Chapter 2

Motion Along a Straight Line

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2-1 Position, Displacement, and Average Velocity

Learning Objectives

2.01 Identify that if all parts of an object move in the same direction at the same rate, we can treat it as a (point-like) particle.

2.02 Identify that the position of a particle is its location on a scaled axis.

2.03 Apply the relationship between a particle's displacement and its initial and final positions.

2.04 Apply the relationship between a particle's average velocity, its displacement, and the time interval.

2.05 Apply the relationship between a particle's average speed, the total distance it moves, and the time interval.

2.06 Given a graph of a particle's position versus time, determine the average velocity between two times.

2-1 Position, Displacement, and Average Velocity

- **Kinematics** is the classification and comparison of motions
- For this chapter, we restrict motion in three ways:
  1. We consider motion along a straight line only
  2. We discuss only the motion itself, not the forces that cause it
  3. We consider the moving object to be a **particle**
- A **particle** is either:
  - A point-like object (such as an electron)
  - Or an object that moves such that each part travels in the same direction at the same rate (no rotation or stretching)

2-1 Position, Displacement, and Average Velocity

- Position is measured relative to a reference point:
  - The **origin**, or zero point, of an axis
- Position has a sign:
  - **Positive direction** is in the direction of increasing numbers
  - **Negative direction** is opposite the positive

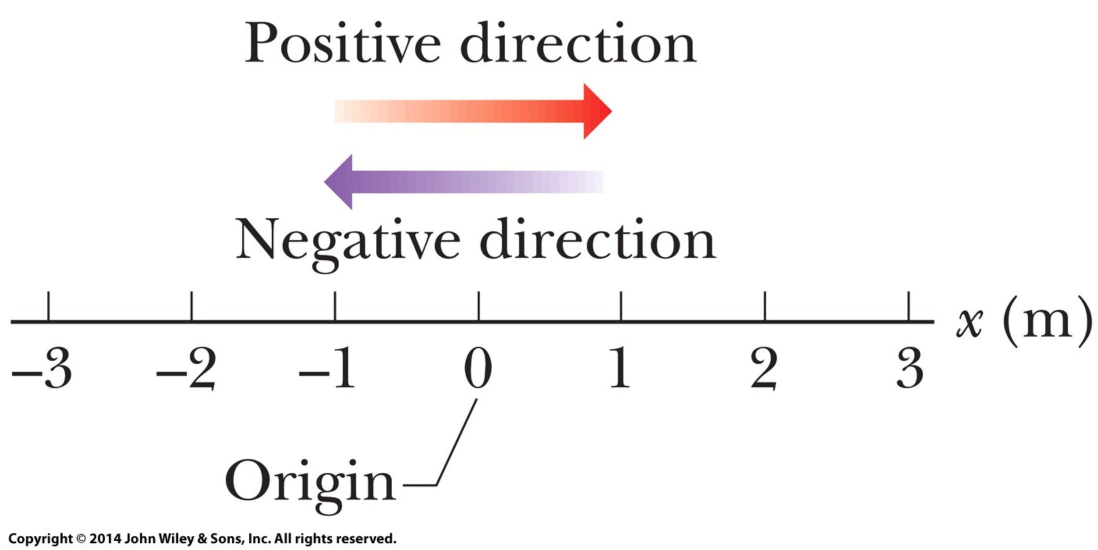

Figure 2-1

2-1 Position, Displacement, and Average Velocity

- A change in position is called **displacement**
  - Δx is the change in x, (*final position*) – (*initial position*)

$$\Delta x = x_2 - x_1 \qquad \text{Eq. (2-1)}$$

Examples A particle moves . . .

- From x = 5 m to x = 12 m: Δx = 7 m (positive direction)
- From x = 5 m to x = 1 m: Δx = -4 m (negative direction)
- From x = 5 m to x = 200 m to x = 5 m: Δx = 0 m

- The actual distance covered is irrelevant

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2-1 Position, Displacement, and Average Velocity

- Displacement is therefore a **vector quantity**
  - Direction: along a single axis, given by sign (+ or -)
  - Magnitude: length or distance, in this case meters or feet
- Ignoring sign, we get its **magnitude** (absolute value)
  - The magnitude of Δx = -4 m is 4 m.

