Q1 Solutions

(a)

(i) $v = \frac{dr}{dt} = -10.0t\hat{j} \quad m/s$

(ii) $a = \frac{dv}{dt} = -10.0\hat{j} \quad m/s^2$

(iii) $r = (4.00\hat{i} - 11.25\hat{j})\ m$
$v = -15.0\hat{j}\ m/s$

(b) $k = \frac{F}{y} = \frac{mg}{y} = \frac{(3.00)(9.80)}{0.015} = 1960 \quad N/m$

(i) For a 4.50 kg mass, $y = \frac{mg}{k} = \frac{(4.50)(9.80)}{1960} = 0.0225\ m$

(ii) The work done in adding the 4.50kg mass is

$$W_{4.5} = \frac{1}{2}ky^2 = \frac{1}{2}\frac{(mg)^2}{k} = \frac{1}{2}\frac{[(4.50)(9.80)]^2}{1960} = 0.496\ J$$

Work done in adding the 3.0kg mass is

$$W_3 = \frac{1}{2}\frac{[(3.00)(9.80)]^2}{1960} = 0.220\ J$$

The net work done in adding the extra 1.5kg mass is

$$W_{4.5} - W_3 = 0.276\ J$$

(c)

(i) Impulse is $I = p_f - p_i = (0.06)(45) - [-(0.06)(50)] = 5.7\ Ns$

(ii) Work is $W = K_f - K_i = \frac{1}{2}(0.06)(45^2 - 50^2) = -14\ J$

(Note only 2 sig. figs. in answer as speeds only given to 2 sig. figs.)

(d) Let the x-axis have its origin at the Earth's centre and point toward the Moon.

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(5.972\times10^{24})(0)+(7.348\times10^{22})(384400\times10^{3})}{(5.972\times10^{24}+7.348\times10^{22})}$$

$$= \frac{2.825\times10^{31}}{6.045\times10^{24}} = 4.672\times10^{6}\ m$$

from the Earth's centre. The centre of mass therefore lies within the Earth, since its radius is 6.37×10^{6} m.

(e) **[1131 only]**

The Earth or Moon period of rotation about the Earth-Moon centre of mass is 2π radians in 27.32 days.

The period is therefore $\omega = 2\pi/(27.32*24*3600) = 2.662 \times 10^{-6}$ radians/second.

The centripetal force associated with the angular motion is $F = ma_c = mr\omega^2$

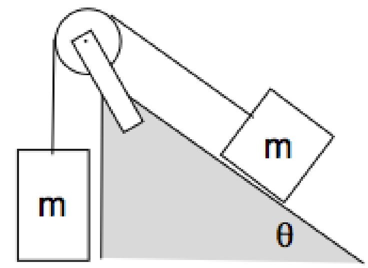
We can work that out for either Earth or Moon but it makes sense to work it out for the Earth since we already calculated the distance from the centre of the Earth to the Earth-Moon CoM. The force therefore is

$$F = M_E r_{E-CoM}\omega^2 = (5.972\times10^{24})(4501\times10^{3})(2.662\times10^{-6})^2 = 1.906\times10^{20} N$$

Note that some students could use the mass of the Moon and the Moon-CoM distance.

Some students might calculate F from GMm/r^2 (which would give a very similar answer) but they were explicitly asked NOT to use G, so no marks if they do that.

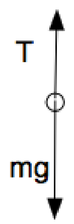
Question 2a (10 marks)



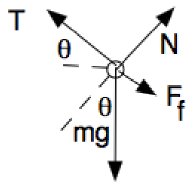
The sketch shows a plane inclined at angle θ to the horizontal. Two equal masses (m) are connected via a light, inextensible string. One of the masses slides on the inclined plane: the coefficients of kinetic and static friction are μ_k and μ_s respectively. The string passes without slipping over a light frictionless pulley. When released, the mass on the left accelerates downwards with acceleration a.

i) Draw free body diagrams for each of the masses m.

ii) Hence or otherwise, derive an expression for a.



i)



ii) Using the free body diagram for the mass on the left.
Newton's second law gives: $ma = mg - T$ (1)

The lower sketch is the FBD for the mass on plane.

