MTRN4010 / ADVANCED AUTONOMOUS SYSTEMS T1.2022

Week 8 tutorial problems. Estimating parameters, via EKF.

Aim of this tutorial / Hands on tutorial:

This set of problems aims to get experience in exploiting the EKF approach for estimating the system's state and certain system parameters which we may not know accurately enough, for being safely used in our process and output models

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Question 1.

Consider that case in which you are estimating the pose of a platform, based on the localization approach we use in Project 2. However, the platform which we use has a kinematic model based on steering angle as one of the model inputs (i.e., the Ackermann model discussed in Lectures 1 and 2) (no gyroscope measurements are used).

We have an extra complication: we do not accurately know the parameter **L**, whose value, we know, is close to L=1.2m, being its possible values in the range from 1.1m to 1.3m.

You are required to propose a way to implement the necessary EKF steps for simultaneously estimating the platform's pose and the parameter L.

Consider that we are sure that the parameter is an actual constant (i.e., it is not time varying).

In your proposal, you must describe the following components:

1.1) Definition of augmented state vector.
1.2) Initial expected value and covariance matrix (in this problem we will assume that the vehicle initial pose is perfectly known).
1.3) Q matrix to be used in the prediction steps.

Assume that the measurements of the steering angle are polluted with GWN, whose standard deviation is 1 degree. Speed measurements are assumed to be polluted by WGN, of standard deviation = 0.1m/s.

Present analytical expressions for any Jacobian matrix we may need in the EKF prediction step.

Propose analytical expressions for any Jacobian matrix we may need in the EKF update step Consider the case in which we process a couple of range and bearing observations, associated to a known landmark whose position, in GCF, is (20m,30m). Assume that the noise that pollutes the range measurements is GWN with standard deviation of 5cm, and that bearing measurements have noise of WGN type, of standard deviation =2 degrees. Based on that, propose a proper R matrix.

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Question 2.

Consider that case in which you are estimating the states of a pendulum, via EKF.

Assume the case in which the model parameters are **a=10**, **c=2** and **b** is not accurately known, being its range of valid values **[0.6 to 0.8]**.

$$\ddot{\varphi} = -a \cdot \sin(\varphi) - b \cdot \dot{\varphi} + c \cdot u(t)$$
$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \varphi \\ \dot{\varphi} \end{bmatrix}$$

In which the variables are expressed in the following engineering units:

Angular position: radians

Angular velocity: radians/second

Input: voltage, in volts.

All the parameters, (a, b, c), are expressed in compatible engineering units, so you do not need to care about scaling them.)

1) Obtain a discrete time model, considering a sample period of T=5ms.

2) You are required to propose how to implement the necessary EKF steps for simultaneously estimating the pendulum's state and parameter **b**.

Consider that we are sure that parameter is **b** well known to be constant (i.e., it is not time varying).

We have measurements of the input voltage, which we know is noisy, having WGN polluting noise, of standard deviation 0.05volts.

We have measurement of the angular position, φ; those measurements are provided by an optical encoder, which, according to maker specifications, introduces a quantization error of 360/4096 degrees, which can be assumed to be WGN when the pendulum is in operation

Like in question 1, propose the necessary components of the prediction step (**J**, **Ju**, **Q**, etc) and those for the update steps (**H**, **R**)

(End to tutorial questions)

Solution question 1

Some relevant components:

The process model we use is the following kinematic model:

$$\mathbf{x}(k+1)=\mathbf{f}(\mathbf{x}(k),\mathbf{u}(k))$$

$$\Downarrow$$

$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ \phi(k+1) \end{bmatrix} = \begin{bmatrix} x(k) \\ y(k) \\ \phi(k) \end{bmatrix} + T\cdot \begin{bmatrix} v(k)\cdot\cos(\phi(k)) \\ v(k)\cdot\sin(\phi(k)) \\ \frac{v(k)}{L}\cdot\tan(\beta(k)) \end{bmatrix}$$

$$\mathbf{x}(k)=\begin{bmatrix} x(k) \\ y(k) \\ \phi(k) \end{bmatrix},\quad \mathbf{u}(k)=\begin{bmatrix} v(k) \\ \beta(k) \end{bmatrix}$$

I prefer to use the inverse of L, a new parameter called **c=1/L**.

