Question 16

Let L be the event that Ramesh is late. Let B be the event that Ramesh bikes to work. Let C be the event that Ramesh drives to work. Let U be the event that Ramesh takes the bus to work.

a)

$$P(C|L) = \frac{P(C) \cdot P(L|C)}{P(C) \cdot P(L|C) + P(B) \cdot P(L|B) \cdot P(U) \cdot P(L|U)}$$

$$= \frac{\frac{1}{3}(\frac{1}{2})}{\frac{1}{3}(\frac{1}{2}) + \frac{1}{3}(\frac{1}{5}) + \frac{1}{3}(\frac{1}{20})}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6} + \frac{1}{15} + \frac{1}{60}}$$

$$= \frac{\frac{1}{6}}{\frac{10}{60} + \frac{4}{60} + \frac{1}{60}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{4}}$$

$$= \frac{4}{6}$$

$$= \frac{2}{3}$$

Under this assumption, there is a $\frac{2}{3} \approx 67\%$ chance that he drove his car, given that he is late.

b)

$$\begin{split} P(C|L) &= \frac{P(C) \cdot P(L|C)}{P(C) \cdot P(L|C) + P(B) \cdot P(L|B) \cdot P(U) \cdot P(L|U)} \\ &= \frac{0.3(0.5)}{0.3(0.5) + 0.6(0.05) + 0.1(0.2)} \\ &= \frac{0.15}{0.15 + 0.03 + 0.02} \\ &= \frac{0.15}{0.2} \\ &= \frac{15}{20} \\ &= \frac{3}{4} \end{split}$$

Using the new information, there is a $\frac{3}{4} = 0.75\%$ chance that he drove his car, given that he is late.