Functional data analysis of stocks during COVID-19

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Vilnius University - Functional Data Analyses project

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Goal

- To study the variability of returns of stocks belonging to different industries during COVID-19 period.
- To study stocks and industries that were affected by COVID-19 and the ones that were not
- To predict the returns by macro-economic events

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Data Presentation

- Stocks data from 2020/01/01 until 2022/12/31
- 8 main industrial sectors were considered:

- automobile
- fashion and clothing
- food and beverage
- healthcare
- tech
- logistic
- oil and gas
- travel and tourism

- For each sector 8 stocks were considered
- The closing price for each end of week was considered
- Total of 157 weeks

The stocks considered

- The main stocks were:
  - **Automobile**: VW - Ferrari - Stellantis - Renault - Mercedes - BMW - Tesla - Toyota
  - **Fashion and clothing**: Kering - Capri - Hermès - LVMH - Richemont - Adidas - Nike - Puma
  - **Food and beverage**: Nestlè - Uniliver - Danone - Bonduelle - Pepsi - McDonalds - Kellogs
  - **Healthcare**: Sanofi - Novartis - Bayer - AstraZeneca - UCB - Merck - Argenx - GSK
  - **Tech**: Spotify - Netflix - Nvidia - Meta - Apple - IBM - Microsoft - Google
  - **Logistic**: Zalando - UPS - Amazon - DHL - FedEx - Maersk - Walmart - SF express
  - **Oil and Gas**: Shell - Eni - Enel - Engie - Orsted - Chevron - Repsol
  - **Travel and tourism**: Trivago - Booking - Ryanair - Lyft - Trip.com - Tripadvisor - Hilton - Uber

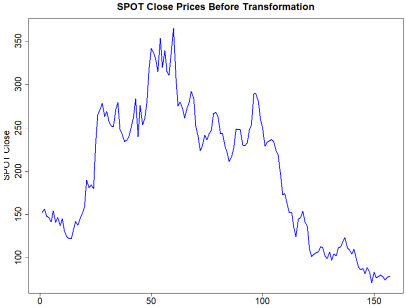
Data transformation

Logarithmic return:

$$R = 100 * \log\left(\frac{P_t}{P_{t-1}}\right)$$

- R is the log return
- P_t is the stock price at time t
- P_{t-1} is the stock price at the previous time period

Before vs. After Transformations



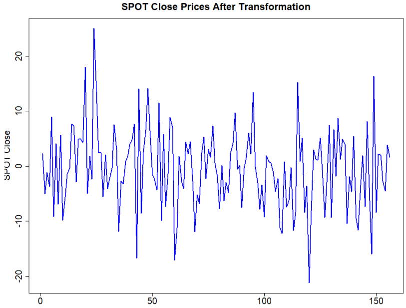


Figure: Before vs. After Transformations.

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Euclidean depth

- Euclidean depth of the deepest point in the dataset: 0.00002917722

```
> st[mdE,]
     ZAL.DE.Close        UPS.Close       AMZN.Close       DHL.DE.Close
         47.25000        207.17999        146.81750           42.44000
        FDX.Close      AMKBY.Close        WMT.Close   X002352.SZ.Close
        219.28000         15.27000         47.54333           53.69000
       SPOT.Close       NFLX.Close       NVDA.Close         META.Close
        148.91000        380.14999         25.93100          231.84000
       AAPL.Close        IBM.Close       MSFT.Close         GOOG.Close
        175.06000        128.89000        310.88000          141.06300
    VOW3.DE.Close    RACE.MI.Close   STLAM.MI.Close       RNO.PA.Close
        151.00000        177.89999         14.15600           22.99000
     MBG.DE.Close     BMW.DE.Close       TSLA.Close           TM.Close
         62.14000         75.32000        267.29666          165.92999
     KER.PA.Close       CPRI.Close     RMS.PA.Close        MC.PA.Close
        557.20001         50.16000       1113.50000          590.00000
     CFR.SW.Close     ADS.DE.Close     NKE.DE.Close       PUM.DE.Close
        104.00000        203.70000        108.44000           71.86000
     SAN.PA.Close    NOVN.SW.Close    BAYN.DE.Close          AZN.Close
         93.46938         75.47266         55.94000           61.37000
     UCB.BR.Close     MRK.DE.Close       ARGX.Close        GSK.L.Close
        100.50000        179.20000        286.47000         1581.55164
    NESN.SW.Close         UL.Close      BN.PA.Close       BON.PA.Close
        115.74000         44.39000         52.76000           17.18000
        PEP.Close        MCD.Close          K.Close          KHC.Close
        159.00000        232.57001         57.57747           37.82000
       SHEL.Close     ENI.MI.Close    ENEL.MI.Close      ENGI.PA.Close
         50.35000         12.92400          5.82600           11.53400
  ORSTED.CO.Close      CHV.F.Close     REP.MC.Close       TTE.PA.Close
        829.59998        144.72000         11.46800           45.75500
     SHEL.Close.1   ENI.MI.Close.1  ENEL.MI.Close.1    ENGI.PA.Close.1
         50.35000         12.92400          5.82600           11.53400
ORSTED.CO.Close.1    CHV.F.Close.1   REP.MC.Close.1     TTE.PA.Close.1
        829.59998        144.72000         11.46800           45.75500
```

```
> apply(st,2,median)
     ZAL.DE.Close        UPS.Close       AMZN.Close       DHL.DE.Close
         66.55000        175.50500        155.11150           40.22500
        FDX.Close      AMKBY.Close        WMT.Close   X002352.SZ.Close
        229.71000         11.60500         46.21500           61.58500
       SPOT.Close       NFLX.Close       NVDA.Close         META.Close
        223.00000        480.54001         14.61325          249.07000
       AAPL.Close        IBM.Close       MSFT.Close         GOOG.Close
        135.38000        128.15134        245.14500          104.76375
    VOW3.DE.Close    RACE.MI.Close   STLAM.MI.Close       RNO.PA.Close
        151.89000        177.42500         13.50200           30.13000
     MBG.DE.Close     BMW.DE.Close       TSLA.Close           TM.Close
         57.85313         76.27000        225.39667          153.31499
     KER.PA.Close       CPRI.Close     RMS.PA.Close        MC.PA.Close
        564.75000         46.55000       1066.25000          601.10001
     CFR.SW.Close     ADS.DE.Close     NKE.DE.Close       PUM.DE.Close
         97.14000        256.25000        110.13000           77.19500
     SAN.PA.Close    NOVN.SW.Close    BAYN.DE.Close          AZN.Close
         86.79584         76.67636         53.35000           56.31500
     UCB.BR.Close     MRK.DE.Close       ARGX.Close        GSK.L.Close
         87.80000        155.32500        293.18500         1503.27991
    NESN.SW.Close         UL.Close      BN.PA.Close       BON.PA.Close
        109.47158         54.25500         56.19500           20.20000
        PEP.Close        MCD.Close          K.Close          KHC.Close
        148.05500        233.28500         60.92958           36.13000
       SHEL.Close     ENI.MI.Close    ENEL.MI.Close      ENGI.PA.Close
         41.60500         10.52400          7.22800           12.13100
  ORSTED.CO.Close      CHV.F.Close     REP.MC.Close       TTE.PA.Close
        842.39999         89.35000         10.63039           39.74750
     SHEL.Close.1   ENI.MI.Close.1  ENEL.MI.Close.1    ENGI.PA.Close.1
         41.60500         10.52400          7.22800           12.13100
ORSTED.CO.Close.1    CHV.F.Close.1   REP.MC.Close.1     TTE.PA.Close.1
        842.39999         89.35000         10.63039           39.74750
```

