Question 1 (19 for 1121, 23 for 1131)

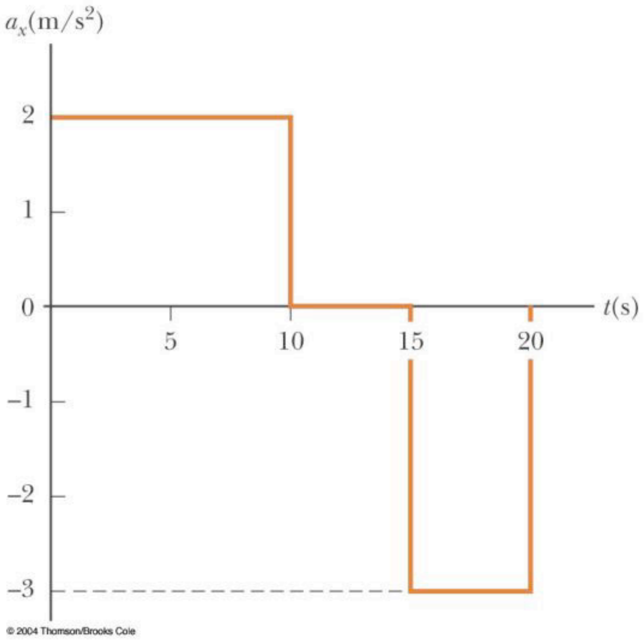
(a)

(i) A particle moves along the x axis. Its position is given by the equation $x = 2.00 + 3.00\,t - 4.00\,t^2$ where x is in meters and t in seconds.
Determine the position of the particle when it changes direction.

(ii) For the same particle as part (i) (i.e. $x = 2.00 + 3.00\,t - 4.00\,t^2$) determine its velocity when it returns to the position it had to begin with, at $t = 0.00$.

(iii) A particle starts from rest and accelerates as shown in the Figure below.

Determine the particle's speed at $t = 10.0$ s and at $t = 20.0$ s, and the distance travelled in the first 15.0 s. Assume you can read the graph to 3 significant figure precision.



b) You are at rest (at the origin, say) when your friend runs by, travelling at speed $v = 0.80$ m.s^{-1} in the x direction. When he is a distance $L = 1.8$ metres ahead of you, you start accelerating with a constant acceleration $a = 1.4$ m.s^{-2}.

i) Sketch a graph of the positions of you and your friend as a function of time t. Include T on the graph.

ii) Determine how long it takes for you to catch your friend.

Part c (1131 only)

You are riding your bicycle towards the North at 20 kilometres per hour (2 sig figs). Your bicycle has a navigation unit that tells you that the wind velocity relative to you on the bicycle is 14 kilometres per hour, coming *from* the direction 45° East of North. Showing all working, determine the 'true wind' velocity, i.e. the velocity of the wind with respect to the ground. (i.e. State the direction that the wind is coming from.)

ANSWER

(i) Differentiate once to get velocity: v=3-8t in SI units.

When it changes velocity, v=0, so t=3/8 s. Substitute t=3/8 into position equation. This gives x = 2 + 3(3/8) - 4(3/8)^2 = 2.56 m to 3 sig figs.

At the start, when t=0, its position was x=2.00. It returns to x=2 at some later time t.
$2 = 2 + 3t - 4t^2$
i.e. $3t - 4t^2 = 0$
so it returns to x=2 at t=3/4 s.
The velocity at this time is therefore v = 3 - 8(3/4) = -3.00 m/s.

ii) Acceleration is constant over the first ten seconds, so at the end,

$$v_f = v_i + at = 0 + \left(2.00\ \text{m/s}^2\right)(10.0\ \text{s}) = \boxed{20.0\ \text{m/s}}\ .$$

Then $a = 0$ so v is constant from $t = 10.0\ \text{s}$ to $t = 15.0\ \text{s}$. And over the last five seconds the velocity changes to

$$v_f = v_i + at = 20.0\ \text{m/s} + \left(3.00\ \text{m/s}^2\right)(5.00\ \text{s}) = \boxed{5.00\ \text{m/s}}\ .$$

