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Senior School Certificate Examination

March 2019

Marking Scheme — Mathematics (041) 65/1/1, 65/1/2, 65/1/3

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled.
5. If a question does not have any parts, marks must be awarded in the left hand margin and encircled.
6. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
7. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
8. A full scale of marks 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
9. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 25 answer books per day.
10. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
11. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as (X) and awarded zero (0) Marks.
12. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
13. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
14. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
15. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 65/1/1
EXPECTED ANSWER/VALUE POINTS
SECTION A

1. $AB = 2I \Rightarrow |AB| = |2I| \Rightarrow |A| \cdot |B| = 2^3|I|$ $\frac{1}{2}$

$$\Rightarrow 2 \times |B| = 8 \Rightarrow |B| = 4$$ $\frac{1}{2}$

2. $(fof)\ (x) = f(x + 1) = x + 2$ $\frac{1}{2}$

$\frac{d}{dx}(fof)(x) = 1$ $\frac{1}{2}$

3. order = 2, degree = 1 $\frac{1}{2}+\frac{1}{2}$

4. d.c.'s = $<\cos 90^\circ, \cos 135^\circ, \cos 45^\circ>$ $\frac{1}{2}$

$= < 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$ $\frac{1}{2}$

OR

$\vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k})$ 1

SECTION B

5. As $a, b \in R \Rightarrow ab \in R \Rightarrow ab + 1 \in R \Rightarrow a*b \in R \Rightarrow *$ is binary. 1

For associative $(a*b)*c = (ab+1)*c = (ab+1)c+1 = abc+c+1$

also, $a*(b*c) = a*(bc+1) = a.(bc+1)+1 = abc+a+1$

In general $(a*b)*c \neq a*(b*c) \Rightarrow *$ is not associative. 1

6. $2A - \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} + \begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 1

$\Rightarrow A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$ 1

7. Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$ $\frac{1}{2}$

$$I = \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2+4}} = \log|t + \sqrt{t^2+4}| + C$$ 1

$$= \log|\tan x + \sqrt{\tan^2 x + 4}| + C$$ $\frac{1}{2}$

8. Let $I = \int \sqrt{1 - \sin 2x}\, dx$

$$= \int (\sin x - \cos x) dx \qquad \text{as } \sin x > \cos x \text{ when } x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$ 1

$$= -\cos x - \sin x + C$$ 1

OR

$$I = \int \sin^{-1}(2x).1\, dx$$

$$= x.\sin^{-1}(2x) - \int \frac{2x}{\sqrt{1-4x^2}} dx$$ 1

$$= x.\sin^{-1}(2x) + \frac{1}{4}\int \frac{-8x}{\sqrt{1-4x^2}} dx = x\sin^{-1}(2x) + \frac{1}{2}.\sqrt{1-4x^2} + C$$ 1

9. $y' = be^{2x} + 2y \Rightarrow b = \frac{y' - 2y}{e^{2x}}$ $\frac{1}{2}$

differentiating again

$$\frac{e^{2x}.(y'' - 2y') - (y' - 2y).2x^{2x}}{(e^{2x})^2} = 0$$ 1

$$\Rightarrow y'' - 4y' + 4y = 0 \text{ or } \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$ $\frac{1}{2}$

10. Given $|\hat{a} + \hat{b}| = 1$

As $|\hat{a} + \hat{b}|^2 + |\hat{a} - \hat{b}|^2 = 2(|\hat{a}|^2 + |\hat{b}|^2)$ 1

$\Rightarrow 1 + |\vec{a} - \vec{b}|^2 = 2(1+1)$

$\Rightarrow |\hat{a}-\hat{b}|^2 = 3 \Rightarrow |\hat{a}-\hat{b}| = \sqrt{3}$ 1

OR

$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$ 1

$= -30$ 1

11. $A = \{2, 4, 6\}$, $B = \{1, 2, 3\}$, $A \cap B = \{2\}$

Now, $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{6}$ 1

as $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$ $\frac{1}{2}$

$\Rightarrow$ A and B are not independent. $\frac{1}{2}$

12. Let X: getting an odd number

$p = \frac{1}{2}$, $q = \frac{1}{2}$, $n = 6$ $\frac{1}{2}$

(i) $P(X = 5) = {}^6C_5 \left(\frac{1}{2}\right)^6 = \frac{3}{32}$ $\frac{1}{2}$

(ii) $P(X \leq 5) = 1 - P(X = 6) = 1 - \frac{1}{64} = \frac{63}{64}$ 1

OR

$k + 2k + 3k = 1$ 1

$\Rightarrow k = \frac{1}{6}$ 1

13. Clearly $a \leq a \ \forall \ a \in \mathbb{R} \Rightarrow (a, a) \in R \Rightarrow R$ is reflexive. 1

For transitive:

Let $(a, b) \in R$ and $(b, c) \in R$, $a, b, c \in \mathbb{R}$

$\Rightarrow$ a $\le$ b and b $\le$ c $\Rightarrow$ a $\le$ c $\Rightarrow$ (a, c) $\in$ R

$\Rightarrow$ R is transitive. $1\frac{1}{2}$

For non-symmetric:

Let a = 1, b = 2. As $1 \le 2 \Rightarrow (1, 2) \in R$ but $2 \not\le 1 \Rightarrow (2,1) \notin R$

$\Rightarrow$ R is non-symmetric. $1\frac{1}{2}$

OR

For one-one. Let $x_1, x_2 \in N$.

$f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow x_1 = x_2$ as $x_1 + x_2 + 1 \ne 0$ $\quad (\because x_1, x_2 \in N)$ $1\frac{1}{2}$

$\Rightarrow$ f is one-one.

For not onto.

for y = 1 $\in$ N, there is no x $\in$ N for which f(x) = 1 $1\frac{1}{2}$

For f^{-1}: $y = f(x) \Rightarrow y = x^2 + x + 1 \Rightarrow y = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}$

$$\Rightarrow x = \frac{\sqrt{4y-3}-1}{2}$$

$\therefore f^{-1}(y) = \dfrac{\sqrt{4y-3}-1}{2}$ or $f^{-1}(x) = \dfrac{\sqrt{4x-3}-1}{2}$ 1

14. $\tan^{-1}\left(\dfrac{4x+6x}{1-(4x)(6x)}\right) = \dfrac{\pi}{4}$ 1

$\Rightarrow \dfrac{10x}{1-24x^2} = 1 \Rightarrow 24x^2 + 10x - 1 = 0$ $1\frac{1}{2}$

$\Rightarrow x = \dfrac{1}{12}$ or $-\dfrac{1}{2}$ 1 as $x = -\frac{1}{2}$ does not satisfy the given equation, so $x = \frac{1}{12}$ $\frac{1}{2}$

15. LHS = $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2(a-1) & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 2

$$= (a-1)^2 \begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 1

Expanding along C_3,

$$= (a-1)^2.(a-1) = (a-1)^3 = \text{RHS.}$$ 1

16. $\log(x^2+y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$

differentiating both sides w.r.t. x,

$$\frac{1}{x^2+y^2}\left(2x+2y\frac{dy}{dx}\right) = 2\cdot\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{x\cdot\frac{dy}{dx}-y\cdot 1}{x^2}\right)$$ 2

$$\Rightarrow \frac{2}{x^2+y^2}\left(x+y\frac{dy}{dx}\right) = \frac{2x^2}{x^2+y^2}\cdot\frac{1}{x^2}\cdot\left(x\frac{dy}{dx}-y\right)$$ 1

$$\Rightarrow (x+y) = (x-y)\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y}$$ 1

OR

Let $u = x^y$, $v = y^x$. Then $u - v = a^b$

$$\Rightarrow \frac{du}{dx} - \frac{dv}{dx} = 0 \quad ...(1) \qquad 1$$

Now, log u = y.logx

$$\Rightarrow \frac{1}{u}\cdot\frac{du}{dx} = y\cdot\frac{1}{x} + \log x\cdot\frac{dy}{dx} \Rightarrow \frac{du}{dx} = x^y\left(\frac{y}{x} + \log x\cdot\frac{dy}{dx}\right) \quad ...(2) \qquad 1$$

Again, log v = x · log y

$$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = x\cdot\frac{1}{y}\cdot\frac{dy}{dx} + \log y\cdot 1 \Rightarrow \frac{dv}{dx} = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) \quad ...(3) \qquad 1$$

From (1), (2) and (3)

$$x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) - y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0 \qquad \frac{1}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^x\cdot\log y - x^{y-1}\cdot y}{x^y\cdot\log x - y^{x-1}\cdot x} \qquad \frac{1}{2}$$

17. $y = (\sin^{-1}x)^2$

$$\Rightarrow y' = 2\cdot\sin^{-1}x\cdot\frac{1}{\sqrt{1-x^2}} \qquad 1$$

$$\Rightarrow \sqrt{1-x^2}\cdot y' = 2\sin^{-1}x$$

$$\Rightarrow \sqrt{1-x^2}\cdot y'' + y'\cdot\frac{1}{2\sqrt{1-x^2}}(-2x) = \frac{2}{\sqrt{1-x^2}} \qquad 2$$

$$\Rightarrow (1-x^2)\cdot y'' - xy' = 2 \text{ or } (1-x^2)\cdot\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0. \qquad 1$$

