Wave speed, reflection and transmission and superposition

Waves/Oscillatory Motion

Textbook sections 16.2-16.5

Last Lecture

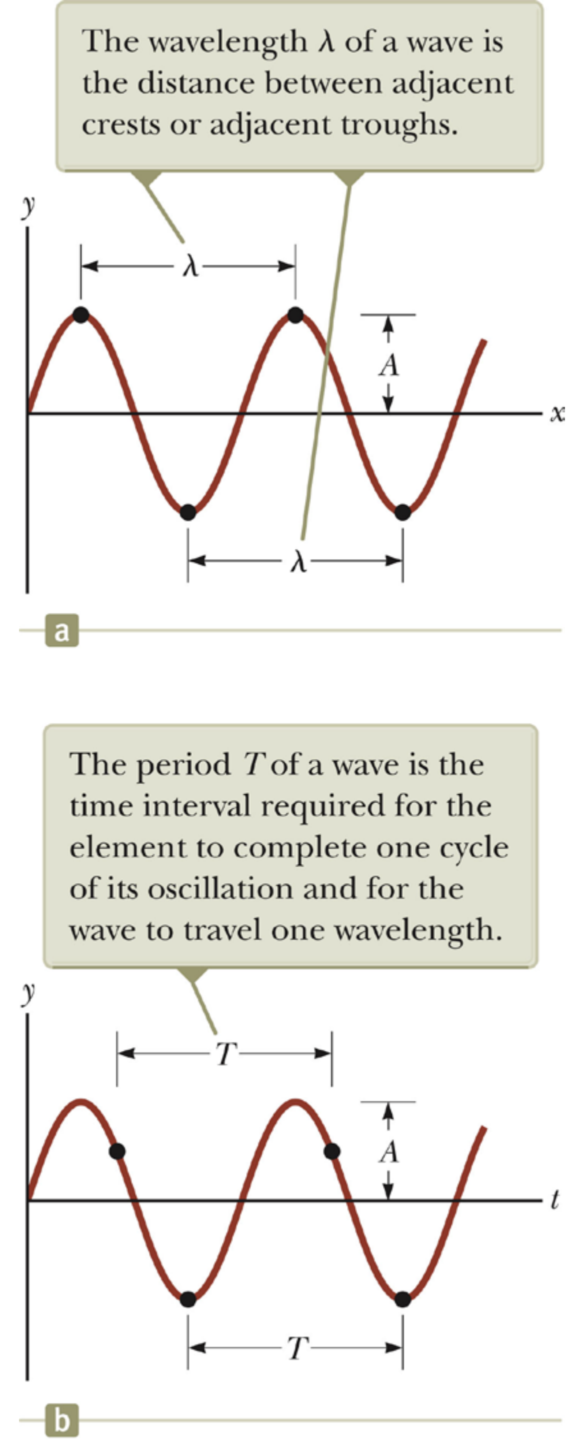
Transverse waves: The direction of the motion of the particles is perpendicular to the direction of motion of the wave.

Longitudinal waves: Particle oscillate in the same direction as the wave travels.

A pulse traveling to the right has a wave function: $y(x,t) = f(x - vt)$

Last Lecture

- Amplitude, A
- Wavelength, λ
- Period, T
- Frequency, $f = \dfrac{1}{T}$
- Velocity, $v = f\lambda$



Last Lecture

Wave equation:

$$y(x,t) = A\sin(kx - \omega t + \phi)$$

Where:

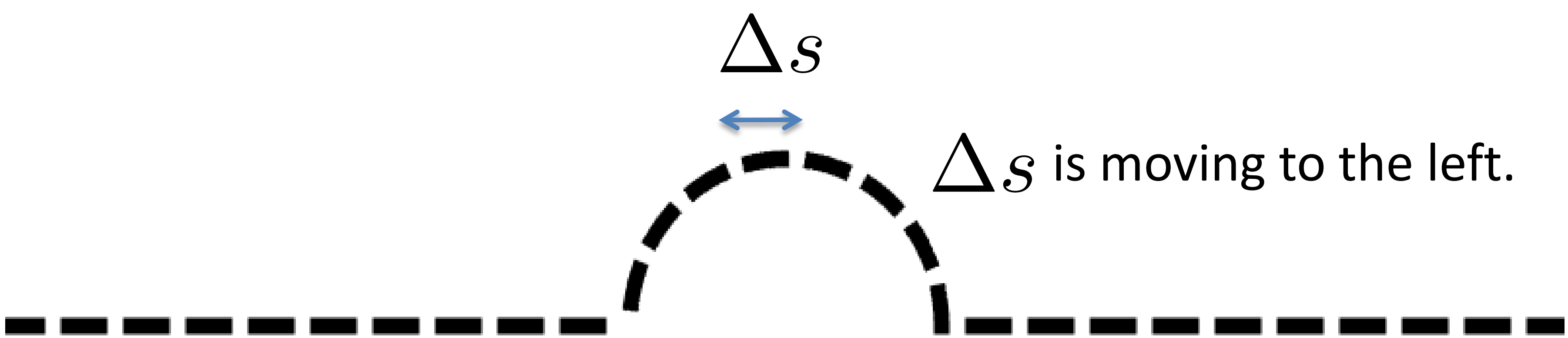
$$k = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi f$$

