Problem A.1 :- Determine A, B, C Such that all of the following functions intersect the point (2,2)

$f_1(x)$ $Ax+1$, $f_2(x) = Bx^2+2$, $f_3(x) = Bx^3+3$

Solution :-

This Question simply says find that A, B and C so that these three functions intersect at (2,2).

So, for this I will equate these ftns at (2,2)

Put (2,2) in all three functions —

$f_1(2) = 2A+1$, $f_2(2) = 4B+2$, $f_3(2) = 8B+3$

Now equate them,

$$2A+1 = 4B+2 \quad \text{——(i)}$$
$$4B+2 = 8B+3 \quad \text{——(ii)}$$
$$8B+3 = 2A+1 \quad \text{——(iii)}$$

$$2A-4B-1=0 \quad \text{——(a)}$$
$$4B-8B-1=0 \quad \text{——(b)}$$
$$8B-2A+2=0 \quad \text{——(c)}$$

By equation (b) $\Rightarrow$ $4B-8B=1$

$-4B=1 \Rightarrow \boxed{B=-1/4}$

Put in equation (a).

$2A = 4B+1$

$A = 4B+1/2 \Rightarrow \dfrac{4(-1/4)+1}{2} = \boxed{0 = A}$

Now If we put these A and B values in any function, they all will give Same Output

$f_1(2) = 2A+1 \Rightarrow$ where $A=0$.

$f_1(2) = 2(0)+1 = \boxed{1}$

$f_2(2) = 4B+2 \Rightarrow$ where $B=-1/4$

$\cancel{4}(-1/\cancel{4})+2 \Rightarrow \boxed{1}$

$f_3(2) = 8B+3$

$2\cancel{8}(-1/\cancel{4})+3 \Rightarrow \boxed{1}$ verified

Problem A.2

Find all $x \in \mathbb{R}$ that are solution of this equation

$$0 = (1 - x - x^2 - \cdots\cdots) \cdot (2 - x - x^2 - \cdots\cdots)$$

As we know these both are power series, or we can say both are polynomials, and if multiplication of both polynomials is equal to zero which tends any one them is zero,

$$1 - x - x^2 - \cdots\cdots = 0 \quad , \quad 2 - x - x^2 - \cdots\cdots = 0$$

As we know $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots\cdots$ (a).

Take (a),

$$\frac{1}{1-x} - 1 = x + x^2 + x^3 + \cdots\cdots$$

Take minus (−) common,

$$1 - \frac{1}{1-x} = -x - x^2 - x^3 - \cdots\cdots \quad (i).$$

If we put this above (i) in given equation -

lets Rewrite above eq (i) $\frac{\cancel{1} - x - \cancel{1}}{1-x} = \frac{-x}{1-x}$.

$$\Rightarrow \quad 0 = (1 - x - x^2 - x^3 - \cdots\cdots)(2 - x - x^2 - x^3 - \cdots\cdots)$$

$$\Rightarrow \quad 0 = \left(1 - \frac{x}{1-x}\right)\left(2 - \frac{x}{1-x}\right)$$

$$\Rightarrow \quad \left(\frac{1 - \cancel{x} - x}{1-x}\right)\left(\frac{2 - 2x - x}{1-x}\right) = 0$$

$$\left(\frac{1-2x}{1-x}\right)\left(\frac{2-3x}{1-x}\right) = 0$$

$$\frac{1-2x}{1-x} = 0 \quad , \quad \frac{2-3x}{1-x} = 0$$

$$1 - 2x = 0 \quad , \quad 2 - 3x = 0$$

$$1 = 2x. \quad , \quad 2 = 3x$$

$$\boxed{\tfrac{1}{2} = x \quad , \quad \tfrac{2}{3} = x}$$

Problem A.3

Find the Derivative $f'(x)$ of the following function with respect to x:

$$f(x) = \sin(\pi^{\sin x} + \pi^{\cos x}).$$

Solution:-

$$f(x) = \sin(\pi^{\sin x} + \pi^{\cos x}) \qquad \because \theta = \pi^{\sin x} + \pi^{\cos x}.$$

Let angle is equal to θ.

