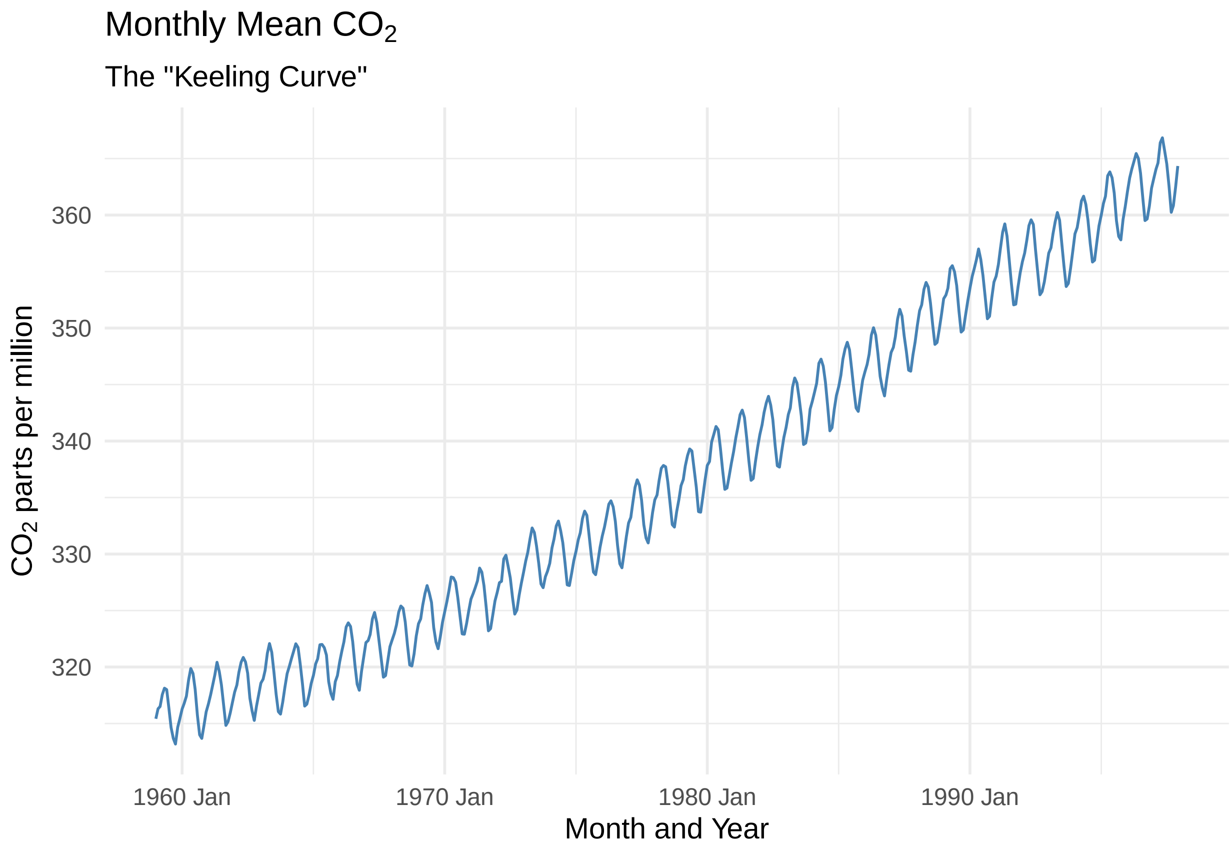
# Statistical Methods for Discrete Response, Time Series, and Panel Data (W271): Lab 2

## The Keeling Curve

In the 1950s, the geochemist Charles David Keeling observed a seasonal pattern in the amount of carbon dioxide present in air samples collected over the course of several years. He was able to attribute this pattern to the variation in global rates of photosynthesis throughout the year, caused by the difference in land area and vegetation cover between the Earth's northern and southern hemispheres.

In 1958 Keeling began continuous monitoring of atmospheric carbon dioxide concentrations from the Mauna Loa Observatory in Hawaii and soon observed a trend increase carbon dioxide levels in addition to the seasonal cycle. He was able to attribute this trend increase to growth in global rates of fossil fuel combustion. This trend has continued to the present, and is known as the "Keeling Curve."


Monthly Mean $CO_2$
The "Keeling Curve"
$CO_2$ parts per million
360
350
340
330
320
1960 Jan
1970 Jan
1980 Jan
1990 Jan
Month and Year

## Your Assignment

Your goal in this assignment is to produce a comprehensive analysis of the Mona Loa CO2 data that you will be read by an interested, supervising data scientist. Rather than this being a final report, you might think of this as being a contribution to your laboratory. You and your group have been initially charged with the task of investigating the trends of global CO2, and told that if you find "anything interesting" that the team may invest more resources into assessing the question.

Because this is the scenario that you are responding to:

1. Your writing needs to be clear, well-reasoned, and concise. Your peers will be reading this, and you have a reputation to maintain.
2. Decisions that you make for your analysis need also be clear and well-reasoned. While the main narrative of your deliverable might only present the modeling choices that you determine are the most appropriate, there might exist supporting materials that examine what the consequences of other choices would be. As a concrete example, if you determine that a series is an AR(1) process your main analysis might provide the results of the critical test that led you to that determination and the results of the rest of the analysis under AR(1) modeling choices. However, in an appendix or separate document that is linked in your main report, you might show what a MA model would have meant for your results instead.
3. Your code and repository are a part of the deliverable. If you were to make a clear argument that this is a question worth pursuing, but then when the team turned to continue the work they found a repository that was a jumble of coding idioms, version-ed or outdated files, and skeletons it would be a disappointment.

## Report from the Point of View of 1997

For the first part of this task, suspend reality for a short period of time and conduct your analysis from the point of view of a data scientist doing their work in the early months of 1998. Do this by using data that is included in *every* R implementation, the `co2` dataset. This dataset is lazily loaded with every R instance, and is stored in an object called `co2`.

```
co2 <- as_tsibble(co2) %>% filter(lubridate::year(index)<1998)
```

### (3 points) Task 0a: Introduction

Introduce the question to your audience. Suppose that they *could* be interested in the question, but they don't have a deep background in the area. What is the question that you are addressing, why is it worth addressing, and what are you going to find at the completion of your analysis. Here are a few resource that you might use to start this motivation.

- Wikipedia
- First Publication
- Autobiography of Keeling

### (3 points) Task 1a: CO2 data

Conduct a comprehensive Exploratory Data Analysis on the `co2` series. This should include (without being limited to) a description of how, where and why the data is generated, a thorough investigation of the trend, seasonal and irregular elements. Trends both in levels and growth rates should be discussed (consider expressing longer-run growth rates as annualized averages).

