* Laplace ~~Inverse~~ One shot

① Change of Scale method $-L[f(at)] = \frac{1}{a}\phi\left(\frac{s}{a}\right)$

② First shifting method : $L[e^{at}f(t)] = \phi(s-a)$

or $L[e^{-at}f(t)] = \phi(s+a)$

③ Multiplication by 't' $- L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}[\phi(s)]$

④ Division by 't' $- L\left[\frac{f(t)}{t}\right] = \int_s^\infty \phi(s)\,ds$

⑤ Basic formulae : $L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$

Q1) LT of $[\cos t \cdot \cos 2t \cdot \cos 3t]$

$\cos A \cdot \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$= f(t) = \cos t \cdot \cos 2t \cdot \cos 3t$

$= \frac{1}{2}[\cos(3t) + \cos(-t)] \cdot \cos 3t$

$\ast \cos(-\theta) = \cos\theta$

$= \frac{1}{2}[\cos 3t + \cos t] \cdot \cos 3t$

$= \frac{1}{2}[\cos^2 3t + \cos t \cdot \cos 3t] = \frac{1}{2}[\cos^2 3t + \cos 4t + \cos 2t]$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$= \frac{1}{2}\left[\frac{1+\cos 2(3t)}{2} + \frac{1}{4}\cos 4t + \frac{1}{2}\cos 2t\right]$

$= \frac{1}{4} + \frac{1}{4} \cdot \cos 6t + \frac{1}{4}\cos 4t + \frac{1}{4}\cos 2t$

$\therefore LT = \cancel{\frac{1}{4}t}$

Q2) $\int_0^\infty e^{-t} \cdot \sinh 2t \cdot \sin 3t\,dt$

$= \int_0^\infty e^{-st} \cdot f(t)\,dt \quad \therefore s = 1, \; f(t) = \sin 3t$

$= \sin 3t \cdot \left(\frac{e^{2t} - e^{-2t}}{2}\right) = \frac{1}{2}[e^{2t}\sin 3t - e^{-2t}\sin 3t]$

Acc to first shifting thm

$= \frac{1}{2}\left[\frac{3}{(s-2)^2+9} - \frac{3}{(s+2)^2+9}\right]$

aplace transform

① $L f(t) = \int_0^{\infty} e^{-st} f(t)\,dt$

② $L[e^{at}] = \frac{1}{s-a}$. ③ $L[e^{-at}] = \frac{1}{s+a}$

* laplace transform of trignometry functions

① ~~sint~~ $[\sin at]$, $L[\cos at]$

$\rightarrow e^{i\theta} = \cos\theta + i\sin\theta$

$e^{iat} = \underbrace{\cos at}_{R\cdot P} + i\underbrace{\sin at}_{I\cdot P}$

Taking L.T of e^{iat} - $L[e^{iat}] = \int_0^{\infty} e^{-st}\cdot e^{iat}\,dt = \int_0^{\infty} e^{-t(s-ia)}\,dt$

$= \left[\frac{e^{-t(s-ai)}}{(-s+ia)}\right]_0^{\infty} = \frac{-1}{(s+ia)}\left[e^{-t(s+ia)}\right]_0^{\infty} = \frac{-1}{s-ia}\left[e^{-\infty} - e^{0}\right]$

$= L[e^{iat}] = \frac{1}{s-ia} \times \frac{s+ia}{s-ia} = \frac{s+ia}{s^2-i^2a^2}$

$L[\cos at + i\sin at] = \frac{s+ia}{s^2+a^2}$

$\therefore L[\cos at] + iL[\sin at] = \frac{s}{s^2+a^2} + i\frac{a}{s^2+a^2}$

Equating R.P of I.P - $L[\cos at] = \frac{s}{s^2+a^2}$, $L[\sin at] = \frac{a}{s^2+a^2}$

① $L[\sinh at]$ - $\boxed{\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}}$, $\boxed{\sinh at = \frac{e^{at} - e^{-at}}{2}}$

Taking L.T on both sides,

$= L[\sinh at] = L\left[\frac{e^{at} - e^{-at}}{2}\right]$

$= \frac{1}{2}\left[L(e^{at}) - L(e^{-at})\right] = \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right] = \frac{1}{2}\left[\frac{(s+a) - (s-a)}{(s-a)(s+a)}\right]$

