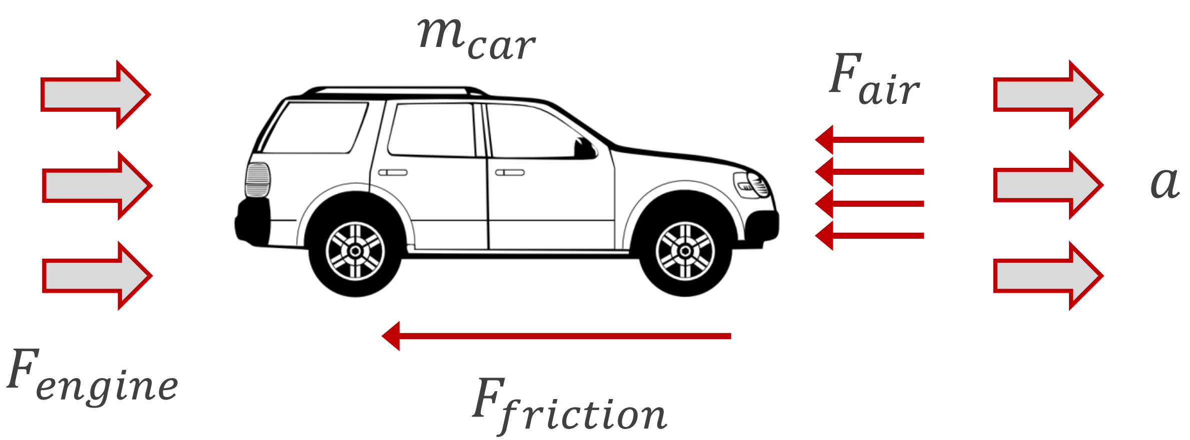
- The acceleration of a particle is proportional to the vector sum of the forces acting on it, and is in the direction of this vector sum

$$F_{net} \propto m_{object} \quad \Rightarrow \quad \sum F = ma$$

- The higher the net force (unbalanced force), the greater will be the acceleration
- The heavier the object, the higher will be the force needed to achieve a certain acceleration

** Can you answer why heavy locomotives need bigger and powerful engines than smaller vehicles?*

Positive acceleration



Object = car

External forces

Forces increased

Acceleration increased

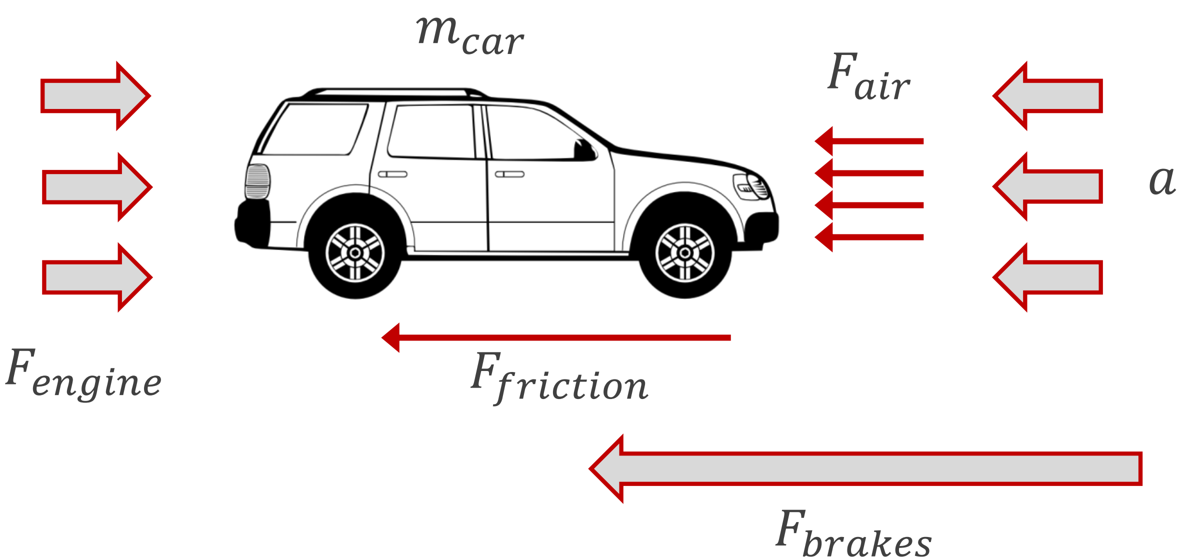
Car speeding up

$F_{net} = m_{car}a$

$F_{engine} - F_{friction} - F_{air} = m_{car}a$

- The higher the net force (unbalanced force), the greater will be the acceleration
- If F_{net} is positive, the acceleration is positive (object speeding up)

Negative acceleration (deceleration)



Object = car

External forces

Brakes applied

Acceleration increased (-ve)

Car slowing down

$$F_{net} = m_{car}a$$

$$F_{engine} - F_{friction} - F_{air} - F_{brakes} = m_{car}a$$

- If F_{net} is negative, the acceleration is negative (object slowing down)

Newton's 2nd Law is a vector equation

- By extension, so is Newton's 1st Law
- This means that when it is useful to do so, we can decompose them into 3 scaler laws
- Both force and acceleration are vector quantities

Newton's 3rd Law

Sometimes referred to as the "First Axiom of Statics", this law is essential in both **Static** and **Dynamic** Analysis

- The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear (they lie on the same line)

If the rocket pushes the propellant backwards, then,

the propellant pushes the rocket forwards.