$$f(x) = \sin\theta$$

$$f'(x) = \cos\theta \quad \text{put } \theta = \pi^{\sin x} + \pi^{\cos x}.$$

$$f'(x) = \cos(\pi^{\sin x} + \pi^{\cos x})$$

according $\boxed{a^x = \ln a \cdot a^x}$ exponential function -

(a) $\frac{d}{dx}(\pi^{\sin x}) \Rightarrow \ln\pi\,(\pi^{\sin x})(\cos x)$ $\quad \because \sin x\, dx = \cos x$

(b) $\frac{d}{dx}(\pi^{\cos x}) \Rightarrow \ln\pi\,(\pi^{\cos x})(-\sin x)$ $\quad \because \cos x\, dx = -\sin x$

put these (a) & (b) in $f'(x)$

$$f'(x) = \cos(\pi^{\sin x} + \pi^{\cos x}) \cdot \frac{d}{dx}(\pi^{\sin x} + \pi^{\cos x})$$

$$f'(x) = \cos(\pi^{\sin x} + \pi^{\cos x})\left(\ln(\pi)(\pi^{\sin x})(\cos x) + \ln(\pi)(\pi^{\cos x})(-\sin x)\right)$$

$$f'(x) = \cos(\pi^{\sin x} + \pi^{\cos x})\, \ln\pi\left(\pi^{\sin x}(\cos x) - \pi^{\cos x}(\sin x)\right)$$

Problem B.1

Let H_n define the sum of reciprocals of all integers from 1 to n,

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \frac{1}{n} \Rightarrow \frac{1}{1-n} := S_n$$

Prove the following identity:

$$H_{2n} - H_n = 1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \pm \cdots + \frac{1}{2n-1} - \frac{1}{2n}.$$

Solution:-

$$H_{2n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \cdots \frac{1}{2n}$$

$$H_{2n} - H_n = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \cdots + \frac{1}{2n}\right) - \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n}\right)$$

above all evens ll be cancel out.

$$H_{2n} - H_n = \ln(2) \cancel{= \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}}$$

$$H_n = 1 - \gamma \qquad \because \gamma = 0.5772\ldots$$

$$H_n = 1 - 0.5772$$

$$H_n = 0.43\ldots$$

This thing gives us odd series But whole even series ll be cancelled out in this way-

$$H_{2n} - H_n = \ln(2).$$

so, I know the series of $\ln(2)$

$$\ln(2) \Rightarrow 1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \cdots + \frac{1}{2n-1} - \frac{1}{2n}$$

so,

$$H_{2n} - H_n = 1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{2n-1} - \frac{1}{2n}$$

Problem B.2

It is well known that squared brackets do not simply square individuals terms:

$$(1+2)^2 \neq 1^2+2^2$$
$$(1+2+3)^2 \neq 1^2+2^2+3^2$$

Instead we add a correction term ψ to make the equations hold true

$$(1+2)^2 = 1^2+2^2+\psi_2 \quad \text{——(i)}$$
$$(1+2+3)^2 = 1^2+2^2+3^2+\psi_3 \quad \text{——(ii)}$$
$$(1+2+3+\cdots n)^2 = 1^2+2^2+3^2+\cdots+n^2+\psi_n$$

Show that correction term ψ_n has the following form and determine the values of α & β.

$$\psi_n = \frac{n^4-n^2}{\alpha} + \frac{n^3-n}{\beta} \quad \text{——(A)}$$

Solution:-

Lets take above equation (i)

$$(1+2)^2 = 1^2+2^2+\psi_2$$
$$1^2+2(1)(2)+2^2 = 1^2+2^2+\psi_2$$
$$\cancel{1+4}+4 = \cancel{1+4}+\psi_2$$
$$\boxed{4=\psi_2}$$

Now put this $\psi_2 = 4$ in eq.(A).

$$\psi_n = \frac{n^4-n^2}{\alpha} + \frac{n^3-n}{\beta}$$
$$\psi_2 = \frac{(2)^4-(2)^2}{\alpha} + \frac{(2)^3-(2)}{\beta}$$
$$4 = \frac{16-4}{\alpha} + \frac{8-2}{\beta}$$
$$4 = \frac{12}{\alpha} + \frac{6}{\beta}$$
$$4 = \frac{12\beta+6\alpha}{\alpha\beta}$$
$$4\alpha\beta = 12\beta + 6\alpha \quad \text{——(a)}$$

