

Q.1 With the help of a diagram, state the algorithm:

1. Rotate Left
2. Rotate Right

Ans :

**Algorithm for Rotate Right:**

Algorithm rotateRight (ref root <tree pointer>)

This algorithm exchanges pointers to rotate the tree right

Pre : roots points to the tree to be rotated

Post :Node rotated and root updated

```
1 tempPtr = root->left
2 root -> left = tempPtr -> right
3 tempPtr -> right = root
4 root = tempPtr
5 return
endrotateRight
```

**Algorithm for Rotate Left:**

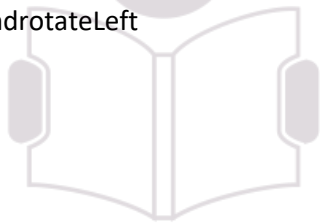
Algorithm rotateLeft (ref root <tree pointer>)

This algorithm exchanges pointers to rotate the tree left

Pre : roots points to the tree to be rotated

Post :Node rotated and root updated

```
1 tempPtr = root->right
2 root -> right = tempPtr -> left
3 tempPtr -> left = root
4 root = tempPtr
5 return
endrotateLeft
```



**E-next**  
THE NEXT LEVEL OF EDUCATION

Example:

The following tree is an example of Right of Left case & Complex double rotation right subcase.

Original Tree:

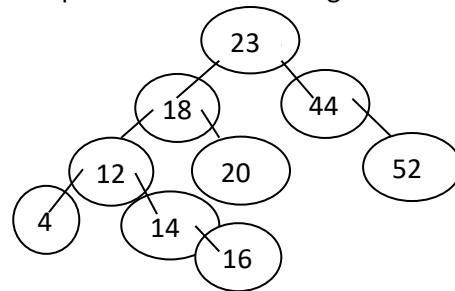
This is imbalanced tree.

$H_L = 4, H_R = 2$

Therefore,  $H_L - H_R = 2$

Effecting node = 12

Therefore, rotating 12 node to left



After Left Rotation:

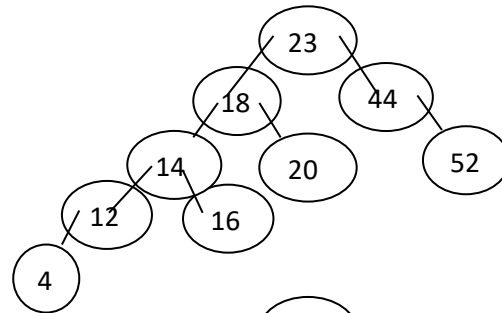
This is imbalanced tree.

$H_L = 4, H_R = 2$

Therefore,  $H_L - H_R = 2$

Effecting node = 18

Therefore, rotating 18 node to right

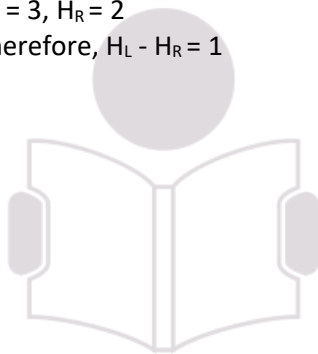
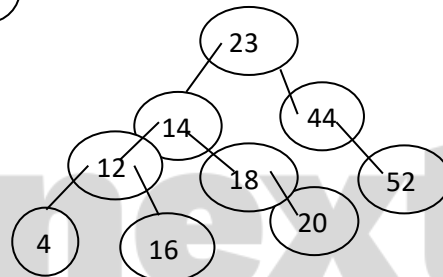


After Right Rotation:

This is balanced tree.

$H_L = 3, H_R = 2$

Therefore,  $H_L - H_R = 1$



E-next  
THE NEXT LEVEL OF EDUCATION

Q.2 In what way is an AVL Tree efficient than a Binary Search Tree? Why is it called a Height Balanced Tree?

Ans :

**AVL Tree :-**

The balanced binary tree structure called the AVL Tree, was designed and named after two Russian mathematicians AdelsonVelskii and Landis.

An AVL Tree is a search tree in which the heights of the subtrees differ by no more than 1. It is thus a balanced binary tree. Of course this advantage comes at a cost of rotating a node each time to maintain the balance. This is time consuming.

**AVL Trees are sufficient than a BST :-**

When the keys come in ordered sequence, the binary search tree can degenerate into a linked list, thereby greatly increasing the search time, whereas an AVL Tree will still be balanced by virtue of its ability to rotate an imbalanced node.

**AVL Tree, an Height Balanced Tree :-**

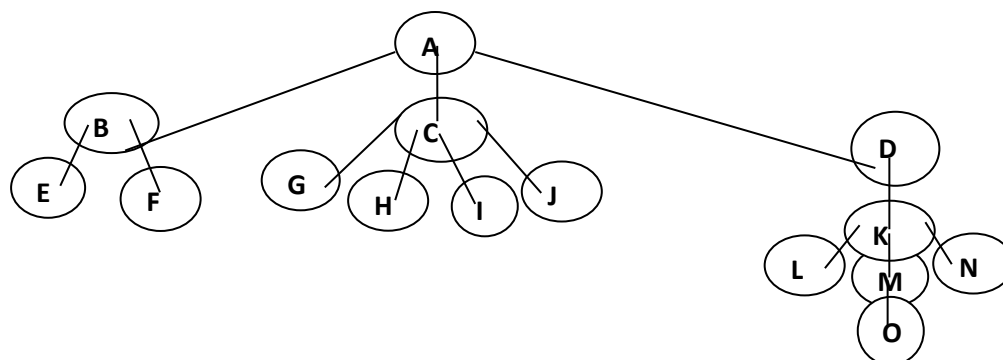
Since the heights of subtrees of every node in an AVL tree differ by no more than 1, the height of the subtree is greatly reduced, thereby shortening the search time. Hence the AVL trees are also known as "Height Balanced Trees".



**E-next**

THE NEXT LEVEL OF EDUCATION

Q.3 Give the definition of a general tree. What are the steps to convert a general tree to a binary tree? Implement the conversions on the given general tree.



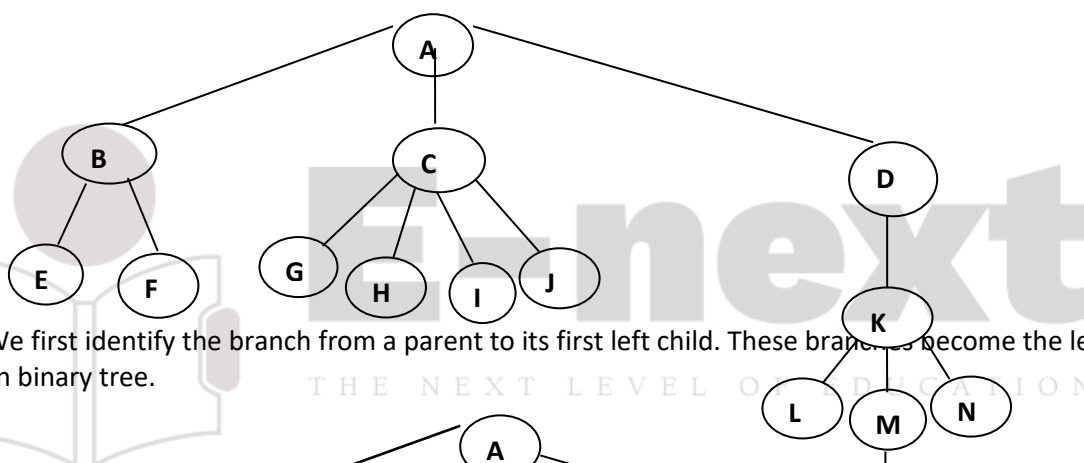
Ans:

### General Tree:

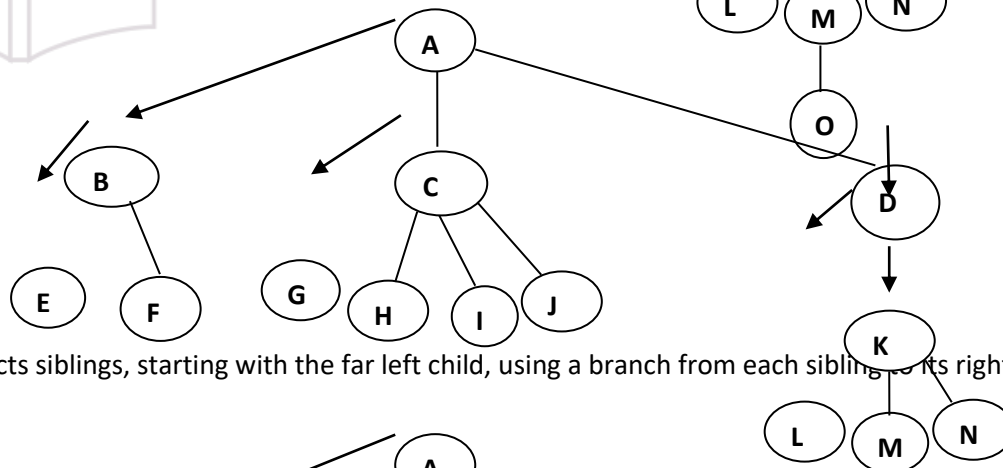
A general tree is a tree in which each node can have an unlimited out degree.