Critical Thinking!

- What causes the planes to cruise in sky?
- What causes a ball thrown at a wall to bounce back?
- What causes the ship to sail while it pushes the water backward?
- Why a book sitting on a table keeps without falling?
- The dynamics of walking?
- The recoil of a gun?
- Numerous other examples ………….

Newton's 3rd Law is a vector equation

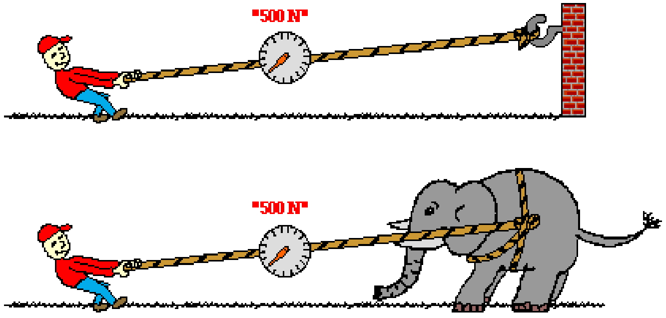
$$F_{12} = -F_{21}$$

- This means that when it is useful, we can decompose it into 3 scalar laws
- The 3rd law is important to the modelling of the systems

  ✓ Forces (contact forces, gravity, etc.) come in equal and opposite pairs

  ✓ This helps us analyse the forces between an object and its surroundings

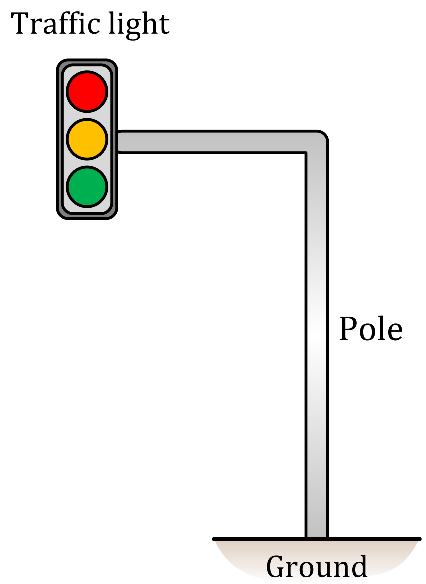
Analysis of Forces



Force Analaysis

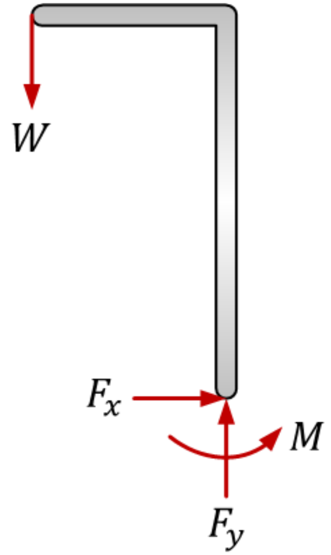
There are forces everywhere in nature

- In order to solve any engineering problem, we need to isolate some small piece of nature and identify the relevant forces
- The small piece of nature is called the system
- The forces we analyse will depend on the system we define



Force analysis of pole?

Isolate the pole from its surroundings



There are no pre-defined systems in nature

- We define systems at our convenience
- The definition of the system depends on the problem we are trying to solve
- Making good choices in system definition is one of the most important parts of solving an engineering problem
- System definition is a key engineering skill!!!!

** We can relate all these points to the traffic light example on previous slide*

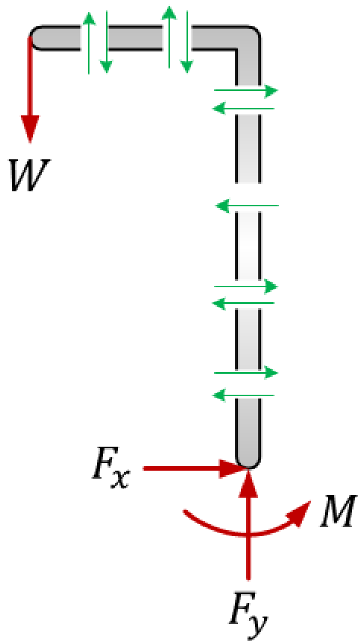
Forces are not internal or external by nature

- Compression, tension and shear only occur inside an object
  - ✓ Sometimes the system of interest does not encompass the whole body
  - ✓ Then these forces may be external

- Likewise, normal and friction forces only occur between bodies
  - ✓ Sometimes the system encompasses more than one body
  - ✓ The these forces may be internal

Only external forces enter into our analysis

- When we draw a free body diagram, we only include external forces (remember the traffic light example?)
- All the internal forces must cancel each other out (by Newton's 3rd Law), and thus have no effect on the answer



