Student Name: Solution and Marking Guide

Student ID: Block Test 3

PSS Room/Demonstrator:

School of Mechanical and Manufacturing Engineering

MMAN1300 – ENGINEERING MECHANICS 1

2018 S1 Block Test 3

Instructions:

- Time allowed: 45 minutes
- Total number of questions: 3
- Answer all the questions in the test
- Answer all questions in the spaces provided
- The 6 marks allocations shown are worth 6% of the course overall
- Candidates may bring drawing instruments, rulers and UNSW approved calculators to the test
- Print your name, student ID and all other requested details above
- Record your answers (with appropriate units) in the ***ANSWER BOXES*** provided

Notes:

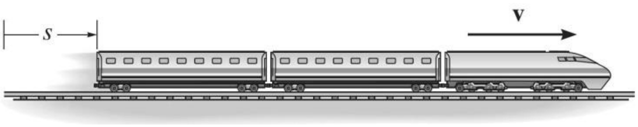
Your work must be complete, clear and logical

Do not skip steps, sign conventions, units and relevant diagrams and clearly state the final answers

No part of this paper is to be retained by candidates until handed back after marking

Question 1: (2 Marks)

Starting at $t = 0$ s, a train travels along a straight track at 2 m/s for 2 s, it then begins to accelerate at $a = (60v^{-4})\ \mathrm{m/s^2}$ where v is in m/s. Determine the following:



Solution:

(a) Determine the velocity (v) of the train 3 s after it started accelerating

$$a = \frac{dv}{dt}$$

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_2^v \frac{dv}{60v^{-4}}$$

$$t = \frac{1}{330}(v^5 - 32)$$

$$v = (300t + 32)^{\frac{1}{5}} \quad \text{---------------------------} \quad (1) \qquad \boxed{0.25}$$

At $t = 3\ s$

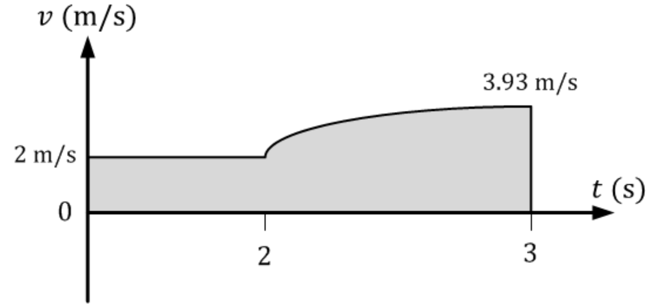
$$v = (300 \times 3 + 32)^{\frac{1}{5}}$$

$$v = 3.925\ \mathrm{m/s} \qquad \boxed{0.25}$$

From $0 \le t < 2\ s$ *the train moved with a constant velocity and from* $2 < t < 5\ s$ *the train moved with a velocity governed by Eq. (1)*

(b) On the axes provided, plot the $v - t$ graph for the train's motion for the interval $0 \le t \le 5$ s. (Cross the attempt you do not want to be marked)

$\boxed{0.5}$



(c) Determine the position (s) of the train 3 s after it started accelerating

$$ads = vdv$$

$$ds = \frac{vdv}{a}$$

$$ds = \frac{v^5 dv}{60}$$

$$\int_0^s ds = \frac{1}{60}\int_2^{3.925} v^5\, dv \qquad \boxed{0.5}$$

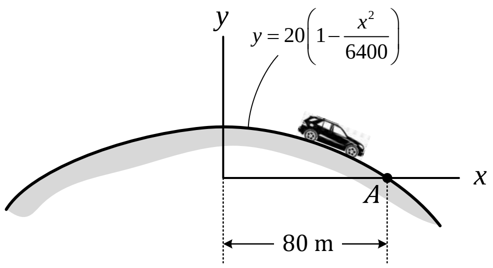
$$s = \frac{1}{60}\left(\frac{v^6}{6}\right)\Bigg]_2^{3.925}$$

$$s = 9.98\ \mathrm{m} \qquad \boxed{0.5}$$

Answers:	$v = 3.925\ \mathrm{m/s}$	$s = 9.98\ \mathrm{m}$

Question 2: (2 Marks)

The 800 kg car travels over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force (N) and the resultant frictional force (F_f) that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.



