① (a)

[20]

Mean Velocity = $\dfrac{\text{Change in Displacement}}{\text{Time Taken}}$

$= \dfrac{-12\text{cm} - (+12\text{cm})}{30 \text{ min}}$

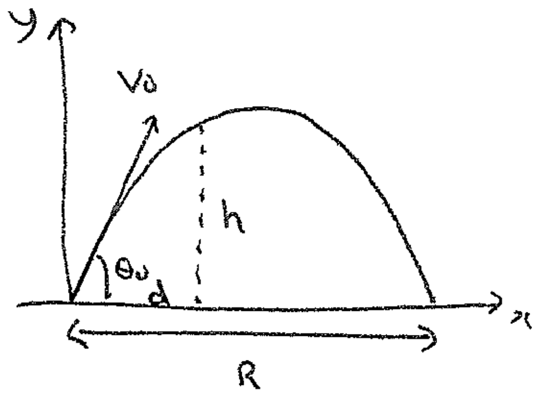
$= \dfrac{-24 \times 10^{-2}}{30 \times 60}$ m/s

$= -1{\cdot}33 \times 10^{-4}$ m/s

ie $\underline{\text{Speed is } 1{\cdot}3 \times 10^{-4} \text{ m/s} = 0{\cdot}13 \text{ mm/s to 2SF}}$

$\underline{\text{Direction is downwards}}$

(b)



Initially Velocity Components

$V_x = V_0 \cos\theta_0$

$V_y = V_0 \sin\theta_0$

(i) x-motion given by $x - x_0 = V_x t$ since no acceleration

ie $\underline{d = V_0 \cos\theta_0 t}$

(ii) In y-direction there is uniform acceleration $-g$

So, applying " $s = ut + \frac{1}{2}at^2$ " we have

$h = y - y_0 = V_y \sin\theta_0 t - \frac{1}{2} g t^2$

ie $\underline{h = V_0 \sin\theta_0 t - \frac{1}{2} g t^2}$

(iii) Range R given by the time t when returns to ground

ie When $h = 0 \Rightarrow V_0 \sin\theta_0 t - \frac{1}{2} g t^2 = 0$

So $t = 0$ [initial condition launch] or $\underline{t = 2 V_0 \sin\theta_0 / g}$

(iv) Substituting into the eqn for the x-distance

$$R = V_0 \cos\theta_0 t \quad \text{with} \quad t = 2 V_0 \sin\theta_0 / g$$
$$= 2 V_0^2 \sin\theta_0 \cos\theta_0 / g$$
$$\underline{R = V_0^2 \sin 2\theta_0 / g} \quad \text{where} \quad \sin 2\theta_0 = 2 \sin\theta_0 \cos\theta_0$$

(c) The range is max when R is at its greatest

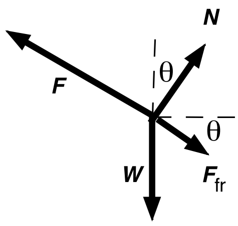
ie $\sin 2\theta_0 = 1$ or $2\theta_0 = 90^\circ \Rightarrow \theta_0 = 45^\circ$

Thus $R_{max} = \dfrac{V_0^2}{g} = \dfrac{9.50^2}{9.8} = 9.21\text{m}$

World Record is 8.95 m

Hence it is $9.21 - 8.95 = \underline{26\text{cm shorter}}$

Question 2



a) (no marks explicitly for the FBD)

i) No acceleration in the normal direction:

$$N - mg\cos\theta = 0 \quad \text{so} \quad N = mg\cos\theta.$$

In the direction up the plane, apply Newton's second law.

$$ma = F - F_{fr} - mg\sin\theta$$

$$ma = F - \mu_k N - mg\sin\theta$$

$$= F - \mu_k mg\cos\theta - mg\sin\theta$$

$$F = m(a + g(\mu_k\cos\theta + \sin\theta))$$

ii) To maximise *F* with respect to θ, take the derivative:

$$dF/d\theta = mg(\cos\theta - \mu_k\sin\theta) = 0$$

Rearrange $\cos\theta = \mu_k \sin\theta$

$$\theta = \tan^{-1}(1/\mu_k)$$

iii) As θ increases, the force up the plane is opposed by an increasing component of mg. However, the force up the plane is also opposed by friction and, as θ increases, the normal force and therefore the friction decrease.

b)

i) At maximal compression, the spring is neither compressing nor lengthening, so the relative velocity is zero.

ii) Let v be the speed at maximum compression.