What you report in the deliverable should not be your own process of discovery, but rather a guided discussion that you have constructed so that your audience can come to an understanding as succinctly and successfully as possible. This means that figures should be thoughtfully constructed and what you learn from them should be discussed in text; to the extent that there is *any* raw output from your analysis, you should intend for people to read and interpret it, and you should write your own interpretation as well.

```
p1 <- autoplot(co2) +geom_smooth(color="lightgrey")+
  ggtitle("Fig.1 Atmospheric CO2 concentration\n monthly average, parts per million (ppm) ") +
  xlab(NULL) + ylab(NULL)
```

```
## Plot variable not specified, automatically selected `.vars = value`
```

```
p2 <- co2 %>% index_by(year = lubridate::year(index)) %>%
  summarise(annual_avg = mean(value)) %>%
  mutate(annual_growth = (annual_avg / lag(annual_avg, 1) - 1) * 100) %>%
  autoplot(.vars = annual_growth) +
  xlab(NULL) + ylab(NULL) +
  ggtitle("Fig.2 Annual growth rate of\n concentration, %")
p3 <- gg_season(co2) +
  xlab(NULL) + ylab(NULL) +
  ggtitle("Fig.3 Seasonal plot of CO2 concentration")
```

```
## Plot variable not specified, automatically selected `y = value`
```

```
p4 <- co2 %>% model(STL(value ~ trend(window = 120) + season(window = "periodic"),
                        robust = TRUE)) %>%
  components() %>% pull(remainder) %>%
  gghistogram() +
  ggtitle("Fig.4 Histogram of irregular\n component by STL")
# p3 <- ggAcf(co2$value)
# p4 <- ggPacf(co2$value)
(p1 | p2) / (p3 | p4)
```

```
## `geom_smooth()` using method = 'loess' and formula = 'y ~ x'
```

```
## Warning: Removed 1 row containing missing values (`geom_line()`).
```

Fig.1 Atmospheric CO2 concentration monthly average, parts per million (ppm)

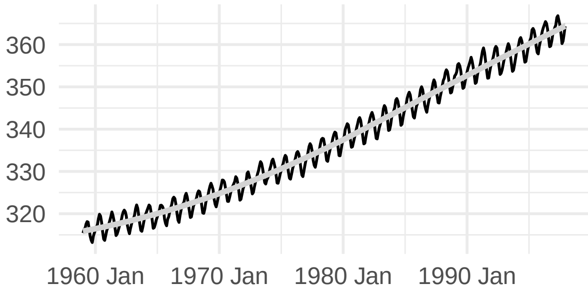



Fig.2 Annual growth rate of concentration, %

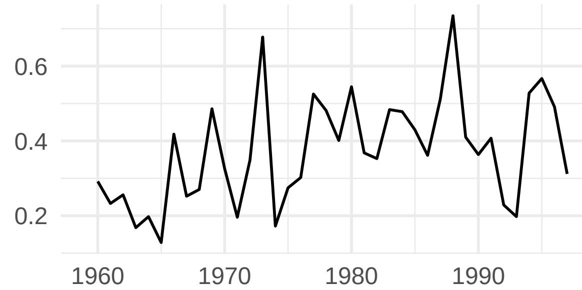



Fig.3 Seasonal plot of CO2 concentration

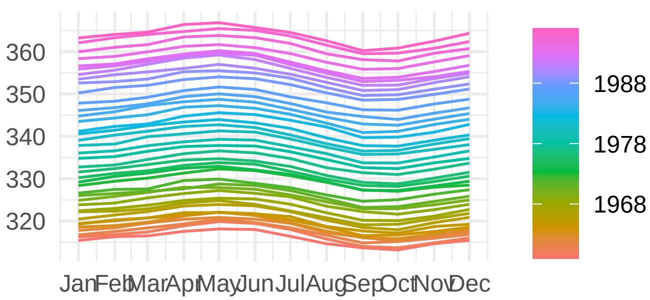
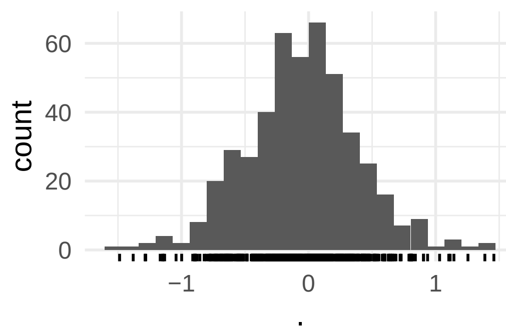



Fig.4 Histogram of irregular component by STL




The data measures the monthly average atmospheric CO2 concentration from 1959 to 1997, expressed in parts per million (ppm). It was initially collected by a infrared gas analyzer installed at Mauna Loa in Hawaii, which was one of the four analyzers installed by Keeling to evaluate whether there was a persistent increase in CO2 concentration.

Fig.1 shows a clear long-term upward trend, which is confirmed by Fig.2 where the growth rate for each year is above zero. Fig.2 also suggests the average growth rate after 1970 is higher than that before 1970, although there's no evidence of accelerating growth.

Another feature of the data is its robust seasonal pattern, with peak in May and bottom in October almost every year (see Fig.3). Keeling believes it was the result of the activity of land plants.

Fig.4 is the histogram of the remaining or irregular components after removing the trend and the seasonal components from the data with STL[1]. It

[1]Cleveland, R. B., Cleveland, W. S., McRae, J. E., & Terpenning, I. J. (1990). STL: A seasonal-trend decomposition procedure based on loess. Journal of Official Statistics,

looks like a normal distribution without obvious outliers.

```
co2 %>%
  features(value, unitroot_kpss)
```

```
## # A tibble: 1 x 2
##   kpss_stat kpss_pvalue
##       <dbl>       <dbl>
## 1      7.82        0.01
```

```
co2 %>%
  mutate(d_value=difference(value)) %>%
  features(d_value, unitroot_kpss)
```

```
## # A tibble: 1 x 2
##   kpss_stat kpss_pvalue
##       <dbl>       <dbl>
## 1    0.0124         0.1
```

```
co2 %>%
  features(value, unitroot_ndiffs)
```

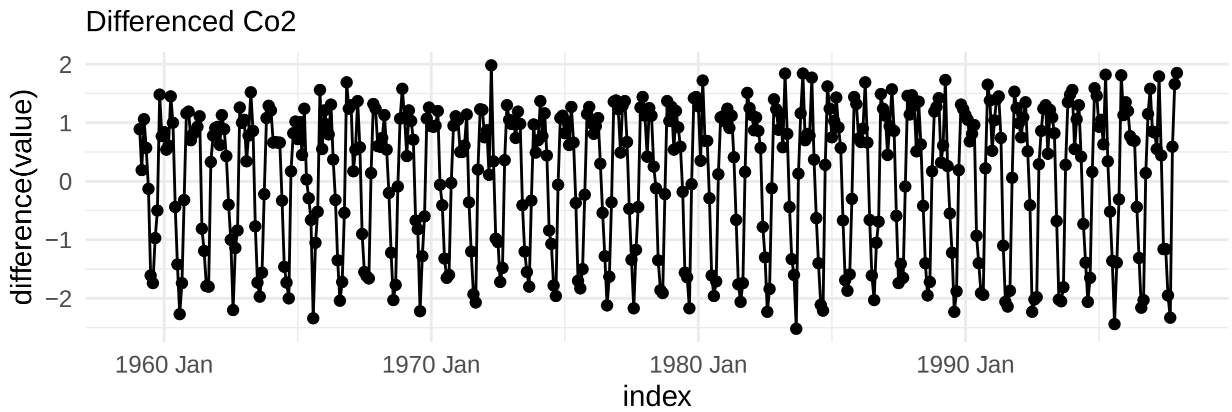
```
## # A tibble: 1 x 1
##   ndiffs
##    <int>
## 1      1
```

