Strictly Confidential — (For Internal and Restricted Use Only)

Senior School Certificate Examination

March 2016

Marking Scheme — Mathematics 65/1/1/D, 65/1/2/D, 65/1/3/D

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/1/1/D

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin\theta & 0 \\ 1 & 0 & \cos\theta \end{vmatrix} = \sin\theta\cos\theta$ $\frac{1}{2}$

$= \frac{1}{2}\sin 2\theta \therefore \text{Max value} = \frac{1}{2}$ $\frac{1}{2}$

2. $(A - I)^3 + (A + I)^3 - 7A, \quad A^2 = I \Rightarrow A^3 = A$ $\frac{1}{2}$

$= 2A - A = A$ $\frac{1}{2}$

3. $\left.\begin{array}{l} 2b = 3 \text{ and } 3a = -2 \\ b = \frac{3}{2} \text{ and } a = -\frac{2}{3} \end{array}\right\}$ $\frac{1}{2} + \frac{1}{2}$

4. Getting position vector as $2(2\vec{a} + \vec{b}) - 1(\vec{a} - 2\vec{b})$ $\frac{1}{2}$

$= 3\vec{a} + 4\vec{b}$ $\frac{1}{2}$

5. $\overrightarrow{AD} = \overrightarrow{AB} + \frac{1}{2}[\overrightarrow{AC} - \overrightarrow{AB}] = \frac{1}{2}(\overrightarrow{AC} + \overrightarrow{AB})$ $\frac{1}{2}$

$|\overrightarrow{AD}| = \frac{1}{2}|3\hat{i} + 5\hat{k}| = \frac{1}{2}\sqrt{34}$ $\frac{1}{2}$

6. $\vec{r}.\frac{(2\hat{i} - 3\hat{j} + 6\hat{k})}{7} = 5$ 1

SECTION B

7. LHS $= \tan^{-1}\left(\dfrac{\frac{1}{5} + \frac{1}{7}}{1 - \frac{1}{5}.\frac{1}{7}}\right) + \tan^{-1}\left(\dfrac{\frac{1}{3} + \frac{1}{8}}{1 - \frac{1}{3}.\frac{1}{8}}\right)$ 1

$= \tan^{-1}\left(\frac{6}{17}\right) + \tan^{-1}\left(\frac{11}{23}\right)$ 1

$= \tan^{-1}\left(\dfrac{\frac{6}{17} + \frac{11}{23}}{1 - \frac{6}{17}.\frac{11}{23}}\right) = \tan^{-1}\left(\frac{325}{325}\right)$ 1

$= \tan^{-1}(1) = \frac{\pi}{4}$ 1

OR

$2\tan^{-1}(\cos x) = \tan^{-1}(2\text{cosec } x)$

$\Rightarrow \quad \tan^{-1}\left(\dfrac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\dfrac{2}{\sin x}\right)$ 2

$\Rightarrow \quad \sin x(\sin x - \cos x) = 0$ 1

$\Rightarrow \quad \sin x = \cos x$ $\frac{1}{2}$

the solution is $x = \dfrac{\pi}{4}$ $\frac{1}{2}$

8. Let the income be 3x, 4x and expenditures, 5y, 7y

$\therefore \quad \left.\begin{matrix} 3x - 5y = 15000 \\ 4x - 7y = 15000 \end{matrix}\right\}$ 1

$$\begin{pmatrix} 3 & -5 \\ 4 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -1\begin{pmatrix} -7 & 5 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$\Rightarrow \quad x = 30000,\ y = 15000$ $1\frac{1}{2}$

$\therefore$ Incomes are ₹ 90000 and ₹ 120000 respectively $\frac{1}{2}$

"Expenditure must be less than income"

(or any other relevant answer) 1

9. Here $x = a\left(\sin 2t + \dfrac{1}{2}\sin 4t\right),\ y = b(\cos 2t - \cos^2 2t)$

$\dfrac{dx}{dt} = 2a[\cos 2t + \cos 4t],\ \dfrac{dy}{dt} = 2b[-\sin 2t + 2\cos 2t \sin 2t] = 2b[\sin 4t - \sin 2t]$ 1 + 1

$\dfrac{dy}{dx} = \dfrac{b}{a}\left[\dfrac{\sin 4t - \sin 2t}{\cos 4t + \cos 2t}\right]$ 1

$\left.\dfrac{dy}{dx}\right]_{t=\frac{\pi}{4}} = \dfrac{b}{a}$ $\frac{1}{2}$

and $\left.\dfrac{dy}{dx}\right]_{t=\frac{\pi}{3}} = \sqrt{3}\dfrac{b}{a}$ $\frac{1}{2}$

OR

$y = x^x \Rightarrow \log y = x.\log x$ $\frac{1}{2}$

$\Rightarrow \quad \dfrac{1}{y}\dfrac{dy}{dx} = (1 + \log x)$ $1\frac{1}{2}$

$$\Rightarrow \quad \frac{1}{y}\frac{d^2y}{dx^2} - \frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 = \frac{1}{x} \qquad 1\frac{1}{2}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0 \qquad \frac{1}{2}$$

10. $\text{LHL} = \lim\limits_{x \to \frac{\pi}{2}^-} \frac{(1-\cancel{\sin x})(1+\sin x+\sin^2 x)}{3(1-\cancel{\sin x})(1+\sin x)}$ 1

$= \frac{1}{2}$ $\frac{1}{2}$

$\therefore\ p = \frac{1}{2}$ $\frac{1}{2}$

$\text{RHL} = \lim\limits_{x \to \frac{\pi}{2}^+} \frac{q(1-\sin x)}{(\pi-2x)^2} = \lim\limits_{h \to 0} \frac{q(1-\cos h)}{(2h)^2}$, where $x - \frac{\pi}{2} = h$ 1

$= \lim\limits_{h \to 0} \frac{2q\sin^2\frac{h}{2}}{4.4.\frac{h^2}{4}} = \frac{q}{8}$ $\frac{1}{2}$

$\therefore\ \frac{q}{8} = \frac{1}{2} \Rightarrow q = 4$ $\frac{1}{2}$

11. $\frac{dx}{dt} = -3\sin t + 3\cos^2 t\ \sin t = -3\ \sin t\ (1 - \cos^2 t) = -3\ \sin^3 t$ 1

$\frac{dy}{dt} = 3\cos t - 3\sin^2 t\ \cos t = 3\cos t\ (1 - \sin^2 t) = 3\cos^3 t$ 1

Slope of normal $= -\frac{dx}{dy} = \frac{\sin^3 t}{\cos^3 t}$ 1

Eqn. of normal is

$y - (3\sin t - \sin^3 t) = \frac{\sin^3 t}{\cos^3 t}[x - (3\cos t - \cos^3 t)]$ $\frac{1}{2}$

$\Rightarrow \quad y\cos^3 t - x\sin^3 t = 3\sin t\cos t\ (\cos^2 t - \sin^2 t)$

$= \frac{3}{4}\sin 4t$ $\frac{1}{2}$

or $\quad 4(y\cos^3 t - x\sin^3 t) = 3\sin 4t$

12. $I = \int \frac{(3\sin\theta - 2)\cos\theta}{5 - (1-\sin^2\theta) - 4\sin\theta}\,d\theta$ $\frac{1}{2}$

$\sin\theta = t \Rightarrow \cos\theta\ d\theta = dt$

$\therefore \quad I = \int \frac{3t-2}{t^2-4t+4}\,dt = \int \frac{3t-2}{(t-2)^2}\,dt$ 1

$$= \int \frac{3(t-2)}{(t-2)^2}\,dt + 4\int \frac{1}{(t-2)^2}\,dt$$ 1

$$= 3\log|t-2| - \frac{4}{(t-2)} + C$$ 1

$$= 3\log|\sin\theta - 2| - \frac{4}{(\sin\theta - 2)} + C$$ $\frac{1}{2}$

OR

Let $I = \int_0^{\pi} \sin\left(\frac{\pi}{4} + x\right) e^{2x}\,dx$

$$= \sin\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}\Bigg]_0^{\pi} - \int_0^{\pi}\cos\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}\,dx$$ 1

$$I = \left[\sin\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}\right\}\right]_0^{\pi} + \frac{1}{2}\int_0^{\pi} -\sin\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}\,dx$$ 1

$$\frac{5}{\cancel{4}} I = \left\{\frac{1}{\cancel{4}}\left[2\sin\left(\frac{\pi}{4}+x\right) - \cos\left(\frac{\pi}{4}+x\right)\right]e^{2x}\right\}_0^{\pi}$$ 1

$$I = \frac{1}{5}\left[\left\{2\left(-\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\right\}e^{2\pi} - \left\{2\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}\right\}\right] = \frac{-1}{5\sqrt{2}}(e^{2\pi}+1)$$ 1

13. $I = \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}}\,dx$

Put $x^{3/2} = t \Rightarrow \frac{3}{2}.\, x^{1/2}\,dx = dt$ or $\sqrt{x}\,dx = \frac{2}{3}dt$ $1\frac{1}{2}$

$$I = \frac{2}{3}\int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}}$$ 1

$$= \frac{2}{3}.\sin^{-1}\left(\frac{t}{a^{3/2}}\right) + C$$ 1

$$= \frac{2}{3}\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right) + C$$ $\frac{1}{2}$

