PHYS1131 Final Exam
2015

Question 1 20 marks

(a) A toy car moves in the x-direction according to the equation
$x = (12t - t^2 - 6)$m, where t is in seconds:

(i) *Calculate the displacement of the car at t = 0s? Give your answer with magnitude and direction, with respect to the positive x-direction.*

ANSWER:

$$x = 12t - t^2 - 6$$

At t = 0s,
$$x = 12(0) - (0)^2 - 6$$

$$x = -6\text{m}$$

The magnitude of the displacement is 6m, in the negative x-direction.

(ii) *Starting with the equation above, derive an expression for the instantaneous velocity of the toy car.*

$$x = 12t - t^2 - 6$$

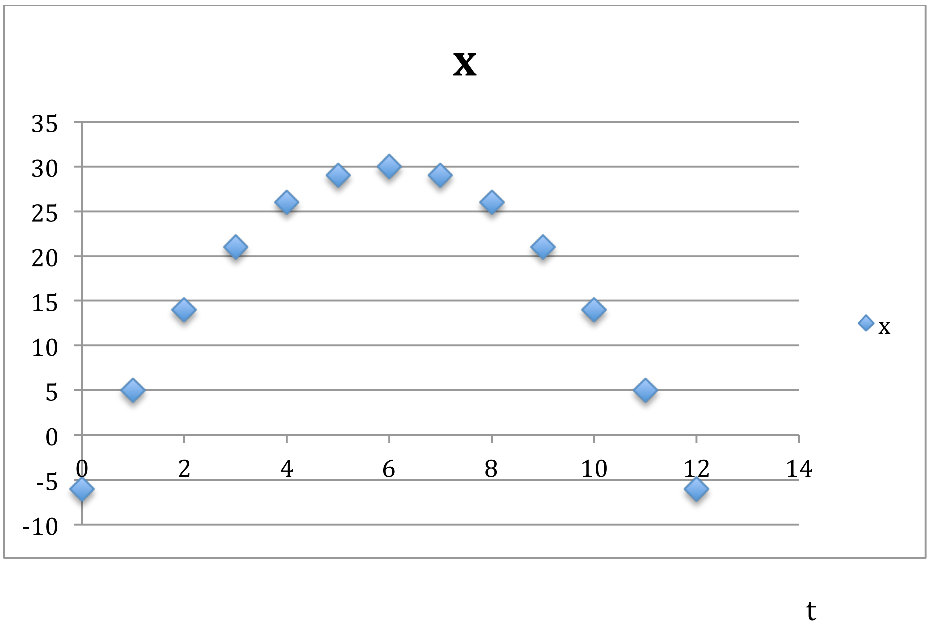
$$v = \frac{dx}{dt} = 12 - 2t \qquad \text{ms}^{-1}$$

(iii) Calculate the acceleration of the car.

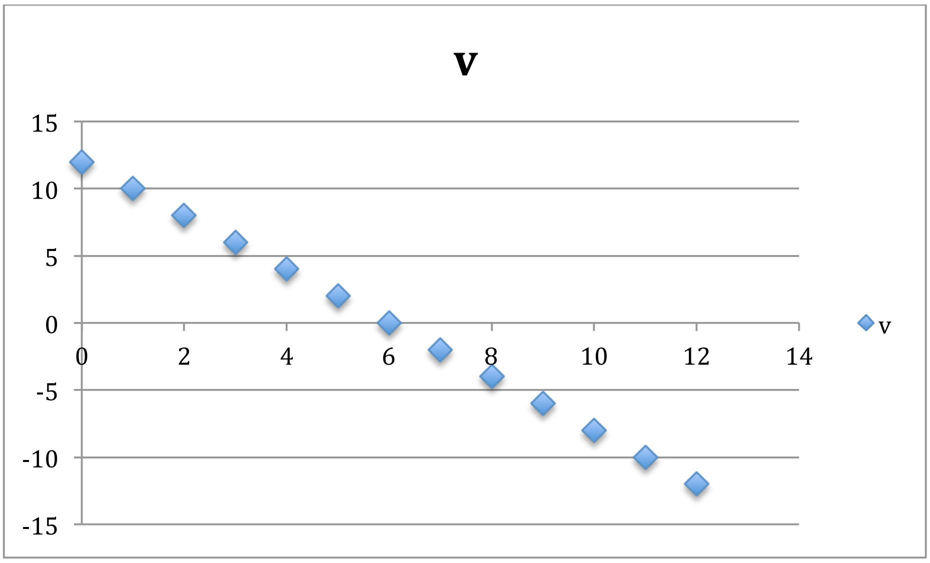
$$v = 12 - 2t$$

$$a = \frac{dv}{dt} = -2ms^{-2}$$

(iv) Sketch a rough graph of the *displacement* of the toy car with time, from t=0 to t=12 s.