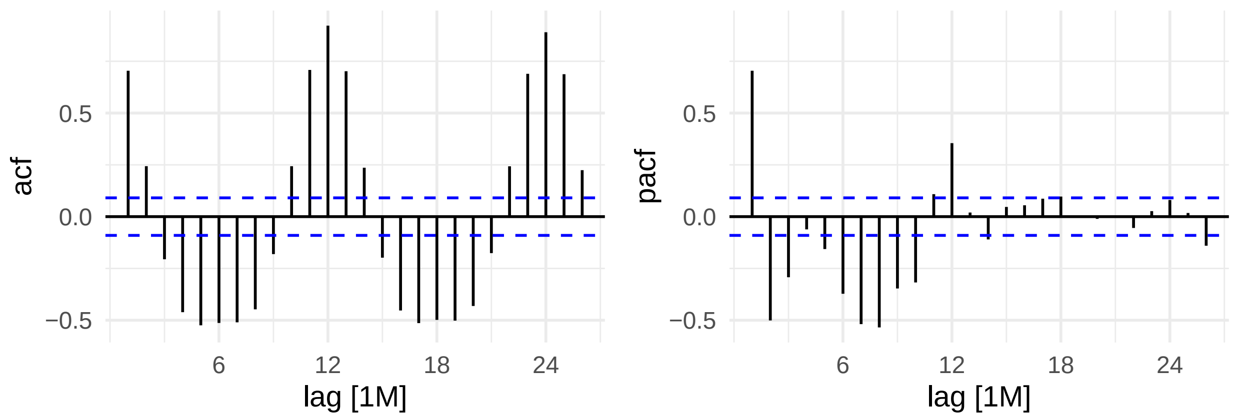
```
co2 %>% gg_tsdisplay(difference(value), plot_type="partial") +labs(subtitle = "Differenced Co2")
```

```
## Warning: Removed 1 row containing missing values (`geom_line()`).
```

```
## Warning: Removed 1 rows containing missing values (`geom_point()`).
```

6(1), 3–33.




### (3 points) Task 2a: Linear time trend model

Fit a linear time trend model to the `co2` series, and examine the characteristics of the residuals. Compare this to a quadratic time trend model. Discuss whether a logarithmic transformation of the data would be appropriate. Fit a polynomial time trend model that incorporates seasonal dummy variables, and use this model to generate forecasts to the year 2020.

```
linear_trend_model<-co2 %>% model(TSLM(value~trend()))
```

Since the long term trend of the $CO_2$ data looks linear and the variation around the trend seems stable, a log transformation of the data is not necessary (which is also supported by the stable and symmetric residuals in Fig.5) and we can fit the original data with a linear time trend model as:

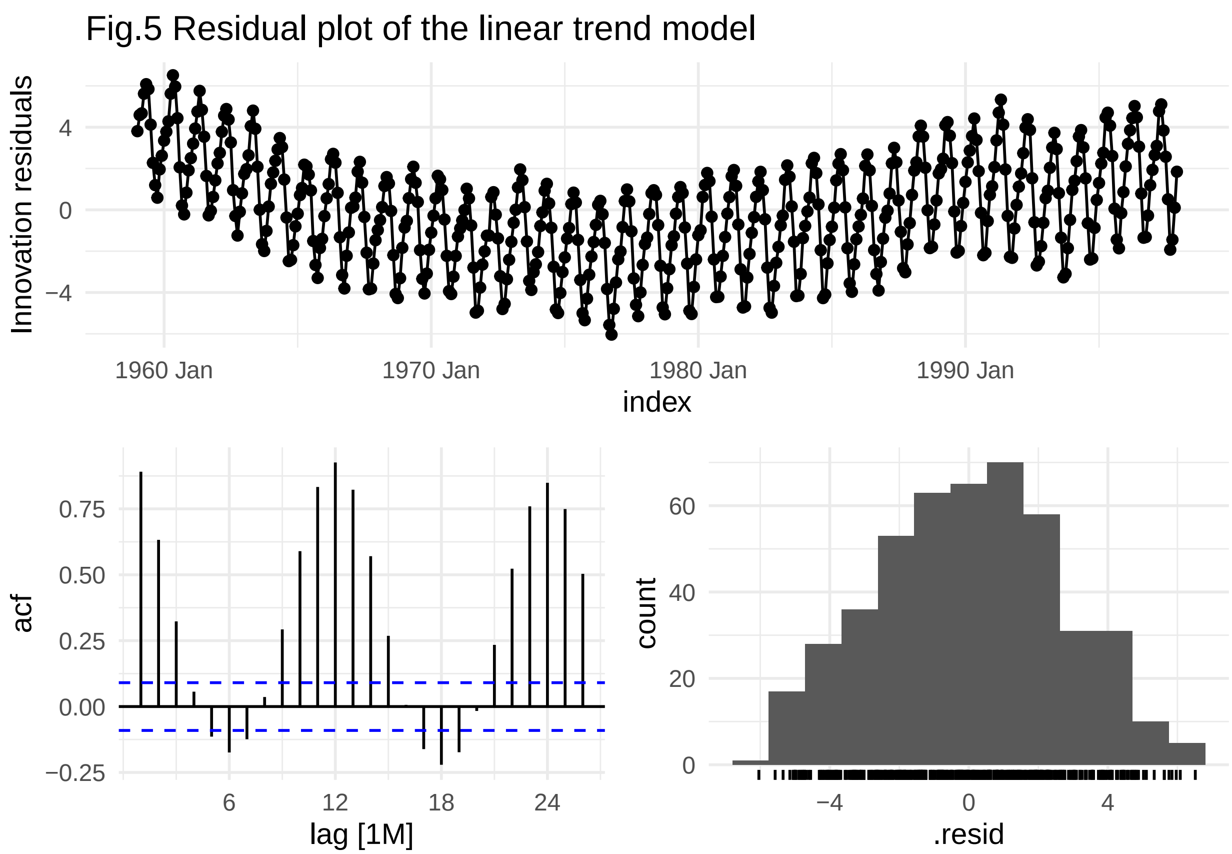
$$\mathrm{CO}_2 = \beta_0 + \beta_1 t + \epsilon_t \tag{1}$$

