Worksheet 6: Knot Theory

MATH 1700: Ideas in Mathematics

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Question 1. Is it possible to switch some crossings in the Borromean rings to obtain a trefoil knot? Why or why not? If so, how many switches are needed? (Note that by switching a crossing, we mean changing which strand goes over/in front and which goes under/behind.)

Answer 1. Yes, it is possible to switch some crossings in the Borromean rings to obtain a trefoil knot. We label the crossings and strands in the Borromean rings as follows:

Question 2. Is it possible to switch some number of crossings in the trefoil knot to obtain an unknot? If so, what is the minimum number of switches needed?

Answer 2.

Question 3. For each of the knot diagrams on page 365 of your book (reproduced on the next page), give a sequence of Reidemeister to obtain another diagram of the same knot which you feel is as simple as possible. In each case, how do you know you can't do any better?

Answer 3.

Question 4. Are the figure-eight, cinquefoil knot, and three-twist knot (entries 41, 51, and 52 on your table, respectively) three-colorable?

Answer 4.

Question 5. Show that for any number $n > 2$, there is a knot diagram of the trefoil with exactly n crossings.

Answer 5.

Question 6. A knot diagram is called *alternating* if you can start anywhere in the knot, and alternate undercrossing and overcrossings until you are back where you started. Show that the usual picture of the trefoil (as shown in the top left corner of the table of knots on Canvas) is alternating. How is this possible for a knot diagram with an odd number of crossings to be alternating?

Answer 6.