Statistical Methods for Discrete Response, Time Series, and Panel Data (W271): Lab 2

Investigating the Keeling Curve and forecasting CO2 levels in Earth's atmosphere

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Abstract

The Keeling curve is a [INTRODUCE THE CONCEPT IN ONE SENTENCE HERE and SHOW IMPORTANCE]. In this paper, the Keeling curve was analyzed from two perspectives, one from a researcher in 1998 and one from today (2023). From the perspective of 1998, EDA was performed from 1959 to 1997 and a linear time trend model was fit. After analysis of the assumptions, a cubic polynomial model was selected. Using the Box-Jenkins method, a SARIMA model was constructed and forecast into 2100. To contrast those models, we present the same analysis from the perspective of 2023. A modern data pipeline was constructed for CO2 data and the linear model and SARIMA model were compared to the actual CO2 levels. Both models under predicted CO2 with [ADD SOMETHING ON RMSE or OTHER HERE]. Finally, new models were fit on the 2023 a forecasted into the future with the goal of reviewing model predictions in a future analysis.

Contents

1 Introduction

We all know the debate about global warming and its connection to human activities. But to study this topic in a scientific way, we need reliable data. The Keeling Curve is a milestone in this aspect. It shows the ongoing increase in atmospheric carbon dioxide (CO2) concentrations over time. It is named after Charles David Keeling, the scientist who initiated and maintained the measurements. Keeling began monitoring atmospheric CO2 levels in 1958 at the Mauna Loa Observatory in Hawaii. He chose this location because it is remote and far from major sources of pollution, providing an ideal site to measure baseline CO2 concentrations. The Keeling Curve graphically represents the seasonal variations in atmospheric CO2 concentrations, as well as the long-term increasing trend. Keeling believes the seasonal pattern is a result of the Earth's vegetation absorbing CO2 during the growing season and releasing it during the dormant period, while the trend is primarily driven by human activities, particularly the burning of fossil fuels such as coal, oil, and natural gas, which release large amounts of CO2 into the atmosphere. The Keeling Curve is an important tool for scientists, policymakers, and the general public to understand the impact of human activities on the Earth's climate. It serves as a stark reminder of the need to reduce greenhouse gas emissions and address the causes and consequences of climate change.

Our research is based on the data from the Keeling Curve above. We first build a model based on data from 1959 to 1997 and make long-term predictions to the present. Then we combine the actual data with our prediction and discuss the implication of this comparison.

2 Report from the Point of View of 1997

2.1 Data

The data measures the monthly average atmospheric CO2 concentration from 1959 to 1997, expressed in parts per million (ppm). It was initially collected by an infrared gas analyzer installed at Mauna Loa in Hawaii, which was one of the four analyzers installed by Keeling to evaluate whether there was a persistent increase in CO2 concentration.

Fig.1 shows a clear long-term upward trend, which is confirmed by Fig.2 where the growth rate for each year is above zero. Fig.2 also suggests the average growth rate after 1970 is higher than that before 1970, although there's no evidence of accelerating growth. The ACF plots in Fig.3 and Fig.4 suggest the original data is non-stationary but its first difference is stationary. More formally, the KPSS tests below confirm the observations above.

Table 1: KPSS test of orignal and 1st difference

	kpss_stat	kpss_pvalue
original	7.8173	0.01
1st_difference	0.0124	0.10

Another feature of the data is its robust seasonal pattern, with the peak in May and the bottom in October almost every year (see Fig.5). This seasonality can also be seen in Fig.4. Keeling believes it was the result of plant photosynthesis absorbing CO2 from the atmosphere.

Fig.4 is the histogram of the remaining or irregular components after removing the trend and the

seasonal components from the data with STL[1]. It looks like a normal distribution without obvious outliers.

Fig.1 Atmospheric CO2 concentration
monthly average, parts per million (ppm)

