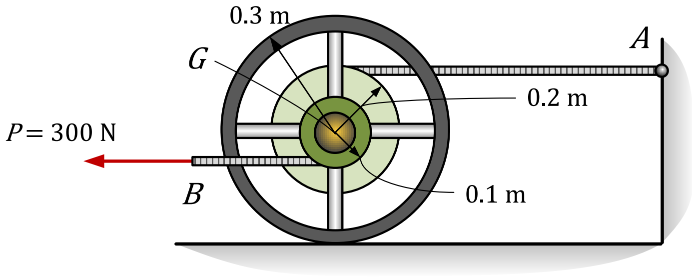
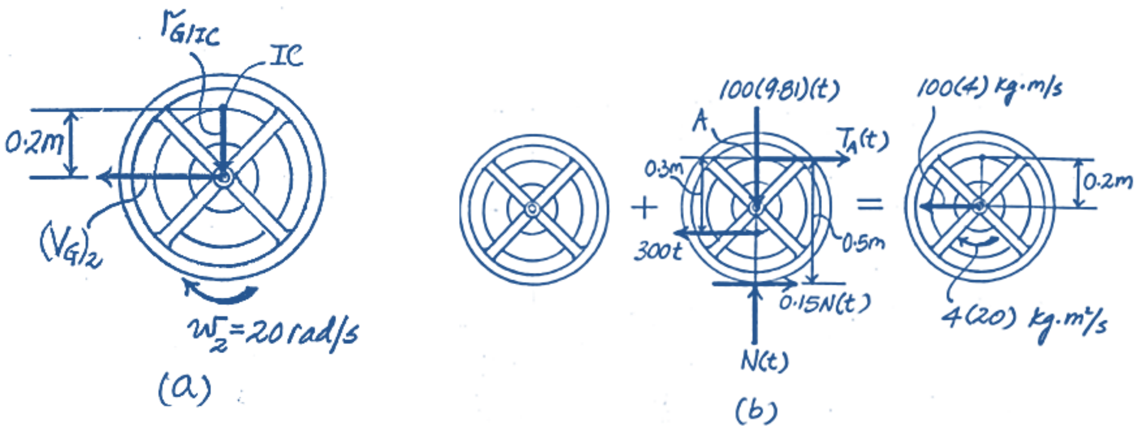
Question 12.6.

The 100 kg reel has a radius of gyration about its center of mass G of $k_G = 200$ mm. If the cable B is subjected to a force of $P = 300$ N, determine the time required for the reel to obtain an angular velocity of 20 rad/s. The coefficient of kinetic friction between the reel and the plane is $\mu_k = 0.15$.



Solution



Referring to Fig. (a), the final velocity of the centre of the spool is,

$$(v_G)_2 = \omega_2 r_{G/IC} = 20(0.2) = 4 \text{ m/s} \quad (\leftarrow)$$

The mass moment of inertia of the spool about its centre of mass is,

$$I_G = m(k_G)^2 = 100(0.2)^2 = 4 \text{ kg.m}^2$$

Applying linear impulse and momentum equation along the y-axis

$(+ \quad \uparrow)$

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$0 + N(t) - 100(9.81)(t) = 0$$

$$N = 981 \text{ N}$$

Using this result to write the angular impulse and momentum equation about the IC

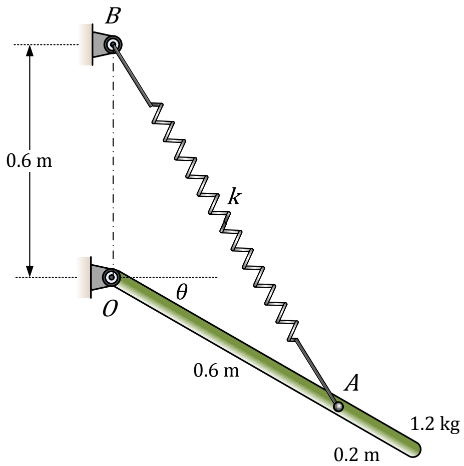
$(+ \quad \circlearrowleft)$

$$(H_{IC})_1 + \sum \int_{t_1}^{t_2} M_{IC} dt = (H_{IC})_2$$

$$0 + 0.15(981)(t)(0.5) - 300t\ (0.3) = -100(4)(0.2) - 4(20)$$

$t = 9.74$ s (Answer)

Question 12.7.



