# Lecture 15: Doppler Effect, Shock Waves and Beats

Green Onions:
https://youtu.be/Pe2K5iT8lwE?t=1m48s

Moanin' - Rhoda Scott 1972
https://youtu.be/oIB2ywz3S9o?list=RDQwLt-VXHxD0

Leslie Speaker:
https://youtu.be/HsLcZkEu4k4

# Last lecture...

- Wave displacement, pressure, power & intensity:

$$s(x,t) = s_{max} cos(kx - \omega t)$$

$$\Delta P(x,t) = B s_{max} k sin(kx - \omega t)$$
$$= \rho v \omega s_{max} sin(kx - \omega t)$$

More useful to relate max change in pressure to **density** than to bulk modulus.

Pressure amplitude

$$\Delta P_{max} = B s_{max} k = (\rho v^2) s_{max} (\frac{\omega}{v}) = \rho v \omega s_{max}$$

- Improved derivation available on Moodle

# Last lecture...

- Wave displacement, pressure, power & intensity:

$$s(x,t) = s_{max} cos(kx - \omega t)$$

$$\Delta P(x,t) = B s_{max} k \sin(kx - \omega t)$$
$$= \rho v \omega s_{max} \sin(kx - \omega t)$$

$$\Delta P_{max} = B s_{max} k = (\rho v^2) s_{max} (\frac{\omega}{v}) = \rho v \omega s_{max}$$

Pressure amplitude

$$(\text{Power})_{avg} = \frac{1}{2} \rho v \omega^2 A s_{max}^2 \qquad I \equiv \frac{(\text{Power})_{avg}}{A}$$

$$I = \frac{1}{2} \rho v (\omega s_{max})^2 \qquad I = \frac{(\Delta P_{max})^2}{2 \rho v}$$

# Last lecture...

- We quantify the **sound level** using a logarithmic scale of the intensity of a sound wave:

$I$ = The actual intensity

$$\beta \equiv 10 log\left(\frac{I}{I_0}\right)$$

In decibels (dB)

Reference intensity

$I_0 = 10^{-12}\ Wm^{-2}$

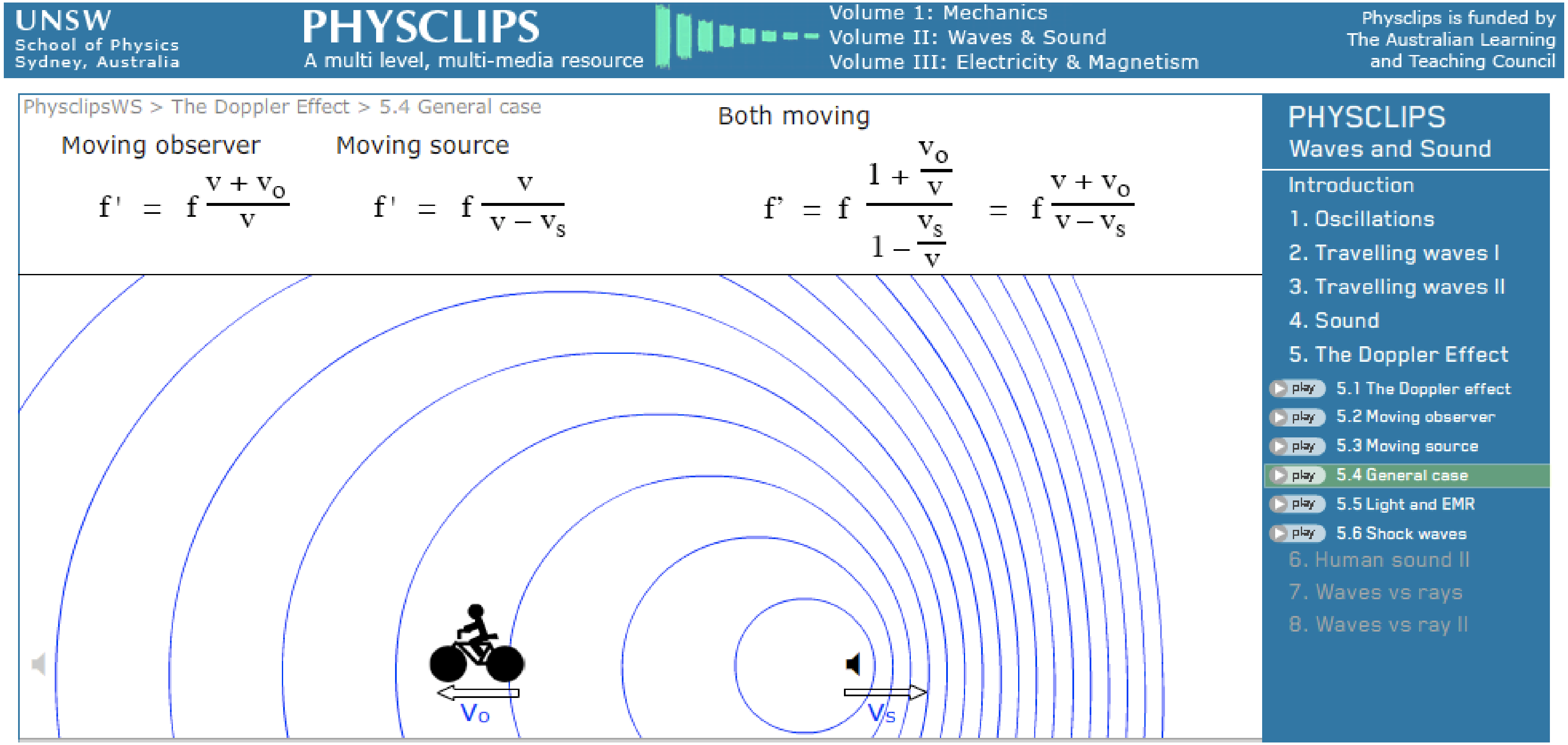
(Threshold of hearing)

# This lecture...

- Doppler
- Shock waves, Mach number and Sonic boom
- Beats (?)

