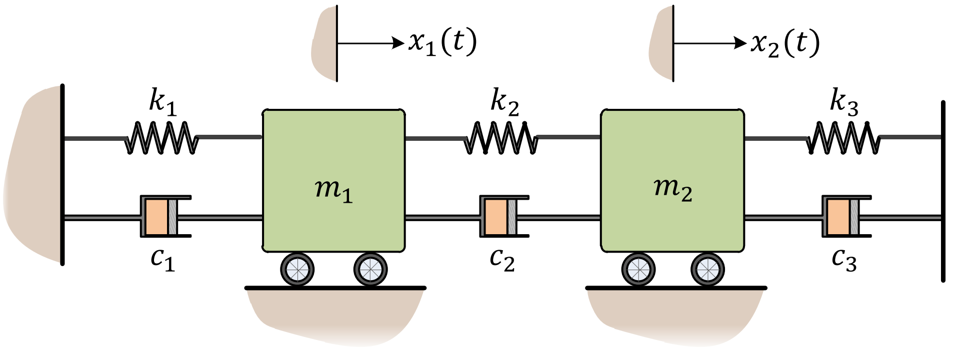
MMAN2300 Engineering Mechanics 2
Part A: Vibration Analysis
Tutorial 3

Question 1

State the equations of motion. Rearrange the equations of motion in matrix form to obtain the mass, damping and stiffness matrices.



Solution

Equations of motion

$$m_1 \ddot{x}_1 = -k_1 x_1 - c_1 \dot{x}_1 - k_2 x_1 - c_2 \dot{x}_1 + k_2 x_2 + c_2 \dot{x}_2$$

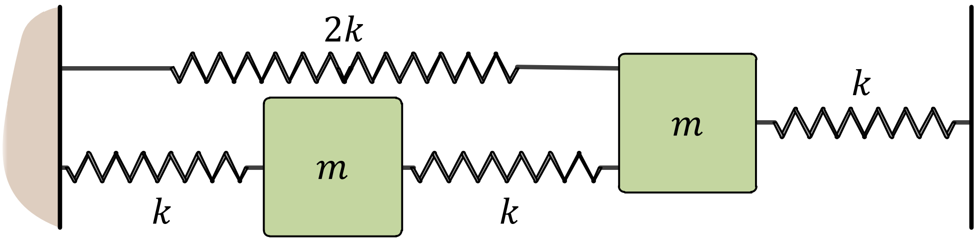
$$m_2 \ddot{x}_2 = -k_2 x_2 - c_2 \dot{x}_2 - k_3 x_2 - c_3 \dot{x}_2 + k_2 x_1 + c_2 \dot{x}_1$$

$$-\omega^2 \underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{\text{mass matrix}} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} + j\omega \underbrace{\begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{bmatrix}}_{\text{damping matrix}} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$$

$$+ \underbrace{\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix}}_{\text{stiffness matrix}} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Question 2

State the equations of motion. Rearrange the equations of motion in matrix form to obtain the mass and stiffness matrices.



Solution

Equations of motion

$$m\ddot{x}_1 = -kx_1 - kx_1 + kx_2$$

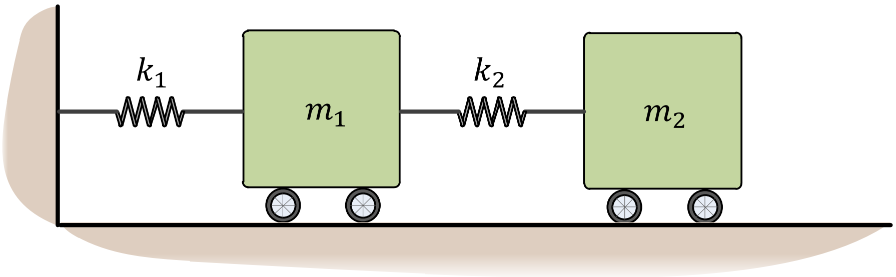
$$m\ddot{x}_2 = -kx_2 - kx_2 - 2kx_2 + kx_1$$

$$-\omega^2 \underbrace{\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}}_{\text{mass matrix}} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} + \underbrace{\begin{bmatrix} 2k & -k \\ -k & 4k \end{bmatrix}}_{\text{stiffness matrix}} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

