Mechanical and Manufacturing Engineering

MMAN2300 – Engineering Mechanics 2

Test 2 2019 T2

Question 1 [25 marks]

The 6-kg disk rotates about a fixed axis through point *O* with a clockwise angular velocity ω_0 = 10 rad/s and a counter-clockwise angular acceleration α_0 = 5 rad/s^2 at the instant shown in Figure Q1. Pin *A* is fixed to the disk but slides freely within the slotted member *BC* with a mass of 2 kg.

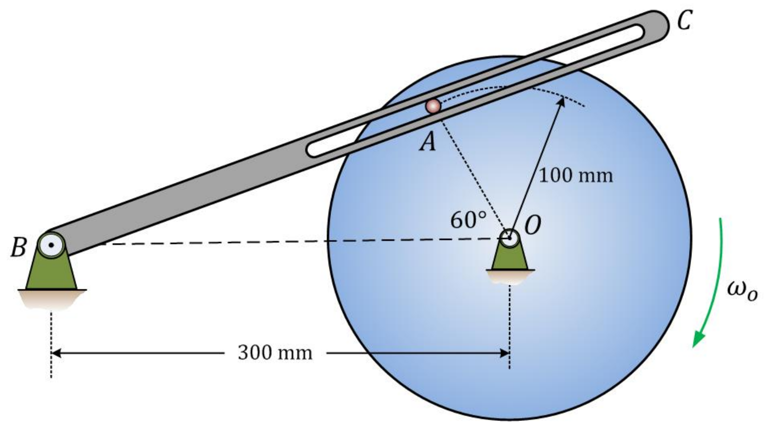


Figure Q1

(a) Determine the angular velocity (magnitude and direction) of *BC* and the velocity (magnitude and direction) of pin *A* relative to slotted member *BC*.

(b) Find the acceleration (magnitude and direction) of pin *A* relative to slotted member *BC*.

(c) Find the reaction force exerted on the disk at point *A* at the instant shown.

Q1 Given: $m_{disk} = 6$ kg

$\omega_0 = 10$ rad/s CW

$\alpha_0 = 5$ rad/s^2 CCW

$m_{BC} = 2$ kg

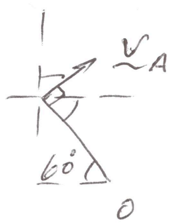
(a) Find ω_{BC} & $V_{A/P}$

10

Let pt A' be on link BC, coincident with pt A on the disk.

Point A is moving on the rotating link BC.

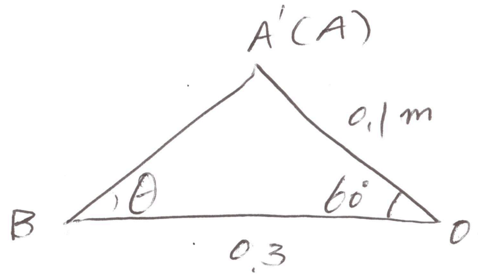
$\underset{\sim}{V}_A = \underset{\sim}{V}_{A'} + \underset{\sim}{V}_{A/A'}$



$V_A = \omega_0 \cdot \overline{OA} = \omega_0 \cdot r = 10(0.1) = 1.0$ m/s

$\underset{\sim}{V}_{A'} = \omega_{BC} \overline{BA'} = \omega_{BC} \cdot \overline{BA}$

Find $\overline{BA'} = \overline{BA}$



$\overline{BA} = \sqrt{\overline{OA}^2 + \overline{OB}^2 - 2\overline{OA}\cdot\overline{OB}\cos 60°}$

$= \sqrt{0.1^2 + 0.3^2 - 2(0.1)(0.3)\cos 60°}$

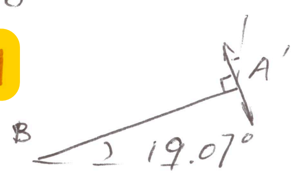
$= \sqrt{0.1 - 0.03} = 0.265$ m

$\dfrac{\sin\theta}{0.1} = \dfrac{\sin 60}{0.265}$

$\Rightarrow \sin\theta = \dfrac{0.1}{0.265}\sin 60 = 0.3268$

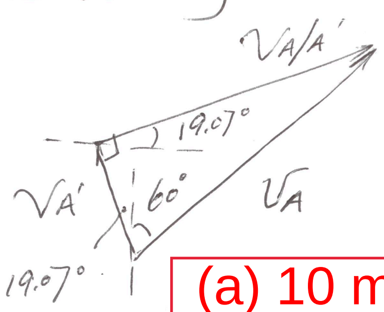
$\theta = 19.07°$

Therefore $V_{A'} = 0.265\,\omega_{BC}$



$V_{A/A'} = ?$

Draw the velocity vecter diagram



$V_{A'} = V_A \cos(19.07 + 60)$

$= 1(\cos 79.07) = 0.190$ m/s

$\omega_{BC} = \dfrac{V_{A'}}{\overline{BA}} = \dfrac{0.190}{0.265} = 0.717$ rad/s CCW

