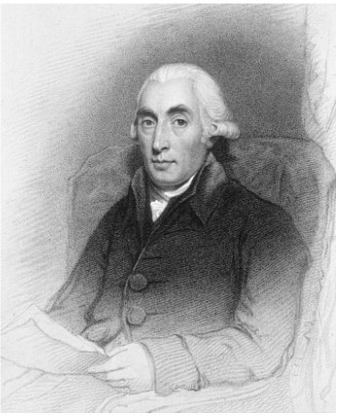
Thermal Physics

Lecture 2 – Pressure, Kinetic Theory, and Ideal Gas

Halliday, Resnick and Walker reference: 18.3, 19.1-19.2

"Under Pressure" – Queen and David Bowie

Last lecture: What is heat?



- Phlogiston theory.
- Joseph Black (1728-1799):
  – Latent heat of melting – adding heat to bucket of ice water doesn't make it warmer! ⇒ heat ≠ temperature
  – Heat *always* flows from hot things to cold things.

- Caloric theory (Antoine Lavoisier, 1770s):
  - ➢ Heat is an invisible fluid, caloric.
    (Like electrical fluid that carries electrical currents)
  - ➢ Amount of caloric is conserved.
  - ➢ You cannot create or destroy caloric – it just moves from hot things to cold ones.
  - ➢ But: How does friction generate heat?
    1798: Count Rumford notices that boring canons makes a *lot* of heat – where is the caloric coming from? (Demo).

- Modern kinetic theory: Heat is vibrations - a form of energy…

Some reading for those suffering from insomnia:

- http://hsm.stackexchange.com/questions/3470/what-are-the-major-flaws-of-the-caloric-theory-of-heat
- http://galileoandeinstein.physics.virginia.edu/more_stuff/TeachingHeat.htm
- Benjamin Count of Rumford, *An inquiry concerning the source of heat which is excited by friction,* Philosophical Transactions of the Royal Society of London, **88**: 80–102 (1798). https://dx.doi.org/10.1098%2Frstl.1798.0006

Last Lecture:

What is temperature?

- Zeroth law of thermodynamics
- Heat flows from hot to cool things
- Temperature scales and absolute zero
- Thermal expansion

$$\Delta L = \alpha L_i \Delta T$$

Linear expansion; for solids

$$\Delta V = \beta V_{\text{ini}} \Delta T$$

Volume expansion;
for solids and liquids

$\beta = 3\alpha$ for solids

This lecture... mainly ideal gases

- Expand on expansion
- Basic concepts:
  - Mole
  - Pressure
- Ideal gas law
- Kinetic theory

Problem

When the temperature of a copper coin is raised by 100^{o}C, its diameter increases by 0.18%. To two significant figures give the percentage increase in (a) the area of the face, (b) the thickness (c) the volume, and (d) the mass of the coin. (e) Calculate the coefficient of linear expansion of the coin.

a) $\frac{\Delta A}{A} = 2\alpha\Delta T$ $\qquad \frac{\Delta d}{d} = \frac{0.18}{100} = \alpha\Delta T$

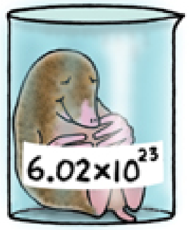
$= 2 \times \frac{0.18}{100} = \frac{0.36}{100} \Rightarrow 0.36\%$

b) $\frac{\Delta t}{t} = \alpha\Delta T = \frac{0.18}{100} \Rightarrow 0.18\%$

c) $\frac{\Delta V}{V} = 3\alpha\Delta T = \frac{3 \times 0.18}{100} \Rightarrow 0.54\%$

d) mass does not change

e) $\frac{0.18}{100} = \alpha \times 100 \Rightarrow \alpha = \frac{0.18}{100^2} = 1.8 \times 10^{-5} K^{-1}$

Basics: Mole...

- A mole of a substance is an Avogadro's number of elementary units of that substance.

$$1\text{mole} = N_{\mathrm{A}} = 6.022 \times 10^{23}\text{units}$$

Avogadro's number

e.g., atoms, molecules

- Examples:
  - 1 mole of O_2 is 6.022 x 10^{23} O_2 molecules.
  - 1 mole of Carbon-12 is 6.022 x 10^{23} carbon atoms.

Mole>Molar mass…

- Molar mass = M = mass per mole

- Put it another way, if I have n mole of a substance X with a total mass m, then

Total mass of X I have

$$n = \frac{m}{M}$$

Number of moles of X

Molar mass of X
(mass per mole)

- What is the molar mass of a substance then?

