.Waves and Oscillations

Lecture 10 – Traveling waves

Textbook reference: 16.1-16.4

Tacoma Narrows bridge collapse 1940. You are doomed to see this in every engineering course.

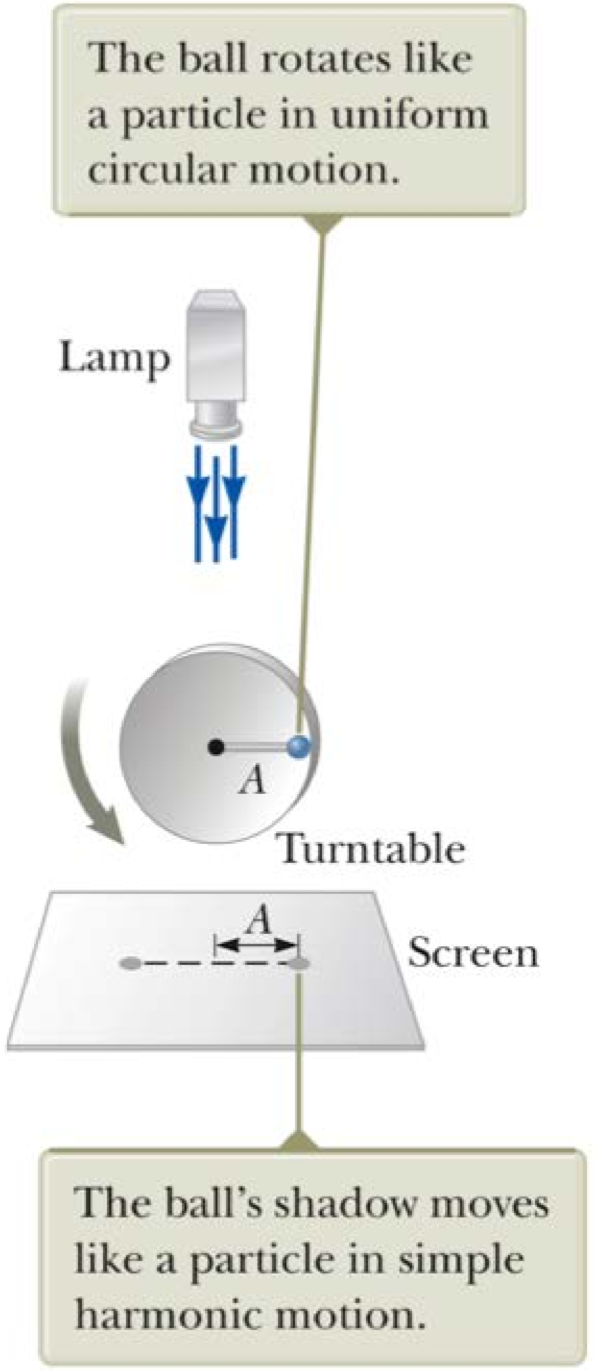
Last lecture...

- **Simple harmonic motion** is the projection of uniform circular motion on a diameter of the circle in which the circular motion occurs.

– Motion of the ball's shadow:

$$x(t) = A\cos(\omega t + \phi)$$

Angular frequency of the SHM
= angular speed of the circular motion



Last lecture...

- The motion of a **simple pendulum** is also SHM, if the displacement angle is small (θ << 1 radian).

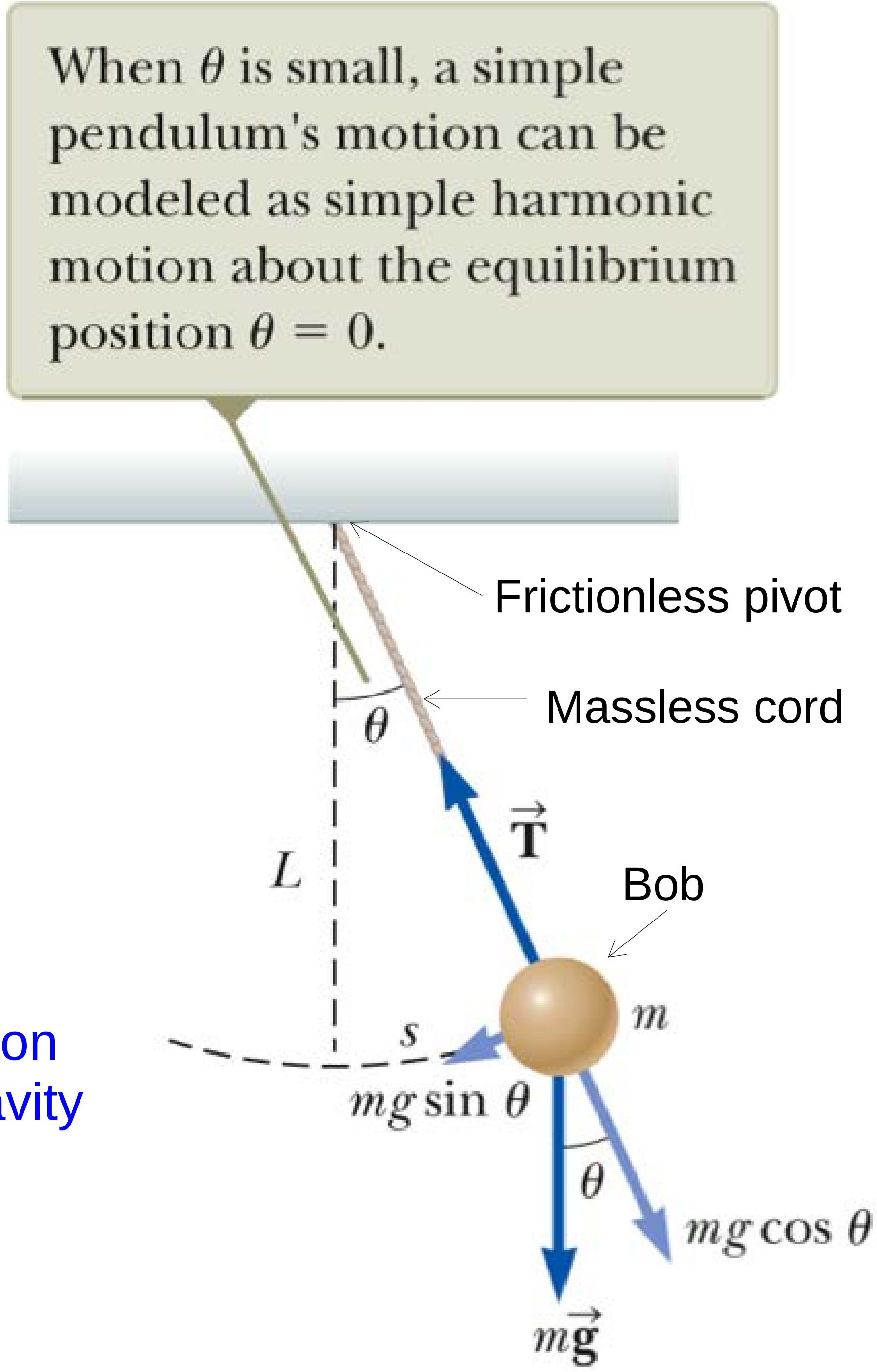
– Period:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

Length of cord

Acceleration due to gravity

Simple pendulum = a bob at the end of a massless cord suspended from a frictionless pivot.



Last lecture...

- A **physical pendulum** swings a pivot O that is not at the centre-of-mass (CM) of the object.

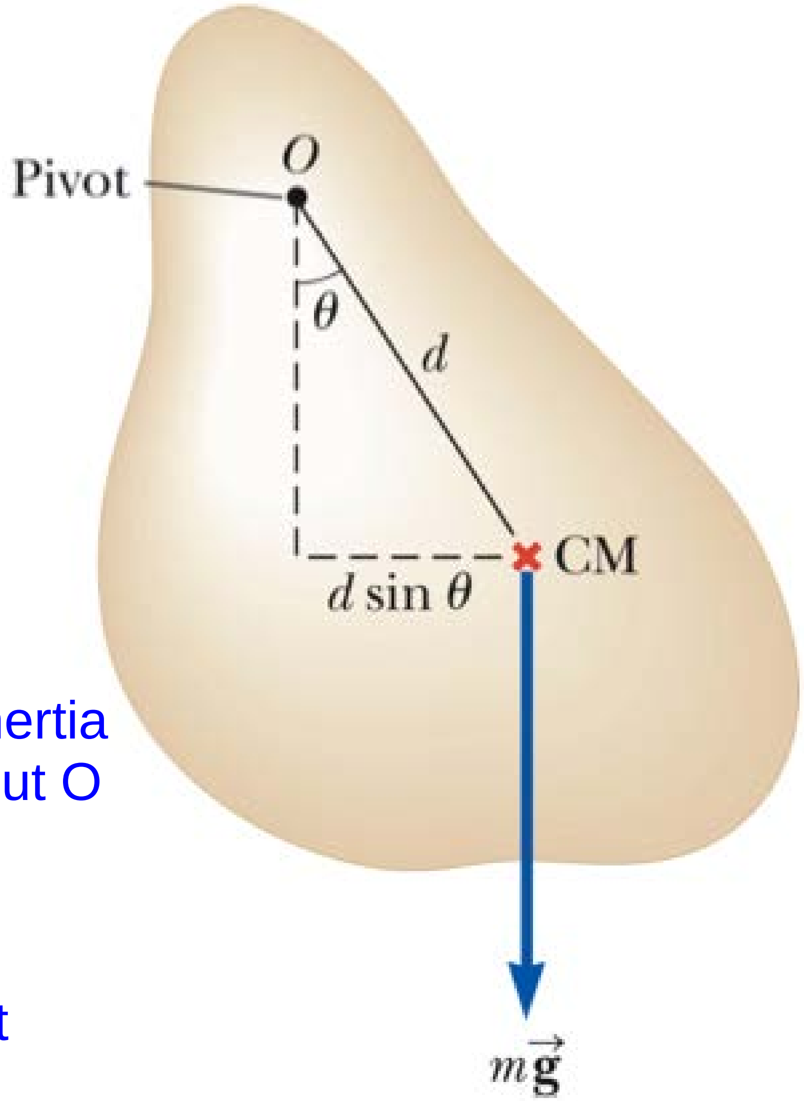
–Its motion is also SHM (for θ << 1 radian).

