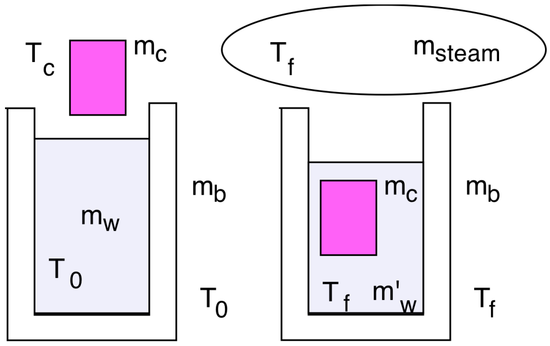
PHYS1131 Higher Physics 1A
Solutions – Homework Problem Set 5

Q1.



$m_b = 0.146kg, m_w = 0.223kg, T_i = 21.0^oC, m_c = 0.314kg, m_{steam} = 4.7 \times 10^{-3}kg, T_f = 100^oC, c_C = 387Jkg^{-1}K^{-1}, c_w = 4200Jkg^{-1}K^{-1}, L_{vap,w} = 2.26\times 10^6$

$a)Q_w = m_w c_w \Delta T_w + \Delta mL$

$= 0.223 \times 4200 \times (100 - 21) + 4.7 \times 10^{-3} \times 2.26 \times 10^6$

$= 84.6kJ$

$b)Q_b = m_b c_C \Delta T_b$

$= 0.146 \times 387 \times (100 - 21)$

$= 4.46kJ$

$c)Q_{cylinder} = -(84.6 + 4.46) = -89.06kJ$

$Q_{cylinder} = m_c c_C \Delta T_C$

$-89.06 \times 10^3 = 0.314 \times 387 \times (100 - T_i)$

$T_i = 837^oC$

Q2.

$m_{tea} = 0.520kg, m_{ice} = 0.520kg, T_{ice} = 0^oC, T_f = ?, \Delta m = ?$

$c_{tea} = c_w = 4186Jkg^{-1}K^{-1}, L_{fus} = 3.33 \times 10^5 Jkg^{-1}$

The heat needed to melt the ice is given by $\Delta mL = 0.520 \times 3.33 \times 10^5 = 173kJ$

$a) -$ Heat lost by tea = Heat gained by ice

$Q_{lostbytea} = -m_{tea}c_w(T_f - T_i) = -0.520 \times 4186 \times (T_f - 90.0)$

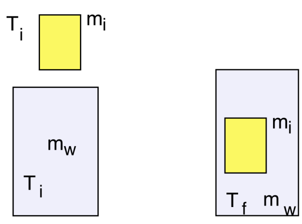
$Q_{gainedbyice} = \Delta mL + m_{ice}c_w(T_f - T_i)$

Assume all the ice melts $\Rightarrow \Delta m = 0.520kg$

$Q_{gainedbyice} = 0.520 \times 3.33 \times 10^5 + 0.520 \times 4186 \times (T_f - T_{i:ice})$

$-2177 \times (T_f - 90) = 173160 + 2177 \times T_f$

$$T_f = \frac{195930 - 173160}{2 \times 2177} = 5.2^oC \text{all the ice is melted}$$



$b)Q_{lostbytea} = -0.520 \times 4186 \times (T_f - 70.0)$

the maximum value is when $T_f = 0, Q_{lostbytea} = 152kJ$

This is not enough to melt all the ice

$T_f = 0.0^oC$

$\Delta mL = 152000$

$\Delta m = 0.458$

So $0.520 - 0.458 = 62g$ of ice remain

Q3. $L_f = 3.4.10^5 J/kg$, $m = 0.1kg$

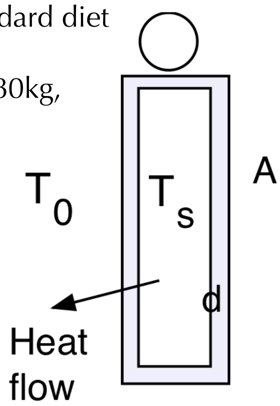
$Q = L_f.m \approx 3.4.10^4 J$, The internal energy of the gas is the same after the cycle So, all work done goes in to melting the ice. i.e. W = Q

Q4. $A = 1.8\ m^2$, $d = 1.0cm$, $T_s = 33^0$, $T_0 = 1.0^0C$, $k = 0.04\ W/m.K$.

a).

$$\frac{dQ}{dt} = k\frac{\Delta T}{d}.A$$

$$\Rightarrow \frac{dQ}{dt} = k\frac{T_s - T_0}{d}.A = 230W$$



As comparison, at rest the human body produces about 80W. The minimum amount of food needed is then 80 * 24hrs = 6900kJ = 1650 cals. Standard diet is 2300-3000 cals.

