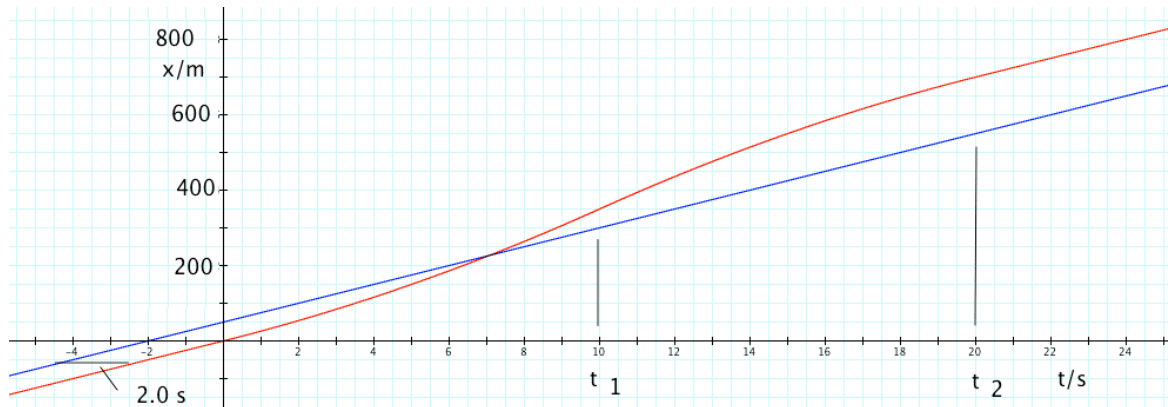
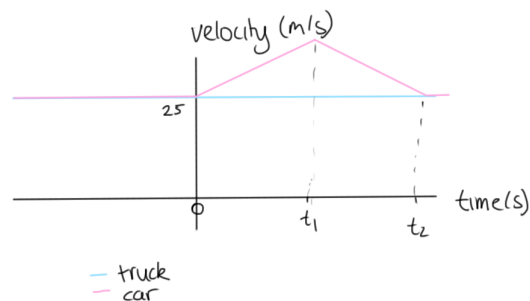


Question 1

i) (a sketch similar to this)



ii)



iii) $v_0 = 90 \text{ k.p.h.} = (90,000 \text{ m} / 3600 \text{ s}) = 25 \text{ m.s}^{-1}$

Car starts at $x = 0$, truck starts at $x = d$

where $d = (2 \text{ s} * v_0) = 2 \text{ s} * 25 \text{ m.s}^{-1} = 50 \text{ m}$.

iv) $x_{\text{truck}} = v_0 t + d$

$x_{\text{car}} = v_0 t + \frac{1}{2} a t^2$ while acceleration is positive

car is 50 m ahead when

$$(v_0 t + \frac{1}{2} a t^2) - (v_0 t + d) = 50 \text{ m}$$

$$\frac{1}{2} a t_1^2 = 100 \text{ m}$$

$$t_1 = \sqrt{2 * 100 \text{ m} / 2.0 \text{ m.s}^{-2}} = 10 \text{ s}$$

v) $x_1 = v_0 t_1 + \frac{1}{2} a t_1^2 = 25 * 10 + \frac{1}{2} * 2 * 10^2 = 350 \text{ m}$

vi) and vii) See graph in part (i) and (ii) working for calculating t_2 and x_2 is below.

For the truck: $x_{2,\text{truck}} = v_0 (t_2 + 2.0 \text{ s})$ (i.e. it arrives 2 seconds later)

For the car: $t_2 = 2t_1 = 20 \text{ s}$ (as it accelerates and decelerates with same magnitude so time spent accelerating is same as time spent decelerating.)

