PHYS1131 HIGHER PHYSICS 1A
SOLUTIONS – Homework Problem Set 2

Q1. Applies N2 in the x direction to the total (m_1+m_2):

(a)
$$F = (m_1 + m_2)a$$
$$\therefore a = F/(m_1 + m_2)$$
Internal force: $F_{int} = m_2 a = \dfrac{m_2}{m_1 + m_2}F = 1.0\text{N}$

(b Here $F'_{int} = \dfrac{m_1}{m_1 + m_2}F$.
Same acceleration, but $m_1 > m_2$ so a larger force required to accelerate m_1.

Q2.

In all cases, the force on the man is $N - mg = ma$ in the up direction.

(a) a = 0, so $N = mg = 980N$ up

(b) a = 0, so $N = mg = 980N$ up

(c) $N - mg = ma \Rightarrow N = m(a + g) = 100 \times 11.8 = 1180N$ up

(d) $\text{N} - \text{mg} = \text{ma} \Rightarrow \text{N} = \text{m(g} - \text{a)} = 680\text{N}$ up

(e) $\text{N} - \text{mg} = \text{ma} \Rightarrow \text{N} = 580\text{N}$ *up*

(f) $N - mg = ma \Rightarrow N = m(g + a) = 1480N$ up

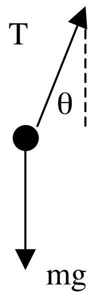
Q3. Assume m_1 accelerates up the slope and m_2 accelerates downwards.

(a) Newton 2 for m_1 in direction of string:

$$m_1 a = T - m_1 g \sin 30$$
and for m_2
$$m_2 a = m_2 g - T$$
now add these equations together
$$(m_1 + m_2)a = m_2 g - m_1 g \sin 30$$
$$a = \frac{m_2 g - m_1 g \sin 30}{m_1 + m_2}$$
$$= 0.98 ms^{-2}$$

(b)
$$T = m_2 g - m_2 a = 2.0(9.80 - 0.98) = 18N$$

Q4.



(a) (1) $T\cos\theta = mg$ (N2 in vertical)

(2) $T\sin\theta = ma$ (N2 in horizontal)

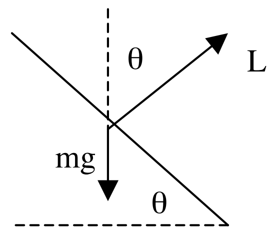
$\tan\theta = a/g$ divide (1) by (2)

$a = g\tan\theta$

(b) substitute θ = 20°, find a = 3.6 ms^{-2}

a = 2 ms^{-2}, find tan θ = a/g, so θ = 12°

Q5.



$$L\cos\theta = mg \qquad \textit{(N2 in vertical)}$$

$$L\sin\theta = \frac{mv^2}{r} \qquad \textit{(N2 in horizontal)}$$

$$\therefore \tan\theta = \frac{v^2}{rg} \qquad \textit{(divide eqns)}$$

$$r = \frac{v^2}{g\tan\theta} = (480/3.6)^2/(9.8\tan 40^o) = 2.2\text{ km}$$

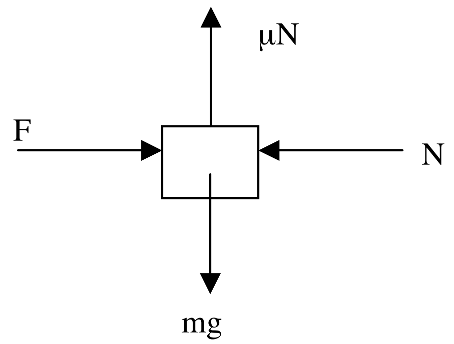
Q6.

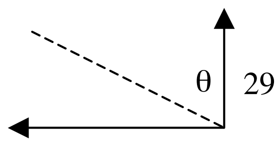
Maximum friction force = $\mu_s N = \mu_s F = 36$ N.
Weight = mg = 29 N.
The block will not move.
Actual friction force = 29 N.

$F = \sqrt{60^2 + 29^2} = 67$ N

$\theta = \tan^{-1}(60/29)$

$\theta = 64^0$





Q7.

