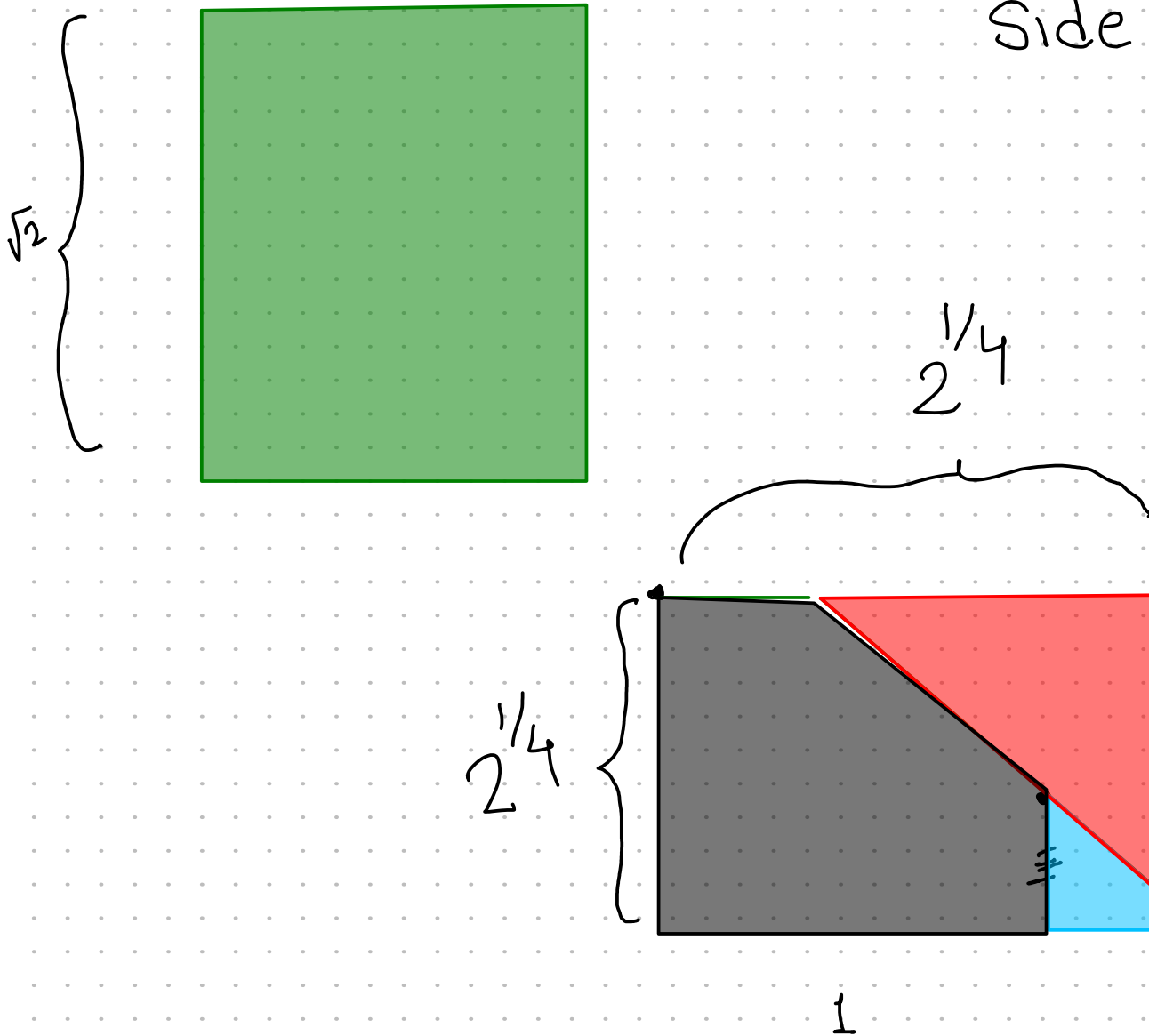


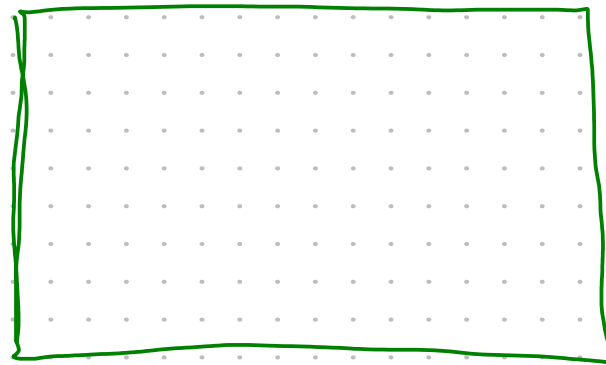
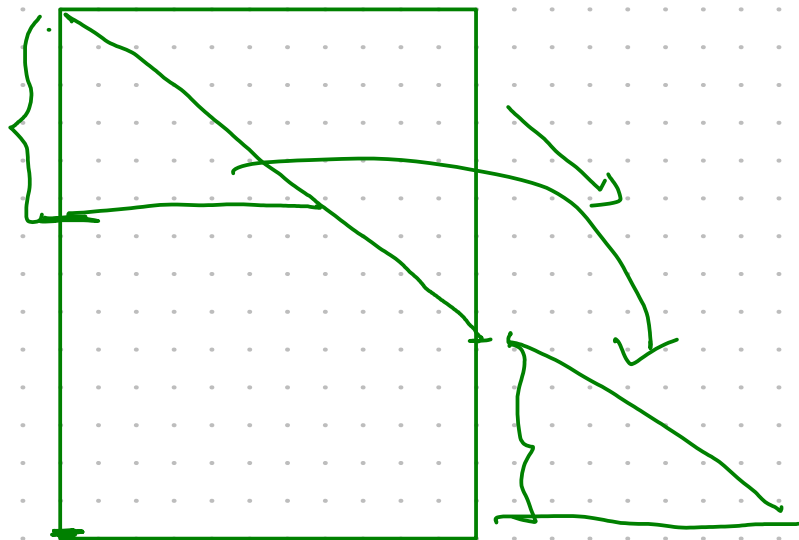
Q: Can we cut / reglue this into a square?

$$\text{Side of Sqr} = 2^{1/4}$$



cut / glue  
and  
translations

Can resize rectangles.

