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Senior School Certificate Examination

March 2018

Marking Scheme — Mathematics 65/1, 65/2, 65/3

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right) = -\frac{\pi}{2}$ $\frac{1}{2}+\frac{1}{2}$

Note: $\frac{1}{2}$m. for any one of the two correct values and $\frac{1}{2}$m. for final answer

2. $a = -2,\ b = 3$ $\frac{1}{2}+\frac{1}{2}$

3. $|\vec{a}| = |\vec{b}| = 3$ $\frac{1}{2}+\frac{1}{2}$

4. $5 o 10 = (5 * 10) + 3 = 10 + 3 = 13$ For $5 * 10 = 10$ $\frac{1}{2}$

For Final Answer = 13 $\frac{1}{2}$

SECTION B

5. In RHS, put $x = \sin\theta$ $\frac{1}{2}$

RHS $= \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta)$ 1

$= 3\theta = 3\sin^{-1}x = \text{LHS}.$ $\frac{1}{2}$

6. $|A| = 2, \quad \therefore\ A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ 1

$\text{LHS} = 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}, \ \text{RHS} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ 1

$\therefore$ LHS = RHS

7. $f(x) = \tan^{-1}\left(\dfrac{1+\cos x}{\sin x}\right) = \tan^{-1}\left(\dfrac{2\cos^2 x\!/\!_2}{2\sin x\!/\!_2 \cos x\!/\!_2}\right)$ 1

$= \tan^{-1}\left(\cot x\!/\!_2\right) = \dfrac{\pi}{2} - \dfrac{x}{2}$ $\frac{1}{2}$

$\therefore \quad f'(x) = -\dfrac{1}{2}$ $\frac{1}{2}$

8. Marginal cost = $C'(x) = 0.015x^2 - 0.04x + 30$ 1

At $x = 3$, $C'(3) = 30.015$ 1

9. $I = \displaystyle\int \frac{1-2\sin^2 x + 2\sin^2 x}{\cos^2 x}\, dx$ $\frac{1}{2}$

$= \displaystyle\int \sec^2 x\, dx$ 1

$= \tan x + C$ $\frac{1}{2}$

10. $\dfrac{dy}{dx} = bae^{bx+5} \Rightarrow \dfrac{dy}{dx} = by$ 1

$\Rightarrow \dfrac{d^2y}{dx^2} = b\dfrac{dy}{dx}$ $\frac{1}{2}$

$\therefore$ The differential equation is: $y\dfrac{d^2y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$ $\frac{1}{2}$

11. $\sin\theta = \dfrac{|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})|}{|\hat{i} - 2\hat{j} + 3\hat{k}|\ |3\hat{i} - 2\hat{j} + \hat{k}|}$ $\frac{1}{2}$

$|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})| = |4\hat{i} + 8\hat{j} + 4\hat{k}| = 4\sqrt{6}$ 1

$\sin\theta = \dfrac{4\sqrt{6}}{14} = \dfrac{2\sqrt{6}}{7}$ $\frac{1}{2}$

12. A: Getting a sum of 8, B: Red die resulted in no. < 4

$P(A/B) = \dfrac{P(A \cap B)}{P(B)}$ 1

$= \dfrac{2/36}{18/36} = \dfrac{1}{9}$ 1

SECTION C

13. LHS $= \begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix}$

$= \begin{vmatrix} 1 & 0 & 3x \\ 1+3y & -3y & -3y \\ 1 & 3z & 0 \end{vmatrix}$ (Using $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$) 1+1 (Any two relevant operations)

$= 1 \times (9yz) + 3x(3z + 9yz + 3y)$ (Expanding along R_1) 1

$= 9(3xyz + xy + yz + zx) = \text{RHS}$ 1

14. Differentiating with respect to 'x'

$2(x^2 + y^2)\left(2x + 2y\dfrac{dy}{dx}\right) = x\dfrac{dy}{dx} + y$ 2

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{y - 4x^3 - 4xy^2}{4x^2y + 4y^3 - x}$ 2

OR

$\dfrac{dx}{d\theta} = a(2 - 2\cos 2\theta) = 4a\sin^2\theta$ 1

$\dfrac{dy}{d\theta} = 2a\sin 2\theta = 4a\sin\theta\cdot\cos\theta$ 1

$\therefore \quad \dfrac{dy}{dx} = \dfrac{4a\sin\theta\cos\theta}{4a\sin^2\theta} = \cot\theta$ 1

$\left.\dfrac{dy}{dx}\right]_{\theta = \frac{\pi}{3}} = \dfrac{1}{\sqrt{3}}$ 1

15. $y = \sin(\sin x) \Rightarrow \dfrac{dy}{dx} = \cos(\sin x)\cdot\cos x$ 1

and $\dfrac{d^2y}{dx^2} = -\sin(\sin x)\cdot\cos^2 x - \sin x\cos(\sin x)$ 1+1

LHS $= -\sin(\sin x)\cos^2 x - \sin x\cos(\sin x) + \dfrac{\sin x}{\cos x}\cos(\sin x)\cos x + \sin(\sin x)\cos^2 x$ 1

$= 0 = \text{RHS}$

16. $x_1 = 2 \Rightarrow y_1 = 3 \quad (\because\ y_1 > 0)$ $\frac{1}{2}$

Differentiating the given equation, we get, $\frac{dy}{dx} = \frac{-16x}{9y}$ $\frac{1}{2}$

Slope of tangent at $(2, 3) = \left.\frac{dy}{dx}\right]_{(2,\,3)} = -\frac{32}{27}$ $\frac{1}{2}$

Slope of Normal at $(2, 3) = \frac{27}{32}$ $\frac{1}{2}$

Equation of tangent: $32x + 27y = 145$ 1

Equation of Normal: $27x - 32y = -42$ 1

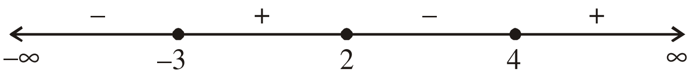
OR

$f'(x) = x^3 - 3x^2 - 10x + 24$ $\frac{1}{2}$

$\quad = (x - 2)(x - 4)(x + 3)$ 1

$f'(x) = 0 \Rightarrow x = -3, 2, 4.$ $\frac{1}{2}$

sign of $f'(x)$:



$\therefore f(x)$ is strictly increasing on $(-3, 2) \cup (4, \infty)$ 1

and $f(x)$ is strictly decreasing on $(-\infty, -3) \cup (2, 4)$ 1

17. Let side of base $= x$ and depth of tank $= y$

$V = x^2y \Rightarrow y = \frac{V}{x^2}$, ($V$ = Quantity of water = constant)

Cost of material is least when area of sheet used is minimum.

