KQ

$kT \ln 2k 1.38 \times 10^{-23} m^2 s^{-2} kg K^{-1} T$

$\delta_\phi \phi = 2^n \delta_\phi n$

$$\langle A, B \rangle \xrightarrow{U} \langle A, f(A, B) \rangle U f(A, B) \langle A, B, 0, 0 \rangle \xrightarrow{U} \langle A, B, A + B, G \rangle ABA + BG$$

$$\langle A, B, 0, 0, 0 \rangle \xrightarrow{U} \langle A', B', f(A, B), G, 0 \rangle$$
$$\xrightarrow{CNOT} \langle A', B', f(A, B), G, f(A, B) \rangle$$
$$\xrightarrow{U^\dagger} \langle A, B, 0, 0, f(A, B) \rangle.$$

$KQGVV^\dagger$

$$VV = V^\dagger V^\dagger = X,$$
$$VV^\dagger = V^\dagger V = I,$$

XV

$$V = \frac{1}{2} \begin{pmatrix} 1 + i & 1 - i \\ 1 - i & 1 + i \end{pmatrix}.$$

$TT|0\rangle |1\rangle T$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}.$$

+
○

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$A \quad\bullet\quad\bullet\quad P = A$$
$$B \quad\bullet\qquad\bullet\quad Q = A \oplus B$$
$$C \quad\boxed{V^\dagger}\boxed{V}\boxed{V}\quad R = AB \oplus C$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$A \quad\bullet\quad\bullet\quad P = A$$
$$B \quad\bullet\qquad\bullet\quad Q = A \oplus B$$
$$C \quad\boxed{V^\dagger}\boxed{V^\dagger}\boxed{V}\quad R = AB \oplus C$$

$$TT^\dagger THCNOTTT^\dagger T$$

$$A \quad\bullet\;\boxed{T}\;\bullet\quad\bullet\quad\bullet\quad P = A$$
$$B \quad\boxed{T^\dagger}\quad\boxed{T}\;\boxed{T^\dagger}\quad\boxed{T}\quad Q = AB + AC$$
$$C \;\bullet\boxed{H}\boxed{T}\bullet\;\boxed{T^\dagger}\quad\bullet\boxed{H}\bullet\; R = AB + AC$$

$sem_{22}m_{21}..m_0$
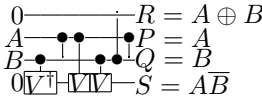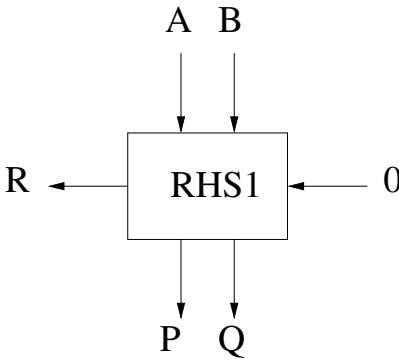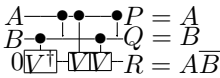$$(-1)^s \times (1.m_{22}m_{21}...m_0) \times 2^{e-127}$$

$$1.000 + 0.0001 = 1.0000.00000.0004$$

AB
$$<$$

$27^{th} 256^{th}$

A
B

Conditional Swap

Y 32 bits
32 bits X

8-bit exponent difference (d)

9 bits

23 bits   40 bits
1   0 0   0
64 bits

24 bits

sign bit

23 bits

Alignment

27 bits   37 bits
G

sign bit
1   0 0 0
8 bits   28 bits

G
28 bits

28-bit Converter
28-bit Converter

28 bits   28 bits

28-bit Reversible Adder

29 bits   28 bits

29-bit Converter
28-bit Converter

29 bits   28 bits

1   0 0 0
28 bits   23 bits   0
8 bits

8-bit exponent   8 CNOTs

Normalization &
Rounding   8-bit exponent

sign bit   sign bit 31 bits   8-bit exponent

31 bits 9 bits G

X+Y   X

8-bit exponent

$A_i^e B_i^e A_i^m B_i^m A^s B^s b_i b_7 b_7 d_7..d_0$

B_7^e B_6^e B_1^e B_0^e

A_7^e 0 A_6^e 0 A_1^e 0 A_0^e 0

b_7 RFS2 b_6 RFS2 b_5 .... b_1 RFS2 b_0 RHS2 ← 0 b_7 d_7 d_6 .... d_1 d_0

A^s B^s A_7^e B_7^e A_6^e B_6^e A_1^e B_1^e A_0^e B_0^e $A_{22}^m B_{22}^m$ $A_1^m B_1^m$ $A_0^m B_0^m$

FRE → FRE → FRE → .... → FRE → FRE → FRE → .... → FRE → FRE

X_{31} Y_{31} X_{30} Y_{30} X_{29} Y_{29} X_{24} Y_{24} X_{23} Y_{23} X_{22} Y_{22} X_1 Y_1 X_0 Y_0

A B

A —•—•—• $P = A$
B —•—•—• $Q = B$
0 —$V^\dagger$—V—V— $R = A\overline{B}$

R ← RHS1 ← 0

P Q

A B 0

0 ——————— $R = A \oplus B$
A —•—•—• $P = A$
B —•—•—• $Q = B$
0 —$V^\dagger$—V—V— $S = A\overline{B}$