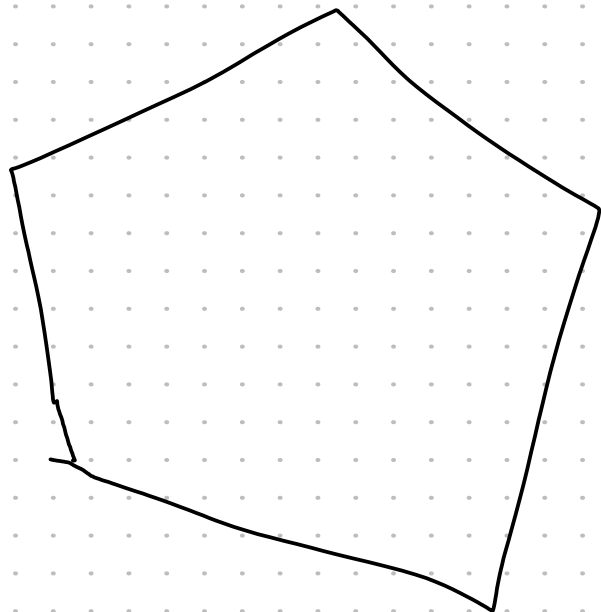


Can take a rectangle of any size

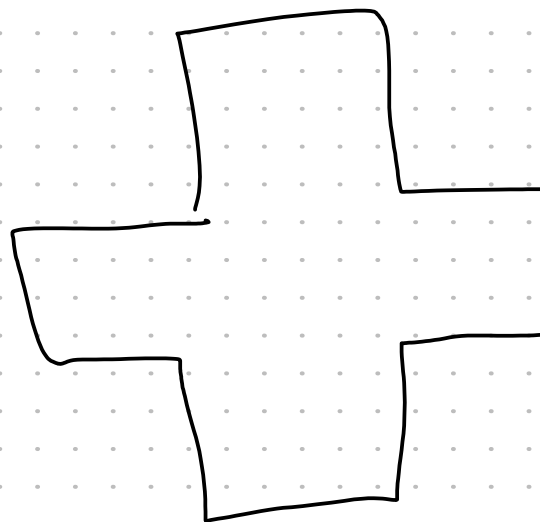
↪ rectangle of size  $1 \times a$  ||


⇒ Can take any rect ↪ Square of same area. ||

Can resize any rectangle  
to any other of same area.

