LINEAR ALGEBRA

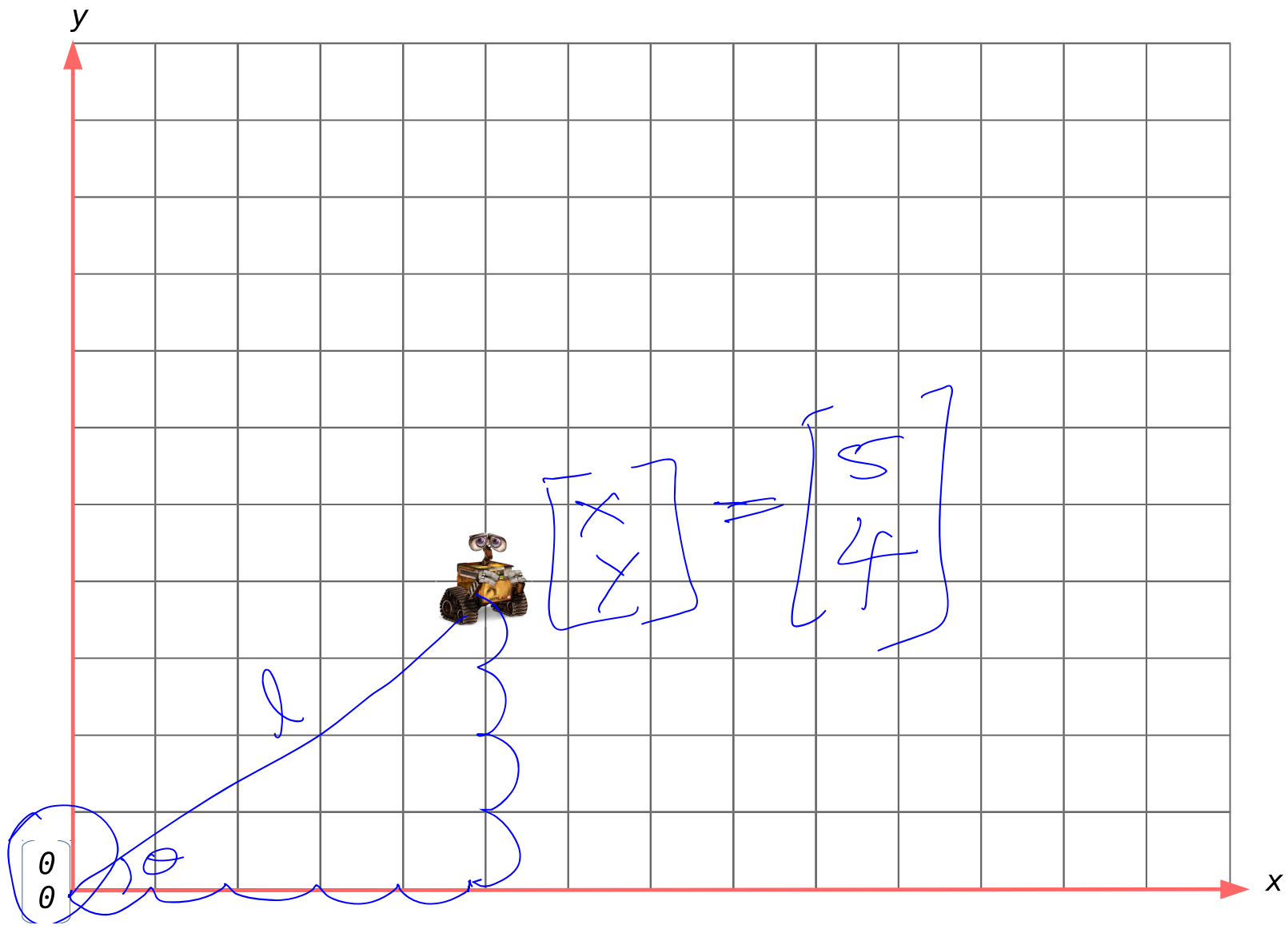
Hochschule Bonn-Rhein-Seig

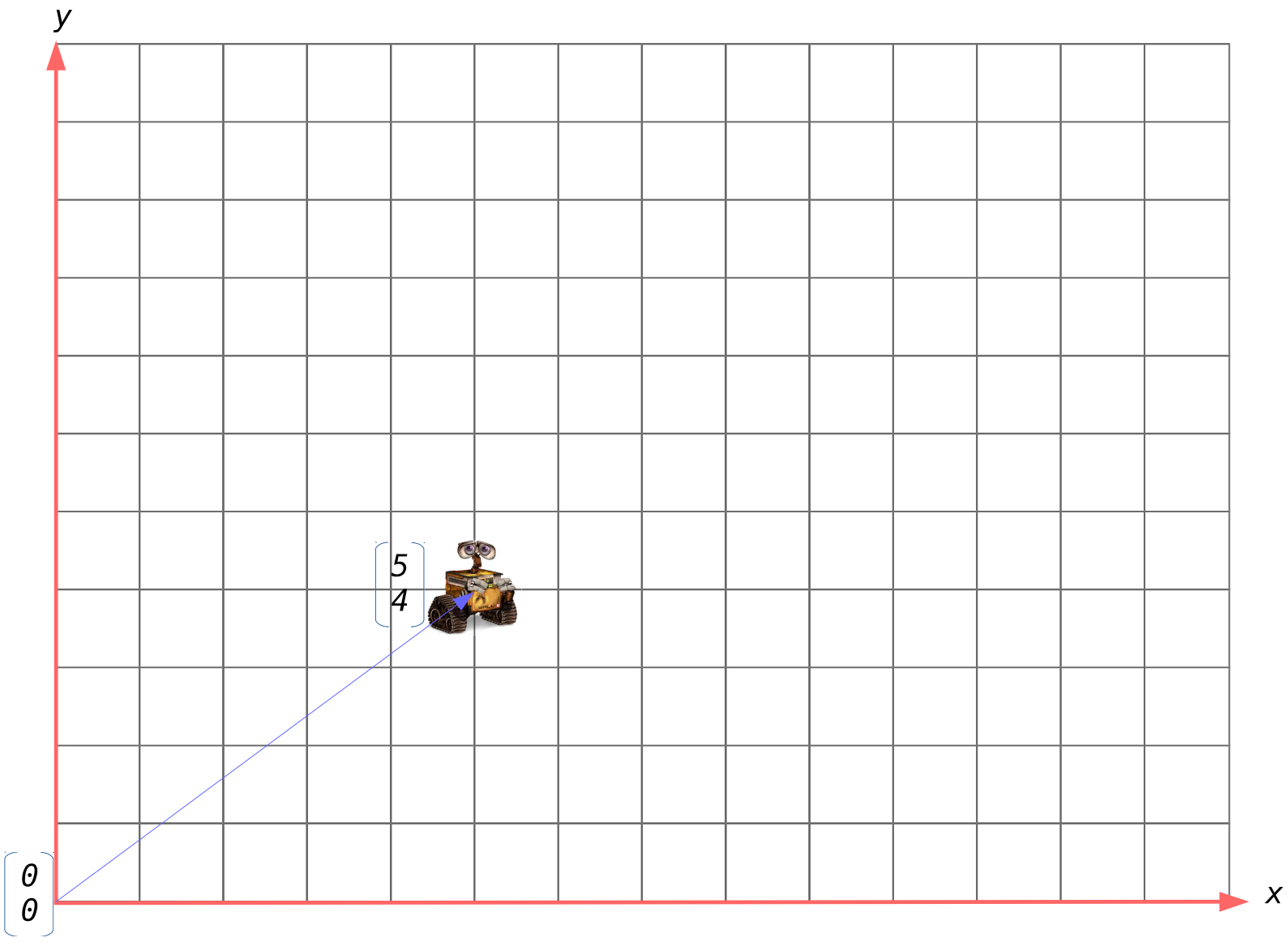
- Divin and Santosh

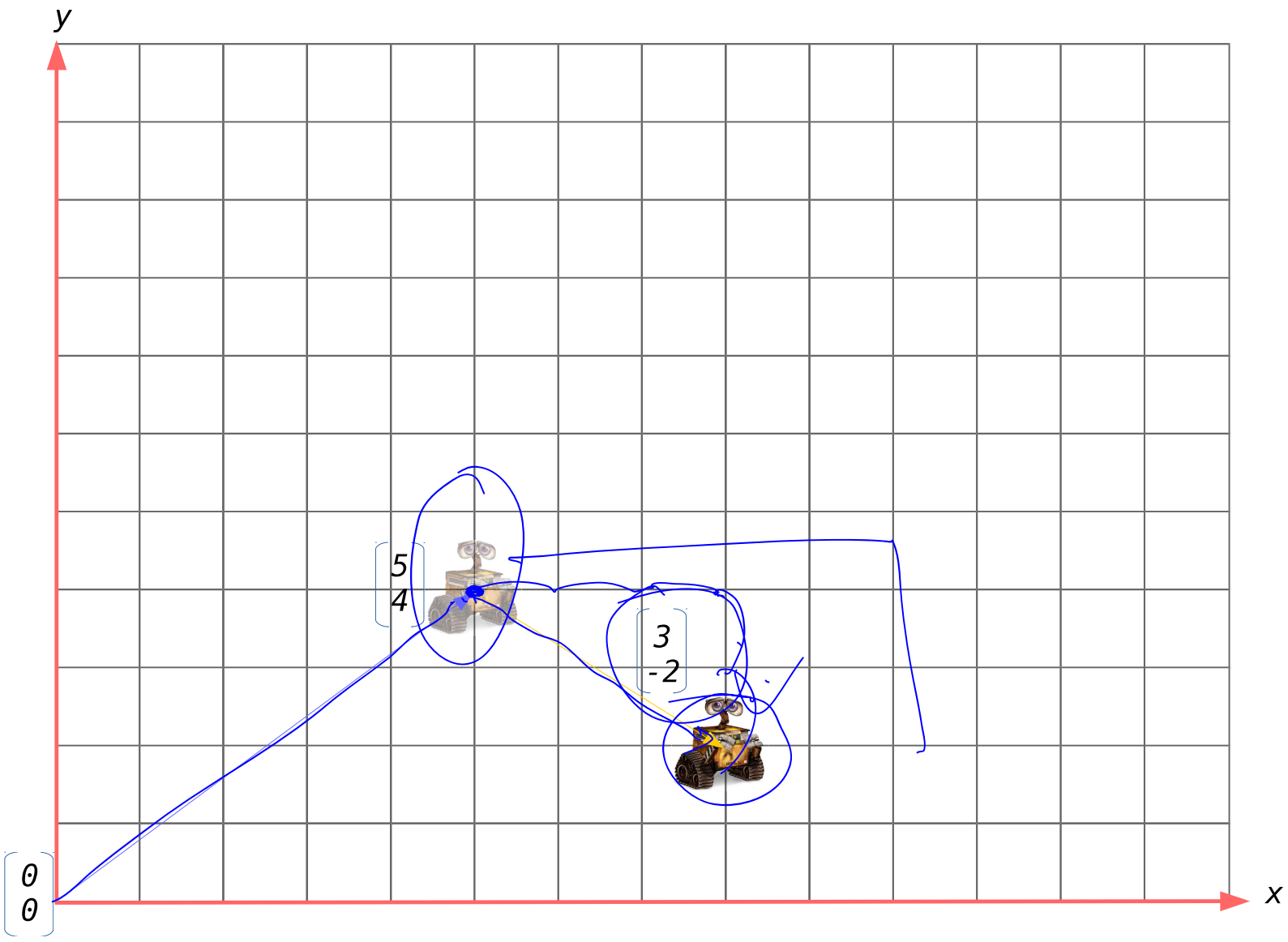
Where is Linear Algebra applied?

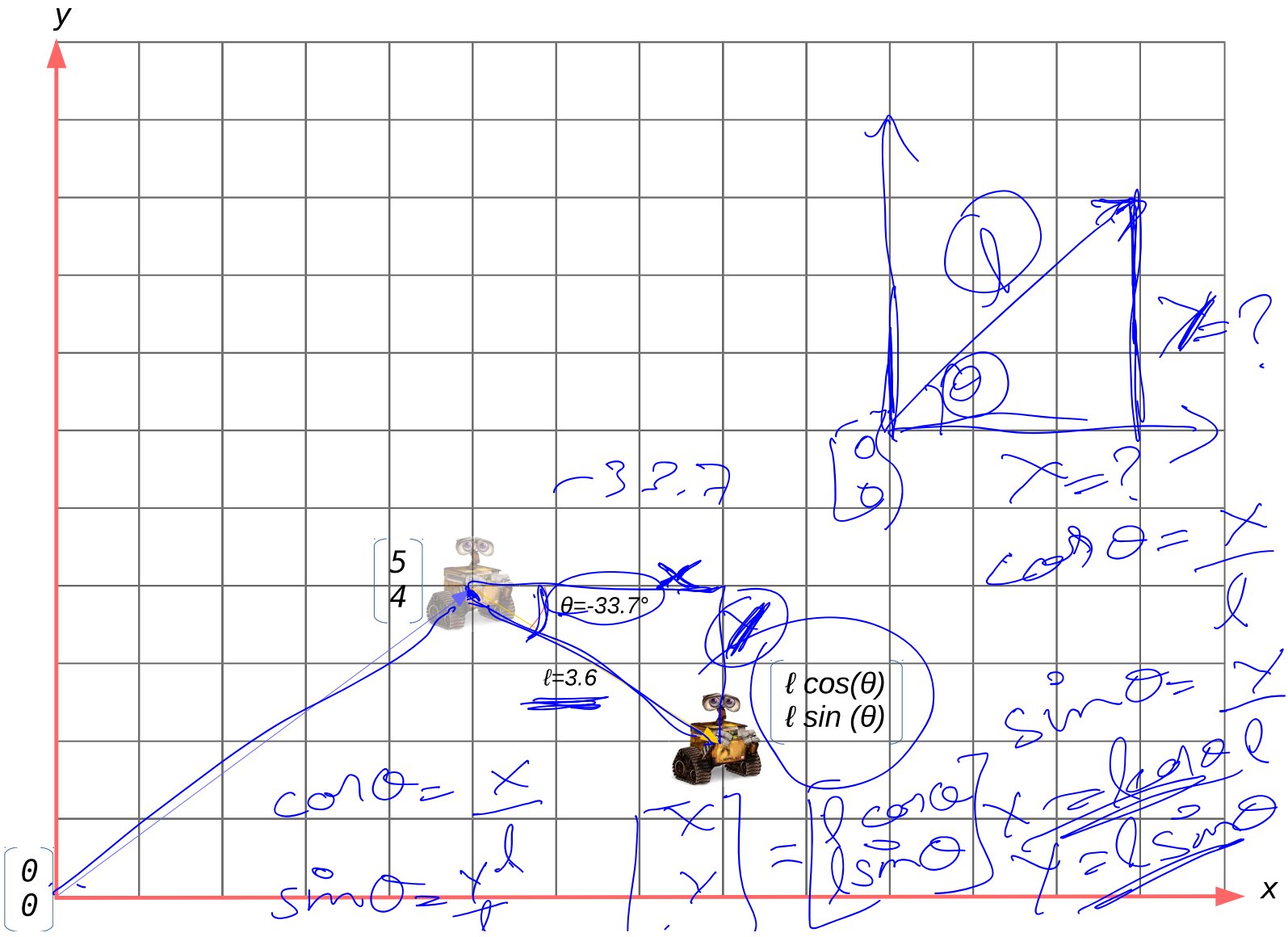
- Robotics 80%
- Data Analysis 5%
- Computer Vision 15%

Vectors

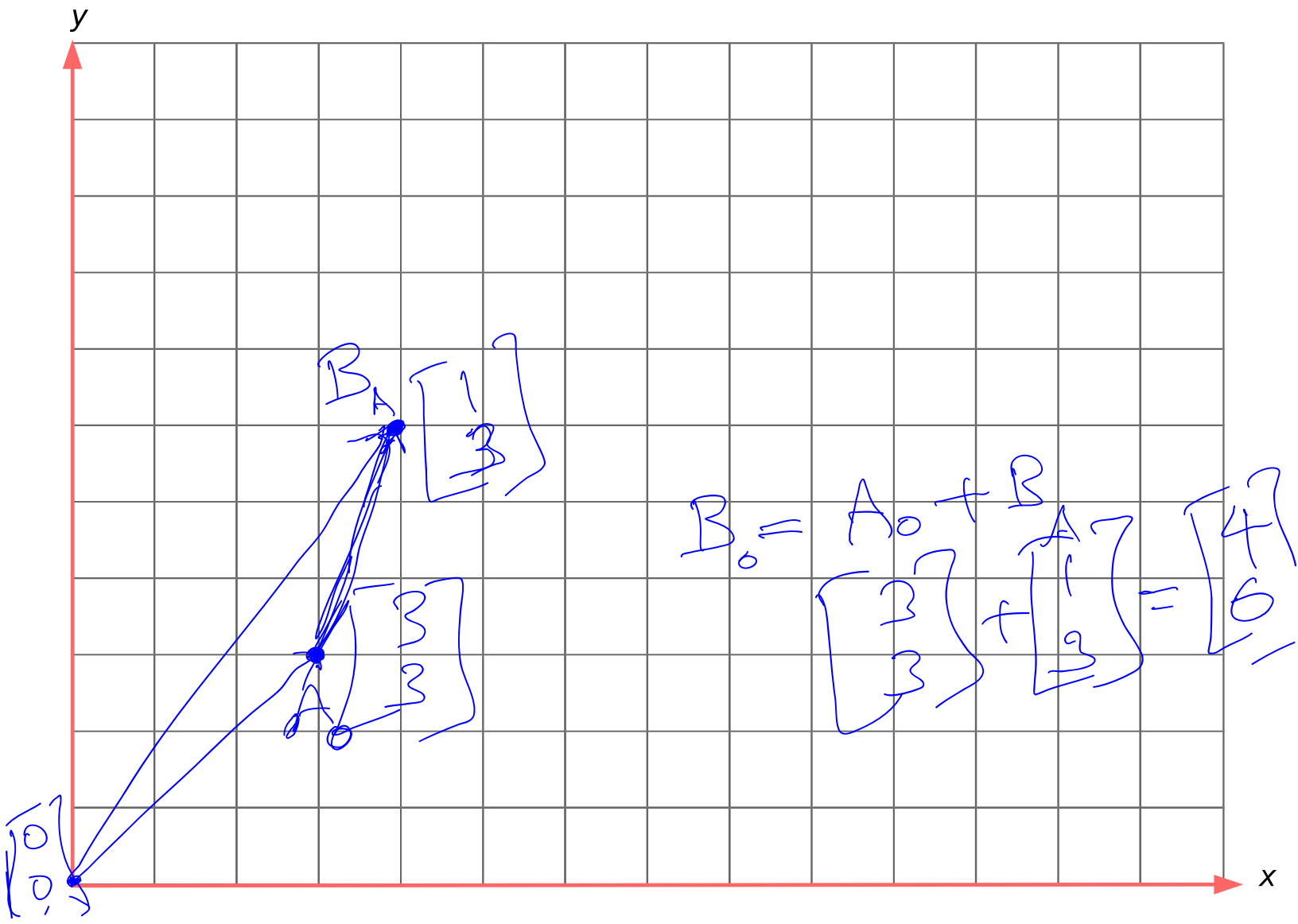

y
x
$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$
l
θ
$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