Solution:

Present your solution to Question - 2 here:

Hint1: the radius of curvature at any point along a curved path is given by: $\rho = \frac{\left[1+(dy/dx)^2\right]^{3/2}}{\left|d^2y/dx^2\right|}$

Hint 2: The slope at any given point on a curve is $\tan\theta = \frac{dy}{dx}$

Geometry:

$$\frac{dy}{dx} = -0.00625x$$

$$\frac{d^2y}{dx^2} = -0.00625 \qquad \boxed{0.25}$$

The slope θ at point A is given by:

$$\tan\theta = \left.\frac{dy}{dx}\right]_{x=80\text{ m}}$$

$$\tan\theta = -0.00625(80)$$

$$\theta = -26.57° \qquad \boxed{0.25}$$

And the radius of curvature of point A is

$$\rho = \frac{\left[1+(dy/dx)^2\right]^{3/2}}{\left|d^2y/dx^2\right|}$$

Continue your solution to Question - 2 here:

$$\rho = \frac{[1 + (-0.00625x)^2]^{3/2}}{|-0.00625|}\Bigg]_{x=80\text{ m}}$$

$\rho = 223.61\text{ m}$ 0.5

Equations of Motion:

Here $a_t = 0$

$\sum F_t = m\, a_t$

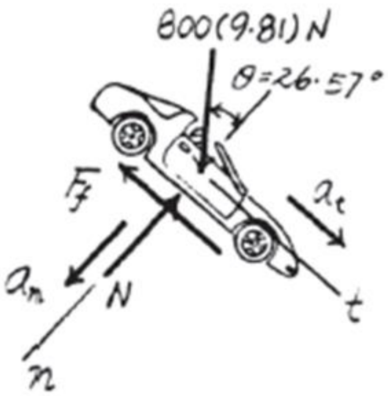
$(800)(9.81)\sin 26.57° - F_f = 800(0)$

$F_f = 3509.73\text{ N} = 3.51\text{ kN}$ 0.25

$\sum F_n = m\, a_n$

$(800)(9.81)\cos 26.57° - N = 800\left(\frac{9^2}{223.61}\right)$

$N = 6729.67\text{ N} = 6.73\text{ kN}$ 0.25

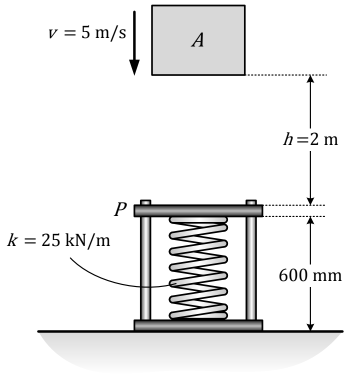


0.5

Answers:	$F_f = 3.51\text{ kN}$	$N = 6.73\text{ kN}$

Question 3: (2 Marks)

A 750-mm-long spring is compressed and confined by the plate P, which can slide freely along the vertical 600-mm-long rods. The 40-kg block is given a speed of $v = 5\ \text{m/s}$ when it is $h = 2\ \text{m}$ above the plate. Determine how far the plate moves downwards (y_{max}) when the block momentarily stops after striking it. Neglect the mass of the plate.