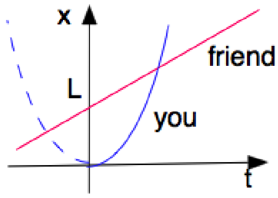
In the first ten seconds,

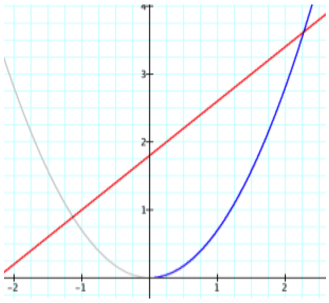
$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}\left(2.00\ \text{m/s}^2\right)(10.0\ \text{s})^2 = 100\ \text{m}\ .$$

Over the next five seconds the position changes to

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 100\ \text{m} + (20.0\ \text{m/s})(5.00\ \text{s}) + 0 = \underline{200\ \text{m.}}$$

b) i)





(*The dashed/ grey section is not required: just added here for interest.*)

ii) *One solution* is:

For my friend,	$x_1 = L + vt$.
For me,	$x_2 = 0 + 0 + \frac{1}{2}at^2$.
I overtake when	$x_2 = x_1$,
i.e.	$L + vt = \frac{1}{2}at^2$
rearranging	$\frac{1}{2}at^2 - vt - L = 0$

$$t = \frac{+v \pm \sqrt{v^2 + 2aL}}{a} \qquad v = 0.80\ \mathrm{m.s^{-1}},\ L = 1.8\ \mathrm{m},\ a = 1.4\ \mathrm{m.s^{-2}}.$$

$$t = \frac{(0.8\mathrm{m.s^{-1}}) \pm \sqrt{(0.8\mathrm{m.s^{-1}})^2 + 2(1.4\mathrm{m.s^{-2}})((1.8\mathrm{m})}}{(1.4\mathrm{m.s^{-2}})}$$ and we take only the positive solution.

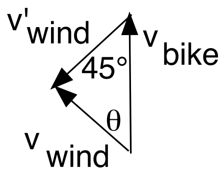
$= 2.3$ s

Alternatively, have the friend start at $t = 0$ and write $x_{\text{friend}} = 0.80t$, *etc.*

c) Let $\boldsymbol{v}$ be the wind velocity with respect to the ground, $\boldsymbol{v}'$ the velocity relative to you on your bicycle, and v_{bike} = be the your velocity with respect to the ground. Then

$$\boldsymbol{v} = \boldsymbol{v}_{\text{frame}} + \boldsymbol{v}'.$$

so $\boldsymbol{v}_{\text{wind}} = \boldsymbol{v}_{\text{bike}} + \boldsymbol{v}'_{\text{wind}}$,



Let the wind have components v_x and v_y with respect to the ground.

$$v_x = -v'_{\text{wind}} \sin 45° = -10 \text{ kph}$$

$$v_y = v_{\text{bike}} - v'_{\text{wind}} \cos 45° = (20 - 10) \text{ kph} = 10 \text{ kph}$$

So $v_{\text{wind}} = \sqrt{10^2 + 10^2}$ kph $= 14$ kph

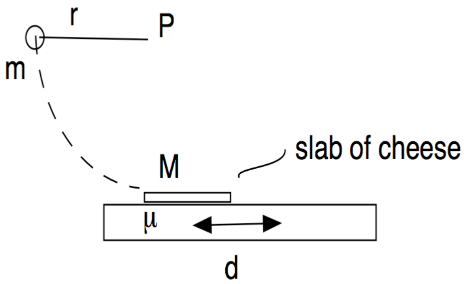
and its direction is from the South West.

(OR: its direction is 225° or 225° East of North, or 45° South of East)

Question 2 (18 for both courses)

a) State the conditions under which the momentum of a system is conserved.

b) State the conditions under which mechanical energy of a system is conserved.



c) A student is training his pet mouse to do circus tricks. The mouse, of mass m = 0.10 kg is holding on to one end of a light, inextensible string with length r = 82 cm. (The mouse is small enough that you can treat it as a particle.) The other end of the string is attached to a fixed point (P in the diagram). Initially, the string is horizontal and a distance r above a slab of cheese (mass M) resting on a table. The coefficients of static and kinetic friction between the slab and the table are μ_s = 0.90 and and μ_k = 0.80 respectively. The mouse is released from rest, swings on the string through an angle of 90° then lets go, landing in a very brief collision on top of the slab. After this collision, the slab of cheese and the animal travel together a distance d = 12 cm as the slab skids on the table.

