Digital filters

① The general distinction of the type of filters used can be
(a) linear filters → 2D-convolution
filter matrices
frequency domain methods

(b) Non-linear filters → morphology
adaptive procedures.

② In general, the digital filters are used for:
(a) Image denoising
(b) image enhancement and restoration
(c) edge enhancement, reconstruction and detection

③ The general form of filter equation is in the form of

$$y(n_1,n_2) = \sum_{k_1=N_{11}}^{N_{12}} \sum_{k_2=21}^{N_{22}} \underbrace{a(k_1,k_2)}_{\text{filter coefficients}} \cdot s(n_1-k_1, n_2-k_2) - \sum_{k_1=m_{11}}^{m_{12}} \sum_{k_2=m_{21}}^{M_{22}} b(k_1,k_2) \cdot y(n_1-k_1, n_2-k_2)$$

} if $b=0$, we obtain an FIR filter.

④ An FIR filter convolves the image with a window. This is the most common operation in image processing.

Types of FIR filters <
- low pass filters (noise suppression)
- High pass filters (Edge enhancement, detection)

⑩ A low pass filter only allows low frequencies to pass through and blocks all the high frequencies. Noise is considered as high frequency therefore low pass filters are used for noise suppression.
Two types (a) box filters
(b) binomial filters.

Box filter → calculates the mean greyvalues specified by the filter.

disadvantage → ① not all ~~comp~~ high frequency components are suppressed.
② direction sensitive. (an-isotropic). Towards rows and columns its okay, but towards diagonal, it behaves differently. (diff. damping behaviour)
③ causes phase shift in certain frequency ranges.

Binomial filter → Basically represented as gaussian function.

$$g(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

But since we are in discrete domain the impulse response is approximated by the binomial coefficients.

$$\binom{a}{b} = \frac{a!}{(a-b)!\,b!}$$

} coefficients can be derived from Pascal's triangle.

$$G_{2N-1|2M-1}(n_1,n_2) = \underbrace{\sum_{m_1=1}^{N}\sum_{m_2=1}^{M} a(m_1,m_2)\cdot\binom{2N-1}{n_1+N}\cdot\binom{2M-1}{n_2+M}}$$ for $|n_1| < N$, $|n_2| < M$

This equation then gives the usual gaussian matrix in which the weight is maximum at the centre and reduces as we go away from it.

advantages ① filters are nearly isotropic
② No phase shift.

* The convolution of a gaussian with another gaussian leads to a gaussian function. This is really a low pass filter.

(ii) High pass filters only allow high frequencies to pass through and blocks almost all the low frequencies.
Edges can be characterised as the sudden change in brightness level. We can use this information to define our high pass filters.

Edges have high differential quotients in their regions. These high filters have a differentiating property.

(a) Difference operator
(b) Sobel operator
(c) Kirsch operator

(a) Difference operator → Simplest form of high pass filter. It uses two filter masks for calculating the differential quotient in the x and y directions.

$$D_1 = \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix}}_{\text{triggers on vertical}} \qquad D_2 = \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{pmatrix}}_{\text{triggers on horizontal}}$$

Disadvantages ① sensitive to noise
② an-isotropic (direction dependent)

(b) Sobel operator → Here, an integration is performed in the direction perpendicular to the direction of differentiation. This decreases the noise sensitivity considerably.

$$S_1 = \frac{\partial x}{\partial s_1} \begin{pmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{pmatrix} \qquad S_2 = \frac{\partial y}{\partial s_2} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix}$$

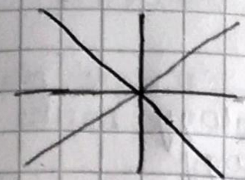
* Advantage of sobel is that we can represent it in a complex form.

$$\boxed{S = S_1 + j S_2}$$

phase → direction information.

(c) Kirsch operator — In this, 8 filter masks are employed · 1 for each direction and the maximum is taken.

* The disadvantage is that the direction information is lost.



with the different coefficients (3, -5)

disadvantage → no direction information. (we should store) information.