14. $I = \int_{-1}^{2} |x^3 - x|\,dx$

$$= \int_{-1}^{0}(x^3 - x)\,dx + \int_0^1 -(x^3 - x)\,dx + \int_1^2 (x^3 - x)\,dx$$ $1\frac{1}{2}$

$$= \frac{x^4}{4} - \frac{x^2}{2}\Bigg]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_1^2$$ $1\frac{1}{2}$

$$= -\left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + (4-2) - \left(\frac{1}{4} - \frac{1}{2}\right)$$

$$= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = \frac{11}{4}$$ 1

15. Given differential equation can be written as

$$\frac{(1+\log x)}{x}\,dx + \frac{2y}{1-y^2}\,dy = 0$$ 1

integrating to get, $\frac{1}{2}(1+\log x)^2 - \log|1-y^2| = C$ 2

$x = 1,\ y = 0 \Rightarrow C = \frac{1}{2}$ $\frac{1}{2}$

$\Rightarrow\quad (1 + \log x)^2 - 2\log|1 - y^2| = 1$ $\frac{1}{2}$

16. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{1}{1+y^2}\,x = \frac{e^{\tan^{-1}y}}{1+y^2}$$ 1

Integrating factor is $e^{\tan^{-1}y}$ 1

$\therefore$ Solution is $x.\,e^{\tan^{-1}y} = \int e^{2\tan^{-1}y}\frac{1}{1+y^2}\,dy$ 1

$\therefore\quad x\,e^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}y} + C$ 1

17. Given, that $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ are coplanar

$\therefore\quad \left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right] = 0$

i.e. $(\vec{a}+\vec{b}).\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\} = 0$ 1

$(\vec{a}+\vec{b}).\{(\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a})\} = 0$ 1

$\Rightarrow\quad \vec{a}.(\vec{b}\times\vec{c}) + \vec{a}.(\vec{b}\times\vec{a}) + \vec{a}.(\vec{c}\times\vec{a}) + \vec{b}.(\vec{b}\times\vec{c}) + \vec{b}.(\vec{b}\times\vec{a}) + \vec{b}.(\vec{c}\times\vec{a}) = 0$ $1\frac{1}{2}$

$\Rightarrow\quad 2[\vec{a}, \vec{b}, \vec{c}] = 0$ or $[\vec{a}, \vec{b}, \vec{c}] = 0$ $\frac{1}{2}$

$\Rightarrow\quad \vec{a}, \vec{b}, \vec{c}$ are coplanar.

18. Vector equation of the required line is

$\vec{r} = (\hat{i}+2\hat{j}-4\hat{k}) + \mu\,[(3\hat{i}-16\hat{j}+7\hat{k})\times(3\hat{i}+8\hat{j}-5\hat{k})]$ 1

$\Rightarrow\quad \vec{r} = (\hat{i}+2\hat{j}-4\hat{k}) + \lambda\,[(2\hat{i}+3\hat{j}+6\hat{k})$ 2

in cartesian form, $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ 1

19. Let events are:

$$\left.\begin{array}{l} E_1 : \text{A is selected} \\ E_2 : \text{B is selected} \\ E_3 : \text{C is selected} \\ A : \text{Change is not introduced} \end{array}\right\}$$

$P(E_1) = \frac{1}{7}, P(E_2) = \frac{2}{7}, P(E_3) = \frac{4}{7}$ 1

$P(A/E_1) = 0.2,\ P(A/E_2) = 0.5,\ P(A/E_3) = 0.7$ 1

$\therefore \quad P(E_3/A) = \dfrac{\frac{4}{7}\times\frac{7}{10}}{\frac{1}{7}\times\frac{2}{10}+\frac{2}{7}\times\frac{5}{10}+\frac{4}{7}\times\frac{7}{10}}$ 1

$= \frac{28}{40} = \frac{7}{10}$ 1

OR

$\left.\begin{array}{l}\text{Prob. of success for A} = \frac{1}{6}\\ \text{Prob. of failure for A} = \frac{5}{6}\\ \text{Prob. of success for B} = \frac{1}{12}\\ \text{Prob. of failure for B} = \frac{11}{12}\end{array}\right\}$ 1

B can win in 2nd or 4th or 6th or....throw 1

$\therefore \quad P(B) = \left(\frac{5}{6}.\frac{1}{12}\right)+\left(\frac{5}{6}.\frac{11}{12}.\frac{5}{6}.\frac{1}{12}\right)+\left(\frac{5}{6}.\frac{11}{12}.\frac{5}{6}.\frac{11}{12}.\frac{5}{6}.\frac{1}{12}\right)+....$ 1

$= \frac{5}{72}\left(1+\frac{55}{72}+\left(\frac{55}{72}\right)^2+....\right)$

$= \frac{5}{72}\times\frac{1}{1-\frac{55}{72}} = \frac{5}{72}\times\frac{72}{17} = \frac{5}{17}$ 1

SECTION C

20. Let $x_1, x_2 \in N$ and $f(x_1) = f(x_2)$

$\Rightarrow \quad 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$_1^2 - x_2^2) + 6(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(9x_1 + 9x_2 + 6) = 0$

$\Rightarrow \quad x_1 - x_2 = 0$ or $x_1 = x_2$ as $(9x_1 + 9x_2 + 6) \neq 0,\ x_1, x_2 \in N$

$\therefore$ f is a one-one function 2

f: $N \to S$ is ONTO as co-domain = Range 1

Hence f is invertible

$y = 9x^2 + 6x - 5 = (3x+1)^2 - 6 \Rightarrow x = \frac{\sqrt{y+6}-1}{3}$

$\therefore \quad f^{-1}(y) = \frac{\sqrt{y+6}-1}{3},\ y \in S$ 2

$f^{-1}(43) = \frac{\sqrt{49}-1}{3} = 2$ $\frac{1}{2}$

$f^{-1}(163) = \frac{\sqrt{169}-1}{3} = 4$ $\frac{1}{2}$

21. Using $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$ we get

$$\Delta = \begin{vmatrix} y(z-x)+z^2-x^2 & x(z-y)+z^2-y^2 & xy-z^2 \\ z(x-y)+x^2-y^2 & y(x-z)+x^2-z^2 & yz-x^2 \\ x(y-z)+y^2-z^2 & z(y-x)+y^2-x^2 & zx-y^2 \end{vmatrix}$$ 2

Taking (x + y + z) common from C_1 & C_2

$$\Rightarrow \quad \Delta = (x+y+z)^2 \begin{vmatrix} z-x & z-y & xy-z^2 \\ x-y & x-z & yz-x^2 \\ y-z & y-x & zx-y^2 \end{vmatrix}$$ 1

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \quad \Delta = (x+y+z)^2 \begin{vmatrix} 0 & 0 & xy+yz+zx-x^2-y^2-z^2 \\ x-y & x-z & yz-x^2 \\ y-z & y-x & zx-y^2 \end{vmatrix}$$ 1

Expanding to get

$\Delta = (x + y + z)^2 (xy + zy + zx - x^2 - y^2 - z^2)^2$ 1

Hence Δ is divisible by (x + y + z) and

the quotient is $(x + y + z) (xy + yz + zx - x^2 - y^2 - z^2)^2$ 1

OR

Writing $\begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1

$R_1 \leftrightarrow R_3$ $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 8 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} A$

$R_1 \to R_1 - 2R_2$
$R_3 \to R_3 - 4R_2$ $\begin{pmatrix} -3 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -2 & 1 \\ 0 & 1 & 0 \\ 1 & -4 & 0 \end{pmatrix} A$

$R_1 \to \frac{1}{3} R_1$
$R_3 \to - R_3$ $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & 1 & 0 \\ -1 & 4 & 0 \end{pmatrix} A$

$R_2 \to R_2 - 2R_1$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & -1/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A$ $2\frac{1}{2}$ marks for operation to get A^{-1}

$R_2 \to R_2 - R_3$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A$

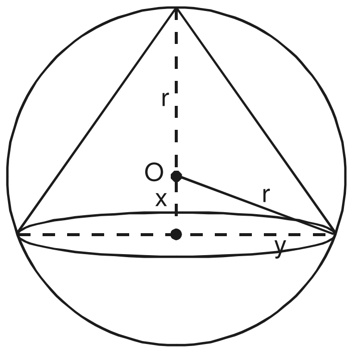
$\therefore \quad A^{-1} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix}$ $\frac{1}{2}$

$AX = B \;\Rightarrow\; X = A^{-1}B$ 1

$\therefore \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} \begin{pmatrix} 19 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$\therefore \quad x = 1, y = 2, z = 1$ 1

22.



