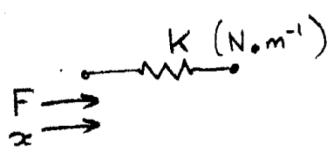
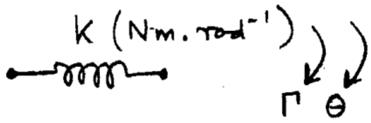
LECTURE 3 – ELEMENTARY BUILDING BLOCKS

The formulation of the equations to describe commonly used engineering components included in electrical, mechanical, fluidic and thermal systems. A discussion of methods of transduction is also given.

1. MECHANICAL COMPONENTS

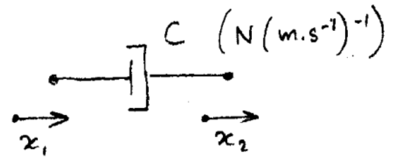
<u>SPRINGS</u>

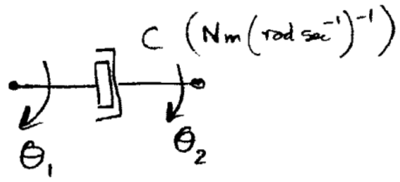




TRANSLATIONAL $F = kx$, ROTATIONAL $\Gamma = k\theta$

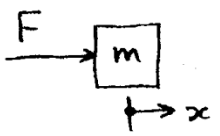
<u>DAMPERS</u> A resistive force is generated by virtue of relative velocity; e.g. a viscous retarding force.

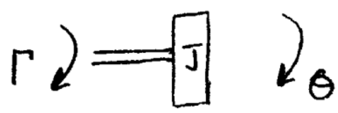




TRANSLATIONAL $F = C(\dot{x}_1 - \dot{x}_2)$, ROTATIONAL $\Gamma = C(\dot{\theta}_1 - \dot{\theta}_2)$

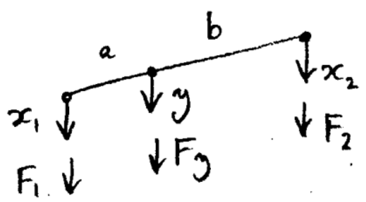
<u>INERTIA</u> A consequence of Newton's 2nd Law





TRANSLATIONAL $F = m\ddot{x}$, ROTATIONAL $\Gamma = J\ddot{\theta}$

<u>LEVERS</u> For small deflections, we may apply SUPERPOSITION to determine y from x_1 and x_2



Thus

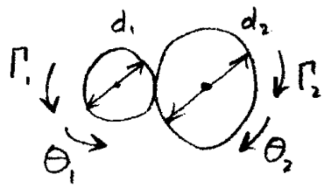
$$y|_{x_2=0} = \frac{b}{a+b}x_1 \,,\; y|_{x_1=0} = \frac{a}{a+b}x_2$$

$$\therefore y = y|_{x_2=0} + y|_{x_1=0} = \frac{b}{a+b}x_1 + \frac{a}{a+b}x_2$$

We can also relate forces in the same way

$$F_y = F_y|_{\substack{x_2 \\ FULCRUM}} + F_y|_{\substack{x_1 \\ FULCRUM}} = -\frac{a+b}{b}\cdot F_1 - \frac{a+b}{a}\cdot F_2$$

<u>GEARS</u> For two inter-meshing gears, we can relate a number of parameters to one another.



$$n = \frac{d_1}{d_2} \equiv \frac{r_1}{r_2}\, , \omega = \dot{\theta} = \frac{\dot{x}}{r}$$

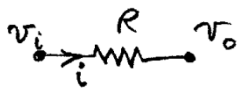
Thus

$$n = \frac{d_1}{d_2} = \frac{\omega_2}{\omega_1} = \frac{\theta_2}{\theta_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\ddot{\theta}_2}{\ddot{\theta}_1} = \frac{\Gamma_1}{\Gamma_2}$$

Note the gear train reverses at each meshing.