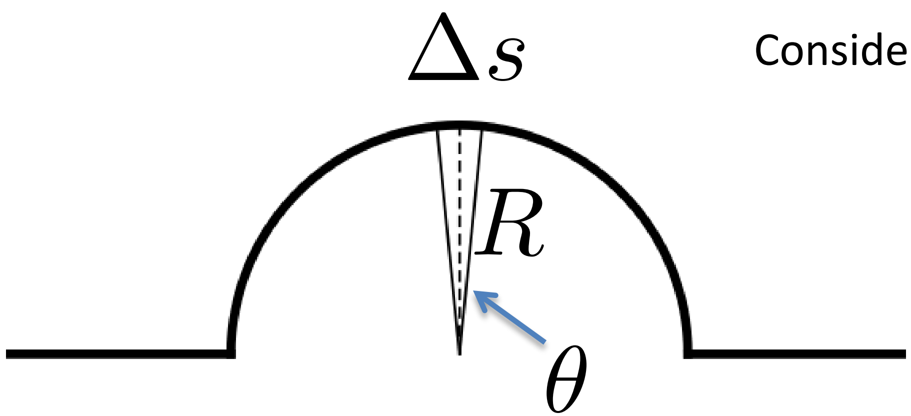
$$\phi = \text{“phase constant”}$$

The speed of waves on string

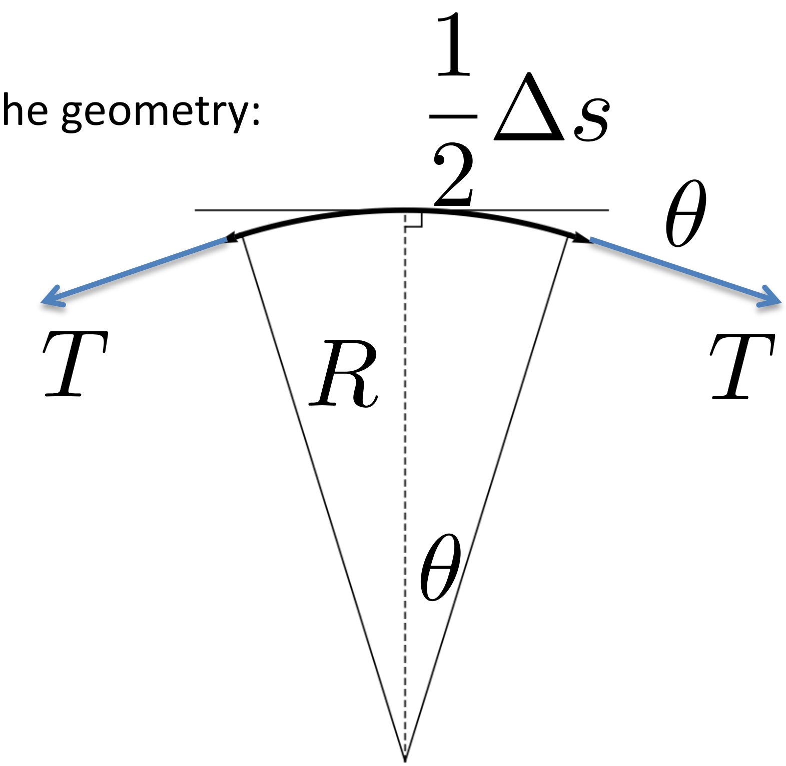
A semi-circular pulse is traveling to the right.
Now switch reference frames to one traveling with the pulse.



The speed of waves on string



Consider the geometry:



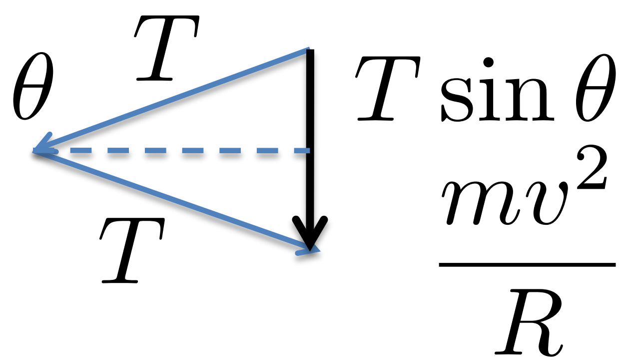
Arc of circle

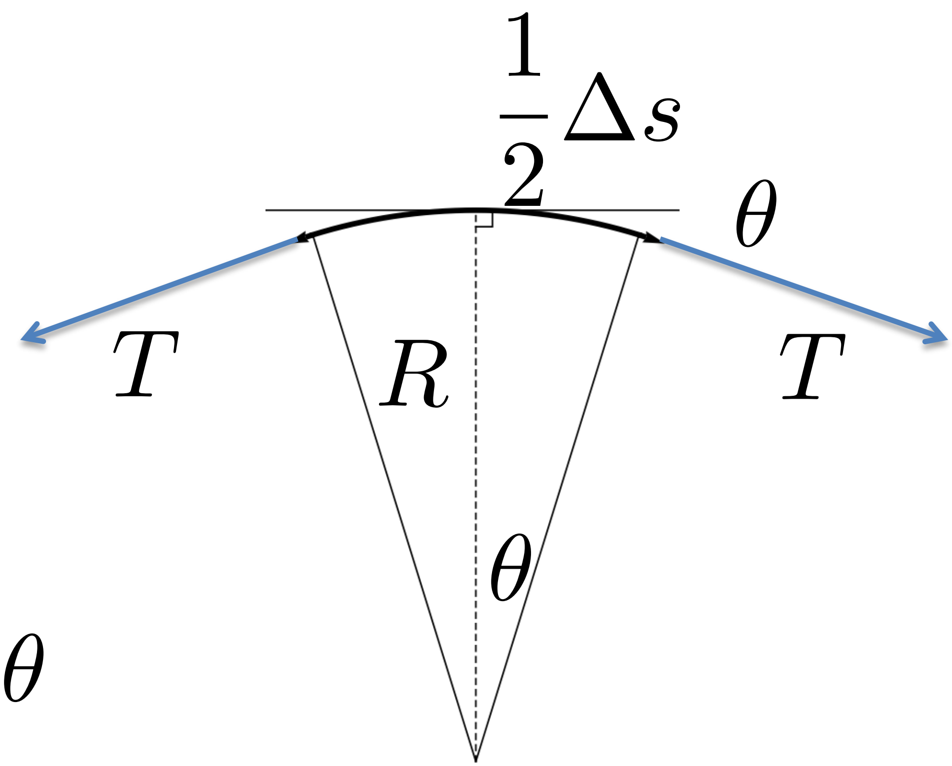
$$\Rightarrow \Delta s = 2R\theta$$

Now consider the forces acting on this system: Tension is the only force

Resultant force = ma_c = centripetal force

The speed of waves on string





$$\Rightarrow \frac{mv^2}{R} = 2T\sin\theta$$

$$\text{Now } \theta \text{ is small } \Rightarrow \quad \sin\theta \approx \theta$$

$$2T\theta = \frac{mv^2}{R}$$

The speed of waves on string

Need m: $m = \mu \Delta s$ but $\Delta s = 2R\theta$

$$= \mu 2R\theta$$

$$\Rightarrow 2T\theta = \frac{\mu 2R\theta v^2}{R}$$

$$\Rightarrow T = \mu v^2$$

$$\Rightarrow v = \sqrt{\frac{T}{\mu}}$$

Homework Set 5:
PHYS 1121:
13

PHYS 1131:
17

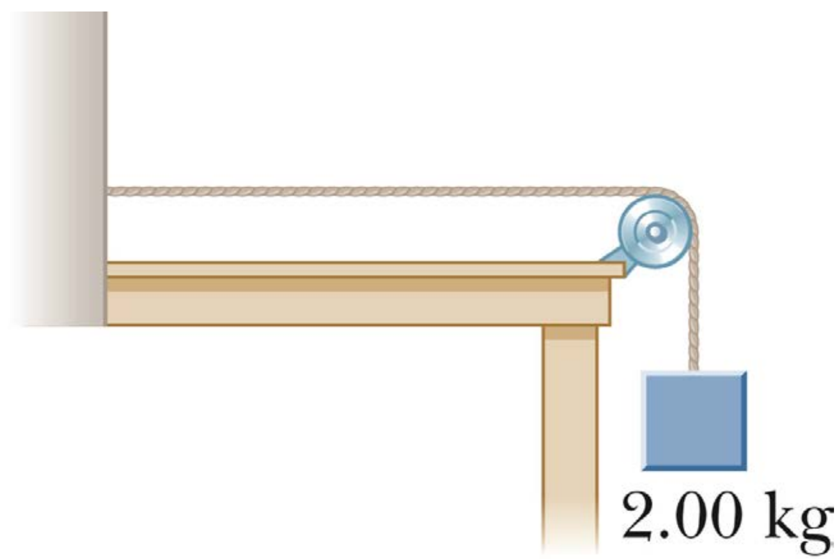
The speed of waves on string

- The pulse shape actually turns out to be unimportant, *any pulse can be approximated as an arc of a circle at the top*.
- We have assumed the tension, *T*, along the string is constant.
- This equation can be generalized to:

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

Question

A uniform string has a mass of 0.300 kg and a length of 6.00 m. The string passes over a pulley and supports a 2.00 kg object. Find the speed of a pulse traveling along the string.



