Question 16

Let L be the even that Ramesh is late. Let B be the event that Ramesh bikes to work. Let C be the event that Ramesh drives to work. Let U be the event that Ramesh takes the bus to work.

a)

$$\begin{aligned}
P(C|L) &= \frac{P(C)\cdot P(L|C)}{P(C)\cdot P(L|C) + P(B)\cdot P(L|B)\cdot P(U)\cdot P(L|U)} \\
&= \frac{\frac{1}{3}\left(\frac{1}{2}\right)}{\frac{1}{3}\left(\frac{1}{2}\right) + \frac{1}{3}\left(\frac{1}{20}\right) + \frac{1}{3}\left(\frac{1}{5}\right)} \\
&= \frac{\frac{1}{6}}{\frac{1}{6} + \frac{1}{60} + \frac{1}{15}} \\
&= \frac{\frac{1}{6}}{\frac{10}{60} + \frac{1}{60} + \frac{4}{60}} \\
&= \frac{\frac{1}{6}}{\frac{1}{4}} \\
&= \frac{4}{6} \\
&= \frac{2}{3}
\end{aligned}$$

Under this assumption, there is a $\frac{2}{3} \approx 67\%$ chance that he drove his car, given that he is late.

b)

$$\begin{aligned}
P(C|L) &= \frac{P(C)\cdot P(L|C)}{P(C)\cdot P(L|C) + P(B)\cdot P(L|B)\cdot P(U)\cdot P(L|U)} \\
&= \frac{0.3(0.5)}{0.3(0.5) + 0.6(0.05) + 0.1(0.2)} \\
&= \frac{0.15}{0.15 + 0.03 + 0.02} \\
&= \frac{0.15}{0.2} \\
&= \frac{15}{20} \\
&= \frac{3}{4}
\end{aligned}$$

Using the new information, there is a $\frac{3}{4} = 0.75\%$ chance that he drove his car, given that he is late.