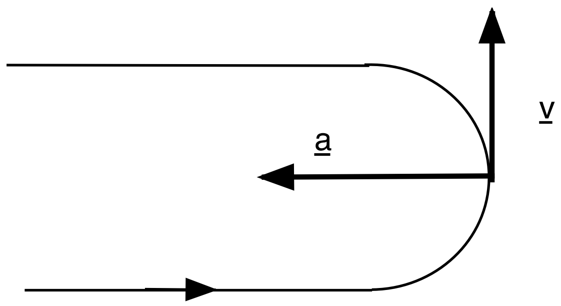
Exam Question 1 PHYS 1131 2013, Session 2



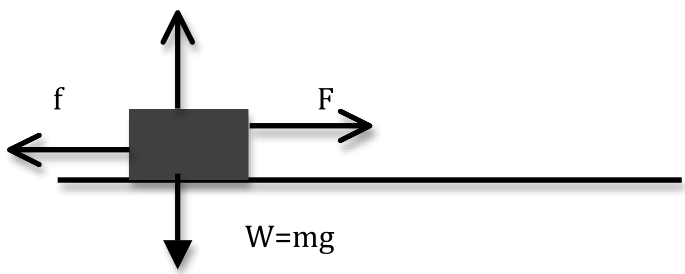
a) i) $\mathbf{v} = v\mathbf{j}$

ii) $\mathbf{a} = -\frac{v^2}{r}\mathbf{i}$

iii) $\mathbf{a}_{av} = (\mathbf{v}_{final} - \mathbf{v}_{initial})/\text{time}$. But time = $\frac{\pi r}{v}$

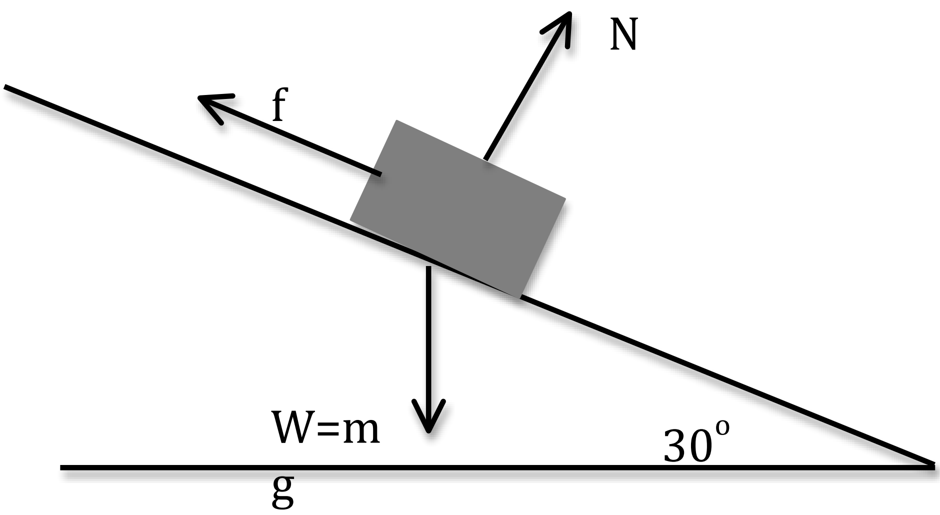
$= \frac{2}{\pi}\frac{v^2}{r}\mathbf{i}$

b)



c) 0 ms^{-2}

d) i)



The direction of the acceleration is parallel to the ramp, in the downward direction.

The forces in the y-dir (perpendicular to ramp) are

N, and mg cos 30°. There is no motion in the y-dir, so

$$\begin{aligned}
\sum F_y &= N - mg\cos\theta \\
0 &= N - mg\cos\theta \\
N &= mg\cos\theta
\end{aligned}$$

In the x-dir

$$\begin{aligned}
\sum F_x &= N - mg\sin\theta \\
ma_x &= mg\sin\theta - f \\
ma_x &= mg\sin\theta - \mu N \\
ma_x &= mg\sin\theta - \mu mg\cos\theta \\
a_x &= g\sin\theta - \mu g\cos\theta \\
&= 9.8\sin(30°) - 0.40(9.8)\cos(30°) \\
&= 1.5ms^{-2}
\end{aligned}$$

As the only acceleration is in the x-direction, this is the magnitude of the total acceleration.

iii) The initial velocity is 1.0 m/s^-1 in the x-dir (down the ramp):

The length of the ramp is 10.0/sin(30°) = 20.0 m

a=1.5 m/s^-2

$$\begin{aligned}
v^2 &= u^2 + 2as \\
v^2 &= 1.0^2 + 2(1.5)(20.0) \\
v^2 &= 61 \\
v &= 7.8m/s
\end{aligned}$$

iv) PE = mgh = 3430 J

v)
Final KE = ½(m)v^2 = 0.5 x 35 x61 = 1068 J

But the crate had a small amount of initial KE from the initial speed of 1 km/s.

