CS 499 Senior Project

Sky Map Utility for Amateur Astronomers

Supplemental Material

Important Astronomical Terms*

Altitude - The angular distance from the observer's horizon, usually taken to be that horizon that is unobstructed by natural or artificial features (such as mountains or buildings), measured directly up from the horizon toward the zenith; positive numbers indicate values of altitude above the horizon, and negative numbers indicate below the horizon --- with negative numbers usually being used in terms of how far below the horizon the sun is situated at a given time [for example, the boundary between civil twilight and nautical twilight is when the sun is at altitude -6 degrees].

Astronomical Unit (AU) - Approximately equal to the mean earth-sun distance, which is about 150,000,000 km or 93,000,000 miles. Formally, the AU is actually slightly less than the earth's mean distance from the sun (semi-major axis) because it is the radius of a circular orbit of negligible mass (and unperturbed by other planets) that revolves about the sun in a specific period of time.

Azimuth - Angular distance measured clockwise around the observer's horizon in units of degrees; astronomers usually take north to be 0 degrees, east to be 90 degrees, south to be 180 degrees, and west to be 270 degrees.

Besselian Year - A quantity introduced by F. W. Bessel in the nineteenth century that has been used into the twentieth century. Bessel introduced a system whereby it would be convenient to identify any instant of time by giving the year and the decimal fraction of the year to a few places, but the starting time of the year was not convenient for dynamical studies that utilize Julian dates (see definition for Julian date), differing by 0.5 day, and the Besselian year varies slowly. The recent change to Julian year usage in dynamical astronomy (and the J2000.0 equinox) took effect in solar-system ephemerides of the Minor Planet Center and Central Bureau for Astronomical Telegrams on Jan. 1, 1992. (See Julian year.)

Celestial Equator - An imaginary line midway between the celestial poles and passing directly above Earth's equator.

Celestial Poles - The points on the Celestial Sphere directly above the geographic North and South poles of the Earth.

Celestial Sphere - An imaginary sphere of great (or infinite) radius that is centered on the earth and is used for practical purposes in astronomical observing. Since stars (other than our own sun!) are very distant from us, they make up a background that is essentially unchanging from year to year; of course, over a period of years, the closer stars will move very slightly and factors such as precession cause a change in the appearance of the stars in our skies over many years. But we create a map grid on the celestial sphere for identifying, referring to, and locating objects in the sky; some of these map grids include equatorial coordinates (right ascension and declination), ecliptic coordinates (ecliptic longitude and latitude), and galactic coordinates (galactic longitude and latitude) -

-- which refer to the earth's rotation, the earth's revolution about the sun, and the Milky Way galaxy's plane, respectively.

Declination - One element of the astronomical coordinate system on the sky that is used by astronomers. Declination, which can be thought of as latitude on the earth projected onto the sky, is usually denoted by the lower-case Greek letter delta and is measured north (+) and south (-) of the celestial equator in degrees, minutes, and seconds of arc. The celestial equator is defined as being at declination zero (0) degrees; the north and south celestial poles are defined as being at +90 and -90 degrees, respectively. When specifying a comet's location on the sky, one must state the right ascension and declination (with equinox), along with date and time (since a comet moves with respect to the background stars).

Eccentricity (of an orbit) - A measure of its departure from a circle. Elliptical orbits have an eccentricity >0 and <1, parabolic paths have an eccentricity =1, and hyperbolic paths have an eccentricity >1.

Ecliptic - The apparent path of the sun against the sky background (celestial sphere); formally, the mean plane of the earth's orbit about the sun.

Ephemeris (plural: ephemerides) - Pronounced ee-FEM-er-is (ef-fi-MARE-uh-deez). A table listing specific data of a moving object, as a function of time. Ephemerides usually contain right ascension ("R.A." in these web pages) and declination ("Decl." in these web pages), apparent angle of elongation ("Elong." in these web pages) from the sun (in degrees), and magnitude (brightness) of the object; other quantities frequently included in ephemerides include the objects distances from the sun and earth (in AU), usually given as Roman letter "r" and Greek letter "Delta", respectively; phase angle; and moon phase.

Ephemeris Time (ET) - Determined in principle from the sun's apparent annual motion, ET is the numerical measure of uniform time, which is the independent variable in the gravitational theory of the earth's orbital motion, coming from Simon Newcomb's *Tables of the Sun*. In practice, ET was obtained by comparing observing positions of the Moon with gravitational ephemerides calculated from theories. In 1992, standard (apparent geocentric) ephemerides of comets and minor planets changed from using Ephemeris Time to Terrestrial Dynamical Time (TDT, or TT).