, which gives the parameters as:

$$\mathrm{CO}_2 = 311.5 + 0.11 * t + \epsilon_t \tag{2}$$

This linear trend model implies that the $CO_2$ concentration increased 0.11/month on average during 1959 to 1997. However, the residuals plots in Fig.5 suggest this simple linear trend model is not adequate since: 1) the mean of the residual forms a "U" shape along time, suggesting a quadratic or higher order polynomial trend model may be better; 2) the ACF plots indicates strong seasonal patterns exists in the residuals, suggesting seasonal dummy variables should be included in the model.

```
gg_tsresiduals(linear_trend_model) + ggtitle("Fig.5 Residual plot of the linear trend model")
```




```
co2_copy <- co2 %>% append_row(600) %>%
  mutate(
    num_index = time(index),
    num_index_qudratic = num_index ^ 2,
    num_index_cubic = num_index ^ 3,
  )
for (i in 1:11) {
  name =
    co2_copy <-
    co2_copy %>% mutate("month_{i}" := ifelse(lubridate::month(index) == i, 1, 0))
}
```

```r
co2_training = co2_copy %>% filter(lubridate::year(index) < 1991)
co2_valid = co2_copy %>% filter(lubridate::year(index) < 1998, lubridate::year(index) >= 1991)
co2_forecast = co2_copy %>% filter(lubridate::year(index) >= 1998)

# stargazer(model_linear,model_quadratic,model_cubic,type="text",
#           add.lines=list(c("AIC", round(AIC(model_linear),1), round(AIC(model_quadratic),1), round(AIC(model_cubic),1)),
#                          c("BIC", round(BIC(model_linear),1), round(BIC(model_quadratic),1), round(BIC(model_cubic),1))))
dummy_name=paste0("month_",1:11,collapse = "+")
fit <- co2_training |>
  model(
    model_linear = TSLM(as.formula(paste0("value ~ num_index   +",dummy_name))),
    model_quadratic = TSLM(as.formula(paste0("value ~ num_index  + num_index_qudratic +",dummy_name))),
    model_cubic = TSLM(as.formula(paste0("value ~ num_index + num_index_qudratic + num_index_cubic +",dummy_name)))
  )
report(fit)
```

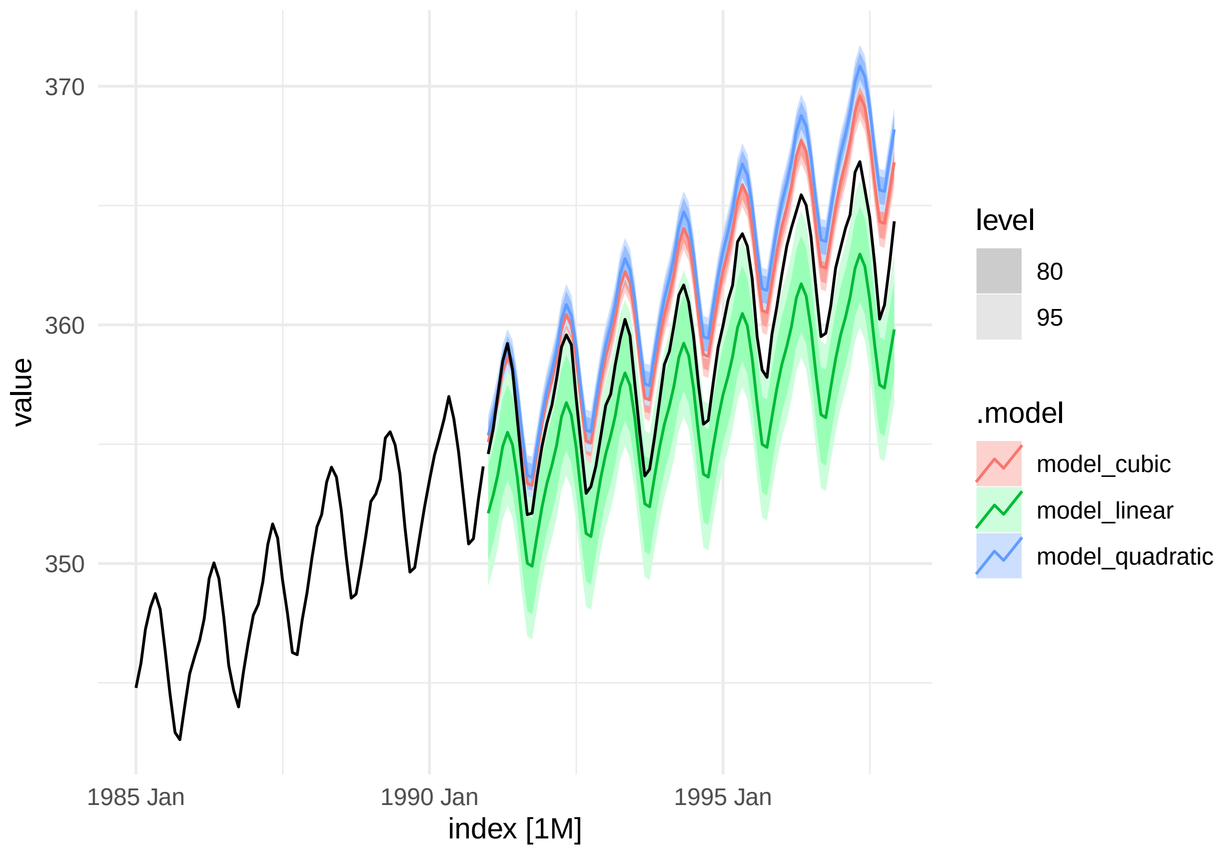
```
## Warning in report.mdl_df(fit): Model reporting is only supported for individual
## models, so a glance will be shown. To see the report for a specific model, use
## `select()` and `filter()` to identify a single model.
```

```
## # A tibble: 3 x 15
##   .model  r_squared adj_r_squared sigma2 statistic   p_value    df log_lik   AIC
##   <chr>       <dbl>         <dbl>  <dbl>     <dbl>     <dbl> <int>   <dbl> <dbl>
## 1 model_~     0.983         0.983  2.35      1842. 4.35e-322    13   -702.  343.
## 2 model_~     0.999         0.999  0.182    22346. 0            14   -210. -639.
## 3 model_~     0.999         0.999  0.172    21942. 0            15   -199. -660.
## # i 6 more variables: AICc <dbl>, BIC <dbl>, CV <dbl>, deviance <dbl>,
## #   df.residual <int>, rank <int>
```

```r
vd <- forecast(fit,new_data = co2_valid)
co2_training %>% filter(index>=yearmonth("1985M01")) %>% autoplot(value,PI = FALSE)+autolayer(vd)+autolayer(co2_valid)
```

```
## Warning in geom_line(...): Ignoring unknown parameters: `PI`
```

```
## Plot variable not specified, automatically selected `.vars = value`
```