18. Let the point of contact be $P(x_1, y_1)$

$$\frac{dy}{dx} = \frac{3}{2\sqrt{3x-2}} \qquad \text{(slope of tangent)}$$

$$\Rightarrow m_1 = \left.\frac{dy}{dx}\right]_{(x_1, y_1)} = \frac{3}{2\sqrt{3x_1-2}} \qquad 1$$

also, slope of given line = 2 = m_2

$m_1 = m_2 \Rightarrow x_1 = \dfrac{41}{48}$ 1

when $x_1 = \dfrac{41}{48}, y_1 = \sqrt{\dfrac{41}{16} - 2} = \dfrac{3}{4}$ $\quad \therefore P\left(\dfrac{41}{48}, \dfrac{3}{4}\right)$ $\frac{1}{2}$

Equation of tangent is: $y - \dfrac{3}{4} = 2\left(x - \dfrac{41}{48}\right)$

$\Rightarrow 48x - 24y = 23$ 1

and, Equation of normal is: $y - \dfrac{3}{4} = \dfrac{-1}{2}\left(x - \dfrac{41}{48}\right)$

$\Rightarrow 48x + 96y = 113$ $\frac{1}{2}$

19. $I = \displaystyle\int \frac{3x+5}{x^2+3x-18}\,dx = \frac{3}{2}\int \frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int \frac{1}{x^2+3x-18}\,dx$ 1

$= \dfrac{3}{2}\displaystyle\int \frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int \frac{1}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\,dx$ 1

$= \dfrac{3}{2}\log|x^2+3x-18| + \dfrac{1}{18}\log\left|\dfrac{x-3}{x+6}\right| + C$ 1 + 1

20. Let $I = \displaystyle\int_0^a f(a-x)\,dx$

Put $a - x = t \Rightarrow -dx = dt$ $\frac{1}{2}$

$I = -\displaystyle\int_a^0 f(t)\,dt = \int_0^a f(t)\,dt = \int_0^a f(x)\,dx$ $\frac{1}{2}$

II part.

$I = \displaystyle\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$

$$\Rightarrow \quad I = \int_0^{\pi} \frac{(\pi - x)\cdot \sin x}{1+\cos^2 x}\,dx$$

$$\Rightarrow \quad 2I = \int_0^{\pi} \frac{\pi \cdot \sin x}{1+\cos^2 x}\,dx$$ $1\frac{1}{2}$

Put cos x = t ⇒ –sinx dx = dt

$$\Rightarrow \; I = -\frac{\pi}{2}\cdot \int_1^{-1} \frac{dt}{1+t^2} = \frac{\pi}{2}\times 2\times \int_0^1 \frac{dt}{1+t^2}$$

$$= \pi[\tan^{-1} t]_0^1 = \frac{\pi^2}{4}$$ $1\frac{1}{2}$

21. Writing $\frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x} = \frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2}$ $\frac{1}{2}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}$

Differential equation becomes $v + x\frac{dv}{dx} = v + \sqrt{1+v^2}$

$$\Rightarrow \; \int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$$ 1

$$\Rightarrow \; \log\left|v+\sqrt{1+v^2}\right| = \log|x| + \log c$$ 1

$$\Rightarrow \; v+\sqrt{1+v^2} = cx \;\Rightarrow\; y+\sqrt{x^2+y^2} = cx^2$$

when x = 1, y = 0 ⇒ c = 1 $\frac{1}{2}$

$$\therefore \; y+\sqrt{x^2+y^2} = x^2$$ $\frac{1}{2}$

OR

Given equation is $\frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{4x^2}{1+x^2}$ $\frac{1}{2}$

I.F. $= e^{\int \frac{2x}{1+x^2} dx} = 1 + x^2$ 1

Solution is given by,

$$y \cdot (1+x^2) = \int \frac{4x^2}{1+x^2} \cdot (1+x^2)\, dx = \int 4x^2\, dx$$ 1

$$\Rightarrow\ y \cdot (1+x^2) = \frac{4x^3}{3} + c$$ $\frac{1}{2}$

when $x = 0,\ y = 0 \Rightarrow c = 0$ $\frac{1}{2}$

$$y \cdot (1+x^2) = \frac{4x^3}{3} \quad \text{or} \quad y = \frac{4x^3}{3(1+x^2)}$$ $\frac{1}{2}$

22. $\overrightarrow{AB} = \hat{i} + 4\hat{j} - \hat{k}$ 1

$\overrightarrow{CD} = -2\hat{i} - 8\hat{j} + 2\hat{k}$ 1

Let required angle be θ.

Then $\cos\theta = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|} = \dfrac{-2-32-2}{\sqrt{18}\,\sqrt{72}} = -1$ 1

$\Rightarrow \theta = 180^\circ$ or π $\frac{1}{2}$

Since $\theta = \pi$ so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. $\frac{1}{2}$

23. Given lines are: $\dfrac{x-1}{-3} = \dfrac{y-2}{\left(\frac{\lambda}{7}\right)} = \dfrac{z-3}{2}$ and $\dfrac{x-1}{\left(\frac{-3\lambda}{7}\right)} = \dfrac{y-5}{1} = \dfrac{z-6}{-5}$ 1

As lines are perpendicular,

$$(-3)\left(\frac{-3\lambda}{7}\right) + \left(\frac{\lambda}{7}\right)(1) + 2(-5) = 0 \Rightarrow \lambda = 7$$ 1

So, lines are

$$\frac{x-1}{-3} = \frac{y-2}{1} = \frac{x-3}{2} \quad \text{and} \quad \frac{x-1}{-3} = \frac{y-5}{1} = \frac{z-6}{-5}$$ $\frac{1}{2}$

Consider $\Delta = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 0 & 3 & 3 \\ -3 & 1 & 2 \\ -3 & 1 & -5 \end{vmatrix} = -63$ 1

as $\Delta \neq 0 \Rightarrow$ lines are not intersecting. $\frac{1}{2}$

SECTION D

24. $|A| = 4 \neq 0 \Rightarrow A^{-1}$ exists. 1

$$\text{adj } A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ 2

$$\therefore \; A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ $\frac{1}{2}$

Given system of equations can be written as AX = B where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$\therefore \; X = A^{-1} \cdot B$ 1

$$= \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$ 1

$\Rightarrow x = 3, \; y = 1, \; z = 2$ $\frac{1}{2}$

OR

A = I.A

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$ 1

$R_2 \to R_2 + R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_2 \to \frac{R_2}{5}$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -2/5 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/5 & 1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 - 2R_2, R_3 \to R_3 + 2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -6/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1/5 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 & 0 \\ 1/5 & 1/5 & 0 \\ 2/5 & 2/5 & 1 \end{bmatrix} \cdot A$$

$R_3 \to 5R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -6/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 & 0 \\ 1/5 & 1/5 & 0 \\ 2 & 2 & 5 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + \frac{6}{5}R_3, R_2 \to R_2 + \frac{2}{5}R_3$

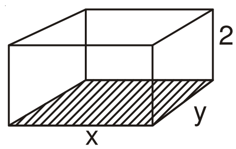
$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \cdot A$$

4

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

1

25.



$V = 2xy \Rightarrow 2xy = 8$ (given) 1

$\Rightarrow y = \frac{4}{x}$

Now, cost, $C = 70xy + 45 \times 2 \times (2x + 2y)$ 1

$= 280 + 180x + \frac{720}{x}$ 1

$\frac{dC}{dx} = 180 - \frac{720}{x^2}$ 1

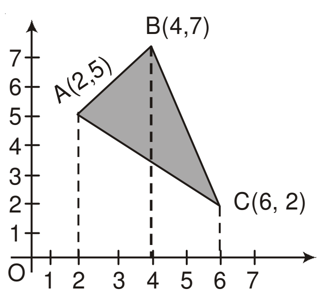
$\frac{dC}{dx} = 0 \Rightarrow x = 2m$ $\frac{1}{2}$

$\frac{d^2C}{dx^2} = \frac{1440}{x^3} = 180 > 0$ at $x = 2$ $\frac{1}{2}$

$\Rightarrow$ C is minimum at $x = 2m$. $\frac{1}{2}$

Minimum cost $= 280 + 180(2) + \frac{720}{2} = ₹\ 1{,}000$ $\frac{1}{2}$

26. Correct Figure 1



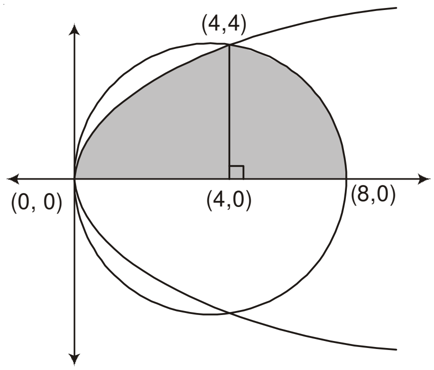
$$\left.\begin{array}{l} \text{Equation of } AB: y = x + 3 \\ \text{Equation of } BC: y = \frac{-5x}{2} + 17 \\ \text{Equation of } AC: y = \frac{-3x}{4} + \frac{13}{2} \end{array}\right\}$$ $1\frac{1}{2}$

