MMAN2300

Engineering Mechanics 2

Part B: Vibration Analysis

Rotating Unbalance

Base Excitation

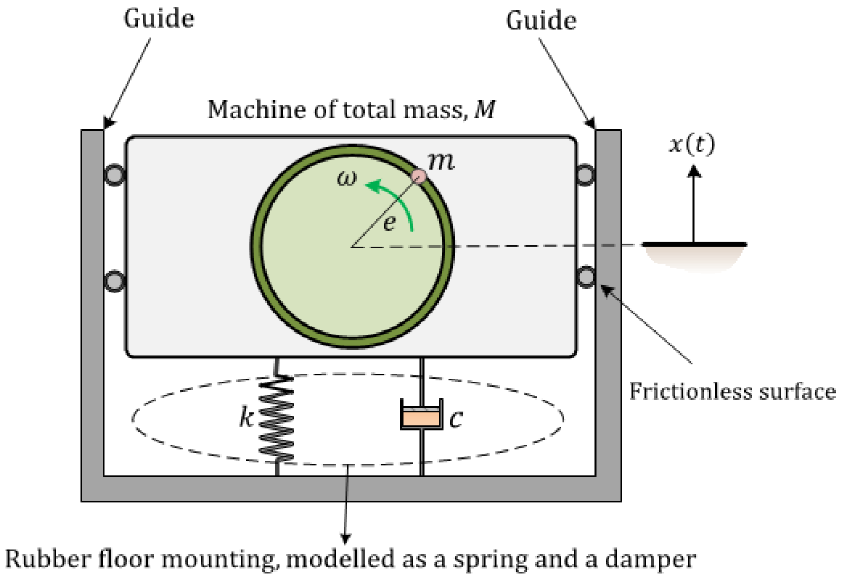
Response of a Damped System under Rotating Unbalance

Section 3.7 Rao

Many machines and devices have rotating components, usually driven by electric motors. Small irregularities in the distribution of the rotating mass can cause substantial vibration. Unbalance in rotating machines is a common source of vibration excitation.

Consider a spring-mass-damper system constrained to move in the vertical direction only, and is excited by a rotating machine that is unbalanced.

The machine of total mass *M* (including the rotor and unbalance mass) rotates with a constant angular velocity ω. The unbalance is represented by an eccentric mass *m* at a distance *e* from the centre of rotation.



Harmonic disturbing force resulting from rotating unbalance

The equation of motion can be obtained as:

$$M\ddot{x} + c\dot{x} + kx = (me\omega^2)\sin \omega t$$

This equation of motion is identical to the equation of motion for damped forced harmonic vibration, where F_o is replaced by $me\omega^2$.

A general solution for the forced response of the mass can be written as:

$$x_p(t) = X \sin(\omega t - \phi)$$

Substituting the general solution into the equation of motion results in the following expressions for the steady-state amplitude and phase:

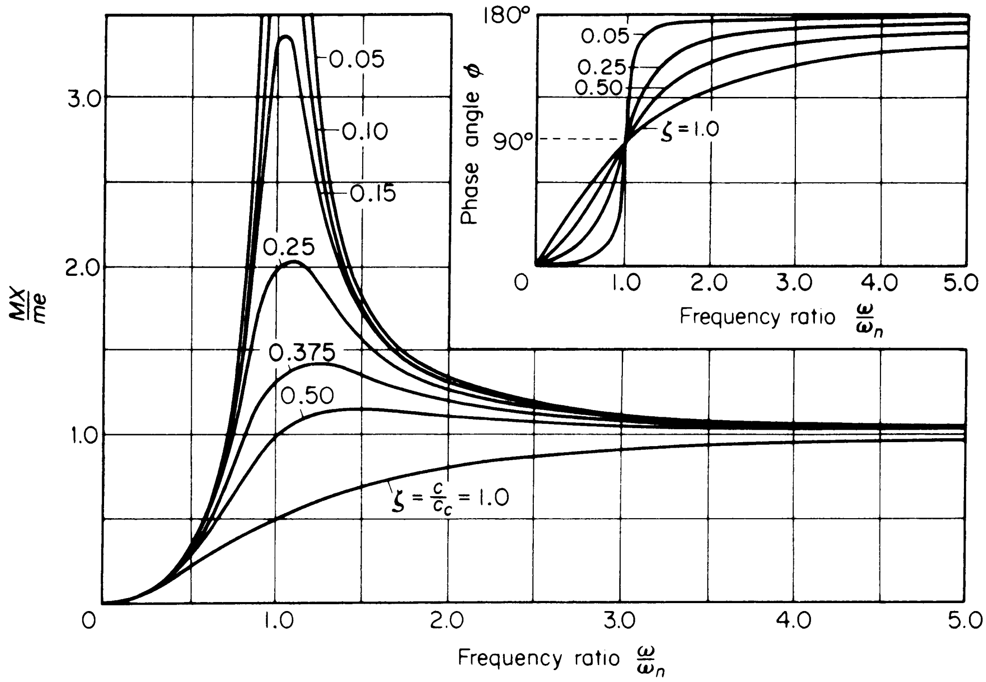
$$X = \frac{me\omega^2}{\sqrt{(k - M\omega^2)^2 + (c\omega)^2}}, \qquad \phi = \tan^{-1}\left(\frac{c\omega}{k - M\omega^2}\right)$$

