The Electricity Sector in Italy: An In-Depth Analysis of Trends and Challenges

Antonio De Patto [1]*

Abstract
Efficient modeling and forecasting of electricity demand and consumption is an important issue in competitive electricity markets. This work investigates the Italian electricity sector from 1931 to 2014 with a focus on energy consumption. More in-depth analyses will follow regarding energy production and electricity price over a shorter period of time, about 30 year.

Keywords
Energy - Electricity - Consumption - Demand - Price

[1] *Department of Data Science, Faculty of Mathematics and Informatics, Vilnius University*
Corresponding author: antonio.de@tprs.stud.vu.lt

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Introduction

This paper first analyzes the composition of the Italian electricity sector from 1931 to 2014 using annual data. After this historical overview, we focus on the past 30 years to examine the electricity market in more detail. We then build a Vector Auto-Regression (VAR) model to explore the relationship between electricity consumption, production, GDP, and CO emissions. Finally, we develop a VARMAX model to analyze electricity and gas prices in relation to two exogenous variables: the Consumer Price Index (CPI) and temperature anomalies.[1] [2] [3]

1. EDA of Italian Electricity Market

In this section, we'll explore how electricity consumption and production have influenced and reflected over 80 years of Italian history. From 1931 to 2014, Italy went through several major shifts, and the energy sector was deeply connected to

them. Key moments during this period include the fascist regime and World War II, the post-war economic boom of the 1960s, the political crisis of the 1990s that led to the birth of the Second Republic, and the 2008 global financial crisis, which was followed by yet another government crisis. These events not only shaped the country's political and economic landscape but also left their mark on how energy was produced and consumed.

The data that will be analyzed in our work come mainly from two sources.

- Historical data from 1931 to 2014 - Produzione lorda e consumo di energia elettrica in Italia - Anni 1883-2014 - come from the National Institute of Statistics (Istituto Nazionale di Statistica - ISTAT). To this data were aggregated data coming from the site Our World in Data, in particular the information related to GDP, population density and data related to annual anomalies in temperatures.

- The most recent data, which consider the period from 1990 to 2022, are instead taken from the site Our World in Data

- The data taken into consideration for the VARMAX model relating to energy prices come from the site FRED (Federal Reserve Economic Data)

1.1 An historic overview
We will now analyze the composition of the Italian electricity sector in detail, comparing the total production and total consumption of electricity. Regarding consumption, we can identify the sectors that are most energy-intensive considering the following variables:
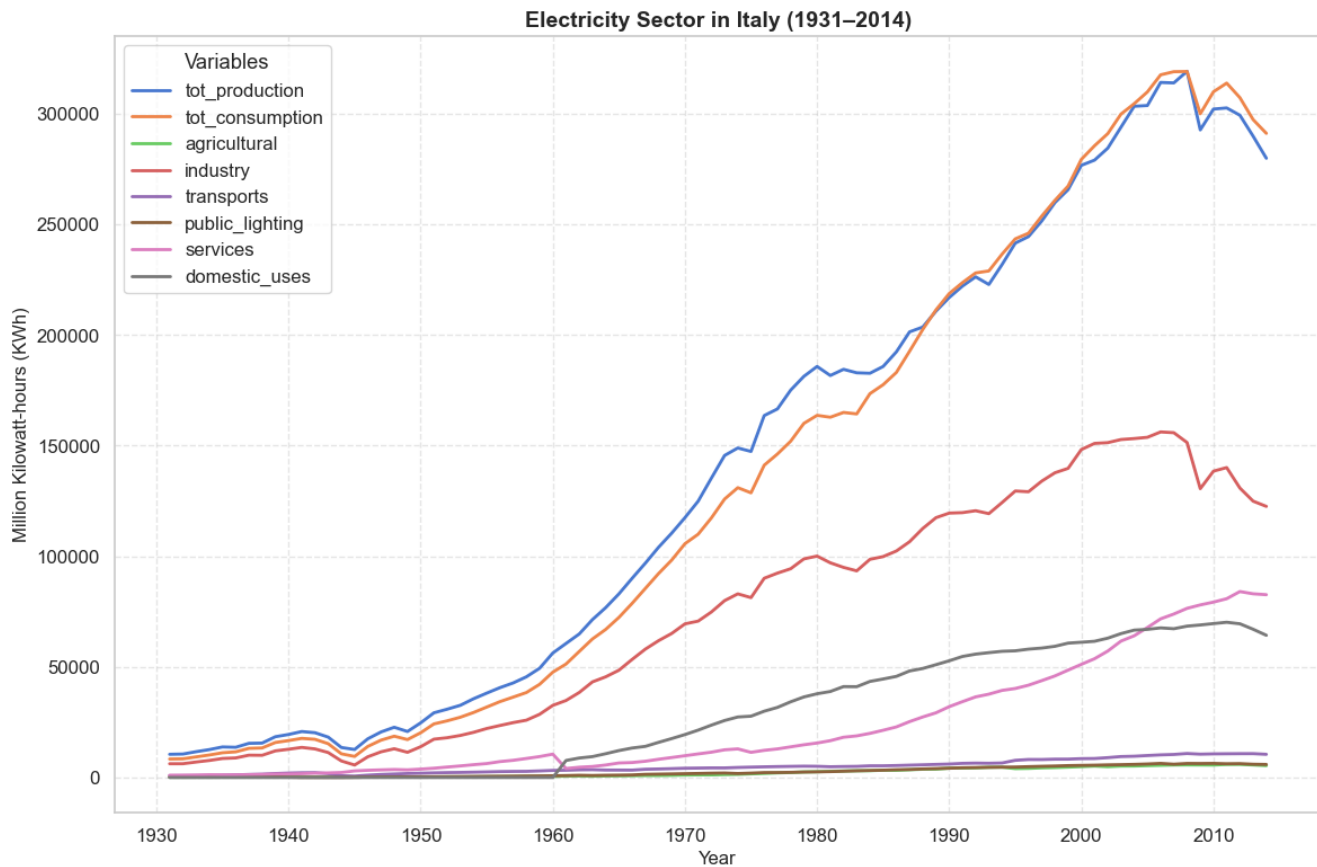
- agricultural sector

- industry

Figure 1. Electricity Sector in Italy (1931-2014) - data taken from ISTAT

- transport

- public lighting

- commerce, services, public administration

- domestic uses

All features taken into consideration are measured in millions of Kilo Watts per hour (kWh). As can be seen from figure 1 the most striking features is the sharp rise in electricity usage and production that began in the post-World War II period, especially from the 1950s onward. This rapid growth mirrors Italy's post-war economic boom, a time marked by industrial expansion and widespread modernization. During this phase, the industrial sector was by far the largest consumer of electricity, as shown by the dominant red line in the graph. Industrial demand continued to rise steadily until the early 2000s, when it began to decline—a trend likely tied to the 2008 financial crisis, the shift toward a service-based economy, and improvements in energy efficiency. While industrial use was leading for much of the century, electricity consumption in domestic and service sectors gradually increased over time. Domestic usage, represented by the black line, shows consistent growth throughout the entire period, reflecting the

electrification of Italian homes, the spread of household appliances, and the general rise in living standards. Similarly, the services sector saw a notable increase, particularly from the 1990s onward, likely influenced by the expansion of office spaces, retail, and digital infrastructure. On the other hand, sectors like agriculture and transport show relatively flat lines throughout the period, suggesting that these areas either relied less on electricity or continued to depend heavily on other energy sources such as fossil fuels. Public lighting also shows only modest growth, remaining one of the smallest categories. What's also notable is the close alignment between total electricity production and consumption. The two lines, blue and orange, track each other almost perfectly over the decades, suggesting that Italy maintained a relatively balanced energy system. However, toward the end of the period—after the 2008 economic downturn—production appears to fall slightly behind consumption, possibly indicating a rise in electricity imports or shifts in energy sourcing. The graph also captures key historical moments that shaped the sector. There's a noticeable dip during the 1940s, corresponding with the impact of World War II, followed by a significant surge through the 1960s and 70s during Italy's economic miracle. Growth begins to level off in the 1990s, coinciding with political and economic turbulence, and eventually shows signs of decline
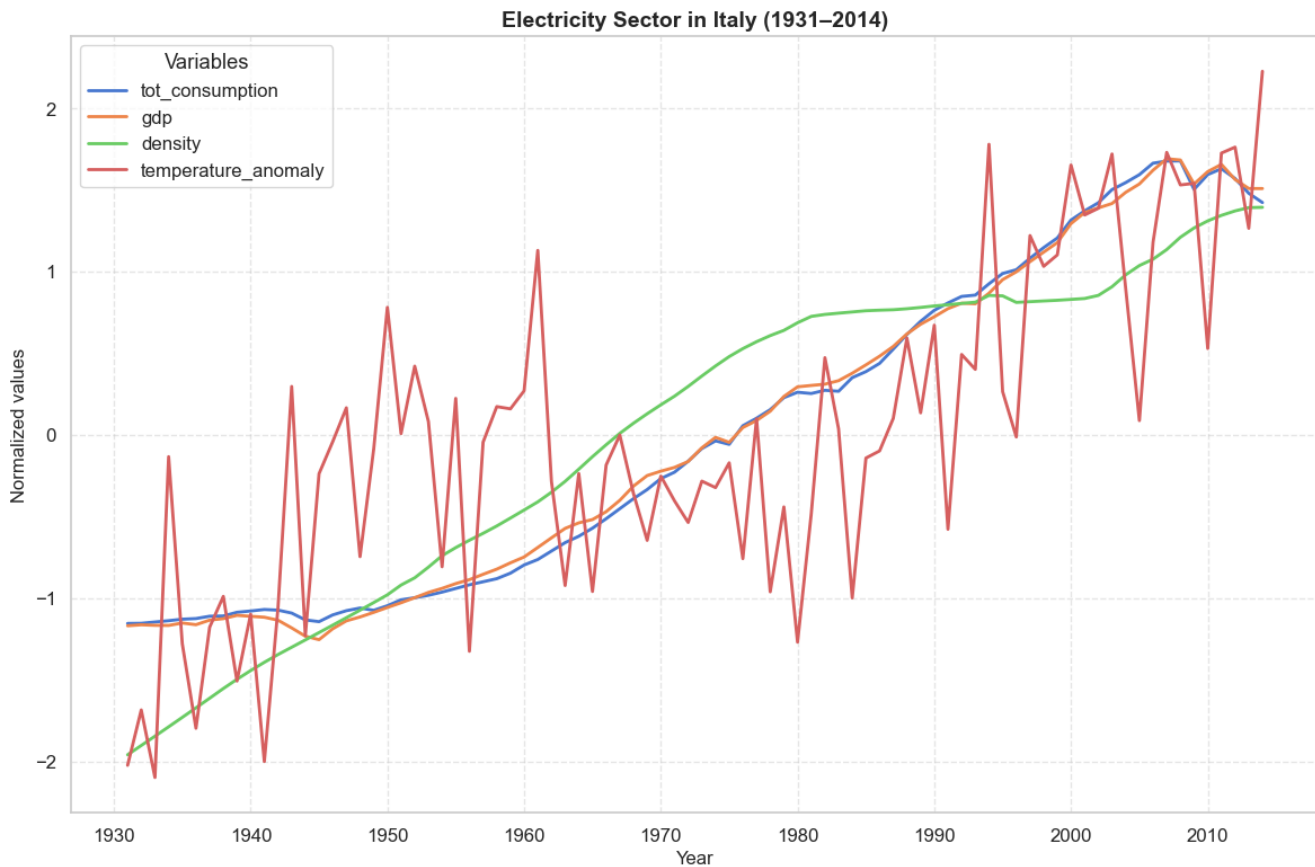
Figure 2. Electricity Sector in Italy (1931-2014) - data taken from ISTAT

following the global financial crisis.

Let's now proceed to the analysis of the variables that can directly influence the consumption of electricity. In this case we will include the variables:

- GDP: Gross Domestic Product

- population density: the number of people per km² of land area

- temperature anomaly: the difference between a year's average surface temperature from the 1991-2020 mean, in degrees Celsius.

These variables are represented in figure 2 where they have been normalized, meaning the variables are scaled to a similar range, making it easier to compare their trends over time. This is because variables like GDP which have values in the order of thousands of billions of dollars makes interpretation with respect to other variables difficult. At first glance, we can see that total electricity consumption(blue line) closely tracks GDP(orange line) throughout the entire period. This suggests a strong relationship between economic activity and energy demand: as the Italian economy grew, so did its need for electricity. The correlation becomes especially apparent in the post-WWII period, where both GDP and electricity

usage rise steeply, particularly from the 1950s through to the early 2000s. There are also synchronized dips during periods of economic slowdown—such as around the 1990s and the 2008 financial crisis. Population density(green line) also rises steadily until about the 1990s, where it starts to flatten. This trend reflects urbanization and demographic changes. Its influence on electricity consumption seems less direct than GDP's, but the gradual upward movement likely contributed to sustained residential and service-sector energy use. The most volatile line is that of temperature anomalies(red line), which fluctuates considerably from year to year. Despite the noise, there is a visible upward trend starting around the 1980s, pointing to a general warming trend consistent with climate change. Its relationship with electricity consumption may not be as tight or obvious, but temperature extremes—especially hotter summers—can impact energy use for things like air conditioning. In that context, the increasing anomalies in recent decades could explain part of the continuing rise in electricity demand even after industrial use began to level off. In sum, this visualization highlights how Italy's electricity consumption has been largely driven by economic performance and population factors, with a possible emerging influence from climate conditions in the later decades.

1.2 What about the last 30 years?

Now that we have a general overview of electricity consumption in Italy, we will analyze its characteristics in terms of production over the last thirty years, a period in which renewable energy has become more common and dependence on fossil fuels and coal is lower than in the twentieth century.

