

Thermal Physics

Lecture 4 – Work, Molar specific heats

Textbook reference: 18.4-18.6, 19.7-19.8



With physics, you can control the lightning!



Ideal Gasses
Work
Molar Specific Heats

Thermal Physics
Dr Elizabeth Angstmann

Textbook

Sections 18.5, 19.2, 19.7

Note: some text books use work done by a gas, we are using work done on a gas.

They have opposite signs!!!!

Last lecture

- Degrees of freedom are ways in which particles can store energy, they are associated with movement: translation, rotation and vibrations.
- Each degree of freedom can store $\frac{1}{2}k_B T$ energy giving a total of:

$$\frac{1}{2} f N k_B T$$

Last lecture

- The mean free path is given by:

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V}$$

- There is a distribution of speeds within an ideal gas

Internal energy

Internal Energy, E_{int} , is all the energy of a system that is associated with its microscopic components – atoms and molecules – when viewed from a reference frame at rest with respect to the center of mass of the system.

$$\overline{v} = 0$$

$$E_{int} = \frac{f}{2} N k_B T = \frac{f}{2} n R T$$

Caution...

Why do we need to say “when viewed from a reference frame at rest with respect to the center of mass of the system” ?

where the observer sees

$$\overline{v_x} = \overline{v_y} = \overline{v_z} = 0$$

Another way to put it: energy associated with the **bulk movement** of the gas is not part of E_{int} .

Question

What is the difference between heat and internal energy?

What is another way of transferring energy to a system?



Heat, Q



Heat is defined as the **transfer of energy** across the boundary of a system due to a temperature difference between the system and its surroundings.

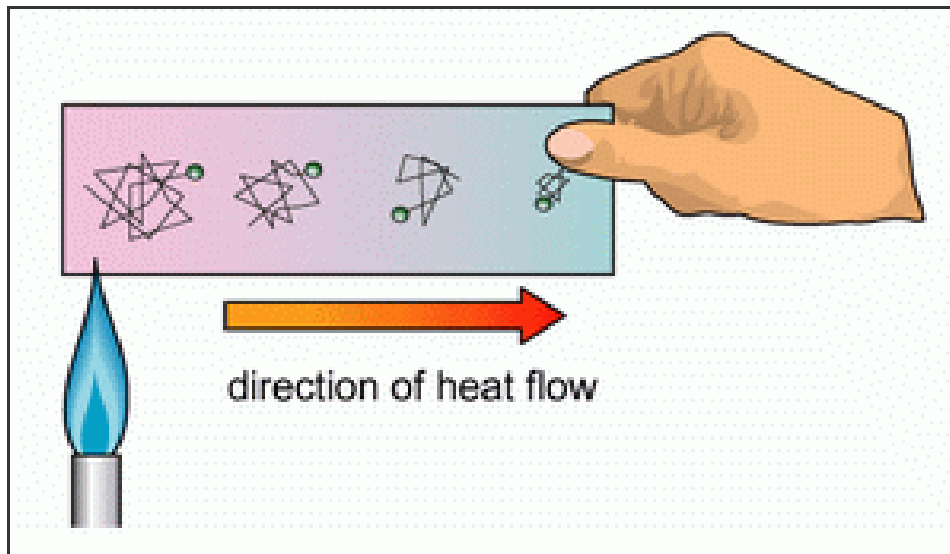
Heat is transferred from the body at the higher temperature to the body at a lower temperature.

Units = Joules, J

Heat does not mean “hot”

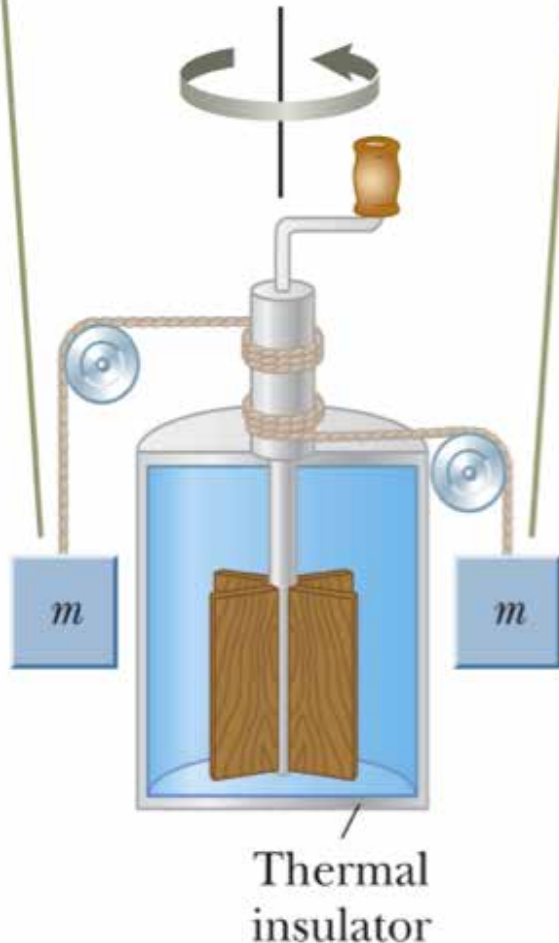
The heat that flows from hot to cold originates in the *internal energy* of the hot substance.

It is not correct to say that a substance contains heat.



"Oh, man! The coffee's cold!
They thought of everything!"

The falling blocks rotate the paddles, causing the temperature of the water to increase.



Joule's experiment to determine the mechanical equivalent of heat.

Gravitational potential energy of the blocks is transferred to the water via the paddles.

Work out the gravitational potential energy transferred to the water - this is how the calorie is defined!

How much energy required to raise temperature of 1 gram of water by 1°C.

$$1 \text{ calorie} = 4.186 \text{ J}$$

Warning!

$$\begin{aligned} \text{Food calorie} &= \text{"Calorie"} = 1000 \text{ calories} \\ &= 1 \text{ kcal} = 4.186 \text{ kJ} \end{aligned}$$

One other concept we need....

Energy (in a system) is always conserved.

Any transfer of energy into or out of a system will result in a change in the internal energy of the system, ΔE_{int} .

There are two ways to transfer energy into or out of a system.....