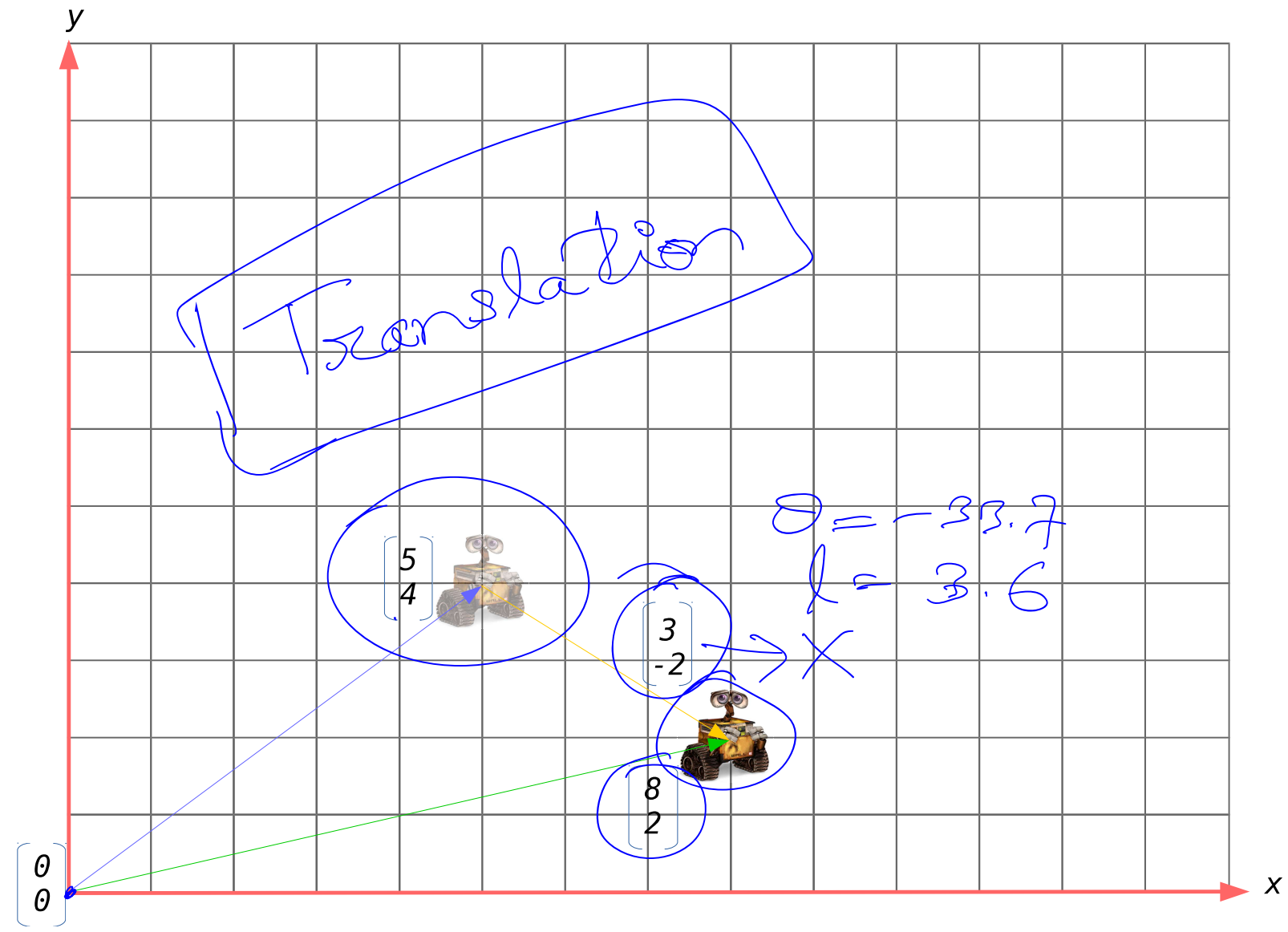

y
5
4
0
0
x

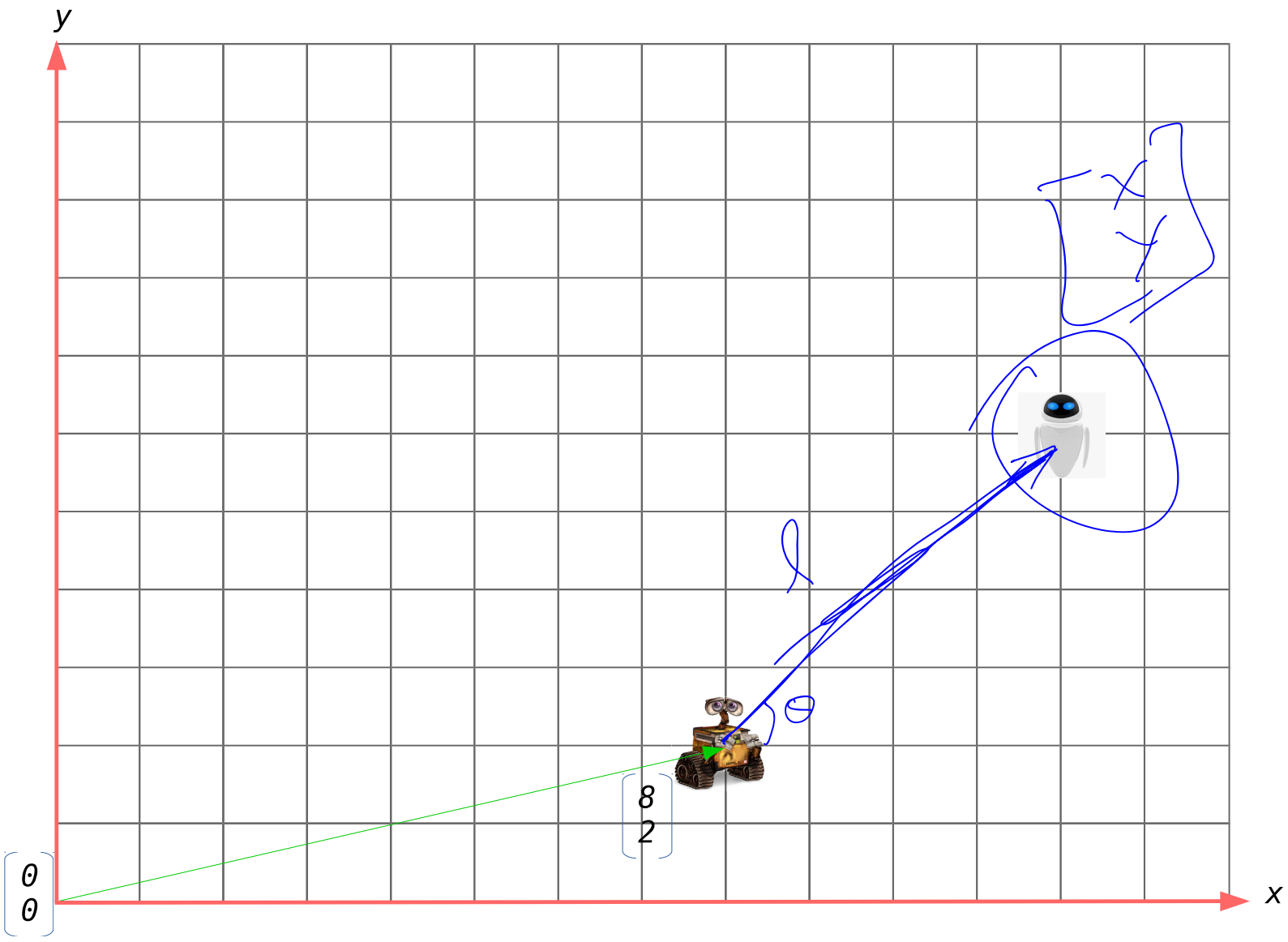

y
x
0
0
5
4
3
-2

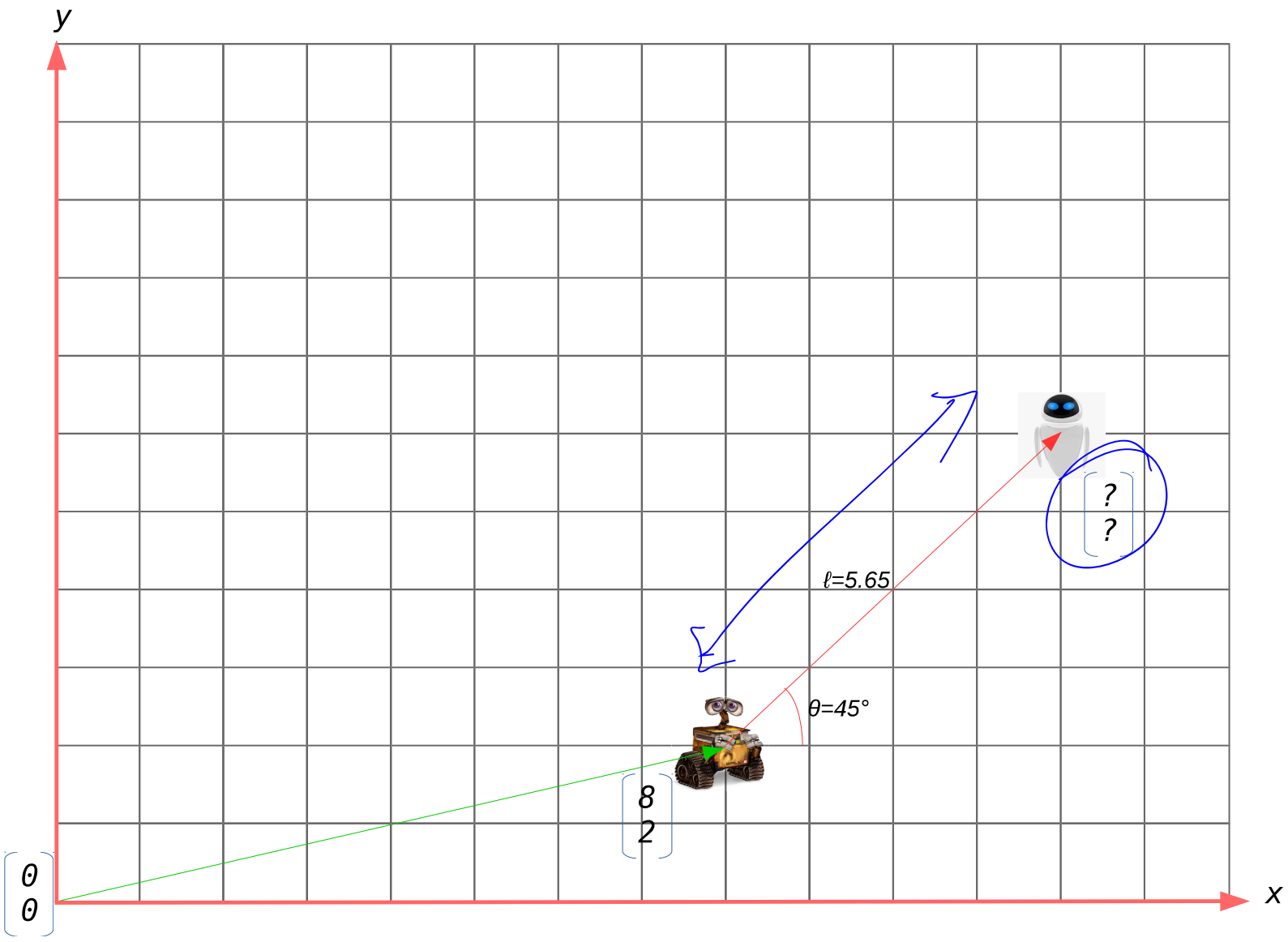

y
x
5
4
θ=-33.7°
ℓ=3.6
ℓ cos(θ)
ℓ sin (θ)
0
0

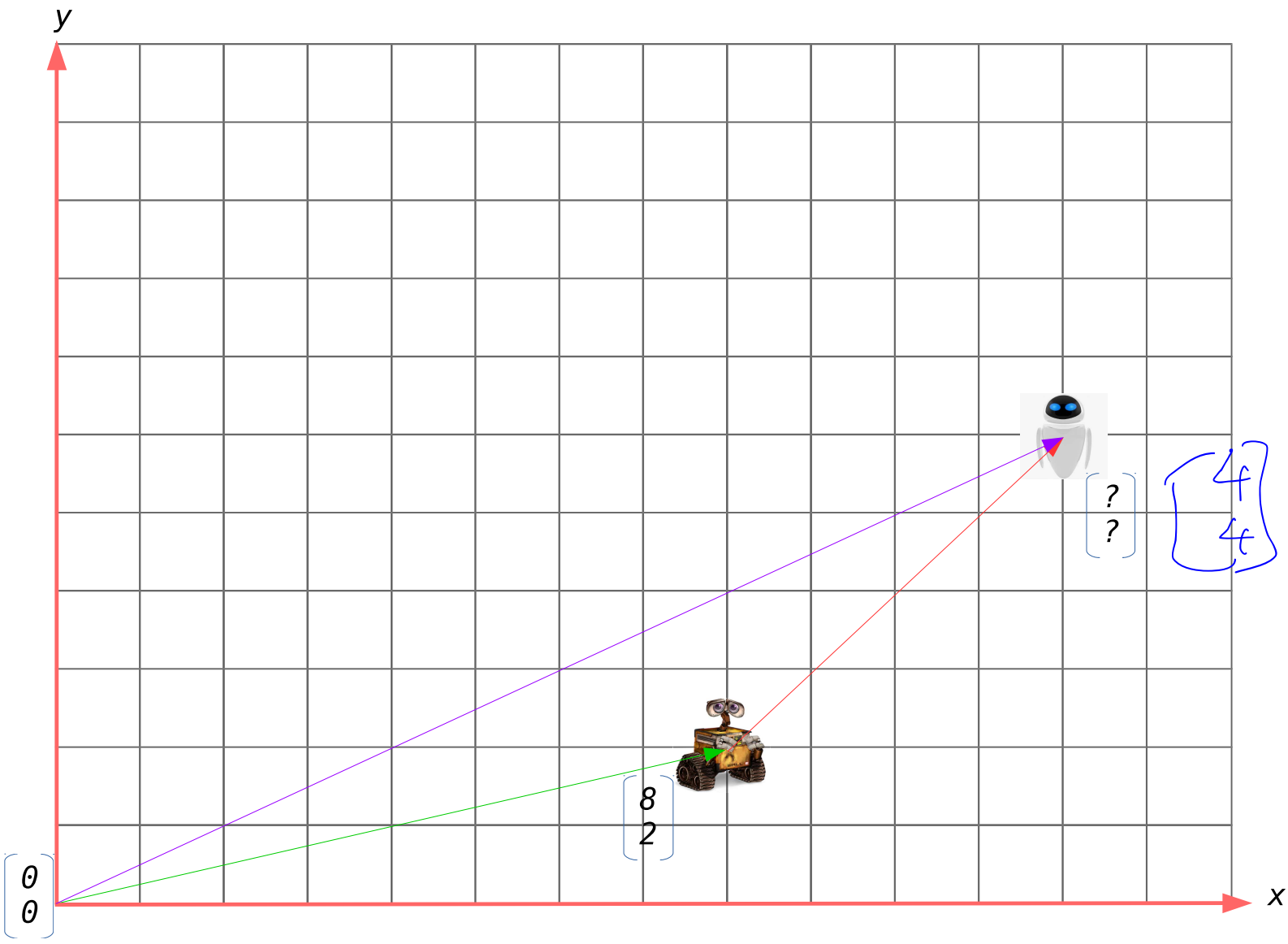
Vector Operations

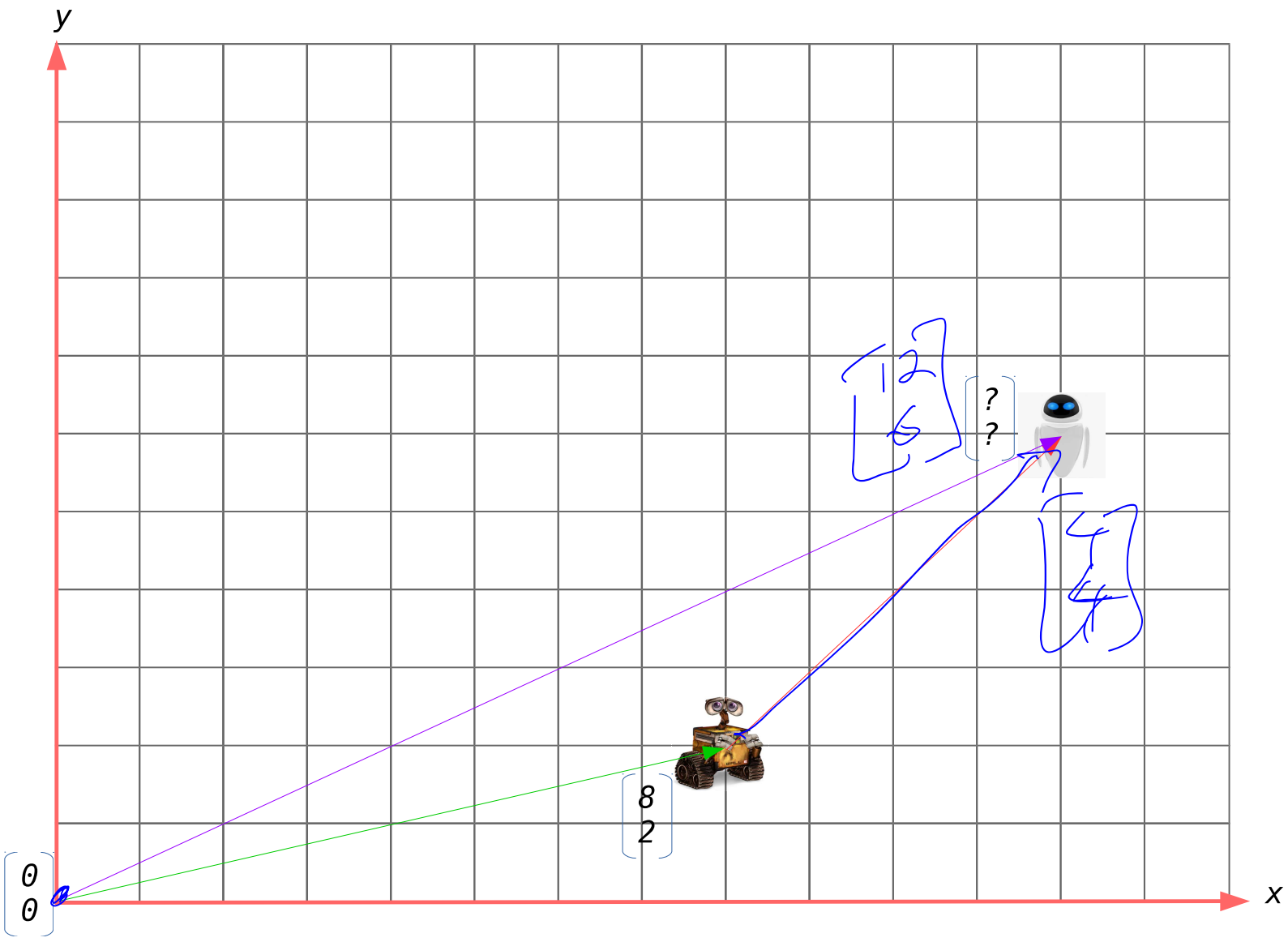

y
x
$B_A \begin{bmatrix} 1 \\ 3 \end{bmatrix}$
$\begin{bmatrix} 3 \\ 3 \end{bmatrix}$
$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
$B_0 = A_0 + B$
$\begin{bmatrix} 3 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

