SOLUTION/ ANSWER KEY OF PRACTICE PAPER-2

CLASS XII MATHEMATICS

2019-20

Q NO	VALUE POINTS
1	(C) AB and BA both are defined
2	(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
3	(D)-5/2
4	(A)1/10
5	(D) $\sqrt{\alpha^2+\gamma^2}$
6	(A) $\frac{\pi}{5}$
7	(C) 15/56
8	(A) $e^x \cos x + c$
9	(D) (α,β,-γ)
10	(B)Parallel
11	Domain = $(-\infty, 1] \cup [2, \infty)$
12	2
13	Maximum value is $\frac{4}{3}$ ORx+y=0
14	$27\|A\|$
15	x+y-z=2 OR $\|\vec{a}\|^2\|\vec{b}\|^2$
16	$\frac{-3}{\sqrt{1-x^2}}$
17	$\log\|2+\log x\|$+c
18	$3\log\|(x+\sin x)\|$+c OR $\log\|x+\sin x\|$+c
19	$\tan x + \cot x + c$
20	y=$\frac{x^4+c}{4x^2}$
21	$(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})=0$ $\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{a}+\vec{b}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{b}+\vec{c}.\vec{c}=0$ $\Rightarrow \|\vec{a}\|^2+\|\vec{b}\|^2+\|\vec{c}\|^2+2(\overline{\vec{a}\vec{b}}+\vec{b}\vec{c}+\vec{c}\vec{a})=0$

<table>
<tr><td></td><td>

⇒ $\overrightarrow{(a}\vec{b}+\vec{b}\vec{c}+\vec{c}\vec{a})=\frac{-83}{2}$

OR

Given vectors are $\vec{a}=3\hat{\imath}-\hat{\jmath}+2\hat{k}$ and $\vec{b}=-2\hat{\imath}+\hat{\jmath}-\hat{k}$, therefore ,

$\vec{a}+\vec{b}.=(3-2)\hat{\imath}+(-1+1)\hat{\jmath}+(2-1)\hat{k}$

⇒ $\vec{a}+\vec{b}=.=1.\,i+\ 0.\hat{\jmath}+1.\hat{k}=\hat{\imath}+\hat{k}$

Hence unit vector in the direction of $(\vec{a}+\vec{b})$ is ,

$\frac{(\vec{a}+\vec{b})}{|(\vec{a}+\vec{b})\,|} = \frac{\hat{\imath}+\hat{k}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\hat{\imath}+\frac{1}{\sqrt{2}}\hat{k}$

</td></tr>
<tr><td>22</td><td>

$$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$$

$=\tan^{-1}\left[\frac{cos^2\frac{x}{2}-sin^2\frac{x}{2}}{cos^2\frac{x}{2}+sin^2\frac{x}{2}-2si\ \ \frac{x}{2}cos\frac{x}{2}}\right]$

$=\tan^{-1}\left(\frac{cos\frac{x}{2}+sin\frac{x}{2}}{cos\frac{x}{2}-sin\frac{x}{2}}\right)$

$=\tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right]$

$=\tan^{-1}[\tan(\frac{\pi}{4}+\frac{x}{2})]$

$=\frac{\pi}{4}+\frac{x}{2}$

OR

f(x)=cosx , g(x)=$3x^2$

<table>
<tr><td>f(x)=cosx

f(x)=cosx⇒f(g(x))=cosg(x)
⇒fog(x)=cos $3x^2$</td><td>g(x)=$3x^2$⇒g(f(x))=3f(x)^2

gof(x)=3cos^{2}x</td></tr>
</table>

Hence gof≠fog

</td></tr>
<tr><td>23</td><td>

$\frac{dy}{dx}$=Acos x+ B(-sinx)=Acos x _Bsin x

$\frac{\mathrm{d2y}}{\mathrm{dx2}}$= A(-sin x) –B cos x=-A sin x –B cos x

</td></tr>
<tr><td>24</td><td>

According to to question,

we have to find out the point on the curve at which the y coordinate is changing 2 times as fast as the x - coordinate.ie ; $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$=2=>$\frac{dy}{dx}$=2 , now equation of the curve 6y = x^3 + 2

