ECON 23620 Inequality: A Perspective from Macroeconomics **Greg Kaplan**
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Problem Set 1
Due: Monday 2 October 2017

Part 1: Exercises

1. Consider the following infinite horizon consumption-savings problem of a household with habits. A household has preferences defined by

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t - \psi c_{t-1}\right)$$

for some positve, strictly increasing and strictly concave function u. The household faces the sequence of budget constraints

$$\begin{aligned} c_t + a_{t+1} &\leq (1+r)\, a_t + y \\ a_{t+1} &\geq 0 \\ a_0 &= 0 \\ \lim_{t\to\infty} a_t &\geq 0 \end{aligned}$$

where a_t is wealth and y is labor income. Write down the household problem in recursive form, making sure to clearly define the state variables for the problem.

Part 2: Coding

1. Consider the following finite horizon consumption-savings problem with indivisible labor supply. A houshelod lives from ages $t = 1$ to $t = T$ and then dies. In every period t, a household chooses consumption c_t and whether or not to work. We denote $h_t = 1$ if the household works in period t and $h_t = 0$ if not. The household receives a wage w_t if working and an unemploymen benefit b if not working. The household's Bellman equation is

$$\begin{aligned} V_t(a) &= \max_{c,h} u(c) - \psi h + \beta V_{t+1}(a') \\ &\text{subject to} \\ c + a' &= (1+r)\, a + w_t h + b\,(1-h) \\ a' &\geq 0 \end{aligned}$$

Assume the following functional forms and parameter values

$$
\begin{aligned}
u(c) &= \frac{c^{1-\gamma}-1}{1-\gamma} \\
\gamma &= 2 \\
\beta &= 0.97 \\
r &= 0.03 \\
\psi &= 0.1 \\
T &= 50 \\
w_t &= \begin{cases} \frac{t}{10} & \text{if } t \leq \frac{T}{2} \\ \frac{T+1-t}{10} & \text{if } t > \frac{T}{2} \end{cases} \\
b &= 0.5
\end{aligned}
$$

(a) Write some computer code that uses value function iteration to solve for the households optimal decisions and policy rules and produce a plot of the optimal paths of consumption (c), work (h) and wealth (a) as a function of the household's age.

(b) Give the economic intuition for why the paths of consumption, work and wealth look the way that they do.

(c) Show how the optimal paths change in each of the following circumstances. In each case, provide the economic intuition behind your result:

i. The interest rate falls to $r = 0.01$.

ii. The household faces a proportional tax $\tau = 40\%$ on their wage income. Unemployment benefits are not taxed.

iii. Unemployment benefits are reduced to $b = 0.1$