Question 1 PHYS 1121/1131

Answers

(a) $u = 0$
$a = 0.5$ m/s
$S = 450$ m

$s = ut + \frac{1}{2}at^2$

$450 = \frac{1}{2}(0.5)t^2$

$t^2 = \frac{450}{(\frac{1}{2})(0.5)}$

$t = 42.4$ s $= 42$ s (2 sig fig).

(b) $v = ?$
$u = 0$
$a = 0.5$ m/s
$t = 42.4$ s

$v = u + at$
$= 0 + (0.5)(42.4)$
$= 21.2$ m/s $= 21$ m/s upwards.

(c) (i) initial vel in y-dir is the same as the velocity of the balloon at that time.

so $u_y = 21.2\ ms^{-1} = 21$ m/s upwards.

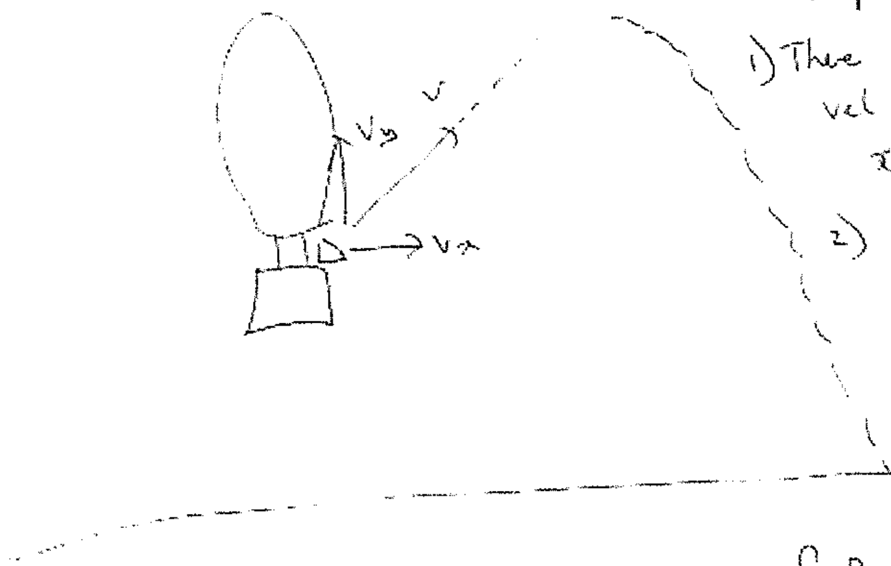
Q1 (2)

(c)

In the x-dir, the sand bag is pushed so that it has $V_x = 2.1\ ms^{-1}$.

so $u_x = 2.1\ ms^{-1}$

(ii)



Important points:

1) There is an initial vel in both the x & y direction.

2) The sand bag rises after it leaves the balloon

3) The sand bag lands on the ground further to the +ve x-direction than directly under the balloon.

(iii) max height above ground sand bag reaches

$u_y = 21.2\ ms^{-2}$

$V_y = 0$

$a_y = -g = -9.8\ ms^{-2}$

$V_y^2 = u_y^2 + 2a_y s_y$

$$s_y = \frac{V_y^2 - u_y^2}{2a_y} = \frac{0-(21.2)^2}{2(-9.8)}$$

$= 22.9\,m$

$\Rightarrow$ final height $= 22.9 + 450 = 473\,m$
$= 470\,m$ (2sig fig).

Q1 (3)

c (iv) How long does the sandbag take to reach the ground?

The upwards direction is positive, and we assume the balloon is at $y = 0$. so

$S_y = -450\,m$
$g_y = -9.8\,ms^{-2}$
$u_y = +21.2\,ms^{-1}$
$t = ?$

$S_y = u_y t + \frac{1}{2} g_y t^2$

$-450 = (21.2)t + \frac{1}{2}(-9.8)(t)^2$

This will be a quadratic equation, and we solve for t by

$-4.9t^2 + 21.2t + 450 = 0$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-21.2 \pm \sqrt{(21.2)^2 - (4)(-4.9)(450)}}{2(-4.9)}$

$= \dfrac{-21.2 \pm 96.3}{-9.8} = \dfrac{-21.2 - 96.3}{-9.8}$ or $\dfrac{-21.2 + 96.3}{-9.8}$

$\Rightarrow 12\,s$ (2 sig fig). $= 12.0\,s$ or $= -7.64$

discard.

