Chapter 8

Potential Energy and Conservation of Energy

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8-1 Potential Energy

Learning Objectives

8.01 Distinguish a conservative force force from a nonconservative force.

8.02 For a particle moving between two points, identify that the work done by a conservative force does not depend on which path the particle takes.

8.03 Calculate the gravitational potential energy of a particle (or, more properly, a particle-Earth system).

8.04 Calculate the elastic potential energy of a block-spring system.

8-1 Potential Energy

- **Potential energy** *U* is energy that can be associated with the configuration of a system of objects that exert forces on one another
- A system of objects may be:
  - Earth and a bungee jumper
  - **Gravitational potential energy** accounts for kinetic energy increase during the fall
  - **Elastic potential energy** accounts for deceleration by the bungee cord
- Physics determines how potential energy is calculated, to account for stored energy

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8-1 Potential Energy

- For an object being raised or lowered:

$$\Delta U = -W. \qquad \text{Eq. (8-1)}$$

- The change in gravitational potential energy is the negative of the work done
- This also applies to an elastic block-spring system

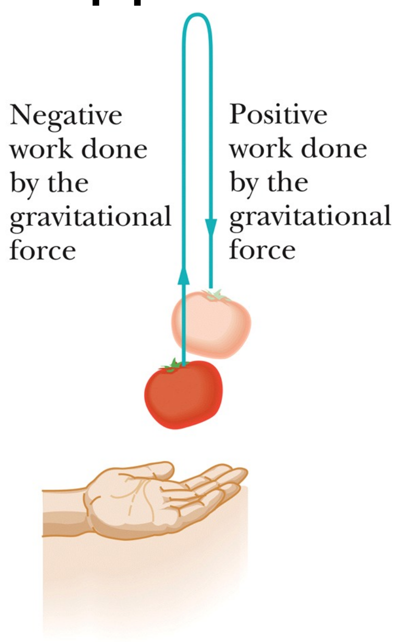


Figure 8-2

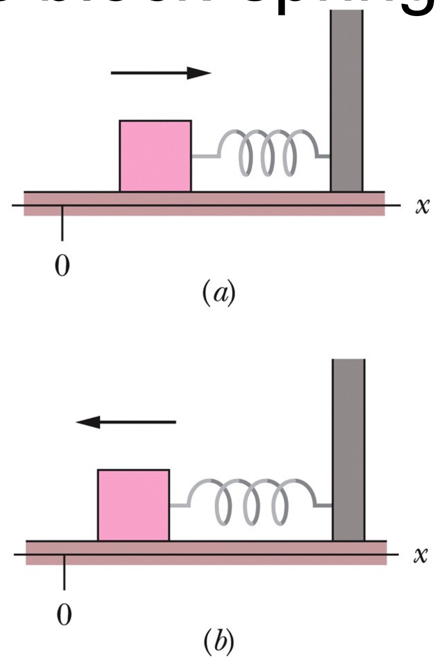


Figure 8-3

8-1 Potential Energy

- Key points:
  1. The *system* consists of two or more objects
  2. A *force* acts between a particle (tomato/block) and the rest of the system
  3. When the configuration changes, the force does *work* W_1, changing kinetic energy to another form
  4. When the configuration change is reversed, the force reverses the energy transfer, doing work W_2
- Thus the kinetic energy of the tomato/block becomes potential energy, and then kinetic energy again

8-1 Potential Energy

- **Conservative forces** are forces for which $W_1 = -W_2$ is always true
  - Examples: gravitational force, spring force
  - Otherwise we could not speak of their potential energies
- **Nonconservative forces** are those for which it is false
  - Examples: kinetic friction force, drag force
  - Kinetic energy of a moving particle is transferred to heat by friction
  - Thermal energy cannot be recovered back into kinetic energy of the object via the friction force
  - Therefore the force is not conservative, thermal energy is not a potential energy

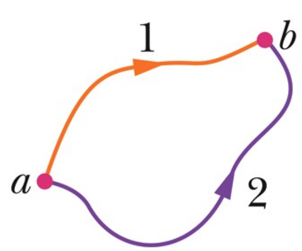
8-1 Potential Energy

- When only conservative forces act on a particle, we find many problems can be simplified:

> The net work done by a conservative force on a particle moving around any closed path is zero.