Checkpoint 1

Here are three pairs of initial and final positions, respectively, along an x axis. Which pairs give a negative displacement: (a) −3 m, +5 m; (b) −3 m, −7 m; (c) 7 m, −3 m?

Answer: pairs (b) and (c)

(b) -7 m – -3 m = -4 m (c) -3 m – 7 m = -10 m

2-1 Position, Displacement, and Average Velocity

- **Average velocity** is the ratio of:
  - A displacement, Δx
  - To the time interval in which the displacement occurred, Δt

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Eq. (2-2)

- Average velocity has units of (*distance*) / (*time*)
  - Meters per second, m/s

2-1 Position, Displacement, and Average Velocity

- On a graph of *x* vs. *t*, the average velocity is the **slope** of the straight line that connects two points
- Average velocity is therefore a vector quantity
  - Positive slope means positive average velocity
  - Negative slope means negative average velocity

This is a graph of position *x* versus time *t*.

To find average velocity, first draw a straight line, start to end, and then find the slope of the line.

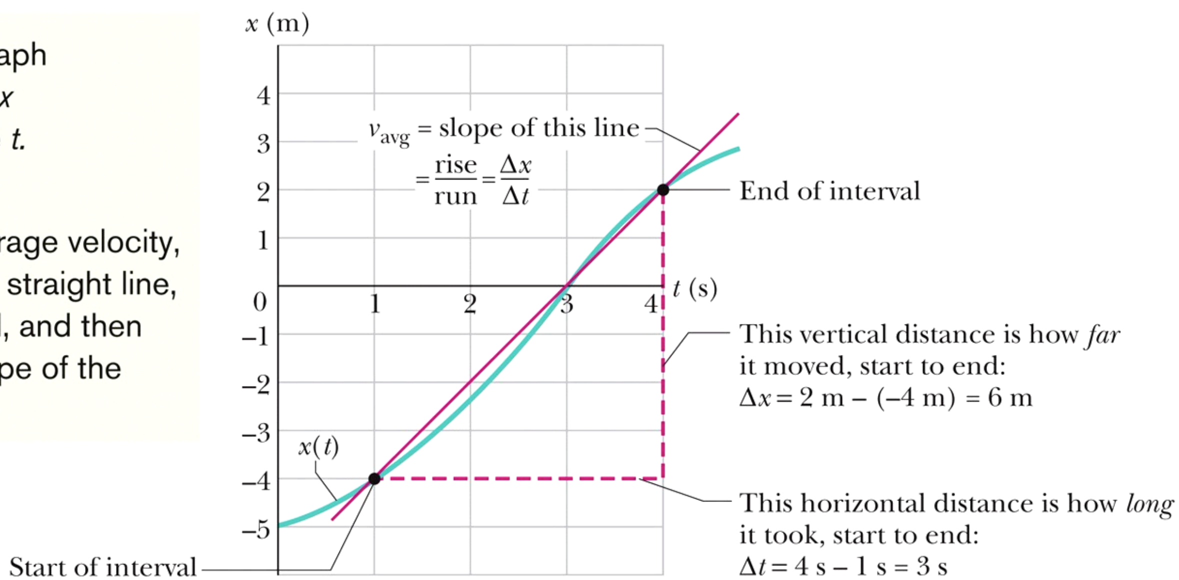

Figure 2-4

2-1 Position, Displacement, and Average Velocity

- **Average speed** is the ratio of:
  - The total distance covered
  - To the time interval in which the distance was covered, Δt

$$s_{avg} = \frac{\text{total distance}}{\Delta t}$$

Eq. (2-3)

- Average speed is always positive (no direction)

Example A particle moves from x = 3 m to x = -3 m in 2 seconds.