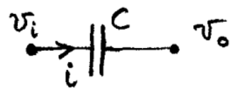
2. ELECTRICAL COMPONENTS

RESISTANCE



$i = \frac{v_i - v_o}{R}$ OHMS LAW

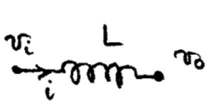
CAPACITANCE



$$i = C\cdot\frac{d}{dt}(v_i - v_o)$$

OR $\quad v_i - v_o = \frac{1}{C}\int i dt$

INDUCTANCE



$$i = \frac{1}{L}\int (v_i - v_o)\, dt$$

OR $\quad v_i - v_o = L\cdot\frac{di}{dt}$

THE CONCEPT OF IMPEDENCE: (this is an interlude)

We may recast all our equations so far into LAPLACE

$F = kx$ becomes $F(s) = k \cdot X(s)$

$F = c(\dot{x}_1 - \dot{x}_2)$ becomes $F(s) = cs(X_1(s) - X_2(s))$

$F = m\ddot{x}$ becomes $F(s) = ms^2 \cdot X(s)$

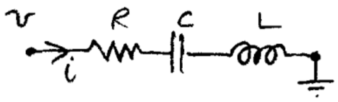
$i = \frac{v_1 - v_o}{R}$ becomes $I(s) = \frac{V_i(s) - V_o(s)}{R}$

$i = C\frac{d}{dt}(v_1 - v_o)$ becomes $I(s) = cs(V_i(s) - V_o(s))$

$i = \frac{1}{L}\int (v_i - v_o)dt$ becomes $I(s) = \frac{1}{LS}(V_1(s) - V_o(s))$

If we consider a VOLTAGE or a FORCE as the initiator and the CURRENT or MOVEMENT as the output, the coefficient that links the two has the form of IMPEDENCE i.e. impeding the outcome → bigger impedance means less output for same force.

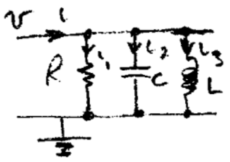
THE SERIES CIRCUIT



$$v = Ri + \frac{1}{C}\int idt + L\frac{di}{dt} \textit{ becomes } V(s) = \left(R + \frac{1}{Cs} + Ls\right) \cdot I(s)$$

SERIES IMPEDENCE

THE PARALLEL CIRCUIT



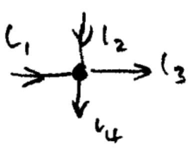
$$i = i_1 + i_2 + i_3$$

$$i = \frac{v}{R} + C\frac{dv}{dt} + \frac{1}{L}\int vdt \;\; \textit{becomes} \;\; I(s) = \left(\frac{I}{R} + Cs + \frac{I}{Ls}\right)V(s)$$

PARALLEL IMPEDENCE

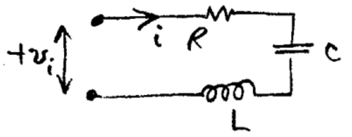
KIRCHOFFS LAWS

The net flow of CURRENT to a junction is ZERO



$i_1 + i_2 - i_3 - i_4 = 0$ or $\sum i = 0$

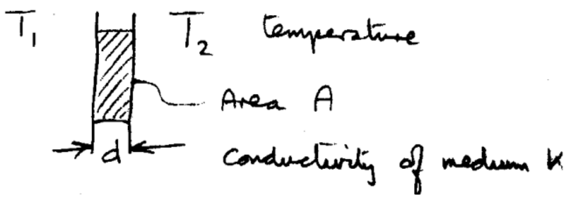
The sum total of VOLTAGE around any loop is ZERO



$$v_i - R_i - \frac{I}{C}\int idt - L\frac{di}{dt} = 0$$

3. THERMAL COMPONENTS

HEAT CONDUCTION



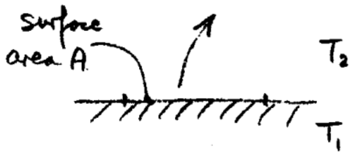
Flow of heat $q = -KA\frac{(T_1 - T_2)}{d}$ [Discrete version]

In terms of a thermal impedance

$$T_1 - T_2 = -\frac{d}{KA} \cdot q$$

$\frac{d}{KA}$ has the units of impedance $°K/(Js^{-1})$

HEAT CONVECTION



Flow of heat $q = h_c A(T_1 - T_2)$ as a simple model

h_c is a surface convective coefficient of heat transfer.