Newton's second law applied in the normal direction, in which acceleration is zero, gives

$0 = N - mg\cos\theta$ so $N = mg\cos\theta$ (2)

Newton's second law applied in the plane gives

$ma = T - F_f - mg\sin\theta$ (3)

Currently, we have four unknowns: a, T, F_f and N. So we need one more equation. Because it is sliding,

$F_f = \mu_k N.$ (4)

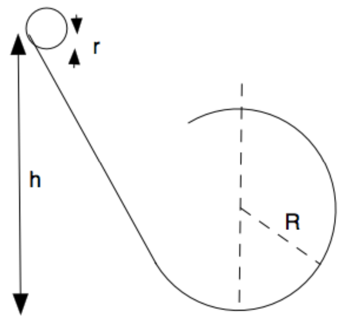
Substitute (4) in (3) gives $ma = T - \mu_k N - mg\sin\theta$

Then substitute (2) $ma = T - \mu_k mg\cos\theta - mg\sin\theta$ (*)

Then add (1) and (*) $2ma = T - \mu_k mg\cos\theta - mg\sin\theta + mg - T$

So $a = g(1 - \sin\theta - \mu_k\cos\theta)/2$

Question 2b (11 marks)



b) A uniform, spherical ball, mass m and radius r, is released from rest and rolls without slipping on a track as shown (but not to scale). The curved section is an arc of a circle with radius R, and R is very much greater than r. The ball is released from rest at a height h above the lowest point of the track. Air resistance is negligible.

i) When the ball reaches the highest point of the curved section of track, it has speed v. Derive an expression for v. State clearly any conservation principles you use, and explain why they are valid.

ii) Derive an expression for the magnitude of the normal force that the track exerts on the ball at this point.

iii) State the direction of the normal force when the ball is at this point.

Hint: Remember that the ball is rolling, not sliding.

Question 2b

b) Although friction is present here, it does no work.

Because non-conservative forces do no work, mechanical energy is conserved.

$U_i + K_i = U_f + K_f$ using i for initial and f for final. Dividing K into rotational and translational terms:

$$U_i + K_{trans,i} + K_{rot,i} = U_f + K_{trans,f} + K_{rot,f}$$

$$mgh + 0 + 0 = mg2R + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

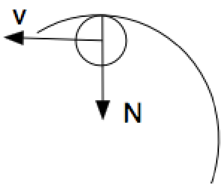
Because it is rolling, $v = r\omega$, so $\omega = v/r$.

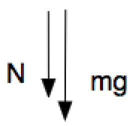
For a uniform sphere, $I = 2/5\, mr^2$ Substituting:

$$mg(h-2R) = \tfrac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mr^2\left(\frac{v}{r}\right)^2$$

$$g(h-2R) = \left(\frac{1}{2}+\frac{1}{5}\right)v^2 = \frac{7}{10}v^2$$

$$v = \sqrt{\frac{10g(h-2R)}{7}}$$





ii) From this sketch and Free Body Diagram, apply Newton's second law $mv^2/R = ma = N + mg$ so

$$N = m(v^2/R - g) = m(10g(h-2R)/7R - g) = mg(10(h-2R)/7R - 1)$$

iii) The direction of N is down (or in the normal direction)

Question 3 (1131)

i) A conservative force does no work around a closed path.

ii) Mechanical energy is conserved if non-conservative forces do no work.

iii) An object with small mass m at distance r from a mass M has escape speed v_e if, in the absence of non-conservative forces it will arrive at a very large distance from M with negligible speed. Here, non-conservative forces do no work so mechanical energy is conserved, which we write thus:

$U_i + K_i = U_f + K_f$ using i for initial and f for final. So

$$-G\frac{Mm}{r} + \tfrac{1}{2} m v_e^2 = 0 + 0$$

so $$\tfrac{1}{2} m v_e^2 = G\frac{Mm}{r} \qquad (*)$$

and $$v_e = \sqrt{\frac{2GM}{r}}$$

iv) For a black hole, set v_e = c. Rearrange (*) to get

$$\tfrac{1}{2} c^2 = G\frac{M}{r_{BH}}$$

$$r_{BH} = \frac{2GM}{c^2}$$

(Comment: surprisingly, this is the same as the result from General Relativity.)