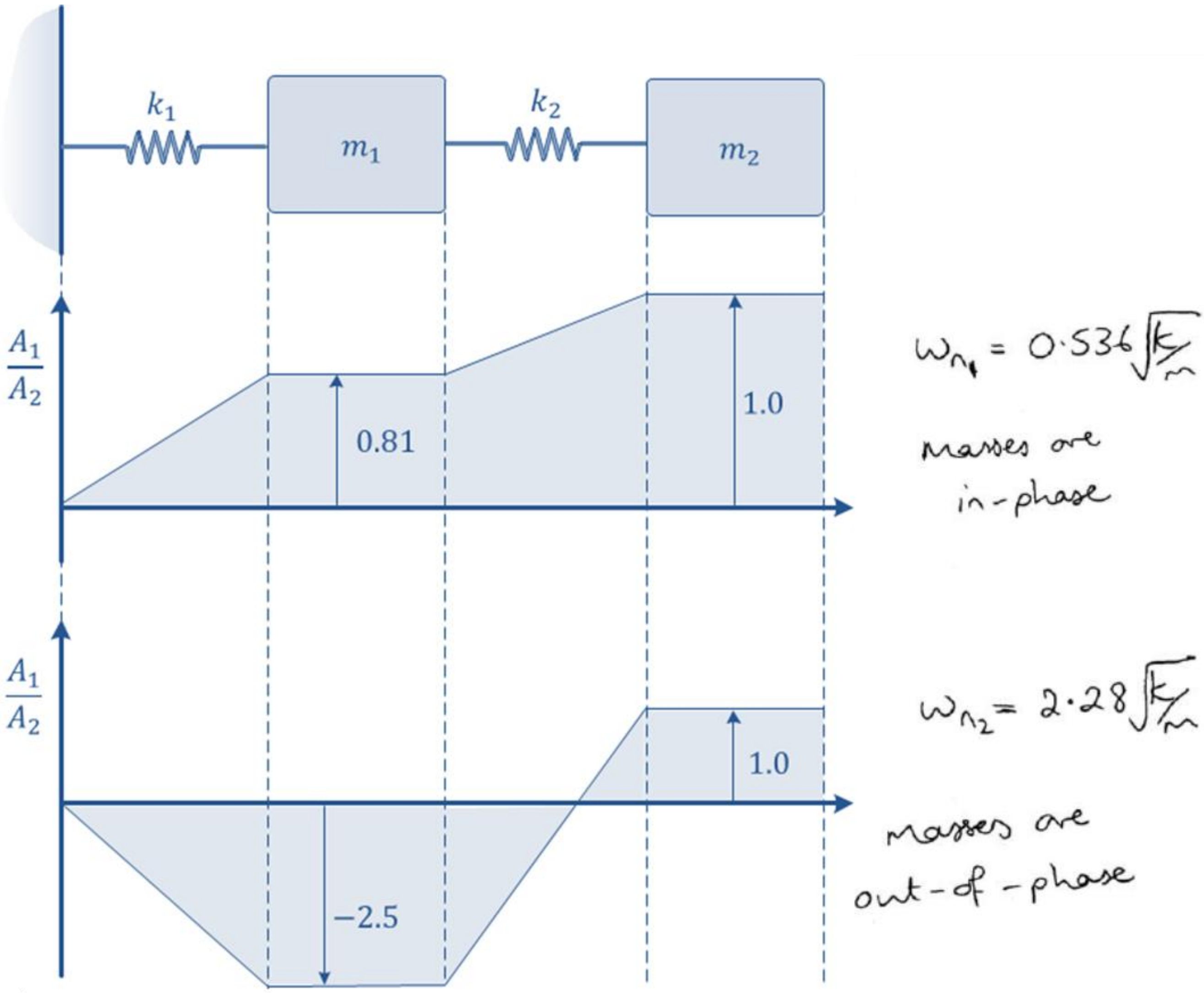
Question 3

The figure below shows two train carriages connected together in series by springs. One spring is fixed to an abutment. Determine the natural frequencies and modeshapes of the system in axial vibration.

