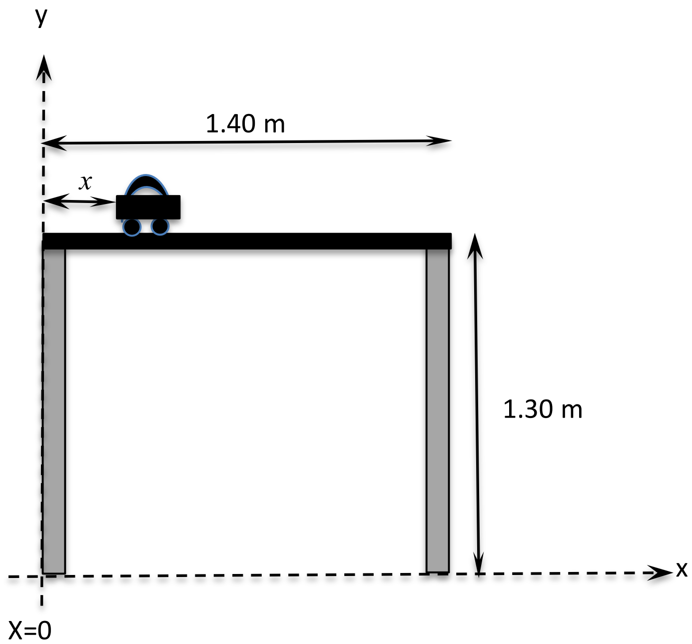
Question 1 (Marks: 20)



A radio controlled toy car, with mass 8.0g, moves along a bench, while on the bench its motion is described by the equation:

$$x = 0.20 + 0.50t^2$$

where x is distance in metres from the left-hand edge of the bench, and t is time in seconds. The top of the bench is 1.40 m long, and the bench is 1.30 metres high. Assume that the positive x-direction is along the positive x-axis as shown in the diagram.

(a) Derive expressions for the

(i) instantaneous velocity, and

(ii) instantaneous acceleration while the car travels along the bench.

(b) At time t = 0, find the

(i) displacement,

(ii) velocity, and

(iii) acceleration.

(c) At what time does the car fall off the right-hand side of the bench? *Treat the car as a particle, i.e. assume its dimensions are very small.*

(d) Calculate the velocity of the car as it leaves the bench.

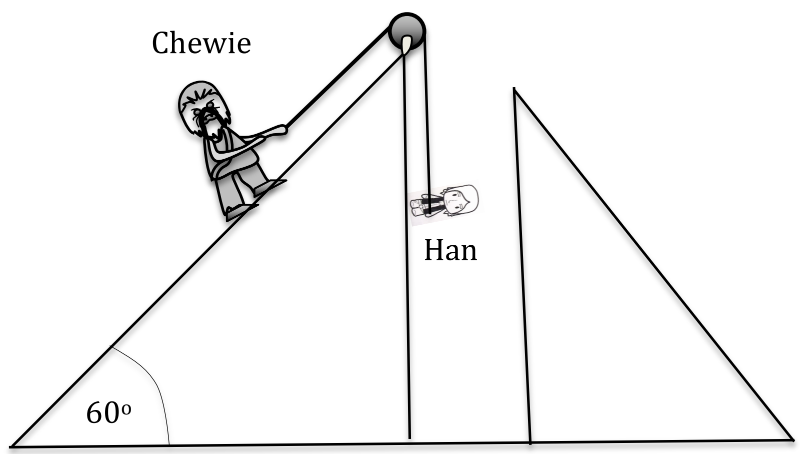
(e) How long does it take the car to hit the ground?

(f) How far from the right-hand edge of the bench does the car hit the ground?

(g) Find the velocity of the car just before it hits the ground. Remember to give both a magnitude and a direction.

(h) Calculate the change in the total energy of the car from when it leaves the bench to just before it hits the ground. Neglect air resistance. Explain your answer in one clear sentence.

Question 2 (Marks: 23)



In a science fiction story, Han and Chewie are attempting to infiltrate an Imperial base situated under a strange conical structure on an ice planet. They have scaled the outer wall (sloped at 60° to the horizontal as shown in the diagram) of the structure. Using a light, inextensible rope and light pulley they are in the process of entering the structure. Inside, there is a vertical shaft. Chewie is near the top of the outer slope, while Han is hanging vertically from the rope without touching the shaft.

The mass of Chewie is 120 kg, the mass of Han is 75 kg, and the local surface gravity on the planet is 8.5 ms^{-2}. Assume that Chewie and Han are at rest.

