

Taylor and Pade` approximations: warming-up exercise

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Taylor approximation

- First task: approximate the natural exponential $e(x)$ with a Taylor approximation, for x in $[-10; 10]$
- We are just going to inspect the result graphically
- Write a program that prints the value of the built-in exponential function and its Taylor approximation on a grid of points between $x=-10$ and $x=10$
- Do it with the Taylor polynomial of degree 5 and of degree 7
- First we have to remember how the Taylor series looks like:

$$e(x) = \sum_{n=0}^{\infty} x^n/n!$$

Taylor approximation

- In this first exercise we just look qualitatively at the results: where does the approximation go REALLY wrong?
- Where does it look like to work best? Why? How is it connected to our choice of the Taylor expansion?
- Increasing further the degree of the polynomial, what do you expect to happen?

Pade` approximation

- Let's now turn to the Pade` approximation

$$f(x) \approx \frac{\sum_{j=0}^m a_j x^j}{\sum_{j=0}^n b_j x^j} = \frac{p(x)}{q(x)}$$

- This is the Pade` approximant of order [m/n]
- We are going to use the Pade` approximant of order [4/4]

Pade` approximation

- For a generic function one should first figure out the coefficients:
- $f(0) = R(0)$
- $f'(0) = R'(0)$
- ...
- But for the exponential there is a known closed formula which we can use: $P(x)/Q(x)$ for order $[p/q]$

$$P_{pq}(x) = \sum_{j=0}^p \frac{(p+q-j)! p!}{(p+q)! j! (p-j)!} x^j$$

$$Q_{pq}(x) = \sum_{j=0}^q \frac{(p+q-j)! q!}{(p+q)! j! (q-j)!} (-x)^j$$

- Remember: we want $p=q=4$

Pade` approximation

- Pade` approximation of order [4/4]:
- Where is it completely wrong?
- Where does it work well?
- Compare it with the Taylor approximation: which works better? Where?