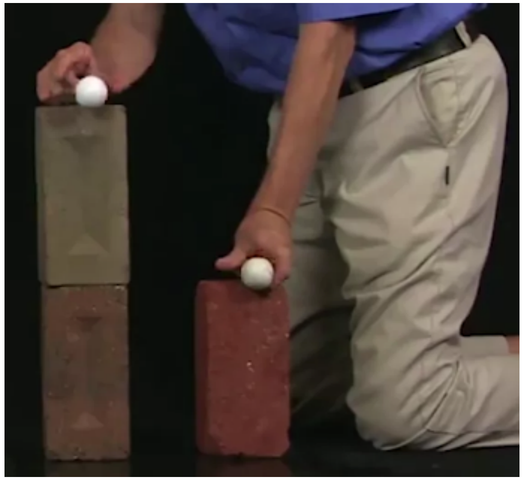
PHYS 1121, 1131, 1141.

Kinematics in one dimension, vectors in 3D.

Introduce yourself to your neighbours and discuss

some high school revision questions:



Question 1: I drop a mass from height L, another from 2L. Does the second ball take

a) less than twice as long to fall

b) twice as long

c) more than twice as long

Question 2: Explain your answer qualitatively to your neighbour, without equations or values

Question 3: (only after you've answered the above): How much longer does the second ball take to fall? $T_2/T_1 = ?$

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$$s - s_0 = v_0 t + \frac{1}{2} a t^2$$

Take s downwards. Set $s_0 = 0$, $v_0 = 0$ and $a = g$ (positive if s downwards).

Distance fallen $= s = \frac{1}{2} g t^2$ rearrange:

$$t = \sqrt{\frac{2s}{g}} \qquad \text{Given } s_1 = L \text{ and } s_2 = 2L$$

$$t_1 = \sqrt{\frac{2s_1}{g}} = \sqrt{\frac{2L}{g}} = T \text{ (say) so}$$

$$t_2 = \sqrt{\frac{2s_2}{g}} = \sqrt{\frac{2*2L}{g}} = \sqrt{2}\sqrt{\frac{2L}{g}} = \sqrt{2}T = 1.4T$$

Question 4: I have a string with masses at heights 0, L, 2 L, 3 L, 4 L. I drop the string.

What will the soundtrack look like?

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What will the soundtrack look like?

$$t_1 = \sqrt{\frac{2s_1}{g}} = \sqrt{\frac{2L}{g}} = T \qquad \text{and} \quad t_n = \sqrt{\frac{2s_n}{g}} = \sqrt{\frac{2nL}{g}} = \sqrt{n}\sqrt{\frac{2L}{g}} = \sqrt{n}\,T$$

Question 5: How do I space the weights so as to get equal times?

i.e. we want $t_n = nt_1 = nT$ where $t_1 = T = \sqrt{\frac{2L}{g}}$

$s_1 = L$ $\qquad s_n = ?$

Question 5: How do I space the weights so as to get equal times?

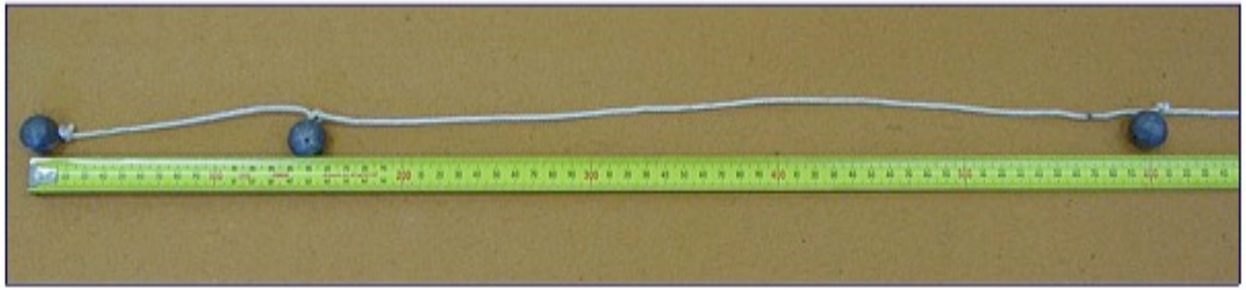
We want $t_n = nt_1 = nT$ where $T = t_1 = \sqrt{\frac{2L}{g}}$ and

Distance fallen $= s = \frac{1}{2}gt^2$ For n[th] weight: $s_n = \frac{1}{2}gt_n^2$

So $s_n = \frac{1}{2}g(nt_1)^2$

$$s_n = \frac{1}{2}g\left(n\sqrt{\frac{2L}{g}}\right)^2 = n^2L$$

Mechanics > Projectiles > 2.2 An experiment with falling bodies



n	0	1	2	3	4
y_0	0	15 cm	60 cm	135 cm	240 cm
y_0 =	0^2 L	1^2 L	2^2 L	3^2 L	4^2 L

where L = 0.15 m

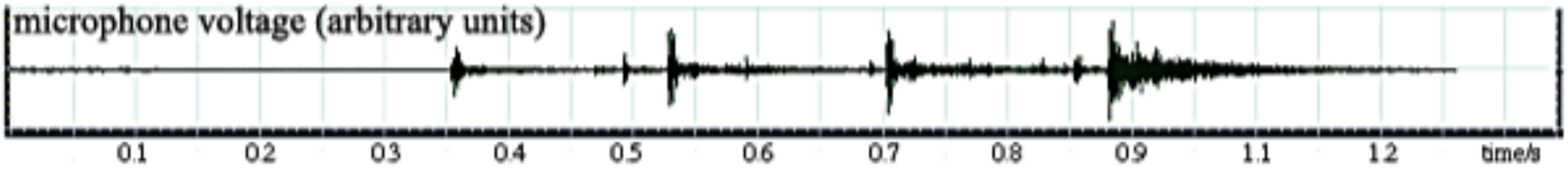
Let’s try it.

We had: $T = t_1 = \sqrt{\frac{2L}{g}}$ and $s_n = \frac{1}{2} g t_n^2 = \frac{1}{2} g (nT)^2 = \frac{1}{2} g \left(n\sqrt{\frac{2L}{g}} \right)^2 = n^2 L$

so $\frac{1}{2} g (nT)^2 = n^2 L$ Rearrange:

so $g = \frac{2L}{T^2}$ *Check it has the right units*

L is 0.15 m. If only we could measure T we'd have g… *Let's try it.*



From Physclips, Chapter 2.2
http://www.animations.physics.unsw.edu.au/

Or
Lesson 3.1 in the web stream of Phys 1121-1131.

Constant acceleration in y direction:

$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2.$$

$$y = y_0 + \frac{1}{2}a_y t^2.$$

Drop from y_0 at $t = 0$. Floor at $y = 0$.

$$0 = y_0 + \frac{1}{2}a_y t^2.$$

$$y_0 = -\frac{1}{2}a_y t^2.$$

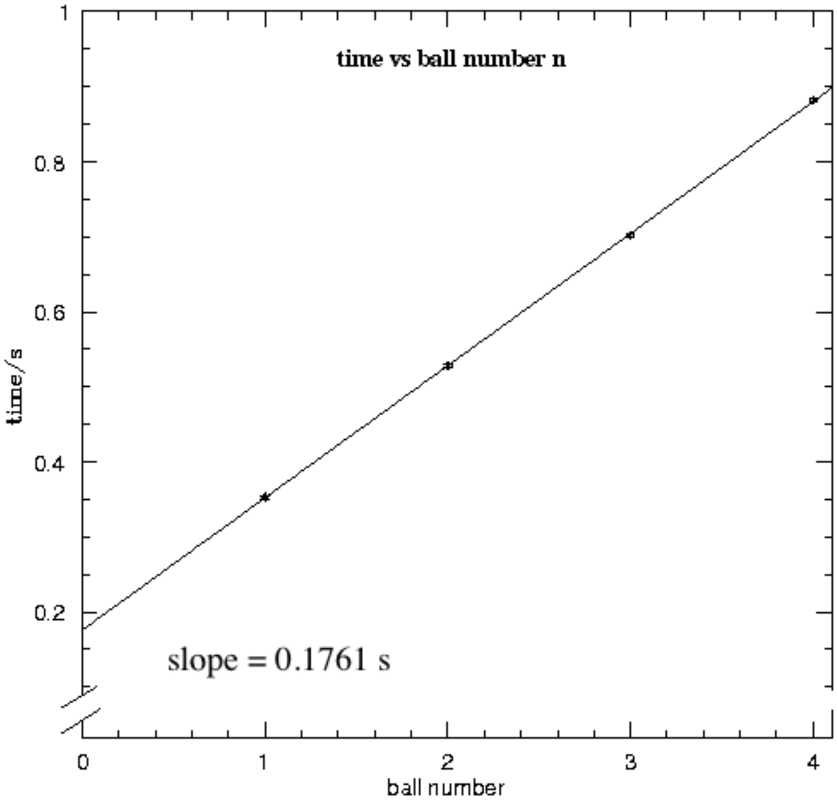
n	0	1	2	3	4
y_0	0	15	60	135	240 cm
	$= 0^2$ L	1^2 L	2^2 L	3^2 L	4^2 L

where $L = 0.15$ m

$$Ln^2 = -\frac{1}{2}a_y t^2$$

rearrange → $t^2 = -\dfrac{2L}{a_y}n^2$

square root → $t = \sqrt{\dfrac{2L}{-a_y}}\ n.$



$$\text{slope} = \sqrt{\frac{2L}{-a_y}}$$

$$a_y = -\frac{2L}{(\text{slope})^2}$$

$$\cong -9.8\ \text{m.s}^{-2}.$$

Physclips version:

Or see Lesson 3.1 in the web version of lectures.

Motion with constant acceleration in one dimension (1D)

Where did $s = s_0 + v_0 t + \frac{1}{2} a t^2$ $\left(\text{or} \quad x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2\right)$ come from?

$a = ?$ $v = ?$ $x = ?$

$a = constant$

$v = \int a dt = at + const.$

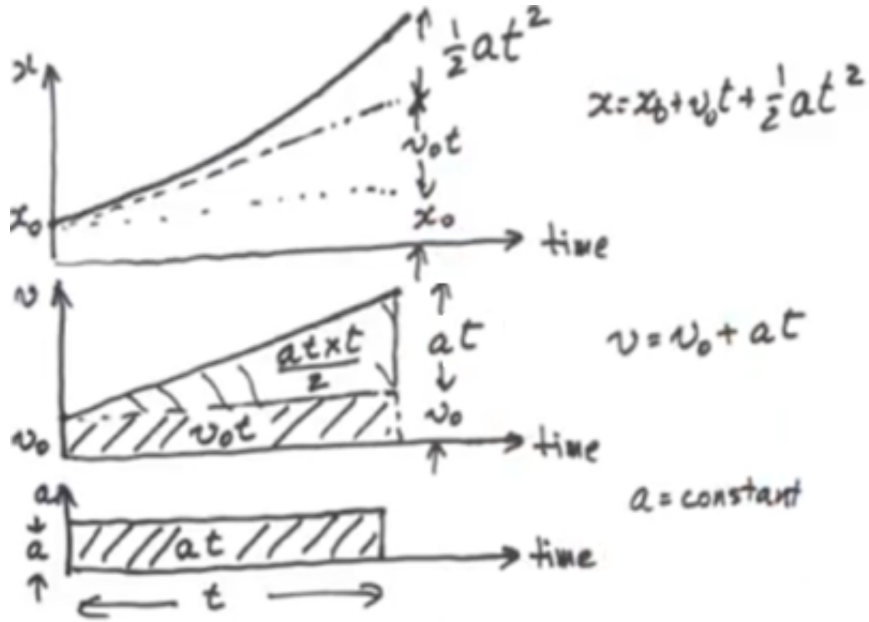
Constant? At $t = 0$, set $v = v_0$ so

$$v = \int a dt = at + v_0$$

$x = \int v dt = \frac{1}{2} a t^2 + v_0 t + const$

$x = \int v dt = \frac{1}{2} a t^2 + v_0 t + x_0$

(See lesson 2.2 in the web lectures)



For constant a, we have $x = x_0 + v_0 t + \frac{1}{2}at^2$ and $v = v_0 + at$

What if we want to relate v, v_0, a and distance travelled $\Delta x = x - x_0$?