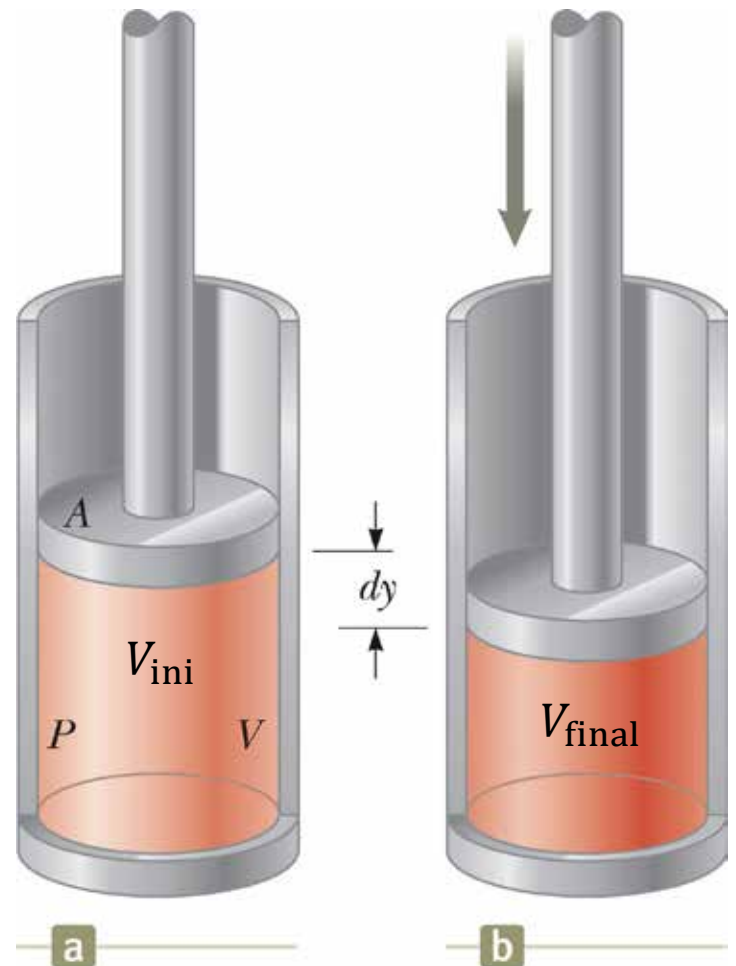
First Law of Thermodynamics

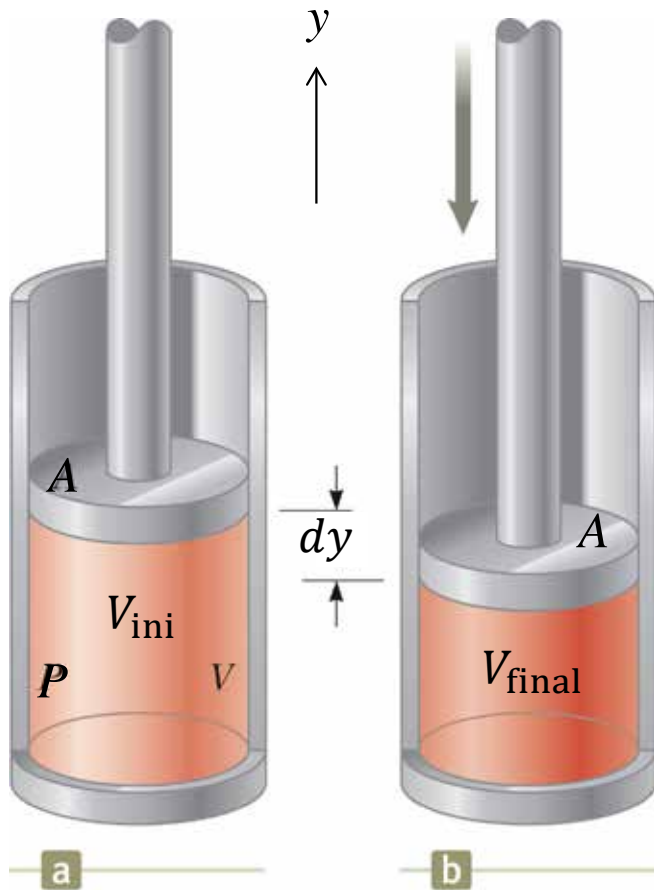
$$\Delta E_{int} = Q + W$$

Work done on a gas...

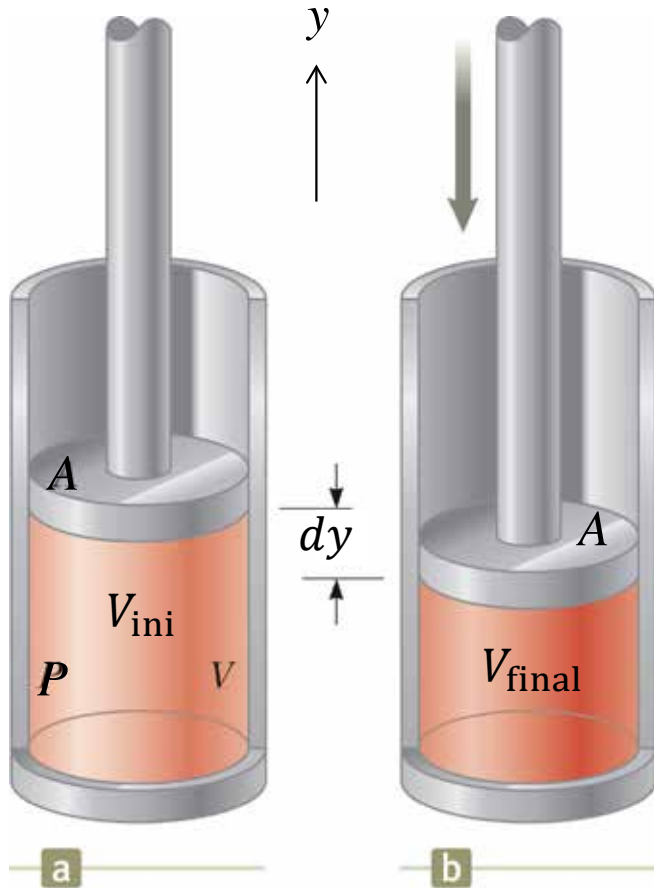
In this course we are mainly concerned with work done on or by a gas via compression or expansion.

How much work is done on the gas when the volume changes from V_{ini} to V_{final} due to the moving piston?





Assumption: the piston moves very slowly (quasi-statically), so that the gas is always in equilibrium within itself, i.e., the same P and T everywhere in the container at any one time.



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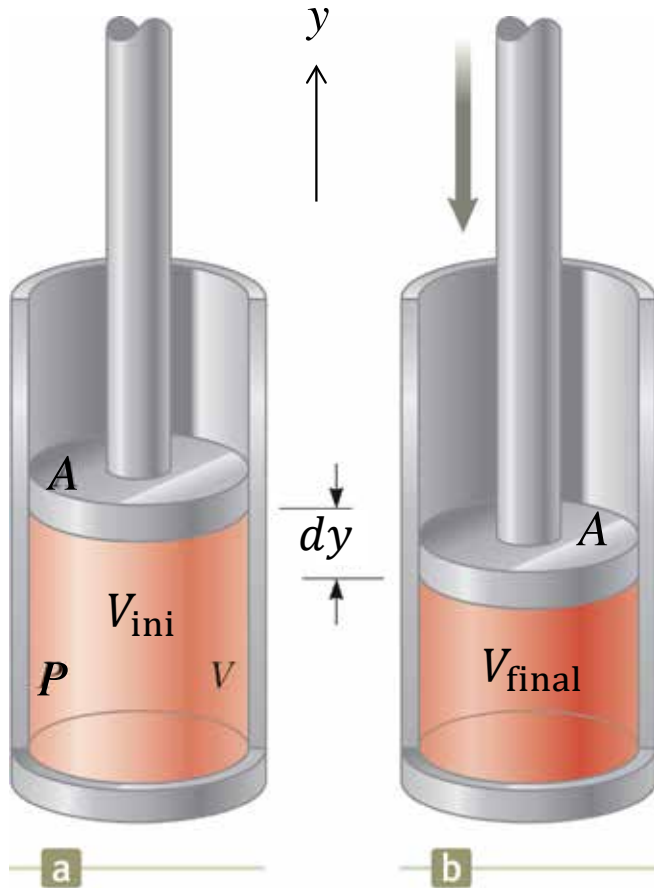
- As we push the piston down, we do **work against an upward force** due to the pressure of the gas:

$$F_y = PA \quad \leftarrow \text{Cross-sectional area of piston}$$

- **Incremental work** done on gas by pushing piston down by dy :

$$dW = -F_y dy = -PA dy$$

↑
Minus sign because the piston moves in the **opposite** direction of the pressure force.