A(Surface area of tank) $= x^2 + 4xy = x^2 + \frac{4V}{x}$ $\frac{1}{2}+\frac{1}{2}$

$\frac{dA}{dx} = 2x - \frac{4V}{x^2}, \frac{dA}{dx} = 0 \Rightarrow x^3 = 2V, y = \frac{x^3}{2x^2} = \frac{x}{2}$ $\frac{1}{2}+\frac{1}{2}$

$\frac{d^2A}{dx^2} = 2 + \frac{8V}{x^3} > 0$, $\therefore$ Area is minimum, thus cost is minimum when $y = \frac{x}{2}$ $\frac{1}{2}+\frac{1}{2}$

Value: Any relevant value. 1

18. Put $\sin x = t \Rightarrow \cos x\, dx = dt$ $\frac{1}{2}$

Let $I = \int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}\, dx = \int \frac{2}{(1-t)(1+t^2)}\, dt$

Let $\frac{2}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2}$, solving we get

$A = 1, B = 1, C = 1$ $1\frac{1}{2}$

$\therefore\ I = \int \frac{1}{1-t}\, dt + \frac{1}{2}\int \frac{2t}{1+t^2} + \int \frac{1}{1+t^2}\, dt$

$= -\log|1-t| + \frac{1}{2}\log|1+t^2| + \tan^{-1} t + C$ $1\frac{1}{2}$

$= -\log(1-\sin x) + \frac{1}{2}\log(1+\sin^2 x) + \tan^{-1}(\sin x) + C$ $\frac{1}{2}$

19. Separating the variables, we get:

$\int \frac{\sec^2 y}{\tan y}\, dy = \int \frac{e^x}{e^x - 2}\, dx$ $1\frac{1}{2}$

$\Rightarrow$ $\log|\tan y| = \log|e^x - 2| + \log C$ $1\frac{1}{2}$

$\Rightarrow$ $\tan y = C(e^x - 2)$, for $x = 0$, $y = \pi/4$, $C = -1$ $\frac{1}{2}$

$\therefore$ Particular solution is: $\tan y = 2 - e^x$. $\frac{1}{2}$

OR

Integrating factor $= e^{\int 2\tan x dx} = \sec^2 x$ 1

$\therefore$ Solution is: $y\cdot\sec^2 x = \int \sin x\cdot\sec^2 x\, dx = \int \sec x\cdot\tan x\, dx$ 1

$\Rightarrow$ $y\cdot\sec^2 x = \sec x + C$, for $x = \frac{\pi}{3}$, $y = 0$, $\therefore\ C = -2$ $1+\frac{1}{2}$

$\therefore$ Particular solution is: $y\cdot\sec^2 x = \sec x - 2$ $\frac{1}{2}$

or $y = \cos x - 2\cos^2 x$

20. $\vec{d} = \lambda(\vec{c} \times \vec{b}) = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 1 & -4 & 5 \end{vmatrix}$ 1

$\therefore \ \vec{d} = \lambda\hat{i} - 16\lambda\hat{j} - 13\lambda\hat{k}$ 1

$\vec{d} \cdot \vec{a} = 21 \Rightarrow 4\lambda - 80\lambda + 13\lambda = 21 \Rightarrow \lambda = -\frac{1}{3}$ 1

$\therefore \ \vec{d} = -\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}$ 1

21. Here $\vec{a}_1 = 4\hat{i} - \hat{j}, \vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}, \vec{a}_2 - \vec{a}_1 = -3\hat{i} + 2\hat{k}$ 1

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i} - \hat{j}$ 1

Shortest distance $= \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$ 1

$= \left|\frac{-6}{\sqrt{5}}\right| = \frac{6}{\sqrt{5}}$ or $\frac{6\sqrt{5}}{5}$ 1

22. E_1 : She gets 1 or 2 on die.

E_2 : She gets 3, 4, 5 or 6 on die.

A: She obtained exactly 1 tail 1

$P(E_1) = \frac{1}{3},\ P(E_2) = \frac{2}{3}$

$P(A/E_1) = \frac{3}{8}, P(A/E_2) = \frac{1}{2}$ 1

$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$ 1

$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}} = \frac{8}{11}$ 1

23. Let X denote the larger of two numbers

X	2	3	4	5	$\frac{1}{2}$
$P(X)$	1/10	2/10	3/10	4/10	1
$X \cdot P(X)$	2/10	6/10	12/10	20/10	$\frac{1}{2}$
$X^2 \cdot P(X)$	4/10	18/10	48/10	100/10	$\frac{1}{2}$

Mean $= \Sigma X \cdot P(X) = \frac{40}{10} = 4$ $\frac{1}{2}$

Variance $= \Sigma X^2 \cdot P(X) - [\Sigma X \cdot P(X)]^2 = \frac{170}{10} - 4^2 = 1$ 1

SECTION D

24. Reflexive: $|a - a| = 0$, which is divisible by 4, $\forall\ a \in A$ 1

$\therefore\quad (a, a) \in R, \forall\ a \in A \quad \therefore\ R$ is reflexive

Symmetric: let $(a, b) \in R$

$\Rightarrow\quad |a - b|$ is divisible by 4

$\Rightarrow\quad |b - a|$ is divisible by 4 ($\because\ |a - b| = |b - a|$)

$\Rightarrow\quad (b, a) \in R \quad \therefore\ R$ is symmetric. 1

Transitive: let $(a, b), (b, c) \in R$

$\Rightarrow\quad |a - b|$ & $|b - c|$ are divisible by 4

$\Rightarrow\quad a - b = \pm 4m,\ b - c = \pm 4n,\ m, n \in Z$

Adding we get, $a - c = 4(\pm m \pm n)$ 2

$\Rightarrow\quad (a - c)$ is divisible by 4

$\Rightarrow\quad |a - c|$ is divisible by 4 $\quad \therefore\ (a, c) \in R$

$\therefore\ R$ is transitive

Hence R is an equivalence relation in A 1

set of elements related to 1 is $\{1, 5, 9\}$

and $[2] = \{2, 6, 10\}$. 1

OR

Here $f(2) = f\left(\frac{1}{2}\right) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$

$\therefore$ f is not 1-1 2

for $y = \frac{1}{\sqrt{2}}$ let $f(x) = \frac{1}{\sqrt{2}} \Rightarrow x^2 - \sqrt{2}x + 1 = 0$

As $D = (-\sqrt{2})^2 - 4(1)(1) < 0$, $\therefore$ No real solution

$\therefore$ $f(x) \neq \frac{1}{\sqrt{2}}$, for any $x \in R(D_f)$ $\therefore f$ is not onto 2

fog $(x) = f(2x - 1) = \frac{2x-1}{(2x-1)^2+1} = \frac{2x-1}{4x^2-4x+2}$ 2

25. $|A| = -1 \neq 0$ $\therefore A^{-1}$ exists 1

Co-factors of A are:

$A_{11} = 0$; $A_{12} = 2$; $A_{13} = 1$

$A_{21} = -1$; $A_{22} = -9$; $A_{23} = -5$

$A_{31} = 2$; $A_{32} = 23$; $A_{33} = 13$

1 m for any 4 correct cofactors 2

$$\text{adj}(A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{|A|} \cdot \text{adj}(A) = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$ $\frac{1}{2}$

For : $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$, the system of equation is $A \cdot X = B$ $\frac{1}{2}$

$$\therefore \quad X = A^{-1} \cdot B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ 1

$\therefore$ $x = 1, y = 2, z = 3$ 1

OR

Using elementary Row operations:

let: $A = IA$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ 1

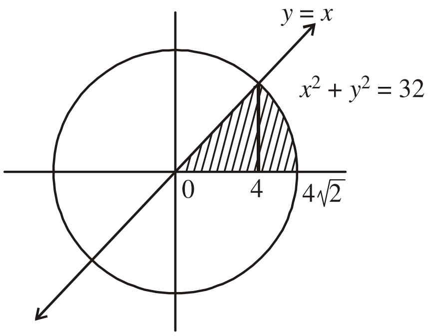
$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \quad \{\text{Using, } R_2 \to R_2 - 2R_1;\ R_3 \to R_3 + 2R_1$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \quad \{\text{Using, } R_1 \to R_1 - 2R_2$$ 4

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A \quad \{\text{Using, } R_1 \to R_1 - R_3;\ R_2 \to R_2 - R_3$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$ 1

26.