i) Determine the speed of the mouse immediately **before** it reaches the cheese. (air resistance is negligible.)

ii) Determine the tension in the string while the mouse is still swinging in the arc, but just **before** it reaches the cheese.

iii) Determine the initial speed V of mouse+plate immediately **after** their collision. (Hint: consider how far do they then travel together and the forces acting on them.)

iv) Determine the mass M of the cheese.

Part d & e for 1131 only

d) State the work-energy theorem.

e) Starting with Newton's second law, prove the work-energy theorem.

Question 2 ANSWERS

a) If external forces are negligible, then momentum is approximately conserved. *OR*
If external forces are zero, then momentum is conserved. *OR*
If the impulse due to external forces is zero (or negligible), then momentum is conserved (or approximately conserved). *OR*
If the impulse due to external forces is zero (or negligible) in one direction, then momentum is conserved (or approximately conserved) in that direction. *Or an equivalent statement.*

b) If non-conservative forces do no work, mechanical energy is conserved.

c) i) We are told that air resistance is negligible, so non-conservative forces do no work, so mechanical energy is conserved. So

$U_i + K_i = U_f + K_f$. So

$mgr + 0 = 0 + \frac{1}{2}mv^2$. Rearranging

$v = \sqrt{2gr} = \sqrt{2 * 9.8\text{m.s}^{-2} * 0.82\text{m}} = 4.0 \text{ m.s}^{-1}$.

ii) The mouse is accelerating in the vertical direction with centripetal acceleration mv^2/r.

So the tension T and the weight mg satisfy: $T - mg = mv^2/r$

$T = mg + mv^2/r = mg + m2gr/r = 3mg = 3*0.1\text{kg}*9.8\text{m.s}^{-2} = 2.9 \text{ N}$

iii) Let the speed after the collision be V. ***Either*** use

work done by friction = change in kinetic energy ***OR*** kinematics $0^2 - V^2 = 2ad$.

$-F_f d = -\frac{1}{2}(m + M)V^2$ ***OR*** $-2(F_f/(m + M))d = -V^2$

$\mu_k(m + M)gd = \frac{1}{2}(m + M)V^2$ ***OR*** $2\mu_k gd = V^2$

$V = \sqrt{2\mu_k gd} = \sqrt{2 * 0.8 * 9.8\text{m.s}^{-2} * 0.12\text{m}} = 1.4 \text{ m.s}^{-1}$.

iv) Although friction acts between the cheese and the table, this horizontal force transmits negligible impulse compared with that of the collision, so horizontal momentum is approximately conserved. Let the horizontal velocity after the collision be V. Setting momenta before and after the collision equal:

$mv = (m + M)V$

$m(v - V) = MV$

$M = \text{m}(v - V)/V = 0.10 \text{ kg}(4.0 - 1.4)/1.4 = 190 \text{ g}$.

Part d & e for 1131 only

d) The total work done by all the forces acting on a body is the change in its kinetic energy.

e) Let the total force be F, so the total work done is

$\mathrm{d}W = F.ds$ and substituting from Newton's second law

$\mathrm{d}W = ma.ds = m\frac{dv}{dt}.ds$ Changing the order of division and mulitiplication:

$\mathrm{d}W = m\frac{ds}{dt}.dv = mv.dv$ but $\mathrm{d}K = \mathrm{d}(\frac{1}{2}mv^2) = mv.dv$ so $\mathrm{d}K = \mathrm{d}W$

The constant force version is true but not general:

Definition of work and Newton's second give:

$W = Fs = mas$

But $v^2 - u^2 = 2as$ so

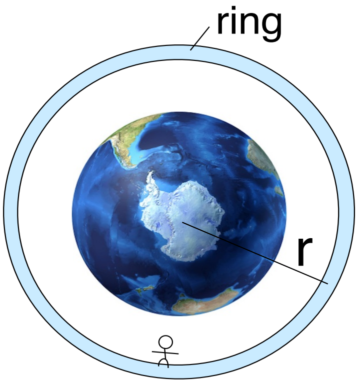
$W = \frac{1}{2}m(v^2 - u^2)$

Question 3 (13 for 1121, 19 for 1131)

a) Most communications satellites are found in an orbit above the equator. They complete one orbit in a period T = 24 hours and so stay above the same point on the equator – we call this a geosynchronous orbit. Beginning with Newton's law for universal gravitation, and showing all working, calculate the radius *R* of the geosynchronous orbit. ($M_{earth} = 5.98*10^{24}$ kg, $G = 6.67*10^{-11}$ $N.m^2.kg^{-2}$.)

b) **Part b is 1131 only**

A recently launched mission to the International Space Station is studying the potential health problems associated with long periods in orbit, having no normal forces from floors, chairs etc. A science fiction author proposes a solution: a new space station in the shape of a uniform ring, radius *r* = 6900 km, could be built above the equator, to replace the International Space Station.



This sketch is not to scale.

i) If the ring is uniform, where is its centre of mass?

ii) If the ring is centred exactly on Earth's centre, what is the total gravitational force (due to the earth) acting on the ring?

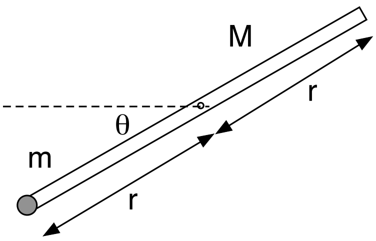
The ring need not be in orbit: in principle, its mechanical strength (if large enough) will hold it up. Because the ring is not in orbit, our author argues that it can turn at any angular speed we like.

iii) Caculate the rotation period at which it would have to turn so that a standing astronaut (mass *m*), her head towards Earth as shown in the sketch, feels as though she is on earth. In other words, she would feel the same normal force on her feet that she would feel when standing on Earth (with her feet towards Earth). You must *draw a free body diagram*. To make the orientation of your diagram clear, draw a figure of the standing astronaut next to your free body diagram.

B or c) **Both courses**

i) Define a conservative force

ii) State the general condition under which mechanical energy is conserved.



A particle of mass *m* is rigidly attached to the end of a uniform rod, mass *M*, which has a length of 2*r* and which turns without friction about a horizontal axis through its centre. It turns in a vertical plane and the sketch shows a side view.

iv) Suppose that the system is released from rest with the rod horizontal, i.e. θ = 0. Derive an expression for the angular speed ω of the system when the rod reaches vertical (θ = π/2). (Neglect air resistance.)

Q3 ANSWERS

a) Newton's law of Universal Gravitation: $|F| = GMm/r^2$. (1)

Newton's second law $F = ma$ (2)

For uniform circular motion, $a = r\omega^2$ (3)

Substitute (1) and (3) in (2) to have

$$G\frac{Mm}{r^2} = mr\left(\frac{2\pi}{T}\right)^2$$ Rearranging:

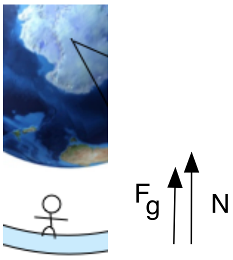
$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$ (which is Kepler's law of periods). Rearranging:

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{6.67*10^{-11}\text{N.m}^2.\text{kg}^{-2}5.98*10^{24}\text{kg}*(24*3600\text{s})^2}{4\pi^2}} = 42{,}000 \text{ km}$$

b) ***THIS ONE FOR 1131 only***

i) From symmetry, its centre of mass is at the centre of the ring.

ii) From symmetry, the total gravitational force on the ring will be zero.

iii) 

On the astronaut, two forces act towards the Earth's centre: the gravitational force on her and N, the normal force exerted by the satellite on her feet. That latter must have magnitude mg so that she feels it as the same normal force she would feel standing on Earth.

Newton's 2nd law: $N + GMm/r^2 = ma$.

$mg + GMm/r^2 = mr\omega^2$.