Epoch - The date at which a the co-ordinates on a star chart will be correct with respect to precession. The date of reference in astronomical calculations.

Equinox - Either of the two points (vernal, autumnal) on the celestial sphere where the ecliptic (which is the apparent path of the sun on the sky) intersects the celestial equator. Due to precession, this point moves over time, so positions of stars in catalogues and on atlases are usually referred to a "mean equator and equinox" of a specified standard epoch. Today the positions are almost always given for "equinox J2000.0", meaning that the reference system is that at the beginning of the year 2000; prior to 1992, most astronomers were using "equinox B1950.0". Many older star atlases and catalogues still in use refer to equinox 1950.0, so observers must be careful when plotting positions (and when reporting positions) to note the proper equinox. (The "B" and "J" preceding the equinox years indicate "Besselian" and "Julian", respectively. See separate definitions for

Besselian year and Julian year.) The differences in an object's position when given in equinoxes 1950.0 and 2000.0 amounts to several arc minutes.

Geocentric Latitude - The acute angle measured perpendicular to the equatorial plane and a line joining the center of the earth and a point on the surface of the reference ellipsoid.

Geographic Latitude - The position on the Earth's surface measured in degrees from the equator. Latitudes north of the equator are measured from 0 to +90 degrees. Latitudes south of the equator are measured from 0 to –90 degrees.

Geographic Longitude - The position on the Earth's surface measured in degrees (or degrees minutes and seconds) from the ***prime meridian*** which runs from pole to pole through Greenwich, England. Longitudes are measured from 0 to +180 degrees moving westward and from 0 to –180 degrees moving eastward from the prime meridian. (see Prime Meridian)

Gregorian Calendar - The most widely used calendar in the world today counting years from the traditional birth of Jesus. Years after this date are designated as A.D. (*anno Domini* meaning "the year of our Lord") or C.E. (Common Era). Years before the date are designated as B.C. (Before Christ) or B.C.E. (Before the Common Era). It was first proposed by the Calabrian doctor Aloysius Lilius and decreed by Pope Gregory XIII, after whom it is named, on February 24, 1582.

Gregorian Calendar Reform - Because of errors in the way dates for the Gregorian Calendar were first calculated in the sixteenth century the exact calendar was out of phase with astronomical events and the Julian calendar. Therefore to correct the discrepancy Thursday, October 4, 1582 was followed by Friday, October 15, which also marked the beginning of the Gregorian Calendar.

Julian Calendar - A system of measuring days, years, and centuries of time in days (and fraction of a day) since Greenwich noon on Jan. 1, 4713 BC.

Julian Century - Exactly 36,525 days. See Julian year.

Julian Year - Exactly 365.25 days, in which a century (100 years) is exactly 36525 days and in which 1900.0 corresponds exactly to 1900 January 0.5 (from the Julian-date system, which is half a day different from civil time or UT). The standard epoch J2000.0, now used for new star-position catalogues and in solar-system-orbital calculations, means 2000 Jan. 1.5 Barycentric Dynamical Time (TDB) = Julian Date 2451545.0 TDB. When this dynamical, artificial "Julian year" is employed, a letter "J" prefixes the year.

Julian Day (JD) - The interval of time in days (and fraction of a day) since Greenwich noon on Jan. 1, 4713 BC. The JD is always half a day off from Universal Time, because the current definition of JD was introduced when the astronomical day was defined to start at noon (prior to 1925) instead of midnight. Thus, 1995 Oct. 10.0 UT = JD 2450000.5.

Magnitude - The units used to describe brightness of astronomical objects. The smaller the numerical value, the brighter the object. The human eye can detect stars to 6th or 7th magnitude on a dark, clear night far from city lights; in suburbs or cities, stars may only be visible to mag 2 or 3 or 4, due to light pollution. The brightest star, Sirius, shines at visual magnitude -1.5. Jupiter can get about as bright as visual magnitude -3 and Venus as bright as -4. The full moon is near magnitude -13, and the sun near mag -26. Comet C/1996 B2 (Hyakutake) reached magnitude about 0 in late March 1996. The magnitude scale is logarithmic, with a difference of one magnitude corresponding to a change of about 2.5 times in brightness; a change of 5 magnitudes is defined as a change of exactly 100 times in brightness.

