UNSW Sydney

School of Mechanical and Manufacturing Engineering

MMAN2300 ENGINEERING MECHANICS 2

Test 1

Question 1 [10 marks]

At the instant shown bar *AB* has a constant angular velocity of 3 rad/s in the clockwise direction.

(a) Using relative velocity analysis, determine the angular velocity (magnitude and direction) of the plate rotating about point *C* and the velocity (magnitude and direction) of point *B* relative to the plate at the instant shown.

(b) Locate all instant centres on Figure Q1 below and identify the instantaneous centres of zero velocity.

(c) Confirm the angular velocity of the plate in (a) using the method of instant centres.

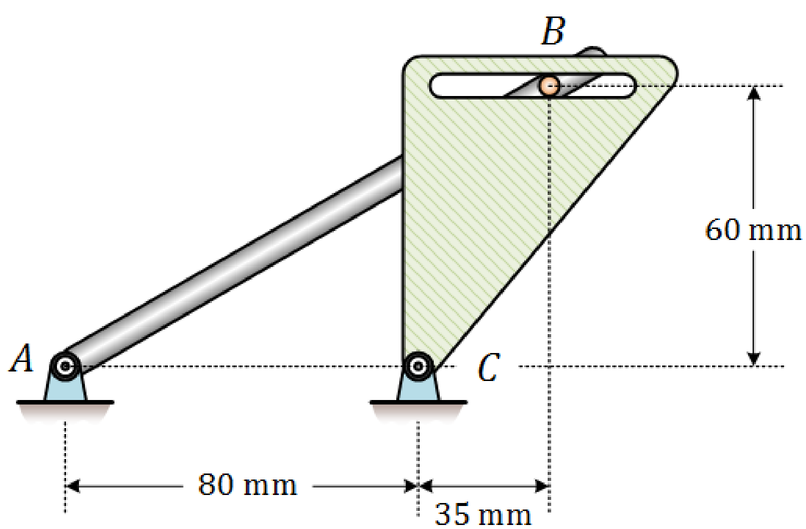


Figure Q1

(a) Using relative velocity analysis, determine the angular velocity (magnitude and direction) of the plate rotating about point C and the velocity (magnitude and direction) of point B relative to the plate at the instant shown.

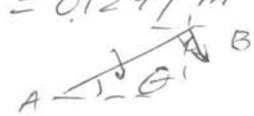
Define point B′ on the plate. Points B & B′ coincident.

$$\underset{\sim}{v}_B = \underset{\sim}{v}_{B'} + \underset{\sim}{v}_{B/B'}$$

$$v_B = \omega_{AB}\,\overline{AB} = 3\,\overline{AB}$$

$$\overline{AB} = \sqrt{(80+35)^2+60^2} = 129.71 \text{ mm} = 0.1297 \text{ m}$$

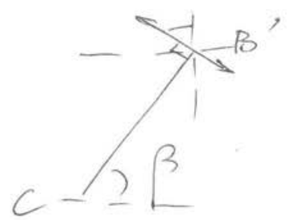
$$v_B = 3(0.1297) = 0.389 \text{ m/s}$$



$$\tan\theta = \frac{60}{80+35} \Rightarrow \theta = 27.55°$$

$$v_{B'} = \omega_{plate}\cdot\overline{B'C} = \omega_{plate}\,\overline{BC}$$

$$\overline{BC} = \sqrt{35^2+60^2} = 69.46 \text{ mm} = 0.0695 \text{ m}$$

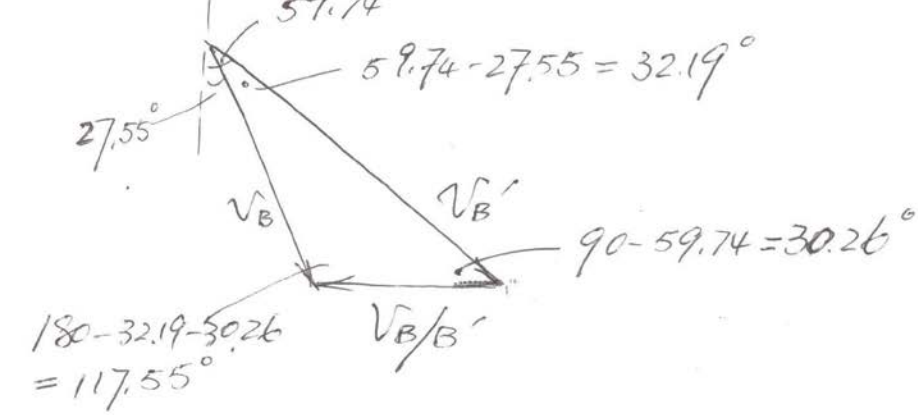


$$\tan\beta = \frac{60}{35}$$

$$\Rightarrow \beta = 59.74°$$

$\underset{\sim}{v}_{B/B'} = ? \quad \longleftrightarrow$

Draw the velocity vector diagram



$$\frac{V_B'}{\sin 117.55} = \frac{V_B}{\sin 30.26}$$

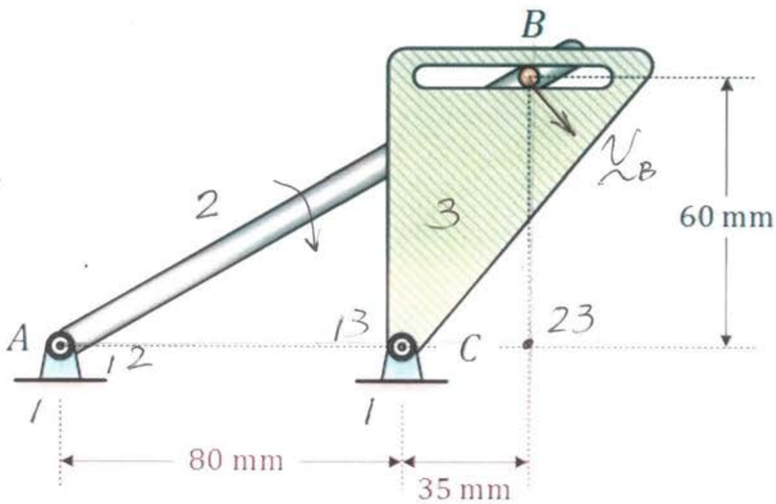
$$\Rightarrow \quad V_B' = V_B \frac{\sin 117.55}{\sin 30.26} = 0.684 \text{ m/s}$$