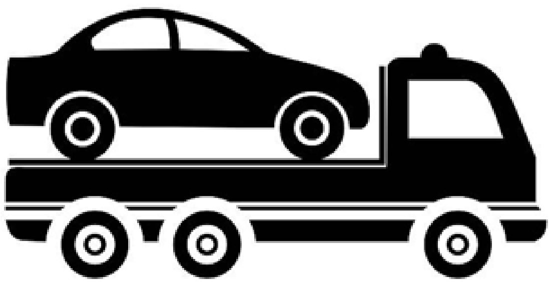
Note: All internal forces (in green) cancel each other

Newton's 3rd Law holds

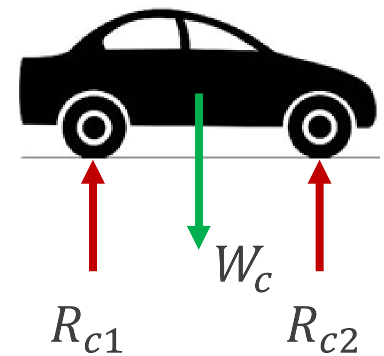
Hence, no effect on the external forces

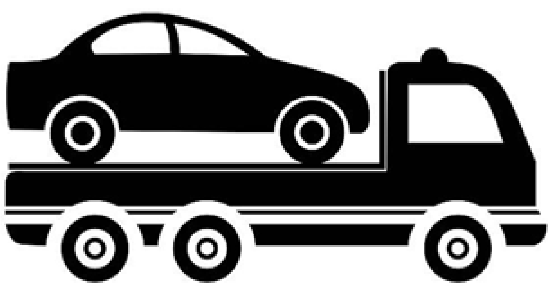
We only draw the chosen system in FBD

- We eliminate, say, contacting bodies outside the system and replace them with the contact forces

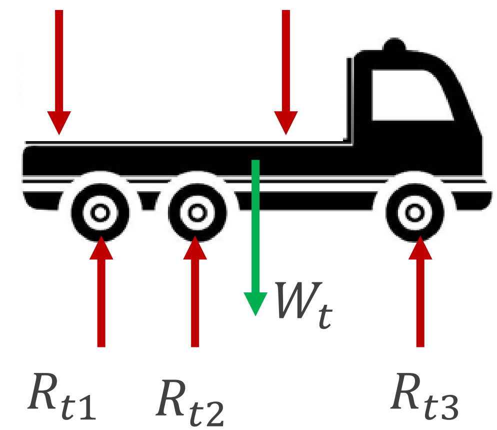


FBD of car →



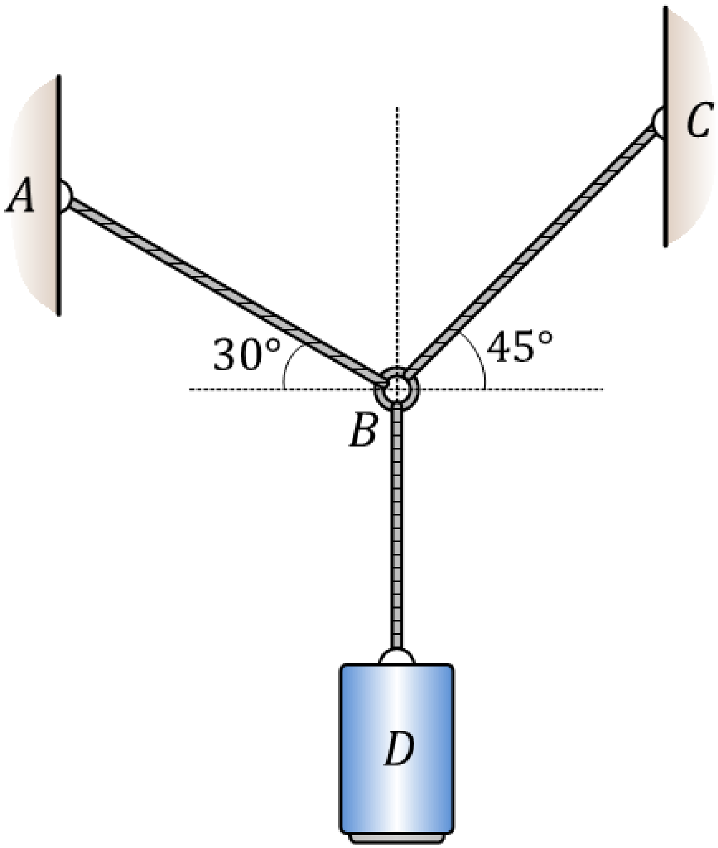


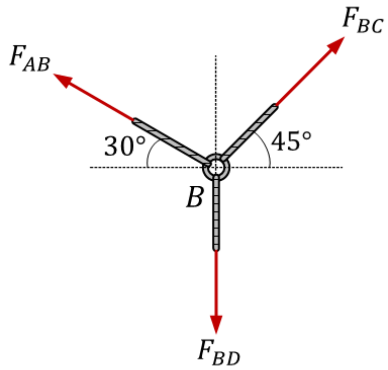
FBD of truck →



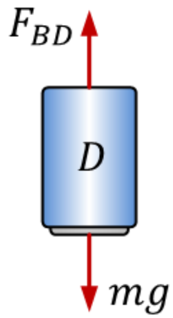
** Note: Action/reaction and system isolation*

