MMAN2300
Engineering Mechanics 2

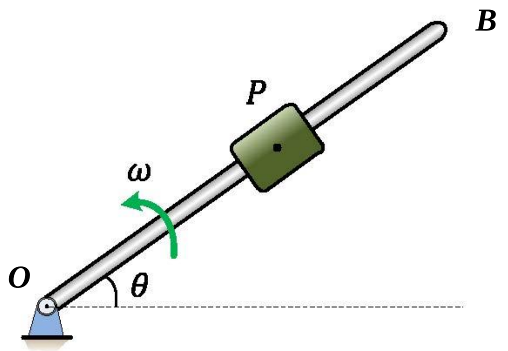
Part A: Week 2

Velocity analysis of rigid bodies to rotating axes

(Chapter 5/7 Meriam and Kraige)

2. Motion relative to a rotating reference frame

- Example



Consider a collar *P* which slides at a relative speed v_{rel} along a rod *OB* rotating at an angular velocity ω about *O* (fixed).

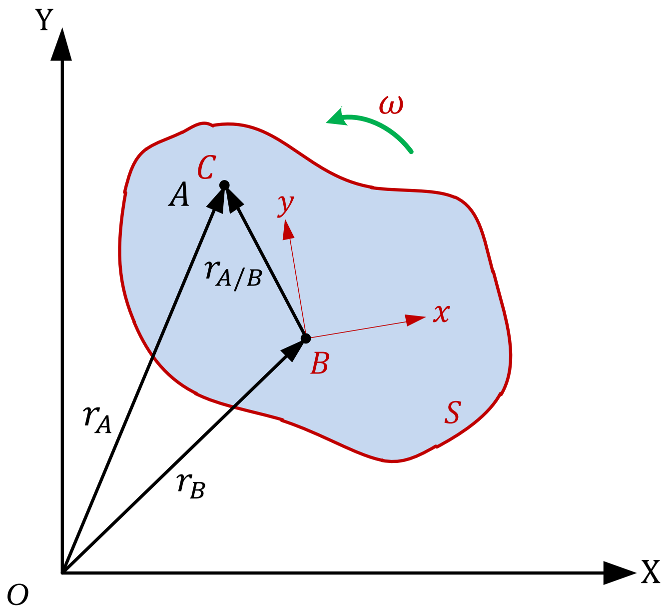
To analyse the motion of point *P* – a moving point on a rotating body *OB*, we need to define a reference point (*P'*) on the rotating body *OB*.

$$\boldsymbol{v}_P = \boldsymbol{v}_{P'} + \boldsymbol{v}_{P/P'}$$

Where $v_{P/P'} = v_{rel}$

- General case

Body *S* has translational and rotational motion; Point *A* is moving on body *S* with a relative velocity



Define two coincident points *A* & *C*:

- Point *A* moves with respect to body *S*;
- Points *B* and *C* are fixed to body *S*, and point *B* is the centre of rotation.

Define two reference systems:

- Red coordinate system fixed to body *S*;
- Fixed reference X-Y

The position of point *A* is expressed as:

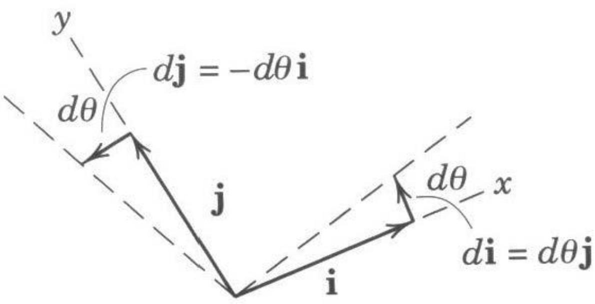
$$\boldsymbol{r}_A = \boldsymbol{r}_B + \boldsymbol{r}_{A/B} \qquad (1)$$

Where $\boldsymbol{r}_{A/B} = x\boldsymbol{i} + y\boldsymbol{j} = \boldsymbol{r}$

Note that

(a) the length of $\boldsymbol{r}_{A/B} = \boldsymbol{r}$ is changing as point *A* is moving on body *S*, and

(b) the unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ are rotating with the x-y axes
Thus, their time derivatives must be evaluated



$d\boldsymbol{i} = d\theta\boldsymbol{j} = \ \omega\boldsymbol{j}$

$d\boldsymbol{j} = -d\theta\boldsymbol{i} = \ -\omega\boldsymbol{i}$

When the cross product is introduced, we have
$\boldsymbol{\omega} \times \boldsymbol{i} = \ \omega\boldsymbol{j}$

