TUTORIAL

SET I – LINEARISATION

1. Linearise the following equations for small variations about the operating points indicated.

   (a) $y = 3x^2 + 2x$; $x_0 = 1$

   (b) $z = x^3 + 2y^2$; $(x_0, y_0) = (1,1)$

   (c) $z = x^3 + 2x^2y + 4xy^2 + 4y^3$; $(x_0, y_0) = (1,1)$

2. The volume of a sphere is:

   $v = \frac{4}{3}\pi r^3$

   Determine the linear approximation of v when $r_0 = 10$.

   What percentage of errors occur if the above approximation is used to find v when:

   (a) $r = 9$ (b) $r = 11$

3. The flow rate of a fluid through an orifice is given by the nonlinear equation:

   $q = Kp^{1/2}$

   where $p = (p_1 - p_2)$ i.e. the difference in pressure across the orifice. K is a constant.

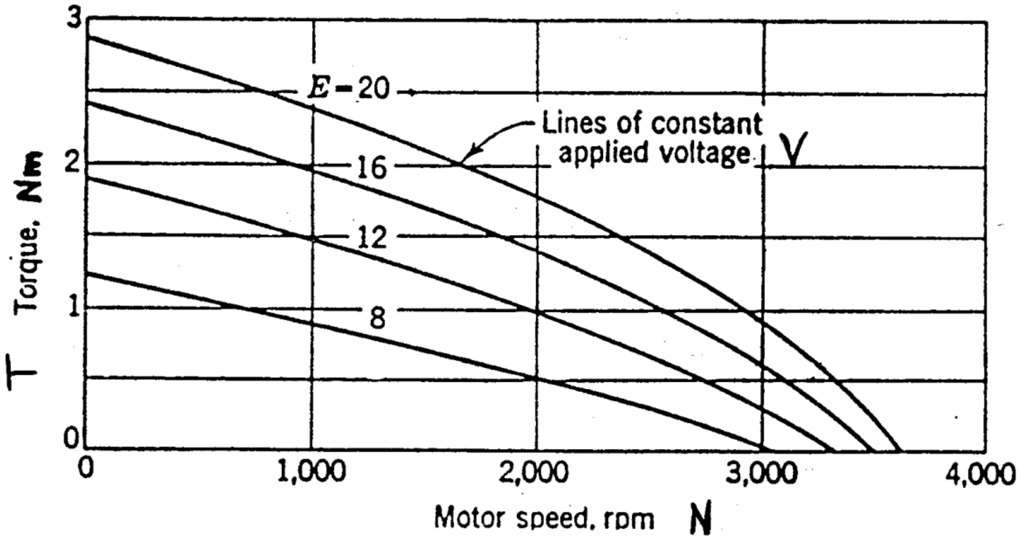
   (a) Determine the linear approximation to the equation at p_0.

   (b) What happens to the approximation obtained when $p_0 \to 0$?

4. Linearise the nonlinear differential equation:

   $M\left(2\dot{R}\dot{\theta} + R\ddot{\theta}\right) = 0$ about the point $\left(\dot{R}_0, \dot{\theta}_0, \ddot{\theta}_0\right)$ where M is a constant.

5. Typical operating curves for a DC motor are shown in the figure below. Effect a linear approximation for the function $N = F(T, E)$ for the operating $T_0 = 1Nm$ and $E_0 = 16\ volts$. (N is the speed in rpm, T is the torque in Nm and E is the applied voltage in volts).



3
2
1
0
Torque, Nm
T
E = 20
16
12
8
Lines of constant applied voltage
V
0
1,000
2,000
3,000
4,000
Motor speed, rpm
N

SET II – LAPLACE TRANSFORMS

1. Find the Laplace transform of the following functions:

   (a) $2e^{-3t}$

   (b) $4e^{-2t}sin8t$

   (c) $3sin4t$

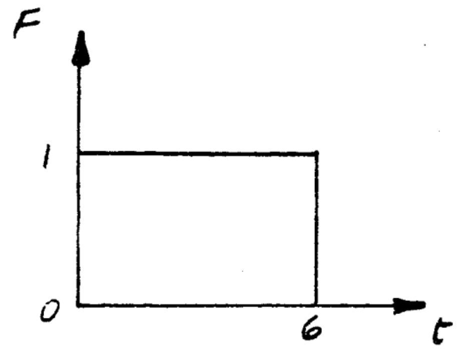
   (d) $2te^{-3t}$

2. Find the inverse Laplace transform of the following functions:

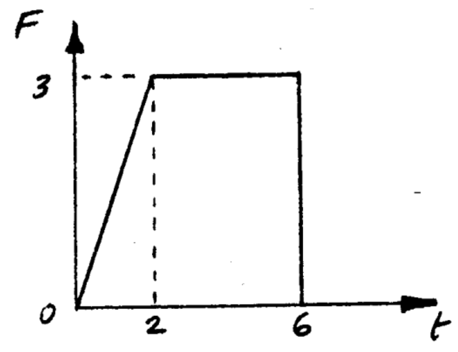
   (a) $\frac{8s-1}{s^2+3s+2}$ (b) $\frac{4}{s^2+2s+17}$ (c) $\frac{s+1}{s^2+2s+5}$

3. Compute the Laplace transforms for the following graphical input excitations:

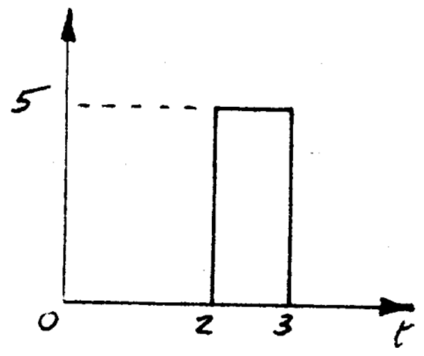
(a)



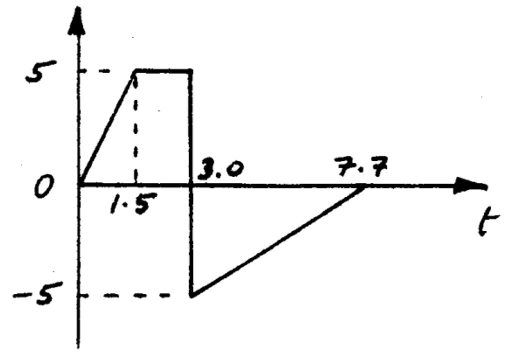
(b)



(c)



(d)



4. Determine the initial and final values of the function $f(t)$ whose Laplace transform is:

$$F(s) = \frac{2}{(s^3+7s^2+15s)}$$

5. Determine the steady state value of the function whose Laplace transform is:

$$\frac{4s^2+5s+6}{s(2s^3+7s^2+13s+2)}$$

6. For the following differential equations and initial conditions, write the transforms of the solutions (i.e. determine the s-domain solutions):

(a) $\ddot{c} + 6\dot{c} + 13c = 5u(t);$ $\quad c(0) = 1, \quad \dot{c}(0) = 4$

(b) $\ddot{c} + 3\dot{c} + 4c = 6sin\omega t;$ $\quad c(0) = 4, \quad \dot{c}(0) = 5$

(c) $\ddot{c} + 2\dot{c} + 4c = u(t);$ $\quad c(0) = 1, \quad \dot{c}(0) = 0$

(d) $\dddot{y} + y = 0;$ $\quad \ddot{y}(0) = \dot{y}(0) = 0, \quad y(0) = 5$

(e) $\frac{d^4\theta}{dt^4} + \frac{d^3\theta}{dt^3} + 6\frac{d^2\theta}{dt^2} + 12\frac{d\theta}{dt} + \theta = (1 + sint)u(t)$ zero initial conditions.

NOTE:

i) $u(t)$ is unit step function

ii) Recall that

$$\mathcal{L}\{\dot{c}(t)\} = sC(s) - c(0)$$

$\mathcal{L}\{\ddot{c}(t)\} = s^2C(s) - sc(0) - \dot{c}(0)$ etc.

7. For the systems described by the following differential equations, input functions r(t) and initial conditions

(a) determine the transfer functions;

(b) find the complete time domain solutions.

(i) $\ddot{c} + 7\dot{c} + 10c = r(t)$ $\quad r(t) = \delta(t)$ $\quad c(0) = 1, \quad \dot{c}(0) = 3$

(ii) $\dot{x} + 12x = r(t)$ $\quad r(t) = sin3t$ zero initial conditions

(iii) $\ddot{x} + 2\dot{x} + 6x = r(t)$ $\quad r(t) = 4\delta(t)$ zero initial conditions

(iv) $\ddot{x} + 6\dot{x} + 25x = r(t)$ $\quad r(t) = e^{-t}$ zero initial conditions

(v) $\ddot{y} + 7\dot{y} + 12y = r(t);$ $\quad r(t) = 2\dot{u} + u$ $\quad$ initial conditions:

$$y(0) = 2, \quad \dot{y}(0) = 3 \quad u(0) = 0$$

NB: u is unit step, δ is unit impulse, $\delta = \dot{u}$.

8. For the system described by its transfer function $G(s) = \frac{1}{(s+1)(s+3)}$ determine:

a) output, if the input is a unit step function;
b) input, if the output is: $y(t) = 2\,(e^{-3t} - e^{-t})$.

