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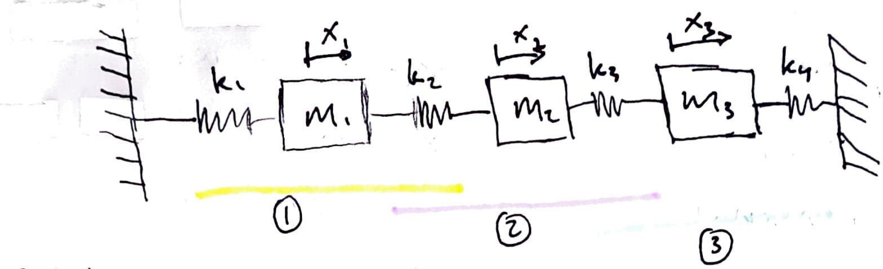
1)

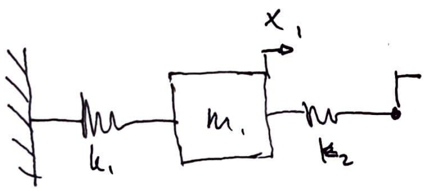


a) Perhitungan matematis

Asumsi $k_1 = k_2 = k_3 = k_4 = 120$ N/m

$m_1 = m_2 = m_3 = 12$ kg

①



$\Sigma F = m.a = m_1 \ddot{x}_1$

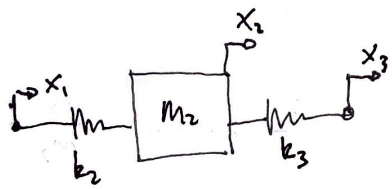
$-k_1 x_1 - k_2(x_1 - x_2) + F_1(t) = m_1 \ddot{x}_1$

$0 = m_1 \ddot{x}_1 + k_1 x_1 + k_2(x_1 - x_2) + F_1(t)$

$0 = m_1 \ddot{x}_1 + (k_1 + k_2) x_1 - k_2 x_2 + F_1(t)$

$x_1 = -\dfrac{(k_1 + k_2)}{m_1} x_1 + \dfrac{k_2}{m_1} x_2 + \dfrac{F_1}{m_1}(t)$

②



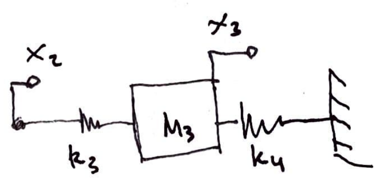
$\Sigma F = m.a = m_2 \ddot{x}_2$

$-k_2(x_2 - x_1) - k_3(x_2 - x_3) + F_2(t) = m_2 \ddot{x}_2$

$-(k_2 + k_3) x_2 + k_2 x_1 + k_3 x_3 + F_2(t) = m_2 \ddot{x}_2$

$x_2 = -\dfrac{(k_2 + k_3)}{m_2} \cdot x_2 + \dfrac{k_2 x_1}{m_2} + \dfrac{k_3}{m_2} x_3 + \dfrac{F_2}{m_2}(t)$

③



$\Sigma F = m.a = m_3 \ddot{x}_3$

$-k_3(x_3 - x_2) - k_4 x_3 + F_2(t) = m_3 \ddot{x}_3$

$m_3 \ddot{x}_3 = -(k_3 + k_4) x_3 + k_3 x_2 + F_3(t)$

$x_3 = -\dfrac{(k_3 + k_4)}{m_3} x_3 + \dfrac{k_3}{m_3} x_2 + \dfrac{F_3}{m_2}(t)$

B) Frequensi natural

$$\ddot{x}_1 = -\frac{(k_1+k_2)}{m_1} x_1 + \frac{k_2}{m_1} x_2 + \frac{F_1}{m_1}(t)$$

$$\ddot{x}_2 = -\frac{(k_2+k_1)}{m_2} x_2 + \frac{k_2}{m_2} x_1 + \frac{k_3}{m_2} x_3 + \frac{F_2}{m_2}(t)$$

Asumsi $x_1 = A \cdot e^{jwt}$

$x_1 = A \sin wt$

$\dot{x}_1 = w A \cos wt$

$\ddot{x}_1 = -w^2 A \sin wt$

$x_2 = B \sin wt$

$\dot{x}_2 = wB \cos wt$

$\ddot{x}_2 = -w^2 B \sin wt$

$$\ddot{x}_3 = -\frac{(k_3+k_4)}{m_3} x_3 + \frac{k_3}{m_3} x_2 + \frac{F_3}{m_3}(t)$$