### Changing a General Tree to a Binary Tree:

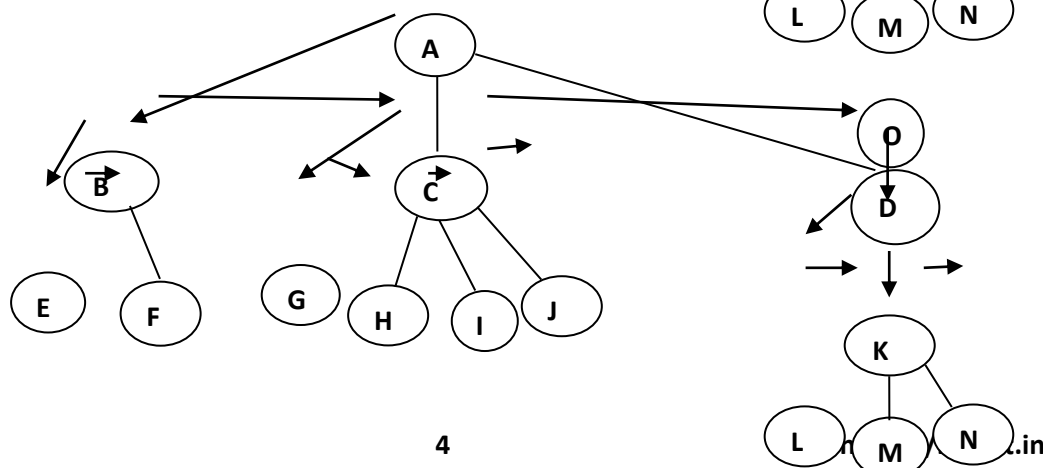
#### A General Tree



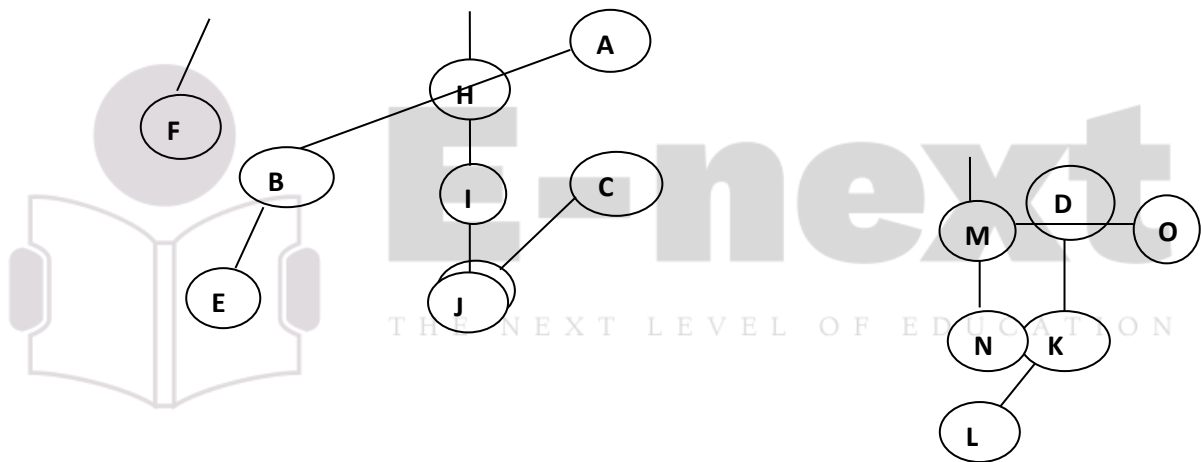
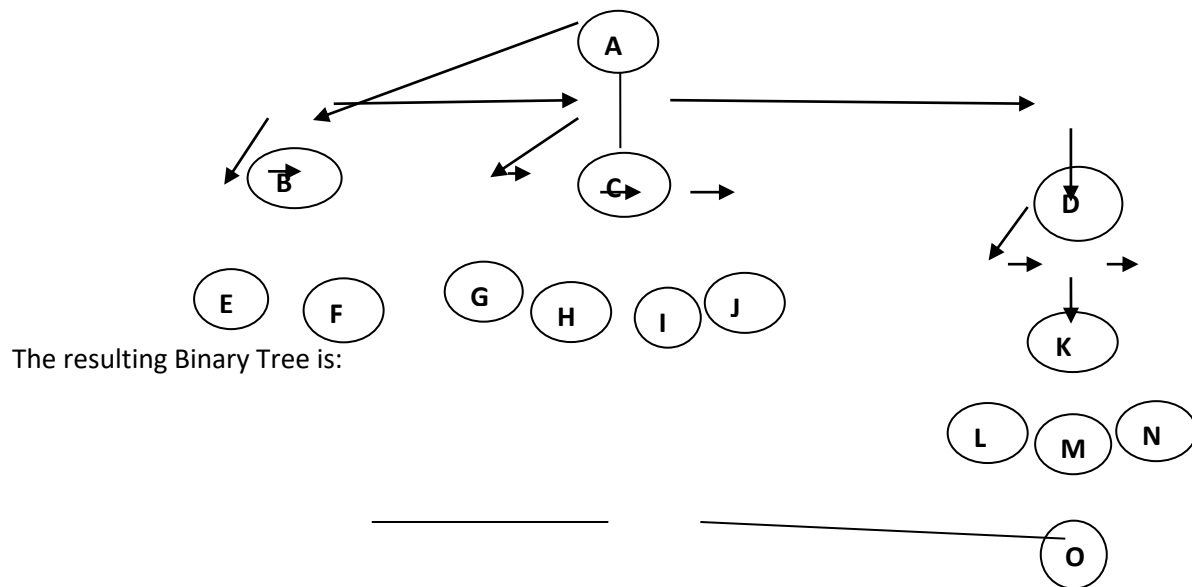
Step 1: We first identify the branch from a parent to its first left child. These branches become the left pointer in binary tree.



Step 2: Connects siblings, starting with the far left child, using a branch from each sibling to its right sibling.



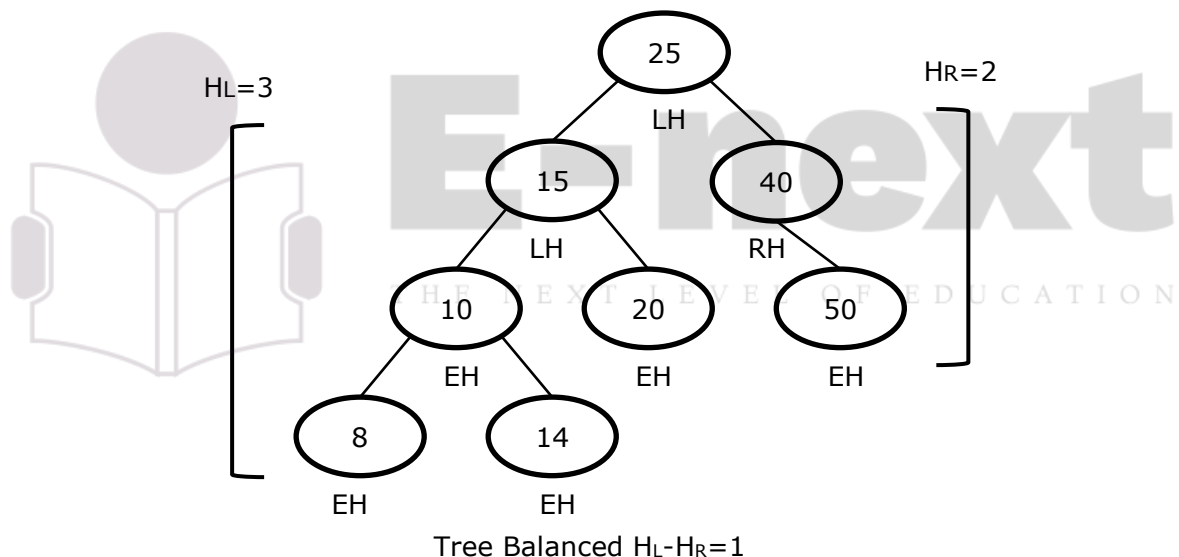
Step 3: Remove all unneeded branches from the parent to its children.



**Q. Define properties of AVL tree.**

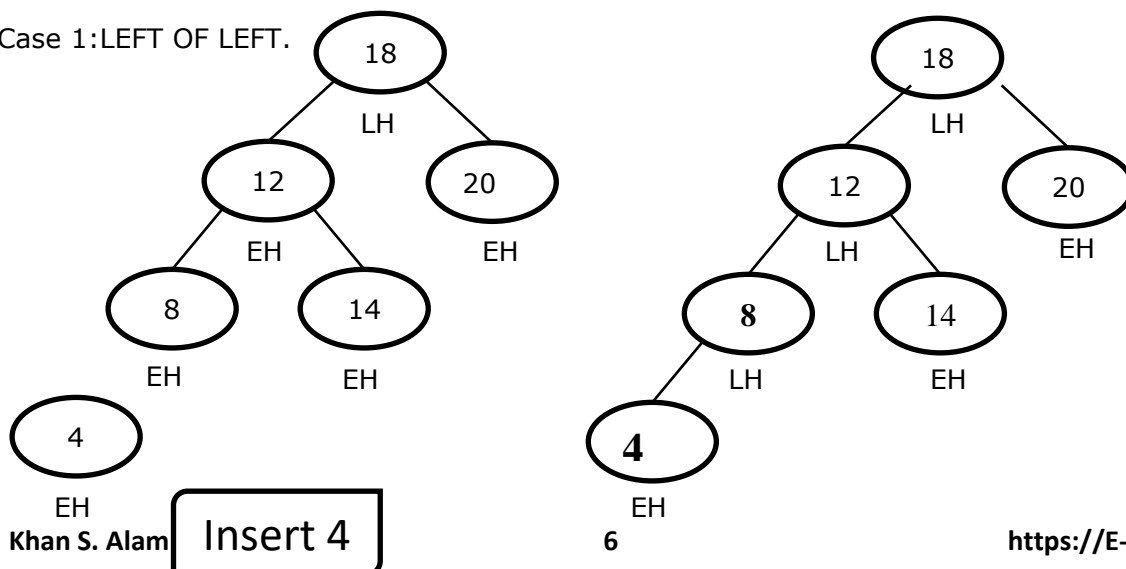
- a) A AVL tree is a tree in which no nodes can have more than two subtrees, the maximum outdegree for a node is two.
- b) These subtree are designated by LEFT subtree and the RIGHT subtree.
- c) Each subtree is a AVL tree itself.
- d) All key values at LEFT side will be less than node n RIGHT side keys are always greater than or equal to value of node.
- e) An AVL tree is a binary search tree in which the balance factor of every node, which is defined as the difference between the heights of the node's left and right subtrees, is either 0 or + 1 or -1.
- f) The balance factor of each node in an AVL tree is calculated as , the height of its left subtree minus the height of its right subtree.  
i.e.  $|H_L - H_R| \leq 1$
- g) If this difference is more than one, then it must so through a rotation so that the difference in their heights is not more than 1.
- h) The node with a balance factor of:
  - i) 1 is called left high or LH
  - 0 is called even high or EH
  - 1 is called right high or RH

