

MATH8705 IN-CLASS WORKSHEET 4

Your name: _____

Improve the following theorem and proofs.

BAD THEOREM. *If $x^2 \neq 0$, then $x^2 > 0$.*

BAD PROOF. If $x > 0$ then $x^2 = xx > 0$. If $x < 0$ then $-x > 0$, so $(-x)(-x) > 0$, i.e., $x^2 > 0$. \square

Bad Theorem: $ab = 0 \implies a = 0$ or $b = 0$.

Bad Proof: If $a \neq 0$ and $b \neq 0$, then $ab \neq 0$. Contradiction.

Bad Theorem: $\frac{x+y}{2} \geq \sqrt{xy}$.

Bad Proof: $\frac{x+y}{2} \geq \sqrt{xy}$

$$\Leftrightarrow x + y \geq 2\sqrt{xy}$$

$$\Leftrightarrow x + y - 2\sqrt{xy} \geq 0$$

$$\Leftrightarrow (\sqrt{x} - \sqrt{y})^2 \geq 0 \text{ (trivial).}$$

Bad theorem: $\log_2 \pi \notin \mathbf{Q}$

Bad proof: Suppose $\log_2 \pi = r \in \mathbf{Q}$. Then $2^r = 2 \times 2 \times \cdots r \text{ times} \in \mathbf{Q}$ (closed under multiplication). But $2^r = \pi \notin \mathbf{Q}$. Contradiction. So $\log_2 \pi = r \notin \mathbf{Q}$.