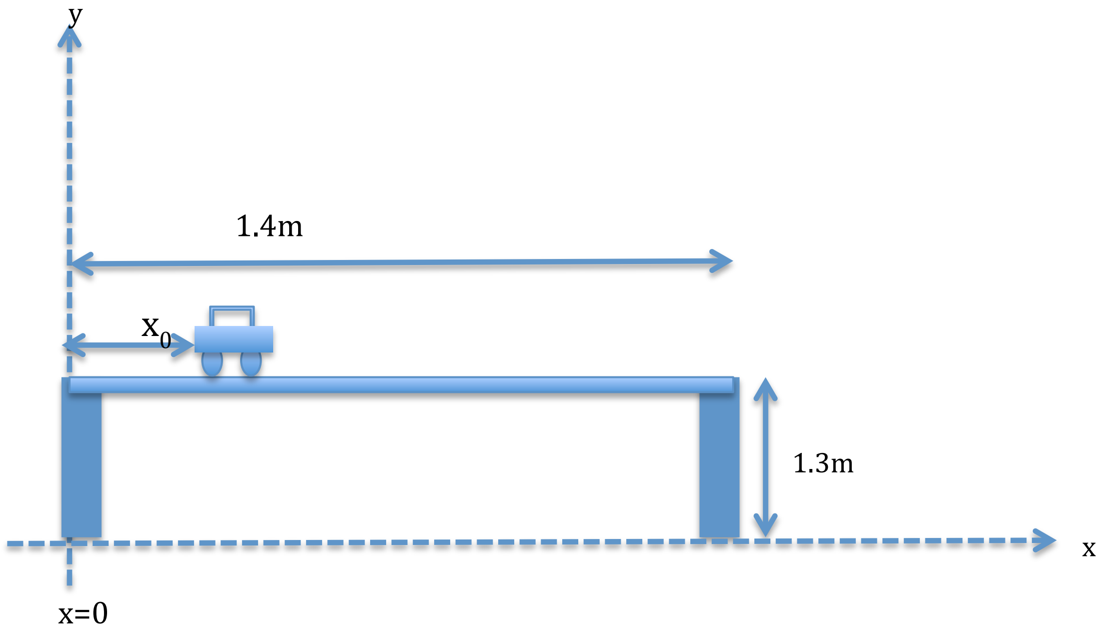
Question 1 (20 marks)



A radio controlled toy car moves along a bench according to the equation

$$\mathbf{x} = 0.2 + 0.5t^2$$

where x is distance in metres from the left-hand edge of the bench, and t is time in seconds. The top of the bench is 1.4 m long, and the bench is 1.3 metres high. Assume that the positive x-direction is along the positive x-axis as shown in the diagram.

(a) Derive expressions for the

(i) instantaneous velocity, and

$$\mathbf{v} = \frac{d\mathbf{x}}{dt} = 1.0t \ ms^{-1}$$

(ii) instantaneous acceleration .

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 1.0 \ ms^{-2}$$

(b) At time t = 0, find the

(i)displacement,

x = 0.20 m

(ii)velocity, and

$$\mathbf{v} = 0 \ ms^{-1}$$

(iii) acceleration.

$\mathbf{a} = 1.0\ ms^{-2}$ a = 1.0 î ms^{-2}

(c) At what time does the car fall off the right-hand side of the bench? [*Hint: how far does the car have to travel from its starting position to reach the right-hand side of the bench?*]

The car starts at position x= 0.2 on the1.4m benchtop. Hance, it travels 1.2m before it falls.

The time taken is found from

$$\begin{aligned}
&\mathbf{x} = 0.2 + 0.5t^2 \\
&1.4 = 0.2 + 0.5t^2 \\
&t^2 = \tfrac{1.4-0.2}{0.5} \\
&t^2 = 2.4 \\
&t = 1.5s
\end{aligned}$$

The answer should be given to 2 significant figures. As long as the answer is correct, there is no need for the students to state explicitly that there is a positive and negative solution.

(d) Calculate the velocity of the car as it just falls off the bench.

The time taken is 1.5 s, so the velocity at this time is given by

$$\begin{aligned}
&\mathbf{v} = 1.0t\ \mathrm{ms}^{-1} \\
&= 1.5\hat{\imath}\ \mathrm{ms}^{-1}
\end{aligned}$$

(e) How long does it take the car to hit the ground?