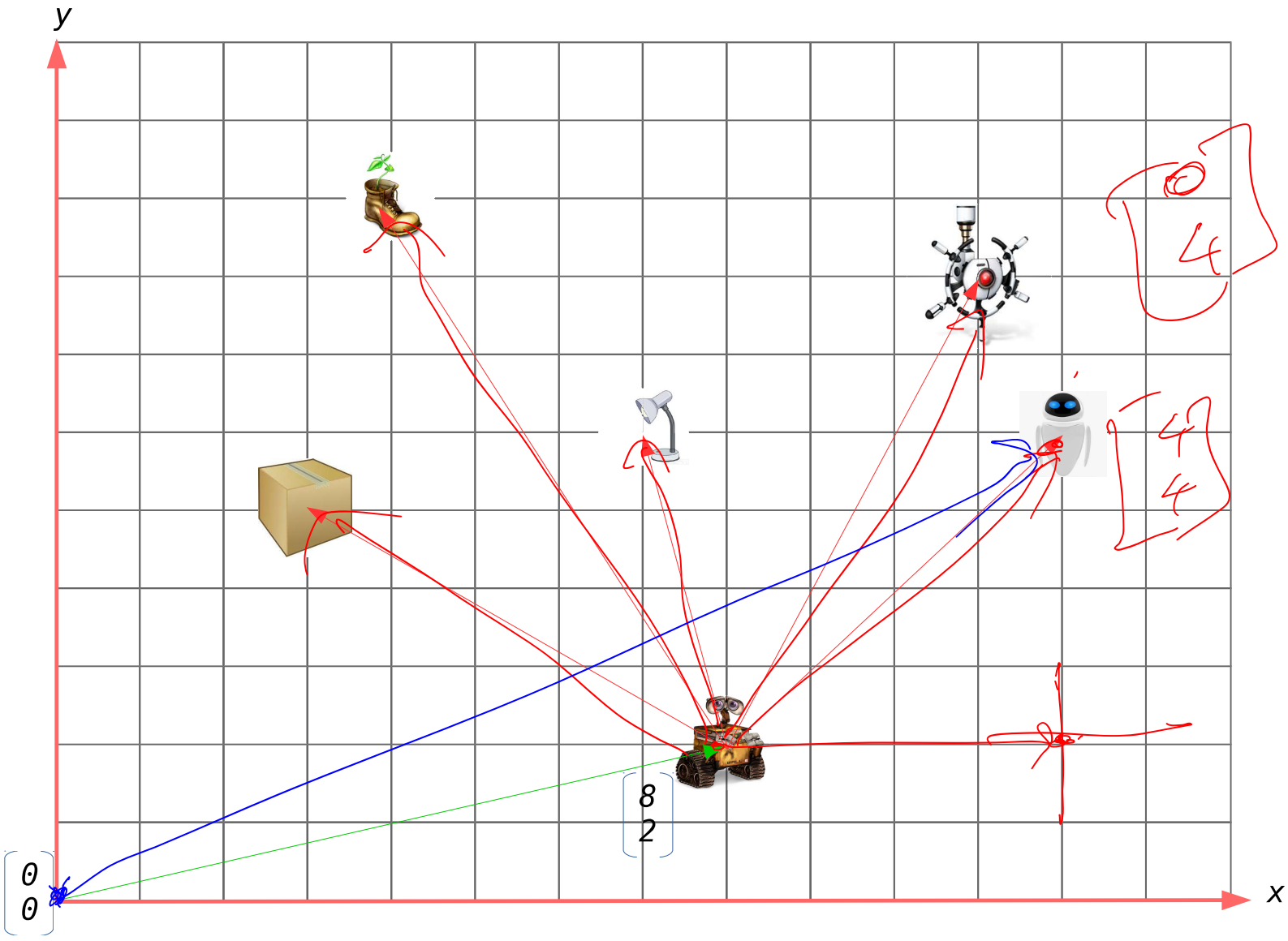

y
x
Translation
5
4
3
-2
8
2
0
0
θ = -33.7
l = 3.6

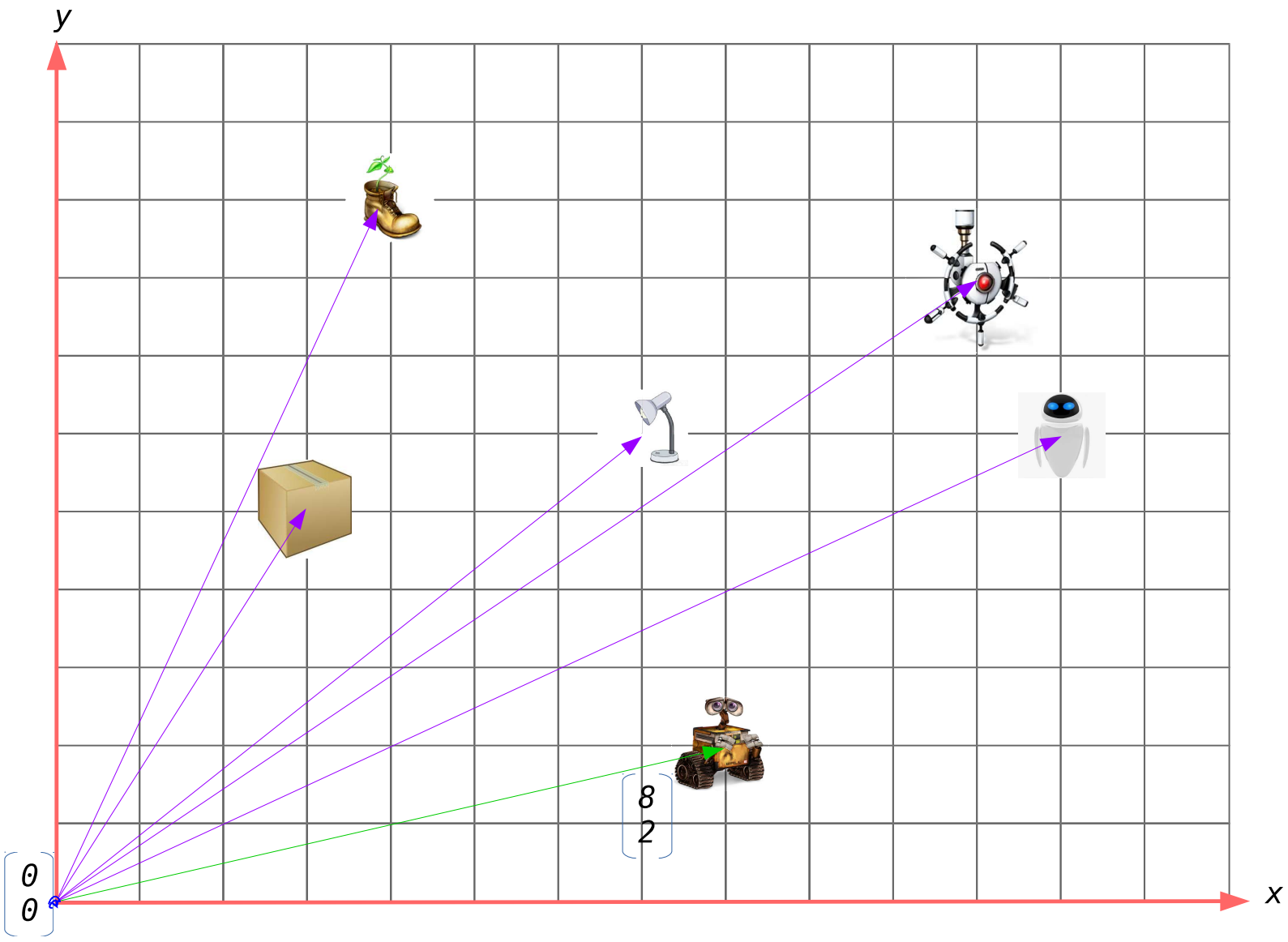

y
x
0
0
8
2


y
x
0
0
8
2
ℓ=5.65
θ=45°
?
?

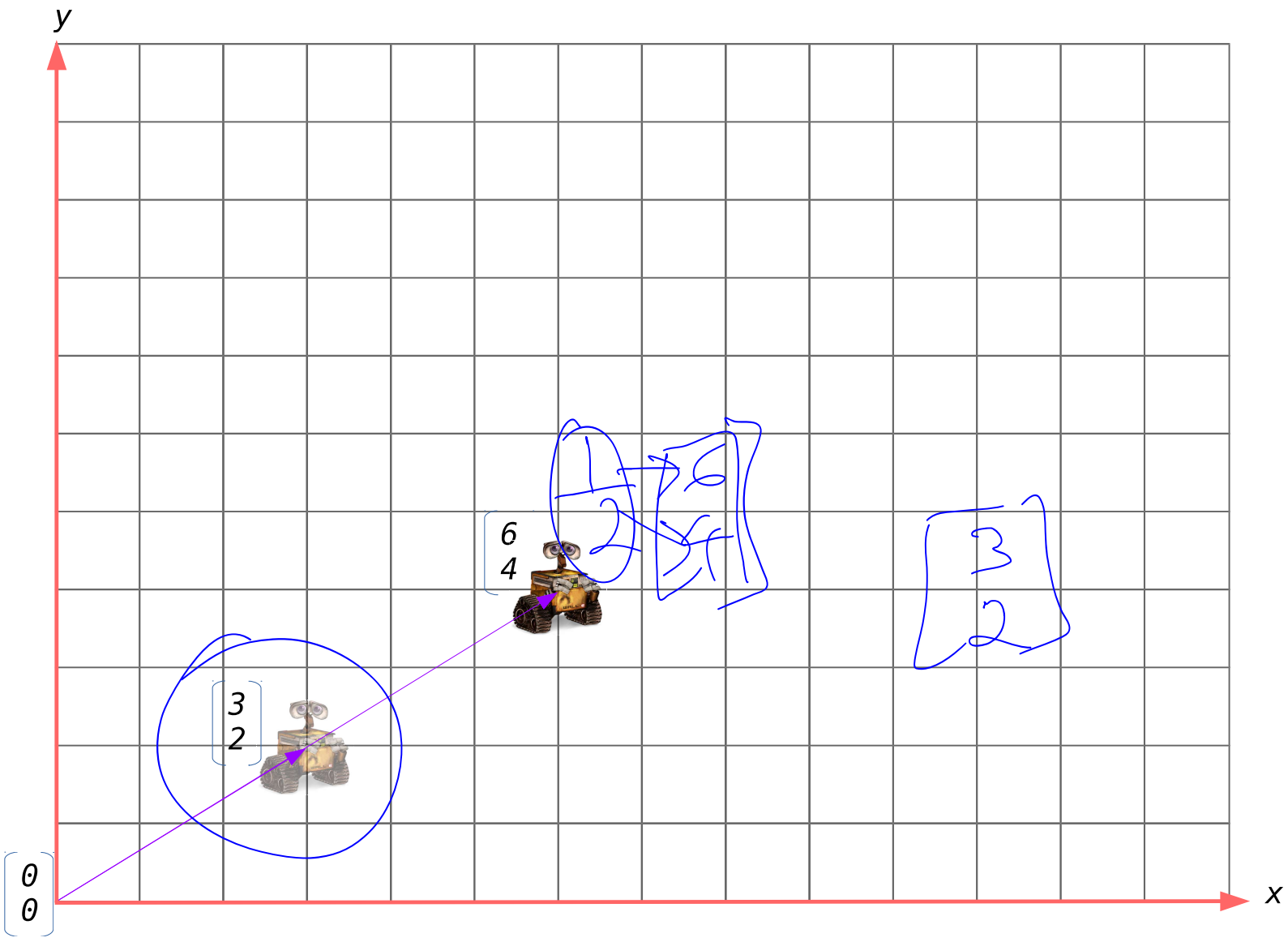

y
x
0
0
8
2
?
?
4
4

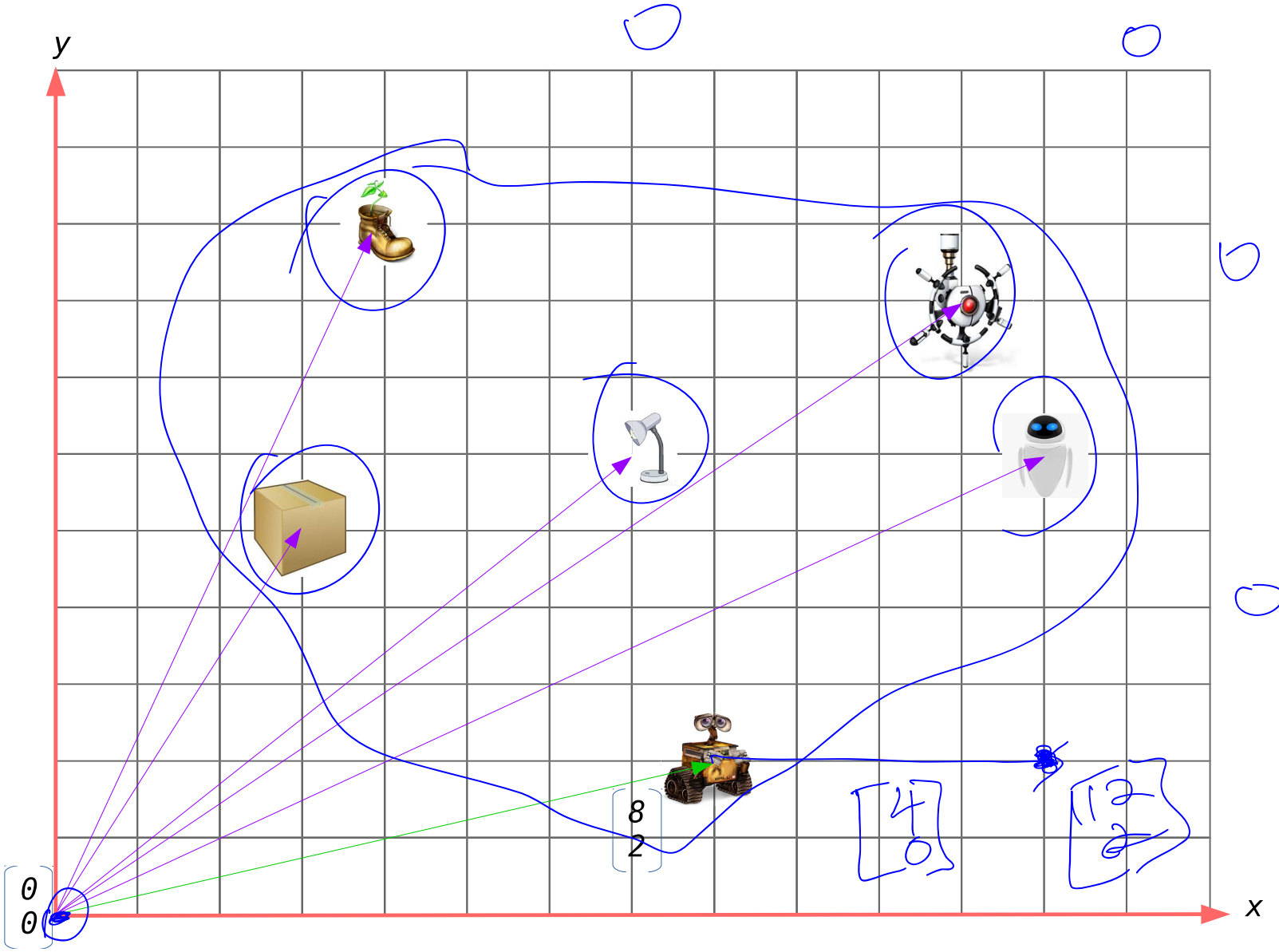

y
x
0
0
8
2
?
?