SET III – MATHEMATICAL MODELLING

1. For each of the three mechanical systems shown, derive the transfer functions relating:

   (a) f and x
   (b) f and y
   (c) x and y

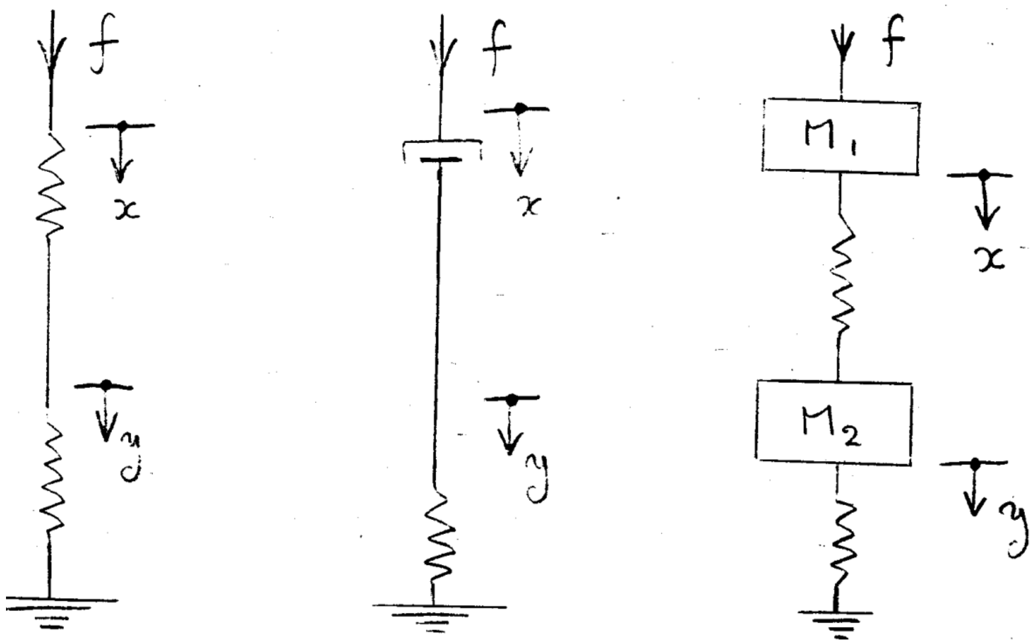


Figure 1.

2. In an accelerometer, the displacement $(x - y)$ of the mass relative to the frame can be a measure of the acceleration $\ddot{x}$ of the machine on which it is mounted. For the accelerometer shown, derive the differential equation and obtain the transfer function Y/X.

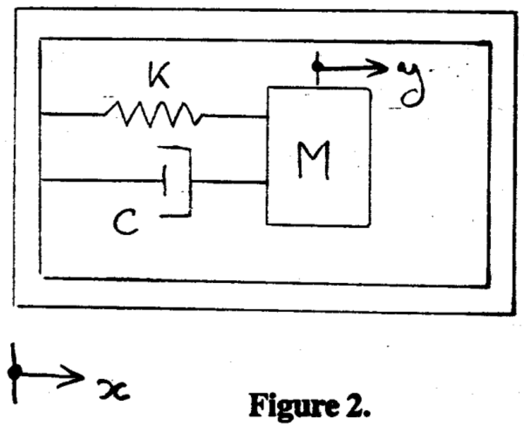


Figure 2.

3. For each of the electrical circuits shown below, derive the transfer function E_0/E_i

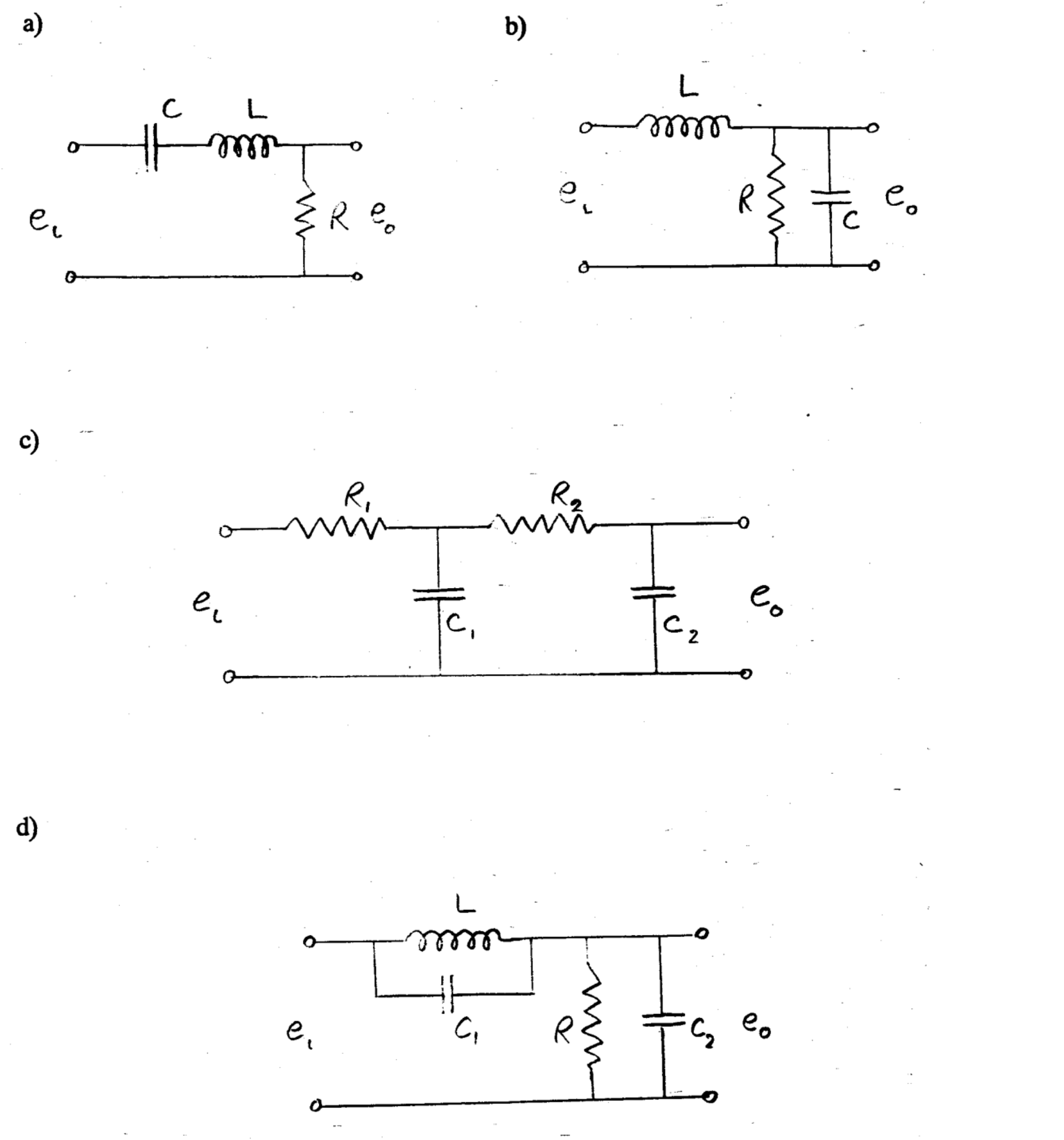


Figure 3.

4. For the liquid level system shown below, obtain the transfer function H_0/Q_i. Each tank has the same area of cross-section A.

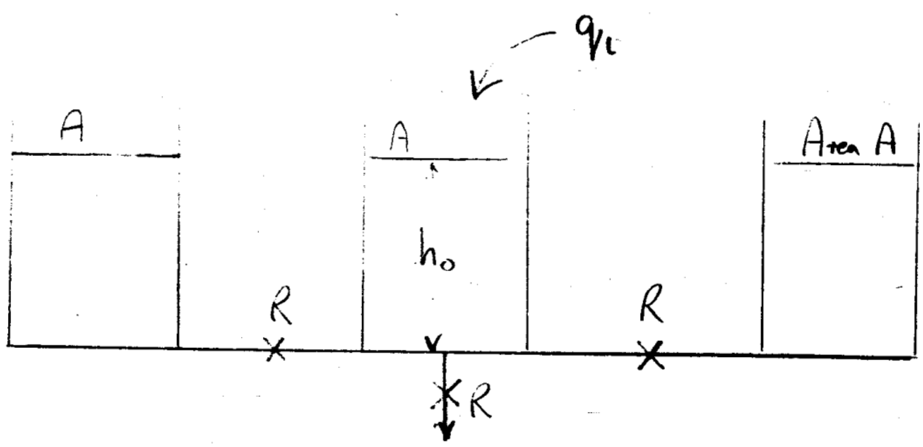


Figure 4.

5. In the heat exchanger in Figure 5, the temperature T_h in the outer chamber can be taken to be constant, because of high flow rate through it. The mass flow rate $\dot{m}$ through the inner chamber is constant. The volume of this chamber is V and its surface area A. Inflow and outflow temperatures are T_i and T_0 respectively. The density of the fluid is ρ and its specific heat c. The surface coefficient of heat transfer is h. All equipment sits at T_i prior to $t = 0$ and instantaneously T_h is establsihed in the outer chamber. Find the transfer function T_0/T_H.

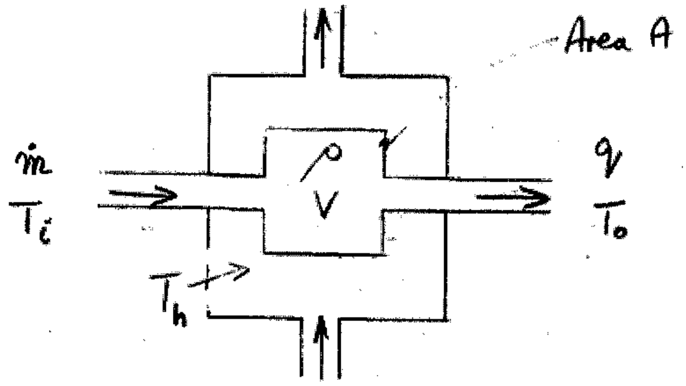


Figure 5.