Is 80W a lot or a little? 736W is one horse power. Running up stairs (80kg, height change = 1ms^{-1}) $\frac{mgh}{\Delta t} = \frac{80.10.1}{1} = 800W$

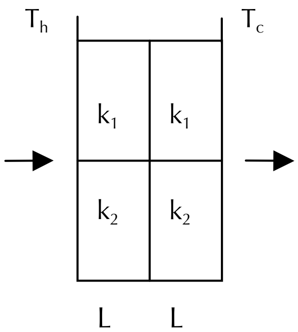
b). $k' = 0.60\ W/m.K$.

$\frac{k'}{k} = 15 \Rightarrow \frac{dQ'}{dt} = 15\frac{dQ}{dt}$, i.e. heat flows 15 times faster.

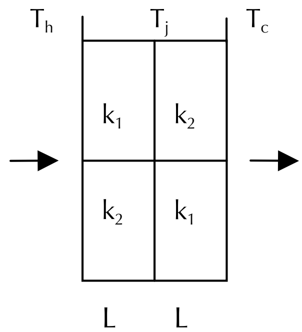
Q5.

$T_h > T_c$

a)



b)



Let the thickness of the materials be L, T_h, be the temperature of the hot side and T_c be the temperature of the cold side
$a)$
Let P_1 be the rate of heat flow through the part made of material with k_1 and P_2 be rate of heat flow through material k_2
$P_1 = k_1 A \frac{T_h - T_c}{2L}$
$P_2 = k_2 A \frac{T_h - T_c}{2L}$
$P_a = P_1 + P_2 = \frac{1}{2}(k_1 + k_2)\frac{A(T_h - T_c)}{L}$
$b)$The heat will flow at the same rate through both halves (the upper and lower half) as the order of the material does not affect the rate of flow of heat
You should prove this as an exercise
The heat will also flow at the same rate through material with k_1 and k_2 as the same amount of heat enters and leaves the material.
Let the temperature at the junction between k_1 and k_2 be T_j.
$\Rightarrow P_b = 2 \times k_1 A \frac{T_h - T_j}{L} = 2 \times k_2 A \frac{T_j - T_c}{L}$
rearrange this to find T_j
$\Rightarrow k_1(T_h - T_j) = k_2(T_j - T_c) \Rightarrow T_j = \frac{k_1 T_h + k_2 T_c}{k_1 + k_2}$
substitute this into P_b
$P_b = \frac{2k_1 A}{L}(T_h - \frac{k_1 T_h + k_2 T_c}{k_1 + k_2})$
$= \frac{2k_1 A}{L} \frac{T_h k_1 + T_h k_2 - k_1 T_h - k_2 T_c}{k_1 + k_2}$
$= \frac{2k_1 k_2 A}{L(k_1 + k_2)}(T_h - T_c)$Now compare P_a and P_b
$P_a = \frac{(k_1 + k_2)^2}{4k_1 k_2} P_b$
Check that when $k_1 = k_2$ these are equivalent
$P_a = \frac{(2k_1)^2}{4k_1^2} P_b = P_b$
For any other values of k_1 and k_2 (substitute any numbers in)
$\frac{(k_1 + k_2)^2}{4k_1 k_2} > 1$
$\Rightarrow P_a > P_b$

Q6. See question sheet for p-V diagram.
For path *iaf*: *Q = 50J, W = -20J*
For path *ibf*: *Q = 36J*

a). $Q + W = \Delta E$, $\Delta E = 30J$
$\Rightarrow W(ibf) = -6J$

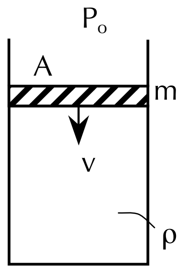
b. (f → i), $Q + W = -\Delta E = -30\ J$ *as return to position i*
$W = 13J \Rightarrow Q = -43J$

c) For $E_{int,i}$=10J, $E_{int,f}$= 10 + 30 = 40 J

d) *ib: Q – 6 = 22 – 10 = 12 ⇒ Q = 18J*

⇒ bf : Q = 18J *since W = 0 along bf as V constant.*

Q7.