Here, no non-conservative forces act, so mechanical energy is conserved.

$$U_i + K_i = U_f + K_f$$

$$0 + \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}kx^2 + \tfrac{1}{2}(m + m)v^2$$

$$kx^2 = mv_1^2 + mv_2^2 - 2mv^2 = m(v_1^2 + v_2^2 - 2v^2)$$

External forces in the x direction are negligible, so momentum in x direction is conserved.

Momentum before collision = momentum at maximum compression

$$mv_1 + mv_2 = 2mv$$

$$v = (v_1 + v_2)/2$$

Combine the two results

$$kx^2 = m(v_1^2 + v_2^2 - \tfrac{1}{2}(v_1 + v_2)^2)$$

$$= \tfrac{1}{2}m(2v_1^2 + 2v_2^2 - v_1^2 - 2v_1v_2 - v_2^2)$$

$$= \tfrac{1}{2}m(v_1^2 + v_2^2 - 2v_1v_2)$$

$$= \tfrac{1}{2}m(v_1 - v_2)^2$$

So, at maximum compression, $x = \sqrt{\dfrac{m}{2k}}(v_1 - v_2)$

Question 3

a) i) $F_g = GMm/R^2$.

ii) $a = R\omega^2 = R(2\pi/T)^2$.

iii) Newton's second law: $F = ma$

$$GMm/R^2 = mR(2\pi/T)^2$$

so $$GMT^2 = 4\pi^2R^3$$

so $$R = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \ldots = 42\ 100 \text{ km}$$

iv) Total mechanical energy of an orbit, with respect to zero at infinite distance, is $-GMm/2r$. On the Earth, and neglecting rotation of the Earth, the spacecraft has only potential energy $-GMm/R_e$.

Easy method

Work energy theorem: total work done by all forces = $\Delta K = ½\, m(R\omega)^2$

$= ½\,(145 \text{ kg})(42\ 100 \text{ km} * 2\pi/23.9 \text{ hr})^2$.

$= 685$ MJ or 6.85×10^8 J

Alternative method:

ΔE = total energy of the orbit – (only potential) energy on earth

But this energy includes the gravitational potential energy ΔU_g. So the total work done by all forces, including gravity, is

Work done = total energy of the orbit – (only potential) energy on earth – ΔU_g,

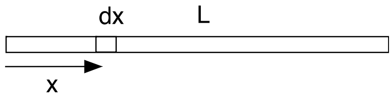
$= -GMm/2R - (-GMm/R_e) - (-GMm/R - (-GMm/R_e))$ *

$= GMm/2R = 685$ MJ or 6.85×10^8 J

(Comment only: In practice, however, the work done is by the rockets is much higher because much energy is wasted lifting and accelerating fuel and rocket stages.)

b) i) Moment of inertia $I = \int_{body} r^2 dm$ where *r* is the perpendicular distance from the axis of mass element dm.

ii)



Let the linear mass density by $\lambda = m/L$, so

$$dm = \lambda\, dx$$

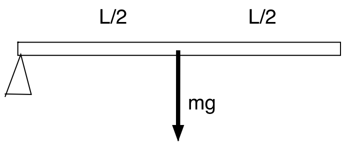
$$I = \int_{body} r^2 dm = \int_{x=0}^{x=L} x^2 dm$$

$$= \int_{x=0}^{x=L} x^2 \lambda dx$$

$$= \frac{\lambda}{3}\left[x^3\right]_{x=0}^{x=L}$$

$$= \frac{\lambda}{3} L^3,$$ but by definition $\lambda L = m$, so

$$I = \frac{1}{3} mL^2.$$



Newton's second law for rotation:

$\Sigma\tau = I\alpha$

Consider rotation about the point of the wedge. The weight provides the only torque, so

$mg.(L/2) = (mL^2/3)\alpha$

$\alpha = (3/2)(g/L)$

Acceleration at the end of the rod: $a = \alpha L = 3g/2$

1131 Solutions Question 4

a) i)

$$T_A = \frac{P_A V_A}{nR} = \frac{5 \times 1.01 \times 10^5 \times 16.46 \times 10^{-3}}{5 \times 8.314} = 200K$$

So as this is a diatomic molecule at 200K it has five degrees of freedom

ii) Three of these degrees of freedom are translational, corresponding to movement in the x, y and z direction.
Two of these degrees of freedom are rotational, corresponding to rotations about two axes.

iii)

$$\begin{aligned} &W = -\int PdV \\ &= -5 \times 1.01 \times 10^5 \times (32 - 16.46) \times 10^{-3} \\ &= -7848J \\ &= -7850J \text{ (3 sig fig)} \end{aligned}$$

iv) 0 J it's isothermal

v)

$$\begin{aligned} &\Delta E_{int} = Q + W \\ &Q = 0 \text{ it's adiabatic} \\ &\text{Two methods to calculate } \Delta E_{int} \\ &\text{method 1 :} \\ &\Delta E_{int} = \frac{f}{2} nR(T_A - T_D) \\ &T_D = \frac{P_d V_D}{nR} = 153.9K \\ &\Rightarrow \Delta E_{int} = \frac{5}{2} \times 5 \times 8.314 \times (200 - 153.9) = 4792J \\ &\text{method 2:} \\ &\text{As A} \rightarrow \text{B} \rightarrow \text{C} \rightarrow \text{D} \rightarrow \text{A is cyclic, total } \Delta E_{int} = 0 \\ &\Delta E_{int A \rightarrow B} = -7.850 + 27.5 = 19.65kJ \\ &\Delta E_{int C \rightarrow D} = -34.2 - 2 \times 1.01 \times 10^5 \times (31.67 - 80) \times 10^{-3} = -24.44kJ \\ &so \Delta E_{int D \rightarrow A} = -(-24.44 + 19.65) = 4.79kJ \\ &\text{As } \Delta E_{int} = W \\ &\Rightarrow W = 4.79kJ \end{aligned}$$

Or it is possible to calculate it directly by integrating PdV using PV^1.4 = constant, but this is harder.

b) i) $$P = kA\frac{\Delta T}{\Delta x}$$
$$= 50.2 \times 100.0 \times 10^{-4} \times \frac{30.00}{0.020}$$
$$= 753W$$

ii) $Q = Pt = mL$
$m = \frac{Pt}{L} = \frac{753 \times 60}{3.33 \times 10^5}$
$= 0.136kg$

iii)

Energy needed to melt ice $= m_i L_{fus}$
$= 0.500 \times 3.33 \times 10^5$
$= 166500J$
energy that water can lose as it goes to $0^oC = m_w c_w \Delta T$
$= 1.000 \times 4186 \times 30 = 125580J$
energy that steel can loose as it goes to $0^oC = m_{st} c_{st} \Delta T$
$= 1.56 \times 456 \times 15 = 10670J$
As the water at the top and steel combined can not lose enough energy to melt all the ice the system has a final temperature of 0^oC.

iv)

$Q = \int_0^t P(t)dt = m_w c_w (T_h - T) + m_{st} c_{st} (\frac{T_h}{2} - \frac{T}{2})$
Note that P and T are functions of time, $P(t)$ is the power through the steel at time t and $T(t)$ is the temperature of the water at time t.
Differentiate both sides with respect to t.
$\Rightarrow P = -m_w c_w \frac{dT}{dt} - \frac{m_{st} c_{st}}{2} \frac{dT}{dt}$
Now: $P = \frac{kAT}{d}$
Substitute this into the differential equation.
$\Rightarrow \frac{kAT}{d} = -m_w c_w \frac{dT}{dt} - \frac{m_{st} c_{st}}{2} \frac{dT}{dt}$
Rearrange putting the T and dT terms on the left hand side, then integrate between t=0 and t=60s T_f is the temperature at 60s.
$\Rightarrow \int_{T_h}^{T_f} \frac{dT}{T} = \int_0^{60} \frac{-kA}{d(m_w c_w + m_{st} c_{st}/2)} dt$
$\ln(\frac{T_f}{T_h}) = \frac{-60kA}{d(m_w c_w + m_{st} c_{st}/2)}$
$\Rightarrow T_f = T_h e^{\frac{-60kA}{d(m_w c_w + m_{st} c_{st}/2)}}$
$\frac{-60kA}{d(m_w c_w + m_{st} c_{st}/2)} = \frac{50.2 \times 100 \times 10^{-4} \times 60}{0.02 \times (4186 \times 1 + 1.56 \times 486/2)} = 0.331595$
$T_f = 30.0 \times e^{-0.331595} = 21.533^oC$
$Q = 1 \times 4186 \times (30 - 21.533) + \frac{1.56 \times 456}{2} \times (30.0 - 21.533)$
$= 38452.9J$
$= 38.5kJ$ (3 sig. fig.)

