Functional data analysis of stocks during COVID-19

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Vilnius University - Functional Data Analyses project

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Table of contents

Goal

- To study the variability of returns of stocks belonging to different industries during COVID-19 period.
- To study stocks and industries that were affected by COVID-19 and the ones that were not

Table of contents

Data Presentation

- Stocks data from 2020/01/01 until 2022/12/31
- 8 main industrial sectors were considered:

- automobile
- fashion and clothing
- food and beverage
- healthcare
- tech
- logistic
- oil and gas
- travel and tourism

- For each sector 8 stocks were considered
- The closing price for each end of week was considered
- Total of 156 weeks

The stocks considered

- The main stocks were:
  - **Automobile**: VW - Ferrari - Stellantis - Renault - Mercedes - BMW - Tesla - Toyota
  - **Fashion and clothing**: Kering - Capri - Hermès - LVMH - Richemont - Adidas - Nike - Puma
  - **Food and beverage**: Nestlè - Uniliver - Danone - Bonduelle - Pepsi - McDonalds - Kellogs
  - **Healthcare**: Sanofi - Novartis - Bayer - AstraZeneca - UCB - Merck - Argenx - GSK
  - **Tech**: Spotify - Netflix - Nvidia - Meta - Apple - IBM - Microsoft - Google
  - **Logistic**: Zalando - UPS - Amazon - DHL - FedEx - Maersk - Walmart - SF express
  - **Oil and Gas**: Shell - Eni - Enel - Engie - Orsted - Chevron - Repsol
  - **Travel and tourism**: Trivago - Booking - Ryanair - Lyft - Trip.com - Tripadvisor - Hilton - Uber

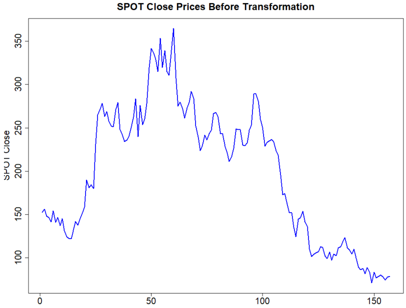
Data transformation

Logarithmic return:

$$R = 100 \cdot \log\left(\frac{P_t}{P_{t-1}}\right)$$

- R is the log return
- P_t is the stock price at time t
- P_{t-1} is the stock price at the previous time period

Before vs. After Transformations



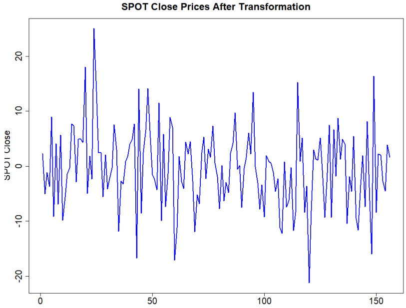


Figure: Before vs. After Transformations.

Table of contents

Euclidean depth

- Euclidean depth of the deepest point in the dataset: 0.00002917722

```
> st[mdE,]
     ZAL.DE.Close          UPS.Close         AMZN.Close        DHL.DE.Close
         47.25000          207.17999          146.81750            42.44000
        FDX.Close        AMKBY.Close          WMT.Close    X002352.SZ.Close
        219.28000           15.27000           47.54333            53.69000
       SPOT.Close         NFLX.Close         NVDA.Close          META.Close
        148.91000          380.14999           25.93100           231.84000
       AAPL.Close          IBM.Close         MSFT.Close          GOOG.Close
        175.06000          128.89000          310.88000           141.06300
    VOW3.DE.Close      RACE.MI.Close     STLAM.MI.Close        RNO.PA.Close
        151.00000          177.89999           14.15600            22.99000
     MBG.DE.Close       BMW.DE.Close         TSLA.Close            TM.Close
         62.14000           75.32000          267.29666           165.92999
     KER.PA.Close         CPRI.Close       RMS.PA.Close         MC.PA.Close
        557.20001           50.16000         1113.50000           590.00000
     CFR.SW.Close       ADS.DE.Close       NKE.DE.Close        PUM.DE.Close
        104.00000          203.70000          108.44000            71.86000
     SAN.PA.Close      NOVN.SW.Close      BAYN.DE.Close           AZN.Close
         93.46938           75.47266           55.94000            61.37000
     UCB.BR.Close       MRK.DE.Close         ARGX.Close         GSK.L.Close
        100.50000          179.20000          286.47000          1581.55164
    NESN.SW.Close           UL.Close        BN.PA.Close        BON.PA.Close
        115.74000           44.39000           52.76000            17.18000
        PEP.Close          MCD.Close            K.Close           KHC.Close
        159.00000          232.57001           57.57747            37.82000
       SHEL.Close       ENI.MI.Close      ENEL.MI.Close       ENGI.PA.Close
         50.35000           12.92400            5.82600            11.53400
  ORSTED.CO.Close        CHV.F.Close       REP.MC.Close        TTE.PA.Close
        829.59998          144.72000           11.46800            45.75500
     SHEL.Close.1     ENI.MI.Close.1    ENEL.MI.Close.1     ENGI.PA.Close.1
         50.35000           12.92400            5.82600            11.53400
ORSTED.CO.Close.1      CHV.F.Close.1     REP.MC.Close.1      TTE.PA.Close.1
        829.59998          144.72000           11.46800            45.75500
```

```
> apply(st,2,median)
     ZAL.DE.Close          UPS.Close         AMZN.Close        DHL.DE.Close
         66.55000          175.50500          155.11150            40.22500
        FDX.Close        AMKBY.Close          WMT.Close    X002352.SZ.Close
        229.71000           11.60500           46.21500            61.58500
       SPOT.Close         NFLX.Close         NVDA.Close          META.Close
        223.00000          480.54001           14.61325           249.07000
       AAPL.Close          IBM.Close         MSFT.Close          GOOG.Close
        135.38000          128.15134          245.14500           104.76375
    VOW3.DE.Close      RACE.MI.Close     STLAM.MI.Close        RNO.PA.Close
        151.89000          177.42500           13.50200            30.13000
     MBG.DE.Close       BMW.DE.Close         TSLA.Close            TM.Close
         57.85313           76.27000          225.39667           153.31499
     KER.PA.Close         CPRI.Close       RMS.PA.Close         MC.PA.Close
        564.75000           46.55000         1066.25000           601.10001
     CFR.SW.Close       ADS.DE.Close       NKE.DE.Close        PUM.DE.Close
         97.14000          256.25000          110.13000            77.19500
     SAN.PA.Close      NOVN.SW.Close      BAYN.DE.Close           AZN.Close
         86.79584           76.67636           53.35000            56.31500
     UCB.BR.Close       MRK.DE.Close         ARGX.Close         GSK.L.Close
         87.80000          155.32500          293.18500          1503.27991
    NESN.SW.Close           UL.Close        BN.PA.Close        BON.PA.Close
        109.47158           54.25500           56.19500            20.20000
        PEP.Close          MCD.Close            K.Close           KHC.Close
        148.05500          233.28500           60.92958            36.13000
       SHEL.Close       ENI.MI.Close      ENEL.MI.Close       ENGI.PA.Close
         41.60500           10.52400            7.22800            12.13100
  ORSTED.CO.Close        CHV.F.Close       REP.MC.Close        TTE.PA.Close
        842.39999           89.35000           10.63039            39.74750
     SHEL.Close.1     ENI.MI.Close.1    ENEL.MI.Close.1     ENGI.PA.Close.1
         41.60500           10.52400            7.22800            12.13100
ORSTED.CO.Close.1      CHV.F.Close.1     REP.MC.Close.1      TTE.PA.Close.1
        842.39999           89.35000           10.63039            39.74750
```