Now lets take eq (ii)

$$(1+2+3)^2 = 1^2+2^2+3^2+\psi_3$$

$$36 = 1+4+9+\psi_3$$

$$36-14 = \boxed{\psi_3 \Rightarrow 22}$$

Put ψ_3 in eq (A).

$$\psi_3 = \frac{(3)^4-(3)^2}{\alpha} + \frac{(3)^3-3}{\beta}$$

$$22 = \frac{81-9}{\alpha} + \frac{27-3}{\beta}$$

$$22 = \frac{72}{\alpha} + \frac{24}{\beta}$$

$$22\alpha\beta = 72\beta + 24\alpha \quad \text{———} (b)$$

As Now we have 2 equations in form of α & B, which Can be Compareable-

so lets Campare eq (a) & (b)

$$12\beta + 6\alpha = 4\alpha\beta$$
$$72\beta + 24\alpha = 22\alpha\beta.$$

$\Rightarrow$ divide both equations with 2 for easiness -

$$6\beta + 3\alpha = 2\alpha\beta \quad \text{—}(3)$$
$$36\beta + 12\alpha = 11\alpha\beta \quad \text{—}(4)$$

from eq (4) drive value for α.

$$12\alpha = 11\alpha\beta - 36\beta$$

$$\alpha = \frac{11\alpha\beta - 36\beta}{12} \rightarrow \text{put this in eq (3)}$$

$$6\beta + \cancel{3}\left(\frac{11\alpha\beta - 36\beta}{\cancel{12}\,4}\right) = 2\alpha\beta$$

$$24\beta + 11\alpha\beta - 36\beta = 8\alpha\beta$$

$$11\alpha\beta - 8\alpha\beta = 36\beta - 24\beta$$

$$3\alpha\beta = 12\beta$$

$$\alpha = 12\beta / 3\beta = 4 \text{ — put}$$

As we know we have $\alpha = \frac{11\alpha\beta - 36\beta}{12}$.

$$12\alpha = 11\alpha\beta - 36\beta \quad \text{where alph } (\alpha) = 4$$
$$12(4) = 11(4)\beta - 36\beta.$$
$$48 = 44\beta - 36\beta.$$
$$48 = 8\beta$$
$$48/8 = \beta = \frac{24}{4} = \frac{12}{2} = 6$$

So, Finally we get $\alpha = 4$, $\beta = 6$.

Lets verify, put α & β in any eq,

I am putting α & β in μ_2 Value –

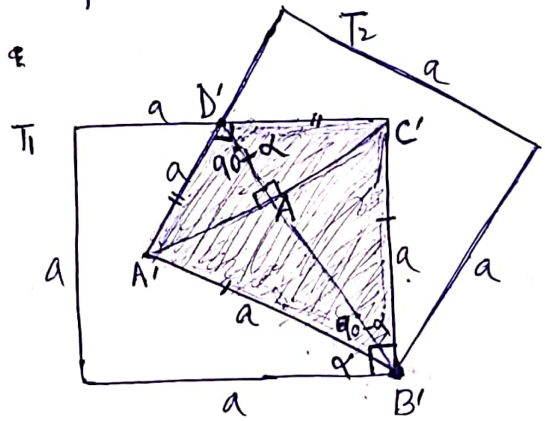
$$\mu_2 = \frac{n^4 - n^2}{\alpha} + \frac{n^3 - n}{\beta}$$

which should be equal to "4".

$$\frac{(2)^4 - (2)^2}{4} + \frac{(2)^3 - 2}{6}$$
$$\frac{16 - 4}{4} + \frac{8 - 2}{6}$$
$$\frac{12}{4} + \frac{6}{6}$$
$$3 + 1 = 4 \quad \underline{\underline{\text{Verified}}}$$

Problem B.3

You are given two Overlaying Squares with Side Length a. One of the Squares is fixed at the bottom right Corner and rotated by an angle of α. Find an expression for the enclosed Area A(α) between the two Squares with respect to rotation angle α



Generally
" This is Square bcz all sides are equal"

" Area of kite = $\frac{d_1 d_2}{2}$ "

Solution

Firstly this picture is Combination of two Squares which has each side of Length "a" and angle 90°

But when we want to get angle of Only region of Area that 'll be 90° − α because α 'll be Subtracted from actual angle of Square which is 90°.