$= \frac{1}{2}\left[\frac{2a}{s^2-a^2}\right] = \frac{a}{s^2-a^2}$

② $L[\cosh at]$ - $\cosh at = \frac{e^{at} + e^{-at}}{2}$

Taking L.T on both side, $= \frac{s}{s^2-a^2}$

③ $L[1]$

$$= \int_0^\infty e^{-st} \cdot 1 \, dt = \left[\frac{e^{-st}}{-s}\right]_0^\infty = -\frac{1}{s}\left[e^{-\infty} - e^{0}\right]$$

$$= -\frac{1}{s}[0-1] \quad \boxed{L[1] = \frac{1}{s}}$$

Function	L.T
① $L[1]$	$\frac{1}{s}$
② $L[e^{at}]$	$\frac{1}{s-a}$
③ $L[e^{-at}]$	$\frac{1}{s+a}$
④ $L[\cos at]$	$\frac{s}{s^2+a^2}$
⑤ $L[\sin at]$	$\frac{a}{s^2+a^2}$
⑥ $L[\cosh at]$	$\frac{s}{s^2-a^2}$
⑦ $L[\sinh at]$	$\frac{a}{s^2-a^2}$

Change of Scale Method

$L[f(t)]$. $L[f(at)]$

$\downarrow$ $\downarrow$

$\phi(s)$ $\frac{1}{a}\phi\left(\frac{s}{a}\right)$

$\therefore L[f(at)] = \frac{1}{a}\phi\left(\frac{s}{a}\right)$.

① $L[f(t)] = \frac{s}{s^2+9}$, $L[f(4t)]$

$= L[f(4t)] = \frac{1}{4}\cdot\frac{s/4}{\left(\frac{s}{4}\right)^2+9} = \frac{s/4}{\frac{s^2}{16}+9} = \frac{1}{16}\times\frac{s}{\frac{s^2}{16}+9}$

$= \frac{s}{s^2+144}$

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* First shifting method

If $L[f(t)] = \phi(s)$

then, $L[e^{-at} f(t)] = \phi(s+a)$

$L[e^{at} f(t)] = \phi(s-a)$

Q1) L.T. $L[e^{-2t}\sin 3t]$

$= f(t) = \sin 3t$

LT on b.s

$L[f(t)] = L[\sin 3t] = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$

$\therefore$ Acc to first shifting method –

$L[e^{-2t}f(t)] = \frac{3}{(s+2)^2+9} = \frac{3}{s^2+4+4s+9} = \frac{3}{s^2+4s+13}$

Q2) first shift + change of scale $L f(t) = \frac{s}{s^2+s+9}$ $L[e^{-3t} f(2t)]$

$\doteq L[f(t)] = \frac{1}{2}\,\frac{s/2}{\left(\frac{s}{2}\right)^2+\left(\frac{s}{2}\right)+9} = \frac{s\times 4}{2\times s^2+s+9}\,2 = \frac{4s}{2s^2+s+4} = \frac{s}{s^2+2s+16}$

Acc to first shift method

$= \frac{s+3}{(s+3)^2+2(s+3)+16} = \frac{s+3}{s^2+6s+9+2s+6+16} = \frac{s+3}{s^2+8s+31}$

① $(\sin 3t + e^{-2t})$

$f(t) = \sin 3t + e^{-2t}$

L-T on b.s,

$L[f(t)] = L[\sin 3t + e^{-2t}] = L[\sin 3t] + L[e^{-2t}]$

$= L[\sin 3t] = \frac{3}{s^2+9} + \frac{1}{s+2}$

② * <u>Change of Scale method</u>

Q1) Evaluate $\int_0^\infty e^{-3t} \cdot \cosh 5t \, dt$

$= \int_0^\infty e^{-st} f(t) dt = L[f(t)]$

$s = 3, \ f(t) = \cosh 5t$

$\therefore$ Taking laplace transform of $f(t)$ -

$= \frac{s}{s^2 - 5^2} = \frac{s}{s^2 - 25}$

$\therefore$ For evaluation $s = 3$, $\int_0^\infty e^{-3t} \cdot \cosh 5t = \frac{3}{3^2 - 25} = \frac{3}{9-25} = \frac{3}{-16}$

