Question 1.

a) i) $a\underset{\sim}{A} + b\underset{\sim}{B} + \underset{\sim}{C} = 0.$

$$(a \times 6.00 - b \times 8.00 + 26.00)\hat{\underset{\sim}{i}} + (-8.00a + 3.00b + 19.00)\hat{\underset{\sim}{j}} = 0.$$

2 simultaneous equations:

$$\begin{aligned} 6a - 8b &= -26. \quad ① \times 4 \\ -8a + 3b &= -19. \quad ② \times 3. \end{aligned} \Rightarrow \begin{aligned} 24a - 32b &= -104. \quad ①' \\ -24a + 9b &= -57 \quad ②' \end{aligned} \; +.$$

$$-23b = -161$$

$$\Rightarrow b = 7.00.$$

$$a = \frac{8 \times 7 - 26}{6} = 5.00.$$

ii) $A = 20.0\hat{\underset{\sim}{j}}$ $\quad B = 40\sin 45\hat{\underset{\sim}{i}} + 40\sin 45\hat{\underset{\sim}{j}}$

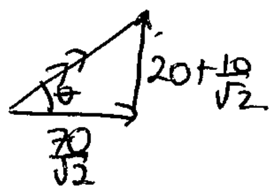
$C = 30\sin 45\hat{\underset{\sim}{i}} - 30\sin 45\hat{\underset{\sim}{j}}.$

$$\Rightarrow \underset{\sim}{A} + \underset{\sim}{B} + \underset{\sim}{C} = 70\sin 45\hat{\underset{\sim}{i}} + (20 + 10\sin 45)\hat{\underset{\sim}{j}}.$$

$$= \frac{70}{\sqrt{2}}\hat{\underset{\sim}{i}} + \left(20 + \frac{10}{\sqrt{2}}\right)\hat{\underset{\sim}{j}} = 49.5\hat{\underset{\sim}{i}} + 27.1\hat{\underset{\sim}{j}}.$$

$$\text{magnitude} = \sqrt{\frac{70^2}{2} + \left(20 + \frac{10}{\sqrt{2}}\right)^2} = 56.4.$$

$$\text{direction: } \tan\theta = \frac{20 + 10/\sqrt{2}}{70/\sqrt{2}}.$$

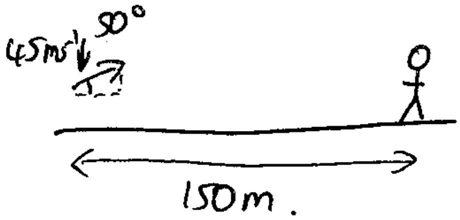


$$= 0.5469.$$

$$\theta = 28.7°.$$

b) i) $v_{xi} = 45\cos 50$

travels with constant horizontal velocity ⇒.



$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{150}{45\cos 50} = 5.19\text{s.} \quad (3 \text{ sig fig}).$$

ii) $y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$.

Assume $y_0 = 0$ $\quad v_{y0} = 45\sin 50$ $\quad a_y = -9.80\,ms^{-2}$.

$\Rightarrow y = 45\sin 50 \times 5.1857 + \frac{1}{2}\times(-9.80)\times 5.1857^2$.

$= 46.9947\,m$.

$= 47.0\,m$ (3 sig fig).

iii) stated in question $\Rightarrow$ 45.0 m/s at 50° to horizontal.

iv). For apple to be thrown with minimum speed maximum height of apple = height of arrow at apple's location

i.e. 46.9947m.

calculate initial speed of apple first:

$0^2 = V_{y0}^2 + 2ay \Rightarrow V_{y0}^2 = 2\times 9.8\times 46.9947$

speed at max height.

$V_{y0} = 30.3496\,m/s$.

$V_{yf} = V_{yi} + at \Rightarrow t = \frac{30.3496}{9.80}$

$= 3.10\,s$ (3 sig fig).

v) (1131 only)

$5.19 - 3.10 = 2.09\,s$ (3 sig fig).

c) i) $KE = \frac{1}{2}mv^2$ $\quad V = 6.00\hat{\underset{\sim}{i}} - 2.00\hat{j}$.

$|v| = \sqrt{6^2 + 2^2} \hat{=} 6.3246\,m/s$.