Po = 1 atm
A = 2.0 cm^2
V = 0.30 cm/s
ρ = 6 x 10^{-4} g cm^{-3}

(a) We have $\rho_{steam} = 6 \times 10^{-4}$ g cm^{-3} constant

Hence rate of change of amount of steam $\dot{m}_{steam} = \rho\ \dot{V}$

Where $\dot{V}$ = rate of change of volume
= Av v the speed of piston

$\dot{m}_{steam} = -\rho$ Av
$= -6 \times 10^{-4} \times 2 \times 0.30$ g/s
$= -3.6 \times 10^{-4}$ g/s

Hence this is rate of condensation steam to water.

(b) Rate of energy lost converting steam to water = Rate of loss of Heat

i.e. $\dot{m}\ L_{Fusion} = \dot{Q}$

$\therefore\ \dot{Q} = 3.6 \times 10^{-7} \times 2.26 \times 10^{6}$ J/s
= 0.81 J/s

This answer is approximately correct. In fact the latent heat of fusion changes with pressure. The pressure inside the piston is ~ 2 atm so L is ~2.20 x 10^6 J/s giving dQ/dt = 0.79 J/s

(c) From First Law = $\frac{dE}{dt} = \frac{dQ}{dt} + \frac{dW}{dt}$

Now dW = -PdV p consists of term from weight of piston plus atmosphere pressure

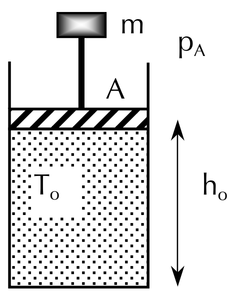
i.e $P = \frac{m_{piston}g}{A} + P_o$ and $\frac{dV}{dt} = Av$

so $\frac{dW}{dt} = -(m_{piston}g + p_o A)v$

$= -(2.0 \times 9.8 + 1.013 \times 10^5 \times 2 \times 10^{-4})(-0.003)$ J/s
= + 0.120 J/s

so $\frac{dE}{dt} = \frac{dQ}{dt} + \frac{dW}{dt} = -0.814 + 0.120 = -0.69$ J/s

PAST EXAM QUESTION



(a) External pressure on air in container is $\frac{mg}{A} + p_a$

within cylinder pV = nRT = constant we allow temperature to equilibrate at T = T_o

so pV = const = $p_a V_o$

Hence $p = \frac{p_a V_o}{V} = \frac{p_a V_o}{A h_o}$

Both $p = \frac{mg}{A} + p_a$ Air $\mu = 0.029$ g/mol

$\Rightarrow \frac{p_a V_o}{A h_o} = \frac{mg}{A} + p_a \quad \therefore \quad \frac{p_a V_o}{A h_o} = \frac{mg + p_a A}{A}$

$\therefore \quad h_o = \frac{p_a V_o}{mg + p_a A}$ i.e. no dependence on μ or T_o!

b)

Stiffness is defined as $|\frac{F}{x}|$, where F is force and displacement is x

For a sudden change no energy is lost/gained (as heat), it is adiabatic

$\Rightarrow PV^\gamma = \text{constant}$

For a slow change it remains at thermal equilibrium with the surrounds, it is isothermal

$\Rightarrow PV = \text{constant}$

$V = hA$

For slow change

Suppose $h \to h - x$ where $x/h << 1$

$\Rightarrow Ph = P'(h - x)$

$P' - P = P(\frac{h}{h-x} - 1) = P[(1 - \frac{x}{h})^{-1} - 1] \approx \frac{Px}{h}$

$\Rightarrow \Delta P = \frac{F}{A} \sim \frac{Px}{h}$

$\Rightarrow \text{Stiffness} = \frac{F}{x} \sim \frac{PA}{h}$

For rapid change

$PV^\gamma = \text{constant}$

$\Rightarrow Ph^\gamma = P'(h - x)^\gamma$

$\Rightarrow \Delta P = P' - P = P[(\frac{h}{h-x})^\gamma - 1]$

$= P[(1 - \frac{x}{h})^{-\gamma} - 1]$

$\approx P[1 + \frac{\gamma x}{h} - 1]$

expand as binomial series and take limit $\frac{x}{h} << 1$

$= \frac{\gamma P x}{h}$

$\text{Stiffness} = \frac{F}{x} \sim \gamma \frac{PA}{h}$

this is γ times larger than isothermal case ($\gamma > 1$)

Hence the system is stiffer for a rapid [adiabatic] than a slower (isothermal) change. To be expected as a slower change will remain in thermal equilibrium, and heat will flow as it occurs.

