MMAN1300
Lab report: Motion of a Rolling Disc

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Abstract

This document is a lab report on the motion of a rolling wheel experiment performed in Willis Annex during Week 9. The report will analyse the motion of a rolling wheel down an inclined plane. The report will also present results obtained from the experiment via theoretical and experimental methods as well as discuss the results, any discrepancy in values, and how these results are related to the motion of the rolling wheel.

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1. Introduction

Calculation of rigid bodies under general plane motion can and should always be decomposed into translational and rotational motion. The motion of a rolling wheel is a type of general plane motion and is the focus of this lab report. To analyse the general plane motion of the wheel subject, an experiment was performed to measure the time for the wheel to be displaced from rolling down an inclined track. Theoretical calculations were performed to confirm the experimental data by deriving a relationship between time and distance and calculating the angular and tangential acceleration of the wheel.

2. Motion of a Rolling Disc Experiment

The aim of the experiment was to measure the time for a wheel to traverse a given distance of an inclined track by allowing the wheel to roll from rest in the groove of the track.

2.1. Apparatus: Motion of a Rolling Disc Experimental Setup

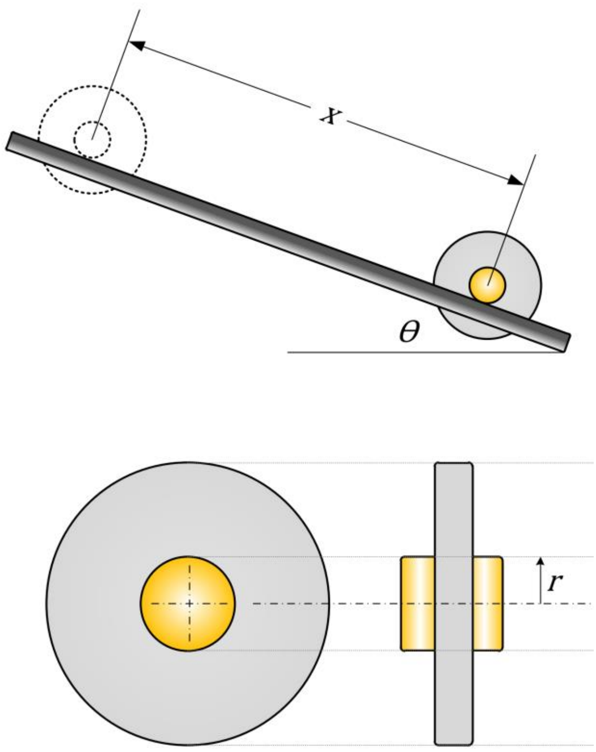


Figure 1. Apparatus *(Source: MMAN1300 Lab 2 – Motion of a Rolling Disk)*

The apparatus used for the experiment *(refer to figure 1)* consisted of:

- 1x steel wheel that is a composite body of an axle and disc
- 1x inclined track with a fixed ruler aligning the track

The equipment required for this experiment was:

- 1x measuring tape
- 1x stopwatch

2.2. Experimental Method

The method to measure the time for the experiment:

1. Place and hold the wheel into the groove of the track – placed at the zero measurement of the fixed ruler on the track.
2. Start the timer when the wheel is let go. The wheel will begin to roll down the track.
3. Stop the timer when the wheel has traversed 0.2 metres. Use the fixed ruler on the track to indicate the distance travelled.
4. Record the time elapsed for the distance travelled. Non-concordant times should be ignored and the trial repeated.
5. Repeat steps 1-4, two more times for a total of three usable trials.
6. Calculate and record the average time for the given distance by summing the three times then dividing by three (i.e. the number of trials).
7. Repeat steps 1-6 increasing the distance by 0.2 metres every time until and the distance of 1.0 metre.

The following method is used to find other information about the experiment:

1. Draw the following diagram of the apparatus *(refer to figure 2 below)*. The inclined track sat on two columns which are denoted by h_1 and h_2 where the distance between the column is denoted by l.

