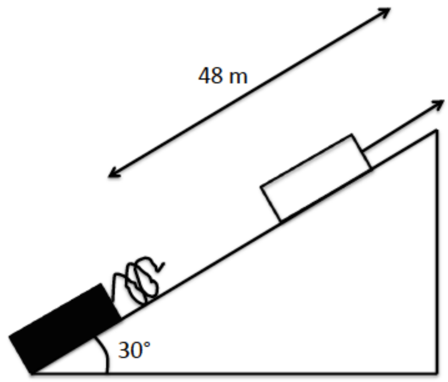
Question 1 (Marks: 20)

A block of mass $m = 5.1$ kg is being pulled by a string along an inclined plane as shown in the figure. The plane has a coefficient of kinetic friction $\mu_k = 0.40$. The angle between the plane and the horizontal is 30° (2 significant figures).



(a) Draw a free-body diagram showing all the forces acting on the block.

(b) The block is accelerated up the plane at a rate of $3.0\ \mathrm{m.s^{-2}}$. Determine the tension in the string.

At the bottom of the plane is a spring. The length of the spring is negligible in comparison with the length and height of the inclined plane. (Air resistance is negligible throughout, but friction is not negligible). The block is pulled up to the top of the inclined plane. At this point the block is released from rest and slides all the way to the bottom of the plane, where it comes into contact with the spring and eventually comes (briefly) to rest, after having slid a total distance of 48 m along the plane.

(c) Determine the work done by the gravitational force on the block between its release at the top and the moment when it comes to rest on the spring.

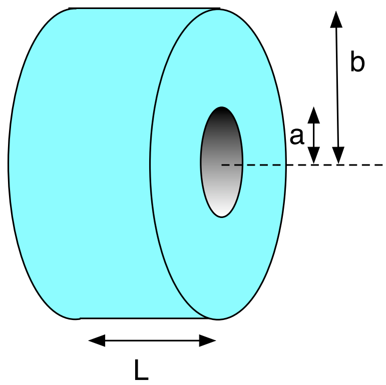
(d) Determine the work done by the frictional force on the block between its release at the top and the moment when it comes to rest on the (light) spring.

(e) The spring has a spring constant $k = 95\ \mathrm{kN.m^{-1}}$. Determine the compression of the spring when the block comes to rest. You can ignore the effects of friction while the block is interacting with the spring.

Question 2 (Marks: 20)

(a) (i) Define moment of inertia (also called rotational inertia). If your definition is an equation, define all terms.

(ii) The hollow disc in the sketch has a thickness L, and inner and outer radii a and b. It is made of homogeneous material of density ρ. From the definition in (i), derive an expression for the moment of inertia of this disc about its axis of symmetry.



(b) State Newton's 2nd law for rotation. If your definition is an equation, define all terms.

(c) The wheel of a light aircraft has radius r = 28 cm and moment of inertia I = 1.1 kg.m^2. It turns freely on its axle. Just before landing, it is a small distance above the runway, and is not rotating. The plane is travelling at 48 m.s^{-1}. The pilot lands surprisingly gently so that the normal force exerted on the runway by the wheel increases very rapidly to 960 N and then remains constant. The velocity of the plane also remains constant for this part of the problem. The coefficients of kinetic and static friction between wheel and tarmac are 0.85 and 0.95 respectively. Determine the length of time for which the wheel skids.

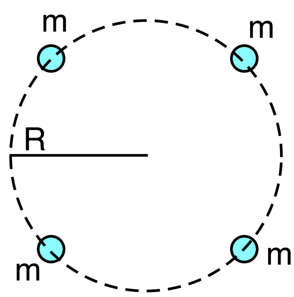
(d) Suppose that, instead of letting the wheel turn freely, the pilot applies the wheel brakes. Does the wheel skid for less time or more time with the brakes on? Explain your answer in two or three brief sentences.

Question 3 (Marks: 20)

(a) From the definition of potential energy and Newton's law of gravitation, derive an algebraic expression for the potential energy of a system comprising a very large mass M and another very much smaller mass m at separation r.

