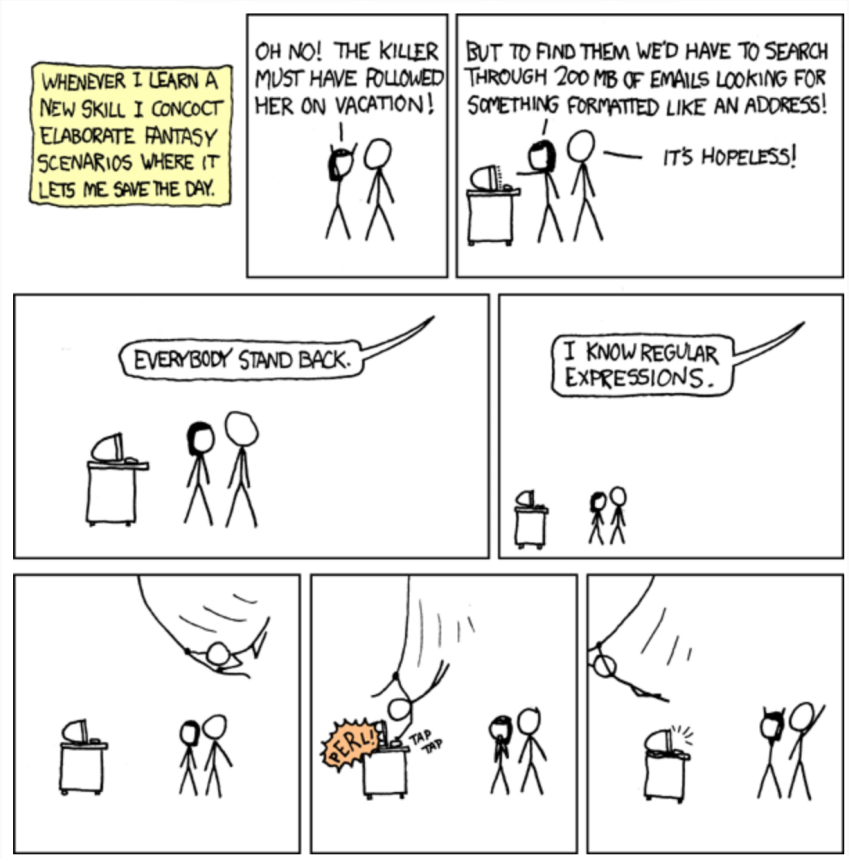
Games, graphs, and machines



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Stars and cats

Alphabet $\Sigma = \{0, 1\}$.
Languages $L = \{0\}$ and $M = \{1, 11, 111, 1111, \cdots\}$.

1. $LM =$
2. $ML =$
3. $L^* =$
4. $M^* =$
5. $L^*M =$

man grep

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REGULAR EXPRESSIONS
       A regular expression is a pattern that describes a set of strings.
       Regular expressions are constructed analogously to arithmetic
       expressions, by using various operators to combine smaller expressions...

Character Classes and Bracket Expressions
       A bracket expression is a list of characters enclosed by [  and  ]...

Anchoring
       The  caret ^ and the dollar sign $ are meta-characters...

The Backslash Character and Special Expressions
       The symbols \< and  \>  respectively  match  the  empty  string...

Repetition
       A  regular  expression  may  be  followed  by  one of several repetition
       operators:
       ?      The preceding item is optional and matched at most once.
       *      The preceding item will be matched zero or more times.
       +      The preceding item will be matched one or more times.
       {n}    The preceding item is matched exactly n times.
       {n,}   The preceding item is matched n or more times.
       {,m}   The preceding item is matched at most m times.   This  is  a  GNU
              extension.
       {n,m}  The preceding item is matched at least n times, but not more than
              m times.

Concatenation
       Two  regular  expressions  may  be  concatenated;  ...

Alternation
       Two  regular  expressions  may  be  joined  by the infix operator |;...
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Our regexps

- $\varnothing$
  ϵ
  0
  1
- Concatenation ab
  alternation $a|b$
  star a^*.

Regular expressions

Explicitly write the language described by the regexp.

1. 01^*
2. $(0|1)^*$
3. $(01)^*$
4. 00^*10^*0

Building regexps

Find regular expressions that describe the following languages.

1. $\emptyset$
2. $\{\epsilon\}$
3. $\{0, 00, 000, \cdots\}$
4. $\{w \mid w \text{ starts with 0 and ends with 1}\}$

Building trickier regexps

Find regular expressions that describe the following languages.

1. $\{w \mid 0 \text{ and } 1 \text{ alternate in } w\}$
2. $\{w \mid \text{every } 0 \text{ in } w \text{ has } 1 \text{ on its left and on its right}\}$.
3. $\{w \mid w \text{ has an even number of } 0\text{s}\}$

Even trickier languages

Can you find regexps that describe the following languages?

1. $\{w \mid w$ has as many 0s as 1s$\}$.
2. $\{w \mid w$ is a palindrome$\}$.