(0.715–0.717) ok

$V_{A/A'} = V_A(\sin 79.07) = 0.982$ m/s, 19.07°

(a) 10 marks

(b) Find $a_{A/A'}$ $(= a_{A/BC})$

10

$a_A = a_{A'} + a_{rel} + a_{cor}$ 1

$a_A = a_{An} + a_{At}$

$a_{An} = \omega_o^2 \cdot \overline{OA} = (10)^2(0.1) = 10 \ m/s^2$ 60° 1

$= 10\cos 60\, i - 10 \sin 60\, j$

$= 5 i - 8.66 j \ m/s^2$

$a_{At} = \alpha_o \cdot \overline{OA} = 5(0.1) = 0.5 \ m/s^2$

$= -0.5 \sin 60\, i - 0.5 \cos 60\, j$ 1

$= -0.433 i - 0.25 j \ m/s^2$

$a_A = (5 - 0.433) i - (8.66 + 0.25) j$

$= 4.567 i - 8.91 j$

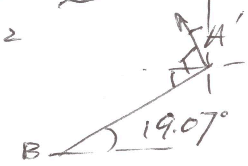
$a_{A'} = a_{A'n} + a_{A't}$

$a_{A'n} = \omega_{BC}^2 \overline{BA} = (0.717)^2 (0.265) = 0.136 \ m/s^2$ 19.07° 1

$= -0.136 \cos 19.07\, i - 0.136 \sin 19.07\, j$

$= -0.129 i - 0.044 j \ m/s^2$

$a_{A't} = \alpha_{BC} \overline{BA} = 0.265 \alpha_{BC}$



$= -0.265 \alpha_{BC} \sin 19.07\, i + 0.265 \alpha_{BC} \cos 19.07\, j$ 1

$= -0.0866 \alpha_{BC} i + 0.25 \alpha_{BC} j$

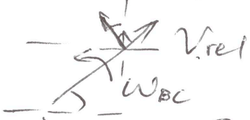
$a_{A'} = (-0.129 - 0.0866 \alpha_{BC}) i + (0.25 \alpha_{BC} - 0.044) j$

$a_{A/A'} = a_{rel} = ?$ 19.07° 1

$= a_{rel} \cos 19.07\, i + a_{rel} \sin 19.07\, j$

$= 0.945 a_{rel} i + 0.327 a_{rel} j$

$a_{cor} = 2 \omega_{BC} V_{A/A'} = 2(0.717)(0.982) = 1.408 \ m/s^2$ 1



1 (direction)

$= -1.408 \sin 19.07\, i + 1.408 \cos 19.07\, j$

$= -0.46 i + 1.331 j$

Group the i terms

$$4.567 = -0.129 - 0.0866\,\alpha_{BC} + 0.945\,a_{rel} - 0.46$$

$$\Rightarrow 0.0866\,\alpha_{BC} = -5.156 + 0.945\,a_{rel}$$

$$\alpha_{BC} = -59.538 + 10.912\,a_{rel} \qquad (1)$$

1 approach

Group the j terms

$$-8.91 = 0.25\,\alpha_{BC} - 0.044 + 0.327\,a_{rel} + 1.331 \qquad (2)$$

Sub (1) into (2) $-10.197 = 0.25(-59.538 + 10.912\,a_{rel}) + 0.327\,a_{rel}$

$$4.6875 = 3.055\,a_{rel}$$

$$\Rightarrow \underset{\sim}{a}_{rel} = \underset{\sim}{a}_{A/A'} = 1.534\ \text{m/s}^2\ \angle 19.07^\circ$$