The problem now becomes one of a projectile with an initial velocity in the x-dir only, of 1.5 ms^{-1}. The car falls through a height of 1.3 m, before hitting the ground.

The time is found from the y-dir, where v_y=0, y= 1.3 m, and a=g.

$$s = ut + \frac{1}{2}at^2$$

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$y = \frac{1}{2}gt^2$$

$$t^2 = \frac{2y}{g}$$

$$t = \sqrt{\frac{2y}{g}}$$

$$= \sqrt{\frac{2(1.3)}{9.8}}$$

$$= 0.52 \ s$$

(f) How far from the right-hand edge of the bench does the car hit the ground?

The initial velocity in the x-dir is 1.5 ms^{-1}, the time of flight is 0.5 s, and the acceleration is 0. So,

$$s = ut + \tfrac{1}{2}at^2$$
$$x = u_x t$$
$$x = 1.5(0.5) = 0.75\text{m}$$

(g) Find the velocity of the car just before it hits the ground. Remember to give both a magnitude and a direction.

The velocity in the x-dir is just 1.5 ms^{-1}, as there is no acceleration in the x-direction

In the y-dir, the velocity just be for the ball hits the ground is found from

(u_y=0, a_y=g, t= 0.5 s)

$$v = u + at$$

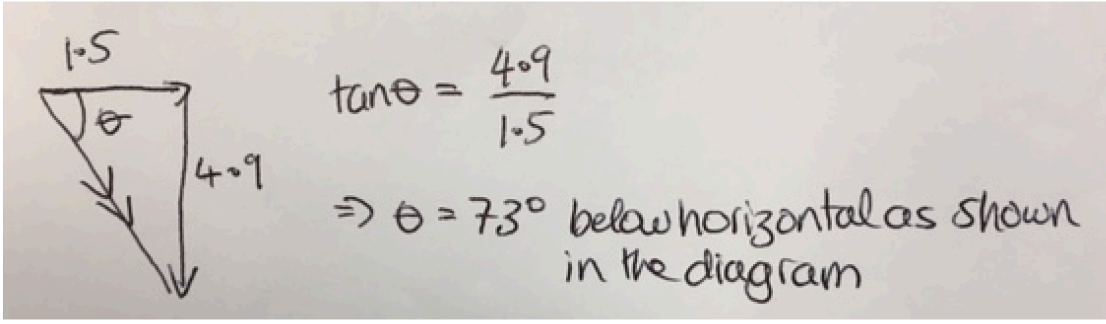
$$= 0 + (9.8)(0.5)$$

$$= 4.9 \ ms^{-1}$$

These need to be added together using pythagoras's theorem to get the magnitude:

$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{1.5^2 + 4.9^2}$$
$$= 5.1 \text{ ms}^{-1}$$

The angle then needs to be calculated.



(h) Calculate the change in the total energy of the car from when it leaves the bench to just before it hits the ground. Neglect air resistance. Explain your answer in 1 clear sentence.

The change in ***total*** *energy of the car will be zero because the only the conservative force of gravity acts on the car as it falls, converting the PE to KE.*

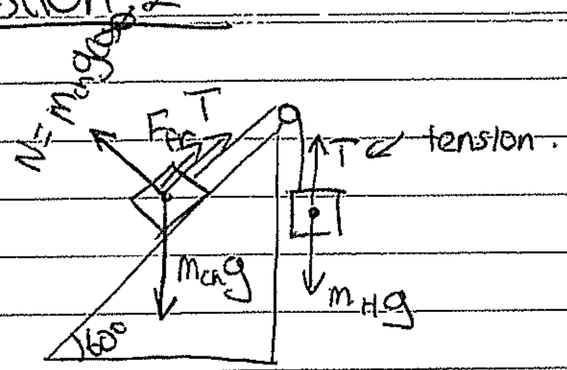
Alternatively as it asks you to calculate you can use:

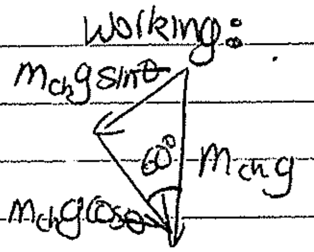
$\frac{1}{2}mv_i^2 + mgh - \frac{1}{2}mv_f^2 = m(\frac{1}{2} \times 1.5^2 + 9.8 \times 1.3 - \frac{1}{2} \times 5.1^2)$
$= 6.88 \times 10^{-3} J$

This is a small number. You would expect it to be zero. This is not zero due to rounding errors.

