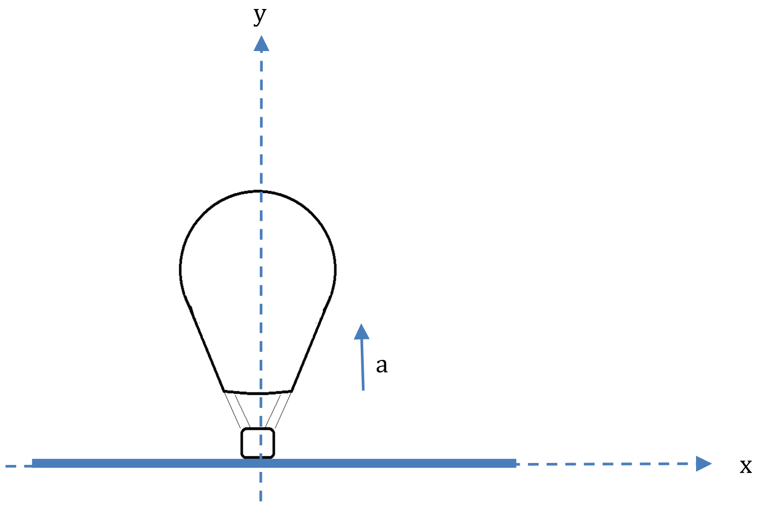
Question 1 (20 Marks)

A hot air balloon starts from rest, on the ground (x=0, y=0), and accelerates vertically at 0.50 ms^{-2} until it reaches a height of 450 m above ground level. The hot air balloon is attached to a basket and there is a person standing on the floor of the basket.



(a) How long does the balloon take to reach a height of 450 m?

(b) Calculate the velocity of the balloon when it reaches this height. Assume that the upwards direction is positive.

(c) When the balloon reaches a height of 450 m, a sand bag is pushed over the side, in a horizontal direction. Assume that the sand bag has an initial horizontal velocity of 2.1 ms^{-1}, in the positive x-direction (direction as shown on the diagram above).

 (i) What will the initial velocity of the sand bag in the y-direction be, immediately after it is pushed from the balloon?

 (ii) Draw a rough sketch showing the path of the sand bag after it leaves the balloon until the time it reaches the ground.

 (iii) Calculate the maximum height above the ground the sand bag reaches.

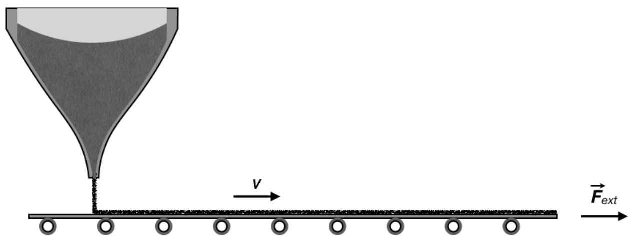
 (iv) How long does the sand bag take to reach the ground?

 (v) how far from the x= 0 position is the sand bag when it lands?

(d) A car is travelling at a constant velocity of 100 km/hr when the driver sees the road is blocked 90 m ahead. Assuming the driver has a human reaction time of 1.0 s, and decelerates at 7.0 ms^{-2}, will the driver be able to stop before hitting the obstacle? Show all working to justify your answer.

Question 2 (20 Marks)

(a) Sand from a stationary hopper falls onto a moving conveyor belt at the rate of 5.00 kg/s as shown below. The conveyor belt is supported by frictionless rollers and moves at a constant speed of $v = 0.750$ m/s under the action of a constant horizontal force.



(i) Find the sand's rate of change of momentum in the horizontal direction.

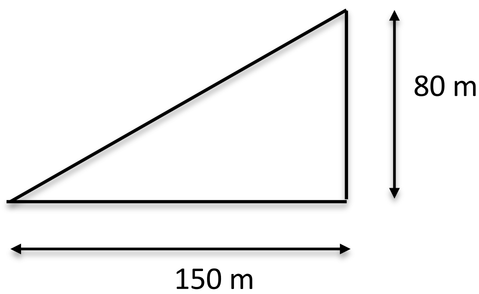
(ii) Find the magnitude of the external force F_{ext}.