$$T = 2\pi\sqrt{\frac{I}{dmg}}$$

Moment of inertia of object about O

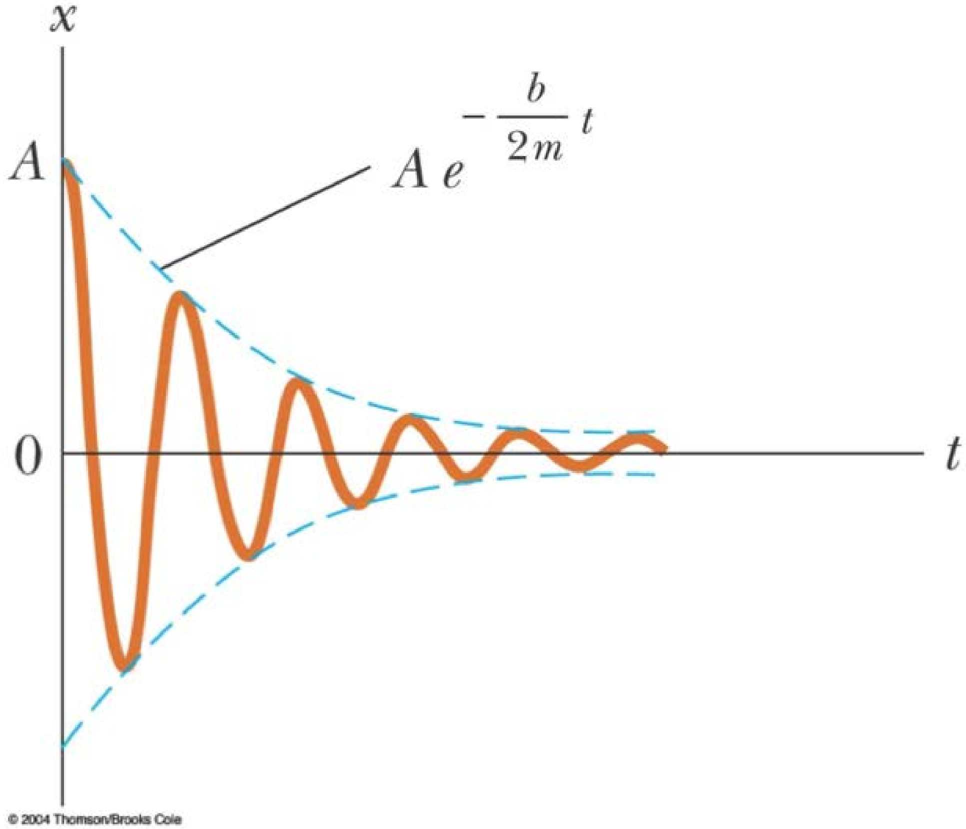
Distance from O to CM

Mass of object

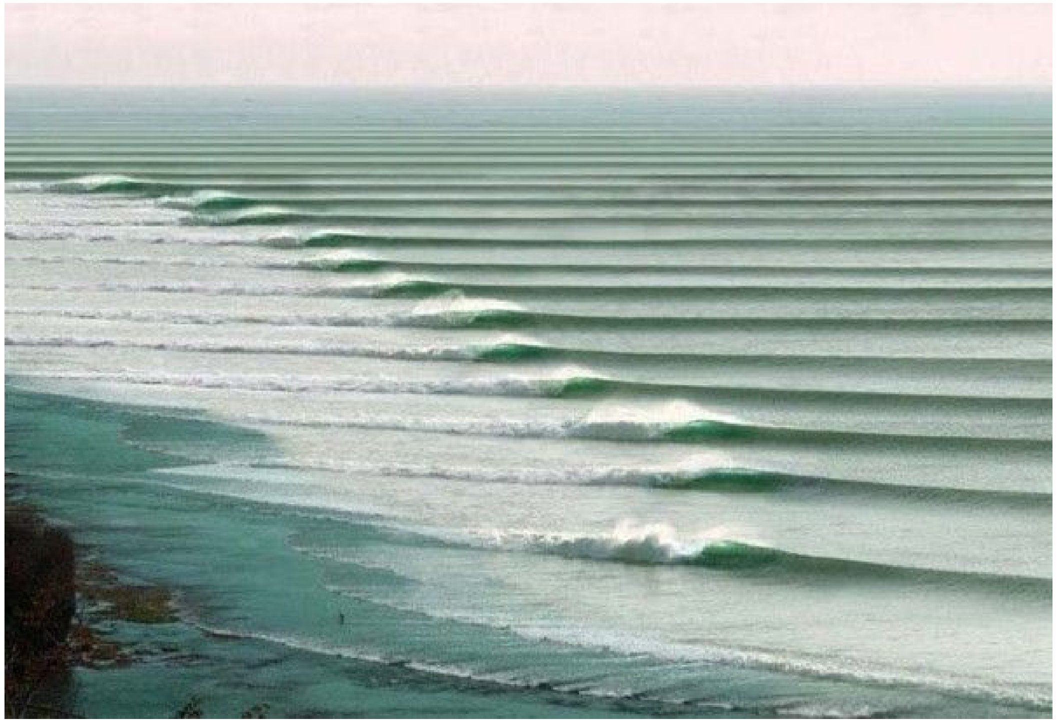


This Lecture

Damped Oscillations



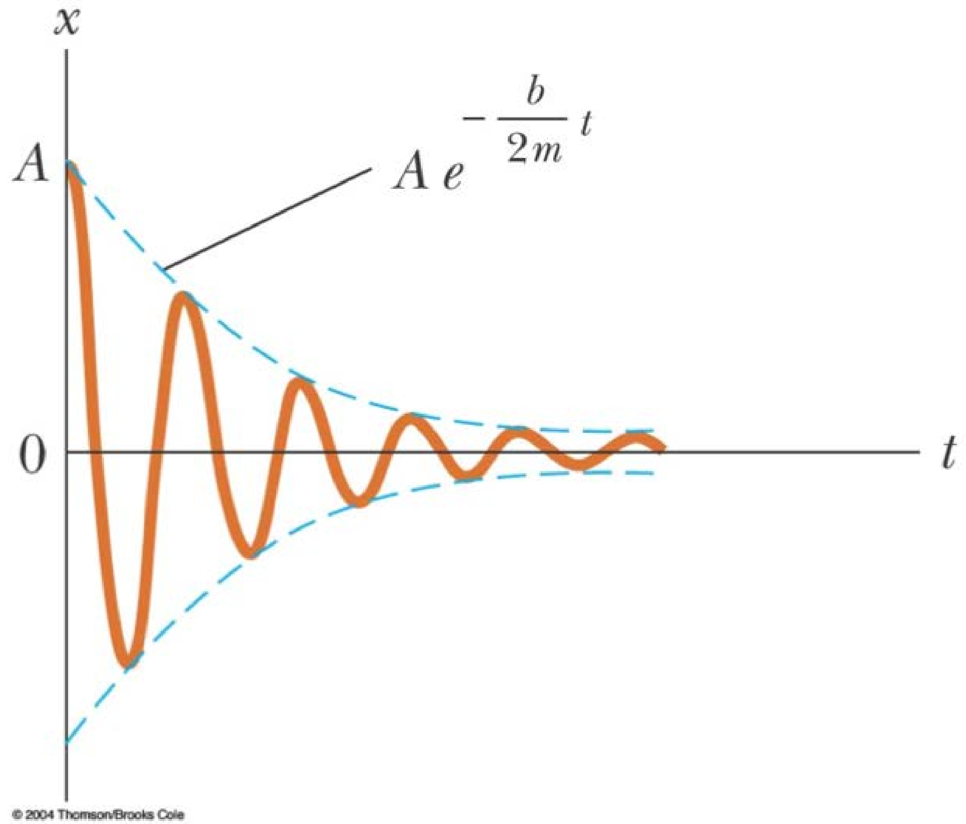
Travelling waves



Damped oscillations...

- In realistic systems, **non-conservative forces** such as friction and air resistance are usually present.

- In such cases, the mechanical energy of the system diminishes in time, and the motion of the oscillator is said to be **damped**.



$$x = Ae^{(-b/2m)t}\cos(\omega t + \phi)$$

Damped oscillations>Example...