Figure 3 provides a detailed look into how Italy's electricity sector evolved from 1990 to 2022, with a specific focus on the composition of electricity generation by source. It reveals important transitions in the country's energy mix over the last three decades, especially the shift away from fossil fuels toward renewable and low-carbon sources.

From the beginning of the period, oil-based electricity(in pink) held a significant share but steadily declined, especially after the early 2000s. This drop coincides with Italy's broader efforts to reduce its dependency on oil for power generation, both for economic and environmental reasons. Gas-generated electricity(in orange), on the other hand, rose rapidly from the late 1990s, peaking around 2008–2010. It became the dominant fossil fuel in the energy mix, likely due to its lower emissions compared to coal and oil and its role as a transitional energy source. After peaking, however, gas usage started to show a slight decline and variability, possibly due to growing investment in renewables and geopolitical pressures affecting

gas imports. Coal electricity(light blue) remained relatively stable but modest over the years, and it too shows a declining trend after 2012, aligning with environmental targets and public pressure to phase out coal.

Looking at renewables, we see a different story. Solar(light peach) and wind energy(pink) began to grow notably in the mid-2000s, with solar in particular experiencing a sharp acceleration after 2010. This surge reflects national and EU incentives that encouraged investment in clean energy. Solar became one of the largest contributors within the renewables category by the end of the period. Hydroelectricity(green) remains a steady and traditional source of power for Italy, although its output shows some fluctuation likely tied to seasonal and climatic factors. It doesn't exhibit dramatic growth, but it holds its ground as a consistent contributor. The category of renewables excluding biofuel(dark brown) and low-carbon electricity (orange-brown) both show a clear upward trajectory, signaling the increasing share of greener energy sources in Italy's total production. These lines synthesize multiple clean sources, indicating a general shift toward de-carbonization. Notably, nuclear energy(red) stays at zero throughout the period, reflecting Italy's long-standing political and public opposition to nuclear power, following the 1987 referendum after the Chernobyl disaster.
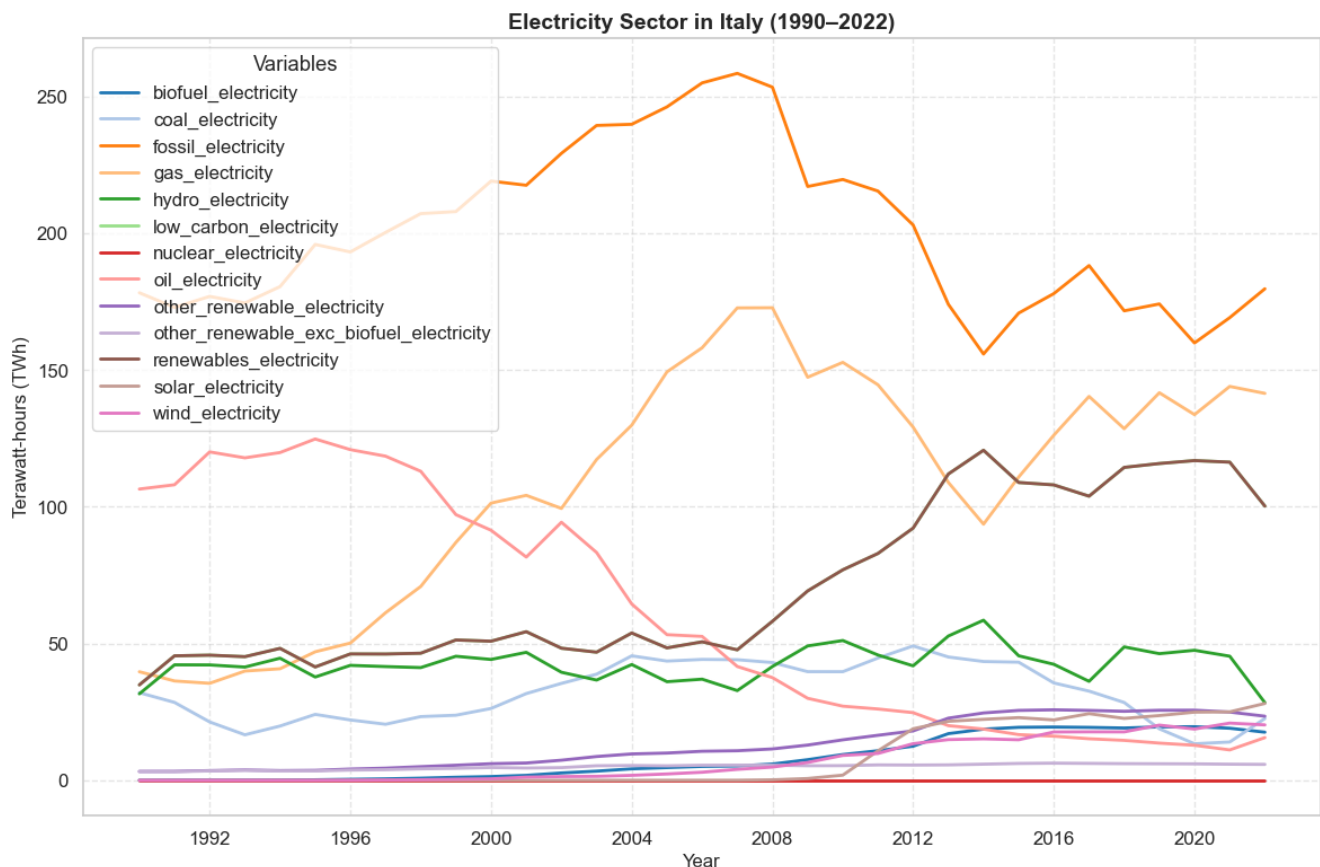

Figure 3. Electricity Sector in Italy (1990–2022) - data taken from Our World in Data

Altogether, this graph captures a critical transformation: Italy's energy policy has moved from reliance on fossil fuels—especially oil and gas—toward a more diversified and sustainable mix, with increasing weight given to solar, wind, and other renewables. The journey is ongoing, but the trend toward de-carbonization is clear and in line with broader European climate goals.

Let's now analyze the Italian electricity sector from a more economic point of view, considering variables such as net electricity imports and carbon intensity, that stands for greenhouse gases emitted per unit of generated electricity, measured in grams of co2 equivalents per kilowatt-hour. Beginning in the early 1990s, we observe a notable upward trend in both electricity demand and generation. This increase can be attributed to Italy's economic growth and the increasing reliance of society on electricity. Domestic generation expanded to fulfill a significant portion of this rising demand. It is important to acknowledge Italy's historical reliance on electricity imports to supplement domestic generation. As illustrated by the trend in "net electricity imports," while relatively small in comparison to overall demand and generation, this reliance has been a consistent characteristic of the country's energy mix.

We now turn our attention to a critical aspect of the sector: carbon intensity. This metric quantifies the volume of green-house gas emissions associated with each unit of electricity produced. The trajectory of carbon intensity is particularly notable. Initially, carbon intensity was elevated, reflecting a substantial dependence on fossil fuels such as coal and oil for power generation. However, the data reveal a marked decline over time. This decline indicates a transition towards cleaner energy sources, including natural gas, renewables such as solar and wind power, and potentially enhanced energy efficiency measures.

The reduction in carbon intensity is a key observation. It suggests that Italy has been undertaking efforts to decarbonize its electricity sector, a crucial step in mitigating climate change and meeting international environmental targets. It is important to note that the specific factors driving this transition, including policy changes, technological advancements, and economic variables, warrant further investigation. In summary, Italy's electricity sector has undergone a substantial transformation. We have observed increased demand and generation, a consistent reliance on electricity imports, and, significantly, a decrease in carbon intensity. This evolution reflects the complex interplay of economic growth, technological progress, and environmental considerations.
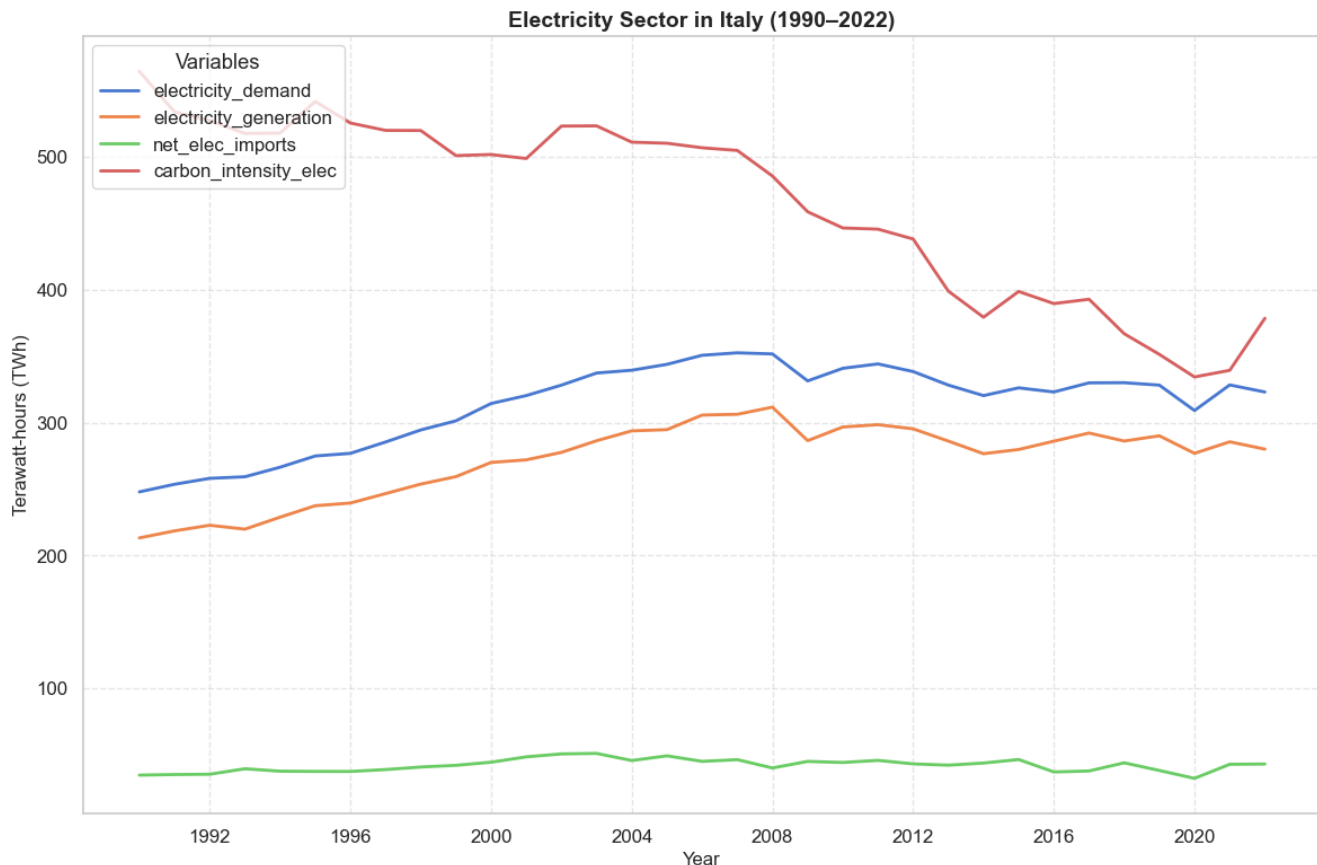

Figure 4. Electricity Sector in Italy (1990–2022) - data taken from Our World in Data

2. VAR model

Before proceeding with the construction of a VAR model, it is essential to first clarify the reasons behind the choice of variables and to examine their key characteristics. This preliminary step is crucial to ensure that the resulting model is both economically meaningful and statistically significant. In the case of the VAR model we are going to build, we will only focus on endogenous variables—that is, variables that influence each other and are determined within the system. The model will capture the dynamic relationships among these variables without including any external (exogenous) factors. At the same time, it is necessary to verify that these variables satisfy the stationarity condition, a fundamental requirement for the stability and reliability of the VAR framework.

By selecting variables based on both theoretical relevance and statistical properties, we ensure that the model we construct is not only capable of capturing meaningful economic relationships, but also robust in its predictions and consistent in its behavior over time. From an economic point of view, the variables included in the model were chosen according to the following criteria:

- **Total Production**: Total production of electricity refers to the total amount of electrical energy generated within a country or region. It includes all forms of energy production, such as from fossil fuels, nuclear, and renewable sources. As the total production of electricity increases, it typically reflects the need to meet rising demand from various sectors, including residential, commercial, and industrial users. Additionally, an increase in electricity production might be driven by the expansion of power plants or the integration of renewable energy sources into the grid. Changes in production levels are crucial for maintaining energy security and meeting the fluctuating demands of the economy.

- **Total Consumption**: Total consumption of electricity refers to the total amount of electricity consumed by all sectors within a country or region, including households, businesses, and industries. Higher consumption levels typically reflect greater economic activity, population growth, or increasing electrification in sectors such as transportation (e.g., electric vehicles) and technology. As the demand for electricity increases, there is often a corresponding rise in prices, especially when consumption surpass the capacity of the power grid. Total consumption is closely linked to factors like GDP growth, technological advancements, and changes in consumer behavior, all of which drive the demand for electricity.

- **GDP**: GDP is a widely recognized indicator of a country's economic activity and overall wealth. A higher GDP typically reflects increased industrial production, commercial activity, and consumer demand. As economies grow, businesses expand, households consume more,

and services proliferate — all of which require energy. Therefore, as GDP rises, electricity consumption tends to increase correspondingly, reflecting the heightened demand for lighting, heating, machinery, electronics, and other power-intensive processes across sectors.

