# Economic Controls for Smart Water Distribution Networks Undergoing Supply Failures

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***Abstract*—Water distribution networks are designed and managed to provide adequate water flow to all users. Unexpected events, such as prolonged dry spells or cyber-physical attacks, can affect this *status quo*. Under water scarcity conditions, users located in high-pressure zones have a competitive advantage at the expense of the other customers, which, if exercised freely, can reduce the overall social welfare of the network, i.e., the value its customer get by using it. These conditions of supply failure are a challenge for water utilities, which can either provide full supply, but only for a limited time, or let the system uncontrolled, resulting in a substantial loss of social welfare. Here, we tackle the problem by adopting a game theoretic approach and develop a uniform pricing mechanism aimed to ensure an equitable and fair supply to all customers. The mechanism takes as input the users' demand—an information nowadays available through smart metering—and provides a pricing scheme that induces customers located in high-pressure zones to limit their demand, thus protecting the remaining users. Results obtained for a typical single-branch water distribution network show that the pricing mechanism attains good performance guarantees relative to the optimum, yet unattainable, water allocation. Our results also show that the performance of our pricing mechanism is often comparable to the one obtained by imposing flow restrictions to all users—whose implementation would require massive infrastructural upgrades. The opportunities we identify for supplying customers in an equitable and fair manner can extend to more complex topologies.**

***Index Terms*—Fair water allocation, game theory, pricing mechanism, smart water metering, water distribution networks.**

## I. Introduction

### A. Preliminaries

WATER distribution systems are lifeline infrastructures that carry potable water from centralized treatment facilities to customers, generally including residential, commercial, and industrial users. From a network modelling perspective, these systems consist of nodes, representing customers/users and hydraulic devices (e.g., pumps, valves), and links, representing pipes. Water is supplied by one or multiple tanks, typically located at a height sufficient to pressurize the network [1]. The reliability of any water distribution network depends on the amount of water available in the elevated tanks: under normal operating conditions, there is enough water to provide adequate flow and pressure to all customers. Yet, there can be instances in which water availability, and thus system's reliability, drops. Such instances can be caused by a multitude of events, such as prolonged droughts [2], structural damages [3], or cyber attacks [4]—which affect water availability by targeting the industrial control system that monitors and controls the water network [5].

Under all these water scarcity conditions, users compete for a limited water resource. Ideally, the role of a network operator (e.g., a water utility) is to maximize the 'social welfare'—i.e., the sum of the value (utility) individual customers obtain by using the network— by allocating enough water to all users. As we shall see later, this means determining the allocation that optimizes the overall user utility. However, the implementation of such plan is generally challenged by two factors. First, network operators can control the amount of water released into a network, but not the one consumed by each customer—water networks include dozens to thousands of customer nodes. Thus, it is technically unfeasible to deploy hydraulic actuators that centrally control the amount of water consumed by each customer. Second, it is practical to consider that customers have an uncontrolled behaviour, meaning that they prioritize their own needs over the benefit of the rest of the society. In a competitive environment, the flow of water a customer gets depends not only on her demand, but also on the topology of the network: customers located in proximity of the water tanks, where the pressure is higher, have a competitive advantage with respect to customers located in peripheral areas. If system pressure at the tanks is low, by consuming a large amount they can completely preclude others from accessing water. Because of these reasons, water utilities often prefer to fully supply all customers but for a limited amount of time. For example, during the 2014–2015 drought in In São Paulo (Brazil), customers received water for only two days a week [6].

Is there a way to ensure that all customers are served in an equitable and fair manner during water scarcity conditions? Is it possible to allocate a given amount of water to all customers without resorting to a massive upgrade of the underpinning

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infrastructure? This paper aims to answer these questions. To this purpose, we bank on recent advantages in smart metering—which provides a nearly real-time estimate of customers' demand within a water network [7], [8]—and look at the demand management aspect of the problem. Specifically, we propose a novel pricing mechanism to indirectly control customers' consumption. To clearly illustrate these ideas we rigorously model the most basic type of a water network: a single tank with a single branch feeding a number of downstream customers. The idea is simple: charge and induce customers located in proximity of the water tanks to reduce their consumption, so as to support customers located in peripheral areas. Since differentiated pricing may be perceived as an unfair strategy, we adopt a uniform pricing mechanism that solely depends on the customers' demand and utility function. The mechanism is designed following a game theoretic approach and, importantly, accounts explicitly for the key physical processes characterizing water distribution networks (i.e., energy losses due to the friction with pipe walls, conservation of mass). Such an approach is motivated by the nature of the problem: individuals make rational decisions on water consumption that affect others, and hence each individual's action depends on the actions of the rest of the individuals and on context parameters set by the utility (prices, restrictions, etc). It is natural to consider the solution of this distributed interaction of customers acting according to their own interest to be a Nash equilibrium, i.e., a set of customer behaviours that no customer wants to deviate from.

### B. Contribution

The main contribution of this study can be summarized as follows:

- For the case of a single branch water network, we derive the optimal water allocation that maximizes the social welfare. From a social planning perspective, this allocation reaches the ideal performance, and thus represents a fundamental benchmark for any water allocation mechanism.
- We calculate the social welfare achieved by the 'uncontrolled allocation' mechanism (i.e., when customers are free to ask for the maximum possible level of water consumption, without any disincentives or extra restrictions from the part of network) and compare it against the one attained by the optimal water allocation. We show that the minimum ratio between the two terms (i.e., the Price of Anarchy) is 0, implying that uncontrolled allocation's performance can be arbitrarily poor.
- We derive the optimal pricing mechanism and prove that it attains good performance guarantee relative to the optimal water allocation. In particular, it always recovers at least half of the amount of the optimum social welfare. We benchmark the optimal pricing mechanism against supply, or flow, restrictions—which, as explained above, would require an upgrade of the physical infrastructure—and prove that, for identical customers, the ratio between the maximum social welfare attained by supply restrictions and pricing ranges between 1 and 2. We also show that there can be instances in which the pricing mechanism largely outperforms supply restrictions.
- We extend our analysis i) to more complex networks, thereby empirically demonstrating that the pricing mechanism, applied to a variety of network topologies including multiple branches and loops, results in a significant improvement of the performance when the system is under failure mode, and ii) to the case of concave utility functions traditionally studied in economics.

### C. Related Work

There are only a handful of works that studied the problem of pricing in urban water systems. In a meta-analysis of 615 estimates of price elasticity of residential water demand, [9] investigated the effect of water scarcity on price elasticity, showing that scarcity is an important driver for price elasticity. Recently, [10] developed dynamic pricing schemes spanning across different timescales. Their analysis, carried out for Greater London (United Kingdom), included sub-daily peak pricing (to reduce water demand during peak hours) as well as weekly pricing used during water scarcity conditions. The authors show that pricing mechanisms could provide a number of benefits for the network operator. The problem of pricing was also considered by [11], who focused on international river basins rather than urban water systems. Specifically, [11] studied how to encourage riparian countries in a river basin to share the water resources. Their approach builds on a compensation scheme used by the downstream countries to encourage the upstream ones to limit their use of the available resources. As we shall see later, our work does not need a bargaining process between multiple agents, but rather relies on the intervention of the system operator—which is tasked with the problem of determining a uniform price for all customers. A fundamental difference between the aforementioned studies and our work stands in the representation of the underlying physical processes: we account explicitly for the physical structure of the network, thereby considering the practical effect of the network's topology and head losses when designing the pricing mechanism.

This paper applies some of the fundamental concepts used in pricing data networks to water networks. The ideas of pricing flows of bits to maximize social welfare were first developed in the late 1990 s (see [12], [13], [14], [15]) and have led to a wide research activity that investigated how pricing affects resource allocation initially in wireline and lately in wireless networks (see [16]–[18]). It is beyond the scope of this paper to summarise all the existing research in pricing communication networks, and since, to our knowledge, this is the first attempt to draw the analogy between data and water networks, we restrict our references to few early papers on data network economics that relate to our work. Specifically, our research uses ideas from [14] regarding social welfare maximization and applies them to a new form of non-elastic utilities that are suitable for water networks. For data networks, a game theoretic analysis was first proposed in [19] and [20] in order to study the equilibrium allocations of bandwidth when network users are uncontrolled. In this paper, we also study the allocation of water resources under such uncontrolled user behaviour and analyze how decentralized pricing mechanisms improve efficiency compared to the uncontrolled outcome.

We must note that pricing in water networks is different from that in data networks because of the friction that makes pressure drop water flows through pipes. When sharing a water tank, upstream customers (close to the tank) experience less friction and hence lower pressure drop compared to downstream customers. Another advantage of upstream customers is that they can always choose to consume their maximum allowable water flow causing a pressure drop that may significantly reduce the consumption capability of downstream customers. Hence, upstream customers have a natural priority in being served by the network. This is not the case in data networks (e.g., [14], [16]–[20]), where information flows share on an equal basis bandwidth and packet loss in the case of congestion. The water network constraints (described in the next section) model fluid dynamics and the network traffic is no longer bits of friction-less information but water. In other words, water is a single indistinguishable commodity and a destination may receive zero supply if water pressure at its location drops significantly due to upstream 'congestion'. Mathematically, the non-linear aspect of the equality constraints that express pressure drop make the optimization problem non-convex even in the case of concave customer utility functions. This makes the analysis from data networks (where constraints are linear and the optimization problem is mostly convex) not any more applicable.

The reminder of this paper is organized as follows. In Section II, we introduce the model of the water network used in this study, along with the utility function characterizing the relation between water supply and customers' utility. In the same section, we also formulate the optimal water allocation problem. In Section III, we introduce the properties of the optimal water allocation and two additional benchmark allocation schemes. In Section IV, we characterize the selfish behaviour of customers by formulating a game whose solution yields the amount of water allocated to each customer. This game theoretical model is used to derive the optimal uniform pricing mechanism in Section V. In Section VI we compare the performance of the optimal pricing mechanism against uniform flow restrictions. The extension to complex water networks is provided in Section VII, while in Section VIII we validate our results for general concave utility functions. Conclusions and future research directions are outlined in Section IX.

## II. System Model and Problem Description

In this section, we set the ground for the problem of deriving pricing mechanisms. Specifically, we introduce a model of the water network in Section II-A, and describe the Utility function in Section II-B. We summarize in Table I the notation adopted within the study.

TABLE I
Notations Adopted in This Study and Their Definitions

| Notations | Definitions |
|---|---|
| $N$ | Number of customers |
| $\mathcal{I}$ | The set of all the $N$ customers |
| $H_0$ | Tank's head |
| $H_i$ | Head of node $i$ |
| $h$ | Minimum pressure at each node |
| $A_i$ | Friction coefficient of pipe $i$ |
| $\alpha$ | Friction parameter in the Hazen-Williams formula |
| $a$ | Fraction of pressure drop at $H_0$ in the case of failure |
| $A$ | Uniform friction coefficient of all pipes |
| $f_i$ | Instantaneous rate of water consumed by customer $i$ |
| $U$ | A customer's utility function |
| $\theta$ | Slope of step utility function |
| $b$ | Minimum instant. demand in step utility function |
| $n_b$ | Maximum number of customers with demand $b$ |
| $d$ | Maximum instant. demand in step utility function |
| $n_d$ | Maximum number of customers with demand $d$ |
| $SW_s$ | Social welfare without network control |
| $\sigma$ | Price of Anarchy |
| $v$ | Vector consisting of all system parameters |
| $SW_p$ | Social welfare under pricing |
| $p$ | Price charged to all customers |
| $p^*$ | Optimal price |
| $SW_p^*$ | Maximum social welfare under pricing |
| $SW_c$ | Social welfare under flow restriction |
| $SW_c^*$ | Maximum social welfare under flow restriction |
| $c$ | Uniform flow restriction to customers |
| $c^*$ | Optimal flow restriction |
| $SW^*$ | Optimal social welfare |
| $f_i^*$ | Optimal instant. water allocation of customer $i$ |

### A. Water Network Model

Our model and analysis are carried out for a typical single-branch water distribution network, which is a case of a drinking water pipeline. This simple, but fundamental, network has been widely adopted in the domain of water distribution system modelling (e.g., [21]–[24]). We will extend the single-branch network to more general network topologies (i.e., water networks with multiple branches, looped water networks) later in Section VII. As illustrated in Figure 1, the single-branch network connects a water tank (i.e., Node 0) to $N$ different customers belonging to the set $\mathcal{I} = \{1, 2, \ldots, N\}$. To feed the customers by gravity, the tank usually has a higher elevation. The network consists of $N+1$ nodes and $N$ pipes. Node 1 to $N$ represent junctions, which distribute part of the water to the customers. Each junction node is connected to a customer at a location.[1] The pressure at the $i$-th node, including the tank (Node 0), is measured with the 'hydraulic head' $H_i$—the value of water pressure above a vertical datum [1]. Only the tank's hydraulic head $H_0$ is fixed; as we shall see next, the head at any other node depends on the amount of water flowing through the pipes as well as the the physical properties of the pipes. The flow, i.e., the instantaneous rate of water that each customer $i$ takes out of the network, is denoted as $f_i$.

[1]Here a customer can be an industrial user or a group of residential users at the same location.

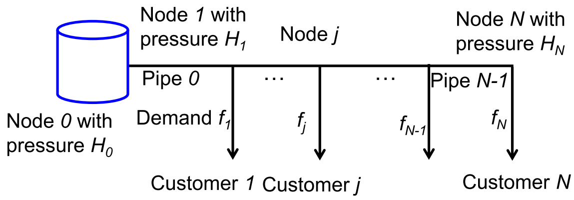


Fig. 1. Topology of a typical single-branch water distribution network.

When water flows through the pipes, it incurs a head loss due to the friction with the pipe wall. To model this loss, we use the Hazen-Williams formula, an empirical relationship relating the head loss due to friction with the water flow and physical properties of the pipes [1]. Specifically, the Hazen-Williams formula states that the pressure difference $H_i - H_j$ between any nodes $i$ and $j$ is equal to $A_{ij}\, f_{ij}^{\alpha}$, where $A_{ij}$ is the friction coefficient of the pipe connecting the two nodes and $f_{ij}$ the water flow between $i$ and $j$. The parameter $\alpha$ appearing in the Hazen-Williams formula is equal to 2.[2] Regarding the distance between customer nodes, we note that this is implicitly contained in the value of the coefficient $A_{ij}$ that models the pipe between the two customers. The longer the distance, the larger $A_{ij}$ will be.

