MMAN1300
Shear Force and Bending Moment Lab Report

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PSS Group: Friday / 13:00 / Quadrangle G031

Abstract

This document is a lab report on the shear force and bending moment experiments performed in Willis Annex during Week 5. The report will consist of a shear force analysis of experiments 1 and 2 (for SF) and a bending moment analysis of experiments 1 and 2 (for BM). The analysis compares and discusses the theoretical and experimental values of the shear forces and bending moments.

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1. Summary

Beams are long members of structures that are assumed to be able to withstand shear deformation, thus are capable of producing bending moments within the structure. This lab report will present four experiments: experiments 1 and 2 for shear force analysis and experiments 1 and 2 for bending moment analysis. The purpose of these experiments are to examine how the values of shear forces (SF) and bending moments (BM) are affected by a varying point load and how SF and BM vary on the beam. The nature of the experimental values will be discussed with the theoretical values with discrepancies and sources of error.

Table 1. Calculation of load and dimensions

W_1 (N)	W_2(N)	a (mm)	b (mm)
0.5×9.81	0.4×9.81	100	340

The variables W_1 , W_2 , a and b *(from table 1 above)* were determined by using the student ID, z5208741, and the legend provided in *"Lab1 – Shear Force and Bending Moment" (refer to Appendix A).* These variables may be used in the following experiments for the calculation of theoretical shear force and bending moment values.

2. Shear Force Analysis

Shear forces are the internal forces of a structure caused by an applied force and act in the direction along the transverse plane to the cross-section of a structure, pushing a part of the body in one direction and another part of the body in the opposite direction *(refer to Appendix B)* [1]. The shear force analysis will calculate and compare experimental and theoretical shear force values. The analysis will also discuss how the shear force varies with point load and location on the beam, and will feature two experiments:

- Experiment 1: Shear Force Variation with an Increasing Point Load
- Experiment 2: Shear Force Variation Away from the Point of Loading

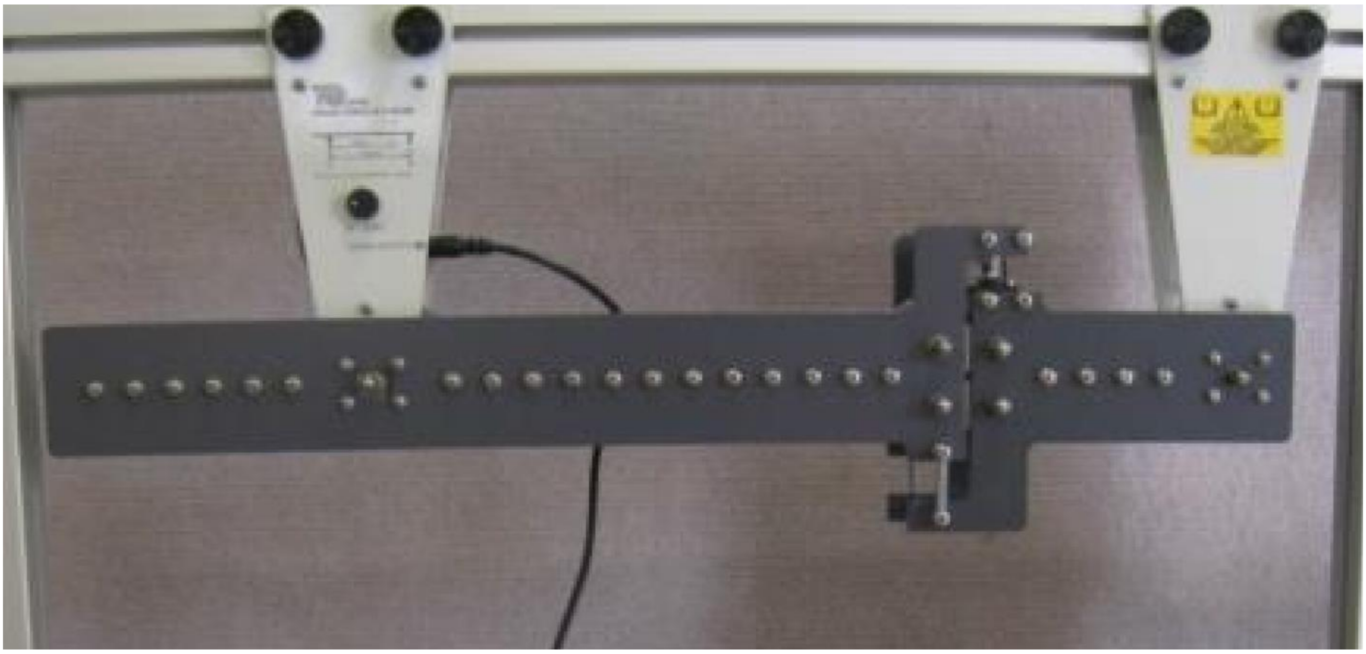


Figure 1. Shear Force Analysis Apparatus *(Source: MMAN1300 Lab 1 - Shear Force and Bending Moment)*

The apparatus *(refer to figure 1)* used for both experiments consisted of a beam with a cut, where a load cell at the cut is read to a digital force display. The digital force display reads (to one decimal place) the experimental shear force applied at the load cell.

2.1. Experiment 1: Shear Force Variation with an Increasing Point Load.

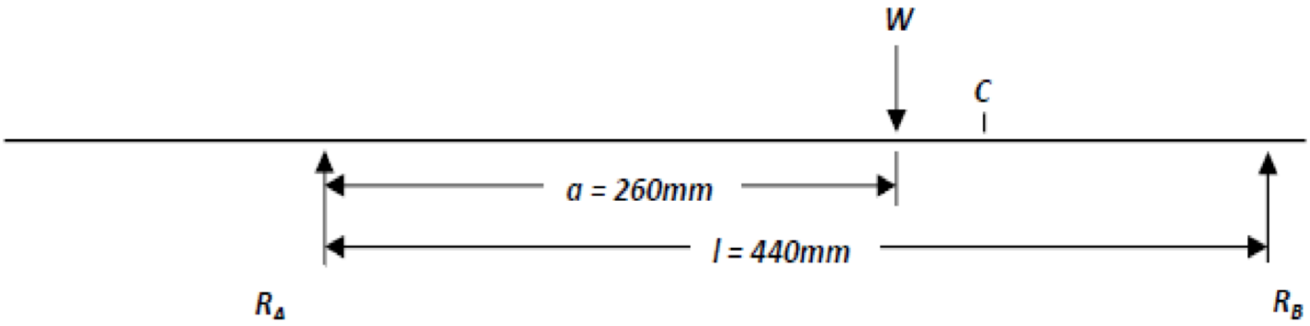


Figure 2. Experiment 1 (SF) Diagram *(Source: MMAN1300 Lab 1 – Shear Force and Bending Moment)*

The aim of experiment 1 was to determine how the shear force at the cut varied with an increasing point load located 300 mm to the right of the left support (i.e. 40 mm left of the cut). The point load (vector W in figure 2) is supplied by the weight of a hanging mass where its mass is increased in increments of 100 g until 500 g. The experimental shear force value was read from the digital force display and recorded in table 2 with each increment of mass.