- **A spring in a viscous fluid**. The fluid opposes the motion of the block by exerting a "retarding force" proportional to its velocity:

Damping constant

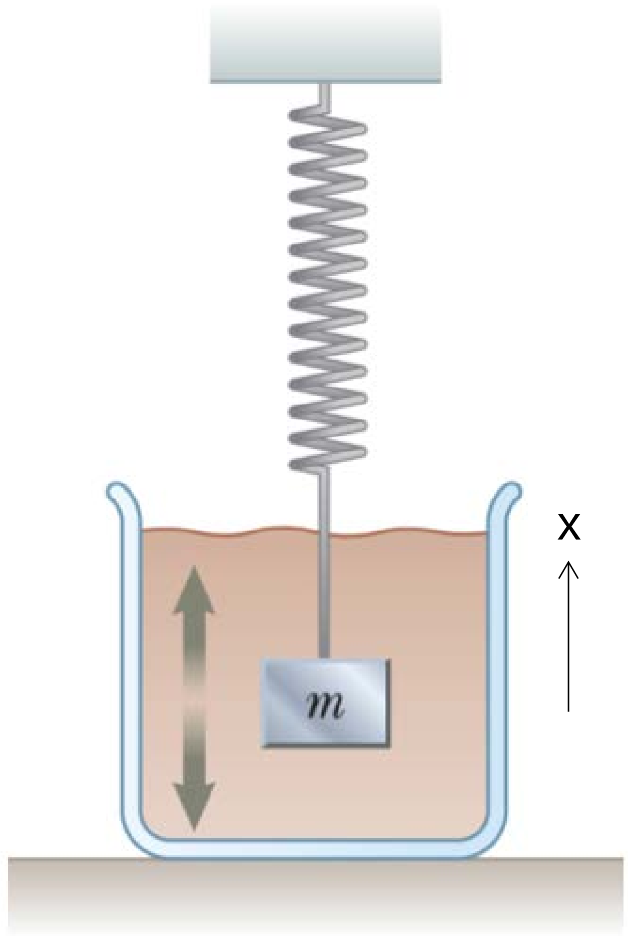
$$\vec{F} = -b\vec{v}$$

  - The **total force** on the block is:

$$F_{\text{total}} = -bv_x - kx$$

From fluid

From spring



- **Equation of motion** for the block:

$$ma_x = -bv_x - kx$$

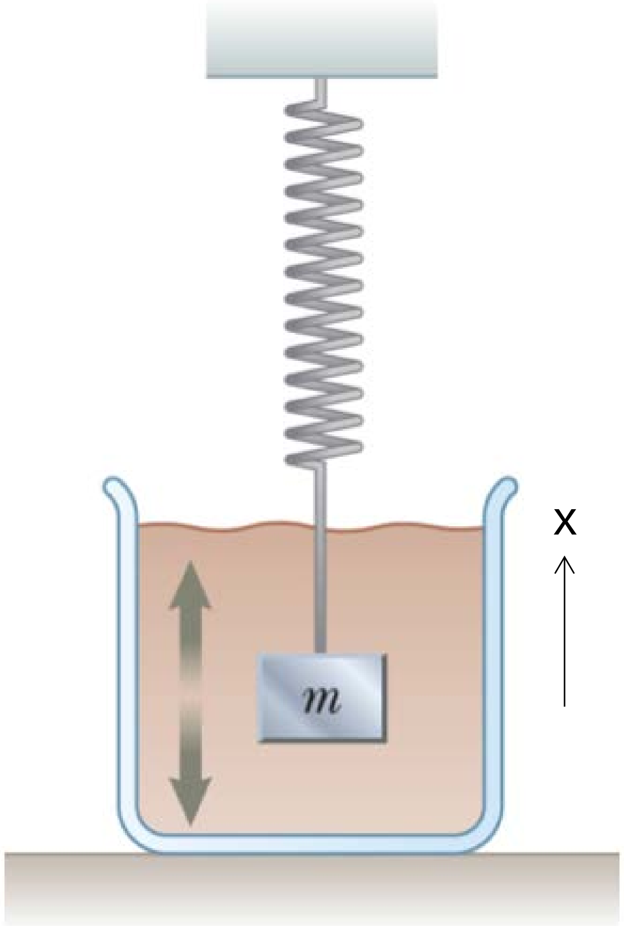
$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{b}{m}\frac{dx}{dt} - \frac{k}{m}x$$

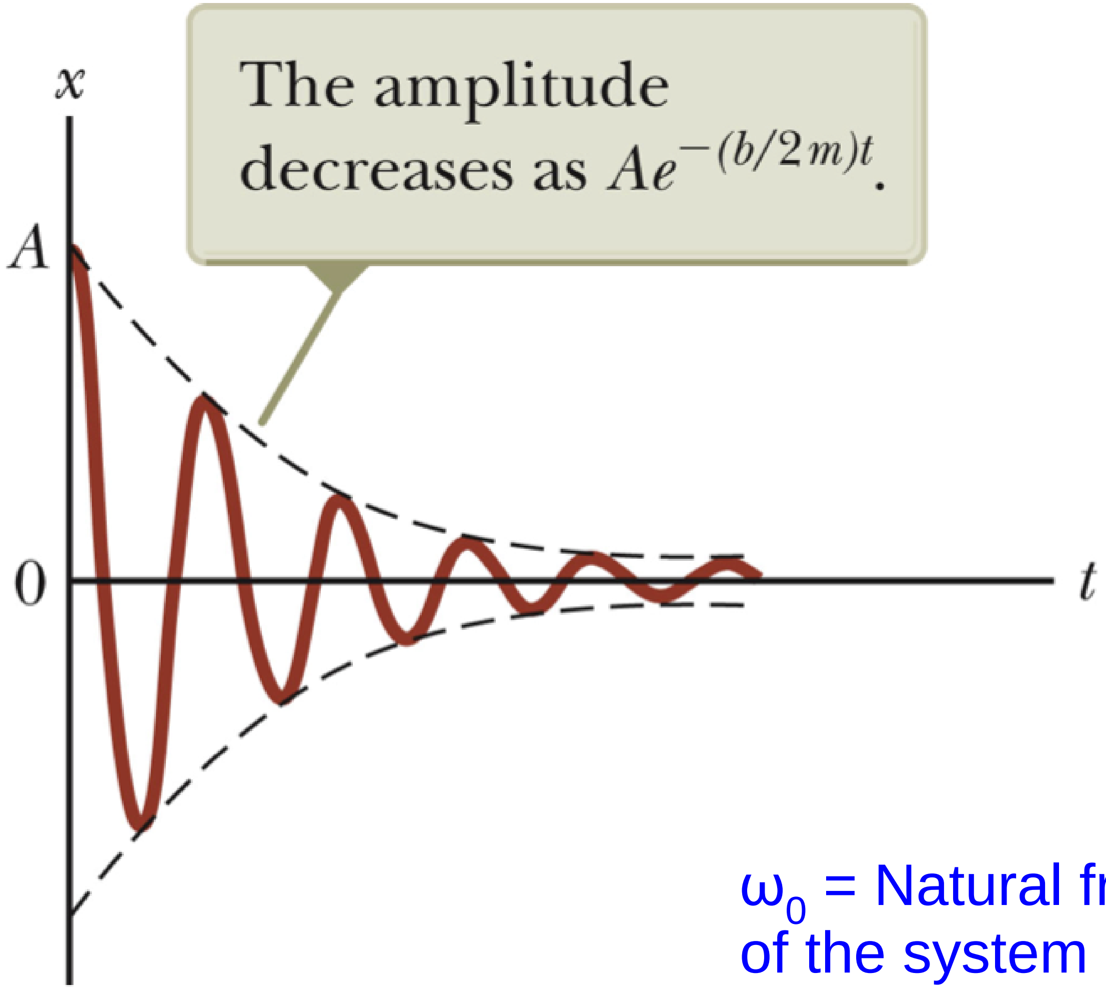
- If the damping constant b is small (I'll tell you what "small" means in a moment), then the **solution** looks like this:

$$x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

Angular frequency





$$x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

- This solution holds when

$$\omega_0^2 \equiv \frac{k}{m} > \left(\frac{b}{2m}\right)^2$$

"Small" damping constant

ω_0 = Natural frequency of the system

- When the retarding force is small, the oscillatory character of the motion is preserved, but the amplitude decreases exponentially with time. What if the retarding force is large?

Types of damping...

- In general, there are **three types** damping:

–Underdamping (we've just seen it):

$$\omega_0^2 > \left(\frac{b}{2m}\right)^2 \Rightarrow \omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} = \text{Real}$$

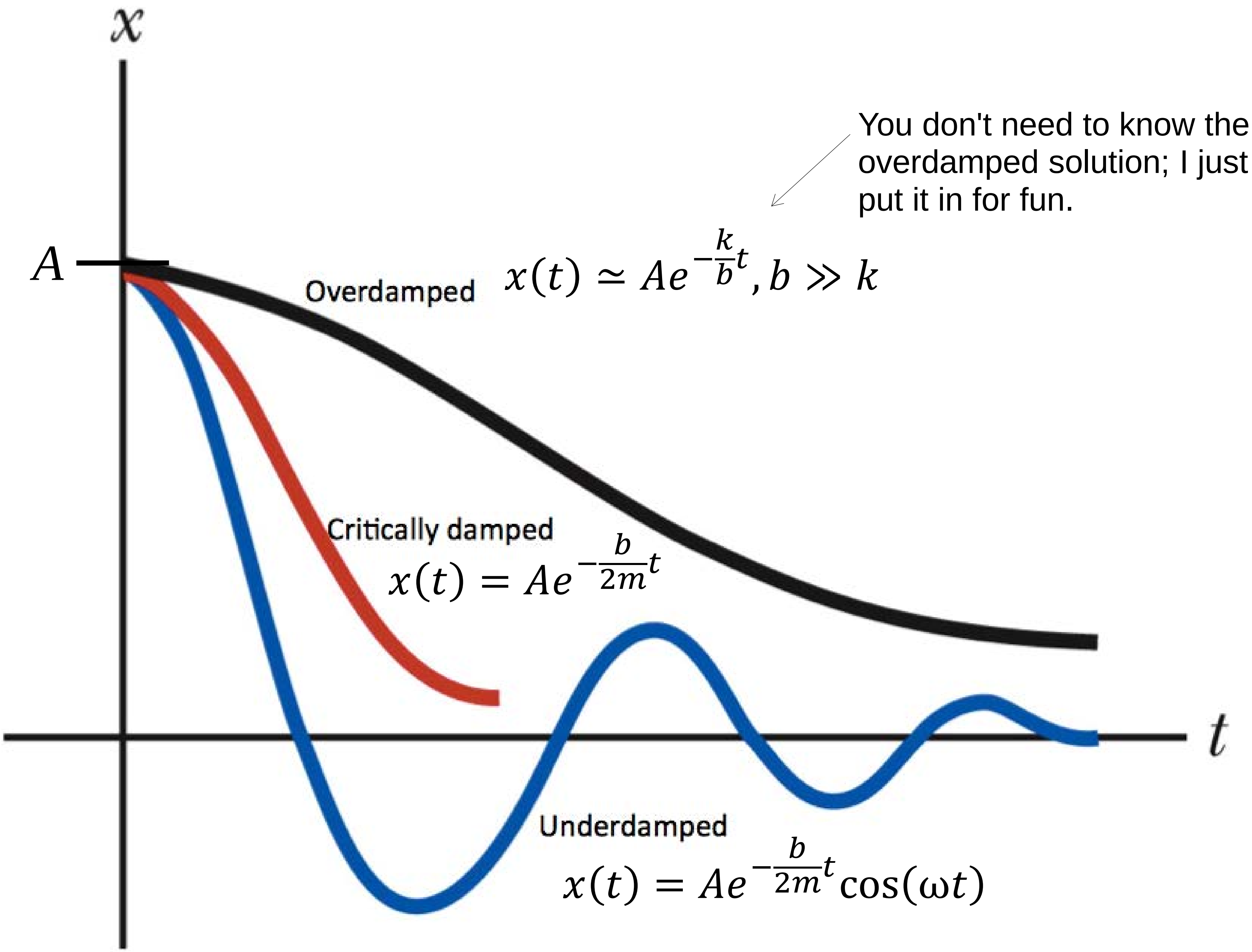
–Critical damping:

$$\omega_0^2 = \left(\frac{b}{2m}\right)^2 \Rightarrow \omega = 0$$

No oscillations

–Overdamping:

$$\omega_0^2 < \left(\frac{b}{2m}\right)^2 \Rightarrow \omega = \text{Imaginary}$$


You don't need to know the overdamped solution; I just put it in for fun.
x
A
Overdamped
$x(t) \simeq Ae^{-\frac{k}{b}t}, b \gg k$
Critically damped
$x(t) = Ae^{-\frac{b}{2m}t}$
Underdamped
$x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega t)$
t

What will happen when the heavy ball is swung?

- All the balls will strat to swing similar amounts
- The black ball will swing the most
- The khaki green ball will swing the most
- The brown ball will swing the most
- The bright green ball will swing the most
- The yellow ball will swing the most
- The red ball will swing the most
- Only the heavy silver ball will swing

https://goo.gl/forms/v4LMBaax2FKzTCYy2

Forced oscillations...

- In a damped oscillator, the system loses mechanical energy.
- It is also possible to add energy to the system by doing work in each cycle.

– For example:

$$\underbrace{F_0 \sin \omega_d t}_{\text{A periodic driving force}} \overbrace{- b\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}}^{\text{Damped oscillator}}$$

– This is very hard to solve...

•…but it will reach a steady state when the energy added each cycle is equal to the energy lost . Then,

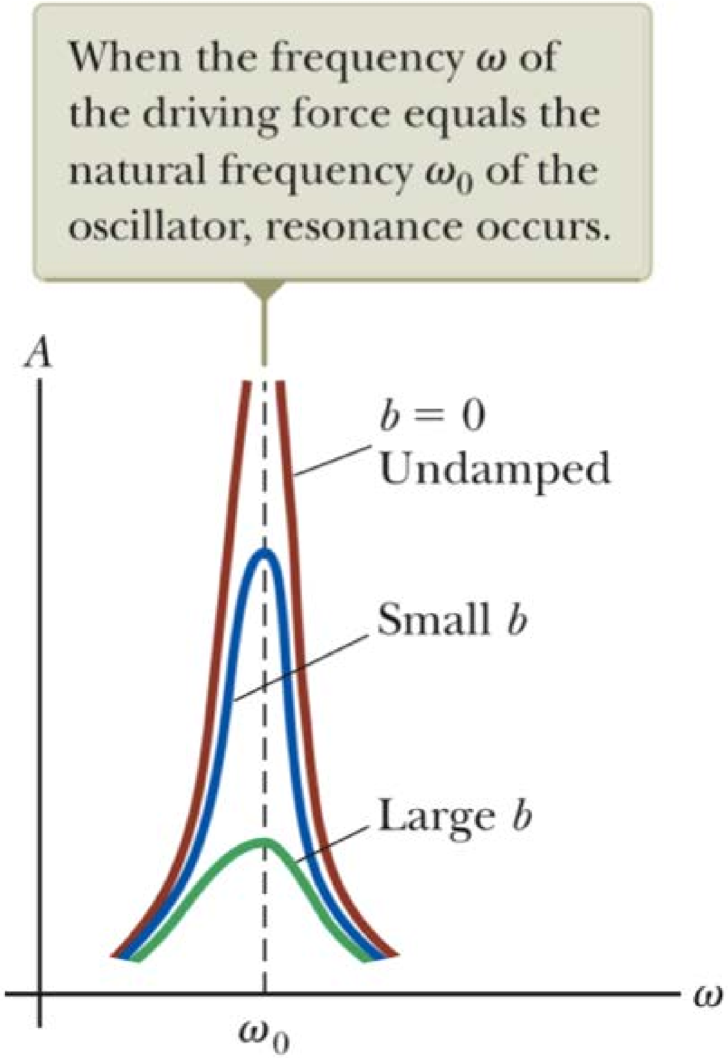
$$x(t) = A\cos(\omega_d t + \phi)$$

Angular frequency of driving force

$$A = \frac{F_0/m}{\sqrt{(\omega_d^2 - \omega_0^2)^2 + b^2/(4m^2)}}$$

Natural frequency of the undamped system

→ **Resonance** happens when ω_d is close to ω_0 → Dramatic increase in the amplitude.



Forced Oscillations

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (\frac{b\omega}{m})^2}}$$

If the wind was **constant**, was the bridge failure due to resonance (forced oscillation)?

A different topic: Travelling waves

Waves and Oscillations topic: Less fire, more math

Waves...

- A wave is a *periodic disturbance* that **transports energy** between two points in space.

- But there is **no** accompanying transfer of matter.

- **All** waves carry energy, but the amount and mechanism of transport depends on the **type of wave**.

Waves>Types of waves...

- **Mechanical waves**, e.g., sounds waves.
  - Propagation requires a medium.

We're mainly dealing with these.

- **Electromagnetic waves**, e.g., visible light, X-rays.
  - What is the medium? The “Aether”? Spacetime?
  - Propagation doesn’t require a medium
- **Matter waves**, i.e., all fundamental particles.
  - Follows from wave-particle duality of quantum mechanics.
  - Seriously cool.

• Wa5: Transverse wave pulses and standing waves

Also notice reflection from fixed end + harmonics up to 4

But first…. A pulse

A pulse is not periodic but is very similar to a wave. It is simpler to visualize so we will occasionally discuss pulses.

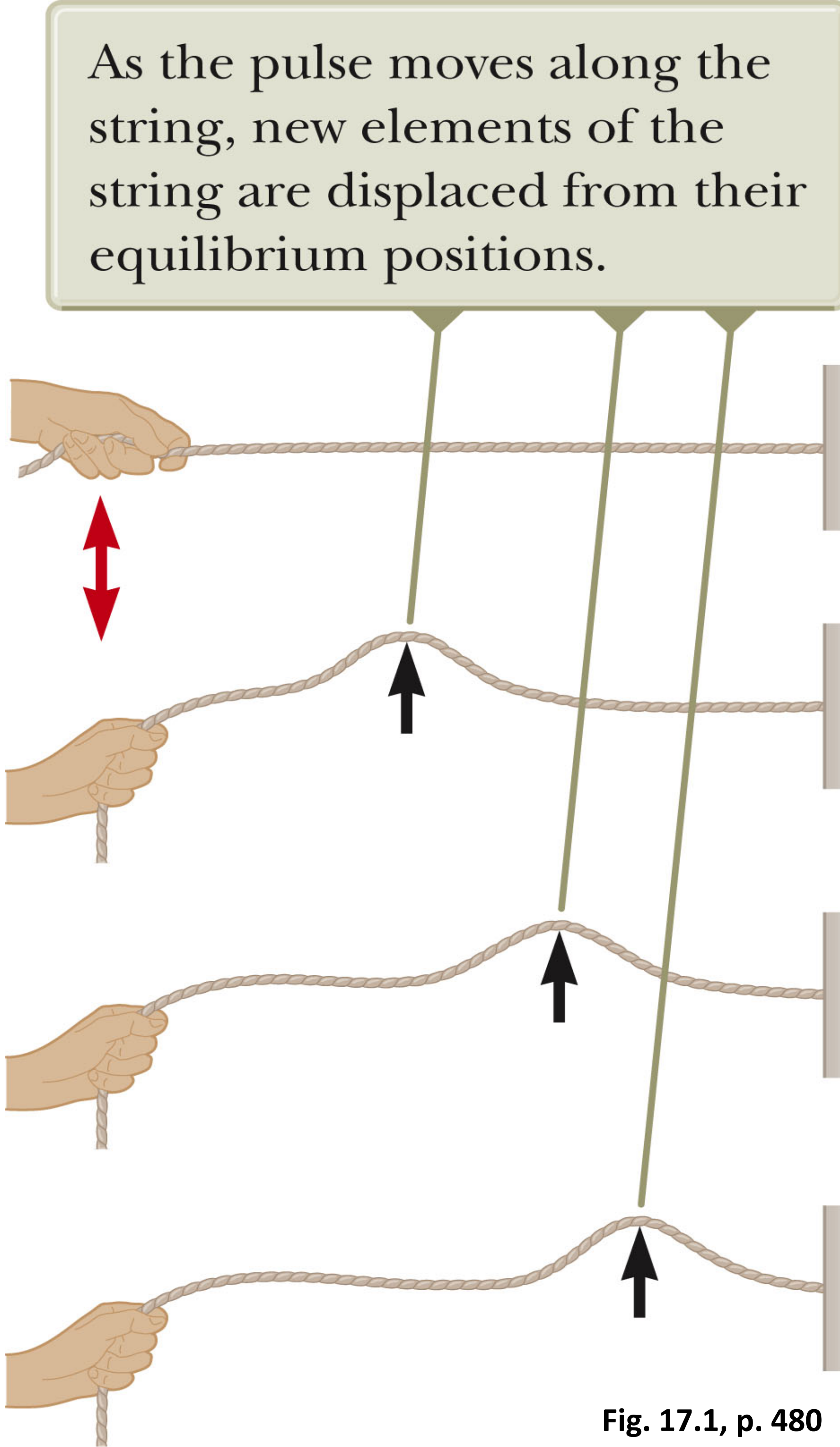


Fig. 17.1, p. 480

Waves>Mechanical waves...

