Worksheet 4: Cardinality First Submission

MATH 1700: Ideas in Mathematics

Professor Rimmer

Due: February 15, 2023 **Denny Cao**

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1 Warm-Up Problems

Question 1. State what it means for sets A and B to have the same cardinality.

Answer 1. Sets A and B have the same cardinality if and only if there exists a bijection between A and B.

Question 2. State what it means for a set A to be *countable*.

Answer 2. A set A is countable if and only if it is finite or if A and $\mathbb{N}$ have the same cardinality—there exists a bijection between A and the set of natural numbers.

2 Some Differences Between Finite Sets and Infinite Sets

Question 3. Give an example of a function between two *infinite* sets with the same cardinality which is injective but not surjective.

Answer 3. Let $f : \mathbb{N} \to \mathbb{Z}$ be defined as

$$f(x) = \begin{cases} x & x \in \{2k \mid k \in \mathbb{N}\} \\ -x & x \in \{2k-1 \mid k \in \mathbb{N}\} \end{cases}$$

f is injective because every element of $\mathbb{Z}^+$ is mapped to a unique element of $\mathbb{Z}$. However, f is not surjective, as $0 \in \mathbb{Z}$ but does not have a preimage in $\mathbb{N}$.

Question 4. Give an example of a function between two *infinite* sets with the same cardinality which is surjective but not injective.