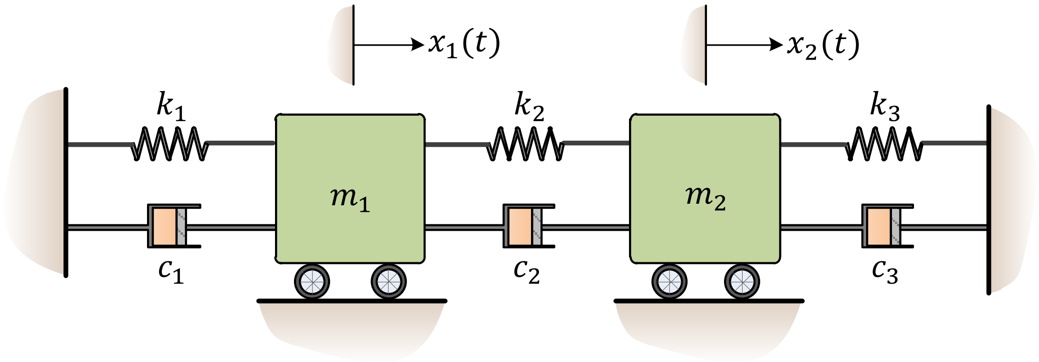
MMAN2300 Engineering Mechanics 2
Part A: Vibration Analysis
Tutorial 6

Question 1-3

State the equations of motion. Rearrange the equations of motion in matrix form to obtain the mass, damping and stiffness matrices.



Solution

Equations of motion

$$m_1\ddot{x}_1 = -k_1x_1 - c_1\dot{x}_1 - k_2x_1 - c_2\dot{x}_1 + k_2x_2 + c_2\dot{x}_2$$

$$m_2\ddot{x}_2 = -k_2x_2 - c_2\dot{x}_2 - k_3x_2 - c_3\dot{x}_2 + k_2x_1 + c_2\dot{x}_1$$

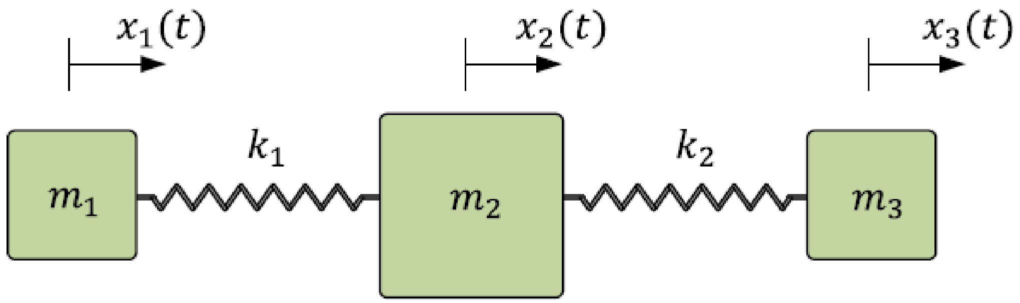
$$-\omega^2\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{\text{mass matrix}}\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} + j\omega\underbrace{\begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2+c_3 \end{bmatrix}}_{\text{damping matrix}}\begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$$

$$+\underbrace{\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}}_{\text{stiffness matrix}}\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Question 4-7

The following system has three masses m_1, m_2, m_3 connected by two springs with stiffness k_1 and k_2. Determine the natural frequencies and mode shapes of the 3DOF system with respect to $\omega_o = \sqrt{\frac{k}{m}}$.

Let $k_1 = 2k$, $k_2 = 2k$, $m_1 = m$, $m_2 = 2m$, $m_3 = m$.