Comparison of the deepest point vs. columns-wise medians

- some values (e.g., UPS.Close, MSFT.Close, GOOG.Close) in the deepest point differ significantly from the median. This may suggest that the dataset contain outliers affecting the distribution

Euclidean depth

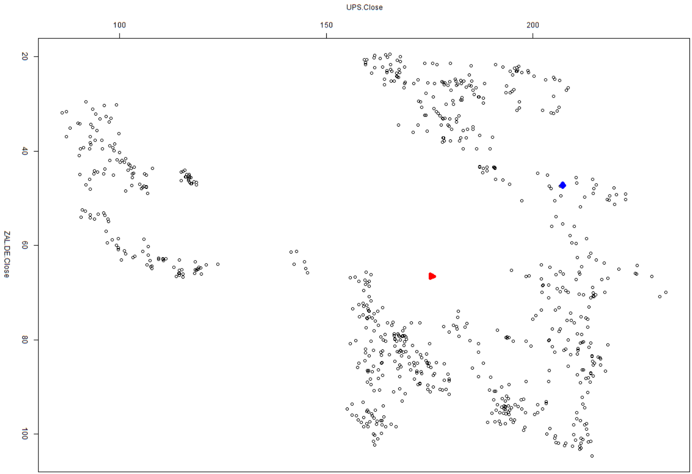


Figure: Comparison between the deepest point(in blue) and the median point(in red) in a scatter plot

Minimum Volume Depth (MBD)

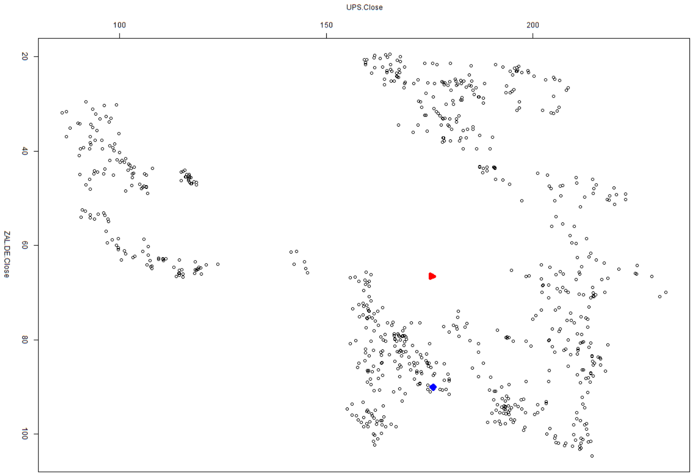


Figure: Comparison between the deepest point(in blue) and the median point(in red) in a scatter plot

Frainman-Muniz depth

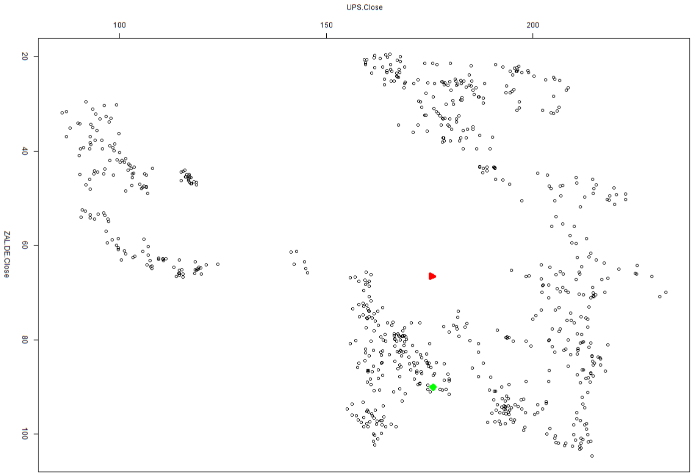


Figure: Comparison between the deepest point(in green) and the median point(in red) in a scatter plot

Table of contents

Smoothing methods

Methods:

- B-splines
- Local regression kernel
- Nadaraya-Watson kernel
- Normal kernel
- Triweight kernel
- Uniform kernel

B-splines results

Smoothing with penalty:

- Generalized cross-validation
  - Optimal lambda $\sim$ 371
  - Number of basis $\sim$ 7
  - GCV $\sim$ 2230
  - SSE $\sim$ 307923

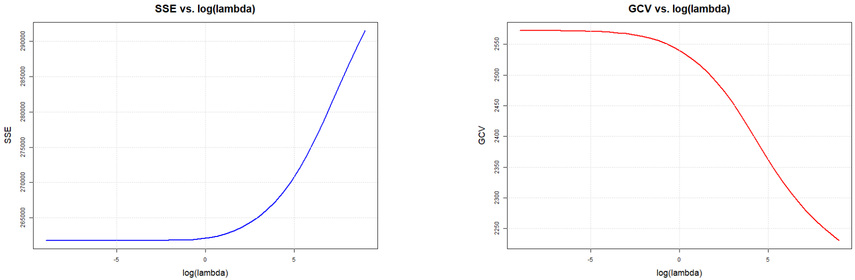


Figure: Comparison of SSE and GCV with respect to lambda.

Kernel smoothing results

Generalized cross-validation

- Sequence of bandwidth between 3 and 70

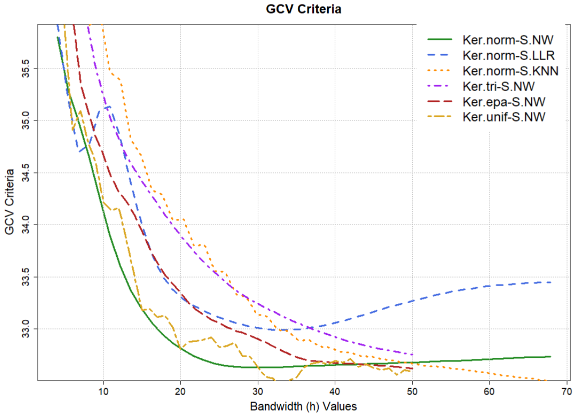


Figure: GCV criteria respect to bandwidth

Comparing optimal GCV and SSE

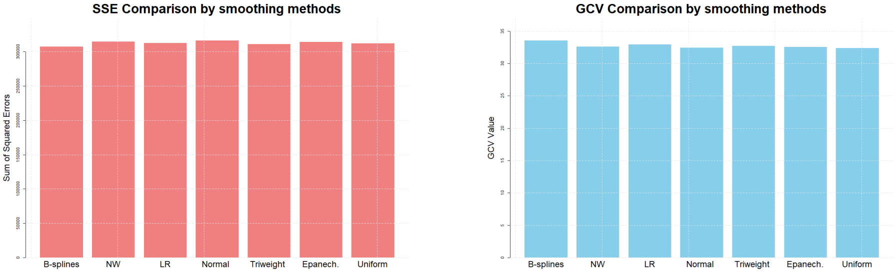


Figure: Comparison of optimal GCV and SSE among smoothing methods.