(v) Sketch a rough graph of the *velocity* of the toy car with time, over the same time interval.



(vi) Calculate the time at which the velocity of the toy car is 0.

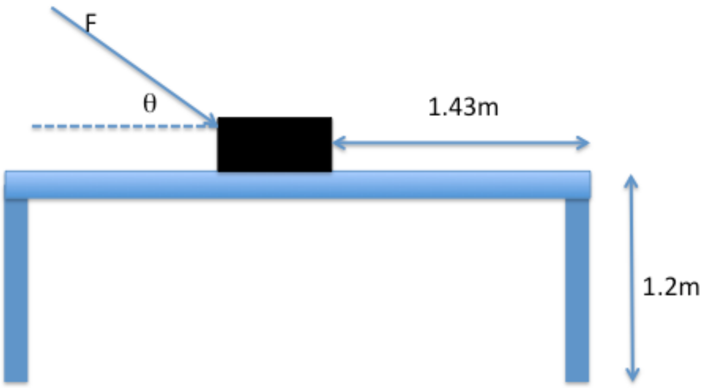
$$v = 12 - 2t$$
$$0 = 12 - 2t$$
$$t = 6s$$

(vii) Find the maximum displacement of the toy car in the positive x-direction.

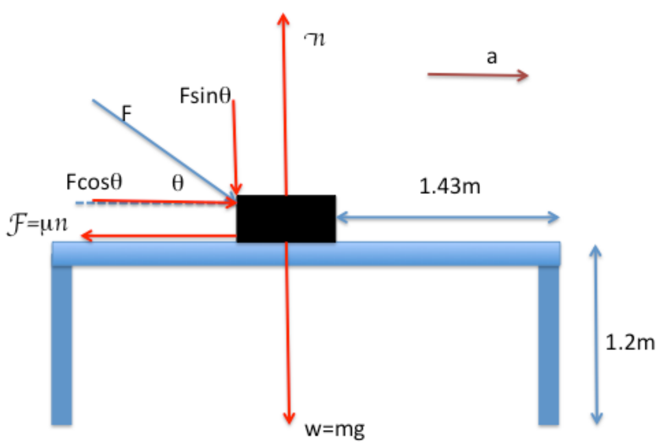
The maximum displacement will be at the time the toy car stops, before it starts moving backwards. This is at 6s. The displacement will be

$$\begin{aligned} x &= 12t - t^2 - 6 \\ x(6) &= (12)(6) - (6)^2 - 6 \\ &= 30m \end{aligned}$$

(b) A wooden block of mass 1.8 kg is pushed along a rough table by a force of 22 N at an angle, θ, of 30.0°. The block starts from rest, and moves a distance of 1.43m, until it reaches the end of the table. The coefficient of kinetic friction between the block and the table is 0.432. When answering the questions below, make sure you give the appropriate number of significant figures in your answer. Use g = 9.8 ms^{-2} where needed.



(i) Draw a free body diagram showing all forces on the block while it is travelling along the table. Make sure you show all vectors resolved into their x and y components. Additionally, show the direction of the acceleration on your diagram.



(ii) Calculate the ***net*** force (sum of forces) acting on the block in the y-direction.

y-direction: There is no motion in the y-direction, therefore the net force in the y-direction = 0

(iii) Calculate the magnitude of the ***normal force*** acting on the block.

$$\sum F_y = ma_y$$
$$\sum F_y = 0$$
$$\sum F_y = n - mg - F\sin\theta$$

$n = mg + F\sin\theta$

$n = (1.8)(9.8) + (22)\sin 30.0^\circ$

$= 28.64N = 29N$ (2 sig fig)

(iv) Calculate the value of the friction force acting between the block and the table. Give a magnitude and a direction.

$$f = \mu n$$
$$= (0.432)(28.64)$$
$$= 12.4N = 12\text{ N (2 sig fig)}$$

in the negative x-direction (opposing the direction of motion)

(v) Calculate the ***net*** force (sum of forces) acting on the block in the x direction. Give both magnitude and direction.

$$\sum F_x = ma_x$$
$$\sum F_x = F\cos\theta - \mu n$$
$$\sum F_x = 22\cos 30^\circ - 0.432(28.6)$$

$= 6.7N$ in the positive x-direction

(vi) Calculate the acceleration of the block as it moves through the 1.42 m distance.

$$\sum F_x = ma_x$$
$$ma_x = 6.7N$$
$$a_x = \frac{6.7}{1.8}$$
$$= 3.7 ms^{-2}$$

(vii) Calculate the velocity of the block just as it reaches the end of the table.

$$v^2 = u^2 + 2as$$
$$= (0)^2 + 2(3.7)(1.43)$$
$$= 10.6$$
$$v = 3.3 ms^{-1}$$

(viii) The block continues in motion off the end of the table. The force stops acting at this point.

