Exam 1 Corrections

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Q1. The domain is the set of all real numbers. Determine the truth value of the following expression: $\forall x \exists y (x < 0 \lor y^2 = x)$

When $x < 0$, it satisfies the first condition, $x < 0$. When $x \geq 0$, $y = \sqrt{x}$ satisfies the second condition, $y^2 = x$.

$\forall x \exists y | x < 0 \to x < 0 \land \forall x \exists y | x \geq 0 \to y = \sqrt{x}$

The expression is true.

Q2. Prove that $\neg[p \lor (s \land (\neg p \to q))] \equiv \neg p \land (\neg s \lor \neg q)$ **using a truth table.**

p	q	s	$\neg p$	$\neg p \to q$	$s \land (\neg p \to q)$	$p \lor (s \land (\neg p \to q))$	$\neg[p \lor (s \land (\neg p \to q))]$
T	T	T	F	T	T	T	F
T	T	F	F	T	F	T	F
T	F	T	F	T	T	T	F
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	F
F	T	F	T	T	F	F	T
F	F	T	T	F	F	F	T
F	F	F	T	F	F	F	T

p	q	s	$\neg s$	$\neg q$	$\neg q \lor \neg s$	$\neg p$	$\neg p \land (\neg q \lor \neg s)$
T	T	T	F	F	F	F	F
T	T	F	T	F	T	F	F
T	F	T	F	T	T	F	F
T	F	F	T	T	T	F	F
F	T	T	F	F	F	T	F
F	T	F	T	F	T	T	T
F	F	T	F	T	T	T	T
F	F	F	T	T	T	T	T

Since the truth values of $\neg[p \vee (s \wedge (\neg p \rightarrow q))]$ are the same as $\neg p \wedge (\neg s \vee \neg q)$, the two statements are equivalent.

Q3. Write the statement:*Everybody loves Raymond* and its negation symbolically using quantifiers. Clearly state what variable represents what statement.
Let $L(x)$ be the statement that x loves Raymond, and the domain consists of everyone.
Original:

$$\forall x(L(x))$$

Negation:

$$\neg(\forall x(L(x)) \equiv \exists x | \neg L(x)$$

Q4. Consider the theorem *The second power of integers end only in the digits* $\{0, 1, 4, 5, 6, 9\}$
a. Write the statement of this theorem symbolically, as a conditional, as if you were to use a direct proof.
Let $P(x)$ be the statement that x is the second power of an integer, and the domain consists of integers.

$$\forall x \in \mathbb{Z}(P(x) \rightarrow x \bmod 10 \in \{0, 1, 4, 5, 6, 9\})$$

b. Write the statement symbolically, as a conditional, as if you were to prove it by contraposition.

$$\forall x \in \mathbb{Z}(x \bmod 10 \notin \{0, 1, 4, 5, 6, 9\} \rightarrow \neg P(x))$$

Q5. Suppose that the universe for x and y is $\{a, b, c, d\}$. Assume that $P(x, y)$ is a predicate that is true in the following cases: $P(a, d), P(b, a), P(b, b), P(b, c), P(c, a), P(c, b), P(c, d), P(d, a), P(d, b), P(d, d)$ and false otherwise.
Determine and justify whether each of the following is true or false:
a. $\forall x(P(x, y) \rightarrow y \neq x$
False. $x = b$. For $P(b, b)$, $y = x$.
b. $\forall x \exists y|((P(x, y) \vee P(y, x))$
True. $\forall x \exists y(P(x, y))$. $x = a, y = d$ $P(a, d)$. $x = b, y = a$ $P(b, a)$. $x = c, y = a$ $P(c, a)$. $x = d, y = a$ $P(d, a)$.
c. $\forall y \exists x|(y \neq x \wedge P(x, y))$
True. $y = a, x = b$ $P(b, a)$. $y = b, x = c$ $P(c, b)$. $y = c, x = b$ $P(c, b)$. $y = d, x = a$ $P(a, d)$.

Q6. Negate this statement symbolically: *No work, no credit without using it is not true that...*
Clearly indicate which variable is which statement.
Let w, c be the propositions that "there is no work", "there is no credit", respectively.
Original:

$$w \to c$$

Negation:

$$\begin{aligned} \neg(w \to c) &\equiv \neg(\neg w \vee c) \\ &\equiv w \wedge \neg c \end{aligned}$$

Q7. The domain of discourse are the students in a class. Define the predicates: $S(x)$ and $A(x)$ as x *received as an A on the test,* respectively.
Write symbolically: *Someone who did not study for the test received an A on the test.*

$$\exists x | (\neg S(x) \wedge A(x))$$

Q8. Determine whether the following argument is valid.

If I get a job then I will buy a new car or a new house.
<u>I won't buy a new house.</u>
$\therefore$ I will not get a job.

Let j, c, h be the propositions that "I get a job", "I will buy a new car", "I will by a new house", respectively.

$j \to (c \vee h)$	Premise	(1)
$\neg h$	Premise	(2)
$\neg(c \vee h) \to \neg j$	Law of Contraposition from (1)	(3)
$\neg c \wedge \neg h \to \neg j$	De Morgan's Laws from (3)	(4)

False. $\neg h$ is not sufficient to prove $\neg j$; we must know the truth value of c as well.

Q9. Prove: *The average of two odd integers is an integer.*

Proof. Direct. Let $a = 2c + 1, b = 2k + 1, c, k \in \mathbb{Z}$

$$\begin{aligned}\frac{a+b}{2} &= \frac{2c+1+2k+1}{2} \\ &= \frac{2c+2k+2}{2} \\ &= c+k+1\end{aligned}$$

The sum of integers is an integer. □

Q10. Prove by contraposition: *For every integer* $n, n^2 - 2n + 7$ *is even, then* n *is odd.*

Proof. Contraposition. For every integer n, if n is not odd, then $n^2 - 2n + 7$ is not even.
If n is not odd, n is even and can be rewritten as $2c, c \in \mathbb{Z}$

$$\begin{aligned}(2c)^2 - 2(2c) + 7 &= 4c^2 - 4c + 7 \\ &= 4c^2 - 4c + 6 + 1 \\ &= 2(2c^2 - 2c + 3) + 1 \\ &\text{Let } 2c^2 - 2c + 3 = k \\ &= 2k + 1\end{aligned}$$

Since $n^2 - 2n + 7$ can be rewritten as $2k + 1$, it is not even; it is odd. □