$= \frac{1}{2}\times 3.00\times (6.3246)^2$

$= 60.0\,J$. (3 sig fig)

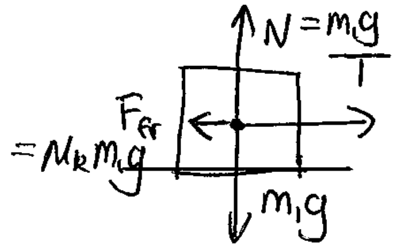
ii) work needs to be done to change KE.

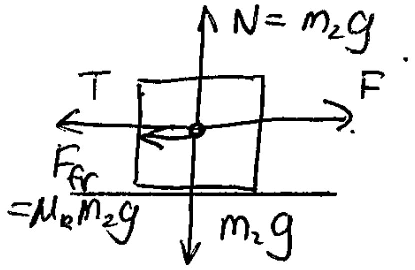
$W = KE_f - KE_i = \frac{1}{2}\times 3\times((8^2+4^2) - (6^2+2^2))$.

$= 60.0\,J$ (3 sig fig).

Question 2.

a) i) for m_1:   for m_2:





ii) Use Newton's second law to write expressions:

$$m_2 a = F - T - \mu_k m_2 g \quad ①$$
$$m_1 a = T - \mu_k m_1 g \quad ②$$

① + ② $\Rightarrow$ $(m_2 + m_1)a = F - \mu_k g(m_2 + m_1)$

$$a = \frac{68.0 - 0.100 \times 9.8(12.0 + 18.0)}{12.0 + 18.0}$$
$$= 1.29\ ms^{-2} \text{ to right.}$$

For T sub into ② $T = m_1 a + \mu_k m_1 g$
$$= 12.0 \times (1.29 + 0.100 \times 9.8)$$
$$= 27.2\ N.$$

1131 only (part (iii))

iii). Newton's second law states that the net force acting on the block is equal to the mass times the acceleration of the block. Vertically the forces cancel $\Rightarrow$ no acceleration vertically; Horizontally T and friction work in opposite directions giving us

$$m_1 a = T - \mu_k m_1 g$$

which we solved in part (ii).

b) Mechanical energy is conserved when there are no non-conservative forces doing work on a body. $W = \underset{\sim}{F} \cdot \underset{\sim}{s}$ so the non-conservative force can not have any component parallel to the displacement.

c) Momentum is conserved when no external forces act on a body.

$$\text{Impulse} = Ft = \Delta p.$$

When the impulse is zero momentum is conserved.

d). i) This is an elastic collision as kinetic energy is conserved. Kinetic energy is conserved as the mass and speed. of the ball do not change.

ii). For the system as a whole (assuming the wall is attatched to the Earth) momentum is conserved. (If you just consider the ball then it is not conserved). During the collision momentum is transfered from the ball to the wall in the horizontal (to the right) direction. The amount of momentum transfered is $2mv_x = 2mv\sin 60$.

iii) $$Ft = 2mv\sin 60.$$

$$F = \frac{2mv\sin 60}{t} = \frac{2mv\sqrt{3}}{2t} = \frac{mv\sqrt{3}}{t}$$

to the left.

Question 3.

i) $\alpha = \frac{\omega}{t}$.

ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\quad \omega_0 = 0$ as it starts from rest.

$\Rightarrow \theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\omega t$.

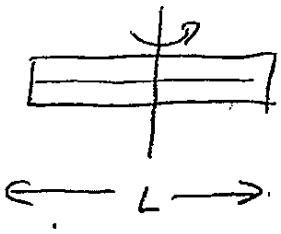
iii) $\tau = r \times F = I\alpha = \frac{1}{12} m L^2 . \frac{\omega}{t} = \frac{mL^2\omega}{12t}$

iv). As it is not gaining potential energy the work done is equal to the change in kinetic energy.

$$K = \frac{1}{2} I \omega^2 = \frac{1}{2} . \frac{1}{12} m L^2 . \omega^2 .$$

$$= \frac{mL^2\omega^2}{24}.$$

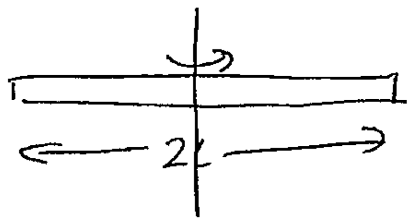
v). The centr of mass of the bar does not gain speed $\Rightarrow$ no net force.