Let's consider an example





(a) FBD of the ring at B



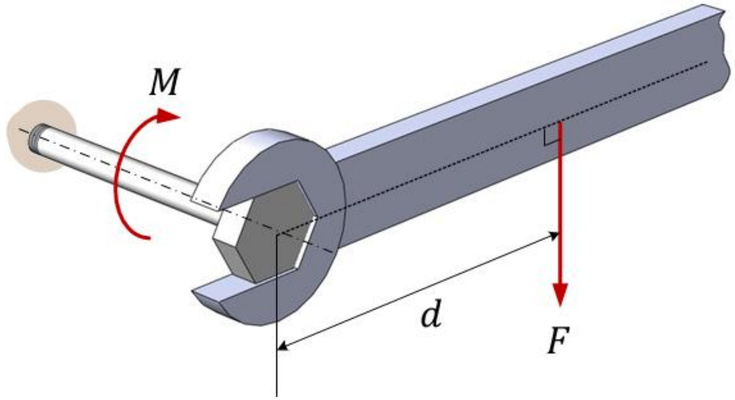
(b) FBD of mass *D*

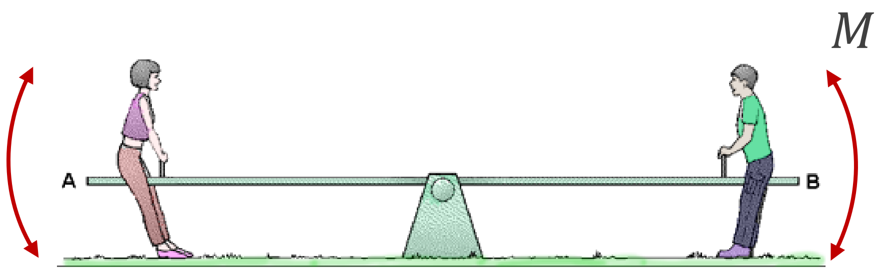
Space Diagram: A sketch showing the physical conditions of the problem

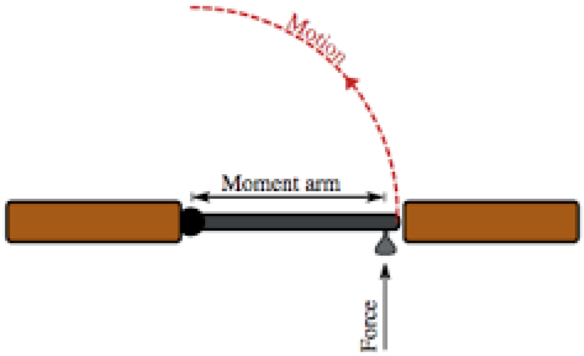
Free-Body Diagram: A sketch showing only the forces on the selected particle

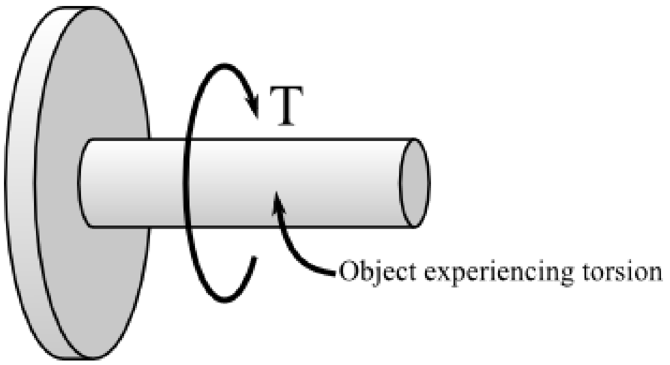
Moments

- In Engineering Mechanics, a moment is our concept of a force that tends to cause turning of an object
- We also sometimes refer to the same concept as torque





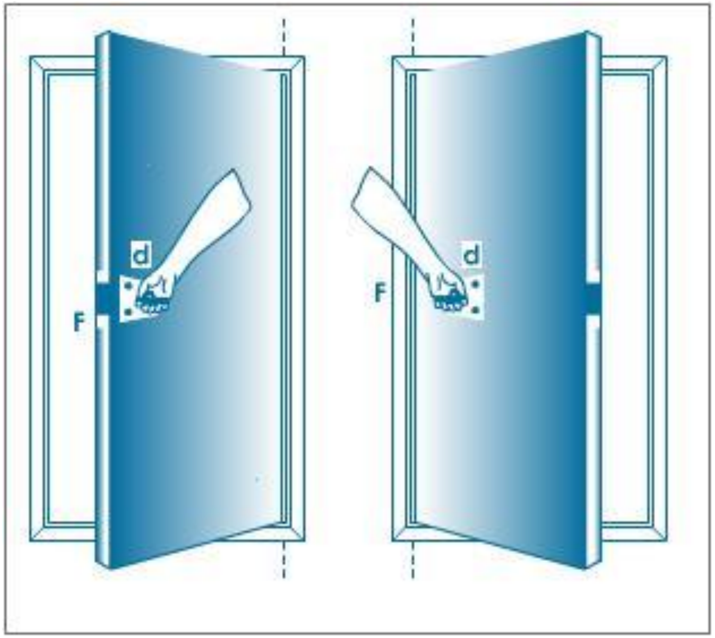




Moments

- Ever wonder why a door know/handle is always at a distance farthest away from where it is hinged?

