Question 1 (Marks: 20)

(a) A toy car moves in the x-direction according to the equation

$$x = (12t - t^2 - 6)\text{m, where t is in seconds:}$$

(i) Calculate the *displacement* of the car at t = 0s.

(ii) Starting with the equation above, derive an expression for the instantaneous velocity of the toy car.

(iii) Write an expression for the acceleration of the toy car at t = 0s.

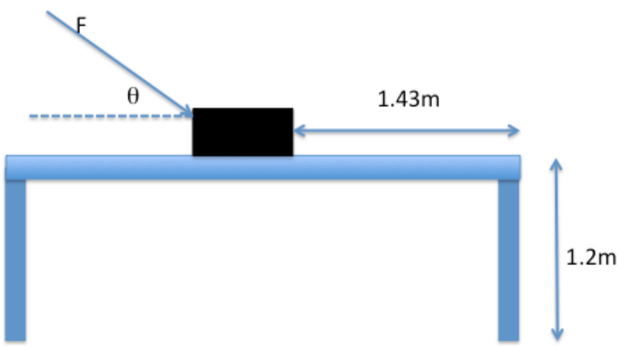
(iv) Sketch a rough graph of the *displacement* of the toy car against *time*, from t=0 to t=12 s.

(v) Sketch a rough graph of the *velocity* of the toy car against *time*, over the same time interval.

(vi) Calculate the time at which the velocity of the toy car is 0.

(vii) Find the maximum displacement of the toy car in the positive x-direction.

(b) A wooden block of mass 1.8 kg is pushed along a rough table by a force of 22 N at an angle, θ, of 30.0° above the horizontal. The block starts from rest, and moves a distance of 1.43m, until it reaches the end of the table.



The coefficient of kinetic friction between the block and the table is 0.432. When answering the questions below, make sure you give the appropriate number of significant figures in your answer. Use $g = 9.8\ ms^{-2}$ where needed.

(i) Draw a free body diagram showing all forces on the block while it is travelling along the table. Make sure you show all vectors resolved into their x and y components. Additionally, show the direction of the acceleration on your diagram.

(ii) Calculate the net force (sum of forces) acting on the block in the y-direction.

(iii) Calculate the magnitude of the normal force acting on the block.

(iv) Calculate the magnitude of the friction force acting between the block and the table.

(v) Calculate the net force (sum of forces) acting on the block in the x direction.

(vi) Calculate the acceleration of the block as it moves through the 1.43 m distance.

(vii) Calculate the velocity of the block at the end of the table.

(viii) The block continues in motion off the end of the table. The force, F, stops acting at this point. The table top is 1.2 m above the floor.

(1) Calculate the time taken for the block to hit the floor.

(2) Calculate the distance the block travels in the x direction before it hits the floor.

(3) Calculate the velocity of the block just as it hits the ground. You should give your answer as a magnitude, and a direction clockwise from the positive x-direction.

Question 2 (Marks: 20)

(a) State the conditions under which the momentum of a system is conserved.

(b) State the conditions under which mechanical energy of a system is conserved.

(c) A car, mass M_c, travelling at a speed v, collides head-on with a truck with mass M_t travelling at the speed $v/2$ in the opposite direction to the car. The two vehicles stick together and skid along the road for a distance D before coming to rest. The coefficients of static and kinetic friction are μ_s and μ_k, respectively.

(i) Derive an expression relating the velocity V_F of the two vehicles immediately after the collision, to v and any other parameters needed. State any assumptions you make.