Eliminate t from equations above

$v = v_0 + at$ gives $t = \frac{v - v_0}{a}$ so substitute in the first equation:

$x - x_0 = v_0 \frac{v - v_0}{a} + \frac{1}{2}a(\frac{v - v_0}{a})^2$ Multiply both sides by $2a$

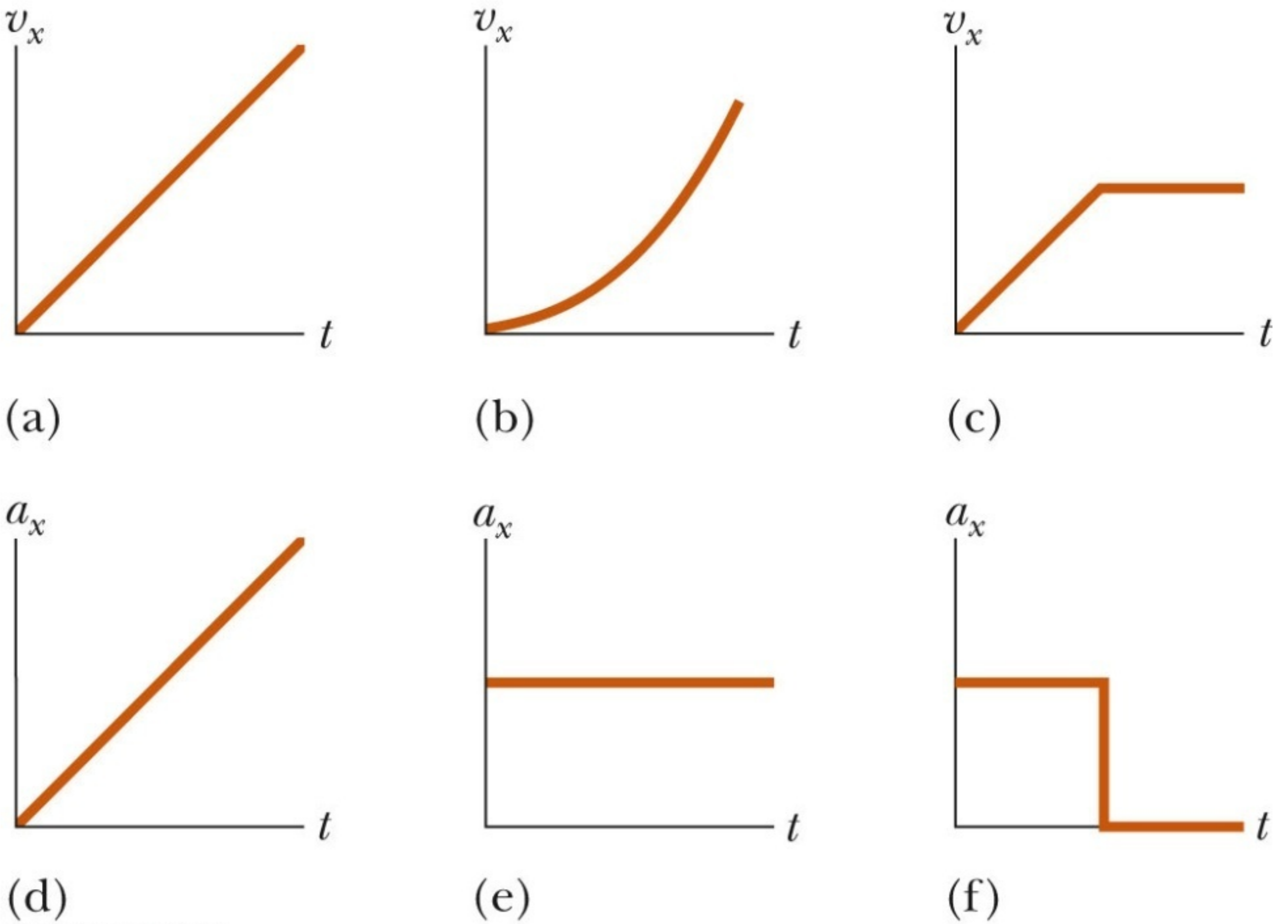
gives $2a(x - x_0) = (2v_0 v - 2v_0^2) + (v^2 - 2vv_0 + v_0^2)$ Cancel and collect terms

gives $2a(x - x_0) = v^2 - v_0^2$

You may remember these as $\Delta s = ut + \frac{1}{2}at^2$ and $v^2 - u^2 = 2a\Delta s$

where $\Delta s = s - s_0$ and $v_0 = u$

Quiz: match v with a



More 1D kinematics

Recall the difference between 'Instantaneous velocity' $\frac{dx}{dt}$ and 'average velocity' $\frac{\Delta x}{\Delta t}$

Recall the difference between acceleration $\frac{dv}{dt}$ and velocity $\frac{dx}{dt}$

Q1 For one dimensional motion, draw $x(t)$ and show a case where

- 'Instantaneous velocity' and 'average velocity' are the same
- 'Instantaneous velocity' and 'average velocity' (over an interval) have opposite signs

Q2 On $x(t)$ and a corresponding $v(t)$ graph, show

- A region where acceleration is positive but velocity is negative, and
- Another region where both are positive

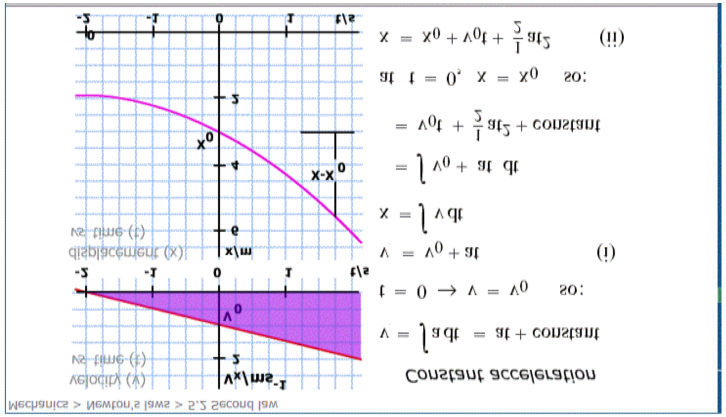
Q3 Object thrown vertically up and then caught (include throw and catch)

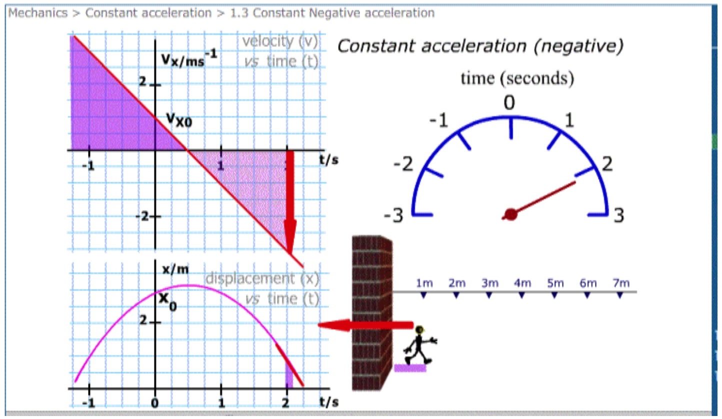
- Where is $|\vec{v}|$ largest? Where is $|\vec{a}|$ largest? (Draw graphs)

If in trouble see

- the web stream videos and quiz on Moodle
- the Constant Acceleration multimedia chapter in Physclips
- Introduction Calculus, a supporting page in Physclips

www.animations.physics.unsw.edu.au





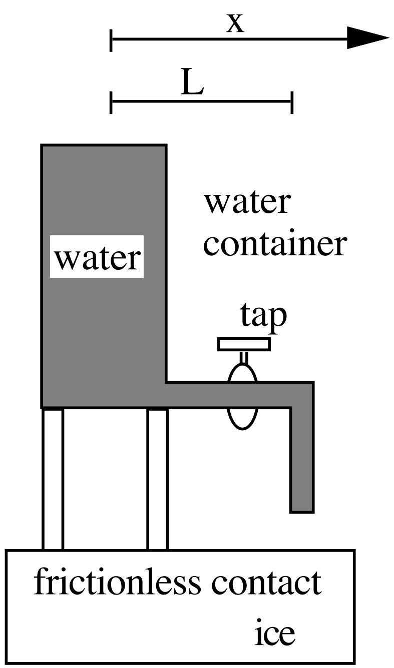
Questions to keep you thinking:

1. Existential question: Why are you here (studying physics)?

2. Fundamental question. Consider $\underline{\mathbf{F}} = m\underline{\mathbf{a}}$

i) Is this a law of physics? ii) Is it a definition of force? (If not, what is?)

iii) Is it a definition of inertial mass? (If not, what is?) *Can it be more than one of these?*



3. Challenge question. A tank (mass M, v= 0) on a frictionless surface contains water (mass m). Distance L from centre of mass of container to pipe. The pipe is small.

i) Open tap. When all the water has left, where is the tank (x = ?) and which way is it moving? (v = ?) Show a clear, quantitative derivation, (no fluid dynamics is required).

ii) *Explain* your answer in terms of forces on the tank.

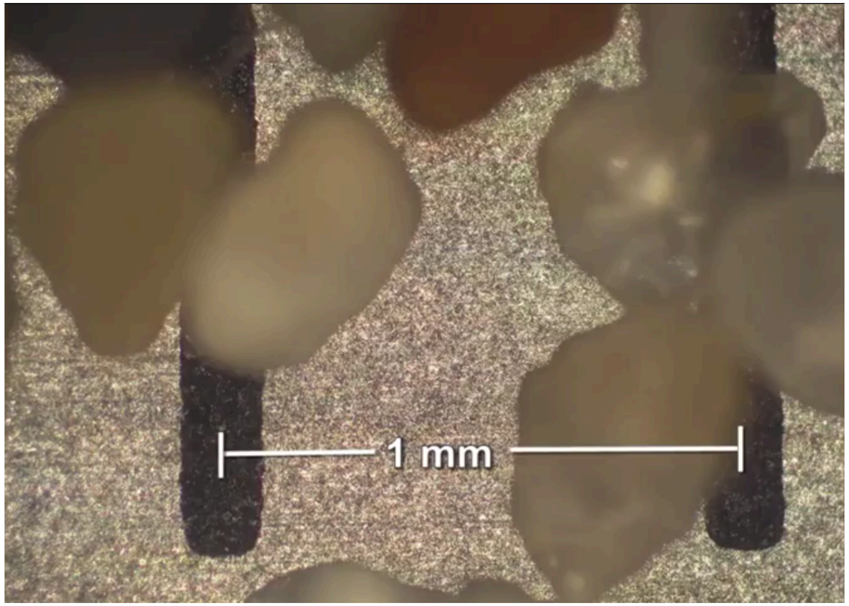
Difficult, so there's a chocolate **prize** for the first clear answer.