  - Average velocity = -3 m/s; average speed = 3 m/s

2-2 Instantaneous Velocity and Speed

Learning Objectives

2.07 Given a particle's position as a function of time, calculate the instantaneous velocity for any particular time.

2.08 Given a graph of a particle's position versus time, determine the instantaneous velocity for any particular time.

2.09 Identify speed as the magnitude of instantaneous velocity.

2-2 Instantaneous Velocity and Speed

- **Instantaneous velocity**, or just **velocity**, *v*, is:
    - At a single moment in time
    - Obtained from average velocity by shrinking Δt
    - The slope of the position-time curve for a particle at an instant (the derivative of position)
    - A vector quantity with units (*distance*) / (*time*)
        - The sign of the velocity represents its direction

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Eq. (2-4)

2-2 Instantaneous Velocity and Speed

- **Speed** is the magnitude of (instantaneous) velocity

Example A velocity of 5 m/s and -5 m/s both have an associated speed of 5 m/s.

Checkpoint 2

The following equations give the position $x(t)$ of a particle in four situations (in each equation, x is in meters, t is in seconds, and $t > 0$): (1) $x = 3t - 2$; (2) $x = -4t^2 - 2$; (3) $x = 2/t^2$; and (4) $x = -2$. (a) In which situation is the velocity v of the particle constant? (b) In which is v in the negative x direction?

Answers:

(a) Situations 1 and 4 (zero)

(b) Situations 2 and 3

2-2 Instantaneous Velocity and Speed

Example

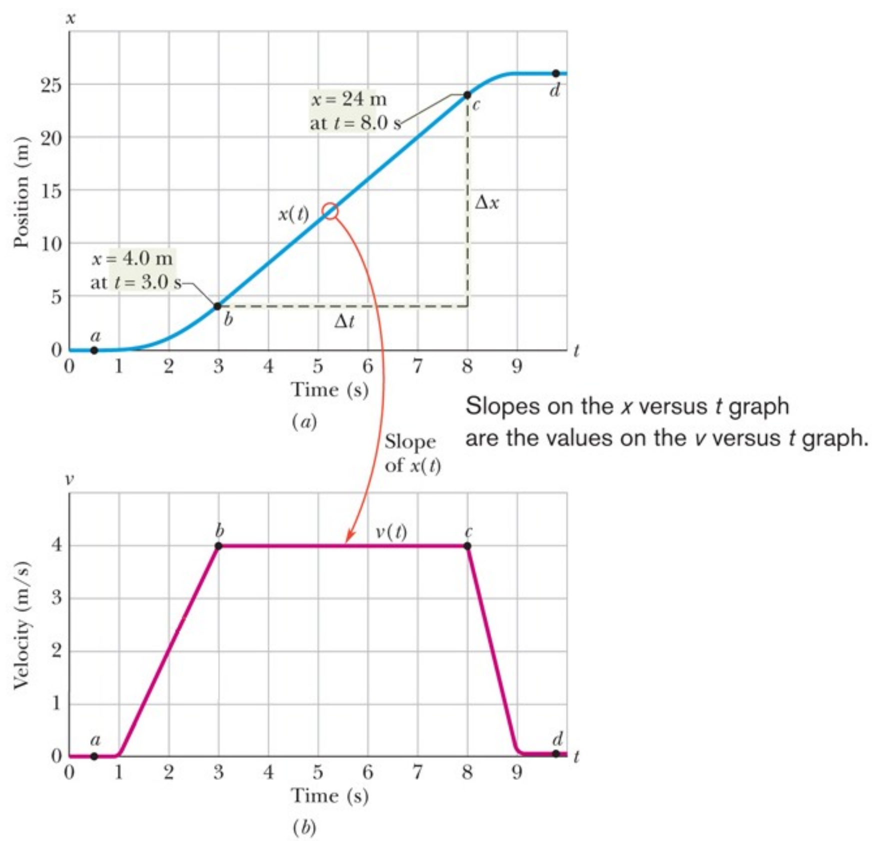

Figure 2-6

- The graph shows the position and velocity of an elevator cab over time.
- The slope of $x(t)$, and so also the velocity v, is zero from 0 to 1 s, and from 9s on.
- During the interval *bc*, the slope is constant and nonzero, so the cab moves with constant velocity (4 m/s).

