Chapter 7

Kinetic Energy and Work

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7-1 Kinetic Energy

Learning Objectives

7.01 Apply the relationship between a particle's kinetic energy, mass, and speed.

7.02 Identify that kinetic energy is a scalar quantity.

7-1 Kinetic Energy

- Energy is required for any sort of motion
- Energy:
  - Is a scalar quantity assigned to an object or a system of objects
  - Can be changed from one form to another
  - Is *conserved* in a closed system, that is the total amount of energy of all types is always the same
- In this chapter we discuss one type of energy (kinetic energy)
- We also discuss one method of transferring energy (work)

7-1 Kinetic Energy

- **Kinetic energy**:
  - The faster an object moves, the greater its kinetic energy
  - Kinetic energy is zero for a stationary object
- For an object with *v* well below the speed of light:

$$K = \tfrac{1}{2}mv^2$$ **Eq. (7-1)**

- The unit of kinetic energy is a **joule** (J)

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2.$$ **Eq. (7-2)**

7-1 Kinetic Energy

Example Energy released by 2 colliding trains with given weight and acceleration from rest:

- Find the final velocity of each locomotive:

$$v^2 = v_0^2 + 2a(x - x_0).$$

$$v^2 = 0 + 2(0.26\ \mathrm{m/s^2})(3.2 \times 10^3\ \mathrm{m}),$$
$$v = 40.8\ \mathrm{m/s} = 147\ \mathrm{km/h}.$$

- Convert weight to mass:

$$m = \frac{1.2 \times 10^6\ \mathrm{N}}{9.8\ \mathrm{m/s^2}} = 1.22 \times 10^5\ \mathrm{kg}.$$

- Find the kinetic energy:

$$K = 2(\tfrac{1}{2}mv^2) = (1.22 \times 10^5\ \mathrm{kg})(40.8\ \mathrm{m/s})^2$$
$$= 2.0 \times 10^8\ \mathrm{J}. \qquad \text{(Answer)}$$

7-2 Work and Kinetic Energy

Learning Objectives

7.03 Apply the relationship between a force (magnitude and direction) and the work done on a particle by the force when the particle undergoes a displacement.

7.04 Calculate work by taking a dot product of the force vector and the displacement vector, in either magnitude-angle or unit-vector notation.

7.05 If multiple forces act on a particle, calculate the net work done by them.

7.06 Apply the work-kinetic energy theorem to relate the work done by a force (or the net work done by multiple forces) and the resulting change in kinetic energy.

7-2 Work and Kinetic Energy

- Account for changes in kinetic energy by saying energy has been transferred *to* or *from* the object
- In a transfer of energy via a force, **work** is:
  - *Done on the object by the force*

Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

- This is not the common meaning of the word "work"
  - To do work on an object, energy must be transferred
  - Throwing a baseball does work
  - Pushing an immovable wall does not do work

7-2 Work and Kinetic Energy

- Start from force equation and 1-dimensional velocity:

$$F_x = ma_x,$$

Eq. (7-3)

$$v^2 = v_0^2 + 2a_x d.$$

Eq. (7-4)

- Rearrange into kinetic energies:

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d.$$

Eq. (7-5)

- The left side is now the change in energy
- Therefore work is:

$$W = F_x d.$$

Eq. (7-6)

7-2 Work and Kinetic Energy

To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.

- For an angle φ between force and displacement:

$$W = Fd\cos\phi \quad \text{Eq. (7-7)}$$

- As vectors we can write:

$$W = \vec{F}\cdot\vec{d} \quad \text{Eq. (7-8)}$$

- Notes on these equations:
  - Force is constant
  - Object is particle-like (rigid)
  - Work can be positive or negative