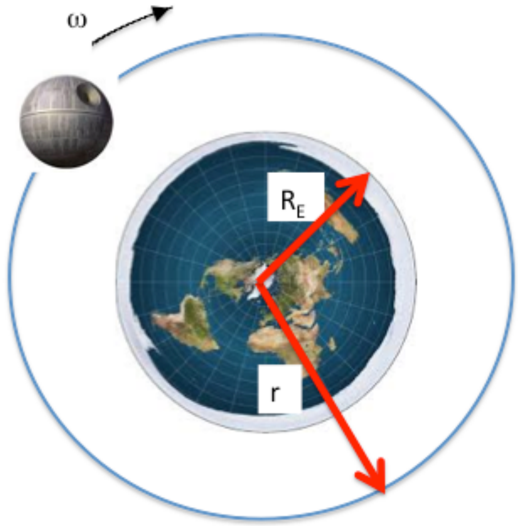
(ii) Derive an expression for V_F in terms of D and other needed parameters, stating the assumptions you are making.

(iii) Derive an expression for the amount of kinetic energy ΔK lost in the collision in terms of M_c, M_t and v only.

(iv) Derive an expression for the power being dissipated due to friction, just after the collision, in terms of v and any other parameters needed.

Question 3 (Marks: 20)

(a) A satellite with a mass of 150 kg is in a geostationary orbit, meaning it orbits the Earth so that it is always above the same position of the Earth's surface.



(i) Find the angular velocity of the satellite as it orbits the Earth in radians per second. You can assume that 1day is exactly 24 hours.

(ii) Find the value of r, the radius of the satellite's orbit.

(iii) Find the velocity, v, of the satellite in its orbit.

(iv) Find the gravitational potential energy of the satellite, with respect to a zero value of P.E at infinity.

(v) Find the total energy of the satellite, considering both gravitational and kinetic energy.

(vi) Is the total energy greater than or less than 0? Considering the orbit of the satellite, is the answer what you expect? Give a reason for your answer.

(b) A car has total mass m (including the wheels), and each of the four wheels have mass m_w, distributed evenly such that one can assume the wheel to be a uniform disc.

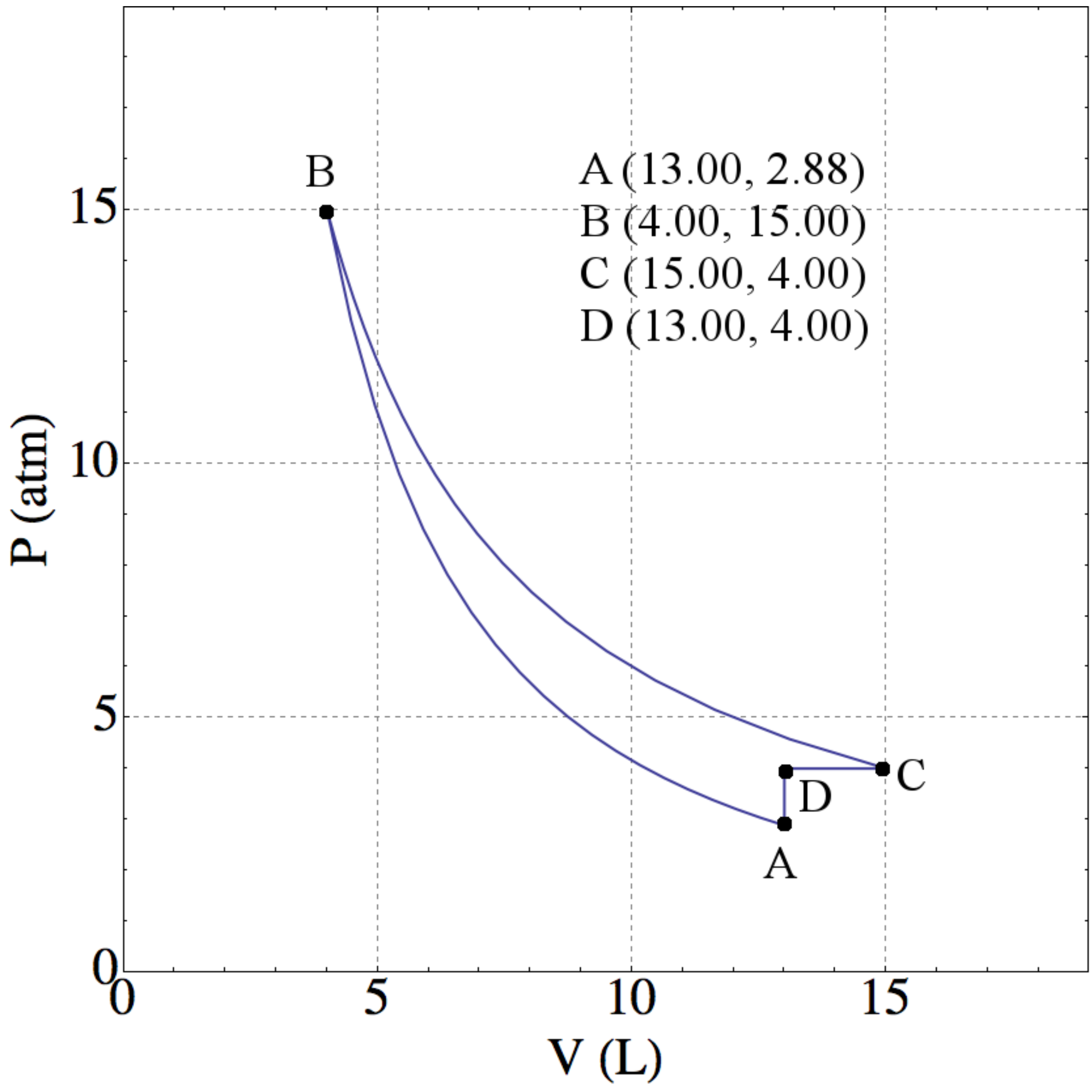
(i) Derive an equation for the moment of inertia of a disc with mass m_w and radius r_w. Show all steps in your working.

