Problem A

What are the roots of function,

$$f(x) = \left(\log(3^x) - 2\log(3)\right) \cdot (x^2-1) \text{ with } x \in R?$$

Solution

Let $f(x) = 0$ for taking roots,

$$\Rightarrow \left(\log(3^x) - \log(3)^2\right) \cdot (x^2-1) = 0 \quad [\because n\log m = \log m^n]$$

$$\Rightarrow \left\{\log(3^x) - \log(3^2)\right\} \cdot (x^2-1) = 0$$

$$\Rightarrow \log\left(\frac{3^x}{3^2}\right) \cdot (x^2-1) = 0 \quad \left[\because \log a - \log b = \log(a/b)\right]$$

$$\Rightarrow \left(\log(3^{x-2})\right) \cdot (x^2-1) = 0$$

$$\log 3^{x-2} = 0$$

convert/take exponential form

$$e^{\log 3^{x-2}} = 0$$

$$\because e^{\log(a)} = 1$$

$$x - 2 = 0$$

$$x = 2$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

Take root

$$x = \pm 1$$

$$\boxed{x = \{2, +1, -1\}}$$

Problem B

Find the Values of following infinite Sum

$$1+\frac{3}{\pi}+\frac{3}{\pi^2}+\frac{3}{\pi^3}+\frac{3}{\pi^4}+\frac{3}{\pi^5}+\cdots\cdots$$

Solution:-

we Can write Series as the form of Summation

$$a_n = \sum_{n=0}^{\infty}\left(\frac{3}{\pi^n}\right) = \sum_{n=0}^{\infty}\left\{\frac{3}{\pi^n}\right.$$

Now Using ratio test, it Used for the Convergence of Series/Sum-

$$\Rightarrow \lim_{n\to\infty} \cdot \frac{a_{n+1}}{a_n}$$

$$\Rightarrow \lim_{n\to\infty} \frac{3}{\pi^{n+1}} \bigg/ \frac{3}{\pi^n}$$

$$\Rightarrow \lim_{n\to\infty} \cdot \frac{\cancel{\pi^n}}{\cancel{\pi^n}\cdot\pi}$$

$$\Rightarrow \lim_{n\to\infty} \frac{1}{\pi}$$

$\Rightarrow$ apply Limit, well there is no need of putting but this is a step we have to follow but it will not effect on Solution-

$\Rightarrow \frac{1}{\pi} \Rightarrow$ (This should must be Less than 1)

which is Less than 1, means Series Converges

Problem C

Determine the neumerical value of the following expression without Use of Calculator,

$$\Rightarrow \text{Log}\left[\text{Log}(3)\cdot\left(\text{Log}(2)\cdot\left(\frac{\sqrt{3}-2\sin(\pi/3)}{\pi^3+1}+1\right)\right)-\text{Log}(2)\,\text{Log}(3)+(-1)^{100}\right]$$

Solution :- $\sin(\pi/3) \Rightarrow \sin 60^\circ = \sqrt{3}/2$

there is One trick I Know to Sin Or Cos Values as well as tangent.

0°	30°	45°	60°	90°	
0	1	2	3	4	$\Rightarrow$ Step 1 write sequence integers 0—4
0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\Rightarrow$ divide all by 4
0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\boxed{\frac{\sqrt{3}}{2}}$	1	$\Rightarrow$ Now take root

$\sin 60^\circ$

Now lets Solve further;

$$\text{Log}\left[\text{Log}(3)\cdot\left(\text{Log}(2)\cdot\left(\frac{\sqrt{3}-\cancel{2}(\sqrt{3}/\cancel{2})}{\pi^3+1}+1\right)\right)-\text{Log}(2)\,\text{Log}(3)+(-1)^{100}\right]$$

