Spinning Things Lab: The Windmill and I

AP Physics C: Mr. Perkins

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1 Observed Acceleration

1.1 Procedure

Using a windmill with 4 rods with weights at their ends, we wounded a string on the rung of the windmill, attaching a 100g to the end of the string. We then dropped the weight 0.77m and measured the time it took to reach the bottom. After the weight dropped, we measured the time it took for the windmill to make 10 revolutions to compute the angular velocity of the windmill. With this, we were able to compute the angular acceleration and moment of inertia of the windmill.

1.2 Data

Measurement	Variable	Value
Distance	h	0.77 m
Mass of dropped weight	m	0.1 kg
Time to drop	t_d	20.08 s
Time to make 10 revolutions	t_r	16.13 s

Figure 1: Recorded Data

1.3 Analysis

Angular Velocity After Weight Dropped:

$$\begin{aligned} \omega &= \frac{\theta}{t} \\ \omega_f &= \frac{2\pi(10)}{t_r} = \frac{20\pi}{16.13} \\ &\approx 3.895\,\frac{\text{rad}}{\text{s}} \end{aligned}$$

Angular Acceleration:

$$\begin{aligned} \alpha &= \frac{\omega_f - \omega_i}{t} \\ &= \frac{3.895 - 0}{20.08} \\ &\approx 0.194\,\frac{\text{rad}}{\text{s}^2} \end{aligned}$$

Moment of Inertia:

$$\begin{aligned} mgh &= \frac{1}{2}I\omega_f^2 \\ I &= \frac{2mgh}{\omega_f^2} \\ &= \frac{2(0.1)(9.81)(0.77)}{(3.895)^2} \\ &\approx 0.099\,58\,\text{kgm}^2 \end{aligned}$$

2 Theoretical Acceleration

2.1 Procedure

We recorded the masses of the rods, weights, and the center of the windmill as well as the lengths of the rods. We then used the Parallel Axis Theorem, combining the moment of inertia of the rods and the moment of inertia of the weights to compute the moment of inertia of the windmill.

2.2 Data

Measurement	Variable	Value
Mass of rod	m_r	0.074 kg
Mass of weight	m_w	0.186 kg
Radius of weight	r_w	0.34 m
Length of rod	l	0.3 m
Distance from center to rod	d_r	0.05 m
Distance from center to weight	d_w	0.33 m
Moment of inertia of pulley assembly alone	I_p	0.000 58 kgm^2

Figure 2: Recorded Data

2.3 Analysis

The moment of inertia of the windmill is given by:

$$I = I_p + 4I_r + 4I_w$$

Inertia of Rod I_r:

$$\begin{aligned} I_r &= \int r^2 dm \\ &= \frac{m_r}{l} \int_{d_r}^{l+d_r} x^2 dx \\ &= \frac{0.074}{0.3} \int_{0.05}^{0.35} x^2 dx \\ &\approx 0.003\,52\,\text{kgm}^2 \end{aligned}$$

Inertia of Weight I_w:

$$\begin{aligned} I_w &= m_w (r_w)^2 \\ &= (0.186)(0.34)^2 \\ &\approx 0.021\,50\,\text{kgm}^2 \end{aligned}$$

Inertia of Windmill:

$$\begin{aligned} I &= I_p + 4I_r + 4I_w \\ &= 0.00058 + 4(0.00352) + 4(0.02150) \\ &\approx 0.100\,66\,\text{kgm}^2 \end{aligned}$$

3 Conclusion

The percent error in the moment of inertia of the windmill is given by:

$$\begin{aligned}\delta &= \frac{|I_A - I_E|}{I_E} \cdot 100\% \\ &= \frac{|0.10066 - 0.09958|}{0.09958} \cdot 100\% \\ &\approx 1.08\%\end{aligned}$$

This error is negligible and can be attributed to how the expected value is calculated without accounting for non-conservative forces such as friction and air resistance. With friction, the angular velocity would be lower than the expected angular velocity, and thus the moment of inertia would be lower than the expected moment of inertia. With air resistance, the time for the 100g weight would be longer than the expected time, and thus the moment of inertia would be lower than the expected moment of inertia. These forces, as well as others, cause a loss in energy. This can be calculated by using the equation for energy conservation:

$$\begin{aligned}E_E &= E_A + W_{\text{lost}} \\ \frac{1}{2}I_E\omega_E^2 &= \frac{1}{2}I_A\omega_A^2 + W_{\text{lost}}\end{aligned}$$