Correct Figure 1

Let radius of cone be y and the altitude be r + x

$\therefore \quad x^2 + y^2 = r^2 \quad \ldots(i)$ $\frac{1}{2}$

$$\text{Volume } V = \frac{1}{3}\pi y^2 (r + x)$$

$$= \frac{1}{3}\pi(r^2 - x^2)(r + x)$$ 1

$$\frac{dV}{dx} = \frac{\pi}{3}[(r^2 - x^2)1 + (r + x)(-2x)] = \frac{\pi}{3}(r + x)(r - 3x)$$ 1

$$\frac{dV}{dx} = 0 \Rightarrow x = \frac{r}{3}$$ $\frac{1}{2}$

$$\therefore \quad \text{Altitude} = r + \frac{r}{3} = \frac{4r}{3}$$ $\frac{1}{2}$

$$\text{and} \quad \frac{d^2V}{dx^2} = \frac{\pi}{3}[(r + x)(-3) + (r - 3x)] = \frac{\pi}{3}[-2r - 6x] < 0$$ 1

$$\therefore \quad \text{Max. Volume} = \frac{\pi}{3}\left(r^2 - \frac{r^2}{9}\right)\left(r + \frac{r}{3}\right) = \frac{8}{27}\left(\frac{4}{3}\pi r^3\right)$$ $\frac{1}{2}$

$$= \frac{8}{27}(\text{Vol. of sphere})$$

OR

$f(x) = \sin 3x - \cos 3x, \ 0 < x < \pi$

$f'(x) = 3\cos 3x + 3\sin 3x$ 1

$f'(x) = 0 \Rightarrow \tan 3x = -1$ $\frac{1}{2}$

$\Rightarrow \quad x = \frac{n\pi}{3} + \frac{\pi}{4}, \ n \in Z$

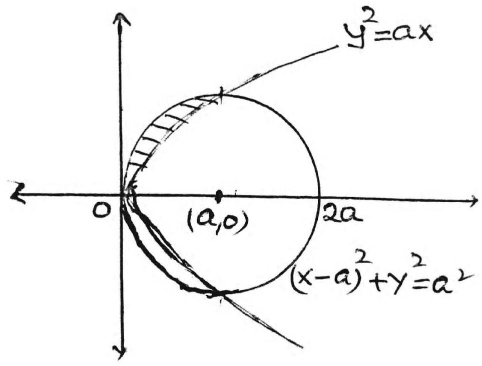
$\Rightarrow \quad x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}$ $1\frac{1}{2}$

Intervals are: $\left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{7\pi}{12}\right), \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right), \left(\frac{11\pi}{12}, \pi\right)$ 1

f(x) is strictly increasing in $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right)$ 1

and strictly decreasing in $\left(\frac{\pi}{4}, \frac{7\pi}{12}\right) \cup \left(\frac{11\pi}{12}, \pi\right)$ 1

23. $y^2 = ax,\ x^2 + y^2 = 2ax \Rightarrow x^2 - ax = 0$



$\Rightarrow \quad x = 0,\ x = a$ 1

Correct Figure 1

Shaded area $= \left[\int_0^a [\sqrt{a^2-(x-a)^2} - \sqrt{a}\sqrt{x}]dx\right.$ 1

$A = \left[\frac{x-a}{2}\sqrt{a^2-(x-a)^2} + \frac{a^2}{2}\sin^{-1}\frac{x-a}{a} - \sqrt{a}\,\frac{2}{3}x^{3/2}\right]_0^a$ 2

$= \left[-\frac{2}{3}a^2 + \frac{a^2}{2}\frac{\pi}{2}\right] = \frac{\pi a^2}{4} - \frac{2a^2}{3}$ sq. units 1

24. Equation of line $AB: \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$ 1

Eqn. of plane LMN: $\begin{vmatrix} x-2 & y-2 & z-1 \\ 1 & -2 & 0 \\ 2 & -3 & -1 \end{vmatrix} = 0$ $1\frac{1}{2}$

$2(x-2) + 1\,(y-2) + 1\,(z-1) = 0$ or $2x + y + z - 7 = 0$ $\frac{1}{2}$

Any point on line AB is $(-\lambda + 3,\ \lambda - 4,\ 6\lambda - 5)$ $\frac{1}{2}$

If this point lies on plane, then $2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 0 \Rightarrow 5\lambda = 10 \Rightarrow \lambda = 2$ 1

$\therefore$ P is (1, –2. 7) $\frac{1}{2}$

let P divides AB in K : 1

$\Rightarrow \quad 1 = \frac{2K+3}{K+1} \Rightarrow K = -2$ i.e. P divides, AB externally in 2 : 1 1

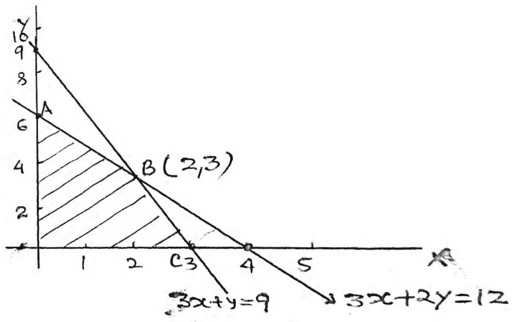
25. X = No. of red

X:	0	1	2	3	4	1
P(X):	${}^4C_0\left(\frac{1}{3}\right)^4$ $= \frac{1}{81}$	${}^4C_1\left(\frac{1}{3}\right)^3\frac{2}{3}$ $= \frac{8}{81}$	${}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2$ $= \frac{24}{81}$	${}^4C_3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3$ $= \frac{32}{81}$	${}^4C_4\left(\frac{2}{3}\right)^4$ $= \frac{16}{81}$	$2\frac{1}{2}$
XP(X):	0	$\frac{8}{81}$	$\frac{48}{81}$	$\frac{96}{81}$	$\frac{64}{81}$	
$X^2P(X)$:	0	$\frac{8}{81}$	$\frac{96}{81}$	$\frac{288}{81}$	$\frac{256}{81}$	

Mean $= \Sigma XP(X) = \frac{216}{81} = \frac{8}{3}$ 1

Variance $= \Sigma X^2P(X) - [\Sigma XP(X)]^2 = \frac{648}{81} - \frac{64}{9} = \frac{8}{9}$ $1\frac{1}{2}$

26. Let production of A, B (per day) be x, y respectively



Maximise P = 7x + 4y 1

Subject to $\left.\begin{array}{l} 3x + 2y \le 12 \\ 3x + y \le 9 \\ x \ge 0,\ y \ge 0 \end{array}\right\}$ 2

Correct Graph 2

P(A) = 24

P(B) = 26

P(C) = 21

$\therefore$ 2 units of product A and 3 units of product B for maximum profit 1

QUESTION PAPER CODE 65/1/2/D

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\left.\begin{array}{l} 2b = 3 \text{ and } 3a = -2 \\ b = \frac{3}{2} \text{ and } a = -\frac{2}{3} \end{array}\right\}$ $\frac{1}{2}+\frac{1}{2}$

2. Getting position vector as $2(2\vec{a}+\vec{b})-1(\vec{a}-2\vec{b})$ $\frac{1}{2}$

$= 3\vec{a}+4\vec{b}$ $\frac{1}{2}$

3. $\overrightarrow{AD} = \overrightarrow{AB}+\frac{1}{2}[\overrightarrow{AC}-\overrightarrow{AB}]=\frac{1}{2}(\overrightarrow{AC}+\overrightarrow{AB})$ $\frac{1}{2}$

$|\overrightarrow{AD}| = \frac{1}{2}|3\hat{i}+5\hat{k}|=\frac{1}{2}\sqrt{34}$ $\frac{1}{2}$

4. $\vec{r}.\frac{(2\hat{i}-3\hat{j}+6\hat{k})}{7}=5$ 1

5. $\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin\theta & 0 \\ 1 & 0 & \cos\theta \end{vmatrix} = \sin\theta\cos\theta$ $\frac{1}{2}$

$= \frac{1}{2}\sin 2\theta \therefore \text{Max value} = \frac{1}{2}$ $\frac{1}{2}$

6. $(A-I)^3 + (A+I)^3 - 7A, \quad A^2 = I \Rightarrow A^3 = A$ $\frac{1}{2}$

$= 2A - A = A$ $\frac{1}{2}$

SECTION B

7. $\frac{dx}{dt} = -3\sin t + 3\cos^2 t \sin t = -3 \sin t\,(1-\cos^2 t) = -3\sin^3 t$ 1

$\frac{dy}{dt} = 3\cos t - 3\sin^2 t \cos t = 3\cos t\,(1-\sin^2 t) = 3\cos^3 t$ 1

Slope of normal $= -\frac{dx}{dy} = \frac{\sin^3 t}{\cos^3 t}$ 1

Eqn. of normal is

$y-(3\sin t - \sin^3 t) = \frac{\sin^3 t}{\cos^3 t}[x-(3\cos t - \cos^3 t)]$ $\frac{1}{2}$

$\Rightarrow \quad y\cos^3 t - x\sin^3 t = 3\sin t\cos t\,(\cos^2 t - \sin^2 t)$

$= \frac{3}{4}\sin 4t$ $\frac{1}{2}$

or $4(y\cos^3 t - x\sin^3 t) = 3\sin 4t$

8. $I = \int \frac{(3\sin\theta - 2)\cos\theta}{5-(1-\sin^2\theta)-4\sin\theta}\,d\theta$ $\frac{1}{2}$