2-3 Acceleration

Learning Objectives

2.10 Apply the relationship between a particle's average acceleration, its change in velocity, and the time interval for that change.

2.11 Given a particle's velocity as a function of time, calculate the instantaneous acceleration for any particular time.

2.12 Given a graph of a particle's velocity versus time, determine the instantaneous acceleration for any particular time and the average acceleration between any two particular times.

2-3 Acceleration

- A change in a particle's velocity is **acceleration**
- **Average acceleration** over a time interval Δt is

$$a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Eq. (2-7)

- **Instantaneous acceleration** (or just **acceleration**), *a*, for a single moment in time is:
  - Slope of velocity vs. time graph

$$a = \frac{dv}{dt}$$

Eq. (2-8)

2-3 Acceleration

- Combining Eqs. 2-8 and 2-4:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \qquad \textbf{Eq. (2-9)}$$

- Acceleration is a vector quantity:
  - Positive sign means in the positive coordinate direction
  - Negative sign means the opposite
  - Units of (*distance*) / (*time squared*)

If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.

2-3 Acceleration

Example If a car with velocity v = -25 m/s is braked to a stop in 5.0 s, then a = + 5.0 m/s^2. Acceleration is positive, but speed has decreased.

- Note: accelerations can be expressed in **units of *g***

$$1g = 9.8 \text{ m/s}^2 \qquad (g \text{ unit})$$ **Eq. (2-10)**

Checkpoint 3

A wombat moves along an x axis. What is the sign of its acceleration if it is moving (a) in the positive direction with increasing speed, (b) in the positive direction with decreasing speed, (c) in the negative direction with increasing speed, and (d) in the negative direction with decreasing speed?