B-splines vs. Normal kernel

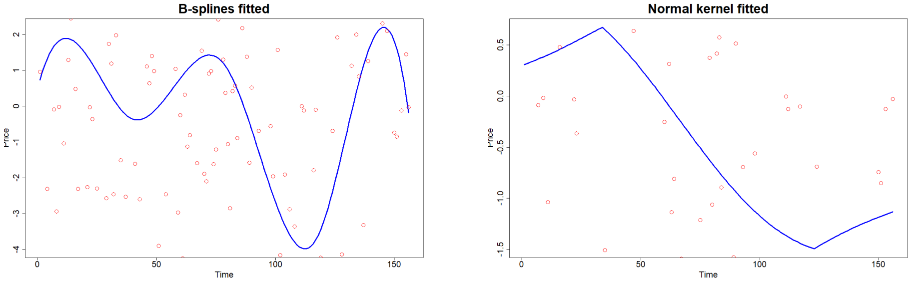


Figure: Comparison of fitted and actual values for B-splines and Normal kernel.

B-splines vs. Triweight kernel

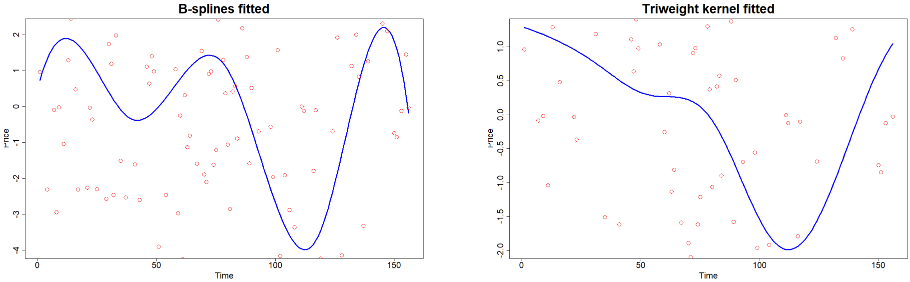


Figure: Comparison of optimal GCV and SSE among smoothing methods.

B-splines fitted

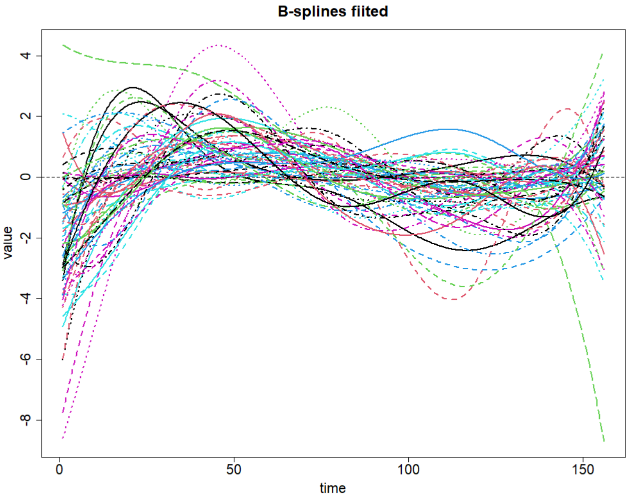


Figure: B-splines fitted for all stocks

Table of contents

Derivatives

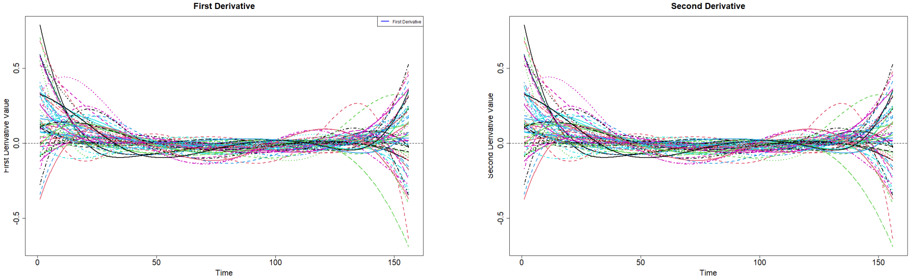


Figure: 1st and 2nd derivatives of B-splines smoothed

Outliers detection

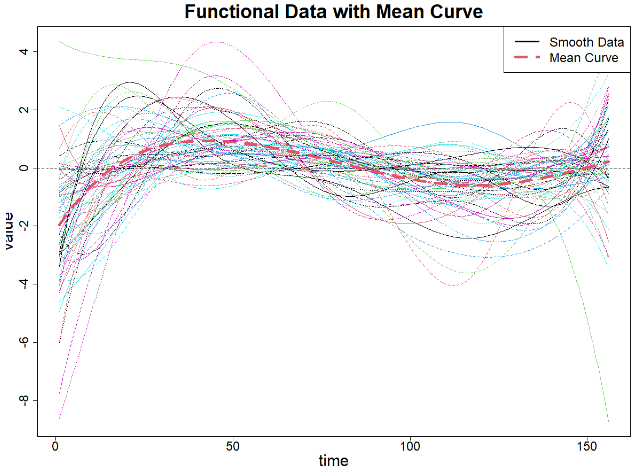


Figure: Mean curve

Euclidean depth

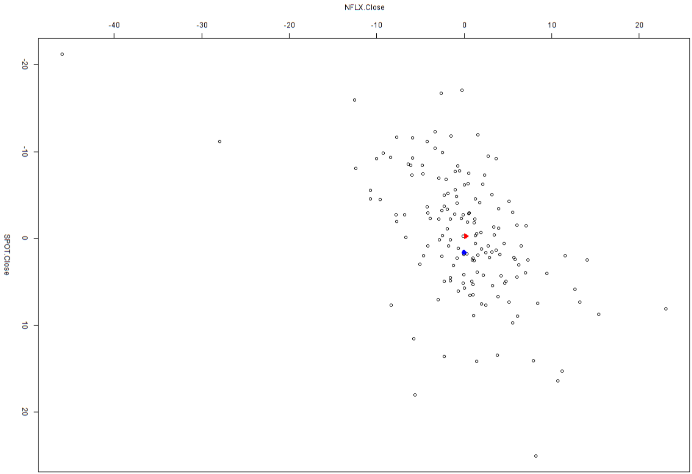
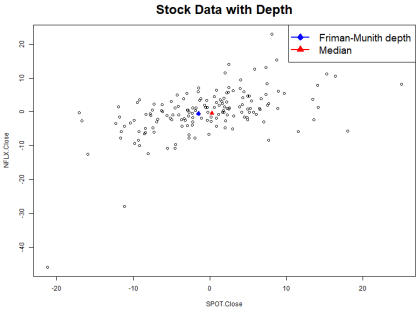


Figure: Comparison between the deepest point(in blue) and the median point(in red) in a scatter plot