The other key physical process occurring in the network is the conservation of mass at each node [1]. As shown in Figure 1, the $i$-th pipe supplies water to all the customers (from customers $i+1$ to $N$) after customer $i$. In the $i$-th pipe, the flow is thus equal to $f_{i+1} + \ldots + f_N$. Combining the Hazen-Williams formula with flow conservation, we obtain the following expression for each pipe in the network:

$$\begin{gathered} H_0 - H_1 = A_0(f_1 + \ldots + f_N)^2, \\ H_1 - H_2 = A_1(f_2 + \ldots + f_N)^2, \\ \ldots \\ H_{N-1} - H_N = A_{N-1} f_N^2. \end{gathered} \tag{1}$$

By summing all the equations in (1), we can calculate the head loss between Nodes 0 and $N$ as follows:

$$H_0 - H_N = \sum_{j=0}^{N-1} A_j \left( \sum_{i=j+1}^{N} f_i \right)^2. \tag{2}$$

To ensure that there is enough pressure within the distribution network to supply all customers, we consider that the pressure $H_N$ at the last node $N$ is larger than a minimum value $h \geq 0$ [1]. With this additional constraint, Equation (2) becomes

$$H_0 - \sum_{j=0}^{N-1} A_j \left( \sum_{i=j+1}^{N} f_i \right)^2 \geq h. \tag{3}$$

[2]The choice of $\alpha$ equal exactly to 2 may reduce the accuracy with which the head loss is estimated, but allows us to work with a more tractable quadratic expression. Though more involved, our main analysis can be extended to other values of $\alpha$.

Suppose that each customer is allocated her maximum allowed flow $d$ and that the friction coefficient of each pipe is identical (i.e., $A_0 = A_1 = \ldots = A_{N-1} = A$), we can calculate the minimum hydraulic head $H_0$ (at the tank) needed to guarantee that $H_N$ is equal to $h$; that is

$$H_0 = \frac{1}{6} N(N+1)(2N+1) A d^2 + h, \tag{4}$$

which is nonlinearly increasing in the number of customers $N$ and their assigned flow $d$ due to head losses in the network.

Under these operating conditions, the customers' maximum demand $d$ is met fully. However, the head $H_0$ may drop below this critical threshold in response to different events, such as prolonged droughts and cyber attacks, thereby causing supply failures. In this work, we focus on these instances of insufficient supply and develop mechanisms for allocating water to customers when not all customers can get the amount $d$. From a social planning perspective, this means determining the water allocation among all the customers that maximizes the total utility generated by water consumption, or 'social welfare' $SW$, subject to the head-loss inequality constraint described in (3). In other words, the network operator aims to solve the following maximization problem:

$$\max_{f_i \geq 0, i \in \mathcal{I}} SW(f_1, \ldots, f_N) = \sum_{i=1}^{N} U(f_i) \tag{5}$$

subject to (3), assuming all customers are identical and hence have the same utility functions, where $U(f_i)$ is the utility measured in \$ of the $i$-th customer when she obtains a flow equal to $f_i$. The optimal water allocation $(f_1^*, \ldots, f_N^*)$ results in the *optimal social welfare* $SW^* = \sum_{i=1}^{N} U(f_i^*)$. We next discuss what a reasonable structure is for utility functions that model water consumption.

## B. Utility Functions

We consider that each customer of the water network is characterized by a utility function that measures the rate at which the customer obtains value from the network when consuming water. Traditional economic modeling considers functions $U(x)$ that have diminishing returns to scale at all consumption levels $x$, i.e., are concave, and $U(x) > 0, x > 0$ with $U(0) = 0$. This implies that customers obtain positive value from the network even when their consumption is arbitrarily small. Although we include the analysis for such functions in Section VIII, we propose a non-concave utility function for water networks. Our 'step utility' function introduced next becomes positive only when a minimum rate of water consumption is possible and is zero for lower rates. This is in line with many practical considerations about how water is utilized by consumers—for example, electrical devices require a minimum water pressure (which translates to water flow) to operate. This is captured by the following utility function:

$$U(f) = \begin{cases} 0 & \text{if } 0 \leq f < b \\ 1 + \theta(f - b) & \text{if } b \leq f \leq d, \end{cases} \tag{6}$$

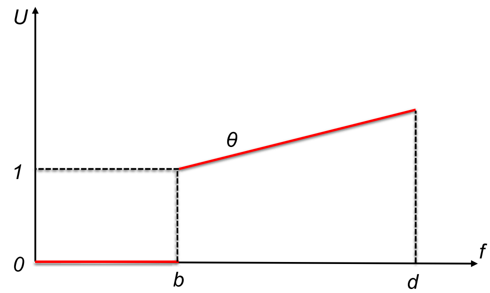


Fig. 2. Step utility function. $f$ represents the customer's flow, i.e., instantaneous rate of water supply (in $m^3$ per unit time), $U$ the resulting utility value, $b$ the minimum demand, $d$ the maximum demand, and $\theta$ the slope of the utility function per flow beyond $b$.

where $f$ is the instantaneous rate of water supply (in $m^3$ per unit time), and $b$, $\theta$ and $d$ are constants satisfying $b > 0$ and $0 \leq \theta < \frac{1}{b}$. This utility function, illustrated in Figure 2, captures three important features. First, it includes a step point at $f = b$, where $b$ expresses the inelasticity of demand and represents the minimum rate of water consumption that creates positive value, i.e, the minimum rate required by customers for their basic needs. Second, the utility does not increase for values of $f$ larger than $d$—that is, the utility expresses saturation effects and reaches a maximum value when the demand for flow $d$ is fully met. Third, the customer's utility increases linearly between $b$ and $d$. This form of a utility function is widely used in economics ([25], [26]), and is the simplest possible when customers are 'inelastic' in terms of some minimum level of consumption.

We normalized the utility function to provide one unit of utility when a customer obtains her minimum amount of water demand $b$. Since the utility is a function of the water demand, it represents the value of the water volume obtained by the customer per unit of time.

Finally, we assume that $\theta < \frac{1}{b}$ such that the utility at the three points $(0, U(0))$, $(b, U(b))$, and $(d, U(d))$ exhibits diminishing returns—a classic assumption in economics [27].

Recall that Equation (4) characterizes the hydraulic conditions needed to meet the customers' demand. When pressure is too low, i.e., $H_0 < Ab^2 + h$, no customer can get her minimum requirement $b$.

Here, we consider the interesting and non-trivial case where

$$Ab^2 + h \leq H_0 < \frac{1}{6}N(N+1)(2N+1)Ad^2 + h. \quad (7)$$

That is, there is not enough pressure to satisfy the maximum requirement $d$ of all the customers, but there is enough pressure to serve some customers by providing at least their minimum requirement $b$.

## III. Optimal and Extreme Water Allocations

In this section we characterize three important allocations to which we refer to in the rest of the paper. The first allocation is 'optimal': it maximizes social welfare as defined in (5) using the step utility functions in (6). The other two allocations are 'extreme': they allocated the maximum possible demand $d$ (minimum possible demand $b$) to as many customers as possible. The optimal allocation serves as a benchmark to compare the efficiency of different allocation mechanisms. The other allocations become relevant in the case of uncontrolled customer behaviour or under certain network controls analyzed in the paper.

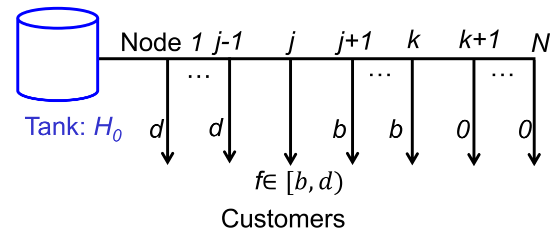


Fig. 3. Illustration of the optimal allocation. $f$ is the water allocation of customer $j$.

### A. Optimal Allocation

We derive a water allocation mechanism that solves Problem (5), thus maximizing the social welfare using the utility functions in (6). We call this the 'SW-Optimal allocation'. This benchmark provides the performance upper bound by ideally deploying hydraulic actuators that centrally control the amount of water consumed by each customer. In practice, however, it is technically unfeasible to deploy such hydraulic actuators and each user is uncontrolled to decide her consumption by herself, as detailed in Section IV.

To exemplify the mechanism, illustrated in Figure 3, suppose that under the optimum allocation the $j$-th customer is allocated an amount of water $f \in [b, d)$. Then, customer $1, 2, \ldots, j-1$ should use an amount of water equal to $d$, while customers $j+1, \ldots, k$ a volume equal to $b$. The customers downstream from $k$ do not receive any supply and $k$ is at most $N$. Such allocation is defined more formally in the following proposition.

*Proposition 3.1:* The SW-Optimal allocation for Problem (5) has the following structure: $f_1 = f_2 = \ldots = f_{j-1} = d, f_j \in [b, d)$, $f_{j+1} = \ldots = f_k = b$, $f_{k+1} = \ldots = f_N = 0$, where $i, j$ are integers such that $j \in [0, N-1]$ and $k \in [j+1, N]$.

We skip the formal proof but the rationale behind Proposition III.1 is as follows. If the $j$-th customer is allocated an amount of water $f_i \in [b, d)$, the upstream customers located closer to the tank should be allocated an amount of water equal to $d$. Indeed, supposing otherwise that a customer upstream from customer $j$ should obtain less than $d$ amount of water, we can find a better allocation. This is because if customer $j$ reduces her water consumption by a small value $\epsilon$, the upstream customer is able to increase her consumption by more than $\epsilon$, since providing an upstream customer with $\epsilon$ additional water causes less head loss compared to providing a downstream customer with $\epsilon$ additional water. Hence, the total water allocated to the customers increases, resulting in higher social welfare.

The allocation of $f_j < d$ and of $f_{j+1} = \cdots = f_k = b$ is justified by noticing that increasing $f_j$ further would result in customer $k$ getting less than $b$, hence losing a whole unit of utility.

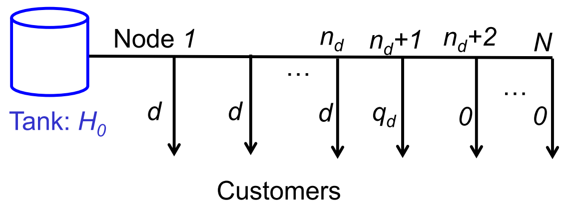


Fig. 4. Illustration of the Max-Allocation scheme.

## B. Two Extreme Allocations

We conclude this section by characterizing two allocations that will be used in the rest of the paper, and which are extreme in nature. The 'Max-Allocation' offers to the largest possible number of customers the maximum desired amount $d$. The 'Min-Allocation' offers to the largest possible number of customers the minimum required amount $b$ in order to get positive utility. It is clear that in both allocations, in order to maximize the number of benefiting customers, we will allocate water with priority to the upstream customers.

We start with the Max-Allocation. As illustrated in Figure 4, only the first $n_d$ customers receive a full supply. The customer located in node $n_d+1$ may receive a smaller volume, denoted as $q_d$, depending on the value of the hydraulic head $H_0$. In other words, we have $\forall i \in [1, n_d]$, $f_i = d$, $f_{n_d+1} = q_d \in (0, d)$, and $\forall i \in [n_d+2, N]$, $f_i = 0$.

To calculate the social welfare corresponding to the Max-Allocation, we must first calculate $n_d$ and $q_d$. Using Equation (3), we know that to meet the demand of the first $k \in [1, N]$ customers (i.e., $\forall i \in [1, k], f_i = d$ and $\forall i \in [k+1, N], f_i = 0$), the head $H_0$ should be at least

$$H_0 = \frac{1}{6}k(k+1)(2k+1)d^2 + h, \tag{8}$$

which is a cubic function of $k$. Thus, given a value of $H_0$, we can solve the cubic equation (8) (with respect to $k$) to know how many customers get a supply equal to $d$. We refer the reader to Appendix A for the detailed derivation of $n_d$. Using Equation (3), and allowing $\forall i \in [n_d+2, N], f_i = 0$, we then derive a closed-form expression for $q_d$, that is:

$$q_d = -\frac{1}{2}dn_d + \frac{\sqrt{-\frac{1}{3}d^2 n_d(n_d+1)^2(n_d+2) + \frac{4(n_d+1)(H_0-h)}{A}}}{2(n_d+1)}. \tag{9}$$

Having derived $n_d$ and $q_d$, we finally calculate the social welfare $SW_d$, which is defined next.

*Definition 3.2:* At Max-Allocation, each customer's water consumption is $\forall i \in [1, n_d]$, $f_i = d$, $f_{n_d+1} = q_d$ given in (9), and $\forall i \in [n_d+2, N]$, $f_i = 0$. The resulting social welfare is:

$$SW_d = \begin{cases} n_d(1+\theta(d-b)) & \text{if } q_d < b \\ n_d(1+\theta(d-b)) + (1+\theta(q_d-b)) & \text{if } q_d \geq b. \end{cases} \tag{10}$$

Note that the expression used to calculate $SW_d$ depends on the relationship between $q_d$ and $b$. This is because the utility functions vary with both $b$ and $d$. It is interesting to note that increasing the maximum water demand $d$ may not increase total social welfare. The reason is that as $d$ increases, while $q_d > b$, the first $n_d$ customers get more flow at the expense of customer $n_d+1$, but this increases the total social welfare: reducing customer $n_d+1$ by $\epsilon$ allows more than $\epsilon$ increase of flows for upstream customers closer to the tank. But when demand $d$ is at the value which makes customer $n_d+1$ drops to $b$, any subsequent increase of $d$ will induce the demand of the above customer $n_d+1$ to become less than $b$. Hence there will be a sudden loss of a whole unit of utility from the social welfare function, leading to a step decrease of the social welfare.