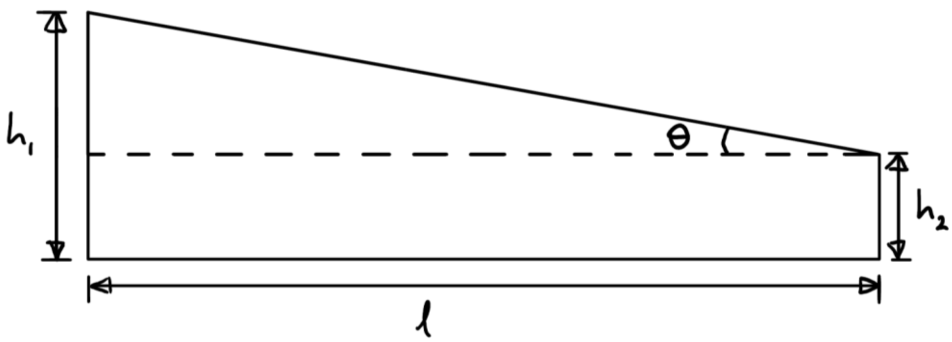


Figure 2. Diagram of Track (1)

2. Measure h_1, h_2 and l with a measuring tape with respect to the diagram.
3. Record the results.

2.3. Wheel Parameters

The wheel is a composite body of two concentric solid discs which will be denoted as: disc and axle *(refer to figure 3 below).* The following information about the wheel is given from *"MMAN1300 Lab 2 – Motion of a Rolling Disk"* [1] and implemented into figure 3 below *(appendix A):*

- $m_{Disc+Axle} = 11.6\ kg$
- $\rho_{Stainless\ Steel} = 7700\ kgm^{-3}$
- $r_{Disc} = 150 \times 10^{-3}\ m$
- $r_{Axle} = 10 \times 10^{-3}\ m$
- $h_{Disc} = 20 \times 10^{-3}\ m$

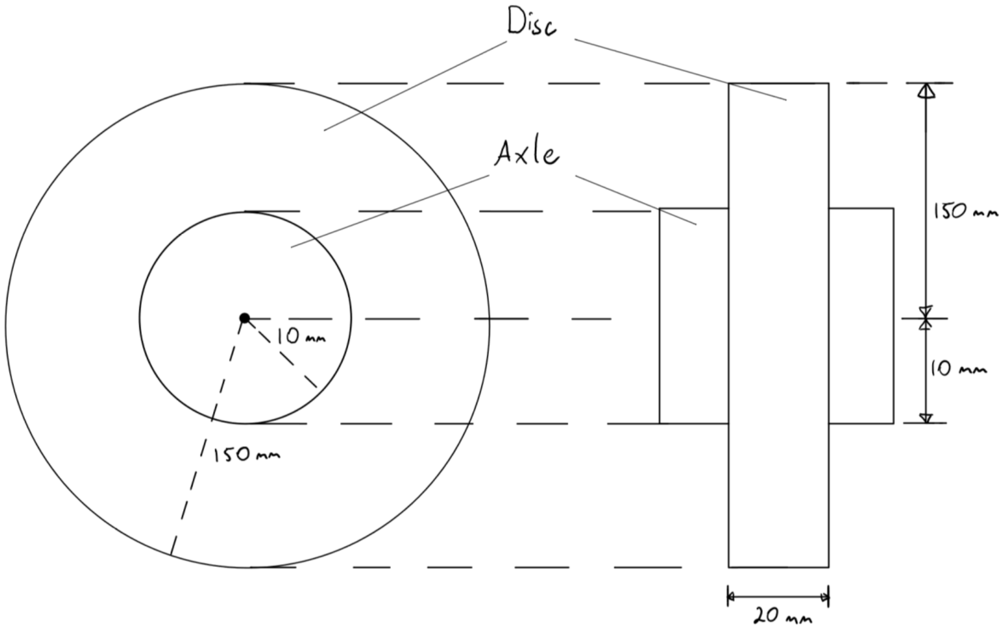


Figure 3. Wheel Geometry

2.4. Experimental Observations

From following the previous experimental method. The experimental time results were recorded in table 1 *(refer below)* with an average time for a given distance.