- **CO2 Emmission**: CO_2 emissions are a direct consequence of burning fossil fuels for energy production, such as coal, oil, and natural gas. As energy consumption increases, particularly from non-renewable sources, CO_2 emissions tend to rise, contributing to climate change and environmental degradation. A higher level of CO_2 emissions often reflects a heavier reliance on traditional energy sources for electricity generation, whereas a shift towards cleaner, renewable energy sources tends to lower emissions. Therefore, changes in CO_2 emissions are not only an indicator of environmental impact but also of the energy mix and its implications for future electricity demand and policy considerations related to climate change.
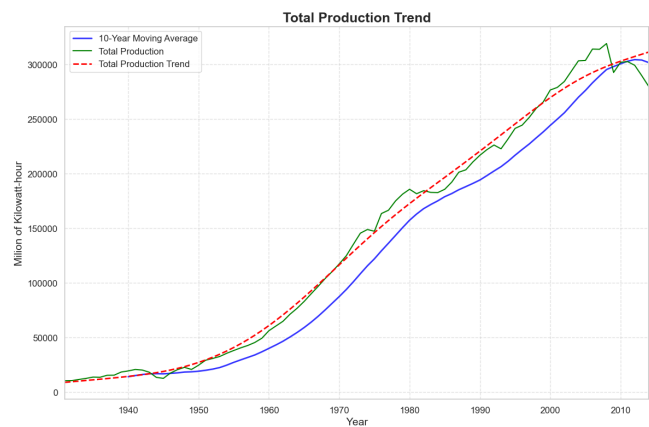

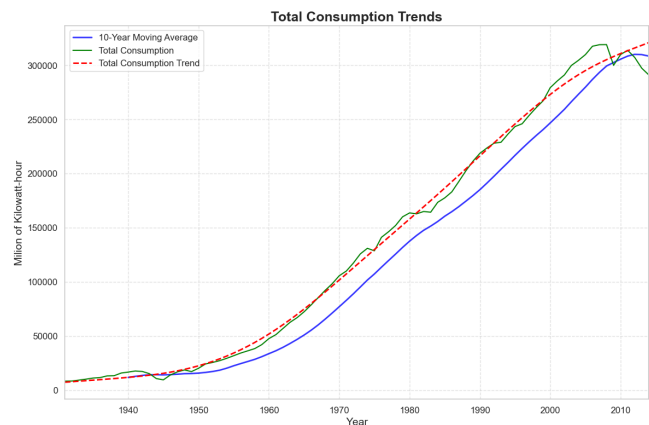**Figure 5.** Total Electricity Production Trend and 10 year Moving Average(MA)

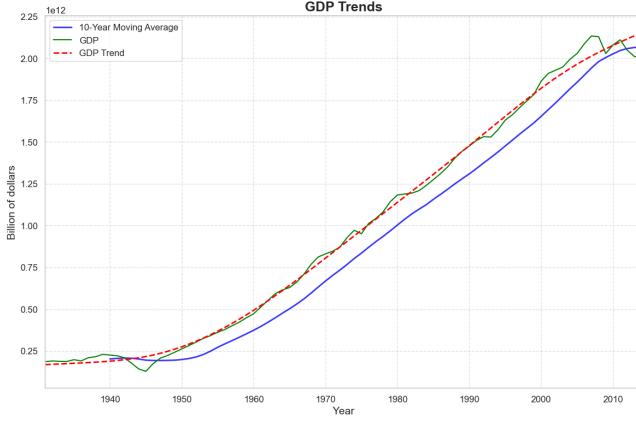**Figure 6.** Total Electricity Consumption Trend and 10 year Moving Average(MA)

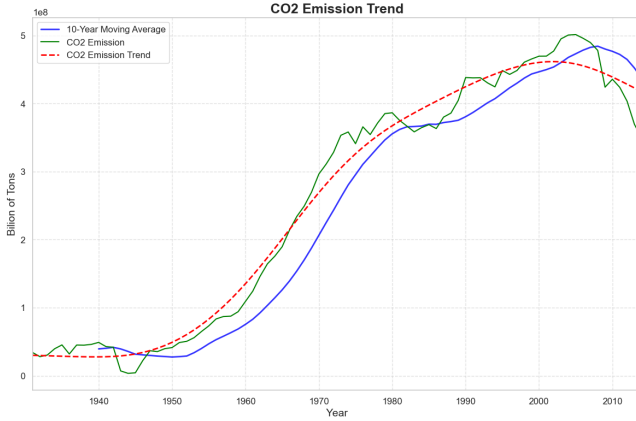**Figure 7.** GDP Trend and 10 year Moving Average(MA)

Figure 8. CO2 Emission Trend and 10 year Moving Average(MA)

Each variable is represented graphically, illustrating both raw data and smoothed trends using a 10-year moving average.

- **Total Production**: figure 5 shows the actual total production, exhibiting variations throughout the years. Focusing on the trend, we observe a general increase in total production over the majority of the period. The trend line shows a gradual rise until around the late 2000s, where it appears to level off and slightly decline.

- **Total Consumption**: figure 6, which depicts the total consumption trend indicates a strong, accelerating rise in total consumption, with the 10-year moving average highlighting this accelerating increase by smoothing out short-term fluctuations.

- **Gross Domestic Product (GDP)**: figure 7 illustrates the trend in GDP, a primary measure of economic output. A clear upward trend is evident, reflecting sustained economic growth. This indicates that the total value of goods and services produced has expanded over the period in question.

- **CO2 Emission**: as depicted in the figure 8, we see a significant increase in CO2 emissions starting around the mid-20th century, peaking around the mid-2000s, and then showing a slight decline towards 2010. The 10-year moving average and the overall trend line both illustrate this general pattern of rising and then slightly falling emissions over the observed period.

Since the variables in our dataset are measured on very different scales, we begin by normalizing them. This step allows for a more balanced comparison and ensures that no single variable influences the model due to its scale. The resulting normalized trends are shown in Figure 2. Normalizing the dataset enhances the stability and robustness of the VAR model by aligning all variables to a common scale while preserving their original temporal patterns and relationships, as previously discussed.

The normalization is performed using the following formula:

$$z_{i,j} = \frac{x_{i,j} - \mu_j}{\sigma_j}$$

where $x_{i,j}$ is the original value, μ_j is the mean of feature j, and σ_j is the standard deviation of feature j.

After that we proceed with the study of the stationarity of the series, which we will carry out using the Augmented Dickey–Fuller test. This test is a widely used statistical test in time series analysis designed to determine whether a given time series is stationary or possesses a unit root, implying non-stationarity. This distinction is fundamental in time series modeling, as many forecasting models, such as the VAR model, assume that the underlying data is stationary. In other words, stationarity implies that the series maintains a constant mean, variance, and auto-covariance structure over time.

The test is built upon the following hypotheses:

- Null hypothesis H_0: the time series has a unit root (*non-stationary*).

- Alternative hypothesis H_1: the time series does not have a unit root (*stationary*).

The ADF test is an extension of the simpler Dickey-Fuller test, enhanced by accounting for higher-order autoregressive processes through the inclusion of lagged differences of the dependent variable. This ensures that autocorrelation in the residuals is controlled.

The model estimated by the ADF test is:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \varepsilon_t$$

Where:

- Δy_t is the first-differenced series,

- α is a constant term (optional),

- βt is the coefficient on a time trend (optional),

- γ is the coefficient we test for a unit root,

- δ_i are the coefficients for the lagged differences,

- ε_t is a white noise error term.

The primary focus is on the coefficient γ. If $\gamma = 0$, the process has a unit root, indicating non-stationarity. Rejecting the null hypothesis implies $\gamma < 0$, and thus the time series is stationary.

The output of the test includes:

1. The ADF test statistic

2. The p-value

3. The number of lags used

4. The number of observations

5. Critical values at 1%, 5%, and 10% significance levels

```
------------------------------------------------
Augmented Dickey-Fuller Test: tot_production
ADF test statistic      -1.351129
p-value                  0.605469
# lags used              4.000000
# observations          79.000000
critical value (1%)     -3.515977
critical value (5%)     -2.898886
critical value (10%)    -2.586694
Weak evidence against the null hypothesis
Fail to reject the null hypothesis
Data has a unit root and is non-stationary
------------------------------------------------
Augmented Dickey-Fuller Test: gdp
ADF test statistic      -0.318433
p-value                  0.922853
# lags used              3.000000
# observations          80.000000
critical value (1%)     -3.514869
critical value (5%)     -2.898409
critical value (10%)    -2.586439
Weak evidence against the null hypothesis
Fail to reject the null hypothesis
Data has a unit root and is non-stationary
```

```
------------------------------------------------
Augmented Dickey-Fuller Test: co2
ADF test statistic      -1.352642
p-value                  0.604758
# lags used              2.000000
# observations          81.000000
critical value (1%)     -3.513790
critical value (5%)     -2.897943
critical value (10%)    -2.586191
Weak evidence against the null hypothesis
Fail to reject the null hypothesis
Data has a unit root and is non-stationary
------------------------------------------------
Augmented Dickey-Fuller Test: tot_consumption
ADF test statistic      -1.627755
p-value                  0.468666
# lags used              5.000000
# observations          78.000000
critical value (1%)     -3.517114
critical value (5%)     -2.899375
critical value (10%)    -2.586955
Weak evidence against the null hypothesis
Fail to reject the null hypothesis
Data has a unit root and is non-stationary
```

Figure 9. Augmented Dickey–Fuller Test Before any Differencing

The interpretation is as follows:

- If the p-value is less than or equal to 0.05, we reject the null hypothesis. This provides strong evidence that the time series is stationary.

- If the p-value is greater than 0.05, we fail to reject the null hypothesis. This suggests the time series is non-stationary, and likely contains a unit root.

If non-stationarity is detected in a time series, the standard practice is to transform the series—most commonly through differencing—until stationarity is achieved. This process is crucial, as non-stationary data can lead to misleading results in VAR models due to trends and evolving variances over time. To confirm the effectiveness of such transformations, the Augmented Dickey-Fuller (ADF) test is reapplied after each differencing step. Upon conducting the ADF test on our normalized series, the results—presented in Figure 9—indicate that none of the original time series are stationary at conventional significance levels. Consequently, we proceed with differencing the data to eliminate unit roots and achieve stationarity.

As shown in Figure 10, the variable representing total electricity production required a second-order differencing to meet stationarity criteria, revealing a strong and persistent trend component in its original form. Likewise, the total electricity

```
tot_production: differenced 1 time(s), p-value = 0.5047
tot_production: differenced 2 time(s), p-value = 0.0000
gdp: differenced 1 time(s), p-value = 0.0383
co2: differenced 1 time(s), p-value = 0.0068
tot_consumption: differenced 1 time(s), p-value = 0.5168
tot_consumption: differenced 2 time(s), p-value = 0.0000

Number of differences applied per variable:
tot_production    2
gdp               1
co2               1
tot_consumption   2
dtype: int64

Stationary DataFrame ready for VAR model:
            tot_production      gdp       co2   tot_consumption
year
1933-01-01        0.008838 -0.003220  0.013688         0.007185
1934-01-01       -0.001034 -0.000713  0.052198        -0.000064
1935-01-01        0.002351  0.014918  0.032472         0.001304
1936-01-01       -0.012712 -0.010294 -0.076058        -0.005371
1937-01-01        0.018184  0.027898  0.074910         0.010504
```

Figure 10. Differencing the Series

consumption series exhibited similar behavior, necessitating two rounds of differencing. This suggests the presence of a structural or long-term trend in energy demand patterns over time. On the other hand, the series for CO2 emissions and GDP displayed lighter non-stationary behavior and reached stationarity after first-order differencing, indicating more stable dynamics in these variables across the observed period.

Following the application of the necessary differencing procedures, and after excluding the missing values introduced by lagging operations, we obtain a refined and fully stationary dataset. This final dataset spans from 1933 to 2014 and consists of 81 valid observations. It provides a robust basis for estimating a VAR model and conducting meaningful impulse response and causality analyses.

2.1 Model Results and Interpretation
Once the initial data analysis process is complete and the data has been normalized and made stationary, we can proceed with the creation of our model. In order to determine the optimal lag length for the VAR model, we evaluated the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) across a range of lag orders from 1 to 7.

As shown in the results, the AIC reaches its lowest value at lag order 3, (AIC=-27.16), indicating that this specification balances model fit and complexity most effectively according to this criterion. The BIC, which penalizes model complexity more heavily, achieves its minimum at lag order 1 (BIC=-26.31), suggesting a more parsimonious model.

After comparing different lag orders using AIC and BIC, we select lag order 3 for the VAR model. While more parsimonious models like VAR(1) may be too simple to capture the complex dynamics among the variables, VAR(3) offers a better balance between fit and model complexity. It is detailed enough to account for meaningful interdependencies without introducing excessive parameters. Therefore, the VAR model

is estimated using a lag order of 3.

```
Order = 1
AIC:  -26.903392001525805
BIC:  -26.312169988026437

Order = 2
AIC:  -27.162511188812264
BIC:  -26.090599203209017

Order = 3
AIC:  -27.567667874887334
BIC:  -26.008031313769802

Order = 4
AIC:  -27.399535678747057
BIC:  -25.34496900932536

Order = 5
AIC:  -27.429565749720165
BIC:  -24.872687107697967

Order = 6
AIC:  -27.347657772008922
BIC:  -24.280903376895328

Order = 7
AIC:  -27.37532200917486
BIC:  -23.790940393572033
```

Figure 12. VAR Model Tuning

We now apply a VAR (Vector Autoregressive) model of order 3 to a system of four variables: total production (tot_production), GDP (gdp), CO2 emissions (co2), and total consumption (tot_consumption). The goal of the model is to analyze the dynamic relationships between these variables over time.

