MATH8705 IN-CLASS WORKSHEET 2

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A recurrent theme of this chapter is that, quite often, concepts of great logical depth are *better explained with words.*

Write the following symbolic expressions in words, using the simplest language you can.

(1) $\forall p, q \in \mathbf{Q}$ with $p < q, \exists z \in \mathbf{R} \setminus \mathbf{Q}$ such that $p < z < q$.
(2) $\exists n \in \mathbf{N} \forall m \in \mathbf{N}, n \leq m$.
(3) $\forall m, n \in \mathbf{N} \exists l \in \mathbf{Z}$ such that $m + l = n$.
(4) $\forall p \in \mathbf{Q}_{>0} \exists q \in \mathbf{Q}$ such that $0 < q < p$.

Write the negation of each sentence in symbols. Then write the sentence and the negation in words.

(1) $\forall n \in \mathbf{N}, 1/n \notin \mathbf{N}$
(2) $\forall x, y \in \mathbf{R}, xy = yx$
(3) $\forall y \in \mathbf{R} \exists x \in \mathbf{R}^{+}, \log(x) = y$.
(4) $\forall \epsilon > 0, \exists r \in \mathbf{Q}, |r - \sqrt{2}| < \epsilon$.

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Convert the following symbolic sentences into words.

1. $\forall x \in \mathbb{Z}, f(x) = 0$

2. $\forall x \in \mathbb{R}, f(f(x)) = x$

3. $\forall x \in \mathbb{R}^+, f(-x) = 0$

4. $\forall x \in [0,1],\ f(x) \neq 0$

5. $\forall x \in \mathbb{Q} \setminus \{0\}, f(x) \neq 0$

6. $\forall x \in \mathbb{Q},\ f(x) \notin \mathbb{Q}$

7. $\forall x \in \mathbb{Z},\ \exists y \in \mathbb{N},\ f(x+y) = 0$

8. $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R}^+,\ |f(x+y)| <$