Study Guide

AP Physics C: Mechanics

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Contents

1 Background

Easy. List of topics:

1. Vectors and Scalars
2. Addition, Subtraction, and Multiplication of Vectors
3. Dimensional Analysis

2 One-Dimensional Kinematics

List of topics:

1. Instantaneous Speed, Velocity, and Acceleration
2. Average Speed, Velocity, and Acceleration
3. Uniformly Accelerated Motion: Freely Falling Objects

Kinematics work **only when acceleration is constant**. Take derivatives and integrals of position, velocity, and acceleration to find the equations of motion. Use the equations of motion to solve problems.

2.1 Instantaneous Velocity

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \iff \Delta x = \int dx = \int_{t_0}^{t_1} v(t)dt \tag{1}$$

2.2 Instantaneous Speed

$$|v(t)| \tag{2}$$

PAY ATTENTION TO QUESTION! SPEED IS ALWAYS POSITIVE!
Speed is related to total distance traveled whereas velocity is related to the displacement vector.

$$\text{Displacement} \leq \text{Distance} \tag{3}$$

2.3 Instantaneous Acceleration

$$a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2} \iff \Delta v = \int dv = \int_{t_0}^{t_1} a(t)dt \tag{4}$$

2.4 Average Velocity

$$\overline{v} = \frac{\int_{t_0}^{t_1} v(t)dt}{t_1 - t_0} = \frac{\int_{t_0}^{t_1} dx}{t_1 - t_0} = \frac{\left[x \Big|_{t_0}^{t_1} \right]}{t_1 - t_0} = \frac{x(t_1) - x(t_0)}{t_1 - t_0} = \frac{\Delta x}{\Delta t} \tag{5}$$

- Derived from calculus average value of a function

2.5 Average Speed

$$\text{Average Speed} = \frac{\int_{t_0}^{t_1} |dx|}{t_1 - t_0} = \frac{\text{total distance}}{\Delta t} \tag{6}$$

- **AVERAGE SPEED IS NOT THE ABSOLUTE VALUE OF THE AVERAGE VELOCITY!**

Average Acceleration

$$\overline{a} = \frac{\int_{t_0}^{t_1} a(t)dt}{t_1 - t_0} = \frac{\int_{t_0}^{t_1} dv}{t_1 - t_0} = \frac{\left[v \Big|_{t_0}^{t_1} \right]}{t_1 - t_0} = \frac{v(t_1) - v(t_0)}{t_1 - t_0} = \frac{\Delta v}{\Delta t} \tag{7}$$

2.6 Graphical Interpretations:

Line connecting the two points is the average velocity (Path independent). The slope of the line is the average acceleration. The absolute value area under the line is the average speed.

2.7 Uniformly Accelerated Motion

By definition:

$$a(t) = \text{constant} = a \tag{8}$$

Velocity is the integral of acceleration:

$$v(t) = \int a(t)dt = at + v_0 \tag{9}$$

Position is the integral of velocity:

$$x(t) = \int v(t)dt = \frac{1}{2}at^2 + v_0 t + x_0 \tag{10}$$

Another unique property of UAM is that velocity is:

$$\overline{v} = \frac{v + v_0}{2} \tag{11}$$

This can be understood by examining how the average of a linear function is the midpoint of the line.

2.7.1 Position Without Reference to Time

From Equation 9:

$$t = \frac{v - v_0}{a}$$

Substitute into Equation 10:

$$v^2 = v_0^2 + 2a(x - x_0) \tag{12}$$

3 Two-Dimensional Kinematics

List of topics:

1. Instantaneous Velocity, Speed, and Acceleration in Two Dimensions
2. Uniformly Accelerated Motion Including Projectile Motion
3. Relative Position, Velocity, and Acceleration
4. Uniform Circular Motion

3.1 Position Vector

We can represent position in two dimensions using a vector. The position vector is defined as:

$$\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix} = x\hat{i} + y\hat{j} \tag{13}$$

3.2 Instantaneous Velocity

$$\vec{v}(t) \equiv \lim_{\Delta t \to 0} = \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \iff \Delta \vec{r} = \int d\vec{r} \equiv \int_{t_0}^{t_1} \vec{v}(t)dt \tag{14}$$

3.3 Instantaneous Speed

$$||v(t)|| = \sqrt{v_x^2 + v_y^2} \tag{15}$$

3.4 Instantaneous Acceleration

$$\vec{a}(t) \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \equiv \frac{d^2\vec{r}}{dt^2} \iff \Delta \vec{v} = \int d\vec{v} \equiv \int_{t_0}^{t_1} \vec{a}(t)dt \tag{16}$$

We can represent two-dimensional vectors by sets of one-dimensional vectors. For example, the velocity vector can be represented by two one-dimensional vectors:

$$\begin{gathered} \vec{v}(t) = \begin{pmatrix} v_x(t) \\ v_y(t) \end{pmatrix} = v_x(t)\hat{i} + v_y(t)\hat{j} \\ \frac{d\vec{r}}{dt} = \begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} \end{gathered} \tag{17}$$

The two dimensions are independent of each other (other than being synchronized by time). We can create a parametric equation for the velocity vector:

$$\begin{cases} v_x(t) = \dfrac{dx}{dt} = v_{x_0} + a_x t \\ v_y(t) = \dfrac{dy}{dt} = v_{y_0} + a_y t \end{cases} \tag{18}$$

3.5 Displacement Vector

Displacement vector points from the initial position to the final position. It is defined as:

$$\Delta\vec{r} = \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \Delta x\hat{i} + \Delta y\hat{j} \tag{19}$$

3.6 Average Velocity

The average velocity during a given time interval is parallel to the displacement vector (Multiplying by a scalar does not change the direction of a vector):

$$\bar{\vec{v}} = \frac{\Delta\vec{r}}{\Delta t} \tag{20}$$

3.7 Projectile Motion

Conversion from rectangular to polar form:

$$v = \sqrt{v_x^2 + v_y^2} \tag{21}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) \tag{22}$$

Conversion from polar to rectangular form:

$$v_x = v\cos\theta \tag{23}$$

$$v_y = v\sin\theta \tag{24}$$

We can separate projectile motion into two components: horizontal and vertical. They are independent from one another.

3.7.1 Horizontal Motion

Horizontal motion is uniform motion in the x-direction. The horizontal velocity is constant. The horizontal acceleration is zero.

$$v_x(t) = v_{x_0} = \text{constant} \tag{25}$$

$$x(t) = x_0 + v_{x_0}t \tag{26}$$

3.7.2 Vertical Motion

The acceleration in the y-direction is constant. The initial velocity in the y-direction is zero. The initial position in the y-direction is zero.

$$a_y = -g \tag{27}$$

$$v_y(t) = v_{y_0} + a_y t \tag{28}$$

$$y(t) = \frac{1}{2}a_y t^2 + v_{y_0}t + y_0 \tag{29}$$

$$v_y^2 = v_{y_0}^2 - 2g(y - y_0) \tag{30}$$