Question 1 [25 marks]

The 6-kg disk rotates about a fixed axis through point O with a clockwise angular velocity $\omega_0 = 10$ rad/s and a counter-clockwise angular acceleration $\alpha_0 = 5$ rad/s^2 at the instant shown in Figure Q1. Pin A is fixed to the disk but slides freely within the slotted member BC with a mass of 2 kg.

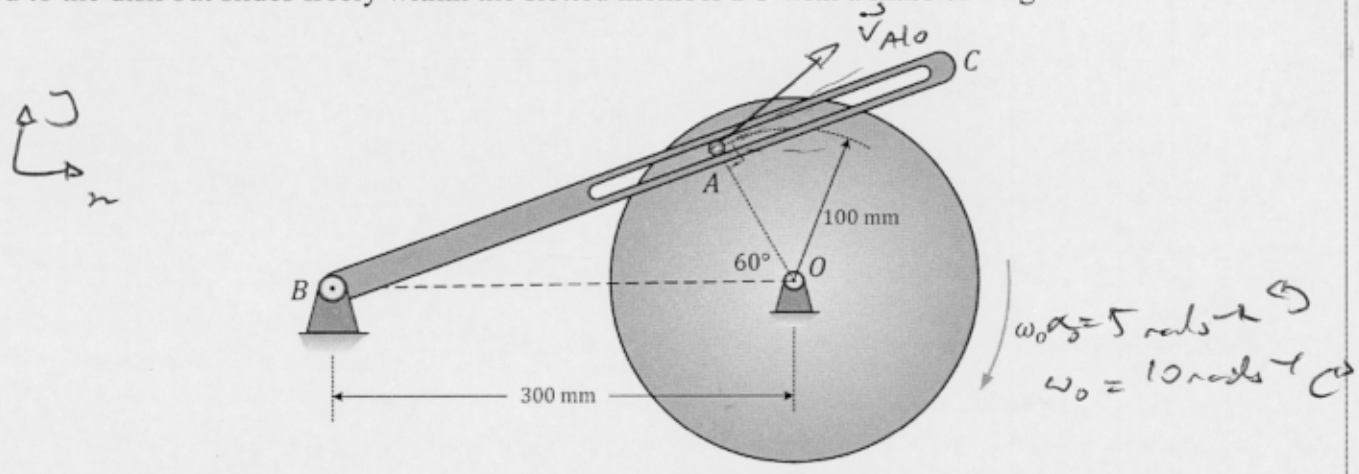

Figure Q1

(a) Determine the angular velocity (magnitude and direction) of BC and the velocity (magnitude and direction) of pin A relative to slotted member BC.

$$\vec{v}_{A'/B} + \vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$$

$$\overline{AB} = \sqrt{0.3^2 + 0.1^2 - 2 \times 0.3 \times 0.1 \cos 60}$$

$$\approx 0.2646 \text{ m}$$

$$\angle ABO = \sin^{-1}\left(\frac{\sin 60}{\overline{AB}} * 0.1\right)$$

$$\approx 19.11^\circ = \phi$$

$$\theta = 90 - \angle ABO \approx 70.89^\circ$$

$$\vec{v}_{A'/B} + \vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$$

$$\begin{pmatrix} -\omega_{AB}\,\overline{AB}\cos\theta \\ \omega_{AB}\,\overline{AB}\sin\theta \end{pmatrix} + \begin{pmatrix} -v_{A/B}\cos\phi \\ -v_{A/B}\sin\phi \end{pmatrix} = \begin{pmatrix} \omega_0\,\overline{OA}\cos 30 \\ \omega_0\,\overline{OA}\sin 30 \end{pmatrix}$$

$$\begin{pmatrix} -\omega_{AB}\,0.2646\cos\theta \\ \omega_{AB}\,0.2646\sin\theta \end{pmatrix} + \begin{pmatrix} -v_{A/B}\cos\phi \\ -v_{A/B}\sin\phi \end{pmatrix} = \begin{pmatrix} 10\times 0.2646\cos 30 \\ 10\times 0.2646\sin 30 \end{pmatrix}$$

$\hat{j}$: $-v_{A/B} = \dfrac{10\times 0.2646\sin 30 - \omega_{AB}\,0.2646\sin\theta}{\sin\phi}$

$\hat{i}$: $-\omega_{AB}\,0.2646\cos\theta + \left(\dfrac{10\times 0.2646\sin 30 - \omega_{AB}\,0.2646\sin\theta}{\sin\phi}\right)\cos\phi = 10\times 0.2646\cos 30$