Correct figure: 1

Pt. of intersection, $x = 4$ 1

Area of shaded region $= \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$ 1

$$= \left.\frac{x^2}{2}\right]_0^4 + \left\{\frac{x}{2}\sqrt{32 - x^2} + 16\sin^{-1}\frac{x}{4\sqrt{2}}\right\}\Bigg]_4^{4\sqrt{2}}$$ 2

$$= 8 + 16\frac{\pi}{2} - 8 - 4\pi = 4\pi$$ 1

27. Put $\sin x - \cos x = t$, $(\cos x + \sin x)\, dx = dt$, $1 - \sin 2x = t^2$ 1

$$\left.\begin{array}{l}\text{when } x = 0, t = -1 \\ \text{and } \quad x = \pi/4, t = 0\end{array}\right\}$$ $\frac{1}{2}$

$$\therefore \quad I = \int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9\sin 2x}\, dx = \int_{-1}^{0} \frac{1}{16 + 9(1 - t^2)}\, dt = \int_{-1}^{0} \frac{1}{25 - 9t^2} dt$$ 2

$$\Rightarrow \quad I = \frac{1}{30} \log \left| \frac{5 + 3t}{5 - 3t} \right| \Bigg]_{-1}^{0}$$ $1\frac{1}{2}$

$$= \frac{1}{30}\left[0 - \log\frac{1}{4}\right] = -\frac{1}{30}\log\frac{1}{4} \text{ or } \frac{1}{15}\log 2$$ 1

OR

Here $f(x) = x^2 + 3x + e^x$, $a = 1$, $b = 3$, $nh = 2$ 1

$$\therefore \quad \int_1^3 (x^2 + 3x + e^x)\, dx = \lim_{h \to 0} [f(1) + f(1 + h) + \ldots + f(1 + \overline{n - 1}h)]$$ 1

$$= \lim_{h \to 0} \left[4(nh) + \frac{(nh - h)(nh)(2nh - h)}{6} + \frac{5(nh - h)(nh)}{2} + \frac{h}{e^h - 1} \times e \times (e^{nh} - 1) \right]$$ 3

$$= 8 + \frac{8}{3} + 10 + e(e^2 - 1) = \frac{62}{3} + e^3 - e$$ 1

28. General point on the line is: $(2 + 3\lambda, -1 + 4\lambda, 2 + 2\lambda)$ $1\frac{1}{2}$

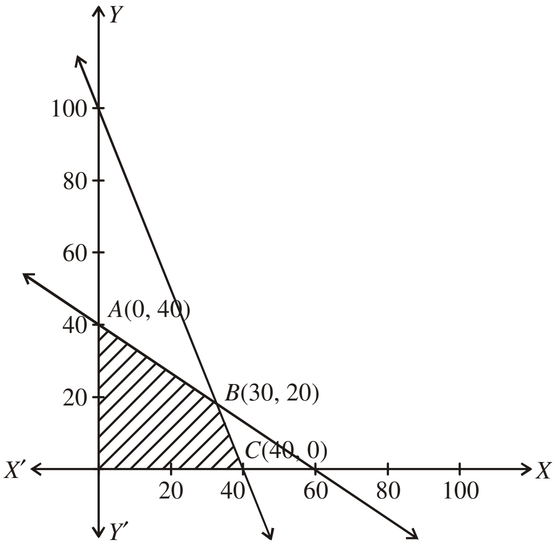
As the point lies on the plane

$\therefore$ $2 + 3\lambda + 1 - 4\lambda + 2 + 2\lambda = 5 \Rightarrow \lambda = 0$ $1\frac{1}{2}$

$\therefore$ Point is $(2, -1, 2)$ 1

Distance $= \sqrt{(2 - (-1))^2 + (-1 - (-5))^2 + (2 - (-10))^2} = 13$ 2

29.



Let number of packets of type $A = x$

and number of packets of type $B = y$

$\therefore$ L.P.P. is: Maximize, $Z = 0.7x + y$ 1

subject to constraints:

$$\left.\begin{array}{lll} 4x + 6y \le 240 & \text{or} & 2x + 3y \le 120 \\ 6x + 3y \le 240 & \text{or} & 2x + y \le 80 \end{array}\right\} \quad 2$$

$x \ge 0,\ y \ge 0$

Correct graph 2

$Z(0, 0) = 0$, $Z(0, 40) = 40$

$Z(40, 0) = 28$, $Z(30, 20) = 41$ (Max.)

$\therefore$ Max. profit is ₹ 41 at $x = 30$, $y = 20$. 1

QUESTION PAPER CODE 65/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. 5o10 = (5 * 10) + 3 = 10 + 3 = 13 For 5 * 10 = 10 $\frac{1}{2}$

For Final Answer = 13 $\frac{1}{2}$

2. $|\vec{a}| = |\vec{b}| = 3$ $\frac{1}{2}+\frac{1}{2}$

3. $a = -2,\ b = 3$ $\frac{1}{2}+\frac{1}{2}$

4. $\frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right) = -\frac{\pi}{2}$ $\frac{1}{2}+\frac{1}{2}$

Note: $\frac{1}{2}$ m. for any one of the two correct values and $\frac{1}{2}$ m. for final answer

SECTION B

5. Marginal cost = $C'(x) = 0.015x^2 - 0.04x + 30$ 1

At $x = 3$, $C'(3) = 30.015$ 1

6. $f(x) = \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right) = \tan^{-1}\left(\frac{2\cos^2 x/2}{2\sin x/2 \cos x/2}\right)$ 1

$= \tan^{-1}\left(\cot x/2\right) = \frac{\pi}{2} - \frac{x}{2}$ $\frac{1}{2}$

$\therefore\ f'(x) = -\frac{1}{2}$ $\frac{1}{2}$

7. $|A| = 2, \quad \therefore\ A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ 1

$\text{LHS} = 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}, \ \text{RHS} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ 1

$\therefore$ LHS = RHS

8. In RHS, put $x = \sin\theta$ $\frac{1}{2}$

RHS $= \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta)$ 1

$= 3\theta = 3\sin^{-1}x =$ LHS. $\frac{1}{2}$

9. A: Getting a sum of 8, B: Red die resulted in no. < 4

$P(A/B) = \dfrac{P(A \cap B)}{P(B)}$ 1

$= \dfrac{2/36}{18/36} = \dfrac{1}{9}$ 1

10. $\sin\theta = \dfrac{|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})|}{|\hat{i} - 2\hat{j} + 3\hat{k}|\,|3\hat{i} - 2\hat{j} + \hat{k}|}$ $\frac{1}{2}$

$|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})| = |4\hat{i} + 8\hat{j} + 4\hat{k}| = 4\sqrt{6}$ 1

