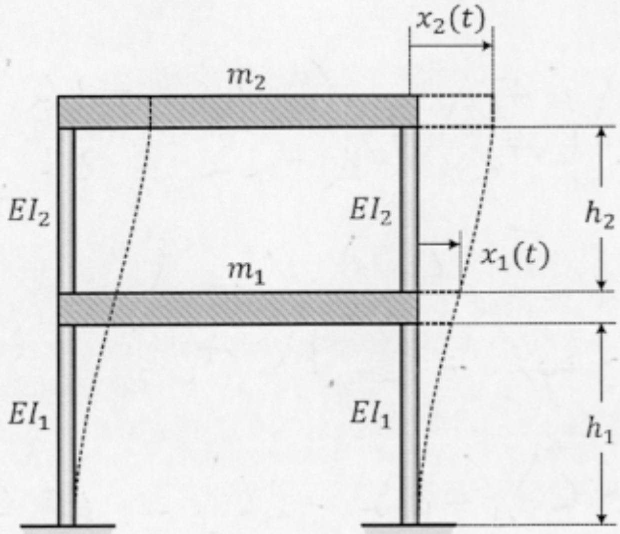
Question 2 [25 marks]

A two-story building frame is modelled as shown in Figure Q2. The floor framing comprises rigid girders of mass m_1 and m_2. The columns have flexural rigidities EI_1 and EI_2 where E is the Young's modulus and I is the second moment of inertia of the beam cross section. The columns are assumed to have negligible mass. The stiffness of each column can be computed as:

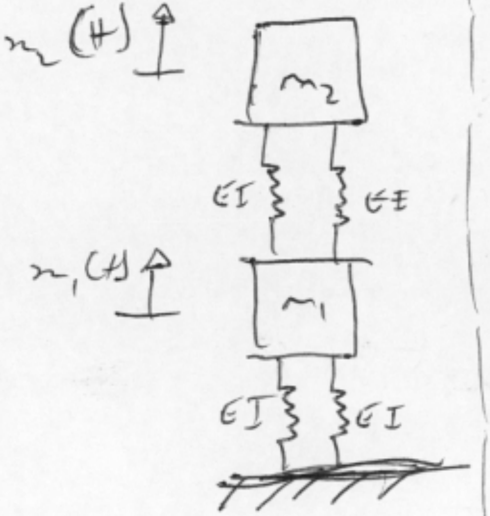
$$k_i = \frac{24EI_i}{h_i^3} \qquad i = 1, 2$$

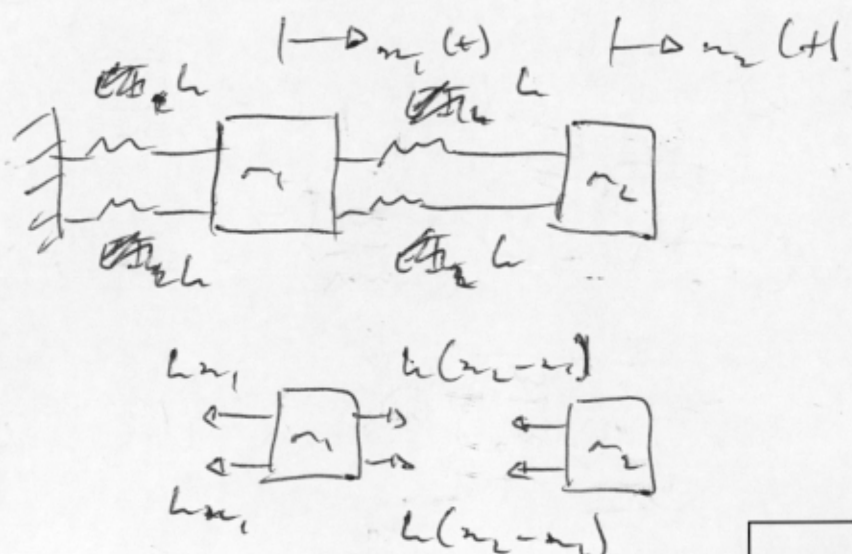
For axial vibration of each mass in the direction shown, sketch an equivalent translational 2-DOF spring-mass system for the two-story building frame. For $m_1 = 2m$, $m_2 = m$, $h_1 = h_2 = h$ and $EI_1 = EI_2 = EI$, determine the natural frequencies and modeshapes of the frame. Sketch the modeshapes in terms of the displacement amplitude of m_1 with respect to the displacement amplitude of m_2.