$\sin\theta = t \Rightarrow \cos\theta\, d\theta = dt$

$\therefore \quad I = \int \frac{3t-2}{t^2-4t+4}\,dt = \int \frac{3t-2}{(t-2)^2}\,dt$ 1

$= \int \frac{3(t-2)}{(t-2)^2}\,dt + 4\int \frac{1}{(t-2)^2}\,dt$ 1

$= 3\log|t-2| - \frac{4}{(t-2)} + C$ 1

$= 3\log|\sin\theta - 2| - \frac{4}{(\sin\theta - 2)} + C$ $\frac{1}{2}$

OR

Let $I = \int_0^{\pi} \sin\left(\frac{\pi}{4}+x\right)e^{2x}\,dx$

$= \sin\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}\Bigg]_0^{\pi} - \int_0^{\pi}\cos\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}\,dx$ 1

$I = \left[\sin\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}\right\}\right]_0^{\pi} + \frac{1}{2}\int_0^{\pi} -\sin\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}\,dx$ 1

$\frac{5}{\cancel{4}} I = \left\{\frac{1}{\cancel{4}}\left[2\sin\left(\frac{\pi}{4}+x\right) - \cos\left(\frac{\pi}{4}+x\right)\right]e^{2x}\right\}_0^{\pi}$ 1

$I = \frac{1}{5}\left[\left\{2\left(-\frac{1}{\sqrt{2}}\right)+\frac{1}{\sqrt{2}}\right\}e^{2\pi} - \left\{2\left(\frac{1}{\sqrt{2}}\right)-\frac{1}{\sqrt{2}}\right\}\right] = \frac{-1}{5\sqrt{2}}(e^{2\pi}+1)$ 1

9. $I = \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}}\,dx$

Put $x^{3/2} = t \Rightarrow \frac{3}{2}.x^{1/2}\,dx = dt$ or $\sqrt{x}\,dx = \frac{2}{3}\,dt$ $1\frac{1}{2}$

$I = \frac{2}{3}\int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}}$ 1

$= \frac{2}{3}.\sin^{-1}\left(\frac{t}{a^{3/2}}\right) + C$ 1

$= \frac{2}{3}\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right) + C$ $\frac{1}{2}$

10. $I = \int_{-1}^{2} |x^3 - x|\,dx$

$= \int_{-1}^{0}(x^3 - x)\,dx + \int_0^1 -(x^3-x)\,dx + \int_1^2 (x^3-x)\,dx$ $1\frac{1}{2}$

$$= \left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}-\frac{x^4}{4}\right]_{0}^{1} + \left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{1}^{2}$$ $1\frac{1}{2}$

$$= -\left(\frac{1}{4}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+(4-2)-\left(\frac{1}{4}-\frac{1}{2}\right)$$

$$= \frac{1}{4}+\frac{1}{4}+2+\frac{1}{4}=\frac{11}{4}$$ 1

11. Given differential equation can be written as

$$\frac{(1+\log x)}{x}\,dx + \frac{2y}{1-y^2}\,dy = 0$$ 1

integrating to get, $\frac{1}{2}(1+\log x)^2 - \log|1-y^2| = C$ 2

$$x = 1,\ y = 0 \Rightarrow C = \frac{1}{2}$$ $\frac{1}{2}$

$$\Rightarrow (1+\log x)^2 - 2\log|1-y^2| = 1$$ $\frac{1}{2}$

12. Given differential equation can be written as

$$\frac{dx}{dy}+\frac{1}{1+y^2}x = \frac{e^{\tan^{-1}y}}{1+y^2}$$ 1

Integrating factor is $e^{\tan^{-1}y}$ 1

$\therefore$ Solution is $x.\,e^{\tan^{-1}y} = \int e^{2\tan^{-1}y}\frac{1}{1+y^2}\,dy$ 1

$\therefore$ $x\,e^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}y}+C$ 1

13. Given, that $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ are coplanar

$\therefore$ $\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right] = 0$

i.e. $(\vec{a}+\vec{b}).\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\} = 0$ 1

$(\vec{a}+\vec{b}).\{(\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a})\} = 0$ 1

$\Rightarrow$ $\vec{a}.(\vec{b}\times\vec{c})+\vec{a}.(\vec{b}\times\vec{a})+\vec{a}.(\vec{c}\times\vec{a})+\vec{b}.(\vec{b}\times\vec{c})+\vec{b}.(\vec{b}\times\vec{a})+\vec{b}.(\vec{c}\times\vec{a}) = 0$ $1\frac{1}{2}$

$\Rightarrow$ $2[\vec{a},\vec{b},\vec{c}] = 0$ or $[\vec{a},\vec{b},\vec{c}]=0$ $\frac{1}{2}$

$\Rightarrow$ $\vec{a},\vec{b},\vec{c}$ are coplanar.

14. Vector equation of the required line is

$\vec{r} = (\hat{i}+2\hat{j}-4\hat{k})+\mu[(3\hat{i}-16\hat{j}+7\hat{k})\times(3\hat{i}+8\hat{j}-5\hat{k})]$ 1

$\Rightarrow$ $\vec{r} = (\hat{i}+2\hat{j}-4\hat{k})+\lambda[(2\hat{i}+3\hat{j}+6\hat{k})$ 2

in cartesian form, $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ 1

15. Let events are:

$$\left.\begin{array}{l} E_1 : \text{A is selected} \\ E_2 : \text{B is selected} \\ E_3 : \text{C is selected} \\ A : \text{Change is not introduced} \end{array}\right\}$$

$$P(E_1) = \frac{1}{7}, P(E_2) = \frac{2}{7}, P(E_3) = \frac{4}{7}$$ 1

$$P(A/E_1) = 0.2,\ P(A/E_2) = 0.5,\ P(A/E_3) = 0.7$$ 1

$$\therefore \quad P(E_3/A) = \frac{\frac{4}{7}\times\frac{7}{10}}{\frac{1}{7}\times\frac{2}{10}+\frac{2}{7}\times\frac{5}{10}+\frac{4}{7}\times\frac{7}{10}}$$ 1

$$= \frac{28}{40} = \frac{7}{10}$$ 1

OR

$$\left.\begin{array}{l} \text{Prob. of success for A} = \frac{1}{6} \\ \text{Prob. of failure for A} = \frac{5}{6} \\ \text{Prob. of success for B} = \frac{1}{12} \\ \text{Prob. of failure for B} = \frac{11}{12} \end{array}\right\}$$ 1

B can win in 2nd or 4th or 6th or....throw 1

$$\therefore \quad P(B) = \left(\frac{5}{6}.\frac{1}{12}\right)+\left(\frac{5}{6}.\frac{11}{12}.\frac{5}{6}.\frac{1}{12}\right)+\left(\frac{5}{6}.\frac{11}{12}.\frac{5}{6}.\frac{11}{12}.\frac{5}{6}.\frac{1}{12}\right)+....$$ 1

$$= \frac{5}{72}\left(1+\frac{55}{72}+\left(\frac{55}{72}\right)^2+....\right)$$

$$= \frac{5}{72}\times\frac{1}{1-\frac{55}{72}} = \frac{5}{72}\times\frac{72}{17} = \frac{5}{17}$$ 1

16. $$\text{LHS} = \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}.\frac{1}{7}}\right)+\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}.\frac{1}{8}}\right)$$ 1

$$= \tan^{-1}\left(\frac{6}{17}\right)+\tan^{-1}\left(\frac{11}{23}\right)$$ 1

$$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}.\frac{11}{23}}\right) = \tan^{-1}\left(\frac{325}{325}\right)$$ 1

$$= \tan^{-1}(1) = \frac{\pi}{4}$$ 1

OR

$2\tan^{-1}(\cos x) = \tan^{-1}(2\text{cosec } x)$

$\Rightarrow \quad \tan^{-1}\left(\dfrac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\dfrac{2}{\sin x}\right)$ 2

$\Rightarrow \quad \sin x(\sin x - \cos x) = 0$ 1

$\Rightarrow \quad \sin x = \cos x$ $\frac{1}{2}$

the solution is $x = \dfrac{\pi}{4}$ $\frac{1}{2}$

17. Let the income be 3x, 4x and expenditures, 5y, 7y

$\therefore \quad \left.\begin{aligned} 3x - 5y &= 15000 \\ 4x - 7y &= 15000 \end{aligned}\right\}$ 1

$$\begin{pmatrix} 3 & -5 \\ 4 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -1\begin{pmatrix} -7 & 5 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$\Rightarrow \quad x = 30000, \; y = 15000$ $1\frac{1}{2}$

$\therefore$ Incomes are ₹ 90000 and ₹ 120000 respectively $\frac{1}{2}$

"Expenditure must be less than income"

(or any other relevant answer) 1

18. Here $x = a\left(\sin 2t + \dfrac{1}{2}\sin 4t\right), y = b(\cos 2t - \cos^2 2t)$

$\dfrac{dx}{dt} = 2a[\cos 2t + \cos 4t], \dfrac{dy}{dt} = 2b[-\sin 2t + 2\cos 2t \sin 2t] = 2b[\sin 4t - \sin 2t]$ 1 + 1