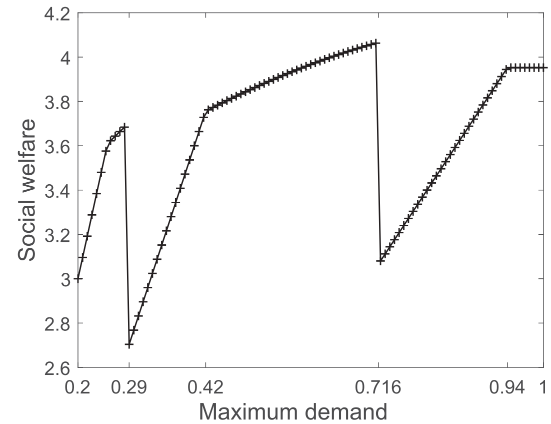


Fig. 5. Social welfare $SW_d$ as a function of the maximum demand $d$. The unit of $d$ is $\mathrm{m}^3/s$.

To illustrate the above arguments, let us consider an example where $N = 3$, $H_0 = 118$ m (i.e., meter), $h = 30$ m, $A = 100$, $\theta = 4$, $b = 0.2$ m$^3$/s. In Figure 5 we observe how $SW_d$ in (10) changes as a function of $d$. Specifically, at $d = 0.29$ m$^3$/s, the first $n_d = 2$ customers get flow of $d$, and customer $n_d + 1 = 3$ gets flow of $b$. After the demand of $d$, we see a step decrease of the social welfare. At $d = 0.716$ m$^3$/s, the first $n_d = 1$ customer gets flow of $d$, and customer $n_d + 1 = 2$ gets flow of $b$. After the demand of $d$, we again see a step decrease of the social welfare.

The second extreme allocation suggests to allocate customers a supply equal to $b$—the minimum amount that makes the utility non-zero. Hereafter, we refer to this allocation as the *Min-Allocation*. We can think of it as an 'altruistic' allocation since customers (that have enough pressure to get more than $b$) do not exercise that possibility. The number of customers $n_b$ receiving a supply equal to $b$ depends on the head $H_0$ of the tank. If $H_0$ is sufficiently large (i.e., $H_0 \geq \frac{1}{6}N(N+1)(2N+1)Ab^2 + h$), all customers get a supply equal to $b$, and so $n_b = N$. If $H_0 < \frac{1}{6}N(N+1)(2N+1)Ab^2 + h$, the first $n_b < N$ customers are supplied a volume equal to $b$ (see the illustration in Figure 6). Note that the tank may have some water left to supply the $(n_b+1)$-th customer, with an amount $q_b \in [0, b)$ but this customer would have a utility equal to 0.

To calculate the social welfare, we must first calculate $n_b$. For this purpose, we adopt an approach similar to the one used

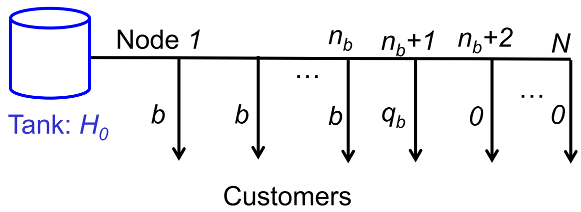


Fig. 6. Illustration of the Min-Allocation scheme.

**Algorithm 1:** Allocation Algorithm For Game $G$.

(1) For customer 1: We find the largest value of $y \leq x_1$ such that $H_0 - A_0(y)^2 \geq h$. Such value always exists. We allocate $f_1 = y$.

(2) For customer $j = 2, \ldots, N$: We assume that the allocations $f_1, \ldots, f_{j-1}$ for customers $1, \ldots, j-1$ have been computed, and we solve for the largest value of $0 \leq y \leq x_j$ such that $H_0 - \sum_{k=0}^{j-1} A_k(\sum_{i=k+1}^{j-1} f_i + y)^2 \geq h$. We allocate $f_j = y$.

for $d$ (please refer to Appendix A for further details). This finally leads to the following definition.

*Definition 3.3:* In the Min-Allocation, each customer's water supply is as follows.

- If $H_0 \geq \frac{1}{6}N(N+1)(2N+1)Ab^2 + h$, then $\forall i \in [1, N]$, $f_i = b$;
- If $H_0 < \frac{1}{6}N(N+1)(2N+1)Ab^2 + h$, then $\forall i \in [1, n_b]$, $f_i = b$, $f_{n_{b+1}} = q_b \in [0, b)$, and for $i > n_b + 1$, $f_i = 0$.

The resulting social welfare is $SW_b = n_b$.

## IV. Game Theoretical Model and Analysis

So far we have been informal about uncontrolled customers. To make the network setup more precise, we use the following formulation of the game $G$,[3] where players correspond to customers that make decisions on their *desired* level of water consumption (i.e., their demand) assuming the network operator charges them for their *actual* consumption. For completeness, we define the general case of a mechanism where the network operator may charge each customer differently. Specifically, we assume that the network operator makes known to each customer at node $j$ a personalized payment function $g_j(f_j) \geq 0$ that defines a monetary transfer from the customer to the network as a function of her water consumption rate $f_j \geq 0$. To make utility and cost of consumption comparable, payments are defined in the same units of value as the utility functions.

We present the optimal uniform pricing mechanism and the corresponding pricing function in Section V.

Given our specific form of utility functions, to make participation in this game $G$ sensible, we must make sure that we protect any customer $j$ from being charged $g_j(f_j) > 0$ when, as a result of the total allocation, the network allocates to her a supply lower than $b$. The customer is not in control of the supply she will finally obtain, which could drop below $b$, and such a small supply has no economic value since $U_j(f_j) = 0$ when $f_j < b$. We can easily correct this technical issue by assuming that $g_j(f_j) = 0$ when $f_j < b$.

In game $G$, each customer $j$'s strategy is to declare her demand $x_j$. A simple corollary of the previous discussion is that the results of the analysis of the game will not change if we restrict $x_j \in \{0, [b, d]\}$, since no customer gains by declaring $0 < x_j < b$. The interpretation of $x_j$ is that she will not be allowed to consume any $f_j > x_j$, even if this is possible due to enough water pressure at her location (or that she will be highly penalized if she consumes more). Since, due to low pressure, the system may not be able to meet fully the demand of all customers, what a customer will end up consuming may be less than her demand. The impact of the strategic customer choices $x = (x_1, \ldots, x_N)$ in game $G$ is reflected to the payoffs of the players through the resulting allocation $f(x)$.

We develop an algorithm to compute $f(x) = (f_1(x), \ldots, f_N(x))$ given demand $x = (x_1, \ldots, x_N)$, subject to the laws of physics in (3) and the constraint that the pressure at any serving node does not drop below $h$ in (7). The idea is to try to meet as close as possible the demand of each customer starting from customer 1 and moving downstream, while allocating $f_j = 0$ if the node requested $x_j = 0$ or the pressure at node $j$ is below $h$. This is summarized in Algorithm 1. We remind the reader that possible decisions for each customer $j$ are $x_j \in \{0, [b, d]\}$.

After Algorithm 1 terminates, in the resulting allocation $f(x)$ we either offer $f_j(x) = x_j$, $j = 1, \ldots, N$, i.e., we meet the demand of all the customers, or we reach the first customer $j$, which, due to the low water pressure, is assigned $0 \leq f_j(x) < x_j$, i.e., a supply strictly smaller than her demand. In such case it is easy to check that all downstream customers $j + 1, \ldots, N$ are assigned zero supply since the pressure at node $j$ has dropped to the minimum allowable level $h$.

The allocation $f(x)$ serves as the basis to determine the payoffs of the customers in the game as functions of their declarations $x$. Given $x$, each customer obtains a payoff

$$\pi_j(x) = U(f_j(x)) - g_j(f_j(x)) \,. \tag{11}$$

For the above game $G$, a Nash equilibrium $(x_1^e, \ldots, x_N^e)$ corresponds to the case where for any customer $j$, deviating unilaterally from her declaration $x_j$ will not improve (strictly) her payoff. Formally, $(x_1^e, \ldots, x_N^e)$ is a Nash equilibrium if $x_j^e \in S_j$ for each $j$, where $S_j = \arg\max_{x_j \in \{0,[b,d]\}} \pi_j(x_1^e, \ldots, x_j, \ldots, x_N^e)$ is the set of values for $x_j$ that maximize the payoff of customer $j$ assuming the rest of the customers choose $(x_1^e, \ldots, x_{j-1}^e, x_{j+1}^e, \ldots, x_N^e)$.

There may be multiple Nash equilibria since customers are charged based on their resulting consumptions $f_j(x)$ and not on their declarations $x_j$. In particular, given a Nash equilibrium $x^e = (x_1^e, \ldots, x_N^e)$, if $f_j(x^e) < x_j^e$ for some customer $j$, then, for any other declaration $x_j'$ of customer $j$ for which $f_j(x^e) < x_j'$, it is true that $x^{e'} = (x_1^e, \ldots, x_j', \ldots, x_N^e)$ is also a Nash equilibrium. The reason is that $f(x^e) = f(x^{e'})$, and hence choosing $x_j'$ instead of $x_j^e$, has no impact on the system by not affecting any of the resulting flows of the participants.

[3]Game theory is the study of mathematical models of strategic interaction among rational decision-makers. For a brief definition of the terminology please refer to https://www.investopedia.com/articles/financial-theory/08/game-theory-basics.asp.

Let's consider the special case where payments are zero, i.e., $g_j(.) = 0$ for all $j$. Then strategy $x_j = d$ dominates (non-strictly) all other strategies for each customer $j$. It is also easy to check that every customer choosing $d$ is the only sub-game perfect Nash equilibrium. This equilibrium $x_s^e = (d, \ldots, d)$ where all customers try to get the maximum possible is referred to as the 'uncontrolled equilibrium'. The next proposition follows directly from the definition of the equilibrium.

*Proposition 4.1:* The uncontrolled Nash equilibrium $x_s^e$ leads to an allocation $f(x_s^e)$ equal to the Max-Allocation in Definition III.2.

To compare the uncontrolled outcome $f(x_e^s)$ against the SW-optimal allocation from Proposition III.1, we use the Price-of-Anarchy (PoA) $\sigma$ to denote the minimum possible percentage of social welfare compared to the optimum that can be achieved due to customers' uncontrolled behaviors. Let $v = (H_0, h, A, N, b, d, \theta) > 0$ be the vector of all the parameters of the water distribution network, and $SW_s(v)$, $SW^*(v)$ be the social welfare of the allocation $f(x_e^s)$ and of the SW-Optimal allocation respectively, in the system parametrized by $v$. We define the PoA $\sigma$ as

$$\sigma = \min_{v>0} \frac{SW_s(v)}{SW^*(v)}. \tag{12}$$

This corresponds to the worst possible ratio of social welfare attained under full decentralization versus perfect network control, when the system parameters are positive. The next proposition suggests that this ratio can be arbitrarily small.

*Proposition 4.2:* At the uncontrolled Nash equilibrium, the resulting PoA is $\sigma = 0$, telling that the customers' uncontrolled demand strategies result in arbitrarily poor social welfare.

Due to the form of our utility functions, the same uncontrolled equilibrium may be achieved even when the payment $g(.) > 0$, if this is not large enough to affect decisions. In particular, we expect that if the price charged per unit of flow is low enough, then the dominant strategy remains equal to $d$. This is typically the case for the 'base contract' price $p_0$ used by the network to charge its customers before the failure. Under normal conditions, customers are willing to consume up to the maximum possible.

The underlying intuition is as follows. Consider the situation in which the slope of the step utility function $\theta$ in (II-B) is very small (e.g., $\theta \to 0$), and $d$ is very large (i.e., $d \geq \sqrt{\frac{H_0-h}{A}}$). We assume $H_0 = \frac{1}{6}n_b(n_b+1)(2n_b+1)b^2 + h$, so that the water distribution network is just able to provide the first $n_b$ customers with $b$ amount of water. At the social optimum, the planner allocates a supply equal to $b$ to the first $n_b$ customers, thereby achieving a social welfare of $n_b$. Given that $d$ is very large, the water distribution network is not able to meet the demand of the first customer. In such case, the first customer uncontrollably consumes all available water in the game $G$, and the resulting social welfare is 1. Note that $n_b$ can be arbitrarily large, as long as $H_0$ is extremely large. Thus, the ratio between the uncontrolled outcome and the optimal social welfare (i.e., $\frac{1}{n_b}$) can approach 0. This motivates the network operator to use economic controls in the form of non-zero prices in order to improve the PoA.

## V. Economic Control Mechanism

It is clear from our analysis in Section IV, that without any network intervention, under conditions of water scarcity, the total value obtained by customers in the uncontrolled equilibrium can be arbitrarily worse than the value obtained when the network is able to exercise full control over the water allocations to its customers. In this section we consider the practical situation where the network is not allowed to exercise this extreme form of control, i.e., force them to accept that the network operator should decide on their behalf and choose the optimal 'personalized' allocations from Proposition III.1. Instead, it is well accepted that the network operator may use economic incentives, e.g., choose the form of the charging functions $g_j(f_j)$ introduced in Section IV.

To be more realistic, we assume that the network is not allowed to discriminate among its customers by charging them differently depending on their location. Indeed, a water utility is expected to charge customers that consume the same amount equally. An implication of this assumption is that the charge $g_j(f_j)$ of any customer $j$ must be equal to $pf_j$, where $p$ is the (uniform) price per unit of supply consumed.[4] Uniform pricing does not differentiate among customers and is considered fair.

We begin by defining the optimal price design problem in Section V-A. Then we derive the optimal pricing policy whose economic performance is theoretically compared against the optimal social welfare in Section V-B. Finally, we conduct a numerical experiment to further analyze how the policy performance varies under different scenarios of water supply pressure drop in Section V-C.

### A. Optimal Price Design Problem

To study the interactions between the network operator and $N$ customers, we use a two-stage Stackelberg game [28]. As the Stackelberg leader, the network operator first decides in Stage I the price $p$ with the goal of maximizing the overall social welfare. In Stage II, each customer $j$ chooses her desired water demand $x_j$ depending on the price $p$, and interacts with each other customer to determine flow allocation takes place as discussed in game $G$ in Section IV.