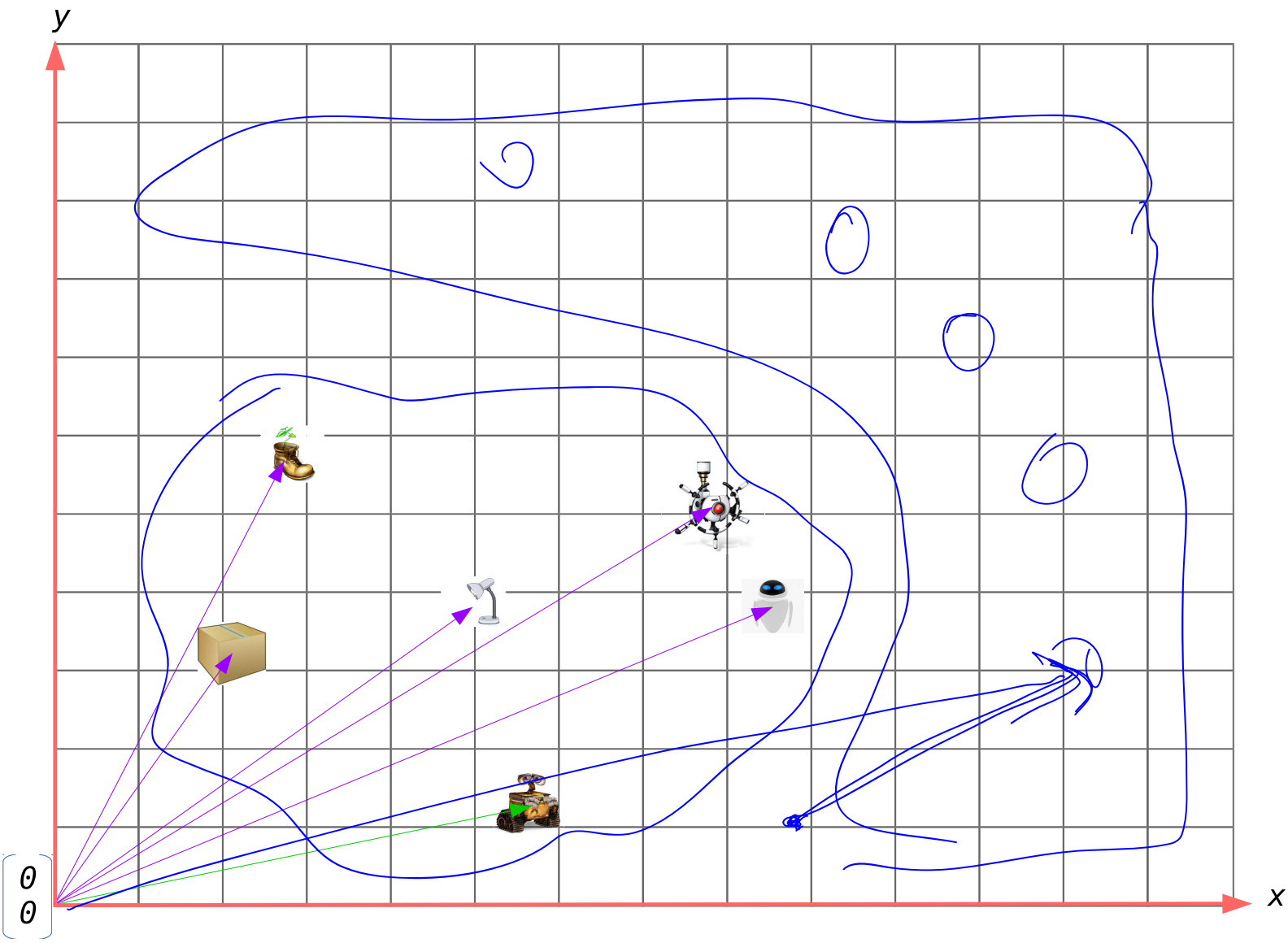

y
x
0
0
8
2
4
4
4


y
x
0
0
8
2

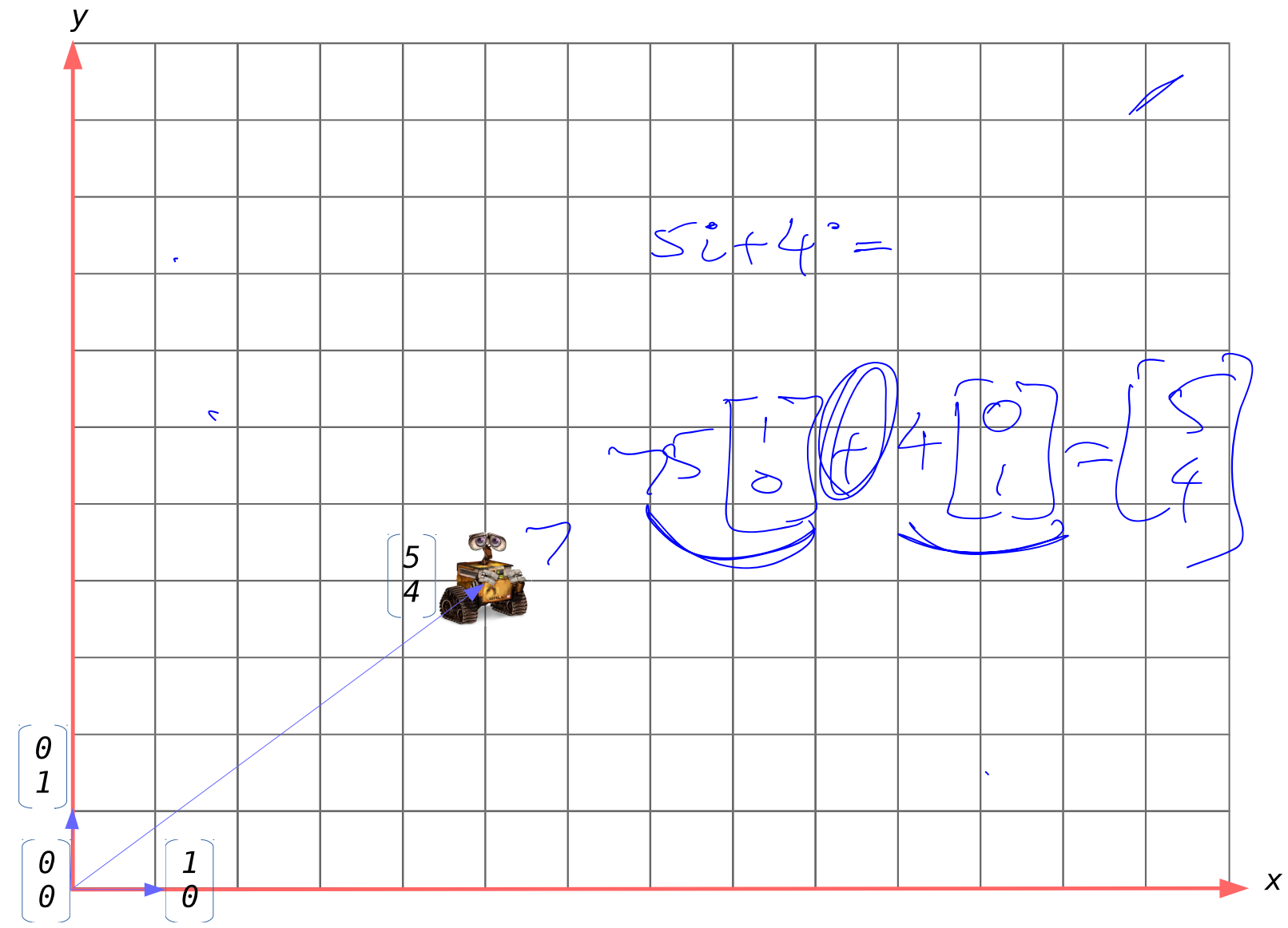
Scaling


y
x
6
4
3
2
0
0


y
x
0
0
8
2

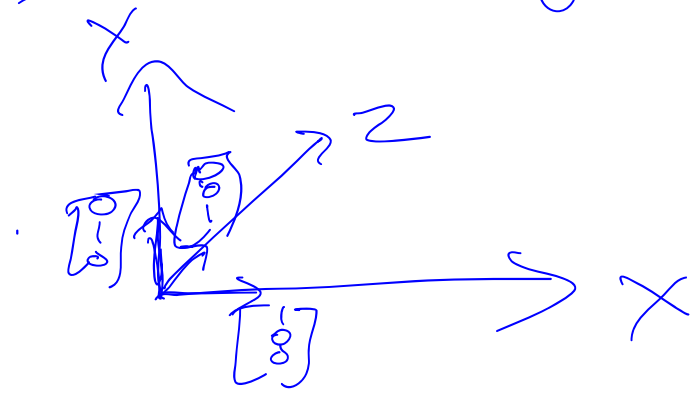

y
x
0
0

Basis Vectors

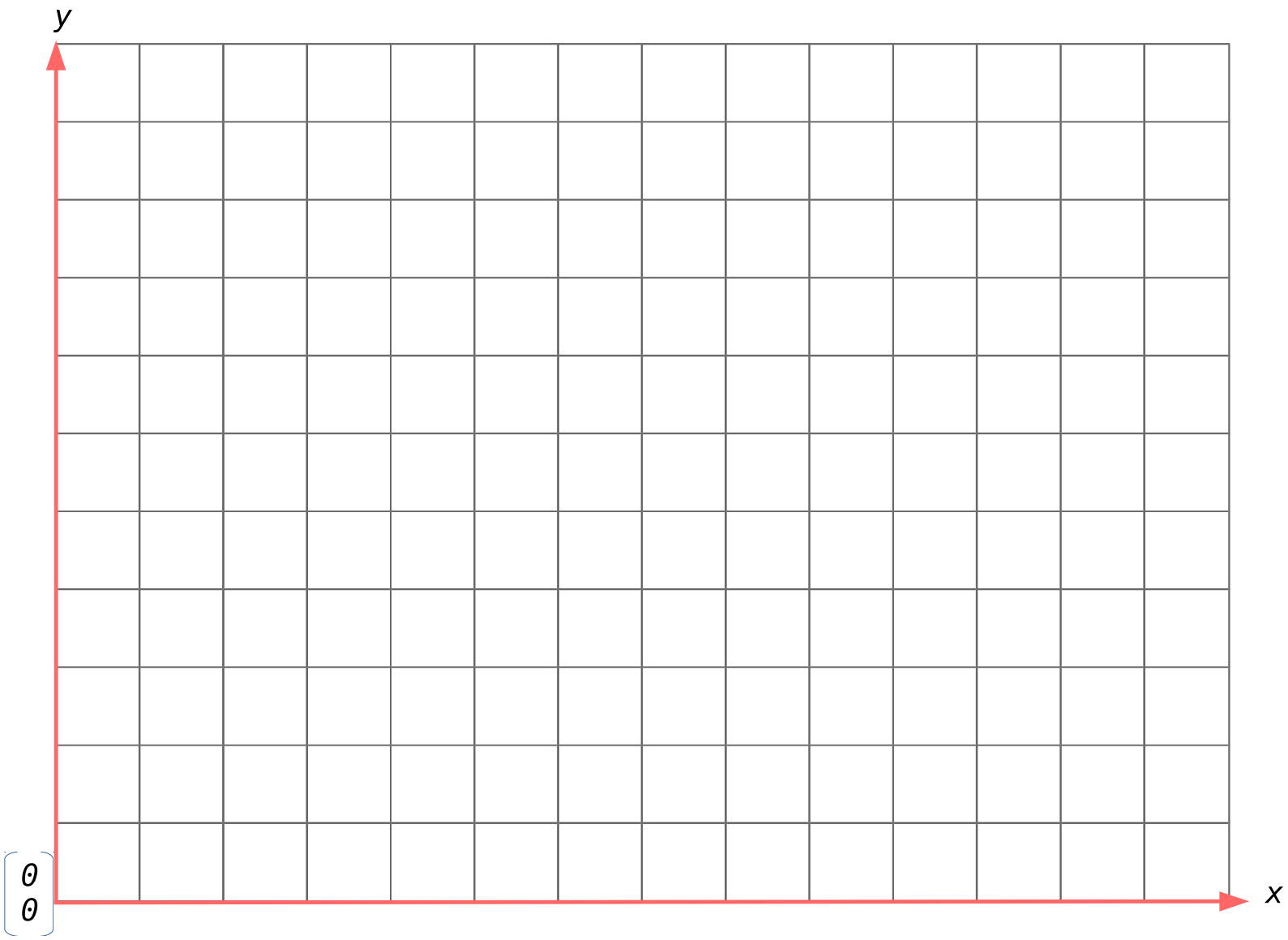

y
x
5i+4j=
5 [1 0] + 4 [0 1] = [5 4]
5
4
0
1
0
0
1
0

2D → $a\hat{i} + b\hat{j} = \begin{bmatrix} x \\ y \end{bmatrix}$

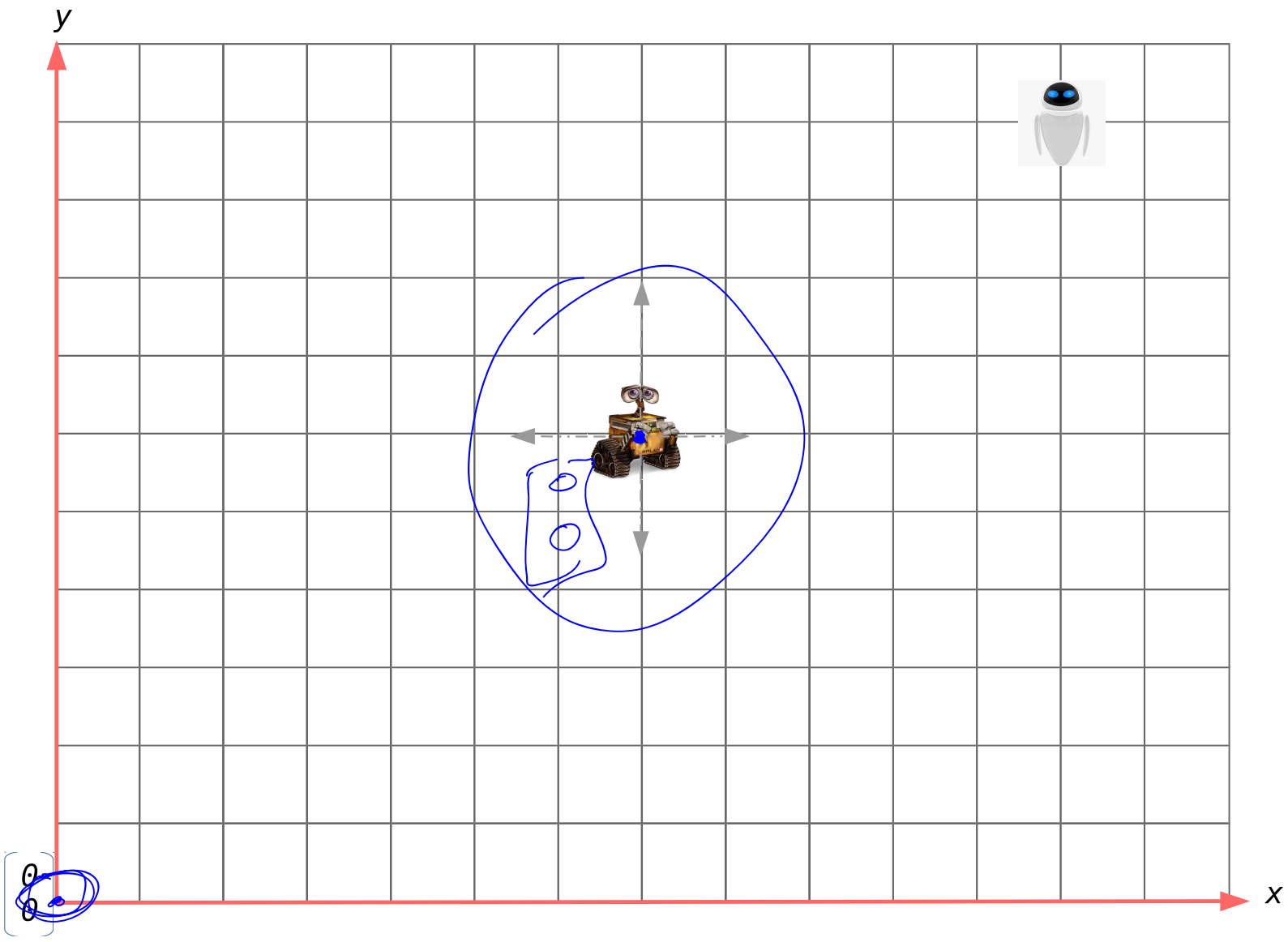
3D → $a\hat{i} + b\hat{j} + c\hat{k} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

