Question 1 (Marks: 20)

(a) (i) The following 3 vectors all have the same units:

$$\vec{\mathbf{A}} = 6.00\hat{\mathbf{i}} - 8.00\hat{\mathbf{j}}$$

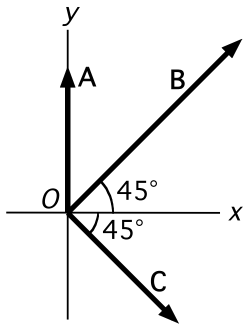
$$\vec{\mathbf{B}} = -8.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}$$

$$\vec{\mathbf{C}} = 26.00\hat{\mathbf{i}} + 19.00\hat{\mathbf{j}}$$

If $a\vec{\mathbf{A}} + b\vec{\mathbf{B}} + \vec{\mathbf{C}} = 0$, determine the values of a and b.

(ii) The figure shown illustrates 3 displacement vectors having magnitudes
$|A| = 20.0\text{m}, |B| = 40.0\text{m}, |C| = 30.0\text{m}$

Find the resultant in unit vector notation and also give the magnitude and direction of the resultant displacement.



(b) An archer shoots an arrow with a velocity of 45.0 m/s at an angle of 50.0 degrees to the horizontal. An assistant standing 150 m (3 sig. fig.) away throws an apple vertically with the minimum initial speed required to meet the path of the arrow. Neglect air resistance and treat both as particles.

(i) Calculate the time interval between the launch of the arrow and the impact with the apple.

(ii) Calculate the height of the arrow above the ground at the moment of impact with the apple.

(iii) Calculate the initial speed of the arrow.

(iv) Calculate the time interval between launch of apple and impact with arrow.

(v) How long after the arrow has been launched should the apple be thrown?

(c) An object with mass 3.00 kg has a velocity given by $(6.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}})$ **m/s**.

(i) Find the kinetic energy of the object.

(ii) Find the total work done on the object if its velocity changes to $(8.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}})$ **m/s**

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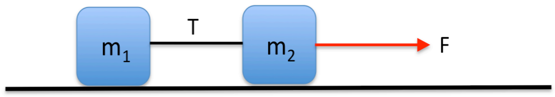
Question 2 (Marks: 20)

(a) Two blocks are connected by a rope of negligible mass. A force F=68.0N is applied as shown in the figure. The masses are m_1=12.0 kg and m_2=18.0kg. The coefficient of kinetic friction between each block and the surface is 0.100.



(i) Draw the free body diagram for each block.

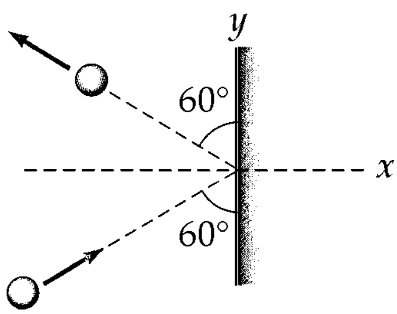
(ii) Determine the tension T and the magnitude of the acceleration of the system.

(iii) Show that your answer to part (ii) is consistent with Newton's second law for the block m_1.

(b) State the conditions under which mechanical energy is conserved.

(c) Under what conditions is momentum conserved?

(d) A ball of mass *m* strikes a wall with an initial speed of *v*. The final speed of the ball is the same as the initial speed.



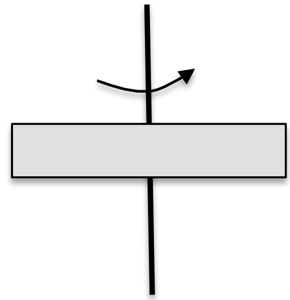
(i) Is this collision elastic, inelastic or perfectly inelastic? Give reasons.

(ii) Is momentum conserved in this collision? If not explain why the momentum is lost. If it is conserved describe any momentum transfers.

(iii) If the ball is in contact with the wall for a time *t* write an expression for the average net force on the ball. Make sure you mention the direction of the force in your answer.

Question 3 (Marks: 20)

A bar starts from rest and rotates with constant angular acceleration about an axis located in the middle of the bar to reach an angular speed of ω radians/s in t seconds. The bar has a moment of inertia I, for a bar of length L and mass m pivoted through its centre the moment of inertia is given by $^{1}/_{12}\, mL^2$.