Question 2

a)





b) As Chewie is at rest the forces parallel to the plane are balanced ⇒

$T - m_{ch} g\sin 60 + F_{fr} = 0.$

$T = m_H g = 8.5 \times 75 = 637.5.$

$m_{ch} g\sin 60 = 120 \times 8.5 \times \sin 60 = 883.34.$

⇒ gravitational force down slope is greater than T up slope, friction opposes this to give net 0 force ⇒ friction is up the slope.

c) $F_{fr} = |637.5 - 883.34| = 250$ N up the slope (2 sig fig).

d). $F = m_{ch} g\sin\theta - \mu_k N = m_{ch} g(\sin\theta - \mu_k\cos\theta)$

$= 120 \times 8.5(\sin 60 - 0.15\cos 60)$

$= 806.84$ N.

$a = 6.7237\ ms^{-1}.$

~~$s = \frac{1}{2}at^2$~~ $v^2 = u^2 + 2as.$

$v^2 = 2 \times 6.7237 \times 110.$

$v = 38.46 = 38$ m/s (2 sig fig).

e) $Ft = \Delta p = mv$ $F_{tot} = F_{shield} - mg = 1200 - 75 \times 8.5 = 562.5$ N up

~~then~~ $t = \frac{mv}{F} = \frac{75 \times 41.02}{562.5}$

$= 5.47$ s

$= 5.5$ s (2 sig fig)

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2.$

$v^2 = 2 \times 8.5 \times 99$

$v = 41.02$ m/s

f) m_s, M_i, R_i

To escape ice planet need enough K to get infinitely far away. (can have zero velocity at ∞).

$$\Rightarrow \frac{1}{2} m_s V_e^2 = U_f - U_i = 0 + \frac{G m_s M_i}{R}$$

change in K = $K_i - K_f$.

$$\Rightarrow \frac{1}{2} V_e^2 = \frac{G M_i}{R}$$

$$V_e = \sqrt{\frac{2GM_i}{R}}$$

Part (g) is for PHYS1131 only

g). $F = \frac{G m_s M_i}{R^2} = m_s \frac{V^2}{R}$

$$\Rightarrow K = \frac{1}{2} m_s V^2 = \frac{1}{2}\left(\frac{G m_s M_i}{R^2}\right) R = \frac{G m_s M_i}{2R}$$

$$U = -\frac{G m_s M_i}{R}$$

$$\Rightarrow E = K + U = \frac{G m_s M_i}{2R} - \frac{G m_s M_i}{R} = -\frac{G m_s M_i}{2R}$$

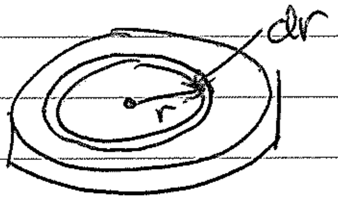
Question 3

a) $\alpha = \frac{\omega}{t}$.

b) $\theta_{TOT} = \frac{1}{2}\alpha t^2 = \frac{1}{2}\frac{\omega}{t}t^2 = \frac{1}{2}\omega t$.

c). $I = \int r^2 \, dm$

Need an expression for dm in terms of dr.