Required Area $= \int_2^4 (x+3)\,dx + \int_4^6 \left(\frac{-5x}{2} + 17\right) dx - \int_2^6 \left(\frac{-3x}{4} + \frac{13}{2}\right) dx$ $1\frac{1}{2}$

$= \left[\frac{(x+3)^2}{2}\right]_2^4 + \left[\frac{-5x^2}{4} + 17x\right]_4^6 - \left[\frac{-3x^2}{8} + \frac{13x}{2}\right]_2^6$ $1\frac{1}{2}$

$= 7$ $\frac{1}{2}$

OR



Correct Figure 1

Given circle $x^2 - 8x + y^2 = 0$

or $(x-4)^2 + y^2 = 4^2$

Point of intersection (0, 0) and (4, 4) 1

Required Area $= \int_0^4 2\sqrt{x}\, dx + \int_4^8 \sqrt{4^2 - (x-4)^2}\, dx$ $1\frac{1}{2}$

$= \left[\frac{4}{3}x^{3/2}\right]_0^4 + \left[\frac{x-4}{2}\sqrt{16-(x-4)^2} + \frac{16}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_4^8$ $1\frac{1}{2}$

$= \left(4\pi + \frac{32}{3}\right)$ 1

27. Equation of plane is $\begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$ 2

$\Rightarrow 5x + 2y - 3z = 17$ (Cartesian equation) 1

Vector equation is $\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17$ 1

Equation of required parallel plane is

$\vec{r}\cdot(5\hat{i} + 2\hat{j} - 3\hat{k}) = (4\hat{i} + 3\hat{j} + \hat{k})\cdot(5\hat{i} + 2\hat{j} - 3\hat{k})$ 1

$\Rightarrow \vec{r}\cdot(5\hat{i} + 2\hat{j} - 3\hat{k}) = 23$ 1

OR

Let required plane be $a(x + 1) + b(y - 3) + c(z + 4) = 0$ ...(1) 1

Plane contains the given line, so it will also contain the point (1, 1, 0).

So, $2a - 2b + 4c = 0$ or $a - b + 2c = 0$ ...(2) 1

Also, $a + 2b - c = 0$ ...(3) 1

From (2) and (3),

$$\frac{a}{-3} = \frac{b}{3} = \frac{c}{3}$$ 1

$\therefore$ Required plane is $-3(x + 1) + 3(y - 3) + 3(z + 4) = 0$

$\therefore$ $-x + y + z = 0$

Also vector equation is: $\vec{r}.(-\hat{i} + \hat{j} + \hat{k}) = 0$ 1

Length of perpendicular from (2, 1, 4) = $\dfrac{|-2+1+4|}{\sqrt{(-1)^2+1^2+1^2}} = \sqrt{3}$ 1

28.

$$\left.\begin{aligned} \text{Let } E_1 &: \text{item is produced by A} \\ E_2 &: \text{item is produced by B} \\ E_3 &: \text{item is produced by C} \\ A &: \text{defective item is found.} \end{aligned}\right\}$$ 1

$$P(E_1) = \frac{50}{100}, P(E_2) = \frac{30}{100}, P(E_3) = \frac{20}{100}$$ 1

$$P(A / E_1) = \frac{1}{100}, P(A / E_2) = \frac{5}{100}, P(A | E_3) = \frac{7}{100}$$ 1

$$P(E_1 | A) = \frac{\frac{50}{100} \times \frac{1}{100}}{\frac{50}{100} \times \frac{1}{100} + \frac{30}{100} \times \frac{5}{100} + \frac{20}{100} \times \frac{7}{100}}$$ 2

$$= \frac{5}{34}$$ 1

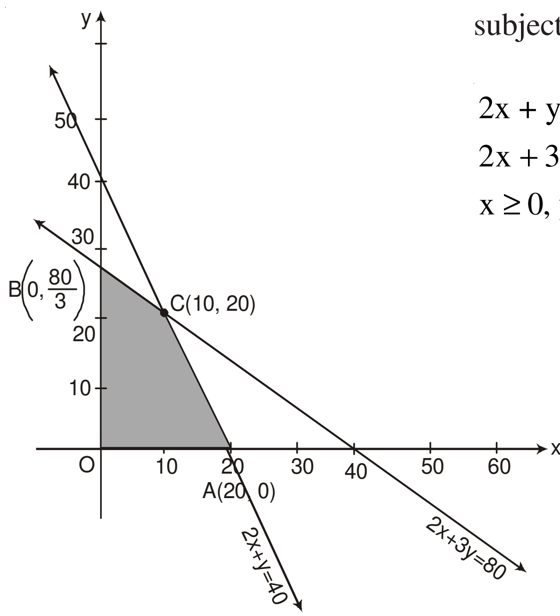
29. Let number of items produced of model A be x and that of model B be y.

LPP is:

Maximize, profit $z = 15x + 10y$ 1

subject to

$$\left.\begin{array}{l} 2x + y \le 5(8) \quad \text{i.e.,} \quad 2x + y \le 40 \\ 2x + 3y \le 10(8) \quad \text{i.e.,} \quad 2x + 3y \le 80 \\ x \ge 0,\ y \ge 0 \end{array}\right\}$$ 2



Correct Figure 2

Corner point	$z = 15x + 10y$	
A(20, 0)	300	
$B\left(0, \frac{80}{3}\right)$	$\frac{800}{3} \approx 266.6$	$\frac{1}{2}$
C(10, 20)	350 ← maximum	

Maximum profit = ₹ 350

when $x = 10$, $y = 20$. $\frac{1}{2}$

If a student has interpreted the language of the question in a different way, then the LPP will be of the type:

Maximise profit $z = 15x + 10y$

Subject to $2x + y \le 8$

$2x + 3y \le 8$

$x \ge 0,\ y \ge 0$

This is be accepted and marks may be given accordingly.

QUESTION PAPER CODE 65/1/2
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. order = 2, degree = 1 $\frac{1}{2}+\frac{1}{2}$

2. (fog) (x) = f(x – 7) = x $\frac{1}{2}$

$\Rightarrow \frac{d}{dx}[(fog)(x)] = 1$ $\frac{1}{2}$

3. $\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ $\frac{1}{2}$

$\Rightarrow$ x = 3, y = 3 $\quad\therefore$ x – y = 0 $\frac{1}{2}$

4. d.c.'s = <cos 90°, cos 135°, cos 45°> $\frac{1}{2}$

$= <0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}>$ $\frac{1}{2}$

OR

$\vec{r} = (3\hat{i}+4\hat{j}+5\hat{k}) + \lambda(2\hat{i}+2\hat{j}-3\hat{k})$ 1

SECTION B

5. As a, b $\in$ R $\Rightarrow$ ab $\in$ R $\Rightarrow$ ab + 1 $\in$ R $\Rightarrow$ a*b $\in$ R $\Rightarrow$ * is binary. 1

For associative $(a*b)*c = (ab+1)*c = (ab+1)c+1 = abc+c+1$

also, $a*(b*c) = a*(bc+1) = a.(bc+1)+1 = abc+a+1$

In general (a*b)*c $\neq$ a*(b*c) $\Rightarrow$ * is not associative. 1

6. $A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$ 1

$$A^2 - 5A = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} = \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix}$$ 1

7. Let $I = \int \sqrt{1 - \sin 2x}\, dx$

$= \int (\sin x - \cos x) dx$ as $\sin x > \cos x$ when $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ 1

$= -\cos x - \sin x + C$ 1

OR

$I = \int \sin^{-1}(2x).1\, dx$

$= x.\sin^{-1}(2x) - \int \frac{2x}{\sqrt{1-4x^2}} dx$ 1

$= x.\sin^{-1}(2x) + \frac{1}{4}\int \frac{-8x}{\sqrt{1-4x^2}} dx = x \sin^{-1}(2x) + \frac{1}{2}.\sqrt{1-4x^2} + C$ 1

8. $y' = be^{2x} + 2y \Rightarrow b = \frac{y' - 2y}{e^{2x}}$ $\frac{1}{2}$

differentiating again

$\frac{e^{2x}.(y'' - 2y') - (y' - 2y).2x^{2x}}{(e^{2x})^2} = 0$ 1

$\Rightarrow y'' - 4y' + 4y = 0$ or $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ $\frac{1}{2}$

9. Let X: getting an odd number

$p = \frac{1}{2}, \; q = \frac{1}{2}, \; n = 6$ $\frac{1}{2}$

(i) $P(X = 5) = {}^6C_5 \left(\frac{1}{2}\right)^6 = \frac{3}{32}$ $\frac{1}{2}$

(ii) $P(X \le 5) = 1 - P(X = 6) = 1 - \frac{1}{64} = \frac{63}{64}$ 1

OR

$k + 2k + 3k = 1$ 1

$\Rightarrow \ k = \frac{1}{6}$ 1

10. $A = \{2, 4, 6\}, \ \ B = \{1, 2, 3\}, \ \ A \cap B = \{2\}$

Now, $P(A) = \frac{1}{2}, \ P(B) = \frac{1}{2}, \ P(A \cap B) = \frac{1}{6}$ 1

as $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$ $\frac{1}{2}$

$\Rightarrow$ A and B are not independent. $\frac{1}{2}$

11. Given $|\hat{a} + \hat{b}| = 1$

As $|\hat{a} + \hat{b}|^2 + |\hat{a} - \hat{b}|^2 = 2(|\hat{a}|^2 + |\hat{b}|^2)$ 1