However, you may keep the model based on L, if you prefer it.

Now, we propose an augmented state vector; we may name it **z**. We also obtain a process model for z(k)

These are the relevant components

$$\mathbf{z}=\begin{bmatrix} x \\ y \\ \phi \\ c \end{bmatrix};\qquad \mathbf{z}(k+1)=\begin{bmatrix} x(k+1) \\ y(k+1) \\ \phi(k+1) \\ c(k+1) \end{bmatrix}=\begin{bmatrix} x(k)+\tau\cdot v(k)\cdot\cos(\phi(k)) \\ y(k)+\tau\cdot v(k)\cdot\sin(\phi(k)) \\ \phi(k)+\tau\cdot v(k)\cdot c(k)\cdot\tan(\beta(k)) \\ c(k)+0 \end{bmatrix}$$

$$\mathbf{f}(\mathbf{z},\mathbf{u})=\begin{bmatrix} x+\tau\cdot v\cdot\cos(\phi) \\ y+\tau\cdot v\cdot\sin(\phi) \\ \phi+\tau\cdot c\cdot v\cdot\tan(\beta) \\ c \end{bmatrix} \Rightarrow \mathbf{J}=\frac{\partial\mathbf{f}}{\partial\mathbf{z}}=\frac{\partial\mathbf{f}}{\partial[\mathrm{x},\mathrm{y},\phi,c]}=\begin{bmatrix} 1 & 0 & -\tau\cdot v\cdot\sin(\phi) & 0 \\ 0 & 1 & +\tau\cdot v\cdot\cos(\phi) & 0 \\ 0 & 0 & 1 & \tau\cdot v\cdot\tan(\beta) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We need Ju as well,

$$\mathbf{J}_{\mathbf{u}}=\left.\frac{\partial\mathbf{f}}{\partial\mathbf{u}}\right|_{\mathbf{x}=\hat{\mathbf{x}},\mathbf{u}=\breve{\mathbf{u}}}=\left.\begin{bmatrix} \tau\cdot\cos(\phi) & 0 \\ \tau\cdot\sin(\phi) & 0 \\ \tau\cdot c\cdot\tan(\beta) & \tau\cdot c\cdot v\cdot(\sec(\beta))^2 \\ 0 & 0 \end{bmatrix}\right|_{\phi=\hat{\phi},\beta=\beta_{measured}}$$

$$\mathbf{P}(0|0)=\begin{bmatrix} .. & .. & .. & 0 \\ .. & .. & .. & 0 \\ .. & .. & .. & 0 \\ 0 & 0 & 0 & p_{4,4}(0) \end{bmatrix}$$

$$var_{L(0)}=(0.15)^2$$

$$\begin{pmatrix} \text{if we have the expected value and variance of L } \Rightarrow \text{ we can also get,} \\ \text{approximately, the expected value and variance of } c=1/L \end{pmatrix}$$

$$var_{c(0)}=\left(-\frac{1}{\hat{L}(0)^2}\right)\cdot var_{L(0)}\cdot\left(-\frac{1}{\hat{L}^2(0)}\right)\cdot var_{L(0)}=\frac{1}{1.2^2}\cdot(0.15)^2\cdot\frac{1}{1.2^2}=..$$

$$p_{4,4}(0)=\frac{1}{1.2^2}\cdot(0.15)^2\cdot\frac{1}{1.2^2}$$

$$(\text{we may"exagetate a bit", to be consevative, specifying twice that value})$$

$$\hat{\mathbf{z}}(0|0)=\begin{bmatrix} .. \\ .. \\ .. \\ \frac{1}{1.2} \end{bmatrix}$$

What about $\mathbf{P_u}$?

$$\mathbf{P_u}=\begin{bmatrix} (0.1)^2 & 0 \\ 0 & \left(\frac{1}{180}\cdot\pi\right)^2 \end{bmatrix}$$

H matrix having column 4 filled with zeros (why?).

