Just Predict the Remainder

Just Predict the remainder

$$\frac{x^3+4x^2-3x+1}{x-2}$$, How long will it take you to find the remainder, Do it within 20 seconds if you can!

$$\frac{x^3+4x^2+7x+2+a}{x-\frac{\pi}{4}} = k,$$ Find a, where ‘a’ is a real number.

$$\frac{\cos x-b}{x-\frac{\pi}{4}} = m,$$ Find b, where b is a real number.

Disclaimer:

I haven't taken any external reference for these problems, it's one of the few ideas that struck my mind while solving Taylor's Series problems given by my Mathematics Teacher. It's a part of the syllabus of Engineering Mathematics for First-Year Engineering Students. I hope my idea, helps the reader.

The above method can be used to verify long divisions, readers (From lower classes) may remember it as a trick, skipping the complete idea. I will begin by Stating Taylor's Series/Theorem and the Division Algorithm and then move on to the idea.

Knowledge share=(knowledge)^2.

Suggestions, Critics as well as Complements are welcomed at
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Idea:

Taylors Series:

$$f(h+x) = f(h) + \frac{x}{1!}f'(h) + \frac{x^2}{2!}f''(h) + \frac{x^3}{3!}f'''(h) + \cdots \frac{x^n}{n!}f^n(x) + \cdots upto\ infinite\ terms$$

Interchanging 'x' and 'h',

$$f(x+h) = f(x) + \frac{h}{1!}f'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + \cdots \frac{h^n}{n!}f^n(x) + \cdots upto\ infinite\ terms$$

Replace 'x' by 'a' and 'h' by 'x-a'

$$f(x) = f(a) + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \cdots \frac{(x-a)^n}{n!}f^n(a) + \cdots upto\ infinite\ terms$$

Replace 'a' by '-a'

$$f(x) = f(-a) + \frac{(x+a)}{1!}f'(-a) + \frac{(x+a)^2}{2!}f''(-a) + \frac{(x+a)^3}{3!}f'''(-a) + \cdots \frac{(x+a)^n}{n!}f^n(-a) + \cdots upto\ infinite\ terms$$

The highlighted form is valid only in cases function f(x) and corresponding derivatives exist at x=a and x=-a respectively.

Division Algorithm:

Dividend = ((Divisor) x (Quotient)) +Remainder

Applying it to polynomial functions:

$$f(x) = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + \ldots\ldots a_nx^n\ , a_n \neq 0, n\ is\ a\ whole\ number.$$

Since polynomial functions have a domain of x belonging to all real numbers and are differentiable for all values of x; We can apply the above idea. The first two problems are based on the application of Taylor's Series to polynomial functions.

$\frac{x^3+4x^2-3x+1}{x-2}$, **How long will it take you to find the remainder, Do it within 20 seconds if you can!**

Standard long Division:

$x-2 \overline{)\, x^3+4x^2-3x+1} \mid x^2+6x+9$

$x^3 - 2x^2$

(-) (+)

$6x^2 - 3x$

$6x^2 - 12x$

(-) (+)

$9x + 1$

$9x - 18$

(-) (+)

19 → Remainder

Shortcut to determine the remainder:

Step 1: Consider the divisor and equate it to 0.

$x-2=0 \rightarrow x=2$

Step 2: Replace x with its value in the dividend.

$2^3+4(2^2)-3(2)+1 = 19 \rightarrow$ Remainder

The way in which the shortcut is observed:

Using Taylor's Series

Let $f(x) = x^3+4x^2-3x+1 \rightarrow$ **Dividend**

Use Taylor Series to write $f(x)$ **in terms of** $(x-2) \rightarrow$ **Divisor**

$$f(x) = f(a) + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \cdots \frac{(x-a)^n}{n!}f^n(a) + \cdots \textit{upto infinite terms}$$

$$x^3+4x^2-3x+1 = 19 + \frac{(x-2)}{1!}(25) + \frac{(x-2)^2}{2!}(20) + \frac{(x-2)^3}{3!}(6)$$

Taking $(x-2)$ **comman**

$$x^3+4x^2-3x+1 = 19 + (x-2)\left[\frac{(25)}{1!} + \frac{(x-2)}{2!}(20) + \frac{(x-2)^2}{3!}(6)\right]$$

After rearrangements we get:

$x^3+4x^2-3x+1 = (x-2)K + 19$

Here:

$x^3+4x^2-3x+1 \rightarrow$ **Dividend**

$(x-2) \rightarrow$ **Divisor**

$K=\left[\frac{(25)}{1!} + \frac{(x-2)}{2!}(20) + \frac{(x-2)^2}{3!}(6)\right] \rightarrow$ **Quotient**

$19 \rightarrow$ **Remainder**

By observation, you will realise that the remainder is $= f(2)$.