Comparison of the deepest point vs. columns-wise medians

- some values (e.g., UPS.Close, MSFT.Close, GOOG.Close) in the deepest point differ significantly from the median. This may suggest that the dataset contain outliers affecting the distribution

Euclidean depth

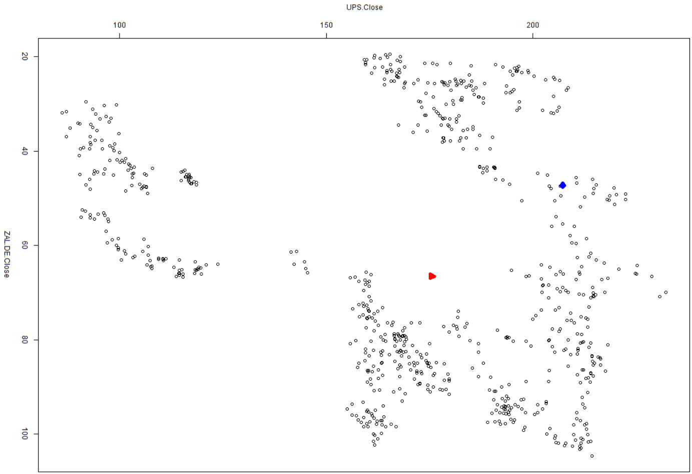

Figure: Comparison between the deepest point(in blue) and the median point(in red) in a scatter plot

Minimum Volume Depth (MBD)

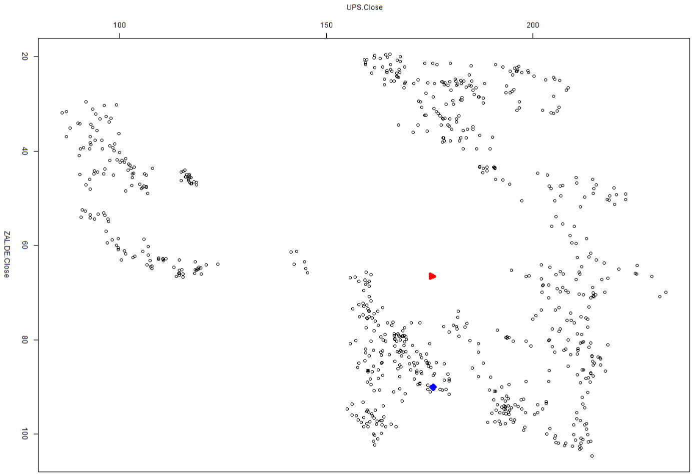

Figure: Comparison between the deepest point(in blue) and the median point(in red) in a scatter plot

Frainman-Muniz depth

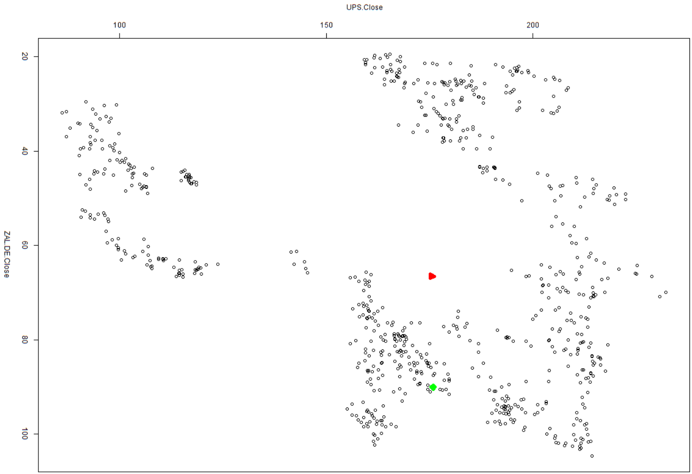

Figure: Comparison between the deepest point(in green) and the median point(in red) in a scatter plot

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Smoothing methods

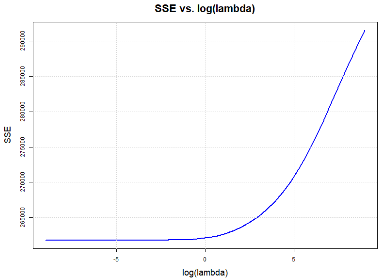
Methods:

- B-splines
- Local regression kernel
- Nadaraya-Watson kernel
- Normal kernel
- Triweight kernel
- Uniform kernel

B-splines results

Smoothing with penalty:

- Generalized cross-validation
    - Optimal lambda $\sim$ 371
    - Number of basis $\sim$ 7
    - GCV $\sim$ 2230
    - SSE $\sim$ 307923



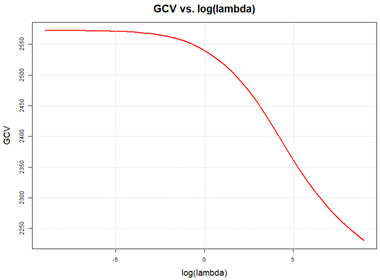


Figure: Comparison of SSE and GCV with respect to lambda.