So, the work done against friction will be

$$W(f) = (PE(i) + KE(i)) - (PE(f) + KE(f))$$
$$= 3430 + 0.5(35)(1^2) - (0 + 1068)$$

$$= 2380J$$

Work done by person pushing involves a non-conservative force. This work is positive.

Gravity is a conservative force. The work it does on the crate as it slides down the ramp is positive.

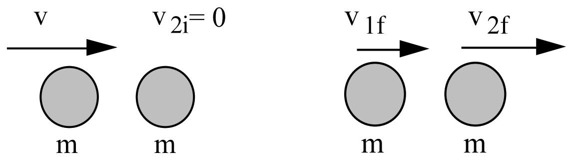
Friction is a non-conservative force. The work friction does on the crate is negative (in both the horizontal slide and the descent).

Q2

a) i) If non-conservative forces do no work, mechanical energy is conserved.

ii) In a completely elastic collision, mechanical energy is conserved.

iii) All initial motion is in x direction, so no y momentum. After the collision the second particle travels in the x direction. So the second particle must also travel in the x direction, if at all.



neglect external forces $\Rightarrow$ $p_i = p_f$

$$mv + 0 = mv_1 + mv_2 \qquad \text{(i)}$$

Elastic collision so mechanical energy conserved

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \qquad \text{(ii)}$$

(i) –> $v_{2f} = v - v_{1f}$ (iii)

substitute in (ii) –>

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}m(v^2 + v_1^2 - 2vv_1)$$

$$\therefore \quad 0 = v_1^2 - vv_1$$

$$0 = v_1(v_1 - v) \qquad \textit{2 solutions}$$

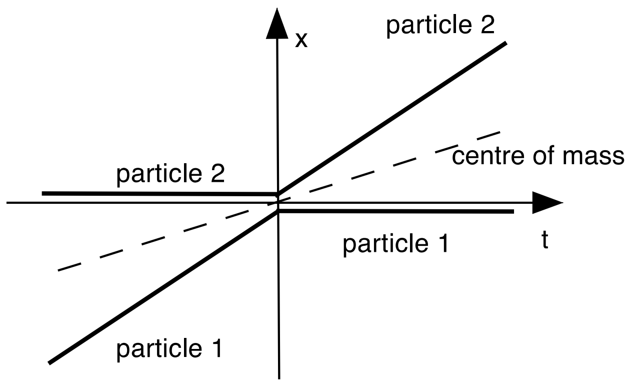
Either: $v_1 = 0$ and (iii)–> $v_2 = v$

i.e. 1st stops dead, all p and K transferred to m_2

or: $v_{1f} = v$ and (iii)–> $v_2 = 0$ i.e. missed it.

We are told that v_2 is non-zero so we keep the first solution: $v_1 = 0$ and $v_2 = v$

iv)



(On this sketch, two lines have been slightly displaced from the x=0 axis, for clarity.)

No external forces, so there is no acceleration of the centre of mass. (So its velocity is constant and lies halfway between the two masses.)

b) It rolls without slipping, so friction does no work, so mechanical energy is conserved.

$K_i + U_i = K_f + U_f$ so

$\frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = 0 + mgh$

Equation sheet has I for a sphere = $2/5\ mr^2$. Rolling so $\omega = v/r$

$\frac{1}{2} mv^2 + \frac{1}{2} (2/5\ mr^2)(v/r)^2 = mgh$

$\frac{1}{2} mv^2 + 1/5\ mv^2 = mgh$

$(7/10)v^2 = gh$

$h = 7v^2/10g$

c) $I = \int_{body} r^2\, dm$

Write $M = \lambda L$ where the mass per unit length $\lambda = M/L$

So for an element of mass $dM = \lambda dx$

$I = \int_{x=0}^{L} x^2 \lambda dx = \frac{1}{3}\lambda(L^3 - 0)$ but $\lambda L = M$ so

$I = \frac{1}{3} ML^2$.