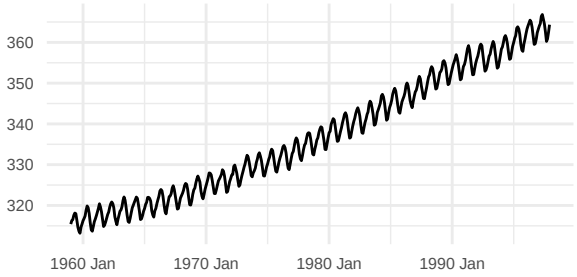


Fig.2 Annual growth rate of concentration, %

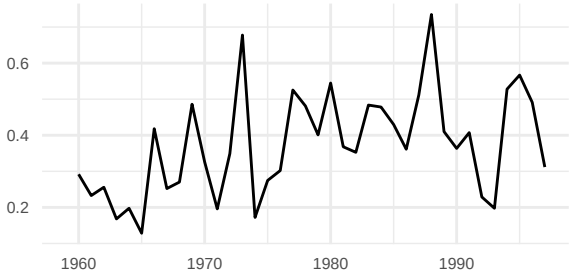


Fig.3 ACF of CO2 concentration

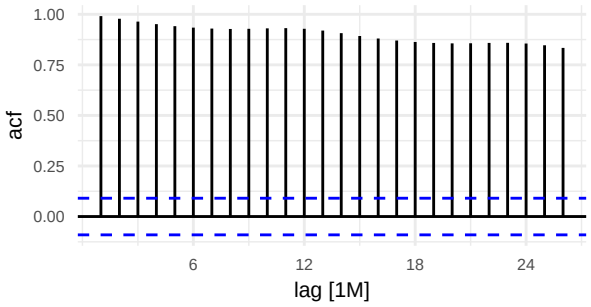


Fig.4 ACF of differenced CO2 concentration

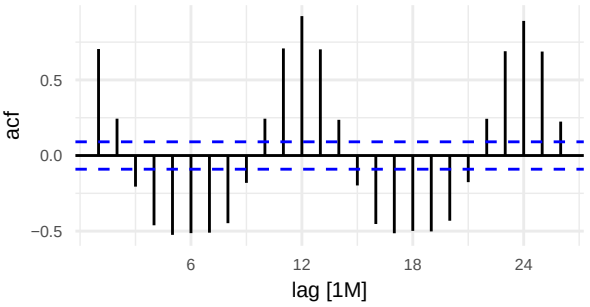


Fig.5 Seasonal plot of CO2 concentration

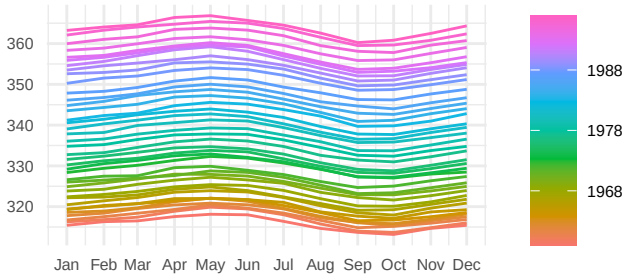
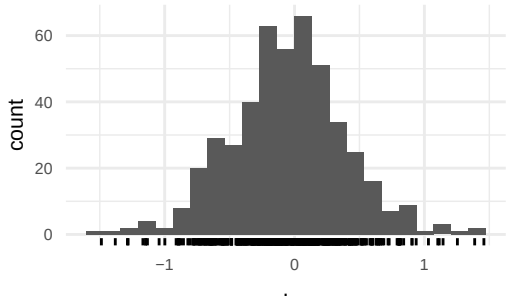


Fig.6 Histogram of irregular
component by STL



2.2 Linear model

Before building the model, we need to consider whether the data need a log transformation. Normally, a log transformation is required when the data shows exponential growth or the variance expands or shrinks over time. From Fig.1 and Fig.2 we can see the slope or the growth rate of the data is stable, which suggests the growth is more close to linear instead of exponential. Also, Fig.5 shows the difference between the annual high and the annual low almost remained the same over the years, suggesting the variance is nearly constant. Therefore, the log transformation is not necessary. We can first fit the original data with a linear time trend model as:

[1]Cleveland, R. B., Cleveland, W. S., McRae, J. E., & Terpenning, I. J. (1990). STL: A seasonal-trend decomposition procedure based on loess. Journal of Official Statistics, 6(1), 3–33.

$$\mathrm{CO}_2 = \beta_0 + \beta_1 t + \epsilon_t, \tag{1}$$

which gives the parameters as:

$$\mathrm{CO}_2 = 311.5 + 0.11t + \epsilon_t \tag{2}$$

This linear trend model implies that the CO_2 concentration increased by 0.11 ppm/month on average from 1959 to 1997. However, the residual plots in Fig.7 to Fig.10 suggest this simple linear trend model is not adequate in the following two aspects.

First, the mean of the residual forms a "U" shape over time, suggesting a quadratic or higher-order polynomial time trend model may be more appropriate. For instance, the residual from a quadratic time trend model shows a more constant mean over time, as shown in Fig.10.

Fig.7 Residual of the linear trend model

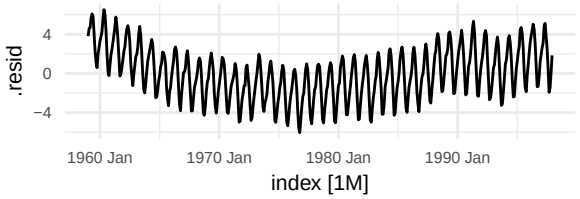


Fig.8 ACF of the linear trend model residuals

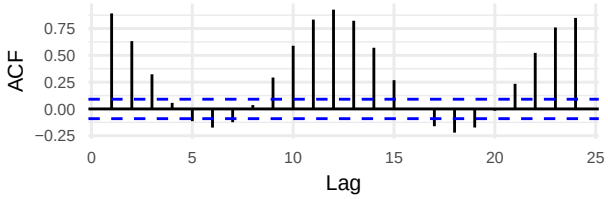


Fig.9 Histogram of the linear trend model residuals

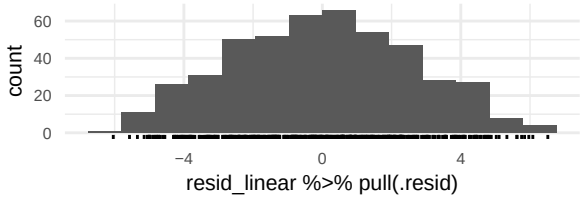
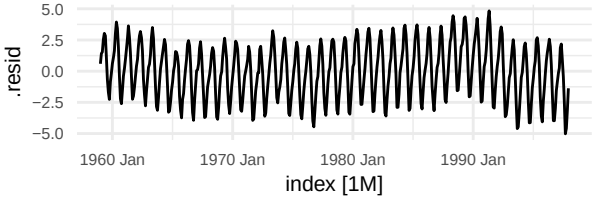


Fig.10 Residual of the quadratic time trend model



In addition, the ACF plot in Fig.6 indicates strong seasonal patterns exist in the residuals, suggesting we should consider seasonal factors in the model. One solution is to include 11 dummy variables in the model to indicate the 12 months.

Based on the two points above, we compare the 2 candidates: a quadratic time trend model and a cubic one, as below.

$$\text{Quadratic time trend: } \mathrm{CO}_2 = \alpha + \beta_0 t + \beta_1 t^2 + \sum_{i=1}^{11} \gamma_i Month_{it} + \epsilon_t \tag{3}$$

$$\text{Cubic time trend: } \mathrm{CO}_2 = \alpha + \beta_0 t + \beta_1 t^2 + \beta_2 t^3 + \sum_{i=1}^{11} \gamma_i Month_{it} + \epsilon_t \tag{4}$$

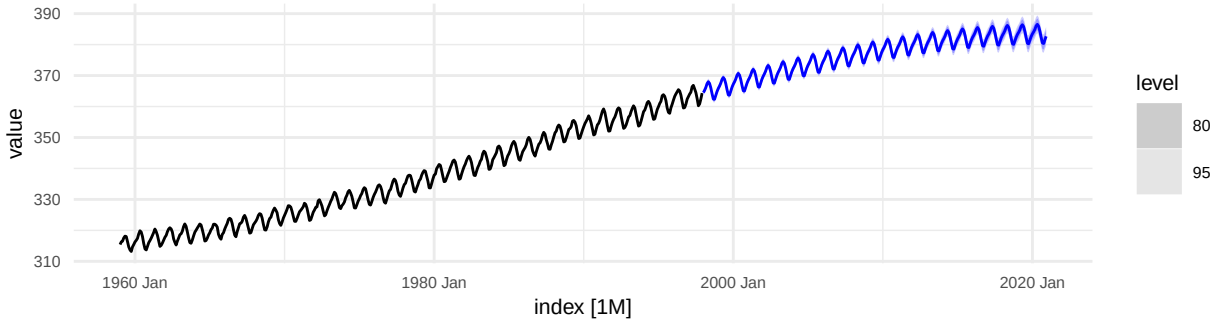
We use the data before 1991 as the training set and the rest as the validation set (similar to an 80-20 split). Our final choice of the model depends on the combination of 2 guidelines: 1) the information criterion (AICc, BIC) from the model fitting process and 2) the root mean square error (RMSE) of predictions on the validation set, which are listed in Table.1. Both information criterion (AICc,