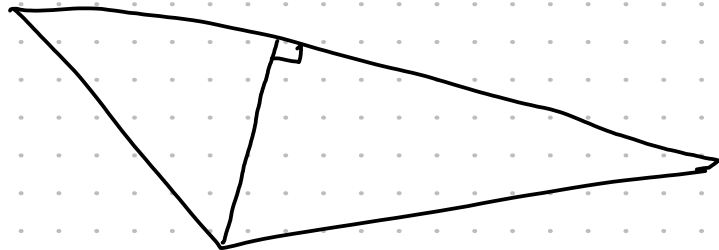


  
 $l \times a$

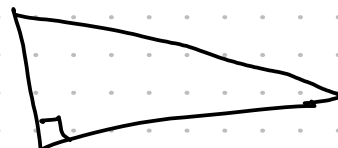


  
 $l \times a$

transl.

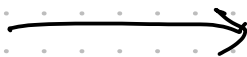
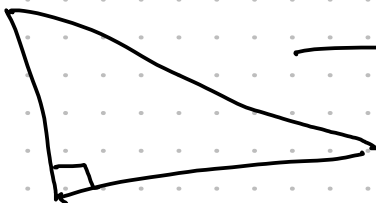
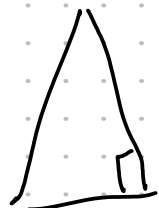
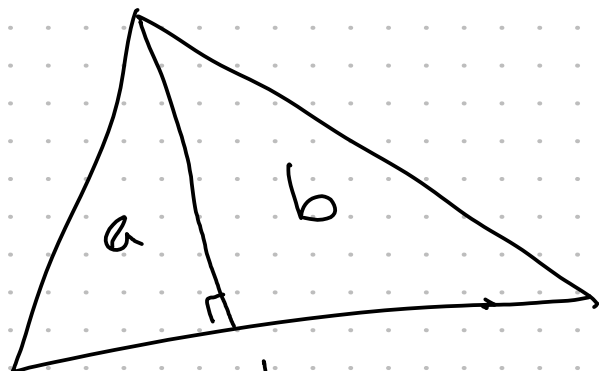
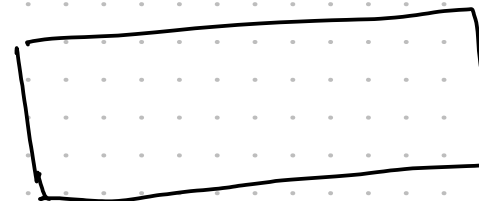
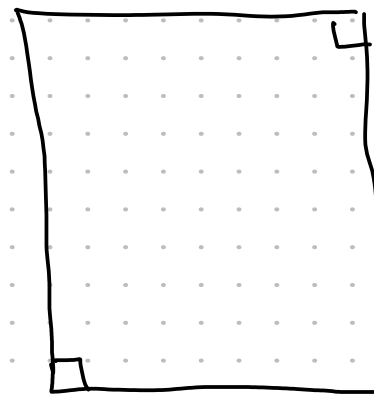
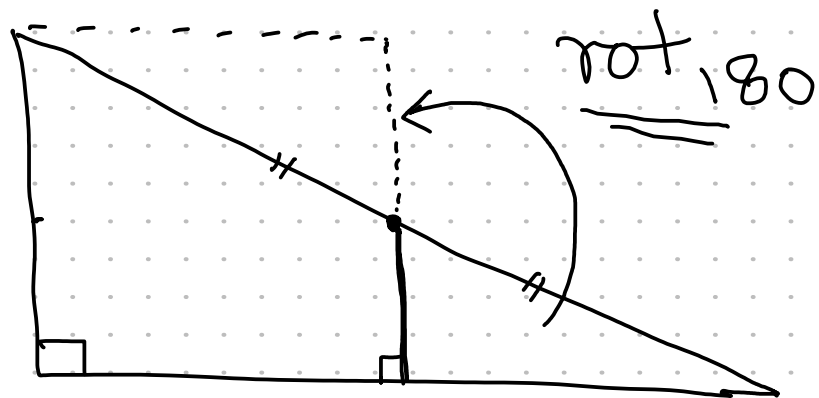