7-2 Work and Kinetic Energy

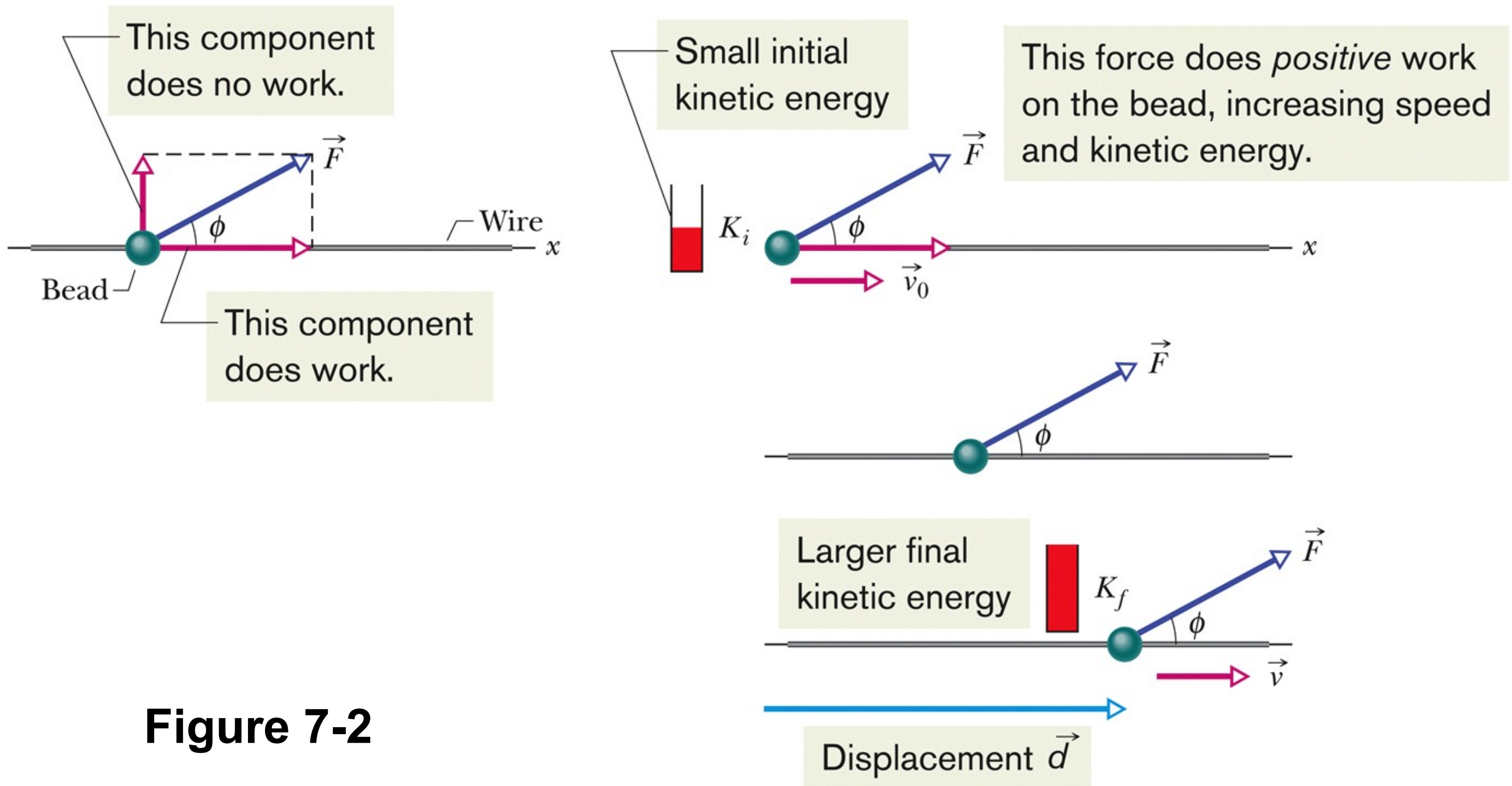

Figure 7-2

- Work has the SI unit of joules (J), the same as energy
- In the British system, the unit is foot-pound (ft lb)

7-2 Work and Kinetic Energy

A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It does zero work when it has no such vector component.