(a) Acceleration of both masses has some magnitude, a (a > 0 if m, accelerates down)

$$\left.\begin{array}{ll}\text{N2 } \textit{on 5 kg (} m_1\textit{)} & m_1 g - T = m_1 a \\ \text{N2 } \textit{on 2 kg} & T - m_2 g = m_2 a\end{array}\right\} (m_1 + m_2)a = (m_1 - m_2)g$$

$$\therefore a = (m_1 - m_2)g/(m_1 + m_2) = 3g/7$$

(b) 2 kg mass:

$$v = 0 + at = 1.26\text{m/s}$$

$$y = 0 + 0 + \frac{1}{2}at^2 = 0.19\text{m}$$

5 kg mass:

$$v = -1.26\text{m/s}$$

$$y = 1.29 - 0.19 = 1.10\text{m}$$

(c) a= -g

(d) Mass A:

$$v_0 = 1.26, y_0 = 0.19$$
$$y = y_0 + v_0 t + 1/2 a t^2$$
$$0 = 0.19 + 1.26t - 4.9t^2$$
$$\therefore t = 0.36\ s$$

Mass B:

$$v_0 = -1.26, y_0 = 1.1.$$
$$y = 1.1 - 1.26t - 4.9t^2$$
$$y = 0 \therefore t = 0.36\ s$$

Q8.

$N2 : F = ma = -\beta v^2$ (a)i)

divide both sides by m:

$$\frac{F}{m} = a = \frac{dv}{dt} = -\frac{\beta}{m} v^2$$

separate variables:

$$dt = -\frac{m}{\beta} \frac{dv}{v^2}$$

Integrate making use of the fact that at $t = 0$, $v = v_0$, and at time t velocity is v:

$$\int_t^0 dt = -\frac{m}{\beta} \int_v^{v_0} \frac{1}{v^2}\, dv$$
$$t = -\frac{m}{\beta}(-\frac{1}{v} + \frac{1}{v_0})$$

Now rearrange to find v:

$$\frac{1}{v} = \beta t m + \frac{1}{v_0}$$
$$v = \frac{m v_0}{t \beta v_0 + m}$$

ii)

$$v = \frac{dx}{dt} = \frac{m v_0}{t\beta + m}$$

Now integrate, at $t = 0$, $x = 0$ and at time t particle is at x

$$\int_0^x dx = \int_0^t \frac{m v_0}{t \beta v_0 + m}\, dt$$
$$x = m v_0 (\frac{1}{\beta v_0} ln|\beta v_0 t + m| - \frac{1}{\beta v_0} ln|m|)$$
$$x = \frac{m}{\beta} ln|\frac{\beta v_0 t}{m} + 1|$$

iii)

$$a = -\frac{\beta v^2}{m} = -\frac{\beta}{m}(\frac{m^2 v_0^2}{(t\beta v_0 + m)^2}$$
$$a = -\frac{\beta m v_0^2}{(t \beta v_0 + m)^2}$$

(b) i) and ii)

rearrange a) part ii)

$$x = \frac{m}{\beta} ln|\frac{\beta v_0 t}{m} + 1| \implies \frac{x\beta}{m} = ln|\frac{\beta v_0 t}{m} + 1|$$
$$e^{\frac{x\beta}{m}} = \frac{\beta v_0 t}{m} + 1$$
$$t = \frac{m}{\beta v_0}(-1 + e^{\frac{x\beta}{m}})$$

Now substitute t into the equations for v and a:

$$v = \frac{mv_0}{t\beta v_0 + m} = v_0 e^{\frac{-x\beta}{m}}$$
$$a = -\frac{\beta m v_0^2}{(t\beta v_0 + m)^2} = -\frac{\beta v_0^2}{m} e^{\frac{-2x\beta}{m}}$$

Past exam question

a) i) Newton's 2nd law in vertical direction: $N = mg$

Newton's 2nd law in horizontal direction: $F_f = |ma|$

greatest frictional force when $F_f = \mu_s N$

$|a| = \mu_s N/m = \mu_s g$

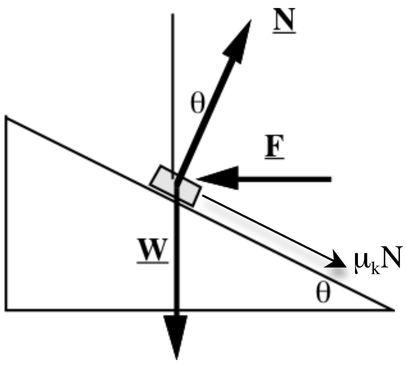
Decellearating with constant acceleration $0^2 - v^2 = 2as$, where $a < 0$.

$s_b = -v^2/2a = v^2/2a = v^2/2\mu_s g$.

ii) For 50 kph, $s_b = 12$ m

For 80 kph, $s_b = 30$ m

b)



i) No acceleration, so Σforces = 0.

