

PHYS1131 HIGHER PHYSICS 1A
SOLUTIONS – Homework Problem Set 1

Q1.

We use equations of uniform accelerated motion with zero initial displacement and velocity.

$$y_1 = -\frac{1}{2}gt^2 \text{ where } t \text{ is measured in seconds}$$

$$y_2 = -\frac{1}{2}gt'^2 = -\frac{1}{2}g(t-1)^2 \text{ where } t' = t-1$$

Then:

$$d = y_2 - y_1 = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2 = gt - \frac{g}{2}$$

To calculate the time taken, we substitute $d = 10\text{m}$ into the above equation:

$$t = \frac{d + \frac{g}{2}}{g} = \frac{14.9}{9.8} = 1.5\text{s}$$

Q2.

Take $y = 0$ as the height of the lift's floor when the bolt starts to drop.

Again, we use the equations of accelerated motion.

$$y = y_0 + v_{y0}t + \frac{1}{2}a_yt^2$$

$$y_{lift} = 0 + 2t + 0.75t^2$$

$$y_{bolt} = 3 + 2t - 4.9t^2$$

The bolt hits the floor when

$$y_{lift} = y_{bolt}$$

Solving these simultaneously

$$3 + 2t - 4.9t^2 = 2t + 0.75t^2$$

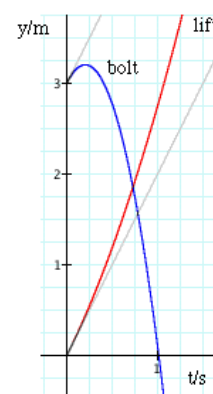
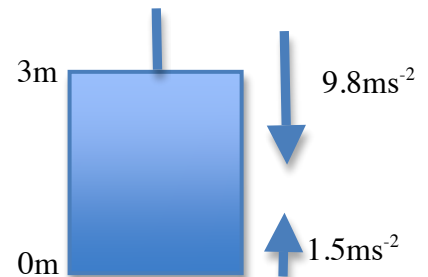
$$\therefore 3 = 5.65t^2$$

$$\therefore t = 0.73\text{s}$$

At this time the lift is at position:

$$y_{lift} = 2(0.73) + 0.75(0.73)^2 = 1.86\text{m}$$

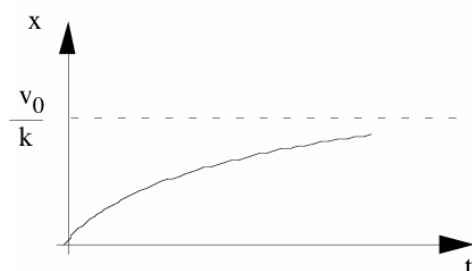
$$\therefore d_{fallen} = (3 - 1.86)\text{m} = 1.1\text{m}$$



Q3.

a)

$$x = \frac{v_0}{k}(1 - e^{-kt})$$



b) $v = \frac{dx}{dt} = v_0 e^{-kt}$

c) $a = \frac{dv}{dt} = -kv_0 e^{-kt} = -kv$

d) c) implies a resistive force proportional to $-v$. This force becomes weaker as the speed becomes lower in such a way that the object never stops. The distance travelled is finite, however, because the integral of $v \cdot dt$ is finite.

Past Exam Question:

i) Depending on the choice of origin, the graph might look like these. The algebra below is for the upper one.

ii)
$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$
$$= h + 0 - \frac{1}{2}gt^2$$