Mole>Molar mass...

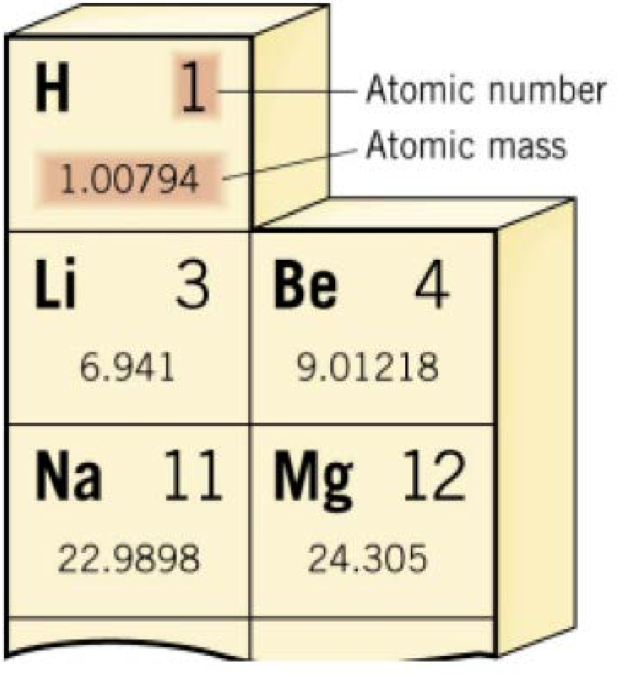
- We will deal mainly with atomic/molecular gases.

– Then, the molar mass of a substance has the same numerical value as the **atomic or molecular mass of that substance in atomic units**.

– Example:

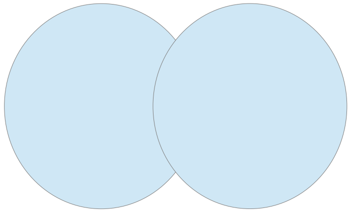
- One Hydrogen atom has mass 1.00794 u.

- The molar mass of atomic Hydrogen is 1.00794 g/mol.



Mole>Molar mass...

- **Caution!**
  - Many gases are in the form of **diatomic molecules**, e.g., hydrogen, nitrogen, oxygen, etc.

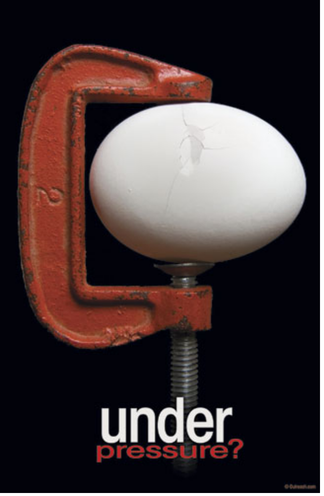


– The elementary unit in these cases is a molecule, and the molar mass is the **molecular mass**.

– Example:

- One mole of atomic Hydrogen has mass 1.00794 g.
- One mole of H_2 gas has mass 2.01588 g.

What about a mole of moles? See http://what-if.xkcd.com/4/



Pressure

Small area=high pressure =PAIN

Force

Pressure

$$P = \frac{F}{A}$$

Area

Can will expand as the force of gas molecules increases.

Pressure in a gas...

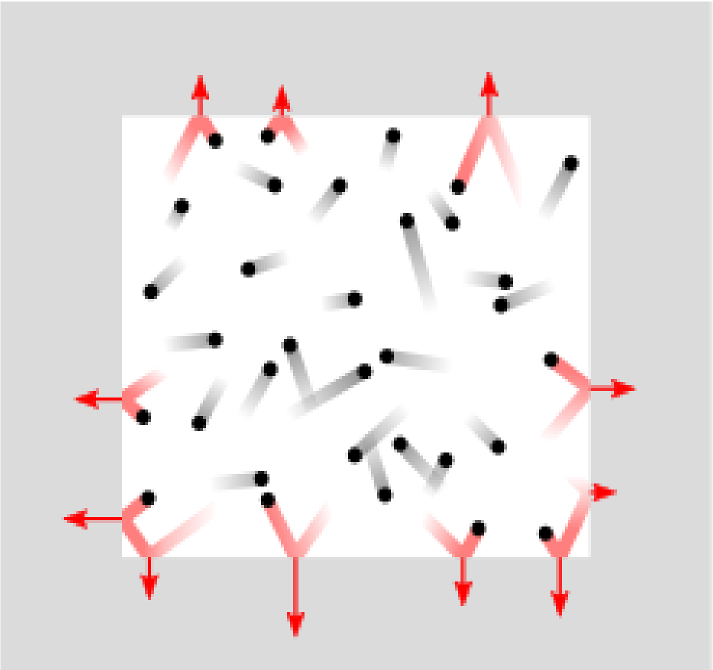
- Microscopically, pressure in a gas is caused by the **collision of gas atoms/molecules** on, e.g., the wall of the container.

