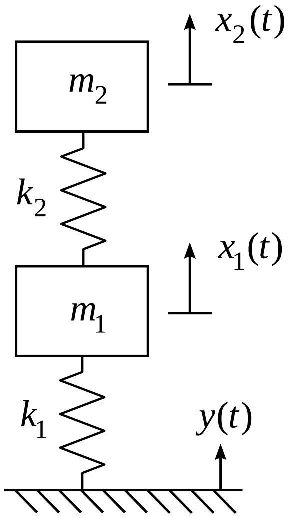
MMAN2300

Engineering Mechanics 2

Part B: Vibration Analysis

Base excitation of a 2DOF spring-mass system

Forced Vibration of a 2DOF Spring-Mass System due to Base Excitation



In base excitation, the input disturbance $y(t)$ is a displacement. Find expressions for the displacement transmissibility of each mass as a function of frequency and sketch the responses.

Equations of motion

$$m_1\ddot{x}_1 = -k_1x_1 - k_2x_1 + k_2x_2 + k_1y$$

$$m_2\ddot{x}_2 = -k_2x_2 + k_2x_1$$

For harmonic motion, use general solutions of the form

$$y(t) = A_o \sin\omega t\,, \quad x_1(t) = A_1 \sin\omega t\,, \quad x_2(t) = A_2 \sin\omega t$$

The displacement transmissibility ratio of each mass is $\dfrac{A_1}{A_o}$, $\dfrac{A_2}{A_o}$ (similar to $\dfrac{X}{Y}$ for a single DOF undamped system).

For $m_1 = m_2 = m$ and $k_1 = k_2 = k$

$$\Rightarrow \quad \omega_{n1} = 0.618\sqrt{k/m}\,, \qquad \frac{A_1}{A_2} = 0.618$$

$$\Rightarrow \quad \omega_{n2} = 1.618\sqrt{k/m}\,, \qquad \frac{A_1}{A_2} = -1.618$$

Substituting the general solutions into the equations of motion results in

$$(2k - m\omega^2)A_1 - kA_2 = kA_o \qquad (1)$$

$$-kA_1 + (k - m\omega^2)A_2 = 0 \qquad (2)$$

From (2),

$$A_2 = \frac{kA_1}{k - m\omega^2} \qquad (3)$$

Substitute (3) in (1) gives

$$(2k - m\omega^2)A_1 - \frac{k^2 A_1}{k - m\omega^2} = kA_o \qquad (4)$$

Rearrange (4)

$$\Rightarrow \qquad \frac{A_1}{A_o} = \frac{k(k - m\omega^2)}{m^2\omega^4 - 3km\omega^2 + k^2} \qquad (5)$$

Note: the denominator corresponds to the characteristic equation of the system.

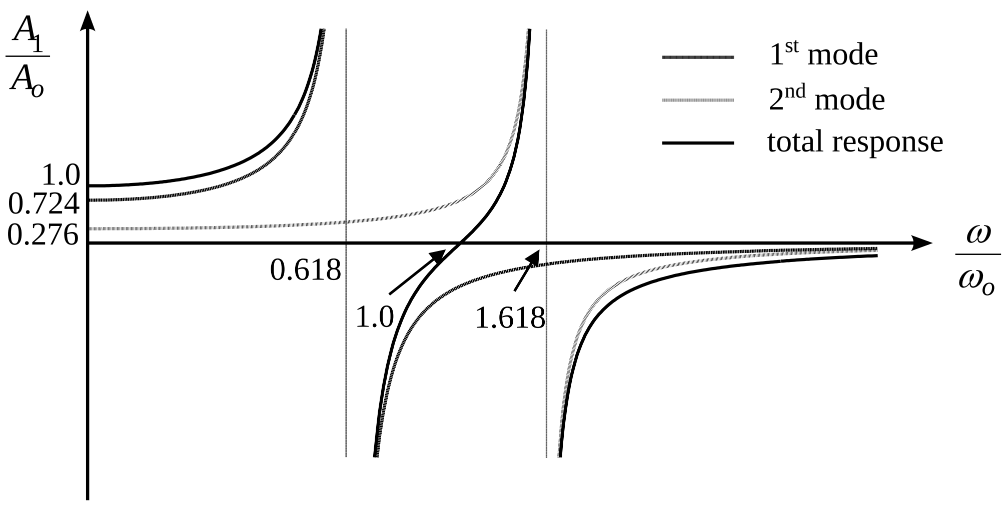
- Divide by k^2 / k^2
- Put $\omega_o = \sqrt{k/m}$ (ω_o is just a reference frequency)
- Expand into partial fractions as a function of the two ω_n's of the system

$$\frac{A_1}{A_o} = \frac{1 - \left(\frac{\omega}{\omega_o}\right)^2}{\left(\frac{\omega}{\omega_o}\right)^4 - 3\left(\frac{\omega}{\omega_o}\right)^2 + 1} = \frac{C_1}{1 - \left(\frac{\omega}{\omega_{n1}}\right)^2} + \frac{D_1}{1 - \left(\frac{\omega}{\omega_{n2}}\right)^2} \qquad (6)$$