(1) Calculate the time taken for the block to hit the floor.

The initial velocity in the y-direction is 0. The time taken to hit the floor is given by

$$s = ut + \frac{1}{2}at^2$$
$$1.2 = 0 + \frac{1}{2}(9.8)t^2$$
$$2.4 = (9.8)t^2$$
$$t^2 = \frac{2.4}{9.8}$$
$$t = 0.5s$$

(2) Calculate the distance the block travels in the x direction before it hits the floor.

The initial velocity in the x-direction is 2.6 m/s. the acceleration in the x-direction =0.

The distance travelled is x=3.26 0.5 = 1.6m.

3.26

(3) Calculate the velocity of the block just as it hits the ground. You should give your answer as a magnitude, and a direction clockwise from the positive x-direction.

$v_x = u_x = 3.3$ m/s
$u_y = 0$ m/s
t= 0.5 s

$$\begin{aligned} v_y &= u_y + a_y t \\ &= 0 + (9.8)(0.5) \\ &= 4.9\,ms^{-1} \end{aligned}$$

v = 5.8 m/s

Direction: 56 degrees clockwise from the positive x-direction.

Question 2

(a) State the conditions under which the momentum of a system is conserved.

ans: When the sum over external forces is zero, then the total momentum of a system is conserved.

(b) State the conditions under which mechanical energy of a system is conserved.

ans: When non-conservative forces do no work, mechanical energy, the sum of potential and kinetic enegies, is conserved.

(c) A car, mass m, travelling at a speed v, collides head-on with a truck with mass M travelling at the speed $v/2$. The two vehicles stick together and skid along the road for a distance D before coming to rest. The coefficients of static and kinetic friction are μ_s and μ_k,respectively.

(i) Derive an expression relating the velocity V of the two vehicles immediately after the collision, to v and any other parameters needed

ans: Conservation of momentum during the collision gives us

$$\begin{aligned} m_c v + M_t \frac{v}{2} &= (m_c + M_t) V \\ \left(m_c + \frac{M_t}{2}\right) v &= (m_c + M_t) V \end{aligned}$$

so

$$V_F = \frac{m_c + \frac{M_t}{2}}{m_c + M_t} v$$

(ii) Derive an expression for V in terms of D and other needed parameters

ans: we write

$$a = \frac{F}{M_t + m_c} = -\mu_k \frac{W}{M_t + m_c} = -\mu_k g$$

$$\begin{aligned} V^2 &= 2aD \\ V_F &= \sqrt{2\mu_k g D} \end{aligned}$$

(iii) Derive an expression for the amount of kinetic energy ΔK lost in the collision in terms of m, M and v only.

ans:

$$v = \frac{m + M}{M - M/2} \sqrt{2\mu_k g D}$$

Change in kinetic energy

$$\Delta K = K_f - K_i$$

$$K_i = \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{v}{2}\right)^2 = \frac{1}{2}\left(m + \frac{M}{4}\right)v^2$$

$$\begin{aligned} K_f &= \frac{1}{2}(M+m)V^2 \\ &= \frac{1}{2}(M+m)\left(\frac{m + \frac{M}{2}}{m + M}\right)^2 v^2 \end{aligned}$$

$$\begin{aligned} K_f - K_i &= \frac{1}{2}(M+m)V^2 - \frac{1}{2}\left(m + \frac{M}{4}\right)v^2 \\ &= \frac{1}{2}(M+m)\left(\frac{m + \frac{M}{2}}{m + M}\right)^2 v^2 - \frac{1}{2}\left(m + \frac{M}{4}\right)v^2 \\ &= -\frac{1}{8}\frac{Mm}{M+m}v^2 \end{aligned}$$

so kinetic energy lost is

$$\frac{1}{8}\frac{Mm}{M+m}v^2$$

.

(iv) Derive an expression for the power being dissipated due to friction, just after the collision, in terms of v and any other parameters needed.

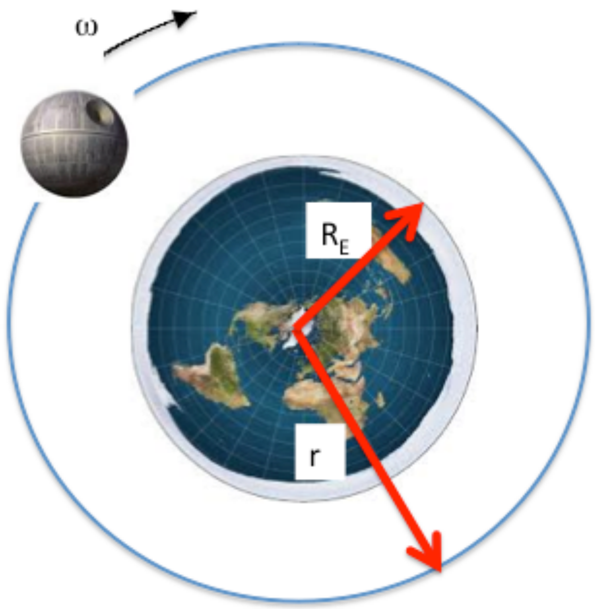
ans: Power is $\frac{dW}{dt}$

$$\begin{aligned} \frac{dW}{dt} &= \frac{d}{dt}Fds = \frac{d}{dt}\mu_k(M+m)gx(t) \\ &= \mu_k(M+m)gV \end{aligned}$$

