MTRN4010

Integrating gyroscope in 2D
Noise in Gyroscope's measurements

When operating in 3D, gyroscopes must be jointly integrated, for estimating the 3D attitude.

In the cases in which the platform does operate in **2D** we can simplify the problem and integrate heading (yaw) rate separately.

If we have interest in estimating the heading, we can do it as follows:

$$\varphi(t) = \varphi(t_0) + \int_{\tau=t_0}^{t} \omega(\tau)\cdot d\tau$$

In which the variable ω is the heading rate, which is provided by a gyroscope properly aligned with the vertical axis of the vehicle.

However, the gyroscope measurements are usually polluted by noise. Gyroscopes' measurements are polluted by randomly fluctuating noise plus by certain offset (bias).
The bias is an unknown value, which is slowly time variant, and that for short period of time can even be considered constant during certain horizon of time.
When the value of that bias is known, that known value can be used to compensate the bias by simply "removing" it from the measurements, as follows

$$\omega_{improved}(t) = \omega_{measured}(t) - B$$

In which $\omega_{measured}(t)$is the measurement from the sensor, B the assumed or known bias, resulting in an approximately "bias free" improved measurement.
Consequently, for estimating the heading we would perform the integration, using the improved version of the heading rate

$$\varphi(t) = \varphi(t_0) + \int_{\tau=t_0}^{t} \omega_{real}(\tau)\cdot d\tau \approx \varphi(t_0) + \int_{\tau=t_0}^{t} \omega_{improved}(\tau)\cdot d\tau = \varphi(t_0) + \int_{\tau=t_0}^{t} \left(\omega_{measured}(\tau) - B\right)\cdot d\tau$$

Estimating the bias
There are various approaches for estimating the bias. A basic one can be applied if the platform stays static for certain horizon of time. We know that during the period of time in which the sensor is kept still, the angular rates must be equal to zero (except the contribution due to the rotation of the planet, but that is too small for a common gyroscope) and so the measurements of those. However, the measurements will be affected by the bias and the fluctuating noise. For estimating the bias, we simply need to average the readings of each of the gyroscopes. We focus here on the one associated to the heading, but we express it in general,

$$B = \frac{1}{T}\int_{\tau=t_1}^{t_1+T} \omega_{measured}(\tau)\cdot d\tau$$

That is because we assume the fluctuating component of the noise is white noise, so it will have zero mean, consequently the calculated B is just due to the bias, so that the calculated average B is the value of the bias!
We will see later, that yes, the fluctuating component of the noise is white noise, in practical terms.

In practical applications, the averaging period T is of few seconds.

Integrating the gyroscopes.

We are not able to sample the angular rate at infinite rate but at a "fast enough" rate. For a mechanical system, like a car, a sampling rate of 200Hz, is adequate, for implementing the time integration in a numerical way.
Given certain rate $\alpha(t) = {dA(t)}/{dt}$, then its associated integral relation is $A(t) = A(t_0) + \int_{\tau=t_0}^{t} \alpha(\tau) \cdot d\tau$. The continuous process can be approximated by the following discrete-time process:

$$A(t_n) = A(t_0) + \sum_{i=1}^{n} \alpha(t_{i-1}) \cdot (t_i - t_{i-1}).$$

This approximation is valid in cases in which $(t_i - t_{i-1})$ are small enough. The integration process can also be expressed in a recursive fashion, as follows,

$$A(t_k) = A(t_{k-1}) + \alpha(t_{k-1}) \cdot (t_k - t_{k-1}).$$

It is also worth noting that the small steps $(t_i - t_{i-1})$ do not need to be constant.

This recursive process can be easily implemented in a computer program, and it can be exploited for integrating the heading rate in our problem. The kinematic model of the platform, which we use in the project, applies that procedure for the prediction of the heading.

