SHORT RESEARCH SUMMARY FOR LETTER WRITERS

Dear Brendan, Davesh, Joe, and Johan,

You already have all my official materials. This is a more personal account of my work: how it came about, what I enjoy doing, and wish could do.

Mathematical tastes and philosophy. I started graduate school trying to decide between number theory or algebraic geometry. With time, I grew more fond of geometry. Joe's amazing style and personality obviously played a great role. Questions in algebraic geometry of a somewhat arithmetic flavor (rationality, rational connectedness, rational points, etc.) still fascinate me. I am organizing an informal reading seminar at Columbia on Voisin's decomposition of the diagonal methods next semester to start thinking seriously about such questions.

I was also influenced early on by algebraic topology, particularly classifying spaces and representability. That perhaps explains why I keep thinking about maps to moduli stacks.

My work so far has been about curves in some way or another. But I have been trying not be pigeon-holed in this way. I have been trying to establish myself as broader in scope than the log MMP for curves. The projects about moduli of surfaces and the broadly stated goals in the research statement are a refection of this.

I like classical and concrete questions; I also like abstraction and sophistication. I can't claim to have deep insights on either front, but I have had a lot of fun straddling the boundary, by playing with fancy toys in the context of classical problems. This is also how I have come to digest abstraction – by looking at how it plays in concrete situations.

Overview of the papers. While reading Abramovich, Corti, and Vistoli's construction of admissible covers as spaces of maps to BS_d and Poonen's paper on the moduli space of finite rank algebras [13], I realized that the proof of the valuative criterion goes through even if we replace BS_d by this Artin stack (and in fact anything of the form affine/GL_n). This idea grew to be my thesis (partly contained in **Compactifications of Hurwitz spaces** [4, *IMRN*]). I realized that the resulting compactifications are smooth for triple covers, so I studied them further. Here another story emerged that involved the Maroni invariant and a similar invariant of a triple-point singularity. This became **Modular compactifications of spaces of marked trigonal curves** [5, *Advances*]. From talking to Polishchuk, it seems that the last space in this sequence of compactifications might be related to his work using the Sato Grassmannian. This will be cool, if it is true.

In an alcove in the Harvard math department, Anand Patel and I had done rough calculations for slopes (λ/δ) for families of trigonal curves. This was in response to questions of Maksym in genus 5. During my first year at Columbia, we realized that other people were also interested in these numbers (by coming across a paper with weaker results). So we wrote them up. Proving that our calculations were sharp required a delicate calculation of a certain divisor class (the Maroni divisor) on the whole admissible cover compactification. Here I again used Abramovich–Vistoli's twisted curves along with a stacky GRR. This was **Sharp slope bounds for sweeping families of trigonal curves** [10, *MRL*].

The idea of using GRR for stacks applied also to a computation in Maksym's paper about cyclic covering morphisms (something he had done indirectly with a lot of work). I wrote it up as an appendix (**Class of the Hodge eigenbundle using orbifold Riemann–Roch** [3]). I also computed his divisors for abelian covers (unpublished), but they didn't give anything new.

Our calculation in [10] of the full Maroni class (pushed forward to $\overline{M}_{0,12}$) gave the degree of the divisor of squares plus cubes, a calculation from one of Ravi's papers [14] (the answer is 3762). He asked if we could generalize it using our method. We couldn't use our method directly, but I found out another solution using branched covers with prescribed ramification. I never actually carried out the computation but went on a different track. I realized that Ravi's picture could be interpreted as the interplay between maps to various related one dimensional stacks: $\overline{M}_{1,1}$, $\overline{M}_{0,4}/S_4$, $\overline{M}_{0,4}/S_3$, $\mathbf{P}(2,3)$, etc. This suggested a strategy to generalize (and compactify) his picture. I also realized that the case of $\overline{M}_{0,4}/S_4$ would apply to plane quintics. This became **Covers of stacky curves and limits of plane quintics** [6, submitted to *Compositio*]. It does not have anything about $\overline{M}_{1,1}$ (or more general genus 1 fibrations) or the enumerative questions—the quintics occupied a lot of time and space. But a connection certainly exists. Hopefully, there's a compelling question about elliptic fibrations that I can solve using this. The realization that I could describe limits of plane quintics was finally what prompted me to write everything up.