Power transmitted by a wave traveling along a string

The power transmitted along a string can be derived from the KE equation. See the derivation in the "interactive lecture".

$$dK = \frac{1}{2} dm u^2$$

Using: $$y(x,t) = A\sin(kx - \omega t + \phi)$$

$$P = \frac{1}{2}\mu v \omega^2 A^2$$

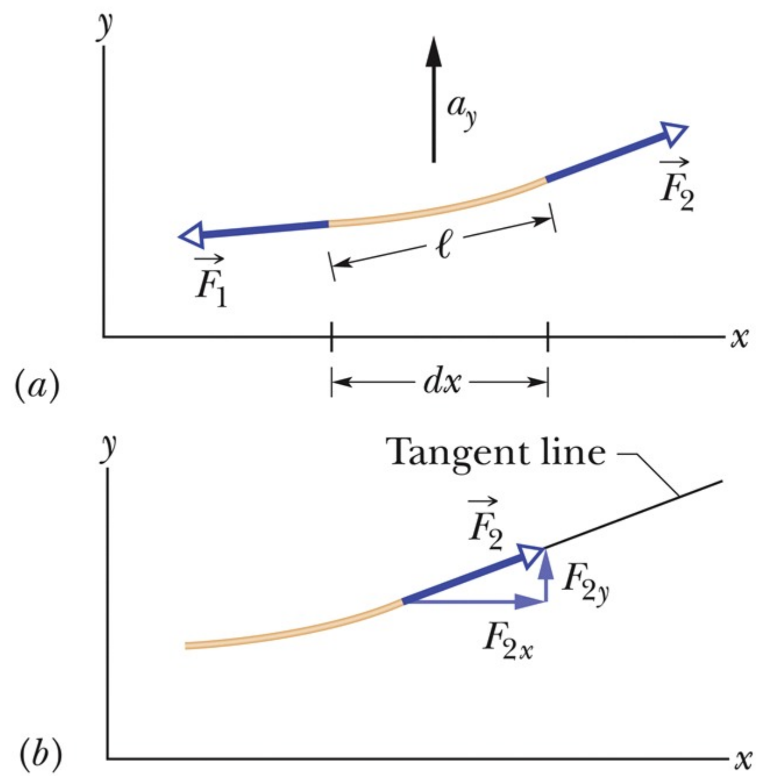
Wave equation

Start with Newton's second law.

We end up with:

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$$

The wave equation



Reflection of a Pulse

- When a pulse is reflected from a fixed end it is inverted.
- This can be explained by Newton’s third.

–When the pulse arrives it exerts an upward force on the wall.

–The wall exerts an equal and opposite force on the string.

–This downward force then generates an inverted pulse on the string.

- For a sine shaped wave we can say that its phase has changed by π.

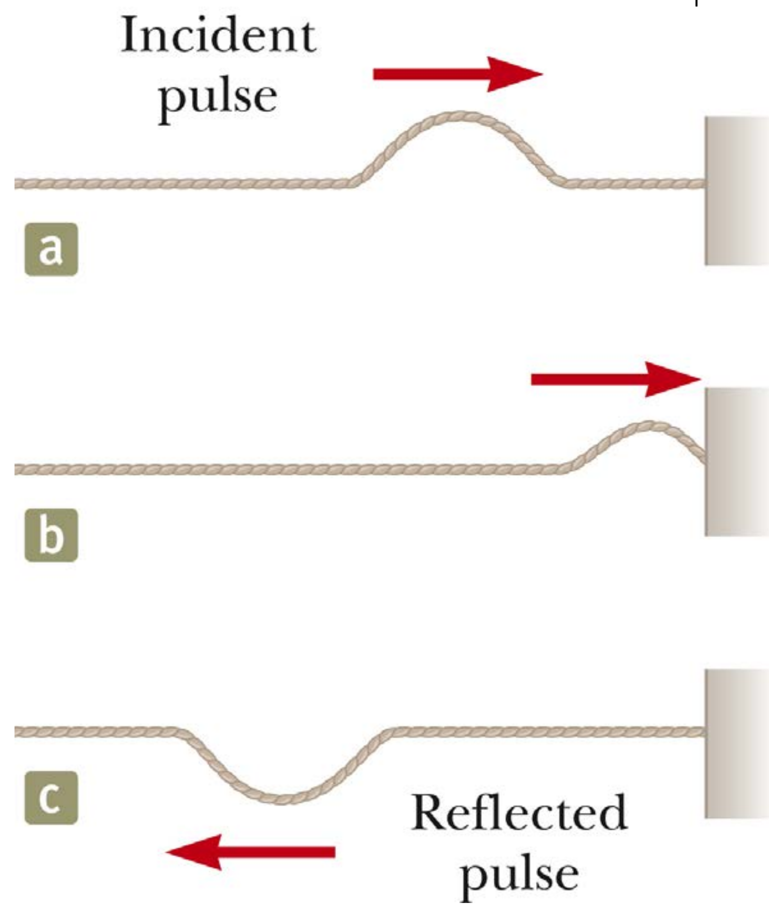


Fig. 18.1, p. 515

Reflection of a Pulse

- When a pulse is reflected from a free end it is **not inverted**.
  - During the reflection, the maximum displacement of the string is twice the amplitude of the propagating pulse.

