Portfolio Resubmission 3

MATH 1700: Ideas in Mathematics

Professor Rimmer

Due: April 26, 2023 **Denny Cao**

Worksheet 10 (Graph Theory I) Question 11

John and Chrissy invite their friends Bey and Jay to dinner. Before dinner, some of them shake hands. Watching the other three people at the party, Chrissy observes that one person shakes no hands, one person shakes exactly one hand, and one person shakes exactly two hands. Nobody shakes their own hand or their partner's hand. How many hands did Chrissy shake, and how do you know?

Final Draft

Chrissy shakes 1 hand.

Proof. Let C denote Chrissy, J_a denote Jay, B denote Bey, and J_o denote John.

Let X be the person who shakes no hands, Y be the person who shakes one hand, and Z be the person who shakes two hands.

As Chrissy observes that no one shakes their partner's hand, we know that Z shakes both people from the other group. Since X shakes no hands, it cannot be John who shakes 2 hands, as it would produce the following situation:

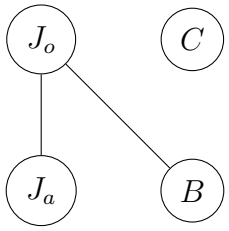

Figure 1: John shakes 2 hands ※

The above situation depicts 4 vertices—people—with one vertex, John, with degree 2—shaking 2 hands. This situation is a contradiction, as Chrissy observed someone shaking no hands, yet there are no vertices other than Chrissy with degree 0. Therefore, Z must be Jay or Bey, producing either of the following situations:

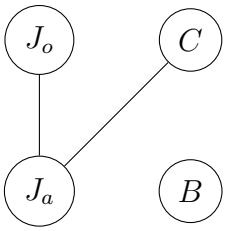

Figure 2: Jay shakes 2 hands

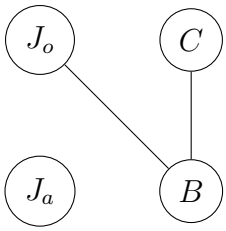

Figure 3: Bey shakes 2 hands

Chrissy's observations are consistent with either of these situations, as in both, there is a vertex incident to 0 edges, someone who shakes one hand, and someone who shakes two hands (or an edge with degree 0, an edge with degree 1, and an edge with degree 2):

Person (Vertex)	Number of Hands Shaken (Degree)
J_a	2
J_o	1
C	1
B	0

Figure 4: Degree of vertices when Jay shakes 2 hands

Person (Vertex)	Number of Hands Shaken (Degree)
B	2
J_o	1
C	1
J_a	0

Figure 5: Degree of vertices when Bey shakes 2 hands

In both cases, Chrissy shakes one hand. □

First Draft

Chrissy shakes 1 hand.

Proof. Let C be Chrissy, J_a be Jay, B be Bey, and J_o be John.

Let X be the person who shakes no hands, Y be the person who shakes one hand, and Z be the person who shakes two hands.

As Chrissy observes that no one shakes their partner's hand, we know that Z shakes both people from the other group. Since X shakes no hands, it cannot be John who shakes 2 hands, as it would produce the following situation:

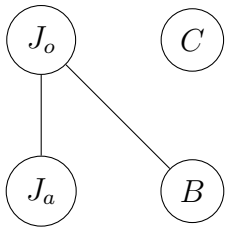

Figure 6: John shakes 2 hands

This contradicts the fact that Chrissy observed someone shaking no hands. Therefore, Z must be Jay or Bey, producing either of the following situations:

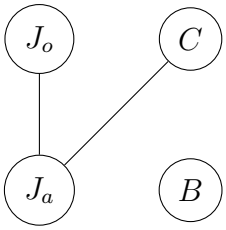

Figure 7: Jay shakes 2 hands

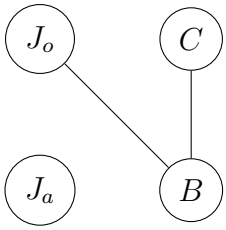

Figure 8: Bey shakes 2 hands

Chrissy's observations are consistent with either of these situations, as in both, there is someone who shakes no hands, someone who shakes one hand, and someone who shakes two hands. In both cases, Chrissy shakes one hand. □