6. In Figure 6 a mechanical brake block, of mass M, is pressed against the drum with force F. The coefficient of friction is μ, so that the friction force is μF. The surface velocity between the block and the drum is V. The friction power is converted into heat. The conversion factor that changes mechanical power into heat power is H. The block loses heat to the ambient T_a through a surface area A with heat transfer coefficient h. Determine the transfer function relating F and the temperature T of the block if all heat power is assumed to enter the block, of which the specific heat is c.

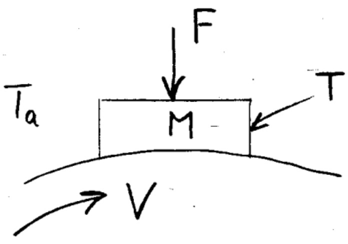


Figure 6.

7. A plastic mould delivery mechanism consists of the components as shown in figure 7. Movement of the end of the lever A causes the piston M to fall and displace a measured quantity of semi molten plastic into the mould below. The piston resists movement both by virtue of movement in the guide which is simulated by K_1 and by virtue of speed of motion with a coeffient C. The lever does not deviate significantly from its horizontal rest position.

Obtain the transfer function relating movement x_2 of the mass M to movement h of the end of the lever at A.

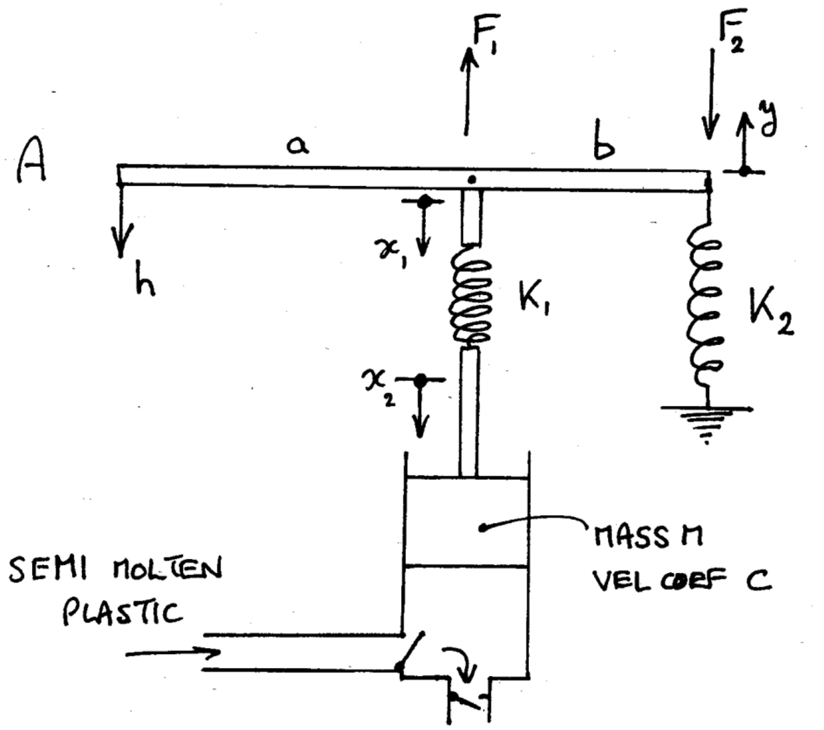


Figure 7.

8. A weir control system consists of an input shaft R which is geared down to drive a rack of mass M_2 . The rack is connected by a lever to a sluicegate of mass M_3 with a fulcrum at A. The layout is shown in Figure 8. Design considerations dictate that the gate should be balanced when motionless and be able to rise at a certain minimum acceleration. Obtain an equation for the acceleration $\ddot{x}_3$ of the sluice gate in terms of all other parameters in the system. The gears are frictionless and the lever does not deviate significantly from its horizontal rest position. What does the first design consideration imply? Use the consideration to full simplify the result of $\ddot{x}_3$.

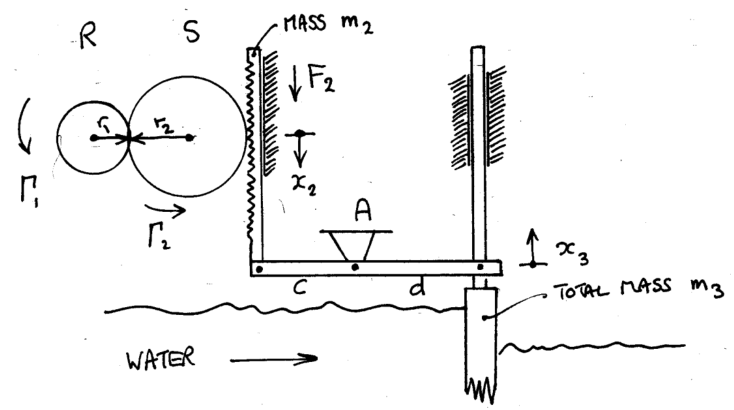


Figure 8.

9. Two heavy rollers of different diameters are used to crush ore. They are geared together at each ends of their cylindrical length. The rollers oscillate back and forth both to crush the ore and to keep freeing the path. The rollers are constrained about an equilibrium position by two springs attached to the second roller. The layout is shown in Figure 9. Find the transfer function X/Γ_1 . [I is the moment of inertia, C is coefficient of resistance due to velocity.]

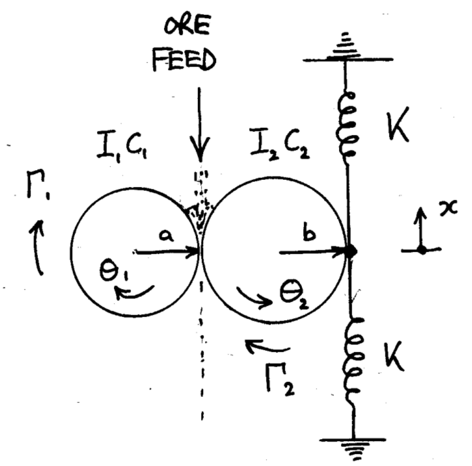


Figure 9.

10. For the lever system shown below, determine the transfer function X_0/X_i .

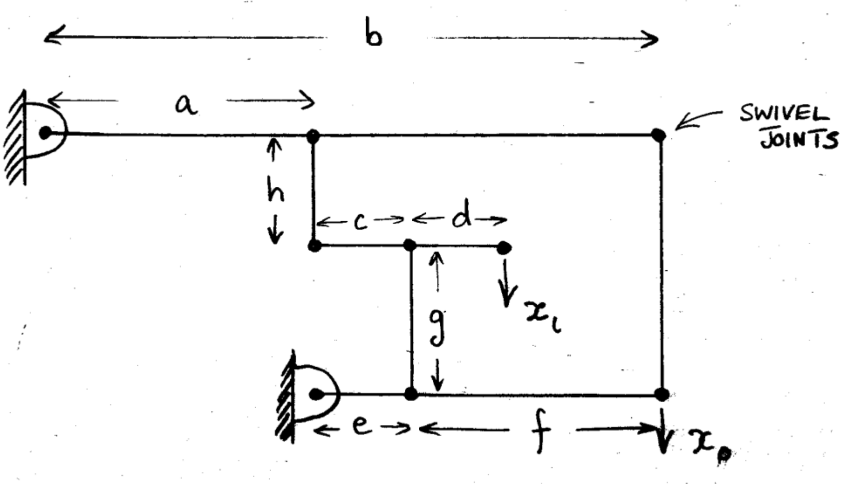


Figure 10

SET IV – BLOCK DIAGRAMS

1. (a) Turn the following equations into block diagram form:

(i) $\dot{y} + 3y = x$ (x input, y output)

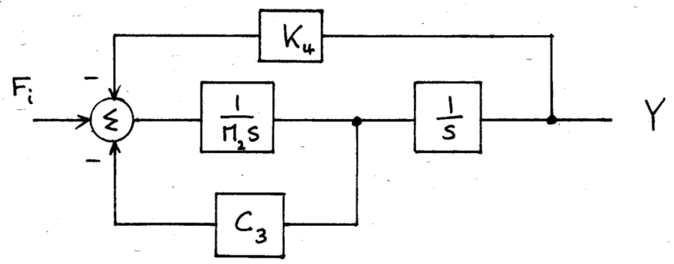
(ii) $3\ddot{a} - 4\dot{a} + 5a = \dot{\mu} + 4\mu$ (μ input, a output)

(iii) $\ddddot{r} + \dddot{r} + \ddot{r} + \dot{r} + 10r - h = 0$ (h input, r output)

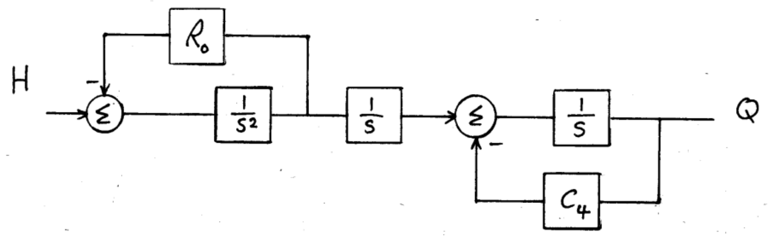
(b) Derive the transfer functions of each of the above systems in 1 (a) both by direct use of Laplace transform and by reduction of the block diagram. Confirm both answers agree.