hits the bottom when $y = 0$, so

$$0 = h - \frac{1}{2}gT_1^2$$

$$\therefore T_1^2 = \frac{2h}{g}$$

$$T_1 = \sqrt{\frac{2h}{g}}$$

iii) $T_1 = 4.0 \text{ s}$

v)
$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

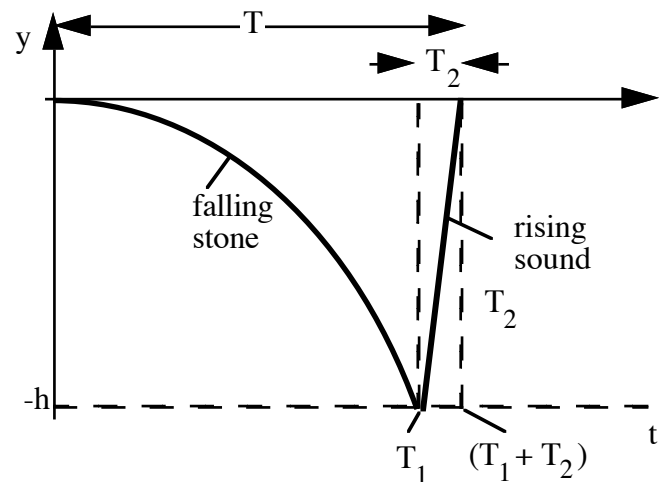
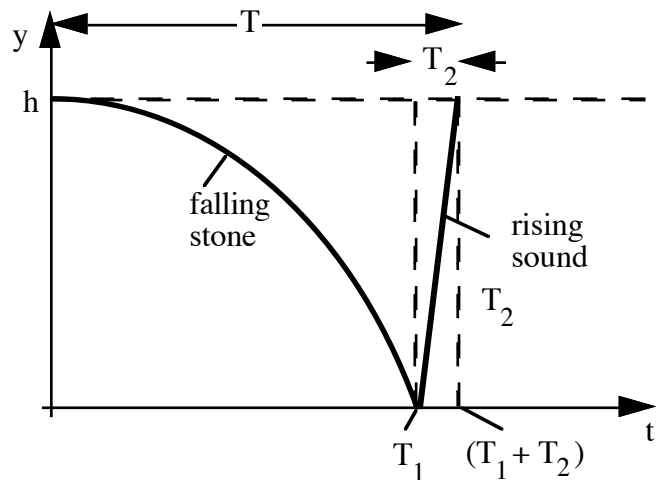
$$T_2 = \frac{h}{v_s} = 0.23 \text{ s}$$

vi) $T = T_1 + T_2 = 3.99 + 0.227 \text{ s}$
 $\therefore T = 4.2 \text{ s}$ (2 sig. fig.)

vii) She says $0 = h - \frac{1}{2}gT_1^2$
 $\therefore h = \frac{1}{2}gT_1^2 \approx \frac{1}{2}gT^2$

So she calculates

$$h \approx \frac{1}{2}(10 \text{ ms}^{-1})(4 \text{ s})^2 = 80 \text{ m}$$



viii) Her estimate is $4.0 \pm 0.5\text{s}$, an error of 13%. Neglecting the time for the sound to travel (6%) and taking $9.8 \approx 10$ (2%) are small errors by comparison. (She is lucky to have worked out an answer so close to the precise one.) *Note: in the marking scheme, any reasonable comment about the accuracy earns two marks.*

Q4.

a) $\vec{r}_1 = (100\text{m})\hat{i}$

$$\vec{r}_2 = -(300\text{m})\hat{j}$$

$$\begin{aligned}\vec{r}_3 &= -(150\text{m})\cos 30^\circ\hat{i} - (150\text{m})\sin 30^\circ\hat{j} \\ &= -(130\text{m})\hat{i} - (75\text{m})\hat{j}\end{aligned}$$

$$\begin{aligned}\vec{r}_4 &= -(200\text{m})\cos 60^\circ\hat{i} + (200\text{m})\sin 60^\circ\hat{j} \\ &= -(100\text{m})\hat{i} + (173\text{m})\hat{j}\end{aligned}$$

b) $\vec{r} = \vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 = -(130\text{m})\hat{i} - (202\text{m})\hat{j}$

c) $r = \sqrt{\vec{r} \cdot \vec{r}} = \sqrt{(130\text{m})^2 + (-202\text{m})^2} = 240\text{m}$

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = 237^\circ$$

Q5.

