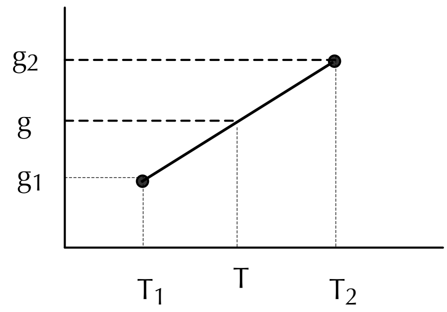
PHYS1131 HIGHER PHYSICS 1A

HOMEWORK PROBLEM SET 4

Temperatures

1. $g(T) = g_1 + \frac{g_2 - g_1}{T_2 - T_1}(T - T_1) \approx 31.2$



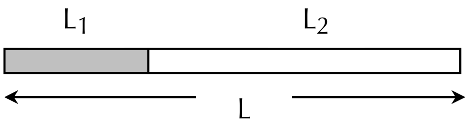
2. Both the ruler (r) and the Rod (R) expand when placed in the oven
Initial is indicated by subscript i, final by subscript f

Start by working out how 1cm of ruler changes when heated:
$\Delta L_r = \alpha_r L_{ri} \Delta T = 1.1 \times 10^{-5} ({}^oC^{-1}) \times 1(cm) \times 250({}^oC)$
$= 2.75 \times 10^{-3} cm$
ie. 1cm of ruler now measures 1.00275cm

Now work out the actual final length of the rod
$L_R f = 20.11 cm \times 1.00275 cm/cm$
$= 20.165 cm$

Now use change in length of Rod to calculate α_R
$\Delta L_R = L_{Rf} - L_{Ri} = 20.165 - 20.05 cm$
$= 0.115 cm$
$$\alpha_R = \frac{\Delta L_R}{L_{Ri} \Delta T} = \frac{0.115 cm}{20.05 cm \times 250 {}^oC} = 2.3 \times 10^{-5} ({}^oC^{-1})(2 sig.fig.)$$

3.



$L = L_1 + L_2$

$\frac{\delta L}{L} = ?$ $\quad L+\delta L = L_1(1+\alpha_1\Delta T) + L_2(1+\alpha_2\Delta T)$

$$= L_1 + L_2 + (L_1\alpha_1 + L_2\alpha_2)\Delta T$$

$$= L\left(1+\frac{L_1\alpha_1 + L_2\alpha_2}{L}\right)\Delta T$$

$$\delta L = L\left(\frac{L_1\alpha_1 + L_2\alpha_2}{L}\right)\Delta T$$

$$\Rightarrow \alpha = \frac{L_1\alpha_1 + L_2\alpha_2}{L}$$

$d_1 = 11.10^{-6}$ $\quad L = 52.4$ cm

$\alpha_2 = 19.10^{-6}$ $\quad d = 13.10^{-6}\ K^{-1}$

$\frac{L_1\alpha_1 + L_2\alpha_2}{L} = \alpha$ $\quad L_1 + L_2 = L$ $\quad L_1 = L - L_2$

$L_1\ \alpha_1 + L_2\ \alpha_2 = L\ \alpha\ (L - L_2)\alpha_1 + L_2\ \alpha_2 = L\ \alpha$

$$L_2 = \frac{L(\alpha - \alpha_1)}{\alpha_2 - \alpha_1} \approx 13 \text{ cm}$$

$$L_1 = \frac{L(\alpha - \alpha_2)}{\alpha_1 - \alpha 2} \approx 39 \text{ cm}$$

Kinetic theory and the ideal gas

4. $P_1 = 1.00$ atm $= 76.0$ cm Hg ($= 1013 \times 10^2$ Pa $= 1.013 \times 10^5$ Pa)

$T_1 = 22.0^oC$ $\quad PV = n\ RT$ (Ideal Gas equation, n# moles)

$V_1 = 3.47\ m^3$

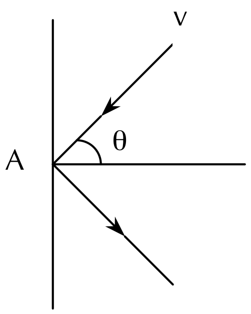
$P_2 = 36.0$ cm Hg $\quad \frac{PV}{T} = \text{const}$

$T_2 = -48.0^oC$

V_2 - ? $\quad \frac{P_1V}{T_1} = \frac{P_2V}{T_2}$

$T_{(k)} = T(^oC) + 273$ $\quad V_2 = \frac{P_1}{P_2}.\frac{T_2}{T_1}.V_1 \approx 5.59\ m^3$

5.