vi) τ is the same $\Rightarrow$.

$$\frac{\cancel{m}\cancel{L^2}\omega}{\cancel{12}\cancel{t}} = \frac{1}{\cancel{12}} \times 2\cancel{m} \times \frac{\cancel{L^2}\omega_{vi}}{\cancel{t}}$$

$\omega_{vi} = \frac{\omega}{2}$. $\quad$ end 1121.



vii) $\frac{m\cancel{L^2}\omega}{\cancel{12}\cancel{t}} = \frac{1}{\cancel{12}} \times 2m \times (2\cancel{L})^2 \frac{\omega_{vii}}{\cancel{t}}$

$\Rightarrow \omega = 8\omega_{vii}$.

$\omega_{vii} = \frac{\omega}{8}$.

viii) moment of inertia is calculated using

$$I = \int r^2 . dm$$

as the "average" distance of the mass elements from the pivot point is greater in this situation the moment of inertia would be greater.

Question 4

Part a

(i) $T = \frac{PV}{nR}$, where $n = 2\,mol$
$R = 0.0821\ L\,atm/mol.K$

$\Rightarrow T_A = \frac{15 \times 4}{2 \times 0.0821} = \underline{365\,K}$

$T_B = T_A = \underline{365\,K}$ $\frac{1}{2}$ because $A \to B$ is isothermal

$T_C = \frac{3 \times 12}{2 \times 0.0821} = \underline{219\,K}$

$T_D = \frac{3 \times 10.51}{2 \times 0.0821} = \underline{192\,K}$

(ii) Work done on gas from A to B:

$$W_{A\to B} = -\int_{V_A}^{V_B} P\,dV = -\int_{V_A}^{V_B} \frac{nRT_A}{V}\,dV$$

$$= -nRT_A \ln\frac{V_B}{V_A} = nRT_A \ln\frac{V_A}{V_B}$$

$$= 2 \times 8.314 \times 365 \times \ln\left(\frac{4}{12}\right) \quad \| \; R = 8.314\ J/mol.K$$

$$= \underline{-6668\ J}$$

(iii) $\Delta E_{B\to C} = \frac{3}{2} nR(T_C - T_B)$ for a monatomic gas

$$= \frac{3}{2} \times 2 \times 8.314 \times (219 - 365)$$

$$= \underline{-3642\ J}$$

(iv) Heat absorbed from C to D:

$$\Delta Q_{C\to D} = \Delta E_{C\to D} - W_{C\to D}$$

where

$$\Delta E_{C\to D} = \frac{3}{2} nR(T_D - T_C)$$

$$= \frac{3}{2} \times 2 \times 8.314 \times (192 - 219)$$

$$= \underline{\underline{-673\text{ J}}} \quad \frac{1}{2}$$

and

$$W_{C\to D} = -\int_{V_C}^{V_D} P_C\, dV = -\underbrace{P_C}_{\text{in Pa}}\underbrace{(V_D - V_C)}_{\text{in m}^3} \Rightarrow W_{C\to D} \text{ in J}$$

$$= -3 \times 1.01 \times 10^5 \times (10.51 - 12)/1000$$

$$= \underline{\underline{451\text{ J}}}$$

Thus: $\Delta Q_{C\to D} = (-673 - 451)\text{J} = \underline{\underline{-1124\text{J}}}$

(v) $$\Delta E_{D\to A} = \underbrace{\Delta Q_{D\to A}}_{=0 \text{ because } D\to A \text{ is adiabatic}} + W_{D\to A}$$

$$\Rightarrow W_{D\to A} = \Delta E_{D\to A}$$

$$= \frac{3}{2} nR(T_A - T_D)$$

$$= \frac{3}{2} \times 2 \times 8.314 \times (365 - 192)$$

$$= \underline{\underline{4315\text{ J}}}$$

(vi) The net work done on the gas in one cycle is negative.

In the process $A \to B$, the gas does work $|W_{A\to B}|$ to bring it from a volume V_A to V_B.

In the process $C \to D \to A$, work $|W_{C\to D\to A}|$ is done on the gas to bring it back from V_B to V_A.

But the process $A \to B$ occurs at higher pressures than the process $C \to D \to A$. Therefore, $|W_{A\to B}| > |W_{C\to D - A}|$, and so the net work done on the gas is negative.