(If the lectures go too slowly for you, work on this problem.)

4. Estimation question. How many grains of sand on a beach? How many atoms in a grain of sand? Which is bigger?

Question. How many grains of sand on a beach? How many atoms in a grain of sand?

Both are order-of-magnitude problems, so rough values will be fine!



(Coogee is close) Let's say 400 * 40 * 3 m.

What do you estimate for a grain?

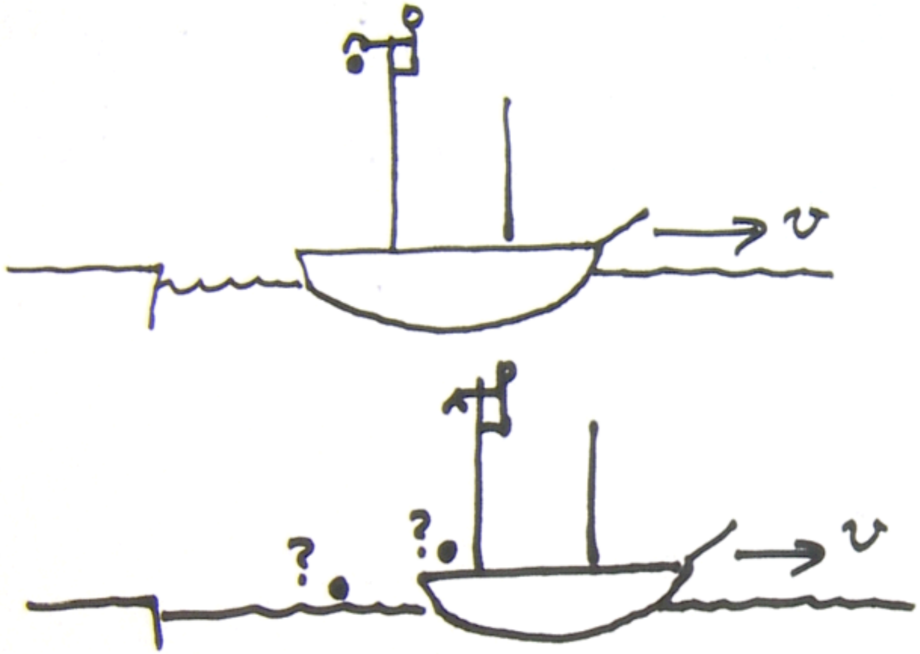
20 grams of sand (SiO_2) contains 6 x 10^{23} atoms*. What about the density of sand? *Guess.* Is it higher or lower than that of water (1 tonne.m^{-3})? Or iron (7 tonne.m^{-3})?

- Silicon: 28 g/mole, Oxygen: 16 g/mole
- See Practice Quiz 1.4 on Moodle for this and several other estimation problems. https://moodle.telt.unsw.edu.au/mod/quiz/view.php?id=1198873

Motion under gravity

Question: Galileo's sailor

drops a mass from the top of the mast of a ship travelling smoothly at $\vec{v}$ = constant (East). Where does the mass land?



Does this tell us the answer?

Galileo's airline passenger

You drop a fork while sitting in a plane travelling at 300 m/s. During the 0.2 seconds it takes to fall 0.2 m, the plane travels 60 m. Does the fork land in your lap or 50 m behind the plane?

There are no instantaneous changes in $\boldsymbol{v}$, because these would be infinite accelerations – prohibited by Newton's second law. Therefore $\vec{v}\,(t)$ curves must be continuous. Therefore position vector curves $\vec{r}(t)$ (and $x(t), y(t)$ etc) must have continuous derivatives.

Question

A lift is ascending at constant v. Its ceiling is h above its floor. At $t = 0$, a screw falls out of the ceiling. When does the screw hit the floor?

Draw a graph of $y(t)$ (showing $t < 0$ as well as $t > 0$) for

- the floor of the lift
- the ceiling of the lift
- the screw

How to solve?

(This is a problem-solving example in the homework book)

- **Vectors, components and unit vectors**
- **Relative motion**

Vectors have direction and magnitude

e.g. displacement, velocity, acceleration, force, spin, electric field...

2 m towards door; 3.7 ms^{-1} at 31° East of North,

– 9.8 ms^{-2} up, 4 N in +ve x direction etc

(compare Scalars: mass, length, heat, temperature...)

Notation: $\vec{a}$ or $\utilde{a}$ or $\underline{a}$ or when hand writing

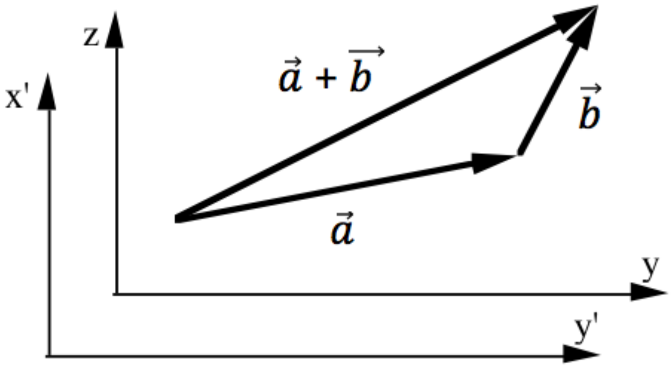
$\vec{a}$ or $\boldsymbol{a}$ in text books

Question:

Which are which

Power? Gravitational field? Sound level? Weight? North?

Addition



put them head to tail to add.

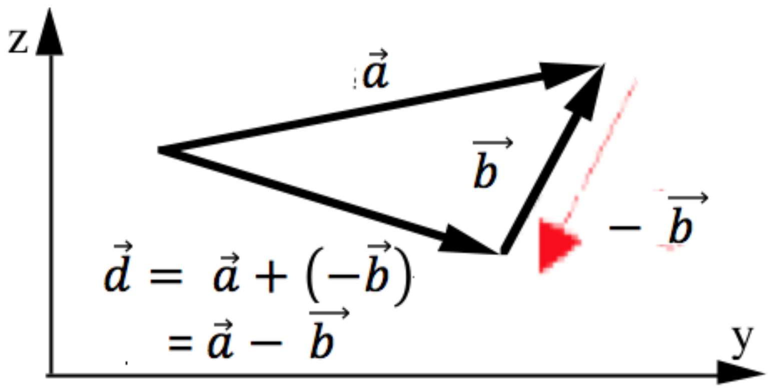
Question:

Does $\vec{a} + \vec{b} = \vec{b} + \vec{a}$?

Subtraction

to subtract, rewrite the equation:

$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$



$$\vec{d} = \vec{a} + \left(-\vec{b}\right) = \vec{a} - \vec{b}$$

(*put them head to head to subtract*)

What do you get when you multiply a vector by a scalar? Some examples:

$$3\vec{r}$$

$$\vec{F} = m\vec{a}$$

Questions

What is the magnitude of 2 metres North?

What direction is (2 metres) North?

What direction is North?

How big is North?

What sort of quantity is North?

North is an example of a unit vector.   Think of 2 metres North

North has magnitude 1. (Not 1 m. Just 1, the number)

It means "in the positive direction of North"

Question: What is North – East (*i.e.* North minus East) ?

We already have a word for "in the positive direction of North" (It's just "North").

Wouldn't it be useful to have a shorthand for "in the positive x direction"

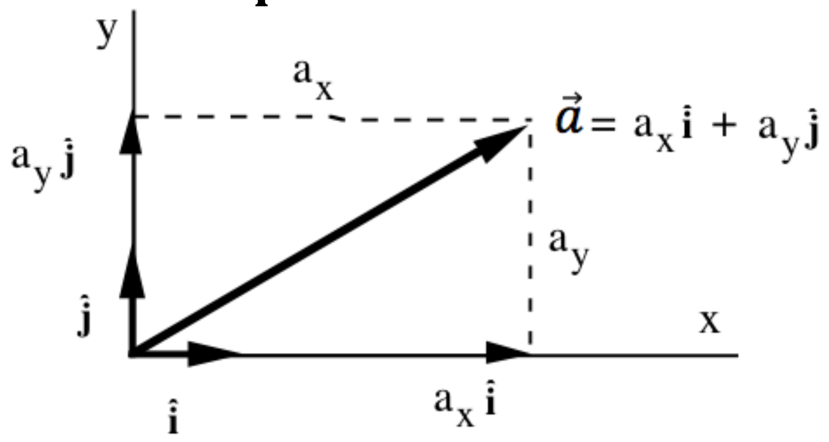
$\hat{\mathbf{i}}$ is a unit vector in the x direction (sometimes written **i**)

$\hat{\mathbf{j}}$ is a unit vector in the y direction (sometimes written **j**)

$\hat{\mathbf{k}}$ is a unit vector in the z direction (sometimes written **k**)

Now we can do useful algebra with vector components

Vector components and unit vectors



$a_x = a \cos\theta$, $a_y = a \sin\theta$

a_x is the **component** of $\vec{a}$ in the x direction: a_x is a *scalar*

remember $\hat{\mathbf{i}}$ is **unit vector**: magnitude of 1 in x direction.

$\vec{a} = a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}}$

$a = \sqrt{a_x^2 + a_y^2}$ and $\theta = \tan^{-1}\frac{a_y}{a_x}$

Addition by components

$$\vec{c} = \vec{a} + \vec{b} = (a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}}) + (b_x \hat{\mathbf{i}} + b_y \hat{\mathbf{j}})$$

$$c_x \hat{\mathbf{i}} + c_y \mathbf{j} = (a_x + b_x) \hat{\mathbf{i}} + (a_y + b_y) \hat{\mathbf{j}}$$

$c_x = a_x + b_x$ and $c_y = a_y + b_y$

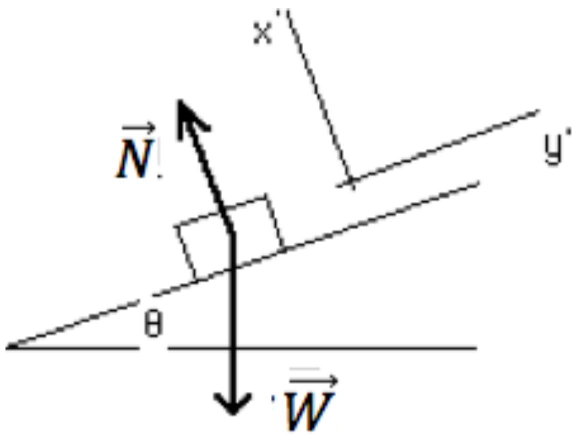
A vector equation in *n* dimensions gives *n* independent scalar equations

important case: $0 = 0 \hat{\mathbf{i}} + 0 \hat{\mathbf{j}}$

if $\vec{a} + \vec{b} = 0$, *e.g.* mechanical equilibrium

then $a_x + b_x = 0$ ***and*** $a_y + b_y = 0$

Resolving vectors is often useful.
Choose ***convenient*** axes:



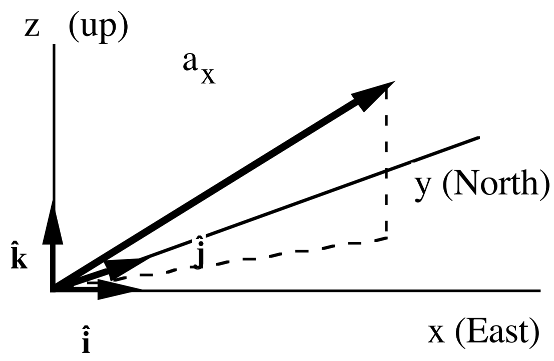
Component of $\vec{W}$ in direction of plane = – $W \sin\theta$

Total force in y' direction = –W sinθ $\left(= m\frac{d^2y'}{d^2x'}\right)$

Components in the normal direction add to zero:

$$N - W\cos\theta = 0$$

In three dimensions:

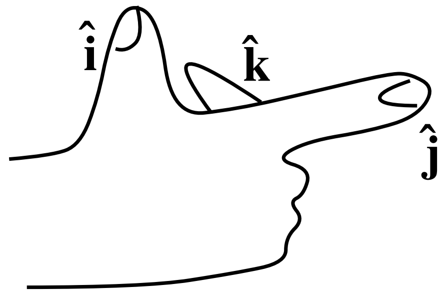


$\vec{r} = \quad r_x\,\hat{\mathbf{i}} + r_y\,\hat{\mathbf{j}} + r_z\,\hat{\mathbf{k}}$ (*sometimes just* **i, j, k**)

right hand convention:

$\hat{\mathbf{i}}$ $\hat{\mathbf{j}}$ $\hat{\mathbf{k}}$ in dir^{n} of

thumb index middle fingers of right hand

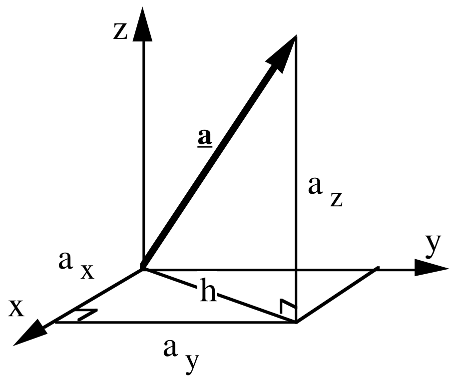


Do and discuss with your neighbours

Using the coordinates and origin near the front bench, estimate the position of your head to a precision of 1 m or so. Write:

$\vec{r}_{\text{you}}$ = $(\ldots)\,\mathbf{i} + (\ldots)\,\mathbf{j} + (\ldots)\,\mathbf{k}$

Pythagoras' theorem in three dimensions



What is magnitude of $\vec{a}$?

Hypotenuse h:

$$h^2 = a_x^2 + a_y^2.$$

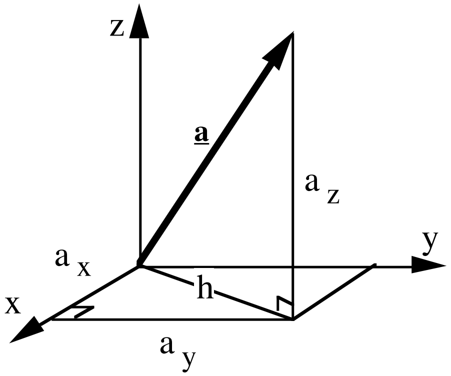
Now look at triangle h, a_z, a:

$$a^2 = h^2 + a_z^2$$

$$a^2 = a_x^2 + a_y^2 + a_z^2$$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Pythagoras' theorem in three dimensions



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$$a^2 = a_x^2 + a_y^2 + a_z^2$$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

In four dimensions (not in our syllabus):

write $j = \sqrt{-1}$

events at (x_1, y_1, z_1, jct_1) *and* (x_2, y_2, z_2, jct_2) *separated by*

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - (ct_2 - ct_1)^2}$$

Question A bus travels North at 100 kph. A car overtakes, travelling North at 110 kph.

i) In the frame of the bus, how fast does the car appear to be travelling and in what direction?

ii) How did you solve this?

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$$\vec{v}_{\text{car with respect to ground}} = \vec{v}_{\text{car with respect to bus}} + \vec{v}_{\text{bus with respect to ground}}$$

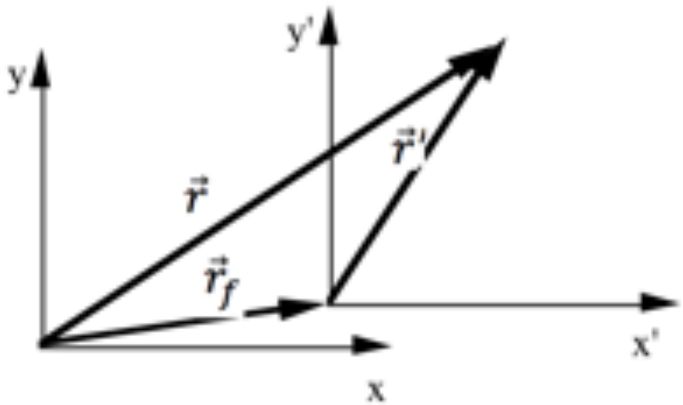
$$\vec{v} = \vec{v}' + \vec{v}_{\text{frame}}$$

$$\vec{v} = \vec{v}' + \vec{v}_{\text{f}}$$

velocity over ground = velocity in frame of refernce + velocity of frame over ground

Relative motion and velocities (*Galilean/ Newtonian relativity: watch for hidden assumptions*)

Origin of frame (x',y') is at $\vec{r}_f$ and moves with $\vec{v}_f$ with respect to (x,y) frame.



$$\vec{r}\,' = \vec{r} - \vec{r}_f$$

$$\vec{v}\,' = \frac{d}{dt}\vec{r}$$

$$= \frac{d}{dt}\vec{r} - \frac{d}{dt}\vec{r}_f$$

$$= \vec{v} - \vec{v}_f$$

Or $\vec{v} = \vec{v}_f + \vec{v}\,'$

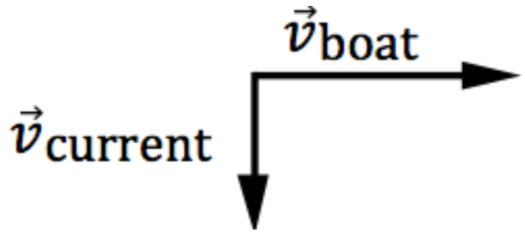
Think: $\vec{v}_{\text{over the ground}} = \vec{v}_{\text{river over ground}} + \vec{v}_{\text{you over river}}$

Question: Explain this to your neighbor:

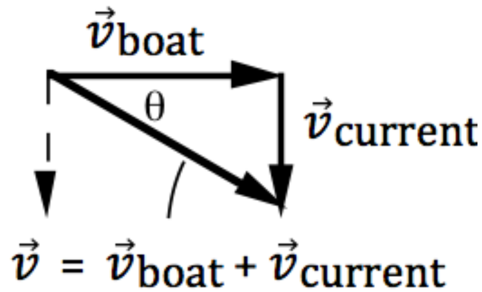
Bicyclist says “you rarely get a tailwind and you never get a tailwind downhill”

Sailor says: “The 18 foot skiffs are always sailing upwind”

Example A boat heads East at 8 km.hr^{-1}. The current flows South at 6 km.hr^{-1}. What is the boat's velocity relative t the earth?



$\vec{v} = \vec{v}_{boat} + \vec{v}_{current}$
To add the vectors, draw them head-to-tail.



magnitude: $v = \sqrt{v_b^2 + v_c^2}$

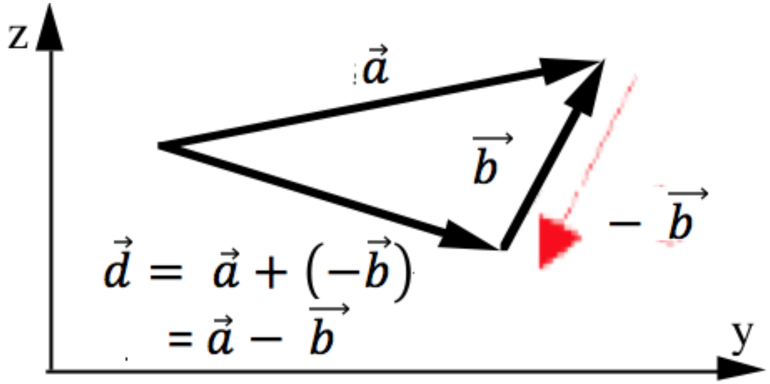
$$= \sqrt{(8\ \text{km.hr}^{-1})^2 + (6\ \text{km.hr}^{-1})^2} = 10\ \text{km.hr}^{-1}$$

direction: $\theta = \tan^{-1} \dfrac{6\ \text{km.hr}^{-1}}{8\ \text{km.hr}^{-1}} = \tan^{-1} 0.75 = 37°$

Answer: 10 km.hr^{-1} at ~~37°~~ 40°South of East (remember significant figures)

Vector subtraction

Draw the vectors head to head to subtract them.



Either draw vectors head to head or write $\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

More on vectors on Physclips and on practice quiz 1.3 and 1.5

Example A sailor wants to travel East at 8 km.hr^{-1}. The current flows South at 6 km.hr^{-1}. What direction must she head, and what speed should she make relative to the water?

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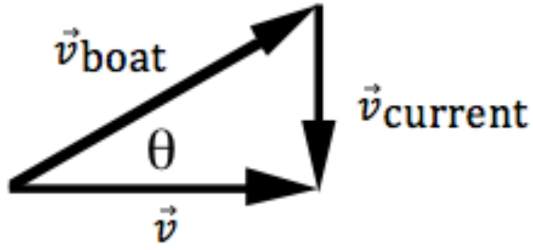


$\vec{v} = \vec{v}\text{boat} + \vec{v}\text{current}$

$\vec{v}\text{boat} = \vec{v} - \vec{v}\text{current}$

To subtract the vectors, draw them head-to-head.

magnitude: $v_b = \sqrt{v^2 + v_c^2}$

$$= \sqrt{(8\text{ km.hr}^{-1})^2 + (6\text{ km.hr}^{-1})^2} = 10\text{ km.hr}^{-1}$$

direction: $\theta = \tan^{-1}\frac{\text{v}_\text{c}}{\text{v}} = \tan^{-1}\frac{6\text{ km.hr}^{-1}}{8\text{ km.hr}^{-1}} = 37°$

She must head 40° North of East and travel at 10 km.hr^{-1} with respect to the water.

Question. A South-East wind* blows at 30 km/hr.

If you are travelling North, how fast must you travel before the wind is coming exactly from your right?

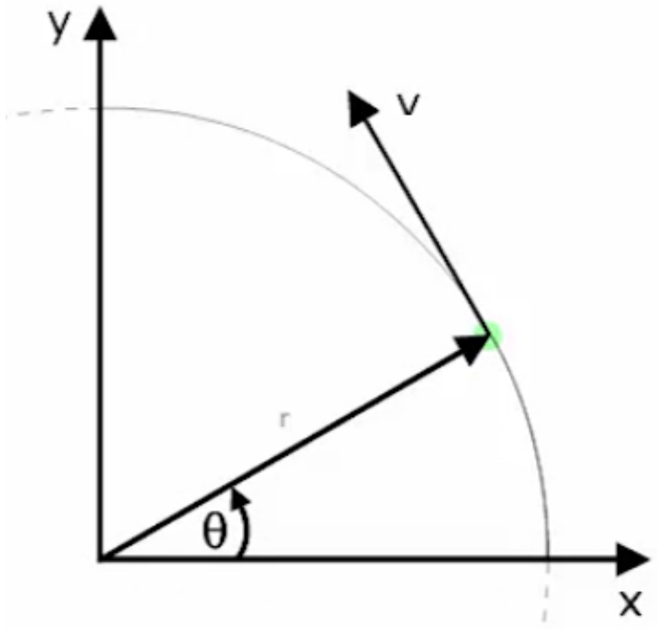
How fast is the apparent wind?