- To produce a mechanical wave, we need

1. Some **source** of disturbance.

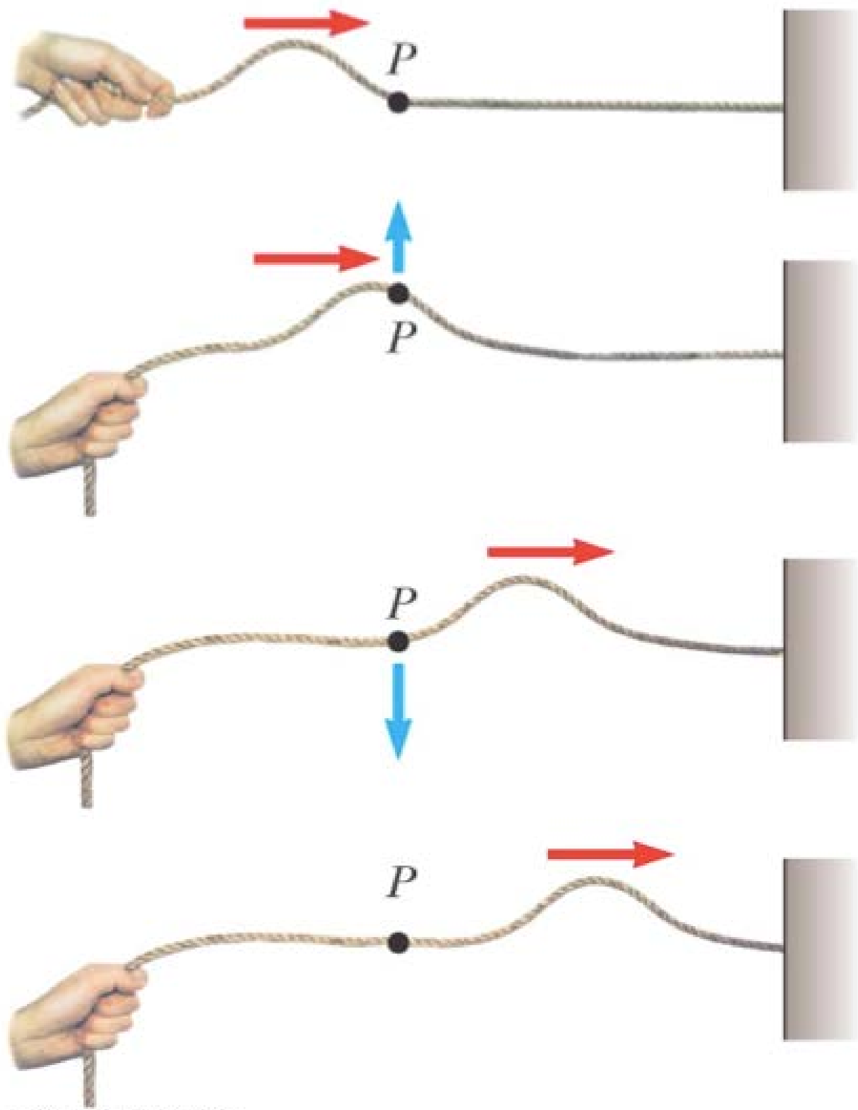
You flick the rope.

2. A **medium** that can be disturbed.

The rope is the medium.

3. Some physical **coupling** mechanism through which elements of the medium can influence each other.

Tension of the rope.



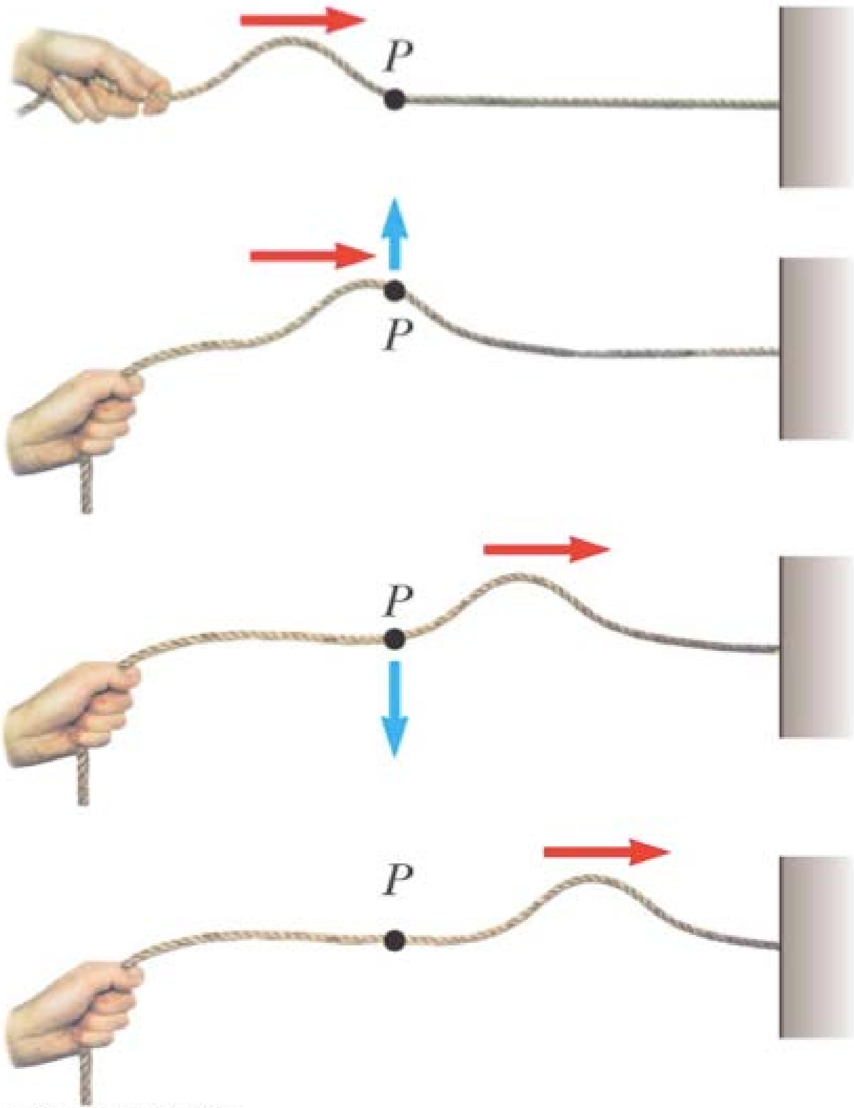
Waves>Pulse on a rope...

The rope is the medium through which the pulse travels

The pulse:

- Carries energy
- Has a definite height
- Has a definite speed of propagation
- Does not change shape as it travels

Continuous flicking would produce a periodic disturbance → A **wave**



Transverse vs longitudinal waves...

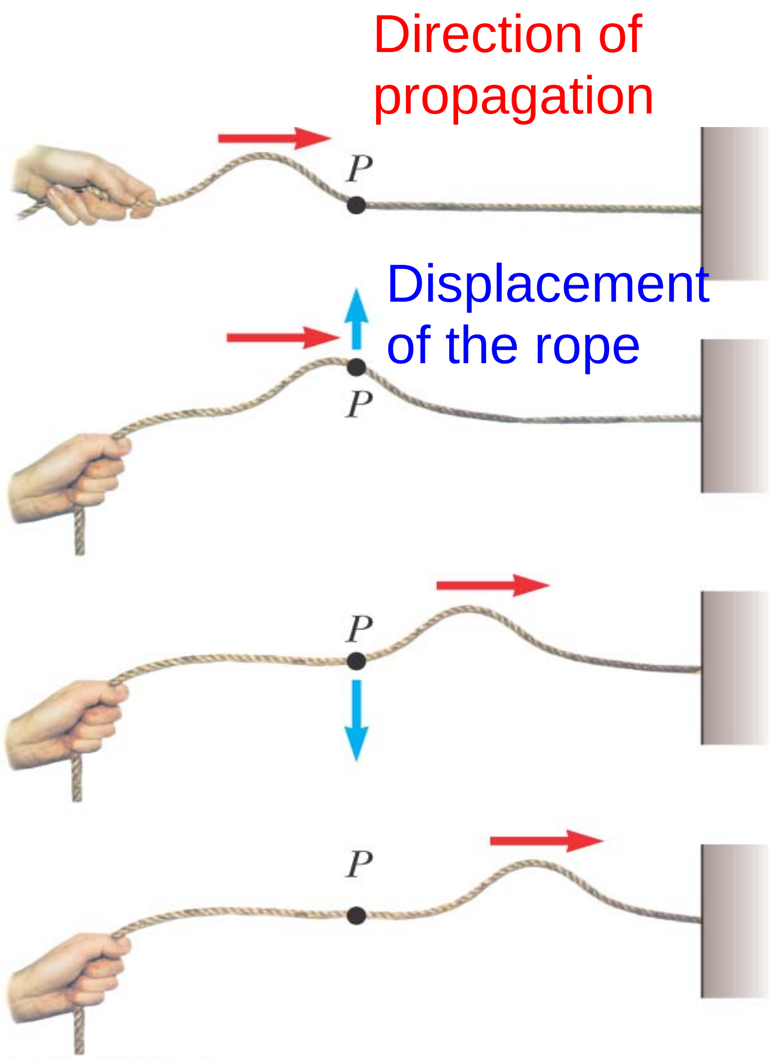
- Two basic propagation modes:

  - **Transverse**: displacement perpendicular to direction of propagation.