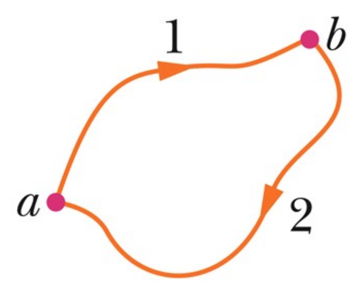
- A result of this is that:

> The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.



(a)

The force is conservative. Any choice of path between the points gives the same amount of work.



(b)

And a round trip gives a total work of zero.

Figure 8-4

8-1 Potential Energy

- Mathematically:

$$W_{ab,1} = W_{ab,2},$$ **Eq. (8-2)**

- This result allows you to substitute a simpler path for a more complex one if only conservative forces are involved

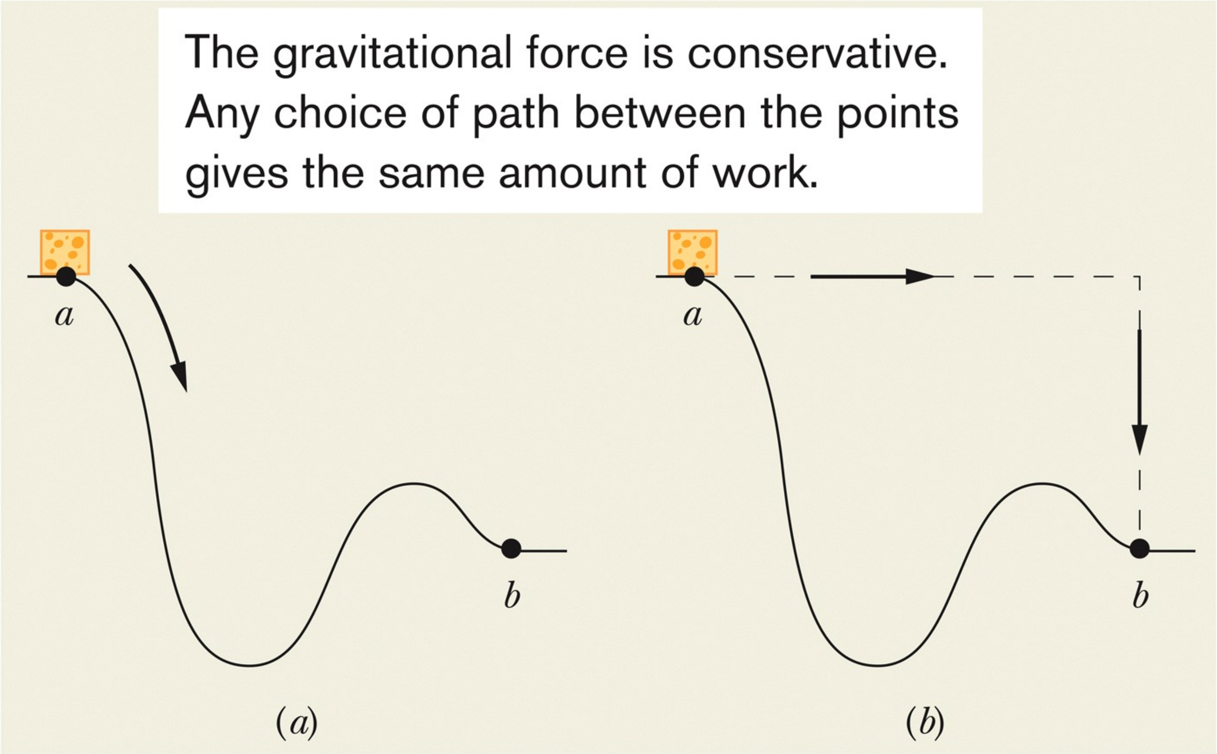
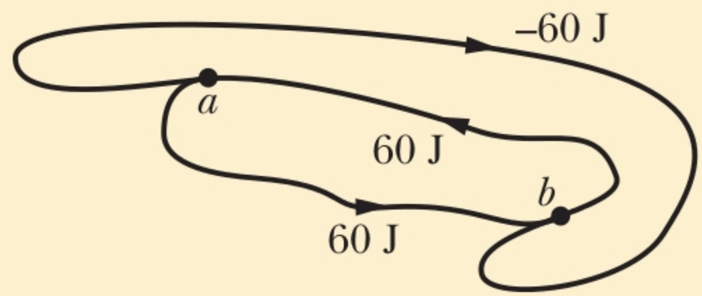