The load was calculated using the following equation for weight force:

$$W = m \times g \ (N)$$

Where:

- W is the applied load
- m is the mass of the point load
- g is the acceleration due to gravity ($g = 9.81 \ ms^{-2} \downarrow$)

Given the values of a and l from figure 2, the theoretical shear force value can be calculated using the following equation:

$$S_T = \frac{Wa}{l} \ (N)$$

Where:

- S_T is the theoretical shear force value at the cut
- a is the distance between the left support and point of applied load *(from figure 1)*
- l is the distance between both supports

To determine the percentage error (denoted by S_{error}) between the theoretical and experimental values, the following equation is used:

$$S_{error} = \left|\frac{S_E - S_T}{S_T}\right| \times 100$$

Working Out: [↓ +]

$m = 100(10^{-3})\ kg$

$a = 260\ mm$

$l = 440\ mm$

Calculating Load:

$$\begin{aligned} W &= m \times g \\ &= 100(10^{-3}) \times 9.81 \\ &= 0.981\ N \downarrow \end{aligned}$$

Substituting Load and Calculating Theoretical SF:

$$\begin{aligned} S_T &= \frac{Wa}{l} \\ &= \frac{0.981 \times 260}{440} \\ &= 0.580\ N \downarrow \end{aligned}$$

Calculating Percentage Error in SF [2]:

$$\begin{aligned} S_{error} &= \left|\frac{S_E - S_T}{S_T}\right| \times 100 \\ &= \left|\frac{0.5 - 0.580}{0.580}\right| \times 100 \\ &= 13.8\ \% \end{aligned}$$

Following the same method of working out for masses up to 500 g – the load, theoretical SF, and percentage error columns of table 2 were filled out.

Table 2. Results for Experiment 1 (SF)

Mass (g)	**Load** (N)	**Experimental SF** (N)	**Theoretical SF** (N)	**Error** (%)
0	0.000	0.0	0.000	0.000
100	0.981	0.5	0.580	13.8
200	1.96	1.1	1.16	5.17
300	2.94	1.6	1.74	8.05
400	3.92	2.2	2.32	5.17
500	4.91	2.8	2.90	3.45

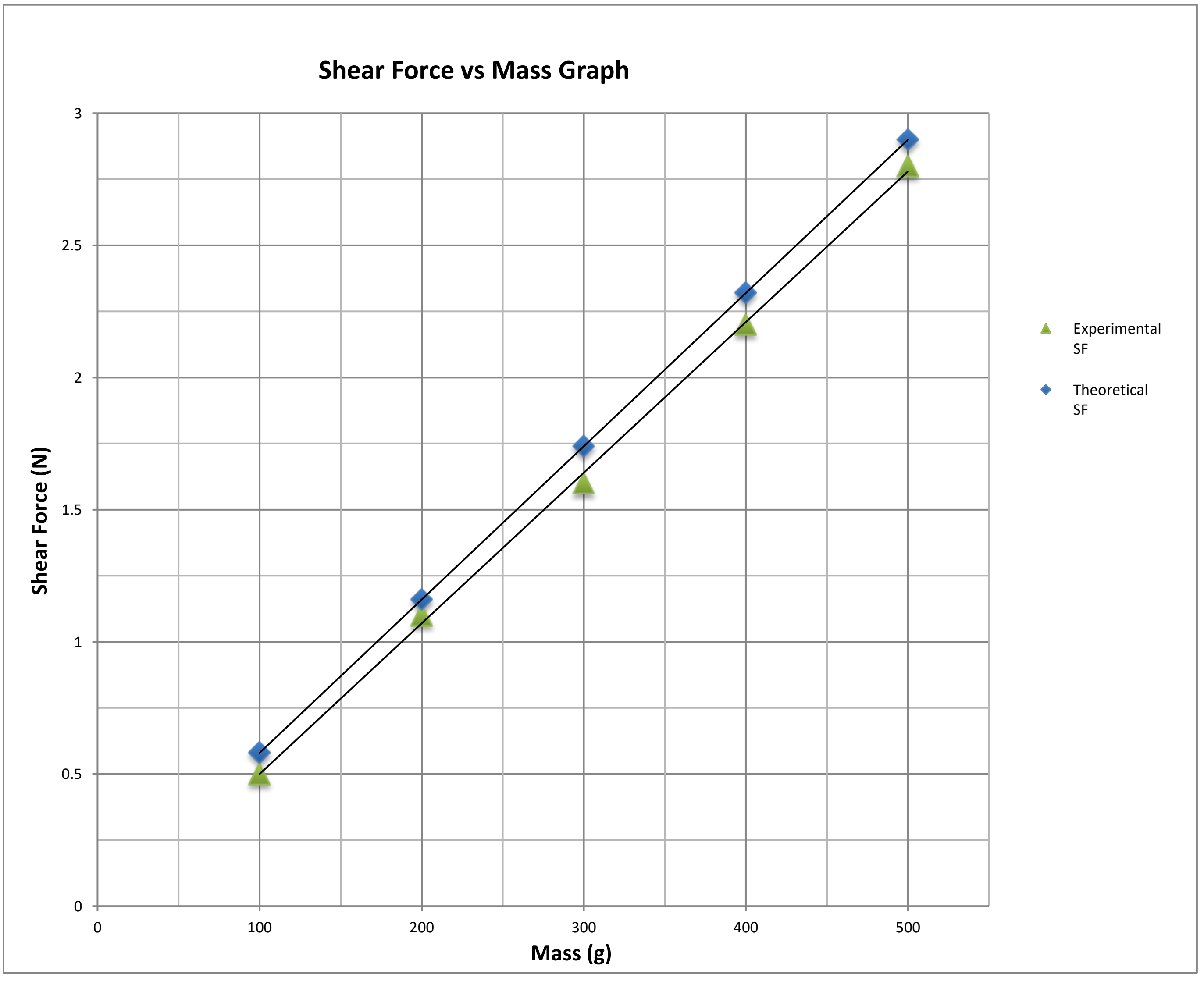


Figure 3. Shear Force Variation with Mass (Theoretical versus experimental)

From table 2, the experimental and theoretical shear force values can be plotted against the mass of the point load in figure 3 with a line of best fit for each data set to show the relationship between data elements.

2.2. Discussion of Results from Experiment 1

By inspection of figure 3, there existed a trend in both theoretical and experimental data where the lines of best fit displayed direct proportionality between the mass and shear force – therefore as the mass of the applied load increased, the shear force increased.

The gradient of lines of best fit of both data sets are similar, however the line of best fit for experimental data was below the theoretical line of best fit. This consistent error may be due to assuming the beam had no mass which shifted the experimental data set.

From table 2, the percentage error between the experimental and theoretical values was consistently below 10% disregarding one data value. By judgement of values, the experimental and theoretical values had less than one decimal place discrepancy. Therefore the experimental data was fairly accurate but was affected by sources of error that may be:

- Accuracy of experimental shear force limited to one decimal place
- Accuracy in mass only known to 10 g
- Pin joints between supports and beam were not frictionless
- Beam assumed to be weightless

2.3. Experiment 2: Shear Force Variation Away from the Point of Loading

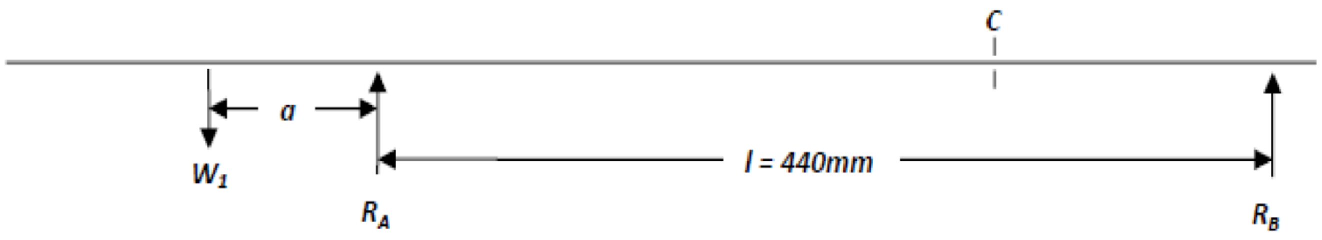


Figure 4. Experiment 2 (SF) Diagram *(Source: MMAN1300 Lab 1 – Shear Force and Bending Moment)*

The aim of experiment 2 was to determine how the shear force varied by location on the beam. As depicted in figure 4, the load was applied a distance a *(variable from table 1)* from R_A. The support reactions, R_A and R_B, both respectively acting at the left and right supports on figure 2 were calculated in order to calculate the theoretical shear force acting at the cut.

Working Out: [↑ +]

$W_1 = 0.5 \times 9.81 = 4.905\ N \downarrow = -4.905\ \text{N} \uparrow$

$a = 100\ mm$

$l = 440\ mm$

A free body diagram of the whole beam (neglecting the cut) is drawn *(refer to figure 5 below).* The supports holding the beam are pin joints and it can be assumed that there are no horizontal reaction forces acting since there are no applied forces in the x-axis.

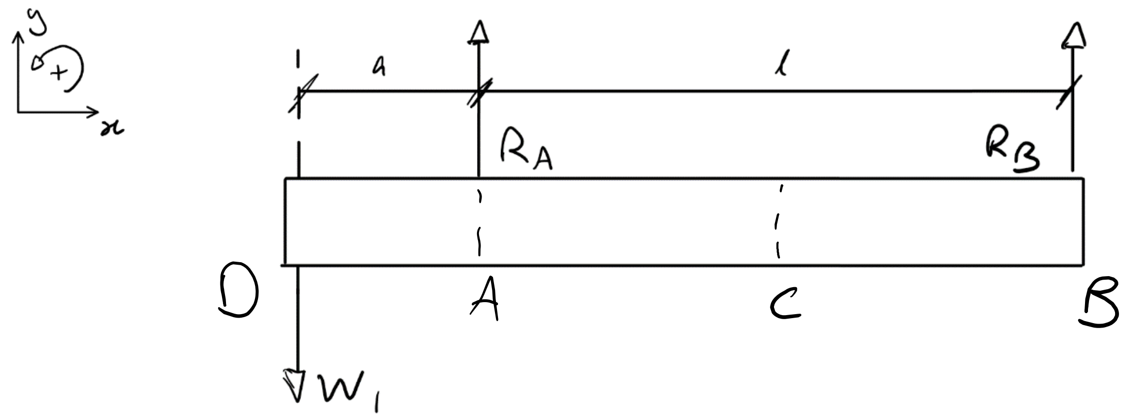


Figure 5. Free Body Diagram 1 for Experiment 2 (SF)

To calculate the unknown reaction forces:

$$\sum M_A = 0$$

$$W_1(a) + R_B(l) = 0$$

$$4.905(100) + R_B(440) = 0$$

$$R_B = -\frac{981}{880}$$

$$\therefore R_B = -1.11\ N \uparrow = 1.11\ N \downarrow$$

$$\sum \overrightarrow{F_y} = 0$$

$$-W_1 + R_A + R_B = 0$$

$$-4.905 + R_A - \frac{981}{880} = 0$$

$$R_A = \frac{26487}{4400}$$

$$\therefore R_A = 6.02\ N \uparrow$$

Consider the free body diagram of the beam left of the cut *(figure 6 below)*. The theoretical shear force, S_T, is acting at the cut and there also exists a moment, M, at the cut which can be ignored for this experiment.

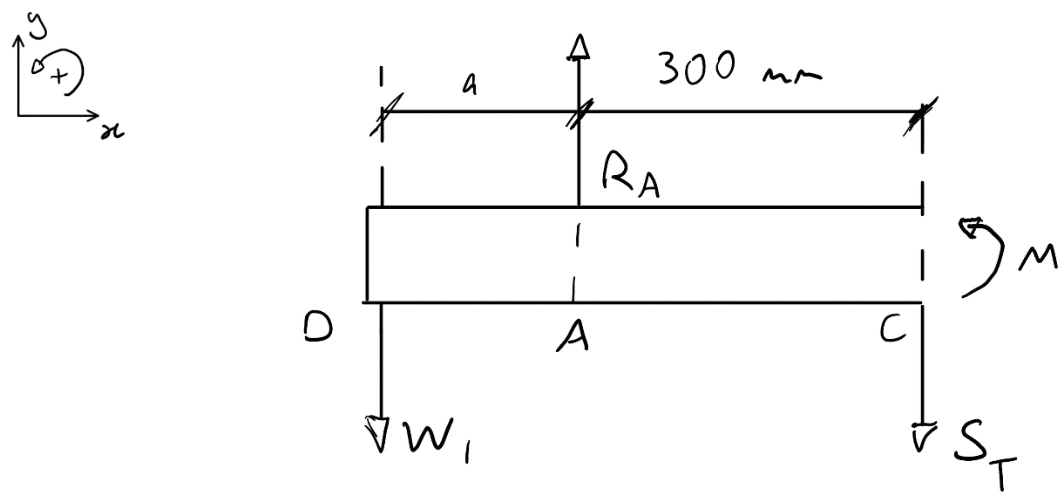


Figure 6. Free Body Diagram of Cut Beam for Experiment 2 (SF)

Calculating Theoretical SF:

$$\sum \overrightarrow{F_y} = 0$$

$$-W_1 + R_A - S_T = 0$$

$$-4.905 + \frac{26487}{4400} - S_T = 0$$

$$\therefore S_T = -1.11\ N \uparrow = 1.11\ N \downarrow$$

The support reactions and shear forces are then recorded in table 3.

Table 3. Results for Experiment 2 (SF)

$\boldsymbol{W_1}$ **(N)**	**Experimental SF (N)**	$\boldsymbol{R_A}$ **(N)**	$\boldsymbol{R_B}$ **(N)**	**Theoretical SF (N)**
-4.91	-1.1	6.02	-1.11	-1.11

Since the shear force is taken on the right side of the cut beam *(from figure 6)*, downwards is positive by shear force sign convention *(refer to Appendix C)* thus positive on the shear force diagram *(figure 7 below)*.

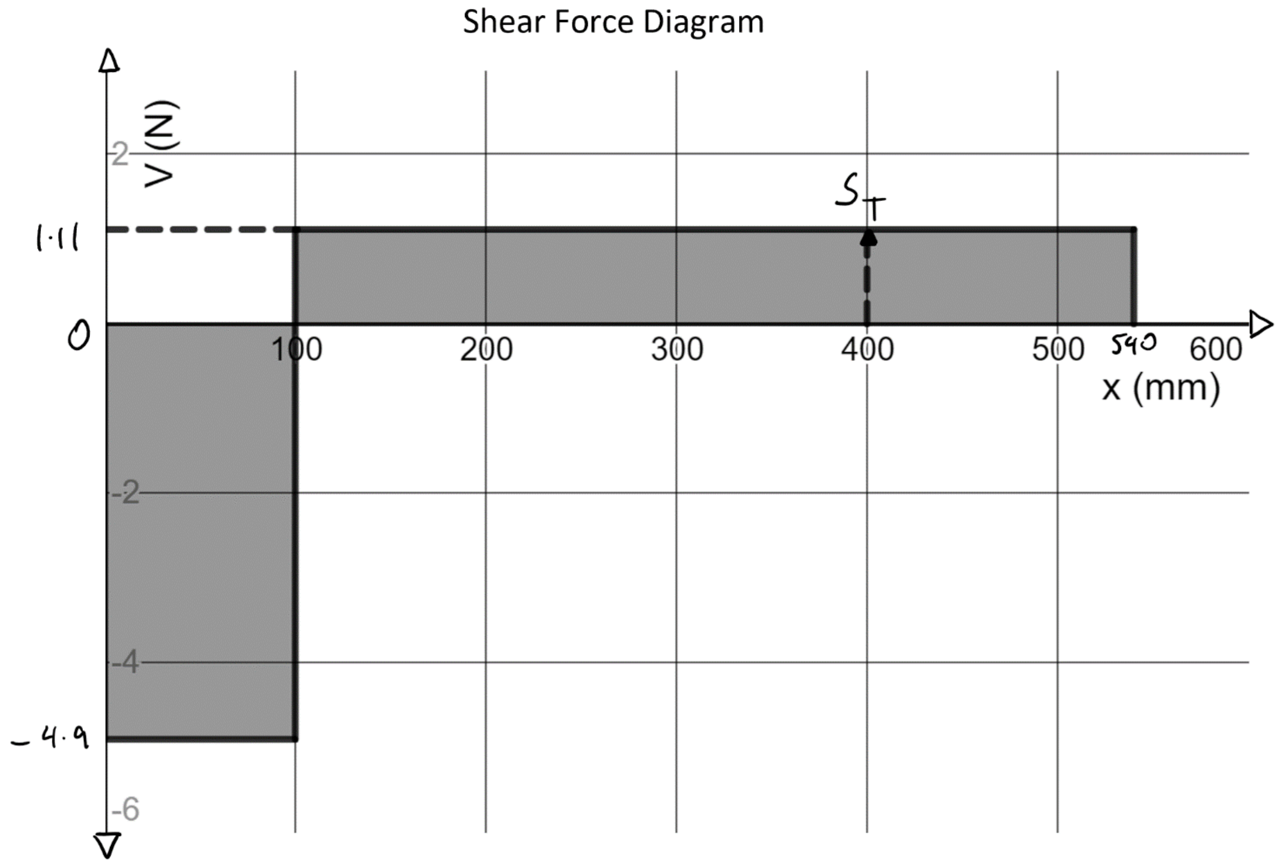


Figure 7. Shear Force Diagram for Experiment 2 (SF) *(computer generated by desmos & edited).*

2.4. *Discussion of Results from Experiment 2*

From figure 7, the shear force will vary from the point of loading only if there exists an external force acting on the beam. At $x = 0$, the point load of $-4.9\ N$ brings the shear force down to a constant $-4.9\ N$ on $0 \leq x < 100$ until a reaction force of 6.02 N at $x = 100\ mm$ acts on the beam bringing the shear force up to 1.1 N. This is the point most likely to have shear deformation due to two shear forces with opposite sense approaching $x = 100\ mm$. For $100 < x \leq 540$, shear force is $1.1\ N$ until a reaction force at $x = 540\ mm$ with value $-1.1\ N$ brings the shear force back to zero. Note that the beam is in equilibrium since the shear force began and ended at zero.

Theoretical and experimental shear force to the least number of significant figures (i.e. two significant figures) showed no discrepancy and can be considered precise to two significant figures:

$$S_T = S_E = 1.1\ N \uparrow$$

Possible sources of errors however are as listed in *'2.2 Discussion of Results from Experiment 1'*.

3. Bending Moment Analysis

Since beams are assumed to have no shear deformation, shear forces will therefore produce internal bending moments [3]. Applied moments can also affect the internal bending moment of the beam. The bending moment analysis will calculate and compare experimental and theoretical bending moment values, and discuss how the bending moment will vary with respect to the following experiments:

- Experiment 1: Bending Moment Variation with an Increasing Point Load
- Experiment 2: Bending Moment Variation Away from the Point of Loading

Figure 8. Bending Moment Analysis Apparatus *(Source: MMAN1300 Lab 1 - Shear Force and Bending Moment)*

The apparatus in figure 8 was used for both experiments in the bending moment analysis and is similar to the setup of the apparatus for shear force analysis.

3.1. Experiment 1: Bending Moment Variation with an Increasing Point Load.

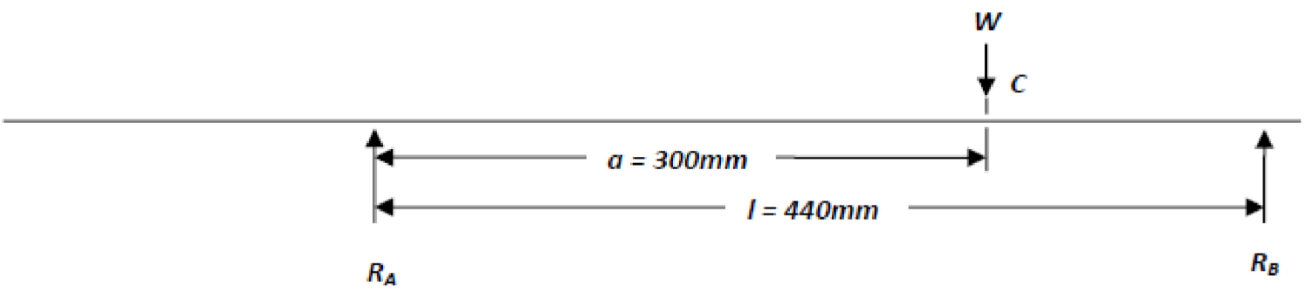


Figure 9. Experiment 1 (BM) Diagram *(Source: MMAN1300 Lab 1 - Shear Force and Bending Moment)*

The aim of experiment 1 for bending moment analysis is to determine the effect that an increasing point load will have on the bending moment within the beam. The point load is placed at the cut *(figure 9)* which is 300 mm from the left support.

The theoretical bending moment, M_T, at the cut can be calculated using the given equation:

$$M_T = \frac{Wa(l-a)}{l} \ (Nm)$$

The experimental bending moment, M_E, at the cut can be calculated using the given equation:

$$M_E = F_D \times 0.125 \ (Nm)$$

Where:

- F_D is the force read from the digital force display

Working Out: [↓ +] **[**↻ +]

$W = 0.981 \ N \downarrow$

$a = 300 \times 10^{-3} \ m$

$l = 440 \times 10^{-3} \ m$

$F_D = 0.6 \ N \downarrow$

Calculating Theoretical BM:

$$\begin{aligned} M_T &= \frac{Wa(l-a)}{l} \\ &= \frac{0.981 \times 300(440-300) \times 10^{-3}}{440} \\ &= 0.0936 \ Nm \ ↻ \end{aligned}$$

Calculating Experimental BM:

$$\begin{aligned} M_E &= F_D \times 0.125 \\ &= 0.6 \times 0.125 \\ &= 0.075 \ Nm \ ↻ \end{aligned}$$

Calculating Percentage Error in BM:

$$\begin{aligned} M_{error} &= \left|\frac{M_E - M_T}{M_T}\right| \times 100 \\ &= \left|\frac{0.075 - 0.0936}{0.0936}\right| \times 100 \\ &= 19.9 \ \% \end{aligned}$$

Following this working out the theoretical and experimental bending moment columns of table 4 *(below)* was filled out for each increment of mass.

Table 4. Results for Experiment 1 (BM)

Mass (g)	Load (N)	Display Force (N)	Experimental BM (N. m)	Theoretical BM (N. m)	Error (%)
0	0.000	0.0	0.000	0.000	0.000
100	0.981	0.6	0.075	0.0936	19.9
200	1.96	1.3	0.163	0.187	12.8
300	2.94	2.0	0.250	0.281	11.0
400	3.92	2.8	0.350	0.375	6.67
500	4.91	3.6	0.450	0.468	3.85

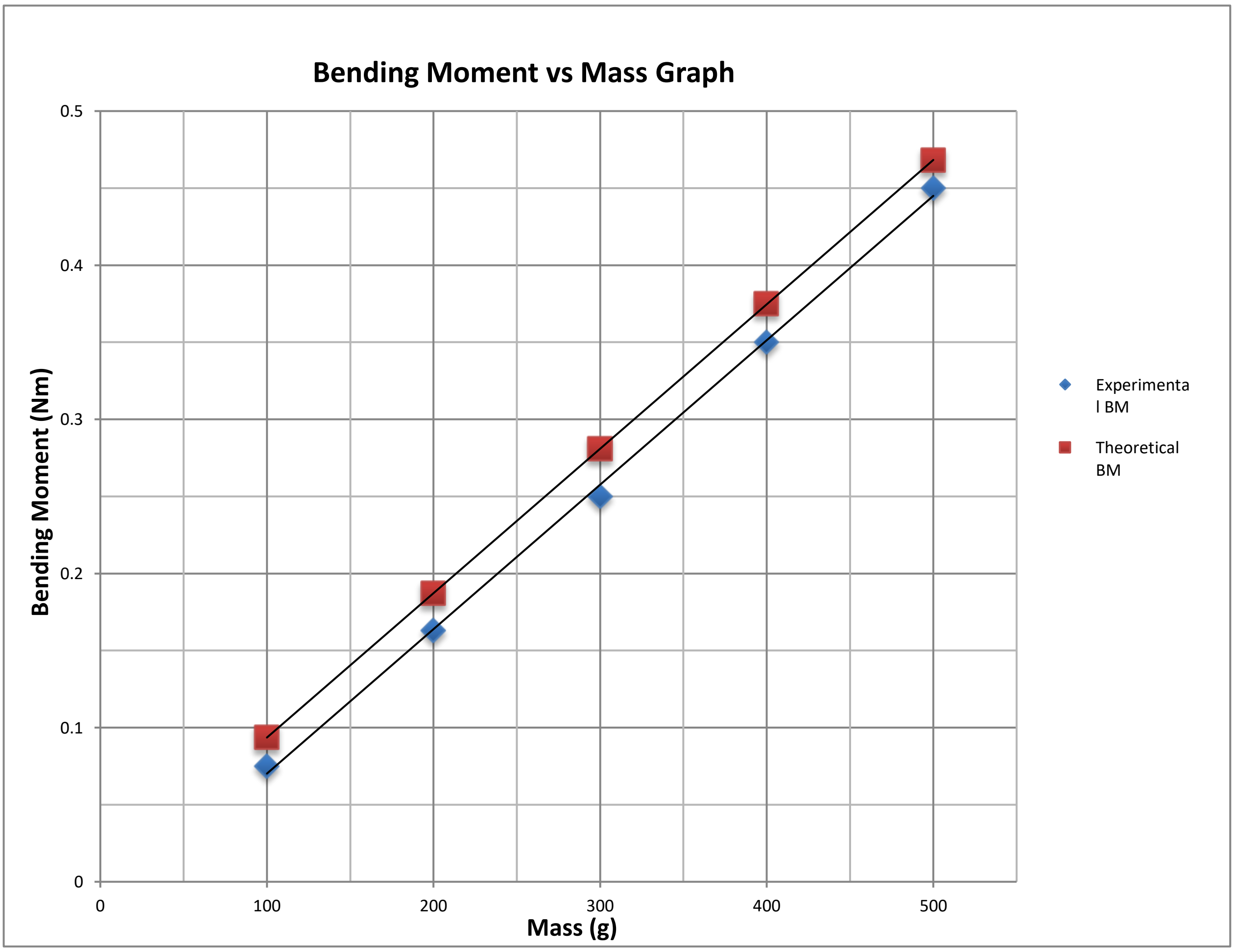


Figure 10. Bending Moment Variation with Mass (Theoretical versus Experimental)

From table 4, the experimental and theoretical bending moments are plotted against the mass of the point load in figure 10.

3.2. Discussion of Results from Experiment 1

By inspection of figure 10, there is a linear relationship between the bending moment and mass of point load. Therefore bending moment will increase as point load increases. The line of best fit of the experimental data is similar in gradient but below the line of best fit for theoretical data. This error may be due to the assumption that the beam had no mass, therefore translating the whole set of experimental data to the right across the axis of mass.

By inspection of table 4, due to the decreasing trend in percentage error as mass of the point load increased, it can be deduced that friction in the pin joint between the supports and beam largely affected the discrepancy of the experimental value. This is because larger masses have a greater weight which will overcome the frictional forces in the pin joint.

3.3. Experiment 2: Bending Moment Variation Away from the Point of Loading

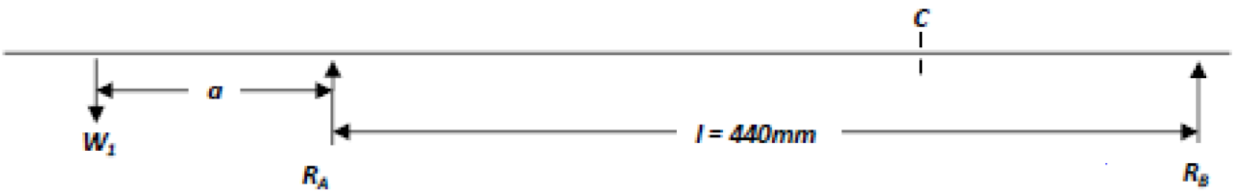


Figure 11. Experiment 2 (BM) Diagram *(Source: MMAN1300 Lab 1 - Shear Force and Bending Moment)*

The aim of experiment 2 was to determine how the bending moment varied with location on the beam. Figure 11 shows the point load applied at vector W_1 that is distance a *(variable from table 1)* away from R_A. Since the system in figure 11 is identical to figure 4 from experiment 2 of the shear force analysis, it is assumed the reaction forces and shear forces will be the same, where: $R_A = 6.02\ N$ and $R_B = -1.11\ N$.

The experimental bending moment, M_E, at the cut can be calculated using the given equation:

$$M_E = F_D \times 0.125\ (Nm)$$

Working Out: $[\uparrow +]\ [\circlearrowleft +]$

$R_A = 6.02\ N \uparrow$

$R_B = -1.11\ N \uparrow$

$W_1 = 0.5 \times 9.81 = 4.905\ N \downarrow = -4.905\ N \uparrow$

$a = 100\ mm$

$l = 440\ mm$

$F_D = -1.2\ N \downarrow = 1.2\ N \uparrow$

Considering the free body diagram of the beam to the right of the cut *(figure 12 below)*, there exists a shear force, S, which can be ignored, and there exists a theoretical bending moment, M_T, that is unknown.

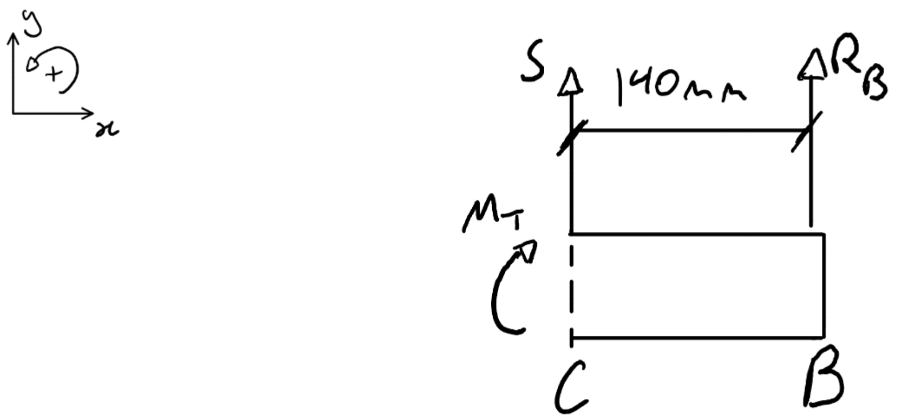


Figure 12. Free Body Diagram (computer generated).

Calculating Theoretical BM:

$$\sum M_C = 0$$

$$-M_T + R_B(140 \times 10^{-3}) = 0$$

$$-M_T - 1.11(140 \times 10^{-3}) = 0$$

$$\therefore M_T = 0.155\ Nm \circlearrowleft$$

Calculating Experimental BM:

$$M_E = F_D \times 0.125$$
$$= 1.2 \times 0.125$$
$$= 0.150\ Nm \ \circlearrowleft$$

Calculating Percentage Error in BM:

$$M_{error} = \left|\frac{M_E - M_T}{M_T}\right| \times 100$$
$$= \left|\frac{0.150 - 0.155}{0.155}\right| \times 100$$
$$= 3.23\ \%$$

However since the bending moments were taken from the left side of the block, due to bending moment sign convention *(refer to Appendix C)*, bending moments will be recorded as negative in table 5.

Table 5. Results for Experiment 2 (BM)

W_1 (N)	Display Force (N)	Experimental BM (Nm)	R_A (N)	R_B (N)	Theoretical BM (Nm)	Error (%)
-4.91	1.2	-0.150	6.02	-1.11	-0.155	3.23

To determine the bending moment diagram, consider the piecewise of the SFD from figure 7:

$$V = \begin{bmatrix} V_1 & for\ 0 \le x \le 100 \\ V_2 & for\ 100 \le x \le 540 \end{bmatrix}$$

Where,

$$V_1 = -4.905$$

$$V_2 = \frac{981}{880} \approx 1.11$$

Integrating with respect to x to obtain the BM equations:

$$M_1 = -4.905x + C_1$$

$$M_2 = \frac{981}{880}x + C_2$$

At $x = 0$, $M_1 = 0$:

$$\therefore C_1 = 0$$

At $x = 540$, $M_2 = 0$:

$$\therefore C_2 = -\frac{26487}{44} \approx -602$$

Therefore, the piecewise of the BMD is:

$$M = \begin{bmatrix} -4.905x & for\ 0 \le x \le 100 \\ \frac{981}{880}x - \frac{26487}{44} & for\ 100 \le x \le 540 \end{bmatrix}$$

The piecewise function for the bending moment diagram is graphed in figure 14 below.

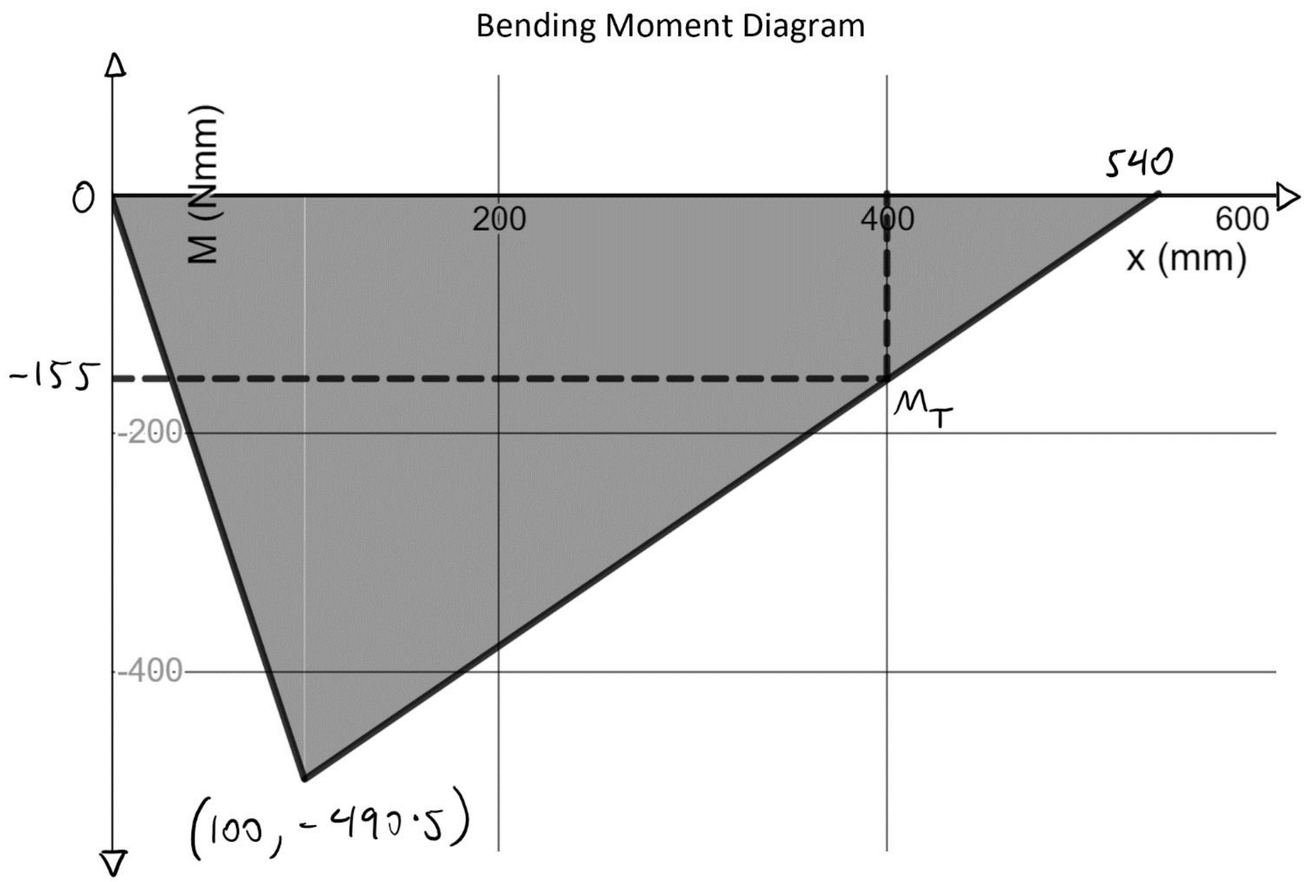


Figure 14. Bending Moment Diagram for Experiment 2 (BM) *(computer generated by desmos & edited).*

3.4. Discussion of Results from Experiment 2

From figure 14, the bending moment will vary with respect to the shear forces acting on the beam – the area under the curve of the shear force diagram is the bending moment. Since the shear force within $0 \leq x < 100$ was a negative constant, the bending moment will decrease at a constant rate from zero until it reaches a global minimum of $-490.5\ Nm$ at $x = 100\ mm$. The shear force within $100 < x \leq 540$ is a positive constant, therefore the bending moment will increase at a constant rate until it is zero at $x = 540$. Note that the bending moment will always begin and end at zero for a length of beam because the beam is in equilibrium.

The theoretical and experimental bending moment had a percentage error of 3.23 % therefore the experimental value of bending moment is fairly accurate. Possible sources of errors may be due to the assumption that the mass of the beam was zero, there was friction in the pin joints, the digital force display was only accurate to one decimal place, and the mass in grams was only accurate to the tens place.

4. Conclusion

Shear forces are internal forces of a beam caused by applied or reaction (external) forces on that beam. Shear forces and applied moments can result in bending moments within the beam since beams are assumed to not be shear deformable.

Experiments 1 for shear force analysis and bending moment analysis respectively demonstrated that the shear force and bending moment within the beam would increase with an increasing point load. This was determined through inspecting the shear force vs mass graph *(figure 3)* and bending moment vs mass graph *(figure 10)* and noting that the experimental and theoretical shear force values both increased at a linear rate and were easily satisfied by lines of best fit.

Experiment 2 for shear force and bending moment analysis demonstrated that the shear force will vary only at the point of where the external force is applied on the beam, and that the bending moment will vary with respect to the area under the curve of the shear force diagram. This was shown on the shear force diagram *(figure 7)* and bending moment diagram respectively (figure 14).

5. References

[1] Unknown. Shear Force. [Internet]. Available at:
https://www.corrosionpedia.com/definition/1596/shear-force [Accessed 11/09/2018]

[2] University of Iowa. Percent Error Formula. [Internet]. Available at:
http://astro.physics.uiowa.edu/ITU/glossary/percent-error-formula/ [Accessed 12/09/2018]

[3] Unknown. The Bernoulli-Euler Beam Theory.[Internet]. Available at:
http://www.learnaboutstructures.com/Bernoulli-Euler-Beam-Theory [13/09/2018]

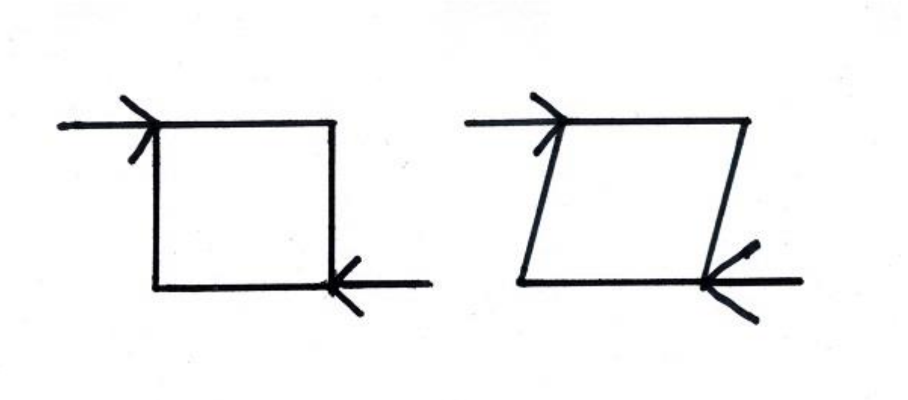
6. Appendix

Appendix A – Load Selection Variables Based on Student Number

Load W_1 (for Exp. 2, 3, 4)	**Load W_2 (for Exp. 3, 4)**	**Distance a (for Exp. 2, 3, 4)**	**Distance b (for Exp. 3)**	**Distance b (for Exp. 4)**
100g, if d=0, 1	100g, if e=0, 1	60mm, if f=0, 1	180mm, if g=0, 1	340mm, if g=0, 1
200g, if d=2, 3	200g, if e=2, 3	80mm, if f=2, 3	200mm, if g=2, 3	340mm, if g=2, 3
300g, if d=4, 5	300g, if e=4, 5	100mm, if f=4, 5	220mm, if g=4, 5	360mm, if g=4, 5
400g, if d=6, 7	400g, if e=6, 7	120mm, if f=6, 7	240mm, if g=6, 7	380mm, if g=6, 7
500g, if d=8, 9	500g, if e=8, 9	140mm, if f=8, 9	260mm, if g=8, 9	400mm, if g=8, 9

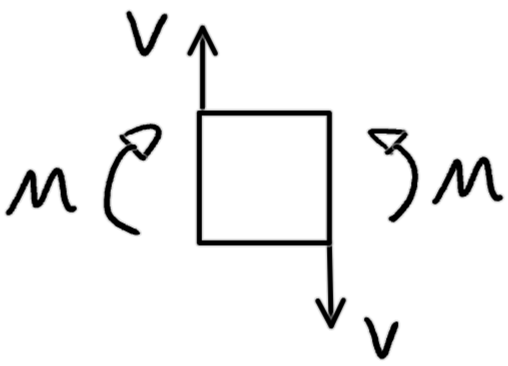
Source: Lab1 – Shear Force and Bending Moments

Appendix B – Shear Forces on an Infinitesimal Block



Source: https://www.designingbuildings.co.uk/wiki/File:Shear.jpg#metadata

Appendix C – Shear Force and Bending Moment Positive Sign Convention



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