SOLUTION/ ANSWER KEY OF PRACTICE PAPER -I

CLASS XII MATHEMATICS

2019-20

Q NO	VALUE POINTS
1	(D) square matrix
2	(C) $A^2 - B^2 + BA - AB$
3	(D)8
4	(C) 1/2
5	(D) (α,β,-γ)
6	(B)$\frac{2}{5}$
7	(B) 9/10
8	(C)$\tan x - \cot x + c$
9	(A) (2,0,0)
10	(D)$\frac{2}{\sqrt{29}}$units
11	R= {(3,8),(6,6).(9,4),(12,2)}
12	a=2
13	X+y=0 OR $(-\infty, -1)$
14	y=2
15	$\left(\frac{\vec{a}.\vec{b}}{\lvert\vec{b}\rvert^2}\right)\vec{b}$ OR 2/3,2/3,-1/3
16	-Interchanging rows and column we get Δ= $\begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$ Taking (-1) common from R_1,R_2,R_3we get Δ=$(-1)^3 \begin{vmatrix} 0 & b-a & c-a \\ a-b & 0 & c-b \\ a-c & b-c & 0 \end{vmatrix}$=- Δ therefore 2Δ=0 ∴ Δ= 0
17	x$\log x - x + c$
18	4 OR

	xtan$\frac{x}{2}$+c
19	$e^x \cos x + c$
20	yx= $\frac{x^2}{2}$+c
21	Let $\vec{c}$ denote the sum of $\vec{a}$&$\vec{b}$ we have $\vec{c}$=($2\hat{\imath}$ -$\hat{\jmath}$ +2$\hat{k}$)+(-$\hat{\imath}$ +$\hat{\jmath}$ +3$\hat{k}$) =$\hat{\imath}$+5$\hat{k}$ now $\left\lvert\hat{\vec{c}}\right\rvert$=$\sqrt{1^2+2^2}$=$\sqrt{26}$ Required unit vector is $\hat{c}$=$\frac{\vec{c}}{\lvert\vec{c}\rvert} = \frac{1}{\sqrt{26}}(\hat{\imath}+5\hat{k})= \frac{1}{\sqrt{26}}\hat{\imath}+\frac{5}{26}\hat{k}$ OR P(2,3,0) and Q(-1,-2,-4) $\overrightarrow{PQ}$=(-1,-2)$\hat{\imath}$+(-2,-3)$\hat{\jmath}$+(-4,-0)$\hat{k}$ = -3$\hat{\imath}$-5$\hat{\jmath}$-4$\hat{k}$ ∴ Vector joining P and Q given by $\overrightarrow{PQ}$=-3$\hat{\imath}$-5$\hat{\jmath}$-4$\hat{k}$
22	a=1/2 not reflexive 1/2≤1/2 so R is not Reflexive A=9, b=4, c=2 , not transitive OR $cos\left[\sin^{-1}\frac{1}{4}+\sec^{-1}\frac{4}{3}\right]$= $cos\left[\sin^{-1}\frac{1}{4}+\cos^{-1}\frac{4}{3}\right]$ =cos (sin$^{-1}\frac{1}{4}$) cos (cos$^{-1}\frac{3}{4}$)-sin sin$^{-1}\frac{1}{4}$ sin cos$^{-1}\frac{3}{4}$ =$\frac{3}{4}\sqrt{1-(\frac{1}{4})^2}$-$\frac{1}{4}\sqrt{1-(\frac{3}{4})^2}$ =$\frac{3\sqrt{15}-\sqrt{7}}{16}$
23	y=$e^{a\cos^{-1}x} \Rightarrow \frac{dy}{dx} = e^{a\cos^{-1}x}$(-a)$\frac{-a}{\sqrt{1-x^2}}$ Therefore ,$\sqrt{1-x^2}\,\frac{dy}{dx}$=-ay------(1) Differentiating again w.r.t x , we get $\sqrt{1-x^2}\,\frac{d^2y}{dx^2} - \frac{-x}{\sqrt{1-x^2}}\frac{dy}{dx} = -a\frac{dy}{dx}$ $\Rightarrow (1-x^2)\ \frac{d^2y}{dx^2}$-x$\frac{dy}{dx}$=-a$\sqrt{1-x^2}\,\frac{dy}{dx}$ =-a(-ay) hence (1-x^2)$\frac{d^2y}{dx^2}$ -x$\frac{dy}{dx}$-a^2y=0
24	Given, diameter of the balloon= $\frac{3}{2}$(2x+1) ∴Radius of the balloon= $\frac{Diameter}{2}$ = $\frac{1}{2}$[$\frac{3}{2}$(2x+1)] = $\frac{3}{4}$(2x+1) For the volume V, the balloon is given by V= $\frac{4}{3}\pi$(radius)^3= $\frac{4}{3}\pi[\frac{3}{4}(2x+1)]^3$ = $\frac{9\pi}{16}(2x+1)^3$ For the rate of change of volume, differentiate w.r.t x, we get

