Lecture 15: Beats, and standing waves in air columns, rods and membranes

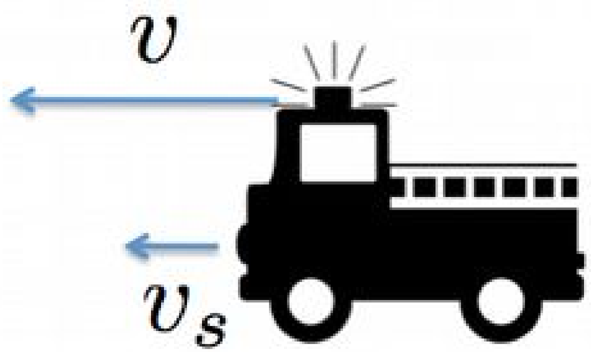
https://youtu.be/Zd_oIFy1mxM?t=29s

https://youtu.be/ho9rZjlsyYY

Last lecture… Doppler Effect

- When **both** the observer and the source are moving, the Doppler shifted frequency is:

$$f' = f\left(\frac{v + v_o}{v - v_s}\right)$$



- **Don't forget to flip the signs** if you switch the direction the source and/or the observer relative to this picture.

Shock waves>Mach number...

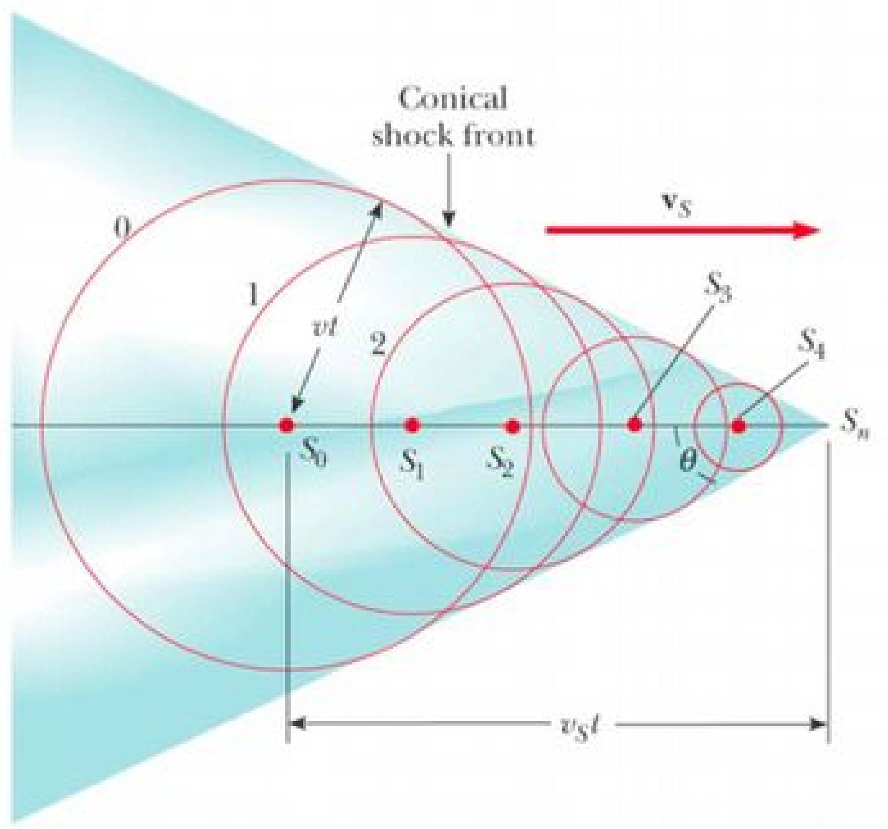
- The **Mach angle** is the apex half-angle of the conical shock wavefront, given by:

BLACKBOARD

Sound speed

$$\boxed{\frac{1}{M} = \sin\theta = \frac{v}{v_s}}$$

Speed of moving source



- The ratio $M = v_s/v$ is called the **Mach number**.

This lecture...