$\Rightarrow \ 1 + |\vec{a} - \vec{b}|^2 = 2(1+1)$

$\Rightarrow \ |\hat{a} - \hat{b}|^2 = 3 \Rightarrow |\hat{a} - \hat{b}| = \sqrt{3}$ 1

OR

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$$ 1

$= -30$ 1

12. $I = \int \frac{\tan^2 x . \sec^2 x}{1 - (\tan^3 x)^2} dx$

Put $\tan^3 x = t \Rightarrow I = \frac{1}{3} \int \frac{dt}{1 - t^2}$ 1

$= \frac{1}{6} \log \left| \frac{1+t}{1-t} \right| + C = \frac{1}{6} \log \left| \frac{1 + \tan^3 x}{1 - \tan^3 x} \right| + C$ $\frac{1}{2} + \frac{1}{2}$

SECTION C

13. $\tan^{-1}\left(\dfrac{2x+3x}{1-(2x)(3x)}\right)=\dfrac{\pi}{4}$ 1

$\Rightarrow \dfrac{5x}{1-6x^2}=\tan\dfrac{\pi}{4}=1 \Rightarrow 6x^2+5x-1=0$ $1\frac{1}{2}$

$\Rightarrow x=-1 \text{ or } x=\dfrac{1}{6}$ 1

as $x=-1$ does not satisfy the given equation,

$\therefore\ x = \dfrac{1}{6}$ $\frac{1}{2}$

14. $\log(x^2+y^2)=2\tan^{-1}\left(\dfrac{y}{x}\right)$

differentiating both sides w.r.t. x,

$\dfrac{1}{x^2+y^2}\left(2x+2y\dfrac{dy}{dx}\right)=2\cdot\dfrac{1}{1+\left(\dfrac{y}{x}\right)^2}\cdot\left(\dfrac{x\cdot\dfrac{dy}{dx}-y\cdot 1}{x^2}\right)$ 2

$\Rightarrow \dfrac{2}{x^2+y^2}\left(x+y\dfrac{dy}{dx}\right)=\dfrac{2x^2}{x^2+y^2}\cdot\dfrac{1}{x^2}\cdot\left(x\dfrac{dy}{dx}-y\right)$ 1

$\Rightarrow (x+y)=(x-y)\dfrac{dy}{dx}\Rightarrow\dfrac{dy}{dx}=\dfrac{x+y}{x-y}$ 1

OR

Let $u = x^y$, $v = y^x$. Then $u - v = a^b$

$\Rightarrow \dfrac{du}{dx}-\dfrac{dv}{dx} = 0$ ...(1) 1

Now, $\log u = y.\log x$

$\Rightarrow \dfrac{1}{u}\cdot\dfrac{du}{dx}=y\cdot\dfrac{1}{x}+\log x\cdot\dfrac{dy}{dx}\Rightarrow\dfrac{du}{dx}=x^y\left(\dfrac{y}{x}+\log x\cdot\dfrac{dy}{dx}\right)$ ...(2) 1

Again, log v = x · log y

$$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = x\cdot\frac{1}{y}\cdot\frac{dy}{dx} + \log y\cdot 1 \Rightarrow \frac{dv}{dx} = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) \quad ...(3)$$ 1

From (1), (2) and (3)

$$x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) - y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$$ $\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{y^x\cdot\log y - x^{y-1}\cdot y}{x^y\cdot\log x - y^{x-1}\cdot x}$$ $\frac{1}{2}$

15. $$I = \int\frac{3x+5}{x^2+3x-18}\,dx = \frac{3}{2}\int\frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int\frac{1}{x^2+3x-18}\,dx$$ 1

$$= \frac{3}{2}\int\frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int\frac{1}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\,dx$$ 1

$$= \frac{3}{2}\log|x^2+3x-18| + \frac{1}{18}\log\left|\frac{x-3}{x+6}\right| + C$$ 1 + 1

16. Let $I = \int_0^a f(a-x)\,dx$

Put a – x = t ⇒ –dx = dt $\frac{1}{2}$

$$I = -\int_a^0 f(t)\,dt = \int_0^a f(t)\,dt = \int_0^a f(x)\,dx$$ $\frac{1}{2}$

II part.

$$I = \int_0^\pi \frac{x\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow \quad I = \int_0^\pi \frac{(\pi - x)\cdot\sin x}{1+\cos^2 x}\,dx$$

$$\Rightarrow \quad 2I = \int_0^{\pi} \frac{\pi \cdot \sin x}{1+\cos^2 x}\,dx$$ $1\frac{1}{2}$

Put $\cos x = t \Rightarrow -\sin x\,dx = dt$

$$\Rightarrow\ I = -\frac{\pi}{2} \cdot \int_1^{-1} \frac{dt}{1+t^2} = \frac{\pi}{2} \times 2 \times \int_0^1 \frac{dt}{1+t^2}$$

$$= \pi[\tan^{-1} t]_0^1 = \frac{\pi^2}{4}$$ $1\frac{1}{2}$

17. $\overrightarrow{AB} = \hat{i} + 4\hat{j} - \hat{k}$ 1

$\overrightarrow{CD} = -2\hat{i} - 8\hat{j} + 2\hat{k}$ 1

Let required angle be θ.

Then $\cos\theta = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|} = \dfrac{-2-32-2}{\sqrt{18}\,\sqrt{72}} = -1$ 1

$\Rightarrow \theta = 180°$ or π $\frac{1}{2}$

Since $\theta = \pi$ so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. $\frac{1}{2}$

18. LHS = $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$

$C_2 \to C_2 + C_1,\ C_3 \to C_3 + C_1$

$$= \begin{vmatrix} a+b+c & a+b & a+c \\ -c & a+b & -(a+c) \\ -b & -(a+b) & (a+c) \end{vmatrix}$$ $1\frac{1}{2}$

$$= (a+b)\,(a+c) \begin{vmatrix} a+b+c & 1 & 1 \\ -c & 1 & -1 \\ -b & -1 & 1 \end{vmatrix}$$ $\frac{1}{2}$

$C_3 \to C_3 + C_2$

$$= (a+b)\,(a+c)\begin{vmatrix} a+b+c & 1 & 2 \\ -c & 1 & 0 \\ -b & -1 & 0 \end{vmatrix}$$ 1

$= 2(a + b)\ (b + c)\ (c + a) = \text{RHS}.$ 1

19. $$\frac{dx}{dt} = -\sin t + \frac{1}{\tan\frac{t}{2}} \times \left(\sec^2\frac{t}{2} \times \frac{1}{2}\right) = \frac{\cos^2 t}{\sin t}$$ 1

$$\frac{dy}{dt} = \cos t$$ $\frac{1}{2}$

$$\frac{d^2y}{dt^2} = -\sin t \Rightarrow \left.\frac{d^2y}{dt^2}\right]_{t=\frac{\pi}{4}} = -\frac{1}{\sqrt{2}}$$ 1

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \tan t$$ $\frac{1}{2}$

$$\frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{dt}{dx} = \sec^4 t \cdot \sin t$$

$$\Rightarrow \left.\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{4}} = 2\sqrt{2}$$ 1

20. Clearly $a \le a\ \forall\ a \in \mathbb{R} \Rightarrow (a, a) \in R \Rightarrow R$ is reflexive. 1

For transitive:

Let $(a, b) \in R$ and $(b, c) \in R$, $a, b, c \in \mathbb{R}$

$\Rightarrow a \le b$ and $b \le c \Rightarrow a \le c \Rightarrow (a, c) \in R$

$\Rightarrow R$ is transitive. $1\frac{1}{2}$

For non-symmetric:

Let $a = 1$, $b = 2$. As $1 \le 2 \Rightarrow (1, 2) \in R$ but $2 \not\le 1 \Rightarrow (2,1) \notin R$

$\Rightarrow R$ is non-symmetric. $1\frac{1}{2}$

OR

For one-one. Let $x_1, x_2 \in N$.

$f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow x_1 = x_2$ as $x_1 + x_2 + 1 \neq 0$ $\quad (\because x_1, x_2 \in N)$ $\quad 1\frac{1}{2}$

$\Rightarrow$ f is one-one.

