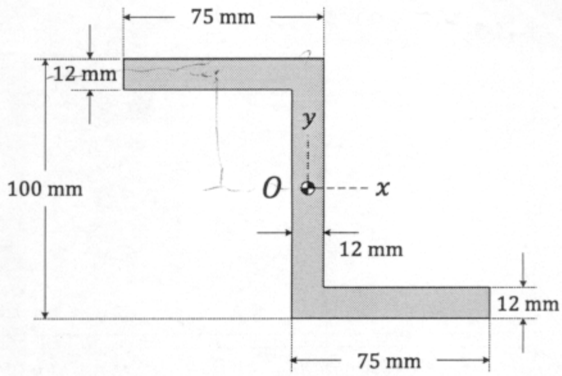
Question 3: (2 Marks)

For the section shown, the moments of inertia about the x-axis is $I_{x'x'} = 3.95\ (10^6)\ \text{mm}^4$ and the moment of inertia about the y-axis is $I_{y'y'} = 2.64\ (10^6)\ \text{mm}^4$, determine the following:

(Proceed according to the steps provided in solution boxes)



Solution:

(a) Determine the product of inertia $I_{x'y'}$ with respect to the xy axes

$$I_{x'y'} = A\, d_x\, d_y$$
$$= 2\,(75\times 12)\left(\frac{75}{2}\right)(50-6)$$
$$= 2.97\times 10^6 \text{ mm}^4$$

$$C = \frac{I_{xx}+I_{yy}}{2}, \quad R = \sqrt{\left(\frac{I_{xx}-I_{yy}}{2}\right)^2 + I_{xy}^2}$$

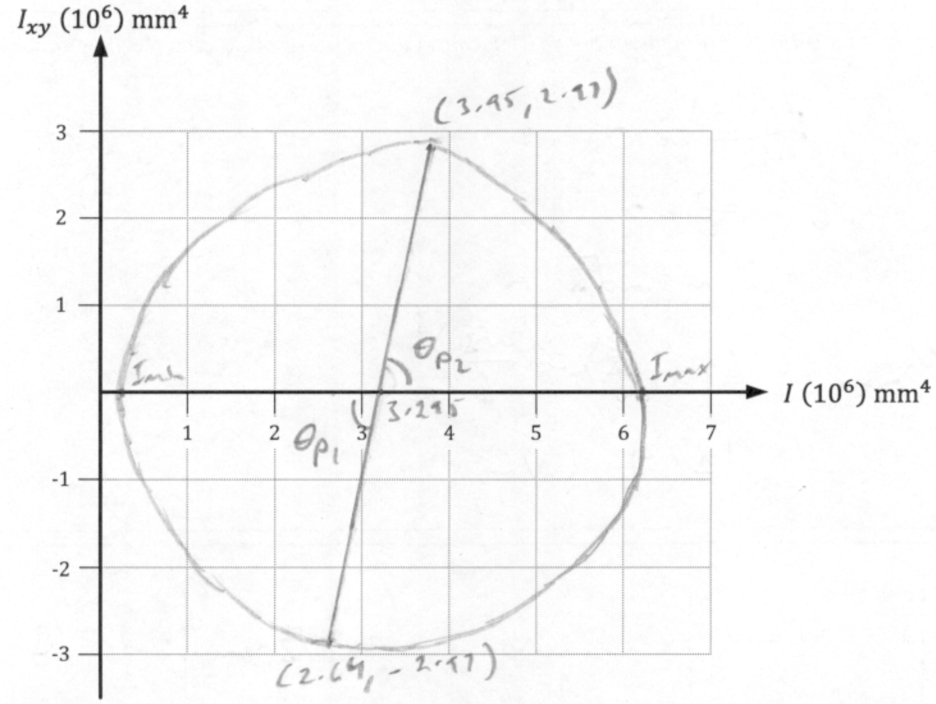
(b) On the axis provided, draw the MOHR's circle for MOI of given cross-section

$C = 3.295$

$R = 3.041$

$I_{max} = 6.336$

$I_{min} = 0.254$



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(c) Using the MOHR's circle determine the magnitude (I_{11} and I_{22}) and orientation (θ_{p1} and θ_{p2}) respectively of the minimum and maximum principle moment of inertias for the cross-section

$I_{11} = I_{min} = C-R = 3.295 - 3.041 = 0.254 \times 10^6 \ mm^4$

$I_{22} = I_{max} = C+R = 3.295 + 3.041 = 6.336 \times 10^6 \ mm^4$

$\theta_{p_2} = \tan^{-1}\left(\frac{2.97}{3.95}\right) = 37°$

$\theta_{p_1} = \theta_{p_2} = 37°$

(d) Calculate the Eigen Values (λ_1 and λ_2) for the matrix I composed of the moments and products of inertia.

$A = \begin{pmatrix} 3.95 & 2.97 \\ -2.97 & 2.64 \end{pmatrix}$

$\det(A - \lambda I) = \det\begin{pmatrix} 3.95-\lambda & 2.97 \\ -2.97 & 2.64-\lambda \end{pmatrix}$

$= \left((3.95-\lambda)(2.64-\lambda) - (2.97)(-2.97)\right)$

$= (10.428 + \lambda^2 - 6.59\lambda + 8.8209)$

$0 = \lambda^2 - 6.59\lambda + 1.6071$

$\therefore \lambda = \cancel{5.8181 \times 10^6 \ mm^4}, \ \cancel{0.7713 \times 10^6} \ mm^4$

$0.254 \times 10^6 \ mm^4 \quad 6.336 \times 10^6 \ mm^4$

What do you notice about the solutions for λ? (Answer below in three lines)

Solutions for λ should be same as I_{11} and I_{22} i.e.

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Answers:	$I_{11} =$ ~~0.7713 ×10⁶ mm⁴~~	$I_{22} =$ ~~5.8181 ×10⁶ mm⁴~~	$\theta_{p1} =$ 37°	$\theta_{p2} =$ 37°	RJBV2OEM 6

$0.254 \times 10^6 \ mm^4 \quad 6.336 \times 10^6 \ mm^4$