SOLUTIONS I

SET I

(1) (a) $8x-3$

(b) $3x+4y-4$

(c) $11x+22y-22$

(2) $V = 400\pi r - \dfrac{8000\pi}{3}$

(a) $\%_{err} = 4\%$ (b) $\%_{err} = 23\%$

(3) (a) $q = \frac{1}{2} K p_0^{-1/2} \cdot p + \frac{1}{2} K p_0^{1/2}$

(b) as $p_0 \to \infty$, slope $\to \infty$

(4) $R = -\dfrac{2\dot{R}_0 \dot{\theta}_0}{\ddot{\theta}_0}\left(1 + \dfrac{\dot{R}-\dot{R}_0}{\dot{R}_0} + \dfrac{\dot{\theta}-\dot{\theta}_0}{\dot{\theta}_0} - \dfrac{\ddot{\theta}-\ddot{\theta}_0}{\ddot{\theta}_0}\right)$

(5) $N = 2600 - (T-1)\cdot 1430 + (E-16)\cdot 120$

(approximate values)

SET II

① (a) $\frac{2}{s+3}$; (b) $\frac{32}{(s+2)^2+64}$; (c) $\frac{12}{s^2+16}$;

(d) $\frac{2}{(s+3)^2}$

② (a) $(17e^{-2t}-9e^{-2t})u(t)$ (b) $[e^{-t}\sin(4t)]u(t)$

(c) $[e^{-t}\cos 2t]u(t)$

③ (a) $\frac{1}{s}(1-e^{-6s})$; (b) $\frac{3}{2s^2}-\frac{3e^{-2s}}{2s^2}-\frac{3e^{-6s}}{s}$

(c) $\frac{5}{s}(e^{-2s}-e^{-3s})$; (d) $\frac{10}{3s^2}-\frac{10e^{-1.5s}}{3s^2}-\frac{10e^{-3s}}{s}+\frac{5e^{-3s}}{4.7s^2}-\frac{5e^{-7.7s}}{4.7s^2}$

④ $f(0)=0$; $f(\infty)=\frac{2}{15}$

⑤ $f(\infty)=3$

⑥ (a) $\frac{s^2+10s+5}{s(s^2+6s+13)}$; (b) $\frac{6\omega+4s(s^2+\omega^2)+17(s^2+\omega^2)}{(s^2+\omega^2)(s^2+3s+4)}$

(c) $\frac{s^2+2s+1}{s(s^2+2s+4)}$; (d) $\frac{5s}{s^3+1}$

(e) $\frac{s^2+s+1}{s(s^2+1)(s^4+s^3+6s^2+12s+1)}$

7. (a) (i) $\dfrac{1}{s^2+7s+10}$; (ii) $\dfrac{1}{s+12}$; (iii) $\dfrac{1}{s^2+2s+6}$;

(iv) $\dfrac{1}{s^2+6s+25}$; (v) $\dfrac{1}{s^2+7s+12}$

(b) (i) $\left(\dfrac{4}{3}e^{-5t}-\dfrac{1}{3}e^{-2t}\right)u(t)$

(ii) $\left[\dfrac{1}{51}e^{-12t}-\dfrac{1}{51}\cos(3t)-\dfrac{4}{51}\sin(3t)\right]u(t)$

(iii) $\left[\dfrac{4}{\sqrt{5}}e^{-t}\sin(\sqrt{5}\,t)\right]u(t)$

(iv) $\left[\dfrac{1}{20}e^{-t}-\dfrac{\sqrt{5}}{40}e^{-3t}\sin(4t+\varphi)\right]u(t)$, $\varphi=\tan^{-1}\dfrac{4}{3}$

(v) $\left(\dfrac{1}{12}-\dfrac{43}{4}e^{-4t}+\dfrac{38}{3}e^{-3t}\right)u(t)$

8. a) $y(t)=\dfrac{1}{3}\left[1-\dfrac{1}{2}\left(3e^{-t}-e^{-3t}\right)\right]u(t)$

b) $x(t)=-4\,\delta(t)$

$\boxed{\text{SET III}}$

1) System I: a) $\dfrac{k_1+k_2}{k_1 k_2}$; b) $\dfrac{1}{k_2}$; c) $\dfrac{k_1}{k_1+k_2}$