It is clear that the price $p$, being a parameter of the mechanism, affects the decisions of the players and hence the resulting Nash equilibrium. Our goal is to find the price that reduces the inefficiency due to customer uncontrolled behavior: at the resulting equilibrium, more customers will have access to water since due to pricing, high-consuming customers may be willing to lower their demand and leave more water resource to downstream customers. Determining the optimal value of $p$ is a non-trivial problem: if $p$ is too large, it may unnecessarily prevent all customers from consuming water, with a negative

[4]We remind the reader of our assumption in game $G$ that if a customer obtains a supply $f_j < b$, then $g_j(f_j) = 0$.

impact on the social welfare. If $p$ is too small, it may encourage overconsumption, with only few customers closer to the tank profiting the most. We denote by $SW_p(p)$ the social welfare achieved at the Nash equilibrium of the Game $G$ under uniform pricing with price $p$. The optimal pricing problem is

$$\max_{p\geq 0} SW_p(p)\,. \tag{13}$$

To determine $SW_p(p)$ in (13) we need to find the flow allocations at the Nash equilibrium of the customers' decisions for price $p$. If customer $j$ chooses to demand $x_j$ and as a result she consumes $f_j$ (determined by Algorithm 1), her payoff in (11) can be rewritten as:

$$\pi_j(f_j) = U(f_j) - pf_jI_1(f_j) \tag{14}$$

$$= \begin{cases} 0 & \text{if } 0 \leq f_j < b \\ 1+\theta(f_j-b)-pf_j & \text{if } b \leq f_j \leq d, \end{cases} \tag{15}$$

where we abused our notation by using $\pi_j(f_j)$ instead of $\pi_j(f_j(x))$, and $I_1(f_j)$ is the indicator function that takes the value one if $b \leq f_j \leq d$,

$$I_1(f_j) = \begin{cases} 1 & \text{if } f_j \in [b,d] \\ 0 & \text{if } f_j < b. \end{cases}$$

Based on the above payoff functions we can prove that players have a unique *dominant strategy* that depends only on the price $p$. This greatly simplifies the solution of the game, since, given $p$, it is reasonable to assume that customers will play their dominant strategies. We remind the reader that the definition of the step utility function in (6) requires $\theta < \frac{1}{b}$.

*Proposition 5.1:* The following is the unique dominant strategy for any customer $j \in \{1,\ldots,N\}$:

$$\text{Strategy of customer } j\text{:}\begin{cases} \text{if } p < \theta, & x_j = d, \\ \text{if } \theta \leq p \leq \frac{1}{b}, & x_j = b, \\ \text{if } p > \frac{1}{b}, & x_j = 0. \end{cases}$$

The reason for this strategy is that, if $p$ is large, i.e., $p > \frac{1}{b}$, then the payoff $\pi_j(f_j) = U(f_j) - pf_jI_1(f_j)$ is non positive for any feasible flow $f_j \in (0,d]$ and hence the customer prefers to abstain from consumption, choosing $x_j = 0$. If $p$ is small, i.e., $p < \theta$, the customer benefits by consuming more flow, and hence demands the maximum possible value $d$. If $\theta < p \leq \frac{1}{b}$, then $\pi_j(f_j)$ is maximized at $f_j = b$, taking the payoff value $1 - pb > 0$. Since the payoff becomes strictly smaller for demands above and below $b$, choosing $x_j = b$ is a dominant strategy for customer $j$. Choosing $x_j < b$ will result in zero payoff, and choosing $x_j > b$ will strictly decrease the payoff except if (by chance) $f_j = b$[5].

Assuming that customers will choose their dominant strategy from Proposition V.1, we can find the resulting flow allocation and the corresponding $SW_p(p)$. We analyze the three different cases as $p$ increases.

If $0 \leq p < \theta$, each customer's water demand is $d$. The resulting allocation is the same attained with the Max-Allocation (see Definition III.2) and the network has higher priority to supply upstream customers. Thus, the social welfare is

$$SW_p(p \in [0,\theta)) = SW_d, \tag{16}$$

where $SW_d$ is given in (10). This social welfare is also equal to the uncontrolled outcome $SW_s$ given in Proposition IV.1. This indicates that the maximum social welfare $SW_p^*$ under the optimal pricing mechanism must be no less than $SW_s$, since the uncontrolled outcome is a special case achieved by the pricing mechanism using any price $0 \leq p \leq \theta$.

If $\theta \leq p \leq \frac{1}{b}$, each customer's water demand is $b$. The resulting allocation is the same attained with Min-Allocation (see Definition III.3). In such case, the social welfare is

$$SW_p\left(p \in [\theta, \frac{1}{b}]\right) = n_b, \tag{17}$$

where $n_b$ is the number of customers receiving a supply equal to $b$.

If $p > \frac{1}{b}$, each customer's water demand is 0 and $f_i = 0$ $\forall i \in \mathcal{I}$. We thus have:

$$SW_p\left(p > \frac{1}{b}\right) = 0. \tag{18}$$

To summarize, we have the three following cases.

*Proposition 5.2:* For price $p$, the social welfare $SW_p(p)$ at the dominant strategy equilibrium is

$$SW_p(p) = \begin{cases} SW_d & \text{if } p \in [0,\theta) \\ n_b & \text{if } p \in [\theta, \frac{1}{b}] \\ 0 & \text{if } p > \frac{1}{b}. \end{cases} \tag{19}$$

In the next subsection we determine the price that maximizes social welfare.

### B. Determining the Optimal Pricing Policy

Having determined the expression of the social welfare function using Proposition V.2, we now determine the solution of the social welfare maximization problem in (13). By observing (19), we note that for any price $p$ chosen by the network, in the dominant strategy equilibrium of the game there are two possible non-trivial allocations (i.e., besides the zero flow allocation when $p > \frac{1}{b}$) that may take place: the Max-Allocation or the Min-Allocation. In these two allocations the values of the allocations to any customer are either $d$ or $b$, respectively, except for the last customer that may obtain a strictly smaller amount denoted by $q_b < b$ and $q_d < d$, respectively. In the case of Min-Allocation, $q_b < b$ is of no value to the customer and can be omitted since it does not add to the social welfare of the system. But in the case of Max-Allocation, if this last customer obtains flow $b \leq q_d < d$, she

[5]Note that the dominant strategy in Proposition V.1 remains the same in the case of $\theta_j, b_j, d_j$ taking different values among customers and being private information for each customer $j$ (i.e., being known only to her). This suggests that the pricing results of the paper hold more generally in the context of partial information.

is still obtaining positive utility, but less than the rest of the upstream customers. We refer to this customer as the 'marginal customer' and her utility must be added to the total amount of social welfare.

We next characterize the price $p^*$ that maximizes the social welfare of the system, where $q_d < d$ is related to the Max-Allocation, see Definition III.2, and $I_2(q_d)$ is the indicator function that takes the value one if $q_d \geq b$.

*Proposition 5.3:*

A price $p^*$ that maximizes the social welfare satisfies

- If $\theta < \frac{n_b - n_d - I_2(q_d)}{n_d(d-b)+\max(q_d-b,0)}$, then $\theta < p^* < \frac{1}{b}$ and $SW_p^* = n_b$.
- Otherwise, $p^* < \theta$ and $SW_p^* = n_d(1+\theta(d-b)) + (I_2(q_d) + \theta\max(q_d - b, 0))$.

Observe that when $\theta$ is small, under the optimal price all customers are incentivized to consume $b$. Otherwise, they are incentivized to consume as much as possible.

The intuition behind Proposition V.3 is as follows. When $\theta$ is small, there is little loss of the social welfare if a customer consumes less than the maximum demand $d$. In particular, in the extreme case of $\theta \to 0$, the best possible allocation for the society would be to serve as many customers as possible by giving them their minimum amount $b$. This is achieved by choosing a price $\theta < p^* < \frac{1}{b}$ so that all customers, including the upstream ones with greater water control, demand at most $b$. Any other price smaller than $\theta$ would induce upstream customers to ask for $d$, taking valuable supply from the downstream customers, even though they don't gain much value by doing so. In the opposite case, when $\theta$ is large, offering more water to the upstream customers is the sensible thing to do. In the extreme case, where there is substantial pressure drop between customers due to high friction coefficients, e.g., when $A$ in (3) is large, it may be preferable to offer as much flow as possible to customer 1.

Using the optimal price in Proposition V.3, we can now present the guaranteed performance of the optimal pricing policy in terms of the PoA $\sigma$.

*Theorem 5.4:* $\sigma = \frac{1}{2}$, i.e., the optimal pricing policy can retrieve at least half of the optimum social welfare.

The intuition behind this theorem may be illustrated as follows (the proof is available in Appendix B). According to Proposition III.1, the optimal water allocation desired by the network operator provides social welfare as the sum of two components, where the first component is the total utility of customers in set $\{1,\ldots,j\}$, and the second component is total utility of customers in set $\{j+1,\ldots,k\}$. Hence $SW^* \leq jd + (k-j)b$. From the definition of the optimal price, the optimum price allocation is the one that achieves the maximum social welfare between the Max-Allocation and the Min-Allocation, i.e., $SW_p^* = \max\{SW_d, SW_b\}$. Since $jd \leq SW_d$ and $(k-j)b \leq SW_b$, it follows that $SW^* \leq SW_d + SW_b \leq 2\max\{SW_d, SW_b\} = 2SW_p^*$.

Comparing Theorem V.4 and Lemma IV.2—which states that the minimum ratio between the uncontrolled outcome and the optimal social welfare is zero— we see that the optimal pricing policy can greatly improve the performance of the existing uncontrolled outcome. Note that the PoA looks at the worst performance achieved by the optimal price, and the optimal price's average performance is actually much better than 1/2 as shown later in simulations.

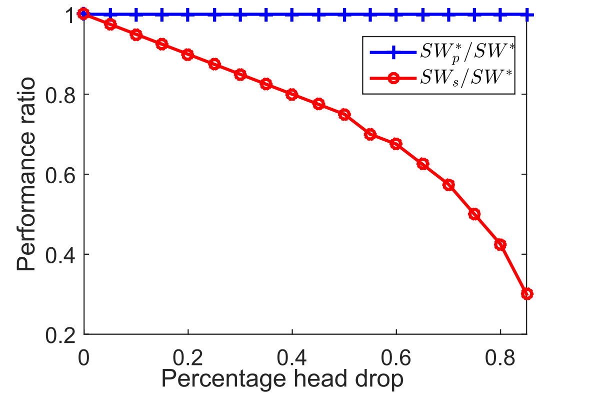


Fig. 7. Ratios $\frac{SW_s}{SW^*}$ and $\frac{SW_p^*}{SW^*}$, where $SW_s$, $SW_p^*$, $SW^*$ are the social welfare of the uncontrolled equilibrium, the maximum social welfare obtained through pricing, and the optimal social welfare respectively, as functions of the tank's head percentage drop $a$, for small $\theta = 0.001$. We assume that for $a = 0$ all the customers can get their maximum demand $d$.

*Remark:* Theorem V.4 is based on the assumption that the water distribution network has homogeneous pipes and single branch. In practice, water distribution networks may have heterogeneous pipes and multiple branches. In Section VII, we empirically demonstrate the good performance of our pricing mechanism for a broader class of water distribution networks.

## C. Numerical Results

To illustrate how the performance of the optimal pricing policy is affected by the failure in supply (drop of pressure), we setup a numerical experiment in which we vary the value of the hydraulic head $H_0$ in node 0. Specifically, we set $N = 40$, $A = 100$, $h = 30$ m, $b = 0.001$ m$^3$/s, $d = 0.01$ m$^3$/s as [1]. Under normal operating conditions, a hydraulic head $H_0 = 251.4$ m would supply a demand $d$ to all the customers. In our experiments, we consider the value $H_0 = (1-a)251.4$ m, where $0 \leq a < 1$ is the percentage drop of $H_0$ and reflects the severity of water resource scarcity (should not be confused with the parameter $\alpha$ in the Hazen-Williams formula). When $a = 0$ all the customers can get their maximum demand $d$. Figures 7 and 8 show how $\frac{SW_s}{SW^*}$ and $\frac{SW_p^*}{SW^*}$ change with $a$, for two different values of the slope $\theta$ of the utility function. We make the following observations.

*Observation 5.5:* From Figures 7 and 8 we note that both for small and large $\theta$s,

1) $\frac{SW_s}{SW^*}$ decreases with $a$, i.e., as the hydraulic pressure $H_0$ drops, the uncontrolled equilibrium becomes more inefficient.
2) $SW_p^*$ in general outperforms $SW_s$, and this becomes very significant for large pressure drops.

The reason behind these observations is that, as $a$ increases and the head $H_0$ drops, the uncontrolled allocation (at the equilibrium of the game $G$) and the socially optimal allocation become very different. The uncontrolled allocation keeps assigning to few upstream customer the maximum demand $d$,

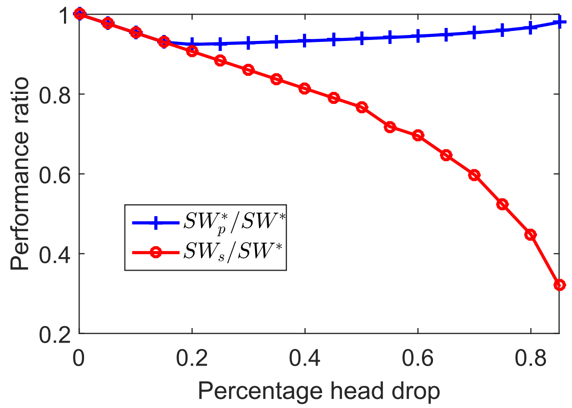


Fig. 8. Same as Figure 7, but with $\theta = 10$. The equilibrium of the system under pricing is more efficient than the equilibrium under no control over customers, and this becomes more prominent for large pressure drops.

whereas the socially optimal allocation assigns to more customers the minimum demand requirement $b$, so that for low enough values of $H_0$ no customer gets $d$. This causes a large discrepancy between the social welfare obtained under the two allocations. Using pricing, we can choose a price so that all customers choose $b$, and hence approximate well the socially optimal allocation for low pressures $H_0$, where few customers are assigned $d$.

