Show all your work. No essays, be concise

Name: ——————————————————— **Due Date**: <u>01/24</u>

Let $R = R : A \to A$ be a relation from a set A to itself then,

$$R^n = \overbrace{R_o R_o \ldots R_o R}^{n}$$

that is, R^n is the composition of R with itself n times.

Q 1) *Give a counter example or prove the following assertions:*
a. if R is reflexive then R^n is reflexive.
b. if R is symmetric then R^n is symmetric.
c. if R is transitive then R^n is transitive.

Q 2) *Suppose that R and S are reflexive relations on a set A. Prove or disprove each of these statements.*
a) $R \cup S$ is reflexive.
b) $R \cap S$ is reflexive.
c) $R \oplus S$ is irreflexive.
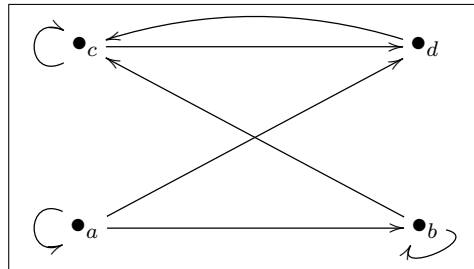*d) $R - S$ is irreflexive.*
e) $S_o R$ (S composed with R) is reflexive.

Q 3) *Find the matrix that represents the relation R on $\{1, 2, 3, 4, 6, 12\}$, where aRb means $a|b$. Use elements in the order given to determine rows and columns of the matrix.*

Q 4) *Draw the directed graph for the relation defined by the matrix:*

$$M = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

Example of a digraph:

Q 5) *A Lemma in the book states:* Let A be a set with n elements, and let R be a relation on A. If there is a path of length at least one in R from a to b, then there is such a path with length not exceeding n. Moreover, when $a \neq b$, if there is a path of length at least one in R from a to b, then there is such a path with length not exceeding $n - 1$. *The book proves for the case that $a = b$. Find the proof for the case that $a \neq b$*

Q 6 *Draw the directed graph that represents the relation* $ARA = \{(a,a),(a,b),(b,c),(c,b),(c,d),(d,a),(d,b)\}$ *where* $A = \{a,b,c,d,e\}$.

Q 7 *Find the matrix of the relation of ARA from question 6 above.*

Q 8 *From the directed graph of question 6 above draw the digraph of $\bar{R}$ (the complement of R).*

Q 9 *Find the matrix of the relation of $A\bar{R}A$ from question 6 above.*

Q 10 *From the directed graph of question 6 above draw the digraph of R^{-1} (the inverse of R).*

Q 11 *Find the matrix of the relation of $AR^{-1}A$ from question 6 above.*

Q 12 *In ARA from question 6 above remove or add the least amount of elements so that ARA represents an equivalence relation.*