Draw a diagram to set this up

* *A South-East wind is one that comes* ***from*** *the South-East.*
Its direction is ∫the North-West.

More kinematics

- **Circular motion**
- **Projectile motion**



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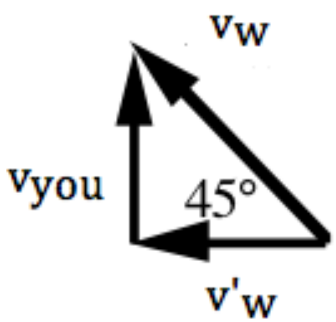
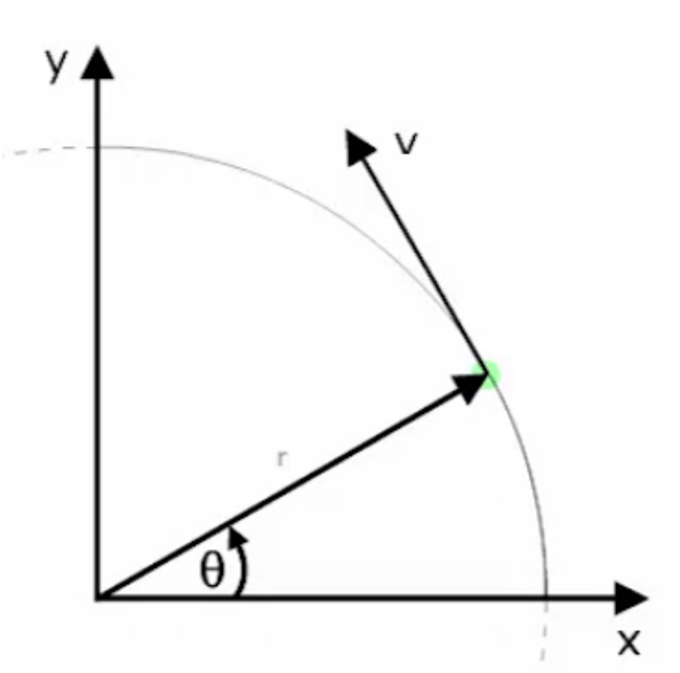


In the North direction: $v_{you} = v_W \sin 45° = 21$ km/hr

In the East direction: $v'_W = -v_W \cos 45° = 21$ km/hr from E

Uniform circular motion



Write $\theta = \omega t$ where ω is the **angular speed** ω = constant for *uniform* circular motion

Don't confuse w *(double U) and* ω *(Greek omega)*

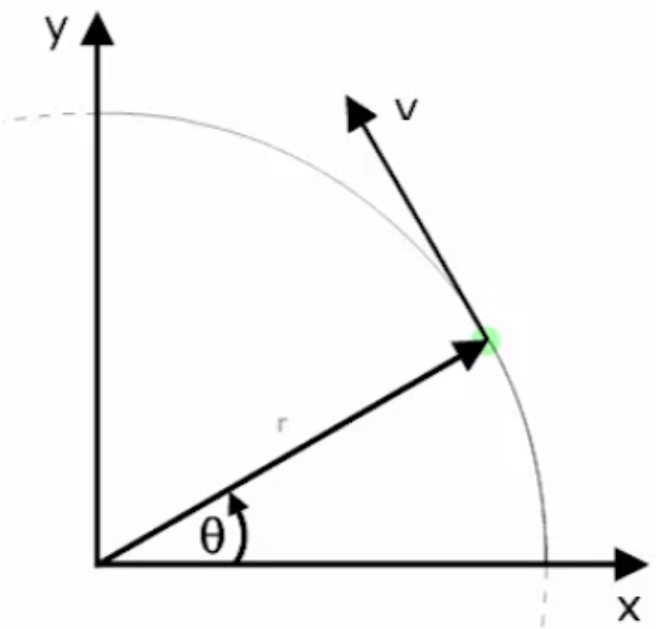
Questions

The period for one circle is T.

- What is v in terms of r and T ?
- What is ω in terms of T ? (answer in radians per second)
- Eliminate T to relate the other variables.

Uniform circular motion

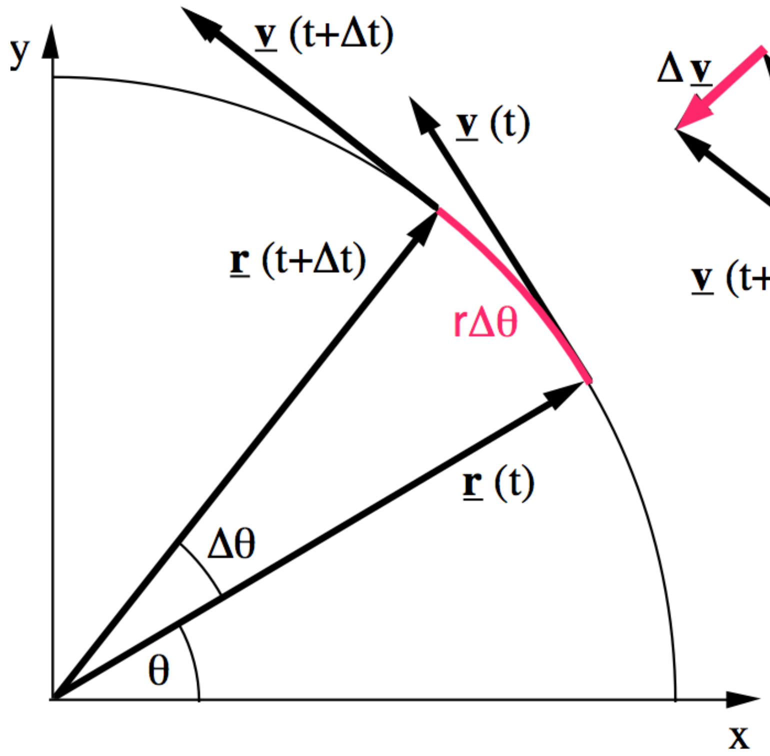
Write $\theta = \omega t$, ω is the **angular speed** (*constant for now*)

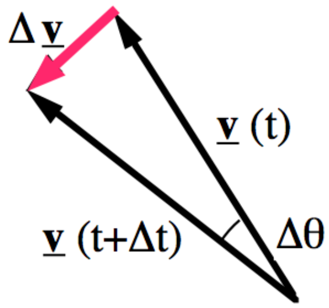


Suppose the period of one circle is T.

- What is v in terms of r and T? $v = 2\pi r/T$
- What is ω in terms of T? $\omega = 2\pi/T$
- Eliminate T to relate the other variable. $v = r\omega$. $\omega = v/r$

Now let's get the speed and acceleration from vector subtraction





(*Make the triangle very thin*)

As Δt and $\Delta\theta$ -> 0, $\Delta\vec{v}$ –> right angles to $\vec{v}$

So $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$ is parallel to $-\vec{r}$

Acceleration towards the centre:

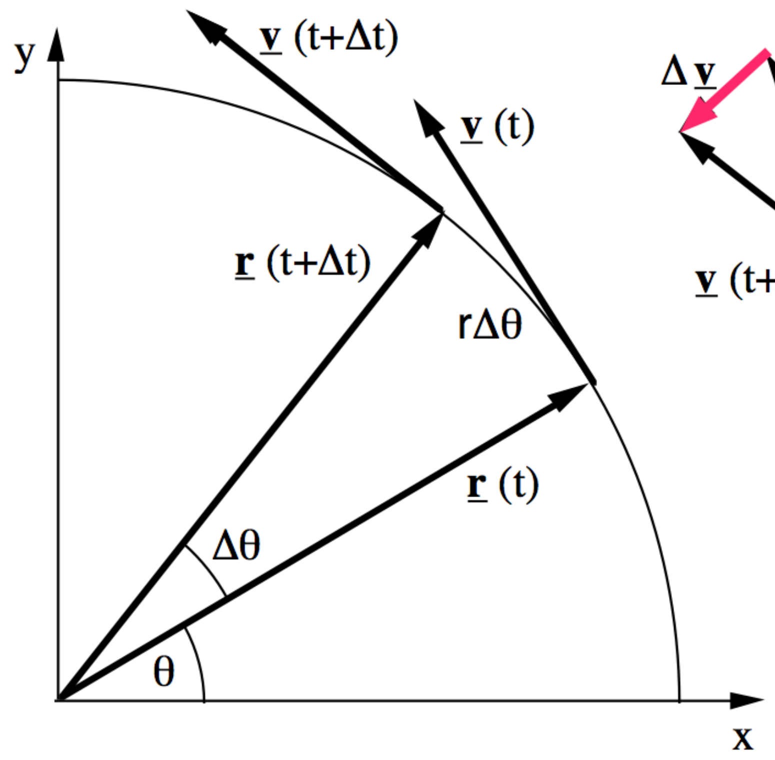
centripetal acceleration

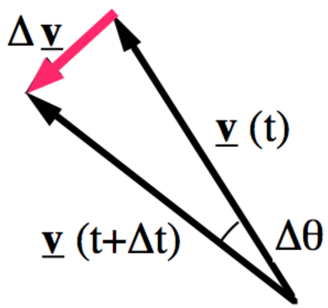
= –1*radial acceleration

Now let's get the magnitude of $\vec{v}$ and $\vec{a}$:

$\Delta s = r\Delta\theta$ (*definition of angle*)

so $v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$ as before





Use arc ~ straight line of triangle, for $\Delta\vec{v}$

$$|\Delta\vec{v}| \cong v\Delta\theta$$

Interesting. Note that $|\Delta\vec{v}| \neq \Delta|\vec{v}|$

In the limit as $\Delta t \to 0$, $\quad dv = v d\theta$

For acceleration: $\qquad |\vec{a}| = \dfrac{|d\vec{v}|}{dt} = v\dfrac{d\theta}{dt} = v\omega$

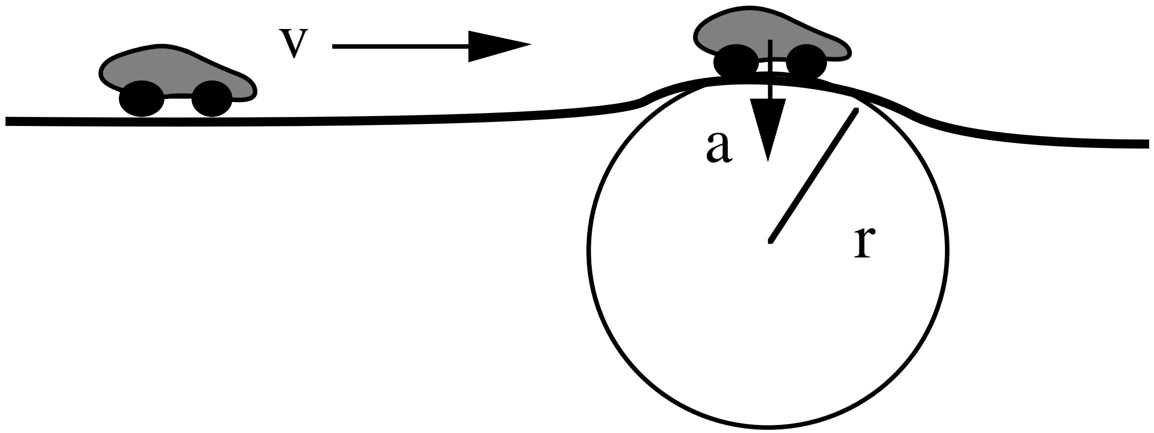
Before, we had $v = r\omega$ $\quad$ so

$$a = \frac{v^2}{r} = \omega^2 r$$

but $\vec{a}$ is towards the centre, parallel to $-\vec{r}$,

so $\qquad \vec{a} = -\omega^2\vec{r} \qquad$ (*vector version*)