a) $\vec{r} = \vec{a} - \vec{b} + \vec{c} = 11.0\hat{i} + 5.0\hat{j} - 7.0\hat{k}$

$$r = \sqrt{r_x^2 + r_y^2 + r_z^2} = 14.0$$

b)

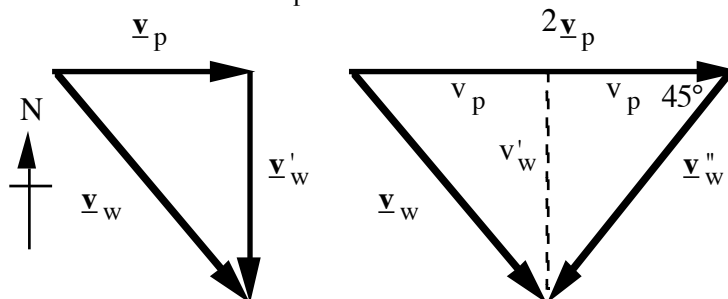
$$\theta_z = \cos^{-1}\left(\frac{r_z}{r}\right) = 120^\circ$$

Q6. With respect to the ground, let the wind have velocity \underline{v}_w and the person (initially) have \underline{v}_p . Let the person's first frame have one dash, and the relative velocity of the wind be \underline{v}_w' .

Let the second frame have two dashes, and the relative velocity of the wind be \underline{v}_w'' .

We have, in the dash frame, $\underline{v}_w = \underline{v}_p + \underline{v}_w'$, which is shown below at left.

We have, in the double dash frame, $\underline{v}_w = 2\underline{v}_p + \underline{v}_w''$, which is shown below at right.



On the right, we are given that the angle at top right is 45° , but because the two lengths on the top side are each v_p , the angle at top left must be 45° too, so the wind must come from NW and $v_w = v_p/\cos 45^\circ = 5.7$ kph.

(It is also possible to do this by vector algebra.)

Q7. The velocity of the rower with respect to the bank is \underline{v}_{rb} . The velocity of the rower with respect to the water is \underline{v}_{rw} . The velocity of the water with respect to the bank is \underline{v}_{wb} . So

$$\begin{aligned} \text{a)} \quad \underline{v}_{rb} &= \underline{v}_{rw} + \underline{v}_{wb} \\ &= -v_{rw} \sin \theta \hat{\mathbf{i}} + v_{rw} \cos \theta \hat{\mathbf{j}} + v_{wb} \hat{\mathbf{i}} \\ &= [(5 - 2 \sin \theta) \hat{\mathbf{i}} + 2 \cos \theta \hat{\mathbf{j}}] ms^{-1} \end{aligned}$$

At this point, some students may choose to use numerical values for the rest of the problem, which would be acceptable in a test. This makes the equations a little shorter. It has two disadvantages: first, one cannot readily do checks for dimensions and limits. Second, an algebraic solution is a solution for all cases, and is also the starting point for further problems.

$$\begin{aligned} \text{b)} \quad \underline{r}_{rb} &= \underline{v}_{rb} t + 0 \\ &= t[-v_{rw} \sin \theta \hat{\mathbf{i}} + v_{rw} \cos \theta \hat{\mathbf{j}} + v_{wb} \hat{\mathbf{i}}] \\ &= t[(5 - 2 \sin \theta) \hat{\mathbf{i}} + 2 \cos \theta \hat{\mathbf{j}}] ms^{-1} \quad (1) \end{aligned}$$

c) At the opposite bank, $y = 40 \text{ m} = w$. Substituting for the y component in (1) gives:

$$\begin{aligned} y &= t[v_{rw} \cos \theta] \\ t &= \frac{w}{v_{rw} \cos \theta} \quad (2) \end{aligned}$$

t is a maximum when $\cos \theta$ is a maximum, ie when $\theta = 0$

$$\text{so here } \theta = 0 \text{ and } t = \frac{40\text{m}}{2ms^{-1}} = 20s$$

Distance D from B on the other bank = the final x component of \underline{r}_{rb} .

