MMAN2300

Engineering Mechanics 2

Part A: Week 8

Plane kinetics of rigid bodies: Work-energy method

(Chapter 6 Meriam & Kraige)

Work-energy method

Work done by a force on a rigid body is: $\boldsymbol{W}_{1-2} = \boldsymbol{F}(\boldsymbol{S}_2 - \boldsymbol{S}_1) = \boldsymbol{F} \cdot \boldsymbol{S}$

- $\boldsymbol{F}$ is an applied external force (N)
- $\boldsymbol{S}$ is the displacement of the body (m)

Work done by a moment on a rigid body is: $\boldsymbol{W}_{1-2} = \boldsymbol{M}(\boldsymbol{\theta}_2 - \boldsymbol{\theta}_1) = \boldsymbol{M} \cdot \boldsymbol{\theta}$

- $\boldsymbol{M}$ is an applied external moment (Nm)
- $\boldsymbol{\theta}$ is the angular displacement of the body (rad)

The rotary motion of a rigid body is *independent* of the translation of the rigid body. Hence, the translation and rotation can be considered separately. The total work done on a rigid body is the sum of the work done by an external force and the work done by an external moment.

$$\boldsymbol{W}_{1-2} = \boldsymbol{F}(\boldsymbol{S}_2 - \boldsymbol{S}_1) + \boldsymbol{M}(\boldsymbol{\theta}_2 - \boldsymbol{\theta}_1) = \boldsymbol{F} \cdot \boldsymbol{S} + \boldsymbol{M} \cdot \boldsymbol{\theta}$$

$$\boldsymbol{W}_{1-2} = \boldsymbol{F} \cdot \boldsymbol{S} + \boldsymbol{M} \cdot \boldsymbol{\theta} = \Delta \boldsymbol{T} + \Delta \boldsymbol{V}_{\boldsymbol{g}} + \Delta \boldsymbol{V}_{\boldsymbol{e}}$$

- $\Delta \boldsymbol{T}$ is the change in kinetic energy
- $\Delta \boldsymbol{V}_{\boldsymbol{g}}$ is the change in potential energy
- $\Delta \boldsymbol{V}_{\boldsymbol{e}}$ is the change in elastic energy

For translational motion: $\Delta T = \frac{1}{2} m (v_2^2 - v_1^2)$

For rotational motion: $\Delta T = \frac{1}{2} I_G (\omega_2^2 - \omega_1^2)$

Hence for a rigid body, the total change in kinetic energy is

$$\Delta T = \underbrace{\frac{1}{2} m (v_2^2 - v_1^2)}_{\text{translation}} + \underbrace{\frac{1}{2} I_G (\omega_2^2 - \omega_1^2)}_{\text{rotation}}$$

- v is the velocity of the centre of mass (v_G)
- I_G is the moment of inertia of the centre of mass

The change in kinetic energy and potential energy are applied to the *centre of mass*.
The change in potential energy is:

$$\Delta V_g = mg(h_2 - h_1)$$

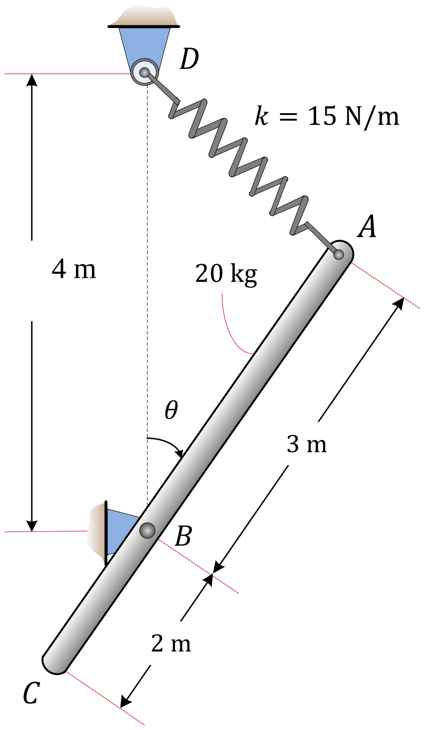
The change in elastic energy is:

$$\Delta V_e = \frac{1}{2} k (x_2^2 - x_1^2)$$

where k is the spring stiffness; x_1 and x_2 are the deformations of the spring at positions 1 and 2, respectively.

Example 1

The uniform 20 kg slender bar *AC* rotates in a vertical plane about the pin at *B*. The ideal spring *AD* has a spring constant k = 15 N/m and an undeformed length L_0 = 2 m. When the bar is at rest in the position θ = 0°, it is given a small angular displacement and released. Find the angular velocity of the bar when it reaches the horizontal position.

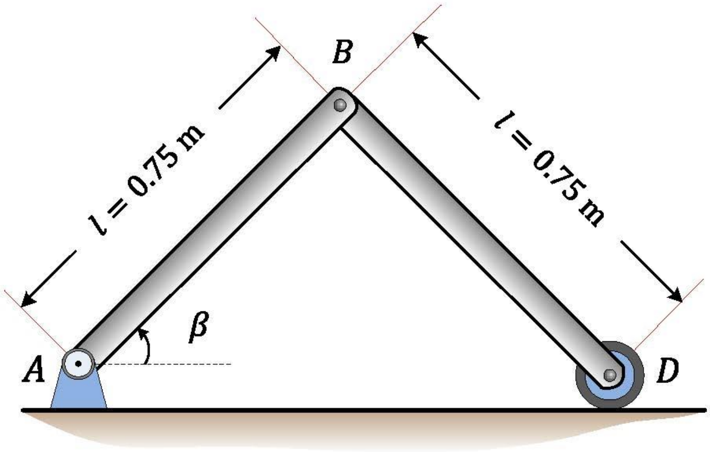


Example 2

Each of the two slender rods shown is 0.75 m long and has a mass of 6 kg. If the system is released from rest with $\beta = 60°$, determine:

(a) the angular velocity of rod *AB* when $\beta = 20°$,

(b) the velocity of the point *D* at the same instant.



Example 3

A slider-crank mechanism is driven by a constant clockwise couple M = 0.5 Nm. All the components are homogeneous, with the mass and dimensions as indicated. When the mechanism is in the position shown, the angular velocity of the crank is ω_1 = 12 rad/s CW. Determine the angular velocity of the crank after it has rotated 90° from the position shown. Neglect friction and assume that motion is in the vertical plane.