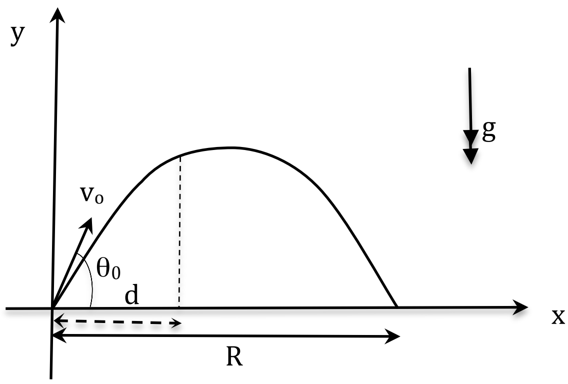
Question 1 (Marks: 20)

(a) The minute hand of a clock is 12cm long, as measured from its tip to the axis about which it rotates. The clock is hanging vertically from a wall. Over the period of time from 9:00am to 9:30am determine the mean velocity of the tip of the minute hand.

(b) A projectile is launched with velocity v_0 at angle θ_0 to the horizontal, as shown in the diagram, with the x-axis indicating the horizontal motion and the y-axis the vertical motion of the projectile. Gravity is acting downwards with magnitude g. You may assume that air resistance is negligible.



(i) By considering the horizontal motion, derive an equation relating horizontal displacement, d, of the projectile to the time t after it has been launched.

(ii) Similarly, considering vertical motion, derive a quadratic equation relating height reached, h, to the time t after launch.

(iii) The range of the projectile, R, is the horizontal distance reached when it strikes the ground. Using the above result in part (ii), derive an expression for the time at which this occurs.

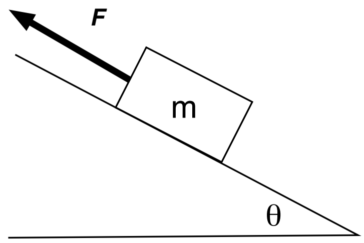
(iv) Use the above results to derive an expression for the range of the projectile as a function of v_0, θ_0 and g.

(c) The world long jump record is 8.95m. Suppose that when this record was set the athlete took off at a speed v_0=9.50 m/s. How much shorter is this distance than the maximum possible range that could be achieved at this take-off speed?

Question 2 (Marks: 20)

(a) A mass m lies on a plane inclined at angle θ. The coefficients of static and kinetic friction between the mass and the plane are μ_s and μ_k respectively.

A force $\boldsymbol{F}$ acting up the plane causes the mass to accelerate up the plane at constant acceleration $\boldsymbol{a}$.

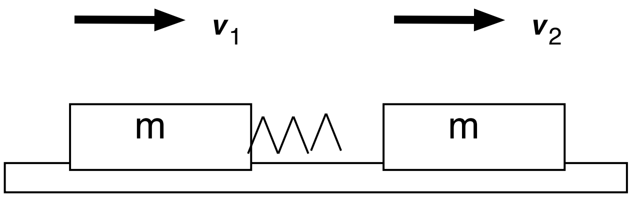


(i) Derive an expression for the magnitude of $\boldsymbol{F}$, in terms of m, a, g, θ and μ_k.

(ii) Derive an expression for the angle θ for which the magnitude of $\boldsymbol{F}$ is maximum for a given m, a, g and μ_k.

(iii) Explain in two short sentences why there is a maximum value of F.

(b) Two sliders, of mass m, travel with negligible friction on an airtrack at speeds v_1 and v_2. A light spring with constant k is attached to one, as shown in the sketch. They collide.



(i) During the collision, when the spring is maximally compressed, what is the relative velocity of the two carts?

(ii) What is the maximal compression of the spring during the collision? Show your reasoning and, if you use any conservation laws, state them carefully, including any conditions. Simplify your answer where possible.

Question 3 (Marks: 20)

(a) The Earth has mass $M = 5.97 \times 10^{24}$ kg and we approximate it as a sphere with radius 6 378 km. A communications satellite has mass m = 145 kg. The satellite is in a circular orbit, above the equator, with radius R from the centre of the Earth. Newton's constant is 6.673×10^{-11} $\mathrm{Nm^2kg^{-2}}$.

(i) Write an expression for the magnitude of the gravitational force that the Earth exerts on the satellite in orbit.

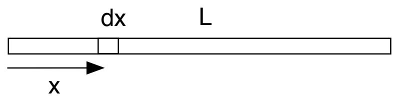
(ii) Write an expression for the acceleration of the satellite in terms of R and its period T.

(iii) Using the answers to (i) and (ii), determine the value of the radius R that will give a period of 23.9 hours.

