PHYS1131 HIGHER PHYSICS 1A
HOMEWORK PROBLEM SET 3

Q1. Definition of centre of mass

$$M\vec{r}_{cm} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3$$
$$6\vec{r}_{cm} = (18t-6)\hat{i} + (18+2t^2)\hat{j} + (12t-6t^3)\hat{k}$$
$$\vec{r}_{cm} = (3t-1, 3+t^2, 2t-t^3)$$
$$\vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt} = (3, 2t, 2-3t^2)$$
$$\vec{v}_{cm}(t=1) = (3,-2,-1)$$
$$\vec{p}_{tot} = M\vec{v}_{cm} = (18,-12,-6)$$

Q2. Measure from N. Let x_H be the distance of the plane of the hydrogen atoms from the nitrogen atom. Note that in the plane of the hydrogen atoms the displacement vectors (from H to X: centre of triangle) cancel out and so do not need to be taken into consideration.

$$x_{cm} = \frac{\sum m x}{m} = \frac{3m_H x_H + m_N 0}{(13.9+3)m_H}$$

Use pythagoras to calculate x_H

$$3.80 = \sqrt{10.14^2 - 9.40^2}$$
$$x_{cm} = \frac{3 \times 3.80 \times 10^{-11}}{16.9} m \quad \therefore \quad x_{cm} = 0.675 \times 10^{-11} m$$

Q3.

(a)
$$MV = m(v-V)$$
$$(M+m)V = mv$$
$$V = \frac{m}{M+m} v \text{ downwards}$$

(b) Put $v = 0$ then $V = 0$

Q4.

$$\Delta p = m\Delta v = 1.0kg \times 35ms^{-1} = 35Ns \text{ upwards}$$
$$Newton\ 2:\ F_{av} = \frac{\Delta p}{t} = 35/0.02 = 1800N \text{ downwards}$$

Q5. Frictionless table $\therefore$ no external forces in x direction
$\therefore$ momentum conserved.

$$m_1u_1 + m_2u_2 = (m_1 + m_2)V$$
$$V = (m_1u_1 + m_2u_2)/(m_1 + m_2) = 5.214 m/s$$
$$\Delta K = K_f - K_i = (92.42 - 126.03)J = -33.61J$$
$$\Delta U = -\Delta K = \frac{1}{2}kx^2 \Rightarrow x = 24.5cm$$

Q6. Suppose that the neutron produced has momentum p_N
Conservation of momentum in the direction of the electron path:

(a)

$p_N \cos(180 - \theta) = -p_N \cos\theta = p_e = 1.2 \times 10^{-22} kg\, ms^{-1}$

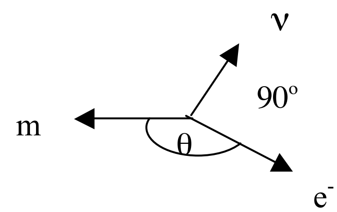
where θ is the angle between electron + nucleus path

Conservation of momentum in the neutrino path direction

$p_N \sin(180 - \theta) = p_N \sin\theta = p_\nu = 0.64 \times 10^{-22} kg\, ms^{-1}$

$\tan\theta = 0.64/1.2 \Rightarrow \theta = 151.9^0$. So direction to neutrino is

$360 - 151.9 - 90 = 118.1^o$



square the first two equations and add to eliminate θ

$p_N^2 = p_e^2 + p_\nu^2$

$p_N = 1.4 \times 10^{-22} kgm/s$

(b) $$K = \frac{p_N^2}{2m} = 1.6 \times 10^{-19} J = 1.0eV$$

Q7. No external forces, $\therefore$ momentum conserved

$mv = MV$ *(1)*

no external forces $\therefore$ *mechanical energy conserved*

$$\frac{1}{2}mv^2 + \frac{1}{2}MV^2 - \frac{GMm}{d} = 0 \quad (2)$$

To substitute (1) in (2), multiply (2) by M

$$mMv^2 + M^2V^2 = \frac{2GM^2m}{d}$$

Substitute

$$m(m+M)v^2 = \frac{2GM^2m}{d}$$

$$v^2 = \frac{2GM^2}{d(m+M)}$$

$$V^2 = \frac{2Gm^2}{d(m+M)}$$

$$v_{rel} = v + V = \sqrt{\frac{2G(M+m)}{d}}$$

PAST EXAM QUESTION

i) The weight of a 60 kg person is 590 N. So 590 N depresses the "spring" by 5 mm, so the spring constant is $k = |F|/x = 120$ kN/m.

ii) $m_{scale} < M$, so the force to accelerate part of it is small so,

the external horizontal forces acting on the block and bullet are negligible,

so their total momentum will be conserved during their collision.

