Question 1 (Marks: 23)

(a) (i) A particle moves along the x axis. Its position is given by the equation

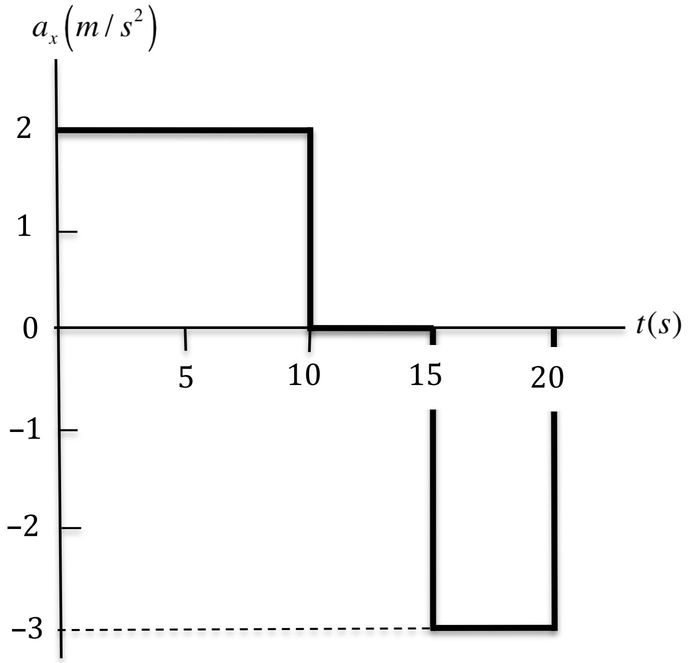
$x = 2.00 + 3.00\,t - 4.00\,t^2$ where x is in metres and t in seconds.

Determine the position of the particle when it changes direction.

(ii) For the same particle as part (i) (i.e. $x = 2.00 + 3.00\,t - 4.00\,t^2$) determine its velocity when it returns to the position it had to begin with, at $t = 0.00$.

(iii) A particle starts from rest and accelerates as shown in the Figure below.

Determine the particle's speed at $t = 10.0$ s and at $t = 20.0$ s, and the distance travelled in the first 15.0 s. Assume you can read the graph to 3 significant figure precision.



(b) You are at rest (at the origin, say) when your friend runs by, travelling at speed $v = 0.80$ m.s^{-1} in the x direction. When he is a distance $L = 1.8$ metres ahead of you, you start accelerating forwards with a constant acceleration $a = 1.4$ m.s^{-2}.

(i) Sketch a graph of the positions of you and your friend as a function of time t. Include L on the graph.

(ii) Determine how long it takes for you to catch your friend.

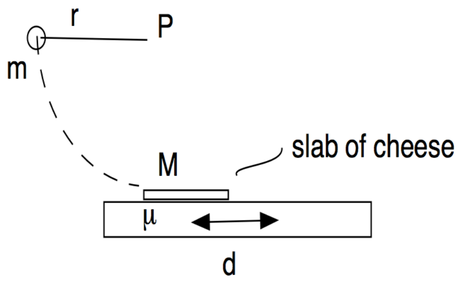
(c) You are riding your bicycle towards the North at 20 kilometres per hour (2 sig figs). Your bicycle has a navigation unit that tells you that the wind velocity relative to you on the bicycle is 14 kilometres per hour, coming *from* the direction 45° East of North. Showing all working, determine the 'true wind' velocity, i.e. the velocity of the wind with respect to the ground. (State the direction in that the wind is coming *from*.)

Question 2 (Marks: 18)

(a) State the conditions under which the momentum of a system is conserved.

(b) State the conditions under which mechanical energy of a system is conserved.

(c) A student is training his pet mouse to do circus tricks. The mouse, of mass m = 0.10 kg is holding on to one end of a light, inextensible string with length r = 82 cm. (The mouse is small enough that you can treat it as a particle.) The other end of the string is attached to a fixed point (P in the diagram). Initially, the string is horizontal and a distance r above a slab of cheese (mass M) resting on a table.



The coefficients of static and kinetic friction between the slab and the table are μ_s = 0.90 and and μ_k = 0.80 respectively. The mouse is released from rest, swings on the string through an angle of 90° then lets go, landing in a very brief collision on top of the slab. After this collision, the slab of cheese and the animal travel together a distance d = 12 cm as the slab skids on the table.

(i) Determine the speed of the mouse immediately **before** it reaches the cheese. (air resistance is negligible.)