There are other ways to calculate this as well such as finding velocity at t_1 and then working out time for car to get back to 25m/s)

$$\begin{aligned}x_{2,\text{car}} &= x_1 + v_0 (t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2 \\&= 350 + 45 \cdot 10 - 0.5 \cdot 2 \cdot 100 = 700 \text{ m}\end{aligned}$$

$$x_{2,\text{truck}} = 25 \cdot 22 = 550 \text{ m}$$

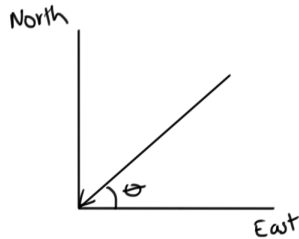
viii) $v_{\text{max}} = v_0 + at_1 = 25 \text{ m.s}^{-1} + (2 \text{ m.s}^{-2})(10 \text{ s}) = 45 \text{ m.s}^{-1} (= 162 \text{ k.p.h.})$

(and lots of points off the licence)

Question 2

a)

i)



ii) $\mathbf{v}_w = -v_w \cos \theta \mathbf{i} - v_w \sin \theta \mathbf{j}$

iii) $\mathbf{v}_w = \mathbf{v} + \mathbf{v}'$ where \mathbf{v}' is the apparent wind velocity

$$\mathbf{v}' = \mathbf{v}_w - \mathbf{v} = -v_w \cos \theta \mathbf{i} - v_w \sin \theta \mathbf{j} - v \mathbf{j}$$

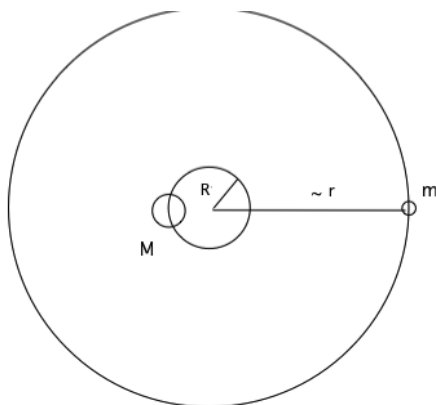
$$\mathbf{v}' = -v_w \cos \theta \mathbf{i} - (v + v_w \sin \theta) \mathbf{j}$$

The magnitude is calculated below, this is not asked for in the question.

$$v' = (v_w^2 \cos^2 \theta + (v^2 + 2vv_w \sin \theta + v_w^2 \sin^2 \theta))^{1/2}$$

$$= (v_w^2 + v^2 + 2vv_w \sin \theta)^{1/2}$$

b)



i)

Note that the radius of the planet's orbit is approximately a circle with radius r , because $r \gg R$

ii) Either:

Newton's 3rd law:

$$|\text{Force on star}| = |\text{force on planet}|$$

or

Newton's 2nd law and law of gravitation

$$M a_{\text{star}} = |F_{\text{grav}}|$$

$$Ma_{\text{star}} = ma_{\text{planet}}$$

$$MR\omega^2 = m r \omega^2$$

$$m = MR/r$$

$$MR\omega^2 = GMm/r^2$$

$$m = r^2 R \omega^2 / G = 4\pi^2 r^2 R / GT^2$$

or anything else correct.

Question 3

- i) Before the collision, there are no non-conservative forces acting, so mechanical energy is conserved.

$$U_i + K_i = U_f + K_f \quad \text{or just} \quad E_i = E_f$$

Immediately before the collision, mass m is travelling horizontally with speed v . So

$$mgR + 0 = 0 + \frac{1}{2}mv^2$$

$$\text{so } v = \sqrt{2gR}$$

During the collision, no external horizontal forces act, therefore momentum is conserved. Take the speed of the combined object as V . So

$$mv = (m + M)V.$$

$$V = mv/(m + M)$$

$$= m\sqrt{2gR}/(m + M) \quad \text{or} \quad = \sqrt{2gR}/(1 + M/m)$$

- ii) During the collision there is no vertical motion so the normal force between the two masses equals the weight of m , i.e. $N = mg$

So, while there is relative motion between m and M , the friction between them is $\mu_k N = \mu_k mg$

Work energy theorem or conservation of energy:

Work done by non-conservative forces = change in mechanical energy = $\Delta U + \Delta K$. Let d be the distance of travel between m and M .

$$- \mu_k mg d = \Delta U + \Delta K$$

$$- \mu_k mg d = \frac{1}{2}(m+M)V^2 - \frac{1}{2}mv^2 \quad \text{OR} \quad = \frac{1}{2}(m+M)V^2 - mgR$$

$$\mu_k mg d = \frac{1}{2}mv^2 - \frac{1}{2}(m+M)(m\sqrt{2gR}/(m+M))^2$$

$$\mu_k mg d = \frac{1}{2}m2gR - (m^2gR/(m+M))$$

$$\mu_k d = R - (mR/(m+M))$$

$$d = R(M/(m+M)\mu_k)$$

4 a) i)