Since F_o is replaced by $me\omega^2$, the non-dimensionalised steady-state amplitude can be written as:

$$\frac{kX}{F_o} = \frac{kX}{me\omega^2} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}.$$

We know $\omega_n = \sqrt{k/M}$, and rearranging as $k = \omega_n^2 M$, the term $\frac{kX}{me\omega^2}$ in the above equation can be written as: $\frac{MX}{me}\left(\frac{\omega_n}{\omega}\right)^2$. The non-dimensionalised steady-state amplitude becomes $\frac{MX}{me}$, and the non-dimensionalised phase variation is the same as for force excitation.

$$\frac{MX}{me} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}, \qquad \phi = \tan^{-1}\left(\frac{2\zeta\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right)$$



- A low frequencies ($\omega << \omega_n$), the amplitude $\frac{MX}{me}$ is near zero (the rotating unbalanced force is negligible).

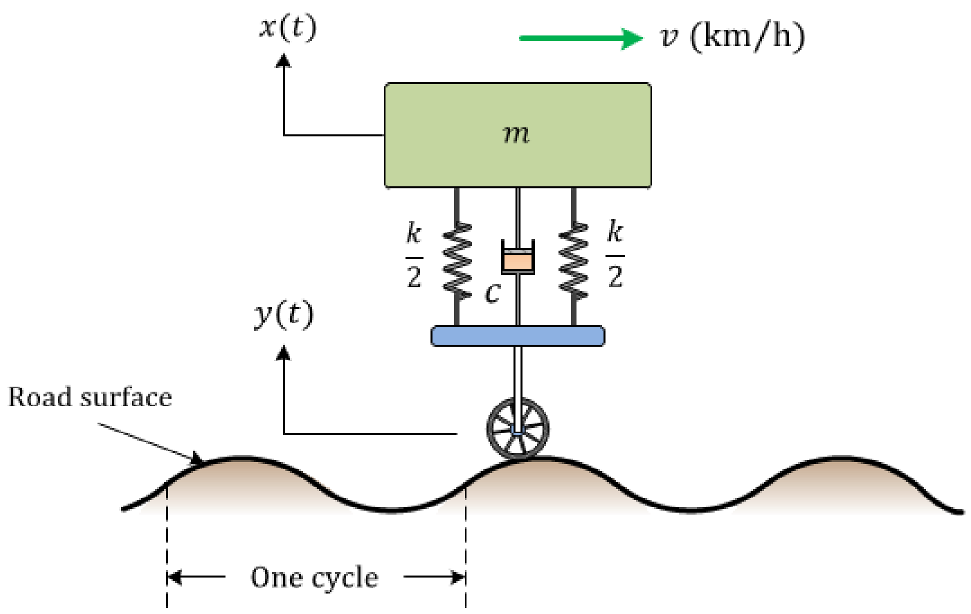
- At high frequencies ($\omega >> \omega_n$), the amplitude $\frac{MX}{me}$ approaches unity ($\frac{MX}{me} \to 1$) regardless of the damping ratio ζ (the magnitude of the displacement X approaches me/M.