Frainman-Muniz and The Most Central Point



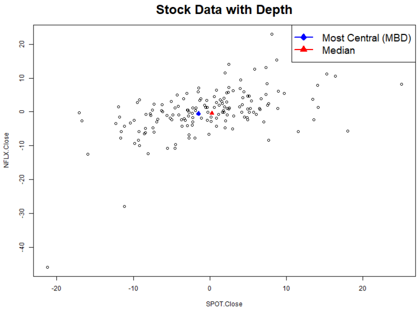


Figure: Friman-Munith depth and MBD

Bivariate Covariance Function

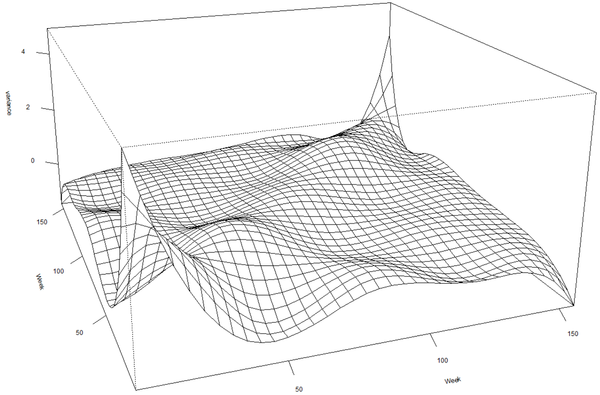


Figure: 3D visualization of the variance function for smoothed stock returns

Contour plot

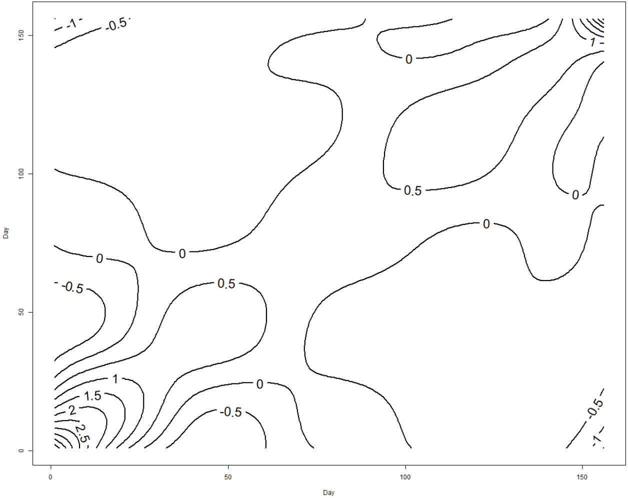


Figure: Contour plot

Table of contents

Principal Component Analysis

FPCA Component	**Percentage Contribution**
PCA Function 1	34%
PCA Function 2	24%
PCA Function 3	15%
PCA Function 4	9%
Total	**82%**

Table: FPCA Component Contributions

VARIMAX Rotation

PCA Function (Varimax Rotation)	Percentage Contribution
PCA Function 1	27.8%
PCA Function 2	18.3%
PCA Function 3	13.0%
PCA Function 4	20.7%
Total	**79.8%**

Table: PCA Function Contributions after Varimax Rotation

VARIMAX Rotation

- PCA 4 captured 20.7% and showed strong variability from the mean among stocks till the 70th week(could explain starting phase)
- PCA 1 may capture the overall market variability across different phases in response to specific events possibly COVID-19 related.

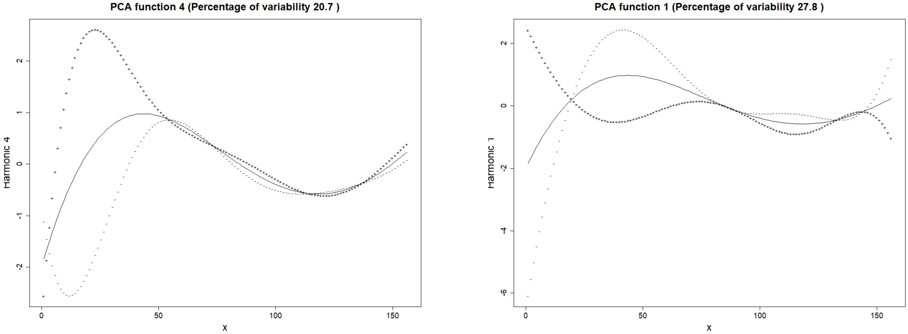


Figure: VARIMAX rotation of FPCA 1 and FPCA 4

VARIMAX Rotation

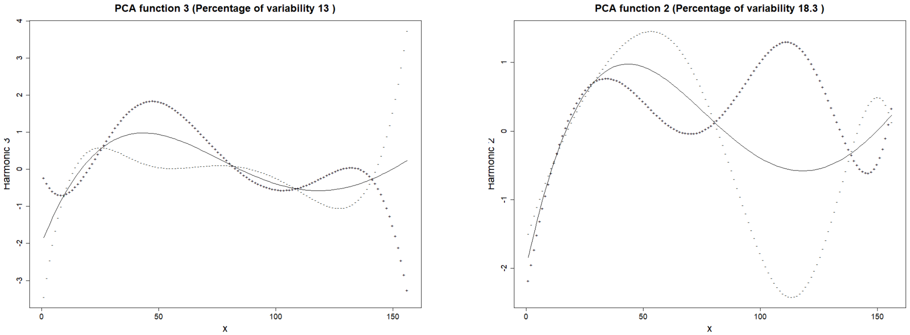


Figure: VARIMAX rotation of FPCA 2 and FPCA 3

- Volatility in stock prices in beginning, mid and end of pandemic
- PCA Functions captured the most volatility in different time periods

Table of contents

The choice of the best number of clusters

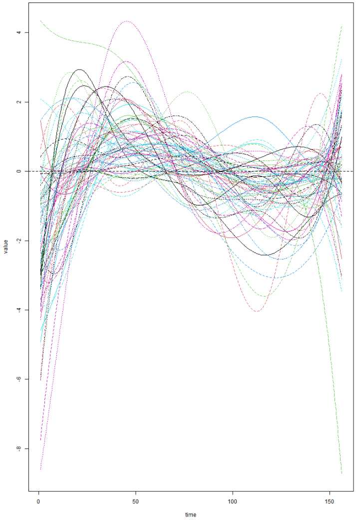
```
>> K = 2
Error in .fstep(fd, T, lambda) : One cluster is almost empty!
>> K = 3
AkjBk   :        bic = -350051.4
>> K = 4
AkjBk   :        bic = -318623.1
>> K = 5
AkjBk   :        bic = -198106.2
>> K = 6
Error in .fstep(fd, T, lambda) : One cluster is almost empty!
>> K = 7
Error in .fstep(fd, T, lambda) : One cluster is almost empty!
>> K = 8
Error in .fstep(fd, T, lambda) : One cluster is almost empty!
>> K = 9
Error in .fstep(fd, T, lambda) : One cluster is almost empty!
>> K = 10
Error in .fstep(fd, T, lambda) : One cluster is almost empty!
The best model is AkjBk with K = 5 ( bic = -198106.2 )
```

	K	model	bic	aic	icl	nbprm	ll
1	2	AkjBk	NA	NA	NA	NA	NA
2	3	AkjBk	-350051.4	-350021.2	-350051.4	28	-349993.2
3	4	AkjBk	-318623.1	-318573.5	-318623.1	46	-318527.5
4	5	AkjBk	-198106.2	-198033.9	-198106.2	67	-197966.9
5	6	AkjBk	NA	NA	NA	NA	NA
6	7	AkjBk	NA	NA	NA	NA	NA
7	8	AkjBk	NA	NA	NA	NA	NA
8	9	AkjBk	NA	NA	NA	NA	NA
9	10	AkjBk	NA	NA	NA	NA	NA

Figure: The choice of the best number of clusters to consider

- The best clustering model selected is AkjBk with $K = 5$ clusters, based on the highest BIC score (-198106.2)
- Models with $K > 5$ resulted in empty clusters.

