MATH 1700: Ideas in Mathematics

Worksheet 2: Numbers and Infinity First Submission

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Due: January 25, 2023

1 Warm-Up Problems

(1) **Briefly summarize the "machine method."**

The "machine model" strategy for arguing that a collection is infinite is based on the idea of a machine that can generate the elements of the collection. If the machine can generate elements indefinitely, then the collection is infinite. For example, consider the collection of all positive integers. A machine that can take any positive integer as input and output the next positive integer can generate all the positive integers indefinitely. Therefore, the collection of all positive integers is infinite.

(2) **Use the machine method to show that there are infinitely many odd natural numbers; i.e., infinitely many numbers that don't have 2 as a factor. Be sure to include an explanation why your machine works as intended.**

(3) **State what it means for a natural number to be prime, and what it means for a natural number to be composite.**

A prime natural number is a number that only has factors of itself and 1. A composite natural number is a number that is the product of two smaller natural numbers.

(4) **You receive the following message from a friend:**

> **There are infinitely many prime numbers. Here's why. We know that every natural number greater than one has a prime factor. There are infinitely many natural numbers greater 1. As the numbers numbers get bigger, their prime factors have to get bigger. Thus, there are infinitely many prime numbers. —- Your friend.**

How would you explain to your friend the flaw in their reasoning?

2 Step Toward a Proof That There Are Infinitely Many Primes