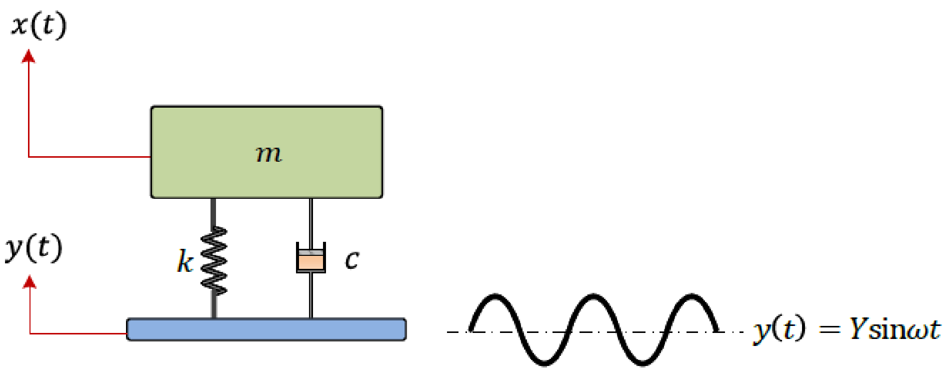
Response of a Damped System under Harmonic Motion of the Base

Section 3.6 Rao

Machines (or parts of) may be excited through elastic mountings, which may be modelled as springs and dashpots. For example, a car suspension is harmonically excited by a road surface through a shock absorber which may be modelled by a linear spring in parallel with a viscous damper.

In this case, the system (mass) is excited by the motion of its support (base) through its elastic mountings (spring and damper).





Note that the input disturbance from the base is a displacement, not a force. The input displacement is given by:

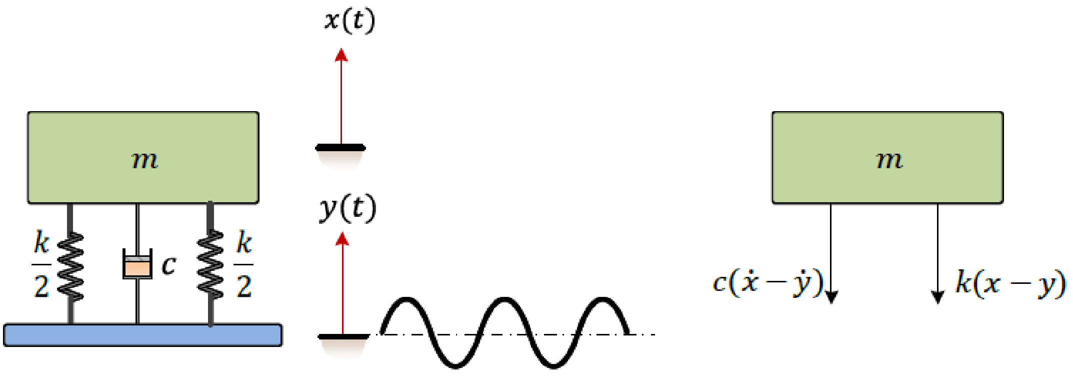
$$y(t) = Y \sin \omega t$$

where Y is the maximum amplitude of the input displacement.

The only forces applied to the mass are through the spring and damper.

Thus, applying a displacement $y(t) = Y\sin\omega t$ to the base is equivalent to applying a harmonic force to the mass given by

$$F(t) = ky(t) + c\dot{y}(t) = kY\sin\omega t + c\omega Y\cos\omega t$$



Spring-mass-damper system excited by motion of the base.

The equation of motion is dependent on the <u>relative</u> displacement and velocity of the spring and damper:

$$m\ddot{x} = -c\dot{x} - kx + c\dot{y} + ky = -c(\dot{x} - \dot{y}) - k(x - y)$$

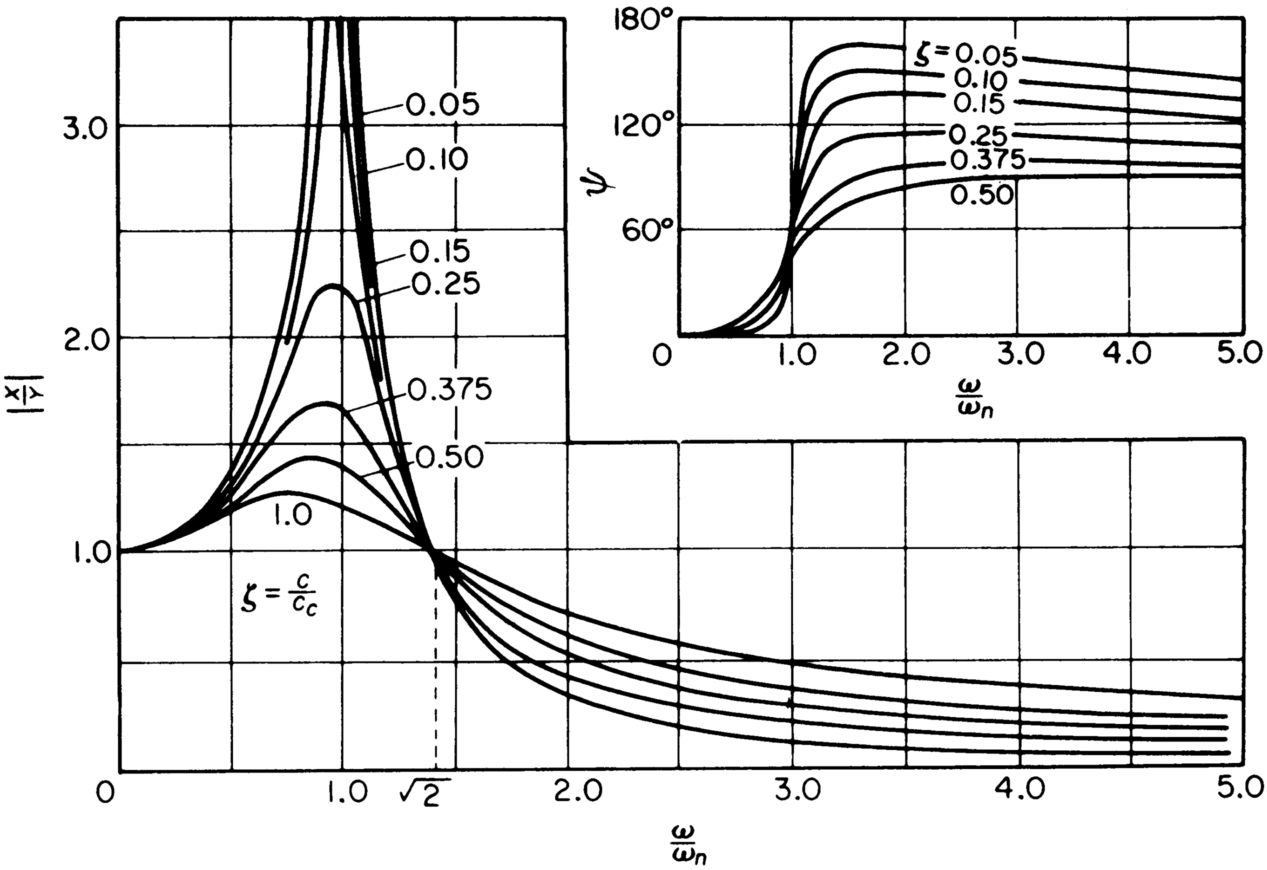
A general solution for the forced response of the mass can be written as:

$$x_p(t) = X\sin(\omega t - \phi)$$

After some derivation, it is possible to obtain a relationship between X (the maximum displacement response amplitude) and Y (the input displacement amplitude).

$$\frac{X}{Y} = \frac{\sqrt{k^2 + (c\omega)^2}}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} = \frac{\sqrt{1 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}.$$

The ratio X/Y is called the <u>displacement transmissibility</u>, and is used to describe how motion is transmitted from the base to the mass as a function of the frequency ratio ω/ω_n.



- At low frequencies ($\omega << \omega_n$), the displacement transmissibility approaches unity regardless of the damping ratio.

- Near resonance ($\omega = \omega_n$), the maximum amount of base motion is transferred to the mass.

- All response curves pass through $\frac{X}{Y} = 1$ when $\frac{\omega}{\omega_n} = \sqrt{2} \approx 1.4$, independent of ζ.

Vibration Isolation

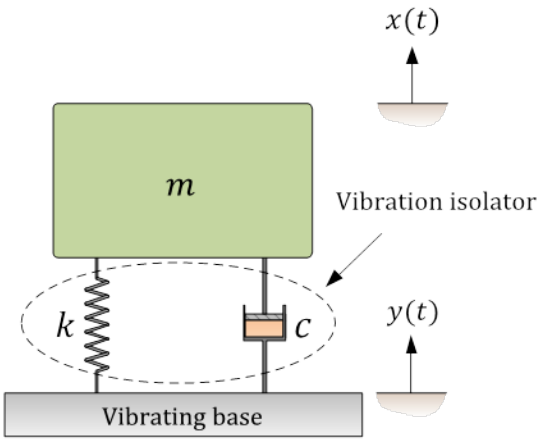
Section 9.10 Rao

Vibrations generated by machines are often unavoidable, but it is desirable to minimise their effects on the dynamic system by isolation. In the previous section, we discussed the motion transmitted from the base to the mass of a spring-mass-damper system. It is equally important to examine the motion transmitted from the mass to the base (foundation).