Hence in such cases of long divisions find $f(a)$ **for remainder.**

Here $a=2$

Therefore:

$f(2) = 19$

$f'(x) = 3x^2+8x-3$

$f'(2) = 25$

$f''(x) = 6x+8$

$f''(2) = 20$

$f'''(x) = 6$

$f'''(2) = 6$

$f''''(x) = 0$

$f''''(2) = 0$

$$\frac{x^3+4x^2+7x+2+a}{x-\frac{\pi}{4}} = k,$$ **Find a, where 'a' is a real number.**

It is not very convenient to proceed with the long division, hence we shall make use of the shortcut observed.

Rearranging the above equation, we get:

$$x^3 + 4x^2 + 7x + 2 = k\left(x - \frac{\pi}{4}\right) - a$$

Comparing the above equation with division algorithm, we get:

$x^3 + 4x^2 + 7x + 2 \rightarrow$ **Dividend**

$\left(x - \frac{\pi}{4}\right) \rightarrow$ **Divisor**

$k \rightarrow$ **Quotient**

(-a) $\rightarrow$ **Remainder**

Step 1: Consider the divisor and equate it to 0.

$$\left(x - \frac{\pi}{4}\right) = 0 \rightarrow x = \frac{\pi}{4}$$

Step 2: Replace x with its value in the dividend.

$$a = -\left[\left(\frac{\pi}{4}\right)^3 + 4\left(\frac{\pi}{4}\right)^2 + 7\left(\frac{\pi}{4}\right) + 2\right]$$

Solving with use of Taylor's Series:

$$x^3 + 4x^2 + 7x + 2 = \left[\left(\frac{\pi}{4}\right)^3 + 4\left(\frac{\pi}{4}\right)^2 + 7\left(\frac{\pi}{4}\right) + 2\right] + \frac{\left(x - \frac{\pi}{4}\right)}{1!}\left[3\left(\frac{\pi}{4}\right)^2 + 8\left(\frac{\pi}{4}\right) + 7\right] + \frac{\left(x - \frac{\pi}{4}\right)^2}{2!}\left[6\left(\frac{\pi}{4}\right) + 8\right] + \frac{\left(x - \frac{\pi}{4}\right)^3}{3!}[6]$$

Taking $\left(x - \frac{\pi}{4}\right)$ comman

$$x^3 + 4x^2 + 7x + 2 = \left[\left(\frac{\pi}{4}\right)^3 + 4\left(\frac{\pi}{4}\right)^2 + 7\left(\frac{\pi}{4}\right) + 2\right] + \left(x - \frac{\pi}{4}\right)\left\{\left[3\left(\frac{\pi}{4}\right)^2 + 8\left(\frac{\pi}{4}\right) + 7\right] + \frac{\left(x - \frac{\pi}{4}\right)}{2!}\left[6\left(\frac{\pi}{4}\right) + 8\right] + \frac{\left(x - \frac{\pi}{4}\right)^2}{3!}[6]\right\}$$

After rearrangements we get:

$$x^3 + 4x^2 + 7x + 2 = \left(x - \frac{\pi}{4}\right)K + \left[\left(\frac{\pi}{4}\right)^3 + 4\left(\frac{\pi}{4}\right)^2 + 7\left(\frac{\pi}{4}\right) + 2\right]$$

$$\frac{x^3 + 4x^2 + 7x + 2 - \left[\left(\frac{\pi}{4}\right)^3 + 4\left(\frac{\pi}{4}\right)^2 + 7\left(\frac{\pi}{4}\right) + 2\right]}{\left(x - \frac{\pi}{4}\right)} = K$$

Comparing above equation with:

$$\frac{x^3 + 4x^2 + 7x + 2 + a}{x - \frac{\pi}{4}} = k$$

We get:

$$a = -\left[\left(\frac{\pi}{4}\right)^3 + 4\left(\frac{\pi}{4}\right)^2 + 7\left(\frac{\pi}{4}\right) + 2\right]$$

Here $a = 2$

Therefore:

$f(2) = 19$

$f'(x) = 3x^2 + 8x - 3$

$f'(2) = 25$

$f''(x) = 6x + 8$

$f''(2) = 20$

$f'''(x) = 6$

$f'''(2) = 6$

$f''''(x) = 0$

$f''''(2) = 0$

An Elegant doubt that came to my mind, which may strike the reader's mind as well.

How can we have remainder as some fraction or floating-point number ?

$$2\overline{)2.037}\,|1$$

$$\begin{array}{r} -\ 2.000 \\ \hline 0.037 \end{array}$$

2.037 → Dividend

2 → Divisor

1 → Quotient

$0.037 = \frac{37}{1000}$ → Remainder

Hence, we can have a fraction or floating-point number as remainder.