From original data to clusters



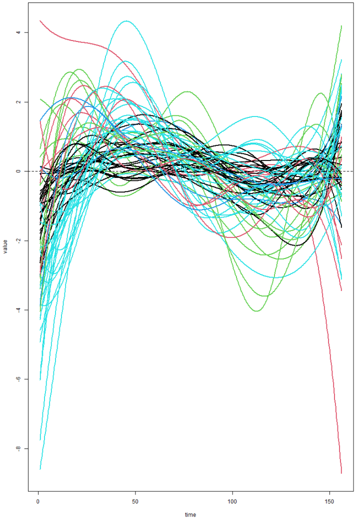
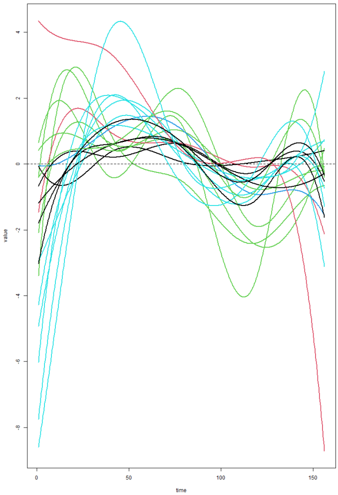


Figure: Comparison between original smoothed data and clusters

Centroids



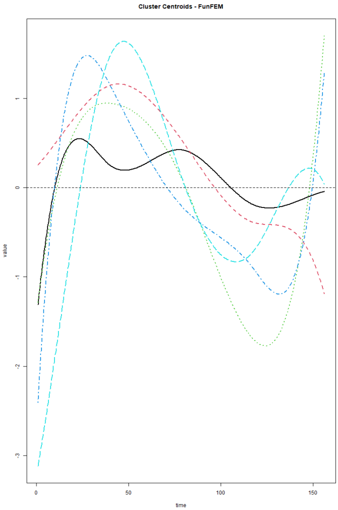
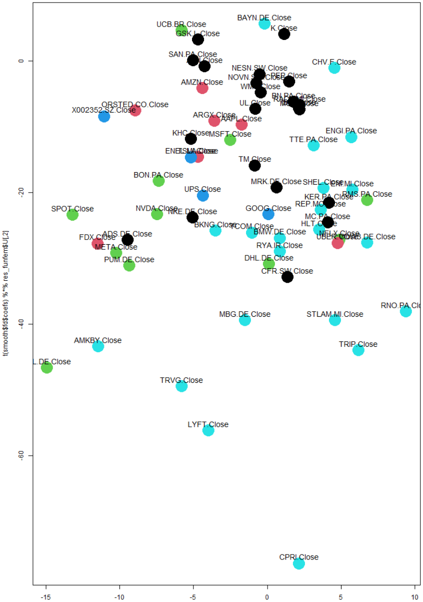
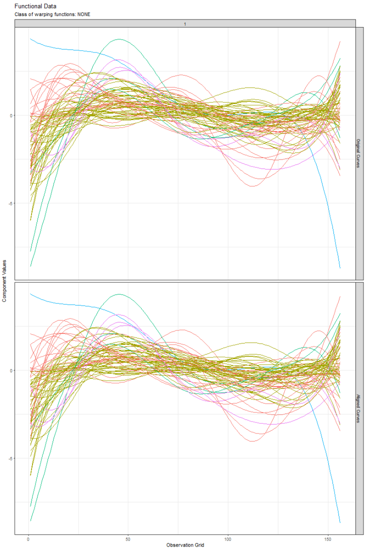


Figure: A more in-depth analyses comparing the first 20 curves(on the left) and the mean of each cluster(on the right)

Discriminative Space Plot



Hierarchical Clustering (HCLUST)



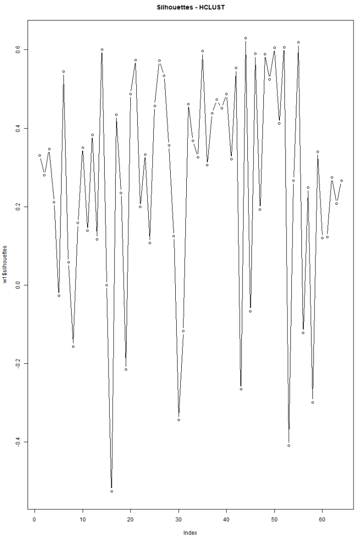


Figure: Hclust with 5 clusters and complete linkage method

HCLUST interpretation

Silhouette Plot for HCLUST Evaluation: assesses the quality of our hierarchical clustering results.

Coefficient Interpretation:

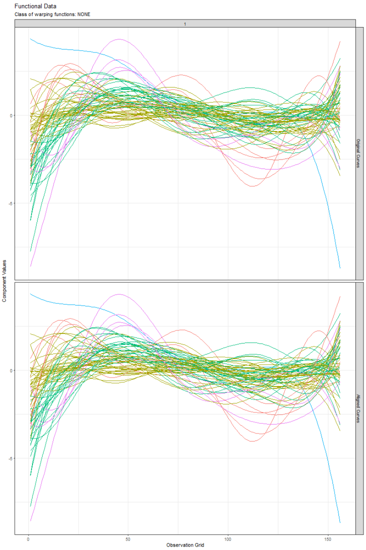
- Values close to +1: Well-clustered data point.
- Values around 0: Data point near the cluster boundary.
- Values close to -1: Potential misclassification.

Key Observations:

- **Variable Coefficients:** Significant differences in silhouette values across data points.
- **Negative Values Present:** Some data points (e.g., around indices 17 and 58) show $s(i)<0$, indicating likely misclassification.
- **Values Near Zero:** Many points have s(i)0, suggesting borderline assignments.
- **Well-Defined Clusters:** Some groups exhibit higher positive s(i) (e.g., indices 45-55), indicating good cluster structure.