Substituting $\omega = 2\pi/T$ and cancelling m,

$r(2\pi/T)^2 = g + GM/r^2$

$$T = \frac{2\pi}{\sqrt{(g + GM/r^2)/r}}$$

$$= \frac{2\pi}{\sqrt{(9.8\text{m.s}^{-2} + 6.67*10^{-11}5.98*10^{24}/(6.9*10^6\text{m})^2))/6.9*10^6\text{m}}}$$

$= 3.9$ ks $= 65$ minutes

Note for students using this for revision. Please make sure that you understand the free body diagram above. The gravitational force of course points towards earth and, because of the altitude, this will be a bit less than mg. The normal force that the station exerts on the feet is also towards the centre. The sum of these two forces provides the centripetal force. Note that centripetal force is not an extra force. For a body in circular motion, whatever forces are acting to produce the centripetal acceleration are the centripetal force. Here, the centripetal force is $g + GM/r^2$.

c) i) A conservative force does no work around a closed path

ii) If non-conservative forces do no work, mechanical energy is conserved.

iii) Because it turns freely and we neglect air resistance, non-conservative forces do no work so mechanical energy is conserved.

$U_i + K_i = U_f + K_f$ where the subscripts indicate initial and final conditions.

$mgr + 0 = 0 + ½mv^2 + ½I\omega^2$ where I is the moment of inertia of the rod* and ω is the angular speed*

The moment of inertia of a rod of length $L = 2r$ about its centre is $ML^2/12 = M(2r)^2/12 = Mr^2/3$

The speed v of m is $r\omega$.

so $mgr = ½mr^2\omega^2 + ½(Mr^2/3)\omega^2$

so $2mg/r = (m + M/3)\omega^2$

so $$\omega = \sqrt{\frac{2mg}{r(m+M/3)}}$$ (*or any equivalent expression*)

Alternatively $mgh + 0 = 0 + ½I\omega^2$ *where I is the moment of inertia of (rod + m)*

* For a rod mass $M = \lambda L = \lambda(2r)$: $I = \int_{body} r^2 dm = \int_{-r}^{r} x^2 \lambda dx = \lambda \left[\frac{x^3}{3}\right]_{-r}^{r} = 2\lambda r\frac{r^2}{3} = Mr^2/3$

PHYS 1121 + 1131 + 1141 2015 - T1

Q4: Thermodynamics

① Isobaric — Pressure constant, $\Delta P = 0$ [A → B, C → D]

Isothermal — Temperature constant, $\Delta T = 0$ [B → C]

Adiabatic — No heat flows into or out of the system

ie $\Delta Q = 0$ [D → A]

② From the formula sheet:

$PV = nRT$ with n # moles

$\therefore \quad T = \frac{PV}{nR}$

For A: $T_A = \frac{1.0 \times 1.01 \times 10^5 \times 0.5}{20 \times 8.314} = 303.7\ K$

$= \underline{304\ K}$ to 3SF

(= 300 K to 2SF)

B: $T_B = \frac{1.0 \times 1.01 \times 10^5 \times 1.0}{20 \times 8.314} = 2T_A = 607.4\ K$

$= \underline{607\ K}$ to 3SF

(= 610 K to 2SF)

C: $T_C = T_B$ as isothermal $= 607\ K$

D. $T_D = \frac{0.25 \times 1.01 \times 10^5 \times 1.15}{20 \times 8.314} = \underline{174\ K}$ 3SF

(170 K 2SF)

(3) Work done **on** the gas given by

$$dW = -P\,dV \qquad \text{[Formula Sheet]}$$

For an isobaric change P = constant,

$$W = \int dW = -P\int_{V_0}^{V_1} dV = -P[V_1 - V_0]$$

$$= -1.0 \times 1.01 \times 10^5 \, [1.0 - 0.5] \text{ J}$$

$$= -5.05 \times 10^4 \text{ J}$$

$$= \underline{-5.1 \times 10^4 \text{ J}} \quad \text{to 2SF}$$

(4) Internal Energy $E_{int} = \frac{f}{2} nRT$ from Formula Sheet

Where $f = 3$ for a monotomic gas

$$\therefore \Delta E_{int} = 3/2 \, nR \, \Delta T = 3/2 \, . \, 20 . \, 8.314 \, [607.4 - 303.7] \text{ J}$$

