HOMEWORK 2

ALGEBRAIC GEOMETRY 2021

(1) Let $\phi\colon X \to Y$ be a regular map between quasi-affine varieties. Prove that ϕ is continuous in the Zariski topology.

(2) Let $U \subset \mathbb{A}^n$ be an open set. Show that the restriction map $k[x_1, \ldots, x_n] \to k[U]$ is injective. Using this, show that we have an injective map
$$k[U] \to k(x_1, \ldots, x_n).$$

(3) Let $f \in k[x_1, \ldots, x_n]$ be non-zero, and let U_f be the complement in $\mathbb{A}^n$ of $V(f)$. Identify the ring $k[U]$ as a sub-ring of the fraction field $k(x_1, \ldots, x_n)$.

(4) Let $U = \mathbb{A}^2 \setminus \{(0,0)\}$. Show that the restriction map $k[x,y] \to k[U]$ is an isomorphism. *Hint: Observe that U is the union $U_x \cup U_y$.*

(5) Let $S \subset \mathbb{P}^3$ be the Fermat cubic
$$S = V(X^3 + Y^3 + Z^3 + W^3).$$
Let $L \subset S$ be the line defined by
$$L = V(X + Y, Z + W).$$
Consider the regular map
$$\phi\colon S \setminus L \to \mathbb{P}^1$$
defined by
$$[X : Y : Z : W] \mapsto [X + Y : Z + W].$$
Prove that ϕ extends to a regular map
$$\phi\colon S \to \mathbb{P}^1.$$

Not to be turned in, but highly recommended.
What are the fibers of the map $\phi\colon S \to \mathbb{P}^1$? Can you picture this over the real numbers?