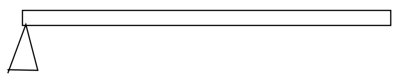
(iv) Determine the total work done on that satellite, by all forces including gravity, to get it from the surface of the Earth into that orbit. (Neglect the motion of the launch site due to the rotation of the Earth.

(b) (i) State the definition of the moment of inertia of an object, defining all terms used.

(ii) By considering a small section of length dx, derive an expression for the moment of inertia of a uniform, thin rod of length L and mass m for rotation about an axis at one end of the rod, at right angles to the rod.



(iii) A uniform, thin rod of mass m and length L is supported at one end by a sharp wedge, as shown.



The friction between the two objects is sufficiently large that the rod does not slip on the wedge. Derive an expression for the downwards acceleration of the point at the unsupported (right hand) end of the rod when the rod is horizontal.

Question 4 (Marks: 30)

In this section make use of the data provided in these tables.

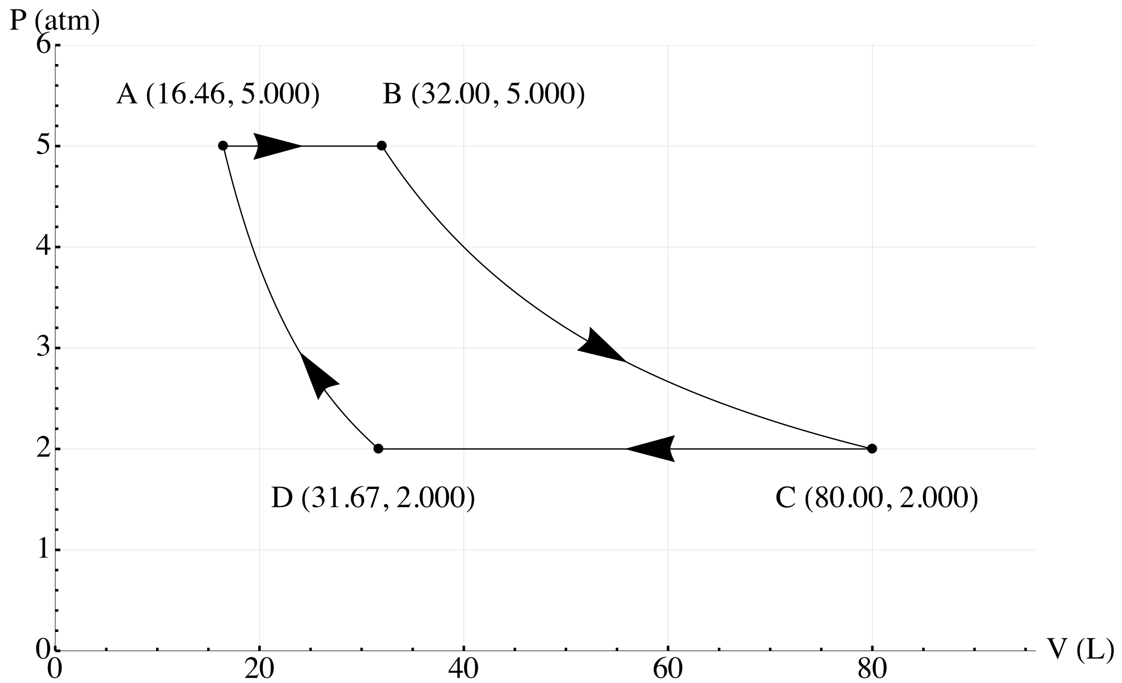
Specific Heats and Thermal conductivities of selected metals

Substance	Specific Heat c, (J kg^{-1} K^{-1})	Thermal conductivity k, (W m^{-1} K^{-1})
Aluminium	910	205.0
Brass	377	109.0
Copper	390	385.0
Lead	130	34.7
Steel	456	50.2

Water

Quantity	**Value**
Specific Heat (liquid)	4186 Jkg^{-1}K^{-1}
Latent heat of Fusion	3.33×10^5 Jkg^{-1}
Latent heat of vapourization	2.26×10^6 Jkg^{-1}
Density (at 4.00^o C)	1000 kgm^{-3}
Melting point (at 1 atm)	0.000 °C
Boiling point (at 1 atm)	100.0 °C
Volume expansion coefficient (β) (at 20^{o}C: you may assume it is constant between 15^{o}C and 100^o C)	207×10^{-6} ($^{\circ}$C)$^{-1}$

(a)



5.00 mols of an ideal diatomic gas undergoes a cyclic process from A → B → C → D → A. The process A → B is isobaric, 27.5 kJ of heat is added to the system during this process, the process B → C is isothermal, the process C → D is isobaric, 34.2 kJ is removed from this system as heat during this process and D → A is adiabatic. The numbers on the diagram represent (V, P) where V is in L and P is in atm.

