Signal theory, linear filters

① In image processing, we have discrete spatial resolution (N^2). Signals are represented by sequences of numbers (not continuous functions)

② Important properties of image signal sequences include:

(i) separability → can be transformed into a matrix only if they are separable.

(ii) Periodicity → we assume periodicity. (signal is infinite)

(iii) causality → we assume causality. i.e. no value <1 $s(n_1, n_2) = 0$ $n_1 < 0$, $n_2 < 0$

③ Convolution is the basis of linear shift invariant systems

2d convolution

$$\underbrace{s(n_1, n_2)}_{\text{image}} ** \underbrace{h(n_1, n_2)}_{\text{impulse response.}} = \sum_{k_1=-\infty}^{+\infty} \sum_{k_2=-\infty}^{+\infty} s(k_1, k_2) \cdot h(n_1 - k_1, n_2 - k_2)$$

$$= \sum_{k_1=-\infty}^{+\infty} \sum_{k_2=-\infty}^{+\infty} h(k_1, k_2) \cdot s(n_1 - k_1, n_2 - k_2)$$

* image is bounded in both ~~directions~~ dimensions signal is limited.

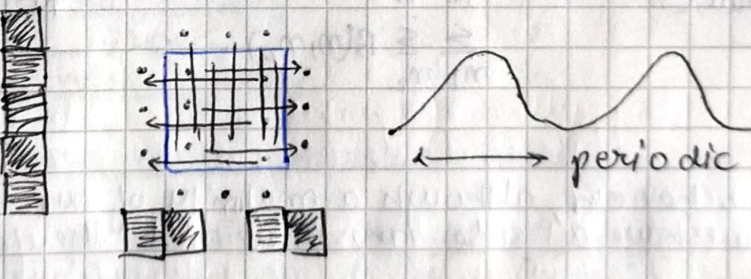
④ we can have different kinds of convolution based on how we simulate infinity. we extend our matrix in different ways.

(a) Linear convolution → continuation of the image with zeroes. Putting 0s above, below, left or right of our image. Not possible to do fourier transform multiplications, because it is not periodic.

(b) Periodic convolution → Periodic continuation of the image. we continue

* we come closer to our signal theory.

* If we want to simulate infinity to infinity, we can use periodic not circular.



(c) ~~Periodic and~~ circular → Periodic continuation of the image with half the filter width.

(d) Reflection → Periodic continuation of the image by is reflection with half the filter width. (left, right, top & bottom)

⑤ when talking about convolutions, we talk about filters. usually we have 2 types of filters.

(a) Infinite impulse response systems (IIR) → implemented as a recursive algorithm; result dependent on the foregoing result).

$$y(n_1, n_2) = \sum_{k_1=-\infty}^{+\infty} \sum_{k_2=-\infty}^{+\infty} x(n_1 - k_1, n_2 - k_2) \cdot a(k_1, k_2) - y(n_1 - k_1, n_2 - k_2) \cdot b(k_1, k_2)$$

(b) finite impulse response systems (FIR) → implemented as a non-recursive algorithm, result independent of the predecessor

$$y(n_1, n_2) = \sum_{k_1=-\infty}^{+\infty} \sum_{k_2=-\infty}^{+\infty} x(n_1 - k_1, n_2 - k_2) \cdot \underbrace{a(k_1, k_2)}_{\text{impulse response}}$$

* These methods are the basis for the various transformations that we have such as DFT, DCT, wavelet transform.

We do linear transformations from spatial domain to frequency domain so that our computations get reduced. Eg. Convolutions in spatial domain gets converted to simple multiplication in frequency domain.

⑥ Discrete Fourier Transform (frequency domain)

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$$B_N = \begin{cases} \alpha & n_1 \leq N;\ n_2 < N \\ 0 & \text{otherwise} \end{cases} \qquad \alpha = \frac{1}{\sum\limits_{n_1}^{N} \sum\limits_{n_2}^{N} B(n_1, n_2)} = \frac{1}{N^2} \qquad Box_3 = \frac{1}{9}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

① Due to sinc behaviour, although a majority of our behaviour has low frequencies, we have a side maximal and therefore we also let some HF pass.