=>6$\frac{dy}{dx}$=3x^2=>2$\frac{dy}{dx}$=x^2

</td></tr>
</table>

	now, put x = 4 in $6y = x^3 + 2$=>$2x2=x^2$ =>x=± 2 When x=2 $6y = 2^3 + 2 = 10$ => y = 10/6 =>y=5/3 hence, a point on the curve is (2,5/3) When x=-2 $6y =(-2)^3 + 2 = -8+2=-6$ => y = -1 =>points on the curve will be (-2,-1), (2,5/3)
25	Given lines are $\vec{r} = (2\hat{\imath} - 5\hat{\jmath} + \hat{k}) + \lambda(3\hat{\imath}+2\hat{\jmath}+6\hat{k})$ and $\vec{r} = (7\hat{\imath}-6\hat{k}) + \mu(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ on compairing with $\vec{r}=\vec{a}_1+ \lambda\vec{b}_1$ and $\vec{a}_2 + \mu\vec{b}_2$ we get $\vec{b}_1=3\hat{\imath}+2\hat{\jmath}+6\hat{k}$ and $\vec{b}_2=\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ angle between the lines is given by $\cos\theta = \left\| \frac{\overrightarrow{b1b2}}{\|\overrightarrow{b1}\|\|\overrightarrow{b2}\|} \right\| = \left\| \frac{(3\hat{\imath}+2\hat{\jmath}+6\hat{k})(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})}{\sqrt{49}\sqrt{9}} \right\| = \frac{19}{\sqrt{49}\sqrt{9}} = \frac{19}{7x3} = \cos^{-1}\frac{19}{21}$
26	*No. of spade cards in a pack of 52 playing cards =13* *Let E: getting a spade* $\therefore$ *P (E) =*$\frac{13}{52}$, *P (E) =*$\frac{39}{52}$ *Therefore, P (only 2 cards are spades) =* ${}^4C_2\frac{13}{52}.\frac{13}{52}.\frac{39}{52}.\frac{39}{52} = \frac{54}{256}$

27	For x_1, x_2 ϵA, let $f(x_1)=f(x_2)$ $\frac{4x_1+3}{6x_1-4}=\frac{4x_2+3}{6x_2-4}$ $(4x_1+3)(6x_2-4)=(6x_1-4)(4x_2+3)$ $\Rightarrow x_1=x_2$ Hence, f is one-one. For any y ϵA s.t $y=\frac{4x+3}{6x-4}$ $\ni$ x such that 6xy-4y=4x+3 $\Rightarrow x=\frac{4y+3}{6y-4} \in A$ Also, $f(x) = f\left(\frac{4y+3}{6y-4}\right) = \frac{4\left(\frac{4y+3}{6y-4}\right)+3}{6\left(\frac{4y+3}{6y-4}\right)-4} = y$ $\Rightarrow$f(x) is onto. Since, f(x) is one-one and onto, therefore f^{-1} exists and $f^{-1}(y) = \frac{4y+3}{6y-4}$.
28	Here $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\cot^{-1} a$ $\Rightarrow\frac{x^2-y^2}{x^2+y^2} = \cos(\cot^{-1} a)$ Applying componendo and dividendo, we get :$\frac{x^2-y^2+x^2+y^2}{x^2-y^2+x^2-y^2}=\frac{\cos(\cot^{-1} a)+1}{\cos(\cot^{-1} a)-1}$ $\Rightarrow\frac{2x^2}{-2y^2}=\frac{\cos(\cot^{-1} a)+1}{\cos(\cot^{-1} a)-1}$ $\Rightarrow \frac{d}{dx}\left(\frac{x^2}{y^2}\right)=\frac{d}{dx}\left(-\frac{\cos(\cot^{-1} a)+1}{\cos(\cot^{-1} a)-1}\right)$ $\Rightarrow \frac{\text{y2x 2x}-\text{x2x2y}\frac{dy}{dx}}{(y^2)^2}=0$