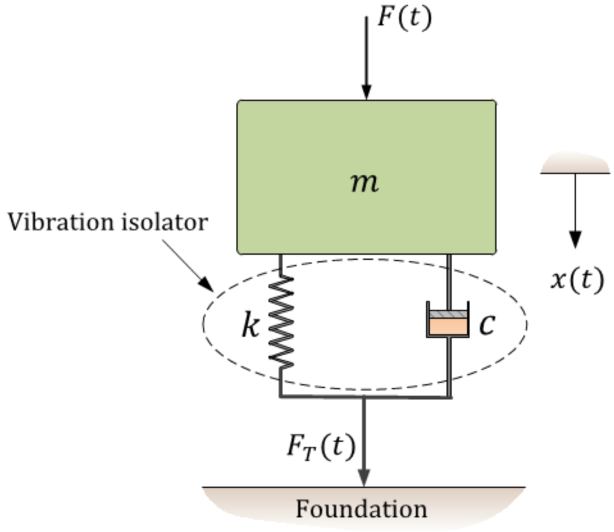
A vibration isolation system attempts to either:

(a) protect an object from excessive vibration transmitted to it from its supporting structure;
(b) prevent vibratory forces generated by a machine from being transmitted to the foundation or base.

In each case, it is desirable to reduce the transmitted force associated with the spring force and the damper force.



(a)



(b)

The harmonic force $F(t) = F_0 \sin \omega t$ in Fig. (b) above is transmitted to the foundation through the spring and damper. Hence, the transmitted force is given by

$$F_T(t) = kx(t) + c\dot{x}(t)$$

The magnitude of the transmitted force can be found as

$$F_T = \sqrt{(kx)^2 + (c\dot{x})^2} = \frac{F_0\sqrt{k^2 + \omega^2 c^2}}{\sqrt{(k - m\omega^2)^2 + \omega^2 c^2}}$$

The transmission ratio (TR) is the ratio of the magnitude of the transmitted force F_T to the magnitude of the exciting force F_0 and is given by

$$TR = \frac{F_T}{F_0} = \frac{\sqrt{k^2 + \omega^2 c^2}}{\sqrt{(k - m\omega^2)^2 + \omega^2 c^2}} = \frac{\sqrt{1 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}}$$

Comparison of the transmission ratio for vibration isolation and the displacement transmissibility in base excitation shows that:

$$TR = \frac{F_T}{F_0} = \frac{X}{Y}$$

Vibration isolators are installed in a wide range of applications including isolated pumps, pipework, generators, gym floors and swimming pools.

Example

Consider the following case study to design a simple vibration isolator.

- An exhaust fan rotates at 1000 rpm and gives rise to a harmonically varying force which is transmitted to the foundation.
- The mass of the exhaust fan is 40 kg.
- The exhaust fan is supported by four springs with total stiffness k.
- Each spring has stiffness k_s.
- Damping is assumed to be negligible.

Design a vibration isolator to be placed between the machine and the rigid foundation such that only 10% of the harmonic force is transmitted to the foundation.

The transmission ratio (TR) is the ratio of the magnitude of the transmitted force F_T to that of the exciting force F_0. For this case study, TR = 0.1. We are also told to neglect damping, that is, ζ =0. We use the following expression for the transmission ratio:

$$TR = \frac{F_T}{F_0} = \frac{\sqrt{1 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}}$$

The excitation frequency is

$$\omega = \frac{1000 \ \times 2\pi}{60} = 104.72 \text{ rad/s}$$

The undamped natural frequency of the system is

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4k_s}{40}} = \frac{\sqrt{k_s}}{3.1623}$$

We are told to neglect damping. For ζ =0 we have

$$TR = \sqrt{\frac{1}{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2}} = 0.1$$

We can find that the stiffness of each spring must be

$$k_s = 9970 \text{ N/m}$$