Assumption: the piston moves very slowly (quasi-statically), so that the gas is always in equilibrium within itself, i.e., the same P and T everywhere in the container at any one time.

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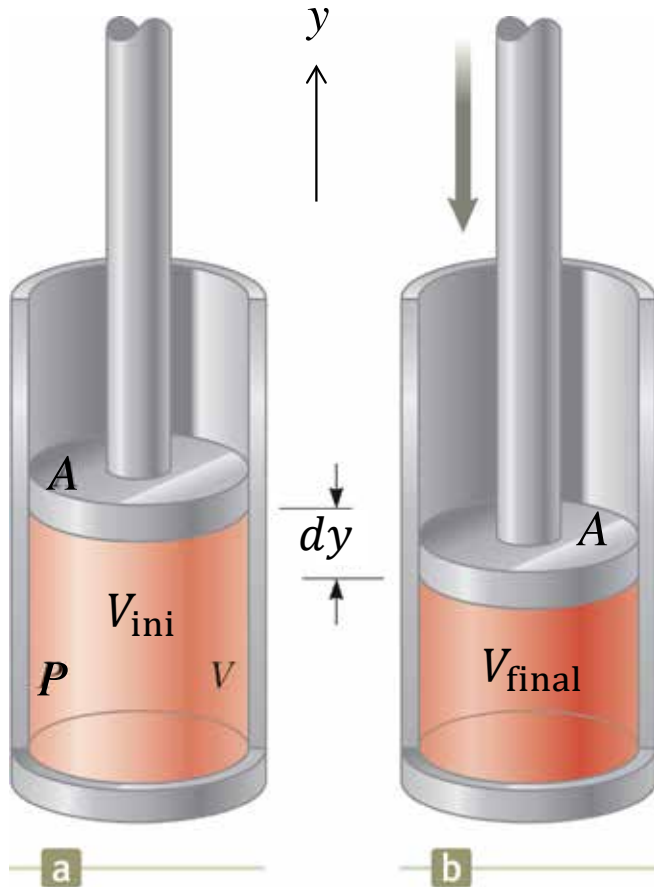
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$$dW = -F_y dy = -PA \underbrace{dy}_{dV}$$

Change in volume

Minus sign because the piston moves in the **opposite** direction of the pressure force.



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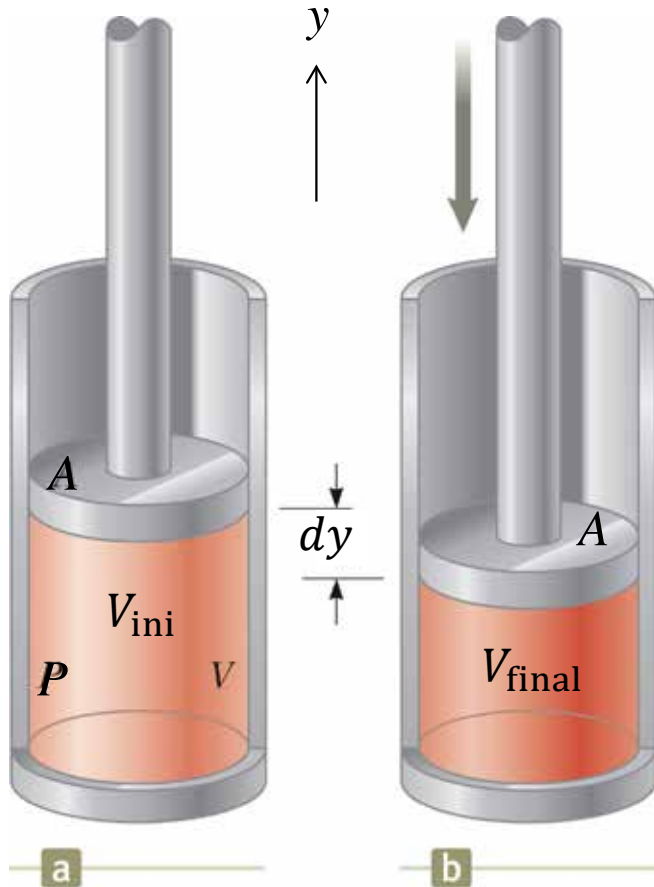
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Assumption: the piston moves very slowly (quasi-statically), so that the gas is always in equilibrium within itself, i.e., the same P and T everywhere in the container at any one time.

- **Incremental work done on the gas:**

$$dW = -PdV$$

If gas is compressed $dV < 0$ and W is positive and *work is done on the gas*.

If gas expands $dV > 0$ and *work is done by the gas*

→ **Total work done on gas:**

$$W = - \int_{V_{ini}}^{V_{final}} P dV$$

Work: A note on sign convention...

.Total work done **on** a gas:

$$W_{\text{ongas}} = - \int_{V_{\text{ini}}}^{V_{\text{final}}} P dV \quad \begin{array}{l} W_{\text{ongas}} > 0 \\ \text{if } V_{\text{final}} < V_{\text{ini}} \end{array}$$

. Total work done **by** a gas:

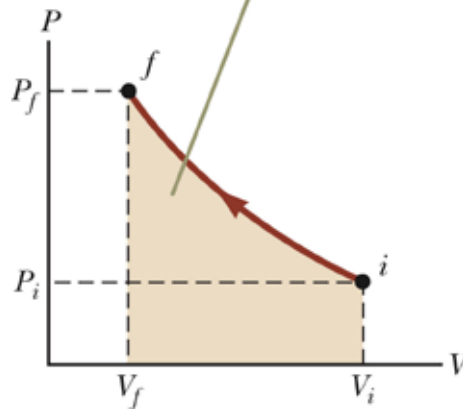
$$W_{\text{bygas}} = \int_{V_{\text{ini}}}^{V_{\text{final}}} P dV \quad \begin{array}{l} W_{\text{bygas}} > 0 \\ \text{if } V_{\text{final}} > V_{\text{ini}} \end{array}$$

→ The upper integration limit is **always** V_{final} , and the lower limit is **always** V_{ini} .

$$dW = -PdV$$

$$W = - \int_{V_i}^{V_f} PdV$$

The work done on a gas equals the negative of the area under the PV curve. The area is negative here because the volume is decreasing, resulting in positive work.



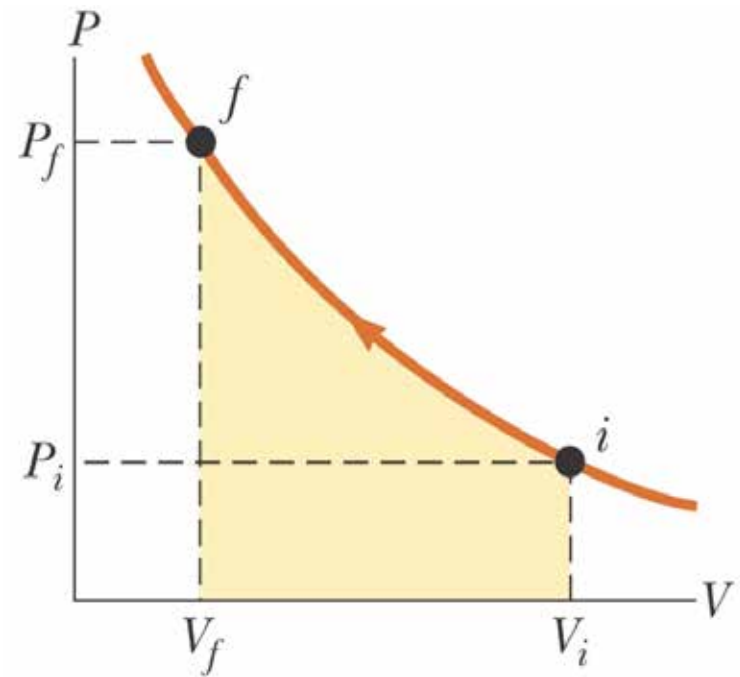
Work: PV diagram...

