MMAN2300

Engineering Mechanics 2

Part A: Week 6

Acceleration analysis of rigid bodies to rotating axes

(Chapter 5/7 Meriam & Kraige)

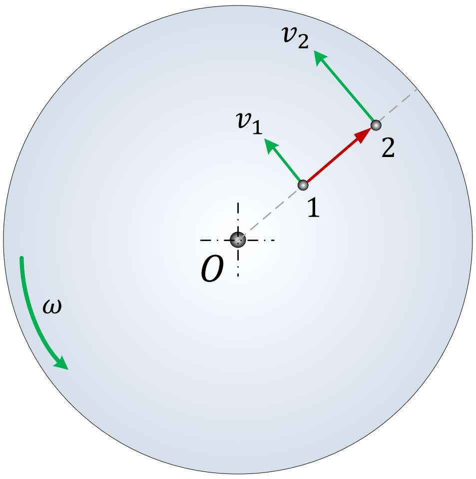
Analysis of accelerations relative to a rotating reference frame

In cases where particles or members move on a rotating body, it is convenient to consider:

- the motion of the member relative to a rotating reference frame fixed to the rotating body, and
- the motion of the moving reference frame (which will, in general, be both rotating and translating) with respect to a fixed frame.

In situations such as this we get an extra component of acceleration of the particle which moves on the rotating body.

To see why this should be the case, consider the following:



A person stands on a rotating disk, initially at point 1.

The person has a velocity $v_1$at point 1

$$v_1 = r_1\omega$$

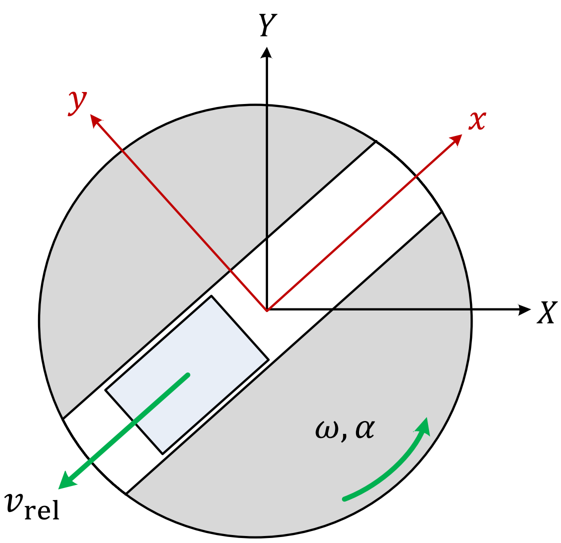
Now the person moves from position 1 to position 2 with a velocity of v_{rel}

$$v_2 = r_2\omega$$

Since $r_2 > r_1$, $v_2 > v_1$.

The person must experience a component of acceleration in this direction (i.e., tangential direction) – “coriolis acceleration component”

For example

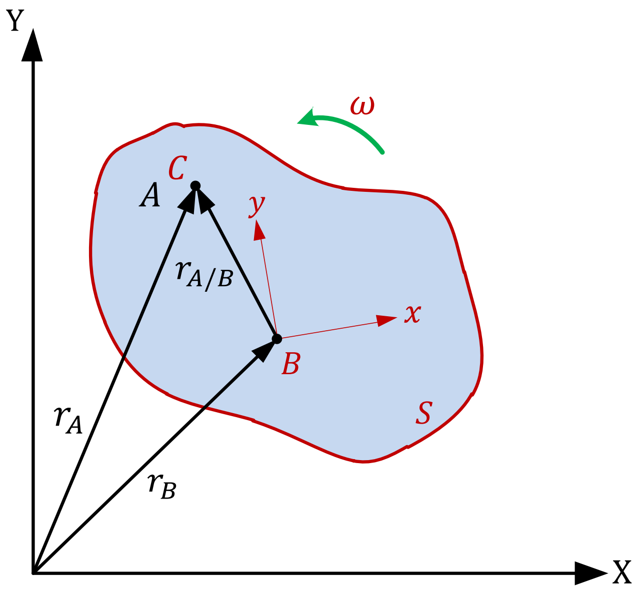


x-y is a rotating reference frame fixed to the rotating body

X-Y is a fixed frame

- General case

Body *S* is both translating and rotating.



