Australian National University

Student Number:

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Mathematical Sciences Institute

EXAMINATION: Mid-semester examination — March 2017

MATH 3345/6215

Exam Duration: 180 minutes.
Reading Time: 0 minutes.

Materials Permitted In The Exam Venue:

- None.
- Unmarked English-to-foreign-language dictionary (no approval from MSI required).

Materials To Be Supplied To Students:

- Scribble Paper.

Instructions To Students:

- You must justify all your answers, except where stated otherwise.

Q1	Q2	Q3	Q4	Q5
30	24	12	12	12

Total / 90

Question 1 **30 pts**

(5 marks, each part) Prove or disprove the following statements.

(a) Every finite extension of fields is algebraic.

(b) For every positive integer n the polynomial $x^n - 7$ is irreducible over $\mathbb{Q}$.

(c) $\pi + \sqrt{5}$ is transcendental over $\mathbb{Q}$.

(d) $\mathbb{Q}(\sqrt{2}) \cong \mathbb{Q}(\sqrt{3})$.

(e) $\cos(\alpha)$ is constructible if and only if $\sin(\alpha)$ is constructible.

(f) A regular 9-gon is constructible.

Question 2 **24 pts**

(8 marks, each part) For every $n \in \mathbb{N}$, let $\zeta_n = e^{2\pi i/n}$.

(a) Find the minimal polynomial of ζ_p over $\mathbb{Q}$ for p prime,

(b) Find the minimal polynomial of ζ_9 over $\mathbb{Q}$,

(c) Prove that $\mathbb{Q}(\zeta_3) = \mathbb{Q}(\sqrt{-3})$.

Question 3 **12 pts**

Prove that a finite integral domain is a field.

Question 4 **12 pts**

Find an element $\alpha \in \mathbb{Q}[\sqrt{3}, \sqrt{7}]$ such that $\mathbb{Q}[\sqrt{3}, \sqrt{7}] = \mathbb{Q}(\alpha)$ and find its minimal polynomial.

Question 5 **12 pts**

Let E denote the splitting field of $x^4 - 3$ in $\mathbb{C}$. Find $[E : \mathbb{Q}]$.