

MMAN2300 Lab 2 Assignment: Coriolis Effect

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Lab Time and Date: Wednesday, 24/07/2019, 14:00

[Total marks: 20]

1. Record the experimental data in the following table. (1 mark)

Deflection of Water Jet, Δ (cm)	Experimental Angular Speed, ω_{exp} (rpm)		
	Trial 1	Trial 2	Trial 3
0	0	0	0
1	8.8	9.5	8.8
2	19.1	18.7	19.0
3	29.2	26.7	27.7
4	36.3	35.5	35.6
5	43.6	43.4	42.2
6	47	49.6	49.5
7	51.6	52.2	52.8

2. Derive equations that relate the deflection of the water jet (Δ) to the angular speed of the rotating arm (ω). Note that relevant diagrams should be included. [HINT: the equations should consist of the terms Δ , t and ω .] (3 marks)

W = Water Particle

N = Nozzle of Pump

R = Radius of Rotating Arm (m)

t = Time for Water Particle to reach Wall (s)

θ = Angle of Rotated Wall (rad)

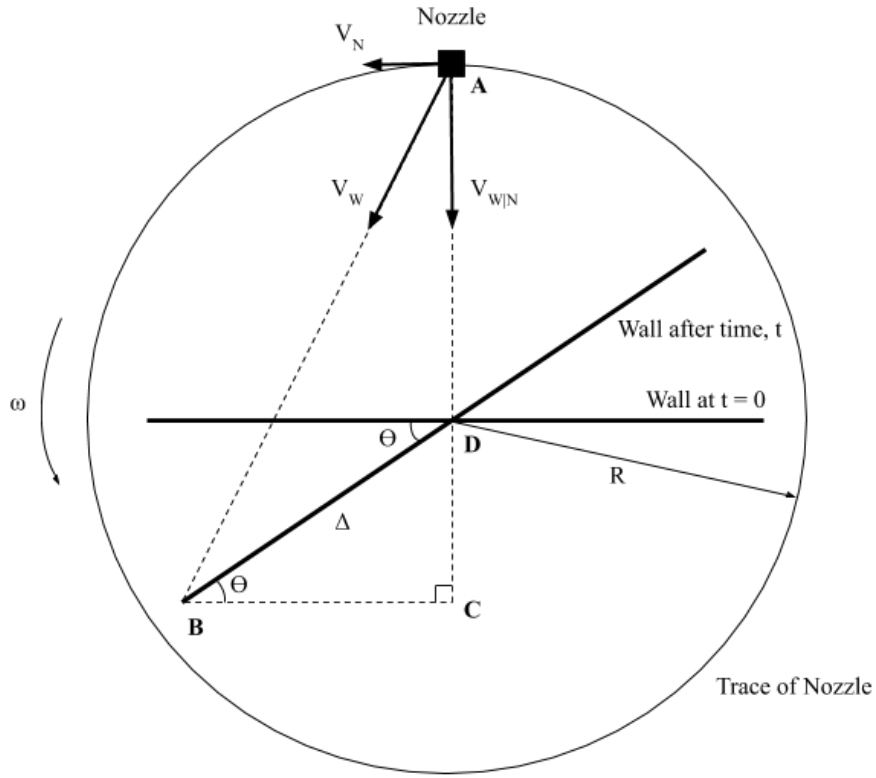


Figure 1 – Top View Diagram of Nozzle Apparatus with Relevant Features

Figure 1 was graphed using Google Drawings and will be used to derive three equations of interest consisting of Δ , t and ω .

$$\overline{AD} = R \quad (1), \quad \overline{DC} = \Delta \sin \theta \quad (2), \quad \overline{AC} = \overline{AD} + \overline{DC} \quad (3)$$

(1), (2) \rightarrow (3):

$$\therefore \overline{AC} = R + \Delta \sin \theta \quad (4)$$

Considering $v_{W|N}$:

$$v_{W|N} = \frac{\overline{AC}}{t} \quad (5)$$

(4) \rightarrow (5):

$$v_{W|N} = \frac{R + \Delta \sin \theta}{t}$$

$$\therefore t = \frac{R + \Delta \sin \theta}{v_{W|N}} \quad (6)$$

$$\overline{BC} = \Delta \cos \theta \quad (7)$$

Considering the tangential velocity, v_N , which has two different expressions:

$$v_N = \omega R = \frac{\overline{BC}}{t} \quad (8)$$

(7) \rightarrow (8):

$$\omega R = \frac{\Delta \cos \theta}{t} \quad (9)$$

Considering ω :

$$\omega = \frac{\theta}{t} \quad (10)$$

(10) \rightarrow (9):

$$\frac{\theta}{t} R = \frac{\Delta \cos \theta}{t}$$

$$\therefore R = \frac{\Delta \cos \theta}{\theta} \quad (11)$$

Therefore, the three equations of interest that consist of the terms Δ , t and ω are (6), (10) and (11).

3. For a flow rate of 0.35 L/min as mentioned in the Lab Handout, calculate the velocity of the water jet relative to the rotating arm. The distance between the nozzle tip of the pump and the centre of rotation is 134 mm and the internal diameter of the nozzle is 2 mm. (1 mark)

$$\text{Water Flow: } Q = 0.35 \times \frac{10^{-3}}{60} \text{ m}^3/\text{s}$$

$$\text{Area of Nozzle: } A = \pi \times (10^{-3})^2 \text{ m}^2/\text{s}$$

$$\vec{v}_{W|N} = \frac{Q}{A}$$

$$= \frac{0.35 \times 10^{-3}}{\pi \times (10^{-3})^2}$$

$$\therefore \vec{v}_{W|N} = 1.857 \text{ ms}^{-1} \downarrow$$

See Appendix C for MATLAB calculations.

4. Calculate the theoretical values of t and ω for the given Δ values below using the equations derived in Q2 and fill in the following table. (3 marks)

Deflection of Water Jet, Δ (cm)	Time for Jet to reach Wall, t (s)	Theoretical Angular Speed, ω_{thr} (rad/s)
0	0.07217	0.000
1	0.07257	1.026
2	0.07375	2.002
3	0.07567	2.888
4	0.07825	3.659
5	0.08141	4.305
6	0.08506	4.826
7	0.08912	5.235

5. Plot the theoretical and experimental angular speeds (ω_{thr} and ω_{exp} in rad/s) against deflection (Δ in cm) in one graph. (2 marks)

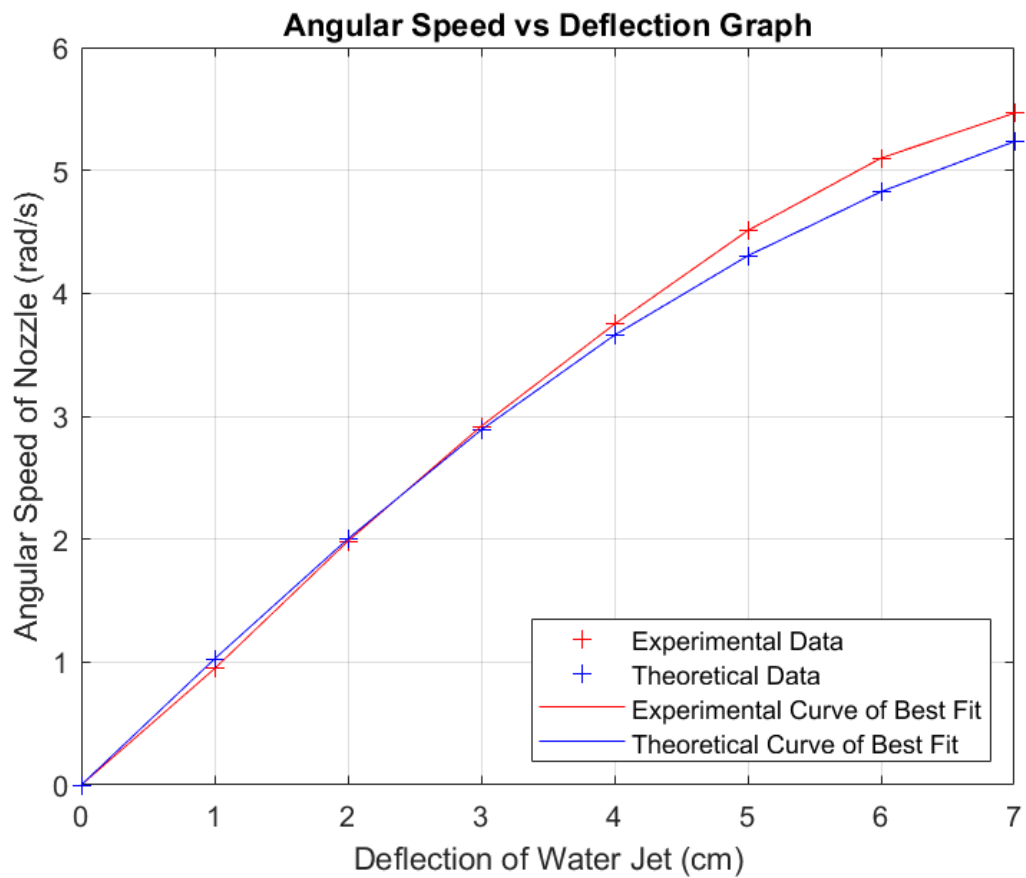


Figure 2 – Graph of Theoretical and Experimental Angular Speed vs Deflection of Water Jet

Figure 2 was graphed using MATLAB (see Appendix E). Due to the difficulty of obtaining a non-parameterised relationship between the angular speed and deflection of the water jet, the curve of best fit was approximated by Lagrange polynomial interpolation.

6. Calculate the percentage error between the theoretical and experimental angular speeds. (2 marks)

Deflection of Water Jet, Δ (cm)	Theoretical Angular Speed, ω_{thr} (rad/s)	Experimental Angular Speed, ω_{exp} (rad/s)	Percentage Error (%)
0	0.000	0.000	-
1	1.026	0.9460	7.8
2	2.002	1.983	0.95
3	2.888	2.918	1.0
4	3.659	3.749	2.4
5	4.305	4.510	4.8
6	4.826	5.100	5.7
7	5.235	5.466	4.4

7. Write a discussion here. (6 marks)

The following aspects should be considered:

- Comment on the effect of angular speed of the rotating arm and its direction on the deflection of the water jet.
- Compare the theoretical and experimental values and discuss the results.
- Identify the sources of errors.

Your answer should not exceed 1.5 page.

The Coriolis effect describes the apparent curvature in the path of a moving object under rotational reference frames. This is due to a fictitious force called the Coriolis force. This experiment investigates the Coriolis effect by examining the relationship between the angular speed of the rotating arm to the deflection in the path of the water jet. The deflection will have the same direction as the rotating arm's angular velocity when observed from the rotating reference frame of the arm. From an inertial reference frame outside the system, the water jet has no deflection.

The deflection of the water jet increased as the angular speed increased as confirmed by theoretical and experimental results from question 6 and *figure 2*. The theoretical and experimental angular speeds are mostly in agreement due to a consistent percentage error of less than 8%, averaging 3.9%. The experimental and theoretical curves of best fit are qualitatively similar in trend. Therefore, the experimental angular speed can be considered accurate to the theoretical values. Due to performing three trials of the experiment, the experimental angular speed is also reliable.

Discrepancies in the experimental angular speed may be attributed to an:

- uncalibrated sensor;

- uncalibrated pump;
- refractive index of the wall;
- parallax error;
- physical marker for the water flow of 0.35 L/min;
- size of water impact;
- water breakup;
- and splash;

The size of water impact will allow the misjudgement of the actual position of the water jet striking the wall and is affected by the water breakup and internal area of the nozzle. The splash may create the illusion of a larger size of water impact. The refractive index of the clear plastic wall and parallax error will also contribute to the misjudgement of the water impact's position. The use of a physical marker to monitor a constant water flow of 0.35 L/min does not account for variations of water flow during arm rotation.

8. Draw a conclusion by summarising the key findings of this experiment and recommendations to improve the experiment. (2 marks)

This experiment visualises the Coriolis effect and investigates the relationship between the angular speed of a rotating arm and deflection of the water jet where an increase in the angular speed increased the deflection. The experimental angular speed was accurate and reliable with an average of 3.9% error. However improvements can be made to the experiment from the use of a wall with a lower refractive index, repetition of experiment to improve reliability, decreasing R to place the nozzle closer to the wall to decrease water breakup, decreasing the internal diameter of the nozzle to decrease the size of water impact, a camera fixed to the rotating arm to monitor the location of the water impact, and the use of a quantitative water flow meter than a physical marker to increase accuracy.

Appendix

Appendix A – coriolisEffect.m

```
% coriolisEffect.m
% LAB 2: CORIOLIS EFFECT
%
% CALLS:
% Q1.m
% Q3.m
% Q4.m
% Q5.m
% Q6.m

format compact

% N = Nozzle
% W = Water Particle
fprintf("\n")

fprintf("Q1\n\n")
[delta, w_exp] = Q1;

fprintf("Q3\n\n")
[v_WrelN] = Q3;

fprintf("Q4\n\n")
[theta, t, w_thr] = Q4(delta, v_WrelN);

fprintf("Q5\n\n")
Q5(delta, w_exp, w_thr);

fprintf("Q6\n\n")
[percentage_error] = Q6(w_exp, w_thr);

% Relevant Answers to 4 Significant Figures
delta = vpa(delta, 4), fprintf("\n")
w_exp = vpa(w_exp, 4), fprintf("\n")
v_WrelN = vpa(v_WrelN, 4), fprintf("\n")
theta = vpa(theta, 4), fprintf("\n")
t = vpa(t, 4), fprintf("\n")
w_thr = vpa(w_thr, 4), fprintf("\n")
percentage_error = vpa(percentage_error, 2), fprintf("\n")
```


Appendix B – Q1.m

```
function [delta, w_exp] = Q1

% Experimental Results for Angular Speed
delta = [0:7]; % cm
w_exp1 = [0 8.8 19.1 29.2 36.3 43.6 47.0 51.6]; % rpm
w_exp2 = [0 9.5 18.7 26.7 35.5 43.4 49.6 52.2]; % rpm
w_exp3 = [0 8.8 19.0 27.7 35.6 42.2 49.5 52.8]; % rpm

% Get Mean of Experimental Angular Speeds
mean_w_exp = mean([w_exp1; w_exp2; w_exp3]);

% Convert RPM to rad/s
w_exp = mean_w_exp*2*pi/60;
```

Appendix C – Q3.m

```
function [v_WrelN] = Q3

% Water Flow
Q = 0.35*10^(-3)/60; % m^3/s

% Area of Nozzle
A = pi*(10^-3)^2; % m^2

% Velocity of Water Jet Relative to Nozzle
v_WrelN = Q/A;
```

Appendix D – Q4.m

```
function [theta, t, w_thr] = Q4(delta, v_WrelN)

% Unknowns to Solve
theta = sym('theta', [1 8]);
t = sym('t', [1 8]);
w = sym('w', [1 8]);

% Radius of Disc
R = 134*10^-3; % m

% Standardise Units
delta = delta*10^-2; % m

% Equation 1
eqn1 = delta.*cos(theta) == R.*theta;

% Solving for theta
theta = struct2array(vpasolve(eqn1, theta));

% Equation 2
eqn2 = R + delta.*sin(theta) == v_WrelN.*t;

% Solving for t
t = struct2array(vpasolve(eqn2, t));

% Equation 3
eqn3 = w == theta./t;

% Solving for w
w_thr = double(struct2array(vpasolve(eqn3, w)));
```

Appendix E – Q5.m

```
function Q5(delta, w_exp, w_thr)

figure(1)

% Plotting Experimental Values
plot(delta, w_exp, '+r');

% Plotting Theoretical Values
hold on
plot(delta, w_thr, '+b');
hold off

% By Lagrange Polynomial Interpolation to approximate w vs delta
L = zeros(length(delta), length(delta));
for m = 1:length(delta)
    P = 1;
    for n = 1:length(delta)
        if m ~= n
            P = conv(P, poly(delta(n)))/(delta(m) - delta(n));
        end
    end
    L(m,:) = P;
end

% Experimental Curve of Best Fit
hold on
y1 = DocPolynom(w_exp*L);
plot(delta, y1(delta), 'r');
hold off

% Theoretical Curve of Best Fit
hold on
y2 = DocPolynom(w_thr*L);
plot(delta, y2(delta), 'b');
hold off

% Decorating Figure
grid on;
xlabel("Deflection of Water Jet (cm)");
ylabel("Angular Speed of Nozzle (rad/s)");
title("Angular Speed vs Deflection Graph");
legend('Experimental Data', 'Theoretical Data', ...
    'Experimental Curve of Best Fit', 'Theoretical Curve of Best Fit');
legend('Location', 'southeast');
```

Appendix F – Q6.m

```
function [percentage_error] = Q6(w_exp, w_thr)

abs_error = abs(w_exp - w_thr);
percentage_error = 100.*abs_error./w_thr;
```