	$\frac{dV}{dx}=\frac{9\pi}{16}\times 3(2x+1)^2\times 2=\frac{27\pi}{8}(2x+1)^2$ Thus, the rate of change of volume is $\frac{27\pi}{8}(2x+1)^2$.
25	Given equations of lines are $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$ here direction ratios of two lines are (2,2,1) and (4,1,8) Let θ be the acute angle between the given lines, then $\cos\theta=\frac{\lvert a_1a_2+b_1b_2+c_1c_2\rvert}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$ $\cos\theta=\frac{\lvert 2\times4+2\times1+1\times8\rvert}{\sqrt{2^2+2^2+1^2}\sqrt{4^2+1^2+8^2}}$ $=\frac{\lvert 8+2+8\rvert}{\sqrt{4+4+1}\sqrt{16+1+64}}$ $=\frac{18}{\sqrt{9}\sqrt{81}}$ $=\frac{18}{3\times9}=\frac{2}{3}\Rightarrow\theta=cos^{-1}\left(\frac{2}{3}\right)$
26	Given n=6 and p= $\frac{Number\ of\ odd\ number\ in\ one\ die}{Total\ number\ in\ one\ die}=\frac{3}{6}=\frac{1}{2}$ $\therefore q=1-p=1-\frac{1}{2}$ So . P(getting a 5 success in six trials)= P(X=5)=$6c_5p^5q^1=1x(\frac{1}{2})^5(\frac{1}{2})^1$=1/64
27	(i) Reflexive: ∀ a ∈ A, |a-a|= 0 which is even ⇒ (a,a) ∈ R, hence R is reflexive. (ii) Symmetric: Let (a,b) ∈ R ⇒|a-b| is even ⇒ |-(b-a)| is even ⇒|(b-a)| is even So, (b,a) ∈ R Hence, R is symmetric. (iii) Transitive: Let (a,b), (b,c) ∈ R So, |a-b| is even and |b-c| is even ⇒a-b=2λ, b-c=2μ where λ, μ ∈ Z Now, a-c= (a-b)+(b-c)=2(λ+μ) ⇒ |a-c| is even, hence R is transitive. Since R is reflexive, symmetric, transitive Therefore, it is an equivalence relation.

28	$\frac{dy}{dx}=\frac{x+y}{x-y}=\frac{1+\frac{y}{x}}{1-\frac{y}{x}}$ Put y/x = v so that $\frac{dy}{dx}=v+x\frac{dv}{dx}$ $\therefore\ v+x\frac{dv}{dx}=\frac{1+v}{1-v}\ \Rightarrow\ x\frac{dv}{dx}=\frac{1+v}{1-v}-v=\frac{1+v-v+v^2}{1-v}$ $\Rightarrow \int\frac{1-v}{1+v^2}dv=\int\frac{dx}{x}\ \Rightarrow\int\frac{1}{1+v^2}dv-\frac{1}{2}\int\frac{2v}{1+v^2}dv=\int\frac{dx}{x}$ $\Rightarrow\ \tan^{-1}v=\frac{1}{2}\log\left\|1+v^2\right\|+\log\|x\|+c$ $\Rightarrow\ \tan^{-1}\left(\frac{y}{x}\right)=\frac{1}{2}\log\left\|\frac{x^2+y^2}{x^2}\right\|+\log\|x\|+c$ or $\tan^{-1}\left(\frac{y}{x}\right)=\frac{1}{2}\log\|x^2+y^2\|+c$ OR $(1+x^2)dy+2xy\ dx=\cot x.dx.$ $\Rightarrow\ \frac{dy}{dx}+\frac{2x}{1+x^2}\cdot y=\frac{\cot x}{1+x^2}$ I.F. $=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=(1+x^2)$ $\therefore$ Solution is, $y\cdot(1+x^2)=\int\cot x\ dx\|=\log\|\sin x\|+c$ or $y=\frac{1}{1+x^2}\cdot\log\|\sin x\|+\frac{c}{1+x^2}$
29	$\Rightarrow\ \frac{dy}{dx}\cdot\log(\cos x)+y(-\tan x)=\log(\sin y)+x\cdot\cot y\cdot\frac{dy}{dx}$ $\Rightarrow\ \frac{dy}{dx}=\frac{y\cdot\tan x+\log(\sin y)}{\log(\cos x)-x\cot y}$

