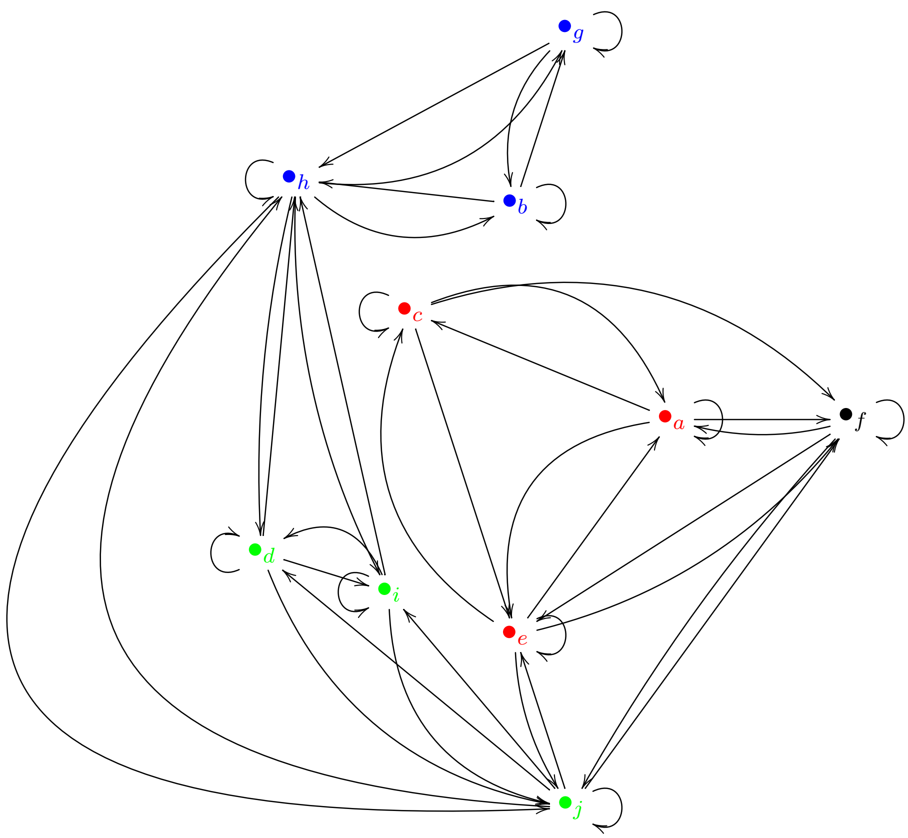
Homework 1B

MATH 263: Discrete Mathematics 2

** Dr. Petrescu

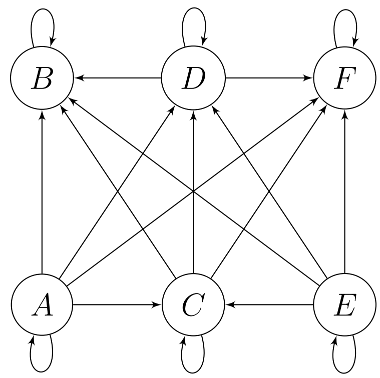
Due: February 10, 2023 **Denny Cao**

Problem 1: Determine whether the relation given by the digraph below is an equivalence relation. Justify your answer.



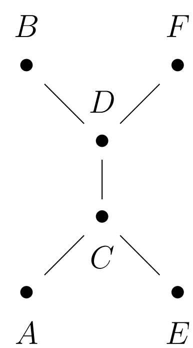
Proof. The relation is not an equivalence relation. aRf and fRj, but $a \not R j$. Thus, the relation is not a transitive relation, and therefore is not an equivalence relation, as equivalence relations must be reflexive, symmetric, and transitive. □

Problem 2: Determine whether the relation given by the digraph below is a partial order. If it is, draw its Hasse diagram.



Proof. The relation is a partial order, as it as reflexive, antisymmetric, and transitive.
The relation is reflexive, as $\forall x \in \{A, B, C, D, E, F\}(xRx)$. This can be seen by observing that all of the vertices are connected to themselves.

The relation is antisymmetric, as $\forall x \forall y \in \{A, B, C, D, E, F\}(xRy \to y \not R x)$. This can be seen by observing that there are no bidirectional edges.
The relation is transitive, as $\forall x \forall y \forall z \in \{A, B, C, D, E, F\}(xRy \wedge yRz \to xRz)$. $\square$



Problem 3: What is the transitive closure of the relation $R = \{(1, 2), (1, 4), (2, 3), (3, 1), (4, 2)\}$?
The original relation can be represented by the following matrix:

$$R^0 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

We apply Warshall's method to find the transitive closure of the relation:

$$R^1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad R^2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$R^3 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \quad R^4 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Problem 4: Show that the relation $R = \{(x, y) | x - y \in \mathbb{Z}\}$ is an equivalence relation on the set of rational numbers. What are the equivalence classes of 0 and $\frac{1}{2}$?