(i) At point A how many degrees of freedom does one molecule of the gas have? Give detailed reasoning about how you know this.

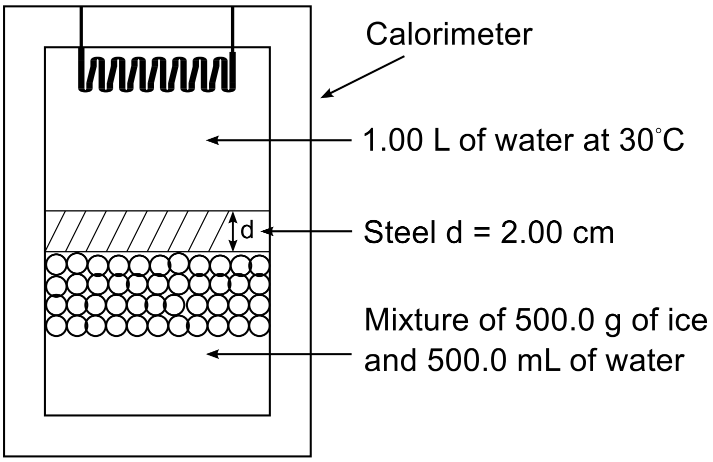
(ii) Describe what type of movement each of these degrees of freedom corresponds to.

(iii) How much work is done on the gas as it goes from A to B?

(iv) What is the change in internal energy as the gas goes from B to C?

(v) How much work is done on the system as the gas goes from D to A?

(b)



A perfect calorimeter is set up as shown in the diagram above. The water at the top has a volume of 1.00 L is kept at a constant 30.0 °C by a heater. At t = 0.00 s the heater is switched off. At t = 0.00 s there is 500.0 g of ice and 500.0 mL of water in the lower part of the calorimeter. The cross sectional area of the calorimeter is 100.0 cm^2 and the steel is 2.00 cm thick.

(i) What is the rate of heat flow through the steel at t = 0.00s?

(ii) What mass of ice melted in the 1.00 minutes before the heater was switched off (ie. from t = -60.0 to 0.00 s)?

(iii) What equilibrium temperature will the system eventually reach? The steel has a mass of 1.56 kg. Show your working and state any assumptions you make.

(iv) How much heat energy will flow through the steel in the first minute after the heater is switched off?

You may find some of the following standard integrals useful:

$$\int \frac{1}{x} dx = \ln|x|$$

$$\int u dv = uv - \int v du$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b|$$

Question 5 (Marks: 30)

Giant alien spiders from the newly-discovered planet GJ 832c have kidnapped Justin Bieber for some reason. (Plot twist: spiders have no proper sense of hearing as humans do) The mass of GJ 832c is 3.225e+25 kg, and the radius is 1.276e+7 m.

(a) Justin is dangling off a cliff holding onto the end of a massless spider-silk thread of the type only found in first-year physics. As he dangles, he swings back and forth. If the period of his swing is 10.0 seconds, how far down the thread is he from the top?

(b) With Justin hanging on the line, the first harmonic frequency of the cable as it vibrates is 4.40 Hz. I feel dirty for having Googled this: Justin has a mass of 66.0 kg. To avoid dividing by zero and destroying the universe, assume the spider-silk thread now has some non-zero mass μ per unit length. What is μ for the silk made by GJ 832c's industrious but tone-deaf spiders?

(c) One of the three strands making up the rope breaks, and it falls away reducing the rope's thickness and mass by one-third. What is the frequency of the first harmonic now?

(d) A hungry pterodactyl clings to the rope and pecks at Justin causing him to fall and cry out. If Justin's voice at rest is 440.0 Hz, derive an expression for the frequency heard by the pterodactyl as a function of time t since dropping from the rope. Assume no wind resistance, and the speed of sound on GJ 832c is 380.0 m/s.

(e) It turns out that the cliff was the edge of a bottomless pit which is drilled through the entire planet. For fun, consider the tunnel to be a vacuum. Make the following simplifying assumptions for this part:

- No air resistance.
- The planet has uniform density and is perfectly spherical.
- The planet has mass M and radius R.
- Justin's motion is simple harmonic as he plunges through the planet back and forth for all eternity.

Derive an expression for the angular frequency, ω, of Justin's motion.

[Hints: (1) Define your coordinate system to get signs to work out, and (2) Use your boundary condition at time t=0.]