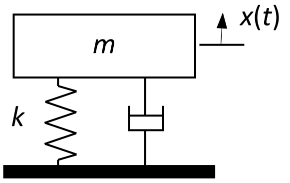
EQUATION SHEET FOR PART A

Spring-Mass-Damper Systems

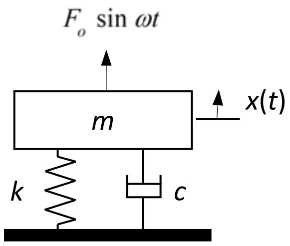
$$\omega = 2\pi f \qquad \omega_n = \sqrt{\frac{k}{m}} \qquad \omega_d = \omega_n\sqrt{1-\zeta^2} \qquad \zeta = \frac{c}{2\sqrt{km}}$$

$$\delta = \ln\left(\frac{x_n}{x_{n+1}}\right) \qquad \delta = \frac{1}{n}\ln\left(\frac{x_1}{x_{n+1}}\right) \qquad \zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} \qquad r = \frac{\omega}{\omega_n}$$



$$m\ddot{x} + c\dot{x} + kx = 0$$

$$x(t) = Xe^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$$

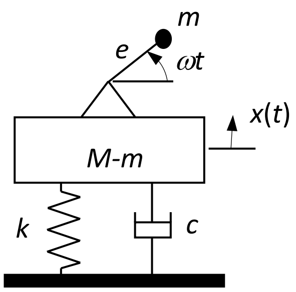


$$m\ddot{x} + c\dot{x} + kx = F_o \sin \omega t$$

$$x(t) = X\sin(\omega t - \phi)$$

$$X = \frac{F_o/k}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}; \qquad \phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$$

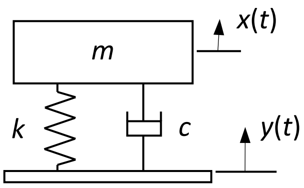
$$TR = \frac{F_T}{F_o} = \frac{\sqrt{1+\left(2\zeta r\right)^2}}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}$$



$$M\ddot{x} + c\dot{x} + kx = me\omega^2 \sin \omega t$$

$$x(t) = X\sin(\omega t - \phi)$$

$$\frac{MX}{me} = \frac{r^2}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}; \qquad \phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$$



$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

$$x(t) = X \sin(\omega t - \phi)$$

$$\frac{X}{Y} = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}}\ ; \quad \frac{F_T}{kY} = r^2 \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

Continuous Systems

<u>Lateral vibration of a taut string</u>

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{c_s^2}\frac{\partial^2 u(x,t)}{\partial t^2}, \qquad c_s = \sqrt{\frac{T}{m_L}}$$

$$u(x,t) = \phi(x)q(t), \qquad \phi(x) = C\sin kx + D\cos kx, \qquad q(t) = A\sin \omega t + B\cos \omega t$$

Boundary Conditions	$x = 0$	$x = L$
Fixed	$u = 0$	$u = 0$

<u>Longitudinal vibrations of rods</u>

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{c_L^2}\frac{\partial^2 u(x,t)}{\partial t^2}, \qquad c_L = \sqrt{\frac{E}{\rho}}$$

Boundary Conditions	$x = 0$	$x = L$
Fixed	$u = 0$	$u = 0$
Free	$\frac{\partial u}{\partial x} = 0$	$\frac{\partial u}{\partial x} = 0$
Spring load	$k_s u = EA\frac{\partial u}{\partial x}$	$k_s u = -EA\frac{\partial u}{\partial x}$
Inertia load	$M\frac{\partial^2 u}{\partial t^2} = EA\frac{\partial u}{\partial x}$	$M\frac{\partial^2 u}{\partial t^2} = -EA\frac{\partial u}{\partial x}$

EQUATION SHEET FOR PART B

Constant Linear Acceleration Equations ($t_o = 0$)

$v = v_o + at$

$v^2 = v_o^2 + 2a(s - s_o)$

$s = s_o + v_o t + \frac{1}{2}at^2$

Angular Motion

$\omega = \frac{d\theta}{dt}$ $\alpha = \frac{d\omega}{dt}$ $\omega d\omega = \alpha d\theta$

Displacement, Velocity and Acceleration Components

(1) Rectangular Co-ordinates

$\boldsymbol{r} = x\mathbf{i} + y\mathbf{j}$

$\boldsymbol{v} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$

$\boldsymbol{a} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$

(2) Normal and Tangential Co-ordinates

$\boldsymbol{v} = v\boldsymbol{e}_t$

$\boldsymbol{a} = a_t\boldsymbol{e}_t + a_n\boldsymbol{e}_n$

$v = \omega r$

$a_t = \dot{v} = \alpha r$

$a_n = \frac{v^2}{\rho} = \omega^2 r$

Relative Motion

$\boldsymbol{r}_A = \boldsymbol{r}_B + \boldsymbol{r}_{A/B}$

$\boldsymbol{v}_A = \boldsymbol{v}_B + \boldsymbol{v}_{A/B}$

$\boldsymbol{a}_A = \boldsymbol{a}_B + \boldsymbol{a}_{A/B}$

Relative Motion for Rotating Axes

$\boldsymbol{v}_{\mathrm{A}} = \boldsymbol{v}_{\mathrm{A'}} + \boldsymbol{v}_{\mathrm{A/A'}}$

$\boldsymbol{a}_{\mathrm{A}} = \boldsymbol{a}_{\mathrm{A'}} + \boldsymbol{a}_{\mathrm{rel}} + \boldsymbol{a}_{\mathrm{cor}}$

Moment of Inertia

Moment of inertia of a uniform disk: $I = \frac{1}{2}mr^2$

Moment of inertia of a uniform slender rod: $I = \frac{1}{12}ml^2$

Parallel axis theorem: $I_O = I_G + md^2$