- Beats
- Standing waves in
  - Air columns
  - Rods
  - Membranes

https://youtu.be/nY3bgZY_nF
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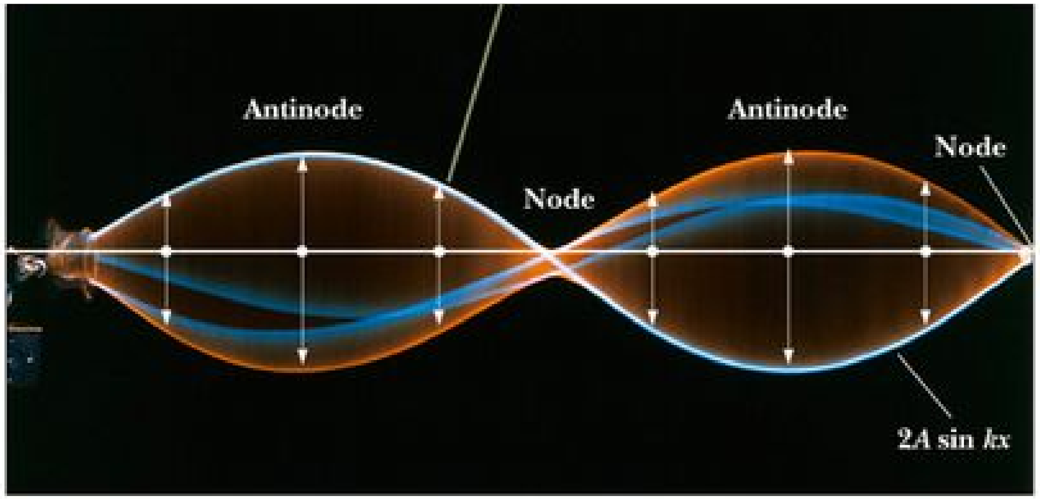
Beats...

- We have seen that in a standing wave, two waves of the same frequency add to give a new wave form whose oscillation amplitude varies with the spatial position → **spatial interference**.

$$y_1 = A\sin(kx - wt + \phi),\ y_2 = A\sin(kx + wt + \phi)$$

$$y = y_1 + y_2 = 2A\sin(kx + \phi)\cos(\omega t)$$



Beats...

- Now suppose we have two waves with slightly different frequencies.

$$y_1(x,t) = A\sin(kx - \omega_1 t + \phi)$$
$$y_2(x,t) = A\sin(kx - \omega_2 t + \phi)$$

For simplicity, assume they have the same phase constant.

- The sum of these two waves will lead to interference in time → **temporal interference**.

$$y = y_1 + y_2 = \ldots$$

$$y = y_1 + y_2 = A\left[\sin(kx - \omega_1 t + \phi) + \sin(kx - \omega_2 t + \phi)\right]$$

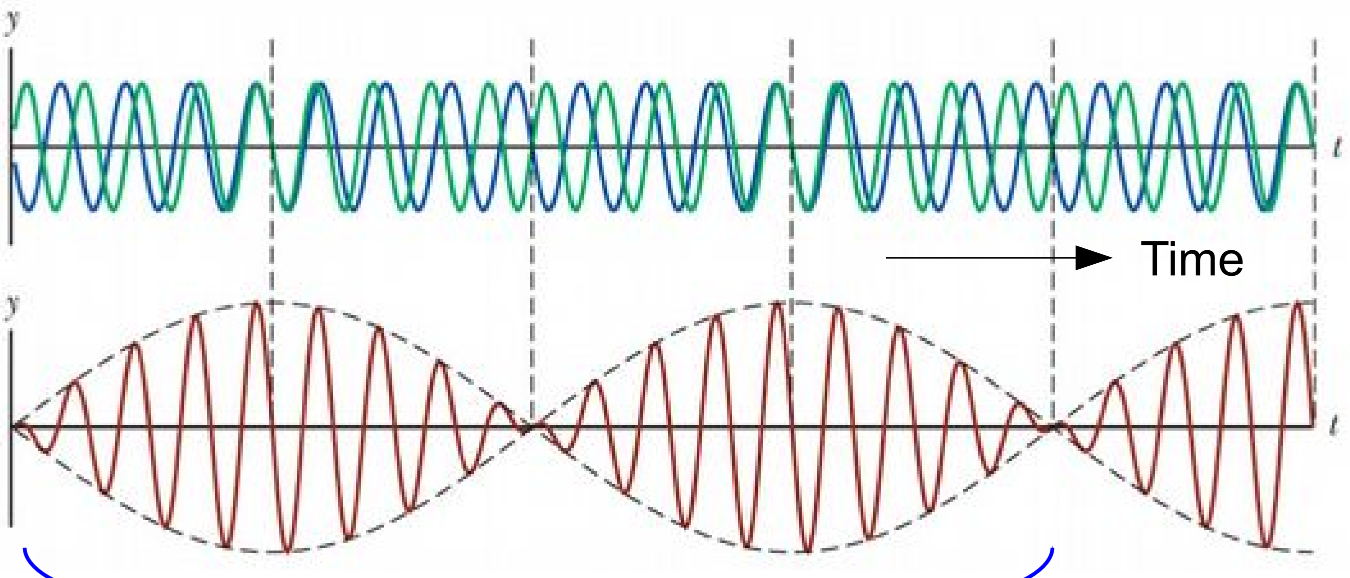
- Use the trigonometric identity:

$$\sin A \pm \sin B = 2\sin\left(\frac{A \pm B}{2}\right)\cos\left(\frac{A \mp B}{2}\right)$$

$$\Rightarrow \boxed{y = \underbrace{2A\cos\left(\frac{\omega_2 - \omega_1}{2}t\right)}_{\text{A time-dependent amplitude determined by the difference between the original two frequencies.}}\underbrace{\sin\left(kx - \frac{\omega_1 + \omega_2}{2}t + \phi\right)}_{\text{A travelling wave, with a new frequency equal to the average of the original two frequencies.}}}$$

A **time-dependent** amplitude determined by the **difference** between the original two frequencies.

A travelling wave, with a **new frequency** equal to the **average** of the original two frequencies.



Time evolution of y_1, y_2 and the sum y **at some fixed position x.**

The amplitude is modulated by the cosine function.

One cycle of the amplitude modulation

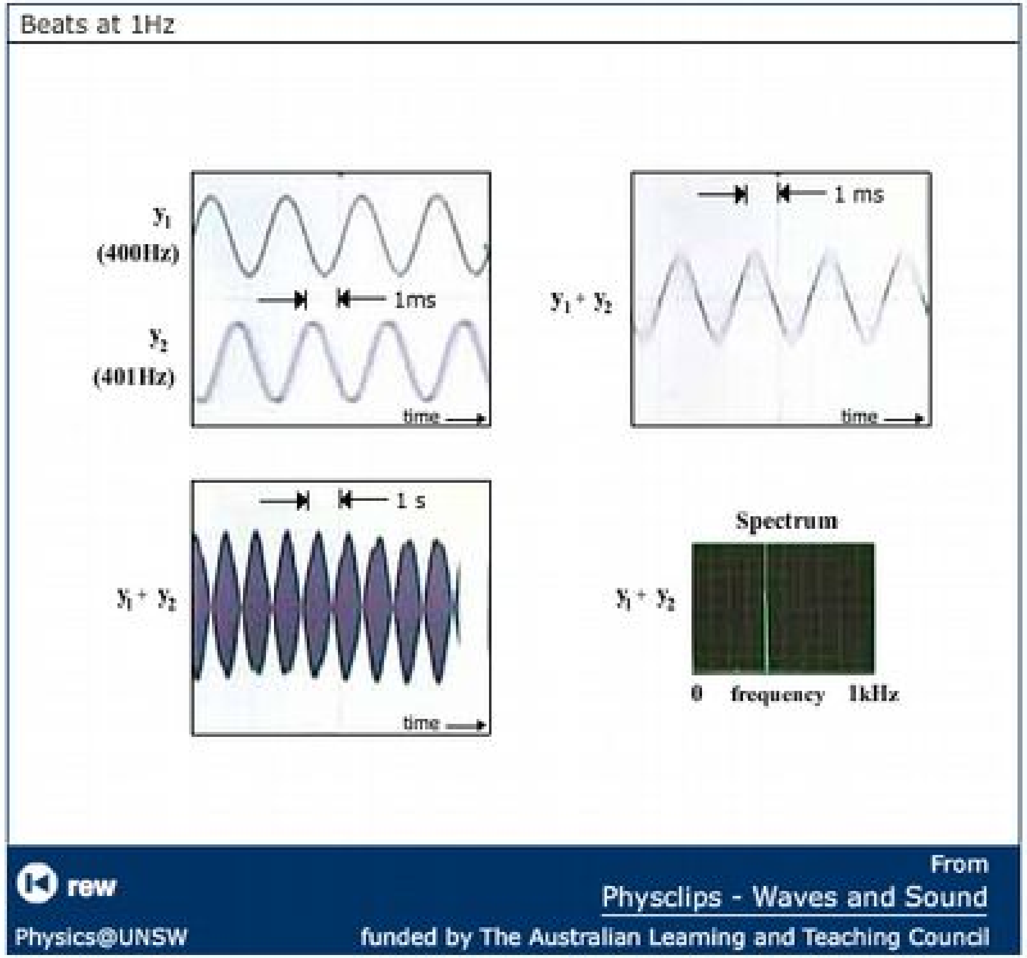
$$y = 2A\cos\left(\frac{\omega_2 - \omega_1}{2} t\right)\sin\left(kx - \frac{\omega_1 + \omega_2}{2} t + \phi\right)$$