Q 1 (4)

(v) How far from the $x = 0$ position does the sand bag land?

$u_x = 2.1\ ms^{-1}$
$t = 12.0\ s$

$s_x = u_x t + \frac{1}{2} a_x t^2$
$= u_x t + \frac{1}{2}(0) t^2$

$= u_x t$
$= 2.1 \times 12.0$
$= 25.2\ m$
$= 25m$ (2 sig fig).

(d) $V = 100\ km/hr$
$= 27.8\ ms^{-1}$

$S_0 = 90\ m$

$t_r = 1.0s$
$a = -7.0\ m/s$

How long does it take the driver to stop?

Q1

(d) Cont..

First convert the 100 km/hr to SI units

$v = 27.8\ ms^{-1}$

Second, because the driver's reaction time is 1s, in the first second, before the driver reacts, the car will travel

$$S_i = 27.8\ m$$

$$90 - S_i = 62.2m \text{ left.}$$

Then, the driver breaks & ~~decel~~ decellerates.

How long does it take the driver to stop?

$v = u + at$ $\qquad v = 0,\ u = 27.8\ m/s$

$\qquad a = -7.0\ ms^{-2}$

$$t = \frac{v - u}{a}$$

$$= \frac{0 - 27.8}{-7.0} = 4.0\ s$$

How ~~long~~ far does the car travel during this time

$s = ut + \frac{1}{2}at^2 \rightarrow s = 27.8t + \frac{1}{2}(-7)t^2$

$= 27.8(4) + \frac{1}{2}(-7)(4)^2$

$= 55.2\ m$

Q1 (6)

So, during the reaction time of 1s, the car travels 27.8m

Once the driver deccellerates, it takes 55.2 m for the car to come to a stop

So, the total distance taken to stop is $(27.8 + 55.2)$m $= 83.0$ m

So, the car is able to stop before the obstacle, which is 90 m distant.

PHYS 1131 Question 2.

a). i) $\frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt}$. as v is constant but m on conveyer belt changes.

$= 0.750 \times 5.00$

$= 3.75 \ kgms^{-2}$ (or N)

to the right.

ii) $F_{ext} = \frac{dp}{dt} = 3.75$ N to the right.

iii) $W = \int F \cdot dx$. in 1 s the sand travels 0.750 m.

$= 3.75 \times 0.750$.

$= 2.81$ J.

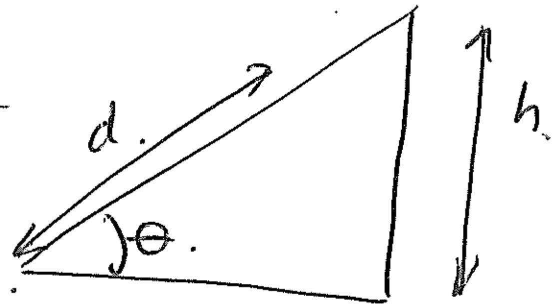
iv). $\frac{dK}{dt} = \frac{d(\frac{1}{2}mv^2)}{dt} = \frac{1}{2}v^2\frac{dm}{dt} = \frac{1}{2}(0.750)^2(5.00)$

$= 1.41$ J.

v) Not all the forces in this case are conservative, there is friction between the sand particles for example.

b). i).

initial K = final P + work done against friction.



$$\frac{1}{2} m v_i^2 = mgh + \mu_k mg\cos\theta . d$$

$$\frac{1}{2}\cancel{m} v_i^2 = \cancel{m} g d\sin\theta + \mu_k \cancel{m} g d\cos\theta .$$

$$\sin\theta = \frac{h}{d}.$$

$$\Rightarrow d = \frac{\frac{1}{2} v_i^2}{g\sin\theta + \mu_k g\cos\theta}.$$

$$\tan\theta = \frac{80}{150} \Rightarrow \theta = 28.072^\circ.$$

$$\Rightarrow d = \frac{\frac{1}{2}\times 20^2}{9.8(\sin 28.072 + 0.20\times\cos 28.072)}$$

$$= 31.5 \text{ m}.$$

This is the energy approach. You can also use a force approach:

$F = ma = mg\sin\theta + \mu_k mg\cos\theta$ down the slope.

then $v^2 = u^2 + 2ad$ where $v = 0$. and $u = 20$ m/s up the slope.

this gives the same equation as above.