System II: a) $\dfrac{s+\frac{k}{c}}{ks}$; b) $\dfrac{1}{k}$; c) $\dfrac{cs}{k+cs}$

System III a) $\dfrac{M_2 s^2+k_1+k_2}{M_1 M_2 s^4+s^2\left[k_1 M_2+(k_1+k_2)M_1\right]+k_1 k_2}$

b) $\dfrac{k_1}{M_1 M_2 s^4+s^2\left[k_1 M_2+(k_1+k_2)M_2\right]+k_1 k_2}$

c) $\dfrac{k_1}{M_2 s^2+k_1+k_2}$

2) $\dfrac{cs+k}{Ms^2+cs+k}$

3) (a) $\dfrac{RCs}{LCs^2+RCs+1}$; (b) $\dfrac{1}{CLs^2+\frac{L}{R}s+1}$

(c) $\dfrac{1}{C_1 C_2 R_1 R_2 s^2+(C_1 R_1+C_2 R_1+C_2 R_2)s+1}$

(d) $\dfrac{(1+C_1 L s^2)R}{(1+C_1 L s^2)R+(1+C_2 R s)Ls}$

(4) $$\frac{R(1+ARs)}{(ARs+3)(ARs+1)-2}$$

(5) $$\frac{Ah}{\rho c V s+(Ah+c\dot{m})}$$

(6) $$\frac{\mu VH}{cMs+Ah}$$

(7) $$\frac{b}{(a+b)(M\alpha s^2+C\alpha s+1)}, \quad \alpha=\frac{1}{K_1}+\frac{a^2}{K_2(a+b)^2}$$

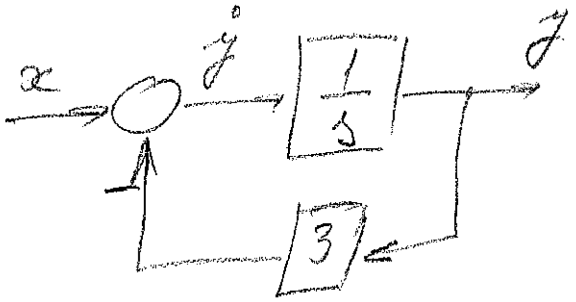
(8) $$\ddot{x}_3=\frac{cd\Gamma}{r_1(m_2c^2+m_3d^2)}$$

(9) $$\frac{1}{s^2\left(\frac{I_1}{a}+I_2\cdot\frac{a}{b^2}\right)+s\left(\frac{c_1}{a}+c_2\cdot\frac{a}{b^2}\right)+2ak}$$

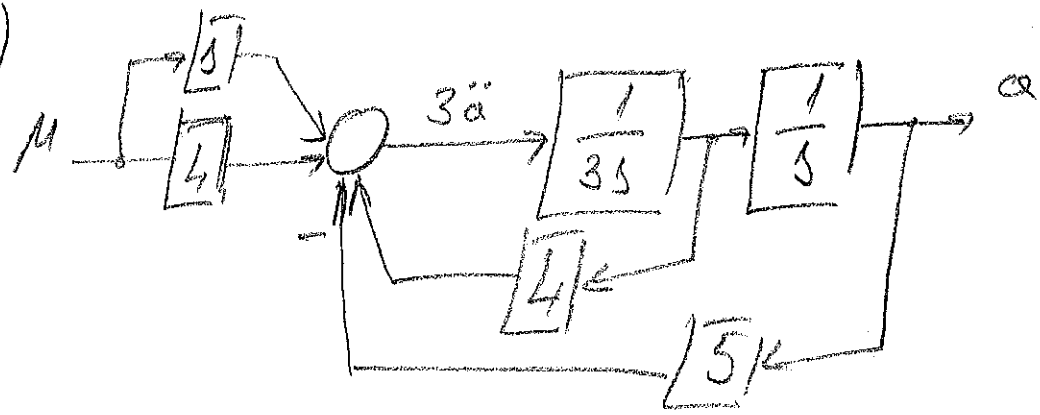
(10) $$\frac{\frac{c}{c+d}}{\frac{e}{e+f}-\frac{a}{b}\cdot\frac{d}{c+d}}$$