(iii) Find the work done by the external force in 1.00 second.

(iv) Find the kinetic energy K acquired by the falling sand each second, due to the change in its horizontal motion.

(v) Why do the answers to (iii) and (iv) differ?

(b) A block is made to slide upwards, from the bottom of an inclined plane of vertical height 80 m, and horizontal length 150 m, with an initial velocity 20 m/sec. The coefficient of kinetic friction, $\mu_k = 0.20$, and the coefficient of static friction, $\mu_s = 0.40$. All data in this question is given with 2 significant figures. The inclined plan is shown in the diagram.



(i) Find how far the block goes up the plane.

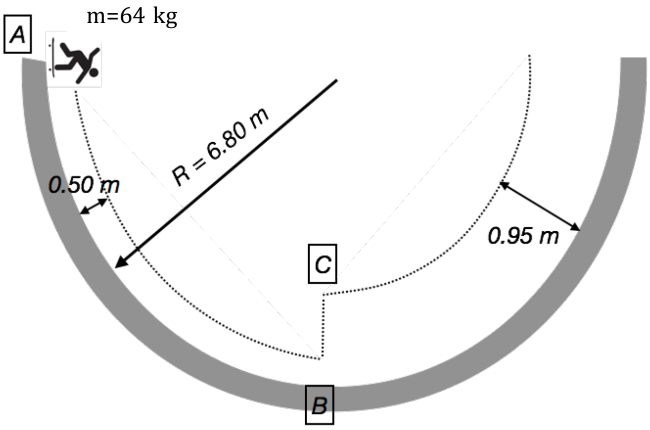
(ii) Will it start sliding back down? Why or why not?

(iii) Assuming is does slide back down, find the total time it takes to go up the plane and come back down to the bottom.

(iv) What is its velocity when it reaches the bottom.

Question 3 (20 Marks)

(a) A skateboarder crouching with her board can be modelled as a particle of mass 64.0 kg located at her centre of mass, 0.50 m above the ground. As shown below, the skateboarder starts from rest in a crouching position at one lip of half-pipe (point A). The half-pipe forms one half of a cylinder of radius 6.8 m with its axis horizontal. On her decent, the skateboarder moves without friction and maintains her crouch to the very bottom of the half-pipe (point B). Ignore the energy of rotation of the skateboard wheels as they are very small and therefore make a negligible contribution.



(i) Find her speed at the bottom of the half-pipe (point B).

(ii) Find her angular momentum about the centre of curvature of the half-pipe at Point B.

(iii) Immediately after passing point B, she stands up and raises her arms (point, changing her centre of gravity to 0.95 m from the concrete. Explain why her angular momentum is conserved, whereas the kinetic energy of her body is not constant.

(iv) Find her speed immediately after she stands up.

(v) How much chemical energy in her legs was converted to mechanical energy when she stood up?

(b) A car has total mass m (including the wheels), and each of the four wheels have mass m_w, distributed evenly such that one can assume the wheel to be a uniform disc.

(i) Derive an expression for the fraction of the total kinetic energy coming from rotational motion of the wheels. Ignore other moving parts such as the engine and transmission.

(ii) Suppose the car is all-wheel drive car with four wheels of radius r. When the car is at rest, what is the maximum torque the engine could supply to the wheels without the wheels skidding? The coefficient of static friction is 1.4. Assume that the weight of the car is distributed evenly over the four wheels and that the engine distributes power equally among the four wheels.

Question 4 (Marks: 30)

(a) A monatomic gas is initially at pressure P_1 and volume V_1 (state 1) and is allowed to isothermally expand from its initial volume to $V_2=2V_1$ (state 2). It is then adiabatically compressed back to volume V_1 (state 3). Finally, the gas is brought back to pressure P_1 and volume V_1 in an isovolumetric process.

(i) Draw a P-V diagram for the steps involved and mark the states 1, 2 and 3 on the graph.

(ii) What is the pressure P_2 after the isothermal expansion in terms of P_1?

(iii) Derive, using the ideal gas equation and the equation for work on a gas, the equation for the work done on the gas during the isothermal expansion. Give your answer in terms of the number of moles n, the temperature T and the gas constant *R*.