– For a fixed volume, the more kinetic energy the atoms/molecules have,

– ⟶ the larger the force,

– ⟶ the larger the pressure.

– We will come back to this later...



Macroscopic properties of a gas...

On the macroscopic level, a gas can be characterised by

– its **temperature T**,

– its **pressure P**, and

– its **volume V**.

How do these properties (P, V, T) relate to one another?

Three empirical relations...

Established by experiments!!

Boyle's law:

Robert Boyle (1627-1691)

- At constant temperature: $P \propto \frac{1}{V}$

Charles's law:

Jacques Alexandre César Charles (1746-1823)

- At constant pressure: $V \propto T$

Demo Unit Hc8: Charles' Law

https://goo.gl/forms/tMTNuzmyWi8DZUMs2

What do you think will happen when liquid nitrogen is poured onto a balloon?

(a) The balloon will expand.

(b) The balloon will contract.

(c) The balloon will become brittle and break.

(d) The balloon will pop.

(e) Nothing will happen.

Three empirical relations...

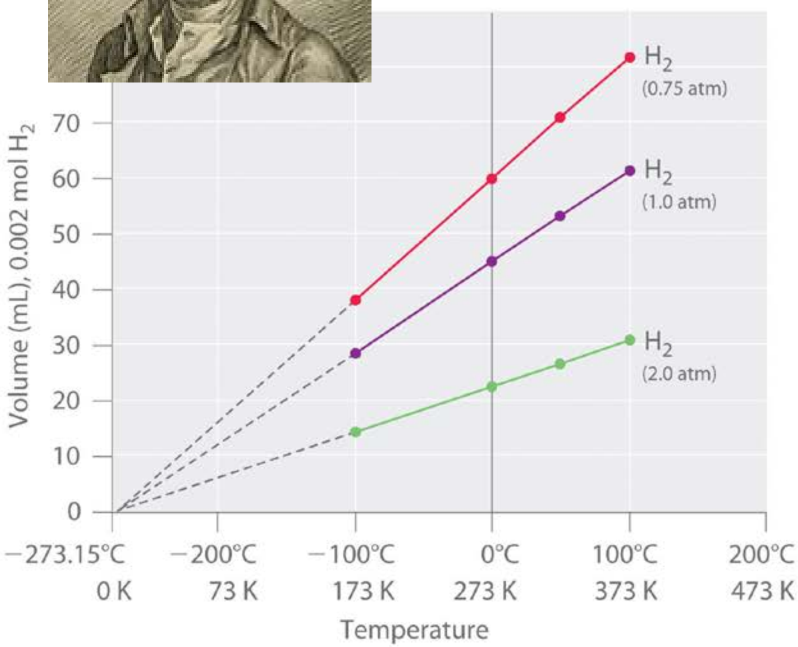
Established by experiments!!

Boyle's law: Robert Boyle (1627-1691)

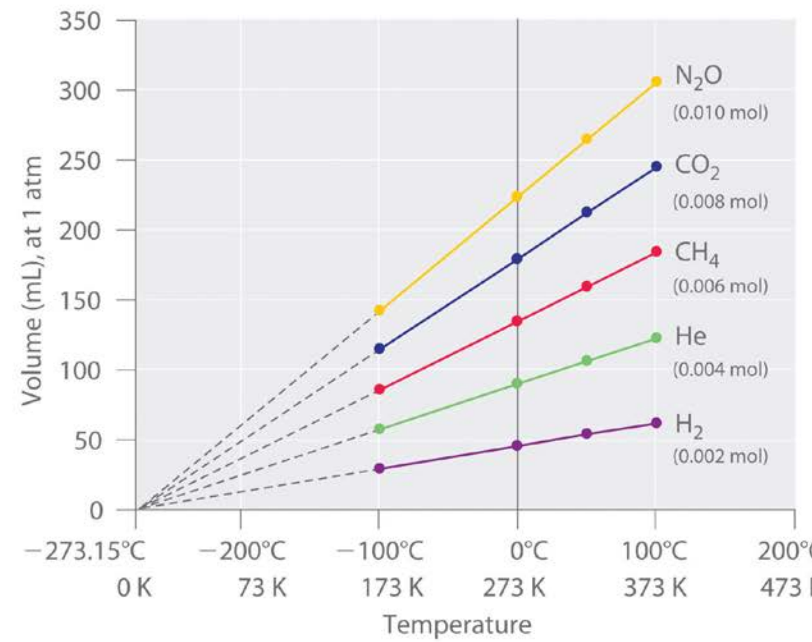
- At constant temperature: $P \propto \frac{1}{V}$