$\sin\theta = \dfrac{4\sqrt{6}}{14} = \dfrac{2\sqrt{6}}{7}$ $\frac{1}{2}$

11. $\dfrac{dy}{dx} = bae^{bx+5} \Rightarrow \dfrac{dy}{dx} = by$ 1

$\Rightarrow \dfrac{d^2y}{dx^2} = b\dfrac{dy}{dx}$ $\frac{1}{2}$

$\therefore$ The differential equation is: $y\dfrac{d^2y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$ $\frac{1}{2}$

12. $I = \displaystyle\int \frac{1 - 2\sin^2 x + 2\sin^2 x}{\cos^2 x}\,dx$ $\frac{1}{2}$

$= \displaystyle\int \sec^2 x\,dx$ 1

$= \tan x + C$ $\frac{1}{2}$

SECTION C

13. $y = \sin(\sin x) \Rightarrow \dfrac{dy}{dx} = \cos(\sin x)\cdot\cos x$ 1

and $\dfrac{d^2y}{dx^2} = -\sin(\sin x)\cdot\cos^2 x - \sin x\cos(\sin x)$ 1+1

LHS = $-\sin(\sin x)\cos^2 x - \sin x\cos(\sin x) + \dfrac{\sin x}{\cos x}\cos(\sin x)\cos x + \sin(\sin x)\cos^2 x$ 1

= 0 = RHS

14. Separating the variables, we get:

$\int \dfrac{\sec^2 y}{\tan y}\,dy = \int \dfrac{e^x}{e^x - 2}\,dx$ $1\frac{1}{2}$

$\Rightarrow$ $\log|\tan y| = \log|e^x - 2| + \log C$ $1\frac{1}{2}$

$\Rightarrow$ $\tan y = C(e^x - 2)$, for $x = 0$, $y = \pi/4$, $C = -1$ $\frac{1}{2}$

$\therefore$ Particular solution is: $\tan y = 2 - e^x$. $\frac{1}{2}$

OR

Integrating factor = $e^{\int 2\tan x dx} = \sec^2 x$ 1

$\therefore$ Solution is: $y\cdot\sec^2 x = \int \sin x\cdot\sec^2 x\,dx = \int \sec x\cdot\tan x\,dx$ 1

$\Rightarrow$ $y\cdot\sec^2 x = \sec x + C$, for $x = \dfrac{\pi}{3}$, $y = 0$, $\therefore$ $C = -2$ $1+\frac{1}{2}$

$\therefore$ Particular solution is: $y\cdot\sec^2 x = \sec x - 2$ $\frac{1}{2}$

or $y = \cos x - 2\cos^2 x$

15. Here $\vec{a}_1 = 4\hat{i} - \hat{j}, \vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}, \vec{a}_2 - \vec{a}_1 = -3\hat{i} + 2\hat{k}$ 1

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i} - \hat{j}$$ 1

Shortest distance $= \dfrac{|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$ 1

$= \left|\dfrac{-6}{\sqrt{5}}\right| = \dfrac{6}{\sqrt{5}}$ or $\dfrac{6\sqrt{5}}{5}$ 1

16. Let X denote the larger of two numbers

X	2	3	4	5	$\frac{1}{2}$
$P(X)$	1/10	2/10	3/10	4/10	1
$X \cdot P(X)$	2/10	6/10	12/10	20/10	$\frac{1}{2}$
$X^2 \cdot P(X)$	4/10	18/10	48/10	100/10	$\frac{1}{2}$

Mean $= \Sigma X \cdot P(X) = \dfrac{40}{10} = 4$ $\frac{1}{2}$

Variance $= \Sigma X^2 \cdot P(X) - [\Sigma X \cdot P(X)]^2 = \dfrac{170}{10} - 4^2 = 1$ 1

17. LHS $= \begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix}$

$= \begin{vmatrix} 1 & 0 & 3x \\ 1+3y & -3y & -3y \\ 1 & 3z & 0 \end{vmatrix}$ (Using $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$) 1+1

(Any two relevant operations)

$= 1 \times (9yz) + 3x(3z + 9yz + 3y)$ (Expanding along R_1) 1

$= 9(3xyz + xy + yz + zx) =$ RHS 1

18. $x_1 = 2 \Rightarrow y_1 = 3 \quad (\because y_1 > 0)$ $\frac{1}{2}$

Differentiating the given equation, we get, $\frac{dy}{dx} = \frac{-16x}{9y}$ $\frac{1}{2}$

Slope of tangent at $(2, 3) = \left.\frac{dy}{dx}\right]_{(2,3)} = -\frac{32}{27}$ $\frac{1}{2}$

Slope of Normal at $(2, 3) = \frac{27}{32}$ $\frac{1}{2}$

Equation of tangent: $32x + 27y = 145$ 1

Equation of Normal: $27x - 32y = -42$ 1

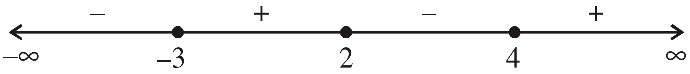
OR

$f'(x) = x^3 - 3x^2 - 10x + 24$ $\frac{1}{2}$

$= (x - 2)(x - 4)(x + 3)$ 1

$f'(x) = 0 \Rightarrow x = -3, 2, 4.$ $\frac{1}{2}$

sign of $f'(x)$:



$\therefore f(x)$ is strictly increasing on $(-3, 2) \cup (4, \infty)$ 1

and $f(x)$ is strictly decreasing on $(-\infty, -3) \cup (2, 4)$ 1

19. Put $\sin x = t \Rightarrow \cos x\, dx = dt$ $\frac{1}{2}$

Let $I = \int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}\, dx = \int \frac{2}{(1-t)(1+t^2)}\, dt$

Let $\frac{2}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2}$, solving we get

$A = 1, B = 1, C = 1$ $1\frac{1}{2}$

$\therefore I = \int \frac{1}{1-t}\, dt + \frac{1}{2}\int \frac{2t}{1+t^2} + \int \frac{1}{1+t^2}\, dt$

$= -\log|1 - t| + \frac{1}{2}\log|1 + t^2| + \tan^{-1} t + C$ $1\frac{1}{2}$

$= -\log(1 - \sin x) + \frac{1}{2}\log(1 + \sin^2 x) + \tan^{-1}(\sin x) + C$ $\frac{1}{2}$

20. E_1 : She gets 1 or 2 on die.

E_2 : She gets 3, 4, 5 or 6 on die. 1

A: She obtained exactly 1 tail

$P(E_1) = \frac{1}{3},\ P(E_2) = \frac{2}{3}$

$P(A/E_1) = \frac{3}{8},\ P(A/E_2) = \frac{1}{2}$ 1

$$P(E_2/A) = \frac{P(E_2)\cdot P(A/E_2)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$ 1

$$= \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}} = \frac{8}{11}$$ 1

21. $\vec{d} = \lambda(\vec{c}\times\vec{b}) = \lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 1 & -4 & 5 \end{vmatrix}$ 1

$\therefore\ \vec{d} = \lambda\hat{i} - 16\lambda\hat{j} - 13\lambda\hat{k}$ 1

$\vec{d}\cdot\vec{a} = 21 \Rightarrow 4\lambda - 80\lambda + 13\lambda = 21 \Rightarrow \lambda = -\frac{1}{3}$ 1

$\therefore\ \vec{d} = -\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}$ 1

22. Let side of base = x and depth of tank = y

$V = x^2 y \Rightarrow y = \frac{V}{x^2}$, ($V$ = Quantity of water = constant)

Cost of material is least when area of sheet used is minimum.