(c) Find the differential equations which gave rise to the following block diagrams:

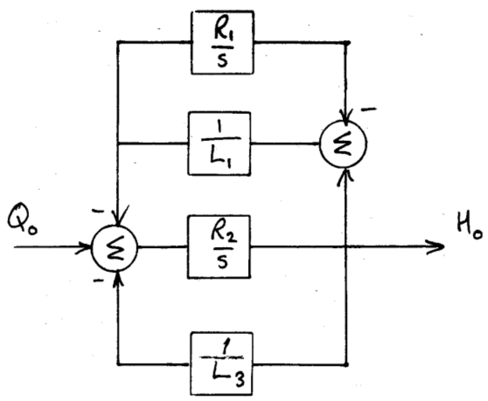
(i)



(ii)



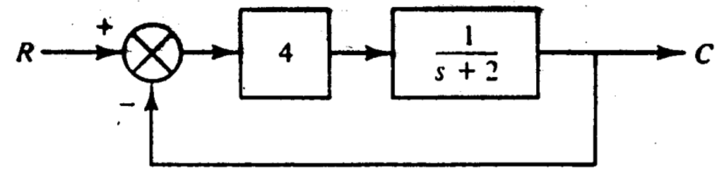
(iii)



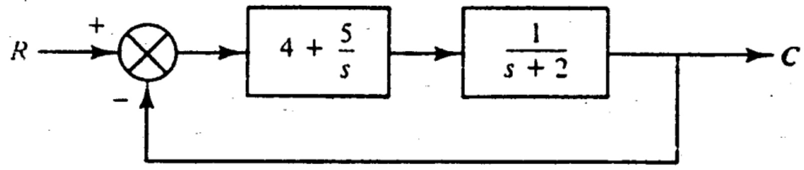
2. For the systems shown in the figure:

(a) Find the closed loop transfer functions.

(b) Calculate the responses to unit step inputs.



(i)



(ii)

3. In the figure shown, R is the input which the output C should follow as closely as possible, and D is the disturbance input to which the output should ideally not respond at all. The total output is the sum of the outputs due to each input separately.

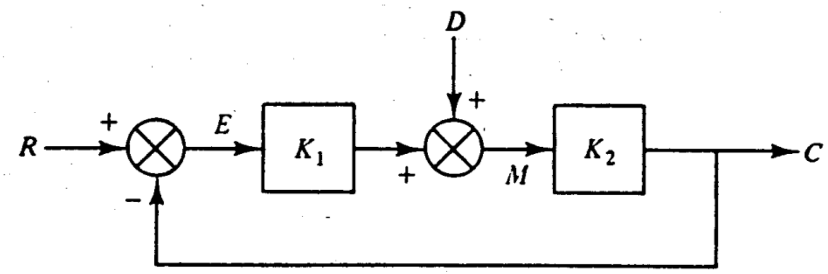
(a) Calculate C/R (with $D = 0$) and C/D (with $R = 0$).

(b) We wish to desensitise the system to D. What values of K_1 and K_2 must be chosen to generate:

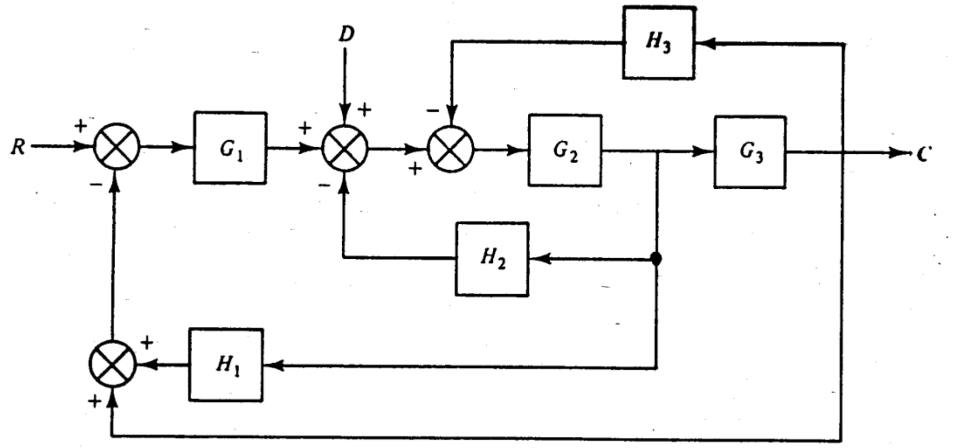
(i) $C/D = 0.1$, $C/R = 0.9$

(ii) $C/D = 0.01$ $C/R = 0.99$

as defined in (a) above.



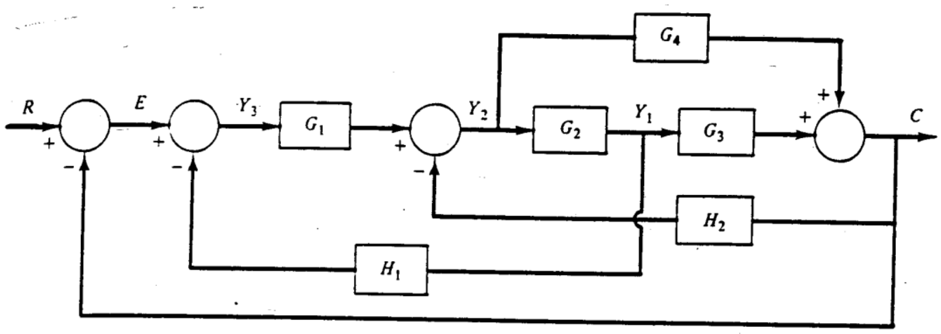
4. Find the transfer functions C/R and C/D for the block diagram as shown below:



5. The figure below shows a block diagram of a system. Obtain the transfer function of the system relating output $C(s)$ to input $R(s)$

(a) using method of block diagrams.

(b) using equivalent algebraic equations.



SET V – TIME DOMAIN ANALYSIS

1. A unit impulse of 1 volt second is applied t an electrical system. From the response record shown in Figure 1 determine the transfer function of the first order system. Note that the initial part of the response has been deleted because it was considered that it had been distorted by the recorder dynamics.

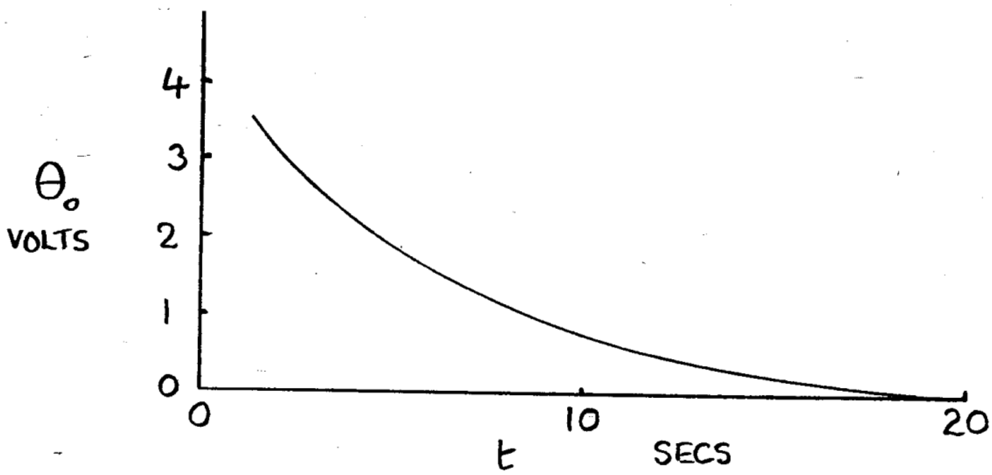


Fig. 1 Impulse response of an electrical system.

2. The transfer function of a pneumatic servomotor is

$$\frac{Y}{P} = \frac{0.05}{2s+1},$$

where $y\ [m]$ is the piston displacement and $P\ [kPa]$ is the input pressure. Sketch the response of the servomotor to a unit step change in the input pressure.

3. In an automotive power steering unit the steering wheel was subjected to a constant angular velocity input of 1 radian per second. The servo cylinder displacement was recorded and the values obtained are tabulated below. Plot the response curve of the servo cylinder displacement versus time. Assuming the system transfer function to be of the form:

$$\frac{X_0}{\Theta_i} = \frac{G}{TS+1},$$

evaluate the gain constant G and the time constant T.

t sec	x_0 mm
0.000	0.000
0.002	0.025
0.004	0.084
0.006	0.168
0.008	0.265
0.010	0.389
0.012	0.518
0.014	0.655
0.018	0.938
0.024	1.376
0.030	1.810

4. The transfer function of a pressure to voltage transducer is

$$\frac{V}{P} = \frac{G}{Ts+1}$$

Figure 2 shows part of the recorded the input, as well as the output steady state sinusoids. From this record evaluate G and T .

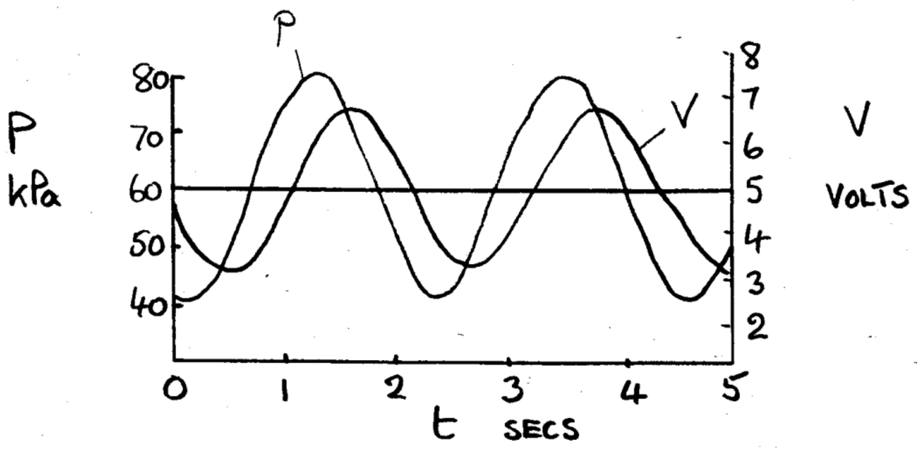


Fig. 2 **Input to and steady state output from a pressure to voltage transducer.**

5. The transfer function of an overdamped second order system is

$$\frac{\theta_0}{\delta} = \frac{1}{(T_1 s+1)(T_2 s+1)} .$$

If the disturbance input δ, is a unit impulse function determine the expressions for the the time to the peak, t_p , of the response function and for the maximum excursion, $\theta_{0_{max}}$ in terms of T_1 and T_2 .

