Texture Analysis

1. Texture →
   1. consists of stochastic or deterministic patterns
   2. elementary structures, also called textons or texels which occur repeatedly in an image. or randomly.
   3. elementary structures which have regular patterns themselves.
   4. micro/macro-textures. (scale, football field)

Texel → smallest distinguishable but describable part of the texture.

2. Tasks of texture analysis →

Texture → Synthesis, Analysis

Synthesis: used in gaming industry

Analysis → modelling, Segmentation

modelling → Synthesis

modelling → classification

Segmentation → classification

classification → identification / selection

Applications:
1. Life sciences
2. Quality control
3. Evaluation of satellite images.

modelling → Analytical description of texture
→ Starting point for texture synthesis

segmentation → division of a textured image into regions which have the same features of texture.

classification → assignment of a texture to one of n texture classes.

3. classes of texture models:
   1. structural texture model
   2. statistical " "
   3. structured stochastic texture model.

1. structural texture models →
   1. made up of exactly defined elementary patterns. (texel)
   2. the configuration follows a rigid scheme (we know its distribution in the image)
   3. can be specified by elementary patterns.
   4. Not suited for modelling natural textures.

$$f(x,y) = \left[\sum_{i=-b}^{+b} \sum_{j=-d}^{+d} \delta(x-N_1 i,\ y-N_2 j) \right] * g(i,j)$$

texel

position coordinates of individual texels.

2. Statistical texture models →
   1. The texture to be analyzed is considered to be a sample realization of a stochastic process.
   2. statistical parameters can be used to characterize the texture.
   3. suitable for describing natural textures.
   4. A visual distinction is possible, only if textures differ wrt. statistics of 2nd ~~lower~~ order or lower.

$S(\xi, n_1, n_2)$ $\qquad f_{n_1 n_2 m_1 m_2}(n_1, n_2, m_1, m_2) = f_{s_1 s_2}(s_1, s_2)$

difficult, as the joint density function is not known and only 1 realization is ~~possible~~ available

solution → using cooccurence matrix
→ Auto correlation function of texture

③ structured stochastic texture models → ① Texture is not specified on the basis of configuration but on that of the relative frequency of occurance of elementary forms.
② A texture which is somewhat deterministic is randomly distributed wrt space coordinates.
③ well suited for correlation analysis

④ Real life textures have characteristics which are partly random and partly deterministic

④ Global texture features with statistical methods

(1) First order stastistics → mean value, variance (not very reliable)

(2) Tool based on 2nd order statistics →

① Auto correlation function $R_{ss}(\Delta m_1, \Delta m_2)$

↳ complete specification for Gauss process
↳ reveals internal structures in the texture
↳ detection of periodicities.

modelling of these textures can be done by:

① Gauss process
② Poisson process → pointwise occurences of features
③ Fractal model → statistical self-similarity at various resolutions (

⑤ Cooccurrence matrix → ① gives an estimate of ~~grey~~ joint density function of the grey values in an image.

② $s(x,y)$ → image with integer grey values
(dx, dy) → shift vector ('dipole vector')

③ matrix

$$C_{dx,dy}(g_1, g_2) = \sum_{i=0}^{N_1-dx-1} \sum_{j=0}^{N_2-dy-1} \delta(s(i,j)-g_1)\cdot\delta(s(i+dx, j+dy)-g_2)$$

① GLCM matrix
② Normalised GLCM matrix

original image → co-occurance matrix → 2nd order stastistics → features: f_1, f_2, ⋮, f_n

Haralick features → "features" reduce the quantity of data necessary for conveying the information contained in the 'raw' data.