- **Total Production Equation:** The model shows that total production is negatively influenced by the lagged value of total production itself, suggesting that an increase in production in the previous period can lead to a decrease in the current period's production. The influence of lagged GDP and CO2 emissions is relatively weak. However, the lagged value of total consumption has a marginally significant positive effect(0.59, p=0.078), suggesting that an increase in consumption can lead to an increase in production in the subsequent period. This could reflect a demand-driven production increase, where higher consumption leads firms to ramp up production to meet consumer demand.

- **GDP Equation:** GDP is positively influenced by both the lagged value of GDP itself (L1: 0.65, p=0.001) and total production (L1: -0.43, p=0.008). The positive effect of GDP on itself suggests some degree of persistence in economic growth, while the negative effect of lagged total production implies that an increase

in production in the prior period can reduce GDP in the current period, possibly due to capacity constraints or diminishing economic returns to production. The relationship between GDP and CO2 emissions is not statistically significant, which suggests that the economic activity, in this case, may not be directly linked to emissions over the short term. Total consumption has a negligible and insignificant effect on GDP, suggesting

that fluctuations in consumption may not immediately impact GDP growth, at least within the short-term dynamics captured by the model.

- **CO2 Emissions Equation:** CO2 emissions are significantly influenced by their own lag (L1: 0.34, p=0.041 and L2: 0.36, p=0.030), showing persistence in emission levels. This means that emissions from the previous periods strongly affect current emissions, which

```
    Summary of Regression Results
==================================
Model:                     VAR
Method:                    OLS
Date:           Thu, 01, May, 2025
Time:                  20:09:06
--------------------------------------------------------------
No. of Equations:      4.00000    BIC:                  -26.0906
Nobs:                  80.0000    HQIC:                 -26.7328
Log likelihood:        668.440    FPE:               1.60374e-12
AIC:                   -27.1625   Det(Omega_mle):    1.04697e-12
--------------------------------------------------------------
Results for equation tot_production
========================================================================
                   coefficient     std. error        t-stat         prob
------------------------------------------------------------------------
const                 0.012466       0.008898         1.401        0.161
L1.tot_production    -1.290451       0.227074        -5.683        0.000
L1.gdp               -0.088090       0.284849        -0.309        0.757
L1.co2                0.097099       0.108929         0.891        0.373
L1.tot_consumption    0.593163       0.336016         1.765        0.078
L2.tot_production    -0.447548       0.234488        -1.909        0.056
L2.gdp               -0.426415       0.264417        -1.613        0.107
L2.co2                0.021036       0.110408         0.191        0.849
L2.tot_consumption    0.273260       0.294536         0.928        0.354
========================================================================


Results for equation gdp
========================================================================
                   coefficient     std. error        t-stat         prob
------------------------------------------------------------------------
const                 0.019974       0.006356         3.143        0.002
L1.tot_production    -0.432818       0.162190        -2.669        0.008
L1.gdp                0.649680       0.203456         3.193        0.001
L1.co2                0.106981       0.077804         1.375        0.169
L1.tot_consumption    0.051341       0.240003         0.214        0.831
L2.tot_production    -0.204690       0.167485        -1.222        0.222
L2.gdp               -0.395794       0.188863        -2.096        0.036
L2.co2                0.076527       0.078860         0.970        0.332
L2.tot_consumption   -0.102649       0.210375        -0.488        0.626
========================================================================


Results for equation co2
========================================================================
                   coefficient     std. error        t-stat         prob
------------------------------------------------------------------------
const                 0.013461       0.013502         0.997        0.319
L1.tot_production    -0.594905       0.344565        -1.727        0.084
L1.gdp                0.105767       0.432233         0.245        0.807
L1.co2                0.337241       0.165291         2.040        0.041
L1.tot_consumption    0.408762       0.509874         0.802        0.423
L2.tot_production    -0.392921       0.355815        -1.104        0.269
L2.gdp               -0.381852       0.401229        -0.952        0.341
L2.co2                0.362672       0.167534         2.165        0.030
L2.tot_consumption    0.207877       0.446932         0.465        0.642
========================================================================
```

```
Results for equation tot_consumption
==============================================================================
                         coefficient      std. error        t-stat          prob
------------------------------------------------------------------------------
const                       0.010423        0.006560         1.589         0.112
L1.tot_production          -0.542026        0.167409        -3.238         0.001
L1.gdp                      0.190370        0.210003         0.907         0.365
L1.co2                      0.070973        0.080307         0.884         0.377
L1.tot_consumption         -0.342063        0.247726        -1.381         0.167
L2.tot_production          -0.162515        0.172875        -0.940         0.347
L2.gdp                     -0.612407        0.194940        -3.142         0.002
L2.co2                      0.010614        0.081398         0.130         0.896
L2.tot_consumption         -0.181802        0.217145        -0.837         0.402
==============================================================================


Correlation matrix of residuals
                  tot_production        gdp        co2   tot_consumption
tot_production          1.000000   0.717630   0.696437          0.865451
gdp                     0.717630   1.000000   0.676748          0.812699
co2                     0.696437   0.676748   1.000000          0.745793
tot_consumption         0.865451   0.812699   0.745793          1.000000
```

Figure 11. Summary Statistics of VAR Model at lag=3

is consistent with the idea that industries may follow carbon-intensive practices that persist over time. The negative relationship between lagged production (L1: -0.59, p=0.084) and CO_2 emissions suggests that reduced production in the previous period may lead to lower emissions, likely due to reduced industrial activity or lower energy consumption. The relationship between CO_2 emissions and GDP is weak, further supporting the idea that emissions may not always respond directly to economic growth in the short term, as economic activities and environmental outcomes can diverge depending on other factors like technological improvements or shifts in industrial practices.

- **Total Consumption Equation:** Total consumption is negatively influenced by the lagged values of both total production (L1: -0.54, p=0.001) and GDP (L2: -0.61, p=0.002), suggesting that when production or GDP growth is high in the past, consumption may decline in the subsequent period. This could be due to increased prices or lower disposable income as a result of higher production costs or economic slowdowns. CO_2 emissions and lagged total consumption have insignificant effects on current consumption, suggesting that environmental factors and past consumption habits do not strongly affect consumer behavior in the short term.

Residual Correlation Matrix: The residual correlation matrix shows high correlation among the variables, particularly between total production and total consumption (0.87), and between GDP and total consumption (0.81). These high correlations suggest a strong interrelationship between the economic and environmental variables. For instance, a change in production levels often leads to a corresponding change in consumption, as production creates jobs and income, which in turn drives consumption. Similarly, GDP and total consumption are highly interconnected, as higher economic output typically supports greater consumption levels.

Economic Implications:

- Production and Consumption Dynamics: The results indicate that production and consumption are tightly linked. An increase in consumption often leads to a boost in production, while higher production levels in the past can suppress both consumption and GDP growth. This underscores the potential for demand-side policies (such as stimulating consumption) to drive economic output in the short term.

- Environmental Impact and Policy Recommendations: The persistence of CO_2 emissions suggests that environmental regulations and technological innovation will be crucial in reducing emissions over time. Policies aimed at reducing production-related emissions, such as carbon pricing or green technology subsidies, could mitigate the long-term environmental impacts of economic growth.

- Short-Term Economic Fluctuations: The weak effects of CO_2 emissions on both total production and GDP suggest that, in the short run, emissions may not be as responsive to changes in economic activity as previously expected. However, their persistence over time highlights the importance of considering long-term sustainability in economic planning.

In conclusion, the VAR(3) model provides valuable insights into the dynamics between economic growth, production, consumption, and environmental emissions.

3. VARMAX(4,3) Model

Before analyzing in depth the VARMAX model relating to electricity and gas prices in Italy, let's examine these variables in relation to the exogenous variables CPI (Consumer Price Index) and temperature anomaly.

The variables and their behavior are presented in Figure 13 over this nearly three-decade span. By observing their trajectories, we can gain insights into potential relationships and significant events that might have influenced the electricity market. The electricity price, represented by the blue line, exhibits a period of relative stability from the late 1990s up until the mid-2000s. Following this, it generally trends upwards with some noticeable volatility. The most striking feature is the dramatic surge observed around 2022, followed by a subsequent decline, although remaining at a higher level compared to the pre-2022 period. The temperature anomaly, depicted in orange, oscillates around the zero mark, indicating years that were warmer or cooler than the average. While there doesn't appear to be a strong long-term trend, the fluctuations suggest the variability in seasonal weather patterns. The gas price, shown in green, mirrors the electricity price to some extent, particularly the significant spike around 2022. This suggests a potential correlation between gas prices and electricity prices, which isn't surprising given that natural gas is a significant

fuel source for electricity generation in many regions. Finally, the CPI, illustrated by the red line, demonstrates a gradual and relatively consistent upward trend over the entire period. This reflects the general inflation experienced in the Italian economy. In summary, the plot highlights the interconnectedness of energy prices, particularly the sharp increase in electricity and gas prices in recent years. It also provides context with the temperature anomaly and the broader economic indicator of the CPI.

A more partial comparison of trend and seasonality between variables can be seen in Figure 14, where the variables have been normalized by removing the mean and dividing by the standard deviation. Looking at the normalized data:

- The normalized electricity price(blue line) shows periods of both relative stability and significant volatility. The sharp spike around 2022 is still evident, indicating a substantial relative increase compared to its historical values within this normalized scale.

- The normalized gas price(green line) also exhibits a notable surge around 2022, similar to the electricity price, reinforcing the idea of a strong relationship between these two energy sources.

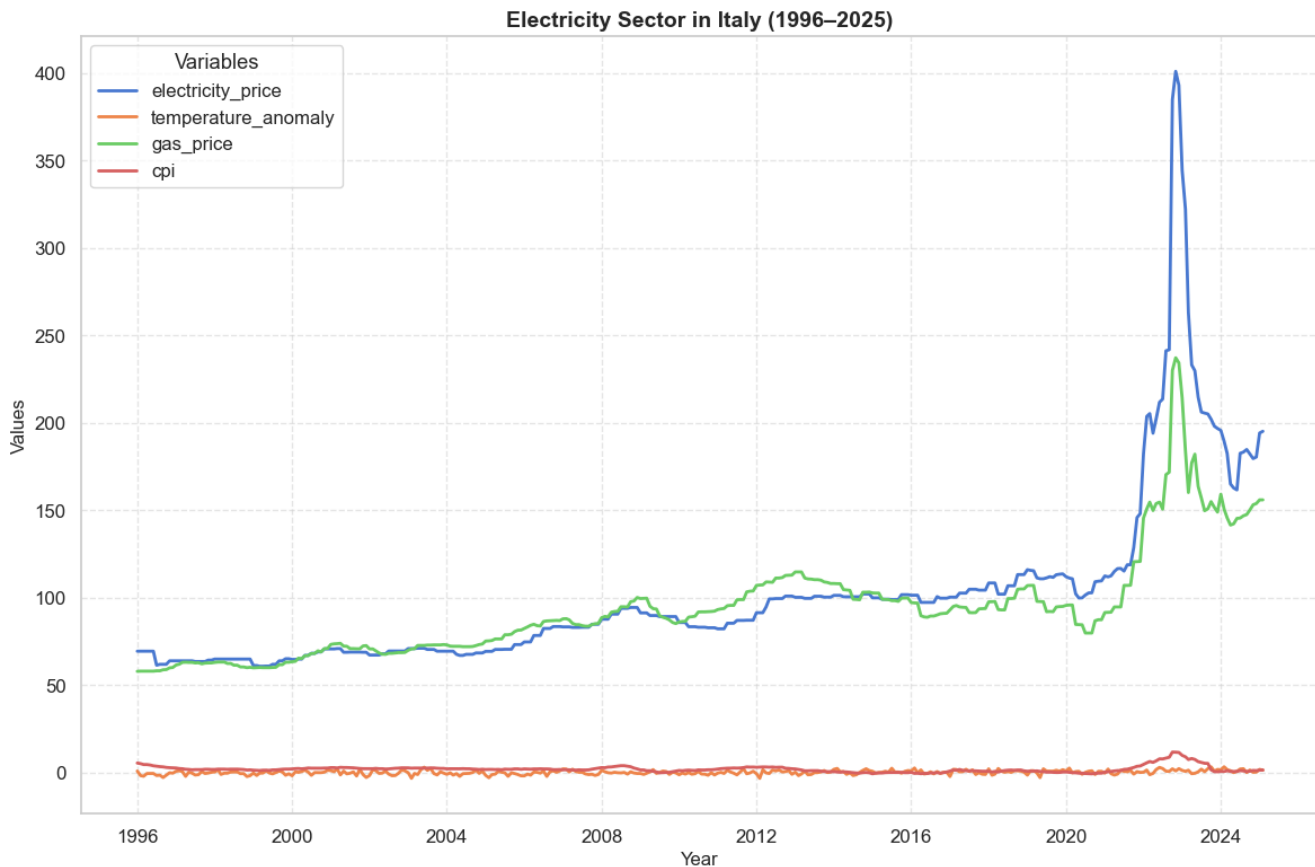- The normalized temperature anomaly(orange line) fluc-