(have a look at your classroom door)



?

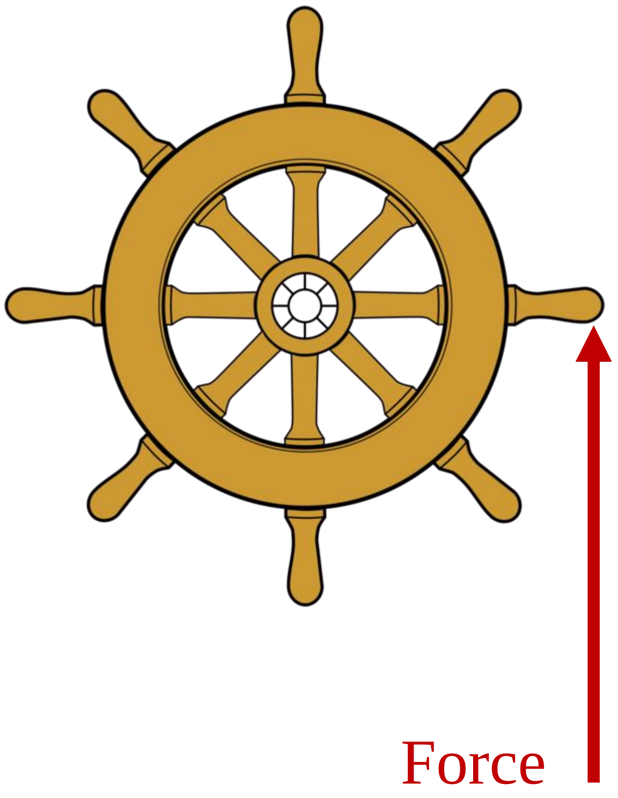
Because:

$$Moment = Force \times perpendicular\ distance$$

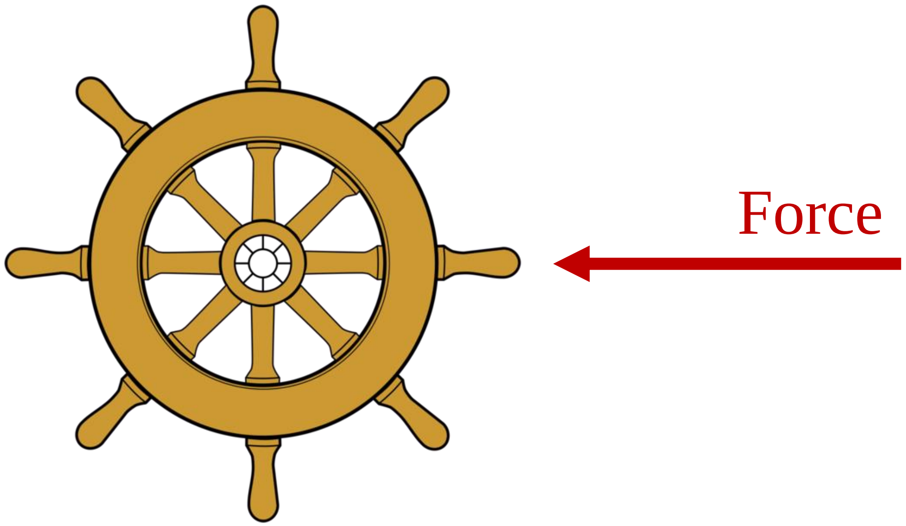
* *We will talk more about mathematical formulation later*

Consider trying to turn this rusty wheel on its axle

- We need to apply a force
- We know that applying the force farther from the axle is better
- Also, a bigger force will tend to work better



The direction also matters



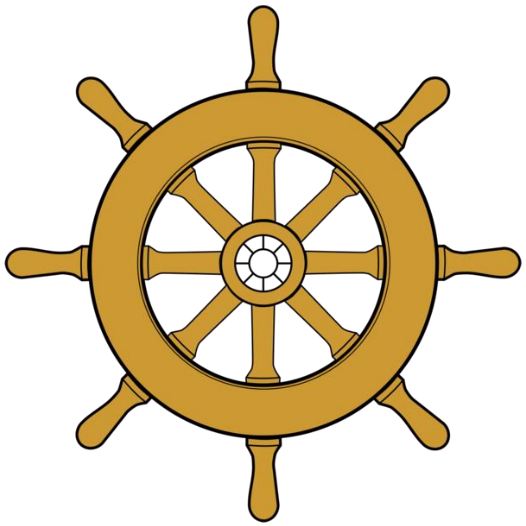
- We know that we cannot turn the wheel if we push directly at the axle
- LESSON: Any force that passes through the centre of rotation doesn't tend to create any moment
- Can you open/close a door by applying a force at the hinge?