(ii) Determine the tension in the string while the mouse is still swinging in the arc, but just **before** it reaches the cheese.

(iii) Determine the initial speed V of mouse+plate immediately **after** their collision. (Hint: consider how far do they then travel together and the forces acting on them.)

(iv) Determine the mass M of the cheese.

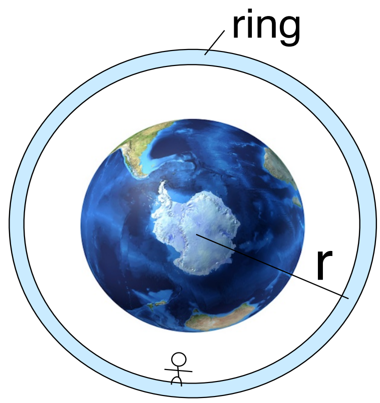
(d) State the work-energy theorem.

(e) Starting with Newton's second law, prove the work-energy theorem.

Question 3 (Marks: 19)

(a) Most communications satellites are found in an orbit above the equator. They complete one orbit in a period T = 24 hours and so stay above the same point on the equator – we call this a geosynchronous orbit. Beginning with Newton's law for universal gravitation, and showing all working, calculate the radius R of the geosynchronous orbit.

(b) A recently launched mission to the International Space Station is studying the potential health problems associated with long periods in orbit, having no normal forces from floors, chairs etc. A science fiction author proposes a solution: a new space station in the shape of a uniform ring, radius r = 6900 km, could be built above the equator, to replace the International Space Station.



This sketch is not to scale.

(i) If the ring is uniform, where is its centre of mass?

(ii) If the ring is centred exactly on Earth's centre, what is the total gravitational force (due to the earth) acting on the ring?

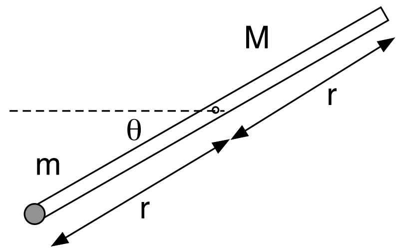
The ring need not be in orbit: in principle, its mechanical strength (if large enough) will hold it up. Because the ring is not in orbit, our author argues that it can turn at any angular speed we like.

(iii) Caculate the rotation period at which it would have to turn so that a standing astronaut (mass m), her head towards Earth as shown in the sketch, feels as though she is on earth. In other words, she would feel the same normal force on her feet that she would feel when standing on Earth (with her feet towards Earth). You must *draw a free body diagram*. To make the orientation of your diagram clear, draw a figure of the standing astronaut next to your free body diagram.

(c) (i) Define a conservative force

(ii) State the general condition under which mechanical energy is conserved.

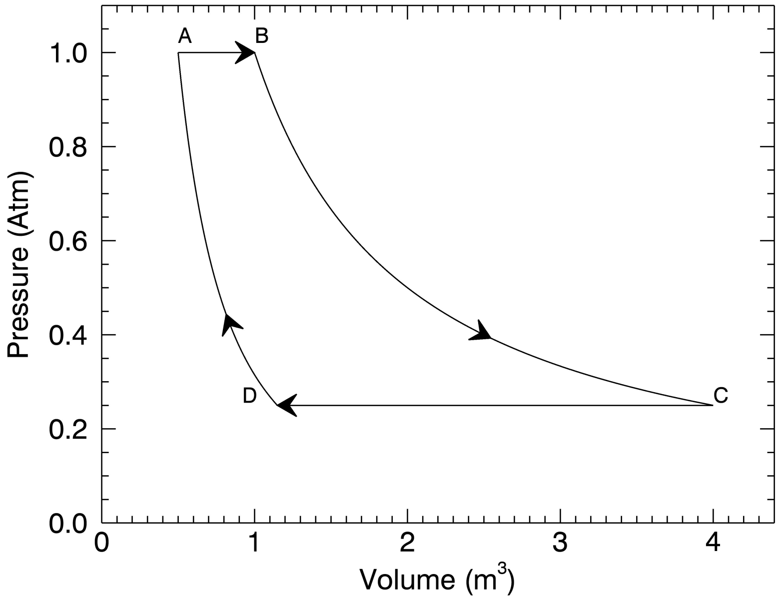
(iii) A particle of mass m is rigidly attached to the end of a uniform rod, mass M, which has a length of $2r$ and which turns without friction about a horizontal axis through its centre. It turns in a vertical plane and the sketch shows a side view.