⑫ Laplacian of Gaussian (LoG) filter is the 2nd derivative of the gaussian
used for finding edges, borders, and edge enhancement.

$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = \frac{1}{\pi\sigma^4}\left(1 - \frac{x^2+y^2}{2\sigma^2}\right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$

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morphology - I

field of digital image processing.

① morphology is a method in which an image is represented as collection of sets.

② Defining image as a set, and then every set has a shape. Image is seen as a set, pixels are seen as elements of this set · used for identifying and segment the object in the image.

③ Non-linear filters → These filters are used in wide range of applications. As we can design our image acquisition systems for specific application, we can also design our own filters to get more accurate results.

* These filters depict a loss in information. This should be taken into account if the image is subjected to further processing.

(i) Threshold operator
(ii) Rank order filters, especially the median filter.
(iii) Morphological operator.

④ Rank order filter → ① As we know that linear filters also take neighbour pixel into consideration, every pixel value (even an outlier) contributes to the result. So some of the outliers influence will always be there in our final result.

② median filter is the idea that before we take the information in a filter matrix, we sort the matrix (ascending / descending) i.e. we set up the Rank order in the filter.

③ additionally, we also have max, min and median filter in Rank order filters.

④ usually we take the odd no. of pixels and select the centre pixel.

Advantages ① "Salt and pepper noise" is eliminated.
② outliers do not have any effect on the result
③ edges remain sharp.

Disadvantages ① Fine structures are eventually lost (in regions smaller than half the ROI width)

$$\begin{pmatrix} 0 & 10 & 0 \\ 0 & 10 & 0 \\ 0 & 10 & 0 \end{pmatrix} \rightarrow (0, 0, 0, 0, \boxed{0}, 0, 10, 10, 10)$$

↳ result.

② The corners gets rounded $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 10 & 10 \\ 0 & 10 & 10 \end{pmatrix} \rightarrow (0,0,0,0,\boxed{0},10,10,10,10)$

③ computationally intensive (search and sort)

⑤ morphology → The study of forms. we can classify
(i) Binary image morphology
(ii) Grey value morphology
(iii) Colored image morphology.

Basic operations $<$ Erosion / dilation

Basic idea → Interpretation of image as a set of structures. The set of coordinates of all the white pixels in a binary image is the complete description of the image.

$$B = \{(n_1, n_2) \mid s(n_1, n_2) = 1\}$$ 0 → Background
1 → foreground.

Sets operations ① Translation → $(A)_x$ → A is shifted by a step size x.

$$(A)_x = \{ y \mid y = a + x,\ a \in A \}$$

Union ② $A \cup B \rightarrow \{ y \mid y \in A \vee y \in B \}$

Intersection ③ $A \cap B \rightarrow \{ y \mid y \in A \wedge y \in B \}$

Compliment ④ $A^c \Rightarrow$ what was inside the set is now outside the set and vice-versa

$$A^c = \{ y \mid y \in M \wedge y \notin A \}$$

Transposition ⑤ $\breve{A} \rightarrow$ change the shape of the set.

Dilation → $A \oplus B = \{ x \mid (\breve{B})_x \cap A \neq 0 \}$

A → Image; B → structuring element → The form of the structuring element influences the result of the operation

① Transpose the structuring element
② Roll over the image.

Results → Dilation generally increases the size of edges (dep. upon the type of mask)

~~Erosion →~~ $\cancel{A \ominus B = \{ x \mid (B)_x \subseteq A \}}$

Once it is used, we cannot revert the dilation effect. Therefore it is non-linear. once changed, its gone.

Erosion → $A \ominus B = \{ x \mid (B)_x \subseteq A \}$

$A \ominus B \neq B \ominus A$ {not commutative}

$(A \ominus B) \ominus C \neq A \ominus (B \ominus C)$ {not associative}

$(A)_x \ominus B = (A \ominus B)_x$ {Translation invariant}

$(A \cup B) \ominus C = (A \ominus C) \cup (B \ominus C)$ {distributive}

Relation b/w dilation and erosion $\boxed{(A \ominus B)^c = A^c \oplus \check{B}}$

Working → The final result will contribute only if the structuring element is a complete part of the image. when structuring element and image match completely.