$$= 7.58 \times 10^4 \text{ J}$$

$$= \underline{7.6 \times 10^4 \text{ J}} \quad \text{to 2SF}$$

(5) Heat flow given by applying the First Law

$$\Delta E_{int} = Q + W$$

Change in Internal Energy = Heat Flow into the Gas + Work done on the Gas

$$\therefore Q = \Delta E_{int} - W$$

$$= 7.58 \times 10^4 - (-5.05 \times 10^4) \text{ J}$$

$$= 1.26 \times 10^5 \text{ J}$$

$$= \underline{1.3 \times 10^5 \text{ J}} \quad \text{to 2SF}$$

⑥ Between $B \to C$ change is isothermal, so no change in E_{int}

$\therefore$ Heat Flow $Q = -$ Work Done, W

$= -(-1.4 \times 10^5)$ J

$= \underline{1.4 \times 10^5 \text{ J}}$ to 2SF

⑦ From $D \to A$ change is adiabatic, hence

Heat Flow $\underline{Q = 0}$

⑧ Thus Work Done on the gas from $D \to A$ $= \Delta E_{int}$,

applying the 1st Law.

$\Delta E_{int} = 3/2 \; n \; R \; \Delta T$

$= 3/2 \;\; 20 \;\; 8.314 \; [303.7 - 174.4]$ J

$= 3.22 \times 10^4$ J

$\Rightarrow$ Work Done $= \Delta E_{int} = \underline{3.2 \times 10^4 \text{ J}}$ to 2SF

PHYS 1131/1141 ONLY

(9) Net Work in one cycle = $WD_{AB+BC+CD+DA}$

$WD_{AB} = -5.05 \times 10^4$ J [Part 3]

$WD_{BC} = -1.40 \times 10^5$ J [Given]

$WD_{DA} = +3.22 \times 10^4$ J [Part 8]

Need WD_{CD}, an isobaric change

Work Done $= -P[V_D - V_C]$ as P constant

$W_{CD} = -0.25 \times 1.01 \times 10^5 [1.15 - 4.0]$ J $= +7.20 \times 10^4$ J

$\sum W = -8.63 \times 10^4$ J

$= \underline{-8.6 \times 10^4}$ J to 2SF

(10) The gas thus does work on the external system

of $+8.6 \times 10^4$ J in one cycle

Since there is no change in internal energy (as the gas returns to its starting point), then an equal amount of heat must flow <u>into</u> the gas.

ie $\underline{8.6 \times 10^4 \text{ J}}$, in order for this work to be extracted

→ This is the principle behind an engine.

PHYS 1121 + 1131 + 1141 2015 - T1

Q5: Oscillations + Waves

(9) (i) The block + bullet moves at speed V, say, after the bullet is embedded in the block. It reaches the spring. Then:

(a) The spring begins to compress, with the block slowing down. KE in the block is converted into PE in compression of the spring.

(b) The spring reaches maximum compression, and the block instantaneously comes to rest. All the KE of motion has been converted into PE in the spring.

(c) The spring begins to expand again, converting PE into KE in the block.

(d) When the spring reaches its uncompressed state all the PE has been transferred back to KE in the block.

(e) Block now moves back along the surface at speed $-V$.

(ii) At maximum compression of the spring:

KE lost by block + bullet = PE gained by spring

ie $\frac{1}{2}(m_b + m_w) V^2 = \frac{1}{2} K x^2$

$$\Rightarrow \quad V^2 = \frac{Kx^2}{m_b + m_w}$$

$$\therefore \quad V = \sqrt{\frac{120 \times 0.16^2}{0.005 + 1.5}} = 1.4287 \text{ m/s}$$

ie $\underline{V = 1.43 \text{ m/s}}$ to 3SF

(iii) Conserving momentum in the collission of bullet with block:

MARKS

$$m_b u = (m_b + m_W) V$$

$$\therefore \quad u = \left(\frac{m_b + m_W}{m_b}\right) V = \left(\frac{0.005 + 1.5}{0.005}\right) . 1.4287$$

$$= 430.04 \text{ m/s}$$

ie $u = 430$ m/s to 3SF

(b)(i) We make use of $f' = f\left(\frac{c \pm V_o}{c \mp V_s}\right)$ (from formula sheet)

with f emitted frequency of source $= 440$ Hz

f' measured frequency of observer $= 575$ Hz

V_o speed observer

V_s speed source

c sound speed $= 340$ m/s

We choose $+V_o$ if observer moving towards source

$-V_s$ if source moving towards observer

(and vice-versa if the other way around)