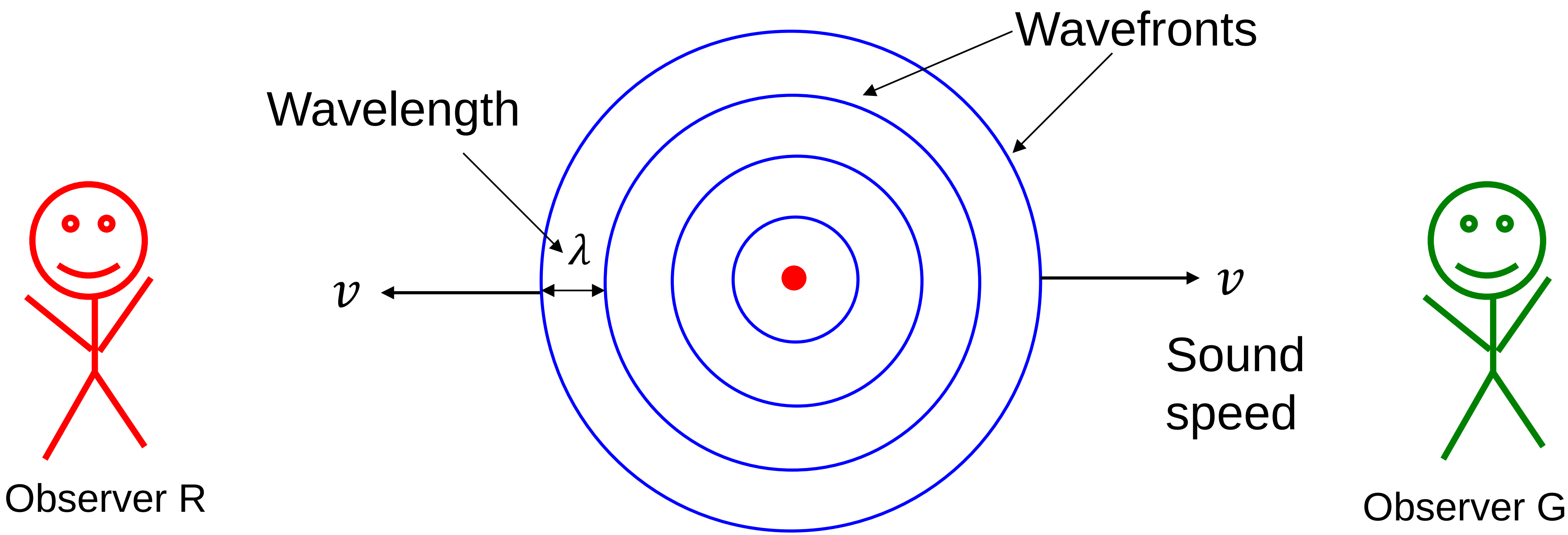
# PHYSCLIPS 5.4 Doppler Effect

## The General Case



# Doppler effect...

- Consider a **stationary sound emitting source**, and two **stationary observers**.
  - The frequency of the emitted sound wave is $f = v/\lambda$.
  - Both observers hear $f$.



# Doppler effect>Moving source...


http://www.animations.physics.unsw.edu.au/jw/doppler.htm

http://www.animations.physics.unsw.edu.au/animation_assets/movingsource_large_Canvas/Assets/movingsource_large_Canvas.html

# Doppler effect>Moving source...

- Suppose now the **observers are stationary**, and only the **sound source moves** to the right with speed $v_s$.

Source moving away from observer

$$\lambda'' = \lambda + v_s T$$

T = Period of emitted wave at the source

Source moving towards observer

$$\lambda' = \lambda - v_s T$$

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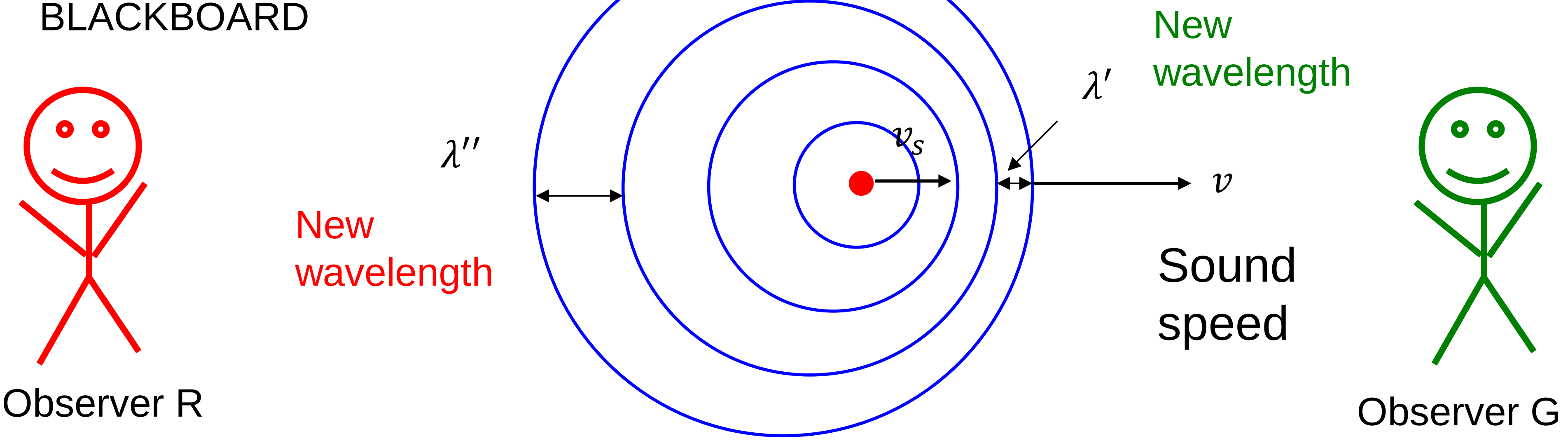


# Doppler effect>Moving source...

- Suppose now the **observers are stationary**, and only the **sound source moves** to the right with speed $v_s$.

Source moving away from observer

$$\lambda'' = \lambda + v_s T$$

T = Period of emitted wave at the source

$$\Rightarrow \frac{v}{f''} = \frac{v}{f} + \frac{v_s}{f}$$

$$\boxed{\Rightarrow f'' = f\left(\frac{v}{v + v_s}\right)}$$

Source moving towards observer

$$\lambda' = \lambda - v_s T$$

$$\Rightarrow \frac{v}{f'} = \frac{v}{f} - \frac{v_s}{f}$$

Apparent frequency detected by observer G

$$\boxed{\Rightarrow f' = f\left(\frac{v}{v - v_s}\right)}$$

# Doppler effect>Moving source recap...

- If a sound source travels at speed $v_s$, the apparent frequency detected by a **stationary observer** is

$$f' = f\left(\frac{v}{v \mp v_s}\right)$$

Sound speed

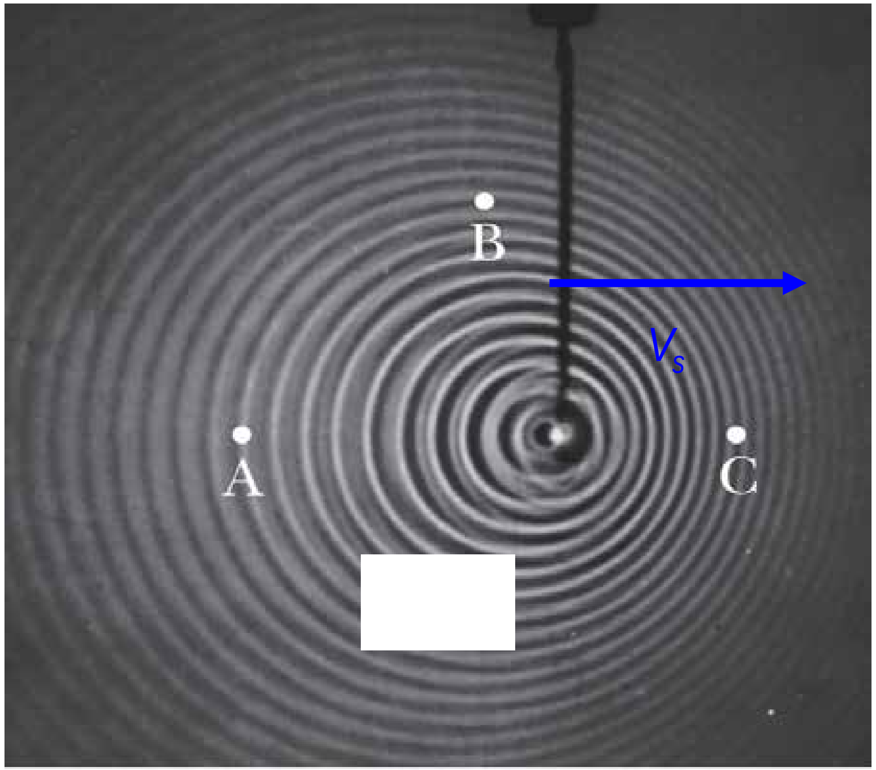
Original frequency of source

- **Minus sign** if source moves towards observer.
- **Plus sign** if source moves away from observer.

