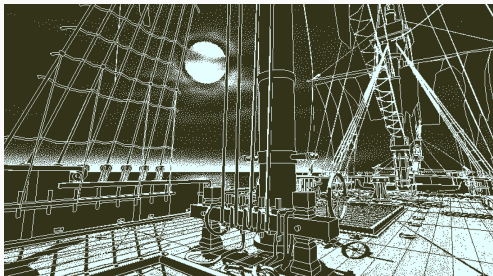# Games, graphs, and machines

October 18, 2024
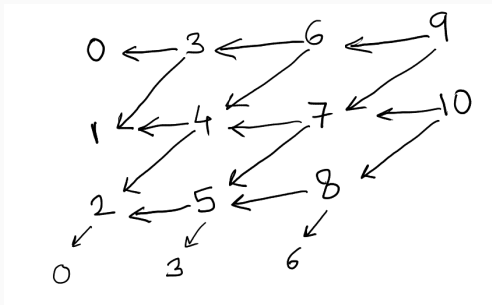
"Nim-sum" for all games: Grundy value

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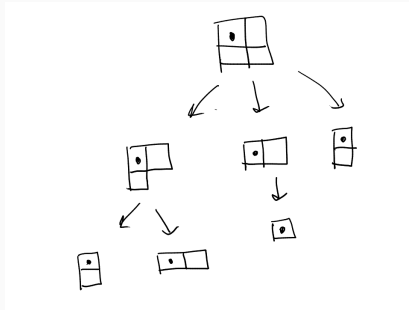1. All sink states get 0
2. Each state gets mex of its children.

"Nim-sum" for all games: Grundy value

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- (2,3) subtraction game

Another example

2×2 Chomp

Key properties

1. Grundy value zero $\iff$ P
2. Grundy value non-zero $\iff$ N
3. Grundy value of $G + H =$ Grundy value of $G \oplus$ Grundy value of H.

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Grundy value of $\text{Chomp}(2, 2) + \text{Nim}(3, 4)$

Big theorem

Theorem: Two games are stably equivalent if and only if they have the same Grundy value.