$$\omega_{plate} = \frac{V_B'}{\overline{BC}} = \frac{0.684}{0.0695} = \underline{9.848 \text{ rad/s CW}}$$

$$\frac{V_{B/B'}}{\sin 32.19} = \frac{V_B}{\sin 30.26}$$

$$\Rightarrow \quad V_{B/B'} = V_B \frac{\sin 32.19}{\sin 30.26} = 0.389 \frac{0.5327}{0.5039} = 0.4112 \text{ m/s} \quad \longleftarrow$$

(b) Locate all instant centres on Figure Q1 below and identify the instantaneous centres of zero velocity.



Instantaneous centres of zero velocity are 12, 13

(c) Confirm the angular velocity of the plate in (a) using the method of instant centres.

At instant centre 23

$$v_{23(2)} = v_{23(3)}$$

$$v_{23(2)} = \omega_2(\overline{12\text{-}23})$$

$$= 3\,(80+35) = 345 \text{ mm/s}$$

$$v_{23(3)} = \omega_3(\overline{13\text{-}23}) = 35\omega_3 \text{ mm/s}$$

$$\Rightarrow 345 = 35\omega_3$$

$$\Rightarrow \omega_3 = \frac{345}{35} = \underline{9.857 \text{ rad/s CW}}$$

Question 2 [10 marks]

A four cylinder automobile engine is to be supported on three shock mounts as shown in the figure below. The engine block assembly has a mass of 225 kg. If the unbalanced force generated by the engine is given by $900 \sin 100\pi t$ N, design the three shock mounts (each of stiffness *k* and viscous damping constant *c*) such that the amplitude of vibration is less than 2.5 mm. Assume a damping ratio of 0.01.

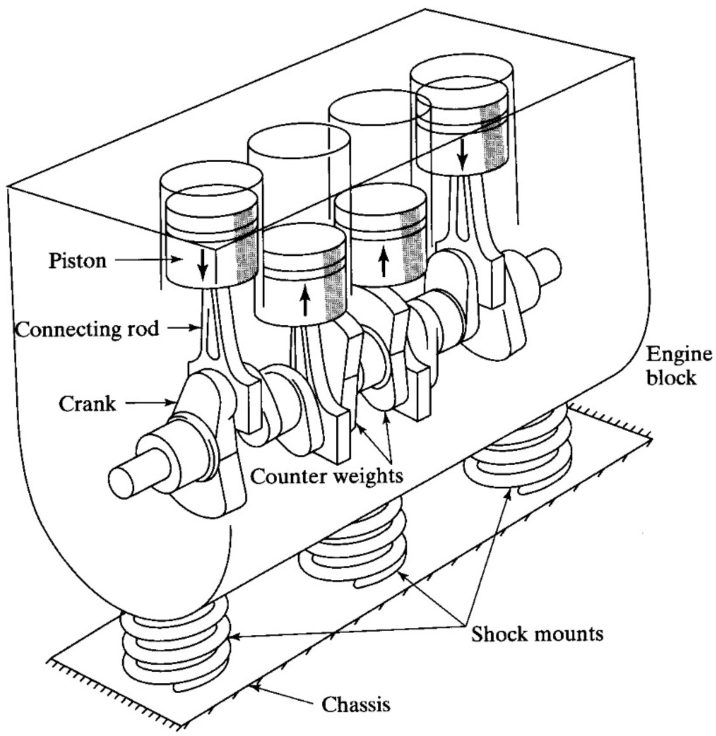


Figure Q2

$F(t) = F_0 \sin \omega t = 900 \sin 100\pi t \quad (N)$

Given $F_0 = 900 \ N$

$\omega = 100\pi \quad rad/s$

$m = 225 \ kg$

$\zeta = 0.01$

Find k_{mount}, C_{mount} such that $X < 2.5 \times 10^{-3} \ m$

X_{max} occurs when $\frac{\omega}{\omega_n} \approx 1$

Assume $X_{max} = 2.5 \times 10^{-3} \ m$

For forced excitation

$$\frac{kX}{F_0} = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}}$$

When $\frac{\omega}{\omega_n} = 1$

$$\frac{kX}{F_0} = \frac{1}{2\zeta}$$

$$\Rightarrow \quad k = \frac{F_0}{2 \times \zeta} = \frac{900}{(2)(0.01)(2.5 \times 10^{-3})}$$

$$= 1.8 \times 10^{7} \text{ N/m}$$

$$c = 2\zeta\sqrt{km}$$

$$= (2)(0.01)\sqrt{(1.8 \times 10^{7})(225)}$$

$$= 1272.8 \text{ kg/s}$$

Each shock mount has stiffness $k/3$ and damping $c/3$ (Since the shock mounts are in parallel).

$$\Rightarrow \quad k_{mount} = k/3 = 6 \times 10^{6} \text{ N/m}$$

$$c_{mount} = c/3 = 424.3 \text{ kg/s}$$