① Entropy → ① measures the disorder or 'complexity' of an image.
② Entropy is large when the image is not texturally uniform.
③ Complex textures → high entropy.

$$m_E = \sum_{g_1=0}^{G} \sum_{g_2=0}^{G} \tilde{C}_{dx,dy}(g_1,g_2) \log(\tilde{C}_{dx,dy}(g_1,g_2))$$

② Contrast → ① measures the spatial frequency of an image and difference moment of GLCM.
② measures the amount of local variations present in the image.

$$m_C = \sum_{g_1=0}^{G} \sum_{g_2=0}^{G} \tilde{C}_{dx,dy}(g_1,g_2)(g_1-g_2)^2$$

③ Homogeneity → ① Also called inverse difference moment
② measures image homogeneity as it assumes larger values for smaller grey tone differences in pair elements.
③ It has maximum values when all elements in the image are same.

$$m_H = \sum_{g_1=0}^{G} \sum_{g_2=0}^{G} (\tilde{C}_{dx,dy}(g_1,g_2))^2$$

⑥ Texture filters → ① Texture filters are employed to highlight certain structures or features in the texture and are useful for feature extraction as a preliminary step for classification.

Types → LAWS (empirical approach)
ADE ("Eigen filter")
CHEN-DAI ("least squares - filter")

LAWS → ① classical approach:
② 5 1-d filters → 25 separable 2D filters
③ typical size → 3x3, better 5x5, because some features cannot be represented by 3x3.
④ By combining, we can have a set of 25 filters.

ADE eigenfilter → These are 3x3 or 5x5 filters, the elements of which are determined from the normalised eigenvectors of the covariance matrix of the image signal in the window.
→ The obtained filters can be considered to be matched to the features of the signal. Filters are orthogonal.

chen-dai filter → ① The original image is assumed to consist of superposition of R different known and mutually disjoint ~~features~~ textures. The filters are designed based on these ~~f~~ textures.

Recognition of local and global features is facilitated by:
① selecting the appropriate filter size
② reducing the image size.

Feature Extraction

① Feature → A quantitative measure of some property of the object.

* Always try to avoid redundant features.

② A feature should have the following properties:

① Separation sharpness → The features should be distinctly diff. from different object.

② Reliability → The feature should vary little for 1 and same object

③ Independence → The variations of the features of 2 "similar" objects should be statistically independent of each other.

④ Low number → Independence decreases with the no. of features, ∴ the no. of features should be small.

③ Features → ① can be defined in original (diameter, area, size, colour) and in frequency (high frequency, low frequency) domain.

② can be based on areas or edges
③ can be derived from texture

* Features are basis for classification.

④ Types of features: ① Geometrical features: Area ✓
Perimeter ✓
compactness ✓
Euler Number ✓

② Features from edges: contour code ✓
Polygon ✓

③ more sophisticated edge oriented features: Fourier descriptors ✓

⑤ Area: The area of an objects is given by the no. of pixels belonging to the area.

$$A = \sum_{x=0}^{N} \sum_{y=0}^{M} \delta(g_L - g(x,y))$$

Perimeter → The perimeter "U" is given by the no. of pixels which constitute the perimeter

$$\boxed{U = K_u + \sqrt{2}\, K_g}$$

4	3	2
5	x	1
6	7	8

Eg. chain code.

$K_u \rightarrow 1, 3, 5, 7$
$K_g \rightarrow 2, 4, 6, 8$

* With these, we cannot reconstruct the image.

Compactness → With compactness, we can derive how round the object is wrt a circle. If $K = 1$, object = circle

$$\boxed{K = \frac{U^2}{4\pi A}}$$

Euler Number → It is a topographical feature, defined as difference b/w connected areas "C" and holes "h".

$$E = C - H$$

$$K - V + F = C - H$$

K → edges
V → corners
F → Areas

For skeletonisation, E should be constant, before and after the operation.

⑥ moments → ① As we previously understood that with geometrical features, we cannot reconstruct the original shape.
② This can be done with the method of moments, as they describe the object completely.

(a) let object be given as function $f(x, y)$ in a finite region.
(b) Then its $(p+q)^{th}$ moment is defined as

$$m_{p,q} = \iint_R f(x, y)\, x^p y^q \, dx\, dy$$

* The moment of 1st order is "centre of gravity"

Advantages of moments: Invariant to changes. Eg.