Mean Equinox - The mean equinox is the position of the equinox corrected for the slight but noticeable changes caused by nutation (a slight irregular motion in the axis of rotation of a planet) the and the Chandler wobble (a small motion in the Earth's axis of rotation, another type of nutation, relative to the Earth's surface, which was discovered by American astronomer Seth Carlo Chandler in 1891.).

Meridian - The imaginary line on the celestial sphere passing through the poles and lying directly overhead for an observer. (see Prime Meridian)

Obliquity – The inclination angle of a planet's rotational axis in relation to the plane of its revolution about the sun. For the earth is approximately 23 degrees. It is this tilt that accounts for our seasons.

Orbital Elements - Parameters (numbers) that determine an object's location and motion in its orbit about another object. In the case of solar-system objects such as comets and planets, one must ultimately account for perturbing gravitational effects of numerous other planets in the solar system (not merely the sun), and when such account is made, one has what are called "osculating elements" (which are always changing with time and which therefore must have a stated epoch of validity). Six elements are usually used to determine uniquely the orbit of an object in orbit about the sun, with a seventh element (the epoch, or time, for which the elements are valid) added when planetary perturbations are allowed for; initial ("preliminary") orbit determinations shortly after the discovery of a new comet or minor planet (when very few observations are available) are usually "two-body determinations", meaning that only the object and the sun are taken into account --- with, of course, the earth in terms of observing perspective. The six orbital elements used for comets are usually the following: time of perihelion passage (T) [sometimes taken instead as an angular measure called "mean anomaly", M]; perihelion distance (q), usually given in AU; eccentricity (e) of the orbit; and three angles (for which the mean equinox must be specified) --- the argument of perihelion (lower-case Greek letter omega), the longitude of the ascending node (upper-case Greek letter Omega), and the inclination (i) of the orbit with respect to the ecliptic.

Parallax - The apparent displacement or the difference in apparent direction of an object as seen from two different points not on a straight line with the object (as from two different observing sites on earth or the same location but at two different time of the year as when the Earth is on opposite sides of its orbit about the sun).

Precession - A slow but relatively uniform motion of the earth's rotational axis that causes changes in the coordinate systems used for mapping the sky. The earth's axis of rotation does not always point in the same direction, due to gravitational tugs by the sun and moon (known as lunisolar precession) and by the major planets (known as planetary precession).

Prime (Greenwich) Meridian - The line of longitude defined as 0 degrees. By international convention, the modern Prime Meridian is one passing through

Greenwich, London, United Kingdom, known as the **International Meridian** or **Greenwich Meridian**. Historically, various meridians have been used, including four different ones through Greenwich. The meridian exactly 180 degrees from the prime meridian is known as the **International Date Line.**

Right Ascension - One element of the astronomical coordinate system on the sky, which can be thought of as longitude on the earth projected onto the sky. Right ascension is usually denoted by the lower-case Greek letter alpha and is measured eastward in hours, minutes, and seconds of time from the vernal equinox. There are 24 hours of right ascension, though the 24-hour line is always taken as 0 hours. More rarely, one sometimes sees right ascension in degrees, in which case there are 360 degrees of right ascension to make a complete circuit of the sky. When specifying a comet's location on the sky, one must state the right ascension and declination (with equinox), along with date and time (since a comet moves with respect to the background stars). Note that, at the celestial equator, there are 15 arc seconds in one second of R.A. (often stated as "one second of time"); as one moves away from the celestial equator, one must multiply this factor of 15 by an additional factor (cosine of the declination), because the lines of right ascension get closer and closer as one nears the celestial poles, to get straight-line distances between two celestial objects that are close to each other (for long distances across the celestial sphere, a more complex formula is used). Thus, when the R.A. is given in h, m, and s, it is usually given in seconds of time to one more significant digit than is the Declination in arcsec (i.e., if R.A. is given to $0^s.01$, the Declination. should be only given to 0".1, though this significant-figures requirement disappears as one approaches the celestial poles).

