Study Guide

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1 Logic and Proofs

1.1 Propositional Logic

1.1.1 Truth Tables of Logical Operators

Proposition: A statement that is either true or false.

p	$\neg p$
T	F
F	T

Figure 1: Truth table for **negation**

p	q	$p \wedge q$	$p \vee q$	$p \oplus q$
T	T	T	T	F
T	F	F	T	T
F	T	F	T	T
F	F	F	F	F

Figure 2: Truth table for **bit operations**

1.1.2 Converse, Contrapositive, and Inverse

Converse: The proposition $q \Rightarrow p$ is the converse of the proposition $p \Rightarrow q$.

p	q	$q \Rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

Figure 3: Truth Table for converse of implication of two propositions p and q

Contrapositive: The proposition $\neg q \Rightarrow \neg p$ is the contrapositive of the proposition $p \Rightarrow q$.

- Same truth value as $p \Rightarrow q$

p	q	$\neg p$	$\neg q$	$\neg q \Rightarrow \neg p$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Figure 4: Truth Table for contrapositive of implication of two propositions p and q

Inverse: The proposition $\neg p \Rightarrow \neg q$ is the inverse of the proposition $p \Rightarrow q$.

p	q	$\neg p$	$\neg q$	$\neg p \Rightarrow \neg q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Figure 5: Truth Table for inverse of implication of two propositions p and q

1.1.3 Precedence of Logical Operators

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\Rightarrow$	4
$\Leftrightarrow$	5

Figure 6: Precedence of Logical Operators

1.2 Applications of Propositional Logic

1. Translating English into Propositional Logic and vice versa
2. Logic Puzzles

Intuitive.

1.3 Propositional Equivalences

1.3.1 Introduction

Tautology: A compound proposition that is always true.
Contradiction: A compound proposition that is always false.
Contingency: A compound proposition that is neither a tautology nor a contradiction.

p	$\neg q$	$p \vee \neg q$	$p \wedge \neg q$
T	T	T	F
T	F	T	F

Figure 7: Truth Table of an example of a Tautology and Contradiction

1.3.2 Logical Equivalences

Two propositions are **logically equivalent** if $p \Leftrightarrow q$ is a tautology. The following are important logical equivalences:

De Morgan's Laws
$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$
$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

Figure 8: De Morgan's Laws

Conditional-disjunction equivalence: $p \vee q \Leftrightarrow \neg p \Rightarrow q$

1.4 Predicates and Quantifiers

1.5 Nested Quantifiers

1.6 Rules of Inference

1.7 Introduction to Proofs

1.8 Proof Methods and Strategy