Q2) Evaluate $\int_0^\infty e^{-2t} \cdot \sin^3 t$

Square etc
× No Laplace

$= \int_0^\infty e^{-st} \cdot f(t) dt$

* $\sin^3 t = \frac{3\sin - \sin 3t}{4}$

$s = (-2)$, $f(t) = \sin^3 t$

$= \frac{3\sin t - \sin 3t}{4}$

$L[f(t)] = L\left[\frac{3\sin t}{4} - \frac{\sin 3t}{4}\right] = \frac{1}{4}\left[3L[\sin t] - L[\sin 3t]\right]$

$= \frac{1}{4}\left[3 \cdot \frac{1}{s^2+1} - \frac{3}{s^2+3^2}\right] = \frac{3}{4(s^2+1)} - \frac{3}{4(s^2+3^2)}$

For evaluation $s = 2$ $= \frac{3}{4(4+1)} - \frac{3}{4(4+9)}$

$= \frac{3}{20} - \frac{3}{52} = \frac{6}{65}$

$(\sin 3t + e^{-2t})$

$) = \sin 3t + e^{-2t}$

Taking laplace transform on b.s,

$\frac{1}{s+a}$

$= L[f(t)] = L[\sin 3t + e^{-2t}] = L[\sin 3t] + L[e^{-2t}]$

$= \frac{3}{s^2+3^2} + \frac{1}{s+2} = \frac{3}{(s^2+9)} + \frac{1}{(s+2)}$

Q2) $(\sin 2t - \cos 2t)^2$

$f(t) = (\sin 2t - \cos 2t)^2 = (\sin^2 2t + \cos^2 2t - 2\sin 2t \cdot \cos 2t)$

$= [1 - \sin 2(2t)] = 1 - \sin 4t$

Taking LT on both sides,

$L[f(t)] = L[1 - \sin 4t] = L[1] - L[\sin 4t]$

$= \frac{1}{s} - \frac{4}{s^2+16}$

$* L[f(t)] = \frac{s^2 - 4s + 16}{s(s^2+16)}$

Q3) $LT(\sin t \cdot \cos 2t) = L(\sin t) \cdot L(\cos 2t)$

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$= \frac{1}{2} \sin(t+2t) + \sin(t-2t) = \frac{1}{2}[\sin 3t + \sin(-t)]$

$= \frac{1}{2}[\sin 3t - \sin t] = \frac{1}{2}[L(\sin 3t) - L(\sin t)]$

$= \frac{1}{2}\left[\frac{3}{s^2+3^2} - \frac{1}{s^2+1^2}\right]$

$= \frac{1}{2}\left[\frac{3}{s^2+9} - \frac{1}{s^2+1}\right] = \frac{1}{2}\left[\frac{2s^2-6}{(s^2+9)(s^2+1)}\right]$

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① $L(\sin^5 t) = \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

$= (\sin t)^5 = \left[\frac{e^{it} - e^{-it}}{2i}\right]^5 = \frac{1}{2i}(e^{it} - e^{-it})^5$

$= (a-b)^5 = a^5 - 5a^4b + 10a^3b^2 + 5ab^4 - b^5$

1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$= \frac{1}{32i}\left[(e^{it})^5 - 5(e^{it})^4 + 10(e^{it})^3(e^{-it})^2 - 10(e^{it})^2(e^{-it})^3 + 5(e^{it})(e^{-it})^4 - (e^{-it})^5\right]$

$= \frac{1}{32i}\left[(e^{5it} - e^{-5it}) - 5(e^{3it} - e^{-3it}) + 10(e^{it} - e^{-it})\right]$

$$= \frac{1}{16}\left[\left(\frac{e^{5it}-e^{-5it}}{2i}\right) - 5\left(\frac{e^{3it}-e^{-3it}}{2i}\right) + 10\left(\frac{e^{it}-e^{-it}}{2i}\right)\right]$$

$$= \frac{1}{16}\left[L(\sin 5t) - 5L(\sin 3t) + 10L(\sin t)\right]$$

$$= \frac{1}{16}\left[\frac{5}{s^2+5^2} - 5\frac{3}{s^2+3^2} + 10\frac{1}{s^2+1^2}\right]$$

$$L[\sin^5 t] = \frac{1}{16}\left[\frac{5}{s^2+25} - \frac{15}{s^2+9} + \frac{10}{s^2+1}\right]$$