Q8

(a) The displacement at t = 2 seconds is obtained by simply substituting t = 2 into the given equation, that is:

$$x = 6.0\cos(3\pi \times 2 + \frac{\pi}{3})$$

$$x = 6.0\cos(\frac{19\pi}{3})$$

$$x = 6.0 \times \frac{1}{2}$$

So x = 3m

(b) To calculate the velocity, we must first differentiate x with respect to time:

$$\frac{dx}{dt} = v = -18\pi\sin(3\pi t + \frac{\pi}{3}) \text{ ms}^{-1}$$

Then we substitute t = 2 into the equation to give $v = -9\pi\sqrt{3}$ ms^{-1}

(c) Acceleration is calculated in a similar fashion, this time by differentiating the velocity equation:

$$\frac{d^2x}{dt^2} = a = -54\pi^2\cos(3\pi t + \frac{\pi}{3}) \text{ ms}^{-2}$$

And substituting t = 2 again gives $a = -27\pi^2$ ms^{-2}

(d) Since the wave equation is already given in the form:

$$x = A\cos(\omega t + \phi) \text{ metres}$$

We can simply read off the phase of the wave. So $\phi = \frac{\pi}{3}$ is the phase constant.

(e) The frequency v is related to the angular frequency ω by the equation:

$$v = \frac{\omega}{2\pi} \text{ Hz}$$

From part (d) we know that $\omega = 3\pi$, so v = 1.5Hz.

(f) The period of motion is given by $T = \frac{1}{f}$. We calculated the frequency in part (e), so the period is simply the reciprocal of this, ie. $T = \frac{2}{3}$ seconds.

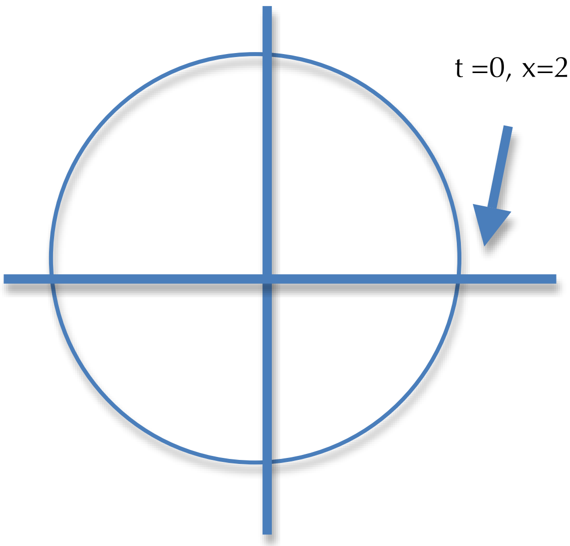
Q 9

a)To find the period we use the equation;

$$T = \frac{2\pi}{\omega}$$
$$\omega = \frac{v}{r}$$
$$\Rightarrow T = \frac{2\pi r}{v}$$
$$= \frac{2 \times 2\pi}{3}$$
$$= 4.19s$$

The frequency $f = \frac{1}{T}$

$f = 0.239Hz$



b)$x = A\cos(\omega t + \phi) = A\cos(2\pi f t + \phi)$

When $t = 0, x = 2 \Rightarrow \phi = 0$

$x = 2\cos(1.5t)$

Q10

(a) The piston moves with simple harmonic motion, therefore:

$$x = A\cos\omega t \qquad \text{(displacement)}$$
$$\frac{dx}{dt} = -\omega A\sin\omega t \qquad \text{(velocity)}$$
$$\frac{d^2x}{dt^2} = -\omega^2 A\cos\omega t \qquad \text{(acceleration)}$$
$$\frac{d^2x}{dt^2} = -\omega^2 x$$

The block and piston will separate when the acceleration of the piston is greater than the acceleration of the block, ie. when $a = g$.

$$g = \omega^2 x$$
$$9.8 = \left(\frac{2\pi}{1.0}\right)^2 x \qquad \left(\text{using } T = \frac{2\pi}{\omega}\right)$$

