

**PHYS 1121: PHYSICS 1 A**

**PHYS 1131: HIGHER PHYSICS 1 A**

Name: \_\_\_\_\_

Student Number: \_\_\_\_\_



**UNSW**  
AUSTRALIA

**If the uncertainties are dependent** (based on the same source of uncertainty) they add linearly:

When	Example	Method
Adding or subtracting	$I = x + x$ $y = x_1 - x_2$ $p = 2(l + h)$ , when the dominant uncertainty is a systematic uncertainty common to the measurement of $l$ and $h$	Add absolute uncertainty $\Delta y = \Delta x + \Delta x = 2\Delta x$ $\Delta y = \Delta x_1 + \Delta x_2$ $\Delta p = 2(\Delta l + \Delta h)$ Note the plus sign
Multiplying or dividing	$a = b \times c$ $v = l \times h \times b$ , where the dominant uncertainty in each is a systematic uncertainty common to $l$ , $h$ , $b$	Add fractional/percentage <sup>1</sup> errors $\frac{\Delta a}{a} = \frac{\Delta b}{b} + \frac{\Delta c}{c}$ $\frac{\Delta v}{v} = \frac{\Delta l}{l} + \frac{\Delta h}{h} + \frac{\Delta b}{b}$
Using formulae	$y = f(x)$	Use the differential method <sup>2</sup> or test the extreme points. $\Delta y = \Delta x \times f'(x) \quad \text{or}$ $\Delta y = \frac{f(x + \Delta x) - f(x - \Delta x)}{2}$

**If the uncertainties are independent** (they have different, uncorrelated sources) they add in quadrature:

Operation	Example	Method
Adding or subtracting	$z = x + y$ $p = 2(l - h)$	Add absolute uncertainties in quadrature. $\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ $\Delta p = 2\sqrt{(\Delta l)^2 + (\Delta h)^2}$
Multiplying or dividing	$z = x \times y$ $v = l \times h \times b$	Add fractional (percentage) uncertainty in quadrature. $\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$ $\frac{\Delta v}{v} = \sqrt{\left(\frac{\Delta l}{l}\right)^2 + \left(\frac{\Delta h}{h}\right)^2 + \left(\frac{\Delta b}{b}\right)^2}$
Using formulae	$z = f(x, y)$	Add partial derivative weighted uncertainties <sup>1</sup> in quadrature. $\Delta z = \sqrt{\left(\Delta x \times \frac{\partial f}{\partial x}\right)^2 + \left(\Delta y \times \frac{\partial f}{\partial y}\right)^2} \quad \text{or}$ $\Delta z = \sqrt{\frac{(f(x + \Delta x, y) - f(x - \Delta x, y))^2}{2} + \frac{(f(x, y + \Delta y) - f(x, y - \Delta y))^2}{2}}$

<sup>1</sup> To get from a fractional uncertainty to a percentage uncertainty multiply by 100

<sup>2</sup> Beware of the possibility of a turning point between  $x + \Delta x$  and  $x - \Delta x$ , which will cause this method to fail.  
(e.g.  $y = \sin(x)$ , where  $x = 90 \pm 10^\circ$ )

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**Warning:** The order in which you undertake experiments may deviate from the order that they are presented in the lab manual

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UNIVERSITY OF NEW SOUTH WALES

# INTRODUCTION AND WELCOME TO FIRST YEAR LAB

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We welcome you to the laboratory component of your First Year Physics Course. Please read through the following introductory pages for important information.

Physics is a science – everything you learn in your lectures is based on, and tested by, observations and experiment. Scientific method is the process of confronting hypotheses and theories with observations: you will learn this method using the experiments described in this lab manual. The aim of the laboratory course is to give you experience in the experimental side of physics: to develop your skills with collecting, analysing and interpreting data, and presenting your results in a way that can be understood by others. These skills are of use to anyone working in a modern, quantitative workplace.

Your course has a number of exercises. You must complete one per lab session. Doing so will help you develop your skills as an experimenter, as a professional, and reinforce lecture material. You will work in small teams of two people, unless told otherwise.

We intend to make the lab work enjoyable and useful to you. You will find the physics lab a relatively easy way to score good marks to add to your total physics score.

We have installed iMacs in the lab and useful software, including graphics and plotting packages and other materials supporting your course.

In designing your lab work, we have taken into account the wishes of employers in the technical and commercial sectors: they often ask for training in quantitative, computing and communication skills. Your ability to work in teams will also be of use to you in higher study and in the workplace.

We hope that you find the physics lab a useful and enjoyable resource and experience – we welcome and encourage you to use the facilities to derive full benefit from them and to give us feedback on your experiences.

## 1. Safety Induction

Anyone who is to work in a laboratory is required to complete a Safety Induction before they are legally allowed to work in that laboratory. This includes all students. The induction is available on your subjects Moodle site and is mandatory. If you do not complete the induction on-line you will not be allowed to enter the laboratory.

Below is a summary of the information given in the induction so that you have a hard copy to refer back to if needed.

### General Comments

First Year Lab (FYL) is already a very safe place – there is therefore little danger to personnel from potential accidents.

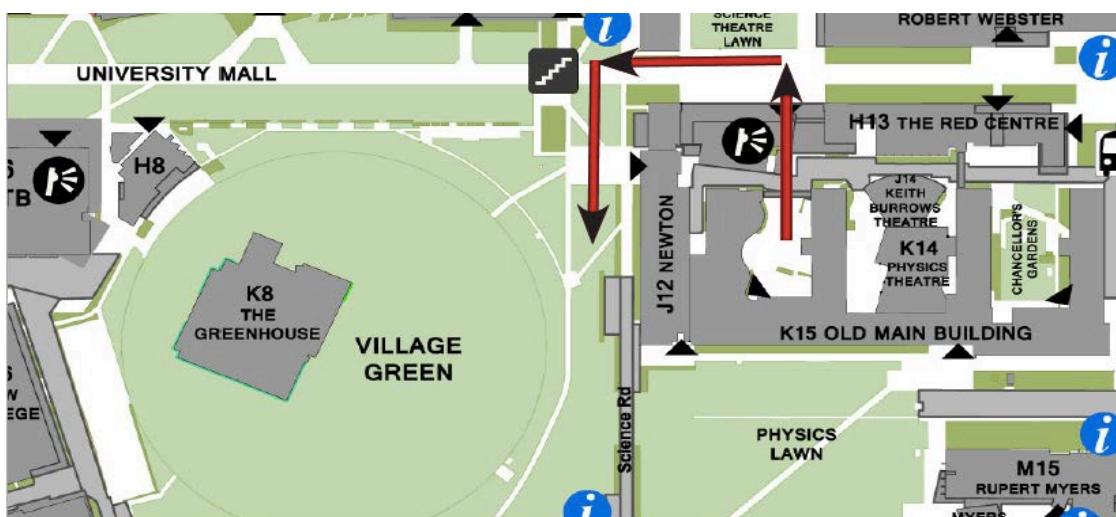
In FYL there are no exercises involving radioactivity, high voltage, harmful chemicals, condensed gases, biohazards or lasers – the level of exposure to accident is therefore little different from what you would find in the home. However, we do expect people using the lab to take a good common sense approach to moving about the lab and using the equipment.

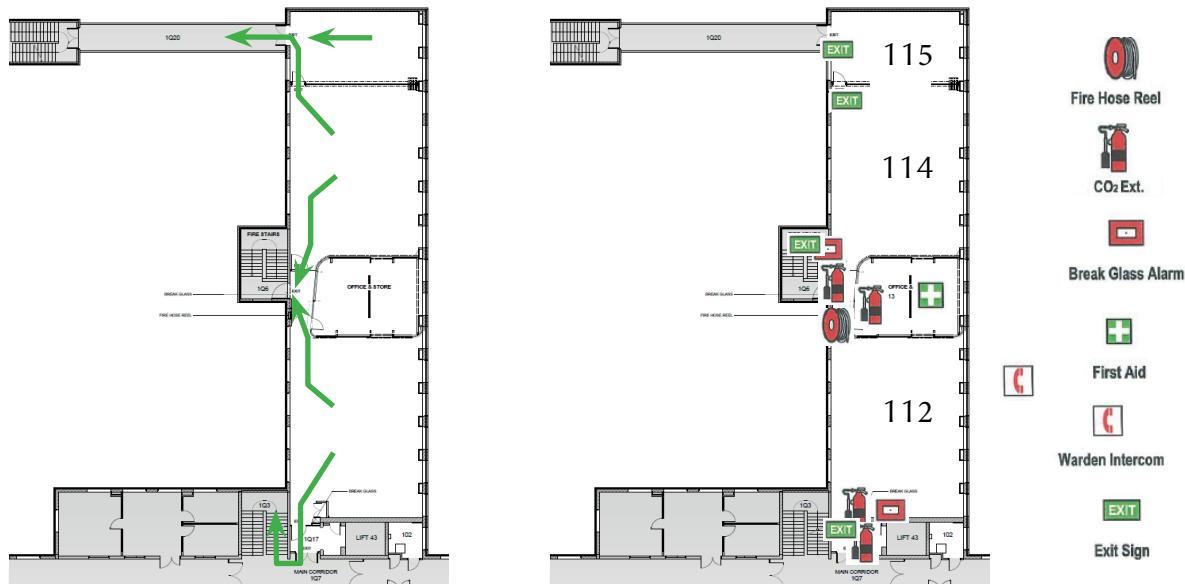
#### 1.1. Emergency Evacuation

The Evacuation Alarm has two distinct tones:

The first tone is called the ALERT tone (Beep-Beep sound and flashing orange lights), which is used to alert the occupants of the building to prepare for an evacuation and to mobilise the evacuation personnel in the building. Upon hearing this alarm turn off any equipment, pack up your personal belongings and wait for the evacuation alarm or further instructions.

The second tone is called the EVACUATION tone (Whoop-Whoop sound, red flashing lights and a recorded voice asking you to evacuate the building). Upon hearing this alarm you MUST EVACUATE the building in an orderly fashion and meet at the marshalling point which is the Village Green (Cricket oval) on the other side of the Newton Building.





There are three emergency exits from the laboratory complex, the usual entrance (or front door), the usual exit (via fire stairs half way down the lab complex) and the emergency exit at the far end of the lab complex from the entry door. In the case of an emergency evacuation DO NOT USE THE LIFT. There is a First Aid Kit, a fire hose and fire extinguishers around the lab, ensure you are aware of their locations (refer to the above map).

## 1.2. Accident and Injury

We have a very good record of safety in the lab. However, if you are nearby when an accident should occur:

- Check the area for any obvious hazards:
  - if there is no obvious hazard, leave the casualty where they are and get help from either a Demonstrator or member of lab staff;
  - if there is an obvious hazard, warn the students around you that there is a hazard and then get help from a Demonstrator or member of staff.
- Give the First Aid Officer plenty of space to work - if you crowd around to watch, you will be asked to assist.

### 1.3. Laboratory Hazards

- The main hazard in the laboratory is the electricity supply. The supply used in the laboratory is the same as you have in the home:

240 VOLTS, ALTERNATING CURRENT, 50 Hz.

As normal, such a supply is hazardous and can cause fatality so take care with mains supplies and mains powered equipment at all times.

You must not attempt to service any electrical equipment yourself. Ask a demonstrator or other member of staff for assistance if you suspect a malfunction, or if you observe that wear-and-tear etc. may be rendering equipment hazardous.

With any electrical hazard, or if electric shock occurs head to the front of the laboratory and alert a demonstrator or other member of staff.

- Another hazard in the lab is a “tripping hazard” – do not leave your bags in the aisles during lab, please put your bags underneath the tables (in the place provided) when not in use and put the seats under the tables when you leave.
- Manual handling is another safety concern – carry equipment with both hands and close to the body. Keep twisting while carrying equipment to a minimum and reduce the distance you reach out to collect or place items.
  - One example is collecting the retort stands – the bases of the retort stands are not always secured tightly to the pole, so you will need to make sure to carry the retort stand in both hands (one on the base and one on the pole).

#### 1.4. Laboratory Rules

There are just a few rules which we request you observe. All of these rules are not put in place randomly but rather form part of the universities Occupational Health And Safety obligations to you and your fellow students. As such should be obeyed at all times and will be strictly enforced by all the FYL staff.

1. *Closed , “sensible” shoes to be worn at all times in the lab (your feet must be covered to the ankle by your shoes - not just the toes), You will **not** be allowed to enter the lab without these!! This rule applies regardless of the type of exercise you will be doing - the room is still a laboratory therefore requires the appropriate safety equipment. The correct footwear is the only personal protective equipment needed, you do not need a lab coat or safety glasses.*

Reason: if you drop a power supply on your big toe it will hurt, and at least spoil your enjoyment of the exercises.

2. If you spill water etc. on the table or floor, please use paper towel to mop it up immediately.
3. Don't run in the lab or throw objects about

Reason: any ‘missile’ is potentially hazardous, even a fast moving human.

4. Eating, drinking and smoking are not allowed in the lab or Study Area. If anyone has ANY type of food or drink container in the laboratory, it will be confiscated and disposed of, regardless of the type of the container.
5. Put your bags in the spaces provided beneath the lab tables to reduce any tripping hazard and any destruction of property.

## 2.Computing/IT facilities

For the computer requirements we have iMacs in the lab. These are used for record keeping and for information resources and data handling in the exercises (data logging, analysis and presentation). There is a section in this lab manual which goes into detail on how to use the iMacs and the programs for each experiment called “Using the Computers” (page xxv of the Introductory section of your lab manual).

### **Online Learning Facilities - Moodle TELT Gateway\***

Your laboratory course will have a UNSW Moodle web site where you can access information and ask questions about the laboratory component of your course. The UNSW Moodle site is where notices about the course are displayed so you should check the UNSW Moodle site regularly for new or updated information.

The Preliminary work for each experiment is also on the Moodle site, this preliminary work is compulsory and must be completed before each lab commences. Your access to the Preliminary work will be closed at the start of your weekly lab so it must be done before the lab. **If you miss a lab and book a catch up lab the pre-work will not be reopened. You should do the preliminary work before your lab is scheduled.**

To access UNSW Moodle, go to <https://moodle.telt.unsw.edu.au> . Click on the UNSW Students and Staff login link to access the Web single sign on page. Use your student number (with a z in front) as your login, and your zpass as your password. After logging in you will see the Moodle home page which will list sites for all courses to which you have access.

(\* TELT, or Technology Enabled Learning and Teaching, is the suite of technologies and applications that support learning and teaching at UNSW. Training resources and support information to help users is also linked to this Gateway. )

**On your course Moodle site there are links to two presentations - a Safety Induction (with a quiz that you must pass) and a Laboratory Introduction (which explains how the labs work and what is expected of you in the Lab) - please watch these presentations before your first lab.**

### 3. Overview of the Session's Labs

There will be a Laboratory Introduction and Safety Induction on Moodle. **Every student must go through the Safety Induction on Moodle in week 1 before commencing the laboratory experiments.**

You will also be given an experiment schedule (available on Moodle) so that you know what you are doing each week. This is important as all experiments have a preliminary work section that must be completed on Moodle before you enter the laboratory. This schedule will inform you as to what you will need to prepare for the next laboratory period.

**The order you are booked in to do the experiments will not necessarily follow the order on the table of contents so do not forget to refer to this schedule regularly.**

Laboratory experiments commence week 2 and generally run for 10 weeks, depending on which subject you are doing. You can find this information on the experiment schedule.

#### a. Laboratory Experiments

Most laboratory exercises are fairly straight forward. In Physics 1A and Higher Physics 1A you will be expected to come up with your own procedures or justify the use of the given procedure for some experiments.

#### b. How the labs are marked

Individual marking schemes apply to each of the laboratory exercises.

The Preliminary work on Moodle will be marked as detailed on Moodle. **You should complete the Preliminary work before your lab commences. The pre-laboratory tests will only be available on Moodle the week prior to when that experiment is scheduled so ensure the quiz is completed when scheduled. The prework tests form 25% of your lab mark.**

Each laboratory will be marked out of 10 marks, according to the marking scheme at the end of each experiment. Please read this table before the lab (for each experiment) so that you know what is expected of you each week. Doing this will help you receive the best marks possible.

#### c. Laboratory Manual

Each exercise will be marked in your own laboratory manual. The laboratory manual must be properly bound (loose pages or sections of the laboratory manual are not acceptable!).

There are some things that can lose you marks. You will lose marks for each of the following:

- **Being late.** Please turn up on time, thus showing regard for your colleagues. **If you are more than 15 minutes late you will not be allowed to commence the lab.**
- **Not finishing the experiment on time.** You must be ready to be marked off by your demonstrator 15 minutes before the end of the laboratory. *All our experiments can be completed in well under 2 hours, IF you complete your preliminary work (Before Lab!) AND read the experiment before attending the laboratory.*

**All experiment marks are included in your final lab mark. It is not the best five or six - all experiments count.**

It is up to you to ensure the Demonstrator enters a mark into an iPad. You can check your mark has appeared on Moodle before your next lab session. If the mark is not there or incorrect, please let your demonstrator know in your next lab session.

d. What you are expected to do each week

There are laboratory exercises for you to **complete at a rate of one per week** for 10 weeks.

You should prepare for each lab period by completing any pre-work on Moodle (there is a video for you to watch and then a test to complete), and reviewing any background and theory associated with the exercise for that week. You must have read the entire lab exercise and have consulted your textbook to fully understand the physics concepts involved. You must bring to the lab your lab manual

**PLEASE TURN UP FOR THE LAB PERIOD ON TIME WITH YOUR OWN LAB MANUAL!**

In the lab you will perform the exercises working in a team of two students, unless otherwise requested by your Demonstrator-In-Charge. Your demonstrator may begin the exercise by reviewing the background and procedure. During the lab period we expect all students to participate fully – this means discussing and planning the practical as an active team member, consulting and discussing aspects of the work with your demonstrator, and asking good questions.

A very important aspect of the lab work is care with numbers: you must **always give the units for any numerical value you write down**. You must also take care to provide reasonable uncertainties wherever appropriate. Marks will be deducted whenever this rule is ignored.

We also mark lab work on the basis of clarity of expression. That is to say, you must always

write clear, complete sentences. Tables, graphs and calculations must always be labelled so that it is obvious what they mean. It is an important part of your general University education that you learn to record your work and communicate results to others.

#### e. Working in Teams

During the session, you are strongly encouraged to work with others in pairs, indeed, in some cases you will be required to, due to limits on quantities of equipment. Feel free to organize your own pair unless otherwise asked to by your Demonstrator-In-Charge.

Also, there is a definite distinction between working actively in a group, and simply taking advantage of other team members to perform all the work. Although you will be sharing equipment, ***it is important to record your own results and conclusions in your laboratory manual.***

Blatant copying of others' work and presenting it as your own is plagiarism, and is considered academic misconduct by the University. Such cases are dealt with severely.

#### f. Laboratory Management Hierarchy

Each laboratory period is managed by a Lab Demonstrator-In-Charge, who will be a member of academic staff or an experienced demonstrator. The Demonstrator-In-Charge is the head demonstrator for any particular lab period and is the Demonstrators' boss. You can also consult the Demonstrator-In-Charge on matters of your progress etc.

#### g. Falling Behind, Missed Labs

We expect you to attend all scheduled, regular laboratory periods in the course. Certain circumstances may cause you to unavoidably miss a lab. The chief reason would be medical, which must be documented. If you have missed a laboratory session and have a medical certificate, you can apply to have an 'average' mark assigned for the missed experiment. At the end of the session we will calculate your average mark per laboratory, and this is the mark that you will get for the missed experiment. You can apply for this option only once, if you are sick more than once you can apply to do a Catch Up Lab.

If you miss a lab for another reason, such as an exam or field trip, having worn the incorrect shoes or not bringing your lab manual, etc., you will need to book a catch-up laboratory using a link on Moodle. There aren't any penalties for doing the experiments in the Catch Up Labs, but you may be doing the experiments alone.

The experimental pre-work **will not be reopened** during the catch up lab week. You must complete it the week you are scheduled to do the laboratory exercise.

h. Laboratory Exemptions

If you are repeating your Physics subject due to failing the theory portion of your course the first time, you may be eligible for a Laboratory Exemption (as long as your lab mark was 15/20 or above). You need to apply for the lab exemption. There is a link through the school of physics website to the online form (there is also a link on Moodle):

***<https://www.physics.unsw.edu.au/content/first-year-teaching-laboratory-exemption-request-form>***

If you are given a laboratory exemption it is expected that you will use your lab time to study Physics. It is a requirement that you see the teaching assistant at least three times during the semester in the study room (room 201A in the old main building). You must show them that you have tried some of the homework problems and have them record your name.

i. Any Questions?

We are here to help you learn and enjoy the labs!

Do you have a question and are not sure who to ask? Ask the Lab Staff, if they cannot answer your question they will direct you to someone who can.

# UNCERTAINTIES AND UNCERTAINTY ESTIMATION

## **Uncertainties, precision and accuracy: why study them?**

People in scientific and technological professions are regularly required to give quantitative answers. How long? How heavy? How loud? What force? What field? Their (and your) answers to such questions should include the value and an uncertainty. Measurements, values plotted on graphs, values determined from calculations: all should tell us how confident you are in the value. In the Physics Laboratories, you will acquire skills in analysing and determining uncertainties. These skills will become automatic and will be valuable to you in almost any career related to science and technology. Uncertainty analysis is quite a sophisticated science. In the First Year Laboratory, we shall introduce only relatively simple techniques, but we expect you to use them in virtually all measurements and analysis.

## **What are uncertainties?**

Uncertainties are a MEASURE OF THE LACK OF CERTAINTY IN A VALUE.

*Example: The width of a piece of A4 paper is  $210.0 \pm 0.5$  mm. I measured it with a ruler divided in units of 1 mm and, taking care with measurements, I estimate that I can determine lengths to about half a division, including the alignments at both ends. Here the uncertainty reflects the limited resolution of the measuring device.*

*Example: An electronic balance is used to measure the weight of drops falling from an outlet. The balance measures accurately to 0.1 mg, but different drops have weights varying by much more than this. Most of the drops weigh between 132 and 139 mg. In this case we could write that the mass of a drop is  $(136 \pm 4)$  mg. Here the uncertainty reflects the variation in the population or fluctuation in the value being measured.*

Uncertainty (often incorrectly still referred to as error) has a technical meaning, which is not the same as the common use. If I say that the width of a sheet of A4 is 210 cm, that is a mistake or blunder, not an uncertainty in the scientific sense. Mistakes, such as reading the wrong value, pressing the wrong buttons on a calculator, or using the wrong formula, will give an answer that is wrong. Uncertainty estimates cannot account for blunders.

## Learning about uncertainties in the lab

The School of Physics First Year Teaching Laboratories are intended to be places of learning through supervised, self-directed experimentation. The demonstrators are there to help you learn. Assessment is secondary to learning. Therefore, do not be afraid of making a poor decision—it's a good way to learn. If you do, then your demonstrator will assist you in making a better decision.

Please avoid asking your demonstrator open ended questions like "How should I estimate the uncertainty". That is a question you are being asked. Instead, try to ask questions such as "Would you agree that this data point is an outlier, and that I should reject it?", to which the demonstrator can begin to answer by saying "yes" or "no". However, do not hesitate in letting your demonstrator know if you are confused, or if you have not understood something.

## Rounding values

During calculations, rounding of numbers during calculations should be avoided, as rounding approximations will accumulate. Carry one or two extra significant figures in all values through the calculations. Present rounded values for intermediate results, but use only non-rounded data for further processing. PRESENT A ROUNDED VALUE FOR YOUR FINAL ANSWER. Your final quoted uncertainties should not have more than two significant figures.

## Some important terms

**Observed/calculated value** A value, either observed or calculated from observations. e.g. the value obtained using a ruler to measure length, or the electronic balance to measure mass, or a calculation of the density based upon these.

**True value** The true value is a philosophically obscure term. According to one view of the world, there exists a true value for any measurable quantity and any attempt to measure the true value will give an observed value that includes inherent, and even unsuspected uncertainties. More practically, an average of many repeated independent measurements is used to replace true value in the following definition.

**Accuracy** A measure of how close the observed value is to the true value. A numerical value of accuracy is given by:

$$\text{Accuracy} = 1 - \left( \frac{\text{observed value} - \text{true value}}{\text{true value}} \right) \times 100\%$$

**Precision** A measure of the detail of the value. This is often taken as the number of meaningful significant figures in the value.

**Significant Figures** Significant figures are defined in your textbook. Look carefully at the following numbers: 5.294,  $3.750 \times 10^7$ , 0.0003593, 0.2740, 30.00. All have four significant figures. A simple measurement, especially with an automatic device, may return a value of many significant figures that include some non-meaningful figures. These non-meaningful significant figures are almost random, in that they will not be reproduced by repeated measurements. When you write down a value and do not put in uncertainties explicitly, it will be assumed that the last digit is meaningful. Thus 5.294 implies  $5.294 \pm \sim 0.005$ .

*For example, I have just used a multimeter to measure the resistance between two points on my skin, and the meter read 564 kΩ—the first time. Try it yourself. Even for the same points on the skin, you will get a wide range of values, so the second or third digits are meaningless. Incidentally, notice that the resistance depends strongly on how hard you press and how sweaty you are, but does not vary so much with which two points you choose. Can you think why this could be?*

**Systematic and random uncertainties**

A systematic uncertainty is one that is reproduced on every simple repeat of the measurement. The uncertainty may be due to a calibration of instrument, a zero reading, a technique uncertainty due to the experimenter, or due to some other cause. A random uncertainty changes on every repeat of the measurement. Random uncertainties are due to some fluctuation or instability in the observed phenomenon, the apparatus, the measuring instrument or the experimenter.

**Independent and dependent uncertainties**

The diameter of a solid spherical object is  $18.0 \pm 0.2$  mm. The volume, calculated from the usual formula, is  $3.1 \pm 0.1$  cm<sup>3</sup> (check this, including the uncertainty). These uncertainties are dependent: each depends on the other. If I overestimate the diameter, I shall calculate a large value of the volume. If I measured a small volume, I would calculate a small diameter. Suppose I measure the mass and find  $13.0 \pm 0.1$  g. This is an **independent uncertainty**, because it comes from a different measurement.

There is a subtle point to make here: if the uncertainty is largely due to resolution uncertainty in the measurement technique, the variables mass measurement and diameter measurement will be **uncorrelated**: a plot of mass vs diameter will have no overall trend. If, on the other hand, the uncertainties are due to population variation, then we expect them to be correlated: larger spheres will probably be more massive and a plot will have positive slope and thus **positive correlation**. Finally, if I found the mass by measuring the diameter, calculating the volume and multiplying by a value for the density, then the mass and size have inter-dependent uncertainties.

**Standard deviation ( $\sigma_{n-1}$ )**

The standard deviation is a common measure of the random uncertainty of a large number of observations. For a very large number of observations, 68% lie within one standard deviation ( $\sigma$ ) of the mean. Alternatively, one might prefer to define their use of the word "uncertainty" to mean two or three standard deviations. The sample standard deviation ( $\sigma_{n-1}$ ) should be used. This quantity is calculated automatically on most scientific calculators when you use the 'σ+' key (see your calculator manual).

**Absolute uncertainty**

The uncertainty expressed in the same dimensions as the value. e.g.  $43 \pm 5$  cm

**Percentage uncertainty**

The uncertainty expressed as a fraction of the value. The fraction is usually presented as a percentage. e.g.  $43$  cm  $\pm 12\%$

## Uncertainty Estimation

We would like you to think about the measurements and to form some opinion as to how to estimate the uncertainty. There will possibly be several acceptable methods. There may be no “best” method. Sometimes “best” is a matter of opinion.

When attempting to estimate the uncertainty of a measurement, it is often important to determine whether the sources of uncertainty are systematic or random. A single measurement may have multiple uncertainty sources, and these may be mixed systematic and random uncertainties.

To identify a random uncertainty, the measurement must be repeated a small number of times. If the observed value changes apparently randomly with each repeated measurement, then there is probably a random uncertainty. The random uncertainty is often quantified by the standard deviation of the measurements. Note that more measurements produce a more precise measure of the random uncertainty.

To detect a systematic uncertainty is more difficult. The method and apparatus should be carefully analysed. Assumptions should be checked. If possible, a measurement of the same quantity, but by a different method, may reveal the existence of a systematic uncertainty. A systematic uncertainty may be specific to the experimenter. Having the measurement repeated by a variety of experimenters would test this.

## Uncertainty Processing

The processing of uncertainties requires the use of some rules or formulae. The rules presented here are based on sound statistical theory, but we are primarily concerned with the applications rather than the statistical theory. It is more important that you learn to appreciate how, in practice, uncertainties tend to behave when combined together. One question, for example, that we hope you will discover through practice, is this: **How large does one uncertainty have to be compared to other uncertainty for that uncertainty to be considered a dominant uncertainty?**

An important decision must be made when uncertainties are to be combined. You must assess whether different uncertainties are **dependent** or **independent**. Dependent and independent uncertainties combine in different ways. When values with uncertainties that are inter-dependent are combined, the uncertainties accumulate in a simple linear way. If the uncertainties are independent, then the randomness of the uncertainties tends, somewhat, to cancel out each other and so they accumulate **in quadrature**, which means that their squares add, as shown in the examples below.

If the uncertainties are dependent (based on the same source of uncertainty) they add linearly:

When	Example	Method
Adding or subtracting	$I = x + x$ $y = x_1 - x_2$ $p = 2(l + h)$ , when the dominant uncertainty is a systematic uncertainty common to the measurement of $l$ and $h$	Add absolute uncertainties $\Delta y = \Delta x + \Delta x = 2\Delta x$ $\Delta y = \Delta x_1 + \Delta x_2$ $\Delta p = 2(\Delta l + \Delta h)$ Note the plus sign
Multiplying or dividing	$a = b \times c$ $v = l \times h \times b$ , where the dominant uncertainty in each is a systematic uncertainty common to $l$ , $h$ , $b$	Add fractional (percentage) <sup>1</sup> uncertainties $\frac{\Delta a}{a} = \frac{\Delta b}{b} + \frac{\Delta c}{c}$ $\frac{\Delta v}{v} = \frac{\Delta l}{l} + \frac{\Delta h}{h} + \frac{\Delta b}{b}$
Using formulae	$y = f(x)$	Use the differential method <sup>2</sup> or test the extreme points. $\Delta y = \Delta x \times f'(x) \text{ or}$ $\Delta y = \frac{f(x + \Delta x) - f(x - \Delta x)}{2}$

If the uncertainties are independent (they have different, uncorrelated sources) they add in quadrature:

Operation	Example	Method
Adding or subtracting	$z = x + y$ $p = 2(l - h)$	Add absolute uncertainties in quadrature. $\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ $\Delta p = 2\sqrt{(\Delta l)^2 + (\Delta h)^2}$
Multiplying or dividing	$z = x \times y$ $v = l \times h \times b$	Add fractional (percentage) uncertainties in quadrature. $\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$ $\frac{\Delta v}{v} = \sqrt{\left(\frac{\Delta l}{l}\right)^2 + \left(\frac{\Delta h}{h}\right)^2 + \left(\frac{\Delta b}{b}\right)^2}$
Using formulae	$z = f(x, y)$	Add partial derivative weighted uncertainties <sup>2</sup> in quadrature. $\Delta z = \sqrt{\left(\Delta x \times \frac{\partial f}{\partial x}\right)^2 + \left(\Delta y \times \frac{\partial f}{\partial y}\right)^2} \text{ or}$ $\Delta z = \sqrt{\left(\frac{f(x + \Delta x, y) - f(x - \Delta x, y)}{2}\right)^2 + \left(\frac{f(x, y + \Delta y) - f(x, y - \Delta y)}{2}\right)^2}$

<sup>1</sup> To get from a fractional uncertainty to a percentage uncertainty multiply by 100.