Because none of the components of the function ***h( )*** is actually function of ***c***

Rest of columns, as usual.

And **R** matrix? (same as usual), e.g.,

$$\mathbf{R} = \begin{bmatrix} (0.05)^2 & 0 \\ 0 & \left(\frac{2}{180} \cdot \pi\right)^2 \end{bmatrix}$$

So, we just need to put those items, accordingly, in our EKF prediction and update steps

(however, we do not implement it here. We simply required to propose these modifications

You will implement something similar in Project 2. Part B).

Solution for Question2:

Original ODE, continuous time process model, approx. discrete time process model, augmented system, etc.

$$\ddot{\varphi} = -10 \cdot \sin(\varphi) - b \cdot \dot{\varphi} + 2 \cdot u(t)$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \varphi \\ \dot{\varphi} \end{bmatrix}; \qquad \frac{d\mathbf{x}}{dt} = \begin{bmatrix} x_2 \\ -10 \cdot \sin(x_1) - b \cdot x_2 + 2 \cdot u(t) \end{bmatrix}$$

(continuous time model)

$$\Downarrow$$

discrete time version, for $\tau = 5ms = 0.005s$

$$\mathbf{x}(k+1) = \mathbf{f}\left(\mathbf{x}(k), \mathbf{u}(k)\right) = \begin{bmatrix} x_1 + \tau \cdot x_2 \\ x_2 + \tau \cdot \left(-10 \cdot \sin(x_1) - b \cdot x_2 + 2 \cdot u\right) \end{bmatrix}$$

augmented state vector

$$\mathbf{z} = \begin{bmatrix} x_1 \\ x_2 \\ b \end{bmatrix}$$

augmented process model

$$\mathbf{z}(k+1) = \mathbf{f}\left(\mathbf{z}(k), \mathbf{u}(k)\right) = \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ b(k+1) \end{bmatrix} = \begin{bmatrix} x_1(k) + \tau \cdot x_2(k) \\ x_2(k) + \tau \cdot \left(-10 \cdot \sin\left(x_1(k)\right) - b(k) \cdot x_2(k) + 2 \cdot u(k)\right) \\ b(k) \end{bmatrix}_{(k)}$$

Now P, covariance matrix, is 3x3. Q as well.

Necessary Jacobian matrixes:

$$\mathbf{J} = \left[\frac{\partial \mathbf{f}(\mathbf{x},\mathbf{u})}{\partial \mathbf{z}}\right] = \begin{bmatrix} 1 & \tau & 0 \\ -\tau \cdot 10 \cdot \cos(x_1) & 1 - b \cdot \tau & -x_2 \cdot \tau \\ 0 & 0 & 1 \end{bmatrix}$$

P(0|0) needs to be properly defined, particularly the parts related to our confidence about the initial guess of **b**.

$$\mathbf{P}(0|0) = \begin{bmatrix} .. & .. & .. \\ .. & .. & .. \\ .. & .. & p_{3,3}(0|0) \end{bmatrix}_{(3\times 3)}$$

$p_{3,3}(0|0) = 0.1^2$ (twice this value would also be adequate, to be conservative)

output equation and $\mathbf{H}$ matrix

We measure ϕ (i.e. x_1) $\Rightarrow \mathbf{h}(\mathbf{z}) = x_1$

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}_{(1\times 3)}$$

$$\mathbf{R} = \left(360/4096 \cdot \frac{\pi}{180}\right)^2$$

comment: scaled to radians, because I assume we are using those units for ϕ

$$\mathbf{J}_{\mathbf{u}} = \left[\frac{\partial \mathbf{f}(\mathbf{x},\mathbf{u})}{\partial \mathbf{u}}\right]_{(3\times 1)} = ? \text{ (you obtain it)}$$

$\mathbf{P}_{\mathbf{u}} = (0.05)^2$ (because noise polluting measurement of inputs is ~N(0, 0.05^2))