Sidereal Time - Time based on the Earth's revolution about the sun as opposed to Solar time which is a measure of time based on the Earth rotation. A Sidereal day lasts $23^h56^m4.09530833^s$

Universal Time (UT or UTC) - A measure of time used by astronomers; UT conforms (within a close approximation) to the mean daily (apparent) motion of the sun. UT is determined from observations of the diurnal (daily) motions of the stars for an observer on the earth. UT is usually used for astronomical observations, while Terrestrial Dynamical Time (TDT, or simply TT) is used in orbital and ephemeris computations that involve geocentric computations. Coordinated Universal Time (UTC) is that used for broadcast time signals (available via shortwave radio, for example), and it is within a second of UT.

Vernal Equinox - The point on the celestial sphere where the sun crosses the celestial equator moving northward, which corresponds to the beginning of spring in the northern hemisphere and the beginning of autumn in the southern hemisphere (in the third week of March). This point corresponds to zero (0) hours of right ascension.

*See: http://www.cfa.harvard.edu/icq/ICQGlossary.html
http://www.astunit.com/tutorials/glossary.htm

Astronomical Symbols used In Formulas

α - (lower case Greek alpha) Right ascension. Usually given in degrees for formulas even though right ascension is usually given in hours-minutes-seconds.

δ - (lower case Greek delta) Declination. Measured in degrees north and south of the celestial equator.

h - (lower case Roman) Altitude. Measured in degrees. If positive the object is above the horizon. If negative the object is below the horizon.

A - (upper case Roman) Azimuth. Measured in degrees westward (CCW) from due south.

ϕ - (lower case Greek phi) Observer's geographic latitude. If positive location is in the northern hemisphere. If negative location is in the southern hemisphere.

ϕ□- (lower case Greek phi prime) Observer's geocentric latitude. If positive location is north of the equatorial plane. If negative location is south of the equatorial.

L - (upper case Roman) Observer's longitude. If positive location is west of Greenwich. If negative location is east of Greenwich.

Θ - (upper case Greek theta) Local sidereal time.

Θ_0 - (upper case Greek theta, subscript 0) Greenwich sidereal time.

e - (lower case Roman) Eccentricity of an orbit.

H - (upper case Roman) Hour angle.

ε - (lower case Greek epsilon) Obliquity of the ecliptic.

π - (lower case Greek pi) Parallax.

ΔT - (upper case Greek delta, upper case Roman **T**) ET – UT (Ephemeris time minus universal time)

JD - (upper case Roman) Julian Day.

Astronomical Formulas

Calculating the exact Julian Day (Must be based on GMT time zone)

YYYY = year (e.g. 2009)
MM = month (January = 01)
DD.dd = day/hour/minutes converted to fractions of a day, e.g. 9:00 AM on the 15th of a month = 15 + 9/24 = 15.375

```
if(MM > 2)
    y = YYYY
    m = MM
else3
    y = YYYY – 1
    m = MM + 12

if(date > October 15, 1582
    A = int(y/100)                Fractions are dropped.  They are
    B = 2 – A + int(A/4)          NOT rounded.
else
    B = 0

JD = int(365.25 * y) + int(30.6001 * (m + 1)) + DD.dd + 1720994.5 + B
```

Example: JD for January 5, 2008, 8:00 PM (20:00 on 24 hour clock)

```
    y = (2008 – 1) = 2007
    m = 1 + 12 = 13
    A = 20
    B = -13
    JD = 733056 + 428 + 5.833 + 1720994.5 – 13
    JD = 2454471.3
```

Calculating the Julian Day relative to 2000 (Must be based on GMT time zone)

YYYY = year (e.g. 2009)
MM = month (January = 01)
HH = hour on 24 hour clock as an integer
MM = minutes as an integer
SS = seconds as an integer
DD = day of the month as an integer

```
JD = (367 * YYYY)
        - (Math.floor(7.0 * (YYYY + Math.floor((MM + 9.0)/12.0))/4.0))
        + (Math.floor(275.0 * MM / 9.0))
        + DD - 730531.5 + HH/24.0;
```

Calculating Elements of the Major Planetary Orbits

The orbits of the major planets can be modeled as ellipses with the Sun at one focus. The effect of gravitational interactions between the planets perturbs these orbits so that an ellipse is not an exact match with a true orbit. Six numbers, the mean orbital elements, specify an elliptical orbit. Mean orbital elements average the effects of gravitational forces between planets. Calculation of a planet's position based on these mean elements can be inaccurate by a few minutes of arc.

The position of a planet (the word originally meant *wandering star*) varies with time. The daily motion changes the mean longitude by the average number of degrees the planet moves in one (mean solar) day. The other elements change slowly with time. They are modeled using power series expansions of centuries from some fundamental epoch. Here, we use the elements with their linear rates of change from the epoch J2000 (12:00 UT, Jan 1, 2000).