|Heat energy gained by water and bucket| - |heat energy lost by axe head| = 0

$$\begin{aligned}m_w c_w (T_f - T_{wi}) + m_b c_s (T_f - T_{wi}) + m_{ah} c_s (T_f - T_{ahi}) &= 0 \\12.5 \times 4186 \times (T_f - 24.3) + 0.500 \times 456 \times (T_f - 24.3) + \\+ 0.755 \times 456 \times (T_f - 337) &= 0 \\ \Rightarrow T_f (344.28 + 52325 + 228) &= 1406830 \\ T_f &= \frac{1406830}{52553} = 26.60 \\ T_f &= 26.6^\circ\text{C} \text{ (3 sig. fig)}\end{aligned}$$

ii)

In this case the total heat loss is given by:

$$\begin{aligned}Q &= 0.755 \times 456 \times (25.4 - 337) + 4186 \times 12.5 \times (25.4 - 24.3) + \\&+ 0.500 \times 456 \times (25.4 - 24.3) \\&= -49469\end{aligned}$$

This heat goes into raising some of the water from 25.4 to 100 °C and then converting this into steam.

$$\begin{aligned}Q &= m_s L_{vap} + m_s c_w \Delta T \\49469 &= m_s (2.26 \times 10^6 + 4186 \times (100 - 25.4)) \\ \Rightarrow m_s &= 0.0192 \text{ kg} \\&= 19.2 \text{ g}\end{aligned}$$

b) i) This is a diatomic gas at approximately room temperature. It has 5 degrees of freedom. 3 of these degrees of freedom correspond to the translational movement of the particles (one in x, one in y and one in z direction) and 2 correspond to the rotational movement of the molecules (about the two axes perpendicular to the line joining the two atoms).

ii) This gas is kept at a constant volume by the air tight seals so:

$$Q = n C_V \Delta T$$

Need to calculate n using the ideal gas law:

$$\begin{aligned}n &= \frac{PV}{RT} \\&= \frac{1.01 \times 10^5 \times 5.00 \times 5.00 \times 2.00}{8.314 \times 273.15} \\&= 2223.72\end{aligned}$$

Alternatively use the fact that at 0.00 °C 1 mol of ideal gas takes up $2.241 \times 10^{-2} \text{ m}^3$ to calculate n , this gives $n = 2\,231$ mols.

$$\begin{aligned}Q &= 2223 \times \left(\frac{1}{2} f R\right) \times 24.5 = 1132 \text{ kJ} \\&= 1130 \text{ kJ (3 sig. fig)}\end{aligned}$$

Or if using n calculated the second way you get $Q = 1136 \text{ kJ} = 1140 \text{ kJ}$ (3 sig fig) which is also correct.

c) i) $W = - \int P.dV$

which is just the area under the rectangle

$$W = -3.00 \times 1.01 \times 10^5 \times (2.000 - 0.6347)$$

$$= -413.685J = -414J \text{ (3 sig fig)}$$

This is work done on the gas, work done by gas is 414 J

ii) $\Delta E_{intB \rightarrow C} = Q_{B \rightarrow C} + W_{B \rightarrow C}$
 $= 1033 - 414 = 619J$

iii) $\Delta E_{intC \rightarrow D} = 0$ isothermal process
 $W_{C \rightarrow D} = \Delta E_{intC \rightarrow D} - Q_{C \rightarrow D}$
 $= 0 - 664 = -664J$

Alternatively could integrate

$$W_{C \rightarrow D} = - \int P.dV$$

With $P = nRT/V$ but this takes a lot longer.....

iv) Around a cycle there is no change in the internal energy:

$$\Delta E_{intA \rightarrow B} + \Delta E_{intB \rightarrow C} + \Delta E_{intC \rightarrow D} + \Delta E_{intD \rightarrow A} = 0$$

$$\Delta E_{intD \rightarrow A} = 1.00 \times 1.01 \times 10^5 \times (6.000 - 1.227) \times 10^{-3} - 1203 = -721J$$

$$\Delta E_{intA \rightarrow B} + 620 + 0 - 721 = 0$$

$$\Delta E_{intA \rightarrow B} = 101J$$

a)

i)

