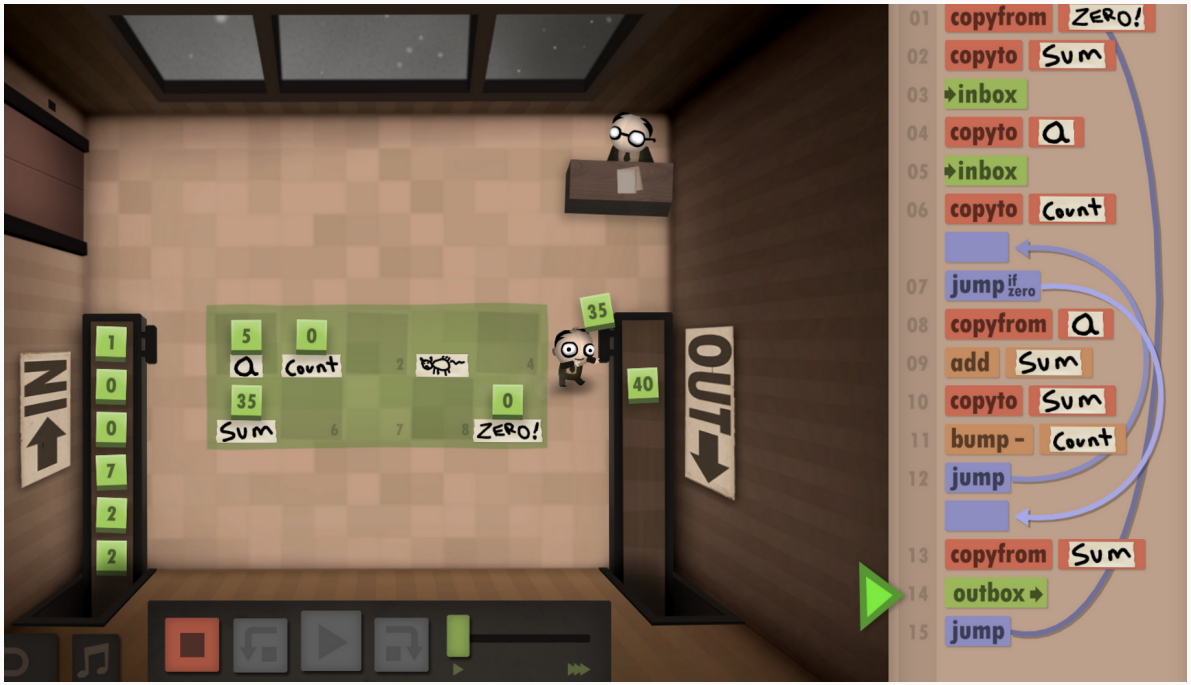
# Games, graphs, and machines




October 23, 2024

# A bit of review: N/P labelling

- $P$ if sink state
- $N$ if an immediate successor that is $P$
- $P$ if all immediate successors are $N$

# A bit of review: Grundy labelling

- 0 if sink state
- mex of immediate successors

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Important properties:

1. Grundy label 0 if and only if $P$ state.
2. $\text{label}(G + H) = \text{label}(G) \oplus \text{label}(H)$.

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- Move to a $P$ position = Move to Grundy label 0.
- For example:
  1. In $G + H$, how to move from an $(N, P)$ position?
  2. What about an $(N, N)$ position?

# Example

What are the winning moves of $\mathrm{Chomp}(3,3) + \mathrm{Nim}(4,5)$ (if any)?

# Fun aside: misere play

The player who cannot make a move wins!

# For more fun and games –