Overall Conclusion: The silhouette plot suggests that while some clusters are well-formed while other need further investigations

K-means Clustering



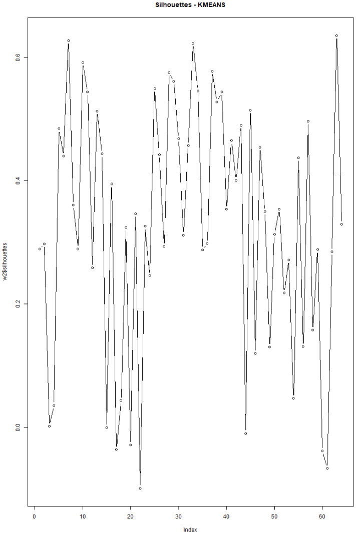


Figure: K-means clustering with 5 clusters and mean used for centroid computation

KMEANS interpretation

Observations from the Plot:

- Predominantly positive silhouette coefficients suggest a reasonable clustering structure.
- Variability in coefficients indicates that some points fit their clusters better than others.
- Several coefficients near zero warrant attention as these points are close to cluster boundaries.
- The absence of strongly negative coefficients implies minimal severe misclassification.

Overall Conclusion: The silhouette plot indicates a moderately good K-Means clustering result. A higher average silhouette score (which can be calculated from these individual values) would signify better overall cluster quality.

DBSCAN Clustering

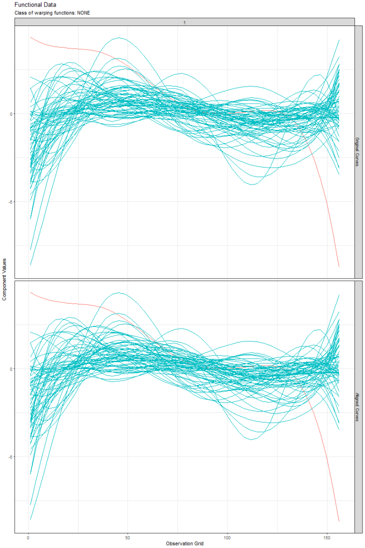


Figure: Results from DBSCAN clustering methods. Red function is Tesla

Interpretations

```
> dt_w[dt_w$FunFEM == 1,]
        StockName FunFEM Hclust Kmeans Dbscan
6       IBM.Close      1      2      2      1
10 RACE.MI.Close      1      2      2      1
16       TM.Close      1      1      2      1
17  KER.PA.Close      1      2      3      1
20   MC.PA.Close      1      2      3      1
21  CFR.SW.Close      1      2      3      1
22  ADS.DE.Close      1      2      3      1
23  NKE.DE.Close      1      2      2      1
25  SAN.PA.Close      1      2      2      1
26 NOVN.SW.Close      1      2      2      1
28      AZN.Close      1      2      2      1
30  MRK.DE.Close      1      1      2      1
32   GSK.L.Close      1      2      2      1
33 NESN.SW.Close      1      2      2      1
34       UL.Close      1      2      2      1
35   BN.PA.Close      1      2      2      1
37      PEP.Close      1      2      2      1
38      MCD.Close      1      2      2      1
39        K.Close      1      2      2      1
40      KHC.Close      1      2      2      1
63      WMT.Close      1      2      2      1
```

Figure: Stocks collected in cluster 1 according to the FunFEM method

Interpretations

```
> dt_w[dt_w$FunFEM == 2,]
          StockName FunFEM Hclust Kmeans Dbscan
5        AAPL.Close      2      1      2      1
15       TSLA.Close      2      4      4      0
31       ARGX.Close      2      1      2      1
45 ORSTED.CO.Close      2      1      2      1
56       UBER.Close      2      1      2      1
59       AMZN.Close      2      1      2      1
61        FDX.Close      2      2      3      1
> dt_w[dt_w$FunFEM == 3,]
      StockName FunFEM Hclust Kmeans Dbscan
1    SPOT.Close      3      1      1      1
2    NFLX.Close      3      1      1      1
3    NVDA.Close      3      1      1      1
4    META.Close      3      1      1      1
7    MSFT.Close      3      1      2      1
19 RMS.PA.Close      3      1      2      1
24 PUM.DE.Close      3      2      2      1
29 UCB.BR.Close      3      1      2      1
36 BON.PA.Close      3      2      2      1
57 ZAL.DE.Close      3      1      1      1
60 DHL.DE.Close      3      2      3      1
> dt_w[dt_w$FunFEM == 4,]
          StockName FunFEM Hclust Kmeans Dbscan
8        GOOG.Close      4      1      2      1
43    ENEL.MI.Close      4      1      2      1
58        UPS.Close      4      1      2      1
64 X002352.SZ.Close      4      1      2      1
```

Figure: Stocks collected in cluster 2-3-4 according to the FunFEM method

```
> dt_w[dt_w$FunFEM == 5,]
```

	StockName	FunFEM	Hclust	Kmeans	Dbscan
9	VOW3.DE.Close	5	2	3	1
11	STLAM.MI.Close	5	2	3	1
12	RNO.PA.Close	5	3	3	1
13	MBG.DE.Close	5	2	3	1
14	BMW.DE.Close	5	2	3	1
18	CPRI.Close	5	3	5	1
27	BAYN.DE.Close	5	2	2	1
41	SHEL.Close	5	2	3	1
42	ENI.MI.Close	5	2	3	1
44	ENGI.PA.Close	5	2	3	1
46	CHV.F.Close	5	2	3	1
47	REP.MC.Close	5	2	3	1
48	TTE.PA.Close	5	2	3	1
49	TRVG.Close	5	5	5	1
50	BKNG.Close	5	2	3	1
51	RYA.IR.Close	5	2	3	1
52	LYFT.Close	5	5	5	1
53	TCOM.Close	5	3	3	1
54	TRIP.Close	5	5	5	1
55	HLT.Close	5	2	3	1
62	AMKBY.Close	5	2	3	1

Figure: Stocks collected in cluster 5 according to the FunFEM method

Interpretations

- Applied FunFEM, hierarchical, k-means, and DBSCAN clustering on smoothed weekly stock price functions.
- **Cluster 1:** Stable, mature companies (e.g., IBM, WMT, MCD) — likely low-volatility, defensive stocks - related to different economic sectors
- **Cluster 2:** High-growth, tech-oriented firms (e.g., AAPL, TSLA, AMZN) — showing dynamic, possibly volatile trends.
- **Cluster 3:** Digital and streaming-focused stocks (e.g., SPOT, NVDA, META) — potentially similar usage trends or momentum patterns.
- **Cluster 4:** Smaller, mixed group — includes GOOG and UPS, may indicate intermediate or hybrid behaviors.
- **Cluster 5:** Automotive, energy, and travel sectors (e.g., BMW, ENI, TRIP) — exhibiting cyclical, macro-sensitive patterns.
- Consistency observed between FunFEM, k-means, and hierarchical clustering, especially for Clusters 1 and 5.
- DBSCAN detected one outlier(Tesla), indicating strong internal cohesion in the smoothed data.