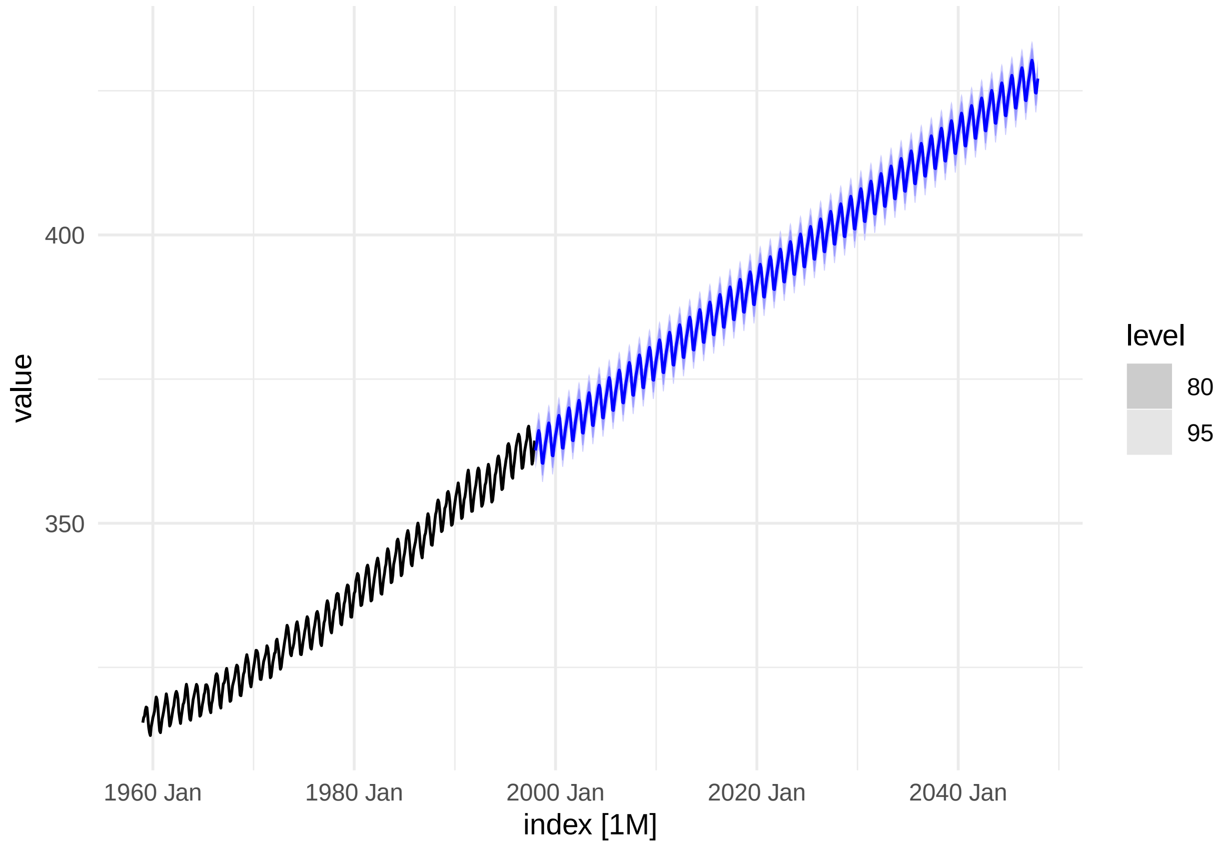



```
fabletools::accuracy(vd,co2_valid)
```

```
## # A tibble: 3 x 10
##   .model          .type    ME  RMSE   MAE    MPE  MAPE  MASE RMSSE  ACF1
##   <chr>           <chr> <dbl> <dbl> <dbl>  <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 model_cubic     Test  -2.09  2.28  2.12 -0.581 0.590   NaN   NaN 0.915
## 2 model_linear    Test   2.83  2.93  2.83  0.786 0.786   NaN   NaN 0.874
## 3 model_quadratic Test  -2.84  3.08  2.85 -0.790 0.792   NaN   NaN 0.937
```

```
# although the result above prefer the cubic model, I would suggest a linear or quadratic one.
final_model_poly <- co2_copy %>% filter(lubridate::year(index)<1998) %>%
  model(TSLM(as.formula(paste0("value ~ num_index + ",dummy_name))))
```

```r
fc_linear <- final_model_poly %>% forecast(co2_forecast)
co2_copy %>% filter(lubridate::year(index)<1998) %>% autoplot(value)+autolayer(fc_linear)
```




### (3 points) Task 3a: ARIMA times series model

We will use the Box Jenkins process to find the best ARIMA model via the following steps:

- Determine the appropriate model from eda
- find the best parameters
- examine the residuals using dianostic plots and statistical tests.

From the initial plots, we saw visual evidence of autoregressive and seasonal components. The ACF plots showed long slow decay of positive lag correlation, evidence of differencing. There is evidence of seasonality as well. We expect a seasonal arima model (SARIMA) with differencing to be best.

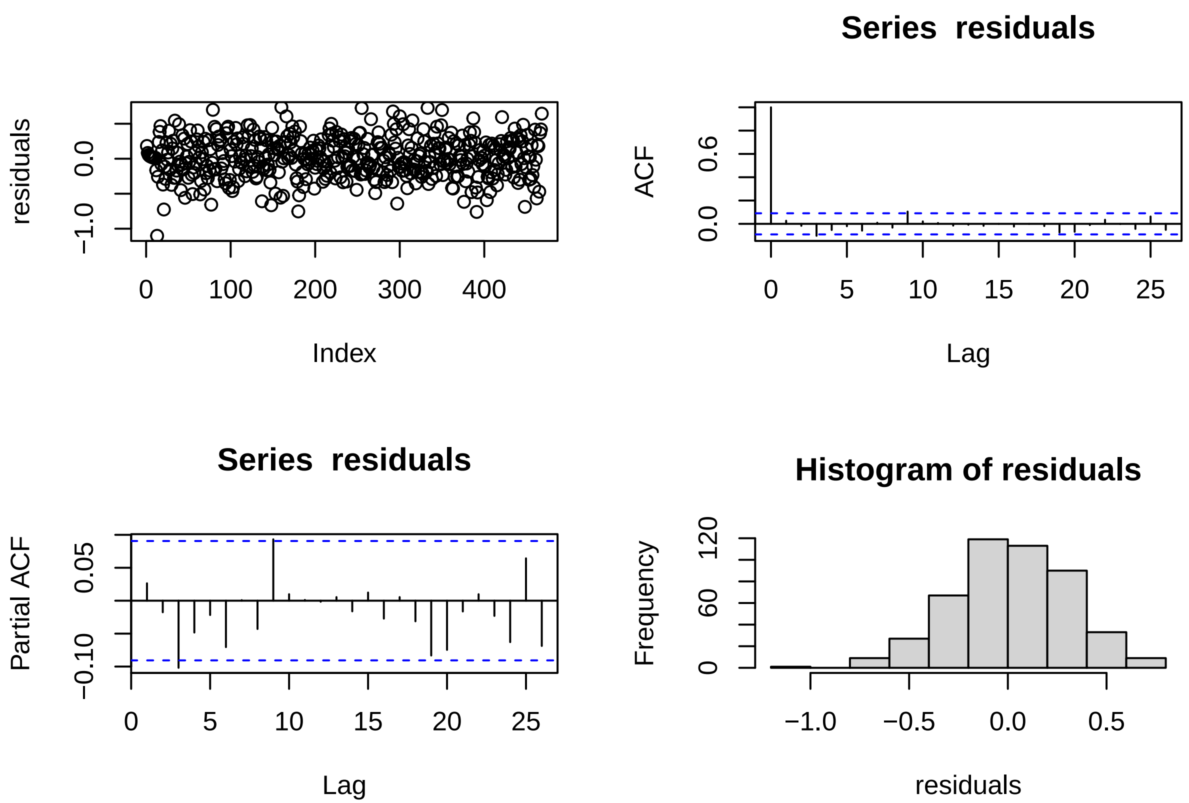
In this section, we fit the best SARIMA model and analyze the results. Simplicity is a desirable property in data science models to help explain the relationship between variables. We choose BIC as our information criteria because it penalizes complex models more than AIC or AICc and therefore selects more simple models with fewer parameters as the best ones. Lower BIC scores are better.

```
model.bic <-df %>%
  model(ARIMA(value ~ 0:1 + pdq(0:8,0:2,0:8) + PDQ(0:12,0:4,0:12), ic="bic", stepwise=F, greedy=F))

model.bic %>%
  report()
```

```
## Series: value
## Model: ARIMA(0,1,1)(1,1,2)[12]
##
## Coefficients:
##           ma1     sar1     sma1     sma2
##       -0.3482  -0.4986  -0.3155  -0.4641
## s.e.   0.0499   0.5282   0.5165   0.4367
##
## sigma^2 estimated as 0.08603:  log likelihood=-85.59
## AIC=181.18   AICc=181.32   BIC=201.78
```

