PHYS1131 Higher Physics 1A
Homework Problem Set 6

Question 1

A wave of frequency 500Hz has a phase velocity of 350ms^{-1}.

(a) How far apart are the two points 60^o out of phase?

(b) What is the phase difference between two displacements at a certain point at times 10^{-3} sec apart?

Solution

The phase velocity $v = \omega / k$ with ω = 2πf

(a) wave function is $y = y_m \sin(kx - \omega t + \phi)$

At a particular time t,

$$(kx_1 - \omega t + \phi) - (kx_2 - \omega t + \phi) = \frac{\pi}{3}$$

$$\Rightarrow k(x_1 - x_2) = \frac{\pi}{3}$$

$$x_1 - x_2 = \frac{\pi}{3k} = \frac{\pi}{3}\frac{v}{\omega}, [\omega = vk]$$

$$= \frac{\pi \times 350}{3 \times 2\pi \times 500}$$

$$= 0.12m$$

(a) At particular <u>position</u> x, phase difference is

$$= (kx - \omega t_1 + \phi) - (kx - \omega t_2 + \phi)$$

$$= \omega(t_2 - t_1)$$

$$= 2\pi \times 500 \times 10^{-3} = \pi = 180^o$$

Question 2

A string vibrates according to the equation:

$$y = 0.5 \sin\frac{\pi x}{3}\cos 40\pi t ,$$

where x and y are in centimetres and t is in seconds.

(a). What are the amplitude and velocity of the component waves whose superposition can give rise to this vibration?

(b). What is the distance between nodes?

(c). What is the velocity of a particle of the string at the position x = 1.5 cm when t = $\frac{9}{8}$ s?

Solution

$$y = 0.5 \sin\frac{\pi x}{\underbrace{0.03}_{0.03m=3cm}}\cos 40\pi t$$

If y = y_1+y_2, what are y_1 and y_2?

Using $[\sin A + \sin B = 2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2})]$

$\Rightarrow \frac{A+B}{2} = \frac{\pi x}{0.03}$ and $\Rightarrow \frac{A-B}{2} = 40\pi t$

$\therefore A = (\frac{\pi x}{0.03} + 40\pi t)$ and $B = (\frac{\pi x}{0.03} - 40\pi t)$

$\therefore y_1 = 0.0025\sin(\frac{\pi x}{0.03} + 40\pi t)$ m

$= 0.0025\sin 2\pi(\frac{x}{0.06} + 20t)$ moving in the negative x direction (right to left).

Similarly,

$y_2 = 0.0025\sin(\frac{\pi x}{0.03} - 40\pi t)$ m

$= 0.0025\sin 2\pi(\frac{x}{0.06} - 20t)$ moving in the positive x direction (left to right).

(a). Amplitudes of y_1 and y_2 equal <u>0.0025m</u>, wave speed: v = λν = 0.06x20=<u>1.2ms^{-1}</u>.

(b). For nodes, y=0 <u>at all times</u> $\Rightarrow \sin\frac{\pi x}{0.03} = 0$.

$$\frac{\pi x}{0.03} = \underbrace{n\pi}_{\sin(n\pi)=0} \Rightarrow x = 0.03n$$

$\Rightarrow$Distance between two adjacent nodes is

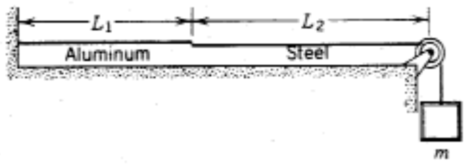
$x = 0.03(n+1) - 0.03n = 0.03m$

(c). $v = \frac{dy}{dt} = 0.5\sin\frac{n\pi}{3}.(-40\pi)\sin 40\pi t$

At t $= \frac{9}{8}$ s,

$$v = 0.5\sin(\frac{x\pi}{3}).(-40\pi)\underbrace{\sin(40\pi\frac{9}{8})}_{\sin(45\pi)} = 0$$

Question 3



An aluminium wire of length l_1 = 60.0 cm and of cross-sectional area 1.00x10^{-2}cm^2 is connected to a steel wire of the same cross-sectional area. The compound wire; loaded with a block m of mass 10.0 kg is arranged as shown in the diagram so that the distance l_2 from the joint to the supporting pulley is 86.6 cm. Transverse waves are set up in the wire by using an external source of variable frequency.