- For two or more forces, the **net work** is the sum of the works done by all the individual forces
- Two methods to calculate net work:
  - We can find all the works and sum the individual work terms.
  - We can take the vector sum of forces (F_{net}) and calculate the net work once

7-2 Work and Kinetic Energy

- The **work-kinetic energy theorem** states:

$$\Delta K = K_f - K_i = W, \qquad \text{Eq. (7-10)}$$

- (*change in kinetic energy*) = (*the net work done*)

- Or we can write it as:

$$K_f = K_i + W, \qquad \text{Eq. (7-11)}$$

- (*final KE*) = (*initial KE*) + (*net work*)

7-2 Work and Kinetic Energy

- The work-kinetic energy theorem holds for positive and negative work

Example If the kinetic energy of a particle is initially 5 J:

- A net transfer of 2 J to the particle (positive work)
  - Final KE = 7 J
- A net transfer of 2 J from the particle (negative work)
  - Final KE = 3 J

7-2 Work and Kinetic Energy

Checkpoint 1

A particle moves along an x axis. Does the kinetic energy of the particle increase, decrease, or remain the same if the particle's velocity changes (a) from -3 m/s to -2 m/s and (b) from -2 m/s to 2 m/s? (c) In each situation, is the work done on the particle positive, negative, or zero?

Answer: (a) energy decreases (b) energy remains the same

(c) work is negative for (a), and work is zero for (b)

7-3 Work Done by the Gravitational Force

Learning Objectives

7.07 Calculate the work done by the gravitational force when an object is lifted or lowered.

7.08 Apply the work-kinetic energy theorem to situations where an object is lifted or lowered.

7-3 Work Done by the Gravitational Force

- We calculate the work as we would for any force
- Our equation is:

$$W_g = mgd\cos\phi$$ Eq. (7-12)

- For a rising object:

$$W_g = mgd\cos 180° = mgd(-1) = -mgd.$$ Eq. (7-13)

- For a falling object:

$$W_g = mgd\cos 0° = mgd(+1) = +mgd.$$ Eq. (7-14)

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7-3 Work Done by the Gravitational Force

- Work done in lifting or lowering an object, applying an upwards force:

$$\Delta K = K_f - K_i = W_a + W_g, \quad \textbf{Eq. (7-15)}$$

- For a stationary object:
  - Kinetic energies are zero
  - We find:

$$W_a + W_g = 0$$

$$W_a = -W_g. \quad \textbf{Eq. (7-16)}$$

- In other words, for an applied lifting force:

$$W_a = -mgd\cos\phi \qquad \text{(work done in lifting and lowering; } K_f = K_i\text{)},$$

Eq. (7-17)

- Applies regardless of path

7-3 Work Done by the Gravitational Force

- Figure 7-7 shows the orientations of forces and their associated works for upward and downward displacement
- Note that the works (in 7-16) need not be equal, they are only equal if the initial and final kinetic energies are equal
- If the works are unequal, you will need to know the difference between initial and final kinetic energy to solve for the work

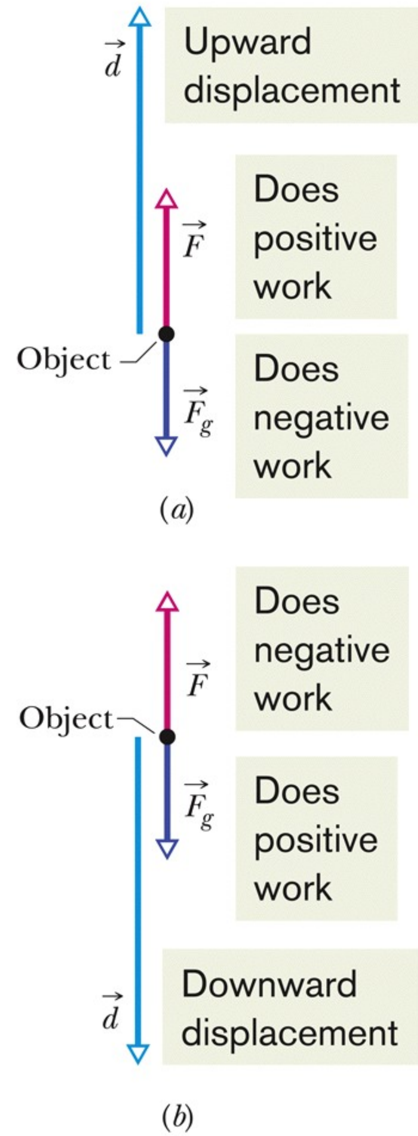

Figure 7-7

7-3 Work Done by the Gravitational Force

Examples You are a passenger:

- Being pulled up a ski-slope
  - Tension does positive work, gravity does negative work