$\Delta p = 2\ mv \cos\theta$ $\quad F = \frac{\Delta p}{\Delta t}$

$\frac{N}{\Delta t}$ collisions/second $= 1.0 \times 10^{23}\ s^{-1}$

$$\text{pressure} = \frac{\sum F}{A} = \frac{\frac{\Delta p}{\Delta t}.N}{A} = \frac{2mv\cos\theta.N/\Delta t}{A} \approx 0.19\ \frac{N}{cm^2}$$
$$= 1900\ \text{Pa}$$

6. (a) $m = 3.3 \times 10^{-27}$ kg
$A = 2.0 \times 10^{-4}\ m^2$
$Q = 55^o$
$v = 1.0 \times 10^3$ m/s

$m = 0.315$ kg
NH_3 $\mu = (14+3).10^{-3}$ kg/mole [Since N + 3 x H]
$P = 1.35 \times 10^6$ Pa

$PV = nRT$ where $n = \frac{m}{\mu}$ number of moles; μ mass for 1 mole

$T = 273 + 77 = 350$ K

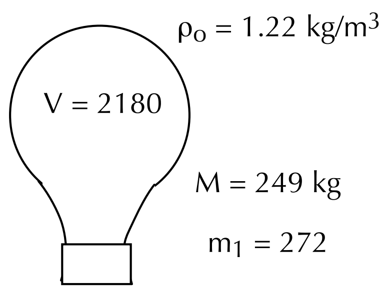
$V = \frac{mRT}{\mu.P} \approx 0.0399\ m^3$ $R = 8.31\ J.K^{-1}/mole$

(b) $p_1 = 8.68\ .\ 10^5$ Pa $T_1 = 295$ K

$m_1 = \frac{\mu P_1 V}{RT_1} = 0.240$ kg $\Rightarrow$ $\Delta m = m - m_1 = 0.075$ kg $= 75$ g

7. Mass of air in the balloon: $\rho_0 V$ in normal conditions
Archimedes principle: Upthrust = Weight of air displaced
We need: $(M + m_1)g = (\rho_0 - \rho_1)Vg$
$\Rightarrow \rho_1 = \rho_0 - \frac{(M + m_1)}{V}$ (1)

To solve for T we need a relationship between ρ and T
Start with the ideal gas law:
$PV = nRT$; $n = \frac{m}{\mu}$ where μ is the molar mass
$\frac{P\mu}{R} = \frac{m}{V}T = \rho T =$ constant since P, μ and R are constant
$\Rightarrow \rho_0 T_0 = \rho_1 T_1$ (2)



Substitute (2) into (1):
$\frac{\rho_0 T_0}{T_1}V = \rho_0 V - (M + m_1)$
$\Rightarrow T_1 = \frac{\rho_0 T_0 V}{\rho_0 V - M - m_1}$
$= \frac{1.22 \times (273 + 18) \times 2180}{1.22 \times 2180 - 249 - 272} = 362K = 89.0^oC$

Past exam question

a) $T_o = 20°C$; Power supplied P; Water boiling F = 0.020 L/min

Supply power at such a rate that amount of water neither increases or decreases; ignore other heat losses

In steady state, Energy supplied = heat to raise to boiling (i.e. ΔT) + heat to boil

ie. $P.t = Mc\Delta T + ML$ c specific heat; L Latent heat; M mass

Flow Rate $F = M/t\rho$ if amount water unchanged

Thus $P = F\rho(c\Delta T + L)$

$$= 0.020\frac{10^{-3}}{60}1000\left(4200(100-20)+2.3\,10^{6}\right)\text{W} = 878\text{ W}$$

$= 880$ W to 2 S. F.

b) (i) Rod A Length $L_A = (L_o+D_o)(1 + \alpha_A(T - T_o))$

Cylinder B length $L_B = L_o(1 + \alpha_B(T - T_o))$

Thus at $T = T_o$ difference is $(L_A - L_D)=D_o$, as required

At T $(L_A - L_B)= (L_o+D_o)(1 + \alpha_A(T - T_o)) - L_o(1 + \alpha_B(T - T_o))$