A nice way to visualise the work integral:

$$W_{\text{ongas}} = - \int_{V_{\text{ini}}}^{V_{\text{final}}} P dV$$

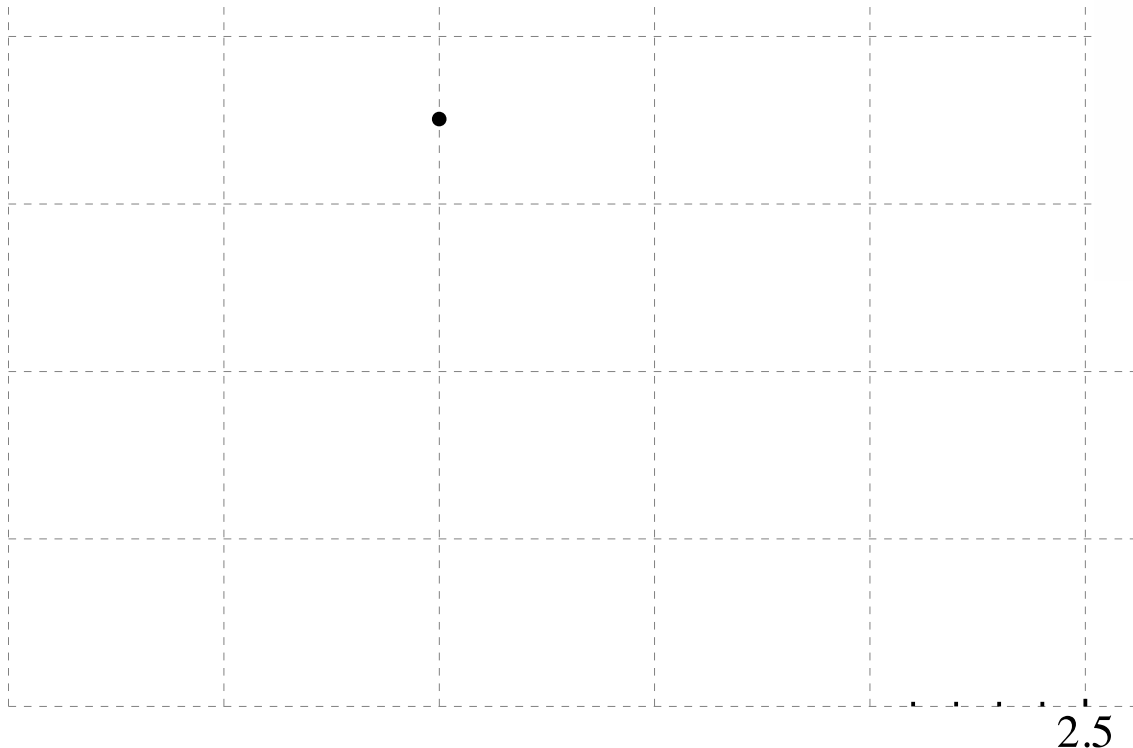
If $V_{\text{ini}} > V_{\text{final}}$, total work done *on the gas* is the **area** underneath the curve.

If $V_{\text{ini}} < V_{\text{final}}$, total work done *on the gas* is **negative** ...



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How much work is done on the ideal gas below?



Does the temperature of the gas change?

<https://goo.gl/forms/pPjl4RtPSavh8fZt2>

How about in this case?



Homework Set 4:

PHYS 1121:

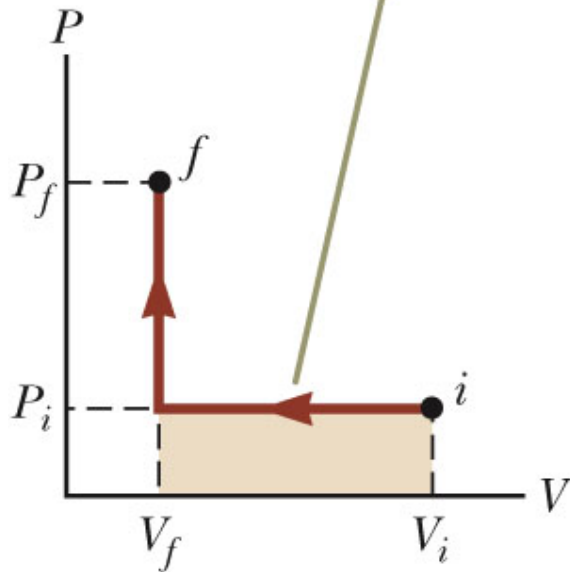
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PHYS 1131:

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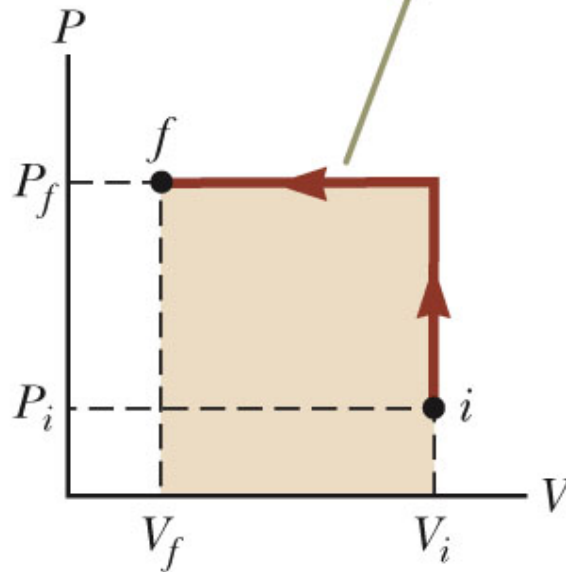
Work is different in these situations