Using the method shown in Week 7 Lecture Notes, we get $C_1 = 0.724$, $D_1 = 0.276$

$$\Rightarrow \qquad \frac{A_1}{A_o} = \frac{1 - \left(\frac{\omega}{\omega_o}\right)^2}{\left(\frac{\omega}{\omega_o}\right)^4 - 3\left(\frac{\omega}{\omega_o}\right)^2 + 1} = \frac{0.724}{1 - \left(\frac{\omega}{\omega_{n1}}\right)^2} + \frac{0.276}{1 - \left(\frac{\omega}{\omega_{n2}}\right)^2}$$

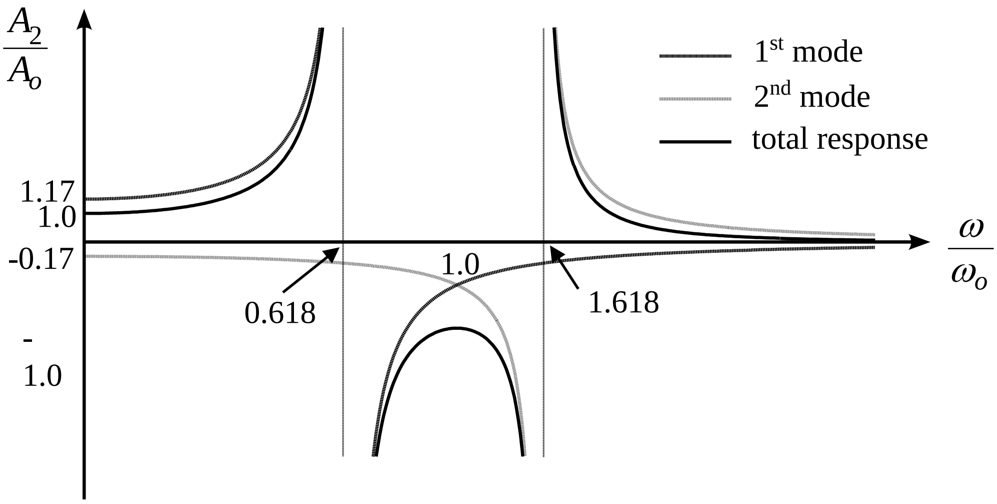
Note $\frac{A_1}{A_o} = 0$ when $\frac{\omega}{\omega_o} = 1$



To find $\dfrac{A_2}{A_o}$

$$\frac{A_2}{A_o}=\frac{A_1}{A_o}.\frac{A_2}{A_1}=\frac{k(k-m\omega^2)}{m^2\omega^4-3km\omega^2+k^2}.\frac{k}{k-m\omega^2}=\frac{k^2}{m^2\omega^4-3km\omega^2+k^2}$$

$$\Rightarrow\qquad \frac{A_2}{A_o}=\frac{1}{\left(\dfrac{\omega}{\omega_o}\right)^4-3\left(\dfrac{\omega}{\omega_o}\right)^2+1}=\frac{1.17}{1-\left(\dfrac{\omega}{\omega_{n1}}\right)^2}-\frac{0.17}{1-\left(\dfrac{\omega}{\omega_{n2}}\right)^2}$$



When $\omega=0$, $\dfrac{A_2}{A_o}=1$

When $\dfrac{\omega}{\omega_o}=1$, $\dfrac{A_2}{A_o}=-1$

We can also check the modeshapes $\dfrac{A_1}{A_2}$ for each ω_n

For $\omega=\omega_{n1}=0.618\sqrt{k/m}$, we know $\dfrac{A_1}{A_2}=0.618$

$$\frac{A_1}{A_2}=\frac{A_1}{A_o}.\frac{A_o}{A_2}=\frac{0.724}{1-\left(\omega/\omega_{n1}\right)^2}.\frac{1-\left(\omega/\omega_{n1}\right)^2}{1.17}=0.618$$

For $\omega=\omega_{n2}=1.618\sqrt{k/m}$, we know $\dfrac{A_1}{A_2}=-1.618$

$$\frac{A_1}{A_2}=\frac{A_1}{A_o}.\frac{A_o}{A_2}=\frac{0.276}{1-\left(\omega/\omega_{n2}\right)^2}.\frac{1-\left(\omega/\omega_{n2}\right)^2}{(-0.17)}=-1.618$$