The 1.2 kg uniform slender bar rotates freely about a horizontal axis through O. The system is released from rest when it is in the horizontal position $\theta = 0°$ where the spring is unstretched. If the bar is observed to momentarily stop in the position $\theta = 50°$ determine the spring constant k. For your computed value of k, what is the angular velocity of the bar when $\theta = 25°$?

Solution

From $\theta = 0°$ to $\theta = 50°$

$W_{1-2} = 0$ (i.e. no external force or moment)

$\Delta T = 0$ (at start and end the rod is at rest)

$x_1 = 0$ (i.e. spring is unstretched at initial position)

$$x_2 = \sqrt{0.6^2 + 0.6^2 - 2(0.6)(0.6)\cos(90° + 50°)} - \sqrt{0.6^2 + 0.6^2} = 0.279 \text{ m}$$

$$\Delta V_e = \frac{1}{2}k(0.279^2) = 0.039k \text{ J}$$

$$\Delta V_g = mg(h_2 - h_1) = 1.2(9.81)(-0.4\sin 50°) = -3.607 \text{ J}$$

$$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

$$0 = 0.039k - 3.607$$

$k = 92.5$ N/m (Answer)

From $\theta = 0°$ to $\theta = 25°$

$$I_O = \frac{(1.2)(0.8)^2}{3} = 0.256 \text{ kg.m}^2$$

$W_{1-2} = 0$ (i.e. no external force or moment)

$$\Delta T = \frac{1}{2}I_O\omega^2 = 0.128\omega^2$$

$x_1 = 0$ (i.e. spring is unstretched at initial position)

$$x_2 = \sqrt{0.6^2 + 0.6^2 - 2(0.6)(0.6)\cos(90° + 25°)} - \sqrt{0.6^2 + 0.6^2} = 0.16354 \text{ m}$$

$$\Delta V_e = \frac{1}{2}(92.5)(0.16354^2) = 1.237 \text{ J}$$

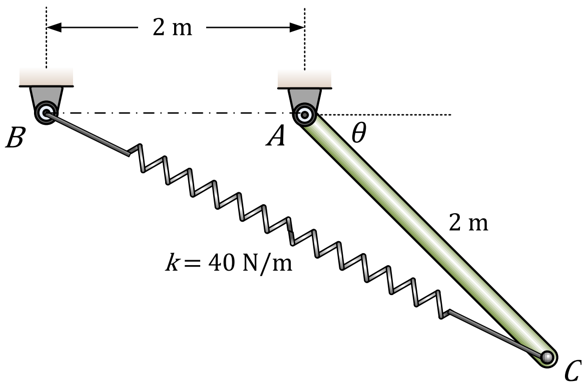
$$\Delta V_g = mg(h_2 - h_1) = 1.2(9.81)(-0.4\sin 25°) = -1.99 \text{ J}$$

$$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

$$0 = 0.128\omega^2 - 1.99 + 1.237$$

$\omega = 2.425$ rad/s (Answer)

Question 12.8.



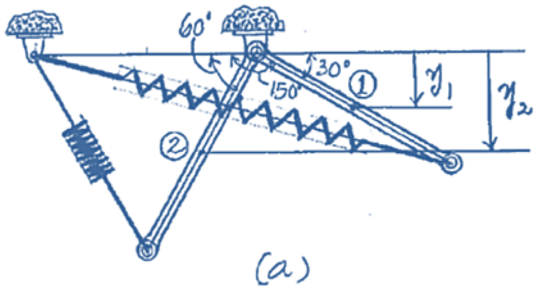
The 12 kg slender rod is attached to a spring, which has an unstretched length of 2 m. If the rod is released from rest when $\theta = 30°$, determine the angular velocity of the rod the instant the spring becomes unstretched.

Solution

$$I_A = \frac{ml^2}{12} + md^2 = \frac{(12)(2)^2}{12} + 12(1)^2 = 16 \text{ kg. m}^2$$

$$\Delta T = \frac{1}{2} I_A \omega^2 = \frac{16}{2}\omega^2 = 8\,\omega^2$$

$W_{1-2} = 0$ (i.e. no external force)



$$\Delta V_g = 12(9.81)(1\sin 30° - 1\sin 60°) = -43.088 \text{ J}$$

The length of the spring in initial and final positions is:

$$\frac{L_1}{\sin 150°} = \frac{2}{\sin 15°}$$

$L_1 = 3.8637$ m *therefore the stretch is* $x_1 = L_1 - L_o = 3.8637 - 2 = 1.8637$ m

$L_2 = 2$ m

$x_2 = L_2 - L_o = 2 - 2 = 0$ m

Thus:

$$\Delta V_e = \frac{1}{2}k\,(x_2^2 - x_1^2) = \frac{1}{2}(40)(-1.8637^2) = -69.467 \text{ J}$$

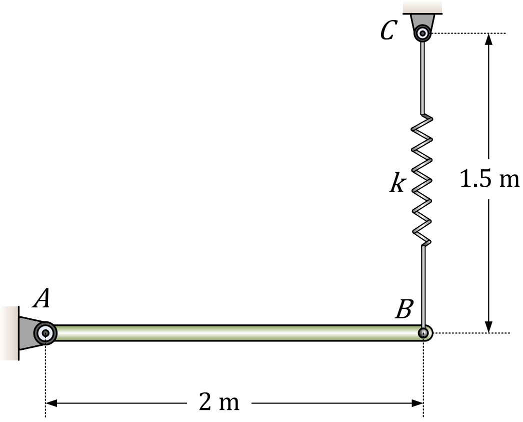
$$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

$$0 = 8\omega^2 - 43.088 - 69.467$$

$\omega = 3.751$ rad/s (Answer)

Question 12.9.

The slender 6 kg bar AB is horizontal and at rest and the spring is unstretched. Determine the angular velocity of the bar when it has rotated clockwise $45°$ after being released. The spring has a stiffness of $k = 12$ N/m.



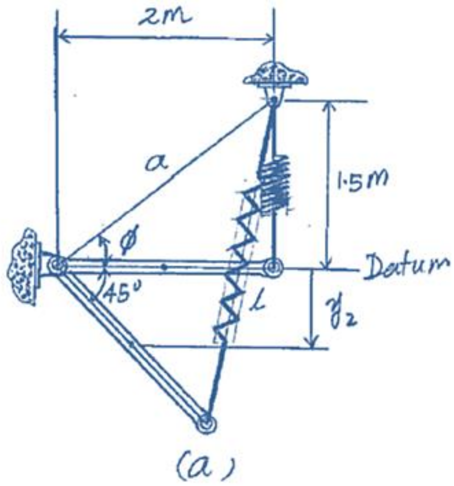
Solution

$$I_A = \frac{ml^2}{12} + md^2 = \frac{(6)(2)^2}{12} + 6(1)^2 = 8\ \text{kg.m}^2$$

$$\Delta T = \frac{1}{2} I_A \omega^2 = \frac{8}{2}\omega^2 = 4\,\omega^2$$

$W_{1-2} = 0$ (i.e. no external force)

$$\Delta V_g = 6(9.81)(0 - 1\sin 45°) = -41.62\ \text{J}$$



From geometry shown,

$a = \sqrt{2^2 + 1.5^2} = 2.5$ m and $\phi = 36.87°$

$x_1 = 0$

$$L_2 = \sqrt{2.5^2 + 2^2 - 2(2)(2.5)\cos(36.87° + 45°)} = 2.9725\ \text{m}$$

therefore

$x_2 = 2.9725 - 1.5 = 1.4725$ m

Thus:

$$\Delta V_e = \frac{1}{2}k\,(x_2^2 - x_1^2) = \frac{1}{2}(12)(1.4725^2 - 0^2) = 13\ \text{J}$$

$$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

$$0 = 4\omega^2 - 41.62 + 13$$

$\omega = 2.67$ rad/s (Answer)