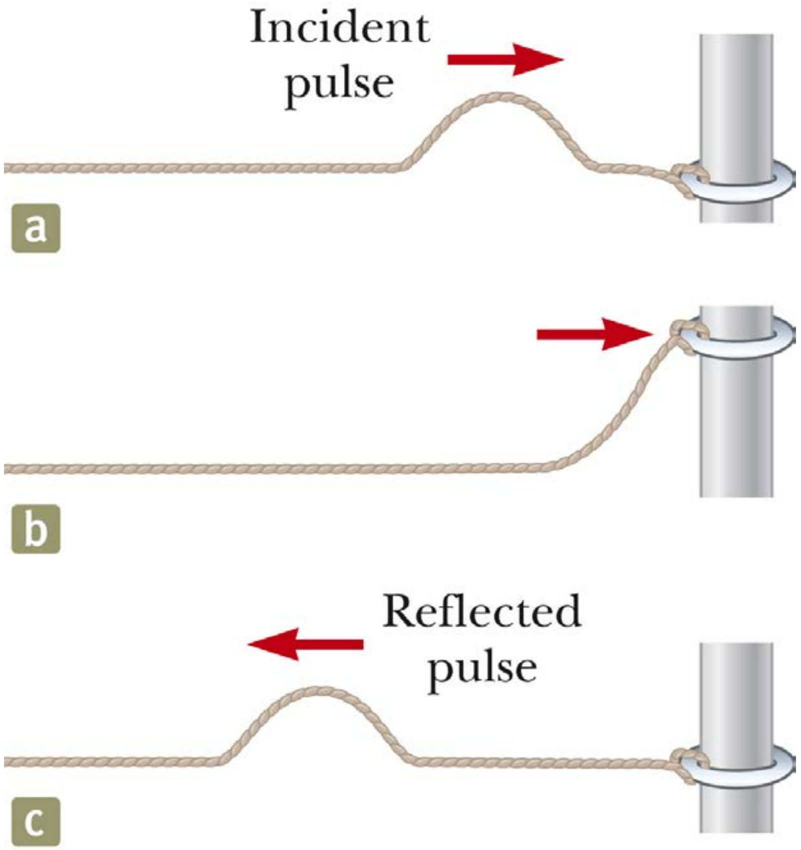
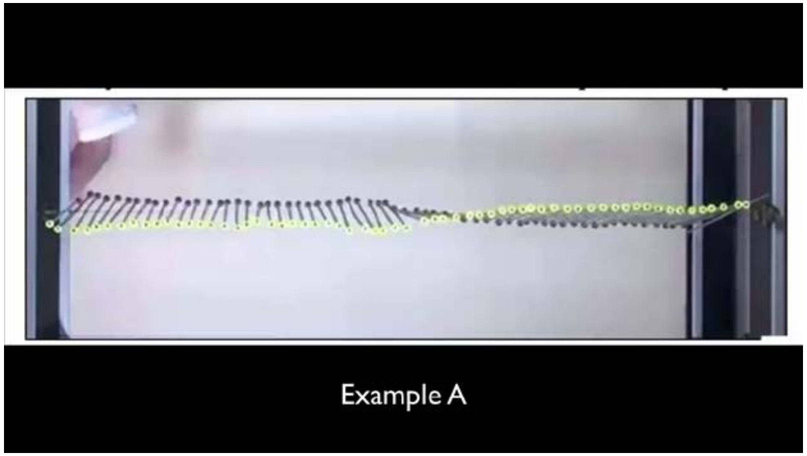
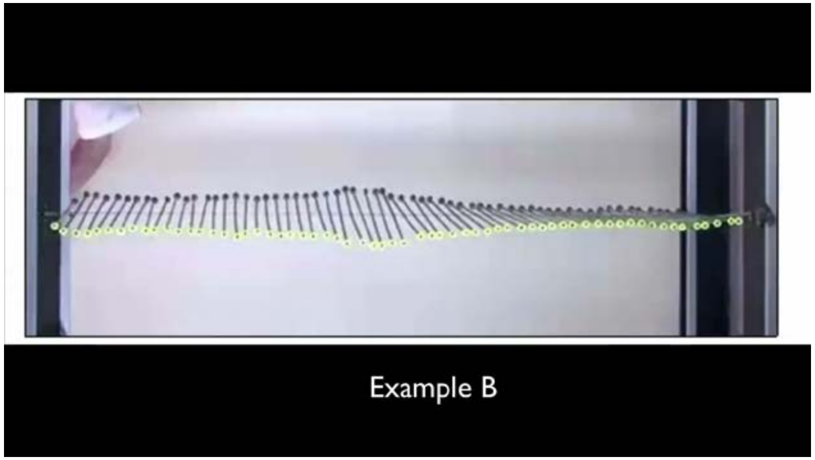


Fig. 18.2, p. 515

Identify these as fixed or loose ends

https://goo.gl/forms/UM18RaQmqL8CjZQh2

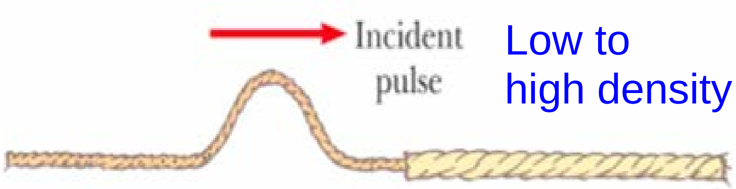




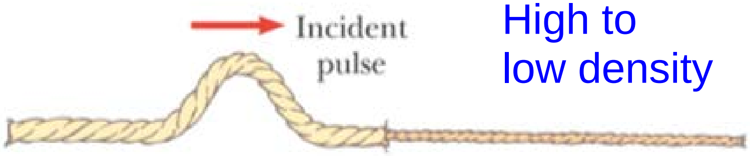
Transmission of a pulse…

- When the boundary is intermediate between the last two extremes (fixed vs free), then
  - Part of the incident pulse is reflected, and
  - Part of it is **transmitted** across the boundary.

- e.g., a pulse crossing from one medium to another.
  - What happens exactly to the pulse depends on the **direction** of the pulse.



Low to
high density



High to
low density

Transmission>Low to high density...

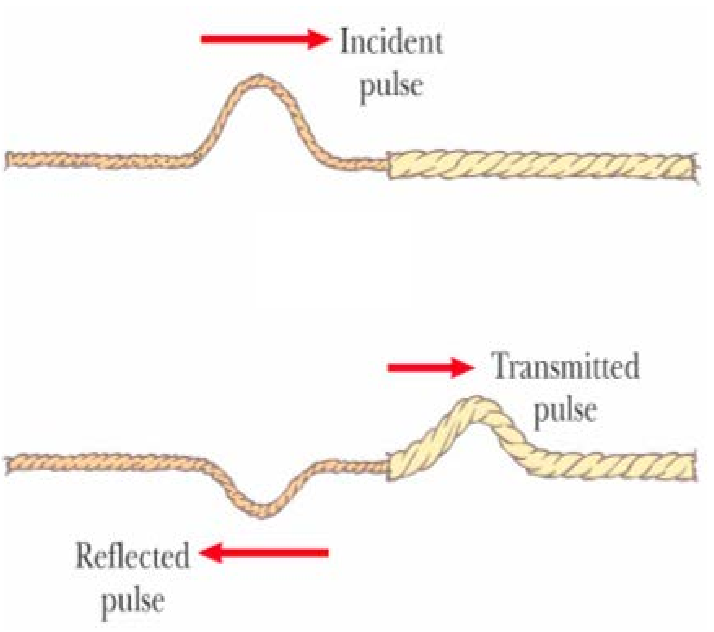
- The **reflected pulse** is inverted (the denser, higher inertia medium behaves like a fixed end).