When $\theta$ is very small in Figure 7, the extra utility from consuming $d$ instead of $b$ is negligible, and hence the socially optimal allocation prefers to assign $b$ to all customers, which coincides with the allocation of the optimal pricing policy which chooses a high price to deter customers from selecting $d$. This is the case where pricing does uniformly better than letting customers make choices under no control. When $\theta$ is large (see Figure 8), for low pressure drops, the optimal policy assigns to few upstream customers the maximum amount $b$. In this case, using a high price to restrict consumption to $b$ for all customers is not the best option and the optimal pricing policy chooses a low price to lead to the same outcome as the uncontrolled allocation policy. For larger pressure drops, the optimal pricing switches to a high price and approximates better the optimal allocation, which offers to more customers the minimum amount $b$.

## VI. Comparison With Mechanical Methods of Restricting Supply

In this section we compare our incentive-based pricing mechanism with traditional methods for restricting water supply to customers (e.g., using valves), even though such methods may be more costly and difficult to implement in large scale. Our goal is to analyze the conditions under which economic mechanisms may perform better in term of social welfare.

### A. *Efficiency of Uniform Flow Restriction*

The simplest 'engineering' method for controlling water supply to customers is for the network operator to deploy hydraulic actuators (e.g., valves) in each customer's premise by which it can restrict the maximum flow $c_j \leq d$ that is allowed to customer $j$. Since the network is expected to fairly treat customers in a non-discriminatory way, we assume, as in the case of pricing, that $c_j = c$ is chosen to be the same for all customers. Similar to Section IV, we next describe the variant of the game $G$ that models the strategic behavior of the customers under such flow restrictions.

Given a choice of $c \leq d$ by the network operator, each customer $j$ chooses her strategy $x_j \leq c$, i.e., her maximum demand. We assume that in this case the price $p$ is equal to the basic pre-existing contract price $p_0$ and that $p_0 < \theta$, since under normal conditions all customers were willing to consume $d$. Using a similar argument as in the case of game G, we observe that the dominant strategy of each customer is to consume the maximum possible, i.e., $x_j = c$.

Given these choices, the network determines the flow allocation by running Algorithm 1. This corresponds to the Max-Allocation (Definition III.2), using $c, n_c$ to replace $d, n_d$, respectively. (We refer the reader to see Appendix A for further details). If there is enough pressure $H_0$, since $c \leq d$, $n_c$ may be equal to $N$, so the welfare $SW_c = n_c(1 + \theta(c - b))$. On the other hand, when $H_0$ is not enough to serve all the customers with $c$, i.e., $n_c < N$, the $(n_c + 1)$ the customer consumes the remaining available water $q_c$, which is calculated by replacing $d$ with $c$ in (9).

The corresponding social welfare denoted by $SW_c$ is given by

$$SW_c = \begin{cases} n_c(1 + \theta(c - b)), & \text{if } n_c < N \& \ q_c < b, \text{ or } n_c = N, \\ n_c(1 + \theta(c - b)) + (1 + \theta(q_c - b)), & \text{if } n_c < N \& q_c \geq b. \end{cases} \tag{20}$$

Note that it is sensible to consider values of $c$ within $[b, d]$: if $c < b$, the social welfare $SW_c$ is equal to 0, while $c > d$ cannot have any affect since the resulting demands are restricted by $d$. The optimal flow restriction design problem is thus to maximize the social welfare $SW_c$ in (20) over $c \in [b, d]$, i.e.,

$$\max_{c \in [b,d]} SW_c \, . \tag{21}$$

To calculate the optimal value of $c$, we first provide an intuition of how $SW_c$ varies with $c \in [b, d]$. To this purpose, consider a numerical example in which $N = 3$, $H_0 = 118$ m, $h = 30$ m, $A = 100$, $\theta = 4$, $b = 0.2$ m$^3$/s, and $d = 1$ m$^3$/s. As shown in Figure 9, the social welfare function can be divided into three (non-decreasing) segments corresponding to the subdomains $[0.2, 0.29], (0.29, 0.716], (0.716, 1]$. In addition, we observe that the last value of $SW_c$ in each segment is a local maximum. To explain this result, let us consider the first segment: initially all customers get $b$ and as the value of $c$ increases, all the customers obtain demands $f_1 = f_2 = f_3 = c$, resulting in an increase of the utility of each customer and hence of the social welfare. At some intermediate point, say $\hat{c} \approx 0.25$, the pressure at the location of customer 3 reaches a value where $f_3 = \hat{c}$, but when $c$ increases further, the pressure drops further and $f_3$ starts decreasing. In this case the total welfare still increases, but at a lower rate, until customers 3 gets $f_3 = b$. With a further increase in $c$, the supply of

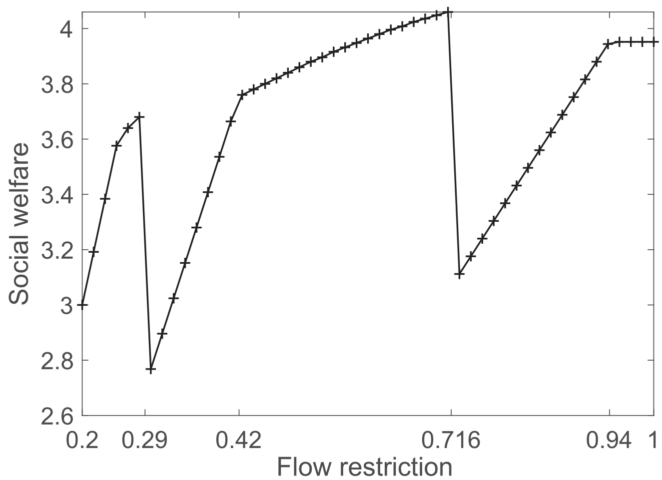


Fig. 9. Social welfare ($SW_c$) as a function of supply restriction $c$. The unit of $c$ is m$^3$/s.

customer 3 drops below $b$, and social welfare decreases by a step reflecting the complete unit utility loss of customer 3.

Figure 9 suggests that the global maximum of $SW_c$ is one among the multiple local maxima that this functions has for $c \in [b, d]$. It turns out that there is no simple analytic expression for characterizing the global maximum, and one has to resort to numerical calculations.

We next analyze the performance of the uniform supply restriction using the concept of the Price-of-Anarchy $\sigma$, similar to the case of Proposition IV.2. This compares the maximum efficiency among equilibria achieved under supply restriction with the allocation that maximizes social welfare $SW^*(v)$, for a set of system parameters $v = (H_0, h, A, N, b, d, \theta)$. In particular, $\sigma = \min_{v>0} \frac{SW_c(v)}{SW^*(v)}$.

*Proposition 6.1:* In systems with supply restrictions, $\sigma = \frac{1}{2}$.

This implies that the social welfare achieved by the optimal supply restriction policy is at least $\frac{1}{2}$ of the optimal social welfare. The reason behind the proposition is similar to that in Theorem V.4. Recall $SW^* \leq SW_d + SW_b \leq 2\max\{SW_d, SW_b\}$ for the reason behind Theorem V.4. Also, we have $SW_c^* \geq \max\{SW_d, SW_b\}$, since the social welfare under Max-Allocation, Min-Allocation are two special cases achieved by supply restriction using $c = d$ and $c = b$, respectively. Hence, we have $SW^* \leq 2SW_c^*$.

## B. Comparing Pricing With Supply Restriction

Under what conditions is pricing a better mechanism to deal with pressure drops compared to the uniform supply restriction? To answer this question we make the following simple observations. First, under (uniform) pricing, given the piecewise linear utility function, we can control customer demand to take any of the three values in the set $\{0, b, d\}$, while the uniform supply restriction allows for a finer control since we can restrict supply to take any value in $[0, d]$. This may suggest the superiority of supply restriction.

But we can also observe that, if customers are heterogeneous in their marginal valuation $\theta$ of using a unit of water flow, i.e., customer $i$ has parameter $\theta_i$, there exists a uniform price $p$ that incentivizes customers that value more large demands (i.e., customers with $\theta_i > p$) to ask for $d$, whereas customers with lower valuations (i.e., customers with $\theta_i < p$) to ask for $b$. This is the great advantage of pricing: under the right price, customers that need more get more, and customers that need less get less. This is not possible under supply restrictions, even if these restrictions were not uniform. Indeed, there is no way for the network to know which customers value demands more than others unless it uses an economic mechanism. Only when customers are charged for their consumption, they become rational consumers. We next elaborate on the above arguments and observe that prices are the best mechanism to control demand when customers are heterogeneous in $\theta$.

We first analyse the performance of restriction compared to our pricing when customers are homogeneous in $\theta$. Similarly to the idea in the definition of the Price-of-Anarchy, for a set of system parameters $v = (H_0, h, A, N, b, d, \theta)$, we consider the ratio of $\frac{SW_c^*(v)}{SW_p^*(v)}$ as an indicator of the relative performance of the optimal use of restrictions compared to an optimal price.

*Proposition 6.2:* Assuming customers are homogeneous in $\theta$, for any set of system parameters $v$, $\frac{SW_c^*(v)}{SW_p^*(v)} \in [1, 2]$.

The above proposition suggests that, as expected, supply restrictions can simulate any pricing policy. In addition, recall that $SW^* \leq 2SW_p^*$ (e.g., see the argument in Theorem V.4). Thus, since $SW_c^* \leq SW^*$, the social welfare achieved by the optimal supply restriction is no larger than twice the social welfare achieved by the optimal pricing policy.

*Remark:* Note that the upper bound of $\frac{SW_c^*(v)}{SW_p^*(v)}$ for identical customers in Proposition VI.2 is a loose upper bound. Extensive simulations carried out by varying the values of all parameters $H_0$, $h$, $A$, $N$, $b$, $d$, and $\theta$ of the water distribution network suggest the conjecture that the ratio is at most 1.5. To show that this upper bound is tight, consider the case in which $N = 2$, $b = 0.1$ m$^3$/s, $d = 0.3$ m$^3$/s, $A = 100$, $h = 30$ m, $\theta = \frac{1}{d-b}$, and $H_0 = 0.999A[(b+d)^2 + b^2] + h$. We have $SW_p^* = 2$, where $f_1 = d, f_2 = 0$, and $SW_c^* = SW^* = 2.9989$, where $c^* = 0.2998$ m$^3$/s, $f_1 = 0.2998 = 0.999\ d$, $f_2 = b = 0.1$ m$^3$/s. Thus, we have $\frac{SW_c^*}{SW_p^*} \approx 1.5$.

We next examine the case of heterogeneous customers in their valuation $\theta$ per unit of obtained flow. Since we are only concerned with determining a rough estimate of the improvement of the social welfare under heterogeneity of customers, we choose a simple setup involving two types of customers where we assume that the two types of customers with $\theta_1 < \theta_2$ are interleaved from upstream to downstream, with $\theta_1$ in the odd-numbered nodes and $\theta_2$ in the even-numbered nodes. It is not possible to obtain a closed-form expression for the social welfare in the general $N$ case, and the analysis is performed numerically in the next subsection. Still, we are able to analyse the case of $N = 2$, and prove that if customers are heterogeneous then the result of Proposition VI.2 does not hold anymore.

*Proposition 6.3:* In the case of $N = 2$ customers with different valuations $\theta_1 < \theta_2$, if $d \geq \sqrt{\frac{H_0 - h}{A}}$, then $\frac{SW_c^*(v)}{SW_p^*(v)} \geq 0.63$, where the lower bound is tight.

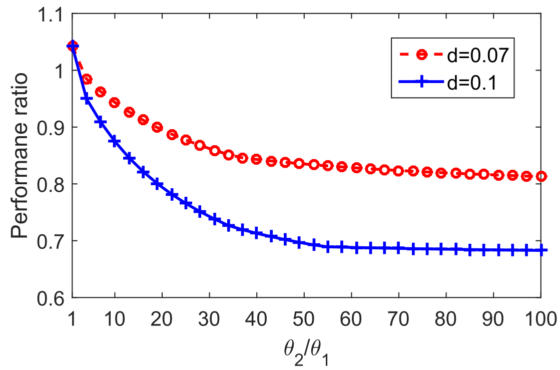


Fig. 10. Ratio of the maximum welfare achieved by supply restriction and optimal pricing as a function of the ratio $\frac{\theta_2}{\theta_1}$, for different values of the maximum demand parameter $d$. The unit of $d$ is $m^3/s$.

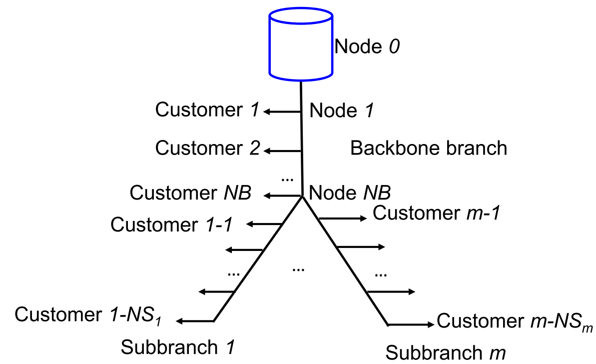


Fig. 11. Topology of the water distribution network with multiple branches (subbranch 1 to $m$).

Note that the above condition of $d \geq \sqrt{\frac{H_0-h}{A}}$ allows the first customer to get all the water demand for herself leaving less than $b$ for the second customer.

The interpretation of this result is that there is a range of system parameters for which the efficiency of the price mechanism is substantially better than using supply restrictions for improving social welfare. The intuition is that, if $\theta_2$ is substantially larger than $\theta_1$, it is more efficient to allocate a larger supply to customer 2 than 1. This type of allocation can be achieved with pricing—for example by charging a price $\theta_1 < p < \theta_2$, so that customer 1's demand for water becomes $b$ and customer 2's demand stays at $d$. Such an allocation where the first customer gets less than the second is not possible by using any restriction $c$. Indeed, using any uniform supply restriction $c$, the first customer will obtain at least as much as the second. But with the right choice of the price $p$, the customer with lower valuation will voluntarily constrain its consumption leaving enough pressure to serve downstream customers that value more water consumption.