v) Write K for the kinetic energy of each body and U for the potential energy of the system of two bodies.

$$U_{\text{mutual}} = -\frac{GM^2}{2R}$$

(Notice the $2R$ because they are separated by $2R$, and remember not to count their mutual potential energy of their interaction twice!) Neglecting relativistic effects, each object has

$$K = \tfrac{1}{2} M v^2$$

Newton's second law: $F = Ma$

The acceleration is centripetal and equals v^2/R. The separation between the two is $2R$, so Newton's law of gravitation gives

$$G\frac{M^2}{(2R)^2} = Ma = M\frac{v^2}{R} \qquad \text{so } Mv^2 = \frac{GM^2}{4R}$$

so $$K_{\text{both}} = 2K_{\text{each}} = Mv^2 = \frac{GM^2}{4R} \qquad \text{and above} \qquad U_{\text{mutual}} = -\frac{GM^2}{2R}$$

So $$E = U_{\text{mutual}} + K_{\text{both}} = -\frac{GM^2}{2R} + \frac{GM^2}{4R} = -\frac{GM^2}{4R}$$

vi) from (iv), $R = 2r_{BH} = \dfrac{4GM}{c^2}$

substitute this in (v) to have

$$E = -\frac{GM^2}{4R} = -\frac{Mc^2}{16}$$

vii) $E = -\dfrac{Mc^2}{16} = -\dfrac{(6\times 10^{31}\text{kg})(3\times 10^{8}\text{m.s}^{-1})^2}{16} = -3.4\times10^{47}$ J (note that this is negative)

and since it's really at best a 1 sig fig problem we could write it as -3×10^{47} J

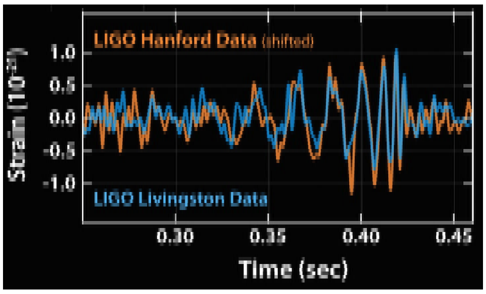
viii) Power out = $(E_i - E_f)/\Delta t$ = $(E/2 - E)/\Delta t$ = $-E/2*0.01\text{s}$ = $1.7\text{x}10^{49}$ W (~$2\text{x}10^{49}$ W)

ix) $$\frac{\text{Power radiated}}{\text{Power of sun}} = \frac{-E/2\Delta t}{(\text{number of galaxies})\dfrac{\text{stars}}{\text{galaxy}}\dfrac{\text{power}}{\text{star}}}$$

$$= \frac{3.4\times10^{47}\,\text{J}/(2*10\text{ms})}{(10^{11}\text{galaxies})\dfrac{10^{11}\text{ stars}}{\text{galaxy}}4\times10^{26}\text{W}} \sim 4$$

(The calculator gives 4, but this is a very rough calculation – there are more approximations than those indicated. Note that this is positive: the orbits go from negative to more negative)

Comments (to be published with the solutions): Why $R = 2r_{BH}$? Because if they get much closer they fuse to become one black hole and stop emitting gravitational waves, as we can see on this plot of the fractional contraction of the interferometer as a function of time as space is contracted and dilated by the gravity waves. See how the period decreases by a factor of two in the last couple of milliseconds (and the last few orbits) before it collapses. (Thanks LIGO for the graph.)



Now this was a *very* rough calculation, but serious relativistic calculations also agree that the total radiation power was (very briefly) greater than that of all the stars in the observable universe. Hooee!

Of course it was 1.3 billion light-years away ($1.3\text{x}10^{25}$ m) so, dividing by 4π*Distance^2, the intensity received here in the Milky Way was ~ 10 mW.m^{-2}. Hey, but that's not small. Light at this intensity is 0.001% of sunlight but you can still read by it. Sound at this intensity is fairly loud (100 dB). So could we have felt these waves?

No, obviously enough, or it wouldn't have taken such a huge effort to detect them. Look at the scale on the plot: a fractional contraction or expansion (strain) in space of 10^{-21}. Gravity is such a weak force that even when the masses are large, the forces produced are weak. Even when the waves have substantial power, the resultant distortion of space is weak.