right after the collision

$$\begin{aligned} P &= \mu_k(M+m)gV \\ &= \mu_k\left(m + \frac{M}{2}\right)gv \end{aligned}$$

Question 3   10 marks



A satellite is in a geostationary orbit, meaning it orbits the Earth so that it is always above the same position of the Earth's surface.

(i) Find the angular velocity of the satellite, ω, as it orbits the earth, in seconds. You can assume that 1day is exactly 24 hours.

The satellite is geostationary, and so it has the same angular velocity as the Earth.

The angular velocity is 1 revolution per day, = (2*pi)/(24*60*60) = 7.27×10^{-5} rad/s

(ii) Find the value of r, the distance from the Earth's centre, of the satellite's orbit.

$$\sum F_g = F_g = \frac{mv^2}{r}$$

$$G\frac{M_E m}{r^2} = \frac{mv^2}{r}$$

$$v^2 = G\frac{M_E}{r}$$

$$v = \sqrt{G\frac{M_E}{r}}$$

$$\tau = \frac{2\pi r}{v}$$

$$v = \frac{2\pi r}{\tau}$$

$$\frac{2\pi r}{\tau} = \sqrt{G\frac{M_E}{r}}$$

$$r = \frac{\tau}{2\pi}\sqrt{G\frac{M_E}{r}}$$

$$r^2 = \frac{\tau^2}{4\pi^2}G\frac{M_E}{r}$$

$$r^3 = GM_E\left(\frac{\tau^2}{4\pi^2}\right)$$

$$= \frac{6.7x10^{-11}(6x10^{24})(8.6x10^4)^2}{4\pi^2}$$

$$= 7.5x10^{22}$$

$$r = 4.2x10^7 m$$

(iii) Find the velocity, v, of the satellite in its orbit.

$$\tau = \frac{2\pi r}{v}$$

$$v = \frac{2\pi r}{\tau}$$

(iv) Find the gravitational potential energy of the satellite, with respect to a zero value of P.E of 0 at infinity.

$$U(r) = -\frac{GM_E m}{r}$$

(v) Find the total energy of the satellite, considering both gravitational and kinetic energy.

$$E = \frac{1}{2}mv^2 - G\frac{Mm}{r}$$

(vi) Is the total energy greater than or less than 0? Considering the orbit of the satellite, is the answer what you expect? Give a reason for your answer.

Less than 0, as expected for a bound orbit.

Question 3 (second part)

(i) A car has total mass m (including the wheels), and each of the four wheels have mass m_{w}, distributed evenly such that one can assume the wheel to be a uniform disc. Derive an expression for the fraction of the total kinetic energy coming from rotational motion of the wheels. Ignore other moving parts such as the engine and transmission.

$$\begin{aligned} K_{\mathrm{rot}} &= 4 \times \frac{1}{2} I\omega^2 = 4\frac{1}{2}\frac{1}{2} m_{\mathrm{w}} r^2\omega^2 \\ &= m_{\mathrm{w}} r^2 \omega^2 = m_{\mathrm{w}} v^2 \end{aligned}$$

$$K_{\mathrm{trans}} = \frac{1}{2} m v^2$$

$$\frac{K_{\mathrm{rot}}}{K_{\mathrm{rot}} + K_{\mathrm{trans}}} = \frac{2m_{\mathrm{w}}}{2m_{\mathrm{w}} + m}$$

(ii) Suppose the car is all wheel drive car with four wheels of radius r. When the car is at rest, what is the maximum torque that the engine could supply to the wheels without the wheels skiding? The coefficient of static friction is $\mu_s = 1.4$. Assume that the weight of the car is distributed evenly over the four wheels and that the engine distributes power equally among the four wheels.

$$\tau = 4rF = 4r\mu_s \frac{mg}{4} = r\mu_s mg$$

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1131 30

Part a

(i) Process A → B is adiabatic, i.e.,

$$P_A V_A^{\gamma} = P_B V_B^{\gamma}$$

$$\Rightarrow \frac{P_A}{P_B} = \left(\frac{V_B}{V_A}\right)^{\gamma}$$

$$\Rightarrow \ln\left(\frac{P_A}{P_B}\right) = \ln\left(\frac{V_B}{V_A}\right)^{\gamma}$$

$$= \gamma \ln\left(\frac{V_B}{V_A}\right)$$

$$\Rightarrow \gamma = \frac{\ln(P_A/P_B)}{\ln(V_B/V_A)}$$

$$= \frac{\ln\left(\frac{2.88}{15}\right)}{\ln(4/13)}$$

$$= \underline{\underline{1.4}}$$

Also:

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} \qquad \Big\| \; C_V = \frac{f}{2}R$$

$$= \frac{f+2}{f}$$

For $\gamma = 1.4$, $f = 5 \Rightarrow$ diatomic gas with 5 degrees of freedom.