SET IV

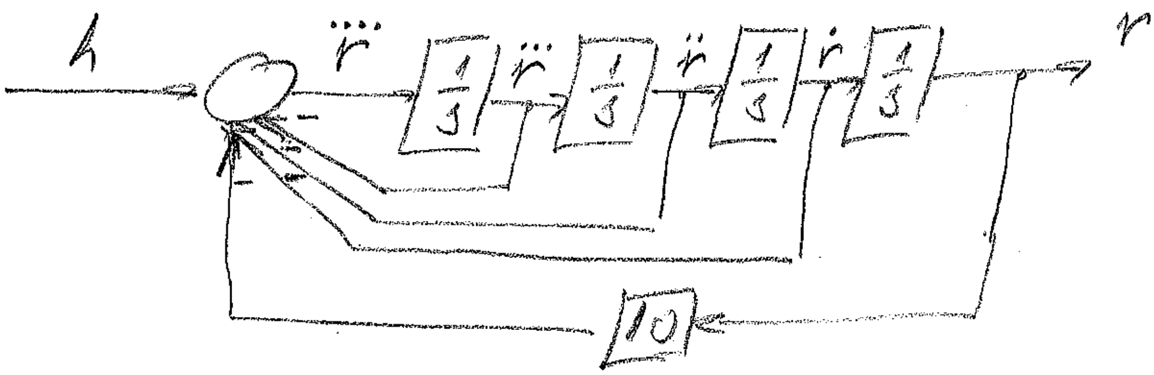
1 (a) (i)



(ii)



(iii)



(b) (i) $\frac{1}{s+3}$; (ii) $\frac{s+4}{3s^2-4s+5}$;

(iii) $\frac{1}{s^4+s^3+s^2+s+10}$

(c) (i) $M_2\ddot{y}+C_3\dot{y}+K_4y=F_i$

(ii) $h=\ddddot{q}+C_4\dddot{q}+R_0\ddot{q}+C_4R_0\dot{q}$

(iii) $L_1L_3\ddot{h}_0+(L_3R_1+L_1R_2+L_3R_2)\dot{h}_0+R_1R_2h_0=$

$=L_1L_3R_2\dot{q}_0+L_3R_1R_2q_0$

② a) (i) $\frac{4}{s+6}$; (ii) $\frac{4s+5}{s^2+6s+5}$

b) (i) $\frac{2}{3}(1-e^{-6t})u(t)$; (ii) $(1-\frac{1}{4}e^{-t}-\frac{3}{4}e^{-5t})u(t)$

③ a) $\frac{C}{R}=\frac{K_1K_2}{1+K_1K_2}$, $\frac{C}{D}=\frac{K_2}{1+K_1K_2}$

b) (i) $K_1=9$, $K_2=1$; (ii) $K_1=99$, $K_2=1$

④ $$\frac{C}{R}=\frac{G_1G_2G_3}{1+G_2H_2+G_2G_3H_3+(G_3+H_1)G_1G_2}$$

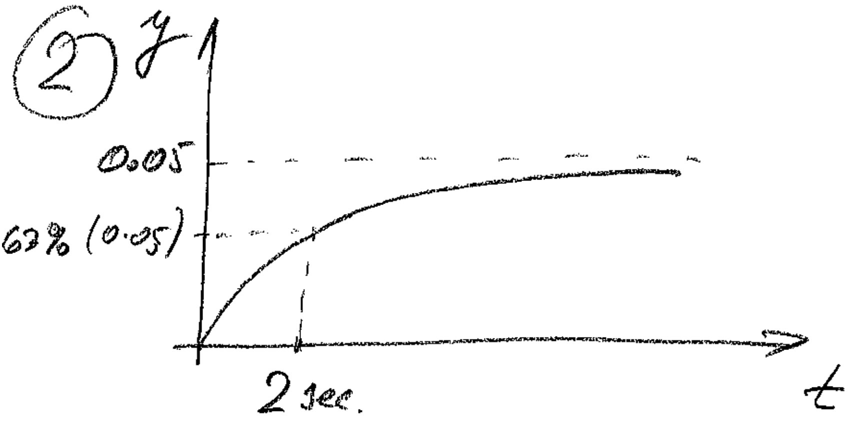
$$\frac{C}{D}=\frac{G_2G_3}{1+G_2H_2+G_2G_3H_3+(G_3+H_1)G_1G_2}$$

