Particle dynamics

Newton's laws:

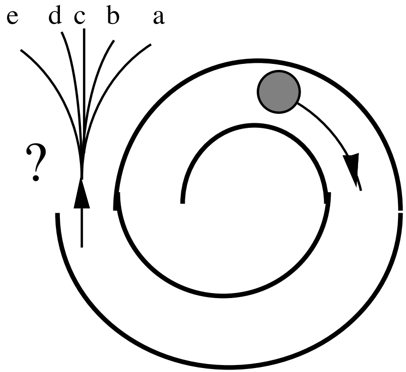
force, mass, acceleration (*don't confuse mass with weight*)

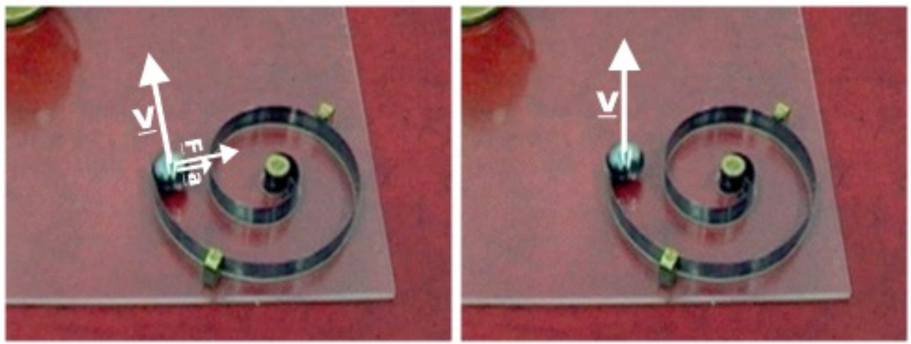
Friction – coefficients of static and kinetic friction

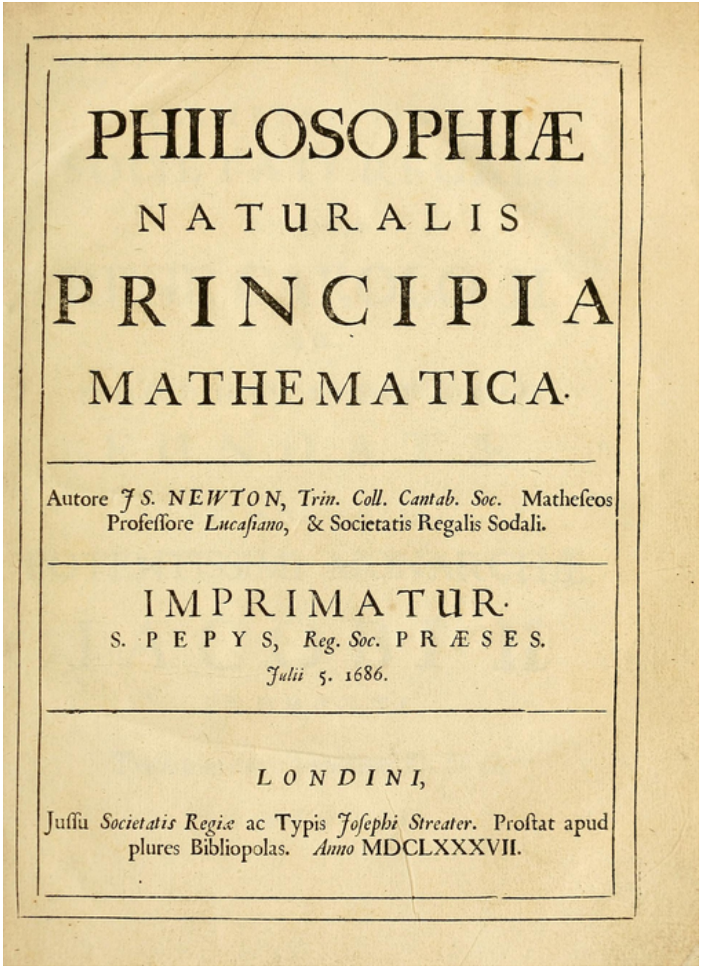
Hooke's Law – restoring force proportional to deformation & opposite direction

Dynamics of circular motion

Question. Top view of ball. What is its trajectory after it leaves the race?






PHILOSOPHIÆ

NATURALIS

PRINCIPIA

MATHEMATICA.

Autore *JS. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheſeos Profeſſore *Lucaſiano*, & Societatis Regalis Sodali.

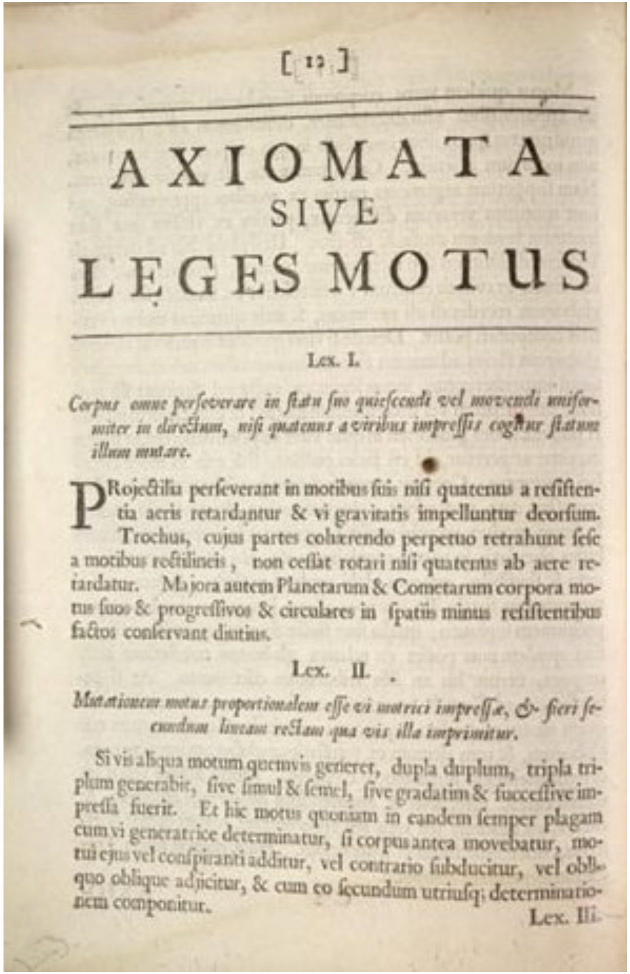
IMPRIMATUR.

S. PEPYS, *Reg. Soc.* PRÆSES.

Julii 5. 1686.

LONDINI,

Juſſu *Societatis Regiæ* ac Typis *Joſephi Streater*. Proſtat apud plures Bibliopolas. *Anno* MDCLXXXVII.


[12]

AXIOMATA
SIVE
LEGES MOTUS

Lex. I.

Corpus omne perſeverare in ſtatu ſuo quieſcendi vel movendi uniformiter in directum, niſi quatenus a viribus impreſſis cogitur ſtatum illum mutare.

Projectilia perſeverant in motibus ſuis niſi quatenus a reſiſtentia aeris retardantur & vi gravitatis impelluntur deorſum. Trochus, cujus partes cohærendo perpetuo retrahunt ſeſe a motibus rectilineis, non ceſſat rotari niſi quatenus ab aere retardatur. Majora autem Planetarum & Cometarum corpora motus ſuos & progreſſivos & circulares in ſpatiis minus reſiſtentibus factos conſervant diutius.

Lex. II.

Mutationem motus proportionalem eſſe vi motrici impreſſæ, & fieri ſecundum lineam rectam qua vis illa imprimitur.

Si vis aliqua motum quemvis generet, dupla duplum, tripla triplum generabit, ſive ſimul & ſemel, ſive gradatim & ſucceſſive impreſſa fuerit. Et hic motus quoniam in eandem ſemper plagam cum vi generatrice determinatur, ſi corpus antea movebatur, motui ejus vel conſpiranti additur, vel contrario ſubducitur, vel oblique obliquo adjicitur, & cum eo ſecundum utriuſq; determinationem componitur.

Lex. III.

Newton calls them "Axioms or laws of motion"

Aristotle: $\vec{v} = 0$ *is "natural" state* *not in syllabus*

Galileo & then Newton: $\vec{a} = 0$ *is "natural" state*

Galileo: ball (almost) regains its original height. But what if we remove one side of the bowl?

Newton's Laws

First law: "Every body persists in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by forces impressed on it."

"zero (total) force ⇒ zero acceleration"

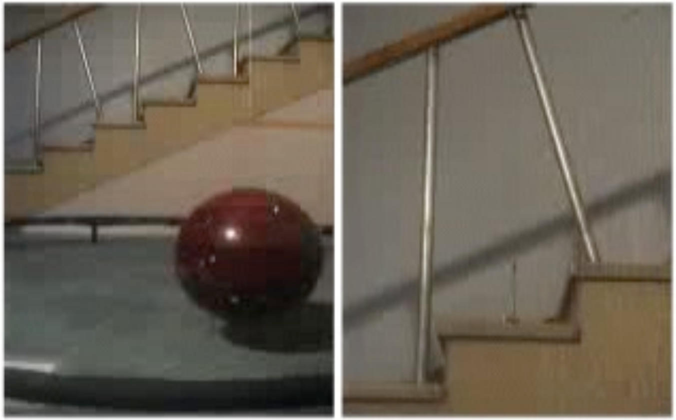
If $\sum \vec{F} = 0$, *then* $\vec{a} = 0$.

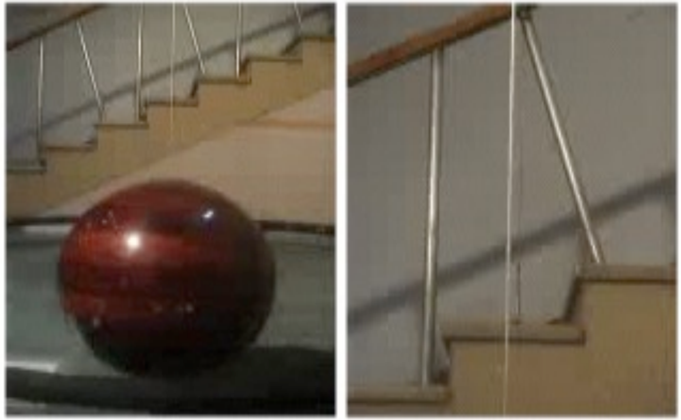
(Applies only in inertial frames of reference, so, more formally:

If $\sum \vec{F} = 0$, *there exist reference frames in which* $\vec{a} = 0$. *These are called* ***inertial frames***

Observation: with respect to these frames, distant stars don't accelerate)

Is the earth an inertial frame? try the experiment in the foyer





In inertial frames:

Second law $\sum \vec{F} = m\vec{a}$ *Σ is important: **Total** force*

*It is a **vector equation**, so*

$\Sigma F_x = ma_x$ $\Sigma F_y = ma_y$ $\Sigma F_z = ma_z$

One vector equation in 3D –> 3 scalar equations

1st law is just a special case of 2nd:

$\sum \vec{F} = m\vec{a}$ *so, if* $\sum \vec{F} = 0$, $\vec{a} = 0$

What does the 2nd law mean?

$$\sum \vec{F} = m\vec{a}$$

$\vec{a}$ is already defined.

i) Does this equation define m?

ii) Does this law define $\vec{F}$?

iii) Is it a physical law?

iv) All of the above?

v) How?