(ii) Work done from A to B.

$$\Delta E_{int} = Q + W$$
$$= W \quad \text{since } Q = 0 \text{ for an adiabatic process}$$

Also:

$$\Delta E_{int} = \frac{f}{2} n R \Delta T \quad \Big\| \quad \text{where } \Delta T = T_B - T_A$$

From the ideal gas law:

$$T_B = \frac{P_B V_B}{nR} \quad ; \quad T_A = \frac{P_A V_A}{nR}$$

$$\Rightarrow \Delta T = \frac{1}{nR}\left[P_B V_B - P_A V_A\right]$$

Thus:

$$\Delta E_{int} = \frac{f}{2} nR \overbrace{\frac{1}{\cancel{nR}}\left[P_B V_B - P_A V_A\right]}^{\Delta T}$$
$$= \frac{f}{2}\left[P_B V_B - P_A V_A\right]$$
$$= \frac{5}{2}\left[15 \times 4 - 2.88 \times 13\right] \underbrace{\times \frac{1}{1000} \times 1.01 \times 10^5}_{\text{unit conversion}}$$
$$= \underline{\underline{5700 \text{ J}}}$$

Alternative solution: use

$$W = -\int_{V_A}^{V_B} P \, dV \qquad \text{with } PV^{\gamma} = P_A V_A^{\gamma}$$
$$\Rightarrow P = P_A \left(\frac{V_A}{V}\right)^{\gamma}$$

$$\Rightarrow W = -P_A V_A^{\gamma} \int_{V_A}^{V_B} \frac{dV}{V^{\gamma}}$$

$$= \frac{P_A V_A^{\gamma}}{(\gamma - 1)} \left[\frac{1}{V^{\gamma-1}} \right]_{V_A}^{V_B}$$

$$= \frac{P_A V_A^{\gamma}}{(\gamma - 1)} \left[\frac{1}{V_B^{\gamma-1}} - \frac{1}{V_A^{\gamma-1}} \right]$$

$$= \frac{2.88 \times 13^{1.4}}{(1.4 - 1)} \left[\frac{1}{4^{0.4}} - \frac{1}{13^{0.4}} \right] \times \overbrace{\frac{1}{1000} \times 1.01 \times 10^{5}}^{\text{unit conversion}}$$

$$= \underline{\underline{5700 \text{ J}}}$$

(iii) Process $B \to C$ is isothermal, i.e.,

$$\Delta E_{int} = Q + W = 0.$$

$$\Rightarrow \quad Q = -W$$

where

$$W = -\int_{V_B}^{V_C} P\,dV$$

from ideal gas law:

$$PV = P_B V_B$$

$$\Rightarrow P = \frac{P_B V_B}{V}$$

$$= -P_B V_B \int_{V_B}^{V_C} \frac{dV}{V}$$

$$= -P_B V_B \ln\left(\frac{V_C}{V_B}\right)$$

$$= -15 \times 4 \ln\left(\frac{15}{4}\right) \times \frac{1}{1000} \times 1.01 \times 10^5$$

$$= -8010\,J$$

Thus

$$Q = \underline{\underline{8010\,J}}$$ is the heat absorbed.

(iv) Change in the internal energy from C to D to A.

Internal energy depends only on the state, not the path. Thus:

$$\Delta E_{int\, C \to D \to A} = \frac{f}{2} nR \left[T_A - T_C \right]$$

$T = \frac{PV}{nR}$ from ideal gas law

$$= \frac{f}{2} \left[P_A V_A - P_C V_C \right]$$

$$= \frac{f}{2} \left[P_A V_A - P_B V_B \right]$$

b/c B→C is isothermal so that $P_B V_B = P_C V_C$.

$$= -\frac{f}{2} \left[P_B V_B - P_A V_A \right]$$

$$= -\Delta E_{int\, A \to B} \text{ from part (ii)}$$

$$= -5700 \text{ J}$$

(v) Work is done on the gas in $A \to B$ and $C \to D$,

$$\Rightarrow \quad W_{A\to B} + W_{C\to D} > 0$$

Work is done by the gas in $B \to C$,

$$\Rightarrow \quad W_{B\to C} < 0$$

But the process $B \to C$ occur in general at higher pressure than $A \to B$ or $C \to D$. Therefore.

$$|W_{B\to C}| > |W_{A\to B} + W_{C\to D}|$$

Thus, the net work done on the gas is negative. Since the internal energy does not change in one full cycle, it follows that

$$\Delta E_{int} = 0 = Q + W \Rightarrow Q = -W > \underline{\underline{0}}$$

The heat absorbed in the full cycle is <u>positive</u>.