BIC) and RMSE favor the cubic model. Therefore, the cubic time trend model becomes our final choice. Its details are in the Appendix. We plot the forecast of this model until 2020 in Fig.9. One thing to note is that because the coefficient of the cubic term is negative, the predicted values will eventually begin to decrease when predicting the far future. In fact, we can see from Fig.11 that the predicted values have almost topped. This may be inappropriate extrapolation behavior. In that case, we should confine our predicting interval to the near term.

Table 2: Information Criterion of model fitting and RMSE of validation

.model	AIC	AICc	BIC	RMSE
cubic	-659.6147	-658.1324	-596.4044	2.112194
quadratic	-639.1525	-637.8481	-579.8928	2.796572

Fig.11 Forecasts of CO2 level Up To 2020 Using Polynomial Trend Time Model



2.3 ARIMA times series model

We will use the Box Jenkins process to find the best ARIMA model via the following steps:

- Determine the appropriate model from EDA
- Find the best parameters
- Examine the residuals using diagnostic plots and statistical tests

The EDA revealed that the time series of CO2 had both autoregressive and seasonal components. Considering the ACF plot's low slow decay of autocorrelation, we expect differencing to be a key part of any time series model. In addition, we predict that the model will require seasonal components to model the 12 month cycle of seasonal variations. Therefore, we expect a seasonal arima model (SARIMA) with differencing and seasonality terms to be best.

In this section, we fit the best SARIMA model and analyze the results. We choose BIC as our information criteria for model selection. Simplicity is a desirable property in data science models to help explain the relationship between variables. We choose BIC as our information criteria because it penalizes complex models more than AIC or AICc and therefore selects more simple models with fewer parameters as the best ones. Lower BIC scores are better.

```
## Series: value
## Model: ARIMA(0,1,1)(1,1,2)[12]
##
```

```
## Coefficients:
##           ma1     sar1     sma1     sma2
##       -0.3482  -0.4986  -0.3155  -0.4641
## s.e.   0.0499   0.5282   0.5165   0.4367
##
## sigma^2 estimated as 0.08603:  log likelihood=-85.59
## AIC=181.18   AICc=181.32   BIC=201.78
```

After searching over seasonal and non-seasonal P, D, and Q variables, the best model was an ARIMA(0,1,1)(1,1,2)[12] model with BIC score of 201.78. Next, we evaluate the model via diagnostic plots and statistical tests, concluding the Box Jenkins process.

Fig.12 Residuals of the SARIMA mode

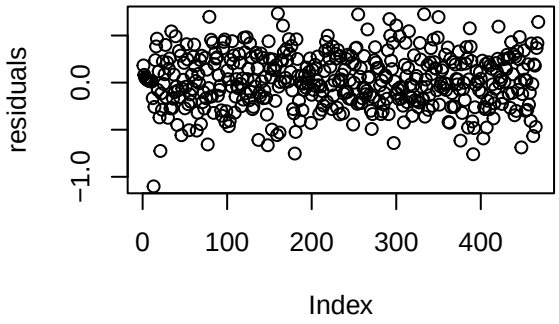


Fig.13 ACF plot of residuals

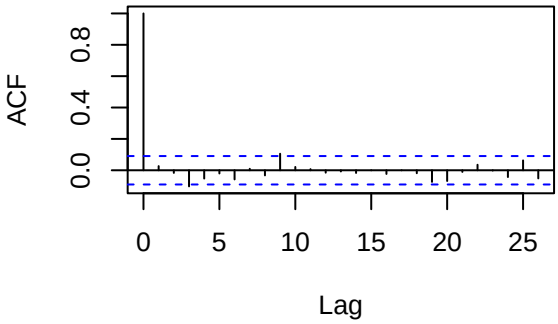


Fig.14 PACF plot of residuals

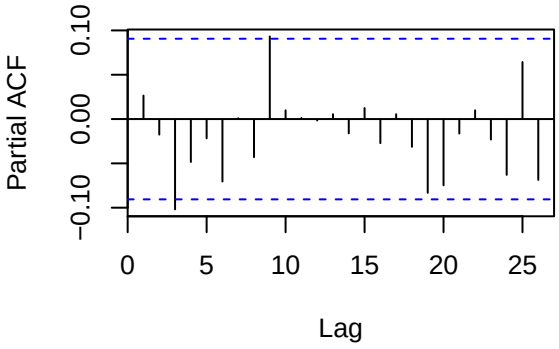
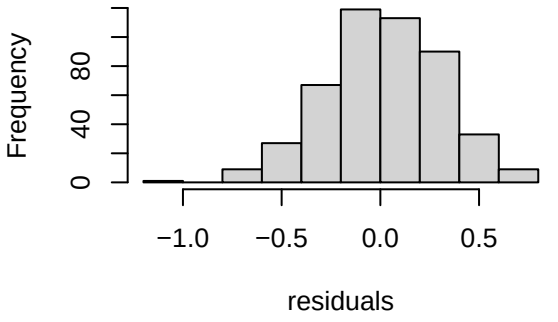


Fig.15 histogram of residuals



The residual plots (Fig 12-15) show that the SARIMA model was effective, with the residuals looking like stationary white noise (Fig 12). The time series has a mean of 0 with about constant variance, the ACF plot (Fig 13) shows no autocorrelation beyond the initial lag value. The PACF plot (Fig 14) appears to have a significant peak around the 3rd lag term, but this may be due to randomness, as it is barely passing the dashed blue line. The histogram (Fig 15) looks normally distributed at 0 with outliers creating a left tail.

We test the residuals for stationarity with the Augmented Dickey Fuller test (ADF). The ADF test has the null hypothesis that the data is non stationary. With a p-value of 0.01, we reject the null hypothesis because there is enough evidence to say that the residuals are stationary.