**Example of AVL tree :**

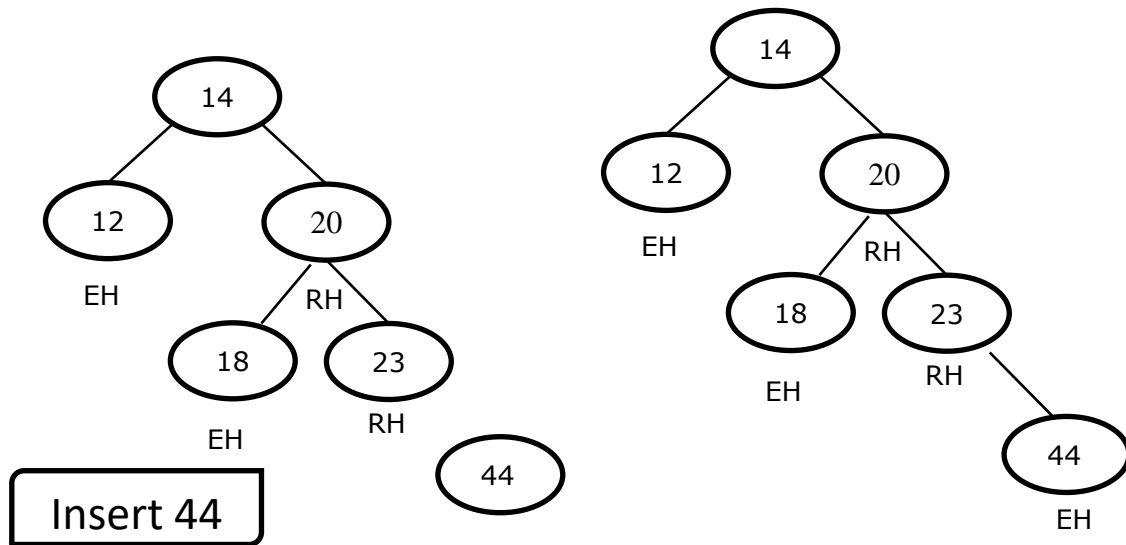


**Q. define 4 major cases and 8 sub cases of AVL tree**

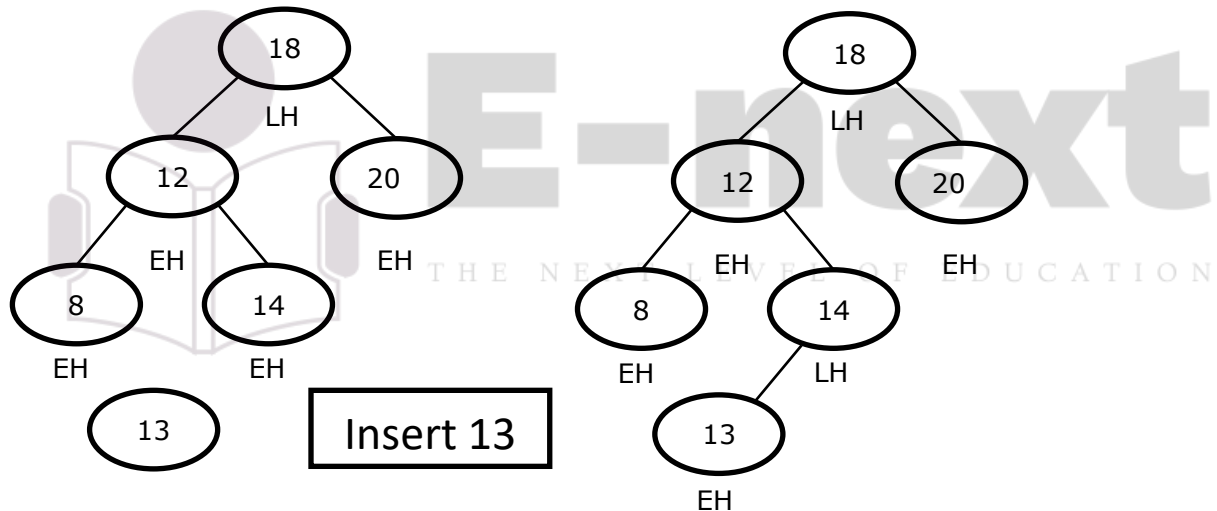
Case 1: LEFT OF LEFT.