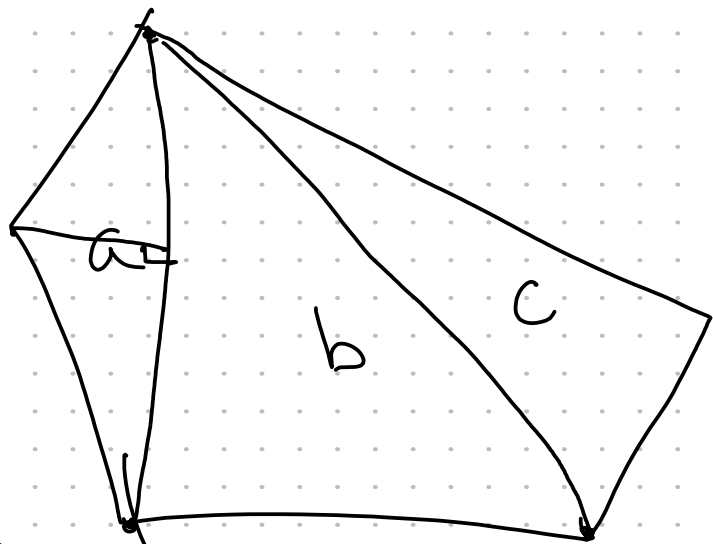




+







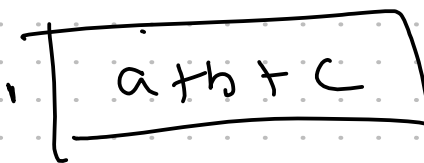
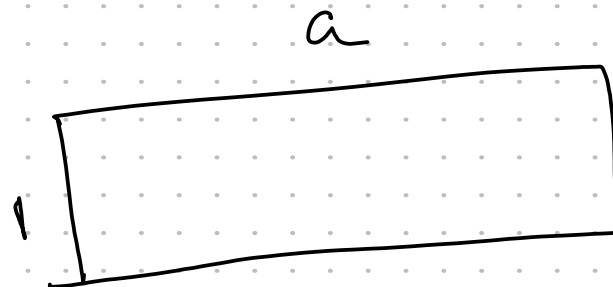
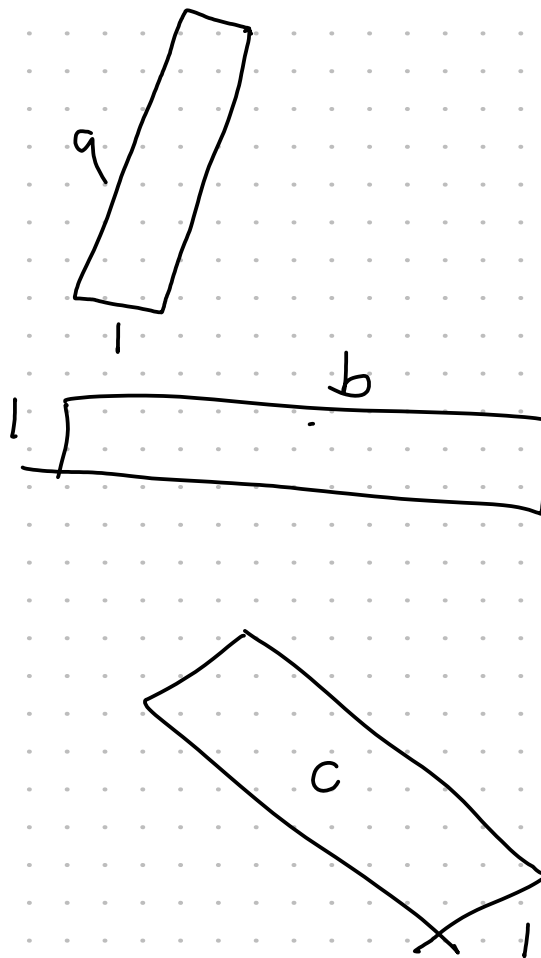
convex  
n-gon