★ Erosion and dilation are dual operators, an erosion does not invert the dilation and vice-versa.

⑥ Combination of dilation and erosion < opening / closing

Both of these have band pass characteristics and are idempotent.

Binary image opening → $A \circ B = (A \ominus B) \oplus B$ {erosion followed by dilation}

It opens the connected structures

Binary image closing → $A \bullet B = (A \oplus B) \ominus B$ {dilation followed by erosion}

It connects the near structures

Properties

$A \circ B = A$	A is open wrt B
$A \bullet B = A$	A " closed " "
$(A \circ B) \circ B = A \circ B$	idempotent
$(A \bullet B) \bullet B = A \bullet B$	
$(A \bullet B)^c = A^c \circ \check{B}$	dual operators

⑦ Hit or miss transformation → ① Extension of base operation (erosion / dilation)

② In addition to the foreground pixels, it also takes the background pixels into account

③ Structuring element → combination of 2 sets < background X_H / foreground X_V

④ A pixel is part of the result of the HMT if at this point, the elements of X_V are completely within the object and the elements of X_H are completely in the background.

$A \odot X = \{ y \mid (X_V)_y \subseteq A, (X_H)_y \subseteq A^c \}$

image, structuring element

OR

$\boxed{A \odot X = (A \ominus X_V) \cap (A^c \ominus X_H)}$ → HOMT as intersection of two erosions.

① HOMT corresponds to an erosion, with X_V, if $X_V = 0$

② " " " " dilation with X_H, if $X_H = 0$

$X_V = 0 \rightarrow A \odot X = A \ominus X$

$X_H = 0 \rightarrow A \odot X = A \oplus X$

End point finder → Can find the end points of the lines.

⑧ Applications of morphology

① morphological gradient → It triggers on gradient in the image.

Two types → Inner morphological gradient :

$$\boxed{\nabla_{m_i}(f) = f - (f \ominus g)}$$ (Taking out the inner part of original image)

with these, we can select either the inner border line or the outer border line to further our task.

→ outer morphological gradient

$$\boxed{\nabla_{m_a}(f) = (f \oplus g) - f}$$ (outer edge of the image)

(g should be symmetric and as small as possible)

→ morphological laplace operator :

$$\underbrace{\Delta_{m_i}(f)}_{\text{functions}} = (f \oplus g) + (f \ominus g) - 2f$$ } suited better for greyscale morphology.

② morphological Reconstruction → we used conditional morphological operators

- It is a dilation, which is only performed on specific predefined image areas.

conditional dilation → $A \oplus_B X = \min\{A \oplus X, B\}$ $\quad \forall x : B(x) \geq A(x)$

based on boundary limits, we can reconstruct the image or preconditions.

Dilation with unity structural element : $\delta_B^{(1)}(A)$

n^{th} geodetic dilation of A under condition B : $\delta_B^{(n)}(A) = \delta_B^{(1)} \circ \delta_B^{(2)} \dots \circ \delta_B^{(1)}(A)$

Reconstruction of Binary Images : $A, B \in Z^2$, $B \subseteq A$

Reconstruction of mask $A \in Z^2$ by marker $B \subseteq A$: $\left\{ \rho_a(B) = \bigcup_{n \geq 1} \delta_n^{(n)}(B) \right\}$

③ Skeletonisation → we derive from an object its inner structure. If we keep doing erosion, a point will come where it cannot be further eroded. (we will have a 1 pixel wide length)

→ Hit or miss transformation can also be used in skeletonisation.

$$S_1 = \begin{pmatrix} 1 & * & 0 \\ 1 & 1 & 0 \\ * & * & 0 \end{pmatrix} \qquad S_2 = \begin{pmatrix} * & 0 & 0 \\ 1 & 1 & 0 \\ * & 1 & * \end{pmatrix}$$

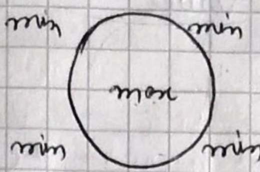
don't care

foreground

background

skeletonisation is also possible with "distance transformation".