30	$\text{RHS} = \int_a^b f(a+b-x)\,dx = -\int_b^a f(t)\,dt$, where $a+b-x=t,\ dx=-dt$ $= \int_a^b f(t)\,dt = \int_a^b f(x)\,dx = \text{LHS}$ Let $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$ ...(i) $= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}}\,dx = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ ...(ii) adding (i) and (ii) to get $2I = \int_{\pi/6}^{\pi/3} 1\cdot dx = x]_{\pi/6}^{\pi/3} = \pi/6.$ $\Rightarrow I = \frac{\pi}{12}$
31	For the first die : P(6)=1/2 , P(6′)=1/2 i.e., P(6′)=P(1)+P(2)+P(3)+P(4)+P(5)=1/2 ⇒ P(1)=1/10, P(1′)=9/10 [$\because P(1)=P(2)=P(3)=P(4)=P(5)$] For the second die: P(1)=2/5 , P(1′)=3/5 Let X: number of ones seen $\therefore X = 0,1,2$ P(X=0)=P(not 1 from 1st die).P(not 1 from 2nd die)= $\frac{9}{10}\times\frac{3}{5}=\frac{27}{50}=0.54$ P(X=1)= P(1 from 1st die) P(not 1 from 2nd die)+ P(not 1 from 1st die) P(1 from 2nd die) $=\frac{1}{10}\times\frac{3}{5}+\frac{9}{10}\times\frac{2}{5}=\frac{21}{50}=0.42$ P(X=2)= P(1 from 1st die)P(1 from 2nd die)=$\frac{1}{10}\times\frac{2}{5}=\frac{2}{50}=0.04$ The table for probability distribution is shown as below: X: 0, 1, 2 P(X): 0.54, 0.42, 0.04

X	0	1	2
P(X)	0.54	0.42	0.04

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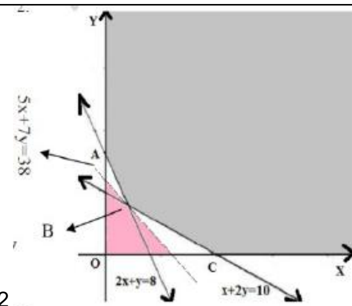

Let x kg of food 1 be mixed with y kg of food 2

To minimize Z = ₹ (50x + 70y)

Subject to the constraints :

$2x + y \geq 8,\ x + 2y \geq 10, x \geq 0, y \geq 0$

Corner Points	Value of Z (in ₹)
A(0, 8)	560
B(2, 4)	380 ← Min. value
C(10, 0)	500

Since feasible region is unbounded so, 380 may or may not be minimum value of z.

To check, draw $50x + 70y < 380$ i.e., $5x + 7y < 38$.

As in the half plane $5x + 7y < 38$, there is no point common with the feasible region.

Hence minimum value of Z is ₹ 380.

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$$\text{Area of ellipse} = 4\left(\frac{b}{a}\int_0^a \sqrt{a^2 - x^2}\, dx\right)$$

$$= 4\left|\left(\frac{b}{a}\left(\frac{x}{2}\sqrt{(a^2 - x^2)} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right)\right)\right|_0^a$$

$$= 4\frac{b}{a}\left(\frac{\pi a^2}{4}\right)$$

$$= \pi ab$$

OR

$a = 1,\ b = 3,\ nh = 2$

$$\int_3^3 (x^2 + x + e^x)\, dx = \lim_{h \to 0} h(f(1) + f(1 + h) + \ldots + f(1 + (n - 1)h)$$

$$= \lim_{h \to 0} = h(2 + e + (1 + h)^2 + (1 + h) + e^{1 + h} + \ldots + (1 + (n - 1)h)^2 + (1 + (n - 1)h) + e^{1 + (n - 1)h})$$

$$= \lim_{h \to 0} = h(2 + e + 2 + 3h + h^2 + e^{1 + h} + \ldots + (1)^2 + (n - 1)^2 h^2 + 2(n - 1)h + 1 + (n - 1)h + e^{1 + (n - 1)h})$$

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$A^3 - 6A^2 + 5A + 11I = O$, Pre-multiplying by A^{-1}

$$\Rightarrow A^2 - 6A + 5I + 11A^{-1} = O \Rightarrow A^{-1} = -\frac{1}{11}(A^2 - 6A + 5I)$$

$$\therefore A^{-1} = \begin{bmatrix} -3/11 & 4/11 & 5/11 \\ 9/11 & -1/11 & -4/11 \\ 5/11 & -3/11 & -1/11 \end{bmatrix}$$

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As the d.r.'s of parallel lines are proportional so, the equation of line passing through (2, 3, 2) and parallel to $\vec{r} = (-2\hat{i}+3\hat{j}) + \lambda(2\hat{i}-3\hat{j}+6\hat{k})$ is : $\vec{r} = (2\hat{i}+3\hat{j}+2\hat{k}) + \lambda(2\hat{i}-3\hat{j}+6\hat{k})$.