Part b

(i) The total heat we need to add is

$$Q = Q_{Al} + Q_{water} + Q_{gas}$$

where

$$Q_{Al} = m_{Al} C_{Al} \Delta T$$

$\Delta T = 30K.$

$$= \rho_{Al} V_{Al} C_{Al} \Delta T.$$

$V_{Al} = 6 \times 1.00m^2 \times 0.01m$
$= 0.06 m^3$

$$= 2750 \times 0.06 \times 910 \times 30$$

$$= \underline{\underline{4.50 \times 10^6 J}} = \underline{\underline{4500 kJ}}$$

$$Q_{water} = \rho_{water} V_{water} C_{water} \Delta T$$

$V_{water} = 0.50m^3$

$$= 998 \times 0.50 \times 4186 \times 30$$

$$= \underline{\underline{6.27 \times 10^7 J}} = \underline{\underline{62700 kJ}}$$

$$Q_{gas} = n C_v \Delta T$$

where $n = \frac{P_i V}{RT_1}$

$$= \frac{P_1 V_1}{\cancel{R}T_1} \frac{3}{2} \cancel{R} \Delta T$$

$C_v = \frac{f}{2} R = \frac{3}{2} R$

for monatomic gas

$$= \frac{1.01 \times 10^5 \times 0.5 \times 3}{293 \times 2} \times 30$$

$P_1 = 1 atm.$
$T_1 = 20°C$
$V = 0.5 m^3$

$$= \underline{\underline{7760 J}}$$

$$m_{Al} = \rho_{Al} V_{Al} = 2750 \times 0.06 = 165 \text{ kg}$$

$$m_{ice} = \rho_{ice} V_{ice} = 917 \times 0.50 = 459 \text{ kg}$$

Then :

because $T_2 = 0°C$

$$T_f = \frac{(499 \times 4186 + 165 \times 910) \times 80 + 0 - 459 \times 3.33 \times 10^5}{(499 + 459) \times 4186 + 165 \times 910}$$

$$= \underline{\underline{6.3 \text{ °C}}}$$

Part c

Power radiated is

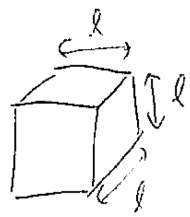
$$P = \sigma A e \left(T_{body}^4 - T_{env}^4 \right)$$

$\sigma = 5.6696 \times 10^{-8}\ W m^{-2} K^{-4}$

$e = 1$ for a blackbody

$T_{body} = 273\ K$

$T_{env} = 263\ K$



$A = 6 \times l^2 \quad \|$ where $l = V^{1/3}$

$= 6 \times V^{2/3}$

$= 6 \times \left(\frac{m}{\rho}\right)^{2/3}$

$= 6 \times \left(\frac{1 kg}{917 kg m^{-3}}\right)^{2/3}$

$= 0.0636\ m^2$

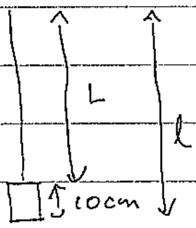
$\Rightarrow \quad P = 5.6696 \times 10^{-8} \times 0.0636 \times 1 \times (273^4 - 263^4)$

$= \underline{\underline{2.78\ W}}$

Phys1121 Q5 S2 2015 Model answer v4

Note Title 7/09/2015

i)



Period: $T = 2\pi\sqrt{\frac{l}{g}} = 3$

$4\pi^2 \frac{l}{g} = 9$

$l = \frac{9}{4}\frac{g}{\pi^2} = \frac{9}{4}\frac{9.81}{(3.14)^2} = 2.24\,m$

But the mass is 10cm high.

So $l = 2.24 - 0.05 = \underline{\underline{2.19m}}$

ii) The cable gets longer, so the period gets longer.

Hence the clock runs <u>slow</u>.

iii) $\Delta T = 8°C$.

Change in length of the pendulum =

$\alpha = \frac{\Delta L}{L} \quad °C^{-1}$

$\Delta L = \alpha L \Delta T$ ~~total length of pendulum~~

$= 100\times10^{-6} \times 2.24 \times 8°C = 1790\times10^{-6}\,m = 1.79\,mm$

New pendulum length $= 2.19 + 0.00179 = 2.1979$ – now either brute force calculate the change...

$T' = 2\pi\sqrt{\frac{l'}{g}} = 2\pi\sqrt{\left(\frac{9}{4}\frac{g}{\pi^2} + 179\times10^{-5}\right)\frac{1}{g}} = 3.0012s$

a day is $24\times60\times60 = 86{,}400$ secs.