Solution

Equations of motion

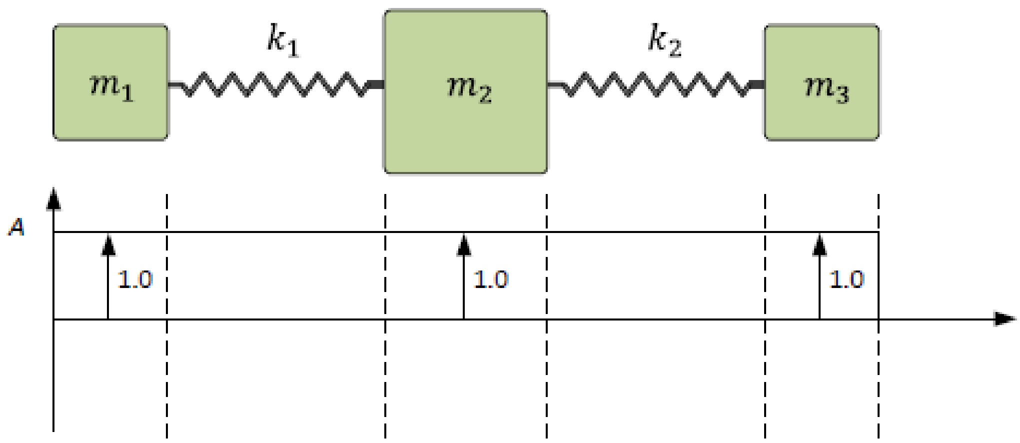
$$m_1\ddot{x}_1 = -k_1x_1 + k_1x_2$$

$$m_2\ddot{x}_2 = -k_1(x_2 - x_1) + k_2(x_3 - x_2)$$

$$m_3\ddot{x}_3 = -k_2(x_3 - x_2)$$

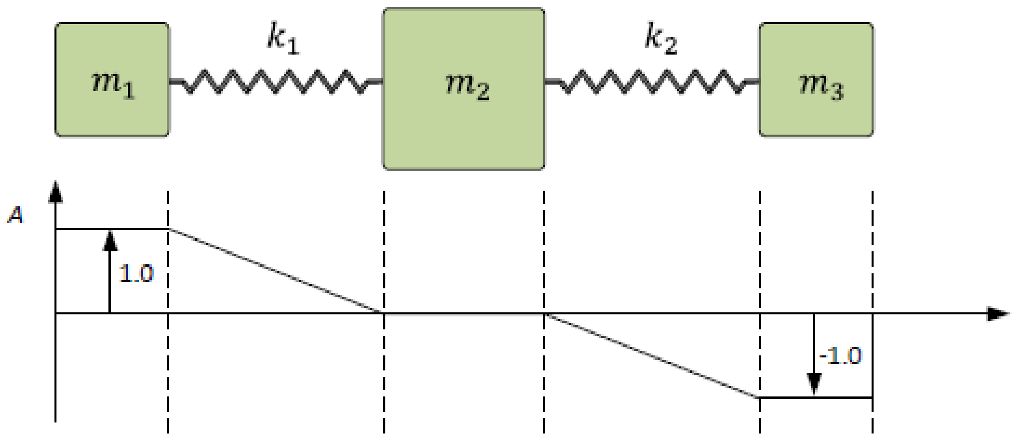
for $\omega = \omega_{n1} = 0$,

$$\frac{A_1}{A_2} = \frac{A_3}{A_2} = 1.0$$



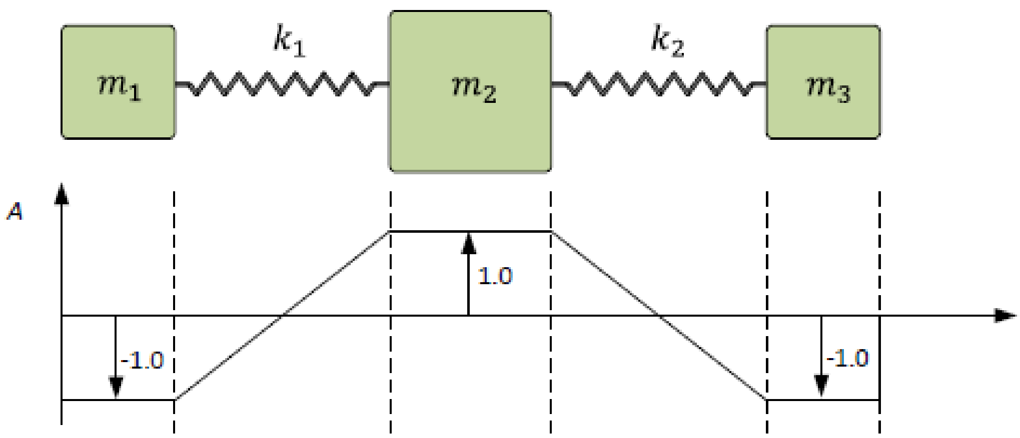
for $\omega = \omega_{n2} = \sqrt{\frac{2k}{m}}$,

$A_1 = 1.0, A_2 = 0, A_3 = -1.0$



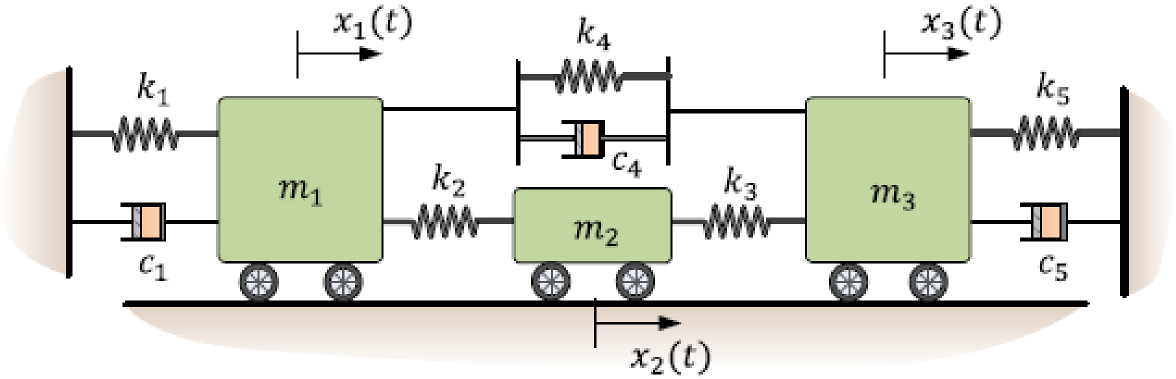
for $\omega = \omega_{n3} = 2\sqrt{\frac{k}{m}}$,

$A_1 = A_3 = -1.0, A_2 = 1.0$



Question 8-10

For the axial vibration system shown in the figure below, state the equations of motion. Using a general solution of the form $x(t) = Ae^{j\omega t}$, rearrange the equations of motion in matrix form to obtain the mass, stiffness and damping matrices.



Solution

$$-\omega^2 \underbrace{\begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix}}_{\text{mass matrix}} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} + j\omega \underbrace{\begin{bmatrix} c_1 + c_4 & 0 & -c_4 \\ 0 & 0 & 0 \\ -c_4 & 0 & c_4 + c_5 \end{bmatrix}}_{\text{damping matrix}} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

$$+ \underbrace{\begin{bmatrix} k_1 + k_2 + k_4 & -k_2 & -k_4 \\ -k_2 & k_2 + k_3 & -k_3 \\ -k_4 & -k_3 & k_3 + k_4 + k_5 \end{bmatrix}}_{\text{stiffness matrix}} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$