① Translation invariance → By calculating the central moments, an invariance to shift is achieved.

$$\mu_{p,q} = \iint_R f(x, y)\,(x - \bar{x})^p (y - \bar{y})^q \, dx\, dy \quad \Big\} \quad \bar{x} = \frac{m_{1,0}}{m_{0,0}} \quad \bar{y} = \frac{m_{0,1}}{m_{0,0}}$$

② Scale invariance → by normalising wrt 0^{th} central moment

$$\eta_{p,q} = \frac{\mu_{p,q}}{(\mu_{0,0})^r}, \quad r = \left(\frac{p+q+2}{2}\right)$$

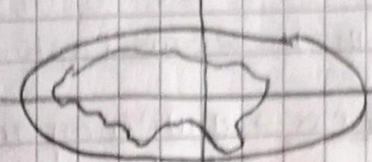
③ Rotation invariant → several rotation invariant moments can be defined.

(7) feature based on moments:

(A) Eccentricity → The eccentricity of an ellipse is the difference b/w semi-axis.
→ It gives an idea, how oval is our object.

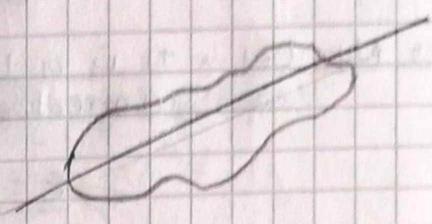
$$\varepsilon = \frac{(\mu_{2,0} - \mu_{0,2})^2 + 4\mu_{1,1}}{A}$$

$A \rightarrow$ Area



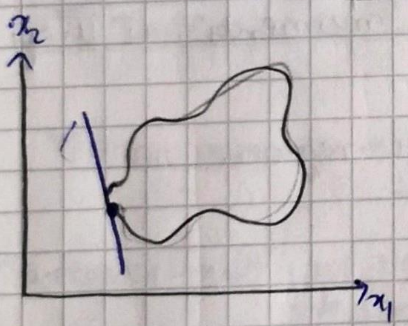
(B) Orientation → The orientation is given by the angular position of the main axis of an object. Defined by

$$\theta = \frac{1}{2}\arctan\left(\frac{2\mu_{1,1}}{\mu_{2,0} - \mu_{0,2}}\right)$$



(8) Edge based features →

(A) ψ - S - Diagram → It is similar to how a chain code is generated
→ we start at a given point, from there, we sum up all the supporting points until we reach 'S' i.e. the length of the curve.



$C = \{x_0, (\psi_0, \psi_1, \ldots \psi_S)\}$

$\psi_S \rightarrow$ Angle b/w the tangent at the point of contact and the reference line.

$S \rightarrow$ length of the curve

$x_0 \rightarrow$ starting point

Normalised Representation:

$$\boxed{\psi^*(s') = \psi\left(\frac{S \cdot s'}{2\pi}\right) + s' \qquad s' = 0 \ldots 2\pi}$$

(B) Polygon → $C = (x_0, x_1 \ldots x_{N-1})$ N contour points $x_i = (x_{i,0}, x_{i,1})$

(B) Area of polygon:

$$A = \frac{1}{2}\sum_{i=0}^{N-1}(x_{i+1,0} \cdot x_{i,1} - x_{i,0} \cdot x_{i+1,1})$$

Advantages: (1) the position information is retained.
(2) contour points do not have to lie in 4 or 8 neighborhood

(C) Fourier descriptors → (1) used as a method for representing a closed curve γ

(2) set up a function θ(l) by traversing the curve in the clockwise direction and assigning to every pixel the angle θ between the tangent to the curve and the

horizontal axis.

~~Rotation invariance~~

Disadvantage → Results strongly depends on the starting point, length and position.

Rotation invariance → $\Theta(l) = \Theta(l) - \Theta(0)$
$\Theta(l)$: end point
$\Theta(0)$: starting point

⇒ for achieving scale invariance, the function is divided by length L.

$$\phi^*(t) = \phi\left(t \frac{L}{2\pi}\right) + t \quad , \quad t = 0 \ldots 2\pi$$

★ The curve is completely specified with parameters which are invariant wrt translation, scaling and rotation.

Disadvantages: ① Reconstruction to be done with numerical integration.
② Not always closed with reduced coefficients
③ Coefficients diminish only propotional to $\frac{1}{n}$

(d) complex fourier descriptors

(5) Texture based features ① Parameters of following attributes can be employed for quantifying texture related features.

Eg. Color mean value — Variance — Higher order statistics (HOS) — Haralick features