## C. Numerical Results

We want to validate the claim of Proposition VI.3 for larger values of $N$. To this purpose, we set $A = 100$, $H_0 = 131$ m, $h = 30$ m, $b = 0.01$ m$^3$/s, and allow $d$ to take two values ($d = 0.07$ m$^3$/s, $d = 0.1$ m$^3$/s). We set $N = 10$ and assume that the utility functions of customers at odd-numbered and even-numbered nodes have parameters $\theta_1 = 1$ (type 1 customers) and $\theta_2 \geq \theta_1$ (type 2 customers), respectively, where $\frac{\theta_2}{\theta_1} \in [1100]$. As in the case of $N = 2$ analyzed earlier, we like odd-numbered customers to voluntarily restrict their consumption so that even-numbered customers consume as much water as possible. Figure 10 shows how the ratio $\frac{SW_c^*}{SW_p^*}$ varies with $\frac{\theta_2}{\theta_1}$ and $d$, for fixed $\theta_1 = 1$.

*Observation 6.4:* From Figure 10 we make the following observations.

- The ratio $\frac{SW_c^*}{SW_p^*}$ is larger than 1 only when $\frac{\theta_2}{\theta_1}$ is small, and decreases below 1 when $\frac{\theta_2}{\theta_1}$ increases.
- For larger values of $d$, the social welfare attained by the pricing mechanism further improves compared to using uniform restrictions given large $\frac{\theta_2}{\theta_1}$.

The first observation is consistent with the intuition developed from Proposition VI.3. As $\frac{\theta_2}{\theta_1}$ increases, it becomes more important for type 2 customers to get larger supply than type 1 customers. Since there is a type 1 customer before (i.e., closer to the tank) each type 2 customer, this control is only possible using pricing. On the other hand, when $\frac{\theta_2}{\theta_1}$ is near one, customers are homogeneous in $\theta$ and Proposition VI.2 suggests that supply restriction is a better mechanism.

For the second observation, we note that as $d$ increases, the negative effect of type 1 nodes consuming more than the socially optimal amount becomes more pronounced, and pricing becomes more efficient compared to supply restrictions. In order to save enough pressure for type 2 customers, the operator must use a low value of $c$ to restrict type 1 nodes, but this unnecessarily restricts type 2 nodes as well and reduces social welfare.

# VII. Extension to More General Water Networks

In this section, we extend the proposed pricing mechanism to two more generous network topologies: a water network with multiple branches and a looped water network. Specifically, we compare the pricing mechanism with the uncontrolled allocation policy, and show that the pricing mechanism outperforms the uncontrolled equilibrium in terms of social welfare. We also compare the pricing mechanism with the supply restriction policy.

## A. Water Network With Multiple Branches

Some real-world water distribution networks consist of multiple branches [23]. Thus, we extend our pricing mechanism for a single-branch water network to a network with multiple branches, illustrated in Figure 11. In the new network, Node 0 is the elevated tank and the other nodes are junctions, each connected to a customer. The network also includes one backbone branch and $m$ sub-branches. In the backbone branch, there are $NB$ nodes, while in each sub-branch $i$ there are $NS_i$ nodes. Each pipe has a different friction coefficient, sampled from a uniform distribution within the range $[r_1, r_2]$. As in the single-branch water distribution network, we assume that each customer has the step utility function defined in (6).

The evaluation of the performance of the pricing mechanism and its comparison with the uncontrolled allocation is now more complex. Although each customer's demand still

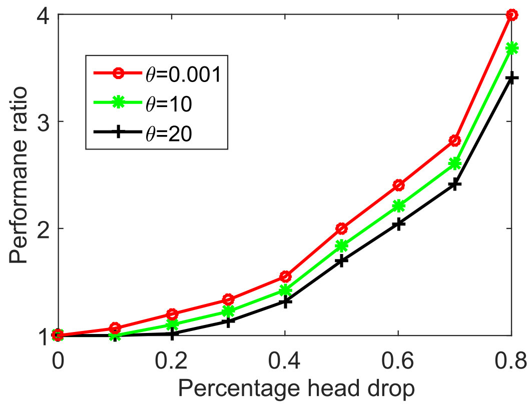


Fig. 12. Ratio between the maximum social welfare obtained through pricing and the social welfare of the uncontrolled equilibrium (i.e., $\frac{SW_p^*}{SW_s}$) for the water network with multiple branches. Results are reported as a function of the percentage $a$ of pressure reduction at $H_0$ due to failure and valuation $\theta$. For $a = 0$, all customers can get the flow $d$.

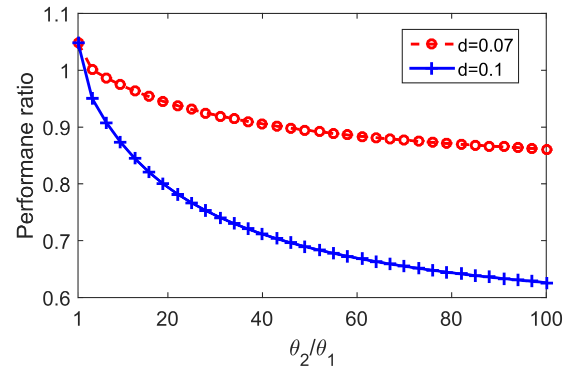


Fig. 13. Ratio of the maximum welfare achieved by supply restriction and optimal pricing policies as a function of the ratio $\frac{\theta_2}{\theta_1}$, for different values of the maximum parameter $d$ (The unit of $d$ is m$^3$/s).

depends on the price as in Proposition V.1, the coupling of the multiple sub-branches through the existence of the common backbone branch does not allow us any more to analyze each branch independently, as in Section V-A. Because of this reason, we cannot obtain a closed-form expression for $SW_p(p)$ and derive the maximum social welfare $SW_p^*$ analytically. Thus, we perform the optimization using numerical analysis. For different values of the price $p$, given the dominant strategy of each customer from Proposition V.1, we compute numerically the resulting water allocation and the corresponding social welfare. This allows us to find the optimal price $p^*$ and the resulting value of $SW_p^*$.

In our numerical simulations we set $NB = 3$, $m = 3$, $NS_1 = 12$, $NS_2 = 15$, $NS_3 = 18$, $h = 30$ m, $b = 0.001$ m$^3$/s, and $d = 0.01$ m$^3$/s. The value of each pipe's friction coefficient is taken from the uniform distribution with range $[r_1, r_2] = [100, 300]$. Under normal operating conditions, the hydraulic head $H_0 = 216$ m would supply the flow $d$ to all the customers. In our experiments, we consider the value $H_0 = (1 - a)216$ m, where $0 \le a < 1$ corresponds to the percentage pressure reduction at $H_0$ due to failure. For simplicity, we consider customers homogeneous in $\theta$. Figure 12 shows how the ratio of the maximum social welfare obtained through pricing and the social welfare at the uncontrolled equilibrium $\frac{SW_p^*}{SW_s}$ changes with $a$ and $\theta$.

From Figure 12, we make the following observations.

*Observation 7.1:* Far all $\theta > 0$, $\frac{SW_p^*}{SW_s} \ge 1$, and is equal to one when $a = 0$. In addition to that, $\frac{SW_p^*}{SW_s}$ is increasing in $a$ and decreasing in $\theta$.

The reason behind the first observation is explained in the following. That the optimal price always outperforms the uncontrolled equilibrium is rather obvious since we can always choose $p = 0$ and induce the uncontrolled allocation. When $a = 0$, the water availability is sufficient to serve each customer with her maximum demand $d$. Thus the optimal price must be zero in order not to affect consumption, leading to the same allocation (and hence social welfare) of the uncontrolled allocation.

As $a$ increases from zero, the number of customers that the water network can supply with the maximum demand $d$ decreases quickly. Yet, the water pressure may be sufficient to supply a large number customers with a flow equal to the minimum demand $b$. Thus, using a price $\theta < p < 1/b$ to induce demand $b$ by all the customers is the best option. The effect of restricting flow from $d$ to $b$ becomes more pronounced as $a$ becomes larger and pressure drop more. Even a single upstream customer consuming $d$ could preclude anyone else from obtaining some useful supply. To see the dependence on $\theta$, observe that as $\theta$ decreases, the difference of the utility of obtaining $d$ instead of $b$ decreases. Thus, the system gains more by serving many more people with rate $b$, which can only be achieved through pricing.

Lastly, we use simulation to compare the optimal pricing policy and the supply restriction policy in terms of social welfare. Similarly to Section VI-C, we consider the case in which customers are heterogeneous in their valuation of $\theta$. Specifically, we assume that the utility functions of customers at odd-numbered and even-numbered nodes in both the backbone and the subbranches in Figure 11 have slope $\theta_1 = 1$ (type 1 customers) and $\theta_2 \ge \theta_1$ (type 2 customers) respectively, where $\frac{\theta_2}{\theta_1} \in [1100]$. To increase social welfare, since $\theta_2 >> \theta_1$, it is desirable that odd-numbered customers restrict their consumption so that even-numbered customers consume as much as possible. This is possible only by using pricing and choosing a price $\theta_1 < p < \theta_2$. Any uniform restriction mechanism would unnecessarily restrict equally both types of customers.

As for the other parameters, we set $NB = 2$, $m = 3$, $NS_1 = 4$, $NS_2 = 4$, $NS_3 = 2$, $H_0 = 230$ m, $h = 30$ m, and $b = 0.01$ m$^3$/s. The friction coefficient of each pipe is taken from a uniform distribution with range $[r_1, r_2] = [100, 300]$. Figure 13 shows how the ratio $\frac{SW_c^*}{SW_p^*}$ between the maximum social welfare under supply restriction and under pricing varies with $\frac{\theta_2}{\theta_1}$ and $d$, for fixed $\theta_1 = 1$.

*Observation 7.2:* Figure 13 validates Observation VI.4.

- $\frac{SW_c^*}{SW_p^*} > 1$ only when $\frac{\theta_2}{\theta_1}$ is small, and decreases below 1 when $\frac{\theta_2}{\theta_1}$ increases.
- $\frac{SW_c^*}{SW_p^*}$ decreases in $d$.

As expected, customer heterogeneity in flow valuation is better addressed using prices. High values of $d$ make pricing an

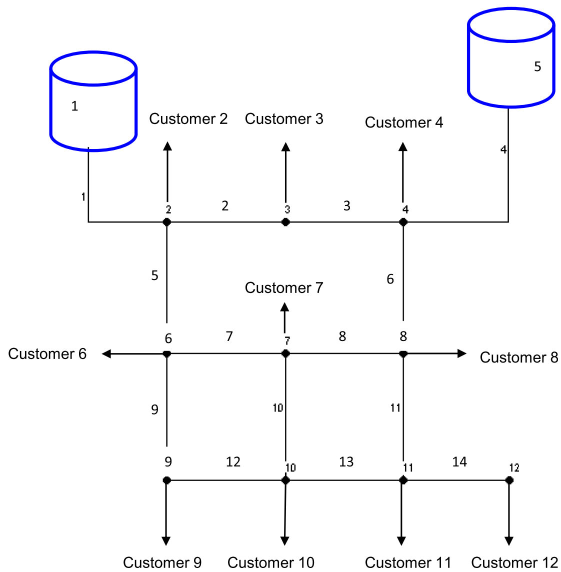


Fig. 14. Topology of the Gessler looped pipe water network [29]. Water supply to the customer is represented by an arrow.

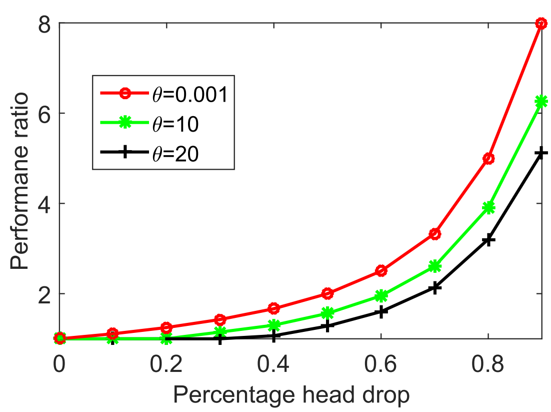


Fig. 15. Ratio between the maximum social welfare obtained through pricing and uncontrolled equilibrium ($\frac{SW_p^*}{SW_s}$) in the Gessler network. Results are reported for different values of percentage $a$ of pressure reduction at $H_0$ and valuation $\theta$.

even more attractive alternative compared to restricting supply uniformly, since incentivizing only low valuation customers to consume $b < d$ has a larger impact to the system efficiency.

### B. Looped Water Network

Some real-world water distribution systems use looped networks [23]. We thus extend our results to such topology using the tyical Gessler network, illustrated in Figure 14 [29]. The network consists of 14 pipes and 12 nodes. The set of nodes includes 2 tanks (Nodes 1 and 5) and 10 junctions, each connected to a customer.

For reasons similar to the ones given in the previous section, the optimum selection of price $p$, restriction parameter $c$, and the correspond social welfare values cannot be determined analytically. Our analysis is therefore carried out numerically.

To illustrate how the performance of the optimal pricing policy varies with the supply failure, we setup a numerical

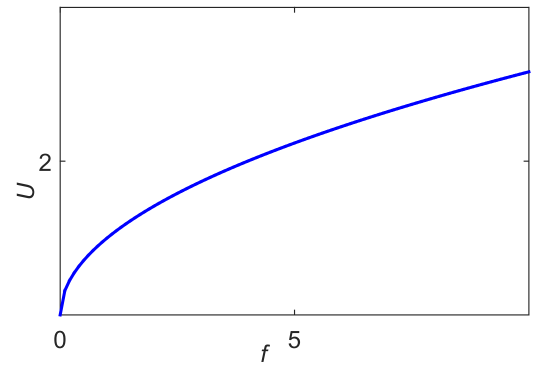


Fig. 16. Customer's square root utility function. The $x$-axis is the flow $f$, and the $y$-axis is the resulting utility $U$.

experiment in which we vary the values of the hydraulic head $H_1$ (in Node 1) and $H_5$ (in Node 5). Similar to [29], the friction coefficients for pipes $1, 2, \ldots, 14$ are 2660, 4070, 4070, 16290, 4070, 23430, 8020, 23430, 4070, 229120, 23430, 8020, 23430, 23430, respectively. We set $h = 30$ m, $b = 0.005$ m$^3$/s, and $d = 0.033$ m$^3$/s. Under normal operating conditions, an hydraulic head of $H_1 = 365.76$ m and $H_5 = 371.86$ m would supply $d$ to all customers [29]. In our experiment, we consider the value $H_1 = (1-a)365.76$ m and $H_5 = (1-a)371.86$ m, where $0 \leq a < 1$ captures the water level drop at $H_0$ due to the failure. For simplicity, we assume that all customers have the same utility function. Figure 15 illustrates the ratio $\frac{SW_p^*}{SW_s}$ as a function of $a$ and $\theta$ and validates the findings of Figure 12 and Observation VII.1.

