

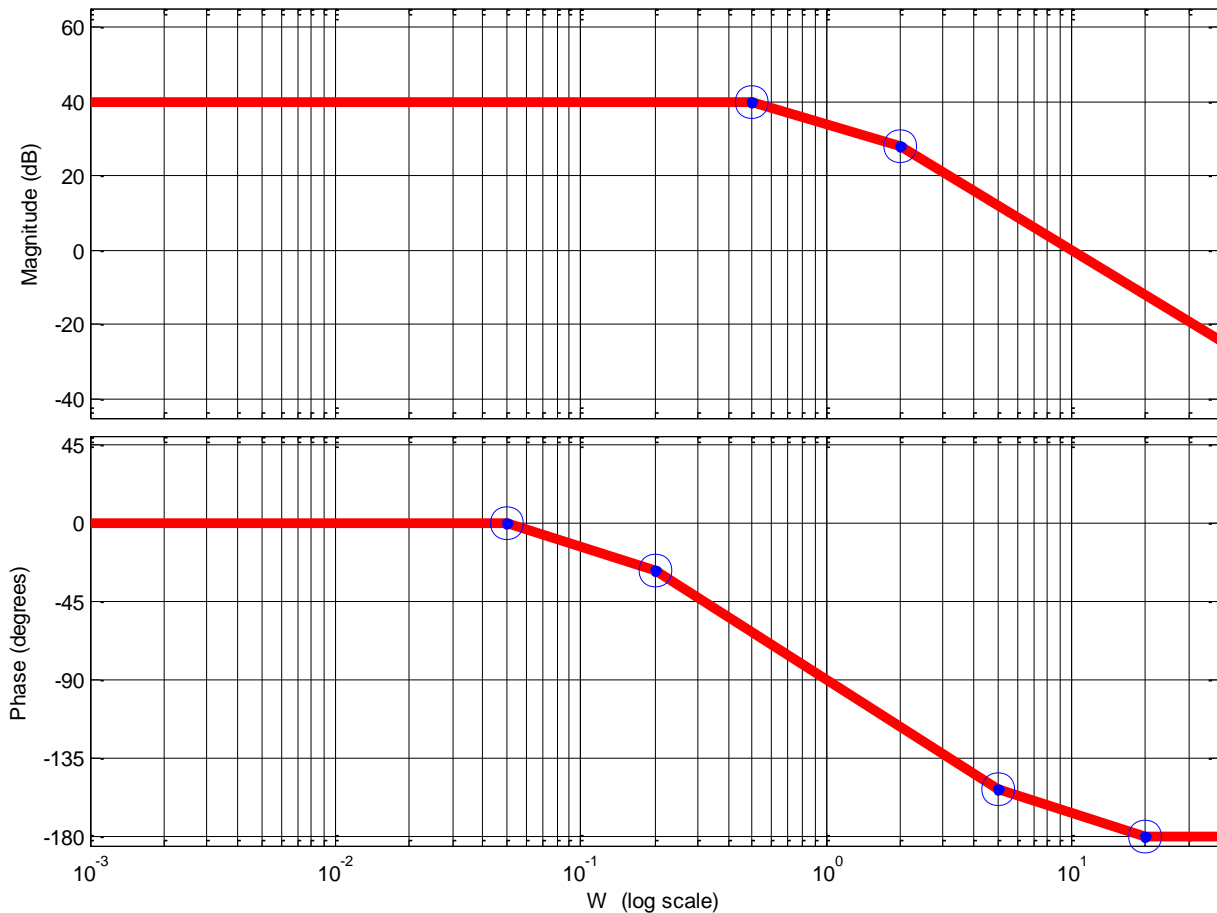
a)
$$H(s) = \frac{100}{(s/2 + 1) \cdot (2s + 1)}$$

Approximate Magnitude:

$\omega \ll 0.5$: $M(\omega) \sim 20 \cdot \log_{10}(100) = 40 \text{ dB}$

$\omega \in [0.5, 2]$: $dM(\omega)/d\omega = -20 \text{ dB/decade}$

$\omega > 2$: $dM(\omega)/d\omega = -40 \text{ dB/decade}$



Approximate Phase:

$\omega \ll 0.5/10$: $\varphi(\omega) \sim 0^\circ$

$\omega \in [0.5/10, 2/10]$: $d\varphi(\omega)/d\omega = -45^\circ/\text{decade}$

$\omega \in [2/10, 0.5 \cdot 10]$: $d\varphi(\omega)/d\omega = -90^\circ/\text{decade}$

$\omega \in [0.5 \cdot 10, 2 \cdot 10]$: $d\varphi(\omega)/d\omega = -45^\circ/\text{decade}$

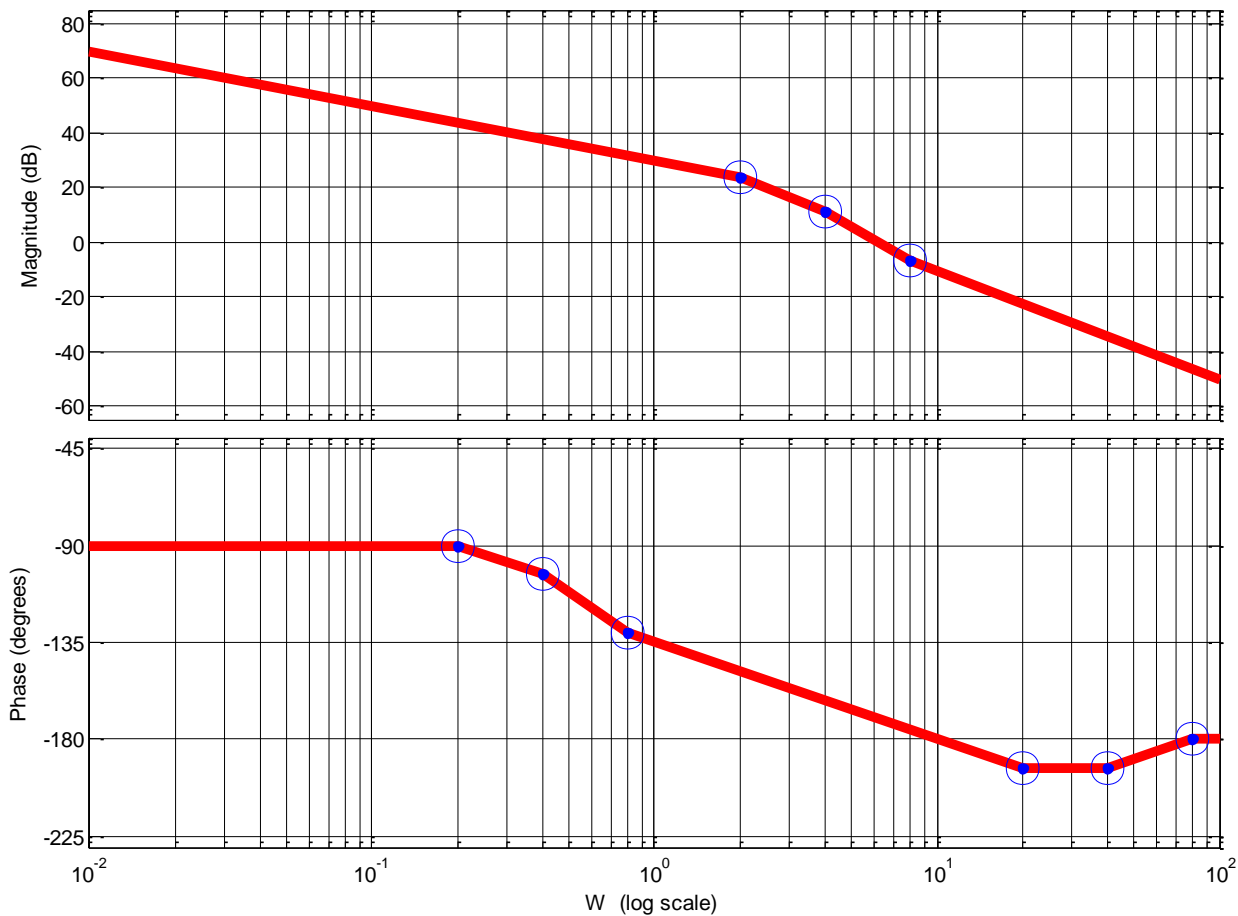
$\omega > 2 \cdot 10$: $d\varphi(\omega)/d\omega = 0^\circ/\text{decade}$, $\varphi(\omega) \sim -180^\circ$

