Tutorial - State Space Modeling

1 Question

Qestion 1.1. Consider the system described by

$$\dddot{y} + 8\ddot{y} + 11\dot{y} + 6y = 6u \tag{1}$$

obtain a state space representation.

Answer

The state equation is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} u \tag{2}$$

The output equation is

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{3}$$

Qestion 1.2. For the RLC network, Fig. 1, write down the state equation with state variables (a) $v_2(t),\ \dot{v}_2(t)$, (b) $v_2(t),\ i(t)$.

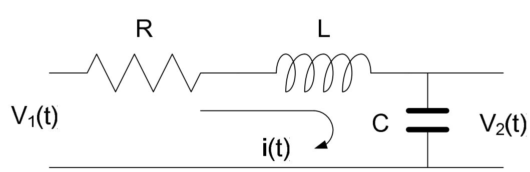

Figure 1: RLC network.

Answer

(a) The state equation is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-1}{LC} & \frac{-R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix} u \tag{4}$$

(b) The state equation is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ \frac{-1}{L} & \frac{-R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u \tag{5}$$

Qestion 1.3. For the dual spring-mass-damper system, Fig. 2, find the state and output equations when the state variables are the position and velocity of each mass.

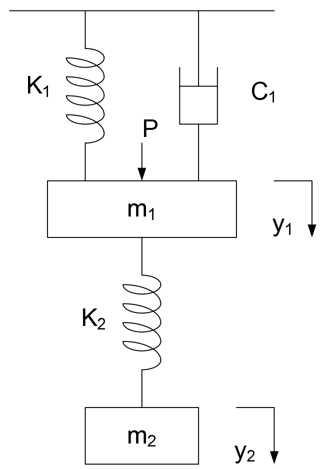

Figure 2: Dual spring-mass-damper system.

Answer

The state equation is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-(K_1+K_2)}{m_1} & \frac{-C}{m_1} & \frac{K_2}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_2}{m_2} & 0 & \frac{-K_2}{m_2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} u \tag{6}$$

The output equation is

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{7}$$