  - **Longitudinal**: displacement in the parallel to direction of propagation.

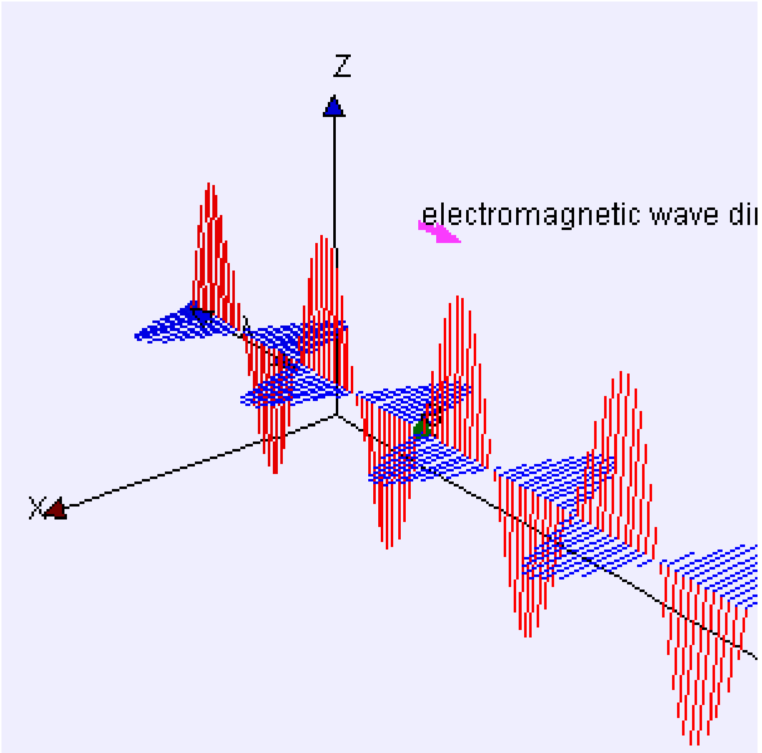
Transverse Wave

- A travelling wave or pulse that causes the elements of the disturbed medium to move **perpendicular** to the direction of propagation is called a **transverse wave**
- The particle motion is shown by the blue arrow
- The direction of propagation is shown by the red arrow
- e.g., wave on a string, ripples in a pond.



Transverse waves>EM waves...

- **Electromagnetic waves** are also transverse waves, although in this case there is **no** disturbed medium.



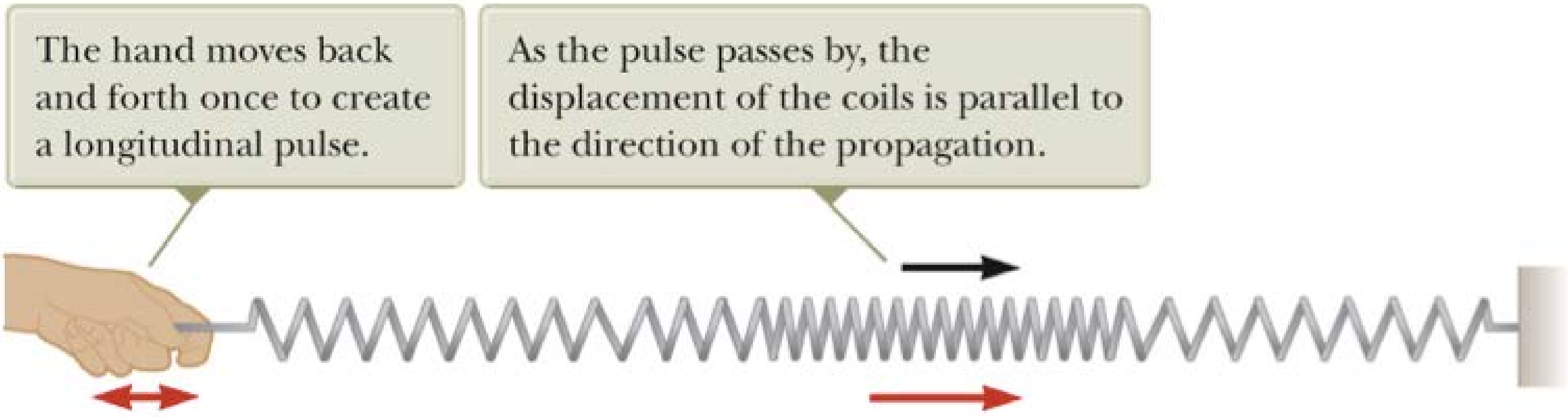
Red = Electric field
Blue = Magnetic field

– It is the electric and magnetic fields that oscillate in directions perpendicular to the direction of propagation.

Longitudinal waves...

- Elements of the disturbed medium move parallel to the direction of propagation.

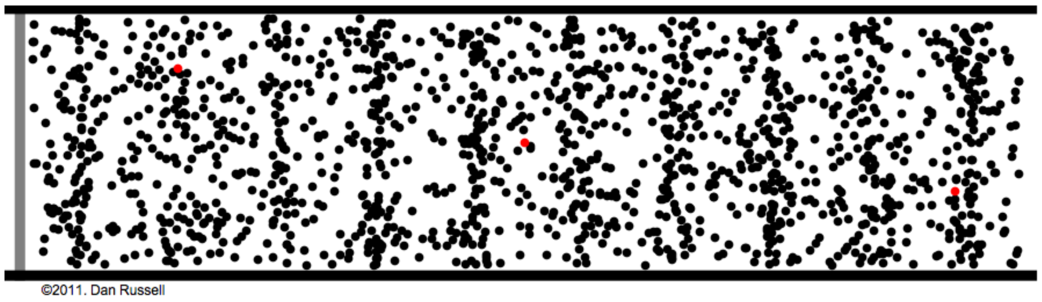
–e.g., wave travelling down a spring coil

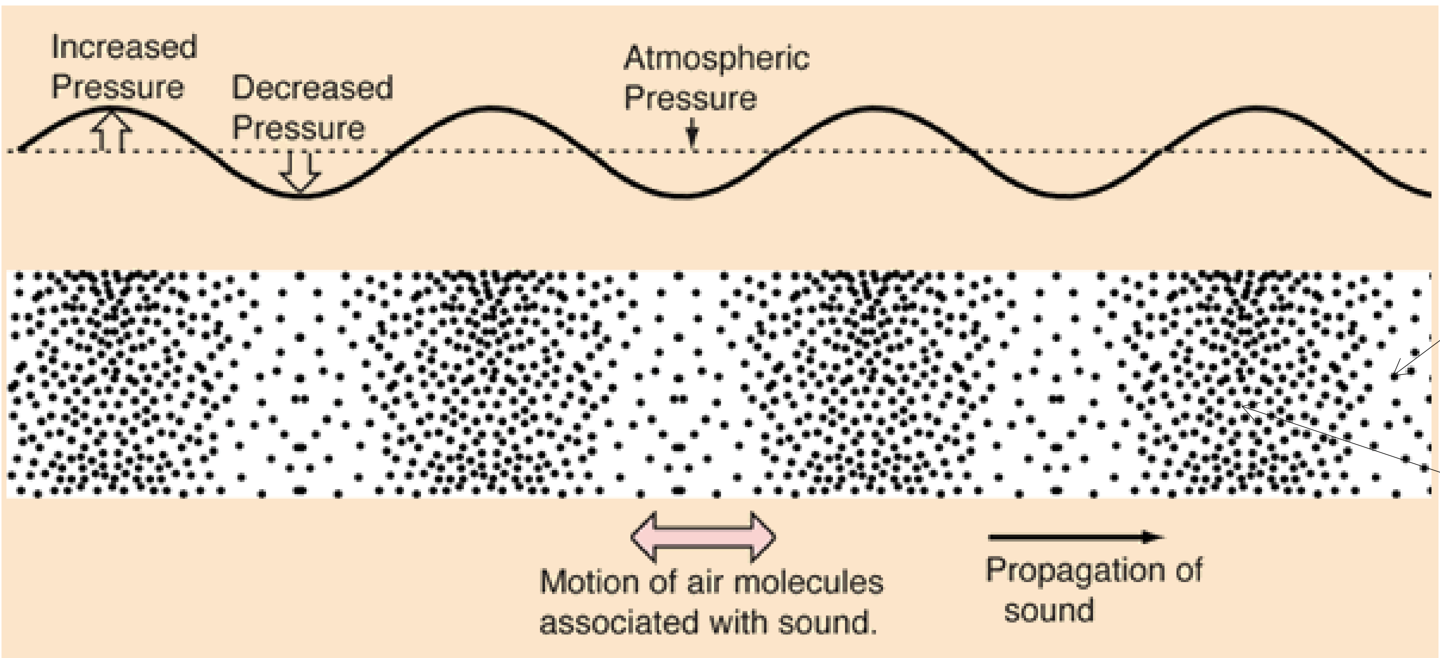


Wa4: The slinky
Longitudinal Waves

Longitudinal waves>Sound waves...

- **Sound waves** are mechanical waves that disturb the air density and pressure.