Answers: (a) + (b) - (c) - (d) +

2-3 Acceleration

Example

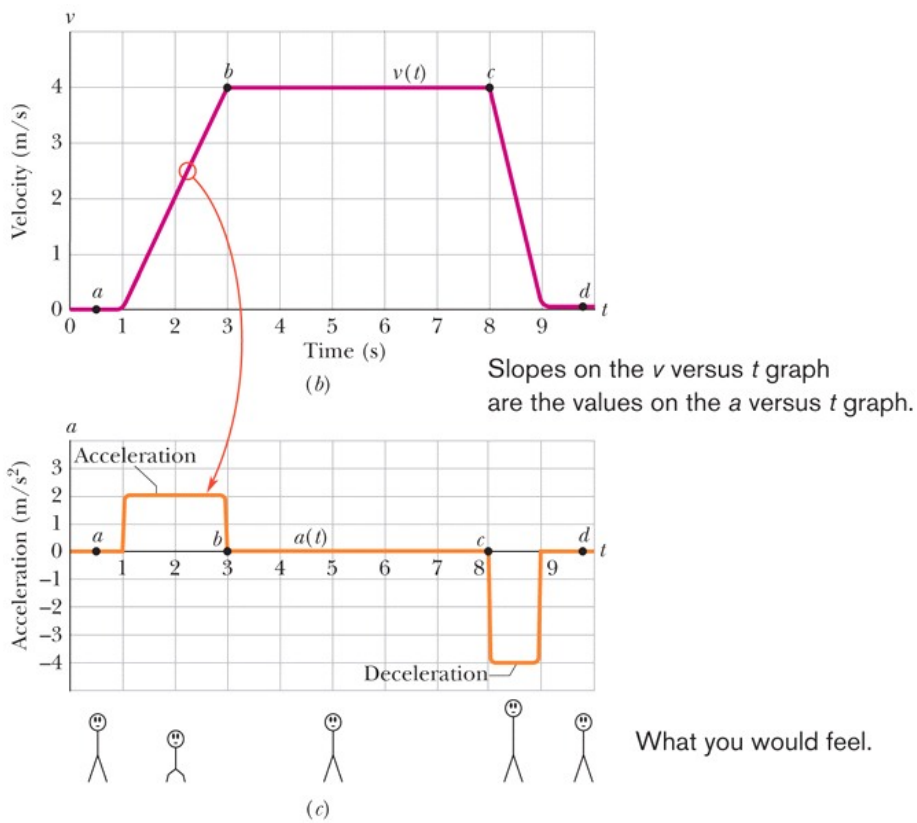

Figure 2-6

- The graph shows the velocity and acceleration of an elevator cab over time.
- When acceleration is 0 (e.g. interval *bc*) velocity is constant.
- When acceleration is positive (*ab)* upward velocity increases.
- When acceleration is negative (*cd)* upward velocity decreases.
- Steeper slope of the velocity-time graph indicates a larger magnitude of acceleration: the cab stops in half the time it takes to get up to speed.

2-4 Constant Acceleration

Learning Objectives

2.13 For constant acceleration, apply the relationships between position, velocity, acceleration, and elapsed time (Table 2-1).

2.14 Calculate a particle's change in velocity by integrating its acceleration function with respect to time.

2.15 Calculate a particle's change in position by integrating its velocity function with respect to time.

2-4 Constant Acceleration

- In many cases acceleration is constant, or nearly so.
- For these cases, **5 special equations** can be used.
- Note that constant acceleration means a velocity with a constant slope, and a position with varying slope (unless *a* = 0).

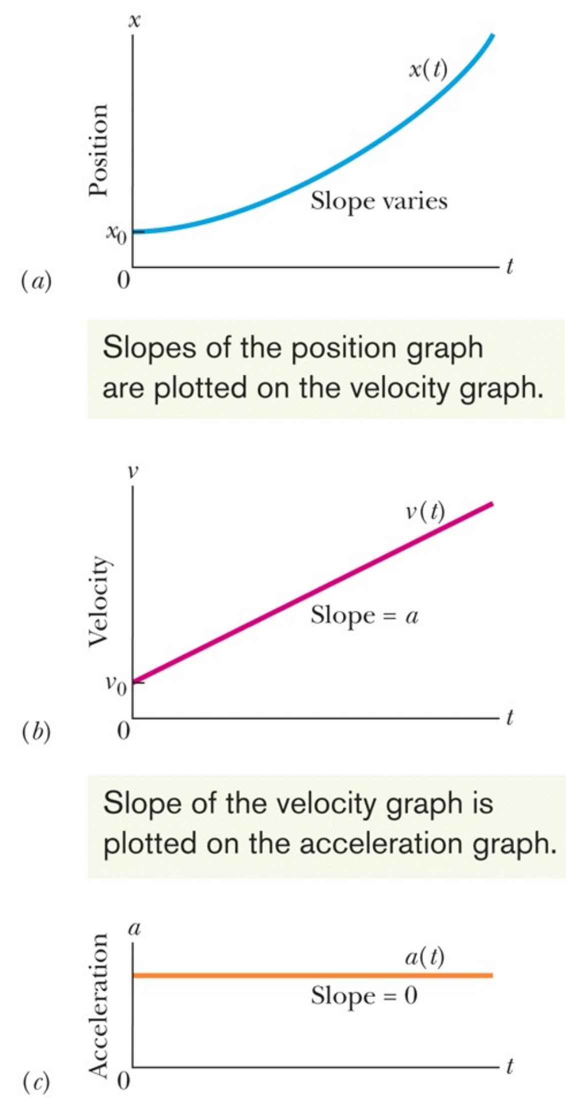

Figure 2-9

2-4 Constant Acceleration

- **First basic equation**
  - When the acceleration is constant, the average and instantaneous accelerations are equal
  - Rewrite Eq. 2-7 and rearrange

$$a = a_{\text{avg}} = \frac{v - v_0}{t - 0}$$

$$\boxed{v = v_0 + at}$$

Eq. (2-11)