The block + bullet has mass M = 10 kg + 6.0 g $\cong$ 10 kg. Let it travel at V, so

$$p_i = p_f$$

$$mv = (M+m)V \cong MV, \text{ so}$$

$$V = mv/M = 0.240 \text{ ms}^{-1}$$

In the compression of the spring in the scale, external forces do negligible work (because it is an undamped spring).

At maximum compression, the block is stationary. Assume that the mechanical energy of the bullet+block is converted into potential energy of the "spring", so

ΔK = -ΔU

so

$$\frac{1}{2}MV^2 = \frac{1}{2}kx^2$$
$$x^2 = \frac{MV^2}{k} = \frac{10\text{ kg} \times (0.24\text{ms}^{-1})^2}{120 \times 10^3\text{ N/m}} = 4.8 \times 10^{-6} m^2$$
$$x = 2.2\text{ mm}$$

iii) for the spring, |F| = kx, so if 60 kg produces a deformation of 5 mm, 2.2 mm will read a "weight" of 27 kg.

Q8.

(a)

$$\frac{dT}{dt} = 2.5 \times 10^{-8}(s/day)$$
$$\omega = \frac{2\pi}{T} \therefore \frac{d\omega}{dt} = -\frac{2\pi}{T^2}\frac{dT}{dt}$$
$$\therefore \alpha = -\frac{2\pi}{T^2}\frac{dT}{dt} = \frac{2\pi\text{ rad}}{(1\text{ day})^2(23.9\text{ hr/day})^2(3600\text{ s/hr})^2}\frac{2.5 \times 10^{-8}\text{ s/day}}{(23.9\text{ hr/day})(3600\text{ s/hr})}$$
$$= -2.5 \times 10^{-22}\text{ rad s}^{-2}$$

(b)

$$\Delta T = (2.5 \times 10^{-8}\text{ s/day}) \times (365\text{ day/yr} \times 10^9\text{ yrs}) = 9125\text{ s} = 2.5\text{ hr}$$

So T = 24 + 2.5 = 26.5 hrs

Q9.

(a)

$$I = 2/5MR^2 = 2/5 \times 6.0 \times 10^{24} \times (6.4 \times 10^6)^2 = 9.8 \times 10^{37}\text{ kgm}^2$$
$$\omega = 2\pi/T = 6.28/(24\text{ hr} \times 3600\text{ s/hr}) = 7.27 \times 10^{-5}\text{ Hz}$$
$$K = 1/2I\omega^2 = 2.6 \times 10^{29} J$$

(b) Energy used per second $= \frac{dW}{dt} \times dt = 10^3 \times 10^{10} = 10^{13}$ J

$T = \frac{2.59 \times 10^{29}}{10^{13}}$ s $\sim 10^9$ years

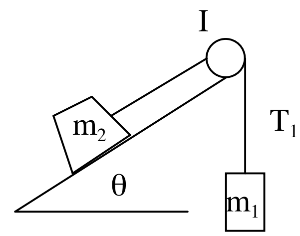
(However, big changes in the day length and the slowing of tides would have serious consequences.)

Q10.

Choose $a > 0$ when m_1 falls.

Newton 2 for m_1:

$m_1 g - T_1 = m_1 a \qquad (1)$



Newton 2 for m_2

$T_2 - m_2 g\sin\theta - \mu m_2 g\cos\theta = m_2 a \qquad (2)$

Newton 2 for the pulley

$$\tau = (T_1 - T_2)R = I\alpha = \frac{1}{2}MR^2 a/R \Rightarrow T_1 - T_2 = 1/2Ma \qquad (3)$$

Adding (1), (2) and (3) we get:

$$m_1 g - T_1 + T_2 - m_2 g\sin\theta - \mu m_2 g\cos\theta + T_1 - T_2 = (m_1 + m_2 + M/2)a$$

$$\Rightarrow a = \frac{m_1 g - m_2 g\sin\theta - \mu m_2 g\cos\theta}{m_1 + m_2 + M/2}$$

$\therefore a = 5.7\ m/s^2$

$\therefore T_1 = m_1(g - a) = 37\ N$

$T_2 = T_1 - Ma/2 = 34\ N$

Q11.