(a) Draw a schematic free body diagram for Han and another for Chewie. You should show all forces with vectors indicating their relative magnitude by the size of the arrow. Label each of the vectors. Pay careful attention to the direction of the vector representing the frictional force.

(b) Justify the direction of the frictional force vector in the diagram above.

(c) Calculate the magnitude of the static friction force between Chewie and the crater's slope.

Suddenly the rope breaks. Chewie and Han start moving. The coefficient of kinetic friction between Chewie and the ice covered sloped surface is 0.15.

(d) Chewie slides down the sloped outer wall of the structure. What is his speed after he has travelled 110m?

There is a "reflector shield" at the bottom of the vertical cliff that absorbs the mechanical energy of incoming projectiles (such as people). This "shield" exerts an average force of 1200 N on Han immediately after he has fallen 99 m from rest near the top of the shaft.

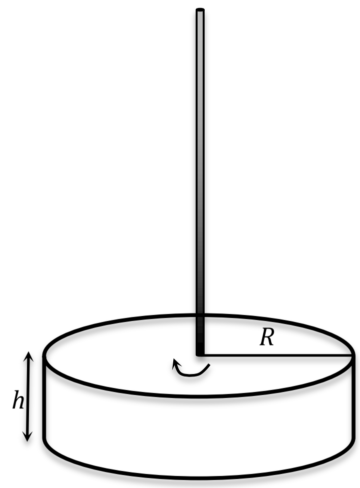
(e) Calculate the duration of the collision as the "shield" brings Han to rest.

After successfully infiltrating the base Chewie and Han escape the ice planet, (which has mass M_i and radius R_i) in their captured shuttle (which has mass m_s).

(f) Derive an expression for the escape velocity from the surface of the planet, showing and explaining all steps and assumptions in your working.

(g) Han and Chewie manoeuvre the shuttle into a circular orbit around the planet. Using Newton's law of gravitation and a suitable expression for centripetal force, derive expressions for the kinetic energy and the total mechanical energy of the orbit in terms of the orbital radius R, G, M_i and m_s. Explain any assumptions you make.

Question 3 (Marks: 17)



A thin, uniform, horizontal solid disk with a radius *R,* and height, *h*, and mass *M* is supported through its centre by a stiff vertical rod as shown in the diagram. At t = 0 the rod starts to rotate the disk from rest with a constant angular acceleration. At time *t* the angular velocity of the rod is give by ω.

(a) In terms of the variables above write an expression for the angular acceleration of the disk.

(b) In terms of the variables above write an expression for the total angle θ_{Tot} through which the disk has rotated since it started moving (each revolution will add 2π radians to θ_{Tot}).

(c) Derive an expression for the moment of inertia of the disk, show all your working.

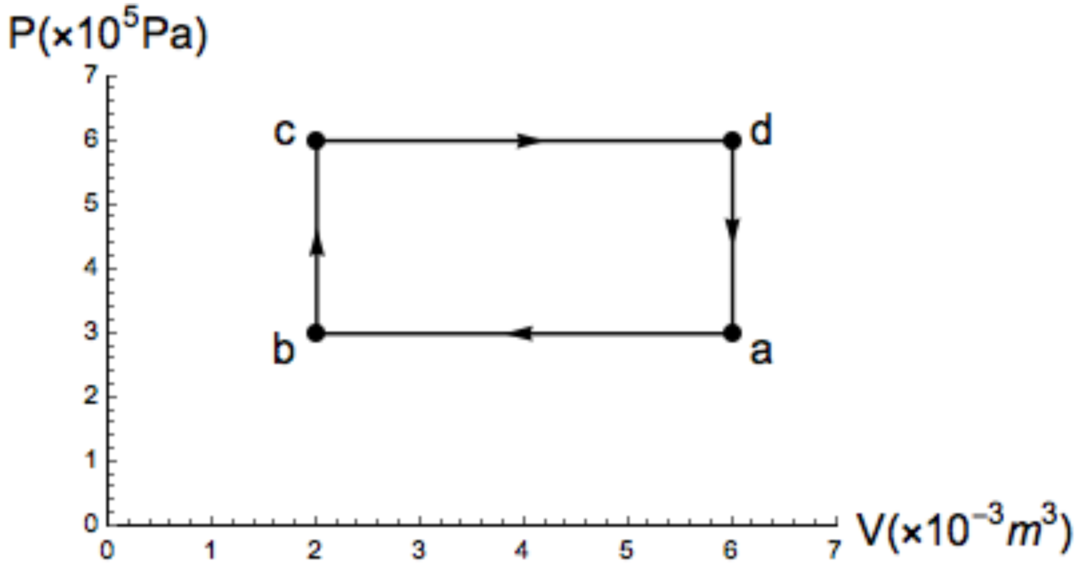
(d) Give an expression for the net torque acting on the disk as it undergoes this angular acceleration.