- The cosine has one max and one minimum per cycle (2 antinodes).
- Careful: human ear hears them as two intensity maxima per cycle.
- These are the **beats**, occurring with frequency:

$$\omega_{beat} = 2 \times \left|\frac{\omega_2 - \omega_1}{2}\right| \Rightarrow \boxed{f_{beat} = |f_2 - f_1|}$$

Beat frequency

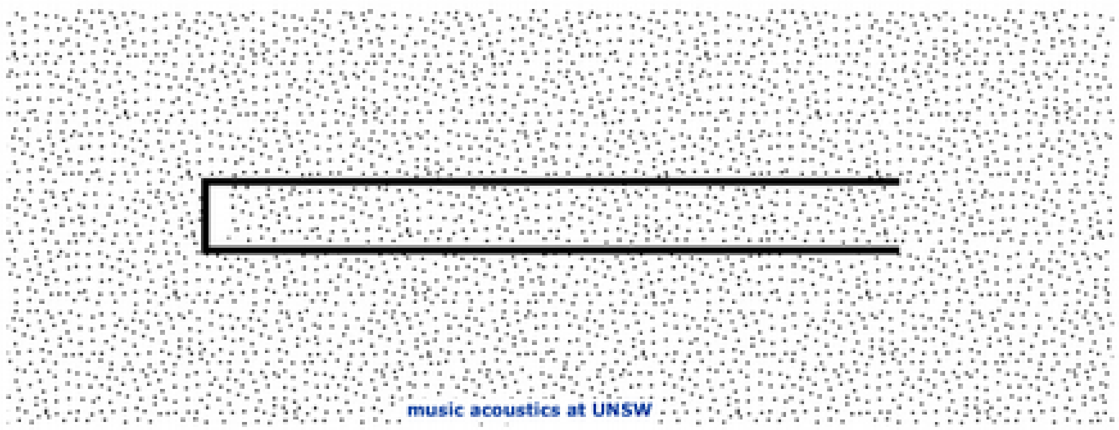
Beats (Demo or animation)...



http://www.animations.physics.unsw.edu.au/jw/beats.htm#varying

Next topic: Standing waves in air columns...

- Like transverse waves on strings, a longitudinal sound wave travelling in an air column is reflected at the boundaries.



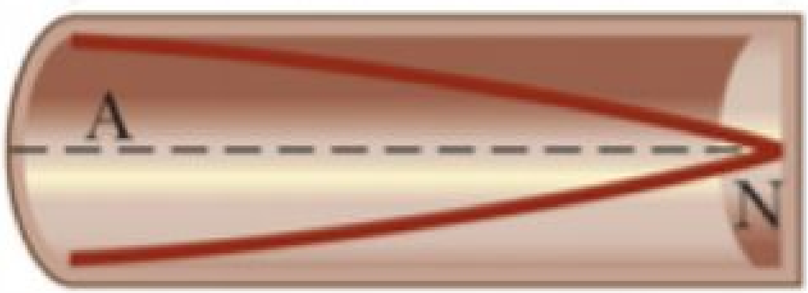
https://youtu.be/N5Ch2NThFvY?t=52s

- **Closed end**: a high/low pressure pulse is reflected as is.
- **Open end**: a high pressure pulse is reflected as a low pressure pulse, and vice versa.

Air columns>Boundary conditions...

- A **closed end** must be a displacement node (a pressure antinode).
  - The wall does not allow longitudinal motion in the air.
- An **open end** must be a displacement antinode (a pressure node).
  - The compressed air is free to expand into the atmosphere.