Kernel smoothing results

Generalized cross-validation

- Sequence of bandwidth between 3 and 70

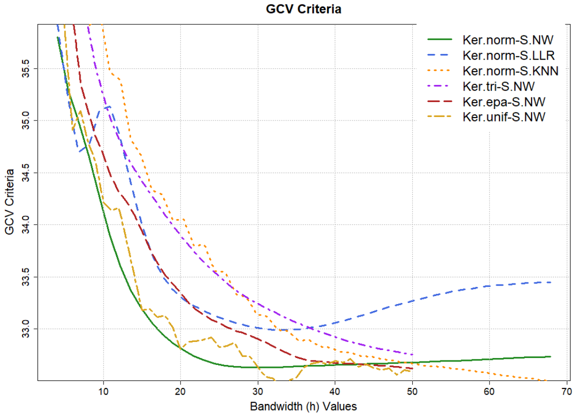


Figure: GCV criteria respect to bandwidth

Comparing optimal GCV and SSE

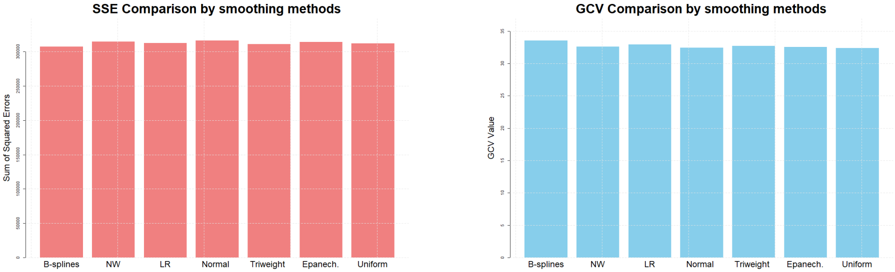
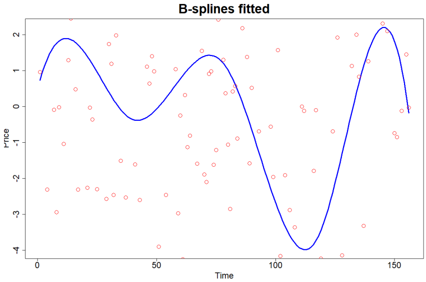


Figure: Comparison of optimal GCV and SSE among smoothing methods.

B-splines vs. Normal kernel



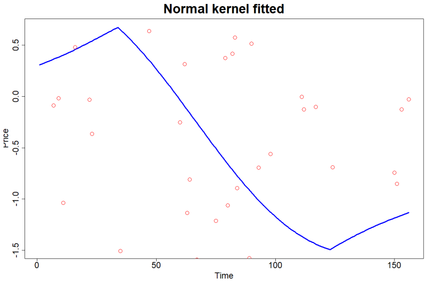


Figure: Comparison of fitted and actual values for B-splines and Normal kernel.

B-splines vs. Triweight kernel

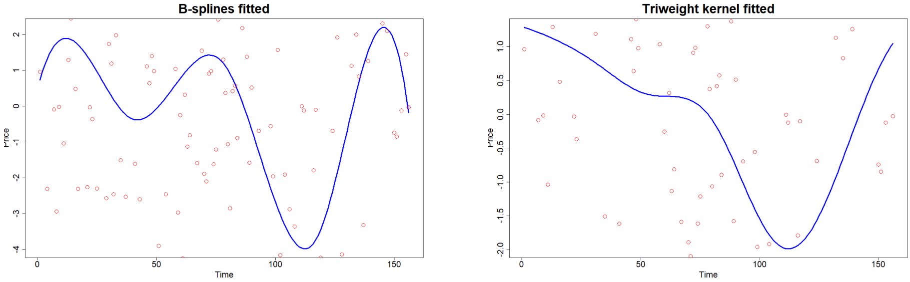


Figure: Comparison of optimal GCV and SSE among smoothing methods.

B-splines fitted

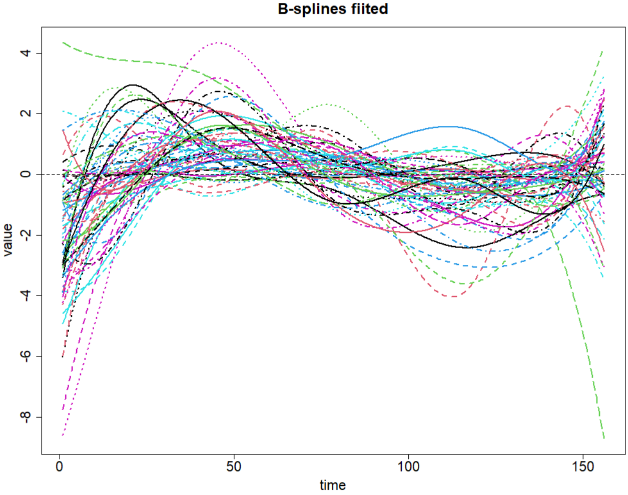


Figure: B-splines fitted for all stocks

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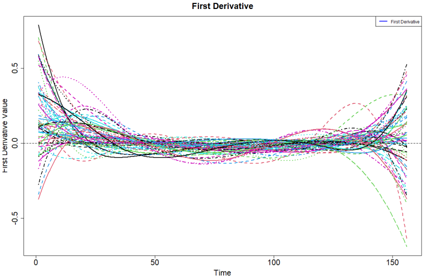
Derivatives