$\dfrac{dy}{dx} = \dfrac{b}{a}\left[\dfrac{\sin 4t - \sin 2t}{\cos 4t + \cos 2t}\right]$ 1

$\left.\dfrac{dy}{dx}\right]_{t=\frac{\pi}{4}} = \dfrac{b}{a}$ $\frac{1}{2}$

and $\left.\dfrac{dy}{dx}\right]_{t=\frac{\pi}{3}} = \sqrt{3}\dfrac{b}{a}$ $\frac{1}{2}$

OR

$y = x^x \Rightarrow \log y = x.\log x$ $\frac{1}{2}$

$\Rightarrow \quad \dfrac{1}{y}\dfrac{dy}{dx} = (1 + \log x)$ $1\frac{1}{2}$

$$\Rightarrow \quad \frac{1}{y}\frac{d^2y}{dx^2} - \frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 = \frac{1}{x}$$ $1\frac{1}{2}$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$ $\frac{1}{2}$

19. $\text{LHL} = \lim_{x \to \frac{\pi}{2}^-} \frac{(1-\cancel{\sin x})(1+\sin x+\sin^2 x)}{3(1-\cancel{\sin x})(1+\sin x)}$ 1

$= \frac{1}{2}$ $\frac{1}{2}$

$\therefore \; p = \frac{1}{2}$ $\frac{1}{2}$

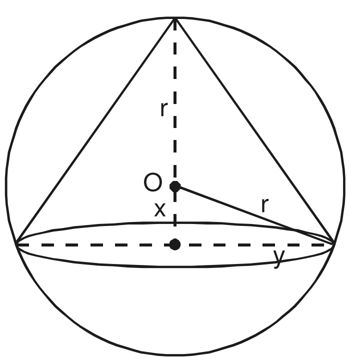
$\text{RHL} = \lim_{x \to \frac{\pi}{2}^+} \frac{q(1-\sin x)}{(\pi-2x)^2} = \lim_{h \to 0} \frac{q(1-\cos h)}{(2h)^2}$, where $x - \frac{\pi}{2} = h$ 1

$= \lim_{h \to 0} \frac{2q\sin^2\frac{h}{2}}{4.4.\frac{h^2}{4}} = \frac{q}{8}$ $\frac{1}{2}$

$\therefore \; \frac{q}{8} = \frac{1}{2} \Rightarrow q = 4$ $\frac{1}{2}$

SECTION C

20. Correct Figure 1



Let radius of cone be y and the altitude be r + x

$\therefore \quad x^2 + y^2 = r^2$ ...(i) $\frac{1}{2}$

Volume $V = \frac{1}{3}\pi y^2(r+x)$

$= \frac{1}{3}\pi(r^2 - x^2)(r+x)$ 1

$\frac{dV}{dx} = \frac{\pi}{3}[(r^2 - x^2)1 + (r+x)(-2x)] = \frac{\pi}{3}(r+x)(r-3x)$ 1

$\frac{dV}{dx} = 0 \Rightarrow x = \frac{r}{3}$ $\frac{1}{2}$

$\therefore \quad$ Altitude $= r + \frac{r}{3} = \frac{4r}{3}$ $\frac{1}{2}$

and $\quad \frac{d^2V}{dx^2} = \frac{\pi}{3}[(r+x)(-3) + (r-3x)] = \frac{\pi}{3}[-2r-6x] < 0$ 1

$\therefore \quad$ Max. Volume $= \frac{\pi}{3}\left(r^2 - \frac{r^2}{9}\right)\left(r + \frac{r}{3}\right) = \frac{8}{27}\left(\frac{4}{3}\pi r^3\right)$ $\frac{1}{2}$

$= \frac{8}{27}$(Vol. of sphere)

OR

$f(x) = \sin 3x - \cos 3x,\ 0 < x < \pi$

$f'(x) = 3\cos 3x + 3\sin 3x$ 1

$f'(x) = 0 \Rightarrow \tan 3x = -1$ $\frac{1}{2}$

$\Rightarrow \quad x = \frac{n\pi}{3} + \frac{\pi}{4},\ n \in Z$

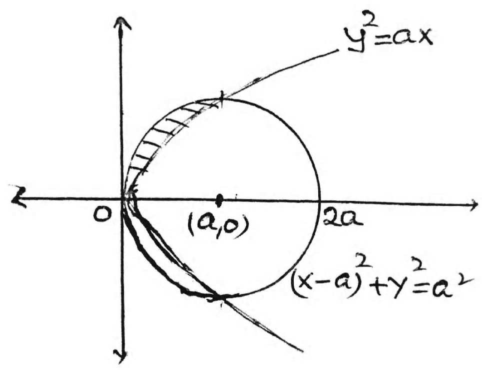
$\Rightarrow \quad x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}$ $1\frac{1}{2}$

Intervals are: $\left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{7\pi}{12}\right), \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right), \left(\frac{11\pi}{12}, \pi\right)$ 1

f(x) is strictly increasing in $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right)$ 1

and strictly decreasing in $\left(\frac{\pi}{4}, \frac{7\pi}{12}\right) \cup \left(\frac{11\pi}{12}, \pi\right)$ 1

21.



$y^2 = ax,\ x^2 + y^2 = 2ax \Rightarrow x^2 - ax = 0$

$\Rightarrow \quad x = 0,\ x = a$ 1

Correct Figure 1

Shaded area $= \left[\int_0^a [\sqrt{a^2 - (x-a)^2} - \sqrt{a}\sqrt{x}]dx\right.$ 1

$$A = \left[\frac{x-a}{2}\sqrt{a^2 - (x-a)^2} + \frac{a^2}{2}\sin^{-1}\frac{x-a}{a} - \sqrt{a}\,\frac{2}{3}x^{3/2}\right]_0^a$$ 2

$$= \left[-\frac{2}{3}a^2 + \frac{a^2}{2}\frac{\pi}{2}\right] = \frac{\pi a^2}{4} - \frac{2a^2}{3} \text{ sq. units}$$ 1

22. Equation of line $AB: \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$ 1

Eqn. of plane LMN: $\begin{vmatrix} x-2 & y-2 & z-1 \\ 1 & -2 & 0 \\ 2 & -3 & -1 \end{vmatrix} = 0$ $1\frac{1}{2}$

$2(x-2) + 1(y-2) + 1(z-1) = 0$ or $2x + y + z - 7 = 0$ $\frac{1}{2}$

Any point on line AB is $(-\lambda + 3, \lambda - 4, 6\lambda - 5)$ $\frac{1}{2}$

If this point lies on plane, then $2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 0 \Rightarrow 5\lambda = 10 \Rightarrow \lambda = 2$ 1

$\therefore \quad$ P is $(1, -2.\ 7)$ $\frac{1}{2}$

let P divides AB in K : 1

$\Rightarrow \quad 1 = \frac{2K+3}{K+1} \Rightarrow K = -2$ i.e. P divides, AB externally in 2 : 1 1

23. X = No. of red

X:	0	1	2	3	4
P(X):	${}^4C_0\left(\frac{1}{3}\right)^4 = \frac{1}{81}$	${}^4C_1\left(\frac{1}{3}\right)^3\frac{2}{3} = \frac{8}{81}$	${}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2 = \frac{24}{81}$	${}^4C_3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 = \frac{32}{81}$	${}^4C_4\left(\frac{2}{3}\right)^4 = \frac{16}{81}$
XP(X):	0	$\frac{8}{81}$	$\frac{48}{81}$	$\frac{96}{81}$	$\frac{64}{81}$
$X^2P(X)$:	0	$\frac{8}{81}$	$\frac{96}{81}$	$\frac{288}{81}$	$\frac{256}{81}$

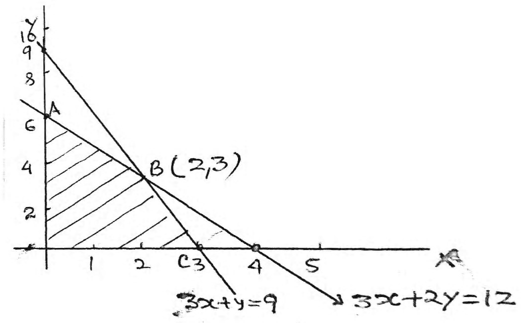
1

$2\frac{1}{2}$

Mean = $\Sigma XP(X) = \frac{216}{81} = \frac{8}{3}$ 1

Variance = $\Sigma X^2P(X) - [\Sigma XP(X)]^2 = \frac{648}{81} - \frac{64}{9} = \frac{8}{9}$ $1\frac{1}{2}$

24.