- This equation reduces to $v = v_0$ for $t = 0$
- Its derivative yields the definition of a, dv/dt

2-4 Constant Acceleration

- **Second basic equation**
  - Rewrite Eq. 2-2 and rearrange

$$v_{avg} = \frac{x - x_0}{t - 0} \qquad x = x_0 + v_{avg} t \qquad \text{Eq. (2-12)}$$

  - Average = ((*initial*) + (*final*)) / 2: $v_{avg} = \frac{1}{2}(v_0 + v)$ **Eq. (2-13)**
  - Substitute 2-11 into 2-13

$$v_{avg} = v_0 + \frac{1}{2} at \qquad \text{Eq. (2-14)}$$

  - Substitute 2-14 into 2-12

$$x - x_0 = v_0 t + \frac{1}{2} at^2 \qquad \text{Eq. (2-15)}$$

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2-4 Constant Acceleration

- These two equations can be obtained by integrating a constant acceleration
- Enough to solve any constant acceleration problem
  - Solve as simultaneous equations
- Additional useful forms:

$$v^2 = v_0^2 + 2a(x - x_0) \qquad \textbf{Eq. (2-16)}$$

$$x - x_0 = \tfrac{1}{2}(v_0 + v)t \qquad \textbf{Eq. (2-17)}$$

$$x - x_0 = vt - \tfrac{1}{2}at^2 \qquad \textbf{Eq. (2-18)}$$

2-4 Constant Acceleration

- Table 2-1 shows the 5 equations and the quantities missing from them.

Table 2-1 Equations for Motion with Constant Acceleration[a]

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

[a]Make sure that the acceleration is indeed constant before using the equations in this table.

Table 2-1

Checkpoint 4

The following equations give the position $x(t)$ of a particle in four situations: (1) $x = 3t - 4$; (2) $x = -5t^3 + 4t^2 + 6$; (3) $x = 2/t^2 - 4/t$; (4) $x = 5t^2 - 3$. To which of these situations do the equations of Table 2-1 apply?

Answer: Situations 1 (a = 0) and 4.

2-5 Free-Fall Acceleration

Learning Objectives

- **2.16** Identify that if a particle is in free flight (whether upward or downward) and if we can neglect the effects of air on its motion, the particle has a constant downward acceleration with a magnitude *g* that we take to be $9.8 m/s^2$.
- **2.17** Apply the constant acceleration equations (Table 2-1) to free-fall motion.