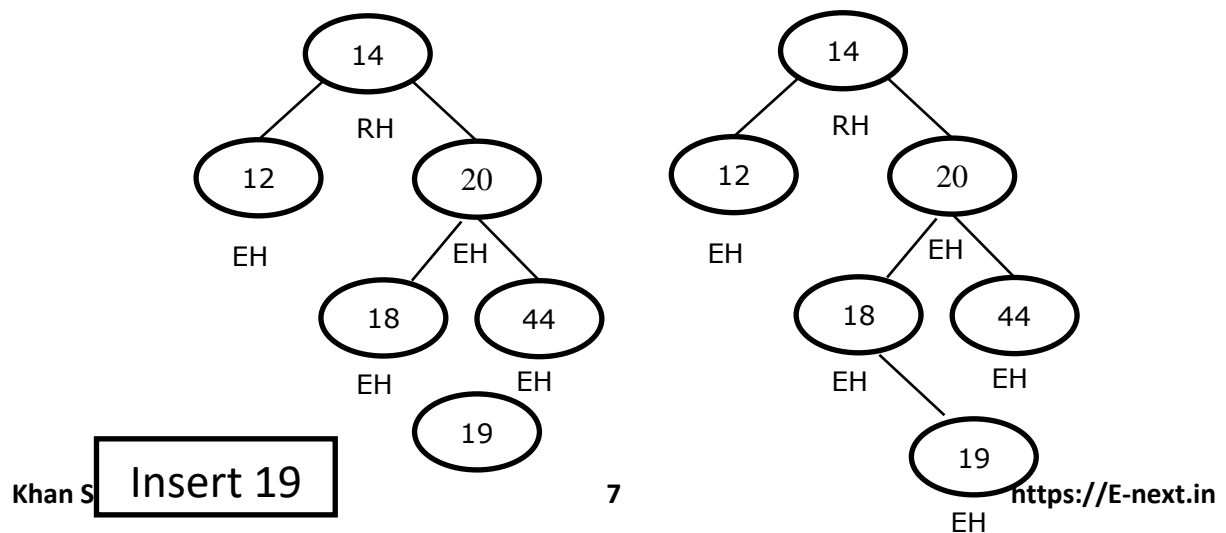
### CASE 2: RIGHT OF RIGHT



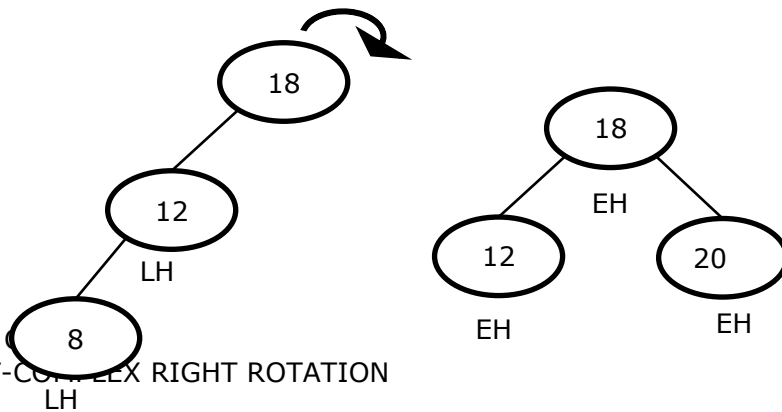
### CASE 3: RIGHT OF LEFT



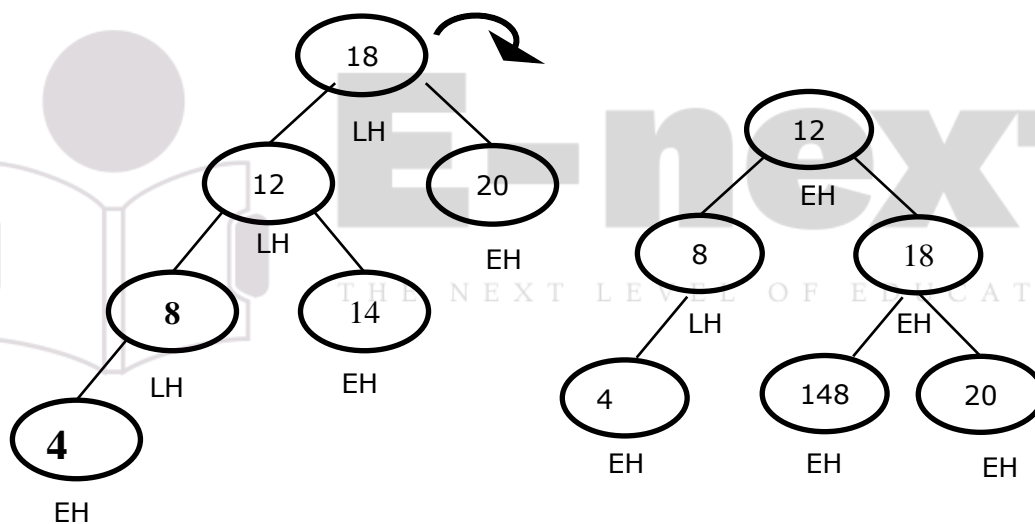
### CASE 4: LEFT OF RIGHT



CASE 1: SUB CASE A)  
LEFT OF LEFT-SIMPLE RIGHT ROTATION

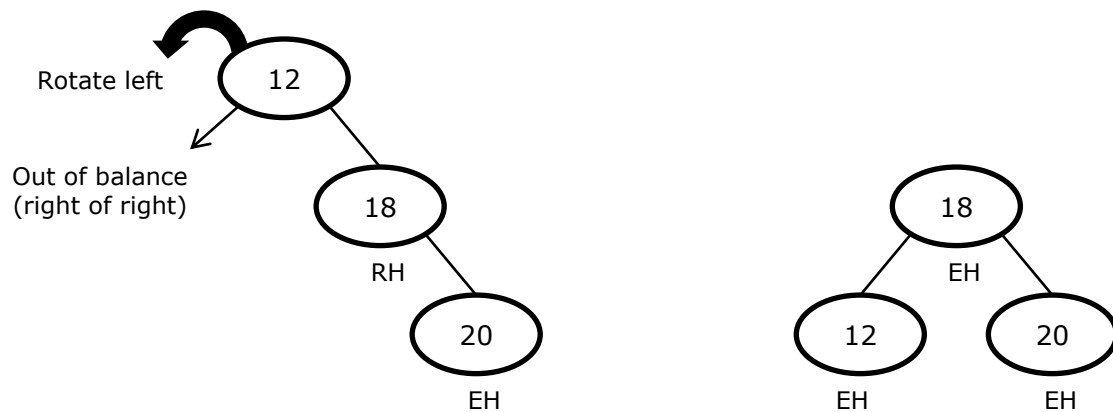


CASE 1: SUB CASE B)  
LEFT OF LEFT-COMPLEX RIGHT ROTATION

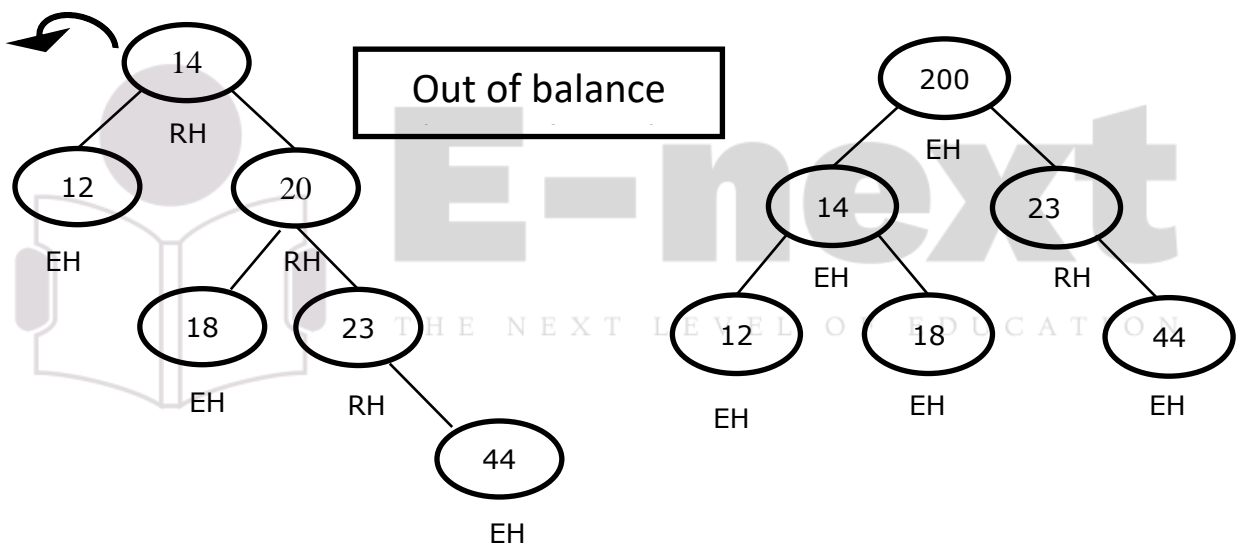




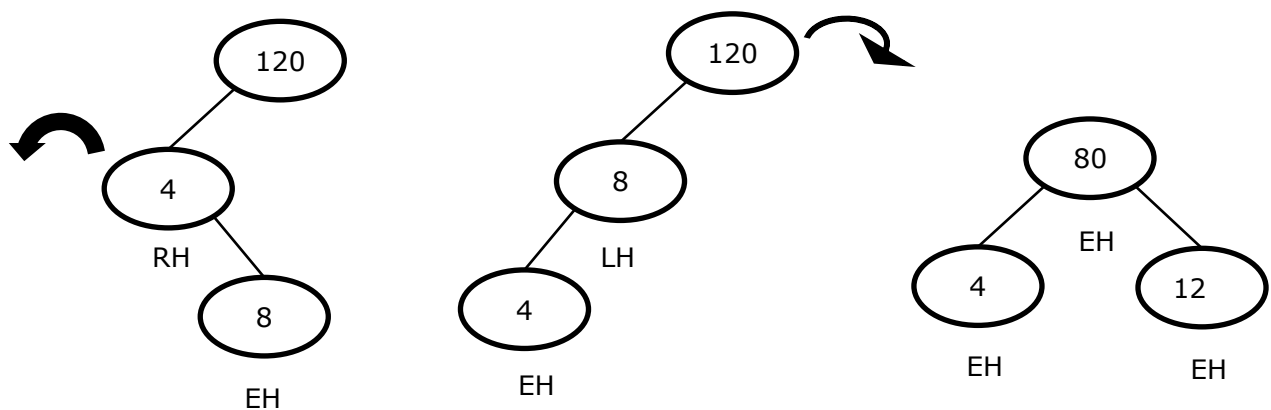
CASE 2: SUB CASE A)  
LEFT OF LEFT-SIMPLE RIGHT ROTATION



CASE 2: SUB CASE B)  
LEFT OF LEFT-COMPLEX RIGHT ROTATION

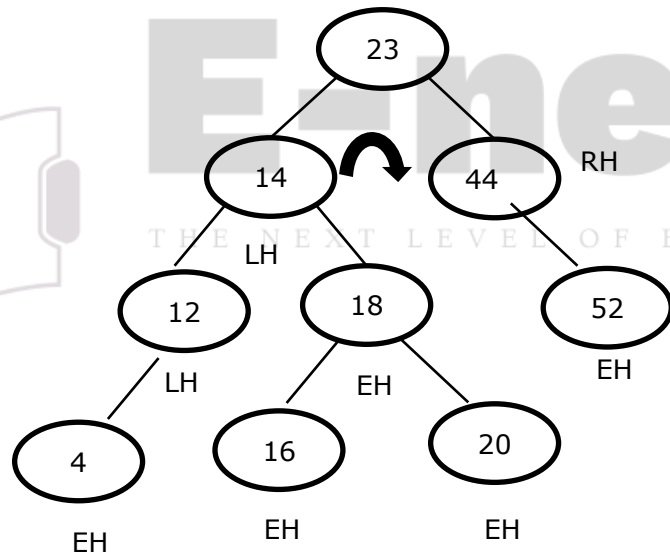
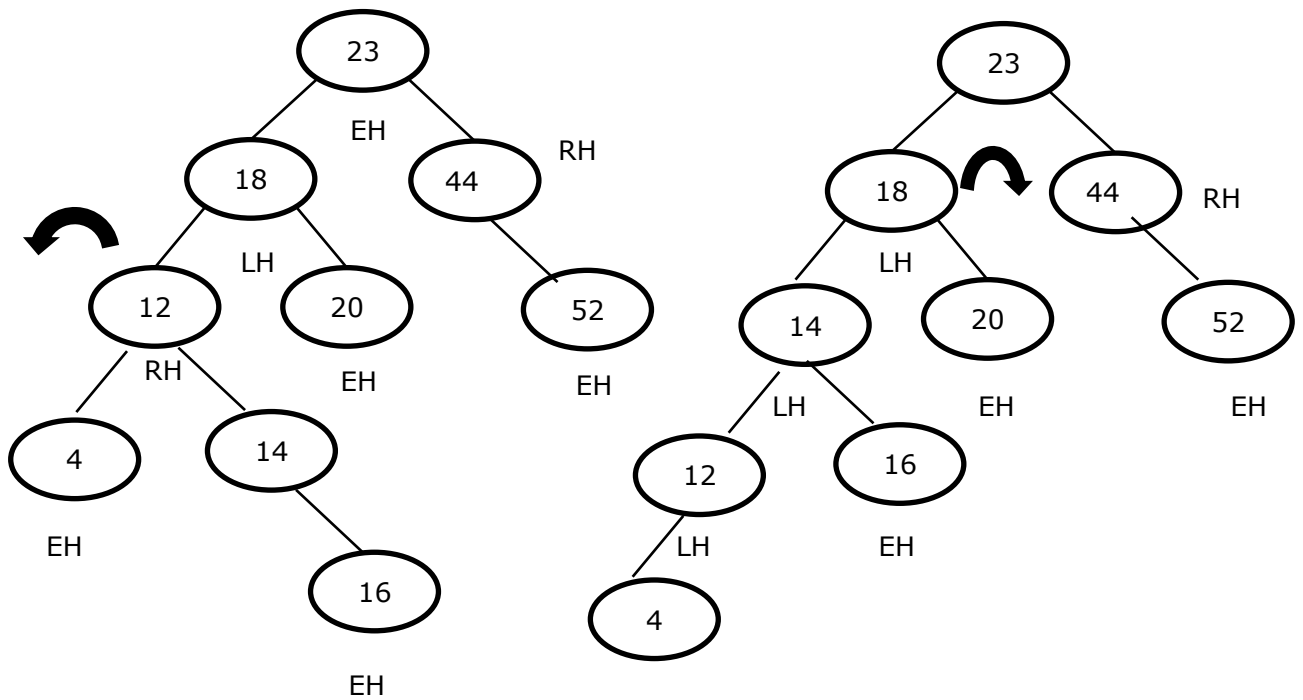


CASE 3: SUB CASE A)  
LEFT OF LEFT-SIMPLE RIGHT ROTATION

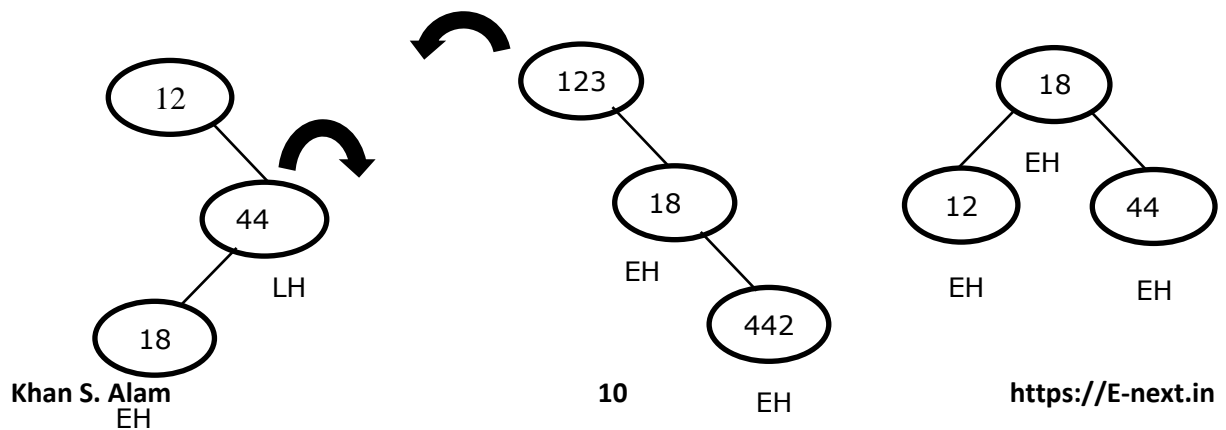


CASE 3: SUB CASE A)

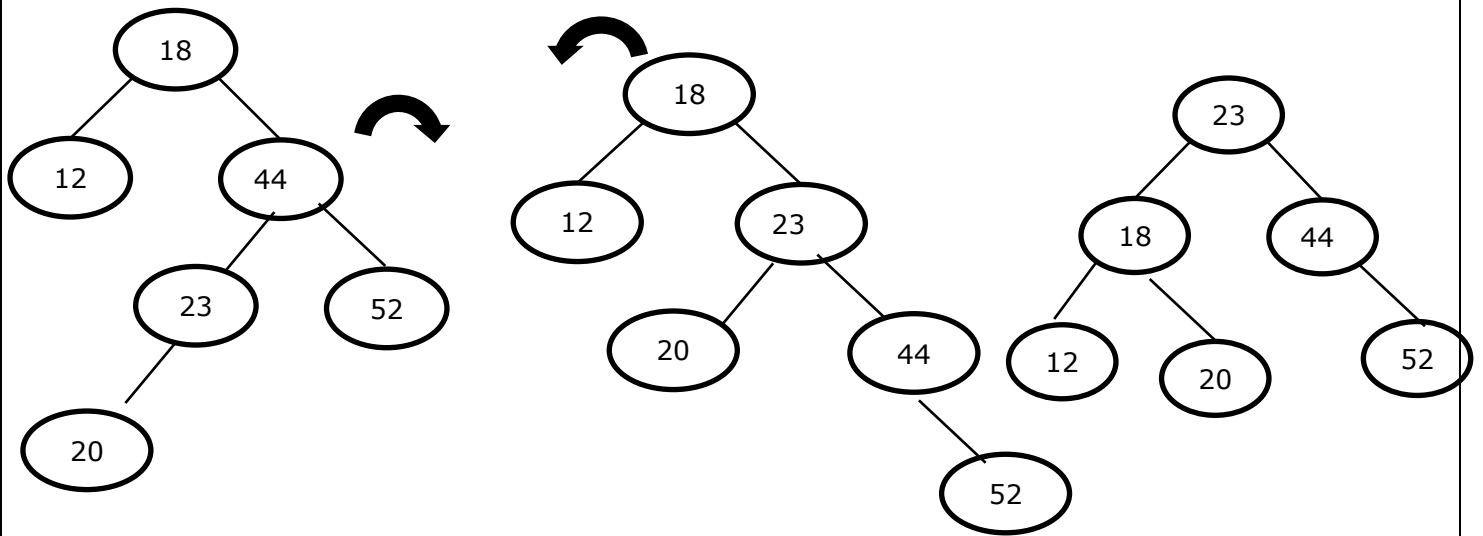
# LEFT OF LEFT-COMPLEX RIGHT ROTATION



## CASE 4: SUB CASE A) LEFT OF LEFT-SIMPLE RIGHT ROTATION



CASE 4: SUB CASE B)  
LEFT OF LEFT-SIMPLE RIGHT ROTATION



**E-next**  
THE NEXT LEVEL OF EDUCATION

**Q.What is meant by Balanced tree(AVL)? Draw an AVL tree for the following data arriving in sequence**

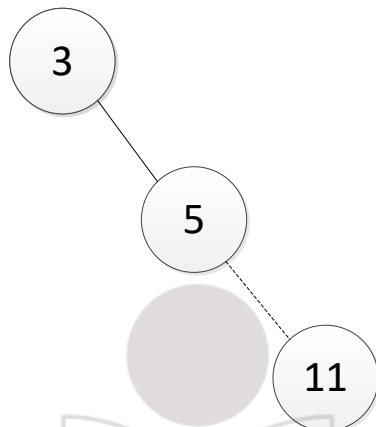
**3, 5, 11, 8, 4, 1, 12, 7, 2, 6, 10**

The balanced binary tree structure called the AVL tree, was designed and named after two Russian mathematicians **AdelsonVelskii and Landis(AVL)**.

