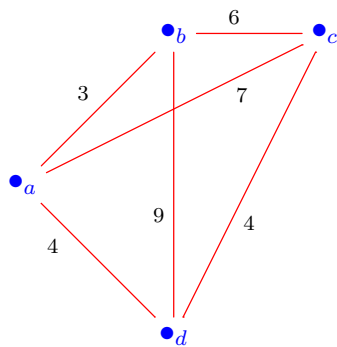
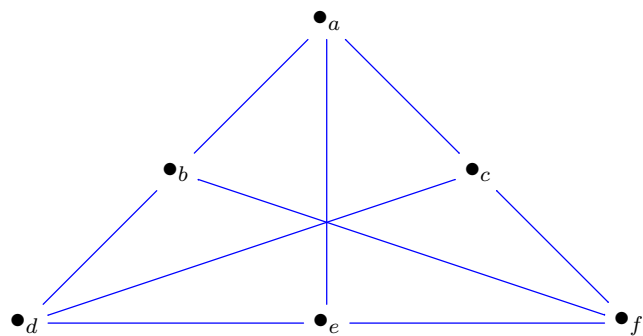


**Q 1.** *Solve the traveling salesperson problem for this graph by finding the total weight of all Hamilton circuits and determining a circuit with minimum total weight.*



**Q 2.** Try to draw the given graph without any crossings. If it is not possible explain why.



**Q 3.** *An edge coloring of a graph is an assignment of colors to edges so that edges incident with a common vertex are assigned different colors. The edge chromatic number of a graph is the smallest number of colors that can be used in an edge coloring of the graph. The edge chromatic number of a graph  $G$  is denoted by  $\chi(G)$ . Find the edge chromatic numbers of:*

a)  $C_n$ , where  $n \geq 3$ .

b)  $W_n$ , where  $n \geq 3$ .

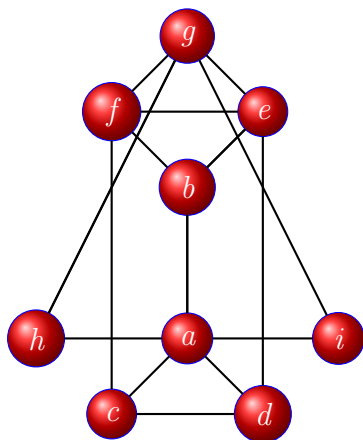
**Q 4.** *Find the edge chromatic number of  $K_n$  when  $n$  is a positive integer.*

**Q 5.** *Show that if  $G$  is a bipartite simple graph with  $v$  vertices and  $e$  edges, then  $e \leq \frac{v^2}{4}$ .*

**Q 6.** *Suppose that you have a three-gallon jug and a five-gallon jug. You may fill either jug with water, you may empty either jug, and you may transfer water from either jug into the other jug. Use a path in a directed graph to show that you can end up with a jug containing exactly one gallon. [Hint: Use an ordered pair  $(a, b)$  to indicate how much water is in each jug. Represent these ordered pairs by vertices. Add an edge for each allowable operation with the jugs.]*

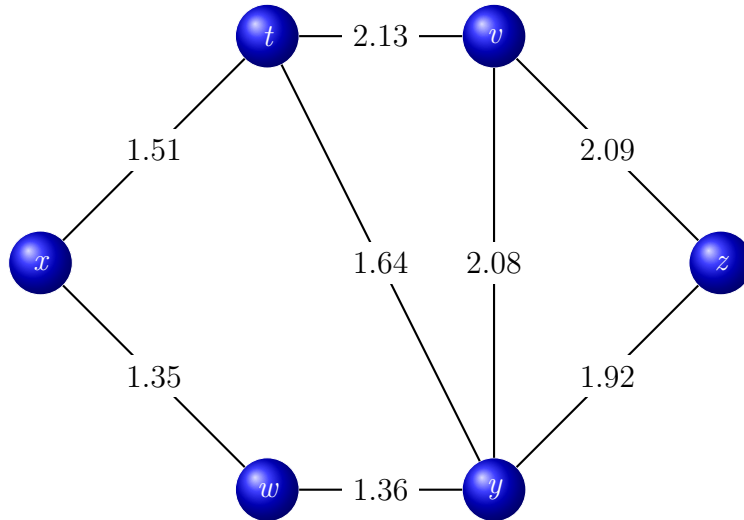
**Q 7.** Find the number of paths of length  $n$  between any two adjacent vertices in  $K_{3,3}$  for the values of  $n$  in  $\{3, 4, 5, 6\}$

**Q 8.** Determine whether (i) Dirac's theorem can be used to show the graphs below have a Hamilton circuit, (ii) whether Ore's theorem can be used and finally (iii) if the graph has a Hamilton circuit.



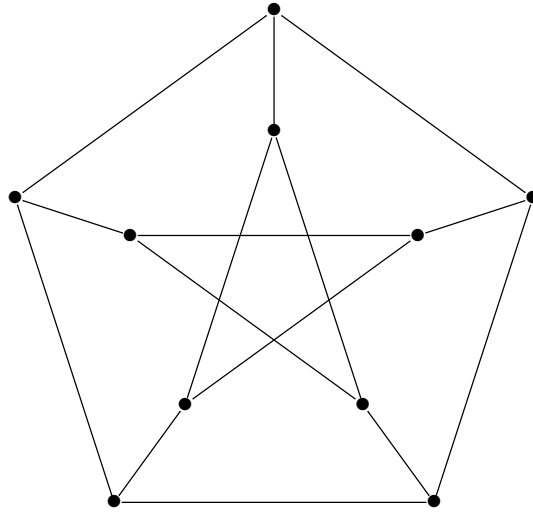


**Q 9.** Find the length of a shortest path between  $\{(x \text{ and } z), (v \text{ and } w), (t \text{ and } z)\}$  in the weighted graph below, using Dijkstra algorithm. Show each step.



**Q 10.** *Prove the following statement: If  $H$  is a subgraph of  $G$  and  $G$  is a planar simple graph, then  $H$  is also planar.*

**Q 11.** Find the chromatic number,  $\chi(G)$ , of the graph below and decide whether or not the graph is planar. Justify your answer.



**Q 12.** *Prove that Dijkstra's Algorithm finds the length of the shortest path between 2 vertices of a connected simple undirected weighted graph.*

**NOTE:** *Check the textbook.*