A(Surface area of tank) = $x^2 + 4xy = x^2 + \frac{4V}{x}$ $\frac{1}{2}+\frac{1}{2}$

$\frac{dA}{dx} = 2x - \frac{4V}{x^2},\ \frac{dA}{dx} = 0 \Rightarrow x^3 = 2V,\ y = \frac{x^3}{2x^2} = \frac{x}{2}$ $\frac{1}{2}+\frac{1}{2}$

$\frac{d^2A}{dx^2} = 2 + \frac{8V}{x^3} > 0$, $\therefore$ Area is minimum, thus cost is minimum when $y = \frac{x}{2}$ $\frac{1}{2}+\frac{1}{2}$

Value: Any relevant value. 1

23. Differentiating with respect to 'x'

$2(x^2+y^2)\left(2x+2y\dfrac{dy}{dx}\right)=x\dfrac{dy}{dx}+y$ 2

$\Rightarrow \quad \dfrac{dy}{dx}=\dfrac{y-4x^3-4xy^2}{4x^2y+4y^3-x}$ 2

OR

$\dfrac{dx}{d\theta}=a(2-2\cos 2\theta)=4a\sin^2\theta$ 1

$\dfrac{dy}{d\theta}=2a\sin 2\theta=4a\sin\theta\cdot\cos\theta$ 1

$\therefore \quad \dfrac{dy}{dx}=\dfrac{4a\sin\theta\cos\theta}{4a\sin^2\theta}=\cot\theta$ 1

$\left.\dfrac{dy}{dx}\right]_{\theta=\frac{\pi}{3}}=\dfrac{1}{\sqrt{3}}$ 1

SECTION D

24. Put $\sin x-\cos x=t$, $(\cos x+\sin x)\,dx=dt$, $1-\sin 2x=t^2$ 1

$\left.\begin{array}{ll}\text{when} & x=0, t=-1\\ \text{and} & x=\pi/4, t=0\end{array}\right\}$ $\frac{1}{2}$

$\therefore \quad I=\displaystyle\int_0^{\pi/4}\frac{\sin x+\cos x}{16+9\sin 2x}\,dx=\int_{-1}^{0}\frac{1}{16+9(1-t^2)}\,dt=\int_{-1}^{0}\frac{1}{25-9t^2}dt$ 2

$\Rightarrow \quad I=\dfrac{1}{30}\log\left|\dfrac{5+3t}{5-3t}\right|\Bigg]_{-1}^{0}$ $1\frac{1}{2}$

$=\dfrac{1}{30}\left[0-\log\dfrac{1}{4}\right]=-\dfrac{1}{30}\log\dfrac{1}{4}$ or $\dfrac{1}{15}\log 2$ 1

OR

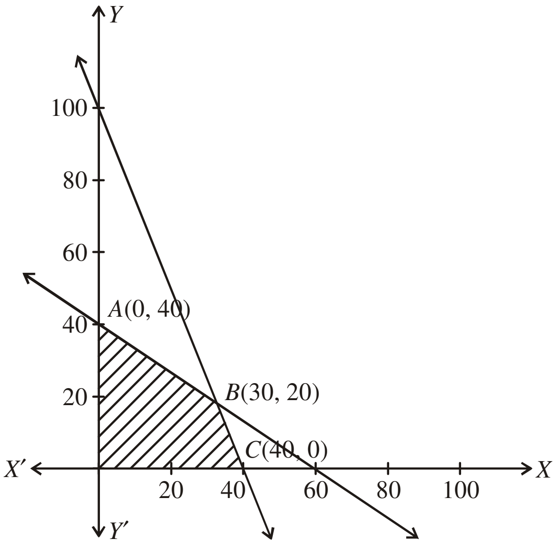
Here $f(x)=x^2+3x+e^x$, $a=1$, $b=3$, $nh=2$ 1

$\therefore \quad \displaystyle\int_1^3(x^2+3x+e^x)\,dx=\lim_{h\to 0}[f(1)+f(1+h)+\ldots+f(1+\overline{n-1}h)]$ 1

$=\displaystyle\lim_{h\to 0}\left[4(nh)+\frac{(nh-h)(nh)(2nh-h)}{6}+\frac{5(nh-h)(nh)}{2}+\frac{h}{e^h-1}\times e\times(e^{nh}-1)\right]$ 3

$=8+\dfrac{8}{3}+10+e(e^2-1)=\dfrac{62}{3}+e^3-e$ 1

25.



Let number of packets of type $A = x$

and number of packets of type $B = y$

$\therefore$ L.P.P. is: Maximize, $Z = 0.7x + y$ 1

subject to constraints:

$4x + 6y \le 240$ or $2x + 3y \le 120$

$6x + 3y \le 240$ or $2x + y \le 80$ 2

$x \ge 0,\ y \ge 0$

Correct graph 2

$Z(0, 0) = 0,\ Z(0, 40) = 40$

$Z(40, 0) = 28,\ Z(30, 20) = 41$ (Max.)

$\therefore$ Max. profit is ₹ 41 at $x = 30,\ y = 20$. 1

26. Reflexive: $|a - a| = 0$, which is divisible by 4, $\forall\ a \in A$ 1

$\therefore\quad (a, a) \in R, \forall\ a \in A\quad \therefore\ R$ is reflexive

Symmetric: let $(a, b) \in R$

$\Rightarrow\quad |a - b|$ is divisible by 4

$\Rightarrow\quad |b - a|$ is divisible by 4 ($\because\ |a - b| = |b - a|$)

$\Rightarrow\quad (b, a) \in R\quad \therefore\ R$ is symmetric. 1

Transitive: let $(a, b), (b, c) \in R$

$\Rightarrow\quad |a - b|$ & $|b - c|$ are divisible by 4

$\Rightarrow\quad a - b = \pm 4m,\ b - c = \pm 4n,\ m, n \in Z$

Adding we get, $a - c = 4(\pm m \pm n)$

$\Rightarrow\quad (a - c)$ is divisible by 4

$\Rightarrow\quad |a - c|$ is divisible by 4 $\quad \therefore\ (a, c) \in R$ 2

$\therefore\ R$ is transitive

Hence R is an equivalence relation in A 1

set of elements related to 1 is $\{1, 5, 9\}$

and $[2] = \{2, 6, 10\}$. 1

OR

Here $f(2) = f\left(\frac{1}{2}\right) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$

$\therefore$ f is not 1-1 2

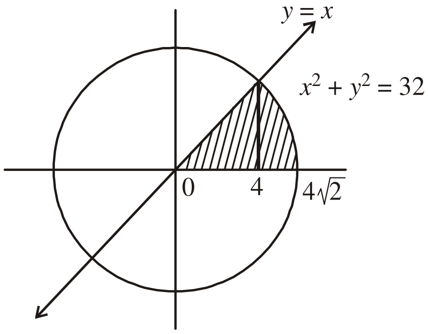
for $y = \frac{1}{\sqrt{2}}$ let $f(x) = \frac{1}{\sqrt{2}} \Rightarrow x^2 - \sqrt{2}x + 1 = 0$

As $D = (-\sqrt{2})^2 - 4(1)(1) < 0$, $\therefore$ No real solution

$\therefore$ $f(x) \neq \frac{1}{\sqrt{2}}$, for any $x \in R(D_f)$ $\therefore f$ is not onto 2

$$\text{fog }(x) = f(2x-1) = \frac{2x-1}{(2x-1)^2+1} = \frac{2x-1}{4x^2-4x+2}$$ 2

27.