Applying it to other functions:

Since many functions like Trigonometric, inverse trigonometric, logarithmic, exponential can be expressed using Taylor's Series. I have tried an approach that may give the remainder of [f(x) /(x-a)] or [f(x)/(x+a)], provided f(x) and its corresponding derivates are defined for x=(a) and x=(-a) respectively. The third problem is based on the above idea.

$$\frac{\cos x - b}{x - \frac{\pi}{4}} = m,$$ **Find b, where b is a real number.**

It is not very convenient to proceed with the long division, hence we shall make use the shortcut observed.

Rearranging the above equation, we get:

$$\cos x = m\left(x - \frac{\pi}{4}\right) + b$$

Comparing the above equation with the division algorithm, we get:

$\cos x$ → Dividend

$\left(x - \frac{\pi}{4}\right)$ → Divisor

m → Quotient

b → Remainder

Step 1: Consider the divisor and equate it to 0.

$$\left(x - \frac{\pi}{4}\right) = 0 \rightarrow x = \frac{\pi}{4}$$

Step 2: Replace x with its value in the dividend.

$$b = \cos\left(\frac{\pi}{4}\right) \rightarrow b = \frac{1}{\sqrt{2}}$$

Solving with use of Taylor's Series:

$$\boldsymbol{cos\,x} = \frac{1}{\sqrt{2}} - \frac{\left(\boldsymbol{x} - \frac{\boldsymbol{\pi}}{4}\right)}{1!}\left[\frac{1}{\sqrt{2}}\right] + \frac{\left(\boldsymbol{x} - \frac{\boldsymbol{\pi}}{4}\right)^2}{2!}\left[\frac{1}{\sqrt{2}}\right] - \cdots\ (-1)^n \left\{\frac{\left(\boldsymbol{x} - \frac{\boldsymbol{\pi}}{4}\right)^n}{n!}\left[\frac{1}{\sqrt{2}}\right]\right\} + \cdots upto\ infinite\ terms$$

Taking $(x - \frac{\pi}{4})$ **comman**

$$\boldsymbol{cos\,x} = \frac{1}{\sqrt{2}} + \left(\boldsymbol{x} - \frac{\boldsymbol{\pi}}{4}\right)\left\{\left[\frac{1}{\sqrt{2}}\right] + \frac{\left(\boldsymbol{x} - \frac{\boldsymbol{\pi}}{4}\right)}{2!}\left[\frac{1}{\sqrt{2}}\right] - \cdots\ (-1)^n \left(\frac{\left(\boldsymbol{x} - \frac{\boldsymbol{\pi}}{4}\right)^{n-1}}{n!}\left[\frac{1}{\sqrt{2}}\right]\right) + \cdots upto\ infinite\ terms\right\}$$

After rearrangements we get:

$$\boldsymbol{cos\,x} = \left(\boldsymbol{x} - \frac{\pi}{4}\right)\boldsymbol{m} + \frac{1}{\sqrt{2}}$$

$$\frac{\boldsymbol{cos\,x} - \left[\frac{1}{\sqrt{2}}\right]}{\left(\boldsymbol{x} - \frac{\boldsymbol{\pi}}{4}\right)} = \boldsymbol{m}$$

Comparing above equation with:

$$\frac{\boldsymbol{cos\,x} - \boldsymbol{b}}{\boldsymbol{x} - \frac{\boldsymbol{\pi}}{4}} = \boldsymbol{m}$$

We get:

$$\boldsymbol{b} = \frac{1}{\sqrt{2}}$$

Here $a = \frac{\pi}{4}$

Therefore:

$$\boldsymbol{f}\left(\frac{\boldsymbol{\pi}}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\mathbf{f}'(\mathbf{x}) = -\sin x$$

$$\boldsymbol{f}'\left(\frac{\boldsymbol{\pi}}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$\boldsymbol{f}''(\boldsymbol{x}) = \cos x$$

$$\boldsymbol{f}''\left(\frac{\boldsymbol{\pi}}{4}\right) = \frac{1}{\sqrt{2}}$$

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$$\boldsymbol{f}^n(\boldsymbol{x}) = (-1)^n\left\{\cos\left(x + n\frac{\pi}{2}\right)\right\}$$

$$\boldsymbol{f}^n\left(\frac{\boldsymbol{\pi}}{4}\right) = (-1)^n\left\{\cos\left(\frac{\boldsymbol{\pi}}{4} + n\frac{\pi}{2}\right)\right\}$$

... and so on upto infinite terms

I hope that I have something new for the reader, you can add your doubts or queries to me via gmail at [mailtovigyannveshi@gmail.com](mailto:vigyannveshi@gmail.com). Till then "Seek Science behind Substances to get Simplified Solution".