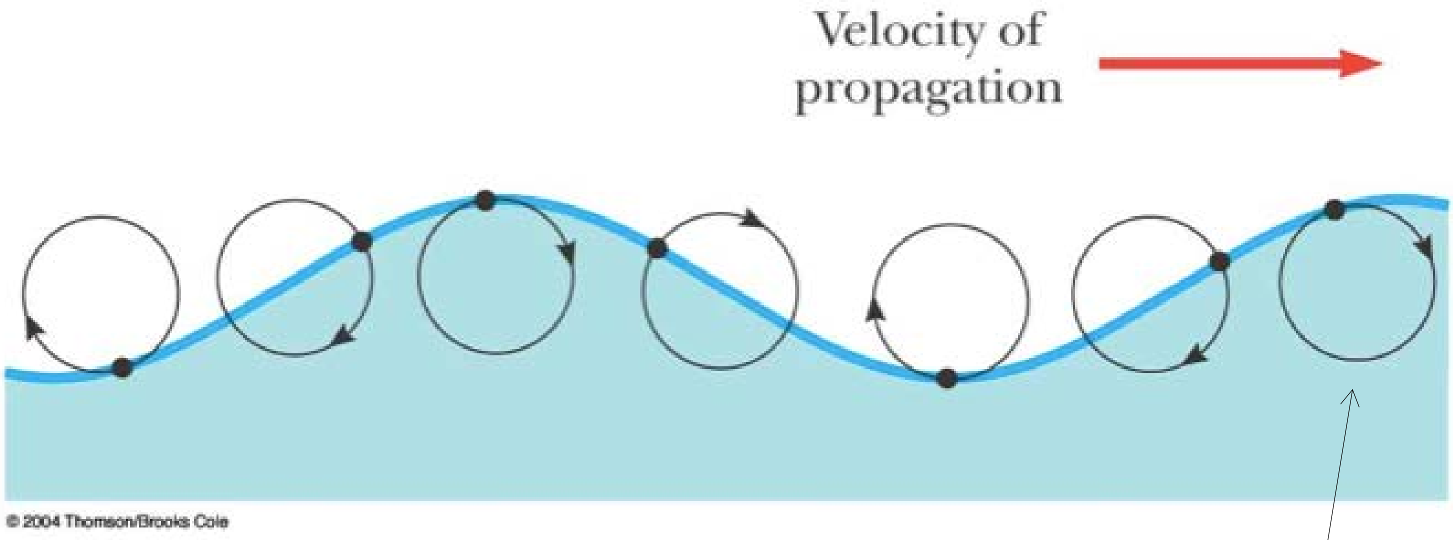


Rarefaction
= low density

Compression
= high density

Complex waves>Examples...

- Surface water waves are a combination of transverse and longitudinal waves.



Water molecules move around in circles.

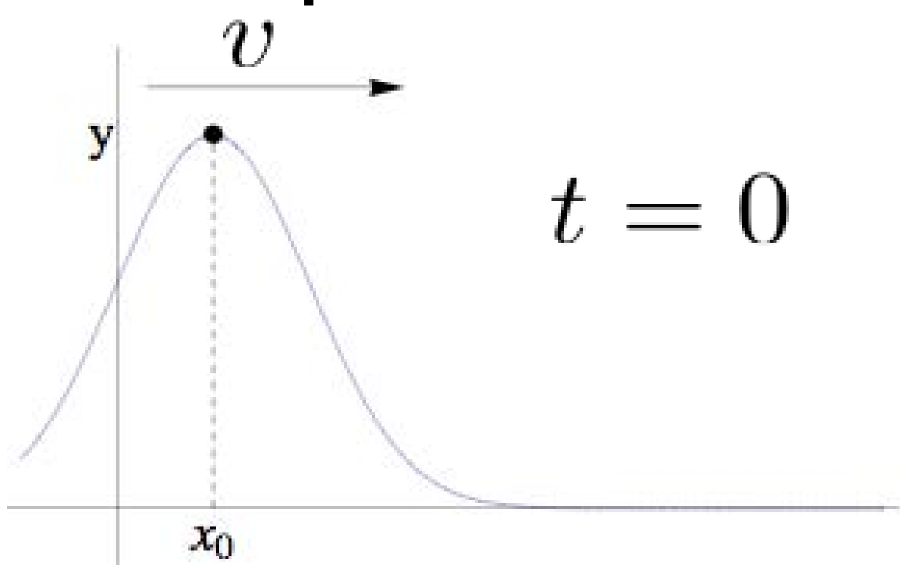
- In a long line of people waiting to buy tickets, the first person leaves and a frantic pulse of motion occurs as people step forward to fill the gap. As each person steps forward, the gap moves through the line. The propagation of this gap is:

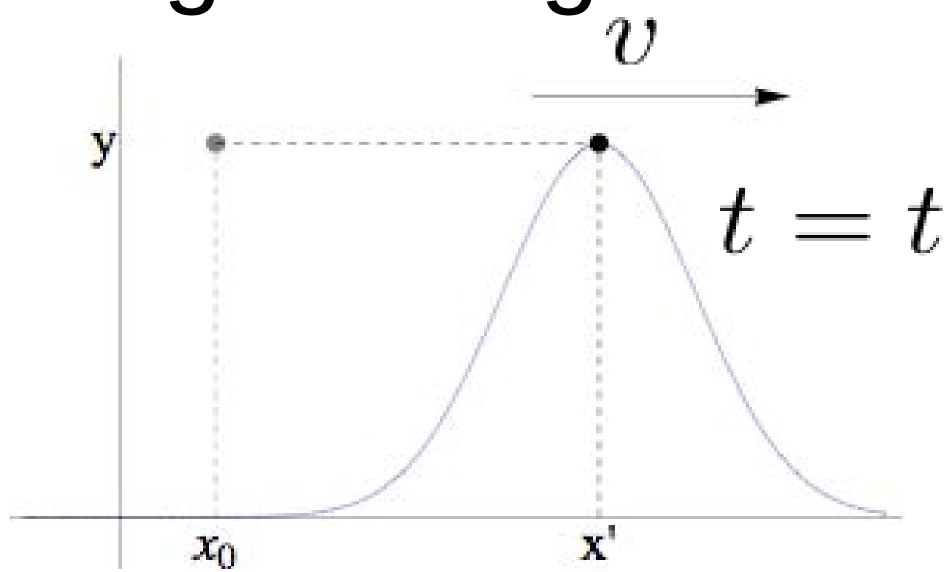
1. transverse.
2. longitudinal.

Consider the (Mexican) “wave” at a football game: people stand up and raise their arms as the wave arrives at their location, and the resultant pulse moves around the stadium. This wave is:

1. transverse.
2. longitudinal.

A pulse traveling along a string





$$y(x_0, 0) = f(x_0)$$

$$x' = x_0 + vt$$

$$y(x', t) = f(x_0) = f(x' - vt)$$

$$f(x) = \text{“height function”}$$

When the wave travels to the right we need to ***subtract*** *vt* from *x*.

A pulse traveling along a string

$$y(x,t) = f(x + vt)$$

Is a pulse traveling to the left.

Sometimes this function is called the “**wave function**” as it describes the shape of the wave.

The “**waveform**” is the shape of the wave, it can be determined at a particular time.

A pulse travelling along a string...

- In general:

– A pulse travelling in the **positive x-direction**:

$$y(x,t) = f(x - vt)$$

– A pulse travelling in the **negative x-direction**:

$$y(x,t) = f(x + vt)$$

- The function $y(x, t)$ is called the **wavefunction**. It describes the transverse displacement.

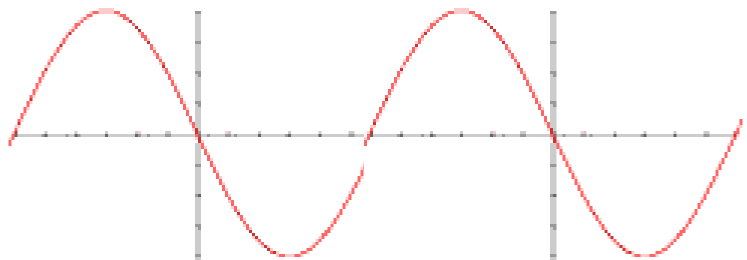
Question

At t = 0, a transverse pulse in a wire is described by the function:

$$y = \frac{6.00}{x^2 + 3.00}$$

Where x and y are in meters. If the pulse is traveling in the positive x direction with a speed of 4.50 m/s, write the function y(x,t) that describes this pulse.

Travelling waves...

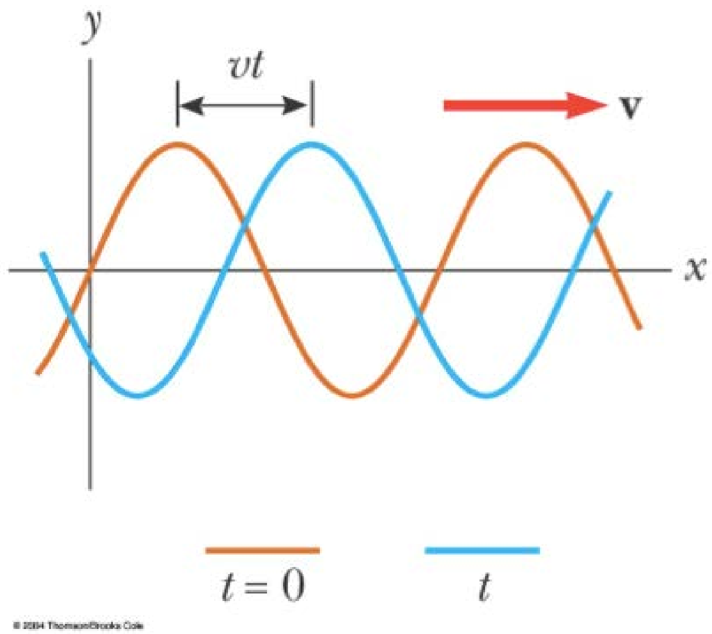


- A sinusoidal wave is a simple type of travelling wave.
  - Take a snapshot at $t = 0$. → The **waveform** is a sine function:

$$f(x) = A\sin(kx + \phi)$$

Some constant
(more on this later)