Define two reference systems:

- Red coordinate system fixed to Body *S*;
- Fixed reference X-Y

Define two coincident points *A* & *C*:

- Point *A* moves with respect to Body *S*;
- Points *B* and *C* are fixed to Body *S*.

The position of point *A* is expressed as:

$$\boldsymbol{r}_A = \boldsymbol{r}_B + \boldsymbol{r}_{A/B} \qquad (1)$$

Where $\boldsymbol{r}_{A/B} = x\boldsymbol{i} + y\boldsymbol{j} = \boldsymbol{r}$

Note that the unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ are rotating with the *x*-*y* axes.

The velocity of point *A* is determined by differentiating (1)

$$\boldsymbol{v}_A = \boldsymbol{v}_C + \boldsymbol{v}_{A/C}$$

Where

$$\boldsymbol{v}_c = \boldsymbol{v}_B + \boldsymbol{v}_{C/B}$$

Therefore

$$\boldsymbol{v}_A = \boldsymbol{v}_B + \boldsymbol{v}_{C/B} + \boldsymbol{v}_{A/C} \qquad (2)$$

Where $\boldsymbol{v}_{A/C} = \boldsymbol{v}_{rel}$

Differentiate (2) to find the acceleration of point A

$$\boldsymbol{a}_A = \boldsymbol{a}_B + \frac{d}{dt}(\boldsymbol{v}_{C/B}) + \frac{d}{dt}(\boldsymbol{v}_{A/C})$$

$$= \boldsymbol{a}_B + \frac{d}{dt}(\boldsymbol{\omega} \times \boldsymbol{r}) + \frac{d}{dt}(\boldsymbol{v}_{A/C})$$

$$\frac{d}{dt}(\boldsymbol{v}_{C/B}) = \frac{d}{dt}(\boldsymbol{\omega} \times \boldsymbol{r}) = \dot{\boldsymbol{\omega}} \times \boldsymbol{r} + \boldsymbol{\omega} \times \dot{\boldsymbol{r}}$$

$$\boldsymbol{\omega} \times \dot{\boldsymbol{r}} = \boldsymbol{\omega} \times \frac{d}{dt}(x\boldsymbol{i} + y\boldsymbol{j}) = \boldsymbol{\omega} \times \boldsymbol{v}_{A/C} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})$$

$$\frac{d}{dt}(\boldsymbol{v}_{A/C}) = \frac{d}{dt}(\dot{x}\boldsymbol{i} + \dot{y}\boldsymbol{j}) = (\ddot{x}\boldsymbol{i} + \ddot{y}\boldsymbol{j}) + (\dot{x}\dot{\boldsymbol{i}} + \dot{y}\dot{\boldsymbol{j}})$$

$$= \boldsymbol{a}_{A/C} + (\boldsymbol{\omega} \times \dot{x}\boldsymbol{i} + \boldsymbol{\omega} \times \dot{y}\boldsymbol{j})$$

$$= \boldsymbol{a}_{A/C} + \boldsymbol{\omega} \times (\dot{x}\boldsymbol{i} + \dot{y}\boldsymbol{j})$$

$$= \boldsymbol{a}_{A/C} + \boldsymbol{\omega} \times \boldsymbol{v}_{A/C}$$

Therefore $\boldsymbol{a}_A = \boldsymbol{a}_B + \dot{\boldsymbol{\omega}} \times \boldsymbol{r} + \boldsymbol{\omega} \times \boldsymbol{v}_{A/C} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) + \boldsymbol{a}_{A/C} + \boldsymbol{\omega} \times \boldsymbol{v}_{A/C}$

$$= \boldsymbol{a}_B + \dot{\boldsymbol{\omega}} \times \boldsymbol{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) + \boldsymbol{a}_{A/C} + 2\boldsymbol{\omega} \times \boldsymbol{v}_{A/C}$$