After searching over seasonal and non seasonal P,D, and Q variables, the best model was an ARIMA(0,1,1)(1,1,2)[12] model with BIC score of 201.78. Next, we conclude the Box Jenkins process to evaluate the model via diagnostic plots and statistical tests.

```
x <- model.bic %>% augment() # tsibble
residuals <- x$.resid # vector

par(mfrow=c(2,2))
plot(residuals)
acf(residuals)
pacf(residuals)
hist(residuals)
```




The residual plots show that the SARIMA model was effective, with the residuals looking like stationary white noise. The time series has a mean of 0 with about constant variance, the ACF plot shows no autocorrelation beyond the initial lag value. The PACF plot appears to have a significant peak around the 3rd lag term, but this may be due to randomness, as it is barely passing the dashed blue line.The histogram looks normally distributed at 0 with outliers creating a left tail.

```
tsresid <- model.bic %>% augment() %>% select(.resid)
# adf test on residuals
dickey <- adf.test(tsresid$.resid, alternative = "stationary", k = 10)
```

```
## Warning in adf.test(tsresid$.resid, alternative = "stationary", k = 10):
## p-value smaller than printed p-value
```

```r
# box-jund test
# null is data is independently distributed
resid.ts<-model.bic %>%
  augment() %>%
  select(.resid) %>%
  as.ts()
box_1 <- Box.test(resid.ts, lag = 1, type = "Ljung-Box")
box_10 <- Box.test(resid.ts, lag = 10, type = "Ljung-Box")

adf.test(tsresid$.resid, alternative = "stationary", k = 10)
```

```
## Warning in adf.test(tsresid$.resid, alternative = "stationary", k = 10):
## p-value smaller than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  tsresid$.resid
## Dickey-Fuller = -6.4994, Lag order = 10, p-value = 0.01
## alternative hypothesis: stationary
```

```r
Box.test(resid.ts, lag = 1, type = "Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  resid.ts
## X-squared = 0.32959, df = 1, p-value = 0.5659
```

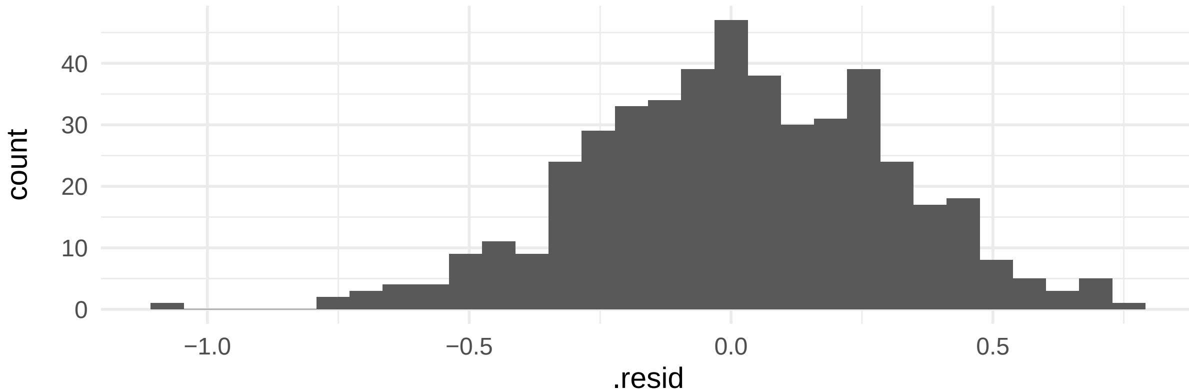
```r
Box.test(resid.ts, lag = 10, type = "Ljung-Box")
```