Consider ring with width dr this has mass. $dm = 2\pi r \rho \, dr$ where $\rho = M/\pi R^2$.

$$\Rightarrow I = \int_0^R r^2 . 2\pi r \left(\frac{M}{\pi R^2}\right) dr.$$

$$= \frac{2M}{R^2}\int_0^R r^3 \, dr.$$

$$= \frac{2M}{R^2}\left[\frac{r^4}{4}\right]_0^R$$

$$= \frac{2M}{R^2}\frac{R^4}{4}$$

$$= \frac{\cancel{2}MR^3}{2}.$$

d) $\tau = I\alpha$ (Newton's 2^{nd} law for rotation).

$$= \frac{MR^2}{2} \times \frac{\omega}{t}.$$

e) Net force = zero as center of mass is stationary.

f) $W = \int_0^{\theta_{TOT}} \tau_0 \, d\theta$

$= \int_0^{\theta_{TOT}} \frac{MR^2}{2} \frac{\omega}{t} \, d\theta$

$= \frac{MR^2}{2} \frac{\omega}{t} \theta_{TOT}$

$= \frac{MR^2 \omega}{2t} \frac{1}{2} \omega t$

$= \frac{MR^2 \omega^2}{4}$

Question 4

a) i) $W = -P\Delta V$. (for paths with constant P).

$W_{ab} = -(3\times10^5)(2\times10^{-3} - 6\times10^{-3}) = +1200J$.

$W_{bc} = W_{da} = 0$.

$W_{cd} = -(6\times10^5)(6\times10^{-3} - 2\times10^{-3}) = -2400J$.

$\Rightarrow W_{total} = -1200\,J$.

ii) $PV = nRT \Rightarrow T = \frac{PV}{nR}$.

$$T_a = \frac{3\times10^5\times6\times10^{-3}}{1\times8.314} = 216.5K.$$

$$T_b = \frac{3\times10^5\times2\times10^{-3}}{8.314} = 72.2K.$$

$$T_c = \frac{6\times10^5\times2\times10^{-3}}{8.314} = 144.3K.$$

$$T_d = \frac{6\times10^5\times6\times10^{-3}}{8.314} = 433.0K.$$

iii) $b \to c$ is constant volume.

$$Q = nC_v\Delta T = 1.00\times\frac{3}{2}\times8.314\times(144.3-72.2)$$
$$= 899\,J.$$

$c \to d$ is constant pressure.

$$Q = nC_p\Delta T = 1\times\frac{5}{2}\times8.314\times(433-144.3)$$
$$= 6000J.$$

iv) $\text{Efficiency} = \frac{\text{work out}}{\text{energy in}} = \frac{2400}{6899} = 34.8\%$.

The question is a bit ambiguous. If we take "work out" for the entire cycle it is 1200 J (part (i)) $\Rightarrow$

$$\text{efficiency} = \frac{1200}{6899} = 17.4\%.$$

b) Total heat transfer = 0.

$Q_{coffee} + Q_{cup} + Q_{sphere} = 0.$

$m_c c_{w}(T_f - T_{ic}) + m_{cup} C_s (T_f - T_{icup}) + m_{sphere} C_s (T_f - T_{isphere}) = 0$

$T_f (m_{coffee} C_w + m_{cup} C_s + m_{sphere} C_s) = T_{ic}(m_{coffee} C_w + m_{cup} C_s)$
$+ T_{isphere}(m_{sphere} C_s).$

$\Rightarrow T_f = \dfrac{25 \times (0.3 \times 4186 + 0.050 \times 490) + 1400 \times (0.10 \times 490)}{0.300 \times 4186 + 0.050 \times 490 + 0.10 \times 490}.$

$= 75.7\ °C$ or $348.8\ K.$

$= 76°C$ or $350\ K$ (2 sig fig).

c). i) $\Delta V = \beta V_0 \Delta T.$

$= 0.0002 \times 5 \times V_0 = 0.001 V_0 = 0.1\%.$

ii) $3688 \times 0.001 = 3.688\ m$

$= 3.7\ m$ rise.

(a) 1131 only.

$PV^{\gamma} =$ constant for adiabatic process.

$\Rightarrow P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$.

$\gamma = \frac{C_p}{C_v}$ $\quad f = 7 \Rightarrow \frac{C_p}{C_v} = \frac{9}{7} = 1.285$.

$(0.1)(1)^{1.285} = (100) V_f^{1.285}$.

$\Rightarrow V_f = 4.63 \times 10^{-3}$ m.