A constant-pressure compression followed by a constant-volume process



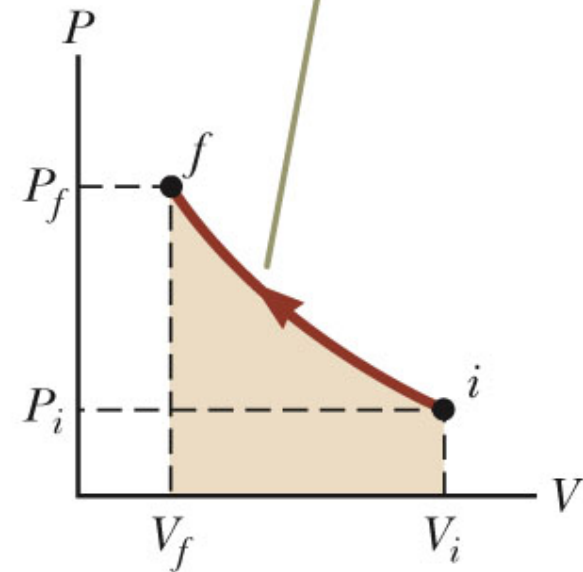
a

A constant-volume process followed by a constant-pressure compression



b

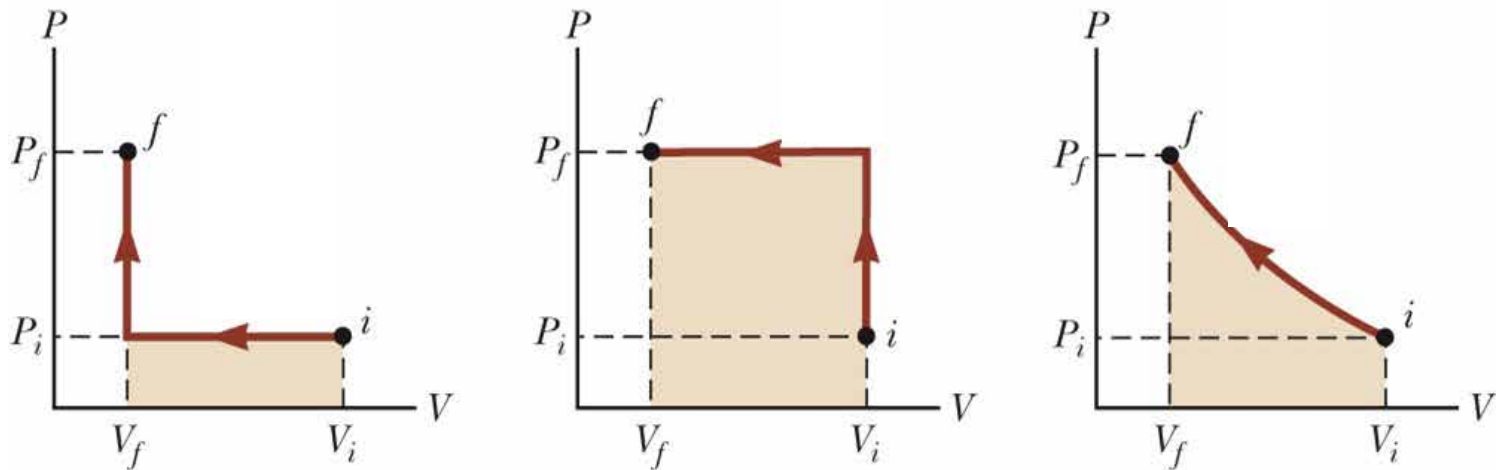
An arbitrary compression



c

Work > Work done via various paths...

How you go from an initial state to a final state on a PV diagram is called a **path**.



- For identical gas samples, these processes all have the same initial and final states.
- But the work done differs (different areas under curve).
- → **Work done depends on the path.**

Work > Work done via various paths...

.Path 1

– From A to B:

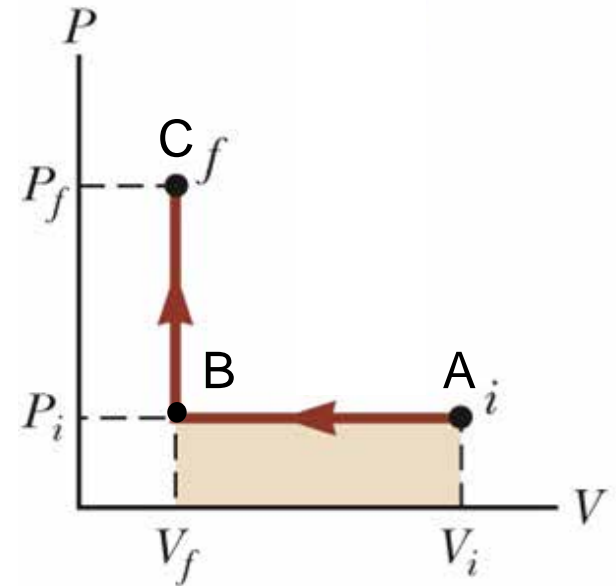
$$\begin{aligned} W_{\text{ongas}}^{A \rightarrow B} &= - \int_{V_i}^{V_f} P_i dV \\ &= -P_i(V_f - V_i) \end{aligned}$$

– From B to C:

$$W_{\text{ongas}}^{B \rightarrow C} = - \int_{V_f}^{V_f} P dV = 0$$

– Total work done on gas from A to B to C:

$$W_{\text{ongas}}^{\text{Path 1}} = W_{\text{ongas}}^{A \rightarrow B} + W_{\text{ongas}}^{B \rightarrow C} = -P_i(V_f - V_i)$$



Work > Work done via various paths...

.Path 2

– From A to B:

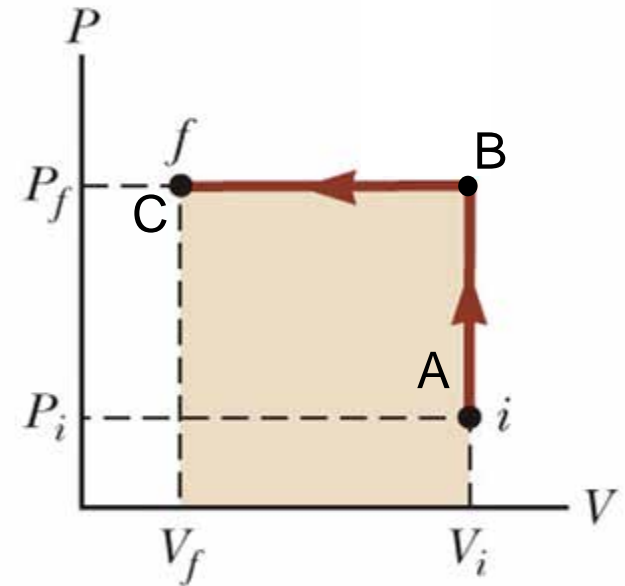
$$W_{\text{ongas}}^{A \rightarrow B} = - \int_{V_i}^{V_i} P dV = 0$$

– From B to C:

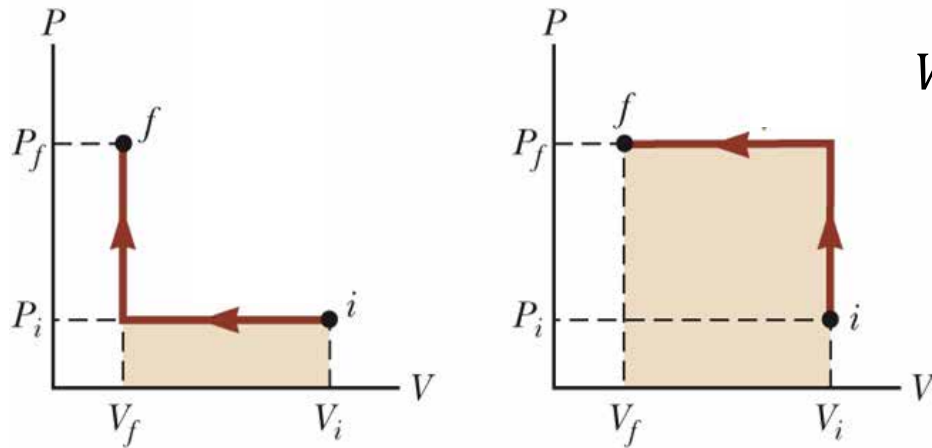
$$\begin{aligned} W_{\text{ongas}}^{B \rightarrow C} &= - \int_{V_i}^{V_f} P_f dV \\ &= -P_f(V_f - V_i) \end{aligned}$$

– Total work done on gas from A to B to C:

$$W_{\text{ongas}}^{\text{Path2}} = W_{\text{ongas}}^{A \rightarrow B} + W_{\text{ongas}}^{B \rightarrow C} = -P_f(V_f - V_i) > W_{\text{ongas}}^{\text{Path1}}$$



What happens to the extra work?



$$W_{\text{ongas}}^{\text{Path2}} > W_{\text{ongas}}^{\text{Path1}}$$

Where does the extra energy go?