Resolve in the plane and perpendicular to it:

// to plane: $F \cos\theta = \mu_k N + W \sin\theta$

perpendicular to plane $N = F\sin\theta + W\cos\theta$

Hence $F\cos\theta = \mu_k F \sin\theta + \mu_k W \cos\theta + W\sin\theta$

$\therefore F[\cos\theta - \mu_k \sin\theta] = W[\sin\theta + \mu_k \cos\theta]$

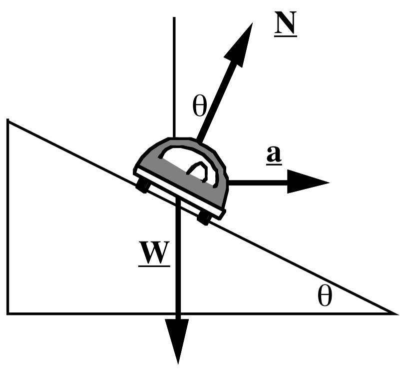
$$\therefore F = W\frac{\sin\theta + \mu_k \cos\theta}{\cos\theta - \mu_k \sin\theta}$$

check: in the limit of no friction, $\mu_k = 0$,

so $F = W \sin\theta/\cos\theta = W \tan\theta$.

also check when $\theta = 0$, $F = \mu_k W$

Exercise for the student: demonstrate this is true.



ii) Let **ds** be the displacement up the plane:

$$Power = \frac{d(work)}{dt} = \frac{\mathbf{F}.\mathbf{ds}}{dt}$$
$$= \frac{F cos\theta ds}{dt} = F v cos\theta$$

iii) If there is no friction force in the plane of the diagram, then the horizontal acceleration **a** satisfies

$$ma = N \sin \theta$$

There is no vertical acceleration so

$$W = N \cos\theta$$

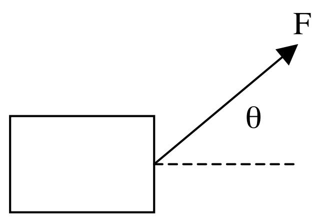
and the centripetal acceleration required is v^2/R, so

$$m\frac{v^2}{R} = N \sin\theta = \frac{W}{\cos\theta}\sin\theta = mg\tan\theta$$

$$\theta = \tan^{-1}(\frac{v^2}{Rg})$$

iv) In the normal direction – the same direction as **N** in the diagram. The driver of the car is also travelling in circular motion with the same speed and radius, so exactly the same equations apply, but with a different value of m.

Q9.



(a) $v = const. \therefore W_{total} = \Delta K = 0$

(b) $W = F\cos\theta = 30.1\,J$

(c) Gravity does no work, total work is zero,
therefore $W_{friction} = -30.1\,J$

(d)

$$F_{friction} = W_{friction}/d = 7.42\text{N} = \mu_k N$$

N2 vertical $\quad N + F\sin\theta = Mg$

N2 horizontal $\quad N = Mg - F\sin\theta = 33\text{N}$

$$\mu_k = 7.42/33 = 0.23$$

Q10.

(a) $$F = -\frac{dU}{dx} = \frac{12A}{x^{13}} - \frac{6B}{x^7}$$

(b) $$F = 0 \therefore x^6 = 2A/B$$
$$\therefore x = (2A/B)^{1/6}$$

Q11.

$$F = 52.8x + 38.4x^3$$

(a) $$W = \int_a^b F dx = \int_a^b (52.8x + 38.4x^3)dx = \left[26.4x^2 + 9.6x^4\right]_a^b = 28.8\text{J}$$

(b) Non conservative forces do no work, so mechanical energy conserved, so

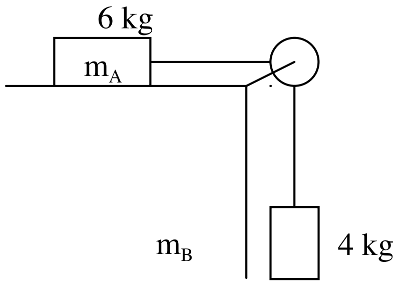
$$\Delta K + \Delta U = 0$$

$$\Delta K = 1/2mv^2 = 28.8J \Rightarrow v = \sqrt{\frac{2 \times 28.8}{2.17}} = 5.15m/s$$

(c) We have shown in (a) that $U = U(x)$, which is the definition of a conservative force.

Q12.

(a) Assume it slides,



$$m_B g - F_{fr} = (m_A + m_B)a$$

$$F_{fr} = \mu N = \mu m_A g < m_B g$$

$$a = \frac{m_B g - \mu_k m_A g}{m_A + m_B} = 2.16 \text{ ms}^{-2}$$

$N2$ for m_B : $m_B g - T = m_B a$

$$\therefore T = m_B(g - a) = 30.6N$$

(b)

$$v^2 = v_0^2 + 2as = 4.32 \times 1.5 = 6.5\ m^2/s^2$$

$$K = 1/2mv^2 = 1/2 \times 10 \times 6.48 = 32\ J$$

(c)

$$\Delta U = -4.0 \times 1.5 \times 9.8 = -59\ J$$

$$\Delta E = -58.8 + 32.4 = -26\ J$$

$$\therefore\ heat\ =\ 26\ J$$

Q13. no non-conservative forces do work so

Mechanical energy is conserved :

$$U_i + K_i = U_f + K_f$$

$$U_i = U_f$$

$$(1/2)kx^2 = mgh$$

$$h = \frac{kx^2}{2mg} = 2.0m$$

$$d = h/\sin\theta = 2h = 4.0m$$

Q14.

(a) Calculate energy loss in each passage over rough surface.

$$-\Delta E = F_{friction}h = \mu mgh = 0.15mgh$$

$$mgh^1 = mgh + \Delta E = 0.85mgh$$

$$h^1 = 0.85h$$

(b) No. of passages = (1/0.15) = 6.7

Since point A is at beginning of 7^{th} passage it passes A 7 times.

Q15. At top of sphere: U = mgh, h=rcosθ

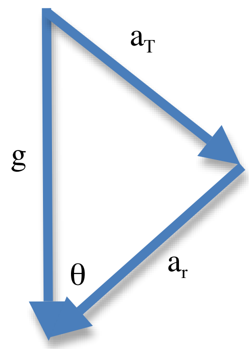
a) $\Delta U = mgr\cos\theta - mgr = -mgr(1-\cos\theta)$

b) Non-conservative forced do no work, so mechanical energy is conserved:

$\Delta K = -\Delta U = mgr(1-\cos\theta)$

c)
$$a_r = \frac{v^2}{r} \implies \frac{1}{2}mv^2 = mgr(1-\cos\theta)$$
$$v^2 = 2gr(1-\cos\theta)$$
$$\implies a_r = 2g(1-\cos\theta)$$
$$a_T = g\sin\theta$$

d)
$$\cos\theta = \frac{a_r}{g}$$
$$\Rightarrow g\cos\theta = 2g(1-\cos\theta)$$
$$3\cos\theta = 2$$
$$\cos\theta = \frac{2}{3}$$
$$\theta = \cos^{-1}\frac{2}{3}$$



Q16.

(a) $F = W_{hanging} = Mg\dfrac{x}{\ell}$

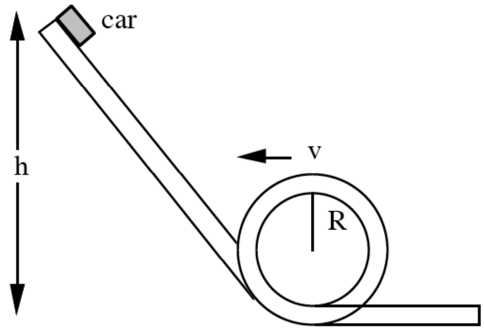
(c) length on table $s = \ell - x$

$$W = \int_{s=0}^{0} F\,ds = -\int_{x=\ell}^{0} F\,ds$$
$$= -\int_{x=\ell}^{0} mg\frac{x}{\ell}\,dx$$
$$= -\frac{mg}{\ell}\left[\frac{x^2}{2}\right]_{\ell}^{0} = \frac{mg\ell}{2}$$

(d) take table as zero of U

$$\Delta U = 0 - \left(-mg\frac{\ell}{2}\right)$$

Past exam question



i) v must be sufficiently great that the centripetal force at the top of the loop at least equals the weight of the car. (Faster than this, a downwards normal force is required.)

No non-conservative forces do work, so conservation of mechanical energy applies:

$$U_i + K_i = U_f + K_f$$

$$mgh + 0 = mg.2R + \frac{1}{2} mv^2$$

$$v^2 = 2g(h - 2R)$$

It loses contact when normal force = 0.

$$N + mg = F_{centrip} = mv^2/R \qquad \text{i.e.} \qquad \text{falls when } mg = mv^2/R$$

$$v^2 = gR$$

Therefore it just falls off if h satisfies

$$gR = v^2 = 2g(h - 2R)$$

$$5gR = 2\ gh \qquad \therefore \qquad h_{min} = 5R/2$$

ii) No. All forces are proportional to the mass and so scale accordingly.

(iii) For the marble, some of the initial potential energy is converted into rotational kinetic energy, so there is proportionally less translational kinetic energy at the top of the loop, so its speed is less. So the centrepital force required is less than the weight, so it falls off the track.