Charles's law: Jacques Alexandre César Charles (1746-1823)

- At constant pressure: $V \propto T$



(a)



(b)

Three empirical relations...

Established by experiments!!

Boyle's law: Robert Boyle (1627-1691)

- At constant temperature: $P \propto \dfrac{1}{V}$

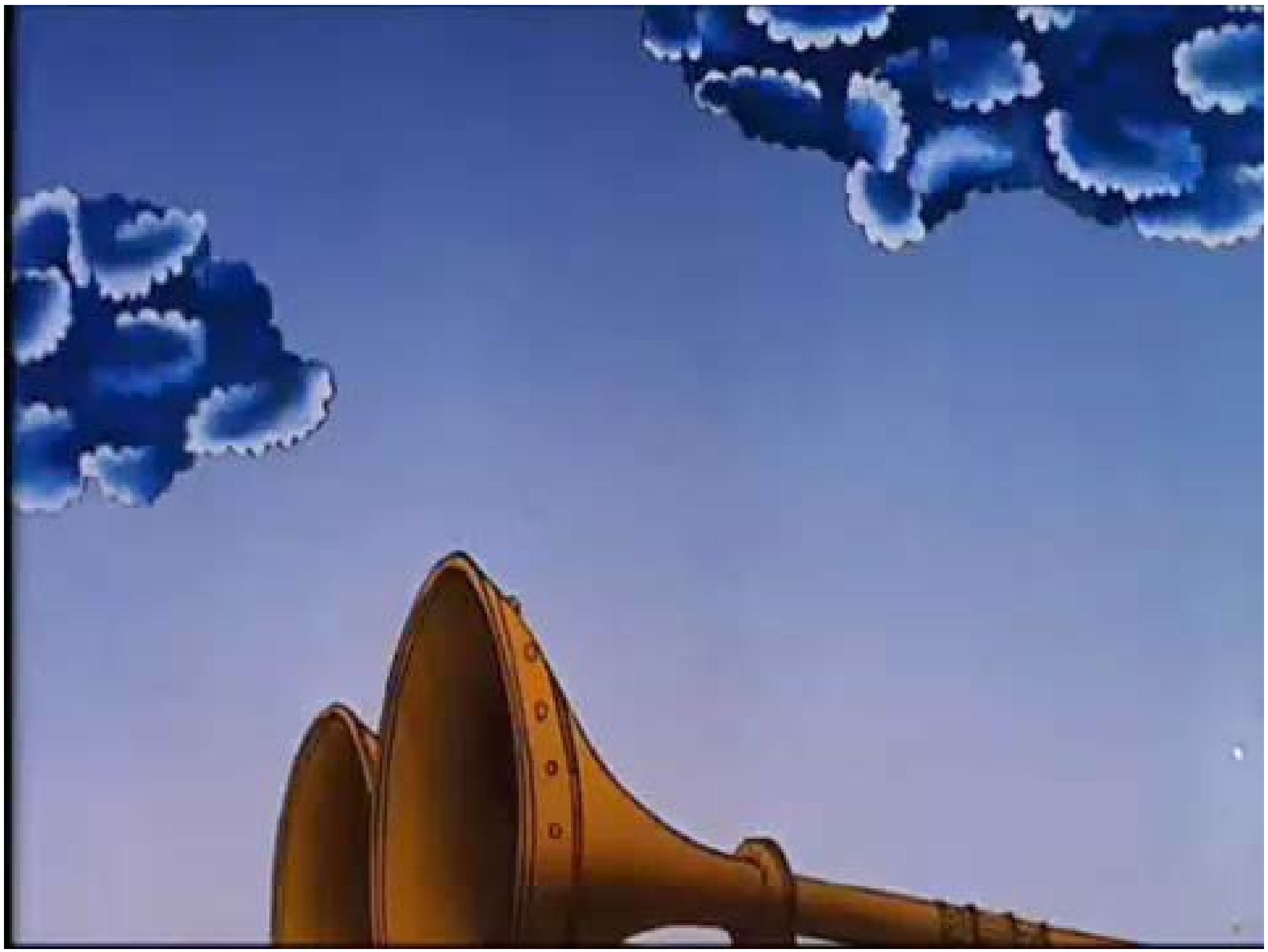
Charles's law: Jacques Alexandre César Charles (1746-1823)