$\boldsymbol{\omega} \times \boldsymbol{j} = \ -\omega\boldsymbol{i}$

Therefore $\quad d\boldsymbol{i} = \omega\boldsymbol{j} = \ \boldsymbol{\omega} \times \boldsymbol{i}$
$\quad d\boldsymbol{j} = -\omega\boldsymbol{i} = \ \boldsymbol{\omega} \times \boldsymbol{j}$

To find velocity of point A, differentiate (1) with respect to time

$$\dot{\boldsymbol{r}}_A = \ \dot{\boldsymbol{r}}_B + \frac{d}{dt}(x\boldsymbol{i} + y\boldsymbol{j})$$
$$= \ \dot{\boldsymbol{r}}_B + \left(x\frac{d\boldsymbol{i}}{dt} + y\frac{d\boldsymbol{j}}{dt}\right) + (\dot{x}\boldsymbol{i} + \dot{y}\boldsymbol{j})$$

Since $\quad x\frac{d\boldsymbol{i}}{dt} + y\frac{d\boldsymbol{j}}{dt} = \boldsymbol{\omega} \times x\boldsymbol{i} + \boldsymbol{\omega} \times y\boldsymbol{j} = \boldsymbol{\omega} \times (x\boldsymbol{i} + y\boldsymbol{j}) = \ \boldsymbol{\omega} \times \boldsymbol{r}$

$\dot{x}\boldsymbol{i} + \dot{y}\boldsymbol{j} = \boldsymbol{v}_{rel}$ the velocity of point A relative to the rotating frame x-y

$$\dot{\boldsymbol{r}}_A = \ \dot{\boldsymbol{r}}_B + \boldsymbol{\omega} \times \boldsymbol{r} + \boldsymbol{v}_{rel}$$

As point B is fixed $\quad \dot{\boldsymbol{r}}_B = 0$

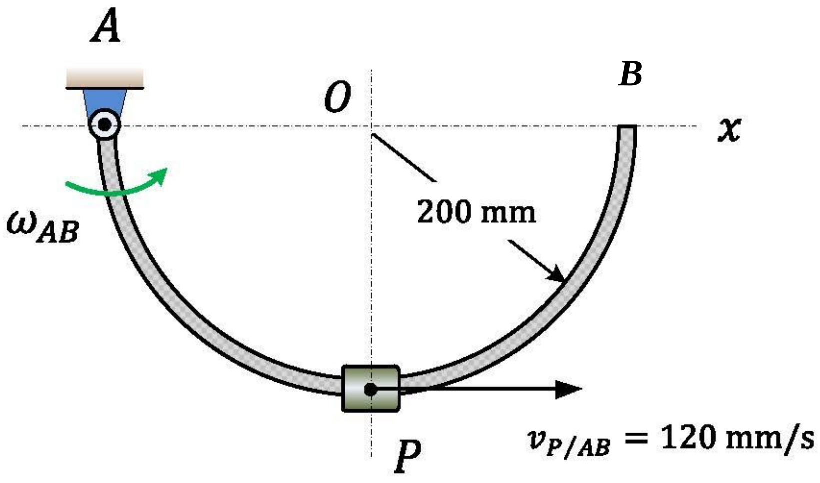
$$\boldsymbol{v}_A = \boldsymbol{\omega} \times \boldsymbol{r} + \boldsymbol{v}_{rel}$$

Where $\boldsymbol{\omega} \times \boldsymbol{r} = \ \boldsymbol{v}_{C/B} = \ \boldsymbol{v}_C$

$$\boldsymbol{v}_A = \boldsymbol{v}_C + \boldsymbol{v}_{rel}$$

Example 1

The collar *P* slides from *A* towards *B* along a semi-circular rod *AB* of radius 200 mm. The rod rotates about the pin at *A*, and the speed of the collar *P* relative to the rod is constant at 120 mm/s. When the system is in the position shown, the angular velocity of the rod is ω_{AB} = 0.8 rad/s counter clockwise. Determine the velocity of *P* at the instant shown.



Example 2

The pin *A* of the hinged link *AC* is confined to move in the rotating slot of link *OD*. The angular velocity of *OD* is ω = 2 rad/s clockwise and is constant for the interval of motion concerned. For the position where θ = 45° with *AC* horizontal, determine the velocity of pin *A* and the velocity of *A* relative to the rotating slot in *OD*.