Observer stationary, source moving towards observer

$$f' = f \frac{c}{c - V_s}$$

$$\Rightarrow \quad c - V_s = \frac{fc}{f'}$$

$$\therefore V_s = c\left[1 - \frac{f}{f'}\right] = 340\left[1 - \frac{440}{575}\right] = 80.01 \text{ m/s}$$

$$= 80.0 \text{ m/s to 3SF}$$

(ii) In this case : Observer moving towards Source

but Source moving away from Observer

$$\therefore f' = f \left(\frac{c + V_0}{c + V_s} \right) = 440 \left(\frac{340 + 40}{340 + 80} \right)$$

$$= 398.1 \text{ Hz}$$

$$f' = \underline{398 \text{ Hz}} \text{ to 3SF}$$

(iii) In this case : Slow Car is the Source, with $V_s = 40$ m/s

Fast Car is the Observer, with $V_0 = 80$ m/s

Source moving towards the Observer,

but Observer moves away from Source

$$\therefore f' = f \left(\frac{c - V_0}{c - V_s} \right) = 440 \left(\frac{340 - 80}{340 - 40} \right)$$

$$= 381.3 \text{ Hz}$$

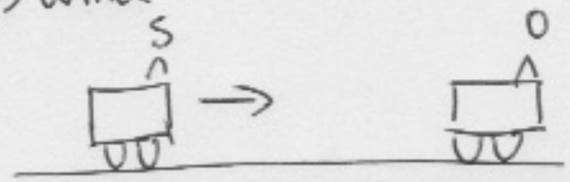
$$= \underline{381 \text{ Hz}} \text{ to 3SF}$$

TI 2015 Question 5 (b) parts (iv)-(vi)

For this part consider everything from the wind's reference frame.

$\xrightarrow{V_w}$ wind.

(iv)



in wind's frame.

$V_o = V_w$ ← towards source

$V_S = V_s - V_w$ ← towards observer.

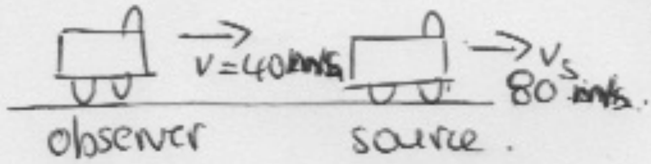
$$\Rightarrow f' = f_0\left(\frac{c+V_w}{c-(v_s-V_w)}\right)$$

$$= f_0\left(\frac{c+V_w}{c-v_s+V_w}\right) = 440\left(\frac{340+10}{340-80+10}\right)$$

$$= 570\ Hz \quad (2\ \text{sig fig}).$$

$\xrightarrow{V_w}$ wind

(v)



in wind's frame.

$V_o' = V_o - V_w$ towards source.

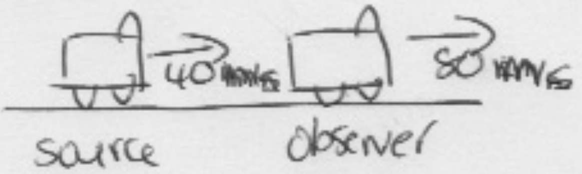
$V_s' = V_s - V_w$ away from observer.

$$f' = f_0\left(\frac{c+V_o-U_w}{c+V_s-V_w}\right) = 440\left(\frac{340+40-10}{340+80-10}\right)$$

$$= 397\ Hz = 400\ Hz \quad (2\ \text{sig fig}).$$

$\xrightarrow{V_{wind}}$ wind

(vi)



in wind's frame.

$V_o' = 80-10 = 70$ m/s. away from source.

$V_s' = 40-10 = 30$ m/s towards source.

$$f' = f_0\left(\frac{c-v_o'}{c-v_s'}\right)$$

$$= 440\left(\frac{340-70}{340-30}\right) = 383\ Hz = 380\ Hz$$

(2 sig fig).