Planet positions are computed in the Equatorial coordinate system as right ascension (RA) and declination (DEC). These give the coordinates of the planet with respect to the fixed stars. The origin for RA is the vernal equinox. Because the orientation of the Earth's axis is changing slowly with time, celestial coordinates must always be referred to an epoch, or date. By using orbital elements referred to epoch J2000, the orbits of the planets are described in a coordinate system that is based on the position the vernal equinox had at J2000. The effect of nutation (the Earth's axis is nodding) is ignored since positions are relative to the mean ecliptic of J2000. The aberration effect caused by the finite speed of light is also ignored.

These elements are used in calculating the location of a planet in its' orbit on a given day and time.

The orbital elements are:

- a = semimajor axis of the orbit (this element is a constant for each planet. See tables following)
- e = eccentricity of the orbit
- i = inclination on the plane of the ecliptic
- ω = argument of perihelion
- Ω = longitude of ascending node
- L = mean longitude of the planet

Calculate the elements as follows:

cy = JD / 36525	Where JD is the Julian Day counted from 2000
RADS = π/ 180.0	Used to convert degrees to radians
DEGS = 180 / π	Used to convert radians to degrees
Mod2Pi(value)	Convert an angle above 360 degrees to one less than 360. Example: the angle 450 degrees on a circle is the same as 90 degrees. Convert using the following algorithm

Mod2Pi Algorithm:

Given angle X in radians

B = X / 2 π

A = (2 π) * (b – abs_floor(b))

where abs_floor(b) is determined by:

if (b >= 0) abs_floor(b) = Math.floor(b)

else abs_floor(b) = Math.ceil(b)

if(A < 0) A = (2 π) + A

Converted angle = A

Precalculated Planetary Elements

Mercury

L = Mod2Pi ((252.25084 + 538101628.29 * cy / 3600) * RADS)

a = 0.38709893 + 0.00000066 * cy

e = 0.20563069 + 0.00002527 * cy

i = (7.00487 - 23.51 * cy / 3600) * RADS

ω = (77.45645 + 573.57 * cy / 3600) * RADS

Ω = (48.33167 - 446.30 * cy / 3600) * RADS

Venus

L = Mod2Pi ((181.97973 + 210664136.06 * cy / 3600) * RADS)

a = 0.72333199 + 0.00000092 * cy

e = 0.00677323 - 0.00004938 * cy

i = (3.39471 - 2.86 * cy / 3600) * RADS

ω = (131.53298 - 108.80 * cy / 3600) * RADS

Ω = (76.68069 - 996.89 * cy / 3600) * RADS

Earth/Sun

L = Mod2Pi ((100.46435 + 129597740.63 * cy / 3600) * RADS)

a = 1.00000011 - 0.00000005 * cy

e = 0.01671022 - 0.00003804 * cy

i = (0.00005 - 46.94 * cy / 3600) * RADS

ω = (102.94719 + 1198.28 * cy / 3600) * RADS

Ω = (-11.26064 - 18228.25 * cy / 3600) * RADS

Mars

L = Mod2Pi ((355.45332 + 68905103.78 * cy / 3600) * RADS)

a = 1.52366231 - 0.00007221 * cy

e = 0.09341233 + 0.00011902 * cy

i = (1.85061 - 25.47 * cy / 3600) * RADS

ω = (336.04084 + 1560.78 * cy / 3600) * RADS

Ω = (49.57854 - 1020.19 * cy / 3600) * RADS

Jupiter

L = Mod2Pi ((34.40438 + 10925078.35 * cy / 3600) * RADS)
a = 5.20336301 + 0.00060737 * cy
e = 0.04839266 - 0.00012880 * cy
i = (1.30530 - 4.15 * cy / 3600) * RADS
ω = (14.75385 + 839.93 * cy / 3600) * RADS
Ω = (100.55615 + 1217.17 * cy / 3600) * RADS

Saturn

L = Mod2Pi ((49.94432 + 4401052.95 * cy / 3600) * RADS)
a = 9.53707032 - 0.00301530 * cy
e = 0.05415060 - 0.00036762 * cy
i = (2.48446 + 6.11 * cy / 3600) * RADS
ω = (92.43194 - 1948.89 * cy / 3600) * RADS
Ω = (113.71504 - 1591.05 * cy / 3600) * RADS