Question 3

i) $a_c = r\omega^2 = r\left(\frac{2\pi}{T}\right)^2$

ii) magnitude of gravitational force between star and planet = centripetal force on the star. Equating magnitudes:

$$Ma_c = |F_g|$$

$$Mr\left(\frac{2\pi}{T}\right)^2 = \frac{GMm}{R^2}$$

$$m = \left(\frac{2\pi}{T}\right)^2 \frac{Rr^2}{G}$$

iii) EITHER From Newton's third law, the gravitational forces on the star and the planet have equal magnitude. They orbit their common centre of mass with the same period T. Hence

$$Mr\left(\frac{2\pi}{T}\right)^2 = mR'\left(\frac{2\pi}{T}\right)^2$$

OR They orbit around their centre of mass. If it is at the origin, then $\Sigma\, m_i r_i = 0$

Both of these give mR' = Mr.

From the diagram, R = r + R' = r(1 + R'/r) = r(1 + M/m)

(iv)
$$\begin{aligned} P &= \sigma A e T^4 \\ &= 5.67\times10^{-8}\times 4\pi(10000\times10^3)^2\times1\times2000^4 \\ &= 1.14\times10^{21}W \\ &= 1\times10^{21}W \text{ (1 sig fig.)} \end{aligned}$$

note: since 2000 K>> 3 K you do not need to do $T^4 - T_0^4$.

(v) Energy falling on planet each second is the power. Will need to consider the geometry in order to work this out.

$$\begin{aligned} P_{planet} &= \frac{\text{area of planet intersecting rays}}{\text{area over which light radiated}}P \\ &= \frac{\pi r_p^2}{4\pi R_{bd}^2}\times P \\ &= \frac{1000^2}{4\times(10^6)^2}\times 1.14\times10^{21} \\ &= 2.85\times10^{14}W \\ &= 3\times10^{14}W \text{ (1 sig fig.)} \end{aligned}$$

(vi)
$$\begin{aligned} P &= \sigma A e(T_p^4 - T_s^4) \\ &= \sigma\times 4\pi r_p^2\times0.9\times(T_p^4 - T_s^4) \end{aligned}$$

(vii) In thermal equilibrium power radiated is equal to power received.

$$\begin{aligned} &2.85\times10^{14} = 5.67\times10^{-8}\times4\pi(1000\times1000)^2\times0.9\times(T_p^4 - 3^4) \\ &\Rightarrow T_p^4 - 3^4 = 4.44\times10^8 \\ &T_p = 145K = 100K \text{ (1 sig fig)} \\ &\text{ OR } 400^oC \end{aligned}$$

It will be a frozen ocean!

Question 4

a) i)
$$\Delta L = \alpha L \Delta T$$
$$= 24 \times 10^{-6} \times 1.00 \times 160$$
$$= 3.84 \times 10^{-3} \text{ cm}$$

ii)
$$\Delta A = 2\alpha A \Delta T$$
$$= 2 \times 24 \times 10^{-6} \times 1.00 \times 160$$
$$= 0.154 \text{ cm}^2$$

iii)
$$A = 2\pi r^2 + 2\pi r h$$
$$20.0 = 2\pi r^2 + 2\pi r \times 1$$
$$3.183 = r^2 + r$$
$$\Rightarrow r = \tfrac{-1 \pm \sqrt{1-4\times -3.183}}{2}$$
$$r = 1.353 cm$$
$$V = \pi r^2 h = 5.751 cm^3$$
$$Q = mc\Delta T = 2.70 \times 10^{-3} \times 5.751 \times 910 \times 160$$
$$= 2260 \text{ (3 sig. fig.)}$$

b) i)
$$PV = nRT$$
$$T = \frac{PV}{nR} = \frac{5.51 \times 1.01 \times 10^5 \times 20 \times 10^{-3}}{2.5 \times 8.314}$$
$$= 535 \text{ K}$$

ii)
$$W = -\int P dV$$
$$= P\Delta V$$
$$= 1.53 \times 1.01 \times 10^5 \times 30 \times 10^{-3}$$
$$= 4636 \text{J}$$
$$= 4640 \text{J is done on the gas}$$