## VIII. Extension to Concave Utility Function

We finally validate our results so far for more general utility functions. For ease of analysis, we consider a square root utility function

$$U(f) = \sqrt{f}, f \geq 0, \tag{22}$$

(see Figure 16). This is the case of an 'elastic' utility (it remains positive as $f \to 0$) that is concave and hence captures the diminishing return property of a user's resource valuation [27].

Although the consideration of this particular utility function may seem restrictive, it allows us to perform an in-depth study of the problem and reveal new insights that are expected to generalize for different concave elastic functions. We again assume for ease of analysis that the friction coefficients of each pipe are identical (i.e., $A_0 = A_1 = \ldots = A_{N-1} = A$). Similarly to the previous analysis, we assume a single-branch water network (Figure 1).

We first determine the uncontrolled allocation and prove that the PoA coefficient $\sigma$ can become arbitrarily small, similarly to Proposition IV.2. We then analyze the properties of optimal pricing and flow restrictions.

### A. Uncontrolled Allocation

Clearly in this case each customer's demand is the largest possible, i.e., $\infty$. Since pressure is not allowed to drop below the value $h$, customer 1 obtains $f_1 = \sqrt{\frac{H_0-h}{A}}$, leaving $f_2 = \ldots = f_N = 0$ for the downstream customers. The resulting social welfare is $SW_s = (\frac{H_0-h}{A})^{\frac{1}{4}}$. We can show that this can be an arbitrarily small fraction of the optimal social welfare. This agrees with the result in Proposition IV.2 concerning the step utility function, although there is a fundamental difference of the uncontrolled allocations in these two cases. Under the step utility function with maximum flow value $d$, if $H_0$ is high enough, all customers can be served, even if they are uncontrolled. This does not hold under $U(x) = \sqrt{x}$, where customer 1 will always consume the largest possible amount of flow, dropping the pressure to the minimum accepted level $h$ and leaving no pressure to serve the rest of the customers.

*Proposition 8.1:* The PoA $\sigma$ defined over all possible values $v = (H_0, h, A, N)$ is 0.

To prove this proposition, consider a flow allocation where each customer's flow is equal and the customers use up all the water supply (i.e., $H_N = h$). Under this allocation, we can calculate that the flow for each customer is $\sqrt{\frac{6(H_0-h)}{AN(N+1)(2N+1)}}$ by equation (2), and the social welfare is $\hat{SW} = (\frac{6(H_0-h)N^3}{A(N+1)(2N+1)})^{\frac{1}{4}}$. Thus, we have $\frac{SW_s}{\hat{SW}} = (\frac{(N+1)(2N+1)}{N^3})^{\frac{1}{4}}$, and $\lim_{N\to+\infty} \frac{SW_s}{\hat{SW}} = 0$. Thus, since $SW^* \geq \hat{SW}$, we also have $\lim_{N\to+\infty} \frac{SW_s}{SW^*} = 0$, and the proposition is proved.

### B. Optimal Price Design

We now formulate the optimal uniform pricing problem. Given a price $p$, if a customer uses $f$ amount of water, her payoff is $\phi(f) = \sqrt{f} - pf$. We can derive each customer's water demand $x$ given a price $p \geq 0$ by considerings $\phi'(x) = 0$, and hence $x = \frac{1}{4p^2}$. As expected, each customer's demand decreases in the price in a continuous fashion, taking any possible positive value. This is in contrast with the case of the step utility function, where demand can take only the values $0, b, d$.

Similarly to the analysis of the step utility function, we can now obtain the total social welfare $SW_p$ as a function of $p$ by replacing each customer's demand in Section 5 with $\frac{1}{4p^2}$. The optimal price design problem is to find a price $p \geq 0$ that maximizes $SW_p$. Unfortunately we cannot obtain a simple condition for the optimal price as before. We can study the properties of $SW_p$ as a function of $p$ in the case of a simple example.

Consider the case with $N = 3$ customers, $H_0 = 200\ m, h = 30\ m, A = 100$ and price $p$. Figure 17 shows how the social welfare $SW_p$ change with price $p$. We observe that $SW_p$ has a different shape in $N + 1 = 4$ price intervals. In the first block where $p \in [0, 0.438)$, the flow allocations are $f_1 = const > 0, f_2 = f_3 = 0$. Customer 1's water demand is $\frac{1}{4p^2}$, which exceeds the maximum possible flow of $\sqrt{\frac{H_0-h}{A}}$, and hence her allocation $f_1$ remains constant and equals the above maximum available amount as $p$ increases. Clearly $f_2, f_3 = 0$.

As $p$ becomes larger and enters $[0.438, 0.655]$, since $\frac{1}{4p^2} < \sqrt{\frac{H_0-h}{A}}$, customer 1 reduces her consumption and leaves

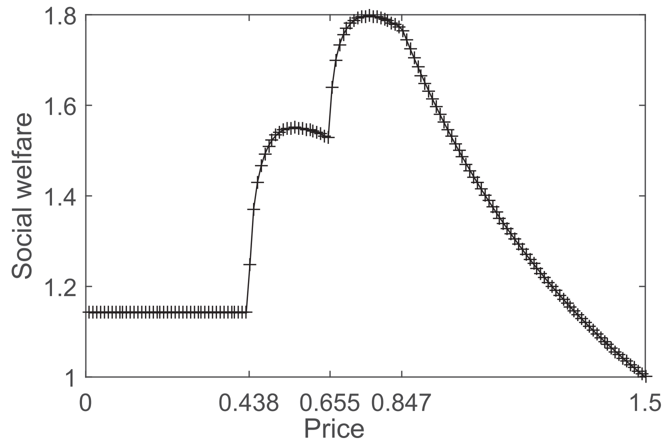


Fig. 17. Social welfare $SW_p$ as function of the price $p$.

some available flow for customer 2 which consumes the maximum that is allocated to her. Since the derivative of the utility function is larger for smaller values of $f$, the marginal gain of utility by serving a new customer is larger than the marginal reduction of the utility of the customer that is already being served and the sum of the utilities initially increases. But after some price $p_1$ it will start decreasing because total consumption is unnecessarily reduced. All this time the customer 2 obtains $f_2$ less than her maximum desired value of $\sqrt{\frac{H_0-h}{A}}$. When the price increases further and enters in the interval $[0.655, 0.847]$, customer 2 requests less than what the network can offer her, and hence customer 3 starts being served. For the same reason as before (i.e., the marginal increase of the utility of the new customer being served is larger than marginal decrease of the total utility of the rest of the customers getting because they get less flow), $SW_p$ increases and after some value $p_2$ starts decreasing. For $p > 0.847$ the price is high enough to make even customer 3 (besides customers 1 and 2) demand lower than what is available to her. This explains the discontinuity of the derivative of $SW_p$ at $p = 0.847$ (besides $p = 0.438, 0.655$).

Figure 17 suggests that the global maximum of $SW_p$ is one among multiple local maxima since, depending on the parameters, the largest local maximum could take place in any of the price intervals discussed earlier. This explains why we cannot obtain a simple condition to characterize the optimum price.

### C. Social Welfare Analysis of Pricing

We analyze the performance of uniform pricing in terms of social welfare for the case of the square root utility function. We next provide a lower bounds for $\frac{SW_p^*}{SW^*}$ in analogy to Proposition V.4 and also show that $\frac{SW_p^*}{SW_s}$ is bounded.

*Proposition 8.2:* The following lower bounds hold for any value of the parameters $H_0, A, h$:

- $\frac{SW_p^*}{SW^*} > \frac{1}{(\ln N+1)^{\frac{1}{2}}}(\frac{6\ N^2}{(N+1)(2N+1)})^{\frac{1}{4}}$.
- $\frac{SW_p^*}{SW_s} > (\frac{6\ N^3}{(N+1)(2N+1)})^{\frac{1}{4}}$

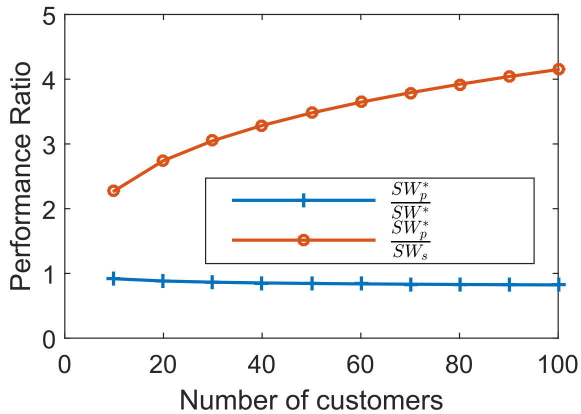


Fig. 18. $\frac{SW_p^*}{SW_s}$ and $\frac{SW_p^*}{SW^*}$ as functions of the number of customers $N$.

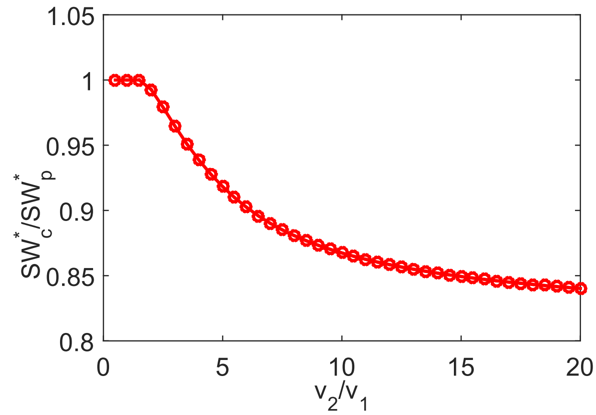


Fig. 19. $\frac{SW_c^*}{SW_p^*}$ as a function of $v_2/v_1$.

The first bound suggests that optimal pricing captures a substantial fraction of the maximum possible social welfare. This is expected since by increasing the price, we can reduce the demand of each customer to any desired level (that is equal for all customers) in order to serve a larger number of customers. Under social welfare maximization a large number of customers receives small flows, but in this case allocations are not equal and upstream customers get more flow. This explains the welfare loss of the optimal pricing. As $N$ increases, this difference in the allocations creates a larger loss that increases very slowly with $N$ as the second bound suggests, since $(\frac{1}{\ln N+1})^{\frac{1}{2}}(\frac{6\ N^2}{(N+1)(2N+1)})^{\frac{1}{4}}$ decreases very slowly with $N$. In particular, if $N = 100$, this term is approximate 0.57; If $N = 10000$, this term is approximate 0.43.

Regarding the second bound, we know that pricing outperforms the uncontrolled allocation since we can always use $p = \infty$, but by how much? This bound answers this question—observe that $(\frac{6\ N^3}{(N+1)(2N+1)})^{\frac{1}{4}}$ increases in $N$. In particular, if $N = 100$, this term is 4.14; if $N = 1000$, it is 7.14; if $N = 10000$, it is 13.16. This suggests that uniform pricing performs significantly better than the uncontrolled allocation in single-branch water networks under the type of utility we consider. Since under the uncontrolled allocation only the first downstream customer is served, as $N$ increases, due to the very steep increase of the utility functions at zero, the allocation under the optimal pricing policy that offers a small amount of flow to $N$ customers produces a much larger total utility. This effect increases with $N$.

It is interesting to examine the empirical performance of the optimal pricing policy in Figure 18. We explore how the number of customers $N$ impacts the performance of the optimal pricing in a setting where $H_0$=130 m, $h = 30$ m, $A = 100$. In practice, there are no more than 100 customers in a single water network branch [1]. Thus, we vary $N$ from 10 to 100. Figure 18 validates the findings in Proposition VIII.2.

### D. Comparing Pricing With Supply Restriction

It is simple to observe that for homogeneous customers uniform supply restrictions and pricing are equivalent. This is because price $p$ induces the same demand as flow restriction with $c = \frac{1}{4p^2}$ when $U(x) = \sqrt{x}$, which can be easily generalized in the case of arbitrary concave elastic utilities. Hence, flow restriction analysis is the same as our analysis in Section VIII-C by rescaling the results of the pricing analysis. For example, social welfare as a function of $p$ in Figure 17 can be redrawn by rescaling the $x$-axis using $c = \frac{1}{4p^2}$. A similar function will appear with large values of $p$ corresponding to small values of $c$.

When utilities are elastic and concave, Proposition VI.2 suggesting that flow restriction is superior to pricing does not hold anymore, since $\frac{SW_c^*}{SW_p^*} = 1$. The advantage of flow restriction vs. pricing in the case of step utilities was that pricing achieves only a small set of discrete demand values (i.e., $0, b, d$) while flow restriction can choose any value on the real axis. But this advantage is lost when utilities are elastic and concave. The superior performance of pricing already observed in Proposition VI.3 remains when customers are heterogeneous. We examine this next.

*Proposition 8.3:*

In the case of two diverse customers with utility functions $U_i(f) = v_i\sqrt{f}, i = 1, 2$, and $v_1 < v_2$, $\frac{SW_c^*}{SW_p^*} \in [0.48, 1]$.

When $v_2 >> v_1$, social welfare is increased when customer 2 gets allocated a larger flow than customer 1. This is not possible using flow restriction, but is possible by pricing since the demand of each customer will be $x_i = \frac{v_i^2}{4p^2}$ and hence $x_1(p) < x_2(p)$.