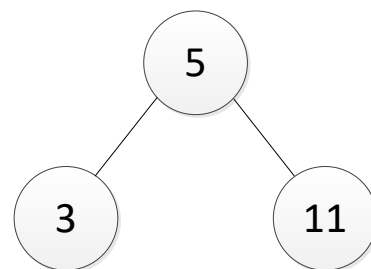
An AVL tree is a search tree in which the heights of the sub-trees differ by no more than 1. It is thus a **balanced binary tree**.

Since the heights of the sub-trees of every node in an AVL tree differ by no more than 1, the height of the tree is greatly reduced, thereby shortening the search time. Hence they are also known as **height balanced trees**.

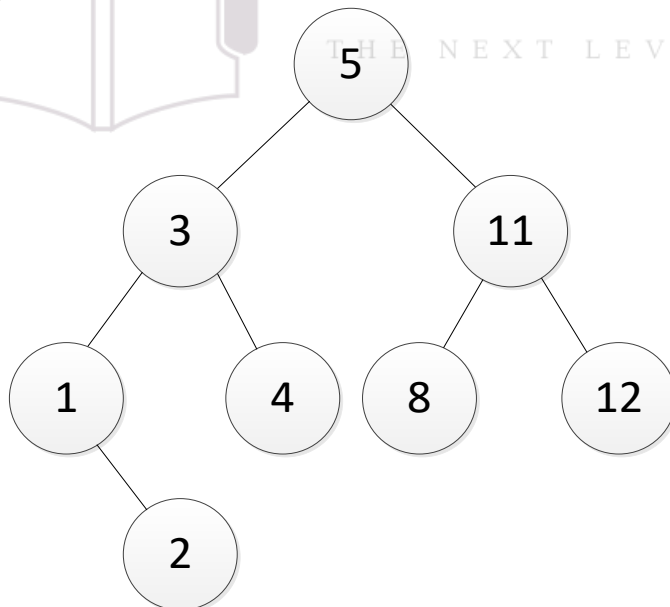
**Step 1:**



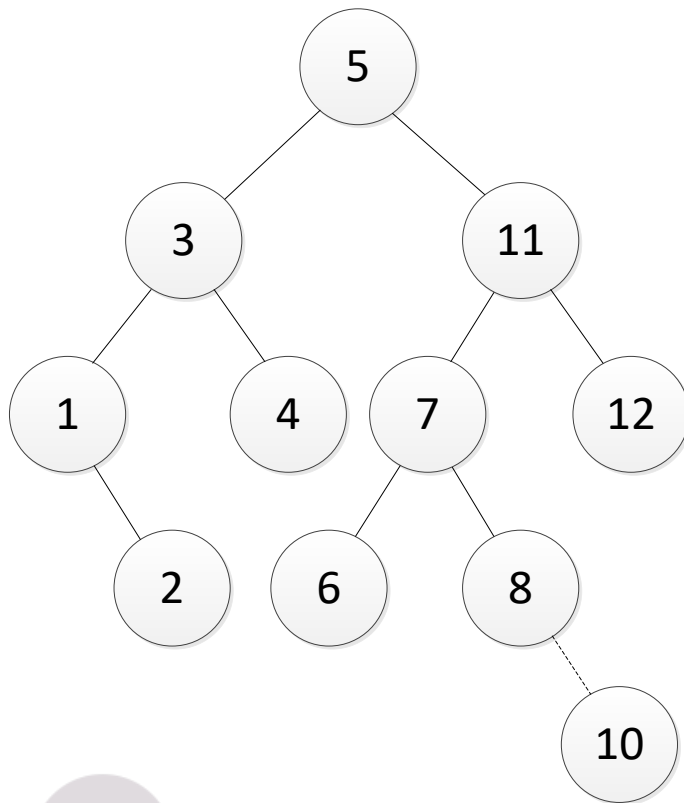
**Step 2:**



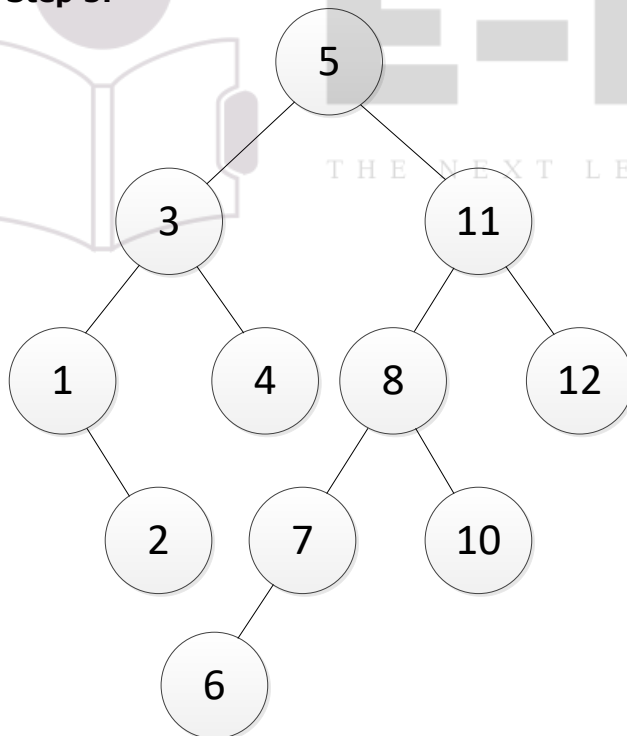
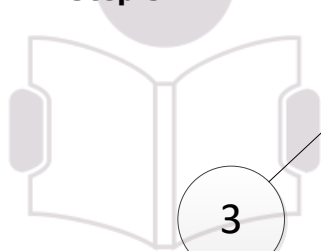
**Step 3:**



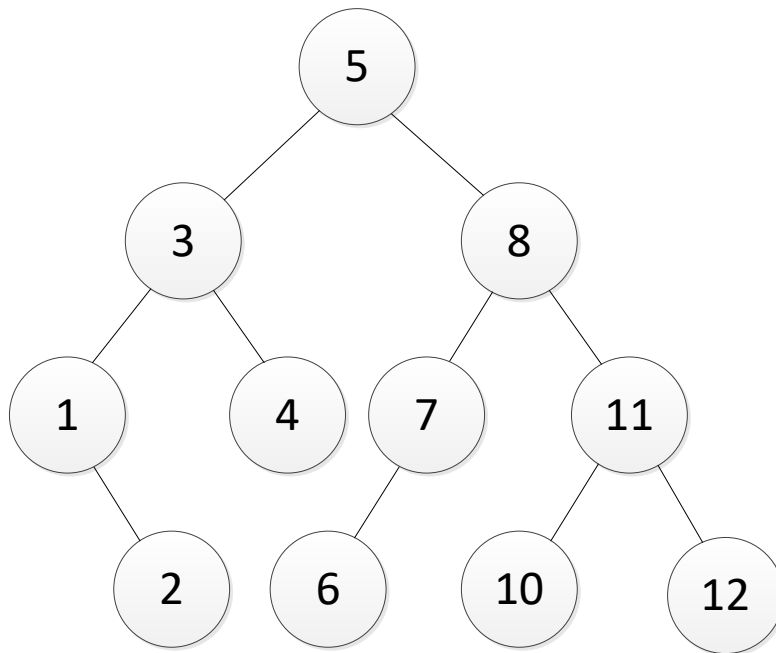
**Step 4:**



**Step 5:**

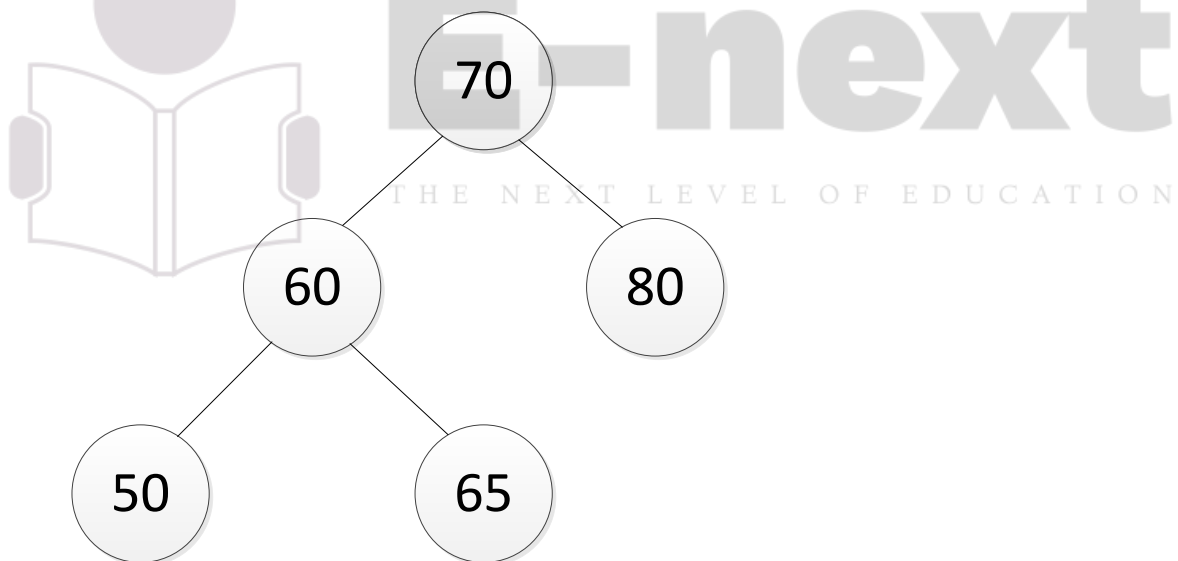


**Step 6:**



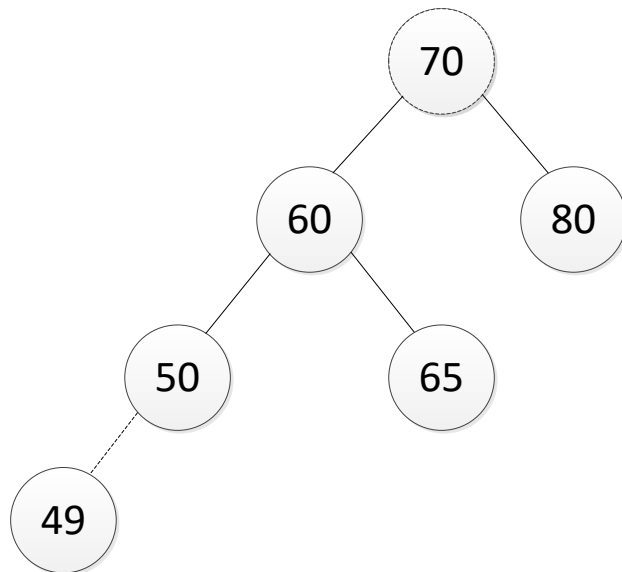
**1. Create an AVL tree for the following data occurring in sequence:**