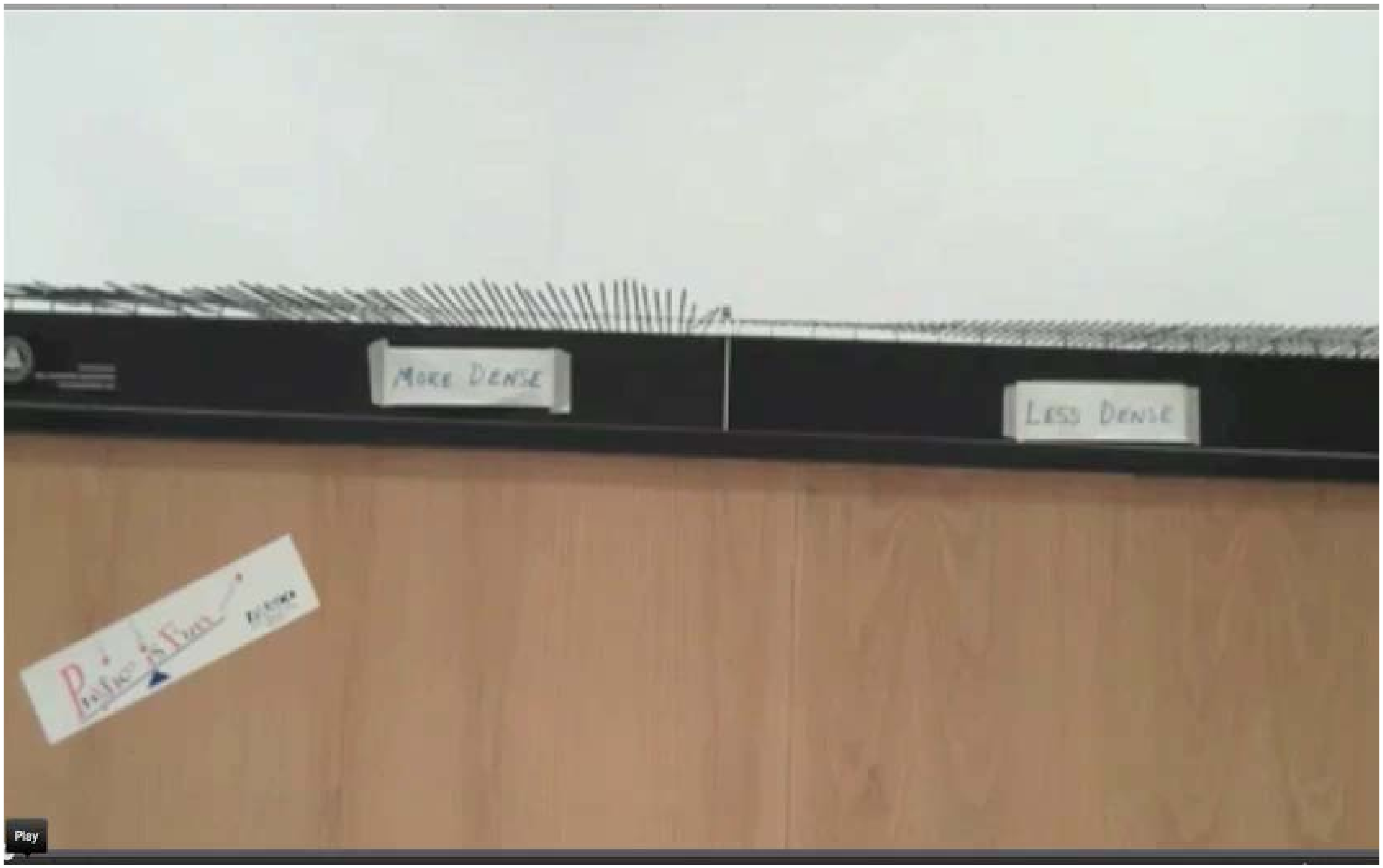
- The **transmitted pulse**
  - Propagates more slowly:

$$v = \sqrt{\frac{T}{\mu}}$$

Mass per unit length of string

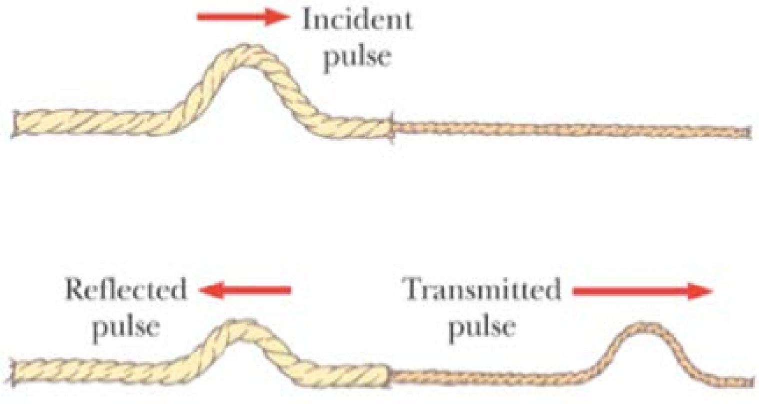
  - Has a shorter wavelength, but frequency is unchanged.




MORE DENSE
LESS DENSE
Play

Transmission>High to low density...

- The **reflected pulse** is **not** inverted (the less dense, lower inertia medium behaves like a free end).



- The **transmitted pulse**
  - Propagates at a higher speed, and
  - Has a longer wavelength, but the frequency is again unchanged.


MORE DENSE
LESS DENSE

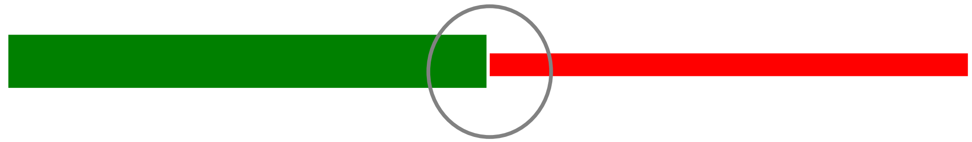
Transmission of a pulse>Summary…

- When a pulse travels from one medium to another:
  - The **reflected pulse** can be inverted or uninverted depending on the relative densities of the media.
  - The **transmitted pulse**
- Propagates at a new speed v.
- The frequency f remains unchanged. **WHY???**
  - The new wavelength is determined from $v = f\lambda$.

- These apply not only to waves on a string, but also to other media, e.g., gases of different densities.

Transmission of a pulse>Frequency...

- Consider a segment **right at the boundar**y between two strings of different densities.

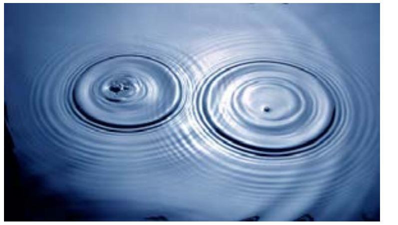


  - When a transverse sine wave passes through, the element oscillates vertically.

– But because the strings are connected, the vertical motion immediately to the left and immediately to the right of the boundary must be identical.

- → The oscillation and hence wave frequency remains the same across the boundary.

Superposition…



- Often there is more than one wave travelling through a medium at the same time.

- The waves pass through each other without being destroyed or altered.

Superposition>Interference…

- The **principle of superposition**: If two or more waves are moving through the same medium, the resultant value of the wavefunction is the algebraic sum of the wavefunctions of the original waves.