(a). Find the lowest frequency of excitation for which standing waves are observed such that the joint in the wire is a node.

(b). What is the total number of nodes observed at this frequency, excluding the two at the ends of the wire? The density of aluminium is 2.60 g cm^{-3}, and that of steel is 7.80 g cm^{-3}.

Solution.

We have $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{mg}{\rho A}}$

Aluminium

$l_1 = \dfrac{n_1}{2}\lambda_1$

$= \dfrac{n_1}{2}\dfrac{\upsilon_1}{f}$

$= \dfrac{n_1}{2f}\sqrt{\dfrac{mg}{\rho_1 A}}$

$f = \dfrac{n_1}{2l_1}\sqrt{\dfrac{mg}{\rho_1 A}}$

←f= freq. same on both wires. →

Steel

$l_2 = \dfrac{n_2}{2}\lambda_2$

$= \dfrac{n_2}{2}\dfrac{\upsilon_2}{f}$

$= \dfrac{n_2}{2f}\sqrt{\dfrac{mg}{\rho_2 A}}$

$f = \dfrac{n_2}{2l_2}\sqrt{\dfrac{mg}{\rho_2 A}}$

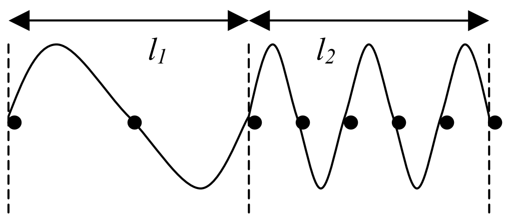
$$\therefore \frac{n_1}{2l_1}\sqrt{\frac{mg}{\rho_1 A}} = \frac{n_2}{2l_2}\sqrt{\frac{mg}{\rho_2 A}}$$

$$\frac{n_2}{n_1} = \frac{l_2}{l_1}\sqrt{\frac{\rho_2}{\rho_1}} = \frac{86.6}{60}\sqrt{\frac{7.80}{2.60}} = 2.5$$

the smallest integer values of n_1 and n_2 are $n_1 = 2$ and $n_2 = 5$.

$$\therefore f = \frac{v_1}{\lambda_1} = \frac{n_1}{2l_1}\sqrt{\frac{mg}{\rho_1 A}} = \frac{2}{2\times 0.60}\sqrt{\frac{10.0\times 9.8}{2.6\times 10^3\times 1.00\times 10^{-6}}} = 324Hz$$

(b). We know $n_1 = 2$ and $n_2 = 5$ which looks like:



⇒ 6 nodes and 7 loops

Question 4

A harmonic wave is given by the function

$$y = (2.0\text{ m})\sin 2\pi/\lambda(x - \mathrm{v}t)$$

where y is the displacement of the wave travelling in the x-direction at speed v. If the frequency of the wave is 2.0 Hz, what is the displacement y at x=0 when t=4.0s?

f = 2.0 Hz and f = v/λ

So $y = 2.0\ sin\left(\dfrac{2\pi x}{\lambda} - 2\pi ft\right)m$

$y(x = 0, t = 4) = 2.0\sin(-2\pi.2.4) = 2.0\sin(16\pi) = 0\text{ m}$

Question 5

Wavelength λ and propagation number (or 'wave number') k are related by $k = 2\pi/\lambda$. Show that a harmonic wave travelling in the positive x-direction at velocity v with wave function

$$y(x,t) = A\sin\frac{2\pi}{\lambda}(x - vt)$$

can be written in the alternative forms

(i) $y = A\sin k(x - vt)$

(ii) $y = A\sin(kx - \omega t)$.

Solution

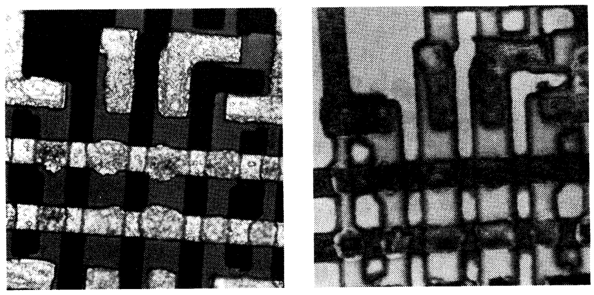
$$y(x, t) = A \sin \frac{2\pi}{\lambda}(x - vt)$$