6. The transfer function of an underdamped second order system is

$$\frac{\theta_0}{\theta_1} = \frac{1}{0.01s^2+0.05s+1}$$

Sketch the response of this system when the input is a unit impulse.

7. The transfer function of a steam boiler operating under automatic control is

$$\frac{P}{\delta} = -10\left[\frac{3.6s}{36s^2+3.96s+1}\right]$$

where the input, $\delta\ [kg/s]$, is the flow rate of steam from the boiler and the output, $p\ [kPa]$, is the steam pressure. If the input is a step change of magnitude $10\ kg/s$ calculate the time to the first peak of the resulting transient, t_p . Also determine the value of the maximum excursion of the steam pressure, p_{max} .

8. Assuming that the response, shown in Figure 3, is the response of a critically damped second order system to a unit step input, determine the transfer function of the system. Using this transfer function check the response function at one or two points and hence state whether or not you consider that the response may be adequately represented by the transfer function.

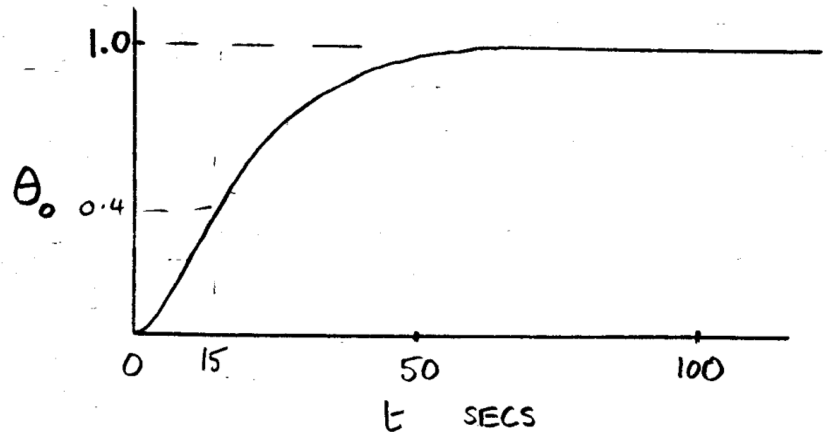


Fig. 3 Response of a system to a unit step change.

9. The time response of a second order controlled system to a unit step change in the set value, θ_i , is shown in Figure 4. Determine the transfer function for the system.

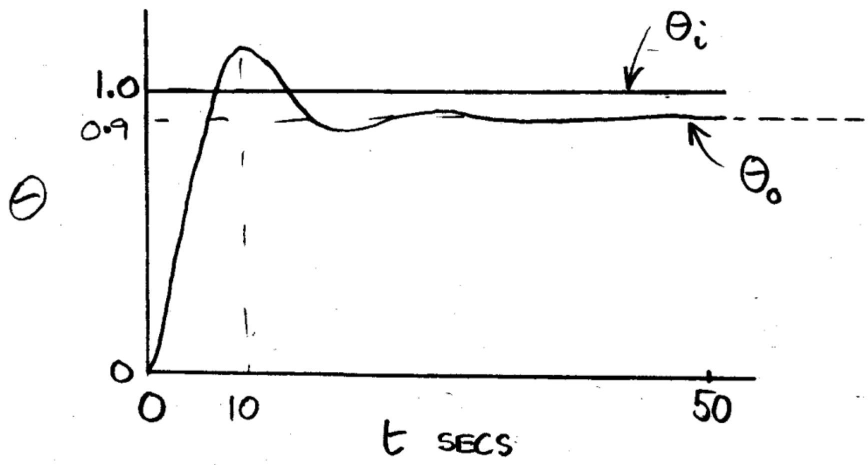


Fig. 4 **Response of a controlled system to a unit step change in the set value.**

SET VI

1. Comment on the stability of the following systems whose transfer functions are as follows:

(a) $\frac{K}{(s+1)}$

(b) $\frac{K(s+2)}{(s+4)}$

(c) $\frac{K}{(s^2+5s+6)}$

(d) $\frac{K}{s^3+6s^2+11s+6}$

(e) $\frac{25s^2+20s+4}{s^3+12s^2+17s-30}$

(f) $\frac{K}{s-1}$

(g) $\frac{K}{(s-3)(s+4)(s+100)(s+2)}$

(h) $\frac{K(s+1)(s+2)}{s^3+6s^2+11s+6}$

(i) $\frac{K(s-2)}{s^2+4s+9}$

(j) $-\frac{4(s-5)}{(s-2)(s-3)(s-4)}$

2. A system is modelled by the equation:

$$\ddot{y}+4\dot{y}+5y=u$$

Is the system inherently stable? By reference to the s plane only, sketch the resulting output if u, the input, is a unit step function.

3. A system's open loop transfer function is

$$G(s)=\frac{K(S+4)}{(S+1)(S+2)}$$

Unity negative feedback is applied to the system.

Determine the range of values of K which will keep the system stable and non oscillatory. Plot the values of s for values of K at the extremes of the ranges discovered.

4. A system is described by the equation:

$$2\dddot{y}+7\ddot{y}+7\dot{y}+2y=\dot{u}+3u$$

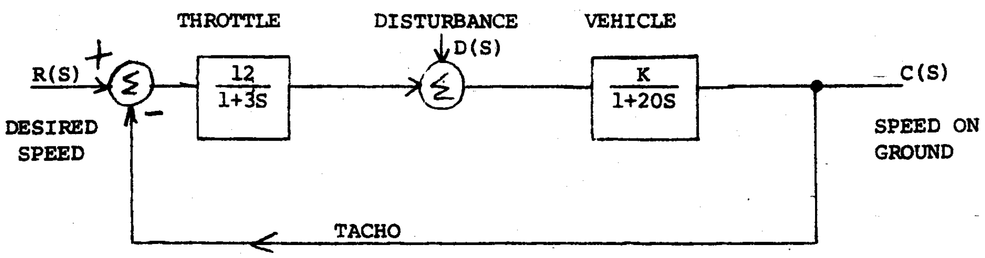
The performance of the system as seen by the outside world is to be modified to give the following transfer function:

$$\frac{1}{S^2+3s+2}$$

by use of a compensator in series with the system, i.e. $\bullet\!-\boxed{G_c}-\boxed{G}-\!\circ$

Define the transfer function of the compensator.

5.



The figure above describes an automatic speed control system. Find the following:

(a) The sensitivity of the transfer function, $C(S)/R(S)$ to changes in the engine speed K .

(b) The transfer function linking load disturbance to speed on the ground $C(S)/D(S)$.

(c) The value of K that ensures that the steady state error of the closed loop system is 5% or less if the input is a unit step signal.

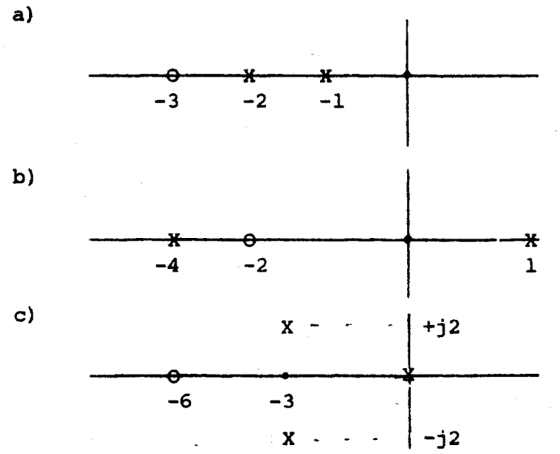
(d) The steady state speed when $r(t)$ is set at 60 Km/h if $K = 100$ and there is no disturbance.

6. The transfer function for a system is given by:

$$G(S) = \frac{K}{(s+2)(s-1)}$$

Unity gain, negative feedback is applied in order to control the system more adequately. By calculating the positions of the roots on the s plane for a number of values of positive K , comment on the adequacy of the method of control. Make reference to what is happening in the time domain solution.