	$\Rightarrow$y-x$\frac{dy}{dx}=0 \therefore \frac{dy}{dx}$=$\frac{y}{x}$---------(i) Now $\frac{d}{dx}\left(\frac{dy}{dx}\right)$=$\frac{d^2\text{y}}{dx^2}$=$\frac{d}{dx}\left(\frac{y}{x}\right)$$\Rightarrow$$\frac{d^2\text{y}}{dx^2}$= $\frac{\text{x}\frac{dy}{dx}-yX1}{\text{x2}}$ $\Rightarrow$ $\frac{d^2\text{y}}{dx^2}$=$\frac{\text{x}\frac{y}{x}-y}{\text{x2}}$ =$\frac{0}{\text{x2}}$ by (i) hence $\frac{d^2\text{y}}{dx^2}$=0 OR sin y = x sin(a+y) $\Rightarrow$ x = $\frac{\sin y}{\sin(a+y)}$ Differentiating w.r.t y, we get, $\frac{dx}{dy}$= $\frac{\sin(a+y)\cos y-\sin y\cos(a+y)}{sin^2(a+y)}$ $\frac{dx}{dy}$ = $\frac{\sin\ (a+y-y)}{sin^2(a+y)}$ = $\frac{\sin a}{sin^2(a+y)}$ $$\therefore \frac{dy}{dx}=\frac{sin^2(a+y}{\sin a}$$
29	The given differential equation can be written as $\frac{dy}{dx}$ = $\frac{y}{x}-tan\left(\frac{y}{x}\right)$ Put $\frac{y}{x}=v$ and$\frac{dy}{dx}=\ v+x\frac{dv}{dx}$, to get $v+x\frac{dv}{dx}=v-tanv$$\Rightarrow x\frac{dv}{dx}=-tanv$ $\Rightarrow$ cot v dv = $\frac{-1}{x}dx, integrating\ both\ sides\ we\ get$ $$\log\lvert sinv\rvert=-\log\lvert x\rvert+logc$$

	$\Rightarrow \log\lvert sinv\rvert = log\left\lvert\frac{c}{x}\right\rvert$ Solution of differential equation is: $sin\left(\frac{y}{x}\right)=\frac{c}{x}$ or $xsin\left(\frac{y}{x}\right) = c$
30	$= \int \frac{3x+5}{x^2+3x-18}dx = \frac{3}{2}\int \frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int \frac{1}{x^2+3x-18}dx$ $$= \frac{3}{2}\int \frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int \frac{1}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}dx$$ $$= \frac{3}{2}\log\lvert x^2+3x-18\rvert + \frac{1}{18}log\left\lvert\frac{x-3}{x+6}\right\rvert + c$$
31	P (at least 3 are diamonds) P(3)+ P(4)= ${}^4C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right) + {}^4C_4\left(\frac{1}{4}\right)^4$ $=\left(\frac{1}{4}\right)^4(12+1) = \frac{13}{256}$ OR P (only one on time) =P (A) P ($\bar{B}$)+P ($\bar{A}$) P (B) $$= \frac{2}{7}\times\frac{3}{7}+\frac{5}{7}\times\frac{4}{7}$$ $$= \frac{26}{49}$$

32

Let no. of packages of nuts be x units and no. of packages of bolts be y units.

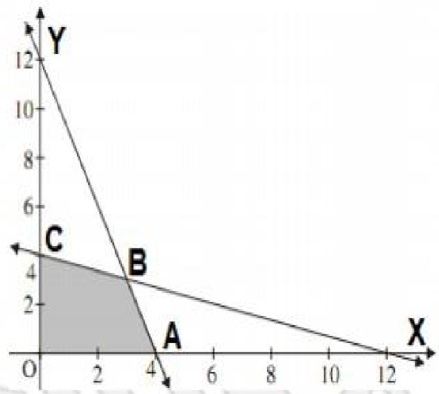
To maximize : $Z = ₹(35x + 14y)$

Subject to constraints :

$x \geq 0,\ y \geq 0,$

$x + 3y \leq 12,$

$3x + y \leq 12$



Corner Points	**Value of Z (in ₹)**
A(4, 0)	140
B(3, 3)	147 ← Maximum
C(0, 4)	56

Hence, maximum profit of ₹147 is obtained when no. of packages of nuts = x = 3 units and, no. of packages of bolts = y = 3 units are produced.

33

$$\text{Let } A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

Then A = IA

$$\Rightarrow \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad (R_1 \to R_1 - R_2)$$

$$\therefore\ A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 9 & -12 & 9 \end{bmatrix}$$

OR

The given system of equations is

AX = B,

$$\text{where } A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$|A| = 1200 \neq 0$

$\Rightarrow A^{-1}$ exists.