We can see that the turning effect of a force depends on 3 things

1. The magnitude of the force

2. The point where the force is applied

3. The direction of the force (relative to the line from the axle to the point where the force is applied)

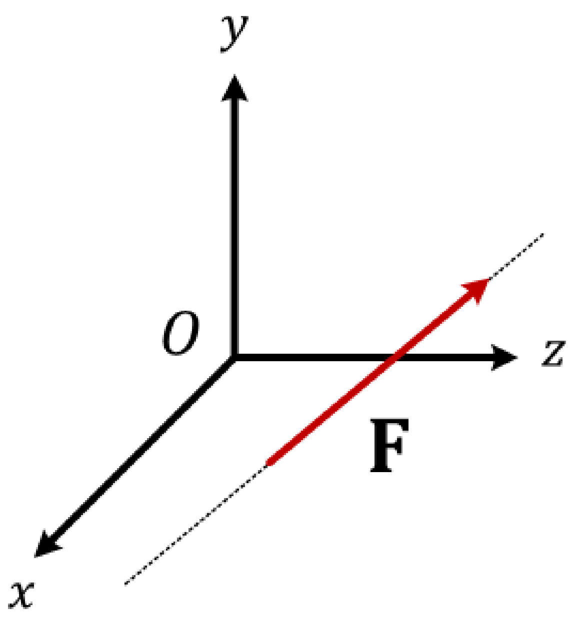


It’s good to have intuition about moments

- But as engineers, we will need to perform calculations about moments
- How do we handle the moments mathematically?

We first need a few preliminary concepts

- The moment about some point in space is created by a force when its line of action is offset from that point
- We refer to that point as **moment centre**
- The moment of the same force will be different about different moment centres



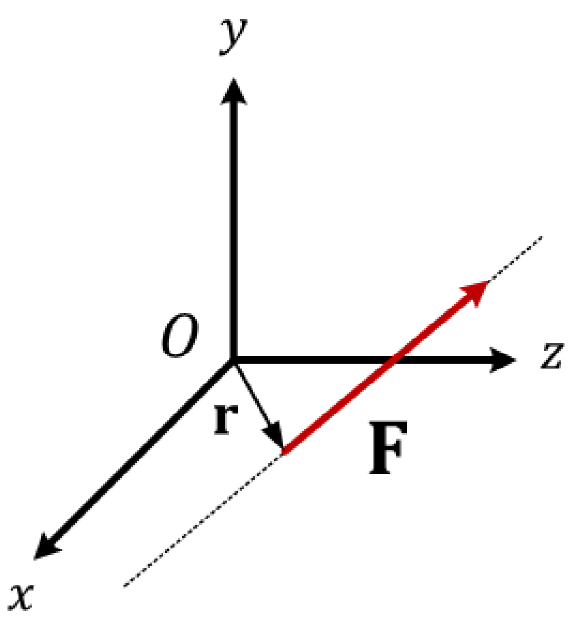
We also need to define a position vector

- The position vector is a vector drawn from the moment centre (*O* at right) to any point on the line of action of the force

- Typically, we choose the point on the body where the force is applied

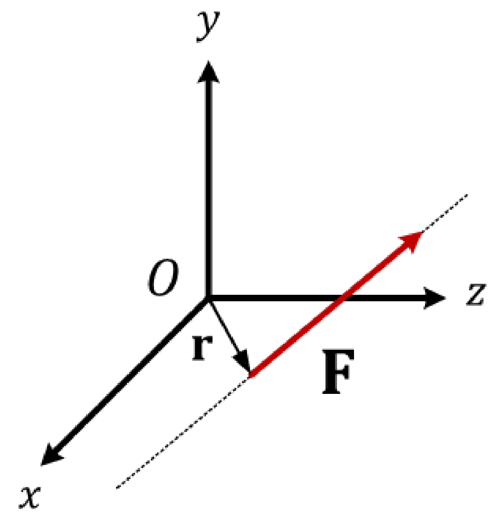
- We usually write the position vector as **r**

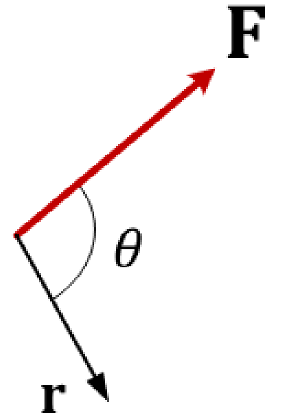
- The position vector is also sometimes referred to as the moment arm



Magnitude of Moment

- The magnitude of the moment $||\mathbf{M}||$
- Is equal to the magnitude of the position vector $||\mathbf{r}||$
- Times the component of the force that is perpendicular to the position vector $||\mathbf{F}|| \sin\theta$
- Where θ is the angle between the two vectors when they are placed tail to tail





Altogether now

- We have

$$\|\mathbf{M}\| = \|\mathbf{r}\|(\|\mathbf{F}\| \sin\theta)$$

- Or

$$M = rF\sin\theta$$

- Does this look familiar?