Part b

(i) The change in the side length of each aluminium sheet is

$$\Delta l = \alpha l_{ini} \Delta T$$

Then, the new volume of the container is

$$\begin{aligned} V_{ini} + \Delta V &= (l_{ini} + \Delta l)^3 \\ &= l_{ini}^3 \left(1 + \frac{\Delta l}{l_{ini}}\right)^3 \\ &\simeq l_{ini}^3 \left(1 + \frac{3\Delta l}{l_{ini}}\right) \\ &= V_{ini} + 3 \Delta l \, l_{ini} \\ &= V_{ini} + 3 \alpha l_{ini}^3 \Delta T \end{aligned}$$

or

$$\begin{aligned} \Delta V &= 3\alpha V_{ini} \Delta T \\ &= 3 \times 24 \times 10^{-6} \times 1 \times (50-20) \\ &= 2.16 \times 10^{-3} \, m^3 \end{aligned}$$

On the other hand, the water expands by

$$\begin{aligned} \Delta V &= \beta V_{ini} \Delta T \\ &= 207 \times 10^{-6} \times 1 \times 30 \\ &= 6.21 \times 10^{-3} m^3 \end{aligned}$$

Thus, there is an excess of water of volume

$$6.21 \times 10^{-3} m^3 - 2.16 \times 10^{-3} m^3 = \underline{\underline{4.05 \times 10^{-3} m^3}}$$

which overflows.

(ii) The total volume of aluminium is

$$V_{al} = 5 \text{ sheets} \times 1m^2 \times 0.01\,m = 0.05\,m^3$$

$$\Rightarrow M_{al} = \rho_{al} V_{al} = 2750 \times 0.05 = \underline{\underline{138\,kg}}$$

The total mass of water is

$$M_{water} = \rho_{water} \times V_{water} = \underline{\underline{1000\,kg}}$$

Thus, the energy added to the system is

$$Q = (M_{water} C_{water} + M_{al} C_{al}) \Delta T$$
$$= (1000 \times 4186 + 138 \times 910) \times 30$$
$$= 129\,347\,400\,J$$
$$= \underline{\underline{129347\,kJ}}$$

PHY113J only

(iii) The energy absorbed by each material is

$$Q_{water} = M_{water} C_{water} (T_f - T_a)$$
$$Q_{al} = M_{al} C_{al} (T_f - T_a)$$
$$Q_{lead} = M_{lead} C_{lead} (T_f - T_b)$$

final temperature

$T_a = T^{ini}_{water} = T^{ini}_{al} = 50°C$

$T_b = T^{ini}_{lead} = 0°C$

In equilibrium,

$$Q_{water} + Q_{al} + Q_{lead} = 0$$

$$\Rightarrow (M_{water} C_{water} + M_{al} C_{al})(T_f - T_a) + M_{lead} C_{lead} (T_f - T_b) = 0$$

$$\Rightarrow \left(M_{water} C_{water} + M_{al} C_{al} + M_{lead} C_{lead}\right) T_f$$

$$= \left(M_{water} C_{water} + M_{al} C_{al}\right) T_a + M_{lead} C_{lead} T_b$$

$$\Rightarrow \quad T_f = \frac{\left(M_{water} C_{water} + M_{al} C_{al}\right) T_a + M_{lead} C_{lead} T_b}{M_{water} C_{water} + M_{al} C_{al} + M_{lead} C_{lead}}$$

$$= \frac{(1000 \times 4186 + 138 \times 910)\,50 + 100 \times 130 \times 0}{1000 \times 4186 + 138 \times 910 + 100 \times 130}$$

$$= \underline{\underline{49.8\,^{\circ}C}}$$

Question: Oscillations

a) Mr. Spock wakes to find he is swinging back and forth on a long rope suspended over a chasm on the Klingon home planet Kronos. Spock knows the acceleration due to gravity on the surface of Kronos is 10.3 m/s^2. If the period of Spock's swing is 12.0s then how far down the rope is he?