Solution:

Present your solution to Question 3 here

$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$ ---- *(1)*

Work done:

$W_{1-2} = 0$

Kinetic Energy: 0.25

$$\Delta T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2}(40)(5)^2$$

$\Delta T_1 = 500\ \text{J}$

$\Delta T_2 = 0$

Gravitational Potential Energy: 0.25

$(V_g)_1 = 0$

$(V_g)_2 = -(40)(9.81)(2 + y)$

$(V_g)_2 = -784.8 - 392.4y$

Continue your solution to Question 3 here

Spring Energy: 0.5

$$(V_e)_1 = \frac{1}{2}ks_1^2 = \frac{1}{2}(25 \times 10^3)(0.75 - 0.6)^2$$

$$(V_e)_1 = 281.25 \text{ J}$$

$$(V_e)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(25 \times 10^3)(0.15 + y)^2$$

Work-energy equation: 0.5

$$0 = 0 - 500 + (-784.8 - 392.4y - 0) + \frac{1}{2}(25 \times 10^3)(0.15 + y)^2 - 281.25$$

$$0 = -1566.05 - 392.4y + 12500(y^2 + 0.3y + 0.0225)$$

$$0 = -1284.8 + 3357.6y + 12500y^2$$

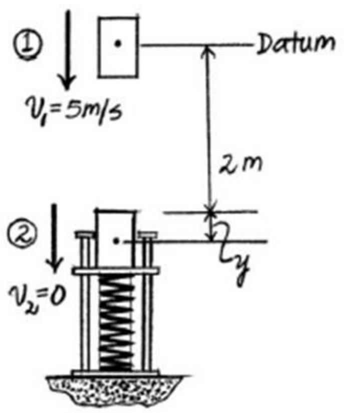
$$12500y^2 + 3357.6y - 1284.8 = 0$$

Solving for the positive root of the equation

$$y_{max} = 0.2133 \text{ m}$$

$$y_{max} = 213.3 \text{ mm}$$ 0.25

0.25



Answers:	$y_{max} = 213.3$ mm

Equation Sheet

Linear motion

$v = \frac{ds}{dt}$ $\quad a = \frac{dv}{dt}$ $\quad v\,dv = a\,ds$

Constant linear acceleration equations $(t_o = 0)$

$v = v_o + at$ $\quad v^2 = v_o^2 + 2a(s - s_o)$ $\quad s = s_o + v_o t + \frac{1}{2}at^2$

Angular motion

$\omega = \frac{d\theta}{dt}$ $\quad \alpha = \frac{d\omega}{dt}$ $\quad \omega\,d\omega = \alpha\,d\theta$

Displacement, velocity and acceleration components

Rectangular coordinates

$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ $\quad \mathbf{v} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$ $\quad \mathbf{a} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$

Normal and tangential coordinates

$\mathbf{v} = v\mathbf{e_t}$ $\quad \mathbf{a} = a_t\mathbf{e_t} + a_n\mathbf{e_n}$ $\quad v = \omega r$ $\quad a_t = \dot{v} = \alpha r$ $\quad a_n = \frac{v^2}{\rho} = \omega^2 r$

Relative motion

$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$ $\quad \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$ $\quad \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$

Equation of motion (Newton's 2nd law)

$\sum \mathbf{F} = m\mathbf{a}$

Work-Energy

$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$ $\quad W_{1-2} = F\Delta s$ and/or $M\Delta\theta$

$\Delta T = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$ and/or $\frac{1}{2}I\left(\omega_2^2 - \omega_1^2\right)$

$\Delta V_g = mg(h_2 - h_1)$

$\Delta V_e = \frac{1}{2}k\left(x_2^2 - x_1^2\right)$ for a linear spring

For a rigid body in plane motion

$\sum \mathbf{F} = m\mathbf{a}$ $\quad \sum M = I\alpha$

Mass moment of inertia $\quad I = \int r^2 dm$

Centroid of a cross-section:

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\sum x_i A_i}{\sum A_i}, \qquad \bar{y} = \frac{\int y\,dA}{\int dA} = \frac{\sum y_i A_i}{\sum A_i}$$

DATA: Acceleration in free fall due to gravity g = 9.81 m/s^2

Quadratic formula:

For: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$