Correct figure: 1

Pt. of intersection, $x = 4$ 1

Area of shaded region $= \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32-x^2}\,dx$ 1

$$= \left.\frac{x^2}{2}\right]_0^4 + \left[\left\{\frac{x}{2}\sqrt{32-x^2} + 16\sin^{-1}\frac{x}{4\sqrt{2}}\right\}\right]_4^{4\sqrt{2}}$$ 2

$$= 8 + 16\frac{\pi}{2} - 8 - 4\pi = 4\pi$$ 1

28. $|A| = -1 \neq 0$ $\therefore A^{-1}$ exists 1

Co-factors of A are:

$A_{11} = 0$; $A_{12} = 2$; $A_{13} = 1$

$A_{21} = -1$; $A_{22} = -9$; $A_{23} = -5$

$A_{31} = 2$; $A_{32} = 23$; $A_{33} = 13$

1 m for any 4 correct cofactors 2

$$\text{adj}(A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{|A|}\cdot\text{adj}(A) = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$ $\frac{1}{2}$

For : $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$, the system of equation is $A \cdot X = B$ $\frac{1}{2}$

$$\therefore \quad X = A^{-1} \cdot B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ 1

$\therefore \quad x = 1, y = 2, z = 3$ 1

OR

Using elementary Row operations:

let: $A = IA$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ 1

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \quad \{\text{Using, } R_2 \to R_2 - 2R_1;\ R_3 \to R_3 + 2R_1$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \quad \{\text{Using, } R_1 \to R_1 - 2R_2$$ 4

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A \quad \{\text{Using, } R_1 \to R_1 - R_3;\ R_2 \to R_2 - R_3$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$ 1

29. General point on the line is: $(2 + 3\lambda, -1 + 4\lambda, 2 + 2\lambda)$ $1\frac{1}{2}$

As the point lies on the plane

$\therefore 2 + 3\lambda + 1 - 4\lambda + 2 + 2\lambda = 5 \Rightarrow \lambda = 0$ $1\frac{1}{2}$

$\therefore$ Point is $(2, -1, 2)$ 1

Distance $= \sqrt{(2-(-1))^2 + (-1-(-5))^2 + (2-(-10))^2} = 13$ 2

QUESTION PAPER CODE 65/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|\vec{a}| = |\vec{b}| = 3$ $\quad \frac{1}{2}+\frac{1}{2}$

2. $\frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right) = -\frac{\pi}{2}$ $\quad \frac{1}{2}+\frac{1}{2}$

Note: $\frac{1}{2}$m. for any one of the two correct values and $\frac{1}{2}$m. for final answer

3. 5o10 = (5 * 10) + 3 = 10 + 3 = 13 — For 5 * 10 = 10 $\quad \frac{1}{2}$

For Final Answer = 13 $\quad \frac{1}{2}$

4. $a = -2$, $b = 3$ $\quad \frac{1}{2}+\frac{1}{2}$

SECTION B

5. A: Getting a sum of 8, B: Red die resulted in no. < 4

$P(A/B) = \frac{P(A \cap B)}{P(B)}$ $\quad 1$

$= \frac{2/36}{18/36} = \frac{1}{9}$ $\quad 1$

6. $\sin\theta = \frac{|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})|}{|\hat{i} - 2\hat{j} + 3\hat{k}|\,|3\hat{i} - 2\hat{j} + \hat{k}|}$ $\quad \frac{1}{2}$

$|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})| = |4\hat{i} + 8\hat{j} + 4\hat{k}| = 4\sqrt{6}$ $\quad 1$

$\sin\theta = \frac{4\sqrt{6}}{14} = \frac{2\sqrt{6}}{7}$ $\quad \frac{1}{2}$

7. $\frac{dy}{dx} = bae^{bx+5} \Rightarrow \frac{dy}{dx} = by$ 1

$\Rightarrow \frac{d^2y}{dx^2} = b\frac{dy}{dx}$ $\frac{1}{2}$

$\therefore$ The differential equation is: $y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ $\frac{1}{2}$

8. $I = \int \frac{1-2\sin^2 x + 2\sin^2 x}{\cos^2 x}\,dx$ $\frac{1}{2}$

$= \int \sec^2 x\,dx$ 1

$= \tan x + C$ $\frac{1}{2}$

9. Marginal cost = $C'(x) = 0.015x^2 - 0.04x + 30$ 1

At $x = 3$, $C'(3) = 30.015$ 1

10. $f(x) = \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right) = \tan^{-1}\left(\frac{2\cos^2 x/2}{2\sin x/2 \cos x/2}\right)$ 1

$= \tan^{-1}\left(\cot x/2\right) = \frac{\pi}{2} - \frac{x}{2}$ $\frac{1}{2}$

$\therefore\ f'(x) = -\frac{1}{2}$ $\frac{1}{2}$

11. $|A| = 2, \quad \therefore\ A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ 1

$\text{LHS} = 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}, \ \text{RHS} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ 1

$\therefore$ LHS = RHS

12. In RHS, put $x = \sin\theta$ $\frac{1}{2}$

RHS $= \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta)$ 1

$= 3\theta = 3\sin^{-1} x$ = LHS. $\frac{1}{2}$

SECTION C

13. Let X denote the larger of two numbers

X	2	3	4	5	$\frac{1}{2}$
$P(X)$	1/10	2/10	3/10	4/10	1
$X \cdot P(X)$	2/10	6/10	12/10	20/10	$\frac{1}{2}$
$X^2 \cdot P(X)$	4/10	18/10	48/10	100/10	$\frac{1}{2}$

Mean = $\Sigma X \cdot P(X) = \dfrac{40}{10} = 4$ $\frac{1}{2}$

Variance = $\Sigma X^2 \cdot P(X) - [\Sigma X \cdot P(X)]^2 = \dfrac{170}{10} - 4^2 = 1$ 1

14. Let side of base = x and depth of tank = y

$V = x^2 y \Rightarrow y = \dfrac{V}{x^2}$, ($V$ = Quantity of water = constant)

Cost of material is least when area of sheet used is minimum.