PHYS 1121 / 1131 / 1141 Exam Q4 2016 - T1

(a) For Volume Expansion we have $\frac{\Delta V}{V} = \beta \Delta T$

Liner - - - - - - - $\frac{\Delta R}{R} = \alpha \Delta T$

with $\beta = 3\alpha$

Thus $\frac{\Delta R}{R} = \frac{\beta}{3} \Delta T$

$= \frac{3.2\ 10^{-5}}{3} (3000 - 300)$

$= 0.0288$

$\therefore$ Radius changed by 2.9% to 2SF

(b) Force = Pressure × Area = Weight =

ie $P.A = mg \Rightarrow m = \frac{P.A}{g}$

Where A surface area of earth $= 4\pi R^2$

$\therefore m_{atmospher} = \frac{P.\ 4\pi R^2}{g}$

$= \frac{101 \times 10^3\ 4\pi (64000 \times 10^3)^2}{9.8}$

$= 5.30 \times 10^{18}$ kg

$m_{earth} = 6.0 \times 10^{24} \Rightarrow$ fraction $= \frac{m_{atm}}{m_{earth}} = \frac{5.30 \times 10^{18}}{6.0 \times 10^{24}} = 8.84\ 10^{-7}$

$= 8.8\ 10^{-7}$

2SF

ie mass of atmosphere is $\simeq 10^{-4}$ % of mass of earth

(C) <u>Any</u> three of:

1 of these

(i) Number of ~~molecules~~ particles large, and average separation between particles large compared to dimensions

<u>or</u>

(i) ~~Molecul~~ Particles occupy negligible volume within the container

<u>or</u>

(i) Particles are point-like

(ii) Obey Newton's Laws but move randomly as a whole

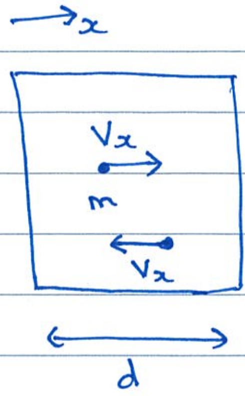
(iii) Particles do not interact except during collisions, which are elastic

(iv) Elastic collisions with walls

(v) Gas is a pure substance; ie all particles are identical

(d)

1131/1141



speed V_x

momentum V_x

Elastic collision with wall $\rightarrow$ rebounds with $-V_x$

Average time between collisions is $\Delta t = \dfrac{L}{V_x} = \dfrac{2d}{V_x}$

where L path travelled $= 2d$

Change in momentum is $mV_x - (-mV_x) = 2mV_x$

Thus Force = Rate of Change of Momentum [Newton's 2nd Law]

$$= \frac{2mV_x}{2d/V_x} = \frac{m\,V_x^2}{d}$$

This is for one partele. Thus for N identical partuls

$$\text{Force} = N \times \frac{mV_x^2}{d}$$

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

(d) 1121 Force exerted on Wall is given by $F = \frac{N m V_x^2}{d}$

(e) 1131/ 1141

We have $V^2 = V_x^2 + V_y^2 + V_z^2$

If moving isotropically then $V_x^2 = V_y^2 = V_z^2$

$$\Rightarrow V^2 = 3 V_x^2$$

Area of Wall is d^2

$$\Rightarrow \text{Pressure} = \frac{F}{A} = \frac{N m}{d^2 \times d} \frac{V^2}{3}$$

But Volume of box $V = d^3$

$$\Rightarrow P = \frac{N m v^2}{3V} = \frac{N}{3V} m v^2$$

Ideal Gas equation $\Rightarrow$

$$PV = nRT \qquad n \quad \text{number moles}$$

f [1131/1141] e [1121] But we have, from previous part

$$PV = \frac{N}{3} m v^2$$

KE of 1 particle $\Rightarrow \frac{1}{2} m v^2$

$\therefore$ KE of N particles $\Rightarrow N . \frac{1}{2} m v^2$ = KE of gas

$$\Rightarrow P.V = \frac{2}{3} . KE = nRT \quad \text{from ideal gas eqn.}$$

$$\text{Thus } KE \cancel{\tfrac{3}{3}} = \underline{\frac{3}{2} nRT}$$

g [1131/1141] f [1121]