MoreOver If we go into deep and focus the region whose Area we need to find Looks Like "Kite"

which also Satisfy property of kite ⇒ " Length of adjacent Sides is equal"

therefore,

$$|A'D'| = |D'C'|$$
$$|A'B'| = |B'C'|$$

Hence we can defined above quadilateral as "Kite"

As we have find angle b/w A'B'C' ⇒ 90° − α so Lets find Area of this triangle

$$\text{Area of } A'B'C' = \frac{1}{2}|A'B'||B'C'| \sin\theta$$
$$= \frac{1}{2}(a)(a)\sin(90° - \alpha)$$
$$= \frac{1}{2}a^2\{\sin 90° \cos\alpha - \cos 90° \sin\alpha\}$$
$$= \frac{1}{2}a^2 \cos\alpha$$

∵ A/c to formula $\sin(a-b) = \sin a \cos b - \cos a \sin b$

Now kite has another property of diagonal angle bisects perpendicular-
so, Lets first find $|A'C'|$

According to Cosine Law we can find side of triangles given:

$$c = \sqrt{a^2 + b^2 - 2ab\cos\theta} \qquad \therefore \theta = 90° - \alpha$$

$$|A'C'| = \sqrt{|A'|^2 + |C|^2 - 2|A||C|\cos(90° - \alpha)}$$

$$= \sqrt{a^2 + a^2 - 2(a)(a)\cos(90° - \alpha)}$$

$$\cos(90° - \alpha) = \cancel{\sin}\,90° \sin\alpha - \cos 90° \cos\alpha$$

$$(1)(\sin\alpha) - 0 \Rightarrow \sin\alpha$$

$$= \sqrt{2a^2 - 2a^2\sin\alpha} \qquad \text{Took Common } 2a^2$$

$$= \sqrt{2a^2(1 - \sin\alpha)}$$

$$d_1 = |A'C'| = \sqrt{2}\,a(1 - \sin\alpha)$$

Now find angle of $\angle D' \Rightarrow$

As we Know Sum of all angles is 360°

$$\angle A' + \angle B' + \angle C' + \angle D' = 360°$$

$$90° + 90° - \alpha + 90° + \angle D' = 360°$$

$$\angle D' + 270° - \alpha = 360°$$

$$\angle D' \cancel{-}\cancel{\alpha} = 360° - 270° = 90°$$

$$\boxed{\angle D' = 90° + \alpha}$$

Now find another side, $|BD'|$ By Sin Law

$$\cancel{B'D'} \Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b} \qquad \because \sin\theta = \frac{\text{Perpendicular}}{\text{Hyptaneus}}$$

where A and B are angles and a, b are sides

$$\cancel{|B'D'| \Rightarrow \frac{a\sin D'}{\sin B'}}$$

$$\frac{\sin D'}{|D'|} = \frac{\sin B'}{|B'|} \qquad \text{By Cross multiplication} =$$

$$|D| = \frac{\sin D'}{\sin B'}|B'| \Rightarrow \frac{\sin(90° + \alpha)}{\sin 90°}(a) \Rightarrow a \cdot \cos\alpha$$