Explanation shouldn't involve weight:

$\sum \vec{F} = m_{inertial}\vec{a}$ and $W = m_{gravitational}\, g$

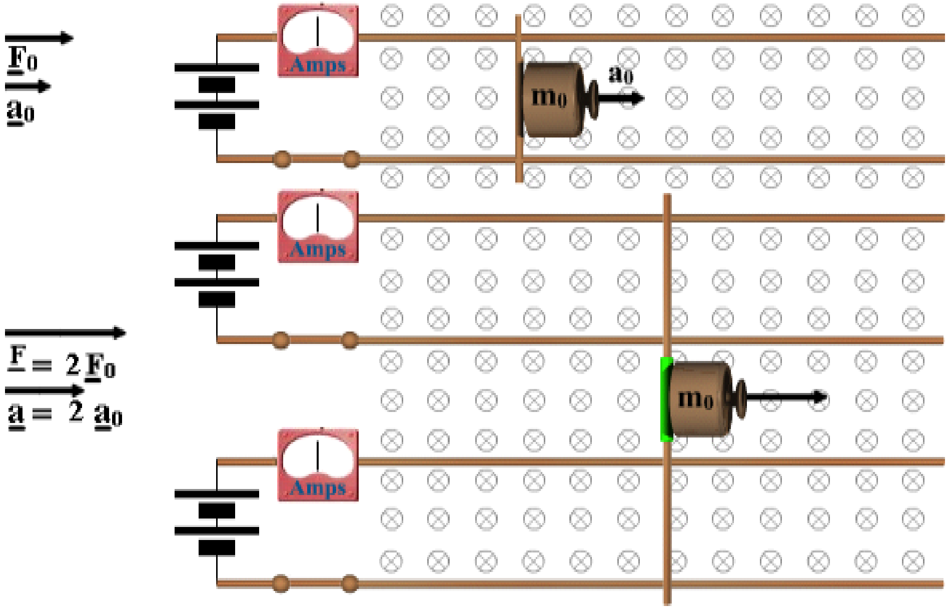
$m_{inertial}$ and $m_{gravitational}$ are not necessarily the same

called inertial and gravitational masses

i) For one mass m, we can calibrate many forces by measuring a.

ii) For any one force F, we can calibrate many masses by measuring a.

iii) 2nd Law is the observation that *all* the m's and F's thus defined are consistent. *e.g.* Having used standard m to calibrate F, now produce $2F$ (eg two identical F systems).



Is $\vec{a}$ now doubled?

Units: Second defined by frequency of an atomic transition, metre by the speed of light. Currently, we use a standard object for the kilogram*,

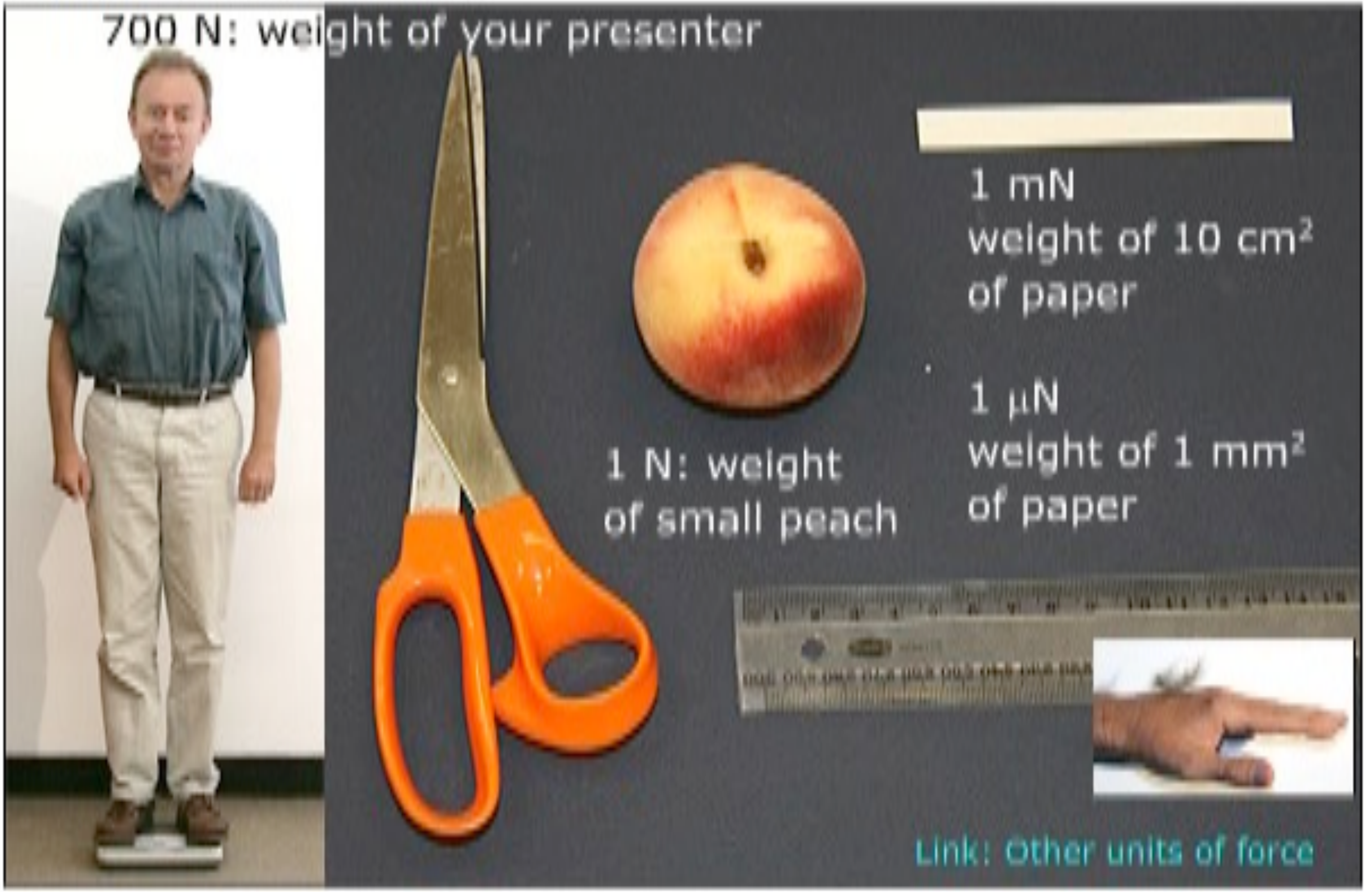
* *For now. In the near future, we'll define the kg in terms of atomic mass*

then choose the unit of force (newtons) such that

$$\sum \vec{F} = m\vec{a}$$

So 1 newton = 1 kg.m.s^{-2}.

But how big is a newton?


700 N: weight of your presenter
1 mN
weight of 10 cm²
of paper
1 μN
weight of 1 mm²
of paper
1 N: weight
of small peach
Link: Other units of force

Questions

What is the weight of

A litre of milk?

A car?

The water in a swimming pool (50m * 10m * 2m at $\rho = 1000\ kg.m^{-3}$)?

What is the maximum force you can exert with your feet?

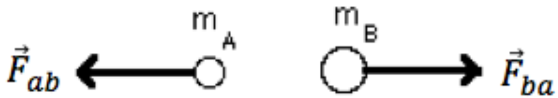
What is the smallest force you can feel?

Newton's 3rd law: "To every action there is always opposed an equal reaction*; or the mutual actions of two bodies upon each other are always equal and directed to contrary parts"

Or Forces always occur in symmetric pairs, $\vec{F}$ and $-\vec{F}$, one acting on each of a pair of interacting bodies. $\vec{F}_{ab} = -\vec{F}_{ba}$

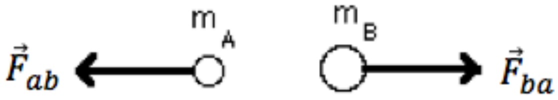
Or Forces come in symmetric pairs that add to zero.

$\vec{F}_{ab} + \vec{F}_{ba} = 0$



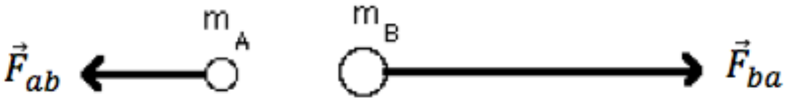
* *I don't like 'reaction' because it might suggest that 'reaction' comes* ***after*** *'action'*

Forces in the 3rd law are completely **symmetrical**. If $\vec{F}_{ab}$ is electric, then $\vec{F}_{ba}$ is electric, etc. For instance, you can't apply the 3rd law to weight and normal force. More on this later.

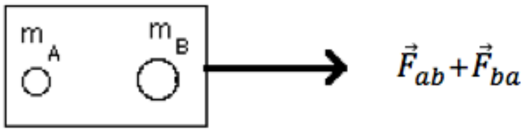


Third law $\quad \vec{F}_{ab} = -\vec{F}_{ba} \qquad\qquad \vec{F}_{ab} + \vec{F}_{ba} = 0$

What if it were not true?



combine the two:



3rd law implies: **internal forces in a system add to zero**.

2nd law $\quad \sum \vec{F} = \vec{F}_{external} + \vec{F}_{internal} = m\vec{a}$

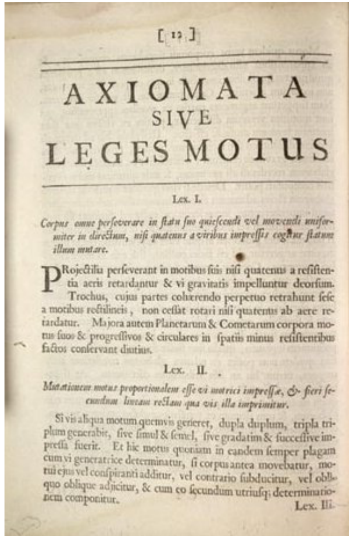
So rewrite 2nd law as $\quad \sum \vec{F}_{external} = m\vec{a}$

First and second laws (and also the definitions of inertial mass and force)**:**

$$\sum \vec{F} = m\vec{a}$$

Third law: $\vec{F}_{ab} = -\vec{F}_{ba}$

Combine the two gives $\sum \vec{F}_{external} = m\vec{a}$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Lex. III.

Question. An insect (mass 2 g) flying South at 3 m.s^{-1} collides with a truck (mass 30 tonnes) travelling North at 20 m.s^{-1}.

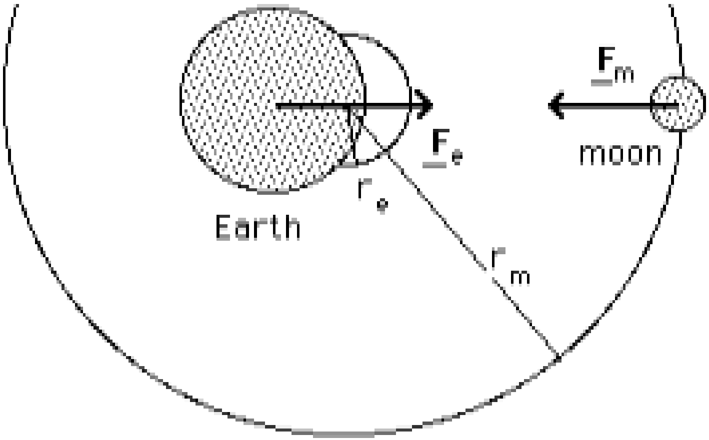
a) During the collision, what is the ratio of the force that the truck exerts on the insect (F_{tl}) to the force the locust exerts on the truck (F_{lt})?

Force on truck/force on insect = >>1 1 <<1

b) During the collision, what is the ratio of the acceleration of the truck (a_{t}) to the acceleration of the insect (a_{l})?