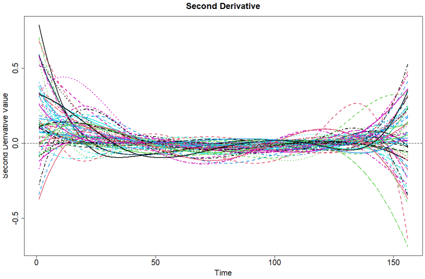


Figure: 1st and 2nd derivatives of B-splines smoothed

Outliers detection

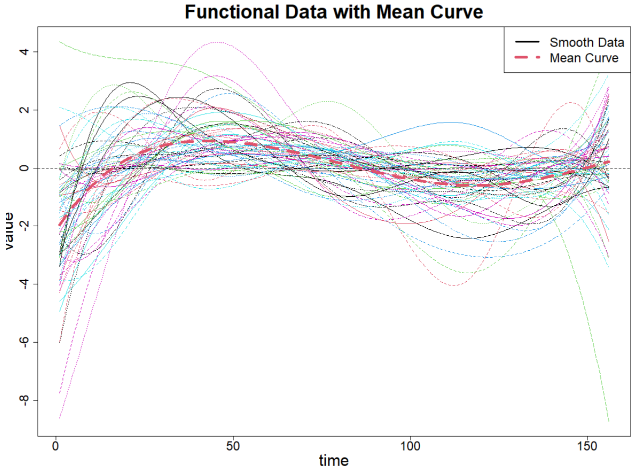


Figure: Mean curve

Euclidean depth

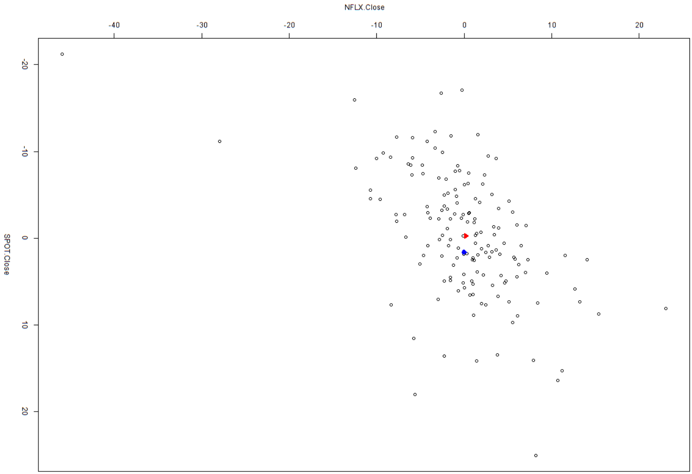
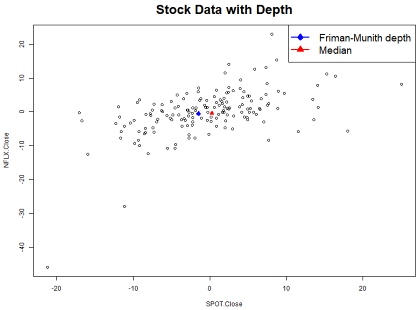


Figure: Comparison between the deepest point(in blue) and the median point(in red) in a scatter plot

Frainman-Muniz and The Most Central Point



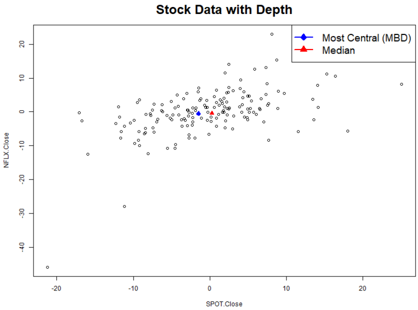


Figure: Friman-Munith depth and MBD

Bivariate Covariance Function

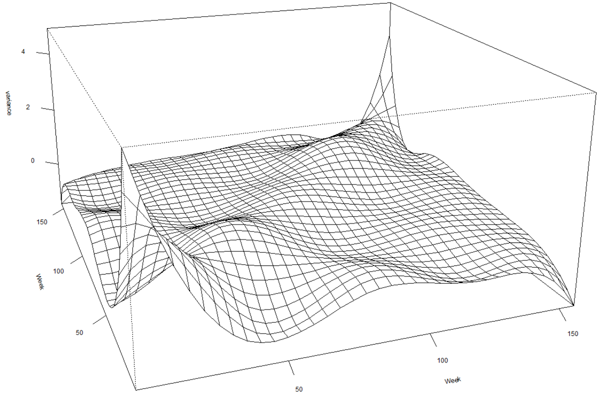


Figure: 3D visualization of the variance function for smoothed stock returns

Contour plot

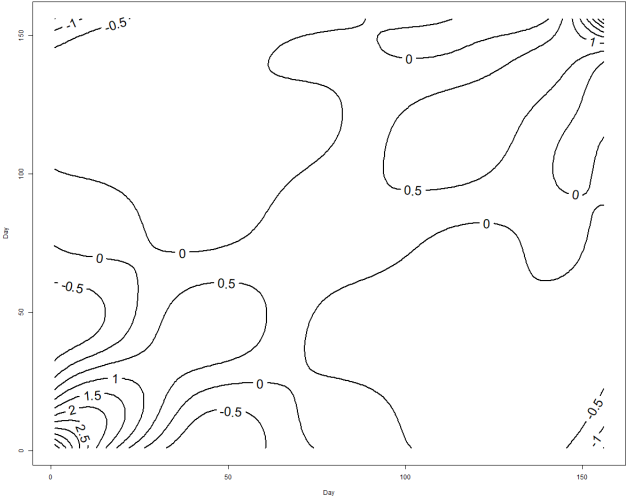


Figure: Contour plot

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Principal Component Analysis

FPCA Component	**Percentage Contribution**
PCA Function 1	34%
PCA Function 2	24%
PCA Function 3	15%
PCA Function 4	9%
Total	**82%**

Table: FPCA Component Contributions

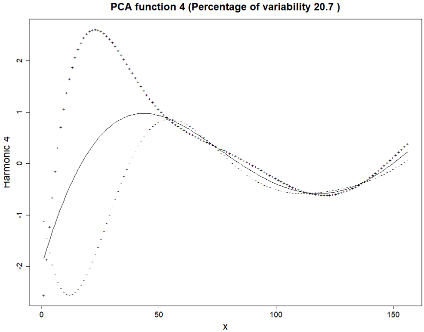
VARIMAX Rotation

PCA Function (Varimax Rotation)	**Percentage Contribution**
PCA Function 1	27.8%
PCA Function 2	18.3%
PCA Function 3	13.0%
PCA Function 4	20.7%
Total	**79.8%**

Table: PCA Function Contributions after Varimax Rotation

VARIMAX Rotation

- PCA 4 captured 20.7% and showed strong variability from the mean among stocks till the 70th week(could explain starting phase)
- PCA 1 may capture the overall market variability across different phases in response to specific events possibly COVID-19 related.