Let $k_1 = k_2/3 = k$ and $m_1 = m_2/2 = m$.

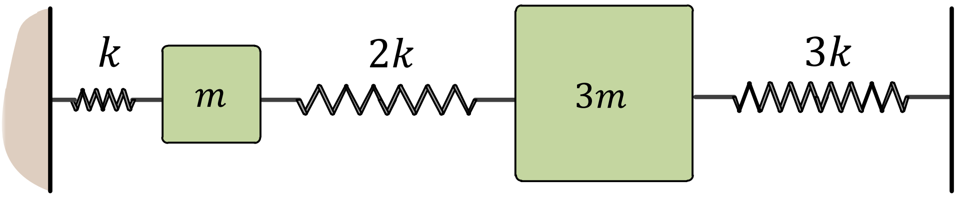


Solution

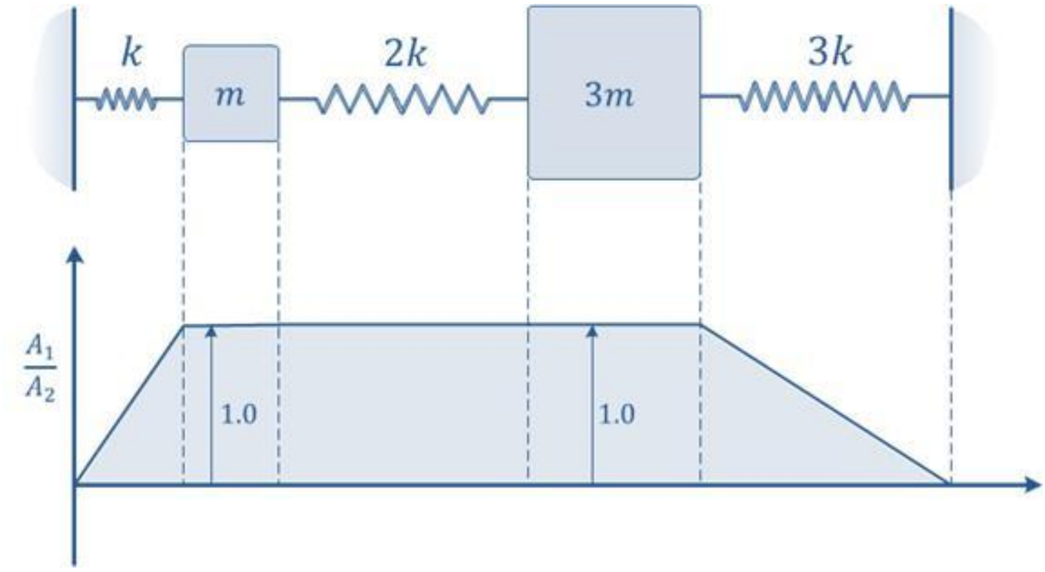


Question 4

Determine the natural frequencies and modes shapes for axial vibration of the system shown below.