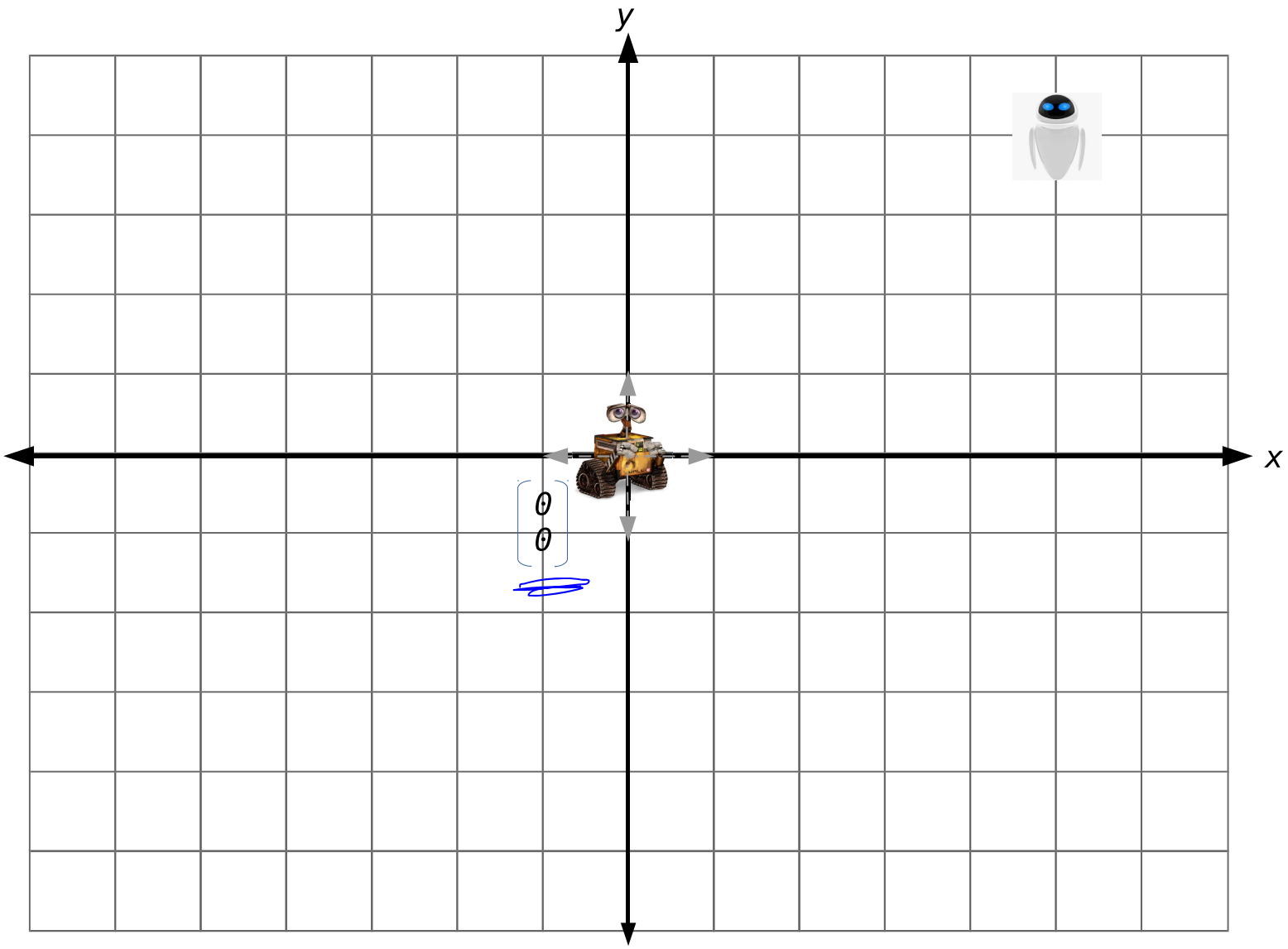


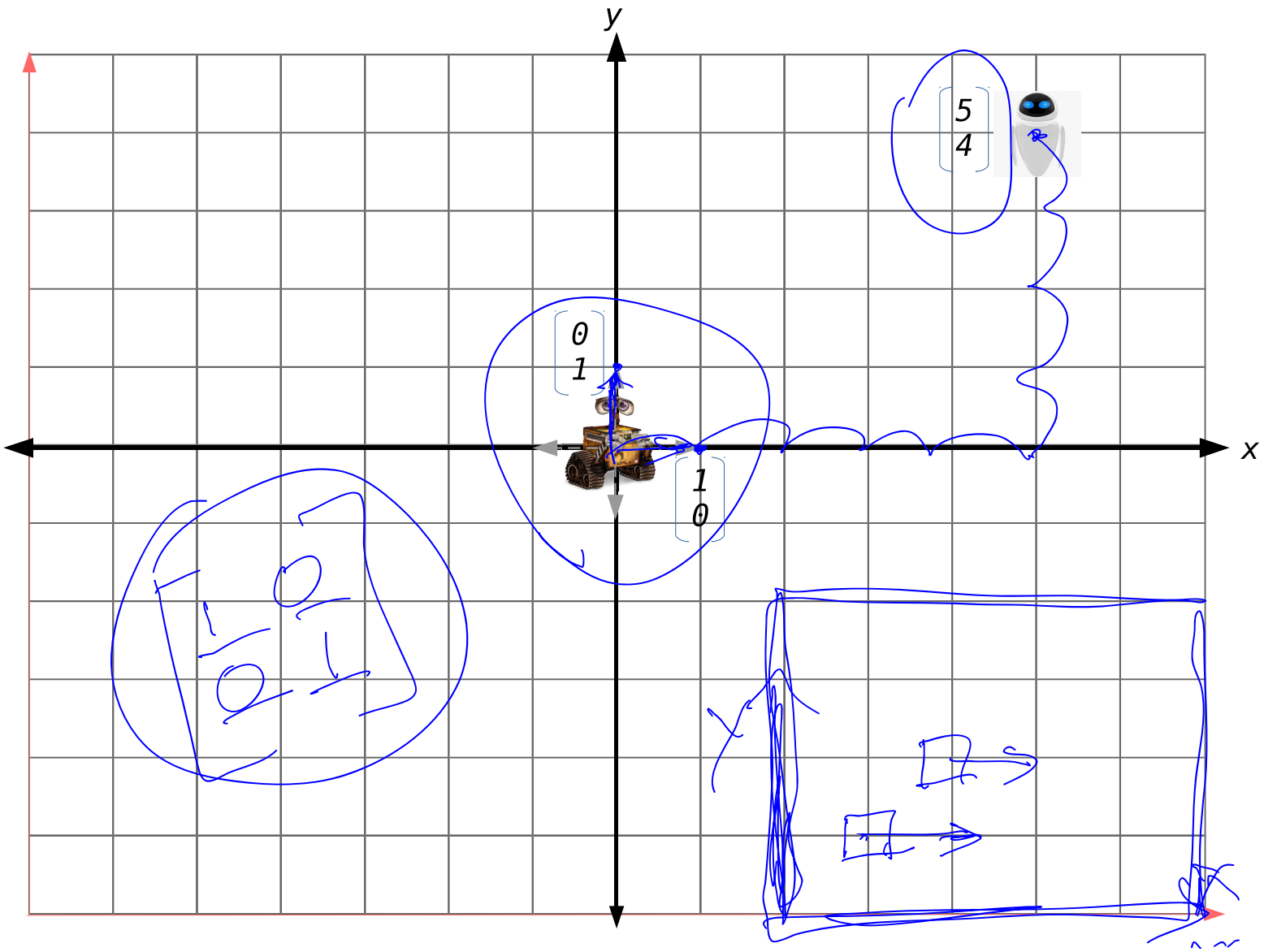
Linear Combination

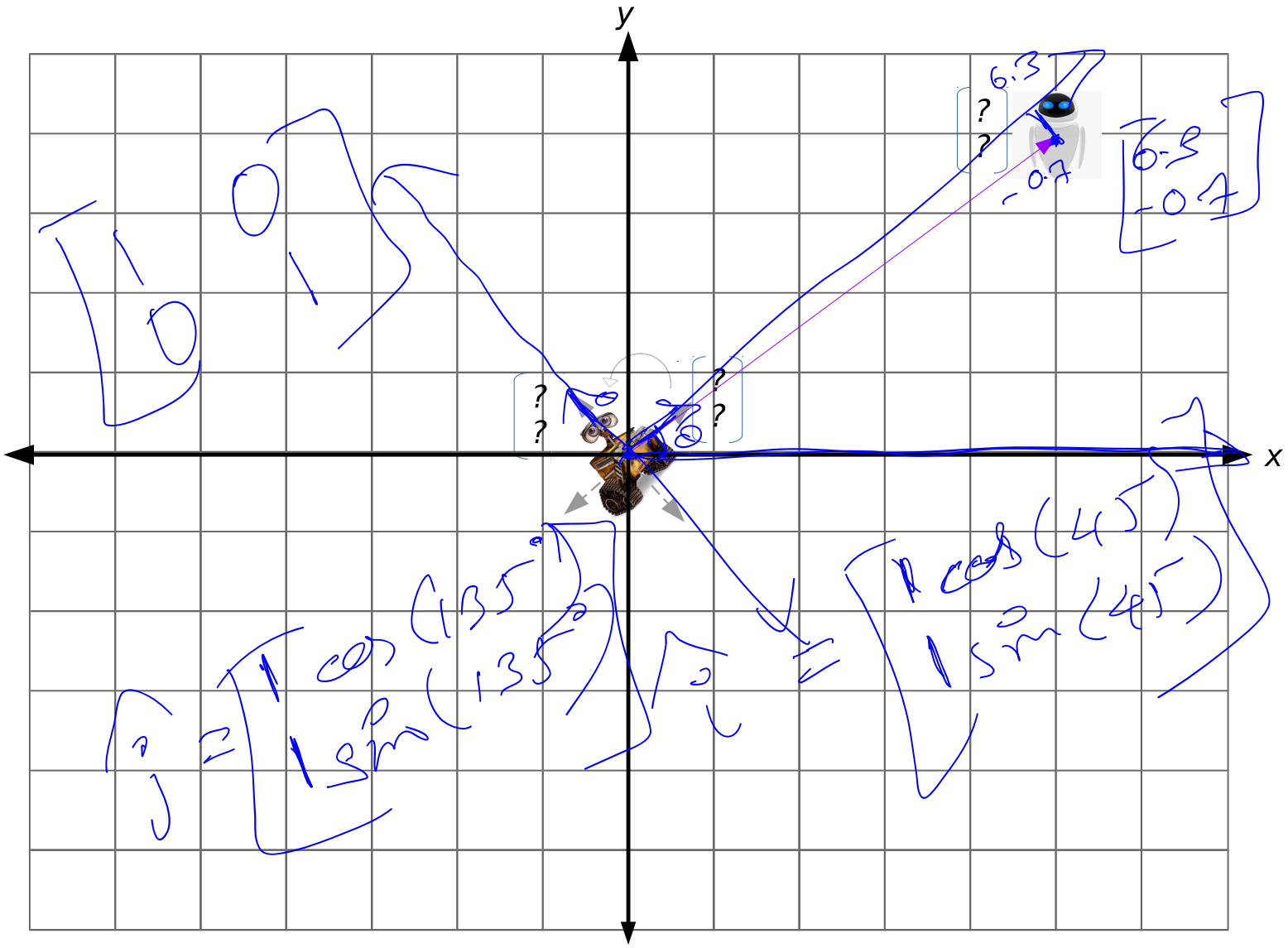

y
x
0
0

Matrix


y
x
0
0


y
x
0
0

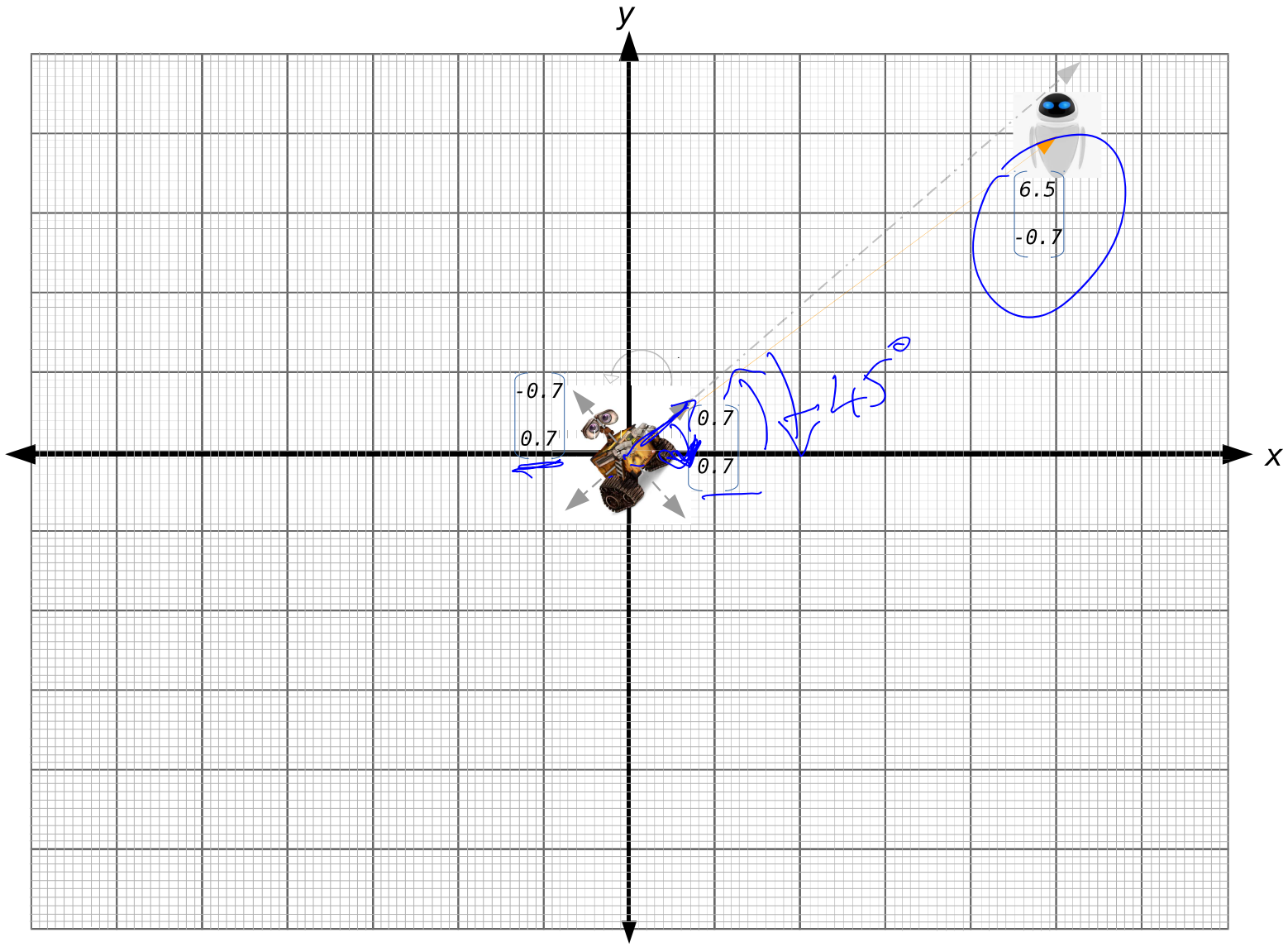

y
x
5
4
0
1
1
0


y
x
?
?
?
?
?
?
6.3
-0.7

Matrix Multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} ax_1 + by_1 \\ cx_1 + dy_1 \end{bmatrix}$$

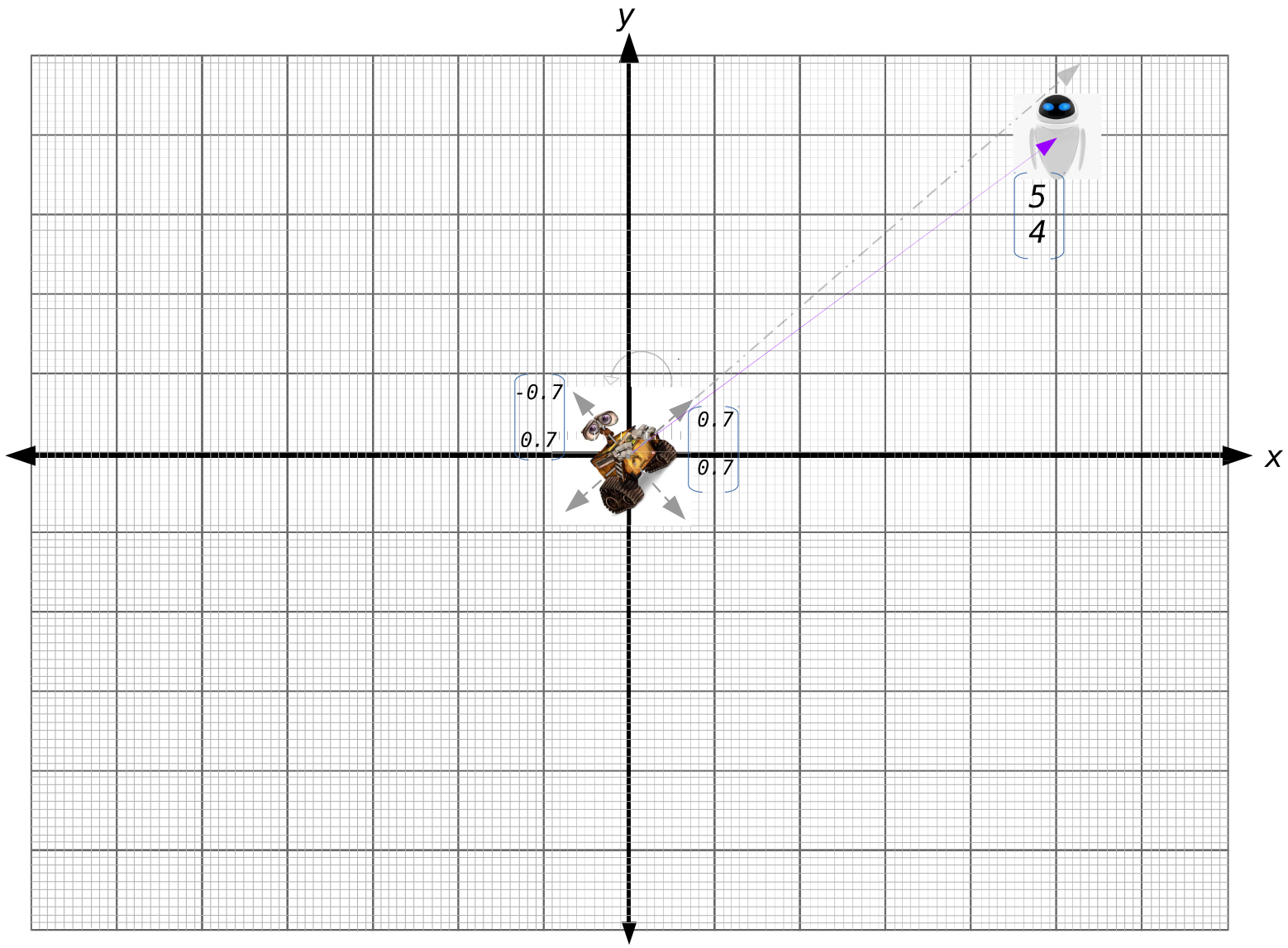
$$\begin{bmatrix} 0.7 & -0.7 \\ 0.7 & 0.7 \end{bmatrix} \begin{bmatrix} 6.3 \\ -0.7 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$