The Box-Ljung test has the null hypothesis that the data presented is independently distributed. When presented with the residuals of the ARIMA model, the test had p-values of 0.566 and 0.144 for lag =1 and lag = 10 respectively. For both of those lags, we fail to reject the null hypothesis and conclude that the data is independently distributed.

Finally, we visually inspect the histogram of the residuals (Fig.16) and the QQ plot (Fig.17) to see if the residuals appear normally distributed. The histogram has the Gaussian bell shaped curve with a few outliers. The QQ plot shows that the data matches up with the normal distribution's quantiles. With these plots, we can confidently say that the residuals are visually normally distributed.

To conclude, both diagnostic plots and statistical tests show that the residuals are stationary with mean 0, constant variance, and no autoregression or seasonality. We forecast our model to the year 2022 (Fig 18).

Fig.17 QQ plot of residuals

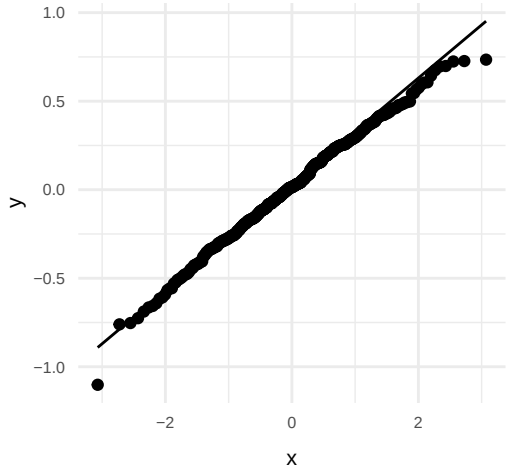
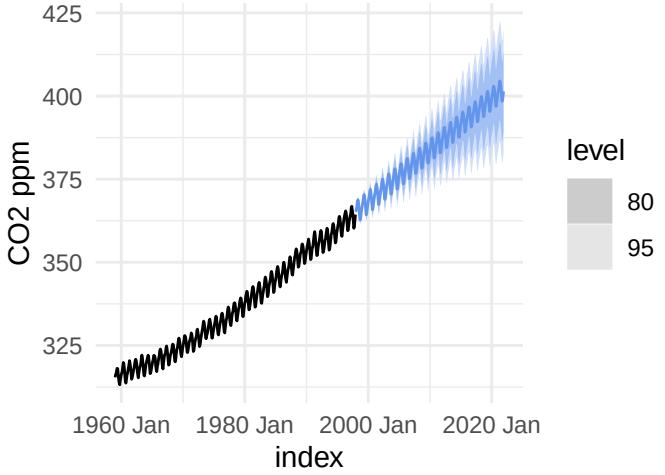


Fig.18 CO2 levels from 1959 to 2022



2.4 Atmospheric CO2 growth Forecast

We use our model to make predictions on future levels of CO2, specifically 420 and 500 ppm. We will investigate the earliest, best guess, and latest ocurrance of these values. The earliest guess will be based on the first time the upper 95% confidence interval (CI) reaches the specified level and the latest guess will be the last time the value is within the lower 95% CI. The best guess will be the point estimate (mean) of the forecast.

Table 3: Predicted occurances of key CO2 levels

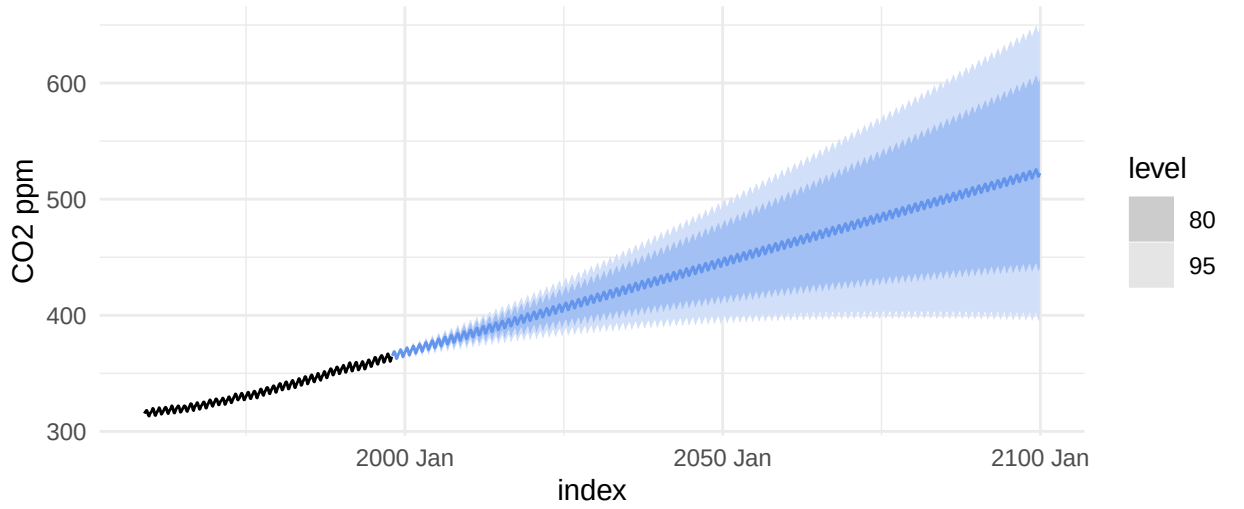
CO2.ppm.level	earliest.occurance	point_estimate	final.occurance
420 ppm	2022 Apr	2031 May	never
500 ppm	2054 Apr	2083 Apr	never

Based on our model, the first time we could potentially see CO2 at 420 ppm is 2022-04-01 because that is when the upper 95% confidence interval (CI) of our model first reaches 420 ppm. The model's lower 95% CI hovers around 420, so there is no predicted final time. Knowing what we know today

in 2023, 419 ppm was reached on May 2021, which was *before* our model's earliest guess. CO2 levels have risen faster than our model anticipated. This is a precursor to the analysis provided later in this paper. The first time our model predicts the earth to reach 500 ppm CO2 on 2054-04-01, which is when the 95% CI reaches 500 ppm. The model's lower 95% CI never reaches 500, so there is no predicted final time.

Below is the prediction of our model to the year 2100. Confidence intervals are shown fanning outward. The error of the predictions compounds overtime which expands the confidence intervals into a funnel shape. The farther out in time from the recorded data points, the less accurate the prediction.

