MMAN2300 Engineering Mechanics 2
Part A Dynamics

Week 1: Velocity analysis of rigid bodies (review)

Week 2: Velocity analysis of rigid bodies to rotating axes

Weeks 3&4: Instant centre method

Week 5: Acceleration analysis of rigid bodies about a fixed reference

Week 6: Acceleration analysis of rigid bodies about a rotating frame

Week 7: Kinetics of rigid bodies – Equations of motion

Week 8: Kinetics of rigid bodies – Work-energy method

Week 9: Gear systems and analysis

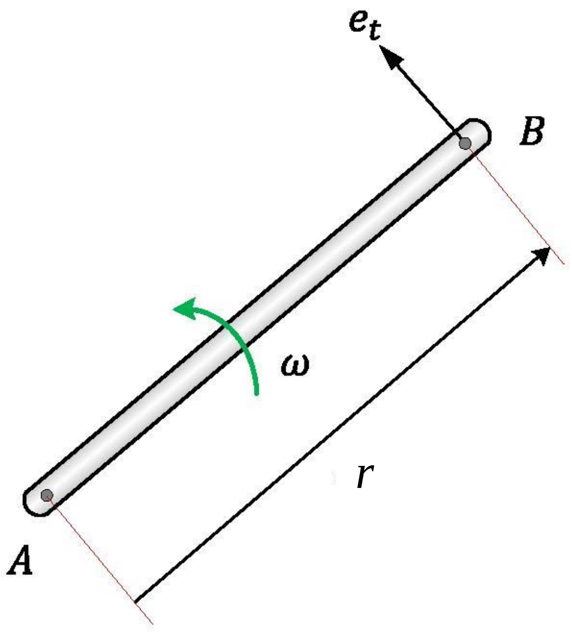
Learning outcomes:

- You are familiar with:
  - plane kinematics of rigid bodies,
  - equations of motion, work and energy for rigid bodies, and
  - the principles and functions of gears and gear trains and gear motion analysis.

- Be able to explain, describe and apply principles and components of Engineering Dynamics to solve problems using a range of techniques

Week 1 - Velocity analysis of rigid bodies (review)

Two points (eg, *A* and *B*) fixed on the same rigid body

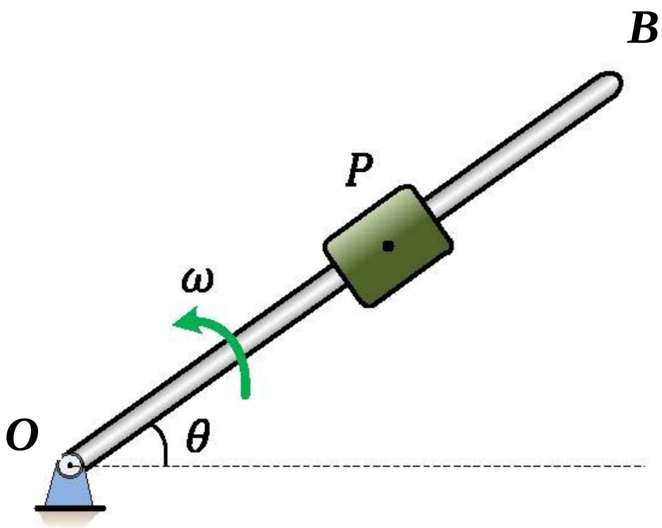


Relative velocity equation:

$$\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{B/A}$$

$$\boldsymbol{v}_{B/A} = \omega r \boldsymbol{e}_t = \boldsymbol{\omega} \times \boldsymbol{r}$$