<sup>2</sup> Beware of the possibility of a turning point between  $x+\Delta x$  and  $x-\Delta x$ , which will cause this method to fail.  
(e.g.  $y = \sin(x)$ , where  $x = 90 \pm 10^\circ$ )

## Averages

When performing an experiment, a common way of obtaining a better estimate of something which you are trying to measure, is to take repeated measurements and calculate the average, or mean, of these measurements. Random uncertainties in the measurements cause them all to be slightly different, so the mean may be thought of as an estimate of the “true value”.

For a set of  $n$  measurements  $(x_1, x_2, x_3, \dots, x_n)$  the mean,  $\bar{x}$ , is given by

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

For example, supposing we measured the time it takes for a ball to fall through a height of 3 metres, and repeat it another six times. The results might look like this:

Time (s)	1.11	1.18	1.02	1.09	1.10	1.13	1.12
----------	------	------	------	------	------	------	------

The mean of this data is given by

$$\frac{1.11 + 1.18 + 1.02 + 1.09 + 1.10 + 1.13 + 1.12}{7} = 1.107 \text{ s}$$

Just as there is uncertainty in each of the measurements, there is also uncertainty in the mean. This uncertainty may be calculated in a variety of ways, which depend on how many measurements have been made.

The simplest estimate of the uncertainty in the mean comes from the range of the data. The range is simply the difference between the most extreme values in the set. In the above example the extreme (highest and lowest) values are 1.18 and 1.02, so the range is 0.16. The uncertainty in the mean, then, is given by the half of the range, that is

$$\Delta\bar{x} = \frac{x^+ - x^-}{2}$$

where  $x^+$  and  $x^-$  are the highest and lowest values in the set, respectively.

This method provides a good estimate when you have less than 10 measurements. So in our example, the uncertainty in the mean time is  $0.16/2 = 0.08$ , so that we can write the mean as  $(1.11 \pm 0.08)$  s. Note that we have dropped the last significant figure in the mean, as the uncertainty is large enough to make it meaningless.

If you have taken a lot of measurements, then the uncertainty in the mean is given by the standard deviation,  $\sigma$  (mentioned earlier on page xvi). Most calculators can calculate the standard deviation, but you can also use the formula:

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

and the uncertainty in the mean is given by

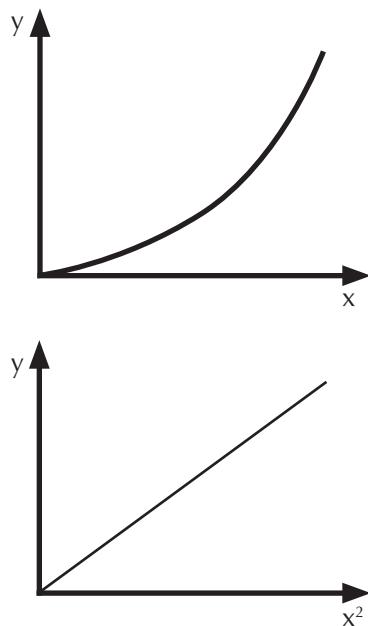
$$\Delta\bar{x} = \sigma_{n-1} = \frac{\sigma}{\sqrt{n-1}}$$

Generally, in First Year Lab, you won't be taking very large sets of measurements, and so will not be using this method. Use half the range for a quick simple uncertainty estimate, taking care to be sure that your extreme values are not outliers (unusually small or large values, usually the result of some one-off mistake). Outliers should generally be discarded when taking averages, and when graphing.

## Graphing

Scientists and technologists very often organise their variables so that a particular theory becomes a straight line on the plot. The reason is that a straight line is very easy to recognize, and even small departures from it can be easily seen. This is harder with curves. For instance, suppose that a theory predicts  $y = ax^2$ . The curve in the upper graph at left looks a bit like  $y = ax^2$ , but it also looks a bit like  $y = a(1 - \cos x)$ , and a bit like  $y = bx^{1.8}$ . It's hard to tell.

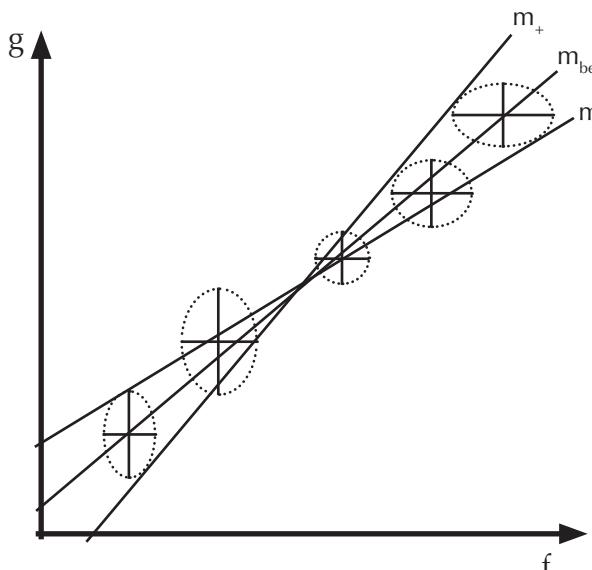
In the lower curve,  $y$  has been plotted against  $x^2$ . Let's call the new variable  $z = x^2$ . This graph does look rather like  $y = az$ , and it doesn't look like  $y = a(1 - \cos \sqrt{z})$ , unless  $z$  is very small, and it doesn't look at all like  $y = bz^{0.9}$ .



For the rest of this section, we shall therefore discuss only graphs in which the variables have been plotted to produce a straight line for the theory being investigated.

## Graphing with Uncertainties

When graphing, **plot uncertainty bars**. A very sharp pencil is good for getting the size just right. If uncertainty bars are less than about 1 mm, then do not try to show them: instead write the size of the uncertainty on the graph and also show your calculation of the uncertainty bar length. If both uncertainty bars (vertical and horizontal) are too small to plot, draw a circle around the experimental point.



Draw a line of *best fit* and lines of *worst fit*. In many cases, your uncertainty bars will be the maximum probable uncertainty, so you should have a high probability that the true value lies within your uncertainty bar.

Sketch (or just imagine) an ellipse whose axes are the uncertainty bars—this is called the *uncertainty ellipse*<sup>3</sup>. In this case, every line should pass through the uncertainty ellipse about every point. A worst fit line necessarily touches the edge of at least two points.

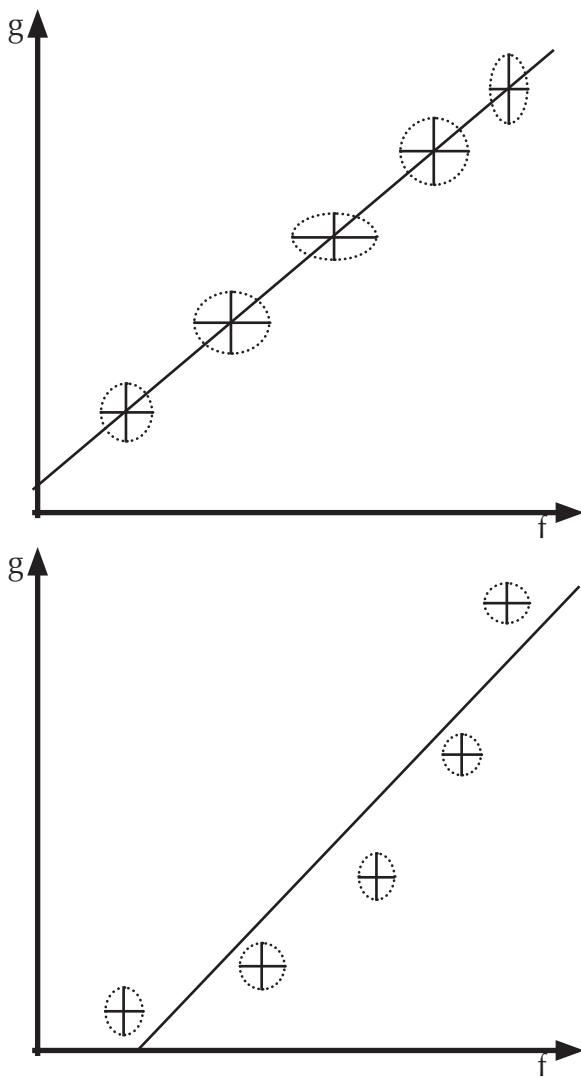
<sup>3</sup>We draw an ellipse, rather than a rectangle, because of the quadrature addition of independent uncertainties.

The uncertainty in the gradient is

$$\Delta m = \frac{m_+ - m_-}{2}$$

To find the y-intercept, other lines of worst fit may have to be drawn. The worst fit that produces the greatest y-intercept, and the worst fit that produces the smallest y-intercept may not necessarily be the same as the worst fits used to find the extremes in gradient. The extremes in the y-intercept may be produced by a combination of rotating the fitted line and moving it without rotation.

In the case shown above, the deviation of the measured values from the fitted line are comparable in size to the uncertainty bars. This is a 'normal' case.



In the graph to the left, the uncertainty bars are large compared with the departure of the measured points from the fitted line. This suggests that the uncertainty estimates are too large: they should be re-examined.

In this graph, the uncertainty bars are small in comparison with the departure of the measured points from the fitted line. It is impossible to fit a straight line without rejecting a substantial fraction of the data as outliers. Such a result suggests either:

- (a) the uncertainty estimates are too small;
- (b) that the measurements were made carelessly;
- (c) that numerical blunders have been made in treating the data; or
- (d) that the relation is better described as non-linear, which means that the theory which gives a straight line in this plot is wrong or inappropriate here.

Further, the general shape of the points suggests that it would be a good idea to try a different plot, such as g vs f or  $\ln g$  vs  $\ln f$ .

### Automatic graphing routines

Most common software packages that graph data and fit lines or curves do not take uncertainties into account. Many do not even plot the uncertainties. Further, they give all points equal weight, even if there is a big variation in the uncertainty bars. Graphing by hand, as described above, you give the points with small uncertainty bars more importance (the statistical term is **weight**) because the line is more tightly constrained to pass through the smaller uncertainty bars. It is possible to include appropriate weighting factors (usually the reciprocal of the uncertainty) in automatic routines.

In the first year lab, you can use the excel template "Linear plot with Errors" (uncertainty is often incorrectly still referred to as error). This will plot the line of best fit and also the maximum and minimum gradient lines that satisfy your data. You can download this template from the "introductory Experimentation" book on Moodle (which you find in the Laboratory section of the Moodle course page).

### References

'Experimental Methods. An Introduction to the Analysis and Presentation of Data.' Les Kirkup, Wiley, (1994).

'Data Reduction and Error Analysis for the Physical Sciences.' Philip R. Bevington, McGraw Hill (1969).

'Statistical Methods in Medical research.' (3rd Edition) P. Armitage & G. Berry, Blackwell: Oxford, (1994).

'Handling Experimental Data.' Mike Pentz, Milo Shott and Francis Aprahamian, Open University Press (1988).

## UNCERTAINTIES AND UNCERTAINTY ESTIMATION

# USING THE COMPUTERS

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## Introduction

In the First Year Lab there are a number of iMac computers that are to be used in the lab exercises. These computers will sometimes be used to take measurements, with the help of a "Lab Pro" interface, and also for processing of data, e.g. graphing.

Whenever an exercise requires the use of the computers you will be told which application to use and there may be a template preconfigured for the exercise.

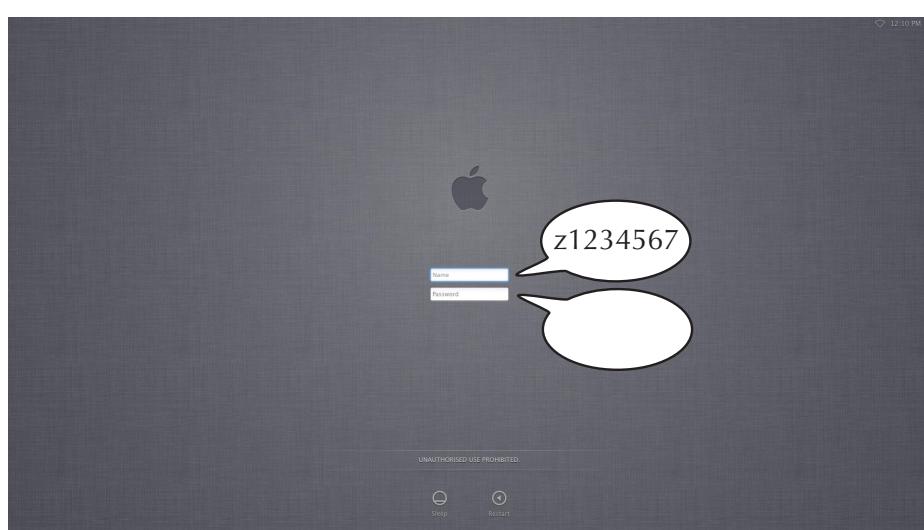
The following notes give an introduction into using the computers in the First Year Lab and also outline the basics of the applications that you will use.

## The Computers

### Logging In

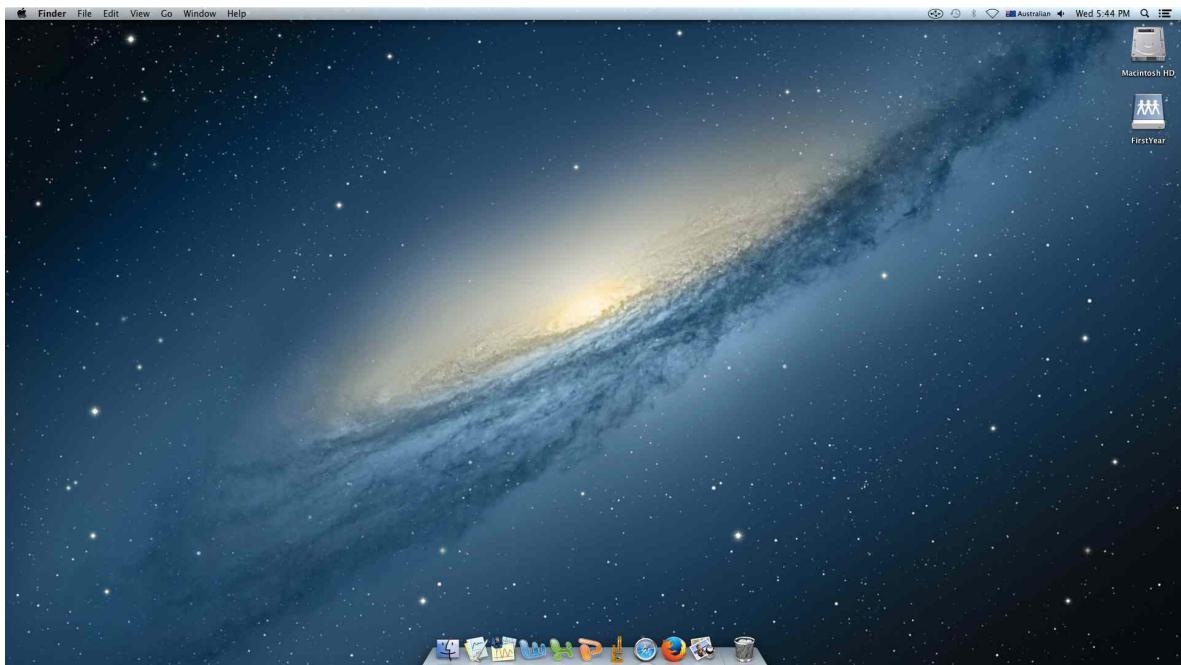
All the computers should already be switched on. If the computer you want to use is not just turn it on. The power button for the iMacs is on the back of the computer on the left hand side. If the computer is asleep then simply moving the mouse should wake it.

The first thing that you should see on the computer will be a box asking for you Name and Password. Your name for the computer is z followed by your student number. i.e. z1234567, where 1234567 is your student number. Your password is the same as your name.



## The Desktop

After you have logged in you will see your desktop. There should be two items on your desktop, the hard disk and a network disk. If you are unfamiliar with Mac OS X then you might also notice the “Dock” at the bottom of the screen and the menus at the top.



The Screen the first time you log in

The Dock allows easy access to programs. Simply clicking on any icon in the Dock will launch the associated program.

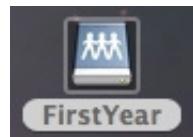


The Dock

The menus at the top of the screen will change depending on what program you are in, although the apple menu, , will always remain and is used for universal things like logging out of the computer.

## Setting Up Your Account

The first time you log in you should drag some files from the network disk to your desktop. Double click on the network disk on the desktop, its the object called “First Year”.



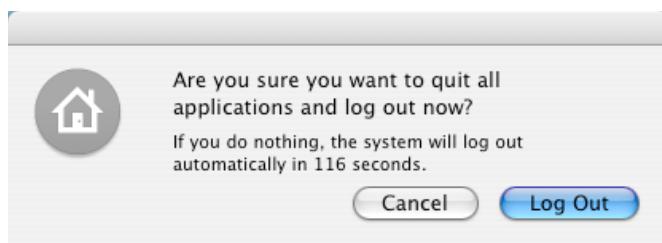
The network disk

This should open up a window that contains two folders. One of these folders is named “Drag to desktop and rename as Experiments” and is highlighted in red. You should do as the name suggests and drag this folder to your desktop. To rename the folder simply click once on the name and then type in the new name. Inside this folder is a folder for your course containing the templates for all you will need to complete the exercises in the lab.



## Logging Out

When you are finished using the computer you should log out. Firstly click on the apple menu, . The last option is the one that you want. Clicking on this will bring up a box confirming your log out, clicking on “log out” will log you out of the computer. If you have any running programs with unsaved data they will ask you to save before the log out completes. If you do not answer the prompts the log out will cancel.



## Apple Computer Keyboard Short-cuts

Key of symbol definitions:

<b>⌘</b>	Command key	<b>⌃</b>	Control key	<b>⌥</b>	Option key
<b>⇧</b>	Shift key	<b>⇪</b>	Caps Lock	<b>⌃</b>	Escape
		<b>⌫</b>	Delete		

List of common Apple keyboard short-cuts:

Action	Shortcut	Details
Force Quit	<b>⌥⌘⏏</b>	Brings up a window where you can select an unresponsive program to force quit.
Quit Program	<b>⌘Q</b>	Must select "Save" or "Don't Save" before program will quit properly. Selecting the red circle at the top of the window will only close that window – it <b>does not</b> close the open program.
Log-out from the computer	<b>⇧⌘Q</b>	Must quit all open programs first before this will work.
Minimise open window to dock	<b>⌘M</b>	Click on program icon on the dock to re-open minimised window.
Close Window	<b>⌘W</b>	Closes open window - does not close open program (if applicable).
Go to the Desktop	<b>⇧⌘D</b>	Straight to the desktop without wasting time minimising open windows.
Empty Trash	<b>⇧⌘⌫</b>	Empties the trash.
Find	<b>⌘F</b>	Finds content.
Select All	<b>⌘A</b>	Selects all items in the foremost open window.
Undo	<b>⌘Z</b>	Undoes last action.
Redo	<b>⇧⌘Z</b>	Redoes last action.
Cut	<b>⌘X</b>	Cuts text/diagrams from content and places it on the clipboard.
Copy	<b>⌘C</b>	Copies text/diagrams and places it on the clipboard.
Save	<b>⌘S</b>	Saves open window.
Save As...	<b>⇧⌘S</b>	Saves open window in the desired file format.
Paste	<b>⌘V</b>	Pastes content from clipboard into the desired location

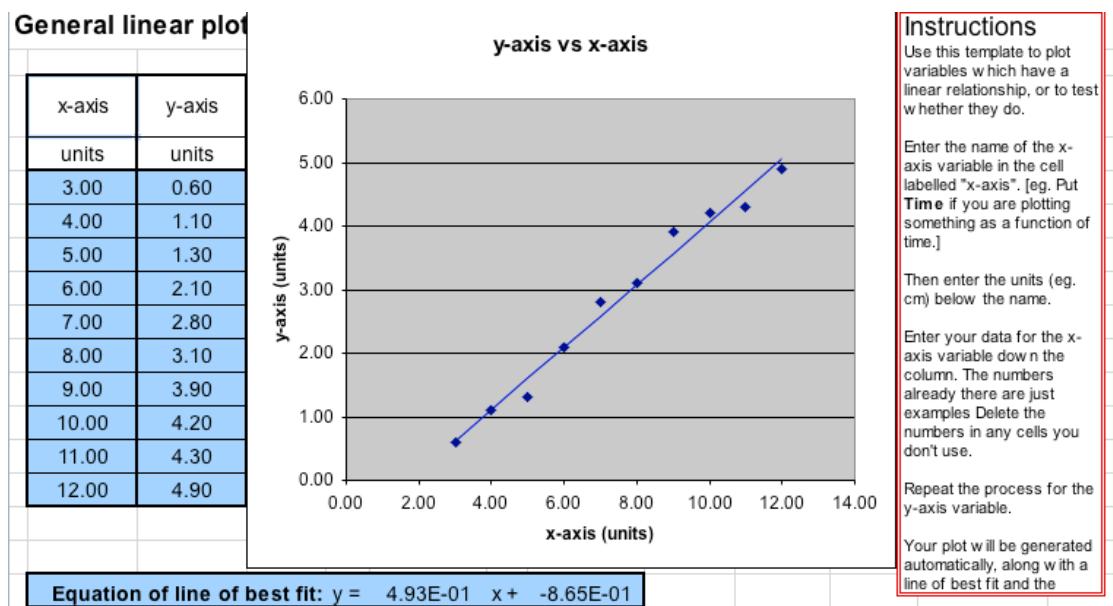
There are more shortcuts than those listed above – please refer to the drop down lists from the menu bar at the top of the desktop for further details.

## Programs

There are two main programs that you will use in the lab. In both cases it is easier to launch the program by double clicking on one of the files needed as this will open the required file as well as the program. Alternatively the programs can be launched from the Dock and the file opened within the program.

### Microsoft Excel

Excel will be used to draw linear plots. There are two Excel templates that you will use in multiple exercises: "General Linear Plot" and "Linear Plot with Errors". These templates are similar in their layout. There are columns for you to enter your data, a place where the graph is displayed, and an equation of the line produced.



This is what you should see when you open the "General Linear Plot" file.

Some exercises, such as "Rotational Inertia", have their own template. These exercises are identified in the laboratory manual. For some exercises such as "The Pendulum" and "Standing Waves on a String" you may find it easier to use excel to calculate the errors, rather than your calculator. If you do use excel, make sure you also record the values in a table in your laboratory manual.

## LoggerPro

The LoggerPro Software is used to collect, display and analyse data collected by the LabPro data interface and the sensors which work with it. The software will generally already be set up if you open the file for the corresponding experiment. A summary of the important features is given below.

## LabPro

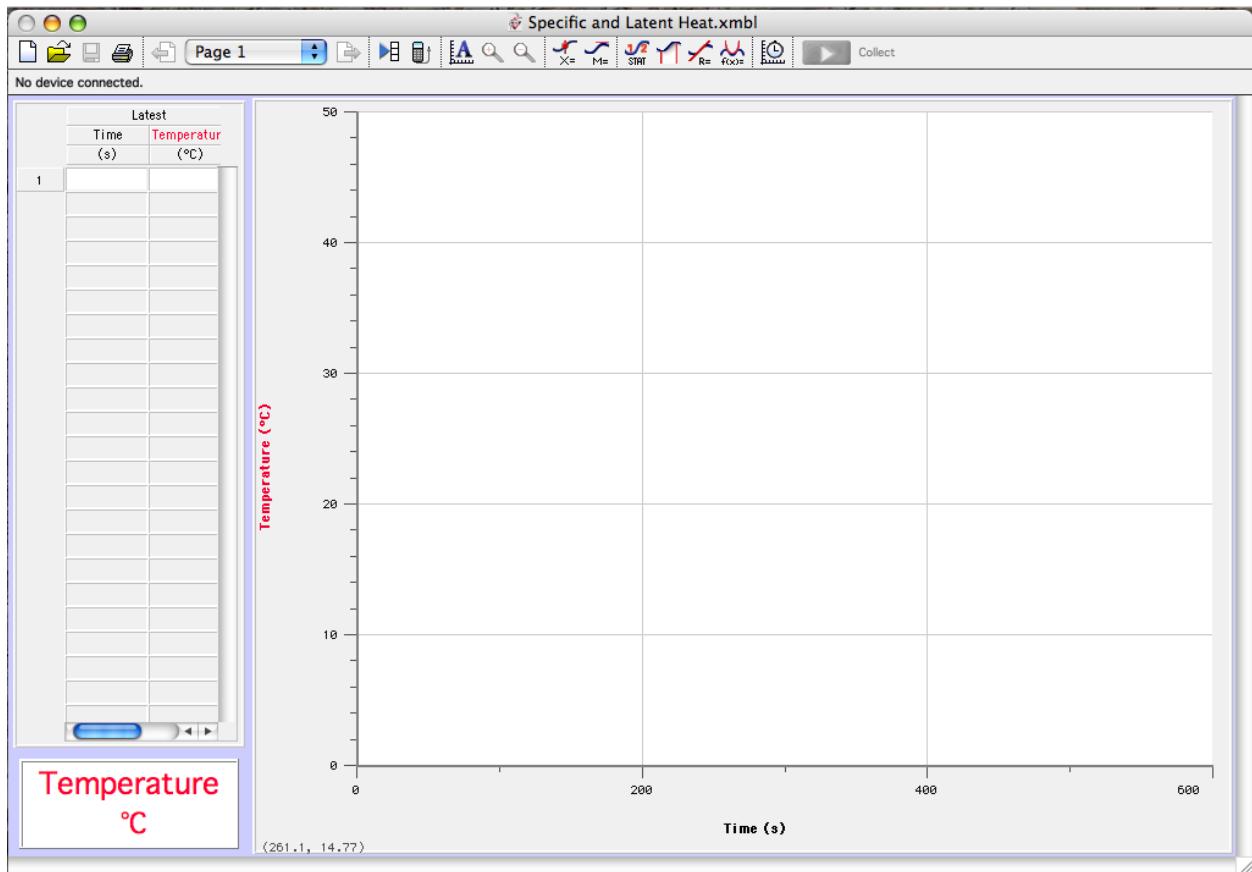
Before using the LoggerPro software you will need to set up and connect a LabPro. A LabPro is relatively easy to setup, firstly plug the power supply into the into the power socket of the LabPro. Next connect the sensor that you wish to use into the appropriate input of the LabPro. Then connect the USB cable to the LabPro (Note: the USB icon on the cable faces down when you plug it into the LabPro) and the back of the computer. Finally switch on the power to the LabPro. Once the power is on the LabPro should make a series of beeping sounds and some lights will flash, if this does not happen then you should make sure that the power is connected properly. Once the LabPro is setup then you should start the LoggerPro software. It is usually easier to start the software by double clicking on the associated file for the exercise that you are doing. Note: The files for each experiment assume that the sensor is connected either in CH 1 (for the temperature probe, accelerometer, or magnetic field probe) or DIG/SONIC 1 (for the motion detector).

**Do not force any cable into the LabPro. All cables should go in easily otherwise see your demonstrator.**



## LoggerPro

When you start the LoggerPro software you should see something that looks like this, although there will be slight differences depending upon which experiment file is opened.

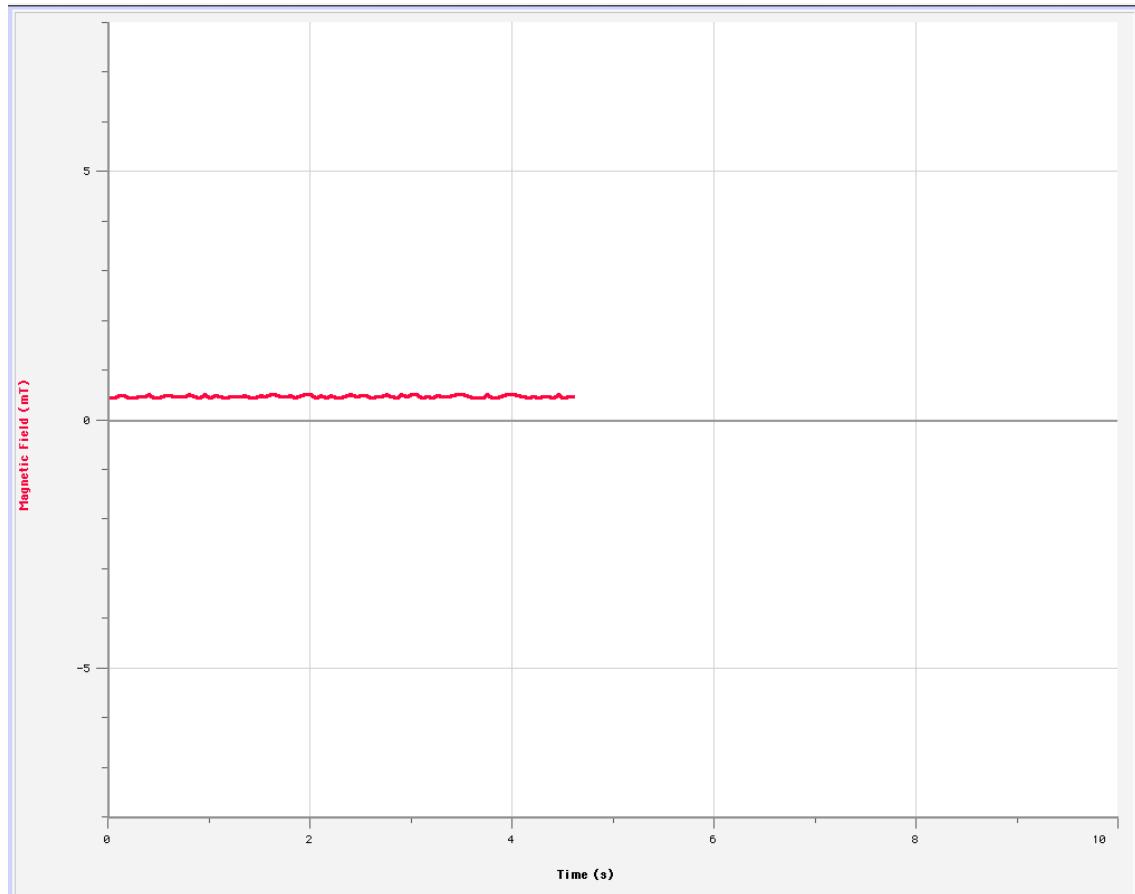


There are three main windows that display data, the graph, the columns of data, and the live read-out. For some parts of some experiments all that you will need will be the live read-out but most of the time you will need to use the program to generate graphs.

You can accomplish almost everything that you will need to do in LoggerPro with the buttons that are at the top of the window.