7. The pole-zero patterns of three systems are given graphically as:



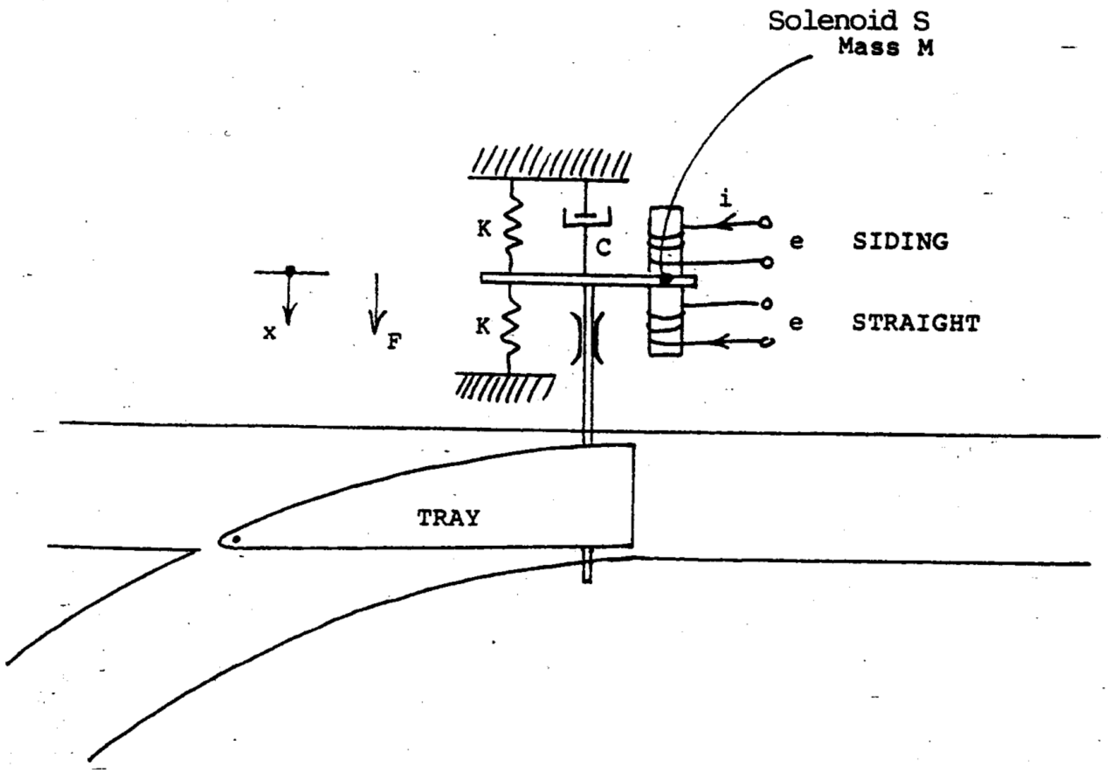
where each system has a d.c. gain K.

In each case determine the form of the transfer function from which they were derived and comment on the stability of the systems. The standard notation for a pole is “x” and for a zero is “o”.

PAST EXAM QUESTIONS

The student is advised it is very much in his/her interest to attempt to complete all these questions.

QUESTION 1 (14 marks)



A railway point's actuator has been constructed as shown above. The linear movement on the changeover tray is accomplished by movement of the solenoid S of inductance, L and series internal resistance, R. Two springs of coefficient K return the tray to a central position if no force from the solenoid operates. The damper C is installed to control the motion by avoiding bounce and percussive damage. The mass of the solenoid core is M [kg] and the Force derived from current flow is αi.

(a) Derive a system differential equation relating the voltage input e to the movement x of the core of the solenoid.

(b) Derive a relationship between M, C and K for the mechanical part of the circuit only to be critically damped.

(c) Prove that the transfer function of the system is

$$\frac{X(s)}{E(s)} = \frac{\alpha}{LMs^3+(RM+LC)s^2+(RC+2LK)s+2RK}$$

[Hint: Relate e to i and i to x separately in LAPLACE TRANSFORM then combine.]

(d) The normal separation of tray rail to fixed rail is 0.2m, the spring coefficients are rated at 1500 $N.m^{-1}$ each, the attractive force coefficient of the solenoid, α is 15 N/A, and its internal resistance, R is 0.5Ω. What is the magnitude of the voltage step required if the design criterion on the force holding the tray to the fixed rail is chosen to be 100 N?

QUESTION 2 (12 marks)

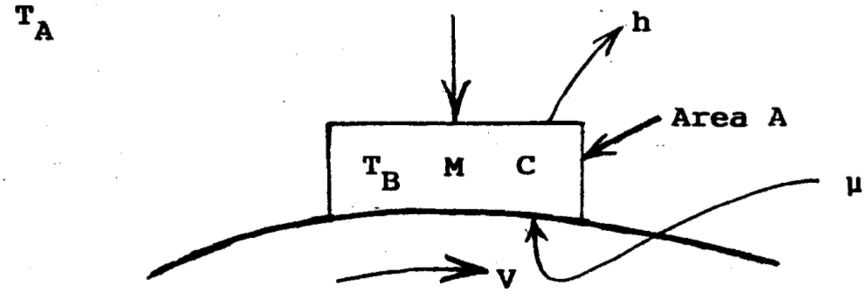
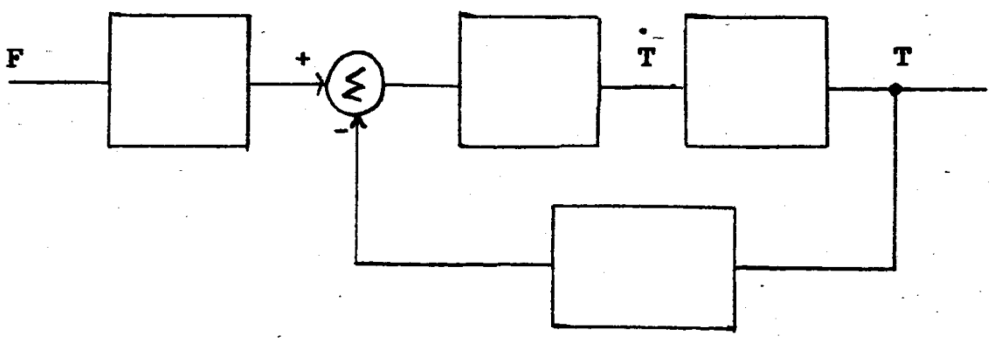


Figure 3.

A brake block of mass M and variable temperature T_B, impinges on the perimeter of a wheel turning with velocity V at the point of contact with a brake. The force applied to the block is F and the coefficient of friction with the wheel is μ. The dead weight of the block Mg is always much less than the force F, and can be ignored. Heat is lost to ambient, T_A through an area A with coefficient h which also accounts for any incidental heat transfer between brake and wheel. The heat capacitance of the block is C [J Kg^{-1}°C^{-1}].

1) Complete the block diagram below (i.e. make your own copy into your answer book filling all boxes provided) using the temperature difference, $T = T_B - T_A$ as the output and force F as the input.



Block Diagram (incomplete).

2) Reduce the block diagram to a single block and thereby prove that the TRANSFER FUNCTION is:

$$\frac{T(s)}{F(s)} = \frac{\mu V}{CM} \frac{1}{\left(s + \frac{hA}{CM}\right)}$$

3) If $\mu = 0.3,\ V = 2ms^{-1},\ C = 10^3 JKg^{-1}°C^{-1}$
$M = 2.5Kg,\ h = 2.5KWm^{-2}°C^{-1},\ A = 100cm^2$

and F is applied as impulses of strength $15 \times 10^4 Ns$, when T_A is $15°C$, what is the maximum temperature the block reaches after the first impulse is applied? Assume all temperatures were in equilibrium prior to applying F.

4) If the impulses identified in (3) continue at 1 minute intervals, what is the temperature of the brake block immediately after the 3rd impulse is applied? [Hint: use the principle of SUPERPOSITION to calculate your answer.]

QUESTION 3 (12 marks)

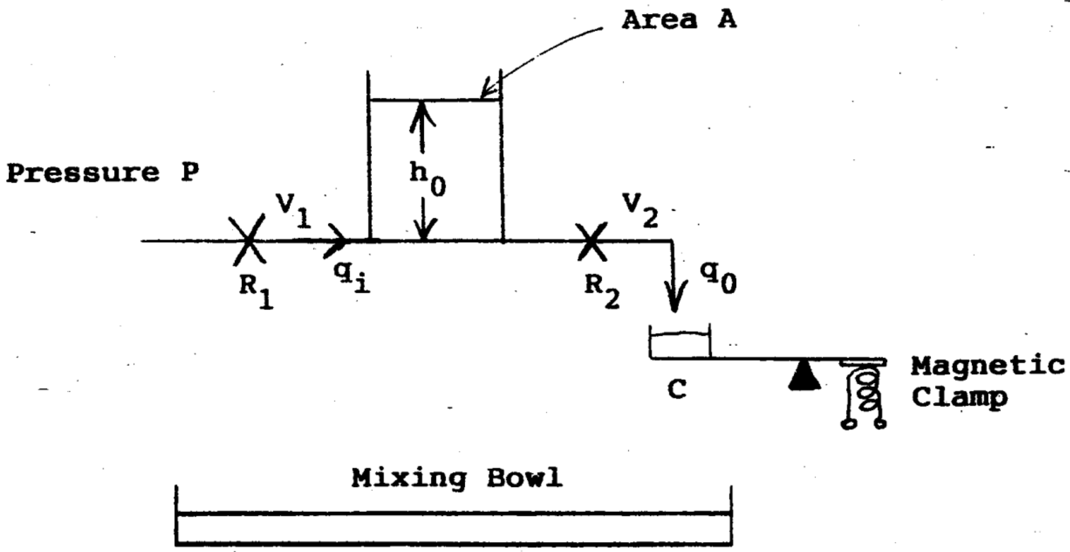


Figure 2

In the diagram above the reservoir of variable height, h_0 supplies fluid in a manner which eventually delivers a measured amount into a mixing bowl passing below. The mixing bowl is fed by a cup C on a balance such that when a particular volume of fluid has accumulated in the cup, it is great enough to break the magnetic clamp at the other end of the balance arm, whereupon the arm delivers the dose, empties and returns to the horizontal, held once more by the magnetic clamp.

The system is supplied with fluid at pressure P and, at time zero with reservoir and cup empty, the supply valve V_1, presumably shut, is immediately cracked open to resistance R_1. The coefficient of valve V_1 is α that flow is equal to $\frac{P\alpha}{R_1}$.

The resistance of the outflow valve V_2 is constant at R_2.

1) Obtain the TRANSFER FUNCTION $\frac{H_0(s)}{Q_i(s)}$ relating flow in q_i to height of the reservoir, h_0.

2) Using the boundary conditions for start up indicated, obtain the time domain solution for flow q_0 through valve V_2.