Accel. of truck/accel. of insect = >>1 1 <<1

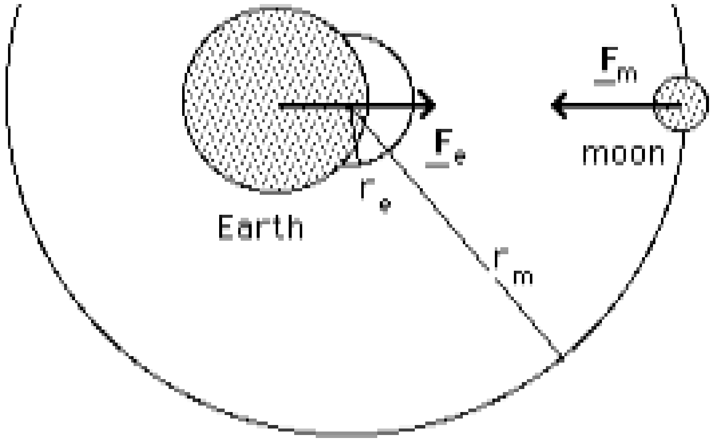
Example Where is centre of earth-moon orbit?



each makes a circle about common centre, so

Using Newton's 2nd and 3rd laws, derive an equation that relates m_m, r_m, m_e and r_e

Example Where is centre of earth-moon orbit?



each makes a circle about common centre, so third law gives

$|F_e| = |F_m| = |F_{grav}|$ equal & opposite

Then the second law gives

$F_{grav} = m_m a_m = m_m \omega^2 r_m$ (centripetal acceleration)

$F_{grav} = m_e a_e = m_e \omega^2 r_e$ Period and therefore ω is the same for both

$$\therefore \quad \frac{r_m}{r_e} = \frac{m_e}{m_m} = \frac{5.98\ 10^{24}\ \text{kg}}{7.36\ 10^{22}\ \text{kg}} = 81.3 \qquad \text{(i)}$$

earth-moon distance $r_e + r_m = 3.85\ 10^8$ m (ii) (*two equations, two unknowns*)

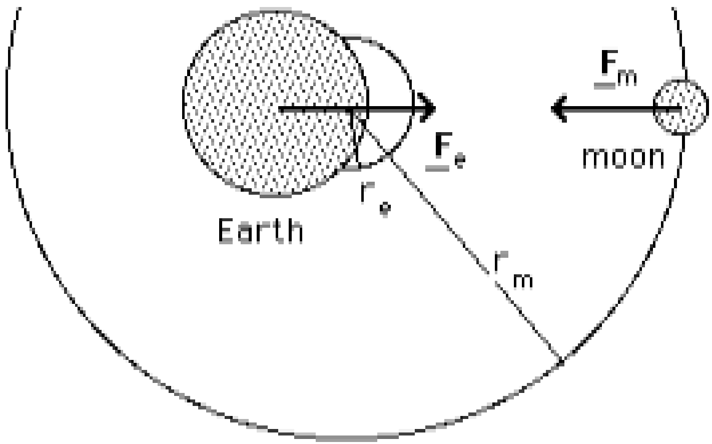
$$\therefore \quad \frac{r_m}{r_e} = \frac{m_e}{m_m} = \frac{5.98\ 10^{24}\ \text{kg}}{7.36\ 10^{22}\ \text{kg}} = 81.3 \qquad \text{(i)}$$

earth-moon distance $\quad r_e + r_m = 3.85\ 10^8$ m $\qquad$ (ii) $\quad$ (*two equations, two unknowns*)

solve -> $\quad r_e + r_m = r_e(1 + 81.3) = 3.85\ 10^8$ m

$r_m = 3.80\ 10^8$ m, $\ r_e = 4.7\ 10^6$ m $= 4700$ km

∴ centre of both orbits is inside earth $\qquad$ *(later, we'll show this point is the centre of mas.*



Using Newton's laws

Newton's 1st & 2nd $\sum \vec{F} = m\vec{a}$ the Σ is important

Newton's 3rd. Forces come in symmetric pairs, $\vec{F}_{ab} = -\vec{F}_{ba}$ or $\vec{F}_{ab} + \vec{F}_{ba} = 0$, so:

- internal forces add to zero. They don't affect motion
- Be careful identifying internal and external forces

Therefore, applying Newton's 2nd, we use $\sum \vec{F}_{external} = m\vec{a}$

- draw diagrams (**free body diagrams**) to show only the external forces on body of interest

Newton's second law is a vector equation, so

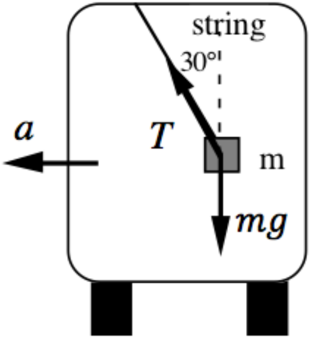
- write components of N2 in 1, 2 or 3 directions
- Then see what else you have/ need and put it in equation form
- Do you have as many equations as variables to solve?

Example. As the bus takes a steady turn with radius 8 m at constant speed, you notice that a mass on a string hangs at 30° to the vertical. How fast is the bus going?

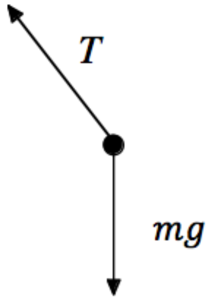
i) *Draw a diagram with the essential physics.*
ii) *Draw a free body diagram.*
iii) *What do we know?*

Example. As the bus takes a steady turn with radius 8 m at constant speed, you notice that a mass on a string hangs at 30° to the vertical. How fast is the bus going?

Diagram with physics



Free body diagram



We know:
tension in direction of string,
weight down,
acceleration horizontal
circular motion

N2* horizontal: write an equation!

N2* vertical: write an equation!

* *I'll use 'N2' as a shorthand for 'Newton's second law'.*

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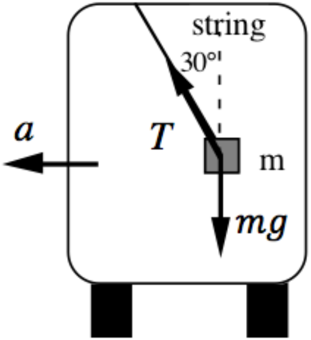
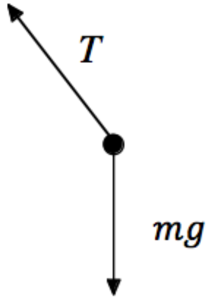
Diagram with physics





We know:
tension in direction of string,
weight down,
acceleration horizontal
circular motion

N2 horizontal: mass in circular motion with bus, so force on m: $\Sigma F_{horiz} = ma = mv^2/r$

Only the tension has a horizontal component, so $\Sigma F_{horiz} = T \sin 30° = mv^2/r$ (i)

Need one more equation: mass is not falling down, ie

N2 vertical: vertical acceleration = 0, so $T \cos 30° - mg = 0$ (ii)

Eliminate T by dividing (i) by (ii): $\tan 30° = m\dfrac{v^2}{r}\dfrac{1}{mg}$

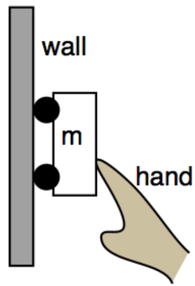
rearrange $v = \sqrt{gr \tan 30°} =$ 6.7 m/s –> 20 kph.

Don't stop yet! Check dimensions. Check limits. Is the answer reasonable?

Example. A toy truck, mass m, has wheels that roll freely. With my finger, I apply a force F at angle $\theta = 30°$ to the vertical to hold it stationary against a vertical wall.

i) Draw a free body diagram

ii) Write two equations for Newton's 1st or 2nd law



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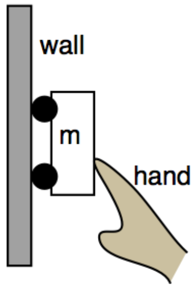
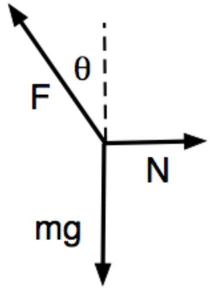


a) We're told that F acts at angle $\theta = 30°$ to the vertical.
b) Because the wheels turn freely, the force exerted by the wall is at right angles to the wall – a **normal force** N.
c) The remaining force is weight mg downwards.

'Stationary' means that acceleration $a_x = 0$ and $a_y = 0$.

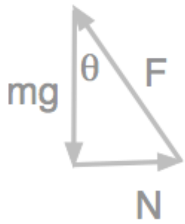


Newton's 2nd law in x: $N - F\sin\theta = ma_x = 0$ (i)

Newton's 2nd law in y: $F\cos\theta - \mathrm{mg} = ma_y = 0$ (ii)

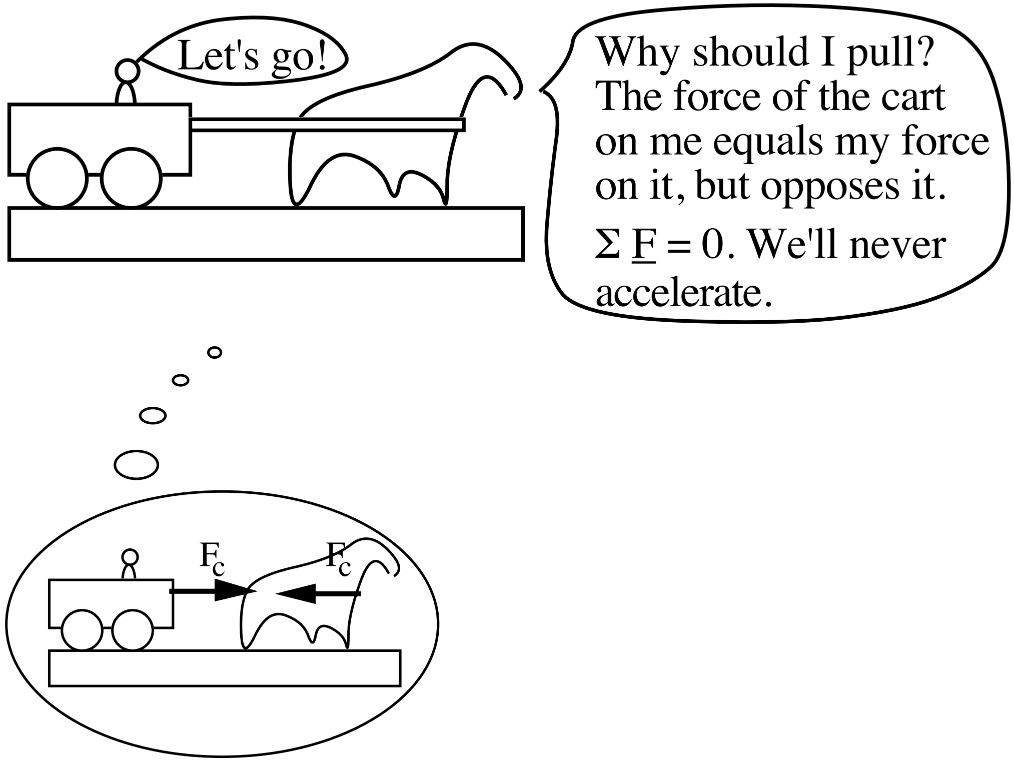
For F, we only need (ii): $F = mg/\cos\theta$

We can substitute in (i) to get $N = F sin\theta = mg\ tan\ \theta$



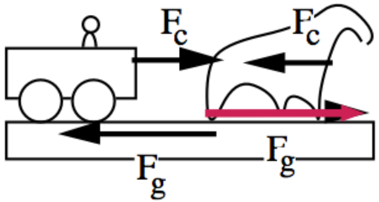
Note that we could also get both from the triangle of Σ force.

Question. Horse and cart. Wheels roll freely.



What would you tell the horse? What diagram(s) would you show her?

Write c for cart, g for ground.