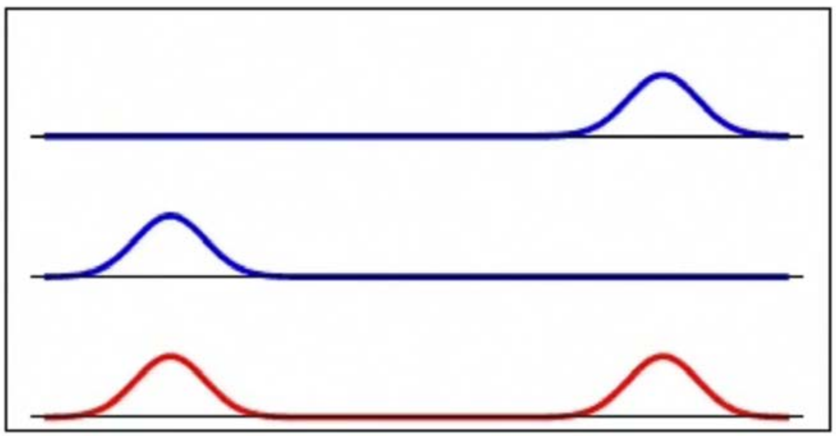
- An **immediate consequence**: The combination of separate waves in the same region of space produces a new waveform that can be of a larger or smaller amplitude compared with the original waves. This is called **interference**.

Principle of Superposition

If two or more waves are moving through the same medium, the resultant value of the wave function is the algebraic sum of the wave functions of the original waves.

Two waves can pass through each other without being destroyed or altered.

Constructive interference



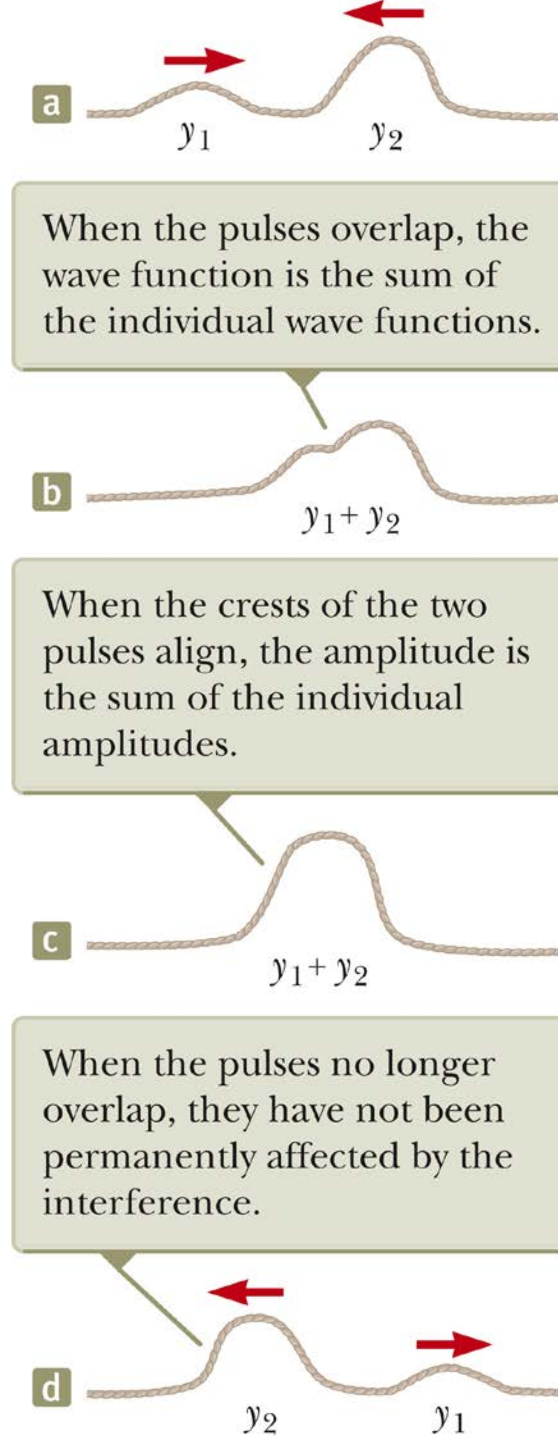
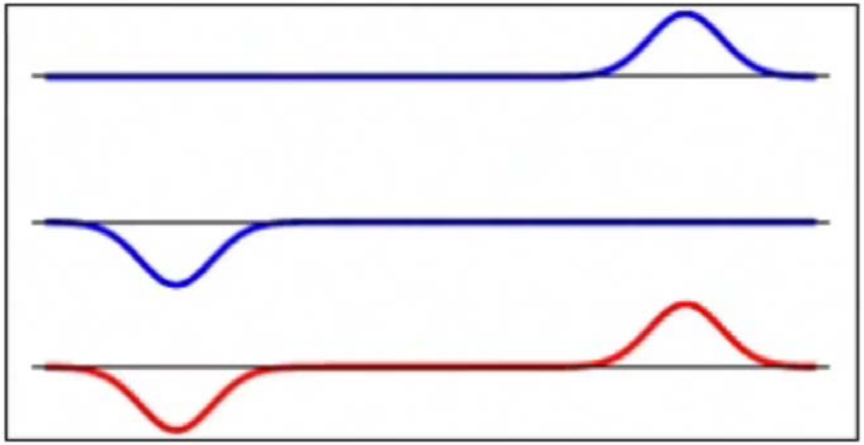


Fig. 18.6, p. 517

Destructive interference



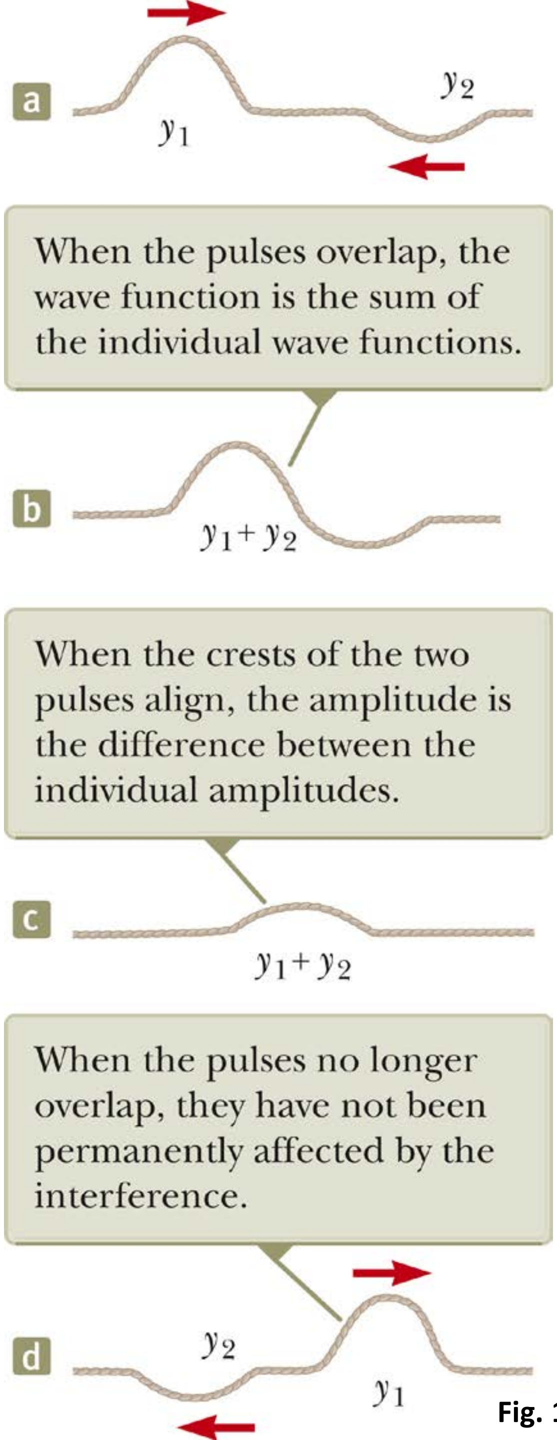


Fig. 18.7, p. 518

And quantitatively

Consider two sine waves that are identical apart from their phase, ϕ, traveling through a medium:

$$y_1 = A\sin(kx - \omega t)$$
$$y_2 = A\sin(kx - \omega t + \phi)$$

The resultant is given by

$$y_1 + y_2 = A[\sin(kx - \omega t) + \sin(kx - \omega t + \phi)]$$

$$y_1 + y_2 = A[\sin(kx - \omega t + \phi_1) + \sin(kx - \omega t + \phi_2)]$$

Remember the trig identities

$$\sin A + \sin B = 2\sin\left(\frac{A \pm B}{2}\right)\cos\left(\frac{A \mp B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A + B}{2}\right)\sin\left(\frac{A - B}{2}\right)$$

You need to be able to use these. They are provided on the formula sheet.

And so…

$$y_1 + y_2 = A[\sin(kx - \omega t) + \sin(kx - \omega t + \phi)]$$

$$\sin A \pm \sin B = 2\sin\left(\frac{A \pm B}{2}\right)\cos\left(\frac{A \mp B}{2}\right)$$

$$= 2A\sin(\frac{2kx - 2\omega t + \phi}{2})\cos(\frac{\phi}{2})$$

$$= 2A\sin(kx - \omega t + \frac{\phi}{2})\cos(\frac{\phi}{2})$$

What does it look like?

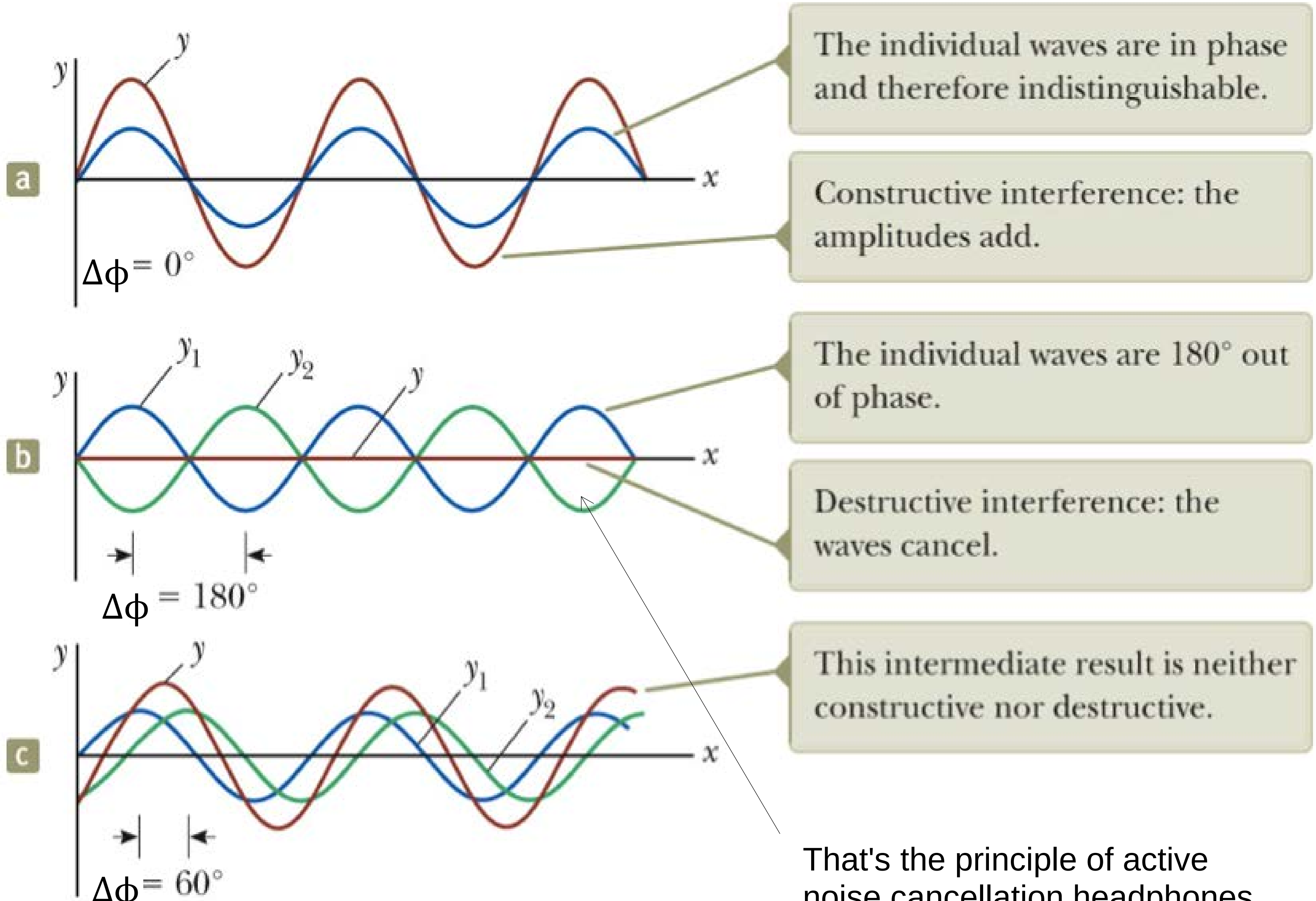
$$y_1 + y_2 = 2A\sin(kx - \omega t + \frac{\phi}{2})\cos(\frac{\phi}{2})$$

This resultant wave is sinusoidal with an amplitude $2A\cos(\frac{\phi}{2})$. ϕ describes the phase difference between the two waves.

$$\phi = 0 \Rightarrow \cos(\frac{\phi}{2}) = 1$$

$$\phi = \pi \Rightarrow \cos(\frac{\phi}{2}) = 0$$

$$\phi = \pi/3 \Rightarrow \cos(\frac{\phi}{2}) = \frac{\sqrt{3}}{2}$$



That's the principle of active noise cancellation headphones.

Interference of sound waves

We have seen that waves interfere constructively if there is a phase difference of 0, 2π, 4π, 6π etc. between them. This is equivalent to having a “path difference” of 0, λ, 2λ, 3λ etc. between them.