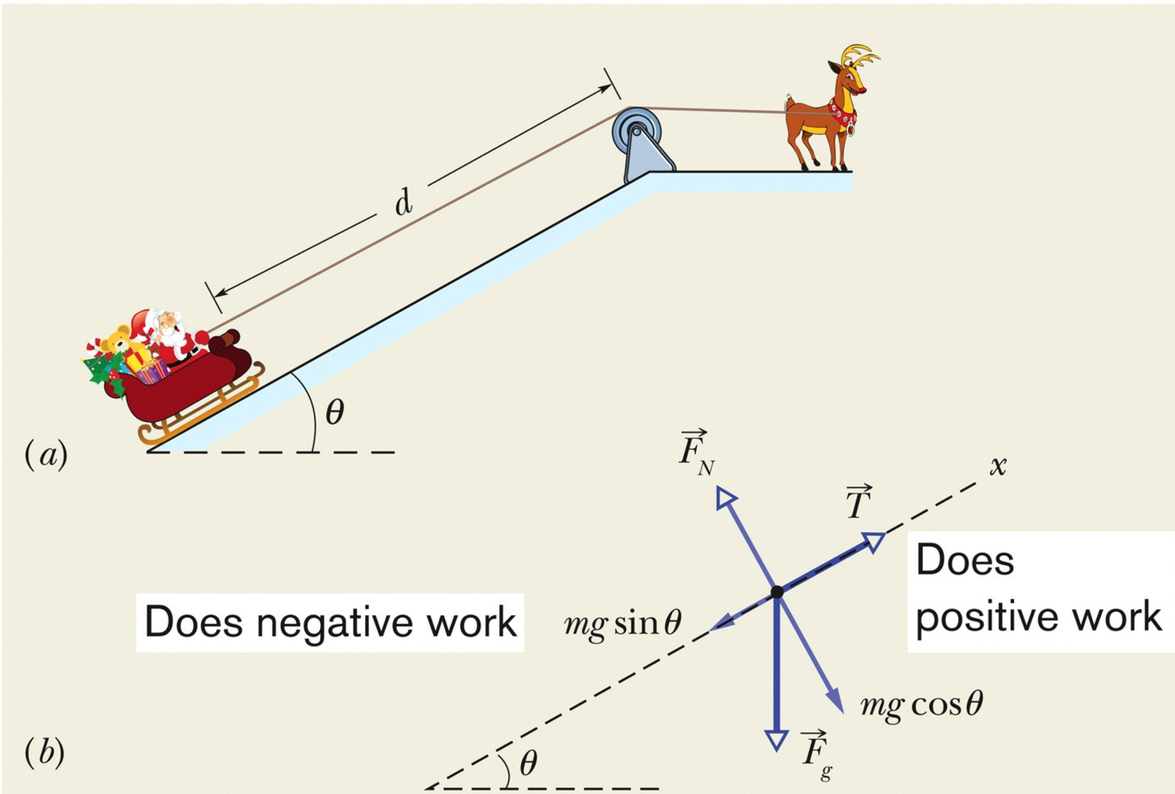

Figure 7-8

7-3 Work Done by the Gravitational Force

Examples You are a passenger:

- Being lowered down in an elevator
  - Tension does negative work, gravity does positive work