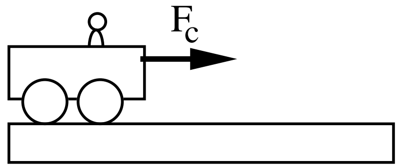
For the system horse+cart, F_c and $-F_c$ are both *internal* forces. The only *external* horizontal force acting on this system is F_g, the force the ground exerts on the horse.

$$\sum \vec{F}_{external} = m\vec{a}$$

$$F_g = m_{both}a_{both} = m_{both}a$$

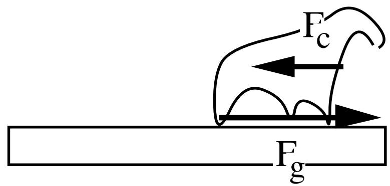
We could, however, define different systems

Horizontal forces on cart (mass m_c):



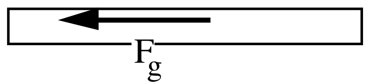
$$\sum \vec{F}_{external} = F_c = m_c a_c = m_c a$$

Horizontal forces on horse (mass m_h):



$$\sum \vec{F}_{external} = F_g - F_c = m_h a$$

Horizontal forces on Earth (mass m_{earth}):



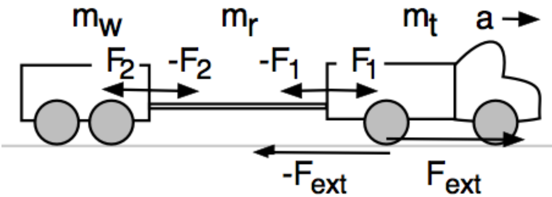
$$m_{earth} >> m_h + m_c$$

so Earth's acceleration is negligible.

A note about "light" strings, ropes etc. *Here, light means m << other masses*

A rope (mass 1 kg) only needs a force of several newtons to produce substantial accelerations, but can resist forces of thousands of newtons. Forces at opposite ends of light ropes etc are equal and opposite. We call this the **tension.**

Truck (m_t) pulls wagon (m_w) with rope (m_r). All have same acceleration a.



i) wagon: $-F_2 = m_w a.$

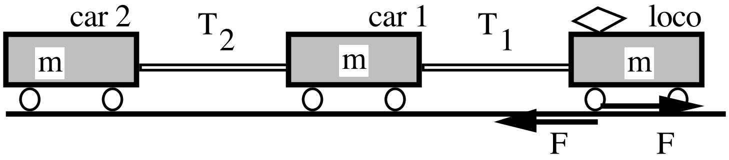
ii) rope: $F_1 - F_2 = m_r a$

iii) truck: $-F_1 + F_{ext} = m_t a$

(ii)/(i) –> $\dfrac{F_1 - F_2}{-F_2} = \dfrac{m_r a}{m_w a}$ *light rope, i.e.* $m_r << m_w$

$\therefore$ if $m_r << m_w$ $F_1 = F_2 = T$ (tension) *A very common assumption*

Question. Train. Wheels roll freely. Loco exerts horizontal force F on the track. What are the tensions T_1 and T_2 in the two (light) couplings?

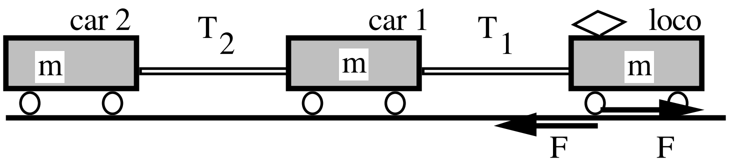


not to scale

Draw diagrams of different *parts* of the train to show F, T_1 and T_2 *as external forces* so that we can calculate them.

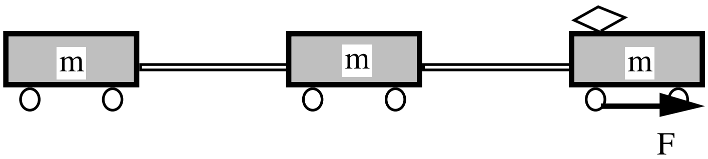
(*No need to show vertical forces, as these add to zero*.)

Example. Train. Wheels roll freely. Loco exerts horizontal force F on the track. What are the tensions T_1 and T_2 in the two couplings?



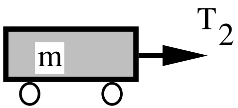
Whole train accelerates together with a.

Look at the external forces acting on the train (horiz. only).



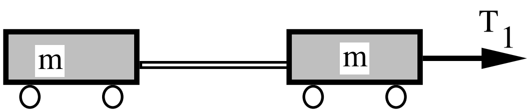
$$F = (m+m+m)a \quad \text{-->} \quad a = F/3m$$

Look at horiz forces on car 2:

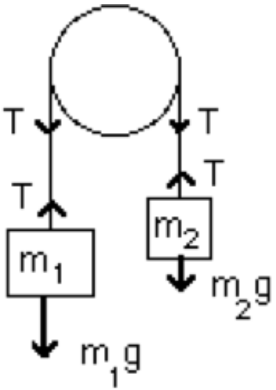


$$T_2 = ma = F/3$$

and on cars 2 and 1 together



$$T_1 = 2ma$$



Example

Light* pulley, inextensible, light string. What is acceleration of the masses?

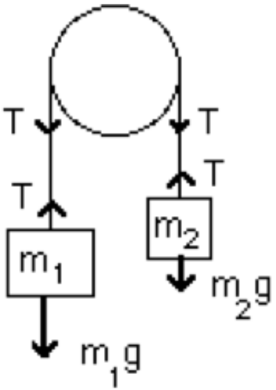
Newton 2 for m_1:

Newton 2 for m_2:

"inextensible" so let a be the downwards acceleration of m_1 and also the upwards acceleration of m_2.

(*If we've guessed the directions wrongly, we'll get a negative answer for a*.)

* "light" *means that for now, it takes no torque to spin the pulley. We'll treat rotation later*



Classic example

Light pulley, inextensible, light string. What is acceleration of the masses?

"inextensible" so let a be the downwards acceleration of m_1 and also the upwards acceleration of m_2.

(*If we've guessed wrongly, we'll get a negative answer*.)

Newton 2 for m_1: $\quad T - m_1 g = -m_1 a$

Newton 2 for m_2: $\quad T - m_2 g = +m_2 a$

Eliminate T to solve for a.

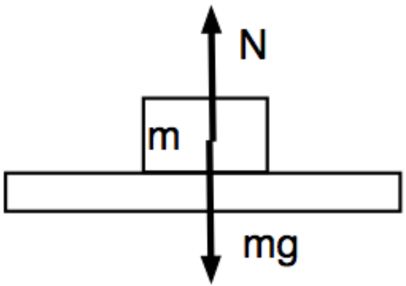
Subtract: $\quad -m_1 g + m_2 g = -m_1 a - m_2 a$

$$a = \frac{m_1 - m_2}{m_1 + m_2}\, g$$

(Check limits: if $m_1 = m_2$, $a = 0$.

If $m_2 = 0$, $a = g$.)

Question. Joe is standing on the floor.



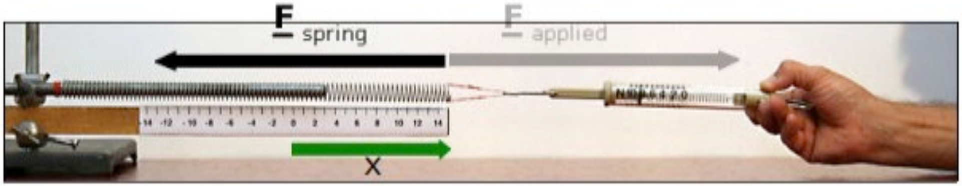
How does floor "know" to exert $N = mg = 700\ N$?

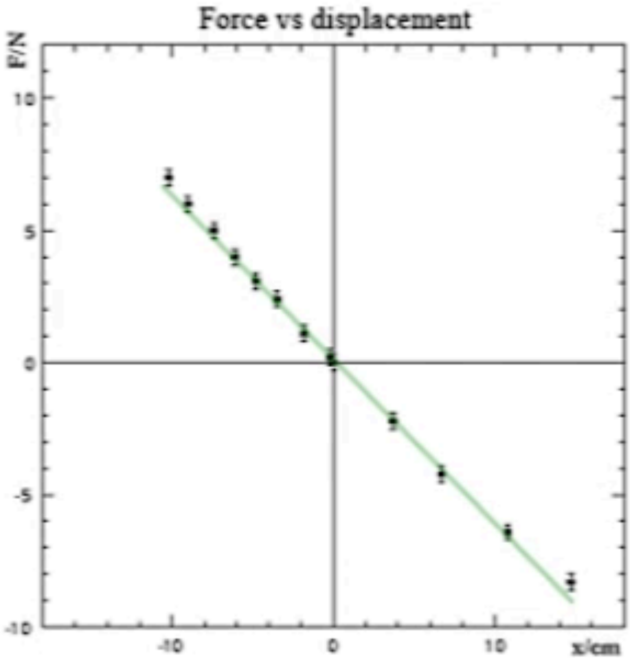
But in general $N \neq mg$. N can be zero (when I'm in the air).

And $N > mg$ when I'm about to take off, and on landing.

Mechanics > Weight contact forces > 6.2 Weight versus mass

Forces associated with deformation. Simple case: Hooke's law





Empirical law:

$$F = -kx$$

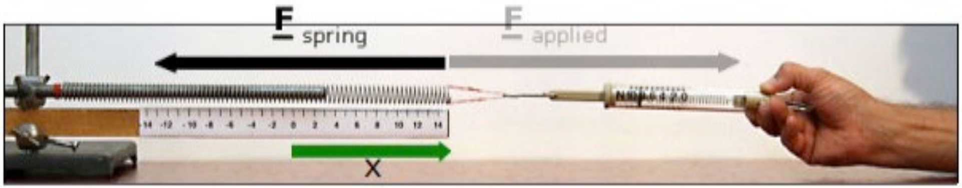
where k is the **spring constant**

k has units of $N.m^{-1}$.

$$k = 61 \pm 1 \ N.m^{-1}$$

Hooke's law applies to many materials – over a small range of deformation.

For our syllabus, we only need apply it to springs, and usually only in the linear range.



F_{spring} is in the opposite direction to x.

Experimentally, F is proportional to x over a small range of x. So we write

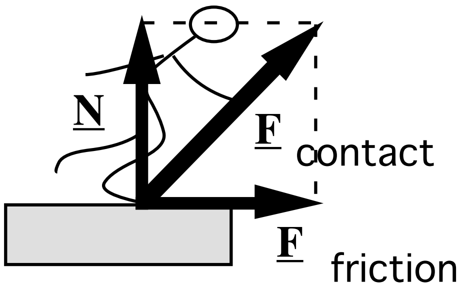
$$F = -kx$$ **Hooke's Law.**

Not needed now, but more generally:

$$\text{strain} \propto \text{stress}$$

i.e. $$\text{fractional deformation} \propto \frac{\text{force}}{\text{area}}$$

Contact forces



Contact forces

The normal component of a contact force is called the **normal force** N. The component in the plane of contact is called the **friction force** F_f.

This division is arbitrary, but useful

Normal force: at right angles to surface.

Friction force: in the plane of the surface.

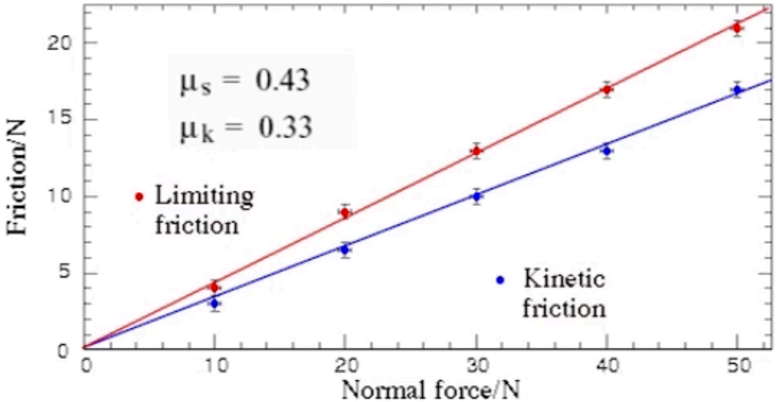
- If there is relative motion, we call it **kinetic** friction, whose direction opposes the relative *motion*)
- If there is *no* relative motion, it is **static** friction, whose direction opposes the applied *force*.