Let production of A, B (per day) be x, y respectively

Maximise P = 7x + 4y 1

Subject to $\left.\begin{array}{l}3x + 2y \le 12\\ 3x + y \le 9\\ x \ge 0,\ y \ge 0\end{array}\right\}$ 2

Correct Graph 2

P(A) = 24

P(B) = 26

P(C) = 21

$\therefore$ 2 units of product A and 3 units of product B for maximum profit 1

25. Let $x_1, x_2 \in N$ and $f(x_1) = f(x_2)$

$\Rightarrow$ $9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow$ $9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(9x_1 + 9x_2 + 6) = 0$

$\Rightarrow$ $x_1 - x_2 = 0$ or $x_1 = x_2$ as $(9x_1 + 9x_2 + 6) \neq 0,\ x_1, x_2 \in N$

$\therefore$ f is a one-one function 2

f: N $\rightarrow$ S is ONTO as co-domain = Range 1

Hence f is invertible

$y = 9x^2 + 6x - 5 = (3x + 1)^2 - 6 \Rightarrow x = \frac{\sqrt{y+6}-1}{3}$

$\therefore$ $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3},\ y \in S$ 2

$f^{-1}(43) = \frac{\sqrt{49}-1}{3} = 2$ $\frac{1}{2}$

$f^{-1}(163) = \frac{\sqrt{169}-1}{3} = 4$ $\frac{1}{2}$

26. Using $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$ we get

$$\Delta = \begin{vmatrix} y(z-x)+z^2-x^2 & x(z-y)+z^2-y^2 & xy-z^2 \\ z(x-y)+x^2-y^2 & y(x-z)+x^2-z^2 & yz-x^2 \\ x(y-z)+y^2-z^2 & z(y-x)+y^2-x^2 & zx-y^2 \end{vmatrix}$$ 2

Taking (x + y + z) common from C_1 & C_2

$$\Rightarrow \quad \Delta = (x+y+z)^2 \begin{vmatrix} z-x & z-y & xy-z^2 \\ x-y & x-z & yz-x^2 \\ y-z & y-x & zx-y^2 \end{vmatrix}$$ 1

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \quad \Delta = (x+y+z)^2 \begin{vmatrix} 0 & 0 & xy+yz+zx-x^2-y^2-z^2 \\ x-y & x-z & yz-x^2 \\ y-z & y-x & zx-y^2 \end{vmatrix}$$ 1

Expanding to get

$\Delta = (x + y + z)^2 (xy + zy + zx - x^2 - y^2 - z^2)^2$ 1

Hence Δ is divisible by (x + y + z) and

the quotient is $(x + y + z) (xy + yz + zx - x^2 - y^2 - z^2)^2$ 1

OR

Writing $\begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1

$R_1 \leftrightarrow R_3$ $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 8 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} A$

$R_1 \to R_1 - 2R_2$, $R_3 \to R_3 - 4R_2$ $\begin{pmatrix} -3 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -2 & 1 \\ 0 & 1 & 0 \\ 1 & -4 & 0 \end{pmatrix} A$

$R_1 \to \frac{1}{3} R_1$, $R_3 \to - R_3$ $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & 1 & 0 \\ -1 & 4 & 0 \end{pmatrix} A$

$R_2 \to R_2 - 2R_1$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & -1/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A$ $2\frac{1}{2}$ marks for operation to get A^{-1}

$R_2 \to R_2 - R_3$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A$

$\therefore \quad A^{-1} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix}$ $\frac{1}{2}$

$AX = B \Rightarrow X = A^{-1}B$ 1

$\therefore \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} \begin{pmatrix} 19 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$\therefore \quad x = 1, y = 2, z = 1$ 1

QUESTION PAPER CODE 65/1/3/D

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\overrightarrow{AD} = \overrightarrow{AB} + \frac{1}{2}[\overrightarrow{AC} - \overrightarrow{AB}] = \frac{1}{2}(\overrightarrow{AC} + \overrightarrow{AB})$ $\frac{1}{2}$

$|\overrightarrow{AD}| = \frac{1}{2}|3\hat{i} + 5\hat{k}| = \frac{1}{2}\sqrt{34}$ $\frac{1}{2}$

2. $\vec{r}.\frac{(2\hat{i} - 3\hat{j} + 6\hat{k})}{7} = 5$ 1

3. $\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin\theta & 0 \\ 1 & 0 & \cos\theta \end{vmatrix} = \sin\theta\cos\theta$ $\frac{1}{2}$

$= \frac{1}{2}\sin 2\theta \therefore \text{Max value} = \frac{1}{2}$ $\frac{1}{2}$

4. $(A - I)^3 + (A + I)^3 - 7A, \quad A^2 = I \Rightarrow A^3 = A$ $\frac{1}{2}$

$= 2A - A = A$ $\frac{1}{2}$

5. $\left.\begin{array}{l} 2b = 3 \text{ and } 3a = -2 \\ b = \frac{3}{2} \text{ and } a = -\frac{2}{3} \end{array}\right\}$ $\frac{1}{2} + \frac{1}{2}$

6. Getting position vector as $2(2\vec{a} + \vec{b}) - 1(\vec{a} - 2\vec{b})$ $\frac{1}{2}$

$= 3\vec{a} + 4\vec{b}$ $\frac{1}{2}$

SECTION B

7. Given differential equation can be written as

$\frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{e^{\tan^{-1}y}}{1+y^2}$ 1

Integrating factor is $e^{\tan^{-1}y}$ 1

$\therefore$ Solution is $x.\, e^{\tan^{-1}y} = \int e^{2\tan^{-1}y}\frac{1}{1+y^2}dy$ 1

$\therefore$ $x\, e^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}y} + C$ 1

8. Given, that $\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}$ are coplanar

$\therefore$ $\left[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}\right] = 0$

i.e. $(\vec{a}+\vec{b}).\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\} = 0$ 1

$(\vec{a}+\vec{b}).\{(\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a})\} = 0$ 1

$\Rightarrow$ $\vec{a}.(\vec{b}\times\vec{c})+\vec{a}.(\vec{b}\times\vec{a})+\vec{a}.(\vec{c}\times\vec{a})+\vec{b}.(\vec{b}\times\vec{c})+\vec{b}.(\vec{b}\times\vec{a})+\vec{b}.(\vec{c}\times\vec{a}) = 0$ $1\frac{1}{2}$

$\Rightarrow$ $2[\vec{a},\vec{b},\vec{c}] = 0$ or $[\vec{a},\vec{b},\vec{c}]=0$ $\frac{1}{2}$

$\Rightarrow$ $\vec{a},\vec{b},\vec{c}$ are coplanar.

9. Vector equation of the required line is

$\vec{r} = (\hat{i}+2\hat{j}-4\hat{k})+\mu[(3\hat{i}-16\hat{j}+7\hat{k})\times(3\hat{i}+8\hat{j}-5\hat{k})]$ 1

$\Rightarrow$ $\vec{r} = (\hat{i}+2\hat{j}-4\hat{k})+\lambda[(2\hat{i}+3\hat{j}+6\hat{k})$ 2

in cartesian form, $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ 1

10. Let events are:

$$\left.\begin{array}{l} E_1 : \text{A is selected} \\ E_2 : \text{B is selected} \\ E_3 : \text{C is selected} \\ A : \text{Change is not introduced} \end{array}\right\}$$

$P(E_1) = \frac{1}{7}, P(E_2) = \frac{2}{7}, P(E_3) = \frac{4}{7}$ 1

$P(A/E_1) = 0.2,\ P(A/E_2) = 0.5,\ P(A/E_3) = 0.7$ 1

$\therefore$ $P(E_3/A) = \dfrac{\frac{4}{7}\times\frac{7}{10}}{\frac{1}{7}\times\frac{2}{10}+\frac{2}{7}\times\frac{5}{10}+\frac{4}{7}\times\frac{7}{10}}$ 1

$= \frac{28}{40} = \frac{7}{10}$ 1

OR

$$\left.\begin{array}{l} \text{Prob. of success for A} = \frac{1}{6} \\ \text{Prob. of failure for A} = \frac{5}{6} \\ \text{Prob. of success for B} = \frac{1}{12} \\ \text{Prob. of failure for B} = \frac{11}{12} \end{array}\right\}$$ 1

B can win in 2nd or 4th or 6th or....throw 1

$\therefore$ $P(B) = \left(\frac{5}{6}.\frac{1}{12}\right)+\left(\frac{5}{6}.\frac{11}{12}.\frac{5}{6}.\frac{1}{12}\right)+\left(\frac{5}{6}.\frac{11}{12}.\frac{5}{6}.\frac{11}{12}.\frac{5}{6}.\frac{1}{12}\right)+....$ 1

$$= \frac{5}{72}\left(1+\frac{55}{72}+\left(\frac{55}{72}\right)^2+....\right)$$

$$= \frac{5}{72}\times\frac{1}{1-\frac{55}{72}} = \frac{5}{72}\times\frac{72}{17} = \frac{5}{17}$$ 1

11. LHS $= \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}.\frac{1}{7}}\right)+\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}.\frac{1}{8}}\right)$ 1

$$= \tan^{-1}\left(\frac{6}{17}\right)+\tan^{-1}\left(\frac{11}{23}\right)$$ 1

$$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}.\frac{11}{23}}\right) = \tan^{-1}\left(\frac{325}{325}\right)$$ 1

$$= \tan^{-1}(1) = \frac{\pi}{4}$$ 1

OR

$2\tan^{-1}(\cos x) = \tan^{-1}(2\text{cosec } x)$

$\Rightarrow \quad \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\frac{2}{\sin x}\right)$ 2