Fig.19 CO2 levels from 1959 to 2100



3 Report from the Point of View of the Present

3.1 0b.Introduction

In our original 1997 paper, we made several predictions on the expected level atmospheric CO2. Currently, we will evaluate the accuracy of those predictions using time series analysis and extrapolate from present data to make predictions about the future.

3.2 1b.Data

A modern data pipeline was constructed to load both weekly and monthly CO2 data from January 1959 to June 2023. This will allow us to compare our forecasts in the previous section to the actual CO2 levels. The code for the pipeline can be found in the appendix.

3.3 MOVE CODE TO THE APPENDIX

4 ADJUSTED THE FIGURE SIZE

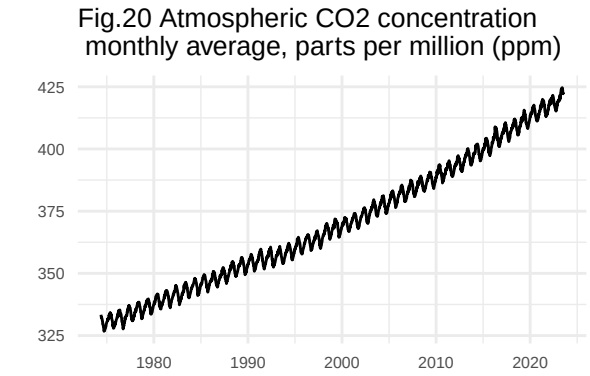
```
p1 <- autoplot(co2_present) +
  ggtitle("Fig.20 Atmospheric CO2 concentration\n monthly average, parts per million (ppm) ") +
```

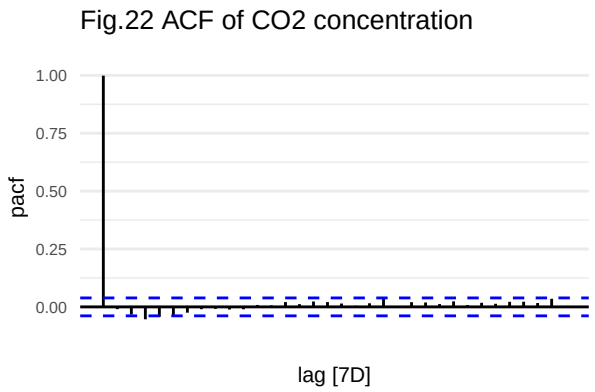
```
  xlab(NULL) + ylab(NULL)+
  theme(text = element_text(size = 8))
```

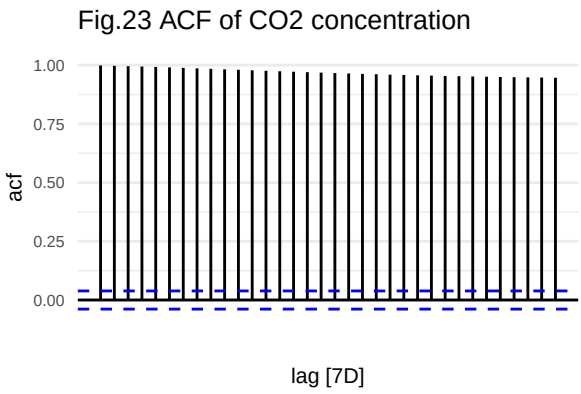
```
## Plot variable not specified, automatically selected `.vars = average`
```