$X = A^{-1}B$

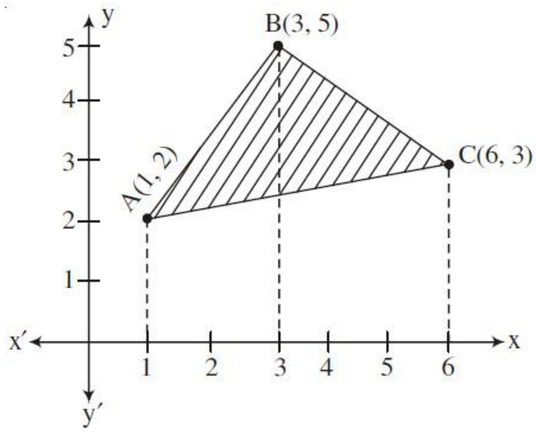
$$\text{adj } A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj}A = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$\therefore\ x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$$

34	 Required Area = $\int_1^3 \frac{3x+1}{2}dx + \int_3^6 \frac{21-2x}{3}dx - \int_1^6 \frac{x+9}{5}dx$ $= \left(\frac{3x^2}{4}+\frac{x}{2}\right)\Big\|_1^3 + \left(7x-\frac{x^2}{3}\right)\Big\|_3^6 - \left(\frac{x^2}{10}+\frac{9x}{5}\right)\Big\|_1^6$ $= 7+12-\frac{25}{2}$ $= \frac{13}{2}$
35	Let Given surface area of open cylinder be S. Then S = $2\pi rh + \pi r^2$ $\Rightarrow h = \frac{S-\pi r^2}{2\pi r}$ Volume V = $\pi r^2 h$ $V = \pi r^2\left[\frac{S-\pi r^2}{2\pi r}\right] = \frac{1}{2}[Sr - \pi r^3]$ $\frac{dV}{dr} = \frac{1}{2}[S - 3\pi r^2]$ $\frac{dV}{dr} = 0 \Rightarrow S = 3\pi r^2$ or $2\pi rh + \pi r^2 = 3\pi r^2$

$$\Rightarrow\ 2\pi rh = 2\pi r^2 \qquad \Rightarrow h = r$$

$$\frac{d^2V}{dr^2} = -6\pi r < 0$$

$\therefore$ For volume to be maximum, height = radius

Let x be the radius of circle and y be the side of square

$$2\pi x + 4y = k$$

$$A = \pi x^2 + y^2$$

$$A = \pi x^2 + \left(\frac{k-2\pi x}{4}\right)^2 = \frac{16\pi x^2 + k^2 + 4\pi^2 x^2 - 4\pi kx}{16}$$

$$\frac{dA}{dx} = \frac{1}{16}(32\pi x + 8\pi^2 x - 4\pi k)$$

$$\frac{dA}{dx} = 0 \Rightarrow 32\pi x + 8\pi^2 x - 4\pi k = 0$$

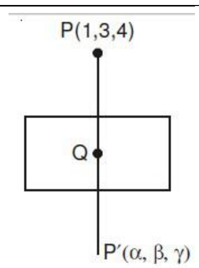
$$\Rightarrow\ x = \frac{k}{8+2\pi}$$

$$\left.\frac{d^2A}{dx^2}\right]_{x=\frac{k}{8+2\pi}} = \frac{1}{16}[32\pi + 8\pi^2] > 0 \Rightarrow \text{Sum of areas is minimum}$$

OR

$$2\pi\left(\frac{k}{8+2\pi}\right) + 4y = k \Rightarrow y = \frac{k}{4+\pi} \Rightarrow y = 2x$$

36



Equation of line PQ is

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda$$

The coordinates of Q are $(2\lambda + 1, -\lambda + 3, \lambda + 4)$

	$\because$ Q lies on plane $2x - y + z + 3 = 0$ $\therefore\ 2(2\lambda + 1) - (-\lambda + 3) + (\lambda + 4) + 3 = 0$ $\Rightarrow 6\lambda + 6 = 0$ i.e., $\lambda = -1$ The coordinates of Q are $(-1, 4, 3)$ Let $P'(\alpha, \beta, \gamma)$ be the image of P. then $\frac{\alpha+1}{2} = -1,\ \frac{\beta+3}{2} = 4,\ \frac{\gamma+4}{2} = 3$ $\Rightarrow \alpha = -3,\ \beta = 5,\ \gamma = 2$ $\therefore$ the image P' is $(-3, 5, 2)$