$$|B'D'| = \sqrt{|B|^2 + |D|^2 - 2|B||D|\cos\theta}$$

Now lets find an Other diagonal side $|B'D'|$ –

$$|B'D'| = \sqrt{|B'|^2 + |D'|^2 - 2|B'||D'|\cos\theta}$$

$$= \sqrt{(a)^2 + (a\cos\alpha)^2 - 2(a)(a\cos\alpha)\cos(90^\circ+\alpha)}$$

$$= \sqrt{a^2 + a^2\cos^2\alpha + 2a^2\cos\alpha\sin\alpha}$$

$$= \sqrt{a^2(1 + \cos^2\alpha + 2\sin\alpha\cos\alpha)}$$

$$\cancel{= a\sqrt{\sin^2\alpha + 2\sin\alpha\cos\alpha}}$$

$$\cancel{\because 1 + 2\sin\alpha\cos\alpha = (\sin\alpha+\cos\alpha)^2}$$

$$\cancel{= \sqrt{a^2(\cos^2\alpha + (\cos\alpha+\sin\alpha)^2)}}$$

$$\cancel{= a\sqrt{\cos^2\alpha + (\cos\alpha+\sin\alpha)^2}}$$

$\sin + \cos = 1 - \cos$

$\sin^2\alpha - 1$

$1 - S = \cos$

Now Lets find an Other Side diagonal bisects prerpendicular

$$|B'D'| = \frac{a}{\sin(90^\circ+\alpha)}$$

$$|B'D'| = \frac{a}{\cos\alpha} = d_2$$

Now we have Both d_1 and d_2 which are diagonals of Kite and follow the Property of Bisects Perpendicularly –

$$\text{Area of kite }(A'B'C'D') \Rightarrow \frac{d_1 \times d_2}{2}$$

Where $d_1 = \sqrt{2}\,a(1-\sin\alpha) \Leftrightarrow |A'C'|$

and $d_2 = \dfrac{a}{\cos\alpha} \Leftrightarrow |B'D'|$

$$\Rightarrow \frac{\left\{\sqrt{2}\,a(1-\sin\alpha)\right\}\left(\dfrac{a}{\cos\alpha}\right)}{2}$$

Area of $(A'B'C'D')$ or shadded Region $\Rightarrow \dfrac{\sqrt{2}}{2}\,a^2\,\dfrac{(1-\sin\alpha)}{\cos\alpha}$

Problem C-1

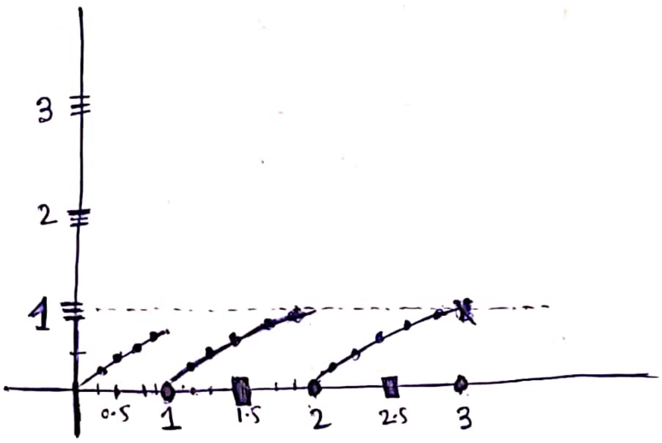
For this problem, we define the fractional part of $x \in \mathbb{R} \geq 0$ as

$$\{x\} = x - \lfloor x \rfloor$$

Where $\lfloor x \rfloor$, is the integer part of x, i-e the greatest integer Less than or equal to x

$\Rightarrow$ Above statment clearly defined as floor ftn $\Rightarrow \lfloor x \rfloor$

(a) Draw the function $\{x\}$ in a Co-ordinate system for $0 \leq x \leq 3$



$0.5 - \lfloor 0.5 \rfloor = 0.5$

$0.7 - \lfloor 0.7 \rfloor = 0.7$

$1 - \lfloor 1 \rfloor = 0$

$2.5 - \lfloor 2.5 \rfloor = 0.5$

$1.8 - \lfloor 1.8 \rfloor = 0.8$

Maximum height is 0.9 and minimum is 0,

(b) find the Area An Under the graph of $\{x\}$ between 0 and $n \in \mathbb{N}$ as given by

$$A_n = \int_0^n \{x\} dx \qquad \because \{x\} = x - \lfloor x \rfloor.$$

Solution :-

$$x - \lfloor x \rfloor = \begin{cases} x, & 0 \leq x \leq 1 \\ x-1, & 1 \leq x \leq 2 \\ x-2, & 2 \leq x \leq 3 \\ \vdots \end{cases}$$

$$\Rightarrow \left[x - (n-1), \quad n-1 \leq x \leq n \right.$$ so we can divide it in Parts,

$$\int_0^n x - \lfloor x \rfloor dx = \int_0^1 x dx + \int_1^2 (x-1) dx + \int_2^3 (x-2) dx + \cdots \int_{n-1}^n (x-n+1) dx$$