(b) Using one of Newton's laws of motion and Newton's law of universal gravitation, derive an algebraic expression for the kinetic energy of small mass m in circular orbit with radius r about a much larger mass M.

(c) An improbable constellation. Four stars, each of mass m, are in a circular orbit with radius R around their centre of mass, as shown. At any instant, the centres of the four stars form a square, as shown in the sketch.



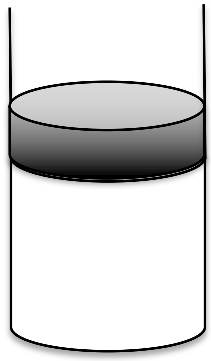
(i) Using Newton's law of universal gravitation, find a simple expression for the magnitude of the gravitational force on one of the stars due to the other three. Write an expression for the force in terms of G, the quantities given in the question, and a numeric factor given to two significant figures. Use this for later parts of the question.

(ii) Using the answer to (i) and Newton's second law of motion, derive an expression for the period of the orbit in terms of m, R and constants.

(iii) The period of the orbit is 20.0 minutes, and light from one of the stars takes 1.0 seconds to reach its nearest neighbour. Determine the mass m.

Question 4 (Marks: 30)

(a) A diatomic gas fills a cylinder topped with a piston which is free to move. The cylinder has a circular cross-sectional area of 50.0 cm^2. Initially the system is at thermal equilibrium with the room temperature surroundings (25.0°C), the pressure inside the cylinder is 1.01 atm and it has a volume of 1.00 L. Lets call this state A.



(i) Describe the number of degrees of freedom per molecule for the gas inside the cylinder. Include how many there are and of which type.

A 5.00 kg mass is now quickly placed on top of the piston. It is now in state B. Immediately after this mass is placed on the piston.

(ii) What is the pressure inside the cylinder?

(iii) What is the height of the air in the cylinder?

You wait for the system to return to equilibrium with the surroundings, this is called state C

(iv) Will the height have changed? If yes what will the new height be?

You now remove the mass and very slowly return the piston to its original position (ie. return it to state A), while doing this you ensure the gas remains at equilibrium with its surroundings.

(v) Sketch a PV plot for the cycle the gas has just completed, include numbers where you can. Label the states A, B and C on your plot.

(vi) On your plot label each of the paths with the type of thermal process they represent (eg. isothermal process).

(vii) Calculate the work done on the gas as it moves from C to A.

Specific Heats, Thermal conductivities and linear expansion coefficients of selected metals

Substance	**Specific Heat** ***c*, (J kg^{-1} K^{-1})**	**Thermal conductivity** ***k*, (W m^{-1} K^{-1})**	**Linear expansion coefficient α, $(^oC)^{-1}$**	**Density (gcm^{-3}) at 0°C**
Aluminium	910	205.0	24×10^{-6}	2.70
Brass	377	109.0	19×10^{-6}	8.40
Copper	390	385.0	17×10^{-6}	8.96
Lead	130	34.7	29×10^{-6}	11.3
Steel	456	50.2	11×10^{-6}	7.85

Water

Quantity	**Value**
Specific Heat (liquid)	4186 $Jkg^{-1}K^{-1}$
Latent heat of Fusion	3.33×10^5 Jkg^{-1}
Latent heat of vapourization	2.26×10^6 Jkg^{-1}
Density (at 4.00° C)	1000 kgm^{-3}
Melting point (at 1 atm)	0.000 °C
Boiling point (at 1 atm)	100.0 °C
Volume expansion coefficient (β) (at 20°C: you may assume it is constant between 15°C and 100° C)	207×10^{-6} $(°C)^{-1}$