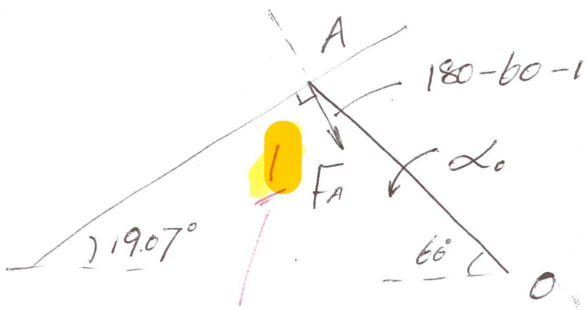
1

(b) 10 marks

(c) Find F_A

5

$$\overset{\curvearrowright}{\Sigma M_O} = I_O \cdot \alpha_O \qquad 1$$



1

FBD & direction of F_A

$$F_A \sin 10.93 \cdot \overline{OA} = I_O \cdot \alpha_O \qquad 1$$

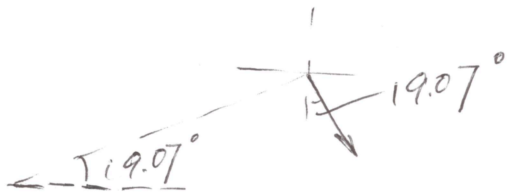
$$I_O = \tfrac{1}{2} m r^2$$

$$= \tfrac{1}{2}(6)(0.1)^2 = 0.03\ \text{kgm}^2$$

$$\alpha_O = 5\ \text{rad/s}^2 \qquad 1$$

$$\Rightarrow F_A = \frac{0.03(5)}{(\sin 10.93)(0.1)} = 7.91\ \text{N} \qquad 1$$

direction



(c) 5 marks

Question 2 [25 marks]

A two-story building frame is modelled as shown in Figure Q2. The floor framing comprises rigid girders of mass m_1 and m_2. The columns have flexural rigidities EI_1 and EI_2 where E is the Young's modulus and I is the second moment of inertia of the beam cross section. The columns are assumed to have negligible mass. The stiffness of each column can be computed as:

$$k_i = \frac{24EI_i}{h_i^3} \qquad\qquad i = 1, 2$$

For axial vibration of each mass in the direction shown, sketch an equivalent translational 2-DOF spring-mass system for the two-story building frame. For $m_1 = 2m$, $m_2 = m$, $h_1 = h_2 = h$ and $EI_1 = EI_2 = EI$, determine the natural frequencies and modeshapes of the frame. Sketch the modeshapes in terms of the displacement amplitude of m_1 with respect to the displacement amplitude of m_2.

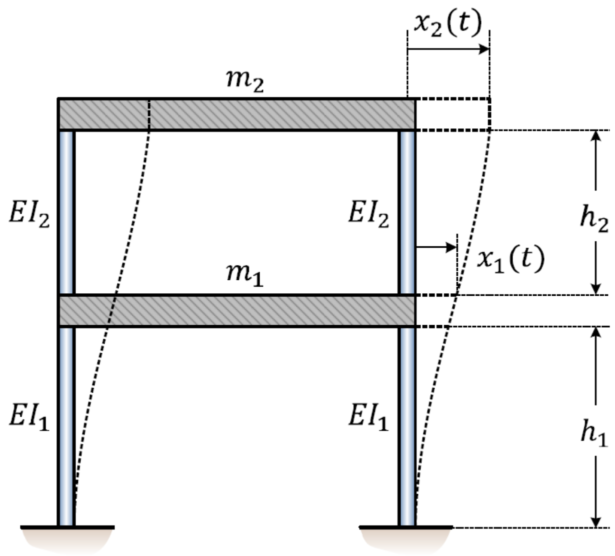
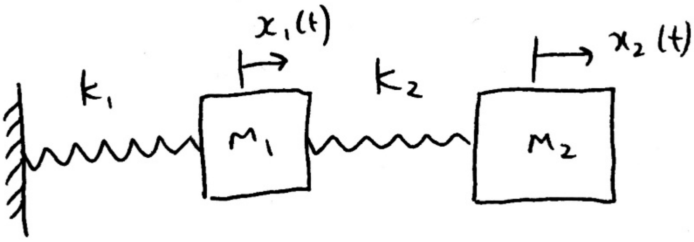


Figure Q2

Q2

Equivalent system is given by



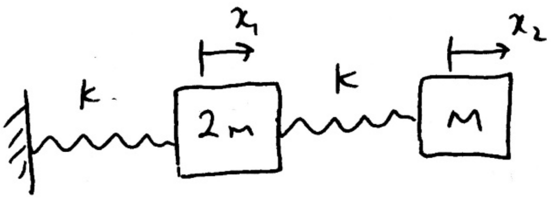
$$k_i = \frac{2(24EI_i)}{h_i^3} \quad \left(\tfrac{1}{2}\right), \quad i = 1, 2$$

(2 columns in parallel for each mass)

$$\Rightarrow \quad k_1 = k_2 = k = \frac{48EI}{h^3} \quad \left(\tfrac{1}{2}\right)$$

Given $M_1 = 2m$, $M_2 = m$

$\Rightarrow$



Equations of motion

$$2m\ddot{x}_1 = -kx_1 - kx_1 + kx_2 \quad (2)$$

$$m\ddot{x}_2 = -kx_2 + kx_1 \quad (2)$$

5 marks

For harmonic motion

$x_1(t) = A_1 \sin\omega t$ (1)