Write an expression for:

(i) the angular acceleration of the bar.

(ii) the angle in radians through which it rotates in this time.

(iii) the torque acting on the bar over this time.

(iv) How much work is done on the bar over time t?

(v) What net force is acting on the bar over this time?

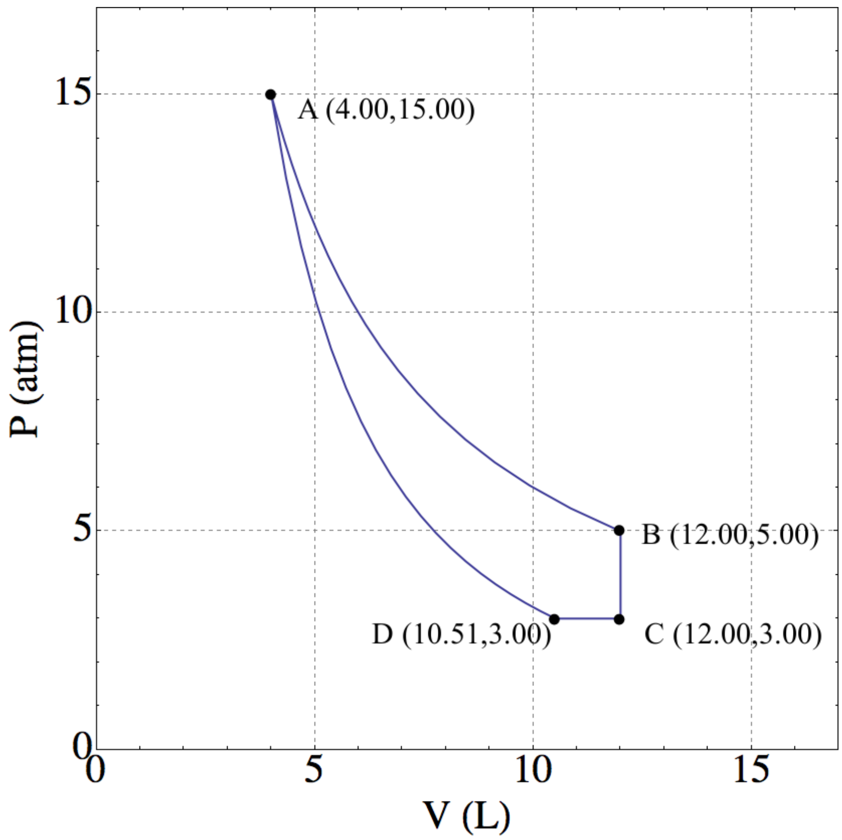
(vi) The mass of the bar is now doubled, keeping the relative mass distribution the same, e.g. by placing two bars against each other on the same axle. The same torque is applied to the combined bar. What is the angular speed after the same time t now? Show all your working.

(vii) With the mass still doubled the length of the bar is now also doubled. The axis is still in the middle of the bar. When the same torque is again applied what is the angular speed after the same time t? Show all your working.

(viii) If the pivot point is moved to the end of the bar would I increase, decrease or stay the same? Give reasons but you do not need to calculate it.

Question 4 (Marks: 30)

(a) 2.00 mols of an ideal monatomic gas undergoes a cyclic process from A → B → C → D → A.



Process A → B is isothermal, while process D → A is adiabatic. (Useful info: 1 m^3 = 1000 L, 1 atm = 1.01×10^5 Pa)

(i) Calculate the temperature of the gas at A, B, C and D.

(ii) Calculate the work done on the gas as it goes from A to B.

(iii) Calculate the change in internal energy as the gas goes from B to C.

(iv) Calculate the heat absorbed as the gas goes from C to D.

(v) Calculate the work done on the gas as it goes from D to A.

(vi) Is the net work done on the gas in one cycle positive or negative? Explain (without calculations) your answer.