Suppose that the system is released from rest with the rod horizontal, i.e. $\theta = 0$. Derive an expression for the angular speed ω of the system when the rod reaches vertical ($\theta = \pi/2$). (Neglect air resistance.)

Question 4 (Marks: 30)

20 moles of an ideal, monotonic gas undergoes the thermodynamic process shown in the diagram below, which shows the variation of pressure, P (measured in Atmospheres) and volume, V (measured in m^3). You may assume that 1 Atmosphere (Atm) = 1.01×10^5 Pa.



From states A➔B the change is isobaric, going from (V,P) = (0.50, 1.00) to (1.00, 1.00).

From states B➔C the change is isothermal, going from (V,P) = (1.00, 1.00) to (4.00, 0.25), and the work done <u>on</u> the gas is -1.40×10^5 J.

From states C➔D the change is isobaric, going from (V,P) = (4.00, 0.25) to (1.15, 0.25).

From states D➔A the change is adiabatic, going from (V,P) = (1.15, 0.25) to (0.50, 1.00).

Answer the following questions about the changes that occur in the system, referring to any relevant formula and physical principles that you apply when doing so:

(i) Explain the terms isobaric, isothermal and adiabatic, with reference to the changes occuring in this system.

(ii) Calculate the temperature of the gas at states A, B, C and D.

(iii) Calculate the work done <u>on</u> the gas as it goes from states A to B.

(iv) Calculate the change in internal energy as the gas goes from states A to B.

(v) Calculate the heat flow into the gas as it goes from states A to B.

(vi) Determine the heat flowing into the gas between states B and C.

(vii) How much heat flows into the gas when moving from state D to state A?

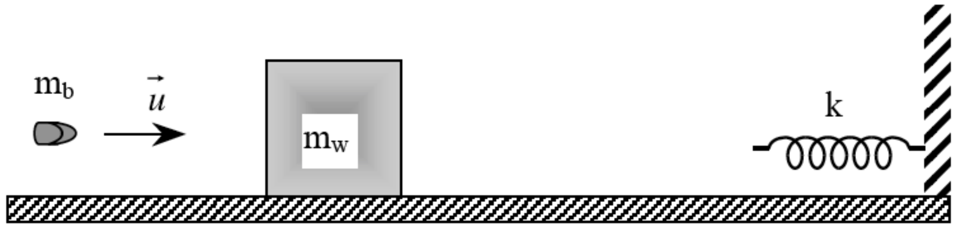
(viii) Hence calculate the work done on the gas moving from state D to state A.

(ix) What is the net work on the gas as it completes one complete cycle, A➔B➔C➔D➔A?

(x) Interpret your answer and relate it to the total heat that flows into the system over one cycle.

Question 5 (Marks: 30)

(a) A bullet is fired into a wooden block, becoming lodged in it with no loss of material. The block + bullet then slides on a frictionless horizontal surface, until it meets a spring, as shown in the diagram. It then compresses the spring.



The masses of the bullet and the wooden block are m_b = 5.00g and m_w = 1.50kg, respectively. After the bullet lodges in the block the block slides a distance of 60.0cm before compressing the spring 16.0cm. The spring obeys Hooke's law and its spring constant, k = 120 N m^{-1}.

(i) Describe qualitatively what happens to the block + bullet system once it reaches the spring, also making reference to the energy of the system.

(ii) What is the speed of the combined block + bullet system immediately before the spring is compressed.

(iii) Thus determine the initial speed of the bullet.

(b) (i) A policeman in a stationary police car hears the siren of an approaching police car and then sees it coming straight towards him from behind. He measures the frequency to be f′ = 575 Hz. Knowing that the standard issue police siren has a frequency of f=440 Hz, determine the speed of the approaching police car. You may assume that the speed of sound in the air is 340 m/s.

(ii) Once the approaching car has overtaken him, the policeman sets off in pursuit. Unfortunately he is driving on old police car, and the maximum speed it can reach is only 40 m/s. When he reaches that speed, at what frequency does he now hear the siren?

(iii) When this occurs he switches on the siren in his police car (which emits at the standard frequency of 440 Hz). At what frequency does the policewoman, driving the faster car, hear this siren emit at?

Suppose now that there is a steady wind blowing at a speed of 10 m/s in the direction of travel of the police cars.

(iv) Determine the frequency with which the stationary policeman first hears the approaching police siren.

(v) At what frequency does he hear the siren when his car reaches its maximum speed?

(vi) At what frequency does the policewoman hear the siren of the following police car?