$x_2(t) = A_2 \sin\omega t$

$$\Rightarrow \quad (2k - 2m\omega^2)A_1 - kA_2 = 0 \quad -(1)$$

(1)

$$-kA_1 + (k - m\omega^2)A_2 = 0 \quad -(2)$$

In matrix form

$$\begin{bmatrix} 2k - 2m\omega^2 & -k \\ -k & k - m\omega^2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (1)$$

Characteristic equation

$$(2k - 2m\omega^2)(k - m\omega^2) - k^2 = 0 \quad (1)$$

$$2m^2\omega^4 - 4km\omega^2 + k^2 = 0$$

put $\omega_0 = \sqrt{k/m}$ (1)

$$2\left(\frac{\omega}{\omega_0}\right)^4 - 4\left(\frac{\omega}{\omega_0}\right)^2 + 1 = 0 \quad (2)$$

$$\Rightarrow \quad \omega_{n_1} = 0.541\,\omega_0 \;\; (1), \qquad \omega_{n_2} = 1.307\,\omega_0 \;\; (1)$$

9 marks

Since $k = \frac{48EI}{h^3}$

$$\Rightarrow \omega_{n_1} = 0.541\sqrt{\frac{48EI}{mh^3}} = 3.75\sqrt{\frac{EI}{mh^3}} \quad (1)$$

$$\omega_{n_2} = 1.307\sqrt{\frac{48EI}{mh^3}} = 9.05\sqrt{\frac{EI}{mh^3}} \quad (1)$$

2 marks

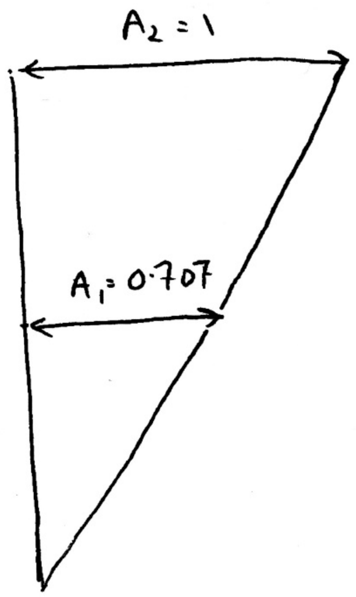
from (1)

$$\frac{A_1}{A_2} = \frac{k}{2k - 2m\omega^2} \quad (1)$$

$$\text{at } \omega_{n_1}, \quad \frac{A_1}{A_2} = \frac{k}{2k - 2m\left(0.541\sqrt{\frac{k}{m}}\right)^2} = 0.707 \quad (1)$$

$$\text{at } \omega_{n_2}, \quad \frac{A_1}{A_2} = \frac{k}{2k - 2m\left(1.307\sqrt{\frac{k}{m}}\right)^2} = -0.707 \quad (1)$$

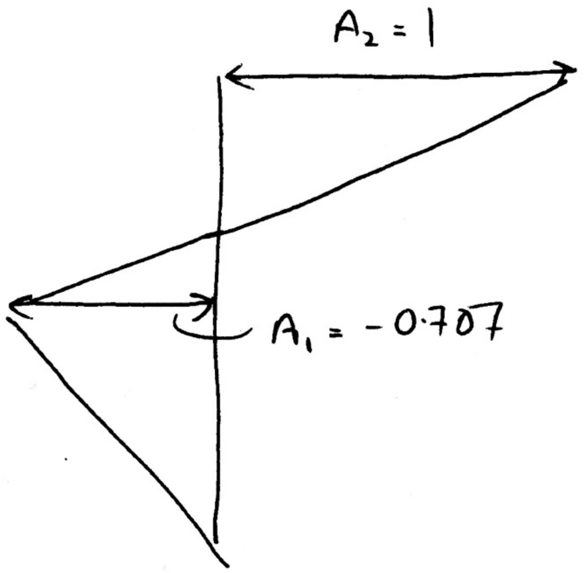
3 marks



3

$$\omega_{n_1} = 0.541\sqrt{\frac{k}{m}}$$

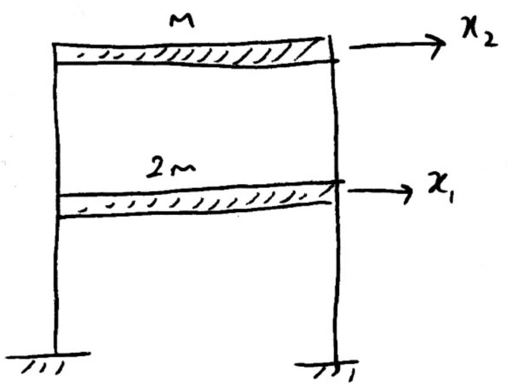
$$= 3.75\sqrt{\frac{EI}{mh^3}}$$



3

$$\omega_{n_2} = 1.307\sqrt{\frac{k}{m}}$$

$$= 9.05\sqrt{\frac{EI}{mh^3}}$$



6 marks