⑤ $$\frac{G_1(G_2G_3+G_4)}{1+H_2(G_2G_3+G_4)+G_1G_2H_2+G_1(G_2G_3+G_4)}$$

SET V

① $\dfrac{5}{s+0.13}$ (approximate values)

②



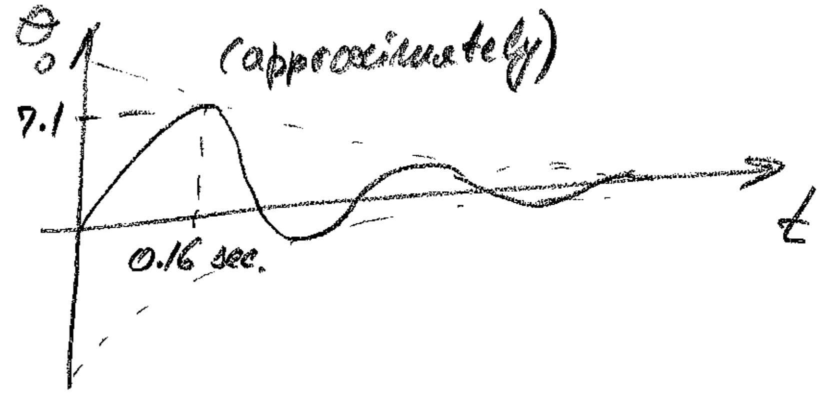
③ $G = 60$, $T = 0.0067$ sec. (approximate values)

④ $G = 0.15$, $T = 0.66$ sec (" ")

⑤ $$t_p = \frac{T_1 T_2}{T_2 - T_1} \cdot \ln\left(\frac{T_2}{T_1}\right)$$

$$\theta_{max} = \frac{1}{T_1 - T_2}\left[\left(\frac{T_2}{T_1}\right)^{\frac{T_2}{T_1 - T_2}} - \left(\frac{T_2}{T_1}\right)^{\frac{T_1}{T_1 - T_2}}\right]$$

⑥ (approximately)



$\boxed{t_p = 0.14 \text{ sec.} \quad \theta_{o_{max}} = 7.51 \quad \text{accurately}}$

⑦ $t_p = 7.86$ sec, $p_{max} = -38.9$ kPa

8 $$\frac{0.01}{(s+0.1)^2}$$

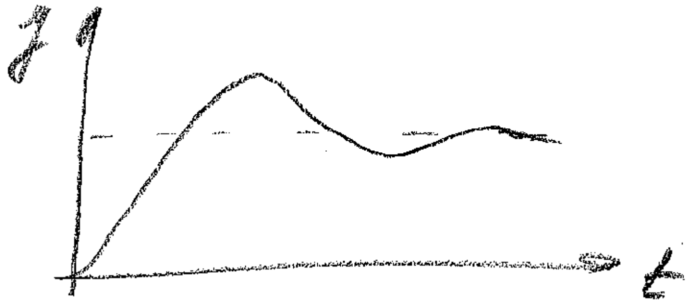
9 $$\frac{0.098}{s^2+0.21s+0.11}$$

SET VI

1) a), b), c), d), h), i) stable

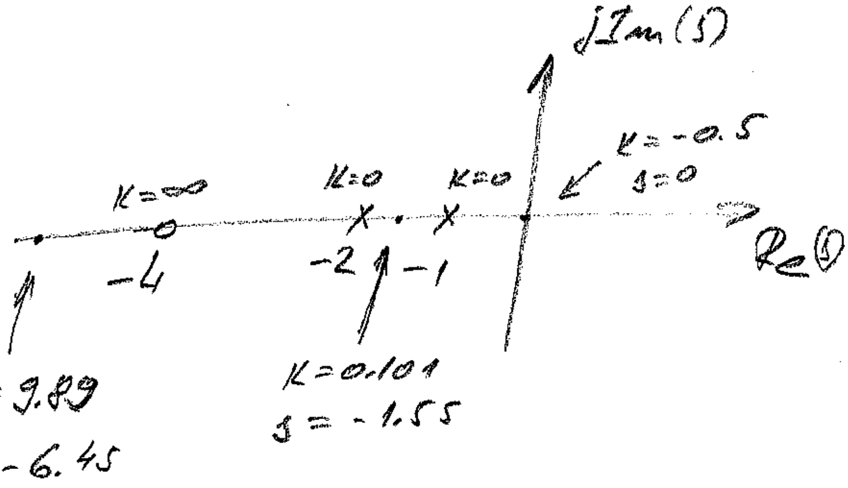
e), f), g), j) unstable

2) yes;



3) $-0.5 < K < 0.101$

and $9.898 < K < +\infty$



4) $\frac{2s+1}{s+3}$

5) b) $\frac{K(3s+1)}{60s^2+23s+12K+1}$; c) $K \geq 1.58$

d) $59.95 \frac{km}{h}$

6) for $0 < K < 2$ unstable

for $K = 2$ neutrally stable

for $2 < K \leq 2.25$ exponentially stable

for $2.25 < K < +\infty$ oscillatory stable

7 a) $\frac{K(s+3)}{(s+2)(s+1)}$ stable

b) $\frac{K(s+2)}{(s-1)(s+4)}$ unstable

c) $\frac{K(s+6)}{s(s^2+6s+13)}$ neutrally stable

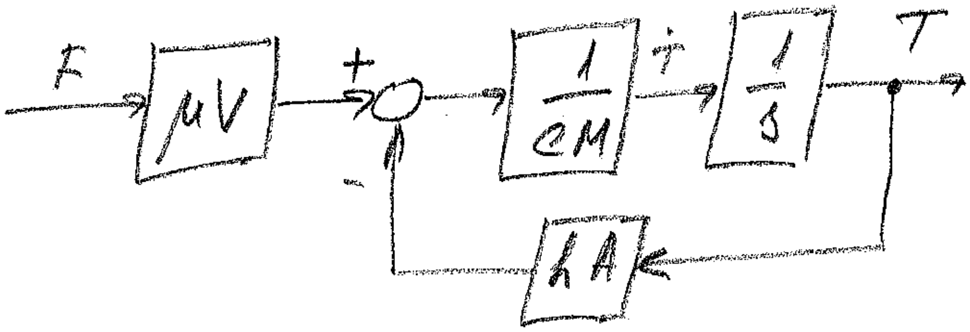
PAST EXAMS

(1) a) $LM\dddot{x} + (RM+LC)\ddot{x} + (RC+2LK)\dot{x} + 2RKx =$
$= \lambda e$

b) $c^2 = 8MK$

d) 23.3V

(2) 1)



3) 51°C

4) 81.6°C

(3) 1) $\dfrac{R_2}{AR_2 s + 1}$; 2) $\dfrac{\alpha p}{R_1}\left(1 - e^{-\frac{t}{AR_2}}\right)$

3) 14.4 lit. 4) 14.4 sec.

(4) a) $\dfrac{V\pi\left(1 + e^{-sT}\right)}{T\left[s^2 + \left(\frac{\pi}{T}\right)^2\right]}$

$\varphi = \tan^{-1} \dfrac{RC\pi}{T}$

b) $\dfrac{V\pi}{T}\left\{ \dfrac{RC}{1+\left(\frac{RC\pi}{T}\right)^2} \cdot e^{-\frac{t}{RC}} + \dfrac{1}{\sqrt{1+\left(\frac{RC\pi}{T}\right)^2}} \cdot \sin\left(\frac{\pi}{T}t - \varphi\right) + \right.$

$\left. + \dfrac{RC}{1+\left(\frac{RC\pi}{T}\right)^2} \cdot e^{-\frac{(t-T)}{RC}} + \dfrac{1}{\sqrt{1+\left(\frac{RC\pi}{T}\right)^2}} \cdot \sin\left[\frac{\pi}{T}(t-T) - \varphi\right] \right\}$

(5) 1) $\frac{K_1 v}{R} = M\ddot{x} + C\dot{x} + Kx$ 2) $\frac{V_1 K_1}{RMs\left(s^2 + \frac{C}{M}s + \frac{K}{M}\right)}$

3) $\omega = \sqrt{\frac{K}{M}}$, $\xi = \frac{C}{2\sqrt{MK}}$

4) (a) 16.3 % (b) $\omega = 72.5 \frac{rad}{sec}$, $K = 52.64$ N/m, $C = 0.92$ $\frac{Ns}{m}$

5) 15.8 N/A

(6) 1) $\frac{K}{Is^2 + Cs + K}$; 2) $\frac{\theta_0 \omega}{\sqrt{1-\xi^2}} e^{-\xi\omega t} \cdot \sin\left(\omega\sqrt{1-\xi^2}\, t\right)$