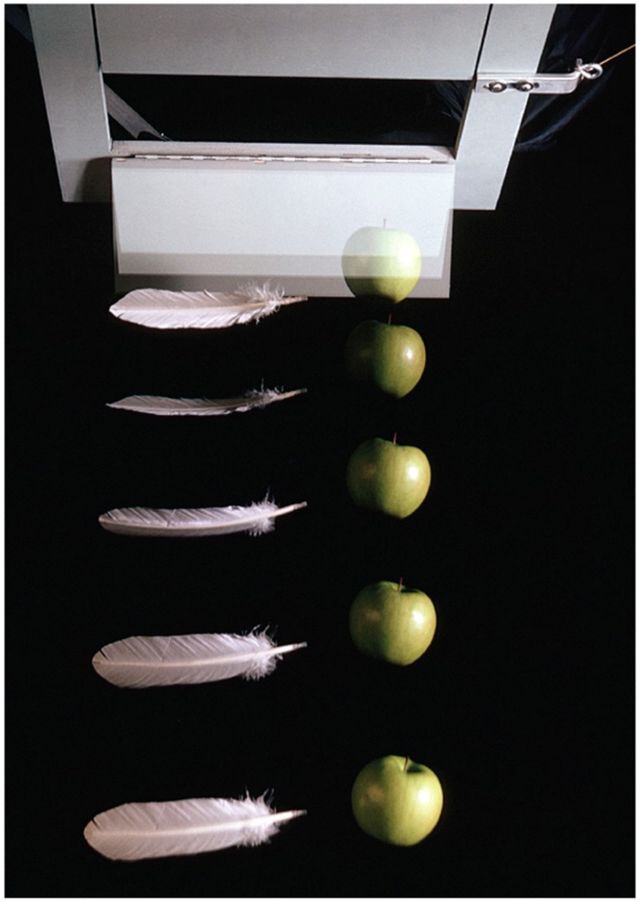

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Figure 2-12

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2-5 Free-Fall Acceleration

- **Free-fall acceleration** is the rate at which an object accelerates downward in the absence of air resistance
  - Varies with latitude and elevation
  - Written as *g*, standard value of 9.8 m/s^2
  - Independent of the properties of the object (mass, density, shape, see Figure 2-12)
- The equations of motion in Table 2-1 apply to objects in free-fall near Earth's surface
  - In vertical flight (along the *y* axis)
  - Where air resistance can be neglected

2-5 Free-Fall Acceleration

- The free-fall acceleration is downward (-*y* direction)
  - Value -*g* in the constant acceleration equations

The free-fall acceleration near Earth's surface is $a = -g = -9.8\ \text{m/s}^2$, and the *magnitude* of the acceleration is $g = 9.8\ \text{m/s}^2$. Do not substitute $-9.8\ \text{m/s}^2$ for g.

Checkpoint 5

(a) If you toss a ball straight up, what is the sign of the ball's displacement for the ascent, from the release point to the highest point? (b) What is it for the descent, from the highest point back to the release point? (c) What is the ball's acceleration at its highest point?

Answers:

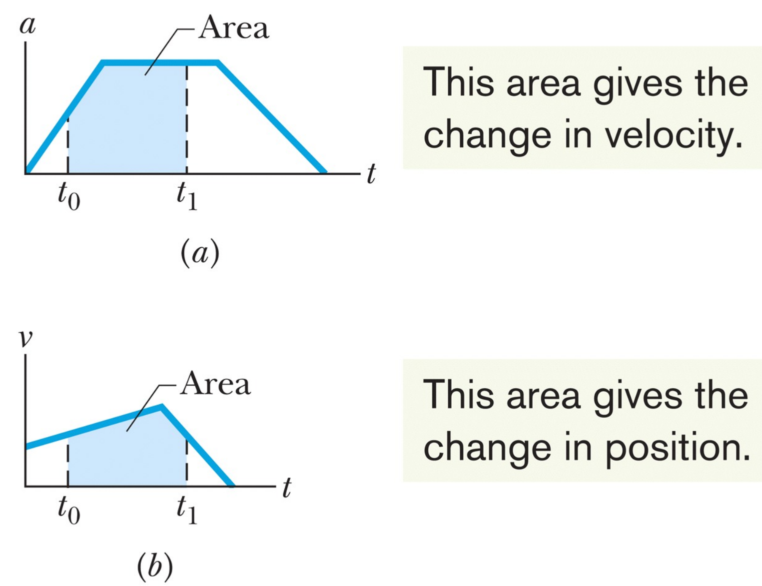
(a) The sign is positive (the ball moves upward); (b) The sign is negative (the ball moves downward); (c) The ball's acceleration is always -9.8 m/s^2 at all points along its trajectory

2-6 Graphical Integration in Motion Analysis

Learning Objectives

2.18 Determine a particle's change in velocity by graphical integration on a graph of acceleration versus time.

2.19 Determine a particle's change in position by graphical integration on a graph of velocity versus time.



2-6 Graphical Integration in Motion Analysis

- **Integrating acceleration**:
  - Given a graph of an object's acceleration *a* versus time *t*, we can integrate to find velocity
- The Fundamental Theorem of Calculus gives:

$$v_1 - v_0 = \int_{t_0}^{t_1} a\, dt$$ **Eq. (2-27)**

- The definite integral on the right can be evaluated from a graph:

$$\int_{t_0}^{t_1} a\, dt = \begin{pmatrix} \text{area between acceleration curve} \\ \text{and time axis, from } t_0 \text{ to } t_1 \end{pmatrix}$$ **Eq. (2-28)**

