The quadratic discriminant

$ax^2 + bxy + cy^2+$ has multiple roots if and only if

The quadratic discriminant

$ax^2 + bxy + cy^2+$ has multiple roots if and only if

$$b^2 - 4ac = 0$$

The cubic discriminant

$ax^3 + bx^2y + cxy^2 + dy^3$ has multiple roots if and only if

The cubic discriminant

$ax^3 + bx^2y + cxy^2 + dy^3$ has multiple roots if and only if

$$b^2c^2 - 4ac^3 - 4b^3d - 27a^2d^2 + 18abcd = 0.$$

The quartic discriminant

$ax^4 + bx^3y + cx^2y^2 + dxy^3 + ey^4$ has multiple roots if and only if

The quartic discriminant

$ax^4 + bx^3y + cx^2y^2 + dxy^3 + ey^4$ has multiple roots if and only if

$$\begin{aligned}
&b^2c^2d^2 - 4ac^3d^2 - 4b^3d^3 + 18abcd^3 - 27a^2d^4 \\
&\quad - 4b^2c^3e + 16ac^4e + 18b^3cde - 80abc^2de \\
&\quad - 6ab^2d^2e + 144a^2cd^2e - 27b^4e^2 + 144ab^2ce^2 \\
&\quad - 128a^2c^2e^2 - 192a^2bde^2 + 256a^3e^3 = 0
\end{aligned}$$

The cubic resolvant

$a_1x^3 + b_1x^2y + c_1xy^2 + d_1y^3$ and $a_2x^3 + b_2x^2y + c_2xy^2 + d_2y^3$ have a common zero if and only if

The cubic resolvant

$a_1x^3 + b_1x^2y + c_1xy^2 + d_1y^3$ and $a_2x^3 + b_2x^2y + c_2xy^2 + d_2y^3$ have a common zero if and only if

$$
\begin{aligned}
& d_1^3a_2^3 - c_1d_1^2a_2^2b_2 + b_1d_1^2a_2b_2^2 - a_1d_1^2b_2^3 \\
& \quad + c_1^2d_1a_2^2c_2 - 2b_1d_1^2a_2^2c_2 - b_1c_1d_1a_2b_2c_2 \\
& \quad + 3a_1d_1^2a_2b_2c_2 + a_1c_1d_1b_2^2c_2 + b_1^2d_1a_2c_2^2 \\
& \quad - 2a_1c_1d_1a_2c_2^2 - a_1b_1d_1b_2c_2^2 + a_1^2d_1c_2^3 - c_1^3a_2^2d_2 \\
& \quad + 3b_1c_1d_1a_2^2d_2 - 3a_1d_1^2a_2^2d_2 + b_1c_1^2a_2b_2d_2 \\
& \quad - 2b_1^2d_1a_2b_2d_2 - a_1c_1d_1a_2b_2d_2 - a_1c_1^2b_2^2d_2 \\
& \quad + 2a_1b_1d_1b_2^2d_2 - b_1^2c_1a_2c_2d_2 + 2a_1c_1^2a_2c_2d_2 \\
& \quad + a_1b_1d_1a_2c_2d_2 + a_1b_1c_1b_2c_2d_2 - 3a_1^2d_1b_2c_2d_2 \\
& \quad - a_1^2c_1c_2^2d_2 + b_1^3a_2d_2^2 - 3a_1b_1c_1a_2d_2^2 + 3a_1^2d_1a_2d_2^2 \\
& \quad - a_1b_1^2b_2d_2^2 + 2a_1^2c_1b_2d_2^2 + a_1^2b_1c_2d_2^2 - a_1^3d_2^3 = 0
\end{aligned}
$$