- At constant pressure: $V \propto T$

Gay-Lussac's law: Joseph Louis Gay-Lussac (1778-1850)

- At constant volume: $P \propto T$

Combining these laws, we arrive at the IDEAL GAS LAW



The ideal gas law…

- No need to remember the three empirical relations separately, because they can be combined into one single relation, called the **ideal gas law**:

$$PV = nRT$$

Homework Set 4:

Lab: "Ideal Gas Law" experiment.

– n = number of moles of gas

– R = universal gas constant
  = 8.314 J/mol.K

The ideal gas law in different forms... (solving ideal gas problems)

If you have *n moles*:

$$PV = nRT = \frac{N}{N_A} RT$$

Avogadro's number

If you have *number of molecules* N:

$$PV = N k_B T$$

Universal gas constant

$$k_B = \frac{R}{N_A} = 1.38 \times 10^{-23}\ \mathrm{J/K}$$

Boltzmann Constant

Avogadro's number

Which form to use depends on what info you're given (moles of gas or number of atoms/molecules).

And if you have neither…

$$\frac{PV}{T} = Nk_B = nR = \text{constant}$$

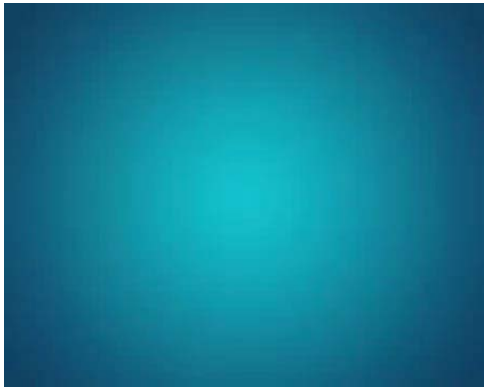
$$\Rightarrow \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Note: Temperature, Pressure and Volume are called macroscopic properties of a gas.

https://goo.gl/forms/tMTNuzmyWi8DZUMs2

Quick Quiz: A common material for cushioning objects in packages is made by trapping bubbles of air between sheets of plastic. This material is more effective at keeping the contents of the package from moving around inside the package on:

1. a hot day.
2. a cold day.
3. either hot or cold days.



Kinetic theory…

The ideal gas law provides a mathematical description of the **macroscopic** behaviour of gases.

Let us now consider what is actually happening on the **microscopic** level to the gas atoms and molecules.

→ **Kinetic theory** (a microscopic description)

Kinetic theory: assumptions...

- All molecules are identical.
- The number of molecules in the gas is large.
  - ⇨ So that we can apply statistics!
- The average separation between molecules is large compared with their dimensions.
  - ⇨ Point particles are easy to deal with.
  - ⇨ No overlapping wavefunctions: no need for quantum mechanics! Molecules obey Newton's laws of motion.
- Molecules move constantly in random directions.

Kinetic theory: assumptions...

- The molecules interact only via short-range forces during elastic collisions.
  - ⇨ No need to worry about inter-molecular force that may affect the motion of the molecules
- The molecules make elastic collisions with the walls and with one another.
  - ⇨ Same kinetic energy before and after a collision
- The timescale of each collision is small compared with the time between successive collisions.
  - ⇨ No need to care about the details of the collision process.