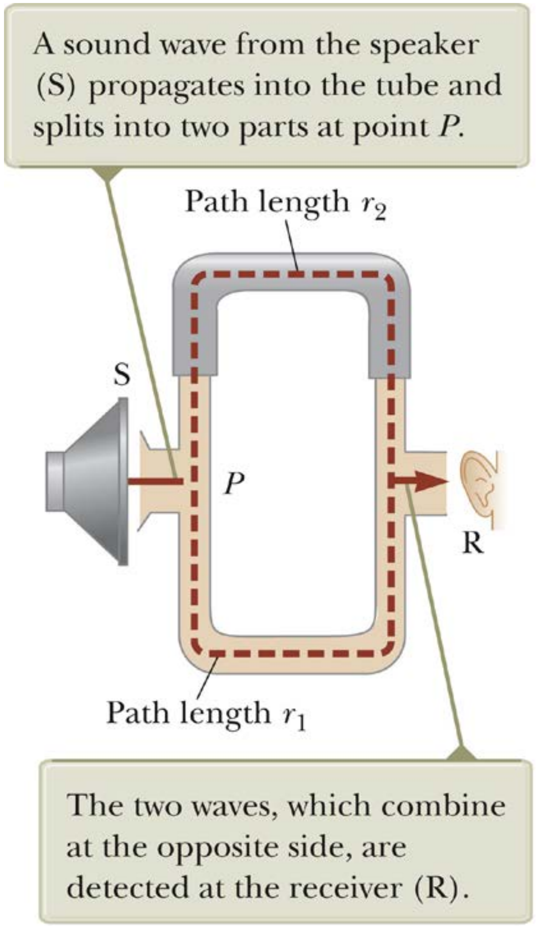


Fig. 18.9, p. 519

Phase difference vs path difference...

- Consider again 2 sine waves with a phase difference:

Phase difference

$$y_1(x,t) = A\sin(kx - \omega t + \phi_1)$$
$$y_2(x,t) = A\sin(kx - \omega t + \phi_1 + \Delta\phi)$$

- We can also write $y_2(x, t)$ this way:

$$y_2(x,t) = A\sin\left[k\left(x + \frac{\Delta\phi}{k}\right) - \omega t + \phi_1\right]$$

Δx can be interpreted as the "extra distance" travelled by wave 2 compared with wave 1.
It is also called the **path difference**.

$$\Delta x \equiv \frac{\Delta\phi}{k} = \frac{\lambda}{2\pi}\Delta\phi$$

As an equation...

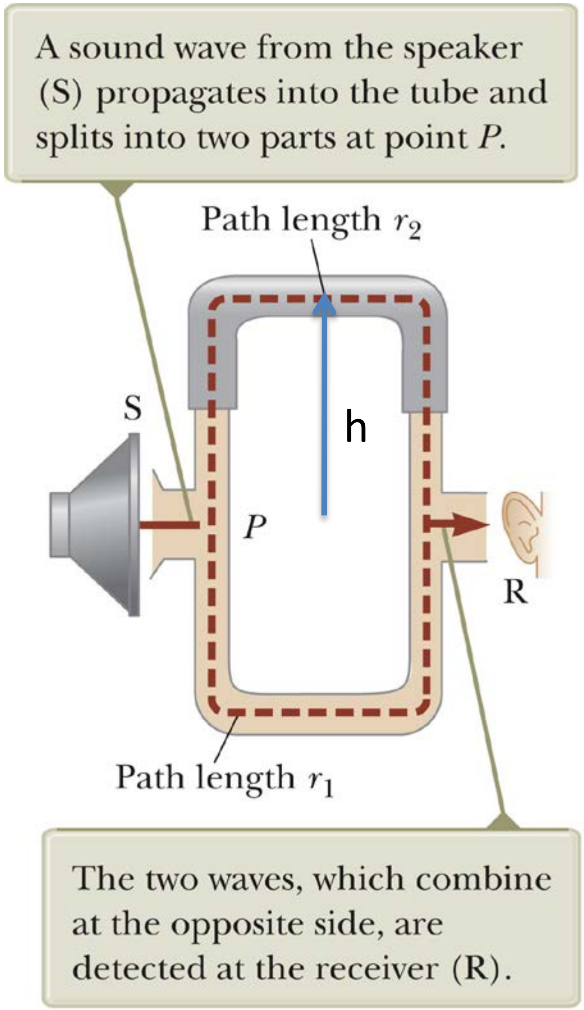
$$\frac{\text{path difference}}{\lambda} \times 2\pi = \text{phase difference}$$

Homework Set 6:

PHYS 1121:
1
PHYS 1131:
1, 8

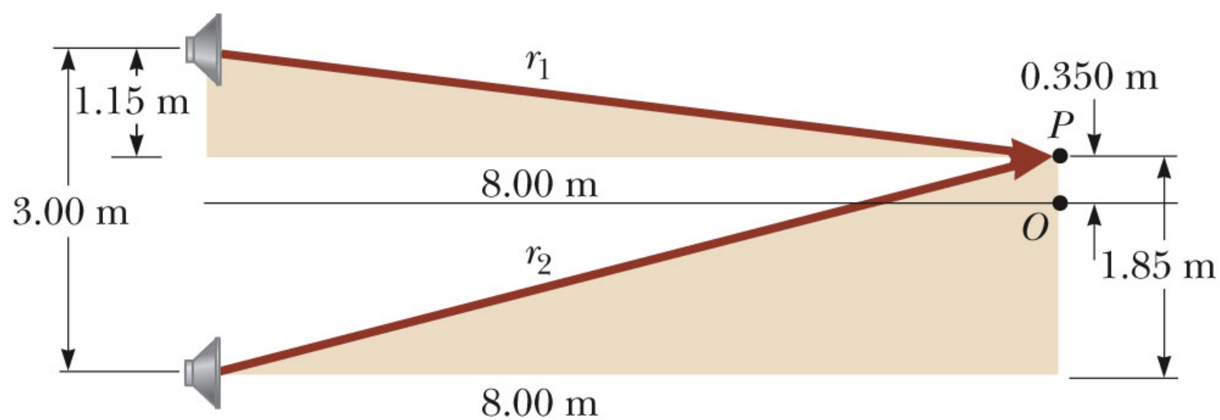
Question

The lower path, r_1, is fixed. The upper path, r_2, can be varied by changing the height, h. Describe the relationship between the height and what you hear. Assume that initially $r_1 = r_2$.



Question

Two identical loud speakers placed 3.00 *m* apart are driven by the same oscillator. A listener is originally at point O, located 8.00 *m* from the center of the line connecting the two speakers. The listener then moves to point P, which is a perpendicular distance 0.350 *m* from O, and experiences the *first minimum* in sound intensity. What is the frequency of the oscillator?



The speed of sound in air is 343 m/s.

Question

An 80.0 kg hiker is trapped on a mountain ledge following a storm. A helicopter rescues the hiker by hovering above him and lowering a cable to him. The mass of the cable is 8.00 kg, and its length is 15.0 m. A sling of mass 70.0 kg is attached to the end of the cable. The hiker attached himself to the sling, and the helicopter then accelerates upward. Terrified by hanging from the cable in midair, the hiker tries to signal the pilot by sending a transverse pulse up the cable. A pulse takes 0.250 s to travel the length of the cable. What is the acceleration of the helicopter? Assume the tension in the cable is uniform.