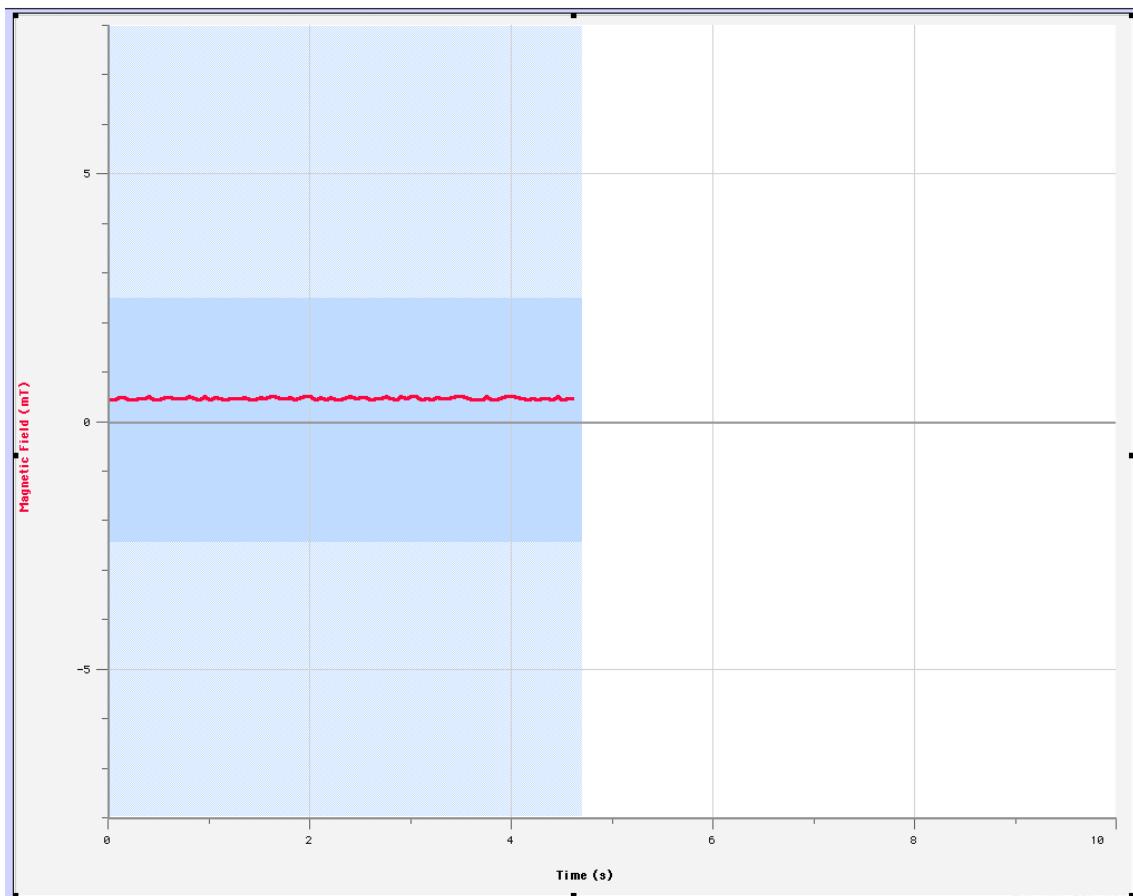


The collect button, , is used to start a collection run. When this is pressed the program will record the readings from the LabPro for a set amount of time and display the results as a graph and also in the data columns. If you want to change the setup of a collection run, e.g. how many samples a second, hitting the data collection button, , will open a dialogue box where this can be done. An example of a graph from the magnetic field sensor is given below.

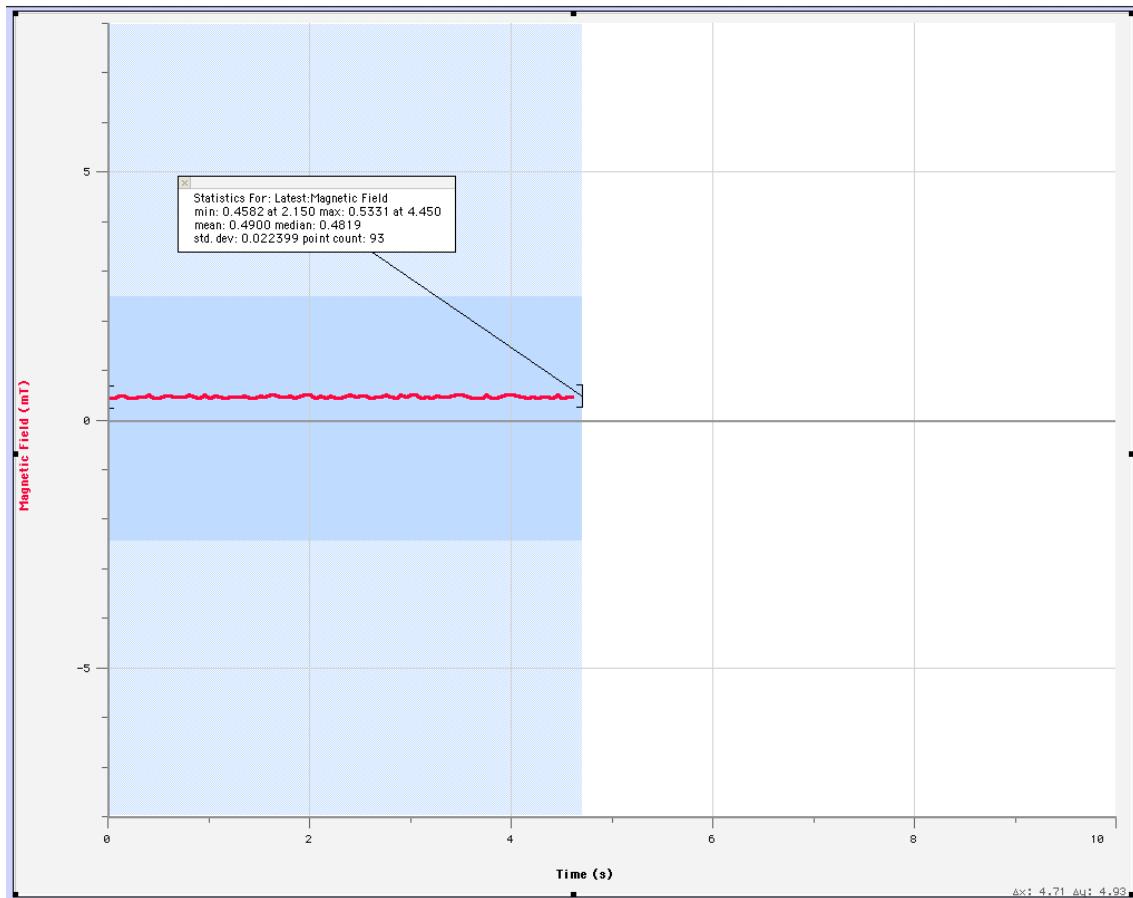


Once you have a graph some other buttons become useful. The first thing that you should do is resize the graph so that it is easy to see. The autoscale button, , automatically resizes the graph so that it best fits into the window. Depending on what you want to know from the graph the computer can tell you just about everything. If you want to know the reading on a particular point in the graph then the examine button, , lets you move the mouse over the graph giving you the value at each point. The tangent button, , does a similar thing but will tell you the value of the tangent at each point.

The following buttons work best if you highlight the region of interest in the graph first. To do this you should click and drag around the region of the graph that you are interested in, you should end up with some thing like this:



The statistics button, , can be quite useful, it brings up a box that gives you the statistics of the highlighted region. This is good if you want to know a mean value and standard deviation. Occasionally it is a good idea to collect data even when you could just read the number you want from the live reading. This is because there can be large fluctuations in the live reading. By taking the mean value with the statistics button you eliminate these fluctuations and also as a bonus the standard deviation gives a good approximation to the uncertainty. For the above graph the statistics button brings up a box like this:



The integral button, , gives the value of the area under the graph. There are two buttons for fitting curves to your graph. The linear fit button, , attempts to fit a straight line to the graph. Make sure that the graph should be straight line before attempting to do a linear fit, otherwise the results will be meaningless. The other fitting button, , is for fitting curves. This button brings up a box that allows you to select the type of curve that you would like to try and fit to your data.

## Other programs

There is an assortment of other programs available to use on the computers. Microsoft Word and Powerpoint are present and may help those who will be making a presentation as part of their lab classes. Some lab classes may be required to use some other software that has not been covered in this section, if this is the case then you will be shown how to use it at the appropriate time.

## Printing

You may NOT print from any computer in the lab. For all the normal lab classes there should be no reason to print anything.



# **INTRODUCTORY EXPERIMENTATION**

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## **Introduction**

In the first week's lab exercise you will learn how to:

- Perform good experiments and make accurate measurements.
- Handle and process measured data.
- Estimate and assign realistic uncertainties (experimental) to your measured values.
- Combine uncertainties arising from different sources in an experiment.
- Use the computers to order, analyse and present the results of your experiments.

The class is divided into the following parts:

- 1) *Logging into your computer account*
  - getting your First Year Lab computer account started so you can use the programs and templates you may need in your course.
- 2) *Sample Experiments and Uncertainty Processing*
  - written exercises with more involved uncertainty handling.
- 3) *Linear Graphing Exercise*
  - written and computer based exercises on graphing data to obtain useful information.
- 4) *Linear Graphing Exercise*
  - written and computer based exercises on graphing data to obtain useful information.
- 5) *Graphing With Uncertainties*
  - written and computer based exercises using data with errors and graphing it manually.

## **What To Do Before You Get To Lab**

- 1) Begin by reading through the Uncertainties section in your Laboratory Manual. You may also want to consult the reference text:  
*Experimental Methods: An Introduction to the Analysis and Presentation of Data*, Les Kirkup, Wiley (1994)
- 2) Read the “Using the Computers” section in the manual. This will explain how to do general tasks such as starting up applications on the computers, once your account has been set up.
- 3) Attend the problem solving workshop in week 1; you will be introduced to the concept of uncertainties in this class.
- 4) Watch the video about the lab; the link can be found on Moodle.
- 5) Complete the preliminary problems and prework test on Moodle, which will introduce the material and concepts upon which the exercises are based.

## In the Laboratory

### Part 1: Logging into your computer account

Login to the computer with zID and then zPASS

### Part 2: Sample Experiments and Uncertainty Processing

Using similar methods to the pre-work problems on Moodle, complete the following exercise. Ask your demonstrator if you need any help. Present your work in a suitable manner.

To determine the density of brass, Judy measures the mass of a small brass sphere using a balance and finds it to be equal to  $(14.40 \pm 0.05)\text{g}$ . She then measures the diameter using a micrometer and finds that the diameter is variable (i.e. that the “sphere” is only approximately spherical). She measures the diameter in a number of different positions and obtains the following results:

Diameter (mm)	15.40	15.32	15.35	15.39	15.33
---------------	-------	-------	-------	-------	-------

What is the density of brass (value and uncertainty) according to her measurements?

Get  
Marked  
Now

### Part 3: Linear Graphing Exercise

As a small ball bearing is allowed to fall from rest, it is illuminated with a strobe light so that it can be photographed and its position determined.

Corresponding values for time,  $t$ , and position,  $S$ , are:

$t$ (sec)	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800
$S$ (m)	5.16	5.00	4.85	4.39	3.98	3.37	2.76	2.00

According to theory, the relationship between  $S$  and  $t$  is of the form:

$$S = S_0 + \frac{1}{2}at^2$$

Use the computer to plot the results and check whether the figures are consistent with this relationship. Use the "General Linear Plot" template (see the 'Using the Computers' section in the front of the manual for information on obtaining templates).

What quantity are you plotting on:

- a) x axi
- b) y axi

Record the equation of the line of best fit here:

If so, evaluate the constants  $S_0$  and  $a$ .

$$S_0 =$$

$$a =$$

Get  
Marked  
Now

## Part 4: Linear Graphing Exercise

Complete the following exercises. Discuss the questions with your teammate and your demonstrators.

1) In an electric circuit, a current,  $I$ , is varying with time,  $t$ .

The current is measured at intervals as follows:

time (ms)	0.0	1.0	2.0	3.0	4.0	5.0	6.0
current (mA)	99	61	37	22	14	8.0	5.0

Check by graphical methods if these figures are compatible with a relationship in the form:

$$I = A e^{-kt}$$

where  $A$  and  $k$  are constants. If so, find the values of  $A$  and  $k$ .

The value of the fundamental constant  $e$  is 2.718. Use the “General Linear Plot” template.

### Hints:

- Don't just plot the time vs current, think about what you should plot instead.
- Rearrange the above equation by taking the logarithm of each side.

Get  
Marked  
Now

## Part 5: Graphing With Uncertainties

Complete the following graphing exercise. Consult the Uncertainties and Uncertainty Estimation notes in the front of your manual for help.

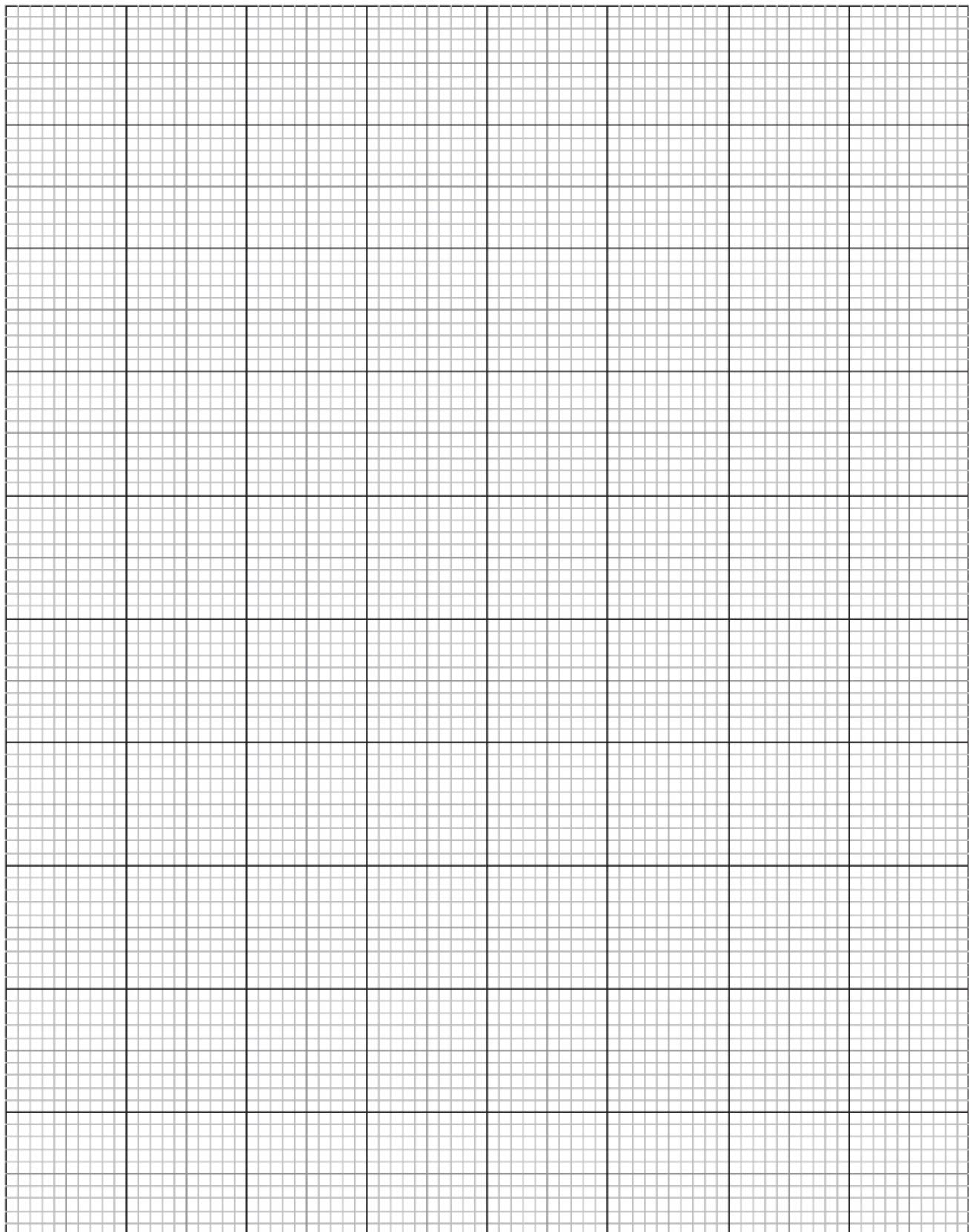
Two students are determining the e.m.f. and internal resistance of a cell from measurements of the potential difference across the cell. The following table gives their results and their estimates of the uncertainties in each measurement.

cell p.d. (V)	$1.98 \pm 0.01$	$1.96 \pm 0.01$	$1.93 \pm 0.01$	$1.90 \pm 0.01$	$1.87 \pm 0.01$
current (mA)	$10.0 \pm 0.5$	$15.0 \pm 0.5$	$20.0 \pm 1.0$	$25.0 \pm 1.0$	$30.0 \pm 1.0$

- a) Using the graph grid on the following page, plot a graph of the cell terminal potential difference against current drawn. Include uncertainty bars to show the uncertainty in the location of each point. Choose your axes so that the gradient is the resistance in ohms. You will need to use  $V=IR=\text{emf}$
- b) Is there a linear relationship amongst the quantities? If so, draw the line of best fit for the points. Then determine the slope and intercept of this line and the uncertainties in the slope and intercept.

Get  
Marked  
Now

## INTRODUCTORY EXPERIMENTATION



(working space)

Get  
Marked  
Now

## Marking Guidelines

Get your Demonstrator to tick the boxes below as you go through the exercise.

Please enter your details and date below before you get your final mark from the Demonstrator.

Student Name: \_\_\_\_\_ Student ID Number: z\_\_\_\_\_

Date: \_\_\_\_\_

Part 2: determined the volume of the brass sphere correctly, including units	
Part 2: calculated density of brass correctly, including units	
Part 2: calculated the uncertainty in the density of brass correctly	
Part 3: drawn an appropriate graph on the computer	
Part 3: found $S_0$ and a, including units	
Part 4: drawn an appropriate linear graph	
Part 4: found the values of A and k, including units	
Part 5: drawn graph on graph paper with error bars	
Part 5: determined slope and intercept, including units	
Part 5: determined the uncertainty in the slope and intercept	

Total: /10

Demonstrator's Signature: \_\_\_\_\_

Demonstrator's Name: \_\_\_\_\_

# THE PENDULUM

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## Introduction

A good physics experimental design will minimise the uncertainties in the quantities measured as much as possible.

This exercise is designed to help you:

- Increase your understanding of the role of uncertainties in the design of experiments

## What To Do Before You Get To Lab

- 1) Read through the Preliminary Information starting on the next page.
- 2) If you are still uncertain about the theory for this experiment consult the relevant sections of your textbook or some other source.
- 3) Answer the preliminary problems and prework test on Moodle.
- 4) Read through the remaining parts of the exercise so that you know what to do when your class begins.
- 5) Watch the video about the experiment online. There is a link to it on the Moodle site.

## Preliminary Information

### The Simple Pendulum

For an idealised pendulum consisting of a point mass,  $m$ , suspended by a weightless, in-extensible and frictionless string of length  $l$ , the force due to gravity will tend to move the pendulum back towards the vertical position (indicated by the dashed line).

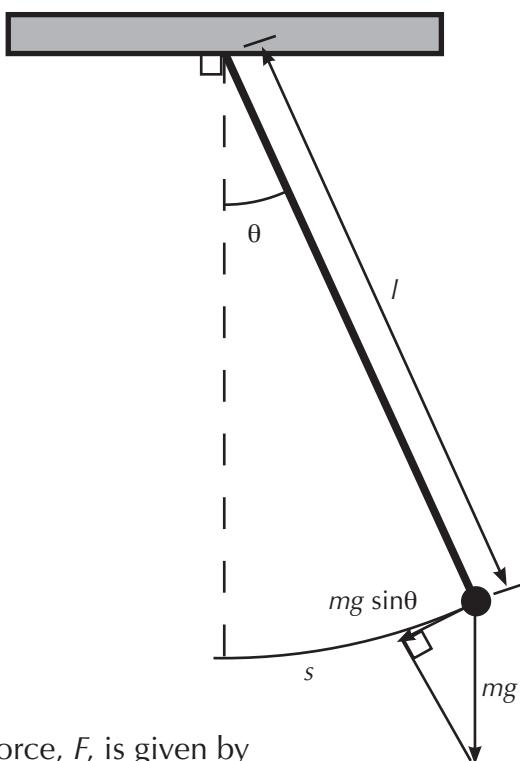
The gravitational force,  $mg$ , acts vertically down, as shown, but may be considered as having components normal and tangential to the direction of motion of the pendulum. It is the tangential component which acts to restore the pendulum to the vertical position, and it is equal to  $mg \sin\theta$ , where  $\theta$  is the angle the pendulum makes with the vertical.

If  $\theta$  is small then we can say that the restoring force,  $F$ , is given by

$$F = mg \sin\theta$$

$$\approx mg\theta$$

$$= mg \frac{s}{l}$$



That is, the restoring force is proportional to the displacement,  $s$ , of the pendulum.

Whenever a body is acted upon by a force which is proportional to its displacement, it will undergo simple harmonic motion (SHM), oscillating back and forth about an equilibrium position (in this case, the vertical position). The period of oscillation,  $T$ , of a body undergoing SHM is given by

$$T = 2\pi \sqrt{\frac{m}{k}}$$

where  $k$  is the constant of proportionality between the force and the displacement. In the case of the simple pendulum described above, this constant is equal to  $(mg/l)$ , so the period of oscillation of the pendulum is given by

$$T = 2\pi \sqrt{\frac{m}{mg/l}} = 2\pi \sqrt{\frac{l}{g}}$$

Note that the period depends upon the length of the pendulum, but not the mass.

The period and length of a pendulum is easily measured in the laboratory, and so we can use these measurements to determine the acceleration due to gravity,  $g$ .

You will derive this equation in lectures in the oscillations part of the course.

### The Physical Pendulum

The derivation above for the period of the pendulum is based on two approximations. Firstly, that the pendulum bob is a point mass, and that the angle through which the pendulum swings is vanishingly small. These approximations place limitations on any experiment we perform using the pendulum. We will examine the effects of these approximations separately.

#### *The effect of a non-vanishing angle*

If we set a (point mass) pendulum swinging by releasing it from an angle  $\theta_0$ , the actual period of oscillation,  $T$ , is given by:

$$T = T_0 \left[ 1 + \frac{\theta_0^2}{16} + \frac{11\theta_0^4}{3072} + \frac{173\theta_0^6}{737280} + \dots \right]^{**} \quad -(1)$$

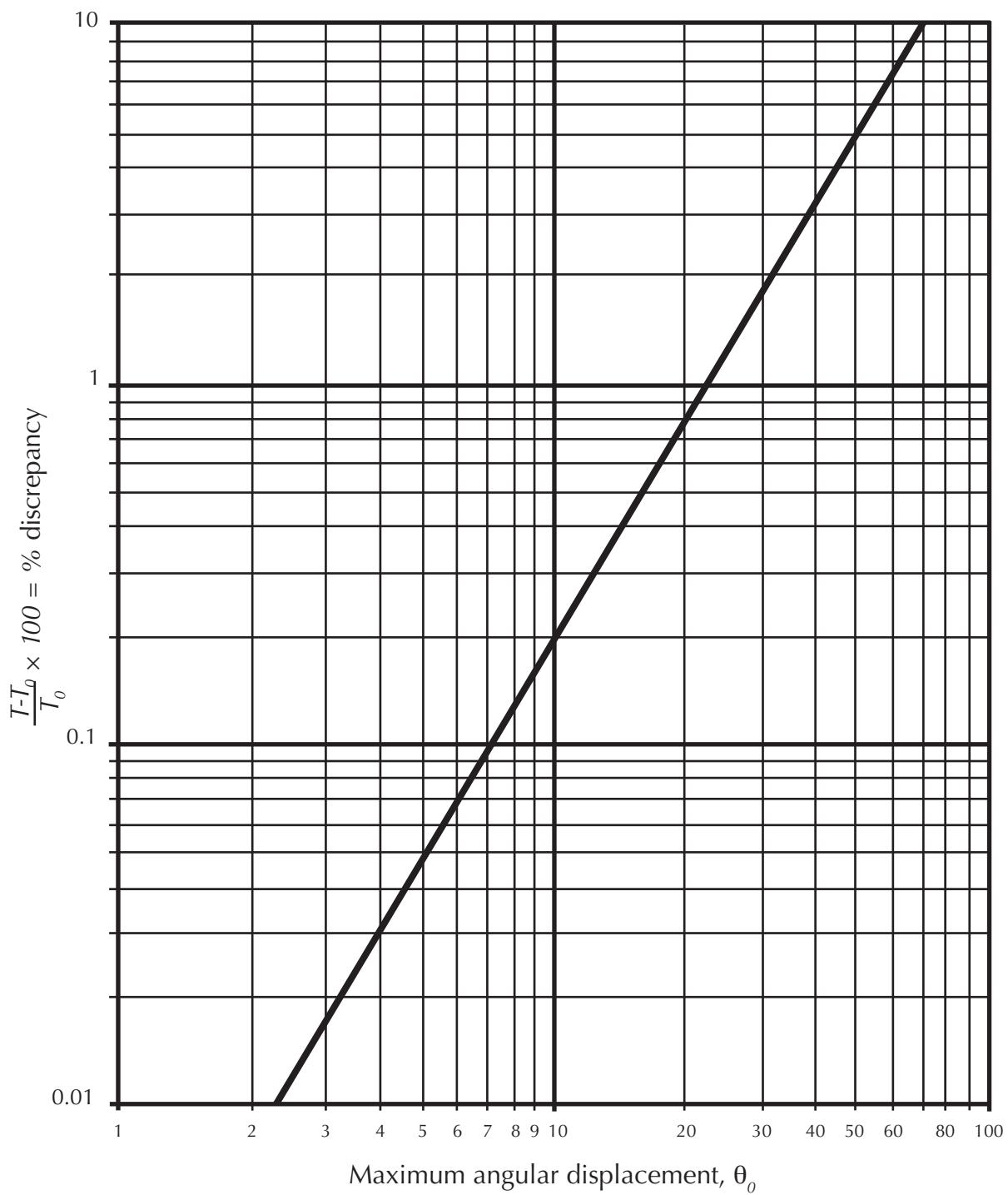
where

$$T_0 = 2\pi \sqrt{\frac{l}{g}} \quad -(2)$$

is the expression previously derived for the ideal case ( $\theta_0 \rightarrow 0$ ).

In all other cases, if expression (2) is used to predict the period of a simple pendulum, then the value obtained will be less than the actual period. The following graph shows the percentage discrepancy between the actual period,  $T$ , and that given by expression (2),  $T_0$ , as a function of the maximum angular displacement,  $\theta_0$ .

\*\* Footnote: This can be derived by solving the equation  $\frac{d^2\theta}{dt^2} + \frac{g}{l} \sin\theta = 0$  exactly. You will see this formula in the oscillations section of the course, but in First Year Physics we use the small angle approximation  $\sin\theta \approx \theta$  to simplify it.

**Discrepancy in period due to nonzero angular displacement**

### The effect of the radius of the bob

This diagram shows a physical pendulum consisting of a frictionless in-extensible string of negligible mass, and length  $(l - R)$ , connected to a spherical bob of uniform density, and radius  $R$ .

For vanishingly small angles, the period of oscillation,  $T'$ , is given by

$$T' = T_0 \left[ 1 + \frac{2}{5} \left( \frac{R}{l} \right)^2 \right]^{1/2} * - (3)$$

where

$$T_0 = 2\pi \sqrt{\frac{l}{g}} \quad - (2)$$

Again, if expression (2) is used to predict the period of this pendulum, then the value obtained will be less than the actual value. The graph opposite shows the percentage discrepancy between the actual period  $T'$ , and  $T_0$ , as a function of the ratio  $(R/l)$ , i.e. (radius of bob/distane from support to centre of the bob).

By combining the effects of both a non-vanishing angular displacement, and the radius of the pendulum bob, the actual period of the pendulum would be given by

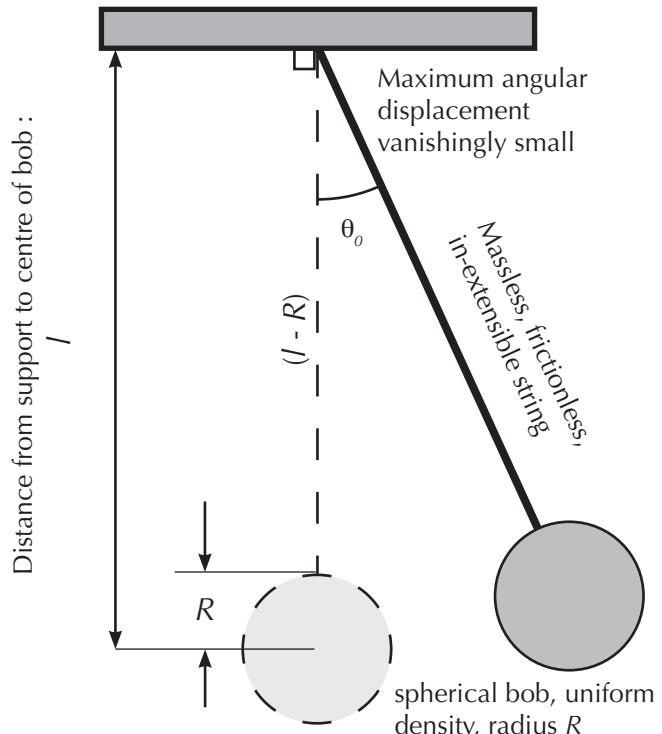
$$T = T_0 \left[ 1 + \frac{2}{5} \left( \frac{R}{l} \right)^2 \right]^{1/2} \left[ 1 + \frac{\theta_0^2}{16} + \frac{11\theta_0^4}{3072} + \frac{173\theta_0^6}{737280} + \dots \right]$$

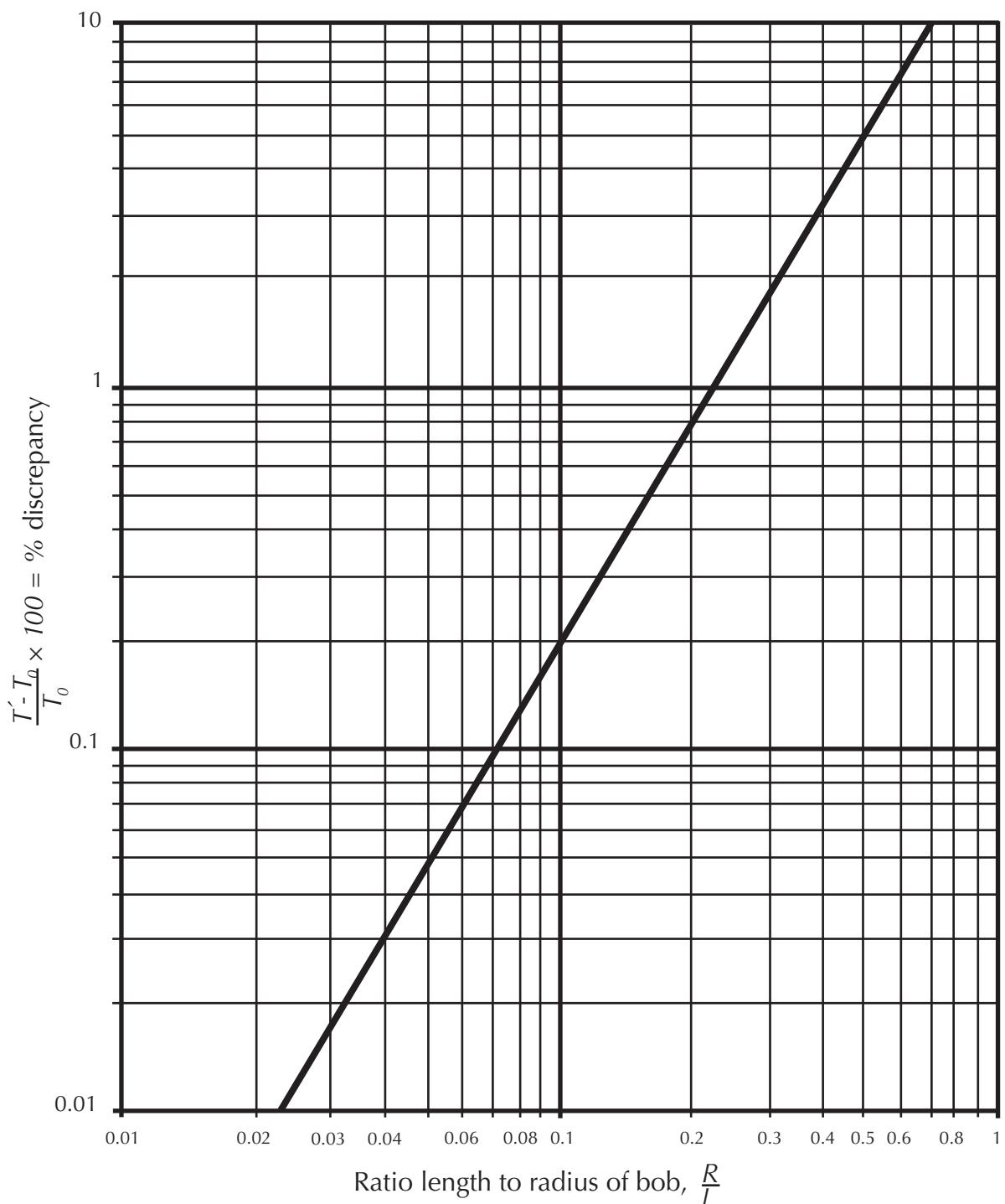
which would be very difficult to use when finding a value for  $g$ .