For not onto.

for $y = 1 \in N$, there is no $x \in N$ for which $f(x) = 1$ $\quad 1\frac{1}{2}$

For f^{-1}: $y = f(x) \Rightarrow y = x^2 + x + 1 \Rightarrow y = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}$

$$\Rightarrow x = \frac{\sqrt{4y-3}-1}{2}$$

$\therefore f^{-1}(y) = \dfrac{\sqrt{4y-3}-1}{2}$ or $f^{-1}(x) = \dfrac{\sqrt{4x-3}-1}{2}$ $\quad 1$

21. Let the point of contact be $P(x_1, y_1)$

$\dfrac{dy}{dx} = \dfrac{3}{2\sqrt{3x-2}}$ (slope of tangent)

$\Rightarrow m_1 = \left.\dfrac{dy}{dx}\right]_{(x_1, y_1)} = \dfrac{3}{2\sqrt{3x_1-2}}$ $\quad 1$

also, slope of given line $= 2 = m_2$

$m_1 = m_2 \Rightarrow x_1 = \dfrac{41}{48}$ $\quad 1$

when $x_1 = \dfrac{41}{48}, y_1 = \sqrt{\dfrac{41}{16}-2} = \dfrac{3}{4}$ $\quad \therefore P\left(\dfrac{41}{48}, \dfrac{3}{4}\right)$ $\quad \frac{1}{2}$

Equation of tangent is: $y - \dfrac{3}{4} = 2\left(x - \dfrac{41}{48}\right)$

$\Rightarrow 48x - 24y = 23$ 1

and, Equation of normal is: $y - \frac{3}{4} = \frac{-1}{2}\left(x - \frac{41}{48}\right)$

$\Rightarrow 48x + 96y = 113$ $\frac{1}{2}$

22. Writing $\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x} = \frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2}$ $\frac{1}{2}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}$

Differential equation becomes $v + x\frac{dv}{dx} = v + \sqrt{1 + v^2}$

$\Rightarrow \int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$ 1

$\Rightarrow \log\left|v + \sqrt{1 + v^2}\right| = \log|x| + \log c$ 1

$\Rightarrow v + \sqrt{1 + v^2} = cx \Rightarrow y + \sqrt{x^2 + y^2} = cx^2$

when $x = 1, y = 0 \Rightarrow c = 1$ $\frac{1}{2}$

$\therefore y + \sqrt{x^2 + y^2} = x^2$ $\frac{1}{2}$

OR

Given equation is $\frac{dy}{dx} + \frac{2x}{1 + x^2} \cdot y = \frac{4x^2}{1 + x^2}$ $\frac{1}{2}$

I.F. $= e^{\int \frac{2x}{1+x^2} dx} = 1 + x^2$ 1

Solution is given by,

$y \cdot (1 + x^2) = \int \frac{4x^2}{1 + x^2} \cdot (1 + x^2)\, dx = \int 4x^2\, dx$ 1

$$\Rightarrow \; y \cdot (1+x^2) = \frac{4x^3}{3} + c$$ $\frac{1}{2}$

when $x = 0, \; y = 0 \Rightarrow c = 0$ $\frac{1}{2}$

$$y \cdot (1+x^2) = \frac{4x^3}{3} \quad \text{or} \quad y = \frac{4x^3}{3(1+x^2)}$$ $\frac{1}{2}$

23. Given lines are: $\dfrac{x-1}{-3} = \dfrac{y-2}{\left(\frac{\lambda}{7}\right)} = \dfrac{z-3}{2}$ and $\dfrac{x-1}{\left(\frac{-3\lambda}{7}\right)} = \dfrac{y-5}{1} = \dfrac{z-6}{-5}$ 1

As lines are perpendicular,

$$(-3)\left(\frac{-3\lambda}{7}\right) + \left(\frac{\lambda}{7}\right)(1) + 2(-5) = 0 \Rightarrow \lambda = 7$$ 1

So, lines are

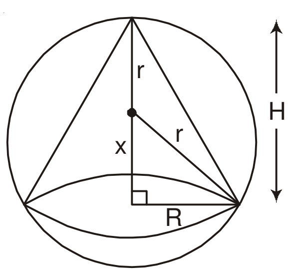
$$\frac{x-1}{-3} = \frac{y-2}{1} = \frac{x-3}{2} \quad \text{and} \quad \frac{x-1}{-3} = \frac{y-5}{1} = \frac{z-6}{-5}$$ $\frac{1}{2}$

Consider $\Delta = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 0 & 3 & 3 \\ -3 & 1 & 2 \\ -3 & 1 & -5 \end{vmatrix} = -63$ 1

as $\Delta \neq 0 \Rightarrow$ lines are not intersecting. $\frac{1}{2}$

SECTION D

24.



Correct Figure 1

$r^2 = x^2 + R^2$

Now, $V = \dfrac{1}{3}\pi R^2 H$

$$= \frac{1}{3}\pi(r^2 - x^2)(r + x)$$

$$= \frac{1}{3}\pi(r + x)^2 \,(r - x)$$ 1

$$\frac{dV}{dx} = \frac{1}{3}\pi\left[(r+x)^2\,(-1) + (r-x).2(r+x)\right]$$

$$= \frac{1}{3}\pi(r+x)\,(r-3x)$$ 1

$$\frac{dV}{dx} = 0 \Rightarrow x = -r \text{ or } x = \frac{r}{3}$$ $\frac{1}{2}$

(Rejected)

$$\frac{d^2V}{dx^2} = \frac{1}{3}\pi\left[(r+x)(-3) + (r-3x)\right] = -\pi H < 0$$ 1

$\Rightarrow$ V is maximum when $x = \frac{r}{3}$.

$H = r + x = r + \frac{r}{3} = \frac{4r}{3}$ $\frac{1}{2}$

Maximum volume $V = \frac{1}{3}\pi\left(r+\frac{r}{3}\right)^2\left(r-\frac{r}{3}\right) = \frac{32}{81}\pi r^3$ 1

25. $|A| = -1 \neq 0 \Rightarrow A^{-1}$ exists. 1

$$\text{adj } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$ 2

$$A^{-1} = \frac{1}{|A|}\cdot \text{adj } A = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$ $\frac{1}{2}$

Given system of equations can be written as AX = B where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Now, $X = A^{-1}B$ 1

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ 1

$\Rightarrow$ x = 1, y = 2, z = 3 $\frac{1}{2}$

OR

A = I.A

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$ 1

$R_1 \leftrightarrow R_2$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_2 \to R_2 + R_1, R_3 \to R_3 - 3R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} \cdot A$$

$R_2 \to \frac{R_2}{3}$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5/3 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1/3 & 1/3 & 0 \\ 0 & -3 & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 - 2R_2, R_3 \to R_3 + 5R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 5/3 \\ 0 & 0 & 1/3 \end{bmatrix} = \begin{bmatrix} -2/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 \\ 5/3 & -4/3 & 1 \end{bmatrix} \cdot A$$ 4

$R_3 \to 3R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 5/3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 \\ 5 & -4 & 3 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + \frac{1}{3}R_3, R_2 \to R_2 - \frac{5}{3}R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} \cdot A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$$ 1

26. $\left.\begin{array}{l} \text{Let } E_1 : \text{item is produced by A} \\ \quad E_2 : \text{item is produced by B} \\ \quad E_3 : \text{item is produced by C} \\ \quad A : \text{defective item is found.} \end{array}\right\}$ 1

$$P(E_1) = \frac{50}{100}, P(E_2) = \frac{30}{100}, P(E_3) = \frac{20}{100}$$ 1

$$P(A / E_1) = \frac{1}{100}, P(A / E_2) = \frac{5}{100}, P(A | E_3) = \frac{7}{100}$$ 1

$$P(E_1 | A) = \frac{\frac{50}{100} \times \frac{1}{100}}{\frac{50}{100} \times \frac{1}{100} + \frac{30}{100} \times \frac{5}{100} + \frac{20}{100} \times \frac{7}{100}}$$ 2

$$= \frac{5}{34}$$ 1

27. Equation of plane is $\begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$ 2

$\Rightarrow 5x + 2y - 3z = 17$ (Cartesian equation) 1

Vector equation is $\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17$ 1

Equation of required parallel plane is

$$\vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = (4\hat{i} + 3\hat{j} + \hat{k}) \cdot (5\hat{i} + 2\hat{j} - 3\hat{k})$$ 1

$$\Rightarrow \vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = 23$$ 1

OR

Let required plane be $a(x + 1) + b(y - 3) + c(z + 4) = 0$ ...(1) 1

Plane contains the given line, so it will also contain the point (1, 1, 0).

So, $2a - 2b + 4c = 0$ or $a - b + 2c = 0$ ...(2) 1

Also, $a + 2b - c = 0$ ...(3) 1

From (2) and (3),

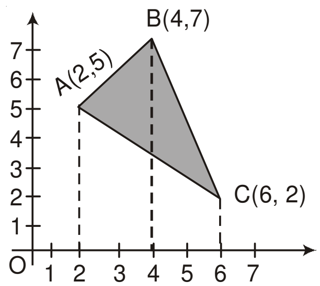
$\frac{a}{-3} = \frac{b}{3} = \frac{c}{3}$ 1

$\therefore$ Required plane is $-3(x + 1) + 3(y - 3) + 3(z + 4) = 0$

$\therefore$ $-x + y + z = 0$

Also vector equation is: $\vec{r}.(-\hat{i} + \hat{j} + \hat{k}) = 0$ 1

Length of perpendicular from (2, 1, 4) = $\frac{|-2+1+4|}{\sqrt{(-1)^2 + 1^2 + 1^2}} = \sqrt{3}$ 1

28. 