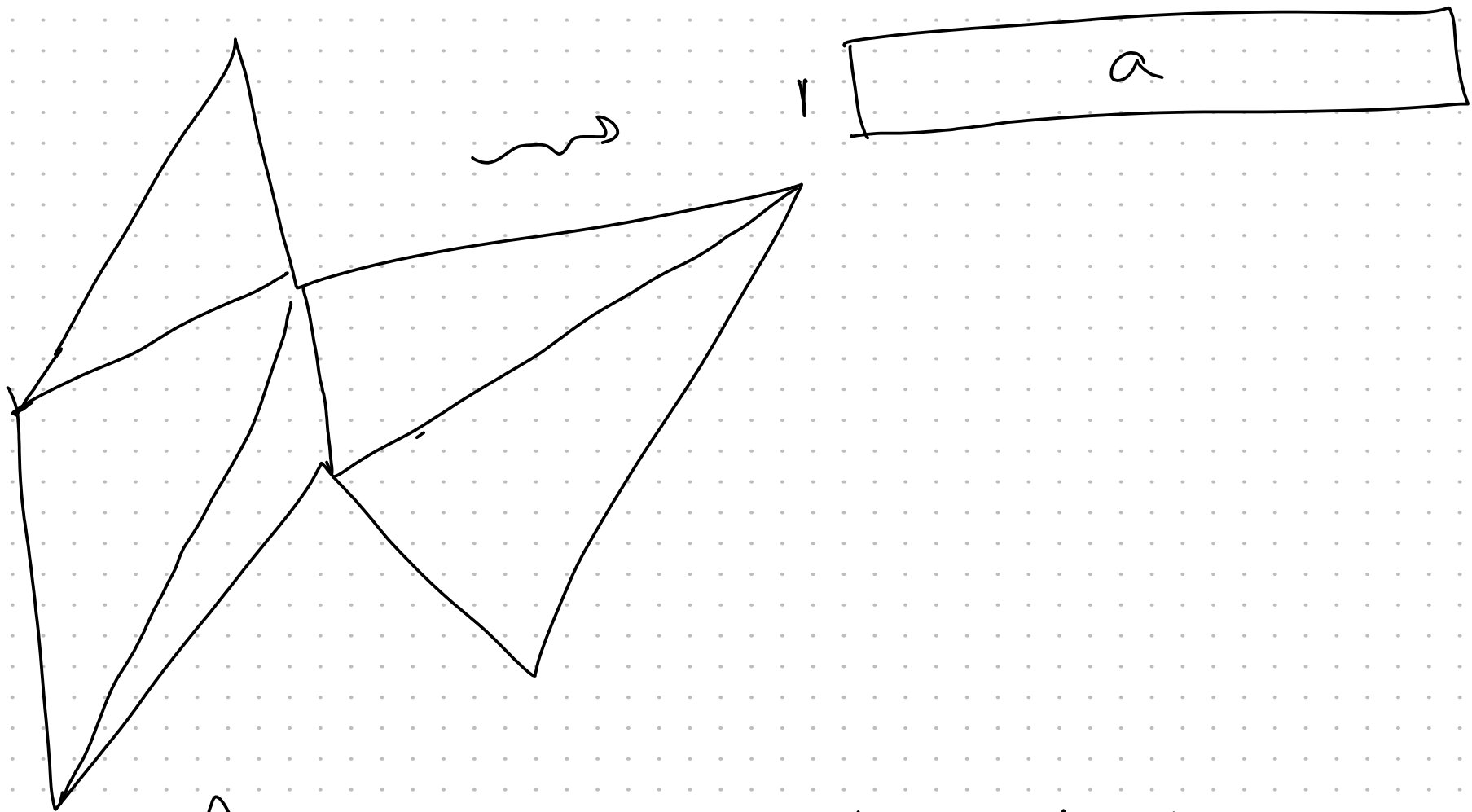
triangulate.

(many ways)

#ways =  $n^{\text{th}}$   
Catalan  
number



arbitrary  
rotate



Thm.: Any polygon can be cut & reglued to a rectangle (square) of the same area.