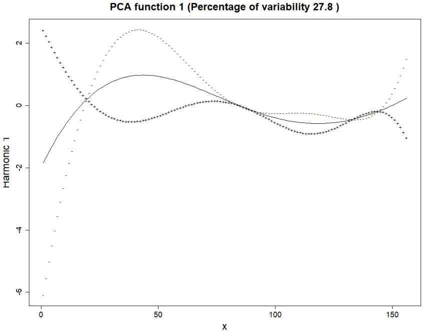


Figure: VARIMAX rotation of FPCA 1 and FPCA 4

VARIMAX Rotation

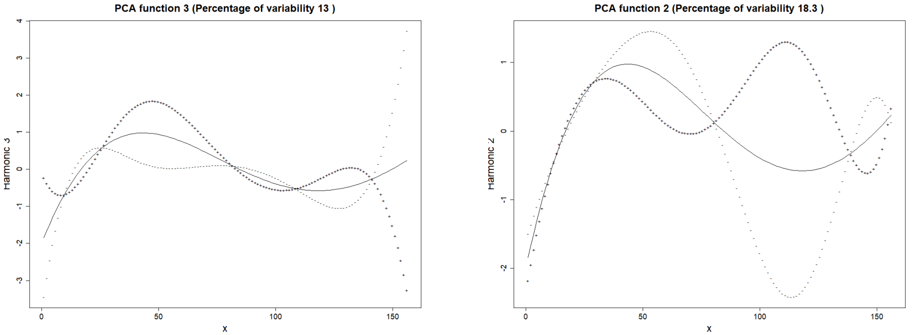


Figure: VARIMAX rotation of FPCA 2 and FPCA 3

- Volatility in stock prices in beginning, mid and end of pandemic
- PCA Functions captured the most volatility in different time periods

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The choice of the best number of clusters

```
>> K = 2
Error in .fstep(fd, T, lambda) : One cluster is almost empty!
>> K = 3
AkjBk   :        bic = -350051.4
>> K = 4
AkjBk   :        bic = -318623.1
>> K = 5
AkjBk   :        bic = -198106.2
>> K = 6
Error in .fstep(fd, T, lambda) : One cluster is almost empty!
>> K = 7
Error in .fstep(fd, T, lambda) : One cluster is almost empty!
>> K = 8
Error in .fstep(fd, T, lambda) : One cluster is almost empty!
>> K = 9
Error in .fstep(fd, T, lambda) : One cluster is almost empty!
>> K = 10
Error in .fstep(fd, T, lambda) : One cluster is almost empty!
The best model is AkjBk with K = 5 ( bic = -198106.2 )
```

	K	model	bic	aic	icl	nbprm	ll
1	2	AkjBk	NA	NA	NA	NA	NA
2	3	AkjBk	-350051.4	-350021.2	-350051.4	28	-349993.2
3	4	AkjBk	-318623.1	-318573.5	-318623.1	46	-318527.5
4	5	AkjBk	-198106.2	-198033.9	-198106.2	67	-197966.9
5	6	AkjBk	NA	NA	NA	NA	NA
6	7	AkjBk	NA	NA	NA	NA	NA
7	8	AkjBk	NA	NA	NA	NA	NA
8	9	AkjBk	NA	NA	NA	NA	NA
9	10	AkjBk	NA	NA	NA	NA	NA

Figure: The choice of the best number of clusters to consider

- The best clustering model selected is AkjBk with $K = 5$ clusters, based on the highest BIC score (-198106.2)
- Models with $K > 5$ resulted in empty clusters.

From original data to clusters

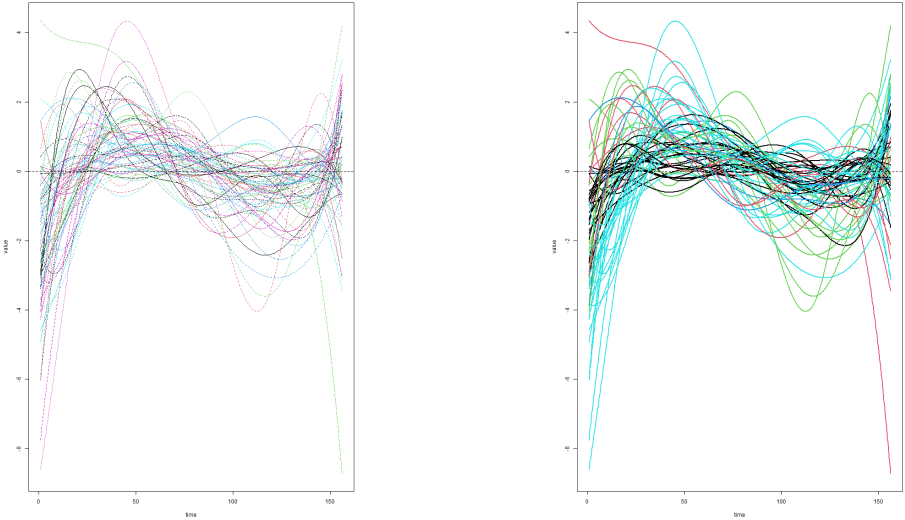
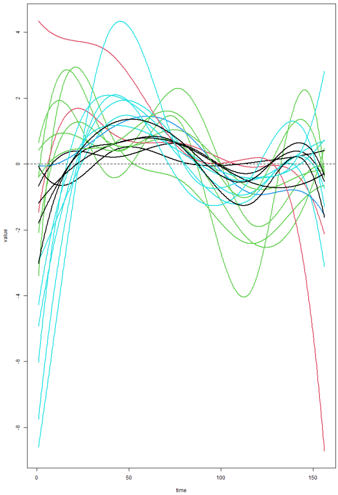