Question: *In terms of friction, what is a wheel for*

For experiments, see Ch 5 in web stream or Physclips Ch 6.4

http://www.animations.physics.unsw.edu.au/mechanics/chapter6_weightandcontactforces.html#6.4

Mechanics > Weight and contact forces > 6.5 Static and kinetic friction



Limiting Friction

Kinetic Friction

Define coefficients of kinetic (k) and static (s) friction:

$|F_f| = \mu_k N$ $|F_f| \leq \mu_s N$ *Note the* $\leq$

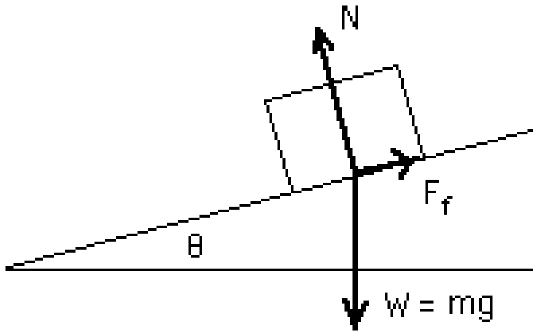
Friction follows this **approximate** empirical law

μ_s and μ_k are approximately independent of N and of contact area

(we'll test this later).

Often, $\mu_k < \mu_s$.

(It takes less force to keep sliding than to start sliding.)



Classic example. θ is gradually increased to θ_c when sliding begins.

What is θ_c? What is a at θ_c?

Newton 2 in normal direction:

$$N - mg\ cos\ \theta = 0 \qquad \text{(i)}$$

Never write N = mg without thinking

Newton 2 in direction down plane:

$$mg\ sin\ \theta - F_f = ma. \qquad \text{(ii)}$$

No sliding: $a = 0$ *Substitute in (ii)*

$$\therefore\ \text{(ii)} \Rightarrow mg\ sin\ \theta = F_f \qquad \text{(ii*)}$$

(i) gives $N = mg\ cos\theta$

definition of μ_s is $F_f \le \mu_s N$ *Note* $\le$

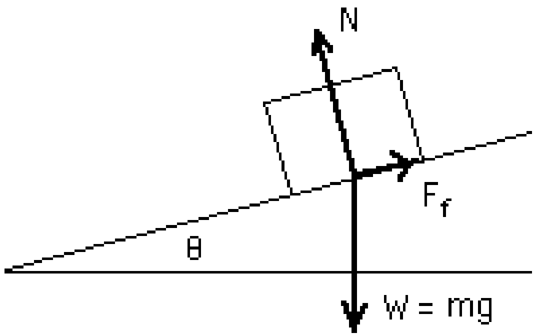
substitute both of these in (ii*) gives (for no sliding

$$mg\ sin\ \theta \le \mu_s\, mg\ cos\theta$$

$$tan\ \theta \le \mu_s, \quad \theta_c = tan^{-1}\mu_s$$

useful technique for finding μ_s

Also use to get $(\mu_s - \mu_k)$



What is a at θ_c?

We had

$$mg \sin\theta - F_f = ma. \qquad \text{(ii)}$$

and

$$N - mg\cos\theta = 0 \qquad \text{(i)}$$

Sliding at $\theta = \theta_c$: $a > 0$

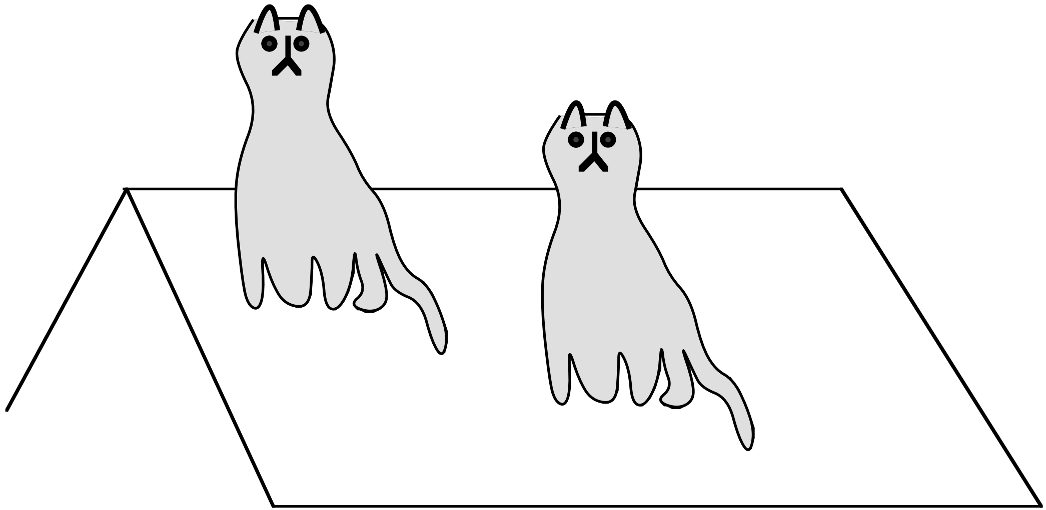
$$\therefore \text{(ii)} \Rightarrow a = g \sin\theta_c - \frac{F_f}{m}$$

$$= g\sin\theta_c - \frac{\mu_k N}{m}$$

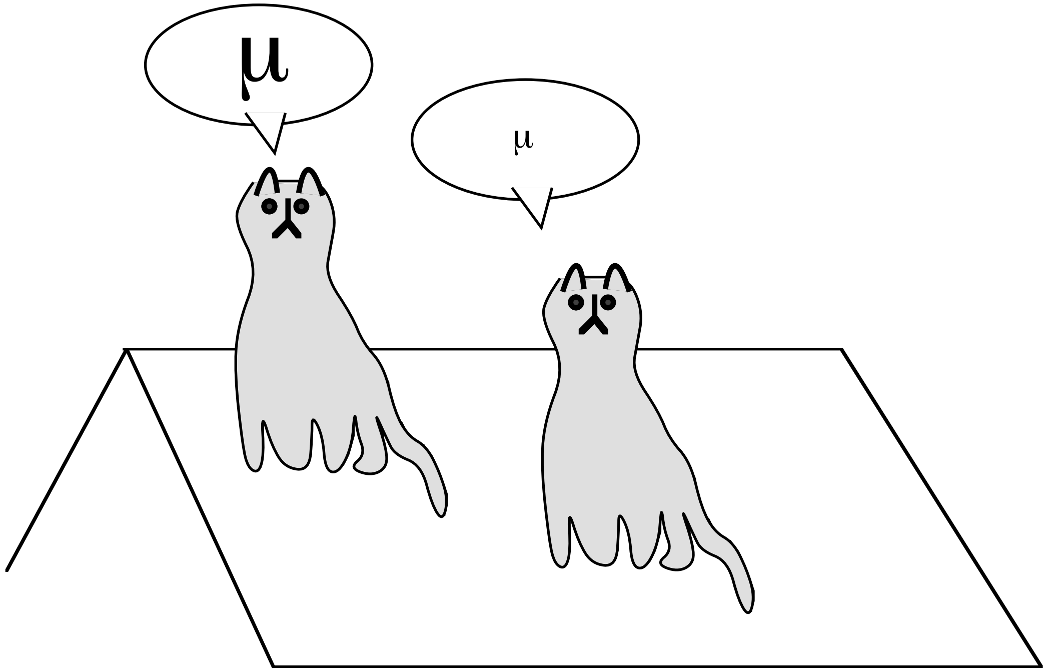
$$\text{(i)} \Rightarrow \quad = g\sin\theta_c - \mu_k g\cos\theta_c$$

we had $\theta_c = \tan^{-1}\mu_s$

so $a = g\cos\theta_c\,(\mu_s - \mu_k)$

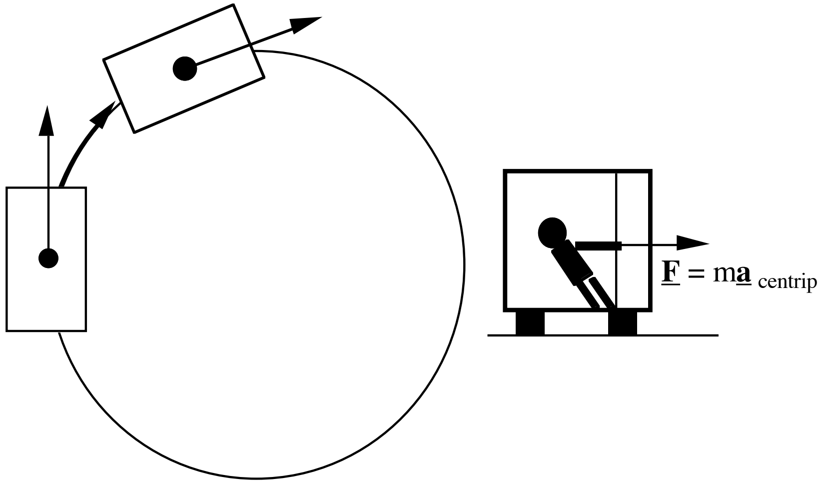


Question: Which cat will fall off first?

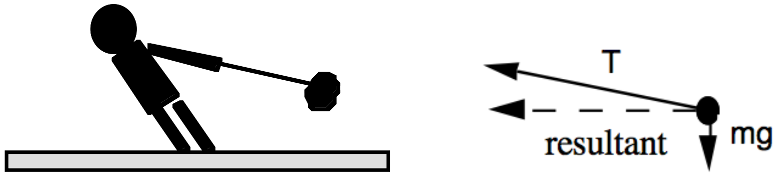

μ
μ

Centripetal acceleration and force (*centripetal = towards the centre*)

Consider circular motion with ω = constant and v constant



e.g. bus going round a corner



Or consider a hammer thrower:

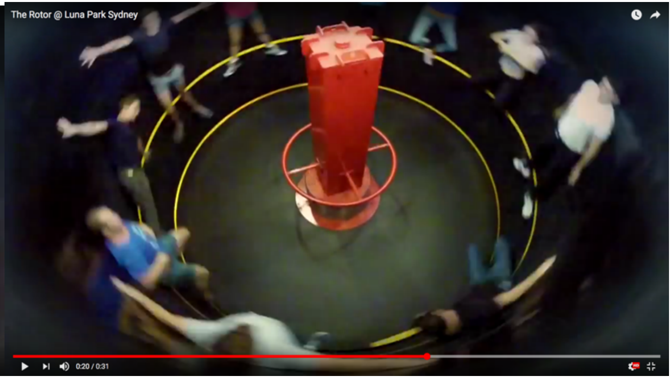
Resultant force produces acceleration in the horizontal direction, towards the centre of the motion.

These are examples of **centripetal force** producing **centripetal acceleration**

About centripetal force and centrifugal force.

i) **Centrifugal force is what physicists call a fictitious force**. It doesn't exist. These people in the Rotor Ride are travelling in circles, so the centripetal force (force pushing them towards the centre of the rotor) is due to the normal force of the wall against their backs.

There is no mysterious force pushing them outwards: they are accelerating *inwards*, towards the centre.



http://www.lunaparksydney.com/roto

i) **Centrifugal force is what physicists call a fictitious force. It doesn't exist**

ii) **Centripetal force is not a mysterious new force**.

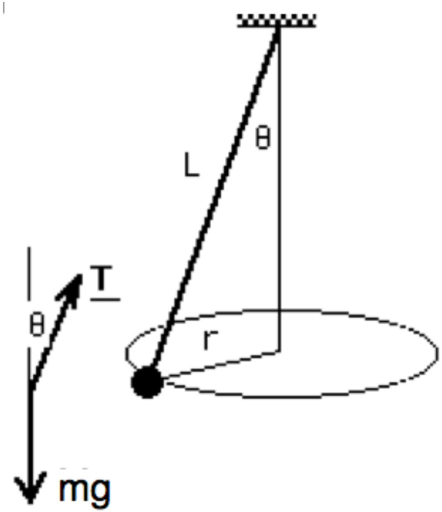
When an object travels in a circle at uniform speed, the total force $\sum \vec{F}$ on them ***is*** the centripetal force (and $\sum \vec{F} = -m\vec{r}\omega^2$). $e.g.$:

- For the earth round the sun, gravity supplies the centripetal force.
- For the bus going round a corner, friction on the tires supplies the centripetal force.
- For the seated passenger in the bus, friction on his bum supplies the centripetal force.
- For the rider in the Rotor, the normal force on his back supplies the centripetal force.

