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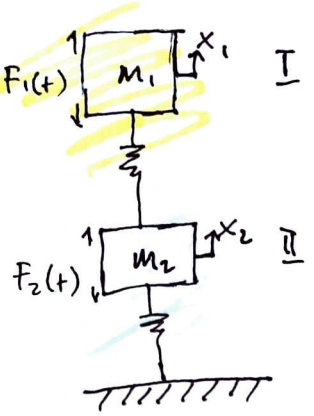
Sistem 2 DOF



$k_1 = 120$ $m_1 = 4$

$k_2 = 144$ $m_2 = 12$

① model matematis

I $m_1 \ddot{x}_1 = \Sigma F = -k_1(x_1 - x_2) + F_1(t)$

$m_1 \cdot \ddot{x}_1 = -k_1 \cdot x_1 + k_1 \cdot x_2 + f_1(t)$

$$\ddot{x}_1 = \frac{-k_1}{m_1} x_1 + \frac{k_1}{m_1} x_2 + \frac{f_1}{m_1}(t)$$

II $m_2 \ddot{x}_2 = \Sigma F = -k_2 x_2 - k_1(x_2 - x_1) + F_2(t)$

$m_2 \ddot{x}_2 = -(k_1 + k_2) x_2 + k_1 x_1 + F_2(t)$

$$\ddot{x}_2 = \frac{-(k_1 + k_2)}{m_2} x_2 + \frac{k_1 x_1}{m_2} + \frac{F_2}{m_2}(t)$$

② frequensi natural

① $\ddot{x}_1 = -\frac{k_1}{m_1} x_1 + \frac{k_2}{m_1} x_2 + \frac{F_1}{m_1}(t) = 0$

$x_1 = A \sin \omega t$

$\dot{x}_1 = \omega A \cos \omega t$

$\ddot{x}_1 = -\omega^2 A \sin \omega t$

② $\ddot{x}_2 = \frac{-(k_1 + k_2)}{m_2} x_2 + \frac{k_1}{m_2} x_1 + \frac{F_2}{m_2}(t) = 0$

$x_2 = B \sin \omega t$

$\dot{x}_2 = \omega B \cos \omega t$

$\ddot{x}_2 = -\omega^2 B \sin \omega t$

Maka

$$-\omega^2 A \sin\omega t = -\frac{k_1}{m_1} A \sin\omega t + \frac{k_1}{m_1} B \sin\omega t$$

$$-\omega^2 A = -\frac{k_1}{m_1} A + \frac{k_1}{m_1} \cdot B$$

$$0 = \omega^2 \cdot A - \frac{k_1}{m_1} A + \frac{k_1}{m_1} B$$

$$0 = \left[\omega^2 - \frac{k_1}{m_1}\right] A + \frac{k_1}{m_1} B$$

$$-\omega^2 B \sin\omega t = \frac{-(k_1+k_2)}{m_2} B \sin\omega t + \frac{k_1}{m_2} A \sin\omega t$$

$$0 = \frac{k_1}{m_2} A + \omega^2 B - \frac{(k_1+k_2)}{m_2} B$$

$$0 = \frac{k_1}{m_2} A + \left[\omega^2 - \frac{(k_1+k_2)}{m_2}\right] B$$

Determinan

$$\begin{bmatrix} \omega^2 - \frac{k_1}{m_1} & \frac{k_1}{m_1} \\ \frac{k_1}{m_2} & \omega^2 - \frac{k_1+k_2}{m_2} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$D = 0$

$$\left(\omega^2 - \frac{k_1}{m_1}\right)\left(\omega^2 - \frac{(k_1+k_2)}{m_2}\right) - \left(\frac{k_1}{m_1}\right)\left(\frac{k_1}{m_1}\right) = 0$$

$$\omega^4 - \frac{k_1}{m_2}\omega^2 - \frac{k_2}{m_2}\omega^2 - \frac{k_1}{m_1}\omega^2 + \frac{k_1 \cdot k_2}{m_1 m_2}$$

$$\omega^4 - \frac{120}{12}\omega^2 - \frac{144}{12}\omega^2 - \frac{120}{4}\omega^2 + \frac{120 \cdot 144}{4 \cdot 12}$$

$$\omega^4 - 10\omega^2 - 12\omega^2 - 30\omega^2 + 360$$

$$\omega^4 - 52\omega^2 + 360$$

$$(\omega^2 - 43.78)(\omega^2 - 8.23) = 0$$

$\omega_1^2 = 43.78$ $\quad$ $\omega_2^2 = 8.23$

$\omega_1 = 6.62$ $\quad$ $\omega_2 = 2.87$

ABC formula

$$x_{1,2} = \frac{-52 \pm \sqrt{(52)^2 - 4.1.360}}{2.1}$$

$x_1 = 43.78$

$x_2 = 8.23$

$D = b^2 - 4ac$

$= 1264$

nilai $D > 0$

$x_1 \neq x_2$

③ state-space

① $\ddot{x}_1 = -\frac{k_1}{m_1}x_1 + \frac{k_1}{m_1}x_2 + \frac{1}{m_1} \cdot f_1$