3) $\frac{\theta_0 \omega}{\sqrt{1-\xi^2}} \cdot e^{-\frac{\xi\pi}{2\sqrt{1-\xi^2}}}$

(7) a) $F = A\rho g h + \mu \tan^2\theta \, \dot{h} + \frac{M}{\cos^2\theta} \ddot{h}$

b) $\frac{\cos^2\theta}{M} \cdot \frac{1}{s^2 + \frac{\mu\tan^2\theta}{M}s + \frac{A\rho g\cos^2\theta}{M}}$

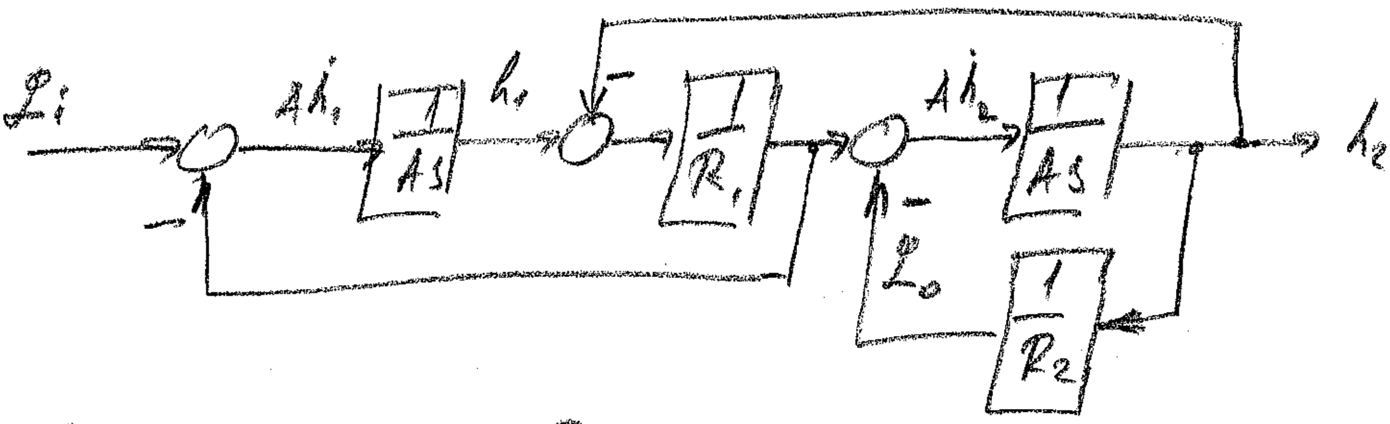
d) 0.0143 kg m^2

e) $\left.\begin{array}{l}\theta = 0 \Rightarrow \text{no viscous drag} \\ \theta = \frac{\pi}{2} \Rightarrow \text{no buoyancy effect}\end{array}\right\} \Rightarrow$ critical damping impossible

(8) $\frac{Z(s)}{\Theta(s)} = \frac{K_2}{Ms^2}$; $\frac{\Theta(s)}{\alpha(s)} = \frac{K_1}{\frac{Is^2}{x} + K_1}$

$$\frac{Z(s)}{\alpha(s)} = \frac{K_1 K_2 x}{Ms^2(Is^2 + K_1 x)}$$

(9)



$$\frac{H_2(s)}{Q_i(s)} = \frac{R_2}{A^2 R_1 R_2 s^2 + (AR_1 + 2AR_2)s + 1}$$

$$s_{1,2} = \frac{-(2R_2 + R_1) \pm \sqrt{4R_2^{\,2} + R_1^{\,2}}}{2AR_1R_2}$$

$$\tau_1 = \frac{1}{s_1} \;, \qquad \tau_2 = \frac{1}{s_2}$$

(10) 1) (a) $M\ddot{x} + C\dot{x} + K_1 x = Wy$

(b) $v_i = A\mu x$; (c) $v_i = iR + L\frac{di}{dt}$

(d) $\alpha i = K_2 y$

2) $\frac{g}{Ms^2 + Cs + K_1}$; $A\mu$; $\frac{1}{R + Ls}$, $\frac{\alpha}{K_2}$

3) $\frac{\alpha A\mu g}{K_2(R+Ls)(Ms^2+Cs+K_1)}$

5) 0.245 m