(iv) Is the work done **on** the gas W positive, negative or zero for each of the processes $1 \rightarrow 2$, $2 \rightarrow 3$, $3 \rightarrow 1$? Explain how you obtained your answer.

(v) Is the heat flow into the gas Q positive, negative or zero for each process? Explain how you obtained your answer.

(vi) What is the value of the gamma (γ) in the adiabatic process?

(vii) What is the pressure P_3 after the adiabatic process in terms of the pressure P_2 just before the adiabatic process?

(b) A person wishes to rapidly decrease the temperature of their 300.0mL coffee from 85.0°C to a more drinkable temperature. To avoid adding too much liquid to the coffee they use water ice cooled to -100.0°C.

How much ice should the person use so that all the ice melts and the final combined liquid is 50.0 °C?

Treat the coffee as if it was water. Take the specific heat of ice as constant at c_{ice} = 1.75 J/g °C

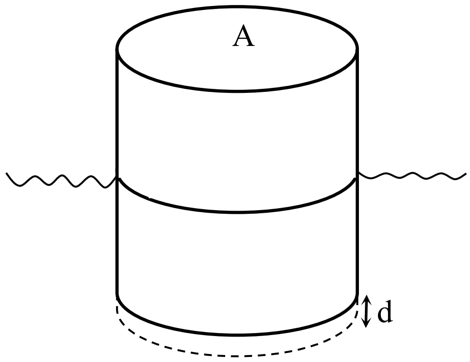
Other constants are on your formula sheet.

(c) (i) Explain with respect to equipartition of energy, the difference in behaviour of a monatomic and diatomic gas' mean particle kinetic energy if a fixed amount of heat is added to each gas. Initially the gases are at room temperature (300K).

(ii) Would the monatomic gas or the diatomic gas have a higher temperature change, or would they have the same temperature change, and why?

Question 5 (Marks: 30)

(a) A cylindrical buoy of mass M = 4 kg and cross-sectional area A = 0.04 m^2 is floating a calm lake, resting at its equilibrium position. Suddenly, a wayward catfish pulls down on a chain that loosely tethers it to the lake bed. It is displaced from its equilibrium position by d cm, and released again at time t = 0.



(i) Archimedes principle states that the buoyancy force is equal to the weight of water displaced, i.e. $F_{up} = \rho V g$, where V is the volume that is below the water's surface and $\rho = 1000$ kg/m^3 is the density of water. Determine the net upward force on the buoy when it is displaced to a depth d below its equilibrium point.

(ii) Use your answer to part (i) and Newton's second law to determine the equation of motion of the buoy.

(iii) When the displacement x is negative (i.e. the buoy is higher than its equilibrium position) then the net force on the buoy is downwards. Explain what forces are acting on the buoy in this case.

(iv) Solve the equation of motion and show that it is simple harmonic.

(v) Derive an expression for the period of the oscillation. Substitute numbers to obtain the period in seconds.

(b) The same buoy, undergoing simple harmonic motion, causes water waves to radiate out in circles along the surface of the lake. The lake is very large and has a constant depth. At a radius of one meter, the waves are 4.0 cm from peak to trough (this is the usual definition of wave height). How high are the waves at a radius of ten meters?

(c) A slide whistle, generously described as a musical instrument, can be modelled as a closed cavity. The frequency it generates can be continuously modified by sliding out a piston which changes the cavity length *L*.

(i) Sketch a diagram of the closed cavity with a standing wave showing the fundamental frequency. Label the nodes and antinodes.

(ii) Determine the frequency of the generated fundamental note as a function of the cavity length, $f(L)$.

(iii) A "musician" is standing at the edge of a circular revolving stage with radius a, rotating with angular frequency ω. He plays the slide whistle at frequency f_0 to an audience that is quite far away (distance $d >> a$). At time $t = 0$ he is moving towards the audience. Determine the frequency that the audience hears as a function of time.

(iv) A second musician stands at the other side of the revolving stage playing the same note. From the perspective of the audience, at time $t = 0$ one musician is receding while the other comes towards the listener.
What is the beat frequency that the audience hears as a function of time?

(v) The second musician wishes to play a note that will sound to the listeners at frequency f_0. How should she manipulate the cavity length L in time?