Table of contents

Mean Trajectories

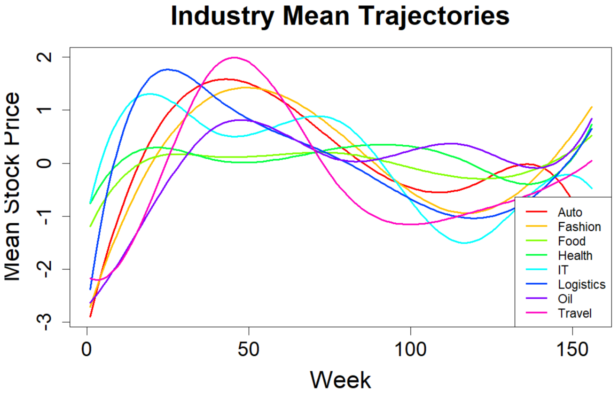


Figure: Mean Trajectories

ANOVA Pointwise

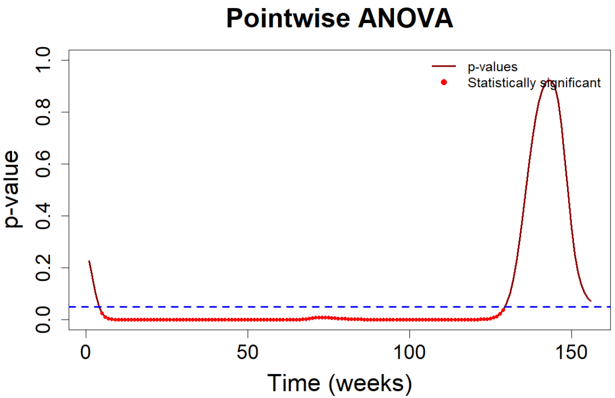


Figure: Pointwise ANOVA: Industry Differences Over Time

Post-Hoc Test 1

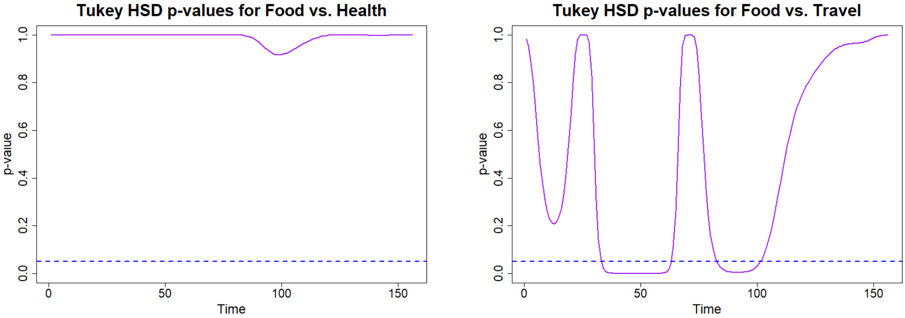
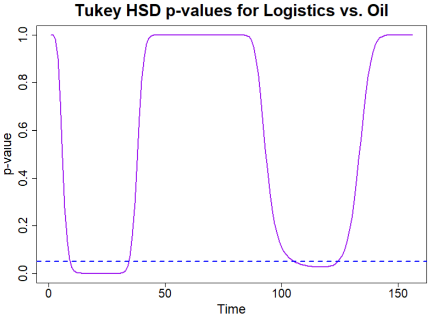


Figure: Tukey p-values

Post-Hoc Test 2



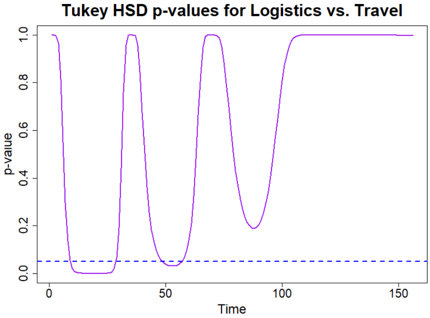


Figure: Tukey p-values

Post-Hoc Test 3

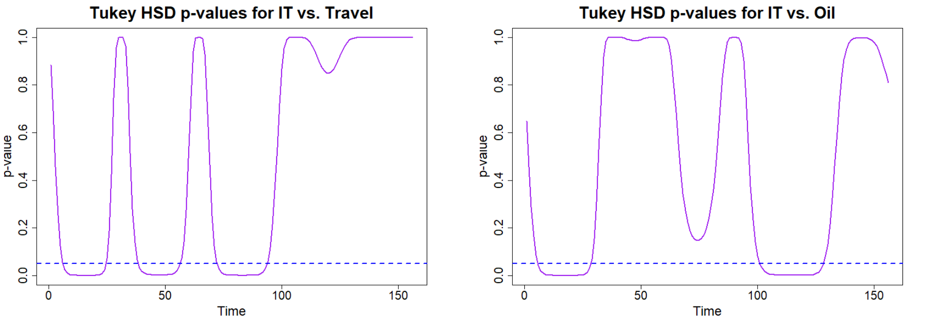


Figure: Tukey p-values

Post-Hoc Test 4

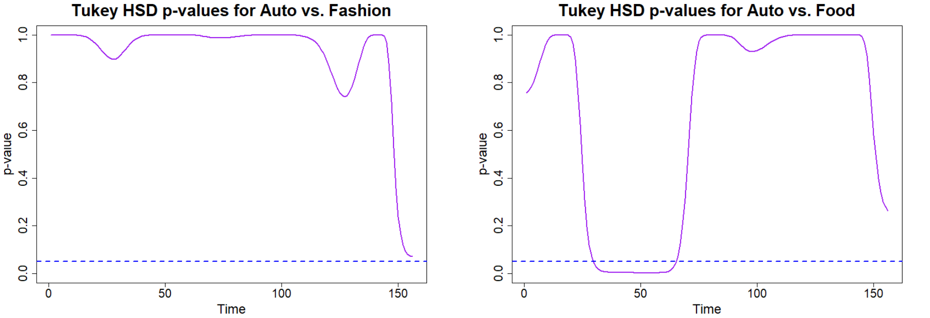


Figure: Tukey p-values

Post-Hoc Test 5

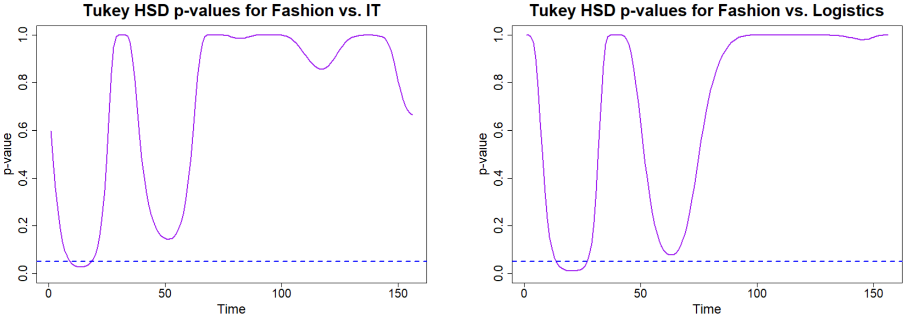


Figure: Tukey p-values

Post-Hoc Test 6

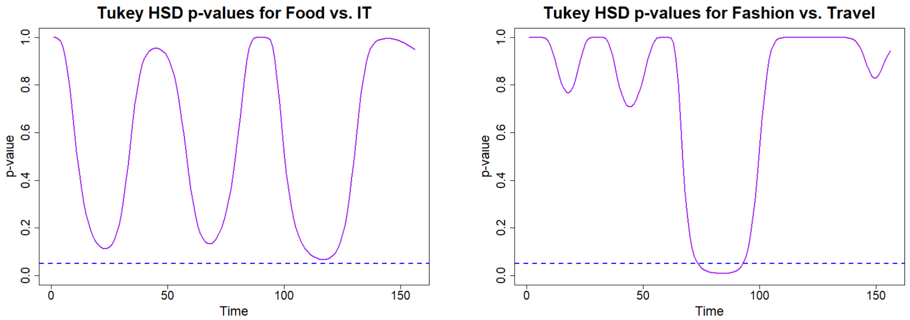
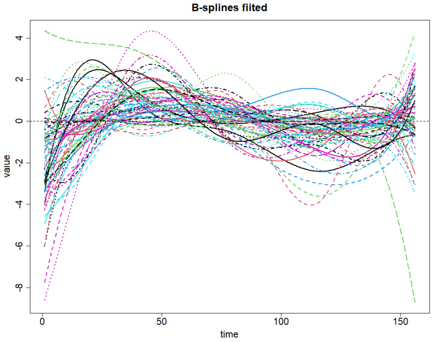