(a)

$$K = \frac{1}{2}(Mv^2 + I\omega^2) = \frac{1}{2}Mv^2 + 1/2 \times 2/5MR^2(v/R)^2 = 7/10Mv^2$$

(b) The sphere rolls without slipping. So, although friction is present, the point of contact is instantaneously stationary. So non conservative forces do no work, so mechanical energy is conserved.

$$\therefore\ \Delta E = 0 = -Mgh + 0.7Mv^2 \Rightarrow v = \sqrt{\frac{10gh}{7}}$$

$$\omega = v/R = \sqrt{\frac{10gh}{7R^2}}$$

(c)

$$\mathrm{v}^2 = \mathrm{v}_0^2 + 2\mathrm{ax} \qquad \mathrm{x/h} = \sin\theta$$

$$\frac{10}{7}\mathrm{gh} = 2\mathrm{ah}/\sin\theta \Rightarrow \mathrm{a} = 5/7\mathrm{g}\sin\theta$$

(d)

$$v = v_0 + at$$

$$t = v/a = \sqrt{\frac{14h}{5g\sin^2\theta}}$$

Q12.

(a)

$$\tau = Mgx\sin\theta$$

(b) Parallel axis theorem:

$$I = I_{cm} + Mx^2 = \frac{1}{12}ML^2 + Mx^2$$

(c) Newton's second law for rotation:

$$\tau = I\alpha$$

$$I\alpha = -Mgx\sin\theta \cong -Mgx\theta \qquad \text{(for small angle } \sin\theta \approx \theta)$$

$$\therefore \alpha = \frac{-Mgx}{I}\theta$$

$$\alpha = \frac{d^2\theta}{dt^2} = -\left(\frac{gx}{x^2 + 1/12L^2}\right)\theta$$

has solutions of the form

$$\theta = \theta_o \sin(\omega t + \phi) \text{ where } \omega = \sqrt{\left(\frac{gx}{x^2 + 1/12L^2}\right)}$$

so the period is

$$T = 2\pi/\omega = 2\pi\sqrt{\frac{x^2 + 1/12L^2}{gx}}$$

(d)

Consider:

$$f(x) = (T/2\pi)^2 = x/g + L^2/(12gx)$$

$$df/dx = 1/g - L^2/(12gx^2) = 0 \text{ when x = L/}\sqrt{12}$$

(e)

$$T = 2\pi\sqrt{\frac{1/12L^2 + 1/12L^2}{gL/\sqrt{12}}} = 1.53s$$

Then $x = \frac{1}{\sqrt{12}} =$ 0.29 m

Q13.

$$g = \frac{GM_e}{r^2} \Rightarrow g/g_0 = (\frac{R_e}{r})^2$$

$$\mathrm{r} = 6.38 \times 10^6 + 10^4 = 6.39 \times 10^6 \ m$$

(a) $\mathrm{R_e / r} = 0.998$

$$\mathrm{g / g_0} = 0.997$$

$$r = 6.38 \times 10^6 + 2.00 \times 10^5 = 6.58 \times 10^6$$
$$R_e / r = 0.970$$
$$g / g_0 = 0.940$$

(b)
$$r = \sqrt{2} R_e$$
$$h = (\sqrt{2} - 1) R_e = 0.414 R_e$$

Q14. Newton 2 at surface for critical case when $|F_g| = |F_{centip}|$

$$\frac{GMm}{r^2} = m\omega^2 r$$
$$M = \frac{\omega^2 r^3}{G} = \frac{4\pi^2 \times (2 \times 10^4)^3}{6.673 \times 10^{-11}} = 4.7 \times 10^{24} \text{ kg}$$
$$\rho = \frac{m}{V} = \frac{4.7 \times 10^{24}}{\frac{4}{3}\pi r^3} = 1.4 \times 10^{11} \text{ kgm}^{-3}$$

Q15.

(a) F is towards Earth's centre, so *a* must be towards Earth's centre.