Week 2 - Velocity analysis of rigid bodies to rotating axes



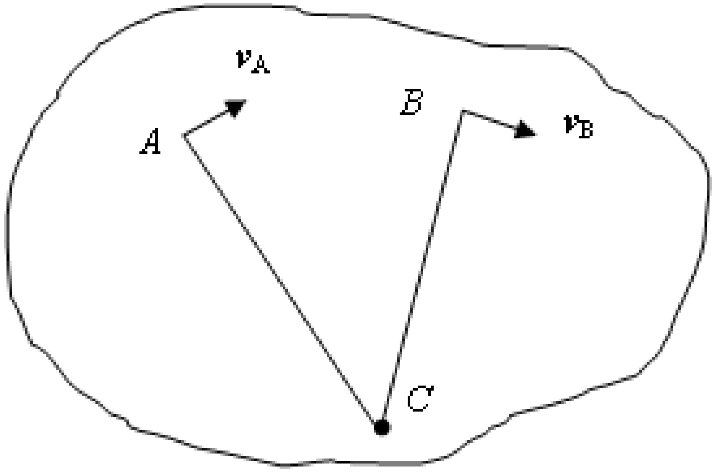
To analyse the motion of point *P* – a moving point on a rotating body *OB*, we need to define a reference point on the rotating body *OB*.

$$\boldsymbol{v}_P = \boldsymbol{v}_{P'} + \boldsymbol{v}_{P/P'}$$

Weeks 3&4 – Method of instant centres

1. Instantaneous centres of zero velocity

At every instant during the motion of a rigid body in a plane there exists a point that is instantaneously at rest. This point is called the instantaneous centre of zero velocity.



The body rotates about C instantaneously.

2. Instant centres

At any instant we can find a common point to two bodies which has the same instantaneous velocity in each body.

3. Three Centre Theorem (Kennedy's Theorem)

Kennedy's Theorem consists of two major components:

(a) If there are *n* bodies and we take them two at a time, the total number of instant centres are:

$$N_{Ic} = \frac{n(n-1)}{2}$$

(b) Three Centre Theorem: The 3 instant centres for 3 independent bodies in plane motion will lie on a straight line.

4. Conservation of Power and Energy

The analysis is based on the assumption that the input power equals the output power.

$$P_{\mathrm{in}} = P_{\mathrm{out}}$$

$$P = T\omega = Fv$$

$$\Rightarrow \quad \frac{T_{in}}{T_{out}} = \frac{\omega_{out}}{\omega_{in}}$$

$$or \quad \frac{F_{in}}{F_{out}} = \frac{v_{out}}{v_{in}}$$

Mechanical Advantage (*M.A.*)

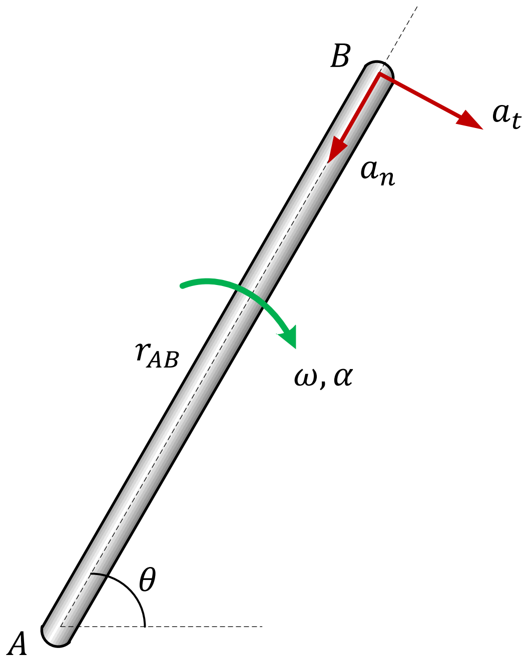
$$M.A. = \frac{F_{out}}{F_{in}} = \frac{v_{in}}{v_{out}}$$

