

# MATH 22A: Vector Calculus and Linear Algebra

Week 1: September 6, 2023

Pset Due: September 13, 2023

Denny Cao

## §1 Administrative Stuff

- **Office Hours:** Sunday – Tuesday (4-6pm) and Thursday (7-9pm) at Northwest B101
- Homework available on Wednesdays and due next Wednesday
  - Submit on Gradescope
- Midterms: October 12 and November 9 at 4-5:30pm
- Finals: December 12

**Remark 1.1** ( $\sqrt{2}$  Myth). Pythagorians discovered that it was not possible to be a rational number in  $\frac{p}{q}$  form, which was against their beliefs of everything being able to be expressed as integers.

## §2 Linear Algebra

### §2.1 Applications of Linear Algebra

We can solve this simple system with matrices and augmented matrices:

$$10x + 5y = 3$$

$$5x - 2y = 7$$

$$\begin{bmatrix} 10 & 5 & 3 \\ 5 & -2 & 7 \end{bmatrix}$$

We can also represent graphs as matrices with adjacency matrix.

**Definition 2.1** (Linear Equation). A **linear equation** in the variables  $x_1, \dots, x_n$  is an equation in the following form:

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = b$$

where  $b$  and the coefficients  $a_1, \dots, a_n$  are real or complex numbers. A **system of linear equations** is a collection of one or more linear equations involving the same variables. A **solution** to the linear system is a list of  $(s_1, \dots, s_n)$  of numbers that are solutions to each of the linear equations in the system.

Fundamental questions for systems:

- Does there exist at least one solution?
- If solution exists, how can we find it?

- If solution exists, is it unique?

**Definition 2.2 (Unique and Inconsistent).** A linear system is **unique** if there is a solution and otherwise, **inconsistent** (if there is no solution).

**Fact 2.3.** A linear system has either no solution, exactly one solution, or infinitely many.

**Definition 2.4 (Elementary Row Operations).** The following elementary row operations preserve solutions:

- Replace one row by the sum of itself and a multiple of another
- Interchange two rows (Swap)
- Multiply all entries in a row by a non-zero constant (Scaling a vector)

**Definition 2.5 (Row Equivalent).** Two matrices are called **row equivalent** if there is a sequence of elementary row operations changing one matrix to another.

**Fact 2.6.** If the augmented matrices of two linear systems are row equivalent, then they have the same solution set.