Do not include centripetal forces (or centrifugal forces) in free body diagrams.

(Or Star Trek force fields, or may-the forces, or …

Example Conical pendulum. (Uniform circular motion.) What is the frequency?



(Here, a component of T is the centripetal force)

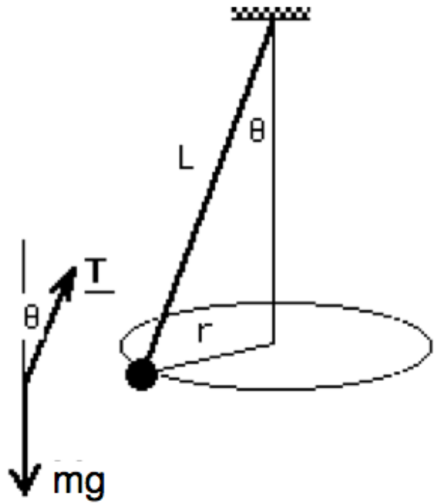
Apply Newton 2 in two directions:

Vertical:

Horizontal:

$\therefore$ frequency = ?

Example Conical pendulum. (Uniform circular motion.) What is the frequency?



Apply Newton 2 in two directions:

Vertical: $a_y = 0 \quad \therefore \quad \Sigma F_y = 0$

$$\therefore \quad \Sigma F_{vert} = T \cos\theta - mg = 0$$

$$T = \frac{mg}{\cos\theta}$$

Horizontal: $\dfrac{mv^2}{r} = ma_c = T \sin\theta$

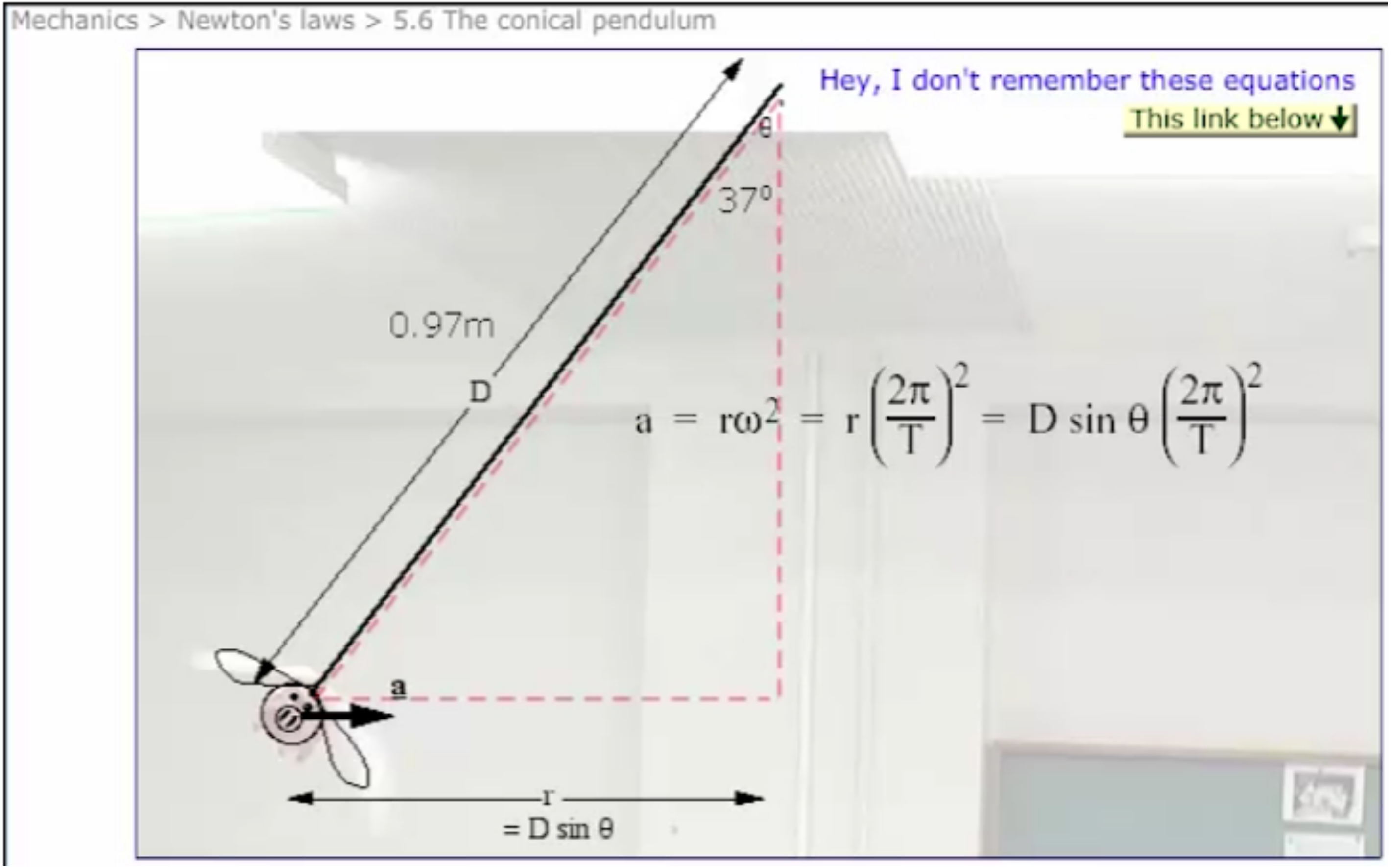
Substitute for T: $\dfrac{mv^2}{r} = \dfrac{mg \sin\theta}{\cos\theta}$

$$\therefore \quad \frac{v^2}{r} = g \tan\theta$$

$$\therefore \quad v = \sqrt{rg \tan\theta}$$

$$\therefore \quad \frac{2\pi \mathrm{r}}{\mathrm{period}} = \sqrt{rg \tan\theta}$$

$$\therefore \quad f = \frac{1}{\mathrm{period}} = \frac{1}{2\pi}\sqrt{\frac{g \tan\theta}{r}}$$



http://www.animations.physics.unsw.edu.au/mechanics/chapter5_newton.html#5.6

Dimensions, units and significant figures

$$T = 2\pi\sqrt{\frac{D\sin\theta}{a}} \quad \text{(i)}$$

Newton 2 (horizontal) : $(F + L)\sin\theta = \text{total force} = ma$ (ii)

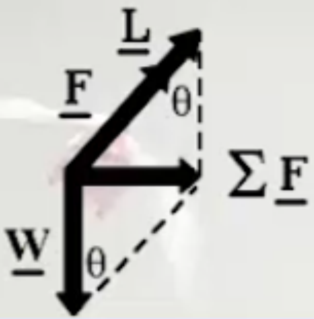
Newton 2 (vertical) : upwards force = downwards force

$$(F + L)\cos\theta = mg \quad \text{(iii)}$$

(ii)/(iii) $\Rightarrow$

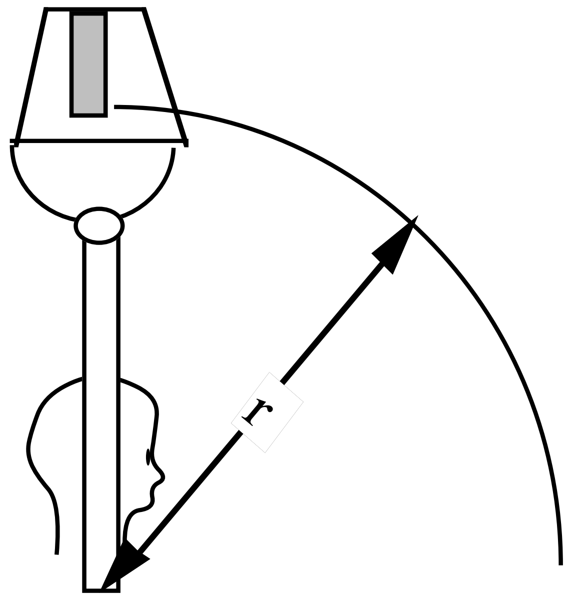
$$\frac{\sin\theta}{\cos\theta} = \frac{a}{g} \quad \text{(iv)}$$

Substitute (iv) in (i)



$$T = 2\pi\sqrt{\frac{D\cos\theta}{g}}$$

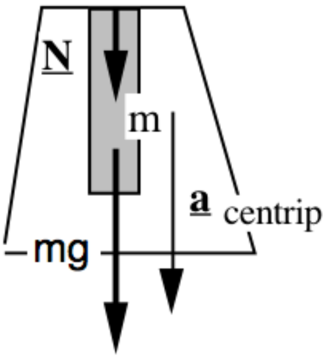
$$= 2\pi\sqrt{\frac{0.97\text{ m}\cos 37^\circ}{9.8\text{ m.s}^{-2}}} = 1.8\text{ s.}$$



Example. Foolhardy lecturer swings a bucket of bricks in a vertical circle. How fast should he swing so that the bricks stay in contact with the bucket at the top of the trajectory?

Draw diagram & identify important variables

Pose the question mathematically



mg and N provide centripetal force.

(Normal forces are a Newton pair:

Bucket pushes brick down

Brick pushes bucket up)

$mg + N = ma_c$

For contact, I want

$N \geq 0$

so $ma_c \geq mg$

how to express a_c?

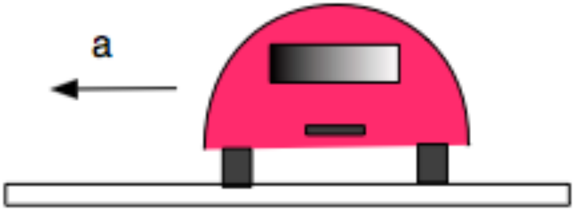
$$a_c = \frac{v^2}{r} = r\omega^2 = r\left(\frac{2\pi}{T}\right)^2$$ *T is easy to measure*

$$T = 2\pi\sqrt{\frac{r}{a_c}} \leq 2\pi\sqrt{\frac{r}{g}}$$

r ~ 1m –> $T <$ 2 s. *please check for errors!*

Question. I am standing on the floor. My weight is 700 N and the coefficients of static and kinetic friction between my shoes and the floor are 0.8 and 0.9 respectively. What is the frictional force between my shoes and the floor?

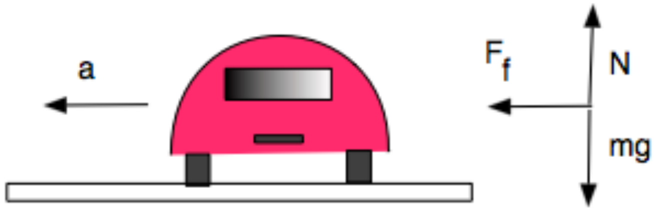
Example. i) Consider a flat road with a curve of radius r = 10 m. Take the coefficients of kinetic and static friction between the tires and the road as 0.8 and 1.0 respectively. What is the maximum speed that a vehicle can take the corner?



Example. i) Consider a flat road with a curve of radius r = 10 m. Take the coefficients of static and kinetic friction between the tires and the road as 0.8 and 1.0 respectively. What is the maximum speed that a vehicle can take the corner?

As usual, $\mu_s > \mu_k$. So we can stop better with wheels turning (μ_s) not skidding (μ_k).

