Exploring Quantum Theory with Qubits

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Abstract

This internship had the broad goal of exploring quantum theory with the help of modern state-of-the-art quantum computers made available to the interested public by IBM. Quantum theory is usually presented as mysterious and challenging in popular media. This internship boldly exhibits that high-school students can master the basics and learn to work with quantum computers.

In this presentation, we will share our results of testing the Nobel-prize-winning phenomenon of Rabi oscillation using a quantum computer made by IBM. Rabi oscillation is a universal phenomenon exhibited by the simplest of quantum systems – a qubit. A qubit is a quantum system that, at any given time, lives in a superposition of two possible states. The qubit has a certain probability of ending up in either of its two possible states. This probability exhibits an oscillatory behavior with time, hence the phenomenon's name.

Rabi oscillation has been known to us since the 1940s. This seemingly simple phenomenon is at the heart of revolutionary technological advancements such as photonics, quantum computers, and even MRI machines that have changed modern medicine!

In our project, we subjected a single qubit IBM quantum computer to three different kinds of time evolution, i.e., we chose three different Hamiltonians. We measured the probabilities for two complete cycles and repeated the experiments 20 times for each of these three cases. This exercise gave us the experimental data, which we compared with the theoretical prediction after we performed its error analysis.

We sincerely hope our project will demystify the unusual world of quantum theory and inspire our fellow students to explore this wondrous subject in the future.

Introduction

During our research project, we focused on reproducing Rabi oscillation in an IBM quantum computer. We were aiming to capture how a qubit evolves under different Hamiltonians. Furthermore, we analyzed and mitigated the effects of error due to the presence of random noises, and applied that to our results.

Code for the Rabi Oscillation is available on [GitHub](), and [notes for the internship]() are also public.

Code on GitHub

Notes on Notion

Methodology

We chose to work with these three Hamiltonians:

- H_1 with 100 as the difference in eigenvalues
- Both real and complex numbers
- Exhibits level crossing

$$H_1 = \begin{pmatrix} 71 & 50i \\ -50i & 71 \end{pmatrix}$$

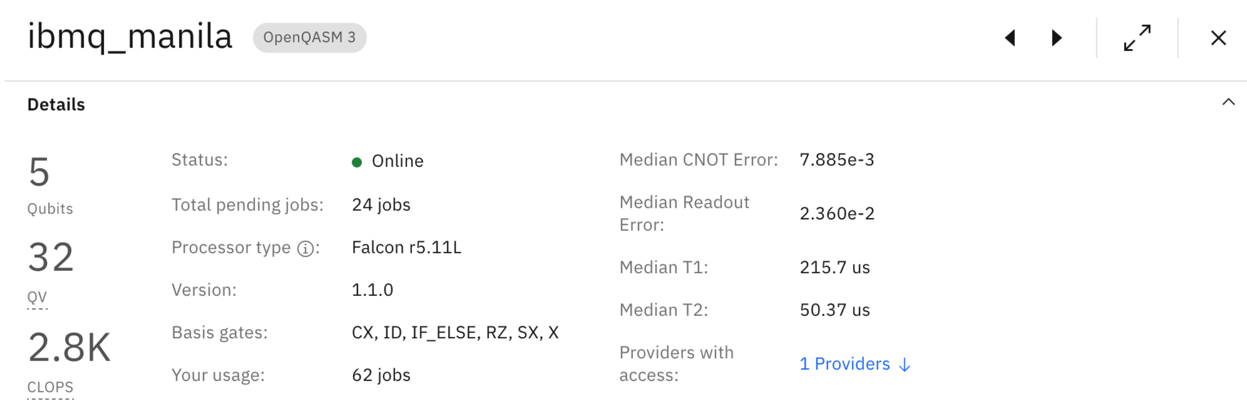
- H_2 with 1.00 as the difference in eigenvalues
- Only real numbers – decimals
- Does not exhibit level crossing

$$H_2 = \begin{pmatrix} 3.2 & 0.14 \\ 0.14 & 4.16 \end{pmatrix}$$

- H_3 with 7.21 as the difference in eigenvalues
- Both real and complex numbers
- Exhibits level crossing

