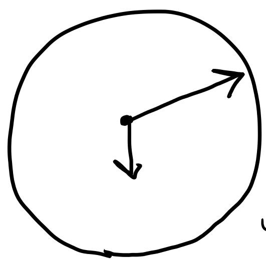
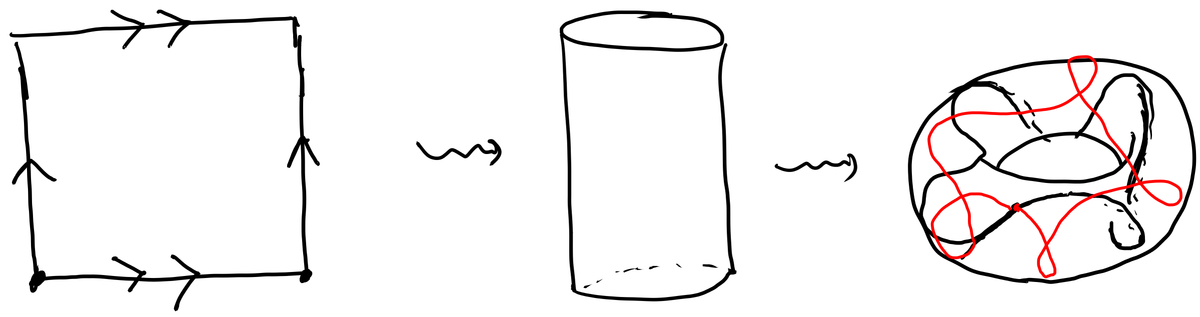
How to count using topology

Q: How many valid clock positions remain valid when the hand positions are switched?

 $\in$ Circle $\times$ Circle

$\rightsquigarrow$ (hour hand, min hand)



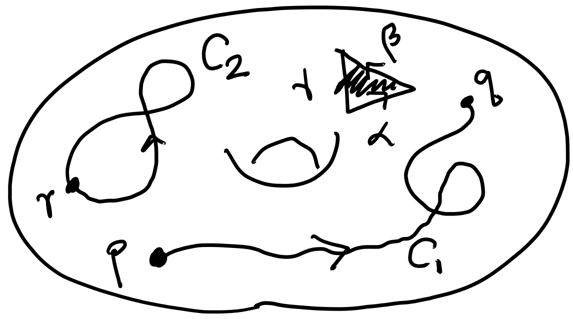
$$C^{\text{lock}} = \{ (\text{hour}, \text{min}) \} \subset \text{Torus}$$

$$A^{\text{nticlock}} = \{ (\text{min}, \text{hour}) \} \subset \text{Torus}$$

Q: Find $\#(C \cap A)$

More generally, (quickly) find the # intersection points of two curves.

Curves on T:



"Curve" is a map $[0,1] \longrightarrow T$

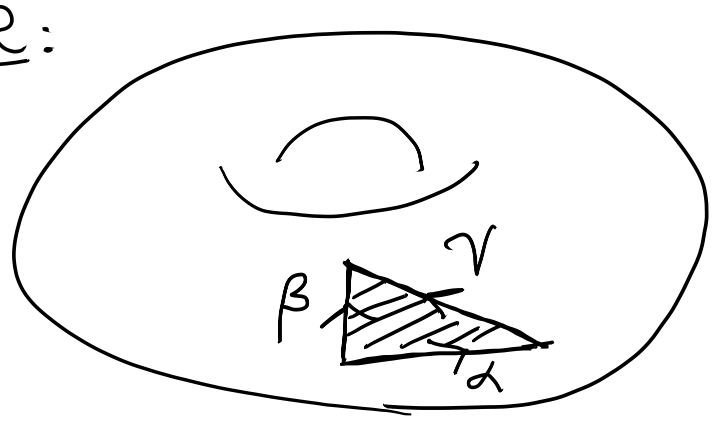
$C_1 + C_2 = C_1 + C_2$, $C_1 + C_2 + C_2 = C_1 + 2C_2$

$C_1 - C_2 = C_1 - C_2$

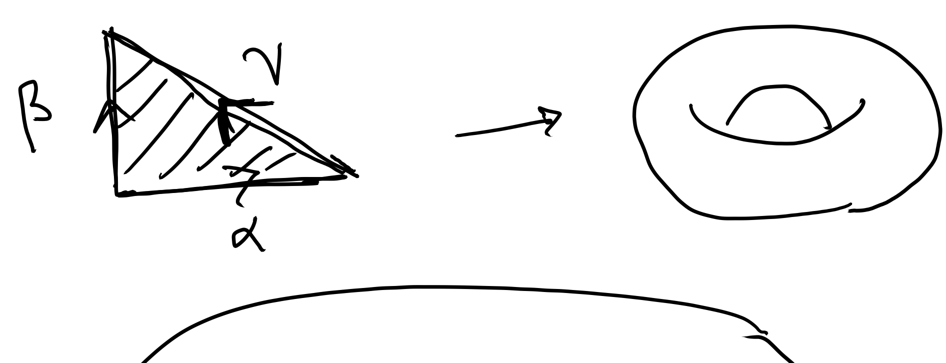
$C_1 - C_2 + C_2 = C_1$

$\mathcal{C}' = \{$Set of expressions $\sum a_i C_i$ $a_i \in \mathbb{Z}$, C_i curves$\}$

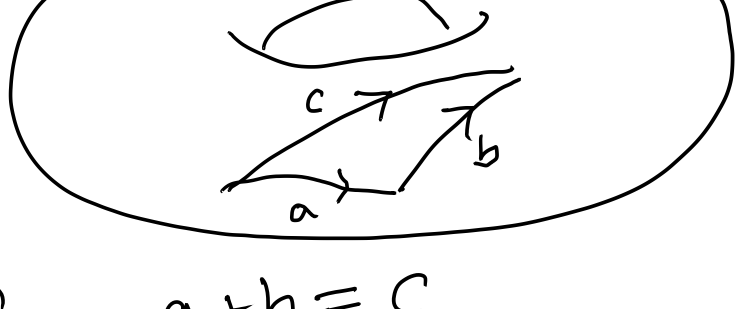
Extra rule:



allowed to "cancel" $\alpha + \gamma - \beta$.



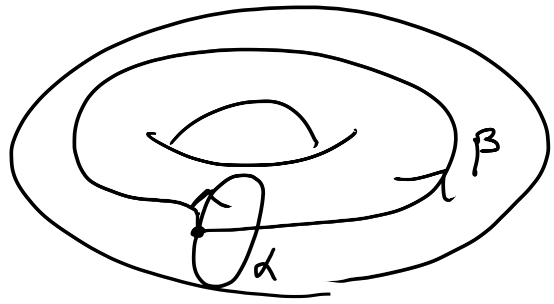
Ex.



Then $a + b = c$.

(why? $a+b-c=0$)

Key Result:



Let C be any closed curve on T. Then

$$C = m\alpha + n\beta$$

for some $m \in \mathbb{Z}$, $n \in \mathbb{Z}$

$$C = m\alpha + n\beta$$

Clock:

$$C = 12\beta + \alpha$$

$$A = \beta + 12\alpha$$

Intersecting