Figure 8-5

8-1 Potential Energy

Checkpoint 1

The figure shows three paths connecting points a and b. A single force $\vec{F}$ does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force $\vec{F}$ conservative?



Answer: No. The paths from $a \rightarrow b$ have different signs. One pair of paths allows the formation of a zero-work loop. The other does not.

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8-1 Potential Energy

- For the general case, we calculate work as:

$$W = \int_{x_i}^{x_f} F(x)\,dx. \qquad \text{Eq. (8-5)}$$

- So we calculate potential energy as:

$$\Delta U = -\int_{x_i}^{x_f} F(x)\,dx. \qquad \text{Eq. (8-6)}$$

- Using this to calculate gravitational PE, relative to a **reference configuration** with **reference point** $y_i = 0$:

$$U(y) = mgy \qquad \text{Eq. (8-9)}$$

The gravitational potential energy associated with a particle–Earth system depends only on the vertical position y (or height) of the particle relative to the reference position $y = 0$, not on the horizontal position.

8-1 Potential Energy

- Use the same process to calculate spring PE:

$$\Delta U = -\int_{x_i}^{x_f} (-kx)\, dx = k \int_{x_i}^{x_f} x\, dx = \tfrac{1}{2}k\left[x^2\right]_{x_i}^{x_f}, \qquad \textbf{Eq. (8-10)}$$

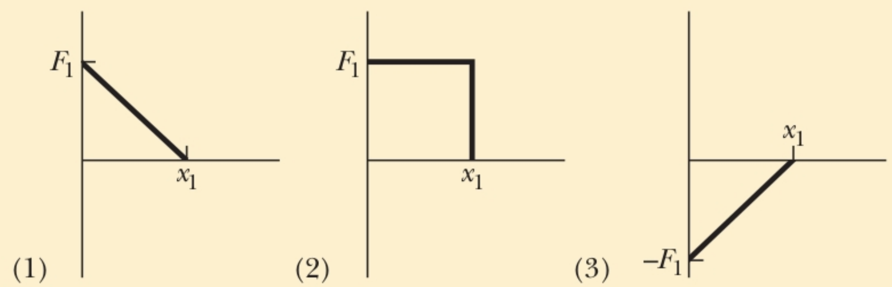
$$\Delta U = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2.$$

- With reference point x_i = 0 for a relaxed spring:

$$\boxed{U(x) = \tfrac{1}{2}kx^2} \qquad \textbf{Eq. (8-11)}$$

Checkpoint 2

A particle is to move along an x axis from $x = 0$ to x_1 while a conservative force, directed along the x axis, acts on the particle. The figure shows three situations in which the x component of that force varies with x. The force has the same maximum magnitude F_1 in all three situations. Rank the situations according to the change in the associated potential energy during the particle's motion, most positive first.



Answer: (3), (1), (2); a positive force does positive work, decreasing the PE; a negative force (e.g., 3) does negative work, increasing the PE

8-2 Conservation of Mechanical Energy

Learning Objectives

8.05 After first clearly defining which objects form a system, identify that the mechanical energy of the system is the sum of the kinetic energies and potential energies of those objects.

8.06 For an isolated system in which only conservative forces act, apply the conservation of mechanical energy to relate the initial potential and kinetic energies to the potential and kinetic energies at a later instant.

8-2 Conservation of Mechanical Energy

- The mechanical energy of a system is the sum of its potential energy *U* and kinetic energy *K*:

$$E_{mec} = K + U$$ Eq. (8-12)

- Work done by conservative forces increases *K* and decreases *U* by that amount, so:

$$\Delta K = -\Delta U.$$ Eq. (8-15)

- Using subscripts to refer to different instants of time:

$$K_2 + U_2 = K_1 + U_1$$ Eq. (8-17)

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

8-2 Conservation of Mechanical Energy

- This is the principle of the **conservation of mechanical energy**:

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0. \quad \textbf{Eq. (8-18)}$$

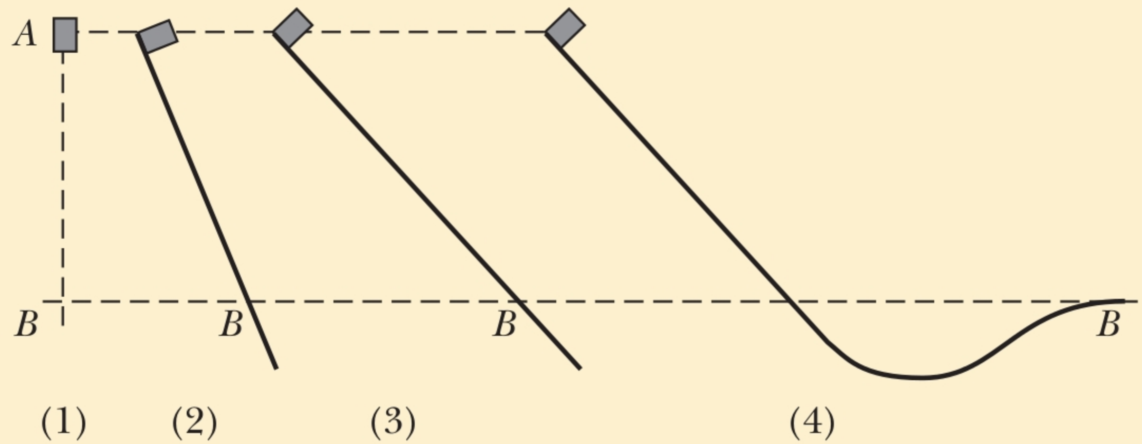
- This is very powerful tool:

> When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant *without considering the intermediate motion* and *without finding the work done by the forces involved.*

- One application:
  - Choose the lowest point in the system as $U = 0$
  - Then at the highest point $U = \max$, and $K = \min$

8-2 Conservation of Mechanical Energy

Checkpoint 3



The figure shows four situations—one in which an initially stationary block is dropped and three in which the block is allowed to slide down frictionless ramps. (a) Rank the situations according to the kinetic energy of the block at point B, greatest first. (b) Rank them according to the speed of the block at point B, greatest first.

Answer: Since there are no nonconservative forces, all of the difference in potential energy must go to kinetic energy. Therefore all are equal in (a). Because of this fact, they are also all equal in (b).

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8-3 Reading a Potential Energy Curve

Learning Objectives

8.07 Given a particle's potential energy as a function of position *x*, determine the force on the particle.

8.08 Given a graph of potential energy versus *x*, determine the force on a particle.

8.09 On a graph of potential energy versus *x*, superimpose a line for a particle's mechanical energy and determine kinetic energy for any given value of *x*.

8.10 If a particle moves along an x axis, use a potential-energy graph for that axis and the conservation of mechanical energy to relate the energy values at one position to those at another position.

8.11 On a potential-energy graph, identify any turning points and any regions where the particle is not allowed because of energy requirements.

8.12 Explain neutral equilibrium, stable equilibrium, and unstable equilibrium.

8-3 Reading a Potential Energy Curve

- For one dimension, force and potential energy are related (by work) as:

$$F(x) = -\frac{dU(x)}{dx}$$ **Eq. (8-22)**

- Therefore we can find the force *F*(*x*) from a plot of the potential energy *U*(*x*), by taking the derivative (slope)
- If we write the mechanical energy out:

$$U(x) + K(x) = E_{\text{mec}}.$$ **Eq. (8-23)**

- We see how *K*(*x*) varies with *U*(*x*):

$$K(x) = E_{\text{mec}} - U(x).$$ **Eq. (8-24)**

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8-3 Reading a Potential Energy Curve

- To find $K(x)$ at any place, take the total mechanical energy (constant) and subtract $U(x)$
- Places where $K = 0$ are **turning points**
  - There, the particle changes direction (K cannot be negative)
- At equilibrium points, the slope of $U(x)$ is 0
- A particle in **neutral equilibrium** is stationary, with potential energy only, and net force = 0
  - If displaced to one side slightly, it would remain in its new position
  - Example: a marble on a flat tabletop

8-3 Reading a Potential Energy Curve

- A particle in **unstable equilibrium** is stationary, with potential energy only, and net force = 0
  - If displaced slightly to one direction, it will feel a force propelling it in that direction
  - Example: a marble balanced on a bowling ball
- A particle in **stable equilibrium** is stationary, with potential energy only, and net force = 0
  - If displaced to one side slightly, it will feel a force returning it to its original position
  - Example: a marble placed at the bottom of a bowl

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8-3 Reading a Potential Energy Curve

- Plot (a) shows the potential $U(x)$
- Plot (b) shows the force $F(x)$
- If we draw a horizontal line, (c) or (f) for example, we can see the range of possible positions
- $x < x_1$ is forbidden for the E_{mec} in (c): the particle does not have the energy to reach those points

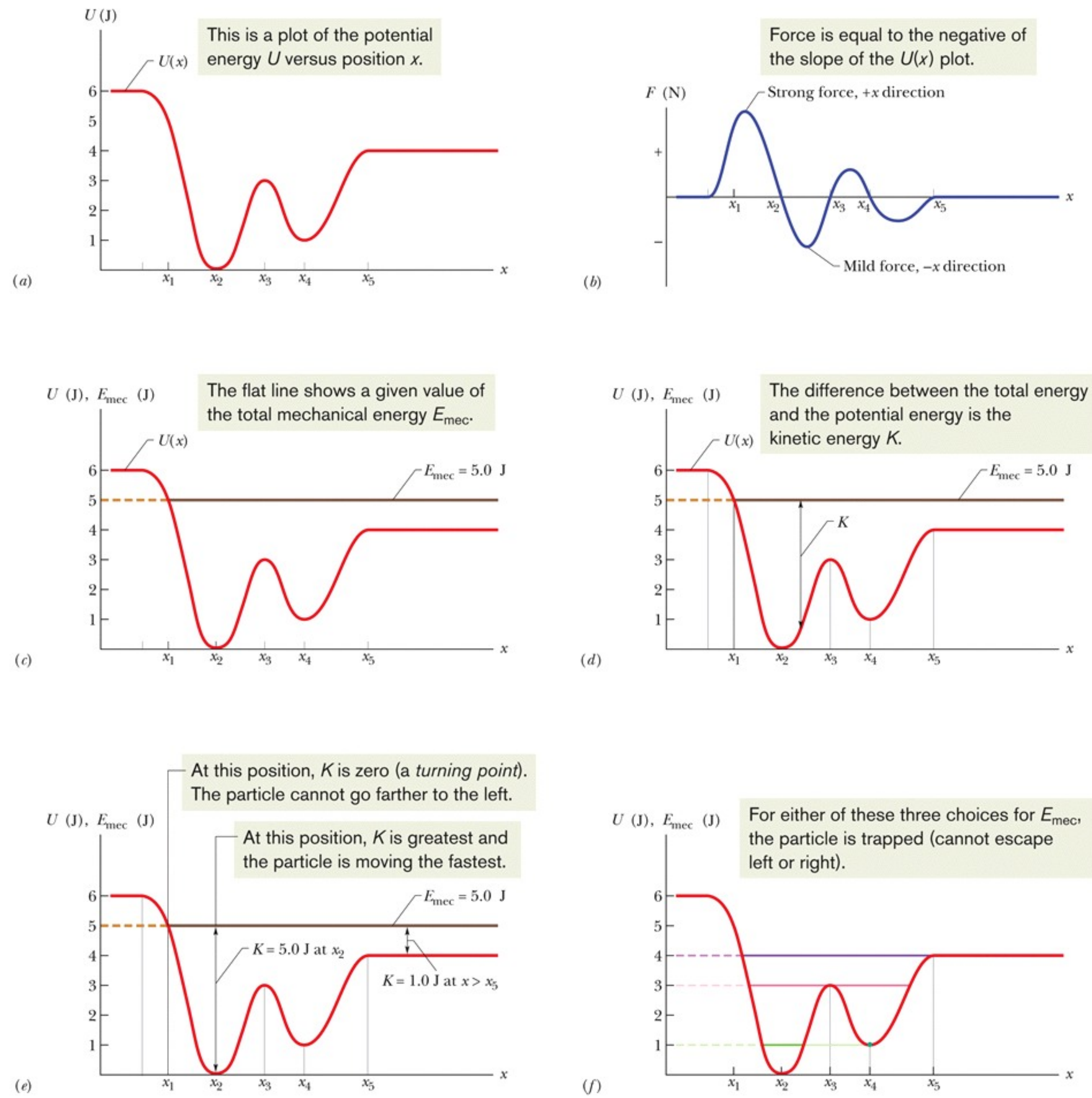
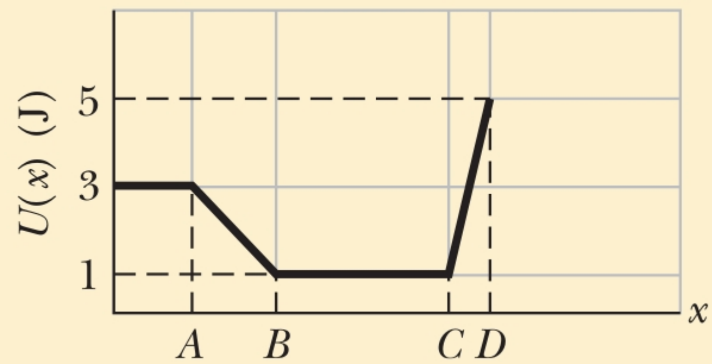


Figure 8-9

8-3 Reading a Potential Energy Curve

Checkpoint 4

The figure gives the potential energy function $U(x)$ for a system in which a particle is in one-dimensional motion. (a) Rank regions AB, BC, and CD according to the magnitude of the force on the particle, greatest first. (b) What is the direction of the force when the particle is in region AB?



Answer: (a) *CD*, *AB*, *BC*   (b) to the right

8-4 Work Done on a System by an External Force

Learning Objectives

8.13 When work is done on a system by an external force with no friction involved, determine the changes in kinetic energy and potential energy.

8.14 When work is done on a system by an external force with friction involved, relate that work to the changes in kinetic energy, potential energy, and thermal energy.

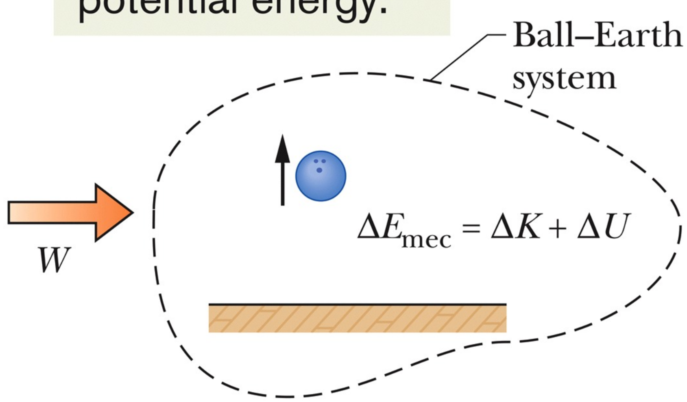
8-4 Work Done on a System by an External Force

- We can extend work on an object to work on a system:

> Work is energy transferred to or from a system by means of an external force acting on that system.

- For a system of more than 1 particle, work can change both *K* and *U*, or other forms of energy of the system
- For a frictionless system:

Your lifting force transfers energy to kinetic energy and potential energy.



$$W = \Delta K + \Delta U, \quad \text{Eq. (8-25)}$$

$$W = \Delta E_{mec} \quad \text{Eq. (8-26)}$$

Figure 8-12

8-4 Work Done on a System by an External Force

- For a system with friction:

$$\Delta E_{\text{th}} = f_k d \quad \text{(increase in thermal energy by sliding).}$$ Eq. (8-31)

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}}$$ Eq. (8-33)

- The thermal energy comes from the forming and breaking of the welds between the sliding surfaces

The applied force supplies energy. The frictional force transfers some of it to thermal energy.

So, the work done by the applied force goes into kinetic energy and also thermal energy.

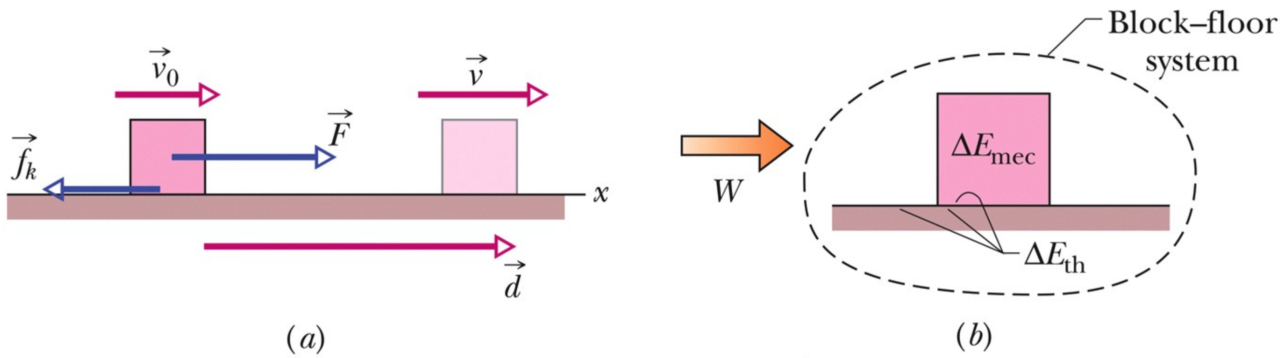


Figure 8-13

8-4 Work Done on a System by an External Force

Checkpoint 5

In three trials, a block is pushed by a horizontal applied force across a floor that is not frictionless, as in Fig. 8-13*a*. The magnitudes F of the applied force and the results of the pushing on the block's speed are given in the table. In all three trials, the block is pushed through the same distance d. Rank the three trials according to the change in the thermal energy of the block and floor that occurs in that distance d, greatest first.

Trial	F	Result on Block's Speed
a	5.0 N	decreases
b	7.0 N	remains constant
c	8.0 N	increases

Answer: All trials result in equal thermal energy change. The value of f_k is the same in all cases, since μ_k has only 1 value.

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8-5 Conservation of Energy

Learning Objectives

8.15 For an isolated system (no net external force), apply the conservation of energy to relate the initial total energy (energies of all kinds) to the total energy at a later instant.

8.16 For a nonisolated system, relate the work done on the system by a net external force to the changes in the various types of energies within the system.

8.17 Apply the relationship between average power, the associated energy transfer, and the time interval in which that transfer is made.

8.18 Given an energy transfer as a function of time (either as an equation or graph), determine the instantaneous power (the transfer at any given instant).

8-5 Conservation of Energy

- Energy transferred between systems can always be accounted for
- The **law of conservation of energy** concerns
  - The **total energy** *E* of a system
  - Which includes mechanical, thermal, and other internal energy

> The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

- Considering only energy transfer through work:

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}},$$

Eq. (8-35)

8-5 Conservation of Energy

- An isolated system is one for which there can be no *external* energy transfer

The total energy E of an isolated system cannot change.

- Energy transfers may happen internal to the system
- We can write: $$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0$$ **Eq. (8-36)**

- Or, for two instants of time:

$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}} - \Delta E_{\text{int}}.$$ **Eq. (8-37)**

In an isolated system, we can relate the total energy at one instant to the total energy at another instant *without considering the energies at intermediate times.*

8-5 Conservation of Energy

- External forces can act on a system without doing work:

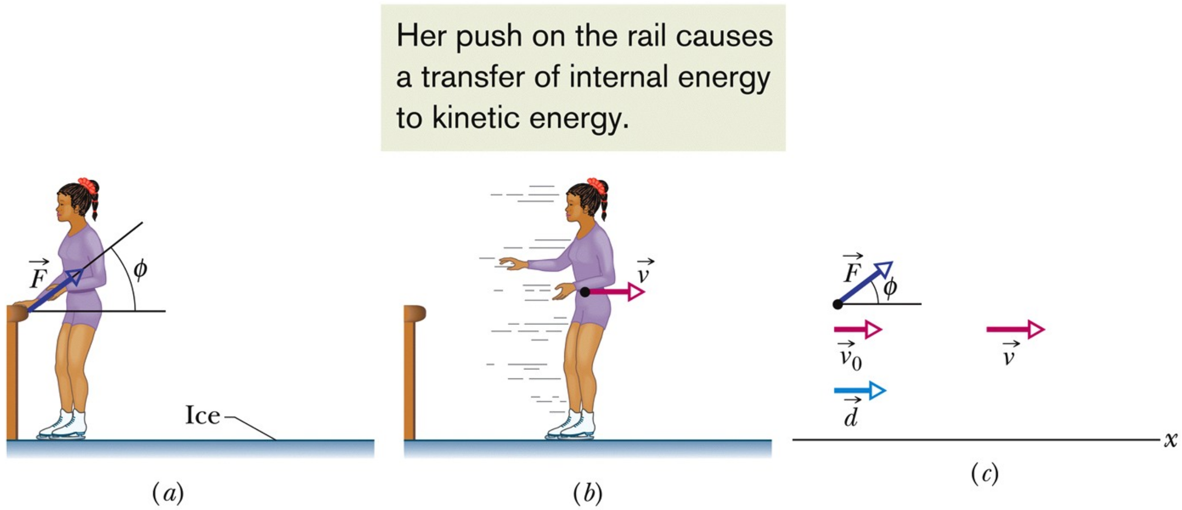

Figure 8-15

- The skater pushes herself away from the wall
- She turns internal chemical energy in her muscles into kinetic energy
- Her *K* change is caused by the force from the wall, but the wall does not provide her any energy

8-5 Conservation of Energy

- We can expand the definition of power
- In general, power is the rate at which energy is transferred by a force from one type to another
- If energy *ΔE* is transferred in time *Δt*, the **average power** is:

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t}. \qquad \textbf{Eq. (8-40)}$$

- And the **instantaneous power** is:

$$P = \frac{dE}{dt}. \qquad \textbf{Eq. (8-41)}$$

8 Summary

Conservative Forces

- Net work on a particle over a closed path is 0

Potential Energy

- Energy associated with the configuration of a system and a conservative force

$$\Delta U = -\int_{x_i}^{x_f} F(x)\, dx. \quad \textbf{Eq. (8-6)}$$

Gravitational Potential Energy

- Energy associated with Earth + a nearby particle

$$U(y) = mgy \quad \textbf{Eq. (8-9)}$$

Elastic Potential Energy

- Energy associated with compression or extension of a spring

$$U(x) = \tfrac{1}{2}kx^2 \quad \textbf{Eq. (8-11)}$$

8 Summary

Mechanical Energy

$$E_{\text{mec}} = K + U$$ **Eq. (8-12)**

- For only conservative forces within an isolated system, mechanical energy is conserved

Potential Energy Curves

$$F(x) = -\frac{dU(x)}{dx}$$ **Eq. (8-22)**

- At turning points a particle reverses direction
- At equilibrium, slope of $U(x)$ is 0

Work Done on a System by an External Force

- Without/with friction:

$$W = \Delta E_{\text{mec}}$$ **Eq. (8-26)**

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}}$$ **Eq. (8-33)**

Conservation of Energy

- The **total energy** can change only by amounts transferred in or out of the system

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}},$$ **Eq. (8-35)**

8 Summary

Power

- The rate at which a force transfers energy
- **Average power**:

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t}. \quad \textbf{Eq. (8-40)}$$

- **Instantaneous power**:

$$P = \frac{dE}{dt}. \quad \textbf{Eq. (8-41)}$$