y
x
6.5
-0.7
-0.7
0.7
0.7
0.7
+45°

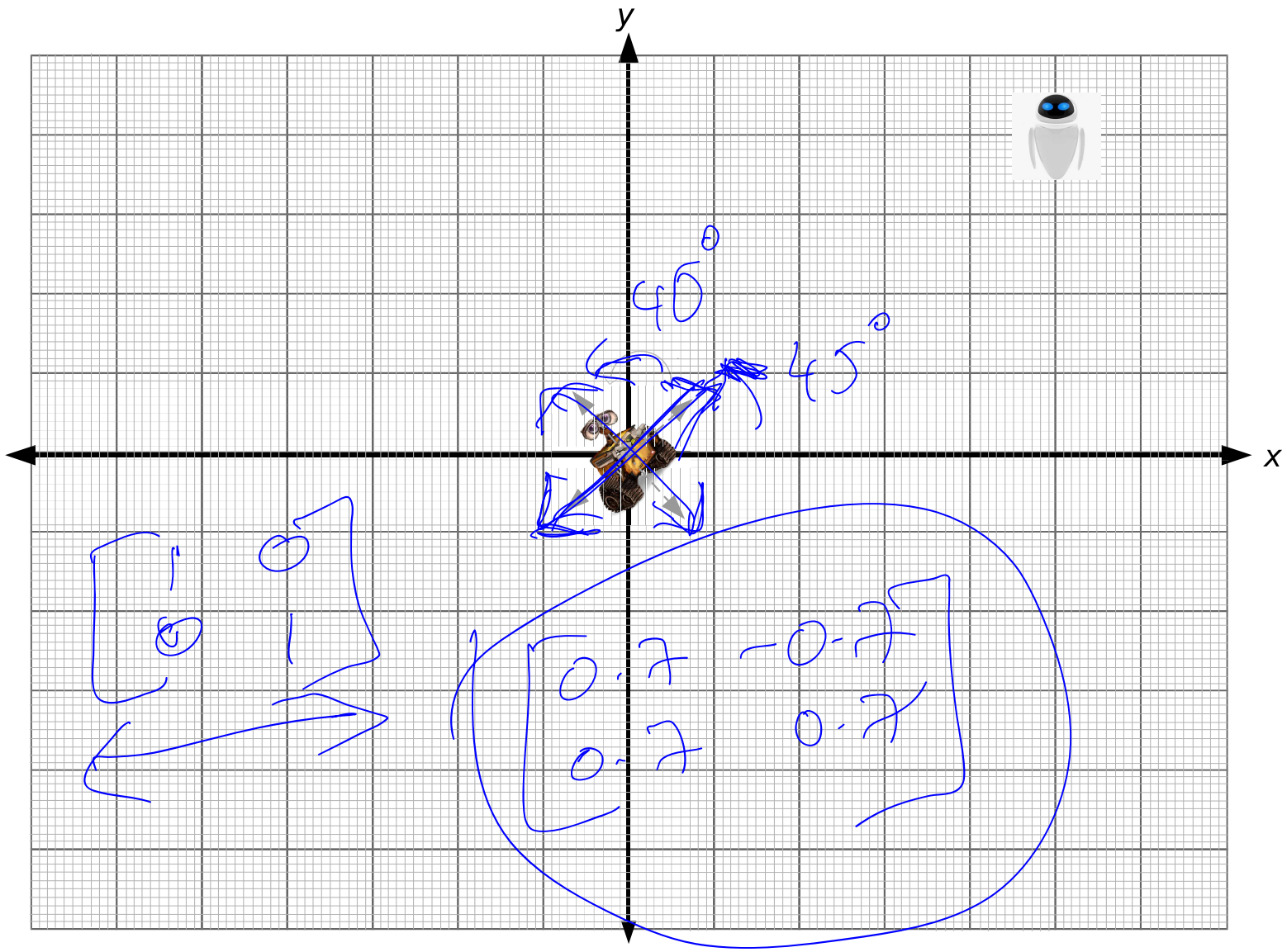
Inverse of a Matrix

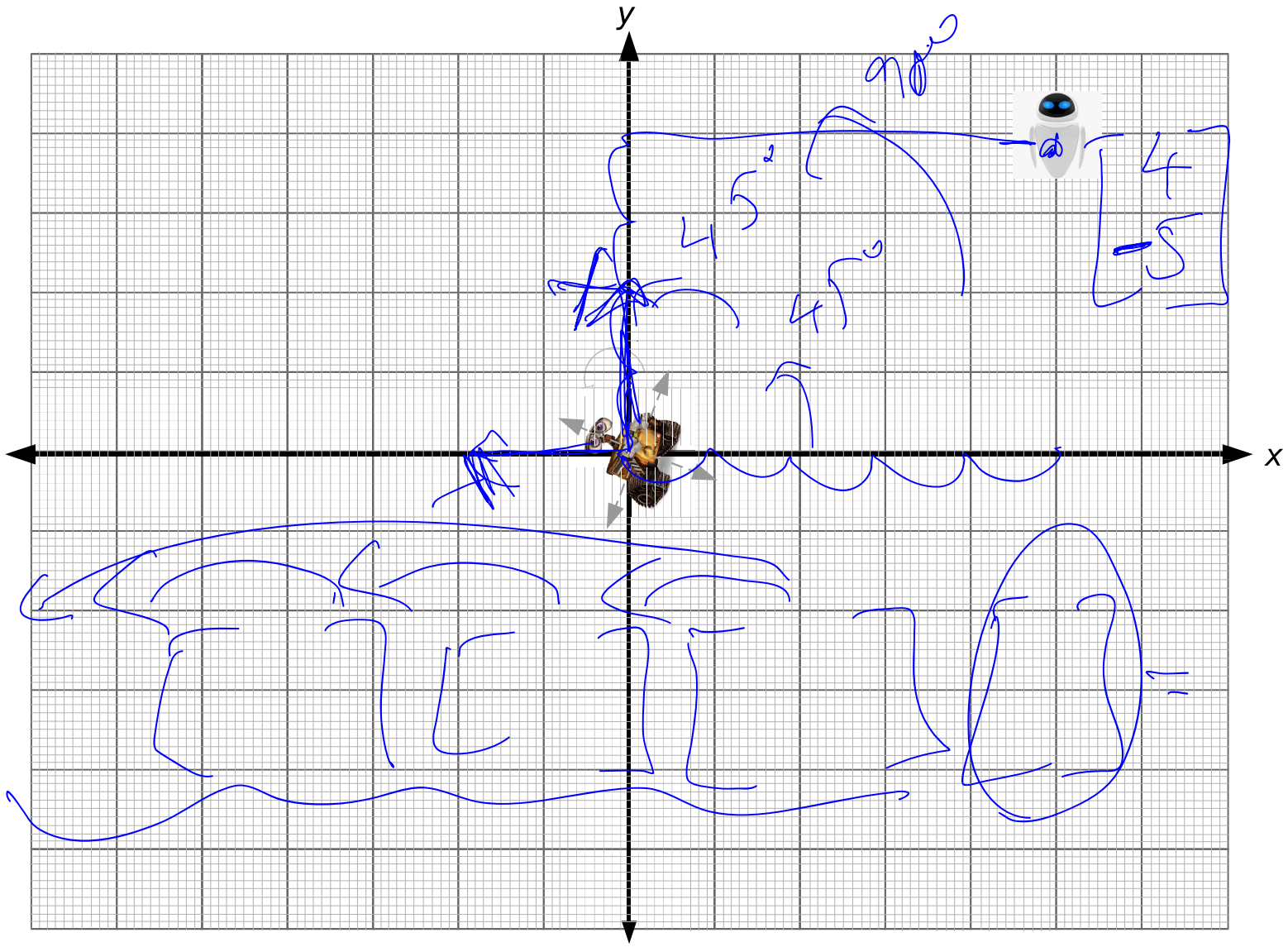
$$A^{-1} = \frac{1}{\det} \operatorname{adj}[A] = \begin{bmatrix} - & - \\ - & - \end{bmatrix}$$

$$\begin{bmatrix} 0.7 & 0.7 \\ -0.7 & 0.7 \end{bmatrix} \begin{bmatrix} 0.7 & -0.7 \\ 0.7 & 0.7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$


y
x
5
4
-0.7
0.7
0.7
0.7

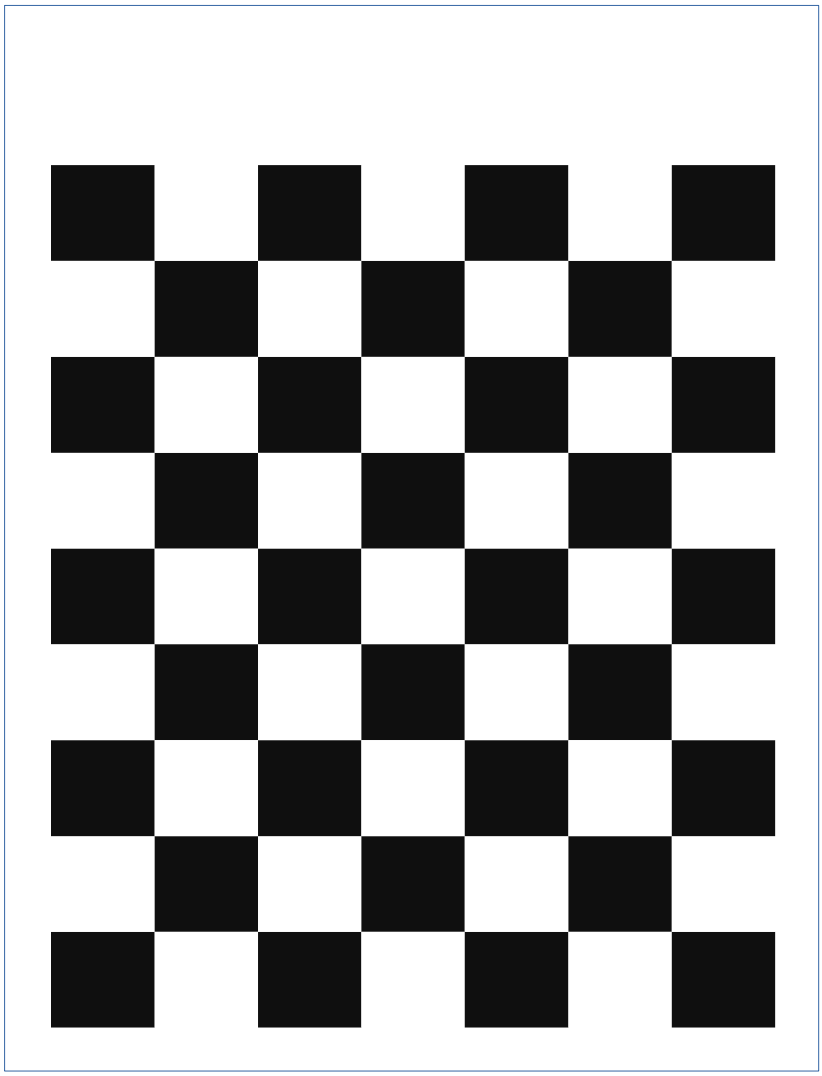
Composition

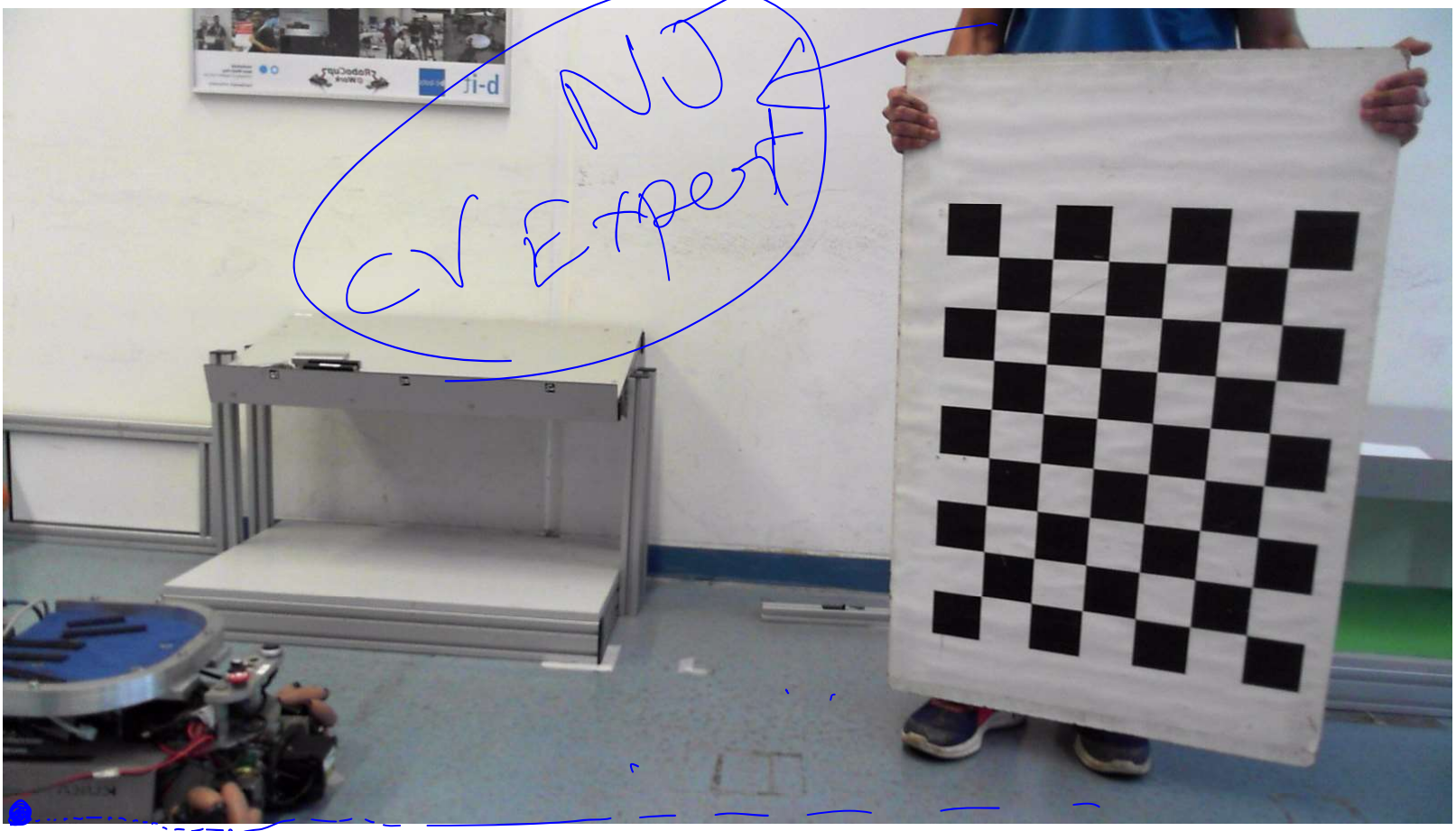

y
x
45°
45°
$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$\begin{bmatrix} 0.7 & -0.7 \\ 0.7 & 0.7 \end{bmatrix}$