2-6 Graphical Integration in Motion Analysis

- **Integrating velocity**:
  - Given a graph of an object's velocity v versus time t, we can integrate to find position
- The Fundamental Theorem of Calculus gives:

$$x_1 - x_0 = \int_{t_0}^{t_1} v \, dt$$

Eq. (2-29)

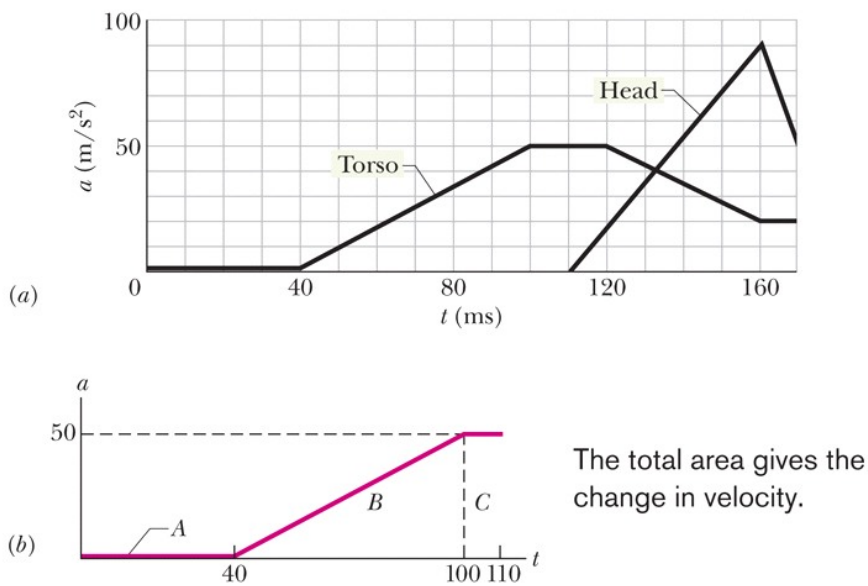
- The definite integral on the right can be evaluated from a graph:

$$\int_{t_0}^{t_1} v \, dt = \begin{pmatrix} \text{area between velocity curve} \\ \text{and time axis, from } t_0 \text{ to } t_1 \end{pmatrix}$$

Eq. (2-30)

2-6 Graphical Integration in Motion Analysis

Example



- The graph shows the acceleration of a person's head and torso in a whiplash incident.
- To calculate the torso speed at t = 0.110 s (assuming an initial speed of 0), find the area under the pink curve:

area A = 0

area B = 0.5 (0.060 s) (50 m/s^2) = 1.5 m/s

area C = (0.010 s) (50 m/s^2) = 0.50 m/s

total area = 2.0 m/s

2 Summary

Position

- Relative to origin
- Positive and negative directions

Displacement

- Change in position (vector)

$$\Delta x = x_2 - x_1$$ **Eq. (2-1)**

Average Velocity

- Displacement / time (vector)

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$ **Eq. (2-2)**

Average Speed

- Distance traveled / time

$$s_{avg} = \frac{\text{total distance}}{\Delta t}$$ **Eq. (2-3)**

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2 Summary

Instantaneous Velocity

- At a moment in time
- Speed is its magnitude

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Eq. (2-4)

Average Acceleration

- Ratio of change in velocity to change in time

$$a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Eq. (2-7)

Instantaneous Acceleration

- First derivative of velocity
- Second derivative of position

$$a = \frac{dv}{dt}$$

Eq. (2-8)

Constant Acceleration

- Includes free-fall, where *a* = -*g* along the vertical axis

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

Tab. (2-1)