A point source is moving to the right.

Consider detectors of water waves at three locations A, B, and C, with the source moving at speed $V_s$ as shown. Which of the following statements is true?

1. The wave speed is highest at location A.
2. The wave speed is highest at location C.
3. The detected wavelength is largest at location B.
4. The detected wavelength is largest at location C.
5. The detected frequency is highest at location C.
6. The detected frequency is highest at location A.



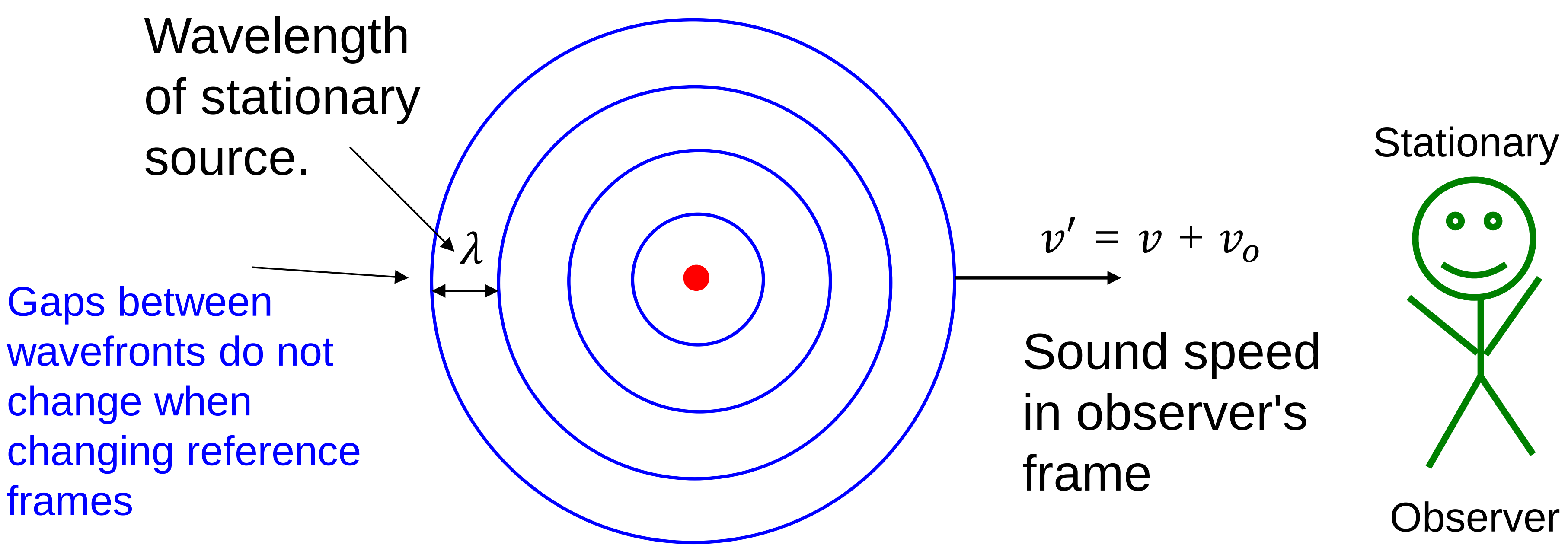
(b)



https://goo.gl/forms/qzCSy4U3gxSlxJ6H3

# Doppler effect>Moving observer...

- Now suppose the **source is stationary**, and the **observer moves** towards it at speed $v_0$.
  - How things look in the observer's reference frame:



# Doppler effect>Moving observer...

- Now suppose the **source is stationary**, and the **observer moves** towards it between speed $v_0$.

Same wavelength λ because the gaps between wavefronts do not change when changing reference frames

$$v' = v + v_o$$

$$\Rightarrow f'\lambda = f\lambda + v_o$$

Apparent frequency detected by observer

$$\boxed{\Rightarrow f' = f\left(1 + \frac{v_o}{v}\right) = f\left(\frac{v + v_o}{v}\right)}$$

# Doppler effect>Moving observer recap...

- If an observer travels at speed $v_0$ , then the frequency he/she perceives emitted from a stationary sound source is:

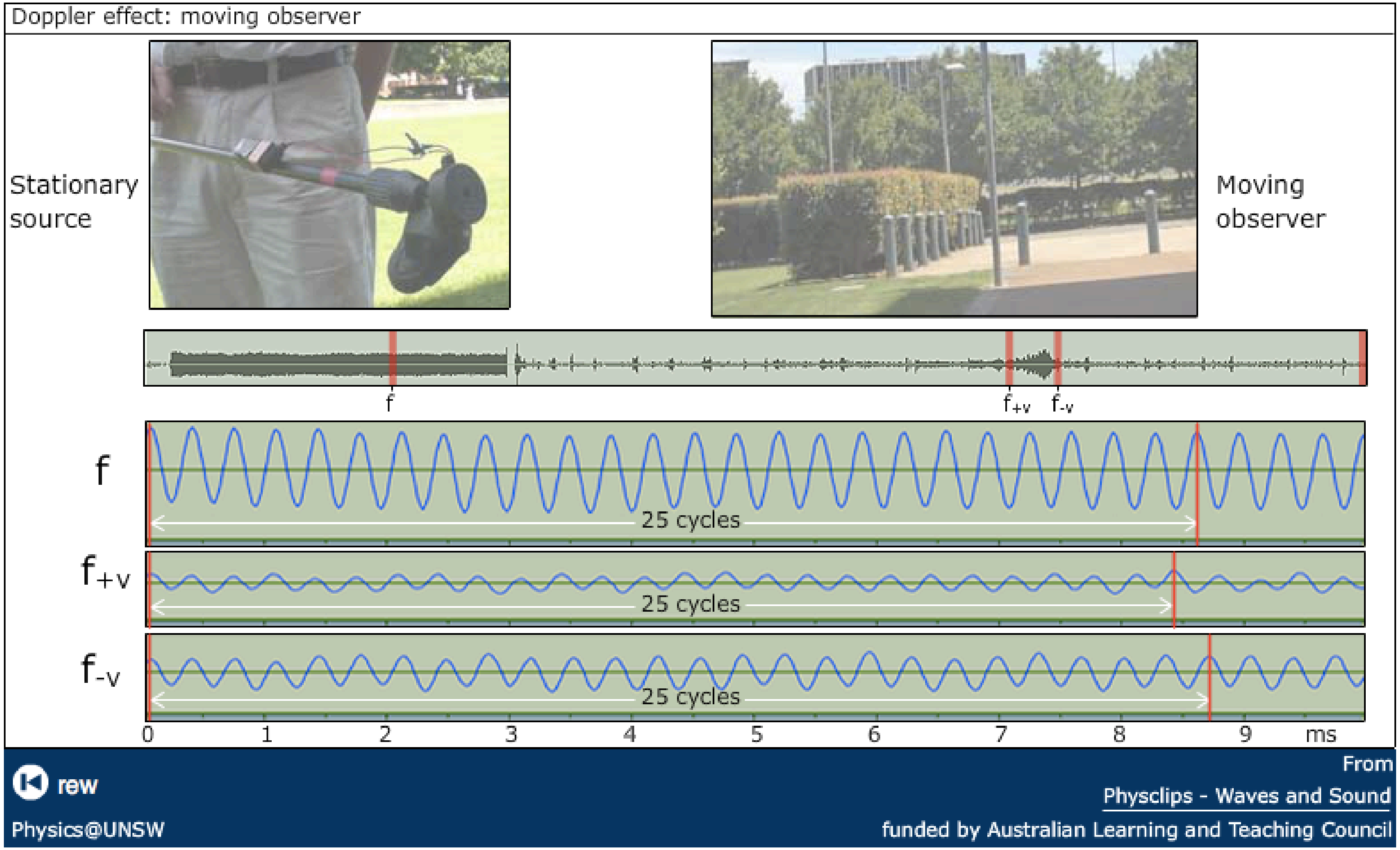
$$f' = f\left(\frac{v \pm v_o}{v}\right)$$

Sound speed

Original frequency of source

  - **Plus sign** if observer moves towards source.
  - **Minus sign** if observer moves away from source.