A(Surface area of tank) = $x^2 + 4xy = x^2 + \dfrac{4V}{x}$ $\frac{1}{2}+\frac{1}{2}$

$\dfrac{dA}{dx} = 2x - \dfrac{4V}{x^2}, \dfrac{dA}{dx} = 0 \Rightarrow x^3 = 2V,\ y = \dfrac{x^3}{2x^2} = \dfrac{x}{2}$ $\frac{1}{2}+\frac{1}{2}$

$\dfrac{d^2A}{dx^2} = 2 + \dfrac{8V}{x^3} > 0$, $\therefore$ Area is minimum, thus cost is minimum when $y = \dfrac{x}{2}$ $\frac{1}{2}+\frac{1}{2}$

Value: Any relevant value. 1

15. $x_1 = 2 \Rightarrow y_1 = 3 \quad (\because\ y_1 > 0)$ $\frac{1}{2}$

Differentiating the given equation, we get, $\dfrac{dy}{dx} = \dfrac{-16x}{9y}$ $\frac{1}{2}$

Slope of tangent at $(2, 3) = \left.\dfrac{dy}{dx}\right]_{(2,3)} = -\dfrac{32}{27}$ $\frac{1}{2}$

Slope of Normal at $(2, 3) = \dfrac{27}{32}$ $\frac{1}{2}$

Equation of tangent: $32x + 27y = 145$ 1

Equation of Normal: $27x - 32y = -42$ 1

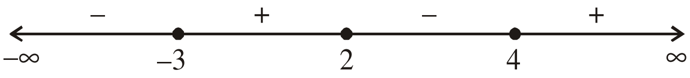
OR

$f'(x) = x^3 - 3x^2 - 10x + 24$ $\frac{1}{2}$

$= (x-2)(x-4)(x+3)$ 1

$f'(x) = 0 \Rightarrow x = -3, 2, 4.$ $\frac{1}{2}$

sign of $f'(x)$:



$\therefore f(x)$ is strictly increasing on $(-3, 2) \cup (4, \infty)$ 1

and $f(x)$ is strictly decreasing on $(-\infty, -3) \cup (2, 4)$ 1

16. Differentiating with respect to 'x'

$$2(x^2 + y^2)\left(2x + 2y\frac{dy}{dx}\right) = x\frac{dy}{dx} + y$$ 2

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y - 4x^3 - 4xy^2}{4x^2y + 4y^3 - x}$$ 2

OR

$$\frac{dx}{d\theta} = a(2 - 2\cos 2\theta) = 4a\sin^2\theta$$ 1

$$\frac{dy}{d\theta} = 2a\sin 2\theta = 4a\sin\theta\cdot\cos\theta$$ 1

$$\therefore \quad \frac{dy}{dx} = \frac{4a\sin\theta\cos\theta}{4a\sin^2\theta} = \cot\theta$$ 1

$$\left.\frac{dy}{dx}\right]_{\theta = \frac{\pi}{3}} = \frac{1}{\sqrt{3}}$$ 1

17. $y = \sin(\sin x) \Rightarrow \frac{dy}{dx} = \cos(\sin x)\cdot\cos x$ 1

and $\frac{d^2y}{dx^2} = -\sin(\sin x)\cdot\cos^2 x - \sin x\cos(\sin x)$ 1+1

$\text{LHS} = -\sin(\sin x)\cos^2 x - \sin x\cos(\sin x) + \frac{\sin x}{\cos x}\cos(\sin x)\cos x + \sin(\sin x)\cos^2 x$ 1

$= 0 = \text{RHS}$

18. Separating the variables, we get:

$$\int \frac{\sec^2 y}{\tan y}\, dy = \int \frac{e^x}{e^x - 2}\, dx \qquad 1\frac{1}{2}$$

$\Rightarrow$ $\log|\tan y| = \log|e^x - 2| + \log C$ $\quad 1\frac{1}{2}$

$\Rightarrow$ $\tan y = C(e^x - 2)$, for $x = 0$, $y = \pi/4$, $C = -1$ $\quad \frac{1}{2}$

$\therefore$ Particular solution is: $\tan y = 2 - e^x$. $\quad \frac{1}{2}$

OR

Integrating factor = $e^{\int 2\tan x dx} = \sec^2 x$ $\quad 1$

$\therefore$ Solution is: $y \cdot \sec^2 x = \int \sin x \cdot \sec^2 x\, dx = \int \sec x \cdot \tan x\, dx$ $\quad 1$

$\Rightarrow$ $y \cdot \sec^2 x = \sec x + C$, for $x = \frac{\pi}{3}$, $y = 0$, $\therefore C = -2$ $\quad 1+\frac{1}{2}$

$\therefore$ Particular solution is: $y \cdot \sec^2 x = \sec x - 2$ $\quad \frac{1}{2}$

or $y = \cos x - 2\cos^2 x$

19. Here $\vec{a}_1 = 4\hat{i} - \hat{j}, \vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}, \vec{a}_2 - \vec{a}_1 = -3\hat{i} + 2\hat{k}$ $\quad 1$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i} - \hat{j} \qquad 1$$

$$\text{Shortest distance} = \frac{|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \qquad 1$$

$$= \left|\frac{-6}{\sqrt{5}}\right| = \frac{6}{\sqrt{5}} \text{ or } \frac{6\sqrt{5}}{5} \qquad 1$$

20. Put $\sin x = t \Rightarrow \cos x \, dx = dt$ $\frac{1}{2}$

$$\text{Let } I = \int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)} dx = \int \frac{2}{(1-t)(1+t^2)} dt$$

Let $\frac{2}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2}$, solving we get

$A = 1, B = 1, C = 1$ $1\frac{1}{2}$

$$\therefore I = \int \frac{1}{1-t} dt + \frac{1}{2}\int \frac{2t}{1+t^2} + \int \frac{1}{1+t^2} dt$$

$= -\log|1-t| + \frac{1}{2}\log|1+t^2| + \tan^{-1} t + C$ $1\frac{1}{2}$

$= -\log(1-\sin x) + \frac{1}{2}\log(1+\sin^2 x) + \tan^{-1}(\sin x) + C$ $\frac{1}{2}$

21. E_1 : She gets 1 or 2 on die.

E_2 : She gets 3, 4, 5 or 6 on die.

A: She obtained exactly 1 tail 1

$P(E_1) = \frac{1}{3}$, $P(E_2) = \frac{2}{3}$

$P(A/E_1) = \frac{3}{8}$, $P(A/E_2) = \frac{1}{2}$ 1

$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$ 1

$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}} = \frac{8}{11}$ 1

22. $\vec{d} = \lambda(\vec{c} \times \vec{b}) = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 1 & -4 & 5 \end{vmatrix}$ 1

$\therefore\ \vec{d} = \lambda\hat{i} - 16\lambda\hat{j} - 13\lambda\hat{k}$ 1

$\vec{d} \cdot \vec{a} = 21 \Rightarrow 4\lambda - 80\lambda + 13\lambda = 21 \Rightarrow \lambda = -\frac{1}{3}$ 1

$\therefore\ \vec{d} = -\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}$ 1

23. LHS $= \begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix}$

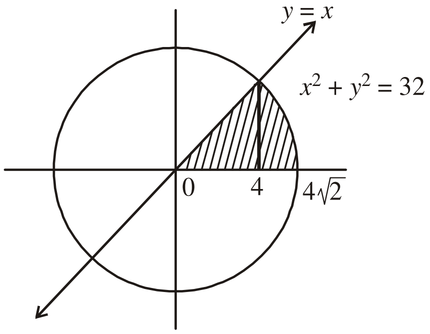
$= \begin{vmatrix} 1 & 0 & 3x \\ 1+3y & -3y & -3y \\ 1 & 3z & 0 \end{vmatrix}$ (Using $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$) 1+1 (Any two relevant operations)

$= 1 \times (9yz) + 3x(3z + 9yz + 3y)$ (Expanding along R_1) 1

$= 9(3xyz + xy + yz + zx) =$ RHS 1

SECTION D

24.