– In a day there should be 28,000 periods. But now there are 28,788 periods. ⇒ We lose 12 periods – i.e. <u>we lose 36 seconds a day</u>

Note: There is an alternative method over the page that gives a slightly different answer of 34s. Both answers were accepted as correct. Keeping more significant figures in the working in the solution on this page gives 34s.

Alternative Derivation:

iii) $\Delta T = 8°C$

Change in length of the pendulum =

$\alpha = \frac{\Delta L}{L}$ $°C^{-1}$

$\Delta L = \alpha L \Delta T$ ~~total length of pendulum~~

$= 100 \times 10^{-6} \times 2.24 \times 8°C = 1790 \times 10^{-6}\ m = 1.79\ mm$

New pendulum length $= 2.19 + 0.00179 = 2.1979$ – now either brute force calculate the change...

An alternative derivation:

period $T = 2\pi\sqrt{\frac{L}{g}}$ $T^2 = 4\pi^2 \frac{l}{g}$ so $\frac{2\Delta T}{T} = \frac{\Delta l}{l}$

proof: $2T\Delta T = \frac{4\pi^2}{g}\Delta l = 4\pi^2 \frac{l}{g}\frac{\Delta l}{l} = T^2\frac{\Delta l}{l}$

$\Delta T = \frac{T}{2}\frac{\Delta l}{l} = \frac{T}{2} \times (8\alpha)$

$= \frac{3}{2} \times (8 \times 100 \times 10^{-6}) = 0.0012s$

So in a day, $60 \times 60 \times 24 = 86,400s$, we lose

$86,400 \times \frac{0.0012}{3} = 34.5$ sec

iv) The pendulum has become longer, due to thermal expansion.

$\therefore$ we need to make it shorter

Add counterweights to reduce the effective length

Another (possibly easier to follow?) solution to part (iii).

$$\text{Period} = 2\pi\sqrt{\frac{l}{g}}$$

$$\frac{d\,\text{Period}}{dl} = \frac{2\pi}{\sqrt{g}}\frac{d\sqrt{l}}{dl} = \frac{2\pi}{\sqrt{g}}\left(\frac{1}{2}\right)l^{-1/2} = \frac{\pi}{\sqrt{gl}}$$

$$\Rightarrow \Delta\,\text{Period} = \frac{\pi\,\Delta L}{\sqrt{gL}}$$

$$\Rightarrow \frac{\Delta\,\text{Period}}{\text{Period}} = \frac{\cancel{\pi}\,\alpha\cancel{L}\Delta T}{\sqrt{\cancel{g}\cancel{L}}}\cdot\frac{\sqrt{\cancel{g}}}{2\cancel{\pi}\sqrt{\cancel{L}}} = \frac{\alpha\Delta T}{2}$$

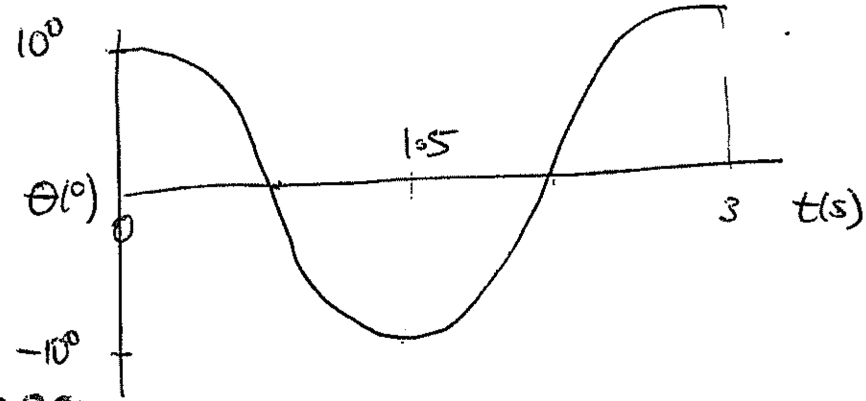
$$= \frac{100\times10^{-6}\times8.00}{2} = 4.00\times10^{-4}.$$

$\Rightarrow$ ~~uncertainty~~ change in 1 day $4\times10^{-4}\times60\times60\times24$

$$= 34.6\,\text{s.}\ (3\text{ sig fig}).$$

could answer to 1, 2 or 3 sig. fig.