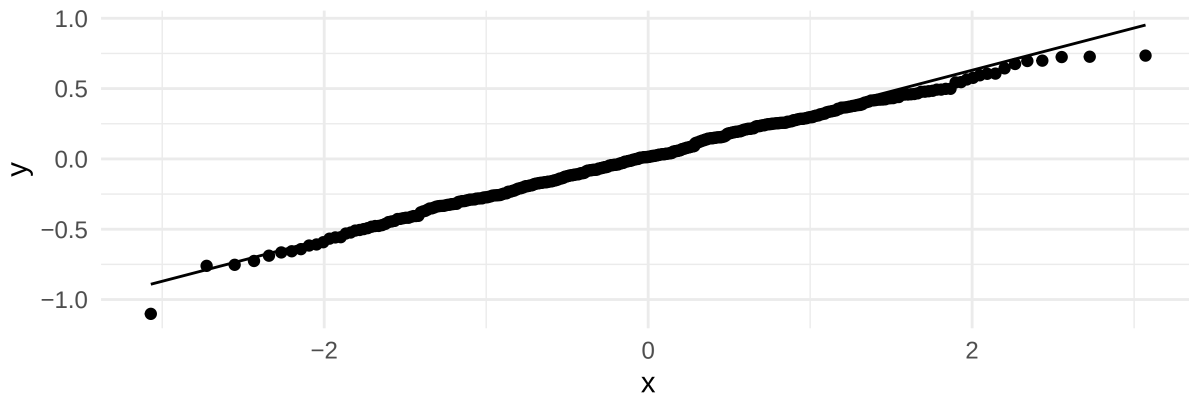
```
##
##  Box-Ljung test
##
## data:  resid.ts
## X-squared = 14.681, df = 10, p-value = 0.1441
```

```r
# qqplot on residuals, histogram on residuals
p1 <- model.bic %>%
  augment() %>%
  select(.resid) %>%
```

```
  ggplot() +
  geom_histogram(aes(x=.resid))

p2 <- model.bic %>%
  augment() %>%
  select(.resid) %>%
  ggplot(aes(sample=.resid)) +
  geom_qq() + stat_qq_line()


p1/p2
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```




We test the residuals for stationarity with the augmented dickey fuller test. The augmented dickey fuller test has the null hypothesis that the data is non stationary. With a p-value of 0.01, we reject the null hypothesis because there is enough evidence to say that the residuals are stationary.

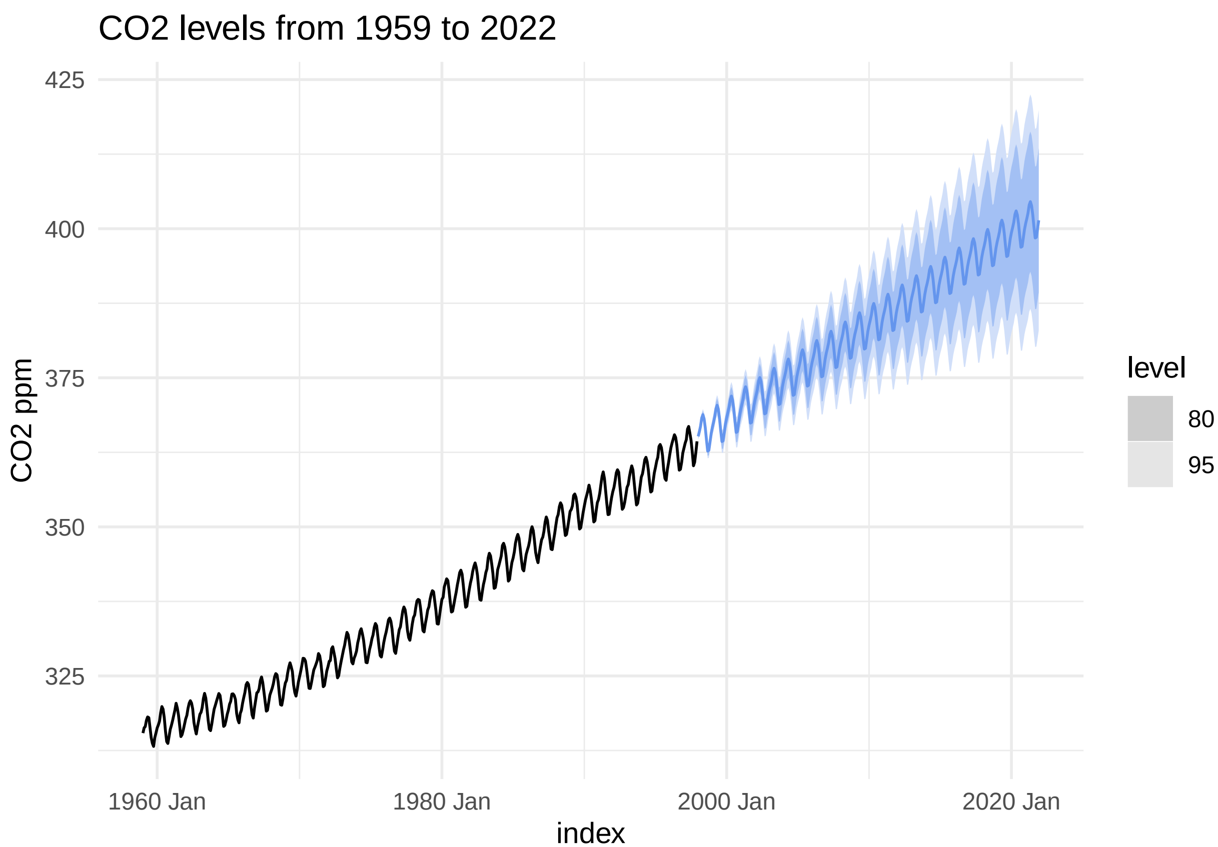
The Box-Ljung test has the null hypothesis that the data presented is independently distributed. When presented with the residuals of the ARIMA model, the test had p-values of 0.566 and 0.144 for lag =1 and lag = 10 respectively. For both of those lags, we fail to reject the null hypothesis and conclude that the data is independently distributed.

Finally, we visually inspect the histogram of the residuals and the qq plot to see if the residuals appear normally distributed. The histogram has the guassian bell shaped curve with a few outliers. The qq plot shows that the data matches up with the normal distribution's quantiles. With these plots, we can confidently say that the residuals are visibly normally distributed.

To conclude, both diagnostic plots and statistical tests show that the residuals are stationary with mean 0, constant variance, and no autoregression or seasonality. We forecast our model to the year 2022.

```
model.bic %>%
  forecast(h=(2022-1998)*12) %>%
autoplot(colour="cornflowerblue") +
autolayer(df, colour="black") +
  labs(y = "CO2 ppm",title = "CO2 levels from 1959 to 2022") +
  guides(colour = guide_legend(title = "Forecast"))
```

```
## Plot variable not specified, automatically selected `.vars = value`
```



### (3 points) Task 4a: Forecast atmospheric CO2 growth

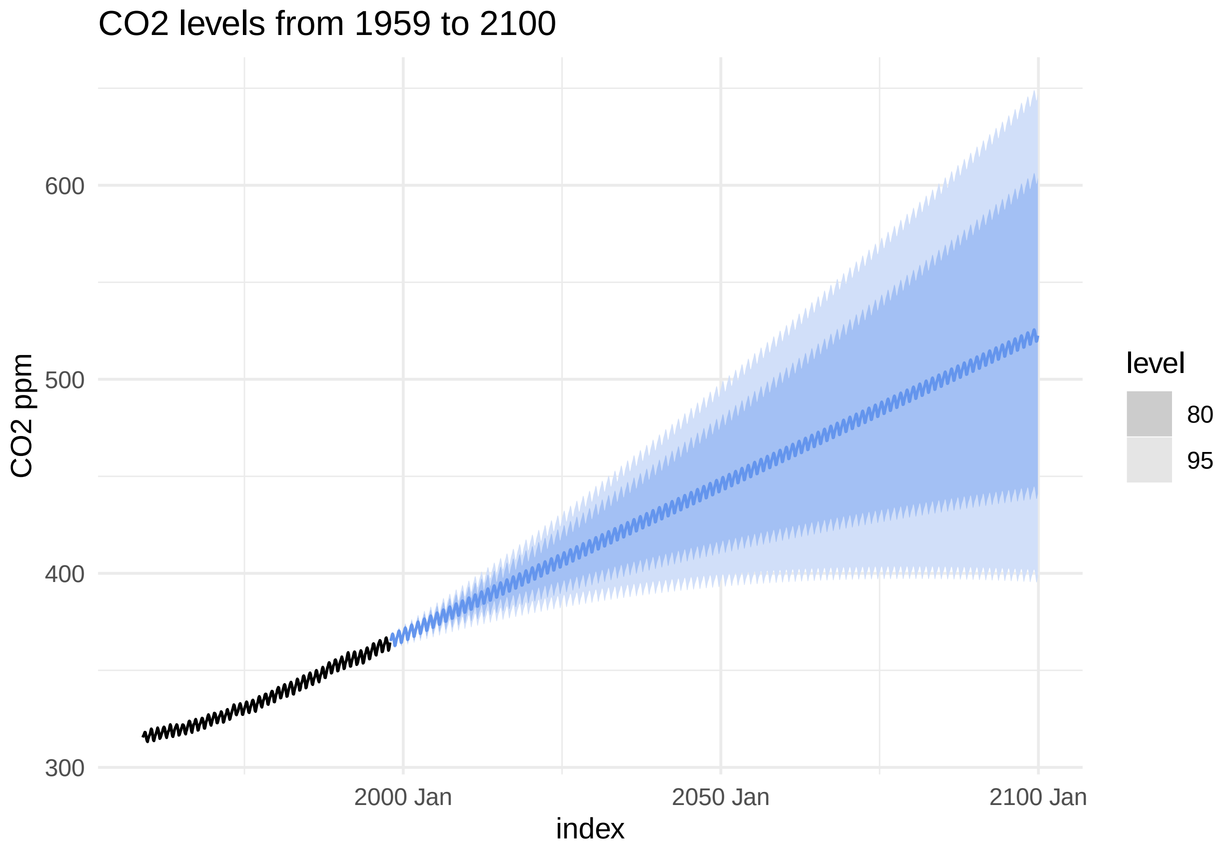
```
fc_arima <- model.bic %>% forecast(h=1900)
fc <-fc_arima %>% mutate(upper=quantile(value,0.95),lower=quantile(value,0.05))
first_420 <- fc %>% filter(upper>=420)
first_420 <- min(first_420$index)
last_420 <- fc %>% filter(lower < 420)
last_420 <- max(last_420$index)