If both input and output link have an angular velocity, then

$$M.A. = \frac{\omega_{in}}{\omega_{out}} \times \frac{r_{in}}{r_{out}}$$

Week 5 – Acceleration analysis of rigid bodies to fixed references

Points A and B are fixed on link AB



Relative velocity equation:

$$\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{B/A}$$

where $\boldsymbol{v}_{B/A} = \omega \overline{AB} \boldsymbol{e}_t = \boldsymbol{\omega} \times \boldsymbol{r}_{AB}$

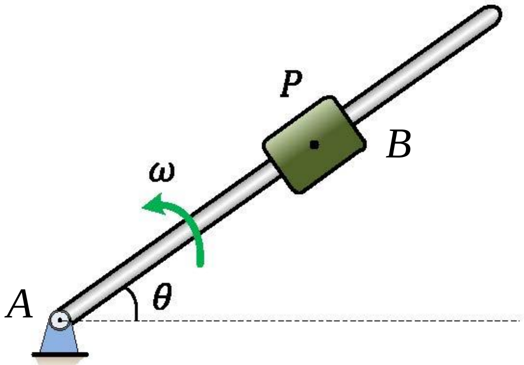
Relative acceleration equation:

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\begin{aligned} \vec{a}_{B/A} &= (a_{B/A})_n \vec{e}_n + (a_{B/A})_t \vec{e}_t \\ &= \omega^2 \overline{AB} \vec{e}_n + \alpha \overline{AB} \vec{e}_t \\ &= \vec{\omega} \times (\vec{\omega} \times \vec{r}_{AB}) + \vec{\alpha} \times \vec{r}_{AB} \end{aligned}$$

Week 6 – Acceleration analysis of rigid bodies to a rotating reference

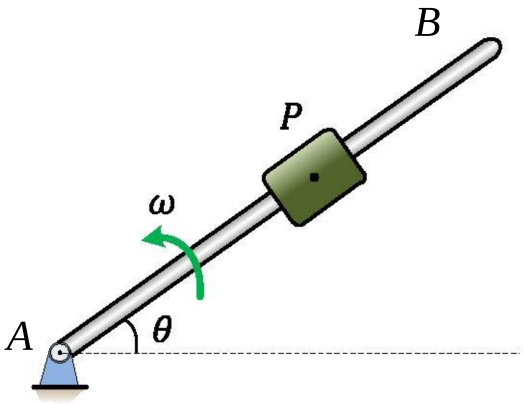
Points *A* and *B* are fixed to rigid link *AB*. Point *P* moves away from *B* with a relative velocity v_{rel}.



This is a coriolis problem (i.e., a moving point on a rotating body)

In cases where particles or members move on a rotating body, it is convenient to consider:

- the motion of the member relative to a rotating reference frame fixed to the rotating body, and
- the motion of the moving reference frame (which will, in general, be both rotating and translating) with respect to a fixed frame.



The velocity of point P is determined using

$$\boldsymbol{v_P} = \boldsymbol{v}_{P'} + \boldsymbol{v}_{P/P'}$$

The acceleration of point P is determined using

$$\boldsymbol{a}_P = \boldsymbol{a}_{P'} + \boldsymbol{a}_{cor} + \boldsymbol{a}_{rel}$$

Week 7 – Plane kinetics of rigid bodies

Equations of motion for a rigid body

$\Sigma \boldsymbol{F} = m\boldsymbol{a}$, where $\boldsymbol{a}$ is the acceleration of the centre of mass

$\Sigma \boldsymbol{M} = I\boldsymbol{\alpha}$, where $\boldsymbol{\alpha}$ is the angular acceleration of the body

Week 8 – Plane kinetics of rigid bodies

Work-energy method

$$W_{1-2} = F \cdot S + M \cdot \boldsymbol{\theta} = \Delta T + \Delta V_g + \Delta V_e$$

$$W_{1-2} = F(S_2 - S_1) + M(\theta_2 - \theta_1) = F \cdot S + M \cdot \theta$$

ΔT is the change in kinetic energy

ΔV_g is the change in potential energy

ΔV_e is the change in elastic energy

The total change in kinetic energy is

$$\Delta T = \frac{1}{2}m(v_2^2 - v_1^2) + \frac{1}{2}I_G(\omega_2^2 - \omega_1^2)$$