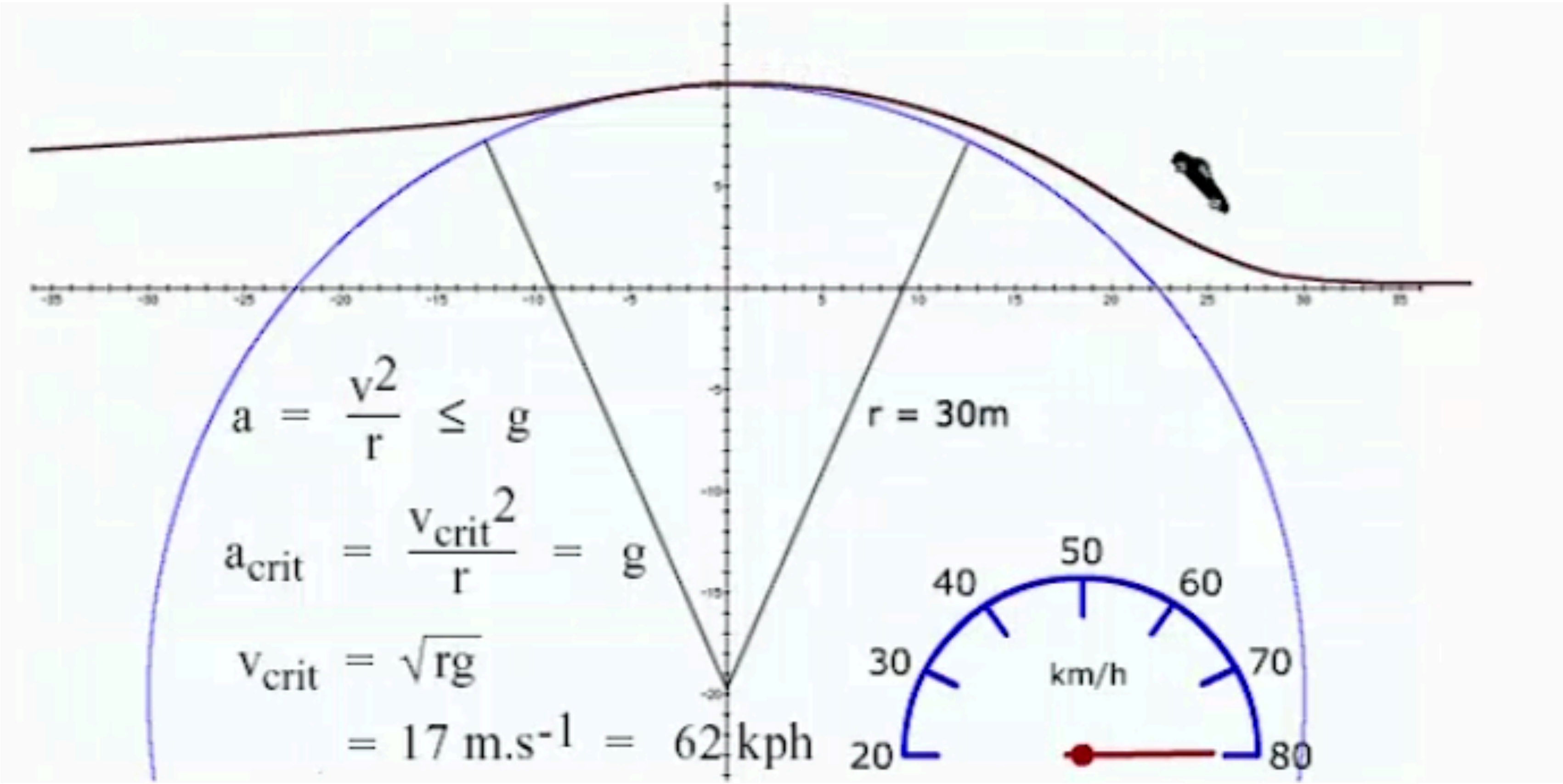
Example. A car travelling at v goes over a hill with vertical radius r = 30 m (>> height of car) at summit. Assume it doesn't slow down. How high must v be for the car to lose contact with the ground at the summit?



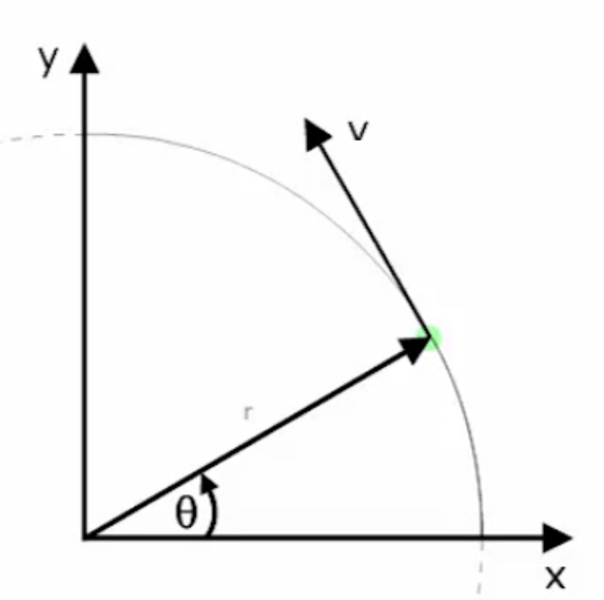
If gravity is only downwards force, then it can't accelerate downwards at greater than g.

if $a_{\text{centrip}} > g$, g is not enough acceleration for circular motion (projectile motion instead).

http://www.animations.physics.unsw.edu.au/mechanics/chapter3_circularmotion.html#3.4


$a = \frac{v^2}{r} \leq g$
$a_{crit} = \frac{v_{crit}^2}{r} = g$
$v_{crit} = \sqrt{rg}$
$= 17\ m.s^{-1} = 62\ kph$
r = 30m
km/h
20
30
40
50
60
70
80

Revision quiz



Uniform circular motion, anticlockwise in *x,y* plane, radius *r*, angular velocity ω.

i) $\vec{r} \;=\; ?\,\hat{\mathbf{i}} + ?\,\hat{\mathbf{j}}$

ii) Using this (i), derive $\vec{v} \;=\; ?\,\hat{\mathbf{i}} + ?\,\hat{\mathbf{j}}$ and $\vec{a} \;=\; ?\,\hat{\mathbf{i}} + ?\,\hat{\mathbf{j}} \;=\; ?$

iii) Using (ii), derive $v \;=\; ?$ $\;a \;=\; ?$

Question 2

$|\vec{a}|\underline{\hat{\boldsymbol{a}}} \;=\; ?$

Example What is the acceleration of this theatre?

Due to rotation of the Earth:

$a_{rot} = \omega^2 r$

$= \left(\frac{2\pi}{T}\right)^2 r$ T is rotation period, r is distance from Sydney to axis

$= \left(\frac{2\pi}{23.9 * 3600\ \mathrm{s}}\right)^2 (5.3 * 10^6 \mathrm{m})$

$= 0.028\ \mathrm{m.s^{-2}}$

Due to Earth's orbit:

$a_{\mathrm{orb}} = \left(\frac{2\pi}{T}\right)^2 r$ *(T_{orbit}, r_{orbit})*

$= \left(\frac{2\pi}{365.24 * 24 * 3600\ \mathrm{s}}\right)^2 (1.5 * 10^{11} \mathrm{m})$ $= 7\ \mathrm{mm.s^{-2}}$

Due to Sun's orbit around galactic centre: $a_{\mathrm{galactic}} \sim 0.1\ \mathrm{nm.s^{-2}}$

Aristotle: if the Earth were moving, we'd feel it.

Example

$\vec{r} = (A \cos \omega t)\,\hat{\mathbf{i}} + (A \sin \omega t)\,\hat{\mathbf{j}} + (Bt)\,\hat{\mathbf{k}}$

where A and B are constants.

What is $\vec{a}$? What shape is $\vec{r}$?

Example

$\vec{r}$ = $(A \cos \omega t)\,\hat{\mathbf{i}} + (A \sin \omega t)\,\hat{\mathbf{j}} + (Bt)\,\hat{\mathbf{k}}$

where A and B are constants.

What is $\vec{a}$? What shape is $\vec{r}$?

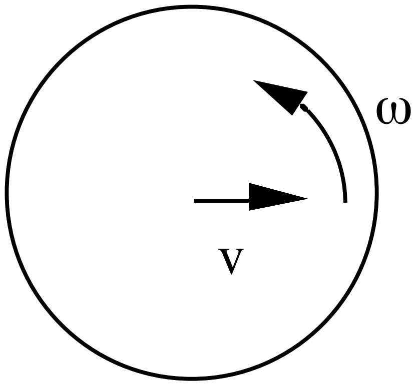
$$\vec{a} = \frac{d^2}{dt^2}\vec{r} \;=\; -(A\omega^2\, sin\ \omega t)\,\hat{\mathbf{i}} \;-\; (A\omega.cos\ \omega t)\,\hat{\mathbf{j}}$$

$\vec{a}$ is in xy plane.

$$a \;=\; \sqrt{a_x{}^2 + a_y{}^2} \;=\; \ldots \;=\; A\omega^2 \;=\; \text{constant}$$

What is the shape?

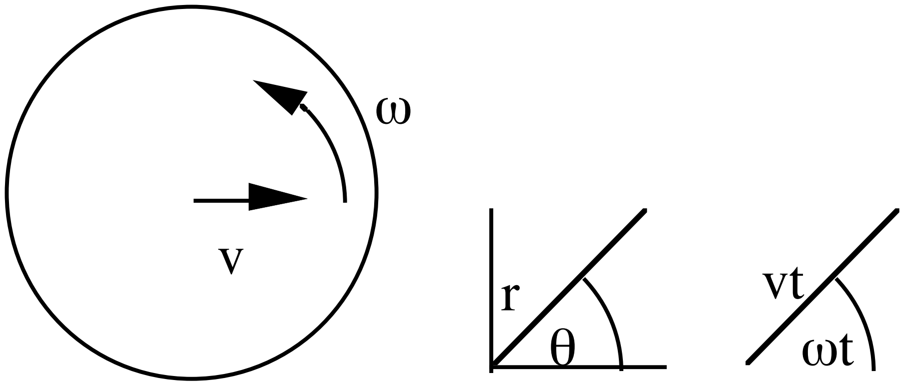
Example A cockroach on a turntable crawls in the radial direction (initially the x direction) at speed v. The turntable rotates at ω anticlockwise. Describe his path in $\underline{\mathbf{i}}$, $\underline{\mathbf{j}}$ coordinates. What is this shape?



$\theta = ?$ $r = ?$

$x = ?,\ y = ?$

Example A cockroach on a turntable crawls in the radial direction (initially the *x* direction) at speed *v*. The turntable rotates at ω anticlockwise. Describe his path in **i**, **j** coordinates. What is this shape?