# Doppler effect>Moving observer...



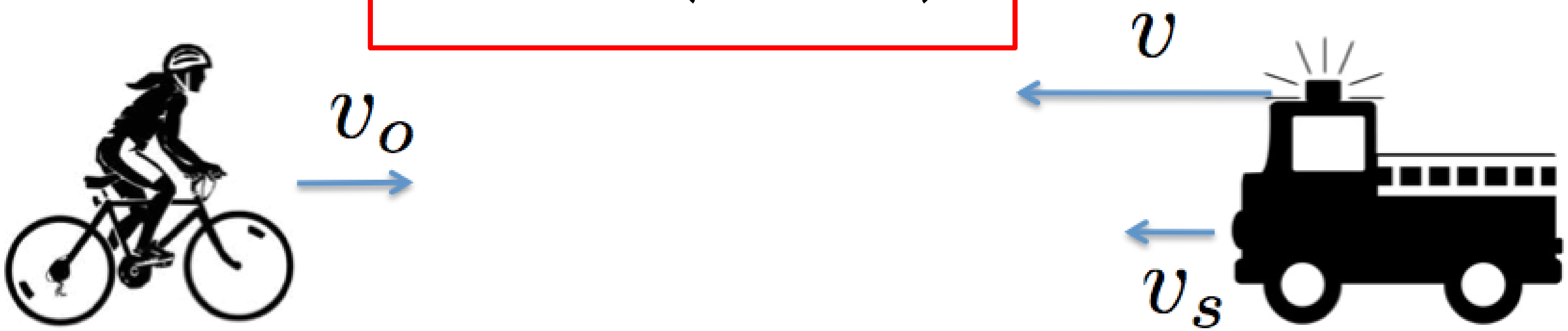

http://www.animations.physics.unsw.edu.au/jw/doppler.htm

http://www.animations.physics.unsw.edu.au/animation_assets/movingobserver_large_Canvas/Assets/movingobserver_large_Canvas.html

# Doppler effect>Both move...

- If **both** the observer and the source are moving, we can combine our previous results to give:

$$\boxed{f' = f\left(\frac{v + v_o}{v - v_s}\right)}$$




- **Don't forget to flip the signs** if you switch the direction the source and/or the observer relative to this picture.

# Doppler effect and wind

**Choose a frame of reference that moves with the wind**.

e.g.


In the situation above assume there is wind with speed $v_w$ traveling in the same direction as the cyclist.

Exercise: Write an expression for *f'* the frequency heard by the cyclist.

# Doppler effect>Final word...

- Convenient rule for signs:
  - **“Toward”** → Increase in the observed frequency.
  - **“Away from”** → Decrease in the observed frequency

- The Doppler effect is common to **all waves**.

- It does **not** depend on the distance between source and observer.

Speed measurements with laser speed guns is based on the Doppler effect.

# An Evil Question

A bat, moving at 5.00 m/s, is chasing a flying insect. If the bat emits a 40.0 kHz chirp and receives back an echo at 40.4 kHz,

a) What is the speed of the insect?

b) Will the bat be able to catch the insect? Explain.

Assume the speed of sound is 343 m/s.

# Doppler effect>EM radiation

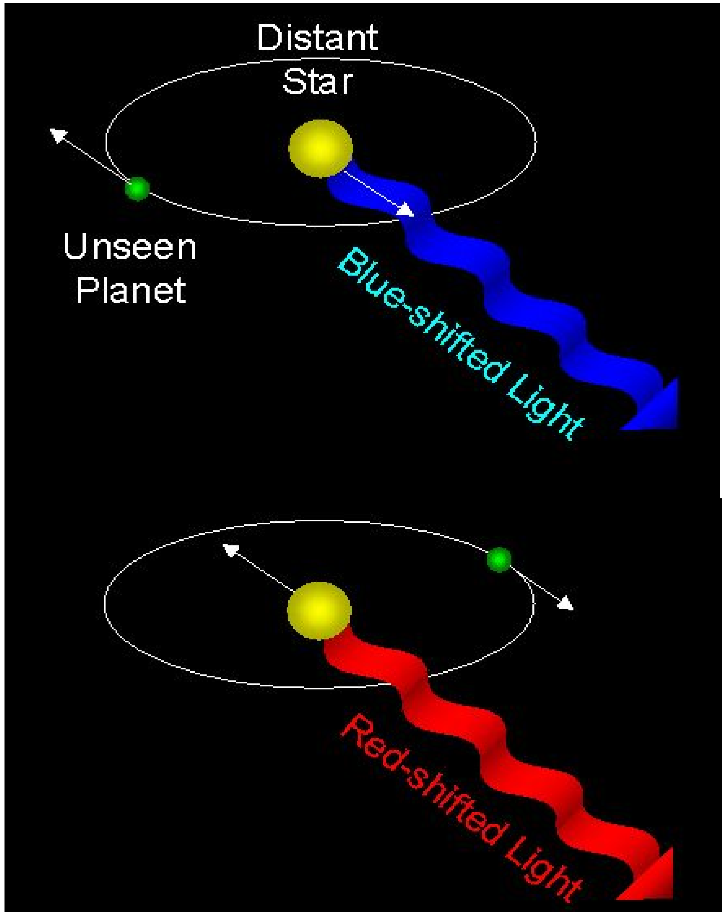
- EM radiation: Special relativity: There is no medium, so can only talk of *relative* velocities. (Einstein). Work in reference frame of observer ($v_o$=0). For v<<c,