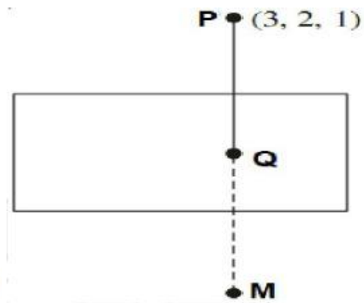
Now $\vec{a}_1 = -2\hat{i}+3\hat{j}$, $\vec{a}_2 = 2\hat{i}+3\hat{j}+2\hat{k}$, $\vec{b} = 2\hat{i}-3\hat{j}+6\hat{k}$

$$\Rightarrow \vec{a}_2 - \vec{a}_1 = (2\hat{i}+3\hat{j}+2\hat{k}) - (-2\hat{i}+3\hat{j}) = 4\hat{i}+2\hat{k} \text{ and } (\vec{a}_2-\vec{a}_1)\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 2 \\ 2 & -3 & 6 \end{vmatrix} = 6\hat{i}-20\hat{j}-12\hat{k}$$

$$\therefore \text{S.D.} = \frac{\left|(\vec{a}_2-\vec{a}_1)\times\vec{b}\right|}{\left|\vec{b}\right|}$$

$$\Rightarrow \quad = \frac{\left|6\hat{i}-20\hat{j}-12\hat{k}\right|}{\left|2\hat{i}-3\hat{j}+6\hat{k}\right|} = \frac{\sqrt{36+400+144}}{\sqrt{4+9+36}} = \frac{\sqrt{580}}{7} \text{ Units}.$$

OR



The d.r.'s of normal to the plane are 2, –1, 1.

Since PQ is perpendicular to the plane so, its equation is

$$\frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1} = \lambda.$$

The coordinates of any random point on the line PQ :

$Q(2\lambda+3, -\lambda+2, \lambda+1)$.

$\because$ Q lies on the plane so, $2(2\lambda+3) - (-\lambda+2) + (\lambda+1) + 1 = 0$

$\Rightarrow 6\lambda + 6 = 0 \quad \therefore \lambda = -1$

$\therefore$ Foot of perpendicular : $Q(1, 3, 0)$.

Distance PQ $\sqrt{(3-1)^2+(2-3)^2+(1-0)^2} = \sqrt{6}$ Units.

Let $M(\alpha, \beta, \gamma)$ be the image of P in the plane.

So, Q will be mid-point of PM.

That is, $Q(1,3,0) = Q\left(\frac{\alpha+3}{2}, \frac{\beta+2}{2}, \frac{\gamma+1}{2}\right)$

On comparing the coordinates, we get: $\alpha = -1$, $\beta = 4$, $\gamma = -1$.

Therefore, the Image is M (–1, 4, –1).

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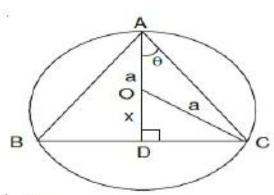

$CD = \sqrt{a^2-x^2}$

Area, $A = \frac{1}{2}\times 2\sqrt{a^2-x^2}\,(a+x)$

$Z = A^2 = (a-x)(a+x)^3$

$\frac{dZ}{dx} = 2(a+x)^2(a-2x)$

$\frac{dZ}{dx} = 0 \Rightarrow x = \frac{a}{2}$

$\frac{d^2Z}{dx^2} = -12(a+x)x$

$\left(\frac{d^2Z}{dx^2}\right)_{x=\frac{a}{2}} = -9a^2 < 0$

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	$\therefore$ Z is maximum when $x = \frac{a}{2}$ i.e., Area is maximum when $x = \frac{a}{2}$ For maximum area $\tan\theta = \frac{CD}{AD} = \frac{\sqrt{a^2 - \frac{a^2}{4}}}{a + \frac{a}{2}} = \frac{1}{\sqrt{3}}$ $\theta = \frac{\pi}{6}$