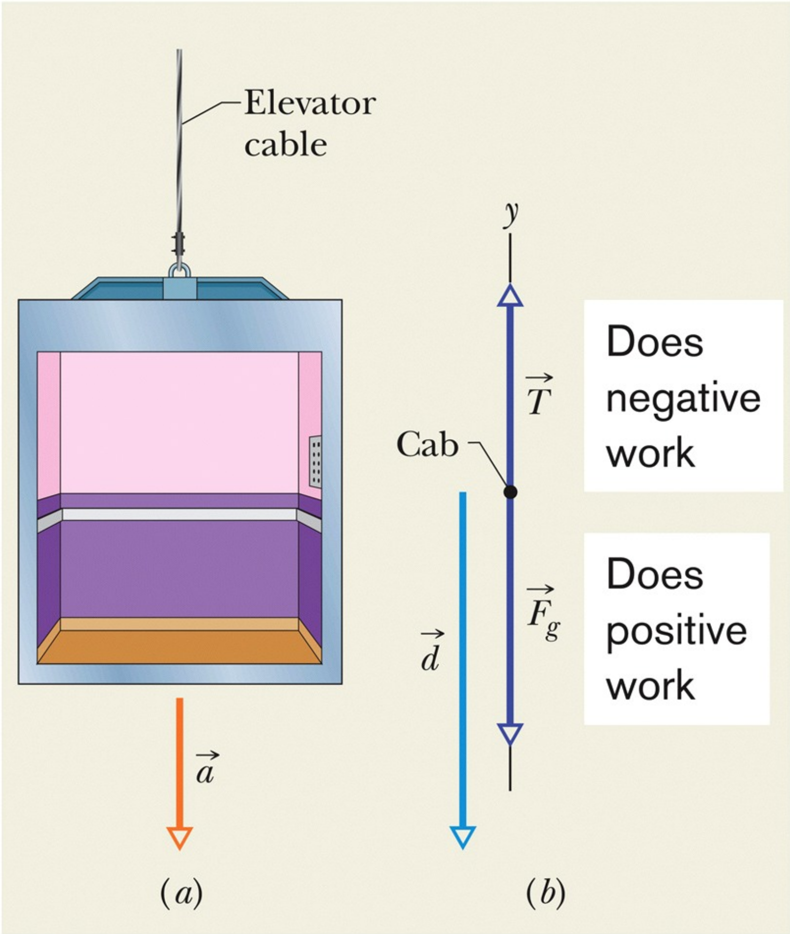

Figure 7-9

7-4 Work Done by a Spring Force

Learning Objectives

7.09 Apply the relationship (Hooke's law) between spring force, the stretch or compression of the spring, and the spring constant.

7.10 Identify that a spring force is a variable force.

7.11 Calculate the work done on an object by a spring force by integrating the force from the initial position to the final position of the object or by using the known generic result of the integration.

7.12 Calculate work by graphically integrating on a graph of force versus position of the object.

7.13 Apply the work-kinetic energy theorem to situations in which an object is moved by a spring force.

7-4 Work Done by a Spring Force

- A **spring force** is the *variable force* from a spring
  - A spring force has a particular mathematical form
  - Many forces in nature have this form

- Figure (a) shows the spring in its **relaxed state**: since it is neither compressed nor extended, no force is applied

- If we stretch or extend the spring it resists, and exerts a *restoring force* that attempts to return the spring to its relaxed state

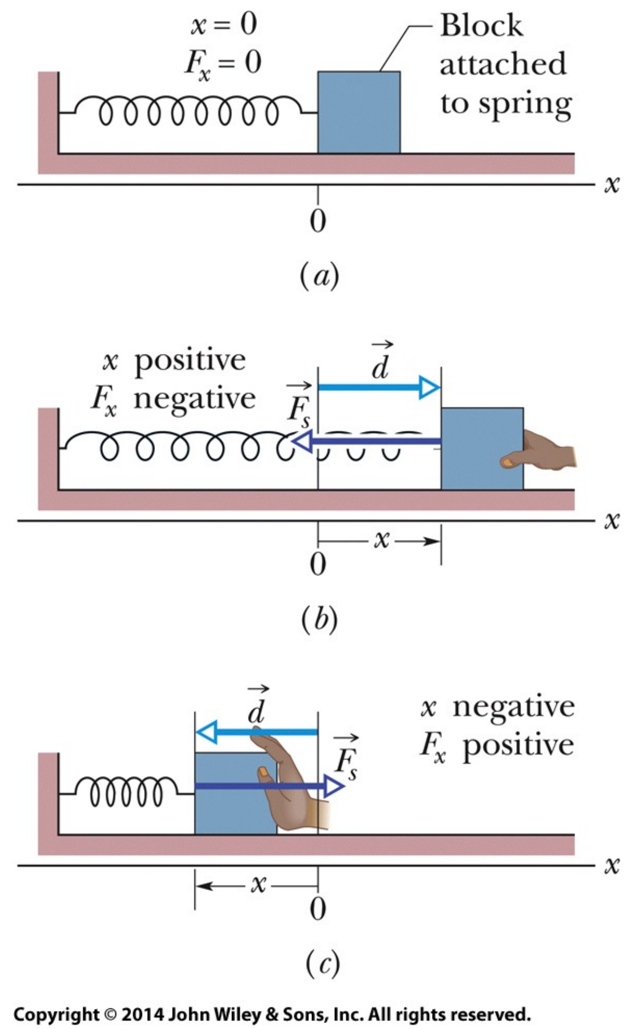

Figure 7-10

7-4 Work Done by a Spring Force

- The spring force is given by **Hooke's law**:

$$\vec{F}_s = -k\vec{d} \qquad \text{Eq. (7-20)}$$

- The negative sign represents that the force always opposes the displacement
- The **spring constant** *k* is a is a measure of the stiffness of the spring
- This is a variable force (function of position) and it exhibits a linear relationship between *F* and *d*
- For a spring along the *x*-axis we can write:

$$F_x = -kx \qquad \text{Eq. (7-21)}$$

7-4 Work Done by a Spring Force

- We can find the work by integrating:

$$W_s = \int_{x_i}^{x_f} -F_x \, dx. \qquad \text{Eq. (7-23)}$$

- Plug kx in for F_x:

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \text{Eq. (7-25)}$$

- The work:
  - Can be positive or negative
  - Depends on the *net* energy transfer

Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

7-4 Work Done by a Spring Force

- For an initial position of $x = 0$:

$$W_s = -\tfrac{1}{2}kx^2 \qquad \textbf{Eq. (7-26)}$$

- For an applied force where the initial and final kinetic energies are zero:

$$W_a = -W_s. \qquad \textbf{Eq. (7-28)}$$

If a block that is attached to a spring is stationary before and after a displacement, then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

7-4 Work Done by a Spring Force

Checkpoint 2

For three situations, the initial and final positions, respectively, along the x axis for the block in Fig. 7-10 are (a) -3 cm, 2 cm; (b) 2 cm, 3 cm; and (c) -2 cm, 2 cm. In each situation, is the work done by the spring force on the block positive, negative, or zero?

Answer: (a) positive

(b) negative

(c) zero

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7-5 Work Done by a General Variable Force

Learning Objectives

7.14 Given a variable force as a function of position, calculate the work done by it on an object by integrating the function from the initial to the final position of the object in one or more dimensions.

7.15 Given a graph of force versus position, calculate the work done by graphically integrating from the initial position to the final position of the object.

7.16 Convert a graph of acceleration versus position to a graph of force versus position.

7.17 Apply the work-kinetic energy theorem to situations where an object is moved by a variable force.

7-5 Work Done by a General Variable Force

- We take a one-dimensional example
- We need to integrate the work equation (which normally applies only for a constant force) over the change in position
- We can show this process by an approximation with rectangles under the curve

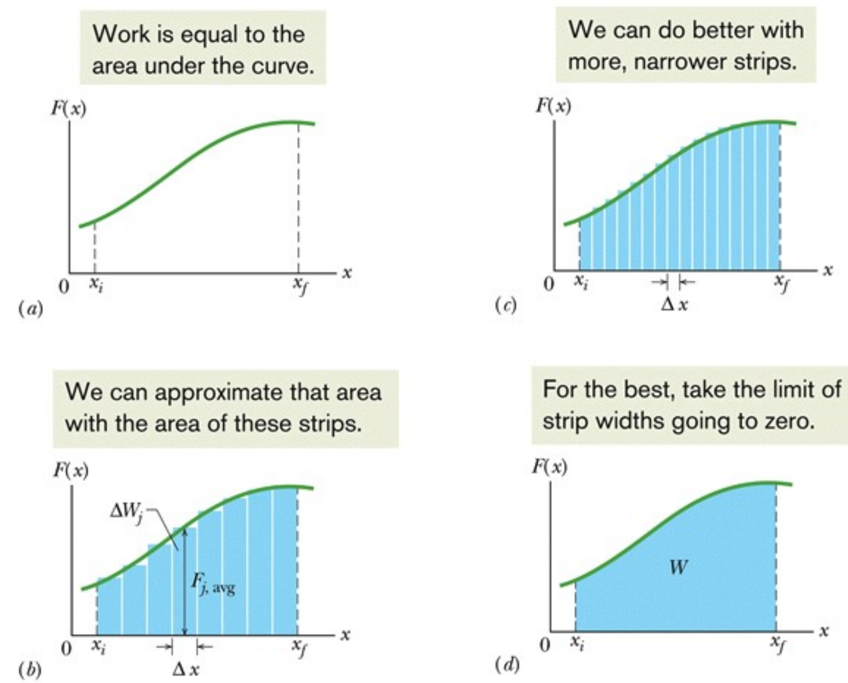

Figure 7-12

7-5 Work Done by a General Variable Force

- Our sum of rectangles would be:

$$W = \lim_{\Delta x \to 0} \sum F_{j,\text{avg}}\, \Delta x. \qquad \textbf{Eq. (7-31)}$$

- As an integral this is:

$$W = \int_{x_i}^{x_f} F(x)\, dx \qquad \textbf{Eq. (7-32)}$$

- In three dimensions, we integrate each separately:

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\, dx + \int_{y_i}^{y_f} F_y\, dy + \int_{z_i}^{z_f} F_z\, dz. \qquad \textbf{Eq. (7-36)}$$

- The work-kinetic energy theorem still applies!

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7-6 Power

Learning Objectives

7.18 Apply the relationship between average power, the work done by a force, and the time interval in which that work is done.

7.19 Given the work as a function of time, find the instantaneous power.

7.20 Determine the instantaneous power by taking a dot product of the force vector and an object's velocity vector, in magnitude-angle and unit-vector notations.

7-6 Power

- **Power** is the time rate at which a force does work
- A force does W work in a time Δt; the **average power** due to the force is:

$$P_{\text{avg}} = \frac{W}{\Delta t} \quad \textbf{Eq. (7-42)}$$

- The **instantaneous power** at a particular time is:

$$P = \frac{dW}{dt} \quad \textbf{Eq. (7-43)}$$

- The SI unit for power is the watt (W): 1 W = 1 J/s
- Therefore work-energy can be written as (*power*) x (*time*) e.g. kWh, the kilowatt-hour

7-6 Power

- Solve for the instantaneous power using the definition of work:

$$P = \frac{dW}{dt} = \frac{F\cos\phi\,dx}{dt} = F\cos\phi\left(\frac{dx}{dt}\right),$$

$$P = Fv\cos\phi. \qquad \textbf{Eq. (7-47)}$$

- Or:

$$P = \vec{F}\cdot\vec{v} \qquad \textbf{Eq. (7-48)}$$

Checkpoint 3

A block moves with uniform circular motion because a cord tied to the block is anchored at the center of a circle. Is the power due to the force on the block from the cord positive, negative, or zero?

Answer: zero (consider $P = Fv\cos\phi$, and note that $\phi = 90°$)

7 Summary

Kinetic Energy

- The energy associated with motion

$$K = \frac{1}{2}mv^2$$ Eq. (7-1)

Work

- Energy transferred to or from an object via a force
- Can be positive or negative

Work Done by a Constant Force

$$W = Fd\cos\phi$$ Eq. (7-7)

$$W = \vec{F}\cdot\vec{d}$$ Eq. (7-8)

- The **net work** is the sum of individual works

Work and Kinetic Energy

$$\Delta K = K_f - K_i = W,$$ Eq. (7-10)

$$K_f = K_i + W,$$ Eq. (7-11)

7 Summary

Work Done by the Gravitational Force

$$W_g = mgd\cos\phi$$ Eq. (7-12)

Work Done in Lifting and Lowering an Object

$$W_a + W_g = 0$$

$$W_a = -W_g.$$ Eq. (7-16)

Spring Force

- Relaxed state: applies no force
- Spring constant *k* measures stiffness

$$\vec{F}_s = -k\vec{d}$$ Eq. (7-20)

Spring Force

- For an initial position *x* = 0:

$$W_s = -\tfrac{1}{2}kx^2$$ Eq. (7-26)

7 Summary

Work Done by a Variable Force

- Found by integrating the constant-force work equation

$$W = \int_{x_i}^{x_f} F(x)\, dx$$

Eq. (7-32)

Power

- The rate at which a force does work on an object
- Average power:

$$P_{\text{avg}} = \frac{W}{\Delta t}$$

Eq. (7-42)

- Instantaneous power:

$$P = \frac{dW}{dt}$$

Eq. (7-43)

- For a force acting on a moving object:

$$P = Fv\cos\phi.$$

Eq. (7-47)

$$P = \vec{F} \cdot \vec{v}$$

Eq. (7-48)