3) If $\alpha = 10^{-6},\ P = 10\ bar,\ R_1 = 1000\ \frac{s}{m^2},\ R_2 = 200\ \frac{s}{m^2},\ A = 0.1\ m^2\ [1\ bar = 10^5 Pa]$ and it takes 30 seconds for the first delivery of a measured amount to take place, what is the size of this measured amount in litres? [HINT: integrate the flow.]

4) Once the system has reached a steady state, what is the regular time between deliveries of the measured amount?

QUESTION 4 (10 marks)

(a)

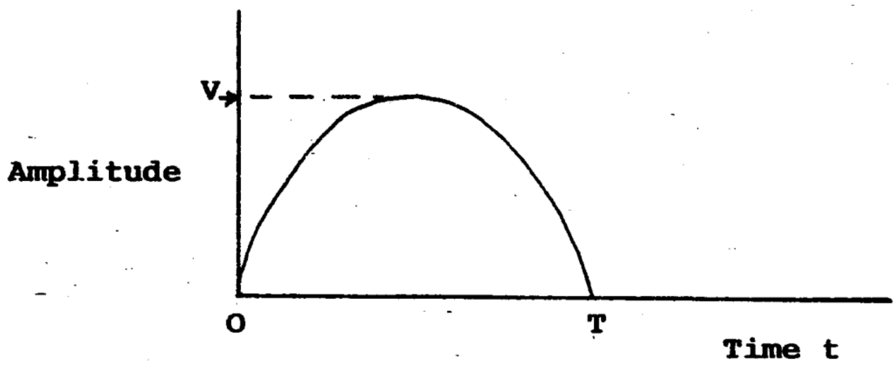


Figure 1(a)

The signal shown in Figure 1(a) is the voltage input to a linear transducer and is a half period of a sinusoid. The signal then returns to zero in the range $T < t < \infty$. Find the LAPLACE TRANSFORM of the signal.

(b) The signal of 1(a) is applied to a low pass filter as shown in Figure 1(b).

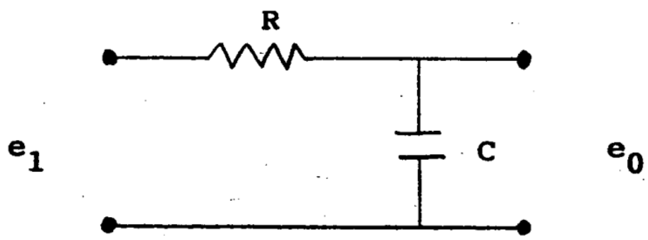


Figure 1(b)

Derive the output waveform in the time domain.

QUESTION 5 (14 marks)

A daisy wheel printer mechanism is described by the diagram in Figure 4.

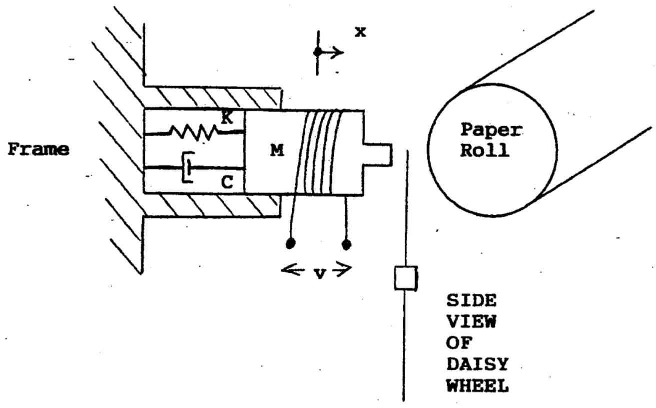


Figure 4.

The solenoid is driven by a voltage v and has pure resistance R only. The force derived from the solenoid is equal to $K_1 i$, where i is the current passing in the circuit. The mass M is pulled outward to strike the edge of the daisy wheel and impress it on the paper roll.

1) Derive the dynamic equation relating voltage input, v to output, x (movement of the mass M). Hence derive the transfer function $\frac{X(s)}{V(s)}$.

2) If a step function V_1 is applied at $t = 0$, derive the LAPLACE TRANSFORM of the output, $X(s)$.

3) After ensuring the transform is expressed in a way that suggests an underdamped system, derive the natural frequency of oscillation, ω, and damping factor, ξ, as functions of K, M and C.

4) If we know the time to the first peak, T_p back in the time domain is $\frac{\pi}{\omega\sqrt{1-\xi^2}}$ and that the percentage overshoot, $M_{\%}$ is $100e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$, and that the design constraints put $T_p = 0.05\,sec,\ M = 0.01\,kg,\ R = 100\,\Omega$ and $\xi = 0.5$ for light damping, find:

   a) the percentage overshoot $M_{\%}$;

   b) ω, K and C.

5) If the method of striking means that, after a 1 volt step function is applied to the solenoid, the hammer must retract and settle 0.49 mm back from its furthest throw (i.e. its overshoot) before the current i is turned off, prove the value of K_1 required is 15.8 NA^{-1}.

QUESTION 6 (12 marks)

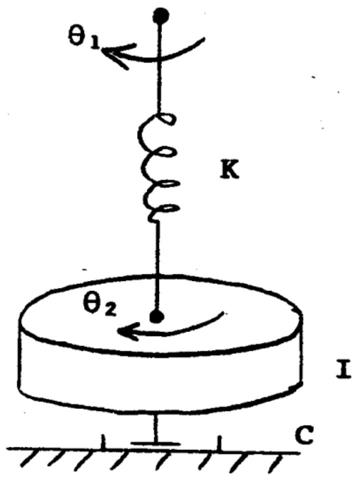


Figure 5.

An impulse meter consists of a torsion spring of coefficient K linked to a disc of rotary inertia I under which is a variable damper of coefficient C.

The impulse is applied as a deflection of magnitude θ_1. The resultant maximum amplitude of θ_2 is measured and regarded as proportional to the magnitude of the impulse applied.

1) Find the TRANSFER FUNCTION $\frac{\theta_2(s)}{\theta_1(s)}$

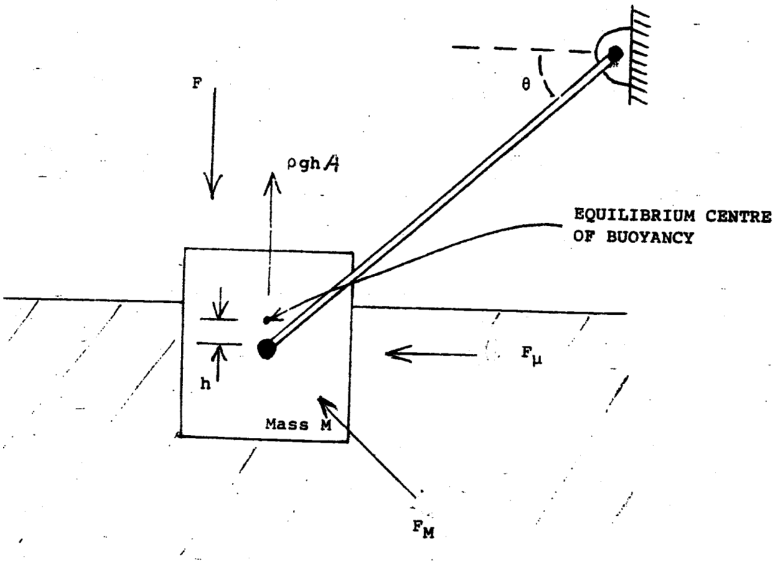
2) Find the time domain solution for an impulse of size θ_0 deg·sec being applied as θ_1, if the system is regarded as UNDERDAMPED. Keep your answer simplified by using ξ and ω_n as variables with definitions supplied.

3) Find the approximate maximum deflection of θ_2 if the system is at rest prior to the impulse being applied.

4) Prove that, if the system is to be critically damped, then $C = 2\sqrt{KI}$.

5) Prove the assumption that the maximum deflection of θ_2 is directly proportional to the magnitude of the impulse applied is true when either critical damping or underdamping is used.

QUESTION 7 (12 marks)



A novel form of viscometer which can measure viscosity over wide ranges has been proposed. The diagram above illustrates the principle whereby a float is constrained to bounce on the surface of a liquid. Observations are made on the frequency and damping of the float's movement.

The force due to buoyancy is ρghA, where ρ is the density of the liquid and h is the vertical static depression of the centre of buoyancy below its equilibrium position. A horizontal force, F_{μ} is derived from viscous drag in the liquid at the surface, and is equal to $\mu\dot{x}$, where μ is the viscosity of the liquid and $\dot{x}$ is the horizontal velocity vector. F_M is the inertial force of the mass of the float which hangs at some angle θ from a pivot point. The weight of the mass M can be changed but not its dimensions.

(a) Derive the dynamic equation relating h, the float's vertical movement, to a general force F which is directed vertically downwards. [HINT: Resolve along a direction perpendicular to the light rod suspending the float.]

(b) Obtain a TRANSFER FUNCTION, $H(s)/F(s)$.

(c) The viscometer is used to find the angle, θ, at which the movement of the float when subjected to an impulse of force, F, will be critically damped. Show that this occurs when:

$$\mu = \left(\frac{4\rho g M A}{tan^2\theta sin^2\theta}\right)^{1/2}$$

(d) The ratio $\mu/\rho^{1/2}$ is approximately known to be 0.5. What approximate value of MA would you choose so that the derived angle would lie somewhere in the region of $\pi/3$?

(e) Why doesn't the meter work at $\theta = 0$ and $\pi/2$? [Not more than 3 sentences.]

