MMAN2300

Engineering Mechanics 2

Part A: Week 9

Gear systems and gear analysis

(Chapters 10-12 Waldron & Kinzel)

1. Introduction
   - Gears are among the oldest devices and inventions.
   - Gears are used to transmit power from one shaft to another, and generally the velocity ratio is constant. Their power transmission efficiency is high.
   - Profiles of meshing gear teeth that give a constant velocity ratio are called conjugate.

   We will introduce gear terminology, gear types and the basic concept of velocity analysis of gears.

2. Gear terminology
   - N: no of teeth on a gear
   - Pitch circle diameter (d): diameter at which the gear meshes (contacts) with another gear
   - Contact point: at which the teeth of 2 gears contact
   - Pitch point (P): as the gears rotate, the common normal to the surfaces at the point of contact always intersects the line of centres at the same point P, called the pitch point.
   - Circular pitch (p_c): the curvilinear distance measured on the pitch circle from a point on one tooth to the corresponding point on the next tooth

     $$p_c = \frac{\pi d}{N}$$

   - Diametral pitch (P): the number of teeth on the gear per unit of pitch diameter

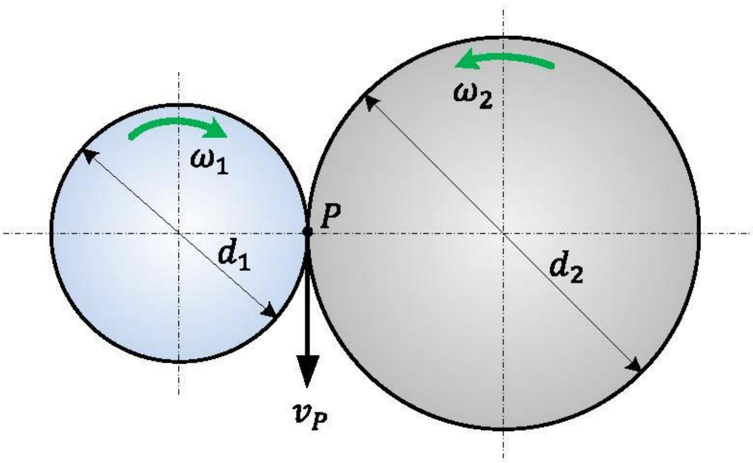
     $$P = \frac{N}{d} = \frac{\pi}{p_c}$$

   - Module (M)

     $$M = \frac{d}{N}$$

     Two gears in mesh must have the same module.

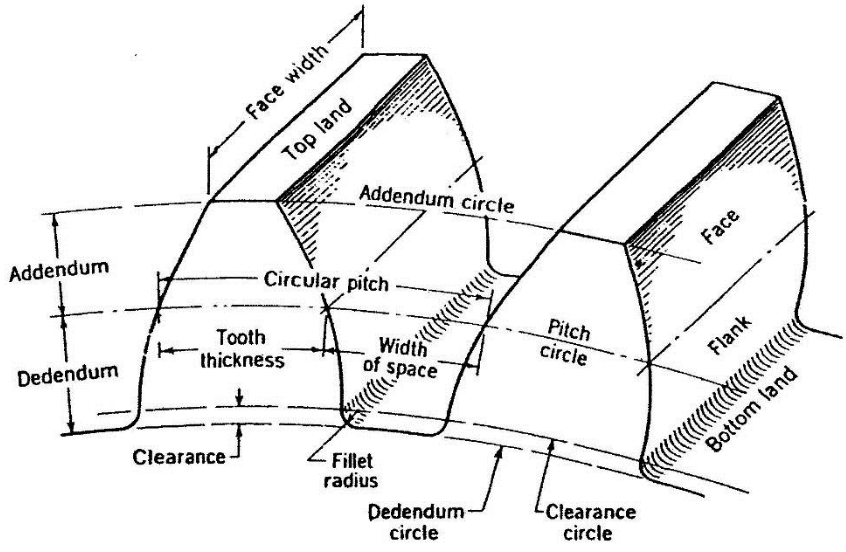
     It is possible to replace two gears by two discs of pitch point diameter.