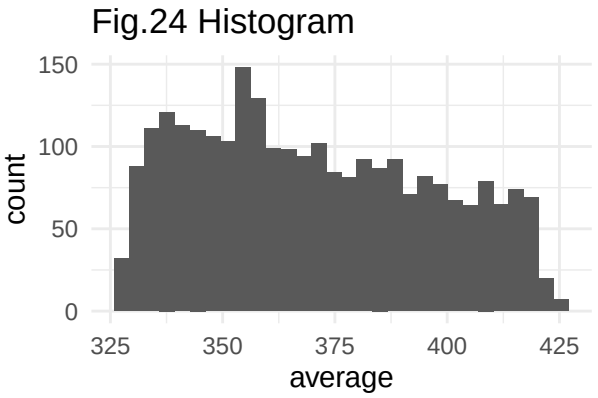
```
p2 <- co2_present %>% index_by(year = lubridate::year(time_index)) %>%
  summarise(annual_avg = mean(average)) %>%
  mutate(annual_growth = (annual_avg / lag(annual_avg, 1) - 1) * 100) %>%
  autoplot(.vars = annual_growth) +
  xlab(NULL) + ylab(NULL) +
  ggtitle("Fig.21 Annual growth rate of concentration, %")+
  theme(text = element_text(size = 8))
p2 <- co2_present %>% PACF(average) %>% autoplot()+
  ggtitle("Fig.22 ACF of CO2 concentration")+
  theme(text = element_text(size = 8))
p3 <- co2_present %>% ACF(average) %>% autoplot()+
  ggtitle("Fig.23 ACF of CO2 concentration")+
  theme(text = element_text(size = 8))
p4 <- co2_present %>% ggplot() +
  geom_histogram(aes(average))+
  ggtitle("Fig.24 Histogram")

(p1|p2)/(p3|p4)
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```









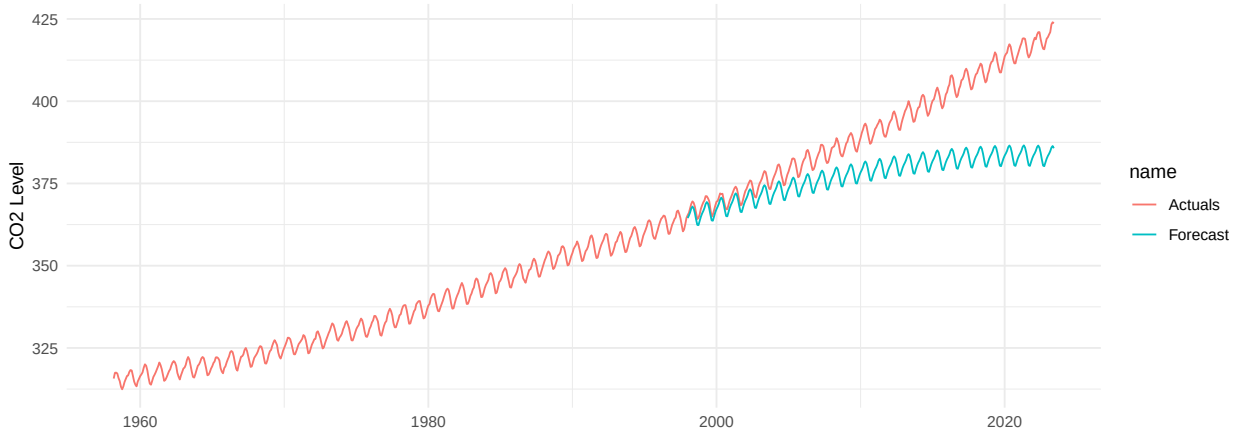
The CO2 levels have continued to grow since 1997, but the growth has not been dramatic. Fig. 20 The time series plot shows that the CO2 levels have increased at a steady rate, with no major spikes or dips.

The most notable difference between the CO2 levels in 1997 and now is the distribution of the data. In 1997, the distribution was almost bimodal, meaning that there were two distinct peaks in the data. The distribution in Fig 24. shows more heavy-tailed pattern, meaning that there are more values at the high end of the distribution. This further suggests that there are more extreme CO2 levels now than there were in 1997.

4.1 2b.Compare linear model forecasts against realized CO2

4.2 I'd recommend starting the forecast in 1998

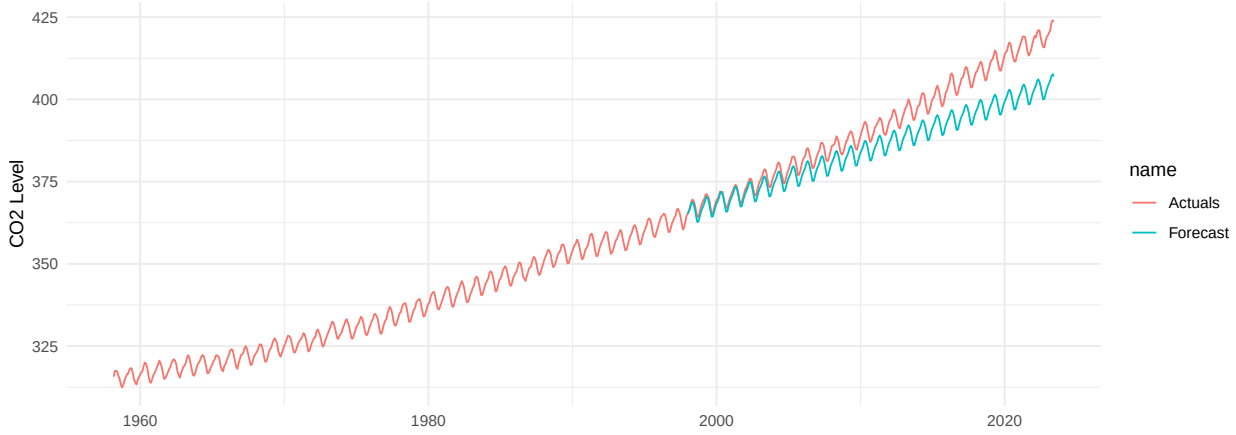
Fig. 25 Linear Model Forecast vs Realized CO2



The linear model in (Fig.25) forecast may not have capture the trend of the realized CO2 levels. The forecast appears to predict a stabilization in the CO2 levels, whereas the actual CO2 level trend increased.

4.3 3b.Compare ARIMA models forecasts against realized CO2

Fig. 26 ARIMA Forecast vs Realized CO2

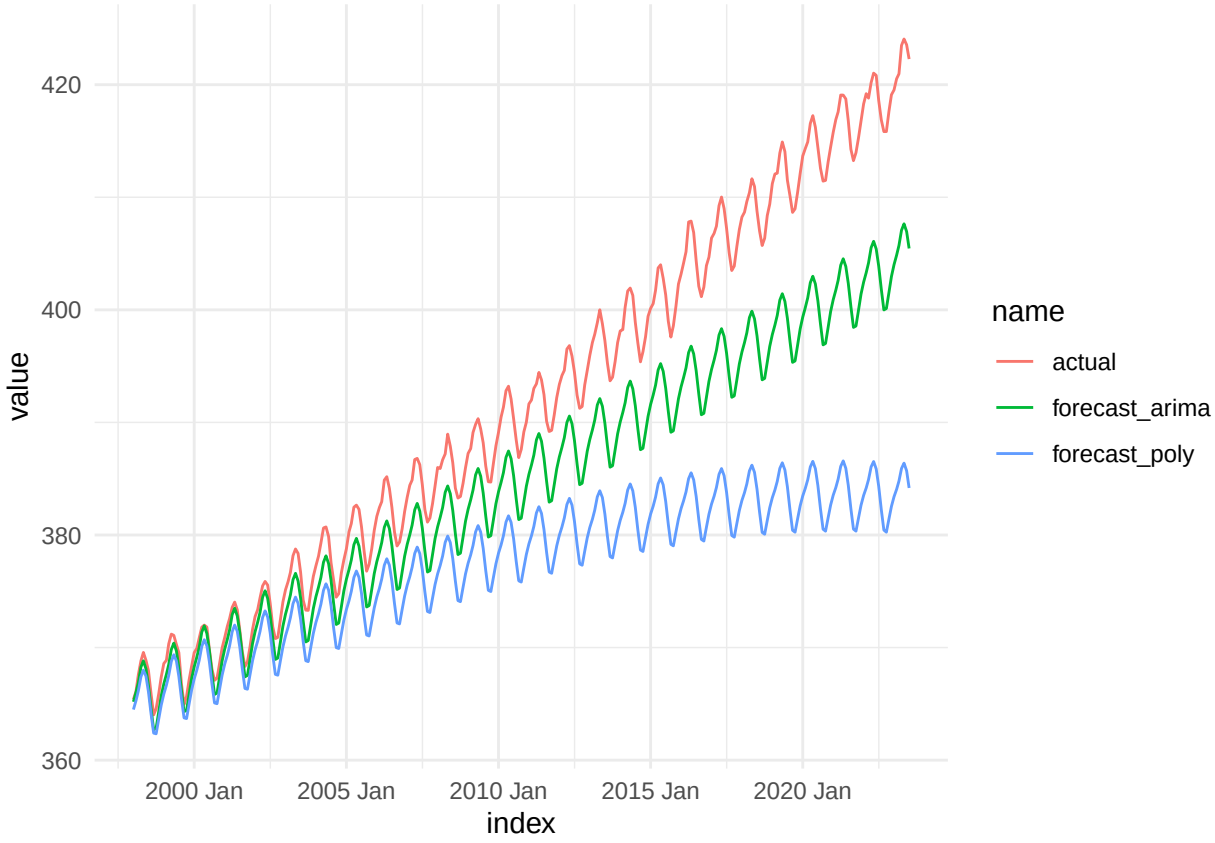


The ARIMA forecast(Fig.26) is much closer to the realized CO2 levels than the Linear Model forecast. The only difference observed, is that the ARIMA model appears to have forecasted a linear trend, while the realized CO2 levels followed an almost exponential growth.

4.4 4b.Evaluate the performance of 1997 linear and ARIMA models