Correct figure: 1

Pt. of intersection, $x = 4$ 1

Area of shaded region $= \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$ 1

$= \left.\frac{x^2}{2}\right]_0^4 + \left.\left\{\frac{x}{2}\sqrt{32-x^2} + 16\sin^{-1}\frac{x}{4\sqrt{2}}\right\}\right]_4^{4\sqrt{2}}$ 2

$= 8 + 16\frac{\pi}{2} - 8 - 4\pi = 4\pi$ 1

25. Reflexive: $|a - a| = 0$, which is divisible by 4, $\forall \ a \in A$ 1

$\therefore \quad (a, a) \in R, \forall a \in A \ \therefore \ R$ is reflexive

Symmetric: let $(a, b) \in R$

$\Rightarrow \quad |a - b|$ is divisible by 4

$\Rightarrow \quad |b - a|$ is divisible by 4 ($\because |a - b| = |b - a|$)

$\Rightarrow \quad (b, a) \in R \quad \therefore R$ is symmetric. 1

Transitive: let $(a, b), (b, c) \in R$

$\Rightarrow \quad |a - b|$ & $|b - c|$ are divisible by 4

$\Rightarrow \quad a - b = \pm 4m, \ b - c = \pm 4n, \ m, n \in Z$

Adding we get, $a - c = 4(\pm m \pm n)$ 2

$\Rightarrow \quad (a - c)$ is divisible by 4

$\Rightarrow \quad |a - c|$ is divisible by 4 $\quad \therefore (a, c) \in R$

$\therefore R$ is transitive

Hence R is an equivalence relation in A 1

set of elements related to 1 is $\{1, 5, 9\}$

and $[2] = \{2, 6, 10\}$. 1

OR

Here $f(2) = f\left(\frac{1}{2}\right) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$

$\therefore \quad f$ is not 1-1 2

for $y = \frac{1}{\sqrt{2}}$ let $f(x) = \frac{1}{\sqrt{2}} \Rightarrow x^2 - \sqrt{2}x + 1 = 0$

As $\quad D = (-\sqrt{2})^2 - 4(1)(1) < 0, \ \therefore$ No real solution

$\therefore \quad f(x) \neq \frac{1}{\sqrt{2}}$, for any $x \in R(D_f) \ \therefore f$ is not onto 2

$$\text{fog } (x) = f(2x - 1) = \frac{2x-1}{(2x-1)^2+1} = \frac{2x-1}{4x^2-4x+2}$$ 2

26. General point on the line is: $(2 + 3\lambda, -1 + 4\lambda, 2 + 2\lambda)$ $1\frac{1}{2}$

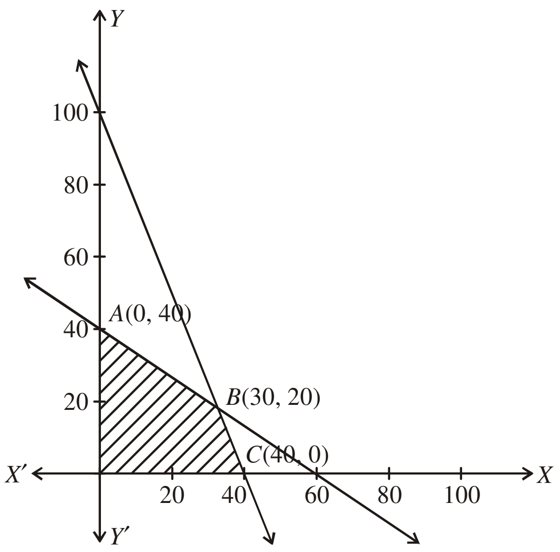
As the point lies on the plane

$\therefore\ 2 + 3\lambda + 1 - 4\lambda + 2 + 2\lambda = 5 \Rightarrow \lambda = 0$ $1\frac{1}{2}$

$\therefore$ Point is (2, –1, 2) 1

Distance = $\sqrt{(2-(-1))^2+(-1-(-5))^2+(2-(-10))^2} = 13$ 2

27.



Let number of packets of type $A = x$

and number of packets of type $B = y$

$\therefore$ L.P.P. is: Maximize, $Z = 0.7x + y$ 1

subject to constraints:

$$\left.\begin{array}{l} 4x + 6y \le 240 \quad \text{or} \quad 2x + 3y \le 120 \\ 6x + 3y \le 240 \quad \text{or} \quad 2x + y \le 80 \end{array}\right\}$$ 2

$x \ge 0,\ y \ge 0$

Correct graph 2

$Z(0, 0) = 0,\ Z(0, 40) = 40$

$Z(40, 0) = 28,\ Z(30, 20) = 41$ (Max.)

$\therefore$ Max. profit is ₹ 41 at $x = 30,\ y = 20$. 1

28. Put $\sin x - \cos x = t$, $(\cos x + \sin x)\, dx = dt$, $1 - \sin 2x = t^2$ 1

$$\left.\begin{array}{ll} \text{when} & x = 0, t = -1 \\ \text{and} & x = \pi/4, t = 0 \end{array}\right\}$$ $\frac{1}{2}$

$$\therefore \quad I = \int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9\sin 2x}\, dx = \int_{-1}^{0} \frac{1}{16 + 9(1 - t^2)}\, dt = \int_{-1}^{0} \frac{1}{25 - 9t^2} dt$$ 2

$$\Rightarrow \quad I = \frac{1}{30} \log \left| \frac{5 + 3t}{5 - 3t} \right| \Bigg]_{-1}^{0}$$ $1\frac{1}{2}$

$$= \frac{1}{30}\left[0 - \log\frac{1}{4}\right] = -\frac{1}{30}\log\frac{1}{4} \text{ or } \frac{1}{15}\log 2$$ 1

OR

Here $f(x) = x^2 + 3x + e^x$, $a = 1$, $b = 3$, $nh = 2$ 1

$$\therefore \int_1^3 (x^2+3x+e^x)\,dx = \lim_{h\to 0}[f(1)+f(1+h)+...+f(1+\overline{n-1}h)]$$ 1

$$= \lim_{h\to 0}\left[4(nh)+\frac{(nh-h)(nh)(2nh-h)}{6}+\frac{5(nh-h)(nh)}{2}+\frac{h}{e^h-1}\times e\times(e^{nh}-1)\right]$$ 3

$$= 8+\frac{8}{3}+10+e(e^2-1)=\frac{62}{3}+e^3-e$$ 1

29. $|A| = -1 \neq 0 \quad \therefore A^{-1}$ exists 1

Co-factors of A are:

$A_{11} = 0$; $A_{12} = 2$; $A_{13} = 1$

$A_{21} = -1$; $A_{22} = -9$; $A_{23} = -5$

$A_{31} = 2$; $A_{32} = 23$; $A_{33} = 13$

1 m for any 4 correct cofactors 2

$$\text{adj}(A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{|A|}\cdot\text{adj}(A) = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$ $\frac{1}{2}$

For : $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$, the system of equation is $A \cdot X = B$ $\frac{1}{2}$

$$\therefore \quad X = A^{-1}\cdot B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ 1

$\therefore \quad x = 1, y = 2, z = 3$ 1

OR

Using elementary Row operations:

let: $A = IA$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ 1

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \quad \{\text{Using, } R_2 \to R_2 - 2R_1;\ R_3 \to R_3 + 2R_1$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \quad \{\text{Using, } R_1 \to R_1 - 2R_2$$ 4

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A \quad \{\text{Using, } R_1 \to R_1 - R_3;\ R_2 \to R_2 - R_3$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$ 1