(ii) Derive an expression for the fraction of the total kinetic energy coming from rotational motion of the wheels. Ignore other moving parts such as the engine and transmission.

(iii) Suppose the car is an all-wheel drive car with four wheels of radius r_w. When the car is at rest, write an expression for the maximum torque the engine could supply to the wheels without the wheels skidding. Assume that the weight of the car is distributed evenly over the four wheels and that the engine distributes power equally among the four wheels, the coefficient of static friction is μ_s and kinetic friction is μ_k between the wheels and the ground.

Question 4 (Marks: 30)

(a) 2.00 mols of an ideal gas undergoes a cyclic process from A → B → C → D → A.



Process A → B is adiabatic, while process B → C is isothermal. (Useful info: 1 m^3 = 1000 L, 1 atm = 1.01×10^5 Pa)

(i) Demonstrate by explicit calculation that this is a diatomic gas.

(ii) Calculate the work done on the gas as it goes from A to B.

(iii) Calculate the heat absorbed as the gas goes from B to C.

(iv) Calculate the change in the internal energy as the gas goes from C to D to A.

Substance	Specific heat c, (J kg^{-1} K^{-1})	Linear thermal expansion coefficient α, (ºC^{-1})	Thermal conductivity k, (W m^{-1} K^{-1})
Aluminium	910	24×10^{-6}	205.0
Brass	377	19×10^{-6}	109.0
Copper	390	17×10^{-6}	385.0
Lead	130	29×10^{-6}	34.7
Steel	456	11×10^{-6}	50.2

Properties of water	Value
Specific heat (liquid)	4186 J kg^{-1} K^{-1}
Latent heat of fusion	3.33×10^{5} J kg^{-1}
Latent heat of vapourisation	2.25×10^{6} J kg^{-1}
Density (at 20.00 ºC)	998 kg m^{-3}
Melting point (at 1 atm)	0.000 ºC
Boiling point (at 1 atm)	100.0 ºC
Volume expansion coefficient β (at 20 ºC; you may assume it is constant between 4 ºC and 100 ºC)	207×10^{-6} ºC^{-1}

(b) Consider a closed, cubic container made of aluminium sheets with side lengths of 1.00 m at a temperature of 20.0 ºC. The container is half filled with water, and half filled with an ideal monatomic gas at atmospheric pressure.