**70, 60, 80, 50, 65**



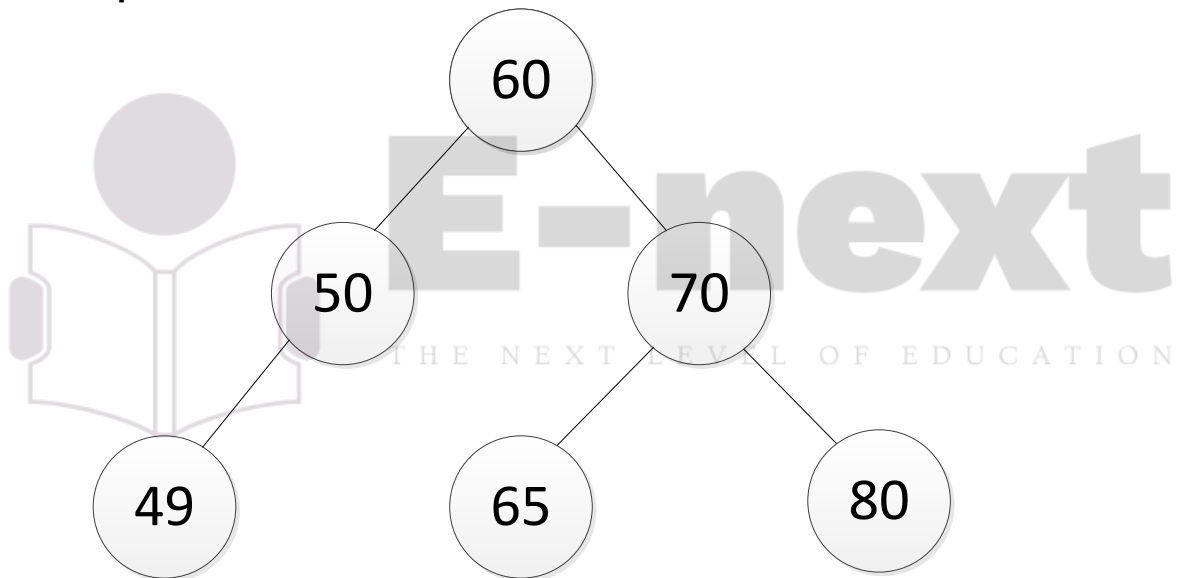
**A] Now Insert 49 and convert it to an AVL tree clearly starting the main case, sub case and the necessary rotations.**

**Step 1:**



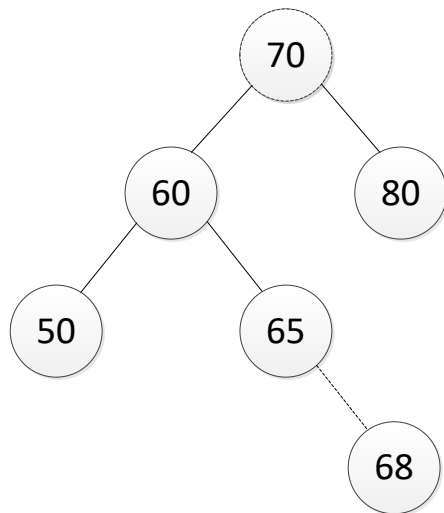
Here, 70 imbalanced  
Left of Right  
Complex Right of Right

**Step 2:**



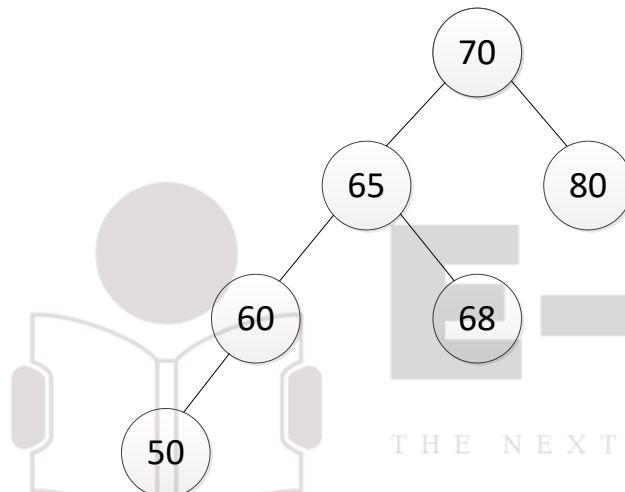
**B] Now insert 68 to the original tree above and do the necessary rotations mentioning all the details.**

**Step 1:**



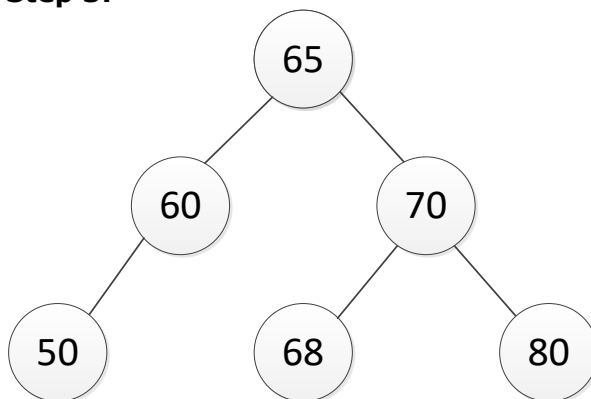
Here, 70 imbalanced  
Right of Left  
Simplex double rotation Right

**Step 2:**



After Left rotation

**Step 3:**

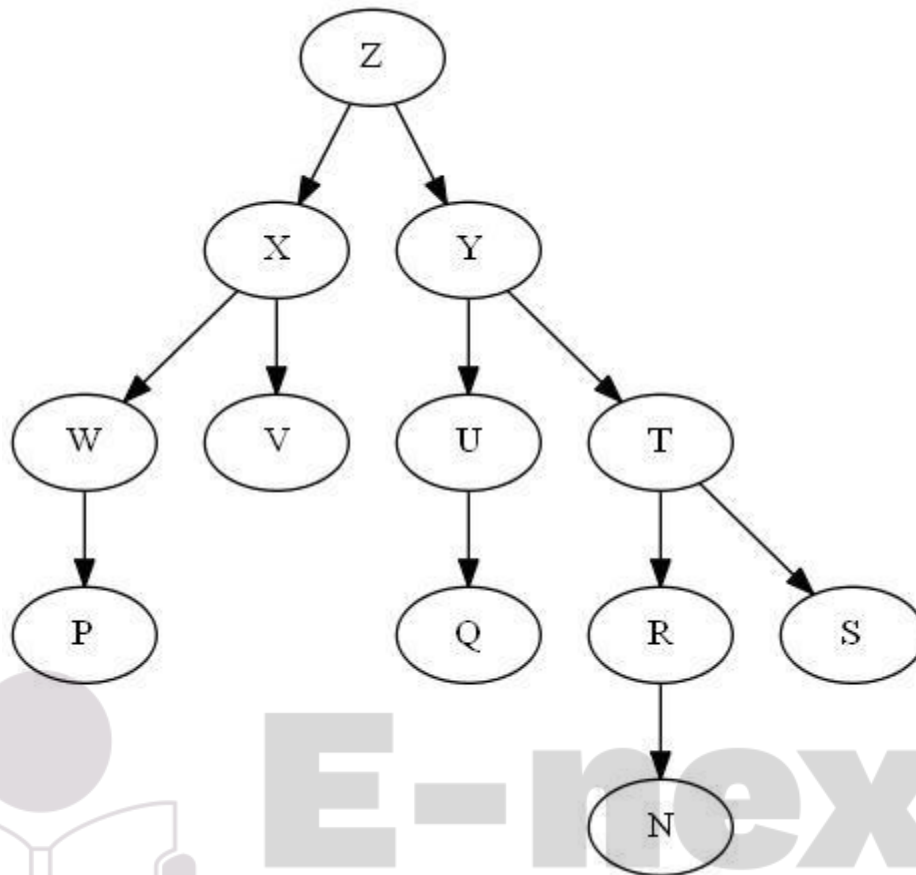


After Right rotation



**May2011– Q6 B– 10 Marks**

Q. Explain the preorder, postorder and inorder traversal of a tree with their algorithms. Give the preorder, postorder and inorder listing of the nodes of the following trees.



**Preorder:**

The depth-first traversal method is called preorder traversal. Preorder traversal is defined recursively as follows. To do a preorder traversal of a general tree:

1. Visit the root first; & then
2. do a preorder traversal each of the subtrees of the root one-by-one in the order given

**Algorithm Prefix(Preorder) Tree Traversal:**

Print the prefix expression for an expression tree.

Pre tree is a pointer to an expression tree.

Post the postfix expression has been printed

1. if (tree not empty)
    - i. print (tree token)
    - ii. prefix (tree left subtree)
    - iii. prefix (tree right subtree)
  2. end if
- end postfix

**Postorder:**

In contrast with preorder traversal, which visits the root first, postorder traversal visits the root last. To do a postorder traversal of a general tree:

1. Do a postorder traversal each of the subtrees of the root one-by-one in the order given; & then
2. Visit the root.

**Algorithm Postfix (Postorder) Tree Traversal:**

Print the postfix expression for an expression tree.

Pre tree is a pointer to an expression tree.

Post the postfix expression has been printed

1. if (tree not empty)
    - i. postfix (tree left subtree)
    - ii. postfix (tree right subtree)
    - iii. print (tree token)
  2. end if
- end postfix

**Inorder:**

Inorder traversal only makes sense for binary trees. Whereas preorder traversal visits the root first and postorder traversal visits the root last, inorder traversal visits the root in between visiting the left and right subtrees:

1. Traverse the left subtree; & then
2. Visit the root; & then
3. Traverse the right subtree.

**Algorithm Infix (Inorder) Tree Traversal:**

Print the infix expression for an expression tree.

Pre tree is a pointer to an expression tree.