S ← RHS2 ← 0

P Q R

C ——— $P = C$
B ——— $Q = B$
A ——— $R = A \oplus B \oplus C$
0 ——— $S = C(A \oplus B) \oplus \overline{A}B$

A   B   0

| | |
S ← | RFS1 | ← C

R   P   Q

C ——— $P = A$
B ——— $Q = B$
A ——— $R = A \oplus B \oplus C$
0 ——— $S = C(A \oplus B) \oplus \overline{A}B$

A   B   0

S ← | RFS2 | ← C

P   Q   R

$ACABR$

A_7 A_6 A_5 A_4 A_3 A_2 A_1 A_0

0 0   0 0   0 0   1 0   1 0   0 0   1 0   0 0

RFS2 RFS2 RFS2 RFS2 RFS2 RFS2 RFS2 RHS1

G G G G G G G

FRE FRE FRE FRE FRE FRE FRE FRE   G

X₇ G   X₆ G   X₅ G   X₄ G   X₃ G   X₂ G   X₁ G   X₀ G

X_7 X_6 X_5 X_4 X_3 X_2 X_1 X_0

n bits | 0 | 0 | | 0 | 1 |

n−bit Converter

n bits | 0 | 0 | | 0 | 1 |

sign I_{n-2} 0 I_{n-3} 0 I_2 0 I_1 1 I_0

G   TR   TR   ·········   TR   TR   0

O_{n-1} O_{n-2} 0 O_{n-3} 0 O_2 0 O_1 1 O_0

n

$nn-2$

$n-1n$

$c_i b_i$

A
B
C
0 $\boxed{V^\dagger}$ $\boxed{V}$ $\boxed{V}$
1 $\boxed{V^\dagger}$ $\boxed{V}$ $\boxed{V}$ $A + B + C$
D
1
1 $\boxed{V^\dagger}$ $\boxed{V}$ $\boxed{V}$ $D + AB'C'$

A
B
G
G

G
G

B   A

D → | RLZC | → A+B+C

C → | | → D+AB'C'

B   A

MSB

27−bit magnitude

conditional right shift

28 bits

E_7 0 E_6 0 E_1 0 E_0 0

0 0 0 0   G   RHA — c_6 — RHA — c_5 ⋯ c_1 — RHA — c_0 — RHA ⊕ 0

G   G   G   G

RLZCU

32 bits

E_7 0 E_6 0 E_1 0 E_0 0

(32, 5) reversible
left barrel shifter

G   RFS1 — b_6 — RFS1 — b_5 ⋯ b_1 — RFS1 — b_0 — RHS2 ← 0

9 garbage bits   23 first bits

E_7 G G E_6 G G E_1 G G E_0 G G

sign bit

$X_7 X_6 ... X_0 00111010010 O_2 O_1 O_0$

X$_7$ B A   X$_7$
X$_6$ X$_6$
X$_5$ B A   X$_5$
X$_4$ X$_4$
X$_3$ B A   X$_3$
X$_2$ X$_2$
X$_1$ B A   X$_1$
X$_0$ X$_0$
0   D   D   D   D   O$_2$
0   C   C   C   C   A   G
0   G
0   B A   G
0   D   O$_1$
0   C   D C B   G
0   A   G
0   B   G
0   D   O$_0$
0   C   G

	$5n-4$	n	n
	$4n-7$	1	$n-1$

$n-1$ $n-2$

n

$$VV^\dagger VV^\dagger$$

$$
\begin{aligned}
P &= A\\
Q &= A \oplus B\\
R &= AB \oplus C
\end{aligned}
$$

$$
\begin{aligned}
P &= A\\
Q &= B\\
R &= AB \oplus C
\end{aligned}
$$

$$
\begin{aligned}
P &= A\\
Q &= A \oplus B\\
R &= A\overline{B} \oplus C
\end{aligned}
$$

$$T$$

$$
\begin{aligned}
&A\\
&B\\
&C\\
&A + B + C\\
&C\\
&D + AB'C'
\end{aligned}
$$

$$VV^\dagger$$
$$VV^\dagger V^\dagger VV$$

$$V$$

$$V^\dagger$$

$$VV^\dagger$$

T

$P = A$
$Q = A \oplus B$
$R = A \oplus B \oplus C$
$S = (A \oplus B) \oplus C \oplus AB$

$VV^\dagger$

$P = A$
$Q = B$
$R = A\overline{B}$

T

$R = A \oplus B$
$P = A$
$Q = B$
$S = A\overline{B}$

T

$KQKQTKQe \ll 1/KQ$
TKQ
$KQKQ$

$\langle A', B', f(A, B), G \rangle A'B'G$

$\Theta(\sqrt{n})$

$$C \qquad\qquad\qquad\qquad\qquad C$$
$$B \qquad\qquad\qquad\qquad\qquad B$$
$$A \qquad\qquad\qquad\qquad\qquad A \oplus B \oplus C$$
$$0 \;\; H \qquad\qquad\qquad\qquad H \;\; C(A \oplus B) \oplus \overline{A}B$$

T

$$C \qquad\qquad\qquad\qquad\qquad C$$
$$B \qquad\qquad\qquad\qquad\qquad B$$
$$A \qquad\qquad\qquad\qquad\qquad A \oplus B \oplus C$$
$$0 \;\; H \qquad\qquad\qquad\qquad H \;\; C(A \oplus B) \oplus \overline{A}B$$

T

	T

T

$\frac{\pi}{8}$