(i) trivially, we know that $k = 2\pi/\lambda$ so
$y(x, t) = A \sin k(x - vt)$

(ii) We know that $\omega = vk$ where ω in the angular velocity and v is the wave propagation velocity.
$v = \omega/k$
and
$y(x, t) = A \sin (kx - \omega t)$

Question 6

The figure here shows part of a computer circuit (the surface of a processor or memory chip) viewed in two different types of microcope. The left figure was obtained by a regular optical light microscope. The view on the right was produced by an acoustic microscope by focussing 3 GHz ($3\text{x}10^9$ Hz) ultrasonic waves through a droplet of water on the chip's surface. The ultrasonic wave speed in water is 1.5 km/s. Find the wavelength of the ultrasonic travelling waves and compare this to the wavelength of green light (approximately middle of the visible spectrum) in air.



(Interested students can find some background at http://www.soest.hawaii.edu/~zinin/Zi-SAM.html)

Solution

For the acoustic microscope, wave propagation speed v = $1.5 \text{ x } 10^3$ ms^{-1}, f = $3 \text{ x } 10^9$ Hz

$$\lambda = \frac{v}{f} = \frac{1.5 \text{ x } 10^3 \text{ ms}^{-1}}{3 \text{ x } 10^9 \text{ Hz}} = 5 \text{ x } 10^{-7} \text{ m} = 500 \text{ nm}$$

The visible spectrum has wavelength range is approximately 400 nm (violet) to 750 nm (red). Green light is in the region 490 – 570 nm.

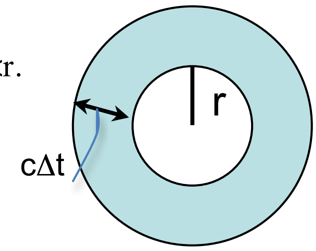
SOUND WAVE

Question 7

Calculate the energy density in a sound wave 4.82km from a 47.5kW siren, assuming the waves to be spherical, the propagation isotropic with no atmospheric absorption, and the speed of sound to be 343m/s.

Solution

In unit time the disturbance travels a distance c, which is <<r.



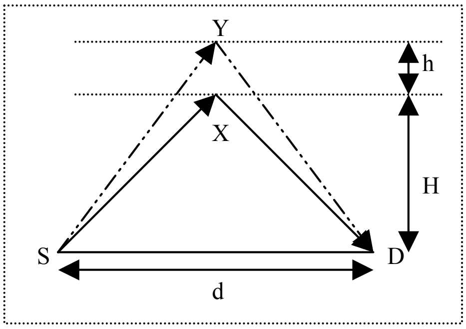
So Energy Density $= \dfrac{P}{4\pi r^2 c}$

Where $4\pi r^2 c$ is the volume increase in that time.

$$= \frac{4.75 \times 10^3}{4\pi \; 4820^2 \; 343} = 4.74 \text{ x } 10^{-7} \text{ J/m}^3 = 474 \text{ nJ/m}^3$$

Question 8

A source S and a detector D of high-frequency waves are a distance d apart on the ground. The direct wave from S is found to be in phase at D with the wave from S that is reflected from a horizontal layer at an altitude H. The incident and reflected rays make the same angle with the reflecting layer. When the layer rises a distance h, no signal is detected at D. Neglect absorption in the atmosphere and find the relation between d, h, H, and the wavelength A. of the waves.



Solution

Constructive interference when p.d = nλ (n integer) (p.d. = path difference)

$$\therefore \underbrace{(SX + XD) - SD}_{\text{p.d reflected \& direct path}} = n\lambda$$

$$2[(\frac{d}{2})^2 + H^2]^{\frac{1}{2}} - d = n\lambda \quad \_\_\_(1)$$

The reflecting layer rises from X to Y ⇒ complete destructive interference

∴ Assume reflected path length increased by λ/2.

$$\Rightarrow \underbrace{(SY + YD) - SD}_{\text{Condition for destructive interference}} = \underbrace{(n + \frac{1}{2})\lambda}_{\text{p.d. new (Y) reflected and direct path}}$$

$$2[(\frac{d}{2})^2 + (H+h)^2]^{\frac{1}{2}} - d = (n + \frac{1}{2})\lambda \quad \_\_\_(2)$$

Take (2) – (1) ⇒

$$\frac{1}{2}\lambda = 2[(\frac{d}{2})^2 + (H+h)^2]^{\frac{1}{2}} - 2[(\frac{d}{2})^2 + H^2]^{\frac{1}{2}}$$

$$\lambda = 2[d^2 + 4(H+h)^2]^{\frac{1}{2}} - 2[d^2 + 4H^2]^{\frac{1}{2}}$$

Question 9

Two waves give rise to pressure variations at a certain point in space given by:

$$P_1 = P\sin 2\pi\nu t, \qquad P_2 = P\sin 2\pi(\nu t - \phi).$$

What is the amplitude of the resultant wave at this point when $\phi=0$, $\phi=1/4$, $\phi=1/6$, $\phi=1/8$?