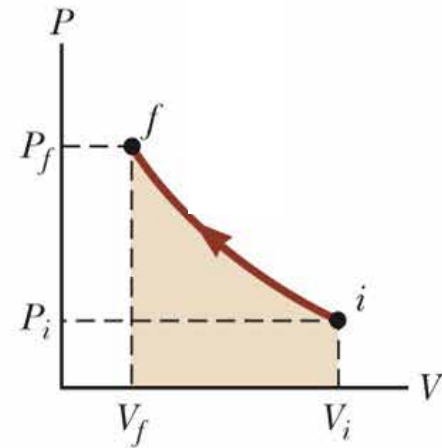
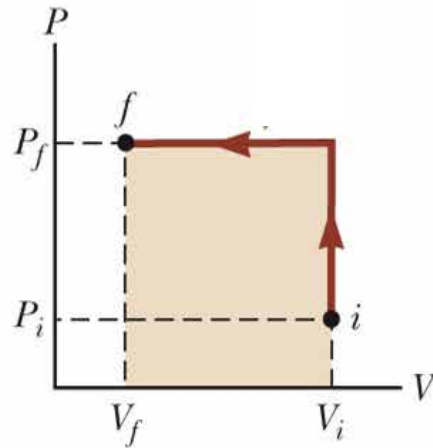
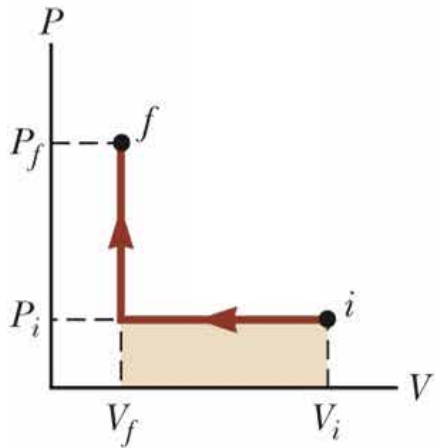
- Makes the gas hotter
- Stored in internal energy of the gas
- Lost as heat
- Lost due to friction



<https://goo.gl/forms/pPjl4RtPSavh8fZt2>

First law of thermodynamics...

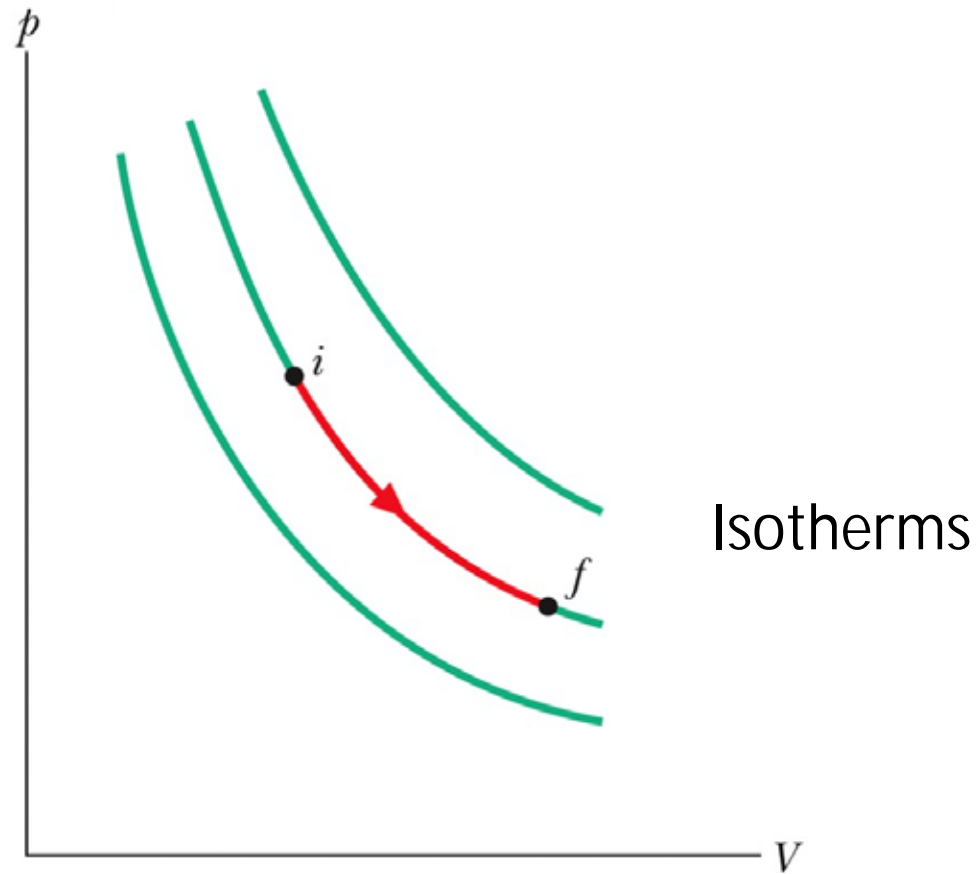
For identical gas samples, the initial **internal energies** of the three processes are identical, as are the final internal energies (regardless of the path taken).



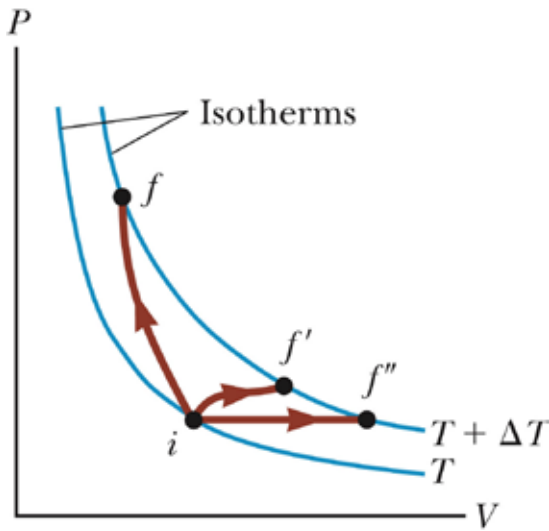
$$E_{int}^{final} = \frac{f}{2} nRT_{final} = \frac{f}{2} P_{final} V_{final}$$

A property of the final state;
no dependence on the path
at all!