$$f' = f\left(\frac{c}{c \mp v_s}\right) = f\left(\frac{1}{1 \mp v_s/c}\right)$$

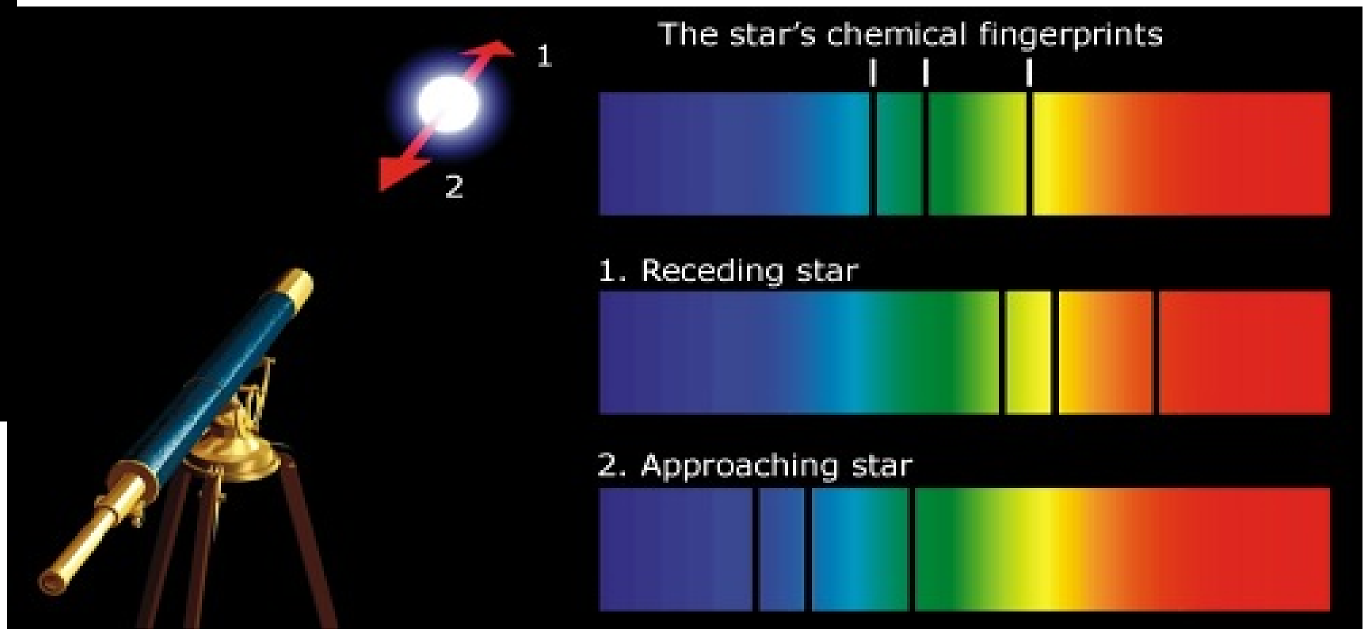
- Wavelength shift: c = constant always (Einstein).

$$\lambda' = \frac{c}{f'} = \lambda\left(\frac{c \mp v_s}{c}\right) = \lambda\left(1 \mp \frac{v_s}{c}\right)$$