A better approach, then, would be to use the simple expression (expression (2) above), while ensuring that the discrepancy between the measured period and that which the equation predicts is kept to a minimum. This means that in our experiment, we will need to keep the angular displacement as low as possible, and make sure that we keep the length of the pendulum large with respect to the radius of the bob.

When deciding upon what maximum angular displacement to allow in an experiment, look at the graphs and decide how much discrepancy you are going to permit. Make this no more than the typical error introduced by the other equipment in the experiment (e.g. the meter rule is graduated in millimetres, so for a typical length of, say, 50 cm, its tolerance is 0.1%; is the string really inextensible or does it stretch and if so by how much?) Then read off the allowable maximum angular displacement, and deduce the minimum pendulum length available to you.

\*Footnote: The correction term in this formula comes about due to the parallel axis theorem. You will learn about this theorem when you are doing rotation at the end of the mechanics section.



**Discrepancy in period due to finite radius of bob**

## Equipment

You will need the following equipment from around the lab:

- a pendulum of adjustable length, with attached protractor, and spherical bob
- a metre rule from the window sill

Please take your student card to the equipment hatch to collect the following:

- Electronic caliper
  - Electronic stopwatch
- } one per group only.

## In the Laboratory

You will need to measure the period of oscillation of the swinging pendulum using an electronic stopwatch.

Give a detailed description in the space below, of a method you could use to obtain the period of a physical pendulum using a stopwatch, taking care to minimise any uncertainties.

Keep in mind the following:

- The electronic stopwatches you will use are graduated to 0.01 s, but your reflexes certainly aren't that good!
- Should you start the stopwatch at the same time as you set the pendulum swinging, or would it be better to start the pendulum swinging and begin timing later?
- Will you measure the time for just one "to-and-fro" swing (i.e. one period), or the time for multiple swings and then divide the time by the number of swings to get the period?
- How many measurements will you make at a particular pendulum length? Would it be better to take the average of several measurements?
- What angle will you be using? Why?
- What is the uncertainty in the length of the pendulum? Is it limited by the precision of your equipment or something else?
- What is the minimum length that you will be using? Why?

## Method: How to measure the period T

Get  
Marked  
Now

Given that the period of oscillation of the simple pendulum,  $T$ , is related to its length,  $l$ , by

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Detail a plan in the space below which you could follow to experimentally determine the value of  $g$  using the equipment available to you (see next section), and a graphical method.

Keep in mind the following:

- The pendulum apparatus you will use is of variable length, what range of lengths of the pendulum should be included and why?
- Should you check the length before and after each period measurement?
- Do you need to measure the bob? If so, how will you do this?
- Remember to take into account the limitations on the experiment described in the Preliminary Information.
- What variables should you plot on a graph?
- What is the uncertainty in your length measurements?

Get  
Marked  
Now

## Method: How to measure g, the acceleration due to gravity

Get  
Marked  
Now

Have a demonstrator check the methods you outlined. Add notes to your work if you need to change anything.

Obtain the equipment you will need from the shelves or cupboards. Use the space below and over the page to record your data as you carry out the experiment according to your plan once your demonstrator has seen it. Make sure you draw a table to record your data in.

You must record all of your data in your lab manual but you may use excel for calculations.

Use the computers to plot your results, making sure you include your uncertainty estimates in the data. You will need to use the "Linear Plot With Errors" (consult the "Using the Computers" section in the front of the manual if you need more info on getting the templates).

Finally, use the information the computers give you to calculate  $g$  and its uncertainty. Show your working.

Get  
Marked  
Now

## Marking Guidelines

Get your Demonstrator to tick the boxes below as you go through the exercise.

Please enter your details and date below before you get your final mark from the Demonstrator.

Student Name: \_\_\_\_\_ Student ID Number: z\_\_\_\_\_

Date: \_\_\_\_\_

Appropriate method to measure T	
Appropriate plan to measure g	
Appropriate table drawn to record values	
Values for T and I recorded	
Uncertainty in T calculated appropriately	
Uncertainty in I measured appropriately	
Uncertainty in $T^2$ calculated	
g calculated from graph, units included	
Uncertainty in g calculated from graph	
g measured accurately (within uncertainties)	

Total: /10

Demonstrator's Signature: \_\_\_\_\_

Demonstrator's Name: \_\_\_\_\_

# **EQUILIBRIUM OF RIGID BODIES**

## **Introduction**

In general, when a number of external forces act on an extended rigid body the body will change its “*state of motion*” - either the velocity of its centre of mass will change (that is, its centre of mass will accelerate), and/or the speed of its rotation about its centre of mass will change (that is, it will have a rotational acceleration).

In the particular case where the velocity of its centre of mass does not change (i.e. there is no net acceleration) the body is said to be in **translational equilibrium**.

Even if a rigid body is in translational equilibrium when acted on by external forces, those forces may still cause it to change its rotational velocity. When there is no change in the rotational velocity the body is in **rotational equilibrium**.

This exercise is concerned with the particular case of a rigid body which is in complete equilibrium i.e. in both translational and rotational equilibrium.

This exercise is designed to help you:

- Determine the resultant of a number of vectors from diagrams drawn to scale.
- Determine the resultant force by resolving the forces into orthogonal components.
- Investigate torque

## **What To Do Before You Get To Lab**

- 1) Read through the preliminary information below.
- 2) If you are still unsure about the theory consult your textbook or another source.
- 3) Complete the preliminary problems and prework test for this exercise on Moodle.
- 4) Read through the rest of the exercise, so that you will know what to do in the laboratory.
- 5) Watch the video about the experiment, there is a link on the Moodle site.

## Preliminary Information

### Scale Vector Diagrams

In a scale vector diagram vectors are represented by arrows in a given direction. The length of the arrow gives the magnitude of the vector according to some scale. Scale vector diagrams can be used to add vectors together by placing the arrows representing the vectors head to tail. The resultant vector is then given by drawing another vector from the tail of the first vector to the head of the last vector. In the diagram below two vectors one 6 Newtons and the other 4.5 Newtons are separated by an angle of  $111^\circ$ . A scale vector diagram has been used to calculate the sum of the two vectors.

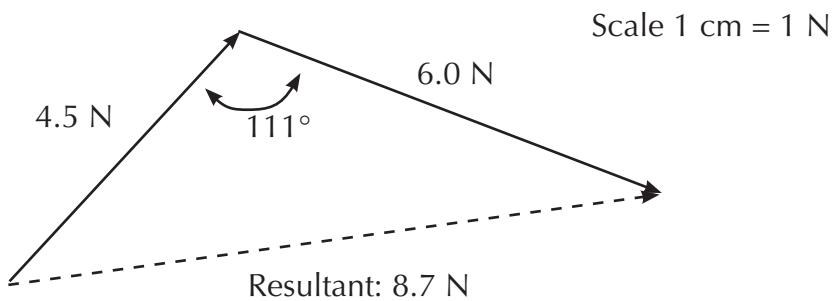


Figure of a scale vector diagram

### Resolving Vectors Into Components

It is possible to replace a vector with two or more different vectors whose sum is equal to the original vector. This is essentially the opposite of what we do when we draw a scale vector diagram. By choosing the replacing vectors to point along the axes of a coordinate system we have resolved the vector into its components in that coordinate system. When we add vectors together we can simply add their components together, that is we add the magnitudes of the vectors that are lying along the same axis to get the magnitude of the resultant vector along that axis, i.e. the vector (3,2) added with the vector (1,7) gives a resultant vector (4,9).

## Vector Products

Torque is a vector product. A vector product is a third vector, **C**, defined by:

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

The magnitude of **C** is given by  $\mathbf{C} = \mathbf{A}\mathbf{B}\sin\theta$  where  $\theta$  is the angle between **A** and **B**. You can find the magnitude by drawing a parallelogram with **A** and **B** as the two sides. The area of the parallelogram gives you the magnitude of **C**.

**C** is in the direction perpendicular to both **A** and **B**, that is if you imagine the plane that **A** and **B** lie in, **C** is perpendicular to this plane. To work out which direction this is in use the right-hand rule.

Wrap the fingers of your right hand from **A** to **B**, your thumb points in the direction **C**.

## Torque

Torque describes the tendency of a force to rotate an object about an axis. Torque is calculated using the formula:

$$\tau = \mathbf{r} \times \mathbf{F}$$

where **r** is the vector from the pivot point to where the force is applied and **F** is the force.

In the diagram of the see-saw we can calculate the torque as follows:

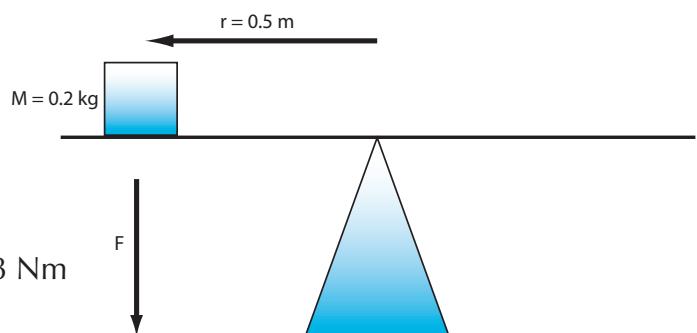
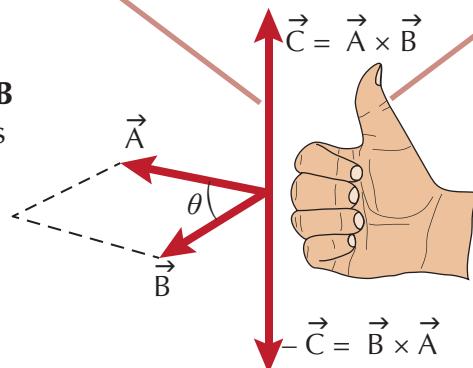
$$\begin{aligned} \tau &= \mathbf{r} \times \mathbf{F} \\ |\tau| &= rF\sin\theta = 0.5 \text{ m} \times 0.2 \text{ kg} \times 9.8 = 0.98 \text{ Nm} \end{aligned}$$

In the direction out of the page.

Torques are in equilibrium when they balance each other out (sum to zero).

On the see saw above placing a 0.4 kg mass 0.25m from the pivot on the other side from the original mass would provide an equal but opposite torque. The see-saw would then be in equilibrium.

The direction of  $\vec{C}$  is perpendicular to the plane formed by  $\vec{A}$  and  $\vec{B}$ ; choose which perpendicular direction using the right-hand rule shown by the hand



## Equipment

You will need the following from around the lab:

- a special vertical “force board” apparatus with clips
- a sheet of A3 paper
- a metal bar with a coloured plastic mesh sleeve
- a short retort stand with clamp

A blue kit, from the hatch (take your ID card), containing:

- 5 mass carriers carrying 4 masses each
- a small, flat, irregularly shaped brass plate with 5 strings attached
- a small plane mirror
- 2 protractors
- a piece of string

### The Vertical “Force Board” Apparatus

Basically this apparatus is a board mounted vertically on a special stand. As shown in figure A (page 27), four relatively large diameter pulleys are positioned in its corners.

Each pulley is on almost frictionless ball race bearing and so can rotate quite freely. Slots in the board allow the pulleys to be moved about in the corners of the board.

**To change the position of a pulley** reach behind the board, loosen the wing nut clamping the pulley shaft, move the pulley along its slot to the required new position, and finally re-tighten the wing nut.

In the exercise the small, flat, irregularly shaped brass plate with its plane vertical is set into equilibrium under the action of up to 5 forces in its plane.

As the figure A shows:

- each force is applied via one of the strings attached to the plate.
- each force is derived from the gravitational force acting on a mass attached to the end of a string [**and in the case of the string attached to the centre of mass, from the total mass of the (mass carrier+the brass plate)**]
- in the case of 4 of the forces the direction of the force is altered from the vertical by passing the string around a pulley.

## In the Laboratory

Obtain the equipment from the hatch and cupboards. Check it against the list above to make sure everything is there. If anything is missing, see one of the lab attendants.

### Preliminaries

Firstly, using one of the mass balances in the laboratory, determine the mass of the small, flat, brass plate.

**Mass of the small, flat, brass plate =** .....

Next, on your special “vertical force board” set the brass plate into equilibrium under the action of 5 forces as in the diagram below.

#### **WARNING**

- ***Be very careful not to allow any of the mass carriers to drop to the floor.***

Start out with only the top two strings loaded with mass carriers, and the other strings hanging loosely. Then add the other mass carriers to the ends of each string one by one. Again, only have a total mass of about 0.150 kg on each carrier to start with.

Once you have all the mass carriers in place, adjust the locations of the pulleys, and the amount of mass on the suspended mass carriers until the plate takes up position in (approximately) the centre of the board.

The final mass of each mass carrier + masses should not be less than 0.200 kg. Some will need a much larger mass.

**Choose your masses to be 200 g or larger. Why is it better to choose larger masses?**

## Part 1: Translational Equilibrium of the Plate

Firstly, in this part you need to obtain an “as accurate as possible” replica of the outline and position of the plate, and the directions of the forces acting on it (i.e. directions of the strings), noting at the same time the magnitude of those forces.  
You do this on an A3 sheet of paper provided.

To obtain this “accurate as possible” replica:

- Using the clips provided on the top of the force board, suspend the A3 page against the board behind the plate and strings, so that the plate is (roughly) in the middle of the page – as shown in figure A.
- Using a sharp pencil place dots on the page directly under each of the vertices of the plate, and 2 or more (widely separated) dots directly under each of the strings making use of the mirror provided (SEE NOTES BELOW).
- Remove the sheet of paper from the board, and using a ruler draw in an outline of the brass plate, and lines along the directions of each of the strings.
- Finally, label each of the lines with the magnitude of the total force applied to the plate along that line.

### Notes:

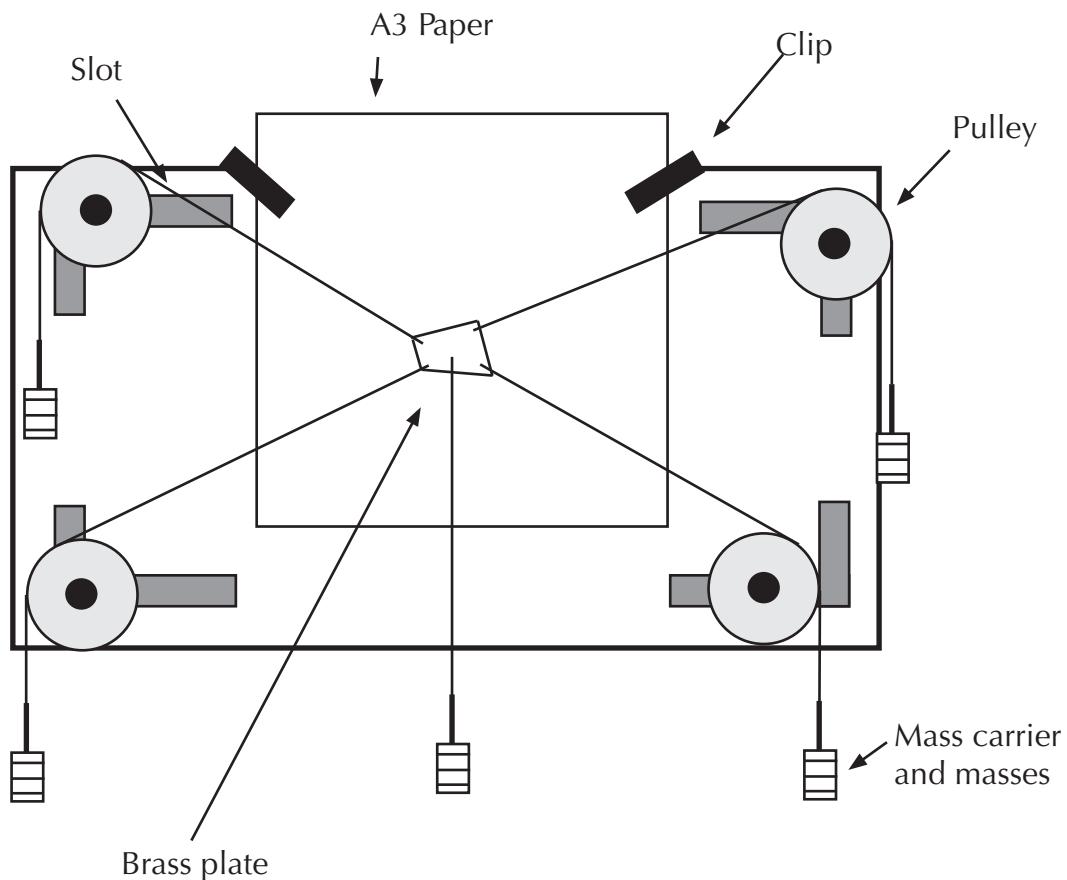
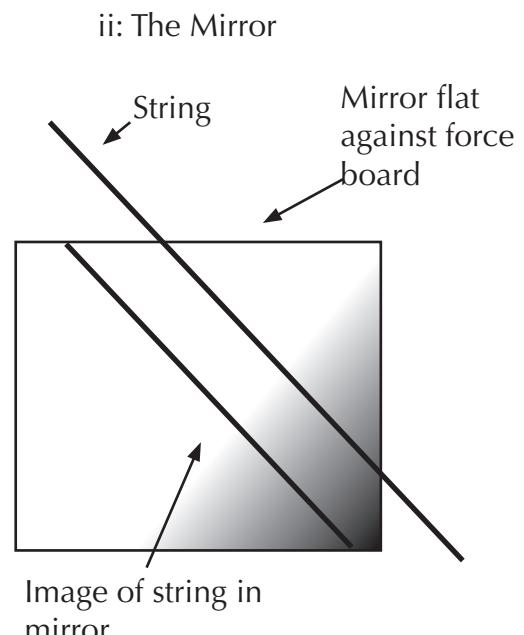
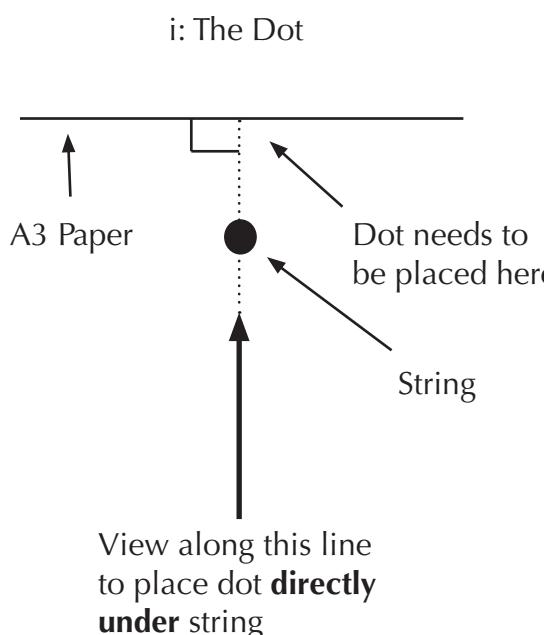
The brass plate and the strings will be a short distance in front of the vertical force board. So to place a dot directly under a string, etc. you will need to view the string along a line at right angles to the force board (and your A3 page), as illustrated in figure B i.

To help you do this you use the small plane mirror provided.

You hold this mirror flat against the A3 page under a string and look at it (figure B ii). In general, you will see both the string and its image in the mirror, this image appearing to be behind the mirror. (If you stand in front of your bathroom plane mirror you see an image of yourself. Your image is positioned behind the mirror!)

However, if you are looking along a line at right angles to the board, the string image will not be visible because it will be obscured by the string itself.

This exercise illustrates a common phenomenon in science: **a little extra care can make a big difference in the quality of the final results.**

**Figure A: The Force Board****Figure B: Parallax Error and the Mirror**

## Results, Discussion, and Analysis

In the space below construct a scale vector diagram to determine the resultant force acting on the plate. The scale should be chosen so that the diagram will fit but should also be as large as possible so that the, hopefully small, resultant force can be read easily.  
Add your vectors head-to-tail. Have a careful think about where on the page to start drawing your diagram. Label the forces  $F_1, F_2, F_3, \dots$  etc.

From your scale vector diagram what was the resultant force acting on the plate:

Magnitude:

Direction:

Is this result consistent with your expectation? Explain.

In practice there may be a discrepancy between your result and the result expected from this condition. Suggest any likely reasons for such a discrepancy.

Get  
Marked  
Now

On your accurate as possible replica on the A3 sheet draw in two mutually perpendicular axes at a convenient orientation. Label one axis **a** and the other **axis b**.

Next, for each of the forces acting on the plate determine the magnitude and direction of its component along these two axes, recording the result in the table given. (**All working must be shown**).

Hence determine the resultant force acting on the plate in each of these axis directions.

Force	Component along <b>axis x</b>	Component along <b>axis y</b>
Resultant		

Total magnitude of resultant:

Direction:

Are your results consistent with your expectations?

Now compare your results for the resultant force calculated from the scale vector diagram and from taking the components of the forces. Which of these methods is more accurate? Why?

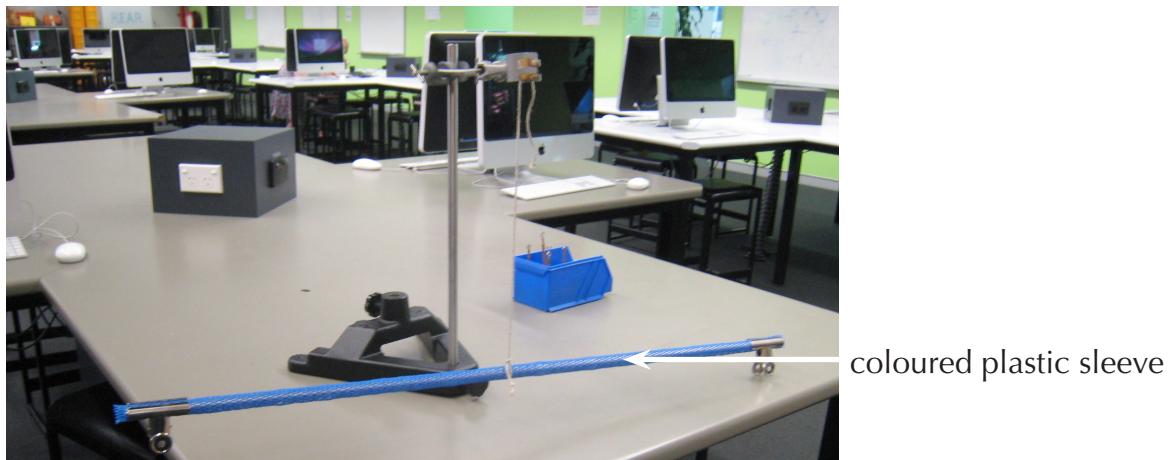
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## Part 2: Rotational Equilibrium of Bar and Masses

In this part of the experiment you will be investigating rotational equilibrium.

### Setup:

Tie the piece of string around the middle of the metal bar which still has the coloured plastic sleeve in place (it gives the string traction). Hang the metal bar from the retort stand by tying the string to a clamp. The photo below shows you how to do this. Slide the string along the metal bar until it will hang horizontally.



Attach the bulldog clips to the metal bar and hang a mass carrier off each bulldog clip.

### Activity:

Place 200g on one of the bulldog clips (clip 1) and 150g on the other (clip 2). Place clip 1 20.0 cm from the pivot point (the point where the string is tied). In the space below calculate the torque generated by clip 1. Include a direction.

## EQUILIBRIUM OF RIGID BODIES

Now calculate where you will need to place clip 2 in order to balance this torque and bring the metal bar into rotational equilibrium.

Now move clip 2 along the bar to bring the body into rotational equilibrium. What distance is clip 2 from the pivot point?

How does this compare with the calculated value? Suggest a reason for any differences.

Complete the table below.

Mass 1 (g)	Distance 1 (cm)	Mass 2 (g)	Calculated distance 2 (cm)	Measured distance 2 (cm)
200	20	100		
200	20	200		
150	15	50		
150	20	100		

Comment on any differences between the calculated values and the measured values.

Get  
Marked  
Now

## Marking Guidelines

Get your Demonstrator to tick the boxes below as you go through the exercise.

Please enter your details and date below before you get your final mark from the Demonstrator.

Student Name: \_\_\_\_\_ Student ID Number: z\_\_\_\_\_

Date: \_\_\_\_\_

Answered question about why final masses should be at least 0.200 kg	
Drawn an accurate representation of strings on the board, including weights	
Drawn an accurate scale diagram of weights in lab book, all weights joined head to tail (each student needs one NOT one per group)	
Calculated resultant force from vector diagram	
Broken vectors into components and added them	
Suggested reasons for discrepancy for scale diagram and breaking into components	
Set up equipment for torque experiment and calculated where to place 150g mass	
Completed table for torques	
Answered all questions about torque	
Explained any discrepancies between calculated and measured value for torque	

Total: /10

Demonstrator's Signature: \_\_\_\_\_

Demonstrator's Name: \_\_\_\_\_

# STATIC FRICTION ON AN INCLINED PLANE

---

## Introduction

In this laboratory exercise you will be placing objects with different surfaces on an inclined plane. You will adjust the angle the plane makes with the horizontal to determine the angle at which the object starts to slide down the plane. You will use this angle to estimate the coefficient of static friction.

The purpose of this experiment is to:

- become familiar with splitting vectors into components parallel and perpendicular to an inclined plane
- investigate factors that influence the coefficient of static friction
- get an understanding of the causes of friction
- practice estimating uncertainties

## What to do before you get to lab

- 1) Read the following preliminary information
- 2) Review the relevant section of your textbook for a more detailed account of the theory
- 3) Complete the preliminary problems and prework test for this exercise on Moodle
- 4) Read through the rest of this exercise so that you will know what to do in the laboratory
- 5) Watch the video about this experiment, the link can be found on Moodle
- 6) You may want to attempt the theoretical problems before the class.

## Preliminary Information

Often when dealing with questions about an inclined plane the question can be simplified by splitting vectors into components parallel to and perpendicular to the plane.

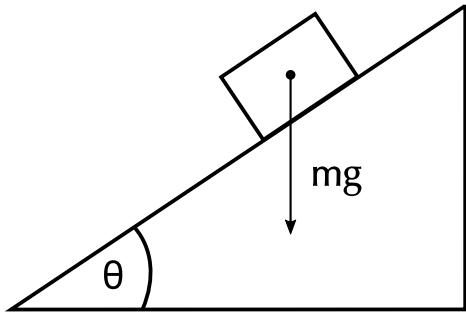


Figure 1

When a mass  $m$  is placed on a plane inclined at an angle  $\theta$  the weight force,  $m \cdot g$ , can be split into a component down the plane,  $m \cdot g \cdot \sin\theta$ , and a component perpendicular to the plane,  $m \cdot g \cdot \cos\theta$ .

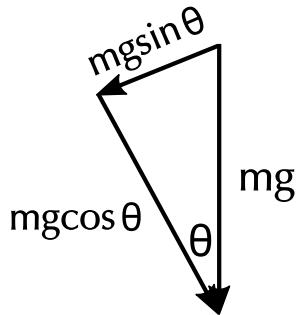


Figure 2

There are two types of friction, static friction and kinetic friction. The static frictional force is the force that needs to be overcome for an object to start moving. The size of the static frictional force for a stationary object will be the same as the net force parallel to the surface that the body is resting on. Static friction has a maximum value,  $f_{s,\max}$ , given by

$$|f_{s,\max}| = \mu_s \cdot |F_N| \quad (1)$$

where  $\mu_s$  is the coefficient of static friction and  $F_N$  is the normal reaction force. In this experiment you will be finding the maximum value of the static friction force by changing the slope of the inclined plane. The static frictional force is directed up the inclined plane. You will measure the angle at which the block just starts to slide. You will then use this to find  $\mu_s$ .

When a body is sliding along a surface a kinetic frictional force opposes the motion. The size of this force can be described by the equation

$$|f_k| = \mu_k \cdot |F_N| \quad (2)$$

where  $f_k$  is the kinetic frictional force and  $\mu_k$  is the coefficient of kinetic friction.

Please note that neither coefficient of friction,  $\mu_s$  or  $\mu_k$  has units, they are unitless quantities. They are equal to the ratio of the frictional force divided by the normal force.

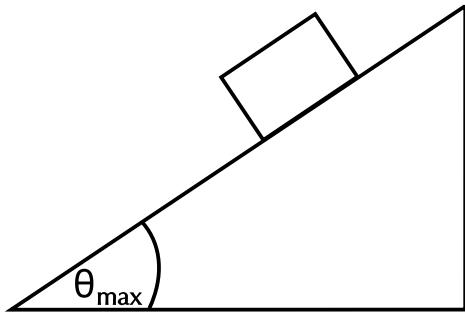
### Theoretical Problems

1. Use geometry to show that the  $\theta$  in Figure 1 is the same as the  $\theta$  in Figure 2.

Get  
Marked  
Now

## STATIC FRICTION ON AN INCLINED PLANE

2. You have a block on an inclined plane and find  $\theta_{\max}$ , the maximum angle for which the block does not slide down the plane.



- Draw arrows on the diagram above to show all the forces acting on the block (ie. Create a free body diagram).
- Write an expression for the gravitational force (weight force) parallel to the slope:
- Write an expression for  $F_N$ , the normal reaction force:
- Use equation (1) along with the expressions above to derive an expression for  $\mu_s$  in terms of  $\theta_{\max}$  and any other required quantities:

Get  
Marked  
Now

## Equipment

For your work in the laboratory you will need the following

- a retort stand, a short rod and pivot clamp
- a ramp
- a metre ruler (get from the window sill)
- one 250 g mass and one 500 g mass
- a set of 4 boxes: one with a felt bottom, one with a cork surface and two with white bases
- a cork mat (place it at the base of the track when needed)

## In the Laboratory

Set up the inclined plane as shown in the photo below.