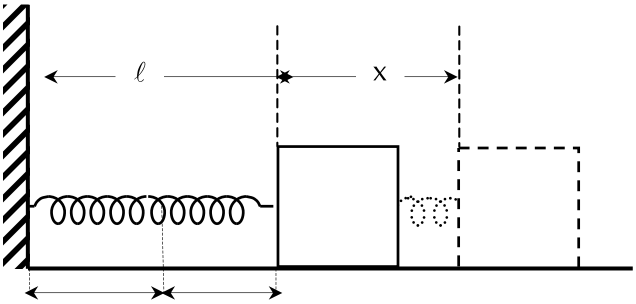
Therefore x = 0.25m is the amplitude

(b) We use the same equation as previously, but this time solve for ω:

$$g = \omega^2 x \qquad \text{(rearranging the equation)}$$
$$\omega = \sqrt{\frac{g}{x}}$$
$$\omega = \sqrt{\frac{9.8}{5 \times 10^{-2}}}$$

This gives ω = 14 rad s^{-1}. Converting to seconds by using the equation $f = \dfrac{\omega}{2\pi}$ gives f = 2.2Hz

Q11



Note: ℓ_1 si ℓ_2 s uniform along the spring.

$= n\ell_2$

$$x = x_1 + x_2$$
$$F = -kx$$
$$= -kx_1 - kx_2$$

We also know that:

$F = -k_1x_1$ and $F = -k_2x_2$

Therefore, setting both values for F equal to each other yields:

$$-k_1x_1 = -kx_1 - kx_2$$
$$k_1 = \frac{k(x_1 + x_2)}{x_1}$$
$$= \frac{k\left(x_1 + \frac{x_1}{n}\right)}{x_1}$$
$$= k\left(1 + \frac{1}{n}\right)$$
$$= k\left(\frac{n+1}{n}\right)$$

And doing similarly for k_2 gives:

$$-k_2x_2 = -kx$$
$$k_2 = \frac{k(x_1 + x_2)}{x_2}$$
$$= \frac{k\left(\frac{nx_2 + x_2}{x_1}\right)}{x_1}$$
$$= k(1 + n)$$

Q12

The total energy is

$$E = \frac{1}{2} kA^2$$

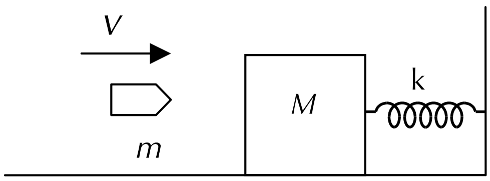
When x = A/3, The potential energy is:

$$PE = \frac{1}{2}kx^2$$
$$= \frac{1}{2}k(\frac{A}{3})^2$$
$$= \frac{1}{18}kA^2$$

And PE + KE = E, so

$$\frac{1}{18}kA^2 + KE = \frac{1}{2}kA^2$$
$$\Rightarrow KE = \frac{4}{9}kA^2 = \frac{8}{9}E$$

Q13



By conservation of momentum, the momentum of the bullet before it strikes the block will be equal to the momentum of the combined bullet-block system after the strike. So:

$$m\mathrm{v} = (M + m)\frac{dx}{dt}$$

For SHM,

$$x = A\cos\omega t$$
$$\frac{dx}{dt} = -\omega A\sin\omega t$$

Substituting this into the original conservation of momentum equation gives us:

$$-\omega A\, sin\omega t\,(M + m) = m\mathrm{v}$$

The maximum amplitude for this system will occur when $\sin\omega t = 1$, thus:

$$A = \frac{mv}{\omega(m + M)}$$ and using $\left(\omega = \sqrt{\frac{k}{m + M}}\right)$,

$$A = \sqrt{\frac{m+M}{k}}\left(\frac{m\mathrm{v}}{m+M}\right)$$
$$= \frac{m\mathrm{v}}{\sqrt{k(m+M)}}$$

Alternative: Momentum mV = (m + M)u

Energy conservation $\frac{1}{2}kx_{max}^2 = \frac{1}{2}(M+m)u^2$ and substitute u to yield the same result.

Q14.

(a) Energy stored in spring is given by:

$$\begin{aligned}E_{\text{spring}} &= \frac{kx^2}{2}\\ &= \frac{(294)(0.239)^2}{2}\\ &= 8.40\text{J}\\ &= U\end{aligned}$$

the kinetic energy is:

$$K = \underbrace{\frac{1}{2}I\omega^2}_{rotation} + \underbrace{\frac{1}{2}Mv^2}_{translation}$$

For a cylinder rolling without slipping, v = ωr, this gives

$$\begin{aligned}&= \frac{1}{2}\left(\frac{MR^2}{2}\right)\left(\frac{v}{R}\right)^2 + \frac{Mv^2}{2}\\ &= \frac{Mv^2}{4} + \frac{Mv^2}{2}\\ &= \frac{3Mv^2}{4}\end{aligned}$$

The proportion of total energy used as translational energy is given by:

$$\frac{\text{Translational Energy}}{\text{Total Energy}} = \frac{\frac{M\text{v}^2}{2}}{\frac{3M\text{v}^2}{4}} = \frac{2}{3}$$

Therefore the total translational energy is 2/3 x 8.40 = 5.60J.

(b) The proportion of total energy that is rotational kinetic energy is given by

$$\frac{\text{Rotational Kinetic Energy}}{\text{Total Energy}} = \frac{\frac{M\text{v}^2}{4}}{\frac{M\text{v}^2}{4} + \frac{M\text{v}^2}{2}} = \frac{1}{3}$$

Therefore the rotational energy is 1/3 x 8.40 = 2.80J

(c) When extension is x then, applying N2L

$$M\ddot{\text{x}} = -\text{kx} - \text{F}_{\text{fr}}$$

where F_{fr} Frictional Force

For the angular acceleration, α, we have

$$\tau = I\alpha$$

$$\text{Torque i.e. } F_{fr}R = \frac{1}{2}MR^2\alpha$$

$$\textit{But } \alpha = \frac{d\omega}{dt} = \frac{1}{R}\frac{dv}{dt} = \frac{\ddot{x}}{R}$$

$$\text{So } F_{fr} = \frac{1}{2}M\ddot{x},$$

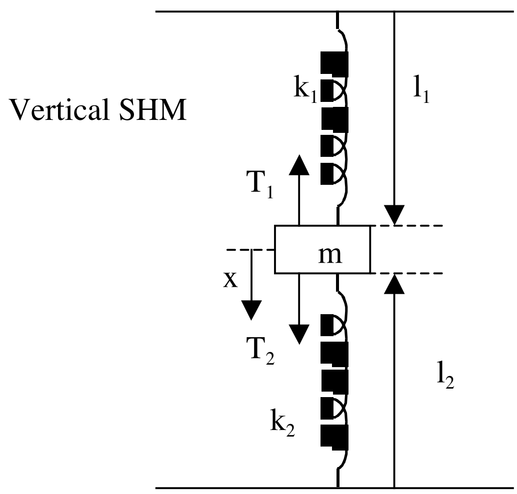
$$\text{i.e. } M\ddot{x} = -kx - \frac{M\ddot{x}}{2}$$

$$\text{Hence } \ddot{x} = -\omega^2 x \text{ with } \omega^2 = \left(\frac{2k}{3M}\right)$$

$$\text{Thus SHM with period } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3M}{2k}}$$

Q16

The system shown undergoes SHM in a vertical direction. Find the equation of motion and the frequency for the system.



Solution:

x is distance of spring from equilibrium

$$ma = T_2 - T_1$$
$$= -k_2x - k_1x$$
$$= -(k_1 + k_2)x$$
$$\Rightarrow \omega^2 = \frac{k_1 + k_2}{m}$$
$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$$

Q16

(a) Standard form is;

$$y = y_m \sin 2\pi(\frac{x}{\lambda} - ft)$$

$$y_m = 0.1m$$

$$f = 1.0Hz$$

so that $\lambda = 2.00m$

$$\upsilon = f\lambda = 1.0Hz * 2.00m = 2ms^{-1}$$

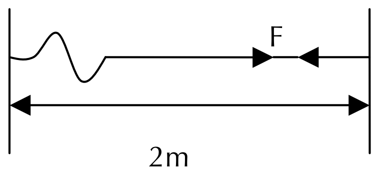
(b)

$$\boxed{\begin{aligned}\upsilon_y &= \frac{dy}{dt} = 2.00\pi * 0.1\cos(x - 2.00t)\\ &= 0.2\pi\cos(x - 2.00t)\end{aligned}}$$

maximum when cosθ=1

$$\Rightarrow \upsilon_y = 0.2\pi = 0.63ms^{-1}$$

Q17



Linear mass density $\mu = \frac{0.060kg}{2.0m} = 0.03kgm^{-1}$

Wave velocity $\upsilon = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{500N}{0.03}} = 130ms^{-1}$