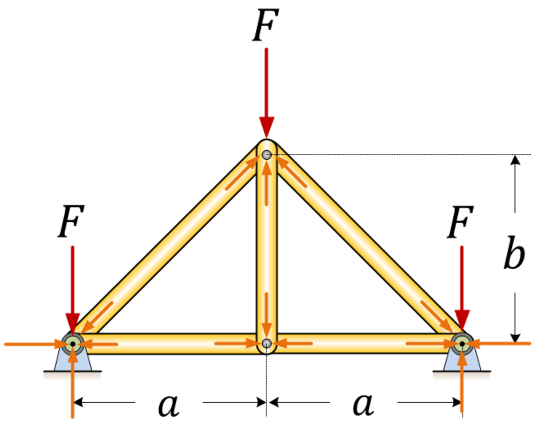
School of Mechanical and Manufacturing Engineering

MMAN1300 Engineering Mechanics 1

Dr. David C. Kellermann



Week 3, L1 – Structures and Trusses

STRUCTURES, TRUSSES AND DETERMINACY

- Equilibrium and FBD summary
- Equivalent systems
- Analysis of Structures
- Trusses - Method of joints
- Static determinacy
- Trusses – Method of sections

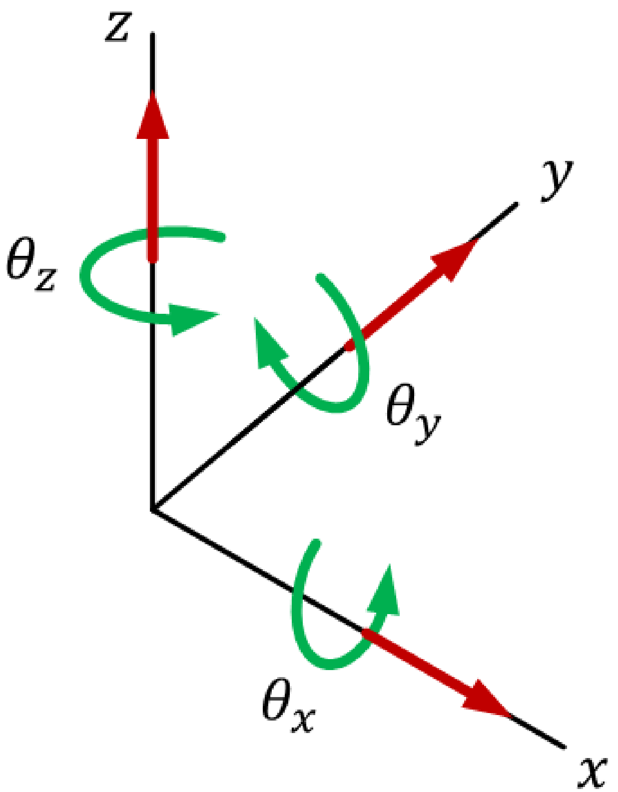
Review:

The six independent, scalar equations of equilibrium

$$\sum F_x = 0, \qquad \sum M_x = 0$$

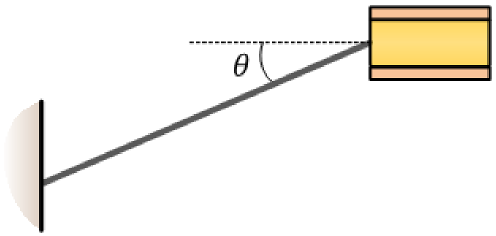
$$\sum F_y = 0, \qquad \sum M_y = 0$$

$$\sum F_z = 0, \qquad \sum M_z = 0$$

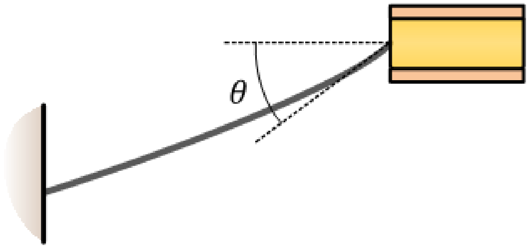


Review (FBDs)

1. Flexible cable, belt, chain, or rope



Weight of cable negligible



Weight of cable not negligible

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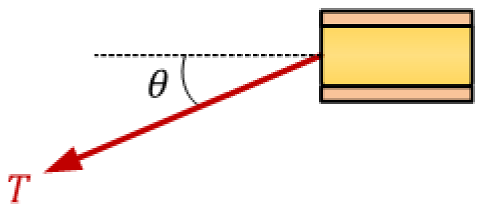
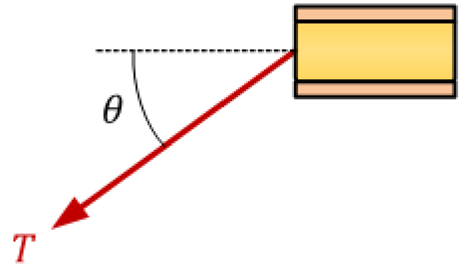
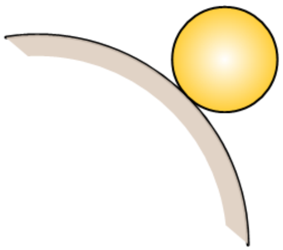




- Force exerted by a flexible cable is always a tension away from the body in the direction of the cable

Review (FBDs)

2. Smooth surfaces



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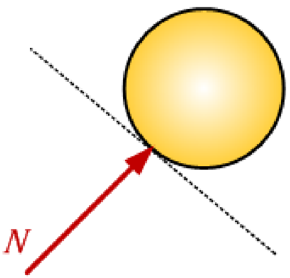
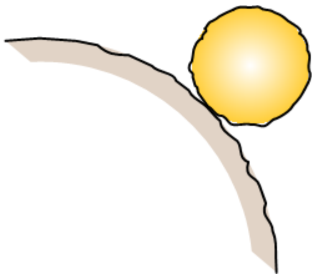


Contact force is compressive and its normal to the surface (plane of contact)

3. Rough surfaces



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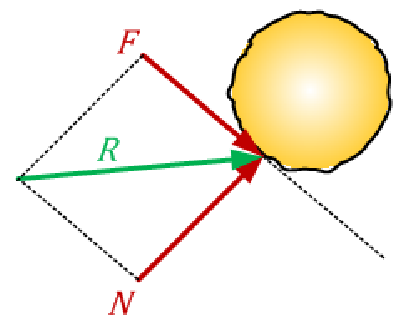
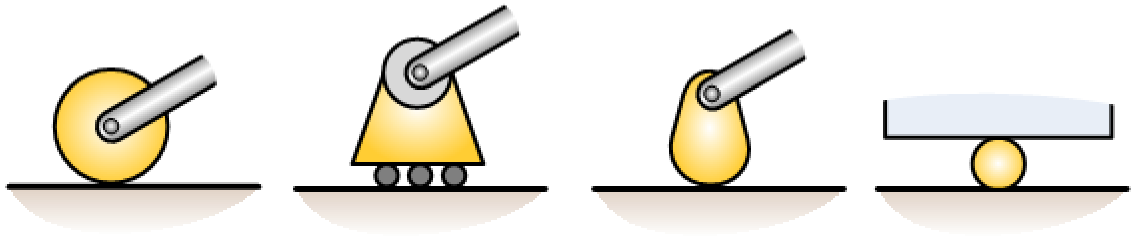


Rough surfaces are capable of supporting a tangential component F as well as a normal component N of the resultant

Review (FBDs)

4. Roller support



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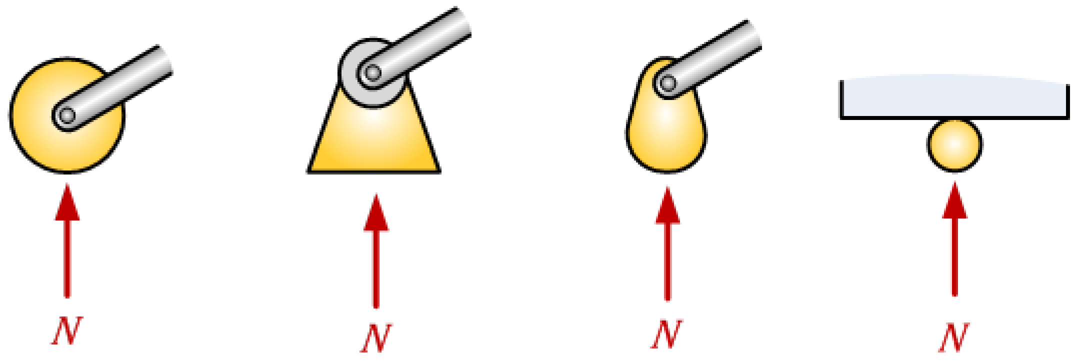
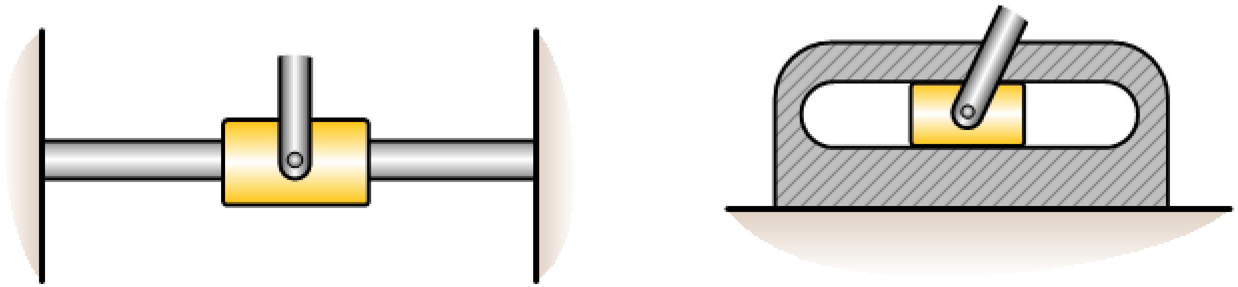


- Roller, rocker, or ball support transmits a compressive force normal to the supporting surface

Review (FBDs)

5. Freely sliding guide



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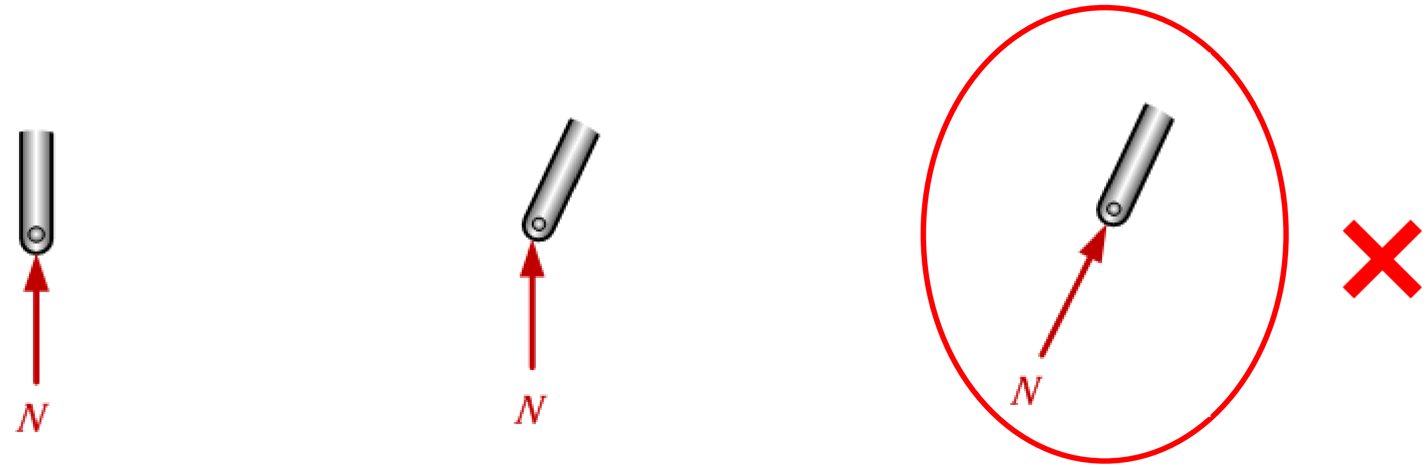
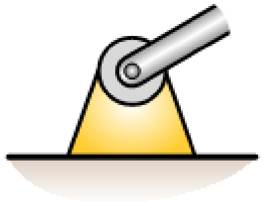


- Collar or slider free to move along smooth guides; can support force normal to guide only

Review (FBDs)

6. Pin connection



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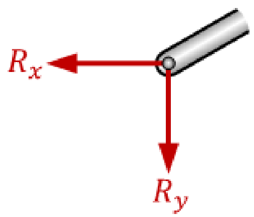
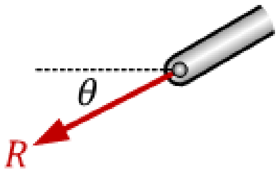
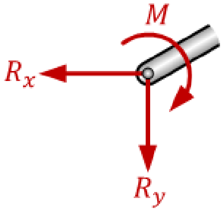