Figure 13. Electricity Sector in Italy (1996–2025) - data taken from FRED(Federal Reserve Economic Data)
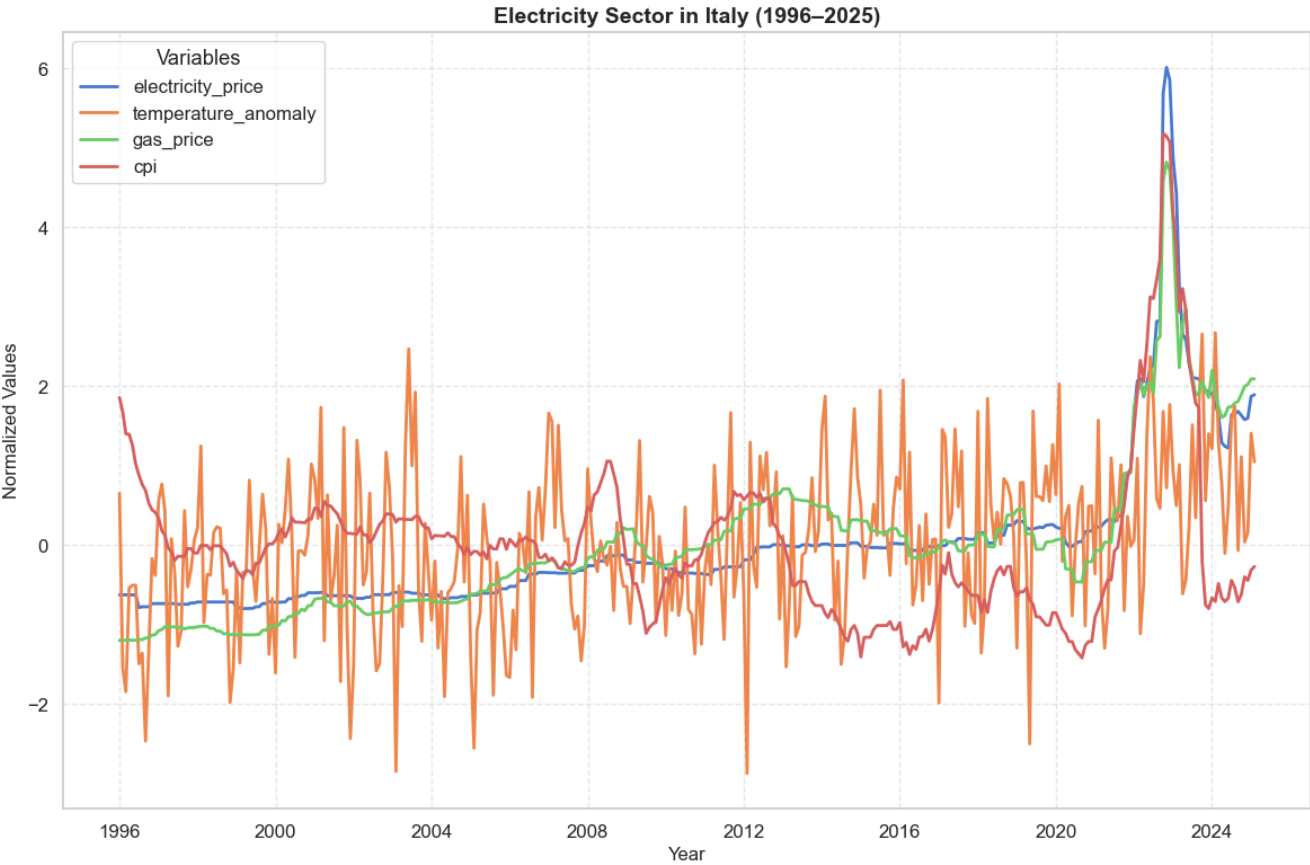
Figure 14. Electricity Sector in Italy (1996–2025) - data taken from FRED(Federal Reserve Economic Data)

tuates around zero, as expected with normalized anomaly data. These variations represent periods that were relatively warmer or cooler compared to the average within the dataset's time-frame.

- The normalized CPI(red line) shows more moderate fluctuations compared to the energy prices, but still captures the relative changes in the overall price level within the normalized scale.

By normalizing the data, we can more readily compare the relative magnitudes of the changes in each variable. For instance, even though the absolute values of electricity and gas prices were much higher than the temperature anomaly or CPI, the normalization allows us to see that the proportional increase in electricity and gas prices around 2022 was quite substantial relative to their own historical fluctuations and also in comparison to the changes in temperature anomaly and CPI within this normalized framework.

We therefore proceed with the creation of the VARMAX model that considers the normalized data presented in figure 14. First of all, we analyze the stationarity of the data to understand how much to differentiate the historical series under examination.

```
Augmented Dickey-Fuller Test: electricity_price
ADF test statistic          -1.911607
p-value                      0.326621
# lags used                  9.000000
# observations             340.000000
critical value (1%)         -3.449730
critical value (5%)         -2.870079
critical value (10%)        -2.571319
Weak evidence against the null hypothesis
Fail to reject the null hypothesis
Data has a unit root and is non-stationary
-------------------------------------------------
Augmented Dickey-Fuller Test: temperature_anomaly
ADF test statistic       -8.023061e+00
p-value                   2.051624e-12
# lags used               2.000000e+00
# observations            3.470000e+02
critical value (1%)      -3.449337e+00
critical value (5%)      -2.869906e+00
critical value (10%)     -2.571227e+00
Strong evidence against the null hypothesis
Reject the null hypothesis
Data has no unit root and is stationary
-------------------------------------------------
```

```
-------------------------------------------
Augmented Dickey-Fuller Test: gas_price
ADF test statistic        -1.045757
p-value                    0.736212
# lags used               17.000000
# observations           332.000000
critical value (1%)       -3.450201
critical value (5%)       -2.870285
critical value (10%)      -2.571429
Weak evidence against the null hypothesis
Fail to reject the null hypothesis
Data has a unit root and is non-stationary
-------------------------------------------
Augmented Dickey-Fuller Test: cpi
ADF test statistic        -3.278165
p-value                    0.015883
# lags used               14.000000
# observations           335.000000
critical value (1%)       -3.450022
critical value (5%)       -2.870207
critical value (10%)      -2.571387
Strong evidence against the null hypothesis
Reject the null hypothesis
Data has no unit root and is stationary
```

Figure 15. Augmented Dickey–Fuller Test before any Differencing

```
electricity_price: differenced 1 time(s), p-value = 0.0000
gas_price: differenced 1 time(s), p-value = 0.0000

Number of differences applied per variable:
electricity_price      1
gas_price              1
temperature_anomaly    0
cpi                    0
dtype: int64

Stationary DataFrame ready for VARMAX model:
                 electricity_price  temperature_anomaly  gas_price      cpi
observation_date
1996-02-01                    0.0            -1.551221    0.00336  1.670550
1996-03-01                    0.0            -1.847361    0.00000  1.399165
1996-04-01                    0.0            -0.541939    0.00000  1.394534
1996-05-01                    0.0            -0.506993    0.00000  1.259119
1996-06-01                    0.0            -0.505565    0.00000  1.019321
```

Figure 16. Differencing the Series

From the graph in Figure 15, we note that the endogenous variables are not stationary, while the exogenous ones exhibit stationarity. Therefore, we proceed by differencing the non-stationary variables until they exhibit constant mean, variance, and auto-covariance over time. As shown in Figure 16, a first-order differencing (lag 1) is sufficient to achieve stationarity for the endogenous variables. We can now divide the normalized and stationary dataset into training and testing sets. Specifically, the last 12 months of the dataset, from '2024-03-01'to '2025-02-01', will be used as the test set.

Subsequently, we select the optimal VARMAX(p,q) model by determining the values of p (the number of auto-regressive lags) and q (the number of moving average lags) that minimize information criteria such as AIC or BIC.

```
Trying order (p=1, q=0) | AIC = -769.0637
Trying order (p=1, q=1) | AIC = -803.1421
Trying order (p=1, q=2) | AIC = -849.7031
Trying order (p=1, q=3) | AIC = -856.2307
Trying order (p=2, q=0) | AIC = -826.9422
Trying order (p=2, q=1) | AIC = -841.9646
Trying order (p=2, q=2) | AIC = -937.0181
Trying order (p=2, q=3) | AIC = -952.2422
Trying order (p=3, q=0) | AIC = -841.0907
Trying order (p=3, q=1) | AIC = -907.0727
Trying order (p=3, q=2) | AIC = -949.3261
Trying order (p=3, q=3) | AIC = -1009.6462
Trying order (p=4, q=0) | AIC = -909.7697
Trying order (p=4, q=1) | AIC = -917.5978
Trying order (p=4, q=2) | AIC = -970.2571
Trying order (p=4, q=3) | AIC = -1045.1780
Trying order (p=5, q=0) | AIC = -936.4276
Trying order (p=5, q=1) | AIC = -955.3404
Trying order (p=5, q=2) | AIC = -1018.7196
Trying order (p=5, q=3) | AIC = -1032.6925

Best order selected: (p=4, q=3) with AIC = -1045.1780
```

Figure 17. Best Model According to AIC

```
Trying order (p=1, q=0) | BIC = -719.4027
Trying order (p=1, q=1) | BIC = -738.2007
Trying order (p=1, q=2) | BIC = -769.4813
Trying order (p=1, q=3) | BIC = -760.7286
Trying order (p=2, q=0) | BIC = -762.0008
Trying order (p=2, q=1) | BIC = -761.7429
Trying order (p=2, q=2) | BIC = -841.5160
Trying order (p=2, q=3) | BIC = -841.4598
Trying order (p=3, q=0) | BIC = -760.8690
Trying order (p=3, q=1) | BIC = -811.5706
Trying order (p=3, q=2) | BIC = -838.5437
Trying order (p=3, q=3) | BIC = -883.5835
Trying order (p=4, q=0) | BIC = -814.2676
Trying order (p=4, q=1) | BIC = -806.8154
Trying order (p=4, q=2) | BIC = -844.1944
Trying order (p=4, q=3) | BIC = -903.8350
Trying order (p=5, q=0) | BIC = -825.6452
Trying order (p=5, q=1) | BIC = -829.2777
Trying order (p=5, q=2) | BIC = -877.3766
Trying order (p=5, q=3) | BIC = -876.0691

Best order selected: (p=4, q=3) with BIC = -903.8350
```

Figure 18. Best Model According to BIC

The VARMAX (Vector Auto-regressive Moving Average with exogenous variables) model extends the VARMA framework by incorporating external predictors, allowing it to capture both the internal dynamics among multiple time

series and the influence of external shocks. Estimating the optimal lag structure ensures that the model balances complexity with predictive power, avoiding overfitting while adequately representing the temporal dependencies in the data.

Looking at Figures 17 and 18, we observe that both the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) identify the model with parameters p = 4 and q = 3 as the optimal configuration. This suggests that a VARMAX(4, 3) model best balances goodness of fit and model complexity, according to both selection criteria.

3.1 Model Results and Interpretation

We therefore proceed with the creation of the VARMAX(4,3) model, the coefficient estimates of which are presented in Figure 19. This model specification includes four lags of the endogenous variables—electricity prices and gas prices—and three lags of their respective moving average (MA) components. This structure allows for the capture of both autoregressive dynamics and shock propagation over time, providing a comprehensive view of the short and medium term interactions between the two energy markets. The inclusion of exogenous regressors, such as temperature anomalies and the Consumer Price Index (CPI), further enhances the model's ability to reveal the effects of broader environmental and macroeconomic trends on price behavior. The overall model fit appears satisfactory. The Akaike Information Criterion (AIC) of -1045.18 supports model parsimony while maintaining explanatory power. Moreover, the Ljung-Box Q-statistics for the residuals suggest that there is no significant autocorrelation remaining, indicating that the model sufficiently captures the temporal structure of the data. However, residual diagnostics point to deviations from normality and the presence of heteroskedasticity, which implies that some aspects of volatility, potentially originating from regime changes, extreme events, or structural breaks, are not fully captured by the linear framework. These findings highlight the limitations of the model in representing certain nonlinearities that may characterize energy markets, particularly during periods of crisis or policy intervention.

Focusing first on the electricity price equation, the model reveals that none of the lagged values of gas prices are statistically significant. This finding implies that, within the temporal window considered, gas prices do not Granger-cause electricity prices. This is somewhat counterintuitive from an economic standpoint, as natural gas is a key input in electricity generation in many economies. However, this result may reflect structural characteristics of electricity markets—such as regulatory price caps, market segmentation, or long-term generation contracts—that isolate electricity prices from short-term volatility in fuel costs. Additionally, exogenous regressors such as temperature anomalies and CPI also exhibit statistically insignificant coefficients, suggesting that, at least in the short run, neither climatic variability nor general price inflation directly influence electricity pricing. This supports the hypothesis that electricity markets, especially in regulated or partially deregulated contexts, are primarily driven by internal market mechanisms and supply-demand balances rather than external shocks.

In contrast, the gas price equation provides a slightly more responsive structure. While most lagged endogenous variables and error terms remain statistically insignificant, the coefficient associated with CPI is marginally significant at the 10% level. Specifically, a one-unit increase in CPI is associated with an approximate 6.9% increase in gas prices, holding all other factors constant. This suggests that gas prices are somewhat more sensitive to macroeconomic conditions, particularly inflationary trends. Such responsiveness is consistent with the nature of gas markets, which tend to be more exposed to global commodity cycles, currency fluctuations, and supply chain disruptions. The CPI effect, although modest, reinforces the role of broader economic forces in shaping gas price dynamics, in contrast to the more insulated electricity sector. Notably, temperature anomalies once again do not emerge as significant, indicating that short-term climatic variations may not play a pivotal role in gas price formation in this context, or that the effect may be more pronounced in specific seasons or geographic sub-markets not captured by the aggregated data.