Answer:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$l = \frac{T^2 g}{4\pi^2}$$

l = 37.8m

b) As he begins to climb up the rope he sees Captain Kirk above him, going up and down attached to the end of a large spring, also suspended over the chasm. If Kirk's mass is 80.0kg and the period of his oscillation is 4.0s then what is the spring constant of the spring? (You can ignore the mass of the spring)

Answer:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T^2 = 4\pi^2 \frac{m}{k}$$

$$k = 4\pi^2 \frac{m}{T^2}$$

***k* = 197 N/m**

c) Kirk is yelling at a frequency of 300Hz, while Spock is below him. Describe in words at which position in Kirk's up and down oscillation Spock will hear the highest frequency sound from Kirk, and why.

Answer:

When Kirk has the highest velocity toward Spock the Doppler shifted frequency Spock hears will be the highest (2 mark).
This will occur when he is moving downward through the rest position of the Kirk-spring system, when he has no net force acting on him. (2 marks)

d) Spock stops climbing to measure Kirk's yelling which varies in frequency as Kirk oscillates. Spock's tricorder tells him the highest frequency he hears coming from Kirk is 309Hz. What is Kirk's maximum speed? (Note: the speed of sound in the air on Kronos is the same as on Earth, $340ms^{-1}$)

Answer:

$$f' = f\left(\frac{c_s \pm v_{observer}}{c_s \pm v_{source}}\right)$$

$v_{observer}$ = 0
c_s = 340 m s^{-1}
***f* = 300Hz**
***f'* = 309Hz**

$$f' = f\left(\frac{c_s}{c_s - v_{source}}\right)$$

$$v_{source} = c_s\left(1 - \frac{f}{f'}\right)$$

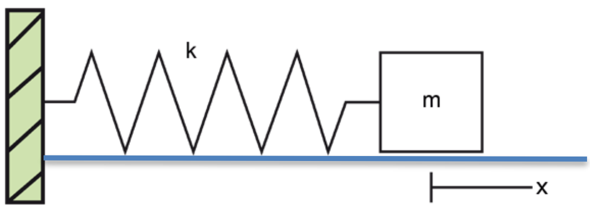
$$v_{source} = 340ms^{-1}\left(1 - \frac{300Hz}{309Hz}\right)$$

v_{source} = 9.90ms^{-1}

e) What is the amplitude of Kirk's oscillation? (Note: the potential energy of a spring is P.E. = ½ kx^2)

Answer:
Mechanical energy ME = kinetic energy + potential energy
At maximum velocity ME = kinetic energy
= ½ mv^2 = ½ 80kg (9.90ms^{-1})^2
= 3920.4 J
The amplitude = maximum displacement
At maximum displacement ME = potential energy = ½ kx^2
x_{max} = A = sqrt(2 ME / k) = 6.31m

f) Later, thanks to an injection of Kironide, Spock is given telekinetic abilities (the ability to move objects with his mind). To test the maximum force Spock's mind can apply he has set up an experiment with a mass (m) on a frictionless surface attached to a horizontal spring that is fixed to a wall (with spring constant k). In words, briefly explain how Spock can measure the maximum force his mind can apply.



__Answer:__

By using his mind to apply a force on the mass against the spring, Spock can measure the strength of that force using Hooke's Law, where the magnitude of the force $|F|=kx$

g) Spock releases the mass attached to the spring, setting up simple harmonic motion in the mass-spring system, write down the equation of displacement for the simple harmonic motion and define all terms.

__Answer:__

$x = A\cos(\omega t + \phi)$

x is the displacement from the rest position of the mass-spring system

A is the amplitude of the oscillation

ϕ is the phase shift of the displacement ($\phi=0$ if at $t=0$ motion is at maximum displacement)

ω is the angular frequency of the simple harmonic motion

PHYS1131 only:

h) Suppose the mass is 10.0kg and has a frequency $f = 0.50$Hz with amplitude $A = 1.0$m, what is the total mechanical energy of the spring–block system?

__Answer:__

Mechanical energy ME = kinetic energy + potential energy

At max displacement ME = PE = $\frac{1}{2} kx^2$

$k = \omega^2 m$

$\omega = 2\pi f$

$k = (2\pi f)^2 m = 98.7$ kg s^{-2}

ME = $\frac{1}{2} kx^2 = 49.3$ J

PHYS1131 only:

i) What is the speed of the block when it is displaced at a position 0.3m from the equilibrium position?

Answer:

ME = $\frac{1}{2} mv^2 + \frac{1}{2} kx^2$

$v = \text{sqrt}(\, 2\,(ME - \frac{1}{2} kx^2) / m\,)$

$v = 3.00$ ms^{-1}