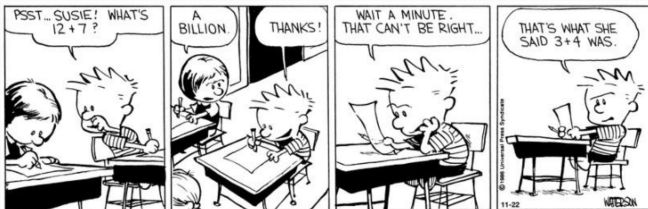


Games, graphs, and machines



August 20, 2024

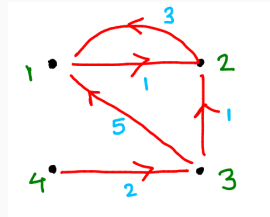
Warm up

Remember that $\oplus = \min$ and $\odot = +$. Find

$$2 \odot (3 \oplus 1) \oplus 3 \odot (\infty \oplus 2).$$

Weighted adjacency matrix

Write the min/plus weighted adjacency matrix of the graph. Assume that the loops have weight 0 (not shown).



Min/plus powers

Find the min/plus square and cube of the adjacency matrix. What do its entries represent?

Why do min/plus powers give shortest paths?

For example, the third power:

$$\begin{aligned} A_{i,j}^3 &= (A_{i,1}^2 \odot A_{1,j}) \oplus (A_{i,2}^2 \odot A_{2,j}) \oplus (A_{i,3}^3 \odot A_{3,j}) \oplus (A_{i,4}^2 \odot A_{4,j}) \\ &= \min(A_{i,1}^2 + A_{1,j}, A_{i,2}^2 + A_{2,j}, A_{i,3}^3 + A_{3,j}, A_{i,4}^2 + A_{4,j}) \end{aligned}$$

When do we stop?

Assume:

1. we have all loops with weight 0,
2. all weights are non-negative.

Theorem

Let n be the number of vertices. Then $A^{\odot(n-1)} = A^{\odot n} = A^{\odot(n+1)} = \dots$.