We explore the empirical ratio between flow restriction and pricing for two diverse customers when $H_0$ = 130 m, $h = 30$ m, $A$ = 100, $v_1$ = 1. We examine in Figure 19 how $\frac{SW_c^*}{SW_p^*}$ varies with $\frac{v_2}{v_1}$.Our results validate the findings in Figure 10.

## IX. Conclusions

Our study extends the current set of tools available to utilities for managing water distribution networks undergoing supply failures. To this purpose, we adopted a game theoretic approach where customers of the water network are uncontrolled and try to maximize their utility from consuming water. We proposed the use of pricing as a mechanism that induces customers located in high-pressure zones to limit their demand, thus protecting the remaining users. Rigorous results obtained for a typical single-branch water distribution network show several attractive features of pricing compared to physical flow restriction mechanisms. In particular, in any such water system, a decentralized control using pricing can recover more than half of the maximum attainable social welfare—maximizing social welfare serves as a benchmark and requires perfect coordination of customer consumption in the network, something that is not practically achievable. In addition, we show that when customers are homogeneous in their valuation for water flow rates, using physical mechanisms to restrict uniformly the supply cannot be more than twice as efficient as pricing. In the more practical case of heterogeneous customers, pricing becomes substantially the most efficient mechanism, over-performing mechanical controls by a large margin.

Overall, these results suggest that there are opportunities for supplying customers in an equitable and fair manner (during water scarcity conditions) without resorting to the infrastructural upgrades required by supply restrictions. We note that such opportunities are not limited to single-branch water distribution networks but rather extend to more complex topologies, as shown by our empirical analysis on multiple branches and looped networks. Our future work will further focus on complex topologies (with multiple tanks and loops) and alternative pricing schemes (e.g., ladder pricing) in the case of larger networks with heterogeneous customers.

## Appendix A

In this appendix, we present the detailed forms of $n_d, n_b, n_c$ in Lemma IX.1, IX.2, IX.3, which are used in the previous sections of the paper. Specifically, Lemma IX.1, IX.2 are used in Section III-B, and Lemma IX.3 is used in Section VI-A.

*Lemma 9.1:* Given $H_0$, the number $n_d$ of customers getting the desired demand $d$ is

$$n_d = \begin{cases} \lfloor y_d \rfloor, \text{if} Ab^2 + h \leq H_0 < \frac{1}{6}N(N+1)(2N+1)d^2 + h; \\ 0, \text{ if } H_0 < Ad^2 + h, \end{cases} \tag{23}$$

where

$$y_d = \left( \frac{3(H_0 - h)}{2Ad^2} + \frac{1}{2}\sqrt{\frac{9(H_0-h)^2}{A^2\ d^4} - \frac{1}{432}} \right)^{\frac{1}{3}} + \left( \frac{3(H_0 - h)}{2Ad^2} - \frac{1}{2}\sqrt{\frac{9(H_0-h)^2}{A^2\ d^4} - \frac{1}{432}} \right)^{\frac{1}{3}} - \frac{1}{2}.$$

Note that $y_d$ is the root of the cubic equation on $k$ in (8). Since the value of $y_d$ may be non-integer, we need to round down $y_d$ to get $n_d$.

*Lemma 9.2:* Given $H_0$, the number of customers $n_b$ getting a supply equal to $b$ is

$$n_b = \begin{cases} \lfloor y_b \rfloor, \text{if} Ab^2 + h \leq H_0 \leq \frac{1}{6}N(N+1)(2N+1)b^2 + h; \\ N, \text{if } H_0 > \frac{1}{6}N(N+1)(2N+1)b^2 + h, \end{cases} \tag{24}$$

where

$$y_b = \left( \frac{3(H_0 - h)}{2Ab^2} + \frac{1}{2}\sqrt{\frac{9(H_0-h)^2}{A^2b^4} - \frac{1}{432}} \right)^{\frac{1}{3}} + \left( \frac{3(H_0 - h)}{2Ab^2} - \frac{1}{2}\sqrt{\frac{9(H_0-h)^2}{A^2b^4} - \frac{1}{432}} \right)^{\frac{1}{3}} - \frac{1}{2}.$$

By replacing $d$ with $c$ in Lemma IX.1, we calculate as follows the number $n_c$ of customers getting a supply equal to $c$.

*Lemma 9.3:* Given $H_0$, the number of customers getting a supply $c$ is

$$n_c = \begin{cases} \lfloor y_c \rfloor, \text{if} Ac^2 + h \leq H_0 \leq \frac{1}{6}N(N+1)(2N+1)c^2 + h; \\ N, \quad \text{if } H_0 > \frac{1}{6}N(N+1)(2N+1)c^2 + h; \\ 0, \quad \text{if} H_0 < Ac^2 + h; \end{cases} \tag{25}$$

where

$$y_c = \left( \frac{3(H_0 - h)}{2Ac^2} + \frac{1}{2}\sqrt{\frac{9(H_0-h)^2}{A^2c^4} - \frac{1}{432}} \right)^{\frac{1}{3}} + \left( \frac{3(H_0 - h)}{2Ac^2} - \frac{1}{2}\sqrt{\frac{9(H_0-h)^2}{A^2c^4} - \frac{1}{432}} \right)^{\frac{1}{3}} - \frac{1}{2}.$$

## Appendix B

In this appendix, we provide the proof of Theorem V.4. Recall that $n_d$ and $n_b$ denote the number of customers getting a supply equal to $d$ and $b$, respectively. $SW_p^*$ is the social welfare under the optimal pricing policy.

*Proof:* By Proposition III.1, we assume that, at the social optimal, the customers $1, 2, \ldots, j-1$ get $d$ amount of water, the subsequent customer $j$ gets $f \in [b, d)$ amount of water, customers $j+1, \ldots, k$ get $b$ amount of water, and, finally, the remaining customers $k+1, \ldots, N$ get 0 amount of water. $SW^*$ can be seen as the sum of two components, where the first is the total utility of the customers $1, 2, \ldots, j$, and the second is the total utility of the subsequent customers $j+1, , \ldots, k$. Let $SW_1$ and $SW_2$ be the first and second component, respectively. We have $SW^* = SW_1 + SW_2$, $SW_1 = jU(d) + U(f)$, and $SW_2 = k - j$, where $k-j$ represents the number of customers getting a supply equal to $b$. We show that both $SW_1$ and $SW_2$ are no larger than $SW_p^*$.

We first show that $SW_1 \leq SW_p^*$. Under price $p \in [0, \theta)$, the water availability is used to serve as many customers as possible with a supply of $d$ (by Proposition V.1)—there may be some capacity left to serve customer $n_d + 1$ with a $q < d$ amount of water. Thus, we have $j \leq n_d$. There can be two cases. In the first case that $j = n_d$, then $f \leq q$, since $q$ is the maximum amount of water available for customer $n_d + 1$. Thus, since $SW_p(p \in [0, \theta)) = n_d U(d) + U(q)$ by (10), and the utility function $U$ is nondecreasing in the demand, we have $SW_1 \leq SW_p(p \in [0, \theta))$. In the second case that $j < n_d$, we then have $j + 1 \leq n_d$. Also, we have $SW_1 \leq (j+1)U(d)$ since $f < d$. Thus, we have $SW_1 \leq n_d U(d)$. Since $SW_p(p \in [0, \theta)) \geq n_d U(d)$ by (10), we have $SW_1 \leq SW_p(p \in [0, \theta))$. Overall we have $SW_1 \leq SW_p(p \in [0, \theta))$. Thus, since $SW_p(p \in [0, \theta)) \leq SW_p^*$, we have $SW_1 \leq SW_p^*$.

We then show that $SW_2 \leq SW_p^*$. Since under price $p \in (\theta, \frac{1}{b})$, the water availability is used to serve as many customers as possible with a supply of $b$ (by Proposition V.1), we have $k - j - 1 \leq n_b$. Thus, since $SW_p(p \in (\theta, \frac{1}{b})) = n_b$ by Proposition V.2, we have $SW_2 \leq SW_p(p \in (\theta, \frac{1}{b}))$. Since $SW_p(p \in (\theta, \frac{1}{b})) \leq SW_p^*$, we have $SW_2 \leq SW_p^*$.

Since $SW^* = SW_1 + SW_2$, we have $SW^* \leq 2SW_p^*$. Thus, we have $\frac{SW_p^*}{SW^*} \geq \frac{1}{2}$. Hence, the theorem holds. ∎

## REFERENCES


[1] H. Methods *et al.*, “Advanced Water Distribution Modeling and Management,” Haestead Press, 2003.

[2] M. Muller, “Cape town’s drought: Don’t blame climate change,” *Nature*, vol. 559, pp. 174–176, 2018.

[3] D. Paez *et al.*, “The battle of postdisaster response and restoration,” *J. Water Resour. Plan. Manage.*, vol. 146, no. 8, Aug. 2020.

[4] A. Hassanzadeh, A. Rasekh, S. Galelli, M. Aghashahi, R. Taormina, A. Ostfeld, and M. K. Banks, “A. review of cybersecurity incidents in the water sector.” *J. Environ. Eng.,* vol. 146, no. 5, pp. 03120003, 2020.

[5] R. Taormina, S. Galelli, N. O. Tippenhauer, E. Salomons, A. Ostfeld, “Characterizing cyber-physical attacks on water distribution systems,” *J. Water Resour. Plan. Manage.*, vol. 143, no. 5, Art. no. 04017009, 2017, doi: 10.1061/(ASCE)WR.1943-5452.0000749.

[6] R. Meganck, K. Havens, R. Pinto-Coelho, “Water: Megacities running dry in Brazil,” *Nature*, vol. 521, Art. no. 289, 2015, doi: 10.1038/521289c.

[7] C. Makropoulos, D. A. Sávic, “Urban hydroinformatics: Past, present and future,” *Water,* vol. 11, no. 10, p. 1959, 2019, doi: 10.3390/w11101959.

[8] J. E. Pesantez, E. Z. Berglunda, N. Kazab. “Smart meters data for modeling and forecasting water demand at the user-level.” *Environ. Model. Softw.,* vol. 125, Mar. 2020, Art. no. 104633.

[9] P. Garronea, L. Grillia, R. Marzanob, “Price elasticity of water demand considering scarcity and attitudes,” *Utilities Policy*, vol. 59, p.100927, 2019.

[10] C. Rougé *et al.*, “Assessment of smart-meter-enabled dynamic pricing at utility and river basin scale,” *J. Water Resource Plan. Manage.*, vol. 144, no. 5, pp. 04018019, 2018.

[11] D. M. Kilgour, A. Dinar, “Flexible water sharing within an international river basin.,” *Environ. Resource Econ.*, 18:43–60, 2001.

[12] F. P. Kelly, *Notes on Effective Bandwidths. Stochastic Networks: Theory and Applications Telecommunications Networks,* Vol. 4, of (ser. Royal Statistical Society Lecture Notes Series), pp. 141–168, London, U.K.: Oxford Univ. Press, 1996.

[13] F. P. Kelly, “Charging and rate control for elastic traffic,” *Eur. Trans. Telecommun.*, vol. 8, no. 1, pp. 33–37, 1997.

[14] C. Courcoubetis, F. P. Kelly, and R. Weber, “Measurement based charging in communication networks,” *Oper. Res.*, vol. 48, no. 4, pp. 535–548, 2000.

[15] C. Courcoubetis and R. Weber, *Pricing Communication Networks: Economics, Technology and Modelling.* (ser. Wiley Interscience Series in Systems and Optimization). Hoboken, NJ, USA: Wiley, 2003.

[16] V. A. Siris and C. Courcoubetis, “Resource control for loss-sensitive traffic in CDMA networks,” in *Proc. IEEE Int. Conf. Comput. Commun.*, 2004.

[17] L. Duan, L. Gao, and J. Huang, “Cooperative spectrum sharing: A contract-based approach,” *IEEE Trans. Mobile Comput.*, vol. 13, no. 1, pp. 174–187, Jan. 2014.

[18] J. Huang and L. Gao, *Wireless Network Pricing.* (ser. Synthesis Lectures on Communication Networks). San Rafael, CA, USA: Morgan and Claypool, 2013.

[19] R. J. Gibbens and F. P. Kelly, “Resource pricing and the evolution of congestion control,” *Automatica,* vol. 35, no. 12, pp. 1969–1985, 1999.

[20] R. Johari and J. Tsitsiklis “Efficiency loss in a network resource allocation game,” *Math. Operations Res.*, vol. 29, no. 3, pp. 407–435, 2004.

[21] Z. Y. Wu, R. H. Wang, T. M. Walski, S. Y. Yang, D. Bowdler, and C. C. Baggett, “Extended global-gradient algorithm for pressure-dependent water distribution analysis,” *J. Water Resource Plan. Manage.*, vol. 135, no. 1, pp. 13–22, 2009.

[22] W. K. Ang, P. W. Jowitt, “Solution for water distribution systems under pressure-deficient conditions,” *J. Water Resour. Plan. Manage.*, vol. 132, no. 3, pp. 175–182, 2006.

[23] P. K. Swamee, A. K. Sharma, *Design of Water Supply Pipe Networks.* Hoboken, NJ, USA: Wiley, 2008.

[24] R. Gupta, P. R. Bhave, “Comparison of methods for predicting deficient-network performance,” *J. Water Resource Plan. Manage.*, vol. 122, no. 3, pp. 214–217, 1996.

[25] K. Jain, “A. polynomial time algorithm for computing an arrow-debreu market equilibrium for linear utilities,” *SIAM J. Comput.*, vol. 37, no. 1, pp. 303–318, 2007.

[26] Y. Ye, “A. path to the arrow-debreu competitive market equilibrium,” *Math. Program.*, vol. 111, no. 1, pp. 315–348, 2008.

[27] P. A. Samuelson, W. D. Nordhaus. *Microeconomics*, 17th ed. New York, NY, USA: McGraw-Hill Education, pp.110, 2001.

[28] A. Mas-Colell, M. D. Whinston, J. R., *Green Microeconomic Theory.* London, U.K.: Oxford Univ. Press, 1995.

[29] Centre for Water Systems. https://emps.exeter.ac.uk/engineering/research/cws/resources/benchmarks/expansion/gessler.php.


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