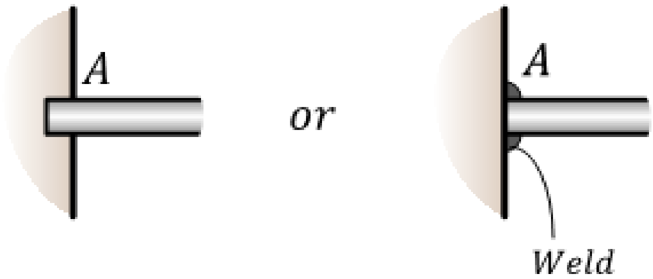


A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and a direction θ

A pin not free to turn also supports a couple M

Review (FBDs)

7. Built-in or fixed support



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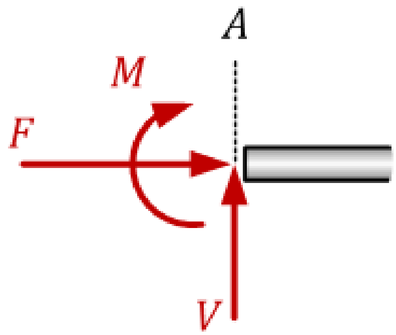
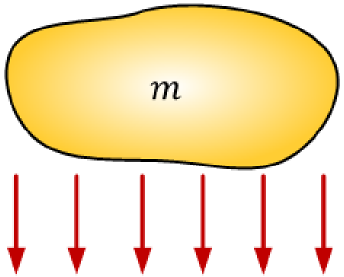


- A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force) and a couple M (bending moment) to prevent rotation

Review (FBDs)

8. Gravitational attraction



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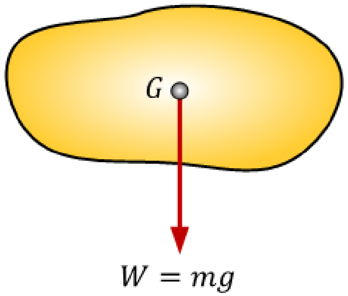
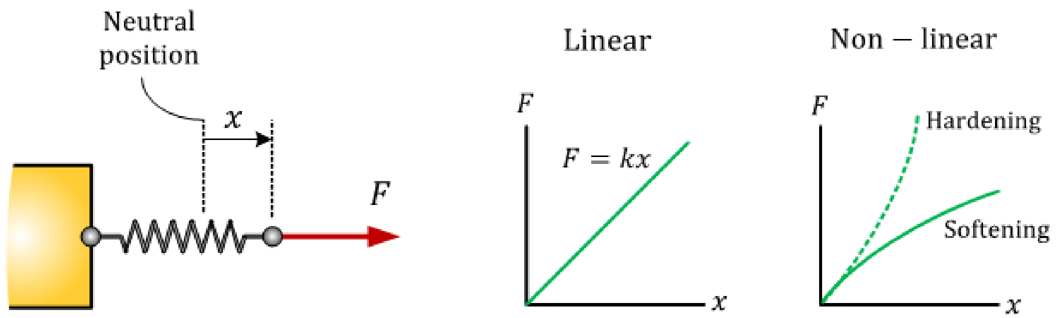


- The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts towards the centre of the earth through the mass centre G

Review (FBDs)

9. Spring action



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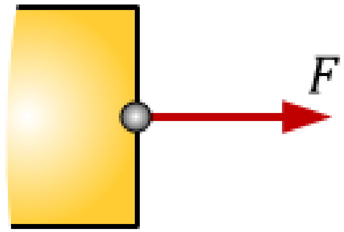
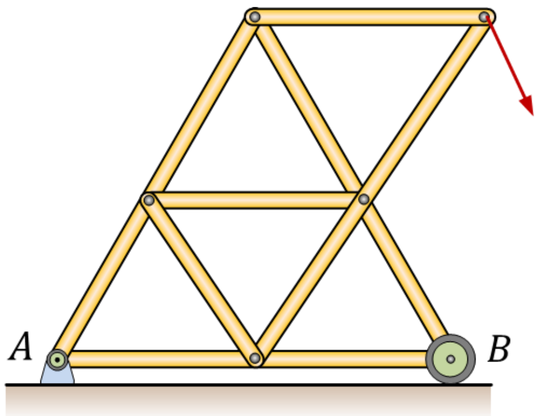


- Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance

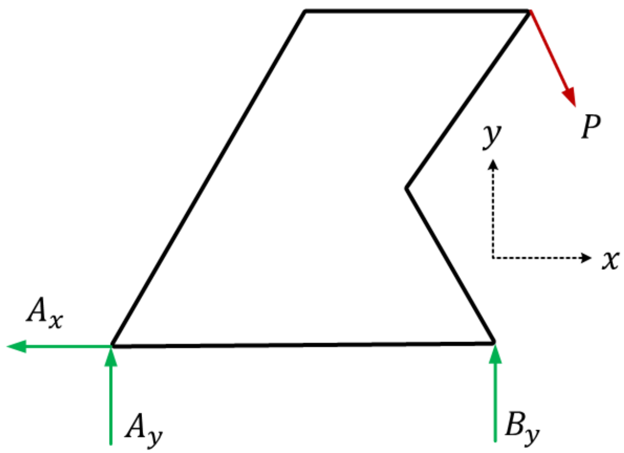
FBD of Structures

1. Plane Truss

System



FBD



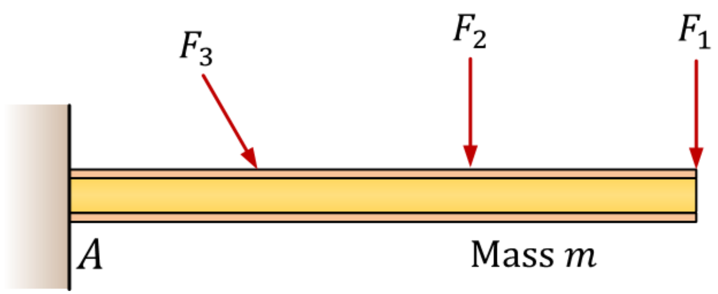
Note:

- Weight of the truss assumed negligible compared to P
- In FBD, we are not interested in what's happening internally (hence members removed and FBD simplified)

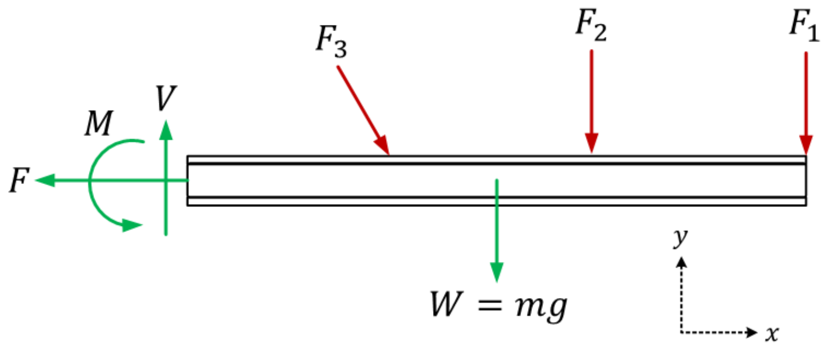
FBD of Structures

2. Cantilever Beam

System



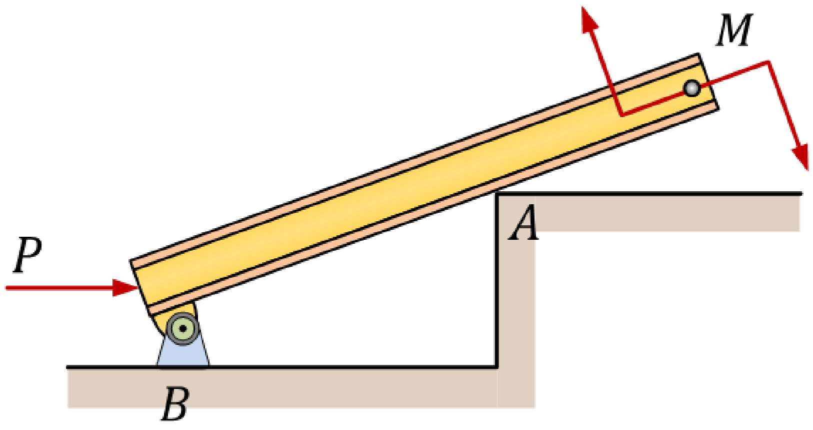
FBD



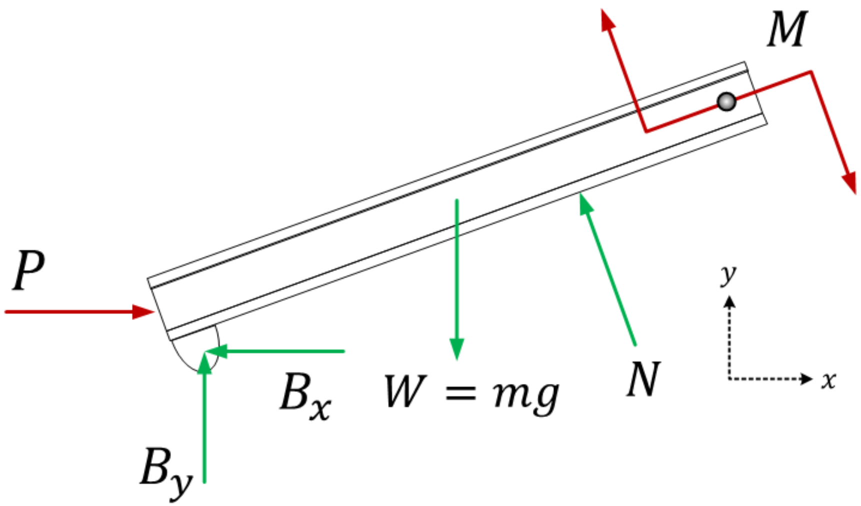
FBD of Structures

3. Beam

System



FBD



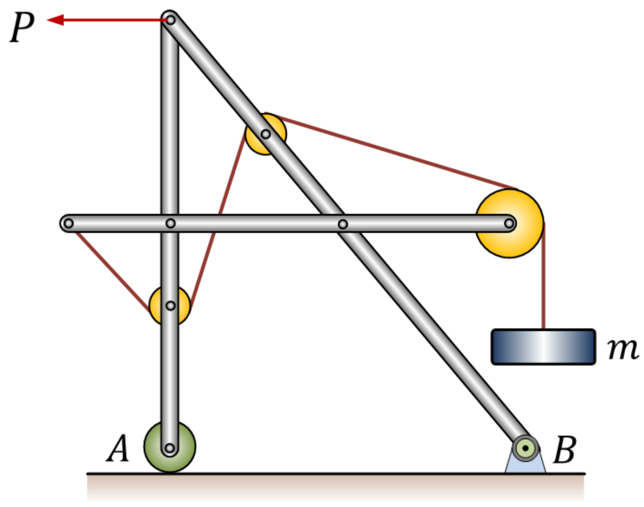
Note:

- Smooth surface contact at A and mass m

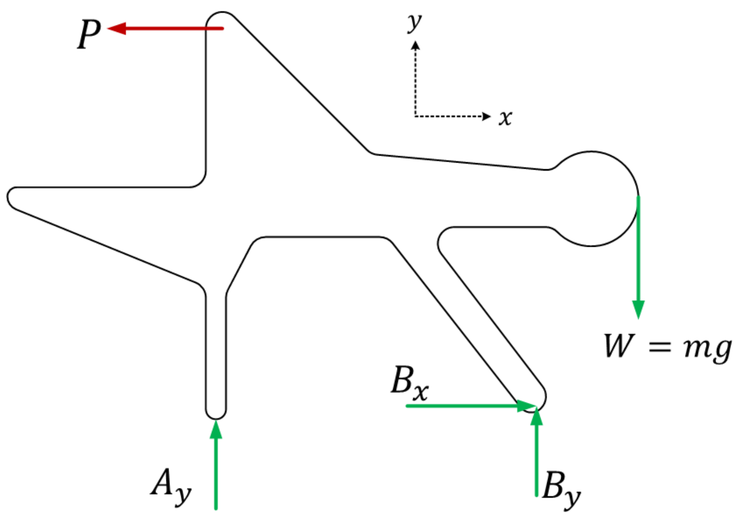
FBD of Structures

4. Rigid system of inter-connected bodies

System



FBD

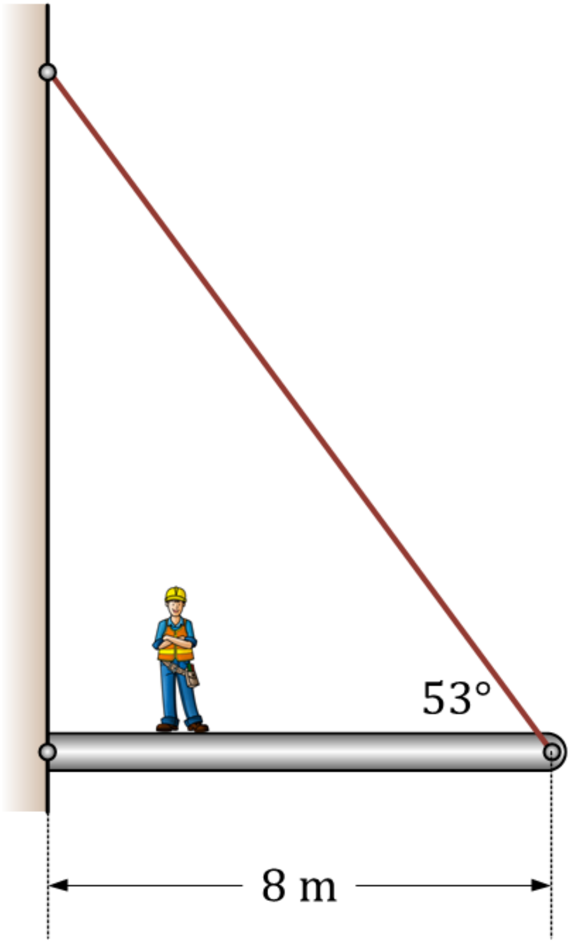


Note:

- Smooth surface contact at A and mass m

Example 1

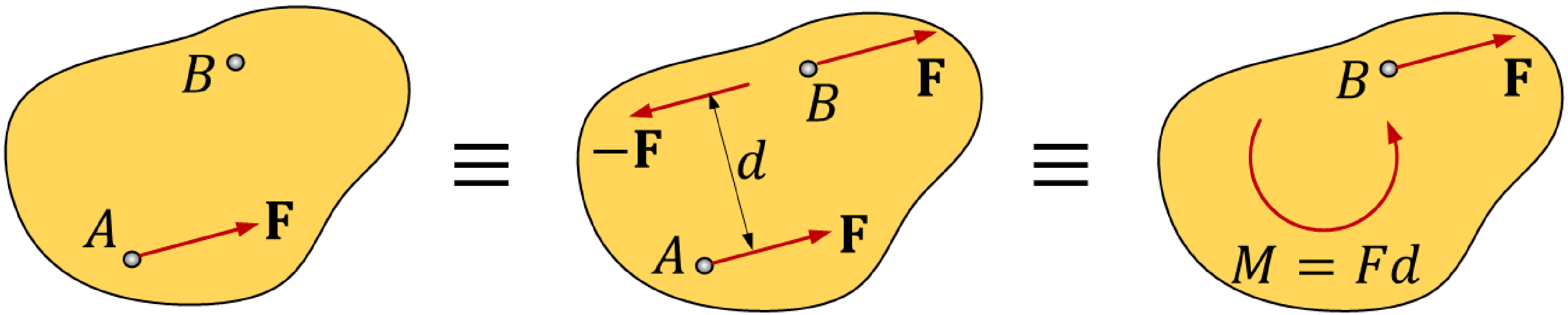
A uniform horizontal beam with a length of $l = 8$ m and a weight of $W_b = 200$ N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of $\theta = 53°$ with the beam. A person of weight $W_p = 600$ N stands a distance $d = 2$ m from the wall. Find the tension in the cable as well as the magnitude and direction of the force exerted by the wall on the beam.



W3 Example 1 (Web view)

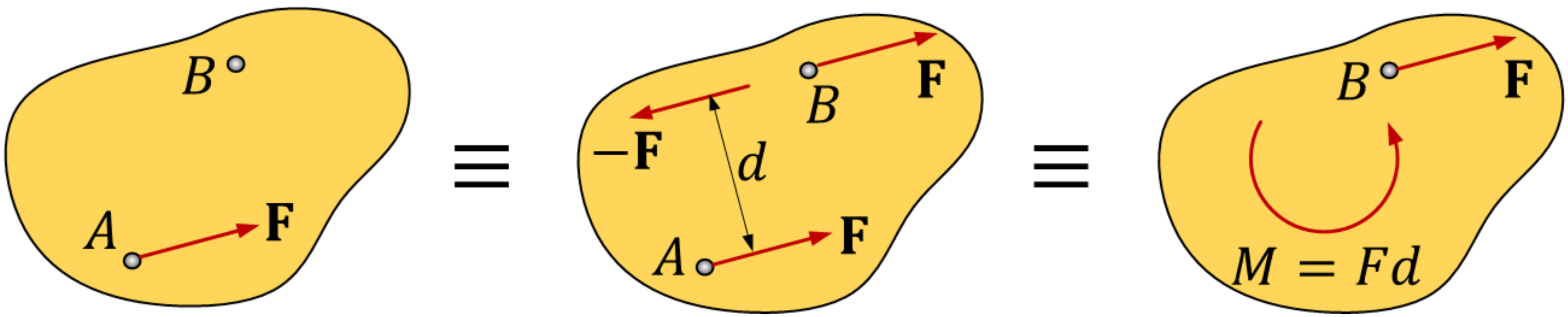
Equivalent Loading Systems

We can add couples to systems in order to create equivalent loading systems that are more convenient for our analysis



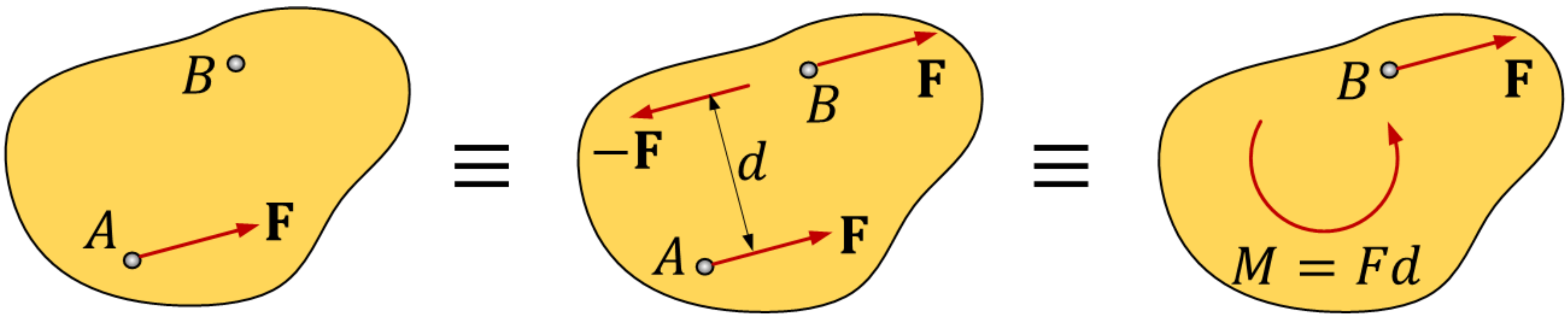
- At left, the force **F** has a tendency to push the bod to the right (and slightly up) as well as rotate it about point B

We can always add zero without changing anything



- By adding both **F** and –**F** acting through *B*, we have not changed any tendency for the body to move
- **F** – **F** = 0
- But we have created a couple (–**F** and the force **F** acting at *A*)
- Since we have only added zero, the load systems at left and right are equivalent

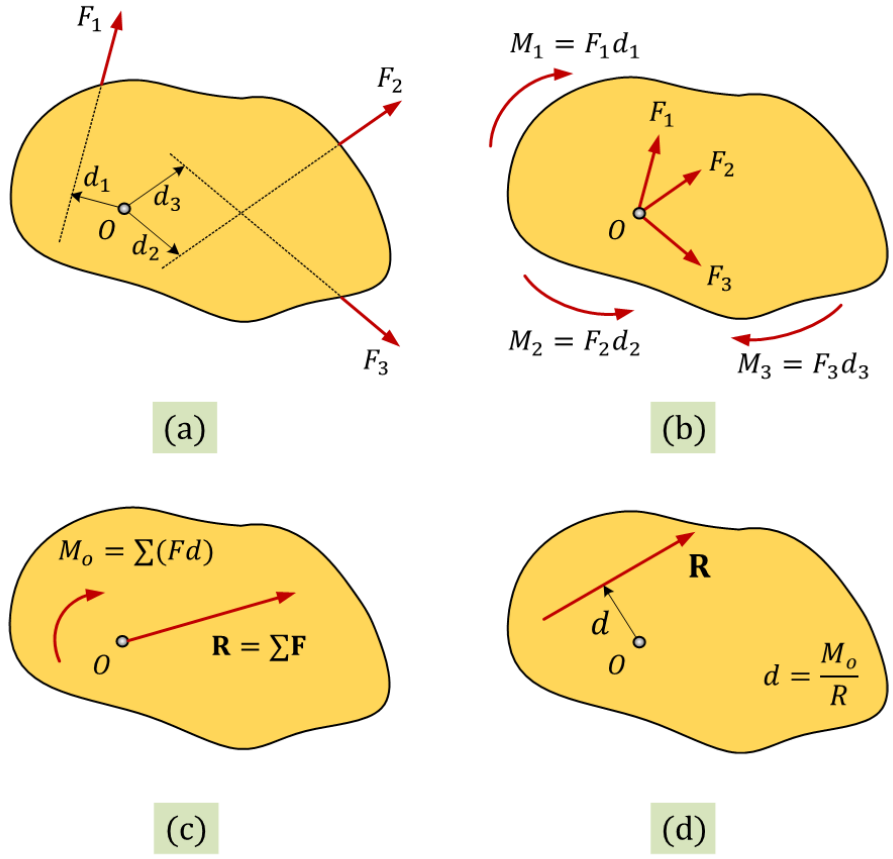
We can also move from right to left



- If we are taking moments about A, the system at left is easier
- We can replace the moment M with an equivalent couple
- Then **F** and –**F** at B cancel
- And we are left with the single force at right

We can use these processes to find equivalent resultants

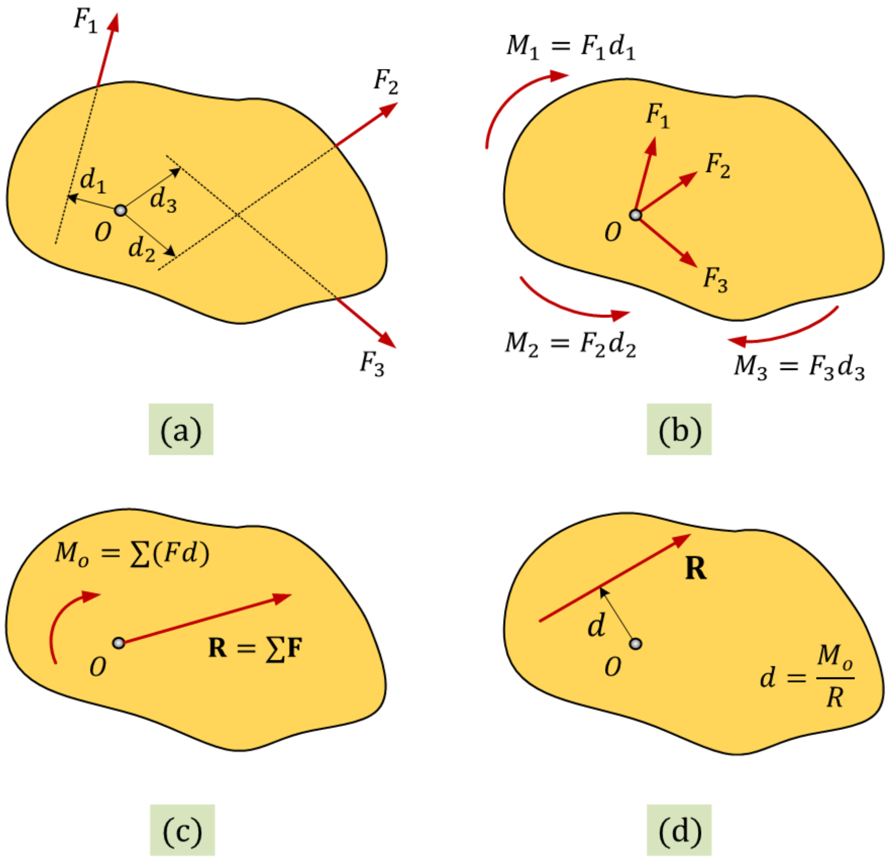
- For any chosen point O, we can add and subtract equivalent couples and moments until we are left with either
  - ✓ A single force **R** at O and a moment M_o about O
  - ✓ A single force **R** whose line of action is offset from O so as to produce the same moment



The force R and the moment M_O are then called the resultants

- Any two load systems that have the same resultant are obviously equivalent

- We can also restate our equilibrium conditions as saying. “a system is in equilibrium when the resultant force and moment are both equal to zero”



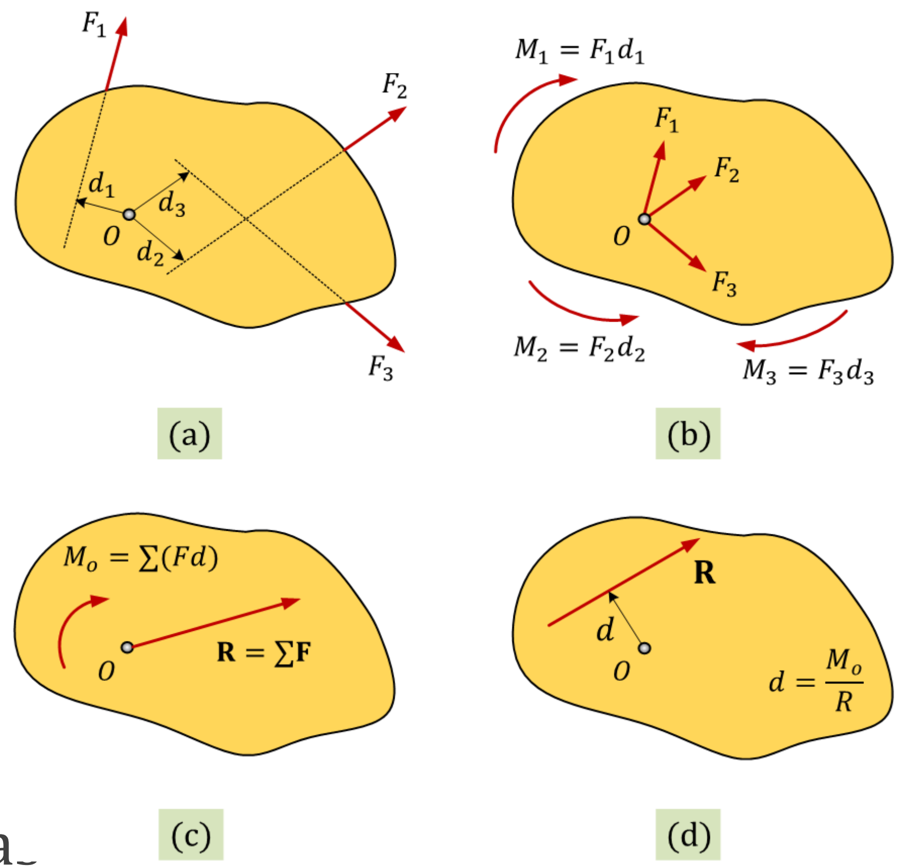
Equivalent Systems

We can write down general definitions for the resultants if we have a system of n forces and m moments

$$\mathbf{F}_R = \sum_{i=1}^{n} \mathbf{F}_i$$

$$\mathbf{M}_{RO} = \sum_{i=1}^{m} \mathbf{M}_i + \sum_{i=1}^{n} \mathbf{r}_i \times \mathbf{F}_i$$

- Where M_{RO} means resultant moment about O and r_i is the position vector of the i$^{\text{th}}$ force a… measured from O



Example 2

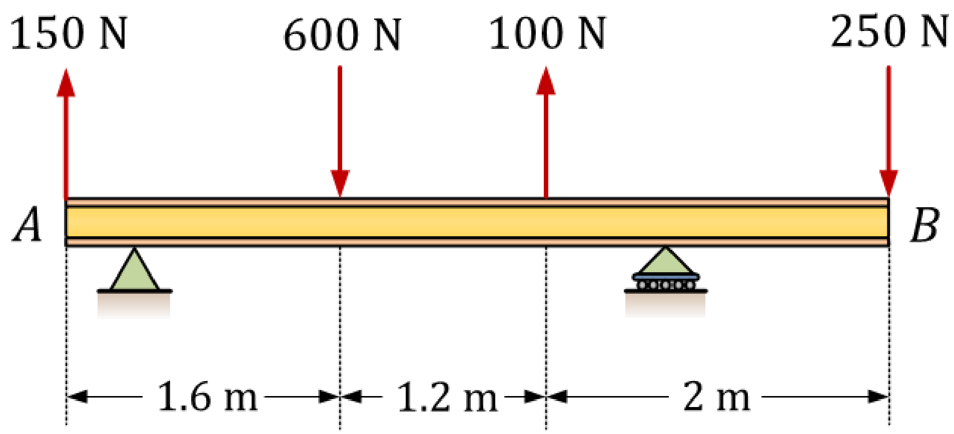
For the beam, reduce the system of forces shown to

(a) an equivalent force-couple system at *A*,

(b) an equivalent force couple system at *B*, and

(c) a single force or resultant

Note: *Since the support reactions are not included, the 'resultant' system will not be in equilibrium*



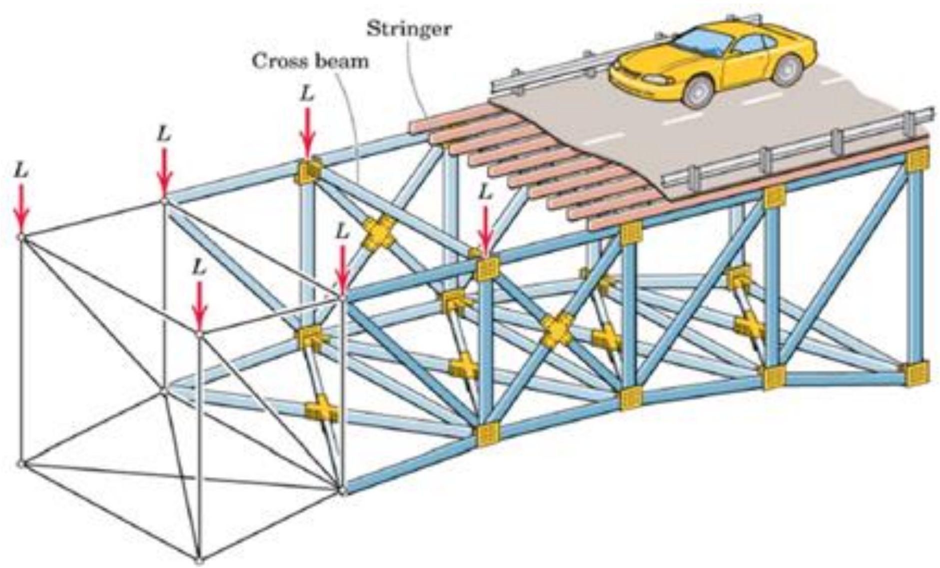
W3 Example 2 (Web view)

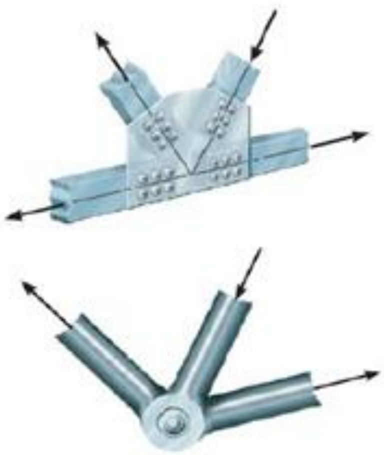
Structural Analysis

Trusses/Frames/Machines/Beams/Cables/

Statically Determinate Structures

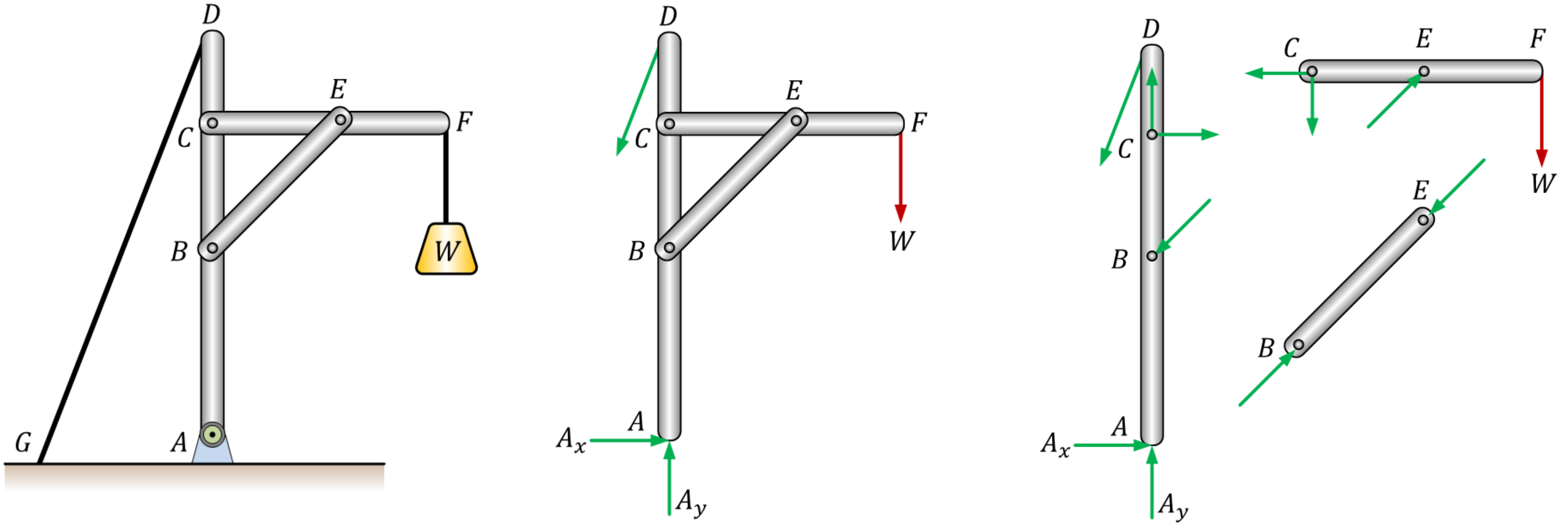
- To determine the internal forces in the structure, dismember the structure and analyse separate free body diagrams of individual members or combination of members





Analysis of Structures (Trusses)

- For the equilibrium of structures made of several connected parts, the *internal forces* as well as *external forces* are considered
- In the interaction between connected parts, Newton's 3rd law states that the *forces of action and reaction* between bodies in contact have the same magnitude, same line of action, and opposite direction



Analysis of Structures (Trusses)

- Three categories of engineering structures are considered:

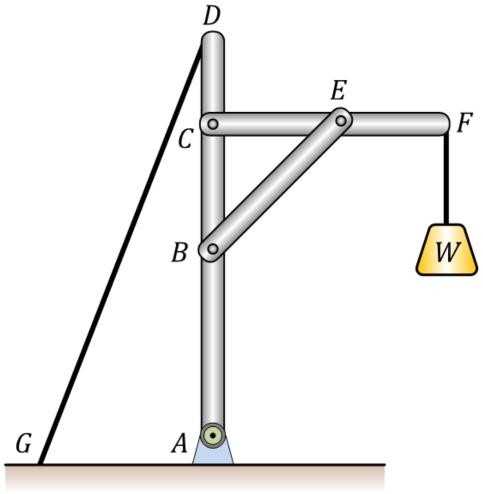
a) Frames: contain at least one multi-force member,

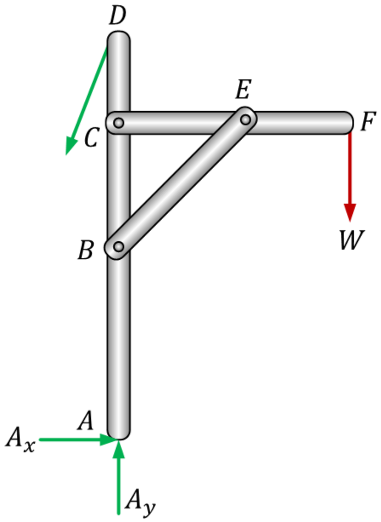
i.e. member acted upon by 3 or more forces

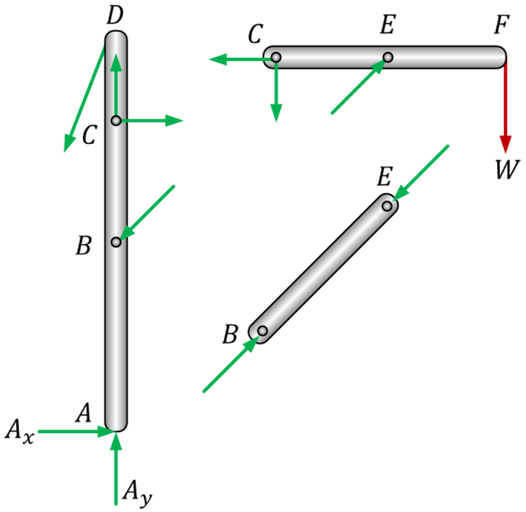
b) Trusses: formed from two-force members

i.e. straight members with end point connections

c) Machines: structures containing moving parts designed to transmit and modify forces

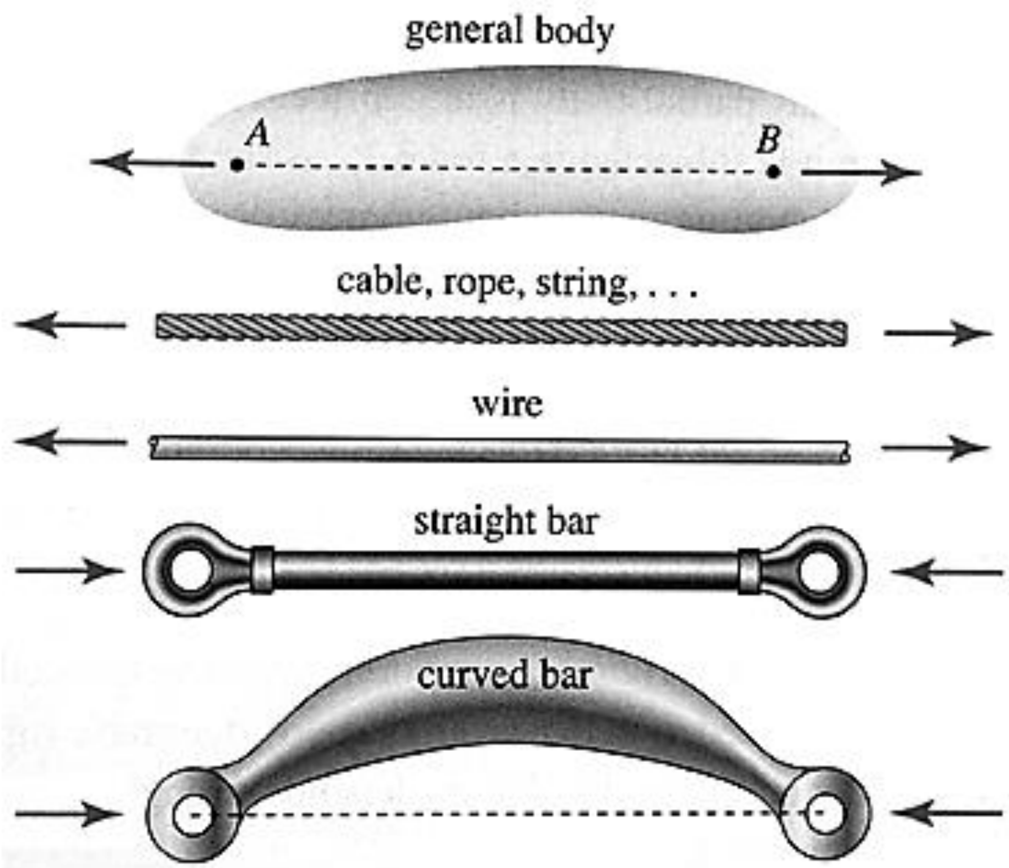






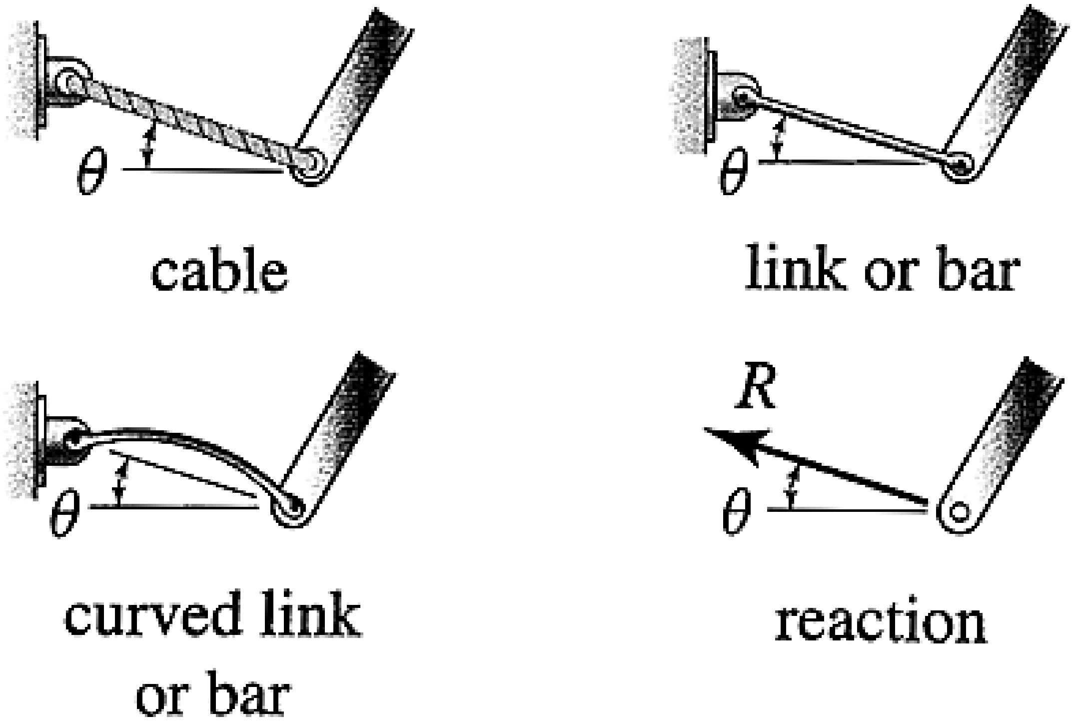
Two force members

- The forces are equal in magnitude and opposite in direction
- Either in *tension* or *compression*

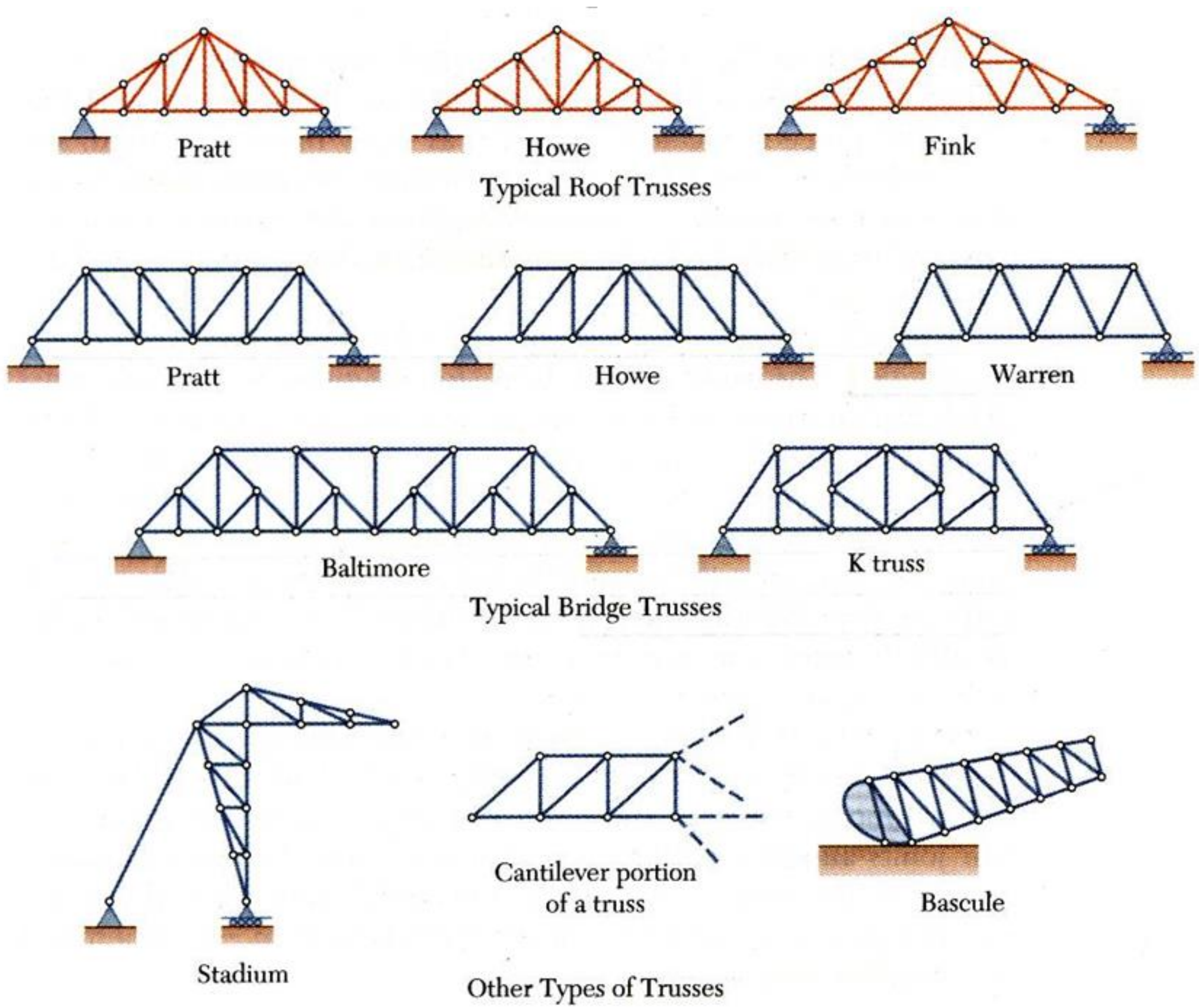


Two force members

- The cable, link or bar and curved link or bar all create a reaction R in the direction θ as there are no distributed forces and no moments applied to the member



Different types of Truss



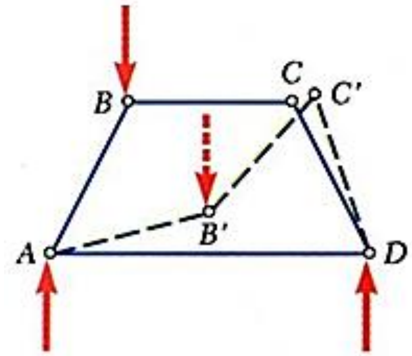
Trusses in real life

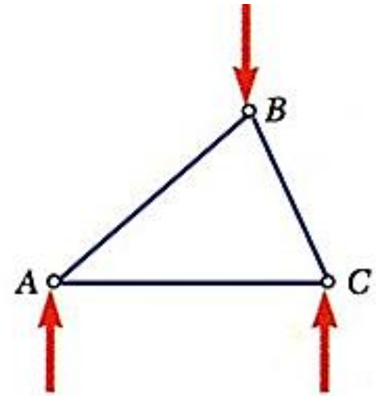
Simple Trusses

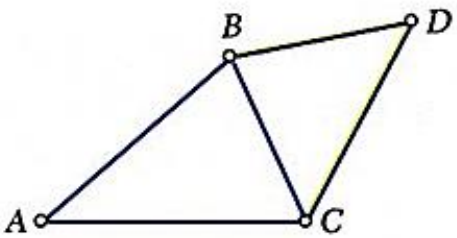
- A *rigid truss* will not collapse under the application of a load

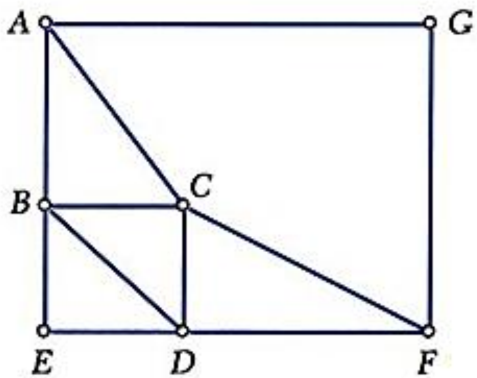
- A simple truss is constructed by successively adding two members and one connection to the basic triangular truss

- In a simple truss $m = 2n - 3$ where m is the total number of members and n is the number of joints







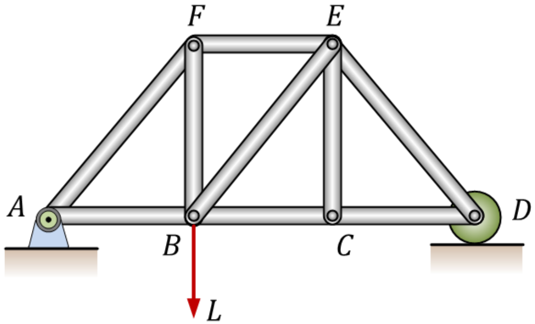


Adopted from J.W. Oler, NCSU, USA

Analysis of Trusses by the Method of Joints

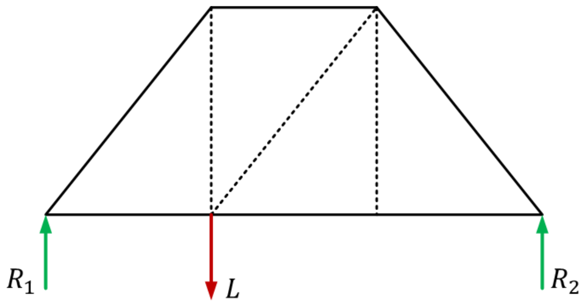
Method of Joints: Conditions of equilibrium are satisfied for the forces at each joint

- ✓ Equilibrium of concurrent forces at each joint
- ✓ Only two independent equilibrium equations are involved



Step of Analysis:

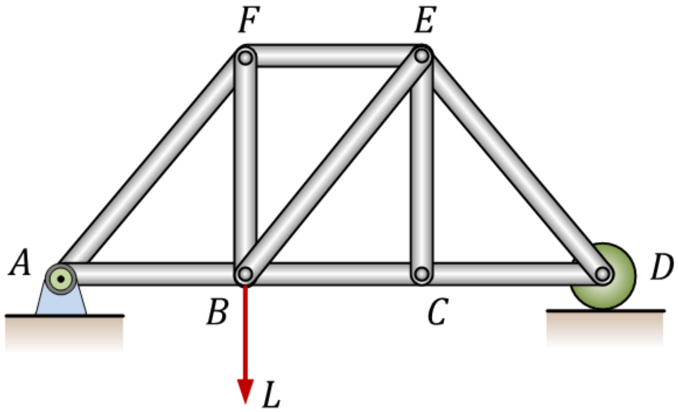
1. Draw FBD of the truss
2. Determine external reaction forces by applying equilibrium equations to the whole truss
3. Perform the force analysis of the remainder of the truss by Method of Joints

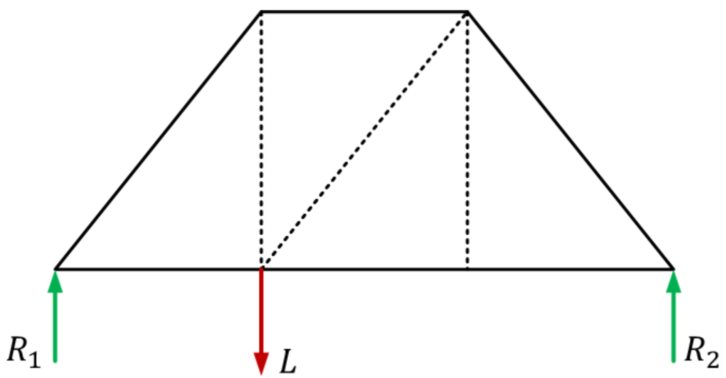


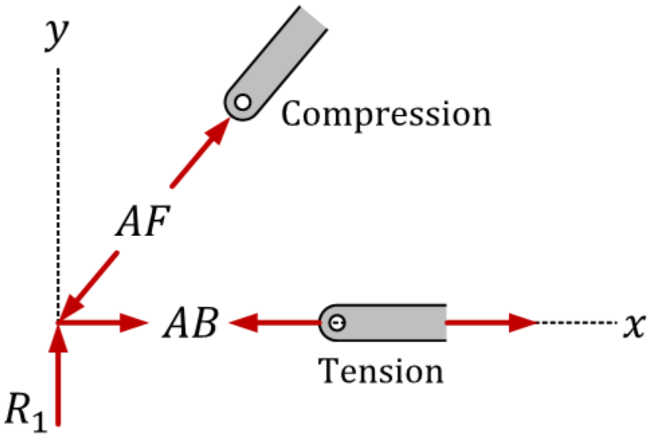
Adopted from Kaustubh Dasgupta

Method of Joints

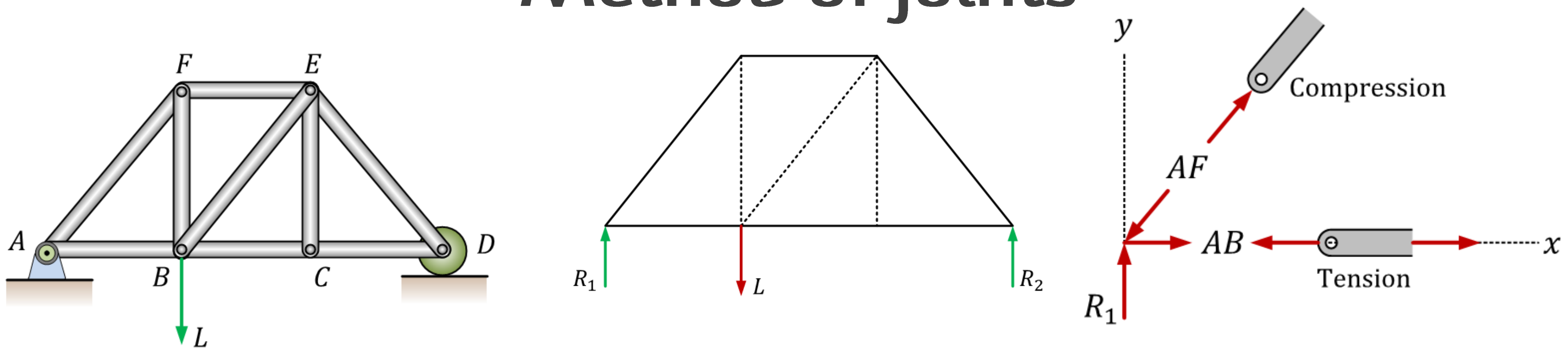
- Start with any joint where at least one known load exists and where not more than two unknown forces are present







Method of Joints



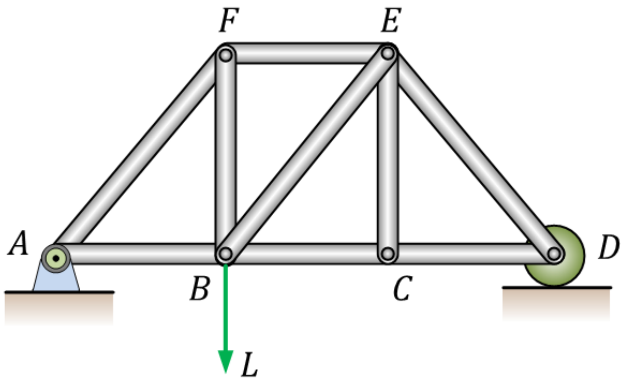
- FBD of joint A and members AB and AF: Magnitude of forces denoted by AB and AF
  - ✓ Tension indicated by an arrow away from the pin
  - ✓ Compression indicated by an arrow towards the pin

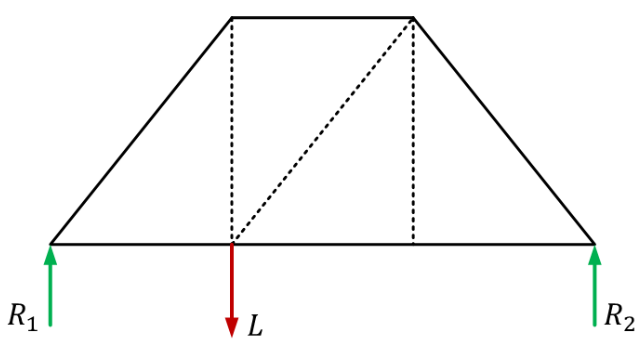
Magnitude of AF from ⟶ $\sum F_y = 0$

Magnitude of AB from ⟶ $\sum F_x = 0$

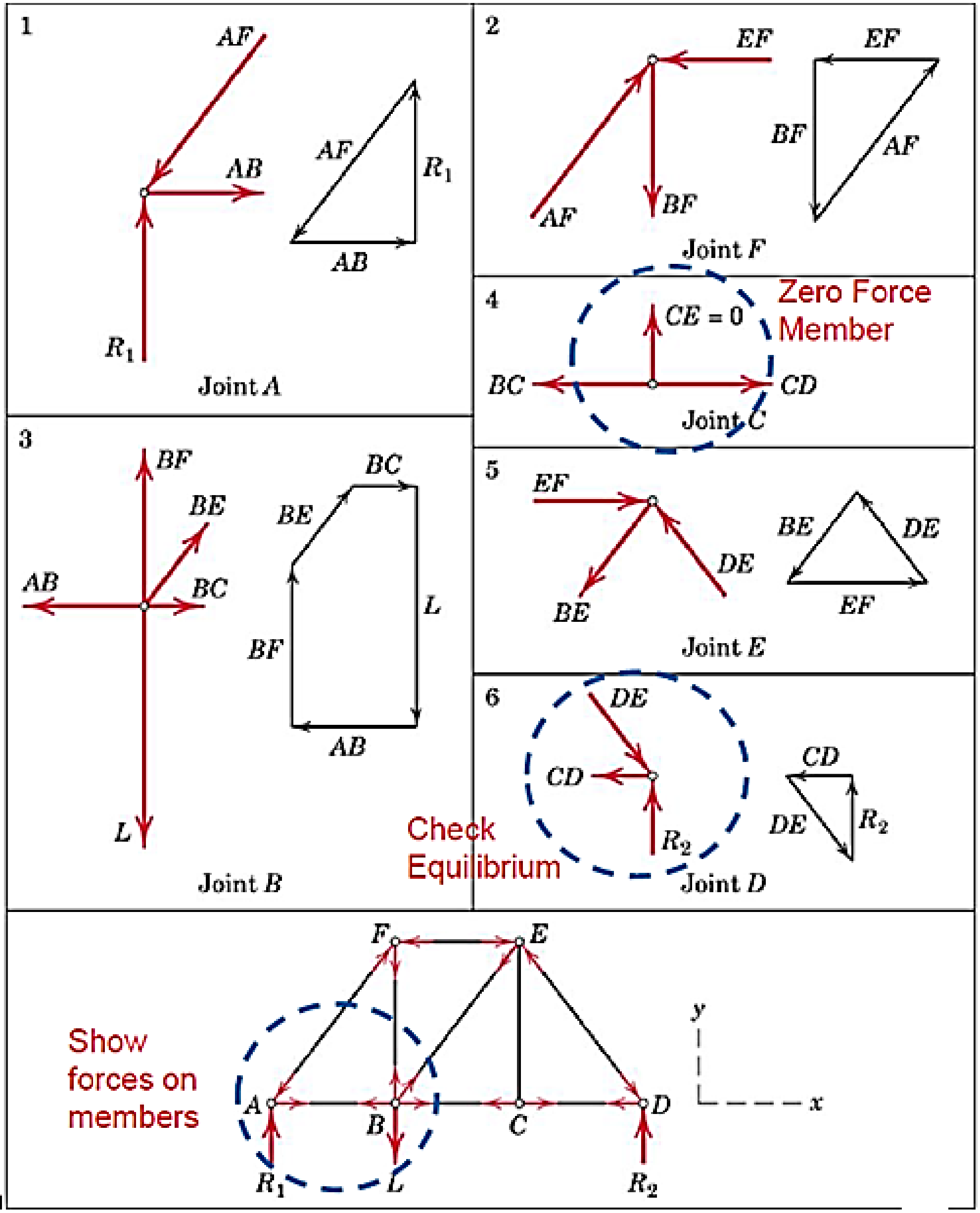
Analyse joints F, B, C, E, and D in that order to complete the analysis

Method of Joints





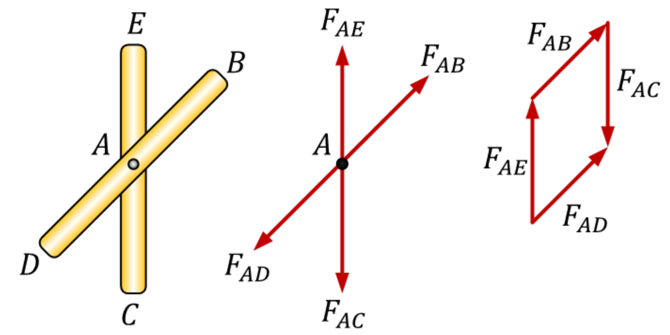
- Negative force if assumed sense is incorrect

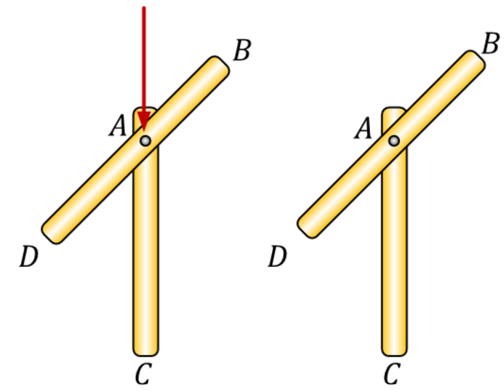


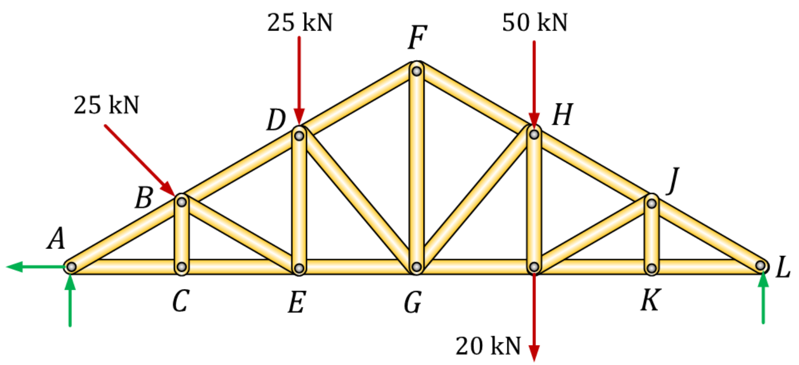
Adopted from Kaust

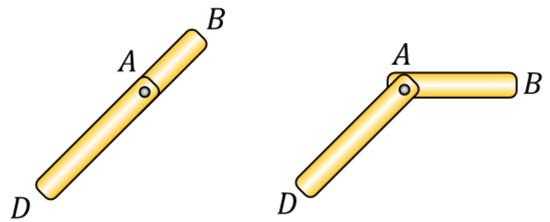
Joints Under Special Loading Conditions

- Forces in opposite members intersecting in two straight lines at a joint are equal
- The forces in two opposite members are equal when a load is aligned with a third member. The third member force is equal to the load
- The forces in two members connected at a joint are equal if the members are aligned and zero otherwise
- Recognition of joints under special loading conditions simplifies a truss analysis



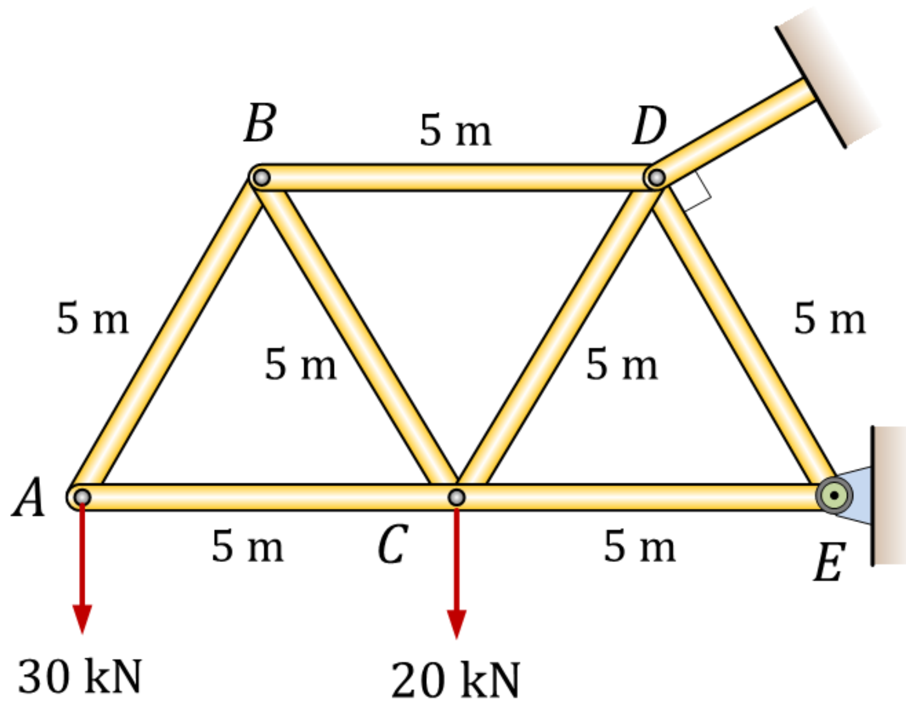






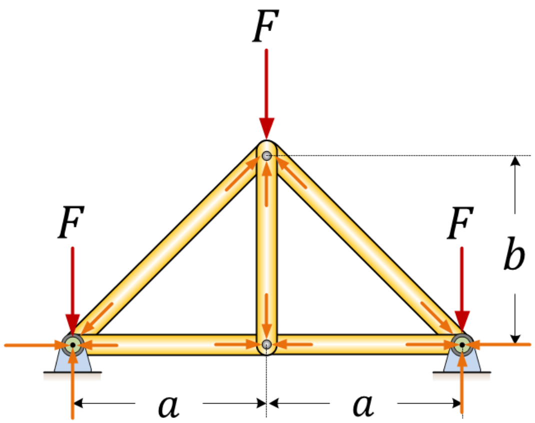
Example 3

Determine the force in member BD and BC of the loaded truss by the **method of joints**



W3 Example 3 (Web view)

Summary

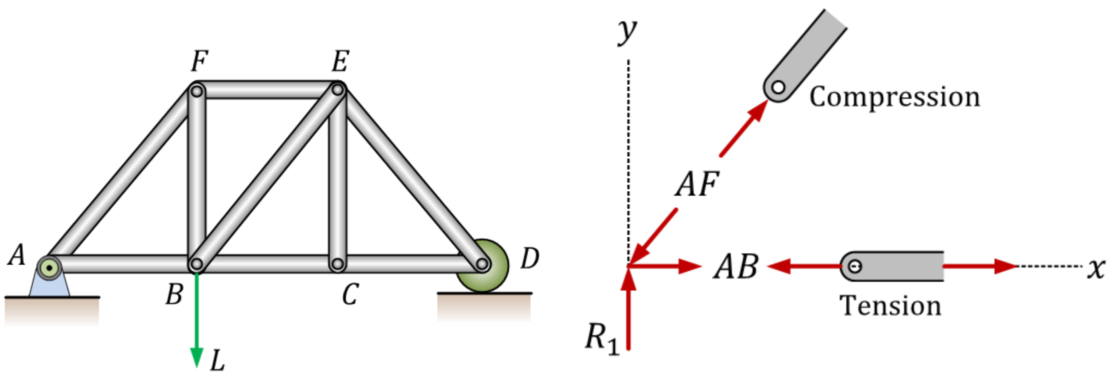


- Equivalent Systems:

$$\mathbf{F}_R = \sum_{i=1}^{n} \mathbf{F}_i$$

$$\mathbf{M}_{RO} = \sum_{i=1}^{m} \mathbf{M}_i + \sum_{i=1}^{n} \mathbf{r}_i \times \mathbf{F}_i$$

- Method of Joints for structures:



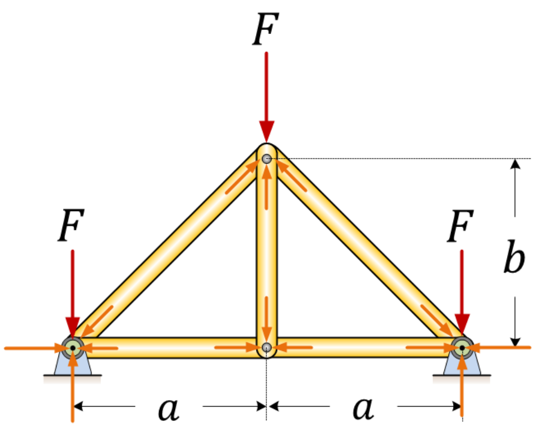
Next Topic:

Determinacy and Method of Sections

School of Mechanical and Manufacturing Engineering

MMAN1300 Engineering Mechanics 1

Dr. David C. Kellermann



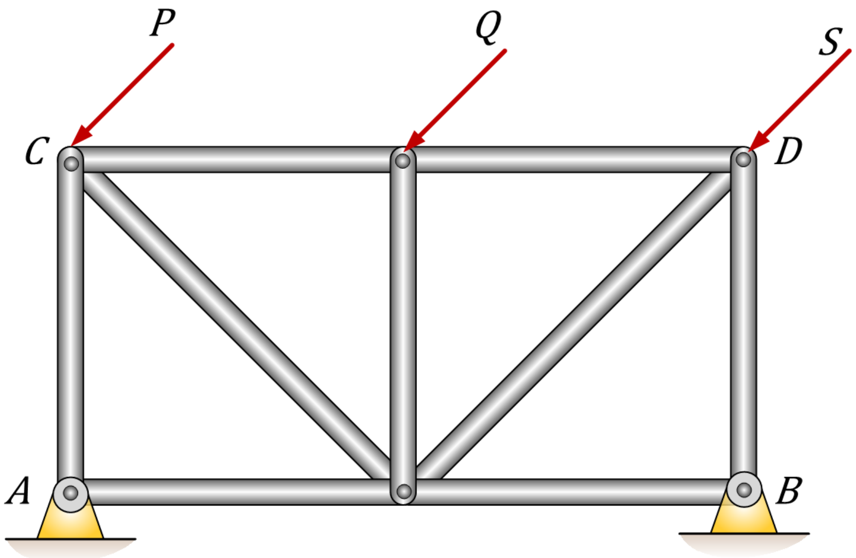
Week 3, L2 – Method of Sections

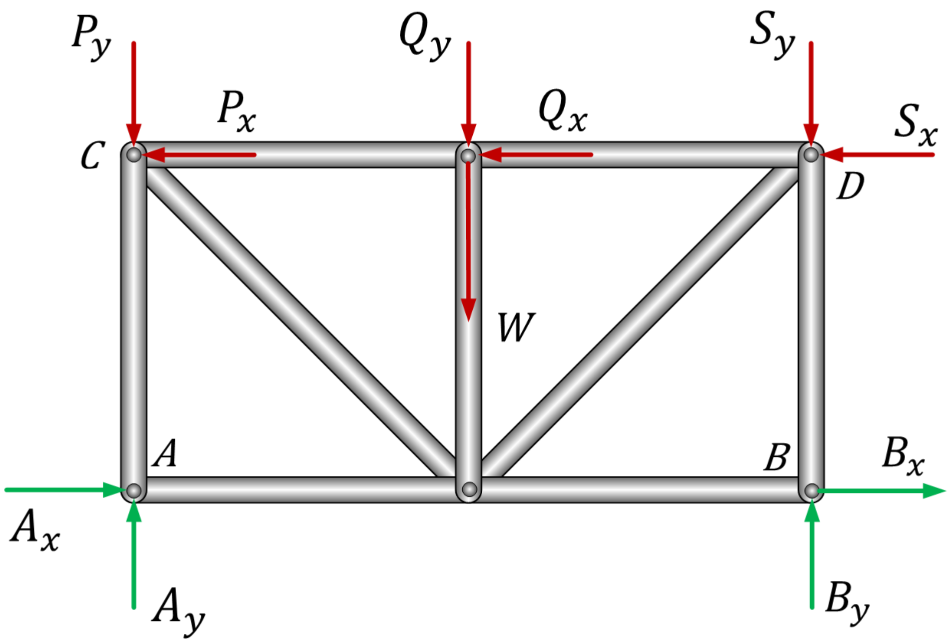
STRUCTURES, TRUSSES AND DETERMINACY

- Equilibrium and FBD summary
- Equivalent systems
- Analysis of Structures
- Trusses - Method of joints
- Static determinacy
- Trusses – Method of sections

Statically Indeterminate Structures

More unknowns than equations: Statically indeterminate



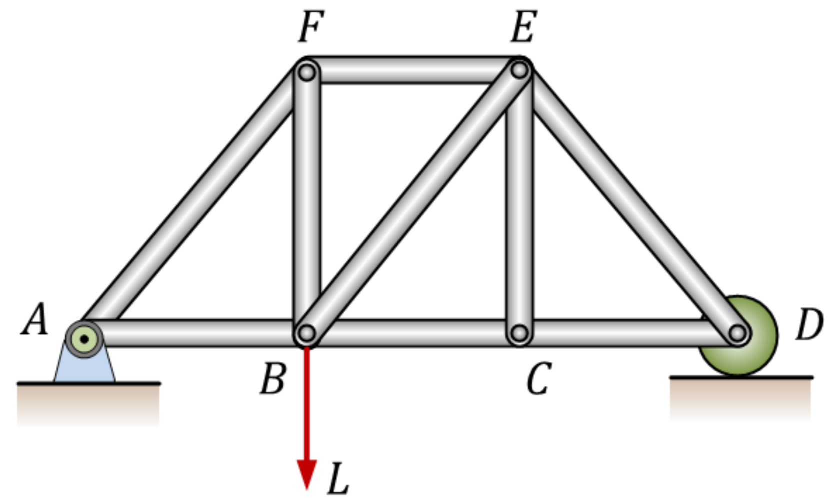


Static Determinacy

No. of unknown reactions = 3

No. of equilibrium equations = 3

> Statically determinate (*External*)



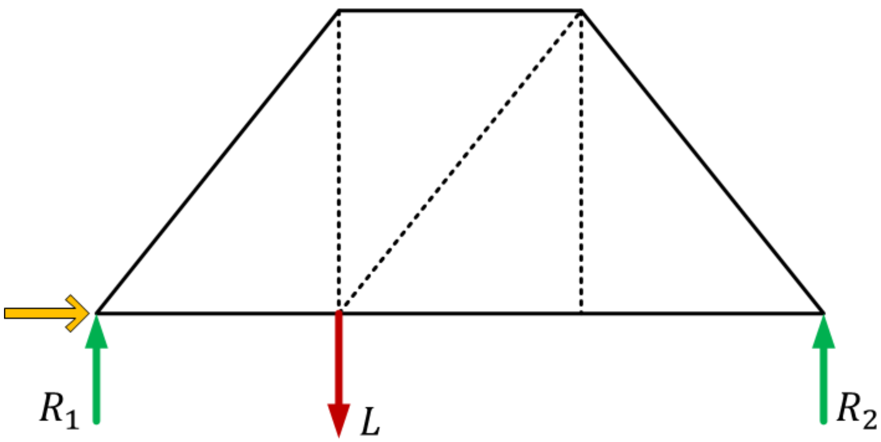
No. of members (m) = 9

No. of joints (j) = 6

No. of unknown reactions (R) = 3

$$m + R = 2j$$

> Statically determinate (*Internal*)



Internal and External Redundancy

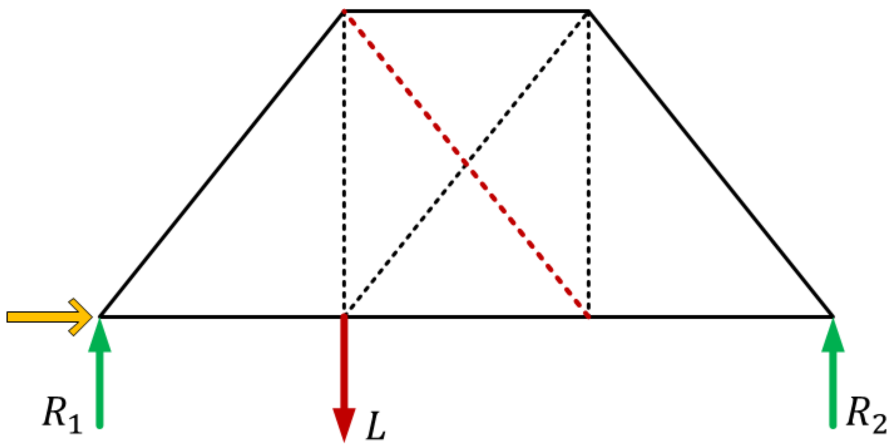
Additional members or supports which are not necessary for maintaining equilibrium configuration are deemed redundant

Extra supports than required

- External redundancy
- Degree of indeterminacy from available equilibrium equations

Extra members than required

- Internal redundancy



Internal Redundancy

Internal redundancy OR

Degree of internal static indeterminacy

Equilibrium of each joint can be specified by two scalar force equations

i.e. $2j$ equations for a truss with j number of joints (known quantities)

For a truss with m number of two force members, and maximum 3 unknown unknown support reactions

Total unknowns = $m + 3$

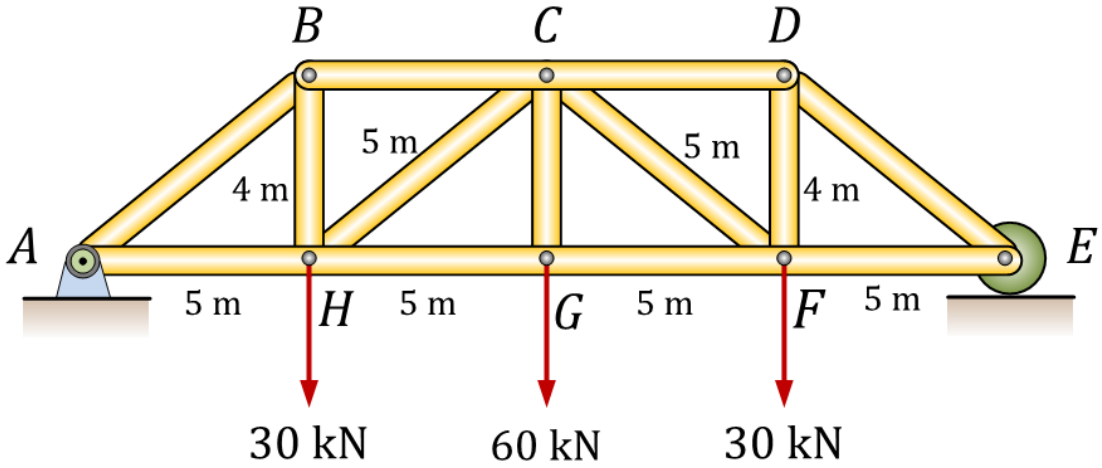
$m + 3 = 2j$ ⟶ Statically determinate internally

$m + 3 > 2j$ ⟶ Statically indeterminate internally

$m + 3 < 2j$ ⟶ Unstable truss

Example 4

Determine the force in member BH of the loaded truss by Method of joints

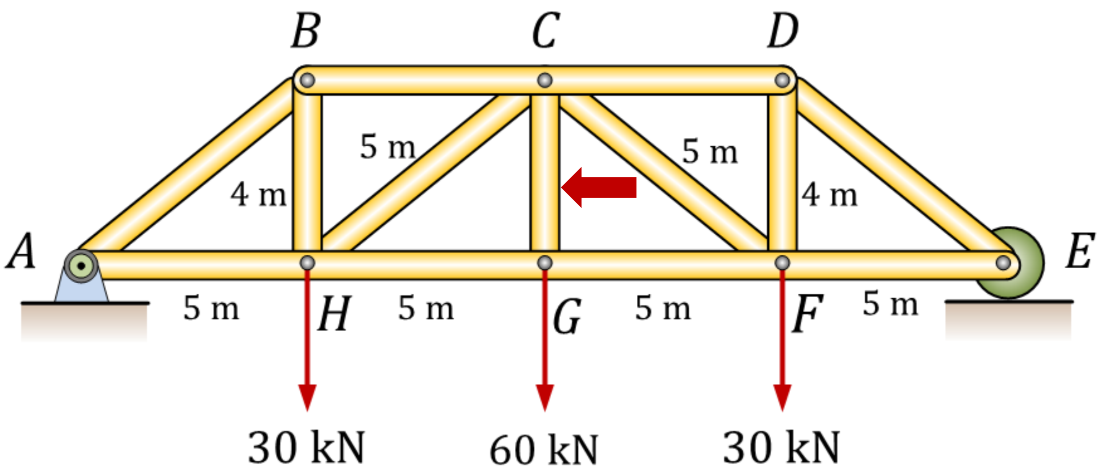


- Is the truss statically determinant externally?
- Is the truss statically determinant internally?
- Are there any zero force members in the truss?

W3 Example 4 (Web view)

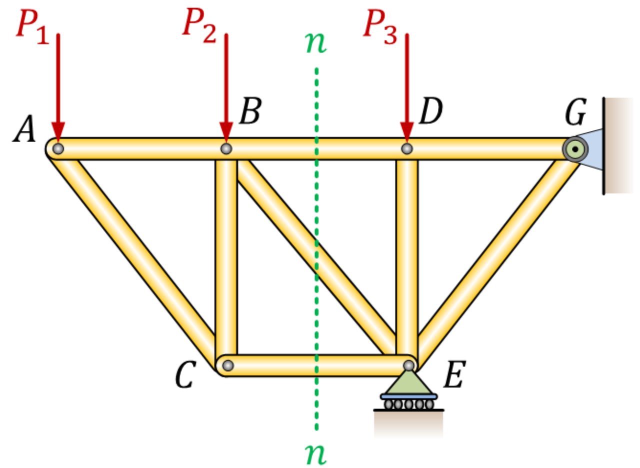
Is there a better way?

What if we wanted to determine the force in member CG?



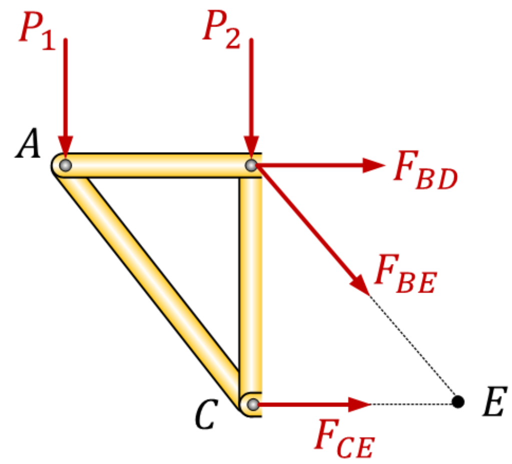
It would be repetitive and time consuming!

Analysis of Trusses by the Method of Sections



- When the force in only one member or the forces in a very few members are desired, the *method of sections* works well

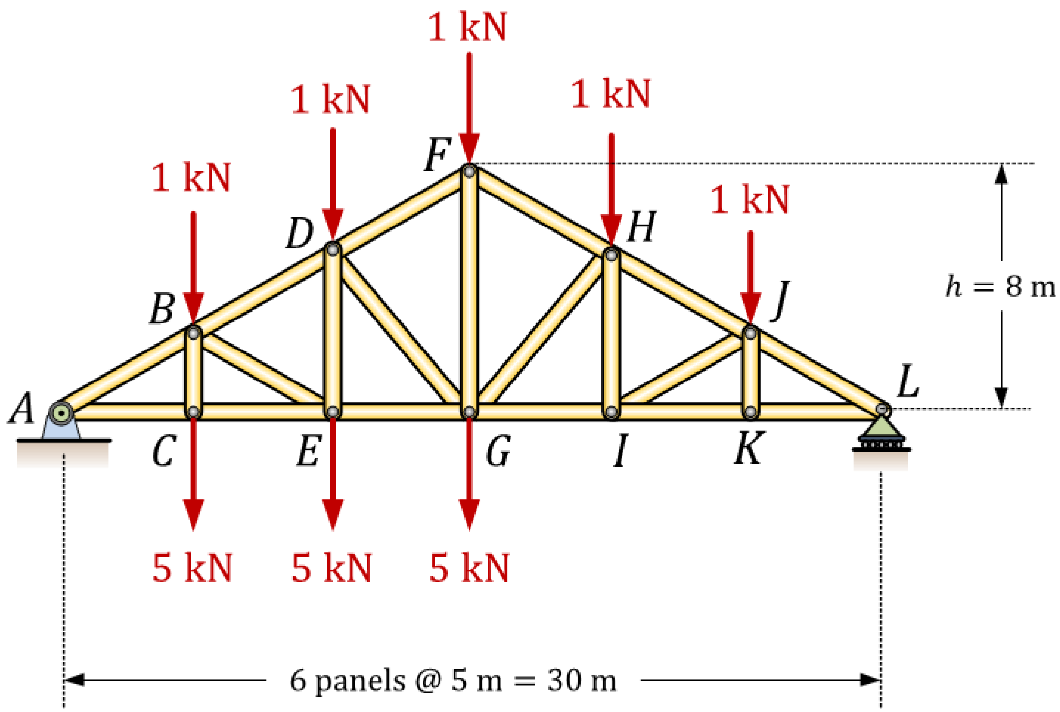
- To determine the force in member *BD*, pass a section through the truss as shown and create a free body diagram for the left side



- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces, including F_{BD}

Example 5

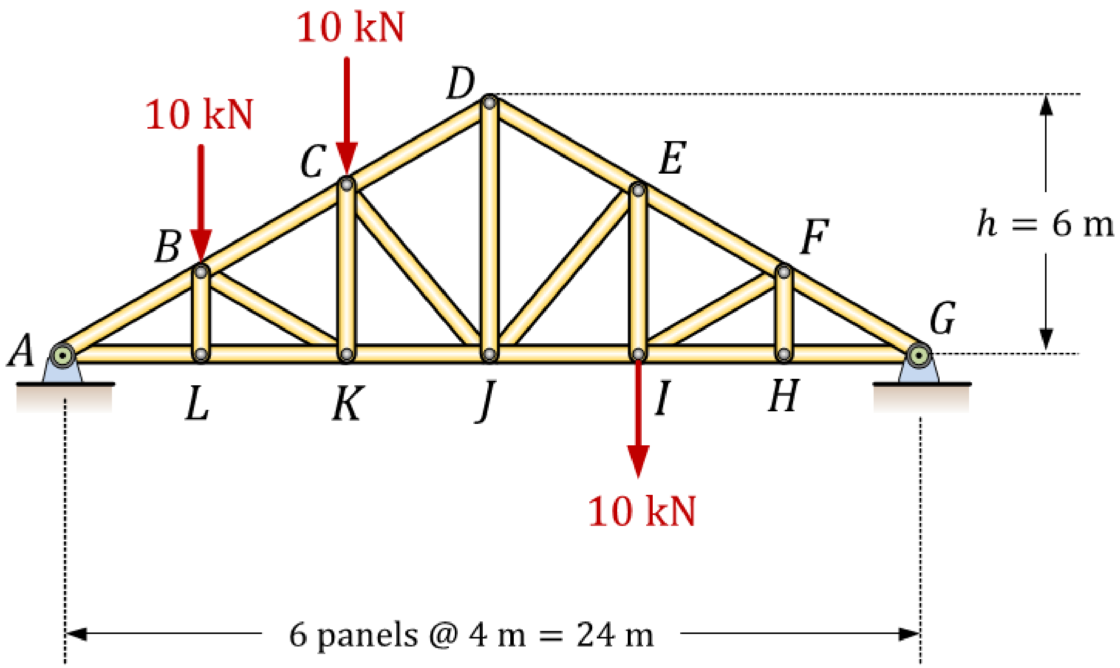
Determine the force in each member *GI* using the method of sections.



W3 Example 5 (Web view)

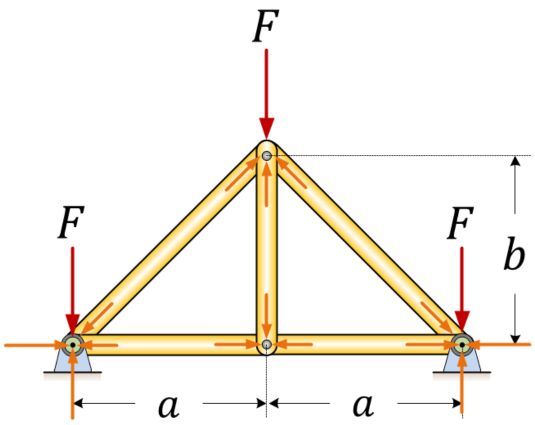
Example 6

Calculate the force in member *DJ* of the loaded truss:



[W3 Example 6](#) ([Web view](#))

Summary



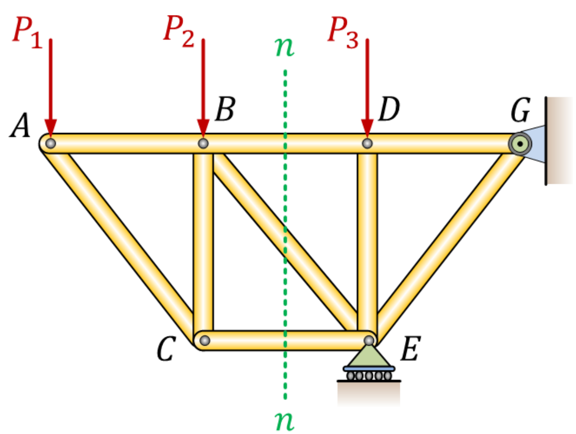
- Static determinacy:

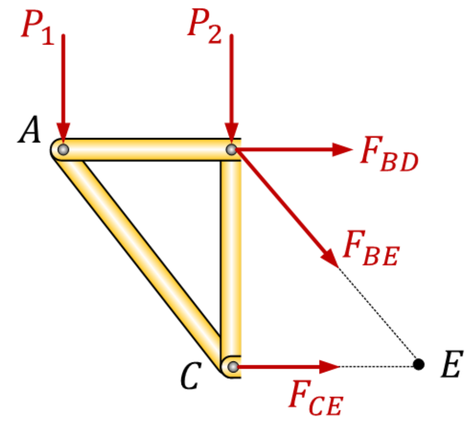
$m + 3 = 2j$ → Determinate internally

$m + 3 > 2j$ → Indeterminate internally

$m + 3 < 2j$ → Unstable truss

- Method of Sections for structures:





Next Topic:

Frames and Machines