iii)
$$\Delta E_{int} = \frac{f}{2} nR\Delta T = 2.50 \times \frac{5}{2} \times 8.314 \times (534.49 - 148.69)$$
$$= 20099 \text{ J}$$
$$= 20100 \text{ J (3 sig fig)}$$

iv)
$$\text{adiabatic } \Rightarrow PV^\gamma = \text{constant}$$
$$\gamma = \frac{7/2}{5/2} = 1.4$$
$$PV^{1.4} = 5.51 \times 1.01 \times 10^5 \times (20 \times 10^{-3})^{1.4}$$
$$= 2328 \text{ Pa m}^{4.2}$$
$$= 2330 \text{ Pa m}^{4.2} \text{ (3 sig fig)}$$

v) One way is to add the changed in internal energy around the cycle:

$$W = \Delta E_{int} = -(\Delta E_{intC\to A} + \Delta E_{intB\to C})$$
$$\Delta E_{intB\to C} = Q_{B\to C} + W_{B\to C} = -16200 + 4640 = -11560$$
$$\Rightarrow W = -(20099 - 11560)$$
$$= -8540 \text{ J}$$

Alternatively you could do the integral (harder though…):

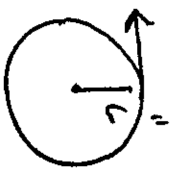
$$\begin{aligned} W &= -\int PdV \\ &= -\int_{20L}^{50L} \frac{2330}{V^{1.4}} dV \\ &= \frac{2330}{0.4}[V^{-0.4}]_{20\times10^{-3}}^{50\times10^{-3}} \\ &= 5825 \times [3.3 - 4.78] \\ &= -8562 \text{ J} \\ &= -8560 \text{ J (3 sig fig)} \end{aligned}$$

The slight difference in the answers is down to how many significant figures were kept in the working. Either answer is acceptable.

PHYS 1131 2013-2 Q5

Total 30 marks

(a)



(2) (i) $T = \frac{2\pi r}{V} = \frac{2\pi \cdot 3}{4} = \frac{3}{2}\pi = 4.7$ m/s to 2SF

$f = \frac{1}{T} = 0.21$ Hz to 2SF

(2) (ii) $(x, y) = (0, 3m)$ at $t = 0s$.

let $x = A \sin(\omega t + \phi_1)$
$y = A \cos(\omega t + \phi_2)$ $\Rightarrow A = 3$ since $x_{max}, y_{max} = 3$

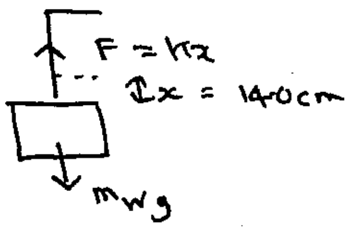
Then $x: 0 = A \sin(\phi_1) \Rightarrow \phi_1 = 0$
$y: 3 = A \cos(\phi_2)$
$= 3 \cos \phi_2$ $\Rightarrow \cos \phi_2 = 1 \Rightarrow \phi_2 = 0$

ie $x = 3 \sin \omega t$
$y = 3 \cos \omega t$ with $\omega = 2\pi f = \frac{2\pi \cdot 2}{3\pi} = \frac{4}{3} = 1.3 s$

(b) $m_b = 4.50g$

$m_w = 1.63kg$

(3) (i)



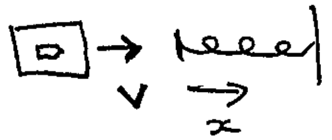
We have $F = kx = m_w g$ in eqvilibrium

$$\Rightarrow \quad k = \frac{m_w g}{x} = \frac{1.63 \times 9.81}{0.140} \text{ N/m}$$

$$= \cancel{112.52}\ 114.22 \text{ N/m}$$

$$\underline{k = 114 \text{ N/m}} \text{ to 3 SF}$$

(ii)
(5)