Place a box with a white base on the plane and carefully determine  $\theta_{\max}$ , the maximum angle at which the cart does not slide down the plane. Do this by gently raising the higher end of the plane.

Determine the angle by measuring the height and length of the inclined plane and then using trigonometry, for example,  $\sin \theta = \text{height}/\text{length}$ .

## STATIC FRICTION ON AN INCLINED PLANE

Use this method to measure  $\mu_s$ . Show all your working.

Place the box with white base at two different positions on the plane (if you have placed it near the top of the plane initially then you could place it in the middle and at the bottom of the plane as the other two positions) and again measure  $\theta_{\max}$  and calculate  $\mu_s$ . Make sure that you always start with a small angle and slowly and smoothly increase the angle until the box just starts to move.

Use these three measurements to calculate an average value for  $\mu_s$  with an uncertainty.

Now you are going to investigate which factors affect the size of  $\mu_s$ .

Get  
Marked  
Now

### Weight Force

Do you expect the size of the weight force to effect  $\mu_s$ ? Use a physical argument to explain why or why not.

Place the 250 g mass in one of the boxes with white bases. Measure  $\mu_s$  with an uncertainty.

Now place the 500 g mass in the box and measure  $\mu_s$  with an uncertainty.

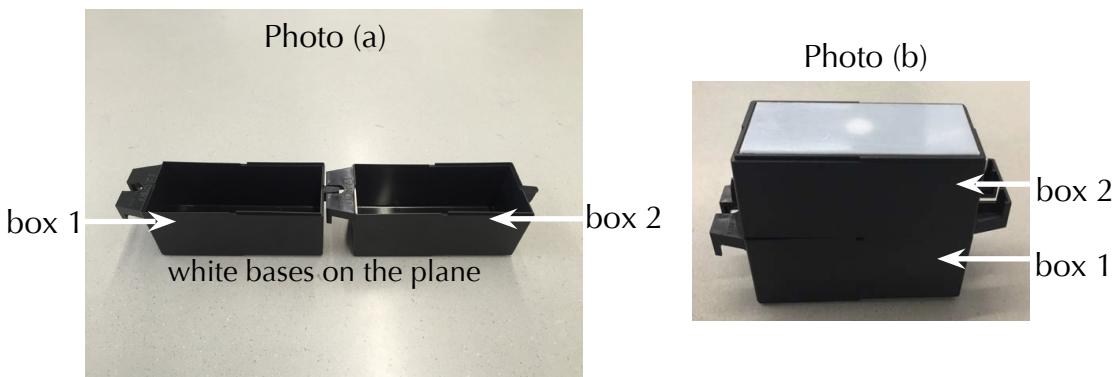
Does  $\mu_s$  depend on the mass in the box? With reference to your measurements explain your result.

Get  
Marked  
Now

**Surface Area with white based box**

Do you expect the surface area of the object to affect the size of  $\mu_s$ ? Use physical arguments to explain why or why not.

Test this prediction by connecting the two boxes together and measuring  $\theta_{\max}$ . The photos below show how to connect the two boxes.



Measure  $\mu_s$  with an uncertainty for each configuration of the boxes shown respectively in photo (a) and (b). Use the same method you have used to determine  $\mu_s$  for one box.

Does the surface area affect  $\mu_s$ ? With reference to your measurements, explain why or why not.

**Material**

Do you expect the surface material of the box to effect  $\mu_s$ ? Explain why or why not.

Measure  $\mu_s$  with an uncertainty for the box with the felt on the bottom.

Measure  $\mu_s$  with an uncertainty for the box with the cork on the bottom.

Does  $\mu_s$  depend on the material? With reference to your measurements explain your result.

What factors affect  $\mu_s$ ? What factors do not affect  $\mu_s$ ? Was there anything in this experiment that you found surprising?

Is  $F = \mu_s N$  an exact equation or an approximation? Include reasons for your answer.

## Marking Guidelines

Get your Demonstrator to tick the boxes below as you go through the exercise.

Please enter your details and date below before you get your final mark from the Demonstrator.

Student Name: \_\_\_\_\_ Student ID Number: z\_\_\_\_\_

Date: \_\_\_\_\_

Used geometry to prove that $\theta$ in Figure 1 and Figure 2 are equivalent	
Derived correct expression for $\mu_s$ involving $\theta_{\max}$	
Give a method to calculate $\theta$ with minimal uncertainty	
Measured $\mu_s$ with an uncertainty for a single plastic box	
Measured $\mu_s$ with an uncertainty for the box containing 250 g	
Measured $\mu_s$ with an uncertainty for the box containing 500 g	
Predicted and then correctly explained how weight affects $\mu_s$	
Measured $\mu_s$ with an uncertainty for two plastic boxes	
Predicted and then correctly explained how surface area affects $\mu_s$	
Measured $\mu_s$ with an uncertainty for felt and cork on slope	
Predicted and then correctly explained how material affects $\mu_s$ and answered the final questions	

Total: /10

Demonstrator's Signature: \_\_\_\_\_

Demonstrator's Name: \_\_\_\_\_

# **COLLISIONS AND CAR CRASHING**

## **Introduction**

The non-technical use of the word “collision” is well known to us long before it is formally studied in a physics course. A ball bouncing on the floor, balls colliding on the pool table, the more serious collision between two cars - all are examples of collisions. The technical sense of a “collision”, however, is a little broader than this.

In this experiment, we will take your general knowledge of collisions and expand upon it by doing some experiments that give you a better view of what is actually happening during a collision. This will help you to understand the background theory.

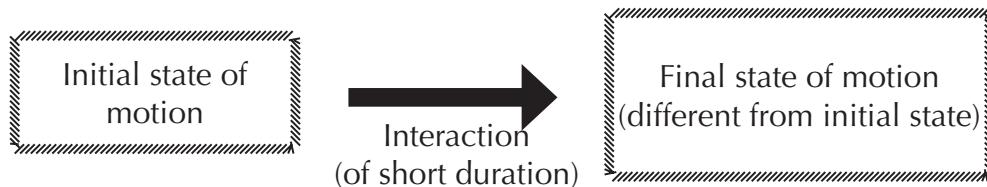
## **What To Do Before You Get To Lab**

- 1) Read the Preliminary Information starting on the next page. This will review the theory you will need.
- 2) Consult the relevant sections of your text books.
- 3) Complete the Preliminary Problems and prework test on Moodle.
- 4) Read through the rest of the experiment notes, so that you will know what to do in the laboratory.
- 5) Watch the video about this experiment. The link can be found on Moodle.
- 5) You may need to attempt the theoretical problems before class.

## Preliminary Information

### Types of Collision

A collision may be defined as an event in which relatively strong forces act on a number of colliding bodies (also called a *system*), over a relatively short time\*. Further, the system will be in *clearly defined states of motion* before and after the collision.



Note that the above definition does not mention the bodies actually touching. This is not a necessary condition for a collision; some of the collisions you'll see in this experiment take place without any physical contact at all. As an example, astronomers may treat the "slingshot" interaction of a space probe around a planet as a collision, even though they never touch, and may take place over several days. It is still a collision because during the interaction, the forces between them have a much larger effect than that of external forces.

Depending on the nature of the collision, the relationship between the initial and final states of motion of the system will be different. In particular the relationships between:

- the relative velocities of the bodies,
- the total momentum of the system, and
- the total kinetic energy of the system;

before and after the collision may be used to define the type of collision which has taken place:

1) *Elastic collisions*. If two bodies collide in such a way that the relative velocity of the bodies is the same before and after the collision, the collision is said to be *elastic*.

Further, the total momentum of the system, as well as the total kinetic energy of the system, is *conserved*. That is, total momentum and total kinetic energy is the same before and after the collision. (The relative velocity being the same before and after the collision is actually a result of these conservation laws, and the neglect\* of external forces acting on the system.)

2) *Partially inelastic collisions*. In some collisions, the kinetic energy of the system will be different after the collision as some energy becomes converted to another form, (to sound, for example). Since kinetic energy is not conserved, these collisions are called *inelastic* collisions. The total momentum of the system is still conserved.

---

\* It is important that the internal forces acting in a collision be strong, and that the collision duration be brief, because collisions are usually analysed by neglecting the impulse due to external forces. For instance, in a collision between motor vehicles, external forces such as each car's weight do act, but over the time of the collision (less than a second) these are neglected in comparison with the forces between the cars, which are enough to bend metal.

3) *Completely inelastic collisions.* If after an inelastic collision, the two colliding bodies are stuck together (and therefore their relative velocity is zero), then the collision is said to be *completely inelastic*. For example, imagine a collision between two droplets of water. A system undergoing a completely inelastic collision will, in fact, have lost all of the kinetic energy it is possible for the system to lose while still obeying the law of conservation of momentum.

### The Coefficient of Restitution

The references to the relative velocity of the colliding bodies in the definitions above, lead us to a simple means of determining the nature of a collision.

Isaac Newton determined that for a collision between two bodies, the ratio of the final and initial relative velocities of the bodies is approximately constant; and that the value of the constant was:

- a) relatively independent of the absolute velocities of the two bodies;
- b) dependent only on the materials the objects were made of (and was different for different materials);
- c) always between 0 and 1, inclusive.

This empirical constant is called *the coefficient of restitution*,  $e$ , and is given by:

$$e = - \frac{(\text{velocity of body A relative to body B after the collision})}{(\text{velocity of body A relative to body B before the collision})}$$

For example, in the one-dimensional system (i.e., in a single line) shown below:

(velocity of body 2 relative to body 1 after collision)

$= -e$  (velocity of body 2 relative to body 1 before the collision)

$$\text{i.e. } \mathbf{v}_2 - \mathbf{v}_1 = -e (\mathbf{u}_2 - \mathbf{u}_1)$$

$$\text{and } e = - \frac{\mathbf{v}_2 - \mathbf{v}_1}{\mathbf{u}_2 - \mathbf{u}_1}$$

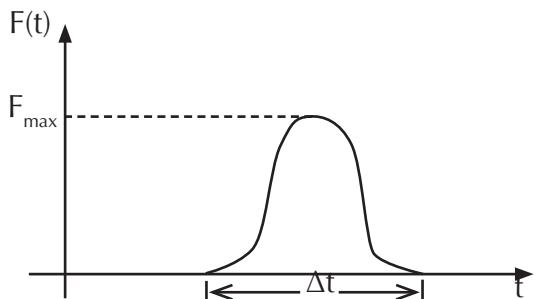
So, for the three types of collision defined earlier we can say:

- *Elastic* collisions have  $e = 1$ ,
- *Inelastic* collisions have  $0 < e < 1$ , and
- *Completely inelastic* collisions have  $e = 0$ .

## Impulse

We now turn to what is taking place during the collision itself. Obviously, because the initial and final velocities of one or more of the bodies are different, there must be forces acting on them.

We know from Newton's third law, that equal and opposite forces act on any pair of bodies involved in a collision. As a function of time, the force acting on the bodies might look something like that shown below:



where  $\Delta t$  is the total time it takes for the collision to take place, and  $F_{\max}$  is the maximum force experienced by the colliding bodies.

According to Newton's second law, the force at a particular time,  $t$ , is related to the change in momentum,  $d\mathbf{p}$ , of the bodies over a small time period,  $dt$  (much less than  $\Delta t$ ), by

$$d\mathbf{p} = \mathbf{F}(t)dt$$

If we integrate both sides of this equation we obtain

$$\mathbf{p}_f - \mathbf{p}_i = \int_{t_i}^{t_f} \mathbf{F}(t) dt = \mathbf{I}$$

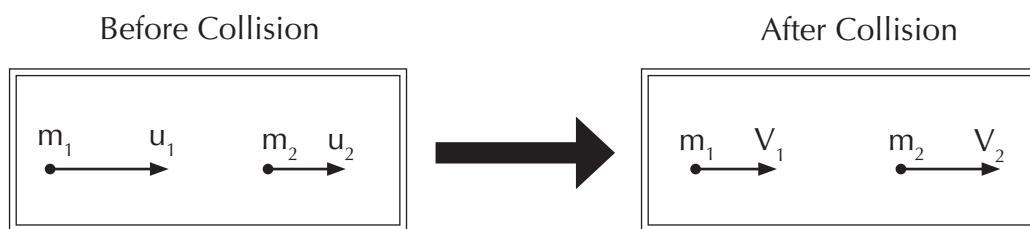
where  $\mathbf{p}_i$  and  $\mathbf{p}_f$  are the initial and final momenta of the body, respectively.

That is, the change in momentum is equal to the area under the curve in the figure above. This change in momentum is called *the impulse*,  $\mathbf{I}$ .

The relationship between the impulse and the force acting during a collision is important. During a car crash, for example, a large factor in determining the likelihood of injury is the maximum force which a passenger experiences. For a collision in which a car is brought to rest against an immovable object (e.g. a wall), the impulse is simply equal to the initial momentum of the car. If we can design cars in such a way that collisions take place over a longer time, the maximum force will be reduced even though the impulse remains the same. This is why modern cars are designed to "crumple".

## Theoretical Problems

Two bodies, of masses  $m_1$  and  $m_2$  respectively, travelling with velocities  $u_1$  and  $u_2$ , are involved in an elastic collision.



If the velocities of the bodies after the collision are  $v_1$  and  $v_2$ , write down expressions showing:

a) Conservation of momentum:

b) Conservation of kinetic energy:

c) Then, using these expressions, show that the relative velocity of body 2 with respect to body 1 is equal and opposite before and after the collision. i.e. show that:

$$u_1 - u_2 = -(v_1 - v_2)$$

Get  
Marked  
Now

## Equipment

1) You will need the following from around the lab:

- a 1.2 m dynamics car track

2) You will need to take your student card to the hatch to collect:

A dynamics car kit containing:

- a dynamics car
- an accelerometer
- a motion detector
- a motion detector cable
- some rubber bands.
- a LabPro interface with a power supply and a USB cable

## In the Laboratory

Connect up your LabPro interface by plugging the USB cable into the USB port of one of the computers. The USB port is on the right hand side of the computer, right near where the keyboard plugs in.

There are specific templates for this experiment called Car Crash Experiment parts 1 and 2. Make a copy of the templates to use in the exercise (see the “Using the Computers” section of the lab manual if you need more info).

### Part 1: The Coefficient of Restitution

We will start by making some measurements of the coefficient of restitution for a few different collisions, which will help us specify the type of each collision.

Tilt the dynamics track so that the end with the magnetic bumper is at the bottom of the slope, about  $2^\circ$  should be enough. If the magnetic bumper is not attached, then attach it to one end by fitting it into the slot on the side of the track (make sure the magnets are on the side facing the track). It's also a good idea to position the track so that the end with the bumper is butted up against something solid.



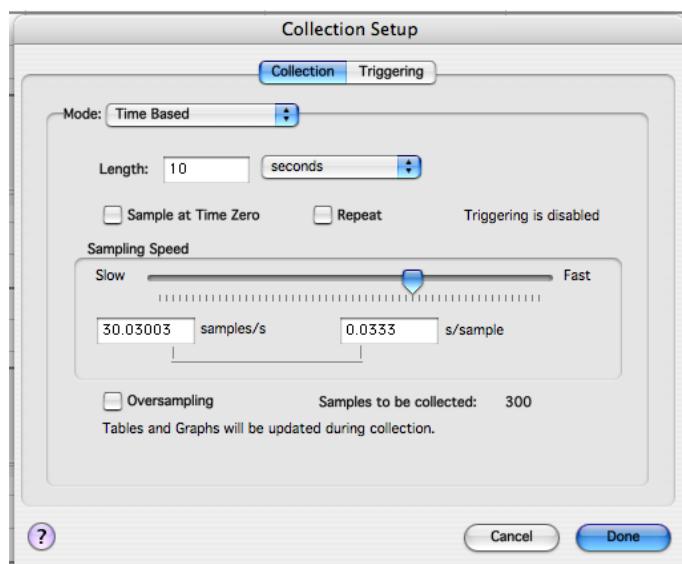
Plug the motion detector into the “DIG/SONIC 1” socket on the side of the LabPro, and then start up the LoggerPro software on the computer by double clicking the “Car Crash Experiment part 1” template file.

Open the motion sensor up so that the sensor face is at 90 degrees, then lay the sensor on its side with the sensor face pointing down the track, about 90cm from the bumper.



The motion detector is very sensitive to interference from reflections off nearby objects, as well as to the signals emitted by other motion detectors. Try to ensure that there are no objects near the track which may interfere with your signal, and that the detector is not pointing towards any other group’s detector.

Open the “Car Crash Experiment part 1” template that is located in the LoggerPro experiments folder. You should see three blank graph grids, one each for the position, velocity, and acceleration of the cart as a function of time.



When you start the software, the sensor is set up to take about 30 readings per second for a period of 10 seconds. Check this in the “Data Collection” section of the “Experiment” menu at the top of the screen. You may change the length of time the sensor will collect data if you like, but leave it at 30 samples per second for best results (the sensor doesn’t work reliably at higher rates).

Hit the “Collect” button and try a few trial experiments, letting the cart roll down the slope and hit the magnetic bumper a few times, to get a feeling for how the sensor operates. Notice that if the cart gets too close to the detector, then it loses track of it. Make sure that during your experiments, the cart is a sufficient distance from the detector to properly provide a signal.

## COLLISIONS AND CAR CRASHING

Using the magnetic bumper, take measurements of the velocity of the cart as it undergoes at least three collisions.

From your measurements, calculate the coefficient of restitution for this collision, include the uncertainty. Record any data, and show your working in the space below.

Magnetic bumper:

Repeat the measurement three more times, but have the cart collide with three different objects. First, turn the cart around so that it is not deflected by the magnetic bumper, but instead smacks hard into it. Also try having it hit other things – a soft pencil case, or one of the plastic lab beakers for example.

Record the coefficients of restitution, with uncertainties, that you measure each time, and classify the type of each collision using your results.

Hard collision (turn magnetic cart around):

Second object to collide with:

Third object to collide with:



## Part 2: Acceleration During Collisions

Here we will be taking a closer look at the forces acting during different types of collisions. To do this, we will use an accelerometer.

The accelerometer you use in this part of the experiment needs to be calibrated. This means for certain measurements you need to tell the computer what the sensor should be measuring as its scale needs to be calibrated against a known value. There are two different ways of calibrating the sensor:

- A one-point calibration: You tell the computer the expected value for one measurement.
- A two-point calibration: You tell the computer the expected values for two measurements.

The accelerometer senses acceleration using an integrated circuit (IC) of a type originally designed to control the release of air bags in an automobile. This IC is micro-machined with very thin “fingers” carved in silicon. These fingers flex when accelerated. They are arranged and connected like the plates of a capacitor. As the fingers flex, the capacitance changes, and a circuit included in the IC monitors the capacitance, converting it into a voltage. An external op-amp circuit amplifies and filters the output from the IC.

The accelerometer measures acceleration along the line marked by the arrow on the label. Plug the accelerometer into the “CH 1” socket on the left side of the LabPro. Open the template “Collisions and Car Crashing Part 2”.

1. Mass of cart and accelerometer: .....

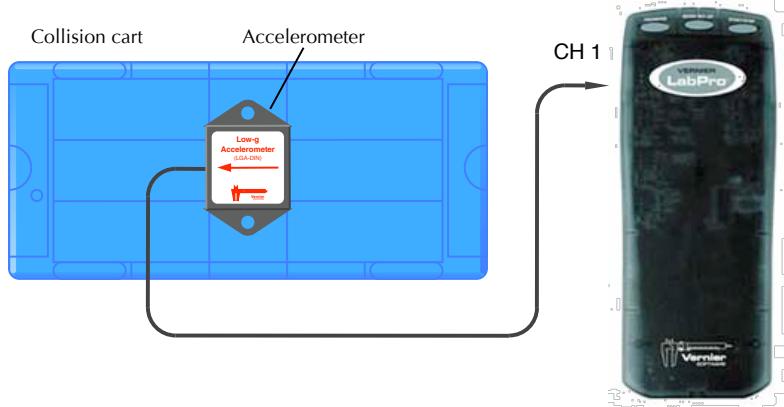
Hold the accelerometer vertically with the arrow pointing upwards.

2. What does the accelerometer read? Why?

Now point the arrow vertically downwards.

3. What reading does the accelerometer give now? Why?

Your accelerometer may need calibrating if these readings are not what you expected. To calibrate the accelerometer, select “Experiment” from the computer navigation bar and then “Calibrate”. Select the “Low-g Accelerometer” and then enter the known values for Reading 1 and Reading 2. Ask your Demonstrator if you need help. Use the rubber band to fix the accelerometer to the top of the cart, the arrow should point along the length of the cart and down the track as shown.



**Set the cart on the track** and make sure that the accelerometer cable doesn't interfere with movement. Check that the accelerometer will take **200 readings each second** (select the “Experiment” menu and choose “Data Collection”). Hold the cart stationary on the track.

4. What does the accelerometer read when you set the cart stationary on the track? Why?

When you are familiar with using the sensor, measure the acceleration experienced by the cart in a collision of the cart with the magnetic bumper. Move the cart so that it is about **30-50 cm** from the bumper and keep this starting point the **same** for all following measurements; click the “collect” button to start the measurement and then release the cart.

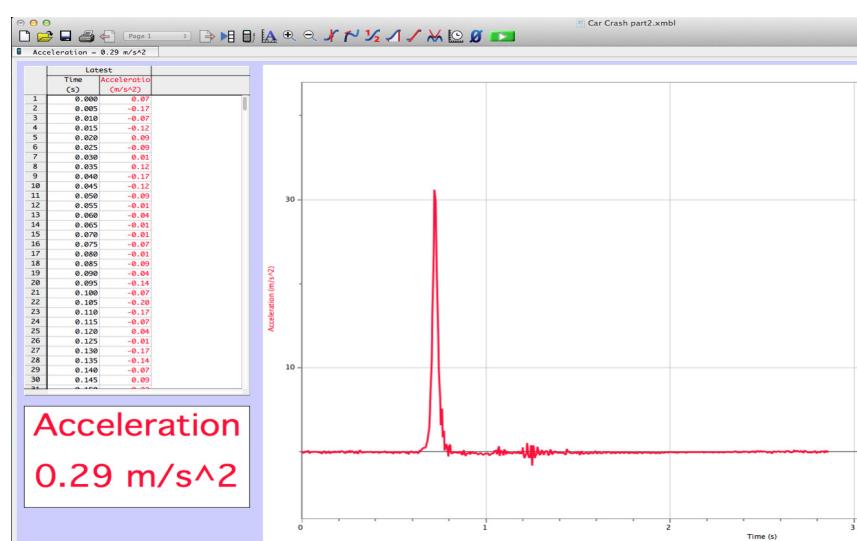
Once you have obtained your plot, you can zoom in on the collision by dragging the mouse cursor around the area you wish to examine and pressing the “zoom in” button,



and you can change the scale of the axes to best fit the data by using the "Autoscale" button .

Complete the tables. In order to do this you will need to find the integral  $\int F(t).dt$ . You can use the "Integral Fit" button to find the integral  $\int a(t).dt$  and then use the mass of the cart and Newton's second law to find the impulse.

The recorded graph on the computer screen should look similar to the one shown below.



	Initial time of collision (s)	Final time of collision (s)	$\Delta t$ (s)	Maximum a (ms <sup>-2</sup> )	$\int a(t).dt$ (ms <sup>-1</sup> )	Impulse (Ns)
Trial 1						
Trial 2						
Trial 3						

Impulse = .....

## COLLISIONS AND CAR CRASHING

Now turn the cart around so that the cart collides hard with the bumper. Release the cart from the same point on the slope. Perform the experiment to complete the table for this case.

	Initial time of collision (s)	Final time of collision (s)	$\Delta t$ (s)	Maximum a ( $\text{ms}^{-2}$ )	$\int a(t) \cdot dt$ ( $\text{m s}^{-1}$ )	Impulse (Ns)
Trial 1						
Trial 2						
Trial 3						

Impulse = .....

Answer the following questions:

5. How does the time of collision change for the two different collisions?

6. How does the maximum acceleration change for the two different collisions?

7. How does the impulse change for the collisions? Give a thorough explanation of which one will be larger, referring to relevant equations and observations. (HINT: Think about the relationship between impulse and momentum.)



You are now going to think of a way to make the collision safer for some imaginary passengers in the cart. As noted in the preliminary information the maximum force experienced by passengers is related to the types of injuries a passenger may receive.

We will set up a simple “crash test” and attempt to make the cart safer for some theoretical passengers.

Assume that the acceleration obtained with the hard bumper is about that which will cause injury to our “passengers”, try to think of some way in which you could reduce this maximum acceleration *by half*, even though the impulse of the collision will stay the same (assuming that the collision is bringing the cart to rest).

Test your ideas and record the measured maximum acceleration each time. You will most

likely need to put something between the cart and the bumper that will alter the dynamics of the collision. In a real car this would usually be attached to the car itself (like a bumper bar, for example), but for our purposes you could just place something in front of the bumper at the end of the track.

	$\Delta t$ (s)	Maximum a ( $m s^{-2}$ )	$\int a(t) \cdot dt$ ( $m s^{-1}$ )	Impulse (Ns)
Trial 1				
Trial 2				
Trial 3				

What solution did you find to this problem?

How does it work (refer to appropriate equations)?

Get  
Marked  
Now

## Marking Guidelines

Get your Demonstrator to tick the boxes below as you go through the exercise.

Please enter your details and date below before you get your final mark from the Demonstrator.

Student Name: \_\_\_\_\_ Student ID Number: z\_\_\_\_\_

Date: \_\_\_\_\_

Answered theoretical problems	
Calculated 'e' for magnetic bumper	
Calculated uncertainty in 'e' for magnetic bumper (using range/2)	
Calculated 'e' with uncertainty for another 3 situations	
Answered all questions 1-4 correctly	
Performed experiment with accelerometer and magnetic bumper	
Completed table for magnetic bumper	
Completed table for hard collision	
Answered questions 5-7 thoroughly	
Come up with a solution for imaginary passengers, demonstrated and explained it	

Total: /10

Demonstrator's Signature: \_\_\_\_\_

Demonstrator's Name: \_\_\_\_\_



# **ROTATIONAL INERTIA**

## **Introduction**

The purpose of this experiment is to find the rotational inertia of a point mass, a ring and a disk experimentally and to verify that these values correspond to the calculated theoretical values.

The exercise is designed to give you an understanding of:

- Angular acceleration
- Torque
- Moments of inertia

What to do before you get to lab

- 1) Read and make notes on the following preliminary information
- 2) Review the relevant section of your textbook for a more detailed account of the theory
- 3) Complete the preliminary problems and prework test for this exercise on Moodle.
- 4) Read through the rest of this exercise so that you will know what to do in the laboratory.
- 5) Watch the video about this experiment. It can be found on Moodle.
- 6) You may want to attempt the theoretical problems before class.

## Preliminary Information

### Background

The moment of inertia of a collection of particles is given by:

$$I = \sum_i m_i r_i^2$$

where  $m_i$  is the mass of the particle and  $r_i$  is the distance from the axis of rotation. For a continuous rigid body (such as a ring or disk) this generalises to:

$$I = \int r^2 dm$$

To determine the moment of inertia experimentally, a known torque is applied to the object and the resulting angular acceleration is measured. Since  $\tau = I\alpha$ ,

$$I = \tau / \alpha$$

where  $\alpha$  is the angular acceleration, which is equal to  $a/r$  ( $a$  = linear acceleration), and  $\tau$  is the torque caused by the weight hanging from the thread that is wrapped around the pulley.

$$\tau = r \times T$$

where  $r$  is the radius of the chosen pulley about which the thread is wound, and  $T$  is the tension in the thread when the apparatus is rotating (see Figure 1a).

Applying Newton's Second Law for the hanging mass,  $m$ , gives

$$\sum F = mg - T = ma$$

Solving for the tension in the thread gives:  $T = m(g - a)$

After the angular acceleration of the mass ( $m$ ) is measured, the torque and the linear acceleration can be obtained for the calculation of the rotational inertia.

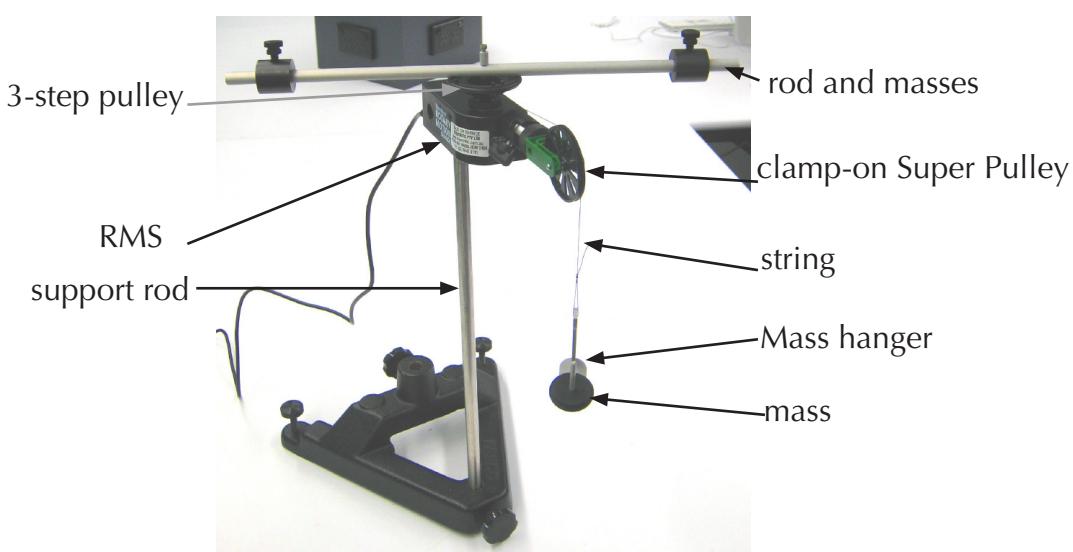


Figure 1a: This shows how the equipment is set up for part 1 of the experiment.

**Theoretical problems:**

Complete the following problems **before** you analyse your results.

