Question 2 [10 marks]

A four cylinder automobile engine is to be supported on three shock mounts as shown in the figure below. The engine block assembly has a mass of 225 kg. If the unbalanced force generated by the engine is given by $900 \sin 100\pi t$ N, design the three shock mounts (each of stiffness k and viscous damping constant c) such that the amplitude of vibration is less than 2.5 mm. Assume a damping ratio of 0.01.

3 springs

$M = 225$ kg

$F_0 = 900 \sin 100\pi t$ N

$\omega = 100\pi$ rads^{-1}

$k = ?$

$c = ?$

$X < 2.5$ mm

$\zeta = 0.01$

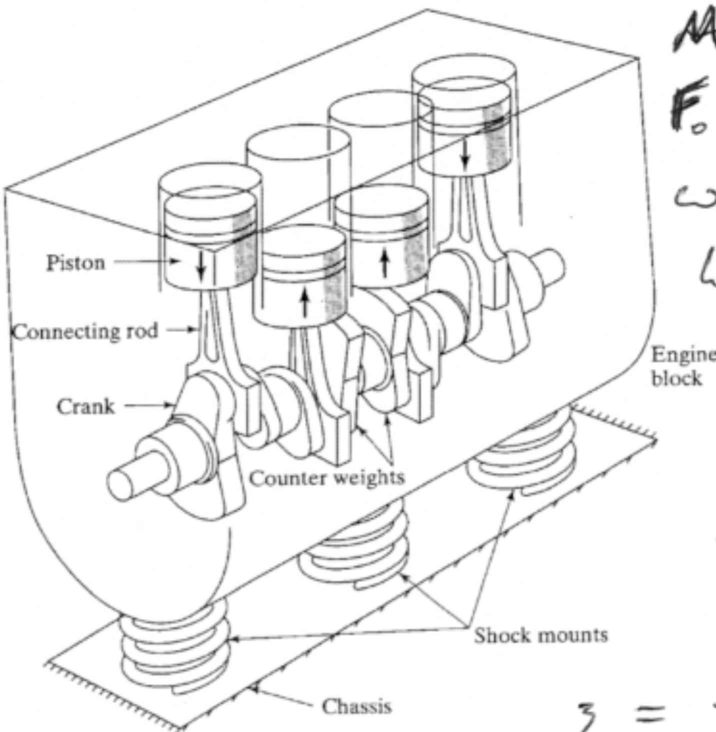


Figure Q2

$$\zeta = \frac{c}{2\sqrt{km}}$$

$$r = \frac{\omega}{\omega_n}$$

$$\cancel{F_0 = me\omega^2 = 900}$$

$$\cancel{225 e \times (100\pi)^2 = 900}$$

$$\cancel{\therefore e = \frac{1}{2500\pi^2}} \quad m ?$$

$$me = \frac{F_0}{\omega^2} = \frac{900}{(100\pi)^2} = \frac{9}{100\pi^2}$$

$$\frac{MX}{me} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$\frac{225X}{\frac{9}{100\pi^2}} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2 \times 0.01 r)^2}}$$

$$\frac{r^2}{\sqrt{1-2r^2+r^4+0.0004r^2}} < \frac{225\times 2.5\times 10^{-3}}{\left(\frac{9}{100\pi^2}\right)}$$

$$\frac{r^4}{r^4+1.9996r^2+1} < 6.25^2 r^4$$

$$r^4 < \frac{625}{64} r^4\left(r^4-1.9996r^2+1\right)$$

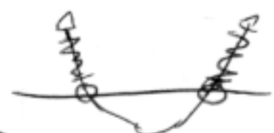
let $r^2 = s$: $s^2 < \frac{625}{64}r^4\left(s^2-1.9996s+1\right)$

$$\left(\frac{625}{64}r^4-1\right)s^2 - 1.9996s + A > 0$$

'A' $\propto$

$$s = \frac{1.9996A \pm \sqrt{1.9996A^2 - 4'A'A}}{2'A'}$$

'A'



let $r^2=s$: $s^2 < \underset{6.25^2}{\frac{25}{4}} r^4\left(s^2-1.9996s+1\right)$

$$s^2 < 6.25^2 r^4 s^2 - 1.9996\times 6.25^2 r^4 + 6.25^2 r^4$$

$0 < (6.25^2 r^4 - 1)s^2 -$ " "

let coefficients be 'A', 'B', 'C'

$$s = \frac{-B - \sqrt{B^2-4AC}}{2A}$$

$=$

Extra Workspace – Question Number 2

~~Since system is under forced response~~
~~with $F_0 = 900 \sin(100\pi t)$ N~~

~~Assume $\omega = \omega_d$~~

~~i.e. $r = \frac{\omega_d}{\omega_n}$, $\frac{\omega_d}{\omega_n} = \sqrt{1-\zeta^2}$~~

~~$\therefore r = \sqrt{1-\zeta^2}$~~

~~$$\frac{MX}{m} = \frac{1-\zeta^2}{\quad}$$~~

$$\frac{r^4}{r^4 - 1.9996r^2 + 1} < (6.25r^2)^2$$

$$\left((6.25r^2)^2 - 1\right)r^4 - (6.25r^2)^2 1.9996r^2 + (6.25r^2)^2 > 0$$