The change in potential energy is

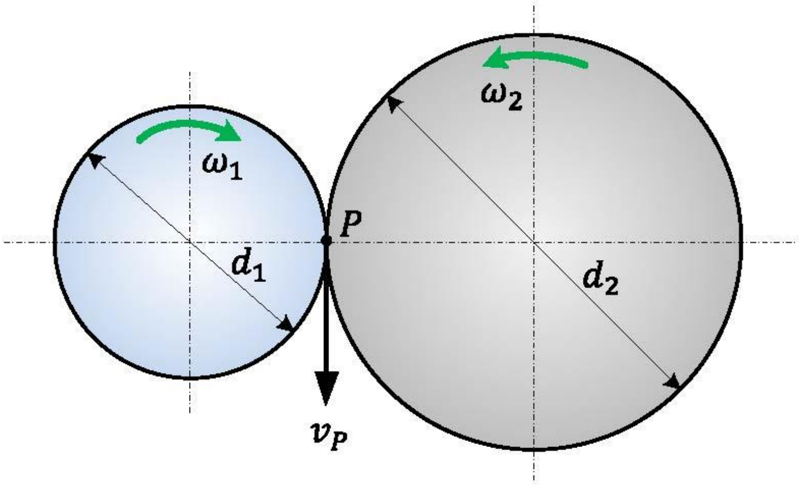
$$\Delta V_g = mg(h_2 - h_1)$$

The change in elastic energy is

$$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2)$$

Week 9: Gear systems and analysis

It is possible to replace two gears by two discs of pitch point diameter.

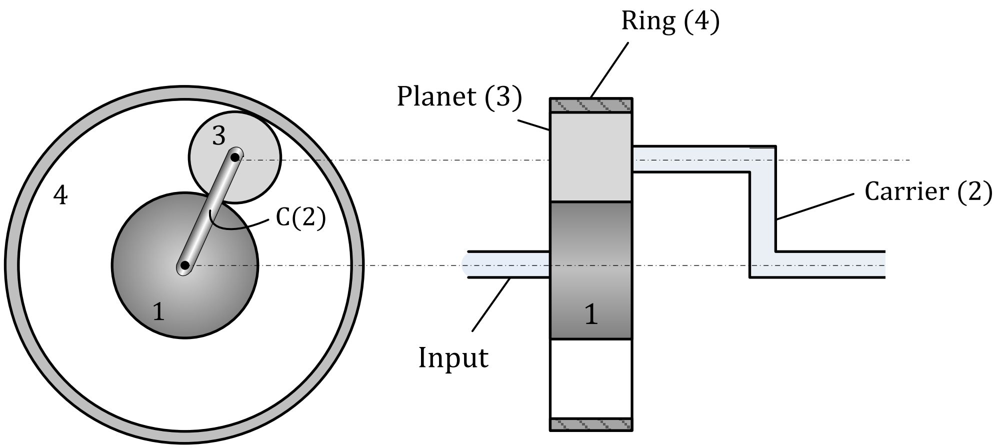


For no slipping, the velocity at the point of contact must be the same on both gears.

$$v_p = \omega_1 r_1 = -\omega_2 r_2$$

Module $M = \frac{d}{N}$ must be the same for both gears

$$\frac{\omega_1}{\omega_2} = -\frac{d_2}{d_1} = -\frac{N_2}{N_1}$$



$$\frac{d_1}{2} + d_3 = \frac{d_4}{2}$$

All gears have the same module

$$\frac{N_1}{2} + N_3 = \frac{N_4}{2}$$

$$\frac{\omega_{sun} - \omega_{carrier}}{\omega_{ring} - \omega_{carrier}} = -\frac{N_{ring}}{N_{sun}}$$