PV plots



Molar specific heats



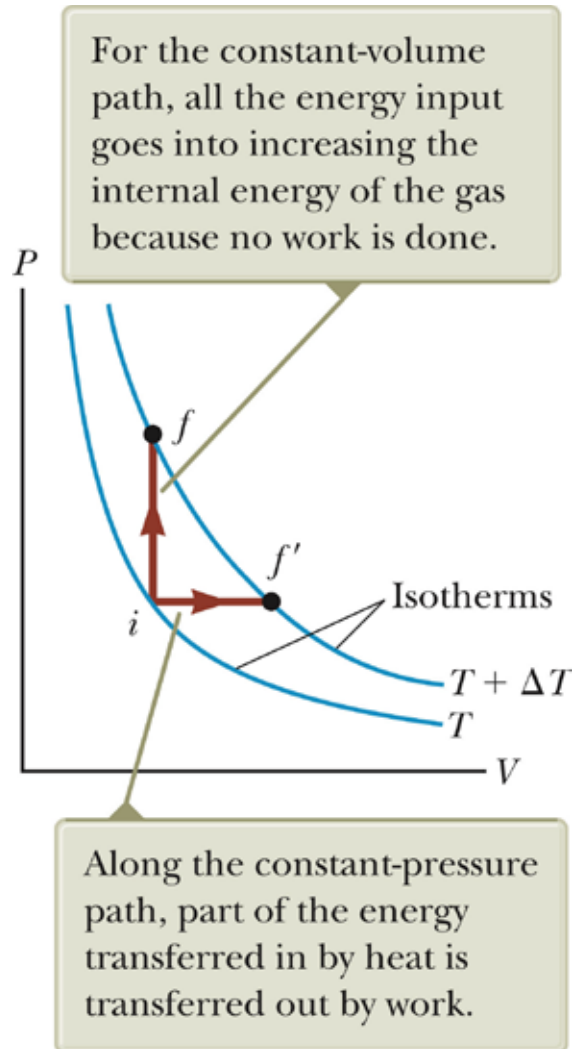
Gasses have a specific heat. We state molar specific heats for gasses, that is the amount of heat needed to raise one mole of a gas by 1 Kelvin.

We can write down a generic equation like this:

$$\begin{array}{ccccc} \text{Heat required} & \longrightarrow & Q = n c_{\text{molar}} \Delta T & \longleftarrow & \text{(Final - initial) temperature} \\ & & \nearrow & & \\ & & \text{Number of moles of gas} & & \end{array}$$

c_{molar} is a property of the type of gas under consideration.
But there's a bit more to it...

But the molar gas constant isn't constant



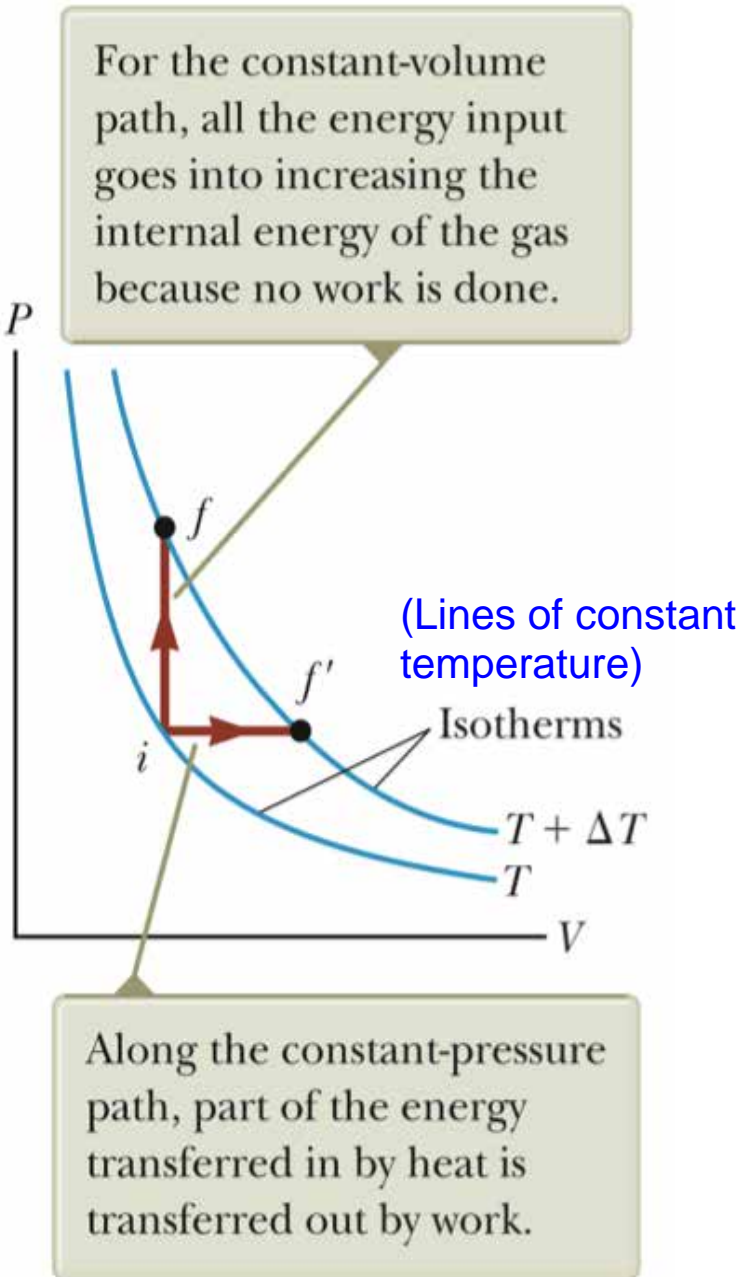
$$\Delta E_{int} = Q + W$$

$$E_{int} = \frac{3}{2} N k_B T$$

Monatomic ideal gas

$$\Rightarrow \Delta E_{int} = \frac{3}{2} N k_B \Delta T$$

If temperature changes a fixed amount the internal energy changes a fixed amount.



.The molar specific heat of gases is path-dependent.

$$\Delta E_{int} = Q + W$$

.Example:

– f and f' have the same internal energy.

– Q needed to raise T by ΔT :

$$Q_{i \rightarrow f} = \Delta E_{int} = \frac{f}{2} n R \Delta T$$

$$\begin{aligned} Q_{i \rightarrow f'} &= \Delta E_{int} - W_{i \rightarrow f'} \\ &= \Delta E_{int} + |W_{i \rightarrow f'}| \end{aligned}$$

Work is done by gas; $W < 0$

$i \rightarrow f'$ requires more heat than $i \rightarrow f$.

Molar specific heat...

.Because of the path dependence, we define **two different molar specific heats**:

- Molar specific heat at **constant volume** c_V :

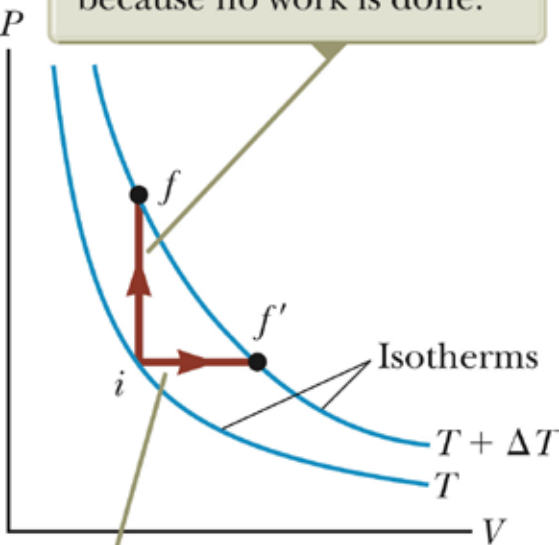
Heat required \rightarrow $Q = nc_V\Delta T$ \leftarrow (Final – initial) temperature

Number of moles of gas

- Molar specific heat at **constant pressure** c_P :

$$Q = nc_P\Delta T$$

For the constant-volume path, all the energy input goes into increasing the internal energy of the gas because no work is done.



Along the constant-pressure path, part of the energy transferred in by heat is transferred out by work.



<https://goo.gl/forms/pPjI4RtPSavh8fZt2>

Question

Which is larger: C_V or C_P ?