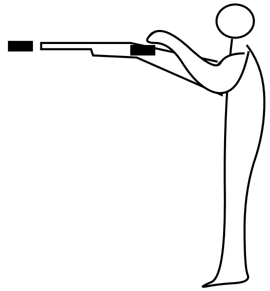
We can specify his position as (r,θ), where $r = vt$, and $\theta = \omega t$.

$x = r \cos \theta, \; y = r \sin \theta$

path is $\underline{\mathbf{r}}(t) = vt \cos \omega t \, \underline{\mathbf{i}} + vt \sin \omega t \, \underline{\mathbf{j}}$

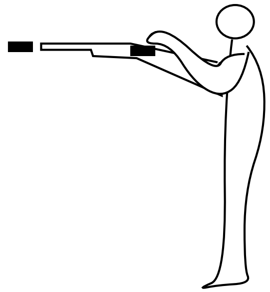
Its shape is ?

Projectiles



Question. A man fires a gun horizontally. At the same time, he drops a bullet. Which hits the ground first? ***Explain your reasoning.***
(To simplify, let's say it happens on flat ground on the moon.)

Projectiles



Question. A man fires a gun horizontally. At the same time he drops a bullet. Which hits the ground first? ***Explain your reasoning.***
(To simplify, let's say it happens on flat ground on the moon.)

Next week: Newton's laws $\Sigma\vec{F} = m\vec{a}$

So a force in y direction does not cause an acceleration in x direction.

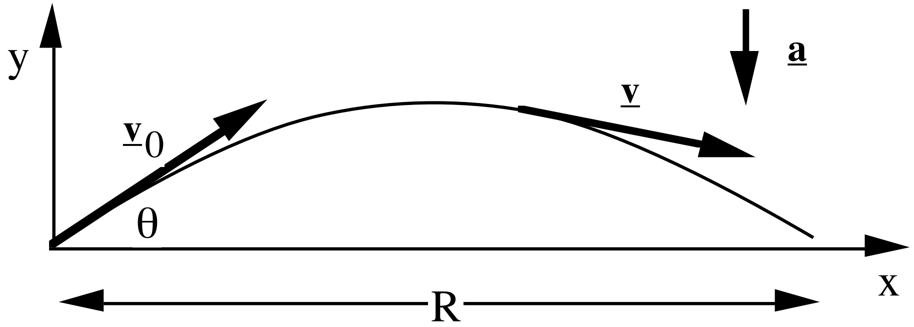
Independence of x and y motion.

When can we neglect air resistance?

We'll be quantitative when we've done work.

Projectiles What is the maximum range?

Find maximum in $R(\theta)$



Without air, $a_y = -g$ = constant. $a_x = 0$

(Galileo: independence of motion)

Strategy: kinematics eqn (ii) gives $y(t)$, $x(t)$:

(ii) –> $y = y_o + v_{yo}t - \frac{1}{2}gt^2$.

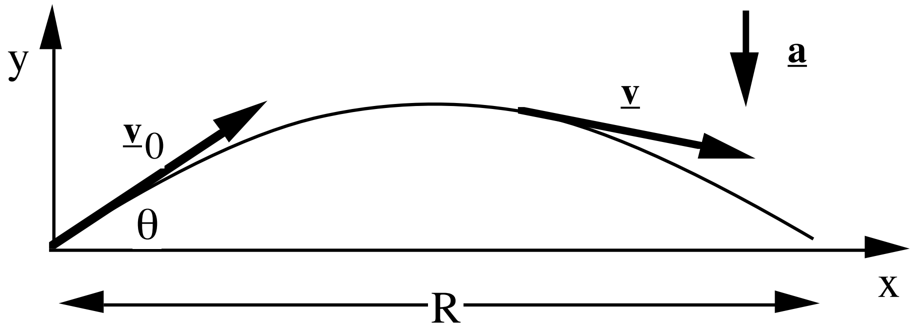
(ii) –> $x = x_o + v_x t + 0$

Eliminate t gives y(x)

Range R defined by altitude $y(R) = 0$

Rearrange to get $R = R(\theta)$

Projectiles What is maximum range?



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(ii) –> $y = y_o + v_{yo}t - \frac{1}{2} gt^2$.

(ii) –> $x = x_o + v_x t$.

Choose axes: $x_o = y_o = 0$ and eliminate t, i.e.
substitute $t = x/v_x$:

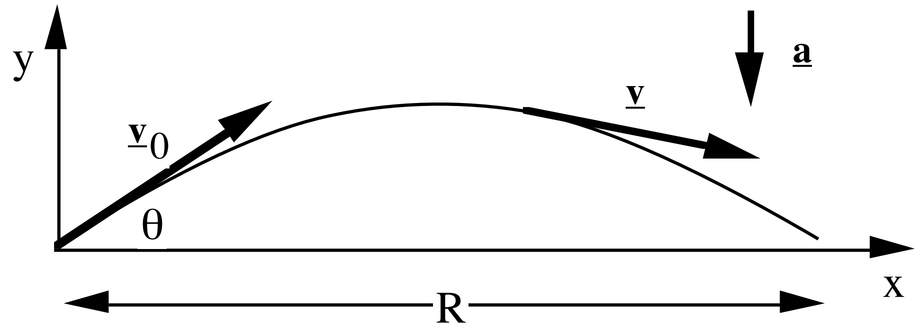
$$y = v_{y0}\left(\frac{x}{v_x}\right) - \frac{1}{2} g \left(\frac{x}{v_x}\right)^2 \qquad (*)$$

For range $y = 0$ when $x = R$

(*) –> $$v_{y0}\left(\frac{R}{v_x}\right) = \frac{1}{2} g \left(\frac{R}{v_x}\right)^2 \qquad (**)$$

solve (**) for R, then find maximum

Projectiles What is maximum range?



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Choose axes: $x_o = y_o = 0$ and eliminate t, *i.e.* substitute $t = x/v_x$ to get

$$y = v_{y0}\left(\frac{x}{v_x}\right) - \frac{1}{2}g\left(\frac{x}{v_x}\right)^2 \qquad (*)$$

For range: $y = 0$ when $x = R$

(*) –> $$v_{y0}\left(\frac{R}{v_x}\right) = \frac{1}{2}g\left(\frac{R}{v_x}\right)^2 \qquad (**)$$

solve (**) for R, then find maximum

cancel R and rearrange:

$$R = \frac{2v_x v_{yo}}{g}$$

Components:

$v_x = v_o \cos\theta$, $v_{yo} = v_o \sin\theta$, so:

$$R = \frac{2v_0 \sin\theta . v_0 \cos\theta}{g}$$

$$= \frac{v_0^2 \sin 2\theta}{g} \qquad (\textit{trigonometric identity})$$

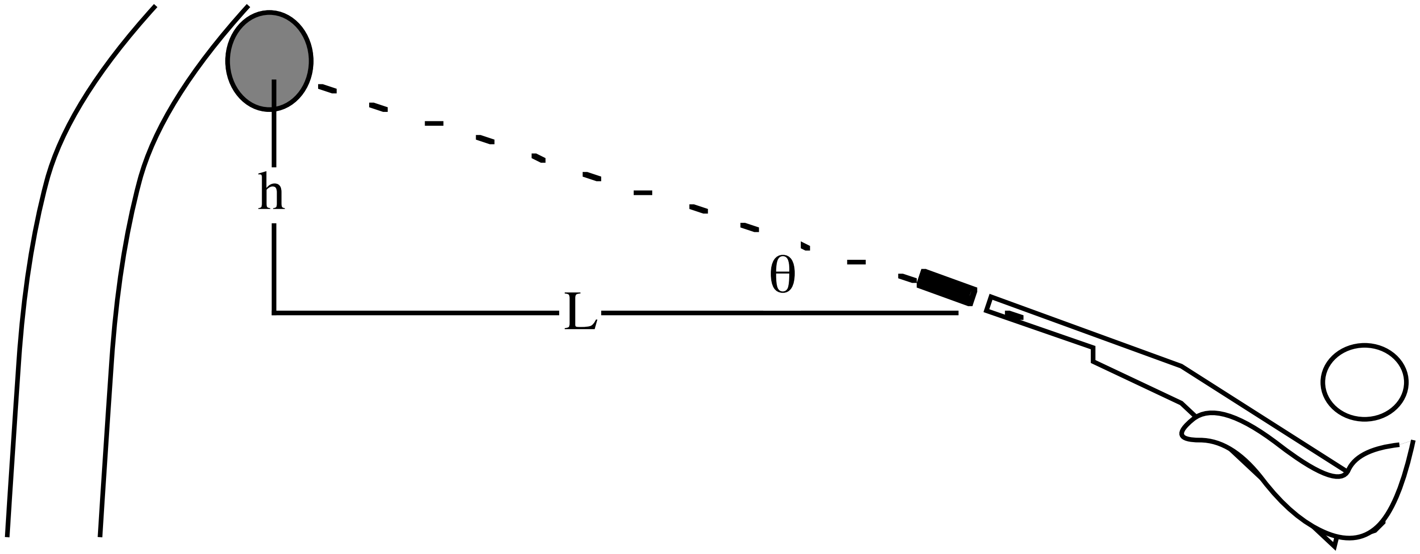
Find maximum in a continuous function:

$$\frac{\partial R}{\partial \theta} = \frac{2v_0^2 \cos 2\theta}{g} = 0$$

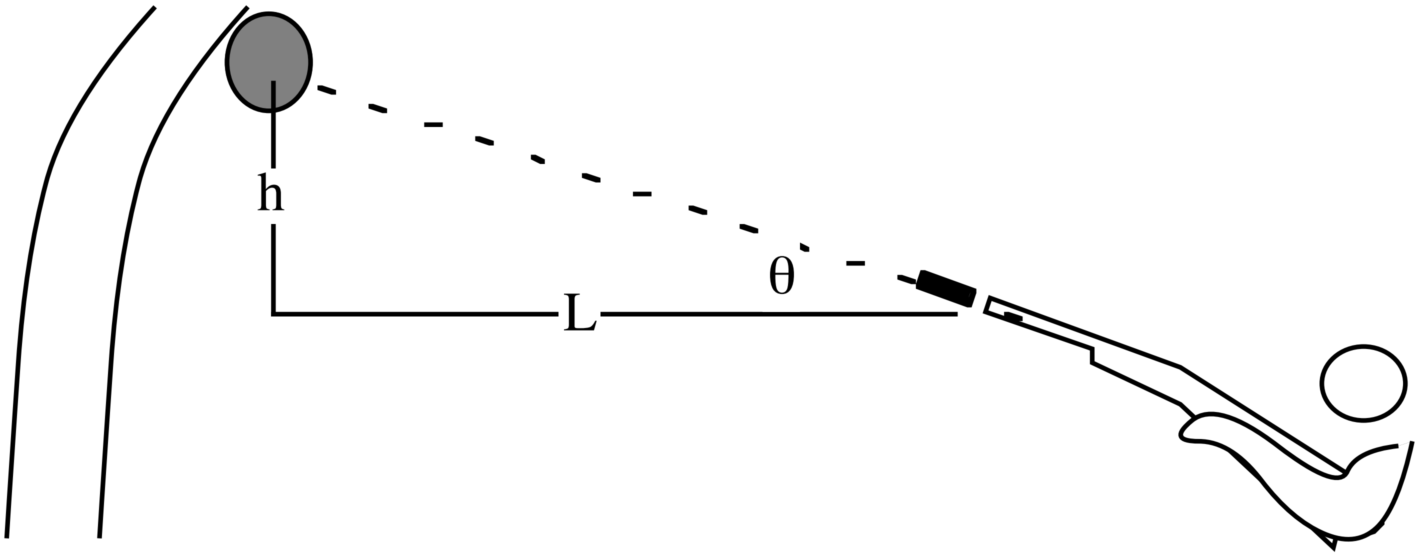
$$\frac{\partial R}{\partial \theta} = 0 \text{ when } 2\theta = 90°$$

so $\theta = 45°$

Question. A man shoots at a coconut*. At the instant that he fires, the coconut falls. Does the bullet (b) hit the coconut (c)?