**Example:**

The moment of inertia,  $I$ , of a disk with radius  $R$  and mass  $M$  can be calculated as follows:

$$I = \int_0^M r^2 dm$$

Now use the fact that  $m = \rho V$   
and  $V = h\pi r^2$

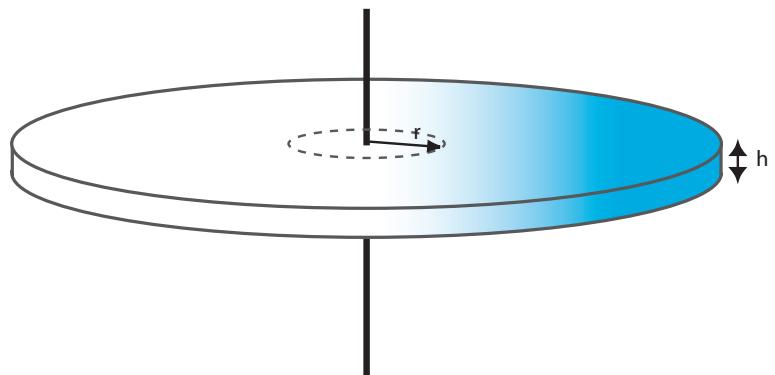
$$\Rightarrow \frac{dV}{dr} = 2h\pi r$$

$$\Rightarrow dm = \rho 2h\pi r dr$$

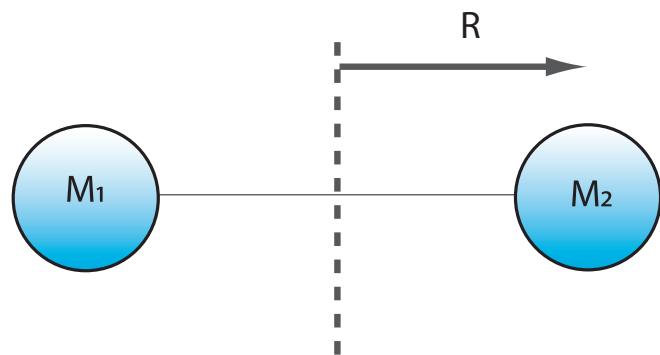
$$\Rightarrow I = 2\pi h \rho \int_0^R r^3 dr$$

$$= 2\pi h \rho \frac{R^4}{4}$$

$$= \frac{1}{2} MR^2$$



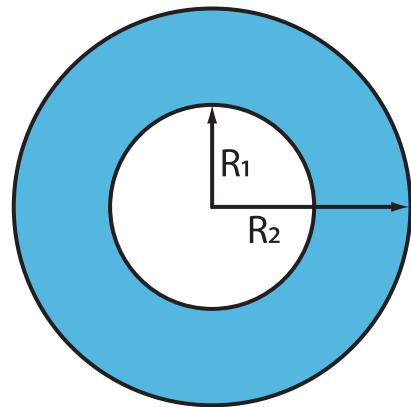
1. Calculate the moment of inertia of two point masses, both located at a distance  $R$  from the axis of location. Assume the total mass ( $M_1 + M_2$ ) is given by  $M$ .



Get  
Marked  
Now

2. Calculate the moment of inertia of a ring of mass,  $M$ , with inner radius  $R_1$  and outer radius  $R_2$ .

Hint: A ring is similar to a disk, the working will be similar but you will need to think about the limits on the integral.



Get  
Marked  
Now

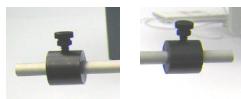
## Equipment

For your work in the laboratory you will need the following

- short retort stand
- rotary motion sensor (RMS)
- a USB cable
- a pulley set (3-step pulley on RMS, an ultra pulley and a screw)
- thread (attached to the pulley)
- a rod with two masses attached
- a disk
- a ring
- a LabPro data interface with a power supply
- a 5 g mass carrier ( $\pm 2.5\%$ )
- a 50 g mass ( $\pm 2.5\%$ )
- a 10 g mass ( $\pm 2.5\%$ )
- electronic caliper

You will also need to use one of the computers and a ruler.

Note: it is easy to get confused between the masses in this experiment - ask your Demonstrator if you are not sure.



Point masses - you have two of these.



Hanging mass carrier with masses (these provide the weight force accelerating the system).

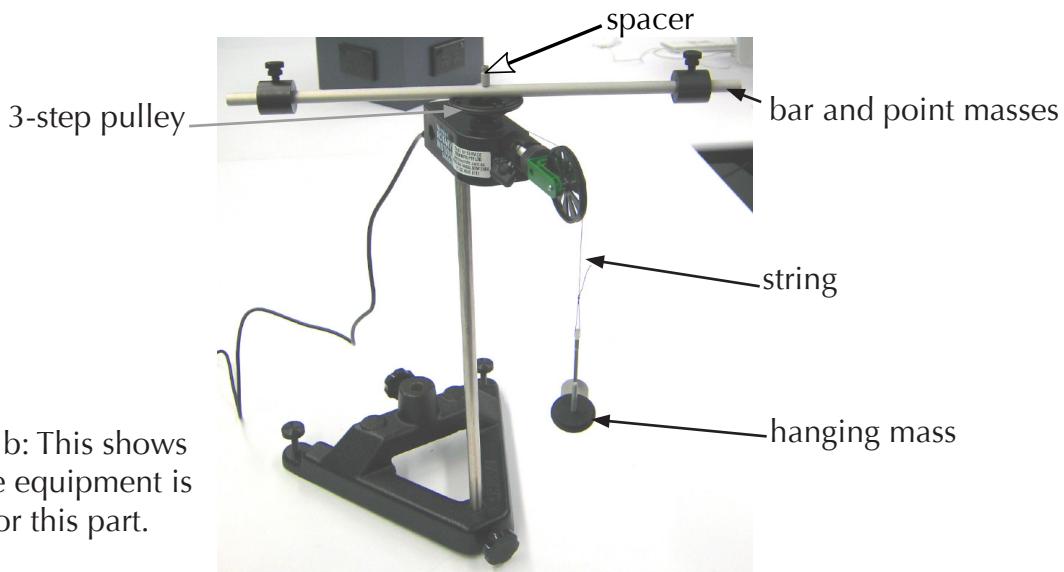
## In the Laboratory

Connect up your LabPro interface by plugging the USB cable into the USB port of one of the iMac computers. The USB port is on the right hand side of the computer, right near where the keyboard plugs in. Connect up the power of the LabPro interface as well. Connect the lead from the rotary motion sensor into "DIG SONIC 1".

There is a template file on the computer for this experiment called "Moment of Inertia". You will need to open this to record data during this experiment.

## Part 1: Rotational Inertia of a Point Mass

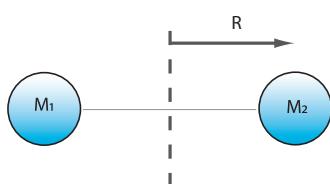
Attach a point mass to each end of the rod. Make sure that the two masses are equidistant from the centre of the rod. You may choose any radius greater than 13 cm. Use a screw to attach the rod to the rotary motion sensor. Make sure that the rod rests inside the slots on the pulley and the spacer goes on the top of the bar.



You will begin by calculating the theoretical moment of inertia of the two point masses. To do this you will need to measure the total mass,  $M_{\text{Total}}$ , of the two masses and also the distance from the centre of the masses to the axis of rotation. Complete the table below, *include uncertainties and units* in your measurements and calculations. There is room for you to show working underneath. Do not include the mass of the rod, just the two point masses.

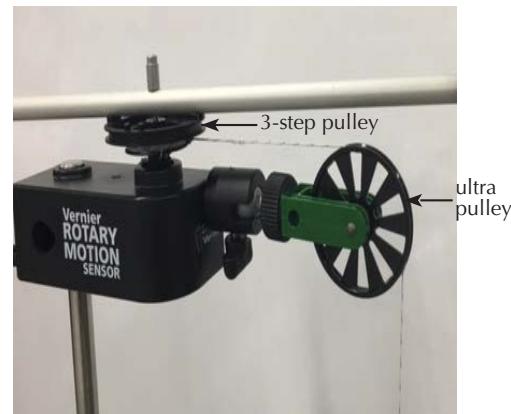
### Theoretical Rotational Inertia Data

Total Mass (g)	
Distance of point masses from pivot, R (mm)	
Moment of Inertia ( $\text{kgm}^2$ )	



You will now measure the moment of inertia experimentally.

One end of the thread should be tied to the pulley on top of the rotary motion sensor. Wind this around the pulley. It is important to always wind it around the same part of the pulley as you will need to use the radius of the pulley in your calculation, the middle part of the pulley works well. Attach the 5g mass carrier with the 50g mass on it to the other end of the thread. Drape the thread from the 3-step pulley over the ultra pulley. Adjust the pulley so that the thread runs in a line tangent to the point where it leaves the 3-step pulley and straight down the middle of the groove on the ultra pulley. Then adjust the height so that the thread is level with 3-step pulley.



Now click on the "Start" button and release the mass. Click the "Stop" button just before the mass reaches the floor to avoid erroneous data. If possible, stop the falling mass before it hits the floor - this will stop the string becoming tangled. Perform a linear fit to the curve on the angular velocity versus time graph in order to find the angular acceleration. Be careful about which data you choose to include in the linear fit. This should be recorded in the table below. You will need to calculate an uncertainty so make sure that you perform the experiment at least 3 times.

Use callipers to measure the *diameter* of the pulley (make sure you measure the appropriate level of the 3-step pulley, not the ultra pulley). The *radius* should be recorded in the table.

The method above allowed you to find the moment of inertia of the apparatus and the point masses. In order to find the moment of inertia of just the point masses you need to find the moment of inertia of the apparatus and subtract this from the moment of inertia of the point masses and the apparatus.

To find the moment of inertia of the apparatus, remove the point masses from the rod and replace the 50 g mass with a 10 g mass. Repeat the experiment.

**Why do you use 10 g instead of 50 g?**

Include units and uncertainties in this table. There is an **Excel template labelled “Rotational Inertia”** that will help you to complete this table. The template will calculate the uncertainties for you but you will need to calculate linear acceleration, tension, torque and moment of inertia.

Experimental Rotational Inertia Data

	<b>Point Mass and Bar</b>	<b>Bar Alone</b>
<b>Hanging Total Mass</b>	50.0 g + 5.0 g	10.0 g + 5.0 g
Slope (angular acceleration)	Trial 1	
	Trial 2	
	Trial 3	
	Average	

Draw a free body diagram showing the forces acting on hanging mass as it falls. Use the free body diagram along with Newton's second law to derive an expression for the tension in the string as the mass falls.

A circular logo containing the text "Get Marked Now". The word "Get" is at the top, "Marked" is in the middle, and "Now" is at the bottom. A red checkmark is positioned to the right of the text.

Diameter of pulley: \_\_\_\_\_

Calculations using Rotational Inertia Data (include uncertainties)

	<b>Point Mass and Bar</b>	<b>Bar Alone</b>
Radius of pulley		
Linear acceleration		
Tension		
Torque		
Moment of inertia		

Moment of inertia of point masses:

Now compare the experimental and theoretical values of the moment of inertia. Comment on the uncertainties and any differences in the values.

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Now

## Part 2: Rotational Inertia of Disk and Ring

Now remove the rod from the rotary motion sensor and replace it with the disk. Be very careful not to loose the screws. The pulleys need to be aligned as before.

Begin by calculating the theoretical moment of inertia of the ring and disk. You will need to measure the mass of each and use callipers to measure the radius of the disk and the inner and outer radii of the ring. Show all working below the table. *Assume that the percentage uncertainty in your measurements is 2%.*

Theoretical Rotational Inertia Data

Disk	Ring
Mass of disk (g):	Mass of ring (g):
Diameter of disk (mm):	Inner diameter of ring (mm):
Radius of disk (mm):	Inner radius of ring (mm):
	Outer diameter of ring (mm):
	Outer radius of ring (mm):
Moment of inertia of disk ( $\text{kgm}^2$ ):	Moment of inertia of ring ( $\text{kgm}^2$ ):
$\pm 5\%^*$	$\pm 5\%^*$

\* Estimated percentage uncertainty

You will now need to perform the experiment for the disk and ring as you did for the two point masses. Initially perform it with the disk in place, then remove the disk and perform it with the ring. Complete the table below, show the working for your calculations and record all uncertainties. The **second sheet in the “Rotational Inertia” template** will help you to complete this table.

Experimental Rotational Inertia Data

	Disk	Ring
<b>Hanging Mass</b>	10.0 g + 5.0 g	10.0 g + 5.0 g
Slope (angular acceleration)	Trial 1	
	Trial 2	
	Trial 3	
	Average	

Diameter of pulley: \_\_\_\_\_

Calculations with Rotational Inertia Data (include uncertainties)

	Disk	Ring
Radius of pulley		
Linear acceleration		
Tension		
Torque		

Moment of inertia of disk:

Moment of inertia of ring:

Now compare the experimental and theoretical values of the moment of inertia. Comment on the uncertainties and any differences in the values.

How could the accuracy/precision of the experiment be improved? Describe the method you could follow to improve this experiment.

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## Marking Guidelines

Get your Demonstrator to tick the boxes below as you go through the exercise.

Please enter your details and date below before you get your final mark from the Demonstrator.

Student Name: \_\_\_\_\_ Student ID Number: z\_\_\_\_\_

Date: \_\_\_\_\_

Completed theoretical problems 1 and 2	
Calculated theoretical I with uncertainty for point masses	
Measured I for point masses accurately, include units	
Calculated uncertainty in I for point masses, compared theory and experiment and commented	
Calculated theoretical I for ring and disk, with units	
Measured I for ring, with units	
Measured I for disk, with units	
I measured for ring and disk is accurate	
Calculated uncertainty in results for ring and disk.	
Compared experimental and theoretical results. Given an improved method for this experiment.	

Total: /10

Demonstrator's Signature: \_\_\_\_\_

Demonstrator's Name: \_\_\_\_\_



# SPECIFIC AND LATENT HEAT

## Introduction

When heat energy is transferred to a body or a quantity of a substance, in general (but not always), the temperature of the body or substance will increase. The exception to this rule occurs when the substance undergoes a *change of state*, such as boiling or melting, which takes place at constant temperature.

Experimentally, for a given quantity of energy (joules) added, the rise in temperature depends both on the mass of the body and the type(s) of material of the body. This thermal property of a substance is quantified by introducing the concept of the *specific heat of a substance*.

When a body undergoes a change of state the temperature remains constant as heat energy is being added or removed. The amount of energy that the body absorbs or emits whilst it undergoes a change of state is called *latent heat*.

In this class, we will measure the specific heat capacity of water as well as the latent heat of fusion for water.

This exercise is designed to help you:

- Develop an understanding of the relationship between heat and temperature.
- Observe a system undergoing a phase change.

## What To Do Before You Get To Lab

- 1) Read through the Preliminary Information starting on the next page.
- 2) If you are still uncertain about the theory involved consult the relevant section in your textbook.
- 3) Complete the preliminary problems and prework test on Moodle.
- 4) Read through the rest of the exercise so that you will know what to do in the laboratory.
- 5) Watch the video about the experiment online. There is a link on the Moodle page.  
You may attempt the theoretical problems before the lab.

## Preliminary Information

### Heat Capacity

The *heat capacity* of a body is the property which determines how much heat energy must be added to it to raise its temperature by a certain amount. That is, if we wish to change the temperature of an object from some initial temperature,  $T_i$ , to some final temperature,  $T_f$ , then we would require an amount of heat,  $Q$ , such that:

$$Q = C(T_f - T_i)$$

Where  $C$  is the heat capacity of the object.

### Specific Heat Capacity

If you have two bodies made of the same material, the heat capacities of the bodies will only be the same if their masses are the same. If one was twice the mass of the other, then you would need to add twice as much energy to it, if you wanted to change the temperatures of both bodies by the same amount.

The *specific* heat capacity,  $c$ , (or simply the ‘specific heat’) is defined as the heat capacity *per unit mass*, of the material (i.e. per kg, in SI units). The specific heat capacity will be the same for any bodies made from the same material, regardless of their mass. Using this definition, we can now write the equation for the amount of heat,  $Q$ , required to raise the temperature of a body from  $T_i$  to  $T_f$  as:

$$Q = mc(T_f - T_i)$$

Where  $m$  is the mass of the body.

### Latent Heat

We are familiar with putting ice into drinks to cool them down and keep them cool. If you put just one ice cube (with a temperature of 0°C) into a warm drink, the ice will melt and the temperature of the drink will go down. In fact, the temperature will fall more than it would if you added the same quantity of water at 0°C. The reason for this is that the melting of the ice required a large amount of heat energy.

Whenever a substance (not only water) melts or vaporises, heat energy is absorbed in the process. Conversely, when a substance condenses (from the gas to the liquid phase or state) or freezes, heat energy is released. The amount of heat energy required to change the state of a substance is called the latent heat. Usually, the latent heat is expressed as a specific quantity - the amount of heat required per unit mass of the substance. Thus, the specific latent heat has units of J.Kg<sup>-1</sup>, and is generally given the symbol  $L$ .

Different kinds of latent heat have been defined to specify the change of state occurring. The latent heat involved in changes between the liquid and solid states is called the latent heat of fusion, while the latent heat involved in changes between the liquid and gaseous states is called the latent heat of vaporisation. Less commonly encountered latent heats include the latent heat of sublimation for changes directly between the solid and gaseous states (e.g. the element Iodine is observed to do this), and the latent heat of atomisation for changes between diatomic and monatomic states of a gas.

## Theoretical Problems

During the exercise you will need to complete the following two problems **while** collecting data.

1) A student uses some equipment that can add energy at a constant rate to some water contained in an aluminium calorimeter can. To keep the temperature of the water uniform, the student stirs the water with an aluminium stirrer. Together, the calorimeter can and the stirrer have a mass of  $m_{Al}$  and a specific heat of  $c_{Al}$ , and the water has a mass of  $m_w$  and specific heat of  $c_w$ .

The initial temperature of the water is  $T_0$ , and the student begins adding energy to the system at a constant rate of  $P$  watts.

*Assuming there is no energy loss from the system,* derive an expression for the temperature,  $T$ , of the system as a function of time,  $t$ .  
 (Hint: the rate at which energy is added,  $P = \frac{dQ}{dt}$  where  $dQ = (m_{Al}c_{Al} + m_w c_w)dT$ . Substitute, rearrange and integrate.)

What is the expression for  $\frac{dT}{dt}$

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2) Consider an aluminium calorimeter can with an aluminium stirrer has a total mass  $m_{Al}$  and specific heat  $c_{Al}$ :

- It contains a mass  $m_w$  of water of specific heat  $c_w$  at a temperature of  $T_i$  °C
- A mass  $m_i$  of ice at 0 °C is added to the calorimeter
- The heat of fusion of ice is  $L_i$
- All the ice melts and the temperature of the can, stirrer, and contents falls to  $T_f$  °C (which is above 0 °C)

(a) Presuming that there is no heat energy gained by or lost from the system, write down an equation representing conservation of energy for the system.  
Indicate clearly what each of the terms in this equation represents

(b) Hence, find an expression for the latent heat of fusion of ice,  $L_i$ , in terms of the other variables in your equation. (Show all steps)

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## Equipment

You will need to collect the following equipment from the hatch, swapping it for your student ID card:

- a calorimeter with heating element and stirrer
- a calorimeter with a stirrer
- a switch mode power supply (**set it to 6V**) and cable
- two digital multimeters
- a LabPro interface with USB cable and a power supply
- a temperature probe,
- a laboratory lead kit
- a switch

You will also need to use:

- a balance for measuring mass
- ice (from the ice machine)
- water
- paper towel
- beakers

## In the Laboratory

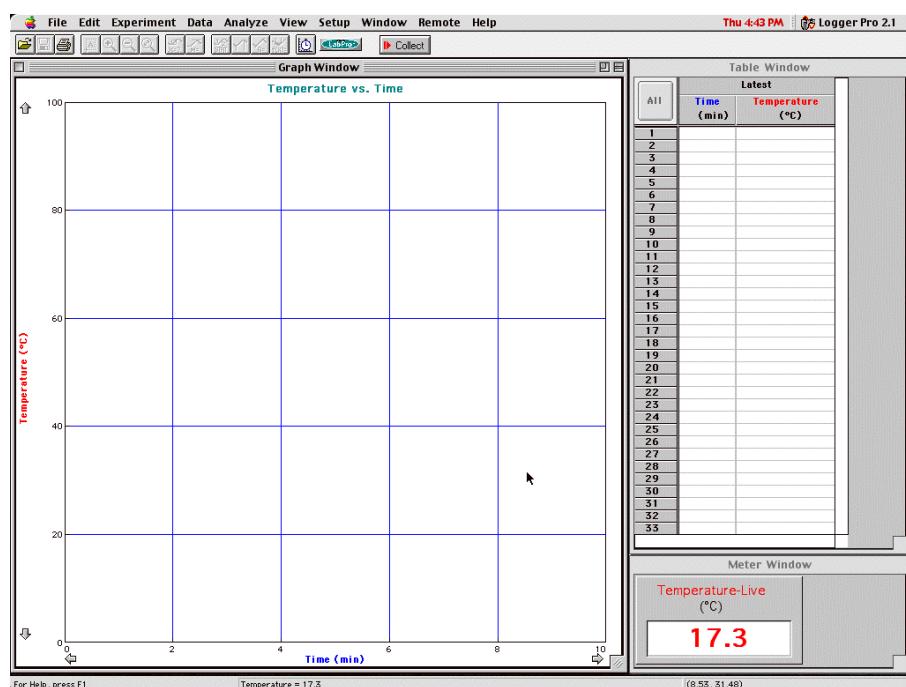
Connect up your LabPro interface by plugging the USB cable into the USB port of one of the iMac computers. The USB port is on the right hand side of the computer, right near where the keyboard plugs in. Connect up the power to the interface as well.

Plug the temperature probe into the "CH1" socket on the left side of the interface.

**Make sure you keep the equipment as far from the computers as the cables will allow!** This prevents damage to the computers from accidental spills of water etc.

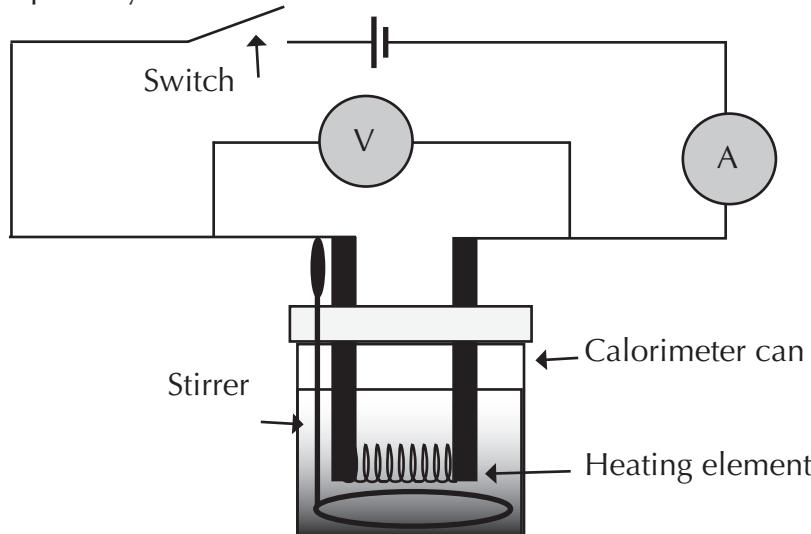
There is a specific template for this experiment called "Specific and Latent Heat". Make a copy of this template to use in the exercise (see the "Using the Computers" section of the lab manual if you need more information).

Double click on your copy of the template to start up the data logging software. You should see an empty temperature versus time graph, and a live temperature readout.



## Part 1: Measuring Heat Capacity

Weigh the calorimeter can (with stirrer), then fill it with water until about two-thirds full, make sure there is enough water that the heating element is completely submerged when you close the calorimeter. Then set up the heating element using the circuit diagram below. Do not turn on the power yet!



You will need to use the ‘unfused’ setting on the multimeter acting as the ammeter and the ‘fused’ setting on the multimeter acting as the voltmeter. Ask your Demonstrator if you need help with this. If you use the fused setting to measure the current, you will blow the fuse.



To use the unfused setting of the multimeter you need to plug a cable into center connection (COM) and one cable into the left hand side connection. And use this multimeter as an ammeter. The multimeter you use as a voltmeter has to be set up in the ‘fused’ setting (center + right hand side connection).

The circuit is now set to supply energy at a constant rate to the water in the calorimeter. Close the calorimeter with the heating element in place.

Insert the temperature probe into the water taking care that it does not rest against the element. Watch the live temperature read out on the screen to see if it changes to the temperature of the water (unless the water is exactly at room temperature!). Switch on the power. Warning - the heating element must be covered with water *before* the circuit is switched on. You should switch off the power before removing the heating element. While the power is on the heating element must be submerged in water, you should not touch it.

Begin your measurement by clicking the “Collect” button at the top of the screen. The software should record the temperature every two seconds for up to 5 minutes. **Make sure you stir the water with the stirrer (not with the temperature probe!) regularly.** If you do

not stir then your results will be distorted.

Once you have finished your measurement, you should have a plot of temperature, T, vs time, t. *Using this data*, obtain an estimate of the specific heat capacity of the water,  $c_w$ , with an uncertainty. Show all working and steps in the space on the next page. Keep in mind the following:

- If the current in the coil is I amperes, and the voltage drop across the coil is V volts, then *Energy is being added at the rate of VI watts*.
- Examine the expression for  $T(t)$  you derived in Theoretical Problem 1.
- You can use the computer to calculate the gradient of any part of your graph by  dragging the mouse cursor around the area and pressing the “Linear fit” button.

Make sure you turn off the heating coil current when you finish your measurements. Weigh the can and stirrer again so as to deduce the mass of water you have.

You will need to know:

$$\text{Specific heat of aluminium, } c_{\text{Al}} = 900 \text{ Jkg}^{-1}\text{K}^{-1}$$

Please do this experiment three times for five minutes each to get three values for  $c_w$  and to be able to calculate an uncertainty for  $c_w$ . Please switch the heating element off between measurements.

1a) If you start and finish with water that is cooler than room temperature, will your measured value for  $c_w$  be higher or lower than the actual value for  $c_w$ ? Why?

1b) What happens if you start and finish with water that is warmer than room temperature? Will the measured value for  $c_w$  be higher or lower than the actual value for  $c_w$ ?

1c) Is your measured value for  $c_w$  higher or lower than the actual value for  $c_w$ ? Why?

1d) If you were to repeat the experiment, how could you change it to improve the accuracy of your result?

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## Part 2: Latent Heat

You will now be performing an experiment to determine the specific latent heat of fusion of water. The method you follow is up to you, so think about your procedure and discuss it with your colleagues. The method should be based upon the situation described in the second theoretical problem. When constructing your method you should consider the uncertainties that you faced in the previous part and think about ways in which you can minimise similar effects in this part. You should also consider the following:

- You do not need the heating element for this exercise. You have been provided with a lid for the calorimeter that does not contain a heating element.
- Ensure that the can plus its contents are in *thermal equilibrium* when determining its temperature (stirring ensures this through thorough mixing).
- If a liquid–calorimeter can system is not at the same temperature as its surroundings (i.e. room ambient temperature) then energy interchange occurs between the system and its surroundings. This generally occurs at a rate *proportional to the difference in temperature* between the calorimeter system and the surroundings. Take steps to minimise the effects of this heat interchange – think carefully about your choice of starting and finishing temperatures. Keep in mind that you don't have to add the ice all at once, but can continue adding ice until a desired final temperature is reached (you might start with 1-2 pieces of ice).
- When adding ice, take steps to ensure that the absolute minimum amount of melted ice (i.e., water!) is added along with it. – Use a piece of paper towel to remove as much water as possible.

**In the space below**, give a description of a method that could be followed to obtain an estimate for the latent heat of fusion of ice,  $L_f$ , using the equipment listed. Please make sure that you collect enough data to be able to obtain an uncertainty for the latent heat of fusion.

Ask your demonstrator if you need any help.

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Once you have completed your plan for your experiment and your demonstrator has approved it, *carry out your experiment as planned.*

Record all of your observations, calculations and anything else relevant to determine your result in the space below.

Write your estimate for the specific latent heat of fusion of water, including uncertainties (use the value for  $c_w$  that you measured in part 1):

*Before you return your equipment to the hatch please dry your calorimeters with paper towels.*

There will almost certainly be some discrepancy between your calculated value and the standard literature value of  $3.35 \times 10^5 \text{ J kg}^{-1}$  for the latent heat of fusion of water. List what you consider to be the main factors which may be contributing towards this discrepancy.

Compare your result to the expected result. Use this to estimate the amount of heat lost/gained by the environment.



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## Marking Guidelines

Get your Demonstrator to tick the boxes below as you go through the exercise.

Please enter your details, Demonstrator's name and date below before you get your final mark from the Demonstrator.

Student Name: \_\_\_\_\_ Student ID Number: z\_\_\_\_\_

Date: \_\_\_\_\_

Theoretical Problems 1 completed	<input type="checkbox"/>
Theoretical Problems 2 completed	<input type="checkbox"/>
Obtained data to calculate $c_w$	<input type="checkbox"/>
Calculated $c_w$ correctly, including units	<input type="checkbox"/>
Suggested improvements and answered all questions about $c_w$	<input type="checkbox"/>
Suggested suitable method for measuring L	<input type="checkbox"/>
Collected data to measure L	<input type="checkbox"/>
Calculated L correctly, with units	<input type="checkbox"/>
Calculated uncertainties in L and $c_w$	<input type="checkbox"/>
Suggested reasons for difference between measured L and accepted value	<input type="checkbox"/>

Total: /10

Demonstrator's Signature: \_\_\_\_\_

Demonstrator's Name: \_\_\_\_\_

# IDEAL GAS LAW

## Introduction

You will begin this laboratory exercise by investigating the relationship between pressure, force, area and volume. You will then look at how pressure, volume and temperature change in adiabatic and isothermal compressions and expansions. You will be considering heat flow into and out of the system and also work done on and by the gas.

**The exercise is designed to give you a deeper understanding of:**

- $P = \frac{F}{A}$
- $PV = nRT$
- Adiabatic compressions ( $PV^\gamma = \text{const}$ )
- The first law of thermodynamics ( $\Delta E_{\text{int}} = Q + W$ )

## What to do before you get to the lab

1. Read and make notes on the preliminary information
2. Review the relevant section of your textbook for a more detailed account of the theory
3. Complete the preliminary problems and prework test for this exercise on Moodle
4. Read through the rest of the exercise so that you will know what to do in the laboratory
5. Watch the video about this experiment; it can be found in Moodle.

## Preliminary Information

When the particles of a gas collide with the walls of a container they exert a force on the wall. We can measure this force as a pressure. Pressure and force are related through the equation:

$$P = \frac{F}{A}$$

Where P is the pressure in Pascals (Pa), F is the force in Newtons (N) and A is the surface area in m<sup>2</sup>. If you exert a force on a syringe the volume inside the syringe will change until the additional force and hence pressure exerted on the gas is the same as the additional pressure the gas exerts on the syringe. In other words the volume will change until equilibrium is reached.

An ideal gas is described by an equation of state:

$$PV = nRT$$

This equation relates the macroscopic properties of the gas, pressure, P, volume, V and temperature, T to the number of moles, n of a gas and the gas constant R = 8.314 J/(mol·K). Air is not quite an ideal gas (why not?) but it is close. You can model air as an ideal diatomic gas. The main components of air are Nitrogen (N<sub>2</sub>, 78%) and Oxygen (O<sub>2</sub>, 21%).

The first law of thermodynamics states that the change in the internal energy of the gas is equal to the heat that is transferred to the gas and the work done on the gas:

$$\Delta E_{int} = Q + W$$

You will be considering how the internal energy of gas in a syringe changes as it undergoes a cycle involving compression, then an isovolumetric process and finally an expansion. The change in the internal energy of a gas is related to the change in temperature through:

$$\Delta E_{int} = \frac{f}{2} nR\Delta T$$

Where f is the number of degrees of freedom of the molecules making up the gas. Degrees of freedom are the ways in which a gas can store energy, there are translational, rotational and vibrational degrees of freedom. One consequence of this equation is that when the temperature of a gas does not change the internal energy of the gas remains constant.