I got involved with the log MMP for $\overline{M}_g$ during an *AIM* workshop. Here, Gabi Farkas explained his old proposal with Keel to construct the canonical model of $\overline{M}_g$ using syzygies. We formalized it using Koszul cohomology but there was absolutely nothing known about these GIT problems. But, there was now a technique to prove generic (semi)stability using curves with automorphisms (by Swinarski, Morrison, Fedorchuk, Smyth, Alper, etc.) and we got it to work for a ribbon in genus 7. (A ribbon is a double structure on $\mathbf{P}^1$). I continued working on it and could show the next few cases by hand. Then Maksym and I really got serious about showing that it works for all (odd) genera. We had to show that there were enough non-zero minors in a matrix of linear forms (of size depending on the genus). We found one non-zero minor fairly quickly (for all genera) but were stuck there for months! After a lot of trial and error, I found another one with an extremely involved proof for its non-vanishing (it involves thinking of syzygies as steps in a Markov chain). This lead to a proof (**Toward GIT stability of syzygies of canonical curves** [8, To appear in *Algebraic Geometry*]). I never thought I could do such clever combinatorial linear algebra (and I have been trying to find a more systematic approach).

Meanwhile, we were trying to bring some order to this kind of combinatorics by more systematically looking at the ribbon and its degenerations under the torus action. We were hoping that we could interpret our proof using Gröbner bases. But it didn't pan out. We still wrote up our efforts and partial progress (**Gröbner techniques and ribbons** [9, *Alb. J. of Math.*]).

We tried to push our methods to higher syzygies. I soon realized that this had to be difficult because we had to prove Green's conjecture for this ribbon (what Bayer and

Eisenbud tried to do in [1]). I believe that there will have to be some new idea if we want to make progress towards GIT for higher syzygies. We need more geometric insight into spaces of higher syzygies and their instability. The whole section in my research statement about GIT and syzygies is motivated by trying to understand such a connection. I am looking at low genus cases, partly with Han-Bom Moon (**On the GIT of syzygies of canonical genus 7 curves** [7]), using tools like Mukai's descriptions.

The project on syzygies was personally satisfying for another reason. I had spent some time in graduate school thinking about different kinds of non-reduced structures on $\mathbf{P}^1$, their moduli spaces, and their possible role in the log MMP (unpublished). So it was nice to have ribbons play a role in my work.

I thought about the unirationality of Hurwitz spaces (who hasn't?!) and realized that you can easily use the unirational parametrization for degree up to five to compute the Picard group. I did the computation with Anand Patel, confirming the conjecture that the simply branched Hurwitz space has no non-torsion line bundles. Stankova-Frenkel had already done something similar for degree 3, so we were surprised to learn that this was not known for $d = 4, 5$. But getting all the dimension estimates took more work than we had imagined. Anand P. also figured out a connection to Severi varieties of Hirzebruch surfaces. We finally wrote everything up in **Picard rank conjecture for Hurwitz spaces of degree up to five** [11, *ANT*]. Anand P. took this much further by computing also the higher codimension classes for $d = 3$. This project raised a lot of questions about the Chow and tautological ring of the Hurwitz spaces (mentioned in the research statement) about which we have been thinking on-and-off.

I got involved with the project with Maria about del Pezzo surfaces to branch out from all of this and do something completely new. She had concrete questions raised by her adviser (Sturmfels) that she wanted to answer about tropical del Pezzo surfaces. I didn't know any tropical geometry but was confident that I could absorb the classical picture of the moduli of del Pezzos and help her on that front. This has lead to my reading about a lot of Dolgachev style classical geometry and invariant theory (like Coble's work from Dolgachev–Ortland [12]). Currently the project is in the form of a lot of sage and polymake computation involving polytopes and hand-written notes about various classical constructions of the moduli of del Pezzo surfaces using root systems (as described in [2], for example).

References

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