HOMEWORK 2

This homework is due by Friday, 22 March, 11:59pm on Gradescope.

1. Problem 1 (15.4.1)

Let $K = \mathbf{Q}(\alpha)$ where α is a complex root of $x^3 - x - 1$. Determine the irreducible polynomial for $\gamma = 1 + \alpha^2$ over $\mathbf{Q}$.

2. Problem 2 (15.5.2(a))

For this problem, first go through Section 5 (Construction with Ruler and Compass) to understand the proof of the following theorem (converse of what we did in class).

Theorem: Suppose the coordinates of a point p lie in a field $F = F_n$ such that there exists a chain of fields

$$\mathbf{Q} = F_0 \subset F_1 \subset \cdots \subset F_n$$

with $\deg(F_{i+1}/F_i) = 2$ for all i. Then p is constructible by ruler and compass starting with $(0, 0)$ and $(0, 1)$.

Prove that a regular 5-gon is constructible by ruler and compass. That is, prove that $(\cos(2\pi/5), \sin(2\pi/5))$ is constructible by ruler and compass starting with $(0, 0)$ and $(0, 1)$.

3. Problem 3 (15.6.2 modified)

For this problem, first understand the proof of Proposition 15.3.3.

Proposition: Let F be a field of characteristic not equal to 2. Then every quadratic extension K/F can be written as $K = F(\delta)$ where $\delta^2 \in F$.

Let $m, n \in \mathbf{Z}$. Determine when $\mathbf{Q}(\sqrt{m})$ and $\mathbf{Q}(\sqrt{n})$ are isomorphic.

4. Problem 4 (15.10.1)

Prove that the subset of $\mathbf{C}$ consisting of the algebraic numbers is algebraically closed.

5. Problem 5 (15.7.8)

The polynomials $f(x) = x^3 + x + 1$ and $g(x) = x^3 + x^2 + 1$ are irreducible over $\mathbf{F}_2$. Let $K = \mathbf{F}_2[x]/(f(x))$ and $L = \mathbf{F}_2[y]/g(y)$. Describe explicitly an isomorphism from $K \to L$. Determine the number of isomorphisms from $K \to L$.