$\rightarrow$ any negative no has power of even no result ll be positive,

$$\Rightarrow \text{Log}\left[\text{Log}(3)\cdot\left(\text{Log}(2)\cdot\left(\frac{0}{\pi^3+1}+1\right)\right)-\text{Log}(2)\,\text{Log}(3)+1\right]$$

$$\Rightarrow \text{Log}\left[\text{Log}(3)\cdot\left(\text{Log}(2)\cdot(1)\right)-\text{Log}(2)\,\text{Log}(3)+1\right]$$

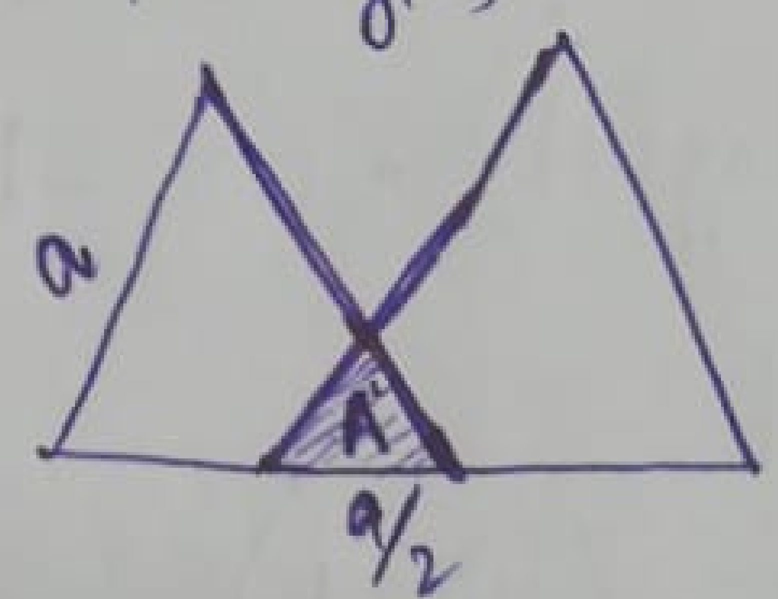
$$\Rightarrow \text{Log}\left[\cancel{\text{Log}(3)\,\text{Log}(2)}-\cancel{\text{Log}(2)\,\text{Log}(3)}+1\right]$$

$$\text{Log}(1) \Rightarrow 0$$

$\Rightarrow \text{Log}(1)=0$
Basic Rule of Logarithms

Problem E

The drawing below shows two equilateral triangle with side length a the triangles are horizantally shifted by $a/2$. Find the intersection area A of the two triangles-



Solution :-

Both are equilateral triangle and its all sides are equal means all sides length is "a" and intersected Area is represented by A Above 2 equilateral triangle make an other isoscelus triangle that has two sides of equal length atleast- which has length $a/2$-

⇒ Now Using Heron's formula for area of shaded triangle-

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

First find s; $s = \dfrac{a+b+c}{2} = \dfrac{a/2 + a/2 + a/2}{2} = 3a/4$

put $s = 3a/4$ in above formula of Area,

$$\text{Area} = \sqrt{\frac{3a}{4}\left(\frac{3a}{4} - \frac{a}{2}\right)\left(\frac{3a}{4} - \frac{a}{2}\right)\left(\frac{3a}{4} - \frac{a}{2}\right)}$$

Take LCM = 4

$$\text{Area} = \sqrt{\frac{3a}{4}\left(\frac{3a-2a}{4}\right)\left(\frac{3a-2a}{4}\right)\left(\frac{3a-2a}{4}\right)}$$

$$\text{Area} = \sqrt{\frac{3a}{4}\left(\frac{a}{4}\right)\left(\frac{a}{4}\right)\left(\frac{a}{4}\right)}$$

$$\sqrt{\frac{3a}{4}\left(\frac{a^3}{64}\right)} = \sqrt{\frac{3a^4}{256}} \Rightarrow \frac{a^2}{16}\sqrt{3}$$

so Area $\Rightarrow A = \dfrac{a^2}{16}\sqrt{3}$