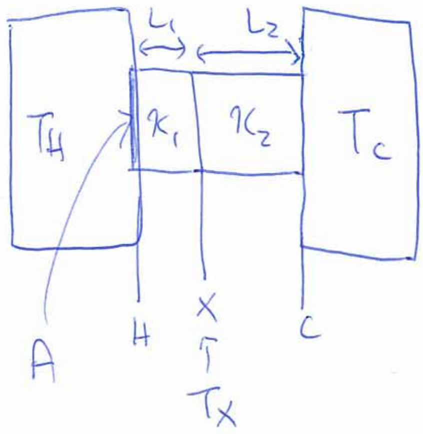
Heat transfer rate by conduction in a composite slab

In a steady state, the heat transfer rate from H to X must be the same as that from X to C, i.e.,



$$P_{cond}^{H\to X} = P_{cond}^{X\to C} \equiv P_{cond}$$

where

$$P_{cond}^{H\to X} = k_1 A \frac{(T_H - T_X)}{L_1} \quad (\star)$$

A: cross-sectional area

|| T_X is some intermediate temperature at X.

$$P_{cond}^{X\to C} = k_2 A \frac{(T_X - T_C)}{L_2} \quad (*)$$

Equating $(\star)$ and $(*)$,

$$k_1 \cancel{A} \frac{(T_H - T_X)}{L_1} = k_2 \cancel{A} \frac{(T_X - T_C)}{L_2}$$

$$\Rightarrow \frac{k_1}{L_1}(T_H - T_X) = \frac{k_2}{L_2}(T_X - T_C)$$

$$\Rightarrow \frac{1}{R_1}(T_H - T_X) = \frac{1}{R_2}(T_X - T_C)$$

|| $R_1 = \frac{L_1}{k_1}$, $R_2 = \frac{L_2}{k_2}$

Multiply both sides by $R_1 R_2$:

$$R_2(T_H - T_X) = R_1(T_X - T_C)$$

$$\Rightarrow T_X(R_1 + R_2) = T_H R_2 + T_C R_1$$

$$\Rightarrow \quad T_x = \frac{T_H R_2 + T_c R_1}{R_1 + R_2}$$

Substitute this into (A):

$$P_{cond} = P_{cond}^{H \to x} = \frac{A}{R_1}\left[T_H - \left(\frac{T_H R_2 + T_c R_1}{R_1 + R_2}\right)\right]$$

$$= \frac{A}{R_1(R_1+R_2)}\left[T_H R_1 + \cancel{T_H R_2} - \cancel{T_H R_2} - T_c R_1\right]$$

$$= A\,\frac{(T_H - T_c)}{R_1 + R_2}$$

This can be easily generalised to the case multiple composites to

$$P_{cond} = A\,\frac{(T_H - T_c)}{\sum_i R_i}$$

Where

$$\sum_i R_i = R_1 + R_2 + R_3 + \ldots$$