$$H_3 = \begin{pmatrix} 5 & 3+2i \\ 3-2i & 5 \end{pmatrix}$$

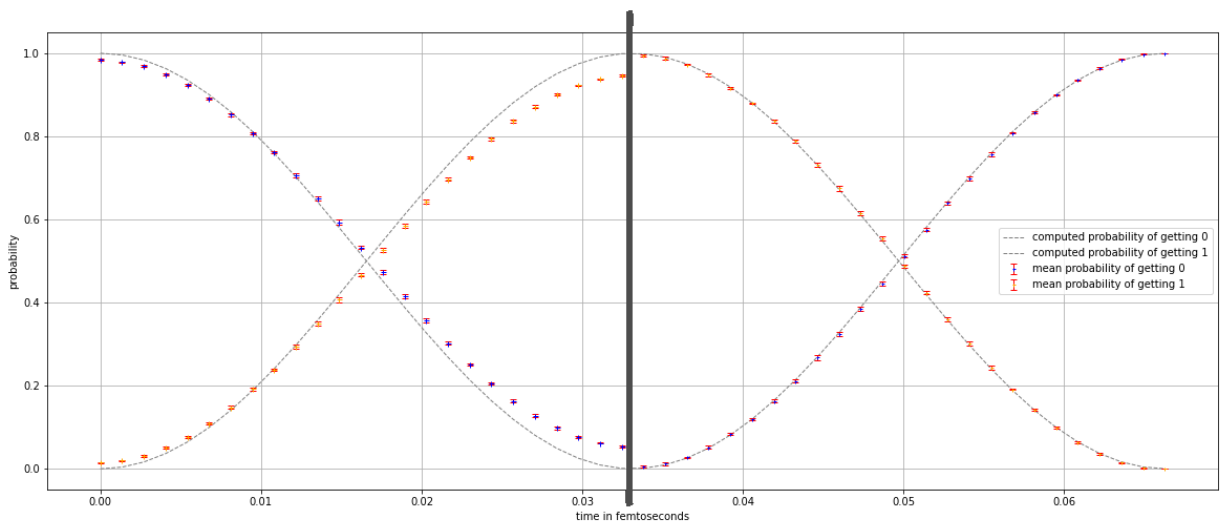
All programming was done in Python using various packages, such as Qiskit. We ran the circuits within the IBM Quantum Experience on a 5-qubit quantum computer named “Manila”. Each circuit was run for 8192 shots to get more accurate outcomes. We also computed the mean from all collected values, which we used for plotting graphs with error bars.



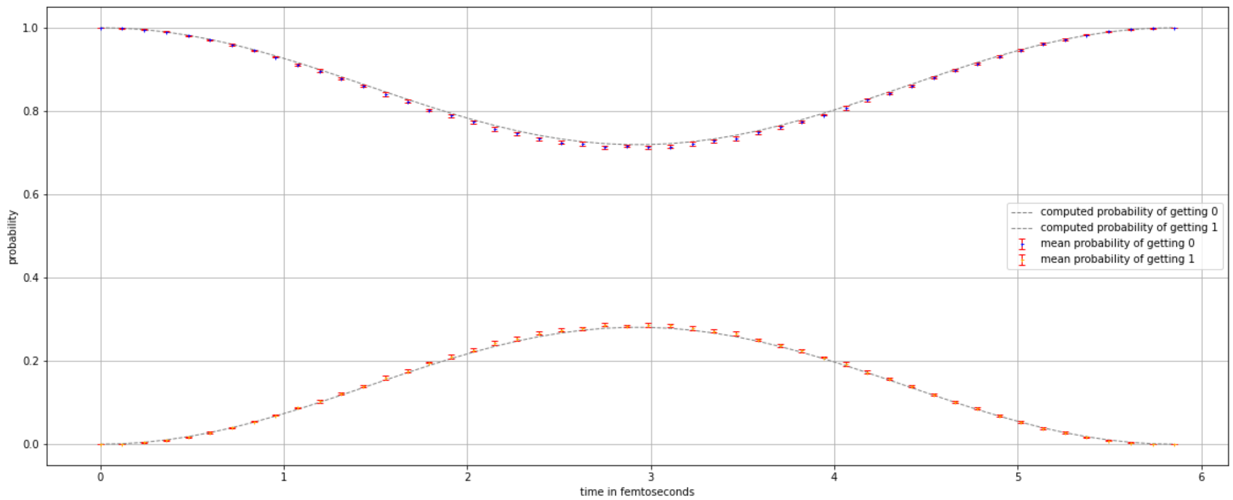
Results

The outcome of our research is a collection of graphs representing the data we collected and analyzed during our study.