$= D_o + L_o (T - T_o)(\alpha_A-\alpha_B) + D_o\alpha_A(T - T_o))$

i.e. $D = D_o(1 + \alpha_A\Delta T) + L_o(\alpha_A-\alpha_B)\Delta T$

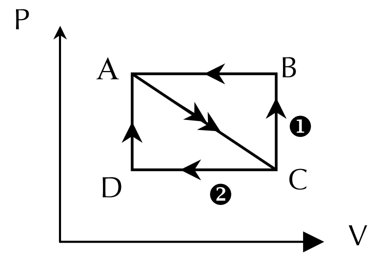
(ii) We require that D is independent of temperature

i.e. $\Delta T\ [D_o\alpha_A + L_o(\alpha_A-\alpha_B)]\ 0$

thus $D_o\alpha_A = L_o(\alpha_B-\alpha_A)$

or $$\frac{D_o}{L_o}=\left(\frac{\alpha_B-\alpha_A}{\alpha_A}\right)=\left(\frac{\alpha_B}{\alpha_A}-1\right)$$

8.



Work done on gas: $dW = -pdV$

Around closed path, therefore, work done
$W = -\int pdV = -$*Area enclosed by path*

Along path ❶ this is negative

Along path ❷ this is positive

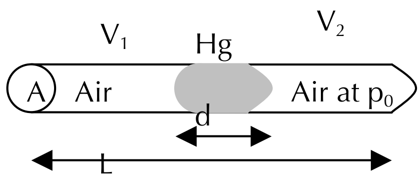
Consider the components to the integrals, to see this

$$Path\ 1:\quad \int_A^C + \int_C^B + \int_B^A$$

$$Path\ 2:\quad \int_A^C + \int_C^D + \int_D^A$$

Area of triangle $\frac{1}{2}$ (8-2)(20-5)x1000 = 45,000
The work done on Path 1 is - - 45 kJ = 45 kJ
Work done on Path 2 is -45 kJ

9.



L = 1.00m, d = 0.100m, p_0 = 1atm = 1013.10^{2}Pa

$$V_1 = A\left(\frac{L-d}{2}\right),\ V_2 = A\left(\frac{L-d}{2}\right)$$

$$V_1' = A\left(\frac{L-d}{2} - x\right),\ V_2' = A\left(\frac{L-d}{2} + x\right),\ x = \text{displacement of Hg}$$

Isothermal:

$$p_1' = \frac{p_0}{1 - \frac{x}{\frac{L-d}{2}}} \quad p_2' = \frac{p_0}{1 + \frac{x}{\frac{L-d}{2}}} \qquad \text{NOTE } \frac{1}{1-\alpha} = 1+\alpha \quad \mathit{if}\ \alpha \ll 1 \ \mathit{with}\ \alpha = \frac{2x}{L-d} \ll 1$$

$$p_2' - p_1' \approx p_0 \frac{2x}{\frac{L-d}{2}} = \rho g d \qquad \text{where } \rho \text{ is density of mercury}$$

$$x_T = \frac{\rho g d \frac{L-d}{2}}{2p_0} \approx 2.96\text{cm}$$

For adiabatic process PV^{γ}=const

$$p_1' = \frac{p_0 A^{\gamma}\left(\frac{L-d}{2}\right)^{\gamma}}{A^{\gamma}\left(\frac{L-d}{2} - x\right)^{\gamma}} = \frac{p_0}{\left(1 - \frac{x}{\frac{L-d}{2}}\right)^{\gamma}}$$

$$p_2' = \frac{p_0}{\left(1 + \frac{x}{\frac{L-d}{2}}\right)^\gamma}$$

$p = \rho g h \Rightarrow p_1' - p_2' = \rho g d$, and expand, assuming x << (L – d)/2

$$p_0 \frac{2\gamma x}{\left(\frac{L-d}{2}\right)} = \rho g d \Rightarrow x_a = \frac{\rho g d\left(\frac{L-d}{2}\right)}{2\gamma p_0} = \frac{x_{isothermal}}{\gamma} = 2.11\text{cm}$$

Process is probably rapid, so an adiabatic change is initially appropriate. After a while, it will approach the isothermal result as the system thermally equilibrates.

10. As P is constant and n is constant we have:

$$\frac{V}{T} = \text{const}$$
$$\Rightarrow \frac{V}{(11 + 273)} = \frac{2V}{T_1}$$
$$\Rightarrow T_1 = 568 \text{ K}$$

Then calculating the heat added:

$$Q = c_P n \Delta T$$
$$= \frac{7}{2} R \times 1.35 \times (568 - 11 - 273)$$
$$= 11.2 \text{ kJ}$$