(b) $$\frac{GMm}{r^2} = m\omega^2 r \Rightarrow r^3 = \frac{GM}{\omega^2} = \frac{GMT^2}{4\pi^2} \Rightarrow r = (\frac{GMT^2}{4\pi^2})^{1/3} = 4.22 \times 10^7 m$$

(c)
$$U = -\frac{GMm}{r} = -2.27 \times 10^9 J$$
$$K = 1/2|U| = 1.13 \times 10^9 J$$
$$E = 1/2U = -1.13GJ$$

$$\frac{1}{2}\text{mv}^2 - \frac{\text{GMm}}{\text{R}_\text{e}} = 0$$

Q16.

$$F = \frac{GM_g M_s}{R_g^2} = M_s \frac{\mathrm{v}^2}{\mathrm{R_g}} \text{ where } v = \frac{2\pi R_g}{T}$$

$$\therefore M_g = \frac{4\pi^2 R_g^3}{T^2 G} = 1.01 \times 10^{41} kg, \text{ mass of Galaxy}$$

$$N = M_g / M_s \text{ with } \mathrm{M_S}, \text{ mass of Sun, } = 2.0 \text{ x } 10^{30} \text{ Kg}$$

$$\Rightarrow \text{ N } = 5.1 \text{ x } 10^{10} \text{ stars}$$

However, the galaxy doesn't have this many stars internal to the sun's orbit. For this very reason, cosmologists propose that most of the gravitational effect is due to dark matter, i.e. matter that isn't visible as a shining star. Note that stars external to the earth's orbit have no net gravitational effect (why?) In reality, the sun is about halfway out from the centre of the galaxy.

Q17.a)

$$dF = \frac{GmdM}{r^2} = \frac{GmdM}{R^2 + x^2}$$

$$dF_x = \cos\theta dF = \frac{GmxdM}{(R^2 + x^2)^{3/2}} \text{ since } \cos\theta = \frac{\mathrm{x}}{\sqrt{\mathrm{R}^2 + x^2}}$$

$$\therefore F = \int_{ring} dF_x = \frac{Gmx}{(R^2 + x^2)^{3/2}} \int_{ring} dM = \frac{GmMx}{(R^2 + x^2)^{3/2}}$$

(b)

$$U = -\int_x^\infty F \bullet dx = -\int_x^\infty \frac{GmMxdx}{(R^2 + x^2)^{3/2}}$$

Integrate this using the substitution $u = R^2 + x^2 \Rightarrow \frac{du}{dx} = 2x \Rightarrow du = 2xdx$

For limits on the integral note that $u = R^2 + x^2$ when x = x and u = ∞ when x = ∞

$$U = -GmM \int_{R^2+x^2}^\infty \frac{\frac{1}{2}du}{u^{3/2}} = -\frac{GmM}{2} \int_{R^2+x^2}^\infty u^{-3/2} du$$

$$= -\frac{GmM}{2}\left[-2u^{-1/2}\right]_{R^2+x^2}^\infty = -\frac{GmM}{\sqrt{R^2 + x^2}}$$

(c)

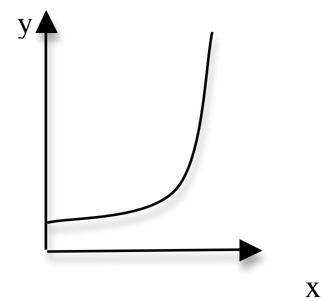
$$U_i = U_f + K_f$$

$$-\frac{GmM}{\sqrt{R^2 + x^2}} = \frac{1}{2}mv^2 - \frac{GmM}{R}$$

$$v^2 = 2GM(\frac{1}{R} - \frac{1}{\sqrt{R^2 + x^2}})$$

$$v = \left[2GM(\frac{1}{R} - \frac{1}{\sqrt{R^2 + x^2}})\right]^{1/2}$$

Q18.



$$\vec{r} = (3t)\hat{i} + (1 + t^2)\hat{j}$$

(a)

$$x = 3t$$

$$y = 1 + t^2 = 1 + 1/9x^2$$

(b)

$$\vec{v} = (3,2t,0)$$

$$\vec{a} = (0,2,0)$$

(c)

$$\vec{F} = m\vec{a} = 4\hat{j}$$

(d)

$$\vec{\tau} = \vec{r} \times \vec{F} = 12t\hat{i} \times \hat{j} = 12t\hat{k}$$

$$\vec{L} = \vec{r} \times m\vec{v} = 6(t^2 - 1)\hat{k}$$

Q19. No external forces act, so momentum is conserved in the direction of the initial motion in the perpendicular direction

(1) $m_\alpha v_\alpha \cos 64^o + m_0 v_0 \cos 51^o = m_\alpha u$

(2) $m_\alpha v_\alpha \sin 64^o = m_0 v_0 \sin 51^o$

(3) $v_\alpha / v_0 = m_0 \sin 51 / m_\alpha \sin 64^o = 3.5$