* *Vegetarian version of the question. In the original, it was a monkey that releases the branch.*



obvious method (c for coconut, b for bullet)

$h = L \tan \theta$ *Will it miss? Let's compare their heights at the same x position:*

coconut $y_c = h - \frac{1}{2} g t^2$ bullet $y_b = v_{yo} t - \frac{1}{2} g t^2$

Difference $y_c - y_b = h - v_{yo} t = h - v_o \sin\theta\, t$ Bullet has no horizontal acceleration, so

$L = t\, v_o \cos\theta$ –> $t = \dfrac{L}{v_o \cos\theta}$ *is the time of flight*

$$y_c - y_b = h - \frac{v_o \sin\theta . L}{v_o \cos\theta} = h - L \tan\theta \qquad = 0$$

video and details at *http://www.animations.physics.unsw.edu.au/jw/monkey_hunter.html*

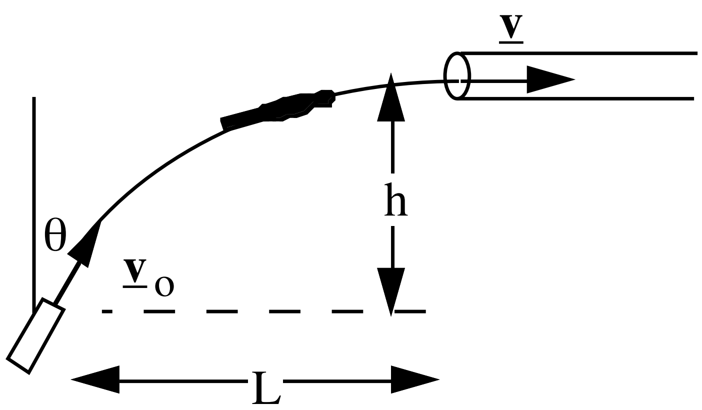
Example. The human cannon of Circus Oz has a muzzle speed v_o. For a proposed trick, they will fire the human canonball into a horizontal teflon tube at height h above the canon mouth. To avoid damage to the canonball, he must arrive with purely horizontal velocity. Calculate the postion of the canon and its angle to the vertical*.

** angle usually given with respect to the horizontal, but here it is declination*

i) draw a diagram

ii) put in symbols for quantities

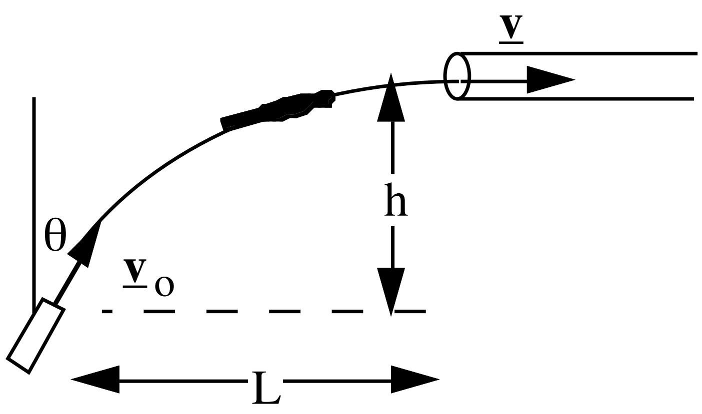
iii) translate the question



v_o is fixed. We must satisfy final h and final $v_y = 0$ (i.e. horizontal motion)

Two constraints, and two variables (θ and L) must be solved to satisfy them.

θ will determine the maximum height.



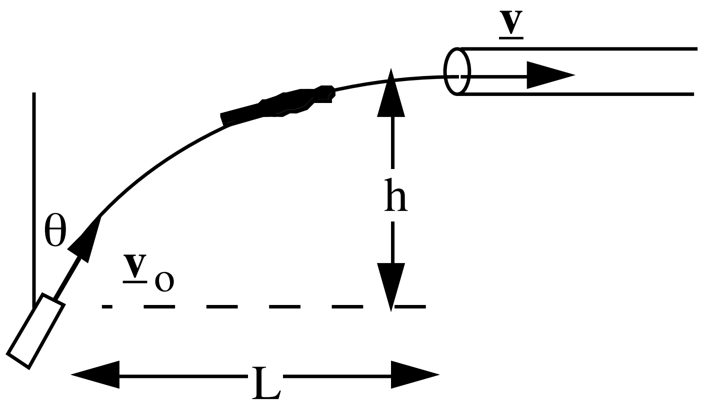
During flight, the acceleration is $-g$ upwards. The desired final v_y is zero, so

$$0 = v_y{}^2 = v_{yo}{}^2 + 2a_y(\Delta y) = v_o{}^2 \cos^2\theta - 2gh$$

$$\therefore \; v_o^2 \cos^2\theta = 2gh$$

$$\therefore \; \cos\theta = \frac{\sqrt{2gh}}{v_o} \qquad \theta = \cos^{-1}\left(\frac{\sqrt{2gh}}{v_o}\right)$$

Now we must find the L required to hit the target. But now we've fixed θ, so we've fixed the time of flight t. So, find t and use it to get L.



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$$0 = v_y^2 = v_{yo}^2 + 2a_y(\Delta y) = v_o^2 \cos^2\theta - 2gh$$

$$\therefore\ v_o^2 \cos^2\theta = 2gh$$

$$\therefore\ \cos\theta = \frac{\sqrt{2gh}}{v_o} \qquad \theta = \cos^{-1}\left(\frac{\sqrt{2gh}}{v_o}\right)$$

Find the time of flight then use $v_x t = L$.

Use $v_y = 0$ at target:

$$0 = v_y = v_{yo} + a_y t$$

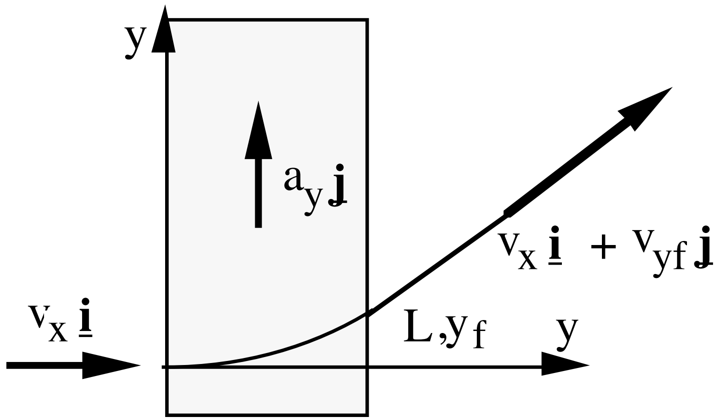
$$\therefore\ t = \frac{v_o \cos\theta}{g}$$

so

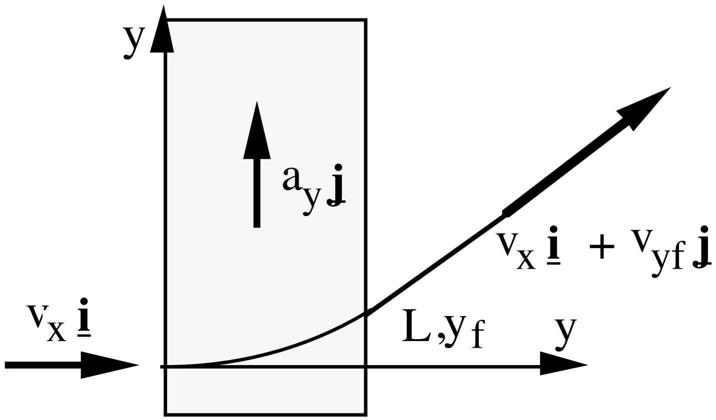
$$L = v_o\, t \sin\theta.$$

$$= v_o \sin\theta\ \frac{v_o \cos\theta}{g}$$

Example. Electron enters a uniform electric field at $(x,y,t)=(0,0,0)$, with velocity $v_x\mathbf{i}$. The field extends from $x = 0$ to $x = L$, but is zero for $x < 0$ and $x > L$. In the field, the electron is accelerated at $a_y\mathbf{j}$. Find $x(t)$, $y(t)$ and the slope of the electron trajectory for $x > L$.



Out of the field, it travels in a straight line. So we need to find the slope of that line and a point on it. First find the point (L,y_f). (Then the slope.)



$x \le 0\ (t \le 0)$ $\qquad\qquad$ $\vec{v} = v_x\boldsymbol{i} =$ constant

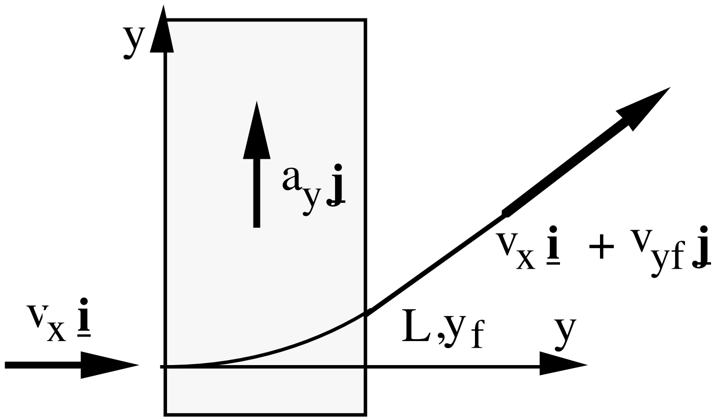
$0 \le x \le L\ (0 \le t \le L/v_x)$

$x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2$ $\qquad\qquad$ $y = y_0 + v_{y0}t + \frac{1}{2}a_yt^2$

this gives us t: $\quad t_f = L/v_x$ $\qquad\qquad$ this gives us $\quad y_f = \frac{1}{2}a_yt_f^2 = \frac{1}{2}a_y\left(\frac{L}{v_x}\right)^2$

$x(t)$ is easy. $\quad x = v_xt$

Find $y(t)$. Say t' is the time since it left the field.



$x \leq 0$ $(t \leq 0)$ $\vec{v} = v_x\boldsymbol{i} =$ constant

$0 \leq x \leq L$ $(0 \leq t \leq L/v_x)$

$x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2$ $y = y_0 + v_{y0}t + \frac{1}{2}a_yt^2$

this gives us *t:* $t_f = L/v_x$ this gives us $y_f = \frac{1}{2}a_yt_f^2 = \frac{1}{2}a_y\left(\frac{L}{v_x}\right)^2$

$v_x = v_{x0} + 0$ (always) $v_y = v_{y0} + a_yt$ *so* $v_{yf} = a_y\frac{L}{v_x}$

$y = y_f + v_{yf}t' = y_f + v_{yf}(t - t_f)$

$= y_f + v_{yf}(t - \frac{L}{v_x})$ and we can substitute for t to have the trajectory.