(e) What is the net force acting on the disk over the same period?

(f) Write an expression for the total work the rod does on the disk as the disk accelerates to angular velocity ω.

Question 4 (Marks: 30)

(a)



This P-V diagram refers to 1.00 mol of a monatomic ideal gas undergoing a cyclic process. I'm sorry, there's really no way to make this more interesting. The pressures and volumes are as follows:

$P_a = P_b = 3.00x10^5$ Pa

$P_c = P_d = 6.00x10^5$ Pa

$V_a = V_d = 6.00x10^{-3}$ m^3

$V_b = V_c = 2.00x10^{-3}$ m^3

(i) Find the total work done on the gas in the complete cycle.

(ii) Calculate the temperatures at each point (a,b,c,d).

(iii) Find the heat Q transferred to the gas for processes b → c and c → d.

(iv) This is known as a "heat engine." Its efficiency is the ratio of the 'work out' and the 'energy in,' which you obtained in the previous parts. Only consider the Q transferred in b → c → d as computed above. What is the efficiency of this engine? Express your answer as a positive number in percent.

(b) *Things fall apart; the centre cannot hold.*

Mere anarchy is loosed upon the world,

... Your beer gets warm and your coffee gets cold.

(with apologies to W.B. Yeats)

Consider a delicious cup of coffee that has gone cold due to the inevitable thermodynamic processes which slowly yet inextricably bring disorder to the universe. As a resourceful yet slightly eccentric physicist, you decide to rage against the dying of the light, and to re-heat the coffee in the most metal way possible: by dropping a heated metal sphere into it.

If you have a 300 g Texas-size serving of coffee at 25°C in a thermally isolated steel cup of mass 0.050 kg, and add a 0.10 kg steel sphere at 1400°C to it, what is the final temperature of the system after it reaches equilibrium? Assume somehow that no energy is lost to the surroundings. Model the coffee as pure water (c_{water} = 4186 J/kg K; c_{steel}= 490 J/kg K).

(c) About 1/3 of the expected sea level rise due to climate change arises from the thermal expansion of seawater. The volume of the oceans is about 1.3×10^{21} m^3.

(i) For a temperature rise of 5K, by what percentage will the volume increase due to thermal expansion alone? ($\beta_{water} = 0.0002\ K^{-1}$)

(ii) The mean depth is 3.7 km. Estimate the sea level rise caused by the volume increase from part (i). *Assume that the area of the oceans remains constant and that therefore expanded water can only go up!*

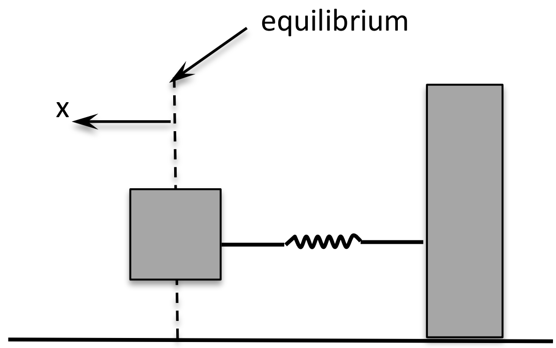
(d) A spacecraft re-enters the atmosphere and adiabatically compresses the air in front of it. Consider the Earth's atmosphere as an ideal diatomic gas with 7 degrees of freedom. A parcel of air in front of the spacecraft has initial volume 1.0 m^3, initial pressure 0.1 atm, and contains 2.0 mol of gas. Its final pressure is 100 atm. What is the final temperature of the gas?

Assume the air remains an ideal gas throughout. (Which it doesn't)

Question 5 (Marks: 30)

(a) A mass, m is attached to an ideal spring with spring constant k. On a frictionless table a student displaces the mass $x = 0.10$ m from its equilibrium position and releases it at $t = 0$.

Air resistance is negligible.



(i) Sketch a displacement-time graph for the mass, including as many details as you can.

(ii) Write down an equation that describes how the displacement for the mass changes with time, include necessary variables given in the question.

