Homework 4

MATH 263: Discrete Mathematics 2

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Due: March 31, 2023 **Denny Cao**

Question 1.

(i) How many nonisomorphic non rooted trees are there with 4 vertices?

(ii) How many nonisomorphic rooted trees are there with 4 vertices?

(iii) How many nonisomorphic **non rooted** trees are there with 5 vertices?

Answer 1.

(i) 2

(ii) 4

(iii) 3

Question 2.

a) How many edges does a tree with 10,000 vertices have?

b) How many vertices does a full 5-ary tree with 100 internal vertices have?

c) How many edges does a full binary tree wtih 1,000 internal vertices have?

d) How many leaves does a full 3-ary tree with 100 vertices have?

Answer 2.

a) A tree with n vertices has $n-1$ edges. Thus, a tree with 10,000 vertices has $10,000-1 = 9,999$ edges.

b) A full m-ary tree with i internal vertices has $n = mi + 1$ vertices. Thus, a full 5-ary tree with 100 internal vertices has $n = 5 \times 100 + 1 = 501$ vertices.

c) A binary tree is a full 2-ary tree. Thus,a full binary tree wtih 1,000 internal vertices has $n = 2 \times 1,000 + 1 = 2,001$ vertices.

d) A full m-ary tree with n vertices has $\ell = \frac{(m-1)n+1}{m}$ leaves. Thus, a full 3-ary tree with 100 vertices has $\ell = \frac{(3-1)\times 100+1}{3} = 67$ leaves.

Question 3. Suppose that the address of the vertex v in the ordered rooted tree T is 3.4.5.2.4.

a) At what level is v?

b) What is the address of the parent of v?

c) What is the least number of siblings v can have?

d) What is the smallest possible number of vertices in T if v has this address?

Answer 3.

a) 5

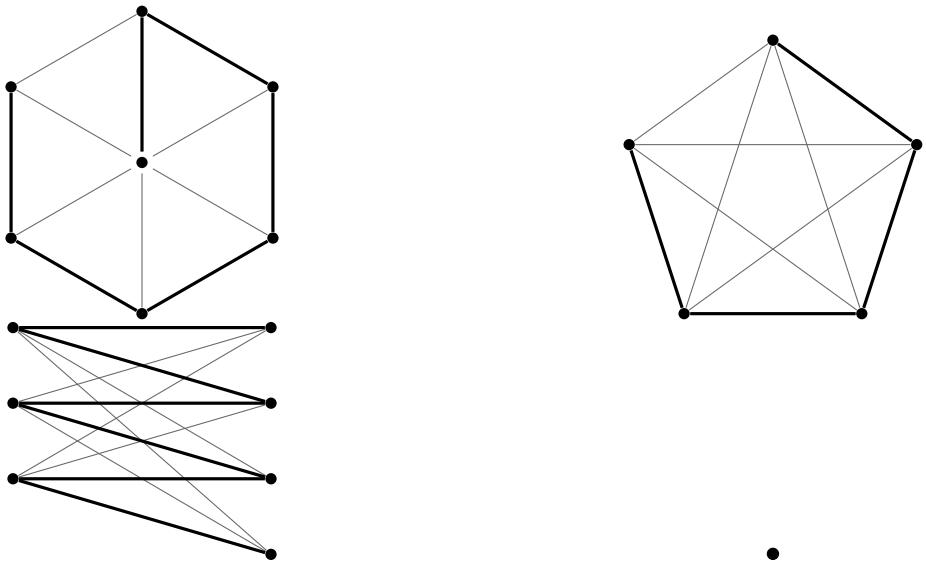
b) 3.4.5.2

c) 3

d) 15

Question 4. Use depth- first search to find a spanning tree of each of these graphs.
a) W_6 , starting at the vertex of degree 6 b) K_5
c) $K_{3,4}$, starting at a vertex of degree 3 d) Q_3