Phase constant in
case f(x) ≠ 0 at x = 0



$$\Rightarrow y(x,t) = f(x - vt) = A\sin[k(x - vt) + \phi]$$

Propagation in the + x-direction

$$y(x,t) = f(x + vt) = A\sin[k(x + vt) + \phi]$$

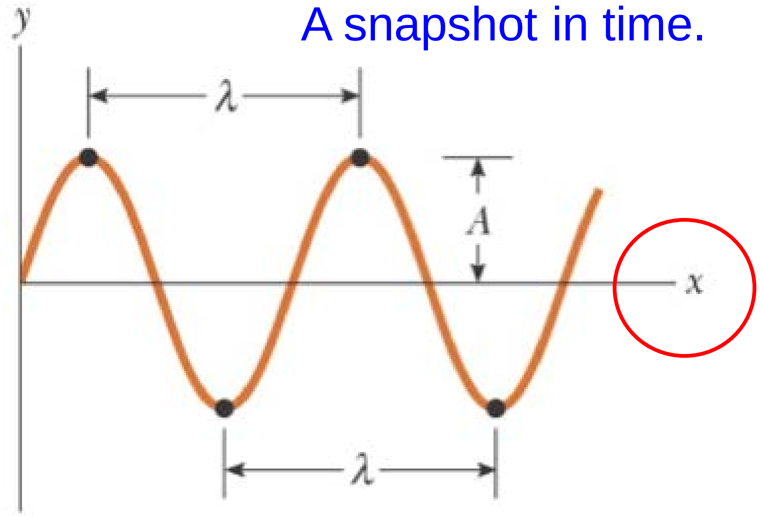
Propagation in the - x-direction

Also check out PhysClips: http://www.animations.physics.unsw.edu.au/

Travelling waves>Some terminology...

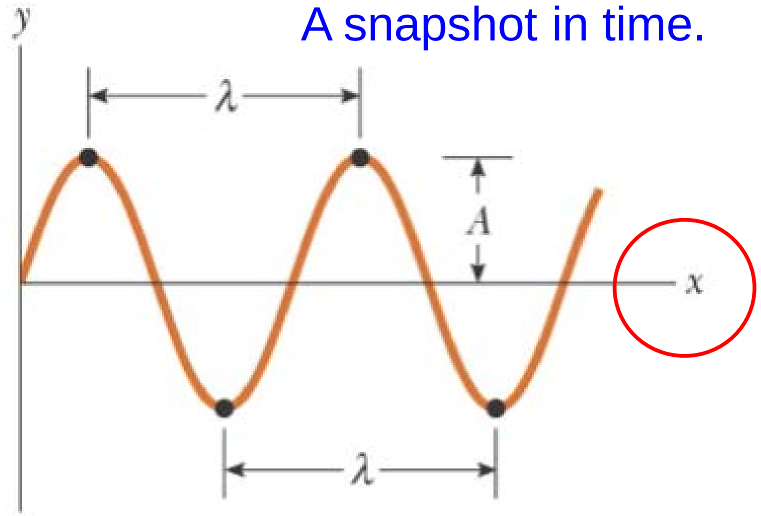
- The **wavelength** λ of a wave is the **distance** between adjacent crests or troughs.

A snapshot in time.

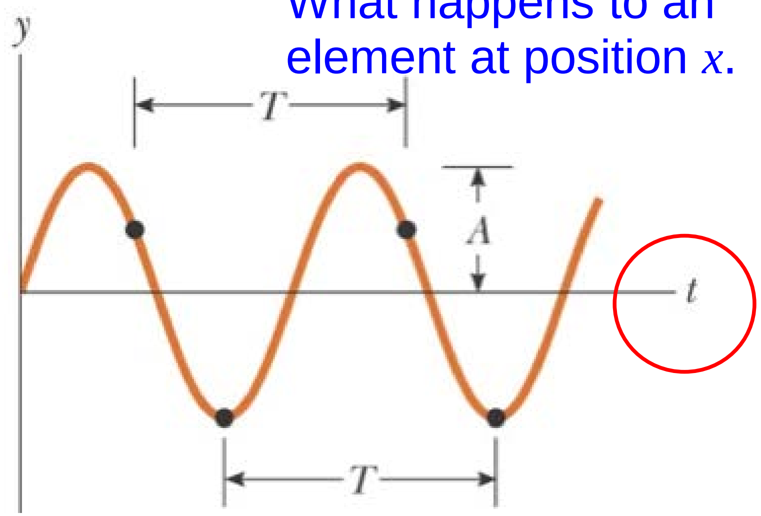


Travelling waves>Some terminology...

- The **wavelength** λ of a wave is the **distance** between adjacent crests or troughs.



- The **period** T is the t**ime interval** needed for an element at position x to complete one cycle of oscillations in the y-direction.



- $\rightarrow$ Also time interval for a wave to trave a distance of one wavelength.

$$f = \frac{1}{T}$$

Travelling waves>Equation of a wave...

- We have worked out that a travelling sine wave is described by

$$y(x,t) = A\sin[k(x - vt) + \phi]$$

Propagation in the + x-direction

$$y(x,t) = A\sin[k(x + vt) + \phi]$$

Propagation in the - x-direction

- What is k? BLACKBOARD

Travelling waves>Equation of a wave...

- We have worked out that a travelling sine wave is described by

$$y(x,t) = A\sin[k(x - vt) + \phi]$$

Propagation in the + x-direction

$$y(x,t) = A\sin[k(x + vt) + \phi]$$

Propagation in the - x-direction

- What is k? BLACKBOARD
  - k is called the **wavenumber**:

$$k = \frac{2\pi}{\lambda}$$

Wavelength

Travelling waves>Equation of a wave...

- This is what we have so far:

$$y(x,t) = A\sin\left[\frac{2\pi}{\lambda}x - \frac{2\pi v}{\lambda}t + \phi\right]$$

Propagation in the + x-direction

- The wave travels one wavelength in a period:

$$v = \frac{\lambda}{T} = f\lambda$$

f = Frequency

$$\Rightarrow y(x,t) = A\sin\left[\frac{2\pi}{\lambda}x - 2\pi f t + \phi\right]$$

$$\Rightarrow y(x,t) = A\sin(kx - \omega t + \phi)$$

$$\omega = 2\pi f$$

Angular frequency

Travelling waves>Summary...

- A sinusoidal wave travelling in the **positive** x-direction is described by

$$y(x,t) = A\sin\left[\frac{2\pi}{\lambda}x - 2\pi f t + \phi\right]$$
$$= A\sin(kx - \omega t + \phi)$$

- A sinusoidal wave travelling in the **negative** x-direction is described by

$$y(x,t) = A\sin\left[\frac{2\pi}{\lambda}x + 2\pi f t + \phi\right]$$
$$= A\sin(kx + \omega t + \phi)$$

Warning!

Frequency of a wave depends on the **source** of the wave.

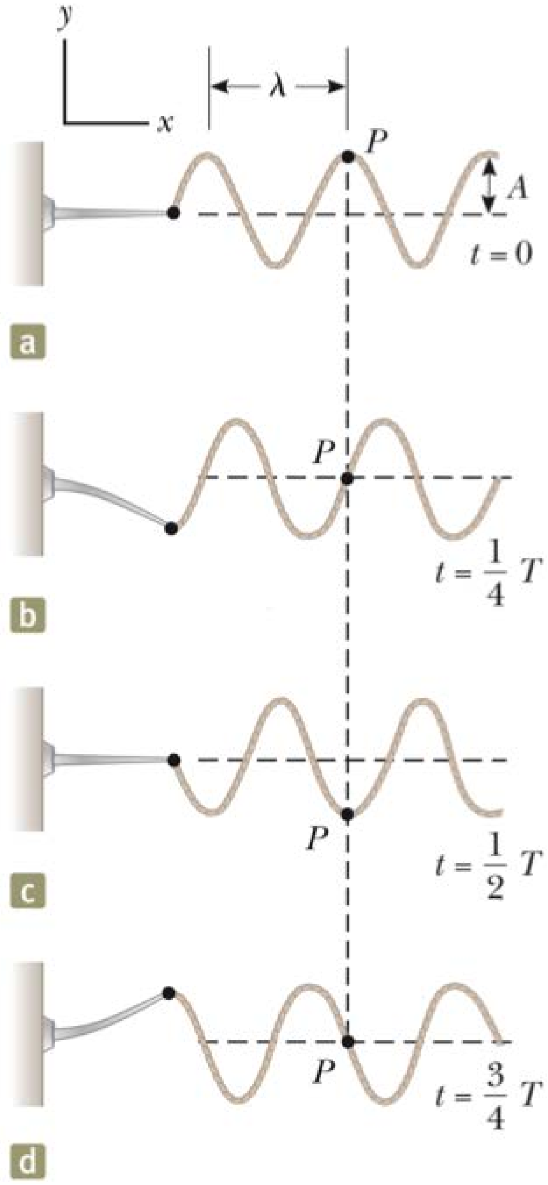
The **speed** depends on the medium, not the frequency or wavelength.

Demo Wa5

Relationship between waves & SHM...

- Sinusoidal waves can be generated by a generators set to oscillate in simple harmonic motion.

–If you consider one point P on the string, then its motion in the y-direction is SHM. (Prove it yourself!)



Question

The wave function for a traveling wave on a taut string is (in SI units)

$$y(x,t) = 0.350\sin(10\pi t - 3\pi x + \frac{\pi}{4})$$

(a) What are the speed and direction of travel of the wave?
(b) What is the vertical position of an element of the string at *t = 0, x = 0.100* m? What are
(c) The wavelength and
(d) The frequency of the wave?
(e) What is the maximum transverse speed of an element of the string?

Question

(a) Write the expression for *y* as a function of *x* and *t* in SI units for a sinusoidal wave traveling along a rope in the negative *x* direction with the following characteristics: A = 8.00 cm, λ = 80.0 cm, f = 3.00 Hz, and $y(0, t) = 0$ at $t = 0$.

(b) Write the expression for *y* as a function of *x* and *t* for the wave in part (a) assuming $y(x,0) = 0$ at the point x = 10.0 cm.