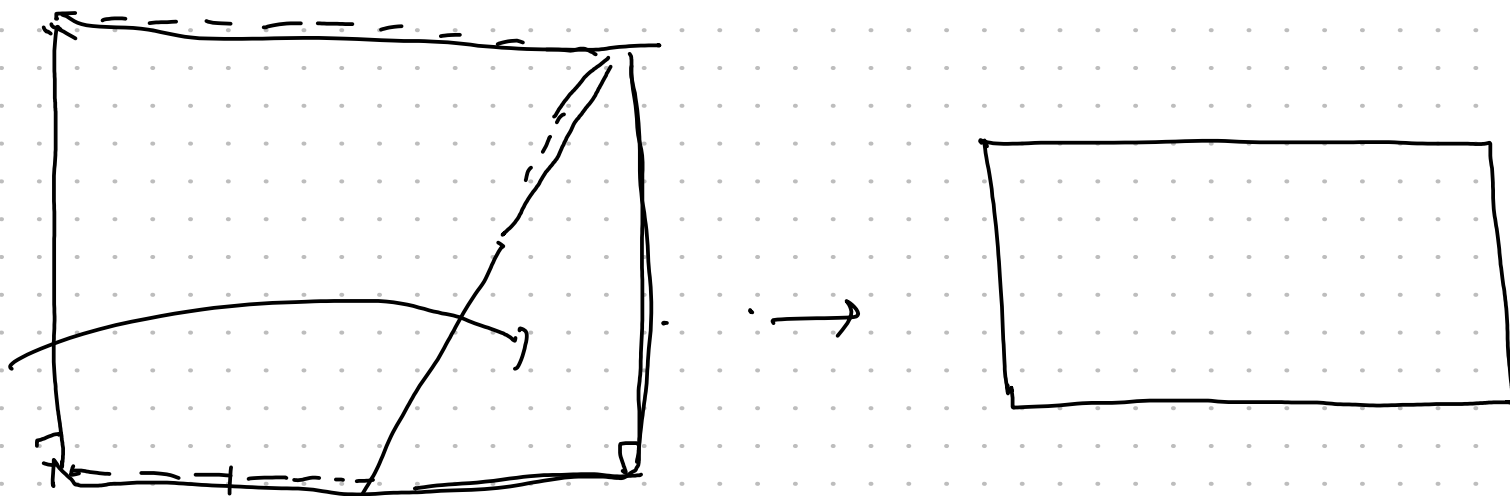
Thm: Given  $P_1$  &  $P_2$  of same area  
Can cut  $P_1$  & reglue to get  $P_2$ .

---

Allowed: cut, move, reglue

↓  
translations,  
(rotations)

Can get away with only  $180^\circ$  - rot.



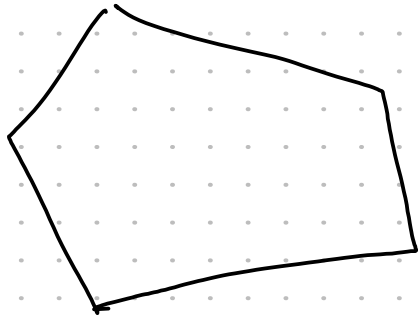
Q: Can we get rid of  $\text{rot}_{180^\circ}$ ?  
only translations!

A: No!  $\exists P_1, P_2$  of same area  
st.  $P_1$  can't be cut & reglved  
to get  $P_2$  without rotating  
the pieces



# Hadwiger Invariant

H-inv.



Quantity invariant  
by cutting /  
transl. / regluing.