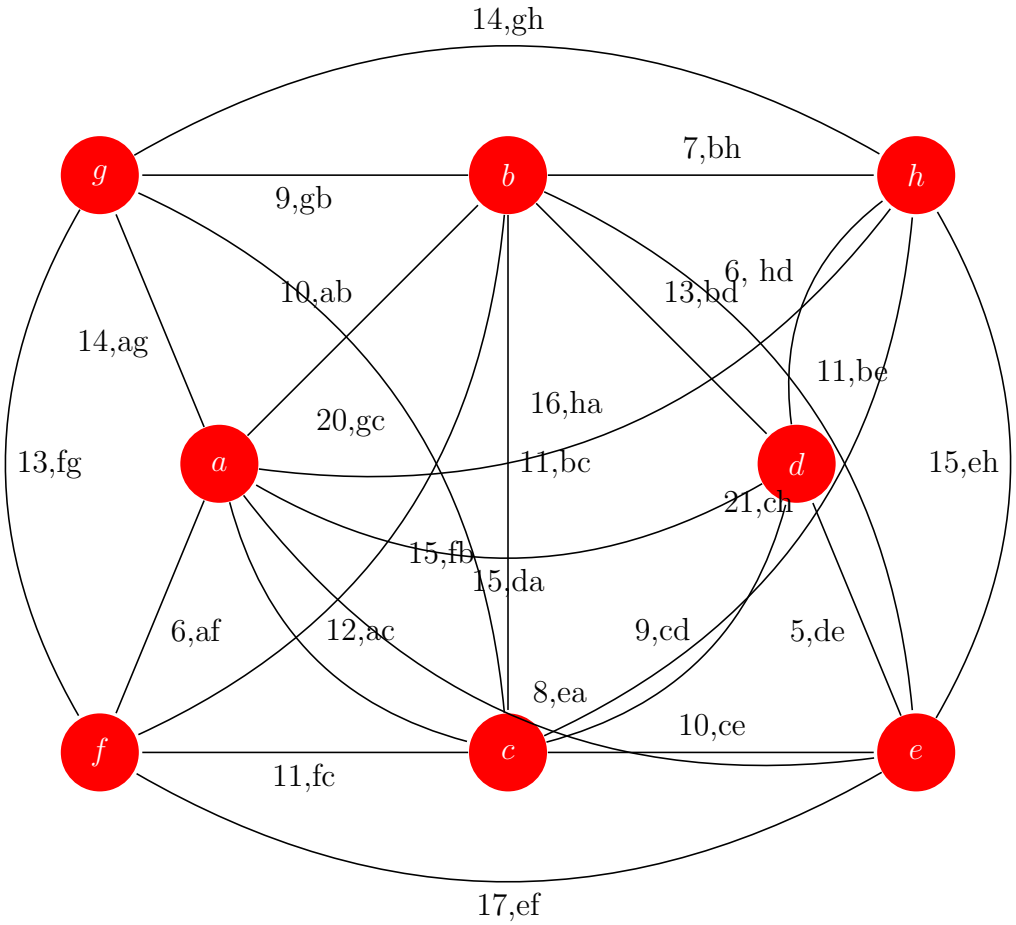
Answer 4.



Question 5. Prove Kruskal's Theorem.

Answer 5.

Question 6. Use Prim-Jarnik's or Kruskal's algorithm to find, step by step, the minimal spanning tree from the graph below. State what method you are using.



Answer 6.

Question 7. Describe the tree produced by breadth-first search and depth-first search for the n-cube graph Q_n, where n is a positive integer.

Answer 7.

Question 8. Build a binary search tree for the words: *oenology, phrenology, campanology, ornithology, ichthyology, limnology, alchemy,* and *astrology* using alphabetical order.

Answer 8.

Question 9. For the tree in Question 8 determine the order in which a inorder traversal visits the vertices of the given ordered rooted tree.

Answer 9.

Question 10. For the tree in Question 8 determine the order in which a postorder traversal visits the vertices of the given ordered rooted tree.

Answer 10.

Question 11. How many nonisomorphic unrooted trees are there with six vertices?

Answer 11.

Question 12. How many nonisomorphic rooted trees are there with six vertices

Answer 12.