QUESTION 8 (12 marks)

A simplistic model of the elevator action in flying an aircraft is given by the two dynamic equations:

$$F_1 = K_1(\alpha - \theta)$$

$$F_2 = K_2\theta$$

where F_1 is the force deflecting the tail of the aircraft, F_2 is the lift force on the body of the aircraft, α is the angle between aircraft horizontal and elevators and θ is the angle between aircraft horizontal and ground horizontal (see Figure 2). M is the mass of the aircraft and I the moment of inertia about the centre of mass. Take α and θ as small angles such that F_1 and F_2 act vertically. Also regard F_1 as giving only a moment acting on the inertia I and F_2 being the only significant force on M.

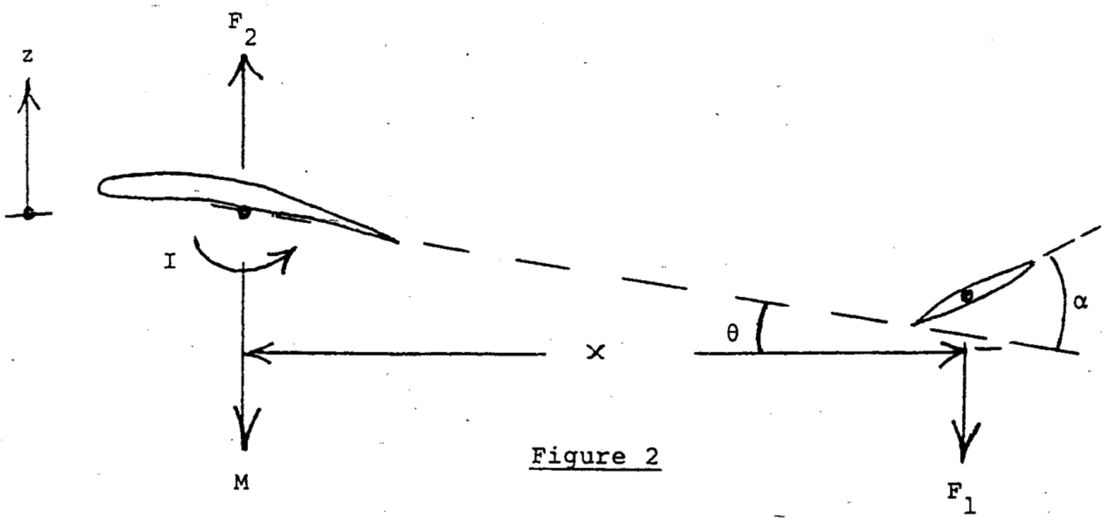


Figure 2

By substituting for F_1 and F_2 find the transfer functions:

$\frac{Z(s)}{\theta(s)}$ and $\frac{\theta(s)}{\alpha(s)}$. Hence find $\frac{Z(s)}{\alpha(s)}$

Show that, by postulating a unit impulse on α, the aircraft would go into a steady climb but with an oscillating motion on the mean climb of $\frac{K_2}{M}\left(\frac{I}{K_1 x}\right)^{1/2} sin\left[\left(\frac{K_1 x}{I}\right)^{1/2} t\right]$. If desired, use partial fractions to revert to the time domain.

QUESTION 9 (12 marks)

In the reservoir feed system as depicted in the diagram below (Figure 3), draw a block diagram showing the essential elements and how they are related to one another.

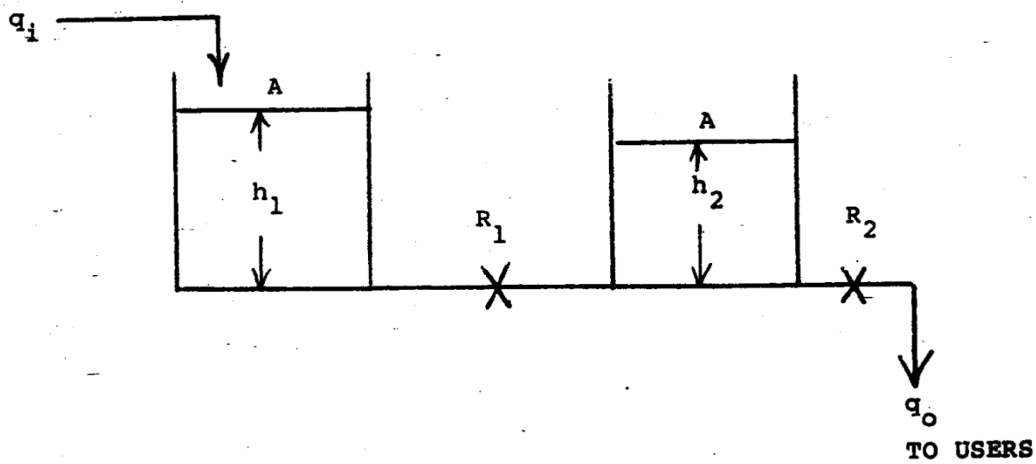


Figure 3

Reduce the block diagram to obtain the transfer function relating q_i to h_2, that is $\frac{H_2(s)}{Q_i(s)}$.

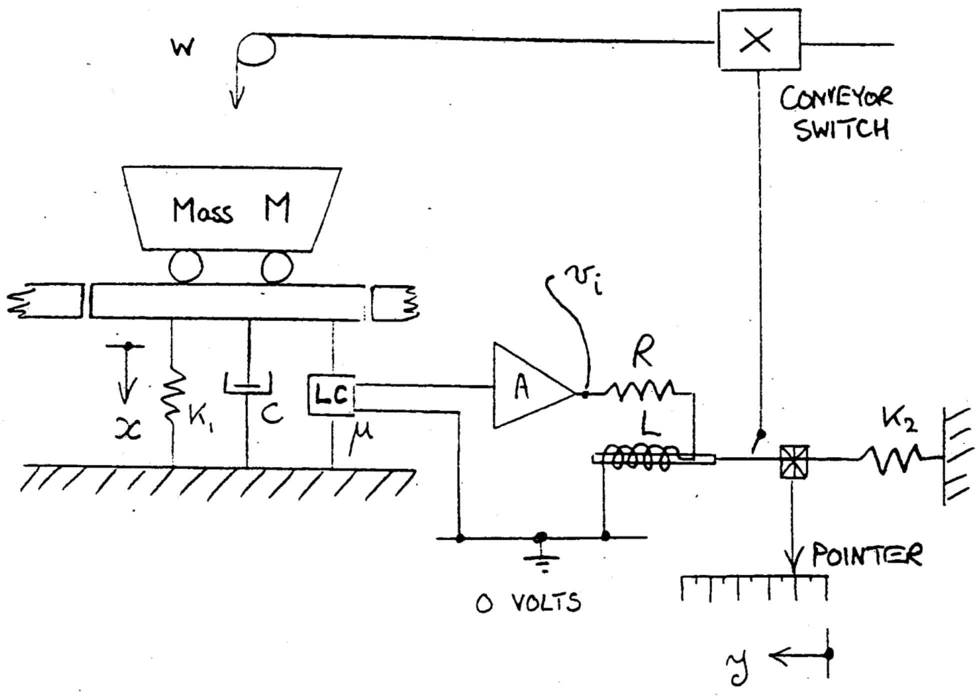
Obtain the time constants of the time domain solution, that is, look for the roots of s in the characteristic equation. Show:

1) That there is no possibility that oscillatory motion will occur when the system is driven with impulses, steps and ramps.

2) That the areas A are important in determining how fast the system settles after being perturbed.

3) An increase in A will lengthen the time to settling after disturbances have occurred, all other factors remaining the same.

QUESTION 10 (15 marks)



A schematic of a coal loader which is automatically controlled to fill coal wagons to a particular weight is shown in figure above.

A load cell weighing platform underneath the rail sleepers can sense the weight of the coal wagon above, including its contents. The load cell LC has a characteristic μ [volts/meter] and does not interfere in any way with the spring damper mechanism of the platform.

The output of the load cell is amplified and fed to a solenoid to deflect a linear pointer on a scale. The electrical circuit has resistance R and inductance L. The solenoid generates a force F_i which is proportional to current flowing via a coefficient α [NA^{-1}].

The pointer on the scale is restrained by a spring K_2 and normally reads zero y co-ordinate with an <u>empty</u> coal truck in position on the weighing platform. The pointer on the scale is also used to throw a micro switch after exceeding a certain deflection. The micro switch then halts the loading action to the coal wagon.

1) At the beginning of loading when any coal mass loaded is much smaller than mass M, form dynamic equations relating:

   a) Cumulative coal input w [Kg] to weighing platform deflection x.

   b) Load cell deflection x to voltage input to electrical circuit v_i .

   c) Voltage v_i to circuit current i.

   d) Current i to pointer deflection y (positive position to left).

2) Translate the system equations into TRANSFER FUNCTIONS:

$$\frac{X(s)}{W(s)}\ ,\ \frac{V_i(s)}{X(s)}\ ,\ \frac{I(s)}{V_i(s)}\ \ and\ \ \frac{Y(s)}{I(s)}$$

3) Obtain the transfer function $\frac{Y(s)}{W(s)}$ relating pointer deflection to coal input.

4) If $\mu = 10\ \mu V/mm,\ A = 1000,\ \alpha = 10\ N/A$

$R = 10\ \Omega,\ L = 100\ mH,\ M = 2\ tonnes$

$C = 4000\ \frac{N}{Ms^{-1}},\ K_1 = 2000\ \frac{N}{m},\ K_2 = 10\ N/m$

find the time constants associated with the weighing platform and the electrical circuit and show that they are 1 sec and 0.01 sec respectively, proving that transients from the mechanical components last much longer than those from the electrical components.

5) If a 50 Kg lump of coal falls into an empty coal truck, what steady state deflection (in mm) eventually appears on the linear pointer scale?