The acceleration of point A is:

$$\boldsymbol{a}_A = \boldsymbol{a}_B + \boldsymbol{a}_{C/B} + \boldsymbol{a}_{rel} + 2\boldsymbol{\omega} \times \boldsymbol{v}_{rel} \qquad (3)$$

Summary of the terms:

$\boldsymbol{a}_A$: acceleration of point A

$\boldsymbol{a}_B$: acceleration of point B

$\boldsymbol{a}_{C/B} = \boldsymbol{a}_{C/Bn} + \boldsymbol{a}_{C/Bt}$

$\boldsymbol{a}_{A/C}$: relative acceleration of point A to point C

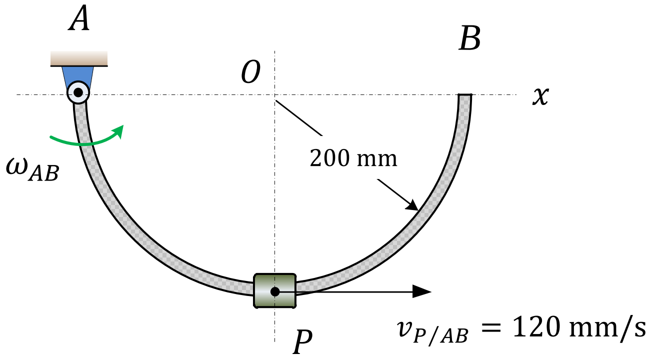
$2\boldsymbol{\omega} \times \boldsymbol{v}_{A/C}$: Coriolis acceleration

If the point of rotation is fixed (eg point B is fixed), then

$$\boldsymbol{a}_A = \boldsymbol{a}_C + \boldsymbol{a}_{rel} + \boldsymbol{a}_{cor}$$

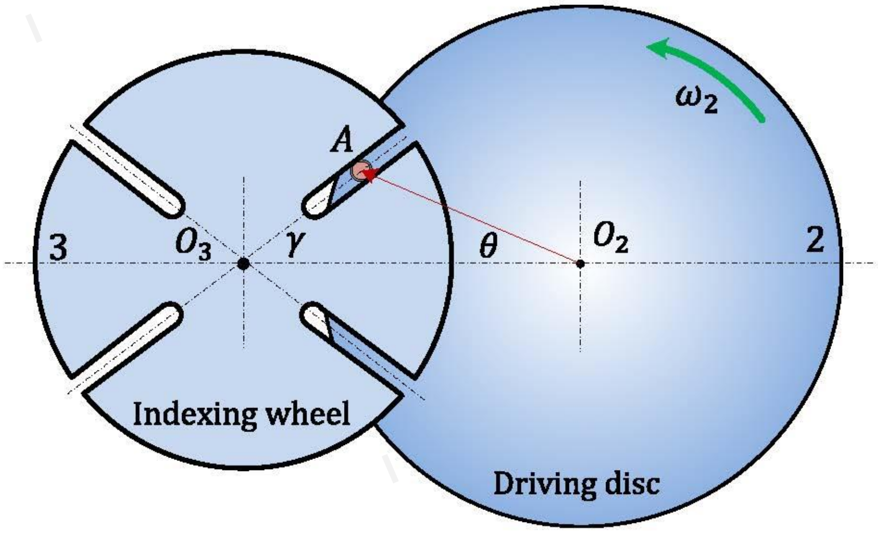
Example 1

The collar P slides from A towards B along a semi-circular rod AB of radius 200 mm. The rod rotates about the pin at A, and the speed of the collar P relative to the rod is constant at 120 mm/s. When the system is in the position shown, the angular velocity of the rod is ω_{AB} = 0.8 rad/s counter clockwise and the angular acceleration of the rod α_{AB} = 0.5 rad/s^2 clockwise. Determine the acceleration of P at the instant shown.



Example 2

For the Geneva mechanism shown below, the driving disk (link 2) rotates with a constant angular velocity ω_2 = 10 rad/s. Find the angular acceleration of link 3 and the velocity of the pin relative to link 3.



$\theta = 30°$

O_2A = 50 mm

O_2O_3 = 70.7 mm