② $\ddot{x}_2 = -\frac{(k_1+k_2)}{m_2}x_2 + \frac{k_1}{m_2}x_1 + \frac{1}{m_1}F_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1}{m_1} & k_1/m_1 & 0 & 0 \\ \frac{k_1}{m_2} & -\frac{k_1+k_2}{m_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/m_1 & 0 \\ 0 & 1/m \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -30 & 30 & 0 & 0 \\ 10 & -22 & 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.25 & 0 \\ 0 & 0.83 \end{bmatrix}}_{B} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{D} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

⑥ FFT

Hasil FFT : 1 Hz dan 0,4 Hz

⑦ membandingkan hasil

FFT

$f_1 = 1,1$

$f_2 = 0,5$

perhitungan

$\omega_1 = 6,62$ $\omega_2 = 2,87$

$\omega_1 = 2\pi f_1$ $\omega_2 = 2\pi f_2$

$6,62 = 2,1 f_1$ $f_2 = 0,44$

$f_1 = 1,05$

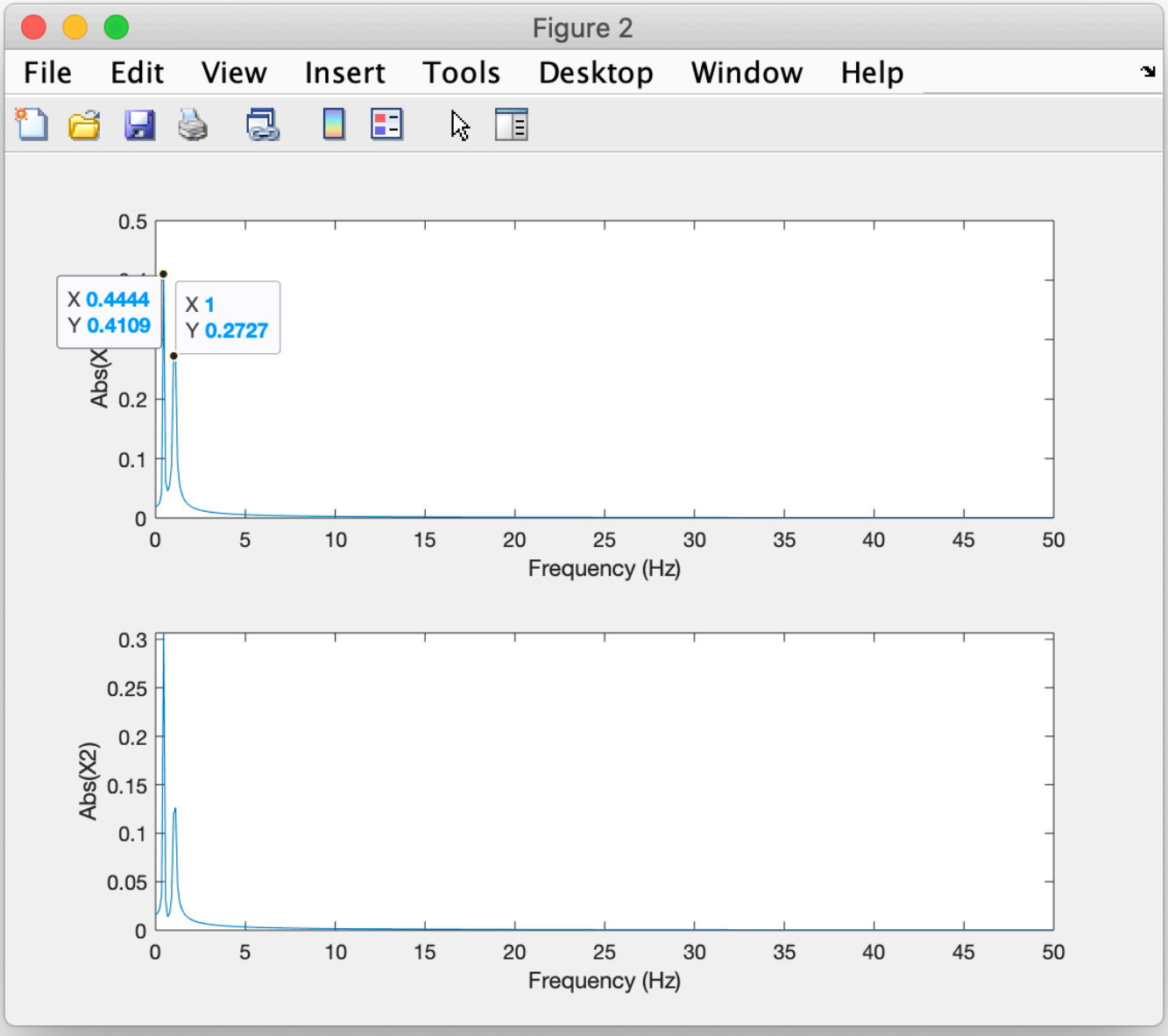


Figure 2
File Edit View Insert Tools Desktop Window Help
X 0.4444
Y 0.4109
X 1
Y 0.2727
Abs(X
0.5
0.2
0.1
0
0 5 10 15 20 25 30 35 40 45 50
Frequency (Hz)
Abs(X2)
0.3
0.25
0.2
0.15
0.1
0.05
0
0 5 10 15 20 25 30 35 40 45 50
Frequency (Hz)