T2 2015 Question 5 (a) v → vii



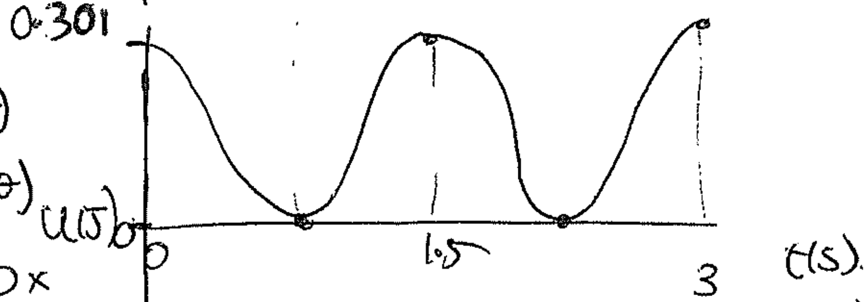
$V = 20h\pi r^2$

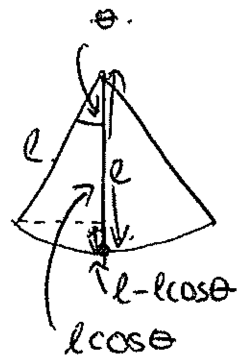
$U = mg(l - l\cos\theta)$

$= \rho V g (l - l\cos\theta)$

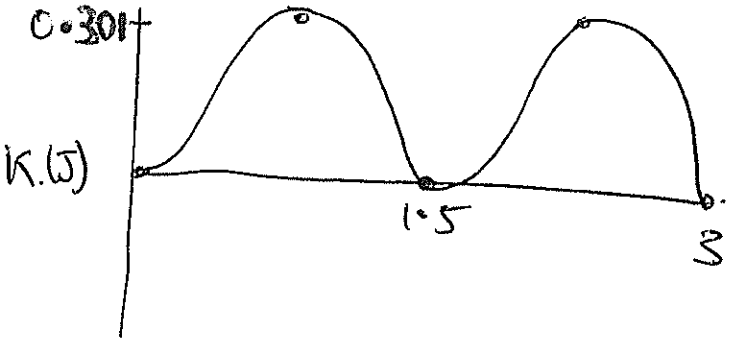
$U_{max} = 1150 \times 20 \times 0.5\times10^{-2} \times \pi \times (5\times10^{-2})^2 \times 9.8 \times 2.24 \times (1-\cos 10)$

$= 0.301\ J.$





ie change in height = $l - l\cos\theta$



vi) The amplitude does not change.

The period of the oscillations gets shorter so the clock gains time.

vii). The amplitude decreases as added block doesn't have KE.

1131 only The period gets shorter and the clock gains time.

1131 T2 2015 Question 5b.

$f_0 = 9GHz$

i) In this situation the car is the observer, the police officer is the source.

$$\Rightarrow f' = f_0\left(\frac{c - v_0}{c}\right)$$

as $v_s = 0$ and observer is moving away from the source.

ii) Now the car is the source emitting waves with frequency f' and the police officer is the observer

$$\Rightarrow f'' = f'\left(\frac{c}{c + v_0}\right)$$

$V_s = V_0$ (velocity of car) now, it is moving away from observer

$$= f_0\left(\frac{c - v_0}{\cancel{c}}\right)\left(\frac{\cancel{c}}{c + v_0}\right)$$

$$= f_0\left(\frac{c - v_0}{c + v_0}\right).$$

iii) We now multiply waves with frequencies f_0 and f'' together. Waves can be written as $y = s\cos(kx - \omega t + \phi)$ we will simplify as much as possible, letting x and $\phi = 0$ and remember $\cos(\theta) = \cos(-\theta)$.

$\Rightarrow$ multiplied waves $= S\cos(\omega_0 t)\cos(\omega'' t)$

Now if we let

$$\omega_0 t = \frac{A+B}{2} \quad \text{and} \quad \omega'' t = \frac{A-B}{2}.$$

we see

$$A = \omega_0 t + \omega'' t \qquad B = \omega_0 t - \omega'' t.$$

is a solution so the multiplied waves

$$= S \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$= \frac{S}{2}\left[\cos(\omega_0 t + \omega'' t) + \cos(\omega_0 t - \omega'' t)\right]$$

But high frequency, $\omega_0 t + \omega'' t$ term, is removed

$\Rightarrow$ wave observed $= \frac{S}{2}\cos(\omega_0 t - \omega'' t)$.

with frequency $f_0 - f''$

$$\Rightarrow f_0 - f'' = 1{\cdot}25 \times 10^3 \, Hz.$$

$$f_0\left(1 - \frac{c - V_0}{c + V_0}\right) = 1{\cdot}25 \times 10^3 \, kHz. \qquad f_0 = 9 \times 10^9.$$

rearrange this

$$1 - \frac{c - V_0}{c + V_0} = \frac{1{\cdot}25}{9 \times 10^6}.$$

$$\Rightarrow V_0 = \frac{c\left(\frac{1{\cdot}25}{9 \times 10^6}\right)}{2 - \frac{1{\cdot}25}{9 \times 10^6}} = 20{\cdot}83 \, m/s.$$

$$= 75 \, km/h.$$