Table 1. Time Recorded for Rolling Wheel in Experiment

Distance (m)	**Time 1 (s)**	**Time 2 (s)**	**Time 3 (s)**	**Average Time (s)**
0.2	7.65	7.52	7.49	7.55
0.4	10.82	10.56	10.55	10.64
0.6	13.05	12.95	12.89	12.96
0.8	15.21	14.93	14.88	15.00
1.0	16.97	16.69	16.68	16.78

3. Theoretical Calculations

The theoretical time for the wheel to traverse the track is calculated in this report section and is required to confirm the experimental times measured. This is done by considering the mass moment of inertia of the wheel and the sum of moments about the wheel's centre of rotation in order to derive the relationship between the distance travelled and time elapsed of the rolling wheel.

Assumptions have been made for the theoretical calculations:

- The wheel does not slip – therefore there is only static friction acting on the wheel and kinetic friction is negligible
- All given information about the wheel's parameters are considered true and very accurate

3.1. Mass Moment of Inertia

The radii and total mass of the composite bodies is given but the individual masses are not given. The mass of the disc and axle is required to calculate the mass moment of inertia of each body.

<u>Consider an Arbitrary Solid Disc:</u>

Where the arbitrary solid disc has:

- mass, M;
- volume, V;
- height, h; and
- radius, R.

$$\rho = \frac{M}{V} = \frac{dm}{dV} \tag{1}$$

$$V = \pi R^2 h \tag{2}$$

Differentiating (2) with respect to r:

$$dV = 2\pi r h \cdot dr \tag{2'}$$

Rearranging (1) for dm:

$$dm = \frac{M \cdot dV}{V} \tag{1}$$

$(2), (2') \rightarrow (1)$:

$$dm = \frac{M \cdot 2\pi rh \cdot dr}{\pi R^2 h}$$

$$= \frac{M \cdot 2\pi rh \cdot dr}{\pi r^2 h}$$

$$\therefore dm = \frac{2Mr}{R^2} dr \tag{3}$$

<u>Deriving Mass Moment of Inertia of a Solid Disc:</u>

Formula for the mass moment of inertia of a continuous body:

$$I = \int_0^R r^2 \, dm \tag{4}$$

$(3) \rightarrow (4)$:

$$I = \int_0^R r^2 \frac{2Mr}{R^2} dr$$

$$= \frac{2M}{R^2} \int_0^R r^3 \, dr$$

$$= \frac{2M}{R^2} \left[\frac{r^4}{4}\right]_0^R$$

$$= \frac{2M}{R^2} \left(\frac{R^4}{4}\right)$$

$$\therefore I_{Solid\ Disc} = \frac{1}{2} MR^2 \tag{5}$$

<u>Determining the Mass of Disc and Axle Independently:</u>

The information given for the wheel parameters can be used to determine the mass of the disc and axle, which are required to calculate the mass moment of inertia of the wheel.

Consider (2) for V_{Disc}:

$$V_{Disc} = \pi r_{Disc}^2 h_{Disc}$$

$$= \pi(150 \times 10^{-3})^2(20 \times 10^{-3})$$

$$\therefore V_{Disc} = (45 \times 10^{-5})\pi m^3 \approx 0.00141\ m^3$$

Consider (1) for m_{Disc}:

$$m_{Disc} = \rho_{Steel} V_{Disc}$$

$$= 7700 \times (45 \times 10^{-5} \times \pi)$$

$$m_{Disc} = 3.465\pi\ kg$$

$$\therefore m_{Disc} \approx 10.89\ kg$$

Considering the mass of whole wheel:

$$m_{Disc+Axle} = m_{Disc} + m_{Axle}$$

$$11.6 = 3.465\pi + m_{Axle}$$

$$m_{Axle} = 11.6 - 3.465\pi\ kg$$

$$\therefore m_{Axle} \approx 0.7144\ kg$$

Calculating Mass Moment of Inertia of Wheel about Axis G:

The mass moment of inertia of the wheel about an axis is the algebraic sum of the mass moment of inertia of the disc and axle through the same axis. Considering the mass moment of inertia about the axis through the centre of mass, *G (refer to appendix B):*

$$I_{Wheel(G)} = I_{Disc(G)} + I_{Axle(G)} \tag{6}$$

For disc and axle, (5) → (6):

$$I_{Wheel(G)} = \frac{1}{2} m_{Disc} r_{Disc}^2 + \frac{1}{2} m_{Axle} r_{Axle}^2$$

$$= \frac{1}{2}(3.465\pi)(150 \times 10^{-3})^2 + \frac{1}{2}(11.6 - 3.465\pi)(10 \times 10^{-3})^2$$

$$I_{Wheel\,(G)} = (38808 \times 10^{-6})\pi + 58 \times 10^{-5}\ kgm^2$$

$$\therefore I_{Wheel\,(G)} \approx 0.1225\ kgm^2$$

Note that the axle can be split into two separate bodies (left and right of the disc from figure 3) each with mass, $\frac{m_{Axle}}{2}$. However, taking the sum of the mass moment of inertia of the axle bodies is equivalent to treating the axle as a single body.

Calculating Mass Moment of Inertia of Wheel about Axis O:

From appendix B, since axis O is parallel to axis G; the parallel axis theorem is used to calculate the mass moment of inertia of the wheel about axis O:

Let perpendicular distance between axes be denoted by d; where $d = r_{Axle} = 10 \times 10^{-3}\ m$.

$$I_{Wheel\,(O)} = I_{Wheel\,(G)} + m_{wheel}d^2$$

$$= \left((38808 \times 10^{-6})\pi + 58 \times 10^{-5}\right) + 11.6(10 \times 10^{-3})^2$$

$$I_{Wheel\,(O)} = (38808 \times 10^{-6})\pi + 174 \times 10^{-5}\ kgm^2$$

$$\therefore I_{Wheel\,(O)} \approx 0.1237\ kgm^2$$

The mass moment of inertia of the wheel about the axis O will be used to calculate time.

3.2. Calculation of Time

To calculate the theoretical time for a wheel to roll down a frictionless inclined track; a relationship between distance and time is derived of which the derivation was given in *"MMAN1300 Engineering Mechanics 1: Lab: 2 – Motion of a Rolling Disk"*.

Calculating the Angle of Inclination of the Track:

The angle of inclination of the track must be calculated first. Using the results of h_1, h_2 and l; and figure 4 below:

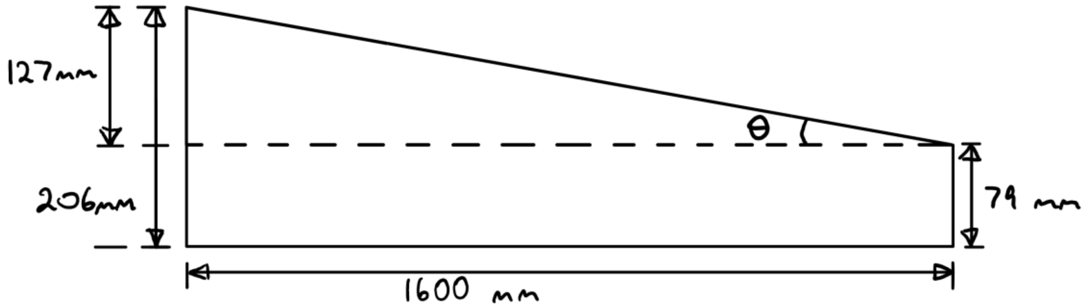


Figure 4. Diagram of Track (2)

The angle theta can be calculated from the following formula:

$$\tan(\theta) = \frac{h_1 - h_2}{l}$$

$$\theta = \tan^{-1}\left(\frac{206 - 79}{1600}\right)$$

$$\therefore \theta \approx 4.5°$$

<u>Derivation for Relationship between Distance and Time:</u>

Considering Newton's Second Law for Moments about Axis O *(with reference to appendix B and C):*

$$\sum M_O = I_O \alpha$$

The tangential component of mg (i.e. $mgsin(\theta)$) is acting from the centre of mass at a distance, d, from and parallel to axis O.

$$mgsin(\theta) \cdot d = I_O \alpha$$

$$\therefore \alpha = \frac{mgsin(\theta) \cdot d}{I_O} \tag{7}$$

Considering acceleration of the wheel:

$$\vec{a}_n = r \cdot \omega^2 = \frac{v^2}{r} \tag{8}$$

$$\vec{a}_t = \alpha \cdot r \tag{9}$$

Since tangential acceleration is constant:

$$x = x_O + v_O t + \frac{at^2}{2} \tag{10}$$

Initially; $x_O = 0$; $v_O = 0$; $(7), (8) \rightarrow (10)$:

$$x = \frac{1}{2} \cdot \left(\frac{mgsin(\theta) \cdot r}{I_O}\right) r \cdot t^2$$

$$\therefore x = \frac{mgsin(\theta) \cdot r^2}{2I_O} \cdot t^2 \tag{11}$$

<u>Calculating Time for Given Distance:</u>

Rearranging (11) for t:

$$t = \sqrt{\frac{2I_O \cdot x}{mgsin(\theta) \cdot r^2}} \tag{11}$$

Substitute $x = 0.2\ m \rightarrow (11)$:

$$t = \sqrt{\frac{2 \cdot \left((38808 \times 10^{-6})\pi + 174 \times 10^{-5}\right) \cdot (0.2)}{11.6 \times 9.81 \times sin(4.5) \times (10 \times 10^{-3})^2}}$$

$$\therefore t \approx 7.44\ s$$

This calculation for time was repeated for every given distance, x, and recorded in table 2 *(refer to end of 3.2: Calculation of Time).*

<u>Calculating Percentage Error in Time:</u>

The percentage error in time (denoted by t_{error}) between the theoretical and experimental values is calculated using:

$$t_{error} = \left|\frac{t_E - t_T}{t_T}\right| \times 100$$

For $x = 0.2\ m$:

$$t_{error} = \left|\frac{7.49 - 7.44}{7.44}\right| \times 100$$

$$\therefore t_{error} = 0.67\%$$

This calculation for the percentage error in time was repeated for every given distance, x, and recorded in table 2 *(refer below).*

Table 2. Theoretical vs Experimental Results

Distance (m)	**Experimental Time (s)**	**Theoretical Time (s)**	**Error (%)**
0.2	7.49	7.44	0.67
0.4	10.55	10.53	0.19
0.6	12.89	12.89	0.00
0.8	14.88	14.89	0.07
1.0	16.68	16.64	0.24

3.3. Calculation of Acceleration

Calculating the angular and tangential acceleration of the wheel can confirm the types of motion the wheel is undergoing in its translational and rotational components. With respect to appendix B, the accelerations can be calculated by using previously derived equations.

Calculating Angular Acceleration:

From (7), consider the angular acceleration:

$$\alpha = \frac{mgsin(\theta) \cdot d}{I_O} \tag{7}$$

$$= \frac{11.6 \times 9.81 \times sin(4.5) \times (10 \times 10^{-3})}{(38808 \times 10^{-6})\pi + 174 \times 10^{-5}}$$

$$\therefore \alpha \approx 0.772\, rads^{-2} \circlearrowright$$

The angular acceleration is constant for the whole motion of the rolling wheel.

Calculating Tangential Acceleration:

From (9), consider the tangential acceleration:

$$\vec{a}_t = \alpha \cdot r \tag{9}$$

$$= 0.772 \cdot (10 \times 10^{-3})$$

$$\therefore \vec{a}_t = 0.00722\, ms^{-2} \text{ with direction parallel and down the plane}$$

The tangential acceleration is constant for the whole motion of the rolling wheel.

4. Discussion and Conclusion

4.1. Comparison of Experimental and Theoretical Values

From table 1, the times for the experimental trial were reliable since the values were similar. From table 2, the experimental and theoretical values of time are taken to two decimal places. The percentage error in values was consistently less than 1%. Therefore the experimental values of time can be considered very accurate. Since the experimental times affirm to the theoretical; the experimental time can be modelled by equation (11):

$$x = \frac{mgsin(\theta) \cdot r^2}{2I_O} \cdot t^2 \tag{11}$$

The translational motion of the wheel also had a constant tangential acceleration of:

$$\vec{a}_t = 0.00722\, ms^{-2}$$

Therefore the relationship between distance and time is quadratic and the translational motion of the wheel is a uniformly accelerated motion.

Potential discrepancy in experimental times however could be due to:

- friction forces acting on the wheel that is not negligible;
- given values of the wheel's parameters that are not accurate;
- reaction time of the user when using the stopwatch; or
- measuring tape having an uncertainty of one millimetre.

The rotational motion of the rolling wheel has a constant angular acceleration as only the tangential component of the wheel's weight acted on the wheel.

The rotational motion of the wheel had a constant angular acceleration of:

$$\alpha \approx 0.772\, rads^{-2}$$

Therefore the rotational motion of the wheel is a uniformly accelerated motion.

Resolving the rotational and translational motions will determine the general plane motion of the wheel; which is simply a wheel translating down the inclined track whilst rotating about an axis perpendicular to the plane of translation that is along axis O *(appendix B).*

4.2. Conclusion

The purpose of this report was to analyse the motion of a wheel rolling down an inclined plane. This was done by performing an experiment to measure the time taken for the distance travelled by the rolling wheel, and conducting theoretical calculations to derive the relationship between distance and time to confirm the experimental values. From *4.1: Discussion*, the theoretical and experimental times were in agreement, therefore suggesting that the general plane motion of the wheel is uniformly accelerated in rotation and translation.

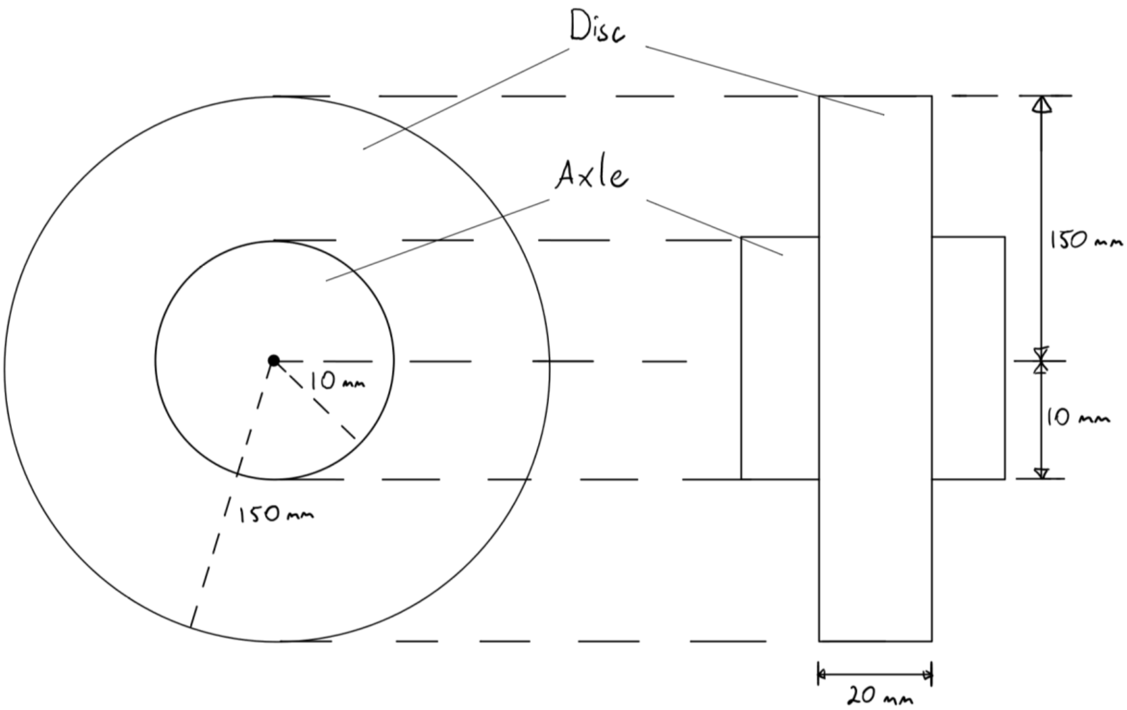
5. References

[1] UNSW (2018). Lab 2 – Motion of a Rolling Disk. MMAN1300 – Engineering Mechanics 1.

6. Appendix

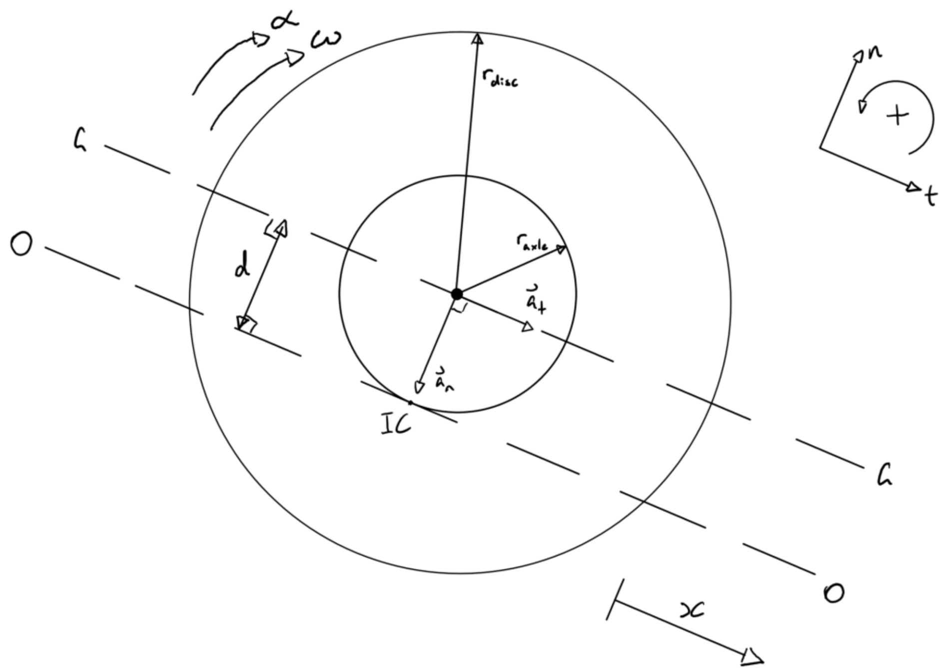
All figures were hand-drawn in OneNote unless specified with a referenced source.

<u>Appendix A – Wheel Geometry</u>

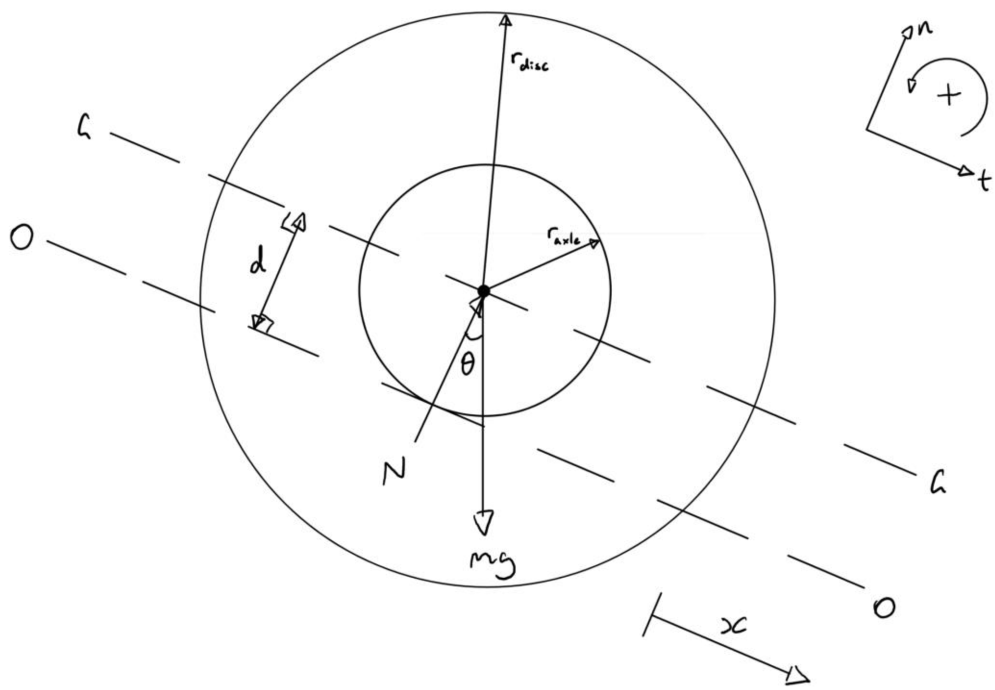


Appendix B – Free Body Diagram of Wheel

The wheel is rotating about the instantaneous centre (denoted by IC). Axis G is the axis along which the centre of mass travels. Axis O is the axis along which the instantaneous centre travels.



Appendix C – Free Body Diagram of Acting Forces on Wheel



Appendix D – Velocity of the Wheel