Figure: Comparison between original smoothed data and clusters

Centroids



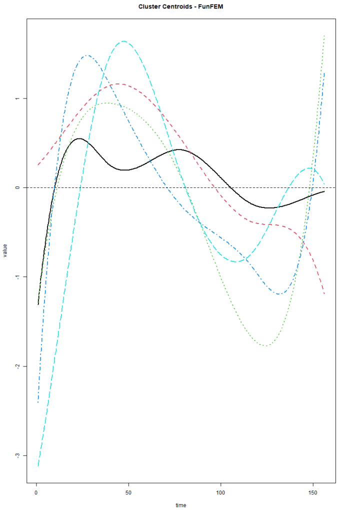
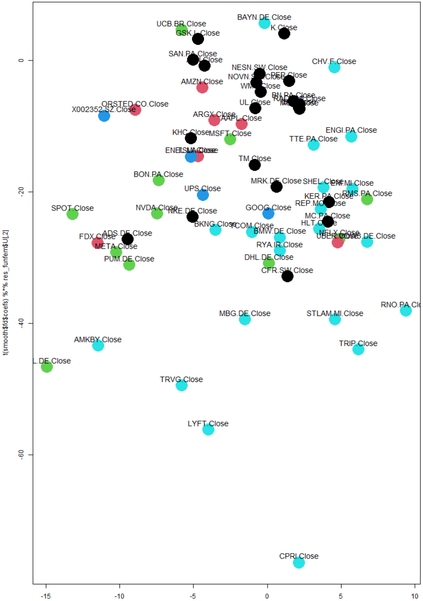
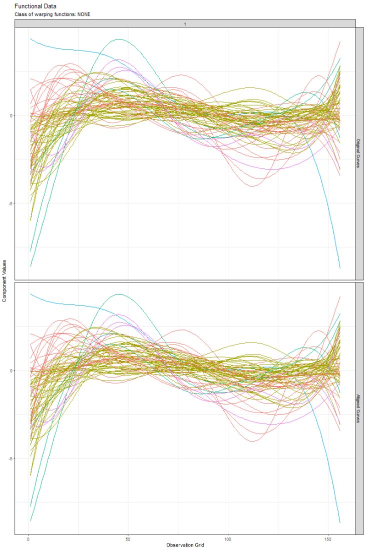


Figure: A more in-depth analyses comparing the first 20 curves(on the left) and the mean of each cluster(on the right)

Discriminative Space Plot



Hierarchical Clustering (HCLUST)



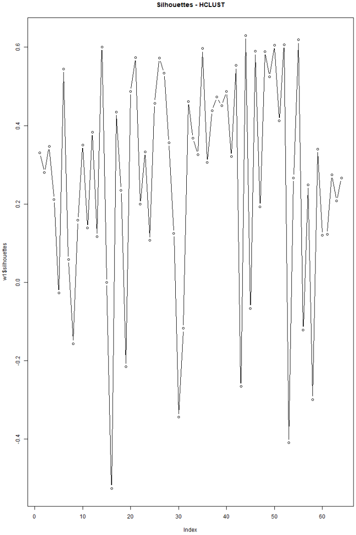


Figure: Hclust with 5 clusters and complete linkage method

HCLUST interpretation

Silhouette Plot for HCLUST Evaluation: assesses the quality of our hierarchical clustering results.

Coefficient Interpretation:

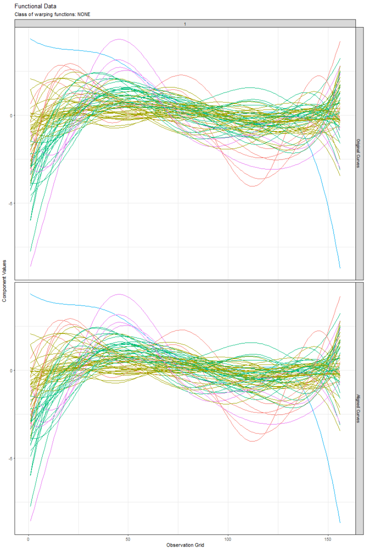
- Values close to $+1$: Well-clustered data point.
- Values around 0: Data point near the cluster boundary.
- Values close to -1: Potential misclassification.

Key Observations:

- **Variable Coefficients:** Significant differences in silhouette values across data points.
- **Negative Values Present:** Some data points (e.g., around indices 17 and 58) show s(i)$<$0, indicating likely misclassification.
- **Values Near Zero:** Many points have s(i)0, suggesting borderline assignments.
- **Well-Defined Clusters:** Some groups exhibit higher positive s(i) (e.g., indices 45-55), indicating good cluster structure.

Overall Conclusion: The silhouette plot suggests that while some clusters are well-formed while other need further investigations

K-means Clustering



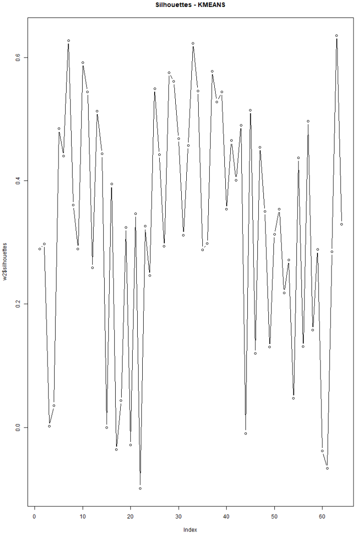


Figure: K-means clustering with 5 clusters and mean used for centroid computation

KMEANS interpretation

Observations from the Plot:

- Predominantly positive silhouette coefficients suggest a reasonable clustering structure.
- Variability in coefficients indicates that some points fit their clusters better than others.
- Several coefficients near zero warrant attention as these points are close to cluster boundaries.
- The absence of strongly negative coefficients implies minimal severe misclassification.

Overall Conclusion: The silhouette plot indicates a moderately good K-Means clustering result. A higher average silhouette score (which can be calculated from these individual values) would signify better overall cluster quality.

DBSCAN Clustering

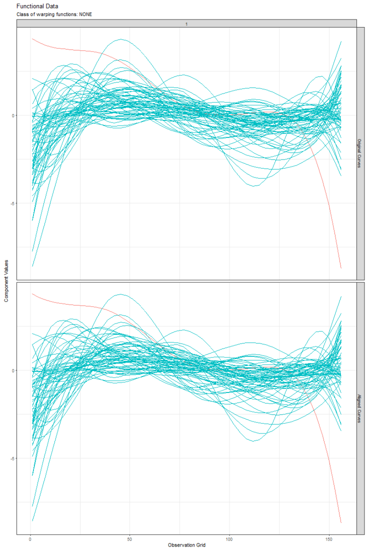


Figure: Results from DBSCAN clustering methods. Red function is Tesla

Interpretations

```
> dt_w[dt_w$FunFEM == 1,]
        StockName FunFEM Hclust Kmeans Dbscan
6       IBM.Close      1      2      2      1
10 RACE.MI.Close      1      2      2      1
16       TM.Close      1      1      2      1
17   KER.PA.Close      1      2      3      1
20    MC.PA.Close      1      2      3      1
21   CFR.SW.Close      1      2      3      1
22   ADS.DE.Close      1      2      3      1
23   NKE.DE.Close      1      2      2      1
25   SAN.PA.Close      1      2      2      1
26 NOVN.SW.Close      1      2      2      1
28      AZN.Close      1      2      2      1
30   MRK.DE.Close      1      1      2      1
32    GSK.L.Close      1      2      2      1
33 NESN.SW.Close      1      2      2      1
34       UL.Close      1      2      2      1
35    BN.PA.Close      1      2      2      1
37      PEP.Close      1      2      2      1
38      MCD.Close      1      2      2      1
39        K.Close      1      2      2      1
40      KHC.Close      1      2      2      1
63      WMT.Close      1      2      2      1
```