$$T = 20°C \equiv 293 \text{ K}$$

$$M_{N_2} = 2 \times 14.0u = 28.0u$$

From Previous Portion

$$KE = \frac{3}{2} nRT = N \tfrac{1}{2} m v^2$$

~~But~~ so $$v^2 = 3 \left(\frac{n}{N}\right) \frac{RT}{m}$$

But $N = n . N_A \qquad N_A$ Avogadros. Number

$$\Rightarrow v^2 = \frac{3RT}{m_{N_2} N_A} = \frac{3 \times 8.314 \times (273+20)}{2 \times 14.0 \times 1.6\cancel{6}\cancel{3} \times 10^{-\cancel{3}27} \times 6.022 \times 10^{23}}$$

$$= \cancel{5.22 \times 10^5 \ (m/s)^2} \qquad 2.61 \ 10^5 \ (m/s)^2$$

$$\Rightarrow v = \cancel{722.6 \ m/s} \qquad \cancel{2.61 \ 10^5} \ 510.97 \ m/s$$

$$= \cancel{720 \ m/s \ to \ 2SF} \qquad \underline{510 \ m/s \ 2SF}$$

Question 5 1121/1131/1141 T1 2016.

$\mu = 0.58 g/m$.

$f = 440 Hz$.

$l = 32 cm$.

$\Rightarrow \lambda = 64 cm$

a) i). $v = f\lambda$

$= 440 \times 2 \times 0.32$

$= 282 m/s$

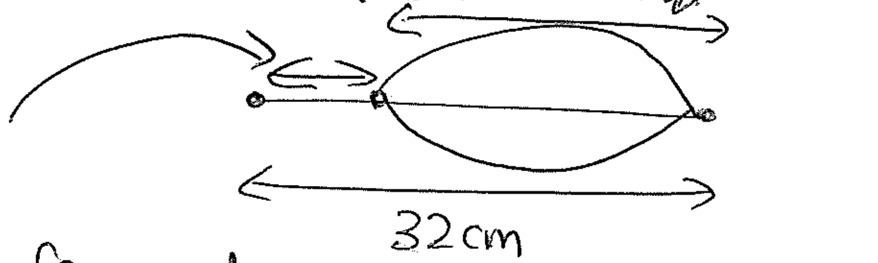
$= 280 m/s$ (2 sig fig).

ii). $v = \sqrt{\frac{T}{\mu}} \Rightarrow T = v^2\mu$.

$= 46 N$ (2 sig fig).

iii). $f = 587 Hz \Rightarrow \lambda = \frac{v}{f} = \frac{282}{587} = 0.480 m$.

$\Rightarrow l = 24.0 cm$ (half of λ)



$= 32 - 24$

$= 8.0 cm$

place finger 8cm from end.

iv) same T, different μ.

$\mu = \frac{mass}{length} = \frac{\rho . l . \pi r^2}{l} = \rho\pi r^2$.

$T = v^2\mu$

$\Rightarrow v_1^2\mu_1 = v_2^2\mu_2$.

$f_1^2\cancel{\lambda_1^2}\mu_1 = f_2^2\cancel{\lambda_2^2}\mu_2$.

$\lambda_1 = \lambda_2$ as same fundamental.

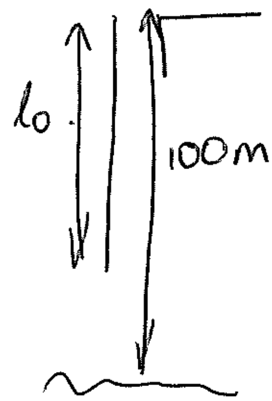
$\Rightarrow \frac{\mu_1}{\mu_2} = \frac{f_2^2}{f_1^2} = \frac{\cancel{\rho\pi}r_1^2}{\cancel{\rho\pi}r_2^2}$.

$\Rightarrow \frac{r_1}{r_2} = \frac{f_2}{f_1} = \frac{587}{440} = 1.3$.

or $= \frac{4}{3}$

b). i) $U = mgh$. as all engergy is PE.

ii). $A + l_0 = 100m$. $\Rightarrow A = 100 - l_0$.