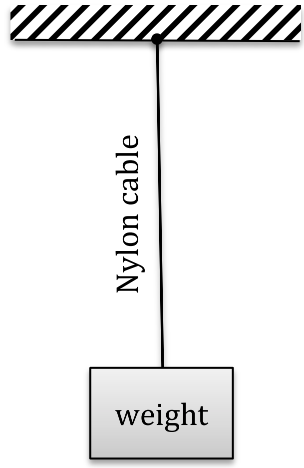
(i) How much heat do we have to add to the system (container + water + gas) in order to raise its temperature to 50.0 ºC? You may take the density of aluminium to be 2750 kg m^{-3}, and the thickness of the sheets to be 1.00 cm.

(ii) Suppose now we have heated the system to a temperature of 80.0 ºC. We let the gas out and fill up the container to the brim with ice of temperature 0.0 ºC. Assuming that the container + water + ice is an isolated system, what is the final temperature of the system when it reaches equilibrium? You may take the density of ice at 0.0 ºC to be 917 kg m^{-3}, and ignore thermal expansion of the water and container.

(c) Consider a 1.00 kg cubic block of ice with a temperature of 0.0 ºC. We place it in a room at a temperature of -10.0 ºC. What is the net power radiated by the ice block, assuming it is a perfect blackbody?

Question 5 (Marks: 30)

(a) A clock in a bell-tower has a long pendulum. Since this is first year physics, we will assume it is a simple pendulum – with a thin nylon cable anchored on a frictionless pivot at the top of the tower holding a weight made of 20 stacked nylon discs, each 0.50 cm high and 10.00 cm in diameter



(i) Taking into account the finite height and centre of mass of the weights at the end of the pendulum, what length (in metres) should the cable be to obtain a period of 3.00 s?

(ii) The clock was calibrated in winter, when the average temperature in the clock-tower was 5.00°C. In summer the average temperature is 13.00°C. Does the clock run faster or slower in summer?

(iii) The thermal expansion coefficient of nylon is $100\text{x}10^{-6}$ °C^{-1}. How many seconds does the clock gain or lose a day?

(iv) To correct for the slow/fast running of the clock, additional 0.50cm high 10.00cm diameter weights can be added on top of the existing discs at the end of the pendulum, which alters the centre of gravity. Should more counterweights be added, or removed, to correct for the change in temperature?
[In practice the clock that inspired this question is corrected by adding or removing old two penny coins].

(v) At time T=0 the pendulum is released from rest at an angle to the vertical θ=10°. Sketch graphs of the angular displacement, potential energy, and kinetic energy vs time from t=0 to t=3.00s. The density of nylon is 1150.00 kg m^{-3}. Include numbers on your axes.

(vi) After a complete period, when the pendulum returns to θ=10°, 1 more disc is quickly added to the pendulum to alter the centre of mass. Describe how the period and amplitude of the oscillation change, and whether the clock loses or gains time.

(vii) Instead of adding the additional disc at θ=10°, it is dropped on top of the other discs when the pendulum is vertical, in the middle of the oscillation. Describe how the period and amplitude of the oscillations change, and whether the clock loses or gains time.

(b) A car is travelling away from a police officer with a radar gun with velocity v_0. The officer sends X-band radio waves (f_0=9GHz) at the car. The car absorbs the radio waves, and re-emits them.

(i) What frequency waves are emitted from the car?

(ii) What frequency radio waves are detected by the officer?

(iii) In the lectures we have learnt about the principle of superposition, where waves are added. But for radar the wave reflected by the car is *multiplied* by the wave sent to the car, using an amplifier and a device called a mixer. The multiplied signal is put through a filter than removes frequencies above 1GHz. The resultant signal contains a single sine wave, oscillating at a frequency of 1.25kHz. How fast is the car going?