Open end



Closed end

Air columns>Closed at one end...

- For one closed and one open end, the **fundamental mode** has

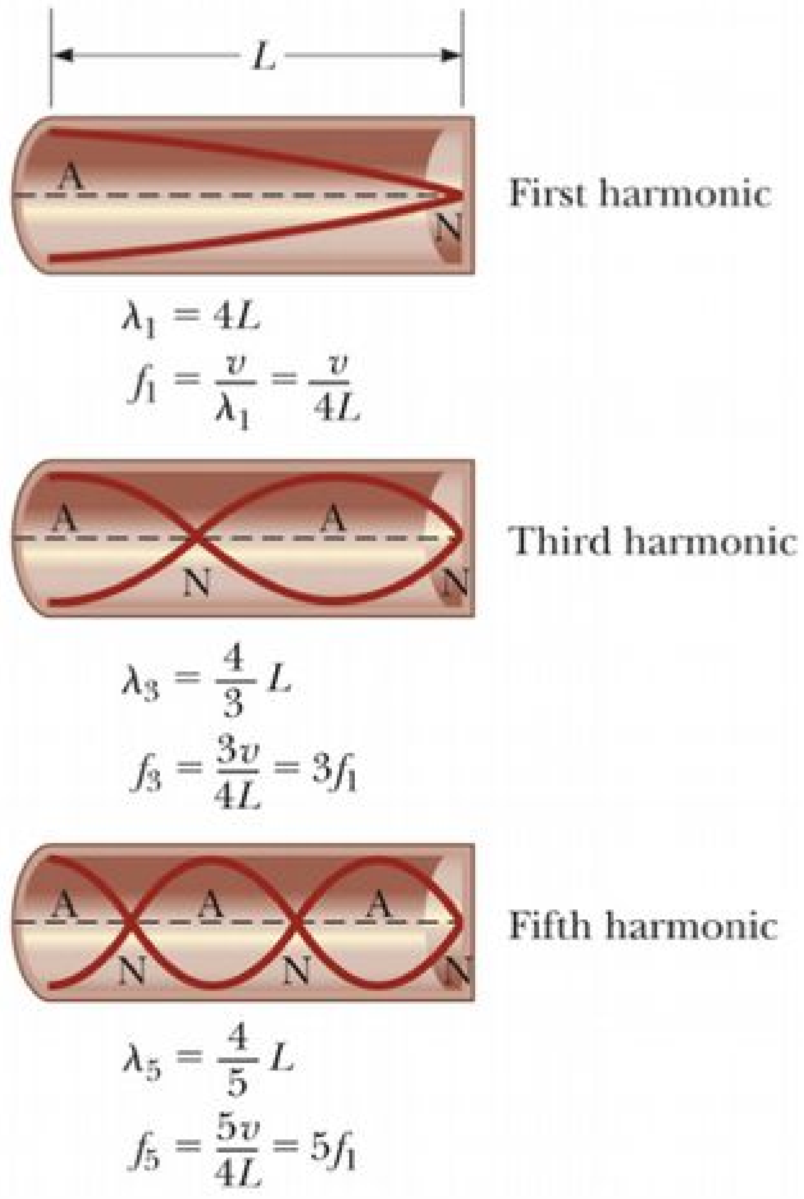
Wavelength $\lambda_1 = 4L$

Frequency $\Rightarrow f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$

- In general, only the odd harmonics can be excited:

Natural frequencies

$$f_n = n\frac{v}{4L}, \quad n = 1, 3, 5, \ldots$$



Air columns>Open at both ends...

