(b)Prove that the real function$x \mapsto 3x^4 + 4x^3 + 6x^2 + 1$is positive.

*Proof.* Let the real function

$$f(x) = 3x^4 + 4x^3 + 6x^2 + 1, \forall x \in R. \tag{1}$$

Then we should prove that $f(x) \geq 0, for \forall x \in R.$
Let take the first and second derivative of f(x):

$$f'(x) = 12x(x^2 + x + 1), \tag{2}$$

$$f''(x) = 12(3x^2 + 2x + 1). \tag{3}$$

Note the equation 3 ,$\triangle = 12^2(2^2 - 4 \times 3 \times 1) < 0$, so it easy to know there doesn't have a solution to $f''(x) = 0$, which implies that $f''(x) > 0, for \forall x \in R$.Hence

1. (b) PROOF. Let $f(x) = 3x^4 + 4x^3 + 6x^2 + 1$, and we need to show that $\forall x \in \mathbb{R}, f(x) > 0$. And denote the derivative function of $f$ as $f'(x)$. By using formula $(x^n)' = nx^{n-1}(n \neq 0)$ and $c' = 0$($c$ is a constant) , we get $f'(x) = 12x^3 + 12x^2 + 12x$, which is also equal to $f'(x) = 12x(x^2 + x + 1)$ according to factorization. And we know that the function $f$ reaches the maximum or minimum value when

$$f'(x) = 0 \tag{1}$$