There is yet a simpler possibility

- If we can easily find the minimum distance from the moment centre to the line of action of the force, we can write

$$M = Fd$$

- Where d is the distance

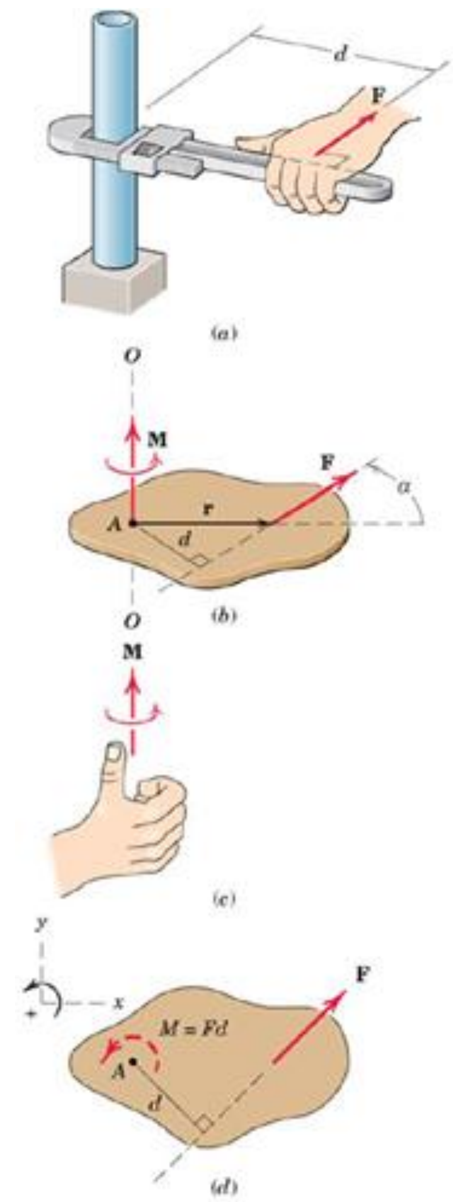
- Of course, this minimum distance runs along a line that is perpendicular to the line of action of the force

Right Hand Rule

We use the right-hand rule to determine the direction

- A moment is perpendicular to the plane in which it tends to cause rotation

- If you curl the fingers on your right hand around so that they point in the sense of the twisting, your thumb points in the direction of the moment vector



Let's review for a moment

- The magnitude of a vector is given by

$$M = rF\sin\theta$$

- The direction is given by the right-hand rule

- This is all very familiar

- We can calculate moments as **cross products**

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

- Recall that the cross products are **NOT** commutative

Units of Measurement

Quantity	SI Units		US Units	
	Unit	**Symbol**	**Unit**	**Symbol**
Mass	kilogram	kg	slug	-
Length	meter	m	foot	ft
Time	second	s	second	sec
Force	newton	N	pound	lb

Addition of Vectors

- Trapeziod rule of vector addition
- Triangle rule of vector addition
- Law of cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos\angle B$$

$$\vec{R} = \vec{P} + \vec{Q}$$

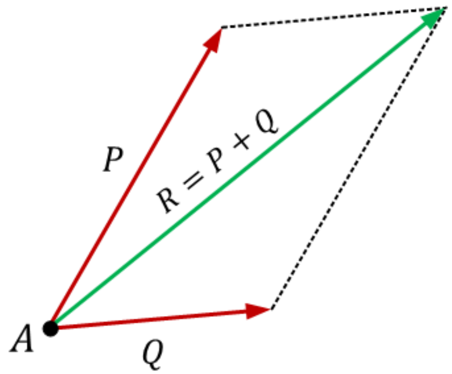
- Law of sines,

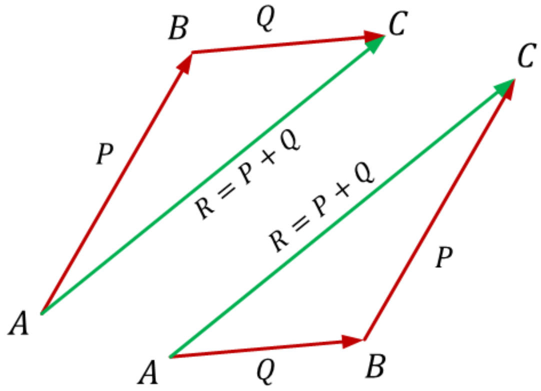
$$\frac{\sin\angle A}{Q} = \frac{\sin\angle B}{R} = \frac{\sin\angle C}{P}$$