ii) To slide down weight component down the slope, $mg\sin\theta$, must be greater than static friction $= \mu_s mg\cos\theta$.

$$mg\sin\theta = m \times 4.611$$

$$\mu_s mg\cos\theta = m \times 3.46$$

$\Rightarrow$ It will slide down the slope.

alternatively it will slide down the slope when

$\tan\theta \geqslant \mu_s$. In this case $\tan\theta = 0.533$

so it will slide down the slope.

iii). $a_{up} = g\sin\theta + \mu_k g\cos\theta$ down the slope.

$$v = u + at.$$

$$\Rightarrow t = \frac{-20}{-9.8(\sin 28.072 + 0.20\cos 28.072)}$$

$$= 3.15\ s.$$

$a_{down} = g\sin\theta - \mu_k g\cos\theta$ down the slope

$$d = \frac{1}{2}at^2.$$

$$\Rightarrow t = \sqrt{\frac{2d}{g\sin\theta - \mu_k g\cos\theta}}$$

$$= 4.68\ s.$$

$\Rightarrow t = 7.83\ s$ (3sig fig).

iv) $v^2 = u^2 + 2ad.$

$= 2 \times 9.8 \times (\sin 28.072 - 0.20 \cos 28.072)$
$\times 31.5$

$= 181.58.$

$\Rightarrow \quad v = 13.5$ m/s down the slope.

1131 Question 3

(a) i) Assume energy is conserved.

$$mgR = mg(0.50) + \frac{1}{2}mv^2.$$

$$\Rightarrow v^2 = 2g(R-0.50) = 2\times 9.8(6.80-0.50)$$

$$v = 11 \text{ m/s} \quad (2 \text{ sig fig}).$$

ii) $L = mvr = 64\times 11.1\times(6.80-0.50)$

$= 4.5\times 10^3$ kg m²/s out of the page

($\underset{\sim}{L} = \underset{\sim}{r}\times \underset{\sim}{p}$ gives the direction).

iii). No tangential forces act, so no torque about the axis, so angular momentum is conserved.

As the skateboarder stands her legs convert chemical energy to gravitational potential energy. (her legs do work), as a result kinetic energy is not conserved.

iv). $L_f = L_i = mvr.$

$$\Rightarrow v_f = \frac{L_i}{mr} = \frac{4.5\times 10^3}{64\times(6.80-0.95)}$$

$$= 12 \text{ m/s}.$$

v). $(K+U)_B + U_{legs} = (K+U_g)_C.$

$$\frac{1}{2}\times 64\times 11.1^2 + U_{legs} = \frac{1}{2}\times 64\times 12.0^2 + 64\times 9.8\times 0.45.$$

$$U_{legs} = 948 \text{ J} = 950 \text{ J} \quad (2 \text{ sig fig})$$

(b) (i). $K_{rot} = 4 \times \frac{1}{2} \times I \omega^2$. $\omega = \frac{v}{r}$.

$I_{disk} = \frac{mr^2}{2}$ (either derived or memorized is fine)

$$\Rightarrow K_{rot} = \cancel{4} \times \frac{1}{\cancel{2}} \times \frac{m_w r^2}{\cancel{2}} \times \left(\frac{v}{r}\right)^2$$

$$= m_w v^2.$$

$$K_{trans} = \frac{1}{2} m v^2.$$

$$\Rightarrow \frac{K_{rot}}{K_{rot} + K_{trans}} = \frac{m_w \cancel{v^2}}{\frac{1}{2} m \cancel{v^2} + m_w \cancel{v^2}}$$

$$= \frac{m_w}{\frac{1}{2} m + m_w}$$

$$= \frac{2m_w}{m + 2m_w}.$$

$$I = \int r^2 dm.$$

$$dm = \rho . 2\pi r . h . dr.$$

$$\Rightarrow I = \int_0^R \rho . 2\pi r^3 h \, dr$$

$$= 2\pi \rho h \left[\frac{r^4}{4}\right]_0^R$$

$$= \frac{2\pi \rho h R^4}{4}$$

$$= \frac{\pi \rho h R^4}{2}$$

$$m = \pi R^2 h \rho.$$

$$\Rightarrow I = \frac{mR^2}{2}.$$

ii). $\tau = 4 \, r \times F$.