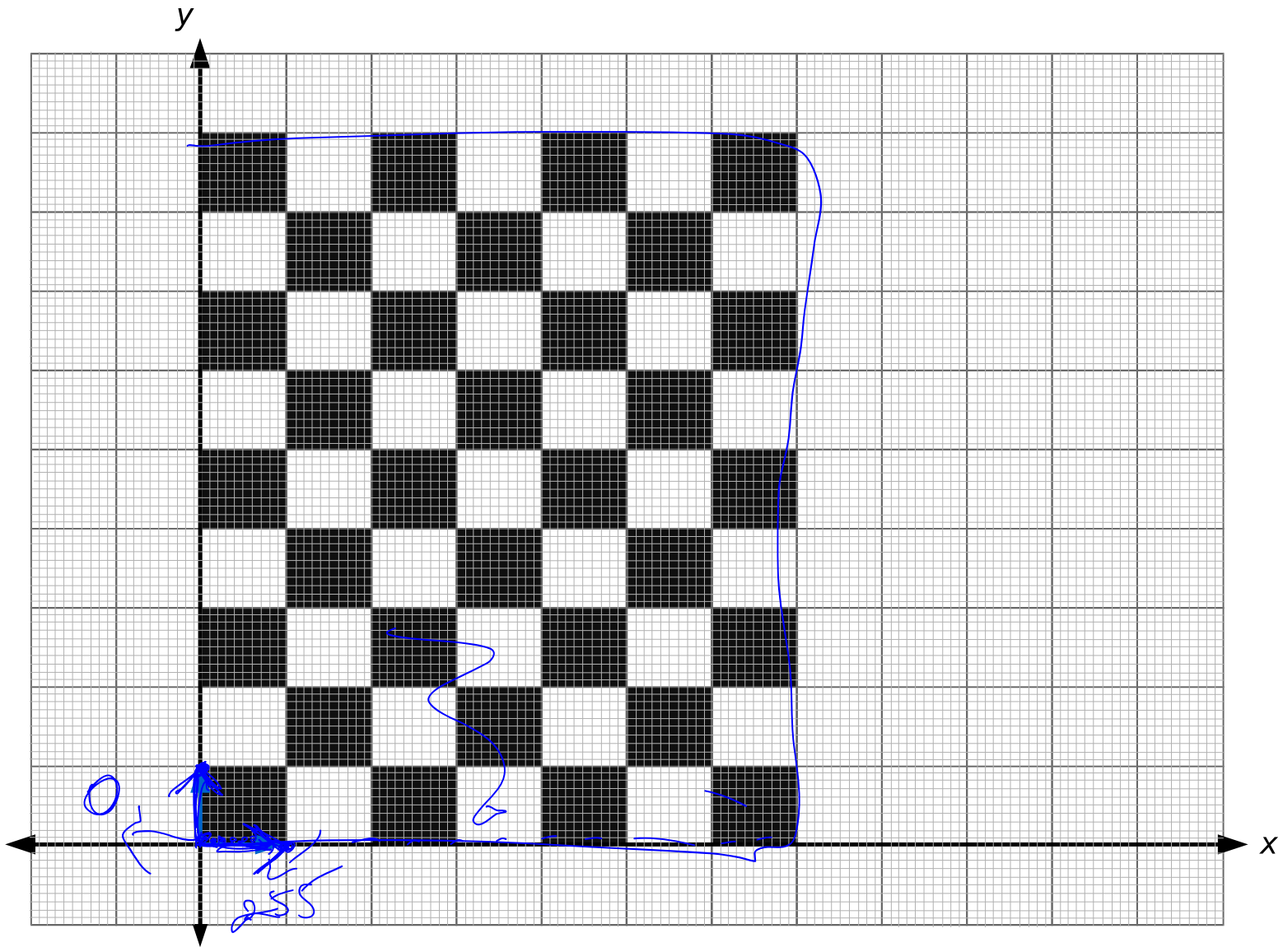

y
x

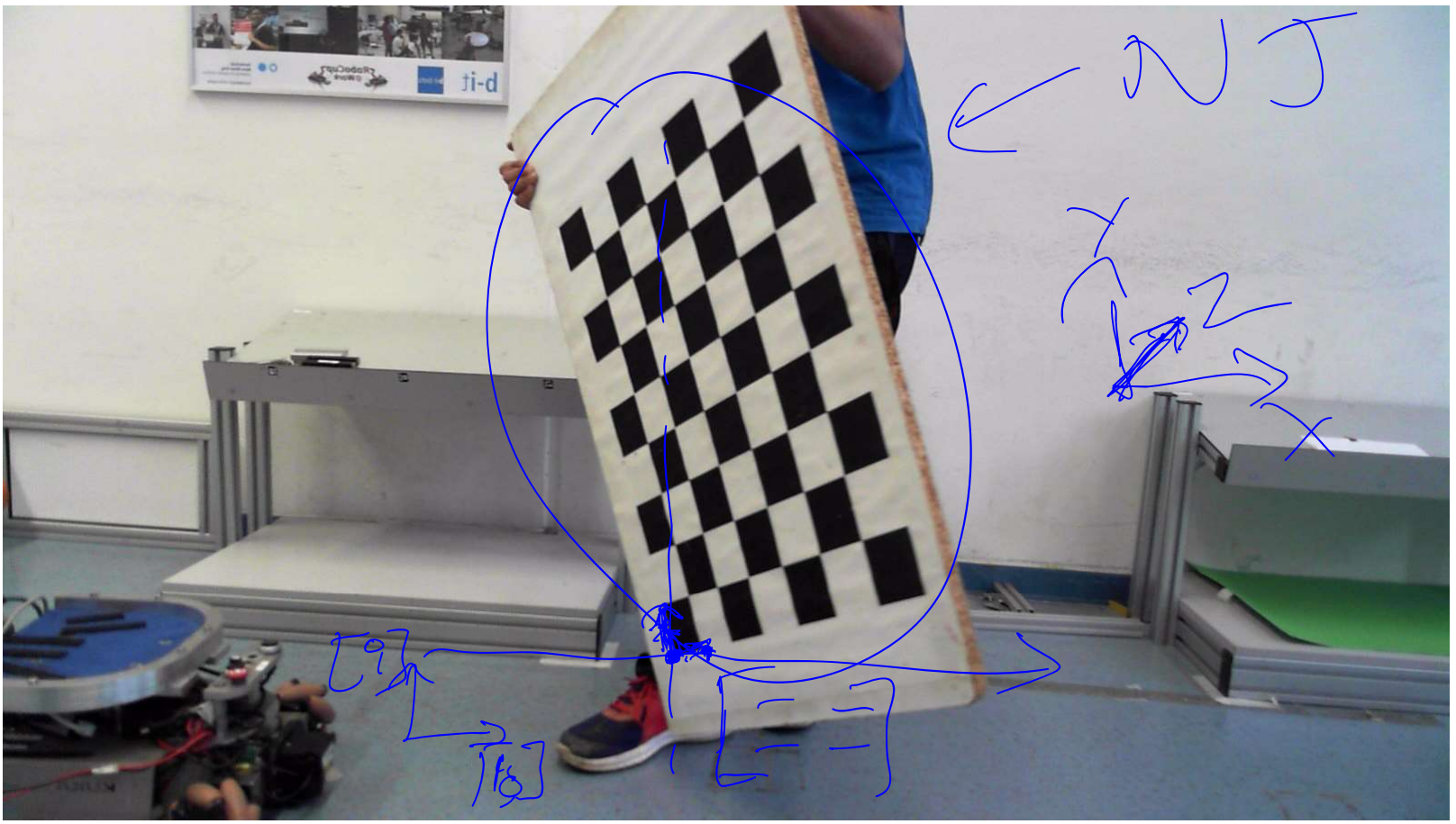
Applications

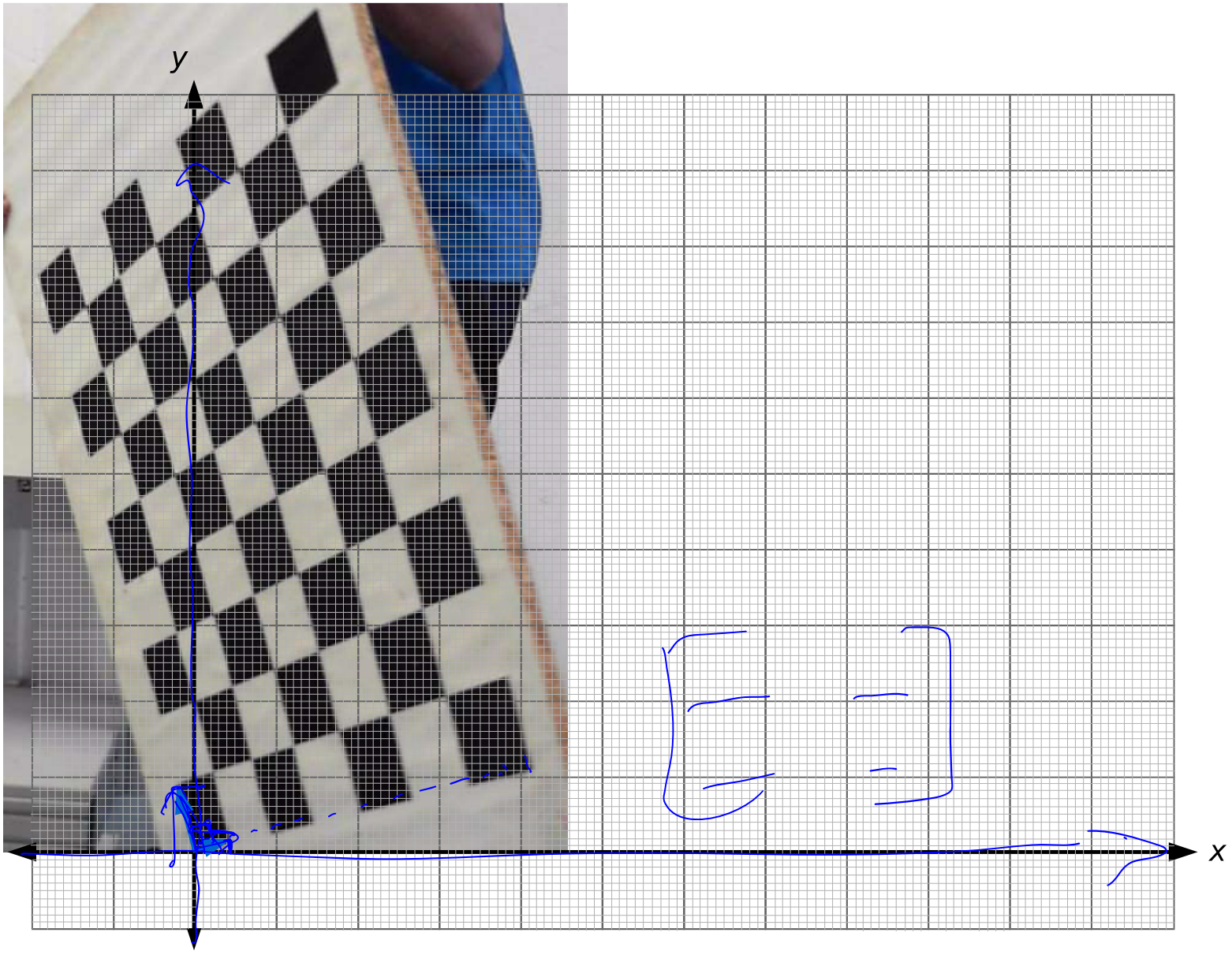
Computer Vision

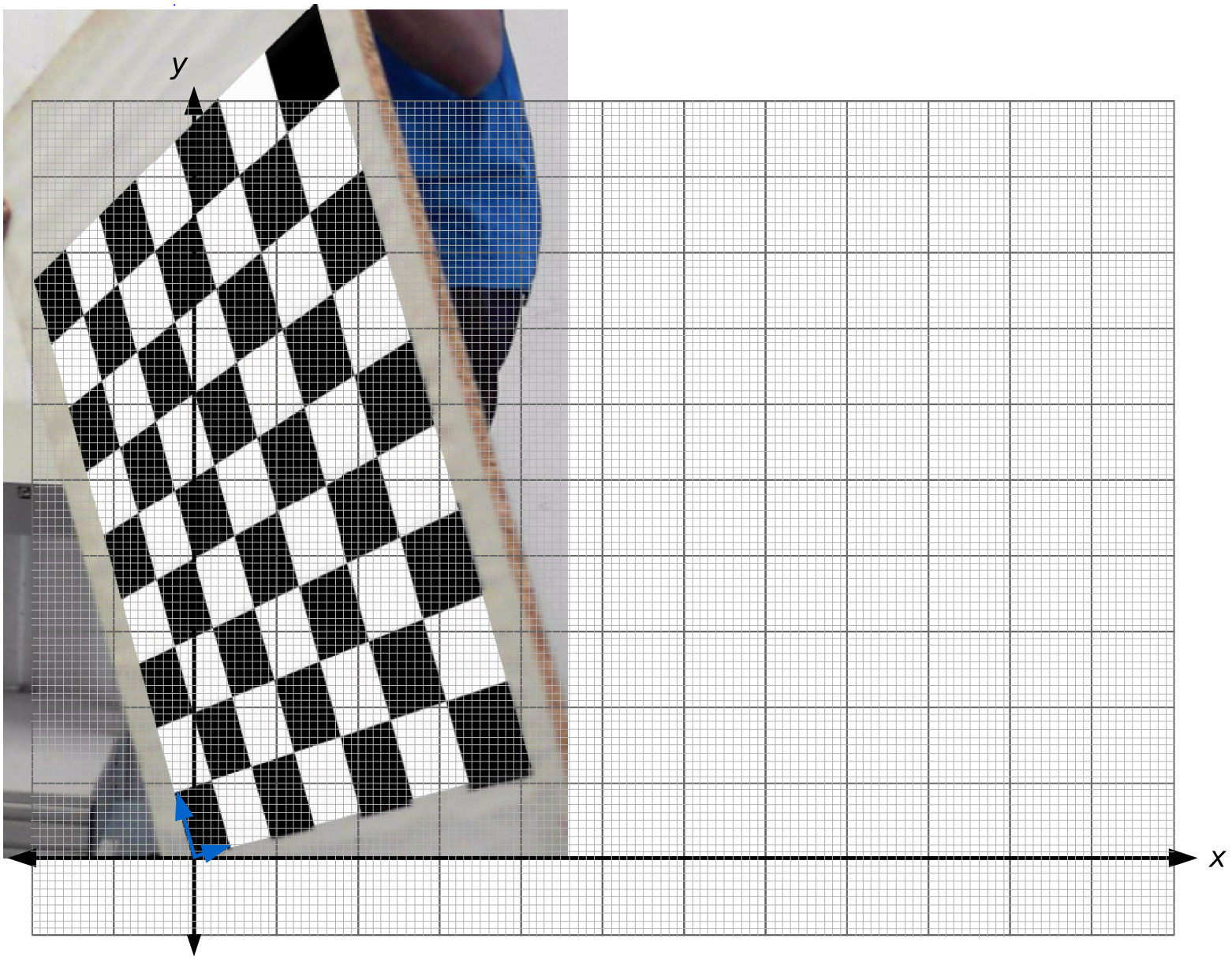


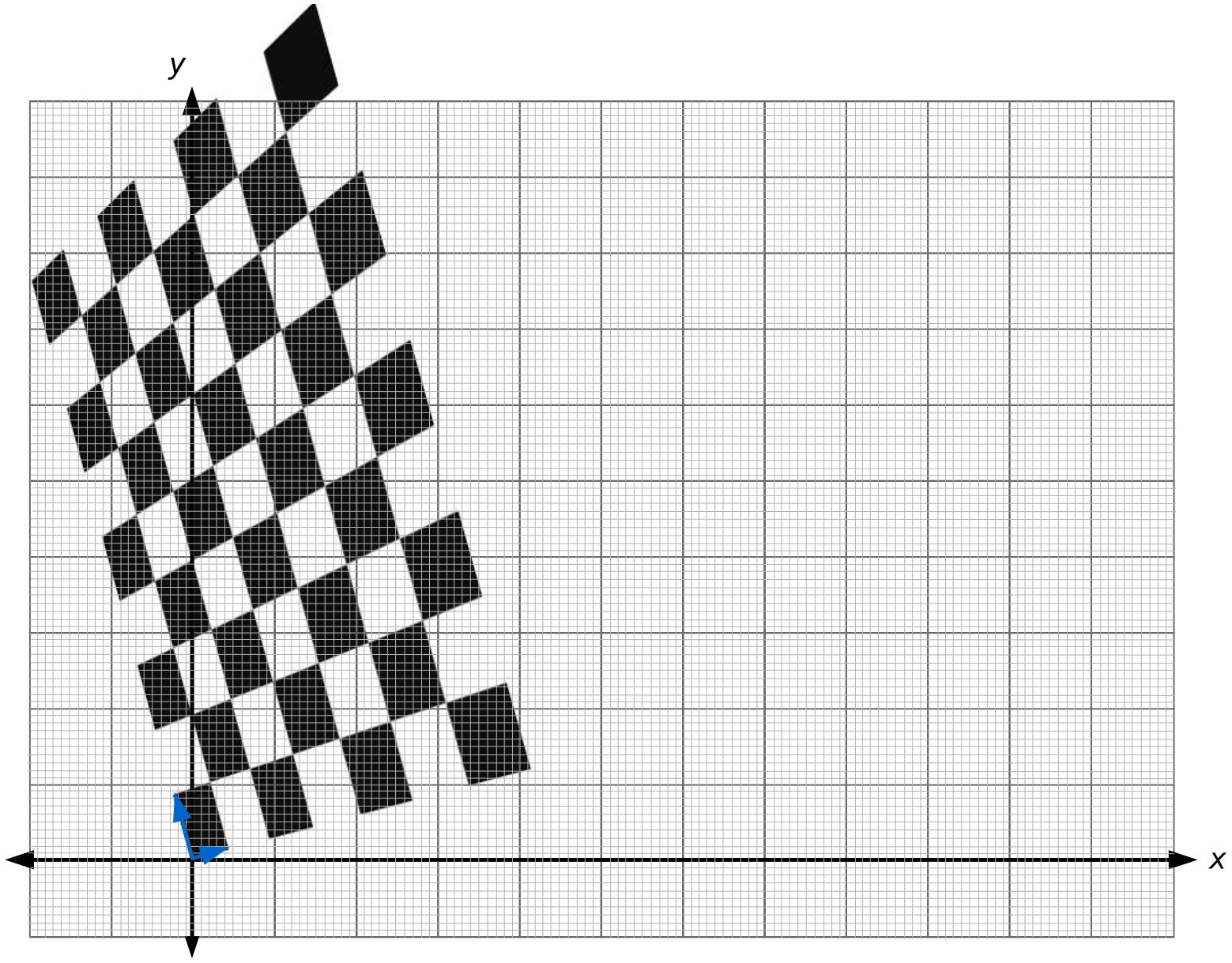

NJ
CV Expert
1920 x 1080


y
x



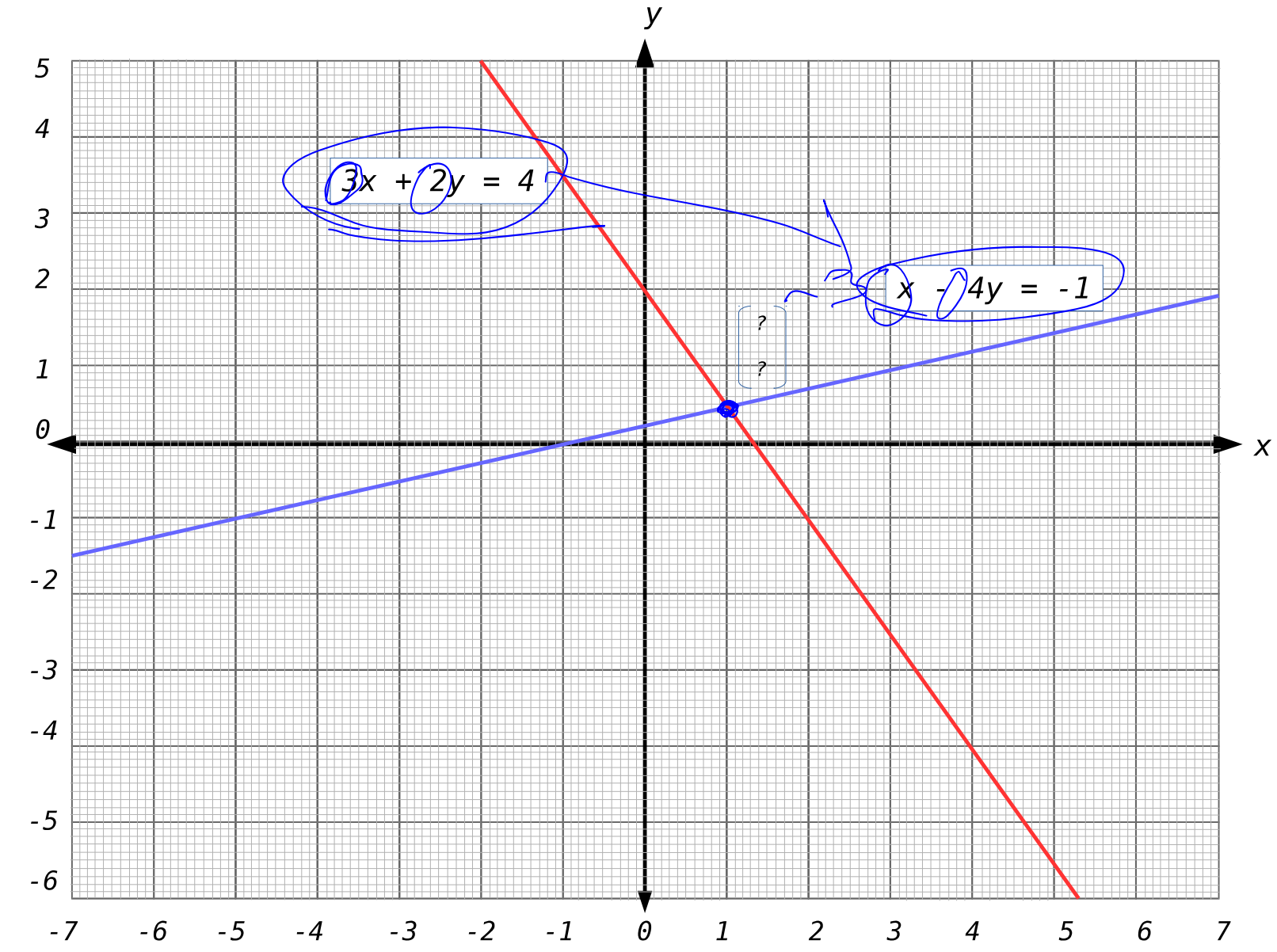

y
x


y
x


y
x

Applications

System of Equations


y
x
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
3x + 2y = 4
x - 4y = -1
?
?