* Gain Margin = INFINITE (The Phase is > -180 for all values of W)

* Phase Margin: Approximately 15 degrees ($M(W) \sim 0 \text{ dB}$ at $W \sim 10 \text{ rad/sec}$, Phase at that W is ~ -165 degrees)

b)

$$H(s) = \frac{30 \cdot (s + 8)}{s \cdot (s + 2) \cdot (s + 4)}$$



Approximate Phase:

$\omega \ll 2/10$: $\varphi(\omega) \sim -90^\circ$ (phase mainly due to pole at 0)

$\omega \in [2/10, 4/10]$: $d\varphi(\omega)/d\omega = -45^\circ / \text{decade}$ (rate mainly due to pole “-2”)

$\omega \in [4/10, 8/10]$: $d\varphi(\omega)/d\omega = -90^\circ / \text{decade}$ (.. to poles “-2,-4”)

$\omega \in [8/10, 2 \cdot 10]$: $d\varphi(\omega)/d\omega = -45^\circ / \text{decade}$ (.. to poles “-2,-4”, zero “-8”)

$\omega \in [2 \cdot 10, 4 \cdot 10]$: $d\varphi(\omega)/d\omega = 0^\circ / \text{decade}$ (.. to pole “-4”, zero “-8”)

$\omega \in [4 \cdot 10, 8 \cdot 10]$: $d\varphi(\omega)/d\omega = +45^\circ / \text{decade}$ (zero “-8”)

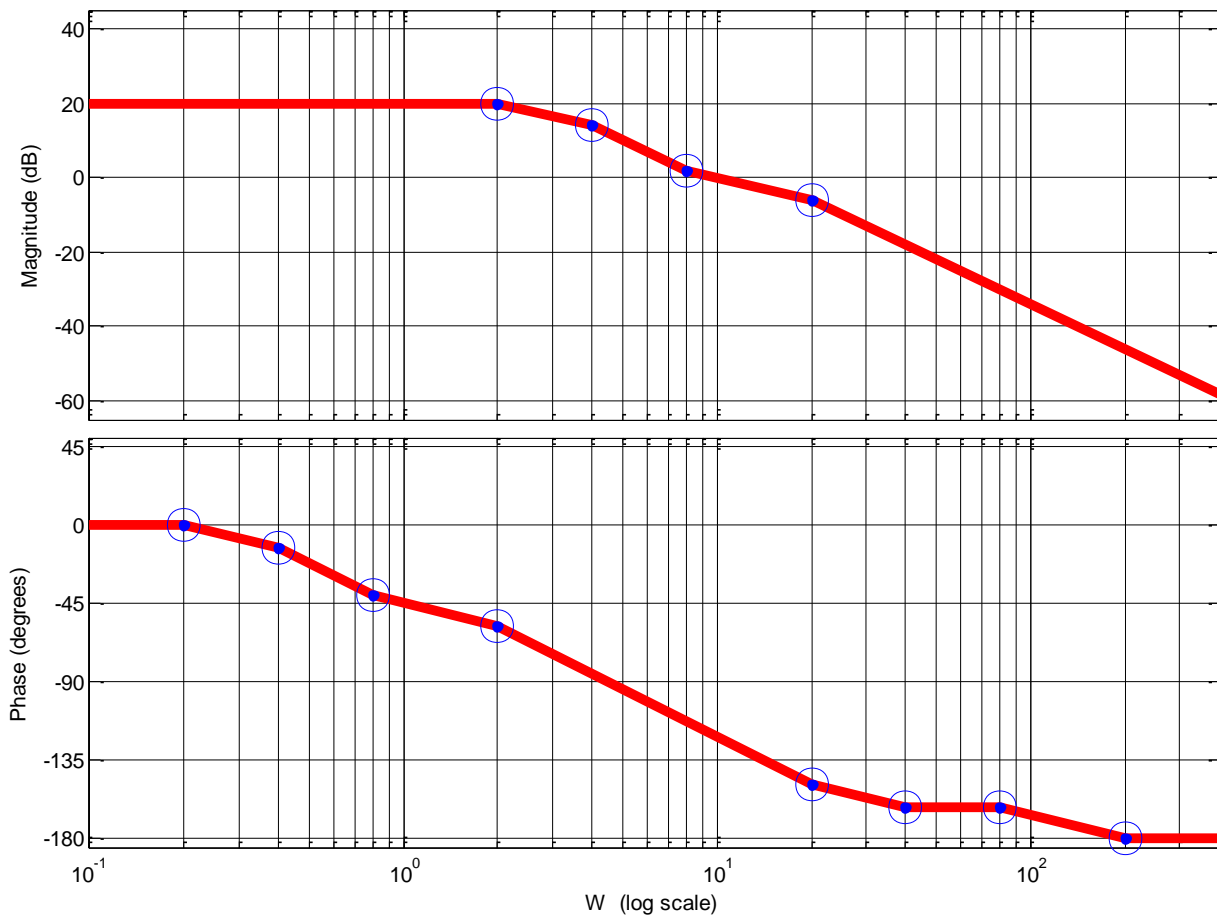
$\omega > 8 \cdot 10$: $d\varphi(\omega)/d\omega = 0^\circ / \text{decade}$, $\varphi(\omega) \sim -180^\circ$