When a gas is compressed quickly there is no time for heat to flow into or out of the system. In this case the compression is said to be adiabatic. For an adiabatic compression the change in internal energy is equal to the work done. During an adiabatic processes on an ideal gas the pressure and volume obey the relationship:

$$PV^\gamma = \text{const}$$

Where  $\gamma$  depends on the gas and is equal to  $\frac{C_p}{C_v} = \frac{f+2}{f}$ .

## Equipment:

For your work in the laboratory you will require the following:

- Tall retort stand
- Ideal gas law syringe
- Set of 8 masses, 0.5 kg each (1% uncertainty) and hanger
- Temperature and pressure sensor with Pasport interface
- 2 bossheads and clamps
- cork board

## In the laboratory

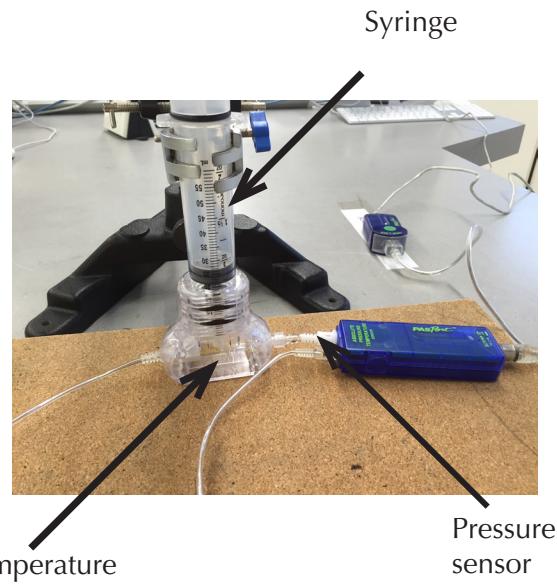
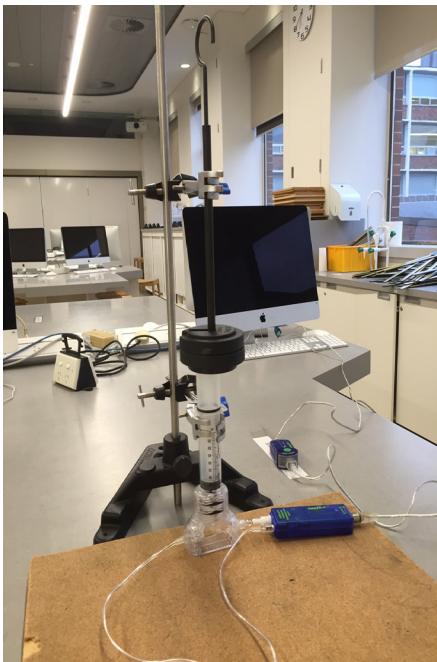
All the clear plastic tubes on the syringe are glued in place. Please do not remove these during the experiment. Connect the temperature probe through the mini stereo jack to the temperature sensor. The pressure probe connects to the pressure sensor through a tubing coupler. You do not need to connect this at present. Plug the USB connector from the sensor into a USB port. Launch the Capstone program by selecting “Ideal Gas Law.cap” from the Physics 1A folder.

1.) The temperature change in the syringe is measured with a low temperature thermistor. The response time on the thermistor is around half a second. Why is it not possible to measure the temperature instantaneously?

Now you are ready to begin the experiment.

### Part 1: Relating Pressure, Force, Area and Volume

In this part of the experiment you will be placing 0.50 kg masses on top of the syringe. You should clamp the syringe to the retort stand using 1 clamp as shown in the picture. The masses will be supported by another clamp, this clamp should not hold the masses firmly; it is to stop the masses falling over *not* to reduce the weight force.



Please answer questions 2-5 before you set up the equipment.

2.) Calculate the cross-sectional area of the syringe. This is the area over which the syringe is applying pressure on the gas inside the cylinder. In order to do this measure the height of the syringe when it contains 60 mL of air. Use this volume with the height to calculate the cross-sectional area. You do not need an uncertainty in this value.

3.) Now calculate the change in pressure you would expect if you were to place a mass of 1.50 kg on the syringe. You do not need to calculate the uncertainty in this value.

4.) Calculate the final volume you would expect after adding 1.50 kg (given that you leave time for the syringe to return to thermal equilibrium with the laboratory). Assume that the initial volume is 60 mL and the initial pressure is 1 atm ( $1.01 \times 10^5$  Pa). Do not calculate an uncertainty.

Move the plunger to the 60mL mark with the pressure sensor disconnected.

5.) Why does the pressure sensor need to be disconnected for this part?

Now connect the pressure sensor to the syringe. Take measurements by hitting the red "Record" button in the bottom left hand corner of the screen. Place the masses on the syringe. Make sure that you leave time for the syringe to reach thermal equilibrium with the laboratory. You can check this by looking at the temperature reading. You need to record the volume of the syringe as you go. The other values can be obtained from your graphs when you have finished taking measurements. Please make sure that the masses are centered on the syringe.

6.) Why is it important that the syringe reaches thermal equilibrium with its surroundings?

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To measure a value from the graph highlight the part of the graph that you are interested in by hitting the highlighter button  and then adjusting the size of the box. To get the value hit the statistics button ( $\Sigma$ ), you can use the mean of the values in this region as the value.

### Results: Mass, pressure, temperature and volume of an air filled syringe

Mass added (kg)	Pressure (kPa)	Temperature (°C)	Volume (mL)
0			
1.50			
2.00			
2.50			
3.00			
3.50			
4.00			

When you have made your measurements take the weights off the syringe.

7.) Compare your predicted value from question 4 with value obtained for volume at 1.5 kg. Comment, suggesting reasons for any discrepancies.

Use “General Linear Plot” to plot a graph showing the relationship between pressure and volume.

Record the details of your graph below:

*y axis (quantity and units):*

*x axis (quantity and units):*

*equation of graph:*

Use this to calculate n,  $R = 8.314 \text{ JK}^{-1}\text{mol}^{-1}$ .

8.) How many mols of gas should be in your syringe and how does this theoretical value compare to the value that you have determined for n experimentally?

Hint: 1 mol of an ideal gas has a volume of 24.5L at room temperature and pressure.

Get  
Marked  
Now

## Part 2: Adiabatic and isothermal compressions/expansions

In this part of the experiment you are going to perform an adiabatic compression and then allow the syringe to come to thermal equilibrium with its surroundings. It is very important that you are a little patient and wait for thermal equilibrium to be reached before you release the plunger. Once the syringe has reached thermal equilibrium you will let the gas expand. It should take much less than a minute to reach thermal equilibrium.

To perform this experiment remove the syringe from the retort stand. You can perform this experiment on the laboratory bench. Disconnect the pressure sensor. Measure the volume of the syringe when the plunger is pushed down as far as it will go. This should be around 20 mL.

Volume (mL) =

Now pull the plunger until the volume of the syringe is 40 mL then reconnect the sensor. Click the “Record” button and then quickly (adiabatically) compress the syringe with the plunger. Hold the plunger in position until thermal equilibrium has been reached and then allow the plunger to slowly return to its initial volume. When the syringe has once again reached equilibrium stop recording the data.

Use the graphs to measure the initial (before compression) pressure, volume and temperature, then the maximum pressure, temperature and the volume when these occur, followed by the pressure and temperature when it has reached thermal equilibrium with its surroundings but before it is released. Check that after the plunger has been released the values return to the initial values. To measure a value from the graph highlight the part

of the graph that you are interested in by hitting the highlight button  and adjusting the size of the box. Then hit the statistics button ( $\Sigma$ ) use whichever statistic is the most appropriate in that region.

**Results: Adiabatic, isovolumetric and isothermal processes**

		Pressure (kPa)	Volume (mL)	Temperature (°C)	Temperature (K)
1	initial				
2	Maximum pressure and temperature				
3	Equilibrium with surrounds				
4	final				

Now it is time to analyse your results.

9.) Which paths 1->2, 2->3 and 3->4 are closest to an adiabatic compression, isothermal expansion and isovolumetric process? Complete the table below:

Path	1->2	2->3	3->4
Process name			

Start by analysing the *isothermal expansion*.

10.) Write down (as a formula) the relationship between pressure and volume in an isothermal expansion.

11.) Do your results agree with this equation with 5%? Give a quantitative answer.

Get  
Marked  
Now

Now consider the *adiabatic compression*. This is from the initial conditions until the maximum pressure and temperature have been reached.

12.) Write down the expression for the relationship between P and V for the adiabatic compression of air (modelling air as a diatomic molecule). You should not include T in this expression. Show and justify all your working.

13.) Do your results agree with this expression with 5%? Make sure you justify your answer quantitatively.

14.) Explain the discrepancy between your results and the expected result.

15.) Now write down the relationship between P, V and T for an ideal gas undergoing an adiabatic compression or expansion.

16.) Quantitatively assess whether your results are consistent with the expression you have written and justify any discrepancies.

17.) Does the syringe return to its initial volume when the plunger is released? Compare your answer with what you expected to happen and suggest reasons for any discrepancies.

Get  
Marked  
Now

In the space below sketch a PV graph.

**PV graph of compression and expansion cycle of the gas**

18.) Label the paths as adiabatic, isothermal and isovolumetric. Above each of the paths on the graph write the value of  $\Delta E_{int}$ , Q and W. Show all your working on the following space.

Hints:

- You will need to calculate  $n$  in order to do calculate  $\Delta E_{int}$  for the adiabatic compression (at least this is a much easier way to do it).
- You will need to do the integral  $W = - \int P.dV$  in order to calculate the work done during the isothermal expansion. You will need to replace P with a function of V before solving the integral.

Working: (make sure you show all steps and reasoning)

19.) Finally, complete the table below:

Path	Units	1->2	2->3	3->4
Process		Adiabatic		
Q		0		
W				
$\Delta E_{int}$				

Get  
Marked  
Now

## Marking Guidelines

Get your Demonstrator to tick the boxes below as you go through the exercise.

Please enter your details, Demonstrator's name and date below before you get your final mark from the Demonstrator.

Student Name: \_\_\_\_\_ Student ID Number: z \_\_\_\_\_

Date: \_\_\_\_\_

Answered question 1 and performed calculations in questions 2 and 3 correctly	
Answered questions 4-6 in detail and correctly	
Completed first results table	
Answered question 7 in detail and correctly	
Calculated and answered question 8	
Completed second results table	
Quantitatively compared results and theory for isothermal expansion (questions 9 to 11)	
Quantitatively compared results and theory for adiabatic compression using P and V and explained any discrepancy (questions 12 to 15)	
Quantitatively compared results and theory for adiabatic compression using P, V and T. Sensible answer given if syringe returns to initial volume (questions 16 to 17)	
Plotted graph with $\Delta E_{int}$ , Q and W calculated for each path and completed table (question 18 to 19)	

Total: /10

Demonstrator's Signature: \_\_\_\_\_

Demonstrator's Name: \_\_\_\_\_

# LINEAR OSCILLATORY MOTION

## Introduction

In this exercise you will determine the “stiffness constant” of a spring using two methods; a direct and an indirect method.

This exercise is designed to give you an understanding of:

- amplitude, frequency, angular frequency and period of a simple harmonic oscillator
- spring constants
- calculating uncertainties

## What To Do Before You Get To Lab

- 1) Read and make notes on the following preliminary information.
- 2) Complete the preliminary problem and prework test on Moodle.
- 3) Read through the rest of the exercise, so that you will know what to do in the laboratory.
- 4) Watch the video about the experiment online. There is a link on the Moodle page.
- 5) If you have time, see the chapters on oscillation and simple harmonic motion in Physclips, at:

[www.animations.physics.unsw.edu.au/](http://www.animations.physics.unsw.edu.au/)

- 6) You may want to attempt the theoretical problems before attending the laboratory.

## Preliminary Information

A periodic or harmonic motion is any motion that repeats itself at regular intervals, the period. The maximum displacement of the oscillating object from its mean position is its amplitude,  $x_m$ . The inverse of the period is the frequency of the motion.

When the motion is in the form of a sinusoidal (or cosinusoidal) function of time, the motion is said to be simple harmonic motion. The displacement from equilibrium of the object undergoing simple harmonic motion as a function of time,  $t$ , can be written as:

$$x(t) = x_m \cos(\omega t + \phi)$$

As the displacement returns to the same value after one period,  $T$ , and the cosine function repeats itself when its argument is increased by  $2\pi$ , it follows that

$$\omega = \frac{2\pi}{T}$$

where  $\omega$  is called the angular frequency.

For a spring, the force,  $F$ , exerted by the spring is proportional to displacement,  $d$ , of the free end from its position when the spring is in a relaxed state. The constant of proportionality,  $k$ , is called the spring constant and is a measure of the stiffness of the spring.

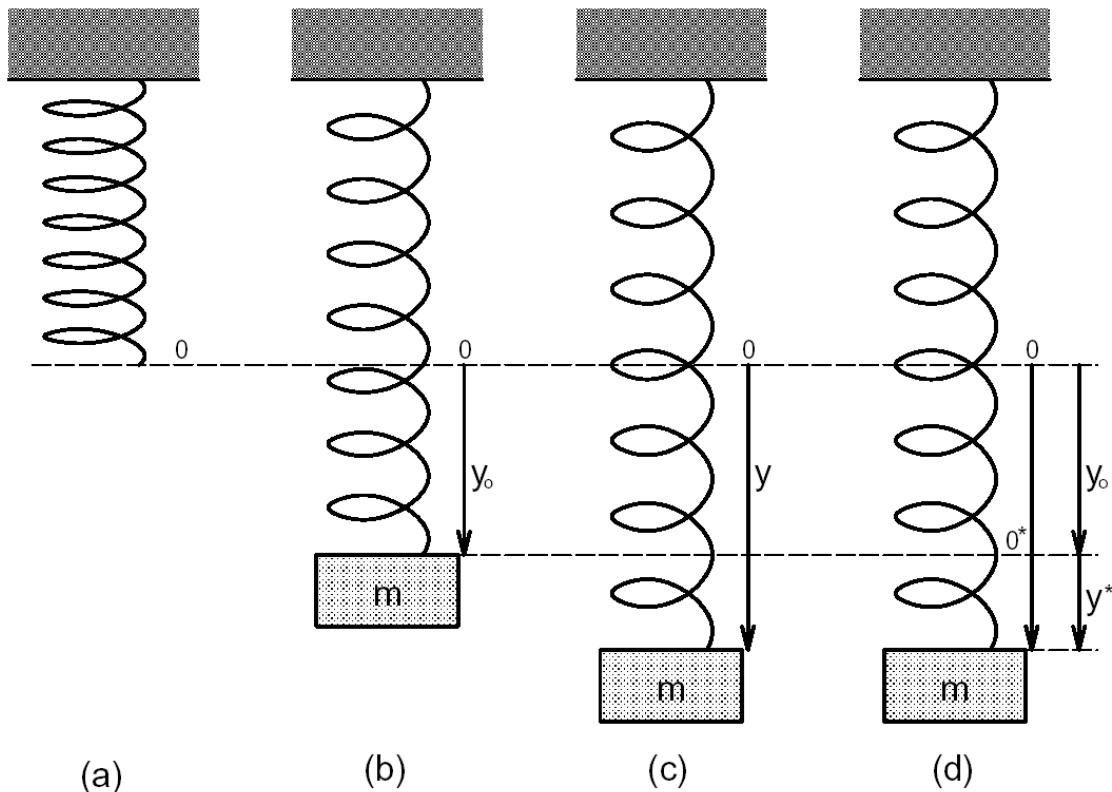
Simple harmonic motion arises when the displacement  $x$  satisfies the differential equation:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

You might like to prove to yourself that  $x_m \cos(\omega t + \phi)$  is indeed a solution to this differential equation, by simply differentiating it twice.

## Theoretical Problems

A massless spring, with spring constant  $k$ , is hung vertically, as shown in figure (a) below.



Consider only vertical motion, and describe the position of the end of the spring, as follows:

- First, set the origin at  $O$ , the end of the hanging spring with no mass attached.
- Let  $y$  be the displacement of the end of the spring from the origin,  $O$ .
- Notice that, in this case, we have chosen the positive direction of  $y$  to be *downwards*

1.) A mass,  $m$ , is hung from the lower end of the spring. When it is at equilibrium, the displacement of the lower end of the spring (and so the upper surface of the mass) is  $y_0$ , as shown in figure (b).

**On figure (b), draw arrows to show the gravitational and spring forces acting on the mass, and give expressions for their values.**

**Hence derive** an expression for the equilibrium displacement,  $y_0$ , of the lower end of the spring, in terms of  $m$ ,  $k$  and any other needed data.

Get  
Marked  
Now

2.) Figure (c) shows the mass  $m$  at a displacement of  $y > y_0$  from the origin. Here, of course, it is not in mechanical equilibrium

- (i) On figure (c) show each of the forces acting on the mass, and give expressions for their values.

For this nonequilibrium case, write an expression for  $\sum F$ , the total force acting on  $m$  *in the positive i.e. downwards direction*:

$$\sum F =$$

- (ii) The acceleration in the positive  $y$  direction,  $a_y = \frac{d^2y}{dt^2}$ , is related to the force in the positive  $y$  direction,  $\sum F$  by Newton's Second Law.

Show that :

$$\frac{d^2y}{dt^2} = g - \left(\frac{k}{m}\right)y$$

- (iii) Now the expression we used at the beginning for simple harmonic motion was for displacement from equilibrium. For that reason, we now introduce a **new displacement variable**,  $y^*$ , which is shown in figure (d).  $y^* = \text{displacement of the mass from its equilibrium position}$ . Simply by looking at figure (d), **write down** an equation for the relationship between  $y^*$ ,  $y$ , and  $y_0$ .

(iv) By differentiating this expression twice with respect to time, show that  $\frac{d^2y}{dt^2} = \frac{d^2y^*}{dt^2}$

[Hint: When you differentiate a constant you get zero]

(v) By substituting for  $y$  from (iii) into the relationship in (ii), **prove** that the differential equation for the motion of the mass,  $m$ , is that for simple harmonic motion, i.e., it has the form:

$$\frac{d^2y^*}{dt^2} = -\left(\frac{k}{m}\right)y^*$$

(vi) Use your result in (v) and some of the preliminary information to **find** an expression for the period of oscillation of the mass,  $m$ . Assume that the mass moves with simple harmonic motion.

Get  
Marked  
Now

## Equipment

The equipment from around the lab for this exercise includes:

- a retort stand fitted with a metre rule and a horizontal bar supporting a red spring and mass carrier. The mass carrier has a mass of 50 g. It should not be disconnected from the spring.
- 9 masses (uncertainty  $\pm 2.5\%$ ).
- a LabPro with a power supply and a USB cable.
- a motion sensor with a cable (make sure it is set to the cart setting)

## In the Laboratory

### Stiffness Constant of the Spring – Direct Method

- In this part you obtain data giving the spring equilibrium extension as a function of the mass load on it, for mass loads from **0.2 kg** to **0.5 kg**.
- Use the following table for recording raw data, calculations, etc.
- Make sure you include an uncertainty in your measurements. How would you measure the uncertainty in the equilibrium extension?

Mass load on spring kg	Position of end of spring* mm	Equilibrium extension ** mm
0.200	$\pm$	0 $\pm$

\* & \*\* Footnotes: these are respectively  $y$  and  $y_0$  in the figures shown in the theoretical problems. They need to be measured from the end of the spring or alternatively from the base of the carrier.

- Plot a graph of the equilibrium extension as a function of the mass load on the spring on one of the computers and an appropriately named copy of the “Linear Plot with Errors” template (see the *Using the Computers* section in the front of the lab manual for more information on obtaining and using the templates).

- An estimate of the stiffness constant for your spring with uncertainty, may be obtained from this graph.

- **In the space below explain how this may be done;**

- You need to state the quantity to be plotted on each axis, and
- Explain in detail how the stiffness constant is to be determined from the resulting graph.
- Record the equations for the lines of best fit and worst fit for your graph.

Follow through to obtain the estimate.

**All working must be shown.**

**Stiffness constant of the spring = .....  
(Don't forget the units)**

Get  
Marked  
Now



## Stiffness Constant of the Spring – Indirect Method

In this part you need to experimentally determine the period of small amplitude (vertical) oscillations *about the equilibrium position* of a mass attached to the end of your spring.

Firstly, *spend some time thinking about how you could go about doing this.*

The following points should be taken into account:

- What constitutes one oscillation?
- It is not possible to accurately determine the period by measuring the time taken for one oscillation. **Why not?**
- Should each time measurement be repeated several times? **Why? Why not?**
- The “Car Crash experiment part 1” is a useful logger pro template to use.
- How are you going to measure the uncertainties?

Then, in the space following, **write out a step-by-step procedure** which you could follow to determine the period of small amplitude vertical oscillations about the equilibrium position of a mass attached to the end of your spring.

**STEP 1:**

Now that you have a method you need to test it for one data point to make sure that the method works well and gives you what you need.

- Put on enough mass to get an extension of about 10 mm
- Follow through with the method that you have outlined on the previous page
- Use the following space for recording all raw data, calculations, etc.

**Mass load = .....**

Period of oscillation = .....

Now take a minute to reflect on the procedure you followed, and your execution of the steps in it (perhaps after reviewing it with one of the demonstrators).

In the space below, list

- Any modifications you will need to make to any of the steps, and/or
- Any additional steps which need to be included in the procedure to improve it.

Get  
Marked  
Now

**Follow through your modified procedure** for each of the cases given in the following table.

Mass load (kg)							
0.30							
0.35							
0.40							
0.45							
0.50							

**By employing an appropriate graphical technique** (refer to Theoretical Problem vi) **and using this data**, obtain another estimate for the stiffness constant of your spring with an uncertainty. Use a computer and a copy of the “Linear Plot with Errors” template.

**A full explanation of the technique to be used must be given in the space below.** In particular,

- You need to state the quantity to be plotted on each axis, and
- Explain in detail how the stiffness constant is to be determined from the resulting graph.

All working must be shown. On the next page record the data about the linear fit.

**Stiffness constant of the spring = .....**

Explain (with a sketch) the relationship between the position, velocity and acceleration versus time graphs.

**Summary of results:**

Method	Stiffness constant
Direct method	
Indirect method	

Are these two estimates consistent?

**Discuss**

Get  
Marked  
Now

## Marking Guidelines

Get your Demonstrator to tick the boxes below as you go through the exercise.

Please enter your details, Demonstrator's name and date below before you get your final mark from the Demonstrator.

Student Name: \_\_\_\_\_ Student ID Number: z\_\_\_\_\_

Date: \_\_\_\_\_

Theoretical problems completed	
Data collected for 'direct method'	
Data plotted on a graph, reasonable k obtained from this graph with units	
Uncertainty in k calculated (direct method)	
Sensible method suggested for 'indirect method'	
Data collected for 'indirect method'	
Appropriate graph plotted and k calculated, with units	
Uncertainty in k calculated (indirect method)	
Relationship between position, velocity and acceleration explained in detail	
Methods compared and discussed intelligently	

Total /10

Demonstrator's Signature: \_\_\_\_\_

Demonstrator's Name: \_\_\_\_\_

# STANDING WAVES ON A STRING

---

## Introduction

In this exercise you will determine the mass per unit length of a string. This is found by establishing a standing wave in the string under different conditions.

The exercise is designed to give you an understanding of

- resonance
- standing waves
- wavelength
- and the superposition principle.

## What To Do Before You Get To Lab

- 1) Read and make notes on the following preliminary information.
- 2) Review the relevant sections in your textbook for a more detailed account of the theory.
- 3) Complete the preliminary problems and prework test for this exercise on Moodle.
- 4) Read through the rest of the exercise, so that you will know what to do in the laboratory.
- 5) Watch the video about the lab. The link can be found on Moodle.
- 6) You may want to attempt the theoretical problems before class.

## Preliminary Information

The speed of propagation of low amplitude travelling waves (disturbances) in a material medium is dependent only on the properties of the medium. The general relationship giving the wave speed in terms of the properties of the medium is:

$$\text{wave speed} = \sqrt{\frac{\text{elastic or stiffness property}}{\text{density or inertia property}}} \quad (*)$$

As examples,

1 For the case of a travelling longitudinal wave in a long metal rod, the stiffness property is Young's Modulus,  $E$ , for the metal, and the inertial property is the density,  $\rho$ , of the metal. Thus the speed of propagation of longitudinal waves in a metal rod is given by:

$$\text{wave speed} = \sqrt{\frac{E}{\rho}}$$

2 For the case of a travelling transverse wave on a flexible string or wire, the stiffness factor is the tension,  $T$ , in the string/wire, and the inertial property is the linear density, i.e. mass per unit length,  $\mu$ , of the string/wire. The speed of propagation of transverse waves on a string/wire is given by:

$$\text{wave speed} = \sqrt{\frac{T}{\mu}}$$

The relationship (\*) above indicates that the value of the stiffness property of a medium may be deduced from measured values of the speed of propagation of waves in the medium, and of the medium density.

Experiments, which give accurate determinations of the speed of propagation of a wave disturbance in a medium, are quite easy to perform.

As a result, the technique of determining the elastic property of a material by measuring *both* the speed of propagation of travelling waves in the material, and the material density, finds quite extensive use.

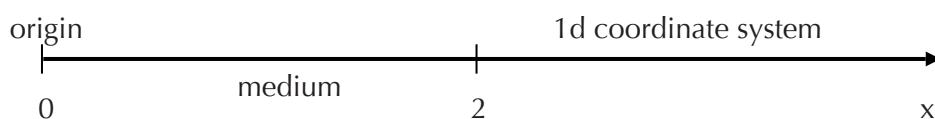
In practice, there are many methods which may be used to determine the speed of propagation of the waves in the material.

One of the simplest of these is the **resonance (standing wave) method**. Here a standing wave is set up on a *known length* of the material, and the frequency of this wave is measured. As both this length (and so the wavelength of the waves giving rise to the standing wave), and the frequency may be determined quite accurately, so too can the speed of propagation of the waves in the material.

## Theoretical Problems

### Background

When a wave disturbance travels through an elastic medium the particles of the medium are displaced from their equilibrium positions.



In the case of a one-dimensional elastic medium we may identify each of the particles in the medium by placing a one-dimensional coordinate system along the medium.  
e.g.  $x=2$  identifies the particle which is at a distance of 2 metres from the origin when in its equilibrium position.

For a travelling sinusoidally shaped disturbance, at any instant of time the displacement of the particles from their equilibrium positions follows a sine curve.

This sinusoidal shape travels through the medium with a constant speed,  $v$ , dependent only on the properties of the medium.

If we denote the displacement of the particle (which is at position  $x$ ) from its equilibrium position by  $\xi$ , then a sinusoidally shaped disturbance travelling in the  $+x$  direction may be represented by:

$$\xi = A \sin \left[ \frac{2\pi}{\lambda} (x - vt) + \Phi \right]$$

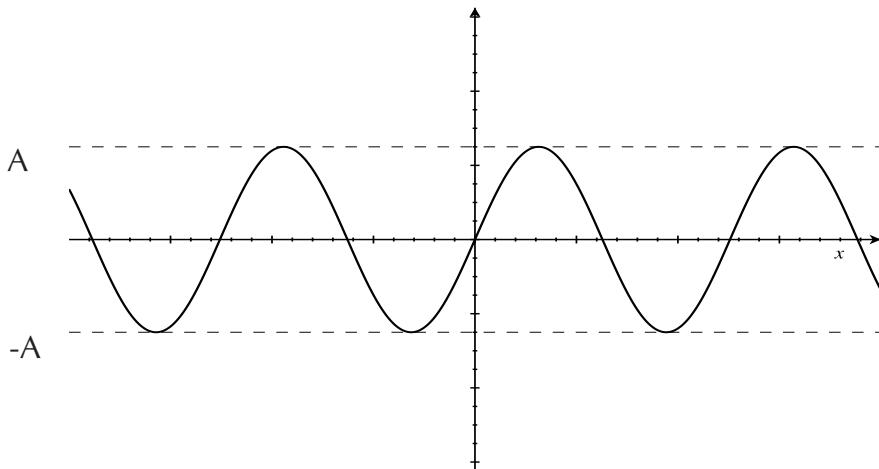
Note that for a "sine" wave travelling in the  $-x$  direction:

$$\xi = A \sin \left[ \frac{2\pi}{\lambda} (x + vt) + \Phi \right]$$

If we look at the situation at a particular instant of time, say  $t = T_0$ , the displacement of the particles will be given by:

$$\xi = A \sin \left[ \frac{2\pi}{\lambda} (x - vT_0) + \Phi \right]$$

The following graph shows the displacement of the particles from their equilibrium positions at that instant. (If we took a flash photograph of the medium at  $t = T_0$ , this is what we would see)



For a *transverse* wave the displacement of the particles of the medium from their equilibrium positions is at right angles to the direction of propagation of the disturbance. For a *longitudinal* wave the displacement of the particles of the medium from their equilibrium positions is along the direction of propagation of the disturbance.

When a transverse sinusoidal wave travelling on a string meets a clamped end a reflection occurs. Ideally there will be no loss of energy on reflection, and the wave incident on the clamp and that reflected from the clamp will have the same amplitude.

This means that in front of the clamp there will be two waves on the string, each of the same amplitude, but travelling in opposite directions.

The resultant disturbance at any given point on the string at any instant may be found by adding:

- The disturbance produced by the “incident” wave if that wave was present on its own, and
- The disturbance produced by the “reflected” wave if that wave was present on its own.

These problems illustrate this process. In this particular case the resultant disturbance on the string is a *standing wave*.

Note that the displacement of the string at the clamp must be zero (i.e. the clamp must be a node of the standing wave). This means that the phase angle of the “incident” wave and the phase angle of the “reflected” wave *at the position of the clamp* must differ by  $\pi$  radian ( $180^\circ$ ).

**Answer the following in the spaces provided, showing all reasoning and working.**

(a) Write down an expression for this sinusoidal wave travelling to the right, also write down an expression for the reflected wave travelling to the left.

(b) Derive the equation for the standing wave by adding the equations for the incident and reflected wave. Use this to find the maximum displacement at the crest and halfway between a node and an antinode. Compare this with your result from the diagram. You may find the following relationship useful:

$$\sin(A) \pm \sin(B) = 2\sin(A/2 \pm B/2) \cos(A/2 \mp B/2)$$

At time  $t = 0$  a point of maximum displacement of the incident wave is at the left hand side of the grid two pages over.