# Doppler effect and planet search

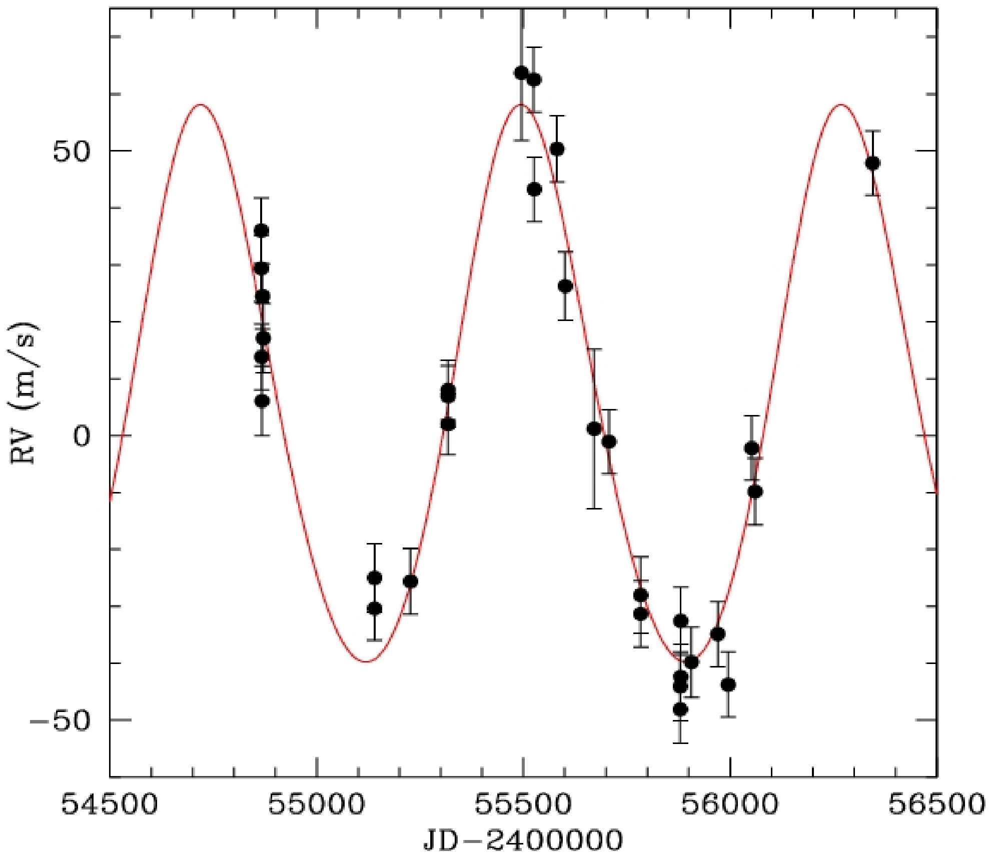



"Radial-velocity" method
Measure velocity of stars using Doppler shift

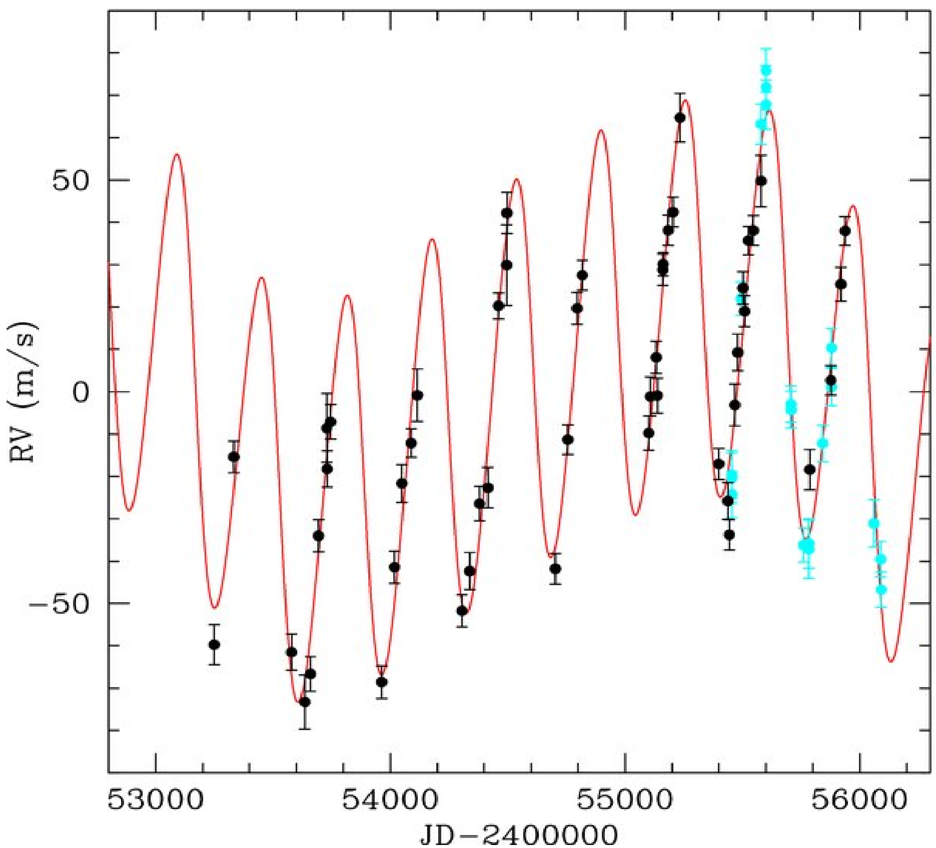


# Doppler velocity method

# The result: variation of radial velocity of star




One planet



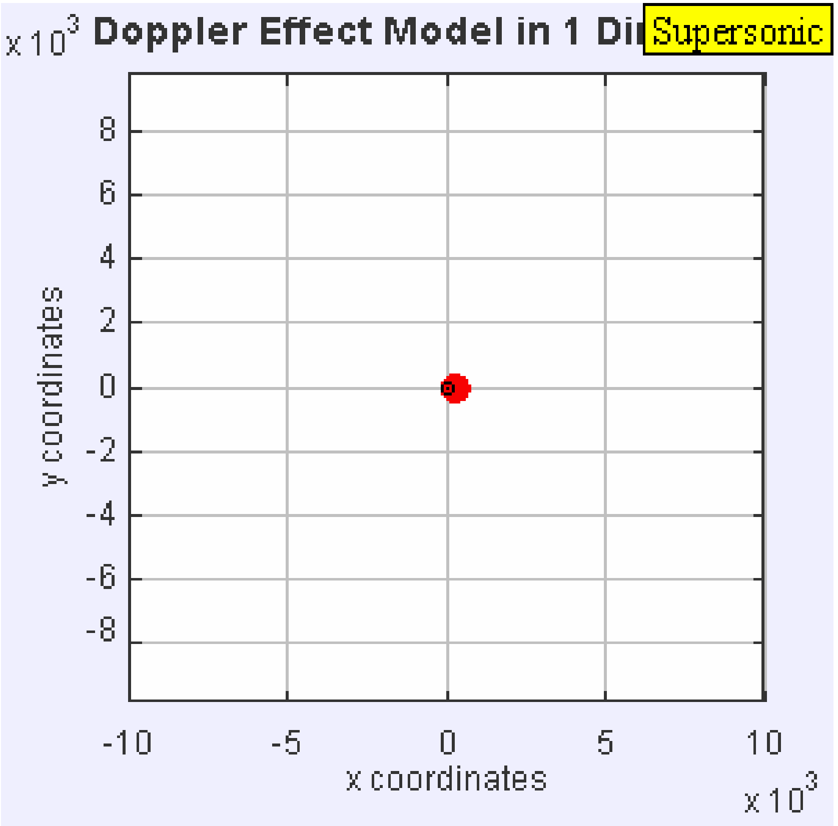

Two planets (superposition!)

# Shock waves...

- What happens if the velocity of the source **exceeds** the speed of sound in the medium?

  - The source is always ahead of the wavefronts.

  - The **envelop of the wavefronts** forms a conical wavefront called a **shock wave**.



# Mach number

The conical wave front is called a *shock wave.*

Using trigonometry the angle, θ, (the Mach angle) is given by:

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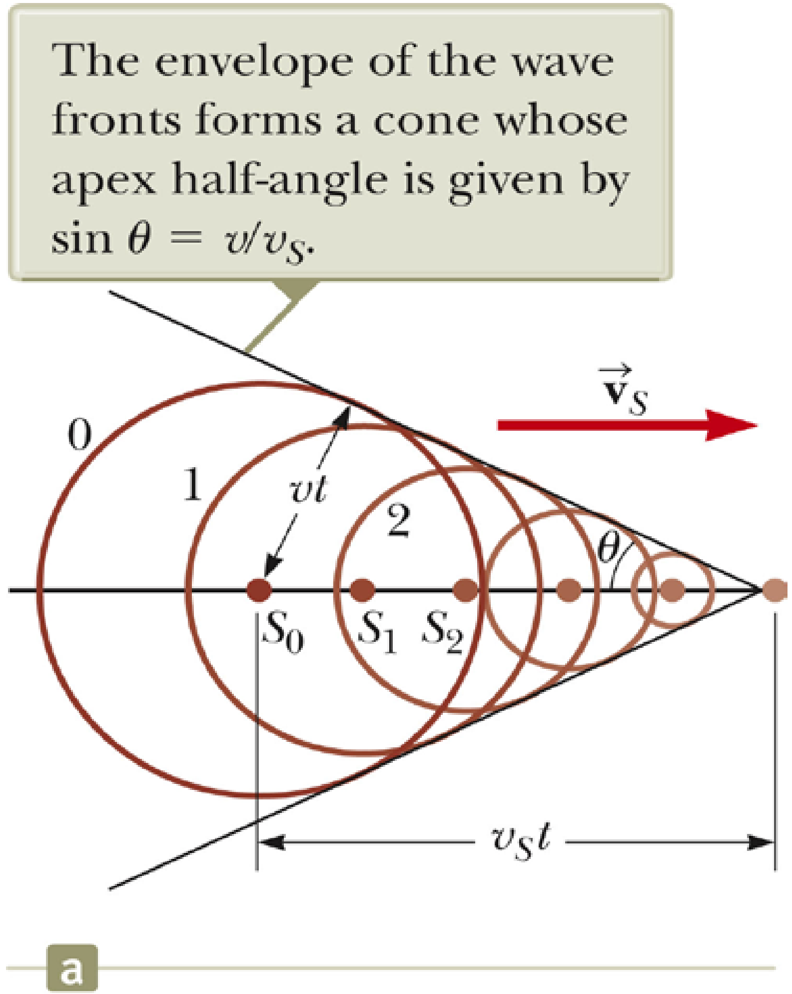
Sound speed

$$\boxed{\frac{1}{M} = sin\theta = \frac{v}{v_s}}$$

Speed of moving source

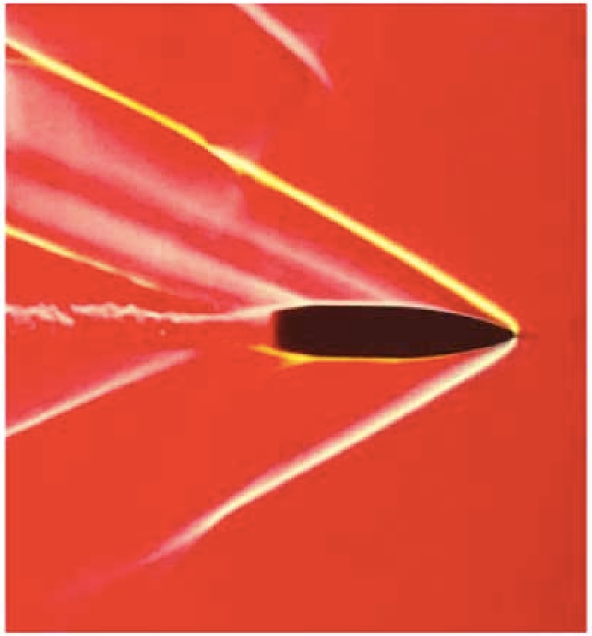
- The ratio $M = v_s/v$ is called the **Mach number**.