From (1)

$$D = t[-v_{rw} \sin \theta + v_{wb}] \quad (3)$$

so, in this case,

$$D = 20s[0 + 5ms^{-1}] = 100m$$

d) To remove t , from (3), substitute from (2):

$$D = \frac{w(-v_{rw} \sin \theta + v_{wb})}{v_{rw} \cos \theta} \quad (4)$$

for which we must find a minimum.

$$\left(\text{using } \frac{d[g(x)h(x)]}{dx} = g \frac{dh}{dx} + h \frac{dg}{dx}\right)$$

$$\frac{dD}{d\theta} = -\frac{wv_{rw} \cos \theta}{v_{rw} \cos^2 \theta} + \frac{w(-v_{rw} \sin \theta + v_{wb})}{v_{rw} \cos^2 \theta} \sin \theta = 0$$

multiply all terms by $\frac{v_{rw} \cos^2 \theta}{v_{rw} w}$ gives

$$-\cos^2 \theta - \sin^2 \theta + \frac{v_{wb}}{v_{rw}} \sin \theta = 0$$

$$\frac{v_{wb}}{v_{rw}} \sin \theta = 1$$

$$\theta = \sin^{-1} \frac{v_{rw}}{v_{wb}} (= 23.6^\circ) = 20^\circ \text{ to one sig. fig.}$$

Substitute (the precise value) into (2) and (3) and (4) to give $t = 20s$ and $D = 90m$.

$$\begin{aligned} \text{Q8.} \quad \Delta \vec{v} &= (100\hat{i} - 75\hat{j}) - (126\hat{i} + 25\hat{j}) ms^{-1} = -25\hat{i} - 100\hat{j} ms^{-1} \\ \vec{a}_{av} &= \frac{\Delta \vec{v}}{\Delta t} = \frac{1}{3}(-25, -100) ms^{-2} = 8.3\hat{i} - 33\hat{j} ms^{-2} \end{aligned}$$

Q9. $\vec{r}(t) = \hat{i} + 4t^2\hat{j} + t\hat{k}$

a) $\vec{v}(t) = \frac{d\vec{r}}{dt} = 8t\hat{j} + \hat{k}$
 $\vec{a}(t) = \frac{d\vec{v}}{dt} = 8\hat{j}$ constant acceleration in the y direction

b) $x = 1; y = 4t^2; z = t$
sub z into y
 $\Rightarrow y = 4z^2$
so the particle moves along a parabolic path in the plane $x = 1$

Q10.

Use the subscript 0 for initial and f for final. We are given the acceleration ($a_y = -g$) and the time of flight t . We are also given the initial speed v_0 and the angle θ , which gives us the initial velocity components:
 $v_{x0} = v_0 \cos \theta$ and $v_{y0} = v_0 \sin \theta$

a) What is the final height $y_f = h$? We know v_{y0} , a_y , and t_f so use

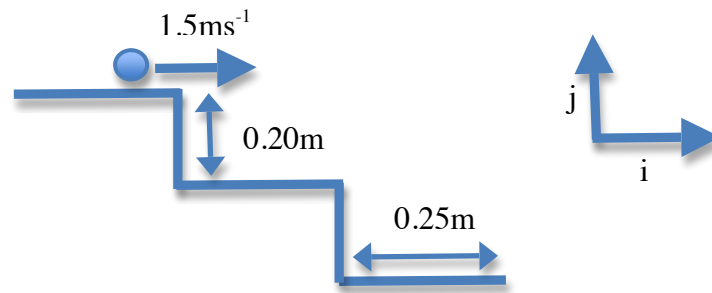
$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

$$h = y_f = 0 + v_0 \sin \theta t_f + \frac{1}{2}a_y t_f^2 = 51.8m$$

b) final speed is $|v_f| = \sqrt{v_{xf}^2 + v_{yf}^2}$
 $a_x = 0$ so $v_{xf} = v_{x0} = v_0 \cos \theta$
 $a_y = -g$ so $v_{yf} = v_{y0} + a_y t_f$
 $|v_f| = \sqrt{(v_0 \cos \theta)^2 + (v_0 \sin \theta - gt_f)^2} = 27.4ms^{-1}$

c) Max height $y_m = H$ occurs when $v_y = 0$
 $v_y^2 - v_{y0}^2 = 2a_y(y - y_0)$ applied here gives
 $0 - (v_0 \sin \theta)^2 = -2gH$
 $H = \frac{(v_0 \sin \theta)^2}{2g} = 67.5m$

Q 11.