Q2) $LT = \sqrt{1+\sin t}$

$$= (1+\sin t) = \left[\sin^2\frac{t}{2} + \cos^2\frac{t}{2} + 2\sin\frac{t}{2}\cos\frac{t}{2}\right]$$

$$= \left[\sin\frac{t}{2} + \cos^2\frac{t}{2}\right]^2 = \sqrt{\left(\sin\frac{t}{2}+\cos\frac{t}{2}\right)^2} = \sin\frac{t}{2} + \cos\frac{t}{2}$$

$$= L\left(\sin\frac{t}{2} + \cos\frac{t}{2}\right) = \frac{1/2}{s^2+1/4} + \frac{s}{s^2+1/4} = \frac{1/2+s}{s^2+1/4} = \frac{s+1/2}{s^2+1/4}$$

Q) $(\cosh^4 t)$ $-\cosh t = \frac{e^{t}+e^{-t}}{2}$

$$= \left(\frac{e^{t}+e^{-t}}{2}\right)^4 = \frac{1}{16}\left(e^{4t} + e^{-4t}\right)$$

ultiplication by T

$L[f(t)] = \phi(s)$

$\text{an } L[t^n f(t)] = (-1)^n \cdot \frac{d^n}{ds^n}[\phi(s)]$

- $L[t.f(t)] = -1 \cdot \frac{d}{ds}\phi(s) = -\phi'(s)$
- $L[t^2.f(t)] = (-1)^2 \frac{d^2}{ds^2}\phi(s) = \phi''(s)$

Q1) LT of $[t.\, e^{at}]$ = $f(t) = e^{at}$

$L[f(t)] = L[e^{at}] = \frac{1}{s-a}$

$\boxed{* \, d\left(\frac{1}{x}\right) = -\frac{1}{x^2}}$

Acc to multiplication by T

$L[t.f(t)] = (-1)^1 \cdot \frac{d}{ds}\left[\frac{1}{s-a}\right]$

$= (-1)\left[\frac{-1}{(s-a)^2}\right] = \frac{1}{(s-a)^2}$

Q2) $[t \cdot \sin at]$

L-T of $- L[f(t)] = \sin at = \frac{a}{s^2+a^2}$

Acc to mult of T prop -

$= (-1)^2 \cdot \frac{d^2}{ds}\left[\frac{a}{s^2+a^2}\right] = \frac{d}{ds}\left[\frac{(-1)(a)}{(s^2+a^2)^2}\right](2s)$

$= \frac{d}{ds}\left[\frac{-2as}{(s^2+a^2)^2}\right] = -2a\frac{d}{ds}\left[\frac{s}{(s^2+a^2)^2}\right].$

$= -2a\left[\frac{(s^2+a^2)(1) - s(s^2+a^2) \cdot 2s}{(s^2+a^2)^4}\right] = -2a\left[\frac{3s^2-a^2}{(s^2+a^2)^3}\right]$

Q3) LT of $[t \cdot e^{-2t} \cdot \sin 3t]$

As $f(t) = \sin 3t$

$= L[f(t)] = L[\sin 3t] = \frac{3}{s^2+9}$

: Acc to first shifting Theorem -

$= \frac{3}{(s+2)^2+9} = L[e^{-2t} \sin 3t] = \frac{3}{s^2+4s+13}$

Acc to mult by T $= (-1)\frac{d}{ds}\left[\frac{3}{s^2+4s+13}\right] = (-1)\left[\frac{(-1)\, 3}{(s^2+4s+13)^2}\right] \cdot (2x+4)$

① $LT\left[t\cdot\sqrt{1+\sin t}\right]$

$= f(t) = \sqrt{1+\sin t} = \sqrt{\sin^2\frac{t}{2}+\cos^2\frac{t}{2}+2\sin\frac{t}{2}\cos\frac{t}{2}}$ $\quad$ * $\sin\theta = 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}$

$= \sqrt{\left(\sin\frac{t}{2}+\cos\frac{t}{2}\right)^2} = \sin\frac{t}{2}\cdot\cos\frac{t}{2}$

$L[f(t)] = \sin\frac{t}{2}+\cos\frac{t}{2} = \frac{1/2}{s^2+(1/2)^2}+\frac{s}{s^2+(1/2)^2} = \frac{2(2s+1)}{4s^2+1}$

* Now acc to multiplication by t

$L\left[t+\sqrt{1+\sin t}\right] = (-1)^1\frac{d}{ds}\left[\frac{2(2s+1)}{4s^2+1}\right] = \frac{16s^2+6s-9}{(4s^2+1)^2}$

Q2) $\int_0^\infty e^{-3t}\cdot t\cos t\,dt \quad - \int_0^\infty e^{-st}\cdot f(t)\,dt$