```
co2_present_monthly<-co2_present %>% index_by(index=yearmonth(time_index))%>%
  summarise(value=mean(average))
```

```
co2_present_monthly_since1998 <-co2_present_monthly%>%filter(lubridate::year(index)>1997)
fc_poly_new <- co2_ts %>%
  model(TSLM(value ~ trend() + I(trend() ^ 2) + I(trend() ^ 3) +
               season())) %>%forecast(h=(2022-1997)*12+7)
fc_arima_new <- model.bic %>% forecast(h=(2022-1997)*12+7)

compared_data=data.frame(index=co2_present_monthly_since1998$index,actual=co2_present_monthly_s

compared_data%>%pivot_longer(cols=c(actual,forecast_poly,forecast_arima)) %>% ggplot(aes(x=inde
```




```
compare_test=rbind(
  fabletools::accuracy(fc_poly_new,co2_present_monthly_since1998),
fabletools::accuracy(fc_arima_new,co2_present_monthly_since1998)

)
compare_test$.model=c("Best Polynomial","Best ARIMA")
kable(compare_test %>% dplyr::select(-.type,-MASE,-RMSSE))
```

.model	ME	RMSE	MAE	MPE	MAPE	ACF1
Best Polynomial	14.395713	18.016760	14.395713	3.568692	3.568692	0.9887861
Best ARIMA	6.808948	8.363163	6.808948	1.690605	1.690605	0.9862961

Now we evaluate the accuracy for the best polynomial and ARIMA models built on the data till 1997. The forecast and actual values are plotted in Fig.X, and a quick glance would tell the both forecast are systematically lower than the actual data. More formally, the RMSE of prediction from the best polynomial model reaches 18.02, and that of the best ARIMA model is 8.36.

4.5 5b. Train best models on present data

In the previous section, we performed a comprehensive evaluation of models trained on 1997 dataset with the focus on long term forecasting 1998 to 2023 and got high residual error. However, for long term forecasting, utilizing seasonally adjusted data becomes more appropriate. To accomplish this we applied the STL method, which effectively decomposes the time series and provides us with seasonal adjusted data. This dataset was subsequently split into training (1997 to 2021) and test (2021 to 2023) sets to assess model performance.

We created both linear regression and ARIMA models and chose the best versions for both seasonally-adjusted and nonseasonally adjusted data.

Consider adding the equations of ARIMA seasonal and non seasonal here Consider adding plots next to each other here @Mingxi

equationhere

Table 5: ARIMA comparison table

model_type	model.equation	BIC.score	test.RMSE
arima seasonal	ARIMA(0,2,2)(4,0,0)[12]	54.82	0.4993524
arima non seasonal	ARIMA(2,1,4) w/ drift	3463.76	2.2039345

add analysis of ARIMA's here. what can we conclude?

Next, we fit a polynomial time-trend model to the seasonally-adjusted series and compare its performance to the ARIMA model. Using the TSLM package, we created our best fit model.

Table 6: linear model comparison

model_type	model.equation	BIC.score	test.RMSE
ARIMA seasonl	equation here	fix	rmse_seasonal_yearly_monthly
linear seasonal	equation here	fix	rmse_weekly_non_seasonal

The ARIMA models were trained on the training dataset and evaluated on the test dataset, yielding an RMSE value of 0.83. Similarly we conducted a parallel analysis on the non seasonal adjusted data, resulting in an RMSE value of 0.11. Interestingly, the non-seasonally adjusted ARIMA model showcased slightly better performance than its seasonally adjusted counterpart.

During the training process, we employed a grid search, exploring various parameter combinations (p, d, q) ranging from (0:8, 0:2, 0:8) and seasonal parameter combinations (P, D, Q) spanning from (0:12, 0:4, 0:12). The final selected ARIMA model was (0,1,1)(4,0,0)[12], indicating the implementation of differencing once for stationarity and incorporating one lag of forecast errors.

This model demonstrated improved performance on the out-of-sample test dataset compared to the ARIMA model built in the previous section. This improvement in predictive capabilities can be attributed to the inclusion of more recent training data from 1997 to 2020, which allowed the model to forecast the immediate future (2021 to 2023) more accurately. Also, the ARIMA model performs well both on in-sample and out-sample, indicating that model is flexible and able to predict accurately and not overfitting. Additionally, we compared the non-seasonally adjusted ARIMA model with a polynomial linear model trained on seasonally adjusted data. The RMSE value