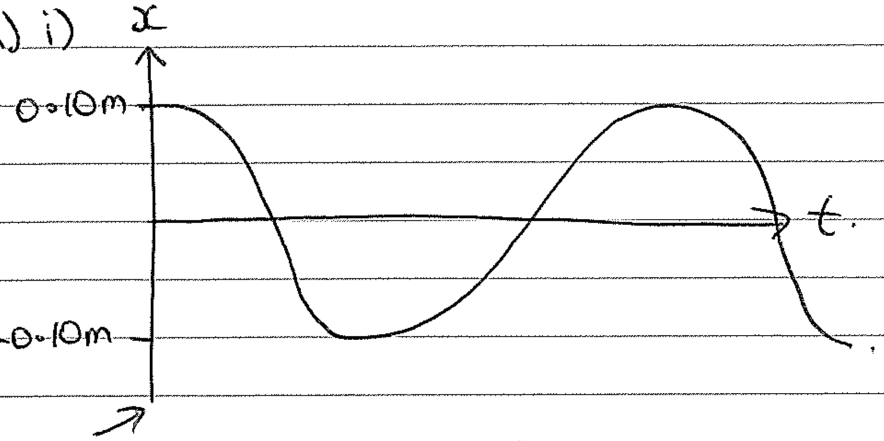
$PV = nRT$

$T = \frac{PV}{nR} = \frac{100 \times 1.01 \times 105 \times 4.63 \times 10^{-3}}{2 \times 8.314}$

$= 2812$ K.

Question 5.

a) i)



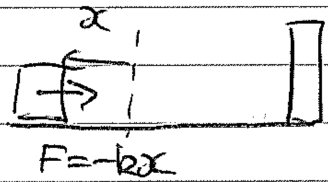
distance from equilibrium.

ii) ~~$x = A\sin(\omega t)$~~ $x = A\cos(\omega t)$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\Rightarrow x = A\cos\left(\sqrt{\frac{k}{m}}t\right).$$

iii)



$F = ma = -m\omega^2 x = -kx$

According to Hooke's law

$F = -kx = ma$ (Newton's 2nd law).

as $x = A\cos\omega t$.