Question 5 T1 2014

a)

For planet GJ832c:

$$g = \frac{GM_{\text{planet}}}{R^2} = 13.22\frac{\text{m}}{\text{s}^2}$$

Simple pendulum:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Find L:

$$T^2 = \frac{4\pi^2 L}{g}$$

$$L = \frac{T^2 g}{4\pi^2} = 33.49 \text{ m} \to 33.5 \text{ m (3 sig. figs)}$$

b)

1st harmonic $\to \lambda = 2L$

$$v = \sqrt{\frac{T}{\mu}} = f\lambda \text{ and } T = mg$$

From a): $L = 33.5$ m

Solve for μ :

$$\sqrt{\frac{T}{\mu}} = 2Lf$$

$$\frac{T}{\mu} = 4L^2f^2$$

$$\mu = \frac{T}{4L^2f^2} = \frac{mg}{4L^2f^2} \; (L = 33.5 \text{ m}, g = 13.22 \; \frac{\text{m}}{\text{s}^2}, f = 4.40 \text{ Hz}, m = 66 \text{ kg})$$

$$\mu = 0.0100 \; \frac{\text{kg}}{\text{m}} \text{ (3 sig. figs)}$$

c)

After rope splits: $\mu^{'} = \frac{2}{3}\mu$

1st harmonic: $f_{\mathrm{n}} = \frac{\mathrm{n}}{2L}\sqrt{\frac{T}{\mu}} \rightarrow f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ and $T = mg$

$f_1' = \frac{1}{2L}\sqrt{\frac{mg \cdot 3}{2\mu}}$, $\mu = 0.0100\frac{\mathrm{kg}}{\mathrm{m}}$ from part b)

$f_1' = \frac{1}{2(33.5)}\sqrt{\frac{66 \cdot 13.22 \cdot 3}{2 \cdot 0.0100}}$ Hz

$f_1' = 5.40$ Hz

d)

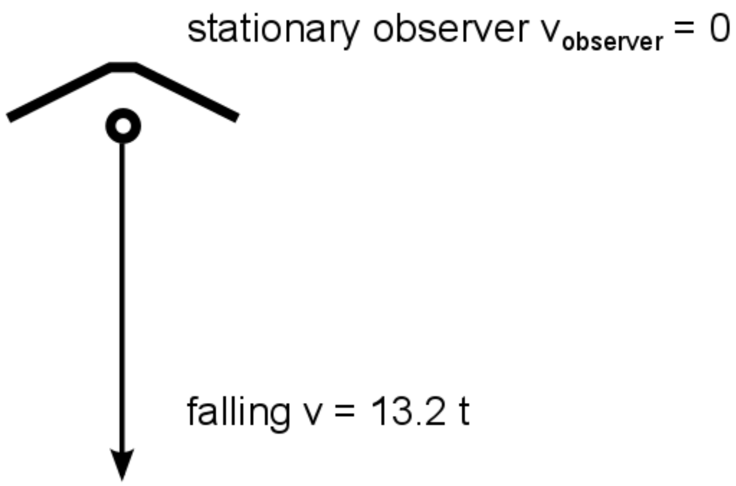
Speed at time t: $v = gt = 13.2t$

Doppler: $v_{\mathrm{source}} = 13.2t$, observer stationary

$f' = \left(\frac{v - v_O}{v + v_S}\right) f$, $f = 440.0$ Hz, $v = 380.0\frac{\mathrm{m}}{\mathrm{s}}$

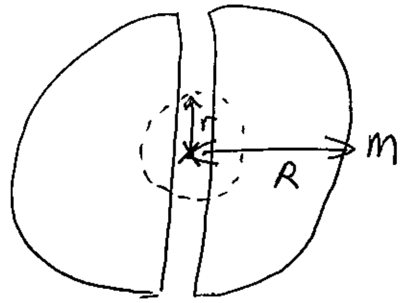
$f' = \left(\frac{380 - 0}{380 + 13.2t}\right) f$

$f' = \frac{167200}{380 + 13.2t}$ Hz



e)

Take center of the Earth as $r=0$.
As the particle undergoes SHM it will oscillate between $+R$ and $-R$.



We need to work out force acting on the particle. The force is the gravitational force so:

$$F = \frac{G m_1 m_2}{r^2}$$

However, only the mass ~~insides~~ inside the radius 'r' will contribute so:

$m_1 = m_0$ = mass of object.

$$m_2 = \rho V = \frac{M}{\cancel{\frac{4}{3}\pi} R^3} \cdot \cancel{\frac{4}{3}\pi} r^3 = \frac{M r^3}{R^3}.$$

$$\Rightarrow \quad F = \frac{G m_0 M r^3}{r^2 R^3} = \frac{G m_0 M r}{R^3} = m_0 a$$

Now for SHM $a = -\omega^2 r$ where $-$ve sign shows acceleration/force and displacement are in the opposite direction to each other so

$$|a| = \omega^2 r = \frac{G M r}{R^3}.$$

$$\Rightarrow \quad \omega^2 = \frac{GM}{R^3}$$

$$\omega = \sqrt{\frac{GM}{R^3}}$$