* Gain Margin $\sim 10\text{dB}$ (Phase crosses -180 degrees at $W \sim 10$ rad/sec, and $M(W) \sim -10\text{dB}$).

Phase Margin: Approximately 10 degrees. $M(W) \sim 0\text{dB}$ at $W \sim 6$ rad/sec, where Phase is ~ -170 degrees ($-170 = -180 + 10$)

c)

$$H(s) = \frac{200 \cdot (s + 8)}{(s + 2) \cdot (s + 4) \cdot (s + 20)}$$



Approximate Phase:

$$\omega \ll 2/10 : \varphi(\omega) \sim 0^\circ$$

$$\omega \in [2/10, 4/10] : d\varphi(\omega)/d\omega = -45^\circ / \text{decade}$$

$$\omega \in [4/10, 8/10] : d\varphi(\omega)/d\omega = -90^\circ / \text{decade}$$

$$\omega \in [8/10, 20/10] : d\varphi(\omega)/d\omega = -45^\circ / \text{decade}$$

$$\omega \in [20/10, 2 \cdot 10] : d\varphi(\omega)/d\omega = -90^\circ / \text{decade}$$

$$\omega \in [2 \cdot 10, 4 \cdot 10] : d\varphi(\omega)/d\omega = -45^\circ / \text{decade}$$

$$\omega \in [4 \cdot 10, 8 \cdot 10] : d\varphi(\omega)/d\omega = +0^\circ / \text{decade}$$

$$\omega \in [8 \cdot 10, 20 \cdot 10] : d\varphi(\omega)/d\omega = -45^\circ / \text{decade}$$