$\therefore \omega_{AB} = 0$

$v_{A/B} = 2.646\ \text{ms}^{-1}\ \angle\phi$

$\therefore \vec{v}_{A/B} = 2.646\ \text{ms}^{-1}\ \angle 19.11°$

$\therefore \vec{\omega}_{AB} = 1.8898\ \text{rads}^{-1}\ \circlearrowleft$

$\therefore \vec{v}_{A/B} = 2.598\ \text{ms}^{-1}\ \angle 19.11°$

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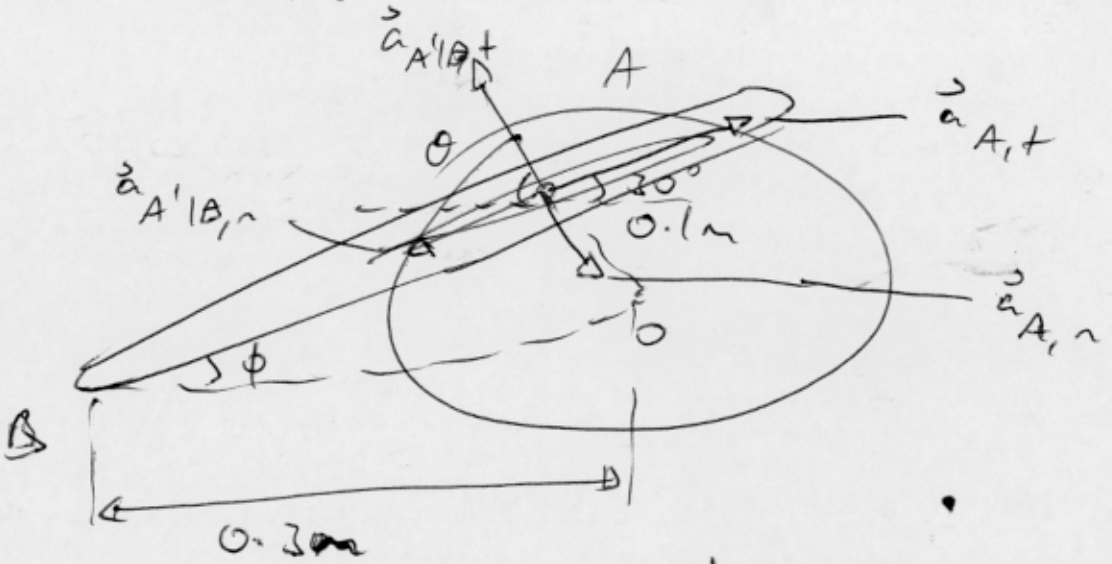
(b) Find the acceleration (magnitude and direction) of pin A relative to slotted member BC.



$$\vec{a}_{A|B,cor} + \vec{a}_{A|B,rel} + \vec{a}_{A'|B} = \vec{a}_A - \vec{a}_B$$

$$2\vec{\omega}_{AB} \times \vec{v}_{A|B} + \begin{pmatrix} a_{A|B}\cos\phi \\ a_{A|B}\sin\phi \end{pmatrix} + \begin{pmatrix} -\alpha_{AB}\overline{AB}\cos\theta \\ \alpha_{AB}\overline{AB}\sin\theta \end{pmatrix} + \begin{pmatrix} -\omega_{AB}^2\overline{AB} \\ -\omega_{AB}^2\overline{AB}\sin\phi \end{pmatrix}$$

$$= \begin{pmatrix} \alpha_0\overline{OA}\cos 30 \\ \alpha_0\overline{OA}\sin 30 \end{pmatrix} + \begin{pmatrix} \omega_0^2\overline{OA}\cos 60 \\ -\omega_0^2\overline{OA}\sin 60 \end{pmatrix}$$

$$2\begin{pmatrix} 0 \\ 0 \\ 1.8898 \end{pmatrix} \times \begin{pmatrix} 2.598\cos\phi \\ 2.598\sin\phi \\ 0 \end{pmatrix} = \begin{pmatrix} 9.2788\ldots \\ -3.21428\ldots \end{pmatrix}$$