Correct Figure 1

Equation of AB : $y = x + 3$

Equation of BC : $y = \frac{-5x}{2} + 17$

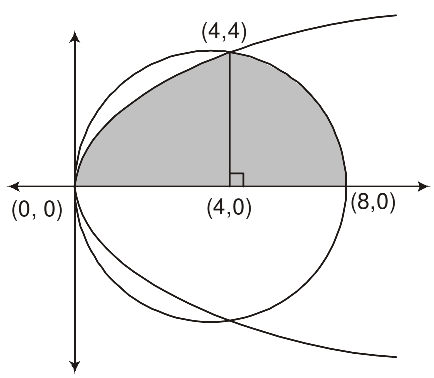
Equation of AC : $y = \frac{-3x}{4} + \frac{13}{2}$ $1\frac{1}{2}$

Required Area $= \int_2^4 (x+3)\, dx + \int_4^6 \left(\frac{-5x}{2} + 17\right) dx - \int_2^6 \left(\frac{-3x}{4} + \frac{13}{2}\right) dx$ $1\frac{1}{2}$

$= \left[\frac{(x+3)^2}{2}\right]_2^4 + \left[\frac{-5x^2}{4} + 17x\right]_4^6 - \left[\frac{-3x^2}{8} + \frac{13x}{2}\right]_2^6$ $1\frac{1}{2}$

$= 7$ $\frac{1}{2}$

OR



Correct Figure 1

Given circle $x^2 - 8x + y^2 = 0$

or $(x-4)^2 + y^2 = 4^2$

Point of intersection (0, 0) and (4, 4) 1

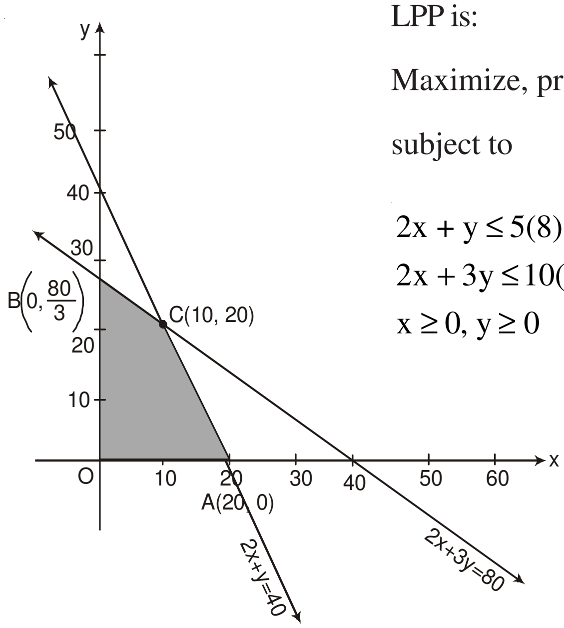
$$\text{Required Area} = \int_0^4 2\sqrt{x}\,dx + \int_4^8 \sqrt{4^2-(x-4)^2}\,dx \qquad 1\frac{1}{2}$$

$$= \left[\frac{4}{3}x^{3/2}\right]_0^4 + \left[\frac{x-4}{2}\sqrt{16-(x-4)^2} + \frac{16}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_4^8 \qquad 1\frac{1}{2}$$

$$= \left(4\pi + \frac{32}{3}\right) \qquad 1$$

Note: A student may also arrive at the answer $\left(8\pi + \frac{64}{3}\right)$ which is double $\left(4\pi + \frac{32}{3}\right)$ because of 'about x-axis'. He/she may be given full marks.

29. Let number of items produced of model A be x and that of model B be y.



LPP is:

Maximize, profit $z = 15x + 10y$ 1

subject to

$$\left.\begin{array}{ll} 2x + y \le 5(8) & \text{i.e., } 2x + y \le 40 \\ 2x + 3y \le 10(8) & \text{i.e., } 2x + 3y \le 80 \\ x \ge 0, y \ge 0 & \end{array}\right\} \qquad 2$$

Correct Figure 2

Corner point	$z = 15x + 10y$	
A(20, 0)	300	
$B\left(0, \frac{80}{3}\right)$	$\frac{800}{3} \approx 266.6$	$\frac{1}{2}$
C(10, 20)	350 ← maximum	

Maximum profit = ₹ 350

when $x = 10$, $y = 20$. $\frac{1}{2}$

If a student has interpreted the language of the question in a different way, then the LPP will be of the type:

Maximise profit $z = 15x + 10y$

Subject to $2x + y \le 8$

$2x + 3y \le 8$

$x \ge 0, \; y \ge 0$

This is be accepted and marks may be given accordingly.

QUESTION PAPER CODE 65/1/3
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $3A = \begin{bmatrix} 9 & 3 \\ 3 & 6 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ $\frac{1}{2}+\frac{1}{2}$

2. Order = 2, degree = 2 $\frac{1}{2}+\frac{1}{2}$

3. (fof) (x) = f(x + 1) = x + 2 $\frac{1}{2}$

$\frac{d}{dx}(\text{fof})(x) = 1$ $\frac{1}{2}$

4. d.c.'s = <cos 90°, cos 135°, cos 45°> $\frac{1}{2}$

$= < 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$ $\frac{1}{2}$

OR

$\vec{r} = (3\hat{i}+4\hat{j}+5\hat{k})+\lambda(2\hat{i}+2\hat{j}-3\hat{k})$ 1

SECTION B

5. $I = \int \sin x \cdot \log(\cos x)\, dx$

$\cos x = t \Rightarrow I = -\int \log t \cdot 1 dt$ 1

$= -\left[t.\log t - \int \frac{1}{t}.t dt \right]$ $\frac{1}{2}$

$= t(1 - \log t) + C = \cos x(1 - \log(\cos x)) + C$ $\frac{1}{2}$

6. Let $f(x) = (1 - x^2) . \sin x \cos^2 x$

as $f(-x) = -f(x) \Rightarrow$ f is odd function. 1

$\therefore I = 0$ 1

OR

$$I = \int_{-1}^{2} \frac{|x|}{x} dx = \int_{-1}^{0} -1 dx + \int_{0}^{2} 1 dx$$ 1

$= -1 + 2 = 1$ 1

7. As a, b $\in$ R $\Rightarrow$ ab $\in$ R $\Rightarrow$ ab + 1 $\in$ R $\Rightarrow$ a*b $\in$ R $\Rightarrow$ * is binary. 1

For associative $(a*b)*c = (ab+1)*c = (ab+1)c+1 = abc+c+1$

also, $a*(b*c) = a*(bc+1) = a.(bc+1)+1 = abc+a+1$

In general $(a*b)*c \neq a*(b*c) \Rightarrow *$ is not associative. 1

8. $$2A - \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} + \begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ 1

$$\Rightarrow A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$$ 1

9. $A = \{2, 4, 6\}$, $B = \{1, 2, 3\}$, $A \cap B = \{2\}$

Now, $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{6}$ 1

as $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$ $\frac{1}{2}$

$\Rightarrow$ A and B are not independent. $\frac{1}{2}$

10. $y' = be^{2x} + 2y \Rightarrow b = \frac{y'-2y}{e^{2x}}$ $\frac{1}{2}$

differentiating again

$$\frac{e^{2x}.(y''-2y')-(y'-2y).2x^{2x}}{(e^{2x})^2} = 0$$ 1

$\Rightarrow y'' - 4y' + 4y = 0$ or $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ $\frac{1}{2}$

11. Let X: getting an odd number

$p = \frac{1}{2},\ q = \frac{1}{2},\ n = 6$ $\frac{1}{2}$

(i) $P(X = 5) = {}^6C_5 \left(\frac{1}{2}\right)^6 = \frac{3}{32}$ $\frac{1}{2}$

(ii) $P(X \le 5) = 1 - P(X = 6) = 1 - \frac{1}{64} = \frac{63}{64}$ 1

OR

$k + 2k + 3k = 1$ 1

$\Rightarrow\ k = \frac{1}{6}$ 1

12. Given $|\hat{a} + \hat{b}| = 1$

As $|\hat{a} + \hat{b}|^2 + |\hat{a} - \hat{b}|^2 = 2(|\hat{a}|^2 + |\hat{b}|^2)$ 1

$\Rightarrow\ 1 + |\vec{a} - \vec{b}|^2 = 2(1 + 1)$

$\Rightarrow\ |\hat{a} - \hat{b}|^2 = 3 \Rightarrow |\hat{a} - \hat{b}| = \sqrt{3}$ 1

OR

$[\vec{a}\ \ \vec{b}\ \ \vec{c}] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$ 1

$= -30$ 1

SECTION C

13. LHS $= \begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$

$R_1 \to R_1 + R_3$ and taking $a + b + c$ common from R_1

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$$ 1

$C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ a-2b+c & b-2c+a & c-a \\ b-a & c-b & a+b \end{vmatrix}$$ 1

$= (a + b + c) [(a - 2b + c) (c - b) - (b - 2c + a) (b - a)]$ 1

$= (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + b^3 + c^3 - 3abc =$ RHS. 1

14. $\tan^{-1}\left(\dfrac{4x+6x}{1-(4x)(6x)}\right) = \dfrac{\pi}{4}$ 1

$\Rightarrow \dfrac{10x}{1-24x^2} = 1 \Rightarrow 24x^2 + 10x - 1 = 0$ $1\frac{1}{2}$

$\Rightarrow x = \dfrac{1}{12}$ or $-\dfrac{1}{2}$ 1

as $x = -\dfrac{1}{2}$ does not satisfy the given equation, so $x = \dfrac{1}{12}$ $\frac{1}{2}$

15. Clearly $a \le a \ \forall\, a \in \mathbb{R} \Rightarrow (a, a) \in R \Rightarrow R$ is reflexive. 1

For transitive:

Let $(a, b) \in R$ and $(b, c) \in R$, $a, b, c \in \mathbb{R}$

$\Rightarrow a \le b$ and $b \le c \Rightarrow a \le c \Rightarrow (a, c) \in R$

$\Rightarrow$ R is transitive. $1\frac{1}{2}$

For non-symmetric:

Let $a = 1$, $b = 2$. As $1 \le 2 \Rightarrow (1, 2) \in R$ but $2 \not\le 1 \Rightarrow (2,1) \notin R$

$\Rightarrow$ R is non-symmetric. $1\frac{1}{2}$

OR

For one-one. Let $x_1, x_2 \in N$.