$3\lambda = 2.40 \text{ m}$ (as there are 6 loops, i.e. 6 half wavelength)

$$\lambda = 0.800 \text{ m}$$

$$v = f\lambda = 50 \times 0.800 = 40 \text{ m s}^{-1}$$

ii)

$$v = \sqrt{\frac{T}{\mu}}$$

$$T = mg = \mu v^2$$

$$\Rightarrow m = \frac{0.234 \times 10^{-3} \times 40^2}{9.8} = 0.0382 \text{ kg} = 38.2 \text{ g}$$

iii) for next harmonic $\lambda = \frac{2.40}{7/2} = 0.6857 \text{ m}$

$$v = 50 \times 0.6857 = 34.2857$$

$$\Rightarrow m = \frac{\mu v^2}{g} = \frac{0.234 \times 10^{-3} \times 34.2857^2}{9.8} = 28.06 \text{ g}$$

so need to remove $38.2 - 28.1 = 10.1 \text{ g}$.

b) i) As observed frequency is less than emitted frequency and the observer is stationary the source (plane) is moving away from Stan, ie. Receding.

ii)

$$f' = f \left(\frac{c}{c+v_s} \right)$$

$$\frac{2}{3}f = f \left(\frac{c}{c+v_s} \right)$$

$$\frac{2}{3} = \frac{c}{c(1+v_s/c)}$$

$$\Rightarrow v_s = \frac{1}{2}c = 170 \text{ ms}^{-1}$$

iii) $40.0 \text{ km/h} = 11.11 \text{ ms}^{-1}$

$$f' = f_p \left(\frac{c+v_o}{c+v_s} \right)$$

$$= f_p \left(\frac{340+11.11}{340+170} \right)$$

$$= 0.688 f_p = 0.69 f_p \text{ (2 sig. fig.) either 2 or 3 sig figs is acceptable.}$$

c)

i)

Make use of conservation of momentum.

$$mv = (m + M)v'$$

$$v' = \frac{10.5 \times 10^{-3} \times 630}{5.7}$$

$$= 1.16 \text{ ms}^{-1} = 1.2 \text{ ms}^{-1} \text{ (2 sig. fig)}$$

ii)

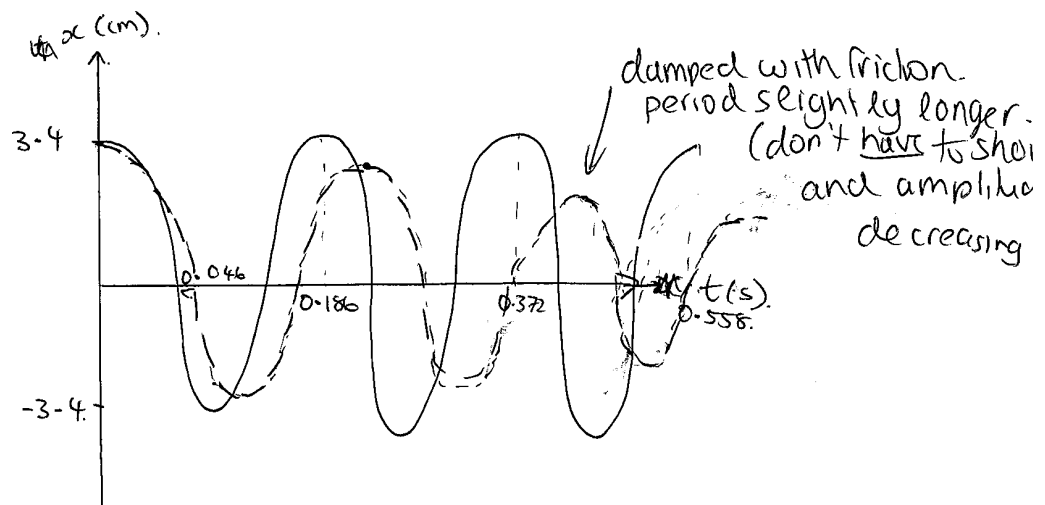
The maximum potential energy is equal to the maximum kinetic energy

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2$$

$$6500 \times A^2 = 5.7 \times 1.16^2$$

$$A = 0.03436 \text{ m} = 3.4 \text{ cm} \text{ (2 sig. fig)}$$

iii) and iv)



$$\omega^2 = \frac{k}{m} = \frac{6500}{5.7} = 1140.$$

$$\Rightarrow f = 5.37 \text{ Hz} \Rightarrow T = 0.186 \text{ s}.$$