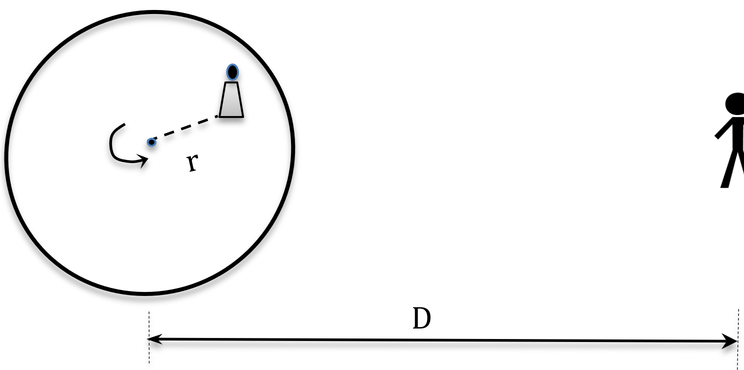
(iii) Showing all working and stating any assumptions derive an expression for ω the angular frequency of the mass in terms of the variables given in the question.

(iv) If the mass were replaced with a new mass, $M = 4m$, and displaced to the same initial displacement what would be the expression for the angular frequency of the motion?

(v) Draw a graph showing the relationship between the kinetic energy, potential energy and total energy of the mass as functions of time. Where possible write expressions for these on the axes. Making any relationships clear show these for the cases with the mass m and the mass M $(=4m)$.

(vi) Under these ideal conditions would the mass ever stop? If yes calculate how long it would take to stop, if no describe what would be needed to make the mass stop.

(b) A bell emitting a frequency f is placed a distance r from the middle of a turntable. The turntable is rotated at an angular frequency ω_t about its center in an anti-clockwise direction when viewed from the above. A person stands a distance D away from the turntable. D is far enough away from the turntable that the person can easily hear the bell but such that $r << D$. Let the speed of sound in air be v_A.

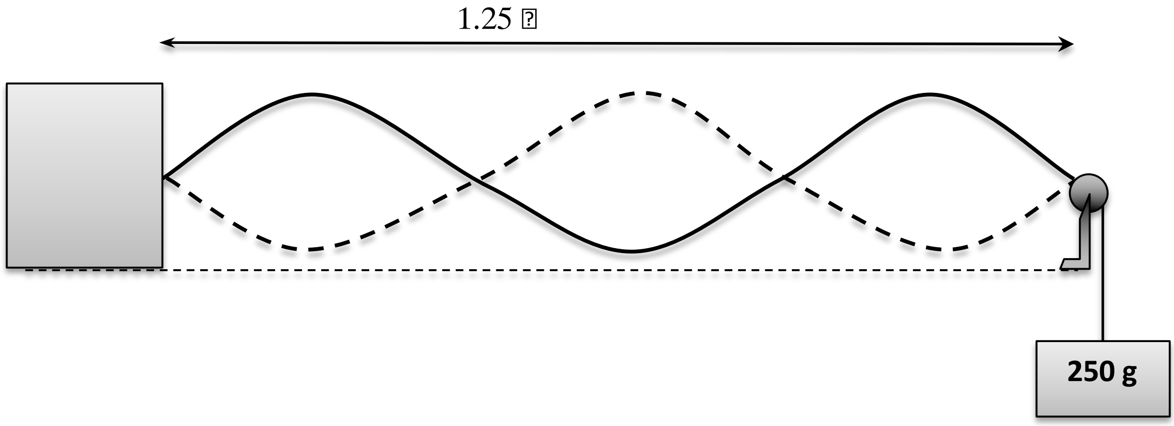


(i) Write an expression for the maximum frequency *f* that the person detects from the bell.

(ii) Sketch a diagram that clearly shows where the bell is relative to the person (and how it is moving) when this maximum frequency is heard.

(iii) Write an expression for the minimum frequency that the person detects from the bell.

(c)



A frequency generator is used to set up the standing wave pattern shown in the diagram above. The vibration amplitude has been exaggerated in the sketch. This pattern is obtained when the frequency generator is set to 77 Hz, there is 250 g hung from the end of the string and the distance between the signal generator and pulley is 1.25 m.

(i) What is the mass per unit length for the piece of string shown in the diagram?

(ii) What frequency will the frequency generator need to be set to in order to observe the next harmonic?

(d) (i) Describe what is meant by the term "resonance".

(ii) Give an example of a system that is undergoing resonance, describe the effects of resonance on the system.

(iii) Sketch a graph showing the relationship between amplitude and angular frequency of a driving force for a system undergoing resonance. Indicate the effect of damping on this sketch.