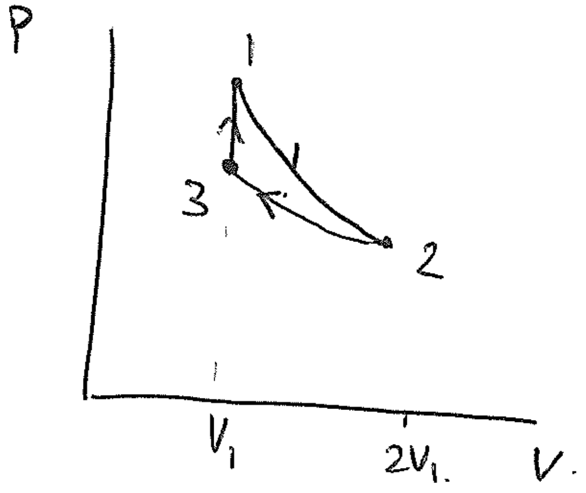
Maximum F without skidding $= \frac{\mu_s mg}{4}$.

as weight spread over 4 wheels.

$$\Rightarrow \tau = 4 r \frac{\mu_s mg}{4} = 1.4 rmg.$$

Question 4.

a) i)



ii) For isothermal

$$P_1V_1 = P_2V_2 \Rightarrow P_2 = \frac{P_1V_1}{V_2} = \frac{P_1V_1}{2V_1} = \frac{1}{2}P_1.$$

iii) $W = -\int P \cdot dV$ $\qquad P = \frac{nRT}{V}.$

$$= -nRT\int_{V_1}^{V_2}\frac{dV}{V}.$$

$$= -nRT\ln(V_2/V_1)$$

iv) $1\to2$: negative as gas is expanding
$2\to3$: positive as gas is contracting
$3\to1$: zero as volume does not change.

v) $1\to2$: $\Delta E_{int} = Q + W = 0 \Rightarrow Q$ is positive as W is negative
$2\to3$: $Q=0$ as adiabatic
$3\to1$: $W=0$; ΔE_{int} is positive as $T\uparrow$ so Q is positive.

vi) $\gamma = \frac{f+2}{f}$ $f = 3$ for monatomic.

$= \frac{5}{3}$.

vii) $P_2 V_2^{\gamma} = P_3 V_3^{\gamma}$.

$$P_3 = P_2 \left(\frac{V_2}{V_3}\right)^{5/3} = P_2 \left(\frac{2V_1}{V_1}\right)^{5/3}$$

$$= 3.17 P_2$$

b) heat gained by coffee + heat lost by ice = 0.

$m_c c_c \Delta T_c - m_i c_i \Delta T_i - m_i L_i - m_i c_w \Delta T_i' = 0$

$0.300 \times 4186 \times 35 - m_i \times 1750 \times 100 - 3.335 \times 10^5 \times m_i$
$- m_i \times 4186 \times 50 = 0.$

$43953 - m_i(717800) = 0.$

$\Rightarrow m_i = 0.0612 \text{ kg}$

$= 61.2 \text{g}.$

c) i) energy added to the monatomic gas goes into the three translational degrees of freedom.

For the diatomic gas there are an additional two rotational degrees of freedom that store the added heat.

The translational KE of the monatomic gas is $\frac{5}{3}$ times that of the diatomic gas.

ii). The ~~diato~~ monatomic gas has the higher temperature change as the temperature is taken as $\frac{3}{2}\left(\frac{1}{2}mv_{rms}^2\right)$, the v_{rms} speed is higher for monatomic as all additional heat went into increasing this..

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QUESTION 5 (30 marks)

a. i. $F_{up} = \rho V g = \rho A d g$

$= 1000 \cdot 0.04 \cdot d \cdot 9.8$

$= 392 d \text{ N}$

ii. When displaced a distance x below the surface

$$F = -\rho A g x = m\ddot{x}$$

$$\therefore \ddot{x} = -\frac{\rho A g}{m} x$$

$$= -98x$$

iii. The buoyancy force acts upwards and gravity pulls down

iv. $\ddot{x} = -98x$

$$x = d \sin(\omega t + \phi)$$

with $\omega = \sqrt{\frac{\rho A g}{m}} = \sqrt{98} = 9.9 \text{ rad s}^{-1}$

(or just s^{-1})

v. $\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{\rho A g}{m}}$

$$\therefore T = \frac{2\pi}{\omega} = 0.63 \text{ s}$$

b. The power of the wave diminishes as it spreads out in 2-dimensions along the surface. It spreads on the circumference of a circle, rather than the surface of a sphere.

$$\therefore I = \frac{P}{2\pi r}$$

Intensity I is proportional to A^2.