Initially $v_{x0} = 1.5 \text{ ms}^{-1}$, $v_{y0} = 0 \text{ ms}^{-1}$

at time t $v_x = 1.5 \text{ ms}^{-1}$, $v_y = -\frac{1}{2}gt^2$

let t_1 = time to drop height of first step

$$y = v_{y0}t + \frac{1}{2}at^2$$

$$-0.20 = \frac{1}{2} \times -9.8 \times t_1^2$$

$$t_1 = 0.20 \text{ s}$$

$$\Rightarrow x = 0.20 \times 1.5 = 0.3 \text{ m} \Rightarrow 5 \text{ cm out from first step}$$

let t_2 = time to drop to height of second step

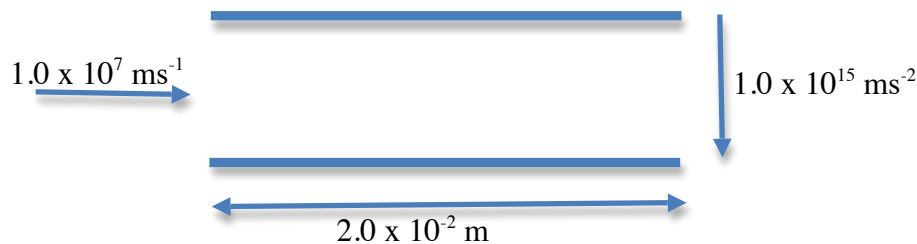
$$-0.40 = \frac{1}{2} \times -9.8 \times t_2^2$$

$$t_2 = 0.286 \text{ s}$$

$$\Rightarrow x = 0.286 \times 1.5 = 0.43 \text{ m} < 0.5 \text{ m}$$

\therefore lands on second step

Q12.



a)

Remember $y = y_0 + v_{y0}t + \frac{1}{2}a_yt^2$; $v_y = v_{y0}t + a_yt$

Find T , time to pass through plates

$$T = \frac{L}{v_x} = 2.0 \times 10^{-2} \times 1.0 \times 10^7 = 2.0 \times 10^{-9} \text{ s}$$

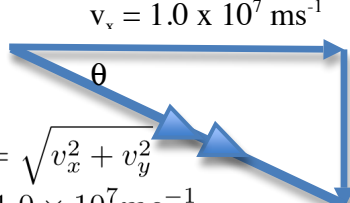
Now find vertical displacement

$$y = 0 + 0 \times T + \frac{1}{2} \times 1.0 \times 10^{15} \times (2.0 \times 10^{-9})^2 = 2.0 \times 10^{-3} \text{ m downwards}$$

b)

$$\tan \theta = \frac{0.20}{1.0}$$

$$\Rightarrow \theta = 11^\circ$$



A vector diagram showing a horizontal vector labeled $v_x = 1.0 \times 10^7 \text{ ms}^{-1}$ and a vertical vector pointing downwards. A resultant vector v is shown at an angle θ below the horizontal. The magnitude of the resultant is given as $v = 1.0 \times 10^7 \text{ ms}^{-1}$.

$$v_y = a_y t$$

$$= 1.0 \times 10^{15} \times 2.0 \times 10^{-9}$$

$$= 2.0 \times 10^6 \text{ ms}^{-1}$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= 1.0 \times 10^7 \text{ ms}^{-1}$$

$\therefore v_f = 1.0 \times 10^7 \text{ ms}^{-1}$ at 11° below horizontal
 3% of c , so $\gamma = 0.9995$ so relativity neglected for 2 sig figs.