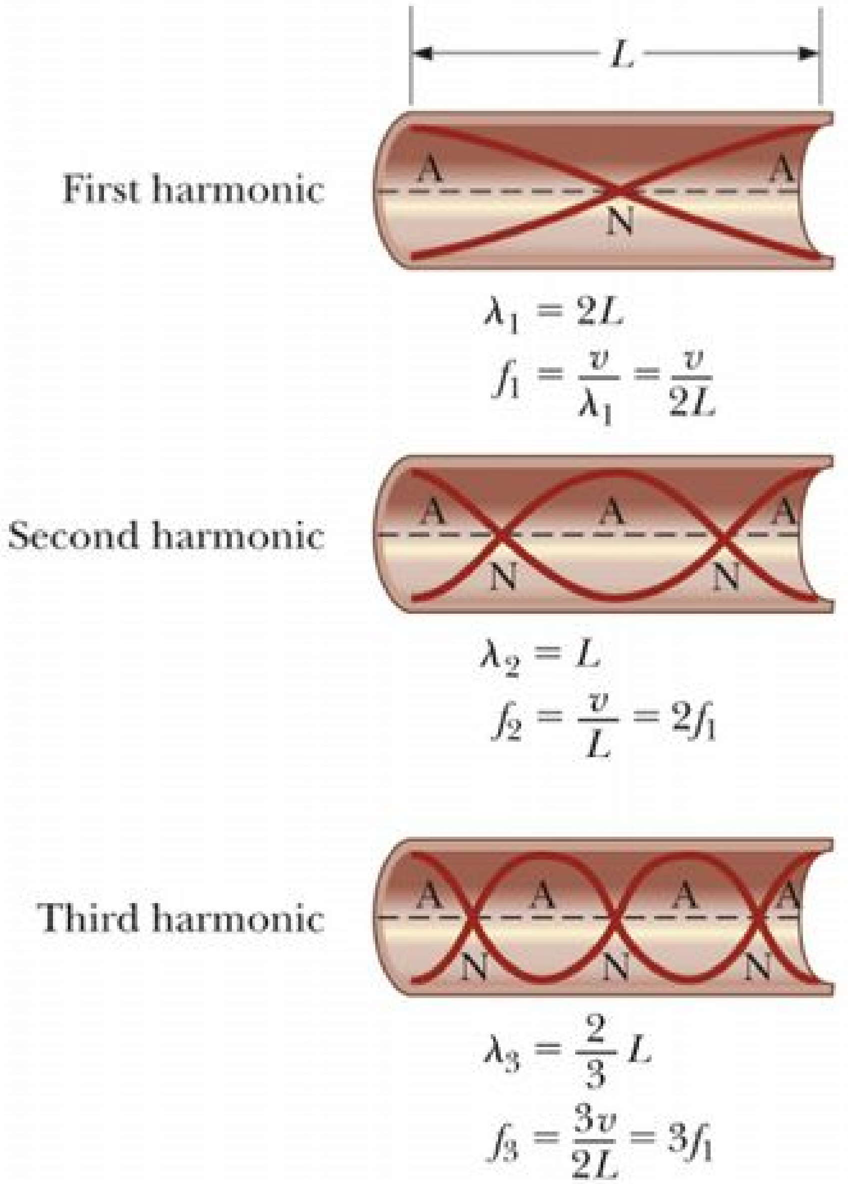
- If both ends are open, the **fundamental mode** has

Wavelength $\lambda_1 = 2L$

Frequency
$$\Rightarrow \quad f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

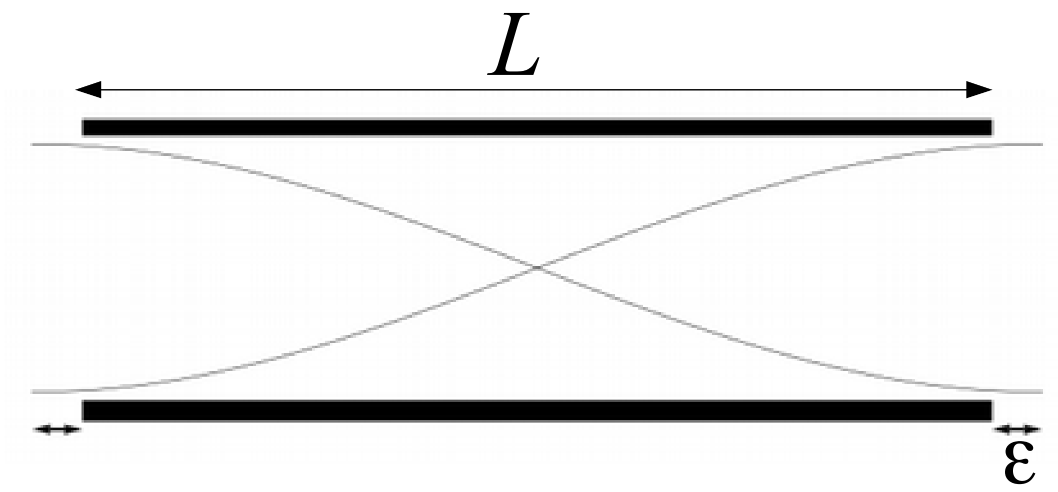
- All harmonics can be excited:

Natural frequencies
$$f_n = n\frac{v}{2L}, \quad n = 1, 2, 3, \ldots$$



Air columns>A small detail...