$-st = 3 \Rightarrow s = 3, \; f(t) = \cos t$

LT on b.s -

$L[F(t)] = \frac{s}{s^2+a^2}$

Acc to multiplication by t

$= (-1)^1\frac{d}{ds}\left[\frac{s}{s^2+1}\right] = \frac{s^2-1}{(s^2+1)^2}$

For evaluation $s=3$ $= \frac{8}{100} = \boxed{2/25}$

=

$L\left[\frac{e^{-2t}\cdot \sin 2t \cdot \cosh t}{t}\right]$ — $\cosh t = \frac{e^{t}+e^{-t}}{2}$

$\therefore e^{-2t}\cdot \cosh t = \frac{1}{2}\left[e^{-2t}\cdot e^{t} + e^{-2t}\cdot e^{-t}\right]$

$F(t) - \sin 2t = \frac{2}{s^2+4}$

$\frac{L\left[e^{-t}\sin 2t + e^{-3t}\cdot \sin 2t\right]}{2} = \frac{1}{2}\left[\frac{2}{(s+1)^2+4} + \frac{2}{(s+3)^2+4}\right]$

$= \frac{1}{2}\left[\frac{2}{s^2+2s+5} + \frac{2}{s^2+6s+13}\right]$

* Acc to div by 't' -

$= \frac{1}{2}\left[L\left(\frac{e^{-t}\sin 2t}{t}\right) + L\left(\frac{e^{-3t}\cdot \sin 2t}{t}\right)\right] = \frac{1}{2}\left[\int_s^\infty \frac{2}{(s+1)^2+4}\,ds + \int_0^\infty \frac{2}{(s+3)^2+4}\,ds\right]$

$= \frac{1}{2}\left[\pi/2 + \pi/2 - \tan^{-1}\left(\frac{s+1}{2}\right) - \tan^{-1}\left(\frac{s+3}{2}\right)\right]$

Q1) $\int_0^\infty \frac{e^{-at}-e^{-bt}}{t}\,dt$

Always observe limits of integration.

$\int_0^\infty e^{-st} f(t)\,dt$

$= \int_0^\infty e^{-st}\cdot f(t)\,dt$

$s = 0,\ f(t) = e^{-at} - e^{-bt}$

Taking L.T on bs - $L[f(t)] = L[e^{-at}] - L[e^{-bt}]$

$= \frac{1}{s+a} - \frac{1}{s+b}$

$\frac{f'(x)}{f(x)} = \log f(x)$

* Acc to div by t - $\int_s^\infty \left[\frac{1}{s+a} - \frac{1}{s+b}\right] ds =$

$= \int_0^\infty \frac{\cancel{t}}{\cancel{st}}\cdot [\log(s+a)]_s^\infty - [\log(s+b)]_s^\infty$

$L[f(t)] = \log\left(\frac{s+b}{s+a}\right)$

For evaluation, put $s=0 = \log\left(\frac{a}{b}\right)$

Q) $L[(t+e^{-t}+\sin t)^2]$

$= (a+b+c)^2 = a^2+b^2+2ab+2bc$

$F(t) = (t+e^{-t}+\sin t)^2$

$= t^2+e^{-2t}+\sin^2 t + 2(t)(e^{-t}) + 2(e^{-t})(\sin t) + 2(t)(\sin t)$

$L[f(t)] = t^2+e^{-2t}+\sin^2 t + 2te^{-t} + 2e^{-t}\sin t + 2t\sin t$

$= \frac{2!}{s} + \frac{1}{s+2} + 2\left[\frac{1-\cos 2\theta}{2}\right] +$

$\frac{1}{2}[L(1) - L(\cos 2t)]$

$= \frac{2!}{s} + \frac{1}{s+2} + \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+9}\right]$

$L[te^{-t}] = (-1)\left[\frac{-1}{(s+1)^2}\right]$

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* Division by t method

If $L[f(t)] = \phi(s)$

then $L\left[\frac{f(t)}{t}\right] = \int_0^\infty \phi(s)\,ds$

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ex

Q) $L\left[\frac{1-\cos t}{t}\right]$

$=$ As, $F(t) = 1-\cos t$

Taking L.T on b.s,

$= L[f(t)] = L[1-\cos t]$

$= \frac{1}{s} - \frac{s}{s^2+1} = \phi(s)$

$\rightarrow$ Acc to div property $= \int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+1}\right) ds$