Substance	**Specific heat c, (J kg^{-1} K^{-1})**	**Linear thermal expansion coefficient α, (°C^{-1})**	**Thermal conductivity k, (W m^{-1} K^{-1})**
Aluminium	910	24×10^{-6}	205.0
Brass	377	19×10^{-6}	109.0
Copper	390	17×10^{-6}	385.0
Lead	130	29×10^{-6}	34.7
Steel	456	11×10^{-6}	50.2

Properties of water	**Value**
Specific heat (liquid)	4186 J kg^{-1} K^{-1}
Latent heat of fusion	3.33×10^{5} J kg^{-1}
Latent heat of vapourisation	2.25×10^{6} J kg^{-1}
Density (at 4.00 ºC)	1000 kg m^{-3}
Melting point (at 1 atm)	0.000 ºC
Boiling point (at 1 atm)	100.0 ºC
Volume expansion coefficient β (at 20 ºC; you may assume it is constant between 4 ºC and 100 ºC)	207×10^{-6} ºC^{-1}

(b) Consider a cubic container made of aluminium sheets with side lengths of 1.00 m at 20.0 ºC. The container has no lid and is filled to the rim with water.

(i) We heat the container (and the water in it) to a temperature to 50.0 ºC. How much water will spill over the container as a result? Here, you may assume the aluminium sheets to have zero thickness.

(ii) Suppose now the aluminium sheets have a thickness of 1.00 cm. By raising the temperature from 20.0 ºC to 50.0 ºC, how much energy have we added to the combined container+water system? You may take the density of aluminium to be 2750 kg m^{-3}.

(iii) Now we drop a 100 kg lump of lead into the water. (Don't worry about the small amount of water that spills over the container as a result.) The lead initially has a temperature of 0.00 ºC. Assuming that the container+water+lead is an isolated system, what is the final temperature of the system when it reaches equilibrium?

Question 5 (Marks: 30)

(a) Mr. Spock wakes to find he is swinging back and forth on a long rope suspended over a chasm on the Klingon home planet Kronos. Spock knows the acceleration due to gravity on the surface of Kronos is 10.3 m/s^2. If the period of Spock's swing is 12.0s then how far down the rope is he?

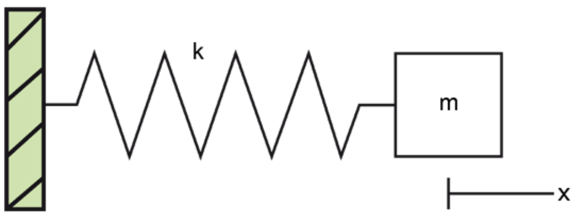
(b) As he begins to climb up the rope he sees Captain Kirk above him, going up and down attached to the end of a large spring, also suspended over the chasm. If Kirk's mass is 80.0kg and the period of his oscillation is 4.0s then what is the spring constant of the spring? (You can ignore the mass of the spring)

(c) Kirk is yelling at a frequency of 300Hz, while Spock is below him. Describe in words at which position in Kirk's up and down oscillation Spock will hear the highest frequency sound from Kirk, and why.

(d) Spock stops climbing (and manages to stop swinging as well) to measure Kirk's yelling which varies in frequency as Kirk oscillates. Spock's tricorder tells him the highest frequency he hears coming from Kirk is 309Hz. What is Kirk's maximum speed? (Note: the speed of sound in the air on Kronos is the same as on Earth, $340ms^{-1}$)

(e) What is the amplitude of Kirk's oscillation? (Note: the potential energy of a spring is P.E. = $\frac{1}{2}kx^2$)

(f) Later, thanks to an injection of Kironide Spock is given telekinetic abilities (the ability to move objects with his mind). To test the maximum force Spock's mind can apply he has set up an experiment with a mass (m) on a frictionless surface attached to a horizontal spring that is fixed to a wall (with spring constant k), as shown in the figure. In words, briefly explain how Spock can measure the maximum force his mind can apply.



(g) Spock releases the mass attached to the spring, setting up simple harmonic motion in the mass-spring system, write down the equation of displacement for the simple harmonic motion and define all terms.

(h) Suppose the mass is 10kg and has a frequency f = 0.50Hz with amplitude A = 1.0m, what is the total mechanical energy of the spring–block system?

(i) What is the speed of the block when it is displaced at a position 0.30m from the equilibrium position?