Portfolio Question 12 Resubmission

MATH 1700: Ideas in Mathematics

Professor Rimmer

Due: February 10, 2023 **Denny Cao**

Question 12

Explain why *each* of the numbers in the list

$$(1 \cdot 2 \cdot 3 \cdot 4) + 2, (1 \cdot 2 \cdot 3 \cdot 4) + 3, (1 \cdot 2 \cdot 3 \cdot 4) + 4$$

is composite. (You can show that they are composite by performing all of the relevant additions and multiplications, but it might help to think about how he way the numbers in the list were constructed guarantees that they are composite.)

Next show that for any natural number n, we can find a natural number m so that the numbers $m+1, m+2, \ldots, m+n$ are all composite. (In other words, for every natural number n, we can find n *consecutive* natural numbers which are all composite.) Be very careful with your choice of m. You might need to try out some examples to make sure your construction really works the way you want it to, even if you don't end up including those extra examples in your write-up.

Part 1

Proof. Let $a = (1 \cdot 2 \cdot 3 \cdot 4)$. For each number in the list, the number added to a is a factor of a. Because of this, we can factor out the the number added to a from both. We can rewrite each number in the list as follows:

$$2(1 \cdot 3 \cdot 4 + 1), 3(1 \cdot 2 \cdot 4 + 1), 4(1 \cdot 2 \cdot 3 + 1)$$

The definition of a composite number is a number that has factors other than 1 and itself. As each of these numbers are multiples of 2,3, and 4 respectively, they are all composite. □

Part 2

When

$$m = (n + 2)! + 2$$

Proof. m is the product of the first $n + 2$ natural numbers plus 2, or $m = (n + 2)! + 2$. Note that, as $(n + 2)!$ is the product of the first $n + 2$ natural numbers, each number from 1 to $n + 2$ is a factor of $(n + 2)!$. As the least value of n is 1, we can always factor out 2 from m.

Consider the list of numbers $m, m+1, m+2, \ldots, m+n$. We can rewrite this list by factoring as follows:

$$\begin{aligned}
m &= (n + 2)! + 2 = 2\left(\frac{(n + 2)!}{2} + 1\right) \\
m + 1 &= (n + 2)! + (2 + 1) = 3\left(\frac{(n + 2)!}{3} + 1\right) \\
m + 2 &= (n + 2)! + (2 + 2) = 4\left(\frac{(n + 2)!}{4} + 1\right) \\
&\vdots \\
m + n &= (n + 2)! + (n + 2) = (n + 2)\left(\frac{(n + 2)!}{n + 2} + 1\right)
\end{aligned}$$

The definition of a composite natural number is a number that has a factor other than 1 and itself. As each number in the list is a multiple of $2, 3, \ldots, n+2$, they are all composite. Thus, for any natural number n, we can find a natural number m so that the numbers $m+1, m+2, \ldots, m+n$ are all composite. $\square$