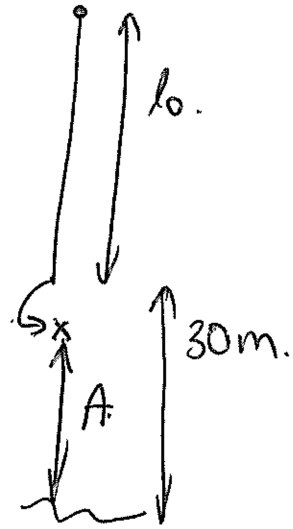
Use conservation of energy.

$$mgh = \frac{1}{2} k A^2.$$

$$\Rightarrow k = \frac{2mgh}{A^2} = \frac{2mgh}{(100 - l_0)^2}.$$

iii). $$k = \frac{2 \times 60.0 \times 9.80 \times 100.}{(100 - 70)^2.}$$

$$= 131 \; Nm^{-1}. \; (3 \text{ sig fig}).$$



iv). The mass of the bungee jumper will move the equilibrium position of the chord.

$$mg = kx \Rightarrow x = \frac{9.80 \times 60.0}{131.}$$

$$= 4.49m.$$

$\Rightarrow A = 25.5m. \Rightarrow 2A = 51.0m$ is the peak-to-peak magnitude.

v). $\cancel{\frac{1}{2}} m v_{max}^2 = \cancel{\frac{1}{2}} k A^2.$

$$v_{max} = \sqrt{\frac{kA^2}{m}} = \sqrt{\frac{131 \times 25.5^2}{60.0}}$$

$$= 37.7 \; m/s.$$

$$= 136 \; km/h.$$

vi). $f = f_0 \left(\frac{c \pm v_0}{c \mp v_s} \right)$ for maximum f need. v_0 max and approaching source.

$$\Rightarrow f = 600 \times \left(\frac{343 + 37.7}{343} \right) \quad v_s = 0.$$

$$= 666 \; Hz.$$

vii). Now the source is moving and observer stationary.

$$f = f_0\left(\frac{c}{c - V_s}\right)$$

Vs needs to be max and approaching observer

$$= 600 \times \left(\frac{343}{343 - 37.7}\right).$$

$$= 674 \text{ Hz}.$$

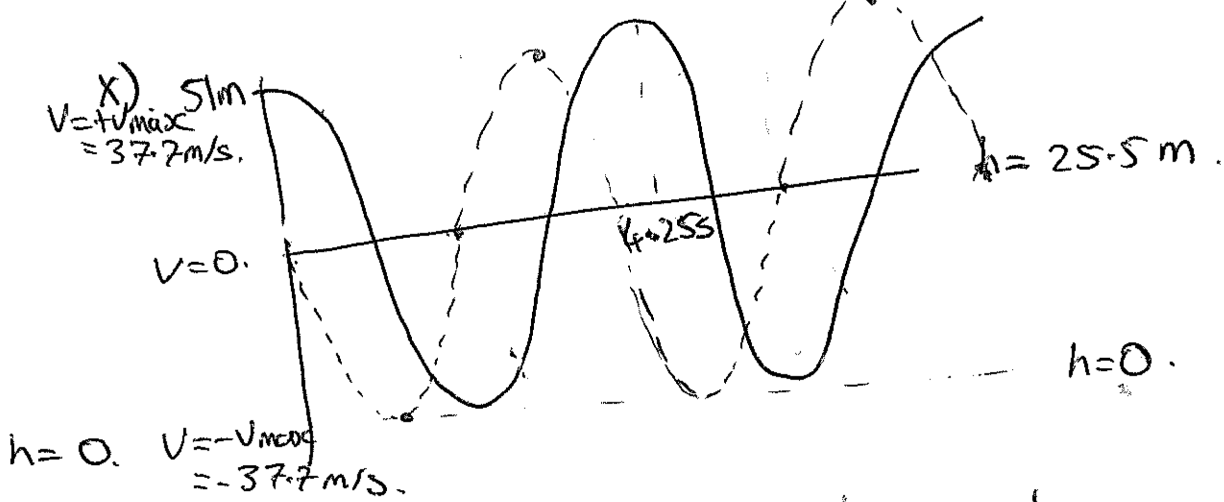
viii). moving together so $f_0 = 600$ Hz.

