CS 120: Intro to Algorithms and their Limitations

Lecture 6: — Thursday September 21, 2023

Pset Due: September 27, 2023 Denny Cao

§1 RAM Model

§1.1 Goals

- Unambiguous: Know what a step is
- Expressivity: Capture what we think of as algorithm
- Mathematical Simplicity: Not too many operations
- Technical Relevances: One model applies to all languages

Definition 1.1 (RAM Model). A program consists of:

- \bullet A set V of variables
- Commands C_0, C_1, \ldots, C_n each:
 - $\operatorname{var}_i = c, \operatorname{var}_i \in V, c \in nats$
 - $\operatorname{var}_0 = \operatorname{var}_1 \operatorname{op} \operatorname{var}_2, \operatorname{var}_0, \operatorname{var}_1, \operatorname{var}_2 \in V, \operatorname{op} \in \{+, -, \times, \div\}$
 - $\operatorname{var}_0 = M[\operatorname{var}_1]$
 - $-M[var_0] = var_1$
 - If var == 0, GO TO $K, K \in \{0_1, ... l\}$

Definition 1.2 (Computation w/RAM Program).

Initialize: Encode input in memory $M[0], \ldots, M[n-1]$ (input is size n) and M[k] = 0 if $k \ge n$.

Execution: Execute C_0, C_1, C_2, \ldots except that GOTO commands change order.

Output: When the program reaches line l, output := $M[\text{output_ptr}], \dots, M[\text{output} + \text{output_len}]$ $P(x) = \bot$ if it halts, $Time_p(x) = \#$ commands executed