*The first grid, two pages over shows the displacement produced by:*

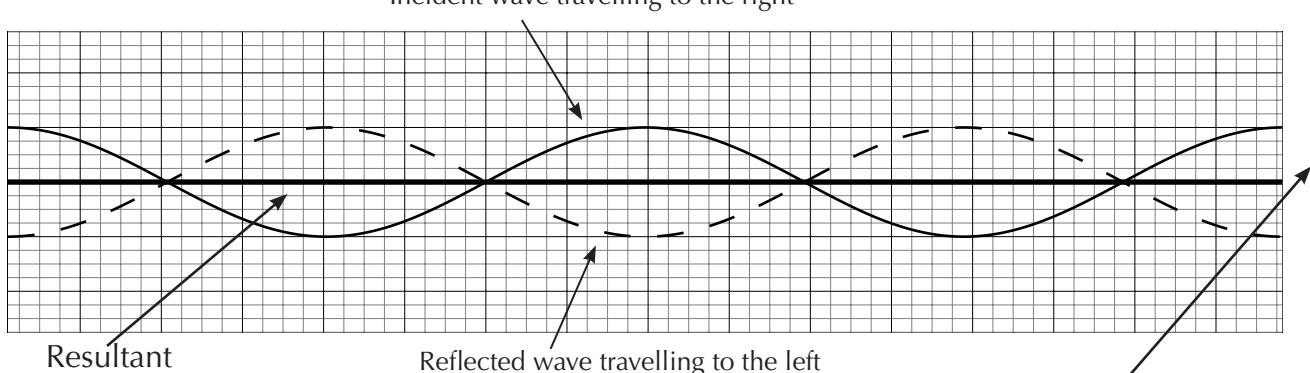
- The incident wave,
  - The reflected wave, and
  - The resultant wave (the wave resulting from the superposition of these) i.e. the standing wave, at time  $t = 0$ .
- (At any particular point the resultant displacement is the sum of the displacements produced by each wave)

(c) On the remaining grids, draw in

- The incident wave,
- The reflected wave (**use a different coloured pencil**), and
- The wave resulting from their superposition (**use a pencil of a third colour**)

## STANDING WAVES ON A STRING

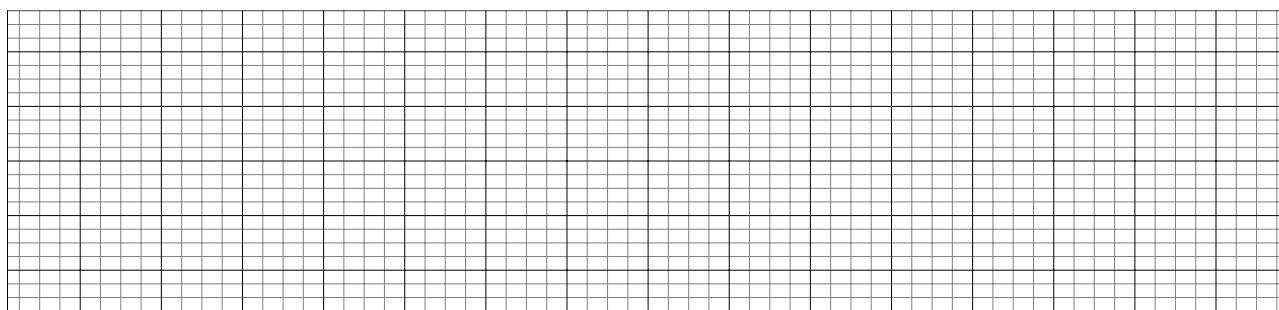
Time  $t=0$



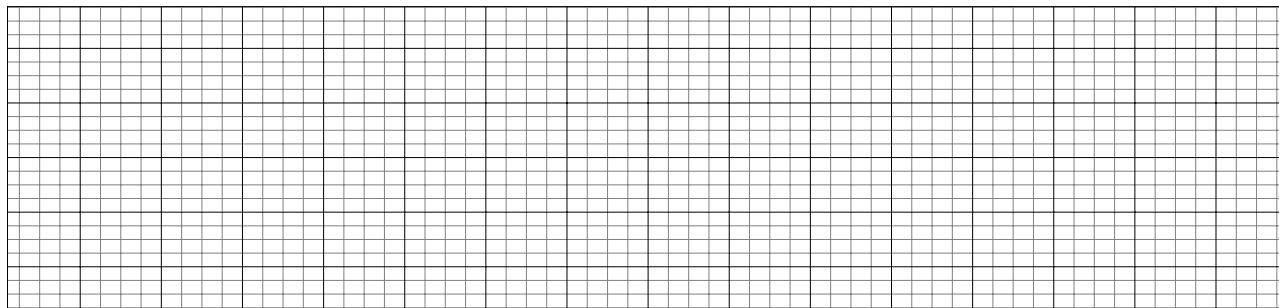
The end of the sketch does not represent the end of the string. The string extends beyond what is shown here

At the end of the string the incident wave is reflected and undergoes a  $180^\circ$  Phase change

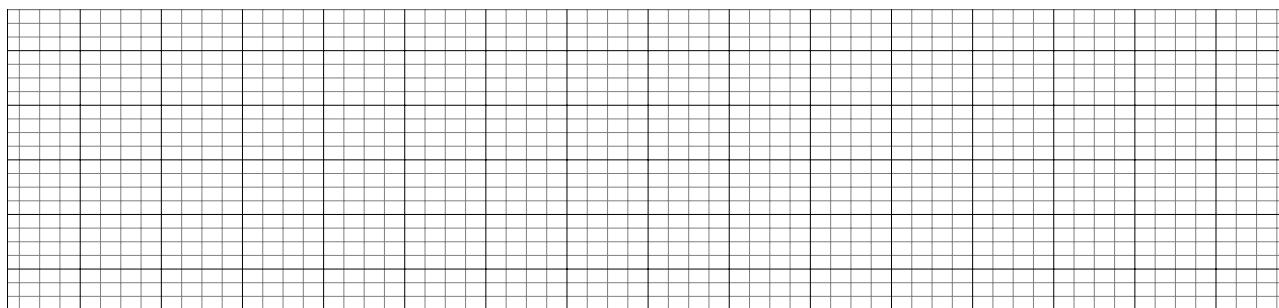
Time  $t=\frac{T}{4}$



Time  $t=\frac{T}{2}$



Time  $t=\frac{3T}{4}$



At the times indicated, namely

$$t = \frac{T}{4}, t = \frac{T}{2}, t = \frac{3T}{4}$$

where  $T$  is the period of the “incident” wave.

Draw the waves before answering part (b).

(d) A transverse sinusoidal progressive wave, wavelength 200mm, amplitude 20mm, and frequency 50 Hz, is reflected from the clamped end of a string. Use your graph to determine the maximum displacement of this standing wave:

- At a crest (i.e. at an antinode)?
- At a point half way between a node and an antinode?

(e) Compare these values to the values you obtain from substituting appropriate values into the equation you derived for a standing wave. Comment.

- Calculate
- Use your graph

## Equipment

You will need the following equipment:

- a string vibrator with string attached
- a measuring tape
- a mass holder with 5 masses
- a pulley with clamp attachment
- a short retort stand
- a sine wave generator with power supply
- two laboratory leads

## In the Laboratory

The aim of this experiment is to use standing waves to measure the density of a piece of string as accurately as possible.

### Uncertainty in masses

Write a method for calculating the uncertainty in the masses provided:

Perform these measurements and record your result:

The uncertainty in the masses is:                   %

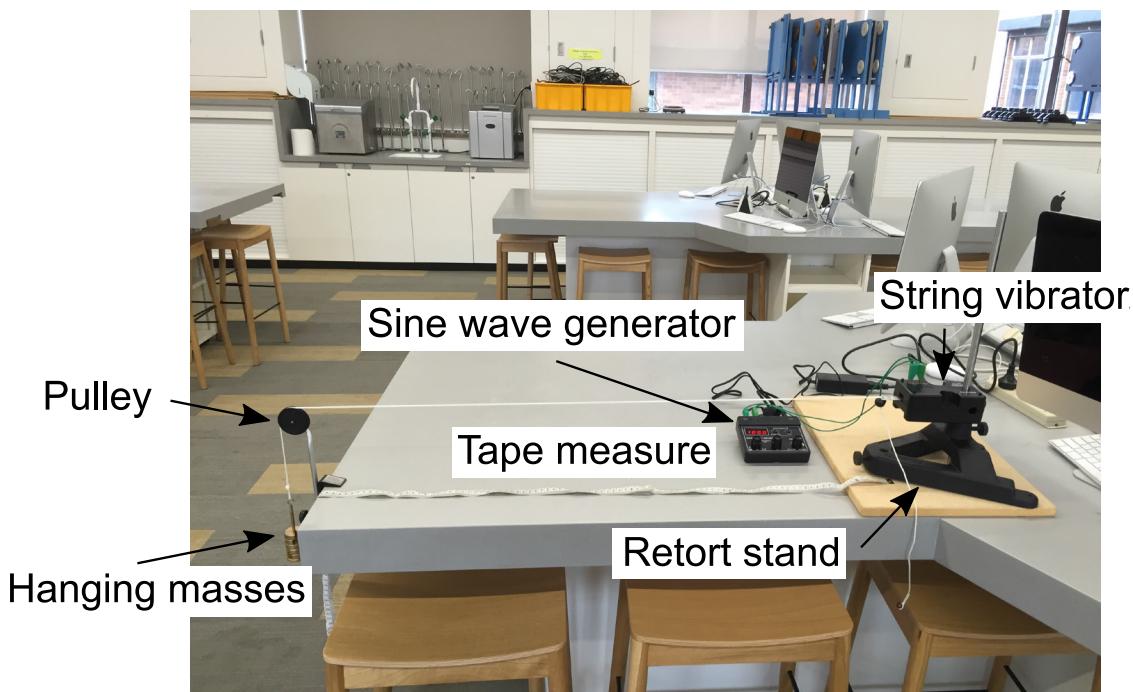
### Uncertainty in wavelength

Write a method for calculating the uncertainty in the wavelength (it will be greater than uncertainty from the precision of the ruler):

Perform these measurements and record your result:

## Resonance and standing waves

You will start by familiarising yourself with the equipment. Set the equipment up as shown in the photo below.



Hang 300 g on the end of the string (250 g on the mass holder + 50 g mass holder). Set the frequency of the sine wave generator to 100 Hz. Adjust the length of the string by sliding the retort stand until you see two complete wavelengths.

Record the wavelength with an uncertainty showing all your working.

$$\text{Wavelength} = \quad \pm$$

Predict the frequency of the harmonic before this one.

$$\text{Frequency } f =$$

## STANDING WAVES ON A STRING

Set the sine wave generator to this frequency. Sketch and comment on what you see.

Predict the frequencies of all earlier harmonics.

Set the sine wave generator to those frequencies and discuss what you see.

**Note:** For the rest of the experiment you will need to vary the length of the string.

You now need to come up with a method to use standing waves to determine  $\mu$ , the mass per unit length, of the string as accurately as possible.

As part of this method you must plot a linear graph with errors using the linear plot with errors template and your graph must contain at least 5 data points. **You must not cut or untie your string.** You should record all your measurements and results in a neat table. Get a demonstrator to check your method before you start carrying it out.

Write an equation relating  $\mu$ ,  $\lambda$ , m and f:

List the variables you will keep constant:

Which variable(s) will you change?

Which variable(s) will you measure?

### Method:

*Think about: What will you put on the x axis and y axis? and What will the gradient be?*

Get  
Marked  
Now

## STANDING WAVES ON A STRING

Perform this method to determine  $\mu$ .



## STANDING WAVES ON A STRING

When you have determined a value for  $\mu$ , use this to predict the mass for 5 m of that string. Let your demonstrator know that you have a prediction. Your demonstrator will then give you 5 m of the string to weigh. How does this mass compare to your prediction? Include the uncertainties.

## Marking Guidelines

Get your Demonstrator to tick the boxes below as you go through the exercise.

Please enter your details, Demonstrator's name and date below before you get your final mark from the Demonstrator.

Student Name: \_\_\_\_\_ Student ID Number: z\_\_\_\_\_

Date: \_\_\_\_\_

Completed Part (a) & (b) of the theoretical problems	
Completed Part (c) of the theoretical problems, including neat, correct diagrams	
Completed Part (d) & (e) of the theoretical problems	
Method of finding uncertainty in masses suggested and carried out	
Recorded wavelength with uncertainty for 300 g and 100 Hz	
Predicted, measured and shown/discussed all earlier harmonics	
Suitable method given for finding $\mu$	
Enough data collected (at least 5 data points) and presented in a neat table	
Graph plotted and gradient used to determine $\mu$	
Accurate value of $\mu$ obtained with uncertainties	
Measured $\mu$ directly and commented	

Total: /10

Demonstrator's Signature: \_\_\_\_\_

Demonstrator's Name: \_\_\_\_\_

## STANDING WAVES ON A STRING

# APPENDIX: CONSTANTS AND OTHER USEFUL DATA

---

## Physical Constants

Quantity	Symbol	Value	Uncertainty	Units
Gravitation Constant	G	$6.6742 \times 10^{-11}$	$0.0010 \times 10^{-11}$	N m <sup>2</sup> /kg <sup>2</sup>
Speed of Light in Vacuum	c	$2.99792458 \times 10^8$	exact	m/s
Electron Charge	e	$1.60217653 \times 10^{-19}$	$0.00000014 \times 10^{-19}$	C
Planck's Constant	h	$6.6260693 \times 10^{-34}$ $4.13566743 \times 10^{-15}$	$0.0000011 \times 10^{-34}$ $0.00000035 \times 10^{-15}$	J s eV s
Universal Gas Constant	R	8.314472	0.000015	J/mol K
Avogadro's Number	N <sub>A</sub>	$6.0221415 \times 10^{23}$	$0.0000010 \times 10^{23}$	mol <sup>-1</sup>
Boltzmann Constant	k <sub>B</sub>	$1.3806505 \times 10^{-23}$ $8.617343 \times 10^{-5}$	$0.0000024 \times 10^{-23}$ $0.000015 \times 10^{-5}$	J/K eV/K
Permittivity of Free Space	ε <sub>0</sub>	$8.854187... \times 10^{-12}$	exact, as $c^2\epsilon_0\mu_0=1$	C <sup>2</sup> /N m <sup>2</sup>
Permeability of Free Space	μ <sub>0</sub>	$4 \pi \times 10^{-7}$	exact	T m/A
Electron Mass	m <sub>e</sub>	$9.1093826 \times 10^{-31}$	$0.0000016 \times 10^{-31}$	kg
Proton Mass	m <sub>p</sub>	$1.67262171 \times 10^{-27}$	$0.00000029 \times 10^{-27}$	kg
Neutron Mass	m <sub>N</sub>	$1.67492728 \times 10^{-27}$	$0.00000029 \times 10^{-27}$	kg
Bohr Magneton	μ <sub>B</sub>	$927.400949 \times 10^{-26}$	$0.000080 \times 10^{-26}$	J/T
Stefan-Boltzmann Constant	σ	$5.670400 \times 10^{-8}$	$0.000040 \times 10^{-8}$	W/m <sup>2</sup> K <sup>4</sup>
Bohr Radius	a <sub>0</sub>	$5.291772108 \times 10^{-11}$	$0.000000018 \times 10^{-11}$	m
Faraday Constant	F	96485.3383	0.0083	C/mol
Fine Structure Constant	α	$7.297352568 \times 10^{-3}$	$0.000000024 \times 10^{-3}$	

This table was composed from values taken from the 2002 CODATA internationally recommended values of the fundamental physical constants. These and other values can be found online at:

<http://physics.nist.gov/cuu/Constants/>

**SI Units**

<b>Quantity</b>	<b>Unit</b>	<b>Symbol</b>
Length	metre	m
Time	second	s
Mass	kilogram	kg
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Amount of Substance	mole	mol
Luminous Intensity	candela	cd

**SI Derived Units**

<b>Quantity</b>	<b>Unit</b>	<b>Symbol</b>	<b>Equivalents</b>
Plane Angle	radian	rad	$\text{m}/\text{m}$
Solid Angle	steradian	sr	$\text{m}^2/\text{m}^2$
Frequency	hertz	Hz	$\text{cycles}/\text{s}$ $\text{s}^{-1}$
Force	newton	N	$\text{J}/\text{m}$ $\text{kg m/s}^2$
Pressure	pascal	Pa	$\text{N}/\text{m}^2$ $\text{kg/m s}^2$
Energy	joule	J	$\text{N m}$ $\text{kg m}^2/\text{s}^2$
Power	watt	W	$\text{J}/\text{s}$ $\text{kg m}^2/\text{s}^3$
Electric Charge	coulomb	C	$\text{A s}$
Electric Potential	volt	V	$\text{J}/\text{C}$ $\text{kg m}^2/\text{A s}^3$
Electric Resistance	ohm	$\Omega$	$\text{V}/\text{A}$ $\text{kg m}^2/\text{A}^2 \text{s}^3$
Capacitance	farad	F	$\text{C}/\text{V}$ $\text{A}^2 \text{s}^4/\text{kg m}^2$
Electric Conductance	siemens	S	$\text{A}/\text{V}$ $\text{A}^2 \text{s}^3/\text{m}^2 \text{kg}$
Luminous Flux	lumen	lm	$\text{cd sr}$ $\text{cd}$
Magnetic Field	tesla	T	$\text{N s/C m}$ $\text{kg/A s}^2$
Magnetic Flux	weber	Wb	$\text{T m}^2$ $\text{kg m}^2/\text{A s}^2$
Inductance	henry	H	$\text{V s/A}$ $\text{kg m}^2/\text{A}^2 \text{s}^2$
Illuminance	lux	lx	$\text{lm}/\text{m}^2$ $\text{cd/m}^2$
Activity (of a radionuclide)	becquerel	Bq	$\text{s}^{-1}$
Absorbed Dose	gray	Gy	$\text{J/kg}$ $\text{m}^2/\text{s}^2$
Dose Equivalent	sievert	Sv	$\text{J/kg}$ $\text{m}^2/\text{s}^2$
Catalytic Activity	katal	kat	$\text{mol/s}$

## SI Prefixes

Factor	Name	Prefix	Factor	Name	Prefix
$10^{24}$	yotta	Y	$10^{-1}$	deci	d
$10^{21}$	zetta	Z	$10^{-2}$	centi	c
$10^{18}$	exa	E	$10^{-3}$	milli	m
$10^{15}$	peta	P	$10^{-6}$	micro	$\mu$
$10^{12}$	tera	T	$10^{-9}$	nano	n
$10^9$	giga	G	$10^{-12}$	pico	p
$10^6$	mega	M	$10^{-15}$	femto	f
$10^3$	kilo	k	$10^{-18}$	atto	a
$10^2$	hecto	h	$10^{-21}$	zepto	z
$10^1$	deka	da	$10^{-24}$	yocto	y

## Other Useful Data

Standard Temperature and Pressure (STP)	273.15 K (0 °C) and 101.325 kPa (1 atm)
Density of Water, relative (4 °C)	$1.000 \times 10^3$ kg/m <sup>3</sup>
Specific Heat Capacity of Water	4.186 kJ/kg K
Standard Acceleration Due to Earth's Gravity	9.80665 m/s <sup>2</sup>
Speed of Sound in Air (20 °C)	343 m/s
Speed of Sound in Air (STP)	331 m/s
Density of Dry Air (STP)	1.29 kg/m <sup>3</sup>
Earth's Magnetic Field Strength	0.05 mT

## The Greek Alphabet

Alpha	A	$\alpha$	Lambda	$\Lambda$	$\lambda$	Phi	$\Phi$	$\phi$
Beta	B	$\beta$	Mu	$\text{M}$	$\mu$	Chi	$\text{X}$	$\chi$
Gamma	$\Gamma$	$\gamma$	Nu	$\text{N}$	$\nu$	Psi	$\Psi$	$\psi$
Delta	$\Delta$	$\delta$	Xi	$\Xi$	$\xi$	Omega	$\Omega$	$\omega$
Epsilon	E	$\epsilon$	Omicron	$\text{O}$	$\circ$			
Zeta	Z	$\zeta$	Pi	$\Pi$	$\pi$			
Eta	H	$\eta$	Rho	$\text{P}$	$\rho$			
Theta	$\Theta$	$\theta$	Sigma	$\Sigma$	$\sigma$			
Iota	I	$\iota$	Tau	$\text{T}$	$\tau$			
Kappa	K	$\kappa$	Upsilon	$\text{Y}$	$\upsilon$			

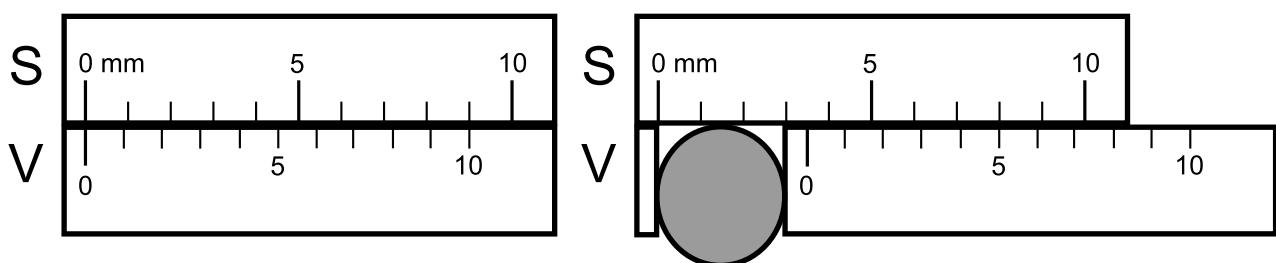
## CONSTANTS AND OTHER USEFUL DATA

# READING A VERNIER SCALE

There are a number of devices in the laboratory that use a vernier scale to obtain an accurate measurement. Such devices include vernier callipers, micrometers and optical spectrometers. A vernier device has two scales. Let's call the graduation (spacing) of the main scale 'S'. We will call the graduation on the vernier scale 'V'. For a vernier device ( $n=10$ ) of the main graduations are equivalent to  $n$  of the vernier scale graduations where  $n$  is an integer:

$$(n-1)S = nV$$

This is illustrated below for a very simple vernier scale.



For the device pictured  $S = 1 \text{ mm}$  and  $n = 10$  giving  $V = 9/10 \text{ mm}$ . When the zeros of the two scales align the 9th marking on the 'S' scale aligns with the 10th marking on the 'V' scale.

When an object such as the ball in the illustration is used to displace the vernier scale we can make use of the human ability to identify lines that are aligned to obtain an accurate measurement.

When the vernier scale is shifted relative to the main scale only one pair of marks will align with each other. If you move the vernier scale  $1/10$ th of a millimetre along, the first line on the main scale aligns with the first line on the vernier scale. The first line on the main scale is at  $1 \text{ mm}$ . The first line on the vernier scale is at  $9/10 \text{ mm}$  and the additional  $1/10 \text{ mm}$  displacement giving  $1 \text{ mm}$ . This first vernier graduation is thus aligned with the first main scale graduation.

If you moved the vernier scale  $2/10 \text{ mm}$  onwards the second line on the vernier scale would align with one of the markings on the main scale as  $2/10 \text{ mm} + 2 \times 9/10 \text{ mm} = 20/10 \text{ mm} = 2 \text{ mm}$ , a whole number of millimetres and hence a mark on the main scale.

If you move the vernier scale  $x/10 \text{ mm}$  along then the  $x$ th line on the vernier scale will align with a line on the main scale. In the illustration above the 5th line on the vernier scale aligns with a mark on the main scale. The 0 mark on the vernier scale is located somewhere

between the mark for 3 mm and 4 mm on the main scale. As the 5th line on the vernier scale aligns with the main scale, we read this as:

$$3 \text{ mm} + 5/10 \text{ mm} = 3.5 \text{ mm}$$

The devices that you will come across in the lab use a variation of this logic.

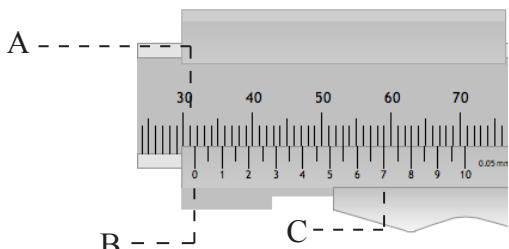
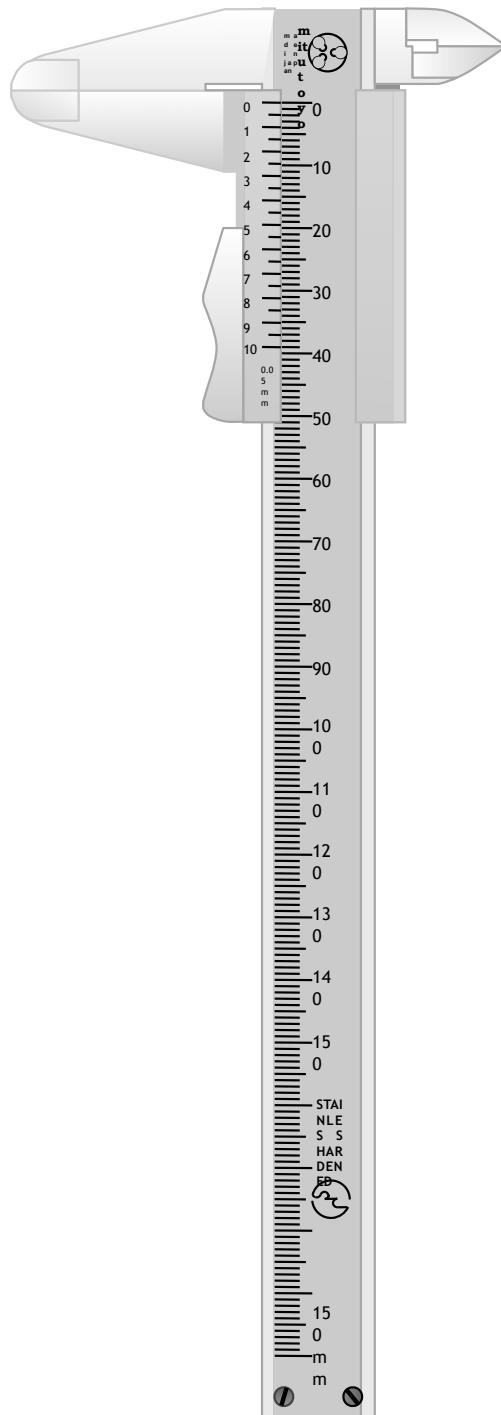
## Vernier Callipers

One instrument with which you will need to be familiar, is the *vernier callipers*. The callipers used in the laboratory, shown at right, have  $50 \mu\text{m}$  graduations (that is, 0.05 mm).

There are two scales on the callipers: the main scale, running the length of the instrument, is graduated in millimetres; while the smaller vernier scale is 39 mm long, and divided into 20 equal parts. That means that the distance between markings on the vernier scale falls just short of 2 mm (it is, in fact, 1.95 mm).

This means that when the callipers are opened to any position, there will always be one vernier scale mark that lies directly opposite one of the marks on the main scale. It is this particular mark on the vernier scale which gives the added accuracy to the main scale.

In the example shown below, the calliper is opened to a fraction of a millimetre over 31 mm, read from the zero on the vernier scale. The 7 on the vernier scale is aligned with the 59 mm mark on the main scale.



Thus, the fraction of a millimetre to which the callipers are open beyond 31 mm (distance AB) is equal to:

$$\begin{aligned} AC - BC &= 28 \text{ mm} - 14 \text{ vernier units} \\ &= (28 - 14 \times 1.95) \text{ mm} \\ &= 0.70 \text{ mm} \end{aligned}$$

Note that this is simply the vernier scale reading divided by 10. So, in this example, the callipers are reading 31.70 mm.

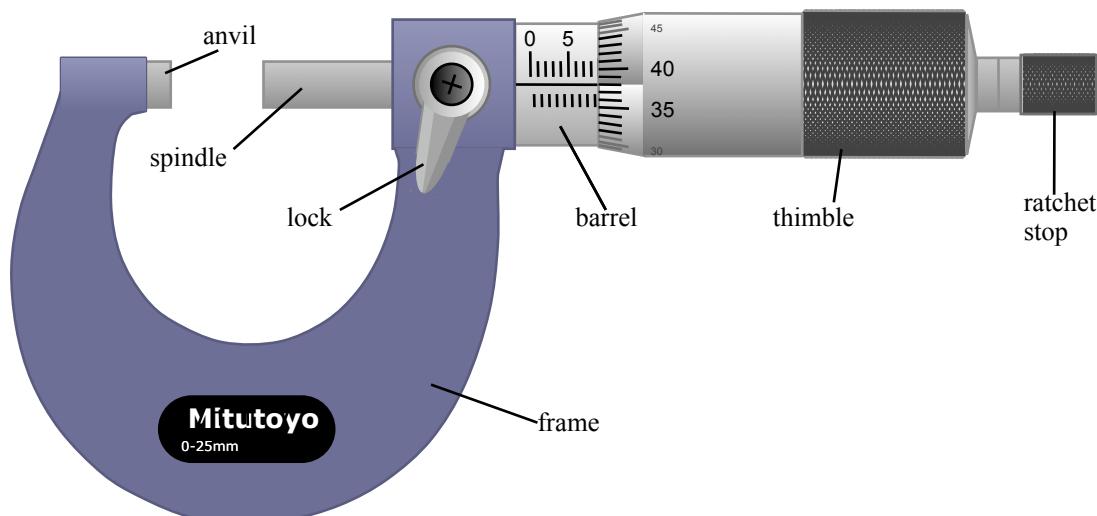
In general, to read the vernier callipers:

- 1) Look for the nearest mark on the main scale below the zero mark of the vernier scale. This gives the millimetre part of the reading. In our example above, this was 31 mm.
- 2) Find the mark on the vernier scale which is exactly aligned with one of the main scale marks. In the example, this was 7.
- 3) Add the vernier scale reading, divided by 10, to the main scale reading. This gave  $31 + 0.70 = 31.70$  mm in the example.

Note that the vernier scale also has half marks, but you treat them the same way. If for example, the 3.5 vernier mark was aligned with the main scale in the example, we would have read  $31 + 0.35 = 31.35$  mm.

## Micrometer

In some situations you will be required to use a micrometer for measurements. The micrometers in the First Year Laboratories, shown below, are graduated down to 0.01 mm (or 10  $\mu\text{m}$ ).



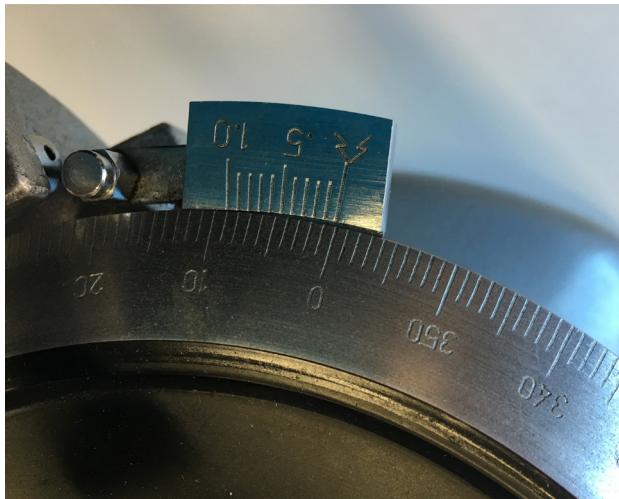
Like the vernier callipers, the micrometer has two scales. The main scale on the barrel is marked with millimetres above the centre line, and half-millimetres below the line. The scale on the thimble is divided into 50 intervals, each representing 0.01 mm on the main scale. That is, if you turn the thimble through one full revolution, the main scale opens by 0.5 mm.

To use the micrometer:

- 1) Open the micrometer and place the object you wish to measure between the anvil and the spindle.
- 2) Begin closing the micrometer by turning the thimble until it is almost closed, then turn the ratchet stop until the micrometer is fully closed and the ratchet clicks a couple of times.
- 3) Read the main scale to the last visible scale mark. In the picture above, this would be 8.5 mm.
- 4) Add to this the reading on the thimble to the nearest scale mark, divided by 100. In the example this would give a final reading of  $8.5 + (38 \div 100) = 8.88$  mm.

## Optical Spectrometers

There are two different types of optical spectrometers used in the lab. The photos below show the main and vernier scale with the zero aligned. Make sure that you know how to read angles on both those scales.



For this spectrometer 10 of the vernier graduations are equal to 9 of the main scale graduations.

For this spectrometer 30 of the vernier graduations are equal to 14.5 of the main scale graduations.