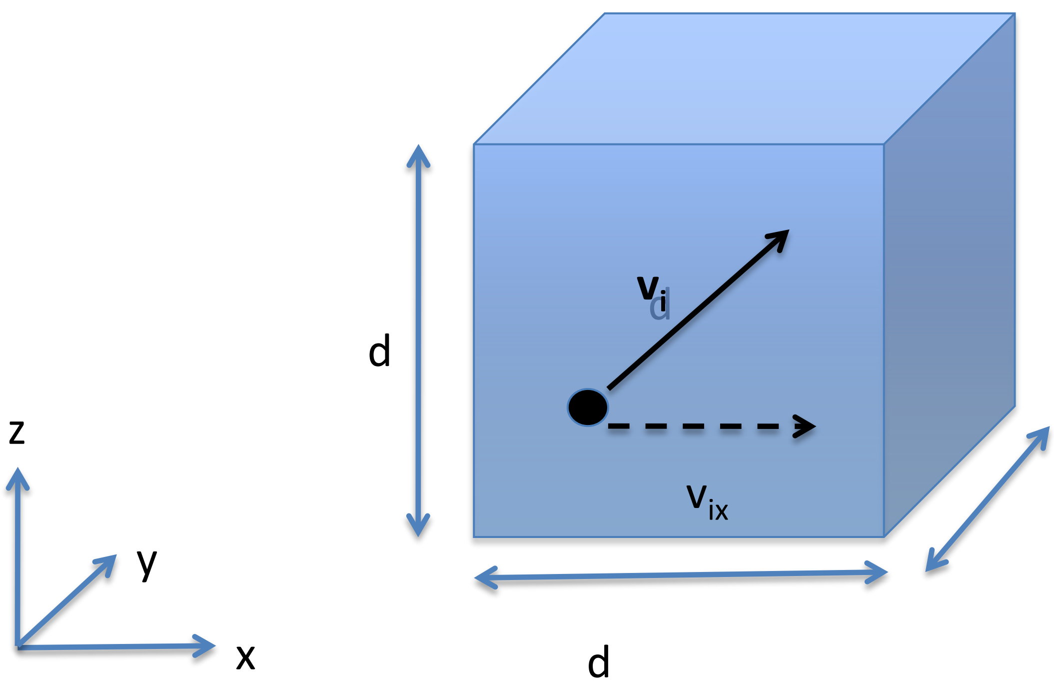
Over the next several slides we are going to derive the relationship between temperature and molecular kinetic energy.

The derivation is long: you do not need to be able to reproduce it but you should understand each step.

I will do this on the board, old-school – you can copy it down (the derivation is on moodle).

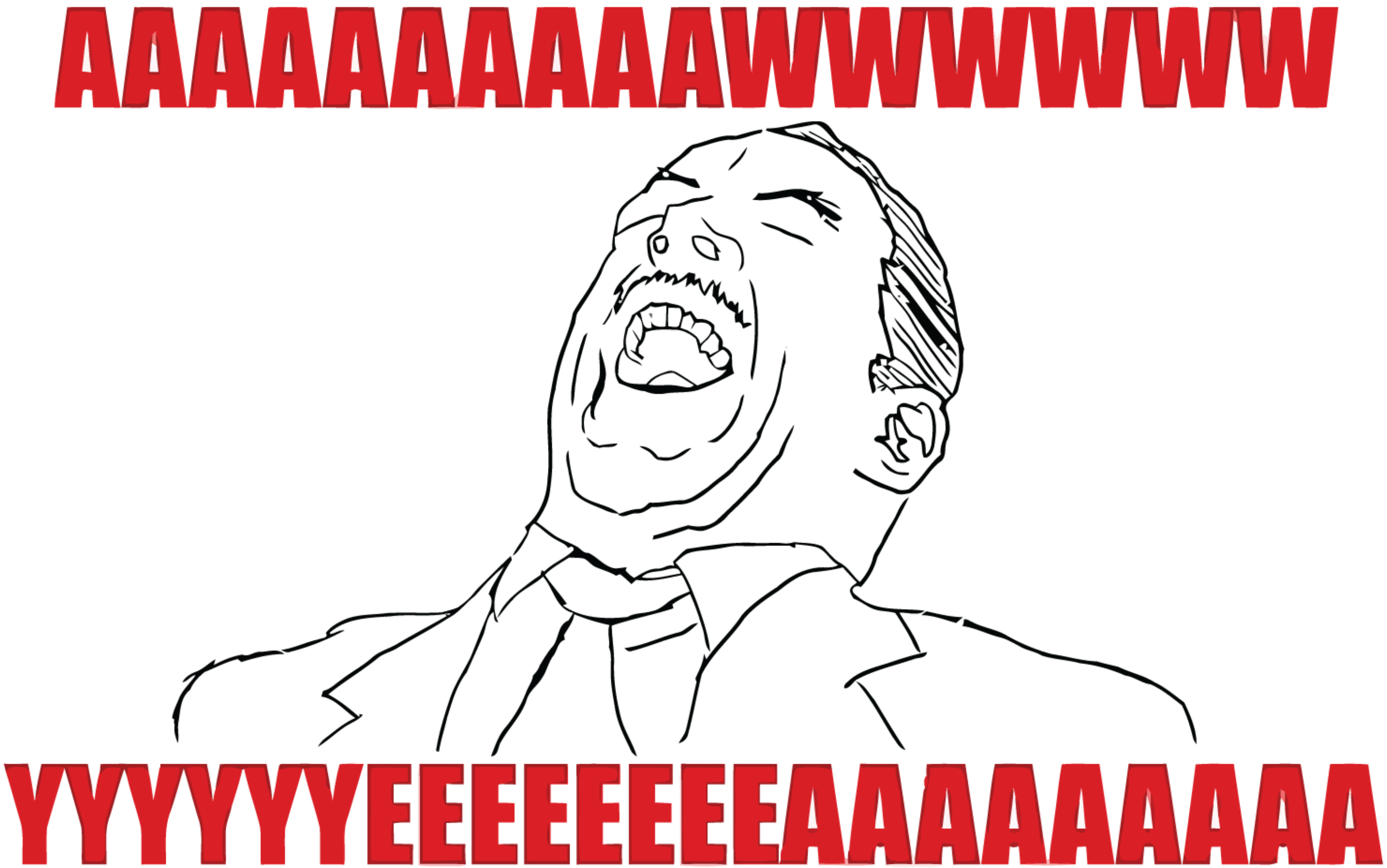
Given these assumptions, kinetic theory provides a relation between the **temperature** and the **molecular kinetic energy** of a gas.