first_500 <- fc %>% filter(upper >= 500)
first_500 <- min(first_500$index)
last_500 <- fc %>% filter(lower<=500)
last_500 <- max(last_500$index)
```

Based on our model, the first time we could potentially see CO2 in 420 ppm is 2022 Apr because that is when the upper 95% confidence interval (CI) of our model first reaches 420 ppm. The last time the model predicts we will see CO2 at 420 ppm is 2156 Apr, which is based on the final time the lower 95% CI is below 420.

The first time our model predicts the earth to reach 500 ppm CO2 on 2054 Apr, which is when the 95% CI reaches 500 ppm. The model's lower 95% CI never reaches 500, so there is no predicted final time. Below is the prediction of our model to the year 2100. Confidence intervals are shown fanning outward. The error of the predictions compounds overtime which expands the confidence intervals into a funnel shape. The farther out in time from the recorded data points, the less accurate the prediction.

```
model.bic %>%
  forecast(h=(2100-1998)*12) %>%
autoplot(colour="cornflowerblue") +
autolayer(df, colour="black") +
  labs(y = "CO2 ppm",title = "CO2 levels from 1959 to 2100") +
  guides(colour = guide_legend(title = "Forecast"))
```

```
## Plot variable not specified, automatically selected `.vars = value`
```




## Report from the Point of View of the Present

One of the very interesting features of Keeling and colleagues' research is that they were able to evaluate, and re-evaluate the data as new series of measurements were released. This permitted the evaluation of previous models' performance and a much more difficult question: If their models' predictions were "off" was this the result of a failure of the model, or a change in the system?

### (1 point) Task 0b: Introduction

In this introduction, you can assume that your reader will have **just** read your 1997 report. In this introduction, **very** briefly pose the question that you are evaluating, and describe what (if anything) has changed in the data generating process between 1997 and the present.

## (3 points) Task 1b: Create a modern data pipeline for Mona Loa CO2 data.

The most current data is provided by the United States' National Oceanic and Atmospheric Administration, on a data page [here]. Gather the most recent weekly data from this page. (A group that is interested in even more data management might choose to work with the hourly data.)

Create a data pipeline that starts by reading from the appropriate URL, and ends by saving an object called `co2_present` that is a suitable time series object.

Conduct the same EDA on this data. Describe how the Keeling Curve evolved from 1997 to the present, noting where the series seems to be following similar trends to the series that you "evaluated in 1997" and where the series seems to be following different trends. This EDA can use the same, or very similar tools and views as you provided in your 1997 report.

```r
# library(zoo)
# co2_present_raw=read.csv("https://gml.noaa.gov/webdata/ccgg/trends/co2/co2_weekly_mlo.csv",skip=51)
# co2_present <- co2_present_raw %>%
#   mutate(time_index=lubridate::make_date(year,month,day)) %>%
#   dplyr::select(time_index,average) %>%
#   as_tsibble(index = time_index) %>%
#   mutate(average =replace(average,average<=-999,NA)) %>%
#   mutate(average = na.approx(average))
```

## (1 point) Task 2b: Compare linear model forecasts against realized CO2

Descriptively compare realized atmospheric CO2 levels to those predicted by your forecast from a linear time model in 1997 (i.e. "Task 2a"). (You do not need to run any formal tests for this task.)

```r
# co2 %>% autoplot(value) + autolayer(fc_linear) + autolayer(co2_present)
```

## (1 point) Task 3b: Compare ARIMA models forecasts against realized CO2

Descriptively compare realized atmospheric CO2 levels to those predicted by your forecast from the ARIMA model that you fitted in 1997 (i.e. "Task 3a"). Describe how the Keeling Curve evolved from 1997 to the present.

```r
# co2 %>% autoplot(value) + autolayer(fc_arima) + autolayer(co2_present)
```

## (3 points) Task 4b: Evaluate the performance of 1997 linear and ARIMA models

In 1997 you made predictions about the first time that CO2 would cross 420 ppm. How close were your models to the truth?

After reflecting on your performance on this threshold-prediction task, continue to use the weekly data to generate a month-average series from 1997 to the present, and compare the overall forecasting performance of your models from Parts 2a and 3b over the entire period. (You should conduct formal tests for this task.)

## (4 points) Task 5b: Train best models on present data

Seasonally adjust the weekly NOAA data, and split both seasonally-adjusted (SA) and non-seasonally-adjusted (NSA) series into training and test sets, using the last two years of observations as the test sets. For both SA and NSA series, fit ARIMA models using all appropriate steps. Measure and discuss how your models perform in-sample and (psuedo-) out-of-sample, comparing candidate models and explaining your choice. In addition, fit a polynomial time-trend model to the seasonally-adjusted series and compare its performance to that of your ARIMA model.

```
# Scott mentioned STL for seasonal adjusment
```

## (3 points) Task Part 6b: How bad could it get?

With the non-seasonally adjusted data series, generate predictions for when atmospheric CO2 is expected to be at 420 ppm and 500 ppm levels for the first and final times (consider prediction intervals as well as point estimates in your answer). Generate a prediction for atmospheric CO2 levels in the year 2122. How confident are you that these will be accurate predictions?