Rotational Momentum

AP Physics C: Mr. Perkins

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Due: January 11, 2023

1 Introduction

2 Data

Measurement	Variable	Value
Moment of Inertia of Windmill	I	$0.100\,66\,\mathrm{kgm}^2$
Mass of Car 1	m_1	0.284 kg
Mass of Car 2	m_2	0.534 kg
Distance Traveled	d	0.5 m
Radius of Windmill	r	0.325 m

Table 1: Measured Constants

Mass of Car (kg): 0.284		
Trial	**Time from Release to Impact (s)**	**Period (s/rev)**
1	0.68	12.99
2	0.81	11.74
3	0.81	12.16
Average	0.767	12.297

Table 2: Car 1 Data

Mass of Car (kg): 0.534		
Trial	**Time from Release to Impact (s)**	**Period (s/rev)**
1	1.01	9.51
2	1.01	9.43
3	1.03	8.26
Average	1.017	9.067

Table 3: Car 2 Data

3 Analysis

Let variables with subscript 1 denote the first car and variables with subscript 2 denote the second car.

3.1 Observational Windmill Speed

The rotational speed of the windmill, ω, is given by:

$$\omega_1 = \frac{2\pi}{T} = \frac{2\pi}{12.297} = 0.511\,\frac{\text{rad}}{\text{s}} \qquad \omega_2 = \frac{2\pi}{9.067} = 0.693\,\frac{\text{rad}}{\text{s}} \tag{1}$$

3.2 Theoretical Windmill Speed

We assume the car is moving on a frictionless surface. Therefore, the acceleration is constant meaning the velocity is given by:

$$v_1 = \frac{\Delta d}{\Delta t} = \frac{0.5}{0.767} = 0.652\,\frac{\mathrm{m}}{\mathrm{s}} \qquad v_2 = \frac{\Delta d}{\Delta t} = \frac{0.5}{1.017} = 0.492\,\frac{\mathrm{m}}{\mathrm{s}} \tag{2}$$

The linear momentum of the car is given by:

$$p_1 = m_1 v_1 = 0.284(0.652) = 0.185\,\mathrm{N\,s} \qquad p_2 = m_2 v_2 = 0.534(0.492) = 0.263\,\mathrm{N\,s} \tag{3}$$

The angle the car makes with the horizontal is $\theta = 90°$. The angular momentum of the car is given by:

$$\begin{aligned} L_1 &= p_1 r \sin\theta = 0.185(0.325) = 0.060\,\mathrm{N\,m\,s} \\ L_2 &= p_2 r \sin\theta = 0.263(0.325) = 0.085\,\mathrm{N\,m\,s} \end{aligned} \tag{4}$$

We assume that momentum is conserved in the system. Thus, after impact and coming to a complete stop, the momentum from the car is transferred to the windmill:

$$L_{\text{car}} = L_{\text{windmill}} = I\omega \tag{5}$$

We can solve for ω:

$$\omega_1 = \frac{L_1}{I} = \frac{0.060}{0.10066} = 0.596\,06\,\frac{\mathrm{rad}}{\mathrm{s}} \qquad \omega_2 = \frac{L_2}{I} = \frac{0.085}{0.10066} = 0.844\,42\,\frac{\mathrm{rad}}{\mathrm{s}} \tag{6}$$

4 Conclusion

The percent loss of momentum from the car to the windmill is given by:

$$\delta = \frac{|L_{\text{car}} - L_{\text{windmill}}|}{L_{\text{car}}} \times 100 \tag{7}$$

where L_{windmill} is the angular momentum using the observational windmill speed from Equation 1.

$$\begin{aligned} \delta_1 &= \frac{|L_1 - I\omega_1|}{L_1} \times 100 = \frac{|0.060 - 0.10066(0.511)|}{0.060} \times 100 = 14.27\% \\ \delta_2 &= \frac{|L_2 - I\omega_2|}{L_2} \times 100 = \frac{|0.085 - 0.10066(0.693)|}{0.085} \times 100 = 17.93\% \end{aligned} \tag{8}$$