An intriguing aspect of the results lies in the interaction between the two price series. The dynamic interdependence between electricity and gas prices appears weak. Cross-lagged coefficients are broadly insignificant, indicating that past movements in one market do not significantly predict future developments in the other. This suggests that, at least within the lag structure modeled, the two markets operate with a degree of temporal independence. However, the estimated covariance between the contemporaneous errors of the two equations is both positive and statistically significant. This indicates that while there is no strong lead-lag relationship, the two price series tend to move together in response to common shocks. Such shocks could include geopolitical events, energy policy announcements, or disruptions in fuel supply chains that simultaneously affect both the gas and electricity markets, although through different transmission channels. The presence of significant contemporaneous correlation highlights the importance of distinguishing between causality and co-movement in multivariate time series modeling.

Taken together, these results reveal a clear asymmetry in how electricity and gas prices respond to both endogenous and exogenous influences. Electricity prices appear predominantly self-driven and shielded from external shocks, while gas prices are more exposed to inflationary pressures and potentially to other macroeconomic forces not captured in the current specification. The weak dynamic linkages, coupled with significant instantaneous co-movement, suggest that while the two markets are affected by some of the same shocks, their internal price formation mechanisms differ substantially. This underscores the importance of modeling electricity and gas prices jointly but also accounting for the structural and institutional features that differentiate their behavior.

```
                           Statespace Model Results
==============================================================================================
Dep. Variable:      ['electricity_price', 'gas_price']   No. Observations:              337
Model:                            VARMAX(4,3)   Log Likelihood              559.589
                                  + intercept   AIC                       -1045.178
Date:                       Fri, 02 May 2025   BIC                        -903.835
Time:                               12:49:50   HQIC                       -988.841
Sample:                           02-01-1996
                                - 02-01-2024
Covariance Type:                         opg
==============================================================================================
Ljung-Box (L1) (Q):              0.67, 0.37   Jarque-Bera (JB):   229614.16, 1053.73
Prob(Q):                         0.41, 0.54   Prob(JB):                   0.00, 0.00
Heteroskedasticity (H):        121.34, 10.87   Skew:                       7.93, 0.63
Prob(H) (two-sided):             0.00, 0.00   Kurtosis:                 129.89, 11.57
```

```
                       Results for equation electricity_price
=============================================================================================
                           coef    std err          z      P>|z|      [0.025      0.975]
---------------------------------------------------------------------------------------------
intercept                 0.0023      0.038      0.059      0.953      -0.073       0.078
L1.electricity_price      0.0436      0.906      0.048      0.962      -1.732       1.819
L1.gas_price              0.2717      0.700      0.388      0.698      -1.099       1.643
L2.electricity_price      0.0944      0.571      0.165      0.869      -1.026       1.214
L2.gas_price              0.2272      0.663      0.342      0.732      -1.073       1.528
L3.electricity_price     -0.2358      0.324     -0.727      0.467      -0.872       0.400
L3.gas_price             -0.1418      0.823     -0.172      0.863      -1.754       1.470
L4.electricity_price     -0.2685      0.359     -0.749      0.454      -0.971       0.434
L4.gas_price              0.2739      0.209      1.308      0.191      -0.136       0.684
L1.e(electricity_price)   0.0210      0.985      0.021      0.983      -1.909       1.951
L1.e(gas_price)          -0.0880      0.829     -0.106      0.916      -1.713       1.537
L2.e(electricity_price)   0.1679      0.662      0.254      0.800      -1.130       1.466
L2.e(gas_price)          -0.3278      0.632     -0.519      0.604      -1.566       0.911
L3.e(electricity_price)   0.1111      0.369      0.301      0.763      -0.612       0.834
L3.e(gas_price)           0.0059      0.642      0.009      0.993      -1.253       1.265
beta.temperature_anomaly  0.0040      0.012      0.343      0.732      -0.019       0.027
beta.cpi                  0.0297      0.043      0.693      0.488      -0.054       0.114
```

```
                           Results for equation gas_price
=============================================================================================
                           coef    std err          z      P>|z|      [0.025      0.975]
---------------------------------------------------------------------------------------------
intercept                 0.0145      0.056      0.258      0.797      -0.096       0.125
L1.electricity_price     -0.0725      0.538     -0.135      0.893      -1.127       0.983
L1.gas_price              0.0019      0.564      0.003      0.997      -1.103       1.107
L2.electricity_price      0.4061      0.462      0.878      0.380      -0.500       1.312
L2.gas_price             -0.5128      0.478     -1.072      0.284      -1.450       0.424
L3.electricity_price     -0.1593      0.280     -0.568      0.570      -0.709       0.390
L3.gas_price             -0.2794      0.625     -0.447      0.655      -1.504       0.946
L4.electricity_price     -0.2135      0.253     -0.845      0.398      -0.708       0.282
L4.gas_price             -0.0790      0.141     -0.559      0.576      -0.356       0.198
L1.e(electricity_price)   0.1988      0.613      0.324      0.746      -1.003       1.400
L1.e(gas_price)          -0.0225      0.554     -0.041      0.968      -1.107       1.062
L2.e(electricity_price)  -0.0459      0.421     -0.109      0.913      -0.872       0.780
L2.e(gas_price)           0.3062      0.453      0.675      0.499      -0.582       1.195
L3.e(electricity_price)  -0.1161      0.254     -0.457      0.648      -0.614       0.382
L3.e(gas_price)           0.5014      0.440      1.140      0.254      -0.360       1.363
beta.temperature_anomaly -0.0023      0.007     -0.319      0.749      -0.016       0.012
beta.cpi                  0.0692      0.037      1.869      0.062      -0.003       0.142
```

```
                              Error covariance matrix
=============================================================================================
                                        coef    std err          z      P>|z|
---------------------------------------------------------------------------------------------
sqrt.var.electricity_price            0.1743      0.008     22.899      0.000
sqrt.cov.electricity_price.gas_price  0.1329      0.011     12.336      0.000
sqrt.var.gas_price                    0.0631      0.002     28.472      0.000
=============================================================================================
```

Figure 19. Summary Statistics of VARMAX(4,3) Model

3.2 Forecasting into the future

With the VARMAX(4,3) model fitted, we now turn to forecasting the next 12 months and evaluating the model's predictive performance. Specifically, we forecast the period from March 2024 to February 2025, using the test set as a benchmark for comparison. It's important to keep in mind that the model was trained on differenced data in order to achieve stationarity, meaning the forecasted values represent first-order differences rather than actual prices. To translate these predictions back to the original scale, we apply a reverse transformation: we take the last known value from the original (non-differenced) training series and add it to the cumulative sum of the forecasted differences. This step reconstructs the level series, allowing us to directly compare the model's output to the actual observed prices.

The plot comparing actual and forecasted electricity prices in Figure 21 provides a clear visual assessment of the model's predictive performance over the period from March 2024 to February 2025. In the early months—particularly from March to June 2024—we observe a notable deviation between the actual and forecasted values, where the model consistently overestimates the electricity prices. This mismatch suggests that certain short-term shocks or seasonal fluctuations, likely not captured by the model's inputs, played a role in driving prices lower than expected. However, moving into the mid-year months—from July through November—the forecasted and actual values begin to align more closely. This improved fit indicates that the model effectively captures the underlying price dynamics when market conditions are relatively stable. Toward the end of the forecasting horizon, particularly in December 2024 and January 2025, we again see a divergence as actual prices rise sharply while the model's predictions remain relatively flat. This under prediction could reflect the model's limited ability to capture structural changes or demand spikes—potentially due to winter energy usage or external market shocks. Moreover, the smoother nature of the forecasted line reflects the model's tendency to average over past behavior, making it less responsive to sudden volatility. Economically, while the model does a good job of approximating electricity price trends under normal conditions, it appears less sensitive to irregular disruptions.

Looking at this plot in Figure 22 we can observe the recent trend of gas prices and how a forecast attempted to predict its path. Initially, the forecast appears to overestimate the actual price. While the actual price dips, the forecast anticipates an increase. Then, for a period around mid-2024, the forecast comes closer to the actual values, even showing a similar upward tendency, although with some underestimation. However, as we move towards the latter part of 2024 and into early 2025, the forecast seems to significantly underestimate the sharp increase that actually occurred in gas prices. This suggests that the model may not have fully captured the factors that led to the later surge in gas prices.

The RMSE values of 0.244 for electricity prices and 0.201 for gas prices from the VARMAX(4,3) model indicate the average prediction error between the actual and forecasted values over the observed period. These relatively low RMSE values suggest that the model performs reasonably well in capturing the underlying dynamics of both electricity and gas prices. The slightly lower RMSE for gas prices implies better predictive accuracy for gas compared to electricity. This means the forecasted gas prices tend to be closer to the actual observed values than the electricity price forecasts. Overall, the model appears effective for short-term forecasting, though
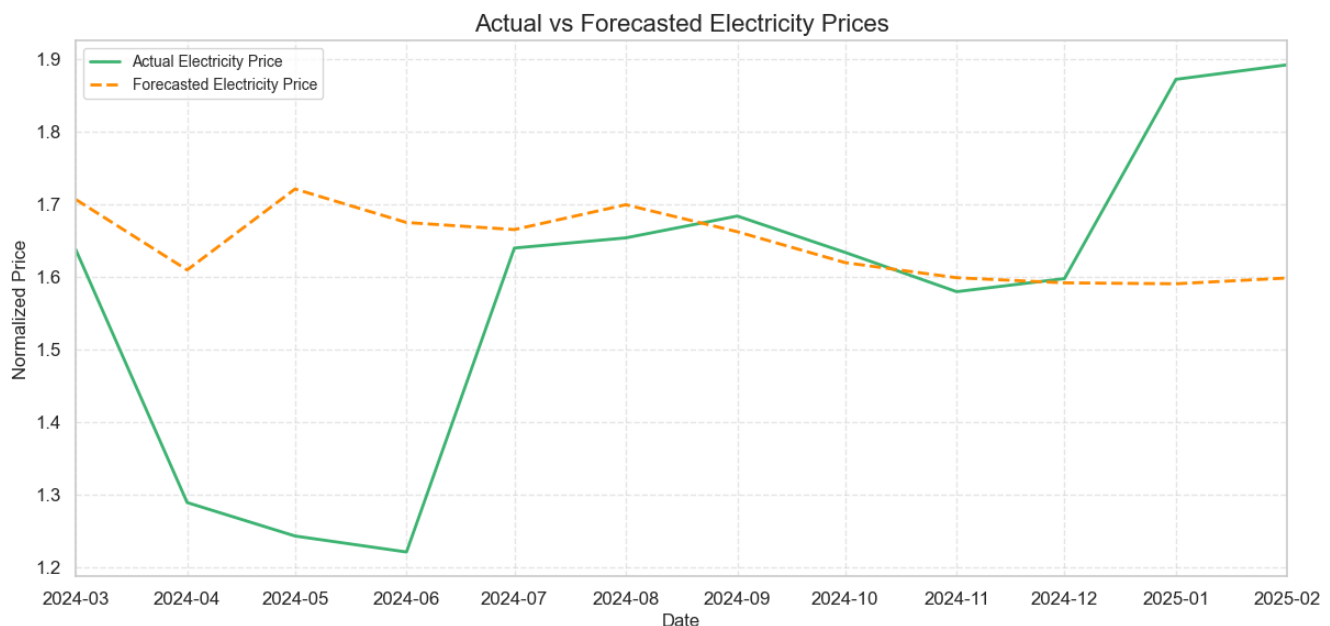

Figure 20. Comparison of Forecasted and Actual Electricity Prices from March 2024 to February 2025
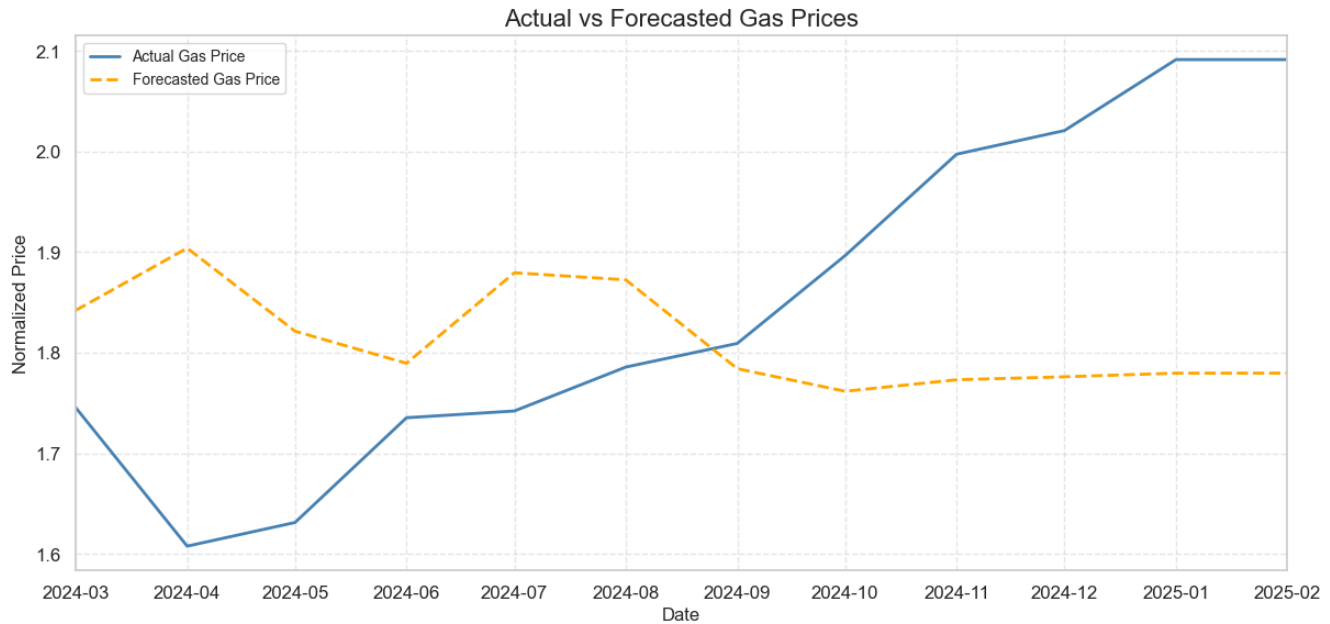
Figure 21. Comparison of Forecasted and Actual Gas Prices from March 2024 to February 2025

residual errors still exist and should be examined for patterns or potential model improvements.