→ Distance transformation represents the mapping of a real numbes which is the minimal distance to the edge - to every pixel.



Since the max value is at the centre, it can be used for skeletonisation.

distance of black pixels to the white pixels. circles of iso distance values.

(4) Granulometry ① with morphology we can easily find out the distribution of particles sizes in an image.

② In this, the image is opened with structuring elements of increasing size. The difference b/w the original and the opened image is calculated after every stage.

③ Finally, the results are normalised and plotted in a "Histogram of Particle size".

(4) useful for: (1) determination of distribution of particle size
(2) determine object size and number.

A granulometry is an operator system (γ_λ) applied to a complete lattice with properties:

① γ_λ is a monoton
② γ_λ is anti-extensive
③ $\gamma_\lambda \gamma_\mu = \gamma_\mu \gamma_\lambda = \gamma_\mu$ for $\mu \geq \lambda$

(I) always the first sieve is effective
(II) from the image of a certain resolution, all images of lower reso. can be obtained.

(III) Requirement "open" → " filter away black areas with a sieve → opening

As an output we get:

(I) Volume spectrum → Ratio of the remaining black areas $F_x(x)$ to the total black in the original image. i.e. distribution function depicted as a function of the sieve size. → SE

$$F_x(x) \approx H(x) = \sum_{-\infty}^{i(x_i)} h(x = x_i)$$

(II) Pattern spectrum →

$$h(x) = \sum_{-\infty}^{i(x_i)} h(x_i)\, \delta(x - x_i)$$

⇒ Grey scale morphology: Basic assumptions:

(1) The image is a function (topographic relief).
(2) The structuring element is a function.

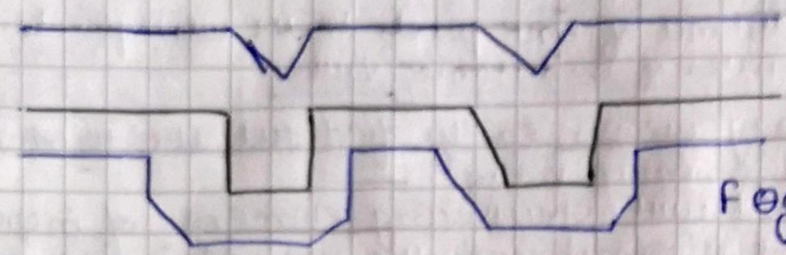
Operations: ① Reflection $\check{g}(x) = -g(-x)$

② Morphological shifting $(f_x + y)(z) = f(z - x) + y$

moving the SE by a distance alongside the greyscale topography (image)

Grey scale dilation → $(f \oplus g)(z) = \max\{f(x) - \breve{g}_z(x) \mid x \in D[f]\}$
→ maximum value

Grey scale erosion → $(f \ominus g)(z) = \min\{f(x) - g_z(x) \mid x \in D[f]\}$
→ minimum value



$f \oplus g$ → increase the brightness level & reducing the dark details. we do not change the image size

$f \ominus g$ → reduced brightness level and widening the dark parts.

→ closing → suppresses small dark features
→ opening → suppresses details.

Applications of morphology

① Top hat transformation → used for emphasizing high frequent structure information in a signal and to remove low frequent information in a signal.

$h(\xi) = p(\xi) - p(\xi) \circ e^r(\xi)$ $e^r(\xi)$ → plane structuring element of size r (no real topography)

$d(\xi) = p(\xi) \bullet e^r(\xi) - p(\xi)$ p → image

decreases the greyscale level overall

$\boxed{q(\xi) = h(\xi) - d(\xi)}$

→ increases greyscale level overall

Result: Suppresses coarse and highlights fine structures.

Note Areas / Regions lost due to erosion cannot be recovered with dilation and vice-versa because the structuring element would not fit anymore. (since there is nothing to fit)

* Additive and multiplicative noise is more related to the acquisition of image time. meaning during image acquisition, due to (eg. illumination changes) we can have additive or multiplicative noise.