Uniform circular motion, so the *total* force must be centripetal



i) In the vertical direction, there is no acceleration so

$$\text{N2:} \qquad ma_y = 0 = N - mg \quad \text{so } N = mg$$

In the horizontal direction,

$$ma = F_f$$

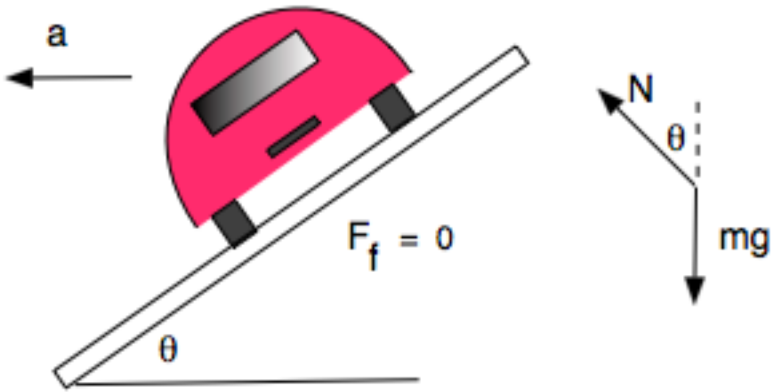
$$mv^2/r = F_f \le \mu_s N = \mu_s mg$$

$$v^2 \le r \mu_s g = 100 \text{ m}^2.\text{s}^{-2}$$

$$v \le 10 \text{ m.s}^{-1} = 36 \text{ kph.}$$

ii) Bank the curve so that vehicles can go around it with no friction at all (*e.g.* icy road). What is the required angle for a given speed?

ii) Bank the curve so that vehicles can go around it with no friction at all ($e.g.$ icy road). What is the required angle for a given speed?

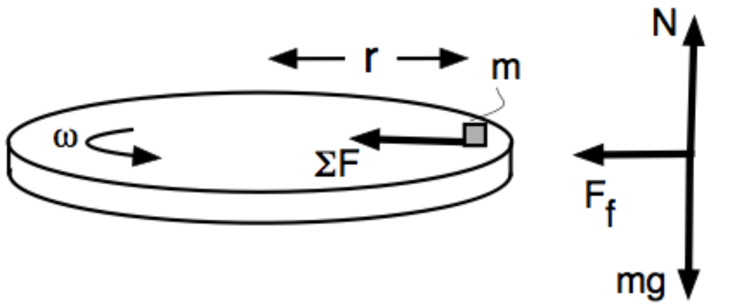


ii) N2 vertical: $N \cos \theta = mg$ *Never write N = mg without thinking*

N2 horizontal $N \sin \theta = m \frac{v^2}{r}$

Divide gives $\tan \theta = \frac{v^2}{rg}$

Classic example. A mass m sits on a horizontal turntable with coefficient of static friction μ_s. The angular speed ω is slowly increased. What is the maximum ω before it slides off?



We know that it isn't accelerating upwards, so N2 in the vertical direction gives $N - mg = 0$

So this time (*but not always*) $N = mg$.

If it is not sliding, the definition of μ_s is

$$F_f \leq \mu_s N = \mu_s mg$$

It's in uniform circular motion, so we know the total force is centripetal. It's a horizontal circle, so the total force is horizontal, and the only horizontal force is friction, so N2 in the vertical direction is

$$\Sigma F_{horiz} = F_f = ma_{centrip} = mr\omega^2$$

But the definition of μ is $F_f \leq \mu_s N$. Substituting all the above

$$mr\omega^2 = ma_{centrip} = F_f \leq \mu_s N = \mu_s mg. \quad \text{Cancel } m \text{ to give}$$

$$r\omega^2 \leq \mu_s g \qquad \text{so} \qquad \omega \leq \sqrt{\mu_s g/r}$$

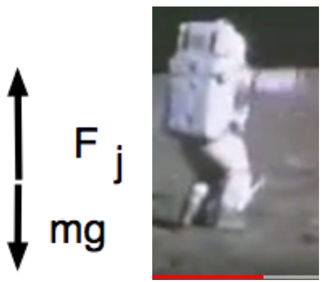
Example. Gravitational field on the moon: $g_m = 1.7\ \text{N.kg}^{-1} = 1.7\ \text{ms}^{-2}$. An astronaut has a mass of 80 kg and his (Apollo) spacesuit also has a mass of 80 kg. While jumping, his feet exert 2 kN (constant) on the ground while his body rises 0.3 m as his knees straighten. How high does he jump on earth (without suit and with an 80 kg suit) and on moon (with an 80 kg suit)?

What does $y(t)$ look like?

What is the ΣF acting on him during the leg-straightening phase of the jump on Earth? On the moon?

Example. Gravitational field on the moon: $g_m = 1.7\ \mathrm{N.kg^{-1}} = 1.7\ \mathrm{ms^{-2}}$. An astronaut has a mass of 80 kg and his (Apollo) spacesuit also has a mass of 80 kg. While jumping, his feet exert 2 kN (constant) on the ground while his body rises 0.3 m as his knees straighten. How high does he jump on earth (without suit and with an 80 kg suit) and on moon (with an 80 kg suit)?

Earth: he weighs 0.8 kN without suit, 1.6 kN with. On moon, he weighs 0.3 kN (with)



Vertical (y) motion with constant acceleration. While feet are on ground,

$\Sigma F = 2\ \mathrm{kN} - mg_E$

$= 1.2\ \mathrm{kN}$ (Earth without suit) or $0.43\ \mathrm{kN}$ with

Moon:

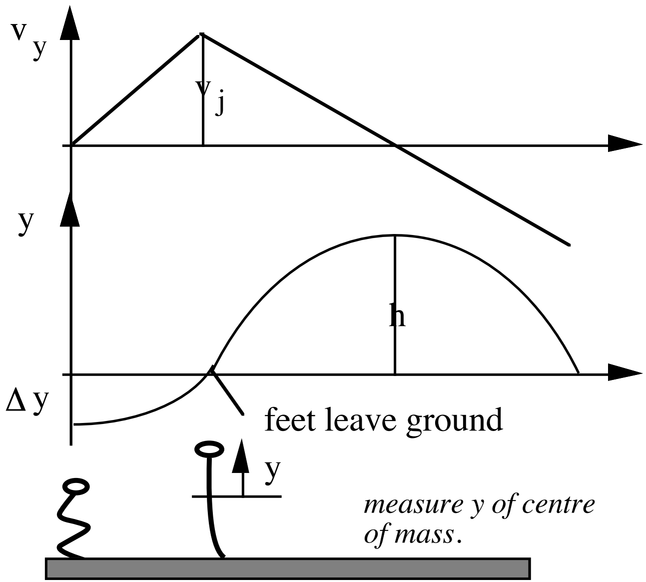
$\Sigma F = 2.0\ \mathrm{kN} - mg_m = 1.7\ \mathrm{kN}$ with

Greater *total* force on moon, therefore greater upwards acceleration, therefore greater take-off speed.

Jump has two parts:

feet on ground $\left(a = \dfrac{\Sigma F}{m}\right)$ $v_i = 0, v_f = v_j$

feet off ground $a = -g$ $v_i = v_j, v_f = 0$



While on ground:

$$v_j^2 - v_o^2 = 2a_j \Delta y = 2\frac{\Sigma F}{m}\Delta y$$

Earth: v_j = 3.0 m/s without, 1.6 m/s with

Moon: v_j = 2.5 m/s with

While above ground:

$$v^2 - v_j^2 = -2gh \quad \text{->}$$

$$h = \frac{v_j^2}{2g}$$ substitute values

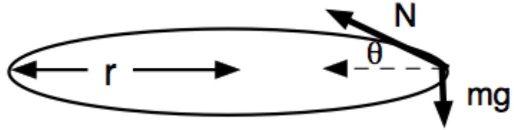
On earth: $h_E = 0.46$ m without.

0.13 with

On moon: 1.8 m with.

https://www.youtube.com/watch?v=16D0hmLt-S0

Example. A plane travels in a horizontal circle, speed v, radius r. For given v, what is the r for which the normal force exerted by the plane on the pilot equals twice her weight? What is the direction of this force?



Centripetal force $\quad F = m\dfrac{v^2}{r} = N\cos\theta$

Vertical forces: $N\sin\theta = mg$ *Never write N = mg without thinking*

eliminate θ: $\quad N^2 = m^2\left(\dfrac{v^4}{r^2} + g^2\right)$

$$\left(\frac{N^2}{m^2} - g^2\right) = \frac{v^4}{r^2} \quad \to \quad r = \frac{v^2}{\sqrt{\dfrac{N^2}{m^2} - g^2}}$$

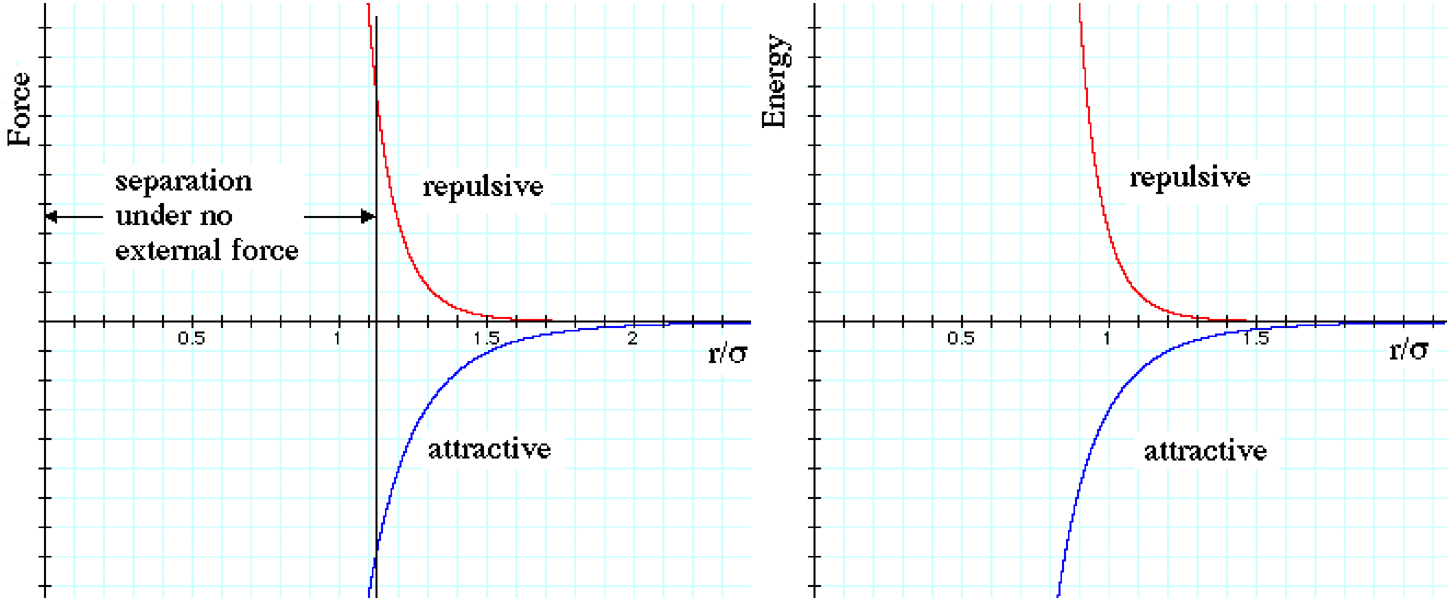
$$\sin\theta = \frac{mg}{N} = \frac{1}{2}$$

$\therefore$ direction of $\vec{N}$ is 30° above horizontal, towards axis of rotation