If we notice the graph clearly we can see pattren of triangle in Part (a) which gives us area of $\frac{1}{2}$ at each interval part

Like lets integrate then put the limit,

$$\int_0^n x-\lfloor x\rfloor \, dx = \int_0^1 x\,dx + \int_1^2 (x-1)\,dx + \cdots\cdots \int_{n-1}^{n} (x-n+1)\,dx$$

$$\Rightarrow \left[\frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2}-x\right]_1^2 + \cdots\cdots \left[\frac{x^2}{2}-nx+x\right]_{n-1}^{n}$$

$$\Rightarrow \left[\frac{1}{2}\right] + \left[\left(\frac{4}{2}-2\right)-\left(\frac{1}{2}-1\right)\right] + \cdots\cdots \left[\left(\frac{n^2}{2}-n^2+n\right)-\left\{\frac{(n-1)^2}{2} - n(n-1)+(n-1)\right.\right.$$

$$\Rightarrow \frac{1}{2} + \left[0+\frac{1}{2}\right] + \left[\frac{n^2-2n^2+2n}{2} - \left\{\frac{n^2-2n^2+1}{2} - (n^2-n)+(n-1)\right\}\right]$$

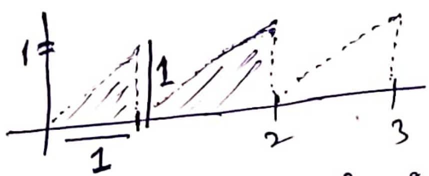
$$\Rightarrow \frac{1}{2} + \frac{1}{2} \cdots + \left[\frac{-n^2+2n}{2} - \left\{\frac{n^2-2n^2+1-2n^2+\cancel{2n}+\cancel{2n}-2}{2}\right\}\right]$$

$$\left\{\frac{\cancel{2n}-\cancel{n^2}-\cancel{n^2}+\cancel{2n^2}-1+2n^2-\cancel{2n}-2n+\cancel{2}}{2}\right\}$$

$$\Rightarrow \frac{1}{2}+\frac{1}{2}+\cdots\cdots+\left\{\frac{2n^2-2n+1}{2}\right\}$$

$$\frac{1}{2}+\frac{1}{2}+\cdots\cdots+\left\{n^2-n+\frac{1}{2}\right\}$$

★ But If we see Logically to the graph it gives us information of



Area of 1 triangle is $\frac{1}{2}$ $\Rightarrow$ so If we are provided with n triangle so area will be "$\frac{n}{2}$" Logically

(c) Use this γ constant to prove bellow identify –

$$\int_1^\infty \frac{\{x\}}{x^2}\,dx = 1-\gamma$$

and $\gamma = \lim_{n\to\infty}\left(H_n - \log(n)\right) = 0.5772\ldots\ldots$

Lets take $1-\gamma \Rightarrow$

as per my research I know, $\gamma = \int_1^\infty \left(\frac{1}{\lfloor x\rfloor} - \frac{1}{x}\right)dx$

solution :-

$\int_1^\infty \frac{\{x\}}{x^2}$

$$\Rightarrow 1 - \int_1^\infty \left(\frac{1}{\lfloor x\rfloor} - \frac{1}{x}\right)dx$$

$$\Rightarrow 1 - \left[\int_0^1 \left(\frac{1}{x-1} - \frac{1}{x}\right)dx + \int_1^2 \left(\frac{1}{x-1} - \frac{1}{x}\right)dx + \cdots\cdots\right]$$

$$\Rightarrow 1 - \left\{\left[\ln x - \ln x\right]_0^1 + \left[\ln(x-1) - \ln x\right]_1^2 + \cdots + \left[\ln(x-n+1) - \ln x\right]_{n-1}^{n}\right.$$

$$\Rightarrow 1 - \left[\{(\ln 1 - \ln 2) - 0\} + \cdots\cdots + \left[\{\ln(\cancel{x}-\cancel{n}+1) - \ln(n)\right.\right.$$

