Spinning Things Lab: The Windmill and I

AP Physics C: Mr. Perkins

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1 Observed Acceleration

1.1 Procedure

Using a windmill with 4 rods with weights at their ends, we wounded a string on the rung of the windmill, attaching a 100g to the end of the string. We then dropped the weight 0.77m and measured the time it took to reach the bottom. After the weight dropped, we measured the time it took for the windmill to make 10 revolutions to compute the angular velocity of the windmill. With this, we were able to compute the angular acceleration and moment of inertia of the windmill.

1.2 Data

Measurement	**Variable**	**Value**
Distance	h	$0.77\,\mathrm{m}$
Mass of dropped weight	m	$0.1\,\mathrm{kg}$
Time to drop	t_d	$20.08\,\mathrm{s}$
Time to make 10 revolutions	t_r	$16.9\,\mathrm{s}$

Figure 1: Recorded Data

1.3 Analysis

Angular Velocity After Weight Dropped:

$$\begin{aligned} \omega &= \frac{\theta}{t} \\ \omega_f &= \frac{2\pi(10)}{t_r} = \frac{20\pi}{16.9} \\ &\approx 3.7179\,\frac{\mathrm{rad}}{\mathrm{s}} \end{aligned}$$

Angular Acceleration:

$$\begin{aligned} \alpha &= \frac{\omega_f - \omega_i}{t} \\ &= \frac{3.7179 - 0}{20.08} \\ &\approx 0.1852\,\frac{\mathrm{rad}}{\mathrm{s}^2} \end{aligned}$$

Moment of Inertia:

$$\begin{aligned} mgh &= \frac{1}{2} I {\omega_f}^2 \\ I &= \frac{2mgh}{{\omega_f}^2} \\ &= \frac{2(0.1)(9.81)(0.77)}{(3.7179)^2} \\ &\approx 0.109\,29\,\mathrm{kgm}^2 \end{aligned}$$

2 Theoretical Acceleration

2.1 Procedure

We recorded the masses of the rods, weights, and the center of the windmill as well as the lengths of the rods. Since the rod is inserted into the pivot of the windmill but does not go to the center, we measured the distance from the pivot to the rod to compute the moment of inertia.

2.2 Data

Measurement	Variable	Value
Mass of rod	m_r	0.074 kg
Mass of weight	m_w	0.186 kg
Radius of weight	r_w	0.34 m
Radius of pulley	r_p	0.02 m
Length of rod	l	0.3 m
Distance from center to rod	d_r	0.05 m
Distance from center to weight	d_w	0.33 m
Moment of inertia of pulley assembly alone	I_p	0.000 58 kgm^2

Figure 2: Recorded Data

2.3 Analysis

The moment of inertia of the windmill is given by:

$$I = I_p + 4I_r + 4I_w$$

Inertia of Rod I_r:

$$\begin{aligned} I_r &= \int r^2 dm \\ &= \frac{m_r}{l} \int_{d_r}^{l+d_r} x^2 dx \\ &= \frac{0.074}{0.3} \int_{0.05}^{0.35} x^2 dx \\ &\approx 0.003\,52\,\mathrm{kgm}^2 \end{aligned}$$

Inertia of Weight I_w:

$$\begin{aligned} I_w &= m_w (r_w)^2 \\ &= (0.186)(0.34)^2 \\ &\approx 0.021\,50\,\mathrm{kgm}^2 \end{aligned}$$

Inertia of Windmill:

$$\begin{aligned} I &= I_p + 4I_r + 4I_w \\ &= 0.00058 + 4(0.00352) + 4(0.02150) \\ &\approx 0.100\,66\,\mathrm{kgm}^2 \end{aligned}$$

3 Conclusion

The percent error in the moment of inertia of the windmill is given by:

$$\begin{aligned}\delta &= \frac{|I_A - I_E|}{I_E} \cdot 100\% \\ &= \frac{|0.10929 - 0.10066|}{0.10066} \cdot 100\% \\ &= 8.57\%\end{aligned}$$

This is a relatively small error that can be attributed to the fact that the theoretical calculations disregarded non-conservative forces such as friction and air resistance. We will compute the energy lost. Let F be the tension force. Let F_W be the force of gravity. We first solve for F:

$$F = F_W - F_{\text{net}}$$

$$\begin{aligned}F_{\text{net}} &= ma \\ a &= \frac{2h}{{t_d}^2} \qquad F_W = mg \\ F_{\text{net}} &= m\frac{2h}{{t_d}^2}\end{aligned}$$

$$\begin{aligned}F &= mg - m\frac{2h}{{t_d}^2} \\ &= (0.1)(9.81) - (0.1)\frac{2(0.77)}{(20.08)^2} \\ &\approx 0.9806\,\text{N}\end{aligned}$$

Let r be the radius of the pulley. We can compute the angular acceleration of the windmill using:

$$\begin{aligned}\tau &= F \times r \\ \tau &= I\alpha \\ F \times r &= I\alpha \\ \alpha &= \frac{F \times r}{I}\end{aligned}$$

$$\begin{aligned}\alpha_A &= \frac{F \times r}{I_A} & \alpha_E &= \frac{F \times r}{I_E} \\ &= \frac{0.9806(0.02)}{0.10929} & &= \frac{0.9806(0.02)}{0.10066} \\ &\approx 0.1794\,\frac{\text{rad}}{\text{s}^2} & &\approx 0.1948\,\frac{\text{rad}}{\text{s}^2}\end{aligned}$$

We can compute the final angular velocity of the windmill using:

$$\omega = \omega_0 + \alpha t$$

$$\omega_A = 0.1794(20.08) \quad \omega_E = 0.1948(20.08)$$
$$\approx 3.6024\,\frac{\text{rad}}{\text{s}} \qquad \approx 3.9116\,\frac{\text{rad}}{\text{s}}$$

We can compute the energy lost using:

$$E_{\text{lost}} = \frac{1}{2}I_E{\omega_E}^2 - \frac{1}{2}I_A{\omega_A}^2$$

$$\begin{aligned} E_{\text{lost}} &= \frac{1}{2}(0.10066)(3.9116)^2 - \frac{1}{2}(0.10929)(3.6024)^2 \\ &\approx 0.0609\,\text{J} \end{aligned}$$

The percentage of energy that is lost is:

$$\begin{aligned} \delta_E &= \frac{E_{\text{lost}}}{E_A} \cdot 100\% \\ &= \frac{0.0609}{\frac{1}{2}(0.10066)(3.9116)^2} \cdot 100\% \\ &= 7.9083\% \end{aligned}$$

Due to this loss of energy in the system, it is reasonable that $I_A > I_E$ as with less energy, the angular velocity is slower meaning the windmill is more "stiff" and less resilient to rotating and thus has a greater moment of inertia. We can verify this by observing that the potential energy should equal the kinetic energy at the end of the experiment due to Conservation of Energy, assuming no energy is lost. $mgh = \frac{1}{2}I_E\omega_E^2$. Since mgh remains constant, with a decrease in ω, I must increase.