$\hat{i}$ $\hat{j}$

$$\begin{pmatrix} \alpha_0\,\overline{OA}\cos 30 \\ \alpha_0\,\overline{OA}\sin 30 \end{pmatrix} = \begin{pmatrix} -1.1458\ldots \\ -0.664437\ldots \end{pmatrix} = \begin{pmatrix} -0.4330\ldots \\ -0.25 \end{pmatrix}$$

$$\begin{pmatrix} \omega_0^2\,\overline{OA}\cos 60 \\ -\omega_0^2\,\overline{OA}\sin 60 \end{pmatrix} = \begin{pmatrix} 5 \\ -5\sqrt{3} \end{pmatrix}$$

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$$\begin{pmatrix} 9.2788 \\ -3.2143 \end{pmatrix} + \begin{pmatrix} a_{AB}\cos\phi \\ a_{AB}\sin\phi \end{pmatrix} + \begin{pmatrix} -\alpha_{AB}\,\overline{AB}\cos\theta \\ \alpha_{AB}\,\overline{AB}\sin\theta \end{pmatrix}$$

$$+ \begin{pmatrix} -0.89286 \\ 0.30924 \end{pmatrix} = \begin{pmatrix} -0.4330 \\ -0.25 \end{pmatrix} + \begin{pmatrix} \frac{5}{5\sqrt{3}} \end{pmatrix}$$

$$\hat{i}: \quad a_{AB} = \frac{-3.81894 + \alpha\, 0.2646\cos\theta}{\cos\phi}$$

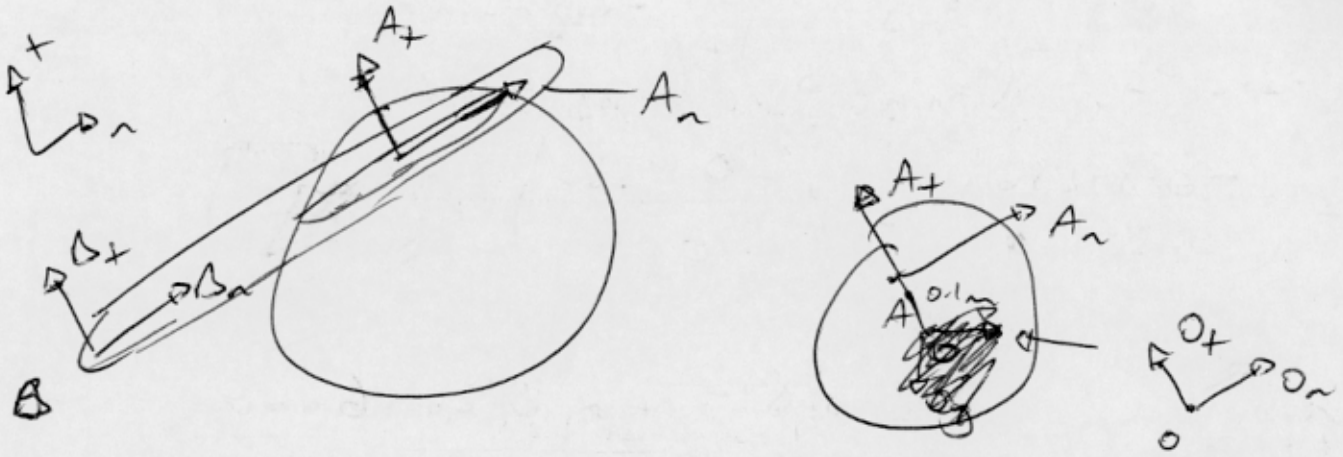
$$\hat{j}: \left(\frac{-3.81894 + \alpha\, 0.2646\cos\theta}{\cos\phi}\right)\sin\phi + \alpha\, 0.2646\sin\theta$$

$$= 11.315$$

$$\therefore \alpha_{AB} = 44.659 \ \text{rad s}^{-2}$$

$$\therefore \vec{a}_{AB} = 0.05145 \ \text{m s}^{-2} \ \angle 19.11^\circ$$

(c) Find the reaction force exerted on the disk at point A at the instant shown.



$$\Sigma M_O = I_O \alpha_{disc}$$

$$-A_n(0.1) = \frac{1}{2} \times 6 \times R^2 \times 5$$

$$\Sigma M_B = I_B \alpha_{AB}$$

$$A_t \times \overline{AB} = \frac{1}{3} \times 2 \times \overline{AB}^2 \times 44.659$$

$$\therefore |A_t| = 7.877 \text{ N}$$

$$|A_n| = 1.5 \text{ N}$$

$$R_A = \sqrt{A_t^2 + A_n^2}$$

$$R_A = \sqrt{1.5^2 + 7.877^2} = 8.019 \text{ N}$$

Extra Workspace – Question Number

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