THERMAL CAPACITANCE

The rate of net heat transfer to a body is equivalent to its rate of increase in internal energy.

Thus $q = \rho c V \frac{dT}{dt}$ where ρ is density, c is specific heat, V is volume

Let $C_t = \rho c V$ and is the thermal capacitance $(J°K^{-1})$ of the body. This is analogous to electical capacitance $i = C\frac{dv}{dt}$

THERMAL RADIATION

Stefan's Law states $q = A\sigma T^4$, σ is Stefan's Constant. Let us invoke linearisation to make the law useable in our systems. T_o is the temperature operating point

$$f(x_o) \equiv A\sigma T_o^4 \ , \ f'(x_o) \equiv 4A\sigma T_o^3$$

$$\therefore q = A\sigma T_o^4 + (T - T_o)4A\sigma T_o^3$$

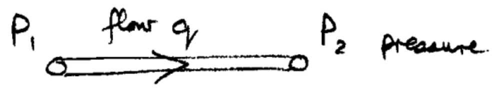
$$\therefore q = 4A\sigma T_o^3 T - 3A\sigma T_o^4 \equiv [y = mx + c]$$

Be careful, the approximation is only valid very close to T_o because of the T^4 power law.

4. FLUIDIC SYSTEMS

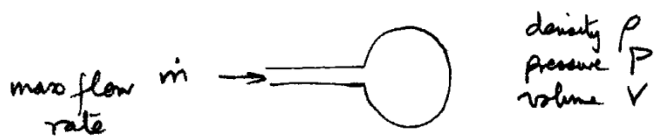
We will show how previous concepts can be derived.

RESISTANCE



$P_1 - P_2 = R_f q$ where R_f is known as a fluidic impedance.

CAPACITANCE



$$\dot{m} = \frac{d}{dt}(\rho V) = V\frac{d\rho}{dt} \text{ if V is assumed a constant}$$

Apply the equation above to a LIQUID.

By definition $p - p_o = \beta \cdot \frac{\rho - \rho_o}{\rho_o}$ at constant volume

Or $\dot{p} = \frac{\beta}{\rho_o} \cdot \dot{\rho}$ but $\dot{\rho} = \frac{\dot{m}}{V}$ $\therefore \dot{m} = \frac{V\rho_o}{\beta} \cdot \dot{p}$

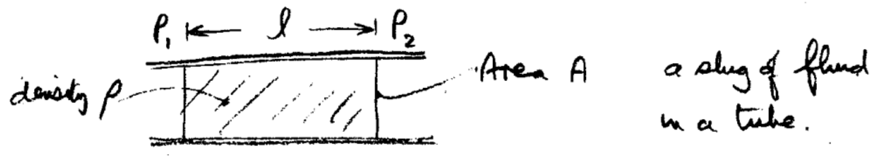
β is known as the bulk modulus. Equation for $\dot{m}$ has similarities to electrical capacitance in that flow is related to rate of charge of driving force.

Apply the equation above to a GAS.

The gas law is: $pV = mRT$ m is in moles, R is universal gas constant.

$$m = \frac{pV}{RT} \quad \therefore \dot{m} = \frac{V}{RT} \cdot \frac{dp}{dt} \text{ or } \frac{V}{RT} \cdot \dot{p} \text{ similarly to above.}$$