for the polynomial linear model was 1.7, whereas the non-seasonally adjusted ARIMA model achieved an RMSE of 0.14. The non-seasonally adjusted ARIMA model showcased superior performance in predicting the immediate future and effectively capturing the seasonal pattern. Also, as seen from the EDA, the seasonal pattern is stable in this data. Conversely, the polynomial linear model on seasonally adjusted data is better for predicting long-term trends and not as effective in capturing seasonal variations. Overall, our analysis highlighted the significance of selecting the appropriate modeling approach based on the forecasting requirements and the nature of the data.

In the previous section, we conducted exploratory data analysis to visually assess the forecast of both the linear model and ARIMA model for atmospheric C02 levels. These models captured the historic trends and patterns effectively up to a certain point 1997. However as we moved from 1997, the forecasted lines started to deviate, making it challenging to determine which model better fits the data. To quantitatively evaluate the accuracy of the models, we conducted a formal evaluation using the Root Mean Squared Error (RMSE) test. Both the ARIMA and linear models were developed using data from 1974 to 1993. We used these models to forecast the atmospheric CO2 levels until 2023. Since models were built on a monthly dataset, the predictions we obtained were also on a monthly basis for the period from 1993 to 2023.

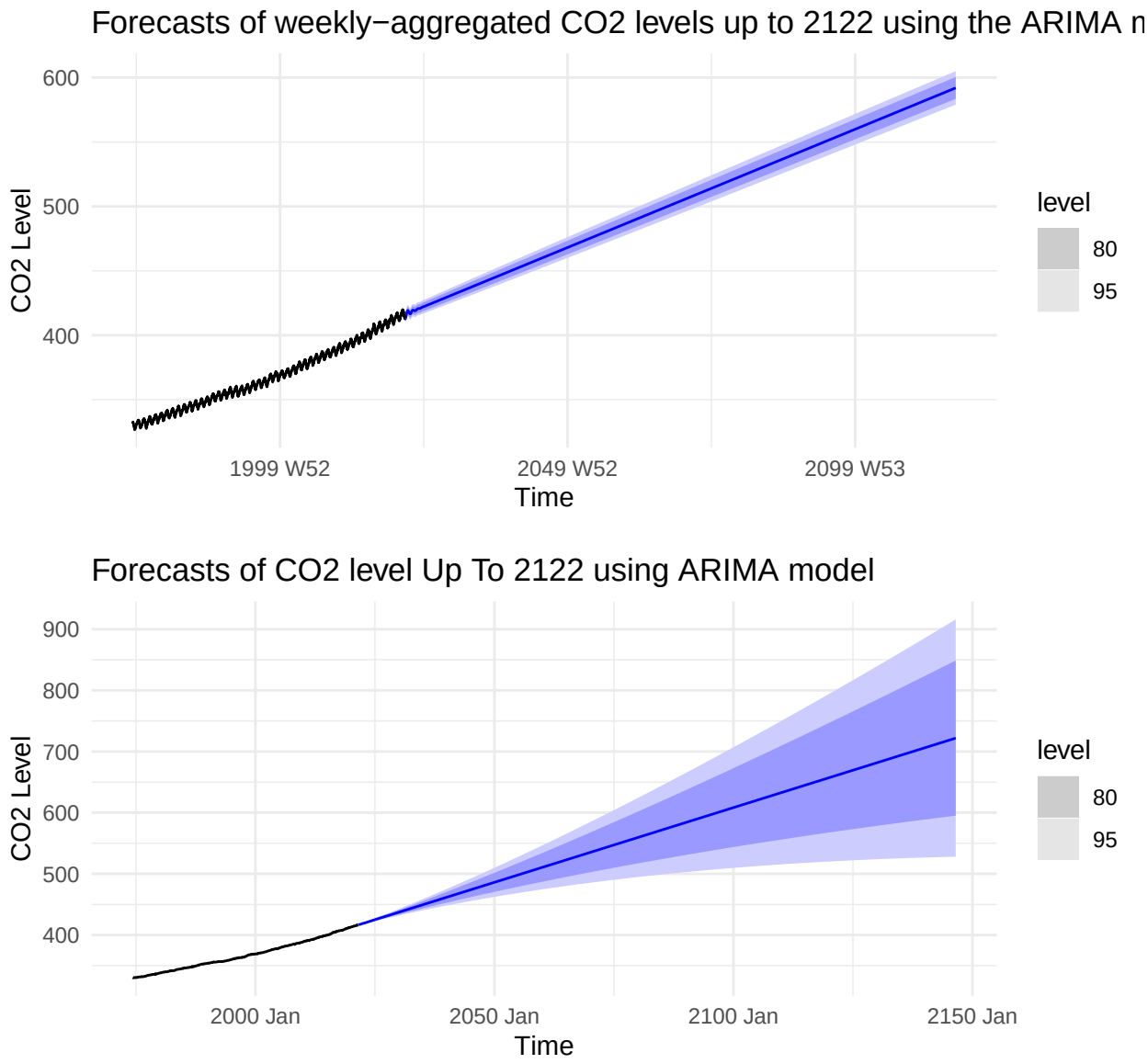
To ensure a fair comparison of these predictions we aggregated the current dataset to a monthly level. The model performance of ARIMA model , after comparing its predictions with the current dataset, resulted in an RMSE of 51.66 On the other hand, the linear model which considered both the seasonal and trend components, had an RMSE of 14.037. It looks like that data has certain trend and pattern which are better captured by linear regression.

4.5.1 (3 points) Task Part 6b: How bad could it get?

The best model we derived from Part 5b was the ARIMA model. Utilizing this model, we projected CO2 levels on the original weekly dataset which is a non-seasonally adjusted data. Our analysis indicated that CO2 levels are predicted to reach 420 ppm in 2023, but are not expected to reach 500 ppm. We observed that in the year 3075, the upper boundary touched 500 ppm, but with low confidence. These predictions can be influenced by the data's aggregation level. For instance, weekly aggregated data may yield more vague predictions because the number of datapoints are larger resulting in higher noise and variability in data, whereas monthly aggregated data can provide slightly improved predictions. On examining the weekly level aggregated data, we noticed that the prediction boundary intervals curve downwards, which is not ideal because it captures too much noise and adds variability. However, with monthly level data, predictions are comparatively better, and the boundary intervals are narrower and upward. According to our analysis, the 500 ppm threshold is projected to be reached by the year 2050.

The predictions for 2122 is 425 with boundary limits of 310-555 in the weekly aggregated dataset. However, in the monthly aggregated dataset, the prediction for 2122 is 650 with narrower boundary levels. Another factor that can impact these predictions is the evaluation metric used. In our

analysis, we employed the Bayesian Information Criterion (BIC). If an alternative metric had been employed, the predictions might have varied. The use of BIC penalizes the data more, leading to a more conservative model selection that favors simpler models. As a result, the chosen model may be too simplistic to accurately capture the underlying patterns and relationships in the future unseen data. Hence, considering alternative evaluation metrics could potentially yield different or more accurate predictions.



A Appendix: Model Robustness

While the most plausible model that we estimate is reported in the main, "Modeling" section, in this appendix to the article we examine alternative models. Here, our intent is to provide a skeptic that does not accept our assessment of this model as an ARIMA of order (1,2,3) an understanding of model forecasts under alternative scenarios.

Modern data pipeline code: