UNSW Sydney

SEMESTER 2 2017 EXAMINATIONS

MMAN2300 Engineering Mechanics 2

1. TIME ALLOWED – 2 hours

2. READING TIME – 10 minutes

3. THIS EXAMINATION PAPER HAS 8 PAGES

4. TOTAL NUMBER OF QUESTIONS – 4 (Part A has 2 questions, Part B has 2 questions)

5. TOTAL MARKS AVAILABLE – 100

6. MARKS AVAILABLE FOR EACH QUESTION ARE SHOWN IN THE EXAMINATION PAPER

7. ALL ANSWERS MUST BE WRITTEN IN INK. EXCEPT WHERE THEY ARE EXPRESSLY REQUIRED, PENCILS MAY BE USED ONLY FOR DRAWING, SKETCHING OR GRAPHICAL WORK

8. THIS PAPER MAY BE RETAINED BY CANDIDATE

9. CANDIDATES MAY BRING TO THE EXAMINATION A UNSW APPROVED CALCULATOR

SPECIAL INSTRUCTIONS

10. THE EXAMINATION PAPER HAS TWO PARTS (Part A and Part B). ANSWER THESE TWO PARTS IN SEPARATE BOOKLETS

Part A – Vibration Analysis

QUESTION 1 [25 marks]

A two-story building frame is modelled as shown in Figure Q1. The girders are assumed to be rigid. The columns have flexural rigidities EI_1 and EI_2, with negligible masses. The stiffness of each column can be computed as:

$$k_i = \frac{24EI_i}{h_i^3} \qquad i = 1, 2$$

For $m_1 = 2m$, $m_2 = m$, $h_1 = h_2 = h$ and $EI_1 = EI_2 = EI$, determine the natural frequencies and modeshapes of the frame. Sketch the modeshapes.

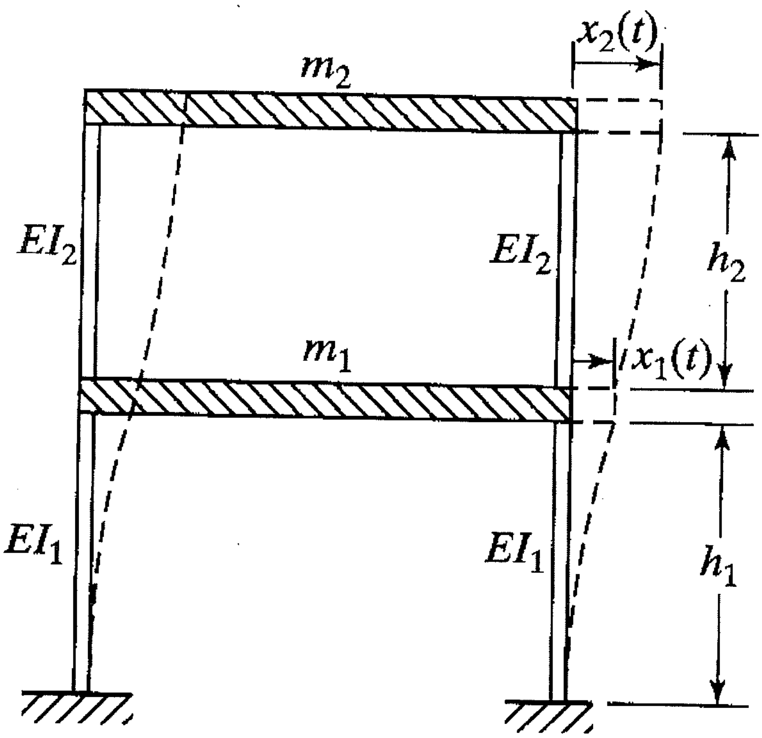


Figure Q1

QUESTION 2 [25 marks]

A cable of length l and mass per unit length m_L is stretched under tension P. One end of the cable is connected to a mass m which can move in a frictionless slot, and the other end is fastened to a spring of stiffness k, as shown in Figure Q2. Derive the frequency equation for the transverse vibration of the cable.

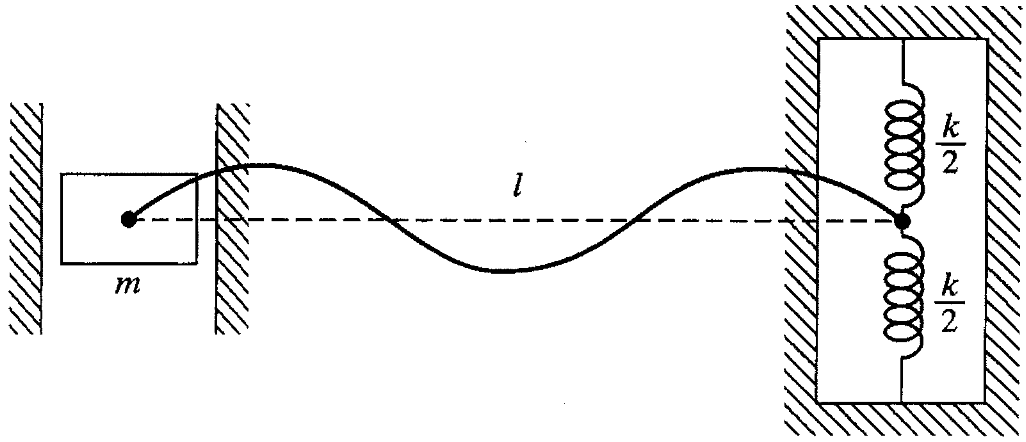


Figure Q2

END of PART A

Part B – Dynamics of Rigid Bodies

Answer Part B in a separate book

QUESTION 3 [25 marks]

The collar at B slides along the circular bar, causing pin B to move at constant speed $v_0 = 1.0$ m/s in the circular part of radius R. Bar BC slides in the collar at A. At the instant shown in Figure Q3, determine:

(a) the angular velocity (magnitude and direction) of bar BC,
(b) the linear velocity (magnitude and direction) of bar BC relative to the collar at A,
(c) the angular acceleration (magnitude and direction) of bar BC.

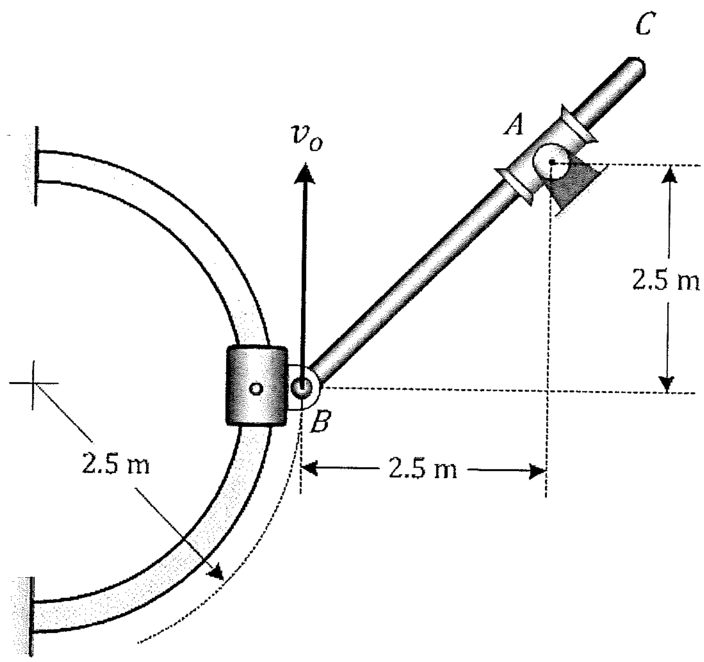


Figure Q3

QUESTION 4 [25 marks]

The slender bar AB weighs 40 kg and the crate C weighs 80 kg. At the instant shown in Figure Q4, the system is at rest and a moment M is applied to the bar AB. The crate C has an acceleration of 2 m/s^2 to the left. The coefficient of kinetic friction between the horizontal surface and the crate is $\mu_k = 0.2$. Determine:

(a) the tension in the rope,
(b) the angular acceleration of the slender bar AB,
(c) the moment M applied to the slender bar AB.

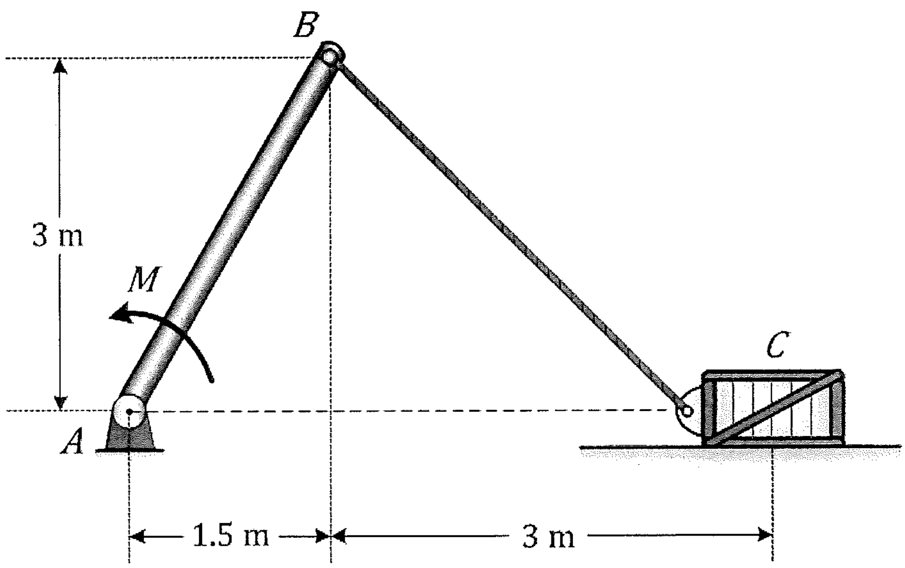


Figure Q4

END of PART B

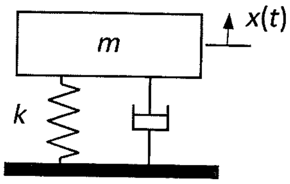
END OF EXAMINATION PAPER

EQUATION SHEET FOR PART A

Spring-Mass-Damper Systems

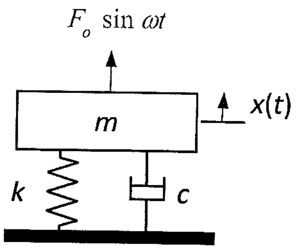
$$\omega = 2\pi f \qquad \omega_n = \sqrt{\frac{k}{m}} \qquad \omega_d = \omega_n\sqrt{1-\zeta^2} \qquad \zeta = \frac{c}{2\sqrt{km}}$$

$$\delta = \ln\left(\frac{x_n}{x_{n+1}}\right) \qquad \delta = \frac{1}{n}\ln\left(\frac{x_1}{x_{n+1}}\right) \qquad \zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} \qquad r = \frac{\omega}{\omega_n}$$



$$m\ddot{x} + c\dot{x} + kx = 0$$

$$x(t) = Xe^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$$

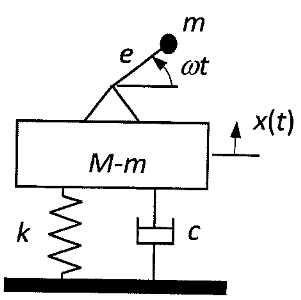


$$m\ddot{x} + c\dot{x} + kx = F_o\sin\omega t$$

$$x(t) = X\sin(\omega t - \phi)$$

$$X = \frac{F_o/k}{\sqrt{\left(1-r^2\right)^2+\left(2\zeta r\right)^2}}; \qquad \phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$$

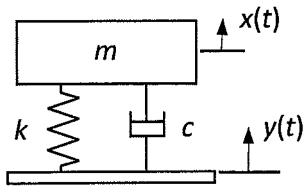
$$TR = \frac{F_T}{F_o} = \frac{\sqrt{1+\left(2\zeta r\right)^2}}{\sqrt{\left(1-r^2\right)^2+\left(2\zeta r\right)^2}}$$



$$M\ddot{x} + c\dot{x} + kx = me\omega^2\sin\omega t$$

$$x(t) = X\sin(\omega t - \phi)$$

$$\frac{MX}{me} = \frac{r^2}{\sqrt{\left(1-r^2\right)^2+\left(2\zeta r\right)^2}}; \qquad \phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$$



$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

$$x(t) = X\sin(\omega t - \phi)$$

$$\frac{X}{Y} = \sqrt{\frac{1+(2\zeta r)^2}{\left(1-r^2\right)^2 + (2\zeta r)^2}}\,; \quad \frac{F_T}{kY} = r^2\frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{\left(1-r^2\right)^2 + (2\zeta r)^2}}$$

Continuous Systems

Lateral vibration of a taut string

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{c_s^2}\frac{\partial^2 u(x,t)}{\partial t^2}, \qquad c_s = \sqrt{\frac{T}{m_L}}$$

$u(x,t) = \phi(x)q(t)$, $\quad \phi(x) = C\sin kx + D\cos kx$, $\quad q(t) = A\sin\omega t + B\cos\omega t$

Boundary Conditions	$x=0$	$x=L$
Fixed	$u=0$	$u=0$

Longitudinal vibrations of rods

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{c_L^2}\frac{\partial^2 u(x,t)}{\partial t^2}, \qquad c_L = \sqrt{\frac{E}{\rho}}$$

Boundary Conditions	$x=0$	$x=L$
Fixed	$u=0$	$u=0$
Free	$\frac{\partial u}{\partial x}=0$	$\frac{\partial u}{\partial x}=0$
Spring load	$k_s u = EA\frac{\partial u}{\partial x}$	$k_s u = -EA\frac{\partial u}{\partial x}$
Inertia load	$M\frac{\partial^2 u}{\partial t^2} = EA\frac{\partial u}{\partial x}$	$M\frac{\partial^2 u}{\partial t^2} = -EA\frac{\partial u}{\partial x}$

Torsional vibration of circular rods

$$\frac{\partial^2\theta(x,t)}{\partial x^2}=\frac{1}{c_T^2}\frac{\partial^2\theta(x,t)}{\partial t^2}, \qquad c_T=\sqrt{\frac{G\gamma}{\rho J_p}}$$

Boundary Conditions	$x=0$	$x=L$
Fixed	$\theta=0$	$\theta=0$
Free	$\dfrac{\partial\theta}{\partial x}=0$	$\dfrac{\partial\theta}{\partial x}=0$
Spring load	$k_t\theta=G\gamma\dfrac{\partial\theta}{\partial x}$	$k_t\theta=-G\gamma\dfrac{\partial\theta}{\partial x}$
Inertia load	$J\dfrac{\partial^2\theta}{\partial t^2}=G\gamma\dfrac{\partial\theta}{\partial x}$	$J\dfrac{\partial^2\theta}{\partial t^2}=-G\gamma\dfrac{\partial\theta}{\partial x}$

EQUATION SHEET FOR PART B

Constant Linear Acceleration Equations $(t_o = 0)$

$v = v_o + at$

$v^2 = v_o^2 + 2a(s - s_o)$

$s = s_o + v_o t + \frac{1}{2}at^2$

Angular Motion

$\omega = \frac{d\theta}{dt}$ $\alpha = \frac{d\omega}{dt}$ $\omega d\omega = \alpha d\theta$

Displacement, Velocity and Acceleration Components

(1) Rectangular Co-ordinates

$\boldsymbol{r} = x\mathbf{i} + y\mathbf{j}$

$\boldsymbol{v} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$

$\boldsymbol{a} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$

(2) Normal and Tangential Co-ordinates

$\boldsymbol{v} = v\boldsymbol{e}_t$

$\boldsymbol{a} = a_t\boldsymbol{e}_t + a_n\boldsymbol{e}_n$

$v = \omega r$

$a_t = \dot{v} = \alpha r$

$a_n = \frac{v^2}{\rho} = \omega^2 r$

Relative Motion

$\boldsymbol{r}_A = \boldsymbol{r}_B + \boldsymbol{r}_{A/B}$

$\boldsymbol{v}_A = \boldsymbol{v}_B + \boldsymbol{v}_{A/B}$

$\boldsymbol{a}_A = \boldsymbol{a}_B + \boldsymbol{a}_{A/B}$

Relative Motion for Rotating Axes

$\boldsymbol{v}_{\mathrm{A}} = \boldsymbol{v}_{\mathrm{A'}} + \boldsymbol{v}_{\mathrm{A/A'}}$

$\boldsymbol{a}_{\mathrm{A}} = \boldsymbol{a}_{\mathrm{A'}} + \boldsymbol{a}_{\mathrm{rel}} + \boldsymbol{a}_{\mathrm{cor}}$

Moment of Inertia

Moment of inertia of a uniform disk: $I = \frac{1}{2}mr^2$

Moment of inertia of a uniform slender rod: $I = \frac{1}{12}ml^2$

Parallel axis theorem: $I_O = I_G + md^2$