INERTIA



Mass of fluid $m = \rho V = \rho A l$

The force acting on the slug by the pressure drop is $F = (p_1 - p_2)A$

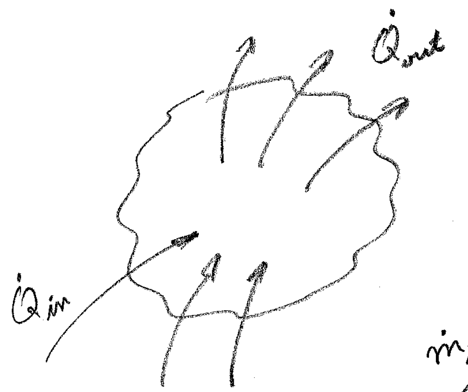
Using Newton's 2nd Law $(F = ma)$

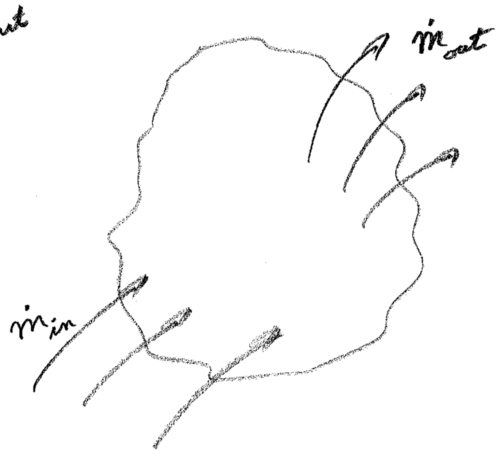
$$(p_1 - p_2)A = \rho A \ell \cdot \ddot{\ell} \quad \text{and} \quad \ddot{\ell} \equiv \dot{v} = \frac{\dot{q}}{A} \quad \text{where } v \text{ is velocity, } q \text{ is flow rate}$$

$$\therefore (p_1 - p_2) = \frac{\rho \ell}{A} \cdot \dot{q}$$

$\frac{\rho \ell}{A}$ is a term similar to inductance in electrical circuits.

5. THERMAL AND FLUID SYSTEMS IN THE NUTSHELL



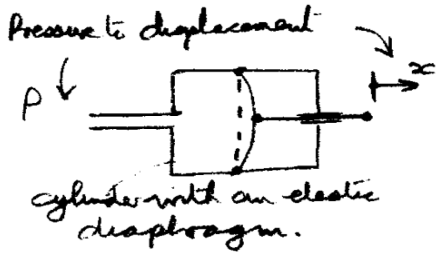


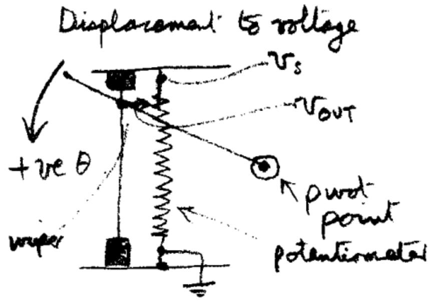
$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{d(mcT)}{dt} = \frac{d(\rho VcT)}{dt} \qquad \dot{m}_{in} - \dot{m}_{out} = \frac{dm}{dt} = \frac{d(\rho V)}{dt}$$

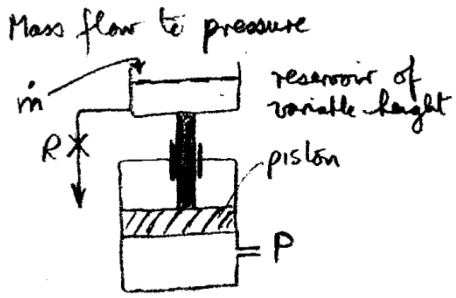
6. TRANSDUCTION

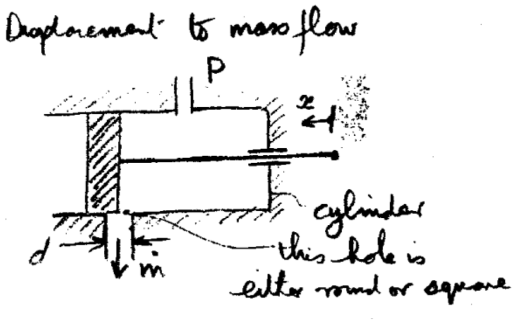
Study the diagrams of the transducers and work out which graphical result is closest to what you would expect to happen.

TRANSDUCER









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