Figure: Tukey p-values

Table of contents

FAR Data Format



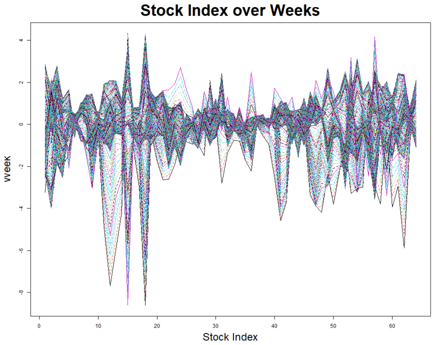


Figure: Change of Data Format

Parameters

Parameters	
Train set	146 weeks
Test set	10 weeks
Optimal K-NN by GCV	4
Past returns lag	1

Table: Parameters

FAR(1) Fitted and Residuals

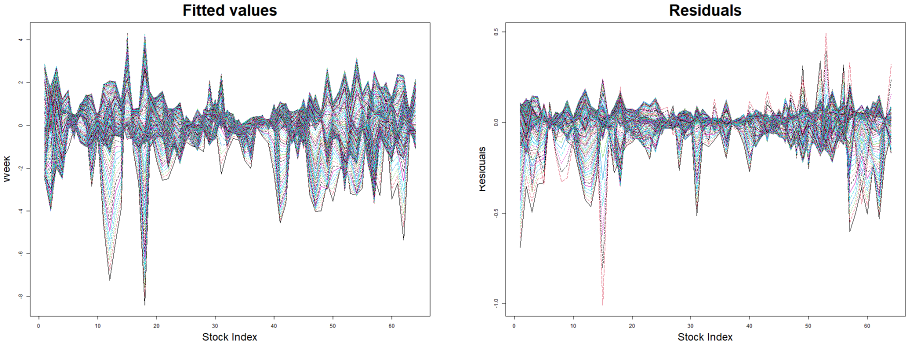


Figure: Fitted and Residuals

Forecasting on Test set

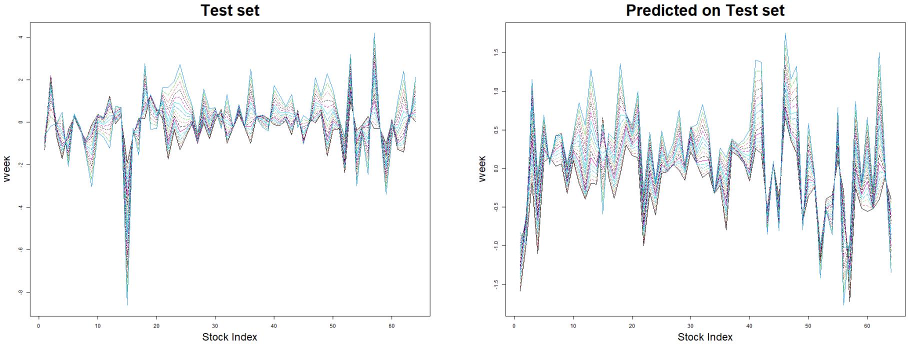


Figure: Prediction on Test set

FPCA + FAR(3)

PCA Function	Percentage Contribution
PCA Function 1	57%
PCA Function 2	26%
PCA Function 3	6%
Total	**89%**

Table: PCA Function Contributions to FAR

Fitted and Residuals

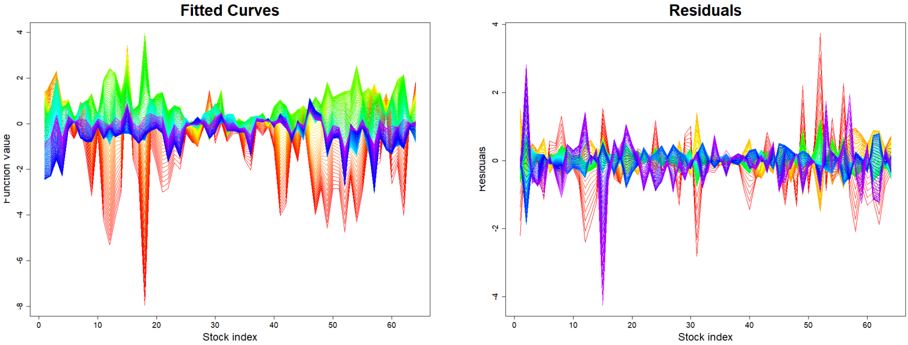


Figure: Fitted and Residuals

Forecasting on Test set

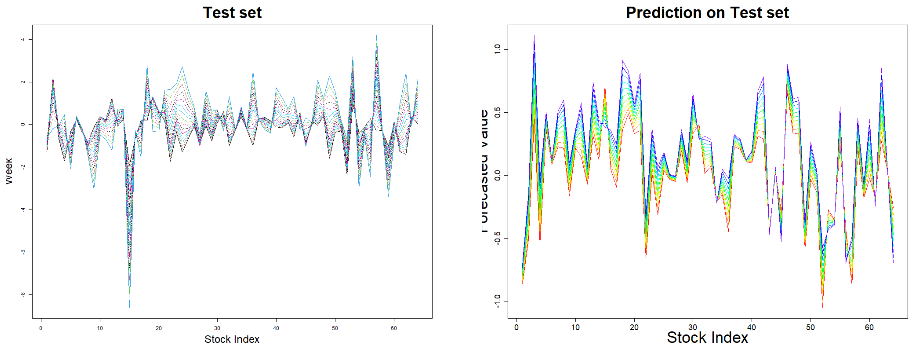


Figure: Prediction on Test set

Results

Model	MSE
FAR(1)	0.05
FPCA + FAR(3)	0.16

Table: MSE Results on Train set

Model	MSE
FAR(1)	1.38
FPCA + FAR(3)	1.82

Table: MSE Results on Test set

Table of contents

Conclusion

- No evident clusters belonging to 8 sectors
- PCA and First Derivative showed clear variation in the beginning and ending of Pandemic
- First constant derivative after the start of Covid-19 and at the end of this indicates how the pandemic has influenced the price movement of stocks
- From Hypothesis only Health and Food industries showed the same pattern
- Both models showed weak predictive accuracy, FAR(1) showed lower MSE than FPCA

Thank you for your attention