Uranus

L = Mod2Pi ((313.23218 + 1542547.79 * cy / 3600) * RADS)
a = 19.19126393 + 0.00152025 * cy
e = 0.04716771 - 0.00019150 * cy
i = (0.76986 - 2.09 * cy / 3600) * RADS
ω = (170.96424 + 1312.56 * cy / 3600) * RADS
Ω = (74.22988 - 1681.40 * cy / 3600) * RADS

Neptune

L = Mod2Pi ((304.88003 + 786449.21 * cy / 3600) * RADS)
a = 30.06896348 - 0.00125196 * cy
e = 0.00858587 + 0.00002510 * cy
i = (1.76917 - 3.64 * cy / 3600) * RADS
ω = (44.97135 - 844.43 * cy / 3600) * RADS
Ω = (131.72169 - 151.25 * cy / 3600) * RADS

Pluto

L = Mod2Pi ((238.92881 + 522747.90 * cy / 3600) * RADS)
a = 39.48168677 - 0.00076912 * cy
e = 0.24880766 + 0.00006465 * cy
i = (17.14175 + 11.07 * cy / 3600) * RADS
ω = (224.06676 - 132.25 * cy / 3600) * RADS
Ω = (110.30347 - 37.33 * cy / 3600) * RADS

Calculating Right Ascension and Declination of a Planet

Step 1: Calculate the elements of the planetary orbit of the planet as shown above. (L_p, a_p, e_p, i_p, ω_p, and Ω_p)

Step 2: Calculate the elements of the planetary orbit of the Earth as shown above. (L_e, a_e, e_e, i_e, ω_e, and Ω_e)

Step 3: Calculate the position of the Earth in its orbit

m_e = Mod2Pi(L_e - ω_e)
v_e = True_Anomoly(m_e, e_e) (See algorithm below)
$r_e = a_e * (1 – e_e^2) / (1 + e_e * \text{Cos}(v_e))$

Step 4: Calculate the heliocentric rectangular coordinates of Earth

$x_e = r_e * \text{Cos}(v_e + \omega_e)$
$y_e = r_e * \text{Sin}(v_e + \omega_e)$
$z_e = 0.0$

Step 5: Calculate the position of the planet in its’ orbit

m_p = Mod2Pi(L_p - ω_p)
v_p = True_Anomoly(m_p, e_p)
$r_p = a_p * (1 – e_p^2) / (1 + e_p * \text{Cos}(v_p))$

Step 6: Calculate the heliocentric rectangular coordinates of the planet

$xh = r_p * (\text{Cos}(\Omega_p) * \text{Cos}(v_p + \omega_p - \Omega_p) – \text{Sin}(\Omega_p) * \text{Sin}(v_p + \omega_p - \Omega_p) * \text{Cos}(i_p))$
$yh = r_p * (\text{Sin}(\Omega_p) * \text{Cos}(v_p + \omega_p - \Omega_p) + \text{Cos}(\Omega_p) * \text{Sin}(v_p + \omega_p - \Omega_p) * \text{Cos}(i_p))$
$zh = r_p * (\text{Sin}(v_p + \omega_p - \Omega_p) * \text{Sin}(i_p))$
If calculating for Earth/Sun xh = yh = zh = 0.0

Step 7: Convert to geocentric rectangular coordinates

$xg = xh – x_e$
$yg = yh – y_e$
$zg = zh – z_e$

Step 8: Rotate around X axis from ecliptic to equatorial coordinates

Ecl = 23.439281 * RADS (value for J2000.0 frame)
xeq = xg
yeq = yg * Cos(Ecl) – zg * Sin(Ecl)
zeq = yg * Sin(Ecl) + zg * Cos(Ecl)

Step 9: Calculate α (right ascension) and δ (declination) from the rectangular equatorial coordinates

α = Mod2Pi(Atan2(yeq, xeq)) * DEGS
$\delta = \text{Atan}(zeq / \text{Sqrt}(xeq^2 + yeq^2)) * \text{DEGS}$
$\text{Dist} = \text{Sqrt}(xeq^2 + yeq^2 + zeq^2)$ (Distance in AUs)

Converting Right Ascension in degrees to Hours:Minutes:Seconds

Right Ascension is always stated in hours, minutes, and seconds. The formula above yields right ascension in degrees. To convert to hours, minutes and seconds use the following:

Hours = int(RA / 15.0)
Minutes = int(((RA / 15.0) – Hours) * 60.0)
Seconds = ((((RA / 15.0) – Hours) * 60.0) – Minutes) * 60.0

Converting Declination in degrees to Degrees:Minutes:Seconds

Declination is always stated in degrees, minutes, and seconds. The formula above yields declination in degrees. To convert to degrees, minutes and seconds use the following:

Degrees = int(dec)
Minutes = int((dec – Degrees) * 60.0)
Seconds = (((dec – Degrees) * 60.0) – Minutes) * 60.0

Algorithm for calculating the true anomaly from the mean anomaly

Given **M** and **e** in radians

```
E = M + e * Sin(M) * (1.0 + e * Cos(M))
do
    E1 = E
    E = E1 – (E1 – e * Sin(E1) – M) / (1 – e * Cos(E1))
while(Abs(E – E1) > (1.0e-12))

V = 2 * Atan(Sqrt((1 + e) / (1-e)) * Tan(0.5 * E))
if(V < 0) V = V + (2 * π)
```

V gives the true anomaly

Calculating Altitude and Azimuth of a Planet given RA and Dec

Given:

Latitude (lat), Longitude (lon), Right Ascension (RA) in degrees, Declination (Dec) in degrees.

```
If (lat is in the southern hemisphere)  lat = lat * -1.0
If (lon is given as West) lon = lon * -1.0
hourAngle = meanSiderealTime() – RA  (See algorithm below)
if(hourAngle < 0) hourAngle += 360
// Convert degrees to radians
decRad = Dec * RADS
latRad = Lat * RADS
hrRad = hourAngle * RADS
```

```
// Calculate altitude in radians
sin_alt = (sin(decRad) * sin(latRad)) + (cos(decRad) * cos(latRad) * cos(hrRad))
alt = asin(sin_alt)

// Calculate azimuth in radians (you will need to handle this inside of a try…catch
//  in case the latitude is at the poles or altitude if 90 degrees
try
    cos_az = (sin(decRad) – sin(alt) * sin(latRad)) / (cos(alt) * cos(latRad))
    az = acos(cos_az)
catch
    az = 0

// Convert altitude and azimuth to degrees
alt *= DEG
az *= DEG

if(sin(hrRad > 0.0) az = 360.0 – az
```

Calculating the Mean Sidereal Time

Given Year (year), Month (month with January = 1), Day (day) of the month, Hour (hour) on a 24 hour clock, Minute (min), Second (sec). All times must be measured from Greenwich mean time (TimeZone = 0).

```
if(month <= 2)          // Adjust month and year if needed
    year = year – 1
    month = month + 12
a = floor(year / 100.0)
b = 2 – a + floor(a / 4)
c = floor(365.25 * year)
d = floor(30.6001 * (month + 1))
// Get days since J2000.0
jd = b + c + d – 730550.5 + day + (hour + min/60 + sec/3600) / 24
// Get Julian centuries since J2000.0
jt = jd / 36525.0
// Calculate initial Mean Sidereal Time (mst)
mst = 280.46061837 + (360.98564736629 * jd) +
    (0.000387933 * jt^2) - (jt^3 / 38710000) + lon
// Clip mst to range 0.0 to 360.0
if(mst > 0.0)
    while(mst > 360.0) mst -= 360.0
else
    while(mst < 0.0) mst += 360.0
```

Result is Mean Sidereal Time for the location given by Lat, Lon

Calculating Lunar Phases

The following formulas can be used to calculate the dates for the mean (approximate) phases of the moon (new, first quarter, full, and last quarter). While not the true times (accurate to the second) the mean time (accurate at least to the day) is adequate for most casual and serious amateur astronomers.

Julian Day of a Given Phase

$$JD = 2415020.75933 + (29.53058868 * k) + (0.0001178 * T^2) – (0.000000155 * T^3) + (0.00033 * \sin((166.56 + (132.87 * T) – (0.009173 * T^2)) * RAD);$$

??? $(0.00033 * \sin(((166.56*RAD) + ((132.87*RAD) * T) – ((0.009173*RAD) * T^2)));$

Convert to RAD before or after???????????????????????????

Note: the value for sin is given in degrees. This must be converted to radians when implemented in Java or C++ code as the Math.sin function expects radians.