- In reality, at the end of an open tube the antinode occurs a small distance ε beyond the end of the tube.



- This extra distance should be added to the length of the tube L when computing the natural frequencies → **End effects**.

More about end effects here:
http://newt.phys.unsw.edu.au/jw/flutes.v.clarinets.html#end

Air columns>Musical instruments...

- Standing waves in air columns are the basis of wind instruments.

- Usually a wide range of frequencies is put into the instrument (by reeds, lips, etc.).
  - Only some frequencies match the natural frequencies of the instrument and resonate.
  - These are the frequencies we hear.

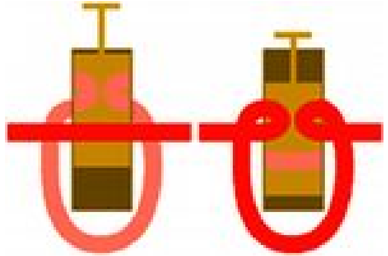
Air columns>Musical instruments...

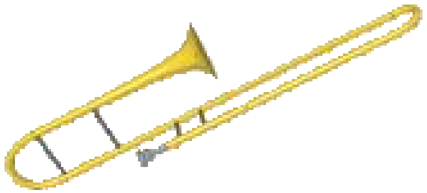
The bugle is the simplest brass instrument with a fixed length.

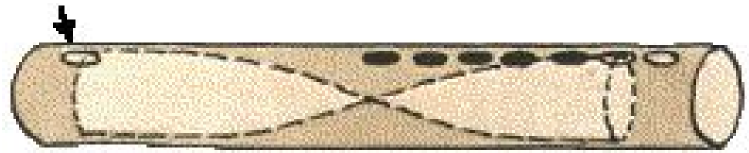
Air columns>Musical instruments...

- In more complicated wind instruments, you can also vary the length of the air column to change the natural frequencies of the instrument.

Brass instruments use a telescopic slide (e.g., trombone) or valves (e.g., trumpet) to lengthen the air column.







Finger holes in woodwind instruments (e.g., flutes) are used to shorten the effective air column

Quick quiz...

- A pipe open at both ends resonates at a fundamental frequency f_{open}. When one end is covered and the pipe is again made to resonate, the fundamental frequency is f_{closed}. Which of the following expressions describes how these two resonant frequencies compare?

1. $f_{\text{closed}} = f_{\text{open}}$
2. $f_{\text{closed}} = f_{\text{open}}/2$
3. $f_{\text{closed}} = 2f_{\text{open}}$
4. $f_{\text{closed}} = (3/2)f_{\text{open}}$

Standing waves in rods...

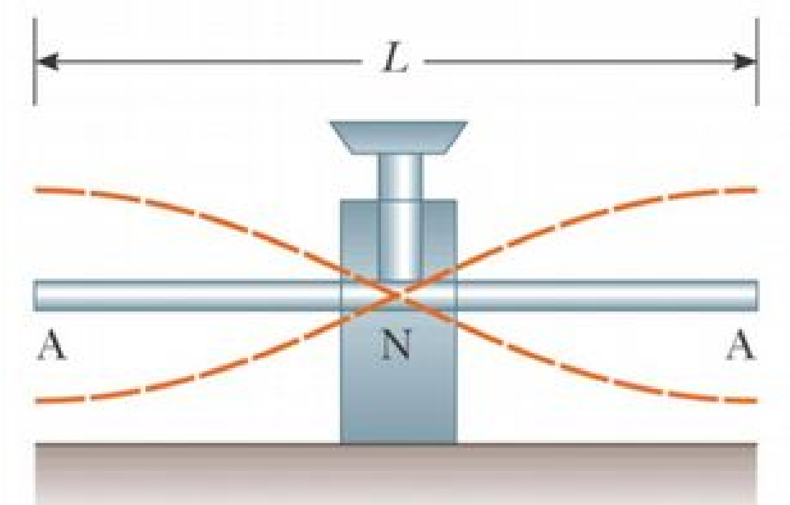
- Clamp a rod at different places to get standing waves of different frequencies.

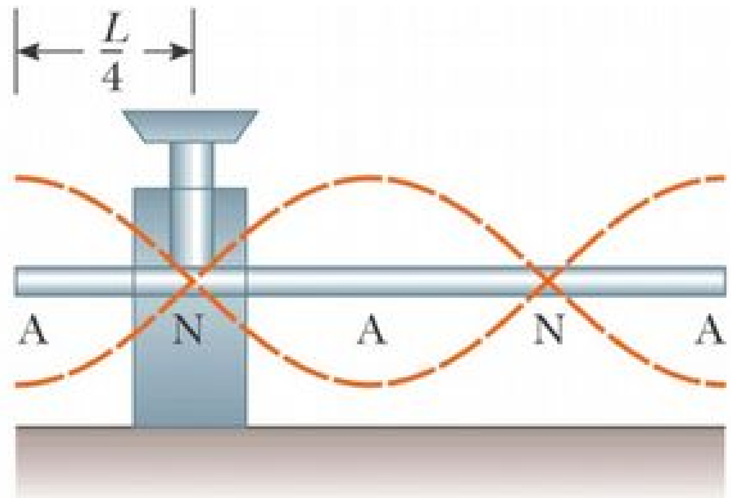
- **Top**: a displacement node in the middle; antinodes at the ends.

$$\lambda_1 = 2L \;\Rightarrow\; f_1 = \frac{v}{2L}$$

- **Bottom**: a displacement node at $L/4$; antinodes at the ends.

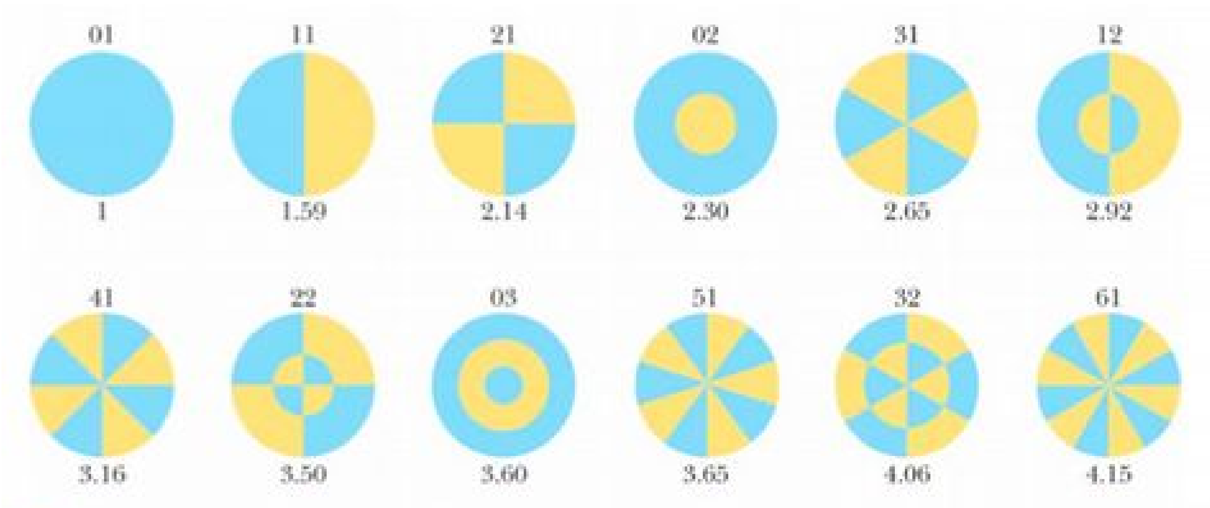
$$\lambda_2 = L \;\Rightarrow\; f_2 = \frac{v}{L} = 2f_1$$





Standing waves in membranes...

- 2D oscillations can be set up in a flexible membrane stretched over a circular hoop



Elements of the medium moving out of the page at an instant of time

Elements of the medium moving into the page at an instant of time

- The resultant sound is not harmonic because the natural frequencies are not related by integers. Instruments like this (e.g., drums) produce ‘noise’ rather than notes/chords.

Standing wave in rigid surfaces...

- **Chladni patterns** (i.e., normal modes) on a violin plate. (The particles gather at the nodes.)

https://www.youtube.com/watch?v=3uMZzVvnSiU