$x_3 = C \sin wt$

$\dot{x}_3 = wC \cos wt$

$\ddot{x}_3 = -w^2 C \sin wt$

① $$-w^2 A \cancel{\sin wt} = -\frac{(k_1+k_2)}{m_1} A \cancel{\sin wt} + \frac{k_2}{m_1} B \cancel{\sin wt}$$

$$0 = \left[w^2 - \frac{(k_1+k_2)}{m_1}\right] A + \frac{k_2}{m_1} \cdot B$$

② $$-w^2 B \cancel{\sin wt} = -\frac{(k_2+k_1)}{m_2} B \cancel{\sin wt} + \frac{k_2}{m_2} A \cancel{\sin wt} + \frac{k_3}{m_3} C \cancel{\sin wt}$$

$$0 = A \frac{k_2}{m_2} + \left[w^2 - \frac{k_2+k_3}{m_2}\right] B + \frac{k_2}{m_2} C$$

③ $$-w^2 C \cancel{\sin wt} = -\frac{(k_3+k_4)}{m_3} C \cancel{\sin wt} + \frac{k_3}{m_3} B \cancel{\sin wt}$$

$$0 = \frac{k_3}{m_3} B + \left[w^2 - \frac{(k_3+k_4)}{m_3}\right] C$$

Determinan

$$\begin{bmatrix} \omega^2 - \frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & 0 \\ \frac{k_2}{m_2} & \omega^2 - \frac{(k_2+k_3)}{m_2} & \frac{k_2}{m_2} \\ 0 & \frac{k_3}{m_3} & \omega^3 - \frac{(k_3+k_4)}{m_3} \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|cc} \omega^2-20 & 10 & 0 & \omega^2-20 & 10 \\ 10 & \omega^2-20 & 10 & 10 & \omega^2-20 \\ 0 & 10 & \omega^2-20 & 0 & 10 \end{array}\right]$$

- $(\omega^2-20)(\omega^2-20)(\omega^2-20) + 0 + 0 - 0 - (100(\omega^2-20)) - (100(\omega^2-20)) = 0$
- $(\omega^4 - 40\omega^2 + 400)(\omega^2-20) - (100\omega^2 - 2000) - (100\omega^2 - 2000) = 0$
- $\omega^6 - 40\omega^4 + 400\omega^2 - 20\omega^4 + 800\omega^2 - 8000 - 100\omega^2$

  $+ 2000 - 100\omega^2 + 2000 = 0$

- $\omega^6 - 60\omega^4 + 1000\omega^2 - 4000 = 0$

$\omega_1 = 5{,}8431 \rightarrow \omega_1 = 2\pi f \rightarrow f_1 = \frac{5{,}8431}{6{,}28} = 0{,}930$

$\omega_2 = 4{,}4721 \rightarrow f_2 = \frac{4{,}4721}{6{,}28} = 0{,}712$

$\omega_3 = 2{,}4203 \rightarrow f_3 = \frac{2{,}4203}{6{,}28} = 0{,}385$

c) State space

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-(k_1+k_2)}{m_1} & \frac{k_2}{m_1} & 0 & 0 & 0 & 0 \\ \frac{k_2}{m_2} & \frac{-(k_2+k_3)}{m_2} & k_3/m_3 & 0 & 0 & 0 \\ 0 & k_3/m_3 & \frac{-k_3+k_4}{m_3} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}$$

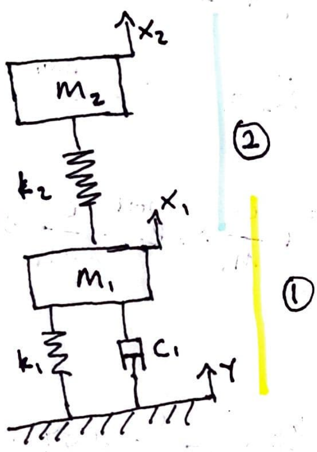
$$+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m_1} & 0 & 1/m_3 \\ 0 & 1/m_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -20 & 10 & 0 & 0 & 0 & 0 \\ 10 & -20 & 10 & 0 & 0 & 0 \\ 0 & 10 & -20 & 0 & 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}$$

$$+ \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/12 & 0 & 1/12 \\ 0 & 1/12 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{B} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{D} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