$\boxed{* \quad \int \frac{f'(x)}{f(x)}\,dx = \log(f(x))}$

$= \int_0^\infty \frac{1}{s}\,ds - \frac{1}{2}\int_0^\infty \frac{2s}{s^2+1}\,ds$

$= [\log s]_0^\infty - \frac{1}{2}[\log(s^2+1)]_0^\infty$

$= \log\infty - \log s - \frac{1}{2}[\log(\infty) - \log(s^2+1)]$

$\boxed{\text{As, } a\log b = \log b^a}$

$= -\log s + \frac{1}{2}\log(s^2+1)$

$= -\log s + \log\sqrt{s^2+1}$

$\int_0^\infty e^{-t} \cdot \frac{\sin^2 t}{t} \, dt = \frac{1}{4} \log 5$

look at int limits first

∴ By comparing,

$\int_0^\infty e^{-st} \cdot f(t) \, dt \quad \therefore s = 1$

$= s = 1, \; f(t) = \sin^2 t \rightarrow \frac{1 - \cos 2t}{2}$

Taking laplace transform on both sides :-

$= L[f(t)] = \frac{1}{2}[L(1) - L(\cos 2t)]$

$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4}\right]$.

$L\left[\frac{f(t)}{t}\right] = \frac{1}{2} \int_s^\infty \left[\frac{1}{s} - \frac{s}{s^2+4} ds\right] = \frac{1}{2}\left[\log s - \frac{1}{2} \int_0^\infty \frac{2s}{s^2+4}\right]$

$= \frac{1}{2}\left[\log s - \frac{1}{2} \log(s^2+4)\right]_s^\infty = \frac{1}{4} \log 5$

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* laplace of derivatives

$\rightarrow L[f'(t)] = -f(0) + sL[f(t)]$

$\rightarrow L[f(t)] = \int_0^\infty e^{-st} f(t) \, dt$

$\int uv = uv_1 - u'v_2 + u''v_3$

$\therefore L[f'(t)] = \int_0^\infty \underset{u}{e^{-st}} \, \underset{v}{f'(t)} \, dt$

Applying Int by parts :-

Final $\rightarrow L[f'(t)] = -f(0) + sL[f(t)]$

$\rightarrow L[f''(t)] = -f'(0) - sf(0) + s^2 L[f(t)]$

$\rightarrow L[f'''(t)] = -f''(0) - sf'(0) - s^2 f(0) + s^3 L[f(t)]$

Ex - Q1) $f(t) = t+1, \; 0 \le t \le 2$ and

$f(t) = 3, \; t > 3$

$L[f(t)], L[f'(t)]$

$= L[f(t)] = \int_0^2 \underset{v}{e^{-st}} \, \underset{u}{(t+1)} + \int_2^\infty e^{-st} \cdot 3 \, dt$

$= \left[(t+1)\left(\frac{e^{-st}}{-s}\right) - (1)\left(\frac{e^{-st}}{s^2}\right)\right] + 3\left[\frac{e^{-st}}{-s}\right]_2^\infty$

$= L[f(t)] = \left[\frac{1}{s} + \frac{1}{s^2}(1 - e^{-2s})\right]$

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$f(t) = t+1$

$f(0) = 0+1 = 1$

$L[f'(t)] = -f(0) + s\,L[f(t)]$

$= -1 + s\left[\frac{1}{s} + \frac{1}{s^2}(1-e^{-2s})\right]$

$= -1 + s \cdot \frac{1}{2} + s \cdot \frac{1}{s^2}(1-e^{-2s}) = \frac{1}{s}(1-e^{-2s})$

* $L[f'(t)]$ always smaller than $L[f(t)]$

* laplace of Integrals

$L\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\,\phi(s)$

① LT $\int_0^t \sin 2u\,du$.

$= f(t) = \sin 2t$

LT of $f(t)$

$= \frac{2}{s^2+4}$

$= \frac{1}{s}\left[\frac{2}{s^2+4}\right]$

Q2] LT $\int_0^t \cos^2 u\,du$

$= \frac{1}{2} L[1+\cos 2t]$

$= \frac{1}{2s^2} + \frac{1}{2(s^2+4)}$

* $\cos 2\theta = 2\cos^2\theta - 1$

$= \frac{1+\cos 2\theta}{2}$

③ LT $\int_0^t u \sin^2 u\,du$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$