$f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow x_1 = x_2$ as $x_1 + x_2 + 1 \neq 0$ $\quad (\because x_1, x_2 \in N)$ $1\frac{1}{2}$

$\Rightarrow$ f is one-one.

For not onto.

for $y = 1 \in N$, there is no $x \in N$ for which $f(x) = 1$ $1\frac{1}{2}$

For f^{-1}: $y = f(x) \Rightarrow y = x^2 + x + 1 \Rightarrow y = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}$

$$\Rightarrow x = \frac{\sqrt{4y-3}-1}{2}$$

$\therefore f^{-1}(y) = \dfrac{\sqrt{4y-3}-1}{2}$ or $f^{-1}(x) = \dfrac{\sqrt{4x-3}-1}{2}$ 1

16. Let the point of contact be $P(x_1, y_1)$

$\dfrac{dy}{dx} = \dfrac{3}{2\sqrt{3x-2}}$ (slope of tangent)

$\Rightarrow m_1 = \left.\dfrac{dy}{dx}\right]_{(x_1,\, y_1)} = \dfrac{3}{2\sqrt{3x_1-2}}$ 1

also, slope of given line $= 2 = m_2$

$m_1 = m_2 \Rightarrow x_1 = \dfrac{41}{48}$ 1

when $x_1 = \dfrac{41}{48}, y_1 = \sqrt{\dfrac{41}{16} - 2} = \dfrac{3}{4}$ $\quad \therefore P\left(\dfrac{41}{48}, \dfrac{3}{4}\right)$ $\frac{1}{2}$

Equation of tangent is: $y-\frac{3}{4}=2\left(x-\frac{41}{48}\right)$

$\Rightarrow 48x - 24y = 23$ 1

and, Equation of normal is: $y-\frac{3}{4}=\frac{-1}{2}\left(x-\frac{41}{48}\right)$

$\Rightarrow 48x + 96y = 113$ $\frac{1}{2}$

17. $\log(x^2+y^2)=2\tan^{-1}\left(\frac{y}{x}\right)$

differentiating both sides w.r.t. x,

$$\frac{1}{x^2+y^2}\left(2x+2y\frac{dy}{dx}\right)=2\cdot\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{x\cdot\frac{dy}{dx}-y\cdot 1}{x^2}\right)$$ 2

$$\Rightarrow \frac{2}{x^2+y^2}\left(x+y\frac{dy}{dx}\right)=\frac{2x^2}{x^2+y^2}\cdot\frac{1}{x^2}\cdot\left(x\frac{dy}{dx}-y\right)$$ 1

$$\Rightarrow (x+y)=(x-y)\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{x+y}{x-y}$$ 1

OR

Let $u = x^y$, $v = y^x$. Then $u - v = a^b$

$$\Rightarrow \frac{du}{dx}-\frac{dv}{dx}=0 \quad ...(1)$$ 1

Now, $\log u = y.\log x$

$$\Rightarrow \frac{1}{u}\cdot\frac{du}{dx}=y\cdot\frac{1}{x}+\log x\cdot\frac{dy}{dx}\Rightarrow\frac{du}{dx}=x^y\left(\frac{y}{x}+\log x\cdot\frac{dy}{dx}\right) \quad ...(2)$$ 1

Again, $\log v = x\cdot\log y$

$$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx}=x\cdot\frac{1}{y}\cdot\frac{dy}{dx}+\log y\cdot 1\Rightarrow\frac{dv}{dx}=y^x\left(\frac{x}{y}\frac{dy}{dx}+\log y\right) \quad ...(3)$$ 1

From (1), (2) and (3)

$$x^y\left(\frac{y}{x}+\log x\frac{dy}{dx}\right)-y^x\left(\frac{x}{y}\frac{dy}{dx}+\log y\right)=0$$ $\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx}=\frac{y^x\cdot\log y-x^{y-1}\cdot y}{x^y\cdot\log x-y^{x-1}\cdot x}$$ $\frac{1}{2}$

18. $y=(\sin^{-1}x)^2$

$$\Rightarrow y'=2\cdot\sin^{-1}x\cdot\frac{1}{\sqrt{1-x^2}}$$ 1

$$\Rightarrow \sqrt{1-x^2}\cdot y'=2\sin^{-1}x$$

$$\Rightarrow \sqrt{1-x^2}\cdot y''+y'\cdot\frac{1}{2\sqrt{1-x^2}}(-2x)=\frac{2}{\sqrt{1-x^2}}$$ 2

$$\Rightarrow (1-x^2)\cdot y''-xy'=2 \text{ or } (1-x^2)\cdot\frac{d^2y}{dx^2}-x\frac{dy}{dx}-2=0.$$ 1

19. Let $I=\int_0^a f(a-x)\,dx$

Put $a-x=t \Rightarrow -dx=dt$ $\frac{1}{2}$

$$I=-\int_a^0 f(t)\,dt=\int_0^a f(t)\,dt=\int_0^a f(x)\,dx$$ $\frac{1}{2}$

II part.

$$I=\int_0^\pi \frac{x\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow \quad I=\int_0^\pi \frac{(\pi-x)\cdot\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow \quad 2I=\int_0^\pi \frac{\pi\cdot\sin x}{1+\cos^2 x}dx$$ $1\frac{1}{2}$

Put $\cos x = t \Rightarrow -\sin x \, dx = dt$

$$\Rightarrow \ I = -\frac{\pi}{2} \cdot \int_{1}^{-1} \frac{dt}{1+t^2} = \frac{\pi}{2} \times 2 \times \int_{0}^{1} \frac{dt}{1+t^2}$$

$$= \pi[\tan^{-1} t]_0^1 = \frac{\pi^2}{4}$$ $1\frac{1}{2}$

20. $I = \int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx.$ Put $\sin x = t$ $\frac{1}{2}$

$$= \int \frac{dt}{(1+t)(2+t)} = \int \left(\frac{1}{1+t} - \frac{1}{2+t} \right) dt$$ 2

$$= \log \left| \frac{1+t}{2+t} \right| + c = \log \left| \frac{1+\sin x}{2+\sin x} \right| + c$$ $1+\frac{1}{2}$

21. $\text{I.F.} = e^{-\int \frac{2x}{1+x^2} dx} = \frac{1}{1+x^2}$ 1

Solution is given by,

$$y \cdot \left(\frac{1}{1+x^2} \right) = \int \frac{x^2+2}{1+x^2} dx$$ $1\frac{1}{2}$

$$y \cdot \frac{1}{1+x^2} = \int \left(1 + \frac{1}{1+x^2} \right) dx = x + \tan^{-1} x + c$$ $1\frac{1}{2}$

or $y = (1 + x^2)(x + \tan^{-1}x + c)$

OR

Given equation can be written as

$$\int \frac{dy}{2e^{-y} - 1} = \int \frac{dx}{x+1}$$

$$\Rightarrow \ \int \frac{e^y}{2-e^y} dy = \int \frac{dx}{x+1}$$ 1

$$\Rightarrow \ -\log|2-e^y| + \log c = \log|x+1|$$ $1\frac{1}{2}$

$\Rightarrow (2 - e^y)(x + 1) = c$

When $x = 0,\ y = 0 \Rightarrow c = 1$ 1

$\therefore$ Solution is $(2 - e^y)(x + 1) = 1$ $\frac{1}{2}$

22. $\overrightarrow{AB} = \hat{i} + 4\hat{j} - \hat{k}$ 1

$\overrightarrow{CD} = -2\hat{i} - 8\hat{j} + 2\hat{k}$ 1

Let required angle be θ.

