Games, graphs, and machines

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Grundy value: warm-up

Find the Grundy value of PosetChomp for the following poset.

The Sprague-Grundy theorem

Theorem Two games have the same Grundy value if and only if they are equivalent.

Why? First a lemma.

Lemma If $G \sim H$, then $G + A \sim H + A$.

Proof of Sprague-Grundy

Suppose $G \sim H$. We want to show that $\text{label}(G) = \text{label}(H)$.

$G \sim H \implies G + H \sim H + H$

But $H + H$ is P, so $G + H$ is P.

So $\text{label}(G + H) = 0$.

But $\text{label}(G + H) = \text{label}(G) \oplus \text{label}(H)$.

So $\text{label}(G) = \text{label}(H)$.

Proof of Sprague-Grundy

Suppose label(G) = label(H).

Then label($G + H$) = 0

So $G + H$ is a P-game.

Then $G + G + H$ is equivalent to both G and H.

So $G \sim H$.