Post the postfix expression has been printed

1. if (tree not empty)
    - i. prefix (tree left subtree)
    - ii. print (tree token)
    - iii. prefix (tree right subtree)
  2. end if
- end infix

**Notations of the tree:**

**Preorder:** Z X WPVYU QT RNS

**Postorder:** PWV XQU NRS TYZ

**Inorder:** PWX VZQ U YNRT S

**May 2011 – Q5 B – 10 Marks**

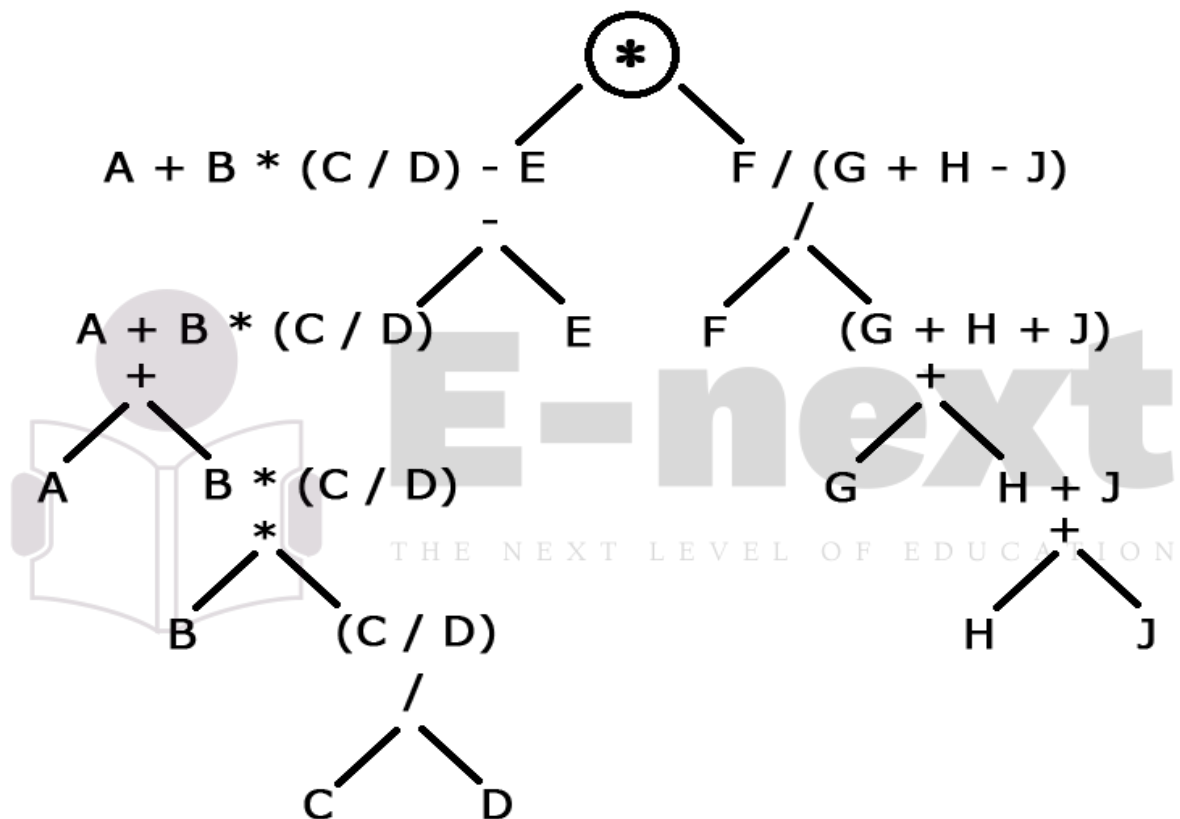
Q. Define an expression tree. The following infix expression is given. Draw the expression tree and find the prefix and the postfix expression.  
(A + B \* (C / D) - E) \* (F / (G + H - J))

**Expression Tree:**

An expression tree is a special kind of tree. In this, the nodes contain one, two or zero children.

Some of the properties of the Expression tree are:

- Each leaf is an operand.
- The root & the internal nodes are operators.
- Subtrees are subexpressions with the root being an operator.



**Prefix**(Preorder) Expression:      \* -+A \* B/C D E/F + -GH J

**Postfix**(Postorder) Expression:      A BCD /\* + E- FGH J -+/\*

**Dec 2010– Q3A– 10 Marks**

A binary tree has 8 nodes. The inorder and the postorder traversal of the tree is given below:

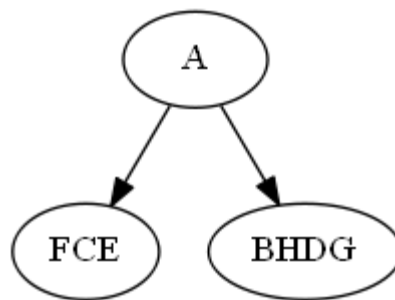
Postorder: F ECHGDBA

Inorder: F CEA BHD G

Show a stepwise reconstruction of the binary tree along with its Preorder traversal

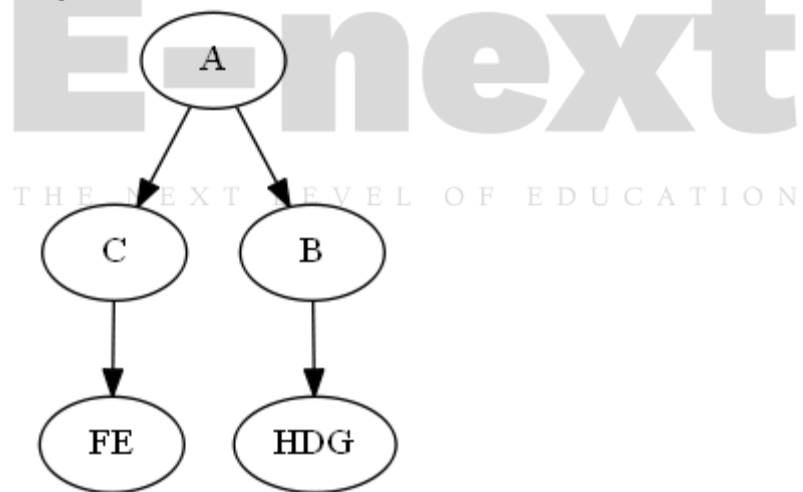
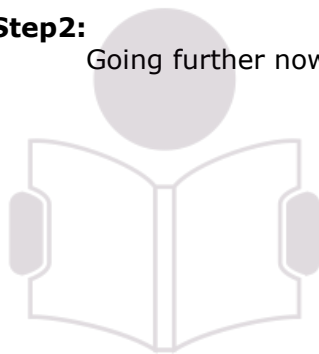
**Step1:**

By the postorder expression we can determine the root. In this case, the root is A. Taking A as the root & seeing the Inorder expression, we get FCE at the left of it & BHDG to its right.



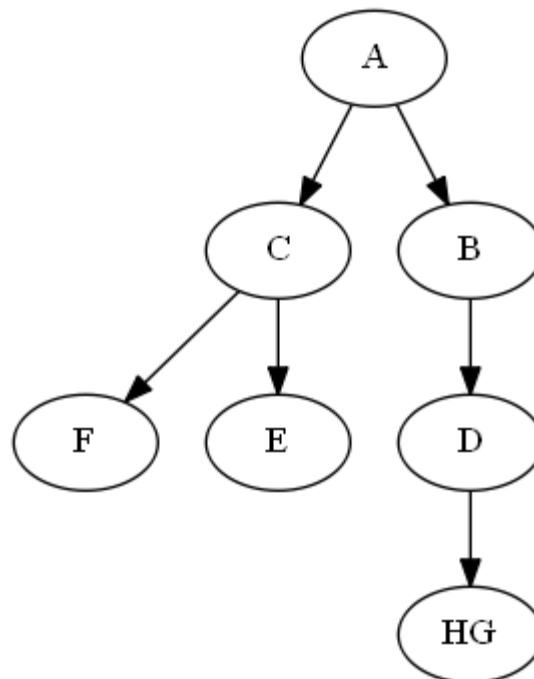
**Step2:**

Going further now, we will get.

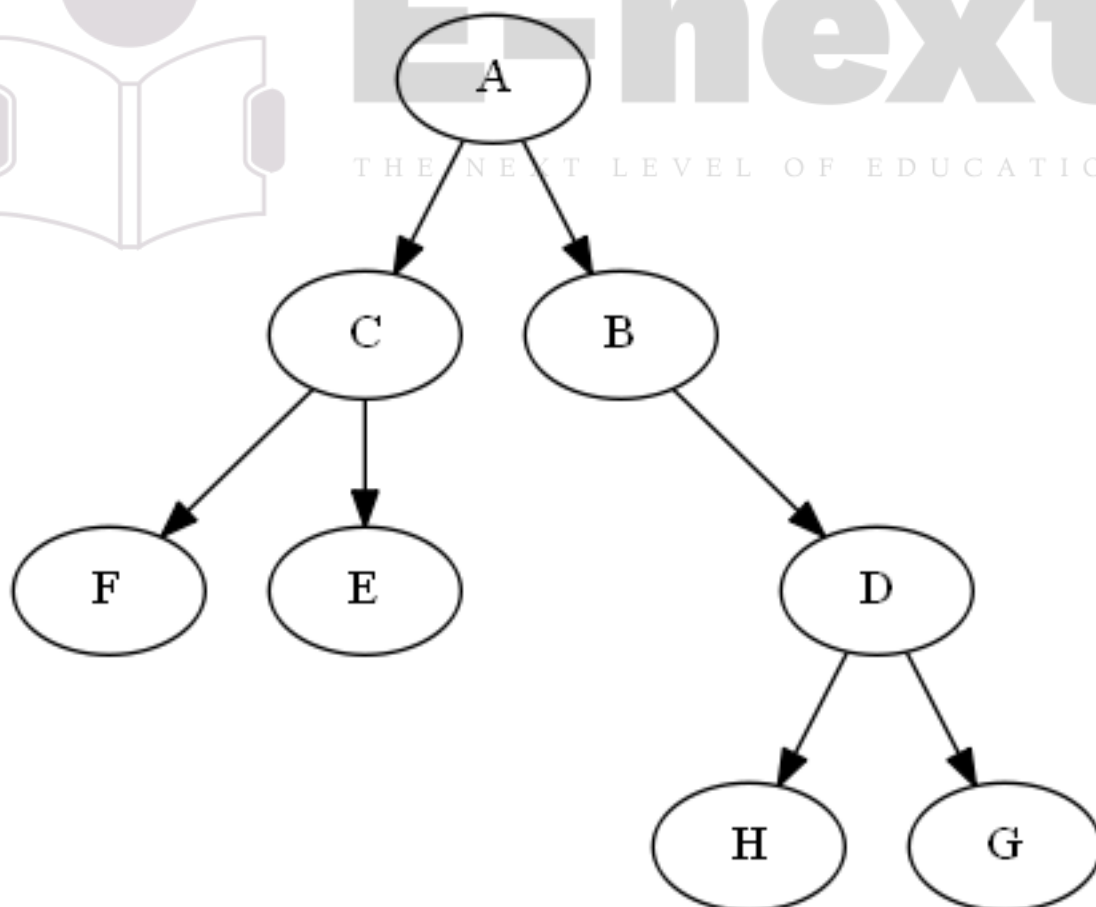


**Step3:**

Splitting the nodes in the left subtree & in the right subtree, we get.