Then $\cos\theta = \dfrac{\overrightarrow{AB}\cdot\overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|} = \dfrac{-2-32-2}{\sqrt{18}\,\sqrt{72}} = -1$ 1

$\Rightarrow \theta = 180°$ or π $\frac{1}{2}$

Since $\theta = \pi$ so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. $\frac{1}{2}$

23. Given lines are: $\dfrac{x-1}{-3} = \dfrac{y-2}{\left(\frac{\lambda}{7}\right)} = \dfrac{z-3}{2}$ and $\dfrac{x-1}{\left(\frac{-3\lambda}{7}\right)} = \dfrac{y-5}{1} = \dfrac{z-6}{-5}$ 1

As lines are perpendicular,

$(-3)\left(\dfrac{-3\lambda}{7}\right) + \left(\dfrac{\lambda}{7}\right)(1) + 2(-5) = 0 \Rightarrow \lambda = 7$ 1

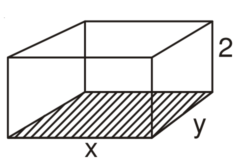
So, lines are

$\dfrac{x-1}{-3} = \dfrac{y-2}{1} = \dfrac{x-3}{2}$ and $\dfrac{x-1}{-3} = \dfrac{y-5}{1} = \dfrac{z-6}{-5}$ $\frac{1}{2}$

Consider $\Delta = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 0 & 3 & 3 \\ -3 & 1 & 2 \\ -3 & 1 & -5 \end{vmatrix} = -63$ 1

as $\Delta \neq 0 \Rightarrow$ lines are not intersecting. $\frac{1}{2}$

24. $V = 2xy \Rightarrow 2xy = 8$ (given) 1



$\Rightarrow y = \dfrac{4}{x}$

Now, cost, $C = 70xy + 45 \times 2 \times (2x + 2y)$ 1

$$= 280 + 180x + \frac{720}{x}$$ 1

$$\frac{dC}{dx} = 180 - \frac{720}{x^2}$$ 1

$$\frac{dC}{dx} = 0 \Rightarrow x = 2m$$ $\frac{1}{2}$

$$\frac{d^2C}{dx^2} = \frac{1440}{x^3} = 180 > 0 \text{ at } x = 2$$ $\frac{1}{2}$

$\Rightarrow$ C is minimum at $x = 2m$. $\frac{1}{2}$

Minimum cost = $280 + 180(2) + \frac{720}{2}$ = ₹ 1,000 $\frac{1}{2}$

25. $|A| = 4 \neq 0 \Rightarrow A^{-1}$ exists. 1

$$\text{adj } A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ 2

$$\therefore \; A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ $\frac{1}{2}$

Given system of equations can be written as AX = B where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$\therefore \; X = A^{-1} \cdot B$ 1

$$= \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$ 1

$\Rightarrow x = 3, \; y = 1, \; z = 2$ $\frac{1}{2}$

OR

A = I.A

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$ 1

$R_2 \to R_2 + R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_2 \to \dfrac{R_2}{5}$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -2/5 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/5 & 1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 - 2R_2, R_3 \to R_3 + 2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -6/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1/5 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 & 0 \\ 1/5 & 1/5 & 0 \\ 2/5 & 2/5 & 1 \end{bmatrix} \cdot A$$ 4

$R_3 \to 5R_3$

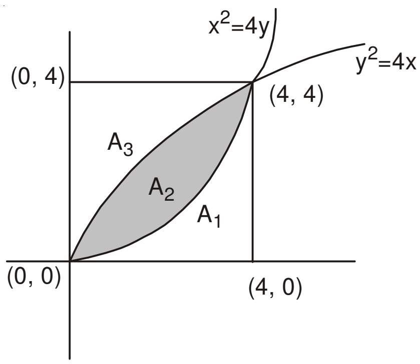
$$\Rightarrow \begin{bmatrix} 1 & 0 & -6/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 & 0 \\ 1/5 & 1/5 & 0 \\ 2 & 2 & 5 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + \dfrac{6}{5}R_3, R_2 \to R_2 + \dfrac{2}{5}R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \cdot A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$ 1

26. Correct Figure 1



Point of intersection are (0, 0) and (4, 4) 1

here, $A_1 = \int_0^4 \frac{x^4}{4}dx = \frac{16}{3}$ ...(1) 1

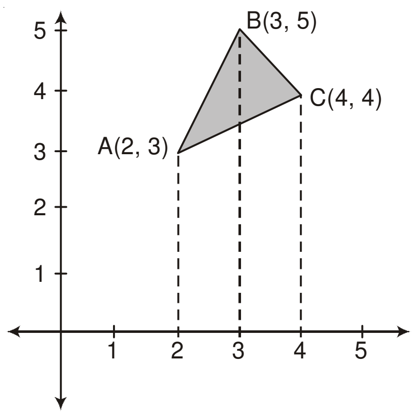
$A_2 = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx = \frac{16}{3}$ ..(2) $1\frac{1}{2}$

$A_3 = \int_0^4 \frac{y^2}{4}dy = \frac{16}{3}$ ...(3) 1

From (1), (2) and (3), $A_1 = A_2 = A_3$. $\frac{1}{2}$

OR

Correct Figure 1



$$\left.\begin{array}{l} \text{Equation of } AB: y = 2x - 1 \\ \text{Equation of } BC: y = -x + 8 \\ \text{Equation of } AC: y = \frac{1}{2}(x+4) \end{array}\right\}$$ $1\frac{1}{2}$

Required Area $= \int_2^3 (2x-1)dx + \int_3^4 (-x+8)dx - \int_2^4 \left(\frac{x+4}{2}\right)dx$ $1\frac{1}{2}$

$= \left[x^2 - x\right]_2^3 + \left[\frac{-x^2}{2} + 8x\right]_3^4 - \frac{1}{2}\left[\frac{x^2}{2} + 4x\right]_2^4$ $1\frac{1}{2}$

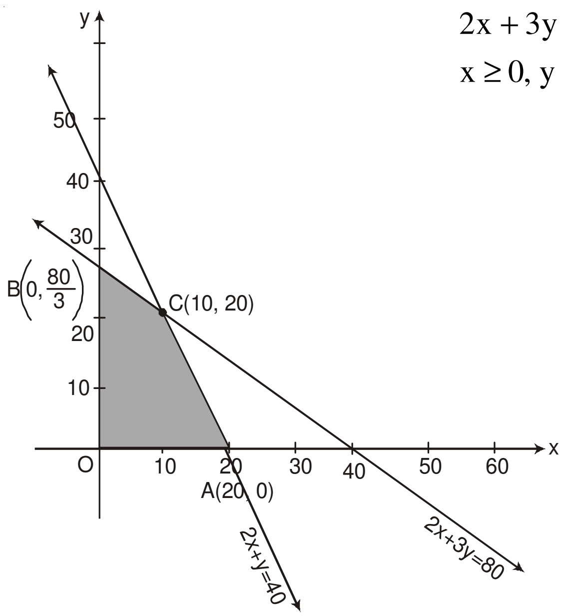
$= 4 + \frac{9}{2} - 7 = \frac{3}{2}$ $\frac{1}{2}$

27. Let number of items produced of model A be x and that of model B be y.

LPP is:

Maximize, profit $z = 15x + 10y$ 1

subject to



$$\left.\begin{array}{ll} 2x + y \le 5(8) & \text{i.e., } 2x + y \le 40 \\ 2x + 3y \le 10(8) & \text{i.e., } 2x + 3y \le 80 \\ x \ge 0,\ y \ge 0 & \end{array}\right\}$$ 2

Correct Figure 2

Corner point	$z = 15x + 10y$	
A(20, 0)	300	
$B\left(0, \frac{80}{3}\right)$	$\frac{800}{3} \approx 266.6$	$\frac{1}{2}$
C(10, 20)	350 ← maximum	

Maximum profit = ₹ 350

when $x = 10,\ y = 20.$ $\frac{1}{2}$

If a student has interpreted the language of the question in a different way, then the LPP will be of the type:

Maximise profit $z = 15x + 10y$

Subject to $2x + y \le 8$

$2x + 3y \le 8$

$x \ge 0,\ y \ge 0$

This is be accepted and marks may be given accordingly.

28. Equation of plane is $\begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$ 2

$\Rightarrow 5x + 2y - 3z = 17$ (Cartesian equation) 1

Vector equation is $\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17$ 1

Equation of required parallel plane is

$\vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = (4\hat{i} + 3\hat{j} + \hat{k}) \cdot (5\hat{i} + 2\hat{j} - 3\hat{k})$ 1

$\Rightarrow \vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = 23$ 1

OR

Let required plane be $a(x + 1) + b(y - 3) + c(z + 4) = 0$ ...(1) 1

Plane contains the given line, so it will also contain the point (1, 1, 0).

So, $2a - 2b + 4c = 0$ or $a - b + 2c = 0$ ...(2) 1

Also, $a + 2b - c = 0$ ...(3) 1

From (2) and (3),

$$\frac{a}{-3} = \frac{b}{3} = \frac{c}{3}$$ 1

$\therefore$ Required plane is $-3(x + 1) + 3(y - 3) + 3(z + 4) = 0$

$\therefore$ $-x + y + z = 0$

Also vector equation is: $\vec{r}.(-\hat{i}+\hat{j}+\hat{k})=0$ 1

Length of perpendicular from (2, 1, 4) = $\frac{|-2+1+4|}{\sqrt{(-1)^2+1^2+1^2}} = \sqrt{3}$ 1

29. X = no. of kings = 0, 1, 2 $\frac{1}{2}$

$$P(X=0) = P(\text{no king}) = \frac{48}{52}\times\frac{47}{51} = \frac{188}{221}$$ 1

$$P(X = 1) = P(\text{one king and one non-king}) = \frac{4}{52}\times\frac{48}{51}\times 2 = \frac{32}{221}$$ 1

$$P(X = 2) = P(\text{two kings}) = \frac{4}{52}\times\frac{3}{51} = \frac{1}{221}$$ 1

Probability distribution is given by

X	0	1	2
P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

$\frac{1}{2}$

Now, Mean = $\Sigma X \cdot P(X) = \frac{34}{221}$ or $\frac{2}{13}$ 1

and $Var(X) = \Sigma X^2 \cdot P(X) - [\Sigma X \cdot P(X)]^2$

$$= \frac{36}{221} - \left(\frac{34}{221}\right)^2 = \frac{6800}{48841} \text{ or } \frac{400}{2873}$$ 1