(cont...)

$\therefore A^2 = \frac{k}{r}$ where k is a constant.

At $1\,m$, $A = 4cm$

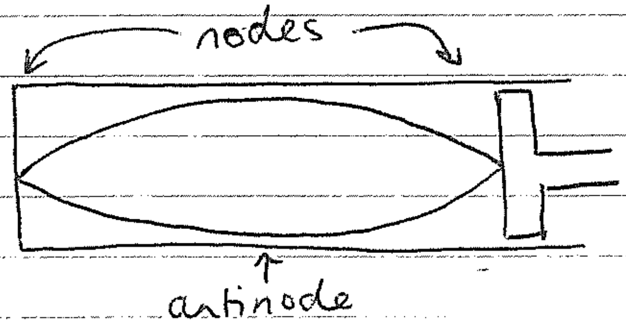
$\therefore k = (4cm)^2 . 1m$

At $10m$ then,

$$A^2 = \frac{(4cm)^2 . 1m}{10m} = \frac{(4cm)^2}{10}$$

$$\therefore A = \frac{4cm}{\sqrt{10}} = 1.3cm$$

c. i.



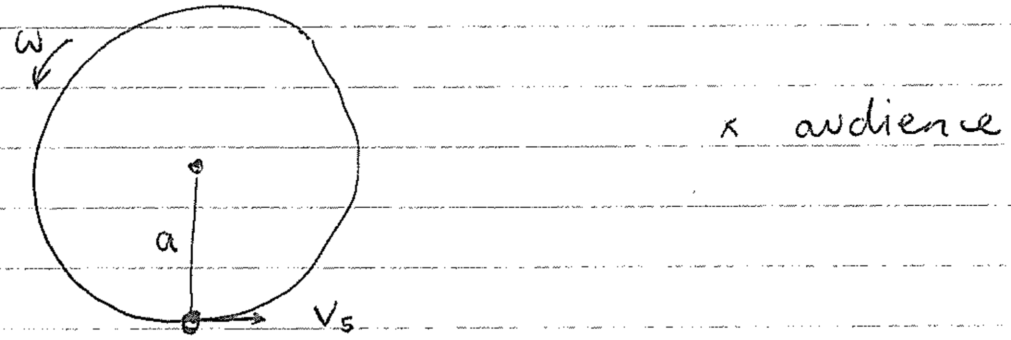
ii. The fundamental note is half wavelength

$$L = \frac{\lambda}{2}$$

$$v = f\lambda$$

$$\therefore f = \frac{v}{\lambda} = \frac{v}{2L}$$

iii.



$t = 0$ shown

From the audience's perspective the whistle comes towards or away with speed

$$v_s = v \cos \omega t$$

and $v = a\omega$

$\therefore \quad V_s = a\omega \cos \omega t$

$\uparrow$

Note that V_s is maximum at $t=0$.

The frequency the audience hears is

$$f' = f_0\left(\frac{c+V_0}{c-V_s}\right) = f_0 \frac{c}{c - a\omega\cos\omega t}$$

iv. The second musician is out of phase by π.

$$V_s = a\omega\cos(\omega t + \pi)$$
$$= -a\omega\cos(\omega t)$$

$\therefore$ Frequency heard by audience from second musician

$$f_2' = f_0 \frac{c}{c + a\omega\cos\omega t}$$

The beat frequency is $f_1' - f_2' = f_{beat}$

$$\therefore f_{beat} = f_0\left(\frac{c}{c - a\omega\cos\omega t} - \frac{c}{c + a\omega\cos\omega t}\right)$$
$$= f_0\left(\frac{1}{1 - \frac{a\omega}{c}\cos\omega t} - \frac{1}{1 + \frac{a\omega}{c}\cos\omega t}\right)$$
$$\simeq f_0 \cdot \frac{2a\omega}{c}\cos\omega t$$

v. Second musician plays $f_2(t)$ so audience hears $f_2'(t) = f_0 = f_2(t) \cdot \dfrac{c}{c + a\omega\cos\omega t}$

$$\therefore f_2(t) = f_0 \frac{c + a\omega\cos\omega t}{c}$$
$$= f_0\left(1 + \frac{a\omega}{c}\cos\omega t\right)$$