**Step4:**

In this we split up the final remaining node HG in the roots of D. The Final Expression tree is given as:



**Prefix**(Preorder) Expression:

**A CF EBDH G**

**May2010– Q7 B– 10 Marks**

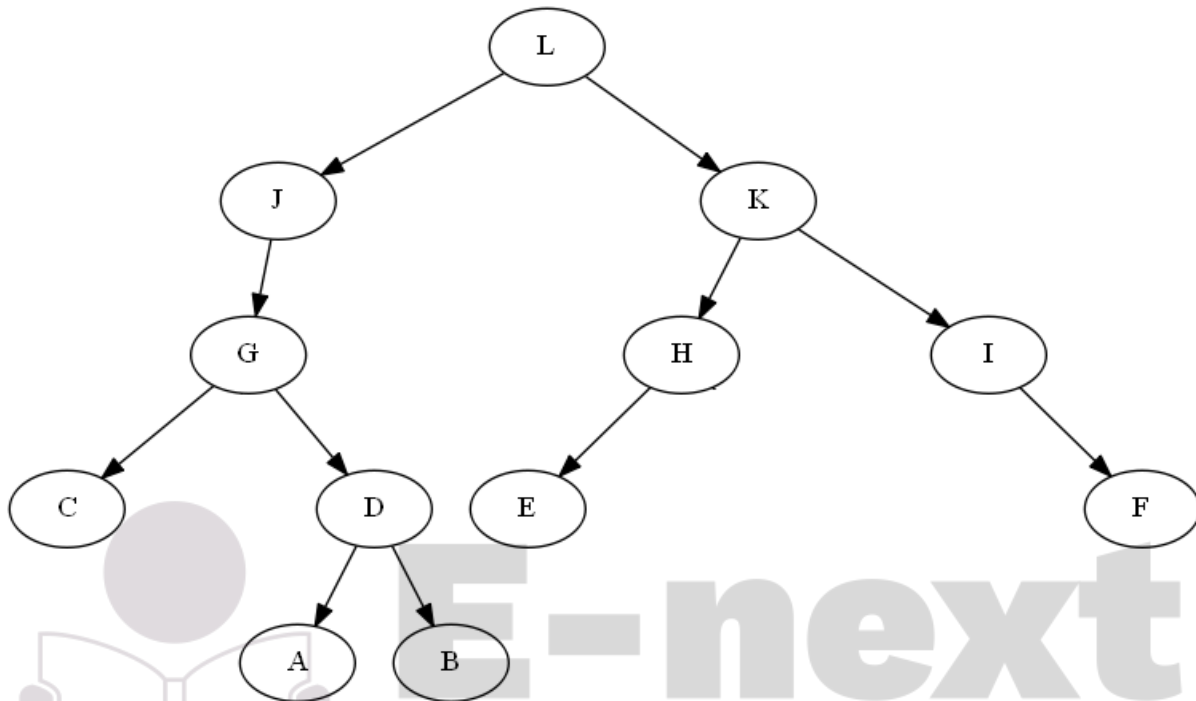
Draw the tree given Preorder & the Inorder traversal below:

Preorder: L J GCDABKHE IF

Inorder: C GADBJLEHK IF

Also Give the Postorder traversal of the tree.

Write Algorithm for traversing a tree.

**Expression Tree:**

**Postfix (Postorder) Expression:** C A BD GJ EHF IKL

**Algorithm Prefix (Preorder) Tree Traversal:**

Print the prefix expression for an expression tree.

Pre tree is a pointer to an expression tree.

Post the postfix expression has been printed

3. if (tree not empty)
  - i. print (tree token)
  - ii. prefix (tree left subtree)
  - iii. prefix (tree right subtree)

4. end if
- end postfix

**Algorithm Postfix (Postorder) Tree Traversal:**

Print the postfix expression for an expression tree.

Pre tree is a pointer to an expression tree.

Post the postfix expression has been printed

- if (tree not empty)
  - i. postfix (tree left subtree)
  - ii. postfix (tree right subtree)
  - iii. print (tree token)

- end if
- end postfix

**Algorithm Infix(Inorder)TreeTraversal:** Print the infix expression for an expression tree. Pre tree is a pointer to an expression tree. Post the postfix expression has been printed

- if (tree not empty)
    - i. prefix (tree left subtree)
    - ii. print (tree token)
    - iii. prefix (tree right subtree)
  - end if
- end infix

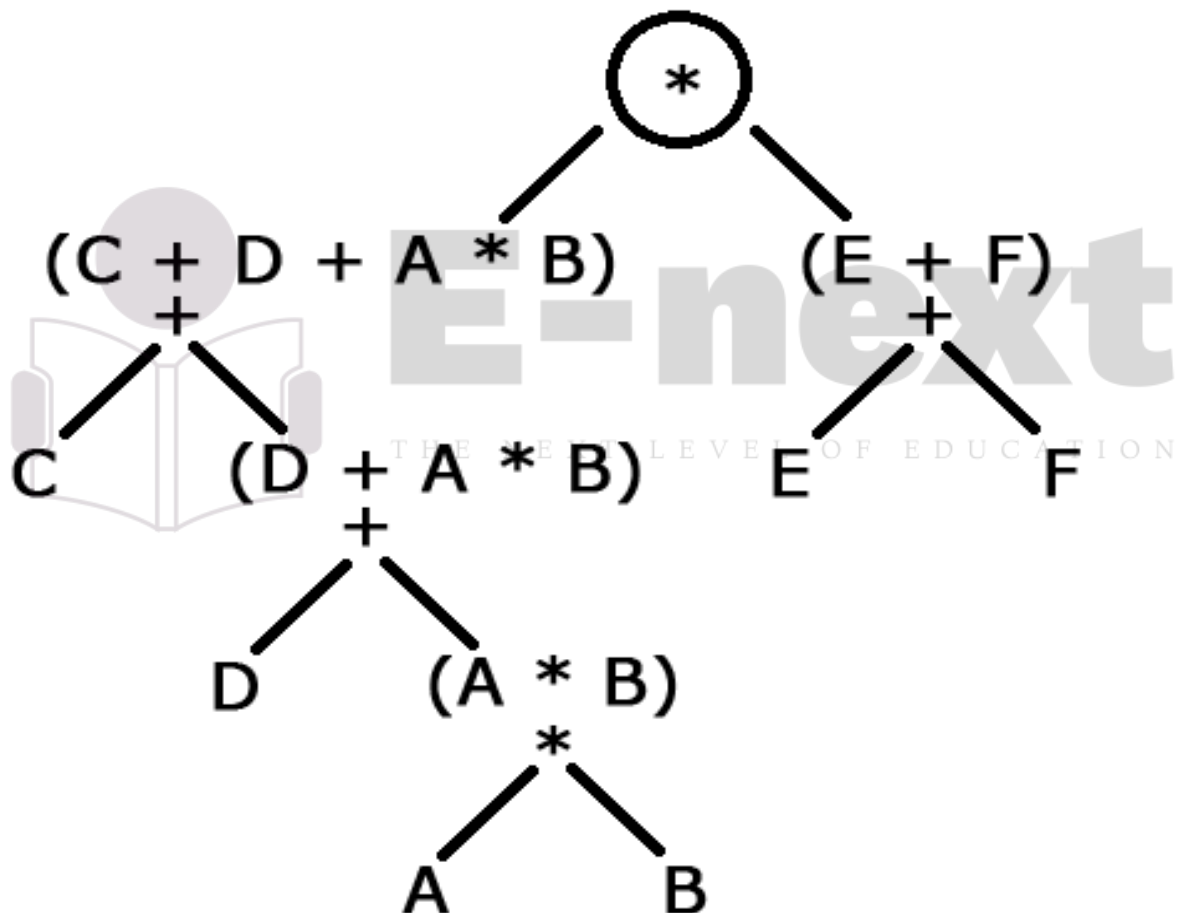
**Dec 2009– Q7A– 10 Marks**

Define an expression tree. Following Infix expression is given. Draw the expression tree.

Find the Prefix & Postfix expression

$(C + D + A * B) * (E + F)$

Expression Tree:



**Prefix**(Preorder) Expression:  $* + C + D * A B + E F$

**Postfix**(Postorder) Expression:  $C D A B * + + E F + *$

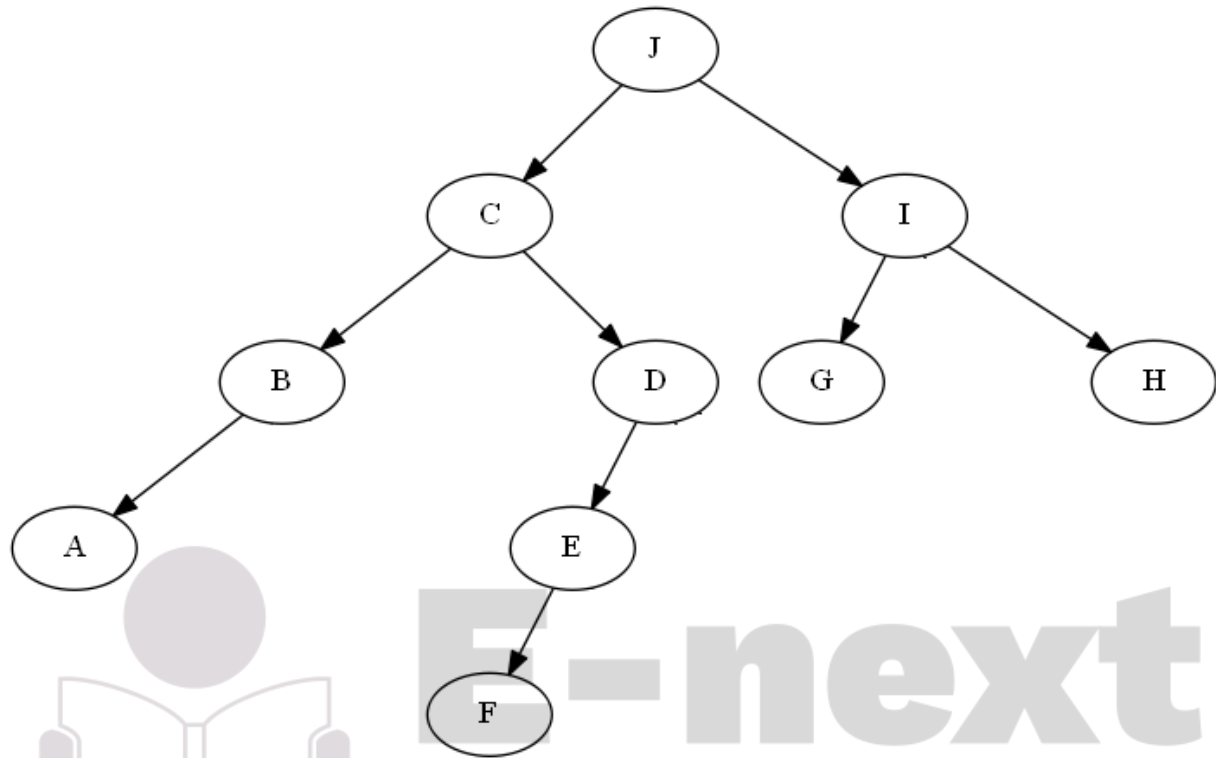
**May 2009– Q5A– 10 Marks& May 2006 – Q1 A – 10 Marks**

A binary tree has 10 nodes. The inorder and preorder traversal are shown below:

Inorder:     A B C D E F J G I H

Preorder:    J C B A D E F I G H

Show the binary tree, along with its Postorder traversal



**Postfix**(Postorder) Expression:   A B F E D C G H I J