The noise which pollutes the gyroscope's measurements is also composed by a fluctuating perturbation, which is assumed to behave as Gaussian white noise (GWN). For verifying it, the lecturer took measurements during a period of time, in which the IMU was completely static. The readings were statistically analyzed, after the bias had been removed. A tool for verifying "whiteness" is the cross-correlation, which indicated there was almost no statistical correlation for shifts in time ~=0 (as it can be seen in Figure 1).
In addition, through applying a histogram, it was verified that the shape was almost Gaussian (as shown in Figure 2). Based on this, we can later use the measurements assuming that the noise present in those is GWN (Gaussian White noise)
Based on these findings we can use the gyroscope in our projects, soon in a deterministic way, after removing the bias by using that basic approach, and later applying an approach which requires noises and uncertainties to be WGN

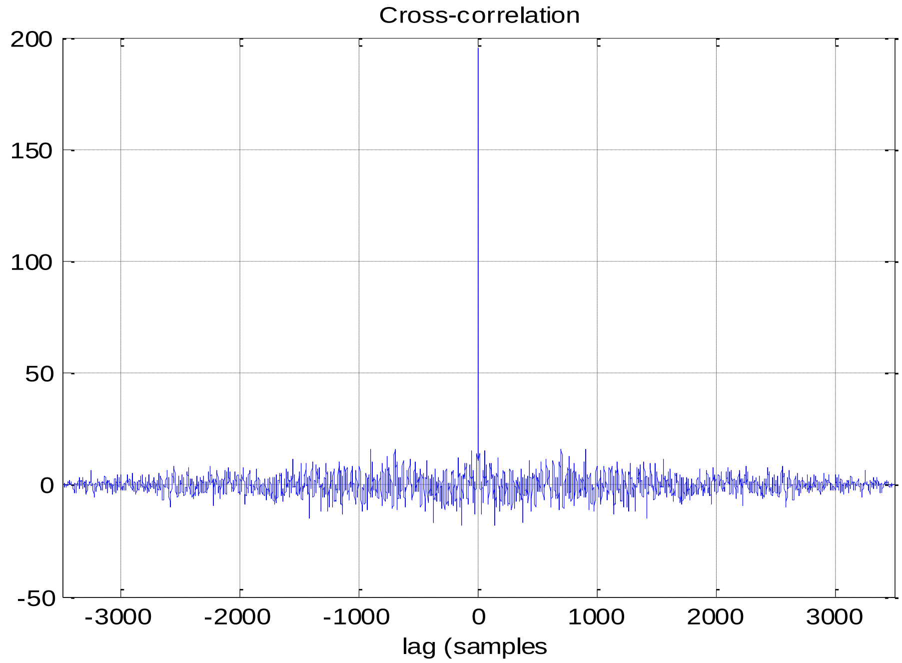

Figure 1: Cross-correlation of the first 3500 samples. It does not indicate statistical dependency between time shifted samples.

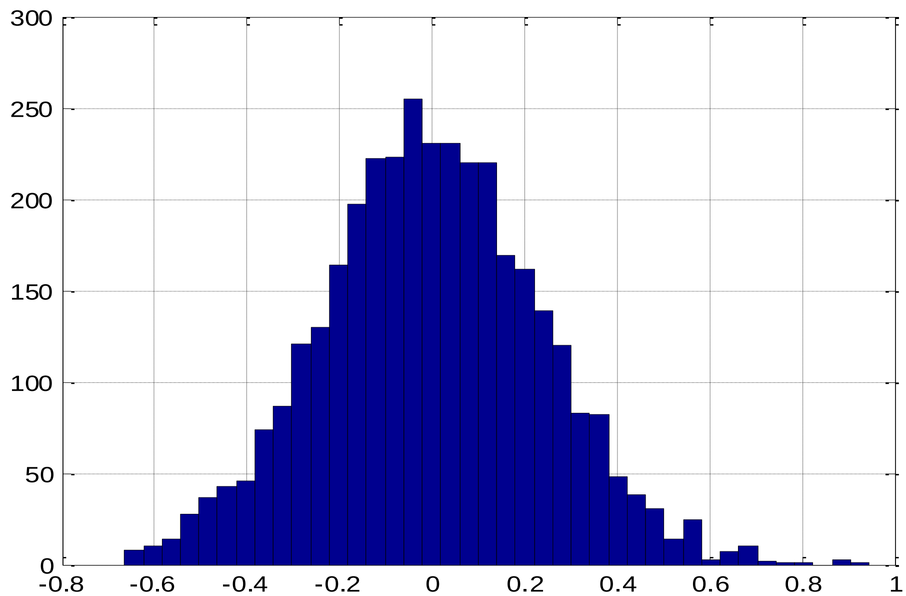

Figure 2: Histogram of the first 3500 samples. It can be seen it has a strong unimodal shape, gaussian-like. Having a standard deviation ~0.25 (expressed in degrees/second)

If you have questions, ask the lecturer via Moodle's Forum, or by email (Email: j.guivant@unsw.edu.au)