# Shock waves & analogous phenomena...

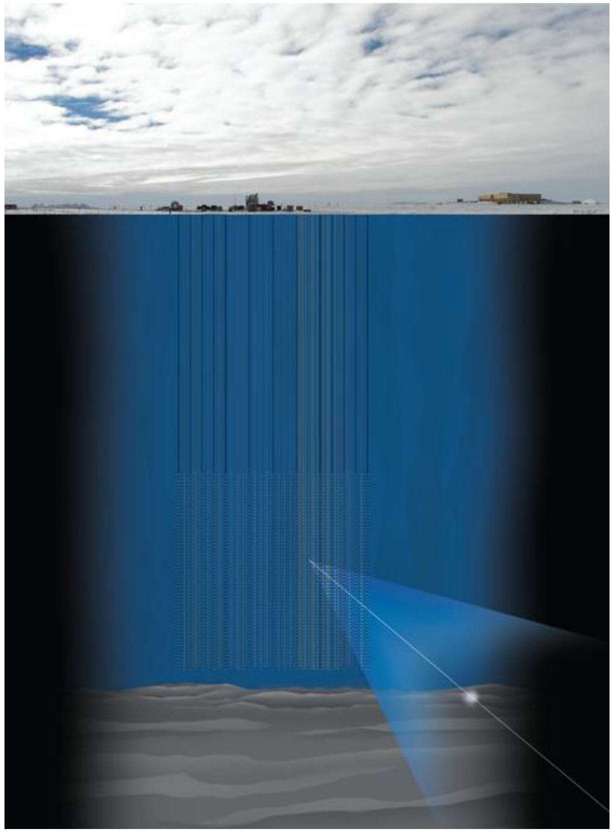
Wake: ducks swimming faster than the speed of water wave propagation

Sonic shock wave: bullet travels faster than the speed of sound in the medium



Cherenkov radiation (EM shock waves) from extremely high energy charged particles moving at faster than the speed of light in the medium.

http://icecube.wisc.edu



# Quick quiz...

- An airplane flying with a constant velocity moves from a cold air mass into a warm air mass. The Mach number:

1. Increases
2. Stays the same
3. Decreases

https://goo.gl/forms/qzCSy4U3gxSlxJ6H3

# Question 1...

- A supersonic jet travelling at Mach 3.00 at an altitude of h = 20 000 m is directly over a person at time t = 0. Assume that the average speed of sound in air is 335 m/s over the path of the sound.

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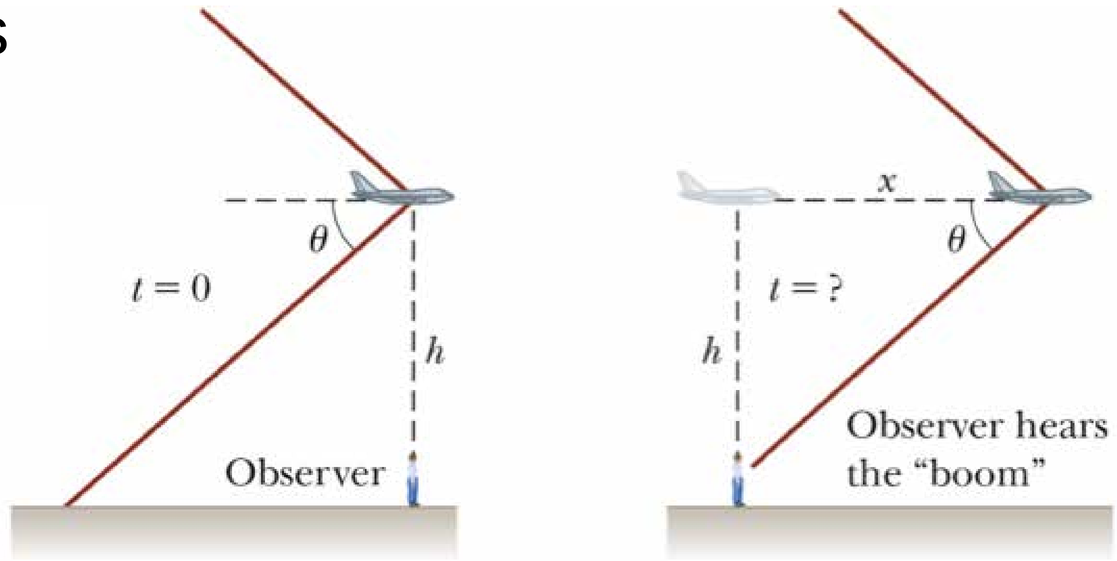
1. Where will the plane be when the person hears the shock wave?

2. What is the time elapsed between t = 0 and when the shock wave is heard?



# Question

A submarine (sub A) travels through water at a speed of 8.0 m/s, emitting a sonar wave at a frequency of 1 400 Hz. The speed of sound in water is 1 533 m/s. A second submarine (sub B) is located such that both submarines are traveling directly towards each other. The other submarine is moving at 9.00 m/s.

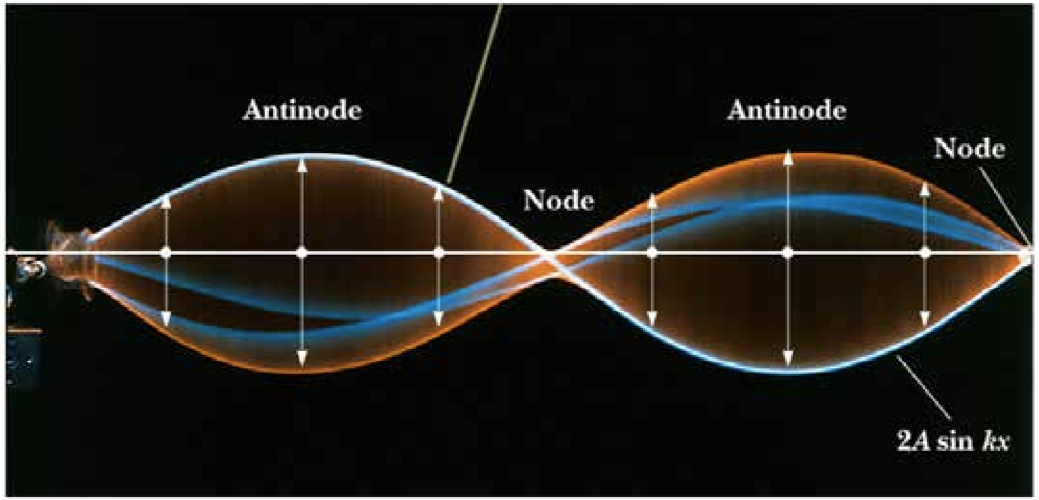
(a) What frequency is detected by an observer riding on sub B as the subs approach each other?

(b) The subs barely miss each other and pass. What frequency is detected by an observer riding on sub B as the subs recede from each other?

# Next topic: Beats...

- We have seen that in a standing wave, two waves of the same frequency add to give a new wave form whose oscillation amplitude varies with the spatial position → **spatial interference**.

$$y = y_1 + y_2 = 2A sin(kx + \phi) cos(\omega t)$$



# Beats...

- Now suppose we have two waves with slightly different frequencies.

$$y_1(x,t) = Asin(kx - \omega_1 t + \phi)$$
$$y_2(x,t) = Asin(kx - \omega_2 t + \phi)$$

For simplicity, assume they have the same phase constant.