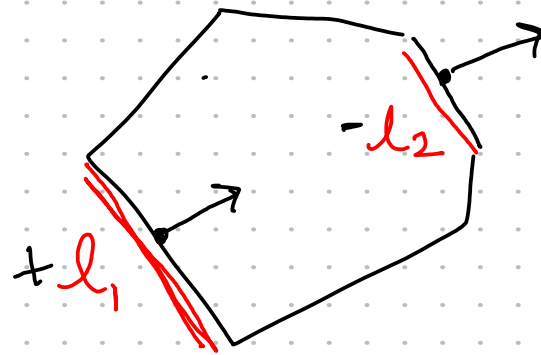
Fix a direction




$h(P)$

$\parallel$

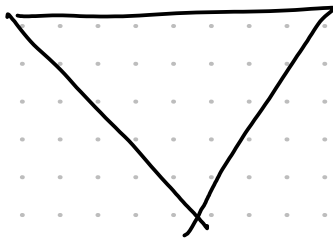
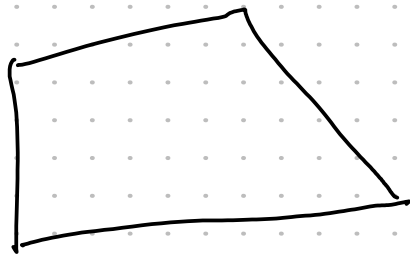
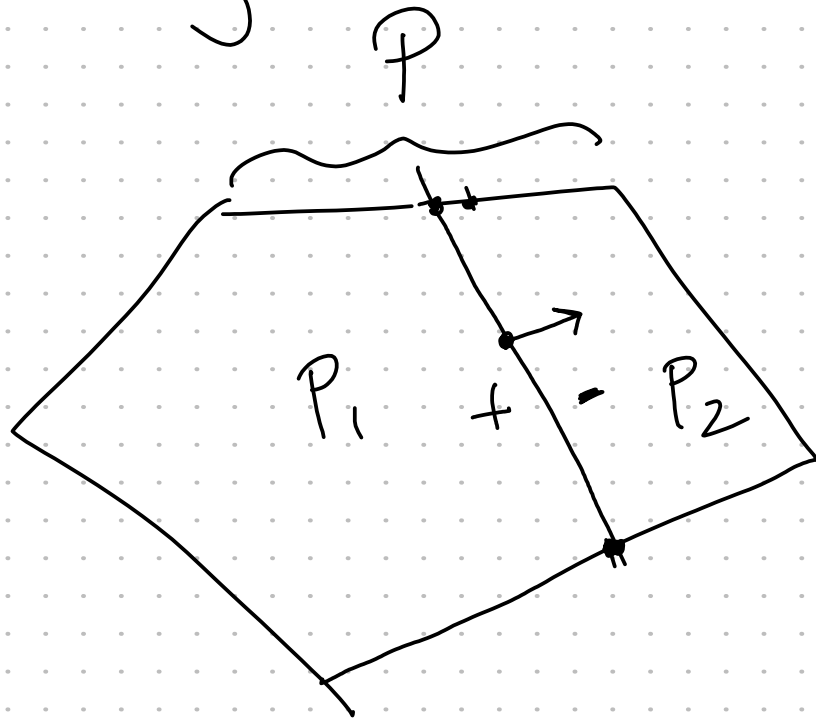
$+l_1 - l_2$



$h(P) = \text{sum of } \pm \text{length of}$   
Sides  $\perp$  to 

Why invariant ?

cutting

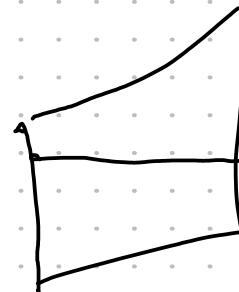
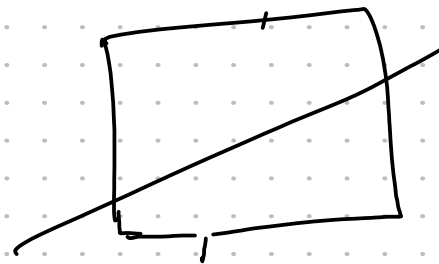


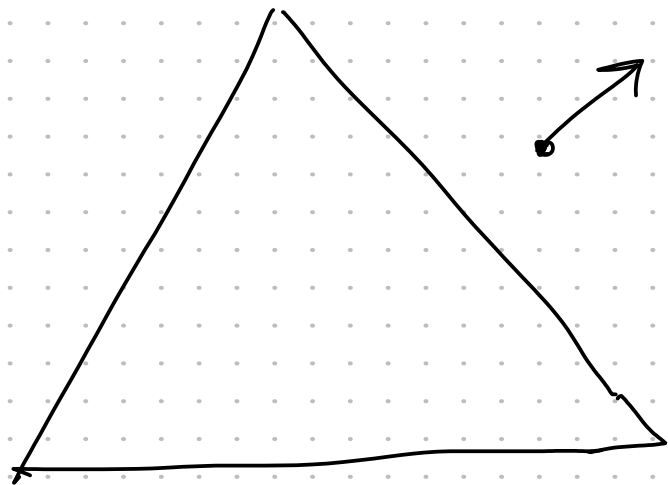
Translations ✓

$$h(P) = h(P_1) + h(P_2)$$



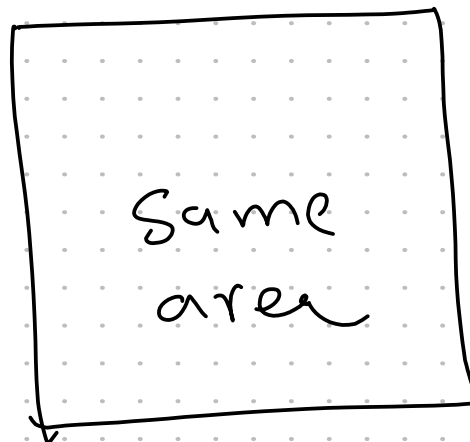
so there!





$\nearrow = \text{dir}$

$$h(\triangle) \neq 0$$



$$h(\square) = 0$$

impossible!



3d:  $P_1, P_2$  in 3d

Same volume

cut / move / reglue

$P_1 \longrightarrow P_2$  ?