(b) A cube of copper with sides 5.00 cm long is heated from 25.0 °C to 275.0 °C

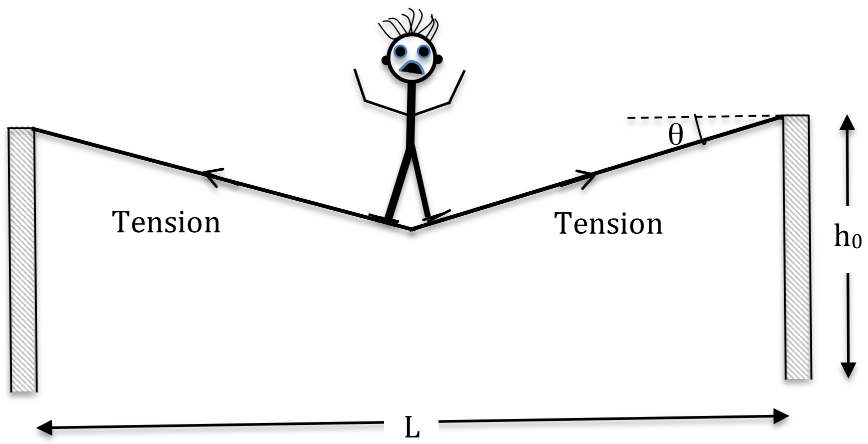
(i) What is the change in volume of the copper block as it is heated?

(ii) The copper block (at 275.0 °C) is now placed in 50.0 kg of water at 20.0°C. The water is in a 0.500 kg aluminium bucket also at 20.0°C. Assuming no heat loss to the surroundings what is the final temperature of the water?

Question 5 (Marks: 30)

(a) A tall building is on fire. Some of the occupants are trapped on the second storey balcony, height *H* above the ground. Assembled onlookers drag a nearby trampoline to catch their fall should they jump. The trampoline has a diameter *L* and is height h_0 above the ground. One of the occupants of mass *m* jumps at time t=0 and lands in the middle of the trampoline. The diagram below is not to scale, it shows a cross-section of the situation.

Make the typical (but unlikely!) first year physics simplification that the person is a particle, the collision is elastic, and the trampoline has very little mass, and that the person sticks to the trampoline (their shoes melted and glued them to the trampoline) on landing.



(i) If the total tension Σtension = Tsinθ. What is the direction of T? Justify your answer.

(ii) Assume that throughout the process θ remains small, so the small angle approximation is valid. After the occupant lands on the trampoline he moves up and down with simple harmonic motion. What is the equilibrium height (h_{eq}) measured from the top of the trampoline? Give an expression in terms of the variables given in the question.

(iii) Defining h as the distance from the equilibrium height prove that the motion of the occupant is simple harmonic. Your proof should include a definition of simple harmonic motion.

(iv) Derive an expression for the frequency of their oscillation ω.

(v) Assume that the person is in free-fall until they reach h_{eq}, what is the amplitude of the simple harmonic motion?

(b) Cave swiftlets are a type of small cave dwelling birds. Like bats, cave swiftlets use echo location to find their way in the the dark. To warn it's family of its approach a certain swiftlet sends out a series of sound pulses with a frequency of 6.11 kHz. The swiftlet if flying towards the nest with a speed of 10.7 m/s. The speed of sound in air is 343 m/s.

(i) What frequency pulse does the family in the nest detect?

(ii) What frequency pulse does the swiftlet detect reflected from the wall on which the nest is built?

(iii) On detecting the pulse the parent waiting in the nest realises that it is its turn to leave to find food. It flies towards the approaching swiftlet with a speed of 14.1 m/s. What frequency pulse does the swiftlet leaving the nest now detect coming from the swiftlet approaching the nest?

(c) The string of a violin is stretched too tight. Beats at frequency f=4.00Hz can be heard when the string is played at the same time as a concert A (440Hz) tuning fork is struck next to it.

(i) What are the possible frequencies of the violin string's oscillation? If there is only one, just give the one.

(ii) If you have two waves travelling in the x-direction with frequencies f_1 and f_2, derive an expression for the superposition of the two waves. What is the beat frequency? Show all your working.

(iii) In a few sentences explain why beats occur. Sketch a rough graph showing amplitude vs time, mark on the beat frequency.