2

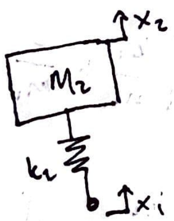


$k_1 = k_2 =$

$m_1 = m_2 =$

$c_1 =$

2

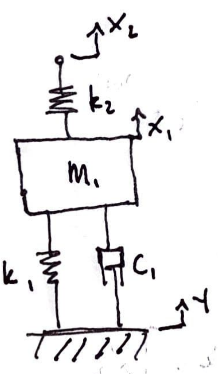


$$M_2 \ddot{x}_2 = \Sigma F = -k_2(x_2 - x_1) + F_2(t)$$

$$M_2 \ddot{x}_2 = -k_2 . x_2 + k_2 x_1 + F_2(t)$$

$$\ddot{x}_2 = -\frac{k_2}{m_2} x_2 + \frac{k_2}{m_2} x_1 + \frac{F_2}{m_2}(t)$$

1



$$m_1 \ddot{x}_1 = F_1(t) - k_1(\dot{y} - x_1) - k_2(x_1 - x_2) - c(\dot{y} - \dot{x}_1)$$

$$m_1 \ddot{x}_1 = F_1(t) + (k_1 - k_2)x_1 - k_1 y + k_2 . x_2 - c\dot{y} + c\dot{x}$$

$$\ddot{x}_1 = \frac{(k_1 - k_2)}{m_1} x_1 - \frac{k_1}{m_1} y + \frac{k_2}{m_1} x_2 - \frac{c}{m_1} \dot{y}$$

$$+ \frac{c}{m_1} \dot{x}_1 + \frac{F_1}{m_1}(t)$$

(b) Motion Transmissibility

$$\ddot{x}_1 = \frac{(k_1-k_2)}{m_1}x_1 - \frac{k_1}{m_1}y + \frac{k_2}{m_1}x_2 - \frac{c}{m_1}\dot{y} + \frac{c}{m_1}\dot{x}_1 + \frac{F_1}{m_1}(t)$$

$$\ddot{x}_2 = -\frac{k_2}{m_2}x_2 + \frac{k_2}{m_2}x_1 + \frac{F_2}{m_2}(t)$$

$$y = Ye^{j\omega t} \rightarrow \dot{y} = Yj\omega e^{j\omega t} \rightarrow \ddot{y} = -\omega^2 Y e^{j\omega t}$$

$$x_1 = X_1 e^{j\omega t} \rightarrow \dot{x}_1 = X_1 j\omega e^{j\omega t} \rightarrow \ddot{x}_1 = -\omega^2 X_1 e^{j\omega t}$$

$$x_2 = X_2 e^{j\omega t} \rightarrow \dot{x}_2 = X_2 j\omega e^{j\omega t} \rightarrow \ddot{x}_2 = -\omega^2 X_2 e^{j\omega t}$$

(1) $$-\omega^2 X_1 \cancel{e^{j\omega t}} = \frac{(k_1-k_2)}{m_1}X_1\cancel{e^{j\omega t}} - \frac{k_1}{m_1}Y\cancel{e^{j\omega t}} + \frac{k_2}{m_1}X_2\cancel{e^{j\omega t}} - \frac{c}{m_1}j\omega Y\cancel{e^{j\omega t}}$$
$$+ \frac{c}{m_1}j\omega X_1 \cancel{e^{j\omega t}}$$

$$-\omega^2 X_1 = \frac{(k_1-k_2)}{m_1}X_1 - \frac{k_1}{m_1}Y + \frac{k_2}{m_1}X_2 - \frac{c}{m_1}j\omega Y + \frac{c}{m_1}j\omega X_1$$

$$0 = \left[\omega^2 + \frac{(k_1-k_2)}{m_1} + \frac{c}{m_1}j\omega\right]X_1 - \left[\frac{k_1}{m_1} + \frac{c}{m_1}j\omega\right]Y + \frac{k_2}{m_1}X_2$$

$$0 = \left[\omega^2 + \frac{(k_1-k_2)}{m_1} + \frac{c}{m_1}j\omega\right]X_1 + \frac{k_2}{m_1}X_2 - \left[\frac{k_1}{m_1} + \frac{c}{m_1}j\omega\right]Y$$