- The sum of these two waves will lead to interference in time → **temporal interference**.

$$y = y_1 + y_2 = A[sin(kx - \omega_1 t + \phi) + sin(kx - \omega_2 t + \phi)]$$

$$y = y_1 + y_2 = A[sin(kx - \omega_1 t + \phi) + sin(kx - \omega_2 t + \phi)]$$

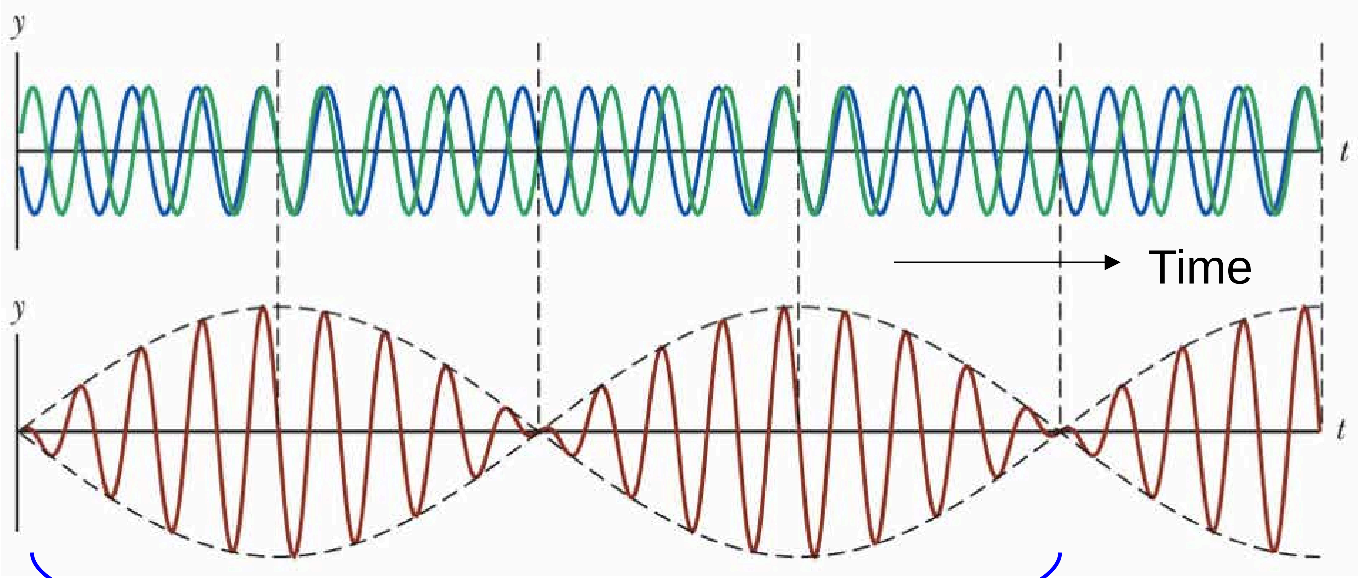
- Use the trigonometric identity:

$$sinA \pm sinB = sin\left(\frac{A \pm B}{2}\right) cos\left(\frac{A \mp B}{2}\right)$$

$$\boxed{\Rightarrow y = 2Acos\left(\frac{\omega_2 - \omega_1}{2} t\right) sin\left(kx - \frac{\omega_1 + \omega_2}{2} t + \phi\right)}$$

A **time-dependent** amplitude determined by the **difference** between the original two frequencies.

A travelling wave, with a **new frequency** equal to the **average** of the original two frequencies.




Time evolution of $y_1$, $y_2$ and the sum $y$ at some fixed position $x$.

The amplitude is modulated by the cosine function.

One cycle of the amplitude modulation

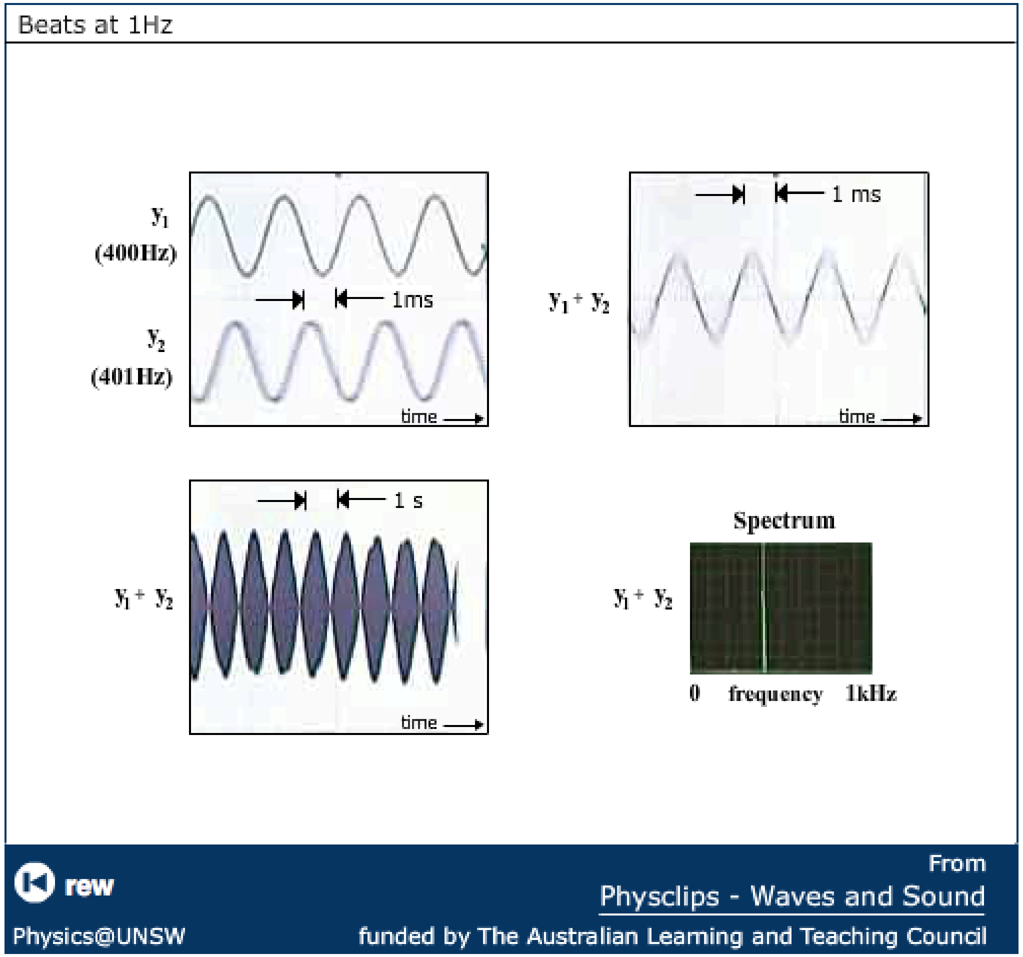
$$y = 2A\cos\left(\frac{\omega_2 - \omega_1}{2}t\right)\sin\left(kx - \frac{\omega_1 + \omega_2}{2}t + \phi\right)$$

- The cosine has one maximum and one minimum per cycle.
- But the human ear hears them as two intensity maxima per cycle. → These are the **beats**, occurring with frequency:

$$\omega_{beat} = 2 \times \left|\frac{\omega_2 - \omega_1}{2}\right| \Rightarrow \boxed{f_{beat} = |f_2 - f_1|}$$

Beat frequency

# Beats...




http://www.animations.physics.unsw.edu.au/jw/beats.htm#varying

# Question 2...

- Two identical piano strings of length 0.750 m are each tuned to exactly 440 Hz. The tension in one of the strings is then increased by 1.0%. If they are now struck, what is the beat frequency between the fundamentals of the two strings?

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