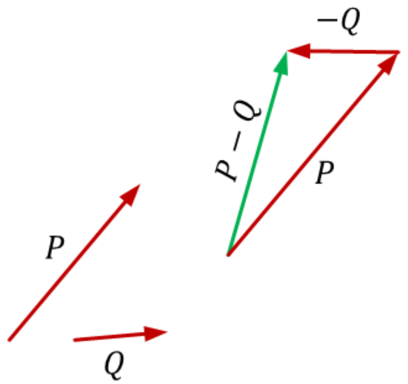
- Vector addition is commutative,

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

- Vector subtraction

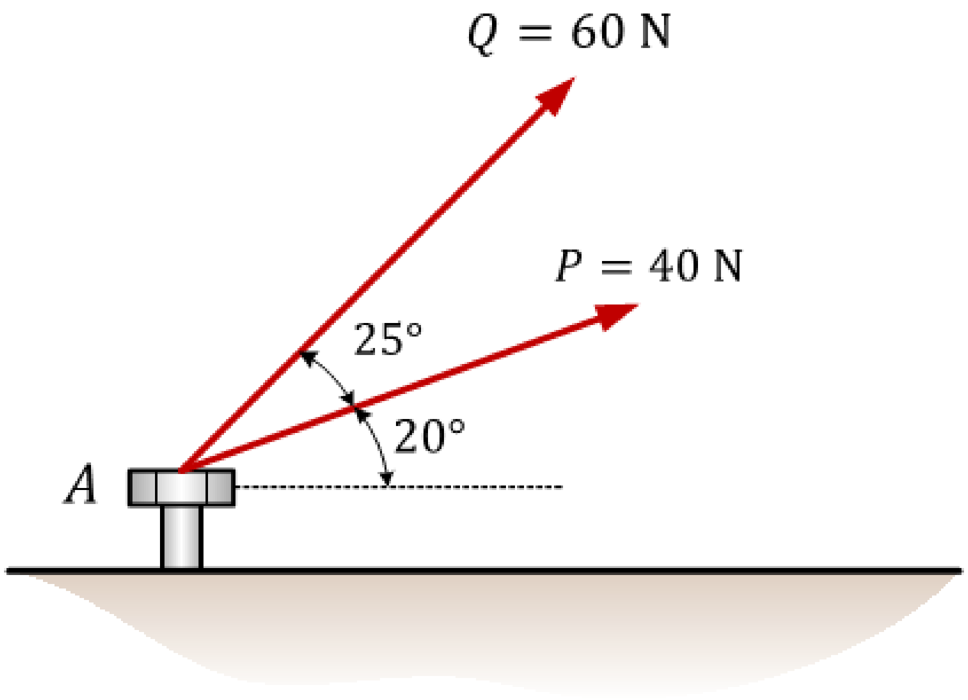






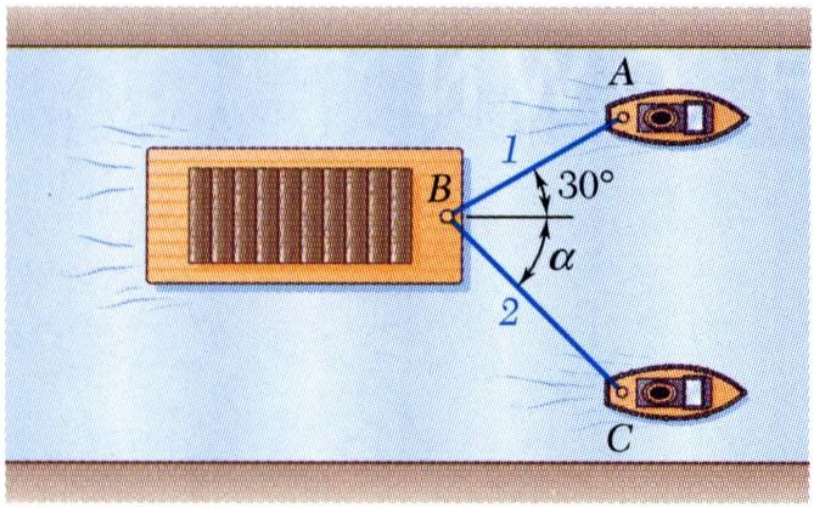
Example 1

The two forces act on a bolt at A. Determine their resultant.



W1 Example 1 (Web view)

Example 2



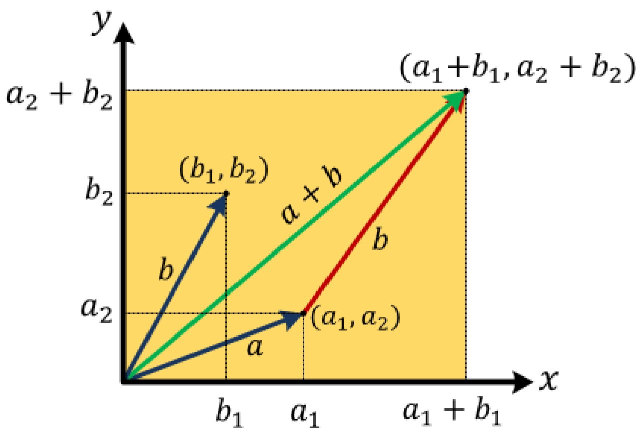
A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5,000 N directed along the axis of the barge, determine

a) the tension in each of the ropes for a = 45°,

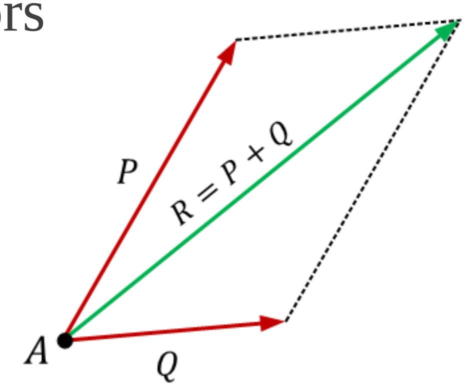
b) the value of a for which the tension in rope 2 is a minimum.

W1 Example 2 (Web view)

Summary



- Primitive concepts
- Scalars and Vectors
- Vector addition
- Forces (N)
- Moments = **r×F**



Next Topic:

FBDs and Equilibrium