$$- (\ln(\cancel{x-1-n+1}) + \ln($$

$$\Rightarrow 1 - \left[\cancel{\ln 1}_{0} - \ln\cancel{2} + \cdots + \cancel{\ln} 1 - \ln(n) - 0 - \ln(n-1)\right.$$

$$\Rightarrow 1 - \left\{\cancel{\ln 2} \cdots\cdots \cancel{\ln} - \ln(n) + \ln(n-1)\right\}$$

$$\Rightarrow 1 + \ln(n) - \ln(n-1) \qquad n > 1$$

Problem C-2

(a) what is difference between twin Primes and Germain primes?
Give Example for both.

Twin primes are either 2 less or 2 more than another Prime
Suppose p is prime, If $P \pm 2$ is also a Prime then Counted as Twin Prime

Example.

$$\underbrace{5 \quad 7}_{+2} \qquad \underbrace{13 \quad 11}_{-2} \qquad \underbrace{41 \quad 43}_{+2} \qquad \underbrace{29 \quad 31}_{+}$$

Germain prime are those prime which makes $2p+1$ is also prime

Like, $P=2 \Rightarrow 2(p)+1$
$2(2)+1=5 \Rightarrow$ Germain prime

$P=3 \Rightarrow 2(3)+1$
$6+1=7 \Rightarrow$ Germain Prime

(b) Which numbers does the set $S_{1,0}$ represents and what is the value of $S'_{1,2}(4 \cdot 10^{18})$?

$S_{1,0} \Rightarrow$ set of all Primes.

Because, from definition $S_{a,b} = \{P, \underline{ap+b} \text{ is a prime}\}$

$S_{1,0} \Rightarrow \{2, 3, 5, 7, 11 \ldots\ldots\}$.

$2(1)+0 \Rightarrow 2 \rightarrow P$
$3(1)+0 \Rightarrow 3 \rightarrow P$
$5(1)+0 \Rightarrow 5 \rightarrow P$
$7(1)+0 \Rightarrow 7 \rightarrow P$

$$S'_{1,2}(4 \cdot 10^{18}) = \left\{\left(\frac{1}{3}+\frac{1}{5}\right)+\left(\frac{1}{5}+\frac{1}{7}\right)+\left(\frac{1}{11}+\frac{1}{13}\right)+\ldots\ldots\right.$$

↳ Based on given table 1, the value will most

* Probably approximately equal to $\boxed{1.8218}$

This is not exact but according to my Logic and Understanding about table 1.

Scanned with CamScanner

(d) Explain difference between table 1 and table 3

Sol:-

Table 1: Represents the value of $S_{1,2}'(x)$ at any given variable x which Table 1 Computes lower bound for Barn's Constant. where Table 3 represents limiting value of $S_{1,2}(x)$ as $x \to \infty$ (approximated value of $S_{1,2}(x)$) in the form of $\pi_{1,2}(x)$ which shows as value of x increases $S_{1,2}(x)$ is approaching towards ∞.

(e) Use theorem 3 to Calculate an upper bound for $\pi_{1,16}(e^{100})$ in orders of magnitude.

Sol:-

$$\pi_{1,16}(e^{100}) < \frac{16 \times 0.66016 \times e^{100}}{(100)(8.37+100)}$$

$$= \frac{e^{100}(16 \times 0.66016)}{10837}$$

$$= (0.0009746) \times e^{100}$$

(f) In this equation (i) Both sides are equal because most of time we use Integration and Sum of any Series are diff Concept but some times they can approximate like here,

In this we have used Stieltjes integration by parts to bound $S_{a,b}(N) - S_{a,b}(M)$ on each interval $[M, N)$.

(C) In the proof of theorem 1, explain why?

$$S_{a,b}(x) = \sum_{p \le x,\, p \in S,\, ab} \frac{1}{p} = \sum_{t=1}^{n} \frac{\pi_{a,b}(t) - \pi(a,b(t-1))}{t=1}$$

Because $S_{a,b}(x)$ is the sum of positive terms we need only show that it is bounded.

Next thing, this article also says–

$$S_{a,b}(x) \cong \pi_{a,b}(x)$$

so we can say that above equations are equal.