$\Rightarrow \quad \sin x(\sin x - \cos x) = 0$ 1

$\Rightarrow \quad \sin x = \cos x$ $\frac{1}{2}$

the solution is $x = \frac{\pi}{4}$ $\frac{1}{2}$

12. Let the income be 3x, 4x and expenditures, 5y, 7y

$\therefore \quad \left.\begin{array}{l} 3x-5y=15000 \\ 4x-7y=15000 \end{array}\right\}$ 1

$$\begin{pmatrix} 3 & -5 \\ 4 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -1\begin{pmatrix} -7 & 5 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$\Rightarrow \quad x = 30000,\ y = 15000$ $1\frac{1}{2}$

$\therefore$ Incomes are ₹ 90000 and ₹ 120000 respectively $\frac{1}{2}$

"Expenditure must be less than income"

(or any other relevant answer) 1

13. Here $x = a\left(\sin 2t + \frac{1}{2}\sin 4t\right)$, $y = b(\cos 2t - \cos^2 2t)$

$$\frac{dx}{dt} = 2a[\cos 2t + \cos 4t], \frac{dy}{dt} = 2b[-\sin 2t + 2\cos 2t \sin 2t] = 2b[\sin 4t - \sin 2t] \quad 1 + 1$$

$$\frac{dy}{dx} = \frac{b}{a}\left[\frac{\sin 4t - \sin 2t}{\cos 4t + \cos 2t}\right] \quad 1$$

$$\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{4}} = \frac{b}{a} \quad \frac{1}{2}$$

and $\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{3}} = \sqrt{3}\frac{b}{a}$ $\quad \frac{1}{2}$

OR

$$y = x^x \Rightarrow \log y = x.\log x \quad \frac{1}{2}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = (1 + \log x) \quad 1\frac{1}{2}$$

$$\Rightarrow \frac{1}{y}\frac{d^2y}{dx^2} - \frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 = \frac{1}{x} \quad 1\frac{1}{2}$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0 \quad \frac{1}{2}$$

14. $\text{LHL} = \lim_{x \to \frac{\pi}{2}^-} \frac{(1 - \cancel{\sin x})(1 + \sin x + \sin^2 x)}{3(1 - \cancel{\sin x})(1 + \sin x)}$ $\quad 1$

$$= \frac{1}{2} \quad \frac{1}{2}$$

$$\therefore p = \frac{1}{2} \quad \frac{1}{2}$$

$\text{RHL} = \lim_{x \to \frac{\pi}{2}^+} \frac{q(1 - \sin x)}{(\pi - 2x)^2} = \lim_{h \to 0} \frac{q(1 - \cos h)}{(2h)^2}$, where $x - \frac{\pi}{2} = h$ $\quad 1$

$$= \lim_{h \to 0} \frac{2q \sin^2 \frac{h}{2}}{4.4.\frac{h^2}{4}} = \frac{q}{8} \quad \frac{1}{2}$$

$$\therefore \frac{q}{8} = \frac{1}{2} \Rightarrow q = 4 \quad \frac{1}{2}$$

15. $\frac{dx}{dt} = -3\sin t + 3\cos^2 t \sin t = -3 \sin t (1 - \cos^2 t) = -3 \sin^3 t$ $\quad 1$

$\frac{dy}{dt} = 3\cos t - 3\sin^2 t \cos t = 3\cos t (1 - \sin^2 t) = 3\cos^3 t$ $\quad 1$

Slope of normal $= -\frac{dx}{dy} = \frac{\sin^3 t}{\cos^3 t}$ 1

Eqn. of normal is

$y - (3\sin t - \sin^3 t) = \frac{\sin^3 t}{\cos^3 t}[x - (3\cos t - \cos^3 t)]$ $\frac{1}{2}$

$\Rightarrow \quad y \cos^3 t - x \sin^3 t = 3\sin t \cos t (\cos^2 t - \sin^2 t)$

$= \frac{3}{4}\sin 4t$ $\frac{1}{2}$

or $\quad 4(y \cos^3 t - x \sin^3 t) = 3 \sin 4t$

16. $I = \int \frac{(3\sin\theta - 2)\cos\theta}{5 - (1 - \sin^2\theta) - 4\sin\theta} d\theta$ $\frac{1}{2}$

$\sin\theta = t \Rightarrow \cos\theta \, d\theta = dt$

$\therefore \quad I = \int \frac{3t - 2}{t^2 - 4t + 4} dt = \int \frac{3t - 2}{(t - 2)^2} dt$ 1

$= \int \frac{3(t - 2)}{(t - 2)^2} dt + 4\int \frac{1}{(t - 2)^2} dt$ 1

$= 3\log|t - 2| - \frac{4}{(t - 2)} + C$ 1

$= 3\log|\sin\theta - 2| - \frac{4}{(\sin\theta - 2)} + C$ $\frac{1}{2}$

OR

Let $I = \int_0^\pi \sin\left(\frac{\pi}{4} + x\right) e^{2x} dx$

$= \sin\left(\frac{\pi}{4} + x\right)\frac{e^{2x}}{2}\Big]_0^\pi - \int_0^\pi \cos\left(\frac{\pi}{4} + x\right)\frac{e^{2x}}{2} dx$ 1

$I = \left[\sin\left(\frac{\pi}{4} + x\right)\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos\left(\frac{\pi}{4} + x\right)\frac{e^{2x}}{2}\right\}\right]_0^\pi + \frac{1}{2}\int_0^\pi -\sin\left(\frac{\pi}{4} + x\right)\frac{e^{2x}}{2} dx$ 1

$\frac{5}{\cancel{4}} I = \left\{\frac{1}{\cancel{4}}\left[2\sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right)\right]e^{2x}\right\}_0^\pi$ 1

$I = \frac{1}{5}\left[\left\{2\left(-\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\right\}e^{2\pi} - \left\{2\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}\right\}\right] = \frac{-1}{5\sqrt{2}}(e^{2\pi} + 1)$ 1

17. $I = \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} dx$

Put $x^{3/2} = t \Rightarrow \frac{3}{2}.x^{1/2} dx = dt$ or $\sqrt{x}\, dx = \frac{2}{3} dt$ $1\frac{1}{2}$

$$I = \frac{2}{3}\int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}}$$ 1

$$= \frac{2}{3}.\sin^{-1}\left(\frac{t}{a^{3/2}}\right) + C$$ 1

$$= \frac{2}{3}\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right) + C$$ $\frac{1}{2}$

18. $I = \int_{-1}^{2} |x^3 - x|\, dx$

$$= \int_{-1}^{0} (x^3 - x)\, dx + \int_{0}^{1} -(x^3 - x)\, dx + \int_{1}^{2} (x^3 - x)\, dx$$ $1\frac{1}{2}$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$$ $1\frac{1}{2}$

$$= -\left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + (4 - 2) - \left(\frac{1}{4} - \frac{1}{2}\right)$$

$$= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = \frac{11}{4}$$ 1

19. Given differential equation can be written as

$$\frac{(1 + \log x)}{x}\, dx + \frac{2y}{1 - y^2}\, dy = 0$$ 1

integrating to get, $\frac{1}{2}(1 + \log x)^2 - \log |1 - y^2| = C$ 2

$x = 1,\ y = 0 \Rightarrow C = \frac{1}{2}$ $\frac{1}{2}$

$\Rightarrow \quad (1 + \log x)^2 - 2 \log |1 - y^2| = 1$ $\frac{1}{2}$

SECTION C

20. Equation of line $AB: \frac{x - 3}{-1} = \frac{y + 4}{1} = \frac{z + 5}{6} = \lambda$ 1

Eqn. of plane LMN: $\begin{vmatrix} x - 2 & y - 2 & z - 1 \\ 1 & -2 & 0 \\ 2 & -3 & -1 \end{vmatrix} = 0$ $1\frac{1}{2}$

$2(x - 2) + 1\,(y - 2) + 1\,(z - 1) = 0$ or $2x + y + z - 7 = 0$ $\frac{1}{2}$

Any point on line AB is $(-\lambda + 3,\ \lambda - 4,\ 6\lambda - 5)$ $\frac{1}{2}$

If this point lies on plane, then $2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 0 \Rightarrow 5\lambda = 10 \Rightarrow \lambda = 2$ 1

$\therefore$ P is (1, –2. 7) $\frac{1}{2}$

let P divides AB in K : 1

$\Rightarrow \quad 1 = \dfrac{2K+3}{K+1} \Rightarrow K = -2$ i.e. P divides, AB externally in $2:1$ 1

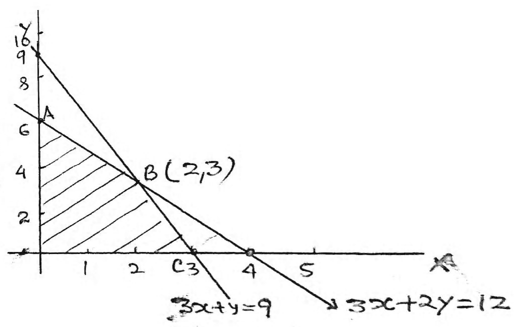
21. X = No. of red

X:	0	1	2	3	4	1
P(X):	${}^4C_0\left(\frac{1}{3}\right)^4 = \frac{1}{81}$	${}^4C_1\left(\frac{1}{3}\right)^3 \frac{2}{3} = \frac{8}{81}$	${}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2 = \frac{24}{81}$	${}^4C_3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 = \frac{32}{81}$	${}^4C_4\left(\frac{2}{3}\right)^4 = \frac{16}{81}$	$2\frac{1}{2}$
XP(X):	0	$\frac{8}{81}$	$\frac{48}{81}$	$\frac{96}{81}$	$\frac{64}{81}$	
X^2P(X):	0	$\frac{8}{81}$	$\frac{96}{81}$	$\frac{288}{81}$	$\frac{256}{81}$	

Mean $= \Sigma XP(X) = \dfrac{216}{81} = \dfrac{8}{3}$ 1

Variance $= \Sigma X^2P(X) - [\Sigma XP(X)]^2 = \dfrac{648}{81} - \dfrac{64}{9} = \dfrac{8}{9}$ $1\frac{1}{2}$

22.