Calculating the value of *k* for the above formula

An apprximate value for **k** accurate enough for lunar phase calculations can be obtained from the formula:

$$k \cong (int)((year – 1900.0) * 12.3685);$$

Where the year is the year with a decimal fraction, e.g. February 1, 2009 would be 2009.087 (2009 + 32/365).

Use the integer value of **k** to calculate the **JD** of a new moon. Using the ceil value (rounding up) gives the **JD** of the next new moon. Using the floor value (rounding down) gives the **JD** of the past new moon.

Note: In any of the formulas where the integer designation *(int)* is used this means to just drop the decimal fraction, i.e. use the Math.floor function to calculate the integer value

For the other phases use the following:

k + 0.25 First quarter
k + 0.50 Full moon
k + 0.75 Last quarter

Calculating the value of *T* for the above formula

An approximate value for T accurate enough for lunar phase calculations can be obtained from the formula:

$$T = k / 1236.85;$$

Converting a Julian Day to a calendar date

```
Step 1:
    Set JD = JD + 0.5;

Step 2:
    Z = (int)JD;
    F = JD – Z (Decimal fraction left after converting JD to an int)
    if( Z < 229161)
        A = Z;
    else
        alpha = (int)((Z – 1867216.25) / 36524.25);
        A = Z + 1 + alpha – (int)(alpha / 4);

Step 3:
    B = A + 1524;
    C = (int)((B – 122.1) / 365.25);
    D = (int)(365.25 * C);
    E = (int)((B – D) / 30.6001);

Step 4:
    Day of the month with decimals = B – D – (int)(30.6001 * E) + F;
        The fraction here can be converted to hours and minutes

    Month number:       if(E < 13.5)
                                m = E – 1;
                        else
                                m = E – 13;

    Year:               if(m > 2.5>
                                y = C – 4716;
                        else
                                y = C – 4715;
```

Calculating Lunar Location

The following formulas can be used to calculate the right ascension and declination or geocentric latitude and longitude for a given date. These formulas have been simplified to give an accuracy of approximately 10” in the longitude and 3” in the latitude of the moon. This is close enough for the Star Map application. Note the geocentric latitude and longitude corresponds to the moon’s location at the zenith above that latitude and longitude.

You will need to calculate the Julian Day using the formula given above. Next calculate T using the formula below:

$$\mathbf{T = (JD - 2415020.0) / 36525}$$

Next calculate the angles **L', M, M', D, F** and **Ω** using the following formulae. Note there are T2 and T3 factors in all the formulae but these do not add a significant accuracy to the calculation for this purpose and so are omitted.

Moon's mean longitude:
L' = 270.434164 + 481267.8831T

Sun's mean anomaly:
M = 358.475833 + 35999.0498T

Moon's mean anomaly:
M' = 296.104608 + 477198.8491T

Moon's mean elongation:
D = 350.737486 + 445267.1142T

Mean distance of Moon from its ascending node:
F = 11.250889 + 483202.0251T

Calculate the moon's geocentric latitude (β) and longitude (λ). Note the formulas below have been greatly simplified but still should give an accuracy within one degree (the apparent diameter of the moon). **Note: in the formulas below angles in the sin calls are given in degrees. THEY MUST BE CONVERTED TO RADIANS BEFORE CALLING THE sin FUNCTION IN math.h OR THE JAVA Math LIBRARY.**

$$\mathbf{e = 1 - 0.002495T - 0.00000752T^2}$$

λ = **L' + (6.288750 * sin(M')) + (1.274018 * sin(2D – M')) +
(0.658309 * sin(2D)) + (0.213616 * sin(2M')) – (0.185596 * sin(M) * e)
- (0.114336 * SIN(2F)) + (0.058793 * SIN (2D – 2M')) +
(0.057212 * sin(2D – M – M') * e) + (0.053320 * sin(2D + M') +
(0.045874 * sin(2D – M) * e)**

β = **(5.128189 * sin(F)) + 0.280606 * sin(M' + F) + (0.277693 * sin(M' – F))
+ (0.173238 * sin(2D – F)) + (0.055413 * sin(2D + F – M')) +
(0.046272 * sin(2D – F – M')) + (0.032573 * sin(2D + F)) +
(0.017198 * sin(2M' + F)) + (0.009267 * sin (2D + M' –F)) +
(0.008823 * sin(2M' – F)**

References

Astronomical Formulae for Calculators, **4th edition, by Jean Meeus,** published by **Willmann-Bell, Inc., © 1988**

Other sources