Study Problems Week 9 – Work-Energy and Impulse Momentum Methods

Question 9.12.

The system is released from rest with no slack in the cable and with the spring unstretched. Determine the distance s travelled by the 10 kg cart before it comes to rest (a) if m approaches zero, and (b) if m = 2 kg. Assume no mechanical interferences.

m k = 125 N/m

Solution

(a) Let 's' be the slant distance down the incline travelled by the 10 kg cart

$$W_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

$$W_{1-2} = 0$$

$$\Delta T = \frac{1}{2}m(v_2^2 - v_1^2)$$

$$v_1 = 0$$
 and $v_2 = 0$

therefore

$$\Delta T = 0$$

$$\Delta V_q = mg(h_2 - h_1)$$

 $\Delta V_{q} = -10(9.81)(s) \sin 25^{\circ}$ i.e. the change in vertical height for 10 kg cart is $(-s) \sin 25^{\circ}$

$$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}k\left[\left(\frac{s}{2}\right)^2\right]$$

According to the Work-energy equation

$$0 = -10(9.81)(s)\sin 25^{\circ} + \frac{1}{2}k\left[\left(\frac{s}{2}\right)^{2}\right]$$

$$10(9.81)(s)\sin 25^\circ = \frac{125}{2} \left[\left(\frac{s}{2}\right)^2 \right]$$

Solving which yields,

$$s = 2.6534 \,\mathrm{m}$$
 (Answer)

(b) Let 's' be the slant distance down the incline travelled by the 10 kg cart

$$W_{1-2} = \Delta T + \Delta V_a + \Delta V_e$$

$$W_{1-2}=0$$

$$\Delta T = \frac{1}{2}m(v_2^2 - v_1^2)$$

$$v_1 = 0$$
 and $v_2 = 0$

therefore

$$\Delta T = 0$$

$$\Delta V_g = mg(h_2 - h_1)$$

$$\Delta V_g = -10(9.81)(s)\sin 25^\circ + 2(9.81)\left(\frac{s}{2}\right)$$

$$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}k\left[\left(\frac{s}{2}\right)^2\right]$$

According to the Work-energy equation

$$0 = -10(9.81)(s)\sin 25^{\circ} + 2(9.81)\left(\frac{s}{2}\right) + \frac{1}{2}k\left[\left(\frac{s}{2}\right)^{2}\right]$$

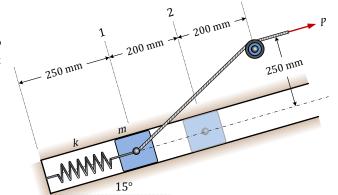
$$10(9.81)(s)\sin 25^{\circ} - 2(9.81)\left(\frac{s}{2}\right) = \frac{125}{2}\left[\left(\frac{s}{2}\right)^{2}\right]$$

Solving which yields,

$$s = 2.025 \,\mathrm{m}$$
 (Answer)

Question 9.13.

Determine the constant force P required to cause the 0.5 kg slider to have a speed $v_2 = 0.8$ m/s at position 2. The slider starts from rest at position 1 and the unstretched length of the spring of modulus k = 250 N/m is 200 mm. Neglect friction.



Solution

$$W_{1-2} = \Delta T + \Delta V_q + \Delta V_e$$

$$W_{1-2} = Pd$$

Where

$$d = \sqrt{0.45^2 + 0.25^2} - \sqrt{0.2^2 + 0.25^2} = 0.1515 \text{ m}$$

therefore,

$$W_{1-2} = 0.1515P$$

$$\Delta T = \frac{1}{2}m(v_2^2 - v_1^2)$$

$$v_1 = 0$$
 and $v_2 = 0.8$ m/s

therefore

$$\Delta T = 0.16 \text{ J}$$

$$\Delta V_a = mg(h_2 - h_1)$$

$$\Delta V_g = 0.5(9.81)(0.2 \sin 15^\circ) = 0.254 \,\mathrm{J}$$

$$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}(250)[0.25^2 - 0.05^2] = 7.5 \text{ J}$$

According to the Work-energy equation

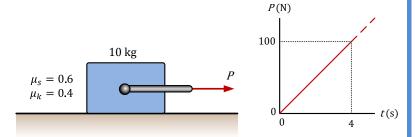
$$0.1515P = 0.16 + 0.254 + 7.5$$

Solving which yields,

$$P = 52.23 \text{ N}$$
 (Answer)

Question 9.14.

The force P, which is applied to the 10 kg block initially at rest, varies linearly with time as indicated. If the coefficients of static and kinetic friction between the block and the horizontal surface are 0.60 and 0.40, respectively, determine the velocity of the block when $t=4\,\mathrm{s}$.



Solution

$$F_s = \mu_s k = (0.6)(98.1) = 58.9 \text{ N}$$

$$F_k = \mu_k k = (0.4)(98.1) = 39.2 \text{ N}$$

The block does not move until,

$$P = F_s$$
 or

$$25t = 58.9$$

$$t = 2.35 \,\mathrm{s}$$

Then F becomes 39.2 N

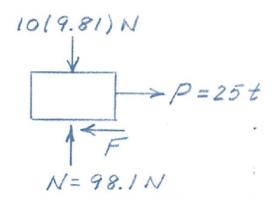
$$\int \sum F \ dt = m\Delta v$$

$$\int_{2.35}^{4} (25t - 39.2) dt = 10(v - 0)$$

$$\frac{25t^2}{2} - 39.2t \bigg|_{2.35}^4 = 10v$$

$$10v = 66.1$$

$$v = 6.61 \,\mathrm{m/s}$$
 (Answer)



Question 9.15.

The two cars collide at right angles in the intersection of two icy roads. Car A has a mass of 1200 kg and car B has a mass of 1600 kg. The cars become entangled and move off together with a common velocity v' in the direction indicated. If car A was traveling 50 km/h at the instant of impact, compute the corresponding velocity of car B just before impact.

v_{B} $\begin{vmatrix} y & v' \\ 30^{\circ} \\ --- x \end{vmatrix}$ $V_{A} = 50 \text{ km/h}$

Solution

$$G_{1_X} = G_{2_X}$$
:
 $m_B v_B + 0 = (m_A + m_B) v' \sin 30^\circ$
 $(1600)(v_B) = 2800 v'(0.5)$ ------(1)

$$G_{1y} = G_{2y}$$
:
 $m_A v_A + 0 = (m_A + m_B) v' \cos 30^\circ$
 $(1200)(50) = 2800 v'(0.866)$ ------ (2)

$$\frac{v_{B}}{v_{A}} = 50 \text{ km/h}$$

Solving (2) and (1), respectively, yields,

$$v' = 24.7 \text{ km/h}$$
 (Answer)
 $v_B = 21.7 \text{ km/h}$ (Answer)