Let production of A, B (per day) be x, y respectively

Maximise P = 7x + 4y 1

Subject to $\left.\begin{array}{l} 3x + 2y \le 12 \\ 3x + y \le 9 \\ x \ge 0,\ y \ge 0 \end{array}\right\}$ 2

Correct Graph 2

P(A) = 24

P(B) = 26

P(C) = 21

$\therefore$ 2 units of product A and 3 units of product B for maximum profit 1

23. Let $x_1, x_2 \in N$ and $f(x_1) = f(x_2)$

$\Rightarrow \quad 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow \quad 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(9x_1 + 9x_2 + 6) = 0$

$\Rightarrow \quad x_1 - x_2 = 0$ or $x_1 = x_2$ as $(9x_1 + 9x_2 + 6) \neq 0,\ x_1, x_2 \in N$

$\therefore$ f is a one-one function 2

f: $N \to S$ is ONTO as co-domain = Range 1

Hence f is invertible

$$y = 9x^2 + 6x - 5 = (3x+1)^2 - 6 \Rightarrow x = \frac{\sqrt{y+6}-1}{3}$$

$\therefore \quad f^{-1}(y) = \dfrac{\sqrt{y+6}-1}{3},\ y \in S$ 2

$$f^{-1}(43) = \frac{\sqrt{49}-1}{3} = 2$$ $\frac{1}{2}$

$$f^{-1}(163) = \frac{\sqrt{169}-1}{3} = 4$$ $\frac{1}{2}$

24. Using $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$ we get

$$\Delta = \begin{vmatrix} y(z-x)+z^2-x^2 & x(z-y)+z^2-y^2 & xy-z^2 \\ z(x-y)+x^2-y^2 & y(x-z)+x^2-z^2 & yz-x^2 \\ x(y-z)+y^2-z^2 & z(y-x)+y^2-x^2 & zx-y^2 \end{vmatrix}$$ 2

Taking (x + y + z) common from C_1 & C_2

$$\Rightarrow \quad \Delta = (x+y+z)^2 \begin{vmatrix} z-x & z-y & xy-z^2 \\ x-y & x-z & yz-x^2 \\ y-z & y-x & zx-y^2 \end{vmatrix}$$ 1

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \quad \Delta = (x+y+z)^2 \begin{vmatrix} 0 & 0 & xy+yz+zx-x^2-y^2-z^2 \\ x-y & x-z & yz-x^2 \\ y-z & y-x & zx-y^2 \end{vmatrix}$$ 1

Expanding to get

$\Delta = (x + y + z)^2 (xy + zy + zx - x^2 - y^2 - z^2)^2$ 1

Hence Δ is divisible by (x + y + z) and

the quotient is $(x + y + z) (xy + yz + zx - x^2 - y^2 - z^2)^2$ 1

OR

Writing $\begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1

$R_1 \leftrightarrow R_3$ $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 8 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} A$

$R_1 \to R_1 - 2R_2$
$R_3 \to R_3 - 4R_2$ $\begin{pmatrix} -3 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -2 & 1 \\ 0 & 1 & 0 \\ 1 & -4 & 0 \end{pmatrix} A$

$R_1 \to \frac{1}{3} R_1$
$R_3 \to - R_3$ $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & 1 & 0 \\ -1 & 4 & 0 \end{pmatrix} A$

$R_2 \to R_2 - 2R_1$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & -1/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A$ $2\frac{1}{2}$ marks for operation to get A^{-1}

$R_2 \to R_2 - R_3$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A$

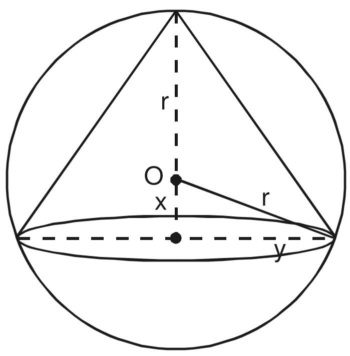
$\therefore \quad A^{-1} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix}$ $\frac{1}{2}$

$AX = B \Rightarrow X = A^{-1}B$ 1

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} \begin{pmatrix} 19 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$\therefore \quad x = 1,\ y = 2,\ z = 1$ 1

25. Correct Figure 1



Let radius of cone be y and the altitude be r + x

$\therefore \quad x^2 + y^2 = r^2 \quad$ ...(i) $\frac{1}{2}$

Volume $V = \frac{1}{3}\pi y^2(r + x)$

$= \frac{1}{3}\pi(r^2 - x^2)(r + x)$ 1

$\frac{dV}{dx} = \frac{\pi}{3}[(r^2 - x^2)1 + (r + x)(-2x)] = \frac{\pi}{3}(r + x)(r - 3x)$ 1

$\frac{dV}{dx} = 0 \Rightarrow x = \frac{r}{3}$ $\frac{1}{2}$

$\therefore \quad$ Altitude $= r + \frac{r}{3} = \frac{4r}{3}$ $\frac{1}{2}$

and $\quad \frac{d^2V}{dx^2} = \frac{\pi}{3}[(r + x)(-3) + (r - 3x)] = \frac{\pi}{3}[-2r - 6x] < 0$ 1

$\therefore \quad$ Max. Volume $= \frac{\pi}{3}\left(r^2 - \frac{r^2}{9}\right)\left(r + \frac{r}{3}\right) = \frac{8}{27}\left(\frac{4}{3}\pi r^3\right)$ $\frac{1}{2}$

$= \frac{8}{27}$(Vol. of sphere)

OR

$f(x) = \sin 3x - \cos 3x,\ 0 < x < \pi$

$f'(x) = 3\cos 3x + 3\sin 3x$ 1

$f'(x) = 0 \Rightarrow \tan 3x = -1$ $\frac{1}{2}$

$\Rightarrow \quad x = \frac{n\pi}{3} + \frac{\pi}{4},\ n \in Z$

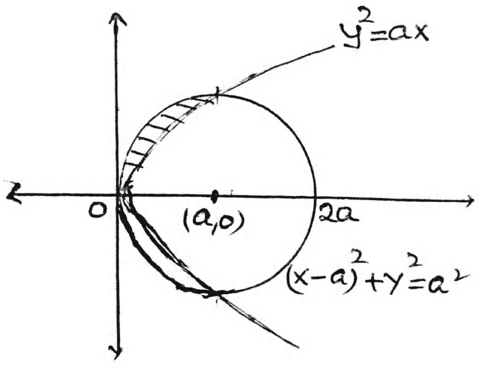
$\Rightarrow \quad x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}$ $1\frac{1}{2}$

Intervals are: $\left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{7\pi}{12}\right), \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right), \left(\frac{11\pi}{12}, \pi\right)$ 1

f(x) is strictly increasing in $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right)$ 1

and strictly decreasing in $\left(\frac{\pi}{4}, \frac{7\pi}{12}\right) \cup \left(\frac{11\pi}{12}, \pi\right)$ 1

26.



$y^2 = ax,\ x^2 + y^2 = 2ax \Rightarrow x^2 - ax = 0$

$\Rightarrow \quad x = 0,\ x = a$ 1

Correct Figure 1

Shaded area $= \left[\int_0^a [\sqrt{a^2 - (x-a)^2} - \sqrt{a}\sqrt{x}]dx\right.$ 1

$$A = \left[\frac{x-a}{2}\sqrt{a^2-(x-a)^2} + \frac{a^2}{2}\sin^{-1}\frac{x-a}{a} - \sqrt{a}\,\frac{2}{3}\,x^{3/2}\right]_0^a$$ 2

$$= \left[-\frac{2}{3}a^2 + \frac{a^2}{2}\frac{\pi}{2}\right] = \frac{\pi a^2}{4} - \frac{2a^2}{3}\text{ sq. units}$$ 1