Solution

When ϕ=0:

$$P_1 + P_2 = P\sin(2\pi ft) + P\sin(2\pi ft)$$
$$= 2P\sin(2\pi ft)$$
$$\Rightarrow \text{Amplitude} = 2P$$

When ϕ=1/4:

$$P_1 + P_2 = P\sin(2\pi ft) + P\sin(2\pi ft - \frac{\pi}{2})$$
$$= 2P\sin(\frac{4\pi ft - \frac{\pi}{2}}{2})\cos(\frac{\pi}{4}) \quad \text{(using } \sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a-b}{2})$$
$$= \frac{2P}{\sqrt{2}}\sin(2\pi ft - \frac{\pi}{4})$$
$$\Rightarrow \text{Amplitude} = \frac{2P}{\sqrt{2}} \approx 1.41P$$

When ϕ=1/6:

$$P_1 + P_2 = P\sin(2\pi ft) + P\sin(2\pi ft - \frac{\pi}{3})$$
$$= 2P\sin(\frac{4\pi ft - \frac{\pi}{3}}{2})\cos(\frac{\pi}{6})$$
$$= \frac{2P \times \sqrt{3}}{2}\sin(2\pi ft - \frac{\pi}{6})$$
$$\Rightarrow \text{Amplitude} = \sqrt{3}P \approx 1.73P$$

When ϕ=1/8:

$$P_1 + P_2 = P\sin(2\pi ft) + P\sin(2\pi ft - \frac{\pi}{4})$$
$$= 2P\sin(\frac{4\pi ft - \frac{\pi}{4}}{2})\cos(\frac{\pi}{8})$$
$$= 2P\cos(\frac{\pi}{8})\sin(2\pi ft - \frac{\pi}{8})$$
$$\Rightarrow \text{Amplitude} \approx 1.85P$$

Question 10

A note of frequency 300 Hz has an intensity of I=1.0μW m^2. What is the amplitude of the air vibrations caused by this sound?

Solution

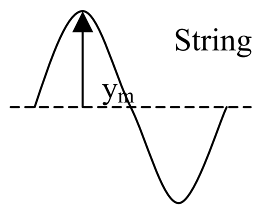
For a wave on a string

$\overline{P} = \frac{1}{2} y_m^2 \mu \upsilon \omega^2$, with y_m = amplitude, μ = mass per unit length of string, υ= wave velocity, ω = angular frequency.

Using ω = 2πf,

We get $\overline{P} = \frac{1}{2} y_m^2 \mu \upsilon (2\pi f)^2$

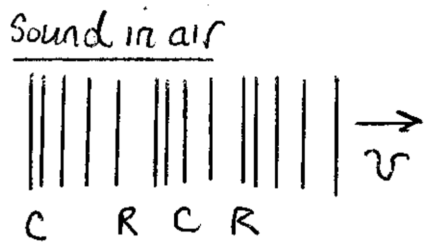
$\overline{P}$ formula true also for <u>sound waves</u> if we set $\mu = \rho_0 A$, ρ_0= equilibrium air density, A = unit area perpendicular to propagation direction.



The average intensity is $\bar{I} = \frac{\overline{P}}{A} = 2\pi^2 f^2 {y_m}^2 \rho_0 v$

∴ amplitude of particle displacement y_m is;

$$y_m = \frac{1}{\pi f} \left(\frac{\bar{I}}{2\rho_0 \upsilon}\right)^{\frac{1}{2}}$$



For air at 20^{0}C & atmospheric P density ρ_0=1.21 kgm^{-3} and sound velocity υ= 343ms^{-1},

$$y_m = \frac{1}{\pi(300Hz)} \left(\frac{1 \times 10^{-6}}{2(1.21kgm^{-3})(343ms^{-1})}\right)^{\frac{1}{2}}$$
$$= 3.7 \times 10^{-8} m$$

At 0^{0}C, y_m=3.7x10^{-8} m

Question 11

A certain sound level is increased by an additional 30 dB. Show that:

(a). Its intensity increases by a factor of 1000; and

(b). Its pressure amplitude increases by a factor of 32.

Solution

(a). Sound Level (SL) is $10 \log(\frac{I}{I_0}) dB$

I_0=10^{-12}Wm^{-2} a standard <u>reference intensity</u>.

$$\Rightarrow 30dB = 10 \log(\frac{I_2}{I_1}) dB$$

$$\log(\frac{I_2}{I_1}) = 3 \Rightarrow \frac{I_2}{I_1} = 1000$$

(b). Intensity I ∝(Δp)^2, (Δp = pressure amp, HRK p449)

$$\frac{I_2}{I_1} = \left(\frac{\Delta p_2}{\Delta p_1}\right)^2$$

$$\frac{\Delta p_2}{\Delta p_1} = \sqrt{10^3} = 32$$

Question 12

Two loudspeakers, S_1 and S_2, each emit sound of frequency 200 vib/sec uniformly in all directions. S_1 has an acoustic output of 1.2×10^{-3} watt and S_2 one of 1.8×10^{-3} watt. S_1 and S_2 vibrate in phase. Consider a point P which is 4.0m from S_1 and 3.0m from S_2.

(a). How are the phases of the two waves arriving at P related?

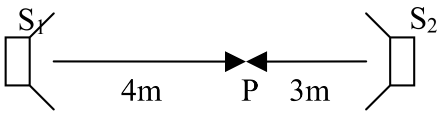
(b). What is the intensity of sound at P if S_1 is turned off (S_2 on)?

(c). What is the intensity of sound at P if S_2 is turned off (S_1 on)?

(d). Describe qualitatively how the intensity at P with both S_1 and S_2 on would compare with the sum of your answers to (b) and (c).

Solution

(a).



The path difference $S_1P - S_2P = (4.0\text{-}3.0)\text{m} = 1\text{m}$

$$\lambda = \frac{\upsilon}{f} = \frac{331ms^{-1}}{200\,Hz} = 1.66m$$

# of wavelengths fitting into p.d. = $\dfrac{1.0}{1.66} = 0.602$

$\therefore$ phase difference = 2π * 0.602 wavelengths = 3.8 radians

(1 complete 'cycle' of oscillation = 2π radians of phase covering 1 wavelength λ in one period T)

(b). I at P if S_1 off, S_2 on.

$$I = \frac{\overline{P}}{A},\ \overline{P} = \text{power in watts W.}$$

If speaker power is W watts at distance r, intensity is $I = \dfrac{W}{4\pi r^2}$ watts per m^2

$\therefore$ Intensity due to S_2 only = I_2 = $\dfrac{W}{4\pi r^2} = \dfrac{1.8 * 10^{-3}}{4\pi(3.0)^2} = 1.6 * 10^{-5}\,Wm^{-2}$

(c). I at P for S_1 only is

$$I_1 = \frac{W_1}{4\pi r_1^{\,2}} = \frac{1.2 * 10^{-3}}{4\pi(4.0)^2} = 6.0 * 10^{-6}\,Wm^{-2}$$

(d). It will be lower. If S_1 and S_2 were perfectly in phase at P they would add constructively and give the maximum intensity, the sum of (b) and (c). However, as shown in part (a) they are not in phase, as a result of this there is some destructive interference and the resulting intensity is less than the sum of (b) and (c).

Question 13

S in figure opposite is a small loudspeaker driven by an audio oscillator and amplifier, adjustable in frequency form 1000 to 2000 Hz only. D is a piece of cylindrical sheet-metal pipe 0.49 m long. Assume that the tube is open at both ends.

(a) If the velocity of sound in air is 339 ms^{-1} at the existing temperature, at what frequencies will resonance occur when the frequency emitted by the speaker is varied from 1000 to 2000 Hz?

(b) Sketch the displacement modes for each. Neglect end effects.

(c) Explain what end effects are and how they would change your results in the real case.

Solution.

Pipe is <u>open at both ends – antinodes</u> at both ends,

$\therefore$ wavelengths of allowed 'stationary' modes are $\lambda_n = \dfrac{2L}{n}$ (n = integer = 1, 2, 3, …)

and frequencies $f_n = \dfrac{nv}{2L}$.

Allowed λ_s and f_s are λ_0, f_0 (fundamental);

λ_1, f_1 (first overtone); λ_2, f_2 (second overtone) and so on.

We know $v=f\lambda$, we calculate

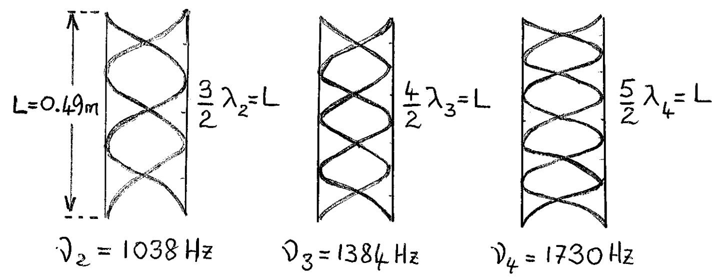
$$f_3 = \frac{v}{\lambda_3} = \frac{3.39*10^2 ms^{-1}}{0.327m} = 1038Hz$$

$$f_4 = \frac{v}{\lambda_4} = \frac{3.39*10^2 ms^{-1}}{0.245m} = 1384Hz$$

$$f_5 = \frac{v}{\lambda_5} = \frac{3.39*10^2 ms^{-1}}{0.196m} = 1730Hz$$

The modes look like (displacement modes);

Other modes give frequencies outside 1000-2000Hz.



Question 14

A tuning fork of unknown frequency makes three beats per second with a standard fork of frequency 384 Hz. The beat frequency decreases when a small piece of wax is put on a prong of the first fork. What is the frequency of this fork?

Solution

The beat frequency is $f_{beat} = |f_1 - f_2|$

3 beats/sec = $|f_2 - 384|$ Hz

Hence f_2=387Hz or 381 Hz

Adding wax will decrease (slow down) f_x. and decreases f_{beat} $\therefore f_x$ must be <u>higher</u> than f_s.

$\therefore f_x$ = 387Hz

SOUND WAVES AND DOPPLER EFFECT

Question 15

Sinusoidal vibrations of 20 Hz propagate along a coil spring. The distance between successive condensations in the spring is 30 cm.

(a) What is the speed of motion of the condensations along the spring?

(b) The maximum longitudinal displacement of a particle of the spring is 4 cm. Write down an equation for this wave motion for waves moving in the positive x direction and which have zero displacement at x =0 at time t =0.

(c) What is the maximum velocity experienced by the particle?

[Ans: (a) 6 ms^{-1} (b) y = 0.04 sin2π(20t-$\frac{x}{0.30}$); (c) 1.6π ms^{-1}.

Solution

(a) The speed of condensations, $\upsilon=\nu\lambda$, ν=20Hz, λ=0.30m
υ=(20)x(0.30)=6m/s

(b) The general travelling wave equation is
$y = y_m \sin 2\pi(\frac{x}{\lambda} - ft + \phi)$, The negative sign means wave is travelling right in positive x direction. The phase constant ϕ is zero because y=0 at x=0, t=0.
The general form becomes $y = 0.04 \sin 2\pi(\frac{x}{0.3} - 20t)$, y_m=amplitude=0.04m, f=20Hz, λ=0.3m

(c) The maximum velocity experienced by the particle (a piece of spring);
$\upsilon_{particle} = \frac{dy}{dt} = 2\pi(0.04)(20)\cos 2\pi(20t - \frac{x}{0.3})$,
which is maximum when $\cos 2\pi(20t - \frac{x}{0.3}) = 1$
$\Rightarrow \upsilon_{particle,max} = 2\pi(0.40)(20) = 1.6\pi$ m/s = 5.03m/s

Question 16

For a sound wave, the pressure amplitude ΔP_{max} is the maximum value of the change in pressure from the ambient pressure (when no wave is present in the medium). ΔP_{max} is related to the wave amplitude A by

$$\Delta P_{max} = (\frac{2\pi}{\lambda})\rho v^2 A$$

where ρ is the density of the medium and v is the wave velocity. Humans can tolerate values of pressure amplitude up to $\Delta P_{max} \sim 30$ Pa ; for these loud sounds, the pressure wave varies by ±30 Pa with respect to the ambient atmospheric pressure P ~ 10^5 Pa . What value of displacement amplitude does this correspond to at a frequency f = 1000Hz ? Take the density of air to be ρ_{air} =1.22 kgm^{-3} and the speed of sound at 37° C to be 353.7 ms^{-1}.

[Ans:0.011 mm]

Solution

We write a travelling sound (compressional) wave in terms of pressure (from lecture notes)

$$\Delta P(x, t) = B\, k\, s_{max} \sin(kx - \omega t)$$
$$= \Delta P_{max} \sin(kx - \omega t)$$

where *B* is the bulk modulus and s_m is the displacement amplitude. The wave propagation velocity is v given by

$$v = \sqrt{\frac{B}{\rho}}$$

for bulk modulus B and medium density ρ, so that

$B = \rho v^2$

Setting s_{max} =A = amplitude and $k = \dfrac{2\pi}{\lambda}$ we have

$$\Delta P_{max} = P_0 = \frac{2\pi}{\lambda} BA = \frac{2\pi}{\lambda}\rho v^2 A$$

Rearranging,

$$A = \frac{P_o \lambda}{2\pi\rho v^2}$$

and since v = fλ, $A = \dfrac{P_o}{2\pi\rho v f}$

Substituting v = 353.7 ms^{-1}, f = 1000 Hz and ρ = 1.22 kg m^{-3} we find

$$A = \frac{30\ \text{Pa}}{2\pi(1.22)\ (353.7)\ (1000)} = 1.1 \times 10^{-5} m$$

so the displacement amplitude A for a very loud sound is

A = 0.011 mm (or 11 μm)

Comment: We see that even for loud sounds the oscillatory displacement of the air molecules at 1 kHz is pretty small. At 40 Hz we find A = 0.27 mm.

Question 17

A siren emitting a sound of frequency 1000 Hz moves away from you towards a cliff at a speed of 10ms^{-1}.

(a) What is the wavelength of the sound you hear coming directly from the siren?

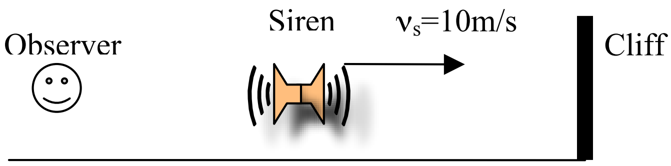
(b) What is the wavelength of the sound you hear reflected from the cliff?

(c) What is the difference in frequency between cases (a) and (b)?

(Velocity of sound in air = 340 ms^{-1})

[Ans: (a) 0.35 m; (b) 0.33 m; (c) 60 Hz]

Solution



(a) The wavelength of siren (source) moving away from stationary observer is:

$$f' = f(1+\frac{\upsilon_s}{\upsilon})^{-1} = 1000(1+\frac{10}{340})^{-1}$$, 340 = speed of sound

$= 971 Hz$

alternative solution

$$\lambda' = \frac{\upsilon}{f} + \frac{\upsilon_s}{f} = \frac{340 ms^{-1}}{1000 Hz} + \frac{10 ms^{-1}}{1000 Hz} = 0.35 m$$

(b) wavelength heard reflected from the cliff?

$$f'' = 1000(1-\frac{10}{340})^{-1} = 1030 Hz$$

$$\lambda'' = \frac{340 ms^{-1}}{1030 Hz} = 0.33 m$$

or $\lambda'' = \dfrac{\upsilon}{v} - \dfrac{\upsilon_s}{v} = \dfrac{340}{1000} - \dfrac{10}{1000} = 0.33 m$

(c) frequency difference $\Delta f = f'' - f' = (1030 - 971) Hz = 59 Hz$

Question 18

A tuning fork, frequency 297 Hz, is used to tune the D-string of two guitars at a temperature of 27°C when the velocity of sound in air is 340 ms^{-1}.

(a) What difference in frequency will the audience detect if one player is stationary and the other is moving towards the audience at 3 ms^{-1}?

(b) What difference in sound would there be if one player plucked the string at the centre point and the other at a point 1/7th the length of the string from one end?

(c) What length of open organ pipe would give a fundamental of 297 Hz at 27ºC? Derive the formula used.

(d) What different frequency would the organ pipe have if the temperature fell to 7ºC?

(e) What changes would occur in the note produced by the organ pipe if it had a hole at its half-way point?

[Ans: (a) increased by 2.62 Hz; (c) 0.572 m; (d) 287 Hz; (e) 594 Hz]

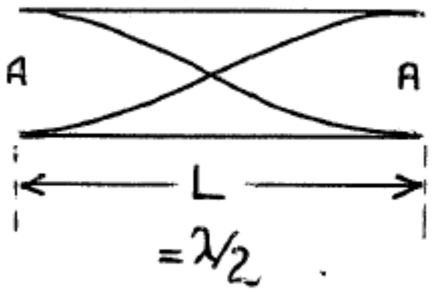
Solution

(a) $f' = f(\frac{\upsilon}{\upsilon - \upsilon_s}) = 297(\frac{340}{337}) = 299.64 Hz$, υ=sound, υ_s=source.

Δf=299.64-297=2.64Hz

(b) Timbre (distinctive character of a musical note) will include higher proportion of higher frequency components in (ii). Antinode occurs at the position where plucked, so that the frequency increases as the wavelength for the standing waves decrease.

(c) The fundamental mode has antinodes at both ends.



f_o=297Hz.

Pipe contains λ/2 when vibrating in fundamental mode.

$f_o = \frac{\upsilon}{\lambda_o} = \frac{\upsilon}{2L} \Rightarrow L = \frac{\upsilon}{2f_o} = \frac{340}{2 \times 297}$, where υ=velocity of sound = 340m/s

L=0.572m

(d) Change in organ pipe frequency if T = 7ºC ⇒ fall in T affects <u>sound velocity</u> (neglect and possible change in pipe length). Sound velocity changes according to;

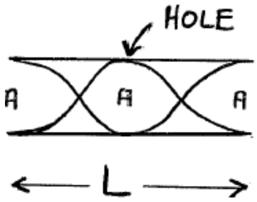
$\upsilon_{T_2} = \upsilon_{T_1}\sqrt{\frac{T_2}{T_1}}$, T_2= 7ºc=280K, T_1=27ºC=300K

$\upsilon_{7C} = \upsilon_{27C}\sqrt{\frac{280}{300}}$

$= 340\sqrt{\frac{280}{300}} = 328.5 m/s$

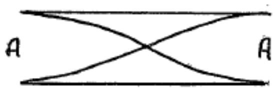
$f'_{7C} = \frac{f_{7C}}{2 \times 0.572} = 287.1 Hz$

(e) The change causes an antinode in the middle of the pipe so that;



(λ in pipe)

instead of



(λ/2 in pipe)

For the pipe with the hole,

$$f_h = \frac{\upsilon}{\lambda} = \frac{\upsilon}{L} = \frac{340}{0.572} = 594.4Hz \text{, (} f_h \text{ is just 2x } f_o = 2 \times 297 \text{ as expected)}$$

Question 19

A girl is sitting near the open window of a train that is moving at a velocity of 10.00 m/s to the east. The girl's uncle stands near the tracks and watches the train move away. The locomotive whistle emits sound at frequency 500.0 Hz. The air is still.

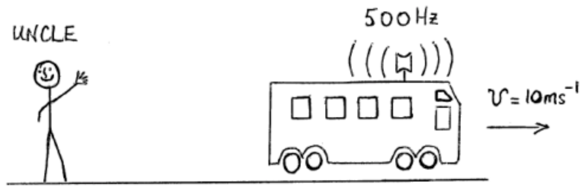
(a) What frequency does the uncle hear?
(b) What frequency does the girl hear?

A wind begins to blow from the east at 10.0 m/s

(c) What frequency does the uncle now hear?
(d) What frequency does the girl now hear?

[Ans (a) 485.8 Hz, (b) 500.0 Hz, (c) 486.2 Hz, (d) 500.0 Hz]

Solution



$f' = \left(\frac{f}{1 + V/c} \right)$ (a) , c = velocity of sound = 343m/s

$\therefore$ frequency observed by uncle = $500 \Big/ \left(1 + \frac{10}{343}\right) = = 485.8Hz$

(b) Girl is moving <u>with whistle</u> – she hears 500Hz

(c) $f' = \frac{f}{1 + \frac{V}{c_m}}$, with c_m velocity of medium (air)

Speed of sound for the observer is now $c_m = c + V_{wind} = 343 + 10 \text{ m/s} = 353 \text{ m/s}$

Uncle hears frequency $f' = 500 \left[\frac{1}{1 + 10/353} \right]$ as now c_m = 343 + 10 = 353 m/s

$= 486.2Hz$

(d) Girl is moving <u>with source</u> – she hears 500Hz.