ix). $\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$

$$= 2\pi\sqrt{\frac{60.0}{131}}$$

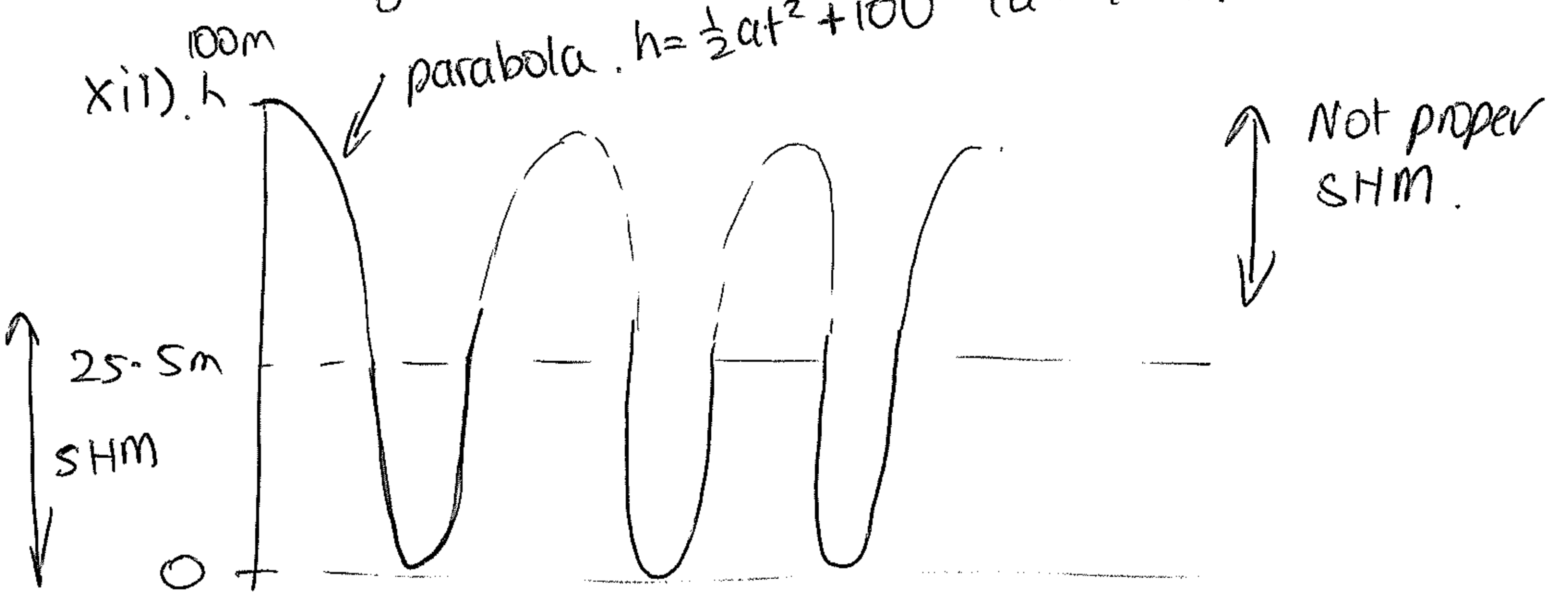
$$= 4.25 \text{ s}.$$

x).



This shows velocity and displacement once the bungee jumper enters SHM.

xi). No, when the chord is undergoing comprssion it will not always obey Hooke's law as it should, the amplitude of the oscillations are too large.

xii). 

xiii). Assume free fall $\Rightarrow$ $100 - 25.5\text{ m} = 74.5\text{ m}$.

$74.5 = \frac{1}{2} \times 9.80 \times t^2$. $\Rightarrow t = 3.90\text{ s}$.

Then it undergoes $\frac{1}{2}T = \frac{1}{2} \times 4.25 = 2.125\text{ s}$.

Then back to top $\Rightarrow$ 3.90 s.

$\Rightarrow T = 3.90 \times 2 + 2.125 = 9.93\text{ s}$.

Though energy will be lost and bungee jumper would not really return to original height giving a lower period.