Figure: Stocks collected in cluster 1 according to the FunFEM method

Interpretations

```
> dt_w[dt_w$FunFEM == 2,]
          StockName FunFEM Hclust Kmeans Dbscan
5        AAPL.Close      2      1      2      1
15       TSLA.Close      2      4      4      0
31       ARGX.Close      2      1      2      1
45 ORSTED.CO.Close      2      1      2      1
56       UBER.Close      2      1      2      1
59       AMZN.Close      2      1      2      1
61        FDX.Close      2      2      3      1
> dt_w[dt_w$FunFEM == 3,]
       StockName FunFEM Hclust Kmeans Dbscan
1     SPOT.Close      3      1      1      1
2     NFLX.Close      3      1      1      1
3     NVDA.Close      3      1      1      1
4     META.Close      3      1      1      1
7     MSFT.Close      3      1      2      1
19 RMS.PA.Close      3      1      2      1
24 PUM.DE.Close      3      2      2      1
29 UCB.BR.Close      3      1      2      1
36 BON.PA.Close      3      2      2      1
57 ZAL.DE.Close      3      1      1      1
60 DHL.DE.Close      3      2      3      1
> dt_w[dt_w$FunFEM == 4,]
           StockName FunFEM Hclust Kmeans Dbscan
8         GOOG.Close      4      1      2      1
43     ENEL.MI.Close      4      1      2      1
58         UPS.Close      4      1      2      1
64 X002352.SZ.Close      4      1      2      1
```

Figure: Stocks collected in cluster 2-3-4 according to the FunFEM method

Interpretations

```
> dt_w[dt_w$FunFEM == 5,]
```

	StockName	FunFEM	Hclust	Kmeans	Dbscan
9	VOW3.DE.Close	5	2	3	1
11	STLAM.MI.Close	5	2	3	1
12	RNO.PA.Close	5	3	3	1
13	MBG.DE.Close	5	2	3	1
14	BMW.DE.Close	5	2	3	1
18	CPRI.Close	5	3	5	1
27	BAYN.DE.Close	5	2	2	1
41	SHEL.Close	5	2	3	1
42	ENI.MI.Close	5	2	3	1
44	ENGI.PA.Close	5	2	3	1
46	CHV.F.Close	5	2	3	1
47	REP.MC.Close	5	2	3	1
48	TTE.PA.Close	5	2	3	1
49	TRVG.Close	5	5	5	1
50	BKNG.Close	5	2	3	1
51	RYA.IR.Close	5	2	3	1
52	LYFT.Close	5	5	5	1
53	TCOM.Close	5	3	3	1
54	TRIP.Close	5	5	5	1
55	HLT.Close	5	2	3	1
62	AMKBY.Close	5	2	3	1

Figure: Stocks collected in cluster 5 according to the FunFEM method

Interpretations

- Applied FunFEM, hierarchical, k-means, and DBSCAN clustering on smoothed weekly stock price functions.
- **Cluster 1:** Stable, mature companies (e.g., IBM, WMT, MCD) — likely low-volatility, defensive stocks - related to different economic sectors
- **Cluster 2:** High-growth, tech-oriented firms (e.g., AAPL, TSLA, AMZN) — showing dynamic, possibly volatile trends.
- **Cluster 3:** Digital and streaming-focused stocks (e.g., SPOT, NVDA, META) — potentially similar usage trends or momentum patterns.
- **Cluster 4:** Smaller, mixed group — includes GOOG and UPS, may indicate intermediate or hybrid behaviors.
- **Cluster 5:** Automotive, energy, and travel sectors (e.g., BMW, ENI, TRIP) — exhibiting cyclical, macro-sensitive patterns.
- Consistency observed between FunFEM, k-means, and hierarchical clustering, especially for Clusters 1 and 5.
- DBSCAN detected one outlier(Tesla), indicating strong internal cohesion in the smoothed data.