$$\frac{dx}{dt} = -A\omega\sin\omega t.$$

$$a = \frac{d^2x}{dt^2} = -A\omega^2\cos\omega t = -\omega^2 x.$$

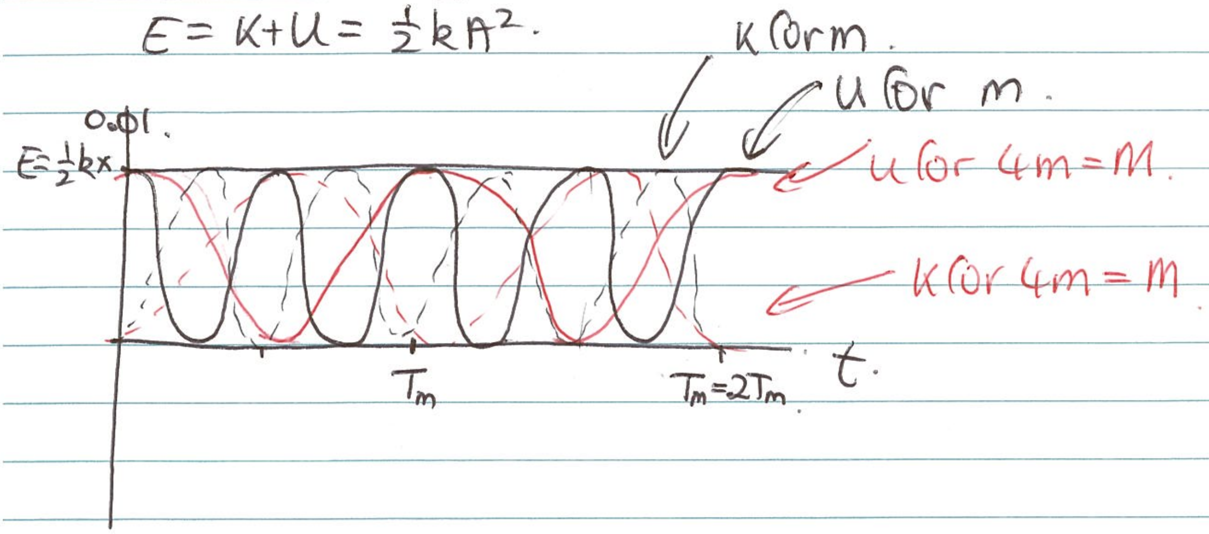
$$\Rightarrow F = -kx = -m\omega^2 x.$$

$$\Rightarrow k = m\omega^2.$$

$$\Rightarrow \omega = \sqrt{\frac{k}{m}}.$$

iv) $\omega = \sqrt{\frac{k}{4m}} = \frac{1}{2}\sqrt{\frac{k}{m}}.$

v) $K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\sin^2\omega t$.
$U = \frac{1}{2}kx^2 = \frac{1}{2}k.A^2\cos^2\omega t$.
$E = K + U = \frac{1}{2}kA^2$.



The period for the mass 4m is double that for m.

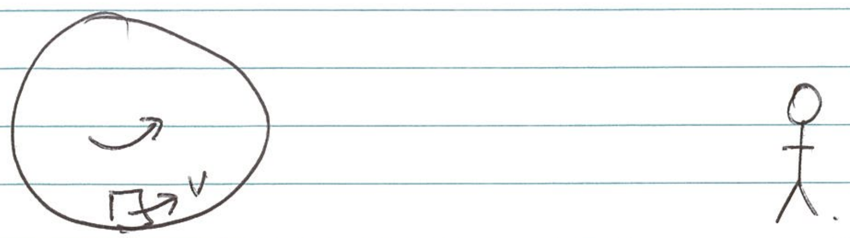
vi) No the mass would never stop, it is not loosing energy. Friction would make it stop as it would then loose energy as work overcoming friction.

b) i) $f' = f\left(\frac{V_a}{V_a - v}\right)$

Observer is stationary
$\Rightarrow V_o = 0$.

$v = \omega r \Rightarrow f' = \frac{f V_a}{V_a - \omega r}$

ii) At this point the bell is moving towards the observer with maximum velocity.



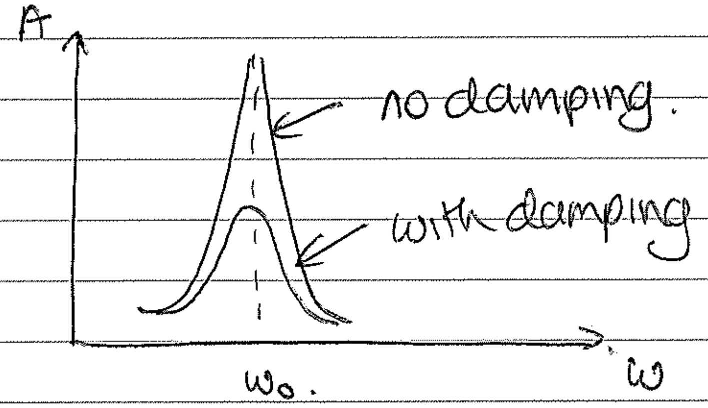
ii) $f'_{min} = \dfrac{f v_a}{V_a + w r}$.

Part (d) is for PHYS1131 students only

d) i) Resonance occurs when the driving frequency of a force is the same as the natural frequency of the system. This leads to large amplitudes.

ii) A mother pushing a swing is an example of resonance, she pushes the child with a force at the same frequency the child is swinnging, this leads to greater amplitudes.

iii).



When $\omega = \omega_0$ $A \uparrow \infty$ if there is no damping. Damping will reduce the amplitude.

Part (c) is for all students

c) i) 3 loops $\Rightarrow$ $\lambda = \frac{2}{3} \times 1.25 = 0.8\dot{3}$ m

$$V = f\lambda = \sqrt{\frac{T}{\mu}} \Rightarrow \mu = \frac{T}{f^2\lambda^2} = \frac{mg}{f^2\lambda^2}$$

$$= \frac{0.250 \times 9.80}{77^2 \times 0.833^2}$$

$$= 0.000595 \text{ kg/m}$$

$$= 0.60 \text{ g/m} \quad (2 \text{ sig fig})$$

ii) Need 4 loops $\Rightarrow \lambda = 0.625$ m

$$\Rightarrow f = \frac{1}{0.625} \times 77 \times 0.8\dot{3} = 103 \text{ Hz}$$

$$= 100 \text{ Hz} \ (2 \text{ sig fig})$$