**MTRN4010 / ADVANCED AUTONOMOUS SYSTEMS T1.2022**

**Week 7 tutorial problems. EKF update and predictions steps.**

**Aim of this tutorial / Hands on tutorial:**

This set of problems aims to get experience in implementing KF and EKF prediction and update steps.

Some of these questions involve direct calculations; some others require implementing small programs.

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**Question 1.**

Consider the case in which we are estimating the state vector, $\mathbf{x}$, of a plant.

At certain time, we have estimates, expressed as expected value and covariance matrix, whose values are

$$\hat{\mathbf{x}} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \ \mathbf{P} = \begin{bmatrix} 0.5 & 0.2 \\ 0.2 & 1 \end{bmatrix}$$

At that time we are able to measure certain output variable of the plant, $\mathbf{w}$, whose functional relationship with the state of the system is $\mathbf{w} = 2 \cdot x_1 + \left(1 + 0.1 \cdot x_2\right)^3$

The measured value of $\mathbf{w}$, at that time, is $\mathbf{w}_{measured} = 5.66$. We know that the measurements generated by the sensor (which is used to measure that system output) are polluted by noise, which behaves as WGN (White Gaussian Noise) and whose standard deviation is 0.1

What would be the posterior estimates of $\mathbf{x}$ if we processed that observation?

Hint: apply an EKF update. Note: we assume that that measurement has not been previously used (in the estimation process which generated the current estimates), so that we are perfectly allowed to exploit the observation.

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**Question 2.**

Consider the case described in question 1, but in which we can measure two variables which are outputs of the system. One is $\mathbf{w}$ ,as in Question 1, and another is $\mathbf{y}$, whose output equation is $\mathbf{y} = x_1 - 0.1 \cdot x_2$

Measurements of both outputs are taken simultaneously, being the measurement of $\mathbf{w}$ as mentioned in question 1, and that of $\mathbf{y}$, $\mathbf{y}_{measured} = 1.9$. We know that measurements of variable $\mathbf{y}$ are polluted by WGN, whose variance is 0.3.

Would we be able to process both observations?

If your answer is YES, implement the necessary calculations for that purpose.

Hint: Based on the description of the problem, the provided measurements of the output variables are independent sources of information, so we can apply a sequence of two Bayes updates (via EKF updates), for processing both measurements.

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**Question 3.**

Suppose we want to implement predictions of a vehicle's pose, using the kinematic model used in Project 1.

We know that the model is a good representation of the plant, but we know that the measurements of the angular rate and speed, which we use as model inputs, are polluted by noises, which behave as WGN. The noise which affects speed measurements has standard deviation 0.1m/second; the standard deviation of the noise which affects angular rate measurements has standard deviation 2 degrees/second.

Assuming that there are no other relevant uncertainties affecting the kinematic model, propose the calculations needed for performing predictions steps based on that model.

Implement a function for that purpose (you may use it, later, in Project 2)

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**Question 4.**

Consider an estimation process for localizing a platform, which has sensor capabilities like those used in Project1.

We want to exploit information provided by its on-board LiDAR. Propose the necessary calculations for performing EKF updates, for processing range measurements associated to valid OOIs (i.e. those which we inferred are associated to known landmarks).

Assume that those range measurements are polluted by errors which behave as WGN, having standard deviation 10cm.

Assume that you are provided with the following data:

A range measurement

The position, in the GCF, of the landmark associated to that range measurement.

The expected value and covariance matrix of the current pose estimates (the "prior").

Assume that the LiDAR is installed at position (Lx,Ly)=(0,0) on the vehicle's coordinate frame.

Your function must return the posterior pose estimates (i.e. after fusing the provided information).

Note: the answer to this question may be useful for solving part of Project 2.

(End to tutorial questions)