$m_1 = 2m$
$m_2 = m$

Figure Q2





$$k = \frac{24EI}{L^3}, \quad x = Ae^{j\omega t}, \quad \ddot{x} = -\omega^2 Ae^{j\omega t}$$

<u>EoM:</u> $$m_1 \ddot{x}_1 = -2kx_1 + 2k(x_2 - x_1)$$

$$-\omega^2 m_1 A_1 + 4kA_1 - 2kA_2 = 0 \quad \text{—} ①$$

$$m_2 \ddot{x}_2 = -2k(x_2 - x_1)$$

$$-\omega^2 m_2 A_2 - 2kA_1 + 2kA_2 = 0 \quad \text{—} ②$$

$$-\omega^2 \begin{pmatrix} 2m & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} + \begin{pmatrix} 4k & -2k \\ -2k & 2k \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -2\omega^2 m + 4k & -2k \\ -2k & -\omega^2 m + 2k \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

<u>Δ:</u> $$(-2\omega^2 m + 4k)(-\omega^2 m + 2k) - (2k)^2 = 0$$

$$2\omega^4 m^2 - 8\omega^2 mk + 4k^2 = 0$$

$\div k^2$, sub $\omega_0 = \sqrt{\frac{k}{m}}$

$$\therefore \left(\frac{\omega}{\omega_0}\right)^4 - 4\left(\frac{\omega}{\omega_0}\right)^2 + 2 = 0$$

$$\left(\frac{\omega}{\omega_0}\right)^2 = 3.414,\ 0.5858$$

$$\therefore \frac{\omega}{\omega_0} = 1.848,\ 0.7654$$

$$\therefore \omega_n = 1.848\sqrt{\frac{k}{m}}, \quad 0.7654\sqrt{\frac{k}{m}}$$

From ①:

$$\frac{A_1}{A_2} = \frac{2k}{-2\omega^2 m + 4k},$$

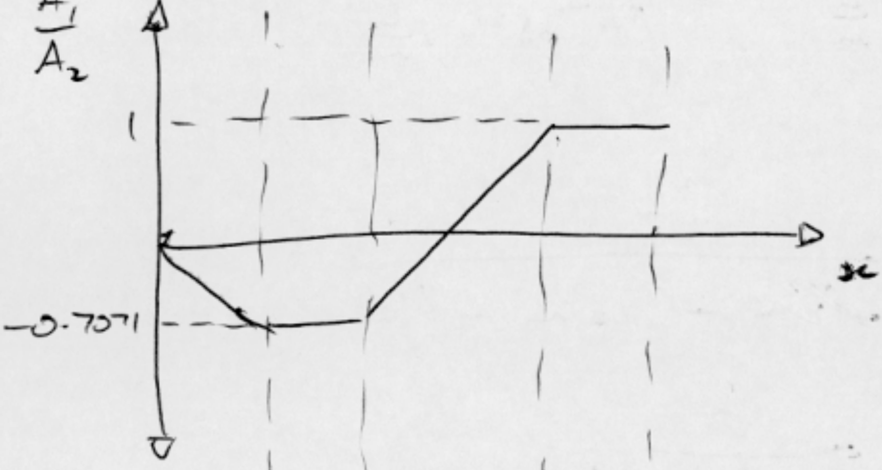
$$= \frac{k}{-\omega^2 m + 2k}$$

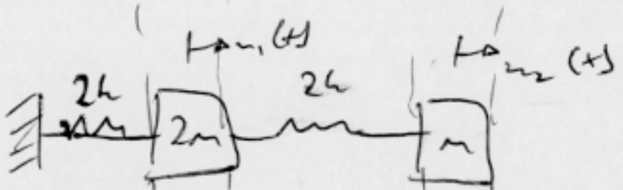
sub ω_n:

$$\left.\frac{A_1}{A_2}\right|_{\omega_n = 1.848\sqrt{\frac{k}{m}}} = -0.7071$$

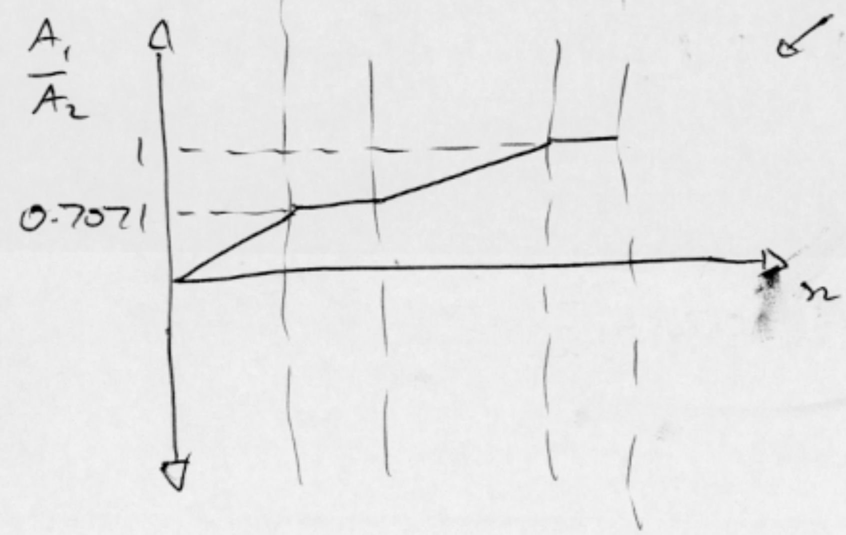
$$\left.\frac{A_1}{A_2}\right|_{\omega_n = 0.7654\sqrt{\frac{k}{m}}} = 0.7071$$

Mode Shape for $\omega_n = -0.7071\sqrt{\frac{k}{m}}$





mode shape for $\omega_n = 0.7071\sqrt{\frac{k}{m}}$



Extra Workspace – Question Number

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