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Mean Trajectories

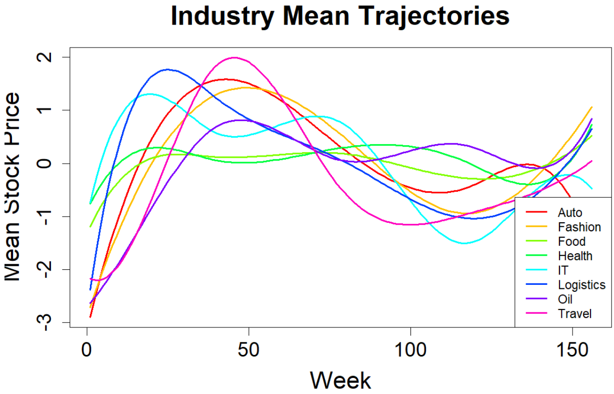

Figure: Mean Trajectories

ANOVA Pointwise

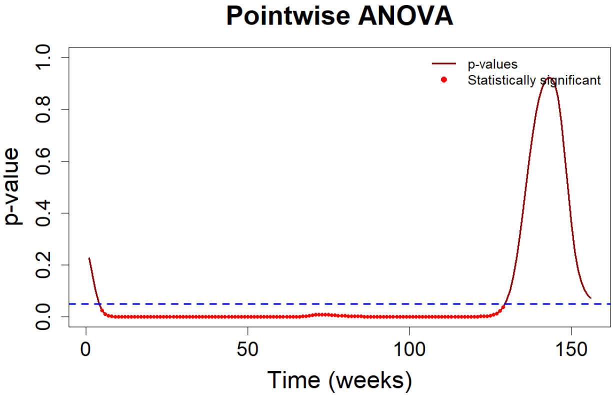

Figure: Pointwise ANOVA: Industry Differences Over Time

Post-Hoc Test 1

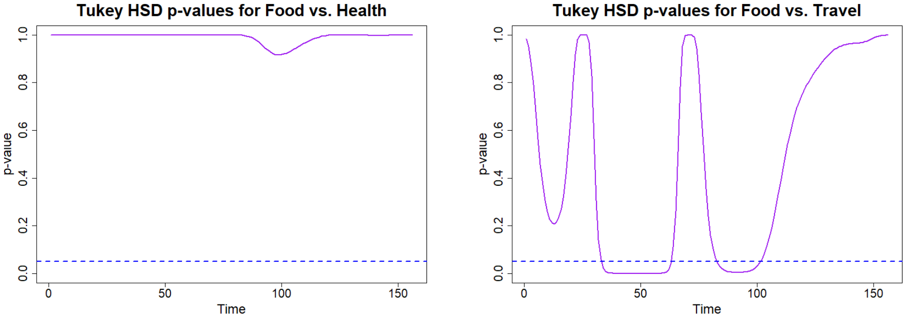

Figure: Tukey p-values

Post-Hoc Test 2

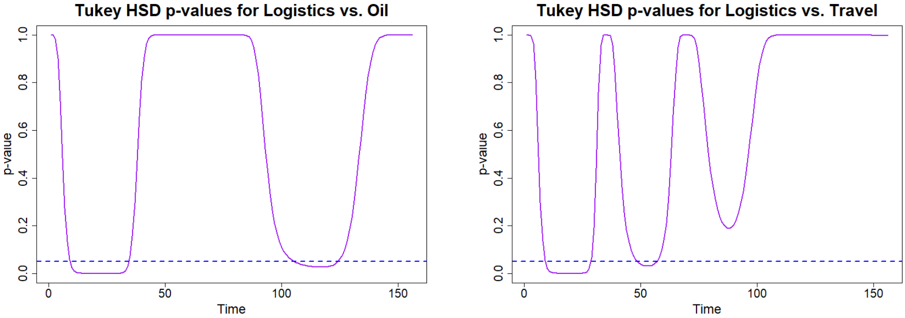

Figure: Tukey p-values

Post-Hoc Test 3

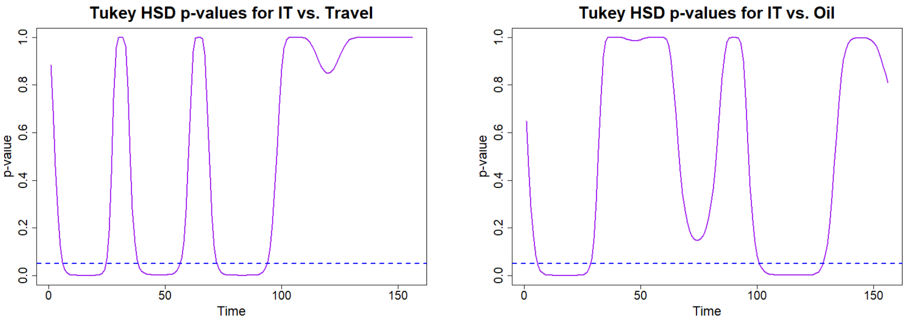

Figure: Tukey p-values

Post-Hoc Test 4

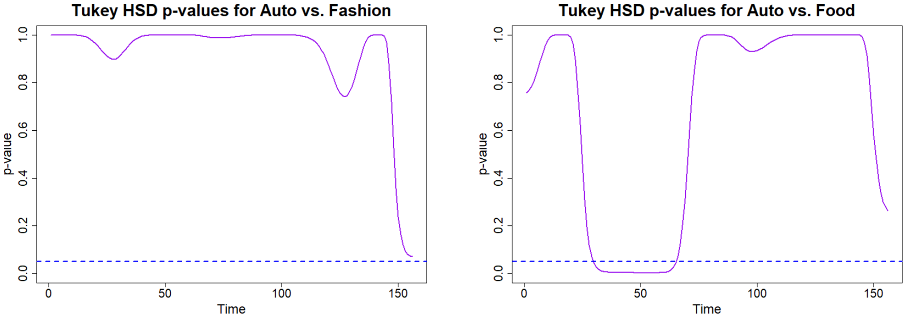

Figure: Tukey p-values

Post-Hoc Test 5

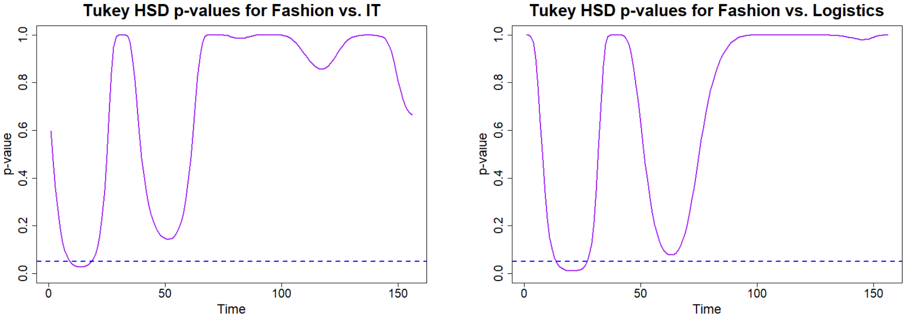

Figure: Tukey p-values

Post-Hoc Test 6

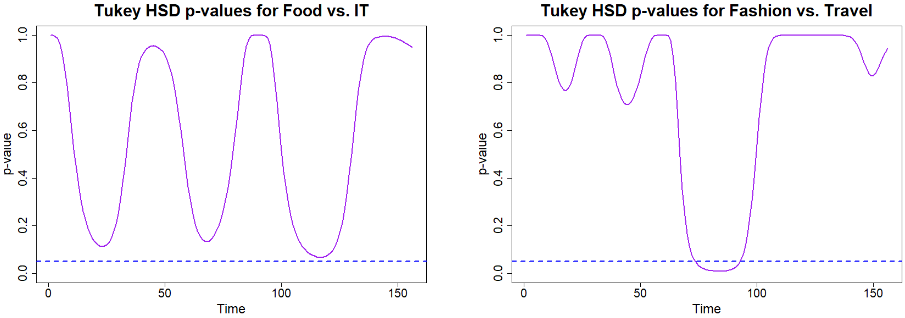

Figure: Tukey p-values

Thank you for your attention