Q13.

a) $a = \frac{v^2}{r} \Rightarrow v = \sqrt{gR} = 19.8 \text{ ms}^{-1}$

b) It will appear “weightless”, but it is not weightless. See discussion ‘weightless’ on the HSC Physics FAQ at www.phys.unsw.edu.au/~jw/FAQ.html .

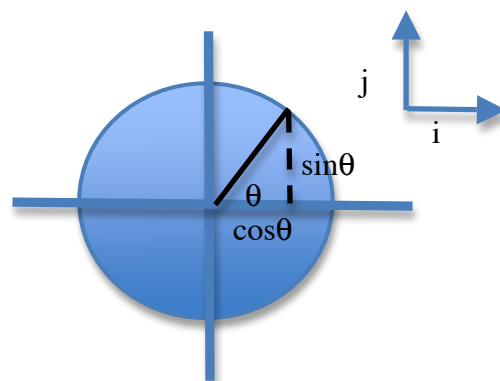
Q14.

a) $\theta = \omega t$
 $\vec{r} = r \cos \omega t \hat{i} + r \sin \omega t \hat{j}$

b)

$$\vec{v} = \frac{d\vec{r}}{dt} = -\omega r \sin \omega t \hat{i} + \omega r \cos \omega t \hat{j}$$

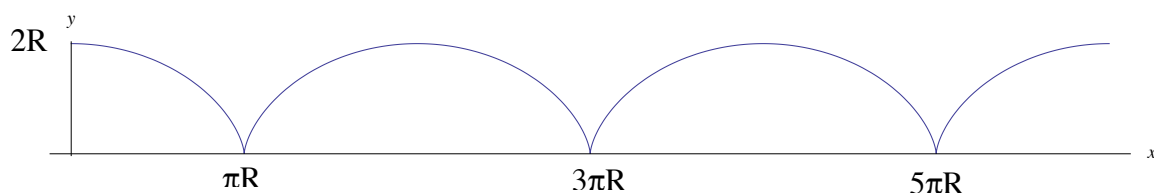
$$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2 r \cos \omega t \hat{i} - \omega^2 r \sin \omega t \hat{j} = -\omega^2 \vec{r}$$



c) Acceleration is always directed in the $-\vec{r}$ direction, ie. towards the centre.

Q15.

a)



b) $x = R \sin \omega t + \omega R t; y = R \cos \omega t + R$

$y_{max} = 2R$ this occurs when $t = 0$

$$v_x = \frac{dx}{dt} = R\omega \cos \omega t + \omega R = 2R\omega$$

$$v_y = \frac{dy}{dt} = -R\omega \sin \omega t = 0$$

$$a_x = \frac{dv_x}{dt} = -R\omega^2 \sin \omega t = 0$$

$$a_y = \frac{dv_y}{dt} = -R\omega^2 \cos \omega t = -R\omega^2$$

$y_{min} = 0$ this occurs when $t = \frac{\pi}{\omega}$

$$v_x = \frac{dx}{dt} = R\omega \cos \omega t + \omega R = 0$$

$$v_y = \frac{dy}{dt} = -R\omega \sin \omega t = 0$$

$$a_x = \frac{dv_x}{dt} = -R\omega^2 \sin \omega t = 0$$

$$a_y = \frac{dv_y}{dt} = -R\omega^2 \cos \omega t = R\omega^2$$

Q16.

a) $x = v_0 \cos \theta \times t$

$$y = h + v_0 \sin \theta \times t - \frac{1}{2}gt^2$$

b) Eliminating t from the above equations, we get:

$$y = h + v_0 \sin \theta \left(\frac{x}{v_0 \cos \theta} \right) - \frac{1}{2}g \left(\frac{x}{v_0 \cos \theta} \right)^2$$

$$\therefore y = h + x \tan \theta - \frac{gx^2 \sec^2 \theta}{2v_0^2}$$

Using $y = 0, x = R$, we obtain:

$$0 = h + R \tan \theta - \frac{gR^2}{2v_0^2} \sec^2 \theta \quad (1) \text{ as required}$$

c) Make use of the standard integrals:

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax + C$$

$$\int \sec^2 x \tan x dx = \frac{1}{2} \sec^2 x + C$$

and the product rule to differentiate (1) with respect to θ :

$$0 = 0 + \frac{\partial R}{\partial \theta} \tan \theta + R \sec^2 \theta - \frac{2gR}{2v_0^2} \frac{\partial R}{\partial \theta} \sec^2 \theta - \frac{2gR^2}{2v_0^2} \sec^2 \theta \tan \theta$$

Setting $\frac{dR}{d\theta} = 0$ for maximum range, R_m and angle, θ_m :

$$0 = R_m \sec^2 \theta_m - \frac{gR_m^2}{v_0^2} \sec^2 \theta_m \tan \theta_m$$

$$\Rightarrow R_m = \frac{v_0^2}{g \tan \theta_m}$$

d)

Substituting this value for R into (1) gives:

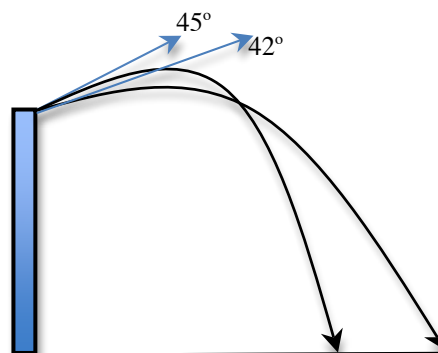
$$\begin{aligned}
 0 &= h + \frac{v_0^2}{g \tan \theta_m} \tan \theta_m - \frac{g}{2v_0^2} \frac{v_0^4}{g^2 \tan^2 \theta_m} \sec^2 \theta_m \\
 &= h + \frac{v_0^2}{g} - \frac{v_0^2}{2g \sin^2 \theta_m} \\
 \therefore \sin^2 \theta_m &= \frac{1}{2} \frac{1}{1 + gh/v_0^2}
 \end{aligned}$$

e) $\sin^2 \theta_m = \frac{1}{2} \frac{1}{(1 + 9.8 \times 2.1/14.0^2)} = 0.4525$

$$\therefore \theta_m = 42.3^\circ$$

$$\therefore R_m = \frac{14.0^2}{9.8 \tan 42.3} = 22.0m$$

f) When $h = 0$ the maximum range is indeed given for $\theta = 45^\circ$ (exercise for the student!). However for $h > 0$ then the projectile has an additional time to travel before it hits the ground, and the extra distance travelled will be maximised if there is additional horizontal speed, i.e. this occurs if $\theta < 45^\circ$.



Past Exam Question:

i) There is no air resistance. Neither the bird's nor the grape's, horizontal velocity changes, so, if they have the same horizontal velocity, they always have it. If they ever have the same horizontal position, they must always have it. So you must throw it when the bird is directly overhead.

ii)

From (i), $v_{xg} = v_0 \cos \theta = v_b$ (a)

Vertical motion under gravity, measured from $y = 0$ at the position of the head:

$$v_y^2 = v_{y0}^2 + 2ay$$

At $y = h$, $v_y = 0$, so

$$0 = v_0^2 \sin^2 \theta - 2gh$$

$$v_0 \sin \theta = \sqrt{2gh} \quad (b)$$

$$(b)/(a) \Rightarrow \tan \theta = \frac{\sqrt{2gh}}{v_b}$$

$$\Rightarrow \theta = \tan^{-1} \frac{\sqrt{2gh}}{v_b} = \tan^{-1} \frac{\sqrt{2 \times 9.8ms^{-2} \times 5m}}{5ms^{-1}} = 63^\circ$$

$$(a) \Rightarrow v_0 = \frac{v_b}{\cos \theta} = \frac{5ms^{-1}}{\cos 63^\circ} = 11ms^{-1}$$

iii) Air resistance would slow the grape during flight. The grape would have greater horizontal velocity until the end of its flight, so it would cover the distance from you to bird faster than the bird would, so you would throw it after it passed overhead.