Games, graphs, and machines

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Function or not?

Do the following rules define functions?

1. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(s) = s^2$.
2. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(s) = s/2$.
3. $f : \mathbb{Z} \to \mathbb{R}$ defined by $f(s) = s/2$.
4. $f : \mathbb{R} \to \mathbb{R}$ defined by $f(s) = \sqrt{s}$.
5. $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(s) = \begin{cases} 1 \text{ if } s > 0 & -1, \\ \text{if } s < 0 & \end{cases}.$$

Number of functions

Suppose $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$.

1. How many functions are there from A to B?
2. How many of these are injective functions?
3. How many of these are surjective functions?

Number of functions (continued)

Suppose A has size n and B has size m.

How many functions are there from A to B?

Suppose $n \leq m$. How many functions are there injective?

The inverse function

Suppose $f\colon S \to T$ is a bijection.

The inverse of f is the function $g\colon T \to S$ defined by the property that if $t = f(s)$ then $s = g(t)$.

1. Suppose $f\colon \{1,2,3\} \to \{1,2,3\}$ sends $1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 1$. Find its inverse g.
2. Find a function $f\colon \{1,2,3\} \to \{1,2,3\}$ which is its own inverse.
3. Find a function $f\colon \mathbb{R} \to \mathbb{R}$ which is its own inverse.

Input/Output relation

Consider $R \subset \mathbb{R} \times \mathbb{R}$ defined by

$$R = \{(x, y) \mid x^3 - xy + x - 1 = 0\}.$$

Is R the input/output relation of a function $f : \mathbb{R} \to \mathbb{R}$?