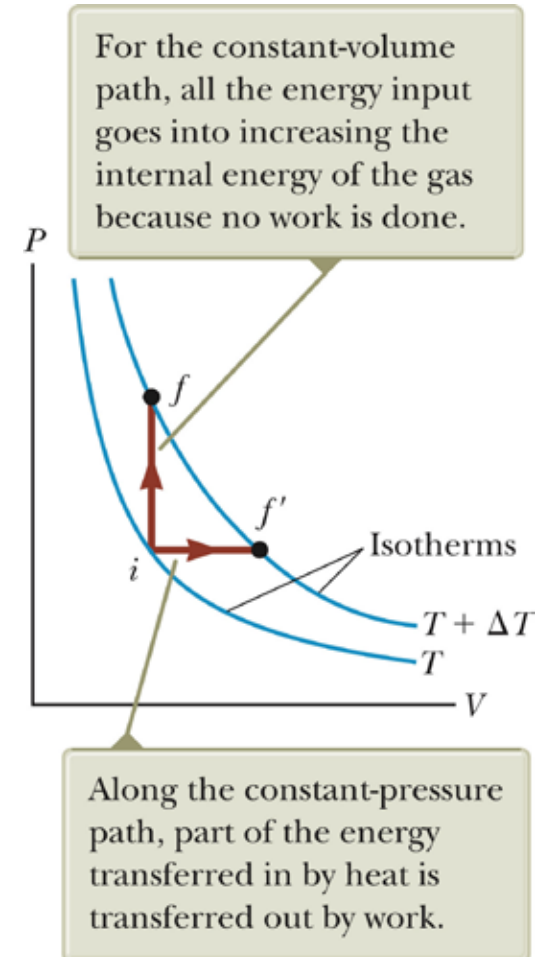
$$\Delta E_{int} = Q + W$$

Hint:

$$Q = nC_V \Delta T$$

$$Q = nC_P \Delta T$$

$$W = - \int_{V_i}^{V_f} P dV$$



Molar specific heat > Constant volume...

.Constant volume means no work is done on or by the gas.

–From the first law of thermodynamics:

$\Delta E_{int} = Q$

$\Delta E_{int} = \frac{f}{2}nR\Delta T$

$Q = \frac{f}{2}nR\Delta T$

$Q = nc_V\Delta T$

$c_V = \frac{f}{2}R$

Molar specific heat at constant volume

Universal gas constant

Gas molecules' number of degrees of freedom, e.g., $f=3$ for monatomic molecules

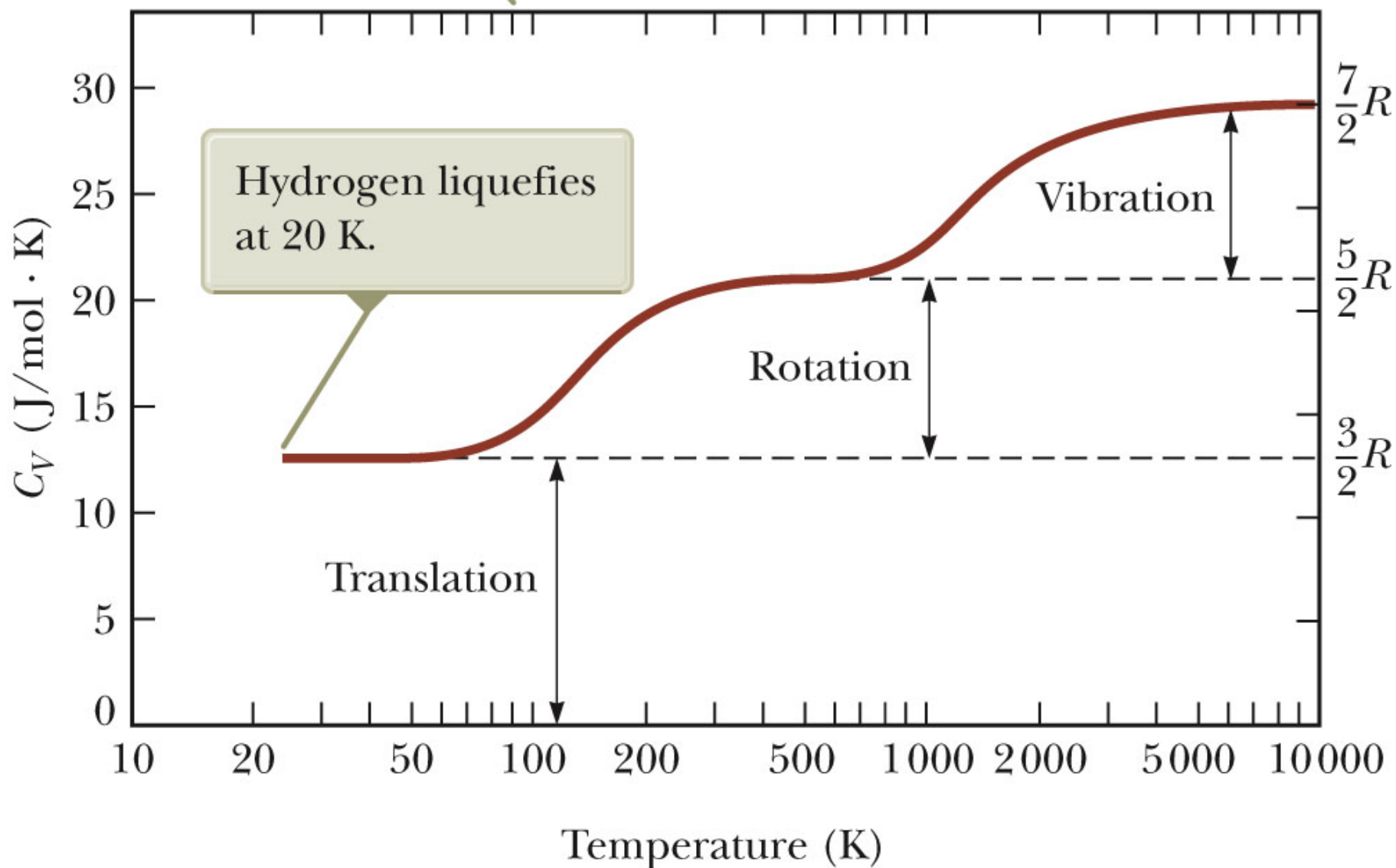
Table 21.1

Molar specific heats of various gases

Molar specific heat (J/mol.K) ^a				
Gas	C_p	C_v	$C_p - C_v$	$\gamma = C_p/C_v$
<i>Monatomic gases</i>				
He	20.8	12.5	8.33	1.67
Ar	20.8	12.5	8.33	1.67
Ne	20.8	12.7	8.12	1.64
<i>Diatomic gases</i>				
H ₂	28.8	20.4	8.33	1.41
O ₂	29.4	21.1	8.33	1.40
Cl ₂	34.7	25.7	8.96	1.35
<i>Polyatomic gases</i>				
CO ₂	37.0	28.5	8.50	1.30
H ₂ O	35.4	27.0	8.37	1.30
CH ₄	35.5	27.1	8.41	1.31

^a All values except that for water were obtained at 300 K.

The horizontal scale is logarithmic.



Molar specific heat: Constant pressure...

.From the first law of thermodynamics:

$$\Delta E_{int} = Q + W$$

.We can show that : BLACKBOARD

$$c_P = \frac{f + 2}{2} R = c_V + R$$