

ALGEBRAIC GEOMETRY: HOMEWORK 9

This homework is due on Friday October 18 by 5pm. Feel free to use the principal ideal theorem and its consequences proved in class.

- (1) Prove that \mathbb{P}^2 and $\mathbb{P}^1 \times \mathbb{P}^1$ are not isomorphic to each other. Do the same for the Fermat cubic surface S and \mathbb{P}^2 .

Challenge (not to be turned in): Do the same for S and $\mathbb{P}^1 \times \mathbb{P}^1$.

- (2) The *Krull dimension* of a ring R is the largest n for which there exists a strictly increasing chain

$$\mathfrak{P}_0 \subset \cdots \subset \mathfrak{P}_n$$

of prime ideals of R .

- (a) What is the Krull dimension of \mathbb{Z} ?
- (b) Let X be an irreducible affine variety. Prove that the Krull dimension of $k[X]$ is equal to the dimension of X .
- (3) Some time ago, we saw the geometric meaning of idempotents of a ring. Here is the analogue for zero-divisors. Let X be an affine variety (not necessarily irreducible). Show that $f \in k[X]$ is a zero-divisor if and only if f vanishes identically on some irreducible component of X . (Recall that an element a of a ring is a zero-divisor if there exists an element b such that $ab = 0$ in the ring.)