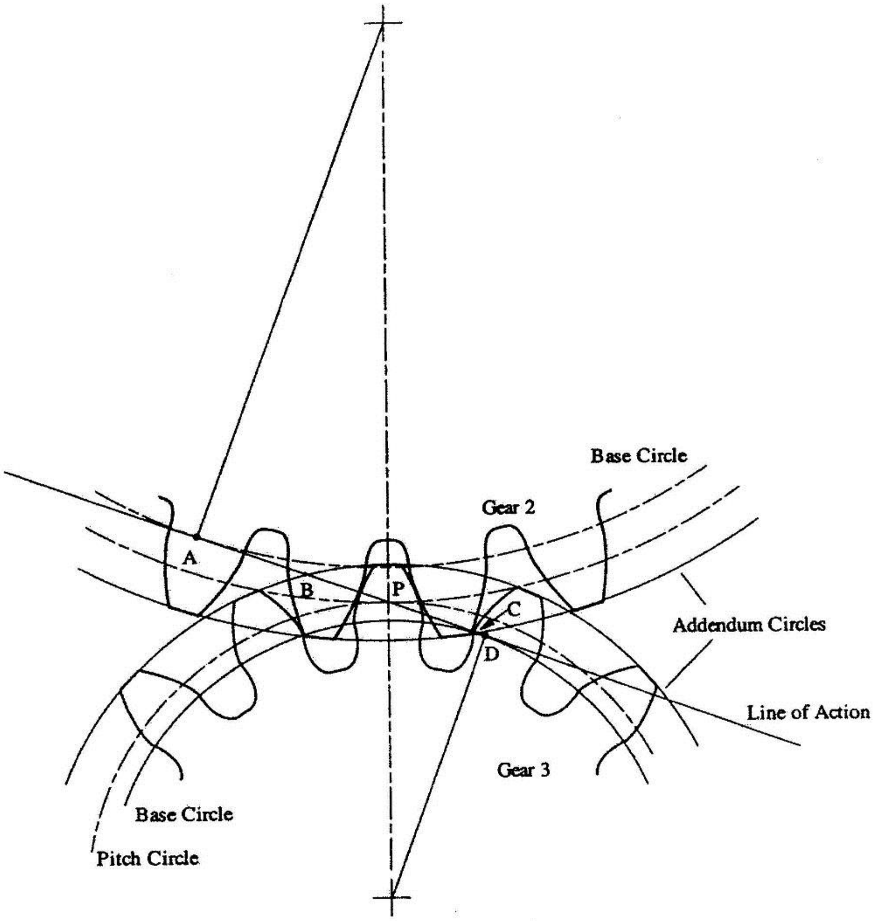
For no slipping, the velocity at the point of contact must be the same on both gears.

$$v_p = \omega_1 r_1 = -\omega_2 r_2$$

Module $M = \frac{d}{N}$ must be the same for both gears $\quad \frac{\omega_1}{\omega_2} = -\frac{d_2}{d_1} = -\frac{N_2}{N_1}$



Nomenclature of gear teeth



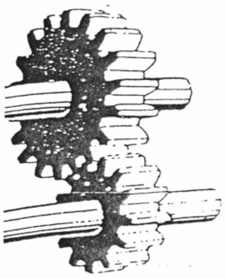
Two spur gears in mesh

3. Types of gears
   - Spur gears
   - Helical gears
   - Bevel gears
   - Idler gears
   - Compound gear trains
   - Epicyclic gear trains

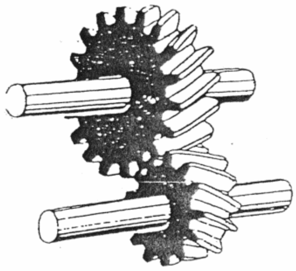
Types of Gears

Gears mounted on parallel shafts

Spur Gears – the teeth are parallel to the shaft axes

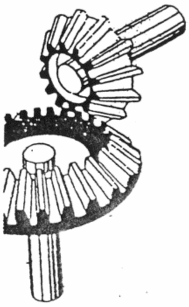


Helical Gears –teeth are inclined to the shaft

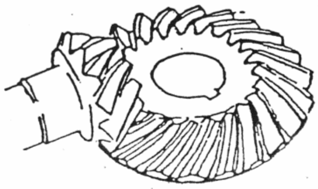


Gears mounted on intersecting shafts

Bevel Gears (bevel spur gears)

