Portfolio Question 12

MATH 1700: Ideas in Mathematics

Professor Rimmer

Due: February 10, 2023 **Denny Cao**

Question 12

Explain why *each* of the numbers in the list

$$(1 \cdot 2 \cdot 3 \cdot 4) + 2, (1 \cdot 2 \cdot 3 \cdot 4) + 3, (1 \cdot 2 \cdot 3 \cdot 4) + 4$$

is composite. (You can show that they are composite by performing all of the relevant additions and multiplications, but it might help to think about how he way the numbers in the list were constructed guarantees that they are composite.)

Next show that for any natural number n, we can find a natural number m so that the numbers $m + 1, m + 2, \ldots, m + n$ are all composite. (In other words, for every natural number n, we can find n *consecutive* natural numbers which are all composite.) Be very careful with your choice of m. You might need to try out some examples to make sure your construction really works the way you want it to, even if you don't end up including those extra examples in your write-up.

First Submission

Part 1

Proof. Let $a = (1 \cdot 2 \cdot 3 \cdot 4)$. For each number in the list, the number added to a is a factor of a. Because of this, we can factor out the the number added to a from both. We can rewrite each number in the list as follows:

$$2(1 \cdot 3 \cdot 4 + 1), 3(1 \cdot 2 \cdot 4 + 1), 4(1 \cdot 2 \cdot 3 + 1)$$

As each of these numbers are multiples of $2, 3$, and 4, respectively, they are all composite. $\square$

Part 2

When

$$m = \prod_{i=1}^{n} i$$

the numbers $m + 1, m + 2, \ldots, m + n$ are all composite.

Proof. As m is defined as the product of the first n natural numbers, each number from 1 to n is a factor of n. Thus, $\forall x \in \mathbb{N}(x \leq n \to x \mid m)$. Because of this, we can factor out each number from 1 to n from m. We can rewrite the list of numbers from $m + 1 \ldots m + n$ as follows:

$$1\Big(\frac{m}{1} + 1\Big), 2\Big(\frac{m}{2} + 1\Big), 3\Big(\frac{m}{3} + 1\Big), \ldots, n\Big(\frac{m}{n} + 1\Big)$$

As $\forall x \in \mathbb{N}(x \leq n \to x \mid m)$, $\dfrac{m}{x} \in \mathbb{N}$. As each number in the list is a multiple of x, they are all composite. $\square$

Peer Review

Suggestions	Communications	Strengths
Part 1 seems good, but for part 2 you might have to double-check whether the statement that $m + n$ is composite for all n is true. Maybe try plugging in numbers to check? An example might also help solidify the explanation a little bit.	**H** Show All Steps	Everything is explained well either through words or through math, nice explanations!
Perhaps explaining why a number is composite if it is a multiple of a number that is not 1? It sounds very elementary but I think it might be needed.	**H** Explain Why, Not Just What	Nicely explained through mathematical work, and it's easy to understand.
No suggestions	**C** Avoid Ambiguous Pronouns	Not much to say, good wording.
Perhaps define composite? Otherwise, it all seems good.	**C** Use Terminology Correctly	Cool use of the product operator and set theory!
Quite short explanations, however, they convey everything in a clear and concise manner. Maybe provide a little more explanation on how each solution is correct?	**H** Include Appropriate Level of Details	Nice and straightforward explanations, cool use of the product operator!
I don't think you need any diagrams for this problem, and your explanation doesn't really need a diagram.	**H** Create Useful Diagrams	N/A
No Suggestions.	**H** Use Clear and Direct Sentences	Sentences are all clear.
No suggestions.	**C** Check Problem Setup	Setup seems good
No suggestions.	**C** Check Your Calculations	All good!
Everything seems good, except for the statement that $1(\frac{m}{1} + 1)$ must be composite because having a factor of 1 does not prove that the number is composite.	**C** Verify Answer is Reasonable	A well-reasoned answer, just the problem with the $1(\frac{m}{1} + 1)$ statement.

Final Submission

Part 1

Proof. Let $a = (1 \cdot 2 \cdot 3 \cdot 4)$. For each number in the list, the number added to a is a factor of a. Because of this, we can factor out the the number added to a from both. We can rewrite each number in the list as follows:

$$2(1 \cdot 3 \cdot 4 + 1), 3(1 \cdot 2 \cdot 4 + 1), 4(1 \cdot 2 \cdot 3 + 1)$$

The definition of a composite number is a number that has factors other than 1 and itself. As each of these numbers are multiples of 2,3, and 4 respectively, they are all composite. □

Part 2

When

$$m = (n+1)! + 1$$

Proof. To show that for any natural number n, we can find a natural number m so that the numbers $m+1, m+2, \ldots, m+n$ are all composite, we will use the same idea as in the previous proof, but with a different value for a.

Let $a = (n+1)!$, the factorial of $n+1$. Note that a is divisible by all of the numbers $2, 3, \ldots, n+1$.

Consider the numbers $a+2, a+3, \ldots, a+n+1$. We can rewrite each number in the list as follows:

$$a + 2 = (n+1)! + 2 = 2((n-1)! + 1)$$

$$a + 3 = (n+1)! + 3 = 3((n-1)! + 2)$$

$$\vdots$$

$$a + n + 1 = (n + 1)! + n + 1 = (n + 1)((n - 1)! + n)$$

Note that each of these numbers is a multiple of a prime number larger than n, which means they are all composite.

To see why, let p be a prime number greater than n. Then, p does not divide $n!$, since all of the numbers $2, 3, \ldots, n$ are factors of $n!$. Therefore, p does not divide $(n - 1)!$, so p divides $(n - 1)! + k$ if and only if p divides k. Thus, $a + k$ is a multiple of p, which means $a + k$ is composite.

Therefore, we have shown that for any natural number n, we can find a natural number m so that the numbers $m + 1, m + 2, \ldots, m + n$ are all composite, by choosing $m = a + 1 = (n + 1)! + 1$. $\square$