(2) $$-\omega^2 X_2 e^{j\omega t} = -\frac{k_2}{m_2}X_2 e^{j\omega t} + \frac{k_2}{m_2}X_1 e^{j\omega t}$$

$$-\omega^2 X_2 = -\frac{k_2}{m_2}X_2 + \frac{k_2}{m_2}X_1$$

$$0 = \frac{k_2}{m_2}X_1 + \left[\omega^2 - \frac{k_2}{m_2}\right]X_2$$

(1)

$$0 = \left[\omega^2 + \frac{(k_1 - k_2)}{m_1} + \frac{c}{m_1} j\omega\right] x_1 + \frac{k_2}{m_1} x_2 - \left[\frac{k_1}{m_1} + \frac{c}{m_1} j\omega\right] Y$$

$$\left[\frac{k_1}{m_1} + \frac{c}{m_1} j\omega\right] y = \left[\omega^2 + \frac{k_1 - k_2}{m_1} + \frac{c}{m_1} j\omega\right] x_1 + \frac{k_2}{m_1} x_2$$

$$Y = \frac{\left[\omega^2 + \frac{(k_1 - k_2)}{m_1} + \frac{c}{m_1} j\omega\right] x_1 + \frac{k_2}{m_1} x_2}{\frac{k_1}{m_1} + \frac{c}{m_1} j\omega}$$

(2)

$$0 = \frac{k_2}{m_2} x_1 + \left[\omega^2 - \frac{k_2}{m_2}\right] x_2$$

$$\left[\omega^2 - \frac{k_2}{m_2}\right] x_2 = -\frac{k_2}{m_2} x_1$$

$$x_2 = \frac{-\frac{k_2}{m_2} x_1}{\omega^2 - \frac{k_2}{m_2}}$$

(1)

$$Y = \frac{\left[\omega^2 + \frac{k-k}{m} + \frac{c}{m} j\omega\right] x_1 + \frac{k}{m}\left(\frac{k}{\omega^2 m - k}\right)}{\frac{k}{m} + \frac{c}{m} j\omega}$$

$$= \frac{\left(\omega^2 + \frac{c}{m} j\omega\right) x_1 - \frac{k_2}{\omega^2 m^2 - km} x_1}{\frac{k}{m} + \frac{c}{m} j\omega}$$

$$x_2 = \frac{-\frac{k}{m} \cdot x_1}{\omega^2 - \frac{k}{m}} = -\frac{k}{m} x_1 \left(\frac{m}{\omega^2 m - k}\right)$$

$$= -\frac{k}{\omega^2 m - k} x_1$$

$$\epsilon = \frac{x_2}{Y} = \frac{\dfrac{k}{w^2 m - k} x_1}{\left[w^2 + \dfrac{c}{m} Jw - \dfrac{k}{w^2 m^2 - km} \right] x_1} \left[\frac{k}{m} + \frac{c}{m} Jw \right]$$

Asumsi $k = 120$, $m = 12$, $c = 1{,}2$

$$\epsilon = \frac{\dfrac{120}{w^2 12 - 120}}{w^2 + \dfrac{1{,}2}{12} Jw - \dfrac{120}{w^2 . 12^2 - 120 . 12}} \left[\frac{120}{12} + \frac{1{,}2}{12} Jw \right]$$

$$\epsilon = \frac{\dfrac{120}{w^2 . 12 - 120}}{w^2 + 0{,}1 Jw - \dfrac{120}{144 w^2 - 1440}} \left[10 + 0{,}1 Jw \right]$$

$$\epsilon = \frac{(1200 + 12 Jw) / (12 w^2 - 120)}{\dfrac{144 w^4 - 1440 w^2 + 14{,}4 w^3 J - 144 Jw - 120}{144 w^2 - 1440}}$$

$$= \frac{(1200 + 12 Jw)}{12 w^2 - 120} \cdot \frac{144 w^2 - 1440}{144 w^4 - 1440 w^2 + 14{,}4 w^3 J - 144 Jw - 120}$$

$$= \frac{\cancel{12} (100 + Jw)}{\cancel{12} \cancel{(w^2 - 10)}} \cdot \frac{\cancel{144} \cancel{(w^2 - 10)}}{\cancel{144} (w^4 - 10 w^2 + 0{,}1 w^3 J - Jw - 0{,}8}$$

$$= \frac{100 + Jw}{w^4 - 10 w^2 + 0{,}1 w^3 J - Jw - 0{,}8}$$