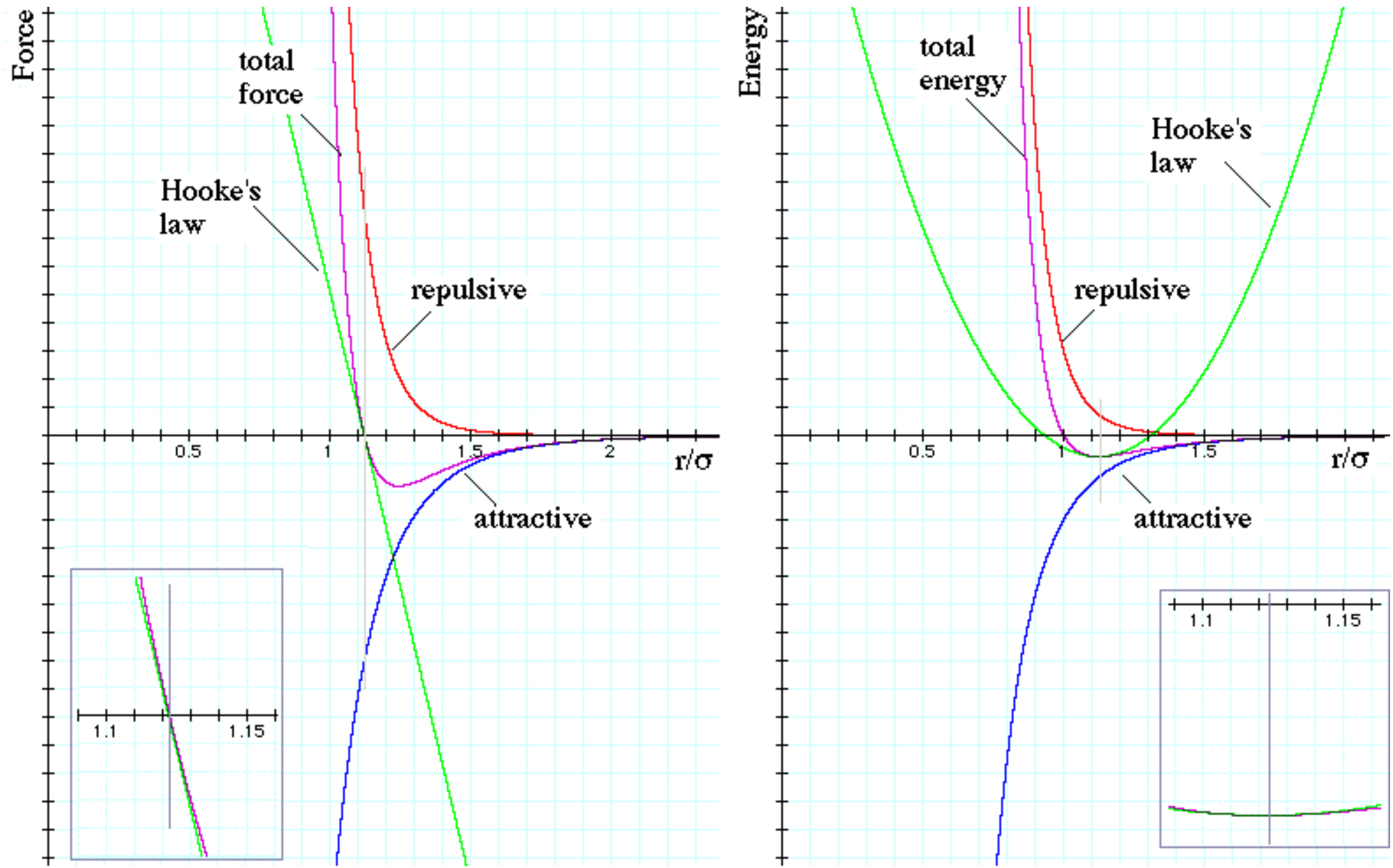
Appendix **Why linear elasticity?**

Intermolecular forces F and energies U:

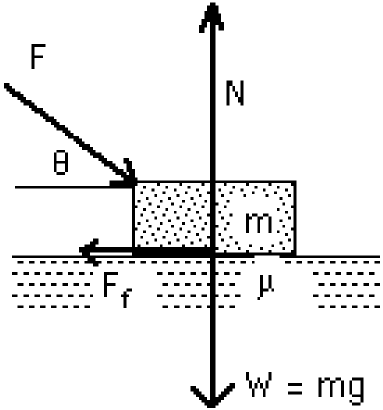
Not in our syllabus, but see homework problem (set 2#10) on interatomic forces



We'll see later that we integrate forces to get energy



The green line is the Hooke's law approximation: linear for F, parabolic for U, *valid only for small proportional deformation*. http://www.animations.physics.unsw.edu.au/jw/elasticity.htm



Difficult example.

Mass m on floor with friction coefficients μ_s and μ_k. We apply force F at θ to horizontal. For any given θ, what force F is required to make the mass move?

Eliminate 2 unknowns N & F_f to get $F(\theta, \mu_s, m, g)$

Stationary if $F_f \le \mu_s N$ (1)

Newton 2 vertical: $N = mg + F \sin\theta$ (2)

Newton 2 horizontal: $F\cos\theta = F_f$ (3)

(1,3) –> stationary if $F\cos\theta \le \mu_s N$

$F\cos\theta \le \mu_s(mg + F\sin\theta)$ *(using (2))*

$F\left(\cos\theta - \mu_s \sin\theta\right) \le \mu_s mg$ (*)

Note the importance of $\left(\cos\theta - \mu_s \sin\theta\right)$

$$\theta = \theta_{crit} = \tan^{-1}(1/\mu_s).$$

If $\theta < \theta_c$, then $\left(\cos\theta - \mu_s \sin\theta\right) > 0$

stationary if $F \le \dfrac{\mu_s mg}{\cos\theta - \mu_s \sin\theta}$

i.e. moves when $F > F_{crit} = \dfrac{\mu_s mg}{\cos\theta - \mu_s \sin}$

What if $\left(\cos\theta - \mu_s \sin\theta\right) = 0$?

F_{crit} goes to ∞

It moves when

$$F \ge \frac{\mu_s mg}{\cos\theta - \mu_s \sin\theta} \to \infty$$

i.e. stationary no matter how large F becomes.

(Good to know for e.g. ladders.)

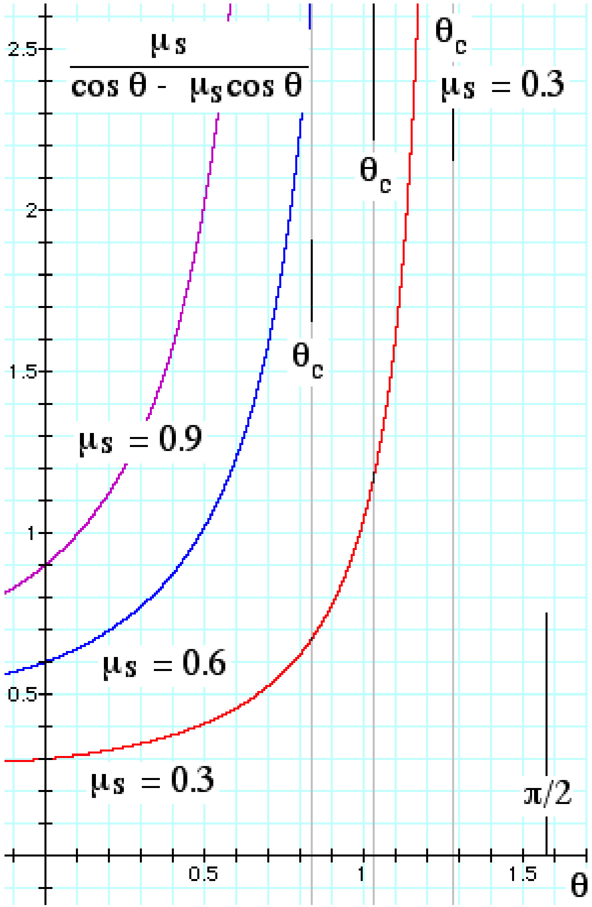
We had:

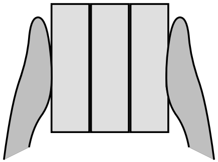
It moves when $F > F_{crit} = \dfrac{\mu_s mg}{\cos\theta - \mu_s \sin\theta}$

What if $(\cos\theta - \mu_s \sin\theta) = 0$?

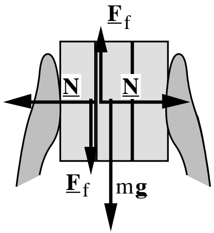
F goes to ∞ at $\theta_{crit} = \tan^{-1}(1/\mu_s)$.

Let's plot F vs θ for different values of μ_s.





Example. Three identical bricks with m and μ_S. What is the minimum force you must apply to hold them like this?



Vertical forces on middle brick add to zero:

$$2\,Ff = mg$$

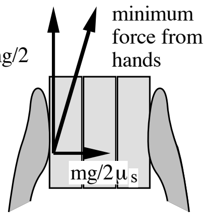
Definition of μ_S

$$Ff \leq \mu_s N$$

$$\therefore \quad N \geq \frac{Ff}{\mu_s} = \frac{mg}{2\mu_s}$$

Bricks not accelerating horizontally, so normal force from hands = normal force between bricks.

$\therefore$ (each) hand must provide $\geq \dfrac{mg}{2\mu_s}$ horizontally.



Vertically, two hands together provide 3mg.

Mass and weight

(inertial) mass m is defined by $F = ma$

weight is the gravitational force on something

observation:

near earth's surface and without air, experimentally we (including Newton) find that all (?) bodies fall with same a ($= -g$)

weight $= mg$

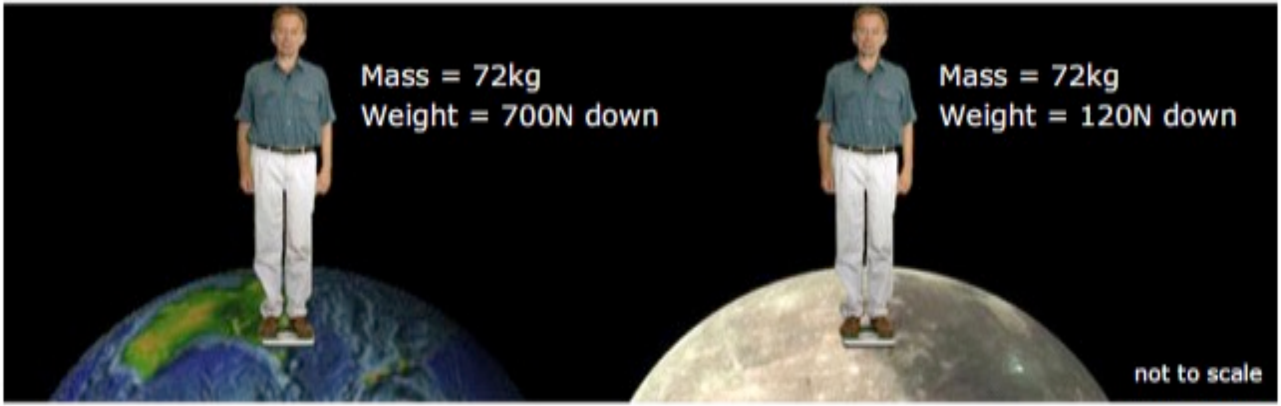
What is your weight?

Mechanics > Weight and contact forces > 6.2 Weight versus mass

mass determines "resistance to accelerate" (inertia)

$$m \equiv \frac{\underline{\mathbf{F}}_{total}}{\underline{\mathbf{a}}}$$ in kilograms (kg) or slugs* (Liberia; Myanmar; USA)

$$\underline{\mathbf{W}} = mg \text{ (down)}$$ in newtons (N) or pounds force* (Liberia; Myanmar; USA)



Warning: do not confuse mass and weight, or their units

kilogram (kg) is the unit of mass newton (N) is the unit of force ($kg.m.s^{-2}$)

A slug is about 14.6 kg

Warning: do not confuse mass and weight, or units

kilogram (kg) is the unit of mass newton (N) is the unit of force (kg.m.s^{-2}).

Interesting puzzle (not in our syllabus)

Why is W proportional to m? *Or,*

Why is $m_{gravitational}$ *proportional to* $m_{inertial}$*?*

Why do bodies fall with the same acceleration?

$$m_{inertial} * a = \Sigma F = W = m_{gravitational}\, g$$

(inertial property of body)*acceleration = (gravitational property of body)*(gravitational field)

Mach's Principle

Principle of General Relativity

Interactions with vacuum field