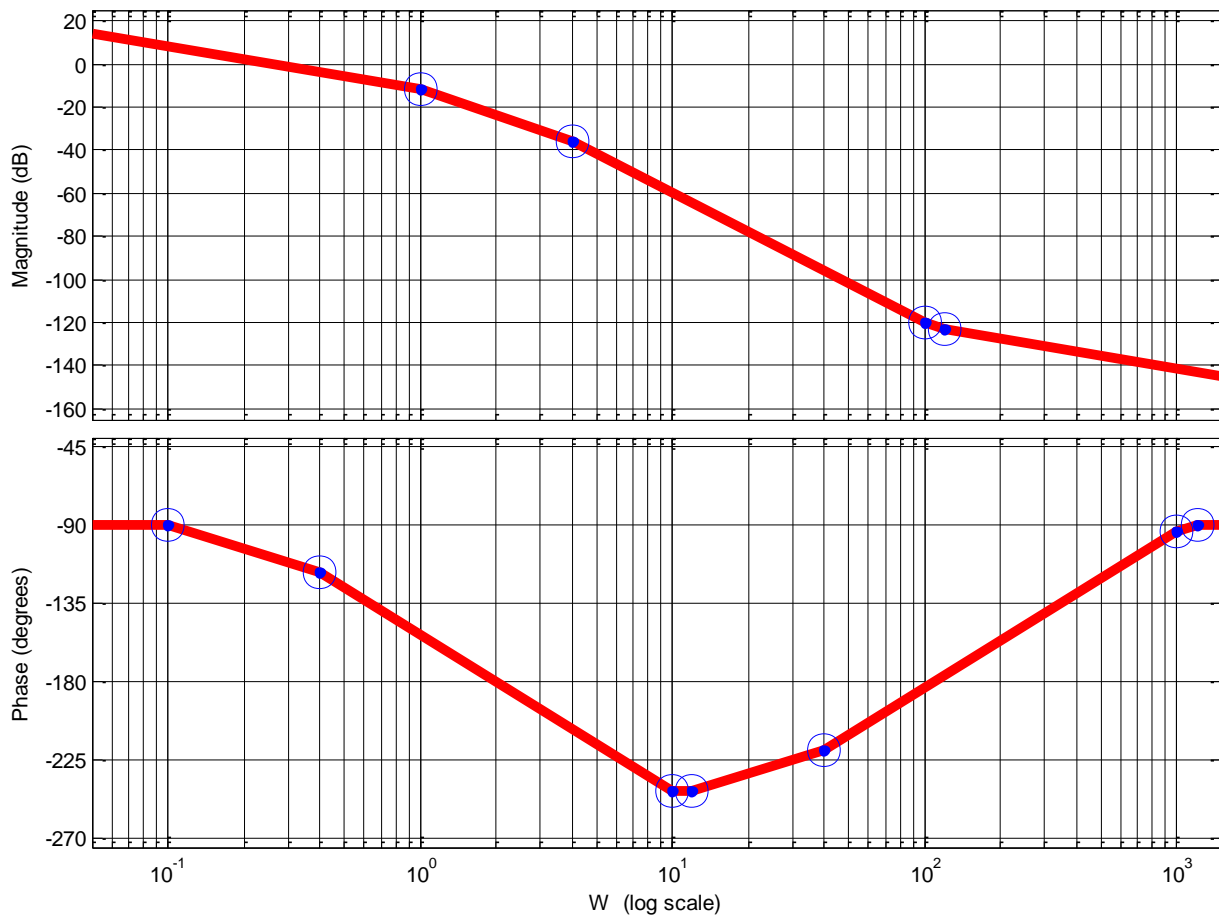
$$\omega > 20 \cdot 10 : d\varphi(\omega)/d\omega = 0^\circ / \text{decade}, \varphi(\omega) \sim -180^\circ$$

* Gain Margin: INFINITE (there is no W where Phase touches -180 degrees)

* Phase Margin: Approximately 60 degrees. $M(W) \sim 0\text{dB}$ at $W \sim 10$ rad/sec, where Phase is ~ -120 degrees ($-120 = -180 + 60$)

d)

$$H(s) = \frac{(s/100 + 1) \cdot (s/120 + 1)}{s \cdot (s + 4) \cdot (s + 1)}$$



Approximate Phase:

$$\omega \ll 0.1 : \varphi(\omega) \sim -90^\circ$$

$$\omega \in [0.1, 0.4] : d\varphi(\omega)/d\omega = -45^\circ / \text{decade}$$

$$\omega \in [0.4, 10] : d\varphi(\omega)/d\omega = -90^\circ / \text{decade}$$

$$\omega \in [10, 12] : d\varphi(\omega)/d\omega = 0^\circ / \text{decade}$$

$$\omega \in [12, 40] : d\varphi(\omega)/d\omega = +45^\circ / \text{decade}$$

$$\omega \in [40, 1000] : d\varphi(\omega)/d\omega = +90^\circ / \text{decade}$$

$$\omega \in [1000, 1200] : d\varphi(\omega)/d\omega = +45^\circ / \text{decade}$$

$$\omega > 1200 : d\varphi(\omega)/d\omega = 0^\circ / \text{decade}, \varphi(\omega) \sim -90^\circ$$

* Gain Margin $\sim 20\text{dB}$ (Phase touches -180 degrees at $W \sim 2$ rad/sec, $\text{Magnitude}(W) \sim -20\text{dB}$.

* Phase Margin: Approximately 75 degrees. $M(W) \sim 0\text{dB}$ at $W \sim 0.2$ rad/sec, where Phase is ~ -105 degrees.

Questions: ask the lecturer; via Moodle Forum or via email.