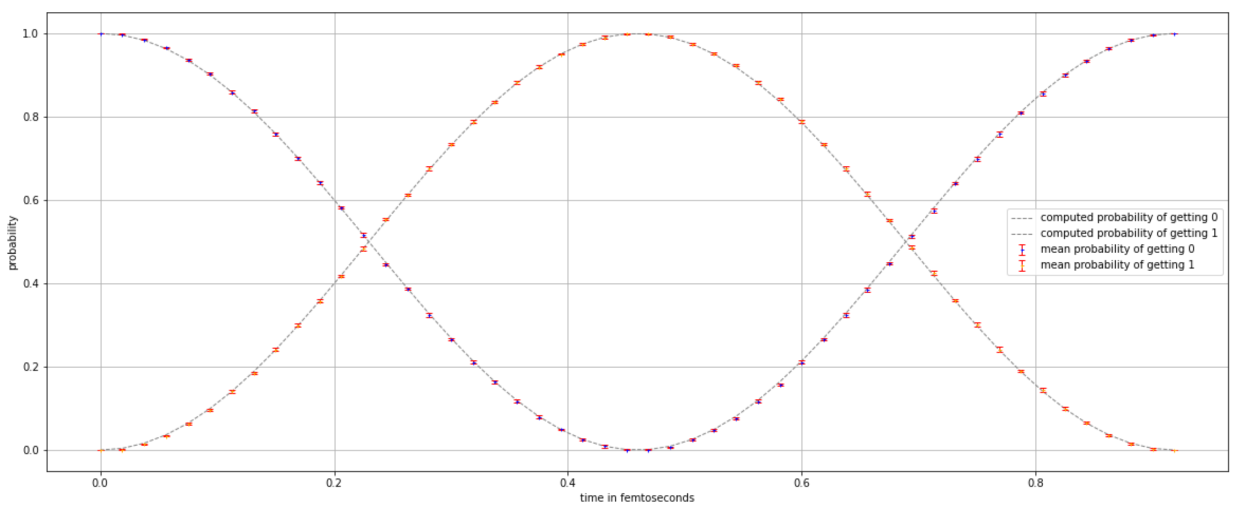
Here you see the result for our experiment with the first Hamiltonian H_1, before error mitigation and after error mitigation (significant reduction of the effects of random noise).



Hamiltonian H_2 does not exhibit any level-crossing, only oscillatory behavior.



Similar to the first Hamiltonian, H_3 also exhibits level-crossing.



And finally, in the table below, we present the errors for each of our experiments.

System	**Mean Error**
H_1 Before Error Mitigation	6.244%
H_1 After Error Mitigation	1.061%
H_2 After Error Mitigation	0.417%
H_3 After Error Mitigation	0.519%

Error Table

Conclusion

We successfully reproduced Rabi oscillations in IBM-Q. It shows good agreement with theoretical predictions prior to error mitigation. After error mitigation, the agreement is excellent.

We also observed that the disagreement between the quantum computer result and the theoretical expectation is more pronounced when the difference in eigenvalues of the Hamiltonian is larger. This can possibly lead to a better insight into pinpointing other sources of errors affecting the quantum computer apart from the effect of random noise. However, such an investigation would require more direct access to the hardware and architecture, which we leave to future projects.

Acknowledgment

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We would also like to thank the Institute of Physics of the Czech Academy of Sciences for providing us with the resources and support necessary to pursue our research.

References

1. Thomas Wong, *Introduction to Classical and Quantum Computing,* Rooted Grove Press, 2022.
2. IBM-Quantum, *Learn Quantum Computation Using Qiskit,* Available online at https://qiskit.org/textbook/preface.html.
3. Martin Laforest, PhD, *The Mathematics of Quantum Mechanics,* University of Waterloo, 2015, Available online at http://www.stat.ucla.edu/~ywu/linear.pdf.