School of Mechanical and Manufacturing Engineering

MMAN1300 – ENGINEERING MECHANICS 1

2017 S2 Block Test 4

Instructions:

- Time allowed: 45 minutes
- Total number of questions: 3
- Answer all the questions in the test
- Answer all questions in the spaces provided
- The 6 marks allocations shown are worth 6% of the course overall
- Candidates may bring drawing instruments, rulers and UNSW approved calculators to the test
- Print your name, student ID and all other requested details above
- Record your answers (with appropriate units) in the ***ANSWER BOXES*** provided

Notes:

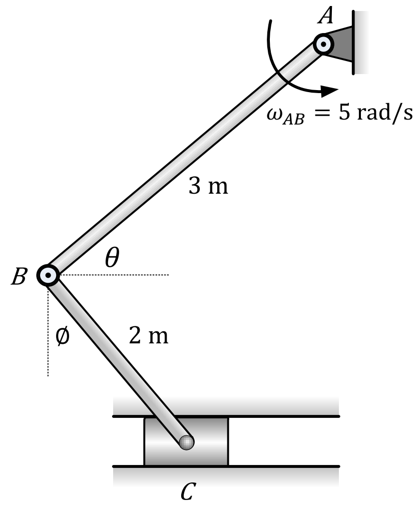
Your work must be complete, clear and logical

Do not skip steps, sign conventions, units and relevant diagrams and clearly state the final answers

No part of this paper is to be retained by candidates until handed back after marking

Question 1: (2 Marks)

The angular velocity of link AB is $\omega_{AB} = 5 \text{ rad/s}$. Determine the velocity of block C and the angular velocity of link BC at the instant $\theta = 40°$ and $\emptyset = 30°$.



Solution:

Perform the relative velocity analysis for link AB:

$$v_B = v_A + v_{B/A}$$

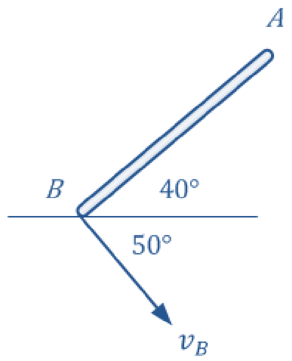
Since A is fixed

$$v_A = 0$$

Therefore

$$v_B = v_{B/A} = \omega_{AB}\overline{AB}$$

$$v_B = v_{B/A} = (5)(3) = 15 \text{ m/s} \ \ (\text{i.e.} \perp AB)$$

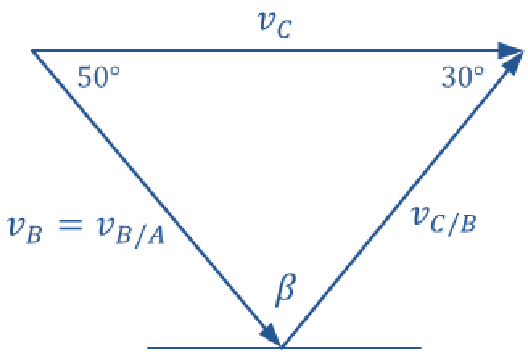


$\boxed{0.5}$

Draw the velocity triangle for the following velocities (give directions for all three velocities):

$$v_C = v_B + v_{C/B}$$

Since the slider is constraint to move in horizontal direction and the direction and magnitude of v_B is known. Also, $v_{C/B}$ is perpendicular to BC. Therefore the velocity triangle can be constructed as follows:



0.5

Where

$$\beta = 180° - 30° - 50° = 100°$$

Obtain the velocity of the slider block C (v_C) and the angular velocity of link BC (ω_{BC}):

Applying law of sine

$$\frac{v_B}{\sin 30°} = \frac{v_C}{\sin 100°}$$

$$\frac{15}{\sin 30°} = \frac{v_C}{\sin 100°}$$

$$v_C = 29.55 \text{ m/s}$$ 0.5

Also by law of sine

$$\frac{15}{\sin 30°} = \frac{v_{C/B}}{\sin 40°}$$

$$v_{C/B} = 19.28 \text{ m/s}$$

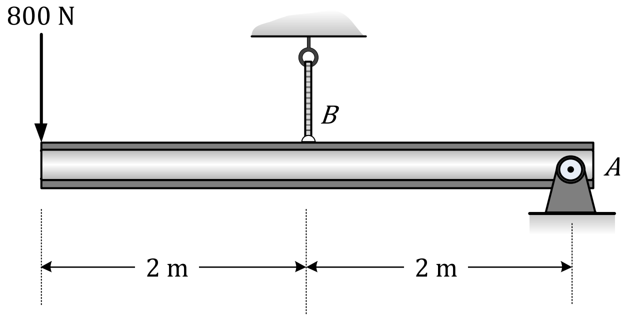
$$\omega_{BC} = \frac{v_{C/B}}{\overline{BC}} = \frac{19.28}{2}$$

$$\omega_{BC} = 9.642 \text{ rad/s (CCW)}$$ 0.5

Answers:	$v_C = 29.55$ m/s	$\omega_{BC} = 9.642$ rad/s (CCW)

Question 2: (2 Marks)

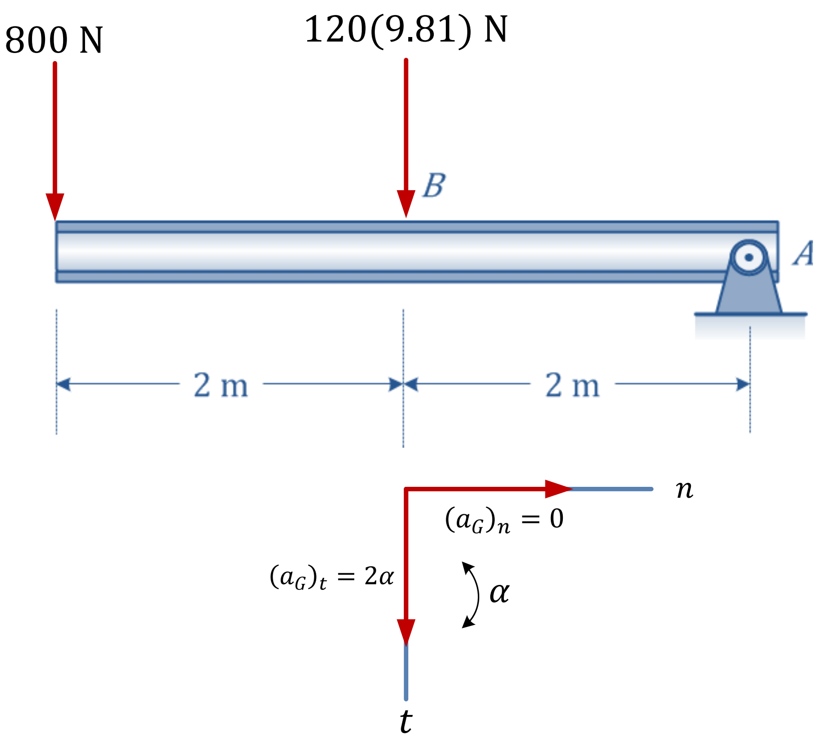
If the cord at *B* suddenly fails, determine the horizontal and vertical components of the initial reaction at the pin *A*, and the angular acceleration of the 120 kg beam. Treat the beam as a uniform slender rod.



Solution:

Present your solution here (including the free-body diagram if needed):
If needed *use the following information:*

- *Normal component of acceleration* = $r\omega^2$
- *Tangential component of acceleration* = αr
- *Mass moment of inertia of a slender rod about its centre of mass* = $ml^2/12$



0.5

Continue your solution (Question 2) here:

Mass moment of inertia of the beam about A is

$$I_A = \frac{ml^2}{12} + md^2$$

$$I_A = \frac{(120)(4)^2}{12} + (120)(2)^2$$

$$I_A = 640\ \mathrm{kg.m^2}$$ 0.25

Initially the beam is at rest, therefore

$$(a_G)_n = (\omega)^2 r_G = 0$$

Also,

$$(a_G)_t = 2\alpha$$

From the FBD,

$$\circlearrowleft +\sum M_A = I_A\alpha$$

$$800(4) + (120 \times 9.81)(2) = (640)\alpha$$

$$\alpha = 8.68\ \mathrm{rad/s^2}$$ 0.5

$$\sum F_n = m\,(a_G)_n$$

$$A_n = 0$$ 0.25

$$\sum F_t = m\,(a_G)_t$$

$$800 + (120 \times 9.81) + A_t = (120)(2\alpha)$$

$$800 + (120 \times 9.81) + A_t = (120)(2 \times 8.68)$$

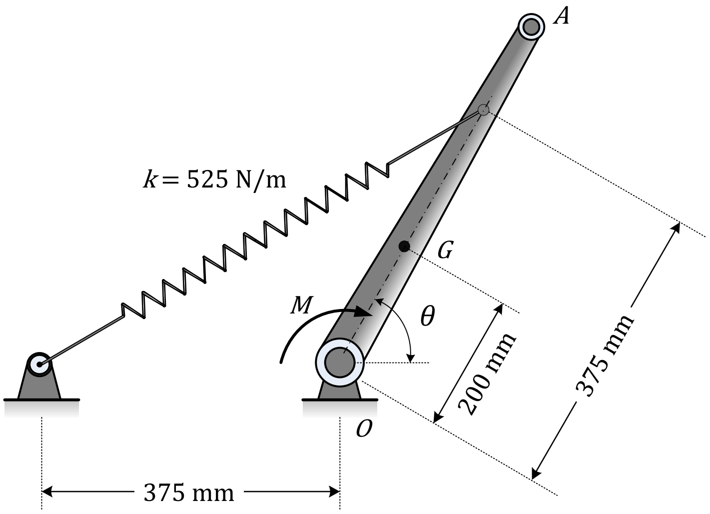
$$A_t = 105.7\ \mathrm{N}$$

0.5

Answers:	$A_x = 0$	$A_y = 105.7\ \mathrm{N}$	$\alpha = 8.68\ \mathrm{rad/s^2}$

Question 3: (2 Marks)

The tapered 5.5 kg lever *OA* with mass moment of inertia of 0.344 kgm^2 about *O*, is initially at rest in the vertical position ($\theta = 90°$), where the attached spring of stiffness $k = 525$ N/m is unstretched. A constant moment *M* is applied to the lever at *O* that will give the lever an angular velocity $\omega = 4$ rad/s as the lever reaches the horizontal position $\theta = 0$.



Solution:

(a) Calculate the change in elastic potential energy (V_e), gravitational potential energy (V_g) resulting from the lever moving from $\theta = 90°$ to $\theta = 0°$:

The change in elastic potential energy (V_e)

$$V_e = \frac{1}{2}k({x_2}^2 - {x_1}^2)$$

Where $x_1 = 0$ (since the spring is unstretched initially), and

$$x_2 = (375 + 375) - \sqrt{(375)^2 + (375)^2}$$

$$x_2 = 220 \text{ mm} = 0.22 \text{ m}$$

Therefore,

$$V_e = \frac{1}{2}(525)(0.22)^2$$

$$\boxed{0.5}$$

$$V_e = 12.705 \text{ J}$$

The change in gravitational potential energy (V_g)

$$V_g = mg(h_2 - h_1)$$

$$V_g = (5.5)(9.81)(0 - 0.2)$$

$$\boxed{0.25}$$

$$V_g = -10.791 \text{ J}$$

(b) Calculate the total kinetic energy T when the lever reaches $\theta = 0°$ and calculate the constant moment M required to achieve the final angular velocity of $\omega = 4$ rad/s :

The change in kinetic energy (T)

$$T = \frac{1}{2}m({v_2}^2 - {v_1}^2) + \frac{1}{2}I_G({\omega_2}^2 - {\omega_1}^2)$$

Where

$$I_G = I_0 - md^2 = 0.344 - (5.5)(0.2)^2 = 0.124 \text{ kg. m}^2$$

$$v_1 = 0 \text{ (since starts from rest)}$$

$$\omega_1 = 0$$

$$v_2 = \omega_2 \overline{OG} = 4(0.2) = 0.8 \text{ m/s}$$

Therefore

$$T = \frac{1}{2}(5.5)(0.8^2 - 0) + \frac{1}{2}(0.124)(4^2 - 0)$$

$$T = 2.75 \text{ J}$$

0.75

Work done

$$W_{1-2} = F.s + M.\theta$$

0.25

$$W_{1-2} = 0 + M.\left(\frac{\pi}{2}\right) = 1.571M \text{ J}$$

Using work energy equation

$$1.571\, M = 2.75 - 10.791 + 12.705$$

$$M = 2.97 \text{ N. m}$$

0.25

Answers:	$V_e = 12.705$ J	$V_g = -10.791$ J	$T = 2.752$ J	$M = 2.97$ N. m

Equation Sheet

<u>Linear motion</u>

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt} \qquad v\,dv = a\,ds$$

Constant linear acceleration equations $(t_o = 0)$

$$v = v_o + at \qquad v^2 = v_o^2 + 2a(s - s_o) \qquad s = s_o + v_o t + \frac{1}{2}at^2$$

Angular motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} \qquad \omega\,d\omega = \alpha\,d\theta$$

<u>Displacement, velocity and acceleration components</u>

Rectangular coordinates

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} \qquad \mathbf{v} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} \qquad \mathbf{a} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$

Normal and tangential coordinates

$$\mathbf{v} = v\mathbf{e_t} \qquad \mathbf{a} = a_t\mathbf{e_t} + a_n\mathbf{e_n} \qquad v = \omega r \qquad a_t = \dot{v} = \alpha r \qquad a_n = \frac{v^2}{\rho} = \omega^2 r$$

<u>Relative motion</u>

$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B} \qquad \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} \qquad \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

<u>Equation of motion (Newton's 2nd law)</u>

$$\sum \mathbf{F} = m\mathbf{a}$$

<u>Work-Energy</u>

$$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e \qquad W_{1-2} = F\Delta s \text{ and/or } M\Delta\theta$$

$$\Delta T = \frac{1}{2}m\left(v_2^2 - v_1^2\right) \text{ and/or } \frac{1}{2}I\left(\omega_2^2 - \omega_1^2\right)$$

$$\Delta V_g = mg(h_2 - h_1)$$

$$\Delta V_e = \frac{1}{2}k\left(x_2^2 - x_1^2\right) \quad \text{for a linear spring}$$

<u>For a rigid body in plane motion</u>

$$\sum \mathbf{F} = m\mathbf{a} \qquad \sum M = I\alpha$$

<u>Mass moment of inertia</u> $I = \int r^2 dm$

<u>Centroid of a cross-section</u>:

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\sum_i x_i A_i}{\sum_i A_i} \quad , \quad \bar{y} = \frac{\int y\,dA}{\int dA} = \frac{\sum_i y_i A_i}{\sum_i A_i}$$

<u>DATA:</u> Acceleration in free fall due to gravity g = 9.81 m/s^2

<u>Quadratic formula:</u>

For: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$