4. VARMAX(2,2) Model

Since the VARMAX(4,3) model yielded statistically insignificant coefficients, we opt for a simpler specification by estimating a VARMAX(2,2) model. In addition to simplifying the model structure, we also shift our approach by using the original, non-normalized data. While normalization(subtracting the mean and dividing by the standard deviation) can sometimes facilitate numerical stability and improve interpretability in certain contexts, it is not always ideal. This is because differencing—often required to achieve stationarity—already transforms the scale of the data, making normalization redundant or even counterproductive. Moreover, normalization can obscure the actual magnitude and interpretation of coefficients, particularly in economic applications where variables like prices, GDP, or emissions have intrinsic scales and economic meaning. By working with the original data, we preserve the interpretability of the model outputs and allow for more meaningful economic inference.

4.1 Model Results and Interpretation

The estimation of the VARMAX(2,2) model offers valuable insights into the dynamic interplay between electricity and gas prices in Italy. The model demonstrates strong statistical significance across many key parameters, suggesting robust dynamic dependencies between the two energy markets. For example, the electricity price is positively and significantly influenced by both the first and second lags of the gas price, with coefficients of 0.7318 and 0.9730 respectively ($p < 0.001$).

This result indicates a delayed but persistent pass-through effect from gas markets to electricity prices, likely due to the structure of Italian electricity generation, which is still heavily reliant on natural gas. Additionally, the second lag of the electricity price itself has a significant negative coefficient (-0.6522), implying a self-correcting behavior in electricity prices over time—possibly reflecting regulatory mechanisms or market equilibrium dynamics that reduce price volatility.

Gas prices are significantly affected by the second lag of electricity prices (-0.4597) and exhibit a strong positive correlation with their own past shocks (as shown in the significance of error lags such as L2.e(electricity_price)). The CPI(Consumer Price Index), included as an exogenous variable, is highly significant for both electricity and gas prices, with coefficients of 9.1743 and 6.7379 respectively, highlighting the impact of inflation on energy costs. This supports the idea that consumer prices and broader macroeconomic conditions play a key role in shaping energy market behavior. Interestingly, temperature anomalies do not display statistical significance, which may imply either that their annualized measure does not adequately capture energy demand fluctuations or that their effect is more nonlinear and indirect. The model also detects significant contemporaneous correlation between the error terms of electricity and gas prices, reinforcing the idea that external shocks, such as geopolitical events or market-wide disruptions, simultaneously influence both price series.

In contrast, the previously estimated VARMAX(4,3) model — although statistically superior in terms of log-likelihood and information criteria (AIC = -1045.178 vs. 3867.322 for the VARMAX(2,2)) — fails to deliver meaningful economic interpretation due to the widespread insignificance of its coeffi-

cients. Despite the improved fit, virtually all lag coefficients in the VARMAX(4,3) model lack statistical significance at conventional thresholds, and even the exogenous variables (temperature anomalies and CPI) show minimal or no explanatory power. This suggests a case of overfitting, where model complexity increases without yielding clearer or more actionable insights. For example, the coefficient of CPI in the electricity price equation is only 0.0297 (p = 0.488), which sharply contrasts with its strong and significant role in the simpler VARMAX(2,2) model. Furthermore, the lagged endogenous variables across both equations in the VARMAX(4,3) model exhibit high standard errors and low z-scores, indicating weak predictive content.

Therefore, despite the higher statistical performance on paper, the VARMAX(4,3) model is less useful for economic interpretation and policy analysis due to the obscurity of its results. The simpler VARMAX(2,2) model strikes a better balance between parsimony and explanatory power, providing clearer, more interpretable dynamics that align with economic theory and the institutional features of the Italian energy system. The effectiveness of fewer lags may also reflect the inertia typical in energy pricing, where short-term shocks

```
                       Statespace Model Results
================================================================================
Dep. Variable:     ['electricity_price', 'gas_price']  No. Observations:        337
Model:                            VARMAX(2,2)  Log Likelihood          -1908.661
                                  + intercept  AIC                      3867.322
Date:                      Tue, 06 May 2025   BIC                      3962.824
Time:                             14:52:44    HQIC                     3905.387
Sample:                         02-01-1996
                              - 02-01-2024
Covariance Type:                        opg
================================================================================
Ljung-Box (L1) (Q):           1.08, 0.01   Jarque-Bera (JB):   114319.64, 256.13
Prob(Q):                      0.30, 0.94   Prob(JB):                  0.00, 0.00
Heteroskedasticity (H):      59.71, 10.70  Skew:                      6.04, 0.28
Prob(H) (two-sided):          0.00, 0.00   Kurtosis:                 92.42, 7.23
                     Results for equation electricity_price
=================================================================================
                         coef    std err          z      P>|z|      [0.025      0.975]
---------------------------------------------------------------------------------
intercept                0.2717      1.826      0.149      0.882      -3.308       3.851
L1.electricity_price    -0.0745      0.177     -0.420      0.674      -0.422       0.273
L1.gas_price             0.7318      0.198      3.702      0.000       0.344       1.119
L2.electricity_price    -0.6522      0.139     -4.678      0.000      -0.925      -0.379
L2.gas_price             0.9730      0.262      3.712      0.000       0.459       1.487
L1.e(electricity_price)  0.1801      0.217      0.831      0.406      -0.245       0.605
L1.e(gas_price)         -0.8420      0.308     -2.732      0.006      -1.446      -0.238
L2.e(electricity_price)  0.9481      0.191      4.968      0.000       0.574       1.322
L2.e(gas_price)         -0.6928      0.366     -1.895      0.058      -1.409       0.024
beta.temperature_anomaly 0.2260      0.188      1.205      0.228      -0.142       0.594
beta.cpi                 9.1743      0.806     11.384      0.000       7.595      10.754

                        Results for equation gas_price
=================================================================================
                         coef    std err          z      P>|z|      [0.025      0.975]
---------------------------------------------------------------------------------
intercept                0.3919      0.843      0.465      0.642      -1.261       2.045
L1.electricity_price     0.1886      0.137      1.382      0.167      -0.079       0.456
L1.gas_price             0.0202      0.148      0.137      0.891      -0.270       0.310
L2.electricity_price    -0.4597      0.107     -4.283      0.000      -0.670      -0.249
L2.gas_price             0.1976      0.186      1.061      0.289      -0.167       0.563
L1.e(electricity_price) -0.1835      0.162     -1.134      0.257      -0.501       0.134
L1.e(gas_price)         -0.0541      0.201     -0.270      0.787      -0.448       0.339
L2.e(electricity_price)  0.7672      0.121      6.354      0.000       0.531       1.004
L2.e(gas_price)         -0.4183      0.232     -1.806      0.071      -0.872       0.036
beta.temperature_anomaly -0.0769     0.151     -0.510      0.610      -0.372       0.219
beta.cpi                 6.7379      0.491     13.714      0.000       5.775       7.701
                             Error covariance matrix
=================================================================================
                                     coef    std err          z      P>|z|
---------------------------------------------------------------------------------
sqrt.var.electricity_price         8.0025      0.221     36.164      0.000
sqrt.cov.electricity_price.gas_price 2.8986    0.259     11.195      0.000
sqrt.var.gas_price                 2.1072      0.065     32.257      0.000
=================================================================================
```

Figure 22. Summary Statistics of VARMAX(2,2) Model

and macroeconomic trends tend to dominate over long memory dynamics. In light of these findings, the VARMAX(2,2) model not only supports more robust forecasting and scenario analysis, but also underscores the value of avoiding over-parameterization, particularly when the goal is to derive economically meaningful conclusions from time series modeling.

4.2 Forecasting into the future

The graphical evaluation of forecast accuracy is presented through two plots comparing actual and predicted values for gas and electricity prices over a 12-month period. These forecasts were generated using the selected VARMAX(2,2) model, and their visual comparison with real data serves as a qualitative validation tool.

The first plot in Figure 23 captures the evolution of electricity prices from March 2024 to February 2025. The x-axis reflects the timeline in monthly intervals, while the y-axis denotes the price level. The solid green line shows actual electricity prices starting at about 182 in March 2024. The price declines steadily to a low of nearly 162 by June, reflecting a seasonal drop. Following this, the trend reverses sharply upward with significant increases in July and again during the winter months, ending at around 195 in February 2025. The forecasted electricity prices, marked by a dashed orange line, begin at a notably higher value of 193 in March 2024—an initial overestimation. The predicted price then declines to about 176 by May, failing to fully anticipate the sharper dip observed in the actual series. However, the model captures the upward reversal starting in mid-year reasonably well. A particularly strong forecasted jump occurs in July, aligning with

the actual spike, and prices peak around 191 in August. After a minor decline in September, the forecast resumes an upward trajectory, closely tracking the actual values from November onward and ending at 197 in February 2025.

The second plot in Figure 24 provides a comparison for gas prices across the same time span. The solid blue line represents actual gas prices. Starting at around 145.5 in March 2024, the actual price dips slightly to a low of approximately 141.5 in April. This is followed by a steady upward trend, with some short-term fluctuations, culminating in a peak value of nearly 156 by January and maintaining that level into February 2025. The forecasted gas prices, illustrated by a dashed orange line, start somewhat higher than actual prices at around 148.5 in March 2024. They then drop sharply to roughly 142.5 by May 2024, undershooting the actual values slightly. Subsequently, the forecast exhibits a pronounced upward movement, peaking around 152 in July–August 2024. After a brief decline to 148 in September, the forecast trajectory gradually converges with the actual price line. From October onward, forecast accuracy improves considerably, with the predicted values closely shadowing actual gas prices and reaching approximately 156 by February 2025. This convergence toward the end of the horizon suggests that the model captures the long-term trend effectively, although short-term fluctuations—especially mid-year—are harder to predict precisely.

In both plots, the forecasts exhibit a general alignment with the broader trends of actual price movements but tend to smooth out short-term volatility. Initial periods show larger forecast errors—especially in March through June—suggesting potential model limitations in responding to sharp market adjustments or external shocks. In contrast, the second half of
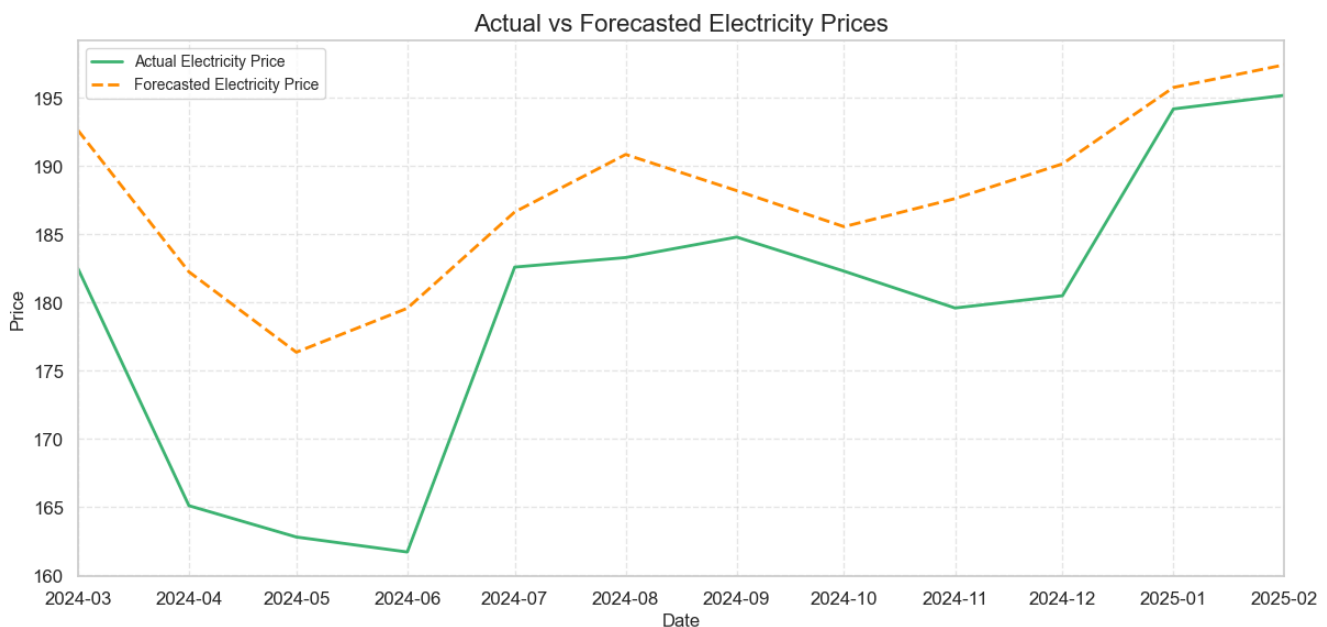

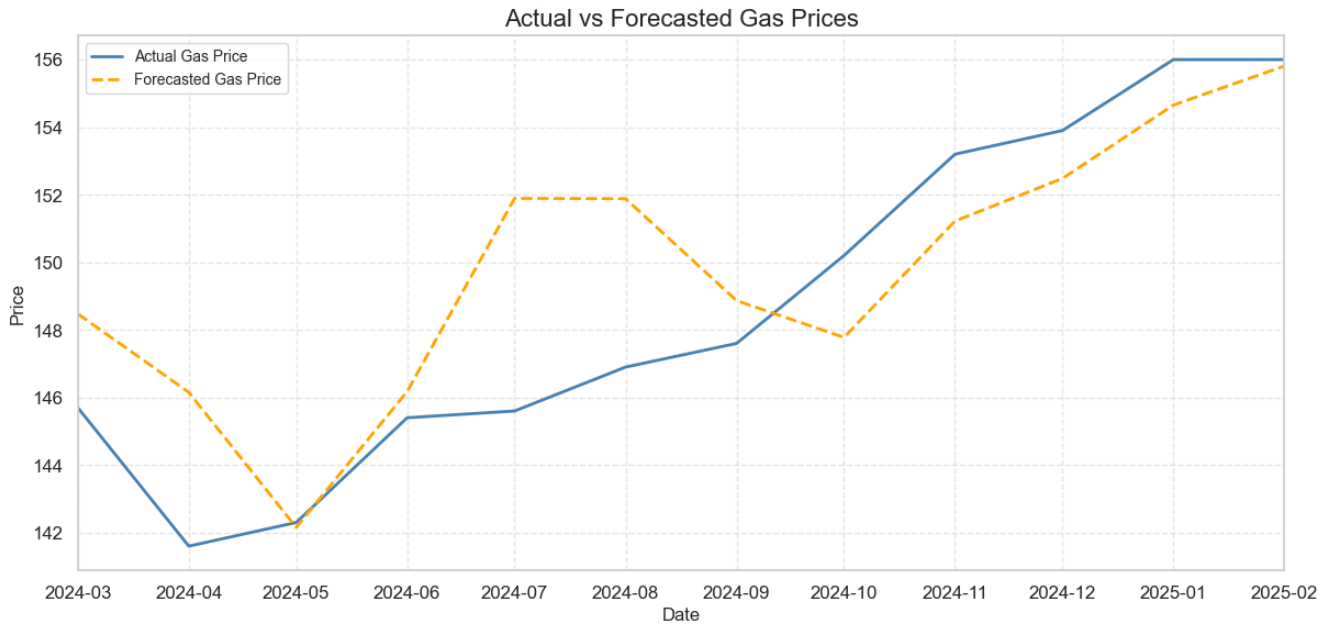**Figure 23.** Comparison of Forecasted and Actual Electricity Prices from March 2024 to February 2025

Figure 24. Comparison of Forecasted and Actual Gas Prices from March 2024 to February 2025

the forecast horizon demonstrates substantial convergence, indicating that the model performs better in capturing long-run co-movements and equilibrium relationships, consistent with the VARMAX framework's strength in modeling joint dynamics and persistence structures.

Overall, these visual results validate the VARMAX(2,2) model's utility for medium- to long-term energy price forecasting, particularly when the goal is to capture directional trends and equilibrium paths. It is therefore possible to state that the model in question performs much better than the previous VARMAX(4,3) model.

4.3 Model Evaluation

To assess the performance and robustness of the fitted VARMAX(2,2) model with monthly data on electricity and gas prices, a series of diagnostic tests were conducted. First, the Ljung-Box test for autocorrelation was applied to the residuals, revealing that the electricity price residuals exhibited significant autocorrelation ($p < 0.001$), suggesting that the model has not fully captured the underlying dynamics of this series. In contrast, the residuals for gas prices showed no autocorrelation (p = 0.526), indicating that the model appropriately accounts for the gas price dynamics. Heteroskedasticity was evaluated using the ARCH test, with results revealing strong ARCH effects in the residuals of gas prices ($p < 0.001$), suggesting volatility clustering that the VARMAX model could not fully account for. On the other hand, marginal evidence of heteroskedasticity was found for electricity prices (p = 0.060), indicating that further exploration into volatility modeling, such as using GARCH models, may be necessary for a more complete understanding of the price dynamics. Normality

of the residuals was tested using the Jarque-Bera test, and both series were found to significantly deviate from normality ($p < 0.001$), with electricity price residuals displaying extreme skewness (6.04) and kurtosis (92.42), suggesting the presence of outliers or other non-linear effects not captured by the model. The CUSUM test for structural change showed no evidence of breaks in the model's parameters over time, as indicated by non-significant results (electricity price p = 0.9163, gas price p = 0.8195), confirming the stability of the estimated coefficients during the period under study. Moreover, Granger causality tests revealed that electricity prices Granger-cause gas prices, as indicated by a significant F-statistic ($p < 0.001$), whereas gas prices do not Granger-cause electricity prices (p = 0.320), highlighting a unidirectional predictive relationship between the two variables. Overall, while the VARMAX model provides a reasonable fit for gas prices, the diagnostics suggest that improvements are needed, particularly for electricity prices, where issues such as autocorrelation, non-normality, and heteroskedasticity should be addressed. Future work may consider incorporating more sophisticated models, such as GARCH for volatility or higher-order VAR models, to capture the underlying dynamics of electricity price fluctuations more accurately.

```
Ljung-Box Test for electricity_price:
        lb_stat      lb_pvalue
12   81.965039   1.738839e-12


Ljung-Box Test for gas_price:
        lb_stat   lb_pvalue
12   11.035634    0.525867
```

Figure 25. Ljung-Box Test for Endogenous Variables

```
ARCH Test for electricity_price:
   LM stat: 20.38, p-value: 0.0602


ARCH Test for gas_price:
   LM stat: 91.37, p-value: 0.0000
```

Figure 26. ARCH Test for Endogenous Variables

```
Normality Test (Jarque-Bera) for electricity_price:
   JB stat: 114319.64, p-value: 0.0000
   Skewness: 6.0411
   Kurtosis: 92.4174


Normality Test (Jarque-Bera) for gas_price:
   JB stat: 256.13, p-value: 0.0000
   Skewness: 0.2798
   Kurtosis: 7.2341
```

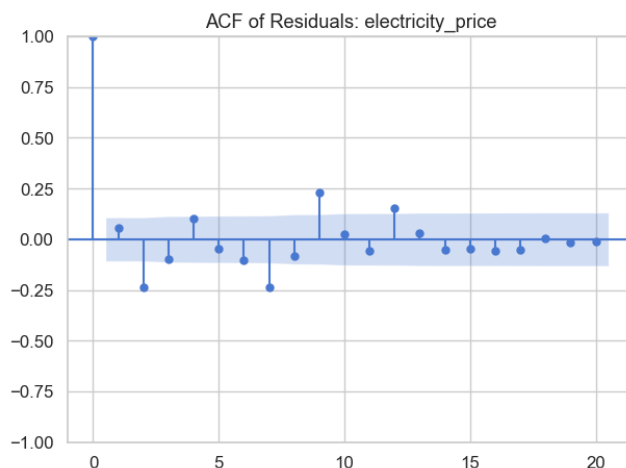**Figure 27.** Normality Test for Endogenous Variables
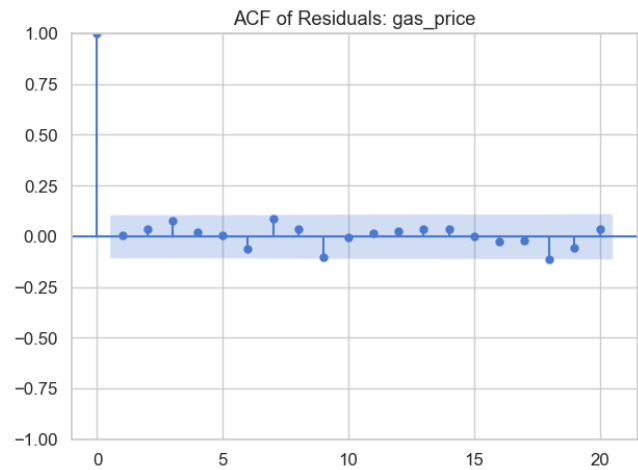

Figure 28. Enter Caption

Figure 29. ACF of Residuals of Endogenous Variables

```
Granger Causality: electricity_price → gas_price

Granger Causality
number of lags (no zero) 1
ssr based F test:          F=7.2878  , p=0.0073  , df_denom=333, df_num=1
ssr based chi2 test:    chi2=7.3535  , p=0.0067  , df=1
likelihood ratio test: chi2=7.2742  , p=0.0070  , df=1
parameter F test:          F=7.2878  , p=0.0073  , df_denom=333, df_num=1

Granger Causality
number of lags (no zero) 2
ssr based F test:          F=5.8969  , p=0.0030  , df_denom=330, df_num=2
ssr based chi2 test:    chi2=11.9724 , p=0.0025  , df=2
likelihood ratio test: chi2=11.7634 , p=0.0028  , df=2
parameter F test:          F=5.8969  , p=0.0030  , df_denom=330, df_num=2
```

```
Granger Causality: gas_price → electricity_price

Granger Causality
number of lags (no zero) 1
ssr based F test:          F=3.0909  , p=0.0796  , df_denom=333, df_num=1
ssr based chi2 test:    chi2=3.1187  , p=0.0774  , df=1
likelihood ratio test: chi2=3.1044  , p=0.0781  , df=1
parameter F test:          F=3.0909  , p=0.0796  , df_denom=333, df_num=1

Granger Causality
number of lags (no zero) 2
ssr based F test:          F=2.6218  , p=0.0742  , df_denom=330, df_num=2
ssr based chi2 test:    chi2=5.3230  , p=0.0698  , df=2
likelihood ratio test: chi2=5.2811  , p=0.0713  , df=2
parameter F test:          F=2.6218  , p=0.0742  , df_denom=330, df_num=2
```
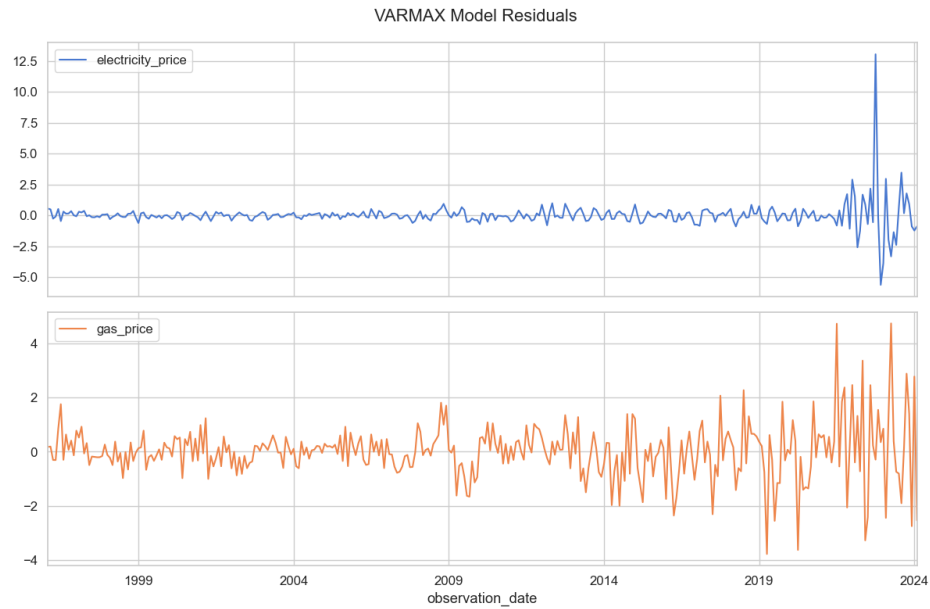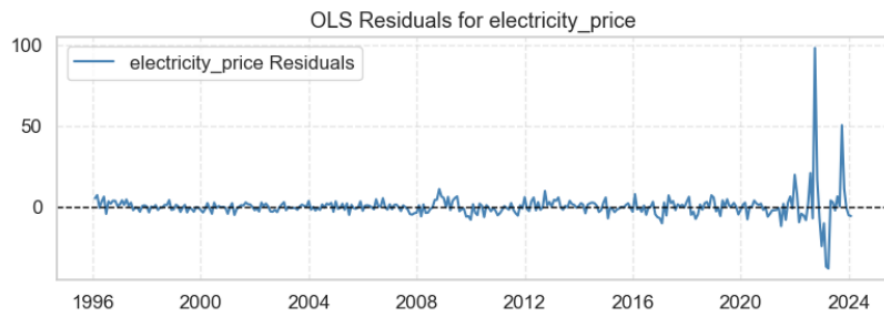
Figure 30. Granger Causality for Endogenous Variables

VARMAX Model Residuals

```
CUSUM Test for Structural Change in: electricity_price
Test Statistic: 0.5563
P-value: 0.9163
Critical Values: [(1, 1.63), (5, 1.36), (10, 1.22)]
```


OLS Residuals for electricity_price

```
CUSUM Test for Structural Change in: gas_price
Test Statistic: 0.6318
P-value: 0.8195
Critical Values: [(1, 1.63), (5, 1.36), (10, 1.22)]
```
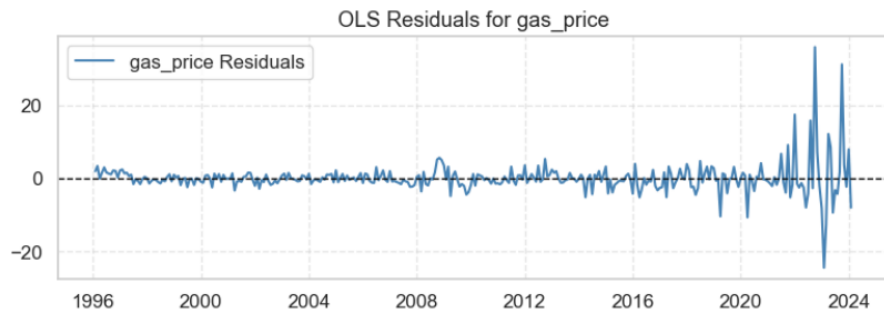

OLS Residuals for gas_price

Figure 31. Residuals Plots

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