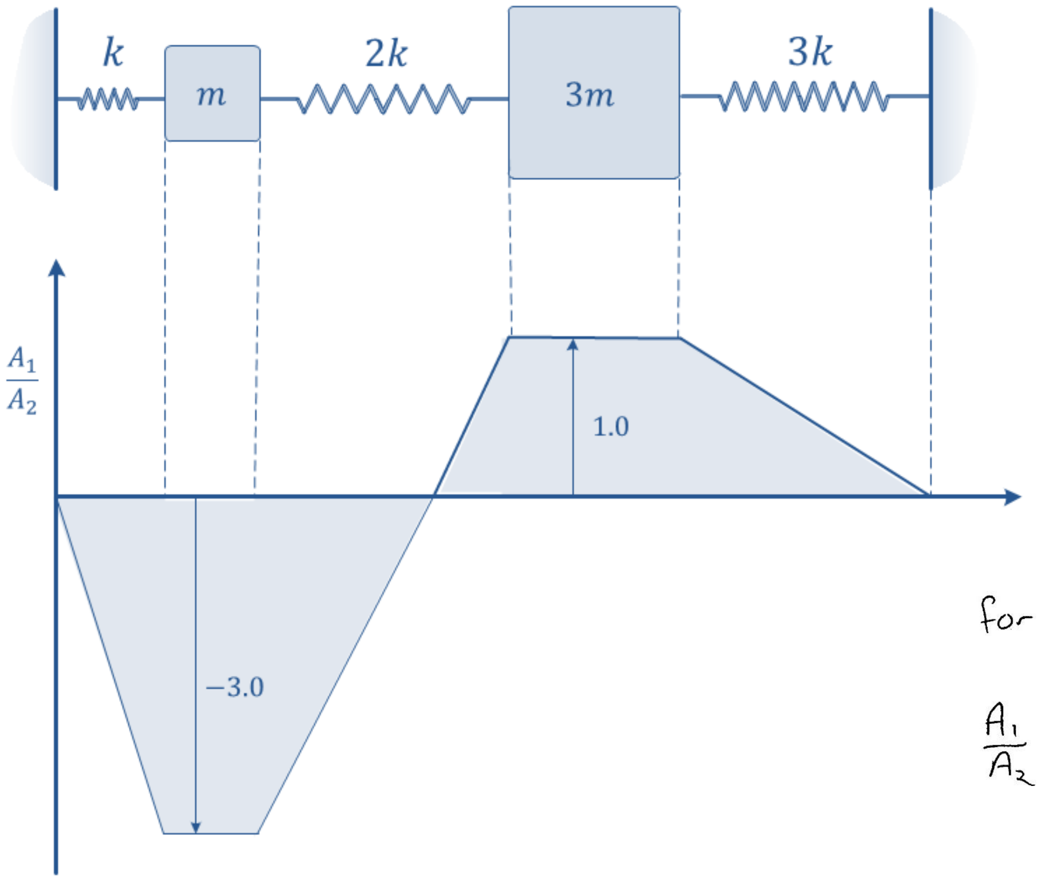


Solution



for $\omega_{n_1} = \sqrt{\frac{k}{m}}$

$\frac{A_1}{A_2} = 1.0$

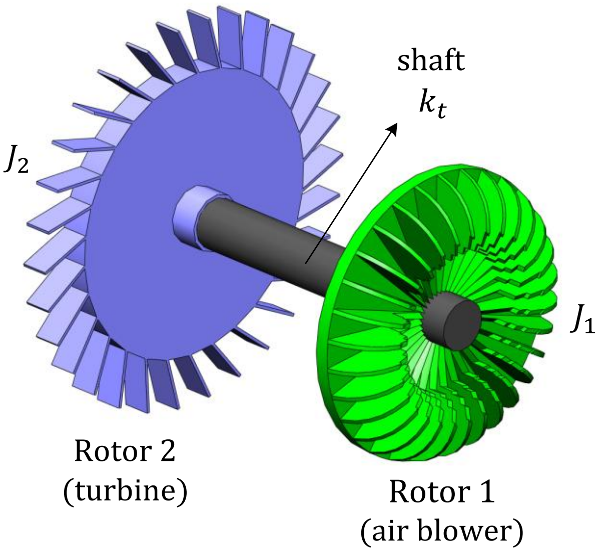


for $\omega_{n_2} = 1.91\sqrt{\frac{k}{m}}$

$\frac{A_1}{A_2} = -3$

Question 5

The figure below shows two rotors of mass moments of inertia J_1 and J_2 connected by a shaft of torsional stiffness k_t. Find expressions for the natural frequencies of the system. Discuss the characteristics of this type of system.



Solution

$$\omega_{n_1} = 0 \qquad \omega_{n_2} = \sqrt{\frac{k_t (J_1 + J_2)}{J_1 J_2}}$$

A free-free system will always have a zeroth natural frequency. At the zero natural frequency, the system is not oscillating, but moves as a whole without any relative motion between the two rotors (rigid body motion).

At the second natural frequency, the rotors are vibrating out-of-phase.