$$3x + 2y = 4$$
$$x - 4y = -1$$

$$\begin{bmatrix} 3 & 2 \\ 1 & -4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

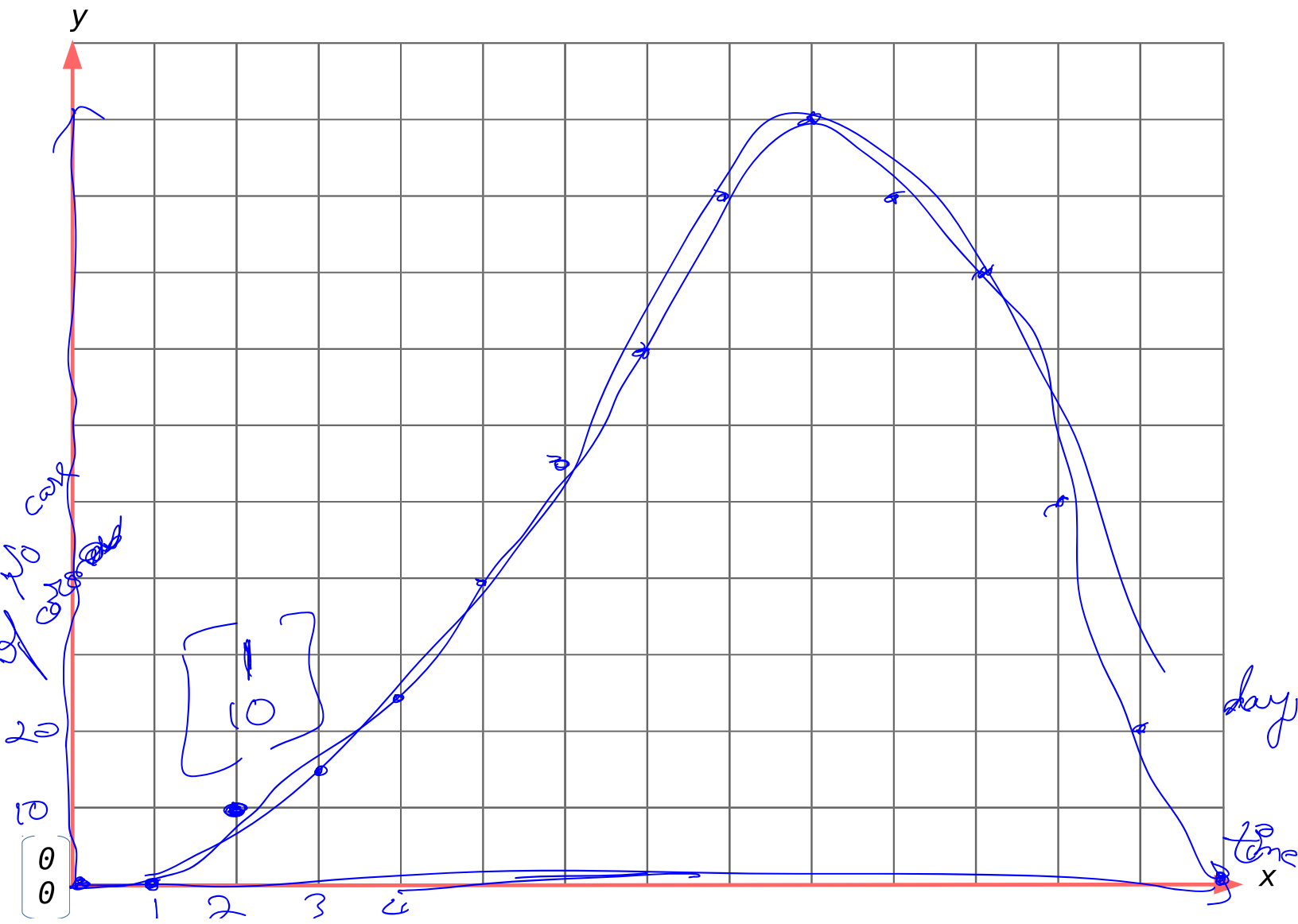
$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B$$

$$AX = B$$

$$(X) = A^{-1}B$$

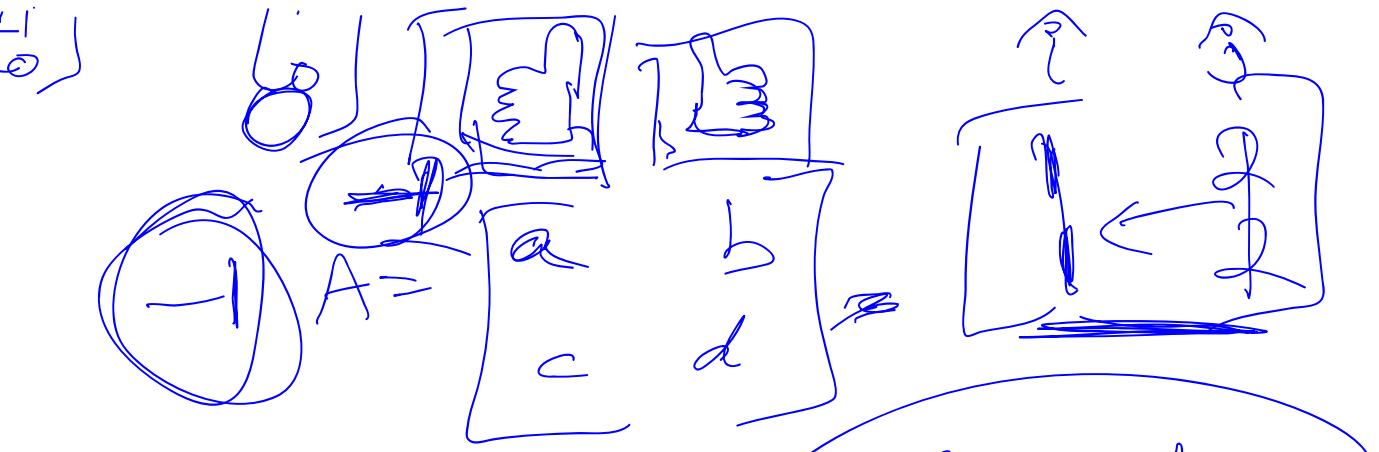
Applications

Data Analysis


y
x
0
0
1
2
3
4
10
20
day
time

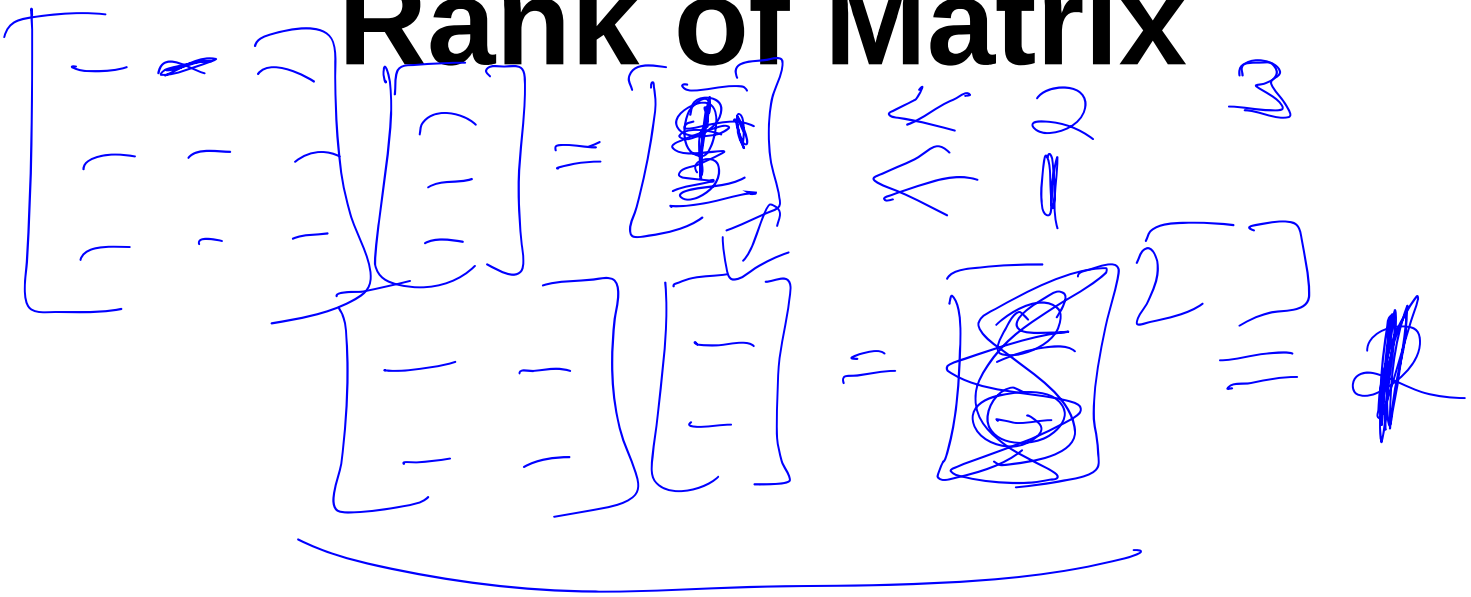
Determinant

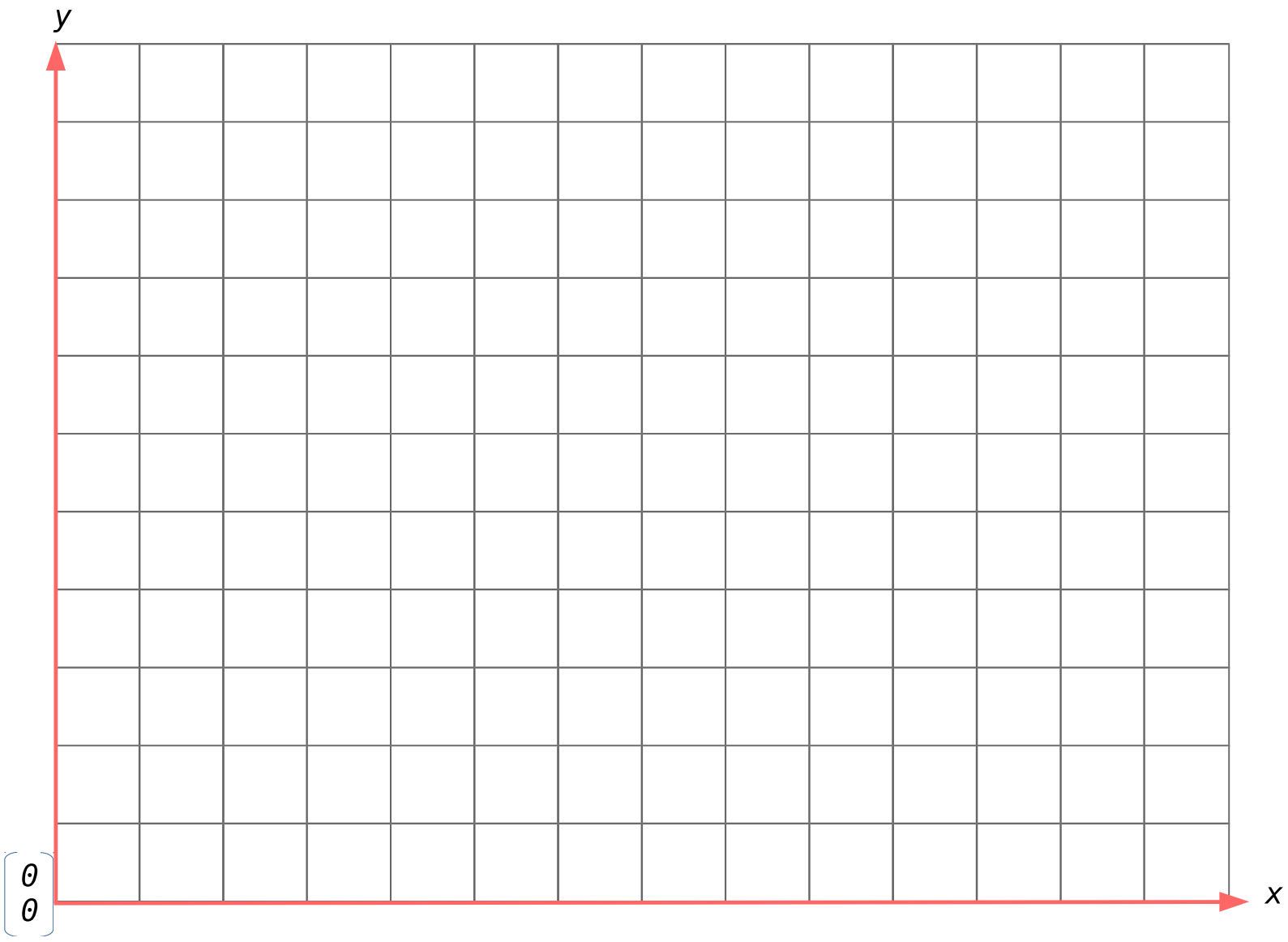

y
x
0
0



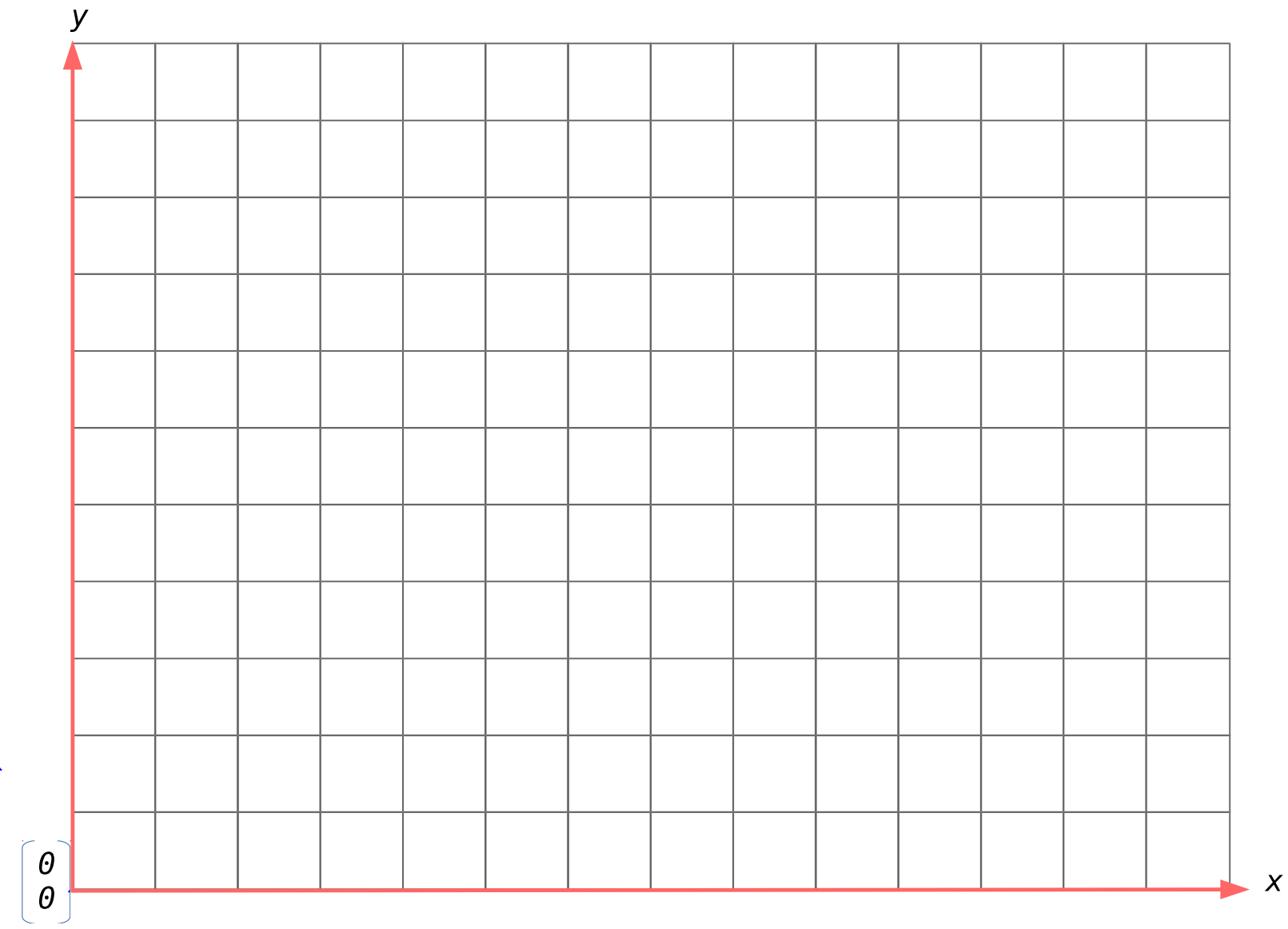
$$\det(A) = ad - cb$$

Rank of Matrix




y
x
0
0

Null and Column Space


y
x
0
0

THE END