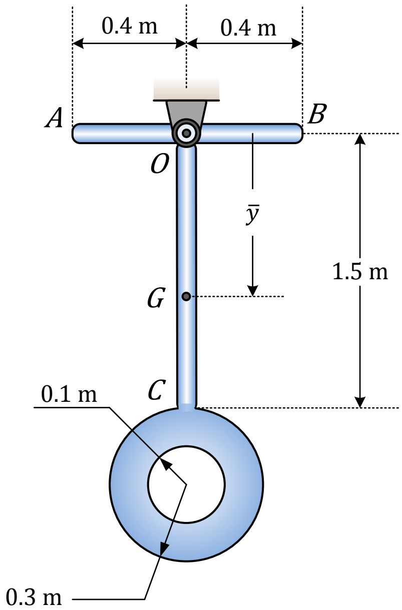
Question 11.11.



The pendulum consists of two slender rods AB and OC which have a mass per unit length of 3 kg/m. The thin circular plate has a mass per unit area of 12 kg/m^2. Determine the location $\bar{y}$ of the center of mass G of the pendulum, then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.

Solution

$$\bar{y} = \frac{1.5(3)(0.75) + \pi(0.3)^2(12)(1.8) - \pi(0.1)^2(12)(1.8)}{1.5(3) + \pi(0.3)^2(12) - \pi(0.1)^2(12) + 0.8(3)}$$

$$\bar{y} = 0.888 \text{ m}$$

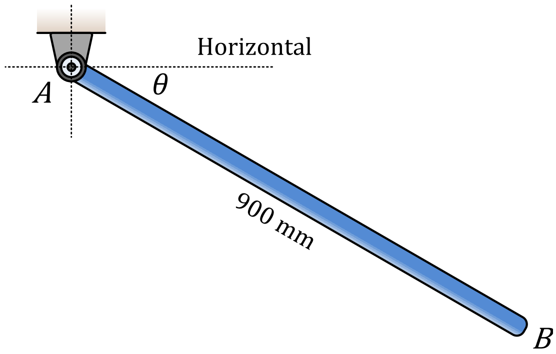
$$I_G = \left[\frac{1}{12}(0.8)(3)(0.8)^2 + 0.8(3)(0.888)^2\right] + \left[\frac{1}{12}(1.5)(3)(1.5)^2 + 1.5(3)(0.75 - 0.888)^2\right]$$

$$+\frac{1}{2}\left[\pi\,(0.3)^2(12)(0.3)^2 + [\pi(0.3)^2(12)](1.8 - 0.888)^2 - \frac{1}{2}\right[\pi\,(0.1)^2(12)(0.1)^2 - [\pi(0.1)^2(12)](1.8 - 0.888)^2$$

$I_G = 5.61$ kg. m^2 (Answer)

Question 11.12.

The uniform slender bar AB has a mass of 8 kg and swings in a vertical plane about the pivot at A. If $\dot{\theta} = 2\ \text{rad/s}$ when $\theta = 30°$, compute the force supported by the pin at A at that instant.

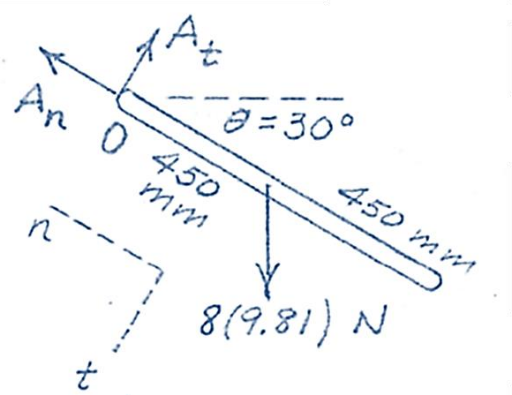


Solution

$$\sum M_O = I_O \alpha$$

$$8(9.81)(0.45\cos 30°) = \frac{1}{3}(8)(0.9)^2\alpha$$

$$\alpha = 14.16\ \text{rad/s}^2$$



$$\sum F_t = m\,\bar{r}\alpha$$

$$8(9.81)\cos 30° - A_t = 8(0.45)(14.16)$$

$$A_t = 16.99\ \text{N}$$

$$\sum F_n = m\,\bar{r}\omega^2$$

$$A_n - 8(9.81)\sin 30° - A_t = 8(0.45)(2)^2$$

$$A_n = 53.64\ \text{N}$$

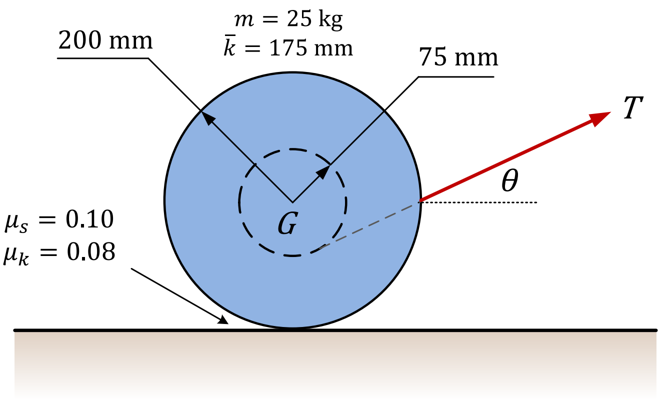
$$A = \sqrt{(A_t)^2 + (A_n)^2}$$

$$A = \sqrt{(16.99)^2 + (53.64)^2}$$

$A = 56.3\ \text{N}$ (Answer)

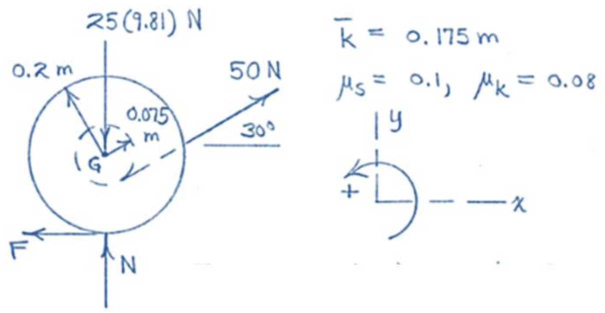
Question 11.13.

The circular disk of 200 mm radius has a mass of 25 kg with centroidal radius of gyration $\bar{k} = 175$ mm and has a concentric circular groove of 75 mm radius cut into it. A steady force T is applied at an angle θ to a cord wrapped around the groove as shown. If $T = 50$ N, $\theta = 30°$, $\mu_s = 0.10$ and $\mu_k = 0.08$, determine the angular acceleration α of the disk, the acceleration a_G of its mass centre G, and the friction force F which the surface exerts on the disk.



Solution

$\bar{k} = 0.175$ m, $\quad \mu_s = 0.10$ and $\quad \mu_k = 0.08$



$+\uparrow \sum F_y = 0$

$N = 25(9.81) - 50 \sin 30° = 220$ N

$+\rightarrow \sum F_x = ma_x$

$50 \cos 30° - F = 25\,a$ -------------- (1)

$+\circlearrowleft \sum M_O = I_G \alpha$

$50(0.075) - F(0.2) = 25(0.175)^2\alpha$ -------------- (2)

Assuming rolling with no slip:

$a = -r\alpha$ -------------- (3)

Solving Eqs. (1) - (3)

$F = 29.4$ N

$a = 0.556 \text{ m/s}^2$

$\alpha = -2.78 \text{ rad/s}^2$

$F_{max} = \mu_s N = (0.1)(220) = 22 \text{ N} < F$ (slips i.e. assumption invalid)

Therefore

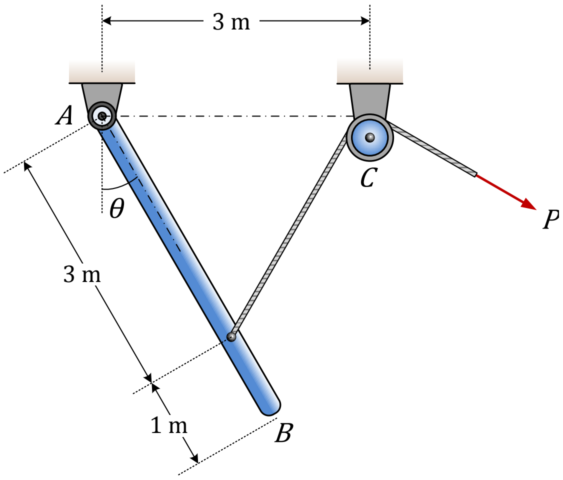
$F = \mu_k N = (0.08)(220) = 17.62$ N (Answer)

From (1) and (2)

$a = 0.556 \text{ m/s}^2$ (Answer)

$\alpha = -2.78 \text{ rad/s}^2$ (Answer)

Question 11.14.



The uniform 100 kg beam is freely hinged about its upper end A and is initially at rest in the vertical position with $\theta = 0°$. Determine the initial angular acceleration α of the beam and the magnitude F_A of the force supported by the pin at A due to the application of a force $P = 300$ N on the attached cable.

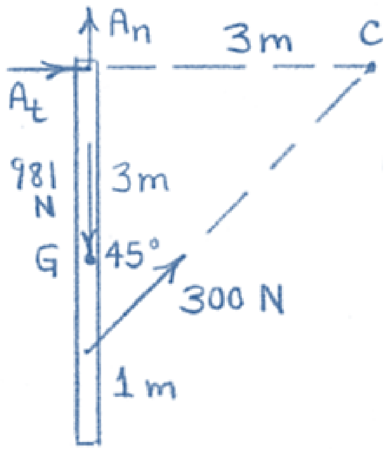
Solution

$$I_A = \frac{1}{3}ml^2 = \frac{100(4)^2}{3} = 533\ \mathrm{kg.m^2}$$

$+\circlearrowleft \sum M_A = I_A\alpha$

$300(3\sin 45°) = (533)\alpha$

$\alpha = 1.193\ \mathrm{rad/s^2}$



$\sum F_n = ma_n$

$A_n + 300\cos 45° - 981 = 0$

$A_n = 769\ \mathrm{N}$

$\sum F_t = ma_t$

$A_t + 300\sin 45° = 100(20(1.193)$

$A_t = 26.5\ \mathrm{N}$

$A = \sqrt{(A_t)^2 + (A_n)^2}$

$A = \sqrt{(26.5)^2 + (769)^2}$

$A = 769.45\ \mathrm{N}$ (Answer)