Derivation: consider a box of volume $V=d^3$ containing N identical molecules of an ideal gas.



Consider one of these molecules of mass m_0

First in-class derivation!



Kinetic theory: temperature...

Kinetic theory tells us that the temperature of a gas is related to the average kinetic energy of the gas molecules via

$$T = \frac{2}{3k_{\mathrm{B}}}\left(\frac{1}{2}m_0\overline{v^2}\right)$$

Average squared speed of the molecules

Mass of each molecule

- If I scared you with the long derivation, don't worry: you don't need to be able to reproduce it in the exam. But you should certainly understand how this relation came about.

Kinetic theory: RMS velocity...

- RMS = root-mean-square
- RMS velocity:

Universal gas constant

$$v_{\mathrm{RMS}} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_{\mathrm{B}}T}{m_0}} = \sqrt{\frac{3RT}{M}}$$

Molar mass of gas

Mass of each molecule

  - M is the molar mass in *<u>kilograms</u>* per mole
  - At given temperature, lighter molecules move faster.

Question

The rms speed of an oxygen molecule (O_2) in a container of oxygen gas is 625 m/s. What is the temperature of the gas?

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_B T}{m_0}} = \sqrt{\frac{3RT}{M}}$$

M is molar mass in <u>kilograms</u> per mole!

Question

One container is filled with helium gas and another with argon gas. Both containers are at the same temperature. Which molecules have the higher rms speed? Explain.

Question

The rms speed of an oxygen molecule (O_2) in a container of oxygen gas is 625 m/s. What is the temperature of the gas?

But first!

$$v_{rms} = \sqrt{\overline{v^2}}$$

Find the rms speed of:

-1ŷ, 2ŷ, 4ŷ, -3ŷ

$$v_{rms} = \sqrt{\frac{1+4+16+9}{4}}$$

$$= 2.74$$

$V_{rms} = 625\, m/s \quad m_0 = \dfrac{32 \times 10^{-3}}{6.022 \times 10^{23}} = 5.31 \times 10^{-26}$

$$T = \frac{2}{3k_B}\left(\frac{1}{2} m_0 \overline{v^2}\right)$$

$$= \frac{2}{3 \times 1.381 \times 10^{-23}} \times \frac{1}{2} \times 5.31 \times 10^{-26} \times 625^2$$

$$= 501\, K$$

Question

A bottle of cold water (5.00^o C) is taken from the fridge, the lid is removed and replaced, and left in a closed car on a hot day. The temperature of the bottle reaches 75.0^o C. Assume that the expansion of the bottle and water is negligible (is this reasonable?). What is the pressure of the gas in the bottle now?