$x = 13.0cm$

Let bullet + block move at V when reachs spring

Assuming energy conservation: [Motion across frictionless not relevant]

i.e. KE lost by bullet/block = PE gained by spring on compression

ie $\frac{1}{2}(m_b+m_w)V^2 = \frac{1}{2}kx^2$

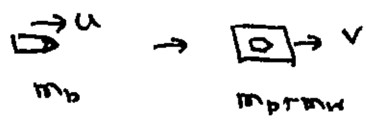
$$\Rightarrow V^2 = \frac{kx^2}{m_b+m_w} = \frac{114 \times (0.13)^2}{[1.63 + 0.0045]}$$

$$= 1.181 \quad (m/s)^2$$

$$\Rightarrow V = 1.087 \quad m/s$$

$$\underline{= 1.09} \quad m/s \text{ to 2SF}$$

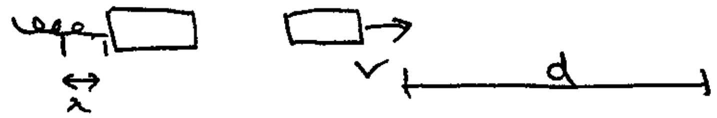
(iii) Now conserving momentum between
(4) bullet and bullet-block



$$m_b u = (m_b+m_w)V$$

$$\Rightarrow u = \left(\frac{m_b+m_w}{m_b}\right)V = \left(\frac{0.0045+1.63}{0.0045}\right)1.087 = 394.7 \text{ m/s}$$

$$= 395 \text{ m/s to 3SF}$$



(c) (i) PE of Spring is converted to Kinetic Energy of Spring/Bullet. This is then lost through friction on the table.

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Initial $PE = \frac{1}{2} kx^2 = \frac{1}{2} m v^2$

x ~~=~~ compression spring

v Speed bullet + block.

~~So~~ Energy lost by friction = Force × Distance

$$= \mu_K (m_b + m_w) g \times d$$

So $\frac{1}{2} k x^2 = \mu_K (m_b + m_w) g d$

$$\Rightarrow \mu_K = \frac{\frac{1}{2} k x^2}{(m_b + m_w) g d}$$

$$= \frac{0.5 \times 1142 \times 0.09^2}{(0.0045 + 1.63)\, 9.81 \,.\, 0.42} = 0.0687$$

$$= 0.07 \text{ to 2SF}$$

(ii) This Mechanical Energy lost is

4 $E = \mu_K (m_b + m_w) g d = 0.07\ (0.0045 + 1.63)\ 9.81\ (0.42 + 0.13)$

$= 0.46\ J = 0.63 J.$

$\underset{\text{Slide}}{0.42}$ $\underset{\text{Compress Spring}}{0.13}$

Initial KE of bullet is $\frac{1}{2} m_B v^2$

$= \frac{1}{2} (0.0045) \times 395^2$

$= 350 J \gg E_{\text{Friction}}$

Thus it is a good approximation to ignore the friction of the block/surface in comparison to initial KE

But KE of block + bullet. $= \frac{1}{2} \times 1.63 \times 1.09^2$

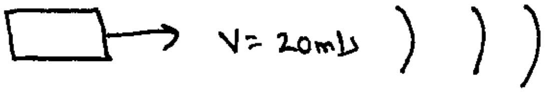
$= 0.968 J \approx$ Mechanical Energy lost

$\Rightarrow$ so not a good approximation in comparison to that transferred on the impact block/bullet

ANY REASONABLE DISCUSSION WITH CORRECT PHYSICS OK

(d)

4



$\leftarrow$ $V_W = 10.0$ m/s

$f = 900$ Hz

$\rightarrow$ $c = 343$ m/s

(i) For the man the frequencies are unchanged as he is at rest wrt to the source

4

ie $f_{man} = 900.0$ Hz

For the Woman apply doppler shift formula for a moving source

$$f' = f\left(\frac{V + V_0}{V - V_s}\right)$$

where V_0 speed observer $= 0$

V_s — source

V — sound

$$= 900\left(\frac{343}{343 - 20}\right) \text{ Hz}$$

$= 955.7$ Hz

$= 956$ Hz to 3SF

(ii) Speed of sound is now effectively $c = V + V_W$

3

(V_W wind speed)

So doppler formula becomes

$$f' = f\left(\frac{V + V_W}{V + V_W - V_s}\right) = 900\left(\frac{343 + 10}{343 + 10 - 20}\right)$$

$= 954.05$ Hz

For woman $= \underline{954 \text{ Hz}}$ to 3SF

For man, frequency is unchanged.