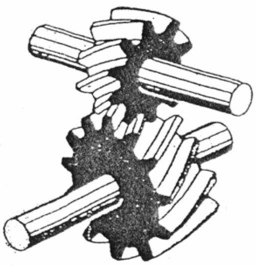


Spiral Bevel Gears (bevel helical gears)

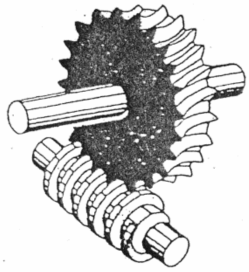


Gears mounted on skew shafts (non-parallel and non-intersection shafts)

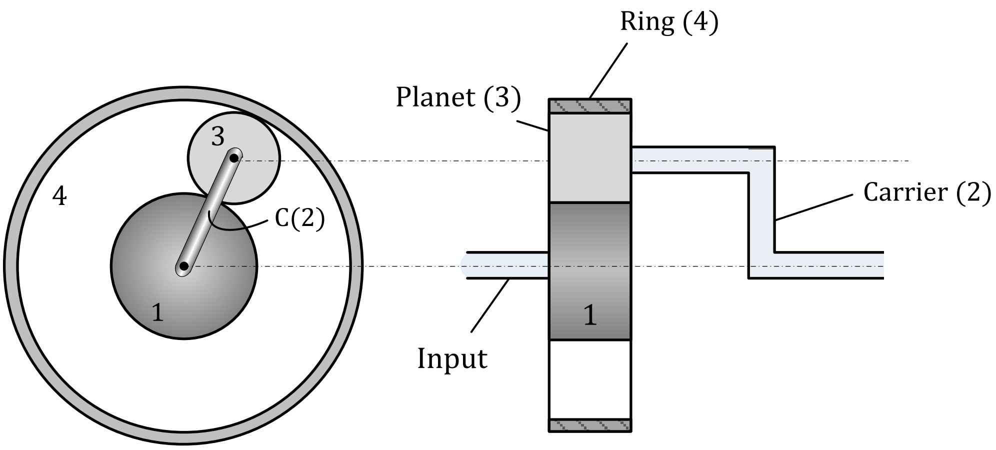
Crossed Helical Gears



Worm Gear and Mating Wheel



4. Simple epicyclic gear trains



1: Sun gear

2: Carrier arm

3: Planet gear

4: Internal ring gear

$$\frac{d_1}{2} + d_3 = \frac{d_4}{2}$$

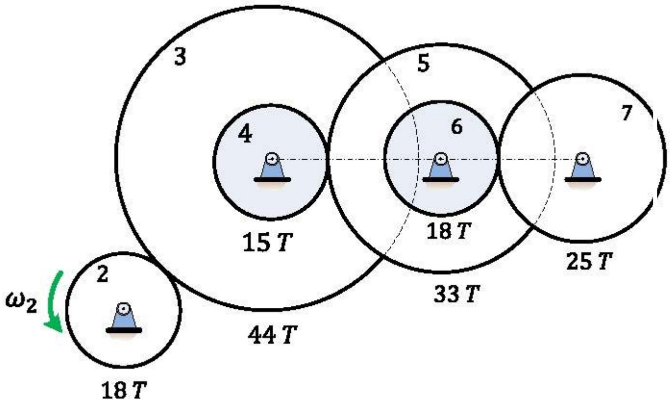
All gears have the same module

$$\frac{N_1}{2} + N_3 = \frac{N_4}{2}$$

$$\Rightarrow \qquad \frac{\omega_{sun} - \omega_{carrier}}{\omega_{ring} - \omega_{carrier}} = -\frac{N_{ring}}{N_{sun}}$$

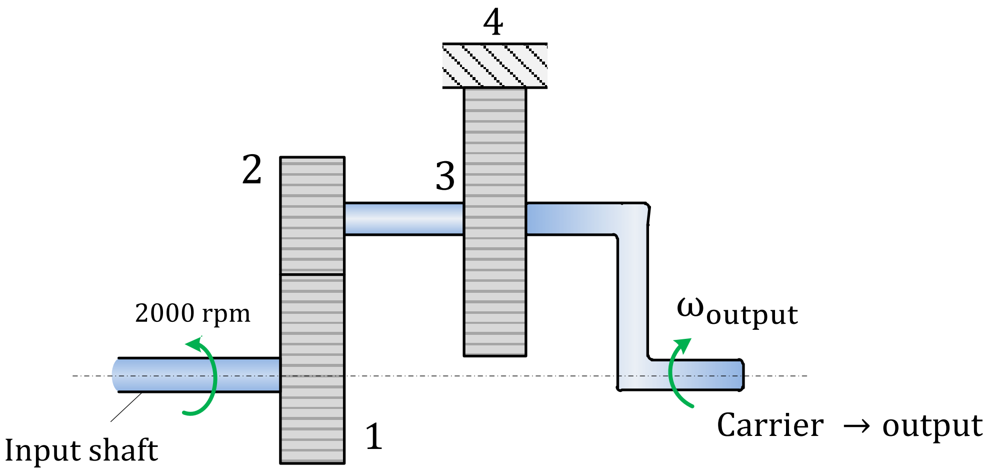
Example 1

Find the angular velocity of gear 7 if the angular velocity of gear 2 is 1000 rpm in the direction shown.



Example 2

For the compound epicyclic gear train shown, all gears have the same module.

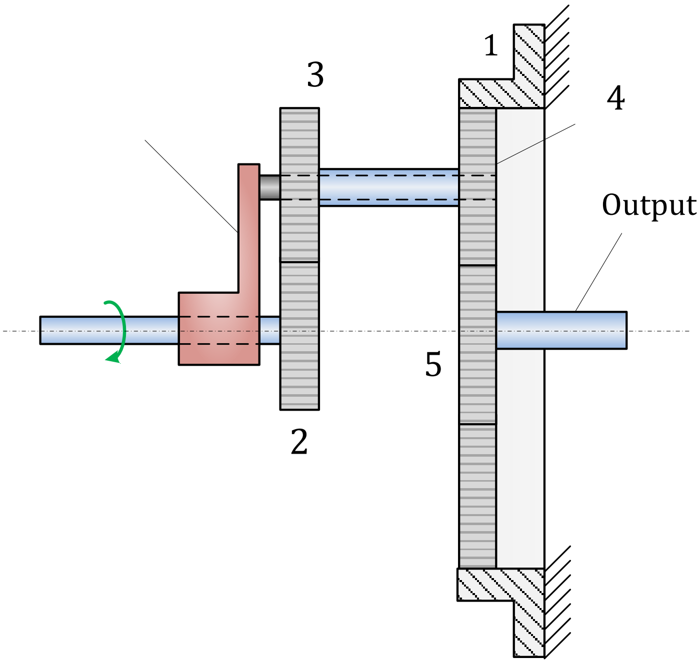


The ring gear is fixed and the input shaft rotates at 2000 rpm CCW.

$N_1 = 10$
$N_2 = 20$
$N_3 = 30$
Find the number of teeth required by the ring gear for the gears to be in mesh. Find the angular velocity of the output shaft.

Example 3

In the planetary reduction unit shown below, gear 2 turns at 300 rpm in the direction shown. Determine the speed and direction of rotation of gear 5 for a